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# The optimal management of flexible nuclear plants in competitive electricity systems: the case of competition with reservoir

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# **The optimal management of flexible nuclear plants in competitive electricity systems: The case of competition with reservoir.**

## **Abstract**

Nuclear power as a generation technology that is widely used in electricity production systems is characterized by high fixed costs and low variable costs. To amortize its fixed costs, nuclear is preferentially used for inflexible baseload operation, i.e. operate at a constant level to meet the non variable part of electricity demand of a system and produce at its maximum capacity. Because of this specificity, the insertion of nuclear production in competitive electricity markets has not been deeply studied so far. Therefore, even in competitive markets, the question of the optimal management of a nuclear generation set has not been raised because nuclear production is supposed to operate continuously (to cover baseline demand). However, there are cases where the management of nuclear generation seems more complex than suggested by this simplified view. Typically, when the proportion of nuclear energy in a production set is high, the nuclear generation output has to adjust to the variations in demand. This raises the question of the optimal way to manage this production technology in that kind of setting. As this question has not been studied so far, there is a need for a theoretical framework that enables an analysis of situations like the French one, with a competitive market and where nuclear represents 80% of generation, i.e. much more than what would be necessary to cover the baseload demand. We place ourselves in a medium-term horizon of the management (1 to 3 years) to take into account the seasonal variation of the demand level. In the medium-term, the manager of a large nuclear set (like the French set) has to set its seasonal variation of output according to the demand level. Since nuclear units have to stop periodically (from 12 to 18 months) to reload their fuel, we can analyze the nuclear fuel as a stock behaving like a reservoir. The operation of the reservoir allows different profiles of nuclear fuel use during the different demand seasons of the year. Thus, we will look at this question as a rational economic analysis of the operation of a nuclear fuel “reservoir”. We then analyze it within a general deterministic dynamic framework with two types of generation: nuclear and thermal non-nuclear. We study the optimal management of the production

in a perfectly competitive market. Then, we establish a numerical model (based on data from the French market) with nuclear plants being not operated strictly as base load power plants but within a flexible dispatch frame (like the French nuclear set).

To build the corresponding modelling, we aim at establishing a microeconomic model of operation of nuclear power stations in a flexible market based operation framework. Within this framework, there are technical production constraints imposed by the flexible operation of nuclear units, generation capacity and nuclear fuel storage that play a central role in determining the equilibrium outcomes in the wholesale electricity market. Additionally, in view of the large proportion of nuclear in the electricity system, the equality between supply and demand is pivotal for the management of market based flexible nuclear since its satisfaction guarantees the prevention of “blackouts”. Different cases regarding the optimal management of flexible nuclear units will be analyzed in this thesis. First, the nuclear operator aims at studying the optimal short-term management of the nuclear fuel reservoir (typically a month). This is a reasonable starting point to determine what the optimal management of a flexible nuclear fuel stock should be in a competitive electricity market. Then, after a while, these basics being well known, the manager proceeds with the inter-temporal optimization of the production in order to determine the optimal operation of the nuclear fuel reservoir in the medium-term (typically 36 months). Finally, we look at the social welfare maximization problem within the same framework. In this case, the optimization no longer only considers the benefits of the generators, it now takes into account the “benefits” for the whole society: social welfare. This is because, the production behaviour of a very large nuclear set that offers the majority of the total electricity generation can have considerable consequences for the whole national electricity system and hence for the welfare of the society.

**Key words:** Electricity market, nuclear generation, electricity fuel mix, flexible management, competition with reservoir, short-term optimal reservoir operation, inter-temporal optimal reservoir operation, supply-demand equilibrium, social welfare, price discontinuity.

# **La gestion optimale des centrales nucléaires flexibles dans des systèmes électriques concurrentiels : Le cas de la concurrence avec réservoir.**

## **Résumé**

L'énergie nucléaire, qui est une technologie de génération largement utilisée dans des systèmes électriques, est caractérisée par des coûts fixes élevés et des coûts variables bas. Pour amortir ses coûts fixes, le nucléaire est préférentiellement utilisé pour une opération en base inflexible, c'est à dire opérer à un niveau constant pour répondre à la partie non variable de la demande d'un système électrique et produire au maximum de sa propre capacité. En raison de cette spécificité, l'insertion de la production nucléaire dans les marchés concurrentiels d'électricité n'a pas été profondément étudiée jusqu'à présent. Par conséquent, même dans des marchés concurrentiels, la question de la gestion optimale d'un parc de production nucléaire n'a pas été soulevée parce que la production nucléaire est censée fonctionner en continu (pour couvrir la demande de base). Cependant, il y a des cas où la gestion de la production nucléaire semble plus complexe que ne le suggère cette vision simplifiée. En règle générale, lorsque la proportion de l'énergie nucléaire dans un parc de production est élevée, la production nucléaire doit s'adapter aux variations de la demande. Cela soulève la question de la façon optimale de gérer cette technologie de production dans ce contexte. Comme cette question n'a pas été étudiée jusqu'à présent, il est nécessaire de proposer un cadre théorique qui permet une analyse des situations comme celle de la France, avec un marché concurrentiel et où le nucléaire représente 80 % de la production, c'est à dire beaucoup plus que ce qui serait nécessaire pour couvrir la demande de base. Nous nous plaçons dans un horizon à moyen terme de la gestion (1 à 3 ans) pour tenir compte de la variation saisonnière de la demande. À moyen terme, le gestionnaire d'un parc nucléaire très large (comme le parc français) doit ajuster sa production selon les variations saisonnières de la demande. Dans ce cadre, le stock de combustible nucléaire peut être analysé comme un réservoir puisque les centrales nucléaires s'arrêtent périodiquement (tous les 12 ou 18 mois) pour recharger leur combustible. La gestion de ce réservoir permet de profils différents d'usages de combustible nucléaire au cours des différentes saisons de l'année. Ainsi, nous nous pencherons sur cette question

comme une analyse économique rationnelle de l’opération d’un “réservoir” de combustible nucléaire. Nous allons ensuite l’analyser dans un cadre général déterministe dynamique avec deux types de production : nucléaire et thermique non-nucléaire. Nous étudions la gestion optimale de la production dans un marché parfaitement concurrentiel. Ensuite, nous établissons un modèle numérique (basé sur les données du marché français) où les centrales nucléaires ne sont pas opérées à production constante, mais dans un cadre de placement flexible (comme le parc nucléaire français).

Pour construire la modélisation correspondante, nous visons à établir un modèle microéconomique de gestion des centrales nucléaires dans un cadre concurrentiel de gestion flexible. Dans ce cadre, il y a des contraintes de production techniques imposées par l’opération flexible des centrales nucléaires, la capacité de production et le stockage de combustible nucléaire qui jouent un rôle central dans la détermination d’équilibre dans le marché de gros d’électricité. En outre, compte tenu de la forte proportion du nucléaire dans le système électrique, l’égalité entre l’offre et la demande est essentielle pour la gestion du nucléaire concurrentiel flexible car sa satisfaction garantit la prévention des “black-outs” i.e. des pannes de courant à large échelle. Des cas différents concernant la gestion optimale des centrales nucléaires flexibles seront analysés dans cette thèse. Tout d’abord, l’opérateur nucléaire vise à étudier la gestion optimale du réservoir de combustible nucléaire à court terme (typiquement un mois). C’est un point de départ raisonnable pour déterminer quelle devrait être la gestion optimale d’un stock de combustible nucléaire flexible dans un marché concurrentiel. Puis, après un certain temps, cet étape basique étant bien connue, le gestionnaire procède à l’optimisation inter-temporelle de la production afin de déterminer la gestion optimale du réservoir de combustible nucléaire à moyen terme (typiquement 36 mois). Enfin, nous étudions le problème de maximisation du bien-être social dans le même cadre. Dans ce cas, l’optimisation n’est plus basée seulement sur les bénéfices des producteurs, il prend désormais en compte les “bénéfices” pour l’ensemble de la société: le bien-être social. Ceci est du au fait que le comportement d’un parc nucléaire très large qui offre la majorité de la production totale d’électricité peut avoir des conséquences considérables pour l’ensemble du système national d’électricité et donc pour le bien-être de la société.

**Mots clés :** Electricité, production nucléaire, opération flexible, concurrence avec

réservoir, gestion optimale des réservoirs à court-terme, gestion optimale inter-temporelle des réservoirs, équilibre offre-demande, bien-être social, discontinuité des prix.

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# General Introduction

Nuclear power as a generation technology that is widely used in electricity production systems is often considered to be a technology which is both low-cost and well-suited for inflexible baseload operation, i.e. operate at a constant level to meet the non variable part of electricity demand of a system. Because of this specificity, the insertion of nuclear production in competitive electricity markets has not been deeply studied so far. Therefore, even in competitive markets, the question of the optimal management of a nuclear generation set has not been raised because nuclear production is supposed to operate continuously (to cover baseload demand). However, there are cases where the management of nuclear generation seems more complex than suggested by this simplified view. Typically, when the proportion of nuclear energy in a production set is high, the nuclear generation output has to adjust to the variations in demand (Regulatory Commission of Energy (2007)). This raises the question of the optimal way to manage this production technology in that kind of setting. As this question has not been studied so far, there is a need for a theoretical framework that enables an analysis of situations like the French one, with a competitive market and where nuclear represents 80% of generation, i.e. much more than what would be necessary to cover the baseload demand (IEA (2008)). Therefore, in the French case, the nuclear production set has to be managed in a flexible manner in order to follow the variations in demand (Nuttall and Pouret (2007), Bruynooghe et al. (2010)).

**Economically**, nuclear is a generation technology with high fixed costs and low variable costs (Bertel and Naudet (2004), DGEMP & DIDEME (2003, 2008), European Economic and Social Committee (2004), MIT (2003, 2009)). To amortize its fixed costs, nuclear produces at a steady rate using its maximum capacity to meet consumer requirements at any time (baseload operation). But in certain situations, when nuclear

represents a high proportion of the generation capacity of a national production system, nuclear generation output needs to adjust in order to address a part of the daily and seasonal variations of demand (load-following operation). In the previously monopolistic and vertically integrated markets, the optimal management of this production technology was mainly a technical issue, as there was a guarantee of selling the whole nuclear production, adjustments of production levels could be done by other even more flexible technologies (e.g. hydro, gas). In a competitive setting, the question of optimal management is not only of a technological nature (Glachant and Finon (2003), Chevalier (2004)). Given the differences in production costs of different technologies and the variations in market prices, the maximization of the economic value of nuclear production becomes a crucial issue for producers.

In the middle of **political and economic** debates around energy policy and electricity (including e.g. security of supply, CO<sub>2</sub> emissions reduction and affordable electricity), several countries are re-examining the benefits of nuclear power. As a consequence, one meets assumptions about the observed or real lack of output flexibility of nuclear power plants, and its relatively weak role in ensuring market equilibrium. **Technically**, this raises the question of whether nuclear production can be a flexible technology for electricity production (Nuttall and Pouret (2007)). In French-like nuclear reactors, the production levels are adjusted (increased and decreased) through the control of neutralizing rods made of chemical elements (e.g. silver, indium and cadmium), the control rods, these control rods enabling the control of the rate of fission of uranium and plutonium and thus the rate of heat released from nuclear fission (Guesdon et al. (1985)). For safety reasons, the ability to realize these adjustments is reserved only to some types of nuclear reactors like the European Pressurized Reactor (EPR) which is conceived to provide load-following operations (AREVA (2005), Goldberg and Rosner (2012)). This implies that some nuclear reactors (like the EPR) technically have the capacity to do safely load-following operations.

If the nuclear production can be relatively flexible, there is an element (which is a constraint) that will interact with how we can implement this flexibility. It is about the production interruptions that are a feature of all nuclear reactors and comes from the fact that reactors must regularly reload their fuel. Here we have two variables: first, the

length of the production period, called a campaign, which occurs between two successive interruptions for reloading (it must meet a number of constraints, but a producer has still some choices) and secondly, the duration of reloading (depending on some constraints). These two variables are another dimension of how to exercise flexibility in time.

Production interrupts periodically so that nuclear reactors are supplied with new nuclear fuel (uranium, plutonium or both in the case of MOX<sup>1</sup> fuel) and neutralizing rods that replace the used ones with the opening of reactor's core. Then a new period of nuclear production, named the campaign, starts. A campaign consists of the maximum number of days during which a nuclear unit produces till the consumption of the reloading fuel. One variable in the optimal management of the nuclear production set is the timing of interruptions of production due to the need to reload nuclear fuel, which is both an economical and technical issue (EDF (2010)). The nuclear manager decides when to shut down nuclear reactors for reloading over a year and as a result is responsible for the general scheduling of reloading. The scheduling of fuel reloading of a nuclear reactor takes into account the level of demand during the different seasons of the year in order to avoid interruptions of supply during seasons of high demand. The timing and the frequency of nuclear fuel reloading determine the length of a campaign which is generally 12 or 18 months. For operational reasons, the length of a campaign needs to be decided in advance in order to obtain a general plan for reloading. This decision is contingent upon many technical factors e.g. the type of reactor, age, size, the choice of a producer to reload the reactor's core per third or quarter of its full capacity, type of nuclear fuel put into the fuel rods, forecasted load and operating factors, regulatory constraints issued by safety inspectors, etc. (World Nuclear Association (2011), CEA (2007, 2008), Bertel and Naudet (2004)). The duration of reloading is conditional on the technical specificities of the reactor; however, a long period of reloading (plus the risk of a supplementary interruption for inspections for safety reasons) could have consequences for the stability of the electricity system. Nevertheless, all decisions regarding the timing, frequency, and duration of reloading of a reactor rely on constraints imposed by safety regulations and have to be consistent with the general scheduling of reloading of all the nuclear reactors of the set. In this context, a producer facing the periodical shut-downs of the nuclear

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<sup>1</sup>MOX fuel is a nuclear fuel consisting of approximately 7% of plutonium and 93% of depleted uranium.

reactors deals with the question of the optimal allocation of the nuclear production to meet demand variations.

The novelty of our work resides in the research assumption that the nuclear fuel works as a “**reservoir**”. **Technically**, this key feature of nuclear fuel as a “reservoir” lies on the discontinuous reloading of nuclear reactors. Following the operation of reloading of a nuclear reactor, the “reservoir” reloads its nuclear fuel every 12 or 18 months and then produces during the period of a campaign. The operation of the reservoir allows different profiles of nuclear fuel allocations in line with the different levels of demand in the course of the campaign.

In **economic** analysis, there is no work yet taking into account these technical aspects of nuclear production, especially the need to replace the fuel periodically, which leads to interruptions of generation while this aspect is crucial for an analysis of flexibility of a nuclear production set in a competitive setting. So we were lacking a theoretical framework that would enable an analysis of situations of nuclear flexibility like those observed in France. The main objective of this thesis is to propose an analytical framework for this. Therefore, we propose to treat the nuclear as a “reservoir”. A reservoir is used to store fuel (e.g. water in the case of an hydro-reservoir). This fuel is reloaded periodically and is then allocated during the different periods of production. In our research, we look at the management of the reservoir of nuclear fuel in the medium-term horizon (1 to 3 years) to take into account the fluctuations of the demand according to the seasons of the year. The short-term horizon associated with the daily or intra-daily variations of demand depending to a large extent on purely technological operational constraints is not treated by us. On the contrary, the medium-term horizon having economical implications regarding the management of a reservoir in a competitive setting deserves more attention than the short-term. In the medium-term, a critical economic feature of market based nuclear is that the nuclear fuel stock can be managed as a “reservoir” of energy. A central economical issue that immediately comes up is the maximization of the value of electricity produced during a campaign of nuclear fuel reservoir by responding to the variations of demand and prices during the different seasons of the campaign. This way, we will analyze the question of the optimal management of the nuclear fuel reservoir of flexible nuclear units in a market based electricity system.

This concept of reservoir is already **traditionally used in the analysis of hydraulic production** (Arellano (2004), Bushnell (1998), Scott and Read (1996), Ambec and Doucet (2003)). We will then demonstrate that the approach used for hydraulic production is transposable to nuclear production which helps to analyze the management of the reservoir of nuclear fuel by analogy with the management of hydro-reservoirs. However, we identify central differences between the hydro and the nuclear reservoir in the modelling of nuclear reservoir's operation. An important difference resides in the questions of when and how often to reload the reservoirs (hydro and nuclear). The timing and the frequency of shut-downs of a nuclear reactor for reloading depend on the producer while capricious rain is only responsible for the determination of these parameters in the case of hydro units. Furthermore, the reloading of an hydro-reservoir does not interrupt production whereas in the case of nuclear, production is interrupted during reloading. Nevertheless, in both cases, the reloading of the units (nuclear and hydro) is realized by taking account of the seasonal variations of demand i.e. more interruptions during low demand seasons and less interruptions during high demand seasons.

The fact that no robust analytical framework for studying nuclear flexibility has been developed so far makes it particularly interesting to develop a **dynamic model of optimal management of flexible nuclear plants taking account of the characteristic of the nuclear fuel reservoir**. The interest becomes even greater when we study the question of the optimal operation of a flexible nuclear set in an open market frame. A historical operator holding the total capacity of a very large nuclear set ensures the majority of the domestic electricity generation and consequently dominates its historical national electricity market (e.g. the case of France). In such situations, the question of the optimal management of the nuclear fleet within a competitive framework combined with the flexible operation of the nuclear plants and the feature of the nuclear fuel reservoir has not been studied yet. Note that, considering the dominant position of the nuclear operator in the electricity system, the decentralization of the management of the nuclear set may be an answer to the market dominance of the nuclear operator and the high concentration of the electricity market (Green and Newbery (1992), London Economics, Global Energy Decision (2007)). In this competitive setting, we bring up the question of the optimal management of the production during a campaign of nuclear fuel reservoir. From a the-



oretical and numerical point of view, we put forward the following questions: what may be the potential stages of optimization of the nuclear production decisions, how the optimal production problem will be modelled in each stage, under which technical-economical constraints all problems will be resolved and by which methods, what could be the potential difficulties of resolving these problems theoretically and numerically, what will be the optimal production levels resulting from their resolution and what properties characterize the optimal solutions. What could be the potential advantages of the decentralization in comparison with the centralized management at the optimum?

In our model, the optimization of the nuclear production decisions takes place on two basic time horizons. **In the early stage of optimization**, the nuclear managers explore how to attain an equilibrium of the optimal production problem in the medium-term. For this reason, the managers choose to restrain the management horizon of the reservoir to a time segment simpler to envisage: the monthly horizon. In this stage of optimization, they examine the existence of an equilibrium on a monthly basis operation of the reservoir (i.e. a feasible production trajectory that maximizes their current monthly profit) and they determine it. This equilibrium will act as a reference for the existence of an equilibrium in the next stage of optimization where we deal with a more complicated optimal production problem. More specifically, **in the late stage of optimization**, a critical question that the nuclear units managers may ask is how to reach an equilibrium of the optimal production problem over a “coherent and longer” time step embodied by one or more campaigns of production (i.e. a feasible production vector that maximizes their inter-temporal profit ). Therefore, following the analytical framework proposed above, we will study: the optimization of production over a short-time horizon of management (typically a month) and the optimization of production over a long-time horizon of management being the entire time horizon of the model (typically 36 months). In the wholesale electricity markets, the price, called merit order price, is determined by the marginal cost of the “last technology” of the merit order (called marginal technology) used to equilibrate supply and demand. The merit order is a way of ranking the available technologies of electricity generation in the same order as their marginal costs of production. This ranking results in a combination of different generation technologies to reach the level of demand at a minimum cost. To take this into account, we assume perfect competition between

producers (Ventosa et al. (2005), Smeers (2007)) in our model. Therefore, we examine the optimal operation of nuclear fuel reservoirs in the case where producers are price-takers and thus the market price is a parameter and not a choice variable. In this case, when producers maximize their profits the price will be equal to marginal cost. In our analysis two flexible types of generation will be presumed: nuclear and non-nuclear thermal (coal, gas).

One can ask if the nuclear producers should be forced to consider the security of supply for all the levels of the domestic demand? This question<sup>2</sup> could be motivated by several arguments: *(i)* electricity is a non-storable good, so demand has to meet supply each moment, *(ii)* instability of the electrical grid because of power failures (“blackouts”) which result from imbalances between supply and demand, *(iii)* the high proportion of nuclear in the national energy mix which makes nuclear producers inevitably responsible for the stability of the grid. The equality between total supply and total demand is not a natural constraint especially if we refer to a market-based electricity system where a number of producers compete with each other to maximize their profits. Here, we are not in the classic competitive case but in a intermediate situation where producers do not look only at their own profits but also at the global supply-demand equilibrium at all times. Blackouts are particularly critical at sites where the environment and public safety are at risk (e.g. hospitals, mines, etc.) and for other systems such as telecommunications. Therefore, the equality between supply and demand is without doubt a constraint intrinsic to social welfare. The responsibility of nuclear operators to meet this constraint is even more important when nuclear is the principal generation technology of the national electricity market. Economically, there exists an externality derived from the consideration of this constraint (positive externality) or not (negative externality) every time. Our model does not take into account the presence of this externality in the profit of a nuclear producer

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<sup>2</sup>The supply-demand equilibrium is obtained by: *(i)* long-terms contracts, *(ii)* the spot electricity market, *(iii)* additional active power capacity available (i.e. when compared to steady state operation prior to a frequency event) from generation units or through reduction in load for the purpose of frequency control which is known as operating reserve. Many different definitions for the categorisation of operating reserve exist: primary, secondary and tertiary operating reserve. Primary operating reserve (POR) is the additional active power available from generators and through reduction of active power consumption of the load which is available between 5 and 15 seconds subsequent to an event on the system. Secondary reserve is defined to be the additional active power available and sustainable for the time period from 15 to 90 seconds after the event. Tertiary reserve is the additional active power available from 90 seconds to 20 minutes subsequent to the event. Finally replacement reserve is the additional active power available from 20 minutes to 4 hours after the event.

since we do not look at the question of compensation or penalties for balancing or not supply and demand. The consideration of this externality would complicate our model since externalities are not in general a simple case to look and examine. Consequently, we conclude that the equality between supply and demand is a pivotal operational constraint that we will integrate in the determination of the optimal management of the nuclear fuel reservoir of market based flexible nuclear plants.

**Logically**, the problem of optimal management of a reservoir (e.g. hydro-reservoir) is principally treated by taking into account constraints regarding the management of the quantity of fuel (e.g. water) stored in the reservoir during a certain period of production. Such a constraint, called fuel storage constraint, imposes that the total production realized during the production period does not exceed the fuel stored in the reservoir. In this case, the optimal production problem would consist of the maximization of producer's profit under fuel storage constraints as well as constraints related to the available generation capacity of a unit. **This kind of optimization problems are frequently resolved in mathematics through the Karush-Kuhn-Tucker (KKT) conditions** which are first order conditions necessary for a solution to be optimal, provided that some regularity conditions (or constraint qualifications) are satisfied (Mas Colell et al. (1995)). Nevertheless, our problem of optimal management of a fuel reservoir has not been studied yet for the case of nuclear with additional constraints regarding social welfare, the satisfaction of all states of demand, as well as the flexible operation of nuclear reactors and is still new in the literature. In this problem, the optimization of the nuclear production will be done with technical-economical constraints imposed by: (a) the inter-temporal management of the nuclear fuel of the reservoir over one or more campaigns of production (storage constraints), (b) the flexible operation of nuclear reactors to load-follow and the available generation capacity of non-nuclear thermal units (minimum/maximum production constraints), (c) the satisfaction of the supply-demand equilibrium constraint at every moment over each month.

**At the beginning of optimization** we will address the question of the optimal management of the nuclear fuel reservoir over a short-time horizon of operation. To answer, we will distinguish two different approaches:

A first approach that may come in mind is the following: every month, each generation units manager looks at the market price and decides on the level of its feasible production (it respects (a), (b)) in order to maximize its profit while taking into consideration the equality between supply and demand (it respects (c)). However, can this approach apply within our model? What are the theoretical and numerical difficulties of computing an equilibrium of the optimal production problem following this approach? What other approach can a producer apply in order to calculate an optimal production level each month?

The resolution of the optimization problem consisting of the maximization of the current monthly profit under production and fuel storage constraints together with the supply-demand equilibrium constraints ((a), (b), (c)) (Lykidi, Glachant and Gourdel (2010, 2012)) could lead to the determination of the optimal short-term production behaviour. We will see that a producer resolves this problem, called the optimal short-term production problem, to determine an optimal production profile during the current month of optimization taking into consideration the optimal production levels achieved the previous months. However, challenges remain in the calculation of an equilibrium of this problem due to the constraint (c) that ensures security of supply for all the monthly levels of demand. More precisely, the question that can be raised is this: the equality between supply and demand should be considered in the determination of the optimal management of the nuclear fuel stock only for a month i.e. the current month of optimization or across the entire time horizon of the model? What could be the consequences of the first, more simplified tactic given that, at the current month of optimization, the producer does not take into account the satisfaction of the future states of demand? We will show that the first tactic, showing “short-sightedness” on behalf of producers with respect to future demand, empirically leads to a dead-end since, following this tactic, demand in future months can not be satisfied without overstepping the constraints (a) and (b). For this reason, we follow a different tactic according to which at the current month of optimization the supply-demand equilibrium constraint is respected throughout the time horizon of the model.

At the end of this first stage of optimization, the generation units managers have determined the optimal management of the nuclear fuel reservoir on a monthly basis in

a competitive market. Therefore, **in the late stage of optimization**, they may want to visualize the optimal profiles of nuclear fuel stock over a longer time step. To do this, they aim at a full optimization of the production all over the time horizon of the model constituted by one or more campaigns (Lykidi, Glachant and Gourdel (2011)). This optimization problem, called the optimal inter-temporal production problem, will consist of the maximization of the inter-temporal profit under constraints identical with those considered in the optimal short-term production problem.

**Finally**, we look at the problem of the maximization of social welfare in exactly the same framework. The consideration of social welfare in the management of a large nuclear fleet is motivated by the abundant proportion of nuclear for electricity production. This means that the nuclear operators have to consider the welfare of the society when they determine the optimal management of the nuclear set even in a competitive market. Constraints intrinsic to social welfare such as those deduced by the equality between supply and demand have already been taken into account in the determination of the optimal management of the fuel reservoir of flexible market based nuclear units in the medium-term. However, the production behaviour of the nuclear operators, having significant effects for the stability of the entire national electricity system, may result in the maximization of the “benefits” of the whole society instead of the maximization of their own “benefits”. The social welfare maximization problem will consist of the complete maximization of social welfare on the whole time horizon of the model under constraints identical with those considered in the optimal short-term production problem and the optimal inter-temporal production problem. We will discover that, in a perfect world where no strategic behaviour is observed, the optimal production behaviour resulting from the maximization of social welfare is very different from the one deduced by the inter-temporal profit maximization problem within our model.

**From a mathematical perspective**, however, we meet difficulties in the determination of an equilibrium of these optimization problems theoretically and numerically because of the complexity of our model (high number of optimization variables, high number of optimization constraints). For this reason, we proceed with the proof of mathematical propositions that we use to simplify the resolution of these problems through

the diminution of their optimization variables. More precisely, we introduce an auxiliary economy with a single producer coming from the “aggregation” of the  $N \geq 2$  producers of the original economy. We show that an equilibrium of the economy with one unique producer can be decentralized as an equilibrium of the economy with  $N$  heterogeneous producers with respect to the amount of capacity (nuclear and non-nuclear thermal) that each of them holds. Reversely, the “aggregation” of an equilibrium of the economy with  $N$  producers constitutes an equilibrium of the economy with a unique producer. Consequently, we establish an “equivalence” of equilibriums between the economy with  $N \geq 2$  and the economy with a single producer. Thanks to this mathematical proposition, we operate the following simplification which decreases the number of optimization variables and hence the number of constraints: the existence of an aggregate producer who maintains the total capacity (nuclear and non-nuclear). This simplification makes coherent the modelling of the optimal short-term production problem and optimal inter-temporal production problem. Logically (mathematically), these simplifications are not fundamental redefinitions of the treated problems. They only serve on reducing the degree of difficulty in resolving these problems. Economically, we compare the decentralization of the nuclear production set and the centralized management with respect to the optimal production behaviour within our model in order to provide insights for policy.

**On a mathematical, economical and technical level**, we deal with another difficulty that has to do with the discontinuity of the merit order price within our model. In reality, we can see in the figure of the merit order price that the problem of price discontinuity exists when a generation technology (hydro, wind, nuclear, coal, gas, oil, etc.) becomes the marginal technology (see Figure 1.4 on page 50, Chapter 1). This discontinuity is intrinsic to our model because we treat the case of nuclear and non-nuclear thermal whose marginal costs have very different values by taking into account the dynamic modelling of the nuclear fuel stock as a reservoir. More precisely, the marginal cost of the non-nuclear thermal production is (significantly) higher than the marginal cost of the nuclear production. In view of the diagram of the merit order price, the modelling of the production costs will consist later of a linear cost function for nuclear and a quadratic cost function for non-nuclear thermal (coal, gas) which results in an increasing marginal cost that takes into consideration the progressiveness of the marginal

costs of coal and gas. The discontinuity of price coming from the non-nuclear thermal generation technologies and in particular the coal and gas can be smoothed through the consideration of a quadratic cost function since the difference between their marginal costs is not significant and therefore, the resulting discontinuity is not brutal. This is not the case for nuclear and non-nuclear thermal firstly because of the important gap between their marginal costs which leads to a brutal discontinuity that can not be smoothed and secondly because our particular interest is to show what is happening with respect to nuclear within our model. Our theoretical and numerical results are specific to the chosen cost functions for nuclear and non-nuclear thermal within our model and thus, they can not be generalized for any convex cost function.

We show that the discontinuity of the merit order price leads to a discontinuous profit (current monthly profit, inter-temporal profit). Particularly, we prove that the reduction of the market price when nuclear becomes the marginal technology, induces a reduction of the profit in both the optimal short-term production problem and the optimal inter-temporal production problem. The discontinuity of the merit order price is something known that does not lead to a lack of solutions for a static model. However, the discontinuity of the merit order price and hence of the profit in combination with the inter-temporal management of the nuclear fuel reservoir makes difficult to resolve numerically these optimization problems and our theoretical approach proves that it may even lead to a lack of solutions. Discontinuous problems have been studied in an economic setting (cf. for example Bich and Laraki (2011)). Numerically, to treat this problem of discontinuity within our model, we propose a “regularization” of the merit order price which permits to achieve a “regularization” of the economical problems (discontinuous problems). This “regularization” of the economical problems will lead to approximate problems called “regularized” problems (continuous problems). Therefore, we focus on the numerical resolution of the corresponding “regularized” optimal production problems (optimal short-term production problem, optimal inter-temporal production problem). The data used for the numerical resolution of these problems is the same and thus, their numerical results can be contrasted. We also proceed with a comparison between the optimal production behaviour that comes from the maximization of social welfare and the optimal production behaviour that results from the maximization of the inter-temporal

profit of producers from a theoretical and numerical point of view.

Now that we exposed the analytical framework and the issues of our thesis we can address the key steps chapter by chapter.



## Organization of the thesis

This thesis includes three chapters.

The **first chapter** explores the principal dimensions of the problem of optimal management of flexible nuclear plants in a competitive setting. First, we start with a presentation of the components of the nuclear production cost. A complete image of the structure of the nuclear production cost will help us to understand why, in spite of the fact that some types of nuclear reactors are technically capable of providing load-follow operations, baseload operation is economically suitable for nuclear plants. However, in the case of a large nuclear set the necessity for adjusting the nuclear output to demand's daily and seasonal variations results from the dominant position of nuclear in the national energy mix. Within this context of flexible management of a nuclear production set, we introduce the notion of the "reservoir" of nuclear fuel. To better understand this notion and the reasons why this concept is pertinent for the study of flexible nuclear plants, we look at the operation of fuel reloading of nuclear reactors. Then, we establish an analytical framework which permits us to study the management of flexible nuclear plants in a competitive electricity system by taking into account the characteristic of the nuclear fuel reservoir. We address the medium-term horizon of operation (1 to 3 years) to take into account the seasonal variations of demand. In the medium-term, a key economic feature of market based nuclear is that the nuclear fuel stock can be managed as a "reservoir" of energy. The nuclear manager allocates the nuclear fuel of the reservoir during a campaign to go along with the seasonal variations of demand so that it maximizes the value of its profit. This concept of reservoir introduced here is mostly applied in the analysis of hydro production. To model the behaviour of a nuclear fuel reservoir, we regard the important literature on the optimal management of hydro reservoirs. At the same time, we underline several differences with respect to the characteristic of the "reservoir" between these two electricity generation technologies (nuclear and hydro). Lastly, within the medium-term horizon, we elaborate our model of optimal management of flexible nuclear plants in a competitive electricity market reflecting on the management of the nuclear fuel reservoir.

The **second chapter** focuses on the question of the optimal management of the nuclear fuel reservoir, at a timescale of operation of one month. At this "exploratory"

stage, the nuclear producers apprehend how to determine the optimal management of the reservoir of nuclear fuel before moving to a medium-term management horizon (yearly or multi-annual). In the previous section, we identified two different approaches in order to deal with the problem of the optimal short-term production behaviour of flexible market based nuclear plants. In this chapter, we prove that the first approach consisting of the computation of a feasible production level that respects the equality between supply and demand each month by looking at the price to maximize profit cannot apply within our model. Therefore, we proceed with the second approach that lies in the resolution of an optimization problem consisting of the maximization of producer's current monthly profit under constraints relative to the flexible management of nuclear units and the non-nuclear thermal generation capacity (minimum/maximum production constraints), the inter-temporal management of the nuclear fuel stock during the whole period of production (storage constraints) and to end, the equality between supply and demand at each moment (supply-demand equilibrium constraints). Here, the principal problem is the calculation of the optimal production during the current month of optimization given the optimal production levels of the previous months. The nuclear fuel stock of the following month is decided on the level of nuclear production realized during the current month of optimization. The price in the market is determined by the marginal cost of the "last technology" of the merit order (marginal technology) used to equilibrate supply and demand. Empirically, we build a simple numerical model and we analyze the production and nuclear fuel decisions resulting from the resolution of optimal short-term production problem within it.

The **first part of the third chapter** examines the question of the optimal inter-temporal management of the nuclear fuel reservoir over several campaigns of production (typically 36 months). At this final stage of optimization, the generation units managers having determined the optimal operation of the fuel reservoir of a flexible nuclear unit on a monthly basis move to a medium-term management horizon of operation (yearly or multi-annual). In this stage, we proceed with the complete maximization of profit under production and storage constraints identical with those considered in the optimal short-term production problem. This problem, named the optimal inter-temporal production problem, permits us to determine the optimal allocation of the nuclear fuel stock over

a longer time horizon of operation than the monthly horizon. Each month, the price results from the equality between supply and demand (merit order price). The quantity of the nuclear fuel stock every month will be determined by the nuclear production of the previous months. We analyze the storage and production behaviour resulting from the theoretical and numerical resolution of the optimal inter-temporal production problem. Numerically, a contrast between the optimal short-term management and the optimal inter-temporal management of the nuclear fuel reservoir is realized in order to identify similarities and differences with respect to the optimal production behaviour.

In the **second and last part of our third chapter**, we study the maximization of social welfare in an identical setting. Within this setting, we analyze the production and storage decisions of generation units managers resulting from the full maximization of social welfare subject to minimum/maximum production constraints, storage constraints and supply-demand equilibrium constraints. Then, we run numerical tests of this optimization problem. The optimal production behaviour derived by this problem may provide some intuition with regards to other behaviours of the units managers (optimal short-term production behaviour, optimal inter-temporal production behaviour). From a theoretical and numerical point of view, we compare the optimal outputs resulting from the social welfare maximization problem and the optimal inter-temporal production problem. We find that the maximization of social welfare induces a production behaviour very different from the one generated by the optimal inter-temporal production problem.

# Chapter 1

## The economics of nuclear production

### 1.1 Introduction

Nuclear generation is characterized by very high fixed costs and low variable costs. This is the main reason why nuclear power is supposed to be used as baseload generation. In a baseload operation, the energy plant produces at a constant rate to meet expected customer requirements at any time (called baseload demand). In view of this operation, the nuclear plants always run at the maximum of their capacity to cover their high fixed costs. However, in some countries nuclear generation is not operated that way. France is distinct from other countries like the UK or Sweden because its far higher generation of nuclear power (80% of electricity) implies not to run nuclear plants strictly as rigid baseload units. More precisely, the high level of nuclear in the energy mix of the country requires nuclear to be flexible, which means adjusting its output in order to follow a part of the daily and seasonal variations of demand. However, in view of the liberalization of the energy sector in Europe and the introduction of competition in electricity markets, the choice and operation of a source (e.g. nuclear, coal, gas, etc.) used for electricity generation may also change. Therefore, in such a competitive regime one can ask what the optimal management of the nuclear generation set is. We place ourselves in a medium-term horizon (1 to 3 years) of the management in order to take into account the variation of the demand level between seasons of high and low demand. A flexible nuclear set

is operated to follow a part of the demand variations. In this context, the nuclear fuel stock can be analyzed as a reservoir since nuclear plants stop periodically (every 12 or 18 months) to reload their fuel. The operation of the reservoir allows different profiles of nuclear fuel uses during the different seasons of the year. In this chapter, we elaborate on this question as follows:

In section 1.2, we analyze the production cost of nuclear energy (investment cost, operation and maintenance cost, fuel cost) and we contrast it with the production cost of other generation technologies (e.g. fossil fuel) in order to better understand why baseload operation is generally preferred for nuclear power plants.

Nevertheless, the operation of nuclear plants can be different when the level of participation of nuclear in the energy mix of a country is important; the operation of the French nuclear set is a textbook example to study. In section 1.3, we look at the case of France where the nuclear plants operate in order to follow-up the variations of energy demand (daily and seasonal) due to the high participation of nuclear in the national energy mix.

We focus on the medium-term horizon of operation to take account of the variation of the seasonal demand between winter (high demand season) and summer (low demand season). Within this time horizon, we make a new research assumption: the nuclear fuel works as a “reservoir” of energy partly similar to a water reservoir for hydro energy (Section 1.4). To better understand the notion of the nuclear fuel “reservoir”, we briefly discuss the operation of fuel reloading of nuclear reactors. In a market based electricity system, the nuclear plant manager aims at allocating the nuclear fuel of the reservoir to follow the seasonal variations of demand in order to maximize its profit. Thus, we address the question of the optimal management of the reservoir of nuclear fuel in a competitive electricity market considering the demand as well as the price characteristics at each period of production. To analyze the management of the nuclear fuel reservoir, we review a few articles that study the optimal management of hydro reservoirs.

Finally, we end with section 1.5 where we start defining a microeconomic deterministic dynamic model of operation of a nuclear set (like the French nuclear set) in a competitive electricity market. Within our model, we take into consideration a key economic feature

of the optimal management of flexible market based nuclear plants in the medium-term: the nuclear fuel reservoir.

## 1.2 Production cost of nuclear energy

In this section, we provide a detailed analysis of all the components of the production cost of nuclear energy in order to obtain a clear vision of its structure. We also look at the differences between the production cost of nuclear and that of other generation technologies (e.g. fossil fuel). This will help in comprehending how nuclear power plants operate and why their operation differs from the operation of other generation technologies (like that of fossil fuel generation technologies e.g. coal, gas). In the next figure, we present a decomposition of the cost of nuclear production where we observe that capital investment represents about 60% of the total production cost, operation and maintenance around 20% and fuel less than 20% (15%).

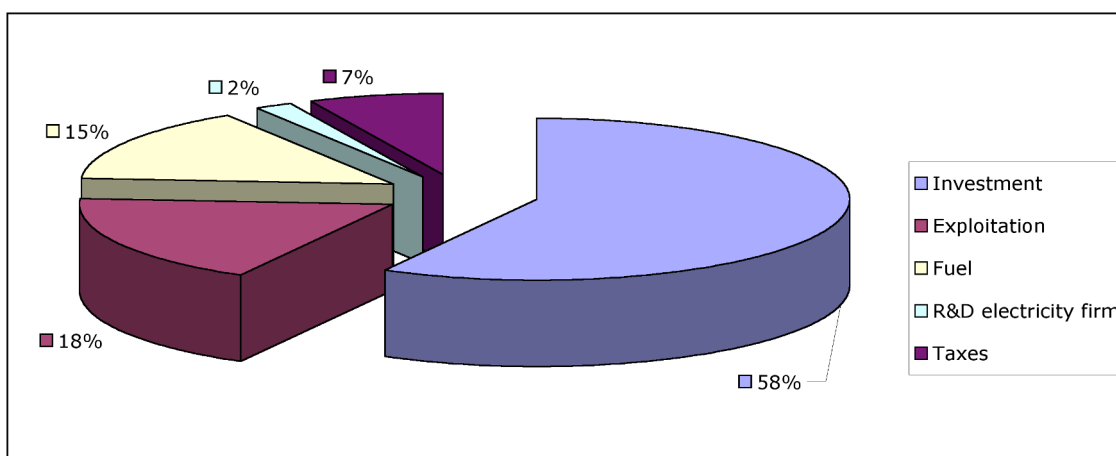


Figure 1.1: Decomposition of nuclear production cost (8% discount rate), Source: DGEMP & DIDEME (2003, 2008)

### 1.2.1 Investment cost

A nuclear unit is the set that consists of two parts: the reactor which produces heat to boil water and make steam and the electricity generation system in which one associates:

the turbine and the generator. The steam drives the turbine which turns the shaft of the generator to produce electricity (Source: SFEN). The investment cost of a nuclear generator unit includes the costs corresponding to the conception and construction of this unit, the major renovations during its life and the decommissioning of the unit at the end of its life. Hence, investment costs are related to direct costs associated with the construction and the deconstruction of the nuclear unit, indirect costs which are the responsibility to the contractor and the operator and insurance costs. We also need to add the costs incurred in order to obtain regulatory approvals for the construction and operation of the units (Bertel and Naudet (2004)).

A nuclear power plant is a thermal power station in which the heat source arises from nuclear reactions. It may contain one or more nuclear reactors and hence one or more nuclear units (e.g. Flamanville Nuclear Power Plant). The investment cost of a nuclear plant in France can be decomposed in the following elements (Bertel and Naudet (2004)):

- direct construction cost (excluding engineering costs).
- engineering costs estimated as a percentage of the direct cost of construction. This percentage depends on the importance of the series of nuclear units which will be constructed.
- pre-operating expenses charged to the prime contractor.
- decommissioning cost of a nuclear plant. Nuclear decommissioning is the dismantling of a nuclear power plant and decontamination of the site to a state no longer requiring protection from radiation for the general public. The main difference from the dismantling of other power plants is the presence of radioactive material that requires special precautions. This is also why the decommissioning cost is more important for nuclear than for other technologies (e. g. coal, gas). In Table 1.1, we can see that the decommissioning cost represents the 0.072% of the investment cost for a series of EPRs (European Pressurized Reactors). Its low level compared to the other components of the investment cost is due to the relatively important value of the discount rate (8%).

- expenses arising from uncertainties related to the construction schedule. More precisely, a possible prolongation of the construction calendar compared to the estimated calendar on which the payment has been scheduled induces additional costs.

In table 1.1, we introduce a decomposition of the investment cost of nuclear production as it appears in the official report, “Reference Costs of Electricity Production”, issued by the French Ministry of the Economy, Finance and Industry (General Directorate for Energy and Raw Materials (DGEMP) and Directorate for Demand and Energy Markets (DIDEME)) in 2003. The DGEMP (recently known as General Directorate for Energy and Climate (DGEC)) undertakes the study of the costs of reference of the electric production every 3 to 5 years which aims at evaluating, within a defined theoretical framework, the full cost of one MWh electric resulting from various means of production to build (and not the average cost of production of the set). However, we choose to present the official report of 2003 and not the other published in 2008 because of the existence of analytical data mentioned in the first report.

Euros/KW	Discounted at 8%
Construction cost	1043.1 (62.73%)
Engineering costs	134.9 (8.11%)
Pre-operating costs	74.3 (4.47%)
Interim interests	379.9 (22.84%)
Decommissioning cost	1.2 (0.072%)
Cost arising from uncertainties related to the construction schedule	29.5 (1.78%)
Total investment cost	1663 (100 %)

Table 1.1: Investment cost per KW evaluated in 2007 for a series of 10 EPRs, Source: DGEMP & DIDEME (2003)

The report of DGEMP & DIDEME (2003, 2008) is based on the costs of the European



Pressurized Reactor (EPR). The EPR is an evolution of the third generation<sup>1</sup> pressurized water reactor (PWR) design with a capacity of 1600 MW. It has been designed and developed mainly by Framatome (now Areva NP), EDF (Électricité de France) in France, and Siemens AG in Germany.

The construction costs, evaluated in constant currency, are distributed according to a payment schedule. This schedule is highly variable from one type of plant to another only because of differences in the length of construction periods. The interim interests calculated on this schedule are determined by the difference between the discounted value of construction expenditures reported on the date of commercial operation and the total construction cost. The amount of the interim interests is based on two parameters, one technical and the other economic: (*i*) the length of the construction period. A long period of construction implies significant interim interests during construction, (*ii*) the discount rate is the parameter which significantly varies the amount of interim interests.

The costs for renovation that may arise during the economic life of a nuclear plant (renovation for e.g. the extension of the production capacity, the improvement of production performance, the improvement of nuclear safety<sup>2</sup>, the increase of the economic viability of facilities, etc.) are not included in the investment cost but in the operation and maintenance costs in France. A major renovation intended to extend the physical life of the plant is not included in the economic evaluation since it concerns the operation of the plant beyond its economic life.

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<sup>1</sup>Generation II refers to a class of commercial reactors designed to be economical and reliable for a typical operational lifetime of 40 years (Goldberg and Rosner (2012)). The pressurized water reactor (PWR) is a prototypical Generation II reactor. Generation II systems began operation in the late 1960s. Generation III nuclear reactors are essentially Generation II reactors with evolutionary, state-of-the-art design improvements. These improvements are in the areas of fuel technology, thermal efficiency, modularized construction, safety systems (especially the use of passive rather than active systems), and standardized design for reduced maintenance and capital costs. Improvements in Generation III reactor technology have aimed at a longer operational life, typically 60 years of operation. Their operation period started in the late 1990s. Generation III+ reactor designs are an evolutionary development of Generation III reactors, operating by the year 2010, offering significant improvements in safety over Generation III reactor designs certified by the NRC (Nuclear Regulatory Commission) in the 1990s. The European Pressurized Reactor (EPR) is a Generation III+ reactor.

<sup>2</sup>All the necessary arrangements in order to ensure the normal operation of a nuclear power plant, to prevent accidents or malicious acts and limit their effects for the workers, the public and the environment. These arrangements have to be done during the design, the construction, the commissioning, the operation, the final shutdown and decommissioning of a nuclear installation or during the transport of radioactive materials.

However, there are factors that can affect the capital<sup>3</sup> costs and decrease them. First, the size effect in plants reflects the economies of scale<sup>4</sup> when the nominal capacity of units with identical technological features (e.g. same type of reactor) increases. Secondly, the effect of duplication of units on a site; the number of units built on the same site is an essential factor in the capital cost for different reasons (e.g. common equipments (technical premises, administrative buildings, etc.), sharing access to the network, systems and technical services, more efficient use of the teams which take part in the construction). Finally, the “effect of series” which occurs when one builds a series of units identical with the “head” of a series, which is the prototype. The “head” of a series represents a significant effort that results in many hours of study and engineering for its conception. The important cost relating to its conception is spread over the entire program made up of units built in the same way as the “head” of a series (“effect of series”). Similarly, the studies regarding the safety, carried out for the “head” of a series, considerably reduce those which will be realized for each unit of the series. The “effect of series” also affects the construction period. For example, within the report of DGEMP & DIDEME (2003, 2008), a period of construction of the first EPR (prototype) of series of 67 months is retained, and a lower duration of 57 months for subsequent EPRs.

Nevertheless, the capital costs may increase because of delays in the construction of the “head” of a series. To illustrate, in the case of the new European Pressurised Reactor (EPR) design, Charpin assumes in its economic study of nuclear power requested by the Prime Minister of France (Charpin (2000)), a surcharge of 30 % for the “head” of a series, a surcharge of 20 % for the following two reactors, a surcharge of 10% for the fourth and a unit cost equal to that proposed in the official report of DGEMP and DIDEME (2003) for the remaining reactors of the series in the case of a series of 10 reactors (e.g. Nuclear

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<sup>3</sup>The capital costs here include the direct construction cost together with engineering costs and pre-operating costs.

<sup>4</sup>Economies of scale, in microeconomics, refers to the cost advantages that a business obtains due to expansion (Sullivan and Sheffrin (2007)).

Power Plant of Flamanville<sup>5</sup>, of Olkiluoto<sup>6</sup>).

### 1.2.2 Operation and maintenance costs

Operation and maintenance (O&M) costs cover all the expenses related to the operation of a nuclear plant except for the investment and the fuel cost (Bertel and Naudet (2004)). We have to mention that the composition of the operating costs presents particularities because of the technical specifications of a nuclear plant associated with the type of the reactor, the size, the age, etc. More precisely, these costs include mostly wages and social expenses for the staff involved in operation and support area, training, security, protection of health and safety as well as in waste disposal operation. They also include the costs of maintenance, of daily and periodical inspections during which nuclear plants usually stop. As we mentioned in the subsection which describes the investment cost of a nuclear plant, the costs of renovation are included in the operation and maintenance costs in France. To end, we note that the factors which affect and more precisely reduce the costs of operation and maintenance are the same as those mentioned in the analysis of the investment cost (i.e. size effect, duplication effect, “effect of series”).

### 1.2.3 Nuclear fuel cost

The estimation of the nuclear fuel cost is not simple compared to the estimation of the fossil fuels cost because of the complexity of its fuel cycle (World Nuclear Association (2011)). The nuclear fuel cycle, also called the nuclear fuel chain, is composed of different stages. It consists of steps in the front end, which are relevant to the preparation of the fuel (mining, milling, conversion, enrichment, fuel fabrication), steps throughout the

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<sup>5</sup>EDF has previously said France’s first EPR would cost 3.3 billion euros and start commercial operations in 2012, after construction lasting 54 months. The estimated cost has now increased to 6 billion euros and the completion of construction is delayed to 2016 i.e. an additional 48 months of construction (World Nuclear News (2007), Nuclear Engineering International (2007), (2011)).

<sup>6</sup>The original commissioning date of the EPR was set to May 2009. However, in May 2009 the plant was at least three and a half years behind schedule and more than 50 % over-budget. The commissioning deadline has been postponed several times and as of November 2011 operation is set to start in 2014 (World Nuclear News (2012), Boxell (2012), BBC (2012)).

service period when the fuel is used during the operation of the reactor, and steps in the back end (transport, used fuel storage, reprocessing, vitrification, final disposal), which are necessary to safely manage the spent nuclear fuel. If the spent fuel is not reprocessed, the fuel cycle is referred to as an open fuel cycle; if the spent fuel is reprocessed, it is referred to as a closed fuel cycle. The industrial operations realized after the use of the nuclear fuel (back end of the cycle) are specific to nuclear because they do not occur in other generation technologies (e.g fossil fuel technologies). France is among the countries that have the necessary knowledge as well as the technical means to realize all the steps of the cycle except the mining of uranium because of the absence of such mines inside these countries.

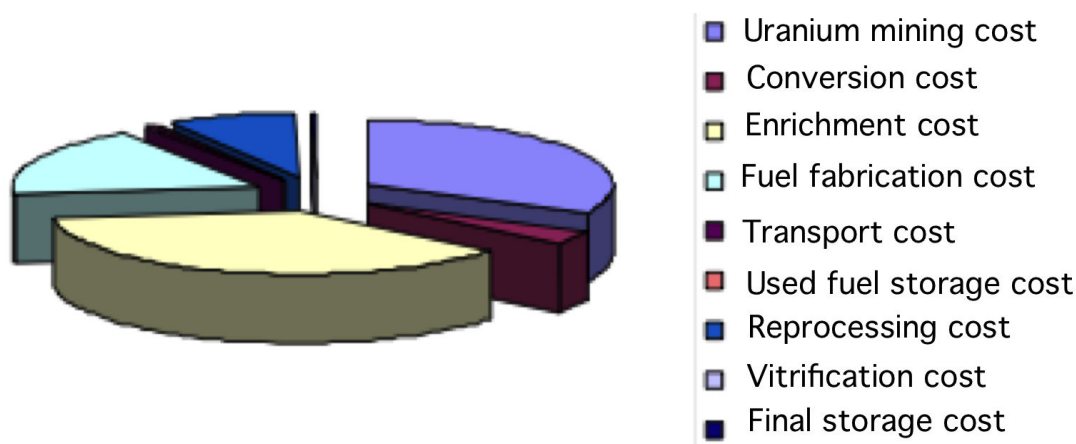


Figure 1.2: Decomposition of nuclear fuel cost (8% discount rate), Source: DGEMP & DIDEME (2003, 2008)

As we can see in figure 1.2 the nuclear fuel cost includes all the costs associated to the front and the back end of nuclear fuel cycle. These costs represent the 15 % of the total electricity cost coming from nuclear, which explains partially why nuclear production cost is not very sensitive to the fluctuations of fuel price (price of uranium) (see Figure 1.1 on page 35). For an EPR the fuel cycle extends up to 24 months while for other types of reactor (e.g. PWR) we meet 12 to 24 months cycles (Source: AREVA (2005)).

#### 1.2.4 Nuclear production cost VS Fossil fuels production cost

In view of the structure of the nuclear production cost, nuclear generation technology is characterized by high fixed costs and long periods of construction which can not be

avoided because of the concept of nuclear reactors and their technological complexities. These main characteristics of nuclear imply financial risks higher than those associated with fossil fuel technologies (e.g. coal, gas, oil) which have lower investment costs and shorter periods of construction than nuclear technology. The production cost of nuclear energy as well as the differences between the production cost of nuclear and of fossil fuel generation technologies can be seen in the following reports.

### **Report of DGEMP & DIDEME (2003, 2008) and of NEA/IEA (2010)**

The study carried out by DGEMP & DIDEME in 2003 and 2008 and the joint report by the International Energy Agency (IEA) and the Nuclear Energy Agency in 2010 (NEA/IEA (2010)) clearly show the great differences in cost structure between nuclear power and fossil fuel generation in France. More precisely, we can see high levels of investment cost and low levels of operation and maintenance and fuel costs for nuclear (see Tables 1.2 and 1.3 on page 43). This is not the case for coal and gas whose investment costs are lower and their operation and maintenance costs as well as their fuel costs are significantly higher than those of nuclear plants. We also observe that the construction period of a unit is more important for nuclear than for coal and gas. Additionally, the lifetime of a nuclear unit is more significant than the lifetime of a coal or gas unit which contributes to the amortization of its high fixed costs. However, we observe negligible differences between nuclear and fossil fuel with regards to the unit capability factor of a unit for a baseload operation (8760 hours of operation) in the report of DGEMP & DIDEME while a standard unit capability factor of 85% has been chosen for nuclear, coal and gas plants in the report of NEA/IEA. The unit capability factor ( $K_d$ ) is the ratio between the available energy during a reference period, and the multiplication of the maximal capacity of the unit by the duration of the same reference period. It illustrates the ability of a reactor to provide energy. This energy is not necessarily called by the electrical grid. The periods of unavailability include the periodical shut-down of nuclear reactors for reloading, maintenance and security inspections as well as the unforeseen shut-down of nuclear reactors in the case of incidents. The unit capability factor is of considerable importance for the economics of power generation, since it defines the amount of electricity

produced per unit of generating capacity that will earn revenues to cover both the capital and the operating costs of a power plant.

Euros/MWh (8760 hours of operation)	Nuclear (EPR)	Coal	Natural Gas
Investment cost	16.3	14.1	6.2
Operation and Maintenance costs	5.1	6.8	4
Fuel cost	4.4	11.7	22.1
Total cost	25.8	32.6	32.3
Total cost + CO <sub>2</sub> cost (20 euros/tonne, 1 dollar = 1 euro)	25.8	48.2	39.6
Percentage			
Investment cost	64%	43%	19%
Operation and Maintenance cost	19%	21%	12%
Fuel cost	17%	36%	69%
Total cost (excluding CO <sub>2</sub> cost)	100%	100%	100%
Construction period (in months)	67	36	26
Lifetime of a unit (in years)	60	35	25
Unit capability factor (in percentage)	90.3%	89.3%	92.2%

Table 1.2: Cost per MWh (8% discount rate), construction period, lifetime, unit capability factor evaluated in 2007, Source: DGEMP & DIDEME (2003)

Euros/MWh (8760 hours of operation)	Nuclear (EPR)	Coal	Natural Gas
Investment cost	67.06	—	—
Operation and Maintenance costs	16	—	—
Fuel cost	9.33	—	—
Total cost	92.38	—	—
Total cost + CO <sub>2</sub> cost (30 euros/tonne, 1 dollar = 1 euro)	92.38	—	—
Percentage			
Investment cost	73%	—	—
Operation and Maintenance cost	17%	—	—
Fuel cost	10%	—	—
Total cost (excluding CO <sub>2</sub> cost)	100%	—	—
Construction period (in months)	84	48	24
Lifetime of a unit (in years)	60	40	30
Unit capability factor (in percentage)	85%	85%	85%

Table 1.3: Cost per MWh (10% discount rate), construction period, lifetime, unit capability factor in 2010, Source: NEA/IEA (2010)

The estimation of the above costs is based on assumptions which have been taken into account by the report of DGEMP & DIDEME and the joint report of NEA/IEA and concern e.g. discount rates (8% for DGEMP & DIDEME and 10% for NEA/IEA), exchange rates (1 dollar = 1 euro), operation hours (8760 hours of operation), CO<sub>2</sub> cost

(20 euros/tonne for DGEMP & DIDEME and 30 euros/tonne for NEA/IEA) whose most likely trend in the long-term is the increase, etc. The omission of the costs for coal and gas in the table 1.3 is due to the absence of French data in the report of NEA/IEA.

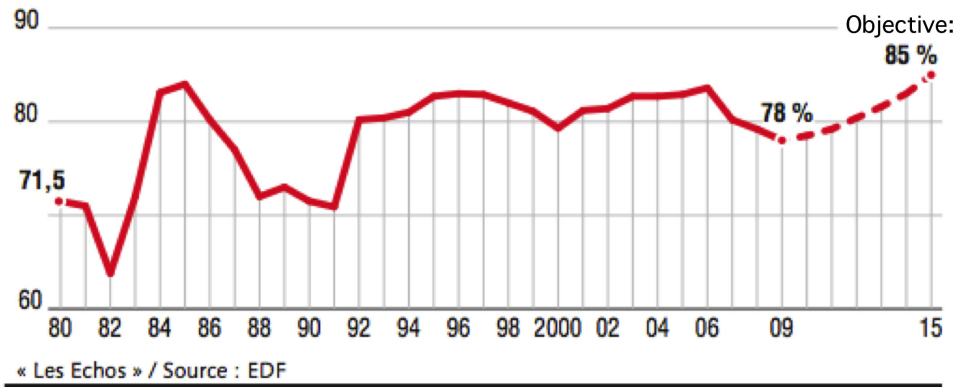


Figure 1.3: Unit capability factor of nuclear plants - France, Source EDF.

This unit capability factor of nuclear (90.3% and 85% respectively) is calculated on a theoretical framework defined by DGEMP & DIDEME and NEA/IEA respectively. As one can see in figure 1.3, the unit capability factor varies every year and in particular it dropped to 78% in 2009, its lowest level since 1993. The operator of the French nuclear set (EDF<sup>7</sup>) aims currently at an unit capability factor of 85% by 2015.

### Sensitivity analysis of the DGEMP & DIDEME report (2003, 2008)

The sensitivity analysis presented in the reference costs of the DGEC (DGEMP & DIDEME (2003, 2008)) makes obvious the higher impact of investment cost on the nuclear production cost than on the production cost of coal and gas (see Table 1.4 on page 45). In particular, we can see that an increase of 10% on the investment cost leads to a more significant augmentation of the nuclear production cost (1.63 Euros/MWh) than of the production cost of coal (1.4 Euros/MWh) and gas (0.7 Euros/MWh). Furthermore, in view of the differences in the lifetime of a unit for nuclear, coal and gas, an important

<sup>7</sup>EDF specialises in electricity, from engineering to distribution. The company's operations include: electricity generation and distribution; power plant design, construction and dismantling; energy trading; transport. The company is characterized by the dominance of nuclear power in its production segment. It operates a set of 58 nuclear reactors in France.

extension of the lifetime of a nuclear unit (10 years) is needed in order to decrease the production cost by 0.1 Euros/MWh while a less important increase of the lifetime of a non-nuclear thermal unit (5 years) is sufficient in order to obtain a more important reduction of the production cost (0.3 Euros/MWh). Symmetrically, a decrease of 10 years of the lifetime of a nuclear unit implies an augmentation of the production cost (0.19 Euros/MWh) less significant than the one (0.5 Euros/MWh) obtained by a decrease of 5 years of the lifetime of a non-nuclear thermal unit. In both cases, the impact of lifetime of a unit on the production cost is more important for coal and gas than for nuclear because of the lower lifetimes of coal and gas units with respect to nuclear units. The impact of the unit capability factor is more important for nuclear and coal than for gas because of the greater fixed production costs of these technologies (nuclear and coal) with respect to gas.

Nuclear	Variation	Impact on production costs (Euros/MWh)
Investment cost	+/- 10%	+/- 1.63
Lifetime of a unit	+/- 10 years	-0.1/ + 0.19
Unit capability factor	+/- 1%	+/- 0.2
Coal	Variation	Impact on production costs (Euros/MWh)
Investment cost	+/- 10%	+/- 1.4
Lifetime of a unit	+/- 5 years	-0.3/ + 0.5
Unit capability factor	+/- 1%	+/- 0.2
Natural Gas	Variation	Impact on production costs (Euros/MWh)
Investment cost	+/- 10%	+/- 0.7
Lifetime of a unit	+/- 5 years	-0.3/ + 0.5
Unit capability factor	+/- 1%	+/- 0.08

Table 1.4: Sensitivity of production cost (Euros/MWh) evaluated in 2007 (8% discount rate, 8760 hours of operation), Source: DGEMP & DIDEME (2003)

### Sensitivity analysis of the NEA/IEA report (2010)

The sensitivity analysis in the report of NEA/IEA tests the sensitivity of the results of the cost calculations to variations in the underlying assumptions on key parameters such as discount rates, construction costs, construction times, fuel and CO<sub>2</sub> prices, lifetime of plants and availability factors (NEA/IEA (2010)). Uncertainties regarding these variables, and their resulting risks, are a reality for energy markets. In addition, all these parameters vary widely across different countries, and even within countries. The economics of nuclear



energy are largely dependent on total investment costs, which are determined by both construction cost and the discount rate. At a 5% discount rate, the key driver of the total production cost of nuclear power is construction costs, while at 10%, discount rates have a larger impact on the production cost than any other parameter. A reduction in construction time also has a significant impact on total costs due to increased interest during construction. Construction delays, on the other hand, have a lower impact on costs given that the total budget remains constant which is generally an unrealistic assumption. In practice, cost delays often entail cost overruns. Early retirement of a nuclear plant has a greater effect on total production cost than its lifetime extension beyond 60 years, mainly due to the discounting effect. Moreover, given the small share of fuel cost in total cost, variations on nuclear fuel prices and services have the least impact on total production. This low degree of exposure to the fuel price risk is one of the advantages of nuclear energy.

The significant impact of discount rates on total generation costs for most technologies has to be taken into consideration in cost calculations. Logically, with an increased cost of capital the total generation cost for all technologies increases. The first observation is the relative stability of the cost of gas-fired power and hence its relative insensitivity to discount rate changes. At the other end of the spectrum, nuclear power is the most sensitive technology to discount rate changes, due to the fact that it has longer construction times than any other technology. Higher discount rates also lower the benefit from longer operating lifetimes of nuclear power plants. Hence, the structure and cost of financing is of considerable importance to investments in nuclear capacity. Total production costs of nuclear are also very sensitive to the construction cost variation since the share of the total investment cost is particularly high for nuclear. In addition, nuclear being a capital-intensive technology for which interests during construction represent a significant cost component demonstrate higher sensitivity to lengthier construction times than other technologies (e.g. gas). Note that construction costs are assumed to be uniformly spread over the construction period within this report. When this is not the case, for instance in the construction of certain nuclear plants where most construction expenditure is in the last four to five years, the impact on cost will be lower than shown in this analysis. Furthermore, since nuclear and coal have much higher fixed costs than alternative fossil-

fuelled baseload generating technologies, their total production cost is most affected by the variation of the unit capability factor (there where fixed costs weigh more heavily). Finally, according to the same report, technologies with longer lifetimes are less affected by relative variations in the operating lifetime of the plant. For example, despite its high fixed costs, which need to be recovered with the revenues produced over the entire lifetime, any extension of the lifetime of a nuclear power plant beyond 60 years has very little impact on the total production cost once costs and revenues for the concerned period are discounted. Once the plant has already recovered the initial capital investment over the original payback period, further extensions will naturally generate additional revenues for the plant; however, due to the discounting effect, revenues accruing far ahead in the future have little impact on production cost after being discounted. In contrast, early retirement significantly increases the total production cost. Indeed, once the plant has been commissioned, and the bulk of the investment cost has been incurred, an early retirement of the plant significantly affects its ability to pay back the initial capital investment.

