



Quantum kelvin : An optomechanical measure of temperature by quantum correlations and metrological validation

Ferhat Loubar

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ÉCOLE DOCTORALE - Sciences et Métiers de l'Ingénieur
Laboratoire Commun de Métrologie LNE-CNAM

THÈSE

présentée par : **Ferhat LOUBAR**
soutenue le : **14 Décembre 2022**

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**Le kelvin quantique : mesure optomécanique de température
par corrélations quantiques et validation métrologique**

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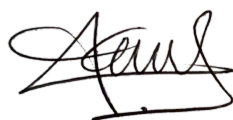
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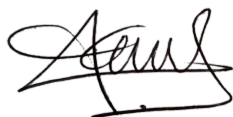
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I, undersigned, Ferhat LOUBAR, hereby declare that the work presented in this manuscript is my own work, carried out under the scientific direction of Mr. Stéphan BRIAUDEAU (thesis director) and of Mr. Tristan BRIANT (co-thesis supervisor), in accordance with the principles of honesty, integrity and responsibility inherent to the research mission. The research work and the writing of this manuscript have been carried out in compliance with the French charter for Research Integrity. This work has not been submitted previously either in France or abroad in the same or in a similar version to any other examination body.

Place Paris, date March 10th 2023

Signature

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Résumé

En 2018, la 26^e Conférence Générale des Poids et Mesures (CGPM) a redéfini les différentes unités du Système International (SI) en particulier le kelvin, unité de température, qui se base désormais sur la valeur fixée de la constante de Boltzmann k_B . Cette redéfinition a suscité le développement de nouveaux capteurs de température primaire permettant la dissémination du nouveau Kelvin. Les capteurs se basant sur les technologies quantiques sont très plébiscités par la communauté de métrologie. Dans ce contexte, nous proposons un capteur de température multimodal dont le fonctionnement repose sur les propriétés optique et optomécanique d'un cristal optomécanique à cristaux photoniques 1D. Sous l'effet de la température le résonateur voit sa fréquence de résonance optique se décaler et le mouvement Brownien induit par le bain thermique environnant varier. Ces deux effets permettent de remonter à la température du résonateur de deux manières différentes, à condition de pouvoir calibrer la chaîne de mesure. Ce type de résonateurs optomécanique ouvre la voie vers des capteurs de température primaires auto-calibrés avec des corrélations quantiques résultantes de la force de pression de radiation exercée par la lumière.

Mots clés : Optomécanique, capteurs de température, cristaux photoniques, cristaux phononiques, corrélations quantiques.

RESUME

Abstract

In 2018, the 26th General Conference on Weights and Measures (CGPM) redefined the various units of the International System (SI), in particular the kelvin, the unit of temperature, which is now based on the fixed value of the Boltzmann constant k_B . This redefinition spurred the development of new primary temperature sensors to disseminate the new Kelvin. Sensors based on quantum technologies are very popular in the metrology community.

In this context, we propose a multimodal temperature sensor whose operation is based on the optical and optomechanical properties of a 1D photonic crystal. Indeed, under the effect of temperature, the resonator sees its optical resonance frequency shift, and the Brownian motion induced by the surrounding thermal bath changes. These two effects allow the temperature of the resonator to be determined in two different ways, provided that the calibration is correct. This type of optomechanical resonator opens the way to self-calibrating primary temperature sensors with quantum correlations resulting from the radiation pressure force exerted by light.

Keywords : Optomechanics, temperature sensors, photonic crystals, phononic crystals, quantum correlations.

ABSTRACT

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Introduction

In the 17th century, German astronomer Johannes Kepler made the first speculation about the existence of a light-mediated force [1], to our knowledge. Kepler noticed, whilst observing the comet that we nowadays know as the Halley comet, that its tail was always pointing away from the sun. He hypothesized that the sun's rays push the tail away from it. It was only until the 19th century that such a force was theoretically predicted by James Clerk Maxwell through his theory of electromagnetic waves [2].

In 1901, the first observation of radiation pressure was made with an experimental apparatus in a light mill configuration, such as that depicted in fig. 1, in Moscow [3] and Dartmouth [4]. It was only 77 years later that Arthur Ashkin demonstrated the first fundamental application of radiation pressure [5] by controlling, trapping, and, surprisingly, cooling the motion of dielectric particles with strongly focused beams. Another study that took place in 1967 by Braginski's group at Moscow state university [6] shed the light on the interaction between an optical field with a mechanical resonator. They have predicted and demonstrated the modification of the dynamics of a mechanical resonator via radiation pressure which changes its damping. *And thus the field of optomechanics was born.*

Subsequently, the attention paid to this emerging field grew rapidly leading to various propositions to explore fundamental aspects of physics ranging



Figure 1: Image of a Crookes radiometer (or light mill) consisting of an airtight glass bulb with a set of vanes mounted on an axis allowing them to revolve around it when light is shone on it. Image copyright Science Museum/SSPL, London

INTRODUCTION

from general relativity through the observation of gravitational waves by the LIGO/Virgo Collaboration in 2015 [7], which involved an interferometric measurement of the displacement of a 40 kg test mass, to quantum physics through various experiments aimed at cooling the motion of different mechanical resonators to their ground state with macroscopic mirrors [8], suspended micro-mirrors [9], microtoroids [10], membrane inside a cavity [11], electromechanical drum resonator [12], levitated nanoparticles [13] and optomechanical crystals [14]. The aforementioned optomechanical systems are depicted in fig. 2.

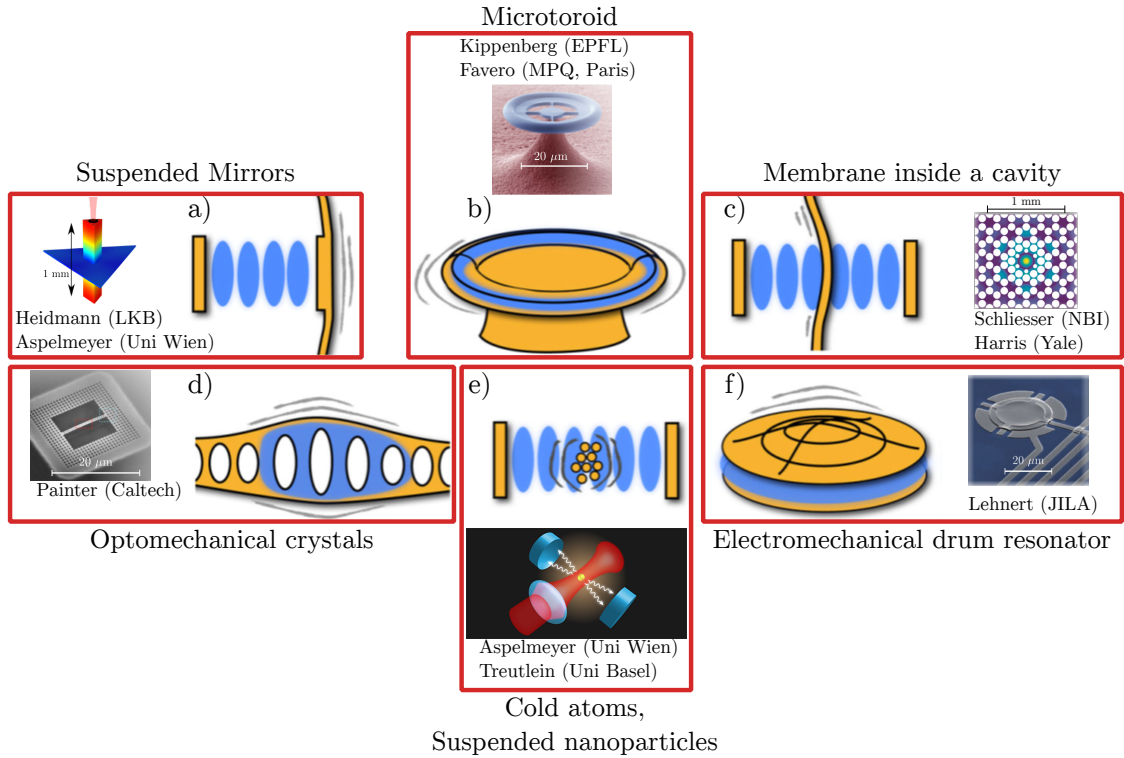


Figure 2: Schematic representation of the most common optomechanical devices, with their optical (in blue) and vibrational (gray lines) modes. Adapted from M. Aspelmeyer *et al.* (2014) [15].

Apart from the fundamental aspects of physical theories, the field of optomechanics has more practical applications such as the development of optomechanical sensors. These sensors cover a wide range of applications such as force sensing [16, 17, 18, 19], mass sensing [20, 21, 22], gravimeters and accelerometers [23, 24] and temperature sensors [25, 26, 27, 28]. In this manuscript, this last application will be explored and we will describe the development and characterization of a multimodal optomechanical temperature sensor based on the one hand, on temperature dependent optical properties of these devices and on the other hand, on the Brownian motion of these optomechanical systems

in response to the thermal bath surrounding them. This project is motivated by the recent redefinition of the thermodynamic temperature unit, the Kelvin, linking it to an energy scale via the Planck and Boltzmann fundamental constants $\hbar$ and k_{B} .

This manuscript is divided into 5 chapters. A theoretical description of the basic concepts of optical and mechanical resonators as well as the interplay between the two through the static and dynamical optomechanical effects will be given in chapter 1. An introduction to temperature metrology through two theoretical examples involving an electrical noise thermometer and an optomechanical self-calibrated thermometer will be described in chapter 2. The design, simulation, and fabrication of the optomechanical crystals allowing the interaction of co-localized THz optical modes and GHz mechanical modes will be explained in chapter 3. In chapter 4, the experimental apparatus including the cryogenic and optical setups will be described as well as the photonic thermometry principle and its results. Finally, chapter 5 will focus on the noise thermometry technique where an homodyne detection is developed and used to probe the phase fluctuations of a beam that has interacted with the optomechanical crystal to extract the mechanical mode's temperature.

Chapter 1

Introduction to cavity optomechanics

The field of cavity optomechanics explores the mutual interaction of an electromagnetic field with a mechanical system endowed with motion. In this chapter, the theoretical framework of electromagnetic waves in optical cavities will first be introduced through the coupled mode theory. Then, the noise spectrum of a harmonic oscillator driven by a Langevin force, introduced through the fluctuation-dissipation theorem, will be derived where we will describe the effects of temperature on the oscillator's dynamics. Finally, we will describe some optomechanical effects that will be of interest to the work presented in the following chapters.

1.1 Optics

1.1.1 Quantum description of electrical fields

The electromagnetic field can be written as the sum of modes comparable to independent harmonic oscillators [29]. A monochromatic field, with a given direction of propagation and a polarization state, can be characterized by introducing the creation and annihilation operators $\hat{\mathbf{a}}$ and $\hat{\mathbf{a}}^\dagger$, respectively. They obey the following commutation relation:

$$[\hat{\mathbf{a}}, \hat{\mathbf{a}}^\dagger] = 1. \quad (1.1)$$

From these operators, one can derive the field quadrature operators $\hat{\mathbf{1}}$ and $\hat{\mathbf{2}}$ related to the real and imaginary parts of the field :

$$\hat{\mathbf{1}} = \hat{\mathbf{a}} + \hat{\mathbf{a}}^\dagger, \quad (1.2)$$

$$\hat{\mathbf{2}} = i(\hat{\mathbf{a}}^\dagger - \hat{\mathbf{a}}). \quad (1.3)$$

Similarly to $\hat{\mathbf{a}}$ and $\hat{\mathbf{a}}^\dagger$, the field quadrature operators do not commute. As a result, the dispersion of these operators $\Delta\hat{\mathbf{1}}$ and $\Delta\hat{\mathbf{2}}$ satisfy a Heisenberg inequality:

$$\Delta\hat{\mathbf{1}}\Delta\hat{\mathbf{2}} \geq 1. \quad (1.4)$$

This relationship implies a minimum variance for both operator $\hat{\mathbf{a}}$ and $\hat{\mathbf{a}}^\dagger$ and then the existence of quantum fluctuations for these variables. A well-suited way of describing these fluctuations and visualizing them in the phase-space is the semi-classical approach that we will present in the following paragraph.

Semi-classical treatment of quantum fluctuations

The semi-classical approach [30] has been proposed for the description of quantum fluctuations evolution after interacting with a given system. The semi-classical approach involve associating to the operators $\hat{\mathbf{a}}$ and $\hat{\mathbf{a}}^\dagger$ two pseudo-random variables α and α^* , whose statistics are described by a Wigner quasi-probability distribution $\mathcal{W}$ such that for any function f , the mean-value of the product $\langle f(\hat{\mathbf{a}}, \hat{\mathbf{a}}^\dagger) \rangle$ is equal to the mean-value of the product $f(\alpha, \alpha^*)$ weighted by the Wigner distribution:

$$\langle f(\hat{\mathbf{a}}, \hat{\mathbf{a}}^\dagger) \rangle = \int d\alpha d\alpha^* \mathcal{W}(\alpha, \alpha^*) f(\alpha, \alpha^*). \quad (1.5)$$

We can thus write the semi-classical field α as the sum of its mean-value $\bar{\alpha}$ that corresponds to the classical value of the field and its fluctuations $\delta\alpha$ that are governed by the semi-classical probability distribution $\mathcal{W}(\alpha, \alpha^*)$:

$$\alpha = \bar{\alpha} + \delta\alpha. \quad (1.6)$$

A representation of the Wigner distribution for a coherent state i.e., a laser field, and its projection on the phase-space, is depicted in fig. 1.1. The axes are formed by the semi-classical quadratures α_1 and α_2 which correspond, respectively, to the real and imaginary parts of the field. The mean intensity of the field is the square modulus of the field amplitude, $|\bar{\alpha}|^2$ and the mean phase of the field $\bar{\varphi}$ corresponds to the angle between $\bar{\alpha}$ and the first quadrature coordinate as shown on fig. 1.1. For a coherent field, The Wigner distribution is strictly positive and takes the form of a Gaussian distribution, centered around the mean-value $\bar{\alpha}$, and its variance is equal to 1 in every direction. We, therefore, have $\Delta\alpha_1 = \Delta\alpha_2 = 1$, thus a coherent state corresponds to a minimum uncertainty state for the inequality 1.4.

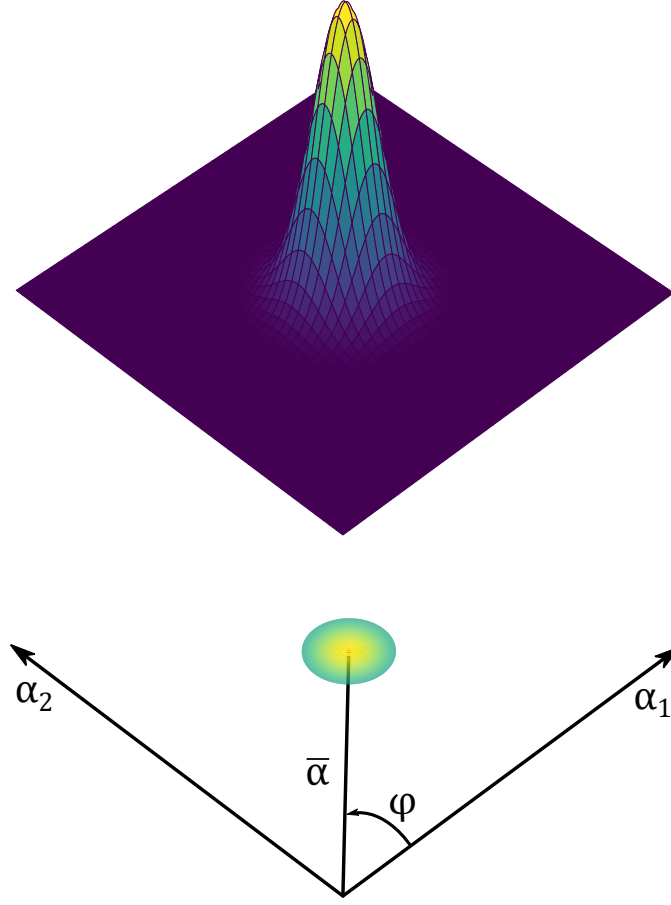


Figure 1.1: A three-dimensional representation of the Wigner quasi-probability distribution and its phase-space projection.

It can be convenient to have access to an arbitrary field quadrature α_θ , by applying a rotation by an angle θ to the axis system of the phase-space defined as follows:

$$\alpha_\theta = e^{-i\theta} \alpha + e^{i\theta} \alpha^*. \quad (1.7)$$

The dispersion of the arbitrary quadrature α_θ can be recovered by projecting the distribution on the axis with angle θ . For a coherent state, the dispersion is unitary in all directions:

$$\Delta\alpha_\theta = 1. \quad (1.8)$$

For a given realization of the field represented by a point in the phase-space, the field's amplitude is depicted by the distance between the point and the axes' origin, whereas the angle relative to the axis α_1 is its phase:

$$\alpha = \sqrt{N} e^{i\varphi}, \quad (1.9)$$

1.1. OPTICS

where $N = |\alpha|^2$ is the photon number and φ the phase of the field. The intensity and phase fluctuations can be estimated by linearizing eq. 1.9 around its mean-value $\bar{\alpha} = \sqrt{N}e^{i\varphi}$. The intensity and phase fluctuations δN and $\delta\varphi$ read:

$$\delta N = |\bar{\alpha}|\delta\alpha_{\bar{\varphi}}, \quad (1.10)$$

$$\delta\varphi = \frac{1}{2|\bar{\alpha}|}\delta\alpha_{\bar{\varphi}+\pi/2}. \quad (1.11)$$

The intensity quadrature $\delta\alpha_{\bar{\varphi}}$, parallel to the mean field is related to the intensity noise, whereas the phase quadrature $\delta\alpha_{\bar{\varphi}+\pi/2}$, orthogonal to $\bar{\alpha}$, describes the phase noise.

The intensity fluctuation of a coherent state can be derived from eqs. 1.8 and 1.10 and are equal to:

$$\Delta N^2 = \bar{N}. \quad (1.12)$$

This relation describes the Poissonian statistical distribution of a coherent photon source. Indeed the variance of a coherent state ΔN^2 scales as the mean number of photons. It is worth noting that the relative fluctuations decrease when the intensity of the field increases.

$$\frac{\Delta N}{\bar{N}} = \frac{1}{\sqrt{\bar{N}}}. \quad (1.13)$$

Similarly, one can derive the phase fluctuations of a coherent state from eqs. 1.8 and 1.11 and find that they are inversely proportional to the mean photon number:

$$\Delta\varphi^2 = \frac{1}{4\bar{N}}. \quad (1.14)$$

The field distribution, depicted in fig. 1.1 is seen with an angle $\Delta\varphi$, from the origin. According to eq. 1.8, this distribution has a unity dispersion regardless of the direction and the field intensity, thus $\Delta\varphi$ decreases when the mean photon number increases.

A Heisenberg inequality can be derived for the quantities N and φ , being two non-commuting quantum variables:

$$\Delta N \Delta\varphi \geq \frac{1}{2}. \quad (1.15)$$

From eqs. 1.12 and 1.14 one finds that for a coherent state $\Delta N \Delta\varphi = \frac{1}{2}$, thus such a state corresponds to a minimum uncertainty state according to eq. 1.15. For classical states of light, these fluctuations define the standard quantum noise where fluctuations are equal on all field quadratures. It is however not the minimal noise one can achieve; for instance, for squeezed states the noise on one quadrature can

be reduced down to the standard quantum noise at the cost of increased noise on the complementary quadrature, keeping the Heisenberg inequality still satisfied.

1.1.2 Coupled mode theory

The time evolution and steady state of the electrical field in a coupled waveguide-resonator system can be described in a variety of theoretical frameworks. The coupled-mode theory [31, 32] is a powerful class of methods that describe a system as an ensemble of ideal components, for instance: waveguides, that are coupled/perturbed to/by some other component (an optical cavity for example). These methods are often used to compute the exact eigenmodes of such ideal systems and provide a numerical result for a given geometry.

In the following, we will consider a more abstract formulation of the coupled-mode theory yielding a universal description of certain devices given a set of assumptions that we will describe later on. This class of methods is called the temporal coupled-mode theory [33, 34].

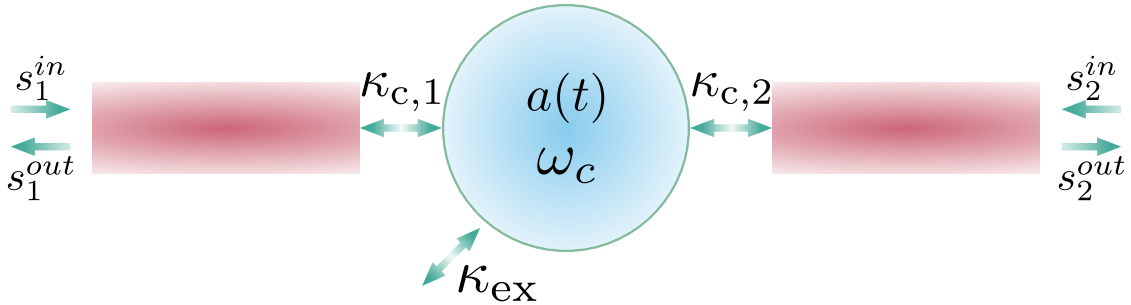


Figure 1.2: Diagram representing a single resonant cavity mode $a(t)$ coupled to two single-mode input-output waveguides with input/output fields $s_{1/2}^{in}/s_{1/2}^{out}$. The cavity mode is represented with a light blue circle and the single-mode waveguide by the red rectangles.

Let us consider the example depicted in fig. 1.2. An optical cavity with a given resonant frequency ω_c is coupled to two single-mode waveguides. The complex intracavity field $\tilde{a}(t)$, normalized such that $|\tilde{a}(t)|^2$ is the number of cavity photons, can decay into either waveguides with lifetimes $\tau_{c,1} = 1/\kappa_{c,1}$ and $\tau_{c,2} = 1/\kappa_{c,2}$, respectively, or to another decay channel, representing the intrinsic losses of the cavity, with lifetime $\tau_{ex} = 1/\kappa_{ex}$. $\kappa_{c,1}$, $\kappa_{c,2}$ and κ_{ex} denote, respectively, the coupling rate to waveguide 1, 2 and the vacuum. We will also define the fields circulating (in and out) the waveguides which are normalized such that $|\tilde{s}_{1/2}^{in}(t)|^2$ is the photon flux going towards the cavity and $|\tilde{s}_{1/2}^{out}(t)|^2$ the photon flux coming out of the cavity. The coupling dynamics can be derived using the following assumptions:

energy conservation and time-reversal invariance [35]. In that framework, the intracavity field's time evolution reads:

$$\dot{\tilde{a}}(t) = (-i\omega_c - \kappa/2) \tilde{a}(t) + \sum_{l=1}^{l=2} \sqrt{\kappa_{c,l}} \tilde{s}_l^{\text{in}}(t) \quad (1.16)$$

where, κ is the total energy loss rate defined as $\kappa = \tau^{-1} = \tau_{\text{ex}}^{-1} + \tau_{c,1}^{-1} + \tau_{c,2}^{-1}$.

The drive field oscillates at an angular frequency ω_L , i.e. $\tilde{s}_l^{\text{in}}(t) \equiv s_l^{\text{in}}(t)e^{-i\omega_L t}$, thus it is more convenient to write the fields into a frame rotating at an angular frequency $\omega = \omega_L$ where we would express the intracavity field as $\tilde{a}(t) \equiv a(t)e^{-i\omega_L t}$. Thus, eq. 1.16 reads:

$$\dot{a}(t) = (i\Delta - \kappa/2) a(t) + \sum_{l=1}^{l=2} \sqrt{\kappa_{c,l}} s_l^{\text{in}}(t) \quad (1.17)$$

where we introduced the frequency detuning $\Delta = \omega_L - \omega_c$ between the drive field and the cavity resonance frequency. Using the semi-classical approach, one can write the drive field as the sum of a mean-field and its fluctuations, $s_l^{\text{in}}(t) = \bar{s}_l^{\text{in}} + \delta s_l^{\text{in}}(t)$. One can, then, derive the steady state solution for the mode amplitude, which reads:

$$\bar{a} = \frac{\sqrt{\kappa_{c,1}} \bar{s}_1^{\text{in}}}{-i\Delta + \kappa/2}. \quad (1.18)$$

The power circulating in the cavity, in the steady state, is defined as $|\bar{s}|^2 = \frac{|\bar{a}|^2}{t_{\text{rt}}}$, where t_{rt} is the

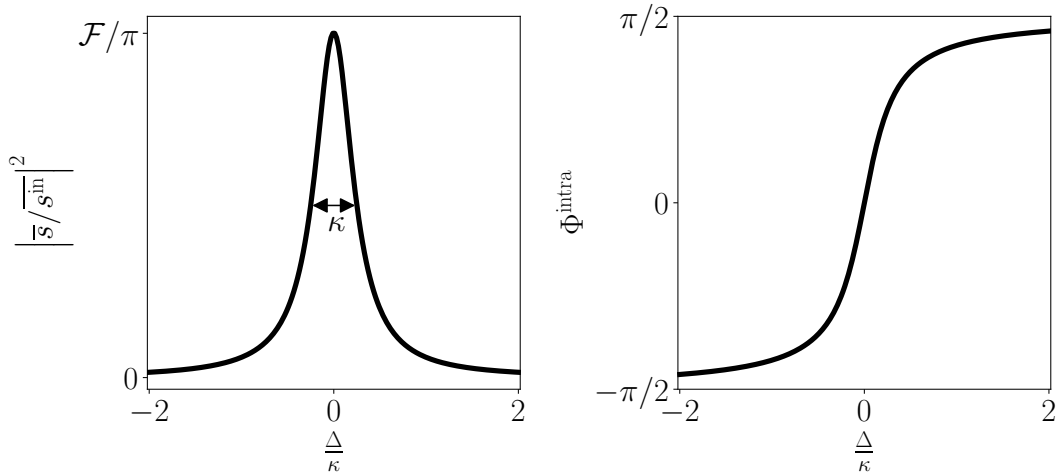


Figure 1.3: **Left:** Intracavity intensity. **Right:** Cavity-induced phase-shift of the intracavity field. Both figures are plotted against the normalized detuning Δ/κ .

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round-trip time of a photon inside the cavity, and can be expressed as follows:

$$|\bar{s}|^2 = \frac{\kappa_{c,1}}{t_{rt}} \frac{|\bar{s}_1^{\text{in}}|^2}{\Delta^2 + (\kappa/2)^2} = 2\eta_c \frac{\mathcal{F}}{\pi} \frac{|\bar{s}_1^{\text{in}}|^2}{1 + 4\Delta^2/\kappa^2}, \quad (1.19)$$

where we introduced the coupling parameter $\eta_c = \kappa_{c,1}/\kappa$. The intracavity field takes the form of a Lorentzian lineshape centered at the resonance frequency ω_c , with a full width at half maximum (FWHM) equal to the total loss rate κ , as depicted in the left panel of fig. 1.3. This figure is obtained for a lossless symmetric cavity ($\eta_c = 1/2$). For a resonant drive field i.e., $\Delta = 0$ the intracavity field is enhanced as compared to the input field by a factor $|\bar{s}/\bar{s}_1^{\text{in}}|^2 = \mathcal{F}/\pi$. $\mathcal{F}$ is known as the finesse and defined as the ratio of the free spectral range and the cavity loss rate:

$$\mathcal{F} = \frac{2\pi/t_{rt}}{\kappa}. \quad (1.20)$$

When the detuning Δ goes from negative to positive values, the cavity induces a π phase shift to the circulating field compared to the input field, as one can see on the left panel of fig. 1.3.

The fields coming out of the cavity can also be expressed using the input-output formalism and read:

$$s_1^{\text{out}}(t) = -s_1^{\text{in}}(t) + \sqrt{\kappa_{c,1}} a(t), \quad (1.21)$$

and

$$s_2^{\text{out}}(t) = \sqrt{\kappa_{c,2}} a(t). \quad (1.22)$$

The coupling geometry depicted in fig. 1.2, which we will refer to as the double-sided end-fire geometry, is one that would describe a Fabry-Perot cavity where one can measure the transmitted and reflected intensities. In the case of nano-scale optical cavities, this kind of coupling scheme is sometimes tricky to achieve. There are, however, other configurations to probe nano-scale cavities that will be described below.

1.1.2.1 Coupling geometries

In the following, we will describe two coupling geometries (fig. 1.4) widely used for coupling nano-scale optical cavities via optical fibers. We will derive the output fields and their intensity.

Bi-directional evanescent coupling geometry In this case, we have a single feed line with two readout ports s_1^{out} and s_2^{out} as one can see on the top panel of fig. 1.4. Assuming a single resonant input field

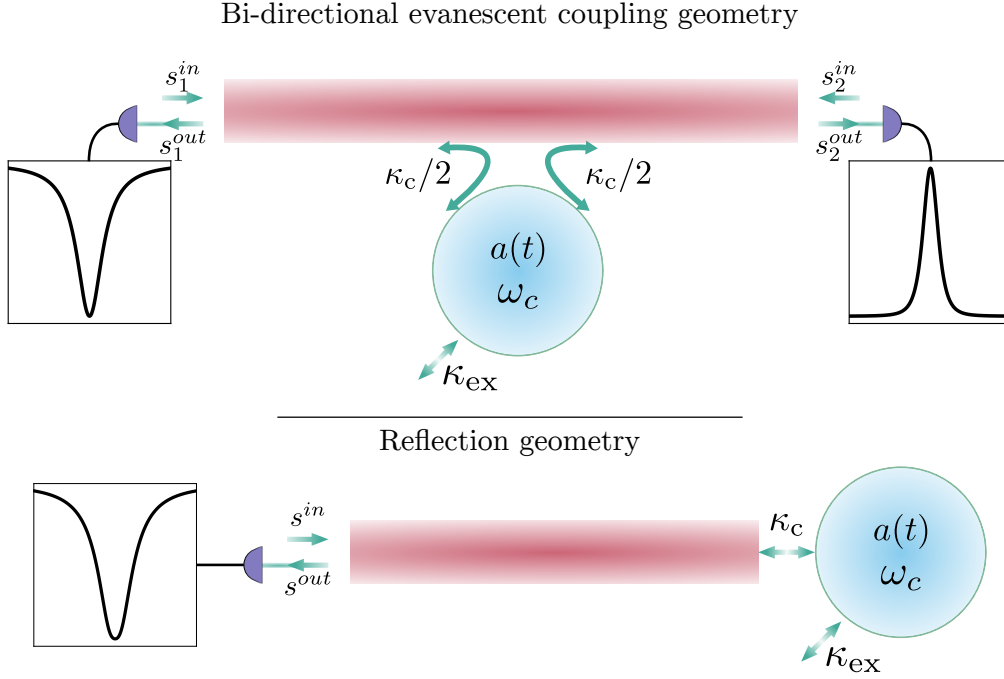


Figure 1.4: Input-output fields in two coupling geometries. **Evanescent coupling:** A single-mode waveguide with two input/output ports channels photons in and out of the cavity at a rate $\kappa_c/2$. **Reflection coupling:** A single-mode waveguide with single input and output ports channels photons in and out of the cavity at a rate κ_c . For both configurations, the output flux monitored with a photodiode, plotted against the detuning, is represented next to each output port.

through s_1^{in} , photons in the cavity that are scattered into the waveguide have two equally probable outcomes, scattering into both readout ports. It is worth noting that because the cavity is as coupled to the input mode (mode 1 with coupling rate $\kappa_{c,1}$) as it is to the output mode (mode 2 with coupling rate $\kappa_{c,2}$), then $\kappa_{c,1} = \kappa_{c,2}$ and the total coupling rate $\kappa_c = \kappa_{c,1} + \kappa_{c,2} = 2\kappa_{c,1} = 2\kappa_{c,2}$. This explains the factor $1/2$ in the expression of the cavity mean field as compared to eq. 1.18.

Using the input-output relations, one can derive the intracavity field and output fields for both readout ports,

$$s_1^{\text{out}}(t) = -s_1^{\text{in}}(t) + \sqrt{\frac{\kappa_c}{2}}a(t), \quad (1.23)$$

$$s_2^{\text{out}}(t) = -s_2^{\text{in}}(t) + \sqrt{\frac{\kappa_c}{2}}a(t), \quad (1.24)$$

$$\dot{a}(t) = (i\Delta - \kappa/2)a(t) + \sum_{l=1,2} \sqrt{\frac{\kappa_c}{2}}s_l^{\text{in}}(t). \quad (1.25)$$

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In the steady-state regime and considering that s_2^{in} is vacuum, these equations read

$$\bar{s}_1^{\text{out}} = \left(\frac{\eta_c}{1 - 2i\Delta/\kappa} - 1 \right) \bar{s}_1^{\text{in}}, \quad (1.26)$$

$$\bar{s}_2^{\text{out}} = \frac{\eta_c}{1 - 2i\Delta/\kappa} \bar{s}_1^{\text{in}}, \quad (1.27)$$

where $\eta_c = \kappa_c/\kappa = \kappa_c/(\kappa_c + \kappa_{\text{ex}})$. Each output field has a Lorentzian lineshape, with the difference that the field $\bar{s}_1^{\text{out}}$ is a Lorentzian dip and $\bar{s}_2^{\text{out}}$ is a Lorentzian peak. Both have FWHM equal to κ and centered around the cavity resonance frequency ($\Delta = 0$).

This geometry is complementary of the geometry described in section 1.1.2: unlike in the double-sided end-fire geometry, the field is completely transmitted by the waveguide far from resonance and partially (completely if $\kappa_{\text{ex}} = 0$) reflected at resonance. However, using both transmission/readout ports required to have access to both ends of the waveguide which is not always the case. When only a single end is available it becomes difficult to couple light correctly in the waveguide as, except at resonance, no light is reflected even with a perfect mode matching.

Reflection geometry In contrast to the geometry described in the previous paragraph, the reflection geometry has a single input and readout port. The transmission line is butt-coupled to the cavity allowing photons to be scattered in and out of the cavity at a loss rate κ_c .

The reflected signal can be expressed as follows:

$$s^{\text{out}}(t) = -s^{\text{in}}(t) + \sqrt{\kappa_c}a(t). \quad (1.28)$$

It follows that the steady-state relation reads:

$$\bar{s}^{\text{out}} = \left(\frac{2\eta_c}{2i\Delta/\kappa} - 1 \right) \bar{s}^{\text{in}} \quad (1.29)$$

The reflected field is maximal far from resonance and presents a Lorentzian dip at resonance. In this configuration, and for a lossless cavity ($\kappa_{\text{ex}} = 0$) all photons incoming in the cavity, eventually go out through the same waveguide whatever is the detuning Δ , which is an advantage either for the alignment procedure and for the detection scheme.

1.1.3 Coupling regimes

The coupling parameter η_c , introduced in the previous section, is a figure of merit of how well the waveguide is coupled to the cavity. Indeed, it compares the waveguide-cavity coupling losses κ_c to the

total losses κ . We can identify three coupling regimes, depicted in fig. 1.5:

- **Critically-coupled cavity:** for $\eta_c = 1/2$. It occurs when the field leaks into the different loss ports at the same rate ($\kappa_c = \kappa_{\text{ex}}$). This situation corresponds to an impedance matching between the injection line and the loss. As a result, the reflected power falls to zero at resonance. From an interferometric point of view, at resonance, the field leaking from the cavity back to the waveguide compensates exactly the field directly reflected by the cavity.
- **Undercoupled cavity:** for $\eta_c < 1/2$. This case occurs when the coupling losses are greater than the cavity's $\kappa_c > \kappa_{\text{ex}}$. The magnitude of the field getting out of the cavity is smaller than the incoming one. The reflected beam is π -shifted as compared to the incoming field, due to the -1 reflection coefficient of the input mirror.
- **Overcoupled cavity:** for $\eta_c > 1/2$. It occurs when the cavity losses exceed the coupling losses $\kappa_c < \kappa_{\text{ex}}$. In this case, the field getting out of the cavity is larger in magnitude than the incoming one. The reflected beam is in phase with the incoming field.

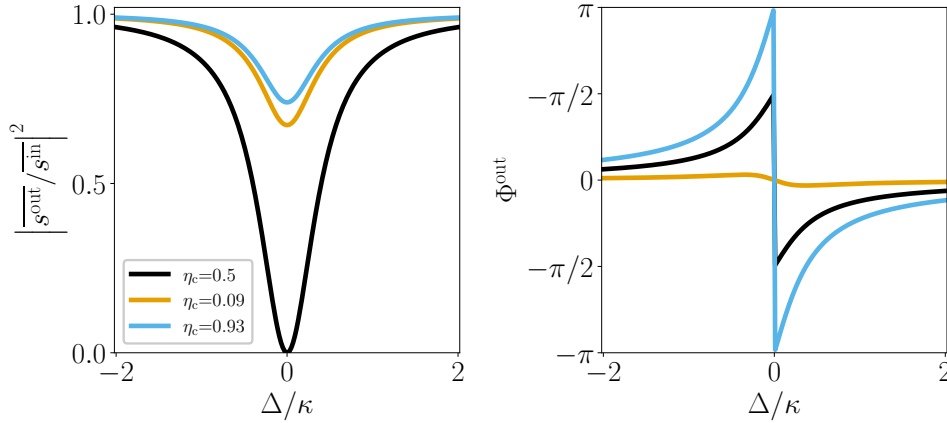


Figure 1.5: Coupling regimes for a cavity coupled in the reflection geometry. **Left:** Reflection spectrum normalized to the input flux. **Right:** Phase of the output flux. Both are plotted against the normalized detuning Δ/κ . Here we consider the undercoupled ($\eta_c = 0.09$), overcoupled ($\eta_c = 0.93$) and the critically coupled ($\eta_c = 0.5$) cases.

In the undercoupled and overcoupled cases, although the reflection spectra can be mistaken for each other, the phase response is on the other hand drastically different. Indeed, in the first case, the

reflected field undergoes a minimal phase-shift whereas in the second one the phase varies maximally, as one can see on the right panel of fig. 1.5. In the critical coupling regime, the phase presents a discontinuity at resonance with a jump from $\pi/2$ to $-\pi/2$ corresponding to the cancellation of the output field.

The fact that the phase is more sensitive to the detuning in the overcoupled case makes it a better coupling situation if one has the choice between an undercoupled and overcoupled cavity.

1.2 Mechanics

1.2.1 Introduction to noise

In optics experiments, light is used to probe a given physical quantity of a certain system. This quantity will be encoded in a given degree of freedom of light, which will eventually be monitored by a photodetector that destructively converts the photons into an electrical photocurrent $I(t)$. The information of interest is usually encoded in the non-zero mean-value of the photocurrent $\overline{I(t)}$ and can be affected by random fluctuations that we will describe as noise.

In optomechanics, the physical property under study is transduced somehow in the movement of a mirror whose displacements are further read by light. Whether it is the response to a monochromatic drive or random forces such as a thermal driving force, generated by the surrounding thermal bath, the signal is superimposed to fluctuations coming from noise measurement or backaction. A careful characterization of this noise is needed to gain access to the mechanical state of the system. To do so, one might want to use the variance of such a quantity $\sigma_I^2 = \overline{I^2(t)} - \overline{I(t)}^2$, however, the variance depends on the intrinsic light fluctuations and the detection characteristics. The autocorrelation function can be used to extract this information regardless of these setbacks.

1.2.1.1 Autocorrelation function

Given a complex random variable $x(t)$, the autocorrelation function C_{xx} describes the linear correlations between values of the stochastic process at different times. It is defined as follows [36]:

$$C_{xx}(t, \tau) = \langle x^*(t)x(t + \tau) \rangle \quad (1.30)$$

where τ is the time lag between two realizations of the process. The notation $\langle \dots \rangle$ refers to an ensemble average.

Using the ergodic hypothesis, the ensemble average is equivalent to a time average. Considering that we will be dealing exclusively with stationary processes, the autocorrelation function is time-independent and therefore only depends on the time difference τ :

$$C_{xx}(t, \tau) = C_{xx}(\tau) = \langle x(t)x(t + \tau) \rangle \quad (1.31)$$

One should note that $C_{xx}(\tau = 0) = \langle |x(0)|^2 \rangle$ is the variance of x .

1.2.1.2 Noise spectral density

From the autocorrelation function C_{xx} , one can directly compute the contribution of the total generated noise at a given frequency span. This quantity, which we will refer to as the *noise spectral density*, can be derived using the *Wiener-Khinchin theorem* stating that the noise spectral density $S_{xx}[\Omega]$ is equal to the Fourier-transform of C_{xx} .

In the rest of this manuscript, two notations will be used for frequency-dependent quantities. Indeed, to emphasize the difference between deterministic frequency-dependent functions, such as the mechanical susceptibility and the Fourier transform of a random variable, the Ω will be enclosed between parentheses for the former case and between square brackets for the latter. We also define the Fourier-transform of a time-dependent variable $\mathcal{A}(t)$ as:

$$\mathcal{A}[\Omega] \equiv \mathcal{F}[\mathcal{A}(t)] \equiv \int_{-\infty}^{+\infty} \mathcal{A}(t) e^{i\Omega t} dt, \quad (1.32)$$

and its inverse Fourier-transform as :

$$\mathcal{A}(t) = \int_{-\infty}^{+\infty} \mathcal{A} e^{-i\Omega t} \frac{d\Omega}{2\pi}. \quad (1.33)$$

The Wiener-Khinchin theorem With the statement of the theorem and the definition of a Fourier-transform, the noise spectral density reads, in units of $[x^2] \cdot \text{Hz}^{-1}$:

$$S_{xx}[\Omega] = \int_{-\infty}^{+\infty} C_{xx}(\tau) e^{i\Omega \tau} d\tau. \quad (1.34)$$

One can also express C_{xx} , if needed, from $S_{xx}[\Omega]$,

$$C_{xx}(\tau) = \int_{-\infty}^{+\infty} S_{xx}[\Omega] e^{-i\Omega \tau} \frac{d\Omega}{2\pi}. \quad (1.35)$$

The variance of x can be found by setting $\tau = 0$ in eq. 1.35 ,

$$\sigma_x^2 = \int_{-\infty}^{+\infty} S_{xx} [\Omega] \frac{d\Omega}{2\pi}. \quad (1.36)$$

Eq. 1.36 highlights that for a given unit frequency range $[\Omega, \Omega + d\Omega]$ [37], the variance gets a contribution equal to $S_{xx} [\Omega] d\Omega$.

Now, let us discuss some very useful properties of noise spectral density.

Properties of $S_{xx} [\Omega]$

1. For a real-valued process, one can easily show from eq. 1.34 the parity of the noise spectral density:

$$S_{xx} [\Omega] = S_{xx} [-\Omega]. \quad (1.37)$$

2. Let us assume two random variables $\zeta(t)$ and $\xi(t)$, the second one is the result of the first undergoing a transduction stage. Assuming a linear transduction relation, through an operator G , between the two i.e., $\xi(t) = G(\zeta(t))$, we obtain in Fourier space:

$$\xi [\Omega] = \chi(\Omega)\zeta [\Omega], \quad (1.38)$$

where $\chi(\Omega)$ is the deterministic frequency-dependent transduction transfer function. From eq. 1.38 we can deduce that the noise spectrum of $\xi [\Omega]$ is related to that of $\zeta [\Omega]$ by the square modulus of the $\chi(\Omega)$,

$$S_{\xi\xi} [\Omega] = |\chi(\Omega)|^2 S_{\zeta\zeta} [\Omega]. \quad (1.39)$$

1.2.2 Harmonic oscillator description of the mechanical motion

The mechanical system studied in this work is a multimode resonator coupled to a thermal bath and subjected to external forces. Each mode can be described as an independent harmonic oscillator and the effect of the thermal bath can be reduced to the action of a random force called the Langevin force. Fig. 1.6 depicts the effect of such a random force F_{th} on the equilibrium position of the oscillator at thermal equilibrium. The response of the mode is influenced by the environment's properties (temperature, pressure) and its mechanical properties (stiffness, mass). One can extract from studying this Brownian motion [38, 39] all the relevant properties of the resonator itself and its environment.

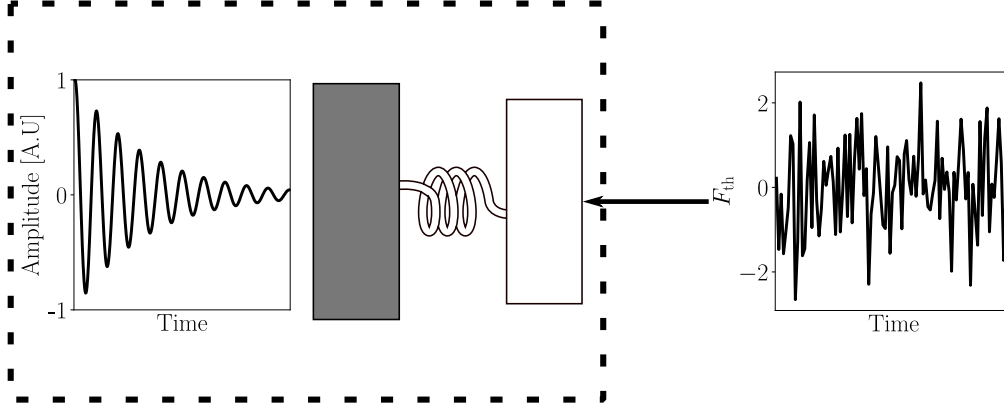


Figure 1.6: Mechanical harmonic oscillator coupled to a thermal bath. The latter drives the oscillator with a random force F_{th} .

In the following, we will consider only one of the modes of the resonator and model it by a damped and driven harmonic oscillator (mass on spring) and we will derive the equation of motion. The dissipation channel can be described as a viscous damping together with a random thermal force mentioned earlier. This force among others such as backaction, excitation, or feedback, are all reduced to an external driving force F_{ext} .

The equation of motion of such a system is quite straightforward to get, and reads:

$$\ddot{x}(t) + \Gamma_m \dot{x}(t) + \Omega_m^2 x(t) = \frac{F_{\text{ext}}}{m_{\text{eff}}}, \quad (1.40)$$

where we define an effective mass m_{eff} which takes into account that the displaced mass is not necessarily the total mass of the resonator, $\Omega_m = \sqrt{\frac{k}{m_{\text{eff}}}}$ is the mechanical resonance frequency with k the spring constant, and Γ_m the energy dissipation rate.

To solve eq. 1.40, one has to express it in the frequency domain using the definition of the Fourier transform in eq. 1.32¹, and thus reads:

$$x[\Omega] = \chi_m(\Omega) F_{\text{ext}}[\Omega], \quad (1.41)$$

where,

$$\chi_m(\Omega) = \frac{1}{m_{\text{eff}} [\Omega_m^2 - \Omega^2 - i\Gamma_m \Omega]}. \quad (1.42)$$

$\chi_m(\Omega)$ is the mechanical susceptibility, and it is a measure of the mechanical response of a given oscillator to external forces acting on it. The imaginary part of which is linked, in a straightforward

¹Useful identities: $\mathcal{F}[\dot{\mathcal{A}}(t)] = -i\Omega \mathcal{A}[\Omega]$ and $\mathcal{F}[\ddot{\mathcal{A}}(t)] = -\Omega^2 \mathcal{A}[\Omega]$.

manner, to the dissipation processes arising from the coupling between the resonator and its environmental thermal bath. This phenomenon is well described by the *Fluctuation-dissipation theorem* [40, 41].

1.2.2.1 Fluctuation-dissipation theorem

To illustrate the principle of the fluctuation-dissipation theorem (FDT), let us imagine a very small object floating in a liquid medium undergoing a random motion known as Brownian motion. Now, assuming an external force acting on the object as a driving force, its motion will endure friction caused by the impacts between the object and the particles of the liquid. The effect of such random impacts is twofold, on the one hand, they act as a random driving force maintaining the continuous Brownian motion, and on the other hand, they generate the frictional force. This suggests that both the **fluctuations** arising from the random force and the **dissipation** due to the frictional force have the same origin.

For our mechanical harmonic oscillator, the FDT states that with the dissipation process, described by the viscous force, comes a fluctuating thermal Langevin force F_T rendering the interaction of the oscillator with a thermal bath. This force has a zero mean-value and its noise spectrum is related to the imaginary part of the mechanical susceptibility:

$$S_{F_T}[\Omega] = -\frac{2k_B T}{\Omega} \text{Im} \left(\frac{1}{\chi(\Omega)} \right) = 2m_{\text{eff}} \Gamma_m k_B T, \quad (1.43)$$

where, k_B is the Boltzmann constant, and T is the oscillator's temperature. For an harmonic oscillator, the Langevin force presents a spectrum flat in frequency.

Having gained knowledge of the driving force, one can express the displacement noise spectrum of our oscillator which will be the cornerstone of the experimental work that will be described later on.

1.2.3 Noise spectrum of a harmonic oscillator

The position of the harmonic oscillator, described in eq. 1.41, takes the form of eq. 1.38 thus its noise spectrum can be deduced straightforwardly using the linear filter property of the noise spectral density (eq. 1.39) and reads:

$$S_{xx}[\Omega] = |\chi(\Omega)|^2 S_{F_T}[\Omega] = 2|\chi(\Omega)|^2 m_{\text{eff}} \Gamma_m k_B T. \quad (1.44)$$

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As depicted in fig. 1.7 the linear filtering of the Langevin thermal force's noise $S_{F_T}[\Omega]$ by the mechanical response function of the resonator $\chi(\Omega)$, generates a Lorentzian-shaped peak centered at the mechanical resonance frequency Ω_m .

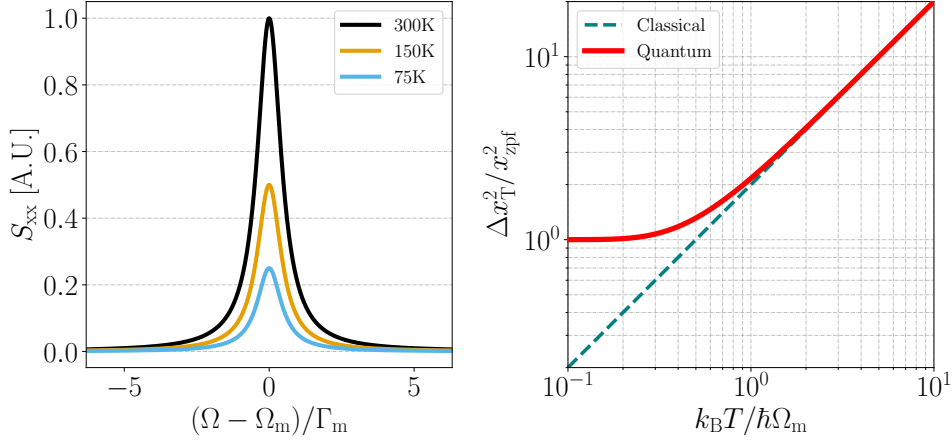


Figure 1.7: **Left:** Generic thermal noise spectra computed according to eq. 1.44, with $\Omega_m = 3$ GHz, $\Gamma_m = 10$ MHz, and $m_{\text{eff}} = 3$ pg. The spectra are plotted against the detuning to the mechanical resonance frequency Ω_m , for several temperatures. **Right:** Evolution of Δx_T^2 as a function of the temperature for the classical and quantum cases.

Using the definition of the variance (eq. 1.36), one can compute the variance of the thermal noise Δx_T^2 for a high Q-factor mechanical resonator i.e., $\Gamma_m \ll \Omega_m$ and it reads:

$$\Delta x_T^2 = \frac{k_B T}{m_{\text{eff}} \Omega_m^2}. \quad (1.45)$$

The above equation allows us to compute the mean potential energy stored in the oscillator as follows, and leads to the equipartition theorem:

$$\frac{1}{2} m_{\text{eff}} \Omega_m^2 \Delta x_T^2 = \frac{1}{2} k_B T. \quad (1.46)$$

Eq. 1.45 suggests that if one has access to the position fluctuations of a mechanical resonator, one can infer the environment temperature to which it is coupled.

Naturally, these relations are only valid in the classical regime i.e. $k_B T \gg \hbar \Omega_m$ or equivalently when n_{th} the mean number of thermal quanta in the system is much greater than 1. n_{th} is defined by the *Bose-Einstein statistics* as follows $n_{\text{th}}(\Omega) \equiv 1/(e^{\hbar \Omega / k_B T} - 1)$.

1.2.4 A quantum harmonic oscillator

The former classical description of the mechanical motion using the classical harmonic oscillator model can be extended to a quantum description of the same oscillator with mass m_{eff} , resonance frequency Ω_m and a decay rate Γ_m . The harmonic oscillator can be described with its position and momentum operators $\hat{x}$ and $\hat{p}$ which do not commute, unlike their classical analogs, and they verify the commutation relation $[\hat{x}, \hat{p}] = i\hbar$. The Heisenberg uncertainty principle dictates that one cannot assign exact simultaneous values to conjugated variables of a physical system which can be translated for the position and momentum operator as $\Delta\hat{x}\Delta\hat{p} \geq \hbar/2$.

