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# Resource Allocation and Pricing in 5G Network Slicing

Mandar Datar

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# THESIS

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in

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*Agrosciences & Sciences*  
*Sciences, Technologies, Santé*  
Speciality : *Computer Science*

By

**Mandar Datar**

## **Resource Allocation and Pricing in 5G Network Slicing**

**Research Unity : EA 4128 LIA – Laboratoire Informatique d'Avignon**

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---

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**Title:** Resource Allocation and Pricing in 5G Network Slicing

**Keywords:** 5G Network Slicing, Kelly mechanism, Coupled constrained game, Normalized Nash equilibrium, Fisher market, Trading post mechanism

**Abstract:** Network slicing is one of the potential technologies to support a higher degree of heterogeneity and flexibility required by next-generation services in 5G networks. In 5G environments, a network slicing is a specific form of virtualisation that allows multiple logical networks (e.g., Mobile Virtual Network Operators (MVNOs)), to run on top of a shared physical infrastructures. In emerging 5G mobile technology, network design also incorporates data-centers into their plan to support computation offloading and network function virtualization. Thus, a slice will often comprise different resource types. (e.g., radio resource, CPU, memory, bandwidth). That implies that a heterogeneous set of resources is shared among Slice tenants or MVNOs, and a portion of them is allocated to each slice to support dedicated service to their customers. The core challenge in this context is to determine at once the price of the available of heterogeneous resources and their assignment across different slices. This thesis presents different novel resource allocation and pricing models for 5G network slicing

First, we devise a flexible sharing mechanism based on a bidding scheme which is provably overbooking-free even though the players' bids are oblivious to infrastructure resources constraints. The proposed scheme can attain desirable fairness and efficiency figures to serve slice tenants and associated mobile users. This goal is attained by designing two coupled games entangled by the same Nash equilibrium. The first is a virtual game that generates the vector of prices of resources, and for which there exists a unique generalised Nash equilibrium. The Infrastructure Provider (InP) can use the price vector to drive the second game, a multidimensional Kelly mechanism based on the so-determined prices, where customers acquire a slice of resources at a price. We finally describe how to attain the Nash equilibrium of the game using an online procedure based on a primal-dual distributed algorithm.

In the second work, we propose a flexible resource allocation and pricing scheme for slicing based on a combination of the Fisher market model and the Trading post mechanism. By

properly pricing network resources, the desired allocation can be attained as a market equilibrium solution that not only maximizes network resource utilization but also assigns slice tenants with their favourite bundle. To make the scheme practically viable and enable tenants to reach market equilibrium in a decentralized manner, we devise budgets distributing rule via trading post mechanisms that hand over tenants direct control to manage their preferences over resources under budget constraints. We theoretically evaluate the efficiency and fairness of the resulting allocations by comparing them with different baseline allocations.

In our third endeavour, we study the business aspect of network slicing with a communication marketplace where slice tenants are in double-sided competition with each other. One competition is in terms of quality of service to attract the end-user to their services, and the second is to access the limited network resources for service provisioning. We model the competitive interaction between service providers (leaders) and end-users (followers) with the Stackelberg game where end-users decide to choose their subscribers through imitation process resulting in competition between the SPs as a multi-resource Tullock rent-seeking game. To determine resource pricing and allocation, we design two innovative market mechanisms. First, we assume that the Service providers (SPs) are pre-assigned with fixed infrastructure shares (budgets) and rely on a trading post mechanism to allocate the resource. Under this mechanism, the SPs can redistribute their budgets in bids and customise their allocations to maximise their profits. We investigate the strategic behaviour of SPs with a noncooperative game, which admits a unique Nash equilibrium when dealing with a single resource. Second, when SPs have no bound on their budget, we cast the problem as a coupled constraints game and show that the market prices can be obtained as the duals of the coupling constraints. Finally, we provide with different learning algorithms to compute solutions to the proposed market mechanisms.

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**Titre :** Allocation des ressources et tarification dans 5G Network Slicing

**Mot clés :** 5G Network Slicing, Mécanisme, Jeu contraint couplé, Équilibre de Nash normalisé, Marché des Fisher, Mécanisme de trading post

**Résumé :** Le "network slicing" est l'une des principales technologies permettant de répondre aux exigences de la vision orientée services des réseaux 5G, à savoir, gérer une hétérogénéité de réseau élevée tout en garantissant certain degré de flexibilité. Dans les applications 5G, le network slicing représente un mode particulier de virtualisation permettant, à titre d'exemple, aux opérateurs de réseau mobile virtuel (MVNOs) et aux fournisseurs de services (SP), etc., de déployer chacun ses propres services au moyen d'une infrastructure physique partagée. Dans la technologie mobile 5G émergente, la mise en place de réseaux prévoit également l'intégration de centres de données afin de prendre en charge les calculs intensifs et la virtualisation des fonctions du réseau. Ainsi, une slice de réseau comprendra souvent différents types de ressources (par exemple, ressources radio, CPU, mémoire, bande passante, etc.), cela sous-entend qu'un ensemble hétérogène de ressources est partagé entre les détenteurs de slices, et une partie de celles-ci est allouée à chaque tranche pour lui permettre de fournir un certain service à ses utilisateurs. Le défi fondamental dans ce contexte est de déterminer à la fois le prix des ressources hétérogènes disponibles et leur affectation entre les différentes slices. Cette thèse présente différents modèles novateurs d'allocation de ressources et de tarification pour le 5G network slicing.

Dans un premier temps, nous élaborons un mécanisme de partage flexible basé sur un système d'enchères qui est démontré sans surréservation et ce même si les offres des joueurs ignorent les contraintes liées aux ressources. Le système proposé peut atteindre une équité et une efficacité adéquates permettant de répondre aux besoins des détenteurs de slices et des utilisateurs mobiles associés. Cet objectif est atteint par la création de deux jeux couplés qui sont liés par le même équilibre de Nash. Le premier est un jeu virtuel qui détermine le vecteur des prix des ressources, et pour lequel il existe un équilibre de Nash généralisé unique. Le fournisseur d'infrastructure peut ainsi utiliser le vecteur de prix pour piloter le deuxième jeu qui est un mécanisme de

blement déterminés. Finalement, nous détaillons la méthode pour atteindre l'équilibre de Nash du jeu en utilisant un algorithme primal-dual distribué.

Ensuite, nous proposons un schéma d'allocation des ressources et de tarification flexible pour le découpage en tranches reposant sur la combinaison du modèle de Fisher market et du mécanisme de trading post. En tarifant correctement les ressources du réseau, l'allocation recherchée peut être atteinte en tant que solution d'équilibre du marché qui non seulement maximise l'utilisation des ressources du réseau, mais aussi d'attribuer aux détenteurs de slices leur forfait (or paquet) de préférence. Afin de rendre le système pratique et de permettre aux détenteurs de slices d'atteindre l'équilibre du marché de façon décentralisée, nous concevons une règle de répartition des budgets via les mécanismes de trading post qui confèrent aux détenteurs de tranches un contrôle direct pour gérer leurs préférences sur les ressources sous contraintes budgétaires. Nous évaluons théoriquement l'efficacité et l'équité des allocations obtenues en les comparant à différentes allocations de référence.

En dernier lieu, nous étudions l'aspect commercial du Network slicing dans le cadre d'un marché de la communication où les détenteurs de tranches, c'est-à-dire les fournisseurs de services, sont en double concurrence les uns avec les autres. Une concurrence se fait en termes de qualité de service pour inciter l'utilisateur final à utiliser leurs services, et la seconde est pour accéder aux ressources limitées du réseau pour la fourniture de services. Nous modélisons l'interaction concurrentielle entre les fournisseurs de services (leaders) et les utilisateurs finaux (suiveurs) par le jeu de Stackelberg où les utilisateurs finaux décident de choisir leurs abonnés par le biais d'un processus d'imitation, ce qui entraîne une concurrence entre les fournisseurs de services sous la forme d'un jeu Tullock rent-seeking multi-ressources. Pour déterminer la tarification et l'allocation des ressources, nous élaborons deux mécanismes de marché innovants. Premièrement, nous supposons que les fournisseurs de services se voient attribuer à l'avance

des parts d'infrastructure fixes (budget) et utilisent un mécanisme de trading post pour allouer les ressources. Suivant ce mécanisme, les fournisseurs de services peuvent redistribuer leurs ressources dans les enchères et modifier leurs allocations pour maximiser leurs profits. Nous étudions le comportement stratégique des fournisseurs de services à l'aide d'un jeu non coopératif qui admet un équilibre de Nash unique lorsqu'il

s'agit d'une seule ressource. Ensuite, lorsque les fournisseurs de services n'ont pas de limite budgétaire, nous considérons le problème comme un jeu à contraintes couplées et montrons ainsi que les prix du marché peuvent être obtenus en tant que duals des contraintes de couplage. Enfin, nous proposons trois algorithmes d'apprentissage différents destinés à déterminer les solutions aux mécanismes de marché proposés.

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# INTRODUCTION

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Communication technology has been playing an essential role in society's digitalization and is a significant contributor to a growing economy worldwide. The unprecedented increase in mobile devices has given rise to the explosive growth of mobile data traffic. Looking towards the future, indeed, mobile communications is expected to grow and extend its utilization in a whole new generation of applications like Virtual Augmented Reality (AR), Virtual Reality (VR) live broadcast [93], Internet of things (IoT) [15], Autonomous driving [64], remote healthcare [2], automated manufacturing based on smart factories [21], sectors where its potential is not fully being realized. Critical challenges in mobile network applicability to the sectors mentioned above are their heterogeneity and conflicting communications needs, that the current monolithic network is insufficient to meet. For example, automotive and healthcare require ultra-reliable services or extremely low latency, whereas the Virtual Reality (VR) live broadcast needs ultra-high-bandwidth communication. The 5G network design should incorporate the capabilities that satisfy all these heterogeneous and conflicting requirements simultaneously. Several new concepts have been proposed for the upcoming 5G network design to satisfy these critical needs. Out of those, probably one of the most important one is "network slicing"[40]. Briefly, network slicing is the concept of running multiple independent logical networks (slice) on top of the common shared physical infrastructure. Each independent logical network (slice) is then explicitly dedicated to meeting each service's needs, contrary to the approach "one-size-fits-all," that was the mainstream approach in the previous mobile generations [40]. Network slicing facilitates slice tenant to share the same physical infrastructure in a flexible and dynamic manner that helps to utilize the resources in a more efficient and economical way

The next-generation mobile applications/services like AR and VR broadcast demand more data-intensive operations than those required by traditional mobile applications. Therefore, to cope with the requirements of additional computational power and memory resources for such services, 5G mobile network design should also incorporate the data-centres or cloud computing capabilities into their plan. However, along with the increasing computational demands on networks, many applications like self-driving cars [114][60], augmented reality, and drones [16] also demand ultra-fast interactions/quick responses. This leads to placing some of the computational resources closer to the base stations, or the edges of the network, in the form of what is known as "multi-access edge computing (MEC) [51] also know as edge computing/fog computing . This means that you will have the service at a location that is closest to where the data is generated and processed. Moreover, it

can significantly lower data transfer costs by processing the information locally. To that end, in next-generation mobile networks, data centres and edge computing are being integrated into the network architecture to provide an integrated service delivery platform for various services. As a result, a network slice is usually composed of diverse heterogeneous resources, including radio access capacity or communication resources, edge storage memory, and computational resources etc. Unlike traditional cloud servers with infinite virtual capacity, the edge servers are capacity-limited. Moreover, the resource requirements of emerging services are diverse and conflicting in nature. Which makes it's challenging for the Infrastructure provider to jointly price and share resources assigned simultaneously across slices.

In this thesis, we study different novel resource allocation and pricing models for network slicing. The pricing and resources allocation scheme have been very well studied in literature [97, 117, 68, 57, 25, 108, 56], assuming agents would demand several resources at once using vectors of bids and so specify their demands. However, compared to standard settings in cloud computing, 5G networks have significant differences, to mention some of them. First, the resources are often scarced in 5G deployment compared to the resources in cloud computing. Second, mobile networks are traditionally designed for fair sharing of resources [97, 117]. Since near-far effects and fading induce different conditions across a deployment in the same cell and across cells, it is more challenging to share resources. Third Load conditions in wireless networks may be both dynamic and heterogeneous. Finally, The joint slice allocation and pricing schemes must adhere to SLAs [43].

The second chapter of the thesis presents a novel theoretical framework for pricing resources for network slices based on the Kelly mechanism [52]. In our framework, slice tenants compete for resources, submit bids for each resource type in slices, and pay depending on how much they bid at a price per resource set by the InP. However, adopting bidding schemes may be prone to overbooking because prices are defined via competitive bidding schemes, and aggregated demand at set prices may exceed available infrastructure capacity. Thus, under such a scenario where the agents act greedily, the InPs must design a proper overbooking pricing scheme to achieve social efficiency. In this chapter, we develop the resource allocation and pricing problem using the game-theoretic framework. Our study demonstrates that overbooking free pricing can be developed based on the concept of the normalized Nash equilibrium [88], or variational equilibrium [88, 33, 55] solution for the coupled constraints non-cooperative game. We observe that agents competing for the resources through the Kelly mechanism induce a non-cooperative game, and the solution for this game may be practically infeasible *i.e.* total demand at Nash equilibrium of game may exceed infrastructure capacity. We develop a new game for resource allocation called "Pricing game ", where agents compete for the available resources taking into account infrastructure capacity constraints. We extend the duality results from standard continuous optimization to non-cooperative games [85], and show that the dual variables of the coupling constraints in the new formulated game can be seen as resource prices. Finally, combining the two games (a game induced through the Kelly mechanism and the

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Pricing game ) into an Extended Pricing Game that also involves the InP maximizing its profit and dictating at once the respect of capacity constraints. The latter the game can be implemented in practice using online learning via a decentralized bidding scheme where the InP refines the prices per resource and a primal-dual converges to the target normalized equilibrium.

The network slicing brings the paradigm shift towards the multitenancy ecosystem, where simultaneously multiple tenants can operate on the shared physical infrastructure to run their services [11]. When the networking architectures are based on shared resources, it may cause security and scalability issues. For example, slice tenants naturally demand logical independent and isolated slices with complete protection of their service level agreement (SLA). That may require a static allocation of resources at the cost of inefficient resource utilization [79]. However, The tenants loads may vary with time and can be spatially inhomogeneous; in such a case, dynamic sharing of resources is regarded as one of the efficient ways to share the resources as this lowers the capital cost and gives better resources utilization [59, 69]. Therefore, one of the critical concerns in 5G network slicing is how to efficiently and dynamically allocate limited resources to tenants with diverse characteristics and services while maintaining the protection of their service level agreements.

To address the above issues, third chapter of the thesis proposes a flexible resource sharing scheme based on a combination of the Fisher market model [17] and the Trading post mechanism [96]. In the proposed scheme, each slice tenant (service provider) is pre-assigned with a fixed share (budget) of infrastructure depending on their service level agreement (SLA) with infrastructure provider (InP). InP dynamically sets the prices for the resources depending on their demand and availability. Given prices announced by InP, slice tenants spend their budgets to purchase the optimal bundle of resources that maximizes their utilities. Where particularly utilities capture the effect of fairness and quality of service employed by the service providers while delivering the service to their users. The proposed allocation scheme aims to obtain resource allocation and prices functioning as market equilibrium, where every slice is satisfied with their assignment, and all the resources get clear.

To obtain the desired resource allocation as the market equilibrium of the the Fisher market model, we leverage the idea of the Esenberg Gale optimization program [31] [32] and frame the equilibrium solution as a convex optimization problem. This problem's primal and dual solution pair provide us with the equilibrium allocation and prices. However, solving this problem in a centralized way demands service providers to disclose their private information (utilities). Thus to solve the equilibrium problem in a decentralized manner and make the proposed resource allocation scheme practically viable, we implement it via the Trading post mechanism. In this mechanism, slices distribute their network shares on different resources in bids depending on their requirements. Each resource's price is set as the total bids submitted for that resource, and the resource is allocated accordingly to SPs bids. This approach regulates the tradeoff between efficient resource utilization and the degree of protection to SLA. On the one hand, it enables dynamic sharing, where tenants

can redistribute their network share based on the active user load; on the other hand, it also provides the slice tenant degree of protection by keeping the preassigned share intact throughout the allocation process.

As discussed earlier, a paradigm shift is brought about by the implication of network slicing, where multiple tenants negotiate with multiple InPs to request resources for their service provision. New players like mobile service providers and virtual network operators are gaining ground by using the existing physical infrastructure to run their virtual, independent business operations. If the service providers want to grow their market share, they need to attract end-users with better quality service and low tariffs. End-users will choose to subscribe to the service provider that provides better service quality and lower tariffs. The resource inventory available with the service provider characterizes their service performance; the more the resources available with the service providers, the better the service they can offer. However, the resources available with the infrastructure provider are limited, resulting in double-sided competitive interaction between the service provider, one interaction due to competition for service provision, and the second interaction due to procurement of resources. Undoubtedly, allocating and pricing resources in such a double sided competitive environment is a significantly challenging task.

In the fourth chapter of the thesis, we propose a novel resource pricing and allocation scheme for the communication marketplace, where service provider *i.e.*, slice tenants relying on slicing access the resources from infrastructure provider and compete to offer a certain communication service to a geographically distributed pool of end-users. First, we model the interaction between end users and service providers as Stackelberg game [99] where service providers act as leaders and end-users as followers. In communication service marketplace, the influence of user's choices on other users is a well-known concept. If users are informed about other user's choices, which is assumed to improve their service satisfaction, they imitate other user decisions. Thus we model the interactions between end-users using the replicator dynamics [94]. In comparison, the competition between service providers results in a Tullock rent-seeking game [102] where service providers anticipate the behaviour of end-users and attempt to attract them to service by investing resources. To that end, we propose two resource allocations and pricing schemes.

The first resource allocation scheme is rooted in a trading post mechanism where service provider distributes their budgets on different resources and the price of each resource is set as the total shares submitted on that resource; resources are allocated accordingly to the service providers shares. Trading post mechanism together with original rent seeing game induces new game. We analyze the existence and uniqueness of the new game and show the presence of pricing and allocation, which satisfies the service provider and utilizes all the resources at once. For the second resource allocation scheme, we formulate the virtual game we call a pricing game, where the service providers take into account the infrastructure constraints while making their decisions. We extend the duality results from standard continuous optimization to the virtual game. The dual

variable linked to coupled constraints in the pricing game can be interpreted as market price, also known as shadow price. The Infrastructure provider uses these shadow prices to charge the service providers depending on how much they contribute to infrastructure utilization. Finally, we provide the three learning algorithms to implement the proposed pricing and allocation schemes. The first proposed algorithm is an exponential distributed algorithm based on dual averaging or mirror descent technique, enabling service providers to get a resource allocation and pricing as the Nash equilibrium of game induced through trading post mechanism. The second learning algorithm is a decentralized algorithm that is based on an extension of the dual averaging technique to coupled constrained game. This algorithm allows service providers and InP to reach stable allocation and market prices as a variational Nash equilibrium of pricing game.

To the best of our knowledge, in the contest of network slicing, the resource allocation and pricing schemes presented in this thesis have not been studied before. The developed schemes/mechanism and algorithms can help the Infrastructure provider and slice tenants/service providers, to efficiently utilize the network resources while maximizing their benefits from the network slicing environment.

## 1.1 Outline of Thesis

The rest of this thesis is organized as follows:

- In Chapter 2, we developed an overbooking-free resource sharing mechanism based on a bidding scheme. This goal is attained by designing two games coupled with the same Nash equilibrium. The first game is induced through a multidimensional Kelly mechanism is called as an Allocation game. This game is prone to overbooking, where total resource demand by the players might violate the infrastructure capacity. The second game is a virtual coupled constraints game called the “Pricing game”, where players consider infrastructure capacity while making decisions.

In the Pricing game, each player’s admissible strategy set is coupled with other players’ strategies through the capacity constraints. The Pricing game is a special case of the generalized Nash equilibrium problem (GNEP). It is a noncooperative game in which each player’s admissible strategy set depends on the other players’ strategies [8]. We show that the Pricing game admits the generalized Nash equilibrium, and the Lagrange multiplier associated with coupled constraints at NE of the Pricing game can be treated as a resources’ price vector. We show that the infrastructure provider can drive the allocation game to a socially optimal allocation by employing a price vector generated by the Pricing game. We provide an online procedure based on a primal-dual distributed algorithm that helps players reach the game’s Nash equilibrium. Finally, numerical results demonstrate the applicability of the described theoretical framework as a general pricing scheme.

- In Chapter 3, we consider a scenario where service providers or slice tenants need heterogeneous resources at geographically distributed locations to support the service for their end-users. We propose a resource sharing scheme based on the Fisher market model and the Trading post mechanism. In the proposed scheme, each slice owns the budget representing its infrastructure share or purchasing power in the market. The slices acquire different resources by spending their budgets to offer the service to different classes of users, which are classified according to their service needs and priorities. We assume that service providers employ the well know  $\alpha$  fairness criteria [73] while delivering the service to their subscribers. The proposed allocation scheme aims to find a market equilibrium that provides allocation and resource pricing where each slice is satisfied with allocation and resources to be fully utilized. We show that the market equilibrium solution problem can be formulated as a convex optimization problem whose primal and dual optimal solution provides equilibrium allocation and pricing. We build a decentralized algorithm based on a convex optimization problem and potential function technique and proportional sharing rule that enables service providers to reach the market equilibrium in a decentralized fashion. We theoretically evaluate the proposed allocation scheme's performance by comparing it with the Social Optimal and Static Proportional allocation schemes. Finally, we run numerical simulations to analyse the fairness and efficiency properties of the proposed scheme.
  
- In Chapter 4, we consider a marketplace where service providers, i.e., slice tenants, are in competition for the access to the network resource owned by an infrastructure provider who relies on network slicing. We model the interactions between the end-users (followers) and the service providers (leaders) as a Stackelberg game. We prove that the competition between the service providers results in a multi-resource Tullock rent-seeking game. To determine resource pricing and allocation, we devise two innovative market mechanisms. In the first mechanism, we assume that the service providers are pre-assigned with fixed shares (budgets) of infrastructure, and rely on a trading post mechanism to allocate the resource. Under this mechanism, the service providers can redistribute their budgets in bids and customise their allocations to maximise their profits. We prove that their decision problems give rise to a noncooperative game, which admits a unique Nash equilibrium when dealing with a single resource. In the second mechanism, we consider that the service providers have no bound on their budget, we formulate the problem as a Pricing game with coupling constraints and derive the market prices as the duals of the coupling constraints. In addition, we prove that the Pricing game admits a unique variational equilibrium. We propose two online learning algorithms to compute solutions to the market mechanisms. A third fully distributed algorithm based on a proximal method is proposed to compute the variational equilibrium solution to the Pricing game. Finally, we run numerical simulations to analyse the economic properties of the market mechanisms and the convergence rates of the algorithms.

- In Chapter 5, summary and concluding remarks are provided and possible future research directions are discussed.

## 1.2 List of publications

This thesis is based on the following publications:

1. **Mandar Datar**, Eitan Altman, Francesco De Pellegrini, Rachid El Azouzi, Corinne Touati. "A Mechanism for Price Differentiation and Slicing in Wireless Networks", *2020 18th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOPT)*, Volos, Greece, 2020, pp. 1-8.
2. Eitan Altman, **Mandar Datar**, Gerard Burnside and Corinne Touati. "Normalized Equilibrium in Tullock Rent Seeking", *Game Theory for Networks. GameNets 2019, Lecture Notes of the Institute for Computer Sciences, Social Informatics and Telecommunications Engineering*, vol 277. Springer, Cham
3. **Mandar Datar**, Eitan Altman. "Strategic Resource Management in 5G Network Slicing", *In 2021 33th International Teletraffic Congress (ITC-33)*, pp. 1-9. IEEE, 2021.
4. **Mandar Datar**, Eitan Altman, H el ene Le Cadre. "Strategic Resource Pricing and Allocation in a 5G Network Slicing Stackelberg Game." *Submitted to IEEE Transactions on Network and Service Management*

Other publications by the author, not included in this thesis, are:

1. **Mandar Datar**, Eitan Altman, Ghilas Ferrat. "Routing into parallel collision channels", *ACM SIGMETRICS Performance Evaluation Review*, September 2020, volume 48,2, 15-17
2. Eitan Altman, **Mandar Datar**, Francesco de Pellegrini, Samir Perlaza, Daniel Sadoc Menasche, "The Mask Game with Multiple Populations", *Dynamic Games and Applications (DGAA)*, 2022
3. Naresh Modina, **Mandar Datar**, Rachid El-Azouzi, Francesco de Pellegrini. "Multi Resource Allocation for Network Slices with Multi-Level fairness", *Accepted at IEEE International Conference on Communications (ICC)*, 2022
4. Francesco De Pellegrini, Francescomaria Faticanti, **Mandar Datar**, Eitan Altman, Domenico Siracusa. "Optimal Blind and Adaptive Fog Orchestration under Local Processor Sharing", *2020 18th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOPT)*, Volos, Greece, 2020, pp. 1-8.
5. Eitan Altman, Ghilas Ferrat, **Mandar Datar**. "Maximizing amount of transferred traffic for battery powered mobiles." *In International Conference on Network Games, Control and Optimization*, pp. 78-84. Springer, Cham, 2021.
6. Eitan Altman, Alexandre Reiffers, Daniel S. Menasche, **Mandar Datar**, Swapnil Dhamal and Corinne Touati "Blockchain Competition Between Miners: A Game Theoretic Perspective" *Frontiers Blockchain*, volume 2, 2019

7. Eitan Altman, Alexandre Reiffers, Daniel S. Menasche, **Mandar Datar**, Swapnil Dhamal and Corinne Touati, "Mining competition in a multicryptocurrency ecosystem at the network edge: a congestion game approach" *Symposium on Cryptocurrency Analysis, 36th International Symposium on Computer Performance, Modeling, Measurements and Evaluation 2018*



# OVERBOOK-FREE PRICING FOR 5G SLICING VIA NORMALISED NASH EQUILIBRIA

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## 2.1 Introduction

Next generation mobile networks will employ slicing in order to provide multiple tenants with a shared resource pool served through a common infrastructure. In this context, it is expected that resources in 5G networks will encompass both computing and communication resources. Slices comprise different resource types, such as fog/edge storage memory or computing power available within the Infrastructure provider's core network, as well as radio access capacity [112]. In fact 5G is integrating Network Function Virtualization (NFV) technologies with new Edge and Fog computing paradigms. Thus, mobile networks will be able grant a whole set of communication and computing resources so as to meet the demands of emerging mobile services. Actually, the most credible business scenarios in 5G networks predict the presence of several mobile virtual network operators (tenant) leveraging a slice of resources obtained from a common Infrastructure Provider (InP). A tenant will offer mobile services to their customers in the form of one or more mobile applications comprising a set of distributed microservices, e.g., for real time gaming, multimedia applications, social networks, etc. Slicing will be key to lower the entry barrier for tenant network operators who do not possess a computing and communication infrastructure.

However, while at the business level high expectations are following slicing standardisation [1], such technology is not yet sustained by a well accepted pricing model for resource slices [10, 107]. In fact, crucial aspect is how to jointly price resources assigned simultaneously across a slice. Actually, in mainstream cloud computing literature, pricing of shared resources is a customary topic [97, 117]. Several works have proposed to use multi-dimensional bidding models to encompass multiple resources and prescribe that customers would demand several resources at once using vectors of bids. Compared to posted prices, bidding is indeed a very flexible scheme suitable for the time-varying demands typical of mobile networks [19, 116]. However, porting bidding schemes to 5G networks poses crucial difficulties for a series of technological reasons. First, traffic in 5G networks

involves invariably radio access and shared radio resources have critical impact on QoE. This is due to performance figures related to delay and throughput experienced by end users. In cloud computing, conversely, the pool of resources is virtually infinite, since datacenters are overprovisioned for both computing and communication tasks. Second, radio resources allocation is prone to near-far effects so that every mobile user is bound to experience very different channel conditions over time. Hence, pricing schemes need to account for fair resources allocation [73, 4] which represents a de-facto standard (up to the point a built-in proportional fair scheduler is considered state-of-art for any modern basestations). Finally, and perhaps more important, all network operators need to comply to standard Service Level Agreements(SLA) [43] which are binding to minimum performance figures – e.g., minimal nominal throughput – whereas in cloud computing availability is the main target [97, 117, 68].

Once an SLA is signed between a tenant and a InP, a tenant could specify a minimum acceptable amount of resources she needs in order to maintain mobile services operational. However, adopting bidding schemes in 5G networks may be prone to overbooking, because they define prices via competitive bidding schemes, and aggregated demand may exceed available resources, that is the infrastructure capacity in terms of bandwidth, memory or storage. In this work we develop a theoretical framework where slices of resources are assigned to tenants based on a Kelly mechanism [48], an aggregate game where players bid for a quantity and obtain an amount of resource proportional to their bidding values. In our context, a bid for a slice is actually a vector of bids for each resource type in a slice (bandwidth, memory or storage), and tenants pay depending on how much they bid at the price per resource set by the InP.

### 2.1.1 Related work

Mechanisms to price and share resources have appeared in literature [97, 117, 68, 57, 25, 108, 56], assuming customers would demand several resources at once using vectors of bids and so specify their demands. In an auction mechanism-based resource allocation scheme, agents bid for the resources and the price of the resource is determined by auction and resources are then allocated accordingly. One of the well-known schemes for resource allocation in communication networks is the Kelly mechanism [53], where agents participate in an auction and submit individual bids for the resources they need. Resources are then allocated to the agents proportional to their bids. The original model of Kelly, though, is not a bidding scheme. Rather, it solves a network flow optimisation problem [52], where the customers of a network provider maximise their own benefit function. Their strategies are coupled only through the price decided by the network provider, and they act as price takers. The work of Kelly proved the existence of a price, in the form of a multiplier – or shadow price – which attains the social optimum, i.e., the maximum possible value of the sum of the benefit functions of the players. However, when agents are price-anticipating and selfish, the authors in [44] showed that induced non-cooperative game has a unique Nash equilibrium (NE).

Article	Bids	Pricing	Efficiency	Behavior	Min. requirement
Kelly in [53]	Monetary	Uniform	100%	Non strategic	No
Johari and Tsitsiklis in [49]	Monetary	Uniform	$\geq 75\%$	Strategic	No
Yang et al. in [111], Tun et al. in [103]	Monetary	Differential	100%	Strategic	No
Our work, Chapter 2	Quantity	Differential	100%	Strategic	Yes

Table 2.1 – Chapter 2, literature review and research contribution positioning

Techniques for explicit evaluation of the equilibrium with the total demand are used in [67] based on implicit function techniques and for general convex functions. The work in [28] described the Stackelberg equilibrium for the Kelly mechanism with linear costs, and characterised the non-increasing dynamics of the Nash equilibrium with the price. Further, it was deduced that the efficiency loss at the NE in the worst case could be at most 25% of the social welfare [49]. To overcome this efficiency loss, the authors in [111] designed a price differentiation in Kelly mechanism, where agents bid and the resources allocated similarly to the Kelly mechanism; however, the price charged for the resources can vary from agent to agent. The authors showed the existence of differentiation pricing, which maximizes social welfare *i.e.*, the sum of utilities of all the agents. To achieve efficient resource allocation and interslice protection, the wireless network slicing problem was decoupled in two sublevel problems in [103]. The two sublevel problems were solved using the generalized Kelly mechanism (GKM) and optimization technique.

In this chapter, we design a price differentiation Kelly mechanism that maximizes social welfare similar to [111, 103]. However, our work departs from above in the following points. First, agents bid for resources in terms of quantity instead of money. Second, the framework also accommodates cases where there might be the minimum requirement by the agents, or there might be the local constraint on the demand for the resources. Third, to build the differential pricing scheme that maximizes social welfare, we rely on the theory of generalized Nash equilibria or normalized Nash equilibrium by Rosen [88] and generalized diagonal strict concavity [6]. Further, the learning techniques for the convex game with variational equilibrium or normalized Nash equilibrium are well developed [71][75]; thus, we adopt them directly in our framework.

### 2.1.2 Main Contribution

1. We propose a resource allocation scheme based on the solution concept of the generalised Nash equilibrium [88], where strategy sets of players, *i.e.*, the tenants, depend on the others through 1) the constraints on the infrastructure capacity per resource and 2) the minimum

quantity of resources tolerated to run tenant services.

2. We decouple the problem into a price-definition, which is defined as the Pricing Game, and the actual Allocation Game, where slice tenants are actually competing for resources.
3. We show that the theory of [88] grants that the unique solution of this Pricing Game, which is a dual virtual game we define for analysis purposes, offers as a byproduct the prices to be exposed to customers.
4. We combine the two games into an extended Pricing Game, which involves also the InP maximising its profit and dictating at once the respect of capacity constraints.
5. We show that the Pricing game can be implemented in practice using online learning via a decentralised bidding scheme where the InP refines the prices per resources and a primal-dual converges to the target normalised equilibrium.

## 2.2 System Model

Table 2.2 – Main notations used throughout the Chapter 2

$\mathcal{S} = \{1, \dots, S\}$	$\triangleq$	a set of tenants (the slices)
$\mathcal{C} := \{1, \dots, C\}$	$\triangleq$	set of resources (cells)
$b_c^s$	$\triangleq$	the amount of resource $c$ to slice $s$
$V_c^s$	$\triangleq$	the benefit function of slice $s$ by using resource $c$
$V^s$	$\triangleq$	the total benefit function of slice $s$
$x_c^s$	$\triangleq$	bid of slice $s$ for resource $c$
$x^s = (x_1^s \dots x_C^s)$	$\triangleq$	vector of bids of slice $s$ for all resources
$\mathcal{X}_s$	$\triangleq$	Constraint set for bid of player $s$
$x_c^{-s} = \sum_{s' \neq s} x_c^{s'}$	$\triangleq$	bid of all other players but $s$ on resource $c$
$\gamma_c^s$	$\triangleq$	unitary price for bids of slice $s$ on resource $c$
$\alpha_s$	$\triangleq$	alpha fair scheduling parameter for slice $s$
$h_u$	$\triangleq$	channel state of user $u$
$\mathbf{h}$	$\triangleq$	the channel state vector for all users
$r_u$	$\triangleq$	the rate attained by a user $u$
$p_u$	$\triangleq$	transmission power of user $u$
$\beta_n$	$\triangleq$	step size for the learning algorithm

We consider a single InP, who leases its physical resources to a set of a slice tenants ,  $\mathcal{S} := \{1, \dots, S\}$ . Each tenant from set  $\mathcal{S}$  needs some specific service with specific requirements in shared physical infrastructure. Specifically, in this work, we consider the slicing concept where a set of network function instances and the required resources (e.g. memory, CPU, storage compute and networking resources) form a deployed Network Slice and each network slice is administrated by

a tenant. This option is coherent with the definition of a network slice instance given in the Third Generation Partnership Project (3GPP) standard<sup>1</sup>.

Each tenant operating on a given slice provides services to its set of users, and for that, it requires several possible types of resources, noted by  $\mathcal{C} := \{1, \dots, C\}$ . In particular, here, we consider that resource slicing is decomposed into two levels. On higher-level InP allocates resources to tenants while on the lower level, each tenant schedules its acquired resources to meet her users' service requirements. We assume that each slice needs a minimum amount of resources to meet the service requirements. Let each tenant  $s \in \mathcal{S}$  needs  $l_c^s \in \mathbb{R}_+$ , a minimum amount of resource  $c$ . We assume each tenant  $s \in \mathcal{S}$  to receive some benefit  $V_c^s(b_c^s)$  by using the amount  $b_c^s$  of resource  $c$ . The total benefit tenant  $s$  get by using vector  $b^s = [b_1^s \dots b_C^s]$  amount of  $C$  resources is  $V^s(b^s) = \sum_{c=1}^C V_c^s(b_c^s)$ . A standard assumption is for  $V_c^s$  to be strictly increasing and concave. In economic theory [70], this assumption is widely used to signify diminishing returns for increasing the value of the attained resource. In our context, such an assumption is further justified by the behavior of technological artifacts such schedulers or orchestrators, which are used to perform resource sharing among users over time. Resources are in fact, divided into a pool of resources units that are then assigned to customers on a time-shared manner. The classical scheme is a time division multiplexing, which assigns units of physical resources such as memory, CPU storage or bandwidth to attain a particular share. Results in scheduling theory connect the design of long term target benefit functions  $V^s$  to a class of algorithms abler to distribute resources over time in the presence of time-varying resources and demands. Since infrastructure updates occur on a much longer timescale than slices' demands, we assume that the total available resources to the InP are inelastic and given in a full capacity  $B_c$  for each resource  $c \in \mathcal{C}$ . The objective for InP is to seek allocation that maximizes the total valuation of all tenant.

$$\text{SW : } \quad \underset{b}{\text{maximize}} \quad \sum_{s \in \mathcal{S}} V^s(b^s) \quad (2.1a)$$

$$\text{subject to} \quad \sum_{s \in \mathcal{S}} b_c^s \leq B_c, \quad \forall c \in \mathcal{C} \quad (2.1b)$$

$$b_c^s \geq l_c^s, \quad \forall c \in \mathcal{C}, \quad \forall s \in \mathcal{S} \quad (2.1c)$$

The problem SW is termed as *social welfare optimization problem*. The InP can solve the problem (SW) with well-known convex optimization methods, provided the valuation functions  $V^s$  of all tenants are known. However, such information is not available to InP when tenants are strategic. In many cases, even if, when the tenants are not strategic, centralized resource allocation schemes can properly address the InP constraints on the resource allocation, but are known to lack scalability and may lead to excessive communication overhead of the number of tenants. The central question we address in this work is how to design a mechanism that solves the problem (SW) in a

1. 3GPP. System architecture for the 5g system. 3GPP TS 23.501 v1.3.0, Tech. Rep., September 2017.

decentralized manner and without requiring full knowledge of the private valuation of each tenant.

## 2.3 Mechanism Design

We consider an auction-based resource allocation mechanism, in which each tenant  $s$  submits the bid  $x^s = (x_1^s, \dots, x_C^s)$  for each of the  $C$  types of resources. The InP collects all bids for each resource and assigns to each user  $b^s$ , a fraction of each resource corresponding to the ratio it attained given the bids received for that resource, namely the quantity

$$b_c^s := B_c \frac{x_c^s}{\sum_{s' \in \mathcal{S}} x_c^{s'}}, \quad (2.2)$$

where  $B_c$  represents the total capacity or availability of resource type  $c$ . We observe that even resources may be divided into relatively small discrete units corresponding to integer resources units, the mechanism allows for a continuous share of resources to be assigned to each customer. Such assumption is reasonable, in our context, since customers are recurrent, in that the share of resources is attained by scheduling repeatedly a certain pool of units over time to each target user, so that any fraction (2.2) can be attained, e.g., scheduling resource units to users with some probability, which actually corresponds to the current technological practice for most resources, including memory, CPU and bandwidth. The advantage of using this proportional rule is to guarantee the mechanism to be scalable when the number of agents increases [49]. Since the allocation of each tenant is proportional to the ratio of its bid with the total amount of bids, the total benefit tenant  $s$  can get by receiving a share encompassing  $b^s$  of  $C$  resources is  $V^s(b^s)$  with

$$V^s(b^s) = \sum_{c \in \mathcal{C}} V_c^s \left( \frac{x_c^s}{\sum_{s' \in \mathcal{S}} x_c^{s'}} B_c \right). \quad (2.3)$$

For each slice  $s$ ,  $V^s$  is an increasing function in  $b^s$ , without any payment slice tenants will always bid as much as possible in order to increase their own benefit. However, after submitting the bids, each customer pays to the InP the cumulative sum of prices for the bids it made. More precisely, let  $\gamma_c^s$  be the unit cost for bidding for one unit (e.g., one PRB) of resource type  $c$  for slice  $s$ . Then, each slice tenant  $s$  pays  $\gamma_c^s x_c^s$  for the resources obtained at base station  $c$ . In turn, the utility of a tenant  $s$  is defined as the difference between the overall benefit it attained by using the different slice resources and the total cost it needs to pay for using them:

$$U^s(x^s, x^{-s}, \gamma^s) = \sum_{c \in \mathcal{C}} V_c^s \left( \frac{x_c^s}{\sum_{s' \in \mathcal{S}} x_c^{s'}} B_c \right) - \gamma_c^s x_c^s. \quad (2.4)$$

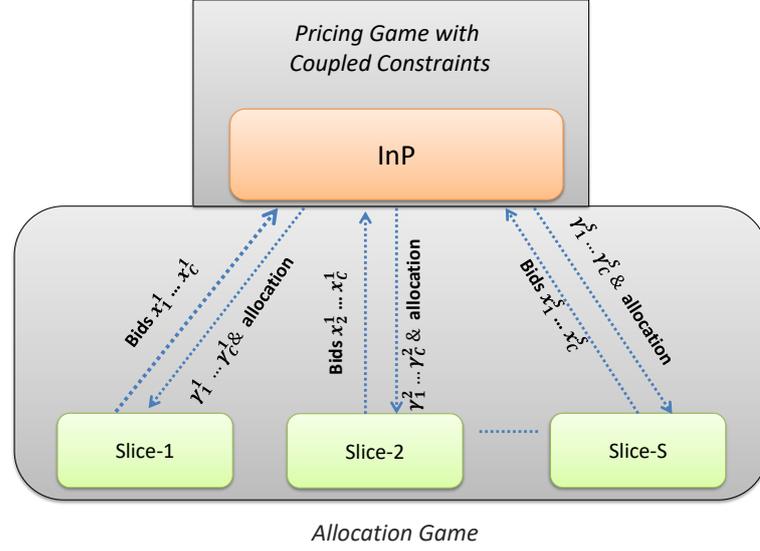


Figure 2.1 – The InP uses the Pricing Game (top) to generate the price vector, whereas the result of the Allocation Game (bottom) decides the actual resource slicing based on the tenants bids.

The tenants are rational players and bid for resources so as to optimise their utility (2.4). Thus, the decision problem of each slice  $s \in \mathcal{S}$  is to find the optimal  $x^s$  optimizing its own utility:

$$Q_s : \quad \begin{aligned} & \underset{x^s}{\text{maximize}} && U^s(x^s, x^{-s}, \gamma^s) \\ & \text{subject to} && x^s \in \mathcal{X}_s \end{aligned}$$

Here set  $\mathcal{X}_s$  is defined by the minimum requirements for resources by tenant  $s$ . Since the utility of each tenant depends on the decisions made by other tenants and they are selfish, the resource allocation mechanism translates as a non-cooperative game where players, i.e., the tenants, compete to acquire resources to increase their utility. We denote this game by the standard notation  $\mathcal{Q} = \{Q_1 \dots Q_S\}$ , and we call it the Allocation game, which represents the first component of our overall the resource allocation mechanism. In the rest of the chapter, we interchangeably use the term player and tenant. To predict the outcome of the allocation mechanism it is customary to use the standard notion of competitive equilibrium, that is the Nash equilibrium

**Definition 2.1.** A multi-strategy  $x^* = (x^{1*}, \dots, x^{S*})$  is called a Nash equilibrium of the game  $\mathcal{Q}$  if

$$\forall s \in \mathcal{S}, U^s(x^{s*}, x^{-s*}) \geq U^s(x^s, x^{-s*}), x_c^s \geq 0, \forall c \in \mathcal{C}. \quad (2.5)$$

with  $\forall s \in \mathcal{S}, x^{s*}, x^s \in \mathcal{X}_s$ , where  $(x^s, x^{-s*})$  is the strategy vector where the  $s^{\text{th}}$  element equals  $x^s$  and

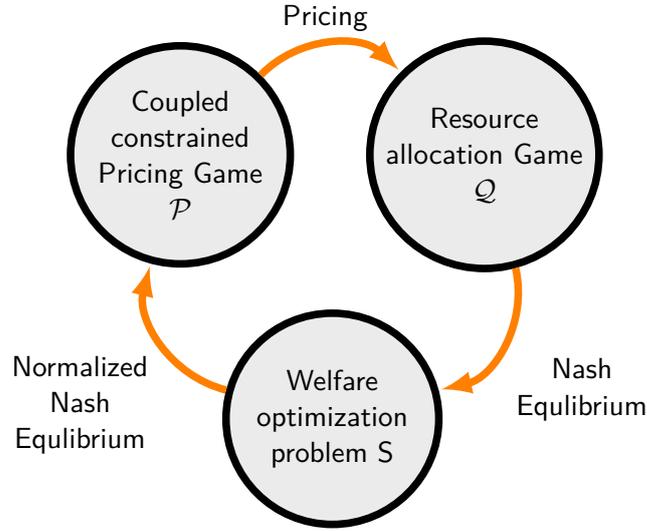


Figure 2.2 – Normalised Nash Equilibria based Mechanism Design

all other elements equal  $x^{s'^*}$  (for any  $s' \neq s$ ).

**Theorem 2.2.** *There exists a unique Nash equilibrium point to an Allocation game for every specified  $\gamma > 0$  where  $\gamma = [\gamma_1 \dots \gamma_S]^T$*

*Proof.* The game induced through Kelly mechanism admits a unique Nash equilibrium [48], the uniqueness result extends immediately to the Allocation Game since the resources are orthogonal.  $\square$

So far, we have presented the first part of the mechanism design and seen that the Allocation Game admits the unique Nash Equilibrium for any value of price vector  $\gamma$ . Now our next objective is to design the pricing scheme for the mechanism so that the resulting Nash equilibrium of Allocation Game is also an optimal solution to the optimization problem SW. We notice that the standard definition of a Nash equilibrium assumes orthogonal constraints, i.e., the strategy set of each player is independent of the actions of the other players; therefore, depending on the prices the optimal strategy for players may be to place bids that exceed the system capacity. This is known as overbooking, and it is a significant concern in any pricing mechanism since the resources assigned to each user do not match their demand. Thus, it becomes imperative to design the bid price so that the resulting Nash equilibrium is the first feasible solution for optimization problem SW. We shall prove that a price vector that avoids overbooking in the Allocation game can be obtained as a byproduct of another game, linked to the Allocation game, whose unique equilibrium is a Nash equilibrium with the resource constraints accounted for. Such a game, which we refer to as the Pricing game, is introduced in the next section.

### 2.3.1 Pricing game

In our scheme, the tenants are strategic; they compete for resource access and do not share information on the amount of resources they request for. Thus, it is unreasonable to expect; the tenants respect the resource capacity constraints while submitting their bid. In turn, we consider a virtual game, where tenants follow the resource capacity constraints while submitting their bid. The InP determines the price vector as the solution of this virtual game, which we called as Pricing game. The InP can use the price as a signal to drive the mechanism to an acceptable operating point. Such prices can be determined using a concept of a normalized Nash equilibrium, a central concept in this work. In the virtual game, each tenant tries to maximise her benefit while obeying the coupled constraints

$$\sum_{s \in \mathcal{S}} x_c^s \leq B_c, \quad \forall c \in \mathcal{C}. \quad (2.6)$$

Thus, the decision problem for a tagged tenant  $s$  writes as

$$\begin{aligned} P_s \quad & \underset{x^s \geq 0}{\text{maximize}} && V^s(x^s, x^{-s}) \\ & \text{subject to} && \sum_{s \in \mathcal{S}} x_c^s \leq B_c, \quad \forall c \in \mathcal{C}, \\ & && x^s \in \mathcal{X}_s. \end{aligned}$$

The system  $\mathcal{P} = \{P_1, \dots, P_S\}$  represents the formalisation of the Pricing game: the notion of an equilibrium for such a continuous game requires to account for the presence of constraints, that is

**Definition 2.3.** A strategy  $x^* = (x^{1*}, \dots, x^{S*})$  is called Nash equilibrium for the game  $\mathcal{P}$  if

$$V^s(x^{s*}, x^{-s*}) \geq V^s(x^s, x^{-s*}) \quad (2.7)$$

for all  $s \in \mathcal{S}$ ,  $x^{s*}, x^s \in \mathcal{X}_s$  and  $\sum_{s \in \mathcal{S}} x_c^s \leq B_c, \forall c \in \mathcal{C}$ .

The fundamental difference between the two games, as anticipated, is that in the Pricing game, the constraints are not orthogonal anymore, but, rather, are coupled constraints. This renders the strategy set of a tagged user dependent on the other users' actions  $x^s \in \mathcal{X}_s$ . While the Pricing game in practice may not be practically viable (indeed it is not reasonable to expect players to respect the aggregate constraint in calculating their best response), the development in this section shows how to map the Pricing game onto the Allocation game.  $\bar{x}$  is a Nash equilibrium of the game  $\mathcal{P}$  if and only

if it satisfies the Karush–Kuhn–Tucker (KKT) conditions conditions, which are:  $\forall c \in \mathcal{C}, \forall s \in \mathcal{S}$ ,

$$\frac{\partial V^s(\bar{x})}{\partial x_c^s} - \lambda_c^s + \xi_c^s = 0 \quad (2.8a)$$

$$\lambda_c^s \left( \sum_{s' \in \mathcal{S}} \bar{x}_c^{s'} - B_c^s \right) = 0 \quad (2.8b)$$

$$\xi_c^s x_c^s = 0 \quad (2.8c)$$

$$\lambda_c^s \geq 0, \xi_c^s \geq 0. \quad (2.8d)$$

In the above KKT conditions,  $\lambda^s = (\lambda_1^s, \dots, \lambda_C^s)^T$  and  $\xi^s = (\xi_c^s, \dots, \xi_C^s)^T$  are Lagrange multiplier vectors, where for each  $c \in \mathcal{C}$ ,  $\lambda_c^s$  and  $\xi_c^s$  are the Lagrange multiplier associated to the  $c^{th}$  resource for the coupled constraint and the non-negative constraint respectively. We are primarily interested in the Lagrange multipliers associated with the coupled constraints, as we can use this as shadow prices for resources. Note that the vector  $\lambda^s$  may be different for different customers  $s \in \mathcal{S}$ , which results in discriminatory shadow pricing. Generally, the uniform pricing scheme is considered as fair and easy to handle over discriminatory pricing. However, in [48] the authors showed that under uniform pricing, there could be up to 25% of efficiency loss of Kelly mechanism at the equilibrium.

**Definition 2.4.** Let  $r \in \mathbb{R}_+^{S \times C}$  be vector.  $\bar{x}$  is said to be  $r$ -normalized Nash equilibrium to pricing game if there exists  $\lambda_c, \forall c \in \mathcal{C}$  such that KKT conditions (2.8) are satisfied with  $\lambda_c^s = \lambda_c / r_c^s$

Here the parameters  $\{r_1 \dots r_S\}$  intuitively show the proportion of a burden on player  $s$  for satisfying the coupled constraints among all other players in the set. Notice that the pricing game has multiple equilibria and in fact, in next the proposition we show that pricing game has infinitely many equilibria.