### MIT reports (2003, 2009)

(MIT 2003)	Nuclear	Coal	Gas
Overnight cost (Dollar/KW)	2000	1300	500
Fuel cost (Dollar/KWh (1 KWh= 0.003412 MMBtu))	0.0016	0.0041	0.012
(MIT 2009)	Nuclear	Coal	Gas
Overnight cost (Dollar/KW)	4000	2300	850
Fuel cost (Dollar/KWh)	0.0023	0.0089	0.024

Table 1.5: Cost evaluated in 2003 and updated in 2009, Source: MIT (2009)

The study realized by MIT in 2003 and its update in 2009 (MIT (2003), MIT (2009)) also show the important differences between the nuclear production cost (higher overnight<sup>8</sup> capital costs – lower fuel costs) and the production cost of coal and gas (lower overnight capital costs – higher fuel costs) (see Table 1.5). Furthermore, we observe the significant increase of all the cost components for both nuclear and fossil generation technologies following the update on the 2003 study. According to this study, in competitive markets,

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<sup>8</sup>In this study, the overnight capital cost represents the initial construction cost plus engineering expenses plus pre-operating costs. Unlike the investment cost, the overnight cost here does not take account the decommissioning cost and the expenses resulting from uncertainties associated with the construction schedule as well as the interim interests.

nuclear power is not a cost competitive choice in comparison to coal and natural gas because of its high fixed costs and the crucial construction time. Moreover, unlike other energy technologies, nuclear power requires significant government involvement because of safety, proliferation and waste concerns. However, as specified by this work, the nuclear option should be retained, precisely because it is an important carbon-free source of power (Percebois (2012)).

### **Report of Cour des Comptes (2012)**

The report undertaken by the Cour des Comptes in 2012 shows that the construction cost per (MW) of nuclear has risen over time (Cour des Comptes (2012)). More precisely, the initial construction cost, including engineering, expressed in terms of reactor power has risen over time from 1.07 million Euros/MW in 1978 to 2.06 million Euros/MW in 2000 (Chooz 1 and 2) or 1.37 million Euros/MW in 2002 (Civaux) with an average of 1.25 million Euros/MW for all 58 reactors. These rising costs can be largely explained by safety baselines, which have become increasingly stringent over time. Furthermore, the Cour des Comptes observes that the construction costs per megawatt continues to rise with the new generation of reactors (Generation III+ reactors e.g. EPR) which must meet extremely strict safety requirements, right from the construction stage. With an estimated construction cost of 6 billion euros for the Flamanville EPR (a first of a kind) and 1.630 MW capacity the cost per MW is 3.7 million euros. With an estimated cost of 5 billion euros for the standard EPR, the cost per MW works out at 3.1 million euros. According to this report the maintenance investments are also set to increase. Maintenance investments are aimed at guaranteeing reactor performance in terms of power generation, gradually enhancing security and safety and, where desirable, extending plant service life. Maintenance investments have been on a downward trend since 2000. This situation has drawn attention to their importance as the drop has had a negative impact on the nuclear fleet's unit availability factor and, consequently, led to a fall in power output. The need to improve safety at nuclear power plants is even greater than before, with the high standard set by the EPR and in light of the accident that occurred at Fukushima in March 2011. The EDF maintenance investment program for 2011 – 2025,

which was prepared in 2010, amounted to 50 billion euros, or an average of 3.3 billion euros per year. This is nearly twice the amount invested in 2010 (1.7 billion euros), a figure that was already up in previous years. The investments needed to meet French Safety Authority (Autorité de Sûreté Nucléaire (ASN)) requirements under the complementary safety assessment initiative following the Fukushima disaster are currently estimated at around 10 billion euros, of which half was already accounted for in the initial 50 billion euros programme. Consequently, maintenance investments should reach a yearly average of 3.7 billion euros, for a programme in the region of 55 billion euros covering the period from 2011 to 2025.

The generating cost provided by this report is not the cost currently calculated in some international comparisons such as those made by the Nuclear Energy Agency (NEA/IEA (2010)), or compared to that of other energy sources, such as in the reference costs of the DGEC (DGEMP & DIDEME (2003, 2008)). In both the above cases, in addition to the capital cost, which can be calculated using still other methods, the cost is calculated for an investor arriving on the market today with new nuclear power plants, such as the EPR in the case of France. This calculation, which simulates the fictitious cost of a fictitious fleet, is very theoretical. At the present time, the Cour des Comptes only knows the estimated construction cost of 6 billion euros for the Flamanville EPR, which gives a generating cost of at least 70 to 90 Euros/MWh, and bearing in mind that this is not the cost of a “standard” EPR. The Cour des Comptes is unable to validate these estimated costs while the site is still under construction. It is therefore far too early for it to calculate and validate a generating cost for an EPR fleet.

### **1.2.5 Operation of nuclear power plants at baseload**

The high fixed costs of nuclear are the main reason why nuclear power plants are preferentially operated as baseload plants. Baseload plants are the production facilities used to meet some or all of a given region’s continuous energy demand, and produce energy at a constant rate, usually at a low cost relative to other production facilities available to the system. The nuclear plants being operated as baseload plants, produce energy at a constant rate using their full capacity in order to cover their fixed costs.

From a market perspective, nuclear is “economically suitable” to operate as a baseload generation technology because of its relatively low variable costs, which imply a low marginal cost (the cost to produce one additional KWh when operating). This can be seen in the merit order, which is a way of ranking the available technologies of electricity generation in the same order as their marginal costs of production. This ranking results in a combination of different generation technologies to reach the level of demand at a minimum cost. The price in the market is therefore determined by the marginal cost of the “last technology” used to equilibrate supply and demand (perfect competitive case<sup>9</sup>). This technology is also called marginal technology. Following the logic of the merit order, the first generation units called are those producing electricity called “fatal”, i.e. the “lost” electricity if it is not used at any given time (run-of river hydro units, renewable energy units (e.g. wind and solar)). Then, the nuclear units, having low marginal costs, are called, before the non-nuclear thermal units (coal, gas or oil-fired) that produce electricity during consumption peaks. Finally, hydroelectric units with possibility to store water within reservoirs, provide reserves of electricity generation capacity.

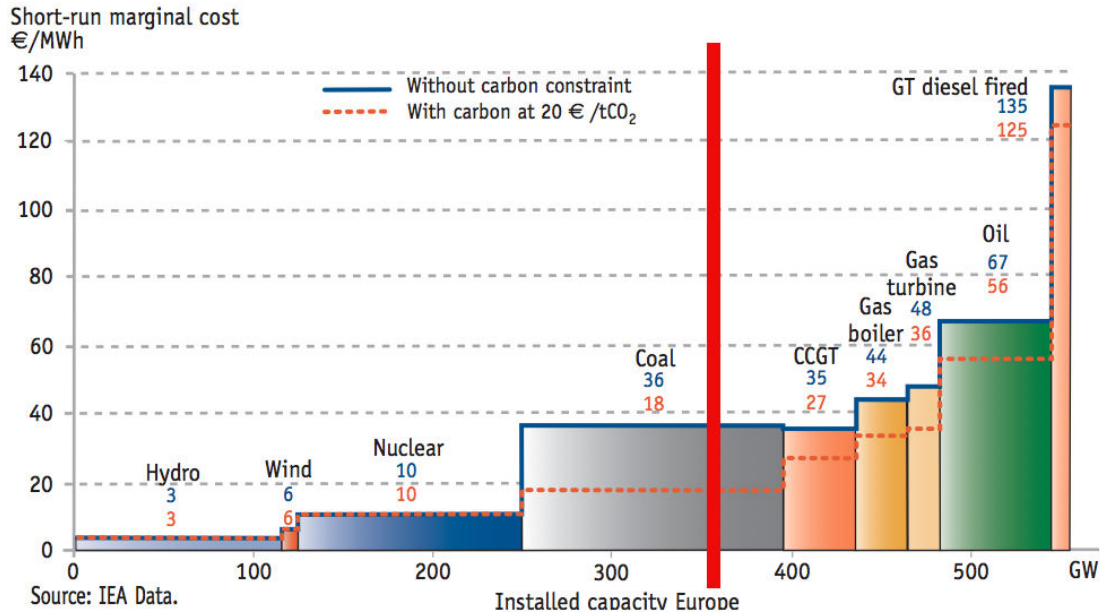


Figure 1.4: European merit order (2005), Source: NEA/IEA

<sup>9</sup>In a perfectly competitive framework, a large number of firms that constitute the supply do not have any affect upon prices. They are considered as “price takers” which means that they “take” the price as it emerges from the equilibrium between supply and demand (Mas Colell et al. (1995)). Thus, they have no market power.

In figure 1.4, we give an illustration<sup>10</sup> of the merit order (see page 50). The marginal costs of the main electricity generation sources in a single EU power market were calculated by the International Energy Agency (IEA) showing clearly that in wholesale electricity markets where competing generators sell their electricity, nuclear power is near the bottom of the merit order and favours baseload operations (NEA/IEA (2005)). It is obvious that the electricity price for consumers is relatively low during periods when nuclear is the marginal technology. This enhances the economic competitiveness of nuclear with respect to other generation technologies and in particular the fossil fuel technologies whose variable costs (and hence their marginal production costs) are higher than those of nuclear (Percebois (2012)).

In conclusion, the nuclear production cost is distinct from the production cost of other generation technologies (e.g. fossil fuel generation technologies) since the exploitation of nuclear energy is based on a relatively complex technology. This technological complexity is reflected in economic terms by investment, operation and maintenance costs being potentially higher than those for energy sources using conventional techniques. In order to give a clear vision of the production cost of nuclear, we look at each component of this cost (investment cost, operation and maintenance costs, nuclear fuel cost) separately. We provide the elements that determine it and differentiate it from the same cost component of other generation technologies (e.g. coal, gas) as well as factors that can affect it. This analysis permits to better a understanding of the specificities of the cost of nuclear production and hence why the baseload operation is preferentially applied to nuclear power plants.

### 1.3 Operation of a nuclear generation set: The case of France

In the previous section, we gave an analytical description of the different costs (investment cost, operation and maintenance cost and fuel cost) that constitute the nuclear production

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<sup>10</sup>In this example, the position of the vertical red line which represents demand implies that coal is the marginal technology. In fact, the installed capacity is equal to the available capacity that serves demand plus the non available capacity.

cost. We also mention that the combination of high fixed costs and of low variable costs leads nuclear to operate as a baseload generation technology. However, the operation of nuclear power plants differs among countries. In this section, we focus on the operation of the nuclear generation set in France which is distinct from other countries (like the UK or Sweden) because of the far higher share of nuclear generation in the energy mix (IEA (2008)). This leads to an operation (called load-following operation) very different from the baseload operation since according to it, the amount of electricity supplied by a generating system at any given time (load) follows the predicted evolutions of the energy demand.

In view of the high fixed costs of nuclear, operators would want their nuclear plants to run constantly at full capacity in order to amortize their fixed costs. Furthermore, within the electricity market, the low fuel and low variable operation and maintenance costs, place nuclear at the bottom of the merit order behind the fossil fuel technologies (i.e. coal, gas, oil, etc.) making nuclear “economically suitable” to operate at full-load. On the contrary, coal, gas and oil having greater variable costs are better suited economically for follow-up load. Consequently, we deduce that nuclear operators would not prefer their units to take part in load-following operations but to be operated as baseload units.

However, when a system has a very large proportion of nuclear power, nuclear power plants must inevitably load-follow. France for instance, constitutes an interesting case very different from other countries like the UK (15.7% of electricity generated by nuclear) or Sweden (38.1% of electricity generated by nuclear) because of the significantly high participation of nuclear in its energy mix. In France, there exist 58 nuclear reactors operated by Electricity of France (Électricité de France (EDF)), being the main producer of electricity, with a total capacity of 63 GW. The share of nuclear in the total electricity generation is around 80% which makes nuclear the main electricity generation technology compared to other countries. Consequently, most nuclear plants have to operate occasionally at semi-base load that corresponds to less than 5000 hours of operation per year and responds to a part of the variable demand. In addition, some plants must be sufficiently flexible (capable to increase or decrease electricity output quickly to follow demand’s variations (daily or intra-daily)) if the French grid operator (RTE) asks them to in order

to balance supply and demand and therefore ensure the stability of the electricity grid (Nuttall and Pouret (2007), Bruynooghe et al. (2010)).

Another motivation for load following with nuclear power plants comes from the large-scale deployment of intermittent electricity sources (like wind power). If there is an important share of intermittent and nuclear power sources on the same electricity grid, the nuclear power plants must be capable to operate in a load following mode to balance the fluctuations of the total power generation, and in this case unexpected large and rapid modulations of the power demand could occur (NEA/AEN (2011)).

From a technical point of view, nuclear reactors of modern design (the third generation and its evolution III+) are capable of a flexible operation (Nuttall and Pouret (2007)). The key to the flexible operation of nuclear power plants is the ability to adjust electricity output quickly, but evenly; that is to say, to adjust output power without overly disturbing the neutron flux distribution within the reactor core (e.g. PWR reactor design (Guesdon et al. (1985))). In fact, this flexibility is primarily due to the new types of fuels which affect the constraints that determine the speed of increase and decrease of production. This type of constraint (called ramping rate constraints) binds the change of operation level of a unit between two successive periods. In principle, all nuclear reactors might reasonably be regarded as having some capacity to follow load. In practice, however, the ability to meet grid needs efficiently and safely is restricted to a certain set of design types (for technical engineering, safety and licensing reasons). The new reactor EPR, which is an evolution of the pressurized water reactor (PWR), is an example of a III+ generation nuclear reactor which is designed to accommodate load-following operation (AREVA (2005), Goldberg and Rosner (2012)).

The monitoring report realized by the French energy regulator (CRE) in 2007 gives an illustration of the operation of the French nuclear set (Regulatory Commission of Energy (2007)). It illustrates the way that the nuclear generation set is managed in France, focusing on the flexibility and the load-following ability which characterize it.

As stated in the report of CRE, the nuclear fleet has been operated at baseload (share of constant consumption throughout the year) and partly at semi-base load (part of the



variable consumption): it followed the modulation of supply between seasons and intraday. In view of the significant share of nuclear energy in the total electricity generation in France, the allocation of the shutdowns of nuclear units between high demand seasons (winter) and low demand seasons (summer) during a year is an important issue for the French operator. Therefore, the majority of shutdowns for the reloading of the fuel in the reactors was programmed during the summer season because of the low levels of demand. This permitted the release of the essential nuclear capacity in the winter, when demand was high. In this way, the nuclear industry has contributed, with cogeneration<sup>11</sup>, to respond to the seasonal variations of demand during the year. They also observed that during periods of low demand, the non-nuclear thermal generation technologies have reduced their production to the minimum (especially at night) and thus, nuclear frequently became the marginal means of production of the French set. As a result, the marginal cost of nuclear determined the market price during periods of low demand and this led to low electricity prices.

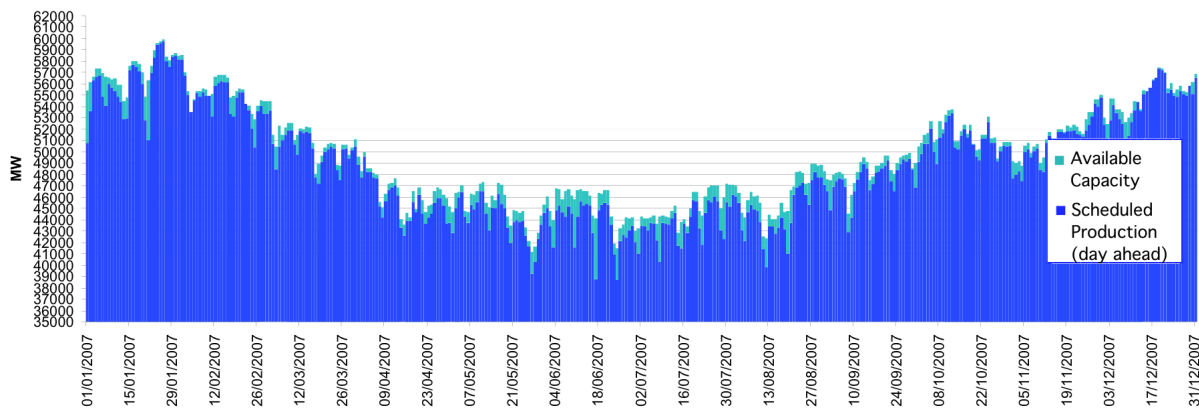


Figure 1.5: Evolution of scheduled nuclear production in 2007/Daily average production levels, Source: RTE – Analysis: CRE

As the figure<sup>12</sup> 1.5 shows, the availability of the nuclear fleet was concentrated in winter. In particular, they observed that the available capacity (after deduction of reserves) was on average more than 57.000 MW in January compared to 45.000 MW in July. Hence, nuclear participated significantly (more than 12 GW gap between January and July) in

<sup>11</sup>The production of electricity and other energy jointly, especially the utilization of the steam left over from electricity generation to produce heat.

<sup>12</sup>In this figure, the available capacity corresponds to the maximum technical capacity from which the capacity used for the “reserve” has been subtracted.

the modulation of supply between winter and summer (seasonality<sup>13</sup>).

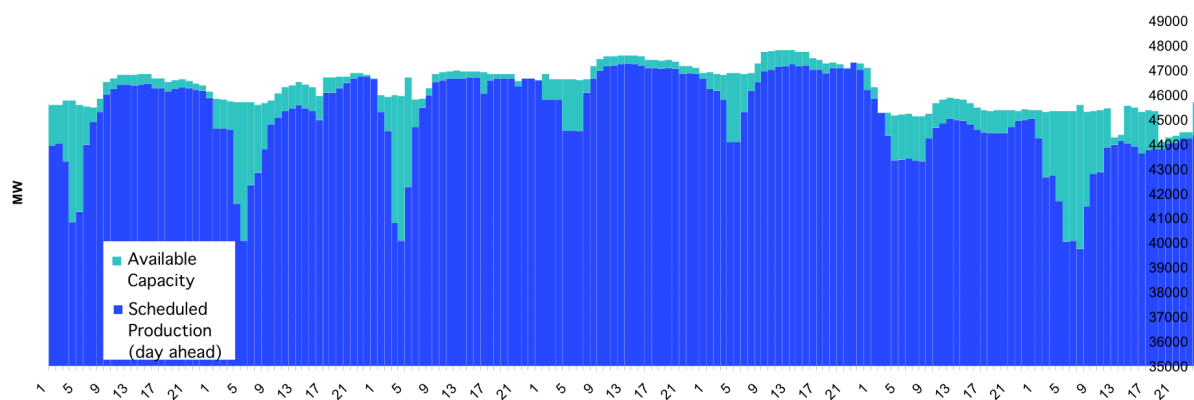


Figure 1.6: Evolution of scheduled nuclear production for a given week/Hourly average production levels, Source: RTE – Analysis: CRE

Figure<sup>14</sup> 1.6, presents the evolution of the scheduled nuclear production for a given week (hourly average production levels). The nuclear set has been operated almost all day at full capacity. However, it was called upon to reduce its production during hours of low consumption (in particular, between 2 and 5 a.m.). During these hours, the nuclear set was able to quickly vary the delivered total capacity to several GW (up to 4750 MW in an hour, maximum observed in 2007) in order to follow to some extent variations in demand, upwards and downwards. According to CRE, it is not uncommon for a nuclear unit to reduce its production to the technical minimum (be several hundreds of MW) for a few hours to respond to a decrease in consumption.

A nuclear unit can vary its capacity level between the nominal capacity and the technical minimum. For example, most PWR nuclear reactors are capable of following loads in a power range of 30 - 100% of nominal capacity at rates from 1 to 5% of nominal capacity per minute (ramping rates) depending on the type of reactor. However, in France, nuclear power plants do not operate in the load-following mode during the first 2 weeks of the fuel cycle and during the last 5 - 20% of the fuel cycle (depending on the reactor type). More precisely, for a PWR reactor whose nominal capacity is 900 MW, the load follow is implemented at a maximum speed<sup>15</sup> of 2% of nominal capacity per minute (till 80% of the fuel cycle) and 0.2% of nominal capacity per minute (after 80% of the fuel cycle). For

<sup>13</sup>Fluctuation according to the season or time of year.

<sup>14</sup>cf. Footnote 12.

<sup>15</sup>This is the maximum rate of change of the operation level of a PWR.

a PWR reactor whose nominal capacity is 1500 MW, the load follow is implemented at a maximum speed of 5% of nominal capacity per minute (till 80% of the fuel cycle) and 2% of nominal capacity per minute (after 80% of the fuel cycle). Finally, for reactors of type N4, the load follow is applied at a maximum speed of 5% of nominal capacity per minute. The load-following ramping rates presented above for PWR and N4 nuclear reactors show the load-following capability of nuclear power plants in France though one may notice a significant improvement of maneuverability capability through time (NEA/AEN (2011)). In the case of an EPR, load follow enables planned variations in energy demand to be followed and can be activated between 25% of nominal capacity (technical minimum) and 100% of nominal capacity (technical maximum) (NEA/AEN (2011)). In particular, two load follow profiles are provided: (a) A “light” load follow, between 60 % of nominal capacity and 100% of nominal capacity at a maximum speed<sup>16</sup> of 5% of nominal capacity per minute (ramping rate), (b) A “deep” load follow between 25% of nominal capacity and 60% of nominal capacity at a maximum speed of 2.5% of nominal capacity per minute (ramping rate).

The minimum requirements for the maneuverability capabilities of modern reactors are defined by the utilities requirements that are based on the requirements of the grid operators. For example, according to the current version of the European Utilities Requirements (EUR) the nuclear power plant must at least be capable of daily load cycling operation between 50% and 100% of its nominal capacity, with a rate of change of electric output of 3 - 5% of nominal capacity per minute (ramping rate). Most of the modern designs implement even higher maneuverability capabilities, with the possibility of planned or unplanned load-following in a wide power range and with ramps of 5% of nominal capacity per minute. Today, some reactors in France operate in the load-following mode with large daily power variations of about 50% of nominal capacity (NEA/AEN (2011)).

The economic consequences of load-following are mainly related to the reduction of the load factor<sup>17</sup> defining the amount of electricity effectively produced by the plant. In the case of nuclear, fuel costs represent a small fraction of the electricity generating cost,

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<sup>16</sup>This is the maximum rate of change of the operation level of an EPR.

<sup>17</sup>Load factor is the ratio between the net energy produced during a reference period, and the energy that could have been produced at maximum capacity during the same reference period.

if compared with fossil fuel technologies (see page 35, Subsection 1.2). Thus, operating at higher load factors is profitable for nuclear power plants, since they cannot make savings on the fuel cost while not producing electricity. In France, the impact of load-following on the operating factor determining the percentage of maximum energy generation that a plant is capable of supplying to the electrical grid, limited only by factors within the control of plant management, is estimated at about 1.2% (NEA/AEN (2011)).

Since most of the currently used nuclear power plants have been designed<sup>18</sup> for strong maneuverability capabilities (except for some very old nuclear power plants), there is no or very small impact (within the design margins) of load-following on acceleration of ageing of large equipment components and hence on the lifetime of nuclear power plants or on the safety of nuclear reactors (i.e. a higher level of maneuverability does not imply a lower lifetime of a plant or a higher number of safety incidents). Consequently, we deduce that the operation in the load-following mode does not lead to any large additional costs attributable to it (e.g. safety related components, NPP lifetime) especially for recent power plants. Hence, the nuclear cost structure is fundamentally unchangeable. However, we observe a loss of revenue due to the modulation of the nuclear production since nuclear plants do not operate uniquely as baseload plants using the maximum of their capacity constantly to generate electricity during a year. This leads to a decrease of the profit (revenue – cost) for the nuclear producer. Furthermore, modulation costs are insignificant on a daily basis since a nuclear manager can not implement very high variations regarding the operation level of a reactor within a day. Even if a significant share of intermittent and nuclear power sources (e.g. wind power) exists on the same electricity grid and therefore nuclear has to contribute to the grid system services in order to balance the fluctuations of the total power generation, nuclear realizes slight variations at rates from 1 to 2% of nominal capacity per minute that last only a few minutes. However, there is some influence of load-following on the aging of some operational components (e.g. valves), and thus one can expect a slight increase of the maintenance costs. Also, for older plants some additional investment could be needed, especially in instrumentation and control, in order to become eligible for operation in a load-following mode (NEA/AEN (2011)).

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<sup>18</sup>The designers of the modern power plants build the maneuverability capabilities into the reactor projects.

To conclude, in view of the cost structure of the nuclear generation technology, nuclear plants are preferentially used for baseload generation. However, if the share of nuclear in the total electricity production is very important then nuclear plants must inevitably participate in load-following operations; the French nuclear set constitutes a typical example<sup>19</sup> of a very large nuclear set which adjusts its production in order to follow (partially) the variations of the energy demand. We keep in mind that modern nuclear reactors are designed to be flexible according to EUR but no important extra costs result from the load-following operation of the nuclear power plants.

## 1.4 The nuclear fuel as a “reservoir”

In section 1.3, we have seen that gas or coal power plants operate a load follow-up, which implies a variable fuel consumption and supply. This is not the case with nuclear power. The existing economics of nuclear power estimate that nuclear plants should always run at full capacity to cover their extremely high fixed costs. In a competitive market, such nuclear plants should roughly be price-takers. This is why nuclear technology is assumed to resemble the hydro run-of-river<sup>20</sup> because the latter does not try to make any follow-up of load. In France, however, the important percentage of electricity generated using nuclear energy implies not running nuclear units strictly as rigid baseload units.

At this point, we assume that we have to distinguish two time horizons of operation of nuclear plants: the short-term and the medium-term. The short-term horizon of operation is related to daily or intra-daily variations of demand while the medium-term horizon of operation is connected to the seasonal variations of demand. The core point of the short-term operation is the daily to intra-daily flexibility of nuclear generation. Can the plant manager adjust its power daily or intra-daily to follow the demand in order to maximize its “costs versus revenue” margin? Of course the nuclear output flexibility depends on the ramping rate constraints that bind the variation of output between two steady production

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<sup>19</sup>Other countries with a high share in nuclear generated electricity, Slovakia (51.8%) and Belgium (51.1%), partly operate in load-following mode.

<sup>20</sup>The run-of-river hydro plants have little or no capacity for energy storage, hence they can not coordinate the output of electricity generation to match consumer demand (Dwivedi et al. (2006)). Consequently, they serve as baseload power plants.

periods. The short-term horizon is therefore organized around a “hard” technological constraint: the operational flexibility of a given nuclear reactor technology. However, different nuclear reactor technologies have different operational flexibilities. In fact, in France that short-term flexibility is quite high for a nuclear set. Optimization problems that take into consideration such operational constraints in the short-term can be found in the articles of Frangioni and Gentile (Frangioni and Gentile (2006)) and of Smeers (Smeers (2007)).

Nevertheless we do believe that the second time horizon - the medium-term - deserves more attention than the short-term. While the short-term horizon is capped by a straight technological constraint, being the operational flexibility of nuclear output, the medium-term horizon appears to be a “pure” question of economic strategy. The core point of the medium-term operation is the seasonal flexibility of nuclear generation. In the medium-term, the nuclear manager has to set its seasonal variation of output according to the demand level. Within this medium-term horizon, the nuclear fuel works as a “reservoir” of energy, partly similar to a water reservoir for hydro energy. In order to understand why nuclear can be viewed as a “reservoir” of energy in the medium-term, it is necessary to know how fuel loading of nuclear reactors is done.

From a technical point of view, the core of a French-like nuclear reactor consists of a bunch of fuel rods containing different types of nuclear fuel (uranium, plutonium or both in the case of MOX<sup>21</sup> fuel) and controlled through neutralizing rods called control rods made of chemical elements (e.g. silver, indium and cadmium). The control rods are used to control the rate of fission of uranium and plutonium and thus the rate of heat released from nuclear fission. These reactors stop periodically to reload their fuel and neutralizing rods with an opening of the core of the reactor. The reloading of a nuclear reactor lasts about 30 days (EDF (2010)). After reloading, a new period (named “campaign”) of nuclear generation starts (see Figure 1.7 on page 60). A campaign of a nuclear unit consists of transforming the potential energy contained in the uranium rods into electricity. The length of a campaign is given by the maximum number of days during which a nuclear unit produces until exhaustion of its fuel of reloading. It generally takes

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<sup>21</sup>We recall that MOX fuel is a nuclear fuel consisting of approximately 7% of plutonium and 93% of depleted uranium.

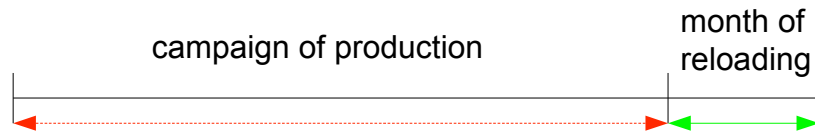


Figure 1.7: Campaign of production

between 12 and 18 months. The general scheduling of fuel reloading of a nuclear unit and hence, the regular length of a campaign depends on many factors: technical specificities of the reactor, size, age, management decisions to reload the reactor’s core per third or quarter of its full capacity (which imply a length of campaign equal to 12 or 18 months (Source: EDF, CEA (2008))), type of nuclear fuel put into the fuel rods, forecasted load and operating factors, regulatory constraints issued by the Nuclear Safety Authority, etc. (World Nuclear Association (2011), CEA (2007), CEA (2008), Bertel and Naudet (2004)). This leads to the following outcomes: *(i)* the normal duration of a campaign of a nuclear unit is determined in advance in order to get an inter-temporal scheduling of reloading which has also to be consistent with the scheduling of all the 58 reactors of the French set, *(ii)* the reloading of a nuclear reactor requires the intervention of many qualified persons external to the nuclear operator. As a consequence, a nuclear unit has a given horizon to manage its fuel stock. The French nuclear operator (EDF), being the manager of the entire nuclear set, is the only responsible for determining the optimal scheduling of fuel reloading (reloading of reactors is to be avoided when the level of demand is high (which is winter in France)) and hence, the length of the campaigns. It does that by using a complex numerical program of high quality called model ORION and analytical internal operational data to which access is not possible for confidentiality reasons.

In view of the operation of reloading of a nuclear reactor, in the medium-term horizon, the nuclear plant manager is allocating a limited and exhaustible amount of nuclear fuel between the different seasons of a campaign, each season having different demand and pricing characteristics. More precisely, we distinguish a season with high demand (respectively high prices) and a season with low demand (respectively low prices). In the european continent, it is correlated respectively with winter and summer. Here a key feature of nuclear fuel as a “reservoir” is based on the discontinuous reloading of the nuclear reactor. Nuclear units stop only periodically to reload their fuel. Then managers have to

determine the length of each campaign of production.

Assuming that nuclear energy has to be sold in the wholesale market, we bet that it will be sold as a stream of “energy blocks”. Energy blocks are fixed quantities sold over a very short period of time at a price determined by the market at each period (then a “spot price”<sup>22</sup> i.e. a market clearing price). The French market has periods of half an hour, which means 48 prices per day, 17520 prices per year. Such spot prices are very volatile from day to day, during the day and throughout the year (see Figure 1.8, Source: Reuters EcoWin). These 17520 prices are essentially determined by three characteristics (hour, work day as opposed to the weekend or at holidays, monthly components). There is a strong seasonal variation described by the monthly components.

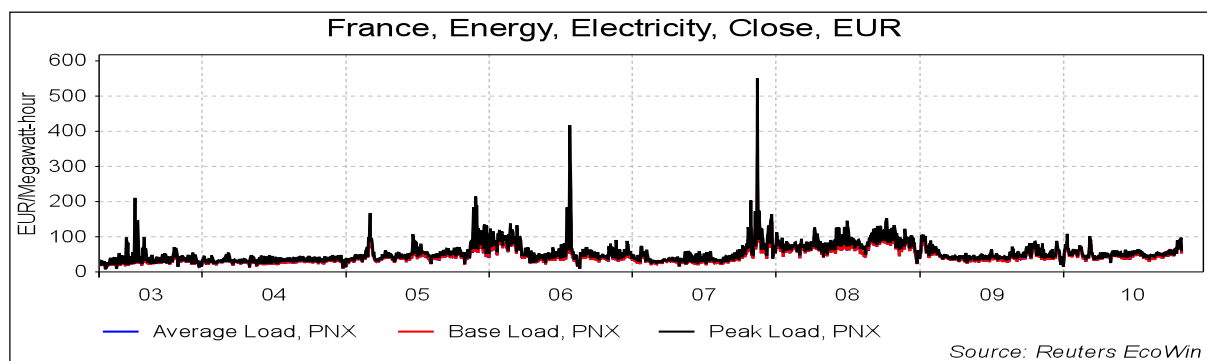


Figure 1.8: Spot prices on the French market during the years 2003 – 2010.

Of course, the total value of the electricity produced during a campaign of nuclear fuel reservoir depends in a crucial way on the temporal profile of generation and how it can respond to the variation of demand and of market price. In a market based electricity industry, the goal of each producer should be the maximization of its profit during a campaign of nuclear fuel reservoir. Can a nuclear producer allocate the nuclear fuel of the reservoir to follow the seasonal demand in order to maximize its profit? To answer this question, we can benefit from an analogy with a hydro producer managing its reservoir<sup>23</sup> and having to allocate the water of its basin between different periods of generation. To

<sup>22</sup>The spot electricity market is actually a day-ahead market, as trading typically terminates the day before delivery. This is due to the fact that the transmission system operator (TSO) needs advanced notice to verify that the schedule is feasible and lies within transmission constraints.

<sup>23</sup>We refer to hydroelectric units which dispose a reservoir to store water. These units run only during periods of high demand (peak load demand) for electricity; the reservoir gives them the possibility to modify the electricity production to balance consumer demand.



model how the reservoir of nuclear fuel will behave at the optimum, we can take into account the existing literature on the optimal management of hydro reservoirs.

The study of the optimal management of a reservoir in a market based hydro-electric system could be useful for the economical modelling of the operation of the nuclear fuel “reservoir” in a competitive electricity market. Articles related to this subject provide us with models that capture the dynamic effect of the hydro-reservoir via multi-period deterministic optimal production problems. A hydro-producer allocates all the water (fuel) stored in the reservoir across periods with different demand and pricing features to maximize its profit. Through the different cases (monopoly/duopoly/oligopoly/perfect competition) considered by the authors we identify the behaviour of a producer who operates its reservoir either strategically to affect the market price or as a price-taker.

To illustrate, we mention a couple of relative articles: Arellano (2004) and Bushnell (1998) analyze the optimal management of a hydro reservoir and the effects of storage in monopolistic and competitive markets. They quantify the inherent advantages that hydro can have for producers in a competitive market because of storage. We mainly study their modelling of the optimal production problem in a mixed hydro-thermal electric system (e.g. market structure, assumptions of the model, exogenous and endogenous variables, production costs, etc.). Arellano (2004) looks at the question of the optimal hydro-scheduling decisions in a monopolistic and a duopolistic power industry and Bushnell (1998) analyzes this question in a deregulated oligopolistic electricity market. The perfect competitive case is also examined by both authors through the existence of small producers acting as price-taking suppliers. Their principal assumption is that the water (fuel) inside the reservoir permits production during a period of time (similar to a campaign of nuclear production). At the end of this period, the water is entirely used and production stops. The hydro-thermal capacities are exogenous variables in their models. We also regard their optimization constraints resulting from the management of the hydro-reservoir and the generation capacity of units as well as the methods used to resolve the optimal production problem for each market structure. In both articles, a producer determines an optimal level of hydro-production at each time period of the game by maximizing its profit over the entire time horizon of the model under minimum

and maximum hydro-thermal generation capacity constraints and a hydro (fuel) storage constraint. According to this constraint, the aggregate hydro-production realized during the entire time period of the model equals the total quantity of water (fuel) stored in the reservoir. The authors apply the Karush-Kuhn-Tucker<sup>24</sup> (KKT) conditions in order to determine a solution of the optimal production problem in each case.

In the same context, we refer to the article of Ambec and Doucet (2003) who suggest a two-period deterministic model in order to study the effects of monopolistic behaviour and of decentralized decision-making in competition on the management of hydro-resources. Here, the optimal production problem consists of maximizing inter-temporal profit discounted between periods subject to supply and storage capacity constraints resulting respectively from the volume of water inflows and the reservoir capacity in a monopolistic market (with one and several hydro-plants) and a competitive market. The Karush-Kuhn-Tucker (KKT) conditions are applied once again in order to resolve the optimal production problem in each type of market.

Scott and Read (1996) propose a multi-period stochastic modelling of optimal operation of hydro reservoirs in a deregulated (oligopolistic) market with both hydro and thermal capacity. In this model, apart from the restrictions on the generation levels (minimum/maximum production constraints), there exist restrictions on the amount of water stored in the reservoir (minimum/maximum water storage constraints) as well as a rule that determines the evolution of water inside the reservoir at each period of production. This rule takes into consideration three different variables in the previous production period to determine the current amount of water stock: (i) the quantity of water stored in the reservoir, (ii) the inflow of water entering in the reservoir and (iii) the production realized during this period. The inflow of water in the reservoir is forecasted with some uncertainty and thus, the optimal production problem is actually stochastic in nature. A generating firm disposing a certain amount of hydro and thermal capacity maximizes its inter-temporal profit under the constraints mentioned above, accounting for contract obligations since each of the generating firms may have contracts with consumers to sell

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<sup>24</sup>In mathematics, the Karush-Kuhn-Tucker (KKT) conditions are first order necessary for a solution in nonlinear programming to be optimal, provided that some regularity conditions (or constraint qualifications) are satisfied (Mas Colell et al. (1995)).

them a pre-arranged quantity of electricity, at a pre-arranged contract price. The authors use the first order optimality conditions to resolve theoretically this problem and the dual dynamic programming approach to resolve it numerically.

Another approach that is not related to the framework of our study but could be considered in the future is the one presented by Crampes and Moreaux (2001 and 2008). The authors focus on the question of how competition in electricity markets works when hydro and thermal units belong to separate owners. They explore the first-best dispatching and the monopoly and the duopoly equilibrium in an economy where the two technologies (hydro - thermal) compete.

There are however differences between the nuclear plants and the hydro storage stations with respect to the characteristic of the “reservoir”. An important point of differentiation is the timing of reloading of the “reservoir”. In the case of nuclear, the timing and the frequency of reloading of the reservoir depends on the producer since it is responsible for the allocation of shutdowns of the nuclear units in order to reload between high and low demand seasons. On the contrary, hydro reservoir stations cannot choose when and how often to reload; only capricious rain will do it when enough has fallen (they have a very typical “seasonal reloading”). Another difference is that during the time of reloading of the nuclear “reservoir”, a unit does not produce. This is because the core of the nuclear reactor opens during the reloading in order to replace some of the used nuclear fuel rods with new ones. Nevertheless, hydro reservoir stations do not stop during the reloading of the reservoir.

A seasonality of reloading has also to be considered in the nuclear case. A “good” seasonal allocation of shutdowns of the nuclear units consists of avoiding shutdowns in high demand periods (winter) and concentrating them as much as possible in low demand periods (between May and September) (see Figure<sup>25</sup> 1.9 on page 65). Thus, the producer takes into account the level of demand when it chooses when to reload the core of the reactor. A fundamental point of the optimization of the French nuclear set is therefore

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<sup>25</sup>Each blue bar shows the number of shut-down units during a week and the red line shows the evolution of the consumption over time. The different levels of consumption are measured on the right axis while the number of shut-down units is reflected on the left axis.

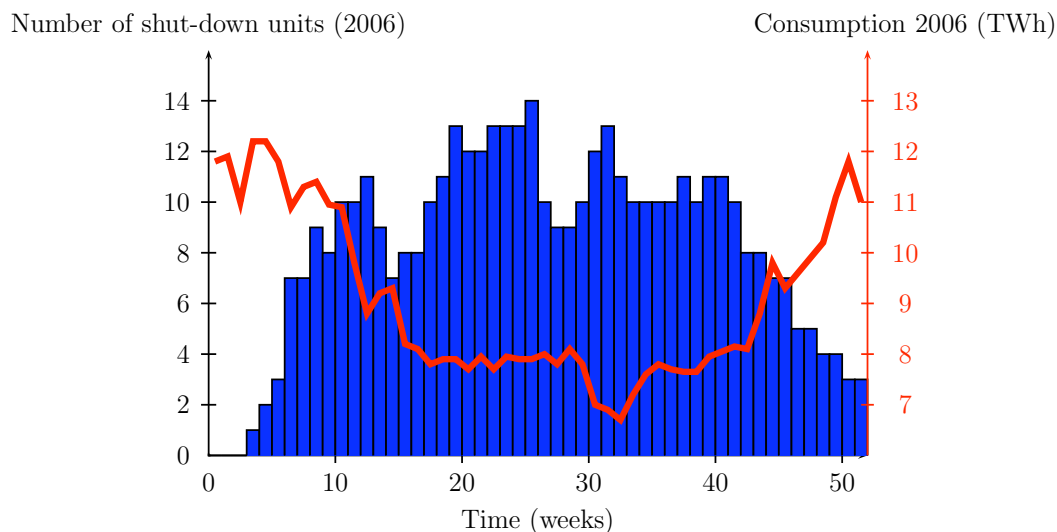


Figure 1.9: Availability of nuclear units, Source: EDF (2006)

the allocation of shutdowns of the nuclear units. Their timing and frequency determine the length of the campaigns.

To conclude, we note that a focus on the medium-term horizon of operation of flexible market based nuclear plants leads us to look at the seasonal flexibility of nuclear generation. In the medium-term, a flexible nuclear set is operated to follow a part of the seasonal variation of the demand level between seasons of high demand and seasons of low demand. In this context, nuclear fuel stock can be analyzed as a “reservoir” given that the nuclear units stop periodically (every 12 or 18 months) to reload their fuel. The operation of the reservoir allows different profiles of nuclear fuel uses during the different seasons of a campaign. A producer would then like to determine the temporal profile of generation to respond to the variation of demand and of price during the campaign in order to maximize its profit. To analyze the management of the nuclear fuel reservoir, we look at the literature related to the optimal scheduling of hydroelectric resources because of its analogy to the hydro reservoir. There exist differences between these two generation technologies (nuclear and hydro) regarding the characteristic of the “reservoir”. These differences mainly concern the timing and the frequency of reloading as well as the fact that production is interrupted during the period of reloading in the case of nuclear. However, a common point is the seasonality of reloading which has to be considered in both cases. Our analysis takes into consideration the inter-temporal management of the nuclear fuel

stock during a period of production that consists of one or more campaigns. It also looks at the constraints imposed by the flexible operation of the nuclear plants, their generation capacity and the balancing of supply and demand in view of the very high reliance on nuclear energy.

## **1.5 A microeconomic model of operation of flexible nuclear plants in a market based electricity system as a case of competition with reservoir**

Over the past two decades a number of countries have moved forward to market liberalization. During the second half of 1996, the adoption of a directive on the creation of an “internal market for electricity” was decisive for the liberalization of the energy sector in Europe. Before the adoption of this directive, other countries like Great Britain and Norway in 1990, as well as Finland and Sweden in 1995 and 1996, had only begun this competitive reform. Three years later, the application of this directive forced a group of 20 countries to simultaneously open their electricity sectors to competition. An economic analysis of the changes in the legislation following the liberalization of the electricity sector can be found in Chevalier (2004), Glachant and Finon (2003), Glachant and Lévêque (2009). This liberalization was also accompanied by the necessity for “unbundling” the vertically integrated monopoly that traditionally managed generation, transport and distribution.

Within this new framework for the electricity production process, the choice and the operation of generation technology may also change. It therefore questions how flexible nuclear plants should be operated in an open market framework. What could be the optimal management of a flexible nuclear set (like the French nuclear set) in a competitive setting? We place ourselves in the medium-term horizon of operation in order to take into account the seasonal variation of the demand level between winter (high demand season like in continental Europe) and summer (low demand season). Within this medium-term horizon, a core feature of the optimal management of flexible market based nuclear is

that nuclear operates as a “reservoir” of energy. Thus, we will look at this question as a rational economic analysis of the operation of a nuclear fuel “reservoir”. While such an analytical frame obviously makes sense from a theoretical point of view, it is both highly unconventional and entirely unexplored. This is mainly due to the existence of a number of difficulties that one faces on a theoretical plan in order to determine the optimal management of flexible nuclear plants in a competitive market.

There are difficulties that result from the relatively high technological complexity that characterizes nuclear as an electricity generation technology. This technological complexity can be seen through the potentially higher investment and operation and maintenance costs than those identified for other energy sources using conventional techniques (e.g. fossil fuel generation technologies). There are several technico-economical constraints that a nuclear producer has to take into account when it searches for an equilibrium of the problem of optimal allocation of the nuclear fuel stock during a campaign of production. These constraints have to do mostly with the flexible operation of nuclear plants (minimum/maximum production constraints) and the operation of the nuclear fuel reservoir (fuel storage constraints).

Another difficult aspect of our problem concerns the implication of competition in the optimal management of a flexible nuclear set. This question of optimal operation of a nuclear set which adjusts its production outcome to the variations of demand (daily and seasonal) has not been raised so far for a competitive market given that France<sup>26</sup> has not fully opened till now its electricity market to competition like other countries e.g. UK, Germany, Austria, etc. (Chevalier (2004)). Indeed, the French historical operator (EDF) holds the total capacity of the nuclear set which offers the majority of the total electricity generation, and thus has a dominant position in its historical national electricity market. Therefore, the creation of a benchmark of optimal operation of a flexible nuclear set in a competitive framework has not been done yet. We build a microeconomic

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<sup>26</sup>The “Nouvelle Organisation du Marché de l’Electricité” (NOME) law indicates the findings of the report of the Commission Champsaur which suggests access to nuclear electricity of the French nuclear operator (EDF) for all producers. Specifically, the NOME law forces EDF to sell at a competitive price to alternative producers of electricity and gas (GDF Suez, E.ON, ENEL, Poweo, Direct Energy, etc.) a quarter of its nuclear production until 2025. This price should include the total cost of the operating nuclear plants.

deterministic dynamic model of optimal management of flexible nuclear plants in a market where producers disposing a certain amount of nuclear capacity compete with each other to maximize their profit and at the same time to even meet energy demand. This benchmark could give some insight with respect to the optimal production behaviour of nuclear producers in a market based electricity system.

The constraints imposed by the equality between supply and demand (called supply-demand equilibrium constraints) play a decisive role in the determination of the optimal management of flexible market based nuclear. In view of the large proportion of nuclear in the electricity system, the global balance between supply and demand depends mainly on the nuclear production. This makes the nuclear set responsible to a large extent for ensuring this balance and preventing potential disruptions of supply which could lead to a “blackout”. As a consequence, each nuclear producer has to take account of constraints intrinsic to the public interest and social welfare. Such a constraint is that of continuous and never ending equality between supply and demand. This constraint must be taken into consideration in the management of market based nuclear so that a failure of production to meet consumption is avoided. Considering these remarks, in the medium-term, the optimal management of flexible nuclear units in a market based electricity system has to take into account the management of the nuclear fuel reservoir so that imbalances between production and consumption are avoided during a campaign of production.

We distinguish two stages regarding the optimal management of the fuel reservoir of a flexible nuclear unit in a competitive setting. We suppose that two flexible types of generation exist: nuclear and non-nuclear thermal. Initially, in the first stage, a producer starting with the optimization of its nuclear generation does not immediately know how to manage all the factors affecting the market equilibrium in the medium-term (all over a fuel campaign). It then reduces the management horizon to that portion of the market that is easier to foresee: the monthly horizon. In this early period of optimization of nuclear generation, each generation units manager playing on a market base aims at the determination of a production profile that: (i) at each month, respects the constraints imposed by the flexible operation of a nuclear unit (e.g. load-following operation of an EPR) and the non-nuclear thermal generation capacity (minimum/maximum production

constraints), *(ii)* respects the constraints imposed by the inter-temporal management of the nuclear fuel stock over the entire time period of production (storage constraints), *(iii)* respects the supply-demand equilibrium constraint at every moment over each month, *(iv)* and maximizes the value of profit for each month. In this stage of optimization, the producers center their interest on the existence of an equilibrium and its determination on a monthly basis. This constitutes a prudent behavior of the producers before they look at a more complex problem which is that of the optimal management of the nuclear fuel reservoir on a yearly or multi-annual basis. This optimization problem will be analyzed later in the second and last stage of optimization of the nuclear production.

In this first stage of optimization, a first approach could be the following: a producer determines each month a feasible production level (it satisfies *(i)*, *(ii)*) by taking account of the equilibrium between supply and demand and by looking at the merit order price during this month to maximize its profit. In this approach, the optimal allocation of nuclear output over time would be extremely simple: produce at full capacity in the months in which prices are highest (and specifically above its variable (or fuel) costs) and produce only at minimum capacity the rest of the time. However, the computation of supply by looking at the merit order price during a month is not possible in our model because of the behaviour of supply with regard to the merit order price. Nuclear production is not a function of the price but a correspondence<sup>27</sup>: one can not associate a single value of nuclear production with the merit order price at each month. In particular, when nuclear is marginal and competitively operated, its own marginal cost is the market price. However, with this constant price, various feasible generation output levels may be associated to maximize profit. Classically, the coordination of producers is realized by the “invisible hand”, a mechanism which is based on the evolution of price and the choice of the optimal production levels according to it. However, it is not the case here considering the fact that nuclear production is a correspondence within our model and thus, price signals can not lead to an equilibrium of the optimal production problem. Inevitably, coordination issues between producers at a production level will come up. Each producer having a range of feasible nuclear production options would have to coordinate with all producers so that the overall equilibrium between demand and production (resulting from all producers) is

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<sup>27</sup>Correspondence is a mathematical concept that extends the concept of function. With each element of the set of departure, we associate a possibly empty subset of the set of arrival.



respected each month and its profit is maximized at the same time. Nevertheless, even in the case that nuclear production was a function and not a correspondence of the merit order price, this approach can not apply because nuclear units managers have to take into account the inter-temporal management of nuclear fuel stock, when they choose how much to produce each month. Therefore, we conclude that producers unable to decide on their optimal production levels by looking at the inter-temporal pattern of prices, they need to look at the pattern of seasonal demand.

In a second approach, we proceed with a short-term optimization of the production subject to constraints imposed by generation capacity, the flexible operation of nuclear reactors, fuel storage and the equality between supply and demand ((*i*), (*ii*), (*iii*)) (Lykidi, Glachant, Gourdel (2010, 2012)). Through this problem, called the optimal short-term production problem, we determine the optimal level of supply of a producer during a month, given the optimal production realized in the previous months. Then, the price is determined by the equality between supply and demand in this month (merit order price rule). The level of stock of the next month is determined by the nuclear supply of the current month. This approach is a prudent research strategy to first see how to analyze the problem on a monthly basis operation's horizon that is easier to apprehend before going to a full time horizon consisting of one or more campaigns of production (typically 36 months).

Then a second stage would be the inter-temporal optimization of the production in the same competitive framework (Lykidi, Glachant, Gourdel (2011)). The producers, being now familiar with the optimal management of nuclear fuel reservoirs on a monthly horizon can be interested in having a complete vision of the optimal allocation of the nuclear fuel stock. Within this approach, the producers realize a full inter-temporal optimization of the production which results from the maximization of the value of profit over the entire time horizon of the model. Within the optimal inter-temporal production problem, we take account of the production constraints (*i*), the storage constraints (*ii*) and the supply-demand equilibrium constraint (*iii*) throughout model's time horizon. The price is given by the equality between supply and demand at each month (merit order price rule). The nuclear supply of the current month determines the level of stock of the following

month. The optimal production decisions resulting from inter-temporal optimization of the production will then be contrasted with the optimal production decisions coming from the short-term optimization.

Finally, we study the social welfare maximization problem in an identical framework. The dominant position of nuclear in the national energy mix makes the nuclear set responsible for the overall equilibrium of the electricity system (balancing supply and demand) which is directly intertwined with social welfare. Therefore, apart from considering the equality between supply and demand in the maximization of the profit, we may choose to proceed with a full optimization of the social welfare. The maximization of social welfare is made under the same constraints with those considered in the optimal short-term production problem and the optimal inter-temporal production problem.

We absolutely do not claim that the French nuclear producers did or do what we are modelling. We only treat a hypothetical case academically while borrowing some key features from the existing world. Of course the French nuclear set is very appealing for us because of the nuclear importance in the energy mix, because the French nuclear set does not entirely operate as baseload, because it has developed a unique load-following management to partly respond to the daily and seasonal variations of demand; also, to end, because the existing economic literature on this precise topic is more than extremely reduced and close to a vacuum.

In conclusion, what we look at is the optimal operation of flexible nuclear plants in a market based operation framework within a medium-term horizon. In this time horizon of operation, we model the nuclear fuel reloaded to the reactor at the beginning of a campaign of production, as a “reservoir” of energy. A reservoir permits different allocations of nuclear fuel between periods of high demand and low demand. Then, we study the optimal management of a nuclear fuel reservoir within a decentralized electricity market where each competitive firm disposes of an amount of nuclear capacity and produces to follow-up the seasonal variations of demand while maximizing its profit. Special attention is given to the supply-demand equilibrium constraints because of the significant size of the nuclear set which is responsible for the stability of the electricity grid. First, the nuclear producers aim at studying the optimal short-term management of the nuclear fuel

reservoir. This is a reasonable starting point to determine what the optimal management of a flexible nuclear fuel stock should be in a competitive electricity market. Then, after a while, these basics being well known, the producers proceed with the inter-temporal optimization of the production in order to determine the optimal operation of the nuclear fuel reservoir in the medium-term. Finally, we look at the social welfare maximization problem within the same framework. The different cases are summarized in Table 1.6 on page 72.

	Objective function	Constraints	Price
Optimal short-term production problem	current monthly profit	<ul style="list-style-type: none"> <li>• nuclear fuel storage constraints</li> </ul>	merit order price
Optimal inter-temporal production problem	inter-temporal profit	<ul style="list-style-type: none"> <li>• minimum/maximum production constraints</li> </ul>	
Social welfare maximization problem	total surplus (surplus of consumer and surplus of producer)	<ul style="list-style-type: none"> <li>• supply-demand equilibrium constraints</li> </ul>	

Table 1.6: Optimization problems

Our model is not based on a classic modelling taken from the economic literature. The optimization of nuclear fuel storage under flexibility of the nuclear production constitutes an entirely new and hence, totally unexplored question which is why no model exists in the literature. Therefore, we add to this literature a novel type of modelling which takes into consideration several parameters such as the operation of flexible nuclear plants, the operation of fuel reloading of a nuclear reactor, the structure of the nuclear production costs, the merit-order price rule, etc. in order to respond to this question.

## 1.6 Conclusion

Since the liberalization of the energy sector, in many countries electricity systems are passing from the vertically integrated monopoly organization to a frame of competitive

markets (e.g. European Union). In such a competitive framework, we addressed the question of the optimal management of a nuclear generation set.

The second section reminds us that nuclear is mainly distinguished from other generation technologies (e.g. fossil fuel) because of its high fixed costs which lead to an operation as baseload generation. Nuclear plants aim at operating at full capacity to cover their high fixed costs. Nuclear has also low fuel and low variable operation and maintenance costs, hence a very low marginal cost. Within the electricity market, nuclear is therefore at the bottom of the merit order and thus is “economically suitable” to operate at its full-load.

However, in the third section we did see that the operation of nuclear plants is not the same for all countries using nuclear power. A characteristic example is that of France. The operation of the French nuclear set is distinct from other countries because of the important contribution of nuclear in the generation (around 80% of electricity comes from the nuclear set). This leads nuclear power plants to operate not strictly at baseload and to demonstrate a load-following ability in order to address the variations (daily or intra-daily and seasonal) of demand.

Then, in the fourth section, we did look at the medium-term horizon of operation to take into account the seasonal variation of the demand level. Within the medium-term horizon, we introduced the notion of nuclear fuel “reservoir” (partly similar to a water reservoir for hydro energy). It is based on the mode of fuel reloading of nuclear reactors. Each nuclear unit stops periodically to reload its fuel and then a campaign of production starts. A nuclear producer allocates a finite and exhaustible amount of nuclear fuel to produce through the different seasons of the campaign. To analyze the management of a reservoir of flexible nuclear units in a competitive market, we reviewed articles on the optimal management of hydro reservoirs. Despite differences between nuclear and hydro storage units regarding the nature of the “reservoir”, a common point is the “seasonality”. It determines the timing and the frequency of reloading during a year.

Finally in our last section, we introduced our model of optimal management of a market based flexible nuclear set considering that nuclear operates as a “reservoir” of

energy. Each nuclear producer aims to maximize its profit under constraints resulting from the flexible operation of nuclear units, the management of its reservoir's nuclear fuel and the equality between supply and demand over the entire time horizon of the model. In the first phase of optimization, the problem is analyzed on a monthly basis in order to understand how to manage the nuclear fuel reservoir in a competitive framework and to examine the existence of an optimal production trajectory before going to the inter-temporal case. Then, in the second and last phase of optimization, we addressed a complete optimization of the production during the whole time horizon of the model. We also raised the issue of maximizing social welfare within an identical framework. These different cases will be studied in the following chapters.

## Chapter 2

# The optimal short-term management of flexible nuclear plants in a market based electricity system

### 2.1 Introduction

As we stated in our first chapter, nuclear generation differentiates itself from other technologies by its important fixed cost and low marginal cost (DGEMP and DIDEME (2003)). Consequently, nuclear is deemed to serve “baseload” to target a constant ribbon of consumption at the bottom of the yearly demand and produce at its maximum capacity. This helps to cover its fixed costs (e.g. United Kingdom, Pouret and Nuttall (2007)). Nevertheless, this mode of operation is not for all countries. An important participation of nuclear in the generation of a country can lead to a different operation. An example of this is France where nuclear generation accounts for 80% of generation. The high share of nuclear in the national mix asks nuclear to not behave as rigid baseload but to adjust its production to (partially) follow the variations of the energy demand (daily and seasonal). This load-following ability of nuclear is crucial in order to ensure the stability of the electricity system.

Furthermore, in numerous countries, electricity systems are quitting the vertically integrated monopoly organization for an operation framed by competitive markets (e.g. European Union). This reopens -both empirically and theoretically- the question of nuclear operation. Economic reasoning supports that in a changing environment, the choice and operation of generation may also change. Consequently, a question arises: what could be the optimal management of a flexible nuclear set in such a competitive setting? Within this new competitive framework, we address the medium-term horizon (1 to 3 years) of management to take into account the fluctuations of demand according to the seasons of year. In the medium-term, the managers of a large nuclear set (like the French) have to set their seasonal variation of output in order to satisfy the seasonal demand. We emphasize two stylized seasons: a season of high demand and a season of low demand. In continental Europe, it corresponds respectively to winter and summer. In this medium-term horizon, a core feature of market based nuclear is that the nuclear fuel works as a “reservoir” of energy - partly similar to a water reservoir of hydro energy. Therefore, we need an economic analysis of the operation of nuclear fuel as a “reservoir” to answer this question. A nuclear “reservoir” contains a limited and exhaustible amount of fuel which has to be allocated between winter (season with high demand) and summer (season with low demand). The reservoir nature of nuclear is based on the discontinuous reloading of the reactor. Nuclear units stop only periodically (from 12 to 18 months) to reload their fuel. Then a new period of production (“campaign” of production) starts. The potential length of each campaign of production is related to the quantity of nuclear fuel stored in the “reservoir” at the beginning of the campaign.

A constraint to take into account for the optimal operation of market based nuclear plants is the equality between supply and demand at any time. This is due to the important size of the nuclear set that makes nuclear the principal generator of electricity and hence responsible for the stability of the electricity system. As a consequence the supply-demand equilibrium constraint is also rooted in social welfare. This has to be considered in the management of market based nuclear. In view of these remarks, in the medium-term, the nuclear fuel reservoirs of flexible market based nuclear units have to be managed so that the equality between supply and demand is respected throughout a campaign of production.

We study the optimal management of the fuel reservoir of a flexible nuclear unit given the decentralization of the nuclear generation. We assume the existence of two flexible types of generation: nuclear and non-nuclear thermal. The optimization of the management of the nuclear fuel reservoir is built in two steps. In this chapter, we focus on the first step towards the monthly optimization of flexible nuclear plants in a market based electricity system while the second step of yearly or multi-annual optimization will be studied in our next chapter (Lykidi, Glachant, Gourdel (2010, 2011, 2012)). At the beginning, the producers do not know immediately how to reach a market equilibrium in the medium term (all over a fuel campaign). For this reason, they reduce their management horizon to that portion of the market being easier to anticipate: the monthly horizon. At this early stage of optimization of nuclear generation, each producer playing in a competitive setting intends to determine a production profile that: *(i)* respects the constraints imposed by the flexible operation of a nuclear unit and the non-nuclear thermal generation capacity for each month (minimum/maximum production constraints), *(ii)* respects the constraints imposed by the inter-temporal management of the nuclear fuel stock over the entire time horizon of production (storage constraints), *(iii)* respects the constraints induced by the overall equality between supply and demand (i.e. total supply equals total demand) at every moment over each month, *(iv)* maximizes the value of profit for each month. The interest of this step is to examine the existence of a possible path of production that verifies the above conditions (*(i)*, *(ii)*, *(iii)*, *(iv)*) on a short-term horizon of operation (the monthly horizon) and determine it. This will serve as a reference for the existence of an equilibrium before the producers proceed with a more complex problem being the inter-temporal optimization of the management of the nuclear fuel reservoir. In the second step, the producers being now acquainted with the optimal operation of the nuclear fuel reservoir on a monthly basis may be interested in increasing their horizon of operation to one or more campaigns of production (typically 36 months). This would permit them to build a medium-term optimal management of the reservoir's nuclear fuel stock.

In the first section (section 2.2), we build a deterministic dynamic model to study the optimal short-term operation of flexible nuclear plants in a perfect competitive setting. The case of perfect competition is presented among other modelling trends of the electrici-



ty market by Ventosa (Ventosa et al. (2005)) and Smeers who uses this context in order to measure market power in restructured electricity systems (Smeers (2007)). We start with a description of the principal assumptions of the model and we then proceed with the modelling of different parameters regarding the electricity demand and the time horizon of the model and of the campaign. In addition, we look at the modelling of the generating units, of the production costs, of the nuclear fuel reservoir and we determine the constraints resulting from the flexible operation of nuclear units, the non-nuclear thermal generation capacity, the nuclear fuel storage and the equality between supply and demand ((i), (ii), (iii)) in the general case of  $N \geq 2$  producers. Producers are mainly differentiated by the amount of the nuclear and non-nuclear thermal capacity that each of them dispose.

In section 2.3, we study the optimal short-term production behaviour. We introduce the notion of the merit order equilibrium in the case of  $N \geq 2$  producers. Then, we present two different approaches to calculate an equilibrium of the optimal short-term management of flexible nuclear plants in a competitive electricity market.

In a first approach, we study the behaviour of supply with respect to the merit order price. We show that, within our model, it is not possible to compute a feasible production vector (it satisfies (i), (ii)) every month that respects the equality between supply and demand by looking at the value of the merit order price in order to maximize profit. This is mainly because nuclear supply is a correspondence<sup>1</sup> and not a function of the merit order price; several feasible production levels (and not a sole feasible production level) may be associated with the merit order price during a month. Thus, every producer decides on its nuclear production level among a continuum of feasible production levels to satisfy the overall equilibrium between supply and demand and maximize its profit. This ambiguity in the individual choice of a single feasible output level would necessitate a potential coordination of the producers in order to achieve maximization of profits and the global equilibrium between supply and demand each month. From the behaviour of supply with respect to the merit order price, we conclude that the nuclear production

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<sup>1</sup>Correspondence is a mathematical concept that extends the notion of function. With each element of the set of departure, we associate a possibly empty subset of the set of arrival. The notion of continuity exists for a correspondence but is significantly more complicated. Numerical optimization of a general correspondence is not possible since numerical software is designed for functions and not for correspondences. Numerical tools are often based on the concept of derivative which is not transposed on the correspondence.

being a correspondence leads to inexistence of equilibrium and hence, the producers need to make their optimal production decisions on the basis of the seasonal variations in demand and not in market price.

In a second approach, we propose a short-term optimization of the production under constraints imposed by the flexible operation of nuclear units, generation capacity, fuel storage and the equality between supply and demand ((i), (ii), (iii)). This optimization problem, called the optimal short-term production problem, consists in determining the level of production that maximizes the profit in the course of a month, given the optimal production realized in the previous months. Then, the price is determined by the equality between supply and demand in this month (merit order price rule). The level of stock of the next month is determined by the nuclear supply of the current month. This approach permits us to see how to optimize production on a monthly horizon of operation that is easier to apprehend before proceeding with an inter-temporal optimization of the production. Within this approach, we distinguish two different tactics of resolving the optimal short-term production problem: according to the first tactic, a producer determines an equilibrium of the optimal short-term production problem by taking into consideration the equality between supply and demand only during the current month of optimization without being concerned about this constraint in future months. This tactic simplifies the resolution of this optimization problem since the supply-demand equilibrium constraint is considered only for a month (current month). However, following this tactic, we see (through a numerical example) that when producers are “short-sighted” with respect to future demand, they can not equilibrate supply and demand in future months respecting at the same time the constraints ((i), (ii)). Consequently, this way to compute an equilibrium of this problem appears empirically as a dead end. For this reason, we carry on with a second and final tactic to calculate an equilibrium of the optimal short-term production problem. According to this tactic, each producer takes into account the supply-demand equilibrium constraint in the optimal short-term management of the reservoir over the entire time horizon of our model.

The marginal cost of nuclear production is (significantly) lower than the one of non-nuclear thermal production which implies a discontinuity of the merit order price on a

mathematical level. We prove that this discontinuity of the merit order price leads to a discontinuous profit. In particular, we show that the decrease of market price during months when nuclear turns into marginal technology induces a decrease in the producer's current monthly profit. This leads to search for an equilibrium of the optimal short-term production problem within a subset of the set of feasible solutions defined by strictly positive non-nuclear thermal production vectors. Within this subset of feasible solutions, the price will be determined by the marginal cost of the non-nuclear thermal generation technology. From a mathematical perspective, we prove that the current monthly profit maximization problem determined on this subset and the current monthly profit maximization problem determined on the whole set of feasible solutions are equivalent (same value and same set of solutions).