The system's Hamiltonian can be expressed as follows:

$$\hat{H} = \frac{\hat{p}^2}{2m_{\text{eff}}} + \frac{m_{\text{eff}}\Omega_m^2}{2}\hat{x}^2. \quad (1.47)$$

It is convenient to introduce a set of new conjugated operators that we will define as a linear combination of our initial operators $\hat{x}$ and $\hat{p}$. These new operators, that we will refer to as the annihilation and creation operators $\hat{b}$ and $\hat{b}^\dagger$, are defined as follows:

$$\hat{b} = \sqrt{\frac{m_{\text{eff}}\Omega_m}{2\hbar}} \left(\hat{x} + i \frac{\hat{p}}{m_{\text{eff}}\Omega_m} \right), \quad (1.48)$$

$$\hat{b}^\dagger = \sqrt{\frac{m_{\text{eff}}\Omega_m}{2\hbar}} \left(\hat{x} - i \frac{\hat{p}}{m_{\text{eff}}\Omega_m} \right). \quad (1.49)$$

These operators satisfy the following commutation relation $[\hat{b}, \hat{b}^\dagger] = 1$. The Hamiltonian can now be expressed in terms of the creation and annihilation operators:

$$\hat{H} = \hbar\Omega_m \left(\hat{b}^\dagger \hat{b} + \frac{1}{2} \right), \quad (1.50)$$

where a constant term appears in the expression and is due to the non-commutativity of the position and momentum operators.

Now let us consider the eigenvectors $|n\rangle$ of the phonon number operator that is defined as $\hat{N} = \hat{b}^\dagger \hat{b}$ associated with the positive integer eigenvalue n . $|n\rangle$ are called the *Fock basis* and, in this basis, the

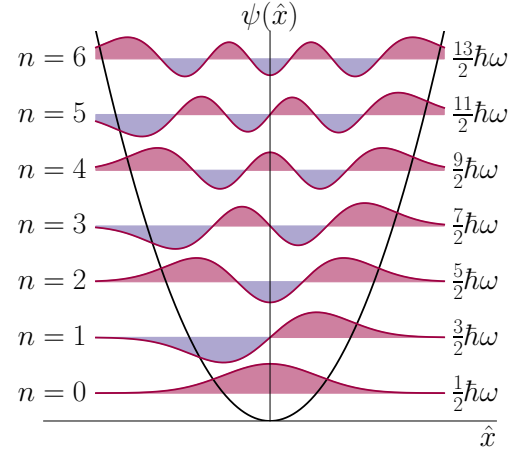


Figure 1.8: Quantized energy levels of a quantum harmonic oscillator.

annihilation and creation operator can be defined as:

$$\hat{\mathbf{b}} |n\rangle = \sqrt{n} |n-1\rangle, \quad (1.51)$$

$$\hat{\mathbf{b}}^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle. \quad (1.52)$$

The Hamiltonian defined in eq. 1.50 is diagonal with eigenenergies defined as:

$$E_n = \hbar\Omega_m \left(n + \frac{1}{2} \right). \quad (1.53)$$

We can see from the eigenenergy expression of a quantum harmonic oscillator that the energy levels are discrete and the lowest achievable energy i.e., in its fundamental state, is non-zero $E_0 = \hbar\Omega_m/2$ as fig. 1.8. This property, stemming from the Uncertainty Principle, implies that even in its lowest energy level such a system is animated with a motion which is referred to as the zero-point motion fluctuations that are defined as $x_{\text{ZPF}} = \sqrt{\frac{\hbar}{2m_{\text{eff}}\Omega_m}}$.

As for the classical case, we will compute the square displacement variance $\Delta\hat{\mathbf{x}}^2$:

$$\begin{aligned} \Delta\hat{\mathbf{x}}^2 &= \langle \hat{\mathbf{x}}^2 \rangle = \frac{\hbar}{2m_{\text{eff}}\Omega_m} \langle \hat{\mathbf{b}}^2 + \hat{\mathbf{b}}^{\dagger 2} + \hat{\mathbf{b}}\hat{\mathbf{b}}^\dagger + \hat{\mathbf{b}}^\dagger\hat{\mathbf{b}} \rangle \\ &= \frac{\hbar}{2m_{\text{eff}}\Omega_m} \langle \hat{\mathbf{b}}\hat{\mathbf{b}}^\dagger + \hat{\mathbf{b}}^\dagger\hat{\mathbf{b}} \rangle \\ &= \frac{\hbar}{m_{\text{eff}}\Omega_m} \left(\langle \hat{\mathbf{b}}^\dagger\hat{\mathbf{b}} \rangle + \frac{1}{2} \right) \\ &= \frac{\hbar}{m_{\text{eff}}\Omega_m} \left(\langle \hat{\mathbf{N}} \rangle + \frac{1}{2} \right) \end{aligned} \quad (1.54)$$

For a harmonic oscillator, composed of *bosonic particles* (phonons, for instance), in thermal equilibrium with its environment at a temperature T , the occupancy probability $p(n)$ of each energy level is given by the *Bose-Einstein statistics*:

$$p(n) = \exp\left(-\frac{\hbar\Omega_m n}{k_B T}\right) \left[1 - \exp\left(-\frac{\hbar\Omega_m}{k_B T}\right) \right]. \quad (1.55)$$

The mean occupancy of the oscillator is

$$n = \langle \hat{\mathbf{N}} \rangle = \sum_{n=0}^{\infty} n p(n) = \left[\exp\left(\frac{\hbar\Omega_m}{k_B T}\right) - 1 \right]^{-1}, \quad (1.56)$$

which allows us to write

$$\Delta\hat{\mathbf{x}}^2 = x_{\text{ZPF}}^2 \left(\frac{2}{\exp\left(\frac{\hbar\Omega_m}{k_B T}\right) - 1} + 1 \right). \quad (1.57)$$

From the last result, one can define a temperature that draws the border between the classical and quantum regimes. The classical result (eq. 1.45) and the above quantum result are compared in the right panel of fig. 1.7 where we can see that both results agree until a critical temperature $T_Q = \frac{\hbar\Omega_m}{k_B}$ where the quantum effects start to become dominant until they fully are for $T \ll T_Q$. Indeed, the variance becomes equal to the zero point fluctuations x_{ZPF} of the mechanical oscillator regardless of the temperature.

1.3 Optomechanical effects

So far we have considered independent optical and mechanical resonators and derived the relevant equations describing them. In the following, we will discuss some interaction phenomena arising from the interaction between optical and mechanical resonators mediated by an optical force. We will first describe its nature before moving into the static and dynamic effects that can occur when such a force is exerted.

1.3.1 Radiation pressure in optical cavities

Optomechanics studies the interaction between an optical field and a mechanical oscillator. On top of the thermal Langevin force F_T , the probe field drives the mechanical oscillator with a force that is referred to as *radiation pressure force* which can lead to interesting phenomena such as cooling of the mechanical motion or on the contrary to an amplification/heating process. This force takes its origin from a momentum transfer from the photon to the mechanical resonator. A single photon at a frequency ω_L has a momentum $\hbar\omega_L/c$, where $\hbar$ is the reduced Planck's constant and c the speed of light. If photons are fully reflected at normal incidence off the mirror then the radiation pressure force F_{RP} will be equal to twice the sum of the momentum of all photons reflected per unit of time and can be expressed as:

$$F_{\text{RP}}(t) = \frac{2\hbar\omega_L}{ct_{\text{rt}}} |a(t)|^2 = -\hbar G |a(t)|^2, \quad (1.58)$$

where the optomechanical coupling constant was introduced as $-\frac{\omega_L}{L}$, L being the cavity length, and where $a(t)$ refers to the intracavity field amplitude. We can consider that $\omega_L = \omega_c$ as photons only enter the cavity close to the optical resonance of the cavity.

For frequencies close to the resonance frequency of a particular mechanical mode, the mirror motion

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is mainly driven by this mode and the dynamics of the mirror subject to the thermal force F_T and radiation pressure force can be described by the following eq. 1.41

$$x[\Omega] = \chi_m(\Omega) (F_T[\Omega] + F_{RP}[\Omega]). \quad (1.59)$$

The radiation pressure coupling in a cavity results in coupled dynamics where the intracavity power depends on the cavity length, and in return, the cavity length depends on the mechanical response to radiation pressure, proportional to the intracavity power.

We can now write a set of coupled equations of motion describing the coupling of the optical and mechanical oscillators through the radiation pressure force. First, we will account for the optical cavity frequency modulation by the mechanical displacement that we can express at first order as

$$\omega_c(x) \approx \omega_c + x \partial\omega_c/\partial x = \omega_c - Gx, \quad (1.60)$$

where we define the optomechanical coupling parameter $G = -\partial\omega_c/\partial x$ as the optical frequency shift per displacement. Using eqs. 1.17 for a single input optical cavity and eq. 1.40 describing the dynamics of a damped and driven mechanical oscillator, the coupled equations of motion read:

$$\dot{a}(t) = [i(\Delta + Gx(t)) - \kappa/2] a(t) + \sqrt{\kappa_c} s^{\text{in}}(t), \quad (1.61)$$

$$m_{\text{eff}} (\ddot{x}(t) + \Gamma_m \dot{x}(t) + \Omega_m^2 x(t)) = F_T(t) + F_{RP}(t). \quad (1.62)$$

In the following, a description of the static and dynamic effects of radiation pressure will be discussed.

1.3.2 Static effects

We will first study the static solution $a(t) = \bar{a}$ and $x(t) = \bar{x}$ of eqs. 1.61 and 1.62 in which all the derivatives are set to zero. The coupled equations describing the mean optical fields and mean displacement of the mirror read

$$\bar{a} = \frac{\sqrt{\kappa_c}}{\kappa/2 - i(\Delta + G\bar{x})} \bar{s}^{\text{in}}, \quad (1.63)$$

$$\bar{x} = \frac{2\hbar\omega_L}{m_{\text{eff}}\Omega_m^2 c} |\bar{a}|^2 = -\hbar G \chi_m(0) |\bar{a}|^2. \quad (1.64)$$

Injecting eq. 1.64 into eq. 1.63 and computing the square modulus of the latter we get the following third-order equation:

$$|\bar{a}|^2 + \frac{4|\bar{a}|^2}{\kappa^2} \left(\Delta - \hbar G^2 \chi_m(0) |\bar{a}|^2 \right)^2 - \kappa_c |\bar{s}^{\text{in}}|^2 = 0 \quad (1.65)$$

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This equation has at least one solution and two additional solutions are possible for high enough

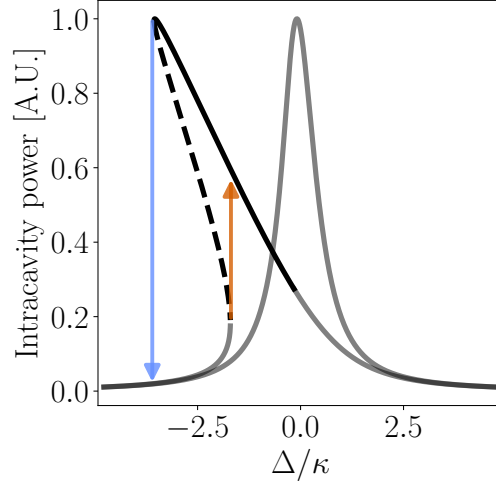


Figure 1.9: Radiation pressure static effect inducing a deformation of the intracavity intensity Lorentzian peak. The solid light black curve and the bistable curve were computed for input powers of 10 nW and 0.4 μ W with the InGaP optomechanical parameters described in table 1.1.

optical power or finesse. Indeed, these extra solutions exist for high enough input photon flux $|\bar{s}^{\text{in}}|^2$: the maximum slope of the square modulus of eq. 1.63, at $\Delta = \pm \frac{\kappa}{2\sqrt{3}}$, should be larger than the slope $[-\hbar G \chi_m(0)]^{-1}$ corresponding to eq. 1.64,

$$|\bar{s}^{\text{in}}|^2 \geq \frac{\sqrt{3}}{9} \frac{\kappa^2}{\eta_c \hbar G^2 \chi_m(0)}. \quad (1.66)$$

When the input power exceeds the above threshold, a bistable behavior [42, 43] of the intracavity power can be observed as depicted in fig. 1.9, where we see the three different solutions of eq. 1.65. Two stable solutions are depicted in two shades of black with solid lines and the unstable one is depicted in a black dashed line. This effect induces a hysteretic behavior where depending on the direction of the detuning sweep one can access different parts of the curve as shown with the blue downgoing arrow (from blue to red detuning) and the orange upgoing arrow (from red to blue detuning).

1.3.3 Dynamical effects

Let us now describe the dynamical effects of radiation pressure force which are due to its delayed nature. For that purpose, let us linearize eqs. 1.61 and 1.62 around their mean-values such that

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$a(t) = \bar{a} + \delta a(t)$ and $x(t) = \bar{x} + \delta x(t)$. We will introduce, for the sake of simplification, a static detuning defined as:

$$\bar{\Delta} = \Delta + G\bar{x}. \quad (1.67)$$

By a global phase redefinition, it is always possible to consider the mean intracavity field $\bar{a}$ as real ($\bar{a} = \bar{a}^*$). The linearized equations then read:

$$\delta \dot{a}(t) = \left(i\bar{\Delta} - \kappa/2 \right) \delta a(t) + iG\bar{a}\delta x(t), \quad (1.68)$$

$$m_{\text{eff}} \left[\delta \ddot{x}(t) + \Gamma_m \delta \dot{x}(t) + \Omega_m^2 \delta x(t) \right] = -\hbar G \bar{a} (\delta a(t) + \delta a^*(t)) + F_T(t), \quad (1.69)$$

where the second-order terms $\delta a(t)\delta x(t)$ and $|\delta a(t)|^2$ were dropped as we have assumed that $|\delta a| \ll |\bar{a}|$.

By applying a Fourier transformation to all time-dependent variables we can solve the above equations in the frequency domain. We obtain the following set of equations

$$-i\Omega \delta a[\Omega] = (i\bar{\Delta} - \kappa/2) \delta a[\Omega] + iG\bar{a}\delta x[\Omega], \quad (1.70)$$

$$-i\Omega \delta a^*[\Omega] = (-i\bar{\Delta} - \kappa/2) \delta a^*[\Omega] - iG\bar{a}\delta x[\Omega], \quad (1.71)$$

$$\chi_m(\Omega)^{-1} \delta x[\Omega] = -\hbar G \bar{a} (\delta a[\Omega] + \delta a^*[\Omega]) + F_T[\Omega]. \quad (1.72)$$

the identity $\delta a^*[\Omega] = (\delta a[-\Omega])^*$ was used to find the complex conjugate of eq. 1.70. We can see that for a non-zero displacement amplitude $\delta x[\Omega]$, sidebands appear at a frequency Ω with amplitudes

$$\delta a[\Omega] = \frac{-iG\bar{a}}{-\left(\bar{\Delta} + \Omega\right) + \kappa/2} \delta x[\Omega], \quad (1.73)$$

$$\delta a^*[\Omega] = \frac{iG\bar{a}}{\left(\bar{\Delta} - \Omega\right) + \kappa/2} \delta x[\Omega], \quad (1.74)$$

where the amplitudes described by eqs. 1.74 and 1.73 are referred to as the *Stokes* and *anti-Stokes* sidebands, respectively. The intracavity field gets modulated by the displacement of the cavity as one can see from the above equations and in turn, it will give rise to an oscillating radiation pressure force that takes the following form

$$\begin{aligned} \delta F_{\text{RP}}[\Omega] &= -\hbar G \bar{a} (\delta a[\Omega] + \delta a^*[\Omega]) \\ &= -\hbar G^2 \bar{a}^2 \left[\frac{\bar{\Delta} + \Omega}{\left(\bar{\Delta} + \Omega\right)^2 + (\kappa/2)^2} + \frac{\bar{\Delta} - \Omega}{\left(\bar{\Delta} - \Omega\right)^2 + (\kappa/2)^2} \right] \delta x[\Omega] \\ &\quad + i\hbar G^2 \bar{a}^2 \left[\frac{\kappa/2}{\left(\bar{\Delta} + \Omega\right)^2 + (\kappa/2)^2} - \frac{\kappa/2}{\left(\bar{\Delta} - \Omega\right)^2 + (\kappa/2)^2} \right] \delta x[\Omega]. \end{aligned} \quad (1.75)$$

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This oscillating force is complex-valued where its real part describes the *optical spring effect* and the imaginary part contributes to the damping of the mechanical oscillator via *optomechanical damping*. In summary, this optical force changes the dynamical response of the mechanical oscillator under the effect of external perturbations. This becomes evident when one substitutes the expression of the oscillating force (eq. 1.75) in eq. 1.72 as one gets

$$\delta x [\Omega] = \chi_{\text{eff}}(\Omega) F_T [\Omega] = \left[\left(\Omega_m^2 + k_{\text{opt}} \right) - \Omega^2 - i (\Gamma_m + \Gamma_{\text{opt}}) \Omega \right]^{-1} F_T [\Omega], \quad (1.76)$$

here we introduced the additional damping and spring constant induced by dynamical backaction and their expression, respectively, are

$$\Gamma_{\text{opt}} = \frac{\hbar G^2 \bar{a}^2}{m_{\text{eff}} \Omega} \left[\frac{\kappa/2}{(\bar{\Delta} + \Omega)^2 + (\kappa/2)^2} - \frac{\kappa/2}{(\bar{\Delta} - \Omega)^2 + (\kappa/2)^2} \right], \quad (1.77)$$

$$k_{\text{opt}} = \hbar G^2 \bar{a}^2 \left[\frac{\bar{\Delta} + \Omega}{(\bar{\Delta} + \Omega)^2 + (\kappa/2)^2} + \frac{\bar{\Delta} - \Omega}{(\bar{\Delta} - \Omega)^2 + (\kappa/2)^2} \right]. \quad (1.78)$$

Finally, the effective damping rate and effective mechanical frequency respectively read for a weak laser drive:

$$\Gamma_{\text{eff}} = \Gamma_m + \frac{\hbar G^2 \bar{a}^2}{m_{\text{eff}} \Omega_m} \left[\frac{\kappa/2}{(\bar{\Delta} + \Omega_m)^2 + (\kappa/2)^2} - \frac{\kappa/2}{(\bar{\Delta} - \Omega_m)^2 + (\kappa/2)^2} \right], \quad (1.79)$$

$$\Omega_{\text{eff}} = \Omega_m + \frac{\hbar G^2 \bar{a}^2}{m_{\text{eff}} \Omega_m} \left[\frac{\bar{\Delta} + \Omega_m}{(\bar{\Delta} + \Omega_m)^2 + (\kappa/2)^2} + \frac{\bar{\Delta} - \Omega_m}{(\bar{\Delta} - \Omega_m)^2 + (\kappa/2)^2} \right]. \quad (1.80)$$

Optical spring effect

Let us first take a look at eq. 1.80 describing the effect of dynamical backaction on the mechanical resonance frequency. In the unresolved sideband regime i.e., $\kappa \gg \Omega_m$, the effective mechanical frequency reads:

$$\Omega_{\text{eff}} = \Omega_m + \frac{\hbar G^2 \bar{a}^2}{m_{\text{eff}} \Omega_m} \frac{2\bar{\Delta}}{(\kappa/2)^2 + \bar{\Delta}^2}. \quad (1.81)$$

Three cases can be distinguished:

- **Red-detuned laser beam** ($\bar{\Delta} < 0$): the mechanical resonator is spring-softened inducing a decrease in the mechanical frequency.

- **Blue-detuned laser beam** ($\bar{\Delta} > 0$): the mechanical resonator is spring-hardened inducing an increase of the mechanical frequency.
- **Resonant laser beam** ($\bar{\Delta} = 0$): the optical spring effect vanishes leaving the mechanical frequency unchanged.

Optomechanical damping

Similarly to the optical spring effect, several cases can be distinguished from eq. 1.79:

- **Red-detuned laser beam** ($\bar{\Delta} < 0$): yielding a positive additional damping $\Gamma_{\text{opt}} > 0$ which increases the effective damping rate of the mechanical oscillator. In other terms, the motion of the mechanical resonator is effectively decreased which in turn leads to a decrease in the effective temperature of the mechanical mode. In this case, dynamical backaction leads to cooling.
- **Blue-detuned laser beam** ($\bar{\Delta} > 0$): conversely, this case introduces anti-damping $\Gamma_{\text{opt}} < 0$ to the system which tends to amplify the thermal fluctuations. The motion of the mechanical resonator is effectively increased, which leads to a heating of the mechanical mode.
- **Resonant laser beam** ($\bar{\Delta} = 0$): The dynamical backaction induced damping cancels out $\Gamma_{\text{opt}} = 0$. At resonance, no optomechanical effect is observed.

In the amplification regime ($\bar{\Delta} > 0, \Gamma_{\text{opt}} < 0$), an instability can occur when the effective damping rate becomes negative $\Gamma_{\text{eff}} < 0$ which leads to an exponential growth of any small fluctuations (mostly, thermal fluctuations) until reaching a steady-state regime. This effect is known as self-induced or backaction-induced optomechanical oscillations and can be described as a lasing effect in a mechanical system where the input laser field acts as the drive pump.

Self-induced optomechanical oscillation

Self-induced optomechanical oscillations were observed on a Gallium Phosphide (GaP) optomechanical system (optomechanical parameters are displayed in table 1.1) using the experimental apparatus described in sec. 5.2.1 where the frequency of a laser is swept across the blue side of the optical resonance ($\bar{\Delta} > 0$) with a measured input laser power of $P_{\text{in}} = 100 \mu\text{W}$. Fig. 1.11.a depicts two mechanical spectra acquired at different detunings. The black curve was acquired close to optical

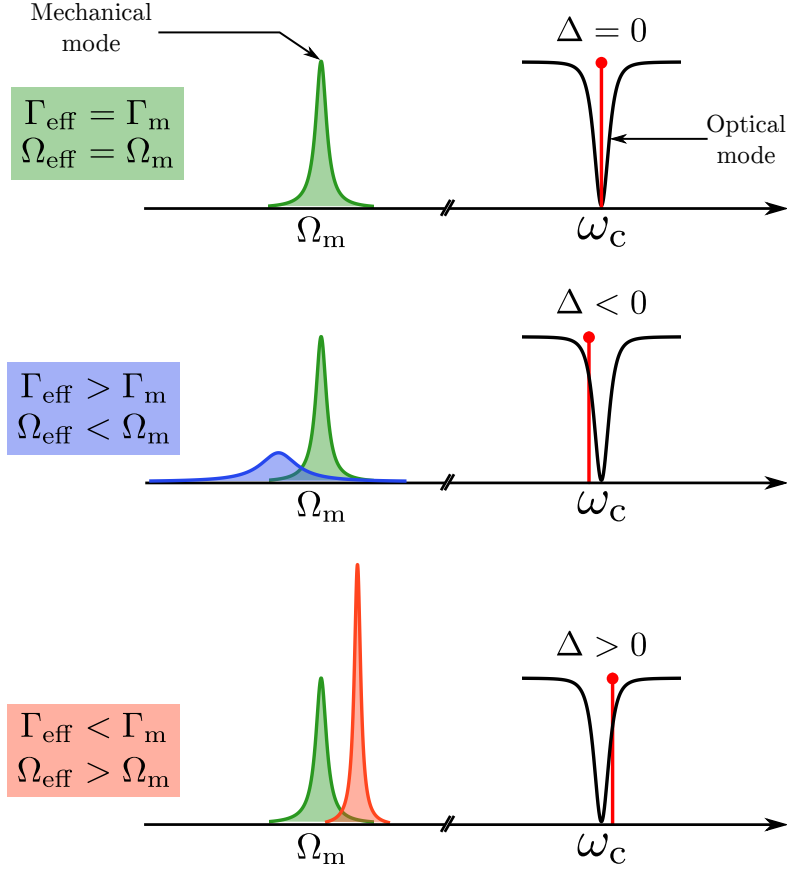


Figure 1.10: Schematic of the dynamical backaction effects as a function of the cavity detuning. **Top:** No dynamical backaction effect at resonance, $\Delta = 0$. **Middle:** The temperature of the mechanical mode is cooled due to radiation pressure induced extra-damping when the laser is red-detuned, $\Delta < 0$. **Bottom:** Amplification of the thermal motion due to radiation pressure induced anti-damping when the laser is blue-detuned, $\Delta > 0$.

resonance, where the dynamical effects of radiation pressure are negligible and translates the bare response of the mechanical oscillator to the thermal force. Its linewidth is proportional to the bare decay rate Γ_m whereas on the orange curve, acquired at a larger detuning, the linewidth is narrower due to the dynamical backaction induced anti-damping ($\Gamma_{\text{opt}} < 0$ i.e., $\Gamma_{\text{eff}} < \Gamma_m$). An amplification effect also occurs, where at the maximum, a four-hundred-fold amplification factor is observed. In the regime of self-oscillations, the motion of the resonator should in principle diverge. Non-linear effects, either mechanical or optical, keep the motion in a finite range. Simultaneously, an increase in the mechanical resonance frequency is also visible, caused by the optical spring effect that increases the effective spring constant of the mechanical oscillator. The two latter effects are displayed for various detuning values in fig. 1.11.b, where we observe a frequency shift of the mechanical resonance

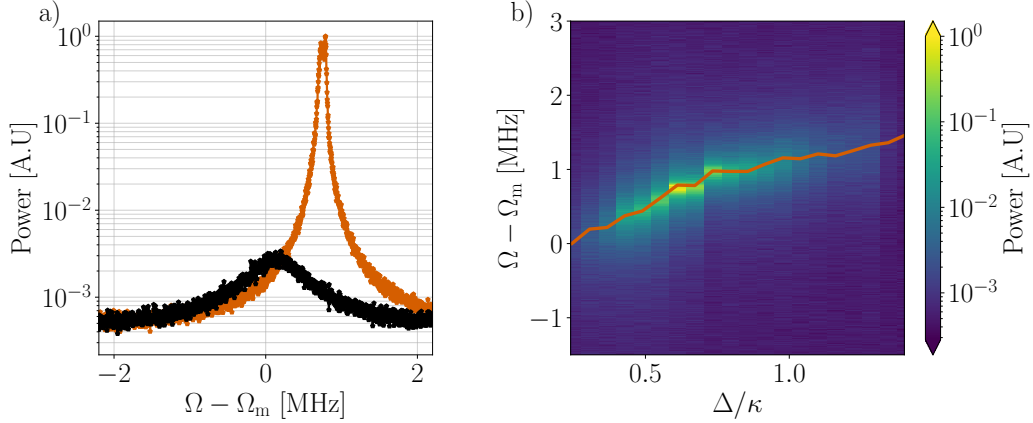


Figure 1.11: Observation of self-induced oscillations in the amplification regime $\Delta > 0$. **a)** Mechanical spectra acquired for two values of detunings. **b)** Evolution of the mechanical frequency and peak amplitude as a function of detuning.

frequency (orange line) $\Omega_{\text{eff}} - \Omega_m = 1.4$ MHz, and where we see the amplification evolution with the detuning with the color scale.

	Parameters						
	Optics		Mechanics			Optomechanics	
Material	κ [GHz]	η_c	m_{eff} [fg]	Ω_m [GHz]	Γ_m [MHz]	G [Hz · m ⁻¹]	x_{ZPF} [fm]
GaP	$2\pi \times 6.5$	0.8	1.07	$2\pi \times 2.92$	$2\pi \times 1.2$	1.47×10^{21}	1.64
InGaP	$2\pi \times 20$	0.48	1.23	$2\pi \times 2.8$	$2\pi \times 8.5$	9.5×10^{20}	1.56

Table 1.1: Table summarizing the relevant optical, mechanical, and optomechanical parameters of two optomechanical systems. A first system made of GaP ² and a second one made of InGaP [44].

These self-induced oscillations occur when input optical power is set at a threshold that can be estimated according to the condition that the mechanical damping rate equates the optomechanical anti-damping i.e., $\Gamma_{\text{eff}} = \Gamma_m + \Gamma_{\text{opt}} = 0$. This input optical threshold can be estimated by introducing the latter condition on the effective damping in eq. 1.79 and has been estimated for two optomechanical systems whose optical, mechanical, and optomechanical parameters are displayed in table 1.1. The optomechanical systems discussed here are 1D optomechanical crystals that will be further introduced in chapter 3. Fig. 1.12 depicts the input optical power needed to put the mechanical system in the unstable regime where these oscillations can be observed for a given detuning. The threshold corresponds to the minimum of both curves, so for the GaP optomechanical system a power threshold

²Its optomechanical parameters are, as of yet, unpublished.

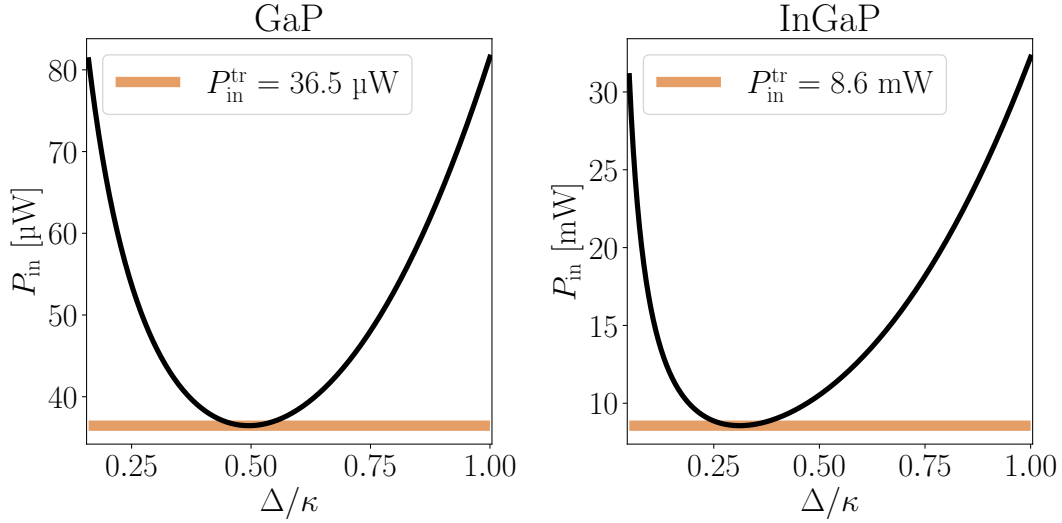


Figure 1.12: Input power threshold needed to set the mechanical system into the self-oscillation regime as a function of detuning for the GaP and InGaP optomechanical systems.

of $P_{\text{in}}^{\text{tr}} = 36.5 \mu\text{W}$ [44] whereas, for an Indium Gallium Phosphide (InGaP) system, that will be used in the rest of this work, the threshold is $P_{\text{in}}^{\text{tr}} = 8.6 \text{ mW}$.

In the following, the temperature will be estimated from the area under the mechanical resonance peak (eq. 1.45) which implies that the dynamical radiation pressure force effects should be minimized as much as technically possible. The detuning should be set close to optical resonance to mainly avoid the optomechanical damping effect and the input optical power should be set below the aforementioned threshold.

Chapter 2

Introduction to temperature metrology

2.1 Introduction

Temperature is one of the most important measurements performed in our daily lives. Whether it is the temperature of the human body which indicates its health state or the weather forecast which indicates several properties and most importantly the temperature of the environment in which we are set into motion.

In our daily lives, depending on the zone of the globe we live in we use a set of temperature scales that are defined relative to a set of reference points. The Fahrenheit scale defined by the physicist Daniel Fahrenheit in 1724, was an improved version of Danish astronomer Ole Christensen Rømer's scale which was defined on two reference points, the first was the boiling point of water and the second was the freezing point of a saltwater mixture. Rømer divided the space between the two points into 60 evenly spaced degrees. Fahrenheit added resolution to Rømer's scale with 4 times the number of degrees separating the boiling and freezing points of water. In its final iteration, the scale was defined such that the freezing point of water and the boiling point of the saltwater solution were set to 32° F and 212° F, respectively. A decade later, Swedish astronomer Anders Celsius developed the second most used temperature scale which bears his name, Celsius. The latter was defined with the same reference points namely the boiling and freezing points of water which were set to 0 °C and 100 °C. After Celsius' death, the points were swapped giving birth to the scale we know nowadays.

An absolute temperature scale was developed by Scots-Irish physicist William Thomson, 1st baron Kelvin, impulsed by the idea that there is a minimum temperature that can be achieved which was referred to at the time as "infinite cold" and became, nowadays, known as the absolute zero. The

2.1. INTRODUCTION

scale bearing his name (Kelvin) has increments with the same magnitude as the Celsius scale's degrees but instead of setting the scale's 0 to an arbitrary reference (freezing point of water), its 0 was set to the coldest point possible for matter i.e. $0\text{ K} = -273.16^\circ\text{C}$. In the development of such an absolute temperature scale, Lord Kelvin defined an absolute temperature which is also called thermodynamic temperature.

The temperature scale T_{90} (expressed in units of kelvin) must follow the updated version of the International Temperature Scale "ITS-90" [45]. ITS-90 defines some practical reproducible temperature references named "fixed points" used to calibrate an interpolating instrument following a procedure defined in the ITS-90 recommendations. By definition, the temperature values of these fixed points of the ITS-90 have no uncertainty whereas their *mise-en-pratique* generates an experimental uncertainty (measurement noise, instrument stability, reference stability, thermal gradient, material purity..). The goal of the temperature scale is to disseminate the kelvin definition to users all around the world at an affordable cost and with a state-of-the-art uncertainty level. As an example, the calibration uncertainty of a contact resistive (platinum) thermometer using state-of-the-art experimental set-up and following ITS-90 recommendations, can stand below 0.5 mK from 24 K to 300 K (room temperature). The temperature values of these fixed points have been fixed by the Comité Consultatif de Thermométrie (CCT) after a deep analysis of their thermodynamic temperatures which have been determined by several National Metrology Institutes (NMI) within the frame of an international key comparison organized by BIPM-CCT. This procedure guarantees independent measurements provided by the NMIs following the protocol defined by the intercomparison. Each NMI must determine the thermodynamic temperature using its own independent primary thermometer.

The International Temperature Scale is periodically revised to fit the thermodynamic temperature values of its fixed points recently determined with improved primary thermometers. The difference between scale temperature and thermodynamic temperature " $T - T_{90}$ " being fitted over the full temperature range covered by the ITS-90. From 4 K up to 300 K, " $T - T_{90}$ " stands within $\pm 10\text{ mK}$ with an attached uncertainty below 2 mK.

2.2 Thermodynamic temperature

2.2.1 Definition

The thermodynamic temperature is defined as a measure of the average energy of matter's degrees of freedom (translational, vibrational, and rotational). Let us consider, for simplicity, a gas formed of point-like particles interacting by elastic collisions, and the mean free path¹ is larger than their diameter. An ideal gas, in substance.

In the case of a monoatomic gas, the kinetic energy of the individual particles of the gas consists only of the energy of the individual translational degrees of freedom. The internal energy $U(n, T)$ of such a gas can be defined as follows:

$$U(n, T) = nc_V T, \quad (2.1)$$

here c_V is the molar heat capacity at constant volume V², n is the number of gas particles, and T is the thermodynamic temperature. For a monoatomic gas, the molar heat capacity is constant $c_V = 3/2R$ with $R = N_A k_B$ is the ideal gas constant (N_A is the Avogadro constant). The internal energy for a monoatomic gas reads:

$$U(n, T) = \frac{3}{2} k_B T. \quad (2.2)$$

This result can also be derived from the equipartition theorem which states that, at thermal equilibrium, each kinetic degree of freedom stores exactly $\frac{1}{2} k_B T$ of average kinetic energy, thus for an ideal gas with three translational degrees of freedom, its average energy will be $3 \times \frac{k_B T}{2}$.

The thermodynamic temperature is an absolute quantity related to a physical phenomenon occurring within a given physical system. Its unit is fixed to be the *kelvin*.

A primary thermometer is based on a well-understood physical system, for which the equation of state describing the relation between thermodynamic temperature T and other independent quantities, such as the ideal gas law or Planck's equation, can be expressed explicitly without unknown or significantly temperature-dependent constants. Thermodynamic temperature can be obtained by measuring the independent quantities. Accurate thermodynamic temperature values require not only accurate measurements of the independent quantities but also a sufficient understanding of the system to enable

¹Average distance over which the particles travel before their direction is changed due to a collision.

² c_V translates the energy that must be added to one mole of a chemical compound such that its temperature increases with one unit.

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a quantitative assessment of departures from the ideal model to apply appropriate corrections.

Absolute primary thermometry allows measuring thermodynamic temperature directly in terms of the definition of the base unit kelvin, i.e. the defined numerical value of the Boltzmann constant. No reference is made to any temperature fixed point ($n = 0$, n is the number of fixed points) and all other parameters specified in the equation of state are measured or otherwise determined.

Relative primary thermometry allows measuring thermodynamic temperature indirectly using a specified equation of state, with one or more key-parameter values determined from temperature fixed points ($n > 0$), for which values for the thermodynamic temperature T and their uncertainties are known a priori from previous absolute or relative primary thermometry.

2.2.2 The kelvin

The kelvin is the standard measurement unit of thermodynamic temperature defined by the International System of units (SI). The 13th *General Conference on Weights and Measures* (CGPM) defined the unit increment of thermodynamic temperature (kelvin) as follows:

” The kelvin, symbol K, unit of thermodynamic temperature, is equal to the fraction $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water ”

This definition of the kelvin prevailed for several decades until it was noted that the isotopic ratio between hydrogen and oxygen making up the water sample influences the triple point which in turn induces variability in the different realization of the water triple point. The *International Committee for Weights and Measures* (CIPM) proposed a redefinition of the different units of the SI system among which the *kelvin*.

In 2019, the kelvin, and the other SI units, were redefined in terms of fundamental constants (see fig. 2.1). In particular, the kelvin was redefined using a fixed value of the Boltzmann constant $k_B = 1.380\,649 \times 10^{-23} \text{ J K}^{-1}$ [46]. The kelvin redefinition reads:

”The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k_B to be $1.380\,649 \times 10^{-23}$ when expressed in the unit J K^{-1} , which is equal to $\text{kg m}^2 \text{s}^{-2} \text{K}^{-1}$, where the kilogram, metre, and second are defined in terms of h , c and $\Delta\nu_{\text{Cs}}$.”

2.2. THERMODYNAMIC TEMPERATURE

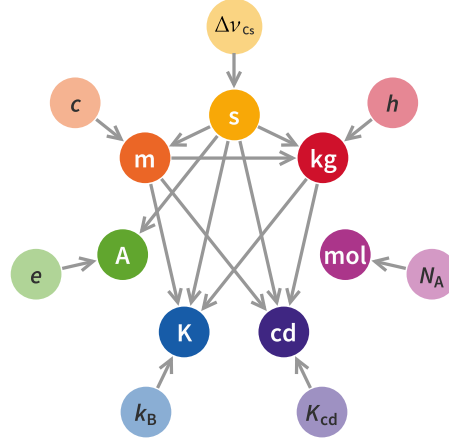


Figure 2.1: SI units with their corresponding fundamental constants. Emilio Pisanty, CC BY-SA 4.0, via Wikimedia Commons

h , c , and $\Delta\nu_{\text{Cs}}$ are, respectively, the Planck constant, the vacuum speed of light, and the groundstate hyperfine transition frequency of cesium 133 atoms.

Having defined the thermodynamic temperature and its measurement unit, one can wonder how to measure this quantity in an absolute way. In the following, a theoretical description of two measurement methods will be given. The first one, in the Johnson noise thermometry which will be the basis of the experiments discussed in chapter 5, and a second method relying on an optomechanical effect induced by the force exerted by light grains on a thermally driven mechanical oscillator.

The first absolute primary thermometer was the constant volume gas thermometer [47] based on the perfect gas equation of state together with Amagat diagram³. The absolute primary thermometer having the highest accuracy has been used for the last determination of the Boltzmann constant: it is the acoustic gas thermometer [46] based on the speed of sound in a resonator filled with gas. A third absolute primary thermometer is the Johnson gas thermometer which is based on the thermal agitation of the charge carriers inside an electrical conductor and the fluctuation-dissipation theorem as its theoretical description.

This work aims to develop and study a primary thermometer based on the optical reading of the thermal motion of an optomechanical resonator and the fluctuation-dissipation theorem as its theoretical description.

³The Amagat diagram describes, for a given fluid at a constant temperature, the variation of the pressure-volume product as a function of the applied pressure.

2.3 Mise en pratique of the kelvin: Johnson noise thermometry

2.3.1 Johnson demonstration

Johnson noise is an electronic noise that is caused by the random thermal motion of charge carriers within electrical conductors. It was first predicted by Einstein [39] whilst making sense of the Brownian motion of suspended particles, then it was measured and reported by Johnson [48], and finally, Nyquist derived the theoretical description of the effect [49].

In his 1928 paper, Nyquist stated that the power spectral density of the noise voltage across a conductor whose complex impedance will be noted $Z(\nu)$, in thermal equilibrium at a temperature T is given by

$$S_V(\nu) = 4h\nu \operatorname{Re}[Z(\nu)] \left[\frac{1}{2} + \frac{1}{e^{h\nu/k_B T} - 1} \right]. \quad (2.3)$$

The first term between the square brackets was added later to account for the zero-point energy fluctuations of the system, which is relevant for low temperatures. Johnson noise is usually characterized by its mean-square voltage $\langle V^2 \rangle$ which for frequencies below 1 MHz and temperatures above 25 K takes the following form

$$\langle V^2 \rangle = 4k_B T R \Delta\nu, \quad (2.4)$$

where $\Delta\nu$ is the bandwidth over which the noise is measured and R is the resistance of the measured system. A generalization of Nyquist's theorem relating thermal fluctuations in a linear dissipative system to a generalized resistance was introduced by Callen and Welton [40] and called the fluctuation-dissipation theorem, already introduced in sec. 1.2.2.1

Thermodynamic temperature can be inferred provided the measurement of the power spectral density $S_V(\nu)$ produced by a resistor of known resistance R making it a good candidate for temperature measurements. *Johnson noise thermometry* (JNT) relies on Nyquist's theorem for the measurement of the thermodynamic temperature. Furthermore, as long as the resistance can be measured precisely (eq. 2.4) and is insensitive to chemical or mechanical changes, this sensor is self-calibrated and is a promising candidate for the dissemination of the newly redefined kelvin.

The main drawback of JNT is that the quantity that is measured, namely the mean-square voltage is very small. For instance, the noise signal produced by a $100\,\Omega$ resistance at 300 K measured with a bandwidth of $\Delta\nu = 100\,\text{kHz}$ is of about $0.4\,\mu\text{V}$ rms. It follows that all extra noises must be

2.3. MISE EN PRATIQUE OF THE KELVIN: JOHNSON NOISE THERMOMETRY

eliminated and the amplifiers used have to be designed with high constraints on their gain, added noise, bandwidth, etc ...

2.3.2 Johnson-noise thermometry

2.3.2.1 Relative Johnson noise thermometry

A simplified schematic diagram of a Johnson noise thermometer is depicted in fig. 2.2. The output signal at the left end of the circuit should ideally be proportional to the mean-square noise voltage produced by the resistor. A noise source consisting of a resistor of resistance R_{ref} at a given temperature T_{ref} is usually used as a reference such that the amplifier's gain and bandwidth, which are hard to quantify precisely, need not be known.

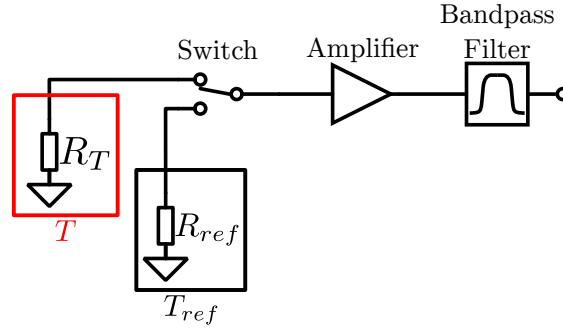


Figure 2.2: Simplified schematic of Johnson-noise thermometry setup.

The noise voltage of the reference resistor is first measured. According to eq. 2.4 this voltage would read:

$$\langle V_{ref}^2 \rangle = 4k_B T_{ref} R_{ref} \Delta\nu. \quad (2.5)$$

The noise voltage of the sensing resistance R_T , at thermal equilibrium with a bath at an unknown temperature T , is then measured and can be expressed as:

$$\langle V_T^2 \rangle = 4k_B T R_T \Delta\nu. \quad (2.6)$$

The temperature T is then inferred from the ratio of eq. 2.6 and eq. 2.5 which, ideally, is independent of the amplifier's gain and bandwidth. The temperature takes the following form:

$$T = T_{ref} \frac{\langle V_T^2 \rangle}{\langle V_{ref}^2 \rangle} \frac{R_{ref}}{R_T}. \quad (2.7)$$

2.3.2.2 Absolute Johnson noise thermometry

In 1959 a method allowing to measure temperature in an absolute way using Johnson noise was introduced by Fink [50] relying on correlation measurements that would allow the elimination of the lead resistance and the amplifier noise.

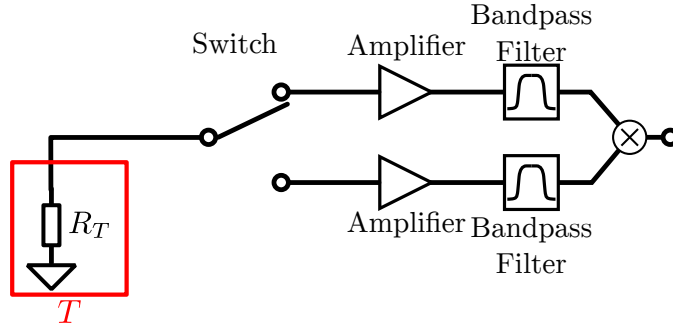


Figure 2.3: Correlation noise thermometry setup.

A simplified schematic is depicted in fig. 2.3 describing the correlation setup consisting of two parallel amplifiers and filter channels, and an electronic mixer. The noise voltage of a resistance, in thermal equilibrium with its environment at a temperature T , is measured through both amplifying channels and then the correlator of these two signals is computed with the mixer. This method allows the rejection of any noise that is not common to both channels. The effect of the noise voltages in the leads and the amplifiers are eliminated leaving only the signal of interest.

2.4 Quantum correlation thermometry

Thermodynamic temperature can also be inferred from optically probed systems, such as optomechanical systems which are driven by an external random thermal force that induces a measurable displacement of the mechanical harmonic oscillator.

The Johnson-noise thermometer described in the previous section has its optomechanical analog. We have shown that the variance of the displacement of an optomechanical system is proportional to the thermodynamic temperature, provided the absence of optomechanical effects (heating or cooling) as described by eq. 1.45. The measurement of temperature through the noise spectrum of an optomechanical system can be challenging as one has to calibrate in absolute terms the optical detection. Several methods can be used to circumvent this issue, one which will be thoroughly described

in chapter 5, relies on imprinting an external phase modulation on the probe field that will be used as a scale to calibrate each measurement. The second one, demonstrated by T. P. Purdy *et. al* [28] in 2016, relies on the observation of optomechanical quantum correlations between the intensity and phase quadratures of the field that interacted with the optomechanical system. They demonstrated a cross-correlation method allowing them to distinguish thermal motion from optically induced motion. The temperature is inferred by calibrating the thermal motion with the scale of the quantum correlations which is defined by fundamental constants. In the following, a theoretical description of the quantum correlation method will be given.

2.4.1 Hamiltonian description

Let us consider a single optical cavity mode coupled to a single mechanical oscillator. The dynamics of such a system can be described by the following Hamiltonian:

$$\hat{H}_{\text{om}} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \left[\frac{\hat{p}^2}{2m_{\text{eff}}} + \frac{m_{\text{eff}}\Omega_m^2}{2} \hat{x}^2 \right] + \hbar G \hat{x} \hat{a}^\dagger \hat{a}, \quad (2.8)$$

where the first term describes the optical cavity through its annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$ as well as the optical resonance frequency ω_c . The second term between square brackets describes the dynamics of the mechanical oscillator where $\hat{x}$ and $\hat{p}$ are, respectively, its position and momentum operators, Ω_m the frequency of the mechanical mode and m_{eff} its effective mass. The third and last term describes the mutual interaction between the optical and mechanical oscillators via the optomechanical coupling constant $G = d\omega_c/dx$. One can also define the optomechanical coupling rate $g_0 = x_{\text{zpf}}G$ with $x_{\text{zpf}} = \sqrt{\hbar/2m_{\text{eff}}\Omega_m}$ is the mechanical zero-point fluctuations.

2.4.1.1 Heisenberg-Langevin equations of motion

From the optomechanical Hamiltonian described in eq. 2.8 one can compute a set of coupled Heisenberg-Langevin equations of motion describing the time-evolution of the optical and mechanical oscillators taking into account fluctuation and dissipation channels in both systems. It is worth reminding that in the Heisenberg picture, for a system described with a Hamiltonian $\hat{H}$, the time-evolution of an operator $\hat{\mathcal{O}}(t)$, describing a system's observable, is defined as follows:

$$d_t \hat{\mathcal{O}}(t) = \frac{i}{\hbar} [\hat{H}, \hat{\mathcal{O}}(t)], \quad (2.9)$$

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where the notation $[\cdot, \cdot]$ denotes the commutator of two operators. The set of coupled equations of motion for a single input and output port optical cavity read:

$$d_t \hat{\mathbf{x}} = \frac{i}{\hbar} [\hat{\mathbf{H}}_{\text{om}}, \hat{\mathbf{x}}(t)] = \hat{\mathbf{p}}/m_{\text{eff}}, \quad (2.10)$$

$$d_t \hat{\mathbf{p}} = \frac{i}{\hbar} [\hat{\mathbf{H}}_{\text{om}}, \hat{\mathbf{p}}(t)] - \Gamma_m \hat{\mathbf{p}} + F_T = -m_{\text{eff}} \Omega_m^2 \hat{\mathbf{x}} - \hbar G \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} - \Gamma_m \hat{\mathbf{p}} + F_T, \quad (2.11)$$

$$d_t \hat{\mathbf{a}} = \frac{i}{\hbar} [\hat{\mathbf{H}}_{\text{om}}, \hat{\mathbf{a}}(t)] - \frac{\kappa}{2} \hat{\mathbf{a}} + \sqrt{\kappa_c} \hat{\mathbf{a}}_{\text{in}} = -i\omega_c \hat{\mathbf{a}} - iG \hat{\mathbf{x}} \hat{\mathbf{a}} - \frac{\kappa}{2} \hat{\mathbf{a}} + \sqrt{\kappa_c} \hat{\mathbf{a}}_{\text{in}}. \quad (2.12)$$

Fluctuation-dissipation terms have been added to the mechanical and optical oscillators through the Heisenberg time-evolution of the momentum and annihilation operators $\hat{\mathbf{p}}$ and $\hat{\mathbf{a}}$, respectively. We introduce Γ_m the mechanical damping rate, and F_T the random Langevin force originating from the thermal bath and acting on the mechanical oscillator. The optical mode has a decay rate defined by κ and the input field, described by the cavity input annihilation operator $\hat{\mathbf{a}}_{\text{in}}$, is coupled into the input port with a decay rate κ_c .

The input port is driven with a coherent state described by, $\hat{\mathbf{a}}_{\text{in}}(t) = (\bar{a}_{\text{in}} + \delta \hat{\mathbf{a}}_{\text{in}}(t)) e^{-i\omega_L t}$, where $\bar{a}_{\text{in}}$ is the coherent state amplitude, ω_L is the laser drive frequency and $\delta \hat{\mathbf{a}}_{\text{in}}(t)$ is the input vacuum noise operator. The intracavity field operator can similarly be expressed as $\hat{\mathbf{a}}(t) = (\bar{a} + \delta \hat{\mathbf{a}}(t)) e^{-i\omega_L t}$ where $\bar{a} = \frac{\sqrt{\kappa_c}}{-i\Delta + \kappa/2}$ as described in sec. 1.1.2.

