

Upgrade of the Near Detector ND280 and its expected impact on the T2K sensitivity and on the study of neutrino-nucleus interactions

Quoc Viet Nguyen

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Quoc Viet Nguyen. Upgrade of the Near Detector ND280 and its expected impact on the T2K sensitivity and on the study of neutrino-nucleus interactions. High Energy Physics - Experiment [hep-ex]. Sorbonne Université, 2022. English. NNT: 2022SORUS348. tel-03935726

HAL Id: tel-03935726 https://theses.hal.science/tel-03935726

Submitted on 12 Jan2023

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Sorbonne Université

École doctorale 560 : Sciences de la Terre et de l'Environnement et Physique de l'Univers de Paris (STEP'UP)

Laboratoire de Physique Nucléaire et de Hautes Energie (LPNHE)

Upgrade of the Near Detector ND280 and its expected impact on the T2K sensitivity and on the study of neutrino-nucleus interactions

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Thése de doctorat de Physique

Présenté et soutenue publiquement le 27/09/2022

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Acknowledgement

Three years at LPNHE were three incredible years that I'll never forget in my life. There were challenges, there were happy moments, and there was covid-19. After all, I am so grateful to everyone who was with me during that time: an amazing neutrino group at LPNHE, an active and helpful T2K collaboration, many nice friends and, of course, my wonderful family. Without these people, it would be impossible to finish this exciting and challenging journey.

First, I express my gratitude to my supervisors, Claudio Giganti and Boris Popov, for their guidance, deep knowledge of physics and encouragement throughout my thesis. I would like to thank Claudio for his fruitful discussions, invaluable advice, and patience while helping me solve the issues. He often pointed out the problems quickly while I was struggling with them. He is so cool to me. I have learned a lot of physics and programming skills from him. I would also like to thank Boris for his nice feedback, encouragement, and guidance during my thesis. He was always there to help me whenever I needed him. I would also like to extend my sincere thanks to Jacques Dumarchez and Mathieu Guigue for their friendliness and incredible support from physics, computer skills to many living problems in France. Jacques, I still keep the "happy new year" letter you wrote to me at the exact lunar new year time in Vietnam. I would like to thank Marco Martini for teaching me the beauty of neutrino interaction from the theoretical side. I had the pleasure of collaborating with him on my thesis's last and exciting project, which is about the Martini et al. model for neutrino interaction. I also want to thank Marco Zito for his discussion and support in physics and paper works. I would be remiss in not mentioning other members of the LPNHE neutrino group: Adrien the coding monster, Sergey the test beam analysis expert, Lucile the smartest and prettiest, Vlada the nicest, Claire the best translator and Ulysse sensei. Thanks for all your help and a wonderful time together. I am pleased to be a member of the neutrino group at LPNHE with these great physicists.

During this thesis, I had a chance to work in a big collaboration. I am thankful to the T2K collaboration, especially the colleagues from the "Physics and Performance" working group. Special thanks to Sara Bolognesi for her discussion, criticism, and vision throughout the projects. This endeavor would not have been possible without Stephen Dolan, an expert in neutrino-nucleus interaction. It is my pleasure to work with him, the supportive one everybody wants to work with. He always responds to my questions very fast, even during the weekend. He is a great and nice colleague. I would like to acknowledge other "Physics and Performance" members: Margherita Buizza Avanzini, Adrien Blanchet, Laura Munteanu, Jaafar Chakrani, and Anna Ershova for all their discussion, support and having shared their experiences. Thanks to them, I greatly enjoyed the "physics studies for upgraded ND280" project. I also want to thank Kenji Yasutome for providing feedback about my work and showing me many things related to oscillation analysis.

I also had the pleasure of working on the HA-TPC test beam projects. I would like to thank Alain DELBART, Samira Hassani, Paul Colas, Gianmaria Collazuol, and Guillaume Eurin for their guidance with the test beam data analysis and support during the test beam time in DESY and CERN. It's my honor to work with many TPC experts. I've learned a lot from them.

Special thanks to all the jury members: Anselmo Meregaglia, Juan Miguel Nieves Pamplona, Sophie Trincaz-Duvoid, Margherita Buizza Avanzini, Dominique Duchesneau, Claudio Giganti and Boris Popov for spending time reading this thesis and giving me a lot of questions and comments about many subjects in this thesis. My thesis is much better with all your feedback. I am also grateful to people in the neutrino group at IFIRSE, Quy Nhon, Vietnam, especially Son Cao, for frequent physics discussions and advice.

It is impossible to thank individual friends since I have so many amazing friends. I would like to thank all for making my life in Paris more joyful. It would be tough to overcome stress and lockdown without them and their beers. Many thanks to Guy Augarde for helping me survive on the French Administration stuff and for picking my family up at the airport on the day I defended my thesis.

Con/minh xin cam on tat ca ban be, nguoi than da ben canh va dong vien con/minh trong suot qua trinh hoan thanh luan an nay. Dac viet hon het la me Phuong va anh Nam, nhung nguoi luon theo sat va luon la dong luc to lon de minh phan dau. Du 3 nam covid khong ve duoc nhung ma me va anh da cho minh thay la gia dinh khong bao gio o xa minh. Con xin cam on ba va ba ngoai, mot lan nua con mong moi nguoi se vui vi nhung gi con lam. Con xin cam on chu Hau da ben canh cham soc me trong luc con vang nha, that su con thay an tam hon rat nhieu.

Minh cung xin gui loi cam on toi nhung nguoi ban da ben canh, chia se va giup do minh trong nhung nam o Paris: anh Hoang, chi Duong, cun, anh Trung, chi Tram, anh Minh, chi Quyen, anh Khanh, chi Suong, Nam, Hoa, Phuong Ngo, Quoc Dinh. Cam on moi nguoi vi nhung ki niem dep cung nhau.

Con xin gui loi cam on den gia dinh bac Vinh, bac Phuoc vi da ho tro, giup do gia dinh con rat nhieu trong thoi gian con di hoc. Con rat tiec vi khong ve tham bac Phuoc vao nhung ngay cuoi duoc. Nhung hinh anh va viec lam cua bac, con se luon tuong nho.

Sau cung minh xin gui loi cam on toi co vo Linh vui tuoi xinh dep cua minh. Cam on vo vi luon ben canh ck, chia se niem vui noi buon. Vk luon la dong luc cho ck phan dau. Nhung nam thang kho khan nay chac chan la ck khong the vuot qua duoc neu thieu vk. Cam on vk vi da luon thong cam, thau hieu, va vi tat ca.

Résumé

La physique des neutrinos est aujourd'hui l'un des domaines les plus attractifs de la physique des particules. La découverte de l'oscillation des neutrinos suggère que les neutrinos doivent avoir une masse et implique une physique au-delà des trois neutrinos sans masse du modèle standard (SM). Ses débuts remontent à la mine Homestake, dans le Dakota du Sud, en 1967, lorsque Ray Davis et John Bahcall ont remarqué pour la première fois un décalage entre le nombre prévu et le nombre observé de neutrinos solaires. L'"anomalie des neutrinos solaires" a fait l'objet de recherches expérimentales et théoriques approfondies. Elle a suscité une attention encore plus grande lorsque les expériences GALLEX et SAGE, se servant de germanium, ont également trouvé moins de neutrinos solaires que prévu. D'autres expériences ont également trouvé des anomalies de neutrinos dans des détecteurs construits pour rechercher un phénomène complètement différent : la désintégration des protons. Bien que la désintégration des protons n'ait pas été détectée, une analyse détaillée de l'arrière-plan de neutrinos de ces mesures a révélé que le nombre de neutrinos muoniques atmosphériques était inférieur aux prévisions, ce qui suggère un phénomène similaire à celui indiqué par l'anomalie solaire.

La théorie privilégiée pour expliquer les anomalies observées des neutrinos a évolué pour devenir l'oscillation des neutrinos. L'idée de l'oscillation de saveur a été développée par Bruno Pontecorvo, Ziro Maki, Masami Nakagawa et Shoichi Sakata (PMNS), afin d'expliquer l'anomalie solaire. Il y a environ 25 ans, l'oscillation des neutrinos a été découverte grâce aux mesures effectuées à Super-Kamiokande et à l'Observatoire de neutrinos de Sudbury, qui ont été récompensées par le prix Nobel de physique 2015. Après diverses découvertes d'envergure, la physique des oscillations de neutrinos est aujourd'hui à l'ère de la précision.

La mesure précise des paramètres d'oscillation des neutrinos et des sections transversales des neutrinos est l'une des plus grandes priorités de la physique des particules élémentaires. Il existe encore de nombreuses propriétés des neutrinos que le modèle standard ne peut expliquer. Les masses non nulles des neutrinos ou la violation potentielle de la symétrie de parité de charge (CP) dans le secteur leptonique sont les plus intéressantes parmi les quelques indices liés à la physique au-delà du SM. La violation de CP est l'une des trois conditions proposées par Andrei Sakharov en 1967 pour expliquer l'asymétrie matière-antimatière, ou en d'autres termes, pour expliquer l'existence de la matière dans l'univers. La violation de CP se retrouve déjà dans le mélange des quarks dans le SM. Cependant, l'asymétrie matière-antimatière observée dans l'univers ne peut pas être entièrement expliquée par une si petite asymétrie dans le secteur des quarks. Heureusement, selon les mesures récentes de l'oscillation des neutrinos, il existe une possibilité potentielle d'une grande violation de CP dans le secteur leptonique. Par exemple, l'expérience T2K - une expérience d'oscillation de neutrinos à longue distance - a contraint la phase δ_{CP} et peut exclure la conservation de CP à 2σ [1].

Sur la base des indices actuels de violation de CP, l'importance de mesures précises de l'oscillation des neutrinos a été démontrée par les deux prochaines expériences de grande envergure : Deep Underground Neutrino Experiment (DUNE) et le projet Tokai-to-Hyper-Kamiokande (T2HK) hébergés par les États-Unis et le Japon, respectivement. Néanmoins, il ne suffit pas d'avoir plus de détecteurs géants ; la mesure précise des neutrinos exige également que les erreurs systématiques soient réduites autant que possible (généralement à quelques pour cent) dans les expériences d'oscillation de neutrinos. L'une des principales erreurs systématiques provient de notre compréhension incomplète des interactions neutrino-noyaux.

Les incertitudes liées à la diffusion neutrino-noyau restent une source importante d'incertitude systématique dans de nombreuses expériences d'oscillation de neutrinos, dont T2K. Pendant de nombreuses années, des mesures de diffusion de neutrinos sur les canaux de courant chargé et de courant neutre ont déjà été accumulées en utilisant de nombreux types de cibles, de méthodologies d'analyse et de technologies de détection. Profitant de l'intensité des sources de neutrinos et de la précision accrue des prévisions de flux de neutrinos dans de nombreuses expériences d'oscillation de neutrinos, les physiciens mesurent actuellement les sections efficaces des interactions de neutrinos afin de mieux comprendre les effets nucléaires.

Cette thèse présente le projet de mise à niveau du détecteur proche du T2K (ND280) et notamment sa capacité à sonder les effets nucléaires, qui peuvent biaiser la reconstruction de la section efficace et de l'énergie des neutrinos. Le ND280 modernisé confrontera notre naïveté sur les interactions entre neutrinos en utilisant une nouvelle configuration de détecteur avec une acceptation totale de l'angle polaire et un seuil de suivi des protons de momentum beaucoup plus bas. ND280 peut mesurer les nouvelles observables en utilisant le déséquilibre cinématique dans le plan perpendiculaire à la direction du neutrino entrant, qui est le seul déséquilibre cinématique transversal. Ces observables sont sensibles à plusieurs effets nucléaires tels que le mouvement de Fermi, la ré-interaction des hadrons dans l'environnement nucléaire, et les corrélations entre les nucléons dans un noyau.

Le premier chapitre de cette thèse aborde l'histoire de la découverte des neutrinos et du formalisme PMNS et fournit une connaissance générale du domaine des neutrinos. Il présente également certaines des expériences critiques sur les neutrinos, les expériences d'oscillation de neutrinos à très longue distance, ainsi que les questions actuelles dans ce domaine.

Le deuxième chapitre est consacré aux interactions entre neutrinos. Il décrit les différents types d'interactions neutrino-nucléon, neutrino-noyaux et les résultats des expériences correspondantes. Plus important encore, tous les effets nucléaires mentionnés ci-dessus sont présentés en détail, ainsi que certains des modèles de noyaux les plus populaires.

Le chapitre 3 détaille l'expérience T2K, y compris les objectifs scientifiques, les ensembles

de detecteurs et certains des résultats récents de T2K. La description des détecteurs part du complexe de l'accélérateur pour produire le faisceau de neutrinos, puis des ensembles de détecteurs proches et lointains. La configuration et les principes de détection sont présentés pour chaque ensembles de détecteurs.

Le chapitre 4 présente le projet de mise à niveau du détecteur proche du T2K pour la phase II du T2K (T2K-II). Le T2K-II est décrit au début, y compris sa motivation, ses améliorations et les résultats de physique attendus. La mise à niveau du ND280 est le cur des projets de mise à niveau pour T2K-II. Les limites du ND280 actuel sont prises en compte par la mise à niveau du ND280, qui est expliquée avec les nouveaux sous-détecteurs de la nouvelle configuration du ND280. Ces nouveaux sous-détecteurs comprennent le temps de vol, le détecteur à grains super fins (Super-FGD) et la chambre de projection temporelle à grand angle (HA-TPC). Les résultats de l'analyse du faisceau d'essai pour le HA-TPC sont présentés en détail à la fin du chapitre.

Le chapitre 5 est l'étude centrale de cette thèse qui présente les études de physique pour la mise à niveau du ND280. Le chapitre commence par l'introduction de l'importance des études physiques, puis la définition de toutes les variables transversales uniques (STV), qui sont utilisées comme nouvelles observables dans l'analyse pour contraindre les effets nucléaires. Un fit de likelihood a été développé pour évaluer la sensibilité du ND280 amélioré au modèle de flux et de section transversale en exploitant les STVs. Tous les détails du fitter sont présentés. Enfin, le fitter a montré les sensibilités quantitatives aux incertitudes systématiques clés telles que CCQE, 2p2h, l'interaction entre les états finaux des nucléons (FSI) et la normalisation de l'hydrogène. Ces résultats constituent une estimation générale de la sensibilité prometteuse des futurs fits à proximité du détecteur en utilisant la mise à niveau ND280 avec une meilleure efficacité en impulsion et en angle.

Le chapitre 6 présente un modèle d'interaction neutrino-noyau de Martini et de ses collaborateurs, qui a été utilisé comme modèle alternatif pour les études de données fictives dans T2K. Ces études de données fictives ont été réalisées pour tester la robustesse de l'analyse des oscillations pour différents modèles d'interaction. Dans cette thèse, le modèle de Martini et al. pour le canal de production de pions est utilisé pour le test. Le chapitre commence par une introduction au modèle de Martini et al. Ensuite, l'implémentation des fausses données et de l'analyse des oscillations dans T2K est présentée. Le test a montré des résultats cohérents avec ceux publiés par T2K.

Introduction

Neutrino physics is one of the most attractive fields in particle physics nowadays. The discovery of neutrino oscillations suggests that neutrinos must have mass and implies physics beyond the three massless neutrinos of the Standard Model (SM). Its beginnings can be traced back to the Homestake Mine in South Dakota in 1967 when Ray Davis and John Bahcall first noticed a mismatch between the anticipated and actual number of solar neutrinos. The "solar neutrino anomaly" was the subject of extensive experimental and theoretical research. It gained even more widespread attention when the Germanium experiments GALLEX and SAGE also found fewer solar neutrinos than predicted. Other experiments also found neutrino anomalies in detectors built to search for a completely unrelated phenomenon: proton decay. Although proton decay was not detected, a detailed analysis of the neutrino background to those measurements revealed that fewer atmospheric muon neutrinos than predicted were found, suggesting a similar phenomenon to that indicated by the solar anomaly.

The preferred theory for the observed neutrino anomalies has evolved to be neutrino oscillations. The idea of flavor oscillation was developed by Bruno Pontecorvo, Ziro Maki, Masami Nakagawa, and Shoichi Sakata (PMNS), which could explain the solar anomaly. Around 25 years ago, neutrino oscillations were discovered thanks to measurements taken at Super-Kamiokande and the Sudbury Neutrino Observatory, which were recognized by the 2015 Nobel prize in physics. After various far-reaching discoveries, neutrino oscillation physics is in the precision era today.

Precise measurement of neutrino oscillation parameters and neutrino cross-sections is one of the highest priorities in elementary particle physics. There are still many properties of neutrinos that the Standard Model can not explain. The nonzero neutrino masses or the potential Charge Parity (CP) symmetry violation in the leptonic sector are the good ones among a few hints related to Physics beyond the SM. CP violation is one of the three conditions proposed by Andrei Sakharov in 1967 to explain the matter-antimatter asymmetry, or in other words, to explain the existence of matter in the universe. The CP violation is already found in the mixing of quarks in the SM. However, the matter-antimatter asymmetry seen in the universe cannot be fully explained by such a small amount of asymmetry in the quark sector. Fortunately, according to recent neutrino oscillation measurements, there is a potential possibility of a large CP violation in the leptonic sector. For example, the T2K experiment - a long baseline neutrino oscillation experiment - has shown the constraint of the phase δ_{CP} where it can exclude the CP conservation at 2σ [1].

Based on the current hint about CP violation, the importance of precise neutrino oscillation measurements has been furthermore proved by the upcoming two extensive experiments: Deep Underground Neutrino Experiment (DUNE) and the Tokai-to-Hyper-Kamiokande project (T2HK) hosted by the US and Japan, respectively. Nevertheless, more giant detectors are not enough; precise neutrino measurement also requires that the systematic errors must be reduced as much as possible (typically down to a few percent) in neutrino oscillation experiments. One of the main systematic errors comes from our incomplete understanding of neutrino-nucleus interactions.

Neutrino-nucleus scattering uncertainties continue to be a significant source of systematic uncertainty in many neutrino oscillation experiments, including T2K. Over many years, neutrino scattering measurements on both charge current and neutral current channels have already been accumulated utilizing many sorts of targets, analysis methodologies, and detector technology. Taking advantage of the intense neutrino sources and more precise neutrino flux predictions in many neutrino oscillation experiments, physicists are presently measuring such neutrino interaction cross-sections to better understand nuclear effects.

This thesis presents the upgrade project of the T2K's near detector (ND280) and especially its capacity to probe the nuclear effects, which can bias neutrino cross-section and neutrino energy reconstruction. The upgraded ND280 will confront our naivety of neutrino interactions using a new detector configuration with full polar angle acceptance and a much lower momentum proton tracking threshold. Thanks to these new properties, ND280 can measure the novel observables using kinematic imbalance in the plane perpendicular to the direction of incoming neutrino, which is the single transverse kinematic imbalance. These observables are sensitive to several nuclear effects such as the Fermi motion, the re-interaction of hadrons within the nuclear environment, and the correlations between nucleons in a nucleus.

The first chapter in this thesis discusses the history of neutrino discovery and the PMNS framework and provides a general knowledge of the neutrino field. It also introduces some of the critical neutrino experiments, exceptionally long baseline neutrino oscillation experiments, together with the current questions in the field.

The second chapter is dedicated to neutrino interactions. The different kinds of neutrinonucleon, neutrino-nucleus interactions, and their relevant experiment results are described. More importantly, all of the nuclear effects mentioned above are presented in detail, together with some of the most popular nucleus models.

Chapter 3 details the T2K experiment, including the scientific goals, the detector complexes, and some of the recent T2K results. The detector description starts from the accelerator complex to produce the neutrino beam, then the near and the far detector complexes. The configuration and detection principles are presented for each detector complex.

Chapter 4 overviews the upgrade project of the T2K's near detector for the T2K phase II

(T2K-II). The T2K-II is described at the beginning, including its motivation, improvements, and expected physics results. The ND280 Upgrade is the core among the upgrade projects for T2K-II. The limitation of the current ND280 is addressed by the upgraded ND280, which is explained together with new sub-detectors in the new ND280 configuration. These new sub-detectors include the Time of Flight, the Super Fine Grained Detector (Super-FGD), and the High Angle Time Projection Chamber (HA-TPC). The test beam analysis results for the HA-TPC are presented in detail at the end of the chapter.

Chapter 5 is the core study of this thesis which presents the physics studies for the ND280 Upgrade. The chapter starts with the introduction of the importance of physics studies, then the definition of all single transverse variables (STVs), which are used as new observables in the analysis to constrain the nuclear effects. A likelihood fit was developed to evaluate the sensitivity of the upgraded ND280 to flux and cross-section model by exploiting STVs. All of the detail of the fitter is presented. Finally, the fitter has shown the quantitative sensitivities to key systematic uncertainties such as CCQE, 2p2h, Nucleon Final State Interaction (FSI), and Hydrogen normalization. These results are the broad estimation of the promising sensitivity of future near detector fits using ND280 Upgrade with better efficiency in momentum and in angle.

Chapter 6 introduces a neutrino-nucleus interaction model from Martini and collaborators, which was used as an alternative model for the fake data studies in T2K. These fake data studies were performed to test the robustness of the oscillation analysis to different interaction models. In this thesis, the Martini et al. model for the pion production channel is used for the test. The chapter starts with an introduction to Martini et al. model. Then, the implementation of the fake data and the oscillation analysis in T2K is presented. The test has shown consistent results with the one published by T2K.

Neutrino physics

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1.1 History of the Neutrino particles

1.1.1 Prehistory of neutrino

Natural radioactivity was discovered by Henri Becquerel in 1896 in Paris by studying the phosphorescence in uranium salts. He put a piece of uranium on a photographic plate, and this uranium piece was emitting something that was leaving an imprint on the photographic plate. This strange thing was later identified as the natural radioactivity and classified as on Tab. 1.1.

Rays	Charge	Bend in Magnetic field	capacity to be absorbed	Spectrum
α rays	+	hard	easy	discrete
β rays	-	easy	harder	continuous
γ rays	0	no	very hard	discrete

Table 1.1: Natural radioactivity status in early state.



Figure 1.1: The β rays spectrum was clearly shown as continuous spectrum in 1914 [2] (left) and then was better measured by Ellis and Wooster[3] (right).

These rays were named by α , β and γ because, at that time, physicists did not know any of the fundamental particles. Nevertheless, not too long later, in 1897, Sir Joseph John Thomson discovered a subatomic particle for the first time, which is the electron particle. It turned out that the β rays are indeed the electrons.

In 1914, Chadwick showed an indisputable evidence for a continuous β ray spectrum (Fig 1.1). That was a big surprise for the physicists since the electron energy should have very well-defined energy in 2-body decays based on the energy-momentum conservation. This result leads to the fact that there is an energy loss mechanism in the system, which was unknown at that time.

To address this, Niels Bohr came up with the idea that energy is not conserved in nuclear physics processes. In other words, energy is just conserved statistically. "At the present stage of atomic theory, however, we may say that we have no argument, either empirical or theoretical, for upholding the energy principle in the case of β -ray disintegrations", said Niels Bohr.

In 1930, Pauli proposed that "there could exist in the nucleus electrically neutral particles", which can solve the β spectra problem. Since the β decay process is the three-body final state process with the assumption that this neutral particle is emitted along with the electron, this new particle was named "neutron" by Pauli since it should be charged neutral, and this "neutron" should have excellent penetrating properties in order not to be observed experimentally. This idea was not published in any paper or conference proceeding because Pauli had many concerns when he had invented a particle that could not be detected.

In 1932: Chadwick discovered the neutron by converting neutrons, produced by α capture on Be, into protons using paraffin wax. This neutron is different from Pauli's neutron because the one proposed by Pauli is much lighter. Then Enrico Fermi re-named Pauli's neutron a "small neutron", which in Italian is a neutrino. The -ino suffix in Italian means small.

In 1934, Fermi postulated the Fermi theory for weak interaction to explain β decay [4]

$$\mathcal{H} \sim G_F(\bar{p}\Gamma n)(\bar{e}\Gamma\nu_e),\tag{1.1}$$

where G_F is the Fermi constant, $\Gamma = \{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$ depending on the currents such as vector, axial vector, pseudo scalar current. That was the first time the possibility of changing particles to other particles after an interaction was allowed. Therefore, the Fermi theory predicts a way to detect neutrino (actually it's anti-neutrino), which is well-known today, the inverse β decay

$$\bar{\nu}_e + p \rightarrow e^+ + n.$$

However, the cross-section followed by Fermi calculation was extremely small $\sigma_{\bar{\nu}p} \sim 5 \times 10^{-44} \text{cm}^2$ for 1MeV anti-neutrino. In comparison, the electromagnetic interactions of similar energies particles have the cross-section $\sigma_{\gamma p} \sim 10^{-25} \text{cm}^2$. Hence people believed that neutrino is unobservable.

Now physicists know that the Fermi theory is an effective theory for the weak interaction with W boson as a propagator. The Lagrangian for this interaction vertex between 4 fermions has dimension six, which is not renormalizable. However, nowadays, this technique is widely used again to probe physics Beyond the Standard Model (BSM), where it considers the Standard Model (SM) as a leading order of a more completed theory [5, 6].

1.1.2 Neutrino discovery

Based on the prediction of Fermi's theory, although neutrino cross-section is tiny, it is possible to detect neutrino interactions if very large neutrino fluxes are available. In 1956, more than 20 years after Pauli's proposal and thanks to the development of nuclear techniques during the World War II, Frederick Reines and Clyde Cowan, Jr., American physicists working at the Los Alamos National Laboratory, finally showed strong evidence of anti-neutrino interactions in their detector using the rich and stable anti-neutrino source from a nuclear reactor at the Savannah River in South Carolina [7].

These anti-neutrino were observed via the inverse beta decay process $(\bar{\nu}_e + p \rightarrow e^+ + n)$ with the help of the intense anti-neutrino flux of $5 \times 10^{13}/cm^2/s$ produced by the reactor. To enhance the chances of capturing a neutron, a big detector filled with liquid scintillator loaded with Cd was constructed. In order to prevent reactor neutrons and gamma rays from reaching the detector and causing undesirable background, the 300-litre detector was covered by handmade boronparaffin shielding mixed up with the lead and recorded by 90 two-inch photomultipliers (PMTs). A coincidence detection of the 511 keV oppositely directed photons produced by positron annihilation $(e^+ + e^- \rightarrow 2\gamma)$ and a neutron absorption event $(n + ^{108} \text{ Cd} \rightarrow ^{109} \text{ Cd}^* \rightarrow ^{109} \text{ Cd} + \gamma)$ a few microseconds later was used as the detection signature. This delayed coincidence is a strong tool for distinguishing the inverse beta decay signal from background. However, the delayed-coincidence background, which was actually present independently of the reactor status, was roughly five events per minute, significantly larger than the anticipated signal rate (0.1 to 0.3 events/minute). Later investigation showed that cosmic rays entered the detector, causing the background. When the reactor was turned on, the little increase in the event rates was insufficient. The first experiment's results were inconclusive.

Despite this, they changed the experiment just after failed trial to better discriminate between events caused by cosmic rays and those caused by reactor neutrinos. Three enormous scintillator detectors with 110 PMTs were placed between the target tanks to capture scintillation light and generate electrical signals. With this upgraded detector, the observed inverse beta decay cross-section was eventually found to match Fermi's prediction, confirming that anti-neutrinos had been discovered. This discovery was honored with the Nobel Prize in Physics in 1995 (Cowan had gone by that time).

1.2 Neutrino sources and their flux

The uncovering of neutrino physics was a major scientific breakthrough in the twentieth century thanks to various available natural and artificial neutrino sources. These sources allow studying diverse physics fields such as nuclear physics, astrophysics, and cosmology. The spectrum representing prominent natural and manufactured neutrino fluxes is illustrated in Fig. 1.2.

The lowest energy from this spectrum is the cosmological neutrino, also known as relic neutrino. They are the relic of the Big Bang. When the universe was only one second old, the relic neutrino was already decoupled from matter. This property of relic neutrino is helpful in studying the early universe because, for reference, the cosmic microwave background radiation (CMB) was produced when the universe was 379,000 years old. The energy range of relic neutrino is from μeV to meV. Although the cosmological neutrino is expected to have the highest flux, their interaction cross-section and the energy of final state particles from their interactions are incredibly tiny; hence, there is no feasible way to detect these neutrinos up to now.

Solar, nuclear reactors, supernovae and the Earth's interior neutrinos are all found in the keV-MeV range. The "atmospheric neutrinos" produced by cosmic ray interactions in the Earth's atmosphere have higher energy, and dominates the energy range from GeV to TeV. Next on the energy scale are neutrinos from sources that are far away, such as Gamma-Ray Bursts, supernova remnants, or Active Galactic Nuclei (AGN). The last one which has the highest energy is the cosmogenic neutrino. These neutrinos are produced by ultra-energetic protons colliding with the cosmic microwave background (CMB) at 2.7 K [9].

Apart from all of the sources above, today physicists can produce a neutrino beam based on accelerator. Since this is an artificial beam, we can choose the energy range and the shape of the spectrum which are well controlled by modern techniques (see Sec. 3.2.1).



Figure 1.2: Observed and predicted neutrino fluxes for currently-known possible types of neutrino sources. Underground detectors operate in the energy spectrum of a few keV to many GeV. Cherenkov light detectors are used underwater and in ice to study the neutrino range from tens of GeV to roughly 100 PeV, which has significantly lower fluxes. The higher neutrino energy to be investigated, the larger detector volumes are required. Plot taken from [8].

1.3 Neutrinos in the Standard Model

Standard Model (SM) is the most successful theory ¹ which is able to describe three of four fundamental interactions: electromagnetic, weak, and strong interactions. Gravity interaction is not considered in this model. This model also helps us to classify all currently known elementary particles, which are shown in Fig.1.3. Depending on their spin, the particles are classified into fermions (non-integer spin²) and bosons (integer spin). There are twelve particles of fermionic matter (quarks and leptons), governed by three forces that are caused by the exchange of four Gauge boson particles (photon, Z, W boson, and gluon). The other particle in SM is the Higgs boson, which gives mass to the other massive particles. In SM, the interaction ranges depend on the interaction propagators' mass except for the gluons due to the gluons' confinement in QCD. The weak interaction propagators W and Z bosons are much more massive than photons or gluons with zero mass, resulting in a concise range of weak interaction. Therefore, the probability of a weak interaction occurring is small, as indicated by its name. The paramount property of SM is using the local

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \tag{1.2}$$

¹The word "model" in Standard Model has been historically and widely used for many decades. Actually, the SM is more like a theory rather than a model.

²spin is the amount of angular momentum associated with a elementary particle or nucleus

gauge symmetry that is spontaneously broken to the subgroup $SU(3)_C \otimes U(1)_{EM}$. Experiments with high-energy particles at accelerators have completed our knowledge about SM with fantastic precision. The Lagrangian of SM is invariant under the transformation in Eq. 1.2 and has the form

$$\mathcal{L} = \mathcal{L}_{fermion} + \mathcal{L}_{gauge} + \mathcal{L}_{qf} + \mathcal{L}_{Hiqgs} + \mathcal{L}_{Yukawa} + \mathcal{L}_{ghost}.$$
(1.3)

Fermions are classified into three types in the SM. Neutrinos are the companions of charged



Standard Model of Elementary Particles

Figure 1.3: Elementary particles in SM. Plot was taken from https://en.wikipedia.org/wiki/S-tandard_Model

leptons, and they are the least understood among the fermions. In SM, neutrinos have zero mass, no electromagnetic or colour charge but only a weak charge. They, therefore, only interact via weak interaction, which is mediated by the W^{\pm} and Z bosons, with an extremely small cross-section. Neutrino forms left-handed weak isospin doublets under the $SU(2)_L$ gauge symmetry. Therefore, a right-handed anti-neutrino with an opposite charge should exist based on the conservation of charge, parity, and time-reversal symmetry (CPT). Nevertheless, the CPT conservation does not imply a right-handed particle existing parallel with a left-handed particle. This interesting case is valid for neutrinos when neutrinos are always "left-handed". However, anti-neutrinos are always "right-handed," as determined by experimental data [10] about the helicity of neutrinos³.

The charged-current (CC) interaction with the associated charged lepton l defines an incoming neutrino of flavour l. In particular, the muon neutrino is always accompanied by a charged muon. The CC interactions between neutrinos and associated charged leptons are provided in

³Helicity is the projection of a particle's spin onto its momentum. Right-handed particles (right helicity) indicate that their spins have the same direction with their momentum, while these directions are opposite for left-handed particles.

SM Lagrangian by^4

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \sum_{l} \bar{\nu}_{Ll} \gamma^{\nu} l_L W^+_{\mu} + h.c.$$
(1.4)

Neutral-current (NC) interactions are also observed in SM neutrinos, as has been shown in

$$-\mathcal{L}_{NC} = \frac{g}{2\cos\theta_W} \sum_l \bar{\nu}_{Ll} \gamma^{\nu} \nu_{Ll} Z^0_{\mu}.$$
 (1.5)

The decay width of the Z^0 boson (i.e., the inverse of its lifetime) to neutrinos may be calculated using Eq. 1.5, which is proportional to the number of left-handed neutrinos generations. By measuring the total Z boson decay width for $Z \to \nu \bar{\nu}$, the number of neutrino flavours involved in this process could be determined. According to the amazing combined precision by four LEP experiment, the invisible Z boson width gives $N_{\nu} = 2.984 \pm 0.008$ [11], shown in Fig. 1.4. This is the number of neutrinos with mass smaller than the one of the Z boson. Beyond the SM (BSM), there are other possible neutrino species that can not couple to the Z boson but can participate in the oscillation process called sterile neutrinos (more details in 1.7).



Figure 1.4: Hadron production cross-section measurements near the Z resonance. The lines show the estimated cross-section for two, three, and four neutrinos with SM couplings and negligible mass. The results are combined by four LEP experiments ALEPH, DELPHI, L3 and OPAL at CERN. Plot taken from [11].

As mentioned before, neutrinos are strictly massless in the SM. However, there is indirect observational data indicating that neutrinos have non-zero masses (more in Sec. 1.5). The SM

⁴The h.c. stands for hermitian conjugate
must be expanded in order to provide mass to the neutrino. This creates some difficulties since, in SM, the mass term of any particle is established by the left- and right-handed particles, while left-handed neutrino does not seem to have a right-handed partner. The SM gauge invariance does not require lepton number symmetry. Depending on the neutrino nature, the total lepton number is not necessarily a symmetry. Theoretically speaking in more detail, any fermion in SM needs to have two helicity states to have mass. The strength of the coupling of these two states represents the mass of the fermion. To add any such neutrinos coupling in the SM, we must first define the right-handed states of the neutrino, which are not present in the SM. There are two options to do so:

• Dirac neutrinos: a right-handed neutrino is introduced in parallel with the lepton number conservation

$$\mathcal{L} = \mathcal{L}_{SM} - M_{\nu} \bar{\nu}_R \nu_L + \text{h.c.}$$
(1.6)

This extension contains three singlets right-handed neutrino states, which are coupled to left-handed ones through neutrino masses. To ensure gauge symmetry, this coupling must be an extremely large Yukawa coupling. Hence, the neutrino masses are proportional to the Higgs field's vacuum expectation value (v). Since all other fermions' masses are also proportional to v, there is still a big hierarchy about the huge gap between neutrinos' masses and other charged leptons' masses.

• Majorana neutrinos: the lepton number conservation is not required; however, the antiparticle of the left-handed neutrino is identified as the right-handed state in the mass term [12]

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{2} M_{\nu} \bar{\nu}_L^C \nu_L + \text{h.c.}, \qquad (1.7)$$

where C is the charge conjugation operator in spinor space. Using this approach does not change the symmetry of SM but the intrinsic properties of neutrinos when they are not Dirac fermions anymore but Majorana fermions (i.e., they are their own anti-particles). The neutrino masses in Majorana picture are given

$$m_{\nu} = \alpha_{\nu} \frac{v^2}{\Lambda}.$$
 (1.8)

Within this picture, the conservation of all fermion charges, even the lepton number, is violated. The genesis scale Λ in the Majorana masses is described in many models. However, in the renowned see-saw mechanism [13], thanks to the tremendous value of the mass eigenvalues scale ($\Lambda >> v$), the hierarchy between neutrino and other lepton masses can be explained spontaneously.

One method to test this hypothesis directly is to seek for neutrino-less double decay $(0\nu\beta\beta)$, which is seen in Fig. 1.5. Observing $0\nu\beta\beta$ would prove that neutrinos are Majorana particles. However, not observing $0\nu\beta\beta$ does not imply that neutrinos are Dirac particles. If the $0\nu\beta\beta$ process exists, its probability of occurring will be very low.



Figure 1.5: Feynman diagrams show double decays under various hypotheses about the nature of the neutrino. (a): In the final state of a two-neutrino double β decay, two anti-neutrinos are produced. (b) depicts the neutrino-less double β decay, which has just one kind of Majorana neutrino, denoted as ν_M . Plot taken from [14].

1.4 Circumstances leading to neutrino oscillation

1.4.1 Problems coming from solar neutrino

Neutrinos have a high penetrating capability since they rarely interact with other particles. As a result, it can travel large distances nearly unaffected and transport information across long distances. This property qualifies them to be an astronomical observable used in multi-messenger astronomy to study cosmological phenomena. Moreover, neutrinos can be an excellent probe of any object or environment that many other particles could not penetrate. For example, to evaluate Bahcall's solar model⁵ [15], neutrinos might be employed to investigate the nuclear reactions occurring beyond the Sun's surface.

In the late 1960s, R. Davis et al. proposed and carried out the first experiment to measure the neutrino flux created in the Sun at the Homestake mine in South Dakota [8]. The Sun's power is generated by several fusion processes (Fig. 1.6), which solely create ν_e with the energy distribution depicted in Fig 1.7. The ν_e flux predicted by Bahcall et al. model reaching the Earth is $10^8 \nu_e/s/m^2$. R. Davis wanted to detect neutrinos created by the Sun and compare the flux to what Bahcall et al. estimated. In this experiment, only ν_e were observed by the inverse beta decay process in a 400 m³ tank of C_2Cl_4 . The electron neutrino flux is then calculated by quantifying the number of radioactive Ar nuclei created by the interaction $Cl + \nu_e \rightarrow Ar + e^+$. Davis et al. discovered that the observed neutrino spectrum was two to three times lower than theoretically predicted by Bahcall's model. The majority of early attempts to explain this difference suggested that the solar model predictions were inaccurate or that something was wrong in the experiment. Both of these explanations have been proved incorrect. Many helioseismology observations and a better understanding of the Sun verified Bahcall's predictions. In the 1990, numerous additional experiments replicated Davis et al discovery using various techniques such as inverse beta decay (Gallex [16], Sage [17], GNO [18]) or water Cherenkov detectors (Kamiokande [19], Super Kamiokande [20]).

⁵It is also called the Standard Solar Model (SSM)



Figure 1.6: The solar proton-proton fusion chain is responsible for 99% of solar energy production [21]. The neutrinos generated in a number of the reactions are highlighted in red. Plot taken from [22].



Figure 1.7: Solar neutrino flux broken down by different fusion chain sources. Plot taken from [22].

The only remaining explanation for the lack of solar neutrinos is that they convert into something else, which an inverse beta decay detector cannot detect. Bruno Pontecorvo proposed an alternate solution to the 'solar neutrino problem' in 1967 [23] which was the expanded version of the original one published in 1957 [24, 25]. He would develop a theory of lepton number violation where it violates individual lepton numbers while preserving overall lepton numbers. This hypothesized that neutrinos might indeed oscillate between flavour states as they travel, which lowers the ν_e rate at Earth by converting a ν_e into a linear superposition of the three neutrino families. However, the overall neutrino flux is conserved. An experiment that could demonstrate that the entire neutrino flux was preserved would be proof of neutrino oscillation.

In 2002, the SNO collaboration showed their first results on the neutrino oscillation hypothesis testing [26]. SNO is a Cherenkov detector filled with heavy water. It is sensitive with not only ν_e charged current interaction (sensitive to electron neutrino flux) but also flavor independent neutral current interactions (sensitive to total neutrino flux). The unbound neutron in the final state is subsequently detected, and the total solar neutrino flux can be determined using this measurement. The flux constraints (illustrated in Fig. 1.8) for charged- and neutral-current



Figure 1.8: Solar neutrino flux was measured using three channels: CC for charged current interactions, NC for neutral current interactions, and ES for elastic scattering off electrons. The dotted lines represent the Standard Solar Model's estimation. Plot taken from [26].

measurements give

$$\phi_{CC} = \phi_{\nu_e} = 1.70 \pm 0.07 (\text{stat})^{+0.09}_{-0.010} (\text{syst}) \cdot 10^6 \frac{\nu}{\text{cm}^2.s}, \tag{1.9}$$

$$\phi_{NC} = \phi_{\nu_e} + \phi_{\nu_{\mu}} + \phi_{\nu_{\tau}} = 4.90 \pm 0.24 (\text{stat})^{+0.29}_{-0.027} (\text{syst}) \cdot 10^6 \frac{\nu}{\text{cm}^2 \cdot s}.$$
 (1.10)

The results revealed that the electron neutrino flux was around one-third of what the solar models predicted, but the overall neutrino flux was in great agreement. It means that solar neutrinos, which are generated in electron flavour, reach the Earth as a mix of 3 flavours, with ν_e accounting for just one-third of the total. This was significant proof that Pontecorvo was right about the flavour change of solar neutrinos. Although the solar neutrino problem is quite old, the answer from SNO in 2002 is not the earliest evidence for neutrino oscillation. The first one had been given four years before SNO results from the atmospheric neutrinos studies.

1.4.2 Atmospheric neutrinos

The Earth's atmosphere is the second biggest natural source of neutrinos after the Sun. These neutrinos are indirectly produced from the interaction between the cosmic rays and the atmosphere. This interaction generates hadronic showers which are mostly composed of pions. These pions later decay into muons and neutrinos. The muons, again, produce neutrino through their decay processes

$$\pi^{\pm} \to \mu^{\pm} + (\nu_{\mu}/\bar{\nu_{\mu}})$$
 (1.11)

$$\mu^{\pm} \to e^{\pm} + (\nu_e/\bar{\nu}_e) + (\nu_{\mu}/\bar{\nu}_{\mu}).$$
 (1.12)

The techniques to measure neutrino from the Sun and the atmosphere are quite different. One thing that makes the atmospheric neutrino oscillation measurement complicated is that



Figure 1.9: Left: different travel lengths to reach Super-Kamiokande for neutrinos created in cosmic ray interactions with the Earth's atmosphere. **Right:** Ratio of ν_{μ} and ν_{e} atmospheric neutrino events observed by Super-Kamiokande to Mont-Carlo non-oscillated prediction as a function of travel distance over neutrino energy. Plot taken from [27].

the neutrino generator sources are not point-like, and the distance from the sources to the detector is not fixed, as was the case for solar neutrinos. The other difficulty is that the range of atmospheric neutrino energy: the broad energy spectrum of atmospheric neutrinos ranges from a few hundred MeV to several GeV. Since neutrinos may be produced at any place in the atmosphere, the travel lengths are different for each event, resulting in varying oscillation probabilities even with the same neutrino energy. Fortunately, since the Earth's atmosphere is isotropic, one can estimate the distance from the neutrino source to the detector based on the incident neutrino direction. Water-Cherenkov detector has been proved to be the most appropriate for this task (more detail about water Cherenkov detector techniques in Sec 3.4).