Finally, in section 2.4, we analyze the production and nuclear fuel storage decisions of our last approach within a simple numerical model. From a purely mathematical point of view, the complexity of our model requires simplifying it through the reduction of its optimization variables. Starting from the original economy with  $N \geq 2$  producers, we can build an auxiliary economy with a unique producer resulting from the "aggregation" of the  $N$  producers. We show that an equilibrium of the economy with an aggregate producer can be decentralized as an equilibrium of the economy with  $N$  producers. Conversely, the "aggregation" of an equilibrium of the economy with  $N$  producers leads to an equilibrium of the economy with a single producer. This mathematical "equivalence" of equilibrium between the economy with  $N$  producers and the economy with one aggregate producer enables us to simplify our model (which is very important for the numerical model) by determining an equilibrium within the economy with a unique producer instead of the economy with  $N$  producers. Therefore, we assume the existence of an aggregate producer that holds the total capacity (nuclear and non-nuclear thermal). We should note that logically (mathematically) these simplifications are not basic redefinitions of the treated economical issues. They are only simplifications realized in order to decrease the level of difficulty in resolving these issues. However, we must not overlook the economic implications of the fact that the equilibrium that we determine in the "aggregate" economy constitutes an equilibrium of the decentralized economy and vice versa. This implies that the decentralized economy can be considered of the "same quality" as the centralized

economy in terms of production since we find that the optimal production behavior is identical in both economies.

The problem of discontinuity mentioned above makes the numerical resolution of the optimal short-term production problem extremely complicated and our theoretical approach shows that it may even lead to a lack of solutions. That is why it is necessary to study an approximate problem (continuous problem) that constitutes a “regularization” of our economical problem (discontinuous problem).

In order to collect some basic data to feed our model, we did significant bibliographic research. However, in order to get a complete data set suitable for the numerical modelling, we proceed with an interpolation of the missing data. Finally, we run numerical tests of the model with that set of data using Scilab<sup>2</sup>.

## 2.2 Model: Perfect competitive case

In this section, we describe our general deterministic dynamic model of a perfectly competitive electricity market where there exist two types of generation: nuclear and thermal non-nuclear. As we showed in our first chapter, nuclear is a complex generation technology given its technical and economical characteristics. Therefore, we deal with a demanding modelling with respect to nuclear technology in order to be realistic and at the same time we make assumptions which permit to our model to be manageable. In our model, the price in the electricity market is determined by the merit order price rule which implies perfect competition according to which firms treat price as a parameter and not as a choice variable. Price taking firms guarantee that when firms maximize their profits (by choosing the quantity they wish to produce and the technology of generation to produce it with) the market price will be equal to marginal cost. This general framework is also characterized by perfect equilibrium between supply and demand and perfect information among producers. First, our modelling aims at determining the optimal short-term

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<sup>2</sup>Scilab is an open source, cross-platform numerical computational package and a high-level, numerically oriented programming language. It can be used for numerical optimization, and modelling, simulation of dynamical systems, statistical analysis etc.

management of a flexible nuclear generation set in that competitive regime. We want to look out to the medium-term horizon which is characterized by the seasonal variation of demand between winter and summer. Second, there are constraints imposed by the flexible operation of nuclear units, production and nuclear fuel storage that play a central role in determining the equilibrium outcomes in this wholesale electricity market.

For simplicity reasons and in the absence of access to detailed data the electricity importations/exportations are not taken into account within our model. However, in the hypothetical case that electricity importations/exportations were part of our modelling, they could be considered either exogenous or endogenous to our model. If they were exogenous then the demand would be translated by the production that is imported/exported. This would modify the value of the demand but it would not affect our modelling. On the contrary, if they were endogenous, the complexity of the modelling would increase since several new parameters have to be taken into account in our model e.g. technical constraints imposed by the transmission power lines, the price elasticity of foreign demand, etc.

Practically, the exportations/importations constitute a very important factor for ensuring the equality between supply and demand on a national level and at the same time for determining the market price. As stated in the report of CRE in 2013, the marginality of the different sectors has changed significantly in 2012 compared to 2011 in France: the time marginality of nuclear power has fallen sharply because of a low availability factor (78% in 2012 against 80,7% in 2011), while the duration of marginality of the borders has seen a sudden increase (Regulatory Commission of Energy 2013). More precisely, we observed a strong increase in the duration of marginality of the borders (from 40% in 2011 to a total of nearly 86% in 2012) and a considerable decrease in the duration of marginality of the nuclear industry which determines the price in 1% of cases (for a production representing 73% of production in 2012) instead of 12% in 2011. Exportations/importations serve also to equilibrate supply and demand and determine the market price outside the French electricity market. For example, in Denmark, the amount of capacity of wind power is significant with respect to the level of demand and thus, given the absence of saturation of the interconnection capacities, exportations permit to smooth peak prices

in a neighbor market and hence, obtain satisfying price convergence rates between these countries. Otherwise, if the available interconnection capacity is saturated, the two markets are found disconnected and high price differentials can then occur between these countries. To illustrate, we present the case between France and Germany during 2012 when a high thermo-sensitivity of the French consumption led to price peaks which were smoothed via importations from Germany but only by using the maximum of the available interconnection capacity between the two countries (Regulatory Commission of Energy 2013). Therefore, the question of exportations/importations is associated with a rather complicated question of the optimal level of interconnection capacities which is correlated to the question of the optimal investment level to build new capacities. A large amount of investments is necessary for an augmentation of these capacities in order to avoid their saturation and hence, achieve security of supply and increase in the economic efficiency of the interconnected countries. It is also a complex question because France exchanges electricity with a relatively important number of countries (Germany, UK, Spain, Swiss, Belgium, Italy, Netherlands) due to its geographical position in Europe. The level of complexity increases if we think that a country can simultaneously export and import electricity production even within the same day mainly during winter as a result of the very important thermo-sensitivity (e.g. France and Germany). We need to stress here that, given the current discussion on the social acceptability of nuclear, French consumers accept the existence of nuclear as an electricity production technology but for a consumption (mainly) within their national electricity market. Consequently, we conclude that exportations/importations is a complex matter which also demands access to analytical data in order to determine several new parameters which will have to interfere with the model and proceed with its numerical elaboration.

Our work centers only on the nuclear fuel storage and the optimal management of the nuclear fuel reservoir without considering the production coming from hydro units with possibility of storage (peaking<sup>3</sup> power plants) because of the additional capacity and storage constraints which would increase the complexity of the model. There exists an extensive literature that studies the optimal management of hydro-reservoirs in mixed hydro-non-nuclear thermal competitive markets and where one can see several modellings

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<sup>3</sup>Peaking power plants are power plants that generally run only when there is a high demand, known as peak demand, for electricity.

of the optimal production problem and notice the increased level of difficulty from a theoretical and numerical point of view (Ambec, Doucet (2003), Arellano (2004), Bushnell (2003)). Identically, we do not take into account other methods used to store electricity on a large scale within an electrical power grid (grid energy storage) such as pumped hydro-storage<sup>4</sup> which is an old method providing a relatively significant amount of capacity in France (Grand'Maison Dam holds a capacity of 1.8 GW), batteries, hydrogen, power to gas, etc.

Similarly, the stochastic nature of the renewable electricity production, from sources such as wind power and solar power would complicate our model which is the main reason why we do not take them into consideration. The electricity production coming from renewable energy plants is variable or intermittent<sup>5</sup> because of the stochastic nature of weather patterns. This means that the renewable energy production should be a stochastic endogenous variable in our model. Therefore, its consideration would impose to realize a radically different modelling, a stochastic modelling, whose nature is not consistent with the deterministic character of our model.

### 2.2.1 Modelling the demand

The demand, being exogenous, is considered perfectly inelastic. It is obviously a simplification. It can nevertheless be motivated by some arguments. In the short-term to medium-term the demand is less sensitive to price because it is already determined by previous investments in electrical devices and ways of life whose evolutions require time. Note that in electricity markets, there exists the possibility of “erasing electricity consumption” or “smoothing the load curve by controlling demand” in the case of an imbalance between supply and demand (during daily or seasonal peaks of electricity demand) in order to

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<sup>4</sup>Pumped-storage hydroelectricity is a type of hydroelectric power generation used by some power plants for load balancing. The method stores energy in the form of potential energy of water, pumped from a lower elevation reservoir to a higher elevation. Low-cost off-peak electric power is used to run the pumps. During periods of high electrical demand, the stored water is released through turbines to produce electric power.

<sup>5</sup>An intermittent energy source is any source of energy that is not continuously available due to some factor outside direct control. The intermittent source may be quite predictable, for example, tidal power, but cannot be dispatched to meet the demand of a power system.

reduce temporarily the consumption of a given site or a group of actors (e.g. households, industries, etc.). We can also consider that, in this time scale, the consumers are not able to observe and respond in price evolutions in real time. Consequently, the sensitivity of demand to price is extremely low. We need to emphasize that the choice of an inelastic demand for electricity has implications for the modelling and the behaviour of the model (see page 205, Subsection 3.5.1, Chapter 3). In the case that we add a price elasticity of demand in our model, it would have an arbitrary value since there are no specific elements that permit to estimate its value.

Electricity is sold to consumers by retailing companies or directly by generators which is however a relatively recent phenomenon. In the French case, most of the consumers and a significant part of the firms have a fixed price contract, being a regulated price contract set by the government and precisely by the French energy regulator (CRE) and the Ministry in charge of economy and energy. Regulated tariffs do not give information regarding the evolution of the spot price and therefore, the evolution of the production costs in the wholesale market which justifies the absence of elasticity of demand within our model. The case of tariffs which take different values for different hours during a day (hours when demand is high and low, respectively) gives access to a price relatively close to the spot price but if one aggregates e.g. on a monthly level the result is not pertinent. Statistics presented by the French energy regulator (CRE) showed that the volumes traded on the French wholesale market (intra-day and day-ahead) represented about 12% of the total volumes in 2011 and around 14% in 2012 (Sources: EPEX Spot, EPD, courtiers). In view of our framework of perfect competition, we focus on the wholesale spot market assuming that there is no bilateral contracting regime between retailers/consumers and producers within our model. The wholesale spot prices are paid by the retailers/consumers directly to the producers.

In order to obtain a more clear vision of the demand served by the nuclear and non-nuclear thermal units, we remove the part of the base load demand served by generation units whose electricity production is “fatal” and they are the first called to meet demand according to the merit order (run-of-river hydro plants, renewable energy plants (e.g. wind and solar)). More specifically, the level of demand observed during a month is translated



by the monthly hydro production coming from the run-of-river hydro plants. The run-of-river hydro plants have little or no capacity for energy storage, hence they can not coordinate the output of electricity generation to match consumer demand. Consequently, they serve as base load power plants. Since the hydro technology with no reservoir (run-of-river) is a base load generation technology which is presumably never marginal, it is necessary to call up nuclear to cover the different levels of demand. As we explained before, we do not consider the production coming from renewable energy plants because of the variability or intermittency that characterizes their production level.

In our model, we do not take into consideration the seasonal variations of hydro production due to precipitation and snow melting because the corresponding data is not available. Therefore, we assume that the monthly run-of-river hydro-production is constant through the entire time horizon of our model. This assumption is also based on the relatively low volatility of the monthly run-of-river hydro-production due to a relatively low standard deviation<sup>6</sup> which results in a smooth evolution of its monthly value close to the mean over a year. This is not the case for the monthly production coming from renewable energy plants since it is significantly volatile (high levels of standard deviation) and is spread out over a large number of values during a year. This does not permit to assume that it is constant over the whole time horizon of the model and take it into account in the modelling of the demand as we did with the run-of-river hydro-production.

## 2.2.2 Modelling the time horizon

The time horizon of the model is  $T= 36$  months<sup>7</sup> beginning by the month of January. We choose a time horizon of 36 months for our modelling because we need a sufficiently long time horizon to follow up the evolution of the optimal levels of production and storage fuel as well as the variations of the price and of the profit. In order to keep our model

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<sup>6</sup>In statistics and probability theory, standard deviation shows how much variation or “dispersion” exists from the average (mean, or expected value). A low standard deviation indicates that the data points tend to be very close to the mean; high standard deviation indicates that the data points are spread out over a large range of values.

<sup>7</sup>The time horizon of the model is a multiplicative of twelve, being expressed in months. Therefore it could be modified.

simple, we assume that the value of profit is not discounted during the period  $T$ . We do not choose a longer time horizon in order to be consistent with this absence of taking into account the discount rate. We can certainly consider longer periods but the model will be less pertinent.

Let us now proceed with the modelling of the time horizon of the campaign of production. A French nuclear producer has two main options with respect to the scheduling of fuel reloading (Source: EDF (2008)):

- per third ( $1/3$ ) of fuel reservoir (representing a reloading of reactor's core per third of its full capacity) that corresponds to 18 months of campaign and 396 days equivalent to full capacity for a unit of 1300 MW,
- per quarter ( $1/4$ ) of fuel reservoir (representing a reloading of reactor's core per quarter of its full capacity) that corresponds to 12 months of campaign and 258 days equivalent to full capacity for a unit of 1500 MW.

For a given nuclear reactor, the choice of normative duration of the campaign can not be changed because: *(i)* any changes on the choice of duration of the campaign have to be authorized by the Nuclear Safety Authority (NSA) which is tasked, on behalf of the state, with regulating nuclear safety in order to protect workers, patients, the public and the environment in France, *(ii)* of the design of the nuclear fuel rods intended for this specific reactor, *(iii)* the optimal allocation of the shutdowns of all 58 nuclear reactors for reloading is decided in advance according to safety rules imposed by NSA. Therefore, we exclude the case of having both a campaign of 12 and of 18 months. Additionally to this, we do not take into account this case to avoid complicate our model. In order to get a tractable model for our numerical simulations, we need a cyclic model for the modelling of the campaign. We do not retain the first modelling, hence a campaign of 18 months because it is not consistent with the “good” seasonal allocation of shutdowns of the nuclear units. Indeed, if the nuclear producer reloads fuel in summer when the demand is low the date of the next reloading will be then in winter when the demand is high. This modelling could be compatible with the “good” seasonal allocation of shutdowns of the nuclear units only in the following cases:

- the nuclear producer may “stretch” the campaign of 18 months because this prolongation of the length of the campaign will shift the date of the next reloading from winter to spring when the demand is less important. However, the “stretch” permits to prolong the campaign only by a few days which is not compatible with our model where we reason on a monthly level. Moreover, this increase of the duration of the campaign results in an additional use and thus an extra degradation of the remaining inside the reactor nuclear fuel (meaning the 2/3 of fuel reservoir) which influence the level of production of the next campaign. This leads to a more complex model since we have to take account of the extra cost related to the further degradation of the remaining nuclear fuel as well as the rate of degradation and its impact to the production of the following campaign,
- the nuclear producer reloads fuel only in spring and hence the date of the next reloading will be in autumn when the levels of demand are lower than those observed in winter. Nevertheless, this constitutes a very particular study case which limits the interest of studying it.

Consequently, we retain a modelling close to the second modelling, thus a duration of campaign equivalent to 12 months to get a cyclic model with a periodicity of one year. The one year period can be then broken down into 11 months being the period of production and 1 month corresponding to the month of reloading of the fuel.

Note that both options<sup>8</sup> of fuel reloading result from the operational schema of EDF (Electricité de France) that is strategically chosen in order to optimize the allocation of shutdowns of nuclear reactors for reloading. We do not deal with the question of the optimal allocation of shutdowns in this thesis for several reasons based on our analysis in chapter 1 (cf. page 60, Subsection 1.4, Chapter 1): (i) lack of operational data for confidentiality reasons, (ii) lack of information with regard to the periodical inspections of nuclear reactors and the inspections imposed by the Nuclear Safety Authority, (iii) it is already determined by the French nuclear operator (EDF) via a high level computational programming (model ORION). For all these reasons which are difficult to control in order

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<sup>8</sup>In the case of a unit of 900 MW, the scheduling of fuel reloading is the following: (i) 1/3 of fuel reservoir that corresponds to 18 months of campaign and 385 days equivalent to full capacity, (ii) 1/4 of fuel reservoir that corresponds to 12 months of campaign and 280 days equivalent to full capacity.

to endogenously determine the optimal point of reloading of nuclear fuel and thus, the duration of the campaign within our model, the scheduling of fuel reloading is entirely exogenous. In our model, our focus is on optimizing the allocation of the nuclear fuel stored in the reservoir during the different campaigns of production for a reloading pattern provided by the French nuclear operator via the model ORION. Ameliorate the model ORION is not among the objectives of this thesis. The optimization given by the ORION model does not respond to our research question of the optimal management of the nuclear production as a result of the optimization of the management of the nuclear fuel reservoir.

### 2.2.3 Modelling the generating units

We study a competitive electricity market with  $N \geq 2$  producers who manage both nuclear and non-nuclear thermal generating units. The capacity of nuclear units as well as the capacity of non-nuclear thermal units are exogenous within our model. A producer  $n = 1, \dots, N$  can operate with all types of nuclear generating units. In addition, each producer disposes of a certain amount of non-nuclear thermal capacity.

#### Concept of type

Among the nuclear generating units, we distinguish several essential intrinsic characteristics:

- available nuclear capacity,
- minimum capacity when in use,
- month of their fuel reloading.

In our model, the minimum capacity is proportional to the available capacity, and this proportion is the same for all “physical” nuclear reactors. Therefore, for each “physical” nuclear reactor, we will focus on the month of fuel reloading, which permits us to define

twelve “types” of nuclear units. Each type indexed by  $j = 1, \dots, 12$  corresponds to a different month of reloading of the nuclear unit. Then, a unit which belongs to the type of unit  $j = 1$  (respectively  $j = 2, \dots, j = 12$ ) shuts down in the month of January (respectively February,  $\dots$ , December).

In the first chapter, we explained that a nuclear plant may contain several “physical” nuclear reactors, which (for operational reasons) do not reload on the same month. The characteristic “type” for the nuclear case is not related to the plant but to the reactor. Each producer  $n = 1, \dots, N$  owns a precise number of “physical” nuclear reactors that are grouped according to the month of reloading (independently of the locations) in order to constitute units. Therefore, it can hold a certain level of capacity from each type of nuclear unit.

For the non-nuclear thermal units, the modelling is the same except that the minimum capacity is equal to zero and that there is no month of reloading. There is a unique type of non-nuclear thermal units.

## Notations

The level of the nuclear production during the month  $t = 1, \dots, T$  for the unit  $j$  of producer  $n$  will be denoted by  $q_{njt}^{nuc}$ . Moreover, the maximum nuclear production that can be realized by the unit  $j$  of producer  $n$  during a month is given by the parameter  $Q_{max}^{n,j,nuc}$ , while the minimum nuclear production is equal to  $Q_{min}^{n,j,nuc}$ .

The level of the non-nuclear thermal production during the month  $t = 1, \dots, T$  for the producer  $n$  will be denoted by  $q_{nt}^{th}$ . Furthermore, the maximum non-nuclear thermal production during a month for the producer  $n$  is given by the parameter  $Q_{max}^{n,th}$  and corresponds to the nominal non-nuclear thermal capacity of producer  $n$ , while there is no minimum for non-nuclear thermal production  $Q_{min}^{n,th} = 0$ .

## Numerical data

In Annex C, we present the values used in our numerical simulation.

The amount of the aggregate nuclear capacity of the units of type  $j$  ( $\text{Capacity}^{j,nuc} = \sum_{n=1}^N \text{Capacity}^{n,j,nuc}$ ) is provided in Table C.1. The level of the aggregate maximum nuclear production ( $Q_{max}^{j,nuc} = \sum_{n=1}^N Q_{max}^{n,j,nuc}$ ) and aggregate minimum nuclear production ( $Q_{min}^{j,nuc} = \sum_{n=1}^N Q_{min}^{n,j,nuc}$ ) that can be realized by the units of type  $j$  during a month is given in Table C.2. Finally, we can read the level of the total maximum non-nuclear thermal production ( $Q_{max}^{th} = \sum_{n=1}^N Q_{max}^{n,th}$ ) in Table C.2.

### 2.2.4 Modelling the production costs

The cost functions of both nuclear and non-nuclear thermal production are common among the different producers. The nuclear cost function is made of a fixed part determined by the cost of investment, the fixed cost of exploitation and taxes and a variable part which corresponds to the variable cost of exploitation and fuel cost (see Subsection 1.2 of Chapter 1 on page 35 and Subsection 2.4.1 on page 117). We assume that the cost function of the nuclear production is affine and defined as

$$C_{n,j}^{nuc}(q_{njt}^{nuc}) = a_{nuc}^{n,j} + b_{nuc} q_{njt}^{nuc}.$$

The non-nuclear thermal cost function is also made of a fixed part which corresponds to the cost of investment, the fixed cost of exploitation and taxes and a variable part covering the variable cost of exploitation, the fuel cost, the cost of CO<sub>2</sub> as well as the taxes on the gas fuel (see page 117, Subsection 2.4.1). We assume that the non-nuclear production has a quadratic cost function  $C_n^{th}(\cdot)$  which is the following:

$$C_n^{th}(q_{nt}^{th}) = a_{th}^n + b_{th} q_{nt}^{th} + c_{th}^n q_{nt}^{th^2}.$$

**Proposition 2.2.1** *The coefficients involved in the cost functions are determined by the capacity.*

- The fixed part  $a_{th}^n$  of the non-nuclear thermal cost function is proportional to the capacity  $Q_{max}^{n,th}$  while the coefficients  $b_{th}, c_{th}^n$  of the variable part of the non-nuclear thermal cost function are such that: (i)  $b_{th}$  does not depend on the capacity  $Q_{max}^{n,th}$ , (ii)  $c_{th}^n$  is inversely proportional to the capacity  $Q_{max}^{n,th}$ .
- The coefficient  $a_{nuc}^{n,j}$  is proportional to the capacity  $Q_{max}^{n,j,nuc}$  since it corresponds to the fixed part of the nuclear cost function.

### Proof

- In order to understand this dependency of the coefficients  $a_{th}^n, b_{th}, c_{th}^n$ , let us consider a particular case. Let us assume that the non-nuclear thermal capacity of producer 2 is twice the non-nuclear thermal capacity of producer 1:  $Q_{max}^{2,th} = 2Q_{max}^{1,th}$ . Therefore, producer 2 can be seen as the aggregation of the identical copies of producer 1. Then, the “total” cost  $C_2^{th}(q_{2t}^{th})$  of production (obtained by the capacity  $Q_{max}^{2,th}$ ) is equal to the minimum of the sum of “individual” costs. More precisely, we have

$$C_2^{th}(q_{2t}^{th}) = \min_{q_{1t}^{th}, \bar{q}_{1t}^{th}} (C_1^{th}(q_{1t}^{th}) + C_1^{th}(\bar{q}_{1t}^{th}))$$

subject to the constraints

$$0 \leq q_{1t}^{th} \leq Q_{max}^{1,th}, \quad \text{for all } t$$

$$0 \leq \bar{q}_{1t}^{th} \leq Q_{max}^{1,th}, \quad \text{for all } t$$

$$q_{1t}^{th} + \bar{q}_{1t}^{th} = q_{2t}^{th}, \quad \text{for all } t$$

The resolution of this convex (and polynomial) optimization problem is very simple and the solution is symmetric:  $q_{1t}^{th} = \bar{q}_{1t}^{th} = \frac{q_{2t}^{th}}{2}$ . Hence, the cost function will be

$$C_2^{th}(q_{2t}^{th}) = 2a_{th}^1 + b_{th}q_{2t}^{th} + \frac{c_{th}^1}{2}q_{2t}^{th2}.$$

Consequently, we conclude that when the capacity doubles, the coefficient  $a_{th}^1$  is multiplied by two, the coefficient  $b_{th}$  is unchanged and the coefficient  $c_{th}^1$  is divided by two.

We showed the dependency of the coefficients of the non-nuclear thermal cost  $C_n^{th}$  from the capacity  $Q_{max}^{n,th}$  in the particular case that  $Q_{max}^{2,th} = \nu Q_{max}^{1,th}$ , where  $\nu \in \mathbb{N}^*$ . Symmetrically, this dependency is also verified in the case that  $Q_{max}^{2,th} = \frac{1}{\nu} Q_{max}^{1,th}$ . Thus, one deduces that a similar result is obtained when<sup>9</sup>  $\nu \in \mathbb{Q}_+^*$  (we remind that a positive rational number is expressed as a fraction  $\frac{a}{b}$ , where  $a, b \in \mathbb{N}^*$ ). However, the set of rational numbers  $\mathbb{Q}$  is dense in  $\mathbb{R}$  which means that all real numbers can be approximated by rational numbers (Pugh (2002)). Hence, this dependency still holds in the general case that<sup>10</sup>  $\nu \in \mathbb{R}_+^*$ .

- In the case of the nuclear units, the proof is similar and even simpler.

□

The nuclear and non-nuclear cost functions are monotone increasing and convex functions of  $q_{njt}^{nuc}$  and  $q_{nt}^{th}$  respectively. We choose a quadratic cost function in the case of non-nuclear thermal because of the increasing marginal cost of the non-nuclear production since it results from different fossil fuel generation technologies (e.g. coal, gas -combined cycle or not-, fuel oil). Furthermore, the non-nuclear thermal production needed a non constant marginal cost function in order to recover its fixed costs. Indeed, if we assume a constant marginal cost function for non-nuclear thermal then the value of the non-nuclear thermal production when non-nuclear is the marginal technology does not permit to recover its fixed costs. This is the simplest cost function that we could choose for non-nuclear thermal in order to not complicate our model. Clearly, the marginal cost of nuclear  $mc^{nuc}(q_{njt}^{nuc})$  is a constant function of  $q_{njt}^{nuc}$  while that of the non-nuclear thermal  $mc_n^{th}(q_{nt}^{th})$  is an increasing function of  $q_{nt}^{th}$ .

<sup>9</sup>The set of positive rational numbers is usually denoted as  $\mathbb{Q}_+^*$ .

<sup>10</sup>The set of positive real numbers is usually denoted as  $\mathbb{R}_+^*$ .



## Numerical data

In table C.4 of Annex C, we present the value of the coefficients of the production cost functions that are used in our numerical simulation. Their calculation is based on the aggregate amount of nuclear and non-nuclear thermal capacity.

### 2.2.5 Modelling the nuclear fuel stock

Following the modelling of the production constraints, we will model the nuclear fuel stock and the associated constraints. We denote  $S_{reload}^{n,j}$  the nuclear fuel stock of reloading available to the unit  $j$  of producer  $n$ . Rather than expressing this stock in kilograms of uranium or number of nuclear fuel rods, we will express it thanks to the conversion<sup>11</sup> between the quantity of energy and the corresponding number of days of operation at full capacity. Note that the number of days of operation equivalent to full capacity is constant for all  $j, n$ . The nuclear fuel stock of reloading  $S_{reload}^{n,j}$  is equal to the corresponding capacity of the units of type  $j$  of producer  $n$  ( $\text{Capacity}^{n,j,nuc}$ ) multiplied by the number of hours equivalent to full capacity during a campaign. More precisely, one has:

$$S_{reload}^{n,j} = 1 \times \text{Capacity}^{n,j,nuc} \times \text{Number of days equivalent to full capacity} \times 24$$

where the number 1 makes clear that we reason over a campaign of production.

Similarly, we will look at the dynamic variable which constitutes the nuclear fuel stock of a unit over time. We will always use implicitly the conversion in terms of potential energy. The notation that we use is  $S_t^{n,j}$  which by convention represents the quantity of fuel stored in the nuclear reservoir and available to the unit  $j$  of producer  $n$  at the beginning of the month  $t$ . Obviously, we have  $S_t^{n,j} \geq 0$ .

---

<sup>11</sup>For a unit of 1500 MW, the quantity of energy resulting from 13728 nuclear fuel rods or equivalently 28002 kilograms of uranium converts to 258 days equivalent to full capacity or equally 6192 hours equivalent to full capacity (corresponding to a reloading per quarter of fuel reservoir). For a unit of 900 MW, the quantity of energy coming from 10560 nuclear fuel rods or equivalently 18468 kilograms of uranium converts to 280 days equivalent to full capacity or equally 6720 hours equivalent to full capacity (corresponding to a reloading per quarter of fuel reservoir) (CEA (2008)).

The evolution of the nuclear fuel stock is classic and is determined by the following rules:

$$S_1^{n,j} \text{ given, } S_{t+1}^{n,j} = \begin{cases} S_t^{n,j} - q_{njt}^{nuc}, & \text{if no reload during month } t \text{ for unit } j \\ S_{reload}^{n,j}, & \text{if unit } j \text{ reloads during month } t \end{cases} \quad (2.1)$$

We have only to take into account the question of reloading. In the case that  $t$  is the month during which the producer  $n$  reloads the fuel of the reactor, the stock at the beginning of the following month (beginning of the campaign) is equal to  $S_{reload}^{n,j}$ .

We also impose

$$S_{T+1}^{n,j} \geq S_1^{n,j} \quad (2.2)$$

The constraint (2.2) implies that a producer must keep its nuclear units at the end of the game at the same storage level as the initial one. A producer has to finish the period  $T$  at least with the same quantity of nuclear fuel as the initial one. In this way the producer has to “spare” its nuclear fuel during the production period. The consideration of this constraint is motivated by some arguments:

- The absence of this constraint could lead to an “over-consumption” of the nuclear fuel stock in order to reach the maximum nuclear production level; this could generate some negative effects (e.g. insufficient level of stock to reach at least the minimum nuclear production level during some months  $t = 37, 38$ , etc. (excluding the month of reloading)).
- The constraint (2.2) guarantees that the producer will start a new cycle of simulations of 36 months with a quantity of stock at least equal to  $S_1^{n,j}$  at the beginning of the game.

Such a constraint is implicit for unit  $j$  if the end of period  $T$  coincides with the end of the campaign of unit  $j$ .

In order to illustrate this constraint we refer to the case of virtual power plants. The operator of the electricity generation set provides access to a number of MW of generation

capacity that can be obtained by producers, suppliers and traders through an auction mechanism. It sells this production capacity in the form of contracts specifying both an energy price and a capacity price. The energy price is fixed in advance and remains fixed for the duration of the contract. The capacity price is determined by the auction mechanism. This capacity price is to be paid every month for each MW purchased and for the duration of the contract. Therefore, the buyers of these options have a drawing right on operator generation capacity, at a pre-defined variable cost, without having to assume all of the engineering and operational risk of plant ownership. Thus, the name of VPP - Virtual Power Plant - was given to the products sold. The VPPs are offered in a variety of durations from 3 to 36 months. One meets VPP auctions in several European countries (e.g. France<sup>12</sup>, Belgium, Netherlands, Denmark, Germany, etc.). In this case, the constraint (2.2) has to be taken into consideration. Alternatively, a system of penalty could be annexed to the constraint (2.2). This however may lead to the violation of this constraint since a producer could choose to pay a certain amount of penalty. Moreover, EDF, being responsible for the operational management of the power plants for the duration of the contract, controls the level of production and thus the amount of generation capacity used over this period. This does not permit to the producer to violate the constraint (2.2). Consequently, we abandon the penalties and we use the constraint (2.2).

The nuclear production  $q_{njt}^{nuc}$  during a campaign (11 months) for the unit  $j$  of producer  $n$  can not exceed the nuclear fuel stock of reloading  $S_{reload}^{n,j}$  available to the unit  $j$  of producer  $n$  at the beginning of the campaign. For example, given the schema stock-production flow represented by (2.1) and the positivity of the nuclear fuel stock ( $S_t^{n,j} \geq 0$ ), the nuclear production  $q_{n1t}^{nuc}$  realized by the unit 1 of producer  $n$  during its first campaign is such that:

$$\sum_{t=2}^{12} q_{n1t}^{nuc} \leq S_{reload}^{n,1} \quad (2.3)$$

---

<sup>12</sup>In France, EDF offer access to 5400 MW of generation capacity.

We recall that the nuclear units of type  $j = 1$  reload their nuclear fuel in the month of January. From the constraint (2.3) and (2.1), we deduce that the producer  $n$  finishes the campaigns with a quantity of stock superior or equal to zero. However, a producer spends all its nuclear fuel stock of reloading  $S_{reload}^{n,j}$  during a campaign, thus it disposes of a quantity of nuclear fuel stock equal to zero at the end of the campaign (beginning of the month of reloading). Several reasons lead us to this ascertainment:

- The length of a campaign is given by the maximum number of days during which a nuclear unit produces until **exhaustion** of its fuel of reloading.
- The evaluation of the variable part ( $b_{nuc}$ ) of the nuclear cost function which partially corresponds to the fuel cost is based on the fact that a producer uses all the available nuclear fuel stock: if a producer keeps paying in order to obtain the fuel stock  $S_{reload}^{n,j}$  even in the case that it does not consume all the stock during a campaign, then this cost should be regarded as a fixed cost which is paid at the beginning of each campaign. Consequently, the fuel cost should be integrated into the fixed part of the nuclear cost function, which means that the coefficient  $a_{nuc}^{n,j}$  and thus the nuclear cost would be modified.
- The cost that a producer undergoes to get rid of the unused nuclear fuel at the end of the campaigns (cost related to the reprocessing of nuclear fuel).

Note also that there exists an obvious analogy with Walras' Law where the inequality budget constraint is represented as an equality.

For these reasons, the constraint (2.3) will now take the form:

$$\sum_{t=2}^{12} q_{n1t}^{nuc} = S_{reload}^{n,1} \quad (2.4)$$

Similarly, the constraint (2.2) can not hold as inequality constraint ( $S_{T+1}^{n,j} > S_1^{n,j}$ ) which means that the surplus of stock at the end of the game is zero. Thus, the constraint (2.2) will become as follows:

$$S_{T+1}^{n,j} = S_1^{n,j} \quad (2.5)$$

Note that if  $j = 1$ , then the condition (2.5) becomes  $S_{T+1}^{n,1} = S_1^{n,1} = 0$  which is obviously true.

We are now ready to define the nuclear fuel constraints for the unit  $j$  of producer  $n$ . In view of the above ascertainment regarding the exhaustion of the nuclear fuel stock at the end of campaigns, the nuclear fuel constraints for the nuclear unit  $j$  of producer  $n$  are defined as follows:

- $j=1$

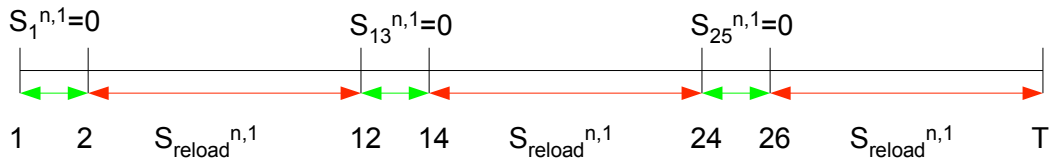


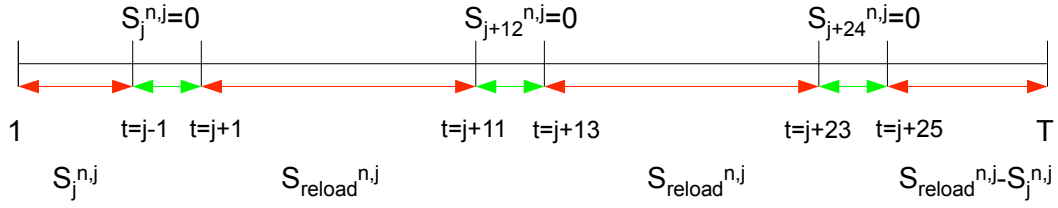
Figure 2.1: Nuclear fuel reloading of unit 1

The nuclear units of type  $j = 1$  reload their nuclear fuel in the month of January (see Figure 2.1).

$$\left\{ \begin{array}{l} \sum_{t=2}^{12} q_{n1t}^{nuc} = S_{reload}^{n,1}, \quad \text{so that unit 1 uses stock reloaded during month 1} \\ \sum_{t=14}^{24} q_{n1t}^{nuc} = S_{reload}^{n,1}, \quad \text{so that unit 1 uses stock reloaded during month 13} \\ \sum_{t=26}^T q_{n1t}^{nuc} = S_{reload}^{n,1}, \quad \text{so that unit 1 uses stock reloaded during month 25} \end{array} \right. \quad (2.6)$$

- $j \in \{2, \dots, 11\}$

The nuclear units of type  $\{2, \dots, 11\}$  reload their fuel in the month of {February,  $\dots$ , November} (see Figure 2.2).

Figure 2.2: Nuclear fuel reloading of unit  $2 \leq j \leq 11$ 

$$\left\{ \begin{array}{ll}
 \sum_{t=1}^{j-1} q_{njt}^{nuc} = S_1^{n,j}, & \text{so that unit } j \text{ uses stock available in month } 1 \\
 \sum_{t=j+1}^{j+12-1} q_{njt}^{nuc} = S_{reload}^{n,j}, & \text{so that unit } j \text{ uses stock reloaded during month } j \\
 \sum_{t=j+12+1}^{j+2 \cdot 12-1} q_{njt}^{nuc} = S_{reload}^{n,j}, & \text{so that unit } j \text{ uses stock reloaded during month} \\
 & \text{ } j + 12 \\
 \sum_{t=j+2 \cdot 12+1}^T q_{njt}^{nuc} = S_{reload}^{n,j} - S_1^{n,j}, & \text{so that unit } j \text{ uses stock reloaded during month} \\
 & \text{ } j + 24 \text{ until the end of the game}
 \end{array} \right. \quad (2.7)$$

- $j=12$

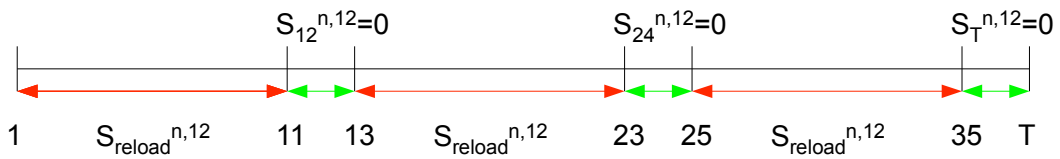


Figure 2.3: Nuclear fuel reloading of unit 12

The nuclear units of type  $j = 12$  reload their nuclear fuel in the month of December (see Figure 2.3).

$$\left\{ \begin{array}{l} \sum_{t=1}^{11} q_{n12t}^{nuc} = S_{reload}^{n,12}, \quad \text{so that unit 12 uses stock of reloading from month 1} \\ \sum_{t=13}^{23} q_{n12t}^{nuc} = S_{reload}^{n,12}, \quad \text{so that unit 12 uses stock reloaded during month 12} \\ \sum_{t=25}^{T-1} q_{n12t}^{nuc} = S_{reload}^{n,12}, \quad \text{so that unit 12 uses stock reloaded during month 24} \end{array} \right. \quad (2.8)$$

We observe that the nuclear units of type  $\{2, \dots, 11\}$  have two additional constraints than those of type 1 and 12. This is due to the presence, at the beginning and end of the game, of a campaign that we will qualify as incomplete.

## Numerical data

As we mentioned in subsection 2.2.3, in Annex C, we provide the value of the aggregate nuclear capacity  $\text{Capacity}^{j,nuc}$  ( $= \sum_{n=1}^N \text{Capacity}^{n,j,nuc}$ ) used in our numerical model for all  $j$ . Those values of the capacity permit us to compute the corresponding stock  $S_{reload}^j = \sum_{n=1}^N S_{reload}^{n,j}$  in Table C.2 and the stock  $S_1^j = \sum_{n=1}^N S_1^{n,j}$  available at the beginning of the month of January ( $t = 1$ ) in Table C.1. We also give in Table C.3 the number of days before reloading as well as the number of days equivalent to full capacity before reloading counting from the beginning of the month of January for all  $j$ .

### 2.2.6 Number of optimization variables and of optimization constraints

In our model, the total number of optimization variables is equal to  $N \cdot (J \cdot T + T) = N \cdot (12 \cdot 36 + 36) = N \cdot 468$ . The number of constraints resulting from the equality between supply and demand is  $T = 36$ . In addition, the number of nuclear fuel constraints is  $N \cdot ((2 \cdot K + 1) \cdot (J - 2) + (2 \cdot K) \cdot 2) = N \cdot ((2 \cdot 3 + 1) \cdot (12 - 2) + (2 \cdot 3) \cdot 2) = N \cdot 82$ ,

where  $K$  represents the number of campaigns within our model. Lastly, the number of minimum and maximum nuclear and non-nuclear thermal production constraints is equal to  $N \cdot (J \cdot T + T) = N \cdot (12 \cdot 36 + 36) = N \cdot 468$ . Hence, the total number of optimization constraints is equal to  $N \cdot 550 + 36$ . Even in the case of a unique producer ( $N = 1$ ), the number of variables (468) and of optimization constraints (586) are quite large which leads to computational difficulties. This is because, the level of difficulty of the numerical program to compute a solution of an optimization problem is increasing with respect to the size of the model (number of optimization variables, number of optimization constraints).

In general, computational difficulties can result from: *(i)* the difficulty of the numerical program in calculating a global optimum since it can stop running when it finds a first solution which could be a local optimum of the optimization problem and not proceeding until it finds a global optimum, *(ii)* the sensibility of calculations with regard to the initial point that one chooses so that the program start running (different initial points can lead to different results), *(iii)* the duration of calculations which is increasing with respect to the size of the model.

## 2.3 Equilibrium and approaches of calculation

In this section, we introduce the notion of a merit order equilibrium in the case of several producers ( $N \geq 2$ ). It consists of finding a feasible production vector (it respects production and nuclear fuel constraints) which satisfies the supply-demand equilibrium constraint each month. The price within the equilibrium is then determined by the marginal cost of the marginal technology. Then, we present our different approaches to calculate a merit order equilibrium. In our first approach, we study the supply behaviour with respect to the merit order price. More precisely, we expose the theoretical and numerical difficulties of computing a merit order equilibrium derived from the impossibility to calculate a feasible production vector that respects the supply-demand equilibrium by looking at the merit order price each month. Therefore, we proceed with our second approach where a producer maximizes its profit in the month  $t$  by taking into consideration production constraints over the entire period  $T$ , the inter-temporal nuclear fuel storage constraints for



all nuclear units  $j$  and the supply-demand equilibrium constraint during the month  $t$ . We show empirically that this approach does not permit the calculation of a merit order equilibrium because the nuclear producers are “short-sighted” with regards to future demand and do not succeed to equilibrate supply and demand in future periods without violating the production constraints. For this reason, we proceed with our third and last attempt to calculate a merit order equilibrium. In this approach, future demand is sufficiently taken into consideration in the optimal short-term management of the nuclear fuel reservoir so that the equality between supply and demand is respected over the entire time horizon of the model.

### 2.3.1 The notion of merit order equilibrium

Let us introduce the definition of a merit order equilibrium with respect to a system of prices  $p \in \mathbb{R}_+^T$ .

**Definition 2.3.1** The production vector  $(\bar{q}_n)_{n=1}^N = (((\bar{q}_{1jt}^{nuc})_{j=1}^J, \bar{q}_{1t}^{th})_{t=1}^T, \dots, ((\bar{q}_{Njt}^{nuc})_{j=1}^J, \bar{q}_{Nt}^{th})_{t=1}^T)$  is a merit order equilibrium with respect to a system of prices  $p \in \mathbb{R}_+^T$  if:

(i) for all  $n$ ,  $\bar{q}_n$  is a feasible production vector: (a) it respects the nuclear fuel constraints, for all  $j$  and (b) it respects the minimum/maximum production constraints, for all  $j, t$ .

(ii) the price, at each month  $t$ , is determined by the marginal cost of the marginal technology. It is called the merit order price associated with the production vector  $(\bar{q}_n)_{n=1}^N$ .

(iii) at each date  $t$ , it respects the equality between supply and demand

$$\sum_{n=1}^N \left( \sum_{j=1}^J \bar{q}_{njt}^{nuc} + \bar{q}_{nt}^{th} \right) = D_t - Q_t^{hyd}. \quad (2.9)$$

Let us now give some precisions to each condition ((i), (ii), (iii)) that makes part of the definition of the merit order equilibrium in order to better understand it. More precisely, we specify that:

(i) The nuclear fuel constraints for the unit  $j$  are provided by (2.6), (2.7), (2.8) in subsection 2.2.5 of this chapter. The minimum/maximum nuclear and non-nuclear thermal production constraints take the form

$$\begin{cases} Q_{min}^{n,j,nuc} \leq q_{njt}^{nuc} \leq Q_{max}^{n,j,nuc}, & \text{if no reload during month } t \text{ for unit } j \\ q_{njt}^{nuc} = 0, & \text{if unit } j \text{ reloads during month } t \end{cases} \quad (2.10)$$

$$0 \leq q_{nt}^{th} \leq Q_{max}^{n,th} \quad (2.11)$$

The constraint (2.10) shows that the nuclear production of each month is bound by the minimum/maximum quantity of nuclear production which can be obtained during a month. If the unit  $j$  shuts down in month  $t$  for reloading, then its production during this month is equal to zero. The non-nuclear thermal production is a non-negative quantity which is bound by the maximum non-nuclear thermal production (constraint (2.11)). A producer may use the non-nuclear thermal resources to produce electricity until it reaches the level of demand of the corresponding month respecting at the same time the constraint (2.11).

(ii) In view of the subsection 2.2.4 where we determine the nuclear and non-nuclear thermal production costs, the merit order price  $p$  associated with a feasible production vector  $(q_n)_{n=1}^N$ ,  $p = (p_t)_{t=1}^T = (\Phi_{nt}(q_{nt}))_{t=1}^T = (\Phi_{nt}(q_{nt}^{nuc}, q_{nt}^{th}))_{t=1}^T = (\Phi_{nt}((q_{njt}^{nuc})_{j=1}^J, q_{nt}^{th}))_{t=1}^T = \Phi_n(q_n)$  is calculated in month  $t$  as follows:

$$p_t = \Phi_{nt}(q_{nt}) = \begin{cases} mc_n^{th}(q_{nt}^{th}), & \text{if } q_{nt}^{th} > 0 \\ mc^{nuc}(q_{njt}^{nuc}), & \text{if } q_{nt}^{th} = 0 \end{cases} = \begin{cases} b_{th} + 2c_{th}^n q_{nt}^{th}, & \text{if } q_{nt}^{th} > 0 \\ b_{nuc}, & \text{if } q_{nt}^{th} = 0 \end{cases} \quad (2.12)$$

If nuclear is the “last technology” of the merit order which is called to equilibrate supply and demand during the month  $t$  ( $q_{nt}^{th} = 0$ ), then the price is determined by the marginal cost of nuclear  $mc^{nuc}(q_{njt}^{nuc})$ . In the case that the non-nuclear thermal production is additionally used to cover the monthly levels of demand ( $q_{nt}^{th} > 0$ ), the price is given by the non-nuclear thermal marginal cost  $mc_n^{th}(q_{nt}^{th})$ . Note that the price is calculated independently of  $n$  because according to Proposition 2.2.1 on page 91, the variable part of the non-nuclear thermal cost  $c_{th}^n$  is inversely proportional to the capacity  $Q_{max}^{n,th}$  which means

that there exists  $\lambda \in \mathbb{R}$  such that  $c_{th}^n = \frac{\lambda}{Q_{max}^{n,th}}$ . Hence, in the case that  $q_{nt}^{th} > 0$ , we have

$$p_t = b_{th} + 2c_{th}^n q_{nt}^{th} = b_{th} + 2 \frac{\lambda}{Q_{max}^{n,th}} q_{nt}^{th} = b_{th} + 2\lambda \frac{q_{nt}^{th}}{Q_{max}^{n,th}} = b_{th} + 2\lambda r_t$$

where  $r_t = \frac{q_{nt}^{th}}{Q_{max}^{n,th}}$  is the rate of use of the non-nuclear thermal capacity at time  $t$ . It varies over time while it is constant from one producer to another. We emphasize that the merit order price  $p_t$  is discontinuous on production vectors whose non-nuclear thermal component  $q_{nt}^{th}$  is equal to zero (see Figure 2.4).

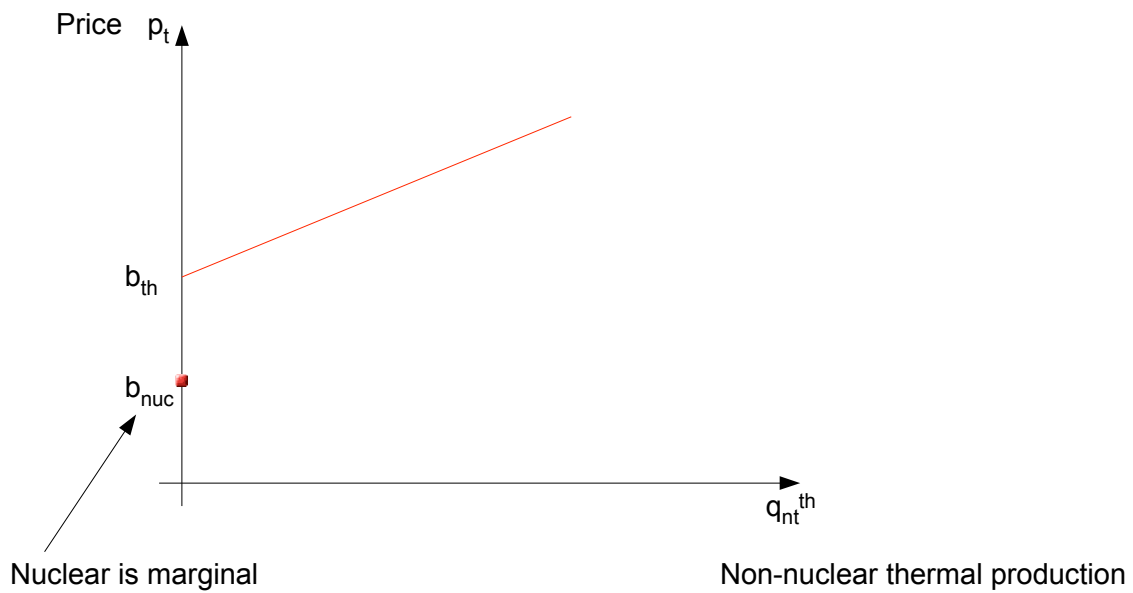


Figure 2.4: Price discontinuity

(iii) The monthly demand which is considered in this model results from the difference between the level of demand  $D_t$  observed in month  $t$  and the hydro production  $Q_t^{hyd}$  provided during the month  $t$  (see page 84, Subsection 2.2.1).

### 2.3.2 Supply behaviour with respect to the merit order price

In this section, we describe the supply behaviour with respect to the merit order price in order to show why producers can not decide on operating the nuclear units given an externally determined intertemporal pattern of prices (e.g. from European Energy

Exchange (EEX<sup>13</sup>) even if one assumes they can perfectly foresee. From a theoretical and numerical perspective, we expose the barriers of computing a merit order equilibrium. These are due to the difficulties of computing a feasible production level during a month that satisfies the equality between supply and demand by looking at the merit order price during this month. This leads producers to make decisions based on the patterns of seasonal demand.

### First theoretical difficulty of computing a merit order equilibrium

Let us denote a feasible production vector  $(q_n)_{n=1}^N = ((q_{nt})_{t=1}^T)_{n=1}^N = ((q_{nt}^{nuc}, q_{nt}^{th})_{t=1}^T)_{n=1}^N = (((q_{njt}^{nuc})_{j=1}^J, q_{nt}^{th}))_{t=1}^T)_{n=1}^N = ((\Phi_{nt}^{-1}(p_t))_{t=1}^T)_{n=1}^N = (\Phi_n^{-1}(p))_{n=1}^N$ . We indicate  $((\Phi_{nt}^{-1})^{nuc}(p_t))_{t=1}^T)_{n=1}^N$  the nuclear component and  $((\Phi_{nt}^{-1})^{th}(p_t))_{t=1}^T)_{n=1}^N$  the non-nuclear thermal component of the function  $((\Phi_{nt}^{-1}(p_t))_{t=1}^T)_{n=1}^N$ . From a theoretical point of view, supply does not behave well with respect to the merit order price within our model because  $\Phi_n^{-1}(\cdot)$  is not a function but a correspondence of the merit order price. This means that a set of different feasible production levels (and not a single feasible production level) are associated with the price.

More precisely, given the relationship (2.12) of subsection 2.3.1 and the figure 2.4, the non-nuclear thermal production of producer  $n$  in the month  $t$  is a function (and not a correspondence) of the form

$$(\Phi_{nt}^{-1})^{th}(p_t) = q_{nt}^{th}(p_t) = \begin{cases} \frac{p_t - b_{th}}{2C_{th}^n}, & \text{if } p_t \geq b_{th} \\ 0, & \text{if } p_t \leq b_{th} \end{cases} \quad (2.13)$$

We can not calculate the nuclear production  $(\Phi_{nt}^{-1})^{nuc}(p_t) = q_{nt}^{nuc}(p_t)$  of producer  $n$  with respect to the price  $p_t$  in the month  $t$  because it is not a function of the price but a correspondence. From a mathematical point of view, if we analyze (2.12), one has  $p_t = \Phi_{nt}(q_{nt}) = \Phi_{nt}(q_{nt}^{nuc}, q_{nt}^{th})$  but in fact, the function  $\Phi_{nt}(\cdot)$  is independent of the first

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<sup>13</sup>EEX operates market platforms for trading in electric energy, natural gas, CO<sub>2</sub> emission allowances and coal.

argument i.e. the nuclear production. Consequently, the function  $\Phi_{nt}(\cdot)$  can not be one-to-one<sup>14</sup>. Therefore, we will never have unicity of the value of the nuclear production of producer  $n$  for a given value of the price  $p_t$  in the month  $t$ . In particular, when nuclear is the marginal technology i.e.  $p_t = b_{nuc}$ , the nuclear production can take any value for this value of price.

### Second theoretical difficulty of computing a merit order equilibrium

In figure 2.4, if  $b_{nuc} < p_t < b_{th}$  then there is no correspondence  $((\Phi_{nt}^{-1}(p_t))_{t=1}^T)_{n=1}^N$  since the nuclear production  $(\Phi_{nt}^{-1})^{nuc}(p_t)$  of producer  $n$  in the month  $t$  is an empty set. Therefore, we can not compute the production  $\Phi_{nt}^{-1}(p_t)$  of producer  $n$  by looking at the merit order price  $p_t$  during the month  $t$ .

### Numerical difficulty of computing a merit order equilibrium

In the case that the nuclear production of producer  $n$   $((q_{njt}^{nuc}(p_t))_{j=1}^J)_{t=1}^T$  was a function of the merit order price and not a correspondence, the problem would be different. More precisely, in view of the inter-temporal management of the nuclear fuel stock (see page 98, Subsection 2.2.5), a producer has to look at the equality between supply and demand over the entire time horizon  $T$  of the model in order to determine the price within the merit order equilibrium. In this typical case of  $T = 36$  months and by taking as example the simplest case of one aggregate producer ( $N = 1$ ), we have to deal with a large non linear system

$$\sum_{j=1}^{12} q_{jt}^{nuc}(p_t) + q_t^{th}(p_t) = D_t - Q_t^{hyd}, \quad \text{for all } t$$

of 36 equations involving 36 unknowns  $((p_t)_{t=1}^T)$  which is difficult to solve numerically.

Note that this system is based on the auxiliary variables  $q_{jt}^{nuc}, q_t^{th}$  whose number is  $J \cdot T + T = 12 \cdot 36 + 36 = 468$ .

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<sup>14</sup>A function  $f(\cdot)$  is called one-to-one or injective if  $f(a) \neq f(b)$  for any two different elements  $a, b$  of the domain of the function.

In view of the above theoretical and numerical difficulties, we conclude that the producer  $n$  can not compute a feasible production vector that satisfies the equality between supply and demand at each month by looking at the merit order price during this month. For this reason, the determination of the price at the merit order equilibrium by calculating the supply  $((q_{njt}^{nuc}(p_t))_{j=1}^J, q_{nt}^{th}(p_t))_{t=1}^T)_{n=1}^N$  and by looking at the equality between supply and demand at each date  $t$

$$\sum_{n=1}^N \left( \sum_{j=1}^{12} q_{njt}^{nuc}(p_t) + q_{nt}^{th}(p_t) \right) = D_t - Q_t^{hyd}$$

is impossible in a theoretical and numerical level. Therefore, we deduce that it is not possible to calculate a merit order equilibrium in this first approach.

### 2.3.3 A second approach to calculate a merit order equilibrium

In this approach, each producer  $n$  operating with a certain level of nuclear capacity of unit  $j$  and an amount of non-nuclear thermal capacity maximizes its profit in the month  $t$  given the optimal production levels realized in the previous months. It takes into account the minimum and maximum production constraints for both nuclear and non-nuclear thermal generation technologies for all months of period  $T$  and the nuclear fuel constraints for every unit  $j$ . In addition, the supply-demand equilibrium constraint has to be satisfied during the month  $t$ .

At time  $t$ , the producer  $n$  could try to solve the following optimal production problem:

$$\max_{(((q_{\nu j\tau}^{nuc})_{j=1}^J, q_{\nu\tau}^{th})_{\tau=1}^T)_{\nu=1}^N \in G^t} p_t \cdot \left( \sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th} \right) - \sum_{j=1}^J C_{nj}^{nuc}(q_{njt}^{nuc}) - C_n^{th}(q_{nt}^{th}) \quad (2.14)$$

where  $p_t$  is a given parameter and  $G^t$  is the set of feasible solutions of the optimization

problem (2.14) defined as

$$G^t = \left\{ \begin{array}{ll} q_{\nu j \tau}^{nuc} = \tilde{q}_{\nu j \tau}^{nuc}, & \text{for all } \nu, j \text{ and for all } \tau < t \\ ((q_{\nu j \tau}^{nuc})_{j=1}^J, q_{\nu \tau}^{th})_{\tau=1}^T)_{\nu=1}^N \in K \text{ s.t.} & q_{\nu \tau}^{th} = \tilde{q}_{\nu \tau}^{th}, \quad \text{for all } \nu \text{ and for all } \tau < t \\ Q_{min}^{\nu, j, nuc} \leq q_{\nu j \tau}^{nuc} \leq Q_{max}^{\nu, j, nuc}, & \text{for all } \nu, j \text{ and for all } \tau \\ 0 \leq q_{\nu \tau}^{th} \leq Q_{max}^{\nu, th}, & \text{for all } \nu \text{ and for all } \tau \end{array} \right\}$$

The set  $K$  is defined by all the production vectors of the form  $q = ((q_{\nu j 1}^{nuc})_{j=1}^J, \dots, (q_{\nu j T}^{nuc})_{j=1}^J, q_{\nu 1}^{th}, \dots, q_{\nu T}^{th})_{\nu=1}^N$  that respect the nuclear fuel constraints (2.6), (2.7), (2.8) for all  $\nu$  as well as the supply-demand equilibrium constraint

$$\sum_{\nu=1}^N (\sum_{j=1}^{12} q_{\nu j t}^{nuc} + q_{\nu t}^{th}) = D_t - Q_t^{hyd}, \quad \text{in month } t.$$

For simplicity reasons, we used the notation  $G^t$  which is the reduced form of the notation  $G^t(((\tilde{q}_{\nu j \tau}^{nuc})_{j=1}^J, \tilde{q}_{\nu \tau}^{th})_{\tau=1}^{t-1})_{\nu=1}^N$  where  $((\tilde{q}_{\nu j \tau}^{nuc})_{j=1}^J, \tilde{q}_{\nu \tau}^{th})_{\tau=1}^{t-1})_{\nu=1}^N$  is the optimal production vector of the months preceding the month  $t$ . In the set of feasible solutions  $G^t$ , we look at the production realized in the months preceding the month  $t$  without focusing only on the production realized in the month  $t$  because of the inter-temporal nature of some optimization constraints. In particular, we refer to the constraints (2.6), (2.7), (2.8) which determine the inter-temporal management of the nuclear fuel stock of all units. In view of the construction of the set of feasible solutions  $G^t$  in the month  $t$ , we deduce that  $G^1 \subseteq G^2 \subseteq \dots \subseteq G^T$ .

The producer  $n$  determines its optimal level of supply  $((q_{n j t}^{nuc})_{j=1}^J, q_{n t}^{th})$  during the month  $t$ , given the optimal production  $((\tilde{q}_{n j \tau}^{nuc})_{j=1}^J, \tilde{q}_{n \tau}^{th})_{\tau=1}^{t-1}$  of the previous months, by solving the optimal short-term production problem (2.14). Then, the price  $p_t$  is determined by the equality between supply and demand in month  $t$ . The solution of this problem determines the new level of stock  $S_{t+1}^{n, j}$ .

A production vector  $(((\bar{q}_{\nu j \tau}^{nuc})_{j=1}^J, \bar{q}_{\nu \tau}^{th})_{\tau=1}^T)_{\nu=1}^N$  is an equilibrium of the optimal short-term optimization problem (2.14) if it is a merit order equilibrium and in addition to this it maximizes the profit of the producer  $\nu$  during the month  $t$  on the set of feasible solutions  $G^t$ , for all  $\nu, t$ .

However, this approach could be qualified as “short sighted” since the equality between supply and demand in future periods is not taken into consideration in the optimization problem (2.14). This may lead to a failure of the system to equilibrate supply and demand in future periods while at the same time respecting the nuclear fuel constraints and the production constraints. In fact, in our numerical example, we find that there exists a month  $t \in \{1, \dots, T\}$  such that the set of feasible solutions  $G^t$  is empty. More precisely, we verify (through a numerical test) the nonexistence of feasible solutions within the set  $G^{16}$  in the month of April ( $t = 16$ ) of the second year of period  $T$ . Therefore, the producers being “short sighted” with respect to future demand are not able to respect the equality between supply and demand during the entire time horizon of the model.

Consequently, we conclude that the calculation of a merit order equilibrium is not possible within this scenario. We intentionally present here a mistaken approach in order to show that too high a level of “short sightedness” with regards to future demand is not bearable. The equality between supply and demand has to be seen with a minimum anticipation in order to manage the current use of the nuclear fuel reservoir. For this reason, we proceed with the next and final approach to calculate a merit order equilibrium.

### 2.3.4 Final approach to calculate a merit order equilibrium

In view of the second approach to calculate a merit order equilibrium, the system operator can not find a feasible production vector which respects the equilibrium between supply and demand during the entire time horizon of the model. This is because producers do not sufficiently take into account the supply-demand equilibrium in future periods within the set of feasible solutions  $G^t$  of the optimal short-term production problem (2.14). This leads to a failure to calculate a merit order equilibrium of this optimization problem since the set  $G^t$  becomes empty ( $G^t = \emptyset$ , for  $t = 16$ ) in our numerical example. Therefore, the nuclear set has to be managed so that the equality between supply and demand is respected over the whole period  $T$ . For this reason, we provide a third and final approach to calculate a merit order equilibrium in order to take into account the supply-demand equilibrium constraint in future periods within the set of feasible solutions of the optimization problem (2.14).



More precisely, at time  $t$ , the producer  $n$  may attempt to solve the following optimal production problem

$$\max_{(((q_{\nu j\tau}^{nuc})_{j=1}^J, q_{\nu\tau}^{th})_{\tau=1}^T)_{\nu=1}^N \in H^t} p_t \cdot \left( \sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th} \right) - \sum_{j=1}^J C_{nj}^{nuc}(q_{jt}^{nuc}) - C_n^{th}(q_t^{th}) \quad (2.15)$$

where  $H^t$  is the set of feasible solutions of the optimization problem (2.15) defined as follows:

$$H^t = \left\{ \begin{array}{ll} q_{\nu j\tau}^{nuc} = \tilde{q}_{\nu j\tau}^{nuc}, & \text{for all } \nu, j \text{ and for all } \tau < t \\ q_{\nu\tau}^{th} = \tilde{q}_{\nu\tau}^{th}, & \text{for all } \nu \text{ and for all } \tau < t \\ Q_{min}^{\nu,j,nuc} \leq q_{\nu j\tau}^{nuc} \leq Q_{max}^{\nu,j,nuc}, & \text{for all } \nu, j \text{ and for all } \tau \\ 0 \leq q_{\nu\tau}^{th} \leq Q_{max}^{\nu,th}, & \text{for all } \nu \text{ and for all } \tau \end{array} \right\}$$

The set  $M$  is defined by all the production vectors of the form  $q = ((q_{\nu j1}^{nuc})_{j=1}^J, \dots, (q_{\nu jT}^{nuc})_{j=1}^J, q_{\nu 1}^{th}, \dots, q_{\nu T}^{th})_{\nu=1}^N$  that respect the nuclear fuel constraints (2.6), (2.7), (2.8) for all  $\nu$  as well as the supply-demand equilibrium constraint

$$\sum_{\nu=1}^N (\sum_{j=1}^J q_{\nu jt}^{nuc} + q_{\nu t}^{th}) = D_t - Q_t^{hyd}, \quad \text{for all } t.$$

The set  $M$  differs from the set  $K$  defined in our second approach to calculate a merit order equilibrium in subsection 2.3.3 over the months during which the equality between supply and demand is respected. Specifically, in our second approach, the producer  $n$  being “short sighted” with respect to future demand takes into account the supply-demand equilibrium constraint only during the current month of optimization. This is not the case in this approach since the producer  $n$  ensures that supply will meet demand during the entire time horizon of the model.

To simplify, the notation  $H^t$  is used for  $H^t(((\tilde{q}_{\nu j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\nu\tau}^{th})_{\tau=1}^{t-1})_{\nu=1}^N$  where  $(\tilde{q}_{\nu j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\nu\tau}^{th})_{\tau=1}^{t-1})_{\nu=1}^N$  is the optimal production realized in the months preceding the month  $t$ . We take into account the production of the months preceding the month  $t$  without centering only on the production of the month  $t$  within the set of feasible solutions  $H^t$ . This is due to the inter-temporal character of the constraints (2.6), (2.7), (2.8) which determine the management of the nuclear fuel stock of all units during the entire time horizon  $T$ .

Considering the structure of the set of feasible solutions  $H^t$  in the month  $t$ , we deduce that  $H^1 \subseteq H^2 \subseteq \dots \subseteq H^T$ .