In the following the equations of motion will be solved for the simple case of a lossless optical cavity $\kappa_c = \kappa$. Furthermore, cavity detuning will be set to $\Delta = 0$ which will be sufficient to describe quantum correlations. By linearizing the equations of motion (eqs. 2.10, 2.11 and 2.12) around the optical steady states, working in a frame rotating at the drive frequency ω_L and solving them in the Fourier domain ⁴ one finds that:

$$\hat{\mathbf{x}}[\Omega] = \chi_m(\Omega) F_T[\Omega] - \hbar G \bar{a} \chi_m(\Omega) \chi_c(\Omega) \sqrt{\kappa} \left(\delta \hat{\mathbf{a}}_{\text{in}}[\Omega] + \delta \hat{\mathbf{a}}_{\text{in}}^\dagger[\Omega] \right), \quad (2.13)$$

$$\delta \hat{\mathbf{a}}[\Omega] = \chi_c(\Omega) \left(-iG \bar{a} \hat{\mathbf{x}}[\Omega] + \sqrt{\kappa} \delta \hat{\mathbf{a}}_{\text{in}}[\Omega] \right). \quad (2.14)$$

Here, we defined $\chi_m(\Omega) = (m(\Omega_m^2 - \Omega^2 - i\Gamma_m \Omega))^{-1}$ and $\chi_c(\Omega) = (\kappa/2 - i(\Omega - \Delta))^{-1}$ with $\Delta = 0$ as the mechanical and optical cavity susceptibility, respectively.

Now that we have these solutions, we can compute the optical quadratures of the output optical field assuming the following input-output relation $\hat{\mathbf{a}}_{\text{out}} = \hat{\mathbf{a}}_{\text{in}} - \sqrt{\kappa} \hat{\mathbf{a}}$, where $\hat{\mathbf{a}}_{\text{out}} = \bar{a}_{\text{out}} + \delta \hat{\mathbf{a}}_{\text{out}}(t)$.

⁴The following convention is used: $\mathcal{F}\{f'(t)\} = -i\Omega f(\Omega)$ where $\mathcal{F}\{\cdot\}$ is the Fourier transform of a time-dependent function.

2.4.1.2 Optical noise quadratures

Assuming a given optical field $\hat{\mathbf{a}}_{\text{x}}$ that can be written as $\hat{\mathbf{a}}_{\text{x}}[\Omega] = \bar{a}_{\text{x}} + \delta\hat{\mathbf{a}}_{\text{x}}[\Omega]$, we can define its intensity $\delta\hat{\mathbf{p}}^{\text{x}}[\Omega]$ and phase $\delta\hat{\mathbf{q}}^{\text{x}}[\Omega]$ quadrature operators as follows:

$$\delta\hat{\mathbf{p}}^{\text{x}}[\Omega] = \left(\delta\hat{\mathbf{a}}_{\text{x}}[\Omega] + \delta\hat{\mathbf{a}}_{\text{x}}^{\dagger}[\Omega] \right), \quad (2.15)$$

$$\delta\hat{\mathbf{q}}^{\text{x}}[\Omega] = -i \left(\delta\hat{\mathbf{a}}_{\text{x}}[\Omega] - \delta\hat{\mathbf{a}}_{\text{x}}^{\dagger}[\Omega] \right). \quad (2.16)$$

The quadrature operators of the input noise fluctuations read:

$$\delta\hat{\mathbf{p}}^{\text{in}}[\Omega] = \left(\delta\hat{\mathbf{a}}_{\text{in}}[\Omega] + \delta\hat{\mathbf{a}}_{\text{in}}^{\dagger}[\Omega] \right), \quad (2.17)$$

$$\delta\hat{\mathbf{q}}^{\text{in}}[\Omega] = -i \left(\delta\hat{\mathbf{a}}_{\text{in}}[\Omega] - \delta\hat{\mathbf{a}}_{\text{in}}^{\dagger}[\Omega] \right). \quad (2.18)$$

We can now compute the quadrature operators of the output noise fluctuations using the solutions of the equations of motion (eqs. 2.13 and 2.14) as well as the definition of the quadrature operators and the input noise quadrature operator defined above. The intensity noise quadrature reads:

$$\delta\hat{\mathbf{p}}^{\text{out}}[\Omega] = \left(\delta\hat{\mathbf{a}}_{\text{out}}[\Omega] + \delta\hat{\mathbf{a}}_{\text{out}}^{\dagger}[\Omega] \right) = \frac{\kappa/2 + i\Omega}{\kappa/2 - i\Omega} \delta\hat{\mathbf{p}}^{\text{in}}[\Omega] = \delta\hat{\mathbf{p}}_0^{\text{in}}[\Omega], \quad (2.19)$$

and the phase noise quadrature reads:

$$\begin{aligned} \delta\hat{\mathbf{q}}^{\text{out}}[\Omega] &= -i \left(\delta\hat{\mathbf{a}}_{\text{out}}[\Omega] - \delta\hat{\mathbf{a}}_{\text{out}}^{\dagger}[\Omega] \right) \\ &= -\frac{\kappa/2 + i\Omega}{\kappa/2 - i\Omega} \delta\hat{\mathbf{q}}^{\text{in}}[\Omega] + 2\sqrt{\kappa}G\bar{a}\chi_{\text{c}}(\Omega)\chi_{\text{m}}(\Omega)F_{\text{T}}[\Omega] - 2\hbar\kappa G^2\bar{a}^2\chi_{\text{c}}^2(\Omega)\chi_{\text{m}}(\Omega)\delta\hat{\mathbf{p}}^{\text{in}}[\Omega] \\ &= -\delta\hat{\mathbf{q}}_0^{\text{in}}[\Omega] + 2\sqrt{\kappa}G\bar{a}\chi_{\text{c}}(\Omega)\chi_{\text{m}}(\Omega)F_{\text{T}}[\Omega] - 2\hbar\kappa G^2\bar{a}^2|\chi_{\text{c}}(\Omega)|^2\chi_{\text{m}}(\Omega)\delta\hat{\mathbf{p}}_0^{\text{in}}[\Omega] \\ &= -\delta\hat{\mathbf{q}}_0^{\text{in}}[\Omega] + \alpha[\Omega]F_{\text{T}}[\Omega] - \beta[\Omega]\delta\hat{\mathbf{p}}_0^{\text{in}}[\Omega]. \end{aligned} \quad (2.20)$$

Cavity, propagation, and detection losses will be accounted for as an additional factor ε in the detection efficiency as they add uncorrelated noise to the light. The added noise will have a zero average for the cross-correlation spectra that will be computed in the next section. The noise quadratures will be modified by the loss factor and they now will read as:

$$\delta\hat{\mathbf{p}}^{\text{out}}[\Omega] : \sqrt{\varepsilon}\delta\hat{\mathbf{p}}^{\text{out}}[\Omega] + \sqrt{1-\varepsilon} \left(\delta\hat{\mathbf{a}}_{\text{v}}[\Omega] + \delta\hat{\mathbf{a}}_{\text{v}}^{\dagger}[\Omega] \right), \quad (2.21)$$

$$\delta\hat{\mathbf{q}}^{\text{out}}[\Omega] : \sqrt{\varepsilon}\delta\hat{\mathbf{q}}^{\text{out}}[\Omega] - i\sqrt{1-\varepsilon} \left(\delta\hat{\mathbf{a}}_{\text{v}}[\Omega] - \delta\hat{\mathbf{a}}_{\text{v}}^{\dagger}[\Omega] \right), \quad (2.22)$$

where $\delta\hat{\mathbf{a}}_{\text{v}}[\Omega]$ is the vacuum noise operator that leaks through the optical loss port.

2.4.1.3 Cross correlation spectra

We can construct any optical quadrature of the output field as a linear combination of the output intensity and phase noise quadratures. We can define any given quadrature by an angle ϕ as follows:

$$\delta \hat{\mathbf{X}}_{\phi}^{\text{out}} [\Omega] = \delta \hat{\mathbf{p}}^{\text{out}} [\Omega] \cos \phi + \delta \hat{\mathbf{q}}^{\text{out}} [\Omega] \sin \phi. \quad (2.23)$$

From this arbitrary quadrature definition we can compute correlation spectra that take the following form:

$$S_{\phi_1, \phi_2} [\Omega] = \int_{-\infty}^{+\infty} \frac{1}{2} \left\langle \delta \hat{\mathbf{X}}_{\phi_1}^{\text{out}} [\Omega'] \delta \hat{\mathbf{X}}_{\phi_2}^{\text{out}} [\Omega] + \delta \hat{\mathbf{X}}_{\phi_2}^{\text{out}} [\Omega] \delta \hat{\mathbf{X}}_{\phi_1}^{\text{out}} [\Omega'] \right\rangle \frac{d\Omega'}{2\pi}, \quad (2.24)$$

if $\phi_1 = \phi_2$ then we are computing the autocorrelation spectrum of the optical quadrature of angle ϕ_1 which corresponds to the power spectral density of the latter whereas if $\phi_1 = \phi_2 + \frac{\pi}{2}[2\pi]$ then one accesses a complex-valued cross-correlation spectrum. The following expectation values of the optical vacuum noise will be used to compute the relevant cross-correlation spectra:

$$\begin{aligned} \left\langle \delta \hat{\mathbf{a}}_{\text{in}} [\Omega'] \delta \hat{\mathbf{a}}_{\text{in}}^{\dagger} [\Omega] \right\rangle &= \left\langle \delta \hat{\mathbf{a}}_{\text{v}} [\Omega'] \delta \hat{\mathbf{a}}_{\text{v}}^{\dagger} [\Omega] \right\rangle = 2\pi \delta (\Omega' + \Omega) \\ \left\langle \delta \hat{\mathbf{a}}_{\text{in}}^{\dagger} [\Omega'] \delta \hat{\mathbf{a}}_{\text{in}} [\Omega] \right\rangle &= \left\langle \delta \hat{\mathbf{a}}_{\text{v}}^{\dagger} [\Omega'] \delta \hat{\mathbf{a}}_{\text{v}} [\Omega] \right\rangle = 0. \end{aligned}$$

The quantum correlations that will be used as a scale for temperature are described by the intensity-phase cross-correlation $S_{0, \frac{\pi}{2}} [\Omega]$ which can be computed as follows:

$$\begin{aligned} S_{0, \frac{\pi}{2}} [\Omega] &= \int_{-\infty}^{+\infty} \frac{1}{2} \left\langle \delta \hat{\mathbf{X}}_0^{\text{out}} [\Omega'] \delta \hat{\mathbf{X}}_{\frac{\pi}{2}}^{\text{out}} [\Omega] + \delta \hat{\mathbf{X}}_{\frac{\pi}{2}}^{\text{out}} [\Omega] \delta \hat{\mathbf{X}}_0^{\text{out}} [\Omega'] \right\rangle \frac{d\Omega'}{2\pi} \\ &= \int_{-\infty}^{+\infty} \frac{1}{2} \left\langle \delta \hat{\mathbf{p}}^{\text{out}} [\Omega'] \delta \hat{\mathbf{q}}^{\text{out}} [\Omega] + \delta \hat{\mathbf{q}}^{\text{out}} [\Omega] \delta \hat{\mathbf{p}}^{\text{out}} [\Omega'] \right\rangle \frac{d\Omega'}{2\pi}, \end{aligned} \quad (2.25)$$

remembering that the contribution of the added noise originating from the optical noise operators averages out to zero in the cross-correlation, the only contribution that is left to this correlation spectrum is that of the quantum correlation between the intensity and phase noise quadratures. Indeed, intensity-phase cross-correlation reads:

$$S_{0, \frac{\pi}{2}} [\Omega] = -2\varepsilon\kappa\hbar G^2 \bar{a}^2 |\chi_{\text{c}}(\Omega)|^2 \chi_{\text{m}}(\Omega) = -D(\Omega) \chi_{\text{m}}(\Omega). \quad (2.26)$$

where $D(\Omega)$ can be seen as an optomechanical transduction strength. We can see that this quantum correlation is a measurement of the linear response of the mechanical resonator to the radiation pressure force applied by the drive field. The thermal noise, which will be scaled by the quantum correlations,

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can be deduced by computing the autocorrelation spectrum of the $\phi = \pi/2$ output field quadrature $S_{\frac{\pi}{2}, \frac{\pi}{2}}[\Omega]$ which will contain the thermal signature and additional noises such as shot noise that have to be subtracted before normalizing by the quantum correlation.

Instead of measuring both the phase noise autocorrelation spectrum and the intensity-phase cross-correlation, Purdy, *et al.* demonstrated that by measuring the cross-correlation defined by the angles $\phi_1 = \pi/4$ and $\phi_2 = 3\pi/4$ a single measurement will yield the thermal signature and the quantum correlations.

$$\begin{aligned} S_{\frac{\pi}{4}, \frac{3\pi}{4}}[\Omega] &= \int_{-\infty}^{+\infty} \frac{1}{2} \left\langle \delta \hat{\mathbf{X}}_{\frac{\pi}{4}}^{\text{out}}[\Omega'] \delta \hat{\mathbf{X}}_{\frac{3\pi}{4}}^{\text{out}}[\Omega] + \delta \hat{\mathbf{X}}_{\frac{3\pi}{4}}^{\text{out}}[\Omega] \delta \hat{\mathbf{X}}_{\frac{\pi}{4}}^{\text{out}}[\Omega'] \right\rangle \frac{d\Omega'}{2\pi} \\ &= \int_{-\infty}^{+\infty} \frac{1}{4} \left\langle \left(\delta \hat{\mathbf{p}}^{\text{out}}[\Omega'] + \delta \hat{\mathbf{q}}^{\text{out}}[\Omega'] \right) \left(\delta \hat{\mathbf{q}}^{\text{out}}[\Omega] - \delta \hat{\mathbf{p}}^{\text{out}}[\Omega] \right) \right\rangle \\ &\quad + \left\langle \left(\delta \hat{\mathbf{q}}^{\text{out}}[\Omega] - \delta \hat{\mathbf{p}}^{\text{out}}[\Omega] \right) \left(\delta \hat{\mathbf{p}}^{\text{out}}[\Omega'] + \delta \hat{\mathbf{q}}^{\text{out}}[\Omega'] \right) \right\rangle \frac{d\Omega'}{2\pi}. \end{aligned} \quad (2.27)$$

Knowing that the spectrum of the random thermal force is given by the fluctuation-dissipation theorem which takes the following form [51]:

$$\int_{-\infty}^{+\infty} \frac{1}{2} \langle F_T[\Omega'] F_T[\Omega] \rangle \frac{d\Omega'}{2\pi} = m_{\text{eff}} \Gamma_m \hbar \Omega \coth \left(\frac{\hbar \Omega}{2k_B T} \right), \quad (2.28)$$

where T is the thermal bath temperature. The expression of this cross-correlation spectrum reads:

$$S_{\frac{\pi}{4}, \frac{3\pi}{4}}[\Omega] = -2\varepsilon \kappa \hbar G^2 \bar{a}^2 |\chi_c(\Omega)|^2 |\chi_m(\Omega)|^2 m_{\text{eff}} \Gamma_m \Omega \coth \left(\frac{\hbar \Omega}{2k_B T} \right) + \varepsilon |\beta[\Omega]|^2 - i \text{Im} \left\{ S_{0, \frac{\pi}{2}}[\Omega] \right\}. \quad (2.29)$$

Assuming a small drive optical power, the second term representing the radiation pressure driven motion is smaller than the thermal motion described by the first term and identifying $|\chi_m(\Omega)|^2 m_{\text{eff}} \Gamma_m \Omega$ as $\text{Im} \{ \chi_m(\Omega) \}$ we get the final expression of the cross-correlation spectrum:

$$S_{\frac{\pi}{4}, \frac{3\pi}{4}}[\Omega] \approx -2\varepsilon \kappa \hbar G^2 \bar{a}^2 |\chi_c(\Omega)|^2 \text{Im} \{ \chi_m(\Omega) \} \coth \left(\frac{\hbar \Omega}{2k_B T} \right) - i \text{Im} \left\{ S_{0, \frac{\pi}{2}}[\Omega] \right\}. \quad (2.30)$$

We can see that this cross-correlation spectrum, in the limit of low optical power, has a real part depicting the pure thermally driven motion of the mechanical oscillator without any additional noise and an imaginary part embedding the quantum correlations.

The temperature is finally inferred by taking the ratio of the real and imaginary parts of the cross-correlation spectrum defined in eq. 2.30 which takes the following form:

$$\frac{\text{Re} \left\{ S_{\frac{\pi}{4}, \frac{3\pi}{4}}[\Omega] \right\}}{\text{Im} \left\{ S_{\frac{\pi}{4}, \frac{3\pi}{4}}[\Omega] \right\}} = \frac{\text{Re} \left\{ S_{\frac{\pi}{4}, \frac{3\pi}{4}}[\Omega] \right\}}{\text{Im} \left\{ S_{0, \frac{\pi}{2}}[\Omega] \right\}} = \coth \left(\frac{\hbar \Omega}{k_B T} \right). \quad (2.31)$$

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This temperature measurement technique allows to have access to the temperature of the thermal bath without detailed knowledge of any experimental parameter or the optomechanical system's properties provided a low probe power and zero detuning. Although it has been shown that the effect of a finite detuning, is equivalent to adding a thermal contribution to the purely quantum correlation $S_{0, \frac{\pi}{2}} [\Omega]$, but can be countered in post-processing by rotating the quadratures by an angle $\phi \sim \Delta/\kappa$.

In sec. 5.6.1, we will describe the optical and electronic requirements allowing the measurement of these cross-correlation spectra.

Chapter 3

Optomechanical crystals (OMCs)

3.1 Context

The mutual coupling between an optical field to the mechanical mode of a mechanically compliant structure is the building block of cavity optomechanics to which the Fabry-Perot cavity with a movable end mirror [8] is the archetypal optomechanical system. Other systems have been developed over the years such as the microtoroid cavity [10], allowing the coupling of a whispering-gallery optical mode to a radial breathing mechanical mode, or square membranes enclosed in a Fabry-Perot cavity [52, 11].

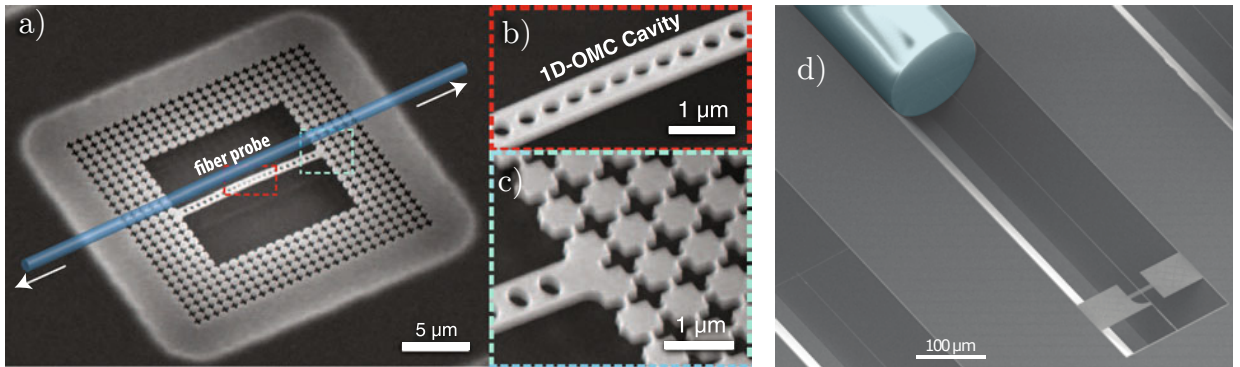


Figure 3.1: Optomechanical crystal cavities. **a)** Optomechanical crystal enclosed in a phononic shield and probed with a tapered fiber (blue). Zooms of the OMC cavity and the bridge between the OMC and the phononic shield are depicted in **b)** and **c)**. **d)** OMC coupled to a tapered waveguide and probed with a cleaved fiber butt coupled to the waveguide. **a)**, **b)**, **c)** are adapted from [14] and **d)** from [53]

In this work, we aim at developing a high signal-to-noise readout optomechanical temperature sensor with a nanoscale spatial resolution, that would allow for device integration, over a wide tem-

perature range. The optomechanical crystal platform [54, 55, 44, 56, 57] is a promising candidate for our purposes as it allows strong confinement of the optical and mechanical modes resulting in high optical quality factors $Q > 10^5$ and large optomechanical coupling rates $g_0 \sim 1$ MHz [14]. Its nanoscale dimensions would allow for device integration but are also suitable for cryogenic environments.

The optomechanical crystal geometry consists of a periodic arrangement of holes perforated in a suspended beam as depicted in fig. 3.1.a where a perturbation of the periodic pattern can be introduced in the middle of the beam (fig. 3.1.b) allowing the formation of small mode volume and high Q-factor nanocavities. These structures can be enclosed in phononic shields to further decouple the localized GHz mechanical modes of interest from low-frequency edge mode (fig. 3.1.a and 3.1.c). These structures can be optically addressed in various ways via optical fibers, a bi-directional coupling using a tapered fiber that is evanescently coupled to the nanocavity or through a single port scheme involving a butt-coupled cleaved or lensed optical fiber [53].

3.2 Optomechanical crystal cavities design

In the following, a theoretical introduction to optomechanical crystal cavities will be given through a description of the optical field in photonic crystal cavities and mechanical vibrations in phononic crystal cavities. Then we will discuss the numerical simulations that led to the fabrication Si_3N_4 OMCs.

3.2.1 Photonic crystal cavity design

3.2.1.1 Maxwell's equations in dielectric media

We will start the description of photonic crystals, as one would treat any electromagnetic problem, by writing down Maxwell's equations. Assuming that the electromagnetic waves are propagating in a lossless¹ isotropic and homogeneous dielectric medium described by a time-independent dielectric function $\epsilon(\mathbf{r})$ and the absence of free charges or currents, Maxwell's equations for an electrical field

¹The dielectric function $\epsilon(\mathbf{r})$ will be treated as strictly real and positive.

$\mathbf{E}(\mathbf{r}, t)^2$ and a magnetic field $\mathbf{H}(\mathbf{r}, t)$ read:

$$\nabla \cdot \mathbf{H}(\mathbf{r}, t) = 0, \quad (3.1)$$

$$\nabla \cdot [\epsilon(\mathbf{r}) \mathbf{E}(\mathbf{r}, t)] = 0, \quad (3.2)$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \epsilon_0 \epsilon(\mathbf{r}) \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t}, \quad (3.3)$$

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu_0 \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t}, \quad (3.4)$$

where ϵ_0 and μ_0 are the vacuum permittivity and permeability, respectively. The linearity of Maxwell's equations allows us to write the solutions as a sum of harmonic modes that take the following form

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r}) e^{-i\omega t}, \quad (3.5)$$

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) e^{-i\omega t}. \quad (3.6)$$

Replacing the harmonic solutions in eq. 3.1 to eq. 3.4 gives the following set of equations

$$\nabla \cdot \mathbf{H}(\mathbf{r}) = 0, \quad (3.7)$$

$$\nabla \cdot [\epsilon(\mathbf{r}) \mathbf{E}(\mathbf{r})] = 0, \quad (3.8)$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = -i\omega \epsilon_0 \epsilon(\mathbf{r}) \mathbf{E}(\mathbf{r}), \quad (3.9)$$

$$\nabla \times \mathbf{E}(\mathbf{r}) = i\omega \mu_0 \mathbf{H}(\mathbf{r}). \quad (3.10)$$

Now, we will take the curl ($\nabla \times \dots$) of eq. 3.9, substitute eq. 3.10 in the resulting and write the following eigenvalue equation

$$\nabla \times \left[\frac{1}{\epsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right] = \frac{\omega^2}{c^2} \mathbf{H}(\mathbf{r}), \quad (3.11)$$

where we introduced the vacuum speed of light as $c^{-1} = \sqrt{\epsilon_0 \mu_0}$. The eigenvalue equation of $\mathbf{H}(\mathbf{r})$ combined with the transversality condition (eq. 3.7) defines fully the magnetic field. Furthermore, the electric field $\mathbf{E}(\mathbf{r})$ can be computed simply with the eigenvalues of the magnetic field using eq. 3.10 without solving its corresponding eigenvalue problem separately.

For simplicity's sake, we will identify the left side of eq. 3.11 as a Hermitian differential operator $\hat{\Xi}$ acting on $\mathbf{H}(\mathbf{r})$ and its eigenvalues are real-valued [35] defined as

$$\hat{\Xi} \mathbf{H}(\mathbf{r}) \equiv \nabla \times \left[\frac{1}{\epsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right] = \frac{\omega^2}{c^2} \mathbf{H}(\mathbf{r}). \quad (3.12)$$

²In the following we will use the bold notation for vectors rather than the arrow notation. This will also be valid for the next chapters.

3.2. OPTOMECHANICAL CRYSTAL CAVITIES DESIGN

It is interesting to note that for photonic crystals and generally for dielectric media, eq. 3.12 is scale-invariant meaning that there is no fundamental length scale defining the problem. We can be curious about what happens when we change the scale of a given system that sees its dielectric function change from $\epsilon(\mathbf{r})$ to $\epsilon'(\mathbf{r}) = \epsilon(\mathbf{r}/s)$ where s is a scaling factor. We rewrite eq. 3.11 by introducing $\mathbf{r}' = s\mathbf{r}$ and $\nabla' = \nabla/s$ such that

$$\begin{aligned} s\nabla' \times \left[\frac{1}{\epsilon(\mathbf{r}'/s)} s\nabla' \times \mathbf{H}(\mathbf{r}'/s) \right] &= \left(\frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r}'/s), \\ \nabla' \times \left[\frac{1}{\epsilon'(\mathbf{r}')} \nabla' \times \mathbf{H}(\mathbf{r}') \right] &= \left(\frac{\omega}{cs} \right)^2 \mathbf{H}(\mathbf{r}'), \end{aligned} \quad (3.13)$$

which is the same equation as eq. 3.11 but describing a re-scaled mode profile $\mathbf{H}'(\mathbf{r}') = \mathbf{H}(\mathbf{r}'/s)$ with a frequency $\omega' = \omega/s$. This statement has a powerful and very practical meaning that the solution of a problem at a given length determines the solutions at any length scale.

3.2.1.2 Photonic band structure

In the following, we will focus on systems exhibiting discrete translational symmetries, such as atomic crystals and photonic crystals. Such a symmetry dictates that the translation invariance is only valid for discrete length scales defined as a multiple of a fixed step length.

Let us take the example of a system that has a one-dimensional periodicity such as that depicted in fig. 3.2.a where dielectric layers ϵ_1 and ϵ_2 , with $\epsilon_1 \neq \epsilon_2$, are stacked alternatively. Such a system has a continuous translational symmetry over the y direction and a discrete translational over the x axis. We can define a unit of this structure, which will be referred to as the *unit cell*, such that the complete structure can be reconstructed by a discrete number of translations of this unit cell along the x direction. We define the *lattice constant* a as the characteristic length of this unit cell along the periodic direction from which the *primitive lattice vector* can be defined as $\mathbf{a} = a\mathbf{e}_x$ such that the dielectric function of the system is translation invariant along the x direction $\epsilon(\mathbf{x}) = \epsilon(\mathbf{x} + \ell\mathbf{a})$ where ℓ is an integer.

We can rewrite eq. 3.12 in this case as

$$\hat{\Xi}\mathbf{H}(\mathbf{x}) \equiv \nabla \times \left[\frac{1}{\epsilon(\mathbf{x})} \nabla \times \mathbf{H}(\mathbf{x}) \right] = \frac{\omega^2}{c^2} \mathbf{H}(\mathbf{x}). \quad (3.14)$$

Furthermore, this one-dimensional translational symmetry allows us to write the electromagnetic

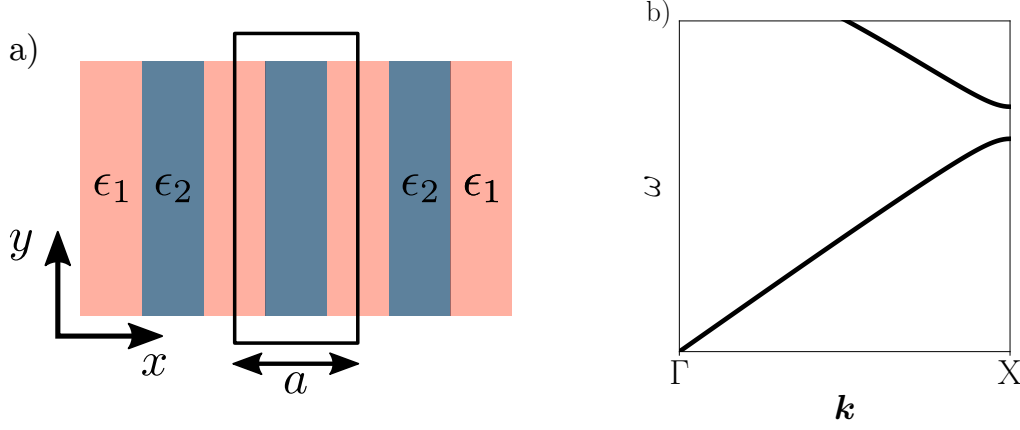


Figure 3.2: Band structure of the stacked thin layers photonic crystal. **a)** Schematic of the thin layer stack with alternating dielectric constant ϵ_1 and ϵ_2 . The black rectangle denotes the unit cell with the characteristic length a corresponding to the lattice constant. **b)** Computed band diagram using MPB for the thin layer stack.

modes inside the system depicted in fig. 3.2 as Bloch states which takes the following form

$$\mathbf{H}_{\mathbf{k}}(\mathbf{x}) = e^{i\mathbf{k} \cdot \mathbf{x}} \mathbf{u}_{\mathbf{k}}(\mathbf{x}), \quad (3.15)$$

where $\mathbf{k}$ is the wave vector and $\mathbf{u}_{\mathbf{k}}(\mathbf{x})$ is a periodic function defined as $\mathbf{u}_{\mathbf{k}}(\mathbf{x}) = \mathbf{u}_{\mathbf{k}}(\mathbf{x} + \ell\mathbf{a})$. By replacing the expression of the Bloch state in eq. 3.14 one gets

$$\begin{aligned} \hat{\Xi} \mathbf{H}_{\mathbf{k}}(\mathbf{x}) &= \frac{\omega(\mathbf{k})^2}{c^2} \mathbf{H}_{\mathbf{k}}(\mathbf{x}) \\ \left[(i\mathbf{k} + \nabla) \times \frac{1}{\epsilon(\mathbf{x})} (i\mathbf{k} + \nabla) \times \right] \mathbf{u}_{\mathbf{k}}(\mathbf{x}) &= \frac{\omega(\mathbf{k})^2}{c^2} \mathbf{u}_{\mathbf{k}}(\mathbf{x}) \\ \hat{\Xi}_{\mathbf{k}} \mathbf{u}_{\mathbf{k}}(\mathbf{x}) &= \frac{\omega(\mathbf{k})^2}{c^2} \mathbf{u}_{\mathbf{k}}(\mathbf{x}), \end{aligned} \quad (3.16)$$

where we defined a new Hermitian operator $\hat{\Xi}_{\mathbf{k}} \equiv (i\mathbf{k} + \nabla) \times \frac{1}{\epsilon(\mathbf{x})} (i\mathbf{k} + \nabla) \times$, which depends on the wave vector $\mathbf{k}$.

The mode profiles and their frequencies can be determined by solving the eigenvalue problem described in the last equation of eq. 3.16 where the function $\mathbf{u}$ is constrained by the following boundary and transversality conditions

$$\mathbf{u}_{\mathbf{k}}(\mathbf{x}) = \mathbf{u}_{\mathbf{k}}(\mathbf{x} + \ell\mathbf{a}) \quad (3.17)$$

$$(i\mathbf{k} + \nabla) \cdot \mathbf{u}_{\mathbf{k}}(\mathbf{x}) = 0. \quad (3.18)$$

3.2. OPTOMECHANICAL CRYSTAL CAVITIES DESIGN

The first boundary conditions (eq. 3.17) allows the eigenvalue problem to be solved for a single unit cell suggesting that for a given value of $\mathbf{k}$ one would expect to find a discretely spaced infinite number of frequencies $\omega_n(\mathbf{k})$ satisfying the eigenvalue problem. This set of indexed frequencies $\omega_n(\mathbf{k})$ describes the *band structure* of a dielectric system such as photonic crystals which supplies us with most of the information needed to describe its properties. Fig. 3.2.b depicts the band structure of the thin layer stack photonic crystal.

3.2.1.3 Unit cell design

The optical properties of our optomechanical crystals will be designed by numerically solving eq. 3.12. The band diagram of a single unit-cell consisting of a Si_3N_4 rectangle with sides of length a along the periodic axis, w in the perpendicular direction, and a thickness h . The lattice parameter a will be chosen such that a band gap appears around the targeted wavelength i.e. 850 nm whereas the thickness h is fixed by the available wafers $h = 200$ nm. Finally, the width is chosen such that a single-mode operation is achieved within the beam. A dimensionless parameter called the *V-number* or *normalized frequency* of a waveguide is used to determine the characteristic length allowing a single mode operation and takes the following form:

$$V = \frac{2\pi w}{\lambda} \sqrt{n_1^2 - n_2^2}, \quad (3.19)$$

where λ is the waveguide's operating wavelength, n_1 and n_2 are the refractive index of the waveguide's material (here Si_3N_4), and the surroundings (here air).

To ensure single-mode operation the normalized frequency should be smaller than 1 ($V < 1$) [58] which in our case yields an upper bound on the width of $w_{\text{lim}} = 760$ nm with $\lambda = 850$ nm, $n_1 = 2.02$ and $n_2 = 1$. Finally, a value of 480 nm has been chosen for the width.

An elliptical hole of semi-major and minor axes, R_y and R_x respectively, is drilled in the center of the rectangle. The hole's shape can be any other shape such as rectangular [59]. We went with an elliptical hole shape because it was demonstrated to yield high-end OMCs [14].

The band structure is numerically computed with the open-source **MPB** (the MIT Photonic-bands) package [60] which solves eq. 3.16 under the periodicity condition (eq. 3.17) and the transversality of the field (eq. 3.18), for given wave vectors $\mathbf{k}_x$. For the sake of notation simplification and because of the translational symmetry along the x axis we will refer to the wave vector $\mathbf{k}_x$ as $\mathbf{k}$.

3.2. OPTOMECHANICAL CRYSTAL CAVITIES DESIGN

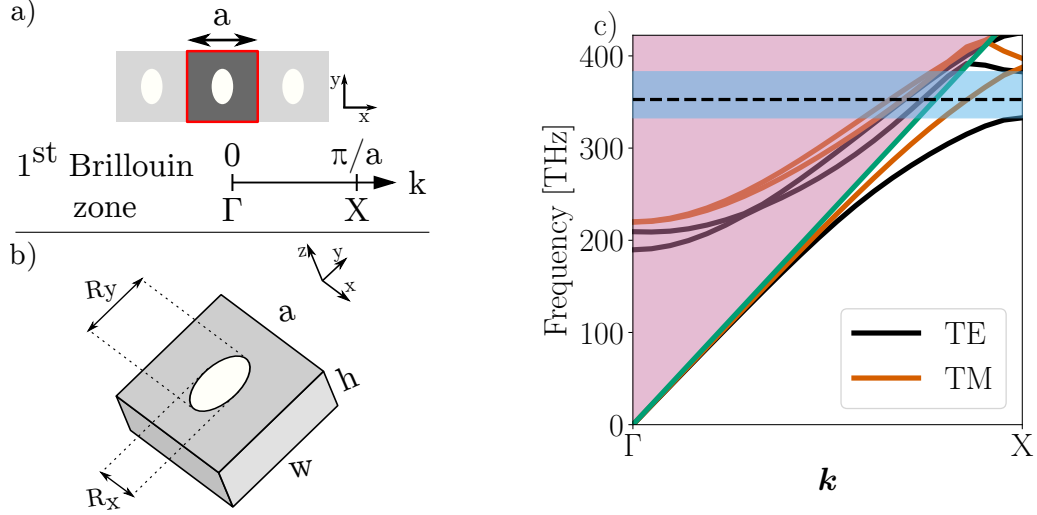


Figure 3.3: Photonic band structure of nanobeam photonic crystal unit cell. **a)** Sketch of the unit cell inside the crystal with a depiction of the FBZ. **b)** Three-dimensional representation of the unit cell. **c)** Band structure computed with the MPB package with the following geometric parameters $a = 330$ nm, $w = 480$ nm, $h = 200$ nm, $R_y = 240$ nm, $R_x = 0.6 \times R_y$ and a Si_3N_4 refractive index of 2.02 at a wavelength of 850 nm. The green solid line represents the light line which defines the light cone highlighted in pink.

In this one-dimensional periodic case exhibiting a single discrete translational symmetry, we will restrain the computation to the *First Brillouin Zone* (FBZ) [35] defined by two points in the reciprocal space, $\Gamma : \mathbf{k} = 0$ and $X : \mathbf{k} = \pi/a$, as depicted on fig. 3.3.a. A mirror symmetry along the plane defined as $z = 0$ allows us to classify the modes into even (**TE-like**) and odd modes (**TM-like**) with respect to the z axis.

Taking into account all the above, the computed band structure over the FBZ ($\Gamma \rightarrow X$) of our OMC unit cell is displayed in fig. 3.3.c where the TE-like and TM-like mode frequencies are depicted in black and orange solid lines, respectively. The green solid line is referred to as the **light line** defined as $\omega = ck$ and separates the band structure into two regions, the first ($\omega < ck$) where only discrete and localized optical modes are allowed and the second ($\omega > ck$), shaded in pink on the band diagram, where a continuum of delocalized "radiation" modes exist.

It appears from the band structure (fig. 3.3.c) that, below the light line, there exist regions where no optical modes are allowed for the TE and TM bands. The band gap of the TE bands is highlighted in a blue rectangle with a dashed line in it depicting the targeted optical resonance frequency of the

OMC, $\nu_0 = 352.7$ THz corresponding to a free-space wavelength of 850 nm.

A defect will be introduced in the crystal to break its periodicity which will allow the existence of localized optical modes inside this optical band gap. The defect region can be built in different manners, for instance, one can remove a certain number of central holes which would resemble a standard free-space cavity or by a variation of one or more crystal parameters i.e. the lattice constant a , a given dimension of the elliptic holes (h_x or h_y) or all of the above.

3.2.2 Phononic crystal cavity design

3.2.2.1 Wave equation of an elastic medium

The deformation of a solid can be described by a wave equation similar to that describing electromagnetic waves in a dielectric medium. Let us assume that our solid can be considered a linear, lossless, and elastic medium. We can define the *displacement field* describing the displacement of a given point when the medium is subject to deformation as

$$\mathbf{Q}(\mathbf{r}, t) = \mathbf{r}'(t) - \mathbf{r}(t), \quad (3.20)$$

where $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\mathbf{r}' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$ are the position of the point of interest prior and after deformation, respectively. Furthermore, such a medium is characterized by its *elasticity tensor* $\mathbf{c}$ which depend on Young's modulus E , Poisson's ratio ν and its components c_{ijkl} .

The deformation gives rise to *strain* $\mathbf{S}$ which can be defined as the local variations of $\mathbf{Q}(\mathbf{r}, t)$ and in the case of a linear solid takes the following form in the Einstein notation³

$$S_{ij} = \frac{1}{2} (\partial_i Q_j + \partial_j Q_i). \quad (3.21)$$

The deformation induces elastic forces in the medium which are characterized by the stress tensor σ_{ij} expressed in units of pressure (GPa) and defined as

$$\sigma_{ij} = c_{ijkl} S_{kl}. \quad (3.22)$$

Under the linear approximation, the strain and stress tensors are both symmetric, implying the fol-

³Summation over repeated indices.

lowing identities

$$S_{ij} = S_{ji} \quad (3.23)$$

$$\sigma_{ij} = \sigma_{ji}. \quad (3.24)$$

We can now write the equation of motion of the displacement field using Newton's law for solids with no external force

$$\begin{aligned} \rho(\mathbf{r}) \frac{\partial^2 Q_i}{\partial t^2} &= \partial_j \sigma_{ji}, \\ &= \frac{1}{2} \partial_j c_{jikl} (\partial_k Q_l + \partial_l Q_k) \\ &= \partial_j c_{ijkl} \partial_l Q_k, \end{aligned} \quad (3.25)$$

where $\rho(\mathbf{r})$ is the solid's mass density. We can write the former equation in its vectorial form that reads

$$\rho(\mathbf{r}) \frac{\partial^2 \mathbf{Q}(\mathbf{r}, t)}{\partial t^2} = \nabla^T (c(\mathbf{r}) \nabla \mathbf{Q}(\mathbf{r}, t)). \quad (3.26)$$

Here, ∇^T is the transpose of the nabla operator. Similarly to the photonic case, we can write the solution of the acoustic wave equation as a linear combination of harmonic solutions of the form $\mathbf{Q}(\mathbf{r}, t) = \mathbf{Q}(\mathbf{r}) e^{-i\Omega t}$. Replacing these solutions in eq. 3.26 gives

$$\hat{\Phi} \mathbf{Q}(\mathbf{r}) = \frac{1}{\rho(\mathbf{r})} \nabla^T (c(\mathbf{r}) \nabla \mathbf{Q}(\mathbf{r})) = -\Omega^2 \mathbf{Q}(\mathbf{r}). \quad (3.27)$$

A more detailed treatment of deformation in elastic media can be found in the first chapter of *Theory of elasticity* by Landau *et al.* [61] and [62, 63].

Similarly to the electromagnetic wave equation, in the presence of discrete translational symmetries, we may express the solutions of eq. 3.27 as Bloch states and numerically solves the wave equation for the phononic band structure.

3.2.2.2 Phononic bandgap simulation

Similarly to the optical eigenvalue problem, the acoustic eigenvalue problem (eq. 3.27) will be initially numerically solved for the unit cell depicted in fig. 3.3.b using the proprietary software *COMSOL Multiphysics* which relies on the **Finite Element Method** (FEM) to solve physics problems ranging from solid mechanics to heat transfer or fluidic flow.

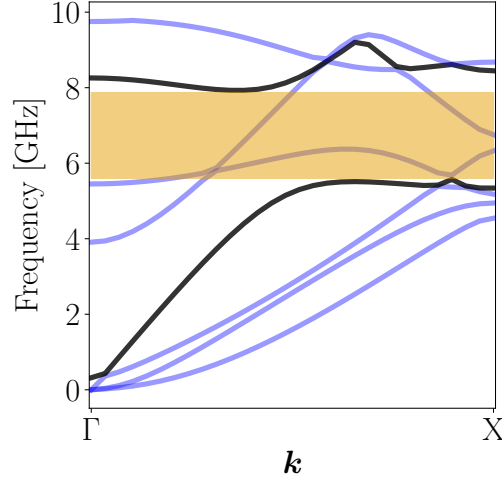


Figure 3.4: Phononic band structure of the Si_3N_4 unit cell depicted in fig. 3.3.b. The following mechanical parameters were used: Young's modulus $E = 300$ GPa and a pre-stress of 1 GPa. The band gap is highlighted in light orange.

Eq. 3.27 is numerically solved using a periodic boundary condition on the sides of the unit cell that are perpendicular to the periodicity axis (x -axis) that is similar to the Bloch state approximation used for the photonic case. This periodic boundary condition applies on the displacement $\mathbf{Q}$ and reads

$$\mathbf{Q}(\mathbf{x}) = e^{i\mathbf{k}\cdot\mathbf{x}} \mathbf{q}_{\mathbf{k}}(\mathbf{x}), \quad (3.28)$$

where $\mathbf{q}_{\mathbf{k}}(\mathbf{x})$ is a periodic function satisfying $\mathbf{q}_{\mathbf{k}}(\mathbf{x}) = \mathbf{q}_{\mathbf{k}}(\mathbf{x} + \ell\mathbf{a})$. A band structure for the displacement modes can be computed for the values of the wave vector $\mathbf{k}$ in the FBZ, for the same reasons as for the photonic case. Fig. 3.4 depicts the resulting phononic band structure for the previously defined unit cell assuming a Young's modulus of $E = 300$ GPa and pre-stressed silicon nitride layer with a stress of 1 GPa. The phononic band structure depicts a band gap spanning from 5.6 GHz to 7.6 GHz, such that an acoustic wave in this range of frequencies will not be able to propagate. A displacement mode localized in this forbidden frequency range can be engineered by introducing a defect in the period array of holes.

3.2.3 Co-localized photonic and phononic modes

The unit cell presented above, if periodically repeated indefinitely, would have photonic and phononic band gaps which are a range of forbidden frequencies for the propagating optical and acoustic

3.2. OPTOMECHANICAL CRYSTAL CAVITIES DESIGN

waves. The optical bandgap ranges from 783.7 nm to 901.27 nm whereas the phononic bandgap lies within the 5.6 GHz to 7.87 GHz frequency range. Furthermore, the targeted photonic and phononic resonance frequencies are in the forbidden frequencies range. Indeed, if one manages to engineer localized photonic and phononic modes [52] inside their respective forbidden frequency band then these modes would be sufficiently decoupled from the rest of the modes, thanks to their respective band gaps, to exhibit high optical and mechanical quality factors. Effectively, this is achieved by introducing a defect in the crystal geometry which breaks the periodic symmetry and introduces localized defect modes into the forbidden band.

In this work, we relied on a quadratic variation of the elliptic holes' semi-major axes in the central region to create the defect region. A schematic of the crystal's design is given in fig. 3.7.a where we can see the crystal's layout and beneath it the evolution of the semi-major axis with respect to the hole number. We can distinguish three regions in the crystal, the first (highlighted in blue) acts as a Bragg-like end mirror whose reflectivity scales with the number of periods, here it scales as the number of holes, making it a highly reflective end mirror. The defect region, where the photonic and phononic modes will be co-localized is highlighted in light orange. Finally, an input mirror, highlighted in green, with a smaller number of holes allows the light to enter the cavity at the resonance frequency.

The optical mode frequencies, profiles, and quality factors are computed using the open-source **Finite-Difference Time-Domain** (FDTD) algorithm **MIT Electromagnetic Equation Propagation** (MEEP) [64]. The algorithm divides the space into a discrete grid throughout which the fields evolve using discrete time steps. Solving Maxwell's equations in the time domain has a tremendous upside which is the ability to obtain the full frequency spectrum of a structure in response to electromagnetic stimulation in a single simulation in a relatively short time.

In practice, a computation cell is defined as containing all the relevant parameters and conditions as depicted in fig. 3.5.a. The dielectric map, describing the structure of interest consisting of a 20 μm Si_3N_4 beam punched with elliptical air holes and surrounded by air, is plugged into the computation cell. A spatially discrete electromagnetic source but spectrally broad, defined by its central frequency and frequency span, is added to the center of the dielectric structure to obtain the resonant frequencies. Spatial boundary conditions are implemented as well in the computation cell, which in this case consists of **Perfectly Matched Layers** (PML) which are defined as an absorbing layer simulating problems with open boundaries. In practice, such absorbing layers are implemented into FDTD simulations of

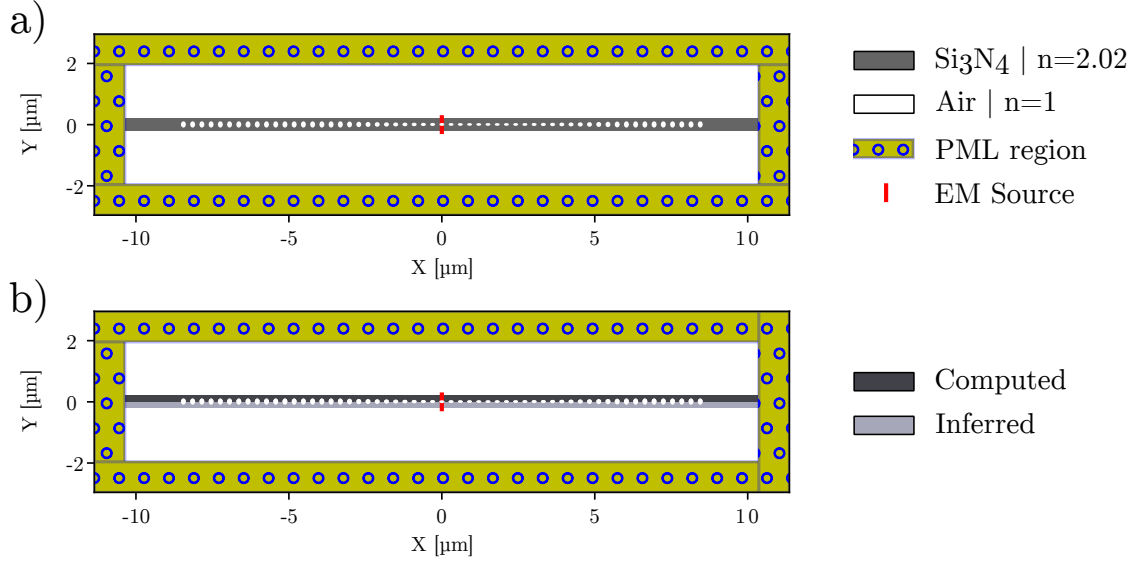


Figure 3.5: Meep computation cell **a)** before symmetry consideration and **b)** after symmetry consideration where only half of the structure is computed and the half is inferred.

Maxwell's equation to avoid reflection of the waves at the interface.

Once all the above is set, one can take advantage of the structure's symmetries to minimize the computation time. In our case, it exhibits a mirror symmetry along the xz plane whereas the yz plane mirror symmetry is broken because of the asymmetric number of holes in the input and end mirrors. The algorithm makes use of the possible symmetries and computes the evolution of the fields on a portion of the structure and infers the remaining by symmetry, as depicted in fig. 3.5.b. The simulation can finally run for a certain time given by a threshold on the field's decay and as a result, one gets the resonant optical modes with their quality factors as well as the mode profiles.

The Python programmatic interface allows us to easily sweep parameters to optimize the structure. An optimization of this nanobeam's design was carried out through a sweep of some relevant geometric parameters. The minimum value of R_y which defines the defect area is swept to roughly $0.4 \times R_x$ following a quadratic trend and the results are depicted in fig. 3.6.a where we can see a strong dependence of the optical quality factor to the minimal value of R_y whereas the resonance frequency does not.

The number of input mirror holes has a very strong effect on the quality factor as well, as depicted in fig. 3.6.b. Although, a trade-off has to be found between a high optical quality factor and ease of coupling. Indeed, a high number of input mirror holes can result in a difficult optical coupling due to

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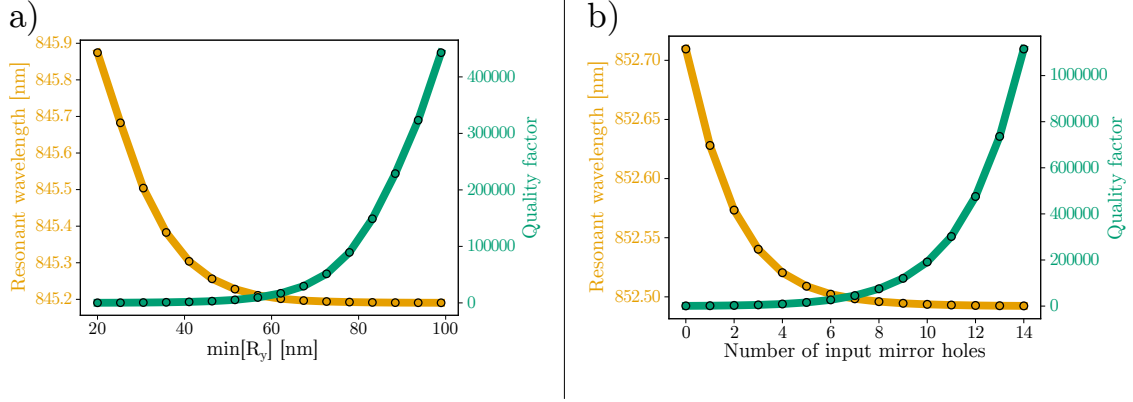


Figure 3.6: Evolution of the optical resonant wavelength and its corresponding quality factor with the parametric sweep of **a)** the minimum value of R_y and **b)** the number of input mirror holes. Here $a = 330$ nm, $w = 480$ nm, $h = 200$ nm, $R_y = 240$ nm and $R_x = 0.6 \times R_y$.

the very small resonance linewidth.

The optimization process led to the design depicted in fig. 3.7.a with the optical mode depicted in fig. 3.7.b exhibiting a resonant wavelength of 850 nm and a quality factor in the order of 3×10^6 .

The optimized structure is then fed into COMSOL to compute the displacement modes frequencies and shapes as well as the optomechanical coupling rate $g_0/2\pi$ of each mode with the simulated optical mode using the perturbation theory of Maxwell's equations for a moving boundary structure described in [59]. The mechanical mode of interest, depicted in fig. 3.7.c, oscillates at an angular frequency $\Omega_m/2\pi = 6.6$ GHz and couples to the optical mode with an optomechanical coupling rate $g_0/2\pi = 4.25$ MHz with an effective mass $m_{\text{eff}} = 170$ fg.