Cherenkov light (or Cherenkov radiation) is electromagnetic radiation emitted when a charged particle passes through a dielectric medium at a speed greater than the phase velocity (speed of propagation of a wavefront in a medium) of light in that medium [28]. A standard water Cherenkov detector is made up of a large water tank (up to a hundred kilotons) with many phototubes installed on its wall that capture the Cerenkov light. In principle, leptons are produced when neutrinos interact with anything via charged current. When the neutrinos have high enough energy, i.e., their interaction in water can produce leptons with energy greater than the water Cherenkov threshold, a cone of light centred on the lepton path is emitted. Based on the Cherenkov light of the lepton, water Cherenkov detectors can reconstruct the direction and the energy of the incoming neutrino.

Super-Kamiokande is the huge water Cherenkov detector [20]. It classifies the atmospheric neutrinos into two kinds based on their direction: down-going neutrinos, which are produced in the atmosphere above the detector, and up-going neutrinos, which are produced in the opposite direction (see Fig. 1.9). The primary distinction between the two kinds of neutrinos in the oscillation process is their travel length. Downgoing neutrinos may travel distances from 10 km to 500 km, while the distances of upgoing neutrinos are on the scale of 10^4 km, indicating that their baselines are substantially different. As a result of the processes in Eq.1.11 and Eq.1.12, it is expected that the number of ν_{μ} is two times larger than ν_{e} in the observed flux. However, Super-Kamiokande discovered that this ratio deviated greatly from the predicted value of 2 depending on the direction of neutrino [27]. The outcome initially reported in 1998 is that the amount of upgoing ν_{μ} , thereby crossing the Earth, is half of that of downgoing ν_{μ} . Furthermore, the reduction was only observed for muon neutrinos, not electron neutrinos. Hence, one could not say the interaction with the Earth is responsible for this ν_{μ} reduction. The remaining potential answer is the same as the solar neutrino problem: muon neutrinos have changed their flavour into a superposition of different flavours. However, this superposition state does not include electron neutrino flavour since there is no excess in the upgoing ν_e flux.

The Super-Kamiokande observations were also matched to the hypothesis of "neutrino flavour oscillations" proposed in 1962 by Ziro Maki, Masami Nakagawa, and Shoichi Sakata [29], which elaborated on Pontecorvo's idea of $\nu \leftrightarrow \bar{\nu}$ oscillations. More than ten years later, Takaaki Kajita from the Super-Kamiokande experiment and Arthur B. McDonald from the SNO experiment shared the 2015 Nobel Prize in Physics for discovering neutrino oscillations.

1.4.3 Reactor anti-neutrinos

Up to now, the richest artificial source of anti-neutrino comes from nuclear reactors. They are antineutrino sources that are intense, pure, isotropic and well-understood. Neutrinos from reactors have also played an essential part in neutrino oscillations. They have contributed to the understanding of the solar anomaly. They have offered unique information on the smallest θ_{13} mixing angle ⁶.

The fission reactions inside nuclear reactors produce many unstable isotopes which eventually produce electron anti-neutrino $(\bar{\nu}_e)$ through β^- decay. ^{235}U , ^{238}U , ^{239}Pu , and ^{241}Pu are the four primary isotopes that contribute to the anti-neutrino emission. The technique to detect these anti-neutrinos is inverse beta decay. When the $\bar{\nu}_e$ interact with a proton, it transforms into a positron (e^+) and a neutron. The e^+ immediately transfers its energy and annihilates into two 511-keV γ -rays. The neutron scatters in the detector until it is grabbed by a proton $\sim 200 \ \mu$ s later, at which point it emits a 2.2-MeV γ -ray. Doping isotopes having very high neutron capture cross-sections, such as gadolinium, can drastically shorten the neutron capture time. The occurrence of this prompt-delayed signal pair reflects the presence of a $\bar{\nu}_e$ candidate. The top plot of Fig. 1.10 shows the schematic of inverse beta decay detection.

Since most of the key factors, such as the reactor power and the composition of the reactor fuel, are well controlled, the neutrino flux can be determined with pretty good precision. Fig. 1.10

⁶This θ_{13} parameter is one of the parameters that characterize the neutrino oscillation properties and will be introduced right after this section.



Figure 1.10: Reactor $\bar{\nu}_e$ detection. **Bottom:** the typical observed anti-neutrino spectrum is illustrated in parallel with the inverse β decay cross-section and anti-neutrino flux broken down by individual isotopes. **Top:** the detecting procedures are illustrated schematically. Plot taken from [30].

shows the typical energy spectrum of antineutrinos from nuclear reactors. However, most of the early reactor neutrino experiments were placed not so far from the nuclear reactor and, hence, do not have good sensitivity to neutrino oscillation.

KamLAND is an exception since it is located at an average distance of 180 km from 55 nuclear reactors in Japan. Thanks to this property, the KamLAND experiment gave the first evidence of reactor neutrinos oscillation in 2002 (see Fig. 1.11). The red point in the left plot of Fig. 1.11 represents the KamLAND finding, which agrees with the estimated disappearance by solar neutrino oscillations. In 2008, KamLAND provided a more precise measurement regarding the disappearance of reactor $\bar{\nu}_e$ [31]. The right plot in Fig. 1.11 shows the consistency between the more precise data from KamLAND and the prediction from the theory of neutrino oscillations. Nowadays, the precise measurement of the value of θ_{13} has been the target of current reactor experiments, which have used the same inverse β decay detection channel but more advanced detection methods.

The evidences for neutrino oscillation are various and clear now, and it is commonly agreed that neutrino oscillations definitely exist. The current emphasis of neutrino oscillation experiments is to examine the accuracy of this theoretical model, i.e. turning neutrino oscillation physics into a precision era. The following section aims to give an outline of neutrino oscillation theory.

1.5 Bird's eye view of neutrino oscillation theory

Different quantum mechanical oscillations in fundamental particle physics have revealed essential physics. In particular, we have learnt many things about CP violation from $(K^0 \leftrightarrow \bar{K^0})$ and



Figure 1.11: Left: the ratio of observed to expected non-oscillated neutrinos as a function of distance for various reactor anti-neutrino experiments including the disappearance $\bar{\nu}_e$ from KamLAND.

Right: Ratio of the background and geo-neutrino-subtracted $\bar{\nu}_e$ spectrum to the expectation for no-oscillation as a function of L_0/E . Plots taken from [31, 32].

 $(B^0 \leftrightarrow \overline{B^0})$ oscillations. The oscillation between the neutral gauge fields $(B \leftrightarrow W_3)$ determines the photon and Z^0 bosons as their mass eigenstates, while the oscillation between the quarks, $(d' \leftrightarrow s')$, permits the decay of s-quark. Because the quark and gauge boson oscillations are just too fast to detect, we can only measure the averaged effects as the Cabbibo and Weinberg angles. The neutrino oscillation is one among these quantum mechanical oscillations. However, the thing that makes neutrino oscillation distinct from the others is that the SM does not account for neutrino oscillations and the oscillation length is extremely long. The other thing that makes neutrinos different is that they can move at ultra-relativistic speeds due to their super small masses. The neutrino oscillation experiment makes it feasible to explore a very low mass scale region, which other experiments fail to attain. Moreover, neutrino physics is a good domain to probe the physics beyond SM (BSM), such as massive neutrinos and sterile neutrinos.

1.5.1 Neutrino mixing

Neutrinos have three flavours which are ν_e, ν_μ, ν_τ and they are massless in SM. A breakthrough came after the discoveries of neutrino oscillations [33] [34] which confirms nonzero neutrino masses. The neutrino flavour eigenstates, $|\nu_{\alpha}\rangle$ produced in a weak interaction, form a complete set $(\langle \nu_{\alpha} | \nu_{\beta} \rangle = \delta_{\alpha\beta})$. Hence, every flavor eigenstate is a superposition of eigenstates from another complete set, such as the mass eigenstates. Writing the flavor eigenstate $|\nu_{\alpha}\rangle$ as a linear combination of the mass eigenstates $|\nu_{j}\rangle$ gives:

$$|\nu_{\alpha}\rangle = \sum_{j=1}^{n} U_{\alpha j}^{*} |\nu_{j}\rangle.$$
(1.13)

In the inverse form, the mass eigenstates are also the combinations of flavor eigenstates

$$|\nu_j\rangle = \sum_{\alpha=1}^n U_{\alpha j} |\nu_\alpha\rangle.$$
(1.14)

Since there are three flavor eigenstates of neutrinos in SM, they are expected to have three mass eigenstates. Then the explicit mixing of neutrino flavour eigenstates ν_e, ν_μ, ν_τ into neutrino mass eigenstates ν_1, ν_2, ν_3 is of the form

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\mu 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$
 (1.15)

where U is the unitary⁷ leptonic mixing matrix and is named Pontecorvo-Maki-Nakagawa-Sakata (PMNS). If this matrix is diagonal, then the flavour eigenstates are identical to the mass eigenstates and the neutrino flavour oscillation does not exist. However, since neutrinos do oscillate, this PMNS is not diagonal and can be parameterized in terms of three mixing angles and one CP violation phase δ_{CP} . It can be split into three terms:

$$U_{\rm PMNS} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix},$$
(1.16)

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ and δ_{CP} is a complex phase. These three terms sub-components can be effectively considered as two-by-two matrices.

- The measurement from the sun dominates the first component in Eq(1.16). That is why we call the mixing angle θ_{12} the solar mixing angle which mixes the electron neutrino coming from the sun with ν_1 and ν_2 .
- The third component is dominated by what we observed with accelerator neutrino or atmospheric neutrino. That results in the name *atmospheric mixing angle* for the mixing angle θ_{23} .
- The second sub-component contains the mixing angle θ_{13} which is better measured with a reactor experiment. This sub-component is the interesting one since it is sensitive to the δ_{CP} value. The sensitivity to δ_{CP} depends on the value of s_{13} . Theoretically, if s_{13} is zero, it is impossible to probe the δ_{CP} value through neutrino oscillation experiments. Fortunately, experimental results showed that θ_{13} is not zero; therefore, the discovery of CP violation is potentially possible by neutrino oscillation experiments.

1.5.2 Neutrino oscillations

Now we know that neutrino oscillation can be parameterized using the PMNS matrix, the next goal is to know the oscillation probability. Starting with the time-dependent Schrödinger equation, we can obtain the neutrino mass eigenstate as a function of time $|\nu_i(t)\rangle^8$

$$i\frac{d}{dt}|\nu_i\rangle = \mathcal{H}|\nu_i(t)\rangle = E_i|\nu_i(t)\rangle, \qquad (1.17)$$

⁷A unitary matrix has to obey $U^{\dagger}U = I_3$

⁸Notice that all the calculations in this section are done in the natural unit (i.e. $\hbar = c = 1$) except for the ones which are clearly mentioned the unit (Eq. 1.28).

where $E_i = \sqrt{p_i^2 + m_i^2}$ denotes the particle energy. Solving the Schrödinger equation above gives us the solution as a plane wave with a complex phase depending on the time

$$|\nu_i(t)\rangle = e^{-iE_it} |\nu_i\rangle \tag{1.18}$$

Since the flavour eigenstate can be written in terms of the mass eigenstate in Eq. 1.13, we can derive the time-dependent flavour eigenstate as follows

$$|\nu_{\alpha}(t)\rangle = \sum_{i} U_{\alpha i}^{*} e^{-iE_{i}t} |\nu_{i}\rangle.$$
(1.19)

By applying the Eq.(1.14) to Eq.(1.19), we can obtain the time-dependent flavor states as a function of the initial flavor states.

$$|\nu_{\alpha}(t)\rangle = \sum_{\beta} \sum_{i} U_{\beta i} U_{\alpha i}^{*} e^{-iE_{i}t} |\nu_{\beta}\rangle.$$
(1.20)

Looking at Eq.(1.20), one can notice that since the PMNS matrix is not diagonal, a neutrino with a flavour β created through a weak interaction can be observed with a new flavour α after travelling during a period of time t. Neutrino creation and detection are always conducted through a weak coupling. Hence, detection techniques are not sensitive to neutrinos mass eigenstates. The probability of observing neutrino with flavour β from an original flavour α after a period of time t is

$$P_{\nu_{\alpha} \to \nu_{\beta}} = |\mathcal{A}_{\nu_{\alpha} \to \nu_{\beta}}|^{2} = |\langle \nu_{\beta} | \nu_{\alpha} \rangle|^{2} = \sum_{ij} U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*} e^{-i(E_{i} - E_{j})t}.$$
(1.21)

In general, the probability $P_{\nu_{\alpha}\to\nu_{\beta}}$ is called the *transition probability* or *oscillation probability*. The probabilities $P_{\nu_{\alpha}\to\nu_{\beta}}$ and $P_{\nu_{\alpha}\to\nu_{\alpha}}$ are known as the *appearance probability* and *survival probability*, respectively. Because U is a unitary matrix, the $\sum_{\beta} P_{\nu_{\alpha}\to\nu_{\beta}}$ is equal to one. This indicates that while the overall neutrino flux remains constant, the relative flavor composition varies over time. Writing the equation above in different way, we have

$$\sum_{\beta}^{\beta \neq \alpha} P_{\nu_{\alpha} \to \nu_{\beta}} + P_{\nu_{\alpha} \to \nu_{\alpha}} = 1, \qquad (1.22)$$

which means the sum of the "disappearance" probability and the survival probability for the flavour α is conserved at one.

Since neutrinos are ultra-relativistic particles $(E \sim p)$, we can use the approximation for all neutrinos' energy as $E_i = p + m_i^2/(2E)$. Moreover, because c = 1 in natural unit, the travelled time of neutrino can be written as travelled distance $t \sim L$. With these approximations, Eq.(1.21) becomes

$$P_{\nu_{\alpha} \to \nu_{\beta}} = \sum_{ij} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* e^{-i\frac{\Delta m_{ij}^2}{2E}L}, \qquad (1.23)$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$ is the squared mass difference between two neutrino mass eigenstates *i* and *j*. This term implies that the neutrino oscillation experiments can only provide information for the squared mass difference, not for the absolute neutrinos masses. The probability $P_{\nu_{\alpha} \to \nu_{\beta}}$ can be rewritten in terms of real and imaginary components separately

$$P_{\nu_{\alpha} \to \nu_{\beta}} = \delta_{\alpha\beta} - 4 \sum_{i>j} \operatorname{Re}(U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin^{2}\left(\frac{\Delta m_{ij}^{2} L}{4E}\right) + 2 \sum_{i>j} \operatorname{Im}(U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin\left(\frac{\Delta m_{ij}^{2} L}{2E}\right).$$
(1.24)

Based on Eq.(1.24), if neutrinos have the same mass, and notably if that mass is zero, the oscillation is nonexistent. Hence, discovering neutrino oscillations implies that neutrinos have masses greater than zero and their masses are different.

To obtain the oscillation probability for anti-neutrino, one needs to simply apply the charge and parity conjugation operators to Eq.(1.20). The result, which is very similar to the one of neutrino, is of the form

$$P_{\bar{\nu}_{\alpha} \to \bar{\nu}_{\beta}} = \delta_{\alpha\beta} - 4 \sum_{i>j} \operatorname{Re}(U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin^{2}\left(\frac{\Delta m_{ij}^{2} L}{4E}\right) - 2 \sum_{i>j} \operatorname{Im}(U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*}) \sin\left(\frac{\Delta m_{ij}^{2} L}{2E}\right).$$
(1.25)

Comparing Eq.(1.24) and Eq.(1.25), the sole distinction between them is the sign of the final term which is related to the imaginary part of the PMNS matrix. If $P_{\nu_{\alpha} \to \nu_{\beta}} \neq P_{\bar{\nu}_{\alpha} \to \bar{\nu}_{\beta}}$, the PMNS matrix must be a complex matrix. Since the imaginary of PMNS matrix is proportional to $\sin \delta_{CP}$, observing different oscillation probabilities of neutrino and anti-neutrino will directly lead to the CP symmetry violation in the leptonic sector. Another consequence from Eq.(1.25) is that CP violation measurement is only feasible in appearance channels ($\beta \neq \alpha$) and not in disappearance channels ($\beta = \alpha$) since the neutrino- and anti-neutrino-survival probabilities are the same

$$P_{\nu_{\alpha}\to\nu_{\alpha}} = P_{\bar{\nu}_{\alpha}\to\bar{\nu}_{\alpha}} = 1 - 4\sum_{i< j} |U_{\alpha i}U_{\alpha j}|^2 \sin^2\left(\frac{\Delta m_{ji}^2 L}{4E}\right).$$
(1.26)

This result is derived from the fact that $\operatorname{Im}(U_{\alpha i}^*U_{\beta i}U_{\alpha j}U_{\beta j}^*) = 0$ for $\alpha = \beta$.

In the two flavour approximation, the PMNS matrix can be parameterized with one parameter only. This parameter is denoted as θ and is called the mixing angle. The PMNS matrix with a rotation form gives

$$U_{2\times 2} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$
(1.27)

$$P(\nu_{\alpha} \to \nu_{\beta}, L, E) = \sin^2(2\theta) \sin^2\left(\frac{1.27\Delta m^2 L}{E}\right), \qquad (1.28)$$

where E is the energy of neutrino in GeV, L is the traveled distance of neutrino in km, and $\Delta m^2 = |m_1 - m_2|^2$ is the difference between squared mass eigenstates in $(eV/c^2)^2$. The factor 1.27 stems from inserting the \hbar and c in the SI unit.

1.6 Matter effects

One thing that should be noticed is that all the neutrino oscillation probabilities mentioned previously are for neutrino oscillation in a vacuum. In reality, when neutrinos travel over long distances in the matter, because of the coherent interactions with the matter it propagates through, effective potentials influence their evolution equation. Notwithstanding neutrinos' tiny interaction cross-section, this effect can modify the neutrino flavour oscillation probability. Therefore, the ultimate neutrino oscillation measurement is also affected by the medium. This effect is essential in neutrino experiments, especially in long-baseline experiments where neutrinos travel a long path through the matter. This matter effect is called MSW (Mikheev, Smirnov, Wolfenstein) effect [35, 36].

There are two types of neutrino interaction with matter, including Charge Current (CC) and Neutral Current (NC), that will be introduced in more detail in Chap. 2. With these two kinds of interaction, there are many final state possibilities with different particles. However, this section will focus on the interaction which maintains a neutrino in the final state since the other processes without neutrinos can not alter the neutrino oscillation probability. These interactions are the coherent scattering of neutrinos with the medium, including coherent forward elastic weak CC and NC interaction (see Fig. 1.12). It is worth noting that neutrinos in the matter are influenced not only by coherent forward elastic scattering but also by incoherent scatterings with atoms in the environment. Nevertheless, in most cases, the proportion of these incoherent scatterings is negligible and may be safely ignored.

As shown in Fig. 1.12, among the processes producing the neutrino in the final state, because NC interactions do not distinguish the interacting neutrino flavour, they have no effect on the neutrino oscillation probability. On the other hand, only electron neutrinos are affected by CC interactions since the natural matter contains only one lepton which is the electron. This impact may be incorporated into the oscillation probability derivation by including an extra effective potential component in the interaction Hamiltonian, which is of the form

$$V_{CC} = \pm \sqrt{2G_F n_e},\tag{1.29}$$

where the Fermi constant is denoted by G_F and the average electron density of the matter medium is denoted by n_e , the positive and negative signs correspond to neutrino and antineutrino, respectively. For reference, the matter potential at the Earth's and solar core are approximately 10^{-13} eV and 10^{-12} eV respectively [37]. Even though the values are relatively



Figure 1.12: Feynman diagram for charged (left) and neutral (right) current neutrino interactions with matter.

small, these influences are not negligible while studying neutrino oscillations. In the two flavours approximation, the Hamiltonian can therefore be written in terms of interaction potential as

$$H = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_V & \sin 2\theta_V \\ \sin 2\theta_V & \cos 2\theta_V \end{pmatrix} + \sqrt{2}G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$
(1.30)

where θ_V stands for the mixing angle in the vacuum.

This MSW effect is essential to study because it affects ν_e appearance probability in competition with δ_{CP} . Eventually, it can bias CP violation measurement in the leptonic sector. Since the matter is only made of particles and not anti-particles, the modifications due to matter effects in neutrino and anti-neutrino oscillation probabilities are different, and they can mimic the CP violation. Another meaningful impact that we should carefully study the matter effects is that they can help to gain sensitivity to the sign of the mass splittings Δm_{ij}^2 and eventually to the mass hierarchy. The comprehensive analysis of matter effects in the three flavour neutrino situation is extensive and carefully presented in numerous publications (e.g. [38]). Fig. 1.13 depicts the influence of matter effects on oscillation probability according to the sign of Δm^2 in a two-flavour neutrino oscillation approximation.

As demonstrated in Sec. 1.5, the oscillation probability is governed by the term $\sin^2[\Delta m^2 L/(4E)]$, which is unaffected by the Δm^2 sign. However, when the matter effects are taken into account, a new term appears in the probability, which is proportional to $\sin[\Delta m^2 L/(4E)]$. This term can help to clarify the sign of the mass difference. Its sensitivity to the mass hierarchy increases with the path length crossed by neutrinos in the matter. As a result, experiments with a very long baseline and neutrinos that transit through a huge quantity of matter have the most exquisite sensitivity to the mass ordering.

1.7 Mass hierarchy

Since the neutrino oscillation discovery, the primary neutrino experiments' aim has been to precisely measure oscillation parameters, including the mixing angles, mass splittings and CPviolation phase. Furthermore, a major unsolved topic is the neutrino mass hierarchy. Because



Figure 1.13: $P_{\nu_e \to \nu_e}$ for vacuum and matter deviation for two neutrino flavors, using a mixing angle $\theta = 22.5^{\circ}$ and a constant baseline L = 5000km. Figure taken from [39].

the oscillation experiments are only sensitive to the squared mass differences, not the absolute masses, the order of masses among three neutrino species is still unknown. The only known order is $m_2 > m_1$ thanks to the solar neutrino experiments, which study the matter effect inside the Sun. This matter effect is one of the important effects in neutrino oscillation experiments and will be discussed more in the following section. Thanks to this result, together with the squared masses differences constrained by the oscillation experiment

$$\Delta m_{32}^2 = 2.52 \times 10^{-3} \left(\text{eV/c}^2 \right)^2 \tag{1.31}$$

$$\Delta m_{21}^2 = 7.42 \times 10^{-5} \left(\text{eV/c}^2 \right)^2, \qquad (1.32)$$

there are two possible mass ordering: Normal Ordering (NO) $(m_3 > m_2 > m_1)$ or Inverted Ordering (IO) $(m_2 > m_1 > m_3)$. Fig.1.14 illustrates these mass ordering possibilities together with the contribution of the flavor eigenstates to each mass eigenstate. The contribution of ν_e , ν_{μ} , and ν_{τ} to the ν_2 state are nearly equivalent. While the ν_1 and ν_3 states are dominated by ν_e and ν_{τ} flavors, respectively.

Figure taken from [40].



Figure 1.14: The two mass hierarchies are depicted. The neutrino mass eigenstates are presented for each hierarchy. The flavour composition is also illustrated for each mass eigenstate.

One should note that there are only two independent parameters of Δm_{ij}^2 . Now we have some experimental observations that there are three experimental mass scales. Thus there must be either a sterile neutrino or the wrong determination of at least one mass scale.

1.8 Long baseline neutrino experiments

After the theoretical development and experimental discovery of neutrino oscillation, the primary goal of the current neutrino oscillations experiment is to precisely measure the oscillation parameters including three mixing angles θ_{12} , θ_{13} , θ_{23} , two mass splittings Δm_{12}^2 , Δm_{23}^2 , the CP-violating phase δ_{CP} , and the neutrino mass hierarchy. It is noticed that the neutrino oscillation experiments can not measure the absolute scale of the neutrino masses.

As proved in Eq. 1.28, the oscillation probability of neutrinos depends on the ratio of their travelled distance and their energy. This ratio is not modifiable for the natural neutrino sources such as solar, atmospheric or supernova neutrino. Modern neutrino oscillation experiments with the artificial sources can alter the L/E ratio to attain as better as possible the sensitivity to the oscillation parameters of interest. They use the accelerated proton to produce neutrino beam via the reaction

$$p + A \to \pi^{\pm} + X \tag{1.33}$$

$$\pi^{\pm} \to \mu^{\pm} + \nu_{\mu}/\bar{\nu}_{\mu} \tag{1.34}$$

The neutrino beam contains mostly muon neutrino since the branching fraction of the $\pi \to \mu + \nu_{\mu}$ decay channel is almost 100%. As a result, the accelerator neutrino beam is appropriate for

investigating the oscillation of muon-type neutrinos. Accelerator neutrinos typically have energies in the GeV range. Then the neutrino flavor change is observed over a few hundred kilometers, where the oscillation probability is maximised. These experiments contain three main parts: a beam production complex, a near detector, located near the beam production point, to measure the neutrino flux before oscillation, and a far detector, located far away where the oscillations are maximal, to measure the neutrino flux after oscillation.

The long-base line technique allows the oscillation measurements in two different channels: neutrino disappearance and neutrino appearance. The disappearance channel was the main focus of the first generation of neutrino oscillation experiments. An example is the KEK experiment [41] in 1999 which was the first experiment using the well-controlled neutrino source. Its goal was confirming the disappearance results previously claimed by Super-Kamiokande experiment. Another one of the first generation experiment is MINOS at Fermilab [42] which had a similar goal of searching for ν_{μ} disappearance. The MINOS experiment used the NuMI beamline [43] and took data from 2005 to 2012. The muon neutrino disappearance channel is particularly sensitive to the two oscillation parameters $\sin^2 \theta_{23}$ and Δm_{23}^2 .

The next generation of neutrino oscillation experiment originally tried to detect the $\nu_{\mu} \rightarrow \nu_{e}$ appearance channel. The T2K experiment (detail in Chap. 3), for the first time ever, observed the ν_{e} appearance with more than 7σ confidence level [44]. This ν_{e} appearance channel is particularly sensitive to the oscillation parameters $\sin^{2} \theta_{13}$ and δ_{CP} . Thanks to the discovery of large value of θ_{13} (cross-checked by precise result from the Daya Bay experiment [45]), the T2K and other current long-baseline experiments have moved their attention to the CP violation in the lepton sector. This is possible thanks to the ability of running with an anti-muon neutrino beam; the δ_{CP} value can be then extracted by comparing the event rate of ν_{e} and $\bar{\nu}_{e}$ appearance. The MINOS, T2K and NO ν A [46] have measured both ν_{μ} disappearance and ν_{e} appearance. The OPERA experiment provided the first and only $\nu_{\mu} \rightarrow \nu_{\tau}$ appearance events [47].

To identify the oscillation parameters, the neutrino events of an interested neutrino flavour are recorded in a histogram of some observables such as neutrino energy or lepton momentum. Let denote this observable Ψ . A predicted distribution of Ψ is provided by a Monte Carlo (MC) simulation with some specific values of oscillation parameters. Then there is a far detector fit whose job is to fit the predicted Ψ distribution to the data by varying the oscillation parameters. The fit with a set of parameters which gives best agreement between MC and data is called the best fit. The oscillation probabilities are significantly affected by oscillation parameters; the number of events, however, depends not only on these parameters but also on the detectors efficiency (ϵ), the neutrino flux (Φ) and neutrino interaction cross-sections (σ). The number of events (N) as a function of the observable Ψ is predicted as following

$$N(\Psi) = \int \Phi(E_{\nu})\sigma(E_{\nu})P_{\nu_{\alpha}\to\nu_{\beta}}(E_{\nu})\epsilon(E_{\nu})R(E_{\nu},\Psi)dE_{\nu}, \qquad (1.35)$$

where E_{ν} is the neutrino energy and $R(E_{\nu}, \Psi)$ is the detector response function. As seen, the number of events depends on many factors apart from the oscillation probability. In order to accurately extract the oscillation parameters, all other factors especially the neutrino flux and neutrino cross section must be adequately comprehended. Now the neutrino community is making great efforts for the next generation of long baseline experiments such as the DUNE [48] and Hyper-Kamiokande (HK) [49] experiments. Apart from precise measurements of the oscillation parameters, these experiments also focus on determining the value of δ_{CP} and the neutrino mass ordering.

Neutrino interaction

2

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2.1 Neutrino-nucleon interactions

As discussed before, Fermi's interaction is an effective theory of the well-known electroweak interaction, which has the W and Z bosons as propagators. Therefore, Fermi's interaction works well if the transferred energy is much smaller than the W mass. The interaction amplitude for the Fermi charged current interaction diagram in Fig 2.1 is of the form

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} [\bar{u}_e \gamma_\mu u_\nu] [\bar{u}_n V^\mu u_p]. \tag{2.1}$$

At first, the Fermi theory successfully explained β decay, allowing the changing of particles' types after an interaction. However, based on Eq.(2.1), the cross-section from Fermi's theory rises linearly with the transferred energy with no limitation. This is still valid if the transferred energy is much smaller than the W boson mass ($q^2 \ll M_W^2$). However, it is not valid anymore with high transferred energy interaction ($q^2 \sim M_W^2$), at least in the case of neutrino. Thus, the neutrino interaction was re-written in the modern Standard Model (SM) as a weak interaction.



Figure 2.1: Neutrino interaction in Fermi's theory is a point-like interaction.

Weak interaction of neutrinos can be classified into two types: Charged Current (CC) interactions via a W-boson and Neutral Current (NC) interactions via a Z-boson. The Feynman diagrams for these two types are shown in Fig.2.2.



Figure 2.2: Charged Current (CC) interactions via a W-boson (left) and Neutral Current (NC) interactions via a Z-boson (right) in Standard Model

It is interesting to look at the neutrino interaction amplitude in SM to see the differences with Fermi's theory. Taking the inverse β decay interaction $\bar{\nu}_e + p \rightarrow e^+ + n$ with W^- as propagator, the amplitude is of the form

$$\mathcal{M} = \frac{g_w^2}{8} \frac{1}{M_W^2 - q^2} \left[\bar{u}_e \gamma_\mu (1 - \gamma_5) u_\nu \right] \left[\bar{u}_n (V^\mu - A^\mu) u_p \right].$$
(2.2)

The resonance term $(M_W^2 - q^2)$ in the denominator will cause the peak in the cross-section at $M_W \sim 80$ GeV; eventually, this term also prevents the cross-section from gradual increasing with energy. The explicit form of the cross-section will be shown and discussed more in the next section.

In most neutrino oscillation experiments, CC interaction events are of interest since the flavour of incoming neutrinos can be determined from the flavour of final state charged leptons. In contrast, this feature can not be achieved with the NC interactions. Almost all of the discussion and analysis in this thesis will be related to CC interaction rather than NC interaction.



Figure 2.3: Total muon neutrino-nucleon interaction cross-section vs neutrino energy split out per interaction mode. Figure taken from [50].

For CC interaction, the neutrino cross-sections are known as the complexity of the processes like quasi-elastic (QE) scattering, single pion production, and deep inelastic scattering (DIS) (see Fig.2.3). Fully understanding the neutrino cross-section is crucial since it directly affects the estimation of event rate in all neutrino experiments. This chapter will describe carefully the neutrino interaction in many aspects including the neutrino-nucleon interaction modes and their measurements. Then the neutrino-nucleus interaction will be introduced together with many nucleus models and nucleus effects.

2.1.1 Quasi Elastic Charge Current Scattering

The quasi-elastic concept in electron/muon scattering refers to the elastic interaction occurring on a confined nucleon inside the nucleus. However, the nomenclature used in the case of neutrino is different. Elastic scattering is a non-charge-exchange interaction (neutral current). The process of elastic scattering without charge exchange (neutral current) called NCQE is as follows:

$$\nu(\bar{\nu}) + p \to \nu(\bar{\nu}) + p \tag{2.3}$$

$$\nu(\bar{\nu}) + n \to \nu(\bar{\nu}) + n \tag{2.4}$$

whereas quasi-elastic scattering is a charge-exchange interaction (charged current) called CCQE:

$$\nu_l + n \to l^- + p \tag{2.5}$$

$$\bar{\nu}_l + p \to l^+ + n \tag{2.6}$$

where $l = e, \mu, \tau$. The major mechanism neutrinos interact with matter at energies below ~ 1.5 GeV is through Charged-current quasi-elastic (CCQE) interactions (as shown in Fig. 2.3). In the CCQE interaction, the propagator for the neutrino and nucleon interaction is the W boson. The final state particles are isospin flipped nucleon and the neutrino flavour corresponding charged lepton (as seen in the left diagram of Fig. 2.2).

Because a nucleon is an extended entity made up of three valence quarks, analytically calculating a cross-section for this interaction is exceptionally challenging. The Llewellyn Smith model [51] could be used to parametrize the cross-section in terms of parameters that beta decay and electron scattering measurements can largely determine. These parameters are known as form factors, and they can be used to describe a nucleon's internal charge distribution and structure. As a function of the four-momentum transfer squared $(Q^2 = (\vec{p}_{initial} - \vec{p}_{final})^2 - (\vec{E}_{initial} - \vec{E}_{final})^2)$, the differential cross-section from this model is of the form [52]:

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dQ^2} = \frac{M^2 G_F^2 \cos^2 \theta_C}{8\pi E_{\nu}^2} \left[A \mp \frac{s-u}{M^2} B + \frac{(s-u)^2}{M^4} C \right].$$
(2.7)

where the negative sign of $B(Q^2)$ is for neutrinos and the positive sign for anti-neutrinos, Mis the nucleon mass, G_F is the Fermi coupling constant, θ_C is the Cabbibo angle, E_{ν} is the neutrino energy. The variables s and u are the Mandlestam kinematic variables corresponding to the square centre of mass-energy (s) and the square four-momentum (u), respectively. The A, B, and C parameters are functions of Q^2 and depend on the vector $F_{1,2}^V(Q^2)$, axial $F_A(Q^2)$ and pseudo-scalar $F_P(Q^2)$ form factors.

$$A = \frac{(m_l^2 + Q^2)}{M^2} \left\{ (1+\tau)F_A^2 - (1-\tau)(F_1^V)^2 + \tau(1-\tau)(F_2^V)^2 + 4\tau F_1^V F_2^V - \frac{m_l^2}{4M^2} \left[(F_1^V + F_2^V)^2 + (F_A + 2F_P)^2 - \left(\frac{Q^2}{M^2} + 4\right)F_P^2 \right] \right\},$$
(2.8)

$$B = \frac{Q^2}{M^2} F_A(F_1^V + F_2^V), \qquad (2.9)$$

$$C = \frac{1}{4} \left[F_A^2 + (F_1^V)^2 + \tau (F_2^V)^2 \right], \qquad (2.10)$$

where $\tau = \frac{Q^2}{4M^2}$ and m_l is the mass of outgoing lepton. With this notation, the (s-u) term in Eq. 2.7 can be rewritten as $(4ME_{\nu} - Q^2 - m_l^2)$. Because of the charge symmetry of the matrix element, the form factors for neutrino and antineutrino scattering are indeed identical. The cross-section is applicable for all flavors because of the given dependency on the lepton mass m_l . Because F_P is multiplied by m_l^2/M^2 , its contribution to ν_{μ} and ν_e is insignificant, but it becomes significant for ν_{τ} . The cross-section is now expressed in terms of four unknown form factors: F_1^V , F_2^V , F_A , and F_P . By assuming Conserved Vector Current (CVC), the vector form factors may now be linked to electron scattering form factors

$$F_{1,2}^V = F_{1,2}^p - F_{1,2}^n, (2.11)$$

where the superscript p, n represents the proton or neutron, $F_{1,2}^p$ and $F_{1,2}^n$ are the Dirac and Pauli form factors of the nucleon. It should be noticed that for CC and NC interaction, the vector $F_{1,2}^V$ and axial F_A form factors are different. However, only form factors of CC interaction will be described in this section. We can express the Sachs electric and magnetic form factors [53] $G_M^{p,n}$ and $G_E^{p,n}$ of the nucleon as:

$$G_M^{p,n} = F_1^{p,n} + F_2^{p,n}, (2.12)$$

$$G_E^{p,n} = F_1^{p,n} - \frac{Q^2}{4M^2} F_2^{p,n}, \qquad (2.13)$$

Then the vector form factors from Eq. 2.11 have been rewritten in terms of Sachs form factors

$$F_1^V(Q^2) = \frac{\left[G_E^p(Q^2) - G_E^n(Q^2)\right] + \frac{Q^2}{4M^2} \left[G_M^p(Q^2) - G_M^n(Q^2)\right]}{1 + \frac{Q^2}{4M^2}},$$
(2.14)

$$F_2^V(Q^2) = \frac{\left[G_M^p(Q^2) - G_M^n(Q^2)\right] - \left[G_E^p(Q^2) - G_E^n(Q^2)\right]}{1 + \frac{Q^2}{4M^2}}.$$
(2.15)

The axial form factor F_A and the pseudoscalar form factor F_P can now be linked [54]

$$F_P(Q^2) = \frac{2M^2}{Q^2 + m_\pi^2} F_A(Q^2).$$
(2.16)

The axial form factor F_A is typically considered to have a dipole form, allowing it to be represented as

$$F_A(Q^2) = \frac{g_A}{(1 + \frac{Q^2}{M_A^2})^2},$$
(2.17)

where $g_A = 1.2670 \pm 0.0035$ is determined based on nucleon decay [55], and M_A is the nucleon axial mass. The fits to bubble chamber $\nu_{\mu} - H_2$ and $\nu_{\mu} - D_2$ measurements, as well as pion electroproduction data, have been used to determine the nucleon axial mass ¹. The global fit value is $M_A = 1.014 \pm 0.014$ GeV [57].

As discussed earlier, the cross section is parametrized in terms of parameters that can largely be determined by beta decay and electron scattering measurements. Starting the cross section in Eq.(2.7) with four unknown form factors F_1^V , F_2^V , F_A , and F_P , we now lead to the Sachs form factors $G_E^{p,n}$ and $G_M^{p,n}$, which can only be determined through electron scattering processes, and the axial form factor F_A , which can only be accessible through weak interaction processes.

2.1.2 Resonance Production

Let us now explore another type of interaction: inelastic interaction. Neutrinos can excite the hit nucleon to an excited state if they have enough energy. In this scenario, the neutrino interaction causes a baryon resonance (N^{*} or Δ). While still in the nuclear medium, the baryon's resonance

¹Further discussion about the axial mass or axial structure of the nucleon can be found in [56].



Figure 2.4: Feynman diagrams of several typical resonance productions for CC interaction.

decays rapidly with the mean lifetime of 0.56×10^{-23} s, mainly to a single nucleon and pion in the final state:

$$\nu_{\mu} + N \to \mu^{-} + N^{*}$$
 (2.18)

$$N^* \to \pi + N' \tag{2.19}$$

where N, N' = n, p. There is also the possibility of more complicated multiplicity decay modes, but it is not discussed in this section. This process is the typical way of producing a single pion in intermediate energy neutrino scattering and it is called *resonant pion production*. Pion production starts at $E_{\nu} \sim 0.4$ GeV and it is the main interaction mode for neutrinos with energies greater than the mass of a delta baryon $\Delta(1232)$ (i.e. between 1.5 and 5 GeV), as illustrated in Fig. 2.3. There are several resonant single pion interaction channels for scattering off unbound nucleons. For charged current:

$$\nu_{\mu} + p \to \mu^{-} + p + \pi^{+}, \qquad \bar{\nu}_{\mu} + p \to \mu^{+} + p + \pi^{-},$$
(2.20)

$$+n \to \mu^{-} + p + \pi^{0}, \qquad \bar{\nu}_{\mu} + p \to \mu^{+} + n + \pi^{0},$$
 (2.21)

$$\nu_{\mu} + n \to \mu^{-} + n + \pi^{+}, \quad \bar{\nu}_{\mu} + n \to \mu^{+} + n + \pi^{-},$$
(2.22)

and for neutral current:

 ν_{μ}

$$\nu_{\mu} + p \to \nu_{\mu} + p + \pi^{0}, \qquad \bar{\nu}_{\mu} + p \to \bar{\nu}_{\mu} + p + \pi^{0}, \qquad (2.23)$$

$$\nu_{\mu} + p \to \nu_{\mu} + n + \pi^{+}, \qquad \bar{\nu}_{\mu} + n \to \bar{\nu}_{\mu} + n + \pi^{0},$$
(2.24)

$$\nu_{\mu} + n \to \nu_{\mu} + n + \pi^{0}, \qquad \bar{\nu}_{\mu} + n \to \bar{\nu}_{\mu} + n + \pi^{0}, \qquad (2.25)$$

$$\nu_{\mu} + n \to \nu_{\mu} + p + \pi^{-}, \qquad \bar{\nu}_{\mu} + n \to \bar{\nu}_{\mu} + p + \pi^{-}.$$
 (2.26)

Fig. 2.4 illustrates some examples of resonance interaction. In many neutrino experiments, the Rein and Sehgal (RS) model [58, 59] is the most commonly used model in Monte Carlo, such as NEUT, GENIE, to describe resonance production processes. In this model, the extended nuclear target was taken into account by using the nuclear form factor which is analogous to the Llewellyn Smith CCQE model. Among the form factors from Rein and Sehgal model, there are two parameters which are not easily measured by electron scattering: C_5^A and M_A^{RES} (C_5^A and g_A are the two equivalent form factors for Resonance and CCQE respectively). Another model which also describe the pion production channel in neutrino interaction is Martini et al.

model. Chap. 6 will compare the prediction from RS model and Martini et al. model.

Apart from pions in the final state, the transitional baryon resonance is also able to decay into other mesons such as kaons, which is known as resonant meson production [60, 61]. This type of interaction has a small cross-section because of the kaon mass and the fact that any dominating resonance does not strengthen the kaon channels. Detecting neutrino-induced kaon production has been the primary goal as a possible background source for proton decay experiments. Numerous SUSY GUT models have high branching ratios for proton decay modes with a final state kaon, $p \to K^+\nu$. Because an atmospheric neutrino interaction has a possible chance of mimicking such a proton decay signal, assessing these background rates has become a progressively essential element of these kinds of proton decay searches [62].