The optimal short-term production problem (2.15) determines the supply of the producer  $n$   $((q_{njt}^{nuc})_{j=1}^J, q_{nt}^{th})$  during the month  $t$ , given the optimal nuclear and non-nuclear thermal production  $((\tilde{q}_{nj\tau}^{nuc})_{j=1}^J, \tilde{q}_{n\tau}^{th})_{\tau=1}^{t-1}$  realized in the previous months. Each month, the price  $p_t$  is determined by the equality between supply and demand. The level of stock  $S_{t+1}^{n,j}$  of the next month is determined by the nuclear supply  $(q_{njt}^{nuc})$  of the current month for the unit  $j$  of producer  $n$ .

A production vector  $(((\bar{q}_{\nu j\tau}^{nuc})_{j=1}^J, \bar{q}_{\nu\tau}^{th})_{\tau=1}^T)_{\nu=1}^N$  is an equilibrium of the optimal short-term production problem (2.15) if it is a merit order equilibrium and it maximizes the profit of the producer  $\nu$  during the month  $t$  on the set of feasible solutions  $H^t$ , for all  $\nu, t$ .

### The decrease of short-term profit in the absence of non-nuclear thermal production

Under some rather mild assumptions (satisfied by our numerical data), we show that the absence of non-nuclear thermal production during the month  $t$  induces a decrease of the profit during this month which results from a decrease of the price.

Let us focus on the set  $H_{th}^t$  defined as

$$H_{th}^t = \left\{ \begin{array}{ll} q_{\nu j\tau}^{nuc} = \tilde{q}_{\nu j\tau}^{nuc}, & \text{for all } \nu, j \text{ and for all } \tau < t \\ q_{\nu\tau}^{th} = \tilde{q}_{\nu\tau}^{th}, & \text{for all } \nu \text{ and for all } \tau < t \\ ((q_{\nu j\tau}^{nuc})_{j=1}^J, q_{\nu\tau}^{th})_{\tau=1}^T)_{\nu=1}^N \in M \text{ s.t.} & Q_{min}^{\nu,j,nuc} \leq q_{\nu j\tau}^{nuc} \leq Q_{max}^{\nu,j,nuc}, \text{ for all } \nu, j \text{ and for all } \tau \\ 0 < q_{\nu t}^{th} \leq Q_{max}^{\nu,th}, & \text{for all } \nu \end{array} \right\}$$

As before, the notation  $H_{th}^t$  is the reduced form of the notation  $H_{th}^t(((\tilde{q}_{\nu j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\nu\tau}^{th})_{\tau=1}^t)_{\nu=1}^N$  where  $((\tilde{q}_{\nu j\tau}^{nuc})_{j=1}^J, \tilde{q}_{\nu\tau}^{th})_{\tau=1}^t)_{\nu=1}^N$  is the optimal production achieved in the months preceding the month  $t$ .

**Remark 2.3.1** For all  $t \in \{1, \dots, T\}$ ,  $H_{th}^t$  is contained in  $H^t$  and  $H^t$  is contained in  $M$  ( $H_{th}^t \subset H^t \subset M$ ).

Since the marginal technology is the non-nuclear thermal on  $H_{th}^t$ , the price is determined by the non-nuclear thermal production during the month  $t$ . We now proceed with Proposition 2.3.1. We will make use of this proposition in order to prove the decrease of the profit at production vectors with zero levels of non-nuclear thermal production at date  $t$ .

**Proposition 2.3.1** *For all  $t \in \{1, \dots, T\}$ , if  $H_{th}^t$  is a non-empty set, then the closure of  $H_{th}^t$  is equal to  $H^t$  ( $\overline{H_{th}^t} = H^t$ ).*

### Proof

First, we show that  $\overline{H_{th}^t} \subset H^t$ . Since  $H_{th}^t$  is contained in  $H^t$  (see Remark 2.3.2) and  $H^t$  is a compact set, we have that

$$H_{th}^t \subset H^t \Rightarrow \overline{H_{th}^t} \subset \overline{H^t} = H^t.$$

Secondly, we prove that  $\overline{H_{th}^t} \supset H^t$ . Let  $q \in H^t$  and  $q' \in H_{th}^t$ . For all  $m \in N$ , there exists a sequence  $z_m = (1 - \frac{1}{m+1})q + \frac{1}{m+1}q'$  belonging to  $H_{th}^t$  such that  $\lim_{m \rightarrow \infty} z_m = \lim_{m \rightarrow \infty} (1 - \frac{1}{m+1})q + \frac{1}{m+1}q' = q$ . Hence,  $q \in \overline{H_{th}^t}$  and the inclusion is proven.

From the first and the second part of the proof, we conclude that  $\overline{H_{th}^t} = H^t$ .

□

From a geometrical point of view, it results from Proposition 2.3.1 that all the points of the set  $H^t$  and as a consequence those which belong to  $H^t \setminus H_{th}^t$  and therefore characterized by zero levels of non-nuclear thermal production in the month  $t$  can be approached by points that belong to  $H_{th}^t$ . This result plays a central role in order to prove the discontinuity and more precisely the decrease of profit at these particular points due to a decrease of the price in the month  $t$  (discontinuous problems have been analyzed in an economic framework (cf. for example Bich and Laraki (2011))). In the next proposition, we show the decrease of the profit during the month  $t$  in the absence of non-nuclear thermal production.

**Proposition 2.3.2** For all  $t \in \{1, \dots, T\}$ , for all  $n \in \{1, \dots, N\}$ , if  $H_{th}^t$  is a non-empty set,  $b_{nuc} < b_{th}$  and  $\bar{q} \in H^t \setminus H_{th}^t$ , then there exists a sequence  $(q_r)_{r \in \mathbb{N}} \in H_{th}^t$  with  $\lim_{r \rightarrow \infty} q_r = \bar{q}$  such that  $\lim_{r \rightarrow \infty} \pi_t^n(q_r) > \pi_t^n(\bar{q})$ .

### Proof

According to the assumptions,  $\bar{q}$  is a production vector which belongs to  $H^t \setminus H_{th}^t \subset H^t$ , for some  $t \in \{1, \dots, T\}$ . Let us denote  $\bar{q} = ((\bar{q}_{\nu j 1}^{nuc})_{j=1}^J, \dots, (\bar{q}_{\nu j T}^{nuc})_{j=1}^J, \bar{q}_{\nu 1}^{th}, \dots, \bar{q}_{\nu T}^{th})_{\nu=1}^N$  while  $\bar{q}_{nt} = ((\bar{q}_{n j t}^{nuc})_{j=1}^J, \bar{q}_{nt}^{th})$  is the production of the producer  $n$  realized in the month  $t$ .

Profit's function  $\pi_t^n : H^t \rightarrow \mathbb{R}$  is defined as

$$p_t \cdot \left( \sum_{j=1}^J q_{n j t}^{nuc} + q_{nt}^{th} \right) - \sum_{j=1}^J C_{n j}^{nuc}(q_{n j t}^{nuc}) - C_n^{th}(q_{nt}^{th})$$

In view of Proposition 2.3.1 on page 112, the production vector  $\bar{q} \in \overline{H_{th}^t}$ . It follows that there exists a sequence  $(q_r)_{r \in \mathbb{N}}$  such that  $(q_r)_{r \in \mathbb{N}} \in H_{th}^t$  and  $\lim_{r \rightarrow \infty} q_r = \bar{q}$ . Let us denote  $q_r = (((q_{\nu j 1}^{nuc})_r, \dots, (q_{\nu j T}^{nuc})_r)_{j=1}^J, (q_{\nu 1}^{th})_r, \dots, (q_{\nu T}^{th})_r)_{\nu=1}^N$  and  $(q_{nt})_r = (((q_{n j t}^{nuc})_r)_{j=1}^J, (q_{nt}^{th})_r)$  the component of the sequence  $q_r$  which corresponds to the month  $t$  and the producer  $n$ . For all  $r$ , we can compute the associated with the production  $(q_{nt})_r$  merit order price  $p_{tr} = mc_n^{th}((q_{nt})_r^{th})$ . The price  $\bar{p}_t$  represents the merit order price associated with the production vector  $\bar{q}_{nt}$ . Since at the limit, the value of  $q_r$  is equal to  $\bar{q}$ , we deduce that the nuclear is the marginal technology during the month  $t$ . Thus, the price  $\bar{p}_t$  in month  $t$  is determined by the nuclear marginal cost  $b_{nuc}$ . It follows that

$$\lim_{r \rightarrow \infty} p_{tr} = \lim_{r \rightarrow \infty} mc_n^{th}((q_{nt})_r^{th}) = mc_n^{th}(\bar{q}_{nt}^{th}) = mc_n^{th}(0) = b_{th} > b_{nuc} = \bar{p}_t. \quad (2.16)$$

At the limit, we obtain

$$\lim_{r \rightarrow \infty} \pi_t^n(q_r) - \pi_t^n(\bar{q}) =$$

$$\lim_{r \rightarrow \infty} (p_{tr} (\sum_{j=1}^J (q_{njt}^{nuc})_r + (q_{nt}^{th})_r) - \bar{p}_t (\sum_{j=1}^J \bar{q}_{njt}^{nuc} + \bar{q}_{nt}^{th})) -$$

$$\lim_{r \rightarrow \infty} ((\sum_{j=1}^J C_{nj}^{nuc}((q_{njt}^{nuc})_r) + C_n^{th}((q_{nt}^{th})_r)) - (\sum_{j=1}^J C_{nj}^{nuc}(\bar{q}_{njt}^{nuc}) + C_n^{th}(\bar{q}_{nt}^{th})))$$

Since  $\lim_{r \rightarrow \infty} q_r = \bar{q}$  and in view of the relationship (2.16) and of the strictly positive nuclear production ( $\sum_{j=1}^J \bar{q}_{njt}^{nuc} \geq \sum_{j=1}^J Q_{min}^{n,j,nuc} > 0$ ) the first term is strictly positive. The second term converges to zero because of the continuity of the production cost functions.

Consequently, we deduce that  $\lim_{r \rightarrow \infty} \pi_t^n(q_r) - \pi_t^n(\bar{q}) > 0 \Leftrightarrow \lim_{r \rightarrow \infty} \pi_t^n(q_r) > \pi_t^n(\bar{q})$  and the proposition is proven.

□

Let us notice that the non-emptiness of the set  $H_{th}^t$  obviously depends on the values of the exogenous variables ( $Q_{max}^{n,j,nuc}, Q_{min}^{n,j,nuc}, Q_{max}^{n,th}, S_{reload}^{n,j}, S_1^{n,j}, D_t, Q_t^{hyd}$ ) of the optimization problem (2.15).

The inequality  $b_{nuc} < b_{th}$  holds within our data; thus according to Proposition 2.3.2, the profit at date  $t$  decreases for all the production vectors whose non-nuclear thermal component at this date is equal to zero. Consequently, it is not profitable for a producer who wants to maximize its profit during the month  $t$  to run only its nuclear units and be remunerated at a price  $p_t = b_{nuc}$ . Therefore, the producer will search for a solution that maximizes its profit in the month  $t$  among the production vectors of the set  $H_{th}^t$ .

Let us mention that if the current monthly profit maximization problem is determined on  $H^t$  which is a compact set, the objective function is not continuous according to Proposition 2.3.2 (see page 113). If the current monthly profit maximization problem is determined on  $H_{th}^t$ , the objective function is continuous in view of Proposition 2.3.2 while the set  $H_{th}^t$  is not compact. Hence, it is not possible to conclude on the existence of solutions of this problem (Varian (1992)).

The following corollary establishes the relation between the optimal short-term production problem on  $H^t$  and the optimal short-term production problem on  $H_{th}^t$ .

**Corollary 2.3.1** *The current monthly profit maximization problem determined on  $H^t$  is equivalent to the current monthly profit maximization problem determined on  $H_{th}^t$  (same set of solutions and same value<sup>15</sup>), for all  $t$ .*

### Proof

This corollary is an obvious consequence of Proposition 2.3.2.

□

It should be mentioned that, for all  $t$ , the value of both optimization problems exists (in the real line) since the objective function (profit function) is polynomial and the set  $H^t$  along with the set  $H_{th}^t$  are bounded. If a solution of the current monthly profit maximization problem on  $H_{th}^t$  exists for all  $t$ , then it constitutes an equilibrium since all the conditions in order to be a merit order equilibrium are met and it maximizes the profit in the month  $t$  (see Corollary 2.3.1 and Definition 2.3.1 on page 102). In the next subsection, we provide a numerical illustration of the optimal short-term production problem (2.15).

### A property of the short-term profit function and its implications

We proceed with the following lemma which shows the concavity of the profit function of the optimal short-term production problem determined on  $H_{th}^t$ .

**Lemma 2.3.1** *For all  $t \in \{1, \dots, T\}$ , for all  $n \in \{1, \dots, N\}$  the profit function of the current monthly profit maximization problem on  $H_{th}^t$  is concave with respect to  $q_{nt}$ .*

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<sup>15</sup>The value of an optimization problem is defined as the upper bound of the set  $\{f(x)|x \in C\}$ , where  $f$  is the objective function and  $C$  is the set of feasible solutions. The value always exists even if the set of solutions is empty. When the set of solutions is nonempty, the value of an optimization problem is the common value  $f(\bar{x})$  for any solution  $\bar{x}$ .

### Proof of Lemma 2.3.1

The profit  $\pi_t^n$  of producer  $n$  at each month  $t$  is

$$p_t \cdot \left( \sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th} \right) - \sum_{j=1}^J C_{nj}^{nuc}(q_{njt}^{nuc}) - C_n^{th}(q_{nt}^{th})$$

We remind that the price  $p_t$  is given by the marginal cost of the non-nuclear thermal production on the set  $H_{th}^t$ . Following some calculations and using the supply-demand equilibrium constraint in the month  $t$  for the producer  $n$ ,

$$\sum_{j=1}^J \bar{q}_{njt}^{nuc} + \bar{q}_{nt}^{th} = D_t - Q_t^{hyd} - \sum_{\nu \neq n} \left( \sum_{j=1}^J q_{\nu jt}^{nuc} + q_{\nu t}^{th} \right)$$

we deduce that producer's  $n$  profit  $\pi_t^n$  is a quadratic function of the non-nuclear thermal production  $q_{nt}^{th}$

$$-c_{th}^n (q_{nt}^{th})^2 + (2c_{th}^n (D_t - Q_t^{hyd} - \sum_{\nu \neq n} \sum_{j=1}^J q_{\nu jt}^{nuc} + q_{\nu t}^{th})) + b_{nuc} - b_{th}) q_{nt}^{th} + c_t^n \quad (2.17)$$

where  $c_t^n = ((b_{th} - b_{nuc})(D_t - Q_t^{hyd} - \sum_{\nu \neq n} (\sum_{j=1}^J q_{\nu jt}^{nuc} + q_{\nu t}^{th})) - a_{th}^n - \sum_{j=1}^J a_{nuc}^{n,j})$  is the constant part of the profit function. The quantity  $D_t - Q_t^{hyd} - \sum_{\nu \neq n} (\sum_{j=1}^J q_{\nu jt}^{nuc} + q_{\nu t}^{th})$  denotes the ‘‘net demand’’ served by the producer  $n$  during the month  $t$ .

We notice that this is a quadratic function of the form  $f_t^n(u_{nt}) = a^n u_{nt}^2 + b_t^n u_{nt} + c_t^n$ , where  $a^n = -c_{th}^n$  and  $b_t^n = (2c_{th}^n (D_t - Q_t^{hyd} - \sum_{\nu \neq n} (\sum_{j=1}^J q_{\nu jt}^{nuc} + q_{\nu t}^{th})) + b_{nuc} - b_{th})$ . Since  $a^n < 0$  for all  $n$ , the function  $f_t^n(u_{nt})$  is concave with respect to  $u_{nt}$ , for all  $n, t$ . In view of the strict negativity of the coefficient  $a^n$ , it is also a strictly concave function with respect to  $u_{nt}$ . As a result, the profit function  $\pi_t^n$  is strictly concave with respect to  $q_{nt}^{th}$  and by taking into account the other variables  $(q_{njt}^{nuc})_{j=1}^J$ , we conclude that the profit function  $\pi_t^n$  is concave with respect to  $q_{nt}$ , for all  $n, t$ .

□

**Remark 2.3.2** *The strict concavity of the profit function  $\pi_t^n$  with regard to the non-nuclear thermal production  $q_{nt}^{th}$  (see Proof of Lemma 2.3.1 on page 115) implies the unicity of solutions with respect to the non-nuclear thermal component for all  $n, t$ . However, if we consider the other variables which do not affect the profit  $\pi_t^n$  and according to the proof of Lemma 2.3.1 the profit function  $\pi_t^n$  is concave with regard to  $q_{nt}$  for all  $n, t$  which does not imply necessarily the unicity of the entire solution.*

## 2.4 Numerical modelling

In this section, we provide an analytical description of our data set. Then, we study the nuclear and non-nuclear thermal production decisions as well as the storage decisions coming from the resolution of the optimal short-term production problem analyzed in the previous section 2.3.4, within a simple numerical model solved with Scilab.

### 2.4.1 Data

The data used in our numerical dynamic model is French. It is of several years due to the difficulty of collection: \* level of French demand during the year 2007, \* generation capacity of hydro (run-of-river), nuclear and non-nuclear thermal, \* nuclear fuel stock of reloading and \* fixed and variable costs of nuclear, coal and gas generation.

Consumption data comes from the French Transmission & System Operator (named RTE). It gives the hourly consumption in MW for the entire year 2007 which we use to determine the monthly consumption. This data takes into account the electricity losses of the network, as estimated by RTE. RTE also provides the annual capacity of nuclear as well as the annual capacity of gas and coal for the year 2009. The annual capacity and production of hydro as well as the nuclear fuel stock of reloading have been provided by Electricité de France.

The costs of production come from the report DGEMP & DIDEME (2003, 2008) which we have already presented in our first chapter (see Subsection 1.2 on page 35).



As we have also mentioned in the first chapter, we choose to use data coming from the official report of 2003 and not the other published in 2008 because a more analytical data is provided in the first report and because we do not expect any radical changes in the cost structure of nuclear and of fossil fuel generation technologies. This report gives the technical characteristics, the costs and a sensitivity analysis for different types of generation technologies (nuclear, coal, gas, oil). It also gives the life duration, the availability of the generating units as well as the typical management of the fuel for nuclear. This data is calculated for the year 2007 and 2015. The report also gives the cost of investment, the variable / fixed cost of operation, the cost of fuel as well as the external costs (e.g. cost of CO<sub>2</sub>, cost of a major nuclear accident, etc.). It also provides the total cost of production for a base load (8760h) and semi-base (3000h) operation. It gives the total cost for each technology as follows: cost of investment, variable and fixed cost of exploitation, fuel cost, taxes, R&D costs for the nuclear and cost of CO<sub>2</sub> per ton in the case of coal and gas for the same levels of operation that were previously mentioned. These costs are estimated for the year 2007 and 2015 with different discount rates (3%, 5%, 8%, 11%) taking into account the influence of exchange rate on the production cost. Finally, a sensitivity analysis links production costs to the main parameters for each technology (e.g. investment cost, availability, life duration, etc.). We remind from our first chapter that this report is based on the costs of the European Pressurized Reactor (EPR).

### **The impact of the discount rate on the calculation of the nuclear cost.**

The role of the discount rate is important in the determination of the nuclear costs for several reasons. We know that the level of the fixed costs depends crucially on the discount rate. For example, the investment cost for nuclear is set at 6,4 Euros/MWh with a discount rate of 3% and at 16,3 Euros/MWh for a discount rate of 8%. In addition to this, the nuclear fuel (being “enriched uranium”) has its own distinctive fuel cycle extremely different than other resources such as coal, oil and natural gas. Nuclear fuel is processed through several steps such as mining and milling, conversion, enrichment, and fuel manufacturing. A last main characteristic of nuclear is the storage of waste and the long process of the dismantling of plants (Source: Brite/Euram III: Projects).

The resulting duration of the nuclear fuel cycle is significantly high. To end, the life duration retained for the non-nuclear thermal units is 30 years, while the new reactor EPR (European Pressurized Reactor) is conceived to operate for 60 years. Therefore, the discount rate is a very critical factor which can significantly affect the nuclear costs.

### **A contrast between nuclear and non-nuclear thermal production cost.**

An important point of differentiation between the nuclear and the thermal non-nuclear production cost that we like to stress here is the degree of their volatility. The cost of non-nuclear thermal production is highly volatile for different reasons. One is the price volatility of CO<sub>2</sub>. In figure<sup>16</sup> 2.5 on page 120, we can see a first period (“Phase I” of the European Union Emissions Trading System (EU ETS)<sup>17</sup>) which was initiated between January 2005 and December 2007 and a second period (“Phase II” of the EU ETS) which covers the years 2008 to 2012. Phase I is characterized by high volatility and a price of CO<sub>2</sub> allowance equal to zero at the end of the period because of the impossibility of using these allowances for the next phase. In Phase II, the price has oscillated around 25 Euros until September 2008, then it decreased significantly (-65% over the period October 2008-February 2009) as a direct result of the economic crisis. Future prices follow the same trend as spot prices in the second period remaining in average higher than 2.23 Euros. Since May 9, 2011, there are no future price 2012. Since June 2011, the price has fallen steadily from 16 Euros/tCO<sub>2</sub> to almost 6 Euros/tCO<sub>2</sub> in July 2012 (Source: Observatoire de l’Industrie Electrique (OIE), Bluenext, July 2012). The other considerable volatility

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<sup>16</sup>The spot price of CO<sub>2</sub> is represented by the blue line in the first period and the white blue line in the second period. In the second period, the green line indicates the future price of CO<sub>2</sub>.

<sup>17</sup>The European Union Emissions Trading System (EU ETS) is a cornerstone of the European Union’s policy to combat climate change and its key tool for reducing industrial greenhouse gas emissions cost-effectively. The first - and still by far the biggest - international system for trading greenhouse gas emission allowances, the EU ETS covers more than 11000 power stations and industrial plants in 31 countries, as well as airlines. Under the cap and trade principle, a cap is set on the total amount of greenhouse gases that can be emitted by all participating installations. Allowances for emissions are then auctioned off or allocated for free, and can subsequently be traded. Installations must monitor and report their CO<sub>2</sub> emissions, ensuring they hand in enough allowances to the authorities to cover their emissions. If emission exceeds what is permitted by its allowances, an installation must purchase allowances from others. Conversely, if an installation has performed well at reducing its emissions, it can sell its leftover credits. This allows the system to find the most cost-effective ways of reducing emissions without significant government intervention (Source: European Commission Climate Action, EU Emissions Trading System. Retrieved 04 January 2013).

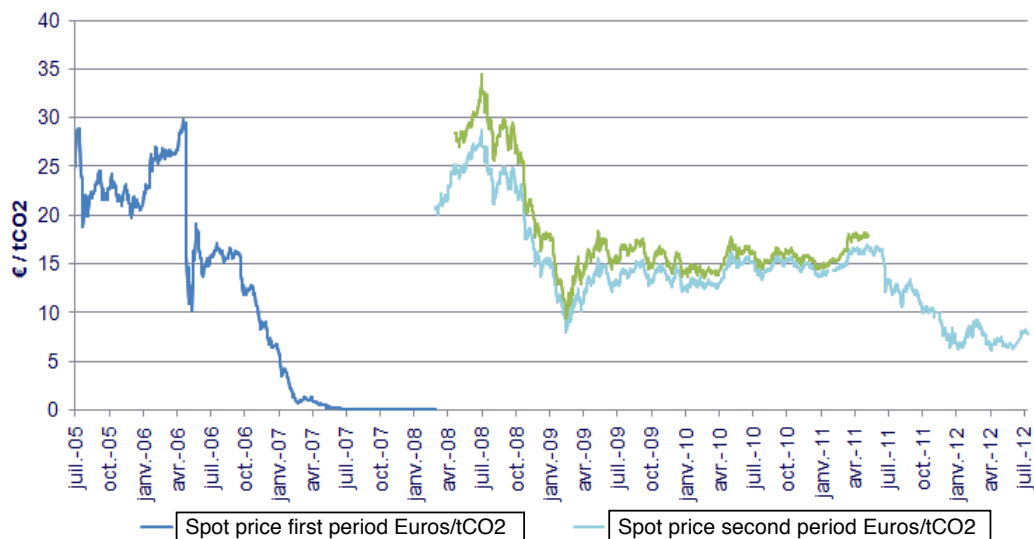


Figure 2.5: Evolution of the CO<sub>2</sub> price from July 2005 until July 2012.

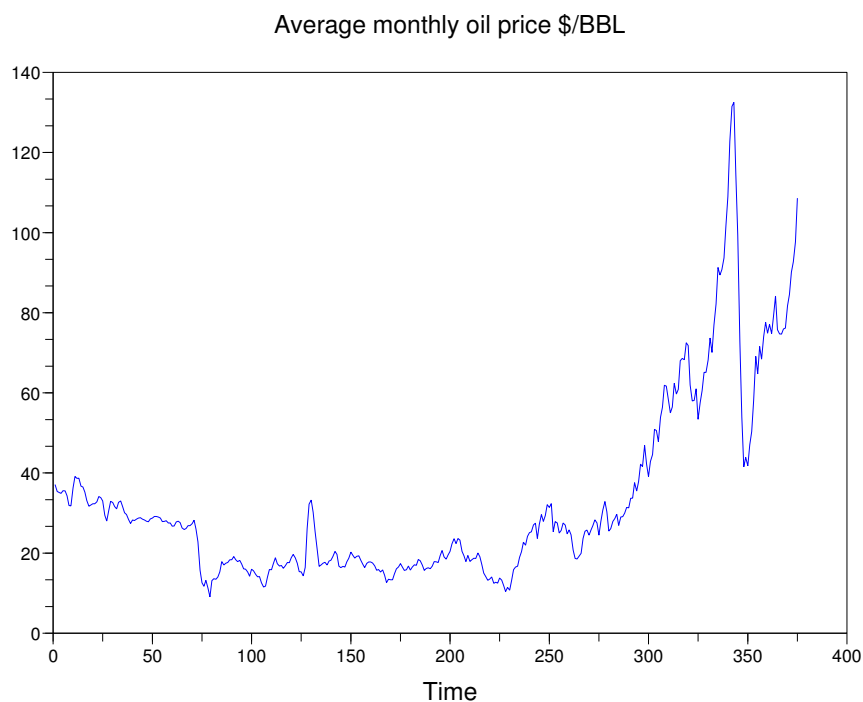


Figure 2.6: Average monthly oil price from January 1980 through March 2011 (\$/BBL).

factor has been the sharp move of oil prices (and gas prices) in these last few years (see Figure 2.6 on page 120, Source: UNCTADstat, CNUCED).

On the contrary, the cost of nuclear fuel and the operation costs<sup>18</sup> are less volatile

<sup>18</sup>The above data refers to fuel plus operation and maintenance costs only. It excludes capital costs since these vary greatly among utilities and states, as well as with the age of the plant.

U.S. Electricity Production Costs (1995–2009, In 2009 cents per kilowatt–hour)

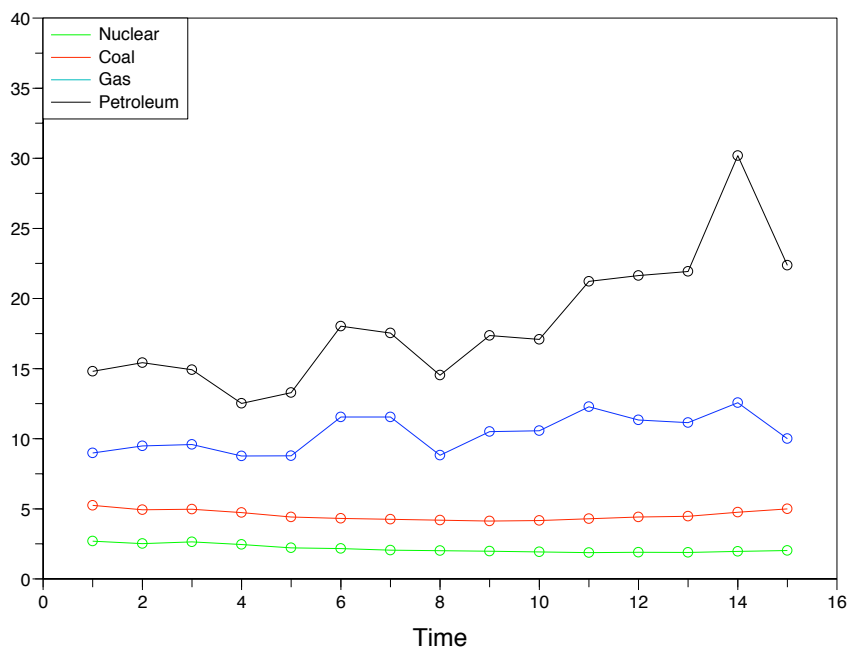


Figure 2.7: Evolution of U.S. electricity production costs (1995-2009, in 2009 cents/KWh).

during that period than the costs of non-nuclear thermal production (see Figure 2.7, Source: Ventyx Velocity Suite, Nuclear Energy Institute). This is mainly due to two reasons; firstly the small level of uranium cost as a component of the final nuclear fuel cost, and secondly a stabilization factor resulting from the delay between the extraction of natural uranium and the final manufacturing of nuclear fuel.

### Specific data assumptions for our numerical modelling

Our modelling is based on a scenario in which one dollar is equal to one euro, the discount rate is 8%, the cost of CO<sub>2</sub> per ton reaches 20 euros, the price of coal is 30 dollars per ton and the price of gas is 3.3 dollars per MBtu (1 MBtu = 0.2931 KWh). The choice of this particular scenario is mainly based on the scenario considered by DGEMP & DIDEME in 2003 for the estimation of costs of the different types of generation technologies in 2007.

In view of the quadratic form of the non-nuclear thermal production cost, we need to specify that the value of the coefficient  $a_{th}$  involved in the non-nuclear thermal cost

function corresponds to the fixed cost provided by the data (investment cost, fixed exploitation cost, taxes), while the other coefficients have been determined by interpolation in order to meet the variable cost of coal and gas provided by our data base (fuel cost, variable exploitation cost, CO<sub>2</sub> cost, taxes on the gas fuel). The consideration of the fixed costs in the production cost of both technologies (nuclear, non-nuclear thermal) permits us to obtain a more realistic vision of the value of profit within our medium-term horizon.

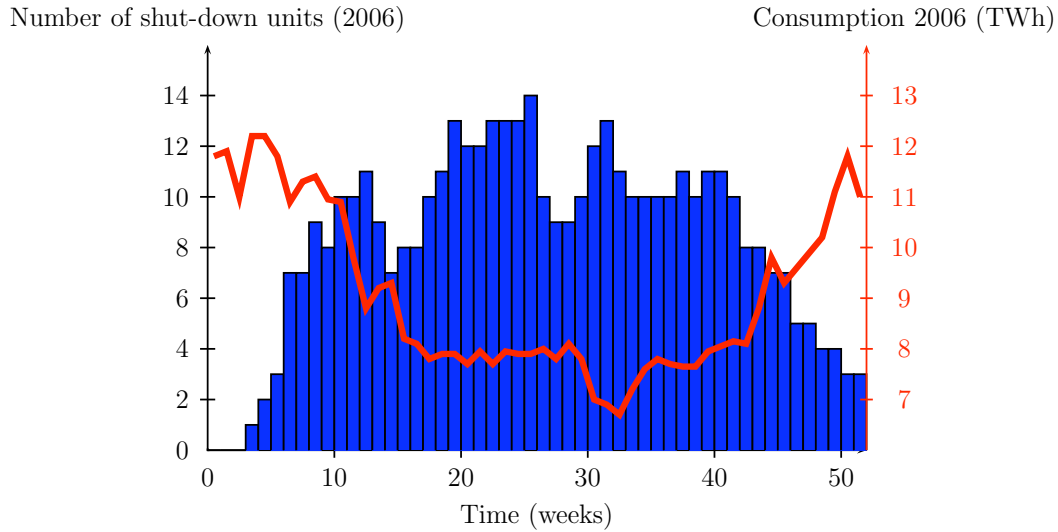


Figure 2.8: Availability of nuclear units.

The capacity of each nuclear unit has been simulated<sup>19</sup> in order to approximate the graph of figure<sup>20</sup> 2.8, which shows the availability of nuclear units per week. For example, the capacity of the nuclear unit  $j = 1$  (respectively  $j = 2, \dots, j = 12$ ) corresponds to the average capacity of shut-down nuclear units in January (respectively February,  $\dots$ , December). A difficulty that interferes with the calculation of the average capacity of shut-down nuclear units during a month, comes from the fact that a month is not composed of an integer of weeks. This has to be taken into account in order to compute the average number of shut-down nuclear units and thus, the average nuclear capacity every month. Note that the french nuclear set is composed of different types of nuclear reactors with different levels of capacity and we do not dispose a detailed data of the information for figure 2.8. The initial value of the nuclear fuel stock ( $S_1^j$ ) has been set by simulating the

<sup>19</sup>Access to detailed nuclear capacity data for each short period of time is not possible due to the confidentiality of such data.

<sup>20</sup>Each blue bar shows the number of shut-down nuclear units during a week and the red line shows the evolution of the consumption over time. The different levels of consumption are measured on the right axis while the number of shut-down nuclear units is reflected on the left axis.

nuclear fuel stock of each unit  $j$  available at the beginning of the time horizon of the model.

The number of days (or number of hours) equivalent to full capacity is calculated within our model as the product of the operating factor of a nuclear reactor ( $K_u$ ) which represents the percentage of time that the reactor is used at its maximum capacity during its availability period and the number of days (or number of hours) of a given year:  $K_u \cdot 365 \approx 0.70 \cdot 365 \approx 256$ . This number is almost identical with the number of days (or number of hours) during which a reactor of 1500 MW operates at full capacity in the case of a campaign of 12 months following the operational schema of the French nuclear operator EDF (see page 86, Subsection 2.2.2).

Finally, as we mentioned in the first chapter, an EPR reactor can maneuver between 25% of nominal capacity and 100% of nominal capacity in order to follow-up load. We take into account these levels of maneuverability within our numerical model to determine the minimum and maximum nuclear production constraints. In particular, the minimum nuclear production is the 25% of the capacity of the units of type  $j$  i.e.  $0.25 \cdot \text{Capacity}^{j,nuc}$  and the maximum nuclear production is the 100% of the capacity of the units of type  $j$  i.e.  $1 \cdot \text{Capacity}^{j,nuc}$  (see Annex C, Table C.2).

### **Numerical simulation of the levels of monthly demand.**

The amounts of monthly demand<sup>21</sup>  $D_t$  obtained for the period January 2007 - December 2009 are presented in figure<sup>22</sup> 2.9 on page 124. In particular, the values of monthly demand during the period January 2007 - December 2007 come from our historical data. Then, we reproduce these values by applying a positive rate on the monthly demand for the years that follow (2008 and 2009). We suppose an augmentation of the demand level by a rate of 1% per year in order to take into account the increasing trend of demand from one year

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<sup>21</sup>Note that we did a rescaling on this data to take into account the diversity on the length of the months.

<sup>22</sup>In our numerical model, the unit of energy used for measuring demand, stock and production is the MWh.

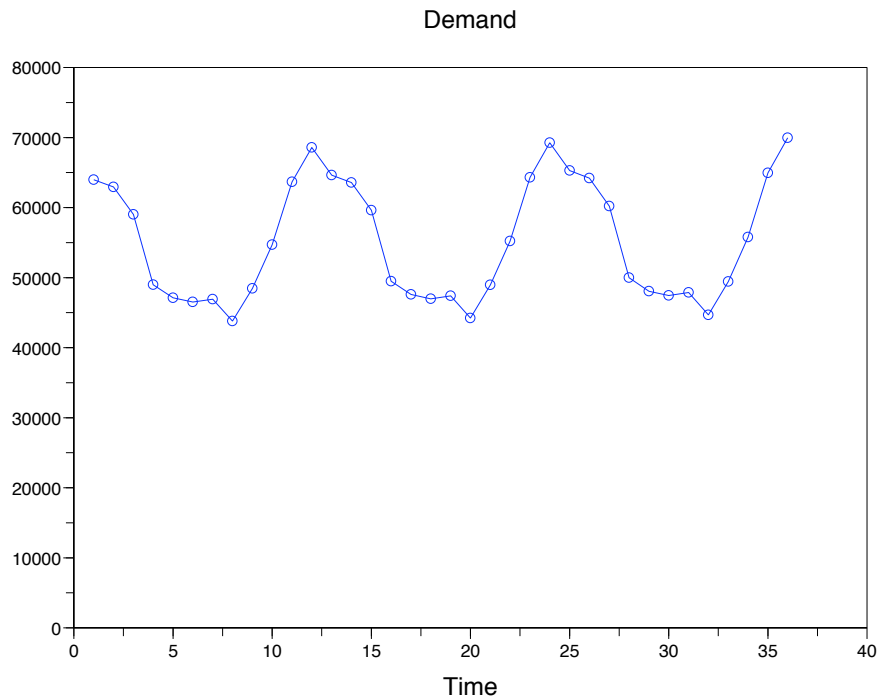


Figure 2.9: Simulated demand (in MW)

to another. This implies that the evolution of demand over time is not periodic<sup>23</sup>. One can see the seasonal variation of demand between winter (high demand) and summer (low demand). We observe high levels of demand during November, December, January and February with demand peaks in December. The demand falls during spring and summer (April - August). We do not notice any demand peaks in summer due to high levels of temperature like in the case of California in 2000 when an unusually hot summer and cold winter over much of the western region in combination with other events led to a severe imbalance between supply and demand and successive blackouts (EPRI (2001)). It should be mentioned that this analysis lacks a certain amount of finesse because it takes into consideration the monthly levels of demand<sup>24</sup> and not the hourly levels of demand. Indeed, if we compare the peak monthly demand with the peak hourly demand for a given year, we could observe a difference between them. For the year 2007, the peak monthly demand which is 68581 GW corresponds to an average hourly demand of 93.9

<sup>23</sup>In mathematics, a periodic function is a function that repeats its values in regular intervals or periods. The most important examples are the trigonometric functions, which repeat over intervals of length  $2\pi$  radians. Periodic functions are used throughout science to describe oscillations, waves and other phenomena that exhibits periodicity.

<sup>24</sup>Note that the monthly demand in 2007 results from the aggregation of the hourly demand found within our historical data.

GW that is superior than 88.2 GW corresponding to the peak hourly demand observed in our historical data.

### **Numerical results deduced from our data set**

Let us now provide a couple of numerical results obtained within our data set:

- According to the “Nouvelle Organisation du Marché de l’Électricité” (NOME) law which was adopted by the parliament in November 2010 within a context of a probable extension of the French nuclear set (increase of nuclear capacity through the construction of new reactors (e.g. EPR reactor), extension of the lifetime of the existing nuclear plants) and it is applied since July 1, 2011, EDF will have to give more access for its competitors. The NOME law is the next step after the “opening” of electricity markets to competition. It largely reflects the findings of the report of the Commission Champsaur in April 2009 which recommends access to nuclear electricity of EDF for all suppliers (Champsaur (2009)). It also moves towards the elimination of regulated tariffs for large industrial firms (yellow and green tariffs) and their maintenance for individuals and small firms. More precisely, the NOME law requires EDF to sell at a competitive price to alternative suppliers of electricity and gas (GDF Suez, E.ON, ENEL, Poweo, Direct Energy, etc.) 100 terawatt hours (1 billion kilowatt hours) per year at most, that is a quarter of its nuclear production until 2025. This price should cover the total cost of the operating plants: investment cost, maintenance cost, capitals return, modernization, waste recycling, decommissioning cost. In 2010, the French energy regulator (CRE) estimated that a price between 37.5 and 38.8 euros per MWh would be justified and consistent with the text of the law (Le Monde (01/02/2011)). Without penalizing EDF, it would permit the development of competition, at least in the market of professionals consumers, according to the regulator. Alternative suppliers argued that the price of nuclear per MWh should be inferior than 35 euros because they feared that a price superior than this would not permit them to compete with the historical operator. Finally, the French government decided that the incumbent (EDF) will sell to its competitors up to a quarter of its nuclear power at a price of 40 euros



per MWh from the first of July 2011, then of 42 euros from the first of January 2012, which corresponds exactly at the price claimed by EDF. According to the French government, these prices will contribute to the security of supply of France and the work that EDF might have to start in order to enhance nuclear safety by ameliorating the security conditions of the operating nuclear plants after Fukushima disaster in Mars 2011 (Les Echos (20/04/2011)). In view of our modelling of the nuclear production cost and the value of its coefficients computed in this data (which is calculated for the year 2007), we determine the average nuclear cost evaluated here at 37.25 euros per MWh. This price is very close to the range of nuclear electricity prices (37.5 - 38.8 euros per MWh) appeared in the analysis of the Regulatory Commission of Energy (CRE) before Fukushima accident. This result shows that within our data the price of the nuclear MWh does not take a value significantly lower than 37.5 euros per MWh as it was asserted by alternative producers in order to compete EDF. However, the price calculated here does not exceed the 38.8 euros per MWh (being eventually close to the price of 42 euros per MWh set by the French government) which is totally understandable since the cost evaluations of DGEMP & DIDEME do not take into account the extra costs resulting from measures intended to improve nuclear safety after Fukushima accident.

- In order to examine whether the non-nuclear thermal production is profitable or not and the potential to pay for the cost coming from it every month, we determine the threshold of profitability of the non-nuclear thermal production realized by  $N$  producers (considered also as one aggregate producer) during the month  $t$  ( $\theta_N$ ). To do this, we take the profit resulting from the aggregate monthly non-nuclear thermal production equal to zero:  $\theta_N = N \sqrt{\frac{a_{th}}{c_{th}}}$ . If the monthly non-nuclear thermal production level realized by the  $N$  producers is higher (lower) than  $\theta_N$ , then the profit is positive (negative). In view of Proposition 2.2.1 on page 91 and for a given total non-nuclear thermal capacity (independent of  $N$ ) divided in  $N$  identical producers, one has  $\theta_1 = \dots = \theta_N$  which means that the threshold of profitability of the non-nuclear thermal production  $\theta_N$  does not depend on  $N$ . The value of the threshold of profitability<sup>25</sup> provided by our numerical model is  $\theta \cong 18$  GW (or

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<sup>25</sup>In figures 2.14 and 3.4, the red crossed line represents the threshold of profitability of the non-nuclear thermal production.

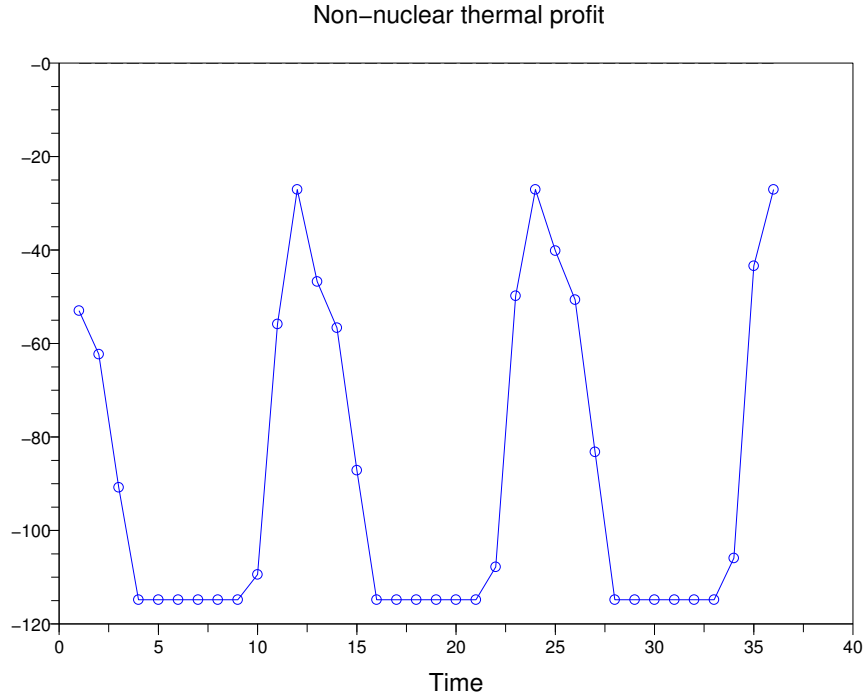


Figure 2.10: Non-nuclear thermal profit (in Euro (million))

equivalently 13 TWh per month) which exceeds the level of the aggregate monthly non-nuclear thermal capacity  $Q_{max}^{th}$  (see Annex C, Table C.2) and thus, it leads always to negative aggregate non-nuclear thermal profits (see Figure 2.10). Consequently, the non-nuclear thermal production is never profitable since the aggregate monthly non-nuclear thermal production cost is never covered. So, we conclude that the quadratic form of the non-nuclear thermal production cost in combination with the pricing of production at marginal cost (merit order price) causes important losses to the aggregate producer in our model. This could be explained by the numerous approximations (in particular the absence of mark-up rate) that we made within our model (see General results on page 142, Subsection 2.4.3). The condition  $\theta > Q_{max}^{th}$  (observed numerically) is not incompatible with positive profits. Indeed, the producer could manage to cover the cost resulting from the non-nuclear thermal production and even obtain a strictly positive profit if another generation technology higher than the non-nuclear thermal in the merit order becomes the marginal technology (hydro technology with storage, oil, etc.). In this case, the non-nuclear thermal production would be remunerated above its own marginal cost which would

help to recuperate its cost. However, we need to emphasize that the marginal cost of the last technology of the merit order used to equilibrate supply and demand (coming after non-nuclear thermal) needs to be non constant so that the non-nuclear thermal production becomes profitable for the producer.

### 2.4.2 “Equivalence” of merit order equilibrium between an economy with $N$ producers and an economy with one aggregate producer

From a purely logical (mathematical) perspective, our numerical model is characterized by a high level of complexity because of the high number of optimization variables and the important amount of operational constraints as we have already explained in subsection 2.2.6 on page 100. Particularly, in the simplified case of a unique producer ( $N = 1$  producer,  $T = 36$  months of optimization,  $K = 3$  campaigns of production,  $J = 12$  types of nuclear units), we have to deal with  $N \cdot (J \cdot T + T) = 1 \cdot (12 \cdot 36 + 36) = 468$  optimization variables. We need also to take into consideration the following optimization constraints: (i)  $N \cdot ((2 \cdot K + 1) \cdot (J - 2) + (2 \cdot K) \cdot 2) = 1 \cdot ((2 \cdot 3 + 1) \cdot (12 - 2) + (2 \cdot 3) \cdot 2) = 82$  nuclear fuel constraints, (ii)  $T = 36$  supply-demand equilibrium constraints and (iii)  $N \cdot (J \cdot T + T) = 1 \cdot (12 \cdot 36 + 36) = 468$  minimum/maximum nuclear and non-nuclear thermal production constraints meaning 586 constraints in total.

To reduce the number of optimization variables and operational constraints of our numerical model in order to facilitate the calculation of a merit order equilibrium, we show that the merit order equilibrium in an economy with  $N \geq 2$  producers is “equivalent” to the merit order equilibrium in an economy with one aggregate producer ( $N = 1$ ). We use this mathematical proposition to calculate an equilibrium of the original economy by working in an auxiliary economy with one aggregate producer from now on. The aggregate producer holds the capacity of all the types of nuclear units as well as the total non-nuclear thermal capacity. In particular, this will allow to simplify the notations  $q_{njt}^{nuc}$  and  $q_{nt}^{th}$  by considering the notations  $q_{jt}^{nuc}$  and  $q_t^{th}$  that represent respectively the level of

the nuclear production during the month  $t$  for the unit  $j$  and the level of the non-nuclear thermal production during the month  $t$  (and similarly for the stock).

### The notion of merit order equilibrium in the case of an aggregate producer

In view of Definition 2.3.1 of merit order equilibrium on page 102, one can develop this model to the case of a unique producer operating with the whole amount of non-nuclear thermal capacity and the aggregate nuclear capacity resulting from all the types of nuclear units. In the context of a unique producer, the Definition 2.3.1 becomes:

**Definition 2.4.1** The production vector  $\bar{q} = ((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$  is a merit order equilibrium with respect to a system of prices  $p \in \mathbb{R}_+^T$  if:

(i)  $\bar{q}$  is a feasible production vector: (a) it respects the nuclear fuel constraints, for all  $j$  and (b) it respects the minimum/maximum production constraints, for all  $j, t$ .

(ii) the price at each month  $t$  is determined by the marginal cost of the marginal technology. It is called the merit order price associated with the production vector  $\bar{q}$ .

(iii) at each date  $t$ , it respects the equality between supply and demand

$$\sum_{j=1}^J \bar{q}_{jt}^{nuc} + \bar{q}_t^{th} = D_t - Q_t^{hyd}. \quad (2.18)$$

In order to clarify the definition of the merit order equilibrium in the case of one aggregate producer, we provide some precisions for each condition ((i), (ii), (iii)) of this definition. More precisely, we have that:

(i) The nuclear fuel constraints for the unit  $j$  are given by (2.6), (2.7), (2.8) in the particular case when  $N = 1$  (see page 98, Subsection 2.2.5). The minimum/maximum nuclear and non-nuclear thermal production constraints have the following form

$$\begin{cases} Q_{min}^{j,nuc} \leq q_{jt}^{nuc} \leq Q_{max}^{j,nuc}, & \text{if no reload during month } t \text{ for unit } j \\ q_{jt}^{nuc} = 0, & \text{if unit } j \text{ reloads during month } t \end{cases} \quad (2.19)$$

$$0 \leq q_t^{th} \leq Q_{max}^{th} \quad (2.20)$$

where  $Q_{max}^{j,nuc}$  is the maximum nuclear production that can be realized by the unit  $j$  during a month and the minimum nuclear production is given by  $Q_{min}^{j,nuc}$ . The parameter  $Q_{max}^{th}$  represents the maximum non-nuclear thermal production during a month while the minimum non-nuclear thermal production is equal to  $Q_{min}^{th} = 0$ .

(ii) In our model, the merit order price  $p$  associated with a feasible production vector  $q$ ,  $p = (p_t)_{t=1}^T = (\Phi_t(q_t))_{t=1}^T = (\Phi_t((q_{jt}^{nuc})_{j=1}^J, q_t^{th}))_{t=1}^T = \Phi(q)$  is calculated in month  $t$  as follows:

$$p_t = \Phi_t(q_t) = \begin{cases} mc^{th}(q_t^{th}), & \text{if } q_t^{th} > 0 \\ mc^{nuc}(q_{jt}^{nuc}), & \text{if } q_t^{th} = 0 \end{cases} = \begin{cases} b_{th} + 2c_{th}q_t^{th}, & \text{if } q_t^{th} > 0 \\ b_{nuc}, & \text{if } q_t^{th} = 0 \end{cases} \quad (2.21)$$

If nuclear is the “last technology” of the merit order which is called to equilibrate supply and demand during the month  $t$  ( $q_t^{th} = 0$ ), then the price is determined by the marginal cost of nuclear  $mc^{nuc}(q_{jt}^{nuc})$ . In the case that the non-nuclear thermal production is additionally used to cover the monthly levels of demand ( $q_t^{th} > 0$ ), the price is given by the non-nuclear thermal marginal cost  $mc^{th}(q_t^{th})$ . We stress that the merit order price  $p_t$  is discontinuous on production vectors whose non-nuclear thermal component  $q_t^{th}$  is equal to zero (see Figure 2.11 on page 131).

(iii) Identically to the condition (iii) of merit order equilibrium in the case of  $N \geq 2$  producers, the monthly demand is determined by the difference between the level of demand  $D_t$  observed in month  $t$  and the hydro production  $Q_t^{hyd}$  provided during the month  $t$  (see page 84, Subsection 2.2.1).

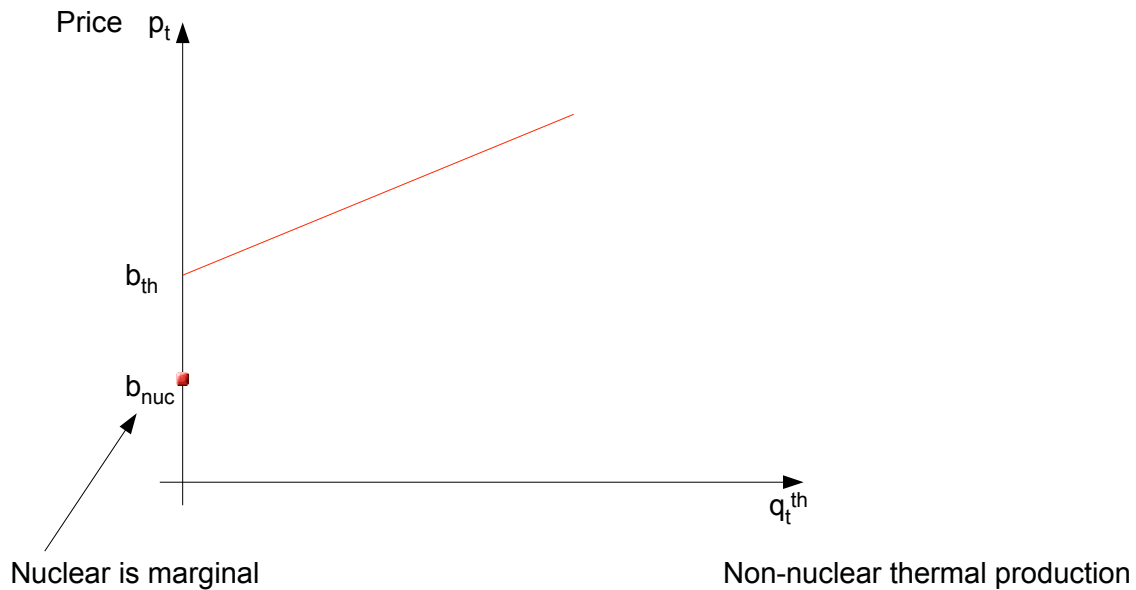


Figure 2.11: Price discontinuity

Let us now proceed with the following proposition which shows the equivalence between the merit order equilibrium of an economy with  $N \geq 2$  producers and the merit order equilibrium of an economy with one aggregate producer.

**Proposition 2.4.1** *Let us consider an economy  $E$  with several producers and let  $\tilde{E}$  be the auxiliary economy with a unique producer obtained by the aggregation of the  $N$  producers of  $E$ .*

- ( $\alpha$ ) *If  $q$  is a merit order equilibrium of  $\tilde{E}$  then it can be decentralized as a merit order equilibrium  $(q_n)_{n=1}^N$  of  $E$  with regard to the same prices.*
- ( $\beta$ ) *Conversely, if  $(q_n)_{n=1}^N$  is a merit order equilibrium of  $E$  then its aggregation defined by  $q = \sum_{n=1}^N q_n$  is a merit order equilibrium of  $\tilde{E}$  for the same prices.*

### Proof

We provide a technical and quite lengthy but not complicated proof.

### Proof of ( $\alpha$ )

Initially, we show that if the production vector  $q = ((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T$  is a merit order equilibrium in an economy with one aggregate producer then the production defined later  $(q_n)_{n=1}^N = (((q_{njt}^{nuc})_{j=1}^J, q_{nt}^{th}))_{t=1}^T)_{n=1}^N$  is a merit order equilibrium in an economy with  $N$  producers. We define the nuclear production  $q_{njt}^{nuc}$  of producer  $n$  as follows:

$$q_{njt}^{nuc} = \frac{Capacity^{j,nuc,n}}{Capacity^{j,nuc}} \cdot q_{jt}^{nuc}$$

where  $Capacity^{j,nuc} = \sum_{n=1}^N Capacity^{j,nuc,n}$ . The first term of this product is the share of the total nuclear capacity  $Capacity^{j,nuc}$  of unit  $j$  obtained by the producer  $n$  whose capacity is  $Capacity^{j,nuc,n}$ . As a consequence, the nuclear production  $q_{njt}^{nuc}$  of producer  $n$  is the part of the aggregate nuclear production  $q_{jt}^{nuc}$  that this producer can realize according to the capacity  $Capacity^{j,nuc,n}$  that it holds. Similarly, we define the non-nuclear thermal production  $q_{nt}^{th}$  of producer  $n$  in the following way:

$$q_{nt}^{th} = \frac{Capacity^{th,n}}{Capacity^{th}} \cdot q_t^{th}$$

where  $Capacity^{th} = \sum_{n=1}^N Capacity^{th,n}$ . The first term of this product is the share of the total non-nuclear thermal capacity  $Capacity^{th}$  obtained by the producer  $n$  whose capacity is  $Capacity^{th,n}$ . Therefore, the non-nuclear thermal production  $q_{nt}^{th}$  of producer  $n$  is the part of the total non-nuclear thermal production  $q_t^{th}$  that this producer can realize with respect to the capacity  $Capacity^{th,n}$  that it holds.

Firstly, we prove that  $(q_n)_{n=1}^N$  is a feasible production vector in the economy with  $N$  producers. Since  $q$  is a feasible production vector, we have that

$$Q_{min}^{j,nuc} \leq q_{jt}^{nuc} \leq Q_{max}^{j,nuc}, \quad \text{for all } j, t$$

and

$$0 \leq q_t^{th} \leq Q_{max}^{th}, \quad \text{for all } t$$

By multiplying the first relationship with the share of the total nuclear capacity of producer  $n$ , one obtains

$$\frac{Capacity^{j,nuc,n}}{Capacity^{j,nuc}} \cdot Q_{min}^{j,nuc} \leq \frac{Capacity^{j,nuc,n}}{Capacity^{j,nuc}} \cdot q_{jt}^{nuc} \leq \frac{Capacity^{j,nuc,n}}{Capacity^{j,nuc}} \cdot Q_{max}^{j,nuc}$$

which can be expressed as follows:

$$Q_{min}^{n,j,nuc} \leq q_{njt}^{nuc} \leq Q_{max}^{n,j,nuc}, \quad \text{for all } n, j, t$$

In the same way, we multiply the second relationship with the share of the total non-nuclear thermal capacity of producer  $n$ . Thus, one has

$$\frac{Capacity^{th,n}}{Capacity^{th}} \cdot 0 \leq \frac{Capacity^{th,n}}{Capacity^{th}} \cdot q_t^{th} \leq \frac{Capacity^{th,n}}{Capacity^{th}} \cdot Q_{max}^{th}$$

or equivalently

$$0 \leq q_{nt}^{th} \leq Q_{max}^{n,th}, \quad \text{for all } n, t$$

Moreover, the production vector  $q$  respects the nuclear fuel constraints (2.6), (2.7), (2.8) in the specific case when  $N = 1$ . In this case, if we multiply all the nuclear fuel constraints with the share of the total nuclear capacity of producer  $n$ , we deduce that the production vector  $(q_n)_{n=1}^N$  satisfies the nuclear fuel constraints (2.6), (2.7), (2.8) in the case of  $N$  producers. Consequently, we conclude that  $(q_n)_{n=1}^N$  is a feasible production vector in an economy with  $N$  producers.

Secondly, we prove that the merit order price associated with the production  $q$  of one aggregate producer and the merit order price associated with the production  $(q_n)_{n=1}^N$  of  $N$  producers coincide. According to the determination of the merit order price in the case of one aggregate producer (2.21) and in the case of  $N$  producers (2.12), if nuclear is the marginal technology then the merit order price is the same in both cases. It remains to show that the merit order price is identical in both the decentralized and the centralized economy when non-nuclear thermal is the marginal technology. In the context of an economy with an aggregate producer and by taking into consideration Proposition 2.2.1 within this context, the variable part of the non-nuclear thermal cost  $c_{th}$  is inversely proportional to the capacity  $Q_{max}^{th}$  which means that there exists  $\lambda \in \mathbb{R}$  such that  $c_{th} =$



$\frac{\lambda}{Q_{max}^{th}}$ . Hence, in the case that  $q_t^{th} > 0$ , we obtain

$$\begin{aligned} p_t &= b_{th} + 2c_{th}q_t^{th} = b_{th} + 2c_{th}\frac{Capacity^{th}}{Capacity^{th,n}}q_{nt}^{th} = b_{th} + 2\frac{\lambda}{Q_{max}^{th}}\frac{Capacity^{th}}{Capacity^{th,n}}q_{nt}^{th} = \\ &= b_{th} + 2\frac{\lambda}{Q_{max}^{th}}\frac{Capacity^{th,n}}{Capacity^{th}}q_{nt}^{th} = b_{th} + 2\frac{\lambda}{Q_{max}^{n,th}}q_{nt}^{th} = b_{th} + 2c_{th}^nq_{nt}^{th} \end{aligned}$$

Therefore, we deduce that the merit order price is identical in both economies.

Finally, we show that at each date  $t$ ,  $(q_n)_{n=1}^N$  respects the equality between supply and demand in the economy with  $N$  producers. More precisely,

$$\begin{aligned} \sum_{n=1}^N \left( \sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th} \right) &= \sum_{n=1}^N \left( \sum_{j=1}^J \frac{Capacity^{j,nuc,n}}{Capacity^{j,nuc}} \cdot q_{jt}^{nuc} + \frac{Capacity^{th,n}}{Capacity^{th}} \cdot q_t^{th} \right) = \\ \sum_{j=1}^J \left( \frac{\sum_{n=1}^N Capacity^{j,nuc,n}}{Capacity^{j,nuc}} \cdot q_{jt}^{nuc} \right) &+ \frac{\sum_{n=1}^N Capacity^{th,n}}{Capacity^{th}} \cdot q_t^{th} = \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} = D_t - Q_t^{hyd} \end{aligned}$$

As a result, we deduce that the production vector  $(q_n)_{n=1}^N$  is a merit order equilibrium in an economy with  $N$  producers.

### Proof of $(\beta)$

Inversely, we show that if the production vector  $(q_n)_{n=1}^N = (((q_{njt}^{nuc})_{j=1}^J, q_{nt}^{th}))_{t=1}^T)_{n=1}^N$  is a merit order equilibrium in an economy with  $N$  producers then the aggregate production vector  $q = ((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T$  is a merit order equilibrium in an economy with one producer.

The production  $(q_n)_{n=1}^N$  is a merit order equilibrium thus, it is a feasible production vector which means that

$$Q_{min}^{n,j,nuc} \leq q_{njt}^{nuc} \leq Q_{max}^{n,j,nuc}, \quad \text{for all } n, j, t$$

and

$$0 \leq q_{nt}^{th} \leq Q_{max}^{n,th}, \quad \text{for all } n, t$$

By taking the sum of the nuclear component with respect to  $n$ , one can easily deduce that

$$\sum_{n=1}^N Q_{min}^{n,j,nuc} \leq \sum_{n=1}^N q_{njt}^{nuc} \leq \sum_{n=1}^N Q_{max}^{n,j,nuc}$$

which can be formulated as follows:

$$Q_{min}^{j,nuc} \leq q_{jt}^{nuc} \leq Q_{max}^{j,nuc}, \quad \text{for all } j, t$$

Similarly, by taking the sum of the non-nuclear component with respect to  $n$ , we deduce that

$$\sum_{n=1}^N 0 \leq \sum_{n=1}^N q_{nt}^{th} \leq \sum_{n=1}^N Q_{max}^{n,th}$$

or equivalently

$$0 \leq q_t^{th} \leq Q_{max}^{th}, \quad \text{for all } t$$

In addition, the production vector  $(q_n)_{n=1}^N$  respects the nuclear fuel constraints (2.6), (2.7), (2.8) in an economy of  $N$  producers. If we take the sum of each of these constraints with respect to  $n$ , we conclude that  $q$  is a production that respects the nuclear fuel constraints in the case of an aggregate producer ( $N = 1$ ). Hence,  $q$  is a feasible production vector in an economy with one aggregate producer.

Furthermore, we prove that the merit order price associated with the production  $(q_n)_{n=1}^N$  of  $N$  producers coincide with the merit order price associated with the production  $q$  of one aggregate producer. As we have already mentioned in the first part of the proof, in view of (2.12) and (2.21), the merit order price is the same in both the decentralized and the aggregate economy when nuclear is the marginal technology. It suffices to come to the same conclusion when non-nuclear thermal is the marginal technology. In view of Proposition 2.2.1 on page 91, the variable part of the non-nuclear thermal cost  $c_{th}^n$  is inversely proportional to the capacity  $Q_{max}^{n,th}$  which means that there exists  $\lambda \in \mathbb{R}$  such

that  $c_{th}^n = \frac{\lambda}{Q_{max}^{n,th}}$ . So, in the case that  $q_{nt}^{th} > 0$ , we obtain

$$\begin{aligned} p_t &= b_{th} + 2c_{th}^n q_{nt}^{th} = b_{th} + 2c_{th}^n \frac{Capacity^{th,n}}{Capacity^{th}} q_t^{th} = b_{th} + 2 \frac{\lambda}{Q_{max}^{n,th}} \frac{Capacity^{th,n}}{Capacity^{th}} q_t^{th} = \\ & b_{th} + 2 \frac{\lambda}{Q_{max}^{n,th} \frac{Capacity^{th}}{Capacity^{th,n}}} q_t^{th} = b_{th} + 2 \frac{\lambda}{Q_{max}^{n,th} \frac{1}{Capacity^{th,n}}} q_t^{th} = b_{th} + 2 \frac{\lambda}{Q_{max}^{th}} q_t^{th} = b_{th} + 2c_{th} q_t^{th} \end{aligned}$$

As a result, the merit order price is exactly the same in both economies.

To end, at each date  $t$ , the production vector  $(q_n)_{n=1}^N$  respects the equality between supply and demand

$$\begin{aligned} \sum_{n=1}^N \left( \sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th} \right) &= D_t - Q_t^{hyd} \Leftrightarrow \sum_{j=1}^J \left( \sum_{n=1}^N q_{njt}^{nuc} \right) + \sum_{n=1}^N q_{nt}^{th} = D_t - Q_t^{hyd} \\ &\Leftrightarrow \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} = D_t - Q_t^{hyd} \end{aligned}$$

Thus, at each date  $t$ , the production vector  $q$  satisfies the supply-demand equilibrium constraint.

Consequently, we conclude that the production vector  $q$  is a merit order equilibrium in an economy with one aggregate producer.

