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Ibrahima Diarra

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Sovereign default and public debt dynamics.

Défaut souverain et dynamique de la dette publique.

Thèse de doctorat de l'université Paris-Saclay

Ecole doctorale n° 630: Droit, Économie, Management- DEM
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Thèse préparée dans le Centre d'Etudes des Politiques Economiques de l'Université d'Evry sous la direction
de Michel GUILLARD (Professeur).

Thèse préparée à Paris-Saclay par

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Titre : Défaut souverain et dynamique de la dette publique

Mots clés : Dette publique, Défaut souverain, Espace fiscal, Changement climatique, Désastres naturels.

Résumé : Cette thèse étudie trois problématiques liées à la soutenabilité de la dette publique, le défaut souverain, et leur lien avec le changement climatique et l'occurrence des désastres naturels. Pour étudier ces questions, je combine modélisation théorique, méthodes économétriques et empiriques. La thèse est composée de trois chapitres.

Le premier chapitre est le fruit d'une collaboration avec Michel Guillard et Hubert Kempf. Nous étudions la relation entre la dynamique de la dette publique d'une part, et le taux de recouvrement de dette applicable en cas de défaut de l'Etat d'autre part. Pour cela, nous développons un modèle stochastique de défaut souverain comportant une règle de recouvrement de dette en cas de défaut de l'Etat. Cette règle dépend d'un paramètre qui permet un recouvrement partiel ou total de la dette après un défaut. Nous montrons que le ratio de dette limite, c'est-à-dire le ratio de dette publique en part de PIB maximum qui peut être soutenu sans faire défaut, est une fonction décroissante et non-linéaire du taux de recouvrement. Avant le défaut, un taux recouvrement élevé se traduit par un espace fiscal plus important, mais cela dégrade la situation financière de l'Etat en cas de défaut. Nous montrons l'importance de prendre en compte ce mécanisme pour une estimation empirique plus précise de l'espace fiscal des pays.

Le deuxième chapitre repose sur un travail commun avec Adham Jaber. Nous estimons l'effet des anomalies de température sur le risque de défaut souverain et explore les canaux de transmission de cet effet. Pour cela, nous utilisons des données de panel portant sur 76 pays durant la période 1999-2017. Nous montrons qu'une augmentation de la température se traduit par une augmentation de la prime de défaut, mesurée par le spread de taux des swaps de défaut (CDS). Partant d'une équation d'évaluation des titres obligataires dérivée des modèles de défaut souverain, nous montrons l'existence d'un canal de dette limite à travers laquelle la température affecte le risque de défaut : un niveau de température plus élevé impacte négativement le taux de croissance du PIB, ce qui diminue le ratio de dette limite. Par conséquent, la probabilité de défaut augmente, ce qui se traduit par une augmentation de la prime de défaut.

Le troisième chapitre s'intéresse à la relation entre le risque de défaut d'une part, et le risque d'occurrence des désastres naturels d'autre part, en particulier ceux qui sont liées au changement climatique. Pour comprendre ce lien, je développe un modèle de défaut souverain comportant une probabilité de désastre qui varie dans le temps. En premier lieu, je montre que la dette limite est une fonction décroissante et non-linéaire de la probabilité de désastre. Ensuite, j'étudie le rôle des anticipations des créanciers par rapport d'éventuels désastres dans le futur. Plus précisément, je compare trois types d'anticipations : anticipations constantes, naïves et rationnelles. Je montre que si les anticipations sont constantes, le ratio de dette limite est également constant. Dans le cas avec anticipations naïves, où les créanciers révisent la probabilité de désastre à chaque période sans tenir compte des variations futures de celle-ci, la dette limite varie dans le temps. Cependant, les créanciers sous-estiment considérablement le risque de défaut comparativement au cas avec anticipations rationnelles. Enfin, je montre qu'en présence du risque de désastre, le défaut peut survenir même dans un contexte très favorable où le taux d'intérêt sans risque reste à un niveau très bas inférieur au taux de croissance du PIB.

Title : Sovereign default and public debt dynamics

Keywords: Public debt, Sovereign default, Fiscal space, Climate change, Natural disasters

Abstract: This Ph.D. thesis studies three different issues related to the topic of public debt sustainability, sovereign default, and their interplay with climate change and the risk of climate-related disasters. To address these issues, I combine theoretical modeling with econometric and simulation methods. The thesis is structured around three chapters.

Chapter 1 "Sovereign Defaults and Debt Sustainability: The Debt Recovery Channel" is the product of a joint work with Michel Guillard and Hubert Kempf. We analyse the debt recovery channel linking the dynamics of public debt to partial sovereign defaults. We build a stochastic model which incorporates sovereign default and a debt recovery rule. This rule depends on a parameter that allows for partial debt recovery. We show that the maximum debt-to-GDP ratio that a country can sustain without defaulting is increasing, nonlinear, and sensitive to the debt-recovery parameter. A higher debt recovery parameter increases the fiscal space but worsens the financial position of a borrowing country after a default episode. We show the empirical relevance of this channel for estimating country-specific fiscal spaces.

Chapter 2 "Sovereign Default Risk and Climate Change: Is it Hot Enough?" is based on a joint work with Adham Jaber. We estimate the effects of temperature anomalies— temperature's deviation from its long-run mean— on sovereign default risk and explore the transmission channels. We use cross-country panel data covering 76 countries over the period 1999-2017. We find that an increase of temperature leads to an increase of the sovereign credit default swap (CDS) premium. Building on an equilibrium bond pricing equation, we document the existence of a "debt limit channel" of temperature: higher temperature, relatively to the long-run mean, has a negative impact on future growth rate, which lowers the country's debt limit— the maximum debt-to-GDP ratio it can sustain without defaulting. As a result, the probability of default increases, leading to a higher CDS spread.

Chapter 3 "Sovereign Defaults in a World of Climatic Disasters: The Expectations Channel" analyzes the expectations channel linking the increasing risk of climatic disasters and the prospects of sovereign defaults. I build a tractable model of sovereign default that allows for time-varying probability of climatic disasters and analyze the role of creditors' expectations on disaster risk. First, I show that the maximum debt-to-GDP ratio that a country can sustain without defaulting is decreasing and nonlinear in the probability of disasters. Second, I compare three types of expectations on disaster risk: the cases of constant, naive, and forward-looking expectations of disaster risk. I show that constant expectations of disaster risk lead to a constant maximum debt ratio. On the other hand, the case with naive expectations of disaster risk— creditors revising the disaster probability in each period while disregarding any future changes— leads to a time dependent maximum debt ratio, but it relatively underestimates default risk compared to the case with forward looking, rational expectations of disasters. Finally, I show that, in the presence of disaster risk, sovereign defaults can occur even in a very favorable environment with low real risk free rate, possibly below the growth rate of output.

Résumé détaillé de la thèse

La crise financière de 2008 et la crise de la dette européenne qui s'en est suivie à remis au centre des débats publics et académiques la question de la soutenabilité de la dette publique des Etats. La grande récession causée par la crise a profondément affecté l'équilibre des finances publiques, aussi bien dans les pays avancés qu'en développement. En réponse à cette récession, la plupart des banques centrales ont mené une politique monétaire d'assouplissement, en ayant souvent recours à des instruments qui étaient jusque là non conventionnels et communément connus sous l'expression « Quantitative Easing ». Cela a conduit, à l'époque, à une baisse généralisée des taux d'intérêt et un écrasement de la prime de risque sur la dette détenue par les Etats. La disparition des primes de risque avait temporairement fait de la question de la soutenabilité de la dette des Etats un problème de second ordre sans réel conséquence à court et moyen terme. Ce point de vue était d'ailleurs largement répandu dans les débats publics, et souvent repris dans les discussions académiques (voir Blanchard, 2019, par exemple) .

Après plusieurs années dans cet environnement de taux bas et d'absence de primes de risque, ce point de vue optimiste est maintenant discutable, du fait notamment des déficits fiscaux records causés par les plans de relance pour faire face aux effets de la Covid 19 et de la crise énergétique et alimentaire due au conflit en Ukraine. Ces chocs multiples et simultanés ont conduit à des taux d'inflation records dans un grand nombre de pays et font planer un risque de récession plus ou moins durable sur leurs économies. Du fait de l'inflation, nous assistons à une remontée graduelle des taux d'intérêt par les banques centrales et à la ré-apparition des primes de risque sur la dette des Etats. Par ailleurs, il y a urgence pour les Etats d'agir et investir pour la transition écologique et faire face aux effets macroéconomiques potentiels du changement climatique et des désastres naturels de plus en plus fréquents. Tout cela met des pressions supplémentaires sur les finances publiques des pays qui, jusque là, ne s'étaient pas complètement remis des effets de la crise de 2008.

Cette thèse a pour objectif d'apporter un nouveau regard sur la question de la soutenabilité de la dette publique et du défaut souverain, dans un contexte de changement climatique et d'augmentation de la fréquence des désastres naturels. La question du défaut souverain n'étant plus seulement un problème des pays en développement, du moins depuis la crise de la dette européenne, nous nous intéressons au risque de défaut souverain à la fois dans les pays en développement que dans les pays avancés.

Contrairement à la majorité des analyses théoriques du défaut souverain qui abordent la question en termes de décision stratégique, cette thèse privilégie l'hypothèse d'un risque de défaut « excusable » au sens de Grossman et Van Huyck (1988). Cet type de défaut apparaît lorsque les Etats ne sont pas en mesure d'obtenir suffisamment de revenu fiscal et/ou de financement sur les marchés financiers pour rembourser leur dette.

La thèse est structurée en trois chapitres et utilise une approche combinant la modélisation théorique avec l'analyse économétrique et empirique. Le premier chapitre part d'une observation empirique. En analysant les données historiques sur les défauts souverains, on constate que la plupart des défauts sont partiels dans le sens où l'Etat rembourse toujours une partie de la dette en cas de défaut. Ce constat est en contraste avec l'hypothèse standard dans les analyses théoriques selon laquelle, en cas de défaut, l'Etat ne rembourse rien à ses créanciers et sa dette est complètement effacée. Nous relâchons cette hypothèse "simpliste" dans le premier chapitre au profit du cas où le défaut souverain peut être partiel. Pour cela, nous développons un modèle stochastique de défaut souverain comportant une règle de recouvrement de dette en cas de défaut. Dans ce modèle l'Etat, représentant un pays, emprunte sur les marchés financiers auprès des créanciers neutres au risque pour combler son déficit budgétaire et/ou faire face à ses obligations financières. L'Etat peut toutefois faire défaut sur la dette contractée, du fait notamment d'un choc de productivité négative, l'incapacité d'augmenter ses revenus fiscaux

ou d'obtenir de nouveau financement sur les marchés financiers. Nous proposons une règle de recouvrement de la dette qui s'applique en cas de défaut. Cette règle dépend d'un paramètre qui permet un recouvrement partiel ou total de la dette par les créanciers après un défaut. Ce paramètre peut prendre des valeurs allant de 0 (aucun recouvrement par les créanciers) à 1 (recouvrement complet).

Nous résolvons le modèle de façon analytique et mettons en évidence plusieurs résultats sur le lien entre la dynamique de la dette publique et le taux de recouvrement. En premier lieu, nous clarifions les concepts de soutenabilité de la dette publique, de défaut souverain et de solvabilité qui sont souvent confondus dans la littérature académique sur le sujet. Nous proposons une définition plus précise et une mesure opérationnelle à ces concepts. Nous montrons que le ratio de dette soutenable est généralement plus faible que le ratio de dette limite, c'est-à-dire le ratio de dette publique en part de PIB maximum qui peut être soutenu sans faire défaut. Ce dernier est également toujours plus faible que le ratio de solvabilité, sauf le cas peu réaliste où l'on suppose un recouvrement complet en cas de défaut. Ensuite, nous montrons que le ratio de dette limite est une fonction décroissante et non-linéaire du taux de recouvrement. Avant le défaut, un taux de recouvrement élevé se traduit par un espace fiscal plus important, mais cela dégrade la situation financière de l'Etat en cas de défaut. Le message clé de ce chapitre est que l'analyse de la soutenabilité de la dette publique dépend énormément du paramètre définissant le taux de recouvrement. Une petite variation de ce paramètre peut avoir des effets très importants sur l'espace fiscal d'un pays et peut donc conduire à des conclusions très différentes quant à la soutenabilité de sa dette publique.

Le deuxième chapitre analyse de façon empirique le lien entre le changement climatique et le risque de défaut souverain. En effet, il y a un intérêt grandissant des économistes et décideurs de politique publique concernant les effets économiques potentiels du changement climatique. De plus, l'importance du rôle des Etats dans le financement de l'adaptation au changement climatique est largement admis dans les débats publics et scientifiques. Etant donné le niveau d'endettement relativement élevé des pays, il est naturel de s'intéresser au lien entre le changement climatique et le risque de défaut souverain.

Pour analyser ce lien, nous estimons l'effet des anomalies de température sur le risque de défaut souverain et explorons les canaux de transmission de cet effet. Pour cela, nous utilisons des données de panel portant sur 76 pays durant la période 1999-2017. Nous utilisons le spread de taux des swaps de défaut (CDS) sur les titres d'obligations d'Etat comme une mesure proxy du risque de défaut. Nous considérons quatre maturités différentes de CDS, à savoir les CDS à 1, 3, 5 et 10 ans. Sur le plan économétrique, nous utilisons différentes méthodes d'estimation, notamment la méthode de régression en panel avec effets fixes.

Nous mettons en évidence plusieurs résultats. D'abord, nous montrons qu'une augmentation de la température se traduit par une augmentation de la prime de défaut, mesurée par les CDS à 3, 5 et 10 ans: une augmentation d'un degré Celsius de la température par rapport à sa moyenne de long terme conduit à une augmentation du CDS de 15.61 à 31.09 points de base. Plus la maturité du CDS est élevée, plus l'effet de la température sur le CDS est important. Ce résultat suggère que les créanciers tiennent compte du risque climatique aussi bien à court qu'à moyen et long terme. Ensuite, nous examinons les mécanismes de transmission des effets de la température sur le CDS. Pour cela, nous utilisons l'équation d'évaluation des titres obligataires dérivée des modèles de défaut souverain. Nous montrons l'existence d'un canal de dette limite à travers laquelle la température affecte le risque de défaut: un niveau de température plus élevé impacte négativement le taux de croissance du PIB, ce qui diminue le ratio de dette limite. Par conséquent, la probabilité de défaut augmente, ce qui se traduit par une augmentation de la prime de défaut. Le message clé de ce chapitre est qu'il existe bien un lien entre le réchauffement climatique et le risque de défaut souverain et ce risque est pris en compte par les créanciers.

Le troisième chapitre s'intéresse à la relation entre le risque de défaut d'une part, et le risque d'occurrence des désastres naturels d'autre part, en particulier ceux qui sont liés au changement climatique. Pour comprendre ce lien, j'étends le modèle de défaut souverain développé dans le premier chapitre en introduisant une

probabilité de désastre qui varie dans le temps. Je résous le modèle analytiquement et fais ensuite des simulations pour illustrer les mécanismes clés. En premier lieu, je montre que le ratio de dette limite est une fonction décroissante et non-linéaire de la probabilité de désastre. Ensuite, j'étudie le rôle des anticipations des créanciers par rapport à d'éventuels désastres dans le futur. Plus précisément, je compare trois types d'anticipations : anticipations constantes, naïves et rationnelles. Je montre que si les anticipations sont constantes, le ratio de dette limite est également constant. Dans le cas avec anticipations naïves, où les créanciers révisent la probabilité de désastre à chaque période sans tenir compte des variations futures de celle-ci, la dette limite varie dans le temps. Cependant, les créanciers sous-estiment considérablement le risque de défaut comparativement au cas avec anticipations rationnelles. Enfin, je montre qu'en présence du risque de désastre, le défaut peut survenir même dans un contexte très favorable où le taux d'intérêt sans risque reste à un niveau très bas inférieur au taux de croissance du PIB. Ce chapitre montre l'importance de prendre en compte le risque lié à l'augmentation de la fréquence des désastres naturels dans l'analyse de la soutenabilité de la dette publique.

Mots clefs : Dette publique, Défaut souverain, Espace fiscal, Changement climatique, Désastres naturels.

Collaborations dans le cadre de la thèse: Adham Jaber (Doctorant, Université Paris 1 Panthéon-Sorbonne), Hubert Kempf (Professeur, Ecole Normale Supérieure Paris-Saclay), Michel Guillard (Professeur, Université d'Evry-Val-d'Essonne).

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Introduction

The Great Financial Crisis of 2007-08 and the ensuing European debt crisis have revived the long dated issue of public debt sustainability and sovereign default. The moderate growth rates following the financial crisis combined with the functioning of automatic stabilizers (reduction of tax revenues) have prevented many economies, both developing and developed ones, in succeeding to reduce the high levels of public debt inherited from the financial crisis (Figure 1). For most developed countries, the implementation of unconventional monetary policies and the resulting incredible low and even negative interest rates on their public debt had temporally made the sustainability of public debt a second order issue with “no real concerns in the short run”. This view was regularly present both in the public debate and among some academics (see, e.g., [Blanchard 2019](#)).

After, a few years in that favorable environment, this optimistic view is now challenged on several grounds. A notable one is the surge in fiscal deficits due, in particular, to fiscal stimulus plans in response to the Covid-19 pandemic (see Figure 1). This led to a complete draining of previous efforts made to reduce the level of public debt in most countries. Moreover, the rapid increase in interest rates by most central banks around the world to fight an ever increasing inflation will increase the burden of public debt for most countries. This situation could be accompanied by a re-emergence of risk premia, particularly for the most heavily indebted countries. Finally, there is an urgent need to mitigate the macroeconomic consequences of climate change, the increasing frequency of climate-related disasters, and to finance the transition towards a more sustainable and environmental-friendly economy.

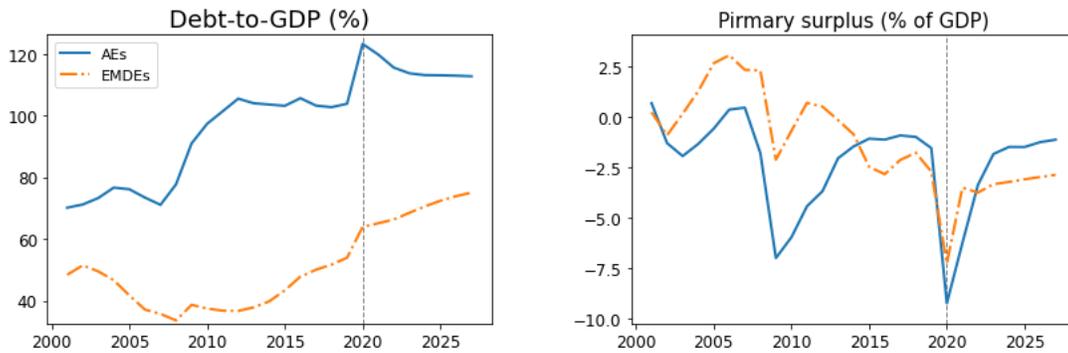


Figure 1: Debt-Surplus dynamics in developing (red) and advanced (blue) countries. Projections start after the vertical line. Sources: IMF WEO (April 2020) and Author’s calculations.

All these factors put additional pressure on the fiscal stance of countries and will potentially lead to the resurgence of sovereign default premia over the coming years. In fact, at the time of writing this thesis, some countries, especially developing ones, are manifesting signs of debt crisis, as reflected in the high yield spreads on their public debt (Figure 2). Even worse is the fact that some of these countries are already in default¹ or on the brink of it.

In this context, the issue of public debt sustainability, which was topical ten years ago from the date of writing this dissertation, is even more topical today.

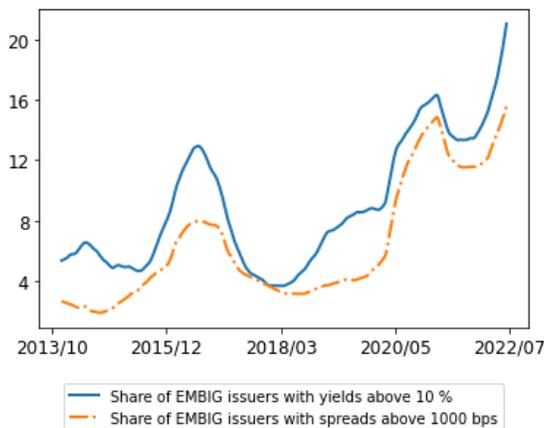


Figure 2: Sovereign yield spread dynamics in emerging countries (year-to-year moving average). Sources: JP. Morgan Emerging Market Bond Index and Author’s calculations.

The aim of this thesis is to revisit the topic of public debt sustainability and

¹This is the case, for example, of Lebanon, Sri Lanka, Suriname and Zambia.

sovereign default, their interlink, and to analyze the role of climate change and related disasters. It addresses three different issues, structured around three chapters.

Briefly, Chapter 1 focuses on the issue of public debt sustainability and sovereign default, and emphasizes the crucial role of the rule governing the rescheduling of public debt after a default. Chapter 2 empirically assesses the impacts of climate change on sovereign default risk and explores the transmission channels, building on the theoretical framework developed in the first chapter. Chapter 3 extends the analysis to investigate the implications of the increasing frequency of climate-related disasters for sovereign default risk.

Defining sovereign default. Throughout the thesis, I focus on “excusable sovereign defaults” in the sense of [Grossman and Huyck \(1988\)](#). This type of defaults are associated with identifiable bad states of the world. They occur only when, following a negative shock, the government is unable to get sufficient fiscal and debt issuance revenue to repay due debt. This definition of default is different from the one used in the literature of strategic default *à la* [Eaton and Gersovitz \(1981\)](#). According to this latter, the government optimally decides to repay or to default on due debt regardless of the realized state of the nature, even if it has the ability to meet its financial obligations. Put bluntly, strategic defaults occur due to the lack of willingness of the government to repay while excusable defaults occur due to its inability to repay.

While the strategic default approach is the most standard framework, recent empirical studies document that countries are in general reluctant to default (see [Yeyati and Panizza 2011](#)). Moreover, the issue of public debt sustainability, which is the focus of this thesis, has little relevance when the government can strategically default each time it finds this optimal.

A key stylized fact in historical sovereign defaults data is that sovereign default is almost always partial in the sense that creditors are able to recover a fraction of the defaulted debt after default. Although it is well supported by recent empirical studies ([Sturzenegger and Zettelmeyer 2008](#); [Cruces and Trebesch 2013](#)), this fact is in contrast with the standard assumption of zero debt recovery found in most

theoretical models of sovereign default.

The first chapter of the thesis, based on a joint work with Michel Guillard and Hubert Kempf, relaxes this unrealistic assumption. We propose a tractable stochastic model of sovereign default that allows for partial debt-recovery after a default. In this model, a country, represented by its government, borrows from risk-neutral creditors on international financial markets to balance its budget constraint, namely the funding it needs to finance fiscal deficit and/or to repay debt contracted in the past. The government may however default on its debt obligations due, in particular, to bad productivity shocks, the inability to raise sufficient fiscal revenue, either by increasing taxes or lowering deficits, or to get sufficient funding from creditors. We propose a simple debt-recovery rule that applies following a default. It depends on a unique parameter, which we refer to as the debt-recovery parameter. This parameter is equal to one minus the “haircut”—the fraction of debt-to-GDP ratio lost by creditors following a sovereign default. It can take any value from 0 to 1. The case with 0 corresponds to a full repudiation of the defaulted debt, while the case with 1 is equivalent to full repayment of public debt.

We solve the model explicitly and analyze its numerical properties and empirical relevance. We find several novel results, which have direct echos with the related literature and profound policy implications.

First, we clarify the notions of the sustainability of public debt, sovereign default, and solvency, which are quite often overlooked by economists and in public debate. We provide more precise definition to these concepts, discuss the importance to distinguish them, and provide operational measure to each of them. We show that the sustainable debt-to-GDP ratio is in general lower than the default ratio—the maximum debt-to-GDP ratio that a country can sustain without defaulting. This latter is itself always lower than the solvency ratio, which obtains under the standard transversality condition on public debt, except in an unrealistic case where the debt-recovery parameter is set to one, its upper limit. In this particular case, the two ratios are equivalent. We find that sustainable and default ratios are both increasing, nonlinear, and sensitive to the debt-recovery parameter:

even a small change in the debt recovery parameter can have substantial effects on the sustainable and default ratios.

The full dynamics of public debt is also shown to depend on the debt recovery parameter, as well as on the realizations of the growth shock. To better understand this dynamics, we resort to the notion of *risky steady state* (RSS) recently used by [Coeurdacier, Rey and Winant \(2011\)](#). This allows us to analyze the impact of a productivity shock when agents form their expectations of relevant variables and take decisions based on the probability distribution of future shocks whereas the realizations of these shocks are equal to their mean values. We show that a RSS debt level does not always exist in this framework. It exists only for sufficiently high values of the debt recovery parameter. In particular, there is no RSS under the no debt recovery assumption found in most models of sovereign default.

Building on these results, we introduce a new definition of debt unsustainability: public debt is unsustainable when its trajectory leads to the default ratio at some finite date, assuming that there is no realization of the growth shock higher than the mean. This allows us to revisit the concept of “fiscal space” introduced by [Ghosh et al. \(2013\)](#). A fiscal space measures the capacity of a country to secure additional borrowing to face bad shocks without defaulting. Here we precisely define the fiscal space as the difference between the actual and RSS debt ratios when this latter exists, or the actual and default ratios when there is no RSS. Since both default and RSS ratios depend positively on the debt recovery parameter, it plays a critical role in the assessment of country fiscal spaces.