2.1.3 Coherent Scattering

When a neutrino scatters coherently with the nucleus, the neutrino interacts with the entire nucleus rather than scattering off of a specific nucleon. This process is called coherent scattering. This scattering is only allowed at deficient Q^2 levels (i.e. the neutrino transfers negligible energy to the target A) and can be totally elastic (coherent elastic scattering) or coherently create a single pion after the collision (coherent pion production). Since the Q^2 is low, this process produces no nuclear recoil and has a tiny interaction cross-section. In coherent pion production, the pion in the final state is differentiated from its resonance-mediated counterparts by its forward orientation. Furthermore, in the coherent pion production, the nucleus remains in its ground state, while in the resonant pion production, one nucleon is kicked out of the nucleus. Both neutral current and charged current interaction are possible to form coherent pion production. The coherent interaction modes for neutrino and antineutrino are of the form:

$$\nu_{\mu} + A \to \nu_{\mu} + A + \pi^{0}, \quad \bar{\nu}_{\mu} + A \to \bar{\nu}_{\mu} + A + \pi^{0},$$
 (2.27)

$$\nu_{\mu} + A \to \mu^{-} + A + \pi^{+}, \quad \bar{\nu}_{\mu} + A \to \mu^{+} + A + \pi^{-}.$$
 (2.28)

Coherent scattering is commonly modeled in neutrino interaction Monte Carlo simulations using the Rein-Seghal coherent model [63, 64]. Even the cross-sections for these events are anticipated to be minimal. Coherent pion production has been seen in both neutral current and charged current interactions of neutrinos and antineutrinos across a wide energy spectrum.

It is noted that not only pions are created in coherent scattering, but also other particles such as kaon or photon can possibly be produced. However, coherent kaon production has a far lower rate than coherent pion production due to the Cabibbo and kinematic suppression produced by the higher kaon mass. There have not been so many experiments currently looking at this specific coherent photon production channel. There is one result from NOMAD experiment at $E_{\nu}=25$ GeV that observed a negligible signal of single γ in the forward orientation, establishing a limit of

$$\frac{\sigma(\text{NC}\gamma, \text{ forward})}{\sigma(\nu_{\mu}A \to \mu^{-}X)} < 1.6 \times 10^{-4}$$
(2.29)

at 90% confidence level [65]. Since the rates of coherent kaon and photon production are much lower than that of coherent pion production, the *coherent scattering* in this thesis will indicate the coherent pion production by convention.

2.1.4 Deep Inelastic Scattering

So far, we have mostly spoken about neutrino scattering from compound objects like nucleons at intermediate energies. The neutrino with enough energy can help to study and resolve the target's internal structure. The neutrino can scatter off a particular quark within the nucleon in the most typical high energy interaction, known as deep inelastic scattering (DIS). This interaction mode causes the nucleon to disintegrate, resulting in a jet of hadrons; since quarks cannot be observed separately, they instantly recombine and form a hadronic shower. In DIS, both W and Z bosons are possible as the exchange particles:

$$\nu_l + N \to l^- + X, \qquad \bar{\nu}_l + N \to l^+ + X,$$
(2.30)

$$\nu_l + N \to \nu_l + X, \qquad \bar{\nu}_l + N \to \bar{\nu}_l + X,$$

$$(2.31)$$

where l stands for three different lepton flavors e, μ, τ . There is no clear distinction in energy between resonant and DIS modes. However, for neutrinos with energies higher than 10 GeV, DIS is the dominant interaction mode based on Fig. 2.3. When a nucleon is confined inside a nucleus, many nuclear medium effects, which will be discussed later in this chapter, impact the interaction. A series of structural functions are used to characterize deep inelastic processes, which are then expressed using Parton Distribution Functions (PDFs). A PDF represents the momentum distribution of the nucleon's quarks and gluons.

Over the years, by using DIS processes, experiments have measured cross-sections, electroweak parameters, coupling constants, nucleon structure functions, and scaling variables [54]. Therefore, one can conclude that DIS is an excellent channel to observe in order to validate the SM and investigate the nucleon structure. There is indeed a lot of high energy neutrino data accessible, allowing for a thorough comprehension of these DIS scattering processes; this will be discussed in Sec. 2.2.5. However, it is not easy to have similar DIS measurements at lower energies relevant to accelerator-based long-baseline neutrino oscillation experiments, and it is doubtful if DIS models can be extended reliably to these energies.

Although it is not relevant to any analysis in this thesis, it is still worth discussing a neutrino interaction sample produced by DIS: multi-pion production. Depending on the neutrino energy, the DIS may contribute as a source of multi-pion production. Another inelastic scattering, such as baryonic resonance production, can probably create multi-pion in the final states. There are few experimental results on this process because of the difficulties in many pions reconstruction in the final states. In the future, when the neutrino oscillation experiments reach higher energy beams such as the DUNE experiment [48], a better constraint on the multi-pion production rate is required since it will help to understand the transition region between resonance and DIS as well as the potential backgrounds for neutrino oscillation analysis.

2.2 Neutrino interaction measurements

2.2.1 Inclusive Scattering

The inclusive scattering for neutrino and antineutrino are of the form:

$$\nu_{\mu} + N \to \mu^{-} + X, \qquad (2.32)$$

$$\bar{\nu}_{\mu} + N \to \mu^+ + X. \tag{2.33}$$

In the past few decades, many experiments have been making great efforts to measure the total inclusive charged current cross-section, which spreads a wide range of neutrino energies. Fig. 2.5 shows the aforementioned cross section for neutrino and antineutrino per nucleon divided by neutrino energy as a function of neutrino energy. Neutrino cross sections are approximately twice as big as antineutrino cross-sections. However, this difference can be higher at lower energies. The reason is that the neutrino-quark scattering is assumed to be the point-like scattering, leading to the linear dependence of inclusive cross-section on neutrino energy. However, this assumption is no more correct at lower energy.

Notice that all the neutrino cross section measurements shown in Fig. 2.5 are for muon flavor. For the electron neutrino cross section, there are some recent results from MicroBooNE [66] and T2K [67–69]. The significant differences between neutrino and antineutrino cross-sections are at lower neutrino energies, in which the dominated interaction in the inclusive cross-section is the mix of quasi-elastic scattering and pion production processes, which we will examine in more detail in the following sections.



Figure 2.5: Measurements of per nucleon ν_{μ} and $\bar{\nu}_{\mu}$ CC inclusive scattering cross-sections divided by neutrino energy as a function of neutrino energy [70].

2.2.2 Quasi-elastic interaction

As previously described in Sec. 2.1.1, the QE scattering is the dominant neutrino interaction process for neutrino energies lower than 1.5 GeV. This interaction mode also significantly contributes to the main sample for oscillation analysis in many neutrino oscillation experiments, including T2K. In the seventies, many experiments were using the bubble chambers [71, 72] to detect the neutrino QE scattering through the inverse beta decay $\nu_{\mu} + n \rightarrow \mu^{-} + p$ or $\bar{\nu}_{\mu} + p \rightarrow \mu^{+} + n$. Since the bubble chamber has light targets (hydrogen or deuterium), we can consider them the free nucleon target. This process mandates the strict detection of a single charged lepton and a single nucleon. As a result, the final state was unambiguous, and elastic kinematic conditions could be confirmed. However, the statistical power of these experiments is quite restricted, and their data cannot be simply reanalyzed using newer methodologies and interaction models.

Modern neutrino experiments are more challenging since the targets are formed of heavier nuclei such as carbon, oxygen, and argon². The purpose of using these heavy targets is partly because of the enhancement in neutrino interaction event rates. However, as discussed previously in this chapter, the nuclear impacts from these targets can influence the cross-section, kinematics or even the final state particles and reconstructed topology of the interaction. Therefore, the QE events selection in modern experiments does not always require the emission of a single lepton and a single nucleon in the final states. Instead, they report the cross-section for reactions that do not have pion in the final states; this reaction is commonly referred to as the CC0 π or QE-like interaction sample. The existing observations of ν_{μ} QE scattering cross-sections as a function of neutrino energy, including historical and contemporary data, are summarized in Fig. 2.6.



Figure 2.6: Cross section measurements per nucleon of the process $\nu_{\mu}n \rightarrow \mu^{-}p$ on different nuclear targets. For comparison, the free nucleon scattering prediction (solid line) is presented with $M_A = 1.0$ GeV. Figure taken from [54].

Historically, cross-sections were measured as a function of calculated variables such as neutrino energy or 4-momentum transfer (Q^2) . Such measurements are easy to compare directly between experiments, but they are also strongly model-dependent. In order to mitigate that strong dependence, modern experiments such as T2K have recently suggested that transverse variables are being used to separate single-nucleon interactions in neutrino and antineutrino

²Many targets are hydrocarbon-based (such as plastic or liquid scintillator), water, iron or liquid argon.

interactions on nuclei [73–77]. We will discuss these variables in Sec. 5.2. However, these more minor model-independent cross-sections are challenging to compare between experiments since there are many heavy-nucleus backgrounds to be controlled³.

The other issue that stems from the difference between the light and the heavy target is the M_A value. The expected M_A value (~1.0 GeV) measured from hydrogen and deuterium interactions does not agree with the result from modern experiments with heavy nuclei as targets. For instance, in Fig. 2.6 the measurements at lower E_{ν} have clear differences between the Carbon target in MiniBooNE (2010) and D_2 target in ANL (1977). The reason behind the higher M_A value in heavy nuclei experiments was the additional effects of nuclear correlations, which was then confirmed by more comparisons using updated theoretical models. Nevertheless, even with many advantages, these updated models are still far from agreement with the data. A number of collaborations such as MINER νA , NOvA and T2K have encountered data-simulation disagreements, including three main properties. At low Q^2 , data appear reduced, whereas at high Q^2 , data appear boosted, and overall data normalization appears to be greater than simulations. The good explanation for that is due to the presence of nuclear correlations and meson-exchange currents [79].

Thanks to the great statistical power of modern experiments, many differential cross sections have formed nice sharp; for the first time, the neutrino CCQE double differential cross section was shown as a function of muon angle and kinetic energy by the MiniBooNE collaboration. T2K also joined the game by measuring the double differential cross-section as a function of the muon momentum and angle for $CC0\pi$ [73] and comparing the results to models proposed by Martini et al. [80] and Nieves et al. [81]. Fig. 2.7 shows the agreement between T2K data with the theoretical predictions.

2.2.3 Pion Production

Pion production from nucleon-neutrino interaction

Modern experiments mostly use heavy nuclei for their targets. Almost all pion production from nucleon-neutrino interaction measurements comes from the bubble chamber experiments conducted in the 1980s. The ANL [82], and BNL [83] are the two experiments that have given the primary data for these processes with neutrino energies less than ~2 GeV. These two data sources are statistically low, but their reconstruction is excellent [79]. Fig. 2.8 shows some historical measurements of $CC1\pi^+$ cross-section as a function of neutrino energy. The statistic for two pion production is very low; it is not discussed in this thesis. However, one should model it carefully if the neutrino energy is higher than ~3 GeV.

Pion production from neutrino-nuclei interaction

MiniBooNE [84] and MINER ν A [85] have shown updated data on one π^+ production crosssection for interaction on similar heavy targets (hydrocarbon) with the neutrino energy spreading from 1 GeV to 4 GeV. However, theoretical calculations have had problems matching the pion kinetic energy spectrum for many models of nucleon production. Fig. 2.9 compares theoretical and event generator calculations with data from MiniBooNE and MINER ν A. With analogous

³There is a current method to address this problem which can have a look in [78].



Figure 2.7: Neutrino CC differential cross-section with no pion in the final state $(CC0\pi)$ as a function of $\cos \theta_{\mu}$ and longitudinal p_{μ} muon momentum measured by T2K [73]. The theoretical predictions are taken from Martini et al. and Nieves et al. models.



Figure 2.8: Measurements of the cross-section for CC process $\nu_{\mu}p \rightarrow \mu^{-}p\pi^{+}$. Almost all of these measurements were conducted on the light targets. Figure taken from [54].

neutrino energy spectrum, T2K has reported the first measurement of ν_{μ} CC1 π^+ interactions on carbon, including the Adler angles data in neutrino-nucleus scattering [86] (see Fig. 2.10) and the first investigation of transverse kinematic imbalances in pion production processes [87]. As mentioned previously, the focus of modern measurements is the differential cross sections as a function of final state particle kinematic rather than the historical function of E_{ν} and Q^2 . This approach can mitigate the model dependence regardless of the interaction channels or targets. Following this path, better modelling of nuclear effects is promising.

Both T2K and MINER ν A are still taking data, and updated results are fascinating. For MINER ν A, the physics community is waiting for the curious results from neutrino's interaction with C, Fe and Pb to model the FSI better. The liquid Ar detectors are the next detector generation also to investigate the nuclear effects with improved low-energy particle detection capabilities. In chapter 5, we are going to discuss the state-of-the-art technique to probe the nuclear effects using imbalance kinematic for final state particles when having the upgraded near detector in the T2K experiment.

2.2.4 Coherent scattering

As mentioned before, the coherent scattering in this thesis indicates the coherent pion production since pions are the dominant particles created in coherent scattering. Based on this fact, the current neutrino event generators have only simulated the coherent pion production.

In 1983, the Aachen-Padova spark-chamber experiment [89] observed coherent pion production for the first time while investigating final state π^0 in ν_{μ} and $\bar{\nu}_{\mu}$ interaction. After this discovery, there are more experiments with several methods such as CHARM, SKAT, and BEBC that detected NC coherent scattering all over a wide range of neutrino energies on various



Figure 2.9: With MiniBooNE $\nu_{\mu}CH_2$ CC π^+ production measurements, a comparison of theoretical and event generator calculations is performed (left). Theoretical and event generator calculations compared to MINERvA $\nu_{\mu}CH$ CC π^+ data (right). Figures taken from [85, 88].



Figure 2.10: Differential cross-sections of ν_{μ} charged-current single π^+ production (CC1 π^+) as a function of π^+ angle (left) and π^+ momentum (right). The cross-sections were measured by T2K [86] and shown together with prediction from NEUT 5.1.4.2 (red) and GENIE 2.6.4 (dashed blue).

nuclear targets. All of these observations, however, were made with neutrino energy greater than 7 GeV.

Nowadays, many accelerator-based neutrino oscillation experiments have used neutrino beams at quite low energy $E_{\nu} < 3.0$ GeV because of the requirement to have maximum oscillation probability. Therefore, the re-measurement of coherent scattering in this low energy range is needed.

KEK and SciBooNE were the first two early modern experiments to quantify coherent scattering at some specific neutrino energy from 1.1 GeV to 2.2 GeV [90, 91]. However, none of these experiments discovered proof for CC coherent scattering. These were unexpected findings, and they could only provide an upper bound on the cross-section. However, NC coherent scattering searches were more encouraging since SciBooNE and MiniBooNE measurements indicated evidence for this interaction mode in the same energy range as CC coherent investigations.

Recent measurements have shown more interesting results. The first experiment that identified limited coherent events is ArgoNeut [92], which employs the NuMi muon neutrino beam at Fermilab and LAr TPC detector. However the used neutrino energy were not exceptionally low, with $E_{\nu} \sim 9.6$ GeV and $E_{\bar{\nu}} \sim 3.6$ GeV. T2K was the first experiment which successfully probed the coherent scattering occurrences at lower muon neutrino energies of 1.5 GeV [93] (see Fig. 2.11). Fig. 2.12 presents a variety of previously measured coherent pion production cross sections for several nuclei.



Figure 2.11: Measurements of coherent π^+ production in low energy (~ 1.5 GeV) neutrino-Carbon scattering by T2K. The Q^2 distribution is shown together with theoretical predictions. The statistical is still insufficient to differentiate the models apart. Figure taken from [93].

2.2.5 Deep Interlastic Scattering

Investigations in the DIS kinematic range necessitate neutrino beams with relatively high energy, multiple GeV and beyond. Whereas targeting this area was traditionally one of the key focuses of early experiments, the neutrino-nucleus interaction in this region is challenging to model.



Figure 2.12: As stated in the legend, measurements of coherent pion production cross-sections were taken from several nuclear targets and materials. Both NC and CC data are presented on the same plot after the CC data has been rescaled using the hypothesis that $\sigma_{NC} = \frac{1}{2}\sigma_{CC}$ [63]. Figure taken from [54].

Hence, the emphasis on measuring neutrino-oscillation characteristics in recent experiments concentrates on lower neutrino energy. As a result, the options to investigate the DIS area in ongoing research are fairly limited. However, in order to significantly improve the statistic and be sensitive to both first and second oscillation maxima, future neutrino oscillation experiments such as DUNE will use a wider band of neutrino energy. This vast range of DUNE neutrino flux would lead to 30% in the DIS region, 40% in the resonance and transition region.

The early bubble chamber experiments ANL [94], BNL [95], BEBC [96], and FNAL continue to provide the sole data from (anti)neutrino DIS off proton and deuterium. Despite the superb resolution of these bubble chamber observations, the aggregate statistics are very restricted $(\mathcal{O}(10^3) \text{ events})$ and utterly inadequate for current purposes. Data from heavier nuclear targets are now more numerous, although they are frequently constrained by experimental precision and resolution. Some of the experiments using big inactive nuclear targets such as CDHS (iron) [97, 98] and CHARM/CHARM II (marble/glass) [99] had successfully gained significant statistics of DIS observations for the first time. Following these results, the first modern experiments like CCFR (1997) [100] and NuTeV (2006) [101] used the same method for an iron target to detect DIS cross-section. A few years later, the NOMAD experiment [102] performed detection of DIS with high experimental resolution from carbon and iron targets. The MINOS [103] also participated in DIS cross-section measurements in 2010 using iron targets. More recently, MINER ν A [104, 105] has provided similar data for various targets such as graphite, iron, lead, and polystyrene. The results of some of the mentioned measurements are summarized in Fig. 2.13.



Figure 2.13: NOMAD, NuTeV, and MINOS measurements of the inclusive CC of DIS crosssection were compared to historical data. As can be witnessed, the CC cross-section in this location is measured at only a few per cent. The dashed lines represent the globally averaged cross-sections. Figure taken from [54].

2.3 Nuclear Effects

The neutrino interaction description in Section 2.1 is for interaction with free nucleons only. This interaction model with free nucleons works well enough for experiments having light nuclear targets such as H_2 - and D_2 -filled bubble chambers. However, in order to increase the target mass and hence the statistics, modern experiments use heavier targets such as Carbon, Oxygen and Argon, which will lead to additional nuclear effects to be addressed. These effects could modify the outgoing particles' kinematics and hence result in many reconstruction difficulties. Fig. 2.14 illustrates the main nuclear effects in neutrino interactions. The chosen example is the CCQE interaction $\nu_{\mu} + n \rightarrow \mu^- + p$. From the left to the right of Fig. 2.14:

- Free nucleon: neutrino interacts with an unbound nucleon, and there are no nuclear effects. There is a limited case that can apply this model is the interaction between anti-neutrino and Hydrogen.
- Initial nuclear state: the nucleon has initial momentum, which can contribute to the energy transfer in the interaction.
- Extra nuclear effects: the neutrino interacts with a nucleon that strongly correlates with another nucleon inside the nucleus. This effect gives one additional nucleon in the final state and changes the interaction topology.
- Final state interaction (FSI): the nucleon in the final state re-interacts with other nucleon in the nucleus. This effect changes totally the kinematics of outgoing particles.

These nuclear effects will be discussed in more detail in the following sections.



Figure 2.14: Some possible nuclear effects in neutrino-nucleus interaction

2.3.1 Neutrino-nucleus cross-section

The charged current interaction $\nu_l(\bar{\nu}_l) + A \rightarrow l^-(l^+) + X$ has a double differential cross section which is of the form

$$\frac{d^2\sigma}{d\Omega_{k'}d\omega} = \frac{G_F^2 \cos^2\theta_C}{32\pi^2} \frac{|\mathbf{k'}|}{|\mathbf{k}|} L_{\mu\nu} W^{\mu\nu}(\mathbf{q},\omega), \qquad (2.34)$$

where $k \equiv (E_{\nu}, \mathbf{k})$ and $k' \equiv (E'_{l}, \mathbf{k}')$ are the initial neutrino and final lepton four momenta respectively. $\omega = E_{\nu} - E'_{l}$ is the energy transferred to the nucleus and $\Omega_{k'}$ is the solid angle in the direction given by the charged lepton momentum k' with respect to the laboratory frame. In general, we denote $q = k - k' \equiv (\omega, \mathbf{q})$ as the four momentum transfer. The weak coupling constant is denoted by G_{F} , θ_{c} is the Cabbibo angle. Whereas $L_{\mu\nu}$ is the leptonic tensor

$$L_{\mu\nu} = 8(k_{\mu}k'_{\nu} + k_{\nu}k'_{\mu} - g_{\mu\nu}k.k' \mp i\epsilon_{\mu\nu\alpha\beta}k^{\alpha}k'^{\beta}), \qquad (2.35)$$

The minus sign or plus sign before the Levi-Civita tensor ϵ in the Eq. 2.35 is for neutrino or antineutrino interaction. This fundamental asymmetry resulting from weak interaction theory has significant implications for the discrepancies in neutrino and antineutrino cross sections.

The $W^{\mu\nu}$ stands for hadronic tensor. It is needed for $W^{\mu\nu}$ to be sum all over the hadronic final states, this sum can be decomposed based on the number of nucleons in the final state: one-particle one-hole (1p1h), two-particles two-holes (2p2h) and more

$$W^{\mu\nu}(\mathbf{q},\omega) = W^{\mu\nu}_{1p1h}(\mathbf{q},\omega) + W^{\mu\nu}_{2p2h}(\mathbf{q},\omega) + \dots$$
(2.36)

The other way to decompose the hadronic tensor is dividing in terms of the different channels of particle-hole excitations

$$W^{\mu\nu}(\mathbf{q},\omega) = W^{\mu\nu}_{NN}(\mathbf{q},\omega) + W^{\mu\nu}_{N\Delta}(\mathbf{q},\omega) + W^{\mu\nu}_{\Delta\Delta}.$$
 (2.37)

2.3.2 Fermi motion

The targets that neutrinos interact with, nucleons, are moving within the nuclei. This initial movement of nucleons inside the nucleus can be described by an isotropic Fermi motion.

Since Fermi motion is an effect inside the nucleus, it is impossible to measure precisely Fermi momentum for each interaction, but one can study this statistically. Table 2.1 shows some of the

Fermi momentum values measured by E.J. Moniz et al. in 1971 [106] in electron scattering experiments. Generally, the more nucleons inside the nucleus, the bigger the Fermi momentum is. The value of Fermi momentum is at the order of 200 MeV, which is undoubtedly not negligible compared to the few-GeV neutrino beam or especially the 600 MeV beam energy peak of T2K (more detail in chapter 3). Hence, the neutrino interaction experiment needs to carefully study how Fermi momentum biases the outgoing particles' momentum to reconstruct neutrino energy better.

Since there are many nucleons inside the nucleus, one always faces the N-body problem while working with the nucleus. That is a pressing problem in modern physics, and it is challenging to have an accurate prediction for the nucleon momentum distribution inside a complex nucleus. Many nuclear models put great effort into making this prediction, and the well-known ones are spectral function models, which are widely used in neutrino-nucleus interaction simulations. These models are described in section 2.4.

Nucleus	Fermi momentum (MeV/c)
⁶ ₃ Li	169
${}^{12}_{6}C$	221
$^{24}_{12}{ m Mg}$	235
$\frac{\overline{40}}{20}$ Ca	251
^{58.7} ₂₈ Ni	260
$\frac{\bar{89}}{39}$ Y	254
$^{208}_{82}$ Pb	265

Table 2.1: Experimental Fermi momentum values for several kinds of nuclei from electron scattering data [106].

2.3.3 Nuclear removal energy

Nucleons are bound inside the nucleus volume due to the strong interaction. To pull them out from the nuclear potential after interaction with neutrino, a minimum amount of energy is required. This amount of energy is called *removal energy*. This removal energy is different for the nucleus and nucleons to be removed. However, the missing energy, the amount of neutrino energy transferring to the nucleus, is not always the same as removal energy, but they are in correlation.

Even for the same nucleus, the removal energy is not the same for each neutrino interaction event. In the Spectral Function model (Sec. 2.4), the removal energy of a particular nucleon depends on the bound state of this nucleon (i.e. the different shell-model levels). Consequently, the missing energy is different for each event and it can cause bias in neutrino energy reconstruction. By definition, the binding energy of the nucleon in the least bound state is equal to the separation energy. The binding potential of nucleons inside nucleus is treated differently with respect to different nuclear models (more in Sec. 2.4).
2.3.4 Nucleon final state interaction

In a typical CC neutrino interaction, an incoming neutrino interacts with a nucleon inside a nucleus resulting in a lepton and a nucleon in the final state. In order to be measured by any current detector, the final state particles must escape the nucleus medium where they are created. However, these final state particles can re-interact with other nucleons in the same nucleus. They can be absorbed, change their kinematics and topologies, or eject new hadrons which do not come from the primary neutrino interaction vertex. These final state re-interactions can change the reconstructed topology of neutrino interaction and outgoing particles' kinematics, possibly causing biases in neutrino energy reconstruction. These final state re-interactions are often called "final state interactions" (FSI) in a shorter version. The FSI effect is not rare. For example, in the case of a few GeV neutrino interacting with the iron nucleus, about 30% of the hadrons produced in the primary vertex will undergo the FSI [107]. It is challenging for theorists to model and experimentalists to constrain this nuclear effect.



Figure 2.15: An example of Final State Interactions scenarios. (taken from [108])

There are some efforts to address this FSI problem, including the intranuclear cascade algorithm. This model treats the particles as classical particles having straight moving paths between collisions. Then the probability for a particle to move a distance λ without any interaction in the nuclear medium is of the form

$$P(\lambda) = e^{-\lambda/\bar{\lambda}}.$$
(2.38)

where $\tilde{\lambda} = (\sigma \rho)^{-1}$ is the mean free path (MFP). The σ is the total cross-section of total hadronhadron interaction, and ρ is the constant nuclear density assuming the travelled distance of the particle λ is small enough. The typical MFP of hadrons inside a nucleus is a few femtometers [107]. However, for the case in which the particles move further, the nuclear density ρ changes concerning the position inside the nucleus medium. This fact leads to many difficulties in solving the problem analytically. In some Monte Carlo methods, to make it more realistic, each hadron from the interaction vertex inside the nucleus medium has the mean free path as follows

$$\tilde{\lambda} = \left[\sigma_p \rho_p(r) + \sigma_n \rho_n(r)\right]^{-1} \tag{2.39}$$

where the indexes p and n stand for proton and neutron, respectively. The r is the distance from the current particle point to the nucleus centre. These FSI are simulated step by step based on the local nuclear density until the particles go out of the nucleus medium. Some possible scenarios of FSI in the cascade model are illustrated in Fig. 2.15. The mean free path scale is adjusted for different FSI interactions since the cross-sections are not the same. The external hadron scattering data also plays a role in constraining the mean free path. More information on FSI implementation in neutrino-nucleus interaction simulations can be found at [107–109].

2.3.5 Nucleon-nucleon correlations

A typical neutrino-nucleon CCQE interaction produces one nucleon in the final states. However, the nucleons interact within the nucleus medium. It is, therefore, not rare that the neutrino may interact with nucleons that bound together to create multinucleon in the final states. The correlations between nucleons inside the nucleus are not something new. In electron-nucleus scattering experiments, these correlations have earlier been detected, and many models were established to characterize the influence of these nucleon correlations on the interaction crosssection. However, one could not simply apply these electron-nucleus interaction models for neutrino-nucleus interaction ones because it is uncertain if the axial vector element in the developed models would be adequately addressed.

Martini et al. [80] and Nieves et al. [110] have presented innovative models for understanding the importance of multinucleon interactions in neutrino-nucleus scattering. There are additional two-particle two-hole (2p2h) excitations in these models, where two nucleons in the final state leave two holes in the nucleus. These nucleons can exit the nucleus but they could be not detected, as in Cherenkov detectors. Therefore these 2p2h events can mimic the genuine CCQE events due to one-particle one-hole (1p1h) excitations.

As mentioned in Sec.2.3.4 for neutrino-nucleus scattering, the nuclear medium could affect the outgoing particles through the FSI processes. However, apart from FSI processes, the nuclear environment could also influence the inner state of neutrino-nucleus scattering which is the electroweak propagator through a nuclear screening effect ⁴. In order to give a prediction of neutrino-nucleus scattering measurements for modern experiments where many heavy targets have been used, the Martini et al. and Nieves et al. models must incorporate the nuclear screening effect, which is commonly known as the random phase approximation (RPA) [80, 110]. Its influence will eventually result in the cross-section deviation: a quenching appears at low Q^2 and for small charged lepton scattering angle.

To prove the power of RPA and the multinucleon correlation models, let us discuss a much debated problem: the nucleon axial mass disagreement between light and heavy targets

 $^{^{4}}$ In case of having nuclear screening effect, the nuclear targets can be shielded by "long-range" interactions between nucleons.



Figure 2.16: Several versions of the CCQE double differential cross section $d^2\sigma/d\cos\theta_{\mu}dT_{\mu}$ on Carbon are displayed with measurements from the MiniBooNE experiment [111]. **Left:** prediction from Martini et al. model [112]. **Right:** prediction from Nieves et al. model [113]. The difference of the data between two plots is because on the right plot the experimental data are multiplied by the factor 0.9.

in neutrino scattering experiments. The nucleon axial mass M_A was previously introduced in Sec. 2.1.1 that has the global fit value of 1.014 ± 0.014 GeV. However, the fit to the Q^2 distribution shape for CCQE measured by MiniBooNE [111] on Carbon gives larger axial mass value, $M_A = 1.35 \pm 0.17$ GeV, which disagrees at 2σ C.L. with the early measurements on lighter targets. With the implementation of multinucleon correlations and RPA, Martini et al. and the Nieves et al. obtain good agreement with MiniBooNE data (see Fig. 2.16) while using the global fit M_A value from bubble chambers.

2.3.6 Pauli blocking

The Pauli principle states that fermions cannot exist in the same quantum state. As a result, interactions that result in a final nucleon in an already occupied state are prohibited. Pauli blocking is the name for this phenomenon. The Pauli Blocking can reduce the neutrino-nucleus cross-section at low energy and momentum transfer.

2.4 Models for the nucleus in its initial state

Several model exist for the purpose of describing the initial state of the nucleus. Here I rapidly review the ones used in NEUT event generator. These models are the Global Fermi Gas, the Local Fermi Gas, and the Spectral Function (SF).

The Fermi gas picture is an easy and convenient model to describe nucleons' momentum and energy distribution, and to calculate the neutrino-nucleus cross-section. The Fermi gas models consists in assuming that nucleons fill all momentum in a potential well. Then, depending on the nucleons density, we get the Global Fermi Gas (GFG) or Local Fermi Gas (LFG). However, the Fermi gas models are far from realistic since they ignore many different nuclear effects. More models have been invented to address these problems, including SF. The cross-section difference between SF and Fermi gas model is not negligible, and it should be carefully studied since it may significantly affect the oscillation analysis results.

2.4.1 Global Fermi Gas

The Global Fermi Gas, and its relativistic extension called Relativistic Fermi Gas (RFG), is one of the simplest models to describe nucleons inside a nucleus. Originally, Fermi investigated the quantistic gas of non-interacting spin 1/2 particles (Fermions). This gas is usually called Fermi Gas, a simple many-body model used in several fields (atomic physics, nuclear physics,...). In the case of the nucleus this model describes the nucleons as freely moving particles inside the nucleus volume. Only statistical correlations (due to the Pauli exclusion principle) are retained. It can be assumed that the potential that every nucleon feels (a superposition of the potential of the other nucleons) has the shape of a well: it is constant inside the nucleus and stops sharply at its edge (see Fig. 2.17).



Figure 2.17: Sketch of the proton and neutron potentials and states in the Global Fermi Gas model (taken from [108])

In the nuclear ground state the nucleons fill all momentum states from zero to the highest momentum state, called Fermi momentum p_F . This Fermi momentum depends on how full the wells are and hence depends on the number of nucleons. Protons and neutrons potential wells are different, which leads to the difference in Fermi momentum. The energy of the highest occupied state, the Fermi energy E_F , is

$$E_F = \frac{p_F^2}{2M},$$
 (2.40)

where M is the nucleon mass. In the case of the Relativistic Fermi Gas the Fermi energy is given by

$$E_F = \sqrt{p_F^2 + M^2}.$$
 (2.41)

The difference between the top of the well and the Fermi level is constant for most nuclei and is just the average binding energy per nucleon $E_B/A \simeq 7-8$ MeV. The formulas to calculate the

proton and neutron Fermi momentum in Global Fermi Gas are

$$p_F^p = \hbar k_F^p = \hbar \left(\frac{3\pi^2 Z}{V}\right)^{1/3} = \frac{\hbar}{r_0} \left(\frac{9\pi Z}{4A}\right)^{1/3} \approx (310 \pm 50) \left(\frac{Z}{A}\right)^{1/3} \text{MeV/c}$$
(2.42)
$$p_F^n = \hbar k_F^n = \hbar \left(\frac{3\pi^2 (A-Z)}{V}\right)^{1/3} = \frac{\hbar}{r_0} \left(\frac{9\pi (A-Z)}{4A}\right)^{1/3} \approx (310 \pm 50) \left(\frac{A-Z}{A}\right)^{1/3} \text{MeV/c},$$

where A is the mass number, Z is the proton number, V is the nuclear volume (assumed to be the same for protons and neutrons), and r_0 is related to the radius R of the nucleus by $R = r_0 A^{\frac{1}{3}}$. The Fermi momentum values of Eqs. 2.42 are in good agreement with experimental results shown in Tab. 2.1.

As already mentioned, in the Fermi gas model every state up to the Fermi level is occupied. In this scenario, Pauli Blocking indicates that any interactions with a final state nucleon with a momentum less than p_F have zero cross-section. At low energy transfer the Fermi gas response is quenched up to the momentum transfer $|\vec{q}| < 2p_F$.

2.4.2 Local Fermi Gas

In the global Fermi gas, the nucleus is a perfect sphere with a uniform nuclear density. In other words, Fermi level are constant inside the nucleus medium. The Local Fermi Gas (LFG) [114] modifies the nuclear potential to the new one, which depends on the nucleon's radial position (r) within the nucleus medium. This modification makes LFG more sophisticated and realistic than the global Fermi Gas since the nuclear density is not uniform inside the nucleus medium but radial dependent. The electron scattering experiment proved this statement [115].

In the LFG, the nuclear density $\rho(r)$ is approximately localised as a function of radial position, and the local Fermi momentum depends on $\rho(r)$ according to:

$$p_{F}^{p}(r) = \hbar k_{F}^{p}(r) = \hbar \left(3\pi^{2}\rho(r)\frac{Z}{A}\right)^{\frac{1}{3}}$$
(2.43)
$$p_{F}^{n}(r) = \hbar k_{F}^{n}(r) = \hbar \left(3\pi^{2}\rho(r)\frac{A-Z}{A}\right)^{\frac{1}{3}}.$$

Of course, it is possible to obtain the GFG from the LFG. If we change the nuclear density to a constant $\rho(r) = A \left(\frac{4}{3}\pi R^3\right)^{-1}$ then the Eq. 2.43 will be identical to the Eq. 2.42. In Fig. 2.18, the distribution of Fermi momentum in LFG is shown in comparison with the one in RFG. The overall effect of LFG is making the initial Fermi momentum smoother.

Also, the difference between Pauli Blocking in RFG and LFG is considerable. Because the LFG has a Fermi level dependent on nuclear density, the influence of Pauli Blocking is more significant near the nucleus's core than at its perimeter. The LFG is the model employed by Martini et al. and Nieves et al. It represents their starting point to calculate QE (1h1p) excitation in RPA, 2p2h excitation and 1π production.



Figure 2.18: Fermi momentum for Global and Local Fermi gas in Carbon nucleus. The distribution in the LFG case is smoother than in GFG (taken from [108]).

2.4.3 Spectral Function

In general, the Fermi Gas models described above are easy to implement into Monte Carlo simulations because of their simplicity. Their predictions are in good agreement with many experimental data, and they are used in many analyses. However, the Fermi Gas models still treat the nucleons as non-interacting fermions (i.e., there is no interaction between nucleons except for the overall nuclear potential). In reality, the nucleon-nucleon interaction plays an essential role in nucleon momentum distribution determination. Many experimental results have proven that statement, especially the exclusive electron scattering data [116, 117]. Spectral Function (SF) turns out to be a better model to describe the initial state of nucleons since SF includes the shell effects and the Short Range Correlation (SRC).

The probability distribution of kicking out a nucleon with the momentum \vec{p} and leaving the excitation energy E in the remaining nucleus is

$$P(\vec{p}, E) = P_{MF}(\vec{p}, E) + P_{corr}(\vec{p}, E), \qquad (2.44)$$

where the first term is the Mean Field (MF) part and the second term describes the correlation part. In the MF part, the nucleons are described as free particles travelling in a mean-field potential. These nucleon's kinematics should obey the shell model picture.

However, the MF part itself is not enough to predict the nuclear dynamic accurately. There is also strong interaction inside the nucleus, and it causes the correlations between nucleons. The nucleons correlations can adjust the shell orbits or supplement more momentum to nucleons for going above the Fermi level. Among all of the SRC, the dominant correlation is the one producing pairs of nucleons [118]. These pairs of nucleons have opposite isospin. Their momentum is relatively large and opposite in direction. As a result, SRC causes the nucleon momentum distribution to have a high-momentum tail (Fig. 2.19).

It is not as simple to apply Pauli blocking to the SF as to the Fermi model because not all states up to Fermi are occupied in SF. There are a few options for dealing with this. The



Figure 2.19: The initial nucleon momentum distributions in spectral function and Fermi gas models are compared. (taken from [108])

Fermi level can be set using the average Fermi momentum, or the local Fermi momentum could be used instead of the average Fermi momentum. The final option is to examine the nucleon momentum distribution and let Monte Carlo determine whether or not the selected momentum is already occupied. The spectral function of Benhar and Fabrocini [119] is implemented in NEUT.

T2K experiment and its off-axis near detector ND280

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3.1 Introduction to T2K

T2K (Tokai to Kamioka) experiment, located in Japan, is a long-baseline neutrino oscillation experiment [120]. The T2K experiment is hosted by the Institute for Cosmic Ray Research (ICRR) for the far detector Super-Kamiokande and the KEK High Energy Accelerator Research Organization for the neutrino source and a set of near detectors. Its international collaboration has about 500 members from 77 institutes in 12 countries Japan, Canada, France, Italy, Korea, Poland, Russia, Spain, Switzerland, the UK, the USA and Vietnam.

The T2K experiment is composed of three main parts: the (anti-)neutrino beam generated at the Japan Proton Accelerator Research Complex (J-PARC) facility [121], the near detector complex, including the INGRID and the ND280 near detectors, located 280m downstream from the target generating the beam, and the water Cherenkov far detector Super-Kamiokande (SK) placed 295 km away from the beam production point. A neutrino beam after production will reach the near and far detector, where the oscillation effect is expected to be negligible and maximum, respectively. By comparing the neutrino interaction rates and neutrino spectrum from these two detectors, the T2K experiment can extract the oscillation parameters. Fig. 3.1 depicts an overview of the experiment.



Figure 3.1: The T2K experiment is illustrated schematically.

T2K launched its first beam in late 2009 and has been collecting physics data since early 2010. This chapter describes the T2K experiment in detail, including its scientific goals, beam, detectors, current status and achievements.

3.1.1 The original scientific goals

When the experiment was officially proposed, the main goals of the T2K experiment were

- The observation of $\nu_{\mu} \rightarrow \nu_{e}$ oscillation (i.e. electron neutrino appearance in a muon neutrino beam). This goal was achieved in 2013 [44].
- Precision measurements of oscillation parameters in ν_{μ} disappearance $(\theta_{23}, \Delta m_{23}^2)$ [122].

There are three mixing angles that were explained in Sec.1.5. At the beginning of T2K, the first two mixing angles (θ_{12} and θ_{23}) were known to be large. The remaining question was whether the third mixing angle θ_{13} value is different from zero. The observation of $\nu_{\mu} \rightarrow \nu_{e}$ oscillation in the T2K experiment leads to the conclusion that $\theta_{13} > 0$ and it is large enough that we can have a promising search for CP violation in future [123].

Because of the fast advancement of neutrino physics worldwide, the physics aims for the current generation neutrino oscillation experiment have to be re-evaluated.

3.1.2 The current scientific goals

Today, in parallel with the precise measurements of oscillation parameters, the current big goals of the T2K experiment are:

- Searching for the CP violation in the lepton sector by comparing the differences between the neutrino and anti-neutrino oscillations probabilities.
- Measurements of neutrino interaction cross-section and testing neutrino-nucleus interaction model.
- Looking for anomalous neutrino physics signals, which include sterile neutrinos.

3.1.3 Design goals of T2K

In order to pursue the scientific goals mentioned above, the T2K collaboration has to meet the following goals

• Neutrino beam energy: By adjusting the neutrino beam energy to the oscillation maximum, the sensitivity of the oscillation measurement may be maximized. As shown in Eq.(1.28), the term depending on the oscillation baseline and neutrino energy is

$$\sin^2\left(\frac{1.27\Delta m^2 L}{E_\nu}\right).\tag{3.1}$$

To be sensitive to the first oscillation maximum with the 295 km baseline of T2K, the neutrino energy has to be ~ 0.6 GeV.

• \mathbf{E}_{ν} reconstruction: In the energy range below 1 GeV, the QE interaction is the dominant interaction (see Fig. 2.3). This allows us to precisely measure the neutrino energy based on the outgoing charged lepton kinematics. The neutrino energy is estimated using the formula

$$E_{\nu} = \frac{m_N E_l - m_l^2 / 2}{m_N - E_l + p_l \cos \theta_l},$$
(3.2)

where N, l stand for neutron and lepton, respectively. Note that the formula above was obtained by neglecting the binding energy and the mass difference between neutron and proton.

• Background suppression: The neutrino energy spectrum, which peaks around 0.6 GeV, could provide the most reliable background suppression. Another benefit of this spectrum is that the ν_e appearance signal must be limited to a specified energy area, which reduces the contaminating ν_e background in the ν_{μ} beam. The narrower band of neutrino energy by using the off-axis beam technique further enhances the precision for the disappearance parameters.

In general, the neutrino energy and the baseline in the T2K experiment are set at the values to be sensitive to both ν_e appearance and ν_{μ} disappearance measurements.