**Proposition 1.** Any  $\bar{x}$  is a Nash equilibrium point to a pricing game if and only if  $\sum_s \bar{x}_c^s = B_c, \forall c \in \mathcal{C}$

*Proof.* let us assume that  $\bar{x}$  is Nash equilibrium to a pricing game and  $\sum_{s \in \mathcal{S}} \bar{x}_c^s < B_c$ , for any  $c \in \mathcal{C}$  as  $V_c^s$  is continuous and increasing in  $\bar{x}_c^s$  player can get more benefit by increase the  $\bar{x}_c^s$ , contradicts  $\bar{x}$  is Nash equilibrium. If  $\sum_{s \in \mathcal{S}} \bar{x}_c^s = B_c$  then no player can increase benefit by deviating from his strategy hence  $\bar{x}$  is a Nash equilibrium.  $\square$

From the above result, we should observe that in a coupled constrained game *i.e* Pricing game, the equilibrium is, in general, non-unique. Actually, by inspection, we note that the Pricing game has a manifold of equilibria. However, we shall be interested in the special type of equilibrium, namely the normalized Nash equilibrium, which, under specific assumptions, results to be unique.

**Theorem 2.5.** There exists a unique  $r$ -normalized equilibrium point to a Pricing game for every specified  $r > 0$

*Proof.* The existence of normalized Nash equilibrium for every specified  $r$  follows from Theorem 2[88]. Let us consider that there exists two normalized Nash  $\bar{x}$  and  $\hat{x}$  for any specified  $r$  such that the Lagrange multiplier for coupled constraint are  $\bar{\lambda}_c^s = \frac{\bar{\lambda}_c}{r_c^s}$  and  $\hat{\lambda}_c^s = \frac{\hat{\lambda}_c}{r_c^s}$ . Then the sufficient condition for uniqueness of normalized Nash equilibrium is

$$\sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}} r_c^s (\bar{x}_c^s - \hat{x}_c^s) \left( \frac{\partial V^s(\bar{x})}{\partial x_c^s} - \frac{\partial V^s(\hat{x})}{\partial x_c^s} \right) < 0 \quad (2.9)$$

The above condition is also referred to as Diagonal strict concavity (DSC) by Rosen [88] and can be interpreted as the generalization of concavity in-game settings.

$$\frac{\partial V^s(x)}{\partial x_i} = \frac{\partial V^s(b_c^s)}{\partial b_c^s} \frac{\partial b_c^s}{\partial x_c^s} = V_c^{s'}(b_c^s) \frac{x_c^{-s}}{(\sum_{s' \in \mathcal{S}} x_c^{s'})^2} B_c$$

where  $V_c^{s'}(b_c^s)$  is derivative of  $V_c^s$  wrt  $b_c^s$  and

at the equilibrium  $\bar{X}_c = \sum_{s' \in \mathcal{S}} \bar{x}_c^{s'} = \sum_{s' \in \mathcal{S}} \hat{x}_c^{s'} = \hat{X}_c$  we can write

$$\sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}} r_c^s (\bar{x}_c^s - \hat{x}_c^s) \left( \frac{\partial V^s(\bar{x})}{\partial x_c^s} - \frac{\partial V^s(\hat{x})}{\partial x_c^s} \right) \quad (2.10)$$

as

$$\sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}} \left( \frac{\bar{x}_c^s}{\bar{X}_c} - \frac{\hat{x}_c^s}{\hat{X}_c} \right) \left( \frac{\bar{x}_c^{-s}}{\bar{X}_c} B_c V_c^{s'} \left( \frac{\bar{x}_c^s}{\bar{X}_c} B_c \right) - \frac{\hat{x}_c^{-s}}{\hat{X}_c} B_c V_c^{s'} \left( \frac{\hat{x}_c^s}{\hat{X}_c} B_c \right) \right) \quad (2.11)$$

and as the above expression is strictly negative [Appendix A.1], the DSC condition holds, and uniqueness of equilibrium follows from Thm.2 [88]  $\square$

### 2.3.2 Characterization of social optimal pricing

In this section, we will show that the proposed mechanism is able to attain the social optimum. The mechanism achieves the desired goal by simply fixing the pricing that can force slices to choose an equilibrium that respects the resources coupled constraint and coincides with (SW) 's optimal solution. The pricing and allocation are performed in a distributed manner using the cascade of both the Pricing game and the Allocation game, with no need to exchange per-resource allocation information. Now to build the differential pricing, let us recall the original problem introduced in Sec. (2.2), where the InP's goal is to solve problem (SW). Concavity of the objective function ensures that there exists a unique allocation  $b^*$  which maximizes the objective function. Let now consider the Lagrangian associated to problem (SW): it writes  $L(b, \mu, \nu) = \sum_s (V^s(b^s) - \sum_c \mu_c (\sum_s b_c^s - B_c) - \sum_c \nu_c^s b_c^s)$ .

Since the problem is feasible and constraints are affine, KKT conditions for (SW) are necessary

and sufficient for optimality of a solution  $(b^*, \mu^*, \nu^*)$  such that  $\forall s \in \mathcal{S} \forall c \in \mathcal{C}$

$$\frac{\partial V_c^s(b^*)}{\partial b_c^s} - \mu_c^* + \nu_c^{*s} = 0 \quad (2.12a)$$

$$\mu_c^* \left( \sum_{s' \in \mathcal{S}} b_c^{*s'} - B_c \right) = 0 \quad (2.12b)$$

$$\nu_c^{*s} b_c^{*s} = 0, \mu_c^{*s} \geq 0, \nu_c^{*s} \geq 0 \quad (2.12c)$$

Where  $\mu = (\mu_1, \dots, \mu_C)$  are the  $C$  Lagrange multipliers for the resources' capacity constraints.

**Theorem 2.6.** *There exists a unique  $r$ -normalized equilibrium point  $x^*$  to a Pricing game with  $r_c^s = \frac{B_c}{x_c^{-s*}}, \forall s \in \mathcal{S}, \forall c \in \mathcal{C}$  such that  $x^*$  is social optimal*

*Proof.* First claim follows from the preceding theorem (2.5) as for every specified  $r$  there exists the unique  $r$ -normalized equilibrium. We will now prove that  $x^*$  is social optimal. For that consider the KKT condition (2.8a), for any normalized equilibrium  $\bar{x}$

$$r_c^s \frac{\partial V^s(\bar{x})}{\partial x_c^s} - \lambda_c + \xi_c^s = 0 \quad (2.13)$$

$$r_c^s \frac{\partial V^s(\bar{b}_c^s)}{\partial b_c^s} \frac{\partial b_c^s(\bar{x})}{\partial x_c^s} - \lambda_c + \xi_c^s = 0 \quad (2.14)$$

As  $\frac{\partial b_c^s}{\partial x_c^s} = \frac{x_c^{-s}}{(\sum_{s' \in \mathcal{S}} x_c^{s'})^2} B_c$  and by Proposition 1, at any equilibrium  $\sum_{s' \in \mathcal{S}} \bar{x}_c^{s'} = B_c$

$$r_c^s \frac{\partial V^s(\bar{b}_c^s)}{\partial b_c^s} \frac{\bar{x}_c^{-s}}{B_c} - \lambda_c + \xi_c^s = 0 \quad (2.15)$$

Now putting  $r_c^s = \frac{B_c}{x_c^{-s*}}, \forall s \in \mathcal{S}, \forall c \in \mathcal{C}$  and for our desired  $r$ -normalized Nash equilibrium  $x^*$ ,

$$\frac{\partial V^s(b_c^s)}{\partial b_c^s} - \lambda_c + \xi_c^s = 0 \quad (2.16)$$

Now as  $x^*$  satisfies the KKT conditions (2.12) and its necessary and sufficient, thus  $x^*$  is social optimum equilibrium.  $\square$

From the result just proved, we obtain the following coupling result:

**Corollary 2.7.** *Every  $r$ -normalised Nash equilibrium of the Pricing game with shadow prices  $\lambda_c^s = \frac{\lambda_c}{r_c^s}$  is a Nash equilibrium for the corresponding Allocation game with  $\gamma_c^s = \frac{\lambda_c}{r_c^s}$ .*

*Proof.* Let us consider a normalised Nash equilibrium  $\bar{x}$  of the Pricing game. From (4.24), we have

that necessary

$$\forall s \in \mathcal{S}, \forall c \in \mathcal{C}, \frac{\partial V^s}{\partial x_c^s}(\bar{x}) = \lambda_c^s - \xi_c^s.$$

If we replace  $\lambda_c^s = \gamma_c^s$  for all  $c \in \mathcal{C}$  we get

$$\frac{\partial V^s}{\partial x_c^s}(\bar{x}) = \gamma_c^s - \xi_c^s.$$

Thus

$$\frac{\partial U_s}{\partial x_c^s}(\bar{x}) = -\xi_c^s.$$

Further,  $\forall s \in \bar{x}$  satisfies the conditions of the optimization problem associated to the Allocation game. Since function  $U_s$  is concave with respect to variable  $x_s$  and the constraints are linear, they are also sufficient and thus  $\bar{x}$  is a Nash equilibrium of the Allocation game.  $\square$

Despite the fact that the above result provides a solid theoretical foundation for the connection of pricing and social optimal resources allocation, it would be of scarce practical utility in the form presented so far. Solving the two games requires perfect information on the players' utilities at the InP side. Such assumption is unrealistic and, as such, cannot be part of a viable pricing and allocation scheme usable in 5G. Hence, in the next section, we will show that the two games can be solved using a learning approach.

## 2.4 Learning and System Stability

In the preceding sections, we have already discussed that the proposed allocation mechanism has a unique equilibrium for any price vector decided by the InP. The InP can fix the price vector in such a way that the equilibrium to the Allocation game is a socially optimum operating point. These price vector can be found by solving the Nash equilibrium problem for the Pricing game. However, the tenants' valuation of resources is generally unknown to the InP. In view of this, we propose a learning algorithm to converge iteratively to the target equilibrium in a decentralized fashion. However, the pricing game has coupled constraints; hence, it is unreasonable to expect players to respect the coupled constraints without communicating with each other. In this spirit, for the learning purpose, we first decouple the constraints with the approach of extended game [95], introduce in the next section. In the proposed solution, the only signal exchanged between the InP and the tenants at each step are the bid vector and the price vector.

### 2.4.1 Extended Pricing Game

We consider a game with  $S + 1$  players, where the first  $S$  players are the slice tenants and the  $(S + 1)^{\text{th}}$  player is the InP, who controls the  $\lambda$  price vector. We define the decision problem of the InP,

$P_{S+1}$  as below

$$P_{S+1} \quad \underset{\lambda \geq 0}{\text{maximize}} \quad \sum_c \lambda_c \left( \sum_s x_c^s - B_c \right). \quad (2.17)$$

The idea behind using  $\sum_c \lambda_c (\sum_s x_c^s - B_c)$  as the utility for InP in the above decision problem, is that it solves complementary condition (2.8b) from KKT conditions (2.8). For the remaining  $S$  players the decision problem is

$$P_s \quad \underset{x_s \geq 0}{\text{maximize}} \quad V^s(x_s, x_{-s}) - \sum_c \frac{\lambda_c}{r_s} x_c^s \quad \forall s \in \mathcal{S}.$$

We call  $\mathcal{P}_+ \triangleq \{P_1, \dots, P_{S+1}\}$  the extended pricing game. The difference between the extended pricing game and the pricing game  $\mathcal{P}$  is that in the former, there are no coupled constraints – complementary conditions are treated as the utility of an additional player (InP).

**Proposition 2.** *If  $\bar{x}$  is a  $r$ -normalized equilibrium of the pricing game then there exist  $\bar{\lambda} \geq 0$  such that  $(\bar{x}, \bar{\lambda})$  is equilibrium of extended pricing game*

*Proof.* We have already proved that the pricing game  $\mathcal{P}$  is monotone, which implies that the extended pricing game is also monotone, the proof follows from [95], Prop.4.  $\square$

## 2.4.2 Learning Algorithm

In the algorithmic game theory literature, mostly gradient-based algorithms [71] have been proposed to compute the Nash equilibrium of the game where players estimate their utility gradient and move along the direction of gradient in feasible action space. We consider a modified gradient descent based scheme. Instead of moving in the gradient direction, players take a step along the present action times gradient.

$$x_c^s(k+1) = x_c^s(k) + \beta(k) b_c^s(k) \left( \nabla \hat{V}_c^s(b_c^s(k)) - X(k) \right) \quad (2.18)$$

In the above equation  $\beta(k)$  is a standard step size, where  $\sum_{k=0}^{\infty} \beta(k) = +\infty$  and  $\sum_{k=0}^{\infty} \beta^2(k) < +\infty$ . To show the proposed scheme's convergence, we first consider the continuous-time version of the proposed scheme and show that it is dynamically stable.

$$\dot{x}_c^s(t) = x_c^s(t) \left( \nabla V_c^s \frac{x_{-s}(t)}{(X_c(t))^2} - \gamma_c^s \right) \quad (2.19)$$

$$\dot{x}_c^s(t) = \frac{x_c^s(t)}{X_c(t)} \left( \frac{1}{\gamma_c^s} \nabla V_c^s \frac{x_c^{-s}(t)}{(X_c(t))} - X_c(t) \right) \quad (2.20)$$

Let the derivative of effective utility  $\frac{1}{\gamma_c^s} V_c^{s'} \frac{x_c^{-s}(t)}{X_c(t)} = \nabla \hat{U}_c^s(b_c^s(t))$

$$\dot{x}_c^s(t) = b_c^s(t) \left( \nabla \hat{U}_c^s(b_c^s(t)) - X_c(t) \right) \quad (2.21)$$

Now consider the  $b_c^s = \frac{x_c^s}{X_c}$  and calculating the time derivative we get

$$\dot{b}_c^s(t) = \frac{X_c - x_c^s(t)}{X_c(t)^2} \dot{x}_c^s(t) - x_c^s(t) \sum_{s' \neq s} \frac{\dot{x}_c^{s'}(t)}{X_c(t)^2} \quad (2.22)$$

$$\dot{b}_c^s(t) = \frac{\dot{x}_c^s(t)}{X_c(t)} - \frac{x_c^s(t)}{X_c(t)^2} \sum_{s'} \dot{x}_c^{s'}(t) \quad (2.23)$$

$$\dot{b}_c^s(t) = \frac{1}{X_c(t)} \left[ \dot{x}_c^s(t) - b_c^s(t) \sum_{s'} \dot{x}_j(t) \right] \quad (2.24)$$

putting in  $\dot{x}_c^s$  from (2.21)

$$\dot{b}_c^s(t) = \frac{1}{X_c(t)} \left[ b_c^s(t) \left( \nabla \hat{U}_c^s(b_c^s(t)) - X_c(t) \right) - b_c^s(t) \sum_{s'} y_j(t) \left( \nabla \hat{U}_c^{s'}(y_c^{s'}(t)) - X_c(t) \right) \right] \quad (2.25)$$

$$\dot{b}_c^s(t) = \frac{b_c^s(t)}{X_c(t)} \left[ \nabla \hat{U}_c^s(b_c^s(t)) - \sum_{s'} b_c^{s'}(t) \nabla \hat{U}_c^{s'}(y_c^{s'}(t)) \right] \quad (2.26)$$

Now to show the stability of above dynamics consider a Lyapunov function  $\mathcal{L}(b) = \sum_s b_c^{s*} \log \frac{b_c^s}{b_c^{s*}}$  taking derivative with respect to time

$$\dot{\mathcal{L}}(b_c) = \frac{1}{X_c(t)} \sum_s \nabla \hat{U}_c^s(b_c^s(t)) (b_c^s(t) - b_c^{s*}) \quad (2.27)$$

$$\dot{\mathcal{L}}(b_c) = \frac{1}{X_c(t)} \sum_s \nabla \hat{U}_c^s(b_c^s(t)) (b_c^s(t) - b_c^{s*}) < 0 \quad (2.28)$$

As  $\sum_s \nabla \hat{U}_c^s(b_c^s(t)) (b_c^s(t) - b_c^{s*}) < 0$  it proves that system is stable. The convergence of  $(x(k))$  algorithm to the unique Nash equilibrium of the Allocation game follows from the standard theory of stochastic approximation, Theorem 6.9 in [12].

On the level of the InP, she updates the prices so that the players are forced to obey coupled constraints. The prices appear as Lagrange multipliers for coupled constraints (capacity constraints). As similar to the players, she takes the step along the gradient of Lagrangian and updates the price per resource:

$$\lambda_c(k+1) = \max \left( 0, \lambda_c(k) + \beta(k) \left( \sum_{s' \in \mathcal{S}} x_c^{s'} - B_c \right) \right) \quad (2.29)$$

This updated value of Lagrangian multipliers act as new prices for all tenants and resources, that is

$$\gamma_c^s(k) = \frac{x_c^{-s}}{B_c} \lambda_c(k)$$

We define  $b_c^s(t) = b_c^s(\gamma_c^s(t))$  as the equilibrium allocation under  $\gamma_c^s$  at time  $t$ . If  $\gamma_c^s$  is governed by the feedback of  $b_c^s(\gamma_c^s(t))$  based on above equation the system is stable by Thm. 9[66]. If all the players and InP simultaneously take action as per the designed algorithm, the proposed algorithm converges to the unique Normalized Nash equilibrium  $(x^*, \lambda^*)$  of the Pricing Game. Moreover, if we fixed the prices  $\gamma$  and players are allowed to play only according to the algorithm, the designed algorithm converges to unique Nash equilibrium of the Allocation Game.

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**Algorithm 1** On-line Decentralized Learning Algorithm
 

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**Require:**  $\sum_{k=0}^{\infty} \beta(k) = \infty, \sum_{k=0}^{\infty} \beta^2(k) < \infty$ , as  $k \rightarrow \infty$

- 1: **repeat** at time step  $k = 1, 2, \dots$ ,
  - 2:   **for each** player  $s \in \mathcal{S}$
  - 3:     **for each** cell  $c \in \mathcal{C}$ , Take action
  - 4:        $x_c^s(k+1) \leftarrow x_c^s(k) + \beta(k) y_c^s(k) \left( \nabla \hat{V}_c^s(b_c^s(k)) - X_c(k) \right)$
  - 5:     **end for**
  - 6:   **end for**
  - 7:   **for each** resource  $c \in \mathcal{C}$  update the price
  - 8:      $\lambda_c(k+1) \leftarrow \max \left[ 0, \lambda_c(k) + \frac{\beta(k)}{K} (\sum_{s \in \mathcal{S}} x_c^s - B_c) \right]$
  - 9:   **end for**
  - 10:   **for each** player  $s \in \mathcal{S}, \forall c \in \mathcal{C}$
  - 11:      $\gamma_c^s \leftarrow \frac{x_c^{-s}}{B_c} \lambda_c$
  - 12:   **end for**
  - 13: **until**  $\|(x(k+1), \lambda(k+1)) - (x(k), \lambda(k))\| \leq \epsilon$
- 

## 2.5 Radio Resource Allocation Problem

In this section, we confine the discussion to a RAN version of the slicing problem, where the InP schedules wireless resources, namely downlink PRBs among multiple tenants. While the RAN resources allocation problem is a known and well studied one, heterogeneity of traffic demands across tenants and cells captures the main features of slice resources allocation, including fairness issues. The case of multi-resource allocation, spanning other types of infrastructure resources beyond PRBs is an immediate extension of the scheme presented for RAN resources and shall be discussed in the next section. Let a single InP owning a set of base stations  $\mathcal{C}$  be shared by a set  $\mathcal{S}$  of tenants that need physical network resources to serve their users. This can be the case of an application provider serving several customers in mobility. The InP assigns to each tenant a slice of resources, and we assume that each tenant proposes a service covered by all base stations in  $\mathcal{C}$ .

Each tenant's users generate demands, and such demands will inevitably depend on their specific location, thus inducing different slice-dependent needs at each base station.

Let each slice tenant  $s$  be associated with users presence vector  $\mathbf{N}^s = (N_1^s, N_2^s, \dots, N_C^s)$  where  $C$  is the total number of cells and  $N_c^s$  is the number of active users on slice  $s$  at base station  $c \in \mathcal{C}$ . Here a base station is modelled as a finite resource shared by its associated users. We observe that the number of active users associated to the same base station vary across slices, and vary across base stations also for same slice. First let us consider some fixed channel condition at all users and at all slices, and let  $r_u$  be the rate attained by a user  $u$  in slice  $s$  at cell  $c$ . The slice benefit function

$$V_c^s(b_c^s) := \sum_{u=1}^{N_c^s} f_s(r_u(b_c^s)) \quad (2.30)$$

where  $b_c^s$  is the amount of resources (bandwidth) allocated to slice  $s$  at base station  $c$  and under  $\alpha$ -fair scheduling it holds

$$f_s(r_u) = \begin{cases} \frac{(r_u)^{1-\alpha_s}}{(1-\alpha_s)} & \text{if } \alpha_s \neq 1 \\ \log(r_u) & \text{if } \alpha_s = 1 \end{cases} \quad (2.31)$$

The meaning of (2.31) is that, when slice  $s$  has received capacity  $b_c^s$ , user  $u$  of slice  $s$  associated to base station  $c$  receives a rate which is the  $\alpha$ -fair share attained with his peer users on the same slice. The average rate  $r_u$  of any user  $u$  is determined by the scheduling policy and by all the specific techniques used at physical layer and MAC layer, such as modulation, coding, scheduling, etc. In the case when the channel per user varies over time, let  $b_c^s \log(1 + \frac{p_u h_u}{N_0})$  the instantaneous rate when tenant's user  $u$  is scheduled, at transmission power  $p_u$ , noise power  $N_0$  and under channel state  $h_u$ , where  $\mathcal{H}_u$  is the finite set of possible channel states of user  $u$ . Vector  $\mathbf{h} = (h_1, \dots, h_{N_c^s})$  is thus the channel state vector for all users in cell  $s$ . Users of slice  $s$  are served under some scheduling policy  $\Pi(\cdot|\cdot)$  at cell  $c$ , which depends on the past and present users' channel state; at each time-slot, the slice scheduler then allocates the channel to a tagged user  $u$  in cell  $c$  with probability  $\Pi(u|\mathbf{h})$ . The average rate achieved by user  $u$  under policy  $\Pi$  is

$$r_u = g_u(b_c^s, \Pi(u|\mathbf{h})) := \mathbb{E}_{\mathbf{h}} \left[ b_c^s \log(1 + h_u \frac{p_u}{N_0}) \Pi(u|\mathbf{h}) \right] \quad (2.32)$$

where the expectation is taken with respect to the channel distribution. We observe that, irrespective of the actual scheduling policy, the average rate for a tagged user  $u$  is linear in the slice bandwidth  $b_c^s$  at cell  $c$ . Once we fixed  $k_u^* = (\mathbb{E}_{\mathbf{h}} [\log(1 + h_u \Lambda_0) \Pi(u|\mathbf{h})])$ , the total benefit function for slice  $s$  writes as

$$V_c^s = \sum_{u=1}^{N_c^s} f_s(k_u^* b_c^s)$$

which is again an increasing concave function of the allocated bandwidth per slice. The classical

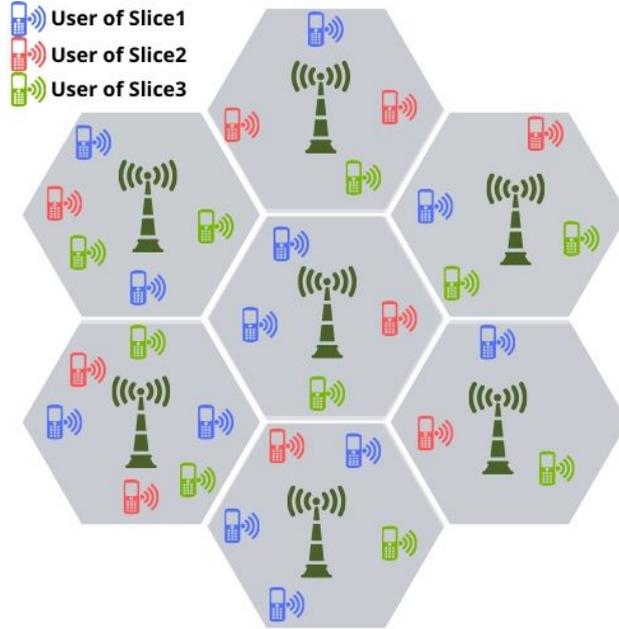


Figure 2.3 – Resource Slicing in 5G Networks: slices are assigned radio resources on a per-cell basis.

optimisation framework for the InP prescribes to provide efficient yet fair allocation for all users belonging to the same slice according to slices' load. Since scheduling is performed per cell, however, it is necessary for the resources allocation to be fair – within the same slice – also across users associated to different base stations. Such a trade-off between efficiency and fairness can be captured by formulating the utility of a given slice as:

$$V^s(b^s) = \sum_{c \in \mathcal{C}} V_c^s(b_c^s) \quad (2.33)$$

For the sake of discussion, we shall assume that the number of users is fixed. Applied at the cell level, utility (2.33), is able to express the customary trade-off between efficiency and fairness among users associated to a tagged slice service. However, it also allows to achieve such a trade-off horizontally, that is across cells. For  $\alpha = 1$ , for instance, the customary log-based proportional-fair utility will severely penalise serving high throughput in a lightly loaded cell while starving slice users in another hot-spot cell. The main objective of the InP is to maximise the total utility of slices, leading to the following 5G resource allocation problem

$$\begin{aligned} P : \quad & \underset{b}{\text{maximize}} && \sum_{s \in \mathcal{S}} V^s(b^s) \\ & \text{subject to} && b_c^s \geq 0 \\ & && \sum_{s \in \mathcal{S}} b_c^s \leq B_c, \quad \forall c \in \mathcal{C} \end{aligned}$$

where  $B_c$  is the total bandwidth available at base station  $c$ . Thus, while (P) still provides reference performance figure for our slicing problem, we shall focus on a decentralised scheme where the resources allocation decision is mediated by a pricing scheme pivoting on the Kelly mechanism.

## 2.6 Numerical Experiments 1

In this section we will provide numerical results to demonstrate the behaviour of the proposed mechanism. For the numerical experiment we considered a system with three slices  $\mathcal{S} = \{1, 2, 3\}$  and two base stations  $\mathcal{C} = \{1, 2\}$ . Tenants of slices 1, 2 and 3 have  $N_1^1 = 3$ ,  $N_1^2 = 5$  and  $N_1^3 = 2$  users, respectively, associated at base station 1. At base station 2 they have  $N_2^1 = 2$ ,  $N_2^2 = 4$  and  $N_2^3 = 6$  users, respectively. The available bandwidth at each base station is 30 MHz and we assume that the SNR of each user lies in the range between 30 and 75 dBs. Every slice uses some scheduling policy to assign the acquired bandwidth among its users: for the purpose of numerical illustration we assume that each slice is served using per-slice proportional fair scheduling. The distributed learning Algorithm-1 is employed in order to determine the socially optimal Nash Equilibrium.

Plots (a) and (b) in Fig. 2.4 show the converging dynamics of the bandwidth bids vector. As seen there, it stabilises at the target Nash equilibrium for both base stations 1 and 2. The distribution of bandwidth allocation at Nash equilibrium is shown in bar graph, Fig.2.6. As it can be clearly seen, the allocation of bandwidth at both base stations is consistent with the number of user per slices. In fact, at base station 2, slice 2 has more users compared to the other two slices; as expected, it attains hence a larger share of the available bandwidth. The target allocation has been achieved by using the pricing vector which is shown in the plot (a) and (b) of Fig.2.7. In those graphs we observe the convergence of prices per slice and per base station. The prices charged by InP for each slice are inversely proportional to number of the users. . Finally, bar graphs (a) and (b) in Fig.2.5 illustrate the throughput achieved per user under the resulting bandwidth allocation; the graphs indicate a mild throughput variation across the users within a slice, a result consistent with the use of PFS at slice level.

## 2.7 Multi-resource allocation Problem

Recall that each network slice  $s$  supports a set of mobile users denote by  $\mathcal{N}_s$ , The resources allocated to a slice are redistributed among the slice's users. Users get some utility by using these resources. Let  $f^u = [f_1^u, \dots, f_C^u]$  be vector of amount of resources allocated to user  $u$ , where  $f_c^u$  is fraction of resource of type  $c$  allocated to user  $u$ . Now we define the utility of each user  $u$

$$U^u(f^u) = \sum_{c \in \mathcal{C}} k_c^u \log(f_c^u) \quad (2.34)$$

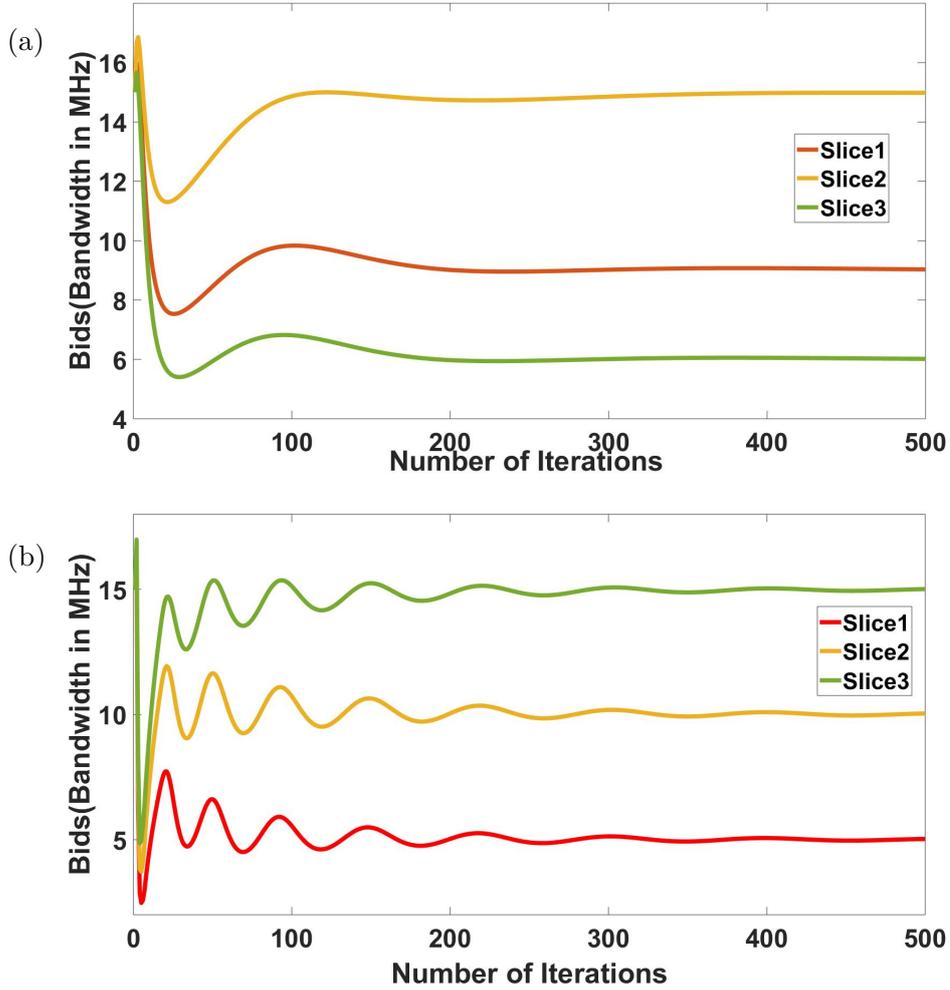


Figure 2.4 – Convergence of the Algorithm to the socially optimal Nash Equilibrium at base station 1 and base station 2

Here we assume that each user associated with the slice possibly have heterogeneous requirements and  $k_c^u \in [0, 1]$  be a set of parameters representing the relative importance of each resource  $c$  for user  $u$ . For example, If in the utility (2.34) we consider only radio resource, and  $f_c^u$  be the the rate achieved by user  $u$ . Then utility defined in (2.34) resembles Proportional Fair Share type utility. PFS-type of utilities are standard for handling radio resources: in LTE, the PFS schedulers onboard of enodeB allocate the rate to their customers such in a way to maximize the sum of the logarithm of the rates made available to each user. In our context, PFS-type has a further advantage: they can bind together different resources involved in the completion of a particular task. For instance, a user evaluates as bad as having much memory but no bandwidth or bandwidth but no memory. A slice can be used with profit if and only if specific subsets of resources, e.g., CPU, are exploited

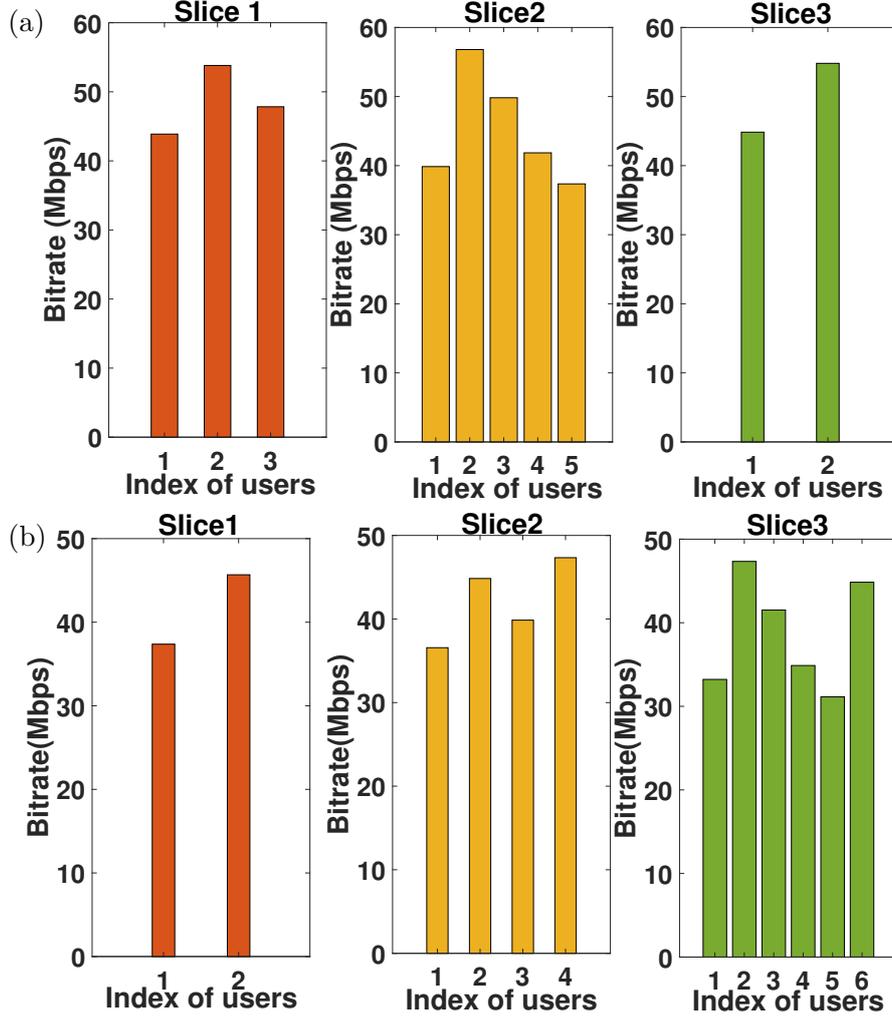


Figure 2.5 – Throughput achieved by the users in base station 1 and base station 2

when other resources are available, e.g., memory and bandwidth. The proportional fairness criterion automatically prevents some resources from vanishing without resorting to multiple additional constraints. The benefit of each tenant is the weighted sum of the utilities of its users. At the sub level, each tenant seeks to maximize its valuation by allocating the acquired resources from the InP. Thus, an optimal value of the following problem defines the valuation of each tenant  $s \in \mathcal{S}$

$$\begin{aligned}
 V^s(b^s) &:= \underset{f_c}{\text{maximize}} && \sum_{u \in \mathcal{N}_s} w^u U^u(f^u) \\
 &\text{subject to} && \sum_{u \in \mathcal{N}_s} f_c^u \leq b_c^s \\
 &&& f_c^u \geq 0, \forall c \in \mathcal{C} \forall u \in \mathcal{N}_s
 \end{aligned} \tag{2.35}$$

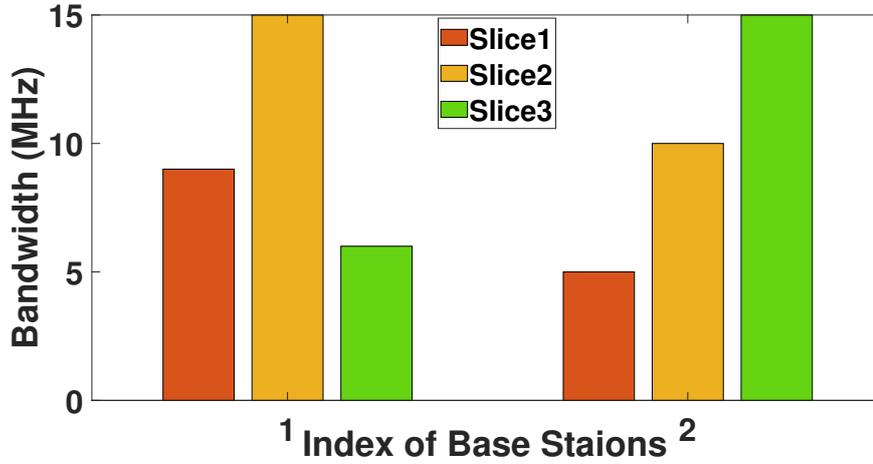


Figure 2.6 – Allocation of bandwidth among the slices at base station 1 and 2.

Here  $w^u$  are the weights or priority assign by tenant to each user  $u \in \mathcal{N}_s$ . In above optimization problem constraints are linear, the feasible set is affine and objective function is concave. Therefore,  $\forall s \in \mathcal{S}$  the optimal value function  $\mathbf{V}^s$  is concave and strictly increasing in the parameter  $b^s$ . The InP aims at maximizing the total sum of utilities of slice tenants and thus apply the proposed price differentiation Kelly mechanism-based resource allocation.

## 2.8 Numerical Experiments 2

Similar to the RAN allocation case, we consider a system with three slices  $\mathcal{S} = \{1, 2, 3\}$ , and they require three types of resources for their service provisioning, namely Bandwidth (BW), virtual-CPU (VCPU) and memory (MEM). Tenants of slices 1, 2 and 3 support the  $N_1 = 12$ ,  $N_2 = 15$  and  $N_3 = 8$  users, respectively. The slices acquire the resources from the InP and redistribute them among their users employing the optimization (2.35). The weights (preferences) of slices over resource types are as mentioned in the Table 2.3. We consider that the available capacities of bandwidth, vcpu and memory are 60 MHz, 120 units and 200 Gb, respectively. We determine the socially optimal Nash Equilibrium employing the distributed learning Algorithm-1 with the differential pricing and then analyze its efficiency gain over uniform pricing. As discussed earlier, the proposed scheme has added advantage over the other Kelly mechanism-based schemes in that it also enables to include the minimum resource requirement of slices. To evaluate the proposed scheme with minimum resource requirements, we consider that each tenant needs the minimum amount of each resource for their service provisioning, details are as mentioned in the Table 2.4. The bar graph in Fig.2.8 and Fig.2.9 describes the allocation of VCPU and memory under the differential and uniform pricing and with no constrained and with minimum requirements. The bar graphs in Fig.2.10 show the benefits gained

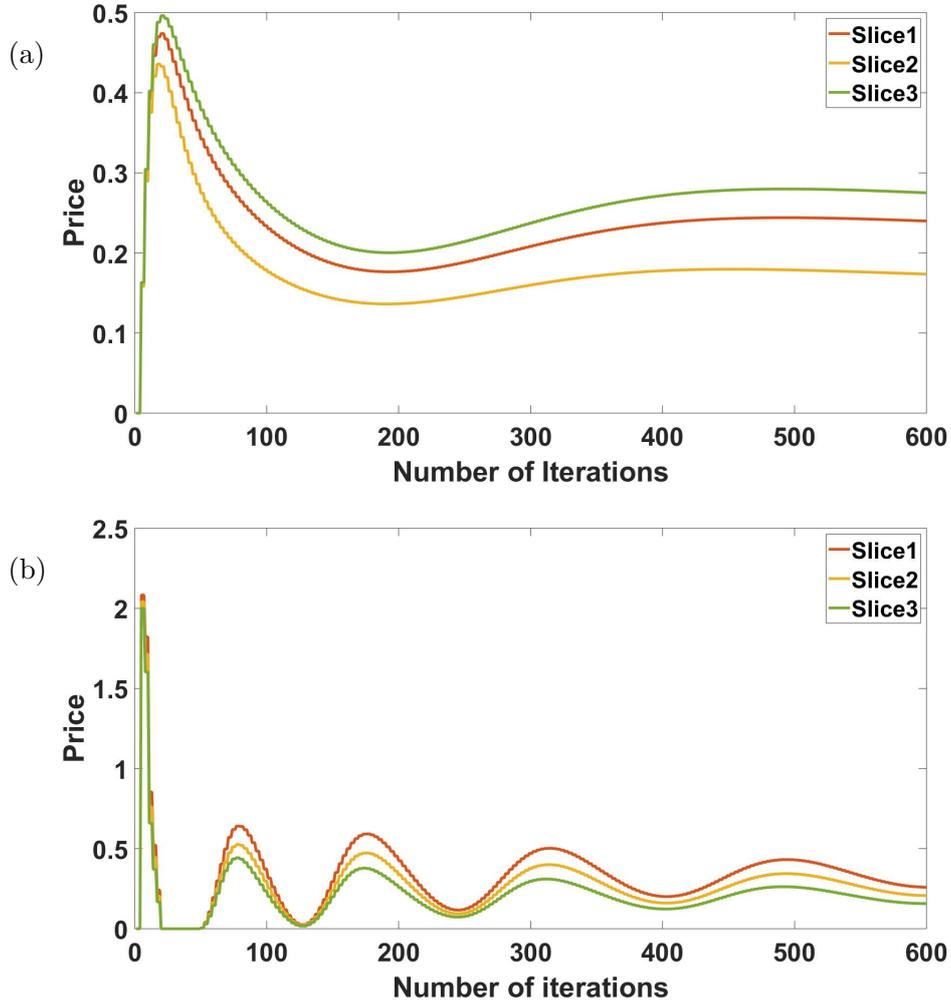


Figure 2.7 – Convergence of price vectors for base station 1 and base station 2

by the slice tenants under the various allocation scheme with the same settings.

We observe that the resources allocated to the slice tenants are in proportion with their number of users and their preference over the resources; for example, slice 2 has both the highest number of (15) subscribers and the highest weight for memory (0.5) and hence receives the highest portion compare to others. The VCPU allocated to slice 2 without minimum requirement under differential pricing is observed to be 47.78 units, but when the minimum requirement for VCPU for slice 2 is considered to be 50 units, the proposed mechanism assign the slice 2 with its minimum requirement. This validates the effectiveness of the proposed mechanism for the case of minimum requirements.

	BW	VCPU	MEM
Slice 1	0.3	0.3	0.4
Slice 2	0.2	0.3	0.5
Slice 3	0.5	0.4	0.1

Table 2.3 – The slices and their weights for different resource types.

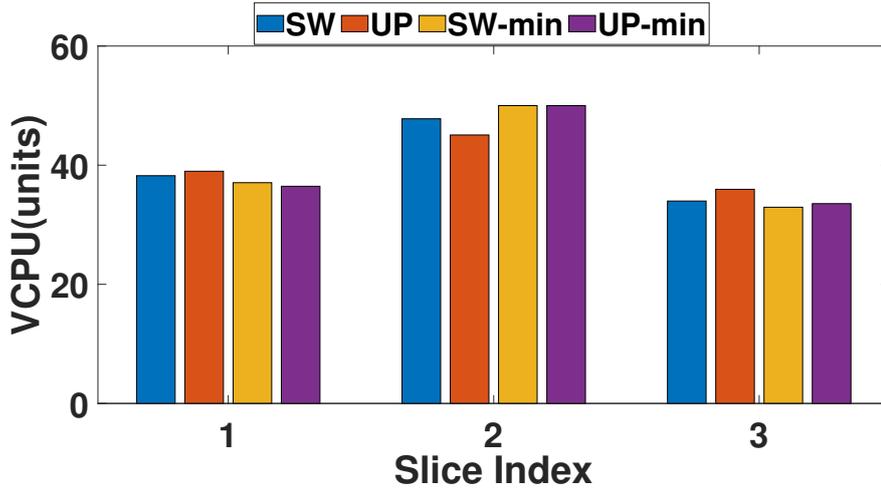


Figure 2.8 – Allocation of of VCPU to the slice tenants under differential pricing (SW), differential pricing with minimum requirements (SW-min), uniform pricing (UP) and uniform pricing with minimum requirements (UP-min).

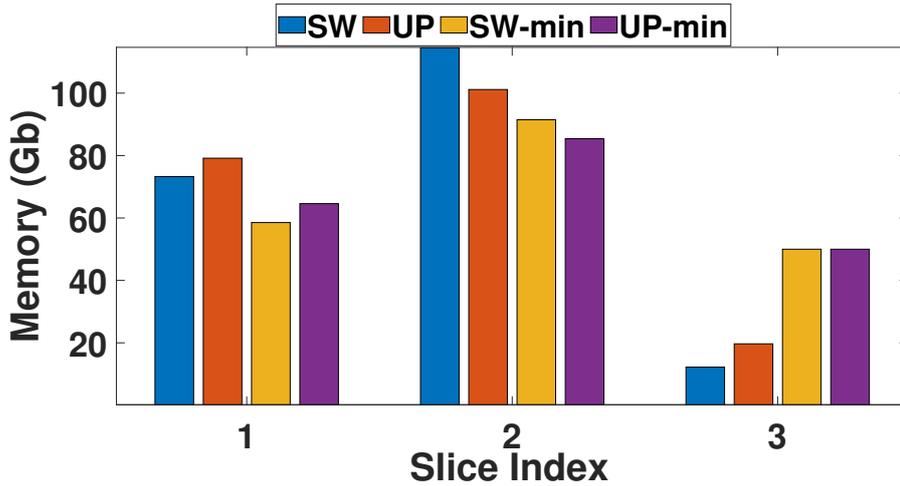


Figure 2.9 – Allocation of of memory to the slice tenants under differential pricing (SW), differential pricing with minimum requirements (SW-min), uniform pricing (UP) and uniform pricing with minimum requirements (UP-min).

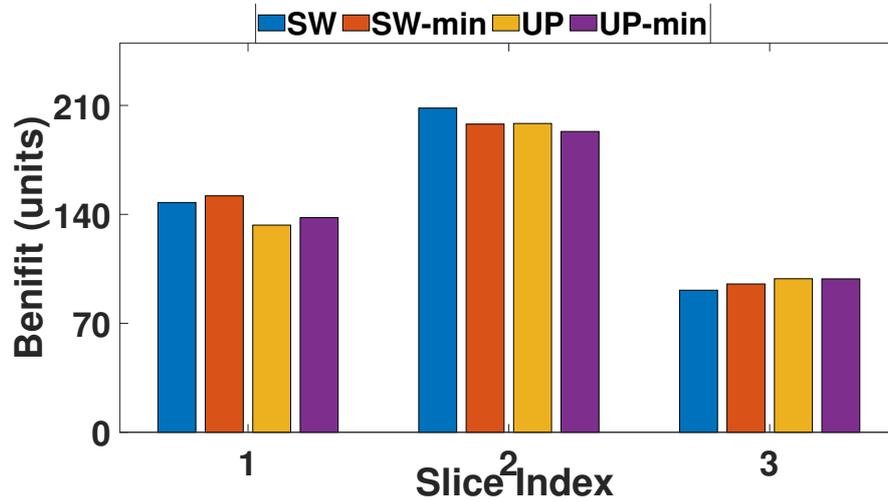


Figure 2.10 – Benefits gained by the slice tenants under differential pricing (SW), differential pricing with minimum requirements (SW-min), uniform pricing (UP) and uniform pricing with minimum requirements (UP-min).

	BW	VCPU	MEM
Slice 1	10	20	20
Slice 2	35	50	20
Slice 3	50	80	50

Table 2.4 – The minimum requirements by slices for different resource types.



# FISHER MARKET MODEL BASED RESOURCE ALLOCATION FOR 5G NETWORK SLICING

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## 3.1 Introduction

The next-generation wireless network is expected to support emerging sectors like Augmented Reality (AR), Virtual Reality (VR) live broadcast [93], Internet of things (IoT) [15], Autonomous driving [64], remote healthcare [2], automated manufacturing based on smart factories [21], etc. Most of these applications or services demand more data-intensive operations than those required by traditional mobile applications. To accommodate the requirements of such services, 5G mobile network design should incorporate the data-centres or cloud computing capabilities into their plan. However, end-to-end latency is also crucial for some of the aforementioned applications, along with the computational capabilities. This requires some of the computational resources to be placed closer to the base stations, leading to the rise of edge computing, also referred to as fog computing [98].

Generally, compared to cloud computing with virtually infinite capacity, the resources available with edge computing facilities are limited. Moreover, the resource requirements of emerging services are diverse and conflicting in nature. Which all together makes the applicability of mobile networks in these emerging sectors challenging. Several new concepts have been introduced for the upcoming 5G network design to satisfy these critical needs. Out of those, probably one of the most important ones is "network slicing" [40]. This is a technique of virtualizing the physical resources and logically partitioning them with the help of technologies like Software-Defined Networking (SDN), and Network Function Virtualization (NFV) [81]. The logical partitioned part of the network is referred to as the slice, which is tailored to meet the specific application/service needs. Generally, Infrastructure providers (InPs) own these resources and provide them to the service providers (SPs) through a dedicated slice.

The slice-based provisioning is at the core of empowering SPs to manage the performance of their own dynamic and mobile user load locally. Network slicing facilitates slice tenants to share the same physical infrastructure flexibly and dynamically, which helps to utilize the resources more effi-

ciently and economically. When SPs or tenants share a common infrastructure to support their service provision, the security and protection of virtual networks are a big concern. Service providers request logical, independent, isolated slices with complete protection of their service level agreements (SLA). One of the most straightforward allocation solutions that offer SPs a guarantee of slice level protection is static partitioning [42], where each resource demanded by the service providers is shared among them depending on their network share or service level agreements. However, this approach lacks in providing load-driven flexibility when the service providers' loads may vary with time and can be spatially inhomogeneous. In this regard, dynamic sharing of resources to slice users is one natural allocation choice that can meet the flexibility of the slices [3]. Moreover, InPs want to maximize their return on investment by employing the dynamic sharing of resources as this lowers the capital cost and gives better resource utilization [59]. However, dynamic sharing of resources can expose the service provider to the risk of violating slice-level agreements. Therefore, one of the critical concerns in 5G network slicing is how to efficiently and dynamically allocate limited resources to slice tenants with diverse characteristics and services while maintaining the protection of their service level agreements.

To address this issue, we propose a Fisher market-based [17] resource allocation scheme, where market agents *i.e.* service providers are assigned with the fixed budgets or share of infrastructure according to their service level agreement. The infrastructure provider sets the prices for the resources. Given prices announced by InP, the SPs distribute their budgets over different resources at different locations to procure the optimal bundle of heterogeneous resources required to support their services. In this work, we use market equilibrium (ME) solution approach to provide stable allocation and resource pricing. At the ME, the market is cleared, *i.e.*, demand meets supply, and every agent is satisfied with allocated resources. To make the proposed resource allocation scheme practically viable, we implement it via the Trading post (TP) mechanism. This type of distributed approach protects the sensitive information of SPs and transfers each SP a direct control to tailor allocation by just adjusting its bids. The required resources are allocated to SPs proportional to their bids. The proposed approach regulates the trade-off between efficient resource utilization and the degree of protection to SLA. On the one hand, it enables dynamic sharing, where tenants can redistribute their network share based on the dynamic load; on the other hand, it also provides the SPs degree of protection by keeping the pre-assigned share intact throughout the allocation process.