Finally, we study the post-default dynamics of public debt. We show that there exists a critical value of the debt recovery parameter such that the post-default debt ratio is “sustainable” if this parameter is below the critical value. Otherwise, the after-default public debt is unsustainable and the defaulting country is exposed to what is known in the literature as “serial defaults”, that is repetitive defaults.

A key message of this chapter is that the assessment of the sustainability of public debt depends crucially on the value of the debt recovery parameter. A small change in this parameter can have substantial effects on the dynamics of public debt, and therefore lead to very different conclusions in terms of debt sustainability

analysis. We illustrate the role of this parameter through calibrations, simulations and estimations of the model using historical data on emerging and advanced countries. We find that the debt-recovery parameter that is implicit in sovereign yield spreads is relatively lower for emerging countries than for advanced ones. Since the fiscal space is positively related to the debt-recovery parameter, this result partly explains the paradox of “debt intolerance”: compared with advanced countries, emerging countries experience both lower default ratios, that is a lower debt tolerance by markets, and higher risk premia.

Accelerating climate change and the increase in the frequency of extreme climate shocks, such as heatwaves, droughts, hurricanes and coastal flooding, have recently received particular attention, both in academia, public debate and the media. A large strand of the literature documents the impacts of climate change on economic growth (Nordhaus 2006; Dell, Jones and Olken 2012; Burke, Hsiang and Miguel 2015) and various economic outcomes (see Dell, Jones and Olken 2014 and Kolstad and Moore 2019 for a survey). Yet, there is little evidence on the link, and the nature of the link, between climate change and sovereign risk, and whether financial markets effectively price climate-related risk.

Chapter 2 of the thesis, which is based on a joint work with Adham Jaber, contributes to the growing literature on the impacts of climate change on the economy. The goal of this chapter is to empirically assess the implications of climate change for sovereign default risk. To address this issue, we use temperature anomalies—temperature’s deviation from its long-run mean—as a proxy for climate change. As for sovereign default risk, we use sovereign Credit Default Swap (CDS) spread as a proxy.

The chapter is organized around its two main contributions. The first part documents the relationship between temperature anomalies and sovereign CDS spreads. We consider sovereign CDS spread at several maturities— one, three, five and ten-year maturities. Our key hypothesis is that financial markets account for climate risk when lending to countries. Econometrically, we address this issue using a large panel dataset that covers 76 developing and advanced countries from 1999 to 2017. The countries are selected based on data availability only.

Using a standard two-way fixed effects estimation method, we document a strong positive impact of temperature on sovereign CDS spread for the three, five and ten-year maturities but not for the one-year one: a one degree Celsius increase in temperature, relatively to the long-run mean, increases CDS spreads by 15.61 to 31.09 basis points. This effect is statistically and economically significant and robust to different alternative measures of temperature anomalies. Moreover, we find that the longer the maturity of the CDS spreads the larger the impact of temperature on spreads. This finding suggests that sovereign creditors price climate risk, not only when investing over a short horizon but also the medium and long ones.

In the second part of our analysis, we investigate the key channels through which the estimated positive effect of temperature on CDS spread may occur. To isolate these channels, we build on an equilibrium bond pricing equation found in most theoretical models of sovereign default, including the one proposed in Chapter 1. This equation relates the spread to its key underlying macroeconomic fundamentals. We document the existence of a “debt limit channel” of temperature: a higher temperature has a negative impact on future growth rate of output, which lowers the country’s debt limit—the maximum debt-to-GDP ratio it can sustain without defaulting. As a result, the probability of default increases, leading to a higher CDS spread. We find that this debt limit channel accounts for the bulk of the estimated effect of temperature on the CDS spread. Interestingly, we find that the debt-to- GDP ratio and the primary balance, which are two macroeconomic determinants present in the basic pricing equation, do not play any role in the transmission of the effect of temperature to CDS spread.

Our findings have interesting implications for the policy responses to climate change in the context of high public debt-to-GDP ratios and limited fiscal space available for countries. Our identification of the key mechanisms suggests that climate risk must be taken into account in the assessment of public debt sustainability.

There is a growing interest of academics and policymakers in the economic impacts of large macroeconomic shocks and the appropriate policy responses. This

interest has been strongly revived recently due to the increase in the frequency and intensity of natural disasters, in particular climate and weather related ones. In its latest Atlas of Mortality and Economic Losses from Weather, Climate and Water Extremes (August 2021),² the World Meteorological Organization shows that the number of climate related disasters, such as floods and extreme temperature, have increased five-fold in 1970-2019, killing more than 2 million people and costing \$3.64 trillion in total losses.

Recent empirical studies show that climate-related disasters have been particularly salient in some recent sovereign default and debt restructuring episodes. Notable examples are Dominican Republic 1998, Grenada 2004, Antigua & Barbuda 2004 and 2009.³ Since countries across the world rely heavily on borrowing from international financial markets, an increase in the frequency of disasters and the associated losses, as predicted by climate scientists, may reinforce their fiscal vulnerabilities and the risk premium on their public debt.

Chapter 3 of the thesis investigates the link between sovereign default risk and the risk of natural disasters, in particular climate-related ones. To address this issue, I expand the framework developed in Chapter 1 and introduce a time-varying probability of disasters. I do that in a tractable way so that the key mechanisms of the model can be analyzed analytically. In the baseline model, the probability of disaster is a deterministic function of time with a linear trend.

Next, I estimate the probability of disaster based on historical disaster occurrences, and calibrate the model to better understand its key properties. Two main findings came out of this analysis. First, I show that the maximum debt-to-ratio that a country can sustain is decreasing and nonlinear in the probability of disasters. Second, in the presence of disaster risk, a sovereign default can occur even in a very favorable environment with a low risk free rate or high growth rate.

I show how these findings may change according to different types of expect-

²Click here: [web link](#)

³See [International Monetary Fund \(1999a\)](#) and [Asonuma et al. \(2018\)](#). Other default episodes related to climatic disasters are Moldova and Suriname which defaulted respectively in 1992 and 1998 following severe droughts ([International Monetary Fund, September 1999](#); [de Jong et al., 2000](#)). Ecuador defaulted in 1997 just a few months after floods caused major power shortages ([Sturzenegger and Zettelmeyer, 2006](#)).

tations that creditors may have about disaster risk. Specifically, I compare three types of expectations that creditors may have about disaster risk: *i)* the case of constant disaster risk; *ii)* the case with “naive” expectations, corresponding to a situation where creditors are short-sighted and revise the disaster probability in each period while ignoring any future changes to this probability; and *iii)* the case with forward-looking, rational expectations about disaster risk. I show that when disaster expectations are constant over time, the maximum debt ratio is also constant. On the other hand, the case with naive expectations of disaster risk leads to a time dependent maximum debt ratio. However, this naive approach relatively underestimates sovereign default risk compared to the case when creditors have forward-looking, rational expectations about disaster risk. In the last part of the chapter, I provide an extension of the model where there is uncertainty about the disaster probability and creditors engage in a Bayesian learning to define this probability.

A nice feature about this model is its relative simplicity. Although it focuses more on climate extreme events, the setup that I develop can be fairly applied to other types of extreme events, not necessarily related to climate, such as major conflicts, the Great Recession, Covid, Ukraine War or any event that can have severe effect on output growth. I plan to adopt this more general approach in a future version of this work.

Chapter 1

Sovereign Defaults and Debt Sustainability: The Debt Recovery Channel

1.1 Introduction.

A stylized fact in historical sovereign defaults data is that default is almost always partial, that is, creditors are able to recover a fraction of the defaulted debt after default.¹ This suggests the existence of a “debt recovery channel” which we define as the link between sovereign defaults, public debt sustainability and the fraction of due debt recovered by lenders after a sovereign default.² In this paper we investigate such a channel and show how it affects the dynamics of public debt, its sustainability, and the occurrence of sovereign defaults. Specifically, we analyze how lenders’ expectations of a debt recovery after a potential default contribute to the “snowball effect” related to the default premium included in the interest rate on public debt. Relying on the concept of “excusable default” (see the seminal paper of [Grossman and Huyck, 1988](#)), we set up a tractable stochastic model of

¹See [Sturzenegger and Zettelmeyer \(2008\)](#), [Cruces and Trebesch \(2013\)](#), and [Arellano, Mateos-Planas and Rios-Rull \(2019\)](#).

²This refers to what is commonly known as an “haircut”. The haircut rate is equal to one minus the fraction of debt-to-GDP recovered by creditors following a sovereign default.

sovereign default with a “debt recovery rule” that allows for partial debt haircuts. We use a simple specification of such a rule which hinges on a unique parameter defined as the expected maximum debt recovery rate.

We solve the model explicitly and uncover the following main findings. First, we show that a country’s default ratio– the maximum debt-to-GDP ratio that can be sustained without default– is increasing, nonlinear and very sensitive to the debt recovery parameter. We show that the default ratio is different from the solvency ratio which obtains under the standard transversality condition on public debt. Second, we resort to the concept of *risky steady state* (RSS) to analyze the dynamics of debt and introduce a new definition of debt unsustainability: public debt is unsustainable when its trajectory leads to the default ratio at some finite date, assuming that there is no realization of the growth shock higher than the mean. The whole dynamics of public debt is shown to depend on the recovery parameter. Third, we use historical data on both advanced and emerging countries and provide estimates of the recovery parameter for both groups. We find that these estimated parameters are markedly lower for emerging countries than for advanced countries. This results sheds light on the evidence of debt intolerance, that is, the fact that countries face different default ratios and experience default at very different debt-to-GDP ratios, consistently with observed risk premia.³ Finally, we reassess the issue of sustainability when the real risk-free interest rate is low, possibly lower than the growth rate. We show that even for high values of the debt recovery parameter, a sovereign default cannot be ruled out as the default ratio is finite, although the solvency ratio– which corresponds to a more classical definition of sustainability– is infinite in this case.

The paper is organized as follows. Section 1.2 provide a brief review of the literature. Section 1.3 presents the model. Section 1.4 addresses the valuation of public debt and its link with the debt recovery rule. Section 1.5 analyzes the dynamics of public debt in the presence of stochastic shocks and addresses the issues of unsustainability. Section 1.6 provides estimations of the debt recovery parameter and compute country default ratios and fiscal spaces associated to these

³The seminal paper on debt intolerance is [Reinhart, Rogoff and Savastano \(2003\)](#).

estimations. Section 1.7 concludes.

1.2 Related literature

Willems and Zettelmeyer (2021) provide a recent and up-to-date survey on sovereign debt sustainability which is a useful introduction to this topic. Sturzenegger and Zettelmeyer (2006), Reinhart and Rogoff (2008) and Das, Papaioannou and Trebesch (2012) provide a comprehensive survey of historical sovereign defaults and restructurings. In a pioneering work, Sturzenegger and Zettelmeyer (2008) introduce a methodology to compute haircuts on defaulted debt. The haircut is defined as the percentage difference between the present value of old and new debt instruments issued during debt restructuring. Using data for 14 debt restructurings in 1998-2005, they document average haircuts ranging from 13% to 73%. Cruces and Trebesch (2013) and, more recently, Meyer, Reinhart and Trebesch (2019) use a similar approach to compute haircuts using data on sovereign default events in a larger number of countries and a time period going back to 1815. They find that debt repudiation and debt cancellations (haircuts of, or close to 100%) are the exception rather than the rule.

Following Eaton and Gersovitz (1981), the bulk of theoretical studies on sovereign default address the issue in a strategic framework. Aguiar and Amador (2014) and Mitchener and Trebesch (2021) provide useful surveys on this topic. This literature focus on solving the puzzle of the existence of sovereign debt contracts between fully rational agents when there is no or limited enforcement capacity. The issue is the designing of efficient contracts taking into account the incentive of the sovereign to default. Important references on the subject are Calvo (1988), Cole and Kehoe (2000), Aguiar and Gopinath (2006) and Arellano (2008). The standard assumption in these papers is a full discharge of public debt after default and a sanction by lenders in the form of complete exclusion from financial markets. These assumptions are in contrast with the empirical studies mentioned above and with our work.⁴ In particular, we allow for a partial hair-

⁴On the assumption of exclusion from financial markets, Gelos, Sahay and Sandleris (2011)

cut on the defaulted debt and the possibility for the government to reenter the markets after default.

A few recent papers depart from the complete default assumption of early papers in the strategic default paradigm. [Yue \(2010\)](#) develops a model of debt renegotiation with Nash bargaining and complete information. In her setting, the government and creditors bargain to a debt haircut that maximizes the total renegotiation surplus. She shows that the renegotiation outcome affects the expected duration of financial exclusion, and therefore the country's incentive to default. In the same spirit, [Benjamin and Wright \(2009\)](#) and [Ghosal, Miller and Thampanishvong \(2018\)](#) consider a model of debt renegotiation with a dynamic alternating offers framework to analyze the delay observed in some historical debt restructurings.⁵

[Arellano, Mateos-Planas and Rios-Rull \(2019\)](#) emphasize the role of missed payments on debt service preceding sovereign default events. In their setting, each period the sovereign strategically decides whether to fully honor its debt payment or to miss a fraction. The amount of payments missed accumulate as arrears and add to future debt. In their model, the government uses missed payments to inter-temporally transfer resources and to smooth consumption.

Following the seminal paper of [Grossman and Huyck \(1988\)](#), a growing strand of the literature takes a different approach and models sovereign defaults as “excusable”. Our paper clearly adopts this approach. An “excusable default” excludes any strategic decision by the sovereign to default and is solely associated to identifiable “bad states of the world”.⁶ Such defaults occur when the government is unable to obtain the necessary funds to refinance its outstanding debt, either by issuing new debt, by decreasing public spending or by raising taxes.⁷ In a model

document that, while the average length of exclusion was 4 years in the 1980s, it drops to 2 years during the 1990s. [Meyer, Reinhart and Trebesch \(2019\)](#) note that, in recent period, defaulting countries managed to place bonds quickly post-default. A notable example is Argentina in 2016. The country re-accessed international markets only months after its 7th default.

⁵See also [Sunder-Plassmann \(2018\)](#), [Asonuma and Joo \(2020\)](#), [Dvorkin et al. \(2021\)](#) and [Amador and Phelan \(2021\)](#).

⁶See [Grossman and Huyck \(1988\)](#), p.1088.

⁷Note that sovereign “excusable defaults” are different from “rollover crises” à la [Cole and Kehoe \(2000\)](#), which are driven by sunspot shocks.

of excusable default, [Bi \(2012\)](#) shows that the existence of fiscal limits drastically modify the conditions on the sustainability of debt and contributes to defaults. [Ghosh et al. \(2013\)](#) relate fiscal fatigue to public default and endogenously derive the “debt limit”. Assuming that default may occur in one period only, [Lorenzoni and Werning \(2019\)](#) investigate the gradual worsening of public debt position which is due to the presence of long-term debt.

Assuming zero debt recovery (haircut of 100%) by investors in case of a sovereign default, [Collard, Habib and Rochet \(2015\)](#) propose a measure of maximum borrowing for advanced economies. This assumption is at odds with the observations on historical sovereign defaults mentioned before. As we shall see below, it substantially underestimates a country’s maximum borrowing, which we find to be a highly non-linear function of (expected) haircut.

Finally, the issue of public debt sustainability has recently been re-examined, taking into account the low risk-free interest rate relative to the growth rate. [Blanchard \(2019\)](#), [Sergeyev and Mehrotra \(2020\)](#) and [Mauro and Zhou \(2020\)](#) suggest that negative $r - g$ differentials⁸ are quite common over the past 200 years and characterize recent years. The authors of these two last papers and [Blanchard, Leandro and Zettelmeyer \(2021\)](#) nevertheless point to the possibility of abrupt bond yield reversals and subsequent reappearances of public debt sustainability issues.

1.3 The model.

We consider a small open economy with international financial markets and perfect diversification of risks. Time is discrete $t = 0, 1, 2, \dots$. In each period t , a quantity Y_t of goods is available and represents the country’s GDP. Let $a_t \equiv Y_t/Y_{t-1}$ be the gross rate of growth of output between $t - 1$ and t .⁹ We assume that a_t evolves randomly across time and follows a probability law with the following characteristics:

⁸ g refers to the real growth rate of GDP, and r is the real risk-free interest rate.

⁹We will often refer to a_t simply as the growth rate and be more precise when necessary.

Assumption 1.

1. a_t is an *i.i.d.* random variable with a density function $g(a)$, denoting by $G(a)$ its cumulative distribution function, both defined on the interval $[0, +\infty)$, and $E(a) \equiv \bar{a} < \beta^{-1}$ where $\beta^{-1} = 1 + r$ is the risk-free real gross interest rate;
2. the hazard function $z(a) = \frac{g(a)}{1-G(a)}$ is monotone and non-decreasing.

Assumption 1.1 makes clear that the productivity follows a random walk and the condition $E(a) < \beta^{-1}$ will guarantee that the long run growth rate is inferior to the risk-free interest rate for this economy. We will relax this assumption in Section 1.6. Assumption 1.2 is a regularity assumption which allows us to exclude the possibility of multiple equilibria as it will be made explicit in Section 1.4.

1.3.1 Private sector.

We assume that international financial markets allow perfect coverage against risk and therefore investors behave as risk-neutral agents. Consider a one-period maturity security offering – in the absence of default – a promise of one unit of goods in $t + 1$. The price at date t , denoted q_t , of such a security satisfies rational expectations if

$$q_t = \beta E_t h_{t+1}, \tag{1.1}$$

where h_{t+1} is the fraction of the end-of-period value that will be repaid in a given state of nature in period $t + 1$, with $h_{t+1} = 1$ if there is no default and $h_{t+1} < 1$ in case of default.

1.3.2 Government.**1.3.2.1 Fiscal rule and fiscal constraint.**

The government generates a sequence of primary fiscal surpluses as fractions of output $\{s_t\}$, representing total taxes collected minus total outlays on government

purchases and transfers. A negative value of s_t corresponds to a primary deficit. The government balances its budget by issuing one-period maturity Treasury bonds of facial value 1 at price q_t . The level of debt (which is also the number of bonds emitted in t) is denoted by B_t . In case of default at t , it reimburses a fraction $h_t < 1$ of its debt contracted at $t-1$, B_{t-1} . The instantaneous government budget constraint writes:

$$q_t B_t = h_t B_{t-1} - s_t Y_t, \quad (1.2)$$

with $h_t \in [0, 1]$. This parameter takes the value of 1 if there is no default in t and a lower value, given by a debt recovery rule, when the government is unable to meet its financial obligations in t and thus defaults.

Following [Davig, Leeper and Walker \(2011\)](#), [Bi \(2012\)](#) and [Daniel and Shiamp-tanis \(2013\)](#), we assume that the primary surplus s_t increases with the *actually redeemed* debt-to-GDP ratio, up to a limit denoted by \hat{s} :

$$s_t = \min \left(\bar{s} + \theta \cdot \left(\frac{h_t B_{t-1}}{Y_t} - \bar{\omega} \right); \hat{s} \right), \quad (1.3)$$

where $\bar{\omega} \geq 0$ is the long run target for the outstanding debt-to-GDP ratio in period t : B_{t-1}/Y_t . Such a limit to the primary surplus can be justified by the coexistence of tax distortions (leading to a Laffer curve) and inelastic public expenditures.

We make the following assumption:

Assumption 2. *The parameters θ , \bar{s} and \hat{s} satisfy:*

$$\theta > 1 - \beta \bar{a}, \text{ and } \hat{s} > \bar{s} \equiv (1 - \beta \bar{a}) \bar{\omega}.$$

The presence of the upper bound \hat{s} captures the maximum fiscal effort the government is able to make in order to repay its debt. When the primary surplus has reached its maximum value \hat{s} , we refer to this situation as *fiscally constrained* and we will say that the economy is in a *constrained fiscal regime*.

1.3.2.2 Default and the debt recovery rule.

Default occurs only when the government does not obtain the necessary funds to refinance its outstanding debt. Let us denote by Ω_t^{def} the maximum (face value of) debt which can be redeemed by the Treasury in t : default occurs when $B_{t-1} > \Omega_t^{\text{def}}$. We refer to Ω_t^{def} as the “default threshold” for period t . As we will see later, this threshold obtains in equilibrium on the financial markets.

We abstract from specifically studying the bargaining process between the defaulting public borrower and its lenders and consider that it is captured by a simple debt recovery rule, contingent on the level of contractual debt B_{t-1} and on the default threshold Ω_t^{def} , is applied. We use the following specification:

$$h_t = \begin{cases} \mathbf{h} \cdot \Omega_t^{\text{def}} / B_{t-1} & \text{if } B_{t-1} > \Omega_t^{\text{def}} \\ 1 & \text{else} \end{cases} \quad (1.4)$$

with $0 \leq \mathbf{h} \leq 1$.¹⁰

According to this rule, any realization of the (stochastic) default threshold Ω_t^{def} below the contractual level of debt triggers default and a rescheduling of public debt. This rescheduling is such that the after-default (redeemed) debt level is a fraction of Ω_t^{def} , i.e. $h_t B_{t-1} = \mathbf{h} \Omega_t^{\text{def}}$. By considering the limit case where the overrun is negligible ($B_{t-1} \rightarrow \Omega_t^{\text{def}+}$), \mathbf{h} can be interpreted as the maximum debt recovery rate in a default episode. By extension, $1 - \mathbf{h}$ is the minimal rate of default, or equivalently and loosely speaking, the lowest possible “haircut”. This rule displays two important features:

1. This debt recovery rule has the property of ensuring that the government is immediately able to re-enter the bond market as its post-default initial debt is below Ω_t^{def} and the economy functions again according to the set of equations characterizing its dynamics.
2. The possibility of future defaults is not ruled out. Nevertheless the rule allows the defaulting government to withstand adverse shocks in the future.

¹⁰Note that, although we use bold notation, \mathbf{h} is a scalar parameter not a vector.

The lower is \mathbf{h} , the more room there is to accommodate future adverse shocks.

1. is meant to simplify the analysis of the dynamics and could be relaxed at the cost of cumbersome analytical complexities. 2. is important as it captures the fact that a debt rescheduling is a temporary arrangement. It does not necessarily provide a definitive solution to a country's fiscal situation which may worsen due to adverse shocks. Cross-country evidence shows that the ratio of recovered debt to due debt h_t is not unique and markedly differs across countries and circumstances.¹¹ This evidence is consistent with (1.4) when considering country-specific values of \mathbf{h} . Moreover the realized values of h_t are affected by macroeconomic shocks.

1.3.2.3 The no-Ponzi condition and the solvency ratio.

The government's budget constraint is subject to a no-Ponzi condition:

$$\lim_{T \rightarrow \infty} E_t \beta^T h_{t+T} B_{t+T-1} \leq 0. \quad (1.5)$$

Using (1.1) in (1.2), one gets:

$$\beta E_t h_{t+1} B_t = h_t B_{t-1} - s_t Y_t.$$

Defining $\omega_t \equiv h_t B_{t-1} / Y_t$, and remembering that $a_{t+1} = Y_{t+1} / Y_t$, we obtain:

$$\beta E_t a_{t+1} \omega_{t+1} = \omega_t - s_t, \quad (1.6)$$

and the no-Ponzi condition (1.5) is equivalent to:

$$\lim_{T \rightarrow \infty} E_t \beta^T \left(\prod_{n=1}^T a_{t+n} \right) \omega_{t+T} \leq 0. \quad (1.7)$$

¹¹See the empirical studies mentioned in Section 1.2.

The no-Ponzi solution is consistent with individual rationality and therefore standard in macro models. In models where the possibility of defaults is a priori excluded, this condition corresponds to a debt sustainability condition. As we shall see below, when taking into account the possibility of defaults and therefore of debt rescheduling, this equivalence does not hold anymore.

Note that ω_t is a stochastic variable which may “jump” in each period according to the growth rate innovation and the possibility of a sovereign default. Using the definition of ω_t , the fiscal rule (1.3) rewrites:

$$s_t = \min(\bar{s} + \theta \cdot (\omega_t - \bar{\omega}); \hat{s}). \quad (1.8)$$

Using (1.8) and the definition of \bar{s} given in Assumption 2, we obtain from (1.6) the following dynamic equation for the expected redeemed debt-to-output ratio:

$$E_t \beta a_{t+1} \omega_{t+1} = \begin{cases} (1 - \theta)(\omega_t - \bar{\omega}) + \beta \bar{a} \bar{\omega} & \text{for } \omega_t < \hat{\omega} \\ \omega_t - \hat{s} & \text{for } \omega_t \geq \hat{\omega} \end{cases} \quad (1.9)$$

with

$$\hat{\omega} \equiv \bar{\omega} + \frac{\hat{s} - \bar{s}}{\theta} > \bar{\omega}, \quad (1.10)$$

where the last inequality comes from Assumption 2.

Equation (1.9) makes clear the consequence of a maximum fiscal surplus \hat{s} . It creates a kink in the dynamics of expected debt-to-output ratio. If the actually redeemed debt-to-output ratio ω_t is sufficiently low (below $\hat{\omega}$), an increase in the public debt ratio can be partially offset by an increase in the primary surplus ratio s_t . Let us consider a deterministic version of this equation by assuming $a_{t+1} = \bar{a}$. The expected debt ratio is obtained from a linear equation. Its slope, equal to $(1 - \theta) / \beta \bar{a}$, is from Assumption 2 less than one. When ω_t is above the debt-to-output ratio $\hat{\omega}$ at which the primary surplus ratio reaches its maximum \hat{s} , the expected actually redeemed debt ratio is obtained from a linear equation the slope of which, $(\beta \bar{a})^{-1}$, is more than one. Hence the kink at $\hat{\omega}$ creates two (deterministic) steady states, the first of which is $\bar{\omega}_1 = \bar{\omega}$, and the second: $\bar{\omega}_2 =$

ω^{sup} , with

$$\omega^{\text{sup}} \equiv \frac{\hat{s}}{1 - \beta\bar{a}}. \quad (1.11)$$

Note that ω^{sup} is equal to the sum of the present and expected discounted primary surpluses (relative to the actual GDP) when they are set at their maximum value. Hence it defines the conventional solvency limit of public debt-to-output ratio in a deterministic environment. It does not depend on the debt recovery parameter. As we will see below this is an important difference with the (equilibrium) default ratio which we find to be very sensitive to the (expected) debt recovery parameter.