3.3 Nanofabrication process

The simulated optomechanical crystals were fabricated following standard nanofabrication techniques that we will briefly describe hereafter. A schematic of the fabrication steps is given in fig. 3.8. To make silicon nitride optomechanical crystals, one starts with $\langle 100 \rangle$ 4" silicon wafers with silicon thickness of 525 ± 25 μm . Both of their sides are coated with 200 nm thick stoichiometric high stress (1 GPa) LPCVD Si_3N_4 (step 1).

A layer of around 450 nm PMMA, acting as the e-Beam resist, is spin-coated on one of the sides of the Si wafer (step 2). A mask containing the details of the structure to be fabricated, designed before

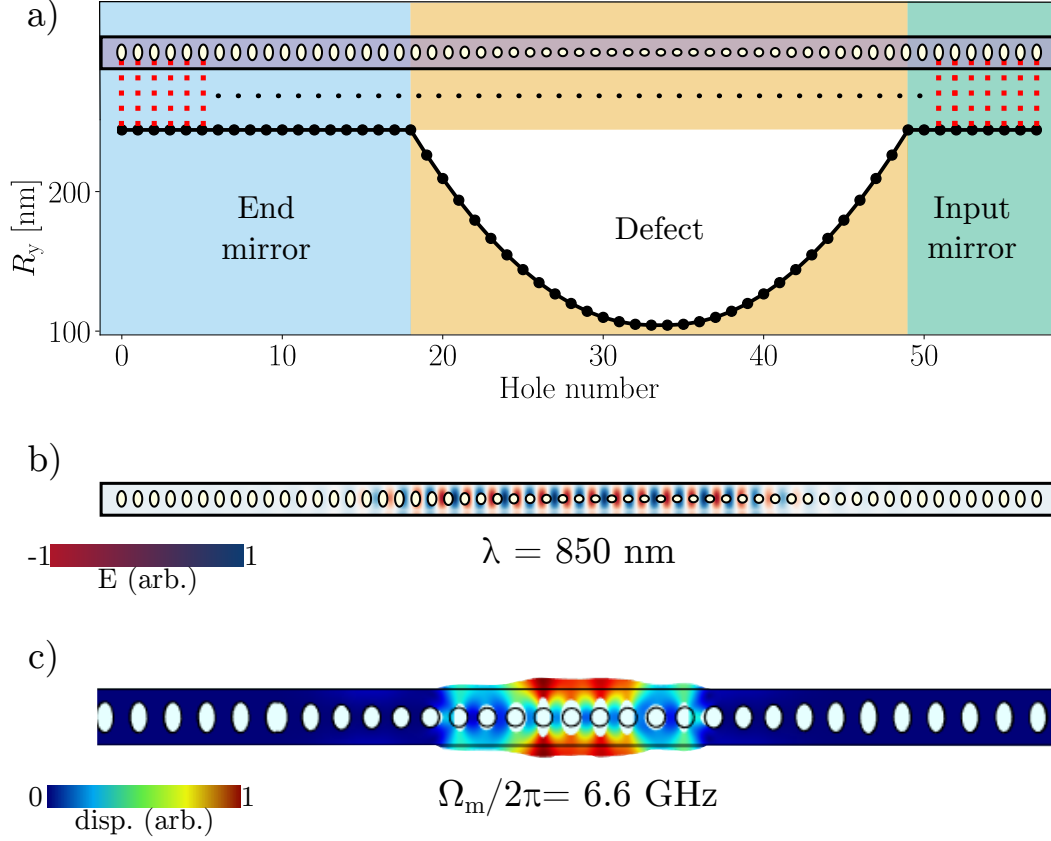


Figure 3.7: OMC design and simulation results. **a)** Schematic of the Si₃N₄ OMC with its Bragg-like input and end mirrors and its defect region defined by quadratically varying the elliptical holes semi-major axes. **b)** Fundamental optical mode at 850 nm ($Q = 3 \times 10^6$) simulated with MEEP. **c)** Fundamental mechanical breathing mode at 6.6 GHz simulated with COMSOL.

this step, is then fed into the e-Beam which insulates the PMMA layer and imprints the details of the mask (step 3).

The insulated PMMA is then developed in a MIBK/IPA solution revealing the imprinted patterns on the PMMA (step 4) which will act as a physical mask for the Si₃N₄ etching. The silicon nitride is then dry etched using a Reactive Ion Etching (RIE) system that creates a fluorine gas plasma that etches the Si₃N₄ and the remaining PMMA is removed through an O₂ plasma etching (step 5).

The last step is to etch a portion of the silicon underneath our newly formed 1D optomechanical crystal to release them and create a suspended structure. The silicon is wet etched with KOH (AZ400K). This etching method is known to be anisotropic creating angled sidewalls that form a 54.7° angle with respect to the surface (step 6).

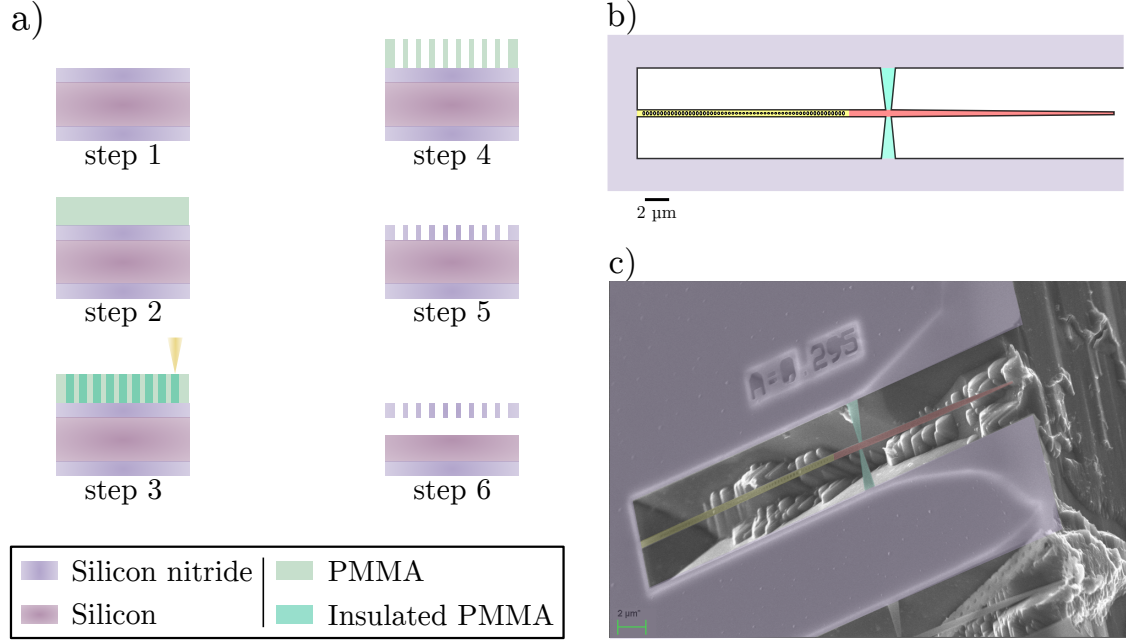


Figure 3.8: Nanofabrication process. **a)** Nanofabrication process steps. **b)** Simplified e-Beam mask depicting the Si_3N_4 layer in purple, the OMC in yellow, a tapered optical waveguide in red, and side clamps in blue. **c)** SEM image of a fabricated suspended optomechanical crystal. The colors are artificial and depict the same elements as **b)**.

The electronic mask that is used in the e-Beam consists of a set of geometric features that will be imprinted on the PMMA. Fig. 3.8.b depicts a simplified version of such a mask where one distinguishes the optomechanical crystal (in yellow) connected to a tapered optical waveguide (red) and this hole structure is held with contact points to the surrounding Si_3N_4 . One is at the left end of the crystal and the remaining two are clamps at the crystal-waveguide junction.

A scanning electron microscope (SEM) image of one of the fabricated structures is shown in fig. 3.8.c. Where we can see the different features described in the previous paragraph with the same color code. All the samples described in this manuscript were fabricated at the *Centre for Nanosciences and Nanotechnologies* (C2N) by Rémy Braive and Théo Martel.

3.4 Gallium Phosphide OMCS

Most of the results that will be discussed in this thesis were performed with different optomechanical crystals than those described in this chapter. These OMCs are made of a 200 nm thick GaP membrane. Lateral confinement of light is performed by structuring the GaP layer into a 700 nm wide

3.4. GALLIUM PHOSPHIDE OMCS

and approximately $20\text{ }\mu\text{m}$ long waveguide. A ladder-like optical cavity is then performed by drilling holes with a diameter of 260 nm periodically positioned along the waveguide. At the center of the nanobeam cavity, the lattice constant ($a_{\text{center}} = 410\text{ nm}$) is chosen to allow a propagating mode at the targeted wavelength (around 1550 nm). From the center to the end of the nanobeam, periodicity is gradually increased until a position where the period is kept constant ($a_{\text{end}} = 480\text{ nm}$).

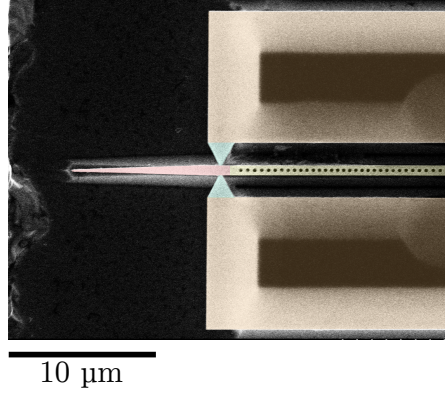


Figure 3.9: SEM image of the Gallium Phosphide (GaP) optomechanical crystal. The colors are artificial.

It is chosen to have the targeted wavelength at the center of the photonic band gap. This method is known as gentle confinement and allows for reaching high optical quality factors. Here, for suspended nanobeam cavities, we achieve a simulated optical quality factor of 10^7 . Beyond the confinement of photons, such structures also display evidence of phononic confinement in the GHz frequency range. With the designed crystal, the fundamental mechanical modes are co-localized with their optical counterparts and are resonating at 2.5 GHz . The coupling between photonic and phononic degrees of freedom can be simulated via finite element modeling, taking into account the two major contributions based on moving boundaries and photo-elastic effects. With the designed structures, the expected optomechanical coupling $g_0/2\pi = 50\text{ kHz}$. Fabrication of such devices has been performed using standard nanofabrication techniques such as those presented in the previous section.

An SEM image is given in fig. 3.9 where we can distinguish the waveguide in red, the clamps in blue, the GaP OMC in yellow, and the GaP layer in light orange.

Chapter 4

Photonic thermometry with GaP OMCs

4.1 Principle

Heat and subsequently temperature gradient can have significant effects on matter either by changing its state, density, or shape for instance. Thermal expansion is one of the most obvious effects with a direct impact on the geometry of a material. Indeed, it can lead to a change in its overall volume generally by contracting when subjected to a decrease in temperature and conversely expanding when the temperature increases. However, other temperature-dependent characteristics of material have a direct impact on the optical properties of the structures as the refractive index of a dielectric material.

Consider an optical cavity of effective length L_{eff} and refractive index n . It presents optical resonances when the optical length is a multiple of the half wavelength λ_m thus the resonance frequency ω_m satisfies:

$$\omega_m(T) = \frac{4\pi n(T) \times L_{\text{eff}}(T)}{c m}, \quad (4.1)$$

where c is the speed of light, T the temperature and $m \in \mathbb{N}$. The dependency of the effective length is directly related to the coefficient of linear thermal expansion α_L defined as follows as the relative length variation per temperature unit:

$$\alpha_L = \frac{1}{L} \frac{\Delta L}{\Delta T}. \quad (4.2)$$

Due to this effect, the last equation shows that the optical cavity resonance frequency shifts with the temperature.

Temperature variations also induce changes in the density of solid dielectrics [65] and in turn modify the refractive index of the dielectric material. The refractive index in eq. 4.1 is thus temperature

4.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

dependent and has the same effect as a thermal expansion on the optical path of light and thus on the resonance frequency of the optical cavity. Similarly to thermal expansion, we can define a coefficient α_n describing the modification of the refractive index with temperature:

$$\alpha_n = \frac{1}{n} \frac{dn}{dT}. \quad (4.3)$$

Our photonic crystal cavities are subject to the interplay of both effects hence for an optical cavity with a resonance frequency ω_c that is subjected to a temperature change ΔT , the variation in resonance frequency reads[10]:

$$\omega_c(T + \Delta T) \simeq \omega_c(T) (1 - (\alpha_L + \alpha_n) \Delta T). \quad (4.4)$$

One might want to estimate the shift coefficient $\beta(\nu_0) = \nu_0 (\alpha_L + \alpha_n)$ for our particular sample's thermal properties. For an initial resonance frequency $\nu_0 = \omega_c(T)/2\pi = 193.4 \text{ THz}$ and for thermal coefficient $\alpha_L = 4 \times 10^{-6} \text{ K}^{-1}$ [66] and $\alpha_n < 5 \times 10^{-5} \text{ K}^{-1}$ [67] one finds that $\beta(\nu_0) < -10.44 \text{ GHz/K}$.

In the following, we will describe the experimental apparatus that will be used to measure this coefficient. Although, in our experiments, we cannot measure the effect of thermal expansion and thermorefractive effects separately as we only are sensitive to the optical length. However, literature shows that α_L is an order of magnitude lower than α_n , and the thermorefractive induced frequency shift is predominant in GaP.

4.2 Description of the experimental setup

4.2.1 Thermostat

In this section, we will describe the developed thermostat, which is based on a commercial ^4He cryostat and commercial Cernox sensors. A description of the cryostat, electrical wiring, and temperature control will be given in the following.

4.2.1.1 Work principle of the cryostat

The cryostat used in this thesis is a commercial ^4He exchange gas cryostat from Oxford Instruments, the Optistat SXM model. The cryostat allows temperature control from room temperature down to 1 K. The cryoprobe is cooled through a process called *static* exchange gas cooling, indeed there is no gas circulation in the experimental chamber in which the cryoprobe is inserted except for gas

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convection. Its compact size and the cooling process generate fairly low vibrations provided that it is well isolated from external vibrations, vacuum pumps vibrations for example.

A (not-to-scale) technical layout of the ^4He cryostat is depicted in fig. 4.1, two radiation shield layers between the insulating vacuum and the helium reservoir are used, instead of a liquid Nitrogen shield, as a first cooling stage. These radiation shields are cooled with the exhaust gas coming from the ^4He reservoir. The latter has a capacity of 4.3 L allowing, according to the manufacturer, a hold time at 4 K of around 8 h.

The cryostat has built-in electrical heater and temperature sensors, located near the heat exchanger with which one can stabilize the temperature at a level of ± 0.1 K over 10 min for the aforementioned temperature range. The temperature sensor probes the temperature around the heat exchanger and not inside the experimental chamber therefore the cryoprobe is equipped with calibrated Cernox sensors, which will be described in more detail in the following sections, to probe the actual temperature within the experimental chamber's walls.

To cool the experimental chamber down to the desired temperature, a couple of vacuum circuits have been connected to the cryostat. One of which is used to pump the insulation vacuum to pressures below 10^{-6} mbar with a turbomolecular pump (Edwards TIC turbomolecular pumping station). The other one is used to pump the experimental chamber, purge the capillary tubes and the chamber, admit the exchange gas into the chamber, and evacuate the evaporated ^4He .

The pressures of different relevant spots (insulating vacuum, experimental chamber, evacuating port, and ^4He circulation port) are monitored using precise Penning gauges for the insulating vacuum and the experimental chamber and mechanical gauges for the rest.

4.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

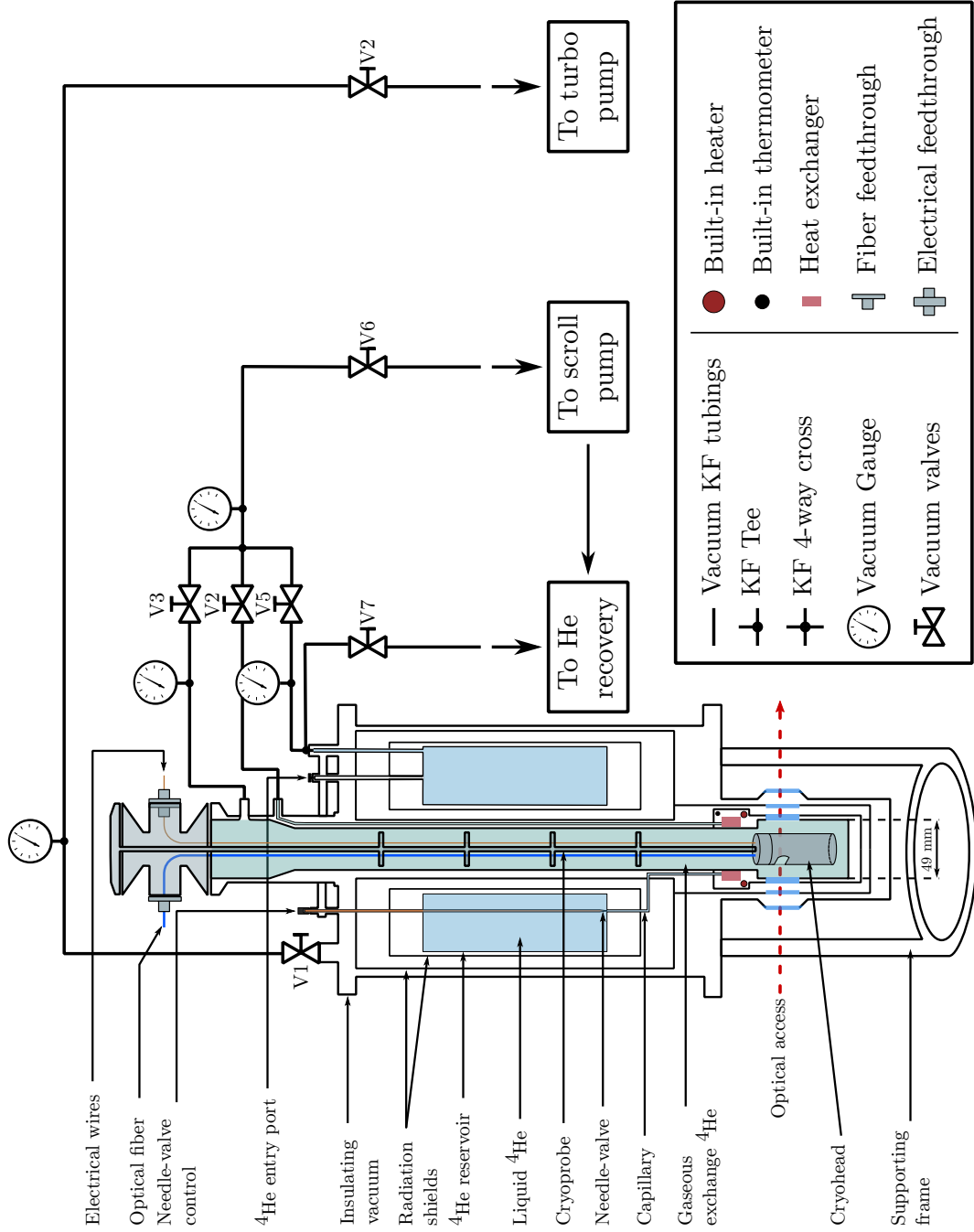


Figure 4.1: (Not-to-scale) Technical schematic of the Oxford Optistat SXM cryostat including the optical fiber and electrical wires feedthroughs layout, the optical access used to image the sample as well as the nearly complete vacuum pumping circuits.

A cooling cycle runs as follows: the experimental chamber is first pumped and purged, as well as the capillary circuit and the ^4He reservoir, before admitting a slight overpressure of clean exchange

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gas, gaseous ^4He for instance. Then, the 4.3 L ^4He reservoir is transferred from an external liquid ^4He Dewar with a transfer tube through the ^4He entry port, as one can see in fig. 4.2. The Dewar is pressurized using a rubber bladder or in our case a home-made heater-tube ¹ consisting of an air-tight aluminum tube with an electric resistance at one of its ends. The latter is connected to a voltage source. The resistance heats the liquid ^4He which results in a slight pressure build-up in the Dewar allowing the liquid ^4He to flow from the Dewar to the reservoir through the vacuum-tight transfer tube ². The helium transfer process takes around 2 hours to complete when the reservoir is at room temperature and can be divided into two steps. Indeed, the reservoir's inner walls have to be cooled down to 4 K before the liquid ^4He starts collecting inside it. During the first step, a large quantity of liquid helium will be evaporated to thermalize the reservoir's inner walls, one should expect the use of roughly 30 L of liquid helium before it starts to collect in the reservoir. A helium level probe from ICEoxford, consisting of a superconducting wire, is used to measure the height of collected helium.

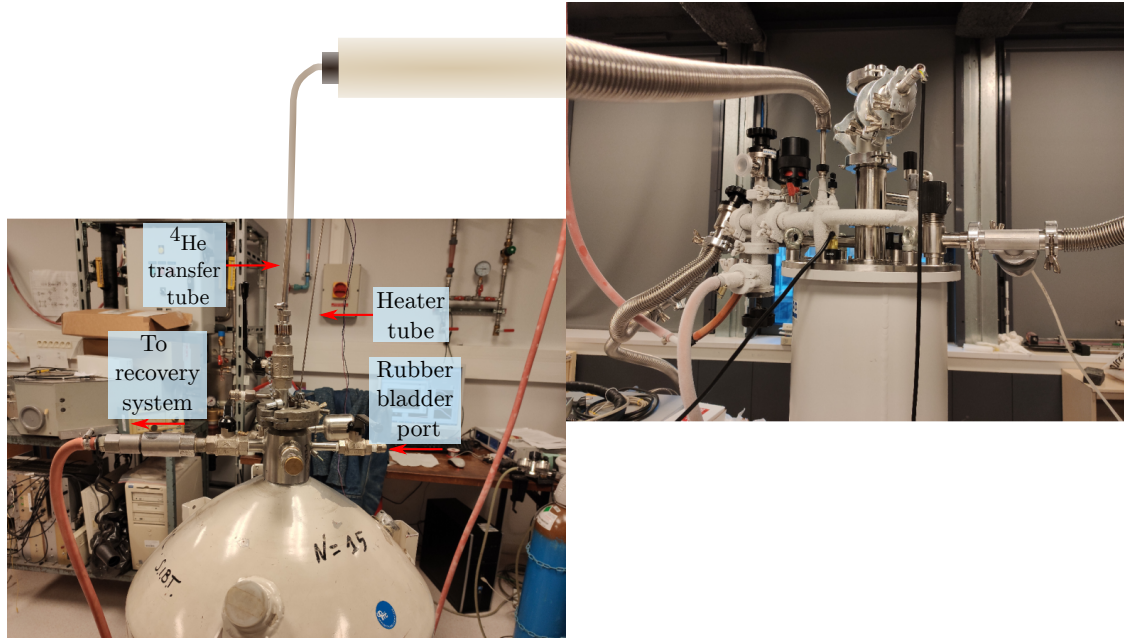


Figure 4.2: Helium transfer from a 100 L Dewar to the cryostat via a transfer tube. **Left:** ^4He Dewar and its different input ports. **Right:** Transfer tube inserted in the cryostat's ^4He entry port.

Once the Helium tank is full, the sample cool-down can start. A (fragile) needle-valve connecting the tank and the capillary is opened and the cooling liquid ^4He is then circulated through the capillary

¹The heater-tube is inserted into the Dewar through its helium level port, not represented in the technical layout, which is positioned next to the helium entry port.

²The overpressure in the Dewar should be kept below 0.5 bar.

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tube and evaporated at an adjustable rate using a dry scroll pump (Edwards nXDS). the needle valve can be used to control and finely vary the liquid's flux. A heat exchanger is used for heat transfer between the liquid coolant and the exchange gas inside the experimental chamber. The evaporated helium is pumped away and directed into the helium recovery system. The sample within the experimental chamber thermalizes with the, now, cold exchange gas.

It is worth noting that both the cryostat and the liquid ^4He Dewar are connected to a recovery system, allowing a quasi-lossless³ helium cycle within the university's premises.

4.2.1.2 Cryoprobe and cryohead

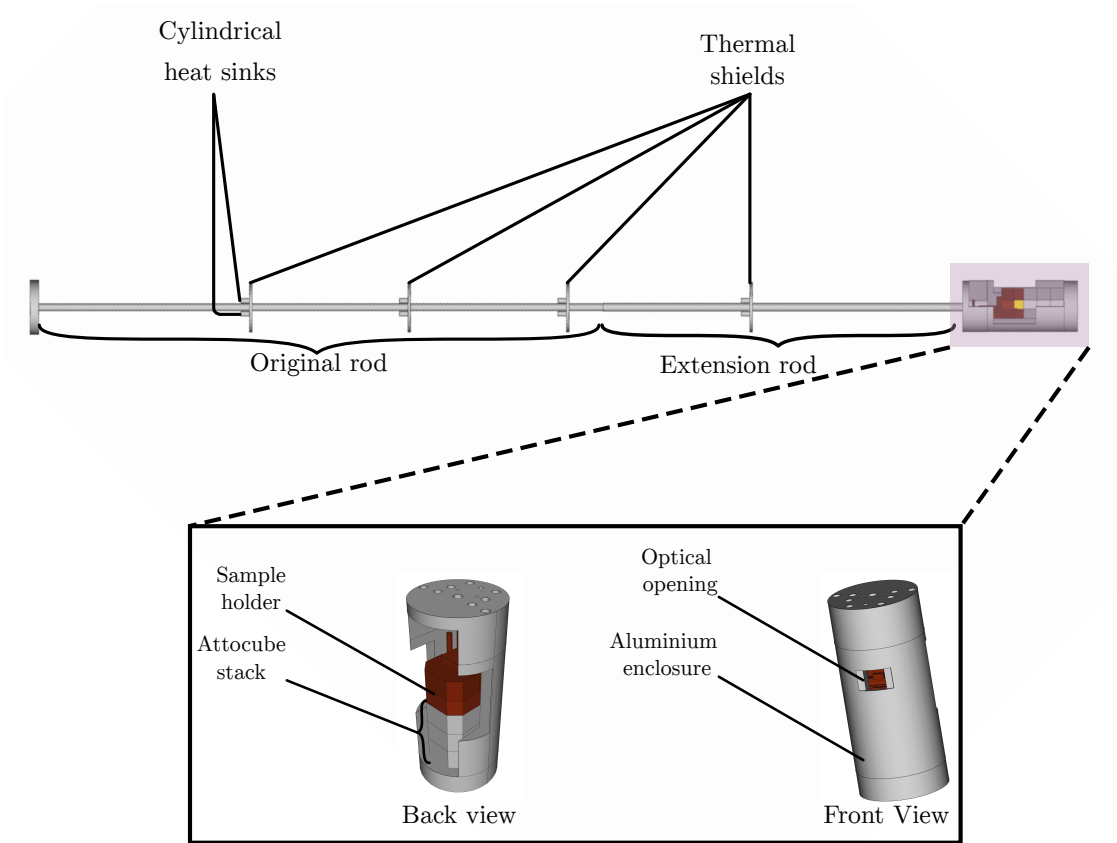


Figure 4.3: 3D rendering of the cryoprobe and cryohead. **Top:** Layout of the original cryoprobe fitted with the extension tube. **Bottom:** Front and back views of the cryohead.

³Of course, one should expect minor losses because of the usual experimental mishaps and/or the occasional forgetting to connect the dry pump to the recovery system.

4.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

As depicted in fig. 4.1, the optical fiber and electrical wires are fed through a KF⁴ 4-way cross placed on the top opening of the cryostat. An extension of the cryoprobe had to be added to the manufacturer's original one. The original cryoprobe consisted of a 60 cm long aluminum tube fitted with 3 thermal shields, each one fitted with 2 cylindrical heat sinks used to thermalize the electrical wire and not to disturb the temperature readings. The extension is a 30 cm long aluminum tube fitted with one radiation shield. The top part of Fig. 4.3 depicts the cryoprobe as described above and at its end, the cryohead is screwed.

The cryohead consists of a hollow cylindrical aluminum enclosure, with a top part drilled with several holes allowing to feed the electrical wires and fiber optic through them. On its front part, a rectangular hole is drilled through which one can visualize the alignment of the fiber with respect to the samples. Inside the cryohead, a stack of three slip-stick motion nanopositioners (Attocube Systems ANPx101/LT and ANPz101/LT, LT for low temperature) are tightly screwed to the bottom plate of the cryoprobe. A set of custom-made copper mechanical parts, that will be described later, are mounted on the attocube stack. These parts are the sample and temperature probe holders. All thermalization-sensitive parts are made of copper because of its good thermal properties at low temperatures.

4.2.1.3 Sample and temperature probes holders

As mentioned before, commercial temperature sensors are embedded in the cryoprobe, and more precisely in or at the surface of the custom-made copper parts, to probe the temperature inside the experimental chamber.

The schematic depicted in fig. 4.4 displays the different parts composing this block and the assembly order. It is composed of 4 main parts: the first one, directly attached to the attocube stack serves as a base to the block and enclosure to the reference temperature sensor. The latter is sandwiched between the first and second copper parts, assuring a good thermal contact with the surrounding thermal bath. The second part is used as a buffer part that will house the last two parts of the block. Two more temperature sensors are fixed in the groove of a U-shaped copper part which is slid and screwed on a rectangular piece that has a step on one of its ends on which the sample is

⁴KF for Klein Flansche is a type of vacuum flange connections, withstanding vacuum pressures on the order of 10⁸ mbar.

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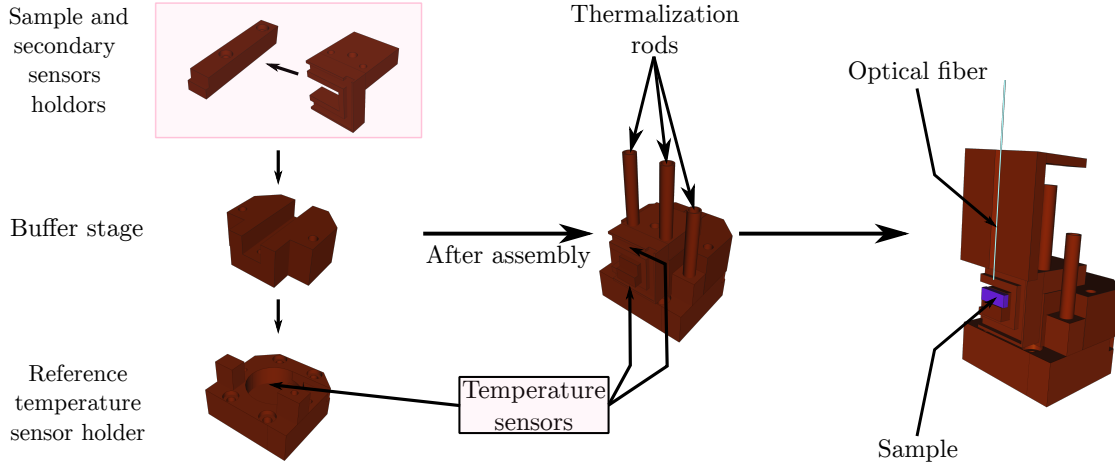


Figure 4.4: 3D rendering of the sample holder assembly. **Left:** The different copper parts composing the overall copper block, enclosing the temperature sensors and sample, and the assembly order. **Middle:** Assembled copper block with its thermalization rods. **Right:** Positioning of the optical fiber on its holder with respect to the sample and the copper block.

glued with *First Contact* Polymer⁵, allowing a robust gluing of the sample and its removal without damage. This assembly places the sample between two temperature sensors allowing it to probe the temperature close to the sample and estimate any temperature vertical gradient that may be caused by gas convection. The different parts of the copper block are fixed with each other using M2 screws. Similarly to the heat sinks present on the thermal shields of the cryoprobe, three copper thermalization rods are mounted on the reference temperature sensor holder and the secondary sensors holder allowing the thermalization of the sensors' wires.

Finally, an L-shaped part fitted with a V-groove fixed on the top plate of the aluminum enclosure mentioned in the previous section is used as a fiber optic holder. The probing fiber optic is glued with the *First Contact* polymer, along the V-groove. Special care should be taken during the gluing process so that the fiber is aligned with the vertical axis and no (or minor) tilt is present. The alignment can be checked using a simple setup composed of a Thorlabs CCD camera mounted on translation stages and a zoom lens assembly as depicted on the right panel of fig. 4.5. The Thorcam software allows to add markers on the video feed depicted in yellow on the left panel of fig. 4.5. By doing so, one can

⁵First contact is primarily used as a strip coat cleaning system allowing the removal of residues from optical systems.

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align the optical fiber (highlighted in blue in fig. 4.5) with the previously set markers. Before applying a few drops of the polymer, using a toothpick, for instance, the fiber is maintained on the fiber holder with tape allowing it to stay in the aligned position.

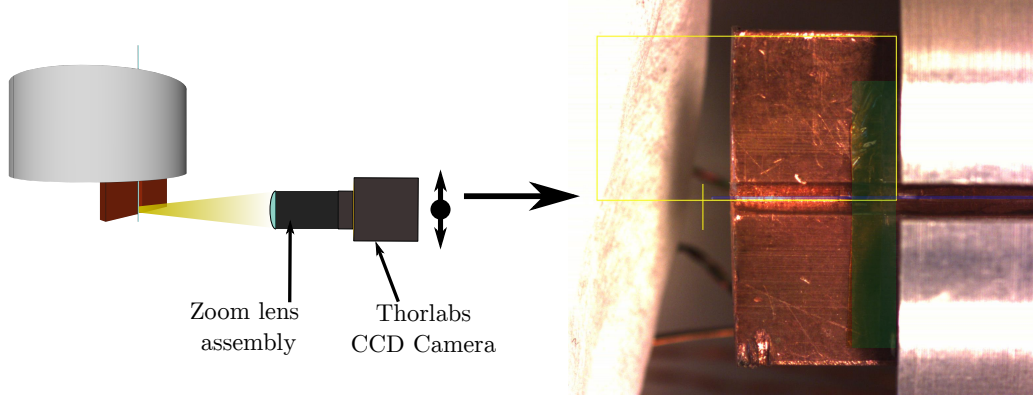


Figure 4.5: Optical fiber alignment before gluing. **Left:** Schematic of the optical setup used. **Right:** Image for the Thorcam software where the optical fiber is highlighted in blue, the tape in green, and the markers are in yellow.

4.2.1.4 Electrical wiring and optical feedthrough

Since the experimental chamber is vacuum sealed, a KF 4-way cross is used to connect the optical fiber and electrical feedthroughs as depicted in fig. 4.6. The optical fiber feedthrough (VACOM KF40-SM850-FCAPC-1) is fixed to one of the cross's ends using a KF-40 to KF-50 adapter, it consists of stainless steel flange with FC/APC connectors on both sides. On the air side, a standard single-mode fiber is connected to it whereas on the vacuum side a stripped cleaved, or lensed optical fiber is plugged and channeled to the fiber holder along the cryoprobe. On the other side of the cross, an electrical feedthrough (VACOM KF16-VB-2B-19) is attached using an assembly equivalent to a KF-50 to KF-16 adapter. It is a 19 contact pins connector both on the air and vacuum sides, into which a standard electrical cable connectorized with a Push-Pull LEMO connector can be plugged. A total of 18 contact pins are used, on both sides, to connect the different temperature sensors and the nanopositioners stack. As depicted on the right panel of fig. 4.6, 12 contact pins are used for the three temperature sensors and 6 for the attocube stack.

4.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

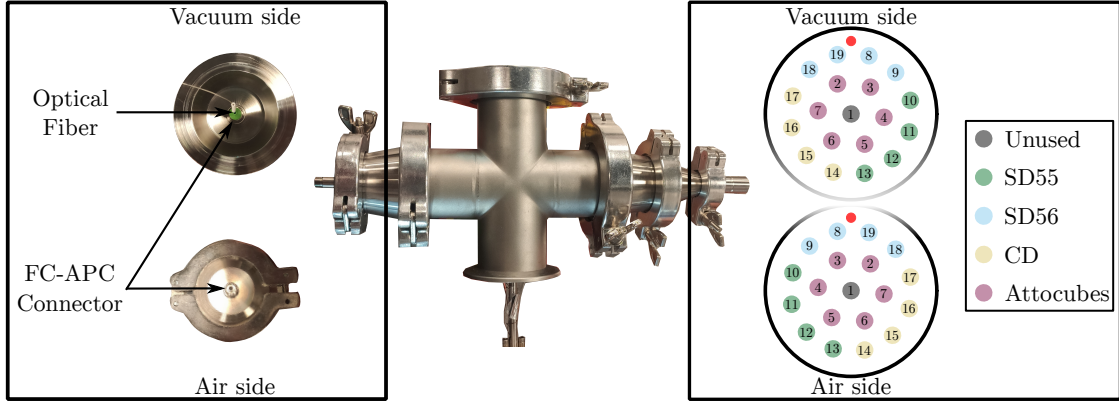


Figure 4.6: The optical and electrical feedthrough system through the 4-way cross. On the left side, two vacuum and air sides of the feedthrough are displayed with the FC/APC connectors. On the right side, a schematic of the contact pins distribution is depicted for the vacuum and air sides. The reference temperature probe is referred to as CD, whereas the vertical gradient temperature sensors as SD65 and SD66.

4.2.1.5 Temperature measurement

The temperature inside the experimental chamber is probed with three commercial Cernox sensors, as mentioned in section 4.2.1.3. The first one probes the temperature in the bulk of the copper block on which the sample is fixed while the two others measure the vertical gradient at the surface.

As depicted on the left panel of fig. 4.7, two Cernox packagings are used. The first one is the Cernox bolt-down packaging (CD packaging), which has a large contact area on both sides ensuring good thermal contact with the copper block, it is used as the reference temperature probe. The vertical gradient is, on the other hand, measured using the SD packaging offering a compact design which is mandatory in our case because of the limited space available. Its base plate is made of Sapphire ensuring good thermal conductivity to its fixing point. These Cernox packagings have, according to the manufacturer, typical uncertainties of ± 5 mK to ± 40 mK for temperatures ranging from 4.2 K to 300 K.

The copper parts housing the sensors (see fig. 4.4) are cleaned in an ultrasonic bath before fixing the sensors. For the SD packaging, the leads are cut to the desired length and a twisted pair of 36 AWG phosphor bronze wires are carefully soldered on the leads' ends. The soldering points are coated with an insulating varnish to avoid unwanted electrical contact between the two leads. The CD packaging is provided with its color-coded 36 AWG phosphor bronze twisted pairs. Finally, the

4.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

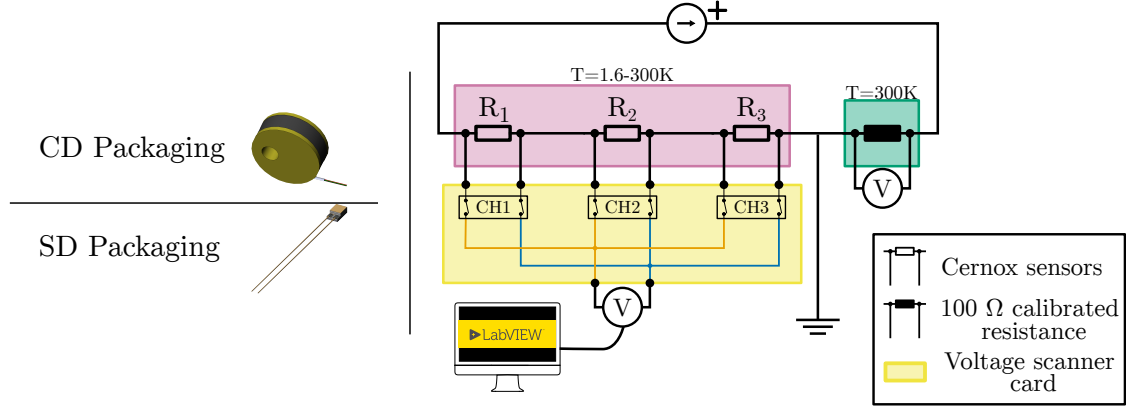


Figure 4.7: Temperature measurement inside the experimental chamber. **Left:** The different Cernox packagings used. **Right:** Temperature measurement electrical diagram representing the four-wire measurement scheme.

sensors are mounted on their housing part after applying thermal grease (Apiezon N Grease) to ensure optimal thermal contact.

The temperature measurement relies on a 4-wire measurement scheme allowing to eliminate the contact and wire resistance and thus measuring the actual sensor's resistance. This measurement scheme uses separate pairs for current-carrying and voltage-sensing electrodes thus the current can flow in the sensor without circulating in the wires used to measure the electric potential difference across the sensor.

The right panel of fig. 4.7 presents the temperature measurement diagram. It is based on a Keithley 6220 Precision Current Source and a Keithley 2002 precision multimeter. This special scheme allows for circumventing measurement errors due to the current fluctuations of the current source. A serial connection is made between the current leads of each sensor and their voltage outputs are connected to a voltage scanner card allowing to cycle between the different sensors and measure the voltage V_x with $x=[1,2,3]$, across each with the precision multimeter. A calibrated $100\ \Omega$ resistance is serially connected to the sensors at room temperature. Its very precise and calibrated resistance value allows us to extract the sensor's resistance without worrying about current fluctuations.

The serial connection ensures that the current intensity is the same at each point of the circuit.

4.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

So for a given sensor, its resistance and that of the reference resistance are given by Ohm's law:

$$R_x = \frac{V_x}{I}, \quad (4.5)$$

$$R_{\text{REF}} = \frac{V_{\text{REF}}}{I}. \quad (4.6)$$

Equalizing the current intensities between eq. 4.5 and 4.6, one finds that:

$$R_x = R_{\text{REF}} \times \frac{V_x}{V_{\text{REF}}}, \quad (4.7)$$

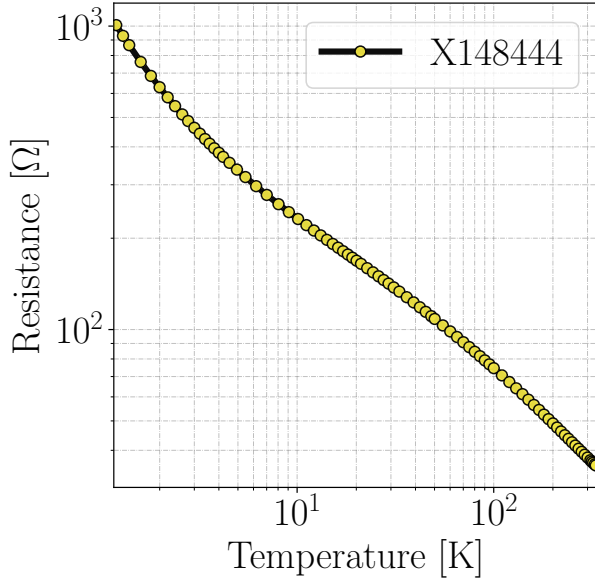


Figure 4.8: Calibration data of the X14844 (SD packaging) sensor for an excitation voltage of 2 mV provided by Lake Shore.

the calibration data), $n = [0, 7]$, X is a normalized variable related to the logarithm of the resistance and $t_n(X)$ represent the Chebychev polynomial that can be generated with the following recurrence relation:

$$t_{n+1}(X) = 2Xt_n(X) - t_{n-1}(X), \quad (4.9)$$

$$t_n(X) = \begin{cases} 1, & \text{if } n = 0, \\ X, & \text{if } n = 1. \end{cases} \quad (4.10)$$

A Labview program has been developed to control the multimeters and the voltage scanner card. The program cycles through the active channels of the card, acquires the voltage value for each sensor and extracts their resistance in a direct (eq. 4.5) and relative (eq. 4.7) manners for a given applied current.

A 7-order Chebychev polynomial fit to the measured calibration data (an example of calibration data for an SD sensor is depicted in fig. 4.8) is used to convert the measured resistance to a temperature. The polynomial equation takes the following form:

$$T(X) = \sum_n a_n t_n(X), \quad (4.8)$$

where $T(X)$ is the temperature in kelvin, a_n are the Chebychev coefficients (provided with

4.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

Finally, the program displays and saves in a text file the measured voltage, resistance, and interpolated temperature (in the direct and relative manners) for each sensor against the elapsed time.

4.2.1.6 Temperature stabilization

The temperature stabilization of the experimental chamber is done by adjusting the ^4He flow and the built-in heater's output. Initially, the former was done manually but we found that it was too cumbersome and can result in excessive use of ^4He . An automated system, provided by Oxford Instruments, was installed on the needle-valve to automatically control the opening of the latter.

The system consists of a stepper motor that is attached to the needle-valve and replaces the manual control knob. The motor is then electrically connected to the Oxford Mercury iTC driver as well as the built-in electrical heater and temperature sensor allowing it to automatically control and stabilize the temperature at a given value. The iTC applies a proportional control with integral and derivative action (PID) on the heater and motor given the temperature measured

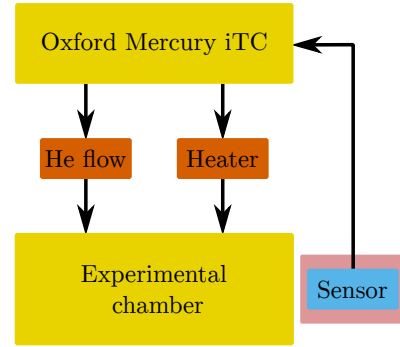


Figure 4.9: Schematic of the temperature stabilization scheme using the automated needle-valve, the electrical heater, and the built-in temperature sensor in the heat exchange area (red rectangle).

by the built-in temperature sensor as depicted in fig. 4.9. It is worth noting that since the temperature feedback is done via the built-in temperature sensor located close to the heat exchanger, a time lag is to be expected between the time the built-in sensor reads the target temperature and the time the experimental chamber reaches that temperature. An experimental upgrade would be to use the reference Cernox sensor to stabilize the temperature via this feedback loop. In the left panel of fig. 4.10 the temperature data from the reference Cernox sensors for two cooldowns from 77 K to 4.1 K are plotted against time. One has been performed with manual control of the ^4He flow and the other with the stepper motor on. The figure shows on the one hand that the automated system reaches 4.1 K faster than the manual control (34 min with feedback and 51 min without) and on the other hand

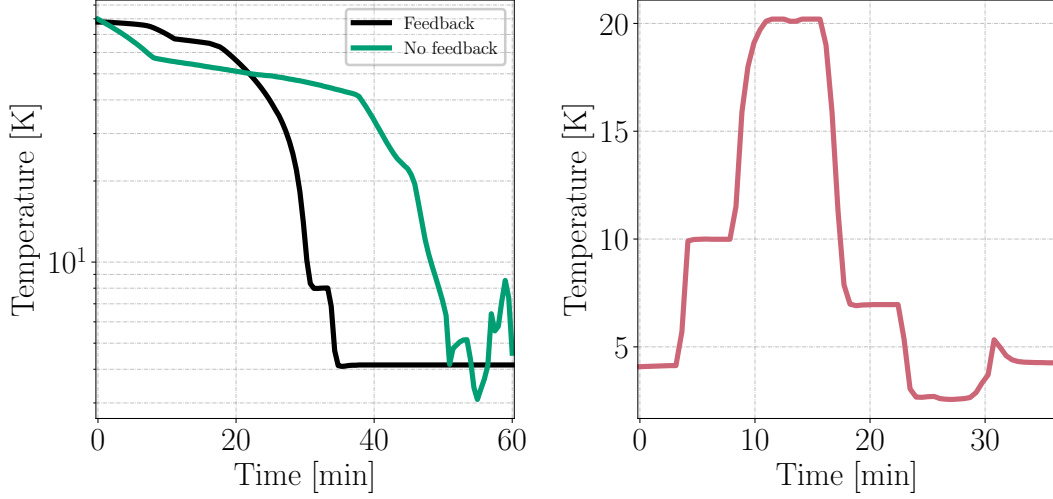


Figure 4.10: Temperature data from the reference Cernox sensor plotted against time. **Left:** Cooldown of the experimental chamber from 77 K to 4.2 K with and without temperature feedback. **Right:** Temperature stabilization of the experimental chamber at various target temperatures.

when reaching the base temperature large temperature fluctuation can be observed on the manual cooldown explained by the constant tweaking of the needle-valve's knob to find a sweet spot whereas the feedback loop finds the optimal parameters to stabilize the temperature rather quickly. The right panel of fig. 4.10 displays the efficiency of the temperature stabilization loop. Indeed, the temperature inside the experimental chamber has been stabilized at different target temperatures between 2.5 K and 20 K. We can note that for temperature variations on the order of 10 K the feedback loop reaches the target temperature in a rather fast time, here it is within the minute. Also, once the target is reached the temperature stays stable for several tens of minutes. Occasionally, undershoots (around the 20 min zone) or overshoots (around the 30 min zone) can be observed due to the response time of the integrator.

4.2.1.7 Alignment procedure

An optical imaging system has been set up allowing us to perform the fiber-to-sample alignment procedure. The system consists of a microscope objective (Mitutoyo MY10X-803) with a 10 $\times$ magnification and a working distance $WD = 35.0$ mm. A long WD is needed in our setup to correctly focus on the sample's focal plane, separated by roughly 3 cm of glass. The microscope objective is

4.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

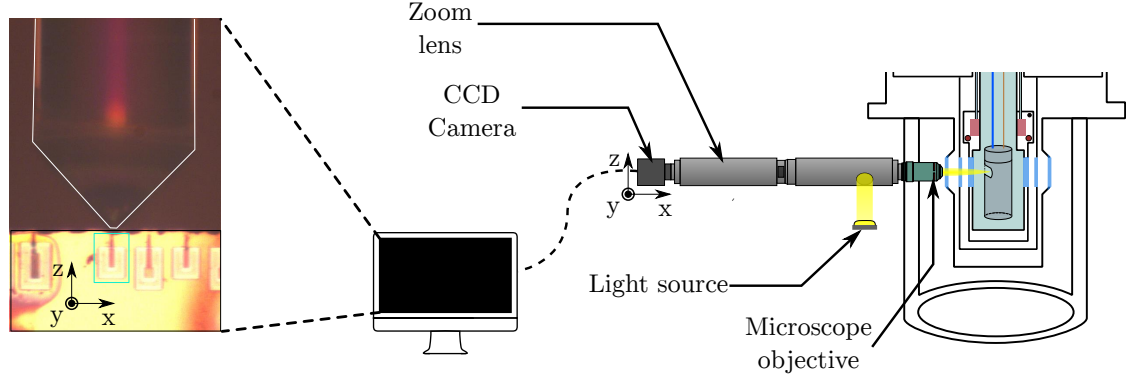


Figure 4.11: Schematic of the optical imaging system next to the cropped version of the cryostat diagram. On the left side, a frame of the video feed is depicted where the optical fiber is framed in white, the sample chip in black, and an individual sample in light blue.

coupled to a $12\times$ zoom lens (Thorlabs MVL12X3Z) with a 3 mm fine focus adjustment knob and an illumination port through which a light beam coming from a Tungsten-Halogen light source (Thorlabs SLS201L/M) is coupled. The light is focused on the sample and is reflected back through the microscope objective which in turn is detected with a Thorlabs CCD camera. The whole system is mounted on xyz translation stages allowing us to align our imaging system with the region of interest and to focus on a given plane, which is, in our case, the optical fiber plane. Fig. 4.11 depicts a schematic version of the imaging system and a frame from the video feed acquired using the CCD camera. This setup allows imaging of the optical fiber, the sample chip, and individual samples quite distinctively although a brightness issue can occur when the sample chip is highly reflective thus rendering the optical fiber's edges relatively dim and hard to distinguish from the dark background.

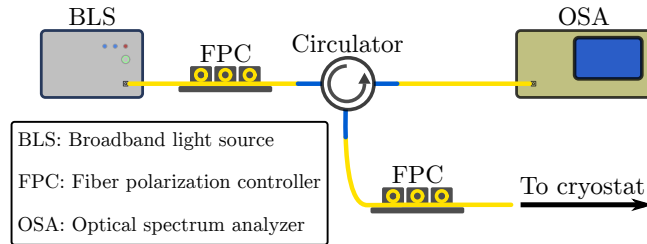


Figure 4.12: Optical alignment setup. Single-mode fibers are depicted in yellow and polarization-maintaining fibers in blue.

Light is routed to the sample through the fibered optical setup depicted in fig. 4.12. The output of a fibered broadband light source (Exalos EBS300006-03) consisting of a 20 mW superluminescent diode (SLED) with a spectrum centered at 1570 ± 110 nm is connected through a single-mode fiber

4.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

to a polarization-dependent fibered optical circulator that transmits light to the sample then directs the reflected light towards an optical spectrum analyzer (ANDO AQ6317-U) which analyzes the reflected light's spectrum. On the optical path, two fiber polarization controllers are used to match the polarization of SLED's output to the input of the circulator and to adjust the polarization of the light impinging on the studied sample.