### 3.1.1 Related work

In this work, we model the resource allocation problem for 5G network slicing as a Fisher market where SPs act as consumers who purchase the different resources available at geographically distributed locations as goods. Computing the equilibrium to the Fisher market is a challenging problem and has been the subject of much interest in the theoretical computer science community [45]. The Eisenberg and Gale in [31] [32] and their generalization [47] showed that if the utilities of agents

in the market are the homogeneous function<sup>1</sup> of degree one. In that case, the market equilibrium can be found by solving a convex optimization programme, also widely known as Esenbeg-Gale (EG) program. It has been observed that the EG program also achieves proportional fairness [50] or optimum Nash social welfare [76] among the market agents

An approach like the EG program provides a centralized solution to find equilibrium; however, they do not justify the markets or the equilibrium concepts where agents practically interact with each other. Thus, the algorithmic game theory community has always been interested in designing algorithms that could plausibly describe the markets and equilibrium concepts and allow agents to reach equilibrium [89, 29, 36, 105, 22]. For example, over a century ago, Walras [106, 24] proposed the most intuitive and natural algorithm, "tâtonnement", to find the market equilibrium of a market. In this algorithm price of resources increases with demand exceeding capacity and vice versa. The shortcomings of this type of approach are that first, they do not provide any causal relation between the prices and demand. Prices depend on agents' demand, and their demand depends on resource prices. Second, total demand by agents may exceed capacity while applying the procedure, so managing the excess demand is critical.

The Shapley and Shubik [96] sought to answer issues through the Trading post mechanism. In the TP-mechanism, market agents submit bids for each resource and then resources are distributed in proportion to their bids; the price per unit of each resource is set to total bids submitted to that resource. The same mechanism has been discovered several times with different application domains, for example, the Kelly mechanism [53] in computer networks, and proportional share scheme by Feldman et al. [34] in computer systems. The TP-mechanism provides an effective answer to many questions, however reaching equilibrium via the TP-mechanism is still challenging. Over the past decade, much attention has been given to designing algorithms to get the Fisher market equilibrium via TP-mechanism. Zang et al. in [113] showed that when CES<sup>2</sup> utilities with substitute relationships determine resource demands of market agents, proportional response dynamics converge to Fisher market equilibrium. Recently, in [23], Cheung et al. extended the above work to a case where any CES utility functions can determine market agents' demand and developed the distributed proportional dynamics to find the market equilibrium of the Fisher market.

The first step toward the multi-resource allocation problem in multi-server computing resources management was made in [109, 37, 72, 110, 92]. All the works just mentioned above proposed Dominant Resource Fairness (DRF) as criteria for multi-resource allocation. Recently, in [35] Fossati et al. studied the multiple resource allocation for network slicing under different fairness criteria such as Ordered weighted averaging (OWA), weighted proportional fairness (WPF), DRF and mood value rule. The authors proposed an Ordered weighted averaging (OWA) as fairness criteria. To balance

1. A function is called as a homogeneous function of any degree 'k' if; when each of its elements is multiplied by any number  $t > 0$ ; then the value of the function is multiplied by  $t^k$ .

2. Constant elasticity of substitution (CES) utility,  $u(x_i) = \left( \sum_j a_{ij} x_{ij}^\rho \right)^{\frac{1}{\rho}}$ , if  $0 < \rho \leq 1$  signifies substitution relationship ;if  $\infty \leq \rho \leq 0$  is complementary relationship

Article	Application	Fairness	Resources	Model	Learning
Caballero et al. in [18]	5G slicing	$\alpha$ -fairness	Radio	Trading Post	Best response
Nguyen et al. in [78, 77]	Fog Computing	None	Multi	Fisher Market	ADMM
Moro et al. in [74]	5G slicing	None	Multi	Fisher Market	None
Our work, Chapter 3	5G slicing	$\alpha$ -fairness	Multi	Fisher Market and Trading Post	Mirror decent

Table 3.1 – Chapter 3, literature review and research contribution positioning.

the trade-off among inter-slice and intra-slice fairness, a new allocation criterion, namely shared constrained slicing (SCS), was proposed by Zheng et al. in [116]

In [78], Nguyen et al. studied edge computing resource allocation problem for service as Fisher market model, where they only dealt with computation resources, considering that the linear function determines the agents' resource demand. Later in [77], they extended the formulation to a multi-resource allocation problem by employing Leontief functions as agents' utilities. Along the same lines, Moro et al. in [74] cast resource allocation problem for 5G network slicing as Fisher market, wherein apart from edge resources like computation and memory, authors also included the radio resource in the model. Above all three works proposed the market equilibrium-based resource allocation as a solution and showed that the desired equilibrium-based allocation can be obtained by solving the EG program.

In-network slicing context, similar to our work previously, in [18] Caballero et al. proposed the TP mechanism for bandwidth allocation problem. In their proposed scheme, tenants can customize their bandwidth demand by splitting their shares based on their geographically distributed user load. In advancement with the above work, in [115] Zheng et al. applied the same resource allocation scheme for statistical multiplexing of stochastic load. They showed that the resource allocation scheme induces a non-cooperative game, and the slices achieve efficient statistical multiplexing at the Nash equilibrium of the game. Further advancing on the same line, Caballero et al. in [20] introduced the admission control over users arrival to ensure the guaranteed service rate for slices' users.

Our work is closely related to [78, 77, 74], we also formulate the resource allocation problem as a Fisher market. However, this work departs from their works in following points. First, this work also accommodates end-user level allocation and fairness in the model. Second, along the lines of [18, 115, 20], we design the distributed resource allocation scheme via a TP-mechanism that allows the service providers to reach the market equilibrium. However above work only dealt with the bandwidth allocation problem we generalize the mechanism for multiple resources type allocation. Finally, our

work also extends the theoretical results from the [23] by providing a TP-mechanism based updating scheme to reach the ME of the Fisher market with complex utility functions.

### 3.1.2 Main Contributions

We list below the key contributions of our work in Chapter 3.

1. In the context of network slicing, we formulate the system where the service providers or slices need heterogeneous resources (Edge resources) at geographically distributed locations to support users from different service classes.
2. We cast the resource allocation problem for the aforementioned system as a Fisher market model and propose a market equilibrium as its stable solution.
3. We build a convex optimization programme whose optimal solutions provide market equilibrium for the formulated market.
4. We devise the bid updating rule via Trading post mechanism that enables SPs to reach the market equilibrium in a decentralized fashion.
5. We investigate the efficiency and fairness properties of the proposed allocation scheme and perform a comparative analysis with two baseline allocation schemes: optimal social allocation and static proportional allocation schemes.

The rest of the chapter is organized as follows: Section 3.2 introduces the system model. In Section 3.3, we cast the resource allocation problem as the Fisher market model. In Section 3.4.1 and Section 3.4.2, we provide centralized and decentralized approaches respectively to compute the market equilibrium of the formulated market. Section 3.5 is specially dedicated to constructing a potential function, which is needed for developing a decentralized allocation scheme. In Section 3.6, we provide with bid updating rule which allows SPs to reach the desired market equilibrium. In Section 3.7, we investigate the fairness and efficiency properties of the proposed allocation scheme. In section 3.8, we validate the performance of the proposed allocation scheme with extensive numerical simulations.

## 3.2 System Model

We consider a system with InP, who owns the physical resources such as CPU, memory, radio resource, etc., at geographically distributed set of locations or cells (macro, micro, small)<sup>3</sup> as represented in Fig. 3.2. Let set of cells be denoted by  $\mathcal{C} = \{1, \dots, C\}$  and  $\mathcal{R}^c = \{1, \dots, R\}$  represents set of resources available at each location  $c$ . A set of SPs denoted by  $\mathcal{S} = \{1, \dots, S\}$  lease available resources from InPs through network slicing, where network slicing is a process in which the

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3. In this work use of 'cell' or 'location' represents the same.

Table 3.2 – Main notations used throughout the Chapter 3

$\mathcal{C} := \{1, \dots, C\}$	$\triangleq$	set of base stations or cells
$\mathcal{S} := \{1, \dots, S\}$	$\triangleq$	set of slices (tenants)
$\mathcal{R}_c$	$\triangleq$	set of resources at base station $c$
$\mathcal{K}^s$	$\triangleq$	set of user class supported by slice $s$
$\mathcal{K}_c^s$	$\triangleq$	set of user class supported by slice $s$ at cell $c$
$B_s$	$\triangleq$	budget or network share of slice $s$
$x_{ckr}^s$	$\triangleq$	amount of resource type $r$ allocated to slice $s$ for users from class $k$ at cell $c$
$x_{ck}^s$	$\triangleq$	spending by slice $s$ on users from class $k$ at cell $c$
$b_{ckr}^s$	$\triangleq$	spending by slice $s$ for users from class $k$ on resource type $r$ at cell $c$
$b_{ck}^s$	$\triangleq$	spending by slice $s$ on users from class $k$ at cell $c$
$n_{ck}^s$	$\triangleq$	number of user from slice $s$ belonging to class $k$ at cell $c$
$D_k = (d_{k1} \dots d_{kR})$	$\triangleq$	base demand vector for user class $k$
$d_{kr}$	$\triangleq$	the minimum amount of resource type $r$ needed by a user from class $k$ to achieve unit service rate
$u_\nu$	$\triangleq$	service rate experienced by user $\nu$
$u_{ck}^s$	$\triangleq$	total service rate experienced by users belonging to class $k$ in cell $c$
$\alpha_s$	$\triangleq$	$\alpha$ fairness parameter for slice $s$
$p_{cr}$	$\triangleq$	price per unit of resource type $r$ at cell $c$ .
$\Phi(b)$	$\triangleq$	potential function to Esenberg-Gale Program
$\Psi(x)$	$\triangleq$	objective function of Esenberg-Gale Program
$\Upsilon(p)$	$\triangleq$	dual function of Esenberg-Gale Program

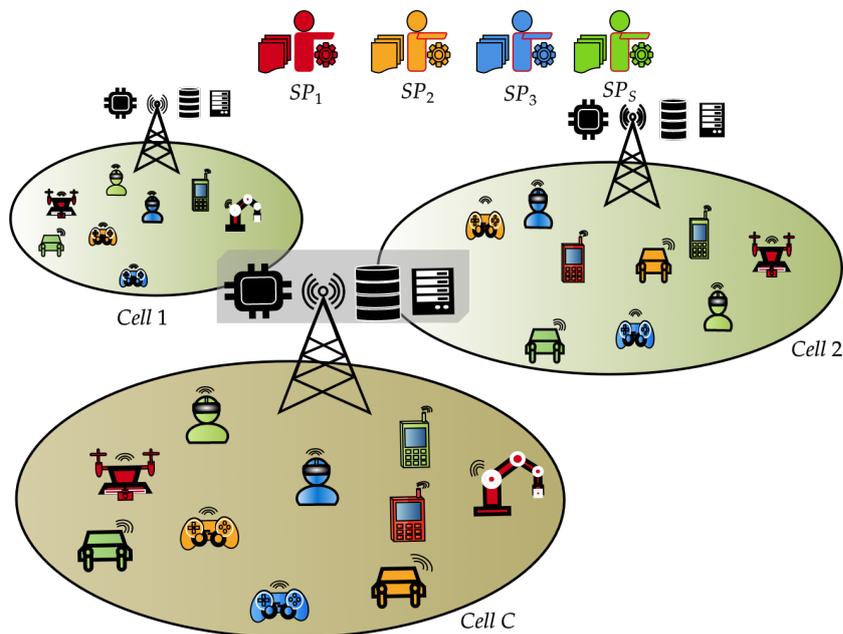


Figure 3.1 – The service providers support the different applications through dedicated slices.

physical network is virtualized and then logically separated to match the SP requirements. The SP then operates on such virtual network (slice<sup>4</sup>) to support different services (e.g., IoT, Virtual Reality (VR), online gaming, autonomous driving and healthcare etc.) for the subscribed users at multiple locations. The resource requirement of SPs in each cell depends on the number of users in that cell and the type of service they provide.

### 3.2.1 User utility model

As shown in Fig. 3.2, a set of users  $\mathcal{V}$  are categorized into a set of classes  $\mathcal{K}$ , where each class represents a different service requirement. Users obtain the resources by subscribing to the services offered by various SPs. The SP needs to provide heterogeneous resources to meet the service rate defined per class. We consider that every user needs a minimum allotment of each resource to meet a certain service rate. Let  $D_k = (d_{k1} \dots d_{kR})$  be the base demand vector, where the element  $d_{kr}$  is the amount of resource type  $r$  required by a user of class  $k$  to achieve a unit service rate. The service rate obtained by the user that belongs to class  $k$  for obtaining a resource bundle  $(x_{k1} \dots x_{kR})$  is defined by the following Leontief function,

$$u_\nu = \min_r \left\{ \frac{x_{kr}}{d_{kr}} \right\} \quad (3.1)$$

4. We refer to network slice as 'slice' for brevity.

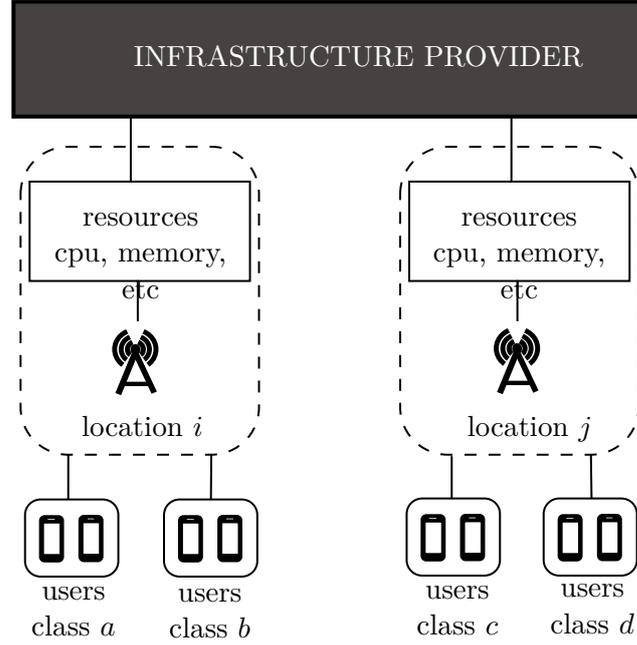


Figure 3.2 – System description: topological view.

where  $x_{kr}$  is the amount of resource type  $r$  allocated to the user of class  $k$ . For instance, consider that a user from class  $k$  with its base demand vector defined as  $d_{k,BW} = 0.2$  and  $d_{k,CPU} = 0.1$  units, receives 0.4 units of bandwidth and 0.2 units of CPU then the service rate achieved by user  $v$  is given by,  $u_v = \min \left\{ \frac{0.4}{0.2}, \frac{0.2}{0.1} \right\}$ . Observe that increment in the bandwidth to 0.6 units does not increase the utility, which highlights the main attribute of the Leontief function, i.e., improvement in the utility is possible only with proportional increment in all the allocated resources.

### 3.2.2 Service Provider Utility Model

Let us consider that each slice supports various classes of users at a given location  $c$ , and this support may vary based on the location. Let a set of users classes supported by a slice  $s$  denoted by  $\mathcal{K}^s$ ,  $n_{kc}^s$  be the number of users from class  $k$  associated to slice  $s$  present in cell  $c$ , and  $\mathcal{K}_c^s$  be the set of classes supported by slice  $s$  at location  $c$ . We assume that each slice treats the users in the same class uniformly, i.e., the service rate provided by the slice  $s$  is equally divided among the users in the same class at location  $c$ , in general, this is valid for all locations. Thus a class-level allocation can easily be treated as to user-level allocation. The utility that signifies the benefit of resource allocation obtained by the slice is defined as follows

$$U_s = \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \frac{n_{ck}^s}{(1 - \alpha_s)} \left( \frac{u_{ck}^s}{n_{ck}^s} \right)^{(1 - \alpha_s)} \quad (3.2)$$

In the above equation  $u_{ck}^s = \min_{r \in \mathcal{R}_c} \left\{ \frac{x_{ckr}^s}{d_{ckr}^s} \right\}$ , where  $u_{ck}^s$  and  $x_{ckr}^s$  represents the total service rate and the amount of resource type  $r$  allocated to set of users belonging to class  $k$  in cell  $c$  respectively and  $d_{ckr}^s = d_{kr}$ . For the sake of convenience, we replace  $(n_{ck}^s)^{\alpha_s}$  with  $w_{ck}^s$  and write the above utility as

$$U_s = \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \frac{w_{ck}^s}{(1 - \alpha_s)} (u_{ck}^s)^{(1 - \alpha_s)} \quad (3.3)$$

The utility (3.3) demonstrates that the slices aim to attain well-known  $\alpha$ -fairness criteria [73] among the classes of users while delivering the service at different cells. The values of  $\alpha \in [0, \infty]$  interpolate between individual fairness among the users and the system's efficiency. The  $\alpha = 0$  corresponds to the utilitarian (average) objective where the goal is to optimize system efficiency, while  $\alpha \rightarrow \infty$  corresponds to max-min fairness (the egalitarian objective). The  $\alpha = 1$  and  $\alpha = 2$  corresponds to the widely established proportional fairness and potential criterion, respectively.

$$U_s = \begin{cases} \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \frac{w_{ck}^s}{(1 - \alpha_s)} (u_{ck}^s)^{(1 - \alpha_s)} & \text{if } \alpha_s = \text{others} \\ \min_{c,k} \left\{ \frac{u_{ck}^s}{n_{ck}^s} \right\} & \text{if } \alpha_s = \infty \\ \prod_{ck} (u_{ck}^s)^{w_{ck}^s} & \text{if } \alpha_s = 1, \end{cases} \quad (3.4)$$

We further consider that each slice  $s \in \mathcal{S}$  is allocated with a finite budget or share  $B_s$  that represents its share of total infrastructure such that  $\sum_{s \in \mathcal{S}} B_s = 1$ , the budget allocated for each slice depends on its service level agreement with the InP.

**Remark 3.1.** *In the above model, we have considered that slices treat users uniformly at each cell while delivering service to users from the same class. However, our framework is more general and applicable to cases where slices may handle each user independently and provide the service according to their preferences and priorities; in such cases, each user will be treated as a separate class.*

**Remark 3.2.** *We use the Cobb-Douglas function instead of the logarithmic function, generally used in  $\alpha$  fairness criteria; however, both the operations perform the identical task of attaining proportional fairness among the users.*

### 3.3 Problem formulation

Given that the resources are available in a limited capacity and more than one SP requires the resources, it is crucial to design a framework that considers the budget constraints of the SP while allocating the resources and adhere to the capacity limit of each resource type as well. We establish such a framework by formulating the resource allocation problem as a Fisher market where  $\mathcal{S}$  slices act as consumers who spend their budget to purchase the resources available at the different cells representing the divisible goods. At the same time, infrastructure providers operate as producers

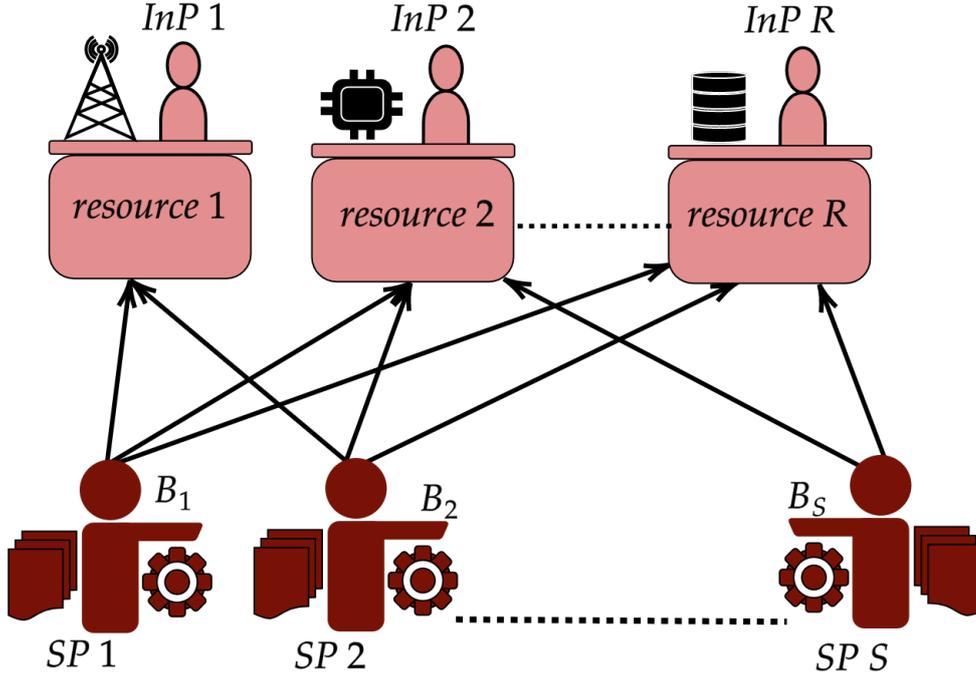


Figure 3.3 – Fisher Market based resource allocation scheme

who sell capacity-constrained resources in exchange for consumers' spending. Each SP spends its budget to obtain the most favoured bundle of resources given the set prices, while the InPs set the prices in the market to optimize resource utilization. Towards this goal, let us consider  $p_c = (p_{c1}, \dots, p_{cR})$  as the vector of prices charged at cell  $c$  where the  $p_{cr}$  is the price per unit of resource type  $r$  at cell  $c$ . Given the prices charged by the infrastructure provider for resources, we anticipate the SPs to act as rational agents and spend their budgets to procure the resources in a manner that maximizes their utility. The decision problem for each service provider  $s$  to find an optimal bundle of resources to be requested is defined by the following optimization problem.

$$P_s : \underset{\mathbf{x}, \mathbf{u}}{\text{Maximize}} : \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \frac{1}{1 - \alpha_s} w_{ck}^s (u_{ck}^s)^{1 - \alpha_s} \quad (3.5a)$$

$$\text{subject to} \quad u_{ck}^s \leq \frac{x_{ckr}^s}{d_{ckr}^s} \quad \forall c \in \mathcal{C}_s, k \in \mathcal{K}_c^s, r \in \mathcal{R}_c \quad (3.5b)$$

$$\sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} p_{cr} x_{ckr}^s \leq B_s \quad (3.5c)$$

Here, the objective is to find an optimal allocation vector  $[x_{ckr}^s]$  that solves the problem (3.5a)-(3.5c). Such an optimal share of the resources are allocated to the SP through the network slicing

process, i.e., each SP is assigned with a slice that contains the resources that were divided optimally among the competing SPs. As mentioned before, using  $\alpha_s$  ensures the fairness criteria among the users of different classes associated with the slice  $s$ . Constraints defined in eq. (3.5b) ensure that the resulting service rate  $u_{ck}^s$  does not consider the excess resources but only the resources that are in proportion. Whereas constraints in eq. (3.5c) guarantee that the resulting allocation adheres to the budget limitations of the SP. As described earlier, this framework is constructed based on the Fisher market model, where the market is formally defined as

**Definition 3.3.**  $\mathcal{M} := \langle \mathcal{S}, (B_s)_{s \in \mathcal{S}}, \cup_{c \in \mathcal{C}} \mathcal{R}_c, (U_s)_{s \in \mathcal{S}}, p \rangle$  as follows:

- *Player set: the set of service providers  $\mathcal{S}$*
- *Budgets :  $B_s$*
- *Resources set:  $\cup_{c \in \mathcal{C}} \mathcal{R}_c$*
- *Utility: The utility of each SP  $s$  is equal to the  $U_s$*
- *Price vector:  $p$*

The primary goal of this work is to provide a Fisher market-based resource allocation scheme that effectively prices and allocates limited physical resources to slices with heterogeneous requirements and preferences. Since resources are allocated to service providers depending on their rational decisions and interactions in the Fisher market, we investigate the outcome of the proposed Fisher market model in light of game theory and recall the definition of market equilibrium.

**Definition 3.4.** A market equilibrium (ME) is defined as  $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$  the prices and resultant allocation, where the market clears its resources and SPs get their favorite resource bundle. Mathematically  $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$  is ME if following two conditions are satisfied.

1. Given the resource price vector, every slice spends its budget such that it receives resource bundle  $\hat{\mathbf{x}}$  that maximizes its utility.

$$\forall s, \hat{\mathbf{x}}_s \in \arg \max_{\sum_c \sum_k \sum_r x_{ckr}^s \hat{p}_{cr} \leq B_s} U_s(x) \quad (3.6)$$

2. Every resource is either fully allocated or has zero price, i.e., we have:  $(\sum_s \sum_k \hat{x}_{ckr}^s - C_{cr}) \hat{p}_{cr} = 0$

In the above definition, the first condition is meant that the equilibrium allocation maximizes satisfaction or the return of market investment of each service provider; no equilibrium could be otherwise established. The second condition represents Walras's law [106], which means that either the total demand of each resource meets the capacity and will be positively priced; otherwise, that resource is allocated free of cost. Undoubtedly, zero-priced resources can be allocated arbitrarily to service providers. However, an additional allocation of these resources will not increase their satisfaction level.

**Assumption 1.** *This work considers that each SP's user load is dynamic and spatially inhomogeneous. The shares (budgets) are allocated to SPs over a long timescale (months/days), and the distribution of shares by the SPs over the resources is performed at a fast time scale (minutes/seconds), such that user load is regarded as stationary while performing the allocation.*

## 3.4 Resource allocation

There are several ways to obtain an optimal solution for the resource allocation problem defined in the previous section. In this section, we explore solutions that mainly use either a centralized or a decentralized approach. In the centralized approach, with the knowledge of SPs utility functions and base demands the InP optimizes the resource allocation by defining the prices and quantity of the resources to each SP such that they satisfies the ME, where as in the decentralized approach that enables SPs to reach the ME with out revealing such private information.

### 3.4.1 Centralized approach

In order to solve the market equilibrium problem, we first introduce the concept of the Esenberg-Gale (EG) optimization problem, which can then be used to find the market equilibrium of the Fisher market  $\mathcal{M}$  under some conditions. Suppose all utilities of consumers in the Fisher market are concave and homogeneous of degree one. In that case, according to [89][31], the market equilibrium of that market can be obtained by solving the below Esenberg-Gale optimization program.

$$\text{EG : - Maximize : } \quad \sum_{s \in \mathcal{S}} B_s \log(U_s) \quad (3.7a)$$

$$\text{subject to} \quad U_s = \left( \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s (u_{ck}^s)^{1-\alpha_s} \right)^{\frac{1}{1-\alpha_s}} \quad \forall s \in \mathcal{S} \quad (3.7b)$$

$$u_{ck}^s \leq \frac{x_{ckr}^s}{d_{ckr}^s} \quad \forall s \in \mathcal{S}, c \in \mathcal{C}_s, k \in \mathcal{K}_c, r \in \mathcal{R} \quad (3.7c)$$

$$\sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_c^s} x_{ckr}^s \leq C_{cr}, \quad \forall c \in \mathcal{C}, r \in \mathcal{R} \quad (3.7d)$$

Suppose  $\mathbf{x}^*$  and  $\mathbf{p}^*$  be the primal and dual optimal solutions of the EG program (3.7), where  $\mathbf{p}^*$  be the dual variable or lagrangian multiplier associated with capacity constraints (3.7d), then  $\mathbf{x}^*$  and  $\mathbf{p}^*$  represent the market equilibrium allocation and prices of market  $\mathcal{M}$ .

**Theorem 3.5.** *Optimal allocation  $\mathbf{x}^*$  and corresponding prices  $\mathbf{p}^*$  (dual variable associated to (3.7d) of optimization program (3.7) represents the market equilibrium (ME)*

*Proof.* In the proposed Fisher market model utility of each service provider is concave and homogeneous of degree one; hence, the result follows from [31][89]. However, for the sake of completeness, we provide a few main steps of the proof.

1. First we show that the optimal solution ( $\mathbf{x}^*$ ) to program and its dual solution ( $\mathbf{p}^*$ ) pair is budgets balancing *i.e.*, at the solution pair the budget of each SP is fully utilize

$$\forall s \in \mathcal{S} \quad \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} p_{cr}^* x_{ckr}^s = B_s$$

the detailed proof is provided in the Appendix B.4

2. Given price vector ( $\mathbf{p}^*$ ) we find best response (demand) by each SP  $s$  by solving (3.5), (the derivation of best response is provided in Appendices B.3) and it gives the same solution allocation  $\mathbf{x}^*$  or consider any possible allocation  $\mathbf{y}$  that can be requested by SP such that

$$\forall s \in \mathcal{S} \quad \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} p_{cr}^* y_{ckr}^s \leq B_s$$

3. then we can show that  $\forall s \in \mathcal{S} \quad U_s(y_s) \leq U_s(x_s^*)$

which proves that ( $\mathbf{x}^*, \mathbf{p}^*$ ) solution to program, is market equilibrium.  $\square$

### 3.4.2 Decentralized approach

We have already discussed in Section 3.4, the market equilibrium solution to the proposed problem (3.5) can be found as a solution to the equivalent convex optimization problem (3.7), which can then utilize by InP to implement proposed allocation scheme. However, such a centralized implementation requires that all the SPs' private utility functions be made available to InP. This is rarely possible, as it is generally not acceptable for SPs to reveal their private data to third parties. In this section, we focus on developing the decentralized algorithm that enables the service provider to reach the market equilibrium of market  $\mathcal{M}$ . One of the possible solutions in this direction is to solve the convex optimization problem using a Walras' tâtonnement-like algorithm where the resource price is raised if the demand for a resource exceeds the resource supply and decreases if the demand for the resource is less than the supply. However, this is not the way how the market generally functions in practice; this type of approach does not always guarantee the ability to satisfy the resource capacity while applying a process.

To deal with this issue, we advocate an alternative approach by Shapley and Shubik, well known as the Trading post mechanism. The proposed method does not require the service providers to reveal their utilities; SPs can distribute their budgets over the resources and customize their allocations.[34] In the Trading post mechanism, slices distribute their budget over their required resources in bids. Once all slices place the bids, each resource type's price is determined by the total bids

submitted for that resource. Let slice  $s$  submits a bid  $b_{cr}^s$  to resource  $r$  at cell  $c$ . The price of resource type  $r$  at cell  $c$  is then set to  $\sum_{s \in \mathcal{S}} b_{cr}^s$ , accordingly slice  $s$  receives a fraction of  $x_{cr}^s$  in return to his spending of  $b_{cr}^s$ .

$$x_{cr}^s = \begin{cases} \frac{b_{cr}^s}{\sum_{s' \in \mathcal{S}} b_{cr}^{s'}} & \text{if } b_{cr}^s > 0, \sum_{s' \in \mathcal{S}} b_{cr}^{s'} > 0 \\ 0 & b_{cr}^s = 0, \sum_{s' \in \mathcal{S}} b_{cr}^{s'} > 0 \\ \text{as per demand} & \sum_{s' \in \mathcal{S}} b_{cr}^{s'} = 0 \end{cases} \quad (3.8)$$

In our framework, the same resource is required by users from different classes; thus, the total budget spent by SPs  $s$  on resource  $r$  is the sum of budgets spent by the SP on resources  $r$  for the set of its users belonging to all possible classes  $b_{cr}^s = \sum_k b_{ckr}^s$ . We assume that the SPs are price takers and request the different amounts of the resources by distributing their budgets over the resources in bids. The InP announces the resources' prices and allocates the resources according to TP-mechanism. If all the SPs are satisfied with the allocation and costs reported by InP, the mechanism has reached market equilibrium; otherwise, SPs might modify their distribution of budgets (demand) depending on the current prices. This brings new the challenge of dynamics or bids updating scheme:(how) do SPs reach a market equilibrium via the TP mechanism. In the coming sections, we focus on developing the bid updating rule that enables the service providers to reach the market equilibrium of the proposed allocation scheme. Particularly we focus on designing for some restricted cases when  $\alpha_s$  parameter applied by each service provider  $s$  takes a value either 0 or in the range of  $[1, \infty]$ . Now, before moving directly to the main results, we build some mathematical tools that will require afterwards to develop the bid updating rule and prove its convergence results.

### 3.5 Potential function

In this section, we construct a potential function to the Eisenberg-Gale program (3.7) and show the optimal solution to the problem (*i.e.* market equilibrium) is an optimal point of the candidate potential function. In this thesis, we restrict our analysis to the case when the  $\alpha_s$  the alpha fairness criteria employed by each service provider  $s$  takes value in  $[1, \infty]$ , for the remaining case when  $\alpha_s \in [0, 1]$  requires complex saddle point analysis and we keep this for future work. Next, we show that when all the SP employ the  $\alpha$  fairness criteria with  $1 \leq \alpha_s \leq \infty$ , our designed potential function is convex, and its minimal point represents the market equilibrium. Now to start with the designing of the potential function, we consider the dual of an optimization problem where the goal is to minimize

$$\Upsilon(p) = \max_{\sum_{c,k,r} x_{ckr}^s p_{cr} = B_s} \left( \sum_s B_s \log(U_s) + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} p_{cr} \left( 1 - \sum_s \sum_{k \in \mathcal{K}_c^s} x_{ckr}^s \right) \right) \quad (3.9)$$

Now we introduce the potential function and write separately depending on the parameter value  $\alpha_s$  used by the service providers. Let  $\Phi(b_{\#})$  denote the potential function of the EG-problem when the alpha fairness parameter  $\alpha$  with the condition  $\#$  has been applied by the service providers. For example,  $\Phi(b_{\geq})$  denotes the potential function when service providers apply the fairness with  $\alpha \geq 1$ . We use the same notation for the remaining article to describe the potential function and its connection with the SPs'  $\alpha$  fairness parameter.

$$\Phi(b_{=1}) = \sum_{s:\alpha_s=1} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} w_{ck}^s b_{ckr}^s \log \left( \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right) \quad (3.10)$$

$$\Phi(b_{>1 \neq \{\infty\}}) = \sum_{s:1 < \alpha_s < \infty} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} b_{ckr}^s \log \left( \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right) - \frac{1}{(1 - \alpha_s)} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} b_{ck}^s \log \left( \frac{b_{ck}^s}{w_{ck}^s} \right) \quad (3.11)$$

$$\Phi(b_{=\infty}) = \sum_{s:\alpha_s=\infty} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} b_{ckr}^s \log \left( \frac{b_{ckr}^s}{w_{ck}^s p_{cr} d_{ckr}^s} \right) \quad (3.12)$$

As all cases provided above are disjoint, combining them all, we write the complete potential function as

$$\Phi(b) = \Phi(b_{=1}) + \Phi(b_{>1 \neq \{\infty\}}) + \Phi(b_{=\infty}) \quad (3.13)$$

In the following theorem, we establish the relationship between the potential function  $\Phi$  and its dual program  $\Upsilon$

**Theorem 3.6.** *Let  $\mathbf{b}$  be the spending of service providers, and  $x(\mathbf{b})$  be the corresponding allocation according to the TP-mechanism, where  $x_{ckr}^s = \frac{b_{ckr}^s}{p_{cr}}$  and  $p_{cr}(\mathbf{b}) = \sum_s \sum_k b_{ckr}^s$  then we have following result*

$$\Upsilon(p(b)) - \Upsilon(p(b^*)) \geq \Phi(b^*) - \Phi(b)$$

where  $b^*$  denotes the market equilibrium of the market  $\mathcal{M}$

*Proof.* The detailed proof is provided in Appendix B.1 □

Moving ahead, we describe some properties of the function  $\Phi$ , which we will need afterwards to prove the convergence of the proposed bid updating scheme. First, we introduce the definition of the L-Bregman convex function and show that our designed potential function  $\Phi$  admits this property.

**Definition 3.7** ([23]). *The function  $f$  is L-Bregman convex w.r.t Bregman divergence  $d_g$  if, for any*

$y \in \text{rint}(C)$  and  $x \in C$ ,

$$f(y) + \langle \nabla f(y), x - y \rangle \leq f(x) \leq f(y) + \langle \nabla f(y), x - y \rangle + L.d_g(x, y) \quad (3.14)$$

In the following lemma, we show that the potential function  $\Phi$  is 1-convex depending on the parameter  $\alpha_s$  employed by the service providers for the fairness criteria.

**Lemma 3.8.** *The potential function  $\Phi(b)$  is 1-Bregman convex concave w.r.t Bregman divergence  $d_g$*

$$d_g = \sum_{s:1 \leq \alpha_s \leq \infty} KL_a(\cdot||\cdot) - \sum_{s:1 < \alpha_s < \infty} \frac{1}{(1 - \alpha_s)} KL_b(\cdot||\cdot) \quad (3.15)$$

where

$$KL_a(x||y) = \sum_c \sum_k \sum_r x_{ckr}^s \log \left( \frac{x_{ckr}^s}{y_{ckr}^s} \right) \text{ and}$$

$$KL_b(x||y) = \sum_c \sum_k x_{ck}^s \log \left( \frac{x_{ck}^s}{y_{ck}^s} \right)$$

*Proof.* We know that the  $\Phi(b)$  is a convex function then by definition of a convex function, we have

$$\Phi(b') + \langle \nabla \Phi(b'), b - b' \rangle \leq \Phi(b) \quad (3.16)$$

Now consider

$$\Phi(b) - \Phi(b') - \langle \nabla \Phi(b'), b - b' \rangle \quad (3.17)$$

putting the value of  $\nabla \Phi(b')$  in above equation and after some calculations, we get

$$\Phi(b) - \Phi(b') - \langle \nabla \Phi(b'), b - b' \rangle = \sum_{s:1 \leq \alpha_s \leq \infty} KL_a(b_s||b'_s) - \sum_{s:1 < \alpha_s < \infty} \frac{1}{(1 - \alpha_s)} KL_b(b^s||b'^s) - KL(p||p') \quad (3.18)$$

and since  $KL(p||p')$  is non negative

$$\Phi(b) - \Phi(b') - \langle \nabla \Phi(b'), b - b' \rangle \leq \sum_{s:1 \leq \alpha_s \leq \infty} KL_a(b_s||b'_s) - \sum_{s:1 < \alpha_s < \infty} \frac{1}{(1 - \alpha_s)} KL_b(b_s||b'_s) \quad (3.19)$$

$$\Phi(b) \leq \Phi(b') + \langle \nabla \Phi(b'), b - b' \rangle + \leq \sum_{s:1 \leq \alpha_s \leq \infty} KL_a(b_s||b'_s) - \sum_{s:1 < \alpha_s < \infty} \frac{1}{(1 - \alpha_s)} KL_b(b_s||b'_s) \quad (3.20)$$

which proves that function  $\Phi(b)$  is 1 Bergman convex wrt  $d_g$  (3.15) □

### 3.6 Bid updating rule

In this section, we provide with bid updating rule which enables the service providers to reach the market equilibrium of the market  $\mathcal{M}$ . We build an analysis of the bid updating rule depending on the service providers'  $\alpha_s$  fairness criteria. We consider a case when all the SPs employ the  $\alpha_s \geq 1$ , calling it a complementary domain; over this domain, as we have shown in the previous section, the potential function  $\Phi$  is convex in its argument, and the minimal point of  $\Phi$  represents the market equilibrium points. As the function,  $\Phi$  is separable in each service provider decision. The SPs can reach the equilibrium by employing a mirror descent update to minimize the potential function  $\Phi$  in a decentralized fashion. Let  $b_{ckr}^s(t)$  represent the bid submitted by SP  $s$  at step  $t$  on the resource type  $r$  in the cell  $c$  for the class of user  $k$ ,  $p_{cr}(t)$  defines the price of the resource set through TP-mechanism in time step  $t$ , where  $p_{cr}(t) = \sum_s \sum_{k \in \mathcal{K}_c^s} b_{ckr}^s(t)$

The bid update for service providers in the time step  $t + 1$  is given as

- if  $\alpha_s = \infty$

$$b_{ckr}^s(t+1) = \frac{B_s w_{ck}^s p_{cr}(t) d_{ckr}^s}{\sum_c \sum_k \sum_r w_{ck}^s p_{cr}(t) d_{ckr}^s} \quad (3.21)$$

- if  $1 \leq \alpha_s < \infty$

$$\frac{B_s \frac{p_{cr}(t) d_{ckr}^s}{\sum_r p_{cr}(t) d_{ckr}^s} (w_{ck}^s)^{\frac{1}{\alpha_s}} (\sum_r p_{cr}(t) d_{ckr}^s)^{\frac{(1-\alpha_s)}{-\alpha_s}}}{\sum_c \sum_k (w_{ck}^s)^{\frac{1}{\alpha_s}} (\sum_r p_{cr}(t) d_{ckr}^s)^{\frac{(1-\alpha_s)}{-\alpha_s}}} \quad (3.22)$$

Following theoretical results show that if the service providers update their bids according to the above-designed rule, then iterative bid updating dynamics of SPs converge to the market equilibrium of market  $\mathcal{M}$

**Theorem 3.9.** *Consider each SP  $s \in \mathcal{S}$  implement the  $\alpha_s$ -fairness with its respective fairness parameter  $\alpha_s \in [1, \infty]$  and repeatedly update their distribution of shares using rule (3.21)-(3.22). Then the potential function  $\Phi$  from (3.13) converges to the market equilibrium as follows*

$$\Phi(b^T) - \Phi(b^*) \leq \frac{1}{T} \sum_s KL_a(b_s^* || b_s^0) - \frac{1}{T} \sum_{s:1 < \alpha_s < \infty} \frac{1}{(1-\alpha_s)} KL_b(b_s^* || b_s^0) \quad (3.23)$$

*Proof.* Steps of proof are as follows

1. we show that if  $\forall s \in \mathcal{S}$  with  $\alpha_s \geq 1$  update rule (3.21) and (3.22) is mirror descent update of potential  $\Phi$  wrt

$$d_g = \sum_{s:1 \leq \alpha_s \leq \infty} KL_a(\cdot || \cdot) - \sum_{s:1 < \alpha_s < \infty} \frac{1}{(1-\alpha_s)} KL_b(\cdot || \cdot) \quad (3.24)$$

the detailed derivation of update rule is provided in the Appendix B.2, where mirror descent update in step  $(t + 1)$  is given as

$$b^s(t + 1) \leftarrow \arg \min_{\sum_c \sum_k \sum_r b_{ckr}^s \leq B_s} \left\{ \nabla_{b^s} \Phi(b^s(t)) (b^s - b^s(t)) + KL_a(b^s || b^s(t)) - \frac{KL_b(b^s || b^s(t))}{(1 - \alpha_s)} \right\} \quad (3.25)$$

2. from Lemma 3.8, we know that  $\Phi(b)$  is 1-Bergman convex function wrt to  $d_g(3.24)$ .
3. Now suppose the  $b^T$  is the point reached after  $T$  applications of the mirror descent update rule then by applying Thm 3.2 [23], we get the desired result (3.23)

□

**Corollary 3.10.** *Consider SPs apply the fairness criteria with  $\alpha_s \geq 1$  and the given price set by the TP-mechanism in each time step they update their distribution of shares in each next time step as the best response. Then the iterative best response dynamics of SPs converges to market equilibrium.*

*Proof.* We show that for all SPs with  $\alpha_s \geq 1$ , given resources prices announced by the TP-mechanism, the bid update rule in the next round is exactly the best response of SPs given resources prices set in the current round. Hence the convergence of best response dynamics follows from the previous theorem. The derivation of the best response of SP given prices is given in Appendix B.3 □

### 3.7 The fairness and efficiency

This section investigates the fairness and efficiency properties of the proposed allocation scheme. We measure the performance of the proposed scheme with the help of the social welfare function; it is a real-valued function that measures the desirability of the allocation outcome. The higher a value it assigns to the outcome, the more desirable the outcomes for a social planner. Various social welfare functions have been mentioned in the literature, the most commonly studied among them are the max-min welfare  $\Phi(x) = \min_s U_s(x_s)$  the Nash welfare  $\Phi(x) = \prod_s U_s(x_s)^{B_s}$  utilitarian welfare  $\Phi(x) = \sum_s U_s(x_s)$ .

As per the result established in the Section 3.4.1, the market equilibrium for market  $\mathcal{M}$  can be computed by solving EG-optimization program (3.7), Eisenberg and Gale showed in their celebrated work [32][31] that allocation under market equilibria achieves optimal Nash welfare. This result has been established based on a relation that the maximization of the objective function in (3.7) is equivalent to the maximization of Nash welfare function.

$$\arg \max_{x \in \mathcal{X}} \prod_s U_s(x_s)^{B_s} = \arg \max_{x \in \mathcal{X}} \sum_{s \in \mathcal{S}} B_s \log(U_s(x_s)) \quad (3.26)$$

Therefore proposed allocation scheme maximizes the Nash welfare or achieves the proportional fair criteria while distributing the resources among the service providers.

### 3.7.1 Baseline resource allocations

This section presents the two baseline allocation schemes to conduct a comparative analysis of the efficiency of our proposed resource allocation scheme. As discussed earlier, one of the goals of our proposed allocation is to achieve a tradeoff between efficiency versus SLA protection, and we know that the optimal social allocation provides better service utilization. In contrast, static proportional allocation (SS) offers complete protection of SLA among slices. Thus we consider the socially optimal allocation and the static proportional sharing scheme as baseline allocation schemes.

**Socially Optimal Allocation (SO):** In this work, we consider that the utility of each SP is its private information and not known to others. However, If the SPs' utilities were known to the InP, the natural choice of allocation scheme InP could have applied is the socially optimal resource allocation scheme. Thus to compare the efficiency of the proposed allocation scheme, we consider the following social welfare optimization problem.

$$\begin{aligned}
& \underset{x}{\text{maximize}} && \sum_{s \in \mathcal{S}} B_s(U_s(x_s)) \\
& \text{subject to} && \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} x_{ckr}^s \leq 1, \forall c \in \mathcal{C}, r \in \mathcal{R} \\
& && x_{ckr}^s \geq 0, \forall c \in \mathcal{C}, r \in \mathcal{R}
\end{aligned} \tag{3.27}$$

**Static Proportional sharing scheme (SS):** It is also known as static proportional splitting. In this resource allocation scheme, resources are partitioned based on SPs' network shares (budgets). To be more precise, every SP is allocated a portion of every demanded resource proportional to its budget or shares *i.e.*,  $\forall s \in \mathcal{S}, \forall c \in \mathcal{C}$  and  $\forall r \in \mathcal{R}$   $x_{cr}^s = \frac{B_s}{\sum_{s' \in \mathcal{S}} B_{s'}}$ .

Now we analyze the efficiency of the proposed scheme *i.e.*, efficiency of ME to the market  $\mathcal{M}$  by comparing it with socially optimal allocation. Let  $U(SO)$  denotes social optimum, an optimal value of optimization problem  $SW$  defined in (3.27), and  $U(ME)$  denotes the value of social welfare under allocation imposed at ME. We consider standard notion of *price of anarchy* defined as  $\text{PoA} = \frac{U(SO) - U(ME)}{U(SO)}$ . To find the PoA of the given market  $\mathcal{M}$ , we first use the result discussed at the beginning of the section that the resource allocation under ME of the market  $\mathcal{M}$  can equivalently be computed by solving EG-program and the problem's optimal solution provides the allocation that attains proportional fairness (PF) among the agents. Generally, the attainment of fairness in allocation results in a decline in the system's efficiency. The trade-off between efficiency and fairness were well studied in the [14] using the notion *price of fairness* (PoF), which is defined as a relative reduction in social welfare under fair allocation compared to the social optimum,  $\text{PoF} = \frac{U(SO) - U(PF)}{U(SO)}$ . Where  $U(SO)$  denoted the value of optimal social welfare function while  $U(PF)$  denotes the value of social

welfare function at proportional fair allocation. In the following theorem, we derive the bound on PoA for the proposed allocation scheme using results on the bound of PoF established in [14].

**Theorem 3.11.** *Let the maximum achievable utility of each SP  $s \in \mathcal{S}$  in market  $\mathcal{M}$  is  $\hat{U}_s > 0$ , then price of anarchy is bounded by  $PoA \leq 1 - \frac{2\sqrt{S}-1}{S} \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\max_{s \in \mathcal{S}} \hat{U}_s} - \frac{1}{S} + \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\sum_{s \in \mathcal{S}} \hat{U}_s}$  and if the maximum achievable utilities of all SPs are equal then  $PoA \leq 1 - \frac{2\sqrt{S}-1}{S}$*

*Proof.* Let the price of fairness (PoF) for proportional fairness criteria is defined as  $PoF = \frac{U(SO)-U(PF)}{U(SO)}$ . According to theorem 2 [14] if maximum achievable utility of each agent  $s \in \mathcal{S}$  in the market  $\mathcal{M}$  is  $\hat{U}_s > 0$  then value of PoF for proportional fairness is bounded by

$$\frac{U(SO) - U(PF)}{U(SO)} \leq 1 - \frac{2\sqrt{S} - 1}{S} \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\max_{s \in \mathcal{S}} \hat{U}_s} - \frac{1}{S} + \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\sum_{s \in \mathcal{S}} \hat{U}_s}$$

We notice that the value of social welfare under proportional fair allocation is equal to the value of social welfare under ME's allocation. Thus replacing  $U(PF)$  by  $U(ME)$  we get

$$\frac{U(SO) - U(ME)}{U(SO)} \leq 1 - \frac{2\sqrt{S} - 1}{S} \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\max_{s \in \mathcal{S}} \hat{U}_s} - \frac{1}{S} + \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\sum_{s \in \mathcal{S}} \hat{U}_s} \quad (3.28)$$

Hence bound on PoA write as

$$PoA \leq 1 - \frac{2\sqrt{S} - 1}{S} \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\max_{s \in \mathcal{S}} \hat{U}_s} - \frac{1}{S} + \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\sum_{s \in \mathcal{S}} \hat{U}_s}$$

Similarly if the maximum achievable utilities of all SPs are equal then by the Thm.1 [14]

$$PoA \leq 1 - \frac{2\sqrt{S} - 1}{S}$$

□

From the above theorem, we can deduce that the efficiency of the proposed resource allocation scheme decreases with the increase in the number of service providers. Nonetheless, the socially optimal allocation offers efficient resource utilization, but at the cost of poor fairness. In the numerical section, we will see that sometimes hardly any resources are allocated for service providers with low marginal gain under the SO allocation scheme. Further, the SO allocation scheme does not guarantee the existence of any market equilibrium or stability in the allocation method. Next, we compare the performance of the proposed scheme with the static proportional allocation scheme

**Theorem 3.12.** *Under the proposed resource allocation scheme, i.e. at the ME of the market  $\mathcal{M}$  each SP achieves the utility higher than or equal to the utility under static proportional allocation (SS).*

*Proof.* Let  $(\hat{\mathbf{x}}, \hat{\mathbf{p}})$  be the market equilibrium of market  $\mathcal{M}$  then by definition of ME

$$\forall s \in \mathcal{S} \quad \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} \hat{p}_{cr} \hat{x}_{ckr}^s = B_s$$

and  $\forall s \in \mathcal{S}, U_s(\hat{x}_s)$  is utility achieved by each SP  $s$  under ME. Let  $\bar{\mathbf{x}}$  be the resource allocation under the static proportional allocation scheme. Then to prove the desired results, we first show that allocation  $\bar{\mathbf{x}}$  is budget exhausting with respect to price vector  $\hat{\mathbf{p}}$  *i.e.*,

$$\forall s \in \mathcal{S} \quad \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} \hat{p}_{cr} \bar{x}_{cr}^s \leq B_s$$

Just by replacing  $\bar{x}_{cr}^s$  with  $\frac{B_s}{\sum_{s' \in \mathcal{S}} B_{s'}}$  in above inequality gives us first result. Now as  $\hat{\mathbf{x}}$  and  $\bar{\mathbf{x}}$  both are feasible and budget exhausting allocation then by definition of market equilibrium

$$U_s(\bar{x}_s) \leq U_s(\hat{x}_s), \forall s \in \mathcal{S}$$

Hence proves the theorem □

The above theoretical result proves that the proposed resource scheme achieves better efficiency than the static proportional allocation scheme. Thus proposed Fisher market-based allocation brings off a better arbitrage between the system efficiency and protection of the service level agreement of SPs

### 3.8 Numerical Experiments

In this section, we numerically evaluate the performance of the proposed allocation scheme. For the simulation purpose, we consider a scenario where the network consists of four cells, and each cell accommodates three types of resources: CPU, RAM, and Bandwidth (BW), and their available capacities at each cell are 30 Units, 126 Gb and 40 MHz, respectively. We assume the four service providers owning an equal share of infrastructure support the four types of service class: CPU-intensive, RAM intensive, BW-intensive, and Balanced class. The base demand vector for each service class is as described in the Table 3.3 [74] For convenience, we assume that each

Service Class	CPU	RAM	BW
BW-Intensive	1CPU <sub>s</sub>	8Gb	10MHz
CPU-Intensive	4 CPU	8Gb	3MHz
RAM-Intensive	1 CPU	32Gb	3MHz
Balanced	5CPU <sub>s</sub>	40Gb	5MHz

Table 3.3 – The base demand vector of service classes

SP supports only a single user class. The SPs’ user load is non-uniform, and the number of users associated with each SP and each cell is described in the Table 3.4. Assuming the SPs employ

class	cell 1	cell2	cell3	cell4
SP1	10	12	7	14
SP2	5	16	11	8
SP3	15	6	10	12
SP4	8	4	15	13

Table 3.4 – Distribution of SPs users across the cells

the different alpha fairness criteria, We calculate the market equilibrium allocations for the given scenario employing the designed bid updating scheme. (3.6). The bar graphs in Fig 3.4 show the service rate or utility achieved by the users at different locations under socially optimal allocation (SO), market equilibrium (ME) and static proportional (SS) allocation schemes.