3.2 T2K Beam

3.2.1 The neutrino beamline production

In T2K, an intense muon neutrino beam is sent through 295 km from Tokai to Kamioka. The key ingredient to produce this neutrino beam is the high-intensity accelerated proton beam produced by the J-PARC beamline [124]. Fig. 3.2 depicts the J-PARC facility's layout, which includes a 400 MeV linac, a 3 GeV 1 MW rapid cycling (25 Hz) synchrotron (RCS), and a 0.75 MW 50 GeV proton synchrotron main ring (MR). A H^- beam is first boosted in the LINAC before being transformed to a H^+ (proton) beam by charge stripping foils when delivered into the RCS injection. The RCS accelerates the beam up to 3 GeV at a rate of 25 Hz. There are two bunches in each RCS cycle. The MR receives about 5% of these bunches, while the remainder will provide the muon and neutron beamline for the Materials and Life Science Facility. The proton beam is accelerated to 30 GeV after being injected into the MR. The MR's harmonic number is nine, and there are eight bunches in the MR. The MR has two extraction positions: slow extraction for the hadron beamline and fast extraction for the neutrino beamline. A set of five kicker magnets extracts all eight bunches in a single cycle for the neutrino beamline (fast extraction). This is called a "spill," and it takes 5.2 μ s. Each spill has ~ 3.3×10^{14} protons. The neutrino beamline is divided into two sequential parts: the primary beamline and the secondary beamline. Fig. 3.3 depicts an overview of the neutrino beamline.



Figure 3.2: Accelerator complex of JPARC.

In the primary beamline, the 30 GeV proton beam is extracted and directed to the far detector located in Kamioka. The beam is adjusted in the preparation section with 11 normal conducting magnets to enter the arc section. In the arc section, there are several superconducting magnets used to bend the direction of the proton beam by 90° to the SK direction. This proton



Figure 3.3: A schematic of the neutrino facility, completed with identified components, in J-PARC.

beam is then shot to a target right after the final focusing section.

In the secondary beamline, after the interaction between the proton beam and the carbon target, secondary mesons (mainly pions and kaons) are then produced as the result of collisions. These mesons are focused and selected in charge by a set of 3 magnetic horns. The first horn is the place where the target is located inside. The target core is a graphite rod with a diameter of 2.6 cm and a length of 91.4 cm. The density of the target is 1.8 g/cm^3 ; the target is not so dense that the pulsed beam heat load would not melt it. The horns are powered by a 250 kA pulsed current, which generates a 1.7 T magnetic field. Thanks to these magnetic horns, the total neutrino flux at SK increases approximately 17 times [121]. Fig. 3.4 gives an overview of the secondary beamline. The secondary pions and kaons then enter a 110-meter long decay volume where they predominantly decay into μ^+ and ν_{μ} (neutrino mode) or μ^- and $\bar{\nu}_{\mu}$ (anti-neutrino mode). At the end of the decay volume, a beam dump plays as a filter to stop all the remaining protons, mesons and almost all other charged particles, except the high-energy muons (> 5)GeV/c) and the neutrinos. The core of the beam dump is comprised of 75 tons of graphite (1.7) g/cm3) with the dimension of 3.174 m (long) $\times 1.94$ m (wide) $\times 4.69$ m (high). The dump core is followed by 2.5 m thick iron plates. To characterize the high-energy muons which pass through the beam dump, a muon monitor named MUMON detector is placed right after the beam dump to measure their intensity and profile. Since the muons and neutrinos are produced in two-body decays, monitoring the muon beam leads to better control of neutrino beam intensity and direction. The neutrino beam direction is determined with better than 0.25 mrad precision using the muon monitor. T2K also utilizes a multi-horn system; therefore, the trajectory of both high and low-energy meson particles should be tracked. It should be noticed that, by changing the polarity of the horn current, positively- or negatively-charged mesons are selected in Forward Horn Current (FHC) or Reverse Horn Current (RHC) mode, respectively. Eventually, T2K can



Figure 3.4: A schematical view of the T2K neutrino beamline and detectors. Figure taken from [125].

T2K began taking physics data in January 2010 and continues to the present day and beyond. The data accumulated by T2K is reported as *protons on target* (POT). Fig. 3.5 shows the T2K history of data-taking. Until the end of 2021, a total of 3.82×10^{21} POT was accumulated, where neutrino and anti-neutrino modes account for 2.17×10^{21} POT and 1.65×10^{21} POT, respectively. The up-to-date maximum beam power that T2K can reach and stably run is 522.6 kW. However, it is not the ultimate number; the beam power is still gradually upgraded.



Figure 3.5: The delivery history of POT to the T2K experiment for analysis. The dots represent the number of POT for every pulse, while the lines represent the total number of POT.

3.2.2 The off-axis technique and advantages

The T2K experiment is the first experiment not using an on-axis beam but an off-axis one. It means that both T2K near and far detectors are not placed on the beam axis but off

produce either a beam of ν_{μ} or $\bar{\nu}_{\mu}$.



Figure 3.6: The oscillation probabilities are shown in parallel with the several angle-dependent neutrino spectra.

the beam axis at the angle of 2.5°. Using the off-axis beam can obtain the optimum possible intensity of low-energy neutrinos while including just a small high-energy portion in the spectrum. Therefore, the neutrino beam energy band is narrower than the on-axis neutrino beam, making the oscillation analysis easier. Fig. 3.6 shows the effects of the beam angle on the T2K flux. Thanks to this estimation, T2K can choose the beam angle of 2.5° since it provides a spectrum peaking at 0.6 GeV where the appearance probability is locally maximum.

To investigate why the beam spectra are so different with respect to the beam angle, let us look at the theoretical prediction of neutrino energy. Since the neutrino is a production of the decay of a secondary meson beam, the neutrino energy can be calculated from the original meson kinematics such as the ones of π

$$E_{\nu}(\pi \to \mu + \nu_{\mu}) = \frac{0.49E_{\pi}}{1 + (\gamma\theta)^2},$$
(3.3)

where E_{π} , θ and γ are the pion energy, the angle between the secondary beam and the outgoing neutrino, and the Lorentz factor of the pion, respectively. A Jacobian peak may be seen in the spectrum at the angle $\theta > 0$. Fig. 3.7 shows the neutrino energy as a function of pion energy with different beam angles. By changing the beam angle, T2K can produce a quasi-monochromatic beam for a certain energy range.



Figure 3.7: The relation between neutrino energy and off-axis angle.

3.2.3 T2K flux predictions

One of the vital things in the accelerator-based neutrino experiment is the neutrino flux which should be precisely characterized. Together with the interaction cross-section and detector efficiency, neutrino flux can strongly affect the event rates of neutrino interactions. There are three stages to derive the T2K flux prediction:

- First stage: this stage starts at the end of the primary beamline where the ~ 30 GeV protons extracted from the MR interact with the graphite target. To predict the secondary mesons' kinematics, a FLUKA simulation package [126, 127] is applied to model the production and interaction of hadrons in the target.
- Second stage: after having the secondary mesons' kinematics prediction, the GEANT3 [128] based simulation is used to trace these hadron particles as they travel through the magnetic field produced by the horns to the near and far detector. These simulations also include the propagation of hadron through the target, cooling envelope, magnetic horns, decay pipe and beam dump. A neutrino is generated if the mesons decay into the tunnel before colliding with one of the tunnel walls. This GEANT3-based software is called JNUBEAM, and it was developed by T2K collaborators. JNUBEAM additionally uses the GCALOR 1.05/04 [129] package to generate re-interactions beyond the target volume.
- Final stage: T2K uses data from NA61/SHINE experiment [130] at CERN to tune the hadronic interaction simulation.

There are two ways to simulate neutrino energy depending on the parent mesons' decay type. If it is a two-body decay, the neutrino energy is obviously defined. In the case of a three-body decay, the neutrino energy is taken randomly from the proper distribution probability. For the SK, it is far enough from the J-PARC to be termed as a point-like detector. The probability of a meson particle emitting a neutrino moving towards SK with determined energy is then calculated. For the Near Detector, it is close enough not to be considered point-like. Hence, T2K collaborators take into account the detector's size. If the neutrino direction crosses the Near Detector, its kinematic will be saved.



The NA61/SHINE experiment to constrain T2K neutrino flux prediction

Figure 3.8: For comparison, total flux uncertainty on the T2K muon neutrino beam in FHC mode, broken down by error source, is presented beside the neutrino flux shape. The black dash and solid lines are different versions of the total flux uncertainty with different tunes using the NA61/SHINE data.

In accelerator-based neutrino oscillation experiments, the lack of understanding about the parent hadrons spectra causes a big uncertainty in the beam prediction, which is required to propagate the observed spectra at the Near Detector to the predicted one at the Far Detector. Fig. 3.8 illustrated the dominant source in T2K total flux uncertainty, which is from the hadron interaction¹. In the T2K experiment, many measurements of the beam (alignment and intensity), and magnetic horn current have been used to tune the simulation. However, a complete simulation requires information on very low energy strong interactions, which is the case of interaction between hadron and the graphite target. J-PARC facility does not have suitable measurements to characterize these hadronic interactions. Due to this reason, T2K has used the NA61/SHINE data to tune such interactions simulation.

NA61/SHINE is a dedicated hadron-production experiment that uses a ~ 30 GeV proton beam extracted from CERN SPS and a replica T2K target to collect useful data of hadron formation that are directly related to the T2K beam [131, 132]. This experiment is an improvement on the NA49 experiment, which was a large-acceptance hadron spectrometer that also used a beam taken from the CERN SPS. Fig. 3.9 shows the layout of NA61 experiment. Upstream of the spectrometer, a series of scintillation and Cherenkov counters, as well as beam position detectors (BPDs), give timing reference, identification, and position measurements of the entering beam particles. The spectrometer's primary tracking devices are large-volume time projection chambers (TPCs). The first two TPCs are operated in the magnetic field (1.5 and 1.1 T). The other two are positioned downstream of the magnets and are symmetrical to the beam direction. There are also three Time Of Flight detectors and several calorimeters.

¹The hadronic uncertainties include the errors from meson multiplicity, meson rescattering, baryon rescattering, and interaction length.



Figure 3.9: The top view of the NA61/SHINE experiment design at CERN SPS.

After all steps above, the T2K flux prediction is illustrated in Fig. 3.10 for different modes at near and far detectors. The T2K neutrino flux is dominated by muon flavour, accompanied by slight contamination of electron neutrino (less than 1%). The main background is the wrong-sign background due to the limitation of the magnetic horn selection. This wrong-sign background is more significant in anti-neutrino mode and becomes dominant at energy larger than 5 GeV. These wrong-sign background differences between neutrino and anti-neutrino modes are due to the higher production multiplicities of positively charged parent particles, which are outgoing mesons after proton-target interactions, compared to negatively charged ones. Because SK's location is far from the neutrino beam production site, the expected unoscillated flux at SK is much lower than that at the near detectors.

3.3 The T2K near detectors

To achieve the physics goals of the long-baseline experiments, a far detector is needed to measure the neutrino flux after the oscillation, and a near detector is required to accurately measure neutrino beam properties, specifically flux and spectra well before oscillation, as well as neutrino cross-sections. This is done using two detectors: an on-axis detector, INGRID, and an off-axis detector complex named ND280. They are both positioned in a pit 280 meters away from the neutrino beam generation source and serve different functions. Fig. 3.11 depicts an overview of the two near detectors and their relative position in the near detector facility.

3.3.1 The On-Axis Detector: INGRID

INGRID (Interactive Neutrino GRID) [120] is the on-axis near detector of the T2K experiment. It was developed with two key specific goals:

- Monitor the neutrino direction.
- Monitor the neutrino flux intensity.



Figure 3.10: Prediction of T2K flux and its composition before oscillation



Figure 3.11: The T2K near detector complex. The off-axis near detector ND280 is located on the top floor, while on the lower floor, there is an on-axis near detector INGRID. The beam center is located where the INGRID horizontal and vertical modules intersect.

INGRID is sensitive to a wide spectrum of neutrinos and is employed to monitor beam characteristics on a daily basis. Ideally, this detector would provide real-time data. Because of the extremely tiny neutrino cross-section, it is not conceivable. As a result, daily information is presented instead. The proton beam measurement and the muon flux observed by the MUMON are then utilized alongside INGRID to provide real-time information. However, due to the high momentum threshold, MUMON is only sensitive to high-energy muons.

INGRID is made up of a set of 7 + 7 identical "standard INGRID" modules arranged in a



Figure 3.12: Front (left) and top (right) views of the INGRID from the incoming neutrino beam.

cross-shape structure, with the neutrino beam center at the core (see Fig. 3.12). The neutrino beam firstly hits the vertical modules, followed by the horizontal modules 4 m downstream. Because the typical beam width at 280 m from the target is 10 m, INGRID should have extensive area coverage to sample the beam shape and center on a 10×10 m². Furthermore, two "shoulder" or "off-cross" standard modules have been added to check the asymmetry of the beam shape. Later, in 2010, a new module called the "Proton Module" (PM) was placed between the core horizontal and vertical INGRID modules to examine neutrino cross-sections.

In each standard module, there are 9 iron planes used as the neutrino interaction's target and 11 plastic scintillating planes, read by Wave-Length-Shifting (WLS) fibers, used to measure muons produced in neutrino interactions (see Fig. 3.13). The iron planes and scintillating planes are sandwiched. Each scintillating plane is split into two sub-planes made up of scintillating rods which are optically isolated. The two scintillating bars are alternately placed horizontally and vertically. Each module is surrounded by an additional set of veto scintillating planes, the purpose of which is to reject interactions that occur outside of a module.

With this configuration, INGRID can monitor the beam direction with a 0.2 mrad resolution or a 5 cm spatial resolution. Hence, the experimental demand to monitor the beam direction



Figure 3.13: A typical INGRID module. The inside part of the module is depicted on the left, with the blue iron planes and the black scintillator planes. The identical module is shown on the right, but with the bottom, top, and side "veto planes" (black) attached.

within 1 mrad was met. Moreover, in parallel with monitoring the neutrino beam, the INGRID also provides several neutrino cross-section measurements [133, 134].

3.3.2 The Off-Axis Detector: ND280

Due to the significant uncertainties (originally $\sim 10\%$) in the neutrino flux, a near detector is necessary to constrain the neutrino flux prior to any oscillation. The off-axis near detector ND280 is conducted to do this job. It can measure the neutrino interaction cross-section, neutrino energy spectrum, and the proportion of flavour content in the neutrino beam before the oscillation. To eliminate the relative systematic error of the flux, this near detector should ideally be a duplicate of the far detector and receive the same flux. In reality, this is clearly challenging with only one neutrino beam: the cylindrical 33 m diameter far detector placed at 295 km would necessitate a water Cherenkov near detector with a 30 cm diameter at 280 m. This kind of detector is too small to obtain physic data due to the statistical problem, the edge effects, and the inability to create the Cherenkov ring due to the small detector size.

Fig. 3.14 shows schematic view of the ND280 detector. The inner parts of the ND280 include an upstream π^0 detector (P0D), a downstream tracker which is comprised of three Time Projection Chambers (TPCs) and two fine-grained scintillating detectors (FGDs) placed alternatively. The tracker and P0D are enclosed by electromagnetic calorimeters (ECals). Then everything is placed in a magnet yoke equipped with a side-muon tracking device (SMRD). These main parts of the ND280 are detailed in the following subsections.

The Magnet

ND280 uses the UA1 magnet operated with a horizontal uniform magnetic field of 0.2 T, perpendicular to the direction of the neutrino beam. The magnet's interior dimensions are $3.5 \times 3.6 \times 7.0$ m³. The magnet is divided into two symmetric halves, each with eight C-shaped flux return yokes composed of low-carbon steel plates. The yoke weighs 850 tons in total. The



Figure 3.14: Cutaway view of the T2K 280 m near detector.

magnetic field is created using aluminium coils with a current of 3 kA.

Based on the simulations, the field's strength and direction are homogeneous, with transverse components surpassing 1% only in places adjacent to the coils. This magnet is used to bend the charged particles produced by neutrino interactions. The momentum of these particles can be determined thanks to these curved trajectories.

The π^0 detector (P0D)

The P0D detector is the first part of ND280 that the neutrino beam encounters, and it is used to measure the neutral current interactions with π^0 in the final state (NC π^0). From that, we are able to estimate the rate of NC π^0 , which is one of the main backgrounds in the ν_e appearance channel.

The P0D detector can be divided into three modules that are linked together. The external modules are electromagnetic calorimeters composed of seven vertical and horizontal scintillator planes interconnected by 4 mm lead planes. The center module includes the water target as well as the tracker. To improve particle energy loss, twenty-five vertical and horizontal scintillator planes are layered with twenty-five (3 cm long) water target planes and twenty-five (1.6 mm long) brass planes in this centre section. The overall weight of the target is 2850 kg. In total, there are 76 tracking planes, and they are perpendicular to the beam direction. The tracking planes are made of polystyrene triangular scintillating bars co-extrusionally formed with a TiO2 reflective coating and a center hole for a WLS fiber.

Since the P0D includes the water layers, these layers can play as an inactive target for monitoring neutrino interaction with oxygen, which would be used to estimate SK systematics. The technique for determining the $NC\pi^0$ cross-section just on water (mainly oxygen) is to compare the cross-sections obtained in the P0D water-filled and water-out to eliminate the contribution of brass and carbon scintillator targets.

The tracker

Downstream of the P0D is the tracker, which is built to measure the momentum of charged particles produced by CC interactions (muons and pions) and to measure the ν_e contamination in the beam. The tracker consists of fine-grained detectors (FGDs) and time projection chambers (TPCs).

• Fine Grained Detectors (FGDs) [135]: The FGDs represent the active target for neutrino. Their interactions will produce charged particles that will be measured by the TPCs. Two FGD modules are placed alternately with TPCs. FGD must provide reconstruction and particle identification of short tracks that stop in FGD, such as the ranges and direction of recoil protons. The FGD can also aid in distinguishing between charged current quasi-elastic (CCQE) and non quasi-elastic processes (CC-nonQE).

Each FGD has an active mass of 1.1 ton and consists of segmented plastic scintillator bars aligned in either x- or y-direction perpendicular to the beam direction². In particular, the upstream FGD comprises thirty horizontal and thirty vertical polystyrene scintillator planes that serve as a hydrocarbon target and a tracker. Each scintillator is $0.96 \times 0.96 \times 184.3$ cm³ in size. The second FGD is a water-loaded detector that alternates each x - y scintillator layer with 3 cm thick layers of passive water. The comparison of neutrino interaction probabilities in the first and second FGDs enables the extraction of cross-sections on carbon and oxygen independently by subtracting the statistic. This helps reduce the uncertainty caused by the target difference between ND280 and SK.

Because FGDs are so thin in the direction parallel to the beam (36.5 cm), the final state particles in neutrino interaction usually pass through the TPCs, which have better momentum resolution and particle identification (PID) abilities. However, reconstructing low momentum or high angle tracks is challenging since they do not enter the TPCs. T2K collaborators use a "momentum-by-range" technique to deal with this problem. By figuring out the starting and stopping point, the particle's momentum can be calculated by adding up all the energy deposits by the track. Thanks to this deposited energy, FGDs can determine the particle ID similarly to the TPCs. However, this method is still limited to high-angle tracks since it requires the particles to stop inside the FGD. Chap. 4 will discuss an upgraded ND280 near detector where it can handle perfectly this issue.

• Time Projection Chambers (TPCs) [136]: The TPCs in the ND280 complex are the gaseous detectors dedicated to reconstructing charged particles with incredibly high resolution. Charged current interactions between neutrino and nuclei in FGDs produce charged leptons. With the determination of the momentum of these leptons, the neutrino energy can be reconstructed. Thus, there are three ND280s argon TPCs to determine these leptons' trajectory in three dimensions and facilitate particles' momentum and particle ID reconstruction. An exemplary particle identification can help better estimate the contamination of ν_e in the ν_{μ} beam. This is very important since, in T2K, the ν_e beam is

 $^{^{2}}$ The direction convention is choosing the z-axis to be the direction of the neutrino beam



Figure 3.15: T2K TPC schematic, with identified components. Figure taken from [136].

currently an irreducible source of background.

Fig. 3.15 shows the schematic view of the TPC. The T2K's TPC is comprised of an inner box, the field cage, enclosed by an outer box. The volume between the inner and outer boxes is filled with CO_2 , a chemical and electrical insulator. The interior box is loaded with a gas composition of $Ar:CF_4:iC_4H_{10} = 95:3:2$, which has been selected because of its high speed and low diffusion. When a charged particle travels through the TPC, it can ionise the gas inside the TPC and produce electrons along its path. These electrons then drift to the two TPC readout planes from the cathode in the center due to the ~ 280 V/cm electric field inside the field cage. An external trigger is applied to indicate the time when the track enters the chamber. Thanks to this trigger, not only the charge but also the arrival time can be measured and combined to reconstruct the whole 3D track. The TPCs are also subjected to a magnetic field of 0.2 T which bends the path of the charged particles. The particles' momentum and charge can be determined based on the track curvature. Moreover, the magnetic field prevents drift electrons from diffusing in the field cage. A micro-mesh is positioned close to the anode, and an extra voltage is provided in between the mesh and the anode. Consequently, the electric signal at the anode is intensified, resulting in an electron avalanche with a gain of about 1500 and a more accurate anode readout. The MicroMegas technique [137] is based on this principle. and T2K is one of the first experiments using this technology.

There are twelve MicroMegas modules installed at the anode end. Each MicroMegas module has the size 342 mm × 359 mm and contains 1728 pads which are the charge-collection element segmentation. The MicroMegas charge-collection element ('pad') segmentation is 70 mm² a sampling length of 700 mm, resulting in a spatial resolution of ~ 0.7 mm. The TPCs can achieve their momentum resolution requirement of < 10%³ [135] thanks to the

³This is for the momentum parallel to the MicroMegas plane. The resolution is limited for particles with



Figure 3.16: Expected energy deposition as a function of the momentum for positive TPC tracks simulated by GENIE 2.8.0. The plot was made by M. Nirkko in T2K internal technical report.



Figure 3.17: An example of a DIS event observed in the ND280 detector's tracking part.

 $0.2~\mathrm{T}$ magnetic field and the previously specified $0.7~\mathrm{mm}$ point spatial resolution.

Furthermore, TPCs can discriminate particle types thanks to the capacity to determine charged particles' kinematics: momentum based on the particle's curvature and energy loss calculated from the ionization rate. The energy loss for a minimum ionizing particle (MIP) in the TPC gas is around 1.2 keV/cm. Fig. 3.16 shows the reconstructed energy loss of charged particles with respect to their reconstructed momentum. As seen, within the specific momentum range, the difference in energy loss is able to distinguish the particle types. This technique is well-known for PID.

To demonstrate the spatial resolution and reconstruction ability of the tracker path in ND280, Fig. 3.17 illustrates the tracks measured for a DIS in the FGD1. By eye, we can clearly determine the interaction point and separate different tracks.

momentum perpendicular to the MicroMegas plane.

The Electromagnetic calorimeter (Ecal)

The Electromagnetic Calorimeter's goal is to measure and analyze the electromagnetic energy of the particles coming from neutrino interactions in the inner detectors (P0D, FGD, TPC); for example, the photons stem from π^0 decay. The ECAL additionally serves as an active veto detector, insulating the internal detectors from neutrino interactions outside of the Ecal as well as detecting charged particles and classifying them based on their shower patterns.

The Ecal used by T2K is a scintillator-based calorimeter consisting of three parts: the barrel ECal surrounding the FGD+TPC area, the downstream ECal located at the downstream end of the tracker, and the P0D ECal surrounding the P0D detector. The ECal is comprised of 1 cm thick, 4 cm wide plastic scintillator bars sandwiched between lead absorber plates. There are two types of Ecal modules; the main difference between them is the number of active layers and their sampling fractions.

The primary module, which is positioned downstream of the tracker system, comprises 34 active layers isolated by 1.75 mm of lead sheet, resulting in a total effective thickness of 11 radiation lengths. The primary module has the active size of $204 \times 204 \times 50$ cm³, and the total active weight is 7.0 ton. The second module is simpler in design, with only six active layers of scintillator split by four centimeter thick lead plates. This module has a total weight of 4.0 ton and an active area of $280 \times 280 \times 50$ cm³.

3.4 The far detector: Super-Kamiokande

The far detector in the T2K experiment is a cylindrical 50 kton water Cherenkov detector named Super-Kamiokande (SK) [62]. It is placed at Kamioka Observatory in Gifu Prefecture, which is ~ 295 km away from the neutrino production point at J-PARC. It is located at a mean depth of 1000 meters in an underground mine beneath the Ikenoyama mountain. The main goal of building the SK under a mountain is to reduce the background from the cosmic rays. The 1000 meters shielded by the mountain is equivalent to 2700 m covered by water. Thanks to this shield, the flux of cosmic rays is decreased by five orders of magnitude compared to the flux on Earth's surface. The SK detector aims to measure neutrinos from many sources such as the Sun, the atmosphere, supernovae, and accelerators. Another mission of SK is searching for proton decay.

3.4.1 Configuration of Super-Kamiokande

Fig. 3.18 depicts a schematic of the detector's location in the mine. The SK tank is made of stainless steel and contains 50 ktons of ultra-pure water in total with a size of 41.4 m in height \times 39.3 m in diameter. The detector is divided into two sections: the inner detector (ID) and the outer detector (OD), which surrounds the ID. The ID is equipped with 11,146 inward-facing 20-inch photomultiplier tubes (PMTs), while the OD is lined with 1,885 outward-facing 8" PMTs. Fig. 3.19 shows a photo and a schematic depiction of a 20-inch PMT. These 20-inch PMTs were built because of their great single photoelectron (p.e.) response. The maximum quantum efficiency at 400 nm is 20% which is fully described in [138].



Figure 3.18: The Super-Kamiokande detector and its location in the Kamioka mine are shown schematically.

In April 1996, the SK detector began collecting data. The first five years of operation are referred to as SK-I. In November 2001, after being paused for maintenance, an accident happened. Sixty per cent of the PMTs were destroyed in the accident. The detector was reconstructed using 50% of the PMTs in December 2002, and the time following the disaster is known as SK-II. In the spring of 2006, the detector was re-instrumented with all of the PMTs (SK-III). The PMT photocathode in SK-II and SK-III is protected by an acrylic cover, while the entire phototube is protected by a fiber reinforced plastic (FRP) shell. This acts as a shock absorber from a chain reaction driven by shock waves following the explosion of a single PMT. The 2001 disaster was induced by this chain reaction.

3.4.2 Cherenkov detector principles

The Cherenkov effect is similar to the supersonic effect for the shock wave of sound. This effect was discovered by Cherenkov in 1937 and then described afterwards by Frank and Tamm. When a charged particle moves faster than the speed of light in a medium, its electric field polarizes the atoms in the medium; these atoms will produce Cherenkov radiation in the shape of a light cone, the base of which is centered on the particle's trajectory. Fig. 3.20 illustrates the procedure of emitting Cherenkov light. The opening angle θ_C of the cone depends on the medium's refraction index n as follows

$$\cos\theta_C = \frac{1}{\beta n},\tag{3.4}$$

where the $\beta = v/c_0$ factor is the ratio between the velocity of a charged particle in medium (v) and light speed in vacuum (c_0) .

From the Eq. 3.4, one can conclude that the opening angle will reach its maximum value $\theta_C = \arccos(1/n)$ if the particles is ultrarelativistic ($\beta \sim 1$). The second conclusion is that



Figure 3.19: Photo (left) and schematic view (right) of a 20 inch PMT.

this formula has proven the existence of the Cherenkov velocity threshold. If $\beta n < 1$ (or v < c, where $c = c_0/n$ is the speed of light in the medium with the refraction index n), there is no Cherenkov light since $\cos \theta_C$ can not be greater than 1. Hence, the Cherenkov velocity threshold for particles (c) depends on the refraction index of the crossed environment. For example, SK contains water with n = 1.33 and hence the velocity threshold is $c_0/n \sim 0.75c_0$. The Cherenkov energy threshold of a charged particle is mass-dependent

$$E_k = \gamma m_k c^2 = \frac{1}{\sqrt{1 - \beta^2}} m_k c^2, \qquad (3.5)$$

where E_k is the energy threshold of a particle with the mass m_k . The condition for the Cherenkov radiation to happen is $\beta n < 1$. To calculate the energy threshold, the threshold of $\beta = 1/n$ should be used. In case of using water Cherenkov detector (n = 1.33), Eq. 3.5 becomes

$$E_k = 1.52m_k.$$
 (3.6)

Based on Eq. 3.6, the threshold in SK is 775 keV for electrons, 160 MeV for muons and 1.4 GeV for protons.

In T2K, a high-energy neutrino from JPARC interacts with the water inside the SK and produces a charged lepton associated with incoming neutrino (μ for ν_{μ} and electron for ν_e in CCQE channel) that moves faster than the speed of light in water, thus creating the Cherenkov radiation. We can then reconstruct the rings of the Cherenkov light thanks to the PMTs surrounding the water tank. The particle types can be determined by the sharpness of the edge of the ring. The electrons' rings are fuzzier than muons because of the large multiple scattering of electrons, while muons travel approximately straight through the water. It is then possible to select samples of ν_{μ} and ν_e interactions that can be used to measure oscillation parameters. Fig. 3.21 displays two types of typical Cherenkov rings used for T2K analysis and detected by SK.

For the hadronic part in the neutrino interaction final state, it is not easy to detect them. The neutrons are neutral particles, so they can not produce Cherenkov radiation. The protons are able to produce Cherenkov light. However, the protons produced by neutrino interaction at T2K



Figure 3.20: The Cherenkov effect principle. Left: the particle's velocity v is smaller than the light's velocity in vacuum c, there is no Cherenkov light. **Right:** photons are emitted in a cone with an angle $\cos_{\theta_C} = c/v = 1/(\beta n)$ by a particle moving at velocity v in a medium where light moves at velocity c.

have the typical energy of $\mathcal{O}(10^2)$ MeV which is lower than their Cherenkov energy threshold (1.4 GeV), and hence cannot be observed by SK. Recently, the SK experiment loaded Gadolinium in its water tank since it helps to detect the neutrons through the capture of Gadolinium. In principle, after the neutron capture, the stimulated Gd nucleus produces de-excitation photons. To identify neutrons, a separate signal of four de-excitation photons happens at a particular time delay (depending on the Gd percentage) after the lepton's signal.

In general, the SK detector has its own physics goals in addition to functioning as the far-detector for the T2K experiment. The SK also measures atmospheric neutrino oscillations, solar neutrino fluxes, and hunts for proton decay and supernova neutrinos.

3.5 Recent results and future

The T2K's primary goal is to study neutrino oscillations. It specifically searches for two oscillation channels: $\nu_{\mu} \rightarrow \nu_{\mu}$ disappearance (primarily ruled by θ_{23} and Δm_{23}^2) and $\nu_{\mu} \rightarrow \nu_e$ appearance (primarily ruled by θ_{13} , θ_{23} and δ_{CP} and mass hierarchy). Many oscillation parameters have been constrained by T2K with a precision below 10%, except the θ_{13} mixing angle has a 15% precision. These results prove that the T2K experiment is one of the leading experiments in the field.

The current constraints on the atmospheric mixing parameters (θ_{23} and Δm_{23}^2) are shown in Fig. 3.22 (left). T2K provides the strongest constraints on these parameters so far. The statistic and systematic model at T2K are continuously improved. Fig. 3.22 (right) show the fits using different atmospheric mixing parameters for $\nu_{\mu}/\bar{\nu}_{\mu}$ that yields consistent outcomes compatible with CPT and standard neutrino interactions.

The constraints for ν_e versus $\bar{\nu}_e$ appearance are still dominated by the statistic uncertainty.



(b) *numu*

Figure 3.21: SK event

	$\sin^2\theta_{23} < 0.5$	$\sin^2\theta_{23} > 0.5$	Line total
Normal ordering	0.236	0.540	0.776
Inverted ordering	0.049	0.174	0.224
Column total	0.285	0.715	1.000

Table 3.1: Posterior probabilities for different mass ordering hypotheses from T2K run 1-10 data and the results of the reactor experiments. **T2K run 1-10**, **2022 preliminary**.

therefore a bi-event plot is shown in Fig. 3.23 (left), which is useful for understanding the origin of multiple parameters. Fig. 3.23 (right) show the constraint on the δ_{CP} . As seen, a wide region of the δ_{CP} space is excluded at 3σ C.L., and CP-conservation corresponding to $\delta_{CP} = 0, \pi$ is excluded at 90% C.L. This δ_{CP} constraint which has used the constraint of θ_{13} from reactor experiment is a world-leading result. The previous analysis with Run 1-9 was published in Nature, in which the evidence for CP violation was shown [1]. Tab. 3.1 shows the posterior probabilities for different hypotheses about mass ordering and $\sin^2 \theta_{23}$ octant. The result indicates a slight preference of normal ordering and $\sin^2 \theta_{23}$ in upper octant.

This section presented the recent neutrino oscillation measurements from T2K employing 3.64×10^{21} POT, with numerous advances at every level of analysis. Good constraints on δ_{CP} and other oscillation parameters are shown. There is a slight tendency for normal ordering and the upper octant. Future constraints on these parameters are promising. Joint fits between NO ν A+T2K and SK+T2K collaborations are actively developed, with the goal of achieving better oscillation parameter constraints as a result of resolved degeneracies. Moreover, the T2K-II plan, as part of the J-PARC program of beam intensity enhancements, calls for an increase in POT exposure by a factor of ten. The far detector mass is also raised by a factor



Figure 3.22: Left: Constraints on $\sin^2 \theta_{23}$ and Δm_{23}^2 parameters, as well as comparisons with NOvA, SK, and IceCube recent results. Right: Constraints on atmospheric parameters using the fit with two samples: neutrinos $(\sin^2 \theta_{23}, \Delta m_{23}^2)$ and anti-neutrinos $(\sin^2 \bar{\theta}_{23}, \Delta \bar{m}_{23}^2)$

of ten in the Hyper-K plan [49]. To take the full advantages from the statistic increase by two orders of magnitude, the near detector is under an upgrade program. The goal of ND280 upgraded is to reduce the statistical and systematic uncertainties to approximately 3-4%. The Chap. 4 will discuss in detail the T2K-II plan together with all the detector upgrade programs in T2K.



Figure 3.23: Left: the ellipses represent predicted event rates for each value of CP, with data points around the maximum CP-violating value of $-\pi/2$. The predictions with different mass ordering, $\sin^2 \theta_{23}$ values were also shown. Right: the distribution of $\Delta \chi^2$ for δ_{CP} in normal (inverted) ordering. The confidence intervals are depicted by areas with different colours.

ND280 upgrade for T2K-II and the High Angle TPC test beam analysis

4

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The observation of ν_e appearance oscillations by the T2K experiment has opened the door to the potential of discovering CP violation in the lepton sector, which would be a key indicator to explain the matter-antimatter asymmetry in the universe [139]. Currently, the best fit of T2K on δ_{CP} assuming the NO (IO) are $-1.89^{+0.70}_{-0.58}$ ($-1.38^{+0.48}_{-0.54}$) which excludes $\delta_{CP} = 0$ at 2σ confident level (C.L.) [1]. These best fit values are very close to the maximum CP violation case ($\delta_{CP} = -\pi/2$). This is an excellent hint for extending the running time at T2K to collect more statistics. That is why the T2K collaboration has begun the studies on "T2K-II", a second phase of the experiment to observe CP violation with more than 3σ C.L. This chapter will introduce the motivation and expected sensitivity to the oscillation parameters of the T2K-II. Then, some sections explain the need for the upgraded near detector ND280 and its upgraded configuration. Finally, I will introduce my analysis of test beam data for the High Angle TPC, which is a new sub-detector in the upgraded ND280 complex.

4.1 T2K phase II

4.1.1 Motivation of T2K-II

At the first stage, the T2K experiment was approved to collect in total 7.8×10^{21} POT. These statistics were primarily motivated by the experiment's sensitivity to θ_{13} . Luckily, the θ_{13} value is large enough that the systematic uncertainties could not dominate it. As a result, the sensitivity of T2K to CP violation is better with more statistics. In 2016, a proposal for an extended run of T2K (T2K-II) was made in which the total of POT at the end of this stage will reach 20×10^{21} [140]. Apart from the time extension, these statistics will be achieved as an outcome of an upgrade to the J-PARC Main Ring power supplies, which will enable T2K to attain ~1 MW of beam power (currently, T2K is stably running at 522.6 kW of beam power).

The next long baseline neutrino oscillation experiments, such as Hyper-Kamiokande (HK) and DUNE, seek to obtain > 3σ sensitivity to CP violation along with a wide range of δ_{CP} values after 2027. The original goal of the T2K-II is to upgrade the beam power and extend data-taking to 2026 so that the new statistics can considerably increase sensitivity to CP violation before reaching the era of HK and DUNE. In particular, more than 3σ sensitivity to CP violation can be accomplished at the end of T2K-II if true $\delta_{CP} = -\pi/2$, and the mass hierarchy can be defined within six years after the first approved running. Moreover, with the beam upgrade, the HK could benefit from higher beam power (expected to exceed 750 kW) from the beginning of data-taking.

To achieve these T2K-II goals, it would necessitate not only data-taking extension and the MR beam power increase but also further improvements in neutrino beamline and systematic uncertainties.

4.1.2 Systematic improvement

Systematic uncertainties in T2K are classified into neutrino flux, neutrino interaction model, and detector model uncertainties. External data constrain the neutrino flux and interaction model's uncertainties, which are further constrained by the near detector fit. If the existing systematic uncertainties are not reduced, the sensitivity to CP violation with the T2K-II statistics will be severely lowered. This section will summarize the critical systematics that needs improvements.

Neutrino flux

The systematics in neutrino flux prediction is presently driven by uncertainties in hadron interaction modelling in the target and surrounding materials in the neutrino beamline (as discussed in Sec. 3.2.3), as well as by proton beam trajectory monitoring. Further constraints on the flux uncertainties are planned with the addition of NA61/SHINE T2K replica target

measurements, improvements in beam direction measurement, and better use of near detector data. For the T2K-II, the absolute flux uncertainty is expected to be improved from $\sim 9\%$ to $\sim 6\%$.

Near detector measurement

Further improvements and development are expected to reduce some systematic uncertainties, such as reconstruction efficiencies and background. However, the most significant uncertainty is from pion secondary interaction uncertainties, which can be improved by external data or more studies on pion interaction inside ND280. The upgraded ND280 (discussed in Sec. 4.2) is expected to be sensitive to the hadronic part and have a lower momentum threshold. As a result, it can help better understand pion interaction. With more statistics, T2K-II expects to reach $\sim 1\%$ overall systematic uncertainty in the ND280 samples.

Neutrino Interaction

One of the main systematic uncertainties comes from our naive understanding of neutrinonucleus interactions. T2K has been continuously developing and improving neutrino-nucleus interaction models as well as the nuclear effects. The current data from ND280 can constrain these models; however, there are still many limitations due to the small angular acceptance and high momentum detection threshold of ND280. This really motivates the T2K collaboration to upgrade the ND280. In Chap. 5, I will present how well we can constrain these nuclear effects with the new configuration of upgraded ND280.

Super-Kamiokande systematic

The present systematic errors in the SK are primarily defined by a fit to the Super-K atmospheric neutrino data and bounds on the energy scale error from cosmic muon control samples. The atmospheric neutrino fit is under modification to use the T2K data cross-section modelling. In the long term, the critical detector parameters will be mainly constrained by calibration, entering muon, and decay electron data instead of just fitting atmospheric neutrino data. This is because the atmospheric neutrino data is subject to atmospheric flux and neutrino cross-section errors. The predicted reduction in Super-K detector uncertainty was not quantitatively apparent when T2K-II was proposed. However, the FSI and neutrino interaction model uncertainties, for sure, will benefit from the ND280 constraints.

4.1.3 Expected physics results

CP violation and oscillation parameters constraints

This section will discuss the sensitivity to CP violation caused by a CP-odd phase in the threeflavour mixing matrix. Fig. 4.1 compares sensitivity to CP violation as a function of true δ_{CP} for two cases: the originally approved T2K statistics (7.8 × 10²¹ POT) with 2016 uncertainties and the complete T2K-II data (20 × 10²¹ POT) with the effective statistic uncertainty improvement. The sensitivity is displayed without systematic errors, and with 2016 T2K systematic errors, these uncertainties cause a large reduction in sensitivity.


Figure 4.1: CP violation sensitivity as a function of true δ_{CP} . The y-axis is the $\Delta \chi^2$ to exclude $\sin \delta_{CP} = 0$. Plot taken from [140].

The predicted change of the sensitivity to CP violation as a function of POT can be seen in Fig. 4.2, given that the T2K-II data are collected in nearly equivalent alternating periods of ν -mode and $\bar{\nu}$ -mode (with true normal mass hierarchy and $\delta_{CP} = -\pi/2$). The sensitivities in Fig. 4.2 are shown with different systematic uncertainties; one can draw a conclusion about the importance of improving systematic uncertainties from this plot. To reach the same 3σ C.L. for resolving $\sin \delta_{CP} = 0$, the improvements in systematic uncertainties help to save $\sim 5 \times 10^{21}$ POT compared to the 2016 systematic uncertainties. For reference, the accumulated statistic of T2K until the end of 2021 is 3.82×10^{21} POT.

Fig. 4.3 depicts the estimated 90 percent C.L. contour for Δm_{32}^2 versus $\sin^2 \theta_{23}$ for the entire T2K-II exposure. If true $\sin^2 \theta_{23} = 0.5$, the projected precision for $\sin^2 \theta_{23}$ is ~ 1.7°, while the expected precision on Δm_{32}^2 is ~ 1% with additional choice of true $\delta_{CP} = -\pi/2$.

Studies of neutrino interaction

The T2K-II extended run will enable better data on neutrino and anti-neutrino interaction, which possibly study the nuclear structure via the axial vector current. Moreover, the upgraded ND280 with better acceptance could be used to study the nuclear effects in CCQE and single pion production channel (CC1 π). This can be done because the better detector acceptance allows for more comprehensive kinematic measurements for all final state particles.

4.2 ND280 upgrade

A deeper understanding of systematic uncertainty will be required to cope with the promised extra statistics. As a result, the T2K collaboration has started an upgrade project for the Near Detector, with the main goal of overcoming the limitations of the present ND280 design in terms of angular acceptance.



Figure 4.2: Sensitivity to CP violation as a function of POT. The results are shown with the same 50% reduction in effective statistics but with different systematic uncertainties. Plot taken from [140].