In view of proof of  $(\alpha)$  and of  $(\beta)$ , the proposition is proven.  $\square$

### Economic implications of Proposition 2.4.1

In view of Proposition 2.4.1, we may say that the merit order equilibrium  $q$  of the economy  $\tilde{E}$  is equivalent to the merit order equilibrium  $(q_n)_{n=1}^N$  of the decentralized economy  $E$ . This proposition permits to determine a merit order equilibrium of the decentralized economy  $E$  by working in the economy  $\tilde{E}$  with one aggregate producer that holds the total nuclear and non-nuclear capacity and aims to satisfy the demand each month. From

an economical point of view, we deduce that at the optimum, the decentralized economy is not “superior” than the economy with a unique producer (centralized economy), in terms of production, in the sense that a merit order equilibrium of the decentralized economy constitutes a merit order equilibrium of the economy with a unique producer and vice versa. Consequently, the decentralization of the nuclear production segment is “neutral” in comparison with the centralized management regarding the optimal production levels obtained in both organizational forms.

### 2.4.3 Simulation results

In order to resolve the optimal short-term production problem numerically within our data set, we deal with the problem of discontinuity of the merit order price which leads to the discontinuity (decrease) of the current monthly profit. To do this, we resolve an approximate problem (continuous problem) that is a “regularization” of our economical problem (discontinuous problem). First, we present some general results coming from the numerical resolution of the “regularized” optimal short-term production problem and then we carry on with a more detailed analysis of the optimal production and storage decisions. We conclude by giving some general remarks in accordance with the simulation outcomes.

#### “Regularization” of the optimal short-term production problem

The hypothesis of Proposition 2.3.2 that  $b_{nuc} < b_{th}$  holds within our data, thus the discontinuity and more precisely the decrease of price at production vectors characterized by zero non-nuclear thermal production during a month induces a discontinuity and specifically a decrease of the value of profit during this month. A point that we need to stress is the nonexistence of an algorithm that maximizes a discontinuous function. Theoretically and numerically, we proceed with a “regularization” of the merit order price in order to treat the problem of discontinuity of the price and resolve the optimal short-production problem. Theoretically, we dealt with the problem of discontinuity through an equivalent

optimization problem being the optimal short-term production problem determined on the subset  $H_{th}^t$  of the set of feasible solutions  $H^t$  (see Corollary 2.3.1 on page 115). We recall that the set  $H_{th}^t$  is characterized by strictly positive non-nuclear thermal production levels and thus, non-nuclear thermal is the marginal technology meaning that its own marginal cost determines the price for all  $t$ . Within this set, the current monthly profit is a continuous function. Numerically, we propose an alternative model, where the price is given by the non-nuclear thermal marginal cost ( $mc^{th}(0) = b_{th}$ ) instead of the nuclear marginal cost ( $b_{nuc}$ ) when nuclear is the marginal technology. Thus, at the date  $t$ , the price  $p_t$  will be

$$p_t = \begin{cases} mc^{th}(q_t^{th}), & \text{if } q_t^{th} > 0 \\ mc^{th}(0), & \text{if } q_t^{th} = 0 \end{cases} = \begin{cases} b_{th} + 2c_{th}q_t^{th}, & \text{if } q_t^{th} > 0 \\ b_{th}, & \text{if } q_t^{th} = 0 \end{cases} \quad (2.22)$$

This means that a producer receives at least  $b_{th}$  (Euros per MWh) when it runs only its nuclear units to cover the monthly demand. The resulting merit order price rule “hides” a compensation for nuclear producers for using nuclear instead of non-nuclear thermal capacities to satisfy customers demand for electricity. In view of this “regularization” of the merit order price, the current monthly profit being now a continuous function is maximized on the entire set of feasible solutions  $H^t$  within our numerical model resulting in a continuous optimization problem, the “regularized” problem. However, the “regularized” problem (continuous problem) and the economical problem (2.15) described in subsection 2.3.4 (discontinuous problem) differ with respect to the objective function which is the profit. More precisely, the profit of the economical problem is smaller than the profit of the “regularized” problem since the value of  $b_{nuc}$  (5.01 Euro/MWh) is less important than the value of  $b_{th}$  (26.24 Euro/MWh). Nevertheless, for all  $t$ , we show that the value of the “regularized” problem and the value of the economical problem are identical, meaning that the optimal value of the profit is the same for both optimization problems (see Annex A, Proposition A.1). Consequently, the “regularized” problem is a “good” approximation of our economical problem (Boyd and Vandenberghe (2004)).

From a theoretical perspective, in view of Proposition A.3 demonstrated in the Annex A, for all  $t$ , if a solution of the “regularized” problem does not belong to the set  $H_{th}^t$ , then the set of solutions of the economical problem is empty. Numerically, the solution of the

“regularized” problem whose results are analyzed in this section does not belong to the set  $H_{th}^t$  since there are months during which the nuclear production is the only one used to cover the corresponding demand implying that the non-nuclear thermal production is zero. Therefore, the set of solutions of the economical problem is empty, hence the interest of focusing on the numerical solution coming from the “regularized” problem. This numerical solution constitutes only an “approximate” solution of our economical problem.

## General results

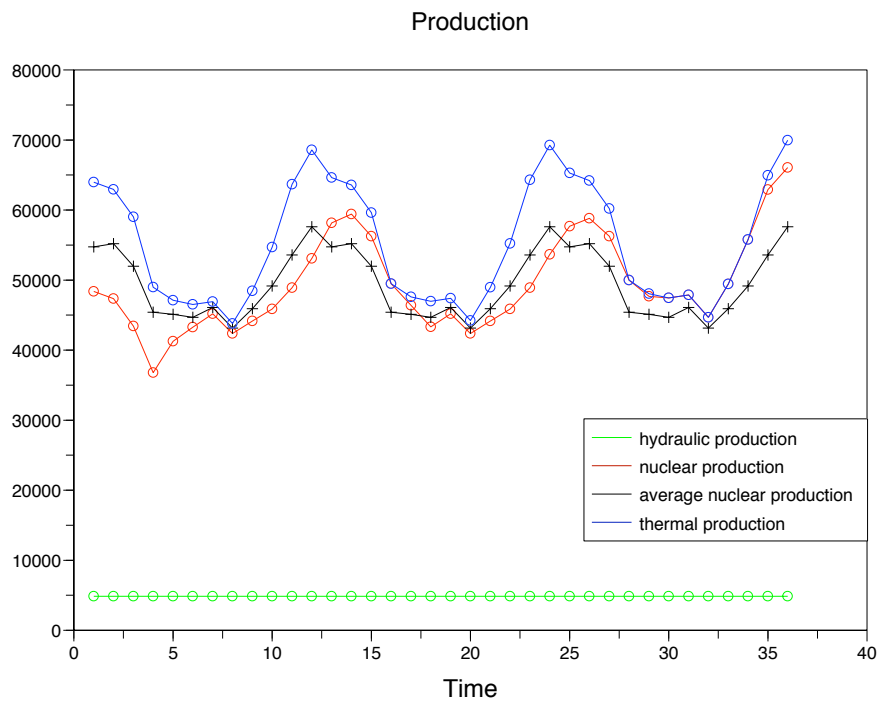


Figure 2.12: Simulated hydro(run-of-river)/nuclear/non-nuclear thermal production (in MW)

Nuclear follows the seasonal variations of demand by decreasing during summer and

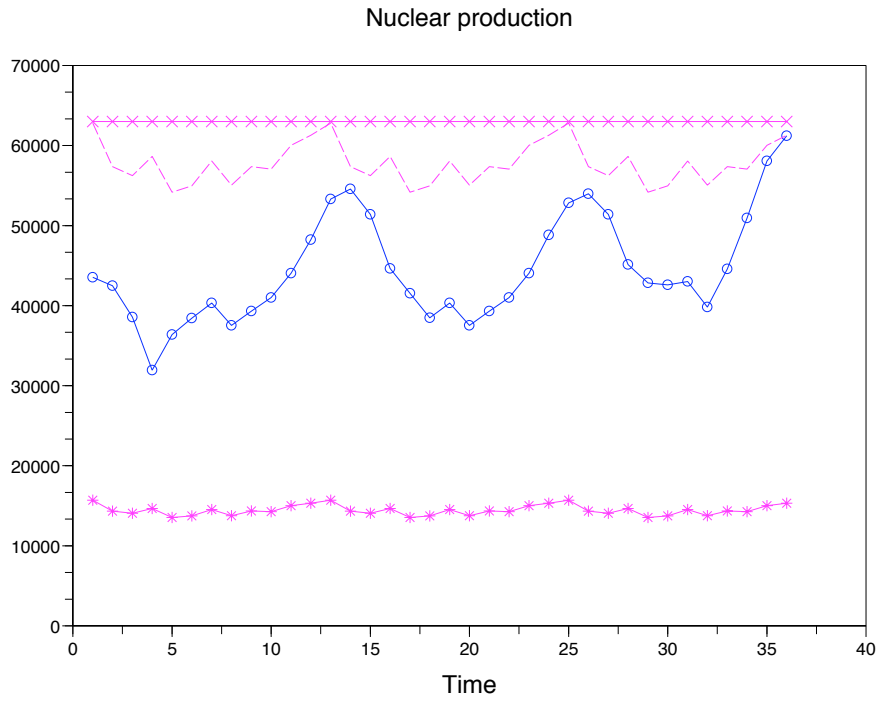


Figure 2.13: Simulated nuclear production (in MW)

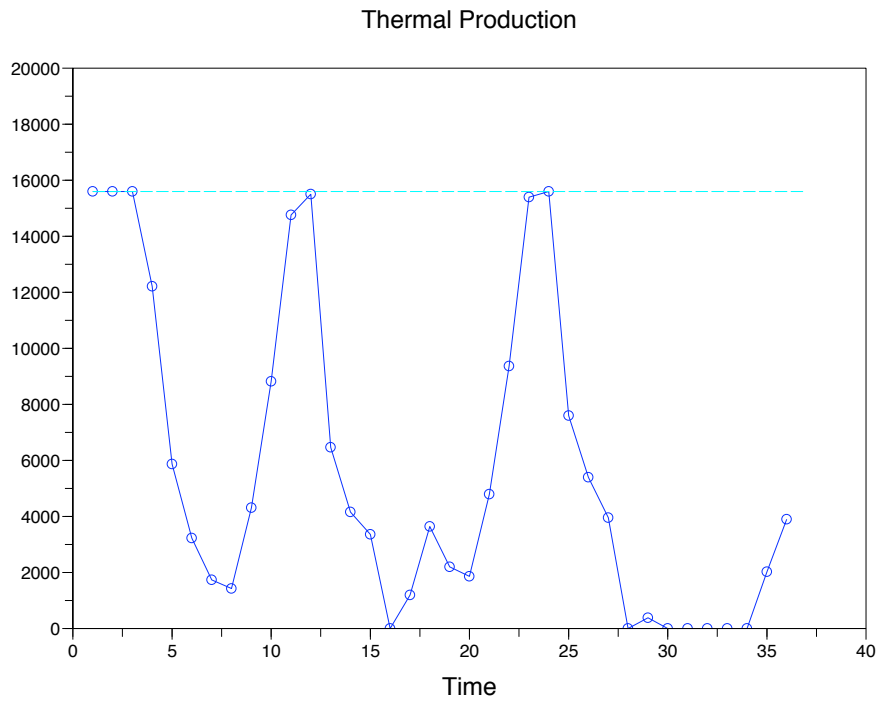


Figure 2.14: Simulated non-nuclear thermal production (in MW)

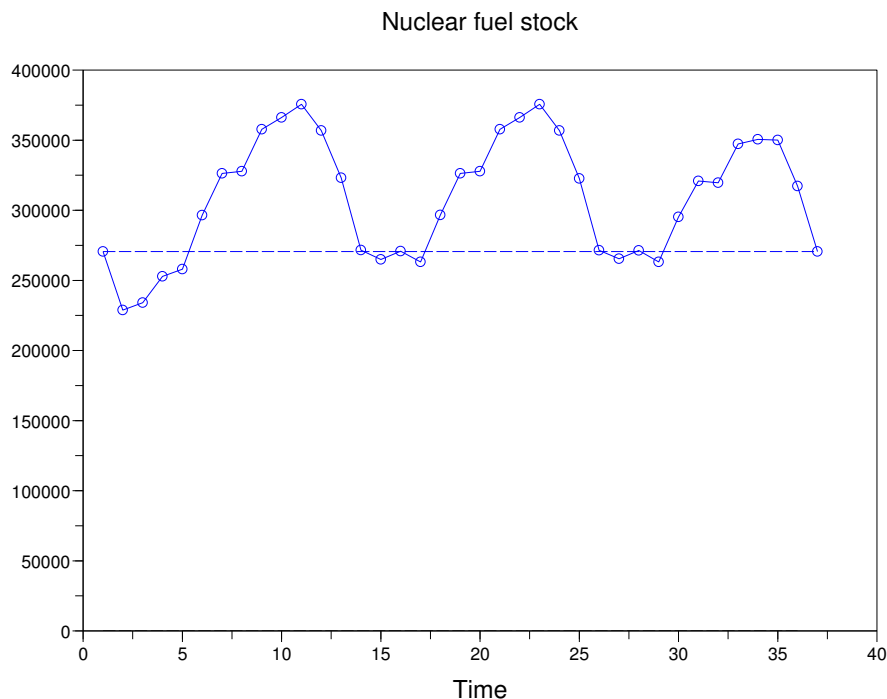


Figure 2.15: Simulated nuclear fuel stock (in MW)

increasing during winter (see Figure<sup>26</sup> 2.12, Figure<sup>27</sup> 2.13). The monthly nuclear production almost never reaches its maximum value (see Figure 2.13).

Similarly, the non-nuclear thermal production adjusts to demand's seasonal variations during the entire time horizon  $T$  (see Figure 2.12, Figure 2.14). The monthly non-nuclear thermal production reaches its maximum value<sup>28</sup> at the beginning of period  $T$  when the nuclear production is significantly low and then during the month of December of 2008 and of 2009 in order to meet the peak levels of demand (see Figure 2.14). However, the non-nuclear thermal production becomes zero reaching this way its minimum value at the end of period  $T$  since the nuclear production is very important and covers entirely the

<sup>26</sup>The average nuclear production in the month  $t$  given that some unit is inactive during this month (month of reloading) is represented by the black crossed line. Its evolution is periodic and is determined by the red line. For example, the average nuclear production in the month of January is the average nuclear production of the months  $t = 1, 13, 25$ .

<sup>27</sup>The maximum nuclear production during the month  $t$  ( $\sum_{j=1}^J Q_{max}^{j,nuc}(t)$ ) given that some unit is inactive during this month (month of reloading) is represented by the purple dotted line. This periodic quantity is obviously below the nominal capacity of the French nuclear set represented by the crossed purple line. The minimum nuclear production during the month  $t$  given that some unit is inactive during this month (month of reloading) is represented by the purple line of asterisks.

<sup>28</sup>The maximum non-nuclear thermal production during a month ( $Q_{max}^{th}$ ) is represented by the white blue dotted line and corresponds to the nominal non-nuclear thermal capacity (including coal, gas, fuel, etc.) of the French set.



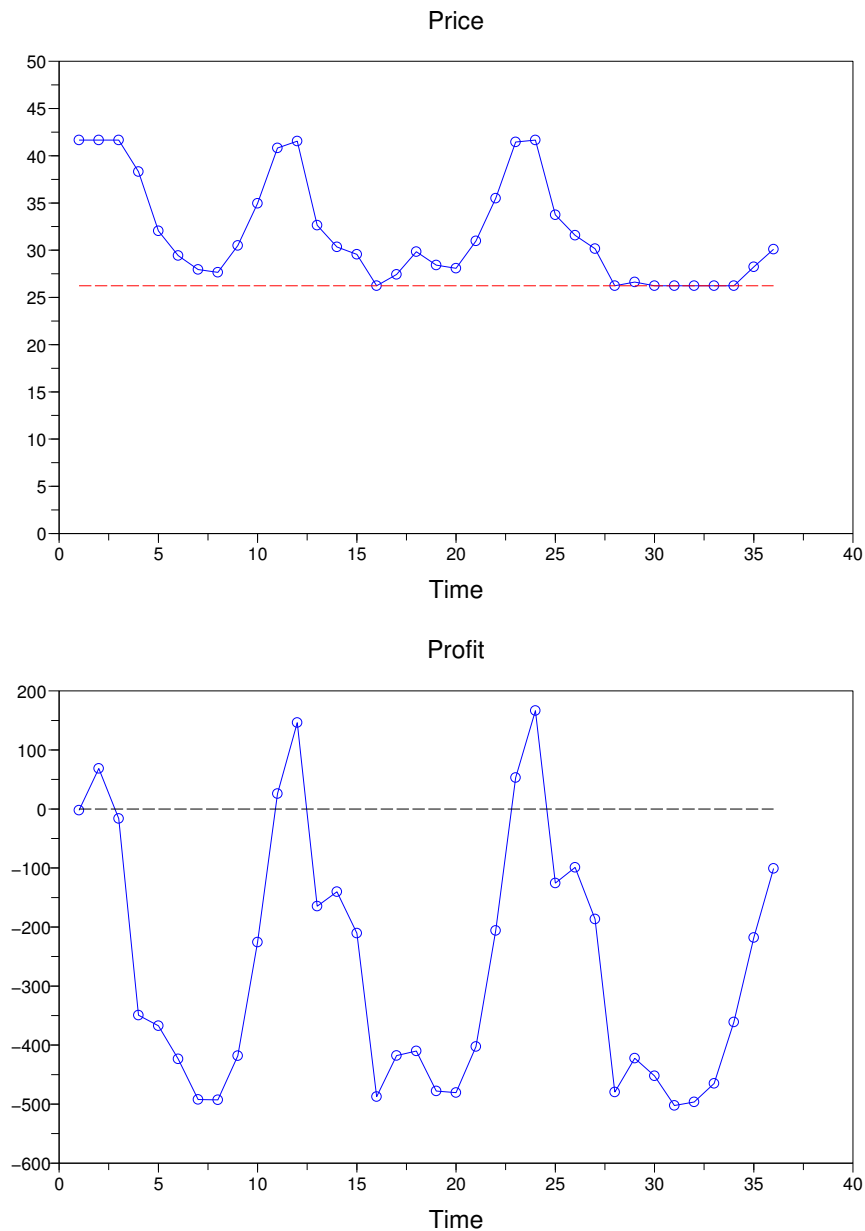


Figure 2.16: Simulated “regularized” price (in Euro/MWh)/Simulated “regularized” profit (excluding profit coming from hydro (run-of-river) generation) (in Euro (million))

monthly demand.

We also observe that the nuclear fuel stock decreases during periods of high demand while increases during periods of low demand (see Figure 2.15). The trend of the stock appears fundamentally above the “stock of reference”<sup>29</sup> during the time horizon  $T$  of the model.

<sup>29</sup>The “stock of reference” is represented by the blue dotted line which shows the value of stock at the beginning, being also the value of stock at the end.

We should note, before we proceed with a more detailed analysis of our simulation results, that the reader should not focus on the precise amount of profit since its level depends on too many of the approximations we did (euro/dollar, oil prices, CO<sub>2</sub> cost, discount rate, no mark-up rate, absence of profits coming from the run-of-river hydro technology, etc.) and because our modelling does not take into account the electricity importations/exportations or the production coming from renewable and hydro storage plants (see Figure 2.16).

### **Analytical results**

We shall separate period  $T$  into three sub-periods to give some structure to the following discussion regarding the evolution of nuclear and non-nuclear thermal production, of stock, of price and of profit. According to figure 2.12, we distinguish first a sub-period during which the nuclear production is below its average value and the non-nuclear thermal is the marginal technology. A medium sub-period with a nuclear production that oscillates around its average value and a basically periodical evolution of the nuclear and non-nuclear thermal production. Finally, a third sub-period during which nuclear production is above its average value and it is mainly the marginal technology.

#### **First sub-period (January 2007 - April 2007)**

The nuclear production decreases from January to April due to the decrease of the demand during this period. We also observe that the nuclear production is significantly low during the first months of the simulation in comparison with the same months of the following years (see Figure 2.12, Figure 2.13).

On the contrary, the non-nuclear thermal production increases significantly because of the important levels of demand and the low levels of the nuclear production of this period (see Figure 2.12, Figure 2.14). In particular, the non-nuclear thermal production reaches its maximum value during the period January - March.

Paradoxically, the amounts of stock observed in this period are less important than those of the corresponding period of the next years. It seems that the nuclear fuel stock is “overused” during the first sub-period and this results in quantities of stock significantly lower than the “stock of reference” (see Figure 2.15). In particular, there is an impulsion to decrease the stock from January to February which drives the trend of the nuclear fuel stock below its reference value over the entire first sub-period. From February to April, the quantity of stock increases progressively following the decrease of the demand and thus the decrease of the nuclear production without however exceeding the “stock of reference”.

The price<sup>30</sup> and the profit detected during the months of the first sub-period are higher than the price and the profit perceived during the same months of the next years because of the maximum levels of non-nuclear thermal production noticed in these months (see Figure 2.16). Consequently, if the nuclear units underproduce then the non-nuclear thermal units need to overproduce to meet the levels of demand of this period. Hence, significant gains<sup>31</sup> are generated if we compare them with the gains obtained during the same months of the years that follow.

### **Medium sub-period (May 2007 - May 2009)**

The nuclear production follows the seasonal variations of demand which means high production during winter and low production during summer (see Figure 2.13). This implies “low” levels of nuclear fuel stock during winter and “high” levels of nuclear fuel stock during summer (see Figure 2.15). Clearly, the essentially periodical evolution of the nuclear production implies a periodical evolution for the nuclear fuel stock too. Note that the trend of the stock is above the “stock of reference” suggesting that the nuclear fuel stock is not “overconsumed” during this period. However, during months of high demand when nuclear production is important, the nuclear fuel stock decreases significantly and reaches its “reference” value.

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<sup>30</sup>The red dotted line indicates the “regularized” price level when nuclear is the marginal technology.

<sup>31</sup>Recall that the mark-up rate is taken equal to zero.

Similarly, the non-nuclear thermal production is high during winter (respectively low during summer) because of the high (respectively low) level of demand. In particular, the non-nuclear thermal production is increasing during winter (beginning from the month of September) until it reaches its peak value during the month of November and of December. It becomes decreasing in summer being a low demand season without however reaching its minimum value because of the very low levels of the nuclear production (see Figure 2.14).

The price is high during months of high demand (winter) by taking its highest value through the period November - December and relatively low during months of low demand (summer). The profit obtained by the producer is high during winter and at the beginning of spring while lower profits are realized during summer. Moreover, we can see that its value can be decomposed in a cyclical component and a linear trend which is slightly increasing (see Figure C.17, Annex C).

### **Last sub-period (June 2009 - December 2009)**

In figures 2.12 and 2.13, we observe that the nuclear production increases significantly during the last sub-period, especially from September to December. Particularly, during the last two months of this period (November, December), nuclear production approaches its maximum level. Inevitably, the nuclear fuel stock decreases without however being lower than the “stock of reference” (see Figure 2.15).

In view of the “overproduction” of the nuclear units, the demand is totally covered by the nuclear production with only exception the month of November, December when demand increases significantly and the participation of the non-nuclear thermal production is necessary. Therefore, the non-nuclear thermal production is very low reaching its minimum value during the majority of the months of the last sub-period except of the period November - December (see Figure 2.12, Figure 2.14).

Nuclear is mainly the marginal technology and determines the price at the last sub-period. For this reason, we see that the price and the profit reach their lowest levels

during this period. This is noticed even at the end of the last sub-period since the low participation of the non-nuclear thermal production leads to a low price and hence to a low profit in the month of November, December (see Figure 2.16).

### **The duration of marginality of nuclear and non-nuclear thermal generation technology**

We deduce that non-nuclear thermal is the marginal technology during most of the months of period  $T$  while nuclear is marginal only at the end of period  $T$  and in particular during almost the entire period of spring, summer and autumn of 2009. The non-nuclear thermal production is used almost always to satisfy demand and maximize the instantaneous monthly profit despite the fact that the nuclear production is remunerated above its “real” marginal cost ( $b_{nuc}$ ) when it is marginal due to the “regularization” of the merit order price. This price “regularization” could imply a more important period of marginality for nuclear since producers are no more penalized by low prices. However, this is not the case in the first stage of optimization when the producers do not know how to manage optimally the nuclear fuel reservoir in a market-based electricity system and hence, they search for an equilibrium of an already complex optimal production problem based on a monthly horizon of operation, the optimal short-term production problem, before they move to a yearly or multi-annual horizon. Therefore, in the optimal short-term production problem, the numerical results regarding the duration of marginality of nuclear provide only an indication of this duration without certainty of how it will evolve in the last step of optimization when producers look at the optimal inter-temporal production problem.

### **General remarks**

In France, given the high participation of nuclear in the domestic electricity production, studies like the one of Spector (2007), Vassilopoulos (2007) and Percebois (2012) pay special attention to the “optimal size” of the French nuclear set in a competitive electricity market. Nuclear plants are preferentially used to cover only the baseload demand and the main reason is the amortization of their high fixed costs. In a competitive market, if

the marginal technology is nuclear all the year, the nuclear producers cannot cover their fixed costs. In such a market, the fixed costs and the variable costs will be covered, say, on a yearly basis only if the nuclear set has its “optimal size” within the whole generation set. In order to better understand this notion, we proceed with a numerical example given by Spector. Following this numerical example the optimal nuclear set for France corresponds to a nuclear marginality of 40%. This means that nuclear plants can cover all their fixed-costs through existing continental Europe market based prices during the 60% of marginality of the other thermal generation technologies (assuming that wind is not taken into account: basically coal, gas and fuel oil) on the basis of marginal costs of the latter. However, the nuclear set could also be “sub-optimal” that is to say, lower than its optimal level. In this case, even in presence of perfect competition, it would be remunerated above its marginal costs more than 60% of time. Consequently, its holders would profit from a scarcity rent, whatever the intensity of competition would be on the wholesale market (Glachant and Finon (2008)). In the case of perfect competition between producers, the causes of the scarcity rent could be found in sudden modifications that affect supply or demand (e.g. increase of the cost of fossil energies, increase of national or foreign demand), because the nuclear set cannot adjust instantaneously to such variations. However, within the time scale of our model ( $T=36$  months), we do not meet Spector’s conclusion about the insufficient size of the French nuclear set which makes the owner of that set (the French state) recipient of a scarcity rent in the case of perfect competition. In our exercise, which focus on the optimal short-term management of the nuclear fuel reservoir, the actual size of the nuclear set does not appear to be significantly below the “optimal size” since the total profit resulting from the aggregation of the current monthly profit during the entire time horizon of the model is not very important (see Figure 2.16 and Table C.5 of Annex C). Therefore, despite the fact that nuclear is almost never the marginal technology during the time horizon  $T$  and hence, the nuclear producer benefits from relatively high prices determined by the marginal cost of non-nuclear thermal technologies (coal and gas), the remuneration of its production does not seem to lead to profits that could justify the existence of a scarcity rent within our numerical model. Particularly, we see in Table C.5 that the total profit coming exclusively from the nuclear production is negative which shows that the fixed costs of nuclear are not recovered over the period  $T$ . Here, an element that we need to draw attention to

is the different methodologies used in our study and the study of Spector regarding the scarcity rent coming from the nuclear production. Its conclusions on the “optimal size” of the French nuclear set are not based on any theoretical or/and numerical modelling that proves the existence of a scarcity rent and calculates it.

In view of Remark 2.3.2, we obtain unicity of solutions with respect to the non-nuclear thermal component but considering the other variables which do not influence the current monthly profit the complete solution is not automatically unique.

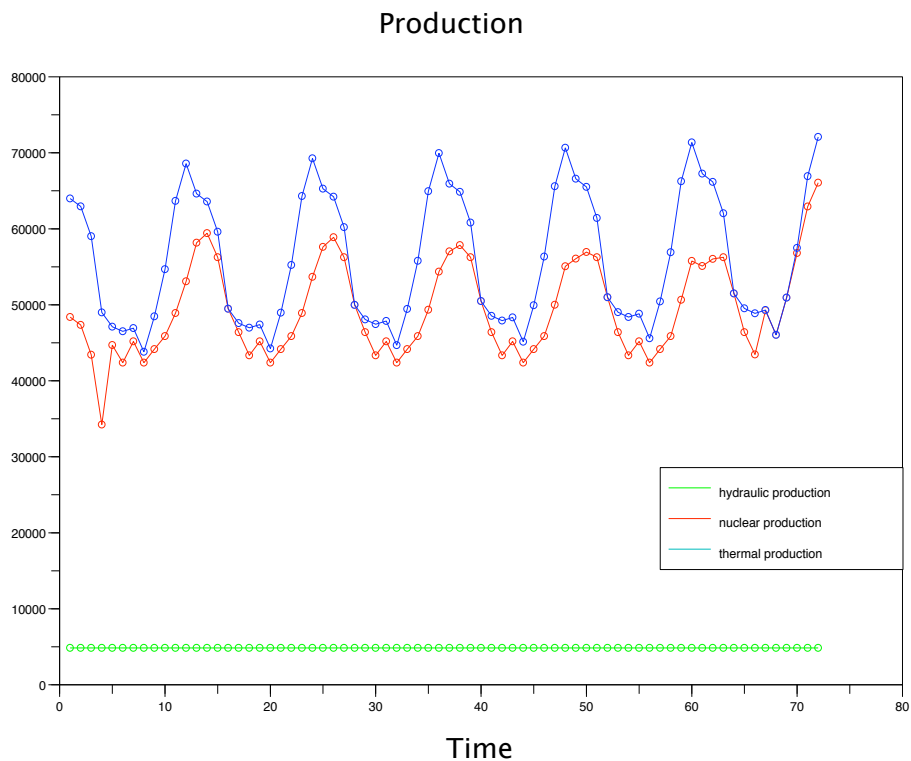


Figure 2.17: Simulated hydro(run-of-river)/nuclear/non-nuclear thermal production (in MW) ( $T=72$ )

To end, we notice that if we modify the length of the time horizon  $T$  of the model (i.e.  $T \geq 36$  months), the behaviour of the producer does not change. The evolution of the nuclear and non-nuclear thermal production during the first and the last sub-period as well as the fundamentally periodical evolution of the production during the medium sub-period are the same (e.g. for  $T = 72$ , see Figure 2.17).

## 2.5 Conclusion

In view of the introduction of competition in an electricity market dominated by the nuclear generation, one can ask what the optimal management of a nuclear set (like the French set) is in a competitive framework. In this chapter, we studied the optimal short-term (monthly) management of flexible nuclear plants which partly respond to the daily and seasonal variations of demand as a result of the significant share of nuclear in a perfectly competitive regime. We focused on the seasonal variation of the demand between high demand (winter) and low demand (summer). The novelty of our model exists in the fact that the nuclear fuel functions as a “reservoir”, which allows different allocations of the nuclear production during the different seasons of a campaign of production. We showed that the nuclear fuel can operate as a “reservoir”. We built a deterministic multi-period model in the general case of  $N \geq 2$  producers in order to study the optimal production behaviour in a perfect competitive market where producers use both nuclear and non-nuclear capacity (section 2.2). We introduced the notion of a merit order equilibrium based on the merit order price rule and then, we presented our different attempts to calculate a merit order equilibrium of the optimal short-term production problem (section 2.3). To end, a numerical simulation was given by taking into account the actual size of a given nuclear set (the French) vis-à-vis the non-nuclear generation set (section 2.4).

Three different approaches in order to determine a merit order equilibrium of the optimal short-term production problem were distinguished. As we mentioned in our first chapter, the nuclear is traditionally used to cover the base load demand by functioning always at its maximum capacity. This is typically not the French case where nuclear is used to meet both the base load and the semi-base. In our first approach, we attempted to calculate a merit order equilibrium by looking at the merit order price every month. However, it is not possible to calculate neither theoretically nor numerically a merit order equilibrium within this approach as a result of the behaviour of supply and especially of nuclear supply with respect to merit order price. Therefore, we proceeded by taking into consideration the inter-temporal behaviour of demand and not that of price in the computation of a merit order equilibrium. In the second approach, every producer determines its production level at each month  $t$  by maximizing its profit in the month  $t$  subject to



production and nuclear fuel storage constraints. In addition to this, the equality between supply and demand is taken into account for the month  $t$  but it is not considered in future months. We observed (via a numerical simulation) that there exists a month during which the production constraints and the supply-demand equilibrium constraint can not be respected simultaneously. Therefore, we deduced that a high level of “short-sightedness” with respect to future demand is intolerable and it leads to a failure to compute a merit order equilibrium of the optimal short-term production problem. In our third approach, we provided a last scenario in which the producers take into consideration the supply-demand equilibrium constraint imposed by the operator of the network for the whole time period  $T$ . In this last approach, the equality between supply and demand is sufficiently anticipated in future periods in order to manage the current use of the nuclear fuel reservoir and a merit order equilibrium of the optimal short-term production problem is calculated within a numerical model.

The marginal cost of nuclear production being (importantly) lower than the marginal cost of non-nuclear thermal production induces a discontinuity of the merit order price from a mathematical point view. We proved that, under some assumptions, the discontinuity and more precisely the decrease of price when nuclear changes into marginal technology implies a decrease of profit (Proposition 2.3.2, page 113). For this reason, the producer will search for a solution that maximizes its profit in the month  $t$  among the production vectors of the set  $H_{th}^t$  which are characterized by strictly positive non-nuclear thermal production levels at date  $t$ . From corollary 2.3.1 on page 115, it results that both the current monthly profit maximization problem defined on the entire set  $H^t$  and the one defined on the set  $H_{th}^t$  are equivalent; they have the same value and the same set of solutions.

In our next section, we proceeded with the analysis of the production decisions resulting from the optimal short-term management of flexible market based nuclear plants within a simple numerical model. From a mathematical perspective, we needed to keep our numerical model simple. For this reason, we focused on an auxiliary economy with one aggregate producer who possesses the total capacity (nuclear and non-nuclear thermal) by proving the equivalence of equilibrium between the original economy with  $N \geq 2$  producers and the auxiliary economy with one aggregate producer ( $N = 1$ ) through Proposition

2.4.1 (see page 131). This permitted the reduction in the number of the optimization variables of the optimal short-term production problem and therefore a simplification of its numerical resolution. Economically, this proposition does not leave any doubts for the “neutrality” of the decentralized economy with respect to the centralized economy since it proves that in terms of production both economies are of the same quality at the optimum level.

The problem of discontinuity mentioned above was treated with the “regularization” of our economical problem (presented in subsection 2.3.4) which is achieved through the “regularization” of the merit order price. In view of this “regularization”, the price in the market is determined by the marginal cost of the non-nuclear thermal production when nuclear is the marginal technology. Consequently, this “regularization” resulted in an approximate problem (continuous problem) which differs from the economical problem (discontinuous problem) by its objective function (profit function). However, we proved that the value of these optimization problems (the “regularized” problem and the economical problem) is the same, which implies that the “regularized” problem is a “good” approximation of our economical problem (see Annex A, Proposition A.1). Within our numerical example, we concluded via Proposition A.3 of Annex A that the set of solutions of the economical problem is empty. This is because there exist months during which the non-nuclear thermal component of the solution obtained by the “regularized” problem is not strictly positive and thus, the solution does not belong to the set  $H_{th}^t$ . This numerical solution is only an “approximate” solution of our economical problem.

In this late frame, we presented the results coming from our numerical problem (“regularized” problem). We did find high levels of nuclear production during months of high demand (winter) and low levels during months of low demand (summer). As expected, the evolution of the nuclear fuel stock is the opposite of the evolution of the nuclear production which means low levels of stock during winter and high levels of stock during summer. Similarly with the nuclear production, the non-nuclear thermal production increases during winter and decreases during summer without however becoming zero, in order to meet the residual levels of the seasonal demand. As a result of this, the price peaks during winter and reaches its lowest point during summer. Accordingly, producers obtain higher profits during winter and lower profits during summer.

In view of the “regularization” of the merit order price, the price is determined by the non-nuclear thermal marginal cost even when nuclear becomes the marginal technology. Thus, the nuclear production is no more penalized by low prices. However, the period during which nuclear is marginal and hence is used to meet demand is not long as one could expect. Basically, we observe that the non-nuclear thermal remains marginal during low and high demand seasons over the time horizon  $T$ . Therefore, the global equilibrium between supply and demand is ensured with the participation of the thermal non-nuclear generation capacity in the electricity generation during almost the entire time horizon  $T$  of the model. In this first stage of optimization of the management of the nuclear fuel reservoir, the producer, being not familiar with the operation of the nuclear fuel reservoir in a competitive market, maximizes its current monthly profit by running the non-nuclear thermal units practically all the time without considering the actual profitability of the nuclear production.

We modelled the optimal production behaviour in a perfectly competitive electricity market as an optimal short-term production problem. This is a problem that consists of the maximization of producer’s profit during the month  $t$  subject to nuclear fuel constraints for each nuclear unit  $j$ , minimum/maximum production constraints and the supply-demand equilibrium constraint at each month  $t$ . Numerically, we determined a merit order equilibrium of this problem. Each month, the merit order equilibrium production levels are feasible production levels that satisfy the equality between supply and demand and the price is given by the marginal cost of the “last technology” of the merit order used to equilibrate supply and demand. This mode of operation is not based on the direct optimization of the production over the entire period  $T$ . It could, however, correspond to the prudent behaviour of a nuclear set quitting the monopoly era and discovering step by step how flexible nuclear plants are operated in a competitive electricity market.

In this first stage of optimization of the nuclear production, we were interested in verifying the existence of an optimal production trajectory on a monthly horizon of management of the nuclear fuel reservoir that is relatively easier to study than a complete horizon of operation composed by a number of campaigns. Practically, this permitted to

formalize the modelling of the management of the nuclear fuel reservoir in presence of fossil fuel technologies (coal, gas), of the generating units, of the production costs, of the optimization constraints as well as the modelling of the exogenous and endogenous variables of our model. We determine the basic assumptions of our modelling and we introduce a novel notion which is that of merit order equilibrium. It also helped to identify and deal with a large number of mathematical, technical and computational difficulties regarding the modelling of the optimal production problem and its theoretical and numerical resolution in order to find an equilibrium being a merit order equilibrium (e.g. number of optimization constraints, number of optimization variables, price discontinuity, data set, computational algorithms, etc.). Consequently, our tactic that consists of starting from an optimization of production over a short-term management horizon of the nuclear fuel reservoir contributed significantly to the analysis of a full inter-temporal optimization as it would result from the optimization of production during the whole time horizon of the model. No doubt that market based management of flexible nuclear plants would then like to look at determining a global optimum of the optimal production problem. This further analysis is to be our work in the following chapter.



# Chapter 3

## An inter-temporal optimization of the management of flexible nuclear plants in a market based electricity system

### 3.1 Introduction

In our previous chapter, we studied the optimal short-term management of the nuclear fuel reservoir of market based flexible nuclear units. In that approach, we assumed that the nuclear producers have just started with the optimization of their generation. In this early period of optimization of nuclear generation, each producer determines its optimal production levels over a month by taking account of the optimal production levels of the previous months and under constraints resulting from the flexible operation of a nuclear unit, the non-nuclear thermal generation capacity, the management of the nuclear fuel stock and finally the equality between supply and demand every month. The importance of this step of optimization consists of finding an equilibrium of the optimal production problem on a short-term time horizon of operation (the monthly horizon) before moving to

a longer time horizon of operation (yearly or multi-annual) which leads to a more complex optimal production problem: the optimal inter-temporal production problem.

In this chapter, we present the second stage of optimization of the management of the nuclear fuel reservoir in a competitive setting. The nuclear units managers, having now learned how to optimally manage the fuel reservoir of their flexible nuclear units on a monthly basis, can aim to optimize the operation of the nuclear fuel reservoir on a full time horizon which consists of one or more campaigns (typically 36 months). Therefore, the producers may choose to extend their management horizon to a medium-term horizon. Within this optimal inter-temporal production problem, each producer maximizes its inter-temporal profit under the same constraints as those considered in the optimal short-term production problem. Consequently, the producers act with a complete vision of the medium-term optimal allocation of the nuclear fuel stock in a market based electricity system, avoiding all types of imbalances between supply and demand. The market price is determined each month by the marginal cost following the merit order price rule (perfect competitive case).

The first section of this chapter (section 3.2) starts with the modelling of the inter-temporal management of the fuel reservoir of nuclear units in a perfect competitive framework. This framework keeps assumptions identical to those made in our chapter 2. The modelling concerning the demand for electricity and the time horizon of the model and of the campaign is itself identical to the optimal short-term production problem. The generating units as well as the production costs are also modelled in the general case of  $N \geq 2$  producers. We recall here the nuclear fuel constraints resulting from the inter-temporal management of the nuclear fuel reservoir, the constraints induced by the load-following ability of nuclear units and the available non-nuclear thermal generation capacity as well as the constraints imposed by the equality between supply and demand. We end this section with the modelling of the optimal inter-temporal production behaviour.

In section 3.3, we proceed with the optimization of the inter-temporal production which consists of the maximization of the inter-temporal profit under constraints identical with those described in section 3.2. However, we meet another difficulty already found in our previous chapter. It is the discontinuity of the merit order price. The marginal cost

of nuclear production being (significantly) lower than the one of non-nuclear induces a discontinuity of producer's inter-temporal profit. A falling price (when nuclear becomes the marginal technology) generates a fall of the inter-temporal profit. For this reason, we search for an equilibrium of the optimal inter-temporal production problem within a subset of the set of feasible solutions defined by strictly positive non-nuclear thermal production vectors. Within this subset of feasible solutions, the price will be determined by the marginal cost of the non-nuclear thermal generation technology. Mathematically, we prove that the inter-temporal profit maximization problem determined on this subset and the inter-temporal profit maximization problem determined on the entire set of feasible solutions are equivalent by having the same value and the same set of solutions. Then, we provide a new property that fully characterizes the optimal solutions of the optimal inter-temporal production problem in the absence of binding production constraints.

We continue with section 3.4 proposing a simple numerical model which analyzes the optimal production decisions resulting from the optimal inter-temporal production problem. From a mathematical perspective, the complexity of the optimal inter-temporal production problem calls for reducing the number of optimization variables. Therefore, we proceed with the proof of "equivalence" of an equilibrium for the original economy with  $N$  producers and for an auxiliary economy with only one aggregate producer (as in the previous chapter). This permits to simplify the optimal inter-temporal production problem by determining an equilibrium in an auxiliary economy with a unique producer who operates the total capacity of nuclear and non-nuclear units. This reduction of the number of producers to a unique producer in both the optimal short-term production problem and the optimal inter-temporal production problem leads to a coherent modelling of these two optimization problems. Economically, we fall on the same conclusion already met in our previous chapter when we compare the decentralization with the centralized management at the optimum. More precisely, we deduce that both the decentralized and centralized economies have the same status in terms of production as a solution of the optimal inter-temporal production problem in the decentralized economy is also a solution of the same optimization problem in the centralized economy and vice versa.

In the same section, we study an economical feature of producers' optimal behaviour: the symmetry of an equilibrium of the optimal inter-temporal production problem. Un-



der the assumption that each producer disposes of the same level of capacity (nuclear and non-nuclear), we show a key economical characteristic of this inter-temporal equilibrium: this equilibrium is “almost” symmetric. It is actually “symmetrisable”. Then we prove that without any loss of generality the analysis may focus on the symmetric equilibrium. Nevertheless, we show that in view of this property the existence of an asymmetric equilibrium is also possible. Another economical feature that we deduce from the symmetrisability of an equilibrium is that the profit of a symmetrisable equilibrium is symmetric.

The discontinuity of the merit order price makes the numerical resolution of this economical problem difficult or results in an absence of solutions. For this reason, we proceed with a “regularization” of our economical problem (discontinuous problem) which leads to an approximate continuous problem. The data used within our numerical model is the same as that described in the previous chapter and we apply Scilab to run our simulations. To end, the numerical results of the optimal inter-temporal production problem are contrasted with the results of the previous chapter (optimal short-term production problem) in order to compare these two approaches.

Finally, in section 3.5, we study the social welfare maximization problem in an identical framework. The optimization no longer only considers the benefits of the generators, it now takes into account the “benefits” for the whole society: social welfare. The nuclear operators being the main producers of electricity in their national electricity market may have to take into consideration constraints inherent in the public interest when they optimize the management of their nuclear fleet even in a competitive market. Such a constraint already covers the equality between supply and demand. It explains why the nuclear producers are taking into account imbalances between supply and demand and therefore the threat of a “blackout”. Indeed, in the medium-term, this constraint is already taken into account by the nuclear producers in the determination of the optimal management of the nuclear fuel reservoir. It covers both the first and the second stage of optimization of the nuclear production (optimal short-term and inter-temporal production problem). However, the production decisions of a very large nuclear set have considerable consequences for the whole national electricity system and hence for the welfare of the

society. This may lead towards a complete optimization of the social welfare instead of the producers own profits. Through this new problem, one determines the optimal production levels that maximize the social welfare in the medium term while taking into account the same constraints as in the optimal inter-temporal production problem. We find that the optimal production behaviour conducting to the maximization of social welfare is very different from the one obtained by the optimal inter-temporal production problem.

## 3.2 Model: Perfect competitive case

In this section, we present our general deterministic dynamic model of a perfectly competitive electricity market where producers operate both with nuclear and thermal non-nuclear plants. The hypothesis considered in this model are identical to those considered in the model of chapter 2. More precisely, we look at the perfect competitive case according to which firms are price-takers<sup>1</sup>: they treat price as a parameter and not as a choice variable. We also assume perfect information among producers. First, our modelling aims at determining the inter-temporal optimal management of the nuclear generation set in that competitive regime. Once again, we focus on the medium-term horizon which is characterized by the seasonal variation of demand between winter and summer. Second, we take into account production and nuclear fuel storage constraints as well as the constraints resulting from the equality between supply and demand and the flexible operation of nuclear units that are decisive for the determination of the equilibrium outcomes in this wholesale electricity market. Finally, we look at the social welfare maximization problem within an identical framework by taking into consideration the constraints mentioned above.

For the same reasons with those exposed in the previous chapter, the importations/exportations as well as the production coming from hydro-storage units and renewables are not considered in our model. Moreover, the modelling regarding the demand for electricity, the time horizon of the model and of the campaign is itself identical to the modelling presented in the second chapter. The generating units as well as the production costs and

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<sup>1</sup>Let us recall that price taking firms guarantee that when firms maximize their profits (by choosing the quantity they wish to produce and the technology of generation to produce it with) the market price will be equal to marginal cost.

the nuclear fuel stock are also modelled in the general case of  $N \geq 2$  producers in the same way as in our previous chapter.

### 3.2.1 The modelling of the optimal inter-temporal production behaviour

The optimal inter-temporal production problem that producer  $n$  resolves is the following:

$$\max_{((q_{njt}^{nuc})_{j=1}^J, (q_{nt}^{th}))_{t=1}^T} \sum_{t=1}^T (p_t \cdot (\sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th}) - \sum_{j=1}^J C_{nj}^{nuc}(q_{njt}^{nuc}) - C_n^{th}(q_{nt}^{th})) \quad (3.1)$$

subject to the nuclear fuel storage constraints (2.6), (2.7), (2.8) and the minimum/maximum production constraints (2.10) and (2.11) (see page 98, Subsection 2.2.5 and page 103, Subsection 2.3.1 of Chapter 2). The price  $p_t$  is given by the marginal cost of the last technology of the merit order used to equilibrate supply and demand (perfect competition). A complete calculation of the price is already given in subsection 2.3.1 of chapter 2 (see page 103).

### 3.2.2 The notion of equilibrium

Let us now give a definition of equilibrium of the optimal inter-temporal production problem with respect to a system of prices  $p \in \mathbb{R}_+^T$

**Definition 3.2.1** The production vector  $(\bar{q}_n)_{n=1}^N = (((\bar{q}_{1jt}^{nuc})_{j=1}^J, \bar{q}_{1t}^{th})_{t=1}^T, \dots, ((\bar{q}_{Njt}^{nuc})_{j=1}^J, \bar{q}_{Nt}^{th})_{t=1}^T)$  is an equilibrium with respect to a system of prices  $p \in \mathbb{R}_+^T$  if:

(i) for all  $n$ ,  $\bar{q}_n$  is a feasible production vector: (a) it respects the nuclear fuel constraints, for all  $j$  and (b) it respects the minimum/maximum production constraints, for all  $j, t$ .

(ii) for all  $n$ , it maximizes the inter-temporal profit of producer  $n$  on the set of feasible solutions.

(iii) the price, at each month  $t$ , is determined by the marginal cost of the marginal technology. It is called the merit order price associated with the production vector  $(\bar{q}_n)_{n=1}^N$ .

(iv) at each date  $t$ , it respects the equality between supply and demand

$$\sum_{n=1}^N \left( \sum_{j=1}^J \bar{q}_{njt}^{nuc} + \bar{q}_{nt}^{th} \right) = D_t - Q_t^{hyd}. \quad (3.2)$$

In view of Definition 3.2.1, the production vector  $(\bar{q}_n)_{n=1}^N$  being an equilibrium of the optimal inter-temporal production problem, for all  $n$ , maximizes the inter-temporal profit of producer  $n$  (condition (ii)) and in addition satisfies the conditions (i), (iii), (iv) which define a merit order equilibrium (see Definition 2.3.1 on page 102). Hence, it constitutes a merit order equilibrium of the optimal inter-temporal production problem.

### 3.3 Optimization of the inter-temporal production

In this section, we study the optimization of the inter-temporal production under production and fuel storage constraints as well as the supply-demand equilibrium constraints. First, we show that under some assumptions, the inter-temporal profit decreases at production vectors characterized by non-nuclear thermal components equal to zero as a result of the discontinuity (decrease) of price at these production vectors (cf. page 104, Subsection 2.3.1 and page 113, Subsection 2.3.4 of Chapter 2). Therefore, we search for a solution of the optimal inter-temporal production problem (3.1) among the feasible solutions which are determined by strictly positive non-nuclear thermal production levels (cf. page 115, Subsection 2.3.4 of Chapter 2). Then, we proceed by giving a property that characterizes the “interior” optimal solutions of problem (3.1) (see page 160). More precisely, we prove that in the absence of binding productions constraints, the solutions of the optimal inter-temporal production problem are fully characterized by a constant nuclear production.

### 3.3.1 The decrease of inter-temporal profit in the absence of non-nuclear thermal production

We define the set of feasible solutions of the optimal production problem (3.1) as

$$C = \left\{ q \in M \text{ s.t. } \begin{array}{ll} Q_{min}^{n,j,nuc} \leq q_{njt}^{nuc} \leq Q_{max}^{n,j,nuc}, & \text{for all } n, j, t \\ 0 \leq q_{nt}^{th} \leq Q_{max}^{n,th}, & \text{for all } n, t \end{array} \right\}$$

where  $M$  is defined by all the production vectors of the form  $q = ((q_{nj1}^{nuc})_{j=1}^J, \dots, (q_{njT}^{nuc})_{j=1}^J, q_{n1}^{th}, \dots, q_{nT}^{th})_{n=1}^N$  that respect the nuclear fuel constraints ((2.6), (2.7), (2.8)) for all  $n$  as well as the supply-demand equilibrium constraint (3.2) for all  $t$ . The set  $M$  is affine and the set  $C$  is compact (closed and bounded) and convex.

Moreover, we define  $F$  as the relative interior<sup>2</sup> of  $C$  ( $F = ri(C)$ ). It has the following form

$$F = \left\{ q \in M \text{ s.t. } \begin{array}{ll} Q_{min}^{n,j,nuc} < q_{njt}^{nuc} < Q_{max}^{n,j,nuc}, & \text{for all } n, j, t \\ 0 < q_{nt}^{th} < Q_{max}^{n,th}, & \text{for all } n, t \end{array} \right\}$$

Notice that if unit  $j$  reloads during month  $t$  then  $q_{njt}^{nuc} = 0$  and thus, the strict inequality constraints that determine the nuclear production  $q_{njt}^{nuc}$  in the set  $F$  are no more valid.

Let us focus on the set  $F^{th}$  defined as

$$F^{th} = \left\{ q \in M \text{ s.t. } \begin{array}{ll} Q_{min}^{n,j,nuc} \leq q_{njt}^{nuc} \leq Q_{max}^{n,j,nuc}, & \text{for all } n, j, t \\ 0 < q_{nt}^{th} \leq Q_{max}^{n,th}, & \text{for all } n, t \end{array} \right\}$$

**Remark 3.3.1**  $F^{th}$  contains  $F$  and is contained in  $C$  and  $C$  is contained in  $M$  ( $F \subset F^{th} \subset C \subset M$ ).

Since the marginal technology is the non-nuclear thermal on  $F^{th}$ , the price is determined by the marginal cost of the non-nuclear thermal production. We now proceed with

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<sup>2</sup>It is important to emphasize that the usual interior of  $C$  is empty since  $M$  is an affine set that is not equal to  $\mathbb{R}^n$ . Consequently, we focus on a classical generalization called relative interior (for the notion of the relative interior of a set cf. for example Florenzano and Le Van (2001), Boyd and Vandenberghe (2004), Pugh (2002)).

Proposition 3.3.1 which we will use to prove the decrease of the inter-temporal profit at production vectors with zero levels of non-nuclear thermal production.

**Proposition 3.3.1** *If  $F^{th}$  is a non-empty set, then the closure of  $F^{th}$  is  $C$  ( $\overline{F^{th}} = C$ ).*

**Proof**

First, we show that  $\overline{F^{th}} \subset C$ . Since  $F^{th}$  is contained to  $C$  (see Remark 3.3.1) and  $C$  is a compact set, we have that

$$F^{th} \subset C \Rightarrow \overline{F^{th}} \subset \overline{C} = C.$$

Secondly, we prove that  $\overline{F^{th}} \supset C$ . Allow  $q_1 \in C$  and  $q_0 \in F^{th}$ . For all  $m \in N$ , there exists a sequence  $z_m = (1 - \frac{1}{m+1})q_1 + \frac{1}{m+1}q_0$  belonging to  $F^{th}$  such that  $\lim_{m \rightarrow \infty} z_m = \lim_{m \rightarrow \infty} (1 - \frac{1}{m+1})q_1 + \frac{1}{m+1}q_0 = q_1$ . Hence,  $q_1 \in \overline{F^{th}}$  and the inclusion is proven.

From the first and the second part of the proof, we conclude that  $\overline{F^{th}} = C$ .

□

From a geometrical point of view one deduces from Proposition 3.3.1 that all the points of the set  $C$  and consequently those which belong to  $C \setminus F^{th}$  and hence contain non-nuclear thermal components equal to zero can be approached by points that belong to  $F^{th}$ . This result is fundamental in order to show in the next proposition the discontinuity and more specifically the decrease of the inter-temporal profit at these particular points which results from a decrease of price (cf. page 113, Subsection 2.3.4 of Chapter 2).

**Proposition 3.3.2** *For all  $n \in \{1, \dots, N\}$ , if  $F^{th}$  is a non-empty set,  $b_{nuc} < b_{th}$  and  $\bar{q} \in C \setminus F^{th}$ , there exists a sequence  $(q_r)_{r \in N} \in F^{th}$  with  $\lim_{r \rightarrow \infty} q_r = \bar{q}$  such that  $\lim_{r \rightarrow \infty} \pi^n(q_r) > \pi^n(\bar{q})$ .*

**Proof**

According to the assumptions,  $\bar{q}$  is a production vector which belongs to  $C \setminus F^{th} \subset C$ . It follows that the set  $S = \{t \in \{1, \dots, T\} \text{ s.t. } (\bar{q}_{\nu t}^{th})_{\nu=1}^N = 0\}$  is a non-empty set. Let us denote  $\bar{q} = ((\bar{q}_{\nu j 1}^{nuc}, \dots, \bar{q}_{\nu j T}^{nuc})_{j=1}^J, \bar{q}_{\nu 1}^{th}, \dots, \bar{q}_{\nu T}^{th})_{\nu=1}^N$  while  $\bar{q}_n = ((\bar{q}_{njt}^{nuc})_{j=1}^J, \bar{q}_{nt}^{th})_{t=1}^T$  is the production of the producer  $n$ .

Profit's function  $\pi^n : C \rightarrow \mathbb{R}$  is defined as

$$\pi^n(q) = \sum_{t=1}^T (p_t (\sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th}) - \sum_{j=1}^J C_{nj}^{nuc}(q_{njt}^{nuc}) - C_n^{th}(q_{nt}^{th}))$$

In view of Proposition 3.3.1 on page 163, the production vector  $\bar{q} \in \overline{F^{th}}$ . It follows that a sequence  $(q_r)_{r \in N}$  exists such that  $q_r \in F^{th}$  and  $\lim_{r \rightarrow \infty} q_r = \bar{q}$ . Let us denote  $q_r = (((q_{\nu j 1}^{nuc})_r, \dots, (q_{\nu j T}^{nuc})_r)_{j=1}^J, (q_{\nu 1}^{th})_r, \dots, (q_{\nu T}^{th})_r)_{\nu=1}^N$  and  $(q_n)_r = (((q_{njt}^{nuc})_r)_{j=1}^J, (q_{nt}^{th})_r)_{t=1}^T$  the component of the sequence  $q_r$  which corresponds to the producer  $n$ . For all  $r$ , we can compute the associated with the production  $(q_n)_r$  merit order price  $p_r = (p_{1r}, \dots, p_{Tr}) = (mc_n^{th}((q_{n1}^{th})_r), \dots, mc_n^{th}((q_{nT}^{th})_r))$ . The price vector  $\bar{p} = (\bar{p}_1, \dots, \dots, \bar{p}_T)$  represents the merit order price associated with the production vector  $\bar{q}_n$ . Since at the limit, the value of  $q_r$  is equal to  $\bar{q}$ , we deduce that the nuclear is the marginal technology during the month  $t$ , for all  $t \in S$ . Thus, the price vector  $\bar{p}$  is such that the price  $\bar{p}_t$  in month  $t \in S$  is determined by the nuclear marginal cost  $b_{nuc}$ . For all  $t \in S$ , it follows that

$$\lim_{r \rightarrow \infty} p_{tr} = \lim_{r \rightarrow \infty} mc_n^{th}((q_{nt}^{th})_r) = mc_n^{th}(\bar{q}_{nt}^{th}) = mc_n^{th}(0) = b_{th} > b_{nuc} = \bar{p}_t. \quad (3.3)$$

For all  $t \notin S$ , the non-nuclear thermal production is strictly positive, thus the price  $\bar{p}_t$  is determined by the non-nuclear thermal marginal cost ( $\bar{p}_t = mc_n^{th}(\bar{q}_{nt}^{th})$ ). So, we have

$$\lim_{r \rightarrow \infty} p_{tr} = \lim_{r \rightarrow \infty} mc_n^{th}((q_{nt}^{th})_r) = mc_n^{th}(\bar{q}_{nt}^{th}) = \bar{p}_t. \quad (3.4)$$

At the limit, we obtain

$$\begin{aligned}
& \lim_{r \rightarrow \infty} \pi^n(q_r) - \pi^n(\bar{q}) = \\
& \sum_{t \notin S} \lim_{r \rightarrow \infty} (p_{tr} (\sum_{j=1}^J (q_{njt}^{nuc})_r + (q_{nt}^{th})_r) - \bar{p}_t (\sum_{j=1}^J \bar{q}_{njt}^{nuc} + \bar{q}_{nt}^{th})) + \\
& \sum_{t \in S} \lim_{r \rightarrow \infty} (p_{tr} (\sum_{j=1}^J (q_{njt}^{nuc})_r + (q_{nt}^{th})_r) - \bar{p}_t (\sum_{j=1}^J \bar{q}_{njt}^{nuc} + \bar{q}_{nt}^{th})) - \\
& \sum_{t=1}^T \lim_{r \rightarrow \infty} ((\sum_{j=1}^J C_{nj}^{nuc} ((q_{njt}^{nuc})_r) + C_n^{th} ((q_{nt}^{th})_r)) - (\sum_{j=1}^J C_{nj}^{nuc} (\bar{q}_{njt}^{nuc}) + C_n^{th} (\bar{q}_{nt}^{th})))
\end{aligned}$$

Since  $\lim_{r \rightarrow \infty} q_r = \bar{q}$  and the price is continuous in the set  $T \setminus S$  (see relationship (3.4)) the first term converges to zero. However, in view of the non-emptiness of  $S$ , of the relationship (3.3) and of the strictly positive nuclear production ( $\sum_{j=1}^J \bar{q}_{njt}^{nuc} \geq \sum_{j=1}^J Q_{min}^{n,j,nuc} > 0$ ) the second term is strictly positive. Finally, the third term converges to zero because of the continuity of the production cost functions.

Consequently, we deduce that  $\lim_{r \rightarrow \infty} \pi^n(q_r) - \pi^n(\bar{q}) > 0 \Leftrightarrow \lim_{r \rightarrow \infty} \pi^n(q_r) > \pi^n(\bar{q})$  and the proposition is proven.

□

Let us remark that the non-emptiness of the set  $F^{th}$  obviously depends on the values of the exogenous variables ( $Q_{max}^{n,j,nuc}$ ,  $Q_{min}^{n,j,nuc}$ ,  $Q_{max}^{n,th}$ ,  $S_{reload}^{n,j}$ ,  $S_1^{n,j}$ ,  $D_t$ ,  $Q_t^{hyd}$ ) of the optimization problem (3.1). From a theoretical point of view, the set  $F$ , which is a subset of  $F^{th}$ , is non-empty. Following some linear transformations in the actual form of the optimization constraints included in the set  $F$  and then using the classical result of non-emptiness of the interior of unit simplex, we find a point contained in  $F$ . Therefore, the assumption of Proposition 3.3.1 and of Proposition 3.3.2 regarding the non-emptiness of the set  $F^{th}$  is satisfied. Numerically, in subsection 3.5.2, we show that  $F$  is a non-empty set for our numerical data, hence the above assumption complies with this particular data.



In view of our data,  $b_{nuc} < b_{th}$  holds, thus according to Proposition 3.3.2, the inter-temporal profit decreases for all production vectors that belong to the subset  $C \setminus F^{th}$  of  $C$  and hence they are characterized by zero levels of non-nuclear thermal production. This leads all producers to search for a solution that maximizes the inter-temporal profit on the set  $F^{th}$  in which the non-nuclear thermal production is strictly positive and the price is provided by the non-nuclear thermal marginal cost.

We notice that if the inter-temporal profit maximization problem is determined on  $C$  which is a compact set, the objective function is not continuous in view of Proposition 3.3.2. If the inter-temporal profit maximization problem is determined on  $F^{th}$ , the objective function is continuous according to Proposition 3.3.2 while  $F^{th}$  is not a compact set. Therefore, it is not possible to conclude on the existence of solutions of this problem (cf. page 115, Subsection 2.3.4 of Chapter 2).

The following corollary shows the relation between the optimal inter-temporal production problem on  $C$  and the optimal inter-temporal production problem on  $F^{th}$ .

**Corollary 3.3.1** *The inter-temporal profit maximization problem determined on  $C$  is equivalent to the inter-temporal profit maximization problem determined on  $F^{th}$  (same set of solutions and same value<sup>3</sup>).*

### Proof

This corollary is an obvious consequence of Proposition 3.3.2.

□

It should be noticed that the value of both optimization problems exists (in the real line) because the objective function (profit function) is polynomial and the set  $C$  together with the set  $F^{th}$  are bounded. If a solution of the inter-temporal profit maximization problem on  $F^{th}$  exists, then it is an equilibrium since all the conditions in order to be an equilibrium are satisfied (see Corollary 3.3.1 and Definition 3.2.1 on page 160).

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<sup>3</sup>The value of an optimization problem is defined as the upper bound of the set  $\{f(x)|x \in C\}$ , where  $f$  is the objective function and  $C$  is the set of feasible solutions. The value always exists even if the set of solutions is empty. When the set of solutions is nonempty, the value of an optimization problem is the common value  $f(\bar{x})$  for any solution  $\bar{x}$ .

### 3.3.2 A property when the optimal solution is “interior”.

In view of Proposition 3.3.2 on page 163, we search for a solution that maximizes the inter-temporal profit within the set  $F^{th}$ . The next proposition gives a property when in addition the solution of the optimal inter-temporal production problem belongs to the set  $F$ .

**Proposition 3.3.3** *For all  $n \in \{1, \dots, N\}$ , if an equilibrium  $((\bar{q}_{njt}^{nuc})_{j=1}^J, \bar{q}_{nt}^{th})_{t=1}^T)_{n=1}^N \in F$  exists such that the inter-temporal profit of the producer  $n$  is maximum on  $C$  and  $((\bar{q}_{nt}^{nuc})_{t=1}^T)_{n=1}^N$  is the corresponding monthly nuclear production vector then  $\bar{q}_{n1}^{nuc} = \bar{q}_{n2}^{nuc} = \dots = \bar{q}_{nT}^{nuc}$ .*

#### Proof

In order to determine a global solution of the inter-temporal profit maximization problem on  $C$ , we choose to apply the Karush - Kuhn - Tucker (KKT) conditions (Mas Colell et al. (1995)). However, the objective function of the inter-temporal profit maximization problem is not continuous on the set  $C$  which prevents the application of KKT conditions (see Proposition 3.3.2 on page 163). For this reason, we make use of Corollary 3.3.1 (see page 166).