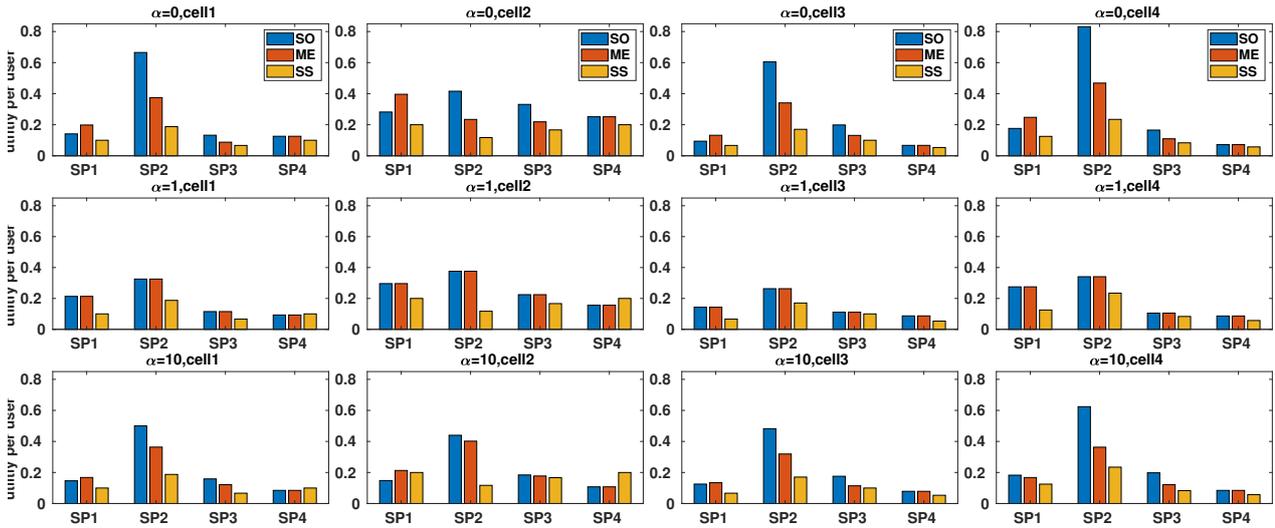


Figure 3.4 – Bar graphs describe the utilities (service rate) under Socially optimal allocation (SO), market equilibrium (ME) and Static proportional (SS) allocation scheme attained by the users associated with the service providers SP1, SP2, SP3 and SP4 in the cell 1-4, when  $\alpha$  fairness criteria with  $\alpha = 0$ ,  $\alpha = 1$  and  $\alpha = 10$  applied by the service providers.

We observe from the graphs that for all the values of  $\alpha$ , the service rate seen by users at each cell is inversely proportional to the number of users associated with the same service provider and the total number of users present in that cell. For instance, in cell 1 SP2 has the least number of users among the other service providers, and the service rate seen by SPs 2 users is high compared to all other users. While in the same cell, SP3 users experience a low service rate. We can also notice the

effect of  $\alpha$  fairness applied by the SPs in the graphs. In the first row of graphs, for service providers, the difference between the highest service rate and the lowest service experienced by their users decreases with the  $\alpha$  in the second and third rows. When SPs employ the proportional fairness criteria *i.e.*,  $\alpha = 1$ , the socially optimal allocation equals the market equilibrium allocation. Moreover, irrespective of fairness applied by the SPs, utilities gained by the SPs under market equilibrium allocation are at least equal to those acquired under the static proportional allocation scheme. The

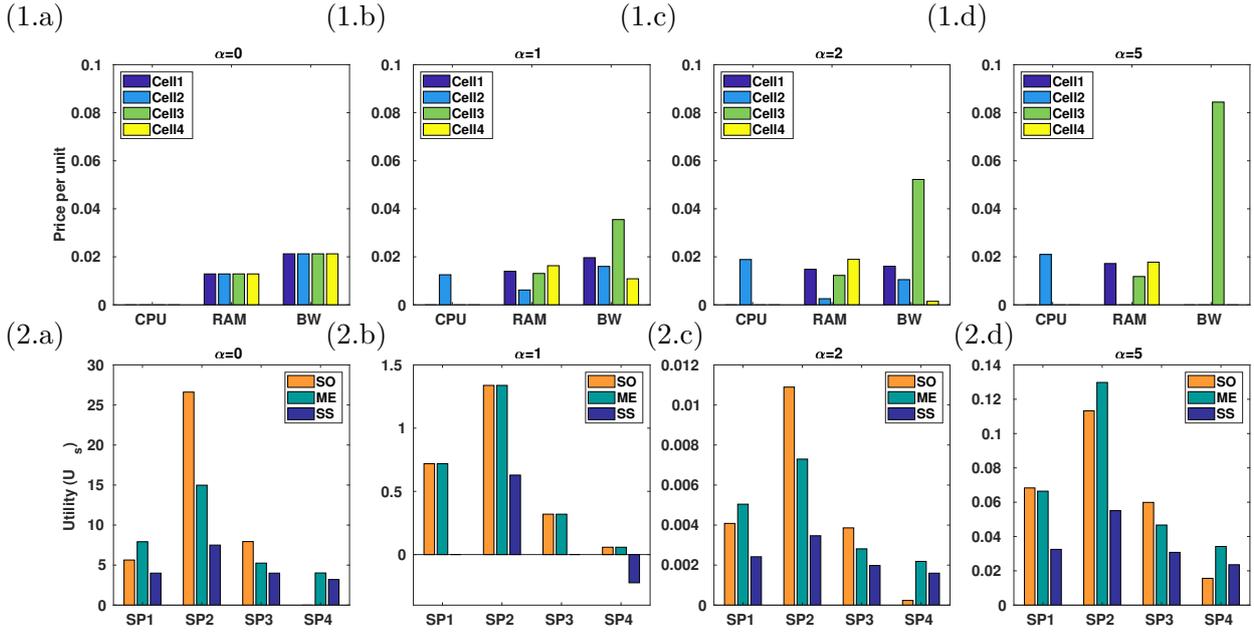


Figure 3.5 – Bar graphs 1. a to 1.d show the price per unit of resources CPU, RAM and Bandwidth (BW) at cells 1 to 4 when alpha fairness criteria with  $\alpha = 0, 1, 2$  and  $5$  applied by the service providers. Each bar graph from 2. a to 2.d describe the total utility achieved by the SPs under optimal social allocation (SO), market equilibrium (ME) allocation, and static proportional allocation (SS) when  $\alpha$  fairness with  $\alpha = 0, 1, 2$  and  $5$  applied by the SPs.

bar graphs (1.a)-(1.d) in the Fig.3.5 depict the effect of the parameter  $\alpha$  applied by SPs on the prices of resources. For instance, the price per unit of BW at cell 3 increases with  $\alpha$ . As the  $\alpha$  increases, the differences between the highest utility and lowest utility achieved by users associated with the SPs decreases. The SPs need to pull more resources at the location where users are experiencing the lowest service rates to achieve the desired results. However, if two or more SPs have the same pattern in service rates achieved by their users, this results in congestion at some locations, which further reflects an increase in pricing. The plot in Fig.3.6 shows the fast convergence of prices to market equilibrium at cell 2 through TP-mechanism.

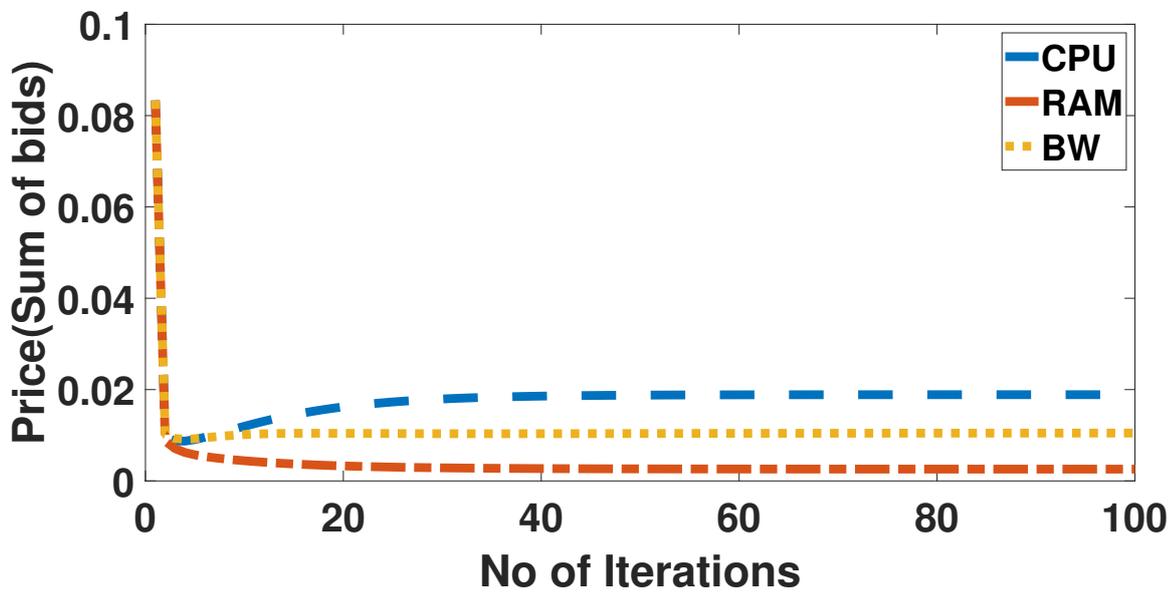


Figure 3.6 – The fast convergence of prices to market equilibrium at cell 2 by SPs via trading post mechanism and when SPs employ proportional fairness criteria.

# STRATEGIC RESOURCE PRICING AND ALLOCATION IN A 5G NETWORK SLICING STACKELBERG GAME

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## 4.1 Introduction

It is predicted that the implication of Network slicing will bring a paradigm shift towards a multi-tenancy ecosystem where multiple tenants owning individual slices negotiate with multiple infrastructure network providers (InPs) to request resources for service provision. In this competitive multi-agent setting, the service providers (SPs), also called slice tenants, generally express a demand for a dedicated virtual network with full ownership of their service level agreement (SLA). On the contrary, InPs aim to maximize their return on investment by enabling the dynamic sharing of the infrastructure, as this lowers their operational and capital costs and allows them to monetize their infrastructure to its fullest potential. However, the sharing of infrastructure may expose the tenants to the risk of violating their SLAs. Hence, one of the fundamental issues in network slicing is an efficient sharing of the network resources, which arbitrages between two conflicting interests, *i.e.*, interslice isolation and efficient network resource utilization. In order to balance the interslice isolation and efficient resource utilization, Caballero et al. in [19] proposed the “share-constrained proportional allocation” (SCPA) scheme where each slice is pre-assigned with a fixed share (budget) of infrastructure. Slices are then allowed to redistribute their shares and customize their allocation according to the load dynamics. In turn, InP allocates each resource to slices in proportion to their shares on that resource. This approach allows a dynamic sharing, where tenants can redistribute their network share based on the load dynamics. At the same time, it provides the slice tenants a degree of protection by keeping the pre-assigned share intact.

Game-theoretic models have been employed for strategic resource allocation in communication networks, power systems, and more generally, a large number of deregulated industries. When dealing with strategic resource allocation, each player’s utility function depends on his own decision variables, and on that of the other players. The players’ feasibility sets are also coupled through some (global/local) coupling constraints, capturing the laws of physics or, simply, shared capac-

ity constraints. Extending duality results from standard continuous optimization to noncooperative games, the dual variables of the coupling constraints can be interpreted as market prices, also called shadow prices or locational marginal prices, capturing the state of the network, *e.g.*, congestion. Applying a similar model to dynamic resource trading in a 5G network, we design a communication service market where the InP charges dynamically the service providers, depending on how much they contribute to the infrastructure utilization. The prices are also locational, being differentiated by cell and resource. Relying on a service market where the prices are automatically adjusted according to supply and demand, Lieto et al. proved that the dynamic trading of resources enables service providers to reduce their costs [63], and therefore maximize their profits.

Relying on network slicing, we consider a market design where a set of SPs lease their respective networks from InP and employ the network slicing mechanism to request the resources required for their service provision. We assume that the SPs offer a particular service to users, and the resources inventory available with SPs characterizes their service performance. The users are free to choose their SP. Their decisions are made based on the service satisfaction attained from SPs. Furthermore, the SPs collect revenue by providing the service to their customers. Assuming a dynamic resource sharing mechanism and that SPs are utility maximizing players, it is highly expected that selfish SPs may exhibit strategic behaviour. For example, they might strategically distribute their shares on the resources conditioned on the trade-off between the quality of service (QoS) they want to offer and the congestion perceived by the users. In this work, we focus on (1) building a game-theoretic model of the communication service market where SPs negotiate with InP to request resources and compete with one another to serve a pool of end-users, (2) developing a dynamic resource allocation and pricing mechanism under a competitive environment.

#### 4.1.1 Related work

There is a large part of the literature dedicated to the design of communication service markets. Broadly, communication service markets have been studied as a two-stage noncooperative game involving three types of participants: Infrastructure provider (InP), Service provider (SP)<sup>1</sup> and End-users (EU). In the first level of the game, SPs (buyers) lease the resources from the InPs (sellers), negotiating for resource prices and resource quantity. In the second level, SPs (buyers) use the acquired resources from InPs to offer a certain service to their end-users (buyers). At this level, SPs decide on their service price and the scheduling of resources, while EUs make their subscription decisions. In [30], SPs' strategic decision over their service pricing scheme has been analyzed as a Cournot game. In [54], Korcak et al. considered that the quality of service (QoS) achieved by the SP's users depends on the number of subscribers of that SP, and users' choice behavior can be analyzed relying on evolutionary game theory (EGT). Li et al. in [62] integrated both the

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1. In many articles, the terms Mobile virtual network operator (MVNO), slice tenant, or Mobile service provider (MSP) are used without distinction to designate SP.

users' choice evolution and the SPs pricing scheme. they formulated the resulting problem as a Stackelberg game. The SPs, interpreted as leaders, strategically decide the price to attract the users, and the users, seen as followers choose the SPs to maximize their service satisfaction level. Also, the number of subscribers of the SPs depends on the perceived QoS and, consequently, on their resources availability. Focusing on competitive aspects, the SPs can act strategically when computing their resource demand, giving rise to a simultaneous noncooperative game [100]. In [26], D'Oro et al. used a similar Stackelberg game formulation for resource allocation and orchestration in the network functions virtualization scenario. In [9], Azouzi et al. considered that the competition between SPs takes place both in prices and in the QoS. In practice, SPs may not have complete information about the other SPs resources. Dealing with such an incomplete information setting, Li et al. in [61] studied SPs' pricing strategies relying on a Bayesian game formulation, where SPs compute their prices based on their beliefs about the resource availability. Li et al. also considered the possibility that the SPs can coordinate and analyzed the impact of cooperation on the pricing scheme. In all the above works, the SPs lease the resources from the InP and compete to serve EUs, which is also the case in our work. However, our work innovates in that the resources are shared using a slice-based dynamic sharing mechanism. Moreover, in our case, resources are spatially distributed, and service offered in a particular cell can only be supported by the resources available within that cell. In communication networks, one of the well-known scheme for resource allocation is the auction-based allocation [25], *e.g.*, Kelly's mechanism. Tun et al. in [103] proposed multi-bidding Kelly's mechanism-based resource allocation for 5G slicing. They showed that Kelly's mechanism leads to a fair and efficient resource allocation both at slices and EUs levels. Our work departs from the auction-based mechanism like [27, 104], where agents' bids are unbounded.

In follow up work to [19], Zheng et al. in [115] considered the network slicing under stochastic loads and applied SCPA based resource sharing scheme. They modeled the resource sharing scheme as a noncooperative game and proved that slices achieve efficient statistical multiplexing at the Nash equilibrium. Guijarro et al. in [41] designed a communication service market where SPs employ the SCPA mechanism to request the resources from InP. They analyzed the economic impact of network slicing on the market. In [63], an automated negotiation mechanism is defined relying on an aggregative game that enables the slice tenants to dynamically trade the radio resources and customize their slices on instantaneous demands, which help tenants achieve higher profits. Our work is closely related to [41]. The main novelty of our work lies in the fact that we consider multi-resource service provisioning, contrary to most articles dealing with communication service market design, which, to the best of our knowledge, only deals with radio resources.

In this work, we leveraged the Tullock contest (TC) framework [5] to model the competition between slices. This framework has been extensively used in the communication network literature, to model the interactions between competitive agents. To mention a few, in [87], the competition between social media users for visibility over the timeline was modeled as a TC. Luo et al. in [65]

proposed a TC based incentive mechanism for crowdsourcing. The TC framework has been applied to the multipath TCP network utility maximization problem [86]. In [7], Altman et al. studied the multi-cryptocurrency blockchain from a game-theoretic perspective, where the competition between the miners is framed as a TC. To the best of our knowledge, the theoretical results on the TC framework and its applications in literature only deal with a single resource case. We extend the TC framework to a multi-resource scenario, and thus our results also contribute to the theoretical literature on the TC framework.

#### 4.1.2 Main Contributions

We list below the key contributions of our work.

1. We propose a business model for the SPs, where the SPs deploy network slices for their business and lease their respective resources through network slicing mechanism (*i.e.*, dynamic sharing). The SPs compete with one another, with respect to their QoS, to serve end-users.
2. We model the interactions between the SPs and users as a Stackelberg game, where SPs act as leaders and users as followers.
3. We model the decision-making process among the users with replicator dynamics.
4. We show that the noncooperative game induced by the competition between SPs (that can be interpreted as a multi-resource Tullock rent-seeking game) admits a unique Nash equilibrium (NE). Thus, our theoretical results also contribute to the study of the Tullock rent-seeking game.
5. In a market design where the InP prices the resources at their optimal values, we implement resource pricing through two different approaches.
6. First, when service providers possess finite monetary budgets, we propose the implementation of pricing through a trading post mechanism.
7. Assuming a single resource is available in the above case, we show analytically that the game induced by the trading post mechanism admits a unique NE.
8. When budgets do not bind service providers for their demand, we prove that resource pricing can be implemented by solving a generalized Nash equilibrium problem, involving coupling constraints capturing the network finite capacity.
9. Finally, we provide two online learning algorithms and a fully distributed proximal based algorithm to reach NE and variational equilibrium solutions of the post trading mechanism and pricing game respectively.

The rest of the chapter is organised as follows: Section 4.2 introduces the system model. Section 4.3 details the game-theoretic model of competition between the SPs. In Section 4.3.2, we describe the Stackelberg game between SPs and EUs. In Section 4.3.3, we analyse the existence

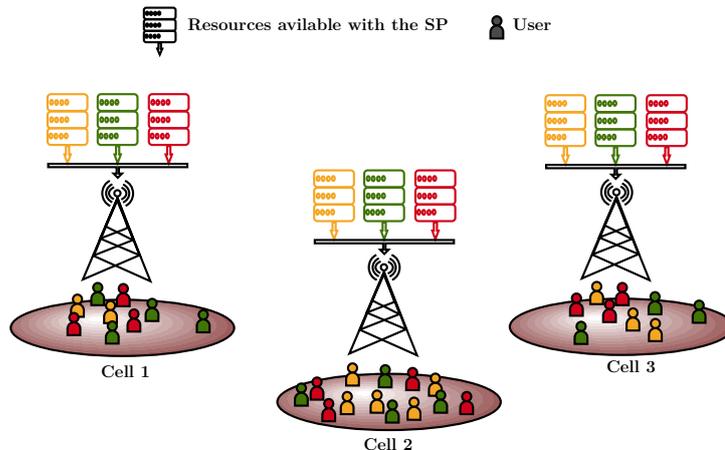


Figure 4.1 – Service providers *i.e.*, (slices) compete to offer a certain service to geographically distributed pool of users

and uniqueness of the NE. Section 4.4 introduces the resource pricing and market equilibrium. In Section 4.5, we provide two (semi-decentralized) online learning algorithms and a fully distributed algorithm to compute market equilibria. In Section 4.6, we report on numerical results. A concluding section ends the chapter.

*Notation:* Let  $\mathbb{R}^n$  indicate the set of  $n$  dimension real vectors, and  $\mathbb{R}_+^n$  its nonnegative orthant.  $\|\cdot\|$  represents the Euclidean norm. Given a vector  $x$ ,  $x^T$  denotes its transpose. Let  $\text{col}(x_1, \dots, x_N) := [x_1^T, \dots, x_N^T]$ . For a closed set  $\mathcal{F} \subseteq \mathbb{R}^n$ , the mapping  $\text{proj}_{\mathcal{F}} : \mathbb{R}^n \rightarrow \mathcal{F}$  denotes the projection onto  $\mathcal{F}$ , *i.e.*,  $\text{proj}_{\mathcal{F}}(x) := \arg \min_{y \in \mathcal{F}} \|y - x\|$ . Depending on the context,  $|\cdot|$  will denote the absolute value of a scalar or the cardinal of a set. Table 4.1 summarizes the main notation used in the chapter.

## 4.2 System Model

We consider a market design, where in the first stage, a set of SPs,  $\mathcal{S}$ , lease their respective networks from the InP and employ a network slicing mechanism to request the resources required for their service provision. In stage two, the SPs (sellers) use the leased resources and compete to maximize their number of end-users (buyers). Specifically, we assume the InP owns a network that consists of a set of base stations or cells,  $\mathcal{C}$ . Each base station at different locations accommodates multiple types of resources such as bandwidth, CPU, memory, etc. Users are spread across the network, let  $N^c$  be the number of users present in cell  $c$ . We assume that the service offered by the SP in a particular cell can only be supported by the resources available within that cell.

Table 4.1 – Main notations used throughout the Chapter 4

$\mathcal{C} := \{1, \dots, C\}$	$\triangleq$	Set of base stations or cells
$\mathcal{S} := \{1, \dots, S\}$	$\triangleq$	Set of slices (tenants)
$\mathcal{M}^c$	$\triangleq$	Set of resources at base station $c$
$N^c$	$\triangleq$	Number of users in cell $c$
$\nu_s^c$	$\triangleq$	Utility of user associated with SP $s$ in cell $c$
$q_c^s$	$\triangleq$	Quality of service offered by SP $s$ in cell $c$
$n_s^c$	$\triangleq$	Number (subscribers) users associated with slice $s$ in cell $c$
$d_s^c := (d_{sm}^c)_{m \in \mathcal{M}^c}$	$\triangleq$	Bundle of resources available with slice $s$ in cell $c$
$d_{sm}^c$	$\triangleq$	Amount of resource type $m$ available with slice $s$ in cell $c$
$D_m^c$	$\triangleq$	Capacity of resource type $m$ at base station $c$
$\omega_m^c$	$\triangleq$	Price per unit resource of type $m$ at base station $c$
$p_s$	$\triangleq$	Service fees charge by slice $s$ to users
$R_s$	$\triangleq$	SP $s$ expected revenue
$U_s$	$\triangleq$	SP $s$ profit
$B_s$	$\triangleq$	Budget available with slice $s$
$K_s$	$\triangleq$	Local constraint function
$\tau_{s,s'}^c(n^c, U^c)$	$\triangleq$	Revision protocol which defines the switching rate at which users in cell $c$ switch their choice from SP $s$ to SP $s'$
$A_s$	$\triangleq$	Slice association probability function
$b_{sm}^c$	$\triangleq$	Bid by SP $s$ to resource $m$ at cell $c$
$\lambda_{sm}^c$	$\triangleq$	Lagrange multipliers of the capacity based coupling constraints
$r_s$	$\triangleq$	Ratio of the coupling constraint dual variable at r-normalized Nash equilibrium ( $\lambda_m^c$ ) over dual variable evaluated by SP $s$ ( $\lambda_{sm}^c$ )
$y_s$	$\triangleq$	Auxiliary variable which accumulates discounted gradient for SP $s$
$\alpha_n$	$\triangleq$	Discounting factor or step size
$h_s(b)$	$\triangleq$	Regularization function or a penalty function
$\zeta$	$\triangleq$	Vanilla ADMM penalty term
$\tilde{\beta}$	$\triangleq$	Proximal approximation penalty term

### 4.2.1 User Model

We assume that all the users need the same type of service, and they achieve their demand by subscribing to one of the SPs. We consider that each user is opportunistic and free to switch from one SP to another, which is equivalent to choose one slice among the set of available slices at his base station. The user chooses the slice (or, equivalently, the SP) that offers the better trade-off, i.e., the higher QoS at the lowest price. We model the utility of each user served by SP  $s \in \mathcal{S}$  in cell  $c$  as [61]

$$\nu_s^c(n_s^c, q_s^c, p_s) = \log\left(\frac{q_s^c}{n_s^c}\right) - p_s, \quad (4.1)$$

where  $q_s^c$  is the QoS of SP  $s$  in cell  $c$ ,  $n_s^c$  is number of users connected to SP  $s$  while  $p_s$  is the subscription fees charged by SP  $s$  for its service. We assume that the service fees charged by each SP is the same across all cells. Here the use of a logarithmic<sup>2</sup>(concave) function as the user's utility in QoS means that the users' satisfaction level saturates as the QoS increases, which is coherent with the economic principle of diminishing marginal returns. In turn, the SP QoS depends on the resources inventory available to him. We assume each SP applies a scheduling policy to distribute his resources among the users, in order to achieve equal QoS among them, in the long run.

### 4.2.2 Service Provider Model

We assume that the SPs aim at maximizing their number of subscribers,  $n_s^c$ , by attracting users with a better QoS and lower price. We assume that the QoS provided by each SP depends on the resource inventory available at the slice and is defined according to the relation  $q_s^c \triangleq q_s^c(d_s^c)$ . Let  $d_s^c \triangleq (d_{sm}^c)_{m \in \mathcal{M}^c}$  denote a bundle of resources available with SP  $s$ .  $d_{sm}^c$  captures the amount of resource type  $m$  acquired by SP  $s$  at cell  $c$ . We assume that for all  $c \in \mathcal{C}$  and  $s \in \mathcal{S}$ , the function  $q_s^c(d_s^c)$  is concave non decreasing in  $d_s^c$ . This assumption is classical in economics, reflecting the principle of diminishing marginal returns.

Each SP collects revenue from the fees paid by his subscribers. The expected revenue of SP  $s$  over the network is defined as

$$R_s = \left( \sum_{c \in \mathcal{C}} p_s n_s^c \right). \quad (4.2)$$

Each SP needs to pay for the resources he leases from the InP. Let  $\omega_m^c$  be the price per unit of resource of type  $m$  charged by the InP, at base station  $c$ . The total cost each SP  $s$  needs to pay to the InP for resource activation is therefore  $\sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}^c} \omega_m^c d_{s,m}^c$ . We define the profit gained by SPs

2. The logarithm function also signifies that the SPs achieve the proportional fair allocation between the user in the long run

as a quasi linear utility function

$$U_s = R_s - \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}^c} \omega_m^c d_{s,m}^c. \quad (4.3)$$

Depending on the SPs' budgets, we consider two possible cases in the following.

In **Case I**, we assume that each SP  $s$  has a finite budget  $B_s$ , which captures the market (purchasing) power of the SP. Another relevant interpretation in the context of network slicing is that it represents the SP's priority or a fixed share of the available resource pool, such that  $\sum_{s \in \mathcal{S}} B_s = 1$ . In this case, each SP  $s$  must satisfy  $\sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}^c} \omega_m^c d_{s,m}^c = B_s$ .

In **Case II**, we assume that SP  $s$  has no bound over his budget. His strategy set is defined as the set of vectors  $d_s$  such that  $d_{s,m}^c \geq 0, \forall m \in \mathcal{M}^c, \forall c \in \mathcal{C}$ . Additional linear local constraints can be included in the form  $K_s(d_s) \leq 0$ .

### 4.3 Game Theoretic Model

We assume that each user is opportunistic, and takes decisions to maximize his utility. From (4.1), we observe that the utility of each user depends on the total number of users of the SP. On the one hand, as the number of users connected to the SP increases, the utility of the user decreases. Therefore, the decision made by each user is also influenced by decisions taken by the other users. On the other hand, SPs maximize their revenues by attracting the maximum number of users. Naturally, each SP anticipates the users' behaviour while computing their strategy. Therefore, it is highly expected that users and SPs exhibit strategic behaviors. In our work, the SPs take selfish decisions while anticipating the rational reactions of the users. We model the interactions between the users and the SPs as a (two-stage) Stackelberg game, where the SPs act as leaders while users react rationally as followers, computing their best responses to the signal sent by the SPs. In the first stage, SPs compete in terms of QoS to attract the maximum number of users. In the second stage, users optimally select their SPs to maximize their utility given prices and QoS offered by SPs.

In classical game theory, Nash equilibrium is the most popular solution concept to analyze non-cooperative game solutions. This concept is based on the assumptions that each player has an exact knowledge about all other players' strategies at the equilibrium, and no player has an incentive to deviate from his own strategy at equilibrium. In many cases, knowing the exact information about all other players equilibrium strategies is a strong assumption, particularly when there are many users, and information about the strategy profile of all opponents is rarely perfectly known. In light of these limitations, we model the interactions between the users as a population game that extends the formulation of a noncooperative game by incorporating the notion of population.

### 4.3.1 Population game $\mathcal{E}$ among Users

The population game provides an alternative to the classical equilibrium approach by involving a dynamic model. Unlike a single-play game or repeated games where all agents take their decisions simultaneously and repetitions occur at regular time periods, in a population game, each agent revises his decision sporadically, and the decision made by the revising agent only depends on the current system state and available payoff opportunities. Now for each cell  $c$ , we define the population game  $\mathcal{E}_c$

- Population: set of users  $\mathcal{N}^c := \{1 \dots N^c\}$  in cell  $c$ .
- Strategy: it is the choice of SP  $s \in \mathcal{S}$  that each user in cell  $c$  opts to join.
- Utility: the utility achieved by each user of slice  $s \in \mathcal{S}$  is equal to  $\nu_s^c$ .

In a population game, each agent revises his decision occasionally after some random duration of time. Whenever an agent reconsiders his decision, it depends on the system state and payoff opportunity available at that time. A general model of decision of the game is based on the concept of revision protocol. It is a mapping that translates the current population state (*i.e.*, distribution of user) and available payoff (*i.e.*, utilities (4.1)) into a switching rate which determines when users might update their choice of provider. Let  $\tau_{s,s'}^c(n^c, U^c)$  be the revision protocol which defines the switching rate at which users switch their choice from SP  $s$  to SP  $s'$  given population state  $n^c = [n_1^c \dots n_S^c]$  and utility vector  $\nu^c = [\nu_1^c \dots \nu_S^c]$ . Let  $\mathbb{N}^c = \{n^c | \sum_{s \in \mathcal{S}} n_s^c = N^c\}$  defines the set of all possible population states. Population game  $\mathcal{E}$  with revision protocol  $\tau$  generates a continuous time evolutionary process on set  $\mathbb{N}^c$  defined as

$$\dot{n}_s^c = \sum_{s'} n_{s'}^c \tau_{s',s}^c - n_s^c \sum_{s'} \tau_{s,s'}^c. \quad (4.4)$$

The first-term in the right-hand side of equation (4.4) measures the rate at which users connect to SP  $s$ . The second term measures the rate at which the portion of the population connected to SP  $s$  disconnects. A different choice of revision protocol results in different dynamics. In this work, we assume that the users follow the pairwise proportional imitation behavior, *e.g.*, after every random interval of time, each user interacts with his opponents (*i.e.*, other users), and only if users meet an opponent with a higher utility than his own, he imitates the opponent with a probability proportional to the utility difference. The switching rate at which users in cell  $c$  switch from SP  $s$  to SP  $s'$  takes the form

$$\tau_{s,s'}^c = \frac{n_{s'}^c}{N^c} [\nu_{s'}^c - \nu_s^c]_+. \quad (4.5)$$

After replacing  $\tau_{s,s'}^c$  in (4.4) with (4.5) and after some analytical calculations detailed in Appendix C.4

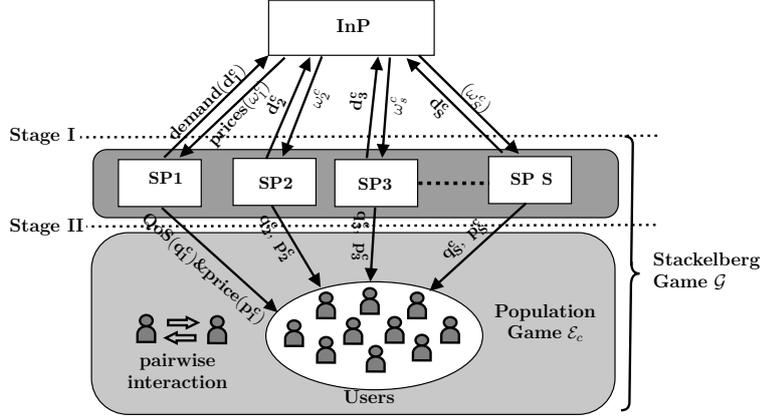


Figure 4.2 – Service providers and end users interactions as a stackelberg game

we get the replicator dynamics

$$\dot{n}_s^c = n_s^c \left[ \nu_s^c - \frac{1}{N^c} \sum_{s'} n_{s'}^c \nu_{s'}^c \right]. \quad (4.6)$$

An Evolutionarily Stable Strategy (ESS) characterizes the equilibrium solution concept for population games. Once the evolutionary process reaches an ESS, the population state will not change. It is defined as the fixed point of the dynamical system defined through equation (4.4).

**Proposition 3.** For all  $c \in \mathcal{C}$  and for any bundle of resources available with SP  $s$ , the replicator equation (4.6) admits a unique globally asymptotically stable equilibrium,  $\hat{n}_s$ . Moreover, the number of users  $\hat{n}_s^c$  in cell  $c$  associated with SP  $s$  at the equilibrium point can be defined as

$$\hat{n}_s^c = \frac{N^c q_s^c e^{-p_s}}{\sum_{s' \in \mathcal{S}} q_{s'}^c e^{-p_{s'}}}. \quad (4.7)$$

*Proof.* The replicator equation (4.6) is nothing but a set of ordinary differential equations (ODE). The equilibrium is the stationary point of ODE. Hence, to show that the replicator dynamics admits a unique equilibrium point, it is sufficient to show that the right-hand side of (4.6) is continuously differentiable and that it admits a unique stationary point [91]. Replacing  $\nu_s^c, \nu_{s'}^c$  from (4.1) in (4.6), we derive analytically the equilibrium point expression (4.7). A detailed proof is provided in Appendix C.1. For the replicator dynamics considering a function  $L(n) := \sum_s \sum_c \hat{n}_s^c \log \left( \frac{\hat{n}_s^c}{n_s^c} \right)$  as the Lyapunov function, we can show the equilibrium point is globally asymptotically stable, see [46] for more details.  $\square$

### 4.3.2 Stackelberg game between Service providers and Users

In our game-theoretic formulation of the communication service market, the SPs are the leaders, and the users are the followers. We have proved in previous section that the equilibrium of the population game  $\mathcal{E}_c$  between the users admits a unique solution, and the distribution of users at the equilibrium point is derived relying on the closed form expression (4.7). In this section, we model the interactions between the SPs as a noncooperative game.

We note that the analytical expression of the number of users  $\hat{n}_s^c$  in cell  $c$  of SP  $s$  at equilibrium is very similar to a contest success function from the well known TC framework [102]. The TC framework is commonly used in the economics literature to model strategic interactions between two or more competing agents. The basic contest framework consists of competing agents who expend costly resources to win a prize (a contest). Given the efforts exerted by all the agents, the probability of an agent  $i$  winning a prize is defined by the contest success function (CSF). Typically, the CSF function is defined as  $\rho_i(x) = \frac{(x_i)^r}{\sum_{i'} (x_{i'})^r}$  where  $x_i$  is the effort made by a agent  $i$  and  $r$  is a parameter. For example,  $r = 1$  is the well know lottery and  $r \rightarrow \infty$  defines the all-pay auction.

In the communication market context, the SPs compete to attract users to their services by exerting effort on costly resources. The resources acquired by SPs further reflect their service quality (a higher QoS is seen as a desirable attribute in the process of SP selection). Thus, in our case, the CSF can be considered as the probability that any SP successfully attracts an end-user. We call it the *slice association probability function*  $A_s$ . It is the probability that given resources expended by all SPs, a user will associate with SP  $s$ . For our model, we rely on a more general and multi-resource CSF function or slice association probability function

$$A_s^c(d^c, p) = \frac{f_s^c(d_s^c, p_s)}{\sum_{s' \in \mathcal{S}} f_{s'}^c(d_{s'}^c, p_{s'})}. \quad (4.8)$$

**Remark 4.1.** *Bernstein and Federgruen proposed a very well known general equilibrium model, named as attraction model, for industries with price and service competition in [13]. It is very similar to our slice association probability function.*

In (4.8), the number of potential users in each cell as well as the slice association probability for each slice, might vary from cell to cell. The expected number of users choosing SP  $s$  is defined as

$$\sum_{c \in \mathcal{C}} N^c A_s^c(d^c, p) = \sum_{c \in \mathcal{C}} \frac{N^c f_s^c(d_s^c, p_s)}{\sum_{s' \in \mathcal{S}} f_{s'}^c(d_{s'}^c, p_{s'})}. \quad (4.9)$$

Incorporating (4.7) and (4.2) in (3), we get

$$U_s(d_s, d_{-s}) = \sum_{c \in \mathcal{C}} p_s \frac{N^c f_s^c(d_s^c, p_s) e^{-p_s}}{\sum_{s' \in \mathcal{S}} f_{s'}^c(d_{s'}^c, p_{s'}) e^{-p_{s'}}} - \sum_{m \in \mathcal{M}^c} \omega_m^c d_{s,m}^c. \quad (4.10)$$

In this work, we set

$$f_s^c(d_s^c, p_s) = q_s^c(d_s^c)e^{-p_s}. \quad (4.11)$$

We assume that SPs are selfish, and that each SP aims at maximizing his profit. They take into account the decisions of the other SPs when computing their own decision. To theoretically analyze the outcome of these strategic interactions, we define the noncooperative game  $\mathcal{G} \triangleq \langle \mathcal{S}, (\mathcal{F}_s)_{s \in \mathcal{S}}, (U_s)_{s \in \mathcal{S}} \rangle$  as follows:

- Player set: the set of service providers  $\mathcal{S}$ .
- Strategy: the vector of resource demand  $d_s = (d_s^1, \dots, d_s^C)$  where  $d_s^c$  is the amount of resource to be requested by each base station  $c$ . The strategy set for each SP  $s$  is  $\mathcal{F}_s$ .
- Utility: the utility of each SP  $s$  is defined as  $U_s$ .

We study the competition between SPs in terms of QoS, *i.e.*, how SPs strategically spend their budget on the resources to attract the maximum number of users and, in turn, maximize their profits. The SPs' profit depends on both their individual decision and the decision taken by the other SPs. Let  $d_s$  be the vector of strategy of SP  $s$ ,  $d_{-s} \triangleq \text{col}((d_{s'})_{s' \neq s})$  is the stack vector which contains the vector of strategies of all the SPs in  $\mathcal{S}$  except  $s$ . The decision problem of each SP  $s$  is defined as

$$Q_s \quad \underset{d_s \in \mathcal{F}_s}{\text{maximize}} \quad U_s(d_s, d_{-s}).$$

To study the outcome of the noncooperative game  $\mathcal{G}$ , we recall the solution concept of Nash equilibrium (NE)

**Definition 4.2.** [82] *A strategy profile  $d^* = (d_1^*, \dots, d_S^*)$  is a Nash equilibrium of the game  $\mathcal{G}$  if*

$$\forall s \in \mathcal{S}, U_s(d_s^*, d_{-s}^*) \geq U_s(d_s, d_{-s}^*), \forall d_s \in \mathcal{F}_s. \quad (4.12)$$

Here,  $(d_s, d_{-s}^*)$  denotes the strategy profile with  $s^{\text{th}}$  element equals  $d_s$  and all other elements equal  $d_{s'}^*$  (for any  $s' \neq s$ ).

In the next section, we analyze the existence and uniqueness of the Nash equilibrium (NE) solution of the noncooperative game  $\mathcal{G}$ .

### 4.3.3 Existence and Uniqueness of the Nash Equilibrium

In this section, we establish the existence and uniqueness of the NE of game  $\mathcal{G}$ . To prove the uniqueness of the NE, we rely on the concept of diagonally strict concavity (DSC) introduced by Rosen [88]. Intuitively, DSC is a generalization of the idea of convexity to a noncooperative game setting.

**Definition 4.3** (Diagonal strict concavity [88]). *A game with profiles of strategies  $d$  and profiles of utility functions  $U$  is called diagonally strict concave (DSC) for a given vector  $r$  if for every distinct  $\bar{d}$  and  $\hat{d}$ ,*

$$\left[ g(\bar{d}, r) - g(\hat{d}, r) \right] (\bar{d} - \hat{d})' < 0, \quad (4.13)$$

with  $g$  the concatenation of the weighted gradients of the players' utility functions

$$g(d, r) = \left[ r_1 \nabla_1 U_1(d), r_2 \nabla_2 U_2(d), \dots, r_S \nabla_S U_S(d) \right], \quad (4.14)$$

where  $\nabla_s U_s(d)$  denotes the gradient of utility of player  $s$  with respect to his own strategy  $d_s$

**Theorem 4.4.** *The game  $\mathcal{G}$  admits a unique NE.*

*Proof.* The utility of each SP in-game  $\mathcal{G}$  is continuous, increasing, and concave, while the strategy space for each SP is convex and compact. Therefore, the existence of an equilibrium for the game follows from [88], Thm.1. To prove the NE uniqueness, we note that if the players' utilities in the game  $\mathcal{G}$  satisfy the DSC property, then  $\mathcal{G}$  admits a unique NE (see [88], Thm.2).

Let  $G(d, r)$  be the Jacobian of  $g(d, r)$  with respect to  $d$ , where  $d$  is any profile of strategies. In order to prove the strict DSC of  $g(d, r)$ , from [88], Thm.6, we note that it is sufficient to prove that the symmetrized version of the pseudo-Jacobian, *i.e.*,  $\widehat{G}(d, r) \triangleq G(d, r) + G(d, r)'$ , is negative definite over the domain of interest. To show that  $\widehat{G}(d, r)$  is negative definite, we must prove that the following three conditions hold simultaneously:

- C 1.** *each  $U_s(d)$  is a regular strictly concave function of  $d_s$  (*i.e.*, its Hessian is negative definite).*
- C 2.** *each  $U_s(d)$  is convex in  $d_{-s}$ .*
- C 3.** *there is some  $r > 0$  such that function  $\sigma(d, r) = \sum_s r_s U_s(d)$  is concave in  $d$ .*

The negative definiteness of  $[G(d, r) + G'(d, r)]$  follows from [39], Lem.1. We first consider the case of a single base station  $c$  and show that  $\widehat{G}^c(d, r)$  is negative definite for this case. We compute the Hessian ( $H_s U_s^c$ ) of utility of any SP  $s$  with respect to SP  $s$  own strategy

$$H_s U_s^c = -2 \frac{p_s \sum_{s' \in \mathcal{S}, s' \neq s} f_{s'}^c}{\left( \sum_{s' \in \mathcal{S}} f_{s'}^c \right)^3} \left[ (\nabla_s f_s^c)^T \nabla f_s^c - H_s(f_s^c) \sum_{s' \in \mathcal{S}} f_{s'}^c \right]. \quad (4.15)$$

On the right hand side of (4.15), matrix  $(\nabla_s f_s^c)^T \nabla f_s^c$  is positive semi-definite, where  $\nabla_s f_s^c$  is the gradient row vector of  $f_s^c$  with respect to SP  $s$ 's own strategy  $d_s^c$ ,  $H_s(f_s^c)$  is the Hessian of  $f_s^c$  with respect to  $d_s^c$  and it is negative definite as  $f_s^c$  is concave. Thus, the Hessian of utility  $H_s U_s^c$  is negative definite and satisfies the first condition **C1**.

We still need to show that the utility of each SP  $s$  is convex in the strategy of all other SPs. For that purpose, consider the Hessian of utility of SP  $s$  with respect to strategy of all other SPs

$$H_{-s}U_s = 2 \frac{f_s^c}{\left(\sum_{s' \in \mathcal{S}} f_{s'}^c\right)^3} [M_s^c - \text{diag}_{-s} \{H(f_u^c)\}], \quad (4.16)$$

where  $M_s^c$  is block matrix and  $uv^{th}$  block is defined as

$$M_{suv}^c = (\nabla_u f_u^c)^T \nabla_v f_v^c \text{ where } u, v \neq s, u, v, s \in \mathcal{S}. \quad (4.17)$$

$\nabla_u f_u^c$  is the gradient row vector of  $f_u^c$  with respect to SP  $s$ 's own strategy and  $\text{diag}_{-s} \{H(f_u^c)\}$  is the block diagonal matrix with block  $u$  where  $H(f_u^c)$  is the Hessian of  $f_u^c$  with respect to strategy vector of  $u$  itself  $\forall u, u \neq s, u \in \mathcal{S}$ . In right hand side of equation (4.16) matrix  $M_s^c$  is positive definite and the block diagonal matrix  $\text{diag}_{-s} \{H(f_u^c)\}$  is negative definite as each diagonal matrix element.  $H(f_u^c)$  is negative definite, thus  $H_{-s}U_s$  is positive definite, which satisfies the condition **C2**.

Finally, by choosing  $r_s = \frac{1}{p_s} \forall s \in \mathcal{S}$  we check that  $\sigma(d, r) \triangleq \sum_s r_s U_s(d)$  is concave in  $d$ , therefore satisfying the condition **C3**.

We now want to extend the previous proof to the multi-base station case. We have already shown that  $\widehat{G}^c$  is negative definite for any single base station  $c$ . For  $C$  base stations consider a  $\widehat{G}$  symmetrized version of the pseudo Jacobian, after arranging columns and rows we get (see [80], Cor.2)

$$(\widehat{G}) = \text{diag} \{ \widehat{G}^1, \dots, \widehat{G}^c, \dots, \widehat{G}^C \}.$$

The above  $\widehat{G}$  matrix is negative definite as each diagonal matrix is negative definite, which proves the DSC property holds for the multi-cell setting. By applying [88], Thm.2, we prove that the NE  $d^*$  solution of the game  $\mathcal{G}$  is unique.  $\square$

## 4.4 Resource pricing and equilibrium

We have shown in the previous section that there exists a unique NE solution of the noncooperative game  $\mathcal{G}$ . We assume that the capacity of the resource released by the InP in each cell is finite. Given the per-unit prices for resources decided by the InP, the total resource requested by the SPs at the NE of  $\mathcal{G}$  may violate the infrastructure capacity. Thus, the InP's primary concern is how to efficiently allocate the finite capacity constrained resources to competing SPs. The desired allocation must satisfy all the SPs' constraints and simultaneously maintain high resource utilization. In this regard, we assume that the InP optimizes the unit price of each resource such that at the NE of the game  $\mathcal{G}$  each SP utilizes his entire budget and no resource remains leftover, *i.e.*, the

total demand of resources matches the available infrastructure capacity. In market economics, this pricing problem is formulated as a market clearing problem, e.g, a Fisher market, where the market prices are settled in such a way that the amount of resources requested by the buyers is equal to the amount of resources supplied by the sellers. We propose two approaches, introduced in Section II B, to deal with this challenge depending on whether the SPs' budget is binding.

One way to compute the market equilibrium is through Walras' "tâtonnement" process, i.e., if the demand exceeds the resource capacity, the market operator increases the resource's price. Conversely, the market operator decreases the resource's price when the demand is smaller than the resource capacity. The process is repeated until demand equals supply (resource capacity). The disadvantage of this approach is that its outcome (known as a general equilibrium) relies on the strong assumption of perfect competition, which in practice does not hold. To overcome this limitation, we use the approach introduced by Shapley and Shubik in their pioneering work [96], also known as trading post share constrained proportional allocation (SCPA) scheme[19]. Now we formally define the trading post mechanism.

#### 4.4.1 Trading post mechanism

In the trading-post mechanism, each player (i.e, SP) places a bid on each type of resource. Once all SPs have placed their bids, each resource type's price is determined by the total bids placed for that resource. Precisely, let SP  $s$  submits a bid  $b_{sm}^c$  to resource  $m$  at cell  $c$ . The price per unit of resource  $m$  at cell  $c$  is then set to  $\frac{\sum_{s \in \mathcal{S}} b_{sm}^c}{D_m^c}$ . Accordingly, SP  $s$  receives a fraction of  $d_{sm}^c$  in return to his spending of  $b_{sm}^c$

$$d_{sm}^c = \begin{cases} \frac{b_{sm}^c D_m^c}{\sum_{u \in \mathcal{S}} b_{um}^c} & \text{if } b_{sm}^c > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (4.18)$$

After replacing  $d_{sm}^c$  in (3) and  $(Q_s)$  in terms of bids, the decision problem of each SP  $s$  can be written as follows

$$\begin{aligned} \hat{Q}_s \quad & \underset{b_s}{\text{maximize}} \quad U_s(b_s, b_{-s}), \\ & \text{subject to} \quad \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}^c} b_{s,m}^c \leq B_s, b_{s,m}^c \geq 0. \end{aligned}$$

We may consider two possible behaviors for the SPs. First, they are *price takers*, i.e., they accept the prices decided by the price setter (market operator), and they only act strategically in terms of QoS by optimizing their demand in the bundle resources. Second, SPs are *price makers*, i.e., they anticipate the effect of their demand on the price of the bundle of resources.