When $\omega_t \geq \hat{\omega}$ we obtain from (1.9):

$$\begin{aligned} \omega_t &= \hat{s} + E_t \beta a_{t+1} \omega_{t+1} \\ &= \frac{\hat{s}}{1 - \beta\bar{a}} + \lim_{T \rightarrow \infty} E_t \beta^T \left(\prod_{n=1}^T a_{t+n} \right) \omega_{t+T}. \end{aligned}$$

Using this last result, the no-Ponzi condition (1.7) implies:

$$\omega_t \leq \omega^{\text{sup}}, \quad (1.12)$$

where ω^{sup} is given by (1.11). This inequality is the solvency condition on government debt in this stochastic environment. In the sequel, we will refer to ω^{sup} as the *solvency ratio* of sovereign debt.

1.3.3 Market equilibrium.

Let us denote by $b_t \equiv B_t/Y_t$ the level of contractual government debt emitted today relative to GDP at t , and

$$\omega_t^{\text{def}} \equiv \Omega_t^{\text{def}}/Y_t, \quad (1.13)$$

the “default threshold” for period t as a percentage of GDP. Using these notations and according to (1.4) default occurs when $b_{t-1} > a_t \omega_t^{\text{def}}$. The market equilibrium

is given by the following equations:

$$q_t b_t = \frac{h_t b_{t-1}}{a_t} - \min \left(\bar{s} + \theta \cdot \left(h_t \frac{b_{t-1}}{a_t} - \bar{\omega} \right); \hat{s} \right) \quad (1.14)$$

$$h_t = \begin{cases} \mathbf{h} \frac{a_t \omega_t^{\text{def}}}{b_{t-1}} & \text{if } b_{t-1} > a_t \omega_t^{\text{def}} \\ 1 & \text{else} \end{cases} \quad (1.15)$$

$$q_t = \beta E_t h_{t+1}, \quad (1.16)$$

together with the no-Ponzi condition (1.7).

Equation (1.14) is the government budget constraint, obtained by using equations (1.2) and (1.3); (1.15) is the debt recovery rule, and (1.16) is the pricing equation. Taking the sequence $\{\omega_t^{\text{def}}\}$ as given, these equations are sufficient to analyze the valuation of public debt and the dynamics of emitted debt-to-output ratio b_t . Of course, the sequence of default ratios $\{\omega_t^{\text{def}}\}$ is endogenous and ultimately needs to be obtained. We will see below that this sequence is actually deterministic in this setting.

1.4 Sovereign default and debt recovery.

In this section, we focus on the study of the functioning of this economy in the fiscal constraint regime.¹² Specifically, we suppose that the economy was in a constrained tax regime in $t - 1$, remains in this regime in t and will be there in $t + 1$. The budget constraint is then written in the following simpler form:

$$q_t b_t = \frac{h_t b_{t-1}}{a_t} - \hat{s}. \quad (1.17)$$

¹²Formally, this leads in particular to neglecting the probability of a shock favorable enough to exit from this regime. Treating this hypothesis more rigorously would require restricting the distribution support of shocks, which would considerably and unnecessarily complicate the analysis (see [Guillard and Kempf 2017](#)).

1.4.1 Debt valuation.

Assuming that $\omega_{t+1}^{\text{def}}$ is known in t and using (1.15) the price of public debt (1.16) rewrites as:

$$q_t = \beta \left[1 - G \left(\frac{b_t}{\omega_{t+1}^{\text{def}}} \right) + \mathbf{h} \frac{\omega_{t+1}^{\text{def}}}{b_t} \int^{\frac{b_t}{\omega_{t+1}^{\text{def}}}} adG(a) \right]. \quad (1.18)$$

Notice that the price of bond is a decreasing function of b_t . Lenders include in the price a risk premium linked to the probabilities of expected future defaults, based on the ratio $b_t/\omega_{t+1}^{\text{def}}$, on the probability law of a_t and the debt recovery parameter in case of default. The market value of public debt in t is denoted by $v_t \equiv q_t b_t$. From (1.18), it is a function of b_t , parameterized by $\omega_{t+1}^{\text{def}}$ and \mathbf{h} :

$$v_t = \beta \left\{ \left[1 - G \left(\frac{b_t}{\omega_{t+1}^{\text{def}}} \right) \right] b_t + \mathbf{h} \omega_{t+1}^{\text{def}} \int^{\frac{b_t}{\omega_{t+1}^{\text{def}}}} adG(a) \right\} \equiv v(b_t; \omega_{t+1}^{\text{def}}, \mathbf{h}). \quad (1.19)$$

The function $v(\cdot)$ is potentially non-monotone. The following proposition formalizes the existence of a unique maximum to this function:

Proposition 1. *Given $\omega_{t+1}^{\text{def}}$, under Assumption 1, the market value of debt v_t reaches a unique maximum v_t^{max} for a quantity of debt $b_t = b_t^{\text{max}}$. Both v_t^{max} and b_t^{max} are linearly increasing in $\omega_{t+1}^{\text{def}}$: $v_t^{\text{max}} = \beta x_{\mathbf{h}} \omega_{t+1}^{\text{def}}$ and $b_t^{\text{max}} = \delta_{\mathbf{h}} \omega_{t+1}^{\text{def}}$ where $\delta_{\mathbf{h}}$ is such that*

$$[1 - G(\delta_{\mathbf{h}})] [1 - (1 - \mathbf{h}) \delta_{\mathbf{h}} z(\delta_{\mathbf{h}})] = 0, \quad (1.20)$$

$z(\delta) = \frac{g(\delta)}{1-G(\delta)}$ being the hazard function and $x_{\mathbf{h}}$ given by

$$x_{\mathbf{h}} = [1 - G(\delta_{\mathbf{h}})] \delta_{\mathbf{h}} + \mathbf{h} \int^{\delta_{\mathbf{h}}} adG(a). \quad (1.21)$$

$\delta_{\mathbf{h}}$ and $x_{\mathbf{h}}$ are increasing functions of \mathbf{h} , with $0 < x_{\mathbf{h}} \leq \bar{a}$ and $0 < \delta_{\mathbf{h}} \leq +\infty$ for $0 \leq \mathbf{h} \leq 1$.

According to this proposition, the maximum value of public debt v_t^{max} and the

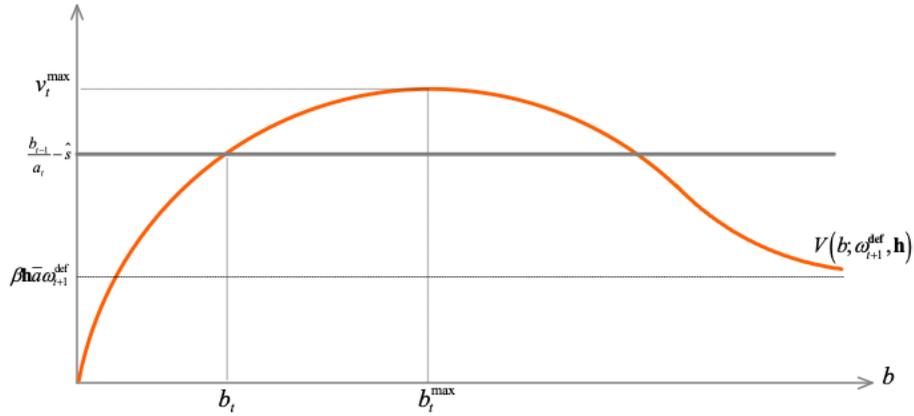


Figure 1.1: Equilibrium debt valuation.

corresponding amount of emitted debt b_t^{max} are increasing functions of the default ratio $\omega_{t+1}^{\text{def}}$ and the debt recovery parameter \mathbf{h} .

The higher the debt recovery parameter \mathbf{h} , the higher the maximal market value: Lenders are ready to lend more as they receive more in case of default. Even in the extreme case of no debt recovery ($\mathbf{h} = 0$), lenders are potentially willing to lend to the government, despite complete loss in case of default, because they are compensated by a positive risk premium. In the extreme case of the highest debt recovery parameter ($\mathbf{h} = 1$), the maximum public debt value is equal to the discounted default ratio, that is: $v_t^{\text{max}} = \beta \bar{a} \omega_{t+1}^{\text{def}}$.¹³

Figure 1.1 illustrates this relation for a given value of \mathbf{h} verifying $0 < \mathbf{h} < 1$. For values of b_t below b_t^{max} , the market value of public debt $v_t = q_t b_t$ is increasing in b_t . Above b_t^{max} , the decreasing effect of bond price overcomes the direct effect of increasing debt and makes the public debt value starting to decrease. Because of its “bell”-shaped form, the function $v(\cdot)$ is referred to as the “debt Laffer curve” in the literature (see D’Erasmus, Mendoza and Zhang 2016, and Lorenzoni and Werning 2019).

An equilibrium debt ratio b_t without default in t is such that (1.17) holds with $h_t = 1$. The equilibrium displayed in Figure 1.1 corresponds to the no-default

¹³Note that, since both v_t^{max} and $\omega_{t+1}^{\text{def}}$ are expressed in terms of output, the discount rate used is the risk-free real interest rate net of the expected growth rate of output.

case. For financing needs $b_{t-1}/a_t - \hat{s}$ between $\beta \mathbf{h} \bar{a} \omega_{t+1}^{\text{def}}$ and v_t^{max} , there are two values of b_t which meet this request (as shown in Figure 1.1). Notice that the equilibrium situated on the decreasing side of the valuation function is “unstable” in the Walrasian sense. In the neighborhood of the high debt equilibrium, in the case of an excess demand a higher bond price increases the gap between demand and supply; the reverse is true in the case of an excess supply.¹⁴ This leads us to select the low debt equilibrium, satisfying $b_t \leq b_t^{\text{max}}$. Excluding the case of default (i.e. assuming $b_{t-1}/a_t \leq \omega_{t+1}^{\text{def}}$), the equilibrium debt-to-output ratio is given by:

$$b_t = \min \left(b \mid v \left(b; \omega_{t+1}^{\text{def}}, \mathbf{h} \right) = -\hat{s} + b_{t-1}/a_t \right). \quad (1.22)$$

1.4.2 Equilibrium default ratio.

Figure 1.1 helps us to graphically understand default as a market event. There is default in t when a sufficiently negative shock heightens the horizontal line above the $v \left(b; \omega_{t+1}^{\text{def}}, \mathbf{h} \right)$ curve, that is, above v_t^{max} . Formally the condition corresponding to default can be written as:

$$\frac{b_{t-1}}{a_t} - \hat{s} > v_t^{\text{max}}. \quad (1.23)$$

The default condition used in (1.15) has been defined as: $b_{t-1} > a_t \omega_t^{\text{def}}$. Thus the default ratio ω_t^{def} is necessarily equal to:

$$\omega_t^{\text{def}} = v_t^{\text{max}} + \hat{s}. \quad (1.24)$$

It is defined as the sum of the maximum value that the government can obtain from the market and the primary surplus of the period.

Since from Proposition 1 we have: $v_t^{\text{max}} = \beta x_{\mathbf{h}} \omega_{t+1}^{\text{def}}$, using (1.24), we get a

¹⁴Lorenzoni and Werning (2019) develop the same argument and give other reasons justifying the discarding of the “unstable” equilibrium.

dynamic expression for ω_t^{def} :

$$\omega_t^{\text{def}} = \beta x_{\mathbf{h}} \omega_{t+1}^{\text{def}} + \hat{s}. \quad (1.25)$$

It is a forward-looking equation: how much can at most be redeemed today depends on how much can at most be redeemed tomorrow, because this last one directly determines the opportunities for public funding.

Denoting by $\omega_{\mathbf{h}}$ the stationary solution of (1.25), the following proposition obtains:

Proposition 2. *The equilibrium default ratio is locally unique and equal to:*

$$\omega_t^{\text{def}} = \frac{\hat{s}}{1 - \beta x_{\mathbf{h}}} \equiv \omega_{\mathbf{h}}, \forall t. \quad (1.26)$$

$\omega_{\mathbf{h}}$ is a strictly increasing function of \hat{s} and \mathbf{h} , with $\omega_{\mathbf{h}} \leq \omega^{\text{sup}}$ for $\mathbf{h} \leq 1$.

Strikingly, even though we reason in a stochastic environment, the default ratio is a constant, $\omega_t^{\text{def}} = \omega_{\mathbf{h}} \forall t$, independent from the dynamics of public debt and thus from the history of shocks. We can deduce from Proposition 1 that

$$b_t^{\text{max}} = \delta_{\mathbf{h}} \omega_{\mathbf{h}} \equiv b_{\mathbf{h}}^{\text{max}}, \forall t, \quad (1.27)$$

and

$$v_t^{\text{max}} = \beta x_{\mathbf{h}} \omega_{\mathbf{h}} \equiv v_{\mathbf{h}}^{\text{max}}, \forall t, \quad (1.28)$$

which denote respectively the maximum quantity of public bonds in percentage of output that can be emitted and the associated maximum public debt value¹⁵ – again in terms of output – where $\delta_{\mathbf{h}}$ and $x_{\mathbf{h}}$ are given by (1.20) and (1.21).

From equation (1.26), we note that, unless $x_{\mathbf{h}}$ is equal to its upper limit \bar{a} corresponding to the case $\mathbf{h} = 1$, the default ratio is lower than the solvency ratio ω^{sup} . Collard, Habib and Rochet (2015) already highlighted the same kind

¹⁵What CHR calls, respectively, the maximum sustainable debt (MSD) and the maximum sustainable borrowing (MSB). We prefer to keep the term "sustainable" for another use, proposed in the next section.

of result in the particular case $\mathbf{h} = 0$. Taking into account a positive recovery parameter allows us to generalize their findings while showing the sensitivity of the default ratio to the recovery parameter \mathbf{h} . Figure 1.2 shows the default ratio $\omega_{\mathbf{h}}$ as a function of the (expected) debt recovery parameter \mathbf{h} , using a baseline calibration proposed in section 1.6.¹⁶

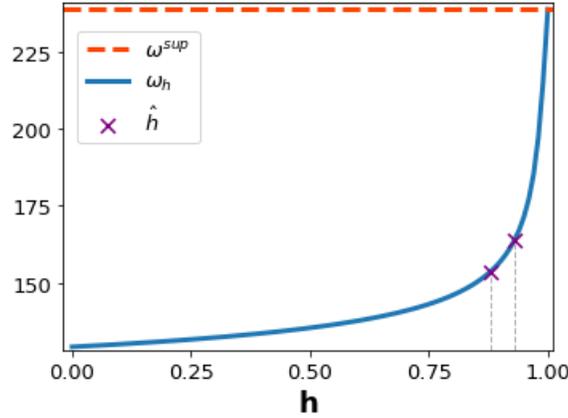


Figure 1.2: Debt recovery parameter and default ratio

The default ratio (blue curve) is an increasing, highly nonlinear function of the debt recovery parameter. Recall that when $\mathbf{h} = 1$, the default ratio is equal to the solvency ratio ω^{sup} , which is evaluated to 238% of GDP (horizontal dash line) with our baseline calibration. As \mathbf{h} moves from 1 to 0.98 the default ratio falls to 197% of GDP and amounts only to 135% at $\mathbf{h} = 0.5$, and 129% when $\mathbf{h} = 0$. The increasing sensitivity of $\omega_{\mathbf{h}}$ to the debt recovery parameter is due to the effect of sovereign risk on debt price: The default premium is decreasing in the debt recovery parameter, the higher \mathbf{h} the lower the prospect of post-default losses and the higher the price of emitted debt. This increases the maximum debt value $v_{\mathbf{h}}^{\text{max}}$ and the implied default ratio: $\omega_{\mathbf{h}} = v_{\mathbf{h}}^{\text{max}} + \hat{s}$. The effect of the debt recovery parameter on the default ratio illuminates the debt recovery channel and shows the limitation of assuming no debt recovery, as it is the case in most

¹⁶To construct Figure 1.2, we set \hat{s} , the maximum primary surplus, to 5%, $\beta = (1 + r)^{-1}$ with a risk free rate r equal to 2.93%, and a log-normal distribution for the gross rate of growth, that is: $\ln a \sim N(\mu, \sigma^2)$ with $\mu = 0.0281$, and $\sigma = 0.0263$. Section 1.6 provides more details on the choice of parameter values.

sovereign default models. It is clear from Figure 1.2 that such an assumption would substantially underestimate a country’s default ratio.

Since this ratio is constant we simplify the notation of the valuation function $v(b_t; \omega_{\mathbf{h}}, \mathbf{h}) \equiv v(b_t; \mathbf{h})$. Equation (1.19) becomes:

$$v(b_t; \mathbf{h}) = \beta \left\{ \left[1 - G\left(\frac{b_t}{\omega_{\mathbf{h}}}\right) \right] b_t + \mathbf{h}\omega_{\mathbf{h}} \int^{\frac{b_t}{\omega_{\mathbf{h}}}} adG(a) \right\}. \quad (1.29)$$

The property of this function is given in the following proposition:

Proposition 3. *The market value of public debt is a strictly increasing function of the debt recovery parameter \mathbf{h} .*

This proposition confirms the intuition that lenders expect to be better covered in case of default when the debt recovery parameter increases and thus value more a given amount of public debt.

1.5 Public debt dynamics and unsustainability.

In this section we address the public debt dynamics when it is subject to market pricing and dependent on the debt recovery rule as explained in the previous section. This dynamics is made complex because it actually depends on many factors: the capacity to proceed to fiscal adjustments, the recurring shocks hitting the economy and, last but not least, the prospects of haircuts to be applied in case of default. This is true even in the constrained fiscal regime. To overcome this difficulty, we exploit the notion of “Risky Steady State” and offer an new approach to the notion of public debt unsustainability in the presence of default. This allows us to reformulate the definition of fiscal space, originally introduced by Ghosh et al. (2013). This notion is central in the management of public debt as it points to the fact that the prospect of default is more or less acute, depending on the capacity of a government to modify its fiscal policy or buffer negative shocks given the probability law governing the relevant random variables. Intuitively, the larger the fiscal space in a given period, the lower the probability of default in the

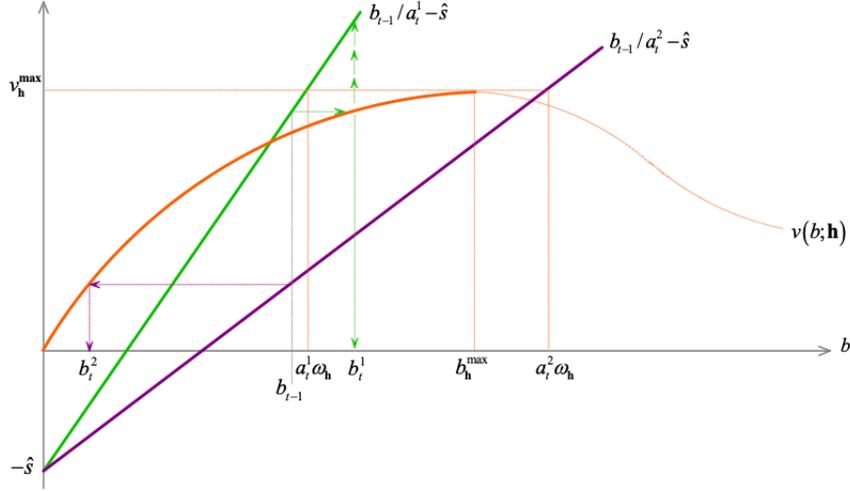


Figure 1.3: The no-default case dynamics

next period. We highlight the impact of the debt recovery rule on the dynamics of public debt and its impact on the fiscal space.

1.5.1 The dynamics of the public debt.

The debt dynamic process can be formally obtained in our model. Consider a period t where the random variable realization a_t and the debt ratio to be redeemed b_{t-1} are such that no default occurs, that is: $b_{t-1}/a_t < \omega_{\mathbf{h}}$, implying $h_t = 1$. The dynamics of public debt defined by the government budget constraint (1.17) expressed in the constrained fiscal regime can be written as:

$$b_t = \min(b | v(b; \mathbf{h}) = -\hat{s} + b_{t-1}/a_t), \quad (1.30)$$

where the function $v(b; \mathbf{h})$ is given by (1.29).

This formula makes clear that the debt dynamics is stochastic and shifts with the realizations of the productivity shock.

Figure (1.3) illustrates the dynamics of the public debt for two possible values of the realized rate of growth a_t^1 and a_t^2 , for which there is no default in t , satisfying: $b_{t-1} < a_t^1 \omega_{\mathbf{h}} < a_t^2 \omega_{\mathbf{h}}$.

For an initial public debt-to-output ratio b_{t-1} , the straight lines $(b_{t-1}/a_t^1 - \hat{s})$ and $(b_{t-1}/a_t^2 - \hat{s})$ give the government’s refinancing requirements in each scenario corresponding to the two states of nature considered. By projecting these values onto the curve $v(b_t; \mathbf{h})$, we get two possible debt-to-output ratios of period t : b_t^1 and b_t^2 . For the higher growth rate, a_t^2 , the service of the maturing debt b_{t-1}/a_t^2 is low, leading to a reduction of the new emitted debt: $b_t^2 < b_{t-1}$. However this is not so for the lower growth rate a_t^1 and the debt ratio increases: $b_t^1 > b_{t-1}$. Interestingly, even if the growth rate a_t^1 is not low enough to lead to an immediate default, it nevertheless leads to a serious deterioration in the government’s financial situation which contributes to a higher default risk premium included in the price of debt. A “snowball effect” comes into play. The increase in a given period t of the amount of emitted debt increases the probability of default and thus the default risk premium. This in turn lowers the price of public bond which increases the quantity of debt to be emitted in the next period for the refinancing of the outstanding debt. This results in a gradual worsening of the financial position of the government. If the same macroeconomic situation is repeated in period $t + 1$, *i.e.* $a_{t+1} = a_t^1$, it leads to a sovereign default since the financial needs in $t + 1$ now exceed the maximum availability of funds $v_{\mathbf{h}}^{\max}$.

1.5.2 The Risky Steady State and the debt recovery rule.

In order to shed more light on the debt dynamics in this stochastic environment, we resort to the concept of “Risky Steady State” (RSS), introduced by [Juillard \(2011\)](#) and [Coerdacier, Rey and Winant \(2011\)](#).¹⁷ This concept makes it possible to study the dynamics of public debt by disregarding the realization of shocks but without eliminating the effect of risk on the debt valuation. Let us consider the following

Definition 1. *A Risky Steady State (RSS) is a stationary equilibrium of the dynamic system when the realization of these shocks are equal to their mean value*

¹⁷An early reference on this notion is [Juillard and Kamenik \(2005\)](#).

and agents form their expectations of relevant variables and make decisions on the basis of the probability distribution of future shocks.

Applying this definition to our problem, the *Risky Steady State* level of debt is the stationary level of the debt-to-output ratio $b_t = b_{t-1}$ in equation (1.30) with $a_t = \bar{a}$. More precisely, denoting by $b_{\mathbf{h}}^{\text{rss}}$ the RSS-debt-to-output ratio, it is such that:

$$v(b_{\mathbf{h}}^{\text{rss}}; \mathbf{h}) = \frac{b_{\mathbf{h}}^{\text{rss}}}{\bar{a}} - \hat{s}. \quad (1.31)$$

The left hand side of (1.31) represents the market value of debt at the RSS, that is what lenders are willing to lend. The right hand side is the financial needs of the sovereign borrower at the RSS. We formalize the existence of the RSS-debt-to-output ratio in the following

Proposition 4. *In the constrained fiscal regime,*

1. *there exists a unique risky-steady-state-debt ratio, $b_{\mathbf{h}}^{\text{rss}}$, satisfying (1.31) and $b_{\mathbf{h}}^{\text{rss}} \leq \bar{a}\omega_{\mathbf{h}} \leq b_{\mathbf{h}}^{\text{max}}$, if and only if $\mathbf{h} \geq \underline{\mathbf{h}} = 1 - \frac{1}{\bar{a}z(\bar{a})}$, with strict equalities for $\mathbf{h} = \underline{\mathbf{h}}$.*
2. *When $\mathbf{h} > \underline{\mathbf{h}}$, $b_{\mathbf{h}}^{\text{rss}}$ and the difference $b_{\mathbf{h}}^{\text{max}} - b_{\mathbf{h}}^{\text{rss}}$ are both increasing in \mathbf{h} .*

Figure (1.4) represents the potential existence and determination of the RSS for different values for the recovery parameter: 0, $\underline{\mathbf{h}}$, 1 and a value $\tilde{\mathbf{h}}$ such that $\underline{\mathbf{h}} < \tilde{\mathbf{h}} < 1$.

A notable result from Proposition 4 is that a RSS does not always exist in this model. Its existence depends on the debt recovery parameter and this parameter must be sufficiently large. In particular a RSS does not exist when $\mathbf{h} = 0$, the case considered for instance by Collard, Habib and Rochet (2015). In this case, and more generally when $\mathbf{h} < \underline{\mathbf{h}}$, defining $b_{\mathbf{h}}^{\text{max}}$ as the debt limit¹⁸ seems to be a good

¹⁸That is, using the definition of Ghosh et al. (2013): “the maximum debt level at which the government can rollover its maturing debt and finance the primary deficit at a finite interest rate”.