4.2.1 Motivation for the near detector upgrade

Before the T2K experiment, the value of θ_{13} mixing angle was undetermined. It was, hence, challenging to estimate the event rate of ν_e appearance in neutrino oscillations. The π^0 production in NC interactions could potentially contribute as the main background for measuring ν_e at the SK. As a result, ND280 was built with the P0D sub-detector to precisely quantify π^0 production in neutrino interactions. However, after determining a relatively large θ_{13} value, better reconstruction algorithms at SK to reduce NC π^0 background in ν_e sample, and the precise knowledge of π^0 cross-section; the P0D did not play an important role anymore.

For long baseline experiments such as T2K, the systematic errors mainly depend on two aspects [40]: the neutrino flux and the interaction cross-section. The role of the ND280 is to reduce these uncertainties by measuring the neutrino spectrum before the oscillation and performing cross-section measurements. Thanks to ND280, systematics uncertainties on the number of expected events in SK are reduced from approximately 13% to around 4%. The main disadvantage of ND280 is the limited angular coverage.

For the detector efficiency, based on the designs of near- and far-detectors, there is a difference in angular acceptance between ND280 and SK (illustrated in Figure 4.4). In ND280, the TPCs are only in the forward direction, while the SK has the full solid angle acceptance. The FGD is made of scintillating bars disposed perpendicularly to the neutrino beam direction; this design makes ND280 ideal for measuring horizontal tracks. However, very high-angle tracks (i.e. charged particles in the final state go perpendicularly to the neutrino beam direction) are inefficiently reconstructed because they do not enter the TPC, and it is difficult to reconstruct in the FGD. Even the tracks with not-so-high angles can be rejected if they only pass a small part of one TPC because the selecting method in T2K necessitates relatively long TPC tracks to reconstruct lepton momentum. Since the current ND280 is not sensitive to the hadrons, most of the events passed the selection contain only one visible track from a lepton. In these circumstances, the time difference between the track edges is utilized to identify whether the



Figure 4.3: Expected constraints on Δm_{32}^2 and $\sin^2 \theta_{23}$ with the 2016 systematic uncertainties. The black line corresponds to 1.9×10^{21} POT. Plot taken from [140].

track is a positively-charged forward-going lepton or a negatively-charged backwards-going lepton. Access to timing information is only possible for FGDs and P0D detectors; thus, the lepton must cross at least two of these detectors in order to reconstruct its charge sign. Usually, the backward tracks have low momentum; therefore, the likelihood of being reconstructed in both detectors resulting in poor detection efficiency. As shown in Fig. 4.4, the current ND280 can reconstruct pretty well high momenta leptons that are primarily emitted in the forward direction. However, low-energy particles are emitted more isotropically; then, the current ND280 is no more suitable to reconstruct these leptons.



Figure 4.4: The comparison of the acceptance between ND280 (left) and SK (right). It is clear that the angular acceptance of current ND280 is limited compared to that of SK. Figures taken from [141].

The other disadvantage of the current ND280 is its high momentum detection threshold. Since the FGDs are constructed from plastic scintillator bars, a charged particle must traverse at least four bars for its track to be reconstructed. Consequently, the reconstruction's momentum threshold is pretty high ($\sim 400 \text{ MeV/c}$). With the T2K neutrino spectrum, most interactions are quasi-elastic; in this case, the neutrino energy can be reconstructed from the lepton's kinematics. However, as explained in Sec. 2.3, the nuclear effects significantly bias the reconstructed neutrino energy because they change the interaction topology as well as the kinematics of final state particles. In order to study of these nuclear effects, it requires the detection of low-momentum hadrons produced in the neutrino CC interactions. Therefore, the newly upgraded detector efficiency must be good enough to measure such low-energy events.

Eventually, the T2K near detector ND280 should meet the main requirements of the upgraded program [142]

- the near detector measurements must cover the full polar angle range for the final state lepton with a well-understood acceptance;
- the near detector must be capable of measuring electron and muon neutrino cross-sections;
- the near detector should be able to address the issue of nuclear effects and their impact on energy reconstruction.

To optimize the capabilities of the ND280 detector and take more advantage of the improved statistics provided by the T2K-II, the ND280 Upgrade project was proposed by the T2K collaboration [143]. The upgraded ND280 will replace the P0D with a new tracker with a horizontal alignment parallel to the neutrino beam and may detect particles perpendicular to the beam, for which the existing tracker has poor acceptance. The configuration of upgraded ND280 is illustrated in Fig. 4.5. The new tracker contains a 3-dimensional scintillator target (Super-FGD) for neutrino interactions. Two High-Angle TPCs (HA-TPCs) will be placed on the top and bottom of the Super-FGD. Finally, six Time-of-Flight (ToF) modules will surround the new tracker system. All of these sub-detectors will be explained in detail in the following sections.



Figure 4.5: ND280 upgrade configuration

4.2.2 Design of new Super-FGD

As mentioned above, the current ND280 tracker has good efficiency for forward-going tracks but not for high-angle or backwards-going tracks. The proposed detector, called Super-FGD [144], has an innovative configuration of fine-grained fully-active plastic scintillator cubes (illustrated in



Figure 4.6: Schematic concept of the Super-FGD structure. The active part of SuperFGD is $192 \times 192 \times 56$ cubes in dimension. The cubes' size is $1 \times 1 \times 1$ cm³. Figure taken from [143]

Fig. 4.6). It consists of 1 cm³ cubes of plastic scintillator. These cubes are optically independent and read out along three orthogonal directions by wavelength shifting (WLS) fibers. The WLS fibers collect light from scintillation and are inserted in each scintillator cube in x, y, and zdirections. One end of them is connected with Multi-Pixel Photon Counter (MPPC)¹, the other end is mirrored.

The scintillator in the cube is manufactured of polystyrene that has been doped with 1.5 per cent paraterphenyl (PTP) and 0.01 per cent POPOP. After production, the cubes are overlaid with a reflecting layer (50-80 μ m thick) made by chemically etching the scintillator surface. Since the number of cubes is so large, if we read out individually each cube, it will cost too much and much dead material will be introduced into the detector. We, therefore, read out for each fiber to keep the number of readout channels at a reasonable value. The total number of cubes is 2,064,384, and the number of readout channels is 58,368.

WLS fibers are widely utilized to capture light from broad areas of scintillators. They are multi-cladding round fibers with a diameter of 1.0 mm. The total length of WLS fiber needed to be produced will be 70 kilometers. After being collected by the WLS fibers, the scintillation light will be detected by the photosensor at one end of the WLS fibers. T2K collaboration has chosen the MPPC for this job. It has a sensitive area of 1.3 mm×1.3 mm, which is the same as the MPPCs used for current ND280 and is corrected to suit the size of the WLS fiber.

The Super-FGD will be built with a mass of 2 tons, and it can collect significantly more data than the current FGD (1 ton). Moreover, the Super-FGD will offer substantially better information on the neutrino interaction than previous FGDs since it will project charged particle tracks onto three planes without inactive portions. By using the scintillator cubes instead of scintillator bars and inserting WLS fibers in three different directions, we are able to reconstruct the trajectory in 3 dimensions. Hence, together with the HA-TPCs, this Super-FGD can efficiently address the issue of tracking ambiguity by providing a 4π acceptance. Super-FGD also has better efficiency for short track. Thanks to the fine granularity with the size of $1 \times 1 \times 1$ cm³, some low momentum nucleons ejected by the nucleus within a few cms can be measured. The Super-FGD is able to measure protons with momenta down to 300 MeV/c [142], and muons with momenta down to 50 MeV/c. This measurement can improve our understanding of the

¹The MPPC is also known as silicon photomultiplier (SiPM)



Figure 4.7: Left: A graphic illustration of the TOF detector planes surrounding the target and TPCs. Right: a technical design demonstrating how the planes are mounted to the ND280 basket's exterior and stabilized by aluminium structures.

nuclear effects and, eventually, the final reconstruction of the neutrino energy.

4.2.3 Time of Flight Detectors

The new tracker in upgraded ND280 will be enclosed by six planes of plastic scintillator bars, read out on both ends by arrays of large-area MPPCs. This detector is called the Time-of-Fight (ToF) detector. Fig. 4.7 illustrates the schematic view of the ToF. The scintillator bars have a length of 2.0 or 2.3 m with a thickness of 1 cm. This size of ToF gives good stability while also being well matched to the light collection with $6 \times 6 \text{ mm}^2$ MPPC.

The TOF system attempts to carefully measure the passing time of charged particles in ND280. When combined with a time measured by the Super-FGD, the ToF enables the separation of neutrino interactions in the target from backgrounds arising outside of the detector. For precise detection of the flight direction of charged particles, a time resolution lower than 500 ps is needed. Furthermore, the ToF detector will improve PID distinction for particles with equivalent energy loss but different masses and hence the different time of flight (e.g. protons and positrons). For particle identification, a resolution better than 100-200 ps is recommended. To check whether the ToF detector can meet these requirements, a ToF prototype was examined using test beams at the CERN PS and obtained a timing resolution of 90 ps [145].

4.2.4 High-Angle time projection chamber (HA-TPC)

As introduced in Sec. 3.3.2, TPC is a type of particle detector that combines electric and magnetic fields with a sensitive volume of gas or liquid to reconstruct a three-dimensional trajectory of a particle. Together with FGDs, TPCs provide the data to constrain the neutrino flux and cross-sections for all the T2K oscillation analyses. The combination of FGDs and TPCs inside the UA1 magnet is the point of difference of the ND280. In general, the ND280 TPCs are especially advantageous in event reconstruction because they allow for track reconstruction in three dimensions, charge measurement, momentum measurement, and particle identification by comparing the dE/dx and measured momentum.

114 ND280 upgrade for T2K-II and the High Angle TPC test beam analysis

The new TPC that will be installed in the upgraded ND280 complex is called High-Angle TPC (HA-TPC). These new HA-TPCs, illustrated in Fig. 4.8, need to have similar performances as the existing TPCs:

- Momentum resolution is better than 10% at 1 GeV/c, motivated by the necessity to reconstruct neutrino energy accurately. For the current TPCs, a 10% momentum resolution equates to a spatial resolution of 600-1000 μ m, giving a benchmark for the spatial resolution of the HA-TPCs.
- dE/dx resolution is better than 10%, which will result in good discrimination between electron and muon. This is a crucial task since it helps to determine the contaminated ν_e in ν_{μ} flux. For reference, the current TPCs have an energy resolution of 8%, enabling electrons and muons separation between a few hundred MeV and ~2 GeV at 4 σ C.L.

To meet these criteria, the new HA-TPCs will be constructed with resistive Micromegas technology for the readout plane. This is the main difference in technology between the current TPCs and HA-TPCs in ND280.



Figure 4.8: Schematic view of the High-Angle TPC [143]

4.2.5 Resistive MicroMegas

Resistive MicroMegas is a new type of detector that was first introduced and successfully tested by the International Linear Collider TPC (ILC-TPC) collaboration [146], and it has new advantages which can not be achieved with the bulk micromegas. The current TPCs used in ND280 have employed the bulk technology for the Micromegas detectors [136]. Actually, the resistive Micromegas technology is analogous to the bulk Micromegas technology, but it includes an extra resistive layer on top of the anode. Fig. 4.9 shows the schematic cross-section view of two types of Micromegas.

In a typical TPC, the electrons are produced by relativistic charged particles and drift towards the anode. After reaching the mesh, they will create an avalanche in the Micromegas amplification zone. This avalanche is caused by a high electric field known as DLC voltage. Producing the avalanche is common for both bulk and resistive Micromegas. However, the pads in the resistive micromegas are covered by a layer of insulating material and a layer of resistive material. The resistive layer forms a 2-dimensional resistive-capacitive network with respect to



Figure 4.9: Schematic of a bulk Micromegas (left) and a resistive Micromegas (right). Compared to the bulk micromegas, the resistive micromegas has in addition resistive and insulator layers in the anode readout structure. Figure taken from [143].

the anode plane. The charge accumulated by the avalanche will normally spread in time and obeys a Gaussian distribution. Because of the spread in the charge on the readout pad plane, determining the charge deposited point could be done more precisely by comparing the signal amplitudes in surrounding pads. This feature allows a reduction in the readout pad density and, therefore, the number of electronic readout channels.

Another improvement is eliminating the sparks and eventually protecting the diodes on the front-end cards. Since the avalanche is extinguished when it reaches the resistive layer. The charge density is then becoming lower, and its function of radius r and time t is of the form

$$\rho(r,t) = \frac{RC}{2t} e^{-r^2 RC/(4t)}$$
(4.1)

where C is the capacitance per unit area and R is the resistivity per unit area. We have chosen the convention that r=0, t=0 at the deposited point. Thanks to this property, the quantity of dead space on the readout plane is decreased since there is no need to install the anti-spark circuitry anymore. Up to now, several HA-TPC prototypes have been produced and tested with many test beams. Some of the HA-TPC performances under these test beams are discussed in Sec. 4.3.

4.3 High Angle TPC testbeam analysis

As shown in Fig. 4.8, eight Micromegas charge readout modules will be installed on each endplate of the new TPCs. As introduced in Sec. 4.2.5, this new micromegas was inherited from the bulk-Micromegas technology and developed with the new technology to form the "Encapsulated Resistive Anode Micromegas" (ERAM). This section will show the performance of one prototype ERAM module which was used for a HA-TPC prototype.

The overall configuration of the ND280 upgrade detector was finalized in the fall of 2018. The sub-detector envelops were specified, and the ERAM module size was determined at $420 \times 340 \text{ mm}^2$ with 32×36 rectangular pads of size $10.09 \times 11.18 \text{ mm}^2$. In 2019, a prototype of the HA-TPC with this ERAM module was tested with an electron beam at the DESY II test

beam facility [147]. This is one of the ongoing series of test beams for the HA-TPC, and the results were also published in [148]. Before this DESY test beam, there was another test beam at CERN to characterize the HA-TPC performance [149].

The prototype was tested inside a 0.2 T magnetic field, the same as the one applied for ND280. The electron beam energy at DESY II spreads from $1 \rightarrow 6$ GeV. The prototype employed a gas mixture consisting of 95% argon, 3% tetrafluoromethane (CF₄), and 2% isobutane (iC₄H₁₀), which is the same gas composition used in the present ND280 TPCs. Furthermore, the field cage direction may be changed to analyze tracks crossing the module at different angles and test alternate clustering topologies. The acquired data is utilized to carefully examine the function of all key factors (electronics peaking time, DLC voltage, drift distance) to completely characterize the charge spreading and resistive foil uniformity, as well as to guarantee that the performance meets the ND280 upgrade criteria.

4.3.1 Charge Spreading

As previously stated, the resistive Micromegas technology causes the deposited charge to spread into nearby pads. For the analysis, the tracks will be divided not into single pads but several pads (clusters). The amount of pads in a cluster determines "cluster multiplicity", which is an essential quantity for characterizing the charge spreading. A schematic view of a typical three-pad cluster for horizontal tracks and the quantity of charge collected as a function of time (waveforms) in these pads are shown in Fig. 4.10. Compared to the signal from direct charge



Figure 4.10: A schematic definition of a pad cluster for a horizontal track (left) and the waveforms of each pad in this cluster (right). The track is purposefully asymmetrical to emphasize the distinction in waveform between the pads above and below the leading pad. Figure taken from [148].

deposition in the leading pad, the signal caused by the resistive layer in the neighbouring pads has a much smaller amplitude, delayed time of the order of a few μ s and is longer-lasting. The maximum of the waveform is utilized as a charge estimator for each pad. The waveform's time delay depends on the distance between the neighbouring pad and the track. As a result, the charge spreading in the longitudinal direction with respect to the track direction is obscured by the direct charge. The spreading charge dominates only the pads in the transverse direction. Therefore, the cluster is defined as a group of adjacent pads in the direction perpendicular to the track.

The signal spread to numerous pads as a result of charge spreading. A higher pad multiplicity is preferable since it provides more accurate spatial reconstruction. Fig. 4.11 shows the cluster multiplicity (i.e. number of pads in each cluster) as well as the charge deposited on the leading pad as a proportion of the entire charge of the cluster $(q_{\text{leading}}/q_{\text{cluster}})$. The pad multiplicity depends on the DLC voltage because this voltage directly causes the avalanche in the readout plane; consequently, the possibility of weak signals in certain pads passing the threshold increases as gain increases. In general, most of the clusters consist of at least three pads, and the leading pad generally holds 80 per cent of the total charge of the cluster.



Figure 4.11: Left: The pad multiplicity per cluster distribution with different DLC high voltage. **Right:** the fraction of the cluster charge collected in the pad with the largest signal at 360 V. Figure taken from [148].

The explanation above is for the horizontal or vertical tracks where the cluster determination is unambiguous. For the inclined/curved tracks and large square pads, it is not obvious to determine which pads are the transverse spreading of a certain leading pad. To separate the longitudinal and transverse spreading topologies, different cluster definitions were used to study the inclined tracks (as shown in Fig. 4.12). These clusters are replicated throughout the ERAM. Note that this method can apply to the squared pad only.

4.3.2 Pattern recognition

Only the through-going tracks (single, straight tracks) are of interest in the test beam analysis since more sophisticated typologies (e.g., showers, multi-particle, curved low-energy tracks) are harder to comprehend. Because of these features, the DB-SCAN [150] method proved adequate for reconstruction. A track is selected if it crosses the whole detector without gaps or splits. A split occurs when there is more than one cluster in a particular column. Hence, any



Figure 4.12: The following cluster patterns can be employed based on the angle of the track (red line) with the pad sides: (a) column, (b) diagonal, (c) 2 by 1, (d) 3 by 1. The coloured pads correspond to one cluster in each example, which is related to the coloured leading pad passed by the track. Figure taken from [148].

multiple-track event with a split is rejected in this analysis. Because of charge spreading, nearby tracks could be reconstructed as a single track. To prevent this, a cut on the pad multiplicity was made for every cluster. Since the pad multiplicity is not an independent variable, the cut value is adjusted regarding different DLC voltages and electronics shaping time. Fig. 4.13 presents a track reconstruction algorithm and its capacity to differentiate tracks.



Figure 4.13: Event displays of a selected single track (left) and a rejected multi-track (right) in the prototype. Figure taken from [148].

4.3.3 dE/dx studies

The majority of current PID algorithms are done by comparing their energy loss (dE/dx) as a function of momentum. A TPC's primary purpose is to provide particle identification (PID), relying on the detection of ionization generated by charged particles passing the gas volume. The resolution of the ionization energy loss data governs the PID capabilities.

The PID is primarily utilized in T2K TPCs to separate electrons from muons, produced by ν_e and ν_{μ} respectively, in CC interaction. The quantity of ionization between electrons and muons varies by ~ 40% in the momentum area of interest for T2K, and a resolution better than 10% enables the effective separation of these two particles. Therefore, it is required that the



Figure 4.14: Charge distribution for one cluster. To have enough statistic, many tracks are used for this plot.

new HA-TPC reclaim a dE/dx resolution equivalent or better to that of present T2K TPCs.

In principle, the resolution is determined by the amount of independent ionization observations (i.e., clusters) and ionization intensity within every cluster. In the current TPCs, integrating data from two Micromegas detectors (72 clusters) yielded a resolution of 8% for electrons. In this analysis, all results are obtained with a single ERAM detector (36×32 pads).

Cluster selection and cluster energy

A horizontal track that crosses the TPC is assigned 36 clusters. However, particles might be released from the original beam direction at different angles. As a result, some tracks contain less than 36 observations of the deposited energy and are rejected.

In the usual case of practically horizontal tracks, each cluster is made up of pads that are close and located on the same Micromegas column. The total energy of the cluster (C_C) is calculated by summing the charges of all the pads making up the cluster.

The truncated mean method

The energy loss in the TPC was calculated using the truncated mean approach. The charges in each cluster of the track are sorted in ascending order, and only a percentage (truncation factor) of the clusters is retained to estimate the mean energy deposition.

The truncated mean energy deposit CT per horizontal cluster is

$$C_T = \frac{1}{\alpha N} \sum_{i}^{\alpha N} C_C(i), \qquad (4.2)$$

where $C_C(i)$ is an array of total energy deposited in cluster *i* and it is ordered by ascending energy, *N* is the total number of cluster of the tracks crossing the TPC and α is the truncation



Figure 4.15: The dE/dx resolution as a function of truncation factor α for horizotal tracks with a magnetic field of 0.2 T and electronics peaking time of 200 ns.

fraction. Normally, the charge distribution follows the Landau distribution which has the long tail because of fluctuations in the ionization processes. These high charge would reduce the relative precision on the mean value and hence the ability to separate various types of particles. The charge distribution in Fig. 4.14 was made by using many tracks with the same energy and detector set up. In principle, if there is enough statistic, one can fit this charge distribution with a Landau fit and then estimate the "mean" of the charge by the most probable value of Landau distribution. However, since the TPC prototype size is limited, there are maximum 36 clusters for each track; hence, they can not form a nice Landau distribution. To calculate the mean charge of each track with less bias, the truncation turns out to be a useful method to cut the long tails of the charge distribution, which will result in a better ionization resolution and a Gaussian distribution of C_T .

Fig. 4.15 shows the dependency of the dE/dx resolution on the truncation fraction. The best resolution is achieved when the truncation fraction is between 50% and 80%. As a result, a truncation factor of 65% is applied for all of the figures provided in this section.

Dependence of the dE/dx resolution on the number of clusters

The prototype used for this test beam includes only one ERAM module which provides maximum 36 clusters for the analysis. In the HA-TPC that will be built for ND280, there are 8 ERAM modules in each readout plane. The majority of the tracks will pass through two ERAM modules, which corresponds to 72 clusters for tracks parallel to the pad plane, before leaving the TPC. The dE/dx resolution as a function of number of clusters can therefore be used to extrapolate the resolution estimated for HA-TPCs. The dE/dx resolution versus the number of clusters is shown in Fig. 4.16.

The deposited energy resolution distribution as a function of the number of clusters N is then fitted with the function: $f(N) = aN^b$, which gives the post-fit parameters' values of $a = 44.36 \pm 0.52$ and $b = -0.414 \pm 0.004$. Notice that the fit range is only from 6 to 35 clusters



Figure 4.16: The dependence of dE/dx resolution on number of cluster.



Figure 4.17: Truncated charge mean for each Micromegas module pad. Figure taken from [149].

because there is fluctuation for the dE/dx resolution with a small number of clusters. With these post-fit parameters, the expected dE/dx resolution for tracks that cross the 2 ERAM modules with 72 clusters is around 7%.

Micromegas gain uniformity

The Micromegas gain and its uniformity are critical factors for PID in the TPC. In the CERN test beam [149], the resistive Micromegas module's gain was examined using a ⁵⁵Fe source producing 5.9 keV gammas and placed in the cathode's center. The gain uniformity was measured using cosmic rays. The result of the gain uniformity test is shown in Fig. 4.17. The gain uniformity is better than 3% except for the pads at the Micromegas's edges.

The cosmic rays are the perfect candidate to test the gain uniformity between the pads since they pass through all the pads of the Micromegas module. There is another way to test the gain uniformity not between the pads but between the clusters by using the test beam. The drawback of using a test beam is that not all the pads can be tested; however, its advantage is



Figure 4.18: The charge distribution for the cluster on the edge (left) and the middle (right) of the Micromegas fitted with the Landau function.



Figure 4.19: The charge gain by each cluster.

having stable tracks in terms of direction and energy.

In this analysis, the detector setup is without the magnetic field, and the electronic peaking time is 412 ns. The horizontal tracks passing the whole 36 clusters were selected. The charge distribution in each cluster is fitted with the Landau distribution (i.e., no truncation is applied in this analysis). Fig. 4.18 shows charge distribution of two out of 36 clusters. The charge of the cluster is then estimated by the most probable value of the Landau distribution. The gain by each cluster is shown in Fig. 4.19. The gain uniformity is good between clusters except for the cluster on the edge of the Micromegas module, which is in agreement with the results of the CERN test beam (see Fig. 4.17). This is because fewer neighbouring leading pads contribute to charge spreading on the measured clusters.



Figure 4.20: The dE/dx resolution versus the track's angle.

Dependence of dE/dx resolution on the angle of the tracks with respect to the pads sides

The dE/dx resolution is stable with the horizontal tracks, which are mostly used in the analysis. However, final state particles in neutrino interaction at the ND280 have a wide range of angles. It is, therefore, crucial to check the stability of dE/dx resolution for tracks with different angles. The charge spreading is isotropic, independent of the track's angle. The detector's performance for dE/dx resolution with different angle tracks depends more on how we reconstruct the charge.

As mentioned in Sec. 4.3.1, various clustering techniques are developed to reconstruct inclined tracks. In this analysis, the simplest cluster pattern was used: the column or the row cluster depending on the track's angle with respect to the pad sides (ϕ). If $\phi < 45^{\circ}$, the tracks are more likely the horizontal ones, and the column cluster pattern is used, while the row one is applied for tracks with $\phi > 45^{\circ}$. Fig. 4.20 shows the dependence of dE/dx resolution on the track's angle. The charge deposited in a unit of length inside the TPC is the same for all track directions. Hence, the dE/dx resolution is expected to be independent of the track's angle. The result at the angle of 40° is worse due to the limitation of the cluster pattern method used in this analysis. A diagonal one (see Fig. 4.12.b) is expected to give better resolution at this angle. The other point which is not good is at 80°. This is not because of the limitation of the cluster pattern method but due to the limitation on the statistic at this angle when many tracks could not pass my selection.

The studies for effects from other parameters

The TPC is expected to perform well under different sets up. In this analysis, particularly, the dE/dx resolution was tested using different electronic peaking time, drift distance and magnetic field.

Fig. 4.21 shows the dependence of dE/dx resolution on the parameters mentioned above. The resolutions for data taken with a magnetic field equal to 0.2 T are clearly better than those without a magnetic field. This result comes from the fact that the magnetic field help to make



Figure 4.21: The dE/dx resolution versus the relative position of the tracks with respect to the anode. The z-axis used in this plot is perpendicular to the anode (readout) plan. z=410 mm and z=550 mm correspond to the tracks very close to the anode and cathode, respectively.

less diffusion when drifting the electron to the anode. Apart from the magnetic field, the longer peaking time (412 ns) also gives resolutions slightly better than those coming from 200 ns. In general, the drift distance does not affect the deposited energy resolution much.

4.3.4 Drift velocity studies

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There are two ways to compute the drift velocity of electrons inside the field cage. The first one employs the arrival time of cosmic tracks passing the cathode or anode on the Micromegas. With the known drift distance from the size of the TPC, one can extract the drift velocity. This technique was already applied in the CERN test beam [149]. The second one is using the test beams at different drift distances. One can obtain the drift velocity by comparing the arrival time of different drift distance beams.

Fig. 4.22 shows the arrival time with respect to the drift distance for different electric fields (E). The stronger the electric field is, the faster the drifted electrons reach the readout. The other things which can be extracted from this figure are the $t_0 \sim 4.56 \ \mu$ s the anode position $(z_0 \sim 411.5 \text{ mm})$. t_0 is the arrival time of the electrons produced at the crossed point between the track and the anode. The drift velocities corresponding to different electric fields are calculated and also shown in Fig. 4.22.

4.3.5 Spatial Resolution

The spatial resolution indicates how well a track position is determined in a pad. The Center of Charge (CoC) or barycenter technique is a simple method to reconstruct the track position. This approach involves weighting the positions of the centers of each pad (x_{pad}) in a cluster by the proportion of the cluster charge deposited in each pad before summing to get the track



Figure 4.22: The arrival time versus the drift distance for various electric fields (E). The data points from the same electric field are linearly fit.

position (x_{track}) :

$$x_{track} = \frac{\sum (x_{pad}Q_{pad})}{\sum Q_{pad}} \tag{4.3}$$

where Q_{pad} is the charge deposited in the considered pad, this method assumes that the charge density in the spreading area is homogeneous. However, the charge measurements are discontinuous in every finite pad size, but the charge in the RC layer spreads continuously. Furthermore, the signal created in the neighbouring pads is not the consequence of a true spreading of an initial charge, and it is detected at separate times, even if it is correlated to the charge detected in the leading pad. As a result, the barycentric approach does not produce an accurate position reconstruction. A novel technique for detecting track position was presented in [151], and it gave enhanced spatial resolution. The technique entails applying the so-called "pad response function" (PFR), which describes the relation between deposited charge ratios and track position with respect to the pad center:

$$PRF(x_{track} - x_{pad}) = \frac{Q_{pad}}{Q_{cluster}}$$

$$\tag{4.4}$$

where x_{track} is the reconstructed track position, x_{pad} is the pad center, Q_{pad} is the charge measured on a specific pad, and $Q_{cluster}$ is the charge measured on the whole cluster. The PRF may be represented analytically [152] by considering the ratio of two 4-th degree polynomials:

$$PRF(x,\Gamma,\Delta,a,b) = \frac{1+a_2x^2+a_4x^4}{1+b_2x^2+b_4x^4}$$
(4.5)

where a_i and b_i can be expressed in terms of the more physical factors: full width at half maximum (Γ), base width (Δ) of the PRF, and two scaling parameters a and b. The track's position is then determined by minimizing the

$$\chi^2 = \sum_{pads} \frac{Q_{pad}/Q_{cluster} - PRF(x_{track} - x_{pad})}{\sigma_{Q_{pad}/Q_{cluster}}}$$
(4.6)

where σ_Q is the charge measurements' error. Assuming Poisson distribution is the charge measurement probability distribution, then $\sigma_{Q_{pad}/Q_{cluster}} = \sqrt{Q_{pad}/Q_{cluster}}$. Minimizing this χ^2 is an iterative method. We utilize the track position reconstruction acquired via the barycentric approach to derive an initial estimation of the PRF parameters. After all the clusters' positions have been determined, the entire track is fitted with a parabola. This result is considered as a true track position since it is fitted with many measurements taken along the track (≥ 34). The scatter plot is filled using this assumed true track position and is shown in Fig. 4.23(a). From this plot, $Q_{pad}/Q_{cluster}$ is extracted for each bin as the mean, and the uncertainty is the full width at half maximum for every $x_{track} - x_{pad}$ bin. This extraction forms a graph which is fitted with the analytical function in Eq. 4.5 and shown in Fig. 4.23(b).



Figure 4.23: The Pad Response Function (PRF) obtained with (a) scatter plot and (b) results of its profile and fit with analytical function. Vertical lines represent the pad borders. From [148]

Physics studies for ND280 upgrade

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5.1 Introduction

Neutrino oscillation physics has now entered the precision era. While the statistic demands for neutrino oscillation experiments are becoming enormous, it is essential to upgrade the beam which is currently used in T2K experiment and to build even more giant detectors such as Hyper Kamiokande [49], DUNE [48]. In parallel with needing larger detectors to collect more data with, future experiments further require a significant reduction of systematic uncertainties with respect to what is currently available. In the neutrino oscillation measurements from the T2K experiment the systematic uncertainties related to neutrino interaction cross sections are currently the most dominant. To reduce this uncertainty a much improved understanding of neutrino-nucleus interactions is required.

Hence, it is crucial to better understand the nuclear effects which can alter the final state topology and kinematics of neutrino interactions in such a way which can bias neutrino energy reconstruction and therefore bias measurements of neutrino oscillations. These nuclear effects contain the effects from nucleon removal energy (E_{rmv}), Short Range Correlation (SRC), or Final State Interaction (FSI) which are described in Sec. 2.3. The T2K far detector SuperK selects single ring events. These events are mostly populated by the Charge Current Quasi-Elastic (CCQE) events and T2K collaboration uses the CCQE kinematic to reconstruct the neutrino energy. The very well known reconstructed neutrino energy formula is of the form

$$E_{QE} = \frac{m_p^2 - m_\mu^2 - (m_n - E_{rmv})^2 + 2E_\mu(m_n - E_{rmv})}{2(m_n - E_{rmv} - E_\mu + p_\mu^z)},$$
(5.1)

where the p, n, and μ stand for proton, neutron and muon, respectively. This reconstructed formula works well for single ring events in SK since it relied on the muon kinematic only. Unfortunately, there are many non-QE interaction events and they have more hadrons in the final state, but sometimes this hadronic part is below Cherenkov threshold and hence not reconstructed. These modes could mimic CCQE events and then the energy reconstruction will give biases in these cases. That is why the near detector ND280 needs to constrain the QE and non-QE interactions carefully.

The upgraded ND280 near detector of T2K will directly confront our naivety of neutrino interactions using a new detector configuration with full polar angle acceptance and a much lower proton tracking threshold shown in Fig. 5.1. With a high-energy neutrino beam, the final state particles are more likely to go forward which are well reconstructed with ND280. However, a lower energy neutrino beam is needed to see more significant effects from nuclear. In this case, the direction of the outgoing particles is more isotropic, and a better angular coverage is required. Therefore the improvement in the polar angle acceptance of upgraded ND280 is vital to study neutrino-nucleus interaction.

Furthermore, neutron tagging capabilities in addition to precision timing information will allow the upgraded detector to estimate neutron kinematics from neutrino interactions [153]. Fig. 5.2 demonstrates how well the upgraded ND280 can reconstruct neutrons using time of flight technique.

Such improvements in hadrons detection permit access to a much larger kinematic phase space which correspondingly allows techniques such as the analysis of transverse kinematic imbalances (TKI) to offer remarkable constraints of the pertinent nuclear physics for T2K analyses. The goal of this chapter is to provide a quantitative estimate of ND280 Upgrade's sensitivity to the most important sources of systematic uncertainty in T2K and Hyper-K oscillation analyses, specifically through its ability to accurately reconstruct nucleons alongside



Figure 5.1: proton reconstruction efficiency as a function of the truth proton momentum. Plot taken from [144].

the charged lepton in charged-current interactions with no mesons in the final state (the primary interaction topology for T2K and Hyper-K). These sensitivities are presented as a function of accumulated statistics, covering both the T2K and Hyper-K eras and given by the number of protons impinging on the target (POT) in the neutrino beamline. For reference, T2K's latest analysis used $1.49 \ (1.64) \times 10^{21}$ POT of neutrino (anti-neutrino) data [154] and will have an ultimate total exposure up to 10×10^{21} POT, whilst the Hyper-K design report [155] considered a total of 27×10^{21} POT (in a 1:3 neutrino over anti-neutrino ratio), corresponding to 10 years of data taking. Sec. 5.2 introduces the Single-transverse variables (STVs) and how they can probe the nuclear effects in neutrino-nucleus interaction. The analysis method is presented in Sec. 5.3 and Sec. 5.4, where the simulation and the reconstruction are described, followed by a discussed in Sec. 5.5 before conclusions are drawn in Sec. 5.6. The results in this chapter come from the paper [156] to which I contributed as the main analyser.

5.2 Single-transverse kinematic imbalance to probe the nuclear effects in neutrino-nucleus interaction

The investigation of neutrino interactions brings novel obstacles. The energy of a neutrino, unlike that of its electroweak charged lepton counterpart, is often challenging to measure and it is crucial in neutrino oscillation measurements. Hence, modern accelerator technologies need to define very well the beam direction as well as energy spectrum of neutrino. In the previous chapter, we learned that the nuclear effects could significantly bias of the reconstructed neutrino energy, topology, final state kinematic, etc.

In T2K, we use only the lepton kinematics in oscillation analysis. Measuring the outgoing lepton kinematics in a $CC0\pi$ topology is a good way for further neutrino energy reconstruction. But it uses only a part of the final state particles. There is also much information coming



Figure 5.2: Neutron reconstruction efficiency as a function of the truth neutron momentum and neutron direction using the upgraded ND280. Plot taken from [153].

from the hadronic part apart from the leptonic one. In the absence of nuclear effects, some equilibrium between the outgoing lepton and outgoing hadron kinematics is predicted. All CCQE interaction imbalances found would seem to be a direct result of nuclear processes. In recent years, many studies have used the transverse kinematic imbalance (TKI) between the final state lepton and hadron to probe the nuclear effects. The approach outlined in this section, in particular, is confined to a $CC0\pi$ topology and the single transverse kinematic imbalance (STKI) between outgoing lepton and the highest momentum nucleon in the plane perpendicular to the incoming neutrino direction. In this transverse plane, a collection of observables will be defined to investigate the described imbalance effectively. The reason for employing STKI to examine nuclear effects has been that it functions as a potent tool to test the nuclear models and the strength of FSI for some exclusive interaction modes.

5.2.1 The single transverse variables (STV)

Let us take the example of a CC interaction on a nucleus. From the most fundamental level, the neutrino ν interacts with a confined nucleon N which is always subject to Fermi motion \vec{p}_N in the rest frame of the considered nucleon. After the interaction, the initial nucleon subsequently transitions to another nucleon state N' and there is also a charged lepton l' in the final state:

$$\nu + \mathcal{N} \to l' + \mathcal{N}'. \tag{5.2}$$

In the case of a CCQE neutrino interaction on a nucleus without nuclear effects (i.e. there are only one lepton and one nucleon in the final state and their momenta are conserved after interaction vertex), the projected momentum of an outgoing lepton on the plane transverse to an incoming neutrino ¹ is precisely equal to and opposite to that of an outgoing nucleon. In the case of having additional nuclear effects, which results in STKI, these transverse momenta are not symmetrical anymore. The combination of three observables known as "Single Transverse variables" (STV) has been used to describe this imbalance completely.

¹Conventionally, I will use the transverse plane to denote the plane transverse to the incoming neutrino.



Figure 5.3: A schematic illustration showing the definition of the variables describing transverse kinematic imbalance employed within this work. The incoming neutrino momentum vector is shown in black (defined to be along the z-axis) whilst the outgoing lepton (ℓ') and highest momentum nucleon (N') momentum vectors are shown as solid blue lines whilst their projection onto the x-y plane, transverse to the incoming neutrino, are shown as dashed lines. The resultant triangle formed from these transverse projections defines the two variables considered within this work: $\delta \alpha_T$ and δp_T (the magnitude of $\delta \vec{p}_T$ shown in the figure). This schematic illustration is taken from [157].

In order to perform measurements of nuclear effects, the energy-momentum transfer inside the nucleus medium turns out to be the perfect observable. Unfortunately, these observables can not be precisely measured by experiments since the initial nucleon momentum and neutrino energy are undetermined. Alternatively, the momentum transfer may very well be effectively established from the STV, which are quantitatively of the forms

$$\delta \vec{p}_T \equiv \vec{p}_T^{\ \ell'} + \vec{p}_T^{N'},\tag{5.3}$$

$$\delta \alpha_T \equiv \arccos \frac{-\vec{p}_T^{\ \ell'} . \delta \vec{p}_T}{\vec{p}_T^{\ \ell'} \delta \vec{p}_T},\tag{5.4}$$

$$\delta\phi_T \equiv \arccos \frac{-\vec{p}_T^{\ \ell'} \cdot \vec{p}_T^{N'}}{\vec{p}_T^{\ \ell'} \vec{p}_T^{N'}}.$$
(5.5)

where $\vec{p}_T^{\ell'}$ and $\vec{p}_T^{N'}$ are the projections of the outgoing lepton and outgoing nucleon on the transverse plane. Figure 5.3 depicts the schematic formulation of these variables from the kinematics of the lepton and nucleon in the final state. The vector $\vec{q}_T = -\vec{p}_T^{\ell'}$ is the transverse component of the momentum \vec{q} carried by a virtual propagator W-boson.



Figure 5.4: The true δp_T distribution for several interaction modes. These events were generated by NEUT generator [159] for neutrino-Carbon CCO π interaction.

Following the Eq. 5.3, the total momentum imbalance in the transverse plane is neutrino energy independent and denoted by δp_T (transverse momentum imbalance). This variable's value would be zero if the initial-state nucleon remained immobile and unbound due to momentum conservation. Smaller δp_T characterizes more horizontally balanced collisions. In reality, even with the case of hydrogen as a target with no nuclear effects, the reconstructed δp_T value can not be exactly zero due to the detector effects and performance. Imagine if there was no FSI, δp_T would be the transverse component of Fermi momentum \vec{p}_N . Apart from the nuclear effects, the other interaction topology, which is not QE, can also make a transverse imbalance system. Hence, δp_T is a good tool to separate the QE and non QE interaction. For example, the NOMAD experiment exploited the transverse momentum imbalance δp_T to improve the quality of the chosen QE sample [158]. Fig. 5.4 illustrates the reconstructed δp_T distribution for several interaction modes. Most of the QE events contribute to the peak at around 200 MeV/c, which is the Fermi momentum value; while the non-QE events dominate the tail of δp_T distribution.

The transverse boosting angle $\delta \alpha_T$ is proposed in [158] for the first time. According to the hypothesis of not having FSI, the $\delta \alpha_T$ would be the angle between \vec{p}_N transverse component and \vec{q}_T . Since the Fermi motion is isotropic, the $\delta \alpha_T$ distribution would be rather flat. The muon particle, which is the primary lepton produced in ν_{μ} CC interaction, is negligibly affected by the FSI. The FSI, on the other hand, generally slows down the outgoing nucleons, resulting in $\delta \alpha_T > \pi/2$. Consequently, the curve of $\delta \alpha_T$ can be used to determine the strength of FSI and the $\delta \alpha_T$ value itself can also be used to categorize a process as 'accelerating' or 'decelerating' on the hadronic system. With a detector that has low momentum threshold enough to measure low p_T' , the Pauli blocking can be constrained by the dependence of $\delta \alpha_T$ on p_T' . Fig. 5.5 shows the $\delta \alpha_T$ distribution for several interaction topologies.

The $\delta \phi_T$ calculates the deflection of N' in the transverse plane with respect to \vec{q} . $\delta \phi_T$ would really be zero if the system is highly balanced where the preliminary nucleon was unvarying and unbound; adding nuclear effects, the deflection, which is generated by the momentum transfer to the medium, adds a spreading to the original distribution of $\delta \phi_T$. This deflection is



Figure 5.5: The true $\delta \alpha_T$ distribution using events generated by NEUT for neutrino-Carbon $CC0\pi$ interaction. Left: The reconstructed $\delta \alpha_T$ with different kinds of interaction topology.

dictated by Fermi momentum \vec{p}_N . However, the trigonometric relationship presented in Fig. 5.3 demonstrates that $\delta \phi_T$ increases with δp_T and decreases with $p_T^{l'}$. Hence, $\delta \phi_T$ is expected to be sensitive to neutrino energy since it depends on the lepton kinematics, which also strongly correlates with neutrino energy. Figure 5.6 demonstrates the variation of $\delta \phi_T$ distribution at various neutrino energies by NuWro event generator [160].