The trading post mechanism induces a new noncooperative game  $\hat{\mathcal{G}}$  defined as follows:

- Player set: the set of SPs  $\mathcal{S}$ .

- **Strategy:** the vector of bids  $b_s = [b_s^1, \dots, b_s^C]$  where  $b_s^c$  is the bid to be submitted to cell  $c$ . SP  $s$  strategy set is  $\mathcal{F}_s \triangleq \left\{ b_s \mid b_{sm}^c \geq 0, \forall m \in \mathcal{M}^c, c \in \mathcal{C}, \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}^c} b_{s,m}^c = B_s \right\}$ .
- **Utility:** The utility of each SP  $s$  is  $U_s$ .

To study the outcome of the mechanism, we consider the standard notion of NE, applied to the trading post mechanism

**Definition 4.5.** A multi-bid strategy  $b^* = (b_1^*, \dots, b_S^*)$  is called a NE of the game  $\widehat{\mathcal{G}}$  if

$$\forall s \in \mathcal{S}, U_s(b_s^*, b_{-s}^*) \geq U_s(b_s, b_{-s}^*), b_s \in \mathcal{F}_s. \quad (4.19)$$

Here,  $(b_s, b_{-s}^*)$  denotes the strategy vector with  $s^{\text{th}}$  element equals  $b_s$  and all other elements equal  $b_v^*$  (for any  $v \neq s$ ).

For the proposed mechanism, a NE solution of game  $\widehat{\mathcal{G}}$  constitutes a stable bidding policy where each SP maximizes his utility and the InP implements the resource allocation mechanism (4.18).

We investigate conditions for the existence and uniqueness of the NE solution of the game  $\widehat{\mathcal{G}}$ . This requires complex calculations. Thus, to keep the analysis tractable, we restrict the problem to a single resource (radio resource). We assume that the QoS provided by SP  $s$  in cell  $c$  is given by  $q_s^c \triangleq (d_s^c)^{\rho_s^c}$  where  $\rho_s^c$  is the sensitivity parameter and  $0 < \rho_s^c \leq 1$ . Such a type of function has been used in [41] to model the effect of users sensity towards their service provider selection. We replace  $q_s^c = (d_s^c)^{\rho_s^c}$  in (4.11) and from (4.8) we get

$$A_s^c(d^c, p) = \frac{(d_s^c)^{\rho_s^c} e^{-p_s}}{\sum_{s' \in \mathcal{S}} (d_{s'}^c)^{\rho_{s'}^c} e^{-p_{s'}}}. \quad (4.20)$$

**Proposition 4.** If for a single resource, the QoS provided by SP  $s$  in cell  $c$  is defined by  $q_s^c = (d_s^c)^{\rho_s^c}$  and  $0 < \rho_s^c \leq 1$ , then the game  $\widehat{\mathcal{G}}$  admits unique NE.

*Proof.* If the QoS provided by SP  $s$  in cell  $c$  is defined by  $q_s^c = (d_s^c)^{\rho_s^c}$  and  $0 < \rho_s^c \leq 1$ , then the SPs' utility functions satisfy the three conditions **C1**, **C2** and **C3**. The detailed proof is provided in Appendix C.2 The rest of the proof is the same as the proof of Theorem 1.  $\square$

Moving ahead, we compare the profit gained by SPs at the NE of the game with the baseline static proportional allocation scheme (SS), i.e., the allocation where each resource is allocated to a SP  $s$  in proportion to his budget, e.g.,  $\frac{B_s}{\sum_{s' \in \mathcal{S}} B_{s'}}$

**Proposition 5.** For two service providers, the revenue gained under a dynamic resource sharing scheme is at least equal to the revenue gained under a proportional allocation scheme

*Proof.* The proof is provided in Appendix C.3.  $\square$

We have seen in the first part of this section, that when SPs are constrained by budgets, the resource pricing can be implemented by a trading post mechanism. However, this mechanism requires a third-party player (market operator) to centralize the bids made by all the SPs, and thus can only lead to semi-decentralized implementations. Furthermore, the network capacity constraints are only implicitly taken into account through the budget constraint.

In the next section, we design a pricing and resource allocation scheme for **Case II** introduced in Section II B, that explicitly takes into account network capacity constraints and can be implemented in a fully distributed way. **Case II** gives rise to a generalized Nash equilibrium problem (GNEP) involving global coupling constraints, which take into account the network finite capacities. To solve the GNEP, we rely on a variational reformulation of the noncooperative game, which leads to a unique variational equilibrium (VE). Using that property, we implement two algorithms to compute the VE: the first one requires an extended game reformulation of the GNEP and is based on asymmetric projected gradient descent methods; the second one relies on an extension of the alternating direction method of multipliers (ADMM).

#### 4.4.2 Pricing Game

We consider a noncooperative game where, similar to game  $\mathcal{G}$ , each SP aims at maximizing his profit by requesting resources under a set of local constraints that are not binded by a finite budget. However, we now assume that the SPs take into account the infrastructure capacity while requesting resources, therefore giving rise to a global coupling constraint for each cell and each resource available within that cell

$$\sum_{s \in \mathcal{S}} d_{sm}^c \leq D_m^c, \forall c \in \mathcal{C}, m \in \mathcal{M}, \quad (4.21)$$

Let  $\tilde{\mathcal{F}}_s \triangleq \{d_s | d_{sm}^c \geq 0, \forall m \in \mathcal{M}^c, c \in \mathcal{C}, K_s(d_s) \leq 0\}$ . The decision problem faced by each SP in this new noncooperative game can be formulated as a parametrized optimization problem with local and global coupling constraints

$$\begin{aligned} Q_s \quad & \underset{d_s \in \tilde{\mathcal{F}}_s}{\text{maximize}} && R_s(d_s, d_{-s}), \\ & \text{subject to} && \sum_{s \in \mathcal{S}} d_{sm}^c \leq D_m^c, \forall c \in \mathcal{C}, m \in \mathcal{M}, (\lambda_{sm}^c) \end{aligned} \quad (4.22)$$

where  $\lambda_{sm}^c$  at the right of (4.22) and between brackets, is the Lagrange multiplier (shadow price) of the coupling constraint (4.22).

We define a new noncooperative game  $\mathcal{G}_p \triangleq \left\langle \mathcal{S}, \left( \tilde{\mathcal{F}}_s \right)_{s \in \mathcal{S}}, (R_s)_{s \in \mathcal{S}} \right\rangle$ , where the set of players and utility is the same as in game  $\mathcal{G}$ . However, the strategy set of the players are coupled through the capacity constraint (4.22), giving rise to a GNEP. Consider the generalized Nash equilibrium (GNE)

as the solution to this game.

**Definition 4.6.** [55] A strategy profile  $d^* = (d_1^*, \dots, d_S^*)$  is called a GNE of the game  $\mathcal{G}_p$  if

$$\forall s \in \mathcal{S}, R_s(d_s^*, d_{-s}^*) \geq R_s(d_s, d_{-s}^*), \quad (4.23)$$

$$d_s \in \tilde{\mathcal{F}}_s, d_{sm}^c \geq 0, \forall m, c \text{ and } \sum_{s \in \mathcal{S}} d_{sm}^c \leq D_m^c, \forall c \in \mathcal{C}, m \in \mathcal{M}$$

Due to coupling, solving directly  $\mathcal{G}_p$  requires coordination among possibly all SPs, which might be hard to enforce in practice. To solve  $\mathcal{G}_p$ , we will make use of the duality approach as a natural way to obtain a hierarchical decomposition of the GNEP. To that purpose, we start by characterizing the GNE solutions of game  $\mathcal{G}_p$  in terms of KKTs [55]: any strategy profile  $\bar{d}$  is a GNE of the game  $\mathcal{G}_p$  if and only if it satisfies the KKT conditions, which are:  $\forall s \in \mathcal{S}, \forall c \in \mathcal{C}, \forall m \in \mathcal{M}^c$ ,

$$\left\{ \begin{array}{l} \frac{\partial R_s}{\partial d_{sm}^c}(\bar{d}) = \lambda_{sm}^c, \\ \lambda_{sm}^c \left( \sum_{s \in \mathcal{S}} \bar{d}_{sm}^c - D_m^c \right) = 0, \\ \text{with } \lambda_{sm}^c \geq 0, \bar{d}_s \in \tilde{\mathcal{F}}_s. \end{array} \right. \quad (4.24)$$

In the above KKT conditions, we are primarily interested in  $\lambda_{sm}^c$ , the Lagrange multipliers of (22), as these Lagrange multipliers can be interpreted as shadow prices for the resource allocation and can be used in the game  $\mathcal{G}_p$  as the evaluations by the SPs of the prices charged by the InP per resource unit. However, notice that if implemented without coordination, the Lagrange multipliers for each SP are different, resulting in possibly discriminatory pricing. Moreover, there can be multiple possible GNEs. In fact, there are infinite GNEs solutions to  $\mathcal{G}_p$  in this case. Nevertheless, in the following discussion, we show that there exists an equilibrium solution to  $\mathcal{G}_p$  with a special characteristic: it is unique and gives rise to the same valuation among the players. Rosen [88] has introduced concept of such equilibrium in his seminal work and called it as normalized Nash equilibrium

**Definition 4.7.** A  $r$ -normalized equilibrium point is such that there exists a  $\lambda_m^c > 0$  associated to each resource at each cell so that for all customers  $\lambda_{sm}^c = \lambda_m^c / r_s$ , for a suitable vector of nonnegative coefficients vector  $r$ .

It is very common in the literature, to relate normalized Nash equilibrium to the concept of variational equilibrium (VE) [55]. A variational equilibrium is a refinement of  $r$  normalized nash equilibrium when the Lagrange multipliers related to coupled constraints are the same for all players. *i.e.*,  $r = 1$  for all players. We will use both concepts without distinction in the following. The parameters  $\{r_1 \dots r_S\}$  intuitively show the proportion of a burden on SP  $s$  for satisfying the coupling constraints among all other service providers in the set. Notice,  $\lambda_m^c$  is the same for all the SP and thus can be treated as the base price. Next, we prove that such  $r$ -normalized Nash equilibrium (variational equilibrium), is unique for game  $\mathcal{G}_p$ .

**Corollary 4.8.** *The Pricing Game  $\mathcal{G}_p$  admits unique normalized equilibrium for  $r_s = 1/p_s$ .*

*Proof.* In the proof of Theorem 4.4, we have shown that the Game  $\mathcal{G}$  has the DSC property  $\forall c \in \mathcal{C}$  and  $\forall c \in \mathcal{M}$  and any  $\omega_m^c \geq 0$ . The utilities of players in the Game  $\mathcal{G}_p$  (i.e. revenues) are the same as in  $\mathcal{G}$  with  $\omega_m^c = 0$ . Hence, the Pricing Game  $\mathcal{G}_p$  also satisfies the DSC property. The proof is a consequence of [88], Thm.4.  $\square$

**Theorem 4.9.** *Every  $r$ -normalized Nash equilibrium of the Pricing Game  $\mathcal{G}_p$  with shadow prices  $\lambda_{sm}^c = \frac{\lambda_m^c}{r_s}$  for all  $s \in \mathcal{S}, m \in \mathcal{M}^c, c \in \mathcal{C}$  is a NE for the corresponding Game  $\mathcal{G}$  with  $\omega_m^c = \lambda_m^c, \forall m \in \mathcal{M}^c, \forall c \in \mathcal{C}$ .*

*Proof.* We reformulate  $\mathcal{G}_p$  using an augmented system-like utility function, that we call the Nash game (NG)-game utility function [85]. The NG utility function is defined as a two argument function

$$\tilde{R}(d; x) \triangleq \sum_{s=1}^S r_s R_s(d_{-s}, x_s), \forall x \in \tilde{\mathcal{F}} \triangleq \prod_s \tilde{\mathcal{F}}_s, \quad (4.25)$$

where  $x \triangleq (x_s)_s$  and  $d_{-s}$  defined as before. We note that Definition 5 can be equivalently formulated with respect to the NG utility function. A vector  $d^* \in \tilde{\mathcal{F}}$  is called a NE solution of this game if its NG utility function  $\tilde{R}$  satisfies

$$\tilde{R}(d^*; d^*) \geq \tilde{R}(d^*; x) \forall x \in \tilde{\mathcal{F}} \quad (4.26)$$

Above condition can equivalently be written as follows for given  $d_{-s}^*$

$$\sum_{s=1}^S r_s R_s(d_{-s}^*; d_s^*) \geq \sum_{s=1}^S r_s R_s(d_{-s}^*; x_s), \forall x \in \tilde{\mathcal{F}}. \quad (4.27)$$

Note that the NG utility function  $\tilde{R}$  is separable in the second argument  $x$  for any given first argument  $d^*$ . The existence of NE is guaranteed by Theorem 2 [85]. Now to extend the NG utility function formulation to coupled constrained game, i.e., pricing game, we use the fact that the pricing game is related to a constrained maximization of NG utility function with respect to the second argument keeping the first argument as a fixed point solution. Consider that the SPs maximize their revenue subject to coupled constraints  $g_i(d) \leq 0, i \in \mathcal{I} \triangleq \mathcal{C} \times \mathcal{M}$  where  $g_i(d) \triangleq D_m^c - \sum_{s'} d_{s'm}^c$ . Then

$$\tilde{R}(d^*; d^*) \geq \tilde{R}(d^*; x) \quad x \in \mathcal{F}, \tilde{g}(d_{-s}^*, x_s) \leq 0, \quad (4.28)$$

where  $\tilde{g}(d^*; x) = \sum_{s=1}^S g(d_{-s}^*, x_s)$ . We introduce the augmented Lagrangian function of the constrained NG utility maximization problem, with  $\lambda$  a Lagrange multiplier vector

$$\tilde{L}(d; x; \lambda) \triangleq \tilde{R}(d; x) + \lambda^T \tilde{g}(d; x). \quad (4.29)$$

In our case  $\forall s \in \mathcal{S}, R_s$  is increasing concave and continuously differential, and  $g$  is affine. Thus, all the constraints are active at the equilibrium. If  $d^*$  is an equilibrium solution of the pricing game  $\mathcal{G}_p$ , then by [85], Lem.2, there exists a unique  $\lambda^* > 0$  such that  $\nabla_d \tilde{L}(d, x, \lambda^*) = 0$  and  $d^*$  maximizes the Lagrangian  $\tilde{L}$ , over  $x \in \tilde{\mathcal{F}}$  as a fixed point.

Relying on the duality framework, we prove that we can decompose the coupled constrained game  $\mathcal{G}_p$  into the equivalent game with no coupled constraints, and indeed the equivalent game coincides with the noncooperative game  $\mathcal{G}$  with  $\omega_m^c = \lambda_m^c$ . To that purpose, we consider the dual cost function  $D(\lambda)$  defined as

$$D(\lambda) \triangleq \tilde{L}(d^*; d^*; \lambda). \quad (4.30)$$

Equivalently, relying on the notion of fixed point, the dual cost can be written as

$$D(\lambda) \triangleq \left[ \max_{x \in \Omega} \tilde{L}(d; x; \lambda) \right] \Big|_{x=d}. \quad (4.31)$$

The dual NG can then be defined as the minimization of the dual cost function

$$D^* = \min_{\lambda \geq 0} D(\lambda). \quad (4.32)$$

The Lagrangian function  $\tilde{L}$  is separable over each SP, thus, the dual function can be separately written for each player as

$$D(\lambda) \triangleq \sum_{s \in \mathcal{S}} \left[ \max_{x_s \in \Omega_s} L_s(d_{-s}; x_s; \lambda) \right] \Big|_{x_s=d_s} \quad (4.33)$$

$$= \sum_{s \in \mathcal{S}} L_s(u_{-s}^*(\lambda), u_s^*(\lambda), \lambda), \quad (4.34)$$

where

$$L_s(d_{-s}; x_s; \lambda) = r_s R_s(d_{-s}; x_s) + \lambda^T g(d_{-s}; x_s). \quad (4.35)$$

From [85], Thm.3, we prove that  $D(\lambda)$  can be obtained by solving the relaxed game with utility function  $L_s$  and no coupled constraints. Indeed, that relaxed game is the game  $\mathcal{G}$  with  $\omega = \lambda$ , which concludes the proof.  $\square$

This approach enables us to reformulate the GNEP  $\mathcal{G}_p$ , as a lower-level noncooperative Nash game with utility function  $L_s(U_s)$  and a higher-level optimization problem for coordination.

#### 4.4.3 Extended Pricing Game

Paccagnan et al. addressed decentralized computation of variational equilibrium (VE) for aggregative games with quadratic utility functions [83, 84]. They relaxed the coupling constraints of the

generalized Nash equilibrium problem by including a penalty term in the original utility functions. A VE is then computed applying asymmetric gradient algorithms with constant step size. The purpose of the penalty term is to assign large penalties to deviations from the constraints. The penalty reformulation helps avoid the high computational complexity of conventional optimization reformulations or the requirement of projection steps. Traditionally, drawbacks is that penalty method convergence might be quite sensitive on selecting penalty parameters. To overcome this issue, we follow the formulation proposed in the [95]. We consider a game with  $S + 1$  players, where the first  $S$  players are the SPs and the  $(S + 1)^{\text{th}}$  player is the InP, who controls the  $\lambda$  price vector. We define the decision problem of the InP,  $Q_{S+1}$  as below

$$Q_{S+1} \quad \underset{\lambda \geq 0}{\text{maximize}} \quad \sum_c \sum_m \lambda_m^c \left( \sum_s d_{sm}^c - D_m^c \right). \quad (4.36)$$

The idea behind using  $\sum_c \sum_m \lambda_m^c (\sum_s d_{sm}^c - D_m^c)$  as the utility for InP in the above decision problem, is that it solves complementary condition from KKT (4.24). For the remaining  $S$  players the decision problem is

$$Q_s \quad \underset{d_s \geq 0}{\text{maximize}} \quad R_s(d_s, d_{-s}) - \frac{1}{r_s} \sum_c \sum_m \lambda_m^c d_{sm}^c \quad \forall s \in \mathcal{S}.$$

We call  $Q_+ \triangleq \{Q_1, \dots, Q_{S+1}\}$  the extended pricing game. The difference between the extended pricing game and the pricing game  $\mathcal{G}_p$  is that in the former, there are no coupled constraints – complementary conditions are treated as the utility of an additional player (InP).

**Proposition 6.** *If  $\bar{d}$  is a  $r$ -normalized equilibrium of the pricing game, then there exists  $\bar{\lambda} \geq 0$  such that  $(\bar{d}, \bar{\lambda})$  is an equilibrium of the extended pricing game.*

*Proof.* We have already proved that the pricing game  $\mathcal{G}_p$  is monotone on  $\tilde{\mathcal{F}}$ , which implies that the extended pricing game is also monotone on  $\tilde{\mathcal{F}} \times \mathbb{R}$ , the proof follows from [95], Prop.4.  $\square$

## 4.5 Algorithms to Compute Market Equilibria

In this section, we introduce two semi-decentralized algorithms to compute the equilibria solutions of the trading post mechanism and extended pricing game, respectively. Computational and privacy issues might limit the implementation of such algorithms on medium to large-scale problems. To mitigate these issues, we propose a fully distributed proximal algorithm, inspired from the inexact-ADMM, to compute the VE of the pricing game  $\mathcal{G}_p$ .

### 4.5.1 Semi-Decentralized Learning Algorithms

We have proved in Section 4.3.3 that  $\mathcal{G}$  admits a unique equilibrium for any price vector decided by the InP. A similar result also holds for 4.4.1 when dealing with a single resource. However, we still

need to check whether tenants can reach this equilibrium in a decentralized fashion. In this regard, we propose the use of the dual averaging or mirror–descent method suggested for continuous action convex games [71]. We proceed by describing the dual averaging method. In the dual averaging method, each player, *i.e.*, SP  $s$  estimates his marginal utility or utility gradient with respect to his own strategy. To increase their utilities, the players need to take action along the direction of their utility gradient while maintaining their action in the feasible action space. In order to achieve this, each player  $s$  at each time step  $n$  accumulates his discounted utility gradient in some auxiliary variable  $y_s$

$$y_s(n+1) = [y_s(n) + \alpha_n \nabla_{b_s} U_s(b_s(n), b_{-s}(n))]. \quad (A1)$$

In the above equation  $\alpha_n$  denotes the discount factor or step size. Once the discounted gradient has been accumulated, every SP  $s$  uses his own updated value of the auxiliary variable,  $y_s$ , to take the next feasible action

$$b_s(n+1) = Q_s(y_s). \quad (4.37)$$

In turn, each SP  $s$  maps the recent value of auxiliary variable  $y_s$  to his decision space  $\mathcal{F}_s$  using the mapping  $Q_s(y_s)$ , *e.g.*,  $Q_s$  can be interpreted as a projection map. The map  $Q_s(y_s)$  is defined more generically as

$$Q_s(y_s) = \operatorname{argmax}_{b_s \in \mathcal{F}_s} \{ \langle y_s(n), b_s \rangle - h_s(b_s) \}, \quad (A2)$$

where  $h_s(b)$  is a regularization function, also called penalty function, over the feasible action set  $\mathcal{F}_s$ . The penalty  $h_s(b)$  aims to force the algorithm to converge within the interior of the feasible domain set. Different definitions of the regularization functions induce different maps. For instance, the use of  $l_2$  norm  $h_s(\cdot) = \|\cdot\|$  as a regularizer, results in the well-known Euclidean projection map.

For the game  $\mathcal{G}$  where service providers actions are bounded by the their budgets, we use the Gibbs entropy function as a regularization function

$$h_s(b_s) \triangleq \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}} b_{sm}^c \log(b_{sm}^c). \quad (4.38)$$

We replace  $h_s(b_s)$  in equation (A2) by the entropic regularization function and after some calculation we get the exponential mapping

$$b_{sm}^c = \frac{B_i \exp(y_{sm}^c)}{\sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{M}} \exp(y_{sk}^c)}. \quad (4.39)$$

The induced map  $Q_s(y_s)$  is similar to the well-know Logit map, where each player distributes his budget (weights) to different resources depending on exponential of accumulated discounted gradients.

**Theorem 4.10.** *If Algorithm 1 satisfies the required conditions for step size sequence, *e.g.*,  $\sum_{n=0}^{\infty} \alpha_n =$*

**Algorithm 2** Online Learning Algorithm for  $\hat{\mathcal{G}}$ **Require:**  $\sum_{n=0}^{+\infty} \alpha_n = +\infty, \alpha_n \rightarrow 0$  as  $n \rightarrow +\infty$ 


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1: repeat  $n = 1, 2, \dots,$ 
2:   for each SP  $s \in \mathcal{S}$ 
3:     Observe gradient of utility and update
4:      $y_s = [y_s + \alpha_n \nabla_{b_s} U_s(b_s, b_{-s})]$ 
5:   end for
6:   for each SP  $s \in \mathcal{S}$ 
7:     for each cell  $x \in \mathcal{C}$  and resource  $m \in \mathcal{M}^c$ 
8:       Play  $b_{sm}^c \leftarrow \frac{B_s \exp(y_{sm}^c)}{\sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{M}} \exp(y_{sk}^c)}$ .
9:     end for
10:  end for
11: until  $\|(b(n) - b(n-1))\| \leq \epsilon$ 

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$+\infty, \alpha_n \rightarrow 0$  as  $n \rightarrow +\infty$ , then it converges to the unique NE of the Game  $\mathcal{G}$ .

*Proof.* The proposed exponential algorithm is the special case of the dual averaging algorithm. If the NE of the any continuous action convex game is strictly r-variationally stable, then the convergence of the dual averaging algorithm to a unique NE of the game is guaranteed by [71], Thm. 4.6. Hence to prove the convergence of the proposed algorithm, it is sufficient to show that the unique NE of game  $\mathcal{G}$  is strictly r-variationally stable. The unique NE  $\hat{b}$  to the any convex game is strictly r-variationally stable if  $\forall b_s \in \mathcal{F}_s$

$$\sum_{s \in \mathcal{S}} r_s \nabla_s U_s(b)(b_s - \hat{b}_s) < 0. \quad (4.40)$$

As we have already shown in section 4.3.3, the SPs' utility functions in game  $\mathcal{G}$  satisfy the DSC for  $r_s = \frac{1}{p_s}, \forall s \in \mathcal{S}$

$$\sum_{s \in \mathcal{S}} r_s [\nabla_s U_s(b) - \nabla_s U_s(\hat{b})] (b_s - \hat{b}_s) < 0. \quad (4.41)$$

We know that for any continuous action convex game, a feasible point  $\hat{b}$  is a NE of the game if and only if

$$\sum_{s \in \mathcal{S}} r_s \nabla_s U_s(\hat{b})(b_s - \hat{b}_s) \leq 0. \quad (4.42)$$

Inequalities (4.42) and (4.41) imply (4.40), which proves that the unique NE of game  $\mathcal{G}$  is strictly r-variationally stable and then by [71], Thm. 4.6, Algorithm 2 converges to the unique NE of game  $\mathcal{G}$ .  $\square$

For **Case II** when the SPs have no bound on their budgets, we have proved in the Section VI B that the resource pricing scheme can be set up by solving the GNEP  $\mathcal{G}_p$ . Furthermore, we have also shown that the VE solution to  $\mathcal{G}_p$  can be computed as the solution of an extended pricing game  $\mathcal{Q}_+$ . Now, we provide an online semi-decentralized learning algorithm that enables the SPs and the InP

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**Algorithm 3** Online Learning Algorithm for  $\mathcal{Q}_+$ 


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**Require:**  $\sum_{n=0}^{+\infty} \alpha_n = +\infty, \alpha_n \rightarrow 0$  as  $n \rightarrow +\infty$

- 1: **repeat**  $n = 1, 2, \dots,$
- 2:   **for each** SP  $s \in \mathcal{S}$
- 3:     Observe gradient of utility and update
- 4:      $y_s \leftarrow [y_s + \alpha_n \nabla_{d_s} U_s(d_s, d_{-s}, \omega_s)]$
- 5:   **end for**
- 6:   **for each** SP  $s \in \mathcal{S}$
- 7:      $d_s \leftarrow \text{proj}_{\mathcal{D}_s} [y_s]$
- 8:   **end for**
- 9: InP update the resource prices
- 10:   **for each** Cell  $c \in \mathcal{C}$
- 11:     **for each** Resource  $m \in \mathcal{M}$  update the base price
- 12:      $\lambda_m^c \leftarrow \max [0, \lambda_m^c + \alpha_n (\sum_{s \in \mathcal{S}} x_{sm}^c - C_m^c)]$
- 13:     **end for**
- 14:   **end for**
- 15: **until**  $\|(x(n), \omega(n)) - (x(n-1), \omega(n-1))\| \leq \epsilon$

---

to reach the VE of  $\mathcal{G}_p$ . In the proposed semi-decentralized algorithm, we leverage on the framework from [101]: the first  $S$  players, *i.e.*, the SPs, follow similar steps as in Algorithm 1. However, an  $(S+1)^{\text{th}}$  player, *i.e.*, the InP, accumulates the augmented discounted gradients of his utility in the auxiliary variable  $y_{S+1}$

$$y_{S+1} = \lambda_m^c + \alpha_n \left[ \left( \sum_{s \in \mathcal{S}} x_{sm}^c - C_m^c \right) - \theta_n \lambda_n \right]. \quad (4.43)$$

Here rationale behind adding an extra term is that the original game is strictly monotone, and thus convergence is guaranteed in that case. However, the extended pricing game is just monotone and therefore, to make the algorithm converge to an equilibrium point, an additional term must be included [101]. InP updates the market price by projecting the stored auxiliary variable on the positive orthant

$$\lambda_m^c \leftarrow \text{proj}_{\mathbb{R}_{\geq 0}} \left( \lambda_m^c + \alpha_n \left[ \left( \sum_{s \in \mathcal{S}} x_{sm}^c - C_m^c \right) - \theta_n \lambda_n \right] \right). \quad (4.44)$$

**Theorem 4.11.** [101] *If Algorithm 2 satisfies the required conditions for step size sequence, e.g.,  $\sum_{n=0}^{+\infty} \alpha_n = +\infty, \alpha_n \rightarrow 0$  as  $n \rightarrow +\infty$  and for an augmented sequence  $\theta_n, \frac{\sum_{n=1}^N \alpha_n \theta_n}{\sum_{n=1}^N \alpha_n} \rightarrow 0, N \rightarrow 0$ , then it converges to the unique equilibrium of the extended pricing game.*

#### 4.5.2 A Distributed Proximal Algorithm

We assume a fully connected communication graph between the SPs, *e.g.*,  $\Gamma_s \triangleq \mathcal{S} \setminus \{s\}, \forall s \in \mathcal{S}$ . We want to compute the  $r$ -normalized Nash equilibrium solution of  $\mathcal{G}_p$  relying on a fully distributed

algorithm. To that purpose, we set  $x_s^s \triangleq d_s$  as SP  $s$ 's own action,  $x_{-s}^s$  as SP  $s$ 's estimate of the other SPs' actions, and  $x^s \triangleq \text{col}(x_s^s, x_{-s}^s)$  as the concatenation of SP  $s$ 's own action and estimate of the others' actions. Let  $\tilde{\mathcal{F}}_s \triangleq \{x_s^s | x_s^s \geq 0, \omega^T x_s^s = B_s\}$  be the strategy set of SP  $s$ . Following [90, 58], we decompose the pricing game  $\mathcal{G}_P$  per agent. Some slack variables  $(v^{ss'})_{s,s'}$  and  $(w_{ss'})_{s,s'}$  are introduced to guarantee the coincidence of the local copies. Let  $M_{S-1}$  be the matrix made of  $S-1$  blocks, each one of them containing the Identity matrix of size  $\sum_c |\mathcal{M}^c| \times \sum_c |\mathcal{M}^c|$ . Each SP  $s$  solves the local optimization problem

$$\min_{\lambda_s \geq 0, (w_{ss'})_{s'}} \max_{x_s^s \in \tilde{\mathcal{F}}_s, (v^{ss'})_{s'}} \left[ \mathcal{R}_s(x_s^s, x_{-s}^s) - \lambda_s^T (x_s^s + M_{S-1} x_{-s}^s - D) \right], \quad (4.45a)$$

$$s.t. \quad x^{s'} = v^{s's''}, \forall s' \in \mathcal{S}, \forall s'' \in \Gamma_{s'}, \quad (\alpha^{s's''}) \quad (4.45b)$$

$$x^{s''} = v^{s's''}, \forall s' \in \mathcal{S}, \forall s'' \in \Gamma_{s'}, \quad (\beta^{s's''}) \quad (4.45c)$$

$$\lambda_{s'} = r_{s'} w_{s's''}, \forall s' \in \mathcal{S}, \forall s'' \in \Gamma_{s'}, \quad (\gamma_{s's''}) \quad (4.45d)$$

$$\lambda_{s''} = r_{s''} w_{s's''}, \forall s' \in \mathcal{S}, \forall s'' \in \Gamma_{s'}. \quad (\delta_{s's''}) \quad (4.45e)$$

where  $r_{s'} = \frac{1}{p_{s'}}$  and  $r_{s''} = \frac{1}{p_{s''}}$ ,  $\forall s' \in \mathcal{S}, s'' \in \Gamma_{s'}$ . Note that we use the convention to have superscript indices for primal variables, and lower script indices for duals of  $\mathcal{G}_p$ . A solution of  $\mathcal{G}_p$  is obtained by assuming that each SP  $s$  solves the partial dual optimization problem (4.45) and by identifying  $x_s^s = d_s$  and  $\lambda_s = \frac{1}{r_s} \lambda$ .

Let  $\zeta > 0$  be a scalar coefficient. We follow the alternating direction method of multipliers (ADMM). To that purpose, we explicit the Lagrangian function associated with (4.45)

$$\begin{aligned} & L_s(x^s, \{v, \alpha, \beta\}, \lambda_s, \{w, \gamma, \delta\}) \\ & := \mathcal{R}_s(x^s) - \lambda_s^T (x_s^s + M_{S-1} x_{-s}^s - D) \\ & \quad - \sum_{s'} \sum_{s'' \in \Gamma_{s'}} \left[ (\alpha^{s's''})^T (x^{s'} - v^{s's''}) + (\beta^{s's''})^T (x^{s''} - v^{s's''}) \right] \\ & \quad + \sum_{s'} \sum_{s'' \in \Gamma_{s'}} \left[ \gamma_{s's''} (\lambda_{s'} - r_{s'} w_{s's''}) + \delta_{s's''} (\lambda_{s''} - r_{s''} w_{s's''}) \right], \end{aligned}$$

and associated KKTs, which give rise to the following relationships:  $\alpha^{ss'} + \beta^{ss'} = 0$  and  $\gamma_{ss'} + \delta_{ss'} = 0, \forall s' \in \Gamma_s$ .

To update the SPs' strategies, we rely on the augmented Lagrangian associated with (4.45):  $\tilde{L}_s(x^s, \{v, \alpha, \beta\}, \lambda_s, \{w, \gamma, \delta\}) \triangleq L_s(x^s, \{v, \alpha, \beta\}, \lambda_s, \{w, \gamma, \delta\}) - \frac{\zeta}{2} \left( \sum_{s'} \sum_{s'' \in \Gamma_{s'}} (\|x^{s'} - v^{s's''}\|^2 + \|x^{s''} - v^{s's''}\|^2) \right) + \frac{\zeta}{2} \left( \sum_{s'} \sum_{s'' \in \Gamma_{s'}} ((\lambda_{s'} - r_{s'} w_{s's''})^2 + (\lambda_{s''} - r_{s''} w_{s's''})^2) \right)$ . Following vanilla ADMM, the duals

in (4.45) are updated according to the rules

$$\alpha^{s's''}(t) = \alpha^{s's''}(t-1) + \frac{\zeta}{2}(x^{s'}(t-1) - x^{s''}(t-1)), \quad (4.46a)$$

$$\beta^{s's''}(t) = \beta^{s's''}(t-1) + \frac{\zeta}{2}(x^{s''}(t-1) - x^{s'}(t-1)), \quad (4.46b)$$

$$\gamma_{s's''}(t) = \gamma_{s's''}(t-1) + \frac{\zeta}{2}\left(\frac{\lambda_{s'}(t-1)}{r_{s'}} - \frac{\lambda_{s''}(t-1)}{r_{s''}}\right), \quad (4.46c)$$

$$\delta_{s's''}(t) = \delta_{s's''}(t-1) + \frac{\zeta}{2}\left(\frac{\lambda_{s''}(t-1)}{r_{s''}} - \frac{\lambda_{s'}(t-1)}{r_{s'}}\right). \quad (4.46d)$$

We update the slacks  $v, w$  by solving the following optimization problems

$$v^{s's'}(t) = \arg \max_{v^{s's'}} \tilde{L}_s(x^s(t-1), \{v, \alpha(t), \beta(t)\}, \lambda_s(t-1), \{w(t-1), \gamma(t), \delta(t)\}), \quad (4.47a)$$

$$w_{s's'}(t) = \arg \min_{w_{s's'}} \tilde{L}_s(x^s(t-1), \{v(t), \alpha(t), \beta(t)\}, \lambda_s(t-1), \{w, \gamma(t), \delta(t)\}). \quad (4.47b)$$

Assuming that  $\alpha^{s's'}(0) = \beta^{s's'}(0) = 0$  and  $\gamma_{s's'}(0) = \delta_{s's'}(0) = 0$  and relying on (4.46a)-(4.46d), the slack update rules (4.47a)-(4.47b) give rise to the following closed form expressions

$$v^{s's''}(t) = \frac{1}{2}(x^{s'}(t-1) + x^{s''}(t-1)), \quad (4.48a)$$

$$w_{s's''}(t) = \frac{1}{2}\left(\frac{\lambda_{s'}(t-1)}{r_{s'}} + \frac{\lambda_{s''}(t-1)}{r_{s''}}\right). \quad (4.48b)$$

Set  $\Phi^s \triangleq \sum_{s' \in \Gamma_s} (\alpha^{s's'} + \beta^{s's'})$  and  $\Psi_s \triangleq \sum_{s' \in \Gamma_s} (\gamma_{s's'} + \delta_{s's'})$ . From (4.46a)-(4.46b) and (4.46c)-(4.46d), we get that  $\Phi$  and  $\Psi$  are updated according to the rules

$$\Phi^s(t) = \Phi^s(t-1) + \zeta \sum_{s' \in \Gamma_s} (x^s(t-1) - x^{s'}(t-1)), \quad (4.49a)$$

$$\Psi_s(t) = \Psi_s(t-1) + \zeta \sum_{s' \in \Gamma_s} \left(\frac{\lambda_s(t-1)}{r_s} - \frac{\lambda_{s'}(t-1)}{r_{s'}}\right). \quad (4.49b)$$

Let  $\tilde{\beta}_{s>0}$  be a penalty factor for the proximal first-order approximation for  $s \in \mathcal{S}$ .

Following [58], from (4.48a)-(4.48b), the primal update rule for SP  $s$  is obtained by solving a local

optimization problem

$$\begin{aligned}
x_s^s(t) = \arg \max_{x_s^s \in \tilde{\mathcal{F}}_s} & \left\{ \nabla_{x_s^s} \mathcal{R}_s(x_s^s(t-1))^T (x_s^s - x_s^s(t-1)) \right. \\
& - \frac{1}{2\zeta|\Gamma_s|} \left[ 2\zeta r_s \sum_{s' \in \Gamma_s} \frac{\lambda_s(t-1) + \lambda_{s'}(t-1)}{r_s + r_{s'}} - \Psi_s(t) \right. \\
& \left. \left. + x_s^s(t-1) + M_{S-1} x_{-s}^s(t-1) - D \right]^T (x_s^s - x_s^s(t-1)) \right. \\
& - \frac{\tilde{\beta}_s}{2} \|x_s^s - x_s^s(t-1)\|^2 - \Phi_s^s(t)^T x_s^s \\
& \left. - \zeta \sum_{s' \in \Gamma_s} \left\| x_s^s - \frac{x_s^s(t-1) + x_{s'}^s(t-1)}{2} \right\|^2 \right\}. \tag{4.50}
\end{aligned}$$

Dual update rule takes the form

$$\begin{aligned}
\lambda_s(t) = \text{proj}_{\mathbb{R}_+^{\sum_c |\mathcal{M}^c|}} & \left( \frac{1}{2\zeta|\Gamma_s|} (x_s^s(t) + M_{S-1} x_{-s}^s(t-1) - D \right. \\
& \left. - \Psi_s(t) + 2\zeta r_s \sum_{s' \in \Gamma_s} \frac{\lambda_s(t-1) + \lambda_{s'}(t-1)}{r_s + r_{s'}} \right). \tag{4.51}
\end{aligned}$$

Let  $\tilde{\mathcal{F}}_{-s} \triangleq \prod_{s' \neq s} \tilde{\mathcal{F}}_{s'} \subseteq \mathbb{R}_+^{(S-1) \sum_c |\mathcal{M}^c|}$ . It is a closed set as the product of closed sets. The mapping  $\text{proj}_{\tilde{\mathcal{F}}_{-s}} : \mathbb{R}_+^{(S-1) \sum_c |\mathcal{M}^c|} \rightarrow \tilde{\mathcal{F}}_{-s}$  denotes the projection onto  $\tilde{\mathcal{F}}_{-s}$ . Update of SP  $s$ 's estimates can be obtained as

$$\begin{aligned}
x_{-s}^s(t) = \text{proj}_{\tilde{\mathcal{F}}_{-s}} & \left( \frac{1}{2} (x_{-s}^s(t-1) + \frac{1}{|\Gamma_s|} \sum_{s' \in \Gamma_s} x_{-s}^{s'}(t-1)) \right. \\
& \left. - \frac{1}{2\zeta|\Gamma_s|} \Phi_{-s}^s(t) \right). \tag{4.52}
\end{aligned}$$

**Theorem 4.12.** *If  $f_s^c(\cdot)$  is  $K_s^c$  Lipschitz continuous for all  $s \in \mathcal{S}, c \in \mathcal{C}$ , Algorithm 4 converges to the  $r$ -normalized Nash equilibrium solution to  $\mathcal{G}_p$ .*

*Proof.* See Appendix C.5. □

## 4.6 Numerical Experiments

In this section, we analyse numerically the dynamic resource allocation schemes proposed in our paper. In our simulations, we primarily focus on a network with two cells, C 1 and C 2, and two service providers SP 1 and SP 2, who request resources for their service provision. This setting allows us to efficiently study the dynamics of interaction between EUs and SPs, and the effect of different system parameters on the outcome of the game  $\mathcal{G}$ .

**Algorithm 4** Distributed Proximal Algorithm for  $\mathcal{G}_p$ **Require:**  $\zeta > 0, \tilde{\beta}_s > 0, \forall s \in \mathcal{S}, \epsilon_{stop}^{primal}, \epsilon_{stop}^{dual}, t_{max}$ 

- 1: ‡ Initialization Step
- 2: Each SP  $s$  builds initial estimate  $x^s(0) \in \tilde{\mathcal{F}}$  and  $\lambda_s(0) \geq 0$
- 3: Set  $\alpha^{ss'} = \beta^{ss'} = 0$  and  $\gamma_{ss'} = \delta_{ss'} = 0, \forall s \in \mathcal{S}, \forall s' \in \Gamma_s$
- 4: **while**  $\epsilon^{primal}(t) \geq \epsilon_{stop}^{primal} \vee \epsilon^{dual}(t) \geq \epsilon_{stop}^{dual} \wedge t \leq t_{max}$
- 5: ‡ Communication Step
- 6: Each SP  $s$  exchanges his previous estimate  $x^s(t-1)$  and his dual Lagrange multiplier  $\lambda_s(t-1)$  with his neighbors  $s' \in \Gamma_s$
- 7: ‡ Action Step Update
- 8: **for each** SP  $s \in \mathcal{S}$
- 9:  $\Phi^s(t)$  is updated according to (4.49a)
- 10:  $\Psi^s(t)$  is updated according to (4.49b)
- 11:  $x_s^s(t)$  is updated by solving (4.50)
- 12:  $\lambda_s(t)$  is updated according to (4.51)
- 13:  $x_{-s}^s(t)$  is updated according to (4.52)
- 14: **end for**
- $t = t + 1$
- 15: **end while**

We assume there are 200 and 300 EUs present in the cell C1 and C2, respectively. First, we consider the simple case of a single resource where the QoS offered by the slices only depends on the radio resource (bandwidth).

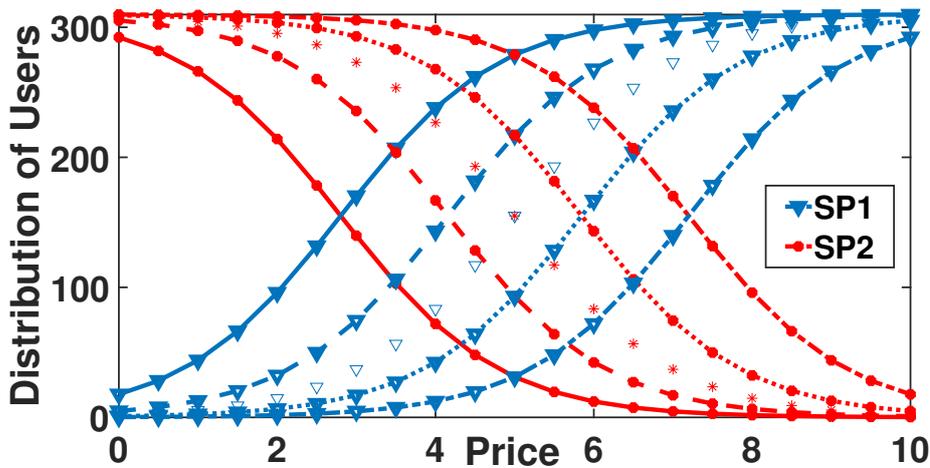


Figure 4.3 – Distribution of users at the NE wrt fees charged by SPs

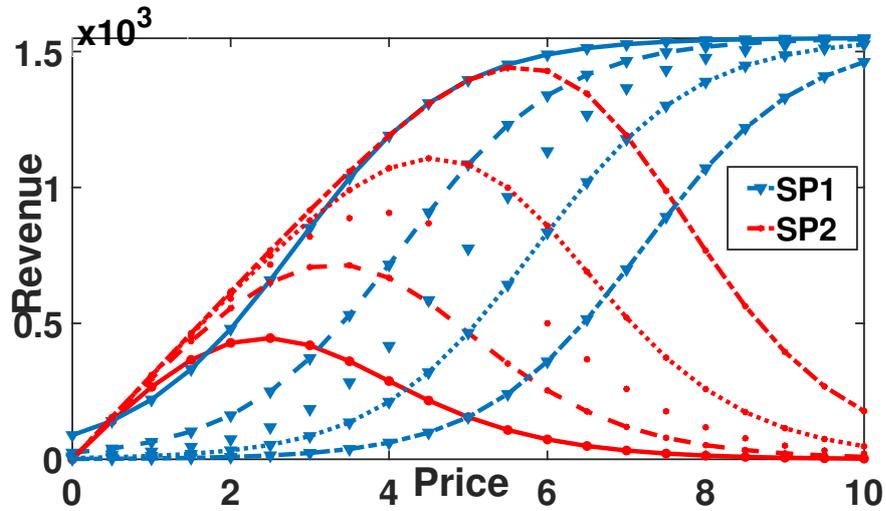


Figure 4.4 – Revenue gained by SPs wrt fees charged by them.

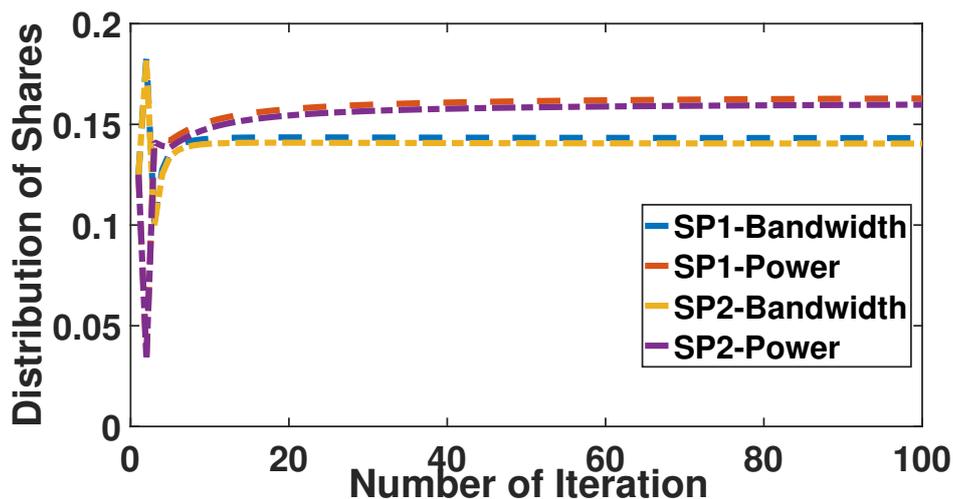


Figure 4.5 – Convergence of Algorithm 1 to the NE

The plot in Fig.4.3 illustrate the impact of the price parameter on the number of EUs in the slices at the NE of  $\mathcal{G}$ . For this simulation, we assume that the price applied by the slice 1 is constant and equal to 5, and we vary the fee applied by SP 2 in the range of 0 to 10. Fig.4.3 shows the change in the distribution of EUs in the slices as function of the price applied by SP 1. In the same figure we analyze the effect of slices shares on the distribution of users at the outcome of the game. The regular line in red and blue shows the distribution of users with slice 1 and slice 2 as a function of the price provided by the slices and when the slices are assigned with 10% share and 90% share of the infrastructure, respectively. The plots with the dash line, dot line and dot-dash line are outcome when 30%, 70% and 90% of share are assigned to SP 1. Under the same settings, Fig.4.4 illustrate

the impact of the price applied by the slices and their infrastructure share on their profits.

As second case, we consider that the QoS provided by SP  $s$  in cell  $c$  is given by  $q_s^c = (d_s^c)^{\rho_s^c}$  where  $\rho_s^c$  is sensitivity parameter and  $0 < \rho_s^c \leq 1$ , we vary  $\rho_2^c$ , *i.e.*, the SP 2 sensitivity parameter in cell C 2 from 0.1 to 1. Fig.4.7 shows the comparison of profit gain by SPs at NE with the profit gained under static resource allocation scheme (SS). For the multi-resource case, we consider that the SP QoS depends on their bandwidth as well as power allocation. To be precise, we assume that the QoS is the maximum possible data rate that SP can achieve. It is given by

$$q_s = B_s \log_2 \left( 1 + \frac{h^2 P_s}{N_0} \right),$$

where  $B_s$  and  $P_s$  is bandwidth and power allocated to SP  $s$  respectively, while  $h$  is the channel gain and  $N_0$  the noise. For simulations purpose, we assume that the availability of maximum bandwidth and transmitting power at each base station is 30 Mhz and 47 dBm respectively. The prices applied by each SP is constant 1, and each SP is assigned with half of the infrastructure share. For the numerical experiments, we vary the channel gain to noise ratio for SP2 at cell C2 from 10db to 50db; for each value of the channel gain to noise ratio, we compute the Nash equilibrium.

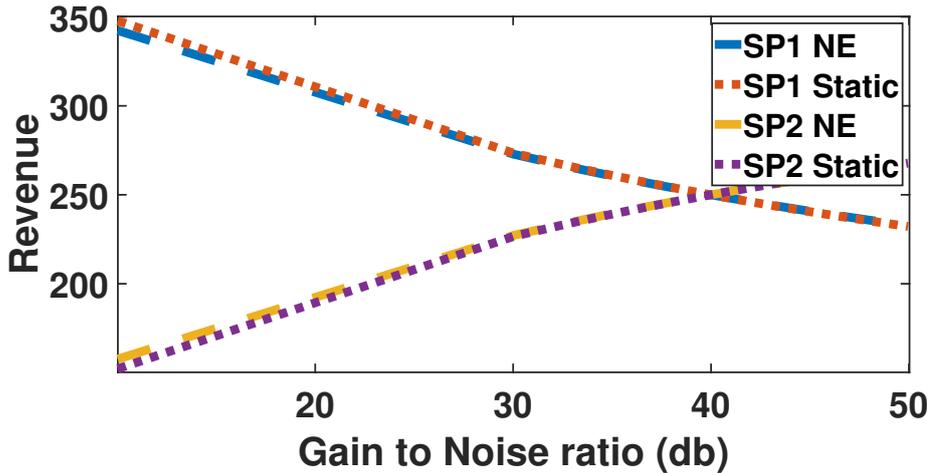


Figure 4.6 – Comparison between the revenue gained by the SPs at the NE of game  $\mathcal{G}$  vs the revenue gain under static proportional allocation scheme (SS) for different value of SP 2 power to noise ratio at cell C2

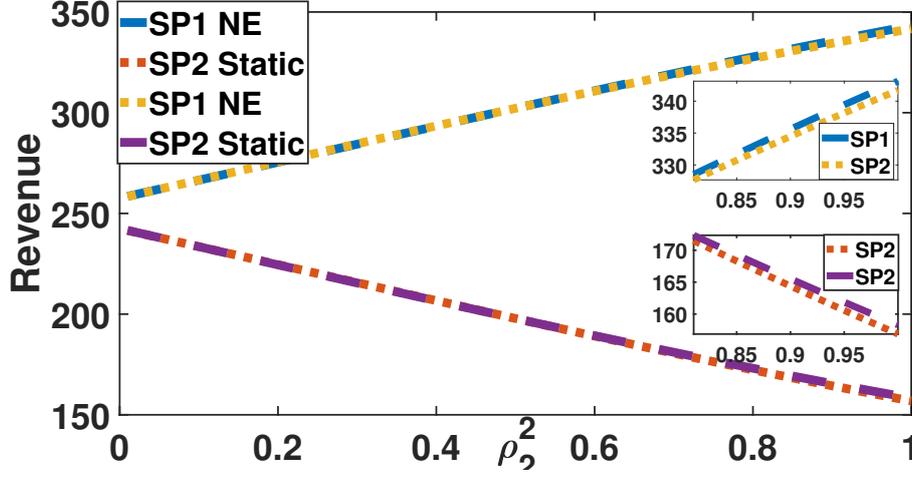


Figure 4.7 – Comparison between the revenue gained by the SPs at the NE of game  $\mathcal{G}$  vs the revenue gain under static proportional allocation scheme (SS) for different values of  $\rho_2^2$ .