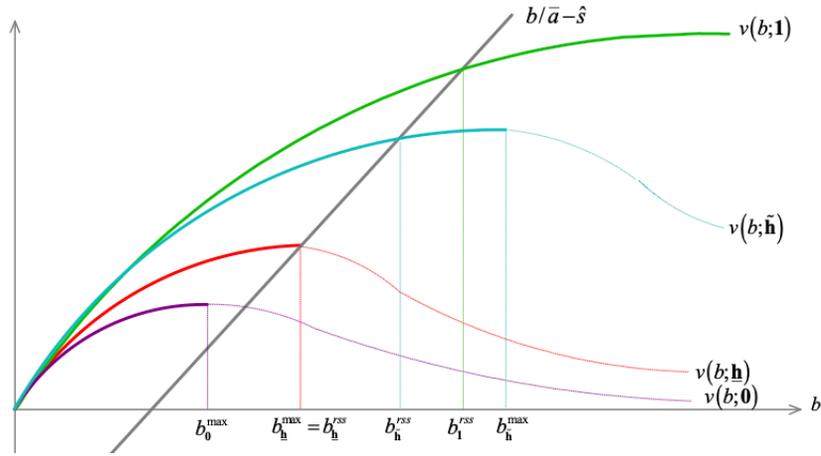


Figure 1.4: (Non-) Existence of a RSS according to \mathbf{h}

choice for the assessment of public debt sustainability. However, when $\underline{\mathbf{h}} \leq \mathbf{h} \leq 1$ a RSS always exists and it is generally below $b_{\mathbf{h}}^{\max}$. We will propose in the section 1.5.3 to consider $b_{\mathbf{h}}^{rss}$ as a relevant alternative candidate to define the debt limit ratio in this case.

Given that the value of an emitted public bond is increasing in the debt recovery parameter, the amount of debt which can be rolled over consistent with the RSS is also increasing in \mathbf{h} . This explains point 2. of Proposition 4.

Figure 1.5 illustrates the implied dynamics of the public debt ratio, given by equation (1.30) when $a_t = \bar{a}$, for the two polar cases $\mathbf{h} = 0$, and $\mathbf{h} = 1$.¹⁹

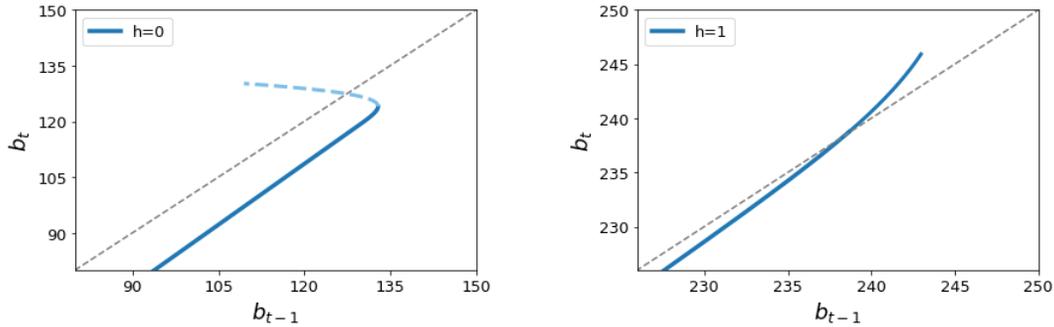


Figure 1.5: Debt dynamics when $a_t = \bar{a}$.

¹⁹We use the same calibration described in the footnote 16. We limit the scale of the axes for ease of display.

When $\mathbf{h} = 0$, a turning point of the curve corresponding to the maximum quantity of public debt exists and is below the 45° degree line. Thus the intersection with the 45° degree line does not define a RSS as the part of the curve above the turning point corresponds to the wrong side of the debt Laffer curve and is discarded.

In the other limit case, $\mathbf{h} = 1$, considered for instance by [Uribe \(2006\)](#) and [Juessen, Linnemann and Schabert \(2016\)](#), there is a RSS but no turning point. The curve is asymptotically vertical and the default ratio is the solvency ratio. In such a configuration, a default makes the post-default indebtedness equal to the solvency ratio. If the post-default value of a_t is at most equal to its mean, this necessarily leads to a renewed default. This captures an extreme case of the feature of serial default.²⁰

There is a value of the debt recovery parameter, denoted by $\underline{\mathbf{h}}$, such that the turning point of the curve is exactly on the 45° line. It is the lowest value of \mathbf{h} for which there exists a RSS. For values of \mathbf{h} higher than $\underline{\mathbf{h}}$ but lower than 1, there exists a RSS which is below the solvency ratio. The level of public debt consistent with the RSS is below the maximum debt level $b_{\mathbf{h}}^{\max}$. Lastly, notice that when it exists, a RSS is unstable as the dynamics of public debt is diverging as long as $b_t > b_{\mathbf{h}}^{\max}$ and $a_{t+\tau} \leq \bar{a}$ (for $\tau \geq 0$).

This makes apparent a striking paradox with respect to the snowball effect (as defined above). The intuition is that the snowball effect, understood as the build-up of public debt possibly leading to default, is large when the risk supported by the lenders is high, that is when the post-default recovered debt is low (due to a low recovery parameter or, loosely speaking, a high haircut). Actually, it happens only when \mathbf{h} is above $\underline{\mathbf{h}}$ and the level of debt is above the RSS: the subsequent debt level is increased and closer to the default ratio (again as long as $a_{t+\tau} \leq \bar{a}$). On the other hand, when \mathbf{h} is below $\underline{\mathbf{h}}$, there is no snowball effect at all: if the due debt level is higher than the level corresponding to the turning point, default is immediate.

²⁰See [Reinhart and Rogoff \(2004\)](#), for instance.

1.5.3 Reassessing unsustainability

Ghosh et al. (2013) define the fiscal space at time t as the difference between the “debt limit”, which corresponds to the maximum level of debt $b_{\mathbf{h}}^{\max}$ in the context of our model, and the current debt ratio b_t . Therefore it depends on the minimum debt recovery parameter \mathbf{h} . This notion is critical for the management of public debt as it points to the fact that the prospect of default is more or less acute, depending on the capacity of a government to modify its fiscal policy²¹ or the capacity to buffer negative shocks given the probability law governing the relevant random variables. The larger the fiscal space, the lower the probability of future default.

However, in line with our discussion in the previous subsection, defining the fiscal space as the difference between $b_{\mathbf{h}}^{\max}$ and the current debt ratio, especially for using it as a criterion of debt sustainability, is of little value when the debt recovery parameter is high and thus a RSS exists. In this case, it may be relevant to more precisely define the fiscal space as the difference between the RSS debt-to-output ratio $b_{\mathbf{h}}^{\text{rss}}$ and the contemporary debt-to-output ratio b_t . This allows to distinguish two very different situations, depending on whether b_t is below or above $b_{\mathbf{h}}^{\text{rss}}$. In the former case, the fiscal situation can be perilous, especially if the debt level is close to $b_{\mathbf{h}}^{\text{rss}}$, but it is “not critical” in the following sense: if the growth rate is not strictly below its average, the share of debt in GDP should decrease over time. In the latter case, the public debt situation is “critical” given the instability of the RSS: the debt sustainability cannot be taken for granted and default looms in even if the growth rate is equal to its mean.

In order to shed some light on this intuition, we first give an original definition of the (un-)sustainability of public debt:

Definition 2. *A public debt is said to be “unsustainable” at date t when its trajectory reaches the default ratio at some finite date, assuming that there is no realization of the (gross) rate of output growth a_{t+s} higher than \bar{a} .*

²¹This is no longer possible in our economy, under the assumption of a constrained fiscal regime.

The case of unsustainability refers to the following “non-optimistic” scenario: no future realizations of the shock will be higher than \bar{a} . The period t public debt is “*unsustainable*” since, under this scenario, a market-triggered default will unavoidably occur in the future.²²

This calls for the redefinition of the notion of “debt limit”. When there exists a RSS (\mathbf{h} above $\underline{\mathbf{h}}$), trespassing this level implies that public debt is unsustainable and leads to future default (assuming that $a_t = \bar{a}$). Thus the RSS should be considered as the debt limit. When it does not exist (\mathbf{h} below $\underline{\mathbf{h}}$), the debt limit is logically the maximum level of debt. Thus we propose the following

Definition 3. *The debt limit and the fiscal space denoted by FS_t in period t are respectively defined as: $b_{\mathbf{h}}^{\text{lim}} = \min(b_{\mathbf{h}}^{\text{max}}, b_{\mathbf{h}}^{\text{rss}})$ and $FS_t = b_{\mathbf{h}}^{\text{lim}} - b_t$.*

As we have just shown that the maximum debt-to-GDP ratio $b_{\mathbf{h}}^{\text{max}}$, and the risky steady state $b_{\mathbf{h}}^{\text{rss}}$ are both increasing functions of the recovery parameter \mathbf{h} , so is the fiscal space FS_t . This comes directly from Proposition 3 and the fact that the value of public debt is increasing in \mathbf{h} .

Figure 1.6 represents $b_{\mathbf{h}}^{\text{max}}$ and $b_{\mathbf{h}}^{\text{rss}}$, and implicitly the debt limit $b_{\mathbf{h}}^{\text{lim}} = \min(b_{\mathbf{h}}^{\text{max}}, b_{\mathbf{h}}^{\text{rss}})$, with the basic calibration already used for Figures 1.2 and 1.5.

We shall see in the next section how this dual definition of the debt limit can be used in empirical analyses to shed light on the public finance positions of different countries, both advanced and emerging.

In line with Definition 2, a worrisome case is when, in the event of a default, the post-default debt ratio is unsustainable. The following proposition establishes that this outcome is possible when the recovery parameter is sufficiently high:

Proposition 5. *When a public default has occurred, the post-default debt-to-GDP ratio $\mathbf{h}\omega_{\mathbf{h}}$ is unsustainable if the debt recovery parameter \mathbf{h} is above a critical value*

²²Symmetrically we could said that a public debt is “sustainable” at date t when its trajectory does not reach the default ratio at any future date, assuming that there is no realization of the (gross) rate of output growth a_{t+s} lower than \bar{a} . It is a very weak definition of sustainability given the very optimistic nature of the considered scenario. We discard this view.

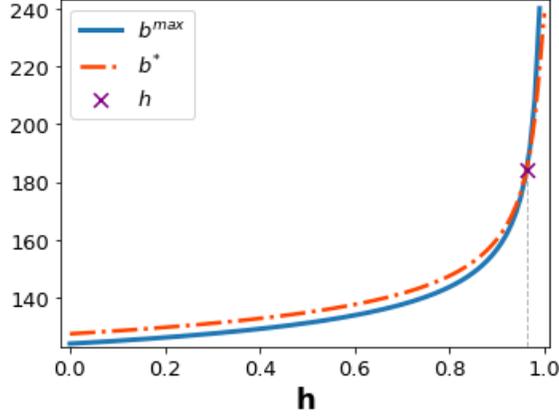


Figure 1.6: Debt limit: $\min(b_h^{\max}, b_h^{\text{rss}})$

$\mathbf{H} : \mathbf{h} > \mathbf{H} > \underline{\mathbf{h}}$, where \mathbf{H} is implicitly defined by:

$$\mathbf{H}\omega_{\mathbf{H}} = \frac{b_{\mathbf{H}}^{\text{rss}}}{\bar{a}}.$$

When $\mathbf{h} > \mathbf{H}$, the post-default debt ratio $\mathbf{h}\omega_{\mathbf{h}}$ is superior to the level $b_{\mathbf{h}}^{\text{rss}}/\bar{a}$ that makes it possible to maintain the debt ratio at its RSS level at the next period when the realization of the shock a_{t+1} is equal to its mean value \bar{a} . In other words, according to Definition 2, public debt is unsustainable. In such a situation, except in the case where a very favorable macroeconomic shock allows the economy to leave the zone of unsustainability, the economy could suffer a series of repeated defaults, i.e. serial defaults. Post default, a higher value of the recovery parameter \mathbf{h} increases the debt burden. Above the threshold value \mathbf{H} , this burden is so high that public debt becomes unsustainable. This is in stark contrast with the ex ante perspective adopted in the previous sub-sections where a high value of \mathbf{h} was viewed as favorable.

1.6 Numerical / Empirical analysis.

The previous analysis provided a better understanding of the dynamics of public debt in a stochastic environment where default is not a priori excluded. It

highlighted the role played by the debt recovery rule on the dynamics of public debt, both before and after default has occurred. This allows us to offer new instruments so as to assess the soundness of the financial position of a country at a given date, by redefining the debt limit and the fiscal space.

In this section, we show how these notions can be put in practical use to empirically investigate the link between public default and the debt recovery parameter.

1.6.1 Data.

We use a dataset that covers two groups of countries over the period 1980-2018. The first one (“Advanced”) contains 31 advanced economies. The second one (“Emerging”) contains 13 emerging economies. We restrict the sample of countries to those with sufficient historical observations for our variables of interest.²³

Appendix 1.8.2 presents descriptive statistics of the data, the definition of the variables, and data sources.²⁴

1.6.2 Baseline calibration.

We consider a log-normal distribution for the growth rate a_t :

$$\ln a \sim N(\mu, \sigma^2).$$

Table 1.1 presents the baseline parameter values used in the calibration exercises to follow.

Growth mean and volatility are computed over the whole country-time sample. The risk-free rate r is set to the average real yield on German Treasury bond.²⁵

²³We limit our analysis to countries with at least ten consecutive years of observations. We use the IMF’s World Economic Outlook definition to classify countries between emerging and advanced groups.

²⁴Table 1.6 presents the definition of the variables, and data sources. Tables 1.7 to 1.11 presents descriptive statistics of the data.

²⁵Calibration results that we shall report below are similar when we use the US Government rate as the risk-free rate. We prefer the German rate as it appears to be a fairly better benchmark over the past few decades than the US rate (see also [Mitchener and Trebesch 2021](#)).

Table 1.1: Baseline calibration (annual basis).

	Emerging	Advanced
Risk-free rate, r	0.0293 ^a	0.0293 ^a
Maximum primary surplus, \hat{s}	0.02 or 0.03 ^b	0.04 or 0.05 ^b
Mean of $\log a_t$, μ	0.0364 ^c	0.0281 ^c
Volatility of $\log a_t$, σ	0.0333 ^c	0.0263 ^c
Length of one period ($t, t + 1$)	4 years ^d	4 years ^d

Notes : a : Annualized rate on 5-year-maturity German bonds (1980-2018), b : IMF(2011; 2018),
 c : Historical (1980-2018), d : US debt duration (2010).

The length of one period in the model is set to 4 years.²⁶ The maximum primary surplus \hat{s} is calibrated following IMF (2011, 2018), with two possible values capturing different degrees of fiscal effort.

1.6.3 Conditional estimates of \mathbf{h} .

We provide conditional estimates of the debt recovery parameter \mathbf{h} for both groups of countries. For this purpose, we examine the relation between a country i 's actual sovereign yield spread in year t , denoted $s_{i,t}$, and its theoretical spread in that same year. Because this theoretical spread is conditional to the assumption concerning the maximum primary surplus, our estimates are conditional to this assumption. Nevertheless, we will see in the next sub-section that the fiscal spaces that we can compute are much less sensitive than the estimates of \mathbf{h} to this assumption.

We compute actual spread as the difference between the country's long-term real interest rate and the German long-term real rate, $s_{i,t} = r_{i,t} - r_{G,t}$. The

²⁶Although the length of one period in the model is 4 years, calibration results reported in the next section are on annual basis. Parameter values in Tables 1.1 are also on annual basis. To obtain parameter values on a 4-year period basis, we follow Collard, Habib and Rochet (2015). The authors assume that the GDP observed in each year corresponds to one fourth of the next (rolling) 4 years. The corresponding growth rate, the mean and the volatility of this growth rate are then computed.

theoretical spread is defined as

$$S(b_{i,t}; \mu_i, \sigma_i, \hat{s}, \mathbf{h}) \equiv \frac{1}{q_{i,t}} - \frac{1}{\beta},$$

where $q_{i,t}$ is the country's bond price, defined in equation (1.18). The term $1/q_{i,t}$ is the gross interest rate on government bonds and $1/\beta$ is the gross risk-free rate, common to all countries. $b_{i,t}$ is the debt-to-GDP ratio observed for country i at date t . μ_i and σ_i are the mean and volatility of the growth rate, respectively, and are calibrated to their sample values at country level. \hat{s} is the maximum primary surplus, which is calibrated according to IMF (2011, 2018).

We estimate the recovery parameter \mathbf{h} by nonlinear least squares, minimizing the sum of squared deviations of theoretical yield spreads from actual spreads.²⁷ That is, our estimated parameter, denoted $\hat{\mathbf{h}}$, solves:

$$\min_{\mathbf{h}} \sum_t \sum_i [S(b_{i,t}; \mu_i, \sigma_i, \hat{s}, \mathbf{h}) - s_{i,t}]^2. \quad (1.32)$$

We estimate equation (1.32) for both country groups separately. Notice that the dataset for each country group is an unbalanced panel because sovereign yields and debt-to-GDP ratios are not available for all countries over the time period considered, 1980-2018.

Table 1.2 reports the obtained values for $\hat{\mathbf{h}}$ for each group of countries, considering the two different values for the primary surplus used in Table 1.1. The last column of Table 1.2 shows the mean absolute deviation of theoretical spreads from actual spreads in percentage point. Overall, the average deviations are small.

We obtain a higher $\hat{\mathbf{h}}$ for advanced countries than for emerging countries, assuming either a high primary surplus or a low one. Notice that the estimated values for both country groups are positive and well above zero, suggesting that *ex-ante* lenders do expect a partial debt recovery should a sovereign default actually occur. This finding is in line with historical estimates of post-default debt haircuts documented in the empirical studies mentioned in Section 1.2. Notice that, in the

²⁷ Arellano and Ramanarayanan (2012) use a similar method to estimate the parameters of the yield curve of long-term Government debt for four emerging countries.

Table 1.2: Debt recovery parameter estimation (Nonlinear Least Squares Method).

	\hat{s}	$\hat{\mathbf{h}}$	Mean absolute yield spread error (%) ^a
Advanced economies	0.05	0.88	0.47
	0.04	0.93	0.46
Emerging economies	0.03	0.42	0.40
	0.02	0.70	0.48

Notes: \hat{s} is calibrated following IMF(2011; 2018). $\hat{\mathbf{h}}$ is the value of \mathbf{h} that solves (1.32).

a: Average (absolute) difference between theoretical spreads and actual spreads when $\mathbf{h} = \hat{\mathbf{h}}$.

case of the group of emerging countries, the estimated value of \mathbf{h} is sensitive to the calibration of the maximum primary surplus \hat{s} : $\hat{\mathbf{h}}$ is equal to 0.70 or 0.42 for \hat{s} equal to 4% or 3% of GDP, respectively. An alternative strategy could be to fix the value of \mathbf{h} and estimate the maximum primary surplus but the same type of sensitivity of the obtained estimates would probably be found.

1.6.4 Sustainability and the debt recovery rule.

Section 1.5.3 introduced a more precise measure of the debt limit than the one proposed by Ghosh et al. (2013). We showed that this measure depends crucially on the debt recovery rule.

In this section, we further illustrate the role of the debt recovery parameter \mathbf{h} by computing debt limits for the Advanced and Emerging groups of countries in our dataset. More precisely, for each country i , we calibrate the mean μ_i and volatility σ_i of log growth rate of GDP to their historical values while setting the risk-free rate r and the primary surplus \hat{s} to their baseline values defined in Table 1.1. We solve the model numerically and compute the debt limit for four different values of \mathbf{h} : the case of no debt recovery $\mathbf{h} = 0$ (haircut of 100%), the case of maximum debt recovery $\mathbf{h} = 1$ (haircut of 0%), an intermediate case: $\mathbf{h} = 0.5$

and the conditional estimated values of $\mathbf{h} = \hat{\mathbf{h}}$ (for each group of countries).

Tables 1.3 and 1.4 present the results of this exercise for advanced and emerging countries, respectively. For comparison, we also report the debt-to-GDP ratio of each country in 2018, the last year in our dataset.

First, consider the group of advanced countries. Assume, as in Collard, Habib and Rochet (2015), zero debt recovery by creditors in case of a sovereign default (that is $\mathbf{h} = 0$) and a primary surplus of 5%. This case corresponds to Column 2 of Table 1.3. Under this assumption, Greece has the lowest debt limit at 82% of GDP, followed by Czech Republic with 98%, and by Latvia with 106%. On the other hand, Singapore presents the highest debt limit at 512% of GDP, followed by Korea and Israel with a debt limit of 395% and 311%, respectively.²⁸

Moving from $\mathbf{h} = 0$ to $\mathbf{h} = 0.5$, again setting $\hat{s} = 5\%$, the debt limit increases by 118% in Singapore, followed by Korea with an increase of 77%. At the same time, the debt limits for Greece, the Czech Republic and Latvia increase only by 11%, 15%, and 28%, respectively. Assuming a maximum debt recovery parameter ($\mathbf{h} = 1$), one would conclude that default is not a concern for any advanced country (including Greece), given their fairly large debt limits. Ten countries are even characterized by an infinite debt limit.²⁹ A similar pattern occurs when we set $\hat{s} = 4\%$.

When \mathbf{h} is equal to its estimated value 0.88, debt limits in advanced countries are reduced by 58% on average with respect to the case $\mathbf{h} = 1$. Two countries only, Singapore and Korea, benefit from an infinite debt limit.

Comparing the debt limit of each country to its debt-to-GDP ratio in 2018 (Table 1.3 Column 1) so as to have a measure of its fiscal space at this date, Greece and Japan are associated with a negative fiscal space when $\mathbf{h} = 0.88$ whereas they benefit from a positive fiscal space when $\mathbf{h} = 1$. This again illustrates the

²⁸The large variation of the debt limit across countries reflects differences in their economic fundamentals, in particular the mean growth rate which is positively related to the debt limit. For instance, over the period 1980-2018, Greece presents an annual growth rate of 0.7% on average while that of Singapore is 8 times larger (6.21% on average). See Table 1.7 in Appendix 1.8.2 for the mean growth of the list of advanced countries.

²⁹For the considered countries, the growth rate is higher than the risk-free interest rate and the solvency ratio is infinite.

Table 1.3: Debt limit, $b_{\mathbf{h}}^{\text{lim}} = \min(b_{\mathbf{h}}^{\text{max}}, b_{\mathbf{h}}^{\text{r}ss})$: Advanced countries.

Country	b_{2018}	$b_{\mathbf{h}}^{\text{lim}} (\hat{s} = 5\%, \hat{\mathbf{h}} = 0.88)$				$b_{\mathbf{h}}^{\text{lim}} (\hat{s} = 4\%, \hat{\mathbf{h}} = 0.93)$			
	(1)	$\mathbf{h} = 0$	$\mathbf{h} = 0.5$	$\mathbf{h} = \hat{\mathbf{h}}_{5\%}$	$\mathbf{h} = 1$	$\mathbf{h} = 0$	$\mathbf{h} = 0.5$	$\mathbf{h} = \hat{\mathbf{h}}_{4\%}$	$\mathbf{h} = 1$
Australia	41.37	306.44	346.56	507.50	∞	245.16	277.25	511.69	∞
Austria	73.75	185.57	200.53	249.44	577.16	148.45	160.42	224.03	461.73
Belgium	102.03	185.10	199.32	245.07	537.45	148.08	159.45	218.54	429.96
Canada	89.94	201.84	227.03	326.07	3122.66	161.48	181.62	323.18	2498.13
Czech Republic	32.56	98.68	113.76	180.35	586.83	78.95	91.01	184.01	469.46
Denmark	34.26	149.12	162.43	207.72	467.46	119.30	129.94	189.80	373.97
Finland	59.26	120.47	136.87	204.73	726.04	96.37	109.49	206.70	580.83
France	98.39	182.98	195.96	236.76	479.41	146.38	156.77	208.92	383.53
Germany	61.69	145.64	158.53	202.30	444.51	116.51	126.82	184.59	355.61
Greece	184.85	82.59	92.31	129.60	246.50	66.07	73.85	122.72	197.20
Hong Kong	0.05	213.35	277.95	995.97	∞	170.68	222.36	∞	∞
Iceland	37.62	150.98	182.13	360.79	∞	120.79	145.70	470.51	∞
Ireland	63.65	164.93	220.18	993.52	∞	131.94	176.15	∞	∞
Israel	60.78	311.72	365.72	630.71	∞	249.37	292.57	741.55	∞
Italy	132.16	129.62	139.51	171.55	313.84	103.69	111.60	153.00	251.07
Japan	237.13	139.95	154.47	207.00	533.35	111.96	123.58	194.91	426.68
Korea	37.92	395.95	702.17	∞	∞	316.76	561.73	∞	∞
Latvia	35.93	106.03	136.17	378.67	∞	84.83	108.93	709.15	∞
Lithuania	34.17	122.43	157.27	456.02	∞	97.94	125.81	1016.01	∞
Luxembourg	21.43	196.29	239.12	504.50	∞	157.03	191.29	748.89	∞
Netherlands	52.39	169.28	185.25	240.59	638.26	135.43	148.20	222.06	510.61
New Zealand	29.84	201.30	224.25	310.42	1675.38	161.04	179.40	299.41	1340.31
Norway	39.97	200.49	221.14	295.08	1116.05	160.39	176.91	277.39	892.84
Portugal	120.13	128.81	143.74	200.58	562.79	103.05	114.99	193.99	450.23
Singapore	113.63	512.60	1119.94	∞	∞	410.08	895.95	∞	∞
Slovak Republic	48.94	201.50	247.04	542.47	∞	161.20	197.63	860.74	∞
Spain	97.09	159.15	176.87	242.94	817.46	127.32	141.50	232.76	653.97
Sweden	38.46	156.85	173.46	233.95	697.75	125.48	138.77	221.35	558.20
Switzerland	40.53	168.85	182.21	225.71	484.54	135.08	145.77	202.20	387.63
United Kingdom	86.82	165.31	181.88	240.78	688.51	132.25	145.50	225.03	550.81
United States	104.26	203.15	226.34	313.46	1737.47	162.52	181.07	302.45	1389.98
Sample average	71.32	188.93	241.62	346.01	822.67	151.15	193.29	349.84	658.14

Notes. ∞ : Cases where $b_{\mathbf{h}}^{\text{max}} = \infty$ and no positive value exists for $b_{\mathbf{h}}^{\text{r}ss}$. $\hat{\mathbf{h}}$ is the estimated value for \mathbf{h} . For each country, the mean μ and volatility σ of the growth rate are calibrated to their historical values. The other parameters (r and \hat{s}) are set to their baseline values in Table 1.1.