NOMAD, MINER ν A and INGRID (the on-axis near detector of T2K experiment) have previously used $\delta\phi_T$ as a selection variable for QE event classifiers [158, 161, 162]. It is inevitable that δp_T gives more details about the transverse imbalance than $\delta\phi_T$. Many experiments still use $\delta\phi_T$ because it just needs the direction of outgoing particles to reconstruct, making it more approachable and allowing for greater resolution reconstruction.

5.2.2 Probing nuclear effects with STV

Fermi motion

As already mentioned, in the case of neutrino-nucleon scattering, where the nucleon stays at rest, the transverse kinematic of all particles in the final state should reach balance (i.e. $\delta p_T = \delta \phi_T = 0$, $\delta \alpha_T$ is undetermined). In a nuclear system, nucleons experience arbitrary motion in direction and up to $\mathcal{O}(10^2)$ MeV in the magnitude of Fermi momentum, which gives the interacting process an unpredictable event-by-event boost.

In the case of QE interaction without the FSI process, the observed momentum imbalance in the transverse plane is caused solely by the Fermi momentum of the hit nucleon. As a result, the Fermi motion's transverse component is the only ingredient that defines the shape of the δp_T distribution. In other words, δp_T is the projection of Fermi momentum on the transverse plane. Fig. 5.7 shows the δp_T and $\delta \phi_T$ distribution for three different nuclear models without FSI. Obviously, δp_T gives a significant distinction between the RFG and the other models. The $\delta \phi_T$ distribution gets considerable width as a result of the unidentified boost by Fermi momentum;



Figure 5.6: The probability density function of $\delta\phi_T$ by NuWro. τ_f quantifies the FSI strength and it is neutrino energy-dependent. The higher neutrino energy gives stronger FSI effects. In contrast, the $\delta\phi_T$ distribution narrows at higher neutrino energies due to an enhancement in $p_T^{l'}$. Figure taken from [157].

the distribution is highly peaked at $\delta \phi_T = 0$.



Figure 5.7: The true distribution δp_T (left) and $\delta \phi_T$ (right) for three different models generated by NuWro for CC interaction using NuMI flux without FSI. Plots taken from [163].

Final state interaction

FSI is one of the main processes that can bias the final state kinematic and topology. It primarily influences the outgoing hadronic part before going out of the nucleus medium. There are many kinds of FSI processes, such as elastic or inelastic re-interaction, hadron absorption or creation, and charge exchange..., which were discussed in detail in Sec. 2.3.4. As a result, the FSI effects are likely to break the transverse kinematic balance and enhance the degree of imbalance more than that caused by Fermi motion. This means that the transverse momentum and angular imbalance are more significant, as seen in Fig. 5.8 where the tails of δp_T and $\delta \phi_T$ are longer than those without FSI.

Fig. 5.8 illustrates the prediction of δp_T and $\delta \phi_T$ distribution of three different nuclear

models. The δp_T distribution is fully dominated by the Fermi motion for δp_T less than the Fermi momentum, k_f . For $\delta p_T > 2k_f$, the distributions are almost independent of the nucleon's initial state, but they strongly depend on the characteristic of the FSI model. The broadening of $\delta \phi_T$, which depends on the neutrino energy and the Fermi momentum, has increased with the FSI effects.



Figure 5.8: The true distribution δp_T (left) and $\delta \phi_T$ (right) for three different models (Local Fermi Gas, Relativistic Fermi Gas, Spectral Function) generated by NEUT for neutrino-Carbon $CC0\pi + 1p$ interaction using T2K flux.

Although there are dependencies of δp_T and $\delta \phi_T$ on the FSI, they are not ideal parameters to select FSI events. This job is better done by $\delta \alpha_T$ since the FSI effects can enhance the $\delta \alpha_T > \pi/2$ part of the distribution. Fig. 5.9 show how much the shape of true $\delta \alpha_T$ distribution changes under the FSI effects when its distribution is separated by events with FSI and those without FSI. In general, FSI processes transfer energy from the hadronic system's vertex to the nuclear medium. It demonstrates that FSIs generate an accumulation of events at $\delta \alpha_T \sim 180^\circ$, which is typical of a decelerated hadronic system. It clearly proves that the $\delta \alpha_T$ may not be an ideal variable to separate QE from non-QE events; however, it is sensitive to the FSI effects.

Pauli blocking

By reducing the phase space of final state nucleon emission, Pauli blocking is able to minimize the interaction cross-section. It is prohibited in quantum mechanics to have two particles in the same quantum state. As a result, there exist some forbidden transferred energy values where the final state nucleon shares the same quantum state with another nucleon. In the RFG, the limitation for Pauli blocking to occur is below Fermi momentum k_F for any final state nucleon. As mentioned before, the low final state nucleon momentum corresponds to the high value of $\delta \alpha_T$. Figure 5.10 demonstrates that Pauli blocking enables the event rate to be suppressed at high $\delta \alpha_T$ and low outgoing lepton momentum $(p_T^{\mu} < k_F)$.

5.3 Simulation

Many subdetectors for the upgraded ND280 such as HA-TPC and SFGD are under testing and development. Hence the complete simulation of the ND280 upgrade is not available at the



Figure 5.9: Comparison of true $\delta \alpha_T$ distribution between FSI and non-FSI interaction using events generated by NEUT for neutrino-Carbon $CC0\pi$ interaction.



Figure 5.10: The prediction of QE event rate as a function of outgoing lepton momentum and $\delta \alpha_T$ for neutrino-Carbon interaction by NuWro without the FSI. The distributions are shown without Pauli blocking (left) and with Pauli blocking (right). Plots taken from [163].

present time because of the lack of precise detector performances and configuration. However, a preliminary simulation was done based on the estimated detector performances coming from many test-beams at CERN [149, 164], LANL and DESY [148]. This data has allowed the development of a preliminary Super-FGD detector simulation and reconstruction framework. From this early simulation, the leading detector effects (smearing and resolution) have been parameterised and applied to a sample of neutrino interactions generated with version 5.4.0 of the NEUT simulation [159] on a hydrocarbon scintillator (CH) target using the T2K flux prediction [165, 166]

The Super-FGD plays a vital role in the identification and reconstruction of the tracks. It can effectively reconstruct not only the leptonic part but also the hadronic part (protons, pions and neutrons) that do not arrive to the HA-TPC. This feature is crucial since these studies highly rely on both leptonic and hadronic parts to exploit the transverse variables. Within the reconstruction framework, charge deposits in the Super-FGD are identified and processed to form tracks. A small region very close to the interaction vertex is willingly neglected in order to remove the impact of energy deposition from other interaction products in the vicinity of the target nucleus. For contained tracks, the charge deposits along most of the track's length are considered to identify the particle energy loss over the whole track range, permitting both momentum reconstruction and particle identification (PID). Tracks with an ambiguous PID and those which undergo apparent secondary interactions (i.e. from an observed deflection of peak in the charge deposits) are rejected². It should be noted that the non-uniformity of energy deposits due to the position of particles within a scintillator cube is not yet modelled (which could worsen the relative momentum resolution for short tracks by ~10%), but it has been ensured that such changes in resolution would not significantly affect the results presented here. Tracks which are not fully contained and enter a TPC are split into two parts: the Super-FGD segment is reconstructed as described above and the TPC segment undergoes a parameterised reconstruction based on known performances of the current T2K TPC [136]. The improved performance expected from the HA-TPCs is not yet taken into account (meaning the detector performance for the TPC segments of high angle tracks is expected to be slightly underestimated).

The input NEUT (version 5.4.0) simulation uses the Spectral Function model [167] for Charged-Current Quasi-Elastic (CCQE) interactions, the Valencia model for a multi-nucleon meson exchange current contribution [110] and the Rein-Seghal model [168] for single-pion production. The Deep-Inelastic-Scattering channel is simulated using the GRV98 [169] parton distribution functions with Bodek-Yang corrections [170] for the cross section, whilst hadron production is modelled by either PYTHIA 5.72 [171] or a custom model based on KNO scaling (see the presentation in Sec. V.C of Ref. [172]) for interactions with a hadronic invariant mass above and below 2 GeV respectively. It should be noted that the CCQE model contains both a mean-field and a "short range correlation" (SRC) component, the latter of which produces two outgoing nucleons. Final state interactions (FSI) of hadrons are described using cascade models tuned to hadron-nucleus scattering data [159, 173]. This is the same neutrino interaction model that is used in the last T2K oscillation analysis and described in details in Ref. [154]. NUISANCE [174] is used to process the NEUT output. In total 6 million neutrino interactions and 2 million anti-neutrino interactions are simulated, corresponding to an estimated 3.0×10^{22} and 4.5×10^{22} POT respectively. Moreover, in order to study the effects of FSI and removal energy on the upgraded ND280, 1 million events were simulated for each of the following modification: strong FSI, weak FSI, high E_{rmv} , low E_{rmv} in both neutrino and anti-neutrino interaction. The simulation is scaled to test sensitivity as a function of accumulated statistics assuming 1.9 tons of mass in the Super-FGD fiducial volume.

The parameterisation of detector effects is applied for protons, neutrons, muons and charged pions on a particle-by-particle basis. For charged particles a Gaussian momentum and angular smearing is applied alongside a probability to not reconstruct the particle (to model inefficiencies). These response functions are applied based on a particle's type and as a function of true momentum and direction. Neutron resolutions and efficiencies are also applied and are handled as described in Ref. [153]. In this analysis no cut is made on the distance the neutron travels from the interaction vertex (i.e. no "lever-arm" cut on the neutron propagation distance is applied), which increases neutrino detection efficiency but also degrades the momentum resolution. The modelled detector performance is summarised in Fig. 5.11 and Fig. 5.12, which describes the momentum resolution and selection efficiency for muons, protons and neutrons.

²Further reconstruction efforts will aim to recover some of this lost selection efficiency.



Figure 5.11: Top Left: the momentum resolution of muons, protons and neutrons as a function of their respective momenta. The neutron momentum resolution ranges from ~15% to 30% and is discussed in more detail in Ref. [153]. Top Right: the zoom of protons and muons momentum resolution. Bottom: the angular resolution as a function of $\cos(\theta)$ where θ is the angle between outgoing particles and incoming neutrino. It is clear that the forward and backward particles have better angle resolution.

The decrease in proton selection efficiency after 500 MeV/c is largely from track rejection due to identified secondary interactions, but at higher momentum (> 1 GeV/c) track rejection from ambiguous PID also plays a role. The parabolic shape of the resolutions stems from difficulties in reconstructing very short tracks, followed by peak performance for fully contained tracks, while higher momentum tracks reach the TPCs with relative resolution worsening at higher momentum (as expected due to the smaller curvature in the magnetic field).



Figure 5.12: The efficiency to detect protons, muons and neutrons as a function of their true momentum estimated by upgraded ND280.

Table 5.1 shows the number of reconstructed events in a sample of Charged-Current interactions without reconstructed pions in the final state (CC0 π) and with at least one reconstructed proton/neutron for neutrino/anti-neutrino interactions. In this study we consider only these CC0 π selections, which is the dominant interaction topology for T2K oscillation analyses. Fig. 5.13 shows the reconstructed momentum and reconstructed $\cos\theta$ distributions by upgraded ND280 for muons, neutrons and protons with the same selection sample as in Table 5.1. For reference, the current ND280 would expect to select ~76,000 neutrino interactions with at least one reconstructed proton in the final state for 1 × 10²² POT [175] and has not been shown to be able to reconstruct neutrons. It should be noted that this simple parameterised approach can

Channel	Neutrino events	Anti-neutrino events
CCQE	299 822	109 750
2p2h	$63 \ 010$	$19\ 116$
Pion absorption	48 966	$6\ 714$
Undetected pions	36 200	5 828

Table 5.1: Number of reconstructed events in $CC0\pi$ channels for 1×10^{22} Protons on Target in neutrino mode and 1×10^{22} in anti-neutrino mode.

not account for all the physics of a full reconstruction and event selection. Most importantly neutral pions are assumed to be always rejected and the possibility of misidentifying one particle type as another is not considered beyond the aforementioned impact on the selection efficiency. Whilst these are important limitations, in general they are subdominant effects for the $CC0\pi$



Figure 5.13: The muons, neutron and protons reconstructed momentum distribution (left) and reconstructed $\cos\theta$ distribution (right) for $CC0\pi$ interaction for 1×10^{22} POT. Neutron can be reconstructed down to very low momentum threshold compared to proton through its time of flight. The events for muons and protons are taken from neutrino interaction while the one for neutron is taken from anti-neutrino interaction, which results in the lower statistic for neutron. The forward neutron suppression on the right plot adhere the momentum conservation resulting from the helicity differences of neutrinos and anti-neutrinos.

selections considered in this paper. Current ND280 CC0 π selections with a reconstructed proton in the final state have around 20% background fully dominated by undetected pions, only a small minority of this background stems from a mis-identification of particle type or from an undetected neutral pion [154, 175].

5.4 Analysis strategy

The T2K oscillation analysis (see e.g. Ref. [154]) relies on a fit of ND280 data to constrain the neutrino flux and neutrino interaction cross sections. In T2K analyses the primary ND280 samples are split depending on whether zero, one or more charged pions are reconstructed in the final state. The distribution of muon kinematics (momentum and angle) in each sample is then compared to a simulated model. The uncertainties on the simulation, related to flux, detector response and neutrino interaction modelling, are parametrised before a likelihood fit is used to provide constraints from the data. The results of applying the same analysis to a sample of events simulated in ND280 Upgrade are shown in Ref. [145]. In this paper a similar strategy is applied for the CC0 π channel only, but now fitting data presented as a function of kinematic variables including lepton and hadron kinematics (protons and neutrons for the CC0 π channel) which better exploits the performance assets of the Super-FGD.

Variables describing the hadronic part of the final state have been investigated in neutrino cross-section measurements at ND280 [175], and other experiments (e.g. MINERvA [176]). These include measurements of proton multiplicity and kinematics as well as of correlations between outgoing protons and muons [157, 177], which are particularly sensitive to the nuclear effects that often drive the dominant uncertainties in neutrino oscillation analyses [178]. The improved performances of ND280 Upgrade in the measurements of such variables are shown

in Ref. [145]. In this analysis we perform a fit to these variables to estimate the sensitivity of ND280 Upgrade to constrain flux and neutrino-interaction systematic uncertainties for future oscillation analyses. As detailed in the following section, the "transverse boosting angle" ($\delta \alpha_T$) and the magnitude of the transverse momentum imbalance between the outgoing lepton and nucleon (δp_T) are chosen for this work, which are defined schematically in Fig. 5.3. These variables analyse the size and direction of the momentum imbalance between the outgoing lepton and nucleon on the plane transverse to the incoming neutrino beam [157].

It should be highlighted that the optimal analysis with ND280 Upgrade would be a multidimensional fit including the muon kinematics, the hadronic kinematics and their correlations. The tools to implement such an approach are under development within T2K. In this paper the study is limited to fitting pairs of variables at once, focusing only on those which exploit the lepton-nucleon correlations. The results are therefore highlighting only the additional sensitivity that such new variables could provide, with respect to the existing T2K analyses.

5.4.1 Fit variables

Currently, the T2K collaboration uses only the lepton kinematic for the neutrino energy reconstruction. With the help of upgraded ND280, nucleon kinematics are better measured and could be used for new neutrino energy reconstruction methods. Nevertheless, before taking them into account, one should clearly understand which process biases the nucleon kinematics. That requires a better-described nucleus model, neutrino-nucleus interaction model and a strategy to keep the nucleon FSI under control. For the nucleus model, in 2021, the T2K collaboration has chosen SF as the nucleus model for its neutrino events generator NEUT. There are many active fake data studies in T2K collaboration for different neutrino-nucleus interaction models. The final goal aims to see how it affects the constraints on neutrino oscillation parameters when we change the fake data based on the currently available neutrino-nucleus interaction models. In the next chapter, one of these fake data studies which is for the Martini et al. interaction model will be introduced. For the nucleon FSI, in future oscillation analyses, this will act as a new nuisance uncertainty that must be understood to propagate constraints on other relevant nuclear effects from the near to far detectors.

In general, the variables used for the fitter should be able to well characterise the nuclear effects as well as separate CCQE and non-QE interaction. One well-known method to investigate the nuclear effects is using the correlation between outgoing lepton and highest momentum outgoing nucleon called the Single Transverse Kinematic Imbalance (STKI) [157] which is well described in Sec. 5.2. The schematic of the STKI is illustrated in Fig.5.3. In parallel with STVs, two other variables will be used in this study which are visible energy (E_{vis}) and the so-called "reconstructed Fermi momentum" p_N ³. They are also served as good neutrino energy estimator and good CCQE separation tool, respectively. This section will describe in details all of the advantages and utilisation of these variables for this analysis.

 $^{{}^{3}}p_{N}$ is calculated based on the final state momentum imbalance dominated by the Fermi motion. However, the FSI also contribute to this imbalance. Hence p_{N} does not solely represent Fermi momentum. Note that a subscript "N" (denoting nucleon) is used rather than "n" (for neutron) since in these studies the variable can be calculated for neutrino or anti-neutrino interactions corresponding to probes of the initial state neutron or proton respectively.

Some of the most important systematic uncertainties for CCQE interactions in neutrino oscillation analyses are those which cause a bias in estimators for reconstructing neutrino energy. Such a bias directly impacts the measurement of neutrino oscillation parameters, notably Δm_{32}^2 and δ_{CP} precision measurements, especially in the presence of large CP violation. The principle cause of this bias stems from: the modelling of the initial target nucleon momentum within the nucleus (Fermi motion); the contribution of CC-non-QE or SRC processes entering CC0 π samples; and the "removal" energy it takes to liberate target nucleons from the nucleus (the latter was responsible for a dominant systematic uncertainty in T2K's latest neutrino oscillation analysis [154]).

To better investigate the contribution from CCQE, the variables to be used in this anal-



Figure 5.14: δp_T distribution in neutrino (left) and anti-neutrino (right) interaction. The CCQE distribution peaks around 200MeV/c which is close to the Fermi momentum. All of the events are $CC0\pi$ interaction with the 10^{22} POT.



Figure 5.15: p_N distribution in neutrino (left) and anti-neutrino (right) interaction. The CCQE distribution peaks around 200MeV/c which is close to the Fermi momentum. All of the events are $CC0\pi$ interaction with the 10^{22} POT.

ysis are δp_T [157], which is the momentum imbalance in the transverse plane, and p_N , which is

an extension to $\delta p_{\rm T}$ considering also an estimation of the longitudinal momentum imbalance in order to estimate the initial state nucleon momentum [177]:

$$p_N = \sqrt{\delta p_L^2 + \delta p_T^2}.$$
(5.6)

 $\delta p_{\rm T}$ and δp_L are transverse- and longitudinal- momentum imbalance, respectively. The way to calculate $\delta p_{\rm T}$ was already introduced in Eq.5.3, whilst the δp_L is not a transverse variable, and its calculation is of the form

$$\delta p_L = \frac{1}{2}R - \frac{M_{A-1}^2 + \delta p_T^2}{2R},\tag{5.7}$$

where

$$R = M_A + p_L^{\mu} + p_L^N - E^{\mu} - E^N, \qquad (5.8)$$

$$M_{A-1} = M_A - M_N + E_{rmv}.$$
 (5.9)

The N letter indicates the nucleon on which (anti-)neutrino interacts. M_A and M_N stand for the mass of the target nucleus and mass of the nucleon, respectively. The typical removal energy value E_{rmv} used for this analysis is 25 MeV/c. If there are no FSI effects, p_N , δp_T and δp_L are the Fermi motion and its transverse and longitudinal projection, respectively. Hence, δp_T and p_N are expected to well characterise the Fermi motion.

The two variables $\delta p_{\rm T}$ and p_N are also known to well separate CCQE from CC-non-QE in CC0 π samples. In anti-neutrino interactions, these variables can also separate the hydrogen and carbon contributions of the CH scintillator, thereby allowing some lifting of the degeneracy between nucleon and nuclear level effects, whilst also providing the opportunity for improved insitu flux constraints [153]. The reconstructed distribution of δp_T for neutrino and anti-neutrino interactions are shown in Fig. 5.14. The neutrino case demonstrates the clear separation of CCQE in the bulk and CC-non-QE in the tail (the small CCQE contribution to the tail is from SRCs and nucleon FSI). The anti-neutrino distributions are broader than the neutrino ones and do not show such good mode separation (due to the relatively poor neutron momentum resolution compared to the proton). Still, the shape difference between the hydrogen and carbon contributions is clearly visible.

The reconstructed initial state nucleon momentum distribution (p_N) for neutrino and antineutrino interactions are shown in Fig. 5.15, its distribution is very similar with δp_T one. The reconstructed (p_N) [177] can be used as an alternative to the transverse momentum imbalance (δp_T) [157] as an input variable in the fit. The former contains more information than that latter, incorporating an inferred longitudinal imbalance, in addition to the transverse imbalance, and so better sensitivity is in principle expected. However, this better sensitivity accompanies the potential need for additional systematic uncertainties. For example, whilst the shape of δp_T is largely independent from neutrino energy and nucleon-level physics [157], this will not be the case for p_N . I therefore choose the more conservative and simple approach of quoting the primary sensitivities using a fit in δp_T but discuss here what would be obtained using p_N instead. The comparison of p_N and δp_T effects on the sensitivity studies will be shown later in this chapter.


Figure 5.16: Left: $\delta \alpha_T$ distribution where the shaded regions show the generator level $\delta \alpha_T$ distribution for all $CC0\pi$ interactions for 1×10^{22} POT split by whether or not the outgoing nucleon underwent FSI. The overlaid solid lines indicate how the total $\delta \alpha_T$ distribution (including the with and without FSI components) changes when imposing the current ND280 FGD proton tracking threshold (450 MeV/c) and expectation from the Super-FGD (300 MeV/c). **Right:** The ratio of events reconstructed by Super-FGD over those by FGD. The Super-FGD increases the events rate at least twice for all $\delta \alpha_T$ value (thanks to the heavier detector target) and significantly improves reconstruction at the high $\delta \alpha_T$ region (thanks to the better reconstruction of lower hadronic part).

It is unambiguous to find a variable that can well control the FSI effects. The shape of the "transverse boosting angle", $\delta \alpha_T$, has been shown to be particularly sensitive to nucleon FSI [157] and is therefore chosen as one of the fit variables for this study. In Fig. 5.16, the $\delta \alpha_T$ distribution for neutrino interactions is shown for CC0 π events with and without FSI. As shown, in the absence of FSI the $\delta \alpha_T$ distribution is expected to be almost flat whilst with FSI an enhancement at large $\delta \alpha_T$ is predicted (stemming from the fact that FSI tends to slow down the outgoing nucleons).

Previous analyses of $\delta \alpha_T$ at ND280 have not been able to clearly identify this FSI induced enhancement because of its high proton tracking threshold (~ 450 MeV/c) which excludes many interactions in which the nucleon underwent FSI [175, 179]. This is illustrated in Fig. 5.16, which shows how the current ND280 tracking threshold flattens $\delta \alpha_T$ to a greater extent than that of ND280 Upgrade. It should also be noted that the neutron in anti-neutrino interactions can be reconstructed, through its time of flight, down to very low momentum threshold. This therefore potentially allows a useful FSI constraint even in the very low neutron momentum regions that cannot be measured for corresponding neutrino interactions [153]. Fig. 5.17 illustrates the $\delta \alpha_T$ distribution samples, split by interaction mode, which will be used as one of the inputs for the fitter.

Whilst variables characterising transverse kinematic imbalance are sensitive to many of the most important systematic uncertainties for neutrino oscillation analyses, they are not particularly sensitive to nuclear removal energy effects. A constraint can instead be established by looking for systematic shifts from expectation in the reconstructed neutrino energy distribution. However, in order to strongly constrain the removal energy uncertainty, a very good resolution



Figure 5.17: The reconstructed $\delta \alpha_T$ distribution for selected $CC0\pi$ neutrino (left) and antineutrino (right) interactions split by interaction mode and target for 1×10^{22} POT.

is necessary in the reconstruction of neutrino energy at the near detector, together with a very good control of energy scale in the detector and of the flux energy peak.

The usual estimator of neutrino energy is based on the very well known (e.g. see Ref. [154]) formula for CCQE events, relying on muon kinematics only, which was shown in Eq. 5.1. This estimator depends on E_B which is some assumed fixed nuclear binding energy of the struck nucleon (which is related to, but not exactly, the removal energy and is usually taken to be ~25 MeV for carbon). A second estimator can be defined as:

$$E_{vis} = E_{\mu} + T_N, \tag{5.10}$$

where T_N is the kinetic energy of the outgoing proton (neutron) in neutrino (anti-neutrino) interactions, E_{μ} is the total energy of the outgoing muon. E_{vis} is the total visible energy of all outgoing particles in CC0 π events with one nucleon in the final state. Such an estimator, before detector smearing, is expected to be slightly smaller than the true neutrino energy in CCQE events due to the need to overcome the nuclear removal energy and the loss of energy through nucleon FSI. Similarly multinucleon interactions and pion absorption will populate the tail of low E_{vis} since the second nucleon or the absorbed pion carry away some of the initial neutrino energy.

Fig. 5.18 compares the neutrino energy resolution for the two estimators alongside the impact of a possible bias due to removal energy. The distributions at generator level and after detector effects are shown. The E_{vis} estimator has a higher peak at very good resolution (< 5%) thus showing an increased sensitivity to possible bias in the removal energy estimation. This feature is preserved at reconstructed level, despite a larger experimental smearing of E_{vis} , due to the inclusion of proton tracking resolution in addition to the muon one. While Fig. 5.18 shows the resolution of two estimators for the CCQE interaction only, Fig. 5.19 shows other bias of these resolution from non-QE interaction modes. Both multinucleon interactions and pion production dominate the left tails of E_{vis} and E_{QE} due to the loss of energy in these process. The E_{QE} estimator not only has worse energy resolution in CCQE mode, but its non-QE components also make contribution to a larger bias in reconstructed neutrino energy compared to E_{vis} .



Figure 5.18: The neutrino energy reconstruction resolution and bias is shown for the two estimators defined in the text (E_{vis} and E_{QE} as solid and dashed lines respectively) for $\pm 10 \text{ MeV}$ shifts to the nominal removal energy (E_{rmv}) distribution (denoted by the red and blue colours respectively). True CCQE interaction mode is chosen for both plots. The left plot does not include the effect of detector smearing such that E_{reco} is constructed using the true muon and proton kinematics directly from the generator. In the righ plot E_{reco} is instead built from the corresponding reconstructed quantities (i.e. with the detector smearing applied).



Figure 5.19: The neutrino energy reconstruction resolution split by interaction mode for E_{vis} (left) and for E_{QE} . Because the second nucleon or the absorbed pion or undetected pion take away some of the original neutrino energy, 2p2h interactions and pion production will dominate the tail of low Evis and low E_{QE} resolution. It is clear by eyes that the level of bias from contamination of non-QE interaction in the E_{QE} estimator is higher than that of E_{vis} . These quantities are reconstructed ones with the detector smearing applied. Note that the plots were done with normal value of removal energy ($E_{rmv} = 25$ MeV).



Figure 5.20: The reconstructed E_{vis} distribution for selected $CC0\pi$ neutrino (left) and antineutrino (right) interactions split by interaction mode and target for 1×10^{22} POT. These distribution start at 200MeV since this is the detection threshold to detect a proton in the final state.

As discussed above, because of the inclusion of proton tracking resolution in addition to the muon one, the observed smearing of E_{vis} in data will be therefore mostly due to detector effects, while the smearing of E_{QE} is dominated by Fermi motion. The smearing induced by tracking resolution can in principle be improved with more performant detectors. Moreover we expect to be able to model quite precisely the detector-induced smearing, thanks to test beam studies and detector simulation, while the smearing due to nuclear effects is typically less well known. The inclusion of both energy estimators and the study of their correlation may also enable future enhanced sensitivity to removal energy.

Fig. 5.20 shows the reconstructed E_{vis} distribution for neutrino and anti-neutrino interaction which will be used as an input for the fitter. Since it is a neutrino energy estimator, the E_{vis} distribution peak is at around 600 MeV which is also the peak of the T2K's neutrino beam fluxes.

5.4.2 Systematic included in fitter

The uncertainty model used in this analysis is designed to offer theory-driven conservative freedoms to modify pertinent aspects of the neutrino interaction model in addition to accounting for flux modelling and detector performance uncertainties. Particular care is taken to allow plausible variations of nucleon kinematics which are especially sensitive to nuclear effects. While this sensitivity is at the core of the improvements driven by ND280 Upgrade, it also requires a parametrisation of nuclear uncertainties beyond what is used by the latest T2K oscillation analyses, which exploit only measurements of lepton kinematics.

Before going to detailed discussion, the list of key parameters taken into this quantitative studies are shown with their notation as follows:

• CCQE: These parameters are modelled in Spectral Function including SRC and shell

parameters which will be described later in this section.

- Npnh: The npnh uncertainty is a combination of 2p2h and SRC uncertainties.
- Pion Production: including the pion absorption FSI contribution (pion absorption) and the Pion production background (undetected pion).
- Nucleon FSI: The strength of nucleon final state interaction.
- E_{rmv} : removal energy shift in SF model.
- H norm: Hydrogen interaction normalisation
- Flux: Flux covariance uncertainty

For NEUT's CCQE interactions the initial nuclear state is characterised by a two-dimensional SF which describes the momentum of initial state nucleons and the energy that is necessary to remove them from the nucleus. In the SF model there is an extended distribution of removal energies rather than a fixed nuclear binding energy. The SF is broadly split into two parts: a part described by mean-field (MF) physics which has a shell structure (for carbon: one sharp p-shell and a diffuse s-shell) and a Short Range Correlations (SRC) part which results in two nucleons in the final state. These two parts were separated with the help of two variables: missing energy (E_{miss}) and missing momentum (p_{miss}), which are calculated in neutrino-carbon interaction as follows:

$$E_{miss} = E_{\nu} + m_n - E_{\mu} - E_p - T_{rem} \tag{5.11}$$

where
$$T_{rem} = \sqrt{P_{N,QE}^2 + M_{^{11}C}^2 - M_{^{11}C}},$$
 (7.10)

$$p_{miss} = p_{\nu} - p_{\mu} - p_{p}. \tag{5.12}$$

The term T_{rem} is the kinetic energy of the remnant nucleus, $P_{N,QE}$ stands for the initial nucleon momentum for QE events. In the case of anti-neutrino interaction, one needs to replace the initial neutron by proton, the final state proton by neutron and the remnant nucleus ${}^{11}C$ by ${}^{11}B$ ⁴ to recalculate the quantities above again. Based on the definition above, the whole phase space of E_{miss} and p_{miss} is divided into three regions with the following constraints

P Shell :
$$E_{miss} < 25 \text{MeV}$$
 and $p_{miss} < 300 \text{MeV}$,
S Shell : $25 \text{MeV} < E_{miss} < 100 \text{MeV}$ and $p_{miss} < 300 \text{MeV}$, (5.13)
SRC : $E_{miss} > 100 \text{MeV}$ or $p_{miss} > 300 \text{MeV}$.

Fig. 5.21 illustrates the distribution of neutrino-carbon interaction events with respect to the E_{miss} and p_{miss} . In the MF part (including p-shell and s-shell), the nucleons are treated as independent nucleons moving in a mean-field potential within the shell model picture. As a result, there is only one nucleon in the final state for interaction in the MF region. On the contrary, the nucleons in SRC part could not move freely since there are strong constraints between them (normally, they are pairs of strongly-correlated nucleons). Eventually, this strong

 $^{{}^{4}}M_{^{11}B} = 10255.097$ MeV is slightly smaller than $M_{^{11}C} = 10257.053$ MeV.



Figure 5.21: Distribution of neutrino-carbon interaction generated by NEUT regarding the missing energy (y-axis) and the missing momentum (x-axis). The bottom left of the phase space is the region for mean-field (MF) physics in which the sharp horizontal band (E_{miss} 20 MeV) is for p-shell events and the diffuse part is for s-shell ones. Outside of mean-field region is the Short Range Correlations (SRC) where a large amount of energy or momentum is missing after interaction. Plot was made by Jaafar Chakrani.

correlation will create two outgoing nucleons from the primary interaction vertex. In order to prove the purity of CCQE selection when applying the variables E_{miss} and p_{miss} to model it, Fig. 5.22 shows the E_{miss} distribution for neutrino and anti-neutrino interaction with carbon broken down by interaction mode. It is clear that the contamination of non-QE interaction in the p-shell and s-shell is negligible for both neutrino and anti-neutrino interaction. Since the non-QE interaction modes will result in a more significant amount of missing energy which is in the region of SRC, that will lead to a combination of SRC parameter and 2p2h parameter uncertainties to form the npnh ⁵ uncertainty later on. At this stage, we are trying to use missing energy to model the 1p1h interaction by the shell model. The processes which have more than one nucleon in the final state lead to larger missing energy and they should be modeled separately.

Based on the separation between SRC and shell events, many histograms were made by modifying the contribution from one of these parameters such as more SRC and less SRC histograms. They are created by scaling the nominal SRC histogram with 30% more or 30% less SRC. The same method is applied for p-shell and s-shell. The uncertainty model used allows a variation of the normalisation of each of the two mean-field shells separately and of the total strength of the SRC component. In other words, each region among SRC, p-shell and s-shell is treated as a normalisation systematics. A wide prior uncertainty of 30% is applied to the SRC and shell parameters. To evaluate the uncertainty of CCQE normalization due to the SF model, many toys are thrown from post fit errors and covariance matrix for shell parameters.

⁵npnh is the extension of 2p2h where there are more than one particle (nucleon in this case) in the final state



Figure 5.22: Missing energy for neutrino (left) and anti-neutrino interaction (right) in $CC0\pi$. The first sharp peak comes from the p-shell interaction and the second broader peak comes from the s-shell interaction.

The number of CCQE is then re-weighted from each toy. After that, the ratio between the number of re-weighted CCQE and the number of original CCQE is filled into a histogram. The standard deviation of this ratio histogram is the uncertainty of CCQE 6 parameter.

One of the biggest systematic uncertainties in the T2K experiment comes from the not well-constrained removal energy. Further freedom is added through an overall removal energy shift parameter (similar to what is described in Ref. [154]) which is built using an interpolation between two alternative simulation histograms that are generated with opposite 10 MeV shifts of the whole removal energy distribution. The histograms are constructed in the same binning used for the fit and in the same variables. The removal parameter is not constrained by any prior uncertainty. Fig. 5.23 shows the deviation of input histograms due to the shift of removal energy for two variables E_{vis} and $\delta p_{\rm T}$. The CCQE mode and the total, which is the sum of all modes, are shown in each plot. The E_{vis} is indeed sensitive to the removal energy as expected when the histogram with high E_{rmv} is shifted about 20 MeV to the left since there is more energy in the final state. On the contrary, the $\delta p_{\rm T}$ distribution does not respond much with the E_{rmv} shift ⁷.

In T2K 2020 oscillation analysis, there were two 2p2h norm parameters depending on the true neutrino energy. There were also parameters called "2p2h energy dependence," which are more sophisticated than just a normalisation parameter. However, in this study, the 2p2h uncertainties are provided solely through two normalisation factors, one for $E_{vis} < 600$ MeV and one for the remaining phase space. This accounts for uncertainties in the 2p2h nucleon ejection model, allowing the 2p2h shape to change as a function of neutrino energy and a consistent treatment between fits with different pairs of variables. The separation of 2p2h is based on the E_{vis} variable since this variable will be a default input for the fitter. In case there is not E_{vis}

⁶It can also be called 1p1h uncertainty since there is no SRC or multinucleon interaction process.

⁷The same property is found with $\delta \alpha_{\rm T}$ and p_N . However, their distributions with different E_{rmv} values are not shown in this thesis.



Figure 5.23: Variation of input histograms caused by removal energy shift for two variables E_{vis} (left) and δp_T (right). The high and low E_{rmv} correspond to 35 MeV and 15 MeV, respectively. The total line is the sum of all interaction modes. As seen, the E_{vis} gives more sensitivity to removal energy compared to δp_T . The CC0 π sample of neutrino interaction is selected for these plots.

in the input, the 2p2h normalisation factors can also be divided in the phase space of p_N or δp_T and it will be noticed. The motivation for choosing the 600 MeV in E_{vis} to separate 2p2h normalisation parameters is shown in Fig. 5.24. The oscillated neutrino spectrum at Super-K is clearly cut at 600 MeV in true neutrino energy and it is good to reclaim that E_{vis} is a good estimator of neutrino energy. The 2p2h uncertainties are not constrained prior to the fit in this fitter.

Since the pion production contribution to the selected samples is low, uncertainties on this are modelled via two parameters: one controlling the normalisation of the pion production background (i.e. events in which a pion was not detected) and another to alter the normalisation of the pion absorption FSI contribution. For a conservative approach, neither are constrained with prior uncertainties. Fig. 5.25 shows the distribution of several variables histograms for pion production interaction mode. The peak in E_{vis} distribution is not at 600 MeV anymore but around 400 MeV since pion absorption and pion background both preoccupy some amount of energy before it is visible in the final state. As seen in the tail of the $\delta p_{\rm T}$ distribution, the pion absorption seems to create more transverse imbalance than the pion background, which also causes more bias in Fermi momentum reconstruction. The pion background, however, behaves a bit like FSI in $\delta \alpha_{\rm T}$ distribution since it increases more events at a high $\delta \alpha_{\rm T}$ value compared to pion absorption⁸. The statistic of pion production is not as high as CCQE and 2p2h due to its lower cross-section; the post-fit uncertainties of pion production processes are not expected to be well constrained among all the parameters in the fitter.

Nucleon FSI is treated similarly to the removal energy shift parameter, relying on an interpolation between histograms generated with NEUT using different nucleon FSI strengths. These are constructed by re-running NEUT with 30% modifications of nucleon mean free paths inside the FSI cascade to cover differences between different FSI models and the variation

⁸This effect is clearly seen in anti-neutrino interaction which is not shown in this thesis.



Figure 5.24: Neutrino cross-section predictions by several models overlaid on top of the oscillated (dark grey) and unoscillated (light grey) neutrino spectrum at Super-K. Plot taken from [180].

in nuclear transparency data [181, 182]. Fig. 5.26 shows how some STV histograms change regarding the strength of nucleon FSI. The distribution of two variables $\delta p_{\rm T}$ and $\delta \alpha_{\rm T}$, which are the most sensitive variables to FSI effects, are illustrated. $\delta \alpha_{\rm T}$ distribution is proved in this plot to have a steeper shape with more FSI effects. The stronger the FSI effects are, the more imbalance caused in the transverse plane. This is indicated in the longer tail and lower peak of more FSI effects in $\delta p_{\rm T}$ distribution.

For anti-neutrino interactions off a hydrogen target nuclear effects are not relevant to removal energy or FSI effects, but a Hydrogen interaction normalisation parameter is considered to account for imperfectly modelled nucleon-level effects. This parameter is valid only in case of anti-neutrino interaction, since neutrino can not interact with proton through CC interaction. The criteria to classify Hydrogen interaction is the extremely small longitudinal imbalance between the momentum of final state and initial state particles

$$\Delta p_L = p_{\nu,L} - (p_{\mu,L} + p_{p,L}). \tag{5.14}$$

In this analysis the longitudinal imbalance limit used to select hydrogen interaction is $\Delta p_L < 5 \times 10^{-4} MeV$. This limit is big enough to cover all Hydrogen interaction, and it is small enough to avoid the misselection of Carbon interaction. A 5% of prior uncertainty is applied to this normalisation parameter. There are many figures (5.14, 5.15, 5.17, 5.20) that show the contribution of hydrogen interaction on the distribution of input variables. Since there are no Fermi motion and FSI effects, the transverse momentum imbalance and the reconstructed Fermi momentum are close to zero and much smaller than those of CC interaction with Carbon.

To have a good fit of flux parameters, it would be better to have some prior constraints for these parameters. There are many sources that cause flux uncertainties such as proton beam and off-axis angle uncertainties, horn current, magnetic field and alignment uncertainties, hadron



Figure 5.25: The distribution of several reconstructed variables for pion production broken down by pion absorption and pion background (undetected pion). All histograms are done with neutrino-carbon interaction and $CC0\pi$ selection.



Figure 5.26: Variation of input histograms caused by different strength of FSI effects for two variables δp_T (left) and $\delta \alpha_T$ (right). The more and less FSI correspond to 30% shorter and 30% longer nucleon mean free paths, respectively. The total line is the sum of all interaction modes. The $CC0\pi$ sample of neutrino interaction is selected for these plots.

production uncertainties. All of these factors are not investigated in this thesis, however, they are carefully taken into account by T2K collaboration. In this analysis, the flux uncertainties are handled using the T2K flux covariance matrix shown in Ref [165], which provides correlated uncertainties on the flux between ranges of neutrino energy. Flux shape uncertainties are a second order effect, and so for computational ease the covariance is applied to bins of visible energy rather than true neutrino energy. In this way only flux uncertainties covering energies between 0.2 and 1.5 GeV are included. The range starts from 0.2 GeV since this is the starting point of E_{vis} histogram, but this covers the ND280 flux peak and the primary region of interest for neutrino oscillation analysis.

5.4.3 Fitter details

A binned likelihood fitter is built in two-dimensions, including statistical and systematic uncertainties. The statistical uncertainty is implemented as a Poisson term in the likelihood whilst the systematic uncertainties are parameterized as a function of the fit variables and implemented as nuisances with priors included mostly as Gaussian penalty terms in the likelihood. The prior uncertainties and fit variables are those discussed in Sec. 5.4.2. One exception to the treatment of the prior uncertainties is an *ad-hoc* bin-to-bin uncorrelated uncertainty (as also detailed in Sec. 5.4.2) which is added directly to the likelihood using the Barlow-Beeston approach [183]. The final χ^2 used in the fitter is therefore defined as:

$$\chi^{2} = \sum_{i=1}^{n\text{-bins}} 2\left(\beta_{i}E_{i} - O_{i} + O_{i}ln\frac{O_{i}}{\beta_{i}E_{i}} + \frac{(\beta_{i} - 1)^{2}}{2\delta^{2}}\right) + \sum_{j}\left(\frac{p_{j}' - p_{j}}{\sigma_{j}}\right)^{2},$$

$$\beta_{j} = \frac{1}{2}\left(-(E_{i}\delta^{2} - 1) + \sqrt{(E_{i}\delta^{2} - 1)^{2} + 4O_{i}\delta^{2}}\right),$$
(5.15)

where the first term is from the Poisson likelihood (with the Barlow-Beeston extension) and the second term is the Gaussian penalty. The definition of the Barlow-Beeston scaling parameter β is also given. O_i and E_i are the observed and expected number of events for bin i, δ is the size of the uncorrelated uncertainty included directly in the Poisson likelihood. The second term is a sum over the systematic parameters in the fit where p'_j and p_j are the value of the parameter and its prior value respectively. σ_j is the prior uncertainty of parameter j.