The Fig.4.6 shows a comparison between the profit gain by SPs at nash equilibrium with profit gain by SPs under a static resource allocation scheme. The numerical results show a tiny difference between the SPs' profits under the dynamic resource sharing scheme and static resource allocation. In Fig.4.5, we observe the fast convergence of distributions of budgets on the different resources at different cells by SPs to a Nash equilibrium of game through an exponential learning scheme.

Next, we present the numerical results to validate the pricing and multi-resource allocation scheme for the second case where SPs have no budgets. In this scenario, particularly to define the QoS as a function of multi resources, we consider a general class of utility function known as CES (constant elasticity of substitution), mathematically defined as

$$q_s(d_s) = \left( \sum_r v_{sr} (d_{sr})^\rho \right)^{1/\rho},$$

where  $\rho \in (-\infty, 0) \cup (0, 1]$  parameterizes the whole family of utility functions. For example  $\rho = 1$  corresponds to linear (additive) valuations  $q_s(d_s) = \sum_r v_{sr} d_{sr}$ ,  $\rho \rightarrow 0$  corresponds to Cobb Douglas function  $q_s(d_s) = \prod_r (d_{sr})^{v_{sr}}$ ,  $\rho \rightarrow -\infty$  correspond to Leontief utilities  $q_s(d_s) = \min_r \left\{ \frac{d_{sr}}{v_{sr}} \right\}$ , and  $D_s = (v_{s1} \dots v_{sr})$  where  $v_{sr}$  is the amount of resource type  $r$  needed by SP  $s$  to support one unit of QoS. Linear valuation signifies the perfect substitutes, representing a scenario where the resources can replace each other in utilization. Contrary, Leontief utilities represent the perfect complement scenario where one resource may have no value without the other. For instance, the CPU and computer memory are both essential for completing a computing task. CSE utility function interpolate between perfect substitutes and the perfect complement through the parameter  $\rho$ . For numerical experiments, we consider that each SP needs three types of resources, namely, Bandwidth (Gbps),

vCPU, Memory (GB). For SP1, we consider  $v_{1,BW} = 10$  and  $v_{1,vCPU} = 32$  and  $v_{1,MEM} = 244$ . Similarly for SP2  $v_{2,BW} = 10$  and  $v_{2,vCPU} = 40$  and  $v_{2,MEM} = 160$ . We consider that the total available capacity of bandwidth and vCPU is fixed at 40 GB and 60 units, respectively. For the memory, we vary the available capacity from 100 GB to 400 GB, and we examine the its effects on the resources' prices.

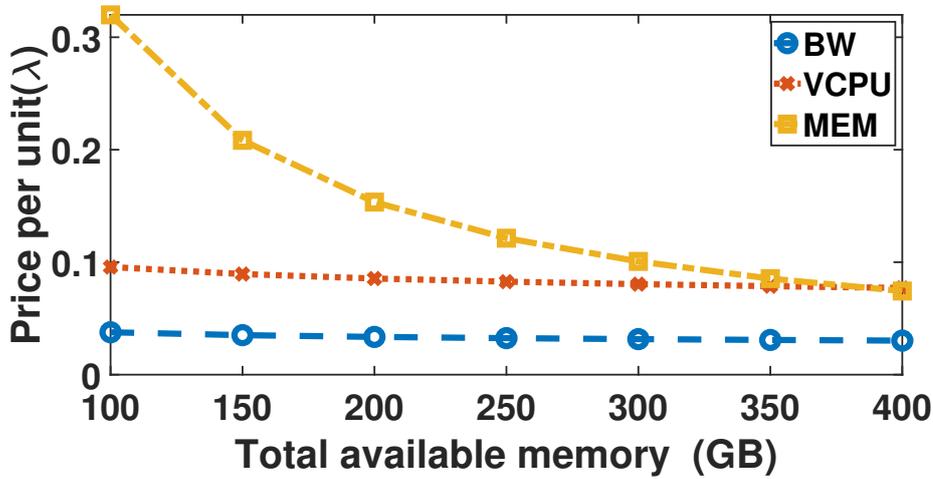


Figure 4.8 – Changes in the price ( $\lambda$ ) with respect to the available memory capacity, with  $\rho = 0.1$ ,

First, we consider the case where the QoS provided by SPs follows a substitutive relationship between resources, e.g., we rely on the CSE function with  $\rho = 0.1$ . Fig.4.8 illustrates the effect of available capacity of resources on the resources price. As the total availability of memory increases. In this case, the cost of memory decreases. However, as the relationship between the resources is substitutive, we observe from the figure that a change in memory availability does not affect the price of the other resources. Next, we consider the scenario where the SPs' QoS is defined by the CSE function with  $\rho = -1.5$ . In this case, the relationship between the resources is more complementary than the previous one.

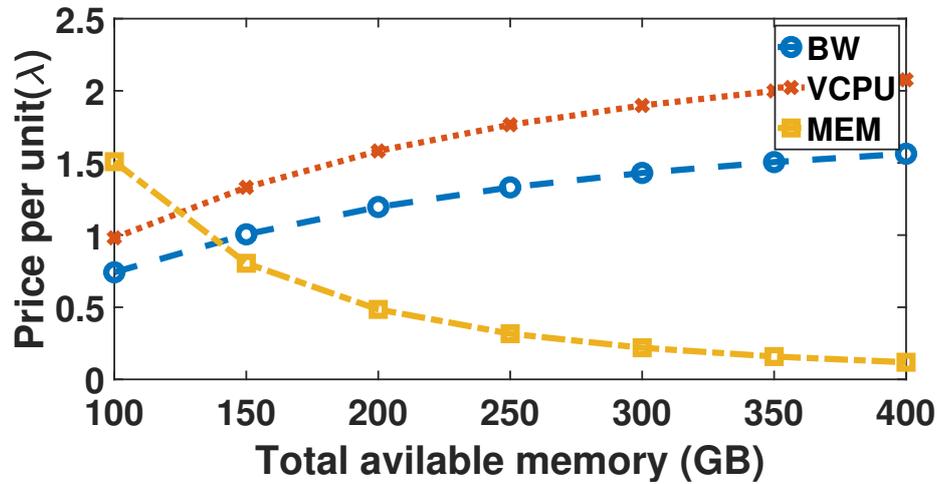


Figure 4.9 – Changes in the price ( $\lambda$ ) with respect to the available memory capacity, with  $\rho = -1.5$ ,

Fig.4.9 illustrates the effect of available capacity of resources on the resources price. As the availability of memory increases, the cost of memory decreases. However, in this case, we observe that a change in memory availability also affects the price of the other resources. The cost of bandwidth and VCPU also increases with a rise in memory's availability. An increase in the capacity of memory gives SPs room to improve their QoS, but at the cost of increasing the other related resources, Bandwidth and VCPU. Thus, it causes congestion at Bandwidth and VCPU hence resulting in a rise in their prices. Similarly, Fig.4.10 presents a change in the prices of the resources with

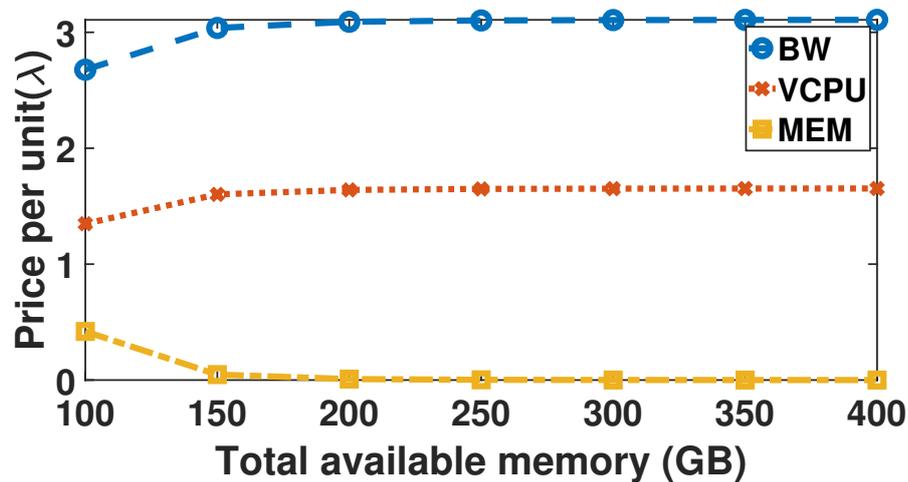


Figure 4.10 – Changes in the price ( $\lambda$ ) with respect to the available memory capacity, with a)  $\rho = -2.5$ ,

respect to the capacity of the memory where QoS is considered a CSE function with  $\rho = -2.5$ . In Fig.4.11 we demonstrate the fast convergence of Algorithm 3. For simulation purposes, we consider

the availability of Bandwidth, VCPU and Memory as 40 Gbps 60 units and 100 GB, respectively. The plot in the figure shows the exact convergence of total demand for all three resources to their available capacity. Fig.4.12 shows the convergence of error in decision variable (resources' demand) by the SPs and the convergence of error in price  $\lambda$ .

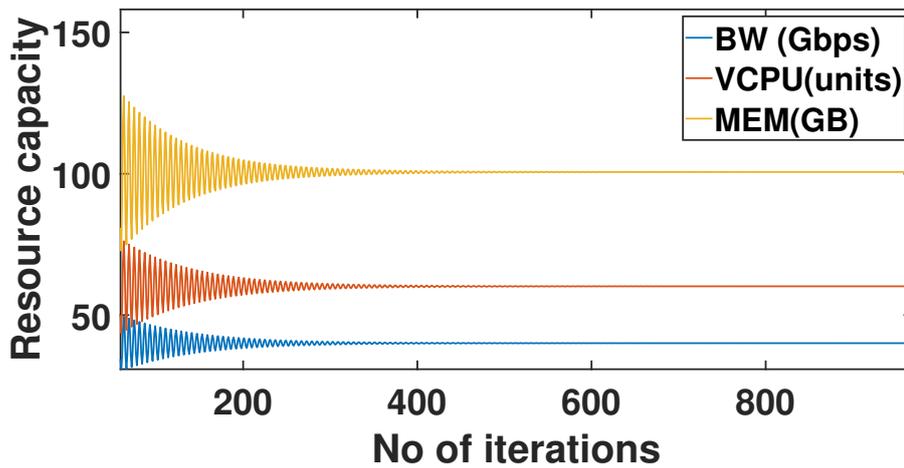


Figure 4.11 – Convergence of Algorithm 3, convergence of total resources' demand to the available capacity

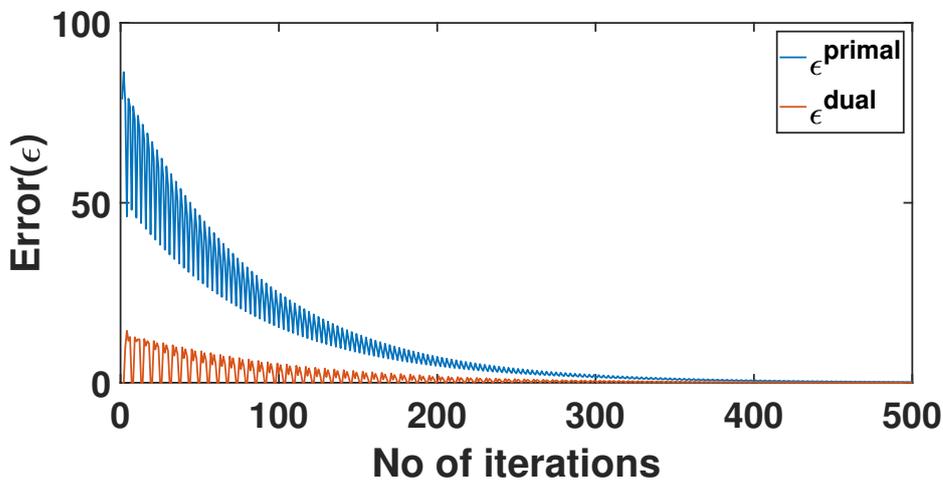


Figure 4.12 – Convergence of the primal and dual errors in Algorithm 3.

# CONCLUSION AND FUTURE DIRECTIONS

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## 5.1 Conclusions

Network slicing is emerging as a promising technique to meet the diverse and conflicting requirements of advance applications like Augmented Reality (AR), Virtual Reality (VR) live broadcast, Internet of things (IoT), Autonomous driving, remote healthcare, etc. Evolving from previous mobile technology, 5G networks will integrate data centre-based cloud/edge/fog architectures into the plan to support the data-intensive demand of applications like AR and VR. Thus, a slice will typically comprise different resource types, such as radio access capacity, edge storage memory and computing power available within the infrastructure. Network slicing enables virtualized and independent logical networks to multiplex over the same physical infrastructure. This also creates a new business opportunity for virtual network operators by enabling them to run their virtualized independent services on the same infrastructure. Network slicing is a potential technology for next-generation mobile networks; however, some key challenges still need to be addressed for its efficient implementation. Challenges include the efficient pricing and allocation of heterogeneous resources to slice tenants or service providers with diverse and conflicting requirements.

In Chapter 2, we have considered a scenario where customers compete to obtain a slice of resources in 5G networks. We employ a mechanism based on a multi-bid Kelly mechanism, using as price vector the one resulting from the normalised Nash equilibrium, which solves a dual game under coupled constraints. The game's solution is obtained via an online learning mechanism that ultimately converges to the social optimum. The key technical challenge overcome by the proposed bidding mechanism is to account for the coupled constraints dictated by the available infrastructure resources. We have also shown that the proposed mechanism accommodates the case where slice tenants can have minimum requirements over resources. This renders the proposed one an interesting candidate mechanism for pricing slicing in 5G networks. In fact, to the best of the authors' knowledge, no suitable learning mechanism is known for Nash equilibria under the coupled resources constraints, which are central in 5G resources slicing.

In Chapter 3, we have considered a resource allocation problem where service providers lease the heterogeneous resources from infrastructure provider through a slicing mechanism to support their geographically distributed pool of users. In particular, we have assumed that the users are classified into different classes depending on their service needs. Service providers seek to be  $\alpha$ -fair

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while providing the service among their users. As a solution, we have proposed the Fisher market based multi-resource allocation scheme that enables flexible slicing. In the proposed method, the service providers can adjust their resources dynamically by distributing their network share over the geographically distributed heterogeneous resources. We have considered that the proposed allocation scheme function at market equilibrium, where each service provider is satisfied with the attained resources and the market gets cleared. We have shown that the desired market equilibrium of the formulated market can be obtained by solving Esineberg -Gale optimization program, whose primal-dual solution pair provides market equilibrium allocation and prices. However, such a centralized approach requires service providers to disclose their sensitive information. Thus, to overcome this issue, we have designed the distributed budget distributing scheme that lets service providers converge to the market equilibrium of the formulated market in a decentralized fashion. We have evaluated the performance of the proposed scheme by comparing it with social optimal and static proportional allocations and shown that the proposed allocation scheme achieves better trade off between efficient resource utilization and interslice protection. Moreover, we have investigated the fairness properties of the proposed market-based scheme and shown that it maximizes well know the Nash welfare function.

In Chapter 4, we have considered a setting where service providers lease resources from an infrastructure provider through a network slicing mechanism and compete with one another to serve a large pool of end-users. We have shown that the interactions between the end-users and service providers can be modelled as a Stackelberg game, where the service providers act as leaders and the end-users as followers. In addition, we have proved that the competition between the service providers results in a multi-resource Tullock rent-seeking game, which admits a unique Nash equilibrium. The market price is computed by the infrastructure provider for each resource, taking into account the finite capacity of the network. To compute the market price and resource allocation, we have proposed two innovative market mechanisms. First, we have implemented a trading post mechanism taking into account the fact that the SPs have bounds on their budgets. We have proved that the noncooperative game induced by the trading post mechanism admits a unique Nash equilibrium in case a single resource is considered. We have implemented a semi-decentralized exponential learning algorithm to compute the unique Nash equilibrium of this game. However, this mechanism does not enable an explicit incorporation of the network finite capacity constraints. To overcome that limitation, in a second design, when SPs have no bound on their budgets but take into account the network finite capacity as a global coupling constraint, we have shown that the market equilibrium can be obtained by solving a generalized Nash equilibrium problem. We have provided a dual averaging-based semi-decentralized algorithm to compute solution of the extended game reformulation of the pricing game, and a proximal inexact- ADMM based distributed algorithm that provably converges to the variational equilibrium of the pricing game. Finally, we have provided numerical results to analyse the economic properties of the two market designs, and confirm the

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fast convergence rate of the inexact-ADMM highlighting its practical applicability.

## 5.2 Future directions

In Chapter 2, we have developed the resource allocation mechanisms based on a bidding scheme that maximizes the utilitarian social welfare function *i.e.* sum of utilities of all agents. While in the Chapter 3, we have shown that the proposed Fisher market-based resource allocation scheme maximizes the Nash welfare function. A question arises, what if the social planner wishes to maximize different welfare functions. For example, if we consider a more general class of welfare function called the "Constant elasticity of substitution " (CES) welfare function.

$$\Psi_{\rho}(d) = \left( \sum_s U_s(d_s)^{\rho} \right)^{1/\rho}$$

In above definition, the values of  $\rho \in [-\infty, 0) \cup (0, 1]$  interpolates between individual fairness and efficiency, small  $\rho$  corresponds to the egalitarian nature of social a planner, where it cares more about individual equality (fairness). In contrast, the larger  $\rho$  corresponds to utilitarian welfare where a social planner cares more about societal good (efficiency). To the best of our knowledge, not much attention has been given in the literature to mechanism designing that maximizes welfare function other than the commonly know utilitarian social welfare, Nash welfare functions or Max-min welfare. Recently, authors in [38] developed a nonlinear pricing-based market mechanism that maximizes all range of CES social welfare functions. However, their proposed mechanism is limited to the special case where market agents' resource demand is defined by the Leontief utility function, and their base demand vector in the utility function is expressed by either 1 or 0. In the future, we plan to extend the current framework to the settings where social planner or Infrastructure provider aims to achieve different fairness levels (CES-welfare) while allocating the resources to service providers.

In Chapter 3, while providing the distributed budget distribution rule for the service providers to converge to a market equilibrium of the formulated market, we have assumed that the service providers act as price takers *i.e.*, they don't anticipate the effect of their decision of future prices. In the proposed rule, the infrastructure provider announces the prices in each round and allocates the resources according to the Trading post mechanism. In each next round, service providers change their bids' distribution depending on the prices reported by the InP in the previous round. However, in practice, service providers might exhibit strategic behaviour while distributing their budgets. They could take a selfish decision anticipating the effect of their decisions on the future price. Such strategic behaviour by service providers could hamper the system's overall efficiency or convergence properties of the proposed scheme. Thus, in such a scenario, it is imperative to study the strategic behaviour of the service provider. In the future, we would like to investigate the strategic behaviour of service providers through the game-theoretic framework.

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While designing all the pricing and resource allocation schemes in the thesis, we have assumed that the utility functions of the slice tenants or service providers are known precisely. However, in practice, the utility function of each slice tenant may not be deterministic. For example, if we consider a case where we allocate the different resources to the service providers in future time slots. The future user load, as well as the demand of each service provider, may not be known accurately. Further, in the radio resource allocation problem knowing exact future channel conditions for users is not possible. Thus one of the interesting directions of research could be to extend the resources allocation and pricing scheme developed in the thesis to a setting where slice tenants' utilities are non-deterministic. In recent years the distributed reinforcement learning algorithms to reach the Nash equilibrium and or generalized Nash equilibrium of convex games when the exact payoff functions of the game players are unknown were developed in [71]. It would be interesting to explore and know whether the algorithm proposed in [71] can be useful for developing pricing and allocation scheme under uncertainty.

# Appendices

# APPENDIX A

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## A.1 Proof of Theorem 2.5

*Proof.* [6]

$$\sum_{s=1}^S \left( \frac{\bar{x}_c^s}{\bar{X}_c} - \frac{\hat{x}_c^s}{\hat{X}_c} \right) \left( \frac{\bar{x}_c^{-s}}{\bar{X}} B_c \nabla V_s \left( \frac{\bar{x}_c^s}{\bar{X}_c} B_c \right) - \frac{\hat{x}_c^{-s}}{\hat{X}} B_c \nabla V_s \left( \frac{\hat{x}_c^s}{\hat{X}_c} B_c \right) \right) \quad (\text{A.1})$$

$$y_c^s = \frac{x_c^s}{X_c}$$

$$\frac{x_c^{-s}}{X_c} B_c \nabla V^s \left( \frac{x_c^s}{X_c} B_c \right) = B_c (1 - x_c^s) \nabla V^s (B_c y_c^s) = \nabla W_c^s (y_c^s) \quad (\text{A.2})$$

$$W_c^s (y_c^s) = (1 - y_c^s) V^s (B_c y_c^s) + \int_0^{y_c^s} V^s (B_c z) dz \quad (\text{A.3})$$

As  $W_c^s (y_c^s)$  is concave function

$$(\bar{y}_c^s - \hat{y}_c^s) (\nabla W^s (\bar{y}_c^s) - \nabla W^s (\hat{y}_c^s)) < 0, \text{ if } \bar{y}_c^s \neq \hat{y}_c^s \quad (\text{A.4})$$

□

# APPENDIX B

## B.1 Construction of potential function and properties

To construct the potential function we consider the objective function for dual of optimization program. Without loss of generality, for all the proofs in Chapter 3, we consider that all resource types at all the base stations have a unit capacity

$$\Upsilon(p) = \max_{\sum_{c,k,r} x_{ckr}^s p_{cr} = B_s} \left( \sum_s B_s \log(U_s) + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} p_{cr} \left( 1 - \sum_s \sum_{k \in \mathcal{K}_c^s} x_{ckr}^s \right) \right) \quad (\text{B.1})$$

and goal is to minimize the  $\Upsilon(p)$

$$\Upsilon(p) = \max_{\sum_{c,k,r} x_{ckr}^s p_{cr} = B_s} \left( \sum_s B_s \log(U_s) + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} p_{cr} \left( 1 - \sum_s \sum_{k \in \mathcal{K}_c^s} x_{ckr}^s \right) \right) \quad (\text{B.2})$$

$$\begin{aligned} \Upsilon(p) = \max_{\sum_{c,k,r} x_{ckr}^s p_{cr} = B_s} & \left( \sum_s \frac{B_s}{(1 - \alpha_s)} \times \log \left( \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s \left( \min_r \left\{ \frac{x_{ckr}^s}{d_{ckr}^s} \right\} \right)^{(1 - \alpha_s)} \right) \right) \\ & + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} p_{cr} \left( 1 - \sum_s \sum_{k \in \mathcal{K}_c^s} x_{ckr}^s \right) \end{aligned} \quad (\text{B.3})$$

$$\Upsilon(p) = \max_{\sum_{c,k,r} b_{ckr}^s p_{cr} = B_s} \left( \sum_s \frac{B_s}{(1 - \alpha_s)} \times \log \left( \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s \left( \min_r \left\{ \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right\} \right)^{(1 - \alpha_s)} \right) + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} p_{cr} - \sum_s B_s \right) \quad (\text{B.4})$$

Let  $b(P)$  be the spending that maximizes subject to constraints  $\forall s (\sum_c \sum_k b_{ckr}^s = B_s)$

$$\Upsilon(p) = \sum_{c,k,r} \max_{b_{ckr}^s = B_s} \left( \sum_s \frac{B_s}{(1-\alpha_s)} \times \log \left( \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s \left( \min_r \left\{ \frac{b_{ckr}^s(p)}{p_{cr} d_{ckr}^s} \right\} \right)^{(1-\alpha_s)} \right) + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} p_{cr} - \sum_s B_s \right) \quad (\text{B.5})$$

$$\begin{aligned} & \sum_s \frac{B_s}{(1-\alpha_s)} \log \left( \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s \left( \min_r \left\{ \frac{b_{ckr}^s(p)}{p_{cr} d_{ckr}^s} \right\} \right)^{(1-\alpha_s)} \right) \\ &= \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} b_{ckr}^s(p) \log \left( \frac{b_{ckr}^s(p)}{p_{cr} d_{ckr}^s} \right) - \frac{1}{(1-\alpha_s)} \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} b_{ck}^s(p) \log \left( \frac{b_{ck}^s(p)}{w_{ck}^s} \right) \end{aligned} \quad (\text{B.6})$$

$$\begin{aligned} & \sum_s \frac{B_s}{(1-\alpha_s)} \log \left( \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s \left( \min_r \left\{ \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right\} \right)^{(1-\alpha_s)} \right) \\ & \leq \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} b_{ckr}^s \log \left( \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right) - \frac{1}{(1-\alpha_s)} \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} b_{ck}^s \log \left( \frac{b_{ck}^s}{w_{ck}^s} \right) \end{aligned} \quad (\text{B.7})$$

The inequality becomes equality if  $b_s = b_s(p)$

$$\begin{aligned} \Upsilon(p(b^*)) &= \sum_s \frac{B_s}{(1-\alpha_s)} \times \log \left( \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s \left( \min_r \left\{ \frac{b_{ckr}^s(p)}{p_{cr} d_{ckr}^s} \right\} \right)^{(1-\alpha_s)} \right) \\ &= \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} b_{ckr}^s(p) \log \left( \frac{b_{ckr}^s(p)}{p_{cr} d_{ckr}^s} \right) - \frac{1}{(1-\alpha_s)} \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} b_{ck}^s(p) \log \left( \frac{b_{ck}^s(p)}{w_{ck}^s} \right) \end{aligned} \quad (\text{B.8})$$

$$\begin{aligned} \Upsilon(p(b)) &= \sum_s \frac{B_s}{(1-\alpha_s)} \log \left( \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s \left( \min_r \left\{ \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right\} \right)^{(1-\alpha_s)} \right) \\ & \leq \Phi_{<}(b) = \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} b_{ckr}^s \log \left( \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right) - \frac{1}{(1-\alpha_s)} \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} b_{ck}^s \log \left( \frac{b_{ck}^s}{w_{ck}^s} \right) \end{aligned} \quad (\text{B.9})$$

$$\Upsilon(p(b)) - \Upsilon(p(b^*)) \geq \Phi(b^*) - \Phi(b) \quad (\text{B.10})$$

---

## B.2 Derivation of update rule for $\alpha_s \geq 1$

$$b_s(t+1) = \arg \min_{\sum_c \sum_k \sum_r b_{ckr}^s \leq B_s} \left\{ \nabla_{b^s} \Phi_p(b^s(t)) (b^s - b^s(t)) + KL_a(b^s || b^s(t)) - \frac{1}{(1-\alpha_s)} KL_b(b^s || b^s(t)) \right\} \quad (\text{B.11})$$

we consider the Lagrangian

$$L_s(b^s, \gamma) = \nabla_{b^s} \Phi_p(b^s(t)) (b^s - b^s(t)) + KL_a(b^s || b^s(t)) - \frac{1}{(1-\alpha_s)} KL_b(b^s || b^s(t)) + \gamma \left( \sum_c \sum_k \sum_r b_{ckr}^s - B_s \right) \quad (\text{B.12})$$

After applying the first order KKT condition we get

$$\left[ 1 + \log \left( \frac{b_{ckr}^s(t)}{p_{cr} d_{ckr}^s} \right) \right] - \frac{1}{(1-\alpha_s)} \left[ 1 + \log \left( \frac{b_{ck}^s(t)}{w_{ck}^s} \right) \right] + 1 + \log \left( \frac{b_{ckr}^s}{b_{ckr}^s(t)} \right) - \frac{1}{(1-\alpha_s)} \left[ 1 + \log \left( \frac{b_{ck}^s}{b_{ck}^s(t)} \right) \right] + \gamma = 0 \quad (\text{B.13})$$

After some calculation we have

$$\log(b_{ckr}^s) = \log(p_{cr} d_{ckr}^s) + \frac{1}{(1-\alpha_s)} \log \left( \frac{b_{ck}^s}{w_{ck}^s} \right) - \gamma \quad (\text{B.14})$$

taking exponentila on both side

$$b_{ckr}^s = e^C p_{cr} d_{ckr}^s \left( \frac{b_{ck}^s}{w_{ck}^s} \right)^{1/(1-\alpha_s)} \quad (\text{B.15})$$

summing over  $r$  on both side

$$b_{ck}^s \left( \frac{b_{ck}^s}{w_{ck}^s} \right)^{-1/(1-\alpha_s)} = e^C \sum_r p_{cr} d_{ckr}^s \quad (\text{B.16})$$

$$(b_{ck}^s)^{\frac{-\alpha_s}{(1-\alpha_s)}} = e^c (w_{ck}^s)^{-1/(1-\alpha_s)} \left( \sum_r p_{cr} d_{ckr}^s \right) \quad (\text{B.17})$$

$$(b_{ck}^s)^{\frac{1}{(1-\alpha_s)}} = e^{c \frac{1}{-\alpha_s}} (w_{ck}^s)^{\frac{-1}{(-\alpha_s)(1-\alpha_s)}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{-\alpha_s}} \quad (\text{B.18})$$

replacing  $(b_{ck}^s)^{\frac{1}{(1-\alpha_s)}}$  with (B.18) in (B.15) we have

$$b_{ckr}^s = e^{c \frac{-\alpha_s}{(1-\alpha_s)}} p_{cr} d_{ckr}^s (w_{ck}^s)^{\frac{-1}{(-\alpha_s)}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{-\alpha_s}} \quad (\text{B.19})$$

summing over all  $r$  across all the classes and cell we have

$$B_s = e^{c \frac{-\alpha_s}{(1-\alpha_s)}} \sum_c \sum_k (w_{ck}^s)^{\frac{-1}{(-\alpha_s)}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1-\alpha_s)}{-\alpha_s}} \quad (\text{B.20})$$

$$b_{ckr}^s = \frac{B_s p_{cr} d_{ckr}^s (w_{ck}^s)^{\frac{-1}{(-\alpha_s)}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{-\alpha_s}}}{\sum_c \sum_k (w_{ck}^s)^{\frac{-1}{(-\alpha_s)}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1-\alpha_s)}{-\alpha_s}}} \quad (\text{B.21})$$

$$b_{ckr}^s = \frac{B_s \frac{p_{cr} d_{ckr}^s}{\sum_r p_{cr} d_{ckr}^s} (w_{ck}^s)^{\frac{1}{\alpha_s}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1-\alpha_s)}{-\alpha_s}}}{\sum_{c' \in \mathcal{C}} \sum_{k' \in \mathcal{K}_{c'}^s} (w_{c'k'}^s)^{\frac{1}{\alpha_s}} \left( \sum_{r'} p_{c'r'} d_{c'k'r'}^s \right)^{\frac{(1-\alpha_s)}{-\alpha_s}}} \quad (\text{B.22})$$

### B.3 Best response

We consider the Lagrangian

$$\frac{1}{(1-\alpha_s)} \sum_c \sum_k w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)} - \sum_c \sum_k \sum_r \lambda_{ckr} \left( u_{ck}^s - \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right) - \gamma \left( \sum_c \sum_k \sum_r b_{ckr}^s - B_s \right) \quad (\text{B.23})$$

After applying the first order KKT conditions

$$w_{ck}^s (u_{ck}^s)^{-\alpha_s} - \sum_r \lambda_{ckr} = 0 \quad (\text{B.24})$$

$$\frac{\lambda_{ckr}}{p_{cr} d_{ckr}^s} = \gamma \quad (\text{B.25})$$

from (B.24) and (B.25)

$$w_{ck}^s (u_{ck}^s)^{-\alpha_s} = \gamma \sum_r p_{cr} d_{ckr}^s \quad (\text{B.26})$$

$$(\text{B.27})$$

$$u_{ck}^s = (\gamma)^{\frac{1}{-\alpha_s}} (w_{ck}^s)^{\frac{1}{\alpha_s}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{-\alpha_s}} \quad (\text{B.28})$$

we know that at best response  $u_{ck}^s = \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s}$

$$\frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} = \gamma^{\frac{1}{-\alpha_s}} (w_{ck}^s)^{\frac{1}{\alpha_s}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{-\alpha_s}} \quad (\text{B.29})$$

$$b_{ckr}^s = \gamma^{\frac{1}{-\alpha_s}} (w_{ck}^s)^{\frac{1}{\alpha_s}} p_{cr} d_{ckr}^s \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{-\alpha_s}} \quad (\text{B.30})$$

summing over  $r$

$$\sum_r b_{ckr}^s = \gamma^{\frac{1}{-\alpha_s}} (w_{ck}^s)^{\frac{1}{\alpha_s}} \sum_r p_{cr} d_{ckr}^s \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{-\alpha_s}} \quad (\text{B.31})$$

$$\sum_c \sum_k \sum_r b_{ckr}^s = \gamma^{\frac{1}{-\alpha_s}} \sum_c \sum_k (w_{ck}^s)^{\frac{1}{\alpha_s}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1-\alpha_s)}{-\alpha_s}} \quad (\text{B.32})$$

Replacing the value of  $\gamma^{\frac{1}{-\alpha_s}}$  from (B.32) in (B.30)

$$b_{ckr}^s = \frac{B_s p_{cr} d_{ckr}^s (w_{ck}^s)^{\frac{1}{\alpha_s}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{-\alpha_s}}}{\sum_c \sum_k (w_{ck}^s)^{\frac{1}{\alpha_s}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1-\alpha_s)}{-\alpha_s}}} \quad (\text{B.33})$$

$$b_{ckr}^s = \frac{B_s \frac{p_{cr} d_{ckr}^s}{\sum_r p_{cr} d_{ckr}^s} (w_{ck}^s)^{\frac{1}{\alpha_s}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1-\alpha_s)}{-\alpha_s}}}{\sum_c \sum_k (w_{ck}^s)^{\frac{1}{\alpha_s}} \left( \sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1-\alpha_s)}{-\alpha_s}}} \quad (\text{B.34})$$

## B.4 Market equilibrium

*Proof.* Consider the Lagrangian of EG problem

$$L(u, x, \lambda, p) = \frac{B_s}{(1 - \alpha_s)} \log \left( \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)} \right) - \sum_s \sum_c \sum_{\mathcal{K}_c} \sum_r \lambda_{ckr}^s \left( u_{ck}^s - \frac{x_{ckr}^s}{d_{kr}^s} \right) - \sum_c \sum_r p_{cr} \left( \sum_s \sum_k x_{ckr}^s - 1 \right) \quad (\text{B.35})$$

After applying first order KKT condition we have  $\forall s \in \mathcal{S}, \forall c \in \mathcal{C}, \forall k \in \mathcal{K}_c^s$

$$B_s \frac{w_{ck}^s (u_{ck}^s)^{-\alpha_s}}{\left( \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)} \right)} - \sum_r \lambda_{ckr}^s = 0 \quad (\text{B.36})$$

$\forall s \in \mathcal{S}, \forall c \in \mathcal{C}, \forall k \in \mathcal{K}_c^s, \forall r \in \mathcal{R}_c$  following condition

$$\frac{\lambda_{ckr}^s}{d_{kr}^s} - p_{cr} = 0 \quad \forall s \in \mathcal{S} \quad (\text{B.37})$$

$$u_{ck}^s = \frac{x_{ckr}^s}{d_{ckr}^s} \quad s \in \mathcal{S}, c \in \mathcal{C}_s, k \in \mathcal{K}_c, r \in \mathcal{R} \quad (\text{B.38})$$

$$p_{cr} \left( \sum_s \sum_k x_{ckr}^s - 1 \right) = 0 \quad \forall c \in \mathcal{C} \quad \forall r \in \mathcal{R}_c \quad (\text{B.39})$$

$$\lambda_{ckr}^s \geq 0, p_{cr} \geq 0 \quad (\text{B.40})$$

$$B_s \frac{w_{ck}^s (u_{ck}^s)^{-\alpha_s}}{\left( \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)} \right)} = \sum_r p_{cr} d_{ckr}^s \quad (\text{B.41})$$

$$B_s \frac{w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)}}{\left( \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)} \right)} = \sum_r p_{cr} x_{ckr}^s \quad (\text{B.42})$$

$$B_s = \sum_c \sum_{k \in \mathcal{K}_c^s} \sum_r p_{cr} x_{ckr}^s \quad (\text{B.43})$$

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summing over the  $s$  we get

$$\sum_s \sum_c \sum_r p_{cr} = 1; \tag{B.44}$$

□

# APPENDIX C

## C.1 Proof of Proposition 3

To find the equilibrium of the replicator dynamics defined in (4.6) consider  $\log\left(\frac{q_s^c}{n_s^c}\right) - p_s = \log\left(\frac{q_{s'}^c}{n_{s'}^c}\right) - p_{s'}$ . Taking the exponential of both sides, we obtain  $\frac{q_s^c}{n_s^c} \frac{n_{s'}^c}{q_{s'}^c} = e^{p_s - p_{s'}} \Leftrightarrow \frac{q_s^c}{n_s^c} n_{s'}^c = q_{s'}^c e^{p_s - p_{s'}}$ . Summing over  $\forall s' \in \mathcal{S}$  gives us  $\sum_{s'} \frac{q_s^c}{n_s^c} n_{s'}^c = \sum_{s'} q_{s'}^c e^{p_s - p_{s'}}$ , which can be rewritten as  $n_s^c = \frac{N^c q_s^c e^{-p_s}}{\sum_{s'} q_{s'}^c e^{-p_{s'}}}$ , *i.e.*,  $n_s^c = \frac{N^c f_s^c(d_s^c) e^{-p_s}}{\sum_{s'} f_{s'}^c(d_{s'}^c) e^{-p_{s'}}}$ .

## C.2 Proof of Proposition 4

First, we show that the utility of each SP is concave in its own decision variable.  $\frac{\partial^2 U_s}{\partial b_1^{c2}} = \frac{A+B}{C} < 0$

$$A = - \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{\rho_2} \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{2\rho_1} \left( (\rho_2^2 + \rho_2) b_1^{c2} + 2\rho_1 b_2^c (\rho_2 + 1) b_1^c + \rho_1 b_2^{c2} (\rho_1 + 1) \right) \quad (C.1)$$

$$B = \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{2\rho_2} \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{\rho_1} \left( (\rho_2^2 - \rho_2) b_1^{c2} + 2\rho_1 b_2^c (\rho_2 - 1) b_1^c + \rho_1 b_2^{c2} (\rho_1 - 1) \right) \quad (C.2)$$

$$C = (b_1^c + b_2^c)^2 b_1^{c2} \left( \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{\rho_1} + \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{\rho_2} \right)^3 \quad (C.3)$$

Next, we show that the utility of each SP is convex with respect to the opponents' decision variable.

$$\frac{\partial^2 U_s}{\partial b_2^{c2}} = \frac{G+H}{I} > 0$$

$$G = - \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{\rho_2} \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{2\rho_1} \left( (\rho_1^2 - \rho_1) b_2^{c2} + 2\rho_2 b_1^c (\rho_1 - 1) b_2^c + \rho_2 b_1^{c2} (\rho_2 - 1) \right) \quad (C.4)$$

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$$H = \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{2\rho_2} \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{\rho_1} \left( (\rho_1^2 + \rho_1) b_2^{c2} + 2\rho_2 b_1^c (\rho_1 + 1) b_2^c + \rho_2 b_1^{c2} (\rho_2 + 1) \right) \quad (\text{C.5})$$

$$I = (b_1^c + b_2^c)^2 b_2^{c2} \left( \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{\rho_1} + \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{\rho_2} \right)^3 \quad (\text{C.6})$$

### C.3 Proof of Proposition 5

Consider that for any bid  $b_2^c > 0$  submitted by SP 2 at cell  $c$ , SP 1 places a bid of  $b_1^c = B_1 \frac{b_2^c}{B_2}$  at cell  $c$ . Then, the quantity of resource received by SP 1 at cell  $c$  is  $d_1^c = \frac{B_1 \frac{b_2^c}{B_2}}{B_1 \frac{b_2^c}{B_2} + b_2^c} = \frac{B_1}{B_1 + B_2}$ . This proves that for any strategy played by SP there exists a strategy for the other SP such that he receives the resource in proportion to his budget.

### C.4 Revision Protocol

Let recall the revision protocol, which defines the switching rate at which users switch their choice from SP  $s$  to SP  $s'$  given population state  $n$   $\tau_{s,s'} = n_{s'} [\nu_{s'} - \nu_s]_+$ . Note that for the sake of simplicity, we omit the cell dependence ( $c$ ).

Relying on the evolutionary process (4) and by substitution of the revision protocol, we get

$$\begin{aligned} \dot{n}_s &= \sum_{s'} n_{s'} \tau_{s',s} - n_s \sum_{s'} \tau_{s,s'}, \\ \dot{n}_s &= \sum_{s'} n_{s'} n_s [\nu_s - \nu_{s'}]_+ - n_s \sum_{s'} n_{s'} [\nu_{s'} - \nu_s]_+, \\ \dot{n}_s &= n_s \sum_{s'} n_{s'} [\nu_s - \nu_{s'}], \\ \dot{n}_s &= n_s \left[ \nu_s - \sum_{s'} n_{s'} \nu_{s'} \right]. \end{aligned}$$

Similarly, considering the revision protocol  $\tau_{s,s'} = \frac{n_{s'}}{N} [U_{s'}^c - U_s^c]_+$ , we get

$$\begin{aligned}\dot{n}_s &= \sum_{s'} n_{s'} \tau_{s',s} - n_s \sum_{s'} \tau_{s,s'}, \\ \dot{n}_s &= \sum_{s'} n_{s'} \frac{n_s}{N} [\nu_s^c - \nu_{s'}^c]_+ - n_s \sum_{s'} \frac{n_{s'}}{N} [\nu_{s'}^c - \nu_s^c]_+, \\ \dot{n}_s &= n_s \sum_{s'} \frac{n_{s'}}{N} [\nu_s - \nu_{s'}], \\ \dot{n}_s^c &= n_s^c \left[ \nu_s^c - \frac{1}{N} \sum_{s'} n_{s'}^c \nu_{s'}^c \right].\end{aligned}$$

## C.5 Proof of Theorem 4.12

Taking the gradient of  $\mathcal{R}_s(\cdot)$  with respect to  $x_s^s := d_s$ , we obtain:

$$\nabla_{x_s^s} \mathcal{R}_s(d) = \sum_c p_s N^c \frac{\nabla_{x_s^s} f_s^c(d_s^c) e^{-p_s}}{\left(\sum_{s'} f_{s'}^c(d_{s'}^c) e^{-p_{s'}}\right)^2} \sum_{s' \neq s} f_{s'}^c(d_{s'}^c) e^{-p_{s'}}.$$

For any  $d, \tilde{d} \in \mathcal{F}$ ,  $\|\nabla_{x_s^s} \mathcal{R}_s(d) - \nabla_{x_s^s} \mathcal{R}_s(\tilde{d})\| \leq \sum_c p_s N^c \max \left\{ \frac{\nabla_{x_s^s} f_s^c(d_s^c) e^{-p_s}}{\left(\sum_{s'} f_{s'}^c(d_{s'}^c) e^{-p_{s'}}\right)^2}; \frac{\nabla_{x_s^s} f_s^c(\tilde{d}_s^c) e^{-p_s}}{\left(\sum_{s'} f_{s'}^c(\tilde{d}_{s'}^c) e^{-p_{s'}}\right)^2} \right\} \cdot \|\sum_{s' \neq s} f_{s'}^c(\tilde{d}_{s'}^c) e^{-p_{s'}} - \sum_{s'} f_{s'}^c(\tilde{d}_{s'}^c) e^{-p_{s'}}\|$  by Hölder inequality. Then, applying Jensen's inequality, we obtain that  $\|\nabla_{x_s^s} \mathcal{R}_s(d) - \nabla_{x_s^s} \mathcal{R}_s(\tilde{d})\| \leq \sum_c p_s N^c \max \left\{ \frac{\nabla_{x_s^s} f_s^c(d_s^c) e^{-p_s}}{\left(\sum_{s'} f_{s'}^c(d_{s'}^c) e^{-p_{s'}}\right)^2}; \frac{\nabla_{x_s^s} f_s^c(\tilde{d}_s^c) e^{-p_s}}{\left(\sum_{s'} f_{s'}^c(\tilde{d}_{s'}^c) e^{-p_{s'}}\right)^2} \right\} \cdot \sum_{s' \neq s} \|f_{s'}^c(d_{s'}^c) - f_{s'}^c(\tilde{d}_{s'}^c)\|$ . If  $f_s^c(\cdot)$  is  $K_s^c$  Lipschitz continuous then by setting  $L_s := \sum_c p_s N^c \max \left\{ \frac{\nabla_{x_s^s} f_s^c(d_s^c) e^{-p_s}}{\left(\sum_{s'} f_{s'}^c(d_{s'}^c) e^{-p_{s'}}\right)^2}; \frac{\nabla_{x_s^s} f_s^c(\tilde{d}_s^c) e^{-p_s}}{\left(\sum_{s'} f_{s'}^c(\tilde{d}_{s'}^c) e^{-p_{s'}}\right)^2} \right\}$  then for any  $d, \tilde{d} \in \mathcal{F}$ ,  $\|\nabla_{x_s^s} \mathcal{R}_s(d) - \nabla_{x_s^s} \mathcal{R}_s(\tilde{d})\| \leq L_s \|d - \tilde{d}\|_1 \leq L_s \cdot \sqrt{S} \cdot \sum_c |\mathcal{M}^c| \cdot \|d - \tilde{d}\|_2$ . This proves that  $\nabla_{x_s^s} \mathcal{R}_s(\cdot)$  is  $L_s \cdot \sqrt{S} \cdot \sum_c |\mathcal{M}^c|$  Lipschitz continuous. In addition, the coupling constraints in the pricing game  $\mathcal{G}_p$  are linear in the SPs' decision variables. Though we introduce projection operators in (4.51), (4.52), Cauchy-Schwarz inequality implies that the norm of the projection matrix can be upper bounded by 1. This enables us to derive the same upper bound and statement as in [58], Thm.1.

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# PUBLICATIONS

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# A Mechanism for Price Differentiation and Slicing in Wireless Networks

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**Abstract**—Slicing has been introduced in 5G networks in order to deliver the higher degree of flexibility and scalability required by future services. Slice tenants such as virtual wireless operators, service providers or smart-city services will be able to book a share of the infrastructure, possibly including storage, computing capacity and link bandwidth. However, 5G slicing is attractive for infrastructure providers as long as they are able to generate revenues, while at once satisfying the tenants' competing and variable demands and coping with resources availability.

This work proposes a flexible mechanism based on a multi-bidding scheme for 5G slice allocation. It is able to attain desirable fairness and efficiency figures in order to serve slice tenants and associated mobile users. Built on the notion of normalised Nash equilibrium, it is also provably overbooking-free even though the players' bids are oblivious to infrastructure resources constraints. Also, it is compatible with standard radio access schedulers used in modern mobile networks.

Finally, a practical algorithm is proposed to drive the system to the socially-optimal operating point via an online procedure rooted on a primal-dual distributed algorithm. Numerical simulations confirm the viability of the mechanism in terms of efficiency and fairness.

**Index Terms**—Game theory, Kelly mechanism, normalised equilibrium, primal-dual algorithms, wireless Network slicing, resource allocation.

## I. INTRODUCTION

In the emerging 5G technology, slicing allows mobile network operators (MNO) to offer differentiated services to their customers using shared resource pools. A slice, in this context, is a share of the mobile network operator infrastructure obtained via Software-Defined Networking (SDN) and Network Function Virtualization (NFV) technologies. A slice forms a logical network on top of the physical one [1], [2]. Evolving from previous mobile technology, the 5G core network architecture integrates data-centers into their architectures to support network function virtualisation and computation offloading. Thus, a slice will typically encompass different resource types, such as radio access capacity, edge storage memory and computing power available within the MNO infrastructure [1].

Slicing techniques entered the standardisation phase recently [3] so that specifications 5G system's slicing architecture and requirements are now available. Some technical aspects such as slice insulation and fair slice allocation are still a key challenge to upgrade LTE technology towards 5G, with large effort by the research community to overcome such technical issues [4], [5], [6], [7], [8]. Using slice insulation,

virtual private networks will be shipped on top of the existing mobile network infrastructure with dedicated customer support. Thus, new emerging service providers will demand a slice to offer dedicated services to their customers on top of the MNO's infrastructure, e.g., for real time gaming, multimedia applications, social networks, etc.

Ultimately, slicing will deeply change the business model in the mobile communication industry [9], [10] and a crucial aspect is how to jointly price and share resources assigned simultaneously across slices. Mechanisms to price and share resources have appeared in literature [11], [12], [13], [14], assuming customers would demand several resources at once using vectors of bids and so specify their demands.

However, compared to standard settings, e.g., in cloud computing [11], [12], slicing in 5G networks has key differences. First, in cloud computing the pool of resources is often over-provisioned, whereas in 5G scarce radio resources are critical for QoE and requires careful resources allocation. Second, mobile networks are traditionally designed for fair resource sharing, since near far effects and fading induce very different conditions across a deployment in the same cell and across cells. Third, load conditions across a 5G deployment may be at once dynamic and heterogeneous. Finally, joint slice allocation and pricing schemes need to adhere to Service Level Agreement (SLA), which are de facto mandatory in the telecommunication industry [15]. This heavily discourages overbooking as a viable option for MNOs willing to increase their revenues by adopting new slicing technology.