Table 1.4: Debt limit, $b_{\mathbf{h}}^{\text{lim}} = \min(b_{\mathbf{h}}^{\text{max}}, b_{\mathbf{h}}^{\text{rss}})$: Emerging countries.

Country	b_{2018}	$b_{\mathbf{h}}^{\text{lim}} (\hat{s} = 3\%, \hat{\mathbf{h}} = 0.42)$				$b_{\mathbf{h}}^{\text{lim}} (\hat{s} = 2\%, \hat{\mathbf{h}} = 0.70)$			
	(1)	$\mathbf{h} = 0$	$\mathbf{h} = 0.5$	$\mathbf{h} = \hat{\mathbf{h}}_{3\%}$	$\mathbf{h} = 1$	$\mathbf{h} = 0$	$\mathbf{h} = 0.5$	$\mathbf{h} = \hat{\mathbf{h}}_{2\%}$	$\mathbf{h} = 1$
Brazil	87.89	72.58	83.38	80.71	621.49	48.39	55.59	63.19	414.33
Chile	25.56	101.46	128.80	121.52	∞	67.64	85.87	109.55	∞
China	50.64	∞	∞	∞	∞	∞	∞	∞	∞
Colombia	52.16	153.08	178.99	172.49	∞	102.05	119.32	138.15	∞
Hungary	70.85	77.45	87.27	84.87	423.49	51.63	58.18	64.83	282.32
Malaysia	55.57	240.04	396.57	346.40	∞	160.03	264.38	553.09	∞
Mexico	53.62	75.47	86.89	84.05	793.18	50.31	57.92	65.99	528.78
Nigeria	27.26	55.64	68.58	65.21	∞	37.09	45.72	56.15	∞
Pakistan	71.69	508.61	873.99	753.70	∞	339.08	582.66	1376.64	∞
Philippines	38.92	105.01	128.16	122.16	∞	70.01	85.44	103.87	∞
Poland	48.89	130.11	156.41	149.68	∞	86.74	104.27	124.55	∞
Russia	14.61	33.59	40.15	38.48	137.64	22.39	26.77	31.72	91.76
South Africa	56.71	92.53	103.00	100.47	475.46	61.69	68.67	75.57	316.97
Sample average	50.34	137.13	194.35	176.65	490.25	91.42	129.57	230.28	326.83

Notes. ∞ : Cases where $b_{\mathbf{h}}^{\text{max}} = \infty$ and no positive value exists for $b_{\mathbf{h}}^{\text{rss}}$. $\hat{\mathbf{h}}$ is the estimated value for \mathbf{h} . For each country, the mean μ and volatility σ of the growth rate are calibrated to their historical values. The other parameters (r and \hat{s}) are set to their baseline values in Table 1.1.

sensitivity of the assessment of public debt sustainability to the debt recovery parameter and the need to improve the estimation of this rate. In the case of Japan, which have not defaulted and doesn't appear on the verge of default, this may be due to a value of \mathbf{h} included in the market risk premium higher than 0.88. In the case of Greece, its negative fiscal space may suggest that its recent default is not completely resolved.

Turning to emerging countries³⁰ (Table 1.4), we observe a pattern similar to advanced countries. Setting $\hat{s} = 3\%$, the debt limit increases on average from 137% when $\mathbf{h} = 0$ to 490% when $\mathbf{h} = 1$. Under the latter assumption, one would conclude that default is not an issue for any emerging country, even if we take into account the debt-to-GDP ratios in 2018 (Column 1) to have a measure of

³⁰See Table 1.8 in Appendix 1.8.2 for the mean growth of the list of emerging countries.

fiscal space.

Comparing the two country groups, although emerging countries have relatively low debt-to-GDP ratios (50% on average) than advanced countries (71% on average), they also have overall lower fiscal spaces. [Reinhart, Rogoff and Savastano \(2003\)](#) refer to this phenomenon as “debt intolerance”, highlighting the fact that developing countries default at relatively low debt levels than what is conventionally considered as prudent. Here we exhibit the link between the unsustainability of public debt and the debt recovery parameter \mathbf{h} , and show that this parameter varies across (groups of) countries, possibly explaining debt intolerance.

Finally, we note that despite the difference between the two conditional estimations of \mathbf{h} , especially for the group of emerging countries, the two computed values for the debt limit are sufficiently close to provide a fairly good approximation or, at least, a reasonable range for this financial sustainability indicator. [Figure 1.7](#) allows us to compare the two evaluations of the debt limit for advanced countries³¹ according to the case $\mathbf{h} = .88$ and $\hat{s} = 5\%$, or $\mathbf{h} = .93$ and $\hat{s} = 4\%$.

This simple numerical/empirical exercise shows how the assessment of public debt sustainability depends crucially on the assumption that one makes about the debt recovery parameter. While the assumption of zero debt recovery parameter ($\mathbf{h} = 0$) prevalent in previous studies appears at odds with historical evidence, assuming a maximum debt recovery by creditors in case of a sovereign default ($\mathbf{h} = 1$) is not realistic and may overestimate a country’s fiscal space

1.6.5 Sovereign default and debt sustainability when $r < g$.

In his presidential lecture to the American Economic Association, [Blanchard \(2019\)](#) argued that “public debt may have no fiscal cost” if interest rates remain below the rate of growth. With close to zero interest rates, governments

³¹Only countries with computed debt limits below 350% are included here. The difference between the two evaluation is greater for countries with a computed debt limit above 400% but the risks associated with these cases are negligible.

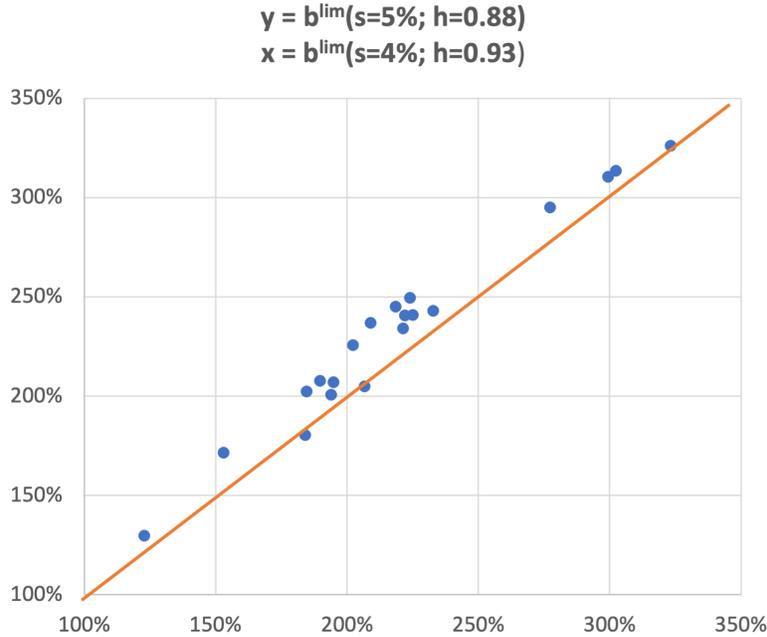


Figure 1.7: Computation of the debt limits of advanced countries for $\hat{s} = 4\%$ and $\hat{s} = 5\%$.

can potentially borrow and roll over their debts despite the existence of an upper bound on future primary surplus. In a more recent contribution with some co-authors [Blanchard, Leandro and Zettelmeyer \(2021\)](#) is more cautious and recognizes the potentially important role of default risk in assessing the sustainability of public debt. In the same vein, [Sergeyev and Mehrotra \(2020\)](#) and [Mauro and Zhou \(2020\)](#) suggest that negative $r - g$ differentials³² are quite common over the past 200 years. But both papers also point to the large uncertainty over future interest rates and shocks, including the possibility of abrupt bond yield reversals and subsequent defaults.

In this sub-section, we relax the condition $\bar{a} < \beta^{-1}$ of Assumption 1 in order to reassess the question of the sustainability of public debt when the risk-free interest rate is lower than the growth rate. To illustrate this point empirically, we

³²Here g refers to the (net) real growth rate of GDP, and r is the real risk-free rate as before.

focus on the situation of the Eurozone countries³³ in recent years, just before the Covid-19 outbreak. More specifically, we consider the average growth rate for each country and the 4-Year German (risk-free) bond rate for the period 2009-2018. We check that the average growth rate is higher than the risk-free interest rate for each country by evaluating the terms $\beta\bar{a}_i$.³⁴ Results are reported in Table 1.9 in Appendix 1.8.2. Except for Greece and Italy, for which $\beta\bar{a}$ is equal to 0.97 and to 0.99, respectively, this term is higher than 1 and the solvency ratio is infinite for all other countries over the considered period. Nevertheless, the default ratio, given by $\omega_{\mathbf{h}} = \frac{\hat{s}}{1-\beta x_{\mathbf{h}}}$, can take a positive and finite value as long as $x_{\mathbf{h}}$ verifies $x_{\mathbf{h}} < \beta^{-1}$. In the same Table 1.9, we compute for each country the critical value of \mathbf{h} , denoted $\tilde{\mathbf{h}}_i$, which verifies $x_{\tilde{\mathbf{h}}_i} = \beta^{-1}$. These critical values are reported on Figure 1.8.

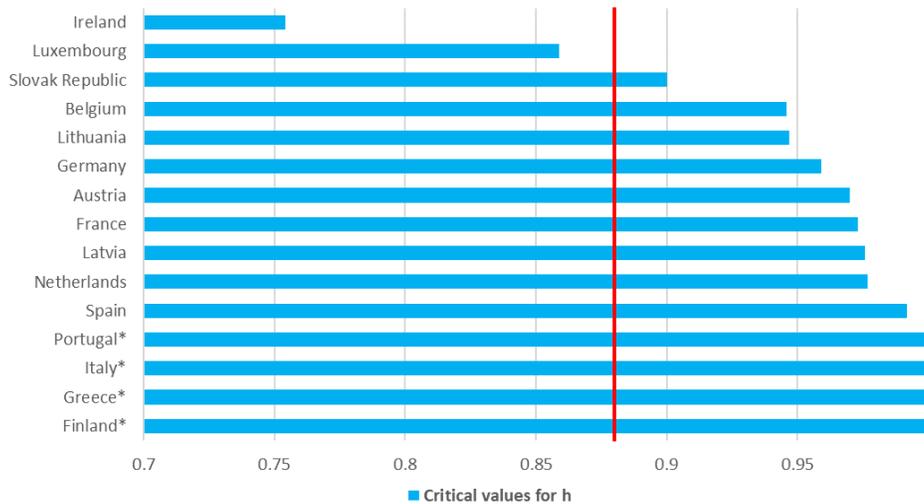


Figure 1.8: Sustainability and low interest interest rate: Euro zone ($r = 0.31\%^a$)

For countries with a value $\tilde{\mathbf{h}}_i$ higher than their own maximum recovery pa-

³³We consider only the 15 countries of the euro zone present in our database of advanced countries which excludes Cyprus, Estonia, Malta and Slovenia for data limitations for these countries.

³⁴Here, the value for β is given by $(1 + r_G)^{-1}$ where $r_G = 0.31\%$ is the annualized German interest rate for the period (2009-2018).

parameter \mathbf{h}_i , there is a positive and finite default ratio. Using the value of the maximum recovery parameter estimated (over the longer period 1980-2018) for the group of advanced countries, $\hat{\mathbf{h}} = 0.88$, as the best proxy³⁵ for any \mathbf{h}_i , and represented by the vertical (red) line on Figure 1.8, it cannot be excluded that 87% of the countries considered, including Germany, admit a finite default ratio despite the non-existence of a finite solvency ratio.

Table 1.5: Debt limit, $b_{\mathbf{h}}^{\text{lim}} = \min(b_{\mathbf{h}}^{\text{max}}, b_{\mathbf{h}}^{\text{r}ss})$, and low interest rates: Euro zone.

Country	Debt-to-GDP (b_t)		$b_{\mathbf{h}}^{\text{lim}} (\hat{s} = 5\%, \mathbf{h} = 0.88)$		
	2018	2022 [†]	$r = 0.31\%$	$r = 1\%$	$r = 1.5\%$
	(1)	(2)	(3)	(4)	(5)
Austria	73.75	85.74	624.01	344.27	261.72
Belgium	102.03	116.23	1751.15	529.98	355.54
Finland	59.26	69.15	235.24	181.46	156.32
France	98.39	114.27	760.44	380.68	281.70
Germany	61.69	67.28	554.10	324.15	250.89
Greece	184.85	200.49	89.78	80.79	75.48
Ireland	63.65	63.16	∞	∞	∞
Italy	132.16	155.51	212.85	166.94	144.95
Latvia	35.93	45.33	249.12	191.90	165.20
Lithuania	34.17	47.67	383.00	262.59	215.03
Luxembourg	21.43	27.30	∞	983.17	521.71
Netherlands	52.39	56.10	517.42	309.25	241.02
Portugal	120.13	125.55	272.72	201.92	170.74
Slovak Republic	48.94	64.29	3094.43	626.79	401.51
Spain	97.09	117.32	301.11	217.16	181.54
Mean	79.06	90.36	695.80	342.93	244.53

Notes. [†]: Projections from IMF World Economic Outlook (April 2021). ∞ : Cases where $b_{\mathbf{h}}^{\text{max}} = \infty$ and no positive value exists for $b_{\mathbf{h}}^{\text{r}ss}$. For each country, the mean μ and volatility σ of the growth rate are calibrated to their historical values in 2009-2018.

Nevertheless, as Table 1.5 shows, the values calculated for the debt limit (in

³⁵The considered time period (2009-2018) and the small sample size of countries do not allow for a new statistically estimate of the parameter \mathbf{h} .

terms of GDP) with a risk-free interest rate of 0.31% (the German rate over the period 2009-2018) are rather high, with the exception of Greece whose debt limit is then 89% of GDP. Columns (4) and (5) give the values of these debt limits for risk-free interest rates of 1% and 1.5% respectively. In the latter case, Italy is also in a particularly dangerous situation, giving reason to [Sergeyev and Mehrotra \(2020\)](#) and [Mauro and Zhou \(2020\)](#) warning about the danger that a rise in risk-free rates would represent in the years to come.

1.7 Conclusion.

We have developed a tractable stochastic model of sovereign default allowing us to highlight the macroeconomic impact of the debt recovery channel, roughly speaking the impact of “haircuts” being applied to the due debt owed by a defaulting country on the whole dynamics of public debt, and in particular on its sustainability and the prospects of defaults. We use a simple specification of such a rule, consistent with empirical evidence, which depends on a single parameter \mathbf{h} , the (maximum) debt recovery rate. We show that the default ratio, namely the maximum debt-to-GDP ratio that a country can reach without defaulting, depends on this debt recovery parameter. It differs from the *solvency ratio* which corresponds to the transversality condition obtained when the possibility of default is neglected. The two quantities are equal only under the extreme, non realistic assumption of debt recovery parameter equal to one.

We provide a new definition of debt unsustainability and a new measure of fiscal space. We show that the assessment of the unsustainability of public debt depends crucially on the debt recovery rule that is applied following a sovereign default. This finding provides some insights on the current debate on the sustainability of public debt in the context of low real interest rates.

Our findings are consistent with the paradox of “debt intolerance”: compared with advanced countries, emerging countries experience both lower default ratios, that is a lower debt tolerance by markets, and higher risk premia.

We illustrate these findings by means of several empirical analyses based on a

dataset covering advanced and emerging countries. First we provide some (admittedly rough given the paucity of data) evaluations of the debt recovery parameter. It appears that its magnitude is higher for advanced countries than for emerging ones. Second we assess the extent of fiscal spaces for the various countries of the dataset. Fiscal spaces for advanced economies are fairly large. Greece and Italy (to a lesser extent and in the event of a future increase in the risk-free interest rate) are notable exceptions. The estimated values of the fiscal spaces for emerging countries are much lower. The sensitivity of the estimated fiscal spaces to the debt recovery parameter shows clearly that it plays a major role in the assessment of the financial position of a country. These analyses illustrate the necessity to take into account the debt recovery channel when studying public debts and their dynamics.

A final word of caution: h itself is not a “deep” parameter. It is the result of a negotiation between lenders and borrowers who carefully take into account the capacity of a defaulting country to correct its fiscal stance, its default record, its expected growth process. Our results show that it is an important factor for understanding various phenomena linked to sovereign default and the capacity to issue sovereign debt. It implies that a better understanding of the debt recovery rules being applied to defaulting countries and their determinants is needed. This is beyond the scope of this paper.

1.8 Appendix.

1.8.1 Proof of Propositions.

1.8.1.1 Proof of Proposition 1

Proposition 1: Given $\omega_{t+1}^{\text{def}}$, under Assumption 1, the market value of debt v_t reaches a unique maximum v_t^{max} for a quantity of debt $b_t = b_t^{\text{max}}$. Both v_t^{max} and b_t^{max} are linearly increasing in $\omega_{t+1}^{\text{def}}$: $v_t^{\text{max}} = \beta x_{\mathbf{h}} \omega_{t+1}^{\text{def}}$ and $b_t^{\text{max}} = \delta_{\mathbf{h}} \omega_{t+1}^{\text{def}}$ where $\delta_{\mathbf{h}}$ is such that

$$[1 - G(\delta_{\mathbf{h}})] [1 - (1 - \mathbf{h}) \delta_{\mathbf{h}} z(\delta_{\mathbf{h}})] = 0,$$

where $z(\delta) = \frac{g(\delta)}{1-G(\delta)}$ is the hazard function, and $x_{\mathbf{h}}$ is given by

$$x_{\mathbf{h}} = [1 - G(\delta_{\mathbf{h}})] \delta_{\mathbf{h}} + \mathbf{h} \int^{\delta_{\mathbf{h}}} a dG(a).$$

These two coefficients are increasing functions of \mathbf{h} , with $0 < x_{\mathbf{h}} \leq \bar{a}$ and $0 < \delta_{\mathbf{h}} \leq +\infty$ for $0 \leq \mathbf{h} \leq 1$.

Proof. By denoting $\delta_t = b_t / \omega_{t+1}^{\text{def}}$, from (1.19) we can rewrite v_t as:

$$v_t = \beta \chi(\delta_t, \mathbf{h}) \omega_{t+1}^{\text{def}}, \quad (1.33)$$

where $\chi(\delta, \mathbf{h})$ is a non-monotonic function defined by:

$$\chi(\delta, \mathbf{h}) \equiv [1 - G(\delta)] \delta + \mathbf{h} \int^{\delta} a \cdot dG(a). \quad (1.34)$$

Let us define $\Phi(\delta, \mathbf{h}) \equiv \partial \chi(\delta, \mathbf{h}) / \partial \delta$, the derivative of $\chi(\delta, \mathbf{h})$ with respect to δ , we get:

$$\Phi(\delta, \mathbf{h}) = [1 - G(\delta)] [1 - (1 - \mathbf{h}) \delta z(\delta)], \quad (1.35)$$

where the function $z(\delta)$ is the hazard function:

$$z(\delta) \equiv \frac{g(\delta)}{1 - G(\delta)}.$$

Assuming that there exists a positive value $\delta_{\mathbf{h}}$ such that:

$$\Phi(\delta_{\mathbf{h}}, \mathbf{h}) = 0, \quad (1.36)$$

we can then define

$$x_{\mathbf{h}} \equiv \chi(\delta_{\mathbf{h}}, \mathbf{h}). \quad (1.37)$$

By denoting $\Phi_z(\delta, \mathbf{h}) \equiv \partial\Phi(\delta, \mathbf{h})/\partial z$, the partial derivatives of $\Phi(\delta, \mathbf{h})$ for $z = \delta, \mathbf{h}$, we get, for any $\mathbf{h} \in [0, 1)$:

$$\Phi_{\mathbf{h}}(\delta_{\mathbf{h}}, \mathbf{h}) = \delta_{\mathbf{h}} g(\delta_{\mathbf{h}}) > 0, \quad (1.38)$$

$$\Phi_{\delta}(\delta_{\mathbf{h}}, \mathbf{h}) = -[1 - G(\delta_{\mathbf{h}})](1 - \mathbf{h})[z(\delta_{\mathbf{h}}) + \delta_{\mathbf{h}} z'(\delta_{\mathbf{h}})] < 0, \quad (1.39)$$

where the last inequality is implied by Assumption 1. Hence, from (1.33), (1.35), (1.36) and (1.39), $v_t^{\max} = \beta\chi(\delta_{\mathbf{h}}, \mathbf{h})\omega_{t+1}^{\text{def}} = \beta x_{\mathbf{h}}\omega_{t+1}^{\text{def}}$ is a maximum reached for $b_t^{\max} = \delta_{\mathbf{h}}\omega_{t+1}^{\text{def}}$. From the definition of $\delta_{\mathbf{h}}$, implicitly given by (1.35) and (1.36), and using (1.34), (1.38) and (1.39), we find that:

$$\frac{\partial\delta_{\mathbf{h}}}{\partial\mathbf{h}} = -\frac{\Phi_{\mathbf{h}}(\delta_{\mathbf{h}}, \mathbf{h})}{\Phi_{\delta}(\delta_{\mathbf{h}}, \mathbf{h})} > 0. \quad (1.40)$$

$$\frac{\partial\chi_{\mathbf{h}}}{\partial\mathbf{h}} = \frac{\partial\chi(\delta, \mathbf{h})}{\partial\mathbf{h}} \Big|_{\delta=\delta_{\mathbf{h}}} = \int^{\delta_{\mathbf{h}}} a \cdot dG(a) > 0. \quad (1.41)$$

Furthermore, from (1.34) we compute:

$$x_{\mathbf{0}} = \chi(\delta_{\mathbf{0}}, \mathbf{0}) = [1 - G(\delta_{\mathbf{0}})]\delta_{\mathbf{0}}$$

where, from (1.35) and (1.36), δ_0 is such that:

$$\delta_0 z(\delta_0) = 1.$$

From the same equations (1.34), (1.35), and (1.36), where $\Phi(\delta, \mathbf{h})$ is given by we get $\delta_1 = +\infty$ and

$$x_1 = \chi(\delta_1, \mathbf{1}) = \int a \cdot dG(a) = \bar{a},$$

which ends the proof of Proposition 1. \square

1.8.1.2 Proof of Proposition 2

Proposition 2: *The equilibrium default ratio ω_t^{\max} is locally unique and equal to:*

$$\omega_t^{\text{def}} = \frac{\hat{s}}{1 - \beta x_{\mathbf{h}}} \equiv \omega_{\mathbf{h}}, \forall t. \quad (1.42)$$

$\omega_{\mathbf{h}}$ is a strictly increasing function of \hat{s} and \mathbf{h} , with $\omega_{\mathbf{h}} \leq \omega^{\text{sup}}$ for $\mathbf{h} \leq 1$.