For all  $n \in \{1, \dots, N\}$ , in view of Corollary 3.3.1,  $\bar{q} = ((\bar{q}_{njt}^{nuc})_{j=1}^J, \bar{q}_{nt}^{th})_{t=1}^T)_{n=1}^N$  solves the following optimal inter-temporal production problem

$$\max_{((q_{njt}^{nuc})_{j=1}^J, q_{nt}^{th})_{t=1}^T)_{n=1}^N \in F^{th}} \sum_{t=1}^T (p_t (\sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th}) - \sum_{j=1}^J C_{nj}^{nuc}(q_{njt}^{nuc}) - C_n^{th}(q_{nt}^{th})) \quad (3.5)$$

The objective function being continuous within  $F^{th}$  permits the application of KKT conditions in order to determine  $\bar{q} = ((\bar{q}_{njt}^{nuc})_{j=1}^J, \bar{q}_{nt}^{th})_{t=1}^T)_{n=1}^N$ . Since  $M$  is an affine set, Slater's condition is satisfied (Bazaraa et al. (1993), Leonard and Van Long (1992)). Therefore, there exists  $((\mu_t^n)_{t=1}^T)_{n=1}^N \in \mathbb{R}^{N \cdot T}$  and  $((\lambda_j^{n,k})_{j=1}^J)_{n=1}^N \in \mathbb{R}^{(3 \cdot 2 + 4 \cdot (J-2)) \cdot N}$  such that the KKT conditions are met, where  $\mu_t^n$  is the Lagrange multiplier associated with the supply-demand

equilibrium constraint at each month  $t$  for the producer  $n$  and  $\lambda_j^{n,k}$  is the Lagrange multiplier for the nuclear fuel constraint of the unit  $j$  during the campaign  $k$  for the producer  $n$  which is defined as

$$\begin{cases} k \in \{1, 2, 3\}, & \text{for } j \in \{1, 12\} \\ k \in \{1, 2, 3, 4\}, & \text{for } j \in \{2, \dots, 11\} \end{cases}$$

In view of KKT conditions of complementarity and since  $\bar{q} \in F$ , all the Lagrange multipliers associated with the minimum/maximum production constraints are equal to zero and they will be omitted in the Lagrangien function of this problem.

Let us call  $L^n$  the reduced Lagrangien of the optimal production problem (3.5)

$$L^n(q) = \pi^n - \sigma^n \cdot (D^n q - E^n)$$

where  $\pi^n$  is the inter-temporal profit of producer  $n$ ,  $\sigma^n = (\lambda_j^{n,k}, \mu_t^n)$  is the vector of the Lagrange multipliers,  $q = (((q_{njt}^{nuc})_{j=1}^J, q_{nt}^{th})_{t=1}^T)_{n=1}^N$  is a production vector of  $F^{th}$ ,  $E^n = (E_j^n)_{j=1}^J$  is the vector that contains the nuclear fuel stock of the unit  $j$  during a campaign for the producer  $n$  and  $D^n$  is the matrix so that the set  $M$  defined in subsection 3.3.1 is equal to  $\{q' \text{ s.t. } D^n q' - E^n = 0\}$ . We remark that the matrix  $D^n$  is “identical” from one producer to another and thus it does not depend on  $n$  any longer.

The inter-temporal profit  $\pi^n$  of the producer  $n$  is

$$\sum_{t=1}^T (p_t (\sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th}) - \sum_{j=1}^J C_{nj}^{nuc}(q_{njt}^{nuc}) - C_n^{th}(q_{nt}^{th}))$$

We remind that the price  $p_t$  is given by the marginal cost of the non-nuclear thermal production in the set  $F^{th}$ . Following some calculations and by using the supply-demand equilibrium constraints, we deduce that the producer’s  $n$  inter-temporal profit is a quadratic function of the non-nuclear thermal production  $q_{nt}^{th}$

$$-c_{th}^n \sum_{t=1}^T (q_{nt}^{th})^2 + 2c_{th}^n \sum_{t=1}^T (D_t - Q_t^{hyd} - \sum_{\nu \neq n} \sum_{j=1}^J q_{\nu jt}^{nuc} + q_{\nu t}^{th}) q_{nt}^{th} + c^n \quad (3.6)$$

where  $D_t - Q_t^{hyd} - \sum_{\nu \neq n} (\sum_{j=1}^J q_{\nu jt}^{nuc} + q_{\nu t}^{th})$  represents the “net demand” served by the producer  $n$  during the month  $t$ . The term  $c^n = (-\sum_{t=1}^T a_{th}^n - \sum_{t=1}^T \sum_{j=1}^J a_{nuc}^{n,j} + (b_{th} - b_{nuc})S_{reload}^n)$  is the constant part of the profit function. Moreover, the quantity  $S_{reload}^n$  denotes the total nuclear fuel stock of reloading that is available to the producer  $n$  over the entire time horizon  $T$ , hence  $\sum_{t=1}^T \sum_{j=1}^J q_{n jt}^{nuc} = S_{reload}^n$ .

According to the KKT conditions, one has

$$\frac{\partial L^n}{\partial q_{n jt}^{nuc}}(\bar{q}) = 0, \text{ for all } n, j, t$$

and

$$\frac{\partial L^n}{\partial q_{nt}^{th}}(\bar{q}) = 0, \text{ for all } n, t.$$

The derivative of Lagrangien with respect to the thermal production  $q_{n1}^{th}$  at month 1 is

$$\begin{aligned} \frac{\partial L^n}{\partial q_{n1}^{th}}(\bar{q}) &= 2c_{th}^n (D_1 - Q_{Tot,1}^{hyd} - \sum_{\nu \neq n} (\sum_{j=1}^J \bar{q}_{\nu j1}^{nuc} + \bar{q}_{\nu 1}^{th})) - 2c_{th}^n \bar{q}_{n1}^{th} - \mu_1^n = 0 \Leftrightarrow \\ & (D_1 - Q_{Tot,1}^{hyd} - \sum_{\nu \neq n} (\sum_{j=1}^J \bar{q}_{\nu j1}^{nuc} + \bar{q}_{\nu 1}^{th})) - \bar{q}_{n1}^{th} = \frac{\mu_1^n}{2c_{th}^n} \end{aligned} \quad (3.7)$$

By a symmetric argument, the derivative of Lagrangien with respect to the thermal production  $q_{n2}^{th}$  at month 2 is

$$\begin{aligned} \frac{\partial L^n}{\partial q_{n2}^{th}}(\bar{q}) &= 2c_{th}^n (D_2 - Q_{Tot,2}^{hyd} - \sum_{\nu \neq n} (\sum_{j=1}^J \bar{q}_{\nu j2}^{nuc} + \bar{q}_{\nu 2}^{th})) - 2c_{th}^n \bar{q}_{n2}^{th} - \mu_2^n = 0 \Leftrightarrow \\ & (D_2 - Q_{Tot,2}^{hyd} - \sum_{\nu \neq n} (\sum_{j=1}^J \bar{q}_{\nu j2}^{nuc} + \bar{q}_{\nu 2}^{th})) - \bar{q}_{n2}^{th} = \frac{\mu_2^n}{2c_{th}^n} \end{aligned} \quad (3.8)$$

In order to compare  $\mu_1^n$  and  $\mu_2^n$ , let us focus on a unit which is active in both month 1 and 2. The derivative of Lagrangien with respect to the nuclear production  $q_{n31}^{nuc}$  of unit 3 at month 1 is

$$\frac{\partial L^n}{\partial q_{n31}^{nuc}}(\bar{q}) = -\mu_1^n - \lambda_3^{n,1} = 0 \Leftrightarrow \mu_1^n = -\lambda_3^{n,1} \quad (3.9)$$

The derivative of Lagrangien with respect to the nuclear production  $q_{n32}^{nuc}$  of unit 3 at month 2 is

$$\frac{\partial L^n}{\partial q_{n32}^{nuc}}(\bar{q}) = -\mu_2^n - \lambda_3^{n,1} = 0 \Leftrightarrow \mu_2^n = -\lambda_3^{n,1} \quad (3.10)$$

From the last two equations, we deduce that, for all  $n$ ,  $\mu_1^n = \mu_2^n$ . In view of the equality between supply and demand, this means that

$$(D_1 - Q_{Tot,1}^{hyd} - \sum_{\nu \neq n} \sum_{j=1}^J (\bar{q}_{\nu j 1}^{nuc} + \bar{q}_{\nu 1}^{th})) - \bar{q}_{n1}^{th} = (D_2 - Q_{Tot,2}^{hyd} - \sum_{\nu \neq n} \sum_{j=1}^J (\bar{q}_{\nu j 2}^{nuc} + \bar{q}_{\nu 2}^{th})) - \bar{q}_{n2}^{th} \Leftrightarrow \bar{q}_{n1}^{nuc} = \bar{q}_{n2}^{nuc}$$

By using a unit available at month  $t$  and  $t + 1$ , we obtain  $\mu_t^n = \mu_{t+1}^n$ , which implies that  $\bar{q}_{nt}^{nuc} = \bar{q}_{n,t+1}^{nuc}$ . Consequently, for all  $n$ ,

$$\bar{q}_{n1}^{nuc} = \bar{q}_{n2}^{nuc} = \dots = \bar{q}_{nT}^{nuc}$$

We conclude that an equilibrium of the optimal production problem (3.5) which belongs to  $F$  is characterized by a constant monthly nuclear production and a variable non-nuclear thermal production which follows demand's seasonal variations.

□

Let us mention that  $F$  is not a compact set, thus the inter-temporal profit maximization problem may not have a solution on  $F$ . Consequently, the existence of a solution of the problem (3.5) on  $F$  takes the form of an assumption in Proposition 3.3.3. Moreover, Proposition 3.3.3 implies that each producer holds a strictly positive level of nuclear capacity from all types of nuclear units (i.e. Capacity $^{n,j,nuc} > 0$  for all  $n, j$ ).

### Economic interpretation of the Lagrange multipliers of the optimal inter-temporal production problem (3.5)

In view of the equality between supply and demand and since  $\bar{q} \in F$ , we deduce from equation (3.7) (respectively (3.8)) that the sign of the multiplier  $\mu_1^n$  (respectively  $\mu_2^n$ ) is strictly positive for all  $n$ . By symmetry, the Lagrange multiplier  $\mu_t^n$  is strictly positive ( $\mu_t^n > 0$ ) for all  $n, t$ . Consequently, in view of equations (3.9) and (3.10), the multiplier  $\lambda_3^{n,1}$  (respectively  $\lambda_j^{n,k}$ ) has a strictly negative sign for all  $n$ . Indeed, if an additional unit of nuclear fuel became available for the unit  $j$  of producer  $n$  during the campaign  $k$ , the non-nuclear thermal production of producer  $n$  and consequently the total non-nuclear thermal production obtained during this campaign would decrease which would lead to a lower market price and thus to a lower revenue for the producer  $n$ . At the same time the nuclear production cost of producer  $n$  would increase while its non-nuclear thermal production cost would decrease. However, the first effect that concerns the decrease of the revenue is the most important. Consequently, the “additional” profit resulting from an additional nuclear fuel unit and thus the value of the multiplier  $\lambda_j^{n,k}$  should be negative. The multiplier  $\lambda_j^{n,k}$  indicates the “marginal value of nuclear fuel stock”, i.e. the additional profit  $|\lambda_j^{n,k}|$  unit  $j$  of producer  $n$  would get if the nuclear fuel stock decreased by one unit during the campaign  $k$ .

We now proceed by showing that a constant monthly nuclear production constitutes a sufficient condition for optimality on  $C$ . Let us state the following proposition:

**Proposition 3.3.4** *For all  $n \in \{1, \dots, N\}$ , if  $((\bar{q}_{njt}^{nuc})_{j=1}^J, \bar{q}_{nt}^{th})_{t=1}^T)_{n=1}^N$  is a production vector belonging to  $F^{th}$  such that  $\bar{q}_{n1}^{nuc} = \bar{q}_{n2}^{nuc} = \dots = \bar{q}_{nT}^{nuc}$ , where  $(\bar{q}_{nt}^{nuc})_{t=1}^T$  is the corresponding monthly nuclear production vector of producer  $n$  then  $((\bar{q}_{njt}^{nuc})_{j=1}^J, \bar{q}_{nt}^{th})_{t=1}^T)_{n=1}^N$  is a solution of the inter-temporal profit maximization problem on  $C$ .*

**Proof**

In view of Corollary 3.3.1 on page 166, it suffices to show that  $\bar{q} = ((\bar{q}_{njt}^{nuc})_{j=1}^J, \bar{q}_{nt}^{th})_{t=1}^T)_{n=1}^N$  is a solution of the inter-temporal profit maximization problem on  $F^{th}$ . More precisely, it is sufficient to prove that there exist Lagrange multipliers such that the KKT conditions associated with this optimization problem are satisfied at  $\bar{q}$  given that the inter-temporal profit function is concave. In order to show the concavity of this function, we proceed with the following Lemma.

**Lemma 3.3.1** *For all  $n \in \{1, \dots, N\}$ , the profit function of the inter-temporal profit maximization problem on  $F^{th}$  is concave with respect to  $q_n$ .*

### Proof of Lemma 3.3.1

The inter-temporal profit  $\pi^n$  of producer  $n$  is a quadratic function of the non-nuclear thermal production  $q_{nt}^{th}$  on  $F^{th}$  given by the function (3.6)

$$-c_{th}^n \sum_{t=1}^T (q_{nt}^{th})^2 + 2c_{th}^n \sum_{t=1}^T (D_t - Q_t^{hyd} - \sum_{\nu \neq n} \sum_{j=1}^J q_{\nu jt}^{nuc} + q_{\nu t}^{th}) q_{nt}^{th} + c^n$$

(see proof of Proposition 3.3.3, page 168). We notice that this is a quadratic function of the form  $f^n(u_n) = a^n(u_n^T \cdot u_n) + b^n u_n + c^n$ , where  $a^n = -c_{th}^n$  and  $b^n = 2c_{th}^n \sum_{t=1}^T (D_t - Q_t^{hyd} - \sum_{\nu \neq n} \sum_{j=1}^J q_{\nu jt}^{nuc} + q_{\nu t}^{th})$ . For all  $n$ , since  $a^n < 0$ , the function  $f^n(u_n)$  is concave with respect to  $u_n$ . In view of the strict negativity of the coefficient  $a^n$ , it is also a strictly concave function with respect to  $u_n$ . As a consequence, the profit function  $\pi^n$  is strictly concave with respect to  $q_n^{th} = (q_{nt}^{th})_{t=1}^T$  for all  $n$  and by taking into account the other variables  $q_n^{nuc} = ((q_{njt}^{nuc})_{j=1}^J)_{t=1}^T$ , we conclude that the profit function  $\pi^n$  is concave with respect to  $q_n$ , for all  $n$ .

□

Let us now continue our proof.

First, we set the Lagrange multipliers associated with the minimum/maximum nuclear production constraints and with the minimum/maximum non-nuclear thermal production constraints equal to zero. Hence, given that the nuclear fuel constraints as well as the

supply-demand equilibrium constraints are pure equalities, KKT complementary conditions are satisfied at  $((\bar{q}_{njt}^{nuc})_{j=1}^J, \bar{q}_{nt}^{th})_{t=1}^T)_{n=1}^N$ . Then, we look at the Lagrange multipliers associated with the supply-demand equilibrium constraint in month  $t$  ( $\tilde{\mu}_t^n$ ) and the nuclear fuel constraint of the unit  $j$  during the campaign  $k$  for the producer  $n$  ( $\tilde{\lambda}_j^{n,k}$ ). We set

$$\tilde{\mu}_t^n = 2c_{th}^n ((D_t - Q_t^{hyd} - \sum_{\nu \neq n} (\sum_{j=1}^J \bar{q}_{\nu jt}^{nuc} + \bar{q}_{\nu t}^{th}) - \bar{q}_{nt}^{th}) = 2c_{th}^n \bar{q}_{nt}^{nuc}, \text{ for all } n, t$$

For all  $n$ , since  $\bar{q}_{n1}^{nuc} = \bar{q}_{n2}^{nuc} = \dots = \bar{q}_{nT}^{nuc}$ , we deduce that  $\tilde{\mu}_1^n = \tilde{\mu}_2^n = \dots = \tilde{\mu}_T^n$ . We call  $\tilde{\mu}^n$  their common value and we set

$$\tilde{\lambda}_j^{n,k} = -\tilde{\mu}^n, \text{ for all } n, j, k$$

For those multipliers the Lagrangian function of the optimal production problem is determined on  $F^{th}$  as follows:

$$H^n(q) = \pi^n - \tilde{\sigma}^n \cdot (Dq - E^n)$$

where  $\tilde{\sigma}^n = (\tilde{\mu}_t^n, \tilde{\lambda}_j^{n,k})$ . We recognize then the reduced Lagrangian function  $L^n$  of the optimal production problem (3.5). By taking the analysis of (3.7) respectively (3.9) and by symmetry, we realize that

$$\frac{\partial L^n}{\partial q_{nt}^{th}}(\bar{q}) = 0, \text{ for all } n, t$$

respectively

$$\frac{\partial L^n}{\partial q_{njt}^{nuc}}(\bar{q}) = 0, \text{ for all } n, j, t.$$

Consequently, the production vector  $((\bar{q}_{njt}^{nuc})_{j=1}^J, \bar{q}_{nt}^{th})_{t=1}^T)_{n=1}^N$  is also a solution of the KKT conditions associated with the inter-temporal profit maximization problem determined on the set  $F^{th}$  for all  $n$ . However, KKT conditions are sufficient for optimality since the objective function (profit function) is concave and the constraints are affine. Thus, for all  $n$ ,  $((\bar{q}_{njt}^{nuc})_{j=1}^J, \bar{q}_{nt}^{th})_{t=1}^T)_{n=1}^N$  is a solution of the inter-temporal profit maximization problem on  $F^{th}$  and the proposition is proven.

□



**Remark 3.3.2** *The strict concavity of the profit function  $\pi^n$  with regard to the non-nuclear thermal production  $q_n^{th}$  (see Proof of Lemma 3.3.1 on page 172) implies the unicity of solutions with respect to the non-nuclear thermal component for all  $n$ . However, if we consider the other variables which do not impact the profit  $\pi^n$  and as specified by the proof of Lemma 3.3.1 the profit function  $\pi^n$  is concave with regard to  $q_n$  for all  $n$ , which does not entail automatically the unicity of the entire solution.*

### Economic analysis of Proposition 3.3.3 and of Proposition 3.3.4

In view of Propositions 3.3.3 and 3.3.4 (pages 167, 171), we conclude that in the absence of binding productions constraints, the solutions of the optimal inter-temporal production problem are fully characterized by a constant nuclear production. Consequently in such situations, from a theoretical point of view, each producer maximizes its inter-temporal profit by using its nuclear units in order to produce at a constant rate while it operates its non-nuclear thermal units to follow-up load so that the global equilibrium between supply and demand is satisfied each month. Hence, at the optimum, the nuclear production does not follow the seasonal variations of demand which means that nuclear units operate only at baseload on a monthly basis. This implies that the amplitude of demand has to be inferior than the non-nuclear thermal capacity  $Q_{max}^{th}$  so that imbalances between supply and demand are avoided every month. Note that the level of the nuclear production of each producer being constant could never reach its maximum value. This is because the nuclear fuel stock of reloading of producer  $n$  corresponds to 258 days equivalent to full capacity which does not permit to reach the maximum level of nuclear production all over the year.

## 3.4 Numerical modelling

In this section, we proceed with an analysis of the nuclear and non-nuclear thermal production decisions as well as the storage decisions obtained by the optimal inter-temporal production problem, within a simple numerical model solved with Scilab. We remind that

the data used in our numerical dynamic model is the same as that used in the previous chapter.

### 3.4.1 Equivalence of equilibrium between an economy with $N$ producers and an economy with one aggregate producer

From a mathematical point of view, the complexity of the optimal inter-temporal production problem even in the simplified case of one aggregate producer (for  $N = 1$ , we obtain 586 operational constraints and 468 optimization variables given the time period of our model ( $T = 36$ ), the number of campaigns ( $K = 3$ ) and the different types of nuclear units ( $J = 12$ )) makes necessary to decrease the number of optimization variables and operational constraints. We show in the next proposition that the equilibrium of the original economy with  $N \geq 2$  producers is “equivalent” to the equilibrium of an alternate economy with one aggregate producer ( $N = 1$ ). Thanks to this mathematical proposition, we simplify the resolution of our optimization problem by determining an equilibrium of the economy with a unique producer operating with the aggregate nuclear and non-nuclear thermal capacity instead of the economy with  $N$  producers. This approach independent of the number of producers (reduction to a unique producer) makes coherent the numerical modelling of the optimal short-term production problem and the optimal inter-temporal production problem.

#### The notion of equilibrium in the case of an aggregate producer

We define the equilibrium of the optimal inter-temporal production problem with respect to a system of prices  $p \in \mathbb{R}_+^T$  in the case of a unique producer as follows:

**Definition 3.4.1** The production vector  $\bar{q} = ((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$  is an equilibrium with respect to a system of prices  $p \in \mathbb{R}_+^T$  if:

- (i)  $\bar{q}$  is a feasible production vector: (a) it respects the nuclear fuel constraints, for all  $j$  and (b) it respects the minimum/maximum production constraints, for all  $j, t$ .

(ii) it maximizes the inter-temporal profit of producer on the set of feasible solutions.

(iii) the price, at each month  $t$ , is determined by the marginal cost of the marginal technology. It is called the merit order price associated with the production vector  $\bar{q}$ .

(iv) at each date  $t$ , it respects the equality between supply and demand

$$\sum_{j=1}^J \bar{q}_{jt}^{nuc} + \bar{q}_t^{th} = D_t - Q_t^{hyd}. \quad (3.11)$$

In view of Definition 3.4.1, the production vector  $\bar{q}$  being an equilibrium of the optimal inter-temporal production problem maximizes the inter-temporal profit of producer (condition (ii)) and fulfills the conditions (i), (iii), (iv) which define a merit order equilibrium (see Definition 2.4.1 on page 129). Therefore, it is a merit order equilibrium of the optimal inter-temporal production problem.

We can now provide the next proposition which shows the equivalence between an equilibrium of an economy with  $N \geq 2$  producers and an equilibrium of an economy with a unique producer.

**Proposition 3.4.1** *Let us consider an economy  $E$  with several producers and let  $\tilde{E}$  be the auxiliary economy with a unique producer obtained by the aggregation of the  $N$  producers of  $E$ .*

( $\alpha$ ) *If  $(q_n)_{n=1}^N$  is an equilibrium of  $E$  then its aggregation defined by  $q = \sum_{n=1}^N q_n$  is an equilibrium of  $\tilde{E}$  with respect to the same prices.*

( $\beta$ ) *Conversely, if  $q$  is an equilibrium of  $\tilde{E}$  it can then be decentralized as an equilibrium  $(q_n)_{n=1}^N$  of  $E$  for the same prices.*

## Proof

Since the proof is technical and quite long but not difficult, it is important to announce that the main step is the use of KKT conditions which are here sufficient for optimality.

### Proof of $(\alpha)$

In view of Corollary 3.3.1 on page 166, it suffices to show that  $q$  is an equilibrium of the inter-temporal profit maximization problem on  $F^{th}$  within the economy  $\tilde{E}$ , given that  $(q_n)_{n=1}^N$  is an equilibrium of the inter-temporal profit maximization problem on  $F^{th}$  within the economy  $E$ .

In order to simplify our proof, we look at the inter-temporal profit maximization problem on  $F$  in both economies  $E$  and  $\tilde{E}$ . In view of Definitions 3.2.1 and 3.4.1 (see pages 160 and 175), a production vector being an equilibrium of the optimal inter-temporal production problem on  $F$  will be also an equilibrium of this problem on  $F^{th}$  because: (i)  $F^{th}$  contains  $F$  (see Remark 3.3.1 on page 162) and thus, a feasible point of the set  $F$  that respects the equality between supply and demand at each month belongs also to the set  $F^{th}$ , (ii) it leads to the same prices since non-nuclear thermal is the marginal technology in both sets (from (i) and (ii), we deduce that it constitutes a merit order equilibrium of the optimal inter-temporal production problem on  $F^{th}$ ) and finally, (iii) the optimal inter-temporal production problem determined on  $F$  and the optimal inter-temporal production problem determined on  $F^{th}$  have the same value.

We recall from Definition 3.2.1 provided on page 160 (respectively Definition 3.4.1 provided on page 175) that a production vector  $(\bar{q}_n)_{n=1}^N$  (respectively  $q$ ) being an equilibrium of the optimal inter-temporal production problem is a merit order equilibrium and in addition to this it maximizes the inter-temporal profit of producer  $n$ , for all  $n$  (respectively it maximizes the inter-temporal profit of the aggregate producer). The proof of the equivalence between the merit order equilibrium of  $E$  and the merit order equilibrium of  $\tilde{E}$  is similar to the proof of Proposition 2.4.1 presented in subsection 2.4.2 of chapter 2 (see page 131).

Since  $M$  is an affine set, Slater's condition is satisfied. Hence, there exist  $\widehat{\lambda}_j^{n,k}$  being the Lagrange multiplier for the nuclear fuel constraint of the unit  $j$  during the campaign  $k$  for the producer  $n$  and  $\widehat{\mu}_t^n$  being the Lagrange multiplier associated with the supply-demand equilibrium constraint at each month  $t$  for the producer  $n$  such that the KKT conditions are met.

In view of KKT conditions of complementarity and since  $(q_n)_{n=1}^N \in F$ , all the Lagrange multipliers associated with the minimum/maximum production constraints are equal to zero and they will be omitted in the Lagrangian function of the inter-temporal profit maximization problem.

We call  $I^n$  the reduced Lagrangian of the inter-temporal profit maximization problem in the economy  $E$

$$I^n(\widehat{q}) = \pi^n - \widehat{\sigma}^n \cdot (D^n \widehat{q} - E^n)$$

where  $\pi^n$  is the inter-temporal profit of producer  $n$ ,  $\widehat{\sigma}^n = (\widehat{\lambda}_j^{n,k}, \widehat{\mu}_t^n)$  is the vector of the Lagrange multipliers,  $\widehat{q} = (((\widehat{q}_{njt}^{nuc})_{j=1}^J, \widehat{q}_{nt}^{th})_{t=1}^T)_{n=1}^N$  is a production vector of  $F$ ,  $E^n$  and  $D^n$  are matrices that have been already defined in the proof of Proposition 3.3.3 (see page 168).

According to the KKT conditions, for all  $n$ , one obtains

$$\frac{\partial I^n}{\partial \widehat{q}}(q) = 0 \tag{3.12}$$

It is sufficient to prove that there exist Lagrange multipliers such that the KKT conditions associated with the inter-temporal profit maximization problem on  $F$  in the economy  $\widetilde{E}$  are satisfied at  $q$  given that the inter-temporal profit function of the aggregate producer is concave.

First, we set the Lagrange multipliers associated with the minimum/maximum production constraints equal to zero. Hence, given that the nuclear fuel constraints as well as the supply-demand equilibrium constraints are pure equalities, KKT complementary conditions are satisfied at  $q$ . Then, we look at the Lagrange multipliers associated with the

nuclear fuel constraint of the unit  $j$  during the campaign  $k$  ( $\tilde{\lambda}_j^k$ ) and the supply-demand equilibrium constraint in month  $t$  ( $\hat{\mu}_t$ ). We set

$$\hat{\mu}_t = \sum_{n=1}^N \hat{\mu}_t^n$$

and

$$\hat{\lambda}_j^k = \sum_{n=1}^N \hat{\lambda}_j^{n,k}$$

For those multipliers the Lagrangian function of the inter-temporal profit maximization problem on  $F$  in the economy  $\tilde{E}$  takes the following form:

$$I(\hat{q}) = \pi - \hat{\sigma} \cdot (D\hat{q} - E)$$

where  $\pi = \sum_{n=1}^N \pi^n$  is the inter-temporal profit of the aggregate producer,  $\hat{\sigma} = (\hat{\lambda}_j^k, \hat{\mu}_t)$  is the vector of the Lagrange multipliers,  $\hat{q} = (((\sum_{n=1}^N \hat{q}_{njt}^{nuc})_{j=1}^J, \sum_{n=1}^N \hat{q}_{nt}^{th})_{t=1}^T) = (((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T)$  is the aggregate production vector which belongs to  $F$ ,  $E$  and  $D$  are such that  $E = \sum_{n=1}^N E^n$  and  $D = \sum_{n=1}^N D^n$ .

By taking into account the relationship (3.12), one has

$$\begin{aligned} \frac{\partial I}{\partial \hat{q}}(q) &= \frac{\partial(\pi - \hat{\sigma} \cdot (D\hat{q} - E))}{\partial \hat{q}}(q) = \frac{\partial(\sum_{n=1}^N (\pi^n - \hat{\sigma}^n \cdot (D^n \hat{q} - E^n)))}{\partial \hat{q}}(q) = \\ &= \frac{\partial(\pi^1 - \hat{\sigma}^1 \cdot (D^1 \hat{q} - E^1))}{\partial \hat{q}}(q) + \dots + \frac{\partial(\pi^N - \hat{\sigma}^N \cdot (D^N \hat{q} - E^N))}{\partial \hat{q}}(q) = \\ &= \frac{\partial I^1}{\partial \hat{q}}(q) + \dots + \frac{\partial I^N}{\partial \hat{q}}(q) = 0 + \dots + 0 = 0 \end{aligned}$$

Thus, the production vector  $q$  is a solution of the KKT conditions associated with the inter-temporal profit maximization problem on  $F$  in the economy  $\tilde{E}$ . The inter-temporal profit function of the aggregate producer is concave and its proof results by analogy to the proof of Lemma 3.3.1 which shows the concavity of the inter-temporal profit function in the case of  $N$  producers (see page 172). Moreover, the production and storage constraints as

well as the supply-demand equilibrium constraints are affine. Therefore, KKT conditions are sufficient for optimality. Consequently, we deduce that  $q$  is an equilibrium of the inter-temporal profit maximization problem on  $F$  in the economy  $\tilde{E}$ .

### **Proof of $(\beta)$**

Conversely, we show that if  $q$  is an equilibrium of the inter-temporal profit maximization problem on  $F$  in the economy  $\tilde{E}$ , then  $(q_n)_{n=1}^N$  is an equilibrium of the inter-temporal profit maximization problem on  $F$  in the economy  $E$ .

Slater's condition is met because  $M$  is an affine set within the economy  $\tilde{E}$ . So, there exist the Lagrange multipliers  $\hat{\lambda}_j^k$  and  $\hat{\mu}_t$  which have been already denoted in the previous part of the proof such that the KKT conditions are satisfied.

In view of KKT conditions of complementarity and since  $q \in F$ , all the Lagrange multipliers associated with the minimum/maximum production constraints are equal to zero and they will be skipped in the Lagrangian function of the inter-temporal profit maximization problem.

Therefore, as we have already seen in the proof of  $(\alpha)$ , the reduced Lagrangian of the inter-temporal profit maximization problem in the economy  $E$  takes the form

$$I(\hat{q}) = \pi - \hat{\sigma} \cdot (D\hat{q} - E)$$

According to the KKT conditions, one gets

$$\frac{\partial I}{\partial \hat{q}}(q) = 0 \tag{3.13}$$

We need to prove that there exist Lagrange multipliers such that the KKT conditions associated with the inter-temporal profit maximization problem on  $F$  in the economy  $E$  are satisfied at  $(q_n)_{n=1}^N$  given that the inter-temporal profit function of producer  $n$  is concave, for all  $n$ .

Initially, we set the Lagrange multipliers associated with the minimum/maximum production constraints equal to zero. Thus, KKT complementary conditions are satisfied at  $(q_n)_{n=1}^N$  given that the nuclear fuel constraints as well as the supply-demand equilibrium constraints are pure equalities. Afterwards, we deal with the Lagrange multipliers associated with the nuclear fuel constraint of the unit  $j$  during the campaign  $k$  for the producer  $n$  ( $\tilde{\lambda}_j^{n,k}$ ) and the supply-demand equilibrium constraint in month  $t$  for the producer  $n$  ( $\hat{\mu}_t^n$ ). We set

$$\hat{\mu}_t^n = \hat{\mu}_t - \sum_{\nu \neq n} \hat{\mu}_t^\nu$$

and

$$\hat{\lambda}_j^{n,k} = \hat{\lambda}_j^k - \sum_{\nu \neq n} \hat{\lambda}_j^{\nu,k}$$

For those multipliers the Lagrangian function of the inter-temporal profit maximization problem on  $F$  in the economy  $E$  becomes

$$I^n(\hat{q}) = \pi^n - \hat{\sigma}^n \cdot (D^n \hat{q} - E^n)$$

By taking into consideration the relationship (3.13), one has

$$\begin{aligned} \frac{\partial I}{\partial \hat{q}}(q) = 0 &\Leftrightarrow \frac{\partial(\pi - \hat{\sigma} \cdot (D\hat{q} - E))}{\partial \hat{q}}(q) = 0 \Leftrightarrow \frac{\partial(\sum_{n=1}^N (\pi^n - \hat{\sigma}^n \cdot (D^n \hat{q} - E^n)))}{\partial \hat{q}}(q) = 0 \Leftrightarrow \\ &\frac{\partial(\pi^1 - \hat{\sigma}^1 \cdot (D^1 \hat{q} - E^1))}{\partial \hat{q}}(q) + \dots + \frac{\partial(\pi^N - \hat{\sigma}^N \cdot (D^N \hat{q} - E^N))}{\partial \hat{q}}(q) = 0 \Leftrightarrow \\ &\frac{\partial I^1}{\partial \hat{q}}(q) + \dots + \frac{\partial I^N}{\partial \hat{q}}(q) = 0 \Leftrightarrow \\ &\frac{\partial I^1}{\partial \hat{q}}(q) = \dots = \frac{\partial I^N}{\partial \hat{q}}(q) = 0 \end{aligned}$$

Hence, the production vector  $(q_n)_{n=1}^N$  is a solution of the KKT conditions associated with the inter-temporal profit maximization problem on  $F$  in the economy  $E$ . The inter-temporal profit function in the case of  $N$  producers is concave (see Lemma 3.3.1 on page



172). Furthermore,  $M$  is an affine set and the production constraints are also affine. Therefore, KKT conditions are sufficient for optimality. As a result, we obtain that  $(q_n)_{n=1}^N$  is an equilibrium of the inter-temporal profit maximization problem on  $F$  in the economy  $E$ .

In view of proof of  $(\alpha)$  and of  $(\beta)$ , the proposition is proven. □

### Economic consequences of Proposition 3.4.1

According to Proposition 3.4.1 on page 176, we can say that the equilibrium  $q$  of the optimal inter-temporal production problem of the auxiliary economy  $\tilde{E}$  is equivalent to the equilibrium  $(q_n)_{n=1}^N$  of the optimal inter-temporal production problem of the decentralized economy  $E$ . In view of this proposition, we consider from now on an economy with a unique producer ( $N = 1$ ) operating with the total nuclear and non-nuclear thermal capacity to meet the monthly demand. For example, the exogenous variable  $Q_{max}^{th}$  will now represent the aggregate maximum non-nuclear thermal production. From an economical perspective, the equivalence of equilibria of the optimal inter-temporal production problem between the economies  $E$  and  $\tilde{E}$  implies that the optimal production behavior in the case of the decentralization of the nuclear generation set is “neutral” with respect to the optimal production behavior observed in the case of the centralized management by an aggregate producer. These results can be also found in the case of the optimal short-term production problem in chapter 2.

### Adaptation of Propositions 3.3.3 and 3.3.4 in the case of an economy with one aggregate producer

In an economy with one aggregate producer who holds the total nuclear and non-nuclear thermal capacity and where  $((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T$  represents the vector of the aggregate nuclear and non-nuclear thermal production, Propositions 3.3.3 and 3.3.4 take the following form:

**Proposition 3.4.2** *If an equilibrium  $((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T \in F$  exists such that the inter-temporal profit of the producer is maximum on  $C$  and  $(\bar{q}_t^{nuc})_{t=1}^T$  is the corresponding monthly nuclear production vector then  $\bar{q}_1^{nuc} = \bar{q}_2^{nuc} = \dots = \bar{q}_T^{nuc}$ .*

**Proof**

This proposition is an obvious corollary of Proposition 3.3.3 since it is a particular situation when  $N = 1$ .

□

**Proposition 3.4.3** *If  $((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$  is a production vector belonging to  $F^{th}$  such that  $\bar{q}_1^{nuc} = \bar{q}_2^{nuc} = \dots = \bar{q}_T^{nuc}$ , where  $(\bar{q}_t^{nuc})_{t=1}^T$  is the corresponding monthly nuclear production vector then  $((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$  is a solution of the inter-temporal profit maximization problem on  $C$ .*

**Proof**

Evidently, this proposition is a corollary of Proposition 3.3.4 because it is a specific case when  $N = 1$ .

□

The Propositions 3.4.2 and 3.4.3 constitute corollaries of Propositions 3.3.3 and 3.3.4 respectively as they refer to the special case of a unique producer ( $N = 1$ ) who holds the total nuclear and non-nuclear thermal capacity. They show that an equilibrium of the optimal inter-temporal production problem is such that the total monthly nuclear production which results from all nuclear units  $j$  ( $(\sum_{j=1}^J \bar{q}_{jt}^{nuc})_{t=1}^T = (\bar{q}_t^{nuc})_{t=1}^T$ ) is constant each month without however attaining its maximum value over the whole period  $T$ . Consequently, the total monthly non-nuclear thermal production has to comply with the variations of the seasonal demand and remains marginal during the entire time horizon of the model.

### 3.4.2 Symmetry of equilibrium of the optimal inter-temporal production problem

In this section, we introduce the notion of symmetrisability within our model in order to provide an economical property of producer's optimal behaviour: under the assumption that each producer disposes of the same level of nuclear and non-nuclear thermal capacity, we show that an equilibrium of the inter-temporal profit maximization problem (3.1) is “almost” symmetric. More precisely, we prove that the non-nuclear thermal component of the equilibrium is symmetric while the nuclear component of the equilibrium is “symmetrisable”, i.e., it can be symmetrised. The interest of showing this property lies in the notion of symmetrisability and its mathematical-economical implications. Indeed, considering the feature of symmetrisability, we can focus on the symmetric nuclear component of the equilibrium and thus obtain a symmetric equilibrium of the optimal inter-temporal production problem. Therefore, under the assumption of symmetry of capacities, we prove the existence of an equilibrium such that all producers realize the same level of nuclear and non-nuclear thermal production. Nevertheless, we will show through a simple example that despite the symmetry of nuclear capacities, the nuclear component of the equilibrium is potentially asymmetric which leads to asymmetric equilibria of the inter-temporal profit maximization problem. From a mathematical point of view, this property permits to concentrate only on the symmetric solution and therefore it may be used to decrease the number of the optimization variables of the optimal inter-temporal production problem (3.1) which simplifies its numerical resolution. In addition, it provides us with an appealing economical feature regarding the profit resulting from a symmetrisable equilibrium.

#### The notion of symmetrisability

In view of the assumption that the nuclear and non-nuclear thermal capacities are symmetric among producers, the exogenous variables  $(Q_{max}^{n,j,nuc}, Q_{min}^{n,j,nuc}, Q_{max}^{n,th}, S_{reload}^{n,j}, S_1^{n,j})$  of the optimization problem (3.1) as well as the production cost functions  $(C_{n,j}^{nuc}, C_n^{th})$  and thus, the marginal non-nuclear thermal cost function  $(mc_n^{th})$  will not depend on  $n$ . If

$((q_{1jt}^{nuc})_{j=1}^J, q_{1t}^{th})_{t=1}^T, \dots, ((q_{Njt}^{nuc})_{j=1}^J, q_{Nt}^{th})_{t=1}^T$ ) is an equilibrium of this problem, we show that the non-nuclear thermal component of the equilibrium is symmetric while the nuclear component of the equilibrium is symmetrisable.

We define a symmetrisable equilibrium as follows:

**Definition 3.4.2** Let  $(q_1, q_2, \dots, q_N)$  be an equilibrium. This equilibrium is called symmetrisable if there exists a symmetric allocation  $(\tilde{q}_1, \tilde{q}_2, \dots, \tilde{q}_N)$ , which is an equilibrium and “leads” to the same prices as the initial allocation  $(q_n)_{n=1}^N$ .

### **An economical property resulting from the notion of symmetrisability of an equilibrium**

The notion of symmetrisability of an equilibrium provides us with an interesting economical feature: the profit of a symmetrisable equilibrium  $(\pi_1, \pi_2, \dots, \pi_N)$  is symmetric. This means that the production levels of a symmetrisable equilibrium are equivalently profitable for all producers. This arises from the fact that the price induced by the symmetrisable equilibrium is equal to the price induced by the symmetric equilibrium. Consequently, the profit  $(\pi_1, \pi_2, \dots, \pi_N)$  coming from a symmetrisable equilibrium is equal to the profit  $(\tilde{\pi}_1, \tilde{\pi}_2, \dots, \tilde{\pi}_N)$  resulting from the deduced symmetric equilibrium. For a symmetric equilibrium, the value of profit is equal among the different producers at the equilibrium state given that the price  $p_t$  as well as the production level are identical for all players and the production cost is symmetric for both technologies.

### **Symmetry of the non-nuclear thermal component and symmetrisability of the nuclear component of equilibrium**

We proceed here with Proposition 3.4.4 which shows the symmetry of the non-nuclear thermal component and the symmetrisability of the nuclear component of equilibrium of the optimal inter-temporal production problem. Before we present this proposition and its proof, let us provide a useful remark:

**Remark 3.4.1** *The optimal inter-temporal production problem (3.1) is additively separable<sup>4</sup> with respect to the couple  $(q_{njt}^{nuc}, q_{nt}^{th})$  (for the notion of separability cf. for example Boyd and Vandenberghe (2004)). This results from the fact that the inter-temporal profit function is an additively separable function since it can be written as the sum of the inter-temporal profit resulting from the nuclear production and the inter-temporal profit coming from the non-nuclear thermal production and because there are no constraints linking the production variables  $(q_{njt}^{nuc})$  and  $(q_{nt}^{th})$ . Consequently, the two inter-temporal profit maximization problems (nuclear and non-nuclear thermal) can be studied independently. First, let us study the non-nuclear thermal inter-temporal profit maximization problem of producer  $n$ . By the same argument, it is additively separable with respect to the time index  $t$  since the inter-temporal profit is additively separable with respect to  $t$  and the minimum and maximum non-nuclear thermal production constraints are mutually “independent” from  $t$ . Then, we look at the nuclear inter-temporal profit maximization problem of producer  $n$ . It is additively separable with respect to the unit index  $j$  because the profit is additively separable with respect to  $j$  and the nuclear production constraints are mutually “independent” from  $j$ . However, this is not the case for the time index  $t$  because the nuclear fuel constraints link the nuclear production variables  $(q_{njt}^{nuc})$  on  $t$  (inside a campaign). Hence, the nuclear optimal production problem of producer  $n$  is not separable with respect to  $t$ .*

We are ready now to state the following proposition:

**Proposition 3.4.4** *Let  $((q_{1jt}^{nuc})_{j=1}^J, q_{1t}^{th})_{t=1}^T, \dots, ((q_{Njt}^{nuc})_{j=1}^J, q_{Nt}^{th})_{t=1}^T$  be an equilibrium of the optimal inter-temporal production problem (3.1). If the nuclear and non-nuclear thermal capacity are symmetric among producers then the non-nuclear thermal component of the equilibrium is symmetric while the nuclear component of the equilibrium is symmetrizable.*

### Proof

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<sup>4</sup>A function of two variables  $F(x, y)$  will be called additively separable if it can be written as  $f(x) + g(y)$  for some single-variable functions  $f(x)$  and  $g(y)$ . For example, any function  $F$  linear is additively separable while  $F(x, y) = xy$  is not.

First, we show that the non-nuclear thermal component of the equilibrium  $(q_{1t}^{th}, q_{2t}^{th}, \dots, q_{Nt}^{th})$  is symmetric. In view of the Remark 3.4.1, it follows that, for all  $t$ , the non-nuclear thermal production  $q_{nt}^{th}$  is a solution of

$$\max_{\psi_{nt}^{th}} p_t \psi_{nt}^{th} - C^{th}(\psi_{nt}^{th})$$

subject to the constraint

$$0 \leq \psi_{nt}^{th} \leq Q_{max}^{th}, \quad \text{for all } t$$

where  $n \in \{1, \dots, N\}$ .

We want to show that  $q_{1t}^{th} = q_{2t}^{th} = \dots = q_{Nt}^{th}$ . One has  $q_{1t}^{th}$  (respectively  $q_{2t}^{th}, \dots, q_{Nt}^{th}$ ) a solution of

$$\max_{\psi_{1t}^{th}} p_t \psi_{1t}^{th} - C^{th}(\psi_{1t}^{th}) \tag{3.14}$$

subject to the constraint

$$0 \leq \psi_{1t}^{th} \leq Q_{max}^{th}$$

In view of the strict concavity of the cost function  $C^{th}$ , the problem (3.14) has a unique solution. Since  $q_{1t}^{th}, q_{2t}^{th}, \dots, q_{Nt}^{th}$  are all solutions of problem (3.14), they are equal and the symmetry is proven.

**Remark 3.4.2** *Note that when the non-nuclear thermal production is positive, the price is determined by the marginal cost of the non-nuclear thermal production ( $p_t = mc^{th}(q_t^{th})$ ).*

Let us now focus on the nuclear component of the equilibrium. We prove that the nuclear component of the equilibrium  $(q_{1jt}^{nuc}, q_{2jt}^{nuc}, \dots, q_{Njt}^{nuc})$  is symmetrisable. By analogy

with the case of the non-nuclear thermal technology, for all  $j$ , the nuclear production  $q_{njt}^{nuc}$  of producer  $n$  is a solution of

$$\max_{\psi_{njt}^{nuc}} \sum_{t=1}^T (p_t \psi_{njt}^{nuc} - C_j^{nuc}(\psi_{njt}^{nuc})) \quad (3.15)$$

subject to the nuclear fuel storage constraints and the minimum/maximum nuclear production constraints.

The constraints of the reduced optimization problem (3.15) determine its domain of definition. It is clear that it is a convex set, since it is defined by affine constraints. Moreover, the objective function of the reduced profit maximization problem (3.15) is affine. Thus, it is a concave function. In view of the convexity of the domain of definition and the concavity of the objective function of the problem (3.15), we conclude that the set of solutions of the optimization problem (3.15) is convex (Mas Colell et al. (1995)). Therefore, the allocation  $(\tilde{q}_{1jt}^{nuc}, \tilde{q}_{2jt}^{nuc}, \dots, \tilde{q}_{Njt}^{nuc})$ , where  $(\tilde{q}_{1jt}^{nuc} = \tilde{q}_{2jt}^{nuc} = \dots = \tilde{q}_{Njt}^{nuc} = \sum_{n=1}^N \frac{q_{njt}^{nuc}}{N}$  for all  $j, t$ ) is also a solution of this problem. In addition, at each date  $t$ , it respects the equality between supply and demand. Consequently, the symmetric allocation  $(\tilde{q}_{1jt}^{nuc}, \tilde{q}_{2jt}^{nuc}, \dots, \tilde{q}_{Njt}^{nuc})$  is an equilibrium. In order to show that it “leads” to the same price as the initial equilibrium  $(q_{1jt}^{nuc}, q_{2jt}^{nuc}, \dots, q_{Njt}^{nuc})$ , we proceed with Lemma 3.4.1.

**Lemma 3.4.1** *The price induced by  $((q_{1jt}^{nuc})_{j=1}^J, q_{1t}^{th}), \dots, ((q_{Njt}^{nuc})_{j=1}^J, q_{Nt}^{th})$  is equal to the price induced by  $((\tilde{q}_{1jt}^{nuc})_{j=1}^J, q_{1t}^{th}), \dots, ((\tilde{q}_{Njt}^{nuc})_{j=1}^J, q_{Nt}^{th})$  for all  $t$ .*

### Proof

Let be  $P^n = \{t \in \{1, \dots, T\} \text{ s.t. } q_{nt}^{th} = 0\}$  for each  $n \in \{1, \dots, N\}$ . Note that in view of the first part of the proof, the set  $P^n$  does not depend on  $n$ . If  $t \notin P$ , there is nothing to prove since the price is determined by the marginal cost of non-nuclear thermal marginal production which is unchanged. If  $t \in P$ , in both cases, the price  $p_t$  is determined by the marginal cost of the nuclear production which is constant ( $p_t = mc^{nuc}(q_{njt}^{nuc}) = b_{nuc}$  for all  $t \in P$ ). Consequently, it is obvious that both prices are equal for all  $t$ .

□

Hence, in view of Lemma 3.4.1, the equilibrium  $(\tilde{q}_{1jt}^{nuc}, \tilde{q}_{2jt}^{nuc}, \dots, \tilde{q}_{Njt}^{nuc})$  “leads” to the same price as the initial one. So, according to the Definition 3.4.2 on page 185, the nuclear component of the equilibrium  $(q_{1jt}^{nuc}, q_{2jt}^{nuc}, \dots, q_{Njt}^{nuc})$  is symmetrisable.

We conclude that the equilibrium  $((q_{1jt}^{nuc})_{j=1}^J, q_{1t}^{th})_{t=1}^T, \dots, ((q_{Njt}^{nuc})_{j=1}^J, q_{Nt}^{th})_{t=1}^T)$  is characterized by a symmetric non-nuclear thermal component and a symmetrisable nuclear component.

□

### Existence of an asymmetric equilibrium of the optimal inter-temporal production problem: An example

It should be noticed that the nuclear component of the equilibrium  $(q_{1jt}^{nuc}, q_{2jt}^{nuc}, \dots, q_{Njt}^{nuc})$  is potentially asymmetric. In order to understand this asymmetry, let us give an example in the case of two producers ( $N = 2$ ). Let  $(\hat{q}_{1jt}^{nuc}, \hat{q}_{2jt}^{nuc})$  be a symmetric equilibrium such that the price is the same during the period 1 and the period 2 (i.e.  $p_1 = p_2$ ). This occurs in particular, if nuclear is the marginal technology in periods 1 and 2. Then, any feasible production realized by the unit 3 of producer 1 (respectively 2) in periods 1, 2 means a solution of the following system:

$$\left\{ \begin{array}{ll} q_{131} + q_{231} = \hat{q}_{131} + \hat{q}_{231} = D_1 - Q_{Tot,1}^{hyd}, & \text{supply - demand equilibrium constraint} \\ & \text{in month 1} \\ q_{132} + q_{232} = \hat{q}_{132} + \hat{q}_{232} = D_2 - Q_{Tot,2}^{hyd}, & \text{supply - demand equilibrium constraint} \\ & \text{in month 2} \\ q_{131} + q_{132} = \hat{q}_{131} + \hat{q}_{132} = S_1^3, & \text{nuclear fuel constraint for unit 3 of producer 1} \\ q_{231} + q_{232} = \hat{q}_{231} + \hat{q}_{232} = S_2^3, & \text{nuclear fuel constraint for unit 3 of producer 2} \end{array} \right.$$

and remains unchanged during the remaining periods ( $q_{njt}^{nuc} = \hat{q}_{njt}^{nuc}$ , for all  $n \in \{1, 2\}$ , for all  $j \neq 3$ , and for  $t \geq 3$ ) will be still an equilibrium. Consequently, there exists at least one asymmetric equilibrium of the optimal production problem (3.15), where for example



unit 3 produces more for producer 1 than for producer 2 in period 1 (and the opposite in period 2 in order to compensate).

### 3.4.3 Simulation results

In this section, we treat the problem of discontinuity of the merit order price which induces a discontinuity (decrease) of the inter-temporal profit in order to resolve numerically the optimal inter-temporal production problem within our data set. More precisely, we go on with a “regularization” of our economical problem (discontinuous problem) which leads to an approximate continuous problem (the “regularized” problem). To begin with, we introduce some general results derived from the numerical resolution of the “regularized” optimal inter-temporal production problem and then we continue with a full analysis of the optimal production and storage decisions. To end, we provide some general remarks in line with the simulation results.

#### “Regularization” of the optimal inter-temporal production problem

Since the hypothesis of Proposition 3.3.2 that  $b_{nuc} < b_{th}$  is valid within our data, the discontinuity and specifically the decrease of price at production vectors with zero levels of non-nuclear thermal production leads to a discontinuity and particularly a reduction in the value of profit. Let us emphasize that it does not exist an algorithm that maximizes a discontinuous function. From both a theoretical and a numerical point of view, we “regularize” the merit order price in order to deal with the problem of discontinuity of the price and solve the optimal inter-temporal production problem. From a theoretical perspective, we treated the problem of discontinuity via the resolution of the optimal inter-temporal production problem on the subset  $F^{th}$  of the set of feasible solutions  $C$  which is an equivalent optimization problem (see Corollary 3.3.1 on page 166). Let us remind that the non-nuclear thermal production is strictly positive in the set  $F^{th}$  and hence, non-nuclear thermal is the marginal technology signifying that the price is set by its own marginal cost for all  $t$ . The inter-temporal profit becomes a continuous function

inside this set. To treat the problem of discontinuity numerically, we suggest an alternative model in which we give to the price the value of the non-nuclear thermal marginal cost ( $mc^{th}(0) = b_{th}$ ) instead of ( $b_{nuc}$ ) during periods when nuclear is the marginal technology. Hence, the price  $p_t$  during the month  $t$  will be

$$p_t = \begin{cases} mc^{th}(q_t^{th}), & \text{if } q_t^{th} > 0 \\ mc^{th}(0), & \text{if } q_t^{th} = 0 \end{cases} = \begin{cases} b_{th} + 2c_{th}q_t^{th}, & \text{if } q_t^{th} > 0 \\ b_{th}, & \text{if } q_t^{th} = 0 \end{cases} \quad (3.16)$$

Therefore, during months when nuclear is the only technology called in order to meet demand, the nuclear production is paid at least  $b_{th}$  (Euros per MWh). Considering this “regularization” of the merit order price, the inter-temporal profit, being now a continuous function, is maximized on the entire set of feasible solutions  $C$  within our numerical model. This constitutes a continuous optimization problem called the “regularized” problem. Nevertheless, the “regularized” problem (continuous problem) is different from the economical problem (3.1) described in subsection 3.2.1 (discontinuous problem) with regard to the objective function i.e. the profit. More precisely, the inter-temporal profit considered in this problem is greater than the inter-temporal profit of the economical problem since the value of  $b_{th}$  (26.24 Euro/MWh) is greater than the value of  $b_{nuc}$  (5.01 Euro/MWh) (see Annex C, Table C.4). However, we prove that the value of the “regularized” problem is the same with the value of the economical problem which means that the value of profit at the optimum is identical for both optimization problems (see Annex B, Proposition B.1). Hence, we deduce that the “regularized” problem is a “good” approximation of our economical problem (Boyd and Vandenberghe (2004)).

Theoretically, in Annex B, Proposition B.7 proves that a solution of the “regularized” problem which is not in the set  $F^{th}$  implies the emptiness of the set of solutions of the economical problem. From a numerical perspective, the solution of the “regularized” problem which is presented in this section is not in the set  $F^{th}$  because the non-nuclear thermal production is not always used to ensure the equality between supply and demand in combination with the nuclear production. This means that the set of solutions of the economical problem is empty which shows the importance of emphasizing the numerical solution resulting from the “regularized” problem. This numerical solution is only an “approximate” solution of our economical problem.

It should be mentioned that the discontinuity of price observed at production vectors with zero levels of non-nuclear thermal production poses an “economical problem”. More precisely, a producer, who covers the monthly levels of demand during summer (low demand season) by running only its nuclear units, is penalized since its nuclear production is evaluated at a low price ( $b_{nuc}$ ). This price does not allow the amortization of the important fixed costs of nuclear. Hence, by realizing an infinitesimal nuclear capacity withholding, the non-nuclear thermal technology becomes the marginal technology that leads prices to a higher level (almost equal to  $b_{th}$ ) which justifies our “regularization” of the merit order price.

## General results

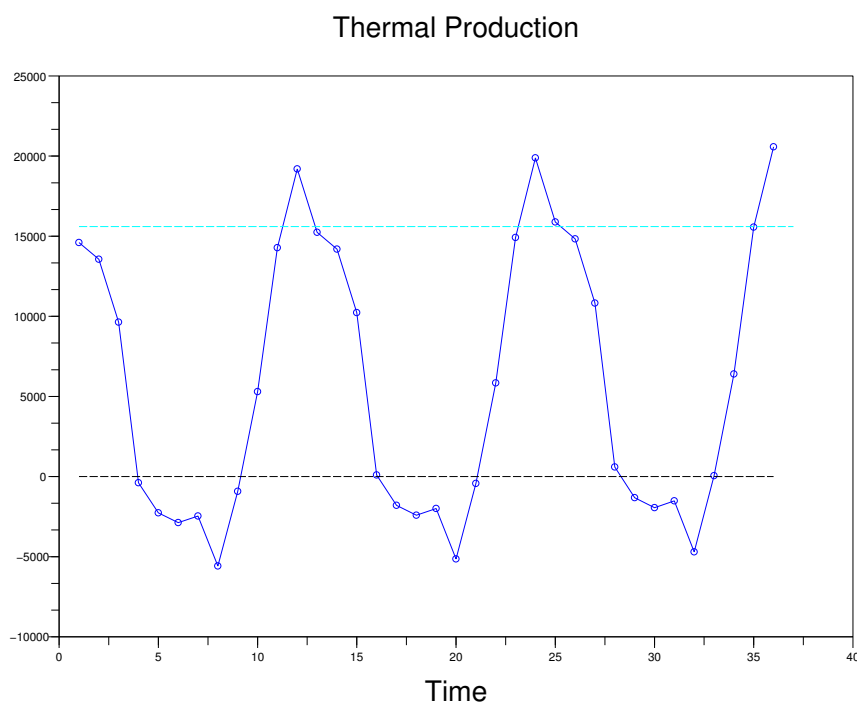


Figure 3.1: Simulated non-nuclear thermal production resulting from a constant nuclear production (in MW)

We recall that under some assumptions, Proposition 3.4.2 and Proposition 3.4.3 on page 183 show that the non-nuclear thermal production is adjusted on the seasonal variations of demand, while the nuclear production remains constant during the entire time horizon of the model. However, in view of our data, a constant nuclear production is not a

feasible solution of the optimal inter-temporal production problem in the case of a flexible nuclear set (like the French nuclear set) because the deduced non-nuclear thermal production violates both minimum and maximum<sup>5</sup> non-nuclear thermal production constraints during some months (see Figure 3.1). In particular, the non-nuclear thermal production does not meet demand during high demand months (winter). As one can see in figure 2.9 of subsection 2.4.1 of chapter 2 and in figure 3.2, the amplitude of demand exceeds the maximum level of non-nuclear thermal production  $Q_{max}^{th}$ . Consequently, given that the nuclear production is constant, the non-nuclear thermal production can not balance supply and demand each month (see page 174). For this reason, the nuclear production has to be flexible and follow the variations of demand in order to avoid imbalances between supply and demand. Consequently, Proposition 3.4.2 says that the inter-temporal profit maximization problem has no solutions on  $F$  within our numerical model.

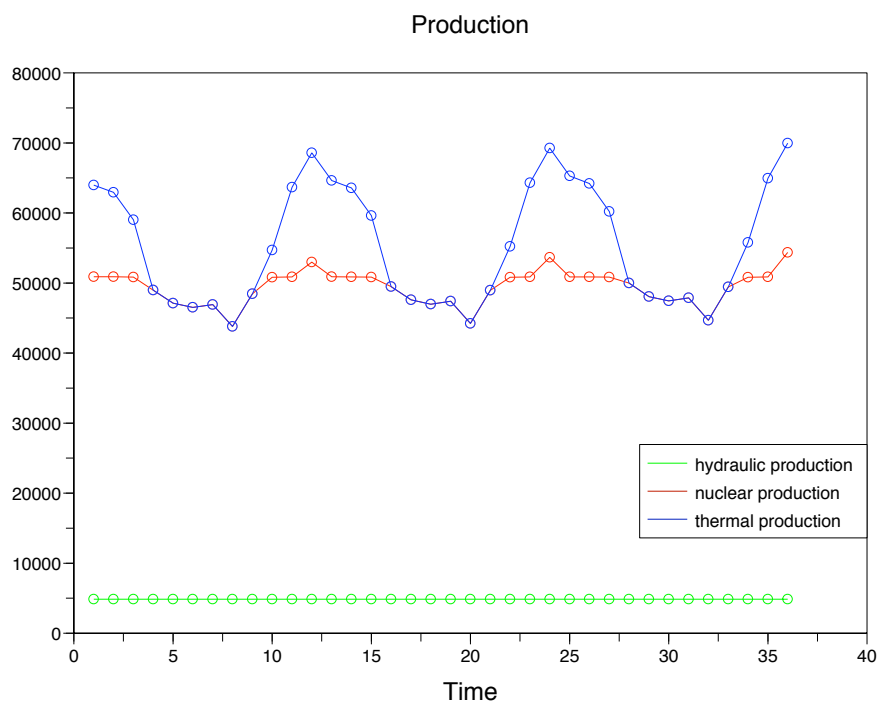


Figure 3.2: Simulated hydro(run-of-river)/nuclear/non-nuclear thermal production (in MW)

We also observe (both graphically and with a numerical test) that the non-nuclear thermal and nuclear production increase (and respectively decrease) simultaneously du-

<sup>5</sup>The maximum non-nuclear thermal production during a month is represented by the white blue dotted line and corresponds to the nominal non-nuclear thermal capacity (including coal, gas, fuel, etc.) of the French set.

ring almost the entire time horizon of our model, which corresponds to the notion of “comonotonicity”<sup>6</sup> introduced by Yaari (1987) (see Figure 3.2). We also deduce, following a theoretical reasoning, that the nuclear production being comonotonic to the non-nuclear thermal production and obviously to itself is comonotonic to the total supply (the sum of nuclear and of non-nuclear thermal production). Consequently, if we take into account the equality between supply and demand, we deduce that the nuclear production is comonotonic to the demand (they increase (and respectively decrease) at the same time). From a theoretical point of view, this result is not trivial and it is particular appealing because: (i) demand is dynamic but is not periodic<sup>7</sup> since we assumed an augmentation of the demand by a rate of 1% per year (see page 123, Subsection 2.4.1, Chapter 2), (ii) there is a third variable which interferes between the demand and the nuclear production and plays a central role as the nuclear production realized every month depends totally on it. This variable represents the level of nuclear capacity available during the month  $t$ ,  $(\sum_{j=1}^J Q_{max}^{j,nuc}(t))$  considering that some unit is inactive during this month (month of reloading). Its evolution over time is periodic as it also appears in figure<sup>8</sup> 3.3. From (i) and (ii) results that despite the fact that the available nuclear capacity is periodic and thus, it repeats its values every year while the demand has an increasing tendency from one year to another, the nuclear production follows constantly the variations of demand i.e. they decrease and increase simultaneously. This shows why the comonotonicity between the demand and the nuclear production is a non-obvious result of particular interest.

## Analytical results

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<sup>6</sup>The vector  $(X_t)_{t=1}^T$  is comonotonic to the vector  $(Y_t)_{t=1}^T$  if  $(X_{t'} - X_t)(Y_{t'} - Y_t) \geq 0$  holds for all  $t, t'$ . It forbids the opposite evolution between two dates for X and Y which means mathematically that it does not exist  $(t, t')$  such that  $X_{t'} > X_t$  and  $Y_{t'} < Y_t$ .

<sup>7</sup>In mathematics, a periodic function is a function that repeats its values in regular intervals or periods. The most important examples are the trigonometric functions, which repeat over intervals of length  $2\pi$  radians. Periodic functions are used throughout science to describe oscillations, waves and other phenomena that exhibits periodicity.

<sup>8</sup>The maximum nuclear production during the month  $t$  given that some unit is inactive during this month (month of reloading) is represented by the purple dotted line. This quantity is obviously below the nominal capacity of the French nuclear set represented by the crossed purple line. The minimum nuclear production during the month  $t$  given that some unit is inactive during this month (month of reloading) is represented by the purple line of asterisks.