In this paper, we propose a new theoretical framework for pricing slices of resources, based on the Kelly mechanism and the concept of the normalized Nash equilibrium. The basic Kelly mechanism is a bidding mechanism where slice tenants submit an individual bid to the resource owner to obtain an amount of resource. They receive a fraction of the whole resource proportionally to the received bids, and they pay depending on how much they bid. Thus, tenants bid strategically to obtain a share of a single resources [16], [17], [18]. We shall consider a multi-bid version of the Kelly mechanism, where the MNO exposes to tenants a vector of prices per resource. Multi-bid auctions are a main line of research in cloud computing, where clients compete to purchase bundles of cloud resources [11], [12]. The case for using the Kelly mechanism in 5G networks comes from the fact that it offers high flexibility: it applies to bundled resources, i.e., pre-defined blocks of computing and communication resources

TABLE I  
MAIN NOTATIONS USED THROUGHOUT THE PAPER

Symbol	Meaning
$\mathcal{C} := \{1, \dots, C\}$	set of base stations
$\mathcal{S} := \{1, \dots, S\}$	set of slices (tenants)
$N_c^s$	number of active users on slice $s$ at base station $c$
$\gamma_c^s$	unitary price for bids of slice $s$ at base station $c$
$B_c$	total available bandwidth at base station $c$
$b_c^s$	the bandwidth allocated to slice $s$ at base station $c$
$V_c^s$	the benefit function of slice $s$ at base station $c$
$\mathbf{V}^s$	the total benefit function of slice $s$
$\alpha_s$	alpha fair scheduling parameter for slice $s$
$h_u$	channel state of user $u$
$\mathbf{h}$	the channel state vector for all users
$r_u$	the rate attained by a user $u$
$p_u$	transmission power of user $u$
$x_c^s$	bid of slice $s$ for bandwidth at base station $c$
$\mathbf{x}^s = (x_1^s \dots x_C^s)$	vector of bids of player $s$ across base stations
$\mathbf{x}_c^{-s} = \sum_{s' \neq s} x_c^{s'}$	bid of all other players but $s$ on base station $c$
$\beta_n$	step size for the learning algorithm

in the form of virtual machines or containers. But it also applies well to elastic radio resources, where it is customary to use utility-based schedulers such as the Proportional Fair Scheduler (PFS) [19].

It is important to notice that, while bidding serves very well the purpose to generate resources demands at the tenants' side, from the MNO's point of view, overbooking may represent the major risk when using a competitive bidding mechanism. In fact, the aggregate behavior of tenants in general will not comply to the MNO's system capacity constraints.

*Main contribution.* We part from standard propositions of joint pricing and resources allocation in mobile networks in literature. We provide an explicit theoretical connection between price definition, bidding mechanism and coupled constraints across slices. Such fundamental problem can be solved by rooting the pricing scheme in the theory of normalised Nash equilibria, according to the seminal work of Rosen [20]. We solve the problem by cascading two coupled games, namely, the Shadow Pricing Game and the Allocation Game. In a fashion which echoes the original ideas of Kelly on shadow prices for multicommodity flow optimisation [21], the Shadow Pricing Game let the MNO settle the price vector via a uniquely determined normalised Nash equilibrium. The resulting price vector induces a Nash equilibrium in the Allocation Game respecting the resources constraint and thus provably overbooking free. Finally, we show that the price vector can be designed to attain the social optimum for the game. In order to render the mechanism practically viable, we provide an online learning procedure based on a primal-dual distributed algorithm able to drive the system to the target socially optimal equilibrium and requiring at each step to disclose solely the price and the bid vectors generated at each step.

## II. OPTIMISATION FRAMEWORK

Let a single MNO having a set of base stations  $\mathcal{C}$  shared by a set  $\mathcal{S}$  of tenants that need physical network resources in order to serve their users. This can be the case of an application provider serving several customers in mobility. The 5G paradigm envisions for MNO resources to be heterogeneous

and include not only standard radio resources such as PRBs, but also storage, CPU and backhauling. The MNO assigns to each tenant a slice of resources, and we assume that each tenant proposes a service covered by all base stations in  $\mathcal{C}$ . Each tenant's users generate demands, and such demands will inevitably depend on their specific location, thus inducing different slice-dependent demand at each base station.

In this section, we confine the discussion to a RAN version of the the slicing problem, where the MNO schedules wireless resources, namely downlink PRBs among multiple tenants. While the RAN resources allocation problem is a known and well studied one, heterogeneity of traffic demands across tenants and cells captures the main features of slice resources allocation, including fairness issues. The case of multi-resources allocation, spanning other type of infrastructure resources beyond PRBs is an immediate extension of the scheme presented for RAN resources.

Let each slice tenant  $s$  be associated with users presence vector  $\mathbf{N}^s = (N_1^s, N_2^s, \dots, N_C^s)$  where  $C$  is the total number of cells and  $N_c^s$  is the number of active users on slice  $s$  at base station  $c \in \mathcal{C}$ . Here a base station is modelled as a finite resource shared by its associated users. We observe that the number of active users associated to the same base station vary across slices, and vary across base stations also for same slice.

First let us consider some fixed channel condition at all users and at all slices, and let  $r_u$  be the rate attained by a user  $u$  in slice  $s$  at cell  $c$ . The slice benefit function

$$V_c^s(b_c^s) := \sum_{u=1}^{N_c^s} f_s(r_u(b_c^s)) \quad (1)$$

where  $b_c^s$  is the amount of resources (bandwidth) allocated to slice  $s$  at base station  $c$  and under  $\alpha$ -fair scheduling it holds

$$f_s(r_u) = \begin{cases} \frac{(r_u)^{1-\alpha_s}}{(1-\alpha_s)} & \text{if } \alpha_s \neq 1 \\ \log(r_u) & \text{if } \alpha_s = 1 \end{cases} \quad (2)$$

The meaning of (2) is that, when slice  $c$  has received capacity  $b_c^s$ , user  $u$  of slice  $s$  associated to base station  $c$  receives a rate which is the  $\alpha$ -fair share attained with his peer users on the same slice. The average rate  $r_u$  of any user  $u$  is determined by the scheduling policy and by all the specific techniques used at physical layer and MAC layer, such as modulation, coding, scheduling, etc.

In the case when the channel per user varies over time, let  $b_c^s \log(1 + \frac{p_u h_u}{N_0})$  the instantaneous rate when tenant's user  $u$  is scheduled, at transmission power  $p_u$ , noise power  $N_0$  and under channel state  $h_u$ , where  $\mathcal{H}_u$  is the finite set of possible channel states of user  $u$ . Vector  $\mathbf{h} = (h_1, \dots, h_{N_c^s})$  is thus the channel state vector for all users in cell  $s$ . Users of slice  $s$  are served under some scheduling policy  $\Pi(\cdot|\cdot)$  at cell  $c$ , which depends on the past and present users' channel state; at each time-slot, the slice scheduler then allocates the channel to a tagged user  $u$  in cell  $c$  with probability  $\Pi(u|\mathbf{h})$ . The average rate achieved by user  $u$  under policy  $\Pi$  is

$$r_u = g_u(b_c^s, \Pi(u|\mathbf{h})) := \mathbb{E}_{\mathbf{h}} \left[ b_c^s \log(1 + h_u \frac{p_u}{N_0}) \Pi(u|\mathbf{h}) \right] \quad (3)$$

where the expectation is taken with respect to the channel distribution. We observe that, irrespective of the actual

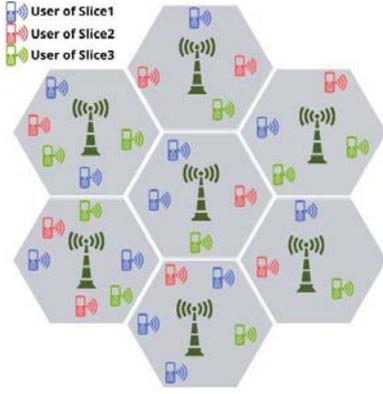


Fig. 1. Resource Slicing in 5G Networks: slices are assigned radio resources on a per-cell basis.

scheduling policy, the average rate for a tagged user  $u$  is linear in the slice bandwidth  $b_c^s$  at cell  $c$ . Once we fixed  $k_u^* = (\mathbb{E}_h [\log(1 + h_u \Lambda_0) \Pi(u|\mathbf{h})])$ , the total benefit function for slice  $s$  writes as

$$V_c^s = \sum_{u=1}^{N_c^s} f_s(k_u^* b_c^s)$$

which is again an increasing concave function of the allocated bandwidth per slice.

The classical optimisation framework for the MNO prescribes to provide efficient yet fair allocation for all users belonging to the same slice according to slices' load. Since scheduling is performed per cell, however, it is necessary for the resources allocation to be fair – within the same slice – also across users associated to different base stations. Such a trade-off between efficiency and fairness can be captured by formulating the utility of a given slice as:

$$\mathbf{V}^s(\mathbf{b}^s) = \sum_{c \in \mathcal{C}} V_c^s(b_c^s) \quad (4)$$

For the sake of discussion, we shall assume that the number of users is fixed. Applied at the cell level, utility (4), is able to express the customary trade-off between efficiency and fairness among users associated to a tagged slice service. However, it also allows to achieve such a trade-off horizontally, that is across cells. For  $\alpha = 1$ , for instance, the customary log-based proportional-fair utility will severely penalise serving high throughput in a lightly loaded cell while starving slice users in another hot-spot cell.

The main objective of the MNO is to maximise the total utility of slices, leading to the following 5G resource allocation problem

$$P : \quad \begin{aligned} & \underset{\mathbf{b}}{\text{maximize}} && \sum_{s \in \mathcal{S}} \mathbf{V}^s(\mathbf{b}^s) \\ & \text{subject to} && b_c^s \geq 0 \\ & && \sum_{s \in \mathcal{S}} b_c^s \leq B_c, \quad \forall c \in \mathcal{C} \end{aligned}$$

where  $B_c$  is the total bandwidth available at base station  $c$ .

This optimisation problem can be solved by well-known convex optimisation methods. However, such a centralised

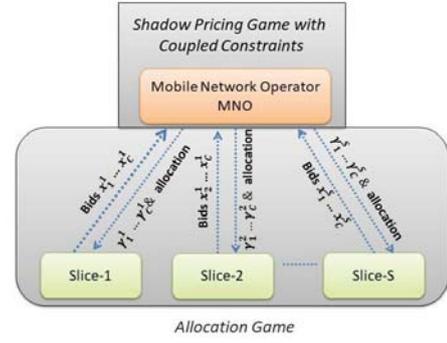


Fig. 2. The MNO uses the Shadow Pricing Game (top) to generate the price vector, whereas the result of the Allocation Game (bottom) decides the actual resource slicing based on the tenants bids.

resource allocation scheme, while addressing properly the MNO constraints on the resources allocation, lacks of scalability, and may lead to excessive communication overhead when the number of slices increases. Furthermore, it is known that such solutions are rarely viable when under dynamic network conditions. Thus, while (P) still provides reference performance figure for our slicing problem, we shall focus on a decentralised scheme where the resources allocation decision is mediated by a pricing scheme pivoting on the Kelly mechanism.

### III. KELLY MECHANISM BASED RESOURCE ALLOCATION

In this section, we design a bidding scheme solving problem (P). The curse of bidding-based schemes is that typically they cannot rule out the possibility of resources overbooking. Indeed, it is unrealistic to assume that, under a competitive pricing scheme, tenants would jointly account for the per-cell *coupled constraint*  $\sum_s b_c^s \leq B_c$ , appearing in (P).

The scheme we propose, conversely, is provably overbooking-free even though the slice bids are oblivious to infrastructure resources constraints. Our design is based on two coupled games, entangled by the same Nash equilibrium: first, a virtual game, namely the *Shadow Pricing Game*, which generates the vector of resource prices, and for which there exist a unique normalised Nash equilibrium, and second the *Allocation Game* where – based on the so-determined prices – the MNO rules a multi-dimensional Kelly mechanism where each tenant acquires a slice of resource in each cell  $c \in \mathcal{C}$  at a price.

Overall, the proposed mechanism can be seen as the cascade of the following two items (see Fig. 2).

- The MNO settles the price obtained by Shadow Game in a way to respect the coupled constraints of resources;
- The prices are announced to tenants in the Allocation game to obtain the Nash equilibrium that respects the resource constraints on resources.
- A specific price will be designed by the MNO to attain the social optimum for the game.

We observe that the proposed scheme requires indeed full information at the MNO side on the tenants' valuations. Thus,

by itself, it does not represent a feasible scheme. We shall relax such demanding request with a learning procedure able to drive the system to the Nash equilibrium resulting from the aforementioned cascade.

#### A. Shadow Pricing Game

The Shadow Pricing Game is a virtual game where the tenants compete for resource access and do not share information on the amount of resource they ask for. The result of this virtual game which matters to the MNO is the resulting price vector. In fact, the seminal work of Rosen [20] ensures the existence of a unique equilibrium vector of this game in the form of a normalised Nash equilibrium, a concept which is pivotal in this paper. Such equilibrium is given by the concatenation of a bid vector and a vector of multipliers. The latter are actually the price vector we are interested in.

This price definition has indeed an algorithmic flavour, reflected in the scheme in Fig. 2: before posting the unit prices for resources, the MNO determines the price vector as the solution of the virtual game. In other words, the Shadow Pricing Game is a virtual game which solves for the optimal price as the signal by which the MNO can drive the Allocation Game to a feasible equilibrium with respect to the capacity constraints in (P).

In the virtual game, each tenant tries to maximise her benefit while obeying the coupled constraints

$$\sum_{s \in \mathcal{S}} b_c^s \leq B_c, \quad \forall c \in \mathcal{C}. \quad (5)$$

Thus, the decision problem for a tagged tenant  $s$  writes as

$$\begin{aligned} Q_s \quad & \text{maximize} \quad \mathbf{V}^s(\mathbf{b}^s, \mathbf{b}^{-s}) \\ & \text{subject to} \quad \sum_{s \in \mathcal{S}} b_c^s \leq B_c, \quad \forall c \in \mathcal{C}. \end{aligned}$$

The system  $\{Q_1, \dots, Q_S\}$  represents the formalisation of the Shadow Pricing Game: the notion of an equilibrium for such a continuous game requires to account for the presence of constraints, that is

**Definition 1.** A strategy  $\mathbf{b}^* = (\mathbf{b}^{1*}, \dots, \mathbf{b}^{S*})$  is called Nash Equilibrium for game  $\{Q_1, \dots, Q_S\}$  if

$$\mathbf{V}^s(\mathbf{b}^{s*}, \mathbf{b}^{-s*}) \geq \mathbf{V}^s(\mathbf{b}^s, \mathbf{b}^{-s*}) \quad (6)$$

for all  $s \in \mathcal{S}$ ,  $b_c^s \geq 0$  and  $\sum_{s \in \mathcal{S}} b_c^s \leq B_c, \forall c \in \mathcal{C}$ .

Here, with standard notation,  $(\mathbf{b}^s, \mathbf{b}^{-s*})$  refers to the multi-strategy vector whose  $s$ -th element equals  $\mathbf{b}^s$  and all other strategy vectors equal  $\mathbf{b}^{-s*}$ .

We should observe that in game with coupled constraints, the equilibrium is, in general, non unique. Actually, by inspection we note that the Shadow Pricing Game has an infinite number of equilibria. Conversely, it is the *normalized* Nash equilibrium that, under specific assumptions, results to be unique. Its definition requires to introduce some further notation.

Because of concavity in players' own strategy [20], a multistrategy vector  $\mathbf{b}^*$  is a Nash Equilibrium for the Shadow Pricing Game if and only if it satisfies simultaneously the

Karush–Kuhn–Tucker (KKT) conditions, which are:  
 $\forall c \in \mathcal{C}, \forall s \in \mathcal{S}$

$$\frac{\partial \mathbf{V}^s(\mathbf{b}^*)}{\partial b_c^s} - \lambda_c^s + \xi_c^s = 0 \quad (7a)$$

$$\lambda_c^s \left( \sum_{s' \in \mathcal{S}} b_c^{s'*} - B_c \right) = 0 \quad (7b)$$

$$\xi_c^s b_c^{s*} = 0 \quad (7c)$$

$$\lambda_c^s \geq 0, \quad \xi_c^s \geq 0. \quad (7d)$$

**Definition 2.** A  $r$ -normalized equilibrium point is such that there exists  $\lambda_c > 0$  associated to each base station so that for all customers  $\lambda_c^s = \lambda_c / r_c^s$ , for a suitable vector of nonnegative vector of coefficients  $r$ .

The important property of normalised Nash equilibria we are leveraging in the rest of the discussion is in the following

**Theorem 1** ([20], Thm. 3). *There exists a unique  $r$ -normalized equilibrium point for the Shadow Pricing Game for every specified  $r > 0$*

While the Pricing Game in practice may not be practically viable (indeed it is not reasonable to expect players to respect the aggregate constraint in calculating their best response), the development in this section has showed how to map the Pricing Game onto the Allocation Game

#### B. Allocation Game

Once the MNO obtained the vector prices, the actual game is an auction-based bandwidth allocation mechanism, in which each slice tenant  $s$  submits bid vector  $x^s = (x_1^s, \dots, x_C^s)$ , one bid for each one of the  $C$  base stations. Bid  $x_c^s$  represents the amount of bandwidth demanded for slice  $s$  at base station  $c$ .

The MNO collects all bids for each base station and assigns to each slice  $s$ , a fraction of each base station corresponding to the ratio he attained given the bids received for that base station, namely the quantity

$$b_c^s := B_c \frac{x_c^s}{\sum_{s' \in \mathcal{S}} x_c^{s'}}, \quad (8)$$

where  $B_c$  represents the total bandwidth available at base station  $c$ . As the valuation of each slice is function of bandwidth received by it, from (1) we write the valuation of slice  $s$  as

$$\mathbf{V}^s(\mathbf{b}^s) = \sum_{c \in \mathcal{C}} V_c^s \left( \frac{x_c^s}{\sum_{s' \in \mathcal{S}} x_c^{s'}} B_c \right). \quad (9)$$

For each slice  $s$ ,  $\mathbf{V}^s$  is an increasing function in  $b_c^s$ : without any payment slice tenants will always bid as much as possible in order to increase their own benefit. However, after submitting the bids, each customer pays to the MNO the cumulative sum of prices for the bids she made. More precisely, let  $\gamma_s^c$  be the unit cost for bidding for one resource unit (e.g., one PRB) at base station  $c$  for slice  $s$ . Then, each slice tenant  $s$  pays  $\gamma_c^s x_c^s$  for the resources obtained at base station  $c$ .

In turn, the utility of a MNO customer is defined as the difference between the overall benefit obtained by using the

portion of bandwidth at different base station and the total cost to pay for using them:

$$U^s(\mathbf{x}^s, \mathbf{x}^{-s}, \gamma^s) = \sum_{c \in \mathcal{C}} V_c^s \left( \frac{x_c^s}{\sum_{s' \in \mathcal{S}} x_c^{s'}} B_c \right) - \gamma_c^s x_c^s. \quad (10)$$

The tenants are rational players and bid for PRBs so as to optimise their utility (10). Thus, the decision problem of each slice  $s \in \mathcal{S}$  is to find the optimal  $x^s$  optimizing its own utility:

$$P_s : \quad \begin{aligned} & \underset{\mathbf{x}^s}{\text{maximize}} && U^s(\mathbf{x}^s, \mathbf{x}^{-s}, \gamma^s) \\ & \text{subject to} && x_c^s \geq 0, \forall c \in \mathcal{C} \end{aligned}$$

Then, the slicing allocation problem can be interpreted as a competitive game where players, i.e. the customers compete to acquire bandwidth for their own slice in order to increase their utility. The standard notation  $\{P_1 \dots P_S\}$  describes formally the *bandwidth allocation game*.

For this game we can consider the standard notion of a Nash equilibrium, where we do not have coupled constraints, that is a multistrategy  $x^* = (x^{1*}, \dots, x^{S*})$  where for all players  $s \in \mathcal{S}$ ,  $U_s(x^{s*}, x^{-s*}) \geq U_s(x^s, x^{-s*})$  with  $x_c^s \geq 0, \forall c \in \mathcal{C}$ . In particular, it is known that in the single resource case

**Theorem 2** ([16]). *The Kelly mechanism has a unique Nash equilibrium.*

Clearly, the uniqueness result extends immediately to the Allocation Game since the resources are orthogonal<sup>1</sup>.

The next result is the central result of this paper, since it provides the connection between the equilibria of the two games:

**Theorem 3.** *Every  $r$ -normalised Nash equilibrium of the Shadow Pricing Game with shadow prices  $\lambda_c$  is a Nash equilibrium for the corresponding Allocation Game with  $\gamma_c^s = \frac{b_c^{-s}}{B_c} \lambda_c$ .*

*Proof.* Let us consider a normalised Nash equilibrium  $\bar{b}$  of the Shadow Pricing Game. From (7c), we have that necessarily

$$\forall s \in \mathcal{S}, \forall c \in \mathcal{C}, \quad \frac{\partial V_s}{\partial b_c^s}(\bar{b}) = \lambda_c^s - \xi_c^s.$$

If we replace  $\lambda_c^s = \frac{B_c}{b_c^{-s}} \gamma_c^s$  and  $\eta_c^s = \frac{B_c}{b_c^{-s}} \xi_c^s$  for all  $c \in \mathcal{C}$  we get

$$\frac{\partial V_s(\bar{x})}{\partial x_c^s} \frac{b_c^{-s}}{B_c} = \frac{\partial U_s}{\partial x_c^s} = \gamma_c^s - \eta_c^s.$$

Thus

$$\begin{cases} \frac{\partial U_s}{\partial x_c^s}(\bar{x}) \leq 0 & \text{if } x_c^s = 0 \\ \frac{\partial U_s}{\partial x_c^s}(\bar{x}) = 0 & \text{if } x_c^s > 0 \end{cases}$$

Thus  $\bar{x}$  satisfies the KKT conditions of the optimization problem associated to the Allocation Game. Since function  $U_s$  is concave with respect to variable  $x^s$  and the constraints are linear, they are also sufficient and thus  $\bar{x}$  is a Nash equilibrium of the Allocation Game.  $\square$

<sup>1</sup>The important case when tenants have a total bidding budget is left as part of future works.

#### IV. SOCIAL OPTIMAL PRICING

In this section we will show that the proposed mechanism is able to attain the social optimum. This mechanism is based on a simple pricing can force slices to choose an equilibrium (respecting the resources coupled constraint) that coincide with the optimal solution of (P). Using the cascade of both the Shadow Pricing game and the Allocation game, the pricing and allocation are performed in a distributed manner with no need to exchange per-bandwidth allocation information.

Let us recall the original problem introduced in Sec. (III), where the MNO's goal is to solve problem (P). Concavity of the objective function ensures that there exists a unique allocation  $\mathbf{b}^*$  which maximizes the objective function. Let now consider the Lagrangian associated to problem (P): it writes  $L(b, \mu, \nu) = \sum_s (V(\mathbf{b}^s) - \sum_c \mu_c (\sum_s b_c^s - B_c) - \sum_c \nu_c^s b_c^s)$ .

Since the problem is feasible and constraints are affine, KKT conditions for (P) are necessary and sufficient for optimality of a solution  $(\mathbf{b}^*, \mu^*, \nu^*)$  such that  $\forall s \in \mathcal{S} \forall c \in \mathcal{C}$

$$\frac{\partial V_s(b^*)}{\partial b_c^s} - \mu_c^* + \nu_c^{*s} = 0 \quad (11a)$$

$$\mu_c^* \left( \sum_{s' \in \mathcal{S}} b_c^{*s'} - B_c \right) = 0 \quad (11b)$$

$$\nu_c^{*s} b_c^{*s} = 0, \mu_c^{*s} \geq 0, \nu_c^{*s} \geq 0 \quad (11c)$$

Where  $\mu = (\mu_1, \dots, \mu_C)$  are the  $C$  Lagrange multipliers for the cells capacity constraints.

Now, if we consider the  $r$ -normalized Nash equilibrium for shadow pricing game with  $r_c^s = 1$ , for  $\forall s \in \mathcal{S}$  and  $\forall c \in \mathcal{C}$ , we obtain  $\lambda_c^1 = \dots = \lambda_c^S = \lambda_c$  for all  $c \in \mathcal{C}$  in the KKT conditions (7c). For  $\mu_c = \lambda_c$  the conditions (7c) and (11) are equivalent and as we have already proved uniqueness of  $r$ -normalized Nash equilibrium in Thm. 1, it holds  $\mu^* = \lambda^*$ .

But then, from Thm. 3 we obtain also that the relation between the Allocation Game and the Shadow Pricing game

$$\gamma_c^s = \frac{b_c^{-s}}{B_c} \lambda_c^* \quad (12)$$

settles the Allocation Game on the social optimum.

**Theorem 4.** *The normalised Nash equilibrium attained for  $r_c^s = 1, \forall s \in \mathcal{S}, \forall c \in \mathcal{C}$  by the Shadow Pricing Game attains the social optimum for problem (P), and so does the Allocation Game under prices  $\gamma_c^s$  as in (12).*

#### V. LEARNING AND SYSTEM STABILITY

We have already seen in the previous section that the proposed mechanism has a unique equilibrium for any price vector decided by the infrastructure owner. However, while the MNO can use the price vector as a signal to drive the system to a socially optimum operating point, the game formulation of the mechanism has scarce practical relevance. In fact, since the tenants' valuation of resources is typically unknown to the MNO. To this respect, we propose a learning algorithm to converge iteratively to the target equilibrium in a distributed fashion. In the proposed solution, the only signal exchanged between the MNO and the tenants at each step are the bid vector and the price vector.

We use the dual averaging or mirror–descent method discussed in [22], [23]. However, those works only considered orthogonal constraints. Thus, we have adapted the original algorithm to tackle the coupled constrained setting. The idea behind the mirror descent is each player estimates his gradient and takes steps along the gradient in dual space (where the gradient lives). The aggregated of the  $s$ -th player’s gradient steps is updated according to equation

$$y^s(n+1) = [y^s(n) + \beta_n \nabla_{x^s} U_s(x^s(n), x^{-s}(n), \gamma^s(n))] \quad (\mathcal{A1})$$

In the above equation  $y^s$  is an auxiliary variable which accumulates the discounted gradient and  $\beta_n$  is a standard step size, where  $\sum_{n=0}^{\infty} \beta_n = +\infty$  and  $\sum_{n=0}^{\infty} \beta_n^2 < +\infty$ . Every player  $s$  uses his own updated output value  $y^s$  to take next action. The technique takes the name mirror–descent because each player  $s$  “mirrors back” the variable  $y^s$  to his action space  $\mathcal{X}_s$  according to the mapping

$$x^s(n+1) = \operatorname{argmax}_{x^s \in \mathcal{X}_s} \{ \langle y^s(n), x^s(n) \rangle - h_s(x^s(n)) \} \quad (\mathcal{A2})$$

Here,  $h_s(x)$  is regularizer, according to definition 3.1 in [22], or rather a penalty function over the feasible action set  $\mathcal{X}_s$ . Penalty  $h_s(x)$  permits convergence within the interior of the domain set, that is, the feasible multistrategy set. In our case we use entropic regularization, also known as Gibbs entropy function; it takes the form,  $\forall c \in \mathcal{C}$

$$h_c^s(x^s) = x_c^s \log(x_c^s) + (1 - x_c^s) \log(1 - x_c^s)$$

over domain  $\mathcal{X}_c^s = \{x : 0 \leq x_c^s \leq 1\}$ . The advantage of this formulation is that it can be easily scaled to original constraints. Furthermore, by applying KKT conditions to maximization problem (A2), after some calculations it produces the exponential mapping for all  $s \in \mathcal{S}$  and  $c \in \mathcal{C}$ :

$$x_c^s(n+1) = \frac{B_c \exp(y_c^s(n))}{1 + \exp(y_c^s(n))} \quad (13)$$

The one above is similar to well-know Logit map, where player gives the weights to different resources depending on exponential of aggregated gradients. The players take the actions (in our case the bids) independently of each other, which could results in violation of the resource capacity constraints. In order to handle this problem, the MNO updates the prices in such a way that, the players are forced to obey coupled constraints. The prices appear as Lagrange multipliers for coupled constraints (capacity constraints). As similar to the players, she takes the step along the negative gradient of Lagrangian and updates the price per resource:

$$\lambda^c(n+1) = \max \left( 0, \lambda^c(n) + \beta_n \left( \sum_{s' \in \mathcal{S}} x_c^{s'} - B_c \right) \right) \quad (14)$$

This updated value of Lagrangian multipliers act as new prices for all tenants and resources, that is

$$\gamma_c^s(n) = \frac{x_c^{-s}}{B_c} \lambda_c(n)$$

If all the players and MNO simultaneously take action as per the designed algorithm, the proposed algorithm converges to the unique Normalized Nash equilibrium  $(x^*, \lambda^*)$  of the

Pricing Game. Moreover, if we fixed the prices  $\gamma$  and players are allowed to play only according to the algorithm, the designed algorithm converges to unique Nash equilibrium of the Allocation Game. Note that the update rule of the MNO corresponds to the choice of vector  $r$  such that  $r_c = 1$  for all  $c \in \mathcal{C}$ . To this respect, we provided a formulation where the  $r$ -normalised Nash equilibrium corresponds to the social optimum. In general, it is possible to set the coefficients of vector  $r$  at will. Clearly, the corresponding solution will not be converging to the social optimum, but this provides some space for tenants’ prioritisation, which we leave as part of future works.

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#### Algorithm 1 On-line Distributed Learning Algorithm

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**Require:**  $\sum_{n=0}^{\infty} \beta_n = \infty, \beta_n \rightarrow 0$  as  $n \rightarrow \infty$

- 1: **repeat** at time step  $n = 1, 2, \dots$ ,
- 2:   **for each** player  $s \in \mathcal{S}$
- 3:     Observe gradient of utility and update
- 4:      $y^{s'} \leftarrow y^s + \beta_n \nabla_{x^s} U_s(x^s, x^{-s}, \gamma)$
- 5:   **end for**
- 6:   **for each** player  $s \in \mathcal{S}$
- 7:     **for each** resource  $c \in \mathcal{C}$
- 8:       Play  $x_c^{s'} \leftarrow \frac{B_c \exp(y_c^s)}{1 + \exp(y_c^s)}$
- 9:     **end for**
- 10:   **end for**
- 11:   **for each** resource  $c \in \mathcal{C}$  update the price
- 12:      $\lambda_c' \leftarrow \max \left[ 0, \lambda_c + \frac{\beta_n}{K} \left( \sum_{s \in \mathcal{S}} x_c^s - B_c \right) \right]$
- 13:   **end for**
- 14:   **for each** player  $s \in \mathcal{S}, \forall c \in \mathcal{C}$
- 15:      $\gamma_c^s \leftarrow \frac{x_c^{-s}}{B_c} \lambda_c$
- 16:   **end for**
- 17: **until**  $\|(x', \lambda') - (x, \lambda)\| \leq \epsilon$

---

**Theorem 5.** *If algorithm satisfies the required conditions for step size sequence,  $\sum_{n=0}^{\infty} \beta_n = \infty, \beta_n \rightarrow 0$  as  $n \rightarrow \infty$  then for sufficiently large  $K$  Algorithm 1 converges to the unique Normalized Nash equilibrium of the Shadow Pricing Game.*

*Proof.* In [24] authors had already proved convergence of algorithm as similar to ours for the single resource Allocation Game. We will use the same technique as discussed in Theorem 2 [24] to prove convergence of the Algorithm 1 to Normalized Nash equilibrium of the Pricing Game. Now to show convergence of the Algorithm we will first show the asymptotic stability of mean dynamics of the algorithm. We write the continuous-time equivalent from steps 4, 8 and 12 of the algorithm. For simplicity of exposition, we consider here the single user case, since the general case follows immediately:

$$\dot{y}_c^s = \frac{\partial V_s(x)}{\partial x_c^s} - \gamma_c^s \quad (15)$$

$$x_c^s = \frac{\exp y_c^s}{1 + \exp y_c^s} \quad (16)$$

$$\dot{\lambda}_c = \left( \sum_{s'} x_c^{s'} - B_c \right) \quad (17)$$

Taking derivative of (16) and replacing in (15) gives

$$\dot{x}_c^s = x_c^s(1 - x_c^s) \left( \frac{\partial V_s(x)}{\partial x_c^s} - \gamma_c^s \right) \quad (18)$$

$$\dot{\lambda}_c = \left( \sum_{s'} x_c^{s'} - B_c \right) \quad (19)$$

Now to show stability of dynamics, let consider Lyapunov function

$$\mathcal{L}(x, \lambda) = \sum_s \left[ x_c^{*s} \log \frac{x_c^{*s}}{x_c^s} - (1 - x_c^{*s}) \log \frac{1 - x_c^{*s}}{1 - x_c^s} \right] + \frac{1}{2} \|\lambda_c - \lambda_c^*\|^2 \quad (20)$$

Taking derivative gives

$$\dot{\mathcal{L}} = \sum_s \frac{x_c^s B_c}{(\sum_{s'} x_c^{s'})^2} \left[ \frac{\partial V_s(y_c^s)}{\partial y_c^s} (x_c^s - x_c^{*s}) - \lambda^* (x_c^s - x_c^{*s}) \right] \quad (21)$$

where at the equilibrium  $\lambda^* = \frac{\partial V_s(y_c^{*s})}{\partial y_c^s} \forall s \in \mathcal{S}$ .

If the pseudo gradient  $(\frac{\partial V_1(x_c)}{\partial x_c^1} \dots \frac{\partial V_S(x_c)}{\partial x_c^S})$  is diagonally strict concave (DSC) (see[20]), then  $\dot{\mathcal{L}}$  is negative. But in our case DSC doesn't hold in general. However it holds for some neighborhood around the Nash equilibrium  $x^*$ , therefore (21) is negative for some  $B_c = B_c'$  which is in neighborhood of  $\sum_{s'} x_c^s$ . In order to overcome this problem, we scale down the step length for Lagrange multiplier update rule by some sufficiently large constant  $K$  which makes (21) negative, thus rendering the dynamics asymptotically stable. The rest of proof follows from the Theorem 2 [24].  $\square$

## VI. NUMERICAL EXPERIMENTS

In this section we will provide numerical results to demonstrate the behaviour of the proposed mechanism. For the numerical experiment we considered a system with three slices  $\mathcal{S} = \{1, 2, 3\}$  and two base stations  $\mathcal{C} = \{1, 2\}$ . Tenants of slices 1, 2 and 3 have  $N_1^1 = 3$ ,  $N_1^2 = 5$  and  $N_1^3 = 2$  users, respectively, associated at base station 1. At base station 2 they have  $N_2^1 = 2$ ,  $N_2^2 = 4$  and  $N_2^3 = 6$  users, respectively. The available bandwidth at each base station is 30 MHz and we assume that the SNR of each user lies in the range between 30 and 75 dBs. Every slice uses some scheduling policy to assign the acquired bandwidth among its users: for the purpose of numerical illustration we assume that each slice is served using per-slice proportional fair scheduling.

The distributed learning Algorithm-1 is employed in order to determine the socially optimal Nash Equilibrium. Plots (a) and (b) in Fig. 3 show the converging dynamics of the bandwidth bids vector. As seen there, it stabilises at the target Nash equilibrium for both base stations 1 and 2. The distribution of bandwidth allocation at Nash equilibrium is shown in bar graph (g). As it can be clearly seen, the allocation of bandwidth at both base stations is consistent with the number of user per slices. In fact, at base station 2, slice 2 has more users compared to the other two slices; as expected, it attains hence a larger share of the available bandwidth. The target allocation has been achieved by using the pricing vector which is shown

in the plot (c) and (d) of Fig.3. In those graphs we observe the convergence of prices per slice and per base station. The prices charged by MNO for each slice are inversely proportional to number of the users. Finally, bar graphs (e) and (f) illustrate the throughput achieved per user under the resulting bandwidth allocation; the graphs indicate a mild throughput variation across the users within a slice, a result consistent with the use of PFS at slice level.

## VII. CONCLUSIONS

In this paper, we have considered a scenario where customers compete to obtain a slice of resource in 5G networks. We employ a mechanism based on a multi-bid Kelly mechanism, using as price vector the one resulting from the normalised Nash equilibrium which solves a dual game under coupled constraints. The solution of the game is obtained via an online learning mechanism which ultimately converges to the social optimum. The key technical challenge overcome by the proposed bidding mechanism is to account for the coupled constraints dictated by the available infrastructure resources. This renders the proposed one an interesting candidate mechanism for pricing slicing in 5G networks. In fact, to the best of the authors' knowledge, no suitable learning mechanism is known for Nash equilibria under the coupled resources constraints which are central in 5G resources slicing.

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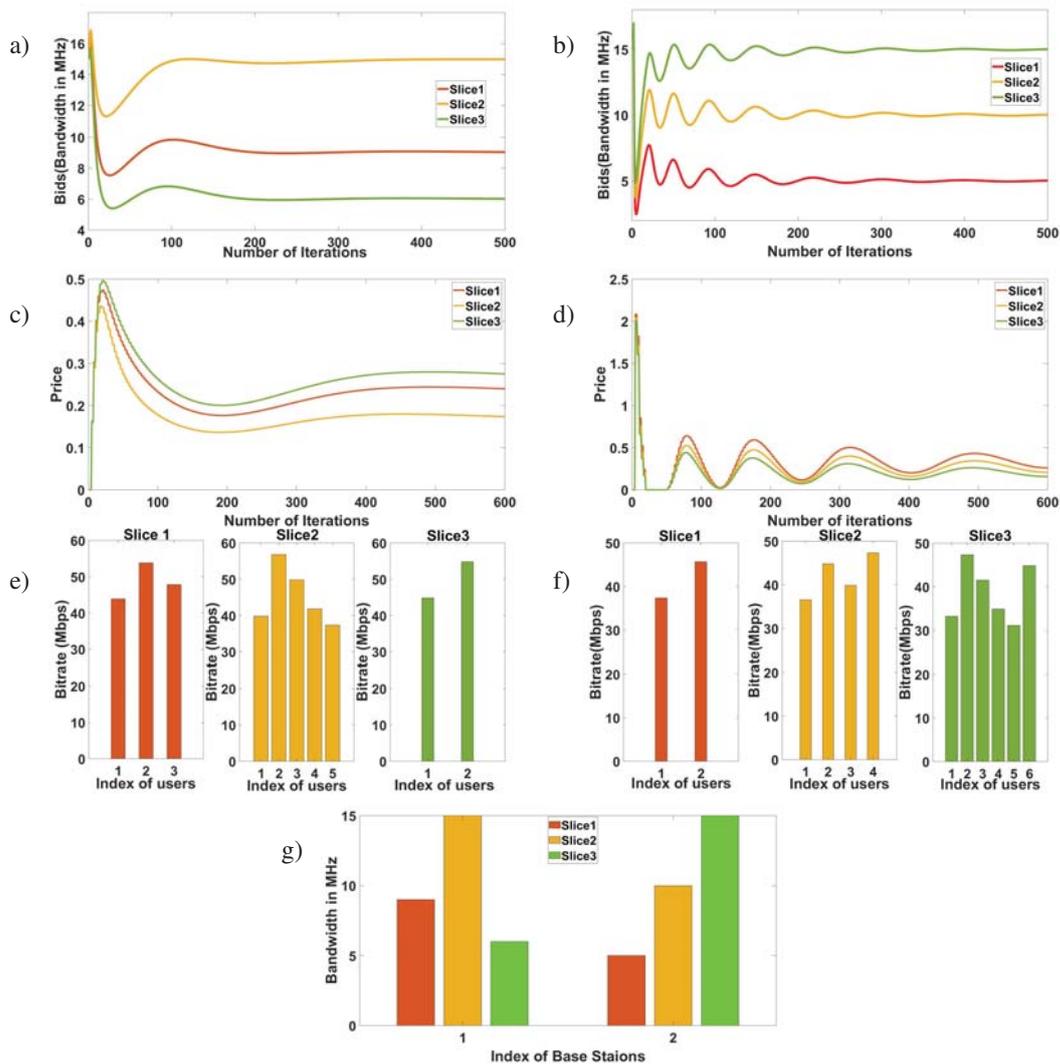


Fig. 3. (a and b) convergence of the Algorithm to the socially optimal Nash Equilibrium at base station 1 and at base station 2 (c and d); convergence of price vectors for base station 1 and base station 2; (e and f) throughput achieved by the users in base station 1, and base station 2; (g) allocation of bandwidth among the slices at base station 1 and 2.

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# Strategic Resource Management in 5G Network Slicing.

## (Invited paper)

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**Abstract**—Network Slicing is one of the essential concepts that has been introduced in 5G networks design to support demand expressed by next generation services. Network slicing will also bring new business opportunities for service providers (SPs) and virtual network operators, allowing them to run their virtual, independent business operations on shared physical infrastructure. We consider a marketplace where service providers (SPs) *i.e.*, slice tenants lease the resources from an infrastructure provider (InP) through a network slicing mechanism. They compete to offer a certain communication service to end-users. We show that the competition between SPs can be model using the multi-resource Tullock contest (TC) framework, where SPs exert effort by expending costly resource to attract users. We study the competition between the SPs under a static and dynamic resource sharing scheme. In a dynamic resource sharing scheme, SPs are pre-assigned with fixed shares (budgets) of infrastructure, and they are allowed to redistribute their shares and customise their allocation to maximise their profit. The decision problem of SPs is analysed using non-cooperative game theory, and it is shown that the resultant game admits a unique Nash Equilibrium (NE). Furthermore, a distributed reinforcement algorithm is proposed that allows each SP to reach the game's unique Nash equilibrium. Finally, simulations results are conducted to analyse the interaction between market players and the economic efficacy of the network sharing mechanism.

**Index Terms**—Communication service market, game theory, tullock contest, trading post mechanism, 5G network slicing, resource allocation.

### I. INTRODUCTION

Next-generation wireless network is expected to spread its applicability and deliver support to emerging sectors like Virtual Reality (VR) live broadcast, automotive, healthcare, manufacturing etc. Critical challenges in mobile network applicability to the sectors mentioned above are their heterogeneity and conflicting communications needs that the current monolithic network is insufficient to meet. Several new concepts have been proposed for the upcoming 5G network design to satisfy these critical needs. Out of those, probably one of the most important concepts in 5G network design is “network slicing”.

Network slicing is the concept of running multiple independent logical networks (slice) on top of the common shared physical infrastructure. Each independent logical network (slice) is then explicitly dedicated to meeting each slice tenant's needs, contrary to the approach “one-size-fits-all” witnessed until previous mobile generations [1].

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The implication of network slicing brings a paradigm shift towards a multitenancy ecosystem where multiple tenants owning individual slices negotiate with multiple InPs to request the resources for their service provision. In this scenario, the SPs or slice tenants generally express a demand for a dedicated isolated (that may need dedicated fixed resources allocation) and independent virtual network with full ownership of their service level agreement (SLA). On the contrary, InPs aim to maximize their return on investment by enabling the dynamic sharing of infrastructure, as this lowers the operational and capital costs and allows InPs to monetize their infrastructure to its fullest potential. However, the sharing of infrastructure may expose the tenants to the risk of violating their SLAs. Hence, one of the fundamental issues in network slicing is an efficient sharing of the network resources, which regulates the trade-off between two conflicting interests, *i.e.*, interslice isolation and efficient network resource utilization.

In order to balance the interslice isolation and efficient resource utilization, authors of [2] suggested the ‘share-constrained proportional allocation’ (SCPA) scheme where each slice is pre-assigned with a fixed share (budget) of infrastructure; slices are allowed to redistribute their shares and customize their allocation according to dynamic load. In turn, InP allocates each resource to slices in proportion to their shares on that resource. This approach allows a dynamic sharing, where tenants can redistribute their network share based on the dynamic load; at the same time, it provides the slice tenants degree of protection by keeping the pre-assigned share intact.

In the context of the above resource sharing mechanism, we consider a market scenario where a set of SPs lease their respective networks from InP and employ the network slicing mechanism to request the resources required for their service provision. We consider the SPs offer a particular service to users, and the resources inventory available with SPs characterizes their service performance. The users are free to choose their SP; their decisions are made based on the service satisfaction attained from SPs. Furthermore, we consider that the SP collects revenue by providing the service to its customer. Under the combined effect of a dynamic resource sharing mechanism and profit-oriented nature of SPs, it is highly expected that selfish SPs may exhibit strategic behaviour. For example, they might strategically distribute their shares on the resources conditioned on the tradeoff between quality of service (QoS) they want to offer and the

congestion perceived at the resources. Such selfish behaviour could hamper the market's economic efficiency or cause instability in the network slicing mechanism. In this work, we focus on (1) building a business model representing the communication service market where SPs negotiate with InP to request resources and compete to serve a pool of end-users. (2) with the help of the proposed model, analyzing the effect of the network slicing mechanism (*i.e.*, SPCA based dynamic resource sharing mechanism) in terms of the economic efficiency and stability of the network.

*Related work:* There is an enormous amount of related work on the communication service market, broadly, the communication service market has been studied as a two-level market where three types of participants: Infrastructure provider (InP), Service Provider (SP<sup>1</sup>) and (EU) End Users, are generally considered. In the first level market, SPs (buyers) leases the resources from the InPs (sellers), negotiating for resource prices and resource quantity. In the second level, SPs (buyers) utilize the acquired resources from InPs to offer a certain service to their end users (buyers). At this level, SPs decide on their service price and scheduling of resources, while EUs make their subscription decisions. In [3], SPs' strategic decision over their service pricing scheme has been analyzed as Cournot game. In [4], authors considered that the Qos achieved by the user from SP depends on the number of subscribers associated with that SP, and users' choice behaviour can be analyzed by evolutionary game theory (EGT). The authors in [5] integrated both the users' choice evolution and the SPs pricing scheme and analyzed it with the Stackelberg game approach. The SPs, the leaders, strategically decide the price to attract the users and the users the followers choose the SPs to maximize their service satisfaction level. Also, the number of subscribers associated with the SPs depends on QoS and consequently the resources available with them; bearing in mind the competition among the SPs, resource demand by SPs can be analyzed with the non-cooperative game [6]. In [7], authors considered that competition between SPs takes place in pricing and quality of service SPs offer. In practice, SPs may not have complete information about the other SPs resources. Keeping this in mind, authors of [8] studied SPs' pricing behaviour with the bayesian game, where SPs decide the pricing schemes based on their belief about the resources available with others. Furthermore, the authors also considered the possibility of cooperation between the SPs and analyzed its impact on the pricing scheme. In all the above work, the SPs lease the resources from the InP and compete to serve end-users, which is also the case in our work. However, our work departs from these works in that resources are shared using a slice-based dynamic sharing mechanism. Moreover, in our case, resources are spatially distributed, and service offered in a particular cellular cell can only be supported by the resources available within that cell. In communication networks, one of the well-known scheme for resource allocation is the auction-based allocation [9] *e.g.*, Kelly mechanism. Authors of [10], [11] proposed multi-bidding Kelly mechanism-based resource allocation for 5G slicing. They showed that mechanism leads

to a fair and efficient resource allocation on the level of both the slices and their end-users. Our work departs from the auction-based mechanism like [10]-[11], where agents' bids are unbounded.

In follow up work to [2], authors in [12] considered the network slicing under stochastic loads and applied SPCA based resource sharing scheme; they modeled resource sharing scheme as a game and showed that slices achieve efficient statistical multiplexing at the Nash equilibrium. The authors of [13] studied the communication service market where SPs employ the SPCA mechanism to request the resources from InP; they analyzed the economic impact of network slicing on the market. In [14], authors designed an automated negotiation mechanism based on the aggregate game that enables the slice tenants to dynamically trade the radio resources and customize their slices on instantaneous demands, which help tenants achieve higher profits. Our work is closely related to [13] however main novelty of our work lies in considering multi-resource service provisioning; at the best of our knowledge, the works related to the communication service market only deals with radio resource.

In this work, we leveraged the TC [15] framework to model the competition between the slices. This framework has been extensively used before in the communication field to model the interaction between competitive agents. To mention a few, in the paper, [16], the competition between social media users for visibility over the timeline has been model as a TC. The authors of [17] proposed the TC based incentive mechanism for crowdsourcing. The Tullock contest framework has been applied to the multipath TCP network utility maximization problem [18]. In the paper [19], authors studied the multi-cryptocurrency blockchain from a game-theoretic perspective, where the competition between the miners is framed as a TC.

*Main Contribution:* The key contributions of this work are the following 1) We proposed the business model for the service providers, where the SPs deploy the network slices for their business and leases their respective resources through network slicing mechanism (*i.e.*, dynamic sharing). The SPs compete to serve end-users in terms of QoS. 2) We model the competition between the SPs as a multi-resource Tullock contest. To the best of our knowledge, this is the first paper where the framework of the multi-resource TC is used. 3) We show that the game induced through competition between the SP *i.e.*, multi-resource Tullock rent-seeking game admits a unique Nash equilibrium (NE). Thus our theoretical results also contribute to the study of the tullock rent-seeking game. 4) We consider that the InP faces with challenge of deciding the resource pricing and we propose the trading post mechanism as its pricing solution 6) For some special case, we show that game induces by trading post mechanism admits unique Nash equilibrium. 7) We also provide the distributed reinforcement learning algorithms that provably converge to the game's unique NE.

The rest of the paper is organised as follows; Section II introduces the system model, Section III present the game-theoretic model of competition between the SPs. In Section IV, we study the existence and uniqueness properties of NE. Section V introduces resource pricing and market equilibrium; in section VI, we provide the distributed learning scheme. In

<sup>1</sup>In many works term Mobile virtual network operator (MVNO), tenant, slice, Mobile service provider (MSP) has been used for SP

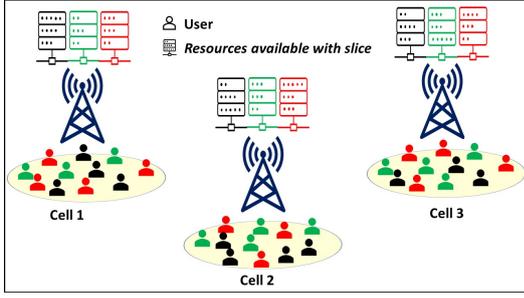


Fig. 1: service providers *i.e.*, (slices) compete to offer a certain service to geographically distributed pool of users

Section VII, we report on numerical results. A concluding section ends the paper.

TABLE I: Main notations used throughout the paper

$\mathcal{C} := \{1, \dots, C\}$	$\triangleq$	set of base stations or cells
$\mathcal{S} := \{1, \dots, S\}$	$\triangleq$	set of slices (tenants)
$\mathcal{M}^c$	$\triangleq$	set of resources at base station $c$
$N^c$	$\triangleq$	number of users in cell $c$
$n_s^c$	$\triangleq$	number (subscribers) users associated with slice $s$ in cell $c$
$d_s^c := (d_{sm}^c, \dots, d_{sM}^c)$	$\triangleq$	bundle of resources available with slice $s$ in cell $c$
$d_{sm}^c$	$\triangleq$	amount of resource type $m$ available with slice $s$ in cell $c$
$D_m^c$	$\triangleq$	the capacity of resource type $m$ at base station $c$
$q_s^c$	$\triangleq$	the quality of service of slice $s$ in cell $c$
$\omega_m^c$	$\triangleq$	the price per unit resource of type $m$ at base station $c$
$p_s$	$\triangleq$	service fees charge by slice $s$ to users
$B_s$	$\triangleq$	budget available with slice $s$

## II. SYSTEM MODEL

We consider a market scenario, where in the first stage, a set of SPs  $\mathcal{S}$  lease their respective networks from InP and employ the network slicing mechanism to request the resources required for their service provision. In stage two, the SPs (sellers) use the leased resources and compete to serve the large set of end-users (buyers). Specifically, as described in Figure 1, we consider InP owns a network that consists of a set of base stations or cells  $\mathcal{C}$ . Each base station at different locations accommodates multiple types of resources such as bandwidth, CPU, memory, etc. Users are spread across the network, let  $N^c$  number of users present in the cell  $c$  and service offered by SP in a particular cell can only be supported by the resources available within that cell.