Proof. Using (1.25)

$$\omega_t^{\text{def}} = \beta x_{\mathbf{h}} \omega_{t+1}^{\text{def}} + \hat{s}, \quad (1.43)$$

we obtain the stationary value for ω_t^{def} that we denote $\omega_{\mathbf{h}}$. It is given by:

$$\omega_{\mathbf{h}} = \frac{\hat{s}}{1 - \beta x_{\mathbf{h}}}. \quad (1.44)$$

From Proposition 1, we know that $x_{\mathbf{h}}$ is an increasing function of \mathbf{h} with a maximum $x_{\mathbf{h}} = \bar{a}$ for $\mathbf{h} = 1$. It immediately follows that $\omega_{\mathbf{h}}$ is a growing function of \mathbf{h} with a maximum

$$\omega_1 = \frac{\hat{s}}{1 - \beta \bar{a}} \equiv \omega^{\text{sup}}.$$

From Assumption 1, we have $\bar{a} < 1 + r$ with $1 + r = \beta^{-1}$ and hence, from Proposition 1, $\beta x_{\mathbf{h}} \leq \beta x_1 = \beta \bar{a} < 1$. This implies that, by rewriting the dynamics of equation (1.43) in a more conventional backward-looking form, it is unstable around the unique stationary equilibrium, $\omega_{\mathbf{h}}$. Since ω_t^{def} is not predetermined, $\omega_{\mathbf{h}}$ is a determinate, *i.e.* locally unique, equilibrium. \square

1.8.1.3 Proof of Proposition 3

Proposition 3: *The market value of public debt is a strictly increasing function of the debt recovery parameter \mathbf{h} .*

Proof. Using equation (1.33) the market value of public debt (1.29) can be rewritten

$$v(b_t; \mathbf{h}) = \beta \chi \left(\frac{b_t}{\omega_{\mathbf{h}}}, \mathbf{h} \right) \omega_{\mathbf{h}}, \quad (1.45)$$

where $\chi(\delta_t, \mathbf{h})$ is given by (1.34). We compute

$$\frac{\partial v(b_t; \mathbf{h})}{\partial \mathbf{h}} = \beta \omega_{\mathbf{h}} \int^{\delta_{\mathbf{h}}} a \cdot dG(a) + \beta \left[\chi \left(\frac{b_t}{\omega_{\mathbf{h}}}, \mathbf{h} \right) - \frac{b_t}{\omega_{\mathbf{h}}} \Phi \left(\frac{b_t}{\omega_{\mathbf{h}}}, \mathbf{h} \right) \right] \frac{\partial \omega_{\mathbf{h}}}{\partial \mathbf{h}},$$

where $\Phi \left(\frac{b_t}{\omega_{\mathbf{h}}}, \mathbf{h} \right)$ given by (1.35) is the derivative of $\chi(\delta, \mathbf{h})$ with respect to δ . Since $\chi \left(\frac{b_t}{\omega_{\mathbf{h}}}, \mathbf{h} \right)$ is strictly concave, with $\chi(0, \mathbf{h}) = 0$, the term in square brackets is strictly positive, as is $\frac{\partial \omega_{\mathbf{h}}}{\partial \mathbf{h}}$ from Proposition 2, which makes it possible to conclude that $\frac{\partial v(b_t; \mathbf{h})}{\partial \mathbf{h}} > 0 \forall b_t$. \square

1.8.1.4 Proof of Proposition 4

Proposition 4: *In the constrained fiscal regime,*

1. *there exists a unique risky-steady-state-debt ratio, $b_{\mathbf{h}}^{rss} = b_{\mathbf{h}}^*$, satisfying (1.31) and $b_{\mathbf{h}}^{rss} \leq \bar{a} \omega_{\mathbf{h}} \leq b_{\mathbf{h}}^{max}$, if and only if $\mathbf{h} \geq \underline{\mathbf{h}} = 1 - \frac{1}{\bar{a}z(\bar{a})}$, with strict equality for $\mathbf{h} = \underline{\mathbf{h}}$.*
2. *when $\mathbf{h} > \underline{\mathbf{h}}$, we have:*
 - (a) *$b_{\mathbf{h}}^{rss}$ is increasing in \mathbf{h} ,*
 - (b) *$b_{\mathbf{h}}^{max} - b_{\mathbf{h}}^{rss}$ is increasing in \mathbf{h} .*

Proof. 1. Recalling equation (1.31) for convenience:

$$v(b_{\mathbf{h}}^*; \mathbf{h}) = \frac{b_{\mathbf{h}}^*}{\bar{a}} - \hat{s}, \quad (1.46)$$

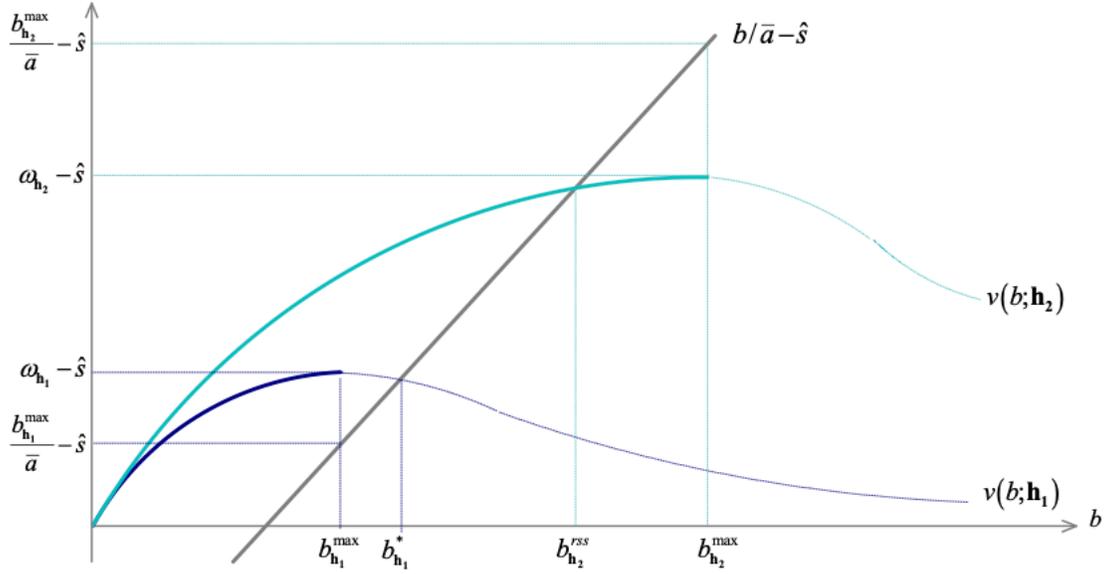


Figure 1.9: Existence of a RSS

a Risky Steady State (RSS) exists and is defined by: $b_{\mathbf{h}}^{\text{RSS}} = b_{\mathbf{h}}^*$, if and only if $b_{\mathbf{h}}^* < b_{\mathbf{h}}^{\text{max}}$, since $b_{\mathbf{h}}^{\text{max}}$ is the maximum level of debt that can be issued on the market. Figure 1.9 represents two curves $v(b; \mathbf{h}_1)$ and $v(b; \mathbf{h}_2)$ corresponding to two different recovery parameters \mathbf{h}_1 and \mathbf{h}_2 , and the line $b/\bar{a} - \hat{s}$. The figure is sufficient to prove the existence of a RSS for $\mathbf{h} = \mathbf{h}_2$, and its non-existence for $\mathbf{h} = \mathbf{h}_1$. In the first case, we observe that $b_{\mathbf{h}_1}^{\text{max}} < b_{\mathbf{h}_1}^*$ and $b_{\mathbf{h}_1}^{\text{max}} < \bar{a}\omega_{\mathbf{h}_1}$, and we also simply check that: $b_{\mathbf{h}_1}^*/\bar{a} - \hat{s} < \omega_{\mathbf{h}_1} - \hat{s}$. This can be summarized as follows:

$$b_{\mathbf{h}_1}^{\text{max}} < b_{\mathbf{h}_1}^* < \bar{a}\omega_{\mathbf{h}_1}.$$

In the other case, we obtain:

$$b_{\mathbf{h}_2}^{\text{RSS}} = b_{\mathbf{h}_2}^* < \bar{a}\omega_{\mathbf{h}_2} < b_{\mathbf{h}_2}^{\text{max}}.$$

It remains to be shown that $\mathbf{h}_1 < \mathbf{h}_2$ and that there exists $\underline{\mathbf{h}}$, verifying $\mathbf{h}_1 < \underline{\mathbf{h}} < \mathbf{h}_2$, and such that $b_{\underline{\mathbf{h}}}^* = \bar{a}\omega_{\underline{\mathbf{h}}} = b_{\underline{\mathbf{h}}}^{\text{max}}$. Note that, from Proposition 1, we can express the difference $b_{\mathbf{h}}^{\text{max}} - \bar{a}\omega_{\mathbf{h}}$ as:

$$b_{\mathbf{h}}^{\text{max}} - \bar{a}\omega_{\mathbf{h}} = (\delta_{\mathbf{h}} - \bar{a})\omega_{\mathbf{h}}. \quad (1.47)$$

This difference is positive for $\mathbf{h} = \mathbf{h}_2$, and negative for $\mathbf{h} = \mathbf{h}_1$. From Proposition 1, we know that $\delta_{\mathbf{h}}$ is an increasing function of \mathbf{h} , which is sufficient to conclude that $\mathbf{h}_1 < \mathbf{h}_2$. Furthermore, when $\delta_{\mathbf{h}} = \bar{a}$, we necessarily have: $b_{\mathbf{h}}^{\max} = b_{\mathbf{h}}^* = \bar{a}\omega_{\mathbf{h}}$, or equivalently $\delta_{\mathbf{h}} = \delta_{\mathbf{h}}^* = \bar{a}$, with $\delta_{\mathbf{h}}^* \equiv b_{\mathbf{h}}^*/\omega_{\mathbf{h}}$. Thus, there is a value $\underline{\mathbf{h}}$ such that $\delta_{\underline{\mathbf{h}}} = \delta_{\underline{\mathbf{h}}}^* = \bar{a}$. From (1.35) and (1.36), $\delta_{\mathbf{h}}$ is implicitly given by: $(1 - \mathbf{h})\delta_{\mathbf{h}}z(\delta_{\mathbf{h}}) = 1$, which implies, when $\delta_{\underline{\mathbf{h}}} = \delta_{\underline{\mathbf{h}}}^* = \bar{a}$:

$$\underline{\mathbf{h}} = 1 - \frac{1}{\bar{a}z(\bar{a})}.$$

A necessary and sufficient condition to have $0 < \underline{\mathbf{h}} < 1$ is therefore $\bar{a}z(\bar{a}) > 1$.

2. a. We now seek to show that, for $\mathbf{h} > \underline{\mathbf{h}}$, the RSS debt ratio, $b_{\mathbf{h}}^{rss} = b_{\mathbf{h}}^*$, is an increasing function of \mathbf{h} . By looking for the derivative $\frac{\partial b_{\mathbf{h}}^*}{\partial \mathbf{h}}$ from equation (1.46), one find:

$$\frac{\partial b_{\mathbf{h}}^*}{\partial \mathbf{h}} = \frac{\bar{a} \frac{\partial v(b_{\mathbf{h}}^*; \mathbf{h})}{\partial \mathbf{h}}}{1 - \bar{a} \frac{\partial v(b_{\mathbf{h}}^*; \mathbf{h})}{\partial b}}.$$

Remembering that $v(b_t; \mathbf{h}) = q_t b_t$ with $\frac{\partial q}{\partial b} < 0$, and $q_t < \beta$, we necessarily have $\frac{\partial v(b_{\mathbf{h}}^*; \mathbf{h})}{\partial b} = q_{\mathbf{h}}^* + \frac{\partial q}{\partial b} b_{\mathbf{h}}^* < \beta$ which implies $1 - \bar{a} \frac{\partial v(b_{\mathbf{h}}^*; \mathbf{h})}{\partial b} > 1 - \beta \bar{a} > 0$, where the last inequality comes from assumption 1. Using this result with $\frac{\partial v(b_{\mathbf{h}}^*; \mathbf{h})}{\partial \mathbf{h}} > 0$, from Proposition 3, we obtain $\frac{\partial b_{\mathbf{h}}^*}{\partial \mathbf{h}} > 0$.

2. b. Finally, we have to prove that $b_{\mathbf{h}}^{\max} - b_{\mathbf{h}}^{rss}$ is increasing in \mathbf{h} when $\mathbf{h} \geq \underline{\mathbf{h}}$. Note first that:

$$b_{\mathbf{h}}^{\max} - b_{\mathbf{h}}^{rss} = (b_{\mathbf{h}}^{\max} - \bar{a}\omega_{\mathbf{h}}) + (\bar{a}\omega_{\mathbf{h}} - b_{\mathbf{h}}^{rss}).$$

From (1.47), the first term of the right-hand side of this equality can be written:

$$b_{\mathbf{h}}^{\max} - \bar{a}\omega_{\mathbf{h}} = (\delta_{\mathbf{h}} - \bar{a})\omega_{\mathbf{h}},$$

where, $\delta_{\mathbf{h}}$ and $\omega_{\mathbf{h}}$ are both increasing in \mathbf{h} , from Propositions 1 and 2 and $\delta_{\mathbf{h}} - \bar{a} > 0$ when $\mathbf{h} \geq \underline{\mathbf{h}}$,

Next, we have to prove that $\bar{a}\omega_{\mathbf{h}} - b_{\mathbf{h}}^{rss}$ is increasing in \mathbf{h} . As $b_{\mathbf{h}}^* = \delta_{\mathbf{h}}^*\omega_{\mathbf{h}}$, and

knowing, from Proposition 2, that $\omega_{\mathbf{h}}$ is a strictly increasing function of \mathbf{h} , we only have to show that $\delta_{\mathbf{h}}^*$ is decreasing in \mathbf{h} for $\mathbf{h} > \underline{\mathbf{h}}$. Using again $\delta = b/\omega_{\mathbf{h}}$, (1.33) and (1.44), we can express the difference between the value function $v(b; \mathbf{h})$ and the refinancing needs, $\frac{b}{\bar{a}} - \hat{s}$, as follows:

$$v(b; \mathbf{h}) - \frac{b}{\bar{a}} - \hat{s} = \left(\frac{\hat{s}}{\bar{a}} \right) \frac{\varphi(\delta; \mathbf{h})}{1 - \beta x_{\mathbf{h}}}, \quad (1.48)$$

where the function $\varphi(\delta; \mathbf{h})$ is defined by

$$\varphi(\delta; \mathbf{h}) \equiv \bar{a} - \delta - \beta \bar{a} [x_{\mathbf{h}} - \chi(\delta, \mathbf{h})], \quad (1.49)$$

and verifies

$$\varphi(\delta_{\mathbf{h}}^*; \mathbf{h}) = 0. \quad (1.50)$$

Let us denote $\varphi_z(\delta, \mathbf{h}) \equiv \partial \varphi(\delta; \mathbf{h}) / \partial z$, the partial derivatives of $\varphi(\delta; \mathbf{h})$ for $z = \delta, \mathbf{h}$. Using again the notation $\Phi(\delta, \mathbf{h}) \equiv \partial \chi(\delta, \mathbf{h}) / \partial \delta$, introduced in Appendix 1.8.1.1, we obtain:

$$\varphi_{\delta}(\delta, \mathbf{h}) = -[1 - \beta \bar{a} \Phi(\delta, \mathbf{h})] < 0, \quad (1.51)$$

$$\varphi_{\mathbf{h}}(\delta, \mathbf{h}) = \beta \bar{a} \left[\int^{\delta} a \cdot dG(a) - \int^{\delta_{\mathbf{h}}} a \cdot dG(a) \right] \geq 0 \quad \text{iff} \quad \delta \geq \delta_{\mathbf{h}}. \quad (1.52)$$

The first derivative is negative since $\Phi(\delta, \mathbf{h})$, given by (1.35), is such that $\Phi(\delta, \mathbf{h}) \leq 1$ for $\delta \geq 0$, and $\beta \bar{a} < 1$ by assumption 1. The second one is negative (respect. positive) if $\delta > \delta_{\mathbf{h}}$ (respect. $\delta < \delta_{\mathbf{h}}$). From (1.50), (1.51), and (1.52) we then obtain:

$$\frac{\partial \delta_{\mathbf{h}}^*}{\partial \mathbf{h}} = -\frac{\varphi_{\mathbf{h}}(\delta_{\mathbf{h}}^*, \mathbf{h})}{\varphi_{\delta}(\delta_{\mathbf{h}}^*, \mathbf{h})} \leq 0 \quad \text{iff} \quad \delta_{\mathbf{h}}^* \leq \delta_{\mathbf{h}}, \quad \text{i.e. iff} \quad \mathbf{h} \geq \underline{\mathbf{h}}, \quad (1.53)$$

which ends the proof. \square

1.8.1.5 Proof of Proposition 5

Proposition 5: *In case of default, the post-default debt-to-GDP ratio $\mathbf{h}\omega_{\mathbf{h}}$ is unsustainable when $\mathbf{h} > \mathbf{H} > \underline{\mathbf{h}}$, where \mathbf{H} is implicitly defined by:*

$$\mathbf{H}\omega_{\mathbf{H}} = \frac{b_{\mathbf{H}}^{r,ss}}{\bar{a}}.$$

Proof. Let us introduce the function $\Delta(\mathbf{h})$ implicitly defined by the condition (1.50): $\varphi(\delta_{\mathbf{h}}^*; \mathbf{h}) = 0$, such that $\delta_{\mathbf{h}}^* = \Delta(\mathbf{h})$. From (1.53), we know that $\Delta'(\mathbf{h}) \leq 0$ for $\mathbf{h} \geq \underline{\mathbf{h}}$. Note that the condition $\mathbf{H}\omega_{\mathbf{H}} = \frac{b_{\mathbf{H}}^{r,ss}}{\bar{a}}$, or equivalently $\frac{\delta_{\mathbf{H}}^*}{\bar{a}} = \mathbf{H}$, is reached when we have $\Delta(\mathbf{h}) = \bar{a}\mathbf{h}$. We represent on Figure 1.10 the functions $\Delta(\mathbf{h})$ and $\bar{a}\mathbf{h}$.

The two functions intersect for $\mathbf{h} = \mathbf{H}$ which unambiguously satisfies: $\underline{\mathbf{h}} < \mathbf{H} < 1$. When $\mathbf{h} > \mathbf{H}$, $\delta_{\mathbf{h}}^* = \Delta(\mathbf{h}) < \bar{a}\mathbf{h}$, or equivalently $\frac{b_{\mathbf{h}}^{r,ss}}{\bar{a}} < \mathbf{h}\omega_{\mathbf{h}}$. \square

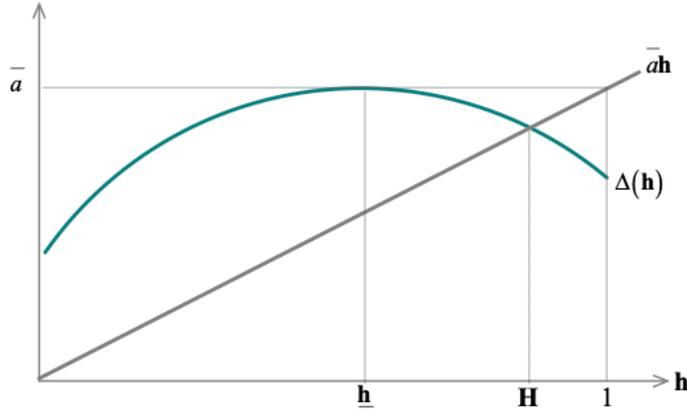


Figure 1.10: $\underline{\mathbf{h}} < \mathbf{H} < 1$

1.8.2 Data and supplementary results.

Table 1.6: Definition of variables and data sources.

Variable	Definition	Source
Debt-to-GDP ratio	General government gross debt to GDP ratio.	World Economic Outlook (IMF, October 2019)
Growth rate	Gross growth rate of real GDP	World Development Indicators (World Bank)
Inflation rate	Consumer price inflation (percent, average)	World Development Indicators (World Bank)
Yield spread	Difference between long term country real interest rate and German rate. Real rate is equal to nominal rate minus three-year average of future consumer price inflation rate.	OECD database and Reuters (for nominal interest rates), World Development Indicators database (for inflation)

Table 1.7: Data : Advanced countries (1980-2018).

Country	μ	σ	\underline{h}
Australia	3.08	1.45	0.964
Austria	1.98	1.47	0.964
Belgium	1.91	1.42	0.965
Canada	2.74	1.95	0.952
Czech Republic	1.98	3.84	0.907
Denmark	1.76	1.88	0.954
Finland	2.16	3.06	0.925
France	1.78	1.34	0.967
Germany	1.70	1.90	0.953
Greece	0.79	3.54	0.914
Hong Kong	4.53	3.58	0.913
Iceland	3.47	3.55	0.914
Ireland	4.76	4.66	0.888
Israel	3.48	1.78	0.956
Italy	1.21	1.84	0.955
Japan	1.90	2.23	0.945
Korea	5.95	3.80	0.908
Latvia	3.90	5.73	0.863
Lithuania	4.10	5.19	0.875
Luxembourg	3.78	3.04	0.926
Netherlands	2.07	1.79	0.956
New Zealand	2.59	1.82	0.955
Norway	2.43	1.69	0.958
Portugal	1.96	2.58	0.937
Singapore	6.21	3.72	0.91
Slovak Republic	3.87	3.05	0.926
Spain	2.25	2.14	0.947
Sweden	2.14	2.08	0.949
Switzerland	1.80	1.56	0.962
United Kingdom	2.13	1.91	0.953
United States	2.60	1.81	0.955
Sample	2.81	2.63	0.936

Notes: μ and σ are the mean and standard deviation of the log gross growth rate of GDP per capita expressed in percentage. \underline{h} is the minimum value of h above which a risky steady state exists (see Proposition 4). Growth data are from the World Bank database and cover the period 1980-2018.

Table 1.8: Data : Emerging countries (1980-2018).

Country	μ	σ	$\underline{\mathbf{h}}$
Brazil	2.37	3.27	0.920
Chile	4.21	3.95	0.904
China	9.07	2.47	0.939
Colombia	3.40	2.05	0.950
Hungary	2.14	2.76	0.932
Malaysia	5.63	3.45	0.916
Mexico	2.48	3.22	0.922
Nigeria	3.01	5.45	0.869
Pakistan	4.80	1.93	0.953
Philippines	3.75	3.34	0.919
Poland	3.66	2.66	0.935
Russia	0.61	6.55	0.844
South Africa	2.23	2.22	0.946
Sample	3.64	3.33	0.919

Notes: μ and σ are the mean and standard deviation of the log gross growth rate of GDP per capita. $\underline{\mathbf{h}}$ is the minimum value of \mathbf{h} above which a risky steady state exists (see Proposition 4). Growth data are from the World Bank database and cover the period 1980-2018.

Table 1.9: Sustainability and low interest interest rate: Euro zone ($r = 0.31\%$).

Country	$\bar{a}(1+r)^{-1}$ (1)	$\tilde{\mathbf{h}}$ (2)
Austria	1.007	0.970
Belgium	1.009	0.946
Finland	1.000	1.000
France	1.006	0.973
Germany	1.010	0.959
Greece*	0.970	1.000
Ireland	1.049	0.754
Italy*	0.994	1.000
Latvia	1.006	0.976
Lithuania	1.014	0.947
Luxembourg	1.021	0.859
Netherlands	1.006	0.977
Portugal	1.000	1.000
Slovak Republic	1.019	0.900
Spain	1.002	0.992

Notes. *: Countries where $\bar{a} < 1 + r$. The time period for the growth rate \bar{a} and the risk-free rate r is 2009-2018.

Table 1.10: Debt-to-GDP ratios and real yield spreads (%): Advanced countries (1980-2018).

Country	Debt-to-GDP ratio (b_t)				Yield spread (\bar{s}_t)			
	Mean	Std	Min	Max	Mean	Std	Min	Max
Australia	23.71	9.68	9.69	41.37	1.36	1.90	-1.03	7.00
Austria	69.1	9.32	55.93	84.4	0.08	0.29	-0.33	1.02
Belgium	110.73	15.37	76.36	138.14	0.38	0.90	-0.79	2.58
Canada	78.24	14.12	44.91	100.25	0.67	1.00	-0.59	3.19
Czech Republic	28.63	10.47	11.65	44.91	0.18	0.96	-1.67	1.46
Denmark	48.9	14.13	27.35	78.63	0.61	1.09	-0.85	4.16
Finland	38.33	17.38	10.89	63.45	1.02	1.87	-1.34	5.50
France	58.83	24.52	20.83	98.42	0.54	0.78	-0.75	2.48
Germany	63.26	11.27	38.99	82.31	–	–	–	–
Greece	101.97	48.44	22.53	184.85	4.10	6.67	-1.78	22.90
Hong Kong	0.97	1.02	0.05	3.52	0.20	3.24	-4.34	7.13
Iceland	47.61	19.64	24.48	92.03	1.46	2.51	-3.77	4.91
Ireland	61.15	30.82	23.62	120.04	1.25	2.35	-2.80	7.60
Israel	74.39	10.74	60.41	92.89	1.86	2.13	-3.70	5.68
Italy	112.52	12.31	92.91	132.16	1.25	1.68	-1.13	4.34
Japan	136.42	66.77	48.81	237.13	-0.72	1.05	-3.46	0.94
Korea	22.64	10.19	7.98	37.92	1.06	1.13	-0.95	3.04
Latvia	26.12	14.09	8.12	46.91	-0.04	4.50	-8.04	8.60
Lithuania	28.59	9.74	14.57	42.58	0.71	3.39	-4.90	9.54
Luxembourg	13.46	6.83	6.49	23.69	-0.58	0.87	-2.39	0.54
Netherlands	60.76	10.56	41.97	76.78	-0.01	0.95	-2.28	1.89
New Zealand	38	14.66	16.3	68.58	1.88	1.60	-0.48	6.74
Norway	36.85	8.09	22.94	52.56	0.60	1.34	-1.27	3.98
Portugal	78.89	30.42	50.34	130.61	1.58	2.90	-2.16	9.56
Singapore	90.48	13.31	69.82	113.63	-0.09	2.23	-3.45	3.55
Slovak Republic	41.59	9.52	21.67	54.74	0.31	1.44	-2.27	3.72
Spain	55.84	22.88	16.58	100.37	1.03	2.02	-1.88	5.18
Sweden	49.68	11.49	37.24	69.15	0.93	1.29	-1.07	4.16
Switzerland	47.69	7	34.35	59.16	-0.48	1.08	-3.21	1.23
United Kingdom	50.4	19.93	28.57	87.91	0.55	0.93	-1.10	2.81
United States	84.29	20.85	53.15	106.82	-0.05	0.93	-1.66	1.96
Sample	60.14	38.13	0.05	237.13	0.68	2.19	-8.04	22.90

Notes: Min, Max and Std are the minimum, maximum and standard deviation. The time period of the sample is 1980-2018. Sources: Debt-to-GDP ratios correspond to general government gross debt from the IMF World Economic Outlook database (October 2019). Yield spreads are the difference between the country's long-term government real interest rates and the German rates, hence the dash (–) in the table for the German spread. Sources: see Table 1.6.

Table 1.11: Debt-to-GDP ratios and real yield spreads (%) : Emerging countries (1980-2018).