Taking into account the imaging brightness issue, an alignment procedure has been developed to couple light into the individual samples and avoid physical contact between the optical fiber and the sample chip that could damage both. The procedure is the following:

Step 1 Using the attocube stacks, the sample is sent far back with respect to the fiber's focal plane allowing to distinguish the fiber's edges, one should expect to see a low signal on the OSA corresponding to the fiber's internal reflections.

Step 2 The position of the imaging system is adjusted on its x-axis such that the focal plane is on the fiber's edges and on its yz-axes such that the image is centered on the fiber end.

Step 3 Virtual markers are placed on the image, to delimit the bottom (horizontal marker) and center (vertical marker) of the optical fiber.

Step 4 The sample chip is brought to the same focal plane as the fiber (along the y-axis) and the desired sample is aligned with the previously set vertical marker (along the x-axis). Doing so, one expects to obtain a higher optical signal on the OSA as compared to **Step 1**. At this point, the polarization of the SLED's output should be optimized using the first FPC by maximizing the signal level on the OSA.

Step 5 The optical resonances dips or peaks should now be visible on the optical spectrum but the polarization on the impinging light might not be optimal and needs to be optimized using the second FPC. Again it is done by maximizing the signal level on the OSA. If the optical resonances are not visible then the polarization must be off and needs to be adjusted.

Step 6 Finally, the sample is brought closer to the fiber (along the z-axis) making sure that the chip's border does not go beyond the horizontal marker set during **Step 3** and the positions on the x and y axes are tweaked to maximize the output signal on the OSA.

4.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

4.2.2 Optical setup

4.2.2.1 850 nm setup

The first generation of Si_3N_4 optomechanical crystals was designed for 850 nm wavelength to be used with a Ti: Sapphire laser source or a tunable Littman/Metcalf laser source (Sacher Lasertechnik TEC 500) available in the laboratory. The injection system combines free space and fiber optical portions. The laser beam is coupled to a single-mode optical fiber using the beam walking process where two mirrors with two degrees of freedom are tuned to maximize the coupling ratio. The power of the output beam can be adjusted with a half-wave plate followed by a polarizing beamsplitter. The latter also ensures that the polarization is linear at its output. The latter also ensures that the polarization is linear at its output.

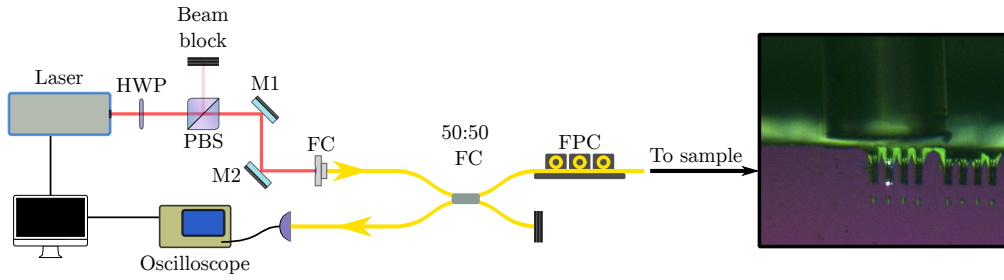


Figure 4.13: 850 nm setup and an image from the imaging feed depicting the cleaved fiber and the samples. **HWP**: Half-wave plate, **PBS**: Polarizing beamsplitter, **M1/2**: Mirror 1/2, **FC**: Fiber coupler and **FPC**: Fiber polarization controller.

A 2×2 single mode (SM) 3 dB fibered bidirectional optical coupler⁶ is used to route the laser light in and out of the sample as depicted in fig. 4.13, and its polarization is tweaked with a fiber polarization controller placed before the sample. The sample is mounted on the copper block described in section 4.2.1.3 and can be positioned, using nanopositioners, with respect to a single-mode cleaved optical fiber. finally, the reflected beam is detected on a photodiode and the electrical signal is acquired on an oscilloscope.

Unlike the setup depicted in the previous section, where the full optical spectrum of the reflected beam can be instantly displayed using the BLS and OSA, the one used for these experiments requires a step-by-step scan of the laser's wavelength through its entire emission range (790 nm to 860 nm) with a given scan speed, on average a single scan takes 10 min to complete with a scan speed of 0.1 nm/s.

⁶The use of such a coupler involves significant losses of optical power. Indeed, half of the incoming power is lost through the unused output port and half of the reflected power goes to the input port.

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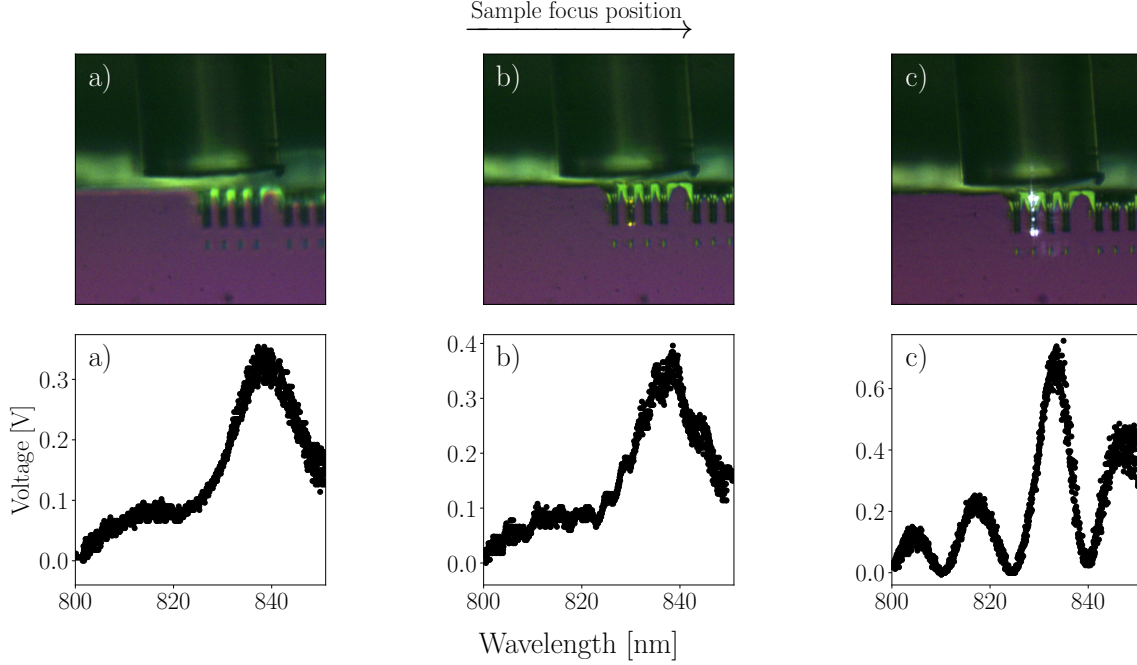


Figure 4.14: Wideband optical spectra of the 850 nm Si_3N_4 photonic crystal cavities. **Top:** Images captured with the imaging system correspond to a fiber-sample alignment. The sample is brought closer to the fiber's plane (from left to right). **Bottom:** Step-by-step optical spectra corresponding to the fiber-sample alignment depicted above it.

On top of that, several degrees of freedom are involved in this optimization problem: for each scan and for a given polarization state⁷ the position of the sample chip has to be fixed along the x and y axes. At these wavelengths, the light scattered by the sample can be detected by the CMOS camera, thus giving us an idea of the alignment along the x and y axes.

Fig. 4.14 shows the step-by-step optical scan acquired with an oscilloscope in the bottom panel and the corresponding image of the position of the sample with respect to the optical fiber in the top panel. The sample is pushed away from the optical fiber plane to acquire its internal reflection spectrum (fig. 4.14.a). The sample is then moved towards the fiber plane (fig. 4.14.b) until we see scattered light on the nearest and farthest edges of the sample⁸. The corresponding spectrum reveals a slight increase in the signal level and a (fast) periodic pattern on top of the internal reflection spectrum suggesting the

⁷Set with the FPC before the sample.

⁸This distance is defined with respect to the cleaved fiber facet

4.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

apparition of a Fabry-Perot cavity formed by the cleaved fiber facet and the sample's farthest edge. Moving the sample even closer (fig. 4.14.c), the scattered light becomes more intense and spread out, the measured signal is twice as high as that of fig. 4.14.a. The periodicity of the Fabry-Perot pattern decreases suggesting a smaller cavity, which might be formed by the cleaved fiber facet and the inverse taper waveguide's tip. At this point, one might expect to see an optical resonance on the spectrum.

Unfortunately, we never observed the optical resonances of these photonic crystal cavities. Several hypotheses were made to explain the inability, of which the most likely:

- The mode field diameter (MFD) of the cleaved optical fiber is, according to the manufacturer, $5.0 \pm 0.5 \mu\text{m}$ at 850 nm and its beam radius $w(z)$ can be described in first approximation by a TEM00 Gaussian beam and verify the following dependency with z the distance to the fiber facet:

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_r}\right)^2} \quad (4.11)$$

where $z_r = \frac{\pi}{\lambda} w_0^2$ is the Rayleigh length and w_0 the minimal radius of the beam also known as waist. Eq. 4.11 suggests that the minimal beam diameter achievable is at $z = 0$ and is equal to the MFD ($w_0 = \text{MFD}/2$), whereas the inverse tapered waveguide's tip is roughly 200 nm wide. The spatial mode matching between these two light-guiding structures is very poor due to a large discrepancy in the mode sizes of both structures.

- The polarization mode matching criteria are not met. Indeed, creating a polarization state with an FPC that matches the one of the photonic crystal cavity is close to impossible in the above-described setup. Before the FC, the polarization state is well-defined and controlled, but after the 3 dB SM fiber coupler, it becomes unknown because stress-induced birefringence causes a drift in the polarization state. Furthermore, the polarization axis of cleaved fiber is also unknown thus rendering the polarization matching process in a step-by-step scan virtually impossible.
- The photonic crystal cavities were designed to have high optical Q-factors on the order of 10^6 with a horizontal polarization axis emphasizing on a very well-controlled probe beam with respect to the aforementioned criteria.

The abovestated hypotheses led us to change our optical coupling setup. Indeed, we introduced the previously described optical alignment setup (fig. 4.12) through which the spectrum can be viewed in

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real-time allowing a fast spatial alignment and polarization match. Furthermore, lensed optical fibers with a waist diameter of $2.0 \pm 0.5 \mu\text{m}$ and a working distance of $12 \pm 2 \mu\text{m}$ were used to couple light into the cavities. This new setup allowed to coupled into the InGaP photonic crystal cavities as one can see in fig. 4.15

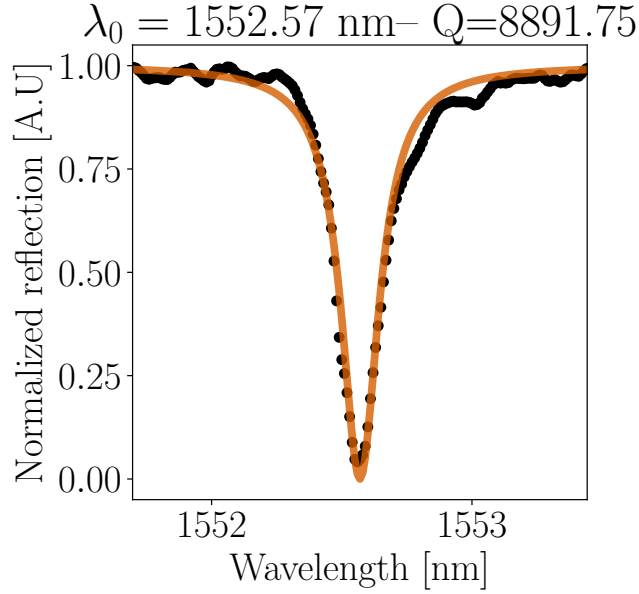


Figure 4.15: Optical resonance dip of an InGaP photonic crystal cavity with a resonant wavelength $\lambda_0 = 1552.52 \text{ nm}$ and a Q-factor of 9000. The spectrum was acquired with the optical alignment setup.

4.2.2.2 Improvement of the optical setup

Initially, the lensed optical fiber that goes down the cryostat to probe our samples was polarization-maintaining (PM). The measured reflection spectra were perturbed by oscillation patterns throughout the whole spectrum as one can see in fig. 4.16.a. This effect was, first attributed to the apparition of a Fabry-Perot cavity formed by the fiber's and waveguide's tips. To confirm this assumption, we investigated the effect of changing the distance between both elements which if the assumption is true would change the free spectral range $\Delta\nu_{\text{FSR}}$ of the unwanted cavity as it is inversely proportional the cavity's length L :

$$\Delta\nu_{\text{FSR}} = \frac{c}{2L}, \quad (4.12)$$

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where c is the speed of light. However, no visible changes were visible suggesting that the effect has another origin.

The source of the problem was later found to be related to polarization. Indeed, PM fibers are designed to have a strong linear birefringence imposed utilizing internal mechanical strain giving rise to two orthogonal axes (slow and fast axes) through which two orthogonal linear states of polarization can travel. The phase velocity along one axis is different from the other.

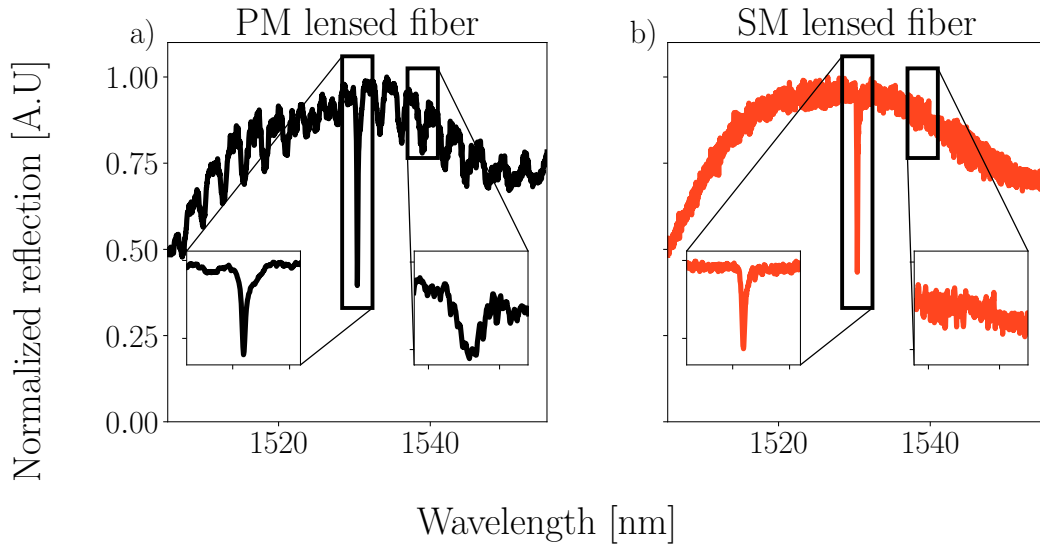


Figure 4.16: Effect of the optical fiber polarization-maintaining ability on the reflected spectra. **a)** Reflection spectrum acquired with PM lensed fiber on which we can observe polarization-induced oscillation patterns around the optical resonance dip at 1530 nm. **b)** Reflection spectrum acquired with non-PM lensed fiber where the oscillations no longer appear.

If a linearly polarized light is injected with its polarization axis aligned on one of the axes of the fiber, this axis will be maintained all along the fiber. However, a light polarized at a random angle will be seen as a linear combination of the two linear polarization states traveling at two different speeds. This leads to a phase difference between the two components and along the fiber, the polarization state varies spatially from linear to elliptical periodically. Eventually, when analyzed by a polarizing component (the optical circulator in our case) it induces interference effects between the two orthogonally polarized waves.

Unfortunately, when gluing the lensed fiber on its holder, as described in section 4.2.1.3, it is impossible to know if the axis of the fiber matches the sample's axis. In this manner, the polarization

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matching that of the photonic crystal cavity can be any linear combination of the two polarization states and thus creates these oscillation patterns when the reflected light passes through the optical circulator.

Replacing these PM-lensed fibers with standard single-mode lensed fibers solved this oscillation issue as depicted in fig. 4.16.b. Standard single-mode fiber exhibit birefringence as well which can be due to external phenomena (bending, heating ..) or to imperfections in the manufacturing process (non-symmetric fiber core) [68] however the polarization change due to this unwanted birefringence can be compensated with fiber polarization controllers. It is done by optimizing our reflection signal as described in section 4.2.1.7.

4.3 Photonic thermometry

4.3.1 Optical thermometry with white light setup

The first temperature measurement has been performed by taking advantage of the thermo-optical properties of Gallium Phosphide, which result from an interplay between the temperature-dependent refractive index $n(t)$ and, to a lesser extent, the thermal expansion of the photonic crystal cavity. As explained in section 4.1 these thermo-optical effects induce a drift of the optical resonance frequency with temperature.

An optical apparatus has been set up to perform this photonic temperature measurement, displayed in fig. 4.18. The setup is similar to the one presented in fig. 4.11 with the exception that a variable optical attenuator (VOA) is placed between the BLS and the first FPC so that the input power can be adjusted. The OSA is controlled with a python program allowing the extraction of the spectra and setting the different parameters (central frequency, span, etc...).

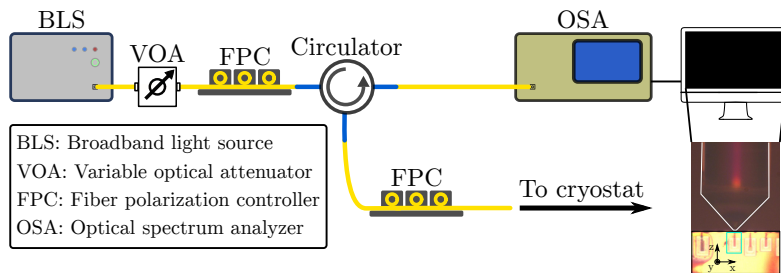


Figure 4.17: Photonic thermometry setup.

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The optimal optical coupling is achieved while the distance between the optical fiber and the probed sample is equal to the former's working distance i.e., 12 μm . In practice, the sample has to be set as close as possible to the optical fiber avoiding contact between the two. Knowing that the copper part on which the fiber and the sample undergo thermal expansions, one has to pull away the sample's position along the y and z axes to avoid unexpected contact thus damaging the fiber and sample chip every time the temperature of the cryostat's sample chamber is changed.

When the cryostat reaches the targeted temperature, the position of the sample and the polarization of the optical probe have to be optimized following the procedure described in section 4.2.1.7. The optical power of the probe is then modified using the VOA such that heating induced by light absorption is minimized thus observing the natural optical resonance frequency before acquiring the optical spectrum. This procedure is performed for each temperature ranging from 296 K to 4.2 K and the resulting spectra are plotted in fig. 4.18.a. Each spectrum is fitted with a Lorentzian lineshape and the resonance frequency ν'_0 is extracted and plotted against the temperature value as depicted in fig. 4.18.b.

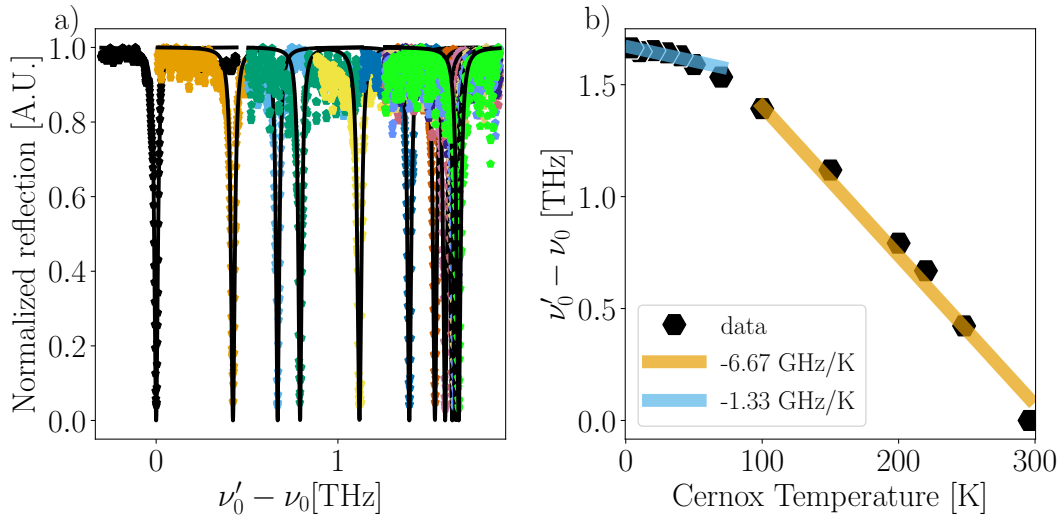


Figure 4.18: Photonic thermometry results. **a)** Optical spectra of the photonic crystal cavity at different temperatures ranging from 296 K, in black, to 4.2 K in light green. Superimposed to the data points in black solid lines are Lorentzian fits. **b)** Fit extracted optical resonance frequency ν'_0 plotted against the Cernox temperature reading. The data points are fitted with two linear fits, in light orange and blue, describing the two different shift regimes.

From fig. 4.18.b, we can distinguish two regimes in the resonance frequency shift process. On the

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one hand, from room temperature to 100 K there is a strong temperature dependency of the resonance frequency and on the other hand, a second regime below 100 K where the temperature dependence is less pronounced suggesting a saturation of the thermodynamical constants at low temperatures.

Both regimes are linearly fitted enabling the extraction of a shift coefficient $\beta(\nu_0)$ from their slopes. The value extracted for the first regime, $\beta(\nu_0) = -6.67 \text{ GHz/K}$ which is in very good agreement with the theoretical estimation made in section 4.1. Furthermore, experiments done at IBM Research-Zürich with one-dimensional GaP photonic crystal cavities [69] where the temperature of their sample holder was varied from 10°C to 40°C found the shift coefficient to be $\beta(\nu_0) = -6.83 \text{ GHz/K}$. Our experiments extended the temperature range over which the frequency shift was probed and shows that over the first regime ($\Delta T = 100 \text{ K}$), the shift coefficient is consistent with the IBM data, however, we unveiled a saturation regime at very low temperatures ($T < 50 \text{ K}$) that has, to our knowledge, never been explored so far and that can be explained by a saturation of the thermodynamical properties of GaP.

P. Deus *et al.* [66] experimentally measured the linear thermal coefficient of GaP from room temperature down to 20 K and they show that it is strongly dependent on temperature from room temperature down to roughly 100 K and show a lower dependence downwards in temperature suggesting a possible plateau. On the other hand, no prior study was found on the temperature dependence of GaP refractive index with temperature. Although, J. Komma *et al.* [70] studied this evolution for Silicon at 1550 nm and show that from room temperature to roughly 100 K the $\frac{dn}{dt}$ coefficient barely decreases ($1 \times 10^{-4} \text{ K}^{-1}$ at 300 K to $8 \times 10^{-5} \text{ K}^{-1}$ at 100 K) and from 100 K downward the decrease becomes more pronounced reaching values of $1 \times 10^{-5} \text{ K}^{-1}$ at 50 K, $2 \times 10^{-6} \text{ K}^{-1}$ at 25 K before dropping to $1 \times 10^{-8} \text{ K}^{-1}$ at 5 K. If this trend is similar for GaP i.e., $\frac{dn}{dt}$ decreases by 4 orders of magnitude, it would suggest that at lower temperatures the frequency shift is mainly due to thermal expansion in GaP which eventually reaches a plateau at low temperature thus the apparition of the second shift regime.

As will be described in the following sections, the results described previously will be used to estimate the effective sample temperature, when one will drive the optical resonance with a laser beam at higher optical powers that may induce an effect called optical bistability.

4.3.2 Optical bistability

4.3.2.1 Physical effect

On top of the thermal effects induced by the thermal contact of the sample with its surrounding thermal bath, the probing laser beam is absorbed within the optical mode volume causing the effective sample's temperature to rise which also causes a frequency shift. This self-heating induced by absorption modifies the shape of the optical resonance Airy peak and can give rise to optical bistability. This well-known phenomenon in optical cavities was first reported in 1983 by A. Dorsel *et al.* [42] and further extensively studied for microresonators [71, 72, 73, 74].

We can give an expression to the absorption-induced temperature change ΔT that appears in eq. 4.4 [10]:

$$\Delta T = \beta_{\text{abs}} \kappa_{\text{abs}} |\bar{a}|^2 \quad (4.13)$$

which gives the temperature increase ΔT for a given mean intracavity power, proportional to $|\bar{a}|^2$ absorbed at a rate κ_{abs} . We define a coefficient β_{abs} , which stands for the temperature increase per absorbed intracavity power.

The power circulating in the cavity (eq. 1.19) reads when taking into account the frequency shift induced by optical absorption (eq. 4.4 and eq. 4.13):

$$|\bar{a}|^2 = \frac{\kappa_c |s_{\text{in}}|^2}{\Delta^2(T) + (\kappa/2)^2} = \frac{\kappa_c |s_{\text{in}}|^2}{(\omega_L - \omega_c + \omega_c (a_L + \alpha_n) \beta_{\text{abs}} \kappa_{\text{abs}} |\bar{a}|^2)^2 + (\kappa/2)^2}. \quad (4.14)$$

This is a third-order polynomial equation with respect to $|\bar{a}|^2$ which can have up to three solutions [75]. At low optical powers, only one solution exists, and the optical resonance exhibits a standard Lorentzian lineshape but the further we increase the optical power the more distorted the lineshape becomes. Indeed, the "natural" resonance frequency is pushed towards the red side when scanned from the blue side until a breaking point where the slope is infinite giving it this very particular triangular shape. In that regime, the equation presents 3 distinct solutions and the system shows a singular signature of a bistable behavior with two turning points corresponding to infinite slopes $\frac{d|\bar{a}|^2}{d\Delta(T)}$. shape⁹. On the other hand, when optical resonance is probed from the red side, the mean intracavity field is pushed towards the second turning point making the optical bistability a hysteretic effect.

⁹An animation showing in detail the bistability effect can be found here[72] <http://www.opticsinfobase.org/oe/viewmedia.cfm?uri=oe-12-20-4742&seq=2>

4.3.2.2 Measurement of the bistability

To measure the coefficient β_{abs} , a fibered optical apparatus has been set up. A tunable laser's (Yenista TUNICS T100S-HP) output is split using a 99:1 fibered splitter, where 1 % of the output light is sent to a fibered wavemeter and the remaining 99 % into the measurement setup depicted in fig. 4.19.

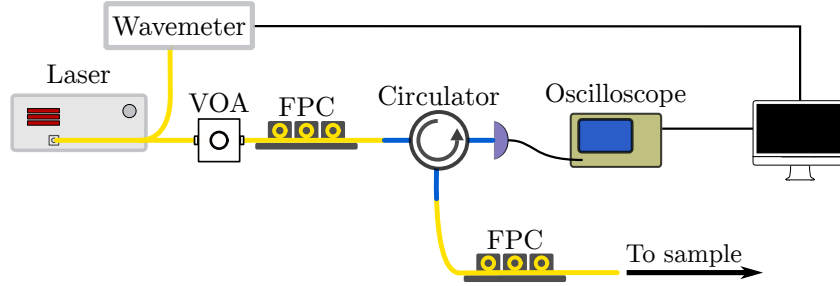


Figure 4.19: Schematic of the measurement setup. Blue lines indicate polarization-maintaining fibers and yellow lines indicate non-polarization-maintaining fibers.

The frequency of the laser is swept from the blue to the red side of the optical resonance. The output signal of a photodiode is read with an oscilloscope for every frequency step which is monitored with the wavemeter. The resulting spectra are plotted against the measured laser frequency. This procedure has been done for various input power beams P_{in} , adjusted with the VOA, ranging from $20 \mu\text{W}$ to $270 \mu\text{W}$.

Fig. 4.20.a shows the reflection optical spectra at different optical powers. A non-distorted lorentzian lineshape can be observed for $P_{\text{in}} = 1 \mu\text{W}$ and increasing the input power results as mentioned before to a distortion of the resonance's lineshape until reaching the triangular shape.

The resonance frequencies¹⁰ are then recovered and plotted, in fig. 4.20.b and plotted against the corresponding input power. A linear fit of these data allows us to extract from its slope a frequency-shift coefficient $\beta_{\nu} = -1.489 \text{ GHz}/\mu\text{W}$. Having the frequency-shift coefficient β_{ν} and previously measured temperature-induced frequency-shift coefficient $\beta(\nu_0)$ one can easily find β_{abs} as it is the ratio of the two. Fig. 4.20.c depicts the temperature change induced by light absorption, computed using the relation $\Delta T = \beta_{\text{abs}} \times P_{\text{in}}$.

¹⁰For the triangular-shaped lineshapes, the breaking point is taken as the resonance frequency.

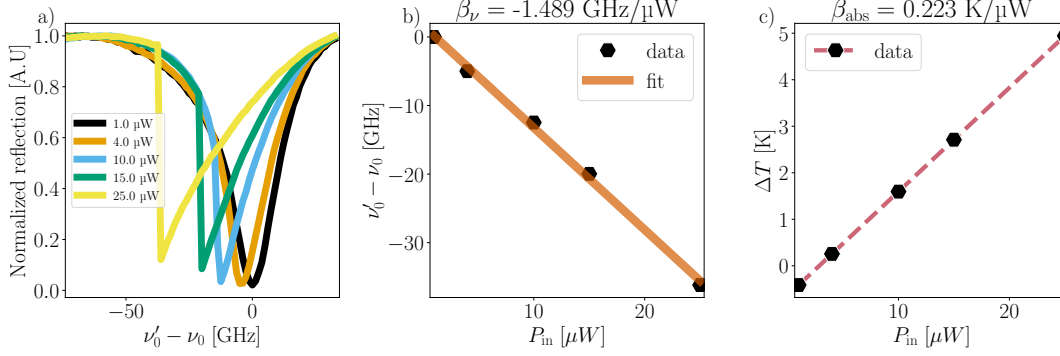


Figure 4.20: Observation of optical bistability and self-heating estimation. **a)** Reflection optical spectra depicting the evolution of the optical resonance with the input power P_{in} . **b)** Evolution of the optical resonance frequency extracted from the previous panel and fitted with a linear fit yielding the frequency-shift coefficient β_ν . **c)** Estimated self-heating from the measured temperature increase per absorbed optical power β_{abs} .

4.3.3 Self-heating estimation

As will be described in the following chapter, noise thermometry is performed by measuring the displacement spectra of our samples with the reflected laser beam. The measurement is done by fixing the frequency of the laser beam close to resonance inducing self-heating of the sample through the effects described above.

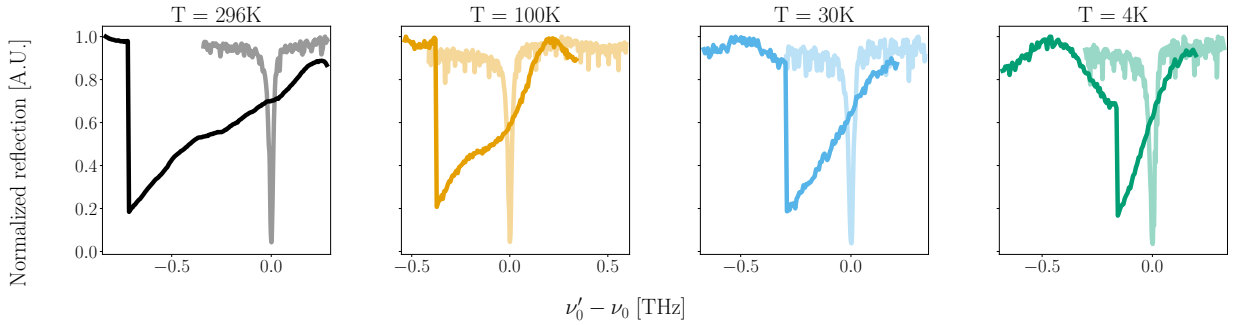


Figure 4.21: Optical reflection spectra acquired with the broadband light setup (transparent) and with the laser setup (non-transparent) for various temperatures. To estimate the on-chip temperature, the frequency difference between the two spectra is extracted.

An estimation of the on-chip temperature is done by acquiring two optical spectra. First, an optical spectrum at low optical power¹¹ is acquired with the broadband light setup (see sec. 4.2.1.7) at every temperature. A second spectrum is acquired by sweeping the laser's frequency and recording on an

¹¹The power is chosen such that the resonance frequency doesn't vary when the optical power is decreased.

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oscilloscope the photocurrent of the reflected beam.

Fig. 4.21, depicts these two optical spectra for different cryostat temperatures¹². The low power spectrum (transparent) is fitted with a lorentzian lineshape and its resonance frequency ν_0 is extracted. For temperatures above $T = 100$ K, corresponding to the linear regime that we can see on fig. 4.18, The resonance frequency of the laser spectra ν'_0 is compared to the low power spectra, and the on-chip temperature is estimated from the bath temperature T_{cryo} as follows:

$$T_{\text{on-chip}} = \frac{\beta_{\text{abs}}}{\beta_{\nu}} \times (\nu'_0 - \nu_0) + T_{\text{cryo}}, \quad (4.15)$$

For temperatures below 100 K, we estimate the on-chip temperature with a quadratic interpolation of the data depicted in fig. 4.18.

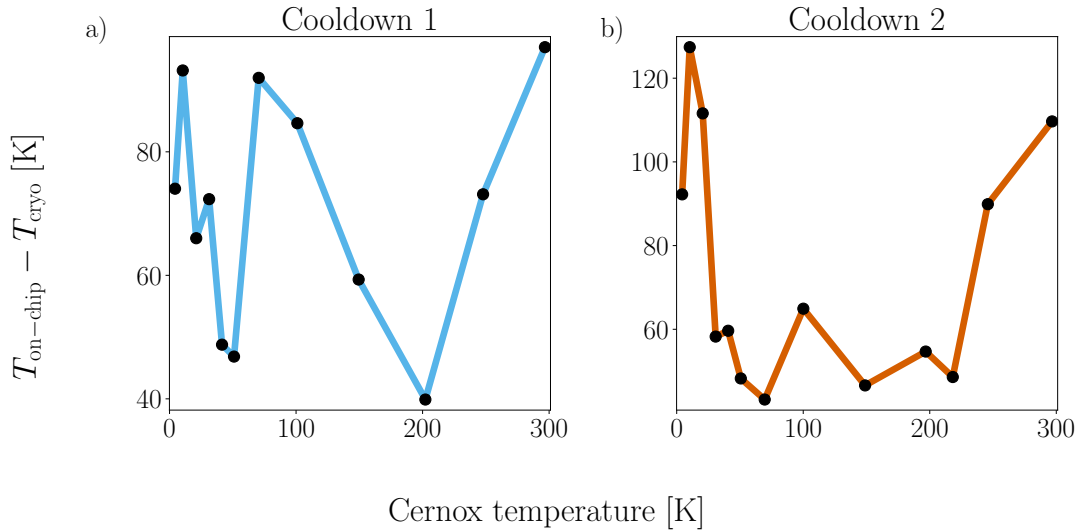


Figure 4.22: Absorption-induced heating estimated eq. 4.15 against the Cernox temperature readings for a constant input optical power $P_{\text{in}} = 350 \mu\text{W}$.

The absorption-induced temperature raise $T_{\text{on-chip}} - T_{\text{cryo}}$ is plotted against the bath temperature measured with the Cernox sensors in fig. 4.22.a and fig. 4.22.b for two distinct cooldowns. They show that for a constant input power, the absorption-induced self-heating varies drastically from one measurement to the other suggesting that the intracavity photon flux varies accordingly. This is not surprising because, as mentioned previously, the optical coupling between the optical fiber and the

¹²It is worth noting that although the input optical power is kept constant, the coupling efficiency from one temperature to another is different.

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sample has to be set at each temperature which makes it a systematic error source that one has to eliminate through the use of an appropriate calibration.

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Chapter 5

Noise thermometry with GaP OMCs

5.1 Principle

The temperature of a given mechanical mode can be inferred by a measurement of its displacement spectrum $S_{xx} [\Omega]$. The variance of the displacement spectrum is proportional to the temperature as one can see through eq. 1.45. In practice, the area under the mechanical mode's peak is proportional to the displacement variance.

The temperature measurements that will be performed in the following are relative to the inferred room temperature variance. A fundamental question that arises from such measurement schemes is how one can calibrate the measured spectra such that for every cryostat temperature, implying a realignment of the fiber's position with respect to the optomechanical crystal thus a change in the cavity coupling rate κ_c , one gets the same transduction between the measured signal and the phase fluctuation induced by the displacement of the optomechanical cavity.

In the following, two calibration methods will be described. The first one involves an external phase modulation of the input optical field and the second relies on the estimation of the coupling parameter $\eta_c = \kappa_c / \kappa$.

5.1.1 Calibration with an external phase modulation

A phase modulation at a frequency Ω_{mod} and depth $\phi(V)$ is applied to the drive laser field $\alpha(t)$ oscillating at ω_L . This field reads:

$$\alpha(t) = \bar{\alpha} e^{i(\omega_L t + \phi(V) \cos(\Omega_{\text{mod}} t))}, \quad (5.1)$$

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which is equivalent to a frequency modulation of the same field

$$\alpha(t) = \bar{\alpha} e^{i \int_0^t \omega_i(t) dt}, \quad (5.2)$$

where $\omega_i(t) = \omega_L + \phi(V) \Omega_{\text{mod}} \sin(\Omega_{\text{mod}} t)$ is defined as the instantaneous frequency where we can see from eqs. 5.1 and 5.2 that a phase modulation at a frequency Ω_{mod} and depth $\phi(V)$ is strictly equivalent to a frequency modulation at the same frequency with modulation depth $\phi(V)$. Furthermore in an optomechanical system, a small modulation δx of the cavity length L is equivalent to a frequency modulation $\delta\nu$ of the drive field at a frequency $\nu_L = \omega_L/2\pi$, such that

$$\left| \frac{\delta x}{L} \right|^2 = \left| \frac{\delta\nu}{\nu_L} \right|^2 = \Omega_{\text{mod}}^2 \frac{\phi^2}{\nu_L^2}. \quad (5.3)$$

The calibration method described here relies on this equivalence, where the relation described by eq. 5.3 is used to convert the measured power spectral densities $S_{\text{II}}[\Omega]$ to displacement spectral densities $S_{\text{xx}}[\Omega]$ and to compare the spectra obtained from one experimental condition to another.

Assuming $S_{\Psi\Psi}[\Omega]$ the spectrum of the phase fluctuations induced by the cavity displacement, the spectrum of the field reflected by the cavity, $S_{\text{II}}[\Omega]$, containing these phase fluctuations measured by a spectrum analyzer (after conversion into a photocurrent by a photodiode) is given by[76]:

$$S_{\text{II}}^{\Psi}[\Omega] = K(\Omega) S_{\Psi\Psi}[\Omega], \quad (5.4)$$

where $K(\Omega)$ is an unknown transduction function taking into account the cavity detuning Δ and the coupling parameter η_c . The full knowledge of this function would give the calibration. This transduction function can be determined with eq. 5.3 that relates a phase modulation of the input field to a cavity displacement. Thus, the spectrum $S_{\phi\phi}[\Omega]$ of the phase modulated field reflected by the cavity and measured by a spectrum analyzer will follow the same transduction as for the cavity phase fluctuations such that:

$$S_{\text{II}}^{\phi}[\Omega] = K(\Omega) S_{\phi\phi}[\Omega]. \quad (5.5)$$

It follows that by taking the ratio $S_{\text{II}}^{\Psi}/S_{\text{II}}^{\phi}$, the knowledge of the transduction function $K(\Omega)$ becomes not necessary.

5.1.2 Loss calibration

The coupling parameter η_c can be inferred from the measurement of the cavity resonance by scanning the laser's frequency. Indeed, from the mean reflected field described in eq. 1.29 one can

5.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

compute the reflected intensity I_{meas} normalized by its maximum value I_{max} reached far from resonance, $I_{\text{meas}}(\Delta)/I_{\text{max}} = |\frac{\bar{s}^{\text{out}}(\Delta)}{\bar{s}^{\text{in}}}|^2$, and obtains:

$$\frac{I_{\text{meas}}(\Delta)}{I_{\text{max}}} = 1 - \frac{4\eta_c(1 - \eta_c)}{1 + (2\Delta/\kappa)^2}. \quad (5.6)$$

The coupling losses can be inferred and used as a calibration for our temperature measurement by measuring the depth of the reflected intensity at resonance $\Delta = 0$ as it would take the following form:

$$1 - \frac{I_{\text{meas}}(\Delta = 0)}{I_{\text{max}}} = 4\eta_c(1 - \eta_c). \quad (5.7)$$

5.2 Description of the experimental setup

5.2.1 Side of fringe setup

The first experiment we developed to perform optomechanical noise thermometry, used a side of the fringe detection scheme. It relies on displacement calibration with a phase modulation. This section first presents the characterization of the phase modulation and then its implementation in the noise thermometry setup.

5.2.1.1 Phase modulator calibration

A phase-modulation imprinted on a laser beam with a modulation amplitude ϕ at a frequency Ω_{mod} can be written as the sum of a carrier and a series of sideband fields using the Jacobi-Anger expansion with the approximation that $\phi \ll 1$:

$$\begin{aligned} \alpha(t) &= \bar{\alpha} e^{i\phi \cos(\Omega_{\text{mod}} t)} = \bar{\alpha} J_0(\phi) + \bar{\alpha} \sum_{n=1}^{\infty} i^n J_n(\phi) \left(e^{i\Omega_{\text{mod}} t} + e^{-i\Omega_{\text{mod}} t} \right) \\ &\approx \bar{\alpha} J_0(\phi) + i\bar{\alpha} J_1(\phi) \left(e^{i\Omega_{\text{mod}} t} + e^{-i\Omega_{\text{mod}} t} \right), \end{aligned} \quad (5.8)$$

where J_n denotes the n -th Bessel function of the first kind. The modulation does not affect the total flux which remains constant $I(t) = \bar{I}$. However, the optical power is redistributed on an infinite number of sidebands with amplitudes given by $J_n(\phi)$. Usually, the depth of the phase modulation is small and at first order, we can consider only the two sidebands at $\pm\Omega_{\text{mod}}$ shifted by $\pi/2$ with respect to the carrier in the complex plane.

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The electro-optic phase modulator used to generate our calibration tone is the model MPZ-LN-20-00-P-P-FA-FA from iXblue. The modulator is equipped with polarization-maintaining FC/APC fibers and has a 3 dB bandwidth of 20 GHz. Electro-optical phase modulation uses the Pockels effect.

This experiment relies on the fact that this phase modulation experiences the same transduction as the thermal noise. It is, thus, necessary to know the induced phase-shift $\phi(V_p)$ defined as:

$$\phi(V_p) = \pi \frac{V_p}{V_\pi}, \quad (5.9)$$

where V_p is the voltage applied on the phase modulator and V_π i.e., the half-wave voltage, is the voltage needed to induce a π phase-shift of the modulated light. The latter has to be determined to use the modulation as a reference.

The half-wave voltage of our modulator has been calibrated using the setup depicted in the left panel of fig. 5.1 where the transmitted phase-modulated laser light is detected on an optical spectrum analyzer. The latter has a sufficiently fine resolution, 0.001 nm, allowing to discriminate between the carrier and phase-modulation-generated optical sidebands at 3 GHz as one can see on the right panel of fig. 5.1. First-order sidebands are clearly visible, whereas the second-order sidebands can be distinguished but are very low thus we decided not to exploit them.

V_π is determined by varying the applied voltage V_p at the wanted frequency (here $\Omega_{\text{mod}} = 2\pi \times 3 \text{ GHz}$), recording the optical spectrum and numerically fitting each peak with a Gaussian lineshape before extracting the amplitudes of the carrier and sidebands for each voltage value. The extracted amplitudes are plotted against V_p and fitted with the corresponding Bessel function¹. The half-wave voltage can then be deduced from the fit knowing the applied voltage. Fig. 5.2 depicts the evolution of the amplitudes with respect to the applied voltage, as well as the Bessel fit and the extracted value of the half-wave voltage for the carrier and the two first-order sidebands. We see from fig. 5.2, that the extracted values of V_π from the carrier and the sidebands agree pretty well.

5.2.1.2 Implementation of the calibration peak

The phase modulation is imprinted on the probe laser beam using the fibered electro-optic phase modulator driven with a low-noise high-frequency signal generator R&S SMB100A described and characterized in the previous section. The phase modulation is set near the mechanical resonance

¹ $J_0(\phi)$ for the carrier and $J_1(\phi)$ for the first-order sidebands.

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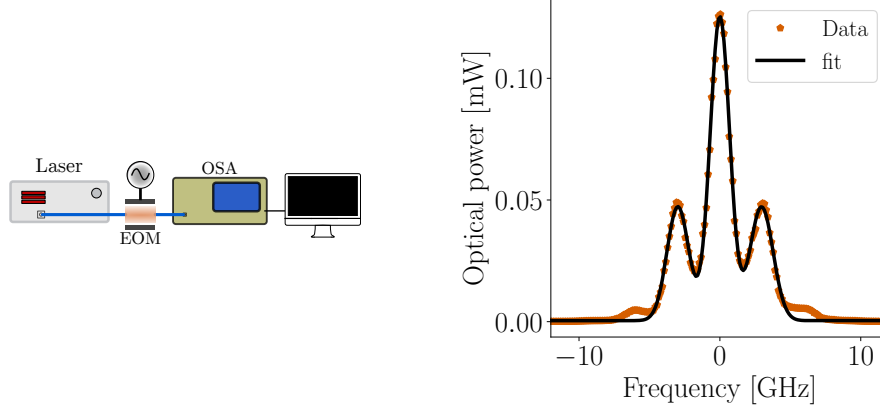


Figure 5.1: Phase-modulator calibration procedure. **Left:** Optical setup allowing the calibration of the phase modulator. The output of a telecom laser source is phase modulated at a frequency $\Omega_{\text{mod}} = 2\pi \times 3 \text{ GHz}$ and later detected on an OSA allowing the resolution of the carrier of the beam and the sidebands generated by the phase-modulation. **Right:** Example of a spectrum acquired with this setup with an applied voltage $V_p = -11 \text{ dBm}$ on which we can distinguish the carrier (the origin of the horizontal axis is set at the carrier frequency) and its two first-order sidebands at frequency $\pm 3 \text{ GHz}$. A fit of the data is superimposed to the data in a black solid line.

frequency around 3 GHz with a given modulation depth. The phase-modulated beam is routed to the sample via a fibered circulator. The power of the incident beam is set with the variable optical attenuator before the circulator.

The reflected signal is amplified with a polarization-maintaining Erbium-Doped Fiber Amplifier (Thorlabs) and finally detected on an ultrafast indium gallium arsenide (InGaAs) photodiode (EOT-3500). The resulting electrical signal is split into DC and RF components using a bias tee. The DC signal is detected on an oscilloscope allowing the acquisition of the optical spectra. The RF signal is amplified with a 20 dB low-noise amplifier and detected on an electrical spectrum analyzer.

The optical amplifier yields gains up to 30 dB for input powers of -20 dBm at 1550 nm that can be adjusted by tuning the pump current. Such amplifiers come with an unwanted effect which is the amplified spontaneous emission (ASE) which tends to limit the achievable gain. In practice, the ASE adds noise to our thermomechanical spectra measurements which in some cases prevents us from observing the Brownian motion signal. A balance between a high enough gain and not too high ASE has to be found for each fiber-sample alignment position to observe the thermomechanical spectra. The effect of the ASE can be mitigated by the use of an optical filter reducing the ASE-ASE beating,

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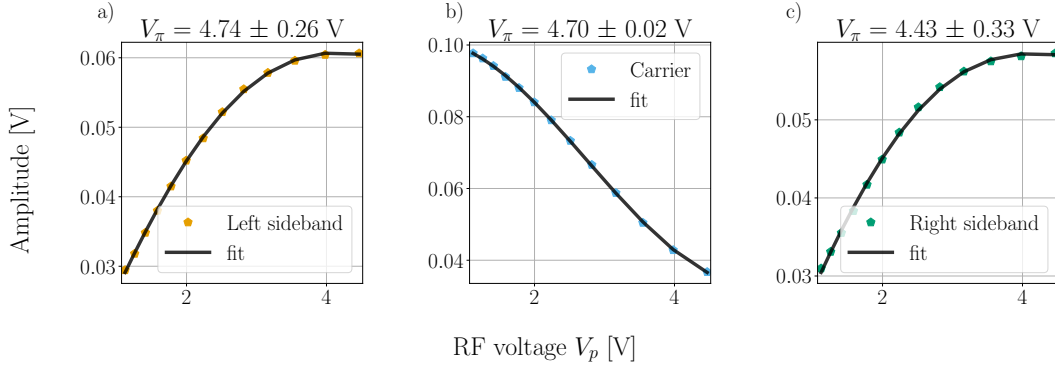


Figure 5.2: Determination of the half-wave voltage of the phase modulator V_π from the evolution of the amplitude of the carrier (b) and first-order sidebands (a and c), depicted in blue, orange and green respectively, fitted with Bessel functions, depicted in solid black line. The extracted values of V_π are in agreement with each other with an error of at most 5 %.

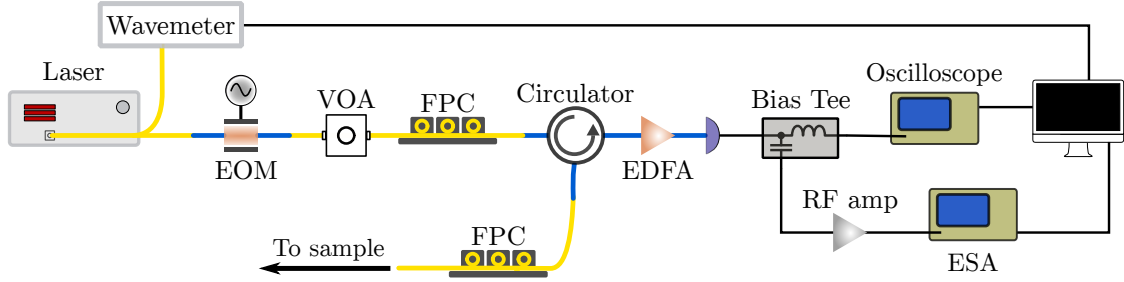


Figure 5.3: Schematic of the setup used to measure the noise spectral densities in a side of the fringe measurement configuration. **EOM**: Electro-optic phase modulator, **VOA**: variable optical attenuator, **FPC**: fibered polarization controller, **EDFA**: Erbium-doped fiber amplifier, **PD**: photodiode, **ESA**: electrical spectrum analyzer.

however, at the time we performed the experiment we didn't get yet such a filter.

5.2.1.3 Direct detection results

The phase-modulation calibration setup depicted in fig. 5.3 was tested for three cryostat temperatures. The purpose of the test is to demonstrate the linearity of the measured noise ratio temperature with the effective cryostat temperature measured with the calibrated sensors.

For each temperature, a noise spectrum is acquired with an optical power of 100 μ W and the following EDFA pump current and temperature values $I_{\text{EDFA}} = 200$ mA, $T_{\text{EDFA}} = 25$ $^{\circ}$ C. The laser frequency is manually tuned to the side of the optical resonance, where the slope is maximal to obtain the best phase-amplitude transduction. The phase modulation is red-detuned with respect to the

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mechanical resonance with an amplitude $\phi = \pi/1000$ i.e., $V_p = V_\pi/1000$. The resolution bandwidth (RBW) of the spectrum analyzer is set to 24 kHz and the spectra are saved after averaging over 400 traces.

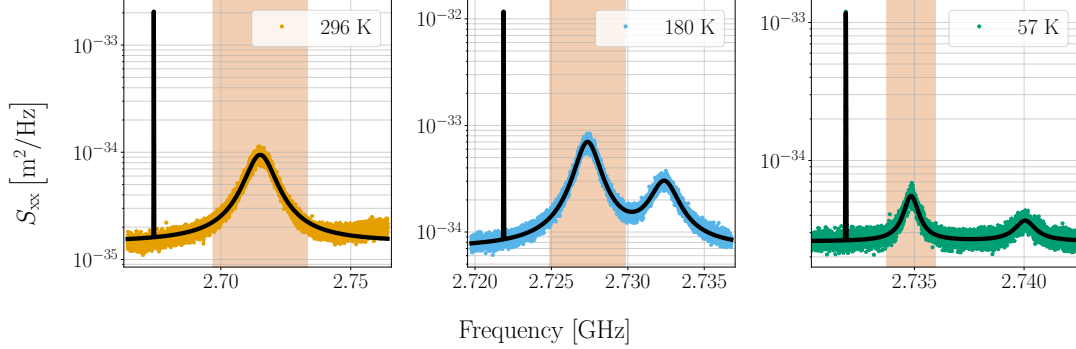


Figure 5.4: Calibrated displacement spectra and the phase modulation calibration tone measured with the side of the fringe scheme at various temperatures. The data is represented in colored points with a numerical fit in a black solid line. The mechanical mode of interest is highlighted in orange for each temperature.