### A. User Model

We assume all the users need the same type of service, and they achieve their demand by subscribing to one of the SPs. We consider each user is opportunistic and free to change its SP, *i.e.*, a slice from available slices at its base station. The user chooses a slice as its SP that offers a better deal, *i.e.*,

higher QoS at a lower price. We model the utility of each user served by SP  $s \in \mathcal{S}$  in cell  $c$  as [8]

$$U_s^c(n_s^c, q_s^c, p_s) = \log\left(\frac{q_s^c}{n_s^c}\right) - p_s \quad (1)$$

Where  $q_s^c$  is the quality of service of SP  $s$  in cell  $c$ ,  $n_s^c$  is number of users connected to SP  $s$  while  $p_s$  is the fees charged by SP for its service. Here the use of a logarithmic<sup>2</sup> function as the user's utility in QoS signifies that the users' satisfaction level saturates as the QoS increases, which is coherent with the economic principal of diminishing marginal returns. The SP's QoS depends on the resources inventory available with it. We assume each SP applies a scheduling policy to distribute its resources among users that achieve equal QoS among users in the long run. From (1) we observe that the utility of each user depends on the total number of users associated with the same SP, as the number of users connected to the same service increases the utility of the user decreases.

**Assumption 1.** *We assume that users revise their choice occasionally. As the users' selection process evolves, the market reaches equilibrium states where none of the users alters their SP choice, and the SPs provide equal utilities to operate with each other.*

This type of assumption is generally used in game theory while analyzing the strategic behaviour of a large number of selfish decision-makers, where for each decision-maker, exact information about all other decision-makers is rarely possible *e.g.*, Evolutionary game theory [20].

**Lemma 1.** *Under assumption 1, the number of users associated with each SP at equilibrium is given by*

$$n_s^c = \frac{N^c q_s^c e^{-p_s}}{\sum_{s' \in \mathcal{S}} q_{s'}^c e^{-p_{s'}}} \quad (2)$$

*Proof.* AppendixA □

### B. Service provider Model

We assume that the service providers aim at maximizing their number of subscribers ( $n_s^c$ ) by attracting users with better QoS and lower price. In (2), the number of users joining a particular SP depends on QoS and the price offered by that SP and QoS and price offered by other SPs. Notice that expression for the number of users associated with SP, in the long run, resembles a contest success function from well know Tullock contest framework [21]. The TC framework is commonly used in economics literature for modeling economic or social interactions between two or more competing agents. The basic contest framework consists of competing agents who expend costly resources to win a prize (a contest); given the efforts exerted by all agents, the probability of an agent winning a prize is defined by the contest success function (CSF). Typically, the CSF function is defined as  $\rho(x) = \frac{(x_i)^r}{\sum_{i'} (x_{i'})^r}$  where  $x_i$  is the effort of agent  $i$  and  $r$  is a parameter, for

<sup>2</sup>The logarithm function also signifies that the SPs achieve the proportional fair allocation between the user in the long run

example  $r = 1$  is the well know lottery and  $r \rightarrow \infty$  defines the all-pay auction.

In the slicing context, we consider that SPs compete to attract users to their services. SPs exert effort by expending costly resources; the resources acquired by SPs further reflect their service quality and help SPs to attract users. Thus, in our case, the contest success function represents the probability that any SP successfully attracts an end-user to its service. Keeping in mind the context of this work, we prefer to call contest success function as *slice association probability function*  $A_s$ , representing the probability that given resources expended by all SPs; a user will associate with a SP  $s$ . For our model, we defined a more general and multi-resource CSF function or slice association probability function

$$A_s^c(d^c, p) = \frac{f_s^c(d_s^c, p_s)}{\sum_{s' \in \mathcal{S}} f_{s'}^c(d_{s'}^c, p_{s'})} \quad (3)$$

Where function  $f_s^c(d_s^c, p_s)$  is concave non decreasing in  $d_s^c$  and convex and decreasing in  $p_s$ . We assume that the QoS provided by SP depends on the resources inventory available to slice and its relation is defined as  $q_s^c := q_s^c(d_s^c)$  where  $d_s^c = (d_{sm}^c, \dots, d_{sM}^c)$  denotes a bundle of resources available with SP  $s$  and element  $d_{sm}^c$  shows amount of resource type  $m$  acquired by SP  $s$  at cell  $c$ . We assume that  $\forall c \in \mathcal{C}$  and  $\forall s \in \mathcal{S}$  function  $q_s^c(d_s^c)$  is concave non decreasing in  $d_s^c$ , this type of assumption is widely use in economics signifying principle of diminishing marginal returns. In this work, we consider  $f_s^c(d_s^c, p_s)$  as  $q_s^c(d_s^c)e^{-p_s}$ . In (3) the number of potential users in each cell as well as the slice association probability for each slice, might vary from cell to cell. The expected number of users associated with SP  $s$  throughout the network is defined as.

$$\sum_{c \in \mathcal{C}} N^c A_s^c(d^c, p) = \sum_{c \in \mathcal{C}} \frac{N^c f_s^c(d_s^c, p_s)}{\sum_{s' \in \mathcal{S}} f_{s'}^c(d_{s'}^c, p_{s'})} \quad (4)$$

Each service provider collects the revenue from the fees paid by its subscribers. The expected revenue generated by SP  $s$  by its subscriber over the network is defined as.

$$R_s(d, p) = p_s \left( \sum_{c \in \mathcal{C}} N^c A_s^c(d^c, p) \right) \quad (5)$$

On the other hand, each SP needs to pay for the resources it leased from the InP. Let  $\omega_m^c$  be the price per unit resource of type  $m$  charge by InP at base station  $c$ . Thus total cost each SP  $s$  needs to pay for its resources is  $\sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}^c} \omega_m^c d_{s,m}^c$ . The profit gained by SPs is defined as

$$U_s(d, p) = p_s \left( \sum_{c \in \mathcal{C}} N^c A_s^c(d^c, p) \right) - \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}^c} \omega_m^c d_{s,m}^c \quad (6)$$

We assume that each SP  $s$  is pre-assigned with a finite budget  $B_s$ , which depends on its service level agreement (SLA) with the InP, and this budget represents the SP's priority or a fixed share of the available resources pool, such that  $\sum_{s \in \mathcal{S}} B_s = 1$ . We observe that the profit gain by the SPs depends not only on their own decisions but also on decisions made by other SPs; in such a scenario, SPs might exhibit strategic behaviour

and face the non-cooperative game.

### III. GAME MODEL

In this section, we model the interaction between the service providers as a non-cooperative game; we assume that the SPs are selfish, and each SP aims at maximizing its profit. We study the competition between the SPs in term of their quality of service, that is, how SPs strategically spend their budget on the resources to attract the users and, in turn, maximize their profits. The profit gain by the SPs depends on both their individual decision and the decision taken by their counterpart. The decision problem of each SP  $s$  is defined as.

$$Q_s \quad \underset{d_s \in \mathcal{B}_s}{\text{maximize}} \quad U_s(d_s, d_{-s})$$

We assume that the service providers are strategic while making a decision; they also take into account the decision of other SPs. To theoretically analyze this strategic interaction, we define the non-cooperative game  $\mathcal{G} := \langle \mathcal{S}, (\mathcal{B}_s)_{s \in \mathcal{S}}, (U_s)_{s \in \mathcal{S}} \rangle$  as follows:

- Player set: the set of service providers  $\mathcal{S}$
- Strategy: the vector of resource demand  $d_s = (d_s^1, \dots, d_s^C)$  where  $d_s^c$  is the amount of resource to be requested to the each base station  $c$ . The strategy set for each SP  $s$  is  $\mathcal{B}_s$
- Utility: The utility of each SP  $s$  is equal to the  $U_s$

To study the outcome of the defined game, we consider the standard notion of a Nash equilibrium,

**Definition 1.** A strategy profile  $d^* = (d_1^*, \dots, d_S^*)$  is called a NE of the game  $\mathcal{G}$  if

$$\forall s \in \mathcal{S}, U_s(d_s^*, d_{-s}^*) \geq U_s(d_s, d_{-s}^*), d_s \in \mathcal{B}_s \quad (7)$$

Here,  $(d_s, d_{-s}^*)$  denotes the strategy profile with  $s^{\text{th}}$  element equals  $d_s$  and all other elements equal  $d_{s'}^*$  (for any  $s' \neq s$ ).

In the next section, we analyze the existence and uniqueness of Nash equilibrium for the game  $\mathcal{G}$

### IV. EXISTENCE AND UNIQUENESS OF THE NASH EQUILIBRIUM

In this section, we establish the existence and the uniqueness of Nash equilibrium of game  $\mathcal{G}$ ; for the proof of the uniqueness of NE, we rely on the concept of diagonally strict concavity (DSC) introduced by Rosen [22]. Intuitively, DSC is a generalization of the idea of convexity to a setting of games.

**Definition 2** (Diagonal strict concavity [22]). A game with strategy vectors  $d$  and utility function  $U$  is called diagonally strict concave (DSC) for a given vector  $r$  if for every distinct  $\bar{d}$  and  $\hat{d}$ ,

$$\left[ g(\bar{d}, r) - g(\hat{d}, r) \right] (\bar{d} - \hat{d})' < 0 \quad (8)$$

with

$$g(d, r) = [r_1 \nabla_1 U_1(d), r_2 \nabla_2 U_2(d), \dots, r_S \nabla_S U_S(d)]. \quad (9)$$

where  $\nabla_s U_s(d)$  denotes the gradient of utility of player  $s$  with respect its won strategy  $d_s$

**Theorem 1.** *The game  $\mathcal{G}$  always admits a unique NE.*

*Proof.* The utility of each SP in-game  $\mathcal{G}$  is continuous, increasing, and concave, while the action space for each SP is convex and compact. Therefore the existence of an equilibrium for the game is followed by (Theorem 1 [22]). Now for the uniqueness of Nash equilibrium, If the utilities of players in the game  $\mathcal{G}$  satisfies the DSC property, then the uniqueness of NE to game  $\mathcal{G}$  follows by (Theorem 2 [22])

Let  $G(d, r)$  be the Jacobian of  $g(d, r)$  with respect to  $d$ , where  $d$  is any multistrategy of the game. In order to prove strict diagonal concavity of  $g(d, r)$ , by (Theorem 6 [22]), it is sufficient to prove that the symmetrized version of the pseudo-jacobian, i.e.,  $\widehat{G}(d, r) = G(d, r) + G(d, r)'$ , is negative definite for all the domain of interest. To show that the  $\widehat{G}(d, r)$  is negative definite it must be shown that following three conditions are satisfied:

**C 1.** *each  $U_s(d)$  is a regular strictly concave function of  $d_s$  (i.e., its Hessian is negative definite)*

**C 2.** *each  $U_s(d)$  is convex in  $d_{-s}$*

**C 3.** *there is some  $r > 0$  such that function  $\sigma(d, r) = \sum_s r_s U_s(d)$  is concave in  $d$*

then negative definiteness of  $[G(d, r) + G'(d, r)]$  follows from Lemma 1 [23]. We first consider a case of single base station  $c$  and show that  $\widehat{G}^c(d, r)$  is negative definite for this case. We calculate the Hessian ( $H_s U_s^c$ ) of utility of any SP  $s$  with respect to SP  $s$  owns strategy.

$$H_s U_s^c = -2 \frac{p_s \sum_{s' \in \mathcal{S}, s' \neq s} f_{s'}^c}{\left( \sum_{s' \in \mathcal{S}} f_{s'}^c \right)^3} \left[ (\nabla_s f_s^c)^T \nabla f_s^c - H_s(f_s^c) \sum_{s' \in \mathcal{S}} f_{s'}^c \right] \quad (10)$$

on the right hand side of (10) matrix  $(\nabla_s f_s^c)^T \nabla f_s^c$  is positive semi-definite, where  $\nabla_s f_s^c$  is gradient row vector of  $f_s^c$  with respect to its own strategy  $d_s^c$ .  $H_s(f_s^c)$  is the Hessian of  $f_s^c$  with respect to  $d_s^c$  and its negative definite as  $f_s^c$  is concave. Thus the Hessian of utility  $H_s U_s^c$  is negative definite and satisfies the first condition **C1**. Now we will show that the utility of each SP  $s$  is convex in the strategy of all other SPs, for that purpose consider the Hessian of utility of SP  $s$  with respect to strategy of all other SPs

$$H_{-s} U_s = 2 \frac{f_s^c}{\left( \sum_{s' \in \mathcal{S}} f_{s'}^c \right)^3} [M_s^c - \text{diag}_{-s} \{H(f_u^c)\}] \quad (11)$$

where is  $M_s^c$  block matrix and  $uv^{th}$  block is defined as

$$M_{suv}^c = (\nabla_u f_u^c)^T \nabla_v f_v^c \text{ where } u, v \neq s, u, v, s \in \mathcal{S} \quad (12)$$

$\nabla_u f_u^c$  is gradient row vector of  $f_u^c$  with respect to its own strategy and  $\text{diag}_{-s} \{H(f_u^c)\}$  is block diagonal matrix with block  $u$  is  $H(f_u^c)$  hessian of  $f_u^c$  with respect strategy vector of  $u$  itself  $\forall u, u \neq s, u \in \mathcal{S}$ . In right hand side of equation (11) matrix  $M_s^c$  is positive definite and the block diagonal matrix  $\text{diag}_{-s} \{H(f_u^c)\}$  is negative definite as the each diagonal

matrix element  $H(f_u^c)$  is negative definite thus  $H_{-s} U_s$  is positive definite, which satisfies the condition **C2**. Now we take the  $r_s = \frac{1}{p_s} \forall s \in \mathcal{S}$  and then  $\sigma(d, r) = \sum_s r_s U_s(d)$  is concave in  $d$

Now we will extend the proof for multi-base station case; we have already shown ( $\widehat{G}^c$ ) is negative definite for any single base station  $c$ . For  $C$  base stations, consider a  $\widehat{G}$  symmetrized version of the pseudo-Jacobian; after arranging columns and rows, we get (see Corollary 2 in [24])

$$(\widehat{G}) = \text{diag} \left\{ \widehat{G}^1, \dots, \widehat{G}^c, \dots, \widehat{G}^C \right\}$$

The above  $\widehat{G}$  matrix is negative definite as each diagonal matrix is negative definite, which proves the DSC property holds for the multi-cell scenario. Then by Theorem 2 [22] the equilibrium point  $d^*$  for the game  $\mathcal{G}$  is unique.  $\square$

## V. RESOURCE PRICING AND EQUILIBRIUM

We have shown in the previous section that there exists a unique NE to game  $\mathcal{G}$ . We assume that the physical resources available with the InP in each cell are finite. Given per-unit prices for resources decided by the infrastructure provider, the total resource demanded by SPs at NE of game  $\mathcal{G}$  may violate the Infrastructure capacity. Thus, InP's primary concern is how to efficiently allocate the limited physical resources to competing SPs with diverse characteristics and preferences. The desired allocation must satisfy all the SPs and simultaneously maintain high resource utilization. In this regard, we assume that InP seeks the pricing scheme (per-unit prices) for each resource such that at the Nash equilibrium of game  $\mathcal{G}$  each SP utilizes its entire budget and no resources remain leftover *i.e.*, the total demand of resources matches the available infrastructure capacity. In economics, such a pricing decision problem has been often studied as a market equilibrium problem *e.g.* Fisher market [25]; market equilibrium is a solution concept where market prices are settled in such a way that the amount of resources requested by buyers is equal to the amount of resources produced or supplied by sellers.

One way to find market equilibrium or pricing scheme is through a tatonnement process, *i.e.*, if the demand for resources exceeds its capacity, increase the resource's price. Contrarily decreases resource's price when the demand is smaller than the capacity. The disadvantage of the above approach is that it does not always guarantee the ability to satisfy the resource capacity while applying such a process. To overcome this limitation, we use the approach introduced by Shapley and Shubik in their pioneer work [26], also known by the various names like Trading Post, share-constrained proportional allocation (SCPA) scheme [2]. Now we formally define the Trading post mechanism.

### A. Trading post mechanism

In the trading post mechanism, each player (*i.e.*, SP) places a bid on each type of resource. Once all SPs place the bids, each resource type's price is determined by the total bids placed for that resource. Precisely, let SP  $s$  submits a bid  $b_{sm}^c$  to resource  $m$  at cell  $c$ . The price per unit of resource  $m$  at cell  $c$  is then

set to  $\frac{\sum_{i \in \mathcal{N}} b_{sm}^c}{D_m^c}$ , accordingly SP  $s$  receives a fraction of  $d_{sm}^c$  in return to his spending of  $b_{sm}^c$ .

$$d_{sm}^c = \begin{cases} \frac{b_{sm}^c D_m^c}{\sum_{u \in \mathcal{S}} b_{um}^c} & \text{if } b_{sm}^c > 0 \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

After replacing  $d_{sm}^c$  in (6) in terms of bids and the decision problem of each SP  $s$  is written as below.

$$\begin{aligned} \hat{Q}_s & \quad \underset{b_s}{\text{maximize}} && U_s(b_s, b_{-s}) \\ & \quad \text{subject to} && \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}^c} b_{s,m}^c \leq B_s, b_{s,m}^c \geq 0. \end{aligned}$$

Here we consider two possible nature of the service providers; first, they are price takers. *i.e.*, they accept the price decided by the market, and they only act strategically in terms of demand for the resources. Second, SPs are price anticipating; they expect the effect of their demand on the price of the resources. Hence they act strategically in term of resource and the congestion on the resources. When SP are strategic in both, the trading post mechanism induces a new non-cooperative game. We define the non-cooperative game  $\hat{\mathcal{G}}$  as follows:

- Player set: the set of SPs  $\mathcal{S}$
- Strategy: the vector of bids  $b_s = [b_s^1, \dots, b_s^C]$  where  $b_s^c$  is the bid to be submitted to the resource cell  $c$ . The strategy set for each SP  $s$  is  $\mathcal{B}_s = \{b_s | \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}^c} b_{s,m}^c = B_s, \}$
- Utility: The utility of each SP  $s$  is equal to the  $U_s$

To study the outcome of the mechanism, we consider the standard notion of NE,

**Definition 3.** A multi-bid strategy  $b^* = (b_1^*, \dots, b_S^*)$  is called a NE of the game  $\hat{\mathcal{G}}$  if

$$\forall i \in \mathcal{N}, U_s(b_s^*, b_{-s}^*) \geq U_s(b_s, b_{-s}^*), b_s \in \mathcal{B}_s \quad (14)$$

Here,  $(b_s, b_{-s}^*)$  denotes the strategy vector with  $s^{\text{th}}$  element equals  $b$  and all other elements equal  $b_v^*$  (for any  $v \neq s$ ).

For the proposed mechanism, interpretation of NE of game  $\hat{\mathcal{G}}$  constitutes a stable bidding policy where each SP is satisfied with its individual utility characteristics and the existing resource allocation mechanism. Now, we investigate the existence and uniqueness properties of NE; showing the uniqueness and existence of multi-resource  $\hat{\mathcal{G}}$  game requires complex calculations; thus, we keep our theoretical analysis of game  $\hat{\mathcal{G}}$  limited to a single resource (radio resource). We assume that the QoS provided by SP  $s$  in cell  $c$  is given by  $q_s^c = (d_s^c)^{\rho_s^c}$  where  $\rho_s^c$  is a sensitivity parameter and  $0 < \rho_s^c \leq 1$ , such type of function has been used in [13] to model the effect of users sensity towards their service provider selection. We replace  $q_s^c = (d_s^c)^{\rho_s^c}$  in (1) and from (3) we get

$$A_s^c(d^c, p) = \frac{(d_s^c)^{\rho_s^c} e^{-p_s}}{\sum_{s' \in \mathcal{S}} (d_{s'}^c)^{\rho_{s'}^c} e^{-p_{s'}}} \quad (15)$$

**Proposition 1.** If for single resource case, the QoS provided by SP  $s$  in cell  $c$  is defined by  $q_s^c = (d_s^c)^{\rho_s^c}$  and  $0 < \rho_s^c \leq 1$  then game  $\hat{\mathcal{G}}$  admits unique NE.

*Proof.* If the the QoS provided by SP  $s$  in cell  $c$  is defined by

$q_s^c = (d_s^c)^{\rho_s^c}$  and  $0 < \rho_s^c \leq 1$  then utilities of SPs satisfies the conditions **C1, C2** and **C3**, rest of proof is same as the proof of theorem 1.  $\square$

Moving ahead, now we compare the profit gain by service providers at the Nash equilibrium of the game with baseline static proportional allocation scheme (SS) *i.e.* allocation where each resource is allocated to a service provider  $s$  in proportion to its budget  $\frac{B_s}{\sum_{s' \in \mathcal{S}} B_{s'}}$

**Proposition 2.** For two service providers, the revenue gain under a dynamic resource sharing scheme at least equal to the revenue gain under a proportional sharing scheme

*Proof.* Appendix C  $\square$

In the next section, we provide the distributed learning algorithm, which provable converge to both  $\mathcal{G}$  and  $\hat{\mathcal{G}}$  games' unique Nash equilibrium.

## VI. DISTRIBUTED LEARNING ALGORITHM

We have already proved in the previous section that the Game  $\mathcal{G}$  admits a unique equilibrium for any price vector decided by the Infrastructure provider. However, we still need to verify whether tenants can reach this equilibrium in a distributed fashion. In this regard, we propose an exponential learning algorithm that allows the tenants to converge to the game's unique NE. The proposed learning algorithm is a special case of dual averaging or mirror-descent method suggested for continuous action convex games [27]. Now, we proceed by describing the dual averaging method; in the dual averaging method, each player *i.e.*, SP  $s$  estimates its marginal utility or utility gradient with respect to its own strategy. To increase their utilities, players need to take action along the direction of their utility gradient while maintaining their action in feasible action space. In order to achieve this, each player  $s$  at each time step  $n$  accumulates its discounted utility gradient in some auxiliary variable  $y_s$ ,

$$y_s(n+1) = [y_s(n) + \alpha_n \nabla_{b_s} U_s(b_s(n), b_{-s}(n))]. \quad (A1)$$

In the above equation  $\alpha_n$  denotes the discount factor or step size. Once the discounted gradient has been accumulated, every SP  $s$  utilize its own updated value of the auxiliary variable  $y_s$  to take the next feasible action.

$$b_s(n+1) = Q_s(y_s). \quad (16)$$

In turn, each SP  $s$  maps the recent value of auxiliary variable  $y_s$  to its decision space  $\mathcal{B}_s$  using the some mapping  $Q_s(y_s)$ , *e.g.*,  $Q_s$  can be projection map. The map  $Q_s(y_s)$  is defined in more general as

$$Q_s(y_s) = \underset{b_s \in \mathcal{B}_s}{\text{argmax}} \{ \langle y_s(n), b_s \rangle - h_s(b_s) \}, \quad (A2)$$

where  $h_s(b)$  is regularization function or a penalty function over the feasible action set  $\mathcal{B}_s$ . Here penalty  $h_s(b)$  helps the convergence of algorithm within the interior of the feasible domain set. The different value regularization functions induce

different maps. We propose using the Gibbs entropy function as a regularization function

$$h_s(b_s) = \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}} b_{sm}^c \log(b_{sm}^c). \quad (17)$$

We replace  $h_s(b_s)$  in equation (A2) by the entropic regularization function and after some calculation we get exponential mapping

$$b_{sm}^c = \frac{B_s \exp(y_{sm}^c)}{\sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{M}} \exp(y_{sk}^c)}. \quad (18)$$

The induced map  $Q_s(y_s)$  is similar to well know Logit map, where each player distributes his budget (weights) to different resources depending on exponential of accumulated discounted gradients.

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**Algorithm 1** On-line Distributed Learning Algorithm

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**Require:**  $\sum_{n=0}^{\infty} \alpha_n = \infty, \alpha_n \rightarrow 0$  as  $n \rightarrow \infty$

- 1: **repeat**  $n = 1, 2, \dots,$
- 2:   **for each** SP  $s \in \mathcal{S}$
- 3:     Observe gradient of utility and update
- 4:      $y_s = [y_s + \alpha_n \nabla_{b_s} U_s(b_s, b_{-s})]$
- 5:   **end for**
- 6:   **for each** player  $s \in \mathcal{S}$
- 7:     **for each** cell  $x \in \mathcal{C}$  and resource  $m \in \mathcal{M}^c$
- 8:       Play  $b_{sm}^c \leftarrow \frac{B_s \exp(y_{sm}^c)}{\sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{M}} \exp(y_{sk}^c)}$ .
- 9:     **end for**
- 10:  **end for**
- 11: **until**  $\|(b(n) - b(n-1))\| \leq \epsilon$

---

**Theorem 2.** *If the Algorithm 1 satisfies the required conditions for step size sequence,  $\sum_{n=0}^{\infty} \alpha_n = \infty, \alpha_n \rightarrow 0$  as  $n \rightarrow \infty$  then distributed Algorithm 1 converges to the unique NE of the Game  $\mathcal{G}$*

*Proof.* As we have already discussed, the proposed exponential algorithm is the special case of the dual averaging algorithm. If the NE of the any continuous action convex game is strictly r-variationally stable, then the converges of the dual averaging algorithm to a unique NE of the game is guaranteed by Theorem 4.6 [27]. Hence to prove the convergence of the proposed algorithm, it is sufficient to show that the unique NE of game  $\mathcal{G}$  is strictly r-variationally stable. The unique NE  $\hat{b}$  to the any convex game is strictly r-variationally stable if  $\forall b_s \in \mathcal{B}_s$

$$\sum_{s \in \mathcal{S}} r_s \nabla_s U_s(b)(b_s - \hat{b}_s) < 0 \quad (19)$$

As we have already shown in section IV that utility of SPs in game  $\mathcal{G}$  satisfies the diagonal strict concavity for  $r_s = \frac{1}{p_s}, \forall s \in \mathcal{S}$

$$\sum_{s \in \mathcal{S}} r_s [\nabla_s U_s(b) - \nabla_s U_s(\hat{b})] (b_s - \hat{b}_s) < 0 \quad (20)$$

Now we know that for any continuous action convex game, a feasible point  $\hat{b}$  is a Nash equilibrium of the game if and only if

$$\sum_{s \in \mathcal{S}} r_s \nabla_s U_s(\hat{b})(b_s - \hat{b}_s) \leq 0 \quad (21)$$

From inequality (21) and (20) implies (19), which proves that the unique NE of game  $\mathcal{G}$  is strictly r-variationally stable and then by Theorem 4.6 [27] Algorithm1 converges to unique NE of game  $\mathcal{G}$   $\square$

## VII. NUMERICAL EXPERIMENTS

In this section, we illustrate an analysis of the dynamic resource allocation scheme with the support of numerical results. Our simulation primarily focuses on a network with two cells, CI and CII, and two service providers SP1 and SP2, who request the resources for their service provision. This setting allows us to efficiently study the dynamics of interaction between users and SPs and the effect of different system parameters on the outcome of the game  $\mathcal{G}$ . We assume there are 200 and 300 users present in the cell CI and CII, respectively. First, we consider the simple case of a single resource where the quality of service offered by the slices only depends on the radio resource (bandwidth). The plot in Figure 2 (c) illustrates the impact of the price parameter on the number of users associated with the slices at the NE of  $\mathcal{G}$ . For this simulation, we assume that the price applied by the SP1 is constant 5, and we vary the fee applied by SP2 in the range of 0 to 10. Figure 2(c) shows the change in the distribution of users associated with the SPs as a function of price applied by SP1. In the same figure, we also analyze the effect of slices shares on the distribution of users at the outcome of the game. The regular lines in red and blue show the distribution of users with SP1 and SP2 as a function of price provided by SP1 and when SP1 and SP2 are assigned with 10% share and 90% share of the infrastructure, respectively. The plots with the dashed line, dot line and dot-dash line are the outcome when 30%, 70% and 90% of share are assigned to SP1. With the same settings, the simulations in Figure 2(d) illustrate the impact of price applied by the slices and their infrastructure share on the revenue gain by them. From Figure 2(c) we can observe that the SPs' subscribers decrease with their offered price, while the rate in the fall in the subscriber's is reducing in their budgets. The Figure 2(d) shows that the revenue gain of SPs is increasing in their budgets. As second case, we consider QoS provided by SP  $s$  in cell  $c$  is given by  $q_s^c = (d_s^c)^{\rho_s^c}$  where  $\rho_s^c$  is sensitivity parameter and  $0 < \rho_s^c \leq 1$ , we vary the  $\rho_2^c$  i.e. the sensitivity parameter for SP2 in cell C2 form 0.1 to 1, the Figure 2(b) shows the comparison of profit gain by SPs at Nash equilibrium with the profit gain under static resource allocation scheme. For the multi-resource case, we consider that the quality of service provided by the SPs depends on their bandwidth as well as power allocation. To be precise, we assume that the QoS is the maximum possible data rate that SP can achieve, given by

$$q_s = B_s \log_2 \left( 1 + \frac{h^2 P_s}{N_0} \right) \quad (22)$$

Where  $B_s$  and  $P_s$  is bandwidth and power allocated to SP  $s$  respectively, while  $h$  is channel gain and  $N_0$  noise, For simulations purpose, we assume that the availability of maximum bandwidth and transmitting power at each base station is 30Mhz and 47dBm, respectively. The prices applied

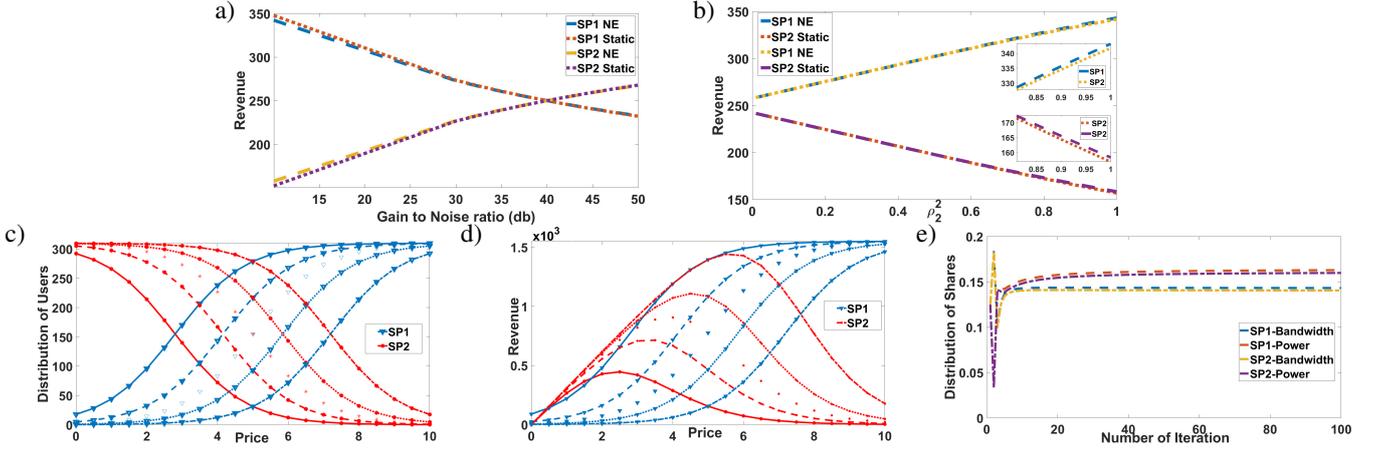


Fig. 2: a) Comparison between the revenue gain by the SPs at NE of game vs the revenue gain under SS for the different value of power to noise ratio of SP2 at C2. b) Comparison between the revenue gain by the SPs at NE of game vs the revenue gain under SS for the different value of the sensitivity parameter  $\rho_2^2$ . c) The distribution of users at NE wrt fees charged by SPs d) The revenue gain by SPs wrt fees charged by them e) Converges of the distributed algorithm 1 to NE

by each SP is constant 1, and each SP is assigned with half of the infrastructure share. For the numerical experiments, we vary the channel gain to noise ratio for SP2 at cell C2 from 10db to 50db; for each value of the channel gain to noise ratio, we compute the Nash equilibrium. The Figure 2(a) shows a comparison between the profit gain by SPs at Nash equilibrium with profit gain by SPs under a static resource allocation scheme. Figures 2(a) and 2(b) show a negligible difference between the revenue gain under the dynamic resource sharing scheme and static resource allocation and dynamic resource sharing scheme. It allows service providers with efficient resource sharing while keeping the revenue of SPs coherent with their SLA.

## VIII. CONCLUSION

In this work, we have considered a communication market scenario where service providers lease resource from infrastructure provider through a network slicing mechanism (SPCA) and compete to serve a large pool of end-users. We have modeled the competition between the service provider as the multi resource tulkock rent-seeking game. We have proved that the resultant game admits a unique Nash equilibrium. We have considered that InP faces the challenge of finding the pricing scheme (per-unit prices) for each resource such that at the Nash equilibrium of the game, total demand satisfies the capacity of the infrastructure. In this regard, we have proposed the trading post-mechanism-based resource allocation. For some limited cases, we have shown that game induced by the trading post mechanism admits a unique Nash equilibrium; thus, resource allocation through the slicing mechanism is provably stable. We have provided the distributed exponential learning algorithm, which allows service providers to reach the unique Nash equilibrium of the game. Our numerical results confirm that under SPCA, the network slicing mechanism enables service providers with stable and economically efficient resource utilization. In the future, we will consider different

types of SLA models for SPs and develop a general resource sharing and resource pricing scheme based on the concept of normalized Nash equilibrium and coupled constrained game [22][15].

## APPENDIX

### A. Proof Of Lemma 1

To find the equilibrium of above dynamics consider

$$\log\left(\frac{q_s^c}{n_s^c}\right) - p_s = \log\left(\frac{q_{s'}^c}{n_{s'}^c}\right) - p_{s'} \quad (23)$$

taking exponential of both sides

$$\frac{q_s^c}{n_s^c} \frac{n_{s'}^c}{q_{s'}^c} = e^{p_s - p_{s'}} \quad (24)$$

$$\frac{q_s^c}{n_s^c} n_{s'}^c = q_{s'}^c e^{p_s - p_{s'}} \quad (25)$$

summing over  $\forall s' \in \mathcal{S}$

$$\sum_{s'} \frac{q_s^c}{n_s^c} n_{s'}^c = \sum_{s'} q_{s'}^c e^{p_s - p_{s'}} \quad (26)$$

$$n_s^c = \frac{N^c q_s^c e^{-p_s}}{\sum_{s'} q_{s'}^c e^{-p_{s'}}} \quad (27)$$

### B. Proof of Proposition 1

$$\frac{\partial^2 U_s}{\partial d_1^2} = \frac{A+B}{C} < 0$$

$$A = - \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{\rho_2} \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{2\rho_1} \left( (\rho_2^2 + \rho_2) b_1^{c^2} + 2\rho_1 b_2^c (\rho_2 + 1) b_1^c + \rho_1 b_2^{c^2} (\rho_1 + 1) \right) \quad (28a)$$

$$B = \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{2\rho_2} \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{\rho_1} \left( (\rho_2^2 - \rho_2) b_1^{c2} + 2\rho_1 d_2^c (\rho_2 - 1) b_1^c + \rho_1 b_2^{c2} (\rho_1 - 1) \right) \quad (28b)$$

$$C = (b_1^c + b_2^c)^2 b_1^{c2} \left( \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{\rho_1} + \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{\rho_2} \right)^3 \quad (28c)$$

$$\frac{\partial^2 U_s}{\partial d_1^2} = \frac{G+H}{I} > 0$$

$$G = - \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{\rho_2} \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{2\rho_1} \left( (\rho_1^2 - \rho_1) b_2^{c2} + 2\rho_2 b_1^c (\rho_1 - 1) b_2^c + \rho_2 b_1^{c2} (\rho_2 - 1) \right) \quad (29a)$$

$$H = \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{2\rho_2} \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{\rho_1} \left( (\rho_1^2 + \rho_1) b_2^{c2} + 2\rho_2 b_1^c (\rho_1 + 1) b_2^c + \rho_2 b_1^{c2} (\rho_2 + 1) \right) \quad (29b)$$

$$I = (b_1^c + b_2^c)^2 b_2^{c2} \left( \left( \frac{b_1^c}{b_1^c + b_2^c} \right)^{\rho_1} + \left( \frac{b_2^c}{b_1^c + b_2^c} \right)^{\rho_2} \right)^3 \quad (29c)$$

### C. Proof of Proposition 2

Consider that for any bid  $b_2^c > 0$  submitted by SP 2 at cell  $c$  SP 1 place a bid of  $b_1^c = B_1 \frac{b_2^c}{B_2}$  at cell  $c$  then quantity of

resource received by SP1 at cell  $c$   $d_1^c = \frac{B_1 \frac{b_2^c}{B_2}}{B_1 \frac{b_2^c}{B_2} + b_2^c} = \frac{B_1}{B_1 + B_2}$

this proves that for any strategy played by service provider there exist strategy for opponent SP such that it receives the resources in proportion to its budget

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# Normalized Equilibrium in Tullock Rent Seeking Game

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**Abstract.** Games with Common Coupled Constraints represent many real-life situations. In these games, if one player fails to satisfy its constraints common to other players, then the other players are also penalized. Therefore these games can be viewed as being cooperative in goals related to meeting the common constraints, and non-cooperative in terms of the utilities. We study in this paper the Tullock rent-seeking game with additional common coupled constraints. We have succeeded in showing that the utilities satisfy the property of diagonal strict concavity (DSC), which can be viewed as an extension of concavity to a game setting. It not only guarantees the uniqueness of the Nash equilibrium but also of the normalized equilibrium.

**Keywords:** Normalized equilibrium · Common Coupled Constraints · Diagonal strict concavity

## 1 Introduction

Games with constraints have long been used for modeling and studying non-cooperative behavior in various areas. This includes road traffic [7,12] and telecommunications [9]. Various types of constraints may appear in everyday game situations; the simplest ones consisting of orthogonal constraints, where the strategies of the players are restricted independently of each other [15]. The second type of constraints are called Common Coupled Constraints (CCC) [3,14,15] in which all players have a common convex non-orthogonal multi-strategy space. This model can be viewed as constraints that are common to all users. A unilateral deviation of a player from some feasible multi-strategy (one that satisfies the constraints) to another strategy that is feasible for that player, does not result, therefore, in the violation of constraints of other users. CCC has often been used in telecommunications networking problems as well as in power transfer over a smart grid, where capacity constraints of links are naturally common. Games

with this type of constraints are a special case of General Constrained Games (GCG) [6], see also [3–5, 8, 10, 16].

In this paper, we study the well known Tullock rent-seeking game with Common Coupled Constraints. This game describes contest over resources. Each player bids an amount that she is ready to pay. She then pays an amount proportional to her bid and receives, in turn, a payoff that is proportional to her bid divided by the sum of bids of all players.

The presence of a capacity constraint results in infinitely many equilibria and we are faced with a question of equilibrium selection. Using Kuhn Tucker conditions to the best response, we can solve a relaxed game instead of the original constrained game, which has, however, the same equilibria as the original game. The Lagrange multipliers can be interpreted as a shadow cost that a manager sets in order to guarantee that the equilibrium achieved satisfies the constraints. This approach may, however, be completely unscalable since KKT Theorem does not guarantee that the price per resource unit is the same for all players. In fact, since the Lagrange multipliers are obtained for the best response function, they could depend not only on the player but also on the policy of all other players, rendering the approach even less scalable. We are interested in finding such shadow cost which is fixed per resource unit. Such an equilibrium along with a fixed shadow price is called a normalized equilibrium.

The Tullock rent-seeking game has been used recently to model and study several game phenomena in networking. It was used to model contests over timelines in social networks for maximizing visibility [17]. Each player  $i$  controls the rate  $\lambda_i a_i$  of a Poisson process of posts that player  $i$  sends into a common timeline of length  $K$ . This rate is given by a basic popularity rate  $\lambda_i$  times the acceleration effort (e.g. through advertisement) given by  $a_i$ . Using basic queueing theory, the authors show that the stationary expected number of posts in the timeline originating from player  $i$  is given by

$$K \frac{\lambda_i a_i}{\sum_{j=1}^N \lambda_j a_j}$$

This visibility measure is the payoff in Tullock’s model, while the cost for acceleration at a rate  $a_i$  is proportional to  $a_i$  as in Tullock’s model.

Another application of the Tullock rent-seeking game is in the study of contests between miners in blockchain [2].

A few words on rent-seeking. According to Wikipedia, “In public choice theory and in economics, rent-seeking involves seeking to increase one’s share of existing wealth without creating new wealth. Rent-seeking results in reduced economic efficiency through the poor allocation of resources, reduced actual wealth-creation, lost government revenue, increased income inequality, and (potentially) national decline.”

Wikipedia further describes the origin of the idea: “The idea of rent-seeking was developed by Gordon Tullock in 1967, while the expression rent-seeking itself was coined in 1974 by Krueger [11]. The word “rent” does not refer specifically to payment on a lease but rather to Adam Smith’s division of incomes into profit,

wage, and rent. The origin of the term refers to gaining control of land or other natural resources.”

Our first contribution is to show that the utilities satisfy a property that extends concavity to games, and is called Diagonally Strict Concavity. This is shown to imply the existence and uniqueness of a normalized equilibrium. We shall then show that this property further extends to the case of contests over several resources.

## 2 A Single Resource

Consider an  $N$  players game. Player  $m$  bids a quantity  $x^m$ . We have minimum constraints  $x^m \geq \epsilon$  for all  $m$ .

The payoff from this contest to player  $m$  is

$$P^m = \frac{x^m}{\sum_{j=1}^M x^j}.$$

This comes at a cost of  $x^m \gamma$  to player  $m$  where  $\gamma$  is a constant. The utility for player  $m$  is thus

$$U^m(x) = \frac{x^m}{\sum_{j=1}^M x^j} - x^m \gamma.$$

**Theorem 1.** (i) *The utility of player  $m$  is concave in its action and is continuous in the actions of other players.*

(ii) *For any strictly positive value of  $\gamma$ , the above game has a unique Nash equilibrium in pure policies.*

**Proof.** Direct calculation leads to (i). The existence then directly follows from [15]. Uniqueness is established in [1], see also [18]. Other related uniqueness results in the asymmetric case can be found in [17,19].

## 3 Normalized Equilibrium

The games we have seen so far involved orthogonal constraints. By that, we mean that the actions that a player can use do not depend on the actions of other players. We next introduce capacity constraint. We require that the following holds for some constant  $V$ :

$$\sum_{j=1}^M x^j \leq V \tag{1}$$

Capacity constraints may represent physical bounds on resources, such as bounded power, or resources that are bounded by regulation. For example, legislation may impose bounds on the power used or on the emission of CO<sub>2</sub> by cars. With the additional capacity constraint, the Nash equilibrium is no more unique and there may, in fact, be an infinite number of equilibria. We call this the game with capacity constraint.

Let  $y$  be an equilibrium in the above game and let  $y_{[-m]}$  denote the action vectors of all players other than  $m$ . By KKT Theorem, since for each  $m$ ,  $U^m$  is concave in  $x^m$ , there is a Lagrange multiplier  $\lambda^m(y_{[-m]})$  such that  $y^m$  maximizes the Lagrangian

$$L^m(x^m) = U^m(x, y_{[-m]}) - \lambda^m(y_{[-m]}) \left( V_k - \sum_{j=1}^M x^j \right)$$

and

$$\lambda^m(y_{[-m]}) \left( V - \sum_{j=1}^M x^j \right) = 0$$

(complementarity property). We call the game with the Lagrangian  $L^m$  replacing the utilities  $U^m$  the relaxed game.

The Lagrange multipliers can be interpreted as shadow prices: if a price is set on player  $m$  such that when other players are at equilibrium, the player pays  $x^m \lambda^m(y_{[-m]})$  for its use of the capacity, then  $y$  is an equilibrium in the game with capacity constraints. Yet this pricing is not scalable since for the same use of the resources it may vary from user to user and it further depends on the chosen equilibrium. For billing purposes, one would prefer  $\lambda^m$  not to depend on  $y$  nor on  $m$ , but to be a constant.

Does there exist a constant Lagrange multiplier  $\lambda$  independent of strategies of the payers and of the identity  $m$  of the player, along with an associated equilibrium  $y$  for the corresponding relaxed game? If the answer is positive then  $y$  is called a *normalized equilibrium* [15].

Our goal is to establish the existence and uniqueness of the normalized equilibrium.

## 4 Diagonal Strict Concavity

For a vector of real non-negative numbers  $r$ , define

$$\sigma(x, r) = \sum_{m=1}^N r_m U^m(x)$$

$$g(x, r) = \begin{bmatrix} r_1 \frac{\partial}{\partial x_1} U^1(x_1, x_{-1}) \\ r_2 \frac{\partial}{\partial x_2} U^2(x_2, x_{-2}) \\ \vdots \\ r_N \frac{\partial}{\partial x_N} U^N(x_N, x_{-N}) \end{bmatrix} \quad (2)$$

$\sigma$  is called diagonally strict concave (DSC) for a given  $r$  if for every distinct  $x^0$  and  $x^1$ ,

$$(x^1 - x^0)' g(x^0, r) + (x^0 - x^1)' g(x^1, r) > 0$$

Let  $G(x, r)$  be the Jacobian of  $g(x, r)$  with respect to  $x$  and let  $G_{i,j}$  be  $i^{th}$  row and  $j^{th}$  column of  $G(x, r)$ . Then a sufficient condition for  $\sigma$  to be diagonally strict concave for some  $r$  is that for all  $x$ ,  $[G(x, r) + G'(x, r)]$  is negative definite.

Our interest in diagonally strict concave utility functions is due to the following properties of games possessing such utilities.

**Theorem 2** (Theorem 4 from [15]). *Let  $\sigma$  be diagonally strict concave for some  $r$ . Then there exists a unique normalized equilibrium.*

## 5 Proof of DSC

In this section we establish that the Tullock game with capacity constraint has a DSC structure and thus has a unique normalized equilibrium.

In our case we have

$$g(x, r) = \begin{bmatrix} \frac{r_1 x_{-1}}{X} \\ \frac{r_2 x_{-2}}{X} \\ \vdots \\ \frac{r_N x_{-N}}{X} \end{bmatrix} \quad (3)$$

where  $X = \sum_{i=1}^N x_i$  and  $x_{-m} = \sum_{i=1, i \neq m}^N x_i$

$$G_{i,j} = \frac{\partial}{\partial x_j} \left( \frac{\partial}{\partial x_i} \frac{r_i x_i}{X} \right) \quad (4)$$

$$r_i \frac{\partial}{\partial x_j} \left( \frac{x_{-i}}{X^2} \right) = \begin{cases} r_i \frac{-2x_i}{X^3} & \text{if } i = j \\ r_i \frac{x_i - x_{-i}}{X^3} & \text{if } i \neq j \end{cases} \quad (5)$$

For  $[G + G']$  consider

$$G_{i,j} + G_{j,i} = \begin{cases} \frac{-4r_i x_{-i}}{X^3} & \text{if } i = j \\ \frac{r_i(x_i - x_{-i}) + r_j(x_j - x_{-j})}{X^3} & \text{if } i \neq j \end{cases} \quad (6)$$

$[G + G']$  is negative definite if  $A' [G + G'] A < 0, \forall A, A \neq 0$  where  $A$  is the column vector

$$A = \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix} \quad (7)$$

$$\begin{aligned} & A' [G + G'] A \\ &= \sum_{i=1}^N \left[ \sum_{j=1, j \neq i}^N a_i a_j \frac{r_i(x_i - x_{-i}) + r_j(x_j - x_{-j})}{X^3} \right] - a_i^2 \frac{4r_i x_{-i}}{X^3} \end{aligned}$$

We choose  $r_i = 1$  for all  $i$ . Then (7) equals  $-Z/X^3$  where  $Z$  is given by

$$\sum_{i=1}^N a_i^2 4x_{-i} + \left[ \sum_{j=1, j \neq i}^N a_i a_j ((x_{-i} - x_i) + (x_{-j} - x_j)) \right] \quad (8)$$

$$= \sum_{i=1}^N a_i^2 4(X - x_i) + \left[ \sum_{j>i}^N 4a_i a_j (X - x_i - x_j) \right] \quad (9)$$

$$= 4 \sum_{i=1}^N a_i^2 (X - x_i) + \left[ \sum_{j>i}^N a_i a_j (X - x_i - x_j) \right] \quad (10)$$

$$= 4 \sum_{i=1}^N \left[ a_i^2 \sum_{j=1, j \neq i}^N x_i + \sum_{j>i}^N a_i a_j \sum_{k=1, k \neq j, k \neq i}^N x_k \right] \quad (11)$$

$$= \sum_{i=1}^N 4x_i \left[ \sum_{j=1, j \neq i}^N a_j^2 + a_j \sum_{k>j, k \neq i}^N a_k \right] \quad (12)$$

Now (12) is positive for any positive value of  $x$  and hence  $[G'+G]$  matrix is negative definite.

## 6 Several Resources

We consider next the following extension to the case of  $K$  resources. Each player  $m$  of the  $M$  players has a budget  $B(m)$  that he can invest by bidding  $x_k^m$  of resource  $k$ . The following (orthogonal) constraint should hold:

$$\sum_{k=1}^K x_k^m \leq B(m).$$

The payoff for player  $m$  is the sum of payoffs in all  $K$  contests, i.e.

$$P^m(x) = \sum_{k=1}^K P_k^m(x_k)$$

where  $x_k$  is the vector  $x_k^1, \dots, x_k^M$  and where

$$P_k^m(x_k) = \frac{x_k^m}{\sum_{j=1}^M x_k^j}.$$

and the cost of a contest  $k$  to player  $m$  is  $\gamma(k)x_k^m$ . Player  $m$ 's utility is thus

$$U^m(x) = \sum_{k=1}^K \left( \frac{x_k^m}{\sum_{j=1}^M x_k^j} - \gamma_k x_k^m \right)$$

For the study of such games, see [17].

We next define capacity constraint on each of the  $K$  resources. Let  $V$  be the column vector with the  $k^{\text{th}}$  entry being a constant  $V_k$ . We then require for each  $k$  that

$$\sum_{m=1}^N x_k^m \leq V_k$$

Note that when applying KKT conditions to the best response at equilibrium, we shall have  $K$  Lagrange multipliers. We wish to find a vector of  $K$  Lagrange multipliers that do not depend on the player nor on the policy of other players, such that the Nash equilibrium for the relaxed game will be an equilibrium for the original constrained game and in particular the constraints would be met and would satisfy the complementarity conditions. This is the vector version of a normalized equilibrium.

According to Theorem 4 of Rosen [15], we have to show that the set of utilities is diagonally strict concave in order to have existence and uniqueness of the normalized equilibrium. The latter follows from the fact that DSC holds for each resource separately and then apply the proof of Corollary 2 in [13].

## 7 Conclusions and Future Work

We have shown that the utilities in the Tullock game are strict diagonal concave. This allows to conclude using Rosen's result that in absence of common correlated constraints, the Nash equilibrium exists and is unique, while in presence of such constraints, the normalized equilibrium exists in pure strategies and is unique. Note that while the statements on the Nash equilibrium have already been available in [1] which proposed an extension to the DSC property, that reference does provide tools to handle the normalized equilibrium.

Another advantage from the derivation of the DSC structure is that one can use dynamic distributed algorithms to converge to the normalized equilibrium and convergence is guaranteed under DSC, see [15].

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# Resource Allocation and Pricing in 5G Network Slicing

by  
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