Country	Debt-to-GDP ratio (b_t)				Yield spread (\tilde{s}_t)			
	Mean	Std	Min	Max	Mean	Std	Min	Max
Brazil	69.32	8.07	60.2	87.89	4.39	3.83	-2.31	13.31
Chile	15.31	8.17	3.88	37.37	1.60	1.54	-1.33	3.65
China	30.59	8.87	20.45	50.64	-0.10	1.92	-3.07	3.16
Colombia	38.63	7.99	23.36	52.16	4.34	2.23	0.19	8.15
Hungary	68.43	9.75	51.58	84.06	1.53	2.31	-1.33	6.84
Malaysia	46.82	11.03	29.62	74.13	0.66	1.87	-1.37	3.96
Mexico	44.21	5.46	37.21	56.76	2.58	1.46	-0.02	6.19
Nigeria	33.64	21.97	7.28	74.96	0.40	3.75	-5.65	5.21
Pakistan	65.62	7.3	52.44	81.23	3.98	2.43	-0.44	7.12
Philippines	55.39	11.14	38.92	76.08	3.04	2.85	-1.74	9.90
Poland	46.85	5.63	36.38	55.69	2.10	1.58	-0.80	5.00
Russia	29.15	31.45	7.44	135.06	0.60	5.97	-6.55	15.84
South Africa	39.67	8.87	26.51	56.71	2.41	2.41	-2.57	7.62
Sample	44.32	20.47	3.88	135.06	2.22	2.84	-5.65	13.31

Notes: Min, Max and Std are the minimum, maximum and standard deviation. The time period of the sample is 1980-2018. Sources: Debt-to-GDP ratios correspond to general government gross debt from the IMF World Economic Outlook database (October 2019). Yield spreads are the difference between the country's long-term real government interest rates and the German rates. See Table 1.6 for data sources.

Chapter 2

Sovereign Default Risk and Climate Change: Is it Hot Enough ?

2.1 Introduction

There is growing interest of academics, investors, and policymakers in the economic impacts of climate change and the appropriate policy responses. This interest has been strongly revived recently due to accelerating increase in global surface temperature (Figure 2.1) and the frequency of extreme climate shocks, such as heatwaves, droughts, wildfires, tropical cyclones, and coastal flooding.¹ According to the latest Assessment Report of the Intergovernmental Panel on Climate Change (IPCC, February 2022),² global surface mean temperature will reach 1.5°C above pre-industrial levels by 2040, even under an optimistic scenario of very low greenhouse gas emissions.³ Above this critical threshold, there will be little room for adaptation and the socioeconomic impacts of climate change and related mitigation costs will increase exponentially. Glaciers will melt at unprece-

¹See Stern (2007), IPCC (2007; 2012; 2021; 2022).

²See <https://www.ipcc.ch/report/ar6/wg2/>

³Green gas emissions have been identified and considered in the scientific community as the main drivers of anthropogenic climate change. See IPCC (2022)

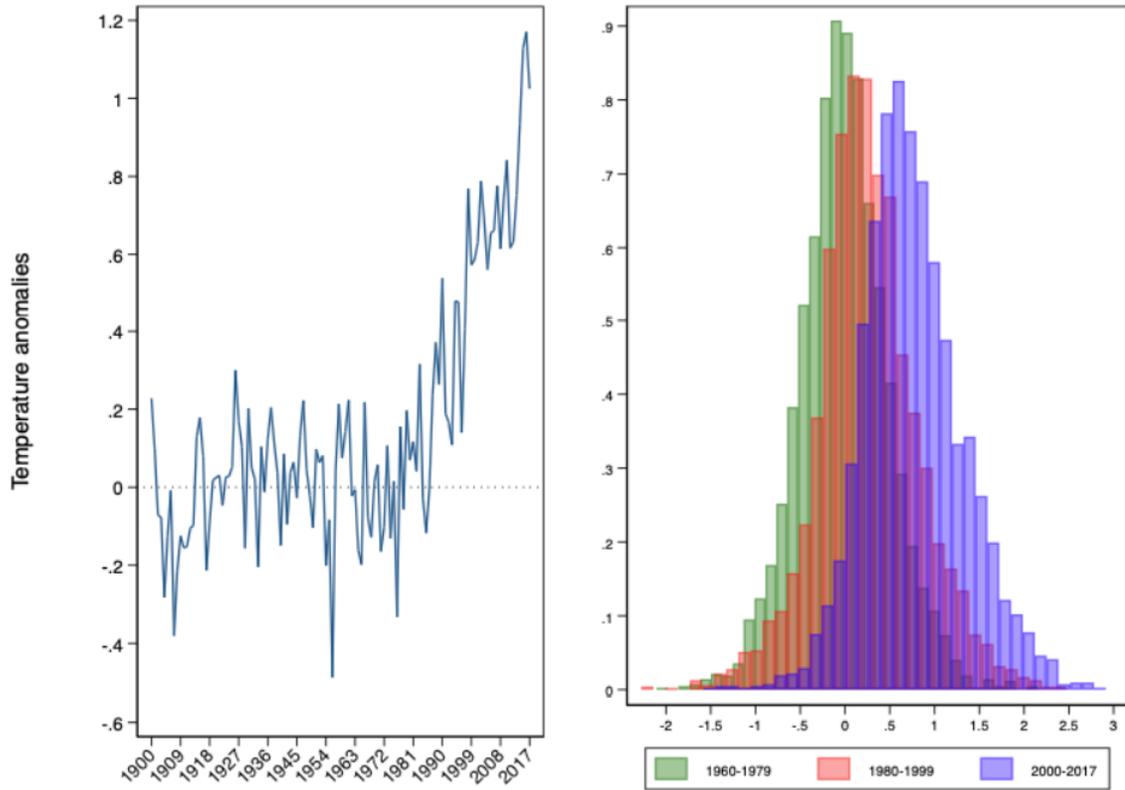


Figure 2.1: Temperature anomalies

dent rates, the impacts of climate-related disasters will be intensified, incidences of climate-borne diseases will increase, human physical and mental health will be severely harmed. Such consequences of climate change will have a doubly negative impact on public finances. First, climate change and related disasters will harm economic growth, reducing opportunities for fiscal revenues. Second, public spending will increase due to automatic stabilizer mechanisms and the funding needs of governments to address the potential damages of climate change and related disasters.

A growing literature documents the impact of climate change on economic growth and various economic outcomes. Yet, there is little evidence on the link and the nature of the link between climate change and sovereign risk and whether

financial markets effectively price climate-related risks.⁴ Although, the sovereign debt market is among the largest asset classes and is considered as the safest one. Conceptually, government bonds are exposed to a range of risks associated with climate change, including both transition risk and physical risk related to climate change. In this paper, we estimate the “physical impact” of climate change⁵ on sovereign default risk, the main determinant of government borrowing cost, and explore the transmission mechanisms.

Measuring Climate change and Sovereign default risk. It is useful to clarify the concepts of climate, climate change, sovereign default risk, and how they can be measured. Climate can be defined as the average and variability of weather over a given period of time. In statistical terms, climate is the distribution of a set of quantities that characterize the weather, such as surface temperature, precipitation and wind. The distribution of these variables can be characterized by their mean and variance. The classical period for averaging the relevant variables is 30 years, as defined by the World Meteorological Organization.

Climate change is defined as a change in the distribution of the climate (Figure 2.1), i.e., changes in the mean and/or the variability of weather variables that persist for an extended period, typically decades or longer. In our case, we focus on changes in the distribution of surface temperature and use temperature anomalies—temperature’s deviation from its long-run mean— as a proxy for climate change.

As for sovereign default risk, we use sovereign Credit Default Swap (CDS) spread on foreign sovereign debt as a proxy for default risk. We consider sovereign CDS spread at one, three, five, and ten-year maturities.

Contribution. In this paper, we estimate the effects of temperature anomalies—temperature’s deviation from its long-run mean— on sovereign credit default swap (CDS) premium and explore the transmission channels. We use a panel dataset covering 76 developing and advanced countries from 1999 to 2017. We consider sovereign CDS spread at several maturities— one, three, five and ten-year

⁴A few exception are [Kling et al. \(2018\)](#), [Agarwala et al. \(2021\)](#) and [Cevik and Jalles \(2022\)](#).

⁵The literature distinguishes between physical risk of climate change, i.e the actual impact of the climate on the economy, and the transition risk, i.e the cost of transition to a green economy. We focus on the first one.

maturities– and estimate the effects of temperature on CDS spread at each one of them. We build on an equilibrium bond pricing equation found in theoretical sovereign default models to isolate the key transmission mechanisms through which temperature can affect CDS spreads.

Succinctly, we find that an increase of temperature leads to an increase of sovereign CDS premium. We document the existence of a “debt limit channel” of temperature: a higher temperature has a negative impact on future growth rate of output, which lowers the country’s debt limit– the maximum debt-to-GDP ratio it can sustain without defaulting. As a result, the probability of default increases, leading to a higher CDS spread. In contrast, we find no evidence of transmission through the primary balance nor the public debt-to-GDP ratio, which are two main determinants of CDS premium.

The paper is structured as follows. Section 2.2 provides a brief review of the literature related to our analysis Section 2.3 presents the dataset we use. Section 2.4 assesses the link between temperature and sovereign CDS spreads and investigates the key transmission mechanisms. Section 2.5 concludes.

2.2 Related literature

The paper combines two strands of literature: the sovereign default risk and the economic impacts of climate change.

The economic impact of climate change literature is clustered into two groups: growth and fiscal impacts. Important references on the growth-climate relationship are Dell, Jones and Olken (2012), Burke, Hsiang and Miguel (2015), Burke, Davis and Diffenbaugh (2018) and Kalkuhl and Wenz (2020).⁶ Dell, Jones and Olken (2012) use a cross country panel data and find evidence for a negative effect of temperature on per capita GDP. Using a similar approach, Burke, Hsiang and Miguel (2015) document the existence of a nonlinear relationship between temperature and GDP per capita. This non-linear relationship is captured considering the square of temperature level as a regressor, along with the level one. Their

⁶Dell, Jones and Olken (2014), Hsiang (2016) and Kolstad and Moore (2019)

findings suggest that countries with low or mild temperature initially experience a positive impact of temperature on output, up to a threshold above which this impact is reversed to a negative one. [Kalkuhl and Wenz \(2020\)](#) use long-difference and cross-sectional regressions to analyze the productivity-temperature relationship at sub-national level and find a negative effect of temperature on the level and growth rate of productivity. In our investigation of key transmission channels through which temperature can affect CDS spreads, we (re)assess the impact of temperature on the growth rate of GDP. A main distinction of our approach is that we use the deviation of temperature from its long-run mean, instead of temperature level, as suggested by recent studies (see [Kahn et al., 2021](#) for a discussion).

Regarding the fiscal impact of climate change, [Jones, Keen and Strand \(2012\)](#) and [Baur, Bruchez and Nicol \(2021\)](#) provide a review of the key channels through which climate change may have fiscal consequences and discuss related mitigation and adaptation mechanisms. These papers highlight the ambiguous potential effects of climate change on fiscal balance. While tax policies aiming at mitigating negative externalities of climate change will likely increase fiscal revenue, adaptation policies in contrast tend to increase public spending. Therefore, the ultimate effect of climate change on the fiscal balance is not *à priori* clear. [Cevik and Nanda \(2021\)](#) and [Khadan \(2019\)](#) reassess the fiscal sustainability in the Caribbean, a region that is highly exposed to climate change. These studies find that fiscal policy in Caribbean is “weakly” sustainable in the sense that the government primary balance is positively related to the debt-to-GDP ratio. We take a more direct approach, in terms of climate change, by estimating the impacts of temperature anomalies on primary balance and find no significant effect.

As for the sovereign default risk literature, only few studies address its link to climate change. [Kling et al. \(2018\)](#) and [Cevik and Jalles \(2022\)](#) use indices from the Notre Dame Global Adaptation Initiative to estimate the link between sovereign yield spreads and the vulnerability and resilience of countries to climate change. Overall, these papers find a positive relationship between yield spreads and countries’ vulnerability to climate change. [Agarwala et al. \(2021\)](#) take a

simulation approach to estimate the potential impacts of various climate scenarios on sovereign credit ratings and find a similar conclusion. Here we assess the link between sovereign CDS spread— which we believe is a better proxy for creditors’ perception of sovereign default risk than the yield spread or credit rating— and temperature anomalies, which are purely exogenous and directly related to climate change.

Our identification of the key mechanisms has direct echoes with the broad literature of fiscal space, public debt sustainability, and sovereign default (Bi 2012; Ghosh et al. 2013; Collard, Habib and Rochet 2015; Lorenzoni and Werning 2019; Diarra, Guillard and Kempf 2022). By identifying the link between temperature, growth and the debt limit, we show that climate risk must be taken into account when assessing public debt sustainability.

Finally, by highlighting the connection between temperature and sovereign CDS spread, our analysis expands the large literature on the determinants of sovereign default risk premia (see Edwards 1984; Eichengreen and Mody 1998).

2.3 Data

2.3.1 CDS spreads on sovereign bonds

We measure the market perception of sovereign default risk by the spread on sovereign credit default swaps (CDS). CDS are financial instruments that are mainly traded in over-the-counter (OTC) derivative markets. The spread represents the periodic payment that the buyer of CDS must pay to the seller for the contingent claim in the case of a credit event, namely a default or restructuring of sovereign debt. Hence, the CDS spread is theoretically related to the probability that a country defaults: the greater this probability the higher the CDS spread. Therefore, we consider it as a good proxy for market-based default risk pricing.

We consider sovereign CDS spreads at four maturities— one, three, five and ten-years, across a sample of 76 advanced countries and developing markets. The spreads are denominated in US dollars and expressed in basis point for all countries

and maturities. The original data is taken from Macrobond and is reported at a daily frequency over the time period 1999-2017. We collapse daily data to yearly frequency by taking the yearly country average.⁷ Table 2.1 provides summary statistics of the CDS spread at the four different maturities.

We do not use interest rate spreads as a proxy of sovereign default risk. There are several reasons why we believe that CDS spreads are more adequate for our analysis than interest rates spreads. First, CDS spreads are a more precise measure of default risk premia than interest rates spreads as they are specifically designed to hedge against the prospects of a sovereign default. Second, CDS spreads are not subject to time-to-maturity issues that occur when using interest rates spreads.⁸ Third, interest rates spreads are subject to a large cross-country heterogeneity due, for example, to differences in the underlying risk-free rates, the level of financial development of the countries, inflation expectations, exchange rate differences, and supply/demand of credit dynamics. All these issues blur the default risk component in interest rate spreads.

2.3.2 Temperature

We use temperature anomalies expressed in degree Celsius as a proxy for climate change. We rely on temperature data from the Terrestrial Air Temperature dataset by [Matsuura and Willmott \(2018\)](#). This dataset contains 0.5 degree gridded monthly mean temperature time series over the period 1900-2017. We use a geocoding procedure to aggregate gridded monthly data to a country-year scale.

Our proxy of climate change defines as the deviation of temperature from its long run mean. Specifically, for each county i and year t we compute

$$\tilde{T}_{i,t} = T_{i,t} - \overline{T_{i,1900-1950}}, \quad (2.1)$$

⁷We are currently working on a new version of the paper that uses monthly CDS spreads as a robustness check of our findings.

⁸See, e.g., [Bank for International Settlements \(2010\)](#). One may argue that it is possible to compute the spread for a given maturity using the yield curve. However, this is not feasible for many (developing) countries where there is a paucity of data on yield at some maturities to construct a reliable yield curve.

Table 2.1: Data: Summary statistics (1999-2017)

Variable	Obs.	Mean	Std. dev.	Min	Max
Year	1444	2008	5.48	1999	2017
CDS 1-year (basis point)	1145	141.75	242.30	0.50	3240.94
CDS 3-year (basis point)	1145	175.45	258.76	0.50	2595.08
CDS 5-year (basis point)	1145	206.96	283.09	0.52	3687.56
CDS 10-year (basis point)	1145	249.40	323.32	0.83	4893.56
$T_{i,t} - \overline{T_{i,1900-1950}}$ (°Celsius)	1444	0.76	0.57	-1.58	2.91
$T_{i,t} - \overline{T_{\text{region},1900-1950}}$ (°Celsius)	1444	-1.20	11.43	-57.07	9.68
$T_{i,t} - \overline{T_{i,1900-1998}}$ (°Celsius)	1444	0.73	0.56	-1.54	2.79
$T_{i,t} - \overline{T_{i,10 \text{ year moving average}}}$ (°Celsius)	1444	0.17	0.51	-2.32	2.00
$T_{i,t} - \overline{T_{i,20 \text{ year moving average}}}$ (°Celsius)	1444	0.31	0.51	-2.14	2.22
$T_{i,t} - \overline{T_{i,30 \text{ year moving average}}}$ (°Celsius)	1444	0.44	0.52	-1.85	2.28
$T_{i,t} - \overline{T_{i,40 \text{ year moving average}}}$ (°Celsius)	1444	0.53	0.53	-1.80	2.40
$T_{i,t} - \overline{T_{i,50 \text{ year moving average}}}$ (°Celsius)	1444	0.58	0.54	-1.68	2.46
$T_{i,t}/sd_{i,1999-2017}$	1444	-4.07	26.00	-103.39	53.99
$T_{i,t}$ (°Celsius)	1444	12.96	12.47	-34.10	28.40
Log of real GDP per capita	1444	9.81	0.92	6.56	11.54
Real GDP Growth rate (%)	1444	2.31	4.01	-48.48	40.20
Debt-to-GDP (%)	1406	53.78	37.52	1.56	344.32
Primary balance (% of GDP)	1429	0.18	4.69	-34.91	28.57
Output gap	1444	-0.25	3.82	-34.23	28.88
Log of consumption	1409	27.84	2.87	22.44	36.38
Inflation rate (%)	1438	6.16	16.76	-8.24	325.03
Reserves	1436	857*10 ⁸	301*10 ⁹	174*10 ⁶	39*10 ¹¹
Credit ratings	1356	13.88	4.91	2.48	21.00
Trade openness (% of GDP)	1429	81.16	37.83	18.35	226.04
FDI inflow (% of GDP)	1435	5.19	14.89	-28.31	280.13
Gross capital formation (% of GDP)	1441	23.58	6.30	0.00	46.66
Population(t-1)	1444	713*10 ⁵	202*10 ⁶	274047	138*10 ⁷
Political stability index	1444	0.06	0.96	-3.18	1.76
Governance effectiveness index	1444	0.47	0.92	-1.98	2.35
Current account (% of GDP)	1428	-0.85	7.55	-28.84	45.46

where $T_{i,t}$ is the mean temperature in year t and $\overline{T_{i,1900-1950}}$ is the long run mean of temperature for country i computed over the period 1900-1950.

The computation choice in (2.1) follows Stock (2020) and Kahn et al. (2021). The authors suggest that the deviation of temperature from the long run mean is a more adequate measure when analyzing the economic impacts of climate change than the standard mean temperature since the latter presents a strong upward trend that may potentially lead to spurious regression.

In our robustness analyses in Section 2.4, we also consider five alternative measures to the one defined in equation (2.1). Table 2.1 summarizes descriptive statistics of the baseline temperature deviation $\tilde{T}_{i,t}$, as defined in (2.1), as well as for the alternative measures.

2.3.3 Macroeconomic variables

To explore the key mechanisms underlying our estimation results in Section 2.4, we use data on country GDP, debt-to-GDP, and primary surplus. We also use others macroeconomic and political variables as control variables in our estimations. Table 2.1 contains summary statistics of the variables considered. Table 1.6 in Appendix 2.6 presents the definition of the variables and data sources.

2.4 Estimation and mechanisms

In this section, we assess the link between temperature anomalies and sovereign default risk. Section 2.4.1 studies the direct impacts of temperature anomalies on sovereign CDS spreads. Section 2.4.3 investigates the key transmission channels of these effects.

2.4.1 Sovereign CDS spreads and temperature

To estimate the impacts of temperature on sovereign CDS spreads, we consider a standard two-way fixed effects model⁹ that accounts for fluctuations in temperature. Specifically, we estimate the following equation:

$$S_{i,t} = \alpha_0 + \alpha_1 \tilde{T}_{i,t} + \beta' X_{i,t} + \omega_i + \eta_t + \varepsilon_{i,t}, \quad (2.2)$$

where $S_{i,t}$ is the CDS spread in basis point observed for country i in year t , and $\tilde{T}_{i,t}$ is the deviation of temperature from its long run mean. $X_{i,t}$ is a vector of economic and political control variables. ω_i and η_t are country and time fixed effects, respectively, which account for unobserved country specific and time-varying factors. $\varepsilon_{i,t}$ is an error term.

Table 2.2 presents estimation results of equation (2.2) using each of the four CDS maturities and our baseline measure of temperature deviation defined in equation (2.1).

We find a positive and statistically significant impact of temperature on CDS spreads for the three, five and ten-year maturities but not for the one-year one: a degree Celsius increase in temperature relative to the long run mean increases CDS spreads by 15.61 to 31.09 basis points. This effect is statistically and economically significant and the longer the maturity of the CDS spreads the larger the impact of temperature on spreads. This suggests that sovereign creditors price climate risk, as measured by increases in temperature, not only when investing over a short horizon but also the medium and long horizons.

Regarding our control variables, they have the expected signs. Unsurprisingly, the ratio of public debt-to-GDP, the growth rate of output, and credit ratings are important determinants of sovereign CDS spreads. In contrast, the other macroeconomic variables, in particular the primary balance and reserves, do not contribute to the CDS spread.

We next investigate whether the geographic location of countries plays a role for the impact of temperature on sovereign CDS spreads. For this purpose, we

⁹See, for example, [Eichengreen and Mody \(1998\)](#) and [Cruces and Trebesch \(2013\)](#).

Table 2.2: CDS spreads and Temperature (baseline, 1999-2017)

Dependent variable:	CDS spreads in basis point with maturity:			
	1-year	3-year	5-year	10-year
Climate				
Temperature	11.68 (7.21)	15.61* (8.22)	22.71** (10.52)	31.09** (13.86)
Controls variables				
Log of GDP	204.77*** (70.16)	147.35** (65.42)	76.06 (66.30)	48.42 (74.24)
Real GDP growth rate	-13.31*** (4.91)	-14.21*** (3.80)	-14.91*** (3.93)	-16.83*** (5.22)
Inflation rate	13.07*** (2.38)	15.00*** (2.41)	16.59*** (3.28)	17.87*** (4.68)
Public debt/GDP	1.13 (0.87)	1.87** (0.81)	2.58*** (0.91)	3.60*** (1.03)
Public debt/GDP squared	-0.00 (0.00)	-0.00 (0.00)	-0.01** (0.00)	-0.01*** (0.00)
Primary balance/GDP	-0.35 (1.29)	0.15 (1.42)	0.80 (1.97)	1.28 (2.73)
Reserves/GDP	-0.00 (0.00)	-0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
Credit ratings	-42.53*** (5.12)	-49.03*** (5.22)	-53.91*** (5.83)	-56.21*** (6.16)
Constant	-1607.14*** (594.95)	-1013.16* (554.51)	-310.51 (557.03)	226.57 (623.46)
Observations	1116	1116	1116	1116
Countries	76	76	76	76
Country fixed effects	Yes	Yes	Yes	Yes
Time fixed effects	Yes	Yes	Yes	Yes
R^2	0.652	0.723	0.732	0.710
Adjusted R^2	0.617	0.696	0.705	0.681
AIC	14,408	14,296	14,462	14,854
BIC	14,895	14,782	14,949	15,336

Notes: Regression results of unbalanced panel data. Robust, country clustered standard errors in parentheses. The dependent variable is the yearly CDS spread measured in basis point. The key explanatory variables is temperature deviation from its long mean in 1900-1950. Temperature is measured in °C. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Source: Authors' estimates.

Table 2.3: Five-year CDS spreads and Temperature (regions, 1999-2017)

Dependent variable:	5-year CDS spreads in basis point						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Climate							
Temperature	19.90*	19.54	23.65**	23.50**	23.09**	22.37**	26.06**
	(10.98)	(14.39)	(10.36)	(11.61)	(11.29)	(10.48)	(11.36)
Interaction terms							
East Asia & Pacific × Temp.	45.38**						
	(19.19)						
Europe & Central Asia × Temp.		5.60					
		(20.74)					
Latin America & Caribbean × Temp.			-12.87				
			(50.76)				
Middle East & North Africa × Temp.				-5.09			
				(27.23)			
North America × Temp.					-6.02		
					(18.58)		
South Asia × Temp.						32.30	
						(98.95)	
Sub-Saharan Africa × Temp.							-46.88**
							(20.65)
Observations	1116	1116	1116	1116	1116	1116	1116
Countries	76	76	76	76	76	76	76
Country fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Control and Constant	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.732	0.732	0.732	0.732	0.732	0.732	0.732
Adjusted R^2	0.705	0.704	0.704	0.704	0.704	0.704	0.705

Notes: Regression results of unbalanced panel data. Robust, country clustered standard errors in parentheses. The dependent variable is the yearly CDS spread measured in basis point. The key explanatory variables are temperature deviation from its long mean in 1900-1950 and the interaction with region dummies. Temperature is measured in °C. Control variables used in Table 2.2 are included but not reported. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Source: Authors' estimates.

classify the countries in our sample in seven geographic regions following the World Bank’s classification methodology. Based on this classification, we expand our baseline specification in (2.2) by including temperature×region interaction terms.

Table 2.3 presents estimations results for the 5-year CDS spreads, which is the most maturity used in empirical studies on sovereign CDS spreads.¹⁰ We find a positive and significant impact of temperature for countries in the East Asia and Pacific region only: one degree Celsius increase in temperature (relative to the long run mean) leads to an increase of spreads by 45.38 basis points if the country is located in the East Asia and Pacific region. This result is in line with the fact that the East Asia and Pacific is one of the regions that are highly exposed to climate change and experienced the fastest temperature increases over the past few decades.