5.5 Results and discussion

The fit described in Sec. 5.4 is performed and the parameter uncertainties obtained are mostof-the-time evaluated as a function of the statistics accumulated (denoted by the simulated POT exposure). Note that all the systematic uncertainties are always fit together, even if the uncertainty on only one parameter is shown.

5.5.1 The Rebinning studies

For the sensitivity study for future detector, the results could be affected a lot by the systematics that is not recently well-understood. For example the wrong estimation of the outgoing

Parameter	Prior Constraint	Notes
p-shell norm.	30%	
s-shell norm.	30%	
SRC strength	30%	
total QE normalisation	10%	
Removal energy shift	Unconstrained	
2p2h, low	Unconstrained	$<\!600~{\rm MeV}$
2p2h, high	Unconstrained	$>600 { m MeV}$
Undetected pions	Unconstrained	
Pion FSI contribution	Unconstrained	
Nucleon FSI strength	30%	
Flux (binned E_{vis})	T2K cov.	
Hydrogen normalisation	5%	$\bar{\nu}$ only
Uncorrelated Uncertainty	11.6%	No parameter fit,
	(at 6 \times 10 ²¹ POT)	POT dependence

Table 5.2: A list of fit parameters, their prior constraints and notes regarding their application. Whilst not a fit parameter, the bin-to-bin uncorrelated uncertainty is also listed.

particle kinematic resolution can bias the bin information of the input histograms. Therefore, the rebinning study is a very useful method to show how sensitivity changes if we use less shape information. For instance, it might be that systematics which have not been included yet (e.g. detector response) means that we are insensitive to variations between 2 bins when we have 100 dpt bins. But then if we only have 50 bins we could be more confident our included errors cover these systematics.

The rebinning results are shown in Fig. 5.27. From that results one can draw a conclusion that the rebin 5 and 10 bias a lot the fitter constraints. While the uncercertainties of all fit-parameters are well controlled with rebin =2 or 30MeV bin width in $\delta p_{\rm T}$ and E_{vis} . With all the current data from dectector performance for upgraded ND280, this bin width is fine enough for high statistic and it is large enough to include some systematics errors inside. Hence, the bin width =30MeV is the default bin width for δ_{P_T} , p_n and E_{vis} for the rest of the studies in this chapter.

5.5.2 $\delta \alpha_T$ and its sensitivity to nucleon FSI

As discussed in Sec. 5.4.1, it is expected that the superFGD should allow a particularly strong constraint on nucleon FSI via a measurement of $\delta \alpha_T$. To evaluate this, Fig. 5.28 shows only the nucleon FSI strength uncertainty as a function of the number of POT following fits using either $\delta \alpha_T$ or δp_T alongside E_{vis} . The extracted uncertainty on the nucleon FSI parameter is of the order of few percent at low statistics and can reach 1% with Hyper-K-era statistics. As can be seen, δp_T is only slightly less sensitive to FSI than $\delta \alpha_T$, but such sensitivity comes from the tail of the distribution with some degeneracy with the impact of non-QE components and SRCs.



Figure 5.27: The rebinning study results for neutrino- (left) and anti-neutrino-interaction (right). The study used δ_{P_T} ; E_{vis} as input with 2×10^{22} POT. The rebin changes the bins for both two observables and rebin=1 equivalent to bin width =15MeV.

As such, the FSI sensitivity in δp_T is more dependent on the shape uncertainties assumed for non-QE and is less robust than the sensitivity from $\delta \alpha_T$. On the other hand we expect a full multi-dimensional fit, using both δp_T and $\delta \alpha_T$, to provide an even more robust constraint on FSI and to be able to cross-check the correctness of FSI simulations through the investigation of possible tensions between the two variables.

For an anti-neutrino fit it is found that the benefit of using $\delta \alpha_T$ rather than δp_T to constrain nucleon FSI is only realised after ~ 6 × 10²¹ POT, and the difference in sensitivity is reduced to the level of only ~ 0.2%. This is likely mainly due to the lower resolution and less statistics per PoT than in the neutrino fit, but it should be noted that using δp_T in the anti-neutrino fit also allows a more independent constraint on the neutrino flux, as discussed in Sec. 5.4.1, and this benefit may partially offset the degeneracies lifted by $\delta \alpha_T$.

Removal energy constraints

The upgraded ND280's ability of hadronic reconstruction is the key factor to constrain better the removal energy. This is because the visible energy (E_{vis}) , which is very sensitive to the removal energy, need the hadronic kinematics to be reconstructed.

Fig. 5.29 shows the constraint on the removal energy parameter from a fit to E_{vis} and δp_T for neutrino and anti-neutrino interactions. In the neutrino case the removal energy shift can be measured at 2 MeV at relatively low statistics and better than 1 MeV with ultimate statistics. The corresponding anti-neutrino constraint is 3 to 4 times worse. As discussed in Sec. 5.5.6, further improved constraints can be obtained by exploiting the p_N variable in place of δp_T , but in this case more longitudinal information is included and thus the constraint is more reliant on the neutrino flux shape prediction.



Figure 5.28: The 1 σ sensitivity to the nucleon FSI parameter as a function of POT for neutrino interactions when fitting the reconstructed CC0 π data binned in δp_T and E_{vis} or in $\delta \alpha_T$ and E_{vis} . The results are shown for neutrino (left) and antineutrino (right) case.

5.5.3 QE and non-QE interaction constraints

As discussed in Sec 5.4.1, non-QE and SRC interactions in the CC0 π channel can induce an important bias in the reconstruction of neutrino energy. Fig. 5.30 shows the constraint on the total normalisation of 1p1h and npnh interactions. As previously noted, relatively advanced shape uncertainties are considered for these processes in the fit but for brevity we show only their combined effect on the total cross sections. The bulk of the δp_T variable is a direct probe of the 1p1h interactions, allowing a constraint as good as $\sim 1.5\%$ ($\sim 2\%$) in neutrino (anti-neutrino) interactions. The tail at high values of δp_T is sensitive to the non-QE component, enabling a constraint better than 5% (10%) in neutrino (anti-neutrino) interactions. The npnh constraint is partially correlated with the constraint on the pion absorption FSI component and the background due to undetected pions in the final state. It should be noted that the effects of such components on the neutrino energy reconstruction are similar so induce similar bias in the neutrino oscillation parameters. Indeed, pion FSI and the 2p2h process including a Δ resonance excitation and a pion production followed by re-absorption, are separately modelled in NEUT but they are fundamentally very similar processes (i.e. they are partially irreducible backgrounds to each other). On the contrary, the background due to the presence of an undetected pion in the final state can be reduced, notably thanks to the lower threshold for pion reconstruction in the Super-FGD.

5.5.4 Hydrogen interaction constraints and its effects on the flux uncertainty

The main effect limiting the precision of the extrapolation of the 1p1h normalisation from the near to the far detector is the degeneracy between the cross section and the neutrino flux. In Ref. [153], a strategy to select an hydrogen enhanced sample in anti-neutrino interactions has been proposed, in order to constrain the flux independently of uncertainties on nuclear effects. Here we consider the hydrogen sample together with the carbon sample in a joint fit to estimate quantitatively its impact on the flux constraints. The contribution of hydrogen interactions in the anti-neutrino sample is illustrated in Fig. 5.14 for the δp_T distribution.



Figure 5.29: The 1 σ sensitivity to the nuclear removal energy shift parameter as a function of POT for neutrino and anti-neutrino interactions when fitting the reconstructed CC0 π data binned in δp_T and E_{vis} .

Fig. 5.31 shows the constraint on the total normalisation of the hydrogen sample, which is reduced by a factor of ~ 2 relative to the prior uncertainty with 2×10^{22} POT worth of data, in addition to the constraint on the flux normalisation. The impact on the flux constraint depends on the prior uncertainties assumed on the anti-neutrino-nucleon cross section, notably due to the nucleon form-factor. It should be noted that this is the result of a complete fit including both anti-neutrino-nucleus and anti-neutrino-nucleon processes, thus showing the *incremental* contribution to the flux precision from the hydrogen enhanced sample, on top of the constraint obtained from the usual fit to carbon interactions. The contribution of the hydrogen-enhanced sample is sizeable, if the prior hydrogen normalisation uncertainty is lower than $\sim 10\%$, improving the relative flux constraint by up to $\sim 20\%$. Note that, whilst not quantified here, the power of the hydrogen-enhanced sample also improves knowledge of the flux shape and better separation between flux and form-factor constraints could be achieved by adding the reconstructed four-momentum (Q^2) from the lepton kinematics as a fit variable. This would allow a separation of the low Q^2 region, in which the form-factor is relatively well understood and so the flux can be constrained, from the high Q^2 region, where the prior flux constraint can instead be leveraged to probe the form-factor details.

5.5.5 Summary of the fit results

A summary of the results are shown in Figs. 5.32 and 5.33, which shows parameter constraints as a function of POT and the correlations between fit parameters (for 10²² POT) respectively. In the former figure the parameters describing CCQE and 2p2h interactions are integrated to provide more meaningful uncertainties on the total one-particle-one-hole (1p1h) final state normalisation (CCQE without SRCs) and an accompanying npnh final state normalisation (2p2h and CCQE with SRCs). SRCs are combined with 2p2h since they have similar kinematic properties (as



Figure 5.30: The 1 σ sensitivity to the 1p1h (top) and npnh (bottom) cross-section normalisations as a function of POT for neutrino and anti-neutrino interactions when fitting the reconstructed $CC0\pi$ data binned in δp_T and E_{vis} .

can be seen by their anti-correlation) and both are responsible for similar neutrino energy reconstruction bias. The uncertainty on the total cross section (integrating all cross-section systematic parameters but not the flux) is also shown. Tabulated sensitivities are shown for two fixed POT values in Tab. 5.4.

5.5.6 Fitting with reconstructed nucleon momentum, the promising variable for future ND280 fit

As discussed in Sec. 5.4.1, the reconstructed initial state nucleon momentum (p_N) [177] can be used as an alternative to the transverse momentum imbalance (δp_T) [157] as an input variable in the fit. The former contains more information than that latter, incorporating an inferred longitudinal imbalance, in addition to the transverse imbalance, and so better sensitivity is in principle expected. However, this better sensitivity accompanies the potential need for additional systematic uncertainties. For example, whilst the shape of δp_T is largely independent from neutrino energy and nucleon-level physics [157], this will not be the case for p_N . We therefore choose the more conservative and simple approach of quoting the primary sensitivities using a fit in δp_T but discuss here what would be obtained using p_N instead.

Generally it is found that using p_N in place of δp_T has a relatively modest impact on the extracted sensitivities shown in Sec. 5.5, although a notable improvement was found for some parameters. These improvements are shown in Fig. 5.34, demonstrating how p_N offers better ability to distinguish npnh, 1p1h and pion production processes compared to δp_T . This behaviour has previously been noted in e.g. Ref. [179].

5.6 Conclusion

It has been demonstrated that the upgraded ND280 detector will allow increasing sensitivity to nuclear-model uncertainties, enabled by the use of observables formed from both lepton and



Figure 5.31: The 1 σ sensitivity to the hydrogen normalisation parameter as a function of POT (left) and the 1 σ sensitivity to the flux normalisation as a function of the hydrogen parameter's prior uncertainty (right) for anti-neutrino interactions when fitting the reconstructed CC0 π data binned in δp_T and E_{vis} for 1×10^{22} POT.

nucleon kinematics, which can complement the usual T2K analysis of only muon kinematics. Importantly, the use of nucleon kinematics also allows less room for incorrect models to describe ND280 Upgrade data and, as such, facilitates more robust constraints.

The inclusion of nucleon kinematics into the analysis brings new systematic uncertainties, notably detector systematics but also new nuclear-model systematics related with nucleon FSI. From simulated studies, benchmarked by test beam data of prototypes and by long-term data-taking experience with ND280, we expect the systematic uncertainties related with detector modelling to be well under control. In this paper we have shown quantitatively, for the first time, that nucleon FSI can also be very well constrained thanks to the use of $\delta \alpha_T$. The precision of the constraint is enabled by the low proton (and neutron) tracking threshold in ND280 Upgrade and the absence of degeneracy (correlation) with other nuclear-model uncertainties in $\delta \alpha_T$.

The use of an improved estimator of neutrino energy, based on the sum of muon energy and nucleon kinetic energy, has been investigated and shows interesting sensitivity to nuclear removal energy shifts. Furthermore, the QE and non-QE components of the cross section have been shown to be well separated by using fits to δp_T . Contrary to an inclusive analysis, these two components can be measured with small degeneracy/correlation, thus reducing ambiguities in the propagation of constraints to the far detector. Finally, the reduction of the flux uncertainty from a hydrogen-enhanced sample, as suggested in Ref. [153], has been quantified for the first time. The relative improvement on the flux normalisation uncertainty can be up to 20%, depending on the prior uncertainty on the nucleon form factors.

Although a quantitative evaluation of the impact of ND280 Upgrade constraints directly on T2K and Hyper-K measurements of PMNS neutrino oscillation parameters is beyond the scope of this work, some qualitative observations can be made. The tight constraints on removal energy and npnh effects indicate a reduction of the uncertainty in both the neutrino energy scale and

$1 \times 10^{22} \text{POT}$	$\delta p_T; E_{vis}$	$\delta \alpha_T; E_{vis}$	$p_N; E_{vis}$
$1p1h(\nu)$	1.9%	1.8%	1.5%
1p1h $(\overline{\nu})$	3.3%	3.9%	2.6%
npnh (ν)	6.5%	13%	5.3%
npnh $(\overline{\nu})$	12%	17%	11%
$E_{rmv}(\nu)$	$0.55 { m MeV}$	$0.38 { m MeV}$	$0.53 { m MeV}$
E_{rmv} $(\overline{\nu})$	$1.3 { m MeV}$	$1.0 { m MeV}$	$1.3 { m MeV}$
Pion FSI (ν)	6.6%	14%	4.8%
Pion FSI $(\overline{\nu})$	34%	35%	30%
Undetected pions (ν)	9.7%	14%	8.2%
Undetected pions $(\overline{\nu})$	37%	36%	31%
Nucleon FSI (ν)	1.1%	0.76%	0.98%
Nucleon FSI $(\overline{\nu})$	2.3%	1.9%	2.4%
Flux (ν)	1.8%	1.9%	1.6%
Flux $(\overline{\nu})$	2.4%	2.3%	2.2%
Total (ν)	1.8%	2.1%	1.6%
Total $(\overline{\nu})$	2.7%	2.7%	2.5%
Hydrogen $(\overline{\nu})$	3.3%	4.0%	2.9%

Table 5.3: Expected 1σ uncertainties on key cross-section parameters at 1×10^{22} POT for neutrino and anti-neutrino interactions with different fit variables.

bias at the far detector. These effects are strongly correlated with the systematic uncertainty on measurements of the neutrino mass splitting (Δm_{32}^2) and so a comparable reduction in the model systematic component of the uncertainty on its measurement might be expected. Measurements of the large mixing angle (θ_{23}) are more dependent on the total cross section and flux uncertainties (and their correlations), whilst measurements of the CP-violating phase (δ_{CP}) additionally require an accurate understanding of the asymmetry in neutrino and anti-neutrino cross sections, and in muon and electron neutrino cross sections. The reported significant reduction in the total cross section and flux uncertainties, and the demonstrated ability of ND280 Upgrade to set powerful constraints from neutrino and anti-neutrino data independently, therefore suggests a correspondingly large constraint on the pertinent model systematic uncertainty on measurements of θ_{23} and δ_{CP} . In particular it can be noted that the (anti) neutrino total cross section uncertainty is reduced to below the $\sim (2.9\%) 2.5\%$ expected statistical uncertainty on the electron neutrino samples at Hyper-K [184]. Although the asymmetry in muon and electron neutrino cross sections is not directly constrained, its root cause can at least partially stem from a mismodelling of the details of the nuclear ground state [185, 186] which has been shown to be well measured, although it can be noted that another significant component of this uncertainty can stem from the modelling of radiative corrections to cross sections [187], which is not considered in this work.

The sensitivity studies presented here are based on the simulation of the detector performances expected with ND280 Upgrade and includes a relatively sophisticated, but still incomplete, set of nuclear uncertainties. Conservative assumptions have been taken when possible, e.g., considering mostly the kinematics projected in the transverse plane to minimise the dependence on flux modelling, leaving parameters unconstrained, and fitting the neutrino and anti-neutrino samples separately, thus not including the correlation between neutrino and

$2 \times 10^{22} \text{POT}$	$\delta p_T; E_{vis}$	$\delta \alpha_T; E_{vis}$	$p_N; E_{vis}$
1p1h (ν)	1.4%	1.2%	1.1%
$1p1h \ (\overline{\nu})$	2.2%	2.5%	1.7%
npnh (ν)	4.7%	8.5%	3.6%
npnh $(\overline{\nu})$	7.5%	10.0%	7.1%
$E_{rmv}(\nu)$	$0.32 { m MeV}$	$0.22 { m MeV}$	$0.3 { m MeV}$
E_{rmv} $(\overline{\nu})$	$0.68 { m MeV}$	$0.57 { m MeV}$	$0.68 { m MeV}$
Pion FSI (ν)	4.2%	8.3%	2.8%
Pion FSI $(\overline{\nu})$	19%	17%	16%
Undetected pions (ν)	6.6%	7.9%	5.4%
Undetected pions $(\overline{\nu})$	21%	19%	18%
Nucleon FSI (ν)	0.62%	0.43%	0.58%
Nucleon FSI $(\overline{\nu})$	1.2%	0.98%	1.2%
Flux (ν)	1.4%	1.4%	1.2%
Flux $(\overline{\nu})$	1.7%	1.6%	1.6%
Total (ν)	1.4%	1.5%	1.2%
Total $(\overline{\nu})$	1.8%	1.7%	1.7%
Hydrogen $(\overline{\nu})$	2.5%	3.1%	2.1%

Table 5.4: Expected 1σ uncertainties on key cross-section parameters at 2×10^{22} POT for neutrino and anti-neutrino interactions with different fit variables.

anti-neutrino cross-section and flux uncertainties. Overall the results demonstrate the interesting potential model constraints enabled by exploiting proton and neutron kinematics measured at the near detector for future oscillation analyses using ND280 Upgrade.

Whilst it is shown that most parameter constraints plateau at an acceptable level for Hyper-K era statistics, some show scope for improvement even beyond this limit. This is particularly true for those relating to the flux constraint from the hydrogen enhanced region of δp_T . At very high statistics improved (and more robust) sensitivities may also be gained by using finer binning or adding additional dimensions to the fit. This could be realised by either additional beam exposure for ND280 Upgrade and/or future further upgrades to increase the target mass, for example by using the methods suggested in Refs. [188, 189].



Figure 5.32: The 1 σ sensitivity to systematic parameters as function of POT in neutrino case (top) and anti-neutrino case (bottom) when fitting the reconstructed $CC0\pi$ data binned in δp_T and E_{vis} . The values in the plot are the ratio of the parameter uncertainty to the parameter nominal value expressed as a percentage.



Figure 5.33: The correlation matrix between constrained parameters following a fit to reconstructed $CC0\pi$ data binned in δp_T and E_{vis} with 1×10^{22} POT. The results following a fit to neutrino (left) and anti-neutrino (right) samples are shown.



Figure 5.34: The 1 σ sensitivity to the 1p1h (top left), npnh (top right) and total (bottom) cross-section normalisations as a function of POT for neutrino interactions when fitting the reconstructed $CC0\pi$ data binned in E_{vis} and either δp_T or p_N .

Neutrino-nucleus Martini interaction model

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This chapter will introduce the neutrino-nucleus interaction model from Martini and collaborators [80, 190], which is capable of describing many interaction topologies over a wide range of neutrino energy. The model is built on a random phase approximation (RPA) approach of nuclear responses in the QE and Delta resonance regions, including nucleon interaction and nucleon kick-off phenomenology inside the nuclear medium. One key advantage of the model is that it can describe not only the pion production but also other interaction channels such as QE, npnh (2p2h or 3p3h) in a single framework and so future fake data studies may aim to make a wholesale reweighting of NEUT to Martini. A major difference to the default NEUT pion production model is that the Martini model includes nuclear effects within the microscopic theoretical calculation whilst the NEUT model is fundamentally a neutrino-*nucleon* interaction model with nuclear effects added in *ad-hoc*. The Martini model describes neutrino-nucleus interactions by employing the concept of nuclear response functions handled using RPA and including Delta-resonance excitations. In the case of incoherent pion production, Delta pionic decay is dominant, which leaves the nucleus in a particle-hole excited state. The variation in Delta width due to nuclear medium effects reduces the pion emission probability compared to that probability coming from an interaction with a free nucleon. Furthermore, the modest RPA effects also add to this lowering. The reduction due to the modification of the Delta width has a counterpart in the presence of the multi-nucleon knock-out component [80].

6.1 Theoretical framework

This introduction to Martini et al. theoretical model was done based on the HDR of Marco Martini [191].

6.1.1 The neutrino-nucleus interaction cross-section

Alternatively to the cross-section form introduced in section 2.3.1, a simplified expression will be presented that excludes the final lepton mass contributions. This can be obtained by retaining only the leading terms for the hadronic tensor. The number of response functions used in the cross section calculation is decreased to three under this approximation. These three response functions include the isospin/isovector R_{τ} , the spin-isospin longitudinal $R_{\sigma\tau(L)}$ and the spinisospin transverse $R_{\sigma\tau(T)}$ responses that contain the information about the nuclear dynamics. Then the cross-section can be rewritten as follows

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2 \cos^2 \theta_c}{\pi} |\mathbf{k'}| E_l' \cos^2 \frac{\theta}{2} \left[\frac{(\mathbf{q}^2 - \omega^2)^2}{\mathbf{q}^4} G_E^2 R_\tau(\mathbf{q}, \omega) + \frac{\omega^2}{\mathbf{q}^2} G_A^2 R_{\sigma\tau(L)}(\mathbf{q}, \omega) + 2 \left(\tan^2 \frac{\theta}{2} + \frac{\mathbf{q}^2 - \omega^2}{2\mathbf{q}^2} \right) \left(G_M^2 \frac{\mathbf{q}^2}{4M_N^2} + G_A^2 \right) R_{\sigma\tau(T)}(\mathbf{q}, \omega) \\
\pm 2 \frac{E_\nu + E_l'}{M_N} \tan^2 \frac{\theta}{2} G_A G_M R_{\sigma\tau(T)}(\mathbf{q}, \omega) \right].$$
(6.1)

In this new form of cross-section, it is clear to see many elements such as lepton kinematic, nucleon electric, nucleon magnetic, axial form factors and nuclear response functions. Hence, this equation is very valuable for illustrating neutrino cross sections. The form factors are determined by the square of the 4-momentum transfer $Q^2 = -q^2$, and one can use the form factors obtained from the electron scattering experiment for neutrino interaction due to the conservation hypothesis for vector current. The axial form factor was already introduced to be dipole parameterized in eq. (2.17), but now we will use other notation for further consistency with the authors' convention.

$$G_A(Q^2) = \frac{g_A}{(1+Q^2/M_A^2)^2}$$
(6.2)

6.1.2 The RPA nuclear response functions

As introduced above, the cross-section in terms of nuclear response functions has some advantages; let us investigate the nuclear response functions that appear in eq. (6.1). The general expression for nuclear response function is of the form

$$R_{\alpha} = \sum_{n \neq 0} |\langle n | \hat{\mathcal{O}}_{(\alpha)} | 0 \rangle|^2 \delta[\omega - (E_n - E_0)]$$
(6.3)

where E_n is the eigenvalue of the full nuclear Hamiltonian operator \hat{H} , $\hat{\mathcal{O}}_{(\alpha)}$ is the second quantized expression of the vertex operator (external field), which can act in different spin and isospin channels. The nuclear response functions, which are shown in Eq. (6.1), are connected to the imaginary component of the entire polarization propagator [191]

$$R(\omega, q) = -\frac{\nu}{\pi} \operatorname{Im}[\Pi(\omega, q, q)], \qquad (6.4)$$

where ν is the nuclear volume that can be calculated as A/ρ (A is the nucleus mass number and ρ is the mass density of the considered nucleus).

All of the mentioned response functions are computed by using the RPA's ring approximation. These calculations are started with the "bare" propagators, which means that the particle-hole interaction has been disabled. In other words, the nuclear correlations have been turned off; however, these bare responses are still subject to the many-body effects and they are the sum of several channels mentioned before in eq. (2.37). These channels include NN: quasielastic, NN: 2p2h, $N\Delta$ and ΔN : 2p2h, $\Delta\Delta$: πN , $\Delta\Delta$: 2p2h, $\Delta\Delta$: 3p3h. The specifics on how they have been determined in the Martini et al. model method are provided below.

The nucleon-hole polarization propagator, which is the typical Lindhard function [192], yields the NN quasielastic

$$\Pi_{N-h}^{0}(\mathbf{q},\omega) = g \int \frac{d\mathbf{k}}{(2\pi)^{3}} \left[\frac{\theta(|\mathbf{k}+\mathbf{q}|-k_{F})\theta(k_{F}-k)}{\omega+\omega_{\mathbf{k}}-\omega_{\mathbf{k}+\mathbf{q}}+i\eta} - \frac{\theta(k_{F}-|\mathbf{k}+\mathbf{q}|)\theta(k-k_{F})}{\omega+\omega_{\mathbf{k}}-\omega_{\mathbf{k}+\mathbf{q}}-i\eta} \right], \quad (6.5)$$

where θ is the Heaviside step function. ω_k is the free-nucleon energy, the k_F is the well-known Fermi momentum, and g is the statistical factor which has the value of 4 coming from the average of spin and isospin.

The relativistic formulation is applied for the bare Delta-hole polarization propagator, which is connected to the $\Delta\Delta\pi N$ channel.

$$\Pi^{0}_{\Delta-h}(q) = \frac{32\tilde{M}_{\Delta}}{9} \int \frac{d^{3}k}{(2\pi)^{3}} \theta(k_{F} - k) \left[\frac{1}{s - \tilde{M}_{\Delta}^{2} + i\tilde{M}_{\Delta}\Gamma_{\Delta}} - \frac{1}{u - \tilde{M}_{\Delta}^{2}} \right],$$
(6.6)

where $\tilde{M}_{\Delta} = M_{\Delta} + 40(MeV)\frac{\rho}{\rho_0}$ and Γ_{Δ} denotes the mass and the width of the Δ in the nuclear medium.

For the 2p2h interaction mode, Martini et al. have used the Fermi gas as the microscopic model to estimate the 2p2h contributions, which means that the estimations are framed based

on uncorrelated nucleons. However, we all know that the nucleons in the nucleus medium are correlated, and in some cases, the correlation between two nucleons is so strong that they function as a singular object under the external field. This strong correlation is called Short Range Correlated (SRC). If an uncorrelated nucleon basis is utilized in the 1p1h sector, the impacts of nucleon-nucleon (NN) correlations must also be examined. These NN correlations are implemented in the framework of free particle models, such as Fermi gas- or mean field-based models, by taking into account an extra two-body current [193], or by adding them into the nuclear wave functions [194]. However, if one would like to use the NN correlations in the correlated-basis concept, then these impacts should be characterized as the one nucleon-two nucleon currents interference.

Among the methods to implement NN correlations, which are just mentioned above, for numerous reasons, a precise relativistic calculation of two-body current contributions in Fermi gas-based models is problematic. For instance, let us begin with the generic form of the 2p2h hadronic tensor

$$W_{2p2h}^{\mu\nu}(\mathbf{q},\omega) = \frac{V}{(2\pi)^9} \int d^3p'_1 d^3p'_2 d^3h_1 d^3h_2 \frac{M_N^4}{E_1 E_2 E_1' E_2'} \\ \theta(p'_2 - k_F) \theta(p'_1 - k_F) \theta(k_F - h_1) \theta(k_F - h_2) \\ \langle 0|J_{TB}^{\mu}|\mathbf{h}_1 \mathbf{h}_2 \mathbf{p}_1' \mathbf{p}_2' \rangle \langle \mathbf{h}_1 \mathbf{h}_2 \mathbf{p}_1' \mathbf{p}_2' | J_{TB}^{\nu}|0 \rangle \\ \delta(E_1' + E_2' - E_1 - E_2 - \omega) \delta(\mathbf{p}_1' + \mathbf{p}_2' - \mathbf{h}_1 - \mathbf{h}_2 - \mathbf{q}),$$
(6.7)

where **p** and **h** stand for nucleon and hole momenta respectively. The first challenge is to calculate very high-dimensional integrals for a vast number of 2p2h response Feynman diagrams; even the integrals above can be reduced to 7-dimensional integrals $\int d^3h_1 d^3h_2 d\theta'_1$ by using the momentum and energy conservation. Another issue is that the NN correlations sector will likely contain the divergences that must be regularized [195]. Moreover, in neutrino experiments, there does not exist only the fixed value of neutrino energy but the neutrino flux with a wide range of energy; hence, neutrino cross-section calculation for all kinematics consistent with the measured neutrino flux should be done. As a result, an accurate relativistic calculation is exceedingly computationally intensive, and several approximations are used to minimize the integrals' complexity and regularize the divergences.

Now, let us go over the Martini model's options for evaluating the multinucleon emission channel. The first thing is that the Martini model has modified the Δ width in the nuclear medium since this modification can count the additional sources for the npnh mode. In 1987, Oset and Sacedo parametrized this phenomenon for the first time when they took into account the 2p2h and 3p3h channels in the assumption of true pion or photon [196]. Martini and his colleagues have taken advantage of this parametrization to alter the Δ width. The Martini model has also included NN correlations, meson exchange current (MEC) contributions, and NN correlations-MEC interference in the 2p2h sector. In terms of the MEC, the Δ -MEC is the main contributor to the MEC; this model solely evaluates the Δ -MEC and excludes the other components from the actual calculation.

In order to have a more visual bare response, Fig. 6.1 shows the components of the bare polarization propagator as a function of transferred energy. It is noticed that this bare polarization



Figure 6.1: The bare response (bare polarization propagator) with various components are plotted for Carbon at q = 300 MeV/c. These components include QE, pion emission, 2p2h and 3p3h. Plot taken from [191]

propagator is density-dependent. In momentum space, $\Pi^0(\omega, \mathbf{q}, \mathbf{q}')$ in a finite system which has finite size effects is non-diagonal. Martini model has analyzed these finite size effects in a semi-classical approximation where the polarization propagator is of the form

$$\Pi^{0}(\omega, \mathbf{q}, \mathbf{q}') = \int d\mathbf{r} e^{-i(\mathbf{q} - \mathbf{q}') \cdot \mathbf{r}} \Pi^{0}\left(\omega, \frac{1}{2}(\mathbf{q} + \mathbf{q}'), \mathbf{r}\right).$$
(6.8)

Then a local density approximation

$$\Pi^{0}\left(\omega, \frac{\mathbf{q} + \mathbf{q}'}{2}, \mathbf{r}\right) = \Pi^{0}_{k_{F}(r)}\left(\omega, \frac{\mathbf{q} + \mathbf{q}'}{2}\right)$$
(6.9)

is used. The $k_F(r)$ is the local Fermi momentum determined by $k_F(r) = (3/2\pi^2 \rho(r))^{1/3}$, where $\rho(r)$ is the experimental density of the nuclei.

However, this semi-classical approximation is unsuitable for assessing the collective effects. In the case of applying the Random Phase Approximation to calculate the polarization propagators, the bare polarization propagator Π^0 is used as an input in a full quantum mechanical resolution of the RPA equations in the ring approximation [191]

$$\Pi^{ring} = \Pi^0 + \Pi^0 V \Pi^{ring}, \tag{6.10}$$

where V stands for the effective interaction between particle-hole excitations and it is parameterized in terms of π , ρ and contact pieces

$$V_{NN} = (f' + V_{\pi} + V_{p} + V_{g'})\boldsymbol{\tau_{1}}.\boldsymbol{\tau_{2}},$$

$$V_{N\Delta} = (V_{\pi} + V_{p} + V_{g'})\boldsymbol{\tau_{1}}.\boldsymbol{T_{2}^{\dagger}},$$

$$V_{\Delta N} = (V_{\pi} + V_{p} + V_{g'})\boldsymbol{T_{1}^{\dagger}}.\boldsymbol{\tau_{2}},$$

$$V_{\Delta \Delta} = (V_{\pi} + V_{p} + V_{g'})\boldsymbol{T_{1}^{\dagger}}.\boldsymbol{T_{2}^{\dagger}}.$$

(6.11)

The notation g' is the Landau-Migdal parameter. One can find more information on this interaction as well as the exact form used in the RPA resolution in [80].

The isolation of individual channels is more difficult in the RPA situation than in the bare one. Consider that the imaginary component could well be expressed generically as:

$$Im\Pi = |\Pi|^2 ImV + |1 + \Pi V|^2 Im\Pi^0.$$
(6.12)

This expression can be divided into terms. The first one, $|\Pi|^2 \text{Im}V$, is missing as long as the effective interaction is turned off. Since the V_{π} is the pion exchange potential, this term reflects coherent pion production in which the nucleus remains in the ground state. For the second term, it is obvious that it corresponds to the bare polarization propagator $\text{Im}\Pi^0$. The bare response is the total of the separate final state components listed as follows: (1) NN quasi-elastic (as described by the standard Lindhard function); (2) NN 2p2h; (3) N Δ and (3') ΔN 2p2h; (4) $\Delta\Delta \pi \text{N}$; (5) $\Delta\Delta 2p2h$; (6) $\Delta\Delta 3p3h$ [190]. Hence this term can indicate the sort of final state, and it can be modified by the collective effects due to the factor $|1 + \Pi V|^2$.

6.2 Motivation for pion production studies

Although the Martini et al. model could provide a unified framework to describe many interaction channels, this study will focus on the pion production channel. The pion production channels have historically been relevant in neutrino oscillation physics. In the case of T2K, the oscillation analyses currently use QE as the main channel, but it is crucial to satisfactorily model the QE and non-QE interaction, primarily single pion production due to its capacity to imitate QE interactions. QE interaction is a great channel to reconstruct neutrino energy by only using lepton kinematics. Unfortunately, the pion production can mimic the QE channel if the pions are below the detection threshold, introducing a significant bias into neutrino oscillation analysis. Moreover, the neutral current π^0 production resulting from Δ -resonance interaction has a high possibility of mimicking the electron signals for electron neutrino appearance oscillation searches in SK. Poor resolution detector could not identify the difference between shower made by electron or gamma-ray very well. In the case of asymmetric π^0 decay in which most of the energy was transferred to the gamma-ray in the final state, there is only one reconstructed electromagnetic shower. Hence this gamma-ray shower can definitely be reconstructed as an electron shower. The inefficiency of detectors that can not reconstruct all of the showers could also contribute to this bias.

There are two channels that contribute to producing pions: coherent and incoherent pion production. Like many other models, the incoherent pion cross-section in the Martini et al. model is substantially larger than the coherent one for the nucleus used in modern experiments. In the case of incoherent pion production, the pion produced by the Delta's pionic decay is dominant; this decay also leaves the nucleus in a particle-hole excited state. In the bare circumstance, the variation in Delta width reduces the pion emission probability compared to that probability coming from an interaction with a free nucleon. Furthermore, the modest RPA effects also add to this lowering. The reduction due to the modification of the Delta width has a counterpart in the presence of the multi-nucleon knock-out component [80].

6.3 Oscillation analysis in T2K

6.3.1 Factors affecting the event rates

The goal of T2K oscillation analysis is to obtain the neutrino oscillation parameters by fitting the oscillated data at the far detector. The event rate at the near and far detector stand for the event rate before and after the oscillation, respectively. In order to achieve the precise measurement of the oscillation parameters, a good estimation of the event rate at ND280 and SK is required. These event rate estimations for ND280 with the ν_{α} flavor $(R_{\nu_{\alpha}}^{ND})$ and SK with the ν_{β} flavor $(R_{\nu_{\beta}}^{FD})$ are as follows:

$$R_{\nu_{\alpha}}^{ND} = \int dE_{\nu} \Phi_{\nu_{\alpha}}^{ND}(E_{\nu}) \times \epsilon^{ND}(E_{\nu}) \times \sigma_{\nu_{\alpha}}^{ND}(E_{\nu})$$
(6.13)

$$R_{\nu_{\beta}}^{FD} = \int dE_{\nu} \Phi_{\nu_{\beta}}^{FD}(E_{\nu}) \times \epsilon^{FD}(E_{\nu}) \times \sigma_{\nu_{\beta}}^{FD}(E_{\nu}) \times P_{\nu_{\alpha} \to \nu_{\beta}}(E_{\nu}).$$
(6.14)

The Near Detector and Far Detector are denoted as the superscripts ND and FD, respectively. $\Phi_{\nu_{\beta}}^{FD}(E_{\nu})$ is the neutrino flux distribution, $\epsilon(E_{\nu})$ stands for the detector efficiency. The $\sigma_{\nu}(E_{\nu})$ is the neutrino cross-section mainly depending on the neutrino energy. The other deviation of neutrino cross-section between far and near detector at T2K comes from the differences in nucleus targets and neutrino flavors.Finally, $P_{\nu_{\alpha} \to \nu_{\beta}}(E_{\nu})$ is the oscillation probability.

It is obvious that the previously specified flux, detector, and cross-section factors are causes of systematic uncertainty that impact the event rate measurement accuracy. The data from the near detector at T2K is used to constrain these uncertainties as much as possible before passing them to the far detector. In order to constrain these uncertainties, they are parametrized by several sets of parameters known as *nuisance parameters*. Unlike the oscillation parameters, these nuisance parameters are not the ultimate goal of the T2K oscillation analysis; however, they got a significant concern since they cause systematic uncertainties on the oscillation parameters' outcome.

After having the flux and cross-section constraints from the ND280 fit, they are propagated to the far detector fit. Since T2K uses a different type of technique for the near and far detectors, the far detector fit has used a different detector model and data samples from the near detector fit. The final results from the far detector fit are the constraints on the oscillation parameters; all details of the oscillation analyses are summarized in fig. 6.2.

6.3.2 Procedure of the fake data studies

The analysis in this chapter concentrates on the neutrino cross-section model. Therefore the flux model and the detector model are not discussed in detail. To parametrize the cross-section uncertainties, the T2K collaboration has used the Spectral Function model [167] for CCQE interactions, the Nieves et al. (Valencia) model for a multi-nucleon meson exchange current contribution (2p2h) [110] and the Rein-Seghal model [168] for single-pion production as explained in section 5.3. The fake data studies performed in this thesis focus on the replacement of the Rein-Seghal model with the Martini et al. model for the single-pion production channel. This fake data study is one among several fake studies of T2K in 2022 where many interaction channels such as "Alternative Axial Form Factors" for CCQE, Radiative Corrections for Associated Real



Figure 6.2: General process of T2K oscillation analysis.

Photon Production. The single-pion production was chosen for this analysis since it contributes to the background of the CCQE events, which is the main sample in the T2K oscillation analysis. Moreover, the T2K collaboration recently started selecting the $CC1\pi$ for its analysis; therefore, it is crucial to model the single-pion production well and test the alternative model's impact on the oscillation parameters. The step-by-step procedure for a general fake data study at T2K is as follows

- Produce the expected spectra of neutrino events at Near and Far Detector (including oscillations) built with an alternate model.
- Fit the Near Detector spectra with the default model and propagate the fitted parameters to the Far Detector.
- Fit the Far Detector spectra, built with the alternate model, using the constraints from the Near Detector fit.
- Check for the biases on the extracted oscillation parameters by comparing them with the results from the normal fit.

6.3.3 Fitting frameworks at T2K

Three distinct analysis codes are employed for the oscillation fits, named P-Theta, MaCh3 and VaLOR. They differ in how they use near detector data. P-Theta and VaLOR are semifrequentist fitters that determine the best-fit values for the oscillation parameters by applying Gradient Descent Minimization (GDM) methods and afterward marginalize across nuisance parameters. The outcomes are reported as frequentist confidence limits on the oscillation parameters. The fundamental distinction between P-Theta and VaLOR is in the variable used in the fit: VaLOR bins data in two dimensions in terms of E_{ν} (reconstructed neutrino energy) and θ (reconstructed final state μ angle with respect to the incoming neutrino direction), while the while P-Theta can use either $p - \theta$ binning (p stands for μ momentum) or the previously described $E_{\nu} - \theta$ binning. MaCh3 is a flexible analysis framework for Bayesian oscillation analyses use Markov Chain Monte Carlo (MCMC) methods to sample the parameter space and provide posterior distributions for the concerned parameters. The values of the oscillation parameter are reported as credible intervals (C.I.). MaCh3 conducts a one-dimensional fit as a function of E_{ν} for the far detector fit. These three fitting frameworks yielded identical outcomes in sensitivity studies and comparable but slightly varied outcomes in data fit. The results of the fake data study in this chapter were extracted using the fit of P-theta.

Among the three aforementioned fitting frameworks, only the MaCh3 has the possibility to fit the data from near and far detectors simultaneously. The other two (P-Theta and VaLOR) need the external fit to have the constraints from the near detector. These near detector constraints are done thanks to the Beam And ND280 Flux extrapolation task Force (BANFF). BANFF is a two-dimensional binned semi-frequentist fitting framework that implements $p - \cos \theta$ binning. The $p - \cos \theta$ is also binning of MaCh3 performs the near detector fit.

6.4 Implementation of fake data

In order to reweight the NEUT single pion prediction into the Martini prediction, and to have the comparison of neutrino cross-section between the Martini model and NEUT, the table of double differential cross section $d^2\sigma/(d\cos(\theta)d\omega)$ theoretically predicted by Martini model has been used. This cross-section is a function of neutrino energy E_{ν} , transferred energy $\omega = E_{\nu} - E_{\mu}$ and the cosine of the angle of the outgoing lepton with respect to the incoming neutrino $\cos(\theta)$. The table covers the full phase space of $\cos(\theta)$, up to 1 GeV for transferred energy and up to 2 GeV in neutrino energy. The tables are generated for carbon and oxygen targets for incoming muon and electron neutrinos and anti-neutrinos. Although only the single pion production tables are used in this work, the tables were provided for many interaction modes: QE, npnh (2p2h+3p3h), Delta resonance, and coherent. For Delta resonance single pion production, all hadronic final states are included, with the incoherent π^+ channel accounting for 5/6 of the total cross section for neutrino interactions. With these tables, it is possible to build 3D histograms of Martini's differential cross section as a function of the aforementioned variables. In order to have precise reweighting, the bins' width are set quite fine for these histograms: 0.05 GeV in E_{ν} , 0.005 GeV in ω and 0.02 in $\cos(\theta)$. The bin contents of these histograms were taken directly from the Martini differential cross-section tables.