Fig. 5.5 depicts the calibrated displacement spectra and the numerical fits at $T_{\text{cryo}} = 296$ K, 180 K and 57 K. Several mechanical modes have been observed in the vicinity of the mode of interest (with the highest thermal peak) that we consider as the fundamental confined mode of the optomechanical crystal even though we have no direct evidence. The central frequency of all the modes slightly changes with temperature due to small thermal dilatation, modification of the young modulus, and modification of the stress in the structure. At some temperatures (for instance 180K), two modes can get very close to each other and one has to be very careful when extracting the thermal noise of one mode without being contaminated by the second.

The noise spectral density $P_{\text{dB}} [\Omega]$ over a given frequency range corresponding to the measurement RBW as is measured with a spectrum analyzer, expressed in dBm, is converted to an amplitude spectral density $V_{\text{AS}} [\Omega]$ expressed in units of $\text{V}/\sqrt{\text{Hz}}$ using the following relations:

$$P_{\text{dB}} [\Omega] = 10 \log \left(20 V_{\text{AS}}^2 [\Omega] \times \text{RBW} \right). \quad (5.10)$$

Thus,

$$V_{\text{AS}}^2 [\Omega] = \frac{10^{P_{\text{dB}}[\Omega]/10}}{20 \times \text{RBW}}. \quad (5.11)$$

The resulting amplitude spectral density is then converted into displacement spectrum $S_{\text{xx}} [\Omega]$

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expressed in m^2/Hz using the known applied phase modulation with the following expression:

$$S_{\text{xx}} [\Omega] = \xi (L, \Omega_{\text{mod}}, \nu_{\text{L}}, V_{\pi}) \times V_{\text{AS}}^2 [\Omega], \quad (5.12)$$

where ξ is a conversion factor expressed in m^2/V^2 that can be derived from eq. 5.3. Indeed, knowing the cavity length from the estimated cavity free spectral range $\Delta\nu_{\text{FSR}} = \frac{c}{2nL}$ where n is the refractive index of the crystal and knowing the phase-shift $\phi(V)$, the conversion coefficient would read:

$$\xi (L, \Omega_{\text{mod}}, \nu_{\text{L}}, V_{\pi}) = \pi^2 \frac{\Omega_{\text{mod}}^2}{\nu_{\text{L}}^2} \frac{L^2}{V_{\pi}^2} \quad (5.13)$$

Finally, a numerical fit of the displacement spectra is performed. The mechanical modes are fitted with Lorentzian lineshapes according to the mechanical response (eq. 1.44) whereas the calibration tone is fitted with a Gaussian lineshape corresponding to the broadening of a monochromatic signal observed with the bandwidth of the spectrum analyzer. The areas of the calibration peak and the highlighted mechanical mode are computed following the fit results. The latter is, according to eq. 1.45, proportional to the mode's temperature T_{mode} . The area of the mechanical mode is normalized by the area of the calibration tone such that any mismatch in the optical coupling and measurement detuning between the different cryostat set temperatures is accounted for through the reference phase modulation. This normalization yields us a ratio that we will refer to as R_{T} , where T is the cryostat set temperature and the ratio at room-temperature $R_{296\text{ K}}$ is used as a reference for computing the other mode temperatures T_{noise} from their corresponding ratios. This computation is summarized in the following relation:

$$T_{\text{noise}} = 296\text{ K} \times \left(\frac{R_{\text{T}}}{R_{296\text{ K}}} \right). \quad (5.14)$$

The results are depicted in fig. 5.5 where the inferred noise temperatures are plotted against the temperature measured by the Cernox temperature sensors that give the bath temperature seen by the sample. A linear fit to the data is visible in orange and the fit error is displayed in light orange. The inferred temperatures show good linearity with respect to the Cernox temperatures, they are all within the fit error except for the second point at 180 K where a discrepancy of 40 K is observed which can partly be explained by hybridization with the second mechanical mode, visible in the corresponding displacement spectrum in fig. 5.4. This hybridization may affect the convergence of the fit thus the corresponding value of R_{T} .

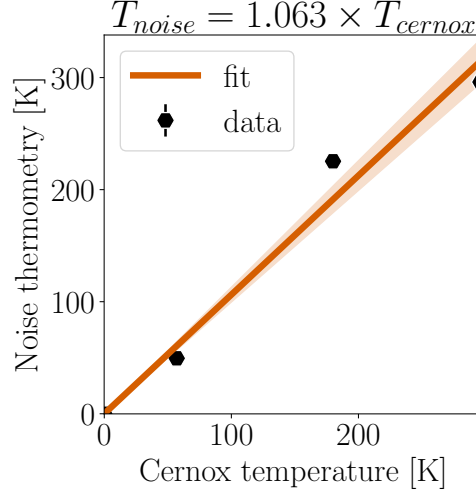


Figure 5.5: Noise thermometry results. Inferred noise temperature is depicted in black pentagons and a linear fit is plotted on top in an orange solid line. The fit error is displayed in light orange. The fit yields a slope of 1.063, suggesting a good linearity of the inferred noise temperature with respect to the Cernox temperatures.

The experiment described above has been performed to test the reliability of the phase modulation as a calibration for our noise thermometry measurements. The extracted temperatures show that this method works and can be used to measure the mechanical mode's temperature.

5.2.2 Homodyne setup

5.2.2.1 Detection principle

A second detection scheme allowing the direct measurement of the phase of the signal beam reflected from the optomechanical crystal by comparing it to a phase reference beam called the local oscillator (LO) beam coming from the same was set up. In practice, a laser beam is split into two separate beams. Part of the beam is sent towards the OMC to probe its mechanical state which after reflection is recombined, on a beamsplitter (BS), with the LO beam shifted by a certain phase delay. The two outputs of the BS are sent onto two photodiodes and finally, the resulting photocurrents are subtracted.

Let a^{sig} and a^{LO} respectively describe the reflected light field containing the signal of interest and the reference LO and φ the relative phase between the two fields, as represented in fig. 5.6. If a^{sig}

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and a^{LO} arrive at the input ports of the BS, the recombined fields exiting the output ports of the BS, $a_{\pm}(t)$ can be expressed as follow:

$$a_{\pm}(t) = \frac{1}{\sqrt{2}} \left[\left(a^{\text{sig}} \pm a^{\text{LO}} e^{i\varphi} \right) \right]. \quad (5.15)$$

The difference of the photocurrents monitored by both photodiodes reads:

$$I_-(t) \propto |a_+(t)|^2 - |a_-(t)|^2. \quad (5.16)$$

where the proportionality coefficient is simply e the fundamental unit of charge, if the square of the fields $|a|^2$ is expressed in terms of photon per second and if the photodiodes are ideal (1 photon gives 1 electron). In this following theoretical study, we take this coefficient equal to unity ($e = 1$) for readability.

We can linearize the expression of $I_-(t)$, eq. 5.16, around the mean-values of the field, assuming the mean amplitudes of the signal and LO, $\bar{a}^{\text{sig}}$ and $\bar{a}^{\text{LO}}$ to be real-valued. We first express the steady-state equation of the difference current $\bar{I}_-$:

$$\bar{I}_- = 2 \bar{a}^{\text{sig}} \bar{a}^{\text{LO}} \cos \varphi = 2 \sqrt{\bar{I}^{\text{sig}} \bar{I}^{\text{LO}}} \cos \varphi. \quad (5.17)$$

We see that the current difference is tied straightforwardly to the relative phase φ between the two fields. Its mean value is, in particular, zero when the fields are in phase quadrature i.e., $\varphi = \pi/2$, and maximal when the fields are in-phase i.e. $\varphi = 0$. This signal can be used as an error signal to control the length of the LO arm, thus controlling the phase difference φ . Now, if we consider a small phase fluctuation $\delta\varphi$ around a mean-value φ , we can see from fig. 5.7 that if φ is a multiple of π the resulting intensity fluctuations are minimal whereas if φ is a multiple of $\pi/2$ then we have maximal fluctuations in the difference current.

On the other hand, the signal of interest will be encoded in the fluctuations of the current difference

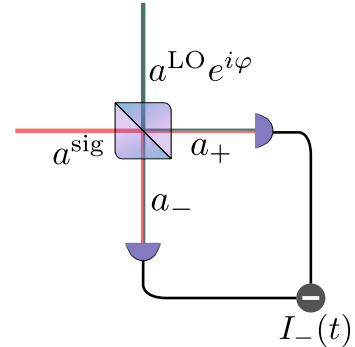


Figure 5.6: Schematic description of a homodyne detection setup. Two light fields coming from the same source are recombined on a BS, and the phase difference φ between the two fields can be controlled. The outputs of the BS are detected on two photodiodes generating two photocurrents that are electronically subtracted.

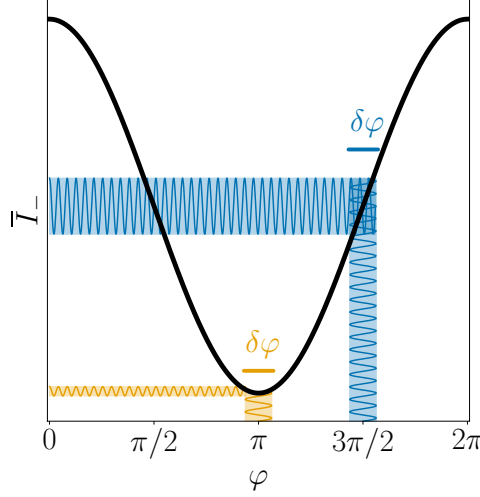


Figure 5.7: Relation between the mean difference current $\bar{I}_-$ and the relative phase φ as expressed in relation 5.17. It can be seen that for a small phase fluctuation $\delta\varphi$ the choice of the detection phase φ is crucial for detection sensitivity. Indeed, if $\varphi = \pi[2\pi]$ (orange) the sensitivity is minimal whereas if $\varphi = \pi/2[2\pi]$ (blue) it is maximal.

I_- which reads:

$$\begin{aligned} \delta I_-(t) = & \bar{a}^{\text{sig}} \left(\delta a^{\text{LO}}(t) e^{i\varphi} + \delta a^{\text{LO}*}(t) e^{-i\varphi} \right) \\ & + \bar{a}^{\text{LO}} \left(\delta a^{\text{sig}}(t) e^{-i\varphi} + \delta a^{\text{sig}*}(t) e^{i\varphi} \right). \end{aligned} \quad (5.18)$$

The fluctuations of I_- are the sum of the fluctuations of the quadrature a_φ^{out} of the reflected field and the quadrature a_φ^{LO} of the local oscillator weighted respectively by the mean fields $\bar{a}^{\text{LO}}$ and $\bar{a}^{\text{sig}}$. In most cases, the first term is neglected with respect to the second one since the intensity of the LO is generally chosen much greater than that of the signal. Using eq. 1.34 and 1.39 we can write the expression of the current difference noise spectrum $S_-[\Omega]$, as follows:

$$S_-[\Omega] = \bar{I}^{\text{LO}} S_\varphi^{\text{sig}}[\Omega], \quad (5.19)$$

where, S_φ^{sig} is noise spectrum of the quadrature of angle φ of the reflected field a^{sig} . Furthermore, from eq. 5.19 we can see that one can access any quadrature of the noise spectrum of the reflected field simply by controlling the relative phase φ and this quadrature is amplified by the mean intensity of the local oscillator $\bar{I}^{\text{LO}}$.

In order to be an efficient phase detection, the homodyne detection that will be described in the next section constituted by two photodetectors has to be balanced on three different aspects. The

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first aspect is the power arriving on the photodetectors. Indeed, after being split on the BS, each field individually must have equal power so that the subtraction process yields results similar to that presented in fig.. 5.7. It requires an as good as possible 50-50% beamsplitter.

The second aspect is balancing the lengths of both arms of the homodyne detection, that is to say, the lengths traveled by light from the location where both beams are created, to the location where they are recombined. Let us now define ϕ_{sig} and ϕ_{LO} as the accumulated phases of the signal and LO arms on their own paths before being recombined, such that $\varphi = \phi_{\text{LO}} - \phi_{\text{sig}}$ (the origin of phase is taken at the point where both beams are separated). The phase of the signal/LO depends on the optical length traveled by light at the laser's wavelength λ such that $\phi_{\text{sig/LO}} = \frac{2\pi}{\lambda} L_{\text{sig/LO}}$, thus the relative phase between the two arms reads:

$$\varphi = \frac{2\pi}{\lambda} (L_{\text{LO}} - L_{\text{sig}}) \quad (5.20)$$

where $L_{\text{sig(LO)}}$ is the effective length of the signal (LO). The desired phase values φ ranging from 0 to $\pi/2$ allowing to access all quadratures require the following condition on the difference of path lengths ΔL , relative to the optical wavelength:

$$\frac{\Delta L}{\lambda} = \frac{L_{\text{LO}} - L_{\text{sig}}}{\lambda} \leq 1. \quad (5.21)$$

This condition is of the utmost importance namely because our laser source has a frequency noise around 400 kHz that is transduced to an intensity noise through the homodyne detection. To illustrate this effect, let us consider a frequency-jitter $\delta\lambda$ of the laser around the center wavelength λ in eq. 5.20 which would give:

$$\frac{\varphi}{2\pi} = \frac{\Delta L + \delta x}{\lambda + \delta\lambda} \simeq \frac{\Delta L}{\lambda} + \frac{\delta x}{\lambda} - \frac{\Delta L}{\lambda^2} \delta\lambda. \quad (5.22)$$

The first term of the expanded expression is the small mismatch that we propose to stabilize with the phase-shifter, the second, related to the displacement δx of the mechanical resonator, is the signal of interest. The last term, however, is an unwanted perturbation that scales with the length mismatch ΔL thus the importance of balancing the arm's length in our setup.

The third, and last, balancing aspect is the polarization of the signal and LO fields. Thus far, we only looked at the scalar components of our optical fields and neglected the vector aspect of our fields. Let us assume that our fields are two monochromatic fields oscillating at a frequency $\omega/2\pi$, propagating along two vectors $\mathbf{k}_{\text{sig}}$ and $\mathbf{k}_{\text{LO}}$ and polarized along $\mathbf{u}_{\text{sig}}$ and $\mathbf{u}_{\text{LO}}$. The resulting electrical

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fields will read:

$$\mathbf{E}^{\text{sig}}(\mathbf{r}, t) = a^{\text{sig}} e^{i(\omega t - \mathbf{k}_{\text{sig}} \cdot \mathbf{r} + \phi_{\text{sig}})} \mathbf{u}_{\text{sig}}, \text{ and } \mathbf{E}^{\text{LO}}(\mathbf{r}, t) = a^{\text{LO}} e^{i(\omega t - \mathbf{k}_{\text{LO}} \cdot \mathbf{r} + \phi_{\text{LO}})} \mathbf{u}_{\text{LO}},$$

where $\mathbf{r}$ is the position vector. Taking into account the vector dimension of the optical fields in the calculation of the steady-state equation of the difference current of our balanced homodyne detection we find that:

$$\bar{I}_- = 2\sqrt{\bar{I}^{\text{sig}} \bar{I}^{\text{LO}}} \cos \varphi (\mathbf{u}_{\text{sig}} \cdot \mathbf{u}_{\text{LO}}). \quad (5.23)$$

It follows from eq. 5.23 that the signal and LO fields interfere only when their respective polarization are collinear to each other.

5.2.2.2 Description of the homodyne setup

In the previous section, we detailed the balanced homodyne detection principle and the different conditions that have to be met to have a phase detection with the suppression of the frequency noise of the laser. In the following, we will describe the experimental realization of our homodyne detection.

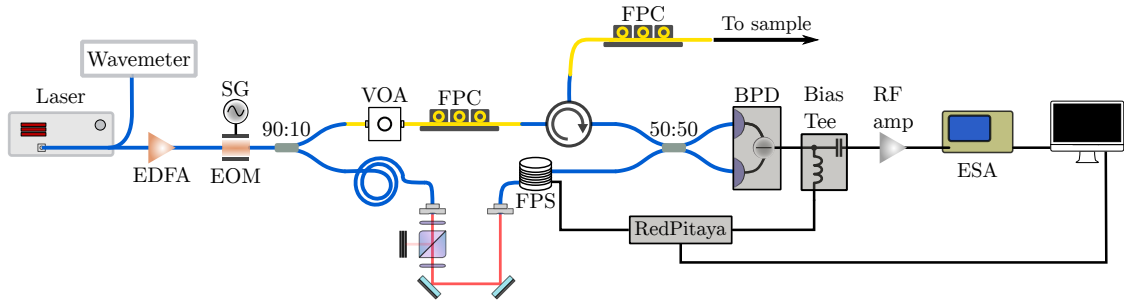


Figure 5.8: Schematic of the developed fibered-homodyne detection setup. **EDFA**: Erbium-doped fiber amplifier, **EOM**: Electro-optic phase modulator, **SG**: Low-noise signal generator, **VOA**: Variable optical attenuator, **FPC**: Fiber polarization controllers, **FPS**: Fibered phase-shifter, **ESA**: Electrical spectrum analyzer. The optical fibers depicted in blue are polarization-maintaining and those in yellow are standard single-mode optical fibers.

The output of a tunable laser (Yenista TUNICS T100S-HP) is amplified using an EDFA before being sent to a fibered electro-optic phase modulator which is driven at a frequency $\Omega_{\text{mod}} = 2.88 \text{ GHz}$ with a low-noise frequency generator. The phase-modulated beam is then split into a signal and LO paths with a 90:10 fibered splitter where 10 % of the light is sent to the local oscillator path and the remaining 90 % into the signal path.

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On the signal path, the laser beam passes through a variable optical attenuator which is used to tune the optical power sent to the sample. An optical circulator routes the incoming beam toward the sample after passing through an FPC and reflected beam towards the detection apparatus. On the local oscillator path, the beam is prepared such that the length it propagates through is equal to that of the signal path using optical fiber spools for coarse adjustments and a free-space delay line for finer adjustments. The latter is also used to match the polarization of the signal beam using free-space polarizing elements (half-wave plates and a polarizing beamsplitter). Finally, a fibered phase shifter (General optics FPS-002) is used to actively stabilize the phase of the LO. A detailed description of the optical setup is given in fig. 5.8.

The two beams are then recombined with 2×2 3 dB splitter and split into two outputs that carry equal power ². Each output is connected to the fiber input of a high-speed balanced photodetector (Discovery Semiconductors DSC710-39-FC/APC-K-2) converting the light impinging on each photodiode into photocurrents that are electronically subtracted within the BPD. The resulting difference current $I_-(t)$ is split into a DC and RF component using a bias tee. The DC signal is used as an error signal to stabilize the relative phase φ whereas the RF signal, carrying the relevant phase fluctuation signal is amplified with a 20 dB low-noise electronic amplifier before sending it to an electrical spectrum analyzer.

Balancing the arm's lengths In this section, we will describe the techniques used to balance the arm lengths of the interferometer. First a description of the coarse balancing procedure $\Delta L \gtrsim 1$ m then the fine-tuning procedure $\Delta L \lesssim 1$ m.

Coarse balancing The procedure for balancing the arm's lengths at a difference $\Delta L \lesssim 1$ m relies on the time-of-flight measurement technique where an amplitude modulation is applied on the laser through its amplitude modulation port. The amplitude-modulated beam travels through the same setup as the one depicted in fig. 5.8 except for the free-space part of the local oscillator's path which will be used for fine-tuning the length difference.

A square-wave signal at a frequency of 1 MHz is generated with a signal generator and fed into the high-speed amplitude modulation input port of the laser that directly modulates the intensity of the

²In practice, the splitting ratio is polarization dependent thus a perfect 50 % splitting ratio is hard to achieve.

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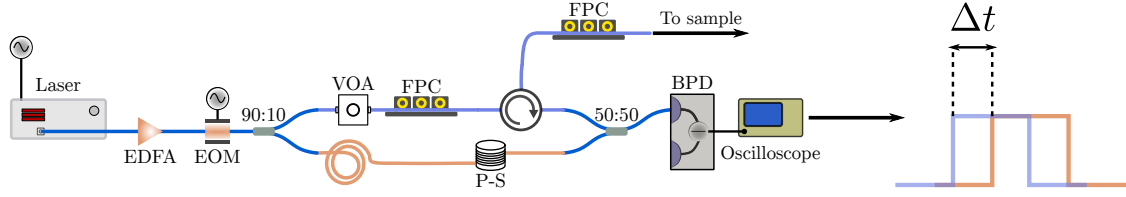


Figure 5.9: Coarse balancing of the arm's lengths. **Left:** Setup used to perform the time-of-flight coarse length balancing using an amplitude-modulated signal. **Right:** Schematic of the signals observed during such experiments, each color corresponds to an interferometer's arm. Purple is for the signal arm and orange is for the LO arm.

laser diode. After traveling through the setup depicted in fig. 5.9, the modulated signal is detected on a single photodiode of the BPD and the resulting photocurrent is read with a high-speed oscilloscope. By letting the beam from the signal arm pass while blocking the other, one can see the modulated signal as a square signal on the oscilloscope, its rising edge is set at the oscilloscope's time origin such that a time reference is set. The signal from the signal is then blocked and the LO is set to pass, we see the amplitude modulation with a time-lag Δt , as depicted on the right panel of fig. 5.9. This time lag is proportional to the length difference ΔL :

$$\Delta L = c n_{\text{silica}} \Delta t, \quad (5.24)$$

where c is the speed of light in vacuum and n_{silica} is the refractive index of silica which is roughly equal to 1.5 at 1550 nm. The time resolution of the used oscilloscope is of 2 ns corresponding to a length resolution of roughly 1 m, according to eq. 5.24.

Fine balancing To finely tune the length difference ΔL below the 1 m another procedure is used and explained hereafter. The optical setup depicted in fig. 5.8 is used without modification: a free-space part mounted on a cage system with a coarse adjustable length of a few centimeters combined with a micrometer translation supporting one of the fiber couplers and additional short optical fibers of 50 cm, 30 cm and 20 cm are used to fine-tune the length difference.

The wavelength of the laser is modulated such that $\lambda(t) = \lambda_l + \delta\lambda(t)$ with $\lambda(t=0) = \lambda_l$. The DC part of the difference current, corresponding to the mean difference current, is monitored on an oscilloscope where interference fringes can be observed. The frequency of these fringes is dependent on the length imbalance ΔL . Let us write the mean difference current that we observe when a frequency

5.2. DESCRIPTION OF THE EXPERIMENTAL SETUP

modulation of the laser is applied using eqs. 5.17 and 5.20:

$$\bar{I}_- = 2\sqrt{\bar{I}^{\text{sig}}\bar{I}^{\text{LO}}} \cos\left(\frac{2\pi\Delta L}{n_{\text{silica}}\lambda_l}\left[1 + \frac{1}{\lambda_l}\frac{d\lambda}{dt}t\right]\right). \quad (5.25)$$

We can see that a frequency modulation adds a time-dependent term to the mean difference current of the homodyne detection. This term corresponds to the interference fringes observed on the oscilloscope.

We can define a frequency ω_{fringe} describing the interference fringes frequency:

$$\omega_{\text{fringe}} = \frac{2\pi\Delta L}{n_{\text{silica}}\lambda_l^2} \frac{d\lambda}{dt}. \quad (5.26)$$

Now that we have a direct relationship between the observed signal $\bar{I}_-$ and the length imbalance ΔL , we start by finding the appropriate combination of short optical fibers to reduce as much as possible ω_{fringe} . The finer tweaks are then made by coarsely adjusting the position of the fiber coupler by sliding it on the cage system's rods while observing the interference signal and looking for the position for which the frequency ω_{fringe} starts to increase. At this point, the coupler is fixed, and using the micrometer translation stage we find the position at which ω_{fringe} is minimal.

Active stabilization of the homodyne phase After balancing the arm's length as close to one another as possible using the aforementioned procedures, one can control the phase φ of the homodyne detection by changing the length of the local oscillator path. Indeed, from eq. 5.25, without the frequency modulation and in the case where $\Delta L \simeq n_{\text{silica}}\lambda_l$, we can see that by varying ΔL from 0 to $n_{\text{silica}}\lambda_l$ one can vary the phase φ from 0 to 2π .

In free-space setups, the fine control of ΔL to lock the homodyne's phase is usually done with a mirror mounted on a piezoelectric actuator. Applying a voltage to the latter induces a displacement of the mirror thus a change in the length difference ΔL . In the setup depicted in fig. 5.8, a commercial fibered phase-shifter is used to control the homodyne's phase. It consists, in our case, of a $\simeq 36 \pm 2$ cm polarization-maintaining fiber mounted on a piezoelectric actuator that stretches the length of optical fiber by an amount ΔL that is proportional to the applied voltage.

The phase-shifter is driven with a field-programmable gate array (FPGA) RedPitaya board equipped with fast analog input and outputs. In particular, a python-based software package PyRPL (Python RedPitaya Lockbox), implemented with different modules that are relevant for optics experiment and created by L. Neuhaus [77], is used to lock the phase of the local oscillator.

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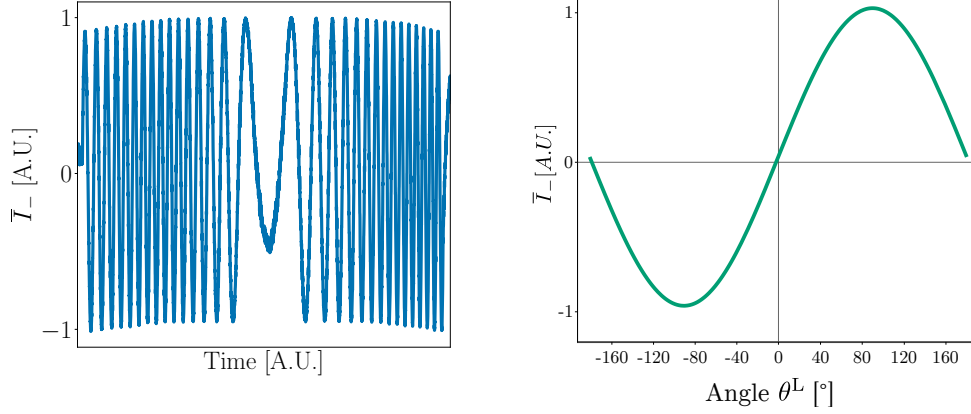


Figure 5.10: Phase lock procedure. **Left:** Interference fringes obtained by sweeping the phase of the local oscillator with the fibered phase-shifter. **Right:** PyRPL phase-lock calibration curve obtained from the measured interferogram (Left).

The DC signal coming from the BPD is fed to one of the RedPitaya's inputs (in1) as an error signal and an output (out1) is plugged into the phase-shifter. The phase-lock procedure is the following:

- A signal generator embedded in the RedPitaya board is set to generate a sine wave signal at a frequency of 10 Hz on the output out1. The sine signal induces a periodic change in the LO length creating interference fringes on the signal read through in1, as depicted on the right panel of fig. 5.10. The offset of input in1 is subtracted.
- The software extracts the parameters of one fringe (minimum, 0, and maximum) and defines the lock angles $\theta^L = \varphi + \pi/2$, such that $\theta^L = 0 [2\pi]$ corresponds to a maximum sensitivity on the phase quadrature and conversely, $\theta^L = \pi/2 [2\pi]$ to a high sensitivity on the amplitude quadrature.
- Finally, one has to tweak the PID parameters such that the phase lock is stable over time.

Polarization matching As mentioned in sec. 5.2.2.1, it is not enough to balance the powers impinging on the photodiodes (through the 2×2 3 dB splitter) and to balance the signal and LO optical lengths to achieve a good phase detection. Another parameter has to be tweaked to observe the interference fringes that would allow the phase-lock to work and it is the polarization of the interfering beams. Indeed, they have to be collinear to see those fringes as eq. 5.23 suggests. Usually, in a free-space setup,

5.3. HOMODYNE DETECTION IN THE SIDE OF THE FRINGE CONFIGURATION

this aspect is automatically realized as the last element used to recombine both beams is a polarization beamsplitter whose outputs are polarized. In our setup, the last element before the photodiodes is a 50%-50% fibered beamsplitter that splits or recombines the beam by evanescent coupling and does not guarantee the output beams to be in the same polarization state.

On this matter, we only have a single degree of freedom on which we can work on. Indeed, the polarization of the signal beam that reaches the 3 dB splitter is set by the optical circulator. The latter allows a single polarization state, aligned with its slow axis, to propagate to the output port. So, the polarization of half our beams is set and is, supposedly, linear therefore the polarization of the LO beam has to be matched to that of the signal.

A set of polarizing elements are placed in the free-space path of the LO beam. Indeed, a first half-wave plate used in combination with a PBS allows the control of the LO optical power and most importantly ensures that the polarization after the PBS is linear. A second half-wave plate, placed after the PBS, is used to rotate the linearly polarized light exiting the PBS with the slow axis of a polarization-maintaining optical fiber which is connected to a fiber coupler. The slow axis of a PM optical fiber is collinear with the axis of its connector key.

Finally, whilst sweeping the phase of the local oscillator, such that one observes interference fringes like those depicted on the left panel of fig. 5.10, the polarization of the LO beam is rotated, using the second half-wave plate, such that the contrast defined as:

$$\mathcal{C} = \frac{\bar{I}_{-}^{\max} - \bar{I}_{-}^{\min}}{\bar{I}_{-}^{\max} + \bar{I}_{-}^{\min}}, \quad (5.27)$$

is maximal, suggesting that the polarizations of the two beams are collinear. The fringe contrast should be minimal when the beams' polarizations are perpendicular i.e., rotating the half-wave plate's axis by an angle of 45° from the position corresponding to the maximal contrast.

5.3 Homodyne detection in the side of the fringe configuration

5.3.1 Experimental limitations

One of the main limitations of our experiment is the low signal-to-noise ratio when measuring the thermal noise of the optomechanical resonator.

The samples used for this study have been designed to have mechanical breathing modes with

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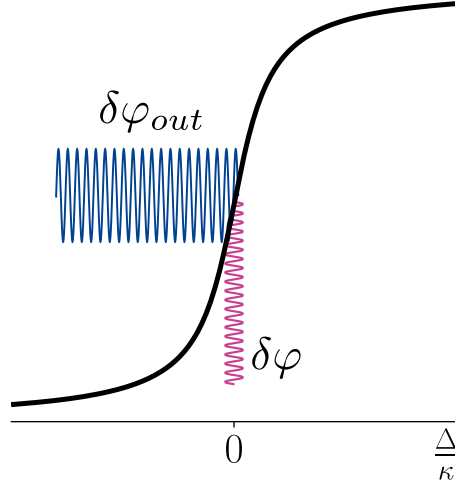


Figure 5.11: Phase response of an optical cavity and optimal transduction of phase fluctuation at zero detuning.

frequencies of a few GHz. The latter have been optimized for the observation of quantum effect whereas for the classical regime lower resonance frequencies (in the MHz regime) would have been more appropriate. The reason for that can be seen through eq. 1.44 evaluated at $\Omega = \Omega_m$ which would take the following form:

$$S_{xx}[\Omega_m] = \frac{2k_B T}{m_{\text{eff}} \Gamma_m \Omega_m^2}, \quad (5.28)$$

so, if we compare a MHz resonator to a GHz one, assuming that the effective mass changes with the same coefficient as the mechanical resonance frequency, one would get a thousandfold improvement of the sensitivity to the displacement of the resonator in the MHz range compared to that in the GHz range. In our case, the use of GHz resonator leads to a very small signal when measuring the thermal noise of the system.

The overall optical losses in our setup, including the losses in the fibered optical paths and the fiber-to-sample coupling losses in addition to the low vibrational signal that we are interested in measuring result in a low signal-to-noise ratio (SNR) of the displacement spectra. Furthermore, other noise sources in our optical setup, the EDFA's ASE for instance, result in a degradation of the already low SNR.

A straightforward solution to increase the SNR is to use higher optical power to probe the mechanical resonator however this simple solution comes with several important drawbacks. The first, as was described in sec.4.3.3, induces a self-heating of the sample, that can be estimated and thus corrected,

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caused by the absorption of optical power within it. The second drawback is that the homodyne measurement can not be performed at zero detuning where the output phase fluctuations are maximal as one can see in fig. 5.11, because an optical bistability appears even at relatively low input powers ($P_{\text{in}} \simeq 10 \mu\text{W}$) prohibiting the measurement at $\Delta = 0$.

5.3.2 Calibration of the optimal lock angle

Ideally at zero detuning, the lock angle θ^L , as defined in fig. 5.10, should be set to a value of 0° such that $\bar{I}_- = 0$ yielding the maximum sensitivity to phase fluctuations. In our setup, as the working point cannot be set at the optical resonance ($\Delta = 0$) due to optical bistability, we combine an homodyne detection with a "side of the fringe" configuration. In that case, the lock angle θ^L of the homodyne detection yielding maximum sensitivity has to be estimated.

The estimation of an optimal lock angle θ^L is done using the setup depicted in fig. 5.12, where the mechanical mode and a phase modulation on the signal arm are measured for different detunings and lock angles θ^L .

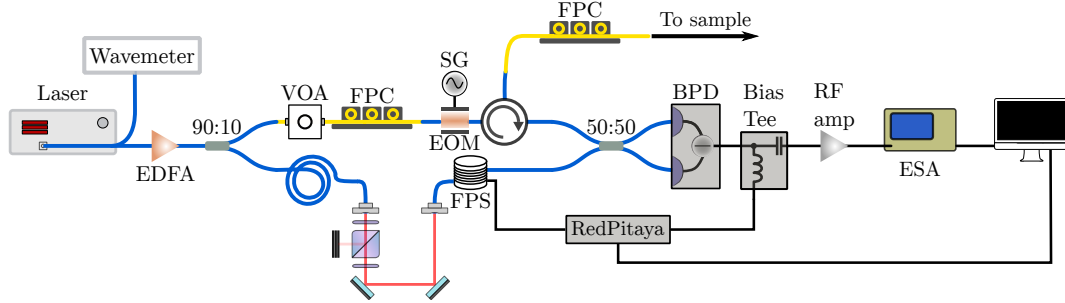


Figure 5.12: Optical setup used to perform the calibration of the lock angle θ^L .

Fig. 5.13, depicts the noise spectra obtained at room temperature with an input optical power $P_{\text{in}} = 260 \mu\text{W}$ at $\Delta = \kappa/5$ for different lock angles ranging from $\theta^L = -60^\circ$ to $\theta^L = 60^\circ$. We didn't investigate angles θ^L higher than $\pm 60^\circ$ because the slope of the fringe becomes very small and the phase lock is hard to achieve. The raw noise spectra are converted to a displacement spectral density $S_{xx} [\Omega]$ which is then numerically fitted with a Lorentzian lineshape for the mechanical mode and a Gaussian lineshape for the phase modulation peak. One can see that amplitude of the mechanical mode's peak increases while the amplitude of the phase modulation peak decreases with increasing lock angle θ^L . The optimal angle allowing to have the maximal sensitivity over the mechanical mode

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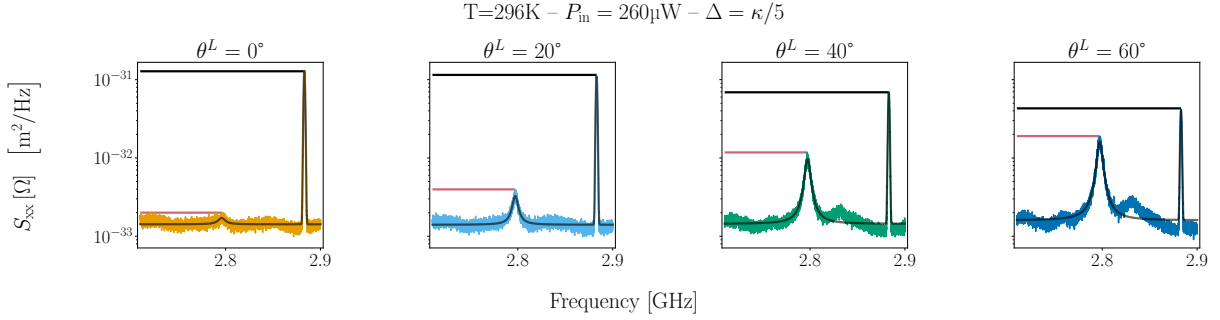


Figure 5.13: Noise spectra depicting the evolution of the mechanical mode and the phase modulation peaks amplitudes with respect to homodyne lock angle. The measurements were performed at room temperature with an input power of $P_{\text{in}} = 260 \mu\text{W}$ and at a detuning $\Delta = \kappa/5$. The pink and black horizontal lines depict the height of the mechanical mode and phase modulation peaks respectively.

peak seems to be at $\theta^L = 60^\circ$.

This experimental procedure has been repeated for various detunings ranging from $\Delta = 3\kappa/2$ to $\Delta = \kappa/10$ and for each detuning the amplitudes of the mechanical mode and phase modulation peaks were extracted from the numerical fit. The results of this experiment are compiled in fig. 5.14 where these amplitudes, normalized such that the maximum value at each detuning is equal to unity, are plotted against the lock angle θ^L .

One would expect the sensitivity of both the thermal noise and the phase modulation applied on the signal arm to follow the same trend. A periodic pattern where they are both maximum at $\theta^L = 0 [2\pi]$ and both minimum at $\theta^L = \pi/2 [2\pi]$.

The calibration phase modulation evolution, on the one hand, is consistent with what was expected. Indeed, for any value of cavity detuning Δ , the sensitivity follows the aforementioned trend as depicted in fig. 5.14.b at the exception of the cases where $\Delta = \kappa/5$ and $\Delta = \kappa/10$ where a deviation can be observed. At this point, we cannot conclude whether it is a measurement artifact or if it has a plausible origin.

On the other hand, the thermal noise sensitivity is strongly dependent on the cavity detuning Δ ; its maximum sensitivity is obtained for a lock angle $\theta^L = 20^\circ$ for $\Delta = 3\kappa/2$ whereas for $\Delta = \kappa/10$ the maximum sensitivity is reached beyond $\theta^L = 60^\circ$, where the phase lock become unstable due to a decreasing of the slope of $\bar{I}_-$ (cf. fig. 5.10). The thermal peak sensitivity follows a sinusoidal profile as expected but shifted as compared to the usual case of a resonant cavity where the maximum sensitivity is at $\theta^L = \pm\pi/2$. We can assume that this sensitivity shift can be caused by detuning dependent phase

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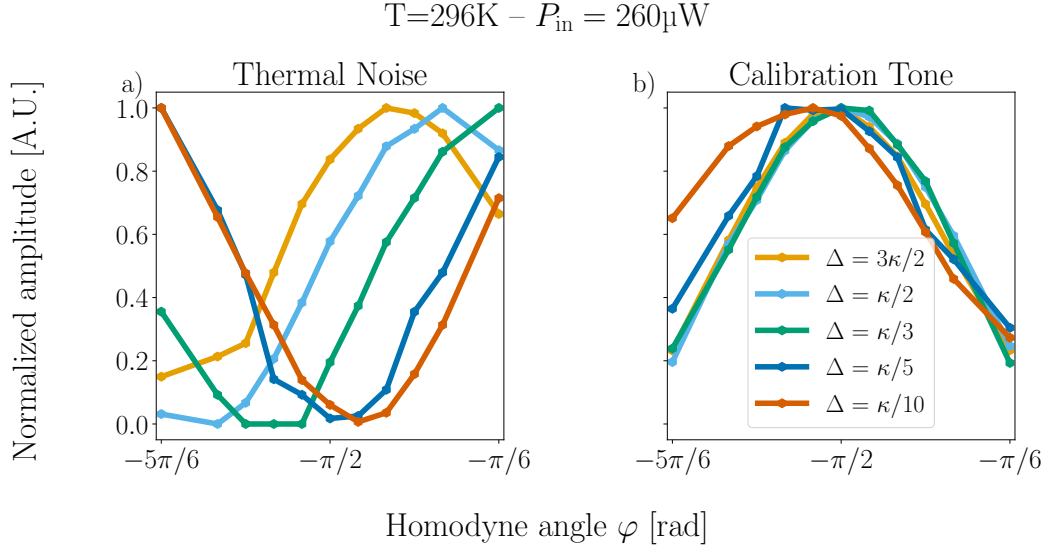


Figure 5.14: Compilation of the lock angle calibration for cavity detunings ranging from $\Delta = 3\kappa/2$ to $\Delta = \kappa/10$ suggesting a strong dependency of the lock angle with the cavity detuning Δ .

sensitivity or an interplay between amplitude and phase fluctuations coming from the side of the fringe measurement.

To understand this detuning-dependent sensitivity in the framework of an homodyne detection, a simplistic theoretical model can be considered. This model will be described in the section below.

5.3.3 Theoretical description of the side of fringe homodyne detection

Let us start with a simple description of the electric field in an optomechanical cavity. As depicted in fig. 5.15, it consists of an optical cavity of length L with an end mirror mounted on a spring allowing it to move around its equilibrium position in response to external forces (thermal force, radiation pressure force, or others), this displacement will be referred to as $\delta x(t)$.

An optical field is coupled in and out of the cavity at a rate κ_c , and the circulating field $a(t)$ in the cavity is radiatively coupled to its environment at a rate κ_{ex} describing the loss on mirrors like absorption or transmission via the end mirror. The total loss rate κ of the intra-cavity field is then $\kappa = \kappa_{\text{ex}} + \kappa_c$. Let us define, the optomechanical coupling $G = \frac{\omega_L}{L}$, where ω_L is the probe laser's frequency. Taking into account all these parameters, the equation of evolution of the intracavity field

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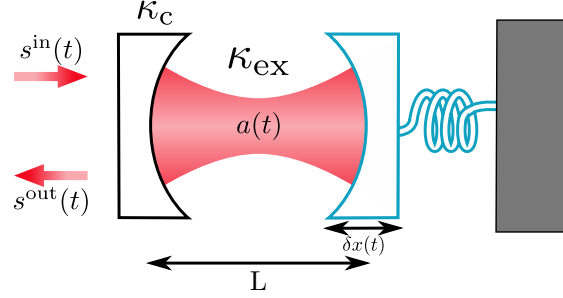


Figure 5.15: Schematic of an optomechanical system consisting of an optical cavity with a moving end mirror.

and the reflected field take the following form:

$$\dot{a}(t) = (i\Delta - \kappa/2) a(t) + \sqrt{\kappa_c} s^{\text{in}}(t) + ia(t)G\delta x(t), \quad (5.29)$$

$$s^{\text{out}}(t) = -s^{\text{in}}(t) + \sqrt{\kappa_c} a(t). \quad (5.30)$$

These fields can be linearized around their mean fields allowing us to define the equations describing the fluctuations of the intracavity and reflected fields. The fluctuation equations in the frequency domain read:

$$\delta a[\Omega] = \frac{\sqrt{\kappa_c} \delta s^{\text{in}}[\Omega] + i\bar{a}G\sqrt{\kappa_c} \delta x[\Omega]}{(\kappa/2 - i\Omega) - i\Delta}, \quad (5.31)$$

$$\delta s^{\text{out}}[\Omega] = \left(\frac{\kappa_c}{(\kappa/2 - i\Omega) - i\Delta} - 1 \right) \delta s^{\text{in}}[\Omega] + \left(\frac{i\bar{a}G\sqrt{\kappa_c}}{(\kappa/2 - i\Omega) - i\Delta} \right) \delta x[\Omega]. \quad (5.32)$$

The mean-values of these fields are described in eq. 1.18 for intracavity field and eq. 1.29 for the reflected field.

The fluctuations of the difference current δI_- in the frequency domain can now be computed by taking the Fourier transform of eq. 5.18. It is worth noting that throughout this calculation, the LO field and the signal field are phase modulated and for the latter, the expressions of the mean-field and fluctuation of the $s^{\text{out}}(t)$ will be used. On top of this, a length mismatch between the arms will be added to the model that will be treated as a time-lag $t' = \Delta L/c$ that will be added to the local oscillator field equations.

The intracavity field is assumed to be complex whereas the mean input field is assumed to be real-valued such that:

$$\bar{a} = |\bar{a}|e^{i\theta_{\text{in}}}, \quad (5.33)$$

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where $\theta_{\text{in}} = \arg\left(\frac{\bar{a}}{\bar{s}^{\text{in}}}\right)$ and the output field is also assumed to be complex thus we can write similarly to the intracavity field:

$$\bar{s}^{\text{out}} = |\bar{s}^{\text{out}}| e^{i\theta_{\text{out}}}, \quad (5.34)$$

where $\theta_{\text{out}} = \arg\left(\frac{\bar{s}^{\text{out}}}{\bar{s}^{\text{in}}}\right)$.

For calculation simplicity, we will define a phase $\Phi = \varphi + \theta_{\text{out}}$ where φ is the phase set between the output field and the local oscillator defined in sec.5.2.2.1. In doing, so the phase differences between the real-valued input field and the complex intracavity and output fields are taken into account in the calculation.

Finally, the calibration phase modulation at Ω_{mod} and of depth $\phi(V)$ is included in the calculation by applying a factor $e^{i\phi(V)\cos(\Omega_{\text{mod}}t)}$ on both the input field and the local oscillator. It acts as a global phase and seems not to play any role in the calculation however as it undergoes the time-lag t' on the LO path and the filtering by the cavity on the probe path, after interference it can be transduced into a visible amplitude modulation.

A tedious calculation leads to the following expression for the difference current $\delta I_- [\Omega]$:

$$\begin{aligned} \delta I_- [\Omega] \propto & \frac{\bar{a}^{\text{LO}}}{(\kappa/2 - i\Omega)^2 + \Delta^2} \left[\left(\kappa_c (\kappa/2 - i\Omega) - \left((\kappa/2 - i\Omega)^2 + \Delta^2 \right) \right) \cos \Phi + \kappa_c \Delta \sin \Phi \right] \delta p^{\text{in}} [\Omega] \\ & - \frac{\bar{a}^{\text{LO}}}{(\kappa/2 - i\Omega)^2 + \Delta^2} \left[- \left(\kappa_c (\kappa/2 - i\Omega) - \left((\kappa/2 - i\Omega)^2 + \Delta^2 \right) \right) \sin \Phi + \kappa_c \Delta \cos \Phi \right] \delta q^{\text{in}} [\Omega] \\ & - \frac{2G\sqrt{\kappa_c}|\bar{a}|}{(\kappa/2 - i\Omega)^2 + \Delta^2} \bar{a}^{\text{LO}} \left[\Delta \cos(\Phi - \theta_{\text{in}}) - (\kappa/2 - i\Omega) \sin(\Phi - \theta_{\text{in}}) \right] \delta x [\Omega] \\ & + 2\phi \bar{s}^{\text{in}} \bar{a}^{\text{LO}} \left\{ \frac{1}{(\kappa/2)^2 + \Delta^2} \left[\kappa_c \Delta \cos(\Phi + \theta') - \left(\kappa_c \kappa/2 - \left((\kappa/2)^2 + \Delta^2 \right) \right) \sin(\Phi + \theta') \right] \right. \\ & \left. - \frac{1}{(\kappa/2 - i\Omega)^2 + \Delta^2} \left[\kappa_c \Delta \cos \Phi - \left(\kappa_c (\kappa/2 - i\Omega) - \left((\kappa/2 - i\Omega)^2 + \Delta^2 \right) \right) \sin \Phi \right] \right\} \delta(\Omega - \Omega_{\text{mod}}), \end{aligned} \quad (5.35)$$

where $\theta' = (\Omega_{\text{m}} - \Omega)t'$ is a phase term that takes into account the length mismatch between the signal and LO arms, ϕ is the phase shift induced by the modulation and is considered < 1 , $\delta x [\Omega]$ the mirror's displacement fluctuations and $\delta(\Omega - \Omega_{\text{mod}})$ is a Dirac function centered at the modulation frequency Ω_{mod} ; and $\delta p [\Omega]$ and $\delta q [\Omega]$ are the intensity and phase quadratures of the input field respectively

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defined as follows

$$\delta p^{\text{in}}[\Omega] = \delta s^{\text{in}*}[\Omega] + \delta s^{\text{in}}[\Omega], \quad (5.36)$$

$$\delta q^{\text{in}}[\Omega] = i \left(\delta s^{\text{in}*}[\Omega] - \delta s^{\text{in}}[\Omega] \right). \quad (5.37)$$

It is worth noting that the field inside the optical cavity is filtered by the cavity bandwidth and this effect is described by the terms in $\kappa/2 - i\Omega$.

The first two terms of eq. 5.35 describing the evolution of the intensity and phase quadratures of the input field inside the cavity are not of importance in the following treatment whereas the third and last terms will be treated extensively hereafter.

The third term of eq. 5.35 describes the transduction of the phase fluctuations induced by the mirror's movement δx . When the cavity detuning $\Delta \neq 0$, we can see that there is an interplay between a phase and intensity modulation carried by the sine and cosine, respectively. It follows that in a phase-sensitive detection such as the homodyne detection, a measure of the phase of an optical resonator at a non-zero detuning leads to contamination of the pure phase fluctuations induced by the position fluctuations of the moving mirror through the transduction of the former phase fluctuation to intensity fluctuations of the reflected field, similar to a side of the fringe measurement in a direct detection scheme.

The fourth term of eq. 5.35, is the sum of two individual terms. The first describes the phase modulation on the LO arm that is not filtered by the optical cavity, it only interacts with mean-field of the reflected field through the homodyne mixing. The second, on the other hand, is a phase modulation that goes through the optical cavity and gets filtered by it. These two phase-modulation terms when summed yield a phase modulation that undergoes the same transduction [76] as the phase fluctuations due to position fluctuations of the moving

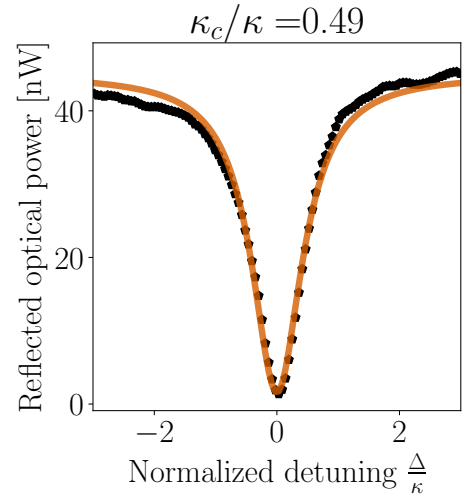


Figure 5.16: Optical resonance on which the sensitivity measurements of the thermal noise and phase modulation, depicted in fig. 5.14 were performed. A lorentzian fit of this resonance is depicted in a solid orange line through which the loss rate κ and κ_c were extracted.

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mirror.

The square modulus of each term of eq. 5.35 is numerically estimated for optical detuning values Δ ranging from $-\kappa$ to $+\kappa$ and homodyne phase values φ ranging from 0 to 2π while using the loss rates estimated from the optical resonance on which the measurement presented in sec. 5.3.2 were performed. Said optical resonance, acquired with the white light setup at low optical power, is depicted in fig. 5.16 with the coupling parameter $\eta_c = \kappa_c/\kappa \simeq 1/2$.