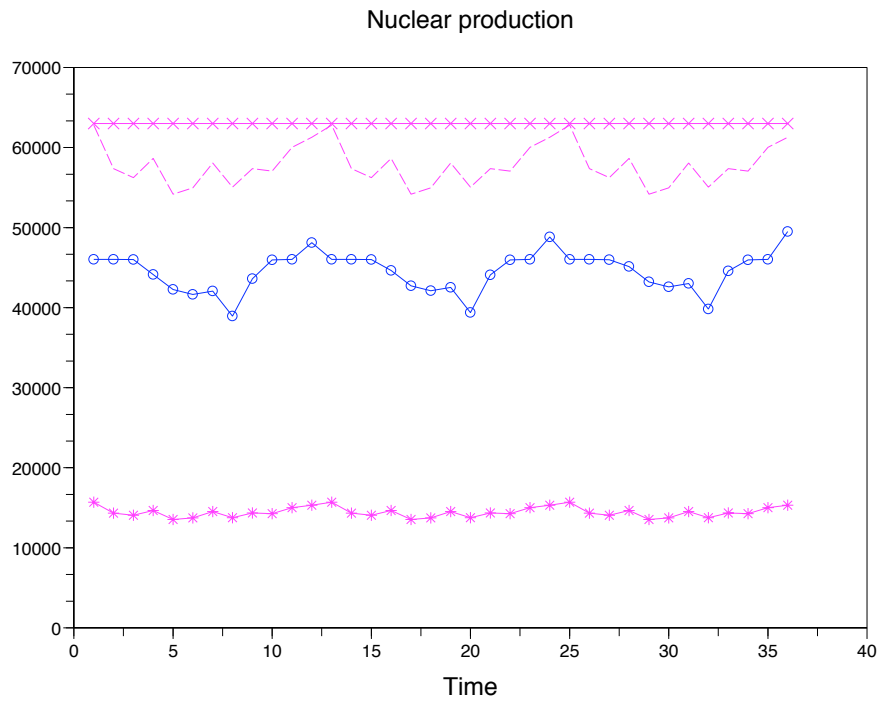


Figure 3.3: Simulated nuclear production (in MW)

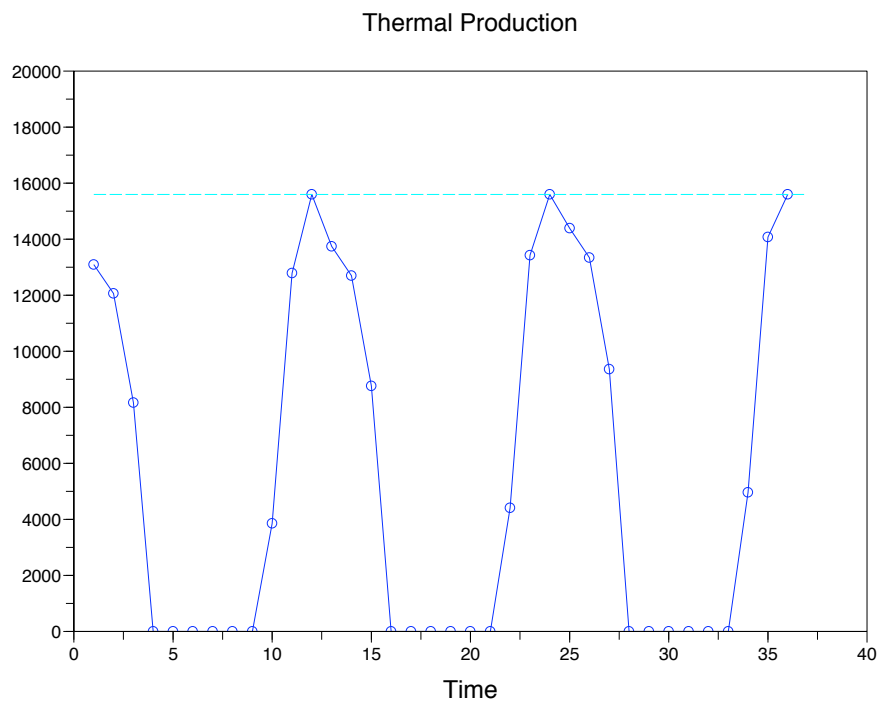


Figure 3.4: Simulated non-nuclear thermal production (in MW)

In figure 3.3, we can see that the nuclear production follows the seasonal variations of demand (high production during winter – low production during summer) but it never

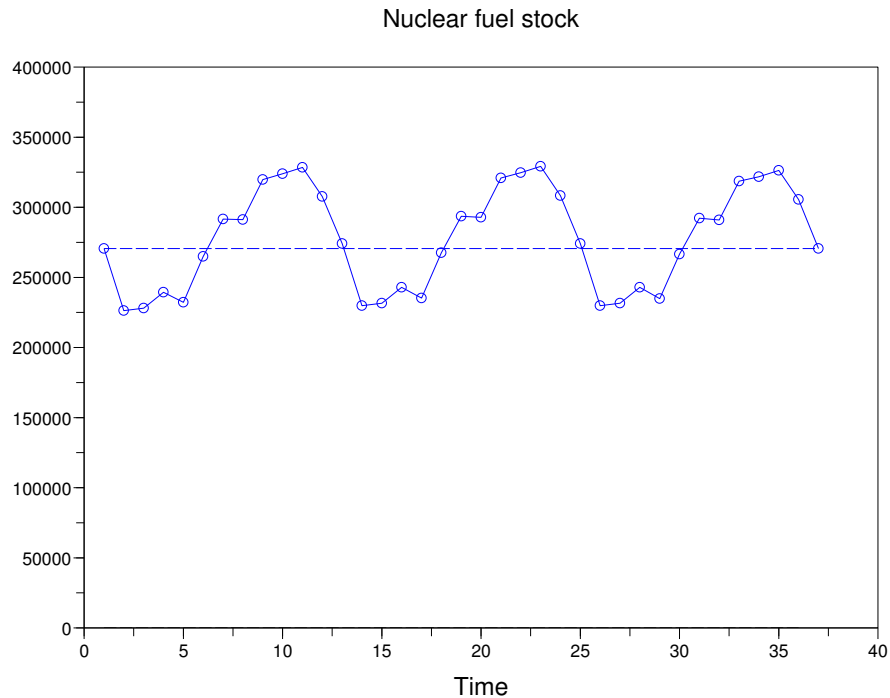


Figure 3.5: Simulated nuclear fuel stock (in MW)

reaches its maximum value<sup>9</sup> over the entire period  $T$ . This evolution of nuclear production means “high” levels of nuclear fuel stock during summer and “low” levels of nuclear fuel stock during winter. Therefore, the essentially periodic evolution of nuclear production leads to a periodic evolution for the nuclear fuel stock. Note that the trend of the stock oscillates around the “stock of reference”<sup>10</sup>. More precisely, the nuclear fuel stock increases during low demand seasons, exceeding the “stock of reference”, while it decreases during high demand seasons falling below the “stock of reference” (see Figure 3.3, Figure 3.5).

Similarly, the non-nuclear thermal production increases during winter and reaches its maximum value<sup>11</sup> when demand peaks in December while it decreases during summer (see Figure 3.4). We also obtain this result from a theoretical thinking using the notion of comonotonicity like we did in the case of nuclear. More precisely, the non-nuclear thermal production is comonotonic to the nuclear production which is itself comonotonic to the demand. Thus, we conclude that the non-nuclear thermal production is essentially

<sup>9</sup>cf. Footnote 8.

<sup>10</sup> The “stock of reference” is represented by the blue dotted line which shows the value of stock at the beginning, being also the value of stock at the end.

<sup>11</sup>cf. Footnote 5.

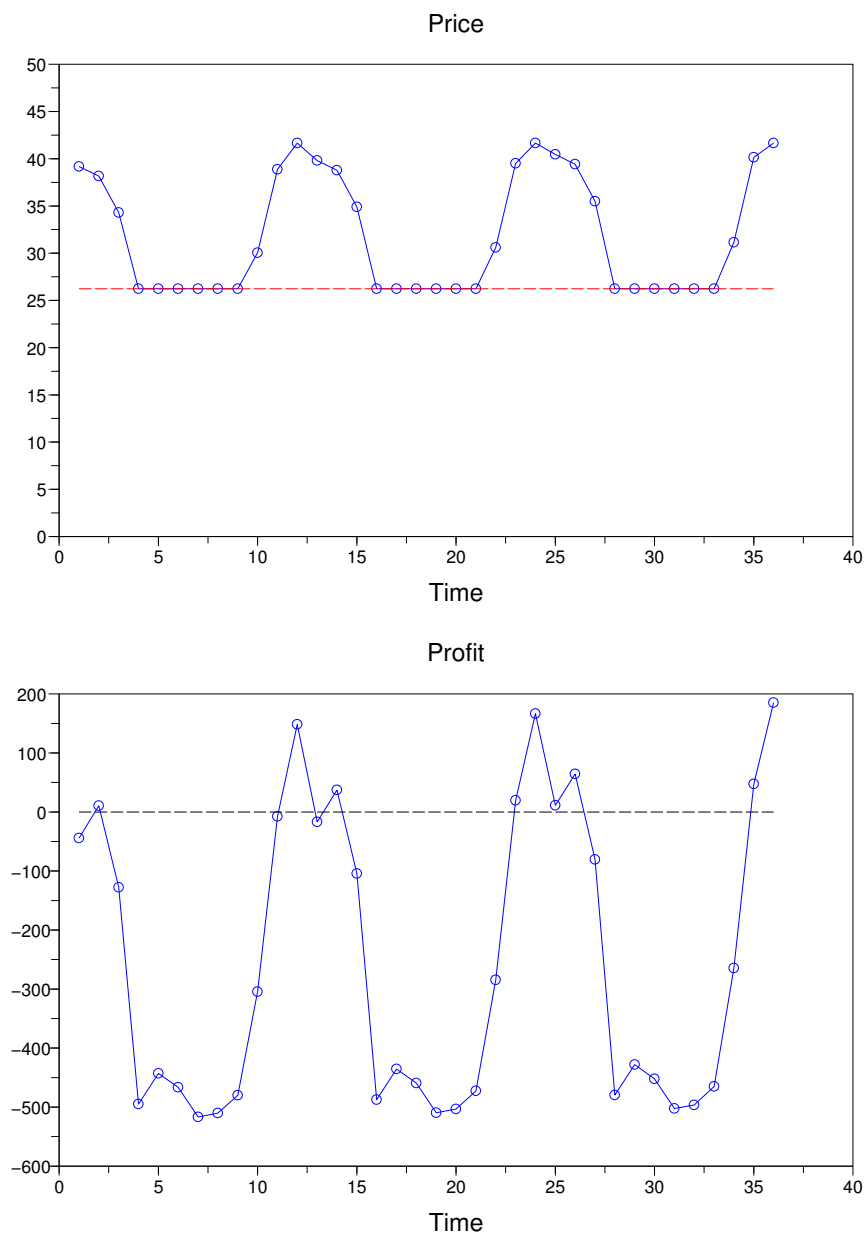


Figure 3.6: Simulated “regularized” price (in Euro/MWh)/Simulated “regularized” profit (omitting profit coming from hydro (run-of-river) generation) (in Euro (million))

comonotonic to the demand. In particular, the non-nuclear thermal production is increasing during winter (beginning from October) until it reaches its peak value in December. Afterwards, it decreases progressively until it takes its lowest value during summer which is a low demand season.

The “regularized” price<sup>12</sup> is high during winter by taking its highest value in December

<sup>12</sup>The red dotted line indicates the level of the “regularized” price when nuclear is the marginal technology.



and relatively low during summer (see Figure 3.6). From a theoretical point of view, this is explained by the fact that the “regularized” price is comonotonic to the non-nuclear thermal production since the price is determined by the non-nuclear thermal marginal cost, which is an increasing function of the non-nuclear thermal production. Additionally to this, in the previous paragraph, we showed that the non-nuclear thermal production is comonotonic to the demand. Hence, the price is basically comonotonic to the demand and, therefore, it follows closely demand’s seasonal variations by increasing during periods of high demand (winter) and decreasing during periods of low demand (summer).

Finally, the inter-temporal profit, being comonotonic to the price within our numerical model, is fundamentally comonotonic to the demand which leads to high profits during winter and at the beginning of spring while lower profits are realized during summer and at the end of spring (see Figure 3.6). We may also observe that its value can break-up in a cyclical component and a linear trend which is slightly increasing (see Figure C.18, Annex C). Once again, the reader should not pay attention to the exact value of profit as it is conditional on the too many approximations we made (euro/dollar, oil prices, CO<sub>2</sub> cost, discount rate, no mark-up rate, absence of profits coming from the hydro technology (run-of-river), etc.) and because our modelling does not take into consideration the importations/exportations of electricity or the production derived from renewable and hydro storage plants.

### **The time period of marginality of nuclear and non-nuclear thermal generation technology**

Simulation results show that the non-nuclear thermal generation technology is marginal during months of high demand while the nuclear technology is marginal during months of low demand. In particular, nuclear stays marginal during almost the entire period of spring and summer (April - September), while non-nuclear thermal is marginal during autumn and winter (October - March) (see Figure 3.2, Figure 3.3, Figure 3.4). Hence, we see that a producer maximizes its inter-temporal profit by running only its nuclear units when the demand is low. Specifically, nuclear is called to meet demand during the half part of the time horizon of the model (18 months over  $T = 36$  months). The high

duration of marginality of nuclear is first met in the optimal solutions deduced from the theoretical resolution of the optimal inter-temporal production problem in the particular case that the production constraints are not saturated. In view of our data, a constant nuclear production determines the price in the market during months of low demand and the length of its period of marginality is almost the same with that resulting from the numerical resolution of the optimal inter-temporal production problem. The time period of marginality of nuclear can be also explained within our numerical model by the fact that the price is given by the non-nuclear thermal marginal cost when nuclear is the marginal technology (via the “regularization” of the merit order price). Consequently, the nuclear production being evaluated in a higher price than its own marginal cost ( $b_{nuc}$ ), is the only generation technology used to satisfy demand during low demand seasons.

### General remarks

In this late stage of optimization of the nuclear production, the inter-temporal optimization of the management of the nuclear fuel reservoir leads to a higher total profit than in the case of the short-term optimization (see Figure 3.6 and Table C.5 of Annex C). However, if we look closely at the value of the inter-temporal profit obtained throughout the period  $T$  within our numerical model, we realize that it is not very significant. In particular, the nuclear profit obtained throughout the time horizon of the model is negative implying that the producer does not cover the fixed costs of nuclear. This does not seem to agree with Spector’s claims that the size of the French nuclear set is considerably below the “optimal size” which makes the nuclear operator beneficiary of a scarcity rent even in a perfect competitive market.

In view of the remark 3.3.2, the solution is unique with respect to the non-nuclear thermal component but bearing in mind the other variables which do not have an impact on the inter-temporal profit the entire solution is not necessarily unique.

Finally, we observe that a variation of the length of the time horizon  $T$  of the model does not lead to different behaviour patterns. The basically periodic evolution of the

nuclear and non-nuclear thermal production is the same during the entire time horizon of the model (e.g. for  $T = 72$ , see Figure 3.7).

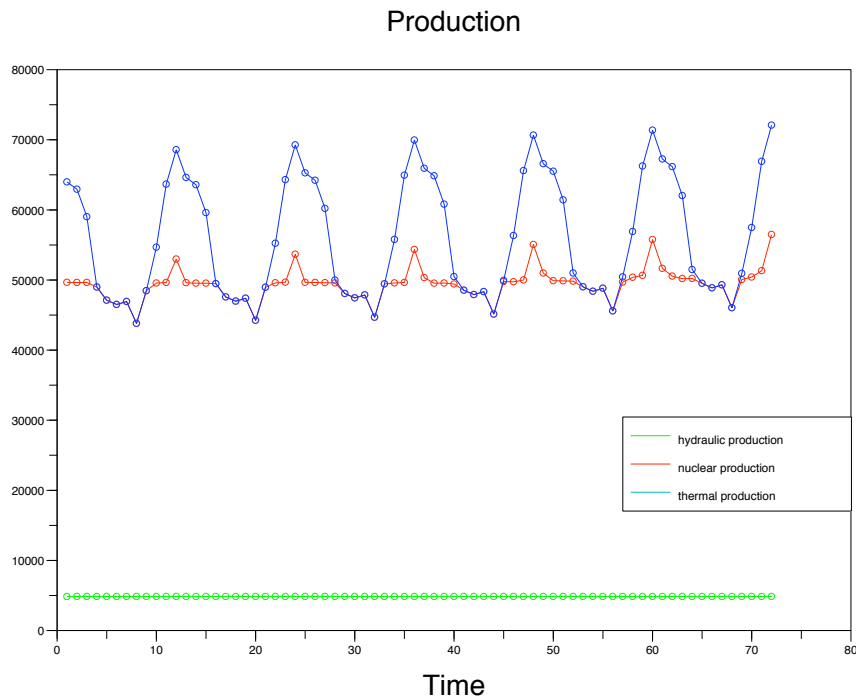


Figure 3.7: Simulated hydro(run-of-river)/nuclear/non-nuclear thermal production (in MW) ( $T=72$ )

### 3.4.4 Optimal inter-temporal production problem VS Optimal short-term production problem

In our previous chapter, we studied the optimal management of a nuclear fuel reservoir in a perfectly competitive market via an optimal short-term production problem, which consists of the optimization of production over a month given the optimal production levels realized in the previous months. In this chapter, we carried on with the inter-temporal optimization based on the direct optimization of production over the entire time horizon  $T$  of the model (36 months). Each optimization problem has been treated within a simple numerical model in which the price is “regularized” (i.e. is determined by the non-nuclear thermal marginal cost when nuclear is the marginal technology). The symmetric approach regarding the “regularization” of these optimization problems makes pertinent the comparison of the two problems.

The optimal short-term production problem provides us with a solution that can be seen as a “local” optimum, since it is optimal within a subset of feasible points. By contrast, the optimal inter-temporal production problem determines a global optimum which is the optimal solution among all the possible solutions. Consequently, from a theoretical point of view, the optimal profit coming from the inter-temporal optimization of production has to be greater than the optimal profit resulting from the short-term optimization of production. Our numerical results show that the value of profit aggregated over the entire period  $T$  is more important when the profit is maximized over the whole time horizon of the model than when it is maximized on a monthly horizon (see Annex C, Table C.5). However, the variation of the profit when we move from the optimal short-term production problem to the optimal inter-temporal production problem (and vice versa) is of the order of 5% which is not significant and is due to the slight variation of the variable cost of production since the initial losses (fixed costs) are identical for both problems (see Annex C, Tables C.5 and C.6). Furthermore, the profit’s value can be decomposed in a cyclical component and a linear trend that increases progressively from one year to another in both optimization problems (see Annex C, Figure C.17, Figure C.18).

An important point of differentiation between the optimal inter-temporal production problem and the optimal short-term production problem consists of the length of the period during which nuclear is the marginal technology and consequently determines the market price. In both numerical models, nuclear is paid at price  $b_{th}$  which is significantly higher than the marginal cost of nuclear  $b_{nuc}$  (see Annex C, Table C.4). This means that no penalty is caused by the exclusive use of nuclear production. Hence, it is profitable for a producer to operate only its nuclear units during periods of low demand. However, we observe in figure 3.8 on page 202 that the non-nuclear thermal technology is marginal during the majority of the months of period  $T$  in the case of the short-term optimization of production (the only exception is the last sub-period when nuclear units “overproduce”). The producer allocates an important amount of nuclear fuel stock during periods of high demand and thus the stock remaining in the reservoir is not sufficient to equilibrate supply and demand throughout low demand periods. Hence, non-nuclear thermal remains almost always the marginal technology. In the optimal inter-temporal production problem, the

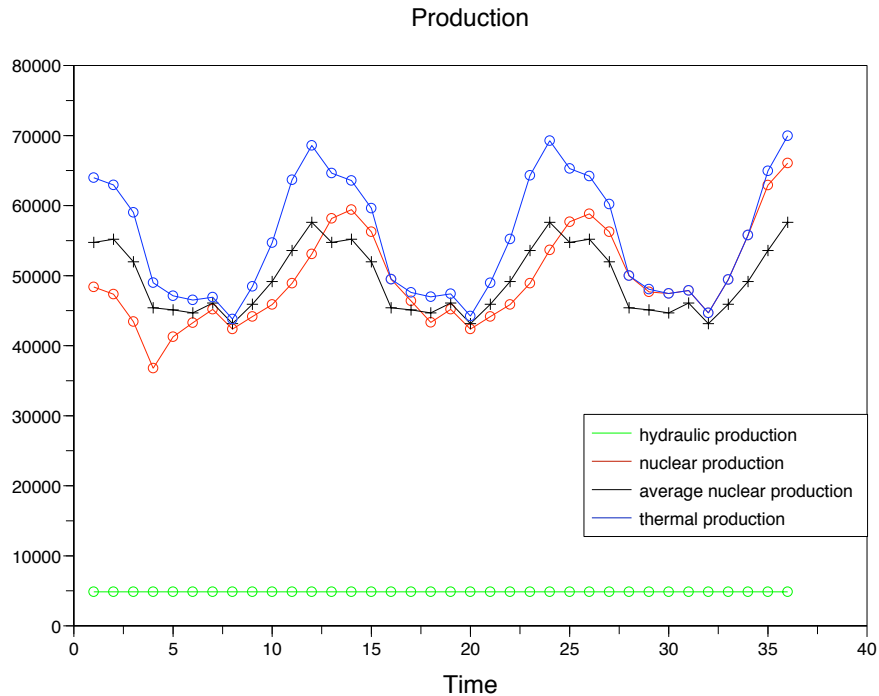


Figure 3.8: Simulated hydro(run-of-river)/nuclear/non-nuclear thermal production (in MW) resulting from the optimal short-term production problem

non-nuclear thermal is marginal only during months of high demand since the nuclear production is sufficient to cover the entire demand during periods of low demand (see Figure 3.2 on page 193). So, we deduce that in the first step of optimization when the producer maximizes its instantaneous profit, the price (respectively the profit) becomes relatively higher during low demand seasons. On the contrary, when the producer looks at the maximization of its inter-temporal profit during the last step of optimization, the price (respectively the profit) is lower during periods of low demand. In the initial stage of optimization when the producer does not know how to reach an equilibrium of the optimal production problem, it searches to determine a possible optimal production path month by month before going to a full optimization of production. In this early stage of optimization, we obtain only a signal regarding the duration of nuclear's marginality. Nevertheless, this signal does not reveal what happens in the final stage of optimization when the producer having determined an equilibrium of the optimal short-term production problem proceeds with the optimal inter-temporal production problem. Indeed, in the final stage, the duration of marginality of nuclear is significantly increased within our numerical model.

Another quite significant point of differentiation regarding the optimal production decisions resulting from the optimal short-term production problem and the optimal inter-temporal production problem is based on the notion of comonotonicity. In the optimal short-term production problem, we observe that nuclear follows the seasonal variations of demand i.e. high levels of nuclear production during winter and low levels during summer. However, we verified through a numerical test that the nuclear production and the demand are not comonotonic meaning that they do not increase (and respectively decrease) at the same time. This could be due to the fact that the available nuclear capacity which determines the level of nuclear production during the time horizon  $T$  of the model is dynamic and periodic while the demand is dynamic but not periodic (it increases from one year to another by a rate of 1% per year within our numerical model). In the case of the optimal inter-temporal production problem however, we do not arrive to the same conclusion. In this problem, it results that despite the periodic evolution of the available nuclear capacity and the non periodic evolution of the demand over time, both the nuclear production and the demand increase and decrease simultaneously and thus they are comonotonic.

A common point between the optimal short-term production problem and the optimal inter-temporal production problem concerns their solutions. More precisely, in view of the remarks 2.3.2 and 3.3.2, we get unicity of solutions with respect to the non-nuclear thermal component in both the optimal short-term production problem and the optimal inter-temporal production problem. However, taking into account the other variables which do not have an effect on the current monthly profit and the inter-temporal profit respectively the full solution is not necessarily unique in each problem. We can also see that, in both optimal production problems, a prolongation of the time horizon of the model  $T$  does not result in different production behaviours (see Figure 3.7 on page 200 and Figure 3.9 on page 204).

We remind that the production levels arising from both “regularized” problems, constitute an “approximate” solution of the corresponding economical problems described in subsections 3.2.1 (optimal inter-temporal production problem) and 2.3.4 (optimal short-term production problem) respectively. Therefore, we do not proceed with an exhaust-

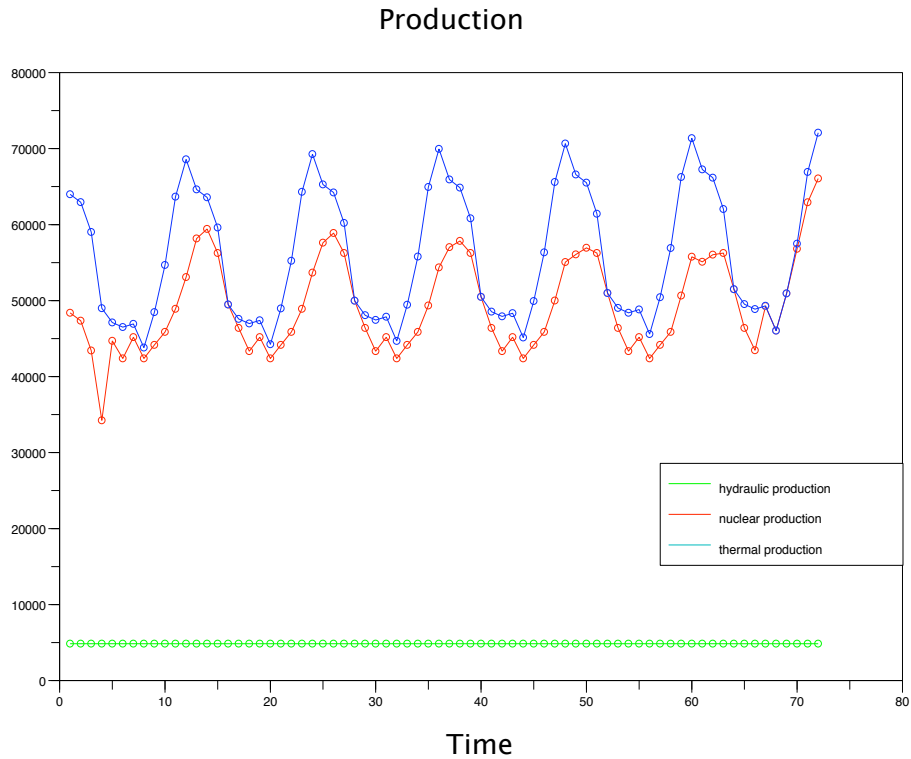


Figure 3.9: Simulated hydro(run-of-river)/nuclear/non-nuclear thermal production (in MW) resulting from the optimal short-term production problem ( $T=72$ )

stive comparison between the production decisions derived from the “regularized” optimal short-term production problem and those resulting from the “regularized” optimal inter-temporal production problem.

### 3.5 Maximization of social welfare

In this section, we study the behaviour of the nuclear set under the maximization of the social welfare in an identical framework. When nuclear dominates the energy mix of a country, then customer needs for electricity are mainly met by the nuclear production. This makes the nuclear set responsible for ensuring the equilibrium in its national electricity market. The constraint of equilibrium between supply and demand, being no doubt inherent in public interest, is already taken into account in the management of flexible market based nuclear. However, in view of the great impact of the production decisions of the dominant nuclear generation technology for the entire national electricity system

and thus for the welfare of the society, one can ask what the management of the nuclear generation set is that maximizes social welfare. This question could be also motivated by the fact that the social acceptability<sup>13</sup> of the nuclear generation technology is never acquired definitively and until now the economic arguments has not given a decisive answer whether or not nuclear has to participate in the energy mix of a country. Therefore, we will study the full optimization of social welfare in place of the maximization of producers own profits given that the non-nuclear thermal capacity as well as the nuclear capacity are exogenous. This problem determines the optimal management of the nuclear fuel reservoir when maximizing social welfare under production and fuel storage constraints and the supply-demand equilibrium constraints within a medium-term horizon. The maximization of social welfare could give some insights with respect to an alternative behaviour of the nuclear production (optimal short-term production behaviour, optimal inter-temporal production behaviour).

### 3.5.1 A property of the “interior” solutions

The maximization of social welfare is an optimization problem which consists in the maximization of the total surplus. Total surplus results from the sum of consumer surplus (denoted by SC) associated with a given level of production and the sum of producer surplus (denoted by SP). Consumer surplus is the difference between the total amount that consumers are willing and able to pay for electricity and the total amount that they actually do pay (electricity evaluated at the market price) (Renshaw (2005)). The surplus of producer is equal to its revenue minus the variable costs or equivalently to the profit increased by the fixed costs (Varian (2006), Renshaw (2005)). Without loss of generality, we may translate producer’s surplus by the fixed costs.

The social welfare maximization problem is

$$\max_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( SC \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) + SP \left( \sum_{j=1}^J q_{jt}^{nuc}, q_t^{th} \right) \right)$$

---

<sup>13</sup>Germany is a typical example of a country which shut-down all its nuclear plants after the Fukushima disaster because nuclear was no more acceptable to society.



Consequently, we have to solve

$$\max_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( \left[ \int_{p_t}^{\infty} D_t(p_t^*) dp_t^* \right] + \left[ p_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th}) \right] \right)$$

where  $D_t(\cdot)$  is the demand function at time  $t$ . We also remind that  $q_{jt}^{nuc} = \sum_{n=1}^N q_{njt}^{nuc}$  is the aggregate nuclear production and  $q_t^{th} = \sum_{n=1}^N q_{nt}^{th}$  is the aggregate non-nuclear thermal production obtained both during month  $t$ . As we can see in the part of the objective function which corresponds to consumer surplus (the indefinite integral of the demand function from the price  $p_t$  to the reservation price), the reservation price of a consumer, i.e. the maximum price that a consumer is willing to pay for electricity, has an infinite value. This is because, in view of the assumption of inelastic demand, the demand function  $D_t(\cdot)$  is constant. We also recall that the price  $p_t$  is given by the equality between supply and demand during the time  $t$ .

However, the formula defining consumer surplus does not make sense in the presence of inelastic demand (infinite value of surplus). In view of this remark, we focus on the variation of consumer surplus. Nevertheless, the infinite value of consumers surplus leads to an indeterminate form of the variation of consumers surplus. In order to avoid this ambiguity, we define explicitly the variation of consumer surplus when the price evolves from  $\bar{p}_t$  which is a level of reference to  $p_t$  by the following formula ( $\Delta$ )

$$\Delta = - \int_{\bar{p}_t}^{p_t} D_t(p_t^*) dp_t^*$$

This definition is coherent with the classical case (finite value of surplus).

We can now start the calculation of the integral to obtain

$$\Delta = -D_t(p_t - \bar{p}_t) = \bar{p}_t D_t - p_t D_t = \bar{p}_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) - p_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) = K_t - p_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right)$$

where  $K_t = \bar{p}_t D_t = \bar{p}_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right)$ . It is obviously a constant that depends on  $t$ .

In view of these remarks, we will maximize the function

$$\sum_{t=1}^T \left( \left[ K_t - p_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) \right] + \left[ p_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th}) \right] \right)$$

Accordingly, the social welfare maximization problem can be written as

$$\max_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( K_t - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th}) \right) \quad (3.17)$$

or equivalently

$$\min_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) + C^{th}(q_t^{th}) \right) \quad (3.18)$$

Therefore, we deduce that the social welfare maximization problem is equivalent to the total cost minimization problem (3.18) (same set of solutions). If the solution of the social welfare maximization problem belongs to the set  $F$  in which the production constraints are not binding, we obtain a property given by the following proposition. Let us notice that the proof of this proposition is similar to the one of Proposition 3.3.3 (see page 167).

**Proposition 3.5.1** *If there exists a solution  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T \in F$  such that the social welfare is maximum on  $C$  then  $\hat{q}_1^{th} = \hat{q}_2^{th} = \dots = \hat{q}_T^{th}$ .*

### Proof

The production vector  $\hat{q} = ((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  solves the equivalent to the social welfare maximization problem, total cost minimization problem (3.18)

$$\min_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) + C^{th}(q_t^{th}) \right)$$

We choose to apply the Karush - Kuhn - Tucker (KKT) conditions in order to determine an optimal solution of the problem (3.18). Let us recall that  $M$  is an affine set, hence

Slater's condition is satisfied. Thus, there exists  $(\bar{\mu}_t)_{t=1}^T \in \mathbb{R}^T$  and  $(\bar{\lambda}_j^k)_{j=1}^J \in \mathbb{R}^{(3 \cdot 2 + 4 \cdot (J-2))}$  such that the KKT conditions are satisfied, where  $\bar{\mu}_t$  denotes the Lagrange multiplier for the supply-demand equilibrium constraint at each month  $t$  and  $\bar{\lambda}_j^k$  is the Lagrange multiplier for the nuclear fuel constraint of the unit  $j$  during the campaign  $k$ . According to the KKT conditions of complementarity and since  $\hat{q} \in F$ , all the Lagrange multipliers associated with the minimum/maximum production constraints are equal to zero and they will be omitted in the Lagrangien function of this problem.

We call  $\bar{L}$  the reduced Lagrangien of the cost minimization problem (3.18)

$$\bar{L}(q) = \sum_{t=1}^T \left( \sum_{j=1}^J (C_j^{nuc}(q_{jt}^{nuc}) + C^{th}(q_t^{th})) - \bar{\sigma}(Dq - E) \right)$$

where  $q = ((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T$  is a production vector belonging to  $C$  and  $\bar{\sigma} = (\bar{\lambda}_j^k, \bar{\mu}_t)$  is the vector of the Lagrange multipliers. We recall from the proof of Proposition 3.3.3 on page 168 that the vector  $E$  and the matrix  $D$  are defined so that  $M = \{q' \text{ s.t. } Dq' - E = 0\}$ .

Following some calculations and with the help of the supply-demand equilibrium constraints, we deduce that the objective function of the cost minimization problem (3.18) is a quadratic function of the non-nuclear thermal production  $q_t^{th}$

$$c_{th} \sum_{t=1}^T (q_t^{th})^2 + d \tag{3.19}$$

where  $d = (\sum_{t=1}^T a_{th} + \sum_{t=1}^T \sum_{j=1}^J a_{nuc}^j + b_{th}((D_t - Q_t^{hyd}) - S_{reload}) + b_{nuc}S_{reload})$  is the constant part of the function.

According to the KKT conditions, we have

$$\frac{\partial \bar{L}}{\partial q_{jt}^{nuc}}(\hat{q}) = 0, \text{ for all } j, t$$

and

$$\frac{\partial \bar{L}}{\partial q_t^{th}}(\hat{q}) = 0, \text{ for all } t.$$

The derivative of Lagrangien with respect to the thermal production  $q_1^{th}$  at month 1 is

$$\frac{\partial \bar{L}}{\partial q_1^{th}}(\hat{q}) = 2c_{th}\hat{q}_1^{th} - \bar{\mu}_1 = 0 \Leftrightarrow \hat{q}_1^{th} = \frac{\bar{\mu}_1}{2c_{th}} \quad (3.20)$$

Symmetrically, the derivative of Lagrangien with respect to the thermal production  $q_2^{th}$  at month 2 is

$$\frac{\partial \bar{L}}{\partial q_2^{th}}(\hat{q}) = 2c_{th}\hat{q}_2^{th} - \bar{\mu}_2 = 0 \Leftrightarrow \hat{q}_2^{th} = \frac{\bar{\mu}_2}{2c_{th}} \quad (3.21)$$

To compare the Lagrange multipliers  $\bar{\mu}_1$  and  $\bar{\mu}_2$ , we focus on a unit that is active during month 1 and 2. The derivative of Lagrangien with respect to the nuclear production  $q_{31}^{nuc}$  of unit 3 at month 1 is

$$\frac{\partial \bar{L}}{\partial q_{31}^{nuc}}(\hat{q}) = -\bar{\mu}_1 - \bar{\lambda}_3^{-1} = 0 \Leftrightarrow \bar{\mu}_1 = -\bar{\lambda}_3^{-1} \quad (3.22)$$

The derivative of Lagrangien with respect to the nuclear production  $q_{32}^{nuc}$  of unit 3 at month 2 is

$$\frac{\partial \bar{L}}{\partial q_{32}^{nuc}}(\hat{q}) = -\bar{\mu}_2 - \bar{\lambda}_3^{-1} = 0 \Leftrightarrow \bar{\mu}_2 = -\bar{\lambda}_3^{-1} \quad (3.23)$$

From the last two equations, we deduce that  $\bar{\mu}_1 = \bar{\mu}_2$ . This means that

$$\hat{q}_1^{th} = \frac{\bar{\mu}_1}{2c_{th}} = \frac{\bar{\mu}_2}{2c_{th}} = \hat{q}_2^{th}$$

By using a unit available at month  $t$  and  $t + 1$ , we obtain  $\bar{\mu}_t = \bar{\mu}_{t+1}$ , which means that  $\hat{q}_t^{th} = \hat{q}_{t+1}^{th}$ . Thus,

$$\hat{q}_1^{th} = \hat{q}_2^{th} = \dots = \hat{q}_T^{th}$$

Consequently, we conclude that a solution of the total cost minimization problem (3.18) which belongs to  $F$  is determined by a non-nuclear thermal production that is

constant during the entire time horizon of our model and a variable nuclear production which entirely follows the seasonal variations of the demand.

□

We remind that  $C$  is a compact set, thus the total cost minimization problem has solutions on  $C$ . Nevertheless, it may not have solutions on the set  $F$  since it is not compact. Hence, the existence of a solution of the problem (3.18) on  $F$  has the form of assumption in Proposition 3.5.1.

### **Economic interpretation of the Lagrange multipliers of the social welfare maximization problem (or the equivalent total cost minimization problem (3.18))**

Since  $\hat{q} \in F$ , the equation (3.20) (respectively (3.21)) implies that the sign of the multiplier  $\bar{\mu}_1$  (respectively  $\bar{\mu}_2$ ) is strictly positive. By a symmetric argument, the Lagrange multiplier  $\bar{\mu}_t$  is strictly positive ( $\bar{\mu}_t > 0$ ) for all  $t$ . Hence, in view of equations (3.22) and (3.23), the multiplier  $\bar{\lambda}_3^1$  (respectively  $\bar{\lambda}_j^k$ ) is strictly negative. Indeed, if an additional unit of nuclear fuel became available for unit  $j$  during campaign  $k$ , the non-nuclear thermal production would decrease which would lead to the augmentation of the nuclear production cost and the diminution of the non-nuclear thermal production cost. However, the second effect that regards the decrease of the non-nuclear thermal production cost is the most important. Consequently, the “additional” cost resulting from an additional nuclear fuel unit and thus the value of the multiplier  $\bar{\lambda}_j^k$  should be negative. The multiplier  $\bar{\lambda}_j^k$  indicates the “marginal value of nuclear fuel stock”, i.e. the additional cost  $|\bar{\lambda}_j^k|$  unit  $j$  would incur if the nuclear fuel stock decreased by one unit during the campaign  $k$ .

Let us now proceed with a proposition which shows that a constant non-nuclear thermal production is a sufficient condition for optimality on  $C$ . The proof of this proposition is similar to the proof of Proposition 3.3.4 (see page 171).

**Proposition 3.5.2** *If  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  is a production vector belonging to  $C$  such that  $\hat{q}_1^{th} = \hat{q}_2^{th} = \dots = \hat{q}_T^{th}$  then  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  is a solution of the social welfare maximization problem (equivalently total cost minimization problem) on  $C$ .*

### Proof

In order to prove Proposition 3.5.2, we need to show that the production vector  $\hat{q} = ((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  is a solution of the total cost minimization problem on  $C$ . Thus, it suffices to show that there exist Lagrange multipliers such that the associated with the problem (3.18) KKT conditions are satisfied at  $\hat{q}$  given that the objective function of this problem is convex. We show the convexity of the total production cost by stating the following Lemma:

**Lemma 3.5.1** *The total production cost function of the total cost minimization problem on  $C$  is convex with respect to  $q$ .*

### Proof of Lemma 3.5.1

The total production cost

$$\sum_{t=1}^T \left( \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) + C^{th}(q_t^{th}) \right)$$

is a quadratic function of the non-nuclear thermal production  $q_t^{th}$  provided by the function (3.19)

$$c_{th} \sum_{t=1}^T (q_t^{th})^2 + d$$

(see proof of Proposition 3.5.1, page 208). We remark that (3.19) is a quadratic function of the form  $g(u) = \alpha(u^T \cdot u) + d$ , where  $\alpha = c_{th}$ . Since  $\alpha > 0$ , the function  $g(u)$  is convex with respect to  $u$ . In view of the strict positivity of the coefficient  $\alpha$ , it is also strictly convex with respect to  $u$ . Thus, the total production cost function is strictly convex with respect to  $q^{th} = (q_t^{th})_{t=1}^T$  and by taking into consideration the other variables  $q^{nuc} = ((q_{jt}^{nuc})_{j=1}^J)_{t=1}^T$ , we conclude that the total production cost function (3.19) is convex with respect to  $q$ .

□

By setting the Lagrange multipliers associated with the minimum/maximum production constraints equal to zero and since the nuclear fuel constraints as well as the supply-demand equilibrium constraints are pure equalities, we deduce that the KKT complementary conditions are satisfied at  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$ . Then, we proceed with the Lagrange multipliers associated with the supply-demand equilibrium constraint in month  $t$  ( $\tilde{\mu}_t$ ) and the nuclear fuel constraints of the unit  $j$  during the campaign  $k$  ( $\tilde{\lambda}_j^k$ ). We set

$$\tilde{\mu}_t = 2c_{th}\hat{q}_t^{th}, \text{ for all } t$$

In view of the constant non-nuclear thermal production ( $\hat{q}_1^{th} = \hat{q}_2^{th} = \dots = \hat{q}_T^{th}$ ), we deduce that  $\tilde{\mu}_1 = \tilde{\mu}_2 = \dots = \tilde{\mu}_T$ . Hence, we set

$$\tilde{\lambda}_j^k = -\tilde{\mu}, \text{ for all } j, k$$

where  $\tilde{\mu}$  is the common value of the multipliers  $(\tilde{\mu}_t)_{t=1}^T$ .

For those multipliers the Lagrangien function of the total cost minimization problem on  $C$  has the following form:

$$\bar{H}(q) = \sum_{t=1}^T \left( \sum_{j=1}^J (C_j^{nuc}(q_{jt}^{nuc}) + C^{th}(q_t^{th})) - \tilde{\sigma}(Dq - E) \right)$$

where  $\tilde{\sigma} = (\tilde{\mu}_t, \tilde{\lambda}_j^k)$ . We can see then that the Lagrangien  $\bar{H}$  coincides with the reduced Lagrangien  $\bar{L}$  of the total cost minimization problem (3.18).

In view of the analysis of (3.20) respectively (3.22) and by a symmetric argument, we realize that

$$\frac{\partial \bar{L}}{\partial q_t^{th}}(\hat{q}) = 0, \text{ for all } t$$

respectively

$$\frac{\partial \bar{L}}{\partial q_{jt}^{nuc}}(\hat{q}) = 0, \text{ for all } j, t.$$

Consequently, the production vector  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  is a solution of the KKT conditions associated with the total cost minimization problem determined on  $C$ . In addition, KKT conditions provide us with solutions that minimize the total production cost on  $C$ , since the objective function (total production cost function) is convex. Thus, a production vector  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  of the set  $C$  that is characterized by a constant non-nuclear thermal production is a solution of the total cost minimization problem on  $C$ .

□

**Remark 3.5.1** *The strict convexity of the total production cost function with regard to the non-nuclear thermal production  $q^{th}$  (see Proof of Lemma 3.5.1 on page 211) implies the unicity of solutions with respect to the non-nuclear thermal component. Nevertheless, if we consider the other variables which do not influence the total production cost function and in light of the proof of Lemma 3.5.1 the total production cost function is convex with regard to  $q$  which does not mean necessarily that the entire solution is unique.*

### Economic analysis of Proposition 3.5.1 and of Proposition 3.5.2

In view of Propositions 3.5.1 and 3.5.2 (pages 207, 211), we conclude that in the absence of binding production constraints, a constant non-nuclear thermal production is a characteristic property of solutions of the social welfare maximization problem. On the contrary, the nuclear production, which is the first that is called to satisfy demand (according to the merit-order rule), is adjusted fully to the seasonal variations of demand. This is a result of the behaviour of producers who use the non-nuclear thermal capacity to produce the same quantity every month in order to meet demand. Note that it means that the amplitude of demand has to be smaller than the nuclear capacity so that the equality between supply and demand is respected each month (more precisely, the amplitude of demand has to be inferior than the amplitude of nuclear production that can be realized  $[\max((\sum_{j=1}^J Q_{max}^{j,nuc}(t))_{t=1}^T) - \min((\sum_{j=1}^J Q_{min}^{j,nuc}(t))_{t=1}^T)]$  given our numerical modelling (see page 91)). Furthermore, this property signifies that non-nuclear thermal is the marginal technology even during seasons of low demand. Consequently, prices are determined permanently by the marginal cost of fossil fuel technologies and hence, they stay constant during the entire time horizon of the model.



### 3.5.2 Numerical Illustration

In this section, we study the nuclear and non-nuclear thermal production levels as well as the storage levels resulting from the social welfare maximization problem, within a simple numerical model solved by using Scilab. Let us notice that the data used in this numerical model is the same as the one used in section 3.5.

#### Simulation results

In our numerical model, we maximize social welfare (equivalently we minimize total production cost) on the entire set of feasible solutions  $C$ .

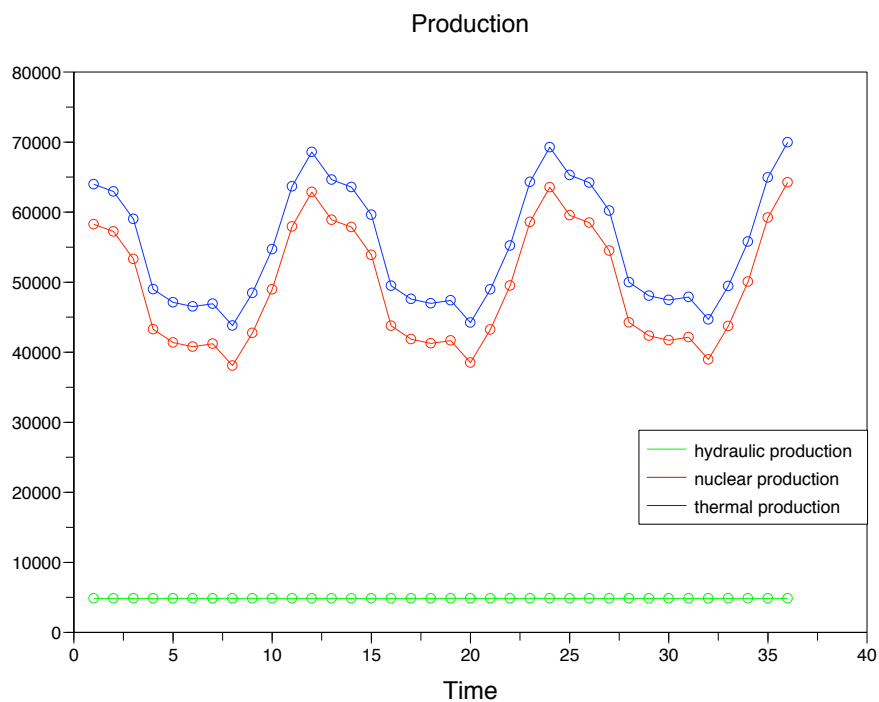


Figure 3.10: Simulated hydro(run-of-river)/nuclear/non-nuclear thermal production (in MW)

Simulation results obtained by our numerical model show that nuclear follows entirely the seasonal variations of demand by decreasing during summer and increasing during winter while non-nuclear thermal has a constant level of production during the entire time horizon of our model. The amount of nuclear capacity is such that the amplitude

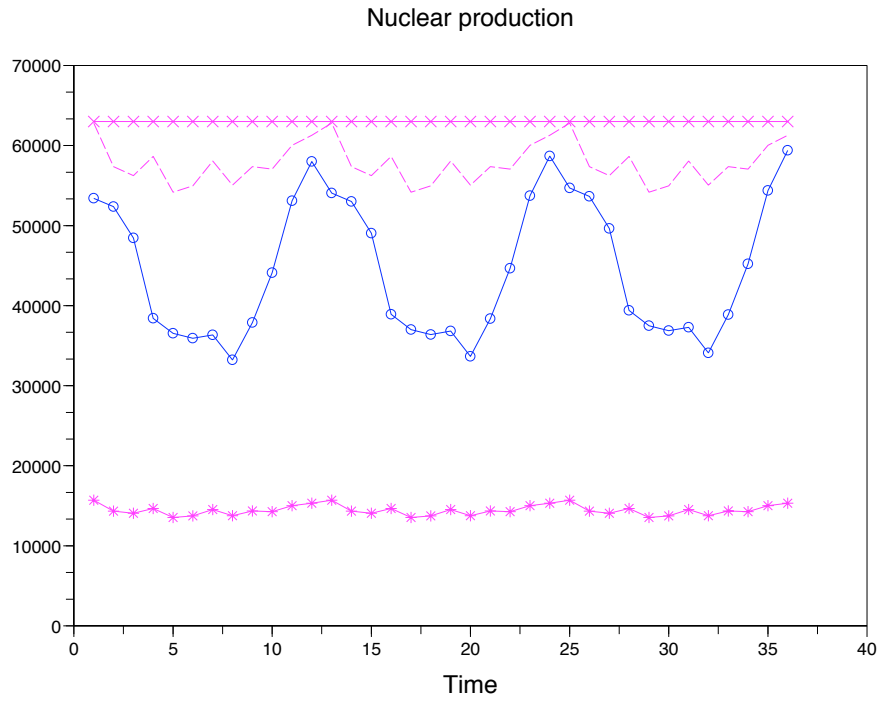


Figure 3.11: Simulated nuclear production (in MW)

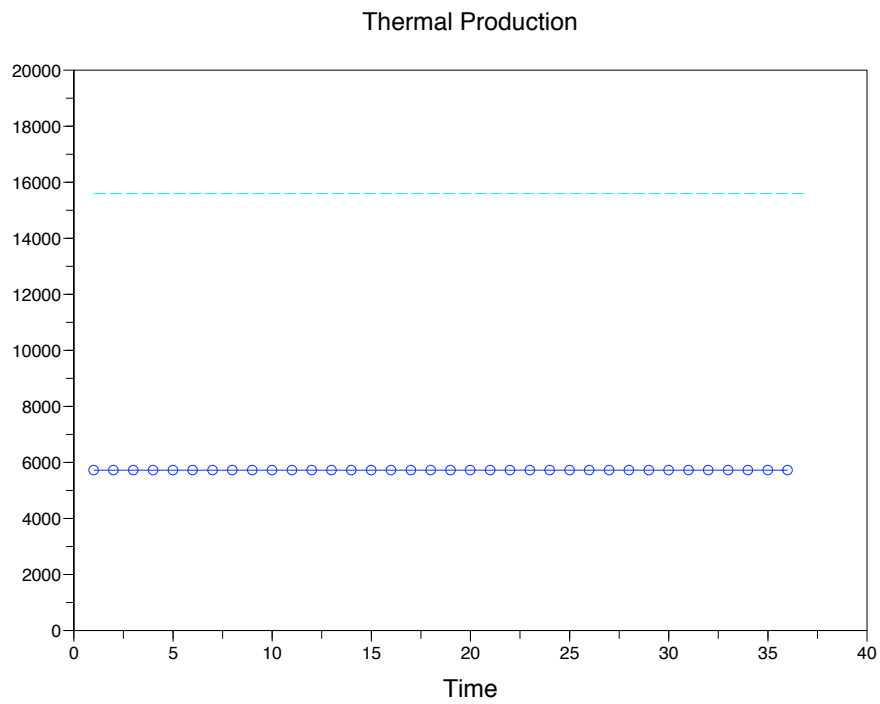


Figure 3.12: Simulated non-nuclear thermal production (in MW)

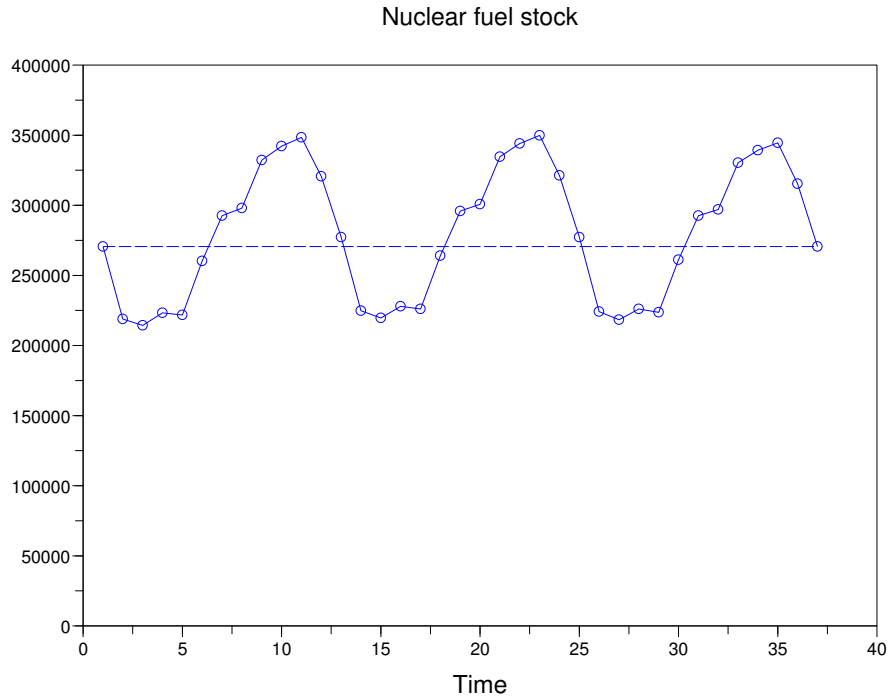


Figure 3.13: Simulated nuclear fuel stock (in MW)

of demand does never exceed it within our numerical model which means that in view of the constant level of non-nuclear thermal production, the equality between supply and demand is always respected with only nuclear operated in load-following mode. Thus, the non-nuclear thermal remains the marginal technology during the entire time horizon of the model while the nuclear technology is never marginal, even during months of low demand (see Figure 3.10, Figure 3.11, Figure 3.12). We also observe that both nuclear and non-nuclear thermal production do not saturate the minimum/maximum nuclear<sup>14</sup> and non-nuclear thermal<sup>15</sup> production constraints. Consequently, we verify (through a numerical test too) that the numerical solution described in this section belongs to  $F$ , therefore our simulation results are in accordance with Proposition 3.5.1 presented on page 207 seeing that the non-nuclear thermal component of this solution is constant. We also deduce that our numerical results are in line with Proposition 3.5.2 appeared on page 211 since a production vector which belongs to  $C$  and is characterized by a constant non-nuclear thermal production constitutes a solution of the social welfare maximization problem within our numerical model.

<sup>14</sup>cf. Footnote 8.

<sup>15</sup>cf. Footnote 5.

The essentially periodic evolution of the nuclear production results in “high” levels of nuclear fuel stock during summer and “low” levels of nuclear fuel stock during winter because of the seasonality that characterizes the variations of the nuclear production (high production during winter – low production during summer). Consequently, we observe a periodic evolution for the nuclear fuel stock as well as an oscillation around the “stock of reference”<sup>16</sup> (see Figure 3.11, Figure 3.13).

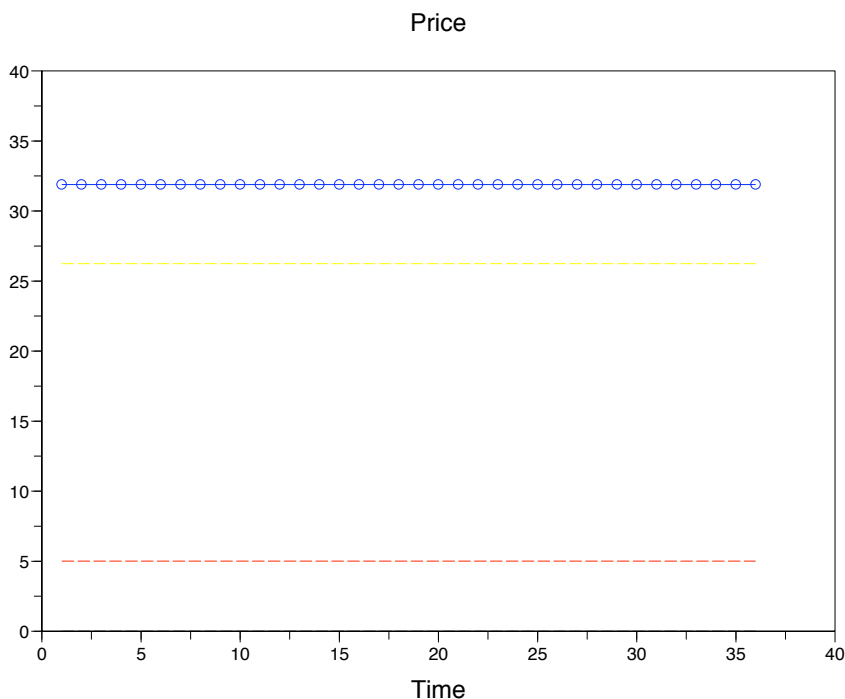


Figure 3.14: Simulated price (in Euro/MWh)

In view of the merit order price rule, the price<sup>17</sup> is determined by the non-nuclear thermal marginal cost since non-nuclear thermal is permanently the marginal technology (see Figure 3.14). Additionally to this, we can see that the price is constant during the entire period  $T$  because non-nuclear thermal is characterized by a constant level of production which leads to a constant non-nuclear thermal marginal cost.

The evolution of total cost is almost periodic and its value increases during winter when we observe high levels of nuclear production and decreases during summer when we notice low levels of nuclear production (see Figure 3.15). As expected, the values of total

<sup>16</sup>cf. Footnote 10.

<sup>17</sup>The red (respectively yellow) dotted line indicates the price level when nuclear (respectively non-nuclear thermal) is the marginal technology.

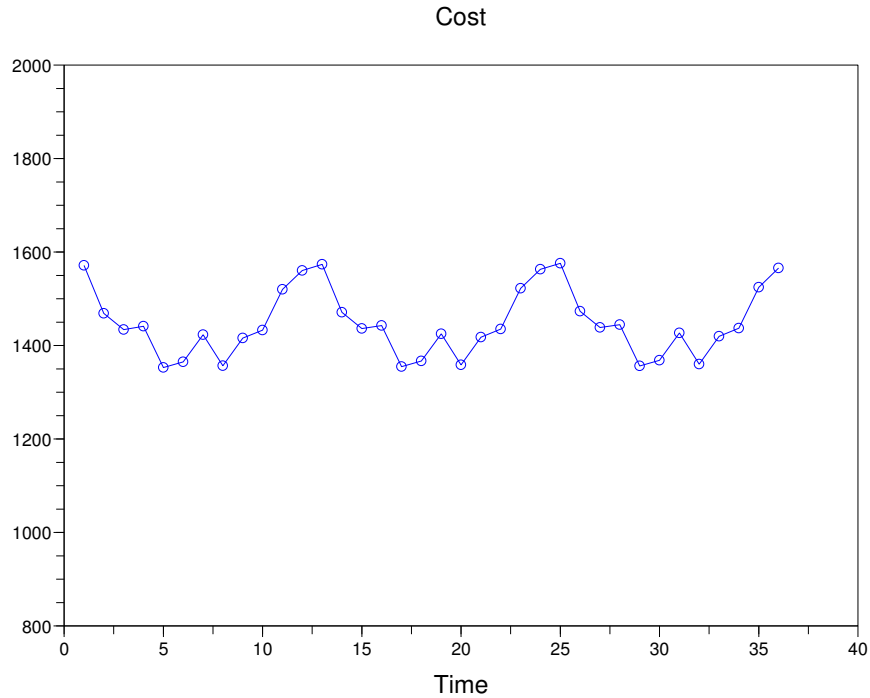


Figure 3.15: Total cost (in Euro (million))

cost (respectively total variable cost) resulting from the optimal short-term production problem and the optimal inter-temporal production problem are higher than the optimal value of total cost (respectively total variable cost) determined in this section (see Annex C, Table C.6). Equivalently, we can say that the values of social welfare obtained in the optimal short-term production problem and the optimal inter-temporal production problem are lower than the optimal value of social welfare given by the resolution of the social welfare maximization problem. To end, we notice that the total cost (respectively total variable cost) resulting from the optimal inter-temporal production problem is relatively higher than the total cost (respectively total variable cost) coming from the optimal short-term production problem. This implies that the value of social welfare is lower when we maximize the inter-temporal profit where we determine a global optimum of the optimal production problem than when we maximize the current monthly profit where we look at “local” solutions of the optimal production problem.

## General remarks

In view of the remark 3.5.1, the solution is unique with respect to the non-nuclear thermal component but considering the other variables which do not act on the total cost the whole solution is not definitively unique.

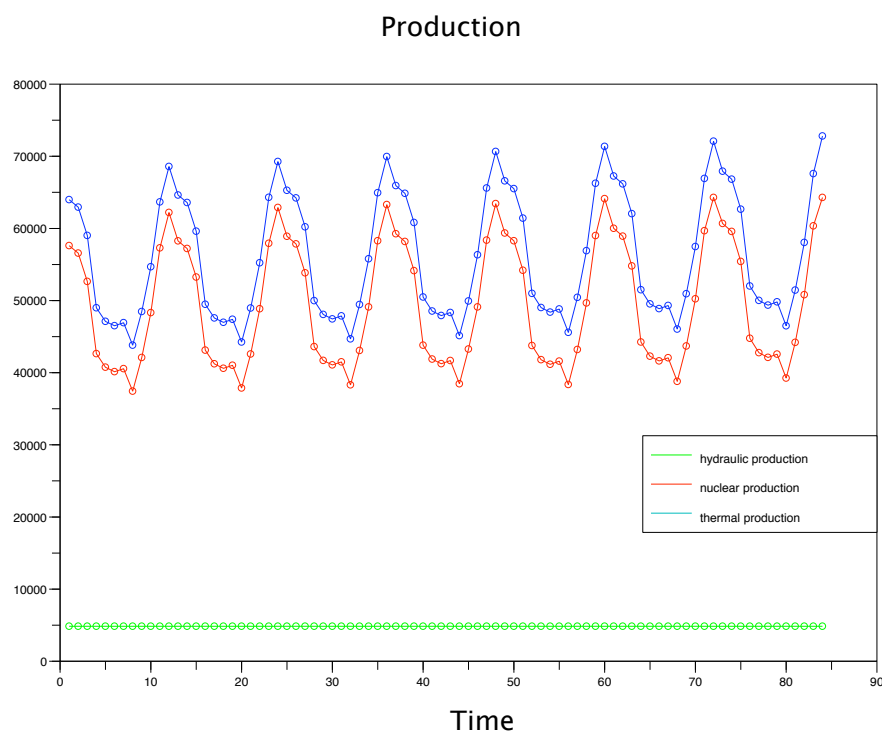


Figure 3.16: Simulated hydro(run-of-river)/nuclear/non-nuclear thermal production (in MW) ( $T=84$ )

Let also us notice that despite the possible modifications of the length of the time horizon  $T$  of the model, we do not obtain different production patterns. Therefore, we do not observe any changes in the periodic evolution of the nuclear and non-nuclear thermal production in the case of a longer time horizon (e.g. for  $T = 84$ , see Figure 3.16).

### 3.5.3 Social welfare maximization problem VS Optimal inter-temporal production problem

In this section, we compare the theoretical and numerical results coming from the resolution of the optimal inter-temporal production problem and the social welfare maximization problem. In the first optimization problem, we determine the production levels that maximize the inter-temporal profit of a producer under production and storage constraints. Particular interest is given to constraints inherent to public interest and social welfare like those imposed by the equality between supply and demand at each month even in a competitive market. These constraints are due to the important size of the french nuclear set which ensures the majority of the domestic demand. The second optimization problem however, gives priority to production decisions that fully maximize social welfare -and not only producers profit- under identical constraints. Here, we take into account that the production decisions of a very large nuclear set have a very significant effect on the equilibrium of the whole national electricity system and consequently on the welfare of the society.

From a theoretical point of view, we find that the optimal production behaviour deduced from the inter-temporal profit maximization problem is diametrically opposite to the optimal production behaviour resulting from the social welfare maximization problem. More precisely, when the minimum and maximum production constraints are not saturated, we observe that the solutions of the optimal inter-temporal production problem are fully characterized by a constant nuclear production while the solutions of the social welfare maximization problem are entirely characterized by a constant non-nuclear thermal production. So, producers maximize their inter-temporal profit by using nuclear as a baseload generation technology that produces always at a steady rate leaving non-nuclear thermal to follow-up the seasonal variations of the demand. On the contrary, social welfare is maximized through a totally flexible management of nuclear production which is adapted completely to demand's seasonal variations resulting from a constant non-nuclear thermal production that covers the residual demand every month. Nevertheless, in both problems, the non-nuclear thermal is always the marginal technology, i.e. the generation technology that determines the market price over the entire time horizon of the

model. Furthermore, in view of the remarks 3.3.2 and 3.5.1, we have unicity of solutions with respect to the non-nuclear thermal component in both the optimal inter-temporal production problem and the social welfare maximization problem. However, taking into consideration the other variables which do not impact the inter-temporal profit and the social welfare (total cost) respectively the total solution is not automatically unique in each problem.

Numerically, the comparison of the optimal production behaviour between the inter-temporal profit maximization problem and the social welfare maximization problem does not provide exactly the same conclusions since the optimal inter-temporal production behaviour is not identical with the one resulted from the theoretical resolution of the problem. In our numerical model, the optimal production deduced from the “regularized” optimal inter-temporal production problem is such that both nuclear and non-nuclear thermal production follow the variations of demand in the medium-term. Optimal solutions of the problem that are characterized by a constant nuclear production do not exist. On the contrary, the optimal solutions derived from the numerical resolution of the social welfare maximization problem are not differentiated from those coming from the theoretical resolution of this problem since in both cases non-nuclear thermal remains constant during the whole period  $T$  and hence nuclear follows entirely the seasonal variations of demand. Consequently, in both numerical problems, nuclear units realize a load-following operation. Non-nuclear thermal units adapt their production output to follow-up load in the inter-temporal profit maximization problem while they produce the same quantity to meet demand every month in the social welfare maximization problem. Regarding the marginality duration of nuclear, we notice that it is significantly increased in the optimal inter-temporal production problem. More precisely, nuclear is the marginal technology during periods of low demand, i.e. during the half of our model’s period  $T$ . This is because of the “regularization” of the merit order price which leads to a higher evaluation of the nuclear production when nuclear is the marginal technology. However, the numerical results derived from the maximization of social welfare show that the non-nuclear thermal is the marginal technology during the total time horizon of our model. We also remark that, in both optimal production problems, the behaviour of producer does not vary with the length of the time horizon of the model  $T$  (see Figure 3.7 on page 200 and Figure 3.16



on page 219). Finally, the remarks 3.3.2 and 3.5.1 lead us to the same conclusions as in the theoretical part regarding the unicity of solutions of these optimization problems.

At this point, it should be noted the paradoxical nature of both theoretical solutions which consists of not modulating production (non-nuclear thermal and nuclear respectively) over the whole time horizon  $T$ . It shows clearly the importance of the choice of capacity (missing here because capacities are exogenous variables within our model) in order to avoid to build and maintain unused capacity taking into consideration the fixed costs of generation technologies. From a theoretical point of view, in the social welfare maximization problem, if the level of nuclear capacity is sufficiently high, we obtain a constant non-nuclear thermal production since nuclear production can adjust fully to the seasonal variations of demand without exceeding its minimum and maximum production levels. In our numerical model, a decrease of 47% in nuclear capacity (compensated by the non-nuclear thermal capacity) would impose to modulate the non-nuclear thermal production in order to balance supply and demand and respect at the same time the production constraints. Similarly, from a theoretical perspective, in the optimal inter-temporal production problem, under the assumption that the amount of non-nuclear thermal capacity is sufficiently important, the non-nuclear thermal units load-follow without violating the minimum and maximum production constraints, thus leaving the nuclear units to produce at a constant rate. However, lower levels of non-nuclear thermal capacity would result in a modulation of nuclear production so that the supply-demand equilibrium constraints and the production constraints are satisfied. Indeed, in our numerical example, we remark that the level of non-nuclear thermal capacity, being lower than the amplitude of demand, results in a modulation of both nuclear and non-nuclear thermal production in order to satisfy the equality between supply and demand.