Similar 3D histograms of the differential cross section were created with identical phase

space and bins width for the nominal NEUT cross-section prediction. The way we calculate the bin content in NEUT histograms is different from the Martini case. The three variables E_{ν} , ω , and $\cos(\theta)$ are determined for each event generated by NEUT and then are filled in the 3D histograms. After filling all of the events, all histograms were scaled by a scale factor to obtain the differential cross section. The files used to generate these NEUT histograms use a flat neutrino energy flux, and so we need very high statistics in order to make the 3D differential cross section at some particular small range of E_{ν} have a reasonable degree of fluctuations. In these studies, for each type of neutrino and each type of target (carbon and oxygen), 100 million events have been generated for the analysis.

To produce fake data, the core idea is to perform a reweighting of each generated single pion production NEUT event which is made based on the ratio of the cross-section given in the two histograms. To deal with the finite bin width, the standard ROOT TH3D::Interpolate() method is used.

Fig. 6.4 and Fig. 6.3 illustrate the difference between NEUT and Martini et al. pion production differential cross section as a function of $\cos \theta$ and transferred energy. The Martini model generally predicts a smaller pion production cross section compared to NEUT. The plots of differential cross-section as a function of transferred energy (ω) show some lack of phase space overlap between the two models. It is worth noticing that these differences are even clearer in the case of QE and npnh interaction (see section A.1) due to the lack of binding energy in the Martini et al. model, which results in additional difficulties for this reweighting technique.

It should not be expected that the two different models have the same predictions. The difference of primary interest between the two comes from the treatment of nuclear effects and, in particular, of the Δ decay width (the Δ decay width in the Martini et al. model is under the effects of the nuclear medium). This is particularly evident in the low energy-transfer region where Martini predicts a larger cross-section than NEUT, likely due to the broadened Delta and alternative Fermi motion treatment allowing the interaction to "switch on" at slightly smaller values.

Despite apparent differences in the models due to nuclear effect treatment, it is worth investigating all reasons behind the observed discrepancies. One of these other reasons concerns differences in the types of pion production events considered within the two models. The NEUT generator accounts for not only the resonant but also the non-resonant interaction that could leave a pion in the final state. Such effects are not included in the Martini et al. model, so one should mitigate the effects of this inconsistency in the reweighting. The best way would be to reweight events according to the relative contribution from resonant and non-resonant amplitudes, but this is not straightforward. Hence, an alternative approximate method is employed where only events with an invariant mass (W) in the Δ -dominated region are reweighted. For this analysis, the W cut is chosen to be 1.3 GeV. Above this, all weights are set to one (such that the nominal NEUT model is used). Fig. 6.5 illustrates this method, the plots in this figure are prepared where every NEUT event that has the invariant mass W above some certain value will be cut from the template used for the reweight. For the Martini model, the invariant mass was calculated not for each event but for the whole bin (since the Martini tables are built from cross-section predictions and not events), which gives rise to the spiky structure seen in the



Figure 6.3: Total and differential cross-section per nucleon of muon neutrino (left) and muon anti-neutrino (right) for a single pion production channel. The comparison was made between Martini et al. model and Nieves et al. model (denoted by NEUT).



Figure 6.4: Total and differential cross-section per nucleon of electron neutrino (left) and electron anti-neutrino (right) for single pion production channel. The comparison was made between Martini et al. model and Nieves et al. model (denoted by NEUT).

plots.



Figure 6.5: Example of applying the invariant mass cut for pion production cross section. The differential cross sections are shown as functions of transferred energy (top plots) and $\cos \theta$ (bottom plots). These plots are for ν_{μ} -Carbon interaction at $E_{\nu} = 1,5 \,\text{GeV}$. There is no cut on the left plots, while all the events with $W > 1.3 \,\text{GeV}$ were cut on the right plots. It is clear that the prediction of the two models becomes closer when the cut is applied (i.e. more background is subtracted) but that substantial differences remain. Note that in the Martini et al. model, the invariant mass was calculated not for each event but for the whole bin (since the Martini tables are built from cross-section predictions and not events), which gives rise to the spiky structure seen in the top right plot.

Another way to make a comparison of the models in the absence of the non-resonant background is to study only the $CC1\pi^+$ (for neutrino interactions), where this contribution is tiny. Fig. 6.6 shows the comparison for pion production and $CC1\pi^+$ cross-section as well as the variation of the ratio between two models when a different type of pion selection has been used.

Overall it is clear that substantial differences remain between the NEUT and Martini resonant single pion production models even when making comparisons in regions where they both "cover" the same nucleon-level physics. The final reweight dial to investigate the impact of these differences uses the cross-section tables to weight NEUT resonant single pion production events



Figure 6.6: Left: the differential cross section as a function of $\cos\theta$ for NEUT and Martini model with different topology selection. Right: the bin-by-bin ratio between NEUT and Martini model for different topology selection. It is clear that $CC1\pi^+$ gives better agreement between the two models.

with W < 1.3 GeV to the predictions of the Martini model as a function of E_{ν} , ω and $\cos(\theta)$. The tables are calculated separately for incoming ν_{μ} , $\bar{\nu}_{\mu}$, ν_{e} , $\bar{\nu}_{e}$ and for carbon and oxygen targets. Similar reweighting dials for npnh, QE and coherent interactions are available but are not used within this work.

6.5 Metrics for evaluating Martini et al. models

In this fake data study, the summary quantities of interest are the range of parameter values encompassed within the one and two sigma intervals, 1σ and 2σ , respectively, and the centerpoint of each interval, $\bar{x}^{1\sigma}$ and $\bar{x}^{2\sigma}$, of the one dimensional $\Delta \chi^2$ distributions obtained from several P-Theta fit simulated SK data. To compute each metric, we take summary quantities from three fits: 1) a full Asimov fit with systematic and statistical uncertainties, 2) a stats-only Asimov fit with only statistical uncertainties, and 3) a fit to fake data provided by the other model under consideration. To be clear, an Asimov fit is one in which the generated data being fit is taken as the precise forecast of the model at the central value of all nuisance parameters, with no further statistical fluctuations. This is characterized by non-integer observed event numbers and $\chi^2 = 0$ at the true point.

6.5.1 Definition of bias and its impact on δ_{CP}

As all other oscillations and systematic parameters are marginalized in each fit, the best-fit point is frequently driven away from the true values, as the parameters of interest might be strongly linked with some of those that are marginalized. As a result, the ability to derive the genuine oscillation parameter values is not used as an indicator that the fit performed as predicted. To examine the extent of bias in the fit results due to tension between the fit model and the fake data, the results of fake data fits are compared to those of the Asimov fit, which is also subject to marginalization effects. The Asimov fit results from a fit to an Asimov data set. The Asimov data set is described as a data set generated using the same underlying model as the fitter but with all parameters set to their previous central values.

The bias of a fake data set (FDS) fit results is calculated as the fractional shift in the center of the 2σ intervals between the full Asimov and the fake data fits, $\Delta^{2\sigma} = \bar{x}_{Asimov}^{2\sigma} - \bar{x}_{FDS}^{2\sigma}$, relative to the 1σ systematic interval, $1\sigma_{syst.}^{Asimov}$. The systematic interval $(1\sigma_{syst.})$ is defined as: $1\sigma_{syst.}^2 = 1\sigma_{tot.}^2 - 1\sigma_{stat.}^2$, where $1\sigma_{tot.}$ and $1\sigma_{stat.}$ are the total and statistical errors, respectively. The relative size of the $2\sigma_{tot.}^{FDS}$ interval to the $2\sigma_{tot.}^{Asimov}$ interval is a secondary metric used to examine a change in uncertainty prediction. In summary, action criteria are determined based on the values of the aforementioned bias $B_x^{syst.}$ and relative size $(R_x^{2\sigma})$:

$$B_x^{\text{syst.}} = \frac{\Delta_x^{2\sigma}}{1\sigma_{\text{syst.}}^{\text{Asimov}}},\tag{6.15}$$

$$R_x^{2\sigma} = \frac{2\sigma_{\text{tot.}}^{\text{FDS}}}{2\sigma_{\text{tot.}}^{\text{Asimov}}}.$$
(6.16)

The 2σ interval is needed to prevent discontinuities in the 1σ intervals on $\sin^2 \theta_{23}$, which are common in Asimov B22 fits (specified in section 6.5.2). Since intervals shorten with larger statistics, this might have to be reconsidered in the future, as we could have a discontinuous 2σ area too. We also provide the bias estimated using the midpoint and size of the 1σ and 2σ intervals relative to 1σ and 2σ , respectively, for completeness.

By the convention of T2K collaboration [197], an observed bias is considered as significant if it is greater than 50%. The sample statistics totally dominate the size of the parameter contours for δ_{CP} ; therefore, $1\sigma_{syst.}^{Asimov}$ may be ignored. It is difficult to compare this contours size to the bias given by fake data sets. We may check how much the data intervals for δ_{CP} shift if the data $\Delta\chi^2$ curves change by an amount equal to the difference between the nominal Monte Carlo $\Delta\chi^2$ curve and the fake data $\Delta\chi^2$ curve. The $\Delta\chi^2$ difference in the FDS is defined as

$$\Delta \chi^2_{diff}(\delta_{cp}) = \Delta \chi^2_{nom}(\delta_{cp}) - \Delta \chi^2_{FDS}(\delta_{cp}), \qquad (6.17)$$

where $\Delta \chi^2_{nom}(\delta_{cp})$ and $\Delta \chi^2_{FDS}(\delta_{cp})$ are the $\Delta \chi^2$ curves coming from the nominal and fake data fits, respectively. From the equation above, the shifted $\Delta \chi^2$ curve for the data is calculated as

$$\Delta \chi^2_{shift}(\delta_{cp}) = \Delta \chi^2_{data}(\delta_{cp}) + \Delta \chi^2_{diff}(\delta_{cp}), \qquad (6.18)$$

where $\Delta \chi^2_{data}(\delta_{cp})$ is the $\Delta \chi^2$ curve from the fit to the data. The confidence intervals are then determined using $\Delta \chi^2_{shift}(\delta_{cp})$ and compared to the data intervals. The change in the interval boundaries for the Martini et al. single pion production $(CC1\pi^+)$ FDS is presented in section 6.6.

The fake data sets, in general, can alter the predicted event rate at the far detector. The statistical uncertainty on these samples will alter as a result, as will the sensitivity contours. This shift in contours will still exist if the near detector fit entirely reproduced the fake data. In this scenario, comparing the fake data sensitivity to the Asimov sensitivity is incorrect. To compensate for this, we can generate a second "Scaled Asimov" result in which the MC used in the fit is also applied to fake data weights to keep the statistical uncertainty unchanged. The BANFF Asimov matrix is used as the input uncertainty in this fit. Because it assumes that the near detector fit completely reproduces the fake data, the Scaled Asimov result is the best possible scenario. Any disparity between the fake data contours and the Scaled Asimov contours is due to near detector misfitting.
6.5.2 Oscillation Parameter Test Values (Asimov Points)

It is not obvious to choose the oscillation parameter values to generate the MC events for the Asimov fit since we do not know their true values. It is, therefore, necessary to test the Asimov fit assuming different prior values of oscillation parameters. The difference between parameter test values depends on how well we can constrain them. table 6.1 reports the two sets of parameters utilized to establish our Asimov fit: Asimov A22 and Asimov B22. Asimov A22 uses the best-fit points obtained by T2K in Run 1-7 analysis as the oscillation parameters. Asimov B22 changes these parameters to conserve CP and alter $\sin^2 \theta_{23}$ to a non-maximal mixing value. Tab. 6.2 specifies the prior distributions used in P-Theta for the oscillation parameters.

Before investigating the bias from the FDS, it is needed to have the post-fit parameter values of the normal fit for reference and for calculating the bias. The results of the two Asimov fits used in this chapter to determine the bias caused by the Martini et al. model FDS are presented in table 6.3. It is worth noting that the Asimov points are used instead of the best-fit data to avoid any potential bias in the results in section 6.6.

Parameter	Asimov A22	Asimov B22
δ_{cp}	-1.601	0.0
$\sin^2 \theta_{13}$	0.0220	0.0220
$\sin^2 \theta_{23}$	0.561	0.45
$\sin^2 \theta_{12}$	0.307	0.307
$ \Delta m_{23}^2 $ (eV ²)	2.494×10^{-3}	2.494×10^{-3}
$ \Delta m_{12}^2 \ (eV^2)$	7.53×10^{-5}	7.53×10^{-5}

Table 6.1: Oscillation parameter values used as priors.

Parameter	Prior
$\sin^2 2\theta_{13}$	Uniform(0, 1)
w/ PDG reactor constraint	Gaussian($\mu = 0.0861, \sigma = 0.0027$)
$\sin^2 heta_{23}$	Uniform(0, 1)
$\delta_{ m CP}$	Uniform $(-\pi, \pi)$
$\Delta m_{32}^2 (\text{NO}) / \Delta m_{31}^2 (\text{IO}) [\text{eV}^2]$	$\text{Uniform}(0, \infty)$

Table 6.2: Oscillation parameter prior distributions.

		$\sin^2 heta_{23}$	Δm_{32}^2	$\delta_{ m CP}$
	Middle of the 1σ interval	0.5577	0.0024918	-1.565
	1σ interval size: 1σ	0.0305	5.23e-05	1.04
	1σ stat-only interval size: $1\sigma_{\rm stat.}$	0.0269	4.81e-05	1.01
Asimov A 22	$1\sigma_{\text{syst.}} = \sqrt{(1\sigma_{\text{tot.}})^2 - (1\sigma_{\text{stat.}})^2}$	0.0144	2.05e-05	0.251
AsimovA22 AsimovB22	Middle of the 2σ interval	0.5212	0.0024919	-1.581
	2σ interval size: 2σ	0.086	0.000105	1.74
	2σ stat-only interval size: $2\sigma_{\text{stat.}}$	0.081	9.62e-05	1.67
	$2\sigma_{\text{syst.}} = \sqrt{(2\sigma_{\text{tot.}})^2 - (2\sigma_{\text{stat.}})^2}$	0.0288	4.12e-05	0.46
	Middle of the 1σ interval	0.453 and 0.57	0.0024976	0.2894 and 2.802
	1σ interval size: 1σ	0.0254 and 0.00553	5.44e-05	0.812 and 0.632
AsimovA22 AsimovB22	1σ stat-only interval size: $1\sigma_{\text{stat.}}$	0.0235 and 0.00753	4.97e-05	0.779 and 0.618
AsimovB22	$1\sigma_{\text{syst.}} = \sqrt{(1\sigma_{\text{tot.}})^2 - (1\sigma_{\text{stat.}})^2}$	0.00957	2.2e-05	0.231 and 0.131
AsimovB22	Middle of the 2σ interval	0.5084	0.0024975	0
	2σ interval size: 2σ	0.0986	0.000109	3.14
	2σ stat-only interval size: $2\sigma_{\text{stat.}}$	0.094	9.94e-05	3.14
	$2\sigma_{\rm syst.} = \sqrt{(2\sigma_{\rm tot.})^2 - (2\sigma_{\rm stat.})^2}$	0.0298	4.38e-05	

Table 6.3: Asimov fit results using as prior the Asimov points defined in table 6.1. For Asimov B22, results at the lower and upper octant minima are shown.

6.6 Fake data study and oscillation analysis results

In this section, the results of the BANFF and P-Theta fits for Martini et al. FDS are shown in detail. The influence on relevant ND280 and SK samples and the consequence on nuisance parameters used in the near detector fit is explored. Only the one-dimensional (1D) $\Delta \chi^2$ contours are presented for the oscillation parameters, together with the changes observed in Martini et al. FDS. It is worth noting that the variables used in the P-Theta are identical to those used in the primary analysis of T2K run 1-10.

6.6.1 ND280 Fit Results

In this FDS, we reweight the default model implemented in NEUT for the single pion production, the one developed by Martini and collaborators as described in section 6.1.

The effect of the reweight applied to the FGD1 $\nu_{\mu} CC1\pi^+$ sample is shown in fig. 6.7. As can be seen from the 2D distribution of the muon $\cos\theta$ versus the momentum before the fit (left plot), the effect of this FDS is a general suppression of the cross-section. This is understandable since the cross-section comparison in section 6.4 shows smaller cross-section of Martini et al. model. The ratio between the BANFF post-fit 2D distribution with the reweighed MC is reported in the right plot in fig. 6.7, showing the largest difference to be below 5% for the sample taken under consideration.



Figure 6.7: Pre- and post-fit ratios to Martini 1π FDS for the FGD1 ν_{μ} CC1 π^{+} sample.

The central values and uncertainty for the cross-section and flux parameters as a result of the BANFF fit to this Martini et al. FDS are reported in figs. 6.8 and 6.9, respectively. They are also compared with the prior central values and uncertainties (red band). Since we are concentrating on the cross-section related parameters, the results on the parameters associated to the detector systematic uncertainties are not reported for brevity.

To find the best fit for this FDS, BANFF had to move the parameters related to single pion production, in particular the non-resonant background, and slightly the shape of the 2p2h cross-section. To compensate for the fake data-MC disagreement, the fit decreases the flux central values at low energy. The results are still in agreement with their prior since the χ^2 associated with the flux penalty is small (1.27).



Figure 6.8: Pre- (red band) and post-fit (blue dots and black error bars) cross-section parameters from the BANFF fit to Martini 1π FDS.



Figure 6.9: Pre- (red band) and post-fit (blue dots and black error bars) flux parameters from the BANFF fit to Martini 1π FDS. Legend is the same with fig. 6.8.

6.6.2 Oscillation Fit Results

The effect of the reweight to generate this Martini et al. FDS on the SK samples is shown in figs. 6.10 and 6.11 for Asimov A22 and B22, respectively. The reconstructed neutrino energy distribution is compared for the nominal SK samples (solid blue line), the SK fake data for Asimov A22 or B22 (solid green line), and the prediction from the BANFF fit to the Martini 1π FDS (red band).

In figs. 6.10 and 6.11, all the samples are with one main ring except for multi-ring and 1d.e samples. The one main ring events are denoted by 1Re and 1R μ for electron and muon ring, respectively. Since 2016, the $CC1\pi$ sample with 1 electron ring and 1 delayed Michel electron has been included in the oscillation analysis, and it is referred to as "1Re 1d.e.". As seen in fig. 6.10 for the Asimov A22 fit, the event rates in multi-ring and one electron ring (1Re)+1d.e. samples is decreased. A small decrease is also present in both FHC and RHC 1Re. The same tendency is observed in Asimov B22. The prediction from the BANFF fit and the FDS generated using SK samples are in agreement within the systematic uncertainty band. A shape-like shift of this kind is likely to have a more significant impact on Δm_{32}^2 .

The 1D likelihood surfaces for Δm_{32}^2 , $\sin^2 \theta_{23}$ and δ_{CP} , with the reactor constraint on $\sin^2 \theta_{13}$ result of the P-Theta fit to Martini 1π FDS, are shown in figs. 6.12 and 6.13 for Asimov A22 and B22 respectively. Both normal and inverted ordering are reported. Also, the results obtained employing the scaled Asimov are included. A small shift in the contour is visible in Δm_{32}^2 and $\sin^2 \theta_{23}$. The metrics used to quantify the impact of this FDS on the oscillation parameters are reported in table 6.4, and in table 6.6 for the scaled Asimov. The values of interest are highlighted in blue.

In general, the bias relative to $\sigma_{syst.}^{Asimov}$ is well below the warning milestone suggested by the T2K collaboration, which is 50%, and the change of the size of the 2σ interval relative to $2\sigma_{tot.}^{Asimov}$ is below 3% for all the oscillation parameters. The fit results from Martini 1π FDS give the most bias in Δm_{32}^2 where its absolute bias in the middle of 2σ interval with respect to the size of the $1\sigma_{syst.}^{ref}$ interval is above 15% and below 19% for all fit results. While the same bias in $\sin^2 \theta_{23}$ and δ_{CP} are below 3.5% and 2%, respectively, for all fit results. Based on these fake data studies results, it is recommended to include the bias on Δm_{32}^2 in the calculation of the smearing. No action is needed for $\sin^2 \theta_{23}$.

For δ_{CP} , it is not practical to derive confidence bounds for this parameter using the standard gaussian approach, in which $\Delta \chi^2$ values equate to the confidence level directly. The Feldman-Cousins technique [198] is employed instead. The Feldman-Cousins approach involves calculating the appropriate critical $\Delta \chi^2$ values to obtain the 90%, 1 σ , and 2 σ confidence levels. The effect on δ_{CP} 1D $\Delta \chi^2$ for the Martini 1 π FDS is shown in fig. 6.14. The impact of such FDS is quantified in table 6.5, where the interval edges for δ_{CP} from the data and Martini 1 π FDS fits and the changes to the edges due to Martini 1 π FDS are shown. Overall, the shift on δ_{CP} 1D $\Delta \chi^2$ distribution caused by Martini et al. FDS is not such that the T2K result on δ_{CP} changes.

The effect on δ_{CP} 1D $\Delta \chi^2$ in case of the scaled Asimov is shown in fig. 6.15. The impact in this case is quantified in table 6.7. Compared with shifts in table 6.5, the scaled Asimov shows a larger size for the shifts of the 1σ interval, and a smaller shift for all intervals.



Figure 6.10: Comparison between the nominal SK samples (blue solid line), the SK fake data for Asimov A22 (green solid line), and the prediction from the BANFF fit to the Martini 1π FDS (red band).



Figure 6.11: Comparison between the nominal SK samples (blue solid line), the SK fake data for Asimov B22 (green solid line), and the prediction from the BANFF fit to the Martini 1π FDS (red band).



Figure 6.12: 1-D likelihood surfaces for Δm_{32}^2 (top left), $\sin^2 \theta_{23}$ (top right) and δ_{CP} (bottom), with the reactor constraint on $\sin^2 \theta_{13}$, for both normal (solid line) and inverted (dashed) ordering, for the Martini 1π FDS and Asimov A22. The contours of the Asimov fit, FDS and scaled Asimov are reported in blue, orange and dark blue respectively.



Figure 6.13: 1-D likelihood surfaces for Δm_{32}^2 (top left), $\sin^2 \theta_{23}$ (top right) and δ_{CP} (bottom), with the reactor constraint on $\sin^2 \theta_{13}$, for both normal (solid line) and inverted (dashed) ordering, for the Martini 1π FDS and Asimov B22. The contours of the Asimov fit, FDS and scaled Asimov are reported in blue, orange and dark blue respectively.

		$\sin^2 \theta_{23}$	Δm^2_{32}	$\delta_{\rm CP}$
	Middle of the 1σ interval	0.5565	0.002488	-1.569
	1σ interval size: 1σ	0.031	5.19e-05	1.06
	1σ interval ratio to Nominal	1.02	0.992	1.03
	Fractional change in the 1σ interval size wrt the syst interval: $\left(1\sigma_{\text{tot.}}^{\text{FDS}} - 1\sigma_{\text{tot.}}^{\text{ref}}\right)/1\sigma_{\text{syst.}}^{\text{ref}}$	3.5%	-2.09%	10.4%
	Bias in the middle of 1σ interval wrt the size of the $1\sigma_{tot.}^{ref}$ interval	-4.03%	-7.37%	-0.328%
AsimovA22	Bias in the middle of 1σ interval wrt the size of the $1\sigma_{\rm syst.}^{\rm ref}$ interval	-8.57%	-18.7%	-1.35%
	Middle of the 2σ interval	0.5207	0.0024881	-1.586
	2σ interval size: 2σ	0.0858	0.000104	1.76
	2σ interval ratio to Nominal	0.998	0.99	1.02
	Fractional change in the 2σ interval size wrt the syst interval: $\left(2\sigma_{\text{tot.}}^{\text{FDS}} - 2\sigma_{\text{tot.}}^{\text{ref}}\right)/2\sigma_{\text{syst.}}^{\text{ref}}$	-0.685%	-2.44%	6.43%
	Bias in the middle of 2σ interval wrt the size of the $2\sigma_{tot.}^{ref}$ interval	-0.527%	-3.63%	-0.241%
	Bias in the middle of 2σ interval wrt the size of the $2\sigma_{syst.}^{ref}$ interval	-1.57%	-9.22%	-0.91%
	Bias in the middle of 2σ interval wrt the size of the $1\sigma_{tot.}^{ref}$ interval	-1.48%	-7.27%	-0.404%
	Bias in the middle of 2σ interval wrt the size of the $1\sigma_{\rm syst.}^{\rm ref}$ interval	-3.15%	-18.5%	-1.67%
	Middle of the 1σ interval	0.4539 and 0.5691	0.002494	0.3373 and 2.744
	1σ interval size: 1σ	0.0256 and 0.00624	5.35e-05	0.833 and 0.649
	1σ interval ratio to Nominal	1.01 and 1.13	0.983	1.03 and 1.03
	Fractional change in the 1σ interval size wrt the syst interval: $\left(1\sigma_{\text{tot.}}^{\text{FDS}} - 1\sigma_{\text{tot.}}^{\text{ref}}\right)/1\sigma_{\text{syst.}}^{\text{ref}}$	2.73%	-4.2%	9.19% and $13.4%$
	Bias in the middle of 1σ interval wrt the size of the $1\sigma_{tot.}^{ref}$ interval	3.79% and $-16.9%$	-6.51%	5.9% and $-9.08%$
	Bias in the middle of 1σ interval wrt the size of the $1\sigma^{\rm ref}_{\rm syst.}$ interval	10%	-16.1%	20.8% and $-43.6%$
AsimovB22	Middle of the 2σ interval	0.5084	0.002494	0
101110/022	2σ interval size: 2σ	0.098	0.000107	3.14
	2σ interval ratio to Nominal	0.994	0.985	1
	Fractional change in the 2σ interval size wrt the syst interval: $\left(2\sigma_{\text{tot.}}^{\text{FDS}} - 2\sigma_{\text{tot.}}^{\text{ref}}\right)/2\sigma_{\text{syst.}}^{\text{ref}}$	-2.01%	-3.74%	
	Bias in the middle of 2σ interval wrt the size of the $2\sigma_{tot.}^{ref}$ interval	0.00282%	-3.24%	0%
	Bias in the middle of 2σ interval wrt the size of the $2\sigma_{\text{syst.}}^{\text{ref}}$ interval	0.00931%	-8.03%	
	Bias in the middle of 2σ interval wrt the size of the $1\sigma_{tot.}^{ref}$ interval	0.0109%	-6.46%	0%
	Bias in the middle of 2σ interval wrt the size of the $1\sigma_{syst.}^{ref}$ interval	0.029%	-16%	0%

Table 6.4: Table of the bias for the Martini 1π FDS for both Asimovs. For Asimov B22, results at the lower and upper octant minima are shown.

	1σ		90CL		2σ		3σ	
Data	-2.704	-0.980	-3.066	-0.425	-3.258	-0.174	-3.839	0.493
FDS edges	-2.719	-0.976	-3.081	-0.414	-3.274	-0.161	-3.860	0.510
Shift	-0.0152	0.0042	-0.0154	0.0111	-0.0157	0.0123	-0.0210	0.0170

Table 6.5: δ_{CP} interval edges from the data and Martini 1π FDS fits and the changes to the edges due to Martini 1π FDS.



Figure 6.14: Comparison of the δ_{CP} 1D $\Delta \chi^2$ from the data and Martini 1 π FDS (left) an the difference between $\Delta \chi^2_{nom.}$ and $\Delta \chi^2_{FDS}$ (right). The Feldman-Cousins critical χ^2 are reported as well in the left plot.



Figure 6.15: Comparison of the δ_{CP} 1D $\Delta \chi^2$ from the data and Martini 1 π FDS (left) an the difference between $\Delta \chi^2_{nom.}$ and $\Delta \chi^2_{FDS}$ (right) when the **scaled Asimov method** is used. The Feldman-Cousins critical χ^2 are reported as well in the left plot.

		$\sin^2 \theta_{23}$	Δm^2_{32}	$\delta_{\rm CP}$
	Middle of the 1σ interval	0.5565	0.002488	-1.569
	1σ interval size: 1σ	0.031	5.19e-05	1.06
	1σ interval ratio to Nominal	1.02	0.998	1.01
	Fractional change in the 1σ interval size wrt the syst interval: $\left(1\sigma_{tot.}^{FDS} - 1\sigma_{tot.}^{ref}\right)/1\sigma_{syst.}^{ref}$	4.59%	-0.59%	5.52%
	Bias in the middle of 1σ interval wrt the size of the $1\sigma_{tot.}^{ref}$ interval	-4.38%	-7.65%	0.0624%
AsimovA22	Bias in the middle of 1σ interval wrt the size of the $1\sigma_{\rm syst.}^{\rm ref}$ interval	-9.43%	-19.3%	0.248%
	Middle of the 2σ interval	0.5207	0.0024881	-1.586
	2σ interval size: 2σ	0.0858	0.000104	1.76
	2σ interval ratio to Nominal	0.997	0.996	1.01
	Fractional change in the 2σ interval size wrt the syst interval: $\left(2\sigma_{\text{tot.}}^{\text{FDS}} - 2\sigma_{\text{tot.}}^{\text{ref}}\right)/2\sigma_{\text{syst.}}^{\text{ref}}$	-0.884%	-1%	2.2%
	Bias in the middle of 2σ interval wrt the size of the $2\sigma_{tot.}^{ref}$ interval	-0.438%	-3.65%	0.0522%
	Bias in the middle of 2σ interval wrt the size of the $2\sigma_{syst.}^{ref}$ interval	-1.34%	-9.18%	0.193%
	Bias in the middle of 2σ interval wrt the size of the $1\sigma_{tot.}^{ref}$ interval	-1.24%	-7.31%	0.0874%
	Bias in the middle of 2σ interval wrt the size of the $1\sigma_{\rm syst.}^{\rm ref}$ interval	-2.67%	-18.4%	0.348%
	Middle of the 1σ interval	0.4539 and 0.5691	0.002494	0.3373 and 2.744
	1σ interval size: 1σ	0.0256 and 0.00624	5.35e-05	0.833 and 0.649
	1σ interval ratio to Nominal	1.01 and 0.924	0.989	1.01 and 1.02
	Fractional change in the 1σ interval size wrt the syst interval: $\left(1\sigma_{\text{tot.}}^{\text{FDS}} - 1\sigma_{\text{tot.}}^{\text{ref}}\right)/1\sigma_{\text{syst.}}^{\text{ref}}$	3.32%	-2.57%	2.49% and $10.2%$
	Bias in the middle of 1σ interval wrt the size of the $1\sigma_{tot.}^{ref}$ interval	3.6% and $-16.1%$	-6.66%	3.96% and $-5.44%$
	Bias in the middle of 1σ interval wrt the size of the $1\sigma_{\rm syst.}^{\rm ref}$ interval	9.38%	-16.2%	13.5% and $-26%$
AsimovB22	Middle of the 2σ interval	0.5084	0.002494	0
10111071522	2σ interval size: 2σ	0.098	0.000107	3.14
	2σ interval ratio to Nominal	0.993	0.991	1
	Fractional change in the 2σ interval size wrt the syst interval: $\left(2\sigma_{\text{tot.}}^{\text{FDS}} - 2\sigma_{\text{tot.}}^{\text{ref}}\right)/2\sigma_{\text{syst.}}^{\text{ref}}$	-2.32%	-2.09%	
	Bias in the middle of 2σ interval wrt the size of the $2\sigma_{tot.}^{ref}$ interval	-0.175%	-3.24%	0%
	Bias in the middle of 2σ interval wrt the size of the $2\sigma_{syst.}^{ref}$ interval	-0.573%	-7.94%	
	Bias in the middle of 2σ interval wrt the size of the $1\sigma_{tot.}^{ref}$ interval	-0.68%	-6.46%	0%
	Bias in the middle of 2σ interval wrt the size of the $1\sigma_{\text{syst.}}^{\text{ref}}$ interval	-1.77%	-15.7%	0%

Table 6.6: Table of the bias for the Martini 1π FDS for both Asimovs when the scaled Asimov method is used. For Asimov B22, results at the lower and upper octant minima are shown.

	1σ		90CL		2σ		3σ	
Data	-2.704	-0.980	-3.066	-0.425	-3.258	-0.174	-3.839	0.493
FDS edges	-2.713	-0.975	-3.073	-0.416	-3.265	-0.165	-3.843	0.501
Shift	-0.0092	0.0050	-0.0073	0.0089	-0.0064	0.0086	-0.0048	0.0075

Table 6.7: δ_{CP} interval edges from the data and Martini 1π FDS fits and the changes to the edges due to Martini 1π FDS when the scaled Asimov method is used.

6.6.3 Conclusion and outlook

The Martini et al. single pion production is an interesting model for studying the bias from the modeling neutrino interaction model. So far, the results from this FDS show consistency with the results from nominal fit. A slight bias in Δm_{23}^2 was reported in this study, and it is suggested to be included in the calculation of the smearing. In general, all the biases found in the reported oscillation parameters are not big enough to change any constraints given by T2K on them. The conclusion of excluding $\delta_{CP} = 0$ at 3σ level is still safe, and the path to the discovery of CP violation in the future is promising for T2K and HK.

Even though there are apparent differences between the cross-section prediction from Nieves et al. and Martini et al. model in the $CC1\pi$ channel, the fit results of the oscillation parameters in the two cases are slightly different. The main reason is that the current primary sample used in T2K oscillation analysis is CCQE, in which the $CC1\pi$ does not significantly contribute to the events sample. Shortly, however, more data samples will be taken into account for the far detector fit, such as the multi-ring sample. At that stage, the single pion production is among the channels that will be more crucial and should be precisely modeled. To prepare for this, a reweight framework was developed to reweight not only the single pion production channel but also the CCQE, 2p2h and coherent channels to the Martini et al. model. This framework produces a complete Martini et al. fake data set for the following potential fake data studies by T2K in the near future.

In general, the fact that FDS do not give large bias indicates also that the T2K's method of doing the oscillation analysis is robust with respect to possible unknowns in the neutrino cross-section models.

Conclusion

Neutrino physics is one of the main fields in particle physics that has gained much attention, especially after the discovery of neutrino oscillation. This discovery has paved the way for the studies of physics beyond the Standard Model, such as the neutrino mass and the potential CP violation in the leptonic sector. The neutrino physics community has made great efforts to study the mechanisms behind neutrino oscillation, particularly by constraining the oscillation parameters that drive this phenomenon. Neutrino physics has entered the precision era; apart from precise measurements of oscillation parameters, the long-baseline experiments also aim to identify the neutrino mass hierarchy and amount of CP violation observed in the neutrino sector. In order to achieve these goals, current and future long-baseline experiments need to reduce the systematic uncertainty to a few per cent. One of the key challenges in reaching such accuracy is our limited understanding of few-GeV neutrino-nucleus interactions. The uncertainties stem from neutrino-nucleus interaction are already the main systematic uncertainties on current neutrino oscillation analysis, such as the one of T2K, and will remain the primary restriction if a better comprehension of the nuclear effect is not attained. Some experiments are dedicated to constraining the neutrino interaction systematic down to 1%, such as the ENUBET experiment. However, these interactions systematic can also be improved by the T2K experiment itself using the upgraded near detector ND280.

The ND280 is designed to constrain the neutrino flux and neutrino cross-section. It decreased systematic uncertainty from 15% to 4% for far-detector event rates. The ND280 is under an upgrade project, and the newly upgraded sub-detectors will be installed at the beginning of 2023. With the new configuration, the target mass of the upgraded ND280 is significantly enhanced with a 2-ton fully-active SuperFGD. Moreover, the upgraded ND280 will become a great tracking device with SuperFGD and High Angle Time Projection Chambers (HA-TPC). These sub-detectors provided the upgraded ND280 with the 4π acceptance, which is equivalent to the far detector. The SuperFGD features a three-dimensional structure made up of scintillating cubes with optical read-out channels, allowing reconstruction of the track in 3D. The SuperFGD will increase particle detection capabilities due to its 3D structure, particularly for low momentum protons and neutrons. The resistive MicroMegas technology is another innovative detection method used by the upgraded ND280. The resistive MicroMegas will be installed in the two horizontal HA-TPCs, which are placed on top and bottom of the SuperFGD. These HA-TPCs are expected to perform well in track reconstruction, especially for the high angle tracks. Many studies and test beam have been done to test the performance of the HA-TPC.

contributed to the test beam results for HA-TPC, such as the gain uniformity, the drift velocity and especially the deposited energy resolution. The test beam at DESY II facility has proved that the spatial resolution is better than 600 μm , and the energy resolution is better than 9% for all incident angles. These results meet the requirements for upgrading the ND280 TPC.

The main concentration of this thesis is on the importance of the upgraded near detector in the future constraints of the systematic uncertainties, especially the ones stemming from nuclear effects. This thesis described a method for studying nuclear effects in neutrino-nucleus interaction by measuring final state particles' kinematics in muon neutrino charged-current interactions with no pions but at least one proton in the final state - $CC0\pi$ +Np interactions. With the upgraded ND280, the ability to reconstruct the hadronic part is significantly improved. Thanks to this future improvement, a likelihood fit was developed to evaluate the sensitivity of the upgrade to the flux and cross-section model by exploiting the new observables called single-transverse variables. These variables are formed from the kinematics of the outgoing lepton and highest momentum nucleon. They can depict the kinematic imbalance in the plane perpendicular to the incoming neutrino, which is sensitive to several nuclear effects. By exploiting proton and neutron kinematics, it is possible to use another variable as an input for the fitter and as a neutrino energy estimator, which is the final state visible energy (E_{vis}). The E_{vis} provides an excellent neutrino energy resolution and, moreover, good sensitivity to removal energy.

Consequently, the fitter has provided good quantitative sensitivities to key uncertainties such as QE and non-QE parameters thanks to the use of the "transverse momentum imbalance" (δp_T) which is able to well separate QE and non-QE events. The nucleon final state interaction (FSI) is among the new parameters coming from the inclusion of nucleon kinematics. This thesis has proved quantitatively, for the first time, that nucleon FSI and removal energy can be well-constrained thanks to the "transverse boosting angle" $\delta \alpha_T$ and E_{vis} variables, respectively. Apart from the cross-section parameters, the reduction of the flux normalisation uncertainty, depending on the prior uncertainty of the nucleon form factors, has been quantified for the first time using a hydrogen-enhanced sample. Overall, the results show the intriguing feasible model constraints allowed by exploiting nucleon kinematics obtained at the upgraded near detector for future oscillation analysis. These promising constraints focus on fitting with variables related to the hadronic part of neutrino interactions, while the current ND280 fit focuses on the muon kinematic. In the future complete upgrade fits, when the optimal analysis with ND280 Upgrade would be a multidimensional fit including the muon and hadronic kinematics and their correlations, the upgraded ND280 is expected to be more powerful in constraining these systematic uncertainties than what has been shown in this thesis.

Since the neutrino-nucleus interaction is the main systematic uncertainty in neutrino oscillation experiments such as T2K, the precision era of neutrino has put nuclear physics to its limitation. Moreover, with the use of nucleon kinematics in upgraded ND280, the demand for a well-described neutrino-nucleus interaction model is more than ever before. Even the future nucleon data from upgraded ND280 facilitates more robust constraints by itself on the interaction model; a fake data study is still beneficial to see the effects of different modelling on the oscillation parameters' values. The chosen sample for this study is from the pion production channel described by Martini et al. model. The Martini et al. model can describe not only the pion production but also other interaction topologies in a single framework, which

can help avoid the risk of double counting between different models for different topologies. This thesis has described a reweight technique to turn the current T2K prediction of neutrino cross-section to Martini et al. model's cross-section. The cross-section sample used in the reweight tool is the differential cross-section as a function of neutrino energy, transferred energy and the angle of outgoing lepton with respect to the incoming neutrino. The fake data is then produced by reweighting each generated single pion production NEUT event based on the cross-section ratio given by the two models (NEUT and Martini et al.). This fake data study aims to evaluate the robustness of the oscillation analysis to modelling in the neutrinonucleus interaction model outside of the recommendation by T2K. The impact of Martini et al. CC1 π fake data study on several oscillation parameters (δ_{CP} , $\sin^2 \theta_{23}$, Δm_{23}^2) is quantified in Chap. 6. One of the main results is that the shift on δ_{CP} 1D $\Delta \chi^2$ distribution caused by this fake data is not significant enough to cause a change in the T2K conclusion of δ_{CP} result. Overall, the result of oscillation fit with the Martini et al. one pion production shows consistent oscillation parameters' values with the published results of T2K. Moreover, a reweight tool has been developed to produce fake data for different interaction channels, which facilitates a full fake data study of the Martini et al. model for all interactions currently used in T2K analysis.

In conclusion, T2K has delivered world-leading measurements of neutrino oscillation parameters. The work given in this thesis has aided in determining the impact of new detecting technologies in upgraded T2K near detector ND280, as well as confronting our limited understanding of neutrino interaction. All these results can help to reduce the systematic uncertainties for current long-baseline oscillation experiments such as T2K and next generation of the same kind such as Hyper-K and DUNE.



A.1 Comparison between Martini et al. and Nieves et al. prediction

This section shows the comparison between the two mentioned model for different interaction modes apart from the single-pion production which is used for the fake data study in chapter 6. Fig. A.1,A.2 show the cross-section for QE channel. While the npnh and coherent cross-section are showed in Fig. A.3,A.4 and Fig. A.5,A.6, respectively.



Figure A.1: Total and differential cross-section per nucleon of muon neutrino (left) and muon anti-neutrino (right) for QE channel. The comparison was done between Martini et al. model and Nieves et al. model (denoted by NEUT).



Figure A.2: Total and differential cross-section per nucleon of electron neutrino (left) and electron anti-neutrino (right) for QE channel. The comparison was done between Martini et al. model and Nieves et al. model (denoted by NEUT).



Figure A.3: Total and differential cross-section per nucleon of muon neutrino (left) and muon anti-neutrino (right) for npnh channel. The comparison was done between Martini et al. model and Nieves et al. model (denoted by NEUT).



Figure A.4: Total and differential cross-section per nucleon of electron neutrino (left) and electron anti-neutrino (right) for npnh channel. The comparison was done between Martini et al. model and Nieves et al. model (denoted by NEUT).



Figure A.5: Total and differential cross-section per nucleon of muon neutrino (left) and muon anti-neutrino (right) for coherent channel. The comparison was done between Martini et al. model and Nieves et al. model (denoted by NEUT).



Figure A.6: Total and differential cross-section per nucleon of electron neutrino (left) and electron anti-neutrino (right) for coherent channel. The comparison was done between Martini et al. model and Nieves et al. model (denoted by NEUT).

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