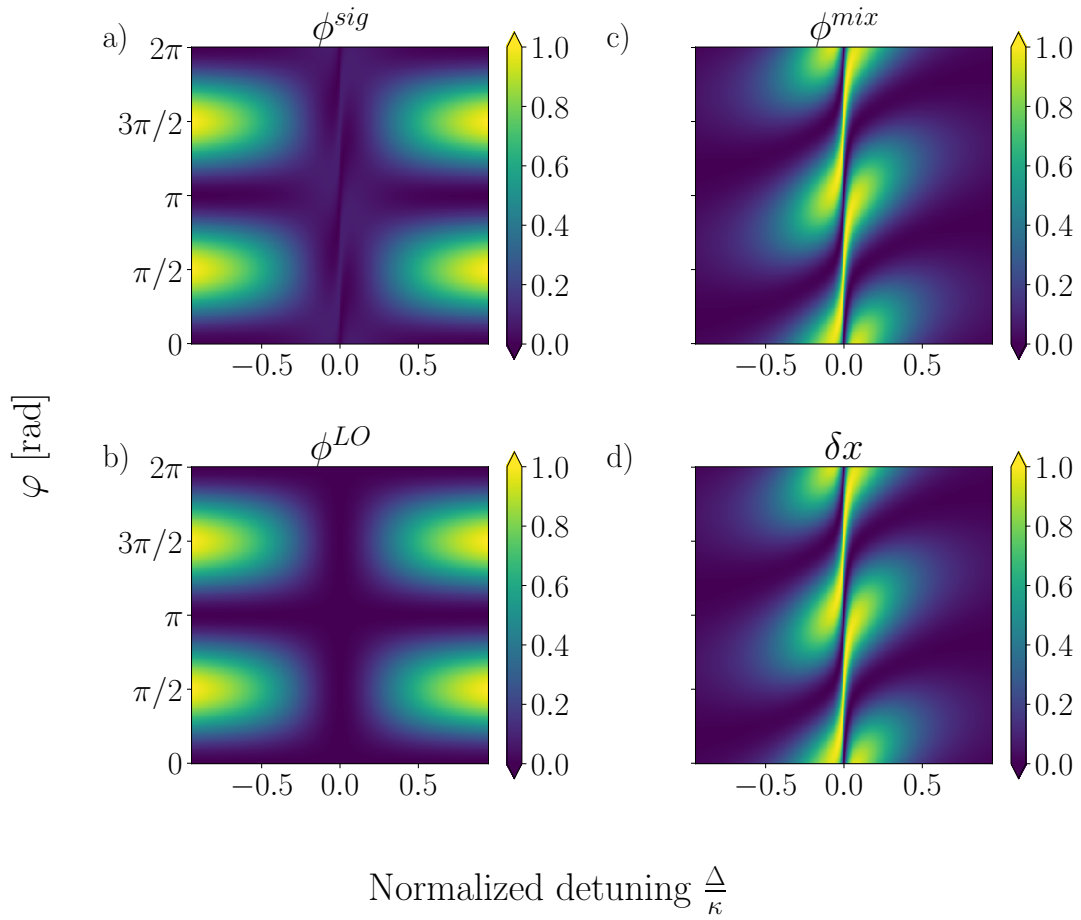


Figure 5.17: Evolution of the sensitivity of the different components of eq. 5.35. **a)**: Phase modulation on the signal field. **b)**: Phase modulation on the local oscillator field. **c)**: Effective phase modulation after mixing the component coming from the signal and LO fields. **d)**: Mirror's position fluctuations.

The resulting square moduli evaluated at $\Omega = \Omega_{\text{mod}} \simeq 3 \text{ GHz}$, for a perfectly balanced arm's length i.e., $\Delta L = 0$, are plotted in 2D plots against the normalized detuning Δ/κ and the homodyne phase φ and are depicted in fig. 5.17 where the amplitudes are normalized such that the maximum value is

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unity for each term. Each plot depicts a specific component of eq. 5.35, the plots represented in fig. 5.17.a and fig. 5.17.b depict the evolution of the phase modulation on one specific arm at a time i.e., the signal and LO arms, respectively, inside the cavity where we can see that one achieves maximum sensitivity over these two components along the phase quadrature of the field i.e. when $\varphi = \pi/2$ $[2\pi]$ as expected.

On the other hand, fig. 5.17.c and fig. 5.17.d depict the phase modulation on both arms obtained by taking the square modulus of the last term of eq. 5.35 in its entirety and which correspond to the effective phase modulation observed after mixing the signal and LO fields and the transduction of the mirror's displacement, respectively. It follows, from these two figures, that both these quantities evolve in the same manner inside the cavity which confirms that phase fluctuations of the cavity and an external phase modulation undergo the same transduction. These figures also, show that in the framework of a phase-sensitive detection, controlling the optical detuning is of the utmost importance as a slight deviation from the zero detuning working point, on which these two quantities can be only detected at $\varphi = \pi/2$ $[2\pi]$, result in contamination of these phase fluctuation/modulation through its transduction into intensity fluctuations on the intensity quadrature.

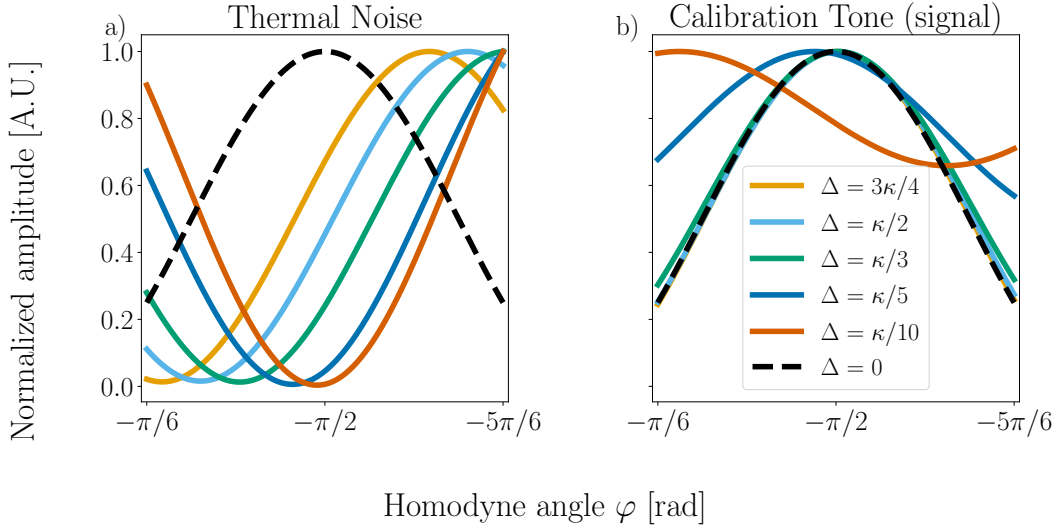


Figure 5.18: Constant detuning slices of fig. 5.17.d and fig. 5.17.a plotted against the homodyne angle φ .

To verify our observations depicted in sec. 5.3.2 with our simple model we will plot different constant detuning slices of the results depicted in fig. 5.17 for the mirror's displacement and the

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phase modulation on the signal to replicate the data depicted in fig. 5.14 and conclude whether this model explains the observed deviation. The values of optical detuning are set to match that of our experimental data, in addition to a zero detuning case to emphasize the expected behavior.

The phase modulation on the signal arm depicted in fig. 5.18.b shows that for detunings $\Delta > \kappa/5$, it evolves as expected with a maximum sensitivity at $\varphi = \pi/2$ [2π] which matches the obtained experimental data, whereas for detuning $\Delta \leq \kappa/5$ although a similar deviation from the $\varphi = \pi/2$ [2π] optimum angle can be seen, the deviation is greater than what is expected especially for the case $\Delta = \kappa/10$. On the other hand, the mirror's displacement depicted in fig. 5.18.a, shows a very good agreement with the experimental data except for a slight shift in the phase φ which can have an origin in the calibration of the phase-locking angle θ^L . Indeed, In most cases, when tuning the polarization of LO in order to center the interference fringes (cf. fig. 5.10) around 0 such that the mean-value of the difference current $\bar{I}_- = 0$ at precisely $\theta^L = 0$, the fringes are in effect not precisely centered around 0 which inevitably leads to slight shift in the optimum lock angle θ^L thus the estimated homodyne angle $\varphi = \theta^L - \pi/2$. The measured phase modulation originating from the mixing of the two phase modulations on the signal and LO follows the same behavior as the mirror's displacement as one can observe in fig. 5.19.

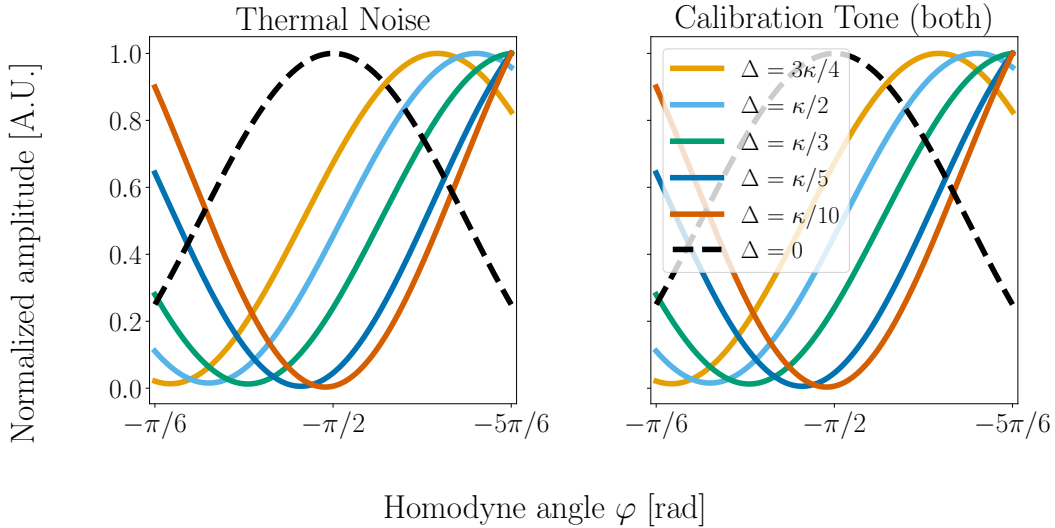


Figure 5.19: Constant detuning slices of fig. 5.17.d and fig. 5.17.c plotted against the homodyne angle φ .

In summary, our model describes well the observed data and confirms that when measuring phase

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fluctuations with a homodyne detection at non-zero detuning, intensity fluctuations produced by the transduction of the phase fluctuation of the resonance's slope inducing a shift of the optimal homodyne angle φ (cf. fig. 5.18). However, in the framework of our experiments where we compare the mirror's displacement's magnitude to that of the overall phase modulation produced by the mixing, it appears that this would not affect our measurement as these two phase components are transduced similarly as depicted in fig. 5.19.

5.3.3.1 Length mismatch effect on the phase modulation

As explained in sec. 5.6, the length mismatch between the signal and local oscillator path must be minimized. If not, the oscillations originating from this mismatch will tend to render the phase lock impossible because the phase will most likely oscillate at a higher frequency (eq. 5.26) than the piezoelectric transducer allowing to tune the relative phase φ . On top of that, the frequency-jitter of the laser will perturb the homodyne measurement introducing intensity noise as described by eq. 5.22.

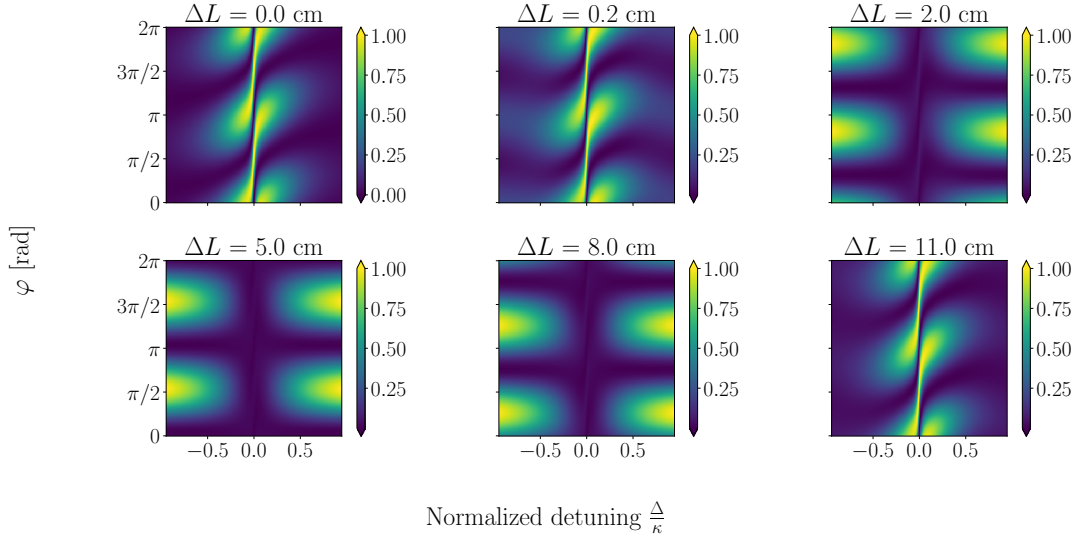


Figure 5.20: Evolution of the sensitivity of the overall phase modulation with the arm's length mismatch.

These unwanted effects are not the only downsides of length mismatch. Indeed, in the calculations performed in the previous section, a length mismatch has been considered which was translated as an additional phase on the local oscillator field θ' , thus affecting only the phase modulation.

Depicted in fig. 5.20, are 2D plots of the evolution of the overall phase modulation resulting from the mixing of the phase modulated signal and LO fields for various lengths ΔL . As one would expect, this length mismatch as it is treated in the calculations will induce a phase-shift on the sensitivity of LO's phase modulation with respect to that of the signal resulting in a loss of the wanted similar transduction as the mirror's motion and this effect becomes visible for lengths mismatch as small as 2 mm constraining us to balance the arm's length to the mm level.

On the other hand, this shift is periodic in phase as one can see from the bottom far right plot where we see that we came back to the same configuration as if the arm's length were perfectly balanced (top far left plot). This effect has a length periodicity of $\Delta L \simeq 11$ cm which converted to a frequency gives a frequency periodicity of roughly 3 GHz corresponding to the frequency of our signal. Although, this periodicity gives us some flexibility over the length balancing the add frequency-jitter noise described in eq. 5.22 constrains us to balance the lengths as best as possible to suppress any common noise between the two fields.

5.4 Noise thermometry

5.4.1 Measurement procedure

We will now describe the procedure that was established to perform the noise thermometry experiments with the balanced homodyne detection, which has been meticulously followed for each cryostat temperature.

First, the alignment of the sample with respect to the fiber is done using the white light setup depicted in fig. 4.17 following the procedure described in sec.4.2.1.7. The input optical power is set such that the resonance frequency doesn't shift anymore with the VOA. A first optical spectrum is acquired with the OSA, an example of such a low optical power spectrum is depicted in fig. 5.21.a. The optical setup is then switched to that depicted in fig. 5.8 to acquire a step-by-step laser spectrum. To do so, the optical power on the signal arm is set at $P_{\text{in}} = 350 \mu\text{W}$, and the LO beam is blocked to see the reflected signal without interfering with the LO beam. At this point, the computer-driven scan is launched and the DC component of BPD's photocurrent is measured on a precision voltmeter (Keithley 2002). Fig. 5.21.b displays a step-by-step laser spectrum at room temperature exhibiting the previously introduced optical bistability (see sec.4.3.2). This bistability prevents the measurement

$$T_{\text{cryo}}=296\text{K} - P_{\text{in}} = 350\mu\text{W}$$

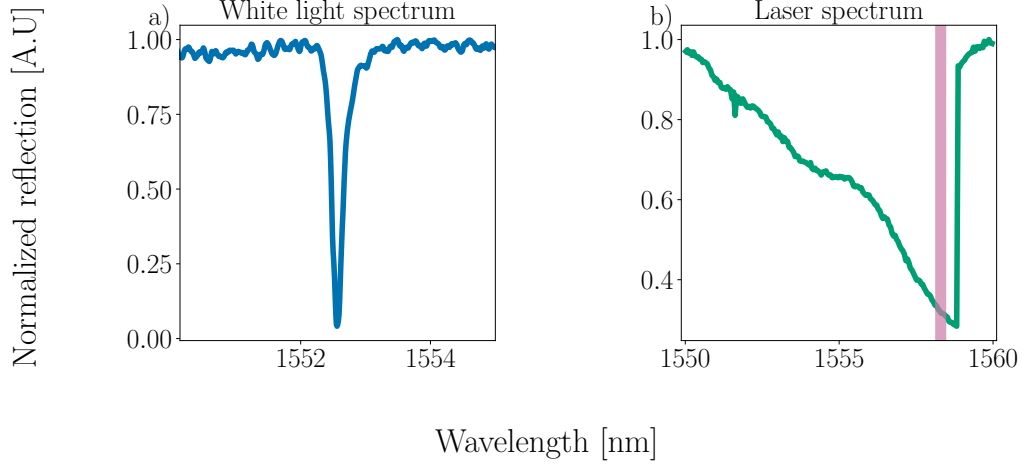


Figure 5.21: Optical spectra of sample:2-test-2. **a)** Optical spectrum acquired with the white light setup at low optical power. **b)** Step-by-step laser scan of the optical resonance displaying optical bistability. The pink vertical line represents the wavelength at which the noise spectra were acquired at room temperature.

of the noise spectra at zero detuning, due to the instability present around the optical resonance frequency. Therefore, the measurement of the noise spectra is performed with a non-zero detuning close to the instability region but sufficiently far so the cavity resonance never jumps from the high-power branch to the low-power branch. In fig. 5.21.b, the measurement wavelength for this specific case is highlighted in pink. The calibration tone produced by the EOM is switched on with the following parameters: $\Omega_{\text{mod}} = 2.88 \text{ GHz}$ and $V_p = \frac{V_\pi}{400}$.

The measurement wavelength being set on the laser, the phase of the LO has to be locked. It is done following the steps described in sec.5.2.2.2. The lock angle θ^L is then optimized by systematically locking the phase of the LO to an angle in the range of -60° and 60° and finding the best trade-off between sensitivity and lock stability over time. During the lock sequence, whilst calibrating the fringes, the polarization of the LO is optimized as described in sec.5.2.2.2 before acquiring 3 successive noise spectra using the ESA. The latter has its RBW set to 820 kHz, the spectra span over 500 MHz and each spectrum is averaged over 1000 traces. The left panel of fig. 5.22 depicts a noise spectrum acquired with a lock angle θ^L of -40° at the acquisition wavelength highlighted in pink in fig. 5.21.b. A series of 4 peaks can be observed, the first three (from low to high frequency) are different mechanical modes of our sample (2-test-2), and the last one is the phase modulation peak. Only the noise spectrum

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and thus the temperature of the highlighted mechanical mode is probed.

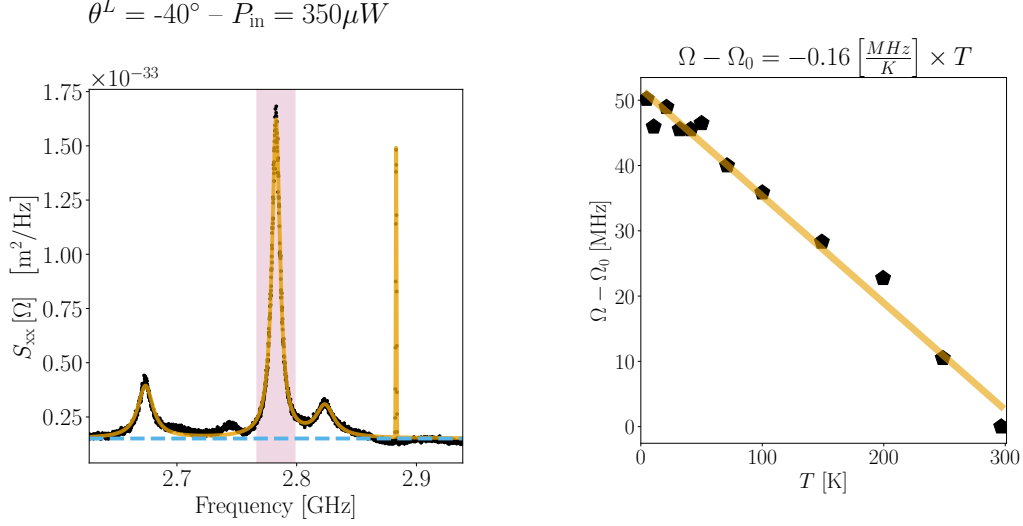


Figure 5.22: **Left:** Displacement spectrum (black) acquired with an input power of 350 μW with a lock angle $\theta_L = -40^\circ$. A multi-lorentzian fit (orange solid line) with a background (blue dashed line) is performed on the data. **Right:** Frequency shift of the highlighted mechanical mode with temperature.

The highlighted mechanical resonance frequency, with a linewidth $\Gamma_m/2\pi = 8$ MHz, of our optomechanical crystal shifts with temperature. Indeed, similarly to the optical resonance, the temperature change induces thermal expansion of the structure and probable modification of the stress. The right panel of fig. 5.22 displays this frequency shift from room temperature to 4 K with a linear fit to the data points from which a shift coefficient is extracted (-0.16 MHz/K). This value suggests that when the cryostat is set at a given temperature with fluctuations on the order of 100 mK produced by the temperature servo loop, the mechanical resonance frequency would only shift by 16 kHz which is negligible with respect to the resonance's linewidth. Such variations won't be detrimental to the spectral noise measurement.

5.4.2 Calibration techniques for noise thermometry

At each temperature, the acquired power spectral densities are converted to displacement spectral densities following the procedure described in sec. 5.2.1.3. The resulting spectra are numerically fitted with the sum of three Lorentzian lineshapes, a Gaussian lineshape, and a constant background.

The areas of the calibration tone and that of the fundamental mechanical mode are numerically

5.4. NOISE THERMOMETRY

computed with the Simpson integration method given the resulting fit parameters for each spectral feature. Fig. 5.23 depicts the calibrated displacement spectra for four different temperatures with the corresponding fits.

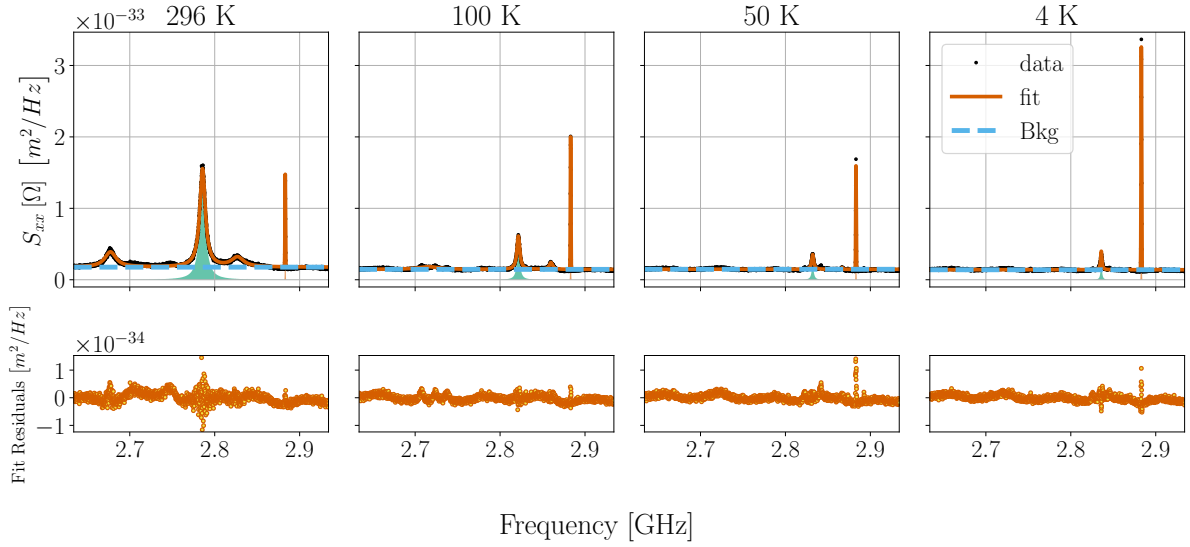


Figure 5.23: Displacement spectra of the studied sample at various temperatures ranging from 296 K to 4 K. The mechanical mode of interest has its area highlighted in green. Fit residuals are displayed below each spectrum.

In the following, we will show the temperature estimated from the displacement spectra calibrated with three different methods: no calibration, calibrated with the external phase modulation and calibrated with losses estimated from the bistable optical reflection spectra. This has been done for two different cryostat cooldowns using the same sample and the noise spectra were probed through the same optical resonance. As mentioned earlier, three different acquisitions of the noise spectra are made for each cryostat temperature. In the above figures, each data point that is displayed corresponds to the mean-value of the computed area over the different acquisitions and the error bars are computed using the propagation of uncertainty method from the standard deviation of the area. A line corresponding to the expected measured temperature i.e., $T_{\text{cryo}} = T_{\text{cryo}}$ is plotted on top of each cooldown dataset.

5.4.2.1 Raw data

Let us first look at the raw mechanical mode areas converted to temperatures following eq. 5.14 and plotted in fig. 5.24.a against the measured cryostat temperature. The trend of the estimated noise

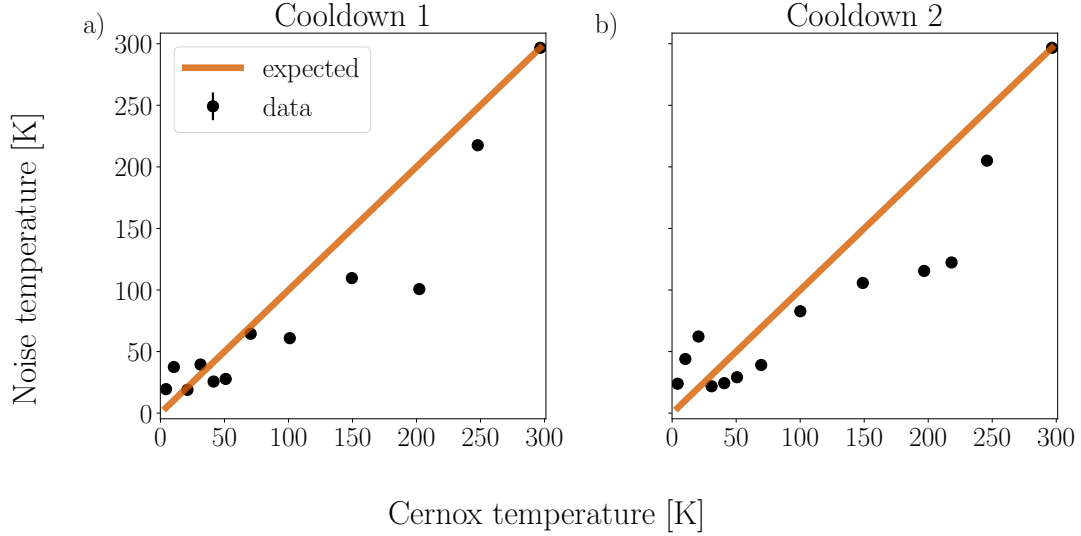


Figure 5.24: Noise thermometry results with no calibration for two different cooldowns **a)** and **b)**. The orange line depicts the expected temperature dependency.

temperature is overall correct meaning that the estimated temperature, which translates the decrease in the area under the mechanical mode peak, decreases with a decreasing cryostat temperature. However, the dispersion of each data point compared to the expected temperature is rather large for both cooldowns suggesting a measurement reproducibility issue from one temperature to the other. A dispersion of the value of a given noise temperature from one cooldown (fig. 5.24.a) to another (fig. 5.24.b) is also observed which emphasizes the measurement reproducibility problem.

As mentioned before, the fiber-to-sample alignment is done for each temperature which affects the coupling losses κ_c of the system. On top of that, the low signal-to-noise ratio constrains us to use high laser power and then a measurement in a bistable regime where the cavity detuning is set to a non-zero value, adding a complication to the measurement reproducibility. Indeed, although the input optical power is set to approximately the same value for each temperature, the optical bistability and the varying coupling losses make it so that the cavity detuning is different from one measurement to another thus the need for absolute calibration of the coupling losses.

5.4.2.2 Calibration with an external phase modulation

A very straightforward way of calibrating our data is by using an external phase modulation of the input light field which as we described in sec. 5.3.3 follows the same transduction as that of the thermal noise regardless of the cavity detuning. In theory, this would correct for the dispersion of the data points from one temperature to the next and from one cooldown to the other. Unfortunately, this

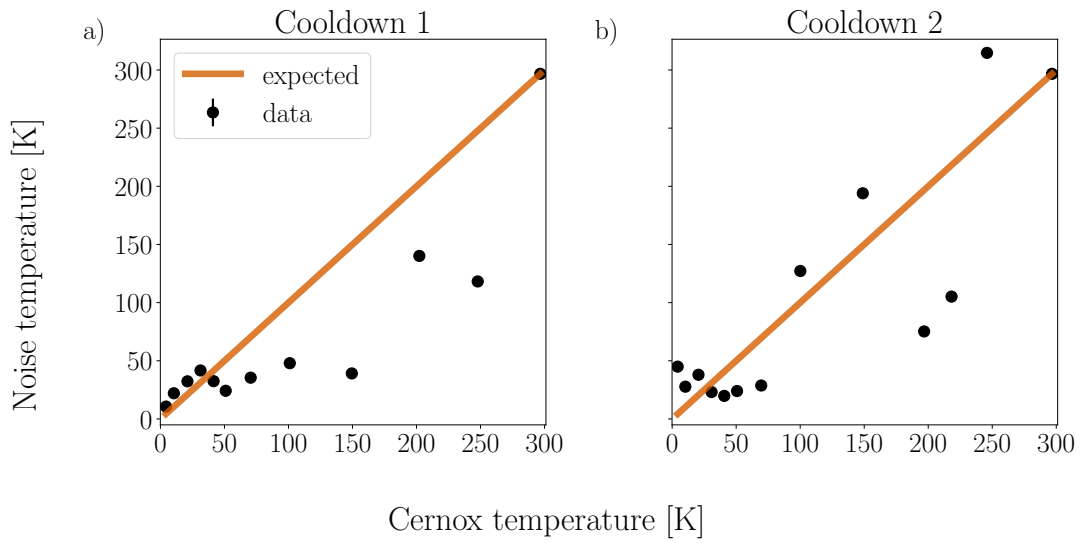


Figure 5.25: Noise thermometry results calibrated with the external phase modulation for two different cooldowns **a)** and **b)**. The orange line depicts the expected temperature dependency.

is not true in the case of this experiment. Indeed as depicted in fig. 5.25, one can see that the effect of this calibration is rather chaotic. For instance, in the first cooldown (fig. 5.25.a) the calibration seems to have, overall, lowered all the raw temperature data points whereas in the second cooldown (fig. 5.25.b) this effect is less obvious suggesting an issue with the phase calibration.

From the theoretical treatment made in sec. 5.3.3 and especially the constant detuning slice of the thermal noise and calibration tone sensitivities for various detunings depicted in fig. 5.19, we can assume that the issue with the phase modulation calibration comes from a source unrelated to the measurement procedure as this theoretical treatment shows that for a given cavity detuning, coupling losses, and homodyne lock angle, the sensitivity of both quantities would have the same transduction and thus account for all the experimental discrepancies between measurements.

Ruling out any measurement-related mechanisms that could interfere with the viability of the

calibration, only one possibility remains and it would come from the origin of the phase modulation. It has been established that for electro-optical phase modulators, phase modulation is never pure. Indeed, it often comes with a residual amplitude modulation (RAM) [78, 79, 80] which superimposes with the phase modulation and thus, in our case, degrades the calibration process. RAM comes from a variety of effects but in the case of our experiment, two inter-dependent effects are generating it, namely the input light field's polarization and temperature variations. Any misalignment of the input field's polarization with respect to one of the polarization axis of the fiber, and thus one of the polarization axis of the electro-optic crystal, will induce a different phase shift for each polarization component which is temperature dependent. Indeed, the fiber and the crystal are made of birefringent materials which induce polarization rotation when subjected to temperature variations or mechanical stress. Any polarizing optical component through which the phase-modulated field passes will convert part of this phase modulation into RAM.

Several procedures have been established to cancel out the effect of the RAM. Wong and Hall [81] have established a method that consists of counteracting the RAM by compensating the natural birefringence of the phase modulator's crystal. It is done by splitting the output of a phase modulator, which has a polarizer on both ends, into two distinct beams. The intensity modulation of the first beam is detected on a photodiode and after demodulation, an error signal is generated which has to be set to zero through a feedback loop. The latter process generated a DC voltage that is, in turn, fed back to the phase modulator counteracting the RAM. The second beam is thus free of RAM and can be used as a pure phase-modulated beam. A similar method but best suited for fibered optical setup has been demonstrated by Descampeaux *et al.* [80] and will be implemented in our setup.

5.4.2.3 Calibration with estimated losses

As described in sec. 5.1.2, the coupling parameter η_c quantifying the ratio of the coupling losses κ_c over to the total cavity losses κ can be estimated from a measure of the depth of the reflected intensity on resonance. It is also worth reminding that, in the case of these measurements and as mentioned in sec. 5.4.1, the noise spectra were acquired on a bistable optical resonance thus the reflected intensity is inferred by measuring the depth of the reflected intensity just before the breaking point i.e., the subtraction of the value of the reflected intensity after the break and that right before it. The area of the fundamental mechanical mode is then divided by this value which is computed for each

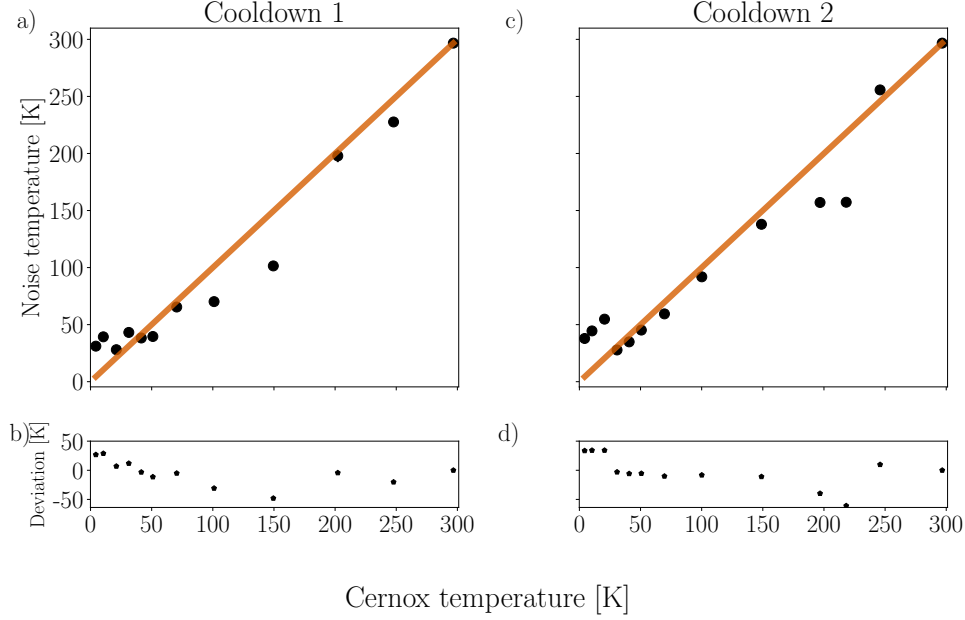


Figure 5.26: Noise thermometry results calibrated with the estimated measurement sensitivity for two different cooldowns **a)** and **c)**. The orange line depicts the expected temperature dependency. Figs. **b)** and **d)** depicts the deviation of the measured noise temperature with respect to the cryostat temperature.

temperature from the corresponding optical resonances acquired by sweeping the laser's frequency. The temperature is then inferred from the calibrated areas using eq. 5.14 and plotted against the measured Cernox temperature as depicted in fig. 5.26.

The inferred noise temperatures for both cooldowns calibrated with the estimated losses give a more satisfying fit to the expected behavior. In the first cooldown, we still observe large dispersion of certain data namely those at $T = 150$ K, 100 K, 10 K and 4 K. On the other hand, for the second cooldown, most data points are within the trend line except for low temperatures ($T = 20$ K to 4 K). Fig. 5.26.b and fig. 5.26.d are the computed deviations from the expected temperature ($T_{\text{noise}} - T_{\text{cryo}}$) and in both case display deviations up to 50 K.

So far we assumed that the mechanical mode is well-thermalized with its environment and that no other heat source interferes with its temperature. However, as described in sec. 4.3.3, the input light power used to measure the noise spectra induces heating of the sample via absorption effects. The effective sample temperature can be inferred by measuring the optical resonance frequency shift at low optical powers and at the measurement optical power as described in sec. 4.3.3.

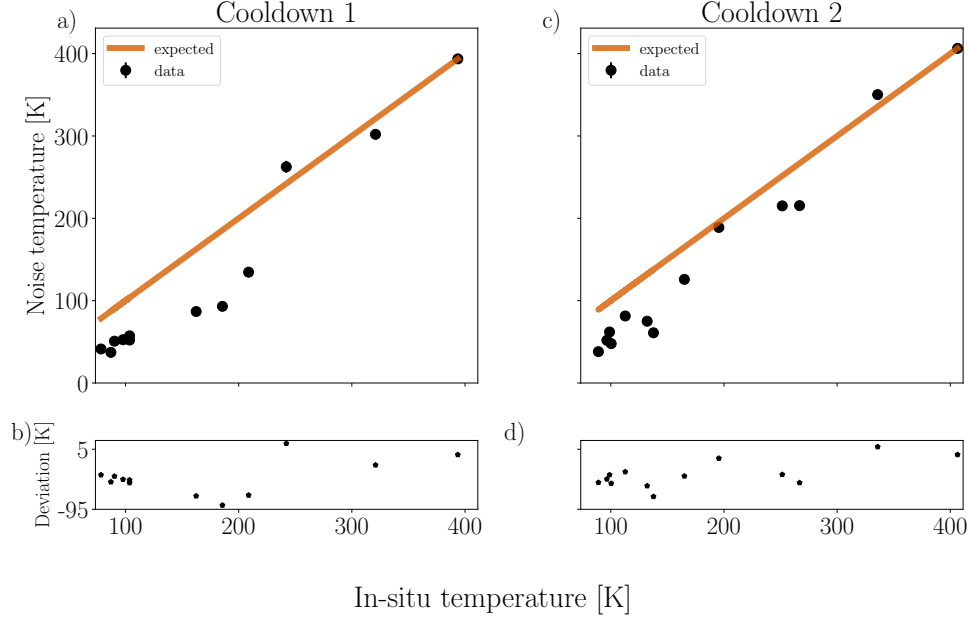


Figure 5.27: Noise thermometry results calibrated with the estimated measurement sensitivity for two different cooldowns **a)** and **c)** corrected with the measured on-chip temperature via the photonic thermometry. The orange line depicts the expected temperature dependency. Figs. **b)** and **d)** depicts the deviation of the measured noise temperature with respect to the cryostat temperature.

We can now compute the noise temperatures with the estimated on-chip temperature. The result is plotted against the temperature of the sample which has been estimated with the photonic method described in chapter 4. We observe a discrepancy between the noise thermometry and photonic thermometry methods as the former measures a lower temperature than the latter.

Let us remind that, the photonic thermometry probes the temperature in the very localized defect region of our OMC method as it relies on a measure of the resonance frequency of the optical mode localized in this defect. Noise thermometry, on the other hand, is a delocalized temperature measurement as it probes the displacement of a mechanical mode which is defined by the fluctuation-dissipation theorem and thus by its loss channels which are not necessarily in the defect region but most likely in the clamping areas. This would suggest that noise thermometry measures an average temperature over the whole beam weighted by the losses. Considering this hypothesis the deviation observed in fig. 5.27 is justified even though it is difficult to estimate precisely where mechanical losses occur and thus compute the correct overlap between temperature map and mechanical losses. However, the temperature measured by noise thermometry should be lower than the photonic temperature as the

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temperature of the beam is higher in the defect but lower around it, especially near the clamps.

In summary, this calibration technique relying on an estimation of the losses from the depth of the optical resonance yields satisfying results albeit with a large error in the estimation of the losses. A more accurate and absolute method is the calibration with the external phase modulation which in light of the issues with the residual amplitude modulation is unusable at this stage.

5.4.3 (A not so) Statistical analysis

Due to technical issues during this thesis, only two cooldowns were performed. A thorough statistical analysis of the result can not be performed with such a low number of iterations. We will, however, analyze two error types related to the measurement repeatability and reproducibility.

5.4.3.1 Measurement repeatability

Measurement repeatability quantifies the agreement level between the results of successive measurements of the same measure, carried under the same conditions. As mentioned in sec.5.4.1, a series

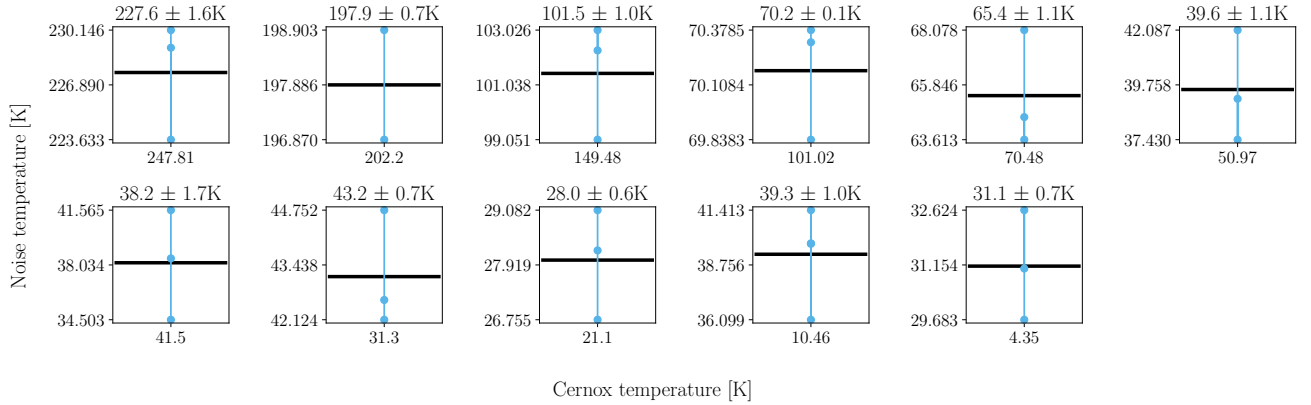


Figure 5.28: Noise temperature estimation for each acquired spectrum at different cryostat temperatures for the first cooldown. The black solid line depicts the average value over the repeated values. The title of each figure described the average value $\bar{T}_r \pm \sigma_{\bar{T}_r}$.

of 3 averaged spectra are acquired for each cryostat temperature. Figs. 5.28 and 5.29, depict the temperature dispersion, calibrated with the estimated losses, for each acquired spectrum at a given temperature (except for room temperature since it is used as a reference) for two cooldowns. Each panel shows the temperature dispersion (in orange or blue dots), the mean-value (black solid line), and

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on top of each panel, we display the mean-value $\bar{T}_r = \frac{1}{n} \sum_{i=0}^n T_i$ and its corresponding standard-error $\sigma_{\bar{T}_r} = \frac{\sigma}{\sqrt{n}}$, where n is the number of spectra per temperature, σ is the standard deviation at each temperature defined as $\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (T_i - \bar{T}_r)^2}$, T_i is the temperature estimated from a given spectrum and the lowercase r indicates repeatability. The standard-error describes the achievable measurement sensitivity with a given number of samples. For instance, if one wants to improve the measurement sensitivity by a factor N , one has to multiply the sample number by N^2 .

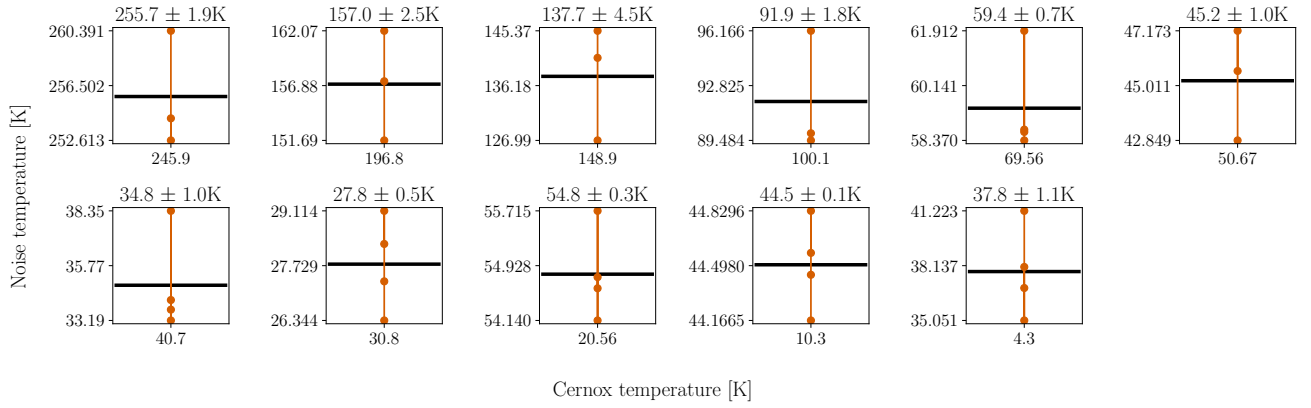


Figure 5.29: Noise temperature estimation for each acquired spectrum at different cryostat temperatures for the second cooldown. The black solid line depicts the average value over the repeated values. The title of each figure described the average value $\bar{T}_r \pm$ the standard-error $\sigma_{\bar{T}_r}$.

At the exception of the 196.8 K and 148.9 K measurements of the second cooldown where a large dispersion of the data points can be observed, all other temperatures in both cooldowns are repeatable with an uncertainty smaller than $\sigma_{\bar{T}_r} = 2$ K. This observation suggests that for a set of initial experimental conditions (detuning, phase lock, optical power ...), the measurement's repeatability is relatively good suggesting that the deviations observed in the calibrated noise thermometry results do not come from this type of systematic error.

5.4.3.2 Measurement reproducibility

We can define measurement reproducibility as the level of agreement between measurements performed in different measuring conditions. In our case, the distinct results from the two cooldowns will be compared to see how well the resulting temperatures agree. We compile in fig. 5.30.a the results from the two cooldowns (blue and orange squares), calibrated to the estimated losses, and

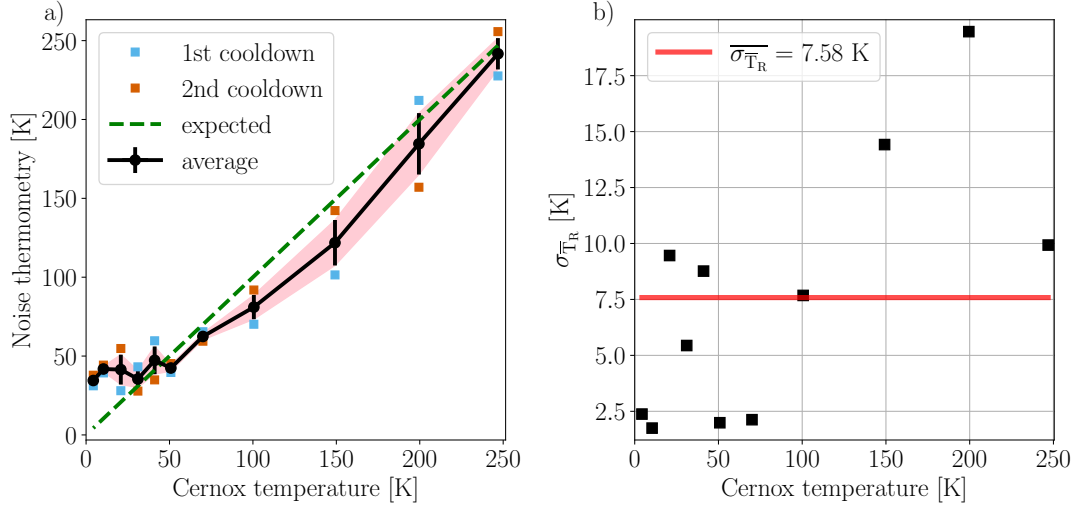


Figure 5.30: Measurement reproducibility. **a)** Compilation of the noise thermometry results, calibrated with the optical losses, for two different cooldowns (blue and orange squares). The average values (black dots) and the standard-error (black bars) are also depicted. **b)** standard-error for each temperature (black squares) and the average error (red solid line).

for each temperature, we compute the mean-value $\overline{T}_R$ (black dot) of the two iterations as well as its standard-error $\sigma_{\overline{T}_R}$ (black error bars), both defined in the previous section, where the capital R stands for reproducibility. The standard-error of each temperature is displayed on fig. 5.30.b with its average value in red solid line.

We observe a large dispersion of the standard-error (fig. 5.30.b) suggesting a bad reproducibility of the measurements. This reproducibility issue with the estimated losses calibration could be explained through experimental considerations. Indeed due to the low SNR, the measurements were acquired at high optical powers inducing a bistable behavior of the optical resonance which in turn prevented us from operating the homodyne detection at zero detuning. The latter was fluctuating from one measurement to the other.

The error related to the estimated temperature calibrated with the cavity losses and depicted in fig. 5.26 is dominant at low temperatures. Indeed, below 50 K the temperature reaches a plateau (cf. fig. 5.30) due to external heat sources such as self-heating induced by the absorption of the laser flux or the imaging lamp flux. Above 50 K the measurement error becomes reasonable but still larger than the reproducibility which we estimate to be of about 8 K.

Conclusion and prospects

5.5 Conclusion

Throughout the work presented in this manuscript, we aimed at developing new types of nano-scale temperature sensors sensitive in a wide range of temperatures (from room temperature down to 4 K) and which have the potential for integration in a "*Lab On Chip*" type of devices. The long-term goal is the development of a self-calibrated optomechanical temperature sensor relying on the quantum correlation method described in sec. 2.4. These quantum correlations are easier to measure when the base temperature is close to a critical temperature defined as $T_c = \hbar\Omega/k_B$ where the fluctuations induced by the probe quantum noise is comparable to that induced by thermal fluctuations. These correlations can be measured up to room temperature, as demonstrated by Purdy *et al.* [28], but it implies very sophisticated post-processing techniques and large integration times thus one has to start at the highest possible T_c . Furthermore, a good thermalization is required to fashion a thermometer that calls for a not-so-high mechanical quality factor i.e. large mechanical damping rates Γ_m which reduces the measurement integration time, by increasing the bandwidth (Γ_m), necessary to resolve the quantum correlations. On top of these mechanical criteria, one needs to have relatively high optical Q-factors and optomechanical coupling rates to probe with low optical power while maintaining a good sensitivity over the thermal noise.

We first designed the optomechanical system answering to the aforementioned mechanical and optical requirements. A natural candidate was the nanobeam optomechanical crystals [14, 82, 44] which can be designed to have GHz mechanical frequencies and allow the co-localization of a high Q-factor optical and mechanical modes in a small volume defect allowing an optomechanical coupling rate in the MHz range. In chapter 3, we report the design and simulation steps to achieve the high Q-factor optical mode ($\sim 10^6$) starting from the design of the unit cell such that a band gap forms

5.5. CONCLUSION

around 850 nm. A localized optical mode with a high optical quality factor is then engineered by introducing a defect to the periodic structure [57, 59]. Finally, The nanofabrication process steps were described.

In chapter 4, we first described the principle of photonic thermometry which relies on the thermal expansion and the thermorefractive effect of the sample subjected to temperature variations. We then thoroughly described the designed thermostat composed of the ^4He exchange gas cryostat whose temperature can be varied and stabilized over a wide range of temperatures (room temperature to 4 K) through the interplay of the ^4He flow and the built-in electrical heater. Cernox sensors, offering traceability to the ITS-90 temperature scale, are mounted on the cryoprobe alongside the sample chip. A brief account of the unsuccessful coupling trials with the fabricated 850 nm OMCs and the hypotheses that could justify it are given. The results obtained with the photonic thermometry experiments, which yielded similar results as the experiments described in ref [83], are discussed next before describing the technique used to estimate the on-chip temperature defined as the temperature of the bath (assuming a good sample thermalization) increased by a temperature amounting for the absorbed optical light.

Finally, chapter 5 describes the noise thermometry experiments. First, a test experimental apparatus in a direct measurement scheme has been set up to ascertain that the calibration method involving an external phase modulation works. Then, a description of the work done to develop a fibered homodyne detection and a theoretical derivation of the homodyne photocurrent in a case where we want to probe optomechanical induced phase fluctuations in the presence of an external phase modulation propagating on one of the interferometer's arm or on both to ensure that the sensitivity over both phase fluctuations is similar regardless of cavity detuning Δ and homodyne phase lock angle φ . Finally, noise thermometry results with the homodyne setup in a side of the fringe configuration ($\Delta \neq 0$) show that the phase calibration doesn't work correctly suggesting the existence of a systematic issue that appears to be Residual Amplitude Modulation (RAM) produced by the phase modulator [81, 78, 80]. An alternative calibration method relying on the estimation of the measurement sensitivity from the optical spectra is presented and used to scale our measurements which yield satisfying results.

In summary, during this thesis, we have developed an optomechanical system allowing the relative measurement of its bath temperature in two distinct ways within the same system. These two techniques are complementary, as the photonic measurement allows the estimation of the on-chip tem-

perature. However, the results presented thus far suffer from several limitations. The measurement of the noise spectra is made at large optical powers and non-zero detuning, both introducing heating through light absorption in the medium and a possible optomechanical amplification process, respectively. This limitation can be circumvented by moving into 2D crystals, as will be described in sec. 5.6.2, expecting less self-heating thanks to a better thermal anchoring of the crystal to the substrate and a satisfactory signal to noise at low input optical powers that would allow to work at resonance.

5.6 Prospects

5.6.1 Towards self-calibrated quantum thermometry

In the main text, we have described two temperature measurement schemes relying on the optical and optomechanical properties of a single optomechanical device. Both schemes need a calibration that is dependent on the experimental parameters (fiber-sample coupling, optical power, optomechanical properties of the devices $\dots$) which vary from one realization of the experiment to another and that have to be estimated with care.