### 3.6 Conclusion

In this chapter, we examined the optimal inter-temporal management of a flexible nuclear generation set in a perfectly competitive regime. Once again, we focused on a medium-term horizon to take into account the seasonal variation of the demand between winter

(high demand) and summer (low demand). As we mentioned in our previous chapter, the nuclear fuel functions as a “reservoir”, which allows different allocations of the nuclear production during the different seasons of a campaign of production. In this chapter, the problem of the optimal management of the nuclear reservoir consists of maximizing the value of profit over the entire time horizon of the model and it leads to the determination of a global optimum of the optimal production problem. In section 3.2, we proposed a deterministic multi-period model to study the optimal inter-temporal production behaviour in a perfect competitive market with  $N \geq 2$  producers who dispose both nuclear and non-nuclear units.

In section 3.3, we proceeded with the inter-temporal optimization of production under production and storage constraints as well as constraints imposed by the equality between supply and demand each month. We showed that, under some assumptions that hold within our model and for our data, the decrease of price when nuclear becomes the marginal technology induces a decrease of the inter-temporal profit of a producer (Proposition 3.3.2, page 163). Therefore, the maximization of the inter-temporal profit is realized on the subset  $F^{th}$  of the set  $C$  of feasible solutions because its production vectors are characterized by strictly positive levels of non-nuclear thermal production. The corollary 3.3.1 on page 166 showed the equivalence between the inter-temporal profit maximization problem determined on the entire set  $C$  and that determined on the set  $F^{th}$  in the sense that both optimal production problems have the same set of solutions and the same value. Then, we looked at a property of the “interior” solutions of the optimal inter-temporal production problem i.e. solutions that do not saturate the minimum and maximum production constraints. From a theoretical point of view, we proved that these solutions are fully characterized by a constant nuclear production while the non-nuclear thermal production is used to meet demand and hence it adjusts entirely to its seasonal variations (Proposition 3.3.3, page 167 and Proposition 3.3.4, page 171).

In the following section (section 3.4), we carried on with the resolution of the optimal inter-temporal production problem within a simple numerical model. We first showed that the equilibrium of an economy with  $N \geq 2$  producers and the equilibrium of an auxiliary economy with one aggregate producer ( $N = 1$ ) are equivalent (Proposition

3.4.1, page 176). We used this proposition in order to simplify the numerical resolution of the optimal inter-temporal production problem by decreasing the number of optimization variables and operational constraints. Therefore, we focused on the economy with a unique producer who holds the full capacity of both the nuclear and the non-nuclear thermal units. Economically, we deduced from this proposition that, in terms of production, a qualitative comparison between the decentralized and the centralized economy has no sense at the optimum since the optimal solutions of both economies are equivalent.

Then, we studied an economical property of the optimal inter-temporal production behaviour which is the symmetry of equilibrium of the optimal inter-temporal production problem. Under the assumption of symmetry of capacity (nuclear and non-nuclear thermal) among producers, we proved that the equilibrium of the optimal inter-temporal production problem is “almost” symmetric. More precisely, we proved that its non-nuclear thermal component is symmetric while its nuclear component is symmetrisable (Proposition 3.4.4, page 186). In view of this property, a focus on the symmetric nuclear component of the equilibrium leads to an equilibrium characterized by symmetric optimal nuclear and non-nuclear thermal production levels. Thanks to this remark, we could diminish the number of optimization variables of the optimal inter-temporal production problem to simplify its numerical resolution. However, we showed via a numerical example that the nuclear component of the equilibrium can be also asymmetric which results in an asymmetric equilibrium of the optimal inter-temporal production problem. To end, an interesting economical feature that resulted from the symmetrisability of an equilibrium is that the profit of a symmetrisable equilibrium is symmetric.

In our numerical model, we dealt with the discontinuity (decrease) of price, which under some assumptions is responsible for the discontinuity (decrease) of the inter-temporal profit when nuclear generation becomes marginal, through the “regularization” of the merit order price. The price “regularization” leads to an alternative problem (continuous problem) different from the economical problem (discontinuous problem presented in subsection 3.2.1) by its objective function (profit function). This is because the price is now determined by the non-nuclear thermal marginal cost even when nuclear is the marginal technology. Nevertheless, as we proved, the value of both optimization problems (the

“regularized” problem and the economical problem) is identical which permits to obtain a “good” approximation of our economical problem via the “regularized problem” (see Annex B, Proposition B.1). Moreover, we deduced from Proposition B.7 that the set of solutions of the economical problem is empty since the solution obtained by the “regularized” problem does not belong to the set  $F^{th}$  within our numerical model (see Annex B, Proposition B.7). The solution of the “regularized” problem constitutes only an “approximate” solution of our economical problem.

After that, we provided an analytical description of the numerical solution. First, we noticed that solutions that are fully characterized by a constant nuclear production do not exist within our numerical model because the non-nuclear thermal capacity being inferior than demand’s amplitude is not sufficient to cover the demand every month. Therefore, the nuclear production has also to adjust to demand’s seasonal variations to ensure the supply-demand equilibrium. We found high levels of nuclear production during months of high demand (winter) and low levels during months of low demand (summer). As expected, this leads to low levels of nuclear fuel stock during winter and high levels of nuclear fuel stock during summer. The evolution of the non-nuclear thermal production is similar with that of nuclear production as we observed important quantities of non-nuclear thermal production during winter and negligible (zero) quantities during summer.

The duration of marginality of nuclear is significant since the nuclear production covers the total monthly demand through the half of the time horizon  $T$  (18 months) and particularly in summer when the demand is low. However, a producer has to take into account the thermal non-nuclear generation capacity during high demand periods to ensure the equilibrium between supply and demand. Therefore, we observed higher prices during winter and lower prices during summer. In the same way, producer’s profit increases during winter and decreases during summer. In view of our data, an important time period of marginality of nuclear is originally observed in the theoretical solutions of the optimal inter-temporal production problem that are characterized by a constant nuclear production in the absence of binding production constraints. It may be also justified by the fact that the producer takes into account that the nuclear production is no more penalized by a low price given the “regularization” of the merit order price within our numerical model.

We showed that, within our numerical model, the evolution of nuclear and non-nuclear thermal production as well as the evolution of price and of profit with respect to the demand can be also deduced by using the notion of comonotonicity. More precisely, we found that the nuclear production is comonotonic to the demand meaning that it decreases and increases simultaneously with the demand. This result becomes even more interesting when we observe that the available nuclear capacity evolves periodically over time while the demand does not since we assumed an augmentation of the demand level by a positive rate per year. We also proved that the non-nuclear thermal production, the price and the profit are essentially comonotonic to the demand.

Numerically, the results of the “regularized” optimal inter-temporal production problem were contrasted with the results of the “regularized” optimal short-term production problem. We did find that the optimal value of total profit resulting from the inter-temporal profit maximization problem is greater than the one coming from the short-term profit maximization problem. This is because in the first problem we find a global optimum of the optimal production problem while the second problem gives a solution that can be seen as a “local” optimum. Moreover, a comparison between the optimal production levels resulting from these optimization problems showed that the period during which nuclear is marginal is significantly shorter in the earliest stage of optimization (optimal short-term production problem) than in the latest stage of optimization (optimal inter-temporal production problem) despite the fact that the exclusive use of nuclear production is no more penalized in both stages. The producer uses practically all the time the non-nuclear thermal production to maximize its current monthly profit driving this way prices (respectively profits) in relatively higher levels during low demand seasons. This is not the case when the producer wants to maximize its inter-temporal profit which leads to use only the nuclear production when the demand is low and this results in lower prices (respectively lower profits). An additional difference is based on the feature of comonotonicity which does not characterize the evolution of production (nuclear and non-nuclear thermal), of price and of profit with regard to demand in the current monthly profit maximization problem while it does in the inter-temporal profit maximization problem.

In the last section, we studied the social welfare maximization problem in exactly the same setting. First, we deduced that the social welfare maximization problem is equiva-

lent to the total cost minimization problem within our model (same set of solutions) in a framework of inelastic demand. Then, we proved that in the absence of binding production constraints a constant non-nuclear thermal production is a property that fully characterizes the solutions of the total cost minimization problem (equivalently the solutions of the social welfare maximization problem). As a result the nuclear production (being the first generation technology that is called to meet demand) is adapted totally to the seasonal variations of demand in order to respect the supply-demand equilibrium constraint every month.

Theoretically, the optimal production behaviour resulting from the social welfare maximization problem is very different from the one derived from the optimal inter-temporal production problem. Indeed, under the assumption that the production constraints are not saturated (implying that capacities are significant), the solutions that maximize social welfare are entirely characterized by a constant non-nuclear thermal production while the solutions that maximize the inter-temporal profit of producers are fully characterized by a constant nuclear production. Therefore, in the first problem, if the nuclear capacity is sufficiently important then nuclear can follow totally the variations of demand and in the second problem, if the non-nuclear thermal capacity is sufficiently high, non-nuclear thermal can adapt to demand's variations so that customers requirements for electricity are always satisfied in both cases. Obviously, non-nuclear thermal is the marginal technology during the entire time horizon  $T$  in both optimal production problems.

Numerically, the non-nuclear thermal units realize the same amount of production each month to cover the residual levels of demand, forcing nuclear to entirely follow the seasonal variations of demand. Here, the amplitude of demand is not greater than the nuclear capacity and hence, given the constant level of non-nuclear thermal production, the load-following operation of nuclear units does not lead to a violation of the minimum and maximum nuclear production constraints in order to equilibrate supply and demand. In particular, we observed high levels of nuclear production during winter and low levels of nuclear production during summer. The essentially periodic evolution of the nuclear production leads to a periodic evolution of the nuclear fuel stock i.e. low quantities of nuclear fuel during winter and high quantities of nuclear fuel during summer over the

whole time period  $T$ . The price is determined exclusively by the marginal cost of non-nuclear thermal production as non-nuclear thermal is the marginal technology all the time. In view of the constant levels of non-nuclear thermal production, the price remains constant over the entire time horizon of the model.

Furthermore, we noticed the basically periodic evolution of total cost which increases during winter and decreases during summer. As expected, the value of total cost (respectively the value of social welfare) coming from the total cost minimization problem (respectively social welfare maximization problem) is lower (respectively higher) than the value of total cost (respectively the value of social welfare) resulting from the current monthly profit maximization problem and the inter-temporal profit maximization problem.

Numerically, the comparison between the optimal inter-temporal production behaviour and the production behaviour arising from the maximization of social welfare showed that the non-nuclear thermal units adjust their production to demand variations to maximize profit but they produce at a constant rate all the time to maximize social welfare. Nuclear units do not produce at a constant rate but are operated to load-follow in the optimal inter-temporal production problem because of the non-nuclear thermal capacity which does not permit to the non-nuclear thermal production to meet demand every month. In the social welfare maximization problem, nuclear production follows entirely the seasonal variations of demand and only a significant reduction (-47%) of the nuclear capacity (compensated by the non-nuclear thermal capacity) would lead to the modulation of non-nuclear thermal production. Nuclear production is marginal during low demand periods in the first case while it is never marginal in the second case.

Finally, from a theoretical and numerical point of view, we detected similarities regarding the solutions coming from these optimization problems: (i) the solution is unique regarding the non-nuclear thermal production, (ii) producer's behaviour does not alter with the prolongation of the time horizon of the model, (iii) important amounts of unused capacity considered at the monthly level may result in contradictory solutions as those found in each optimization problem (constant nuclear production, constant non-nuclear

thermal production) which shows that choosing capacity can influence significantly the actions of producers at the optimum.





# General Conclusion

In many countries, competition is replacing the vertically integrated monopoly within electricity markets. In such a competitive setting, our research focused on the optimal management of a large nuclear generation set which is operated in order to adapt its generation output according to the variations of energy demand (daily and seasonal). We placed ourselves in a medium-term horizon of management in order to take into account the seasonal variation of the demand level between winter (high demand) and summer (low demand). The novelty of our research consists of the assumption that the nuclear fuel functions as a “reservoir” of energy in the medium-term. This key feature of nuclear fuel as a “reservoir” is based on the periodical shut-down of nuclear reactors for fuel reloading. The consideration of the periodical interruptions of production to reload nuclear reactors with fuel makes immediately our model complex. Their timing and frequency determine the modelling of the generating units by specifying their essential inherent characteristics (available nuclear capacity, month of their fuel reloading) that play a crucial role in the choice of the optimal production. Following the reloading of nuclear reactors, a nuclear manager disposes a finite and exhaustible amount of nuclear fuel which allocates during a campaign of production to respond to the seasonal variations of demand so that it maximizes its profit. A priori, the existence of reservoir (e.g. hydro-reservoir, nuclear reservoir) is a favorable point for responding to the necessity of “storing” electric power but poses problems in terms of coordination between the different producers at a production level in order to find feasible production levels which respect the global equilibrium between supply and demand each month and maximize their profits at the same moment (cf. page 69). Our analysis took place in a perfectly competitive framework where two types of generation exist: nuclear and non-nuclear thermal. We distinguished two stages of optimization

of the nuclear production decisions corresponding to different time horizons of operation of the nuclear fuel reservoir: a short-term (a month) and a medium-term (1 to 3 years). A tremendous work regarding the general modelling (e.g. modelling the demand, the time horizon of the model and of the campaign, the generating units, the production costs, the nuclear fuel stock, the optimization constraints (production and storage constraints, supply-demand equilibrium constraints)) and the numerical calibration of our model has been realized in this thesis. The choice of reasoning in months rather than weeks within our model is a compromise between refinement of the model and computational capacity. Note also that, in this specific context, the production costs are to be seen as short-term production costs since they consist of a fixed and of a variable part. The fixed part is mainly due to investment in capacity within our model. The theoretical and numerical results are highly related to the structure of the cost functions assumed within our model (linear cost function for nuclear and quadratic cost function for non-nuclear thermal). In the hypothetical case that the concept of long-term costs is considered, the level of investment should be variable and its optimal value should be determined through our model. In this case, all parts of the production costs are variable (fixed costs are zero in the long-term) which leads to a different profit function and thus, a different optimal production problem in each stage of optimization of the nuclear production decisions (Varian Hal R. (1992)). Consequently, this may lead to different optimal production and storage decisions than these coming from our concept of short-term production costs. We choose the concept of short-term costs to determine the production cost functions within our model because the consideration of the fixed costs in the production cost of both technologies (nuclear, non-nuclear thermal) permits us to obtain a more realistic vision of the value of profit within our medium-term horizon (see page 121). The concept of long-term costs would lead to an even more complex model with a very high number of optimization variables. Moreover, investment costs of nuclear are affected by a number of factors that are difficult to take into account because of the absence of relevant data (see pages 39 and 118). Among these factors, the choice of the discount rate and the potential risks specific to the nuclear investment project which both help to measure its profitability on a socio-economic level play a central role on the determination of the investments costs (Commissariat général à la stratégie et à la prospective (2013)).

In the initial stage of optimization, we determined the optimal management of the nuclear fuel reservoir on a monthly horizon of operation under constraints imposed by the flexible operation of a nuclear reactor and in particular of the EPR reactor as well as the available generation capacity of non-nuclear thermal units (minimum/maximum production constraints) and others coming from the inter-temporal management of the nuclear fuel reservoir (storage constraints). This stage helped to find an optimal production trajectory in an already complex optimal production problem (high number of optimization variables, high number of optimization constraints) before moving to a yearly or multi-annual horizon of operation. Furthermore, a vital constraint that all the producers have to take into account every month is the global equilibrium between supply and demand (supply-demand equilibrium constraints). This is because nuclear, being the main source of electricity generation in the domestic electricity market, ensures the largest part of national demand.

In a first approach, we observed that a producer may attempt to calculate a merit order equilibrium (i.e. a production vector that respects production and nuclear fuel storage constraints and satisfies the supply-demand equilibrium constraint each month with the price being determined by the marginal cost of the marginal technology within the equilibrium) by looking at the merit order price. However, we showed that there exist theoretical and numerical difficulties which do not permit to calculate it. The main theoretical difficulty is that the nuclear production is a correspondence and not a function of the merit order price i.e. a continuum of feasible production vectors is associated with the merit order price during a month. Even in the case that one could associate a unique feasible production vector with the merit order price during a month, it is impossible numerically to compute a merit order equilibrium. This is because, given the inter-temporal management of the nuclear fuel stock, the equality between supply and demand has to be considered during the entire time horizon of the model in order to calculate the price within a merit order equilibrium. This leads to a large non-linear system of equations which is difficult to resolve numerically. Consequently, we concluded that a producer has to look at the evolution of seasonal demand and not of seasonal price in order to determine its optimal production each month.

In a second approach, we proposed the optimization of production on a monthly

basis under identical constraints (the production and storage constraints and the supply-demand equilibrium constraints). Thanks to this optimization problem, called the optimal short-term production problem, a producer determines the optimal production level through the maximization of its profit during the current month of optimization under constraints given the optimal production levels calculated in the previous months. The price is an outcome of the equilibrium between supply and demand following the merit order price rule. In this early stage of optimization, the producer determines an optimal production path which serves as a reference for the existence of an equilibrium in the late stage of optimization when it deals with a more complex optimal production problem based on a yearly or multi-annual horizon of operation. In order to simplify the resolution of the optimal short-term production problem, the producer follows a tactic whereby the supply-demand equilibrium constraint is only considered during a month: the current month of optimization. However, we noticed, via a numerical example, that this tactic leads to production levels that do not respect the equality between supply and demand in future months. This result showed that a “short-sightedness” on behalf of producer with respect to the future states of demand is not tolerable for the determination of the optimal management of the nuclear fuel reservoir. For this reason, we proposed another tactic by which the producer considers the supply-demand equilibrium constraint not only during the current month of optimization but throughout the time horizon of the model (typically 36 months).

From a mathematical perspective, we showed that the discontinuity of the merit order price induced by the high marginal cost of non-nuclear thermal and the low marginal cost of nuclear results in a discontinuity of producer’s current monthly profit. In particular, we proved that, under some assumptions, the value of profit decreases when nuclear comes to be the marginal technology and thus, its low marginal cost determines the price in the market (absence of non-nuclear thermal production). The discontinuity of the merit order price is well-known and does not lead to an absence of equilibrium in the case of a static model. Here, the dynamic character of our model resulting from the modelling of the nuclear fuel stock as a reservoir brings up the difficulty of finding an equilibrium theoretically and numerically in view of the discontinuity of the price and hence of the profit. To deal with this difficulty in a theoretical level, we proceeded by

looking at the current monthly profit maximization problem on the set of feasible solutions characterized by strictly positive levels of non-nuclear thermal production. In this subset of feasible solutions, non-nuclear thermal is the marginal technology. Mathematically, we showed that this problem is equivalent to the current monthly profit maximization problem determined on the entire set of feasible solutions.

In the last part of the second chapter, we analyzed the optimal production decisions coming from the optimal short-term production problem within a simple numerical model. Mathematically, we proved that the merit order equilibrium of an economy with  $N$  producers is “equivalent” with the merit order equilibrium of an economy with an aggregate producer. From an economical point of view, we deduced from this proposition that the decentralized economy is neither “inferior” nor “superior” than the aggregate economy in terms of optimal production. In order to simplify the numerical resolution of the optimal short-term production problem, we worked in the economy with an aggregate producer who holds the total nuclear and non-nuclear thermal capacity.

To treat the problem of discontinuity of the merit order price within our numerical model, we “regularized” the merit order price. According to that, the merit order price is determined by the non-nuclear thermal marginal cost in place of the nuclear marginal cost during the time that nuclear is the marginal technology. So, this difficulty to find theoretically and numerically an equilibrium has economic consequences since it leads to overpay electricity during months of nuclear marginality. The “regularization” of the merit order price refers to an “opportunity” cost which is added to the marginal cost of nuclear. Particularly, within our model, the marginal cost of non-nuclear thermal during months when the non-nuclear thermal production is zero may be seen as the sum of the marginal cost of nuclear and of the “opportunity” cost of using nuclear capacities rather than non-nuclear thermal capacities in order to satisfy demand. The “opportunity” cost then represents a compensation of producers for using only their nuclear capacities to meet demand instead of calling non-nuclear thermal generation technologies (coal, gas). Therefore, they benefit from a higher market price which helps to amortize their high fixed costs. This can be seen in the monitoring report of the French energy regulator (CRE) in 2007 (Regulatory Commission of Energy (2007)) which validates the use of nuclear

capacities given that the use is based on a calculation of “opportunity” cost (which is not accessible/verifiable). According to this report, on the day-ahead market, the price reflected the valorization, decided by the historical operator (EDF), of nuclear production when nuclear is the marginal technology. The level of this valorization is generally higher than the marginal cost of nuclear production. Thus, a producer, even dominant, may legitimately seek the optimization of its income given that it does not constitute an abuse of dominant position or price manipulation.

In our numerical model, the “regularization” of the merit order price erased “irregularities” i.e. the discontinuity of the price and of the profit and led to a “regularized” optimal production problem, in which we found an equilibrium in both phases of optimization. We need to remind here that, numerically, the economical problem (discontinuous optimal production problem) has no solutions within our model. Thereby, we reached a satisfying situation which permits to obtain an equilibrium. The fact that if an economic phenomenon is not “regular”, it is not intellectually satisfying can be found for example in Balasko’s work regarding the theory of general equilibrium (Balasko (1988)). More precisely, Balasko mentions that, within a “regular” economy, we can find several properties of an equilibrium (e.g. continuity, existence and stability of equilibrium with respect to the parameters that define the economy) that we hope to verify in all economies since they are desirable for an equilibrium (insofar as they are not verified, infinitely more complex and also difficult to manage phenomena occur).

The “regularization” of the merit order price engendered the “regularization” of our economical problem (discontinuous problem) which led to an approximative continuous problem: the “regularized” problem. We showed that both optimization problems (the economical and the “regularized” problem) have the same value which permits to characterize the “regularized” problem as a “good” approximation of the economical problem. Within our numerical model, we deduced that the set of solutions of the economical problem is empty. Thus, we focused on the numerical solution obtained by the “regularized” problem which constitutes only an approximative solution of our economical problem. In view of this numerical solution, the nuclear production follows the seasonal variations of demand. We found high levels of nuclear production (low levels of nuclear fuel stock)

during winter and low levels of nuclear production (high levels of nuclear fuel stock) during summer. The producer uses the non-nuclear thermal capacity to balance supply and demand and to maximize its current monthly profit during seasons of low and of high demand. Despite the “regularization” of the merit order price, the producer does not choose to use only the nuclear production to meet demand through low demand seasons in this early stage of optimization. Consequently, non-nuclear thermal is the marginal technology and hence it determines prices almost all along the time horizon of our model (only exception is the last sub-period). We observed high levels of price and of profit during winter and low levels of price and of profit during summer. The duration of marginality of nuclear appeared at this stage of optimization constitutes only an indication before going to the final stage of optimization.

In our third and last chapter, we focused on the final stage of optimization when the producers are familiarized with the management of nuclear fuel reservoirs in a competitive market having found an equilibrium of the current monthly profit maximization problem. In this later stage, we extended the management horizon to a medium-term horizon including several campaigns of production (typically 3 years). Hence, we proceeded with a full optimization of the production following from the maximization of the inter-temporal profit under constraints identical with those considered in the optimal short-term production problem. Therefore, the resolution of this problem (optimal inter-temporal production problem) provided us with a global optimum while the second problem (optimal short-term production problem) determined a solution that can be thought as a “local” optimum. In this way, we managed to get a complete profile of the optimal allocation of the nuclear fuel stock of flexible nuclear units in a competitive market ensuring at the same time the security of supply at all months.

However, during the resolution of the optimal inter-temporal production problem, we encountered the same type of difficulty already met in the second chapter: the discontinuity of the merit order price. We proved that, under some assumptions, this price discontinuity leads to a discontinuity of the inter-temporal profit. Specifically, the decrease of the price when nuclear changes into marginal technology causes a decrease of the inter-temporal profit.



Therefore, we resolved theoretically the optimal inter-temporal production problem on a subset of the set of feasible solutions defined by strictly positive amounts of non-nuclear thermal production. Mathematically, this problem and the optimal inter-temporal production problem defined on the whole set of feasible solutions are equivalent. Among the different theoretical approaches to resolve the optimal inter-temporal production problem, we looked at a particular case which regards the “interior” optimal solutions. More precisely, we showed that, in the absence of binding production constraints, the nuclear units always produce at a constant rate while non-nuclear thermal is the only generation technology used to load follow during the whole production period  $T$ . Consequently, if the non-nuclear thermal capacity is sufficiently important then it permits to the non-nuclear thermal production to follow totally the seasonal variations of demand without the need to modulate the nuclear production.

Then, we proceeded with the numerical resolution of the optimal inter-temporal production problem. From a mathematical point of view, we showed the “equivalence” of equilibrium between a decentralized economy with  $N$  producers and an aggregate economy with a unique producer. Economically, this proposition permitted us to conclude that the decentralization of the nuclear production is neither “better” nor “worst” than the centralized management of the nuclear production regarding the optimal production levels. Thanks to this proposition, we reduced the degree of complexity of the inter-temporal profit maximization problem by searching for a numerical solution in the economy with an aggregate producer (as we did in the case of the optimal short-term production problem). This helped to reduce the number of optimization variables of the problem.

Another economical property of an equilibrium of the optimal inter-temporal production problem is related to the notion of symmetry. Specifically, under the assumption that all producers hold the same amount of capacity (nuclear and non-nuclear thermal), we showed that an equilibrium is “almost” symmetric what we called “symmetrisable”. However, it was found that there exist asymmetric equilibriums. Through the property of “symmetrisability”, it is possible to derive a symmetric equilibrium from an asymmetric equilibrium and thus to focus on the symmetric equilibrium.

Once again, we tackled the problem of discontinuity via the “regularization” of the

merit order price. This “regularization” is identical to the one realized in our previous chapter. Consequently, nuclear is no more penalized by a low price (being its constant marginal cost) when it is the only technology used to meet demand. The “regularization” of the economical problem (discontinuous problem) introduced an alternative continuous problem called also “regularized problem”. We proved that the “regularized” problem is a “good” approximation of our economical problem (discontinuous problem) since both optimization problems have the same value. Then, we deduced that the economical problem has no solutions within our numerical example. Therefore, we looked at the numerical solution resulting from the “regularized” problem which is only an approximative solution of our economical problem. At this point, it should be noticed that solutions characterized by a constant nuclear production do not occur within our numerical example. Here, the nuclear production is modulated since the non-nuclear thermal capacity is not sufficient so that the non-nuclear thermal production follows completely the variations of demand without violating production constraints that are active at the optimum. We observed high levels of nuclear production (low levels of nuclear fuel stock) during winter and low levels of nuclear production (high levels of nuclear fuel stock) during summer. The producer maximizes its profit ensuring at the same time the equilibrium between supply and demand by using the non-nuclear thermal capacity during periods of high demand and the nuclear capacity during periods of low demand. So, we found that non-nuclear thermal is the marginal technology during winter while nuclear becomes the marginal technology during summer. High prices are observed in winter and low prices in summer. Similarly, high profits are obtained in winter and low profits in summer. However, the price (respectively the profit) during low demand seasons is lower than the corresponding price (respectively the corresponding profit) observed in the optimal short-term production problem where the non-nuclear thermal production is used to meet demand and maximize profit during these seasons. Therefore, the length of the period of marginality of nuclear is more important in the optimal inter-temporal production problem than in the optimal short-term production problem. In this stage of optimization being the final one, we observed that, structurally, phenomena of marginality of nuclear may occur. More precisely, within our data set, a property of both theoretical and numerical solutions of the optimal inter-temporal production problem is the significant time period during which nuclear determines the price in the market. The high duration of margina-

lity of nuclear can be also explicated by the fact that the exclusive use of nuclear to meet customers needs for electricity is no more penalized thanks to the “regularization” of the price within our numerical example. Nevertheless, the total profit resulting from the optimal inter-temporal production problem is higher than the total profit coming from the optimal short-term production problem since, as we have mentioned before, the solution of the first optimization problem constitutes a global optimum.

The evolution of the nuclear production with respect to the demand can be analyzed through the notion of comonotonicity. In our numerical example, we found that the nuclear production is comonotonic to the demand (it increases and decreases simultaneously with the demand). More precisely, the non-nuclear thermal production, the price and the profit are basically comonotonic to the demand. In general, the solutions of the optimal short-term production problem are not comonotonic to the demand. In the case of a larger time scale, a specificity of the model intervenes, the exponential increase of the demand (1% per year), and therefore, we have to consider an augmentation of the capacities and in particular of the nuclear capacities in order to keep the comonotonic character of the model.

In the last part of our third chapter, we studied the problem of social welfare maximization in exactly the same framework. The social welfare maximization problem determined production and storage decisions when the nuclear operators decide to take full account of the welfare of the society. We maximized social welfare under the same constraints as those considered in the optimal production problems studied previously (optimal short-term production problem and optimal inter-temporal production problem).

Initially, we showed that the problem of maximization of social welfare can not be resolved within a framework of inelastic demand. To overcome this difficulty issued from this basic assumption of our model, we focused on the resolution of the total cost minimization problem by proving that it is equivalent to the problem of maximization of social welfare (same set of solutions).

On a theoretical level, we resolved the social welfare maximization problem in the absence of binding production constraints which is a specific case. In this case, we proved

that the optimal solutions are fully characterized by a constant non-nuclear thermal production. Therefore, under the assumption that the nuclear capacity is significant, the nuclear production follows fully the variations of demand permitting to the non-nuclear thermal production to be constant. Obviously, the optimal production scheduling determined by the social welfare maximization problem (equivalently total cost minimization problem) and the optimal inter-temporal production problem are completely opposite. Social welfare (respectively inter-temporal profit) is maximized when the non-nuclear thermal (respectively nuclear) units produce at a constant rate.

From an empirical point of view, we found that the non-nuclear thermal units produce the same amount of electricity during the entire time horizon of the model in order to meet demand. So nuclear is the only one used to follow the seasonal variations of demand which coincides with the theoretical approach. In our numerical example, a significant decrease of 47% of the nuclear capacity (counterbalanced by the non-nuclear thermal capacity) would necessitate the simultaneous modulation of the non-nuclear thermal production in order to balance supply and demand every month. This has shown the robustness of our results since they would change only if an important reduction of nuclear capacities occurred. We obtained high levels of nuclear production during winter and low levels during summer and as a consequence we observed low levels of nuclear fuel stock during winter and high levels during summer. Since non-nuclear thermal is always the marginal technology, the price is determined by its marginal cost and therefore remains constant. To end, we noticed that the value of total cost is high during winter when the nuclear production increases and low during summer when the nuclear production decreases following an almost periodical evolution.

It should be noted that the solutions of all the optimization problems treated in this thesis have the following characteristics: (i) the non-nuclear thermal component of the optimal production vectors is unique and (ii) production behaviour patterns do not change in the case of an extension of the time horizon of the model. We also deduced that the choice of the amount of capacity to build and maintain (nuclear and non-nuclear thermal) may have important economical implications in the optimal production behaviour since significant levels of unused capacity (viewed at the monthly level) could lead to

paradoxical solutions as those observed theoretically in both the optimal inter-temporal production problem (constant nuclear production) and the social welfare maximization problem (constant non-nuclear thermal production). Lastly, we mention a couple of economical results which can be inferred within our data base: (i) the average nuclear cost calculated here (37.25 euros per MWh) is close to the range of nuclear electricity prices (37.5 - 38.8 euros per MWh) estimated for the NOME law (Commission for Energy Regulation (CRE) estimated this range of prices in 2010 (before Fukushima accident in 2011) in order to propose to EDF a fair price for selling nuclear capacity to alternative producers), (ii) the total monthly non-nuclear thermal production cost is never covered except if other generation technologies with higher marginal costs are called to meet demand (e.g. hydro-storage units).

**What are the main characteristics of our model and what do they tell us about its nature and consequently the nature of the deduced optimization problems?**

In our model, we dealt with an original and especially complex question regarding the optimal operation of flexible nuclear plants in a competitive electricity market. The difficulty of this question and thus of our model is based on the features of the French nuclear energy and of the French market. The first feature regards the flexibility that characterizes the nuclear production in France where the national energy mix is dominated by nuclear power (minimum/maximum production constraints). The second one has to do with the characteristic of the reservoir, a new idea based on the periodical shutdowns of nuclear reactors to reload their fuel and allocate it during a period of campaign (storage constraints). The third feature is related to the significant share of nuclear energy in the total electricity generation (it represents the 80% of electricity production) which makes it responsible for ensuring the equilibrium between supply and demand (supply-demand equilibrium constraints). The fourth and final one concerns the structure of the French electricity market chosen to be fully (and not partly) open to competition within our exercise (perfect competitive case). All these have been the key characteristics of our novel deterministic dynamic model which is developed in the general case of  $N \geq 2$  producers and of  $T \geq 36$  time periods (months) in order to study this question. However, in the real

world, the consideration of all these components within our model is not straightforward because of the complexity as regards the modelling of the parameters (e.g. modelling the demand, the time horizon of the model and of the campaign, the generating units, the production costs, the nuclear fuel stock, the optimization constraints (production and storage constraints, supply-demand equilibrium constraints)) and the numerical calibration of the model. Therefore, a number of assumptions and simplifications (e.g. electricity importations/exportations, price elasticity of demand, renewable energies, discount and mark-up rate, etc.) needed to be made in order to overcome the potential difficulties concerning the model itself and the resolution of the optimization problems resulted in it. Obviously, these assumptions limited our modelling to some extent and driven us relatively away from reality, however they were necessary in order to find an equilibrium. Additionally to this, we need to mention the “regularization” of situations which permitted to obtain desirable properties or cure “irregular” phenomena (e.g. price and profit discontinuity) and thus, reach to the existence of solutions. Consequently, the creation of such an abstract model helped us first to identify the extremely complex nature of our question and the difficulties of responding to it (theoretically and numerically) and then to establish several optimal production problems within it in order to respond from different perspectives (short-term optimization, inter-temporal optimization, social welfare maximization).

One of the central elements mentioned above that characterizes our model and differentiates it from real life to some degree regards the structure of the French electricity market and the method of pricing within it. This market, dominated by a state-owned company (EDF), is marked by the importance of the nuclear sector and the ongoing process of opening to competition promoted by the European Commission. The progressive introduction of competition in the French electricity market was marked by the adoption and the application of NOME law applied since July 1, 2011. The NOME law provides a sharing of “nuclear scarcity” between EDF and alternative electricity suppliers, forcing EDF to sell up to 100 TWh of electricity per year to its competitors at a competitive price that represents the economic conditions of electricity production by its nuclear plants, conditions assessed by the French energy regulator (CRE) until 2015. More precisely, this price should designate the total cost of the operating nuclear plants. In our model, we considered an electricity market that is fully opened to competition and thus, regulated

access to historical nuclear capacity (ARENH) is not considered. The nuclear set is decentralized and a finite number of producers who are price-takers and dispose a certain amount of nuclear capacity, compete with each other in order to maximize their profit and to even satisfy energy demand. The price (merit order price) is determined directly by the equality between supply and demand in the electricity wholesale market. Hence, we worked in a particular setting of perfect competition which does not correspond totally to the French reality but from which we can draw some interesting economical conclusions for industry and policy with respect to the optimal operation of flexible nuclear plants within this setting. This is the main purpose of this exercise. The functioning of competitive markets is not examined within this thesis since it is completely determined within our model. Note that, we did not consider in our model the case of fixed price contracts in order to obtain a regulated price set by the government and specifically by the French energy regulator (CRE) and the Ministry in charge of economy and energy and proposed exclusively by EDF (see Subsection 2.2.1, page 84).

### **What kind of conclusions for industry and policy can be drawn?**

The nuclear fuel reservoir is a novel mechanism which permitted to analyze in an innovative way the dynamic evolution of the nuclear fuel of flexible nuclear units in a competitive environment and determine its optimal allocation over time so that customers needs for electricity are met. It revealed us the difficulties of calculating an equilibrium and thus, it contributed to understand and overcoming them. Several economic conclusions have been deduced from the modelling of nuclear fuel as a reservoir.

A first conclusion is that producers can not decide on the optimal level of their outputs by looking at the inter-temporal pattern of prices (even if we assume that prices can be totally anticipated) given the theoretical and numerical difficulties of calculating a merit order equilibrium due to the behaviour of supply with respect to the merit order price within our model (see page 104). We do not claim that this behaviour will persist if we choose marginal cost functions different than those of our model. However, our choice of marginal cost functions (nuclear and non-nuclear thermal) being a satisfactory approximation of the marginal costs of generation technologies as they appear in the merit order figure (see Figure 1.4 on page 50) could give some insights with respect to the

potential barriers of determining an equilibrium. Here, it results that the inter-temporal pattern of seasonal demand should be the criterion for selecting the optimal production levels which satisfy demand every month. For this reason, we focused on the resolution of “classical” optimization problems which could provide us an equilibrium; they consisted of the maximization of producer’s profit under production and storage constraints as well as the equality between supply and demand every month.

Equilibrium is calculated by taking into account two distinctive time horizons of management of the nuclear fuel reservoir: (i) a month, (ii) one or more campaigns of production. This led to examine different but coherent optimal production problems: the optimal short-term production problem and the optimal inter-temporal production problem resolved respectively in the initial and final step of optimization given the decentralization of the nuclear set. In view of the complexity of our model, the first step of optimization gave us the possibility to explore all the aspects of the optimal production problem of a producer and therefore, to discover and treat potential difficulties (e.g. number of optimization variables, number of optimization constraints, discontinuity of price and of profit, existence of equilibrium, symmetry of equilibrium, etc.) in order to find an optimal production path before we move to the core problem of the optimal allocation of reservoir’s nuclear fuel: the optimal inter-temporal production problem. Consequently, it needs time and knowledge of the management of the nuclear fuel reservoir of a flexible unit in a competitive market dominated by the nuclear production so that producers attain a global optimum of their optimal production problem.

From the first stage of optimization, we concluded that nuclear producers can not be “short-sighted” with respect to the future states of demand in a market where nuclear has a dominant position. Each producer has to take into account the equality between supply and demand during the entire period of operation of its units (nuclear and non-nuclear thermal) and not only during the current month of optimization in order to avoid violating production constraints and hence find an equilibrium of its optimal production problem. This also implies that any level of “short-sightedness” regarding nuclear capacity investments in future months is not acceptable in order to avoid future disruptions on supply.



Then, the following conclusion consists of the fact that, numerically, the optimal production behaviour derived from the maximization of the instantaneous profit is relatively different from the one obtained by the maximization of the inter-temporal profit. It has to do mostly with the time period of marginality of nuclear. More precisely, during the earliest stage of optimization, the “learning” period, producers use non-nuclear thermal capacities to maximize their current monthly profit and meet demand during almost the whole time horizon of the model. Despite the fact that the nuclear production is valorized at a price that is higher than its marginal cost when it is the marginal technology, market price is determined almost always, even during seasons of low demand (summer), by the marginal cost of non-nuclear thermal. So, the partial “short-sightedness” that characterizes producers with regard to future profit optimization does not permit them to use nuclear capacities more efficiently. This is not the case for the inter-temporal profit maximization problem resolved in the latest optimization stage. Its solutions are characterized by a more important duration of marginality of nuclear from a theoretical and a numerical perspective (marginal nuclear during low demand seasons - marginal thermal non-nuclear during high demand seasons). This resulted in a higher total profit in the late phase of optimization than in the initial phase. For this reason, we concluded that, in the earliest optimization stage, the numerical results concerning the time period of marginality of nuclear give not more than a hint about its duration.

We continued our research with the study of the social welfare maximization problem within the same setting. In a time period during which nuclear as an electricity generation technology is questioned significantly given the accident of Fukushima, we decided to determine the optimal production levels that result from the maximization of social welfare in a country where the global equilibrium between supply and demand depends totally on nuclear generation. The maximization of social “benefits” and the maximization of producers profits only consisted two different approaches that the social planner could look at and they led to very different optimal production behaviours.

Theoretically, we found that the optimal inter-temporal production behaviour is determined by a constant nuclear production if the non-nuclear thermal capacity is sufficient in order to follow entirely the variations of demand which implies a baseload operation

for nuclear plants. On the contrary, the optimal production behaviour that maximizes social welfare resulted in a constant non-nuclear thermal production by leaving nuclear production to load follow which means a totally flexible operation of nuclear plants, if nuclear capacity is sufficient.

Numerically, we did not arrive at the same conclusions with respect to the inter-temporal profit maximization problem. This is because in France where nuclear power is the principal electricity generation technology the non-nuclear thermal capacity is not enough in order to manage nuclear uniquely as a baseload generation technology at the optimum. It has also to operate at semi-base load following a part of the variable demand, therefore, the flexible operation of nuclear units is inevitable. As for the social welfare maximization problem, its numerical resolution shows a production behaviour that is identical with the one resulting from its theoretical resolution. The fact that nuclear capacities are very important with respect to non-nuclear thermal capacities in the French electricity market leads to a “paradoxical” optimal production behaviour being that of a steady non-nuclear thermal production.

Our theoretical and numerical results showed that social optimum is ensured within our model by investing significantly in nuclear capacity which shows clearly the necessity of nuclear in the energy mix of a country from the social welfare perspective. On the contrary, in a decentralized market, producers optimum is obtained by making important investments in non-nuclear thermal capacity from a theoretical and numerical point of view. Consequently, we conclude that the level of investments in order to build new capacities of a generation technology (e.g. nuclear) plays a major role in the determination of the production behaviour at equilibrium not only of the corresponding technology but also of the other generation technologies (e.g. non-nuclear thermal) that participate in the energy mix of a country.

Nevertheless, no behaviour coming from the resolution of the optimal short-term production problem, optimal inter-temporal production problem and social welfare maximization problem is fully observed in the real French electricity market. For example, as stated in the report of CRE in 2007, nuclear has been the marginal technology during periods of low demand meaning that non-nuclear thermal has not been the marginal

technology over the entire year as it is suggested by the numerical results of the optimal short-term production problem. Note that, in the optimal short-term production problem, a finer management horizon of the reservoir e.g. a week instead of a month could show periods during which nuclear is the marginal technology. Moreover, the duration of marginality of nuclear indicated by the report was not so important (a total of 1 or 2 months per year) as it resulted theoretically and numerically from the optimal intertemporal production behaviour (the half part of the year) (see page 54). In addition, the non-nuclear thermal units do not always produce at a constant rate as we derived from the production behaviour that maximizes social welfare. These deviations from reality are observed mainly because the obtained results are intrinsic to our model and nuclear managers and in particular the French nuclear operator (EDF) does not take into account our assumptions and considerations that limit our model to a certain extent but help to overcome difficulties so that an equilibrium for each of the above optimization problems is found. Economically, the theoretical and numerical results obtained by all these different approaches proposed in our model and in which the system operator may be interested provide insights in order to supply conclusions for policy and industry.

If the nuclear operator is private then it will inevitably focus on the maximization of its own profit. Currently, the character of the French nuclear operator is essentially public, thus, it arises the question of its behaviour and therefore the place of politics that have not chosen between the competitive model (why not have increased its private character) and the state model (100% public). The complexity that ran this thesis is to come closer to the difficulty of actually introducing competition in France. The analysis of the NOME law is introduced late, after the liberalization of markets, and looks like more to a temporary, ad hoc solution than a law based on an economic analysis.

In all cases, nuclear fuel modelled as a “reservoir” of energy follows the seasonal variations of demand in a competitive electricity market where nuclear capacity exceeds non-nuclear thermal capacity to a significant degree. But even if nuclear power does not possess the greatest part of the energy mix of a country (like France), it can be still operated at semi-base load following a part of demand’s variations because, technically, modern nuclear reactors are capable of flexible operation (see Subsection 1.3). This could lead to

a more significant use of nuclear in the electricity production of a country and therefore a higher share of nuclear power as a percentage of its national energy production especially since nuclear promotes: *(i)* reduction of CO<sub>2</sub> emissions, *(ii)* energy independence from fossil fuel generation technologies, *(iii)* large-scale deployment of intermittent electricity sources (renewable energy), *(iv)* economic competitiveness of a country's energy sector. All these factors play a very important role in the future of nuclear energy worldwide.

**Which questions could be considered for future research on the basis of this work?**

A number of questions that lead to different research paths could also be examined following this work. A first question could be the following: the theoretical and numerical results of our thesis depend fully on the specific assumptions of our model and thus, they will change if the assumptions of the model change (without however moving away from reality) or they will be generalized? For example, the cost functions are convex but they can take other forms e.g. we could suppose a quadratic cost function for nuclear production or a cubic cost function for non-nuclear thermal production. In this case, how the optimal solutions and their characteristics would change? In our thesis, we showed that the determination of the optimal management of flexible nuclear plants in a perfectly competitive market has been already a very complex work. This work was necessary, if we want to proceed with more complex market structures which leave room for strategic behaviour and are closer to reality. In particular, it may be interesting to study the question of the optimal operation of flexible nuclear plants considering the characteristic of the nuclear fuel reservoir in the case of monopoly and of oligopoly respectively. What would result from the maximization of the monopolistic profit in the short-term and intertemporarily under optimization constraints identical with those considered in the perfect competitive case? What solutions would be provided by the Cournot equilibrium in the case of oligopoly? To what extent these solutions are differentiated from those obtained under perfect competition and what will be the implications of this from an economical perspective? Note that different model assumptions regarding e.g. the inverse demand function, the elasticity of demand, etc. may be needed in order to study these cases. Another type of questions could focus on externalities e.g. externalities generated by

the CO<sub>2</sub> emissions for non-nuclear thermal and/or the probability of an accident or the emission of radioactive wastes for nuclear. What will be the form of the externality costs and how they will affect the optimum of producers? Moreover, consumer behaviour is relatively simplified in our model which is why consumer preferences and a price elasticity of demand as well as the demand side management (e.g. use of smart grids to improve the efficiency, reliability and sustainability of the production and distribution of electricity) could be incorporated in a future work. Numerically, a sensitivity analysis with respect to the parameters of our model could take place in order to examine the existence of equilibrium and the induced production behaviour which could lead to additional insights and conclusions from an economical point of view. However, this may prove especially difficult to achieve because the collection of new data coming from the French nuclear operator (EDF), the French Transmission & System operator (RTE), the Regulatory Commission of Energy (CRE), etc. is not straightforward due to confidentiality. To conclude, all these questions show clearly the different approaches that one can consider, based on our model and on the methods applied within it as well as the numerous possibilities of extension of this research in the future.

# Annex A

Let us introduce some notations used in this annex. We call  $P_1^t$  the economical problem (presented in subsection 2.3.4)

$$\max_{q \in H^t} \pi_t(q)$$

and  $P_2^t$  the “regularized” problem (presented in subsection 2.4.3)

$$\max_{q \in H^t} \psi_t(q)$$

where  $\pi_t$  and  $\psi_t$  represent the current monthly profit functions of problem  $P_1^t$  and  $P_2^t$  respectively.

We will recall the properties proven in the chapter 2 in order to allow a self-content annex to the maximum extent. Functions  $\pi_t$  and  $\psi_t$  are such that  $\pi_t \leq \psi_t$  on  $H^t$ . This is because  $\pi_t = \psi_t$  on  $H_{th}^t$  which is a subset of  $H^t$  since the price is determined by the non-nuclear thermal marginal cost in both problems and  $\pi_t < \psi_t$  on  $H^t \setminus H_{th}^t$  because when nuclear is the marginal technology, the nuclear production is paid at price  $b_{th}$  within the “regularized” problem while it is paid  $b_{nuc}$  inside the economical problem and  $b_{nuc} < b_{th}$  (see Proposition 2.3.2). Let us also mention that  $\psi_t$  is a continuous function which, in addition, is strictly concave with respect to the non-nuclear thermal production  $q_t^{th}$  (see proof of Lemma 2.3.1). Moreover, the set  $H_{th}^t$  is dense in  $H^t$  (see Proposition 2.3.1).

We are now ready to state the following propositions:

**Proposition A.1** *For all  $t \in \{1, \dots, T\}$ , the value of the economical problem ( $\text{val}(P_1^t)$ ) and the value of the “regularized” problem ( $\text{val}(P_2^t)$ ) are the same ( i.e.  $\text{val}(P_1^t) = \text{val}(P_2^t)$ ).*

**Proposition A.2** *For all  $t \in \{1, \dots, T\}$ , the set of solutions of the economical problem ( $\text{Sol}(P_1^t)$ ) and the set of solutions of the “regularized” problem ( $\text{Sol}(P_2^t)$ ) are such that  $\text{Sol}(P_1^t) = \text{Sol}(P_2^t) \cap H_{th}^t$ .*

**Proposition A.3** *For all  $t \in \{1, \dots, T\}$ , if  $q_t$  is a production vector that does not belong to  $H_{th}^t$  and  $q_t \in \text{Sol}(P_2^t)$  (it constitutes a solution of the “regularized” problem), then  $\text{Sol}(P_1^t) = \emptyset$ .*

We do not provide a proof for each of the above propositions since they are simple adaptations of the corresponding Propositions *B.1*, *B.6* and *B.7* presented and proven in the annex of the following chapter.

# Annex B

In this annex, we call  $P_1$  the economical problem (presented in subsection 3.2.1)

$$\max_{q \in C} \pi(q)$$

and  $P_2$  the “regularized” problem (presented in subsection 3.4.3)

$$\max_{q \in C} \psi(q)$$

where  $\pi$  and  $\psi$  represent the inter-temporal profit functions of problem  $P_1$  and  $P_2$  respectively.

Let us now recall the properties proven in the chapter 3 in order to allow a self-contained annex to the greatest extent. Functions  $\pi$  and  $\psi$  are such that  $\pi \leq \psi$  on  $C$ . This is because  $\pi = \psi$  on  $F^{th}$  which is a subset of  $C$  since the price is determined by the non-nuclear thermal production in both problems and  $\pi < \psi$  on  $C \setminus F^{th}$  because when nuclear is the marginal technology, it is paid at price  $b_{th}$  within the “regularized” problem, while it is paid  $b_{nuc}$  inside the economical problem and  $b_{nuc} < b_{th}$  (see Proposition 3.3.2). Let us also mention that  $\psi$  is a continuous function which, in addition, is strictly concave with respect to the non-nuclear thermal production  $q_t^{th}$  (see proof of Lemma 3.3.1). Moreover, the set  $F^{th}$  is dense in  $C$  (see Proposition 3.3.1).

We are now ready to state the following propositions:

**Proposition B.1** *The value of the economical problem ( $val(P_1)$ ) and the value of the “regularized” problem ( $val(P_2)$ ) are the same ( i.e.  $val(P_1) = val(P_2)$ ).*



**Proof**

First, since  $\pi \leq \psi$ , we deduce that  $val(P_1) \leq val(P_2)$ .

Secondly, we prove that  $val(P_1) \geq val(P_2)$ . The set  $C$  is compact, hence the set of solutions of the “regularized” problem is non empty. This means that there exists  $q \in C$  such that  $\psi(q) = val(P_2)$ . In view of the density of  $F^{th}$  in  $C$ , the production vector  $q \in \overline{F^{th}}$ . It follows that there exists a sequence  $(q_h)_{h \in \mathbb{N}}$  such that  $q_h \in F^{th}$  and  $\lim_{h \rightarrow \infty} q_h = q$ . Since  $\pi = \psi$  on  $F^{th}$  and  $\pi(q_h) \leq val(P_1)$ , at the limit we obtain

$$val(P_2) = \psi(q) = \lim_{h \rightarrow \infty} \psi(q_h) = \lim_{h \rightarrow \infty} \pi(q_h) \leq val(P_1)$$

Consequently, the inequality  $val(P_1) \geq val(P_2)$  is proven.

From the first and the second part of the proof, we conclude that  $val(P_1) = val(P_2)$ .

□

**Proposition B.2** *The set of solutions of the economical problem ( $Sol(P_1)$ ) is contained in the set  $F^{th}$  ( i.e.  $Sol(P_1) \subset F^{th}$ ).*

**Proof**

If  $Sol(P_1) = \emptyset$  then there is nothing to prove. Let  $\hat{q} \in Sol(P_1)$ . This means that for all  $q \in C$ ,  $\pi(q) \leq \pi(\hat{q})$ . Especially, for all  $q \in F^{th}$ ,  $\psi(q) = \pi(q) \leq \pi(\hat{q}) \Rightarrow \psi(q) \leq \pi(\hat{q})$ . In particular, for  $q = \hat{q}$ , we obtain  $\psi(\hat{q}) \leq \pi(\hat{q})$  which means that  $\hat{q} \notin C \setminus F^{th}$  and thus,  $\hat{q} \in F^{th}$ . Hence, we deduce that  $Sol(P_1) \subset F^{th}$ .

□

**Proposition B.3** *The set of solutions of the economical problem ( $Sol(P_1)$ ) is contained on the set of solutions of the “regularized” problem ( $Sol(P_2)$ ) ( i.e.  $Sol(P_1) \subset Sol(P_2)$ ).*

**Proof**

In the case that  $Sol(P_1) = \emptyset$ , there is nothing to prove. If  $Sol(P_1)$  is a non-empty set then there exists  $\tilde{q} \in Sol(P_1)$ , which means that: (i)  $val(P_1) = \pi(\tilde{q})$ , (ii) in view of Proposition B.2,  $\tilde{q} \in F^{th}$ , hence  $\psi(\tilde{q}) = \pi(\tilde{q})$ . Consequently, we obtain

$$val(P_1) = \pi(\tilde{q}) = \psi(\tilde{q}) \leq val(P_2) = val(P_1) \Rightarrow \psi(\tilde{q}) = val(P_2)$$

Thus,  $\tilde{q} \in Sol(P_2)$  and the proposition is proven. □

We will reinforce the last two propositions.

**Proposition B.4** *The set of solutions of the economical problem ( $Sol(P_1)$ ) is determined as follows:*

$$Sol(P_1) = \begin{cases} Sol(P_2), & \text{if } Sol(P_2) \cap F^{th} \neq \emptyset \\ \emptyset, & \text{otherwise} \end{cases}$$

**Proof**

If  $Sol(P_2) \cap F^{th} \neq \emptyset$ , then there exists a production vector  $\bar{q}$  such that  $\bar{q} \in F^{th}$  and  $\bar{q} \in Sol(P_2)$ . Since  $\bar{q} \in Sol(P_2)$  one has, for all  $q \in C$ ,  $\psi(q) \leq \psi(\bar{q})$ . If  $q \in F^{th}$ , then  $\pi(q) = \psi(q) \leq \psi(\bar{q}) = \pi(\bar{q})$ . If  $q \in C \setminus F^{th}$ , then  $\pi(q) < \psi(q) \leq \psi(\bar{q}) = \pi(\bar{q})$ . Hence, we conclude that  $\bar{q}$  corresponds to a solution of the problem  $P_1$ , which means that  $Sol(P_2) \subset Sol(P_1)$ . Together with Proposition B.3, we obtain  $Sol(P_1) = Sol(P_2)$ .

In view of Proposition B.2, one has  $Sol(P_1) \subset F^{th}$ . In addition,  $Sol(P_1) \subset Sol(P_2)$  (see Proposition B.3). Consequently, we deduce that  $Sol(P_1) \subset Sol(P_2) \cap F^{th}$ . If  $Sol(P_2) \cap F^{th} = \emptyset$  then  $Sol(P_1) \subset \emptyset \Leftrightarrow Sol(P_1) = \emptyset$ , thus our economical problem  $P_1$  has no solutions, which concludes our proof. □

**Proposition B.5** *The set of solutions of the “regularized” problem ( $Sol(P_2)$ ) is such that either  $Sol(P_2) \subset F^{th}$  or  $Sol(P_2) \subset C \setminus F^{th}$ .*

**Proof**

In view of the strict concavity of the inter-temporal profit function  $\psi$  with respect to  $q_t^{th}$ , the non-nuclear thermal component of the solutions of the problem  $P_2$  is unique. Consequently, either  $Sol(P_2) \subset F^{th}$  or  $Sol(P_2) \subset C \setminus F^{th}$ .

□

**Proposition B.6** *The set of solutions of the economical problem ( $Sol(P_1)$ ) and the set of solutions of the “regularized” problem ( $Sol(P_2)$ ) are such that  $Sol(P_1) = Sol(P_2) \cap F^{th}$ .*

**Proof**

If  $Sol(P_2) \cap F^{th} = \emptyset$ , then, according to Proposition B.4,  $Sol(P_1) = \emptyset$ . Consequently,  $Sol(P_1) = Sol(P_2) \cap F^{th}$ .

If  $Sol(P_2) \cap F^{th} \neq \emptyset$ , then, in view of Proposition B.4, one has  $Sol(P_1) = Sol(P_2)$ .

However, from Proposition B.5, we obtain  $Sol(P_2) \subset F^{th}$ . Thus,  $Sol(P_1) = Sol(P_2) \cap F^{th}$ .

□

**Proposition B.7** *If  $q$  is a production vector that does not belong to  $F^{th}$  and  $q \in Sol(P_2)$  (it constitutes a solution of the “regularized” problem), then  $Sol(P_1) = \emptyset$ .*

**Proof**

In view of the hypothesis,  $q \in Sol(P_2)$  but it is not a production vector of the set  $F^{th}$ . Thus, according to Proposition B.5,  $Sol(P_2) \subset C \setminus F^{th}$ . This means that  $Sol(P_2) \cap F^{th} = \emptyset$ . Hence, in view of Proposition B.4 (or equivalently Proposition B.6), the set of solutions of the economical problem ( $Sol(P_1)$ ) is empty, which proves our proposition.

□

# Annex C: Tables

In this section, we present the values of the exogenous variables of our numerical model. We also provide the value of the total “regularized” profit obtained by the optimal inter-temporal production problem as well as by the optimal short-term production problem. In addition, we present the value of the total “regularized” nuclear and non-nuclear thermal profit. Finally, we give the total cost and the total variable and fixed cost resulting from all three optimization problems (optimal inter-temporal production problem, optimal short-term production problem and the social welfare maximization problem).

Type of units	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$	
Nuclear capacity Capacity $^{j,nuc}$ of unit $j$ (in MW)	201	5641	6758	4359	8838	8045	
Nuclear fuel stock $S_1^j$ available to the unit $j$ at the beginning of January (t=1) (in MW)	0	167634	401656	351075	898350	987543	
	$j = 7$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$	Total
	4923	7956	5634	5950	2972	1723	63000
	708216	1312459	1048322	1232702	678457	444534	8230949

Table C.1

Capacity (in MW)	Nuclear	Non-nuclear thermal	Hydro
$Q_{min}^{j,nuc}$	$0.25 \times \text{Capacity}^{j,nuc}$	—	—
$Q_{max}^{j,nuc}$	$1 \times \text{Capacity}^{j,nuc}$	—	—
$Q_{min}^{th}$	—	0	—
$Q_{max}^{th}$	—	15600	—
$Q_t^{hyd}$	—	—	4851.6
Stock of reloading $S_{reload}^j$ of unit $j$ (in MWh)	$1 \times \text{Capacity}^{j,nuc} \times 258 \times 24$	—	—

Table C.2

where 258 corresponds to the number of days during which a nuclear unit operates at full capacity in the course of a campaign consisting of 12 months.

Type of units	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$
Number of days before reloading for the unit $j$ counting from the beginning of January ( $t=1$ )	0	30.5	61	91.5	122	152.5
Number of days equivalent to full capacity before reloading for the unit $j$ counting from the beginning of January ( $t=1$ )	0	30.5	61	79.5	98	116.5
	$j = 7$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$
	183	213.5	244	274.5	305	335.5
	135	153.5	172	190.5	209	227.5

Table C.3

In order to simplify our calculations in table C.3, we follow the convention that one month is equivalent to 30.5 days.

Parameters	Nuclear	Non-nuclear thermal
$a_{nuc}^j$ (in Euro)	$22.79 \times \text{Capacity}^{j,nuc} \times (8760/11)$	—
$b_{nuc}$ (in Euro/MWh)	5.01	—
$a_{th}$ (in Euro)	—	$11.5 \times 10^7$
$b_{th}$ (in Euro/MWh)	—	26.24
$c_{th}$ (in Euro/MWh <sup>2</sup> )	—	$6.76 \times 10^{-7}$

Table C.4

where 22.79 represents the total fixed cost of nuclear in Euro/MWh.

	Optimal inter-temporal production problem	Optimal short-term production problem
Total “regularized” profit (in Euro)	$-9.147 \times 10^9$	$-9.636 \times 10^9$
Total “regularized” nuclear profit (in Euro)	$-5.957 \times 10^9$	$-6.332 \times 10^9$
Total “regularized” non-nuclear thermal profit (in Euro)	$-3.189 \times 10^9$	$-3.304 \times 10^9$

Table C.5

	Social welfare maximization problem	Optimal inter-temporal production problem	Optimal short-term production problem
Total cost (in Euro)	$5.209 \times 10^{10}$	$5.261 \times 10^{10}$	$5.250 \times 10^{10}$
Total variable cost (in Euro)	$1.023 \times 10^{10}$	$1.075 \times 10^{10}$	$1.064 \times 10^{10}$
Total fixed cost (in Euro)	$4.186 \times 10^{10}$	$4.186 \times 10^{10}$	$4.186 \times 10^{10}$

Table C.6

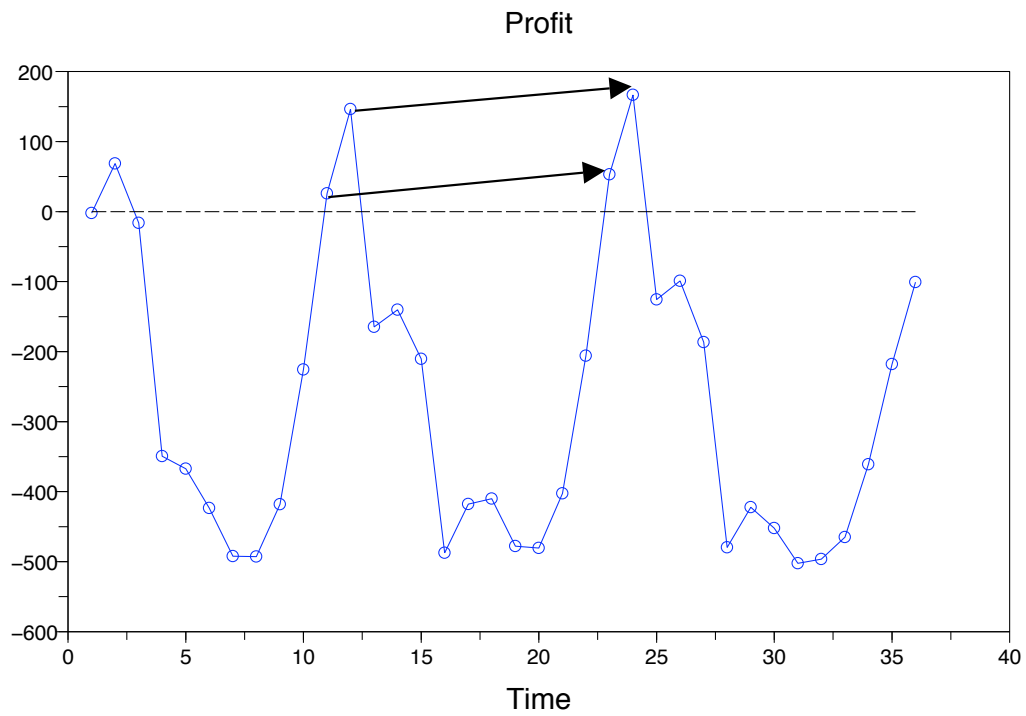


Figure C.17: Simulated “regularized” current monthly profit (excluding profit coming from hydro (run-of-river) production) (in Euro (million))

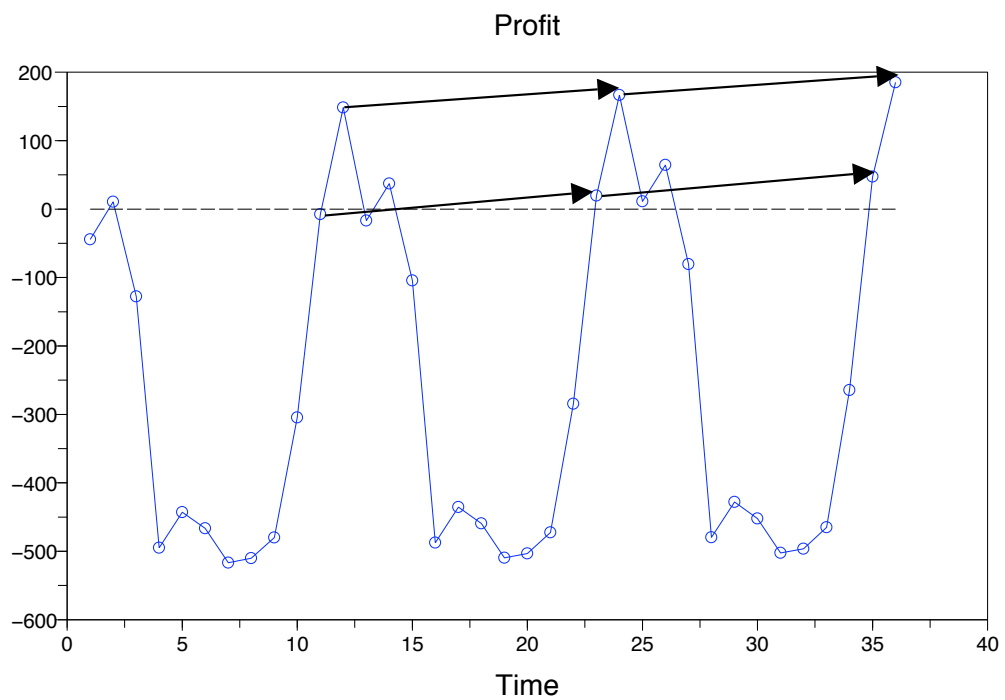


Figure C.18: Simulated “regularized” inter-temporal profit (excluding profit coming from hydro (run-of-river) production) (in Euro (million))





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