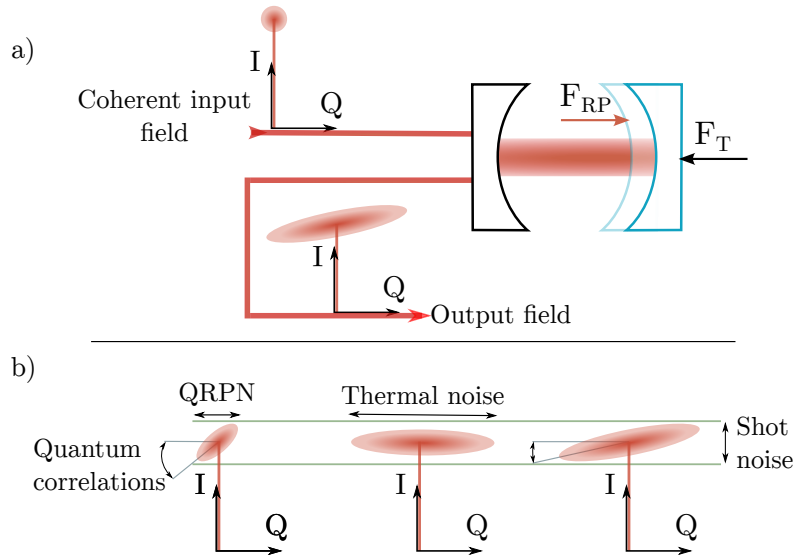


Figure 5.31: Quantum correlation measurement principle. **a)** Schematic representation of the quantum correlations appearing after the interaction of the coherent field with an optomechanical system driven by a thermal force F_T and radiation pressure force F_{RP} . **Phase-space representation of the output field broken down to its main thermal and radiation pressure contributions.**

In chapter 2, we have introduced the theoretical framework (originally described in ref. [28]) of a

5.6. PROSPECTS

self-calibrated optomechanical temperature sensor relying on the use of quantum radiation pressure noise as a temperature scale. The method relies on the measurement of given quadratures of the output field and computing the correlations between them as it has been shown that (eq. 2.19 and eq. 2.20) following the interaction of the coherent input field with an optomechanical system, correlations between the intensity and phase quadrature, which has a quantum origin, arises due to the radiation pressure force. Fig. 5.31.a depicts a phase-space representation of the input and output fields where the fluctuations of the output field are elongated along the phase quadrature and tilted with respect to the horizontal. We can break down these fluctuations to the sum of two contributions, a classical contribution of the thermal Langevin force F_T translated as an elongation along the phase quadrature and a quantum contribution of radiation pressure force F_{RP} introducing a Quantum Radiation Pressure Noise (QRPN) giving rise to the phase-intensity correlations (tilt of the ellipse). These contributions are represented in fig. 5.31.b.

5.6.1.1 Improvement of the optical setup

Starting from the current homodyne setup (cf. sec. 5.2.2.2), the quantum correlations can be measured continuously by introducing a frequency shift Δ_{LO} between the probe and local oscillator fields, which by definition turns the homodyne detection into a heterodyne detection. A simplified schematic representation of this setup is depicted in fig. 5.32.a. In effect, as depicted in fig. 5.32.b,

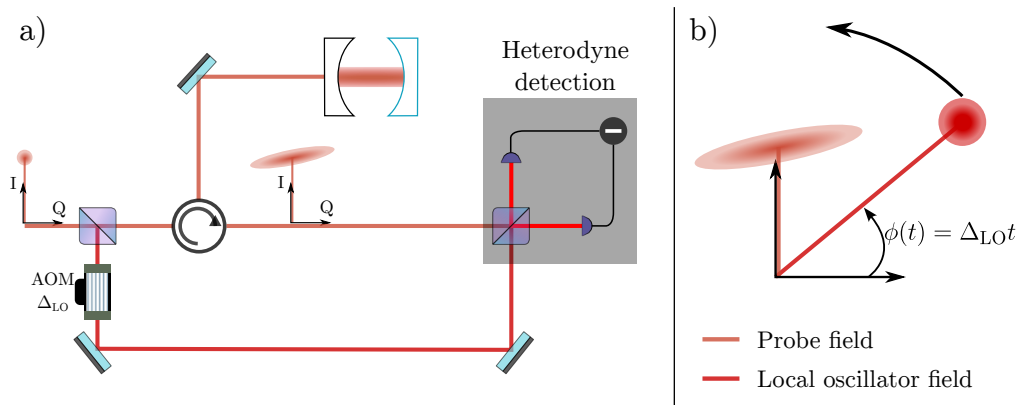


Figure 5.32: Quantum correlations optical setup. **a)** Simplified schematic of the optical heterodyne setup that will be used to measure the quantum correlations. **b)** Description of the heterodyne principle where the LO field is used to probe different optical quadratures of the output probe field.

the local oscillator will rotate in phase space at the heterodyne frequency Δ_{LO} defining the angle $\phi(t)$.

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This rotation will allow the measurement of arbitrary quadratures $\delta\hat{\mathbf{X}}_\phi^{\text{out}}[\Omega]$ which (as a reminder) are expressed as

$$\delta\hat{\mathbf{X}}_\phi^{\text{out}}[\Omega] = \delta\hat{\mathbf{p}}^{\text{out}}[\Omega] \cos \phi + \delta\hat{\mathbf{q}}^{\text{out}}[\Omega] \sin \phi. \quad (5.38)$$

Ultimately the correlations between the $\phi = \pi/4$ and $\phi = 3\pi/4$ optical quadratures will be computed and the temperature will be estimated by taking the ratio of its real part (thermal noise) and its imaginary part (QRPN) as

$$\frac{\text{Re} \left\{ S_{\frac{\pi}{4}, \frac{3\pi}{4}}[\Omega] \right\}}{\text{Im} \left\{ S_{\frac{\pi}{4}, \frac{3\pi}{4}}[\Omega] \right\}} = \frac{\text{Re} \left\{ S_{\frac{\pi}{4}, \frac{3\pi}{4}}[\Omega] \right\}}{\text{Im} \left\{ S_{0, \frac{\pi}{2}}[\Omega] \right\}} = \coth \left(\frac{\hbar\Omega}{k_{\text{B}}T} \right). \quad (5.39)$$

This relation, as stated in the theoretical derivation in sec. 2.4, is only valid when the cavity detuning Δ is strictly zero. This condition should be experimentally met otherwise contamination of the calibration quantum signature by a thermal contribution renders this scheme inefficient. Active stabilization of the probe laser's frequency should also be implemented in the current setup through, for instance, the implementation of a Pound-Drever-Hall frequency stabilization technique [84].

5.6.1.2 Cross correlation measurement

The heterodyne detection scheme allows the continuous measurement of the optical quadratures of the output field defined by the angle $\phi(t)$. The challenge is to measure the cross correlation between two optical quadratures delayed by a $\pi/2$ phase between them.

Fig. 5.33 describes the envisioned electronic setup that will allow the measurement of the cross-correlation spectra. The difference photocurrent coming out of the heterodyne detection will be split into a DC and RF signal with a bias-tee. The DC signal containing the beat signal between the LO and probe field at a frequency Δ_{LO} will allow the computation of the time dependant beat note phase $\phi(t)$ with which we will ensure that we are measuring the right quadrature. The RF signal will be demodulated at a frequency close to the mechanical frequency Ω_{m} with an I-Q demodulator, after passing through a high-pass filter that ensures that only the relevant signal is let through to the next stage. The I-Q demodulator will generate the real and imaginary parts of the optical quadrature $\delta\hat{\mathbf{X}}_{\phi(t)}^{\text{out}}[\Omega]$ which will be filtered with a low-pass filter.

Finally, by implementing splitting these real and imaginary parts into two signals delayed by a time τ defined such that $\phi(t + \tau) = \phi(t) + \pi/2$, the cross-correlations are computed by a series of

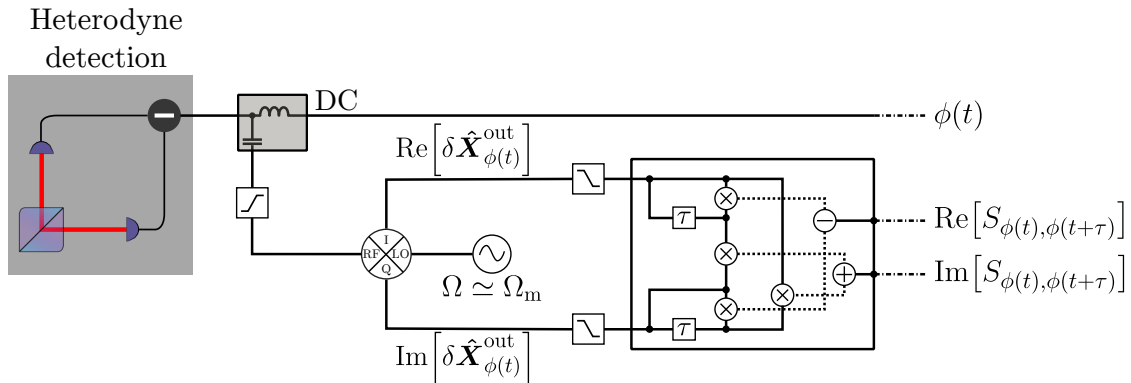


Figure 5.33: Schematic of the electronic setup that will be used to compute correlations between the desired optical quadratures.

electronic operations on the generated signals as schematically described in fig. 5.33. The real part of the cross-correlation spectrum $S_{\phi(t), \phi(t+\tau)}$, containing the thermal signature, will be computed as

$$\left(\text{Re} \left[\delta \hat{\mathbf{X}}_{\phi(t)}^{\text{out}} \right] \times \text{Re} \left[\delta \hat{\mathbf{X}}_{\phi(t+\tau)}^{\text{out}} \right] \right) - \left(\text{Im} \left[\delta \hat{\mathbf{X}}_{\phi(t)}^{\text{out}} \right] \times \text{Im} \left[\delta \hat{\mathbf{X}}_{\phi(t+\tau)}^{\text{out}} \right] \right) \longrightarrow \text{Re} \left[S_{\phi(t), \phi(t+\tau)} \right],$$

whereas the imaginary part of the cross-correlation spectrum $S_{\phi(t), \phi(t+\tau)}$, containing the purely quantum correlation signature, will be computed as

$$\left(\text{Re} \left[\delta \hat{\mathbf{X}}_{\phi(t+\tau)}^{\text{out}} \right] \times \text{Im} \left[\delta \hat{\mathbf{X}}_{\phi(t)}^{\text{out}} \right] \right) + \left(\text{Re} \left[\delta \hat{\mathbf{X}}_{\phi(t)}^{\text{out}} \right] \times \text{Im} \left[\delta \hat{\mathbf{X}}_{\phi(t+\tau)}^{\text{out}} \right] \right) \longrightarrow \text{Im} \left[S_{\phi(t), \phi(t+\tau)} \right].$$

A proof of principle of this measurement technique has been demonstrated by Purdy *et al.* [28] with a Si_3N_4 optomechanical crystal (fig. 5.34.a). They have been able to measure the thermal cross-correlations (fig. 5.34.c) and the quantum cross-correlations (fig. 5.34.d) from cryogenic up to room temperature. The temperature measured by this self-calibrated thermometer shows good linearity with respect to the commercial resistive thermometers embedded near the sample (fig. 5.34.e). The effect of the cavity detuning on the measured quantum correlations was also investigated for two detunings differing by about 0.0005κ (fig. 5.34.b) where we see that even a residual detuning induces a dramatic change in the quantum correlation signal because of the additional thermal contribution.

The implementation of such a technique in our case requires, in addition to the above-stated optical and electronic implementations, an improvement of the overall stability and reproducibility of our optical coupling. Indeed, a single data point from fig. 5.34.e required Purdy's team an integration time ranging from several hundred to one thousand seconds implying an improvement of the mechanical stability of our coupling through different coupling schemes. Another challenge is to limit the

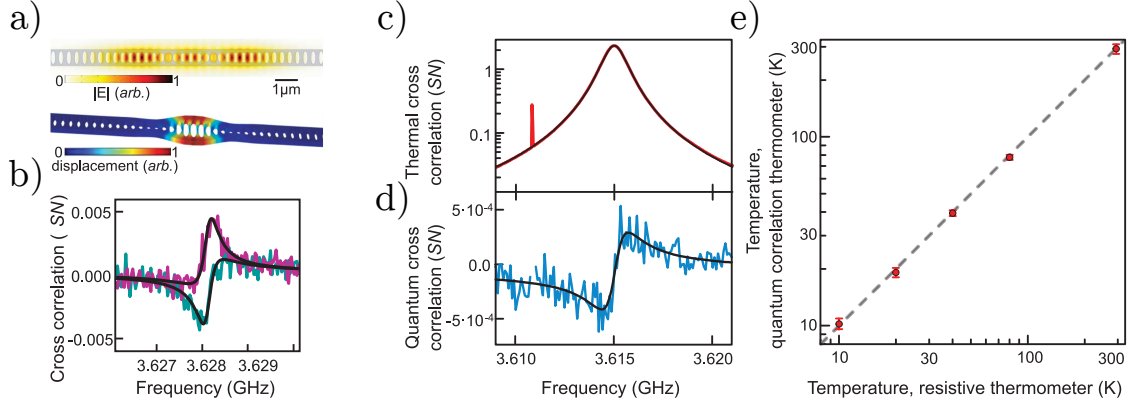


Figure 5.34: Proof of concept of the quantum correlation calibration technique. **a)** Optomechanical crystal with an optical resonance near 990 nm (top) and a mechanical mode at $\Omega_m/2\pi = 3.62$ GHz (bottom). **b)** Detuning effects of the probed quantum correlations. The two curves differ by about 0.0005κ . Thermal **c)** and quantum **d)** cross-correlations acquired at room temperature and atmospheric pressure. **e)** Temperature inferred with quantum correlation thermometry plotted against a resistive measurement of the sample's temperature. Figure adapted from [28].

absorption-induced self-heating, which was one of the main limitations in the experiments described in this manuscript by designing new optomechanical systems with a better optical quality factor and better thermal dissipation.

5.6.2 Towards square membranes

The absorption-induced self-heating, in the optomechanical thermometry experiments, will be addressed by changing the geometry of the optomechanical system. Indeed, the thermal dissipation in the current optomechanical crystal suffers from bad thermal anchoring to the chip's substrate as it is only achieved via small side clamps. This thermal issue should be minimized by moving towards square membranes allowing the co-localization of high-Q factor optical mode and a GHz mechanical mode in an L3 cavity [85, 86]. A finite-element simulation allowed us to compare the heating induced by a $1 \mu\text{W}$ optical source placed at the center of the cavities for both geometries. The results depicted in fig. 5.35 show an almost tenfold reduction of the heating as it is dissipated toward the edges of the membrane.

As for the improvement of the mechanical stability of the optical coupling, a new coupling scheme is considered where the 2D membrane is suspended over an SOI waveguide [87] which will be terminated by a grating coupler on both sides allowing a bi-directional evanescent coupling to and from the

5.6. PROSPECTS

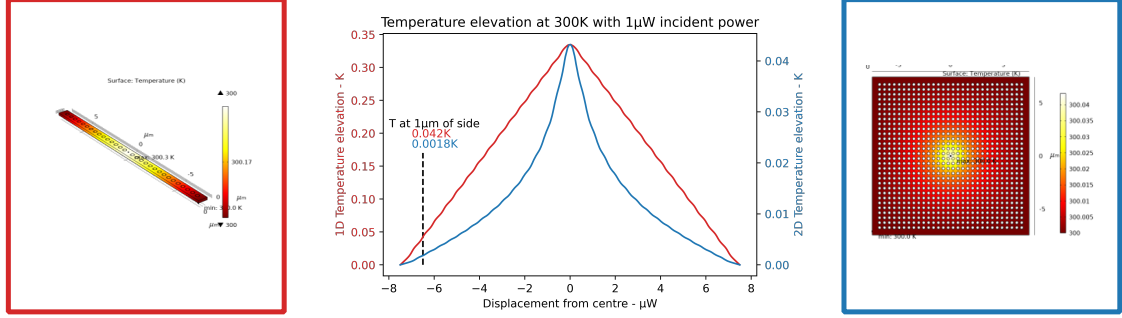


Figure 5.35: Comparison between the absorption-induced self-heating for a nanobeam geometry and a 2D membrane. The right scale of the figure in the middle, corresponding to the 2D structure is almost 10 times lower than the left scale corresponding to the 1D beam.

membrane. One can imagine setting up a room temperature characterization setup to test different membranes and after selecting one with suitable optical and mechanical properties, input and output optical fibers can be physically fixed using special optical adhesives [88] or through *Van der Waals* forces [89]. The chip and fiber assembly will ensure a roughly constant coupling throughout the temperature range as well as better mechanical stability.

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Chapter 6

Résumé en Français

6.1 Introduction

Au XVII^e siècle, l'astronome allemand Johannes Kepler a émis la première hypothèse sur l'existence d'une force induite par la lumière, à notre connaissance. Kepler a remarqué, en observant la comète que nous connaissons aujourd'hui sous le nom de comète de Halley, que sa queue s'éloignait toujours du soleil. Il a émis l'hypothèse que les rayons du soleil repoussaient la queue. Ce n'est qu'au XIX^e siècle que James Clerk Maxwell a prédit théoriquement une telle force grâce à sa théorie des ondes électromagnétiques [2].

En 1901, la première observation de la pression de radiation a été réalisée à l'aide d'un appareil expérimental en configuration de moulin à lumière, tel que celui illustré à la figure 6.1, à Moscou [3] et à Dartmouth [4]. Ce n'est que 77 ans plus tard qu'Arthur Ashkin a démontré la première application fondamentale de la pression de radiation [5] en contrôlant, en piégeant et, étonnamment, en refroidissant le mouvement des particules diélectriques à l'aide de faisceaux fortement focalisés. Une autre étude réalisée en 1967 par le groupe de Braginski à l'université d'État de Moscou [6] a mis en lumière l'interaction entre un champ optique et un résonateur mécanique. Ils ont prédit et démontré la modification de la dynamique d'un résonateur mécanique par la pression de radiation qui change son amortissement. C'est ainsi qu'est né le domaine de l'optomécanique.

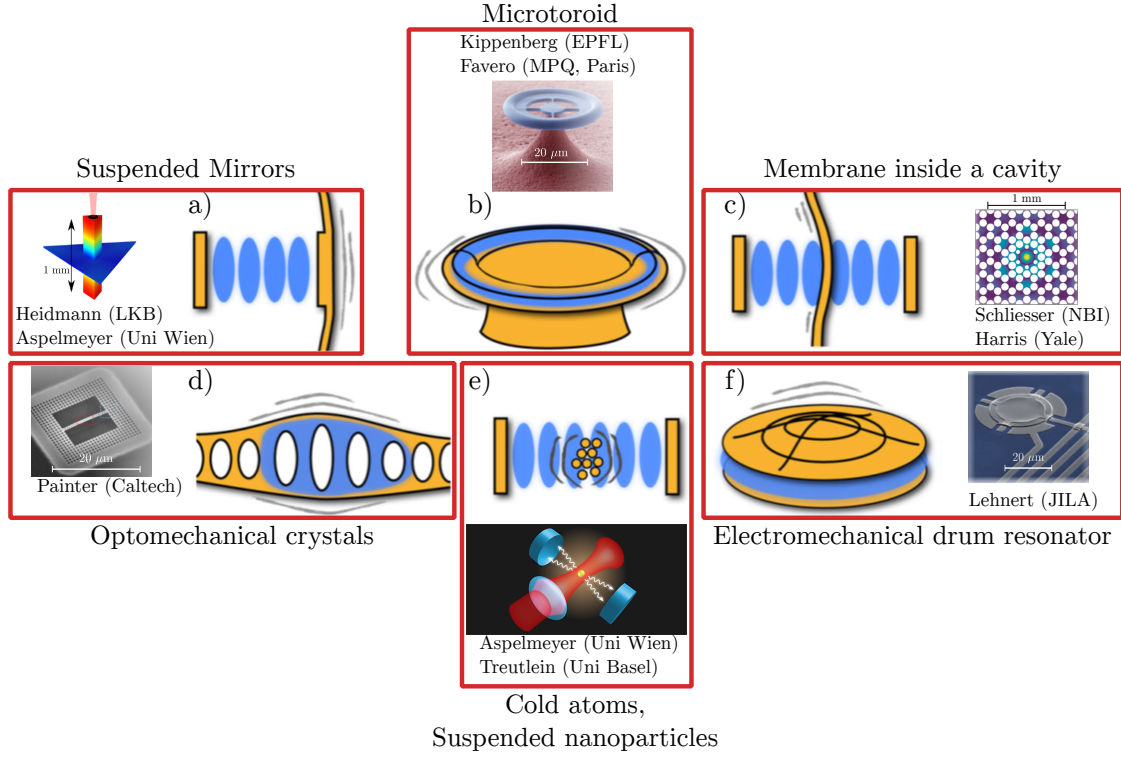


Figure 6.1: Image d'un radiomètre de Crookes (ou moulin à lumière) composé d'une ampoule de verre étanche à l'air et d'un ensemble d'ailettes montées sur un axe leur permettant de tourner autour d'elle lorsque la lumière est projetée dessus. Image copyright Science Museum/SSPL, Londres

Par la suite, l'attention portée à ce domaine émergent s'est rapidement accrue, conduisant à diverses propositions d'exploration d'aspects fondamentaux de la physique allant de la relativité générale à l'observation d'ondes gravitationnelles par la collaboration LIGO/Virgo en 2015 [7], qui impliquait une mesure interférométrique du déplacement d'une masse test de 40 kg, à la physique quantique par le biais de diverses expériences visant à refroidir le mouvement de différents résonateurs mécaniques jusqu'à leur état fondamental à l'aide de miroirs macroscopiques [8], de micromiroirs suspendus [9],

6.1. INTRODUCTION

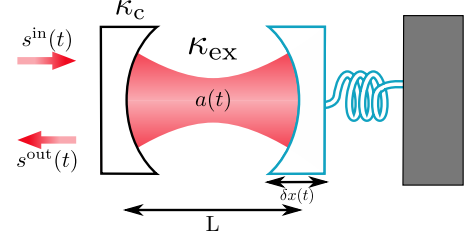
microtoroïdes [10], membranes à l'intérieur d'une cavité [11], résonateurs électromécaniques à tambour [12], nanoparticules en lévitation [13] et cristaux optomécanique [14]. Les systèmes optomécanique susmentionnés sont décrits dans la figure 6.2.



Outre les aspects fondamentaux des théories physiques, le domaine de l'optomécanique a des applications plus pratiques telles que le développement de capteurs optomécanique. Ces capteurs couvrent un large éventail d'applications telles que la détection de force [16, 17, 18, 19], la détection de masse [20, 21, 22], gravimètres et accéléromètres [23, 24] et capteurs de température [25, 26, 27, 28]. Dans ce manuscrit, cette dernière application sera explorée et nous décrirons le développement et la caractérisation d'un capteur de température optomécanique multimodal basé d'une part, sur les propriétés optiques dépendantes de la température de ces dispositifs et d'autre part, sur le mouvement brownien de ces systèmes optomécanique en réponse au bain thermique qui les entoure. Ce projet est motivé par la récente redéfinition de l'unité de température thermodynamique, le Kelvin, qui la relie à une échelle d'énergie via les constantes fondamentales de Planck et de Boltzmann $\hbar$ and k_B .

6.2 Introduction à l'optomécanique

Le domaine de l'optomécanique étudie l'interaction entre de la lumière et des systèmes mécaniques dont une représentation canonique d'un système optomécanique est donnée en figure 6.3. Une cavité Fabry-Pérot constituée d'un miroir arrière mobile et totalement réfléchissant et d'un miroir d'entrée supposé immobile avec une transmission non nulle.



Le déplacement du miroir mobile induit un déphasage du faisceau lumineux entrant dans la cavité et ce dernier agit par pression de radiation sur la surface du miroir. Le premier effet permet de sonder la position du miroir en détectant le déphasage induit par son mouvement. En effet, le miroir mobile est soumis à des forces extérieures qui induisent ce mouvement, notamment une force de Langevin fluctuante d'origine thermique F_T . Le second effet vient perturber le mouvement du miroir qu'on cherche à mesurer.

Figure 6.3: Système optomécanique canonique. Ici, κ_c représente le taux de perte du miroir d'entrée, κ_{ex} le taux de perte intrinsèque de la cavité, $a(t)$ le champ intracavité, $s^{in/out}$ les flux de photons entrant/sortant de la cavité, L représente la longueur de la cavité et $\delta x(t)$ les fluctuations de position du miroir.

Mouvement du miroir

Le mouvement du miroir peut être décrit par la théorie de la réponse linéaire si on se restreint à des petits déplacements. Les forces extérieures $F_{ext}(t)$ qui s'appliquent sur le miroir, et notamment la force de Langevin F_T , peuvent être reliées aux déplacements du miroir à travers la susceptibilité mécanique $\chi_m(\Omega)$. On peut ainsi exprimer la transformée de Fourier de la position du miroir $x[\Omega]$, en supposant que le miroir peut être modélisé par un oscillateur harmonique, comme suit :

$$x[\Omega] = \chi_m(\Omega) F_{ext}[\Omega], \quad (6.1)$$

avec la susceptibilité qui prend la forme suivant dans l'hypothèse de l'oscillateur harmonique

$$\chi_m(\Omega) = \frac{1}{m_{eff} [\Omega_m^2 - \Omega^2 - i\Gamma_m\Omega]}, \quad (6.2)$$

où m_{eff} est la masse effective de l'oscillateur, Ω_m sa fréquence de résonance et Γ_m son taux de dissipation mécanique.

Comme mentionné précédemment, ce déplacement est transduit en déphasage du faisceau lumineux sonde. Ainsi dans le cas d'une cavité de grande finesse, un déplacement δx du miroir induit un déphasage du champ sortant :

$$\delta\phi^{\text{out}} = 8\mathcal{F}\frac{\delta x}{\lambda}, \quad (6.3)$$

où, $\mathcal{F}$ est la finesse de la cavité, et λ la longueur d'onde de la sonde. Ainsi en mesurant les fluctuations de phase du faisceau sortant de la cavité, on peut remonter au déplacement et ainsi à la température. En effet, la force de Langevin a un spectre de bruit qui prend la forme suivante :

$$S_{F_T}[\Omega] = -\frac{2k_B T}{\Omega} \text{Im} \left(\frac{1}{\chi_m(\Omega)} \right) = 2m_{\text{eff}}\Gamma_m k_B T, \quad (6.4)$$

où $k_B T$ est la constante de Boltzmann et T la température de l'oscillateur.

Effets optomécaniques

La lumière sonde vient perturber le déplacement du miroir en appliquant une force de pression de radiation, qui correspond au transfert d'impulsion d'un photon à une fréquence ω_L au résonateur mécanique, $F_{RP}(t) = -\hbar G|a(t)|^2$ avec $G = -\frac{\omega_L}{L}$ le couplage optomécanique.

De ce couplage optomécanique peuvent naître des effets statiques ou dynamiques, qui dans notre cas sont indésirables car ils viennent modifier la température de l'oscillateur mécanique. Une description détaillée de ces effets est donnée dans le corps de la thèse, cependant nous pouvons les citer ici pour les introduire. La bistabilité optique (effet statique), le ressort optique et les effets de frictions optiques (effets dynamiques). Il sera important de minimiser ces effets autant que possible pour avoir une mesure de température non altérée.

6.3 Cristaux optomécanique unidimensionnels

Durant cette thèse nous avons développé des résonateurs optomécanique qu'on identifiera comme des cristaux optomécanique [54, 55, 44, 56, 57]. Cette catégorie de résonateurs optomécanique est caractérisée par des forts confinements des modes optiques et mécaniques de la structure permettant un large couplage optomécanique entre les deux [14].

Ces structures optomécanique combinent un cristal photonique qui sert à confiner les photons dans un défaut en son centre et un cristal phononique qui confine les phonons dans la même région de

6.3. CRISTAUX OPTOMÉCANIQUE UNIDIMENSIONNELS

défaut. Les deux types de cristaux sont régis par des équations maîtresses similaires. En effet, la partie photonique de la structure est décrite par les équations de Maxwell dans les milieux diélectriques [2] dont l'équation maîtresse décrivant le champ magnétique $\mathbf{H}(\mathbf{x})$ est donnée ci-dessous :

$$\hat{\mathbf{E}}\mathbf{H}(\mathbf{x}) \equiv \nabla \times \left[\frac{1}{\epsilon(\mathbf{x})} \nabla \times \mathbf{H}(\mathbf{x}) \right] = \frac{\omega^2}{c^2} \mathbf{H}(\mathbf{x}), \quad (6.5)$$

où $\epsilon(\mathbf{x})$ représente la fonction diélectrique de la structure.

Pour la partie mécanique une équation similaire peut être dérivée à partir des équations d'onde des milieux élastiques [61] reliant le champ de déplacement $\mathbf{Q}(\mathbf{r})$ de la structure à son tenseur d'élasticité $\mathbf{c}(\mathbf{r})$ ¹ comme suit :

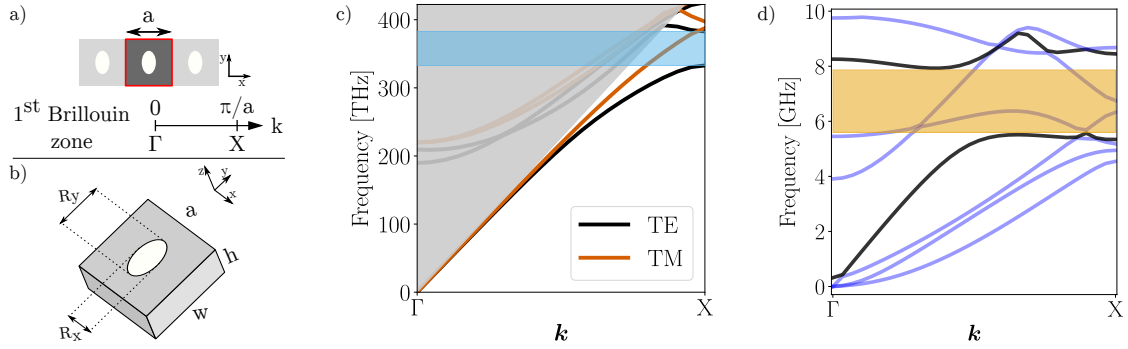


Figure 6.4: Structures de bandes photonique et phononique de la cellule unitaire d'un cristal. **a)** Esquisse de la cellule unitaire à l'intérieur du cristal avec une représentation de la première zone de Brillouin. **b)** Représentation tridimensionnelle de la cellule unitaire. **c)** Structure de bande calculée avec le package MPB avec les paramètres géométriques suivants : $a = 330$ nm, $w = 480$ nm, $h = 200$ nm, $R_y = 240$ nm, $R_x = 0.6 \times R_y$ et un indice de réfraction Si_3N_4 de 2.02 à une longueur d'onde de 850 nm. La ligne continue verte représente la ligne de lumière qui définit le cône de lumière mis en évidence en rose. **d)** Structure de la bande phononique. Les paramètres mécaniques suivants ont été utilisés : Module d'Young $E = 300$ GPa et une précontrainte de 1 GPa. La bande interdite est mise en évidence en orange clair.

$$\hat{\Phi}\mathbf{Q}(\mathbf{r}) = \frac{1}{\rho(\mathbf{r})} \nabla^T (\mathbf{c}(\mathbf{r}) \nabla \mathbf{Q}(\mathbf{r})) = -\Omega^2 \mathbf{Q}(\mathbf{r}). \quad (6.6)$$

Les équations 6.5 et 6.6 sont, dans un premier temps, résolues pour la maille élémentaire de la structure représentée en figure 6.4. b afin de calculer sa structure de bandes photonique et phononique et déterminer les bandes interdites photonique et phononique. Ce calcul est fait sur la première zone de Brillouin (figure 6.4. a) et pour des modes qui satisfont les critères des états de Bloch. Les structures

¹Ce tenseur dépend du module d'Young E et du coefficient de Poisson ν de la structure

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de bande photonique et phononique sont représentées sur les figures 6.4. c et 6.4. d où l'on voit les bandes interdites photonique et phononique.

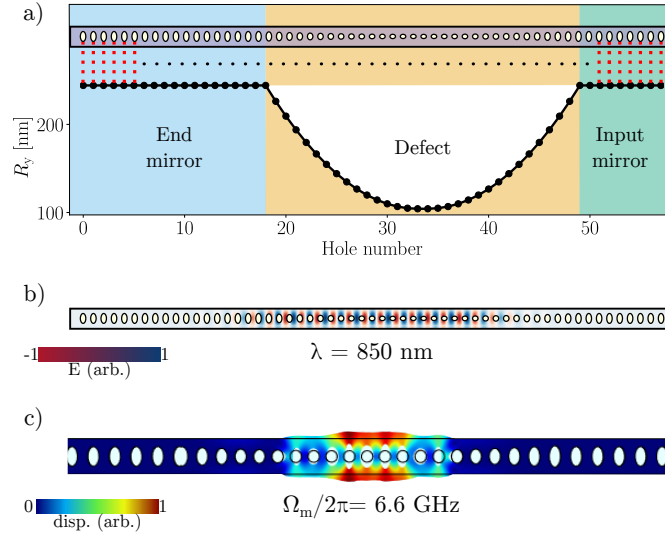


Figure 6.5: Conception du cristal optomécanique et résultats de la simulation. **a)** Schéma d'un cristal optomécanique Si_3N_4 avec ses miroirs d'entrée et de fin de type Bragg et sa région de défauts définie par la variation quadratique des demi-grands axes des trous elliptiques. **b)** Mode optique fondamental à 850 nm ($Q = 3 \times 10^6$) simulé avec MEEP. **c)** Mode mécanique fondamental de respiration à 6.6 GHz simulé avec COMSOL.

À partir de cette maille élémentaire, nous pouvons construire les cristaux optomécanique en introduisant un défaut dans la structure cristalline. Ce défaut brisera la symétrie de translation du cristal et en jouant dessus nous pouvons insérer des modes photonique et phononique localisés dans la structure de bande. Ce défaut consiste en une variation adiabatique d'une grandeur des trous composant la structure comme on peut le voir sur la figure 6.5.a. Une simulation par différences finies dans le domaine temporel (FDTD) permet d'extraire les profils spatiaux et fréquences des modes optiques localisés (figure 6.5. b). Une simulation par éléments finis permet de trouver les profils spatiaux et fréquences des modes de vibration localisés (figure 6.5. c).

Ces cristaux optomécanique ont finalement été fabriqués par Rémy Braive et Théo Martel au Centre de Nanosciences et de Nanotechnologies (C2N) suivant des techniques de nanofabrication classiques. Un exemple de structure est représenté sur la figure 6.6.

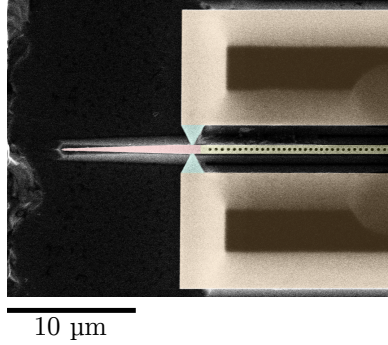


Figure 6.6: Image SEM du cristal optomécanique en phosphure de gallium (GaP). Les couleurs sont artificielles.

6.4 Montage expérimental

6.4.1 Thermostat

Un thermostat permettant le contrôle, la stabilisation et la mesure de sa température a été mis au point durant cette thèse. Un cryostat commercial a été adapté pour être utilisé comme thermostat pour nos expériences de thermométrie.

Le cryostat provient de chez Oxford Instruments modèle Optistat SXM, dont un schéma est représenté en figure 6.7, reposant sur le principe de gaz d'échange pour refroidir le "doigt froid" sur lequel les échantillons de cristaux optomécanique sont fixés.

Le cryostat est équipé d'une résistance chauffante ainsi que d'un capteur de température au niveau de l'échangeur de chaleur permettant la stabilisation de la température dans la chambre expérimentale avec une précision de $\pm 0.1\text{K}$ pendant 10 min. De plus, nous avons équipé le doigt froid de capteurs de températures commerciaux Cernox calibrés par le NIST et ayant une traçabilité à l'Échelle internationale de température de

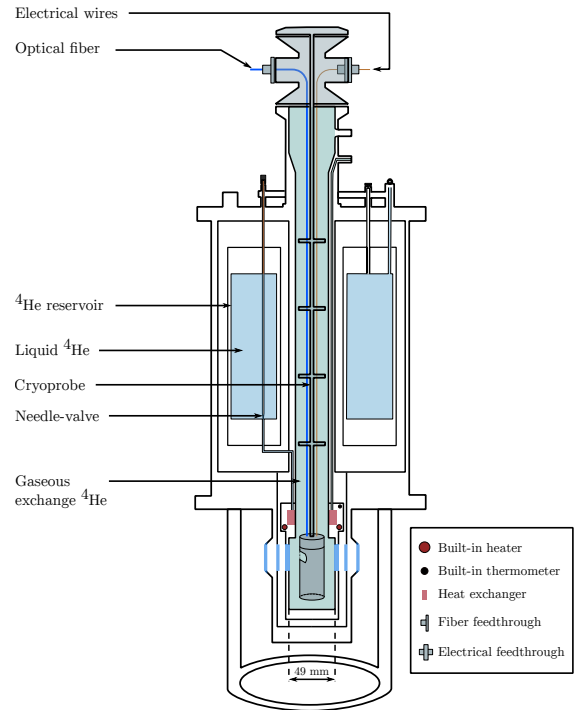


Figure 6.7: Schéma du cryostat représentant ces composantes principales.

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1990 (EIT-90).

6.4.2 Injection optique

L'injection de la lumière dans les échantillons, fixés au niveau du doigt froid, est faite par voie de fibres optique. En effet, une traversée optique a été adaptée sur le cryostat afin de faire passer une fibre optique lentillée dans la chambre expérimentale sous-vide. La fibre lentillée est fixée sur un support en cuivre au-dessus de l'échantillon. Ce dernier est monté sur des translations piézoélectriques qui permettent l'alignement de la fibre avec l'échantillon.

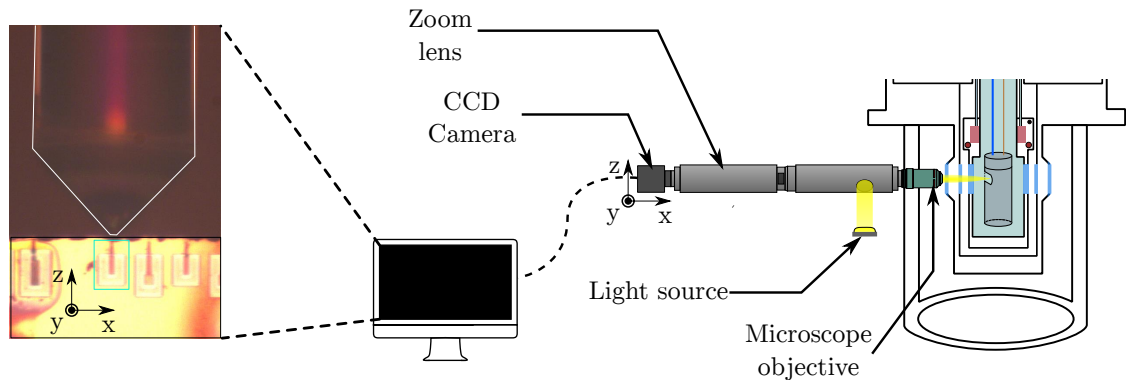


Figure 6.8: Schéma du système d'imagerie optique à côté de la version recadrée du diagramme du cryostat. Sur le côté gauche, une image du flux vidéo est représentée où la fibre optique est encadrée en blanc, la puce de l'échantillon en noir et un échantillon individuel en bleu clair.

Un système d'imagerie constitué d'un objectif de microscope, d'un zoom optique et d'un capteur CCD nous permet de visualiser l'alignement de la fibre avec l'échantillon comme le montre la figure 6.8. Avec un montage fibré en lumière blanche nous arrivons à observer les résonances optiques des cristaux optomécanique comme le montre la figure 6.9.

Les résonances sont centrées autour de 1550 nm et ont des facteurs de qualité d'environ 9000. Les résonances optiques sont caractérisées par un creux dans le signal réfléchi, qui prend la forme de Lorentzienne inversée.

6.5 Résultats

Dans cette section, nous allons décrire sommairement des résultats importants obtenus durant cette thèse. Dans un premier temps nous allons voir que la fréquence de résonance optique des cristaux

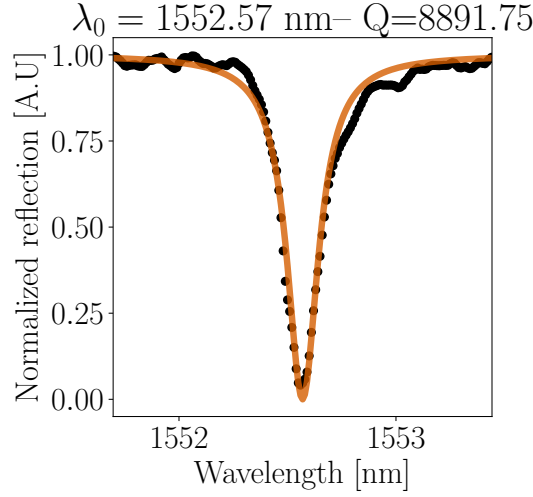


Figure 6.9: Creux de résonance optique d'une cavité à cristal photonique InGaP avec une longueur d'onde de résonance $\lambda_0 = 1552.52$ nm et un facteur de qualité de 9000.

optomécanique dépend de la température. Finalement, les mesures de thermométrie à bruit seront discutées.

6.5.1 Thermométrie photonique

Les variations de température induisent deux effets qui changent la fréquence de résonance optique des cristaux optomécanique. Le premier étant la dilatation thermique que nous allons décrire avec le coefficient d'expansion thermique linéaire $\alpha_L = \frac{1}{L} \frac{\Delta L}{\Delta T}$ qui correspond à la variation relative de la longueur de la cavité L par unité de température T . Le second est l'effet thermoréfractif qu'on va décrire avec le coefficient $\alpha_n = \frac{1}{n} \frac{\Delta n}{\Delta T}$ correspondant à la variation relative de l'indice de réfraction n par unité de température T . Finalement, la variation de la fréquence de résonance peut être décrite comme suit :

$$\omega_c(T + \Delta T) \simeq \omega_c(T) (1 - (\alpha_L + \alpha_n) \Delta T). \quad (6.7)$$

Si on définit un coefficient $\beta(\nu_0) = \nu_0 (\alpha_L + \alpha_n)$ décrivant la variation de fréquence à partir d'une valeur initiale de cette dernière $\nu_0 = \omega_c/2\pi = 193.4$ THz. On peut estimer sa valeur en utilisant des valeurs de littérature des coefficients α_L et α_n pour des cristaux optomécanique en phosphure de gallium (GaP). On trouve $\beta(\nu_0) < -10.44$ GHz/K pour $\alpha_L = 4 \times 10^{-6} \text{ K}^{-1}$ [66] et $\alpha_n < 5 \times 10^{-5} \text{ K}^{-1}$

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[67].

Nous avons mesuré cette variation de fréquence de résonance optique avec la température en utilisant un montage fibré en lumière blanche. Les spectres optiques pour les différentes températures sont représentés en figure 6.10. a où l'on voit clairement la dépendance de la fréquence avec la température. Nous représentons ensuite, en figure 6.10. b, les fréquences de résonance en fonction de la température. Sur cette figure, nous remarquons l'existence de deux régimes. Le premier fortement dépendant de la température ($T > 100$ K) qui est consistant avec la littérature [69] ainsi qu'avec l'estimation théorique. Un deuxième régime dit de saturation est observé pour des températures inférieures à 100 K.

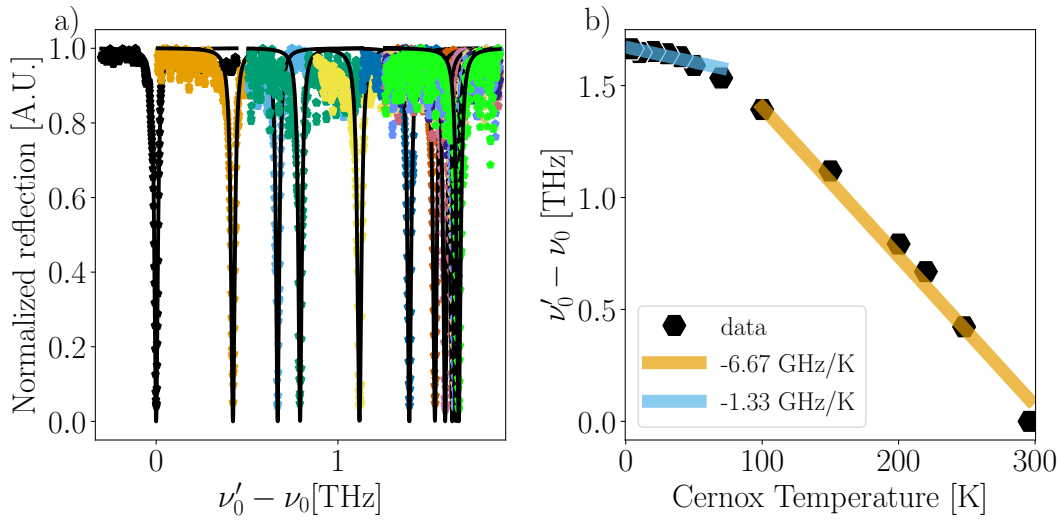


Figure 6.10: Résultats de la thermométrie photonique. **a)** Spectres optiques de la cavité à cristal photonique à différentes températures allant de 296 K, en noir, à 4.2 K en vert clair. Les lignes noires pleines superposées aux points de données sont des ajustements avec des fonctionne lorentziennes. **b)** Fréquence de résonance optique extraite ν'_0 tracée en fonction de la température relevée au Cernox. Les points de données sont ajustés avec deux ajustements linéaires, en orange clair et en bleu, décrivant les deux différents régimes de décalage.

6.5.2 Thermométrie à bruit

Les mesures de température par thermométrie à bruit qui reposent sur la mesure du déplacement induit par la vibration du cristal optomécanique ont été faites en utilisant un montage optique appelé détection homodyne. Ce montage décrit en détail dans le corps du texte est basé sur un faisceau laser

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monochromatique interagissant avec le cristal. La phase du signal réfléchi est comparée à celle d'un faisceau de référence appelé oscillateur local. Cette méthode permet de mesurer le déphasage induit par les vibrations du cristal et ainsi remonter à la température.

Cette mesure nécessite la calibration de la sensibilité de la détection afin de comparer les différentes mesures entre elles. Une première calibration consistant en l'utilisation d'une modulation de phase du faisceau laser qui après interaction avec le cristal, est transduite de la même manière que les vibrations de ce dernier. Cependant, des problèmes expérimentaux n'ont pas permis l'utilisation de cette méthode de calibration. Une autre calibration a donc été faite avec l'estimation de la sensibilité à partir des spectres optiques acquis lors de chaque mesure.

Deux cycles de refroidissement (de 300 K à 4 K) durant lesquels des séries de données ont été acquises. Ces séries de données sont représentées en figure 6.11. a où l'on distingue que la tendance décroissante correspond à ce qui est attendu. On peut également voir un effet de saturation aux basses températures lié aux effets d'échauffement induits par l'absorption du flux lumineux.

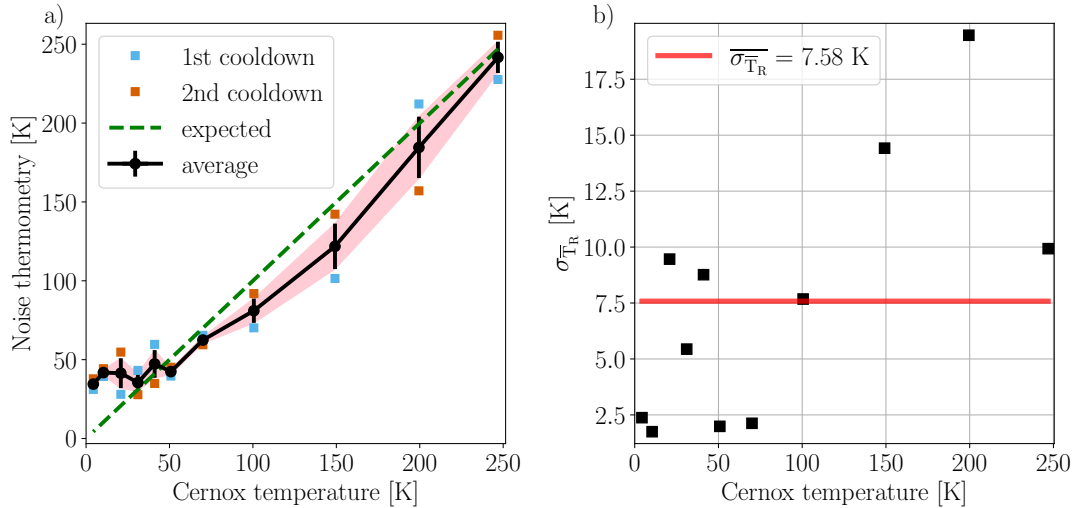


Figure 6.11: Reproductibilité des mesures. **a)** Compilation des résultats de la thermométrie du bruit, calibrés avec les pertes optiques, pour deux refroidissements différents (carrés bleus et orange). Les valeurs moyennes (points noirs) et l'erreur standard (barres noires) sont également représentées. **b)** erreur standard pour chaque température (carrés noirs) et l'erreur moyenne (ligne continue rouge).

On peut également s'intéresser à la reproductibilité de la mesure sur les deux cycles de refroidissement en calculant l'erreur type qui décrit la dispersion des températures moyenne. La figure 6.11. b

montre l'évolution de cette erreur type avec la température ou l'on remarque que la reproductibilité de notre mesure est mauvaise qu'on peut estimer à environ 8 K.

6.6 Conclusion et perspectives

Au cours de cette thèse, nous avons développé un système optomécanique permettant la mesure relative de la température de son bain de deux manières distinctes au sein d'un même système. Ces deux techniques sont complémentaires, puisque la mesure photonique permet d'estimer la température sur la puce. Cependant, les résultats présentés jusqu'à présent souffrent de plusieurs limitations. La mesure des spectres de bruit est effectuée à des puissances optiques élevées et à un désaccord non nul, ce qui introduit un échauffement par absorption de la lumière dans le milieu et un processus d'amplification optomécanique possible, respectivement. Cette limitation peut être contournée en passant à des cristaux 2D, en espérant moins d'auto-échauffement grâce à un meilleur ancrage thermique du cristal au substrat et un rapport signal/bruit satisfaisant à de faibles puissances optiques d'entrée qui permettraient de travailler à la résonance.

Résumé : En 2018, la 26^e Conférence Générale des Poids et Mesures (CGPM) a redéfini les différentes unités du Système International (SI) en particulier le kelvin, unité de température, qui se base désormais sur la valeur fixée de la constante de Boltzmann k_B . Cette redéfinition a suscité le développement de nouveaux capteurs de température primaire permettant la dissémination du nouveau Kelvin. Les capteurs se basant sur les technologies quantiques sont très plébiscités par la communauté de métrologie. Dans ce contexte, nous proposons un capteur de température multimodal dont le fonctionnement repose sur les propriétés optique et optomécanique d'un cristal optomécanique à cristaux photoniques 1D. Sous l'effet de la température le résonateur voit sa fréquence de résonance optique se décaler et le mouvement Brownien induit par le bain thermique environnant varier. Ces deux effets permettent de remonter à la température du résonateur de deux manières différentes, à condition de pouvoir calibrer la chaîne de mesure. Ce type de résonateurs optomécaniques ouvre la voie vers des capteurs de température primaires auto-calibrés avec des corrélations quantiques résultantes de la force de pression de radiation exercée par la lumière.

Mots clés : Optomécanique, capteurs de température, cristaux photoniques, cristaux phononiques, corrélations quantiques.

Abstract : In 2018, the 26th General Conference on Weights and Measures (CGPM) redefined the various units of the International System (SI), in particular the kelvin, the unit of temperature, which is now based on the fixed value of the Boltzmann constant k_B .

This redefinition spurred the development of new primary temperature sensors to disseminate the new Kelvin. Sensors based on quantum technologies are very popular in the metrology community. In this context, we propose a multimodal temperature sensor whose operation is based on the optical and optomechanical properties of a 1D photonic crystal. Indeed, under the effect of temperature, the resonator sees its optical resonance frequency shift, and the Brownian motion induced by the surrounding thermal bath changes. These two effects allow the temperature of the resonator to be determined in two different ways, provided that the calibration is correct. This type of optomechanical resonator opens the way to self-calibrating primary temperature sensors with quantum correlations resulting from the radiation pressure force exerted by light.

Keywords : Optomechanics, temperature sensors, photonic crystals, phononic crystals, quantum correlations.