We do not find any significant effect of temperature for the other regions, except the Sub-Saharan Africa region where we find a negative impact on spreads. One possible explanation of this finding is that countries in this region are exposed to others exogenous shocks and factors that dominate the effect of climate change on CDS spreads.

2.4.2 Robustness to alternative measures of temperature

We conduct two sets of robustness analyses to check the sensitivity of our results regarding the measurement of temperature.

In the first set, we consider four alternatives to our baseline measure of temperature deviation defined in equation (2.1). First, we change the reference period used to compute the long run mean of temperature in the baseline specification in equation (2.1) from 1900-1950 to 1900-1998. This corresponds to the time period before the start of our sample (i.e., 1999-2017). Second, we compute the deviation of temperature with respect to the long run mean observed in the geographical region of the country, that is, we redefine equation (2.1) as:

¹⁰We find similar results for the other maturities but do not report them for parsimony.

Table 2.4: CDS estimation (5 years) – Temperature robustness (1)

Dependent variable:	5-year CDS spreads in basis point			
	(1)	(2)	(3)	(4)
Climate				
$T_{i,t} - \overline{T_{i,1900-1950}}$	22.71** (10.52)			
$T_{i,t} - \overline{T_{\text{region},1900-1950}}$		22.71** (10.52)		
$\tilde{T}_{i,t}/sd_{i,1999-2017}$			12.90** (5.81)	
$T_{i,t}$				22.71** (10.52)
Observations	1116	1116	1116	1116
Countries	76	76	76	76
Country fixed effects	Yes	Yes	Yes	Yes
Time fixed effects	Yes	Yes	Yes	Yes
Control and Constant	Yes	Yes	Yes	Yes
R^2	0.732	0.732	0.732	0.732
Adjusted R^2	0.705	0.705	0.705	0.705
AIC	14,460	14,462	14,462	14,459
BIC	14,942	14,949	14,949	14,940

Notes: Regression results of unbalanced panel data. Robust, country clustered standard errors in parentheses. The dependent variable is the yearly CDS spread measured in basis point. Temperature measures are in °C. Control variables used in Table 2.2.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Source: Authors' estimates.

$\tilde{T}_{i,t} = T_{i,t} - \overline{T_{\text{region},1900-1950}}$. Third, we explicitly account for the volatility of temperature by dividing our baseline measure in (2.1) by its country standard deviation computed over the period 1999-2017, that is: $\tilde{T}_{i,t}/\sigma(\tilde{T}_{i,1999-2017})$. Finally, we consider the standard temperature level, $T_{i,t}$, used in most previous studies.

Table 2.4 reports estimations results using the four alternative measures. We find a significant positive impact of temperature on CDS spreads for all alternative measures, with estimated coefficients comparable to those from the baseline regression (Table 2.2).

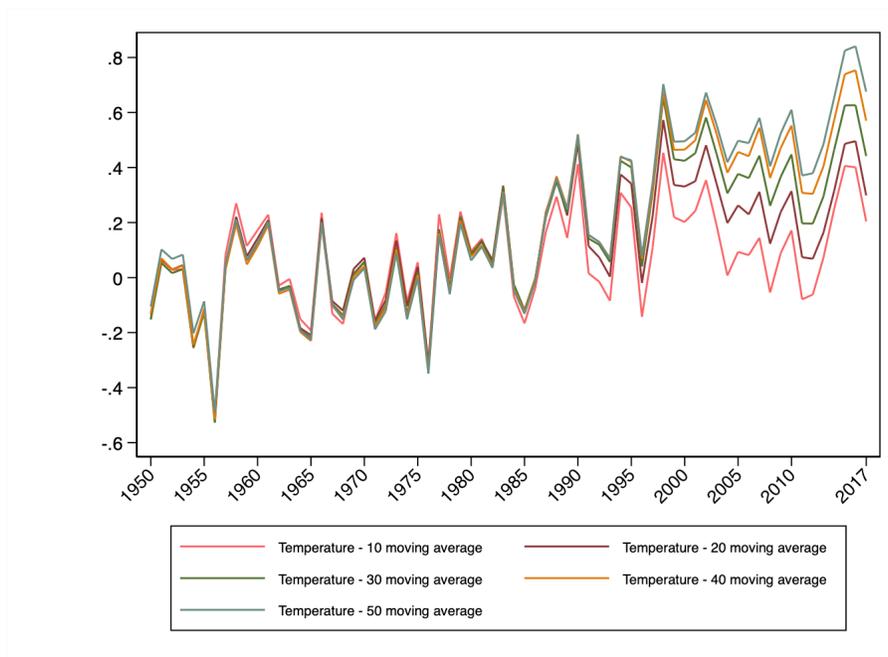


Figure 2.2: Temperature anomalies (moving average)

In the second set of robustness checks, we consider the deviation of temperature from the moving average computed over a time window of 10, 20, 30, 40 and 50 years separately. Figure 2.2 presents the computed temperature measure for each time window. Inspecting this figure, we notice that the five measures closely move together but start to deviate in the 90's. Starting from 1999, we observe relatively important differences between temperature measures with small (10, 20 years) and large (30, 40, 50 years) windows. This is mainly due to an acceleration in the increase of temperature over the recent period as shown in Figure 2.1.

Table 2.5 reports estimation results using each measure. We find a significant positive effect of temperature on CDS spreads for all time windows, except the for the shortest window (i.e., 10-year). We also notice that the effect of temperature on spreads is mainly observed for larger windows, in particular above 30 years, which is the standard time span used to define and measure climate change in the literature (see Dell, Jones and Olken 2014). This result reinforces the relevance of our approach to consider the deviation of temperature from its long run mean.

In Table 2.5, we also report results using the cyclical and trend components

Table 2.5: CDS estimation (5 years) – Temperature robustness (2)

Dependent variable:	5-year CDS spreads in basis point					
	(1)	(2)	(3)	(4)	(5)	(6)
Climate						
Temperature (10-y mov av)	10.03 (8.70)					
Temperature (20-y mov av)		18.53* (9.93)				
Temperature (30-y mov av)			20.71** (10.19)			
Temperature (40-y mov av)				21.93** (10.22)		
Temperature (50-y mov av)					21.23** (10.13)	
Temperature (Trend)						90.37** (38.68)
Temperature (cycle)						12.80 (9.57)
Observations	1116	1116	1116	1116	1116	1116
Countries	76	76	76	76	76	76
Country fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Time fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Control and Constant	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.731	0.731	0.732	0.732	0.732	0.733
Adjusted R^2	0.704	0.704	0.705	0.705	0.705	0.706
AIC	14,464	14,464	14,465	14,462	14,461	14,457
BIC	14,946	14,950	14,957	14,949	14,942	14,944

Notes: Regression results of unbalanced panel data. Robust, country clustered standard errors in parentheses. The dependent variable is the yearly CDS spread measured in basis point. Temperature measures are in °C. Control variables used in Table 2.2. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Source: Authors' estimates.

of temperature in level.¹¹ We find significant effect only for the latter, which is in line with the results presented above.

2.4.3 Mechanisms

In the previous section, we estimated the impacts of temperature anomalies on sovereign default risk, proxied by CDS spreads. We have documented robust positive impact of temperature on sovereign default risk. A natural question therefore is: what are the transmission mechanisms of this effect ? We address this question in this section.

Our main goal is to isolate the key channels through which temperature anomalies can lead to an increase in sovereign CDS spreads. As a starting point of our investigation, we build on the basic equilibrium pricing equation of public debt that is found in most theoretical models of sovereign default (Bi 2012; Ghosh et al. 2013; Collard, Habib and Rochet 2015; Lorenzoni and Werning 2019). This equation relates a country’s spread to its key macroeconomic fundamentals that are relevant for the pricing of sovereign debt on financial markets.

According to this framework, the CDS spread of country i in year t can be defined as follows

$$S_{i,t} = f(b_{i,t}, b_{i,t}^{\text{lim}}), \quad (2.3)$$

where $f(\cdot)$ is a potentially non-linear function, $S_{i,t}$ is the CDS spread observed for country i in period t and $b_{i,t}$ is the debt-to-GDP ratio. $b_{i,t}^{\text{lim}}$ is the “debt limit” of the country, that is the maximum debt-to-GDP ratio that the country can sustain without defaulting.

Notice that, in equation (2.3), the debt limit of the country is time dependent. This is not the case in most models of sovereign default, where the debt limit is constant over time: $b_{i,t}^{\text{lim}} = b_i^{\text{lim}} \forall t$. This is because these models make simplifying assumptions on the maximum primary surplus (as a fraction of out-

¹¹For the decomposition into trend and cycle, we use the standard Hodrick–Prescott filter. We set the smoothing parameter $\lambda = 100$.

put) and the growth rate of output, which are the main determinants of the debt limit. Specifically, these models assume that the maximum primary surplus is constant over time and that the growth rate is i.i.d (Collard, Habib and Rochet 2015) or constant (Ghosh et al. 2013).¹² However, one can have a time-varying debt limit either by assuming a time-varying maximum primary surplus and/or an autocorrelated growth rate.

Since there is no particular economic justification for the simplifying assumptions mentioned above, we do not impose them here. Instead, we consider a more general formulation that allows for time-varying maximum primary surplus and autocorrelated growth rate. Let $a_{i,t}$ denote the real GDP growth rate of country i in period t , which may be autocorrelated, and $s_{i,t}^{\max}$ the maximum primary surplus. We consider the following reduced form for the debt limit:

$$b_{i,t}^{\text{lim}} = g\left(a_{i,t}, s_{i,t}^{\max}\right), \quad (2.4)$$

where the function $g(\cdot)$ depends positively on $a_{i,t}$ and $s_{i,t}^{\max}$.¹³

Figure 2.3 illustrates the key transmission channels of the effects of temperature on sovereign CDS spreads. The straight arrow in the middle indicates the positive effect of temperature on CDS spreads identified in the previous section. This positive effect can be transmitted through two main channels. First, it can be related to the increase in public debt-to-GDP ratio following a temperature shock, which in turn increases the probability of default and thus the CDS spread. This potential channel is described in the upper branch of the figure. Second, temperature can also lowers the country's debt limit due to its negative effects on the growth rate and/or the maximum primary surplus, which in turn increases the country's probability of default and the spread. This second potential channel is described in the lower branch of the figure.¹⁴

¹²One exception is Bi (2012), who develops a framework that generates a time-varying maximum primary surplus from endogenous Laffer-curves.

¹³This is in line with the positive link between the debt limit, the maximum primary surplus and output growth present in the papers mentioned previously.

¹⁴Note that we use $+/-$ on the lower branch of the figure to indicate the fact that, *à priori*, temperature can have both negative and positive effects on growth and the maximum primary

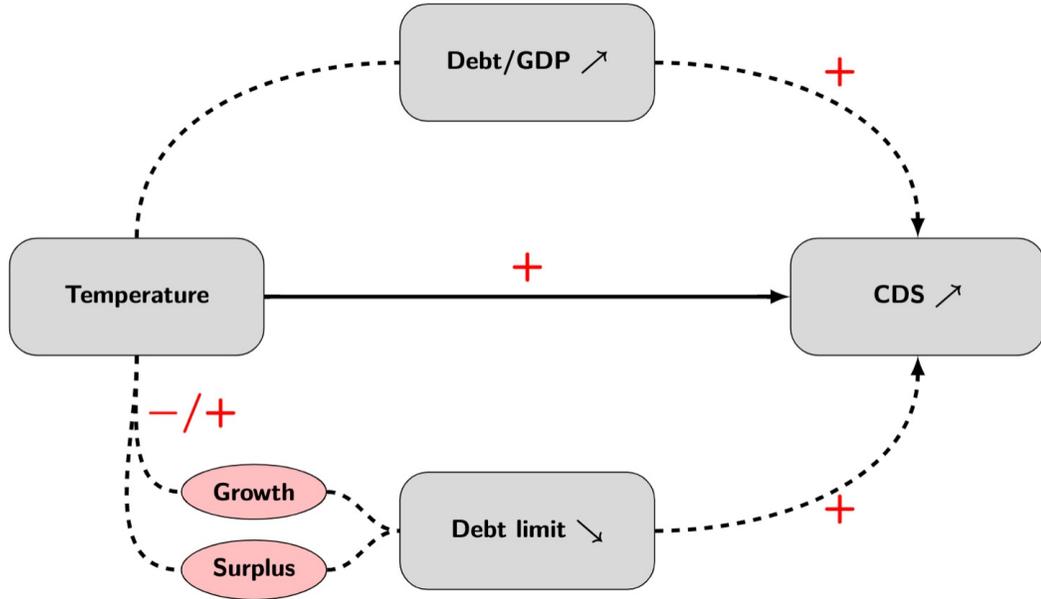


Figure 2.3: Temperature shocks and CDS spreads: Transmission Mechanisms.

Discussion

The CDS spread defined in equation (2.3) is a reduced form that emerges under rational expectations in theoretical models of sovereign default. It relates the spread to key macroeconomic fundamentals of the country, namely the debt-to-GDP ratio and the debt limit, which are the two transmission mechanisms in our analysis. One may ask whether these are the only possible channels through which temperature can affect the CDS spread.

Another potential transmission channel is the (expected) “haircut”, that is the fraction of debt that creditors lose in case of a sovereign default, which is found to be an important determinant of the debt limit (Diarra, Guillard and Kempf 2022) and the spreads (Cruces and Trebesch 2013). Therefore, the CDS spread could vary across countries and over time if creditors believe that temperature can affect the haircut. This is not however an issue in our investigation of key mechanisms surplus. Our empirical investigation will later illuminate the sign of these effects.

because theoretical models of sovereign default suggest that the haircut is itself a function of the defaulting country’s fundamentals, in particular the debt-to-GDP ratio, the maximum primary surplus and the growth rate of output (Yue 2010; Lorenzoni and Werning 2019). Therefore, if temperature has any effect on the haircut, this effect should transmit through the underlying macroeconomic variables that we consider. For this reason, we do not explicitly explore this (potential) haircut channel here.

In addition, since equation (2.3) arises under rational expectations it does not account for potential behavioral mechanisms of creditors. While behavioral arguments are often used in the media and by some academics to rationalize fluctuations in country spreads, in particular during periods of debt crisis,¹⁵ recent studies show that these fluctuations are actually in accordance with standard default models with rational expectations (see Bi and Traum 2012, 2014). This is the view that we adopt here.

2.4.4 Identification of the mechanisms

In this section, we investigate the transmission mechanisms through which temperature affects sovereign default risk, as presented in Figure 2.3. Only two channels are involved: the debt-to-GDP channel and the debt limit channel.

2.4.4.1 Debt-to-GDP channel

The first part of our identification exercise aims to isolate the potential increase of the CDS spread due to increased debt-to-GDP ratio following a temperature shock. Specifically, we ask whether the statistically significant impacts of temperature on CDS spreads found in Table 2.2 can be attributed to the (potential) increase in the debt-to-GDP ratio following a temperature shock, or if the debt limit also plays an important role.

To address this matter, we follow a two-step estimation approach. In the first step, we take advantage of the orthogonality property between residuals and

¹⁵See DeGrauwe and Ji (2012).

regressors in an OLS estimation in order to separate the two channels. Precisely, we first estimate the following equation:

$$S_{i,t} = \gamma_0 + \gamma_1 b_{i,t} + \varepsilon_{i,t}, \quad (2.5)$$

where $S_{i,t}$ is the CDS spread in basis point observed for country i in year t , $b_{i,t}$ is the debt-to-GDP ratio observed in the same period and $\varepsilon_{i,t}$ is an error term.

From the OLS estimation of equation (2.5), we define

$$\hat{S}_{i,t} \equiv \hat{\gamma}_0 + \hat{\gamma}_1 b_{i,t} \quad (2.6)$$

as the part of the CDS spread that is predicted by the debt-to-GDP ratio.

Let $\hat{\varepsilon}_{i,t}$ denote the residual from the OLS estimation. Therefore, from (2.5) and (2.6), we have, by definition,

$$S_{i,t} \equiv \hat{S}_{i,t} + \hat{\varepsilon}_{i,t}. \quad (2.7)$$

Notice that $\hat{S}_{i,t}$ and $\hat{\varepsilon}_{i,t}$ are orthogonal. Therefore, equation (2.7) decomposes the CDS spread in two components: the predicted spread, $\hat{S}_{i,t}$, which is related to the debt-to-GDP ratio, and the residual, $\hat{\varepsilon}_{i,t}$, which is not related to debt-to-GDP.

Recall that, as illustrated in Figure 2.3, temperature can affect the CDS spreads either through the debt-to-GDP ratio or through the debt limit. Therefore, since $\hat{\varepsilon}_{i,t}$ is orthogonal to $\hat{S}_{i,t}$, and thus to $b_{i,t}$, one can interpret $\hat{\varepsilon}_{i,t}$ as the variation in CDS spreads that is explained by the debt limit.

Our second step estimation regresses $\hat{S}_{i,t}$ and $\hat{\varepsilon}_{i,t}$ on temperature, while accounting for the standard control variables included in the baseline specification defined in (2.2). Specifically, we separately estimate the following two equations:

$$\hat{S}_{i,t} = \theta_0 + \theta_1 \tilde{T}_{i,t} + \beta' X_{i,t} + \omega_i + \eta_t + \nu_{i,t}, \quad (2.8)$$

$$\hat{\varepsilon}_{i,t} = \phi_0 + \phi_1 \tilde{T}_{i,t} + \beta' X_{i,t} + \omega_i + \eta_t + u_{i,t}, \quad (2.9)$$

Table 2.6: Debt channel: First step regression

Dependent variable: 5-year CDS spread (basis point)	
Public debt/GDP	−0.87 (0.88)
Public debt/GDP square	0.02*** (0.01)
Constant	183.79*** (23.76)
Observations	1097
Countries	75
R^2	0.041
Adjusted R^2	0.039

Notes: The table reports OLS estimates of equation (2.5). The dependent variable is the yearly CDS spread measured in basis point. Japan is excluded from this regression. Standard errors in parentheses. *** $p < 0.01$. Source: Authors' estimates.

where $\tilde{T}_{i,t}$ is the deviation of temperature from its long run mean (see equation 2.1) and $X_{i,t}$ is the vector of economic and political control variables included in equation (2.2).¹⁶ The terms ω_i and η_t are country and time fixed effects, respectively. $\nu_{i,t}$ and $u_{i,t}$ are error terms.

The intuition behind this two-step identification approach is related to the basic equation of the CDS spread defined in equation(2.3): If temperature affects sovereign CDS spreads through the debt-to-GDP channel, then the coefficient θ_1 in equation (2.8) should be statistically significant. On the other hand, if the debt limit plays a role in the transmission of the effects of temperature to spreads, then the coefficient ϕ_1 in equation (2.9) should be statistically significant. Of course, both θ_1 and ϕ_1 can be statistically significant, in which case both the debt-to-GDP ratio and the debt limit will be important transmission channels.

Tables 2.6 and 2.7 report estimation results of the first step and second step regressions, respectively. In the first step regression (Table 2.6), we find a positive

¹⁶We exclude the debt-to-GDP ratio from the vector of controls to avoid reverse causality bias in Table 2.7 Column 1. We also do not include it in Column 2 for homogeneity purposes.

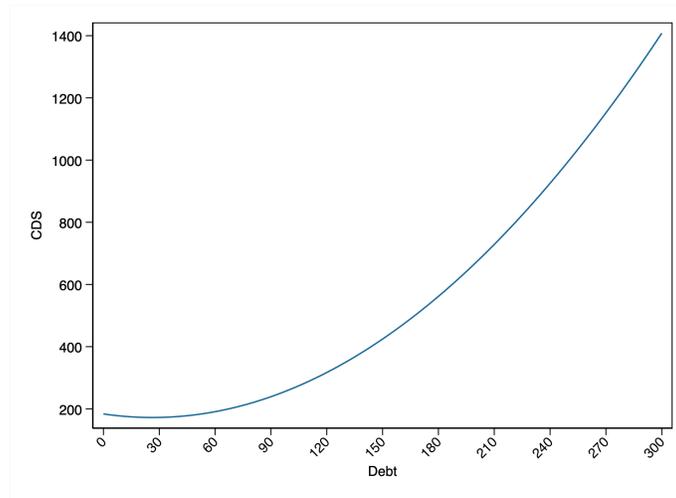


Figure 2.4: CDS - Debt relationship

and non linear significant effect of public debt-to-GDP ratio on CDS spreads. The estimated relationship between the two variables is illustrated in Figure 2.4.

In the second step regression (Table 2.7 Column 1), we find no significant estimated value for the coefficient θ_1 (i.e., -1.69). This indicates that the estimated effect of debt-to-GDP ratio on CDS spread obtained in the first step regression (Table 2.6) is not related to temperature. Therefore, there must exist a debt limit channel through which temperature affects CDS spreads.

The regression of the spread residual (Table 2.7 Column 2) shows that this is indeed the case. We find a positive and significant effect of temperature anomalies on the spread residual. A degree Celsius upward shift in temperature, relatively to the long run mean, increases the CDS spread residual by 21.27 basis points, which is quite close to the estimated effect of temperature on spreads reported in Table 2.2 (i.e., 22.71 basis points). Two differences must be reminded: Table 2.7 excludes Japan and does not include neither the Debt-to-GDP ratio nor its squared value as it is the case in Table 2.2 leading to a difference in the estimated coefficients.

These findings show that increases in temperature lead to increased sovereign CDS spreads through the debt limit channel but not the debt-to-GDP one. In the next section, we investigate how the debt limit channel is impacted following

Table 2.7: Debt channel: Second step regression

Dependent variable:	Predicted CDS spread ($\hat{S}_{i,t}$)	CDS residual ($\hat{\varepsilon}_{i,t}$)
	(1)	(2)
Climate		
Temperature	-1.69 (1.21)	21.27** (10.66)
Controls		
Log of GDP	-40.98*** (9.38)	82.32 (65.40)
Real GDP growth rate	-0.35 (0.34)	-15.23*** (3.96)
Inflation rate	-0.27* (0.14)	17.23*** (3.39)
Primary balance/GDP	0.39 (0.24)	0.14 (2.17)
Reserves/GDP	0.00*** (0.00)	0.00 (0.00)
Credit ratings	-9.25*** (0.75)	-49.37*** (5.28)
Constant	607.82*** (77.97)	-477.94 (538.78)
Observations	1097	1097
Countries	75	75
Country fixed effects	Yes	Yes
Time fixed effects	Yes	Yes
R^2	0.897	0.713
Adjusted R^2	0.887	0.685

Notes: The table reports estimation results of equations (2.8) and (2.9). Robust, country clustered standard errors in parentheses. The key explanatory variables is temperature deviation from its long mean in 1900-1950. Temperature is measured in °C. Japan is excluded from this regression. We do not use the debt-to-GDP ratio neither its squared value as controls to avoid reverse causality bias.* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Source: Authors' estimates.

a temperature shock.

2.4.4.2 Debt limit channel

The identification of the debt limit channel of the effects of temperature on CDS spreads is quite challenging because the debt limit is a theoretical construct that is not observed in the data. However, from equation (2.4), we know that the debt limit, $b_{i,t}^{\text{lim}}$, depends positively on both the growth rate, $a_{i,t}$, and the maximum primary surplus, $s_{i,t}^{\text{max}}$. Therefore, we can assess the effects of temperature on these two variables. The debt limit channel of temperature can thus be decomposed in two parts, as indicated in Figure 2.3: a growth effect and a maximum surplus effect.

Growth effects of temperature

We are interested in the (potential) effects of temperature on the growth rate of real GDP. To address this issue, we do not make any theoretical assumption on the (aggregate) output function *à priori*, nor how this function is related to changes in temperature. Instead, we take an agnostic approach, guided by econometric methods. Theoretically, temperature can have various effects on output, which can be both positive or negative, transitory or permanent.¹⁷

We start our analysis by testing the presence of unit roots in the log GDP. We use the Levin- Lin-Chu test to detect unit roots at the panel level.¹⁸ The tests suggest that log GDP is integrated of order one, that is, I(1). Therefore, we take the first difference, which corresponds to the growth rate, and find evidence for stationarity of this later.

We proceed and estimate a dynamic growth model:

$$a_{i,t} = \rho_0 + \rho_1 a_{i,t-1} + \rho_2 \tilde{T}_{i,t} + \phi' X_{i,t} + \omega_i + \varepsilon_{i,t}, \quad (2.10)$$

where $a_{i,t} \equiv \Delta \ln \text{real GDP}_{i,t} \times 100$ is the percentage growth rate of real GDP

¹⁷See [Burke, Hsiang and Miguel \(2015\)](#) for the possibility of positive and negative effects of temperature on output.

¹⁸Results available upon request.

Table 2.8: GDP growth rate and Temperature (Arellano-Bond)

Dependent variable:	Real GDP growth rate (%)			
	(1)	(2)	(3)	(4)
Climate				
Temperature	−0.488*** (0.160)	−0.489*** (0.164)	−0.475*** (0.165)	−0.437** (0.180)
Controls				
Growth rate (t-1)	0.174** (0.068)	0.174** (0.069)	0.169** (0.070)	0.178** (0.069)
Growth rate (t-2)		−0.011 (0.054)	−0.011 (0.054)	−0.011 (0.057)
Growth rate (t-3)			−0.003 (0.024)	0.007 (0.025)
Growth rate (t-4)				−0.025 (0.031)
Inflation	−0.051** (0.022)	−0.051** (0.022)	−0.053** (0.023)	−0.050** (0.021)
Trade openness	0.033** (0.013)	0.033** (0.013)	0.033** (0.014)	0.033** (0.014)
Foreign direct investment	−0.003 (0.013)	−0.003 (0.013)	−0.003 (0.012)	−0.004 (0.012)
Gross capital formation	0.297*** (0.068)	0.302*** (0.077)	0.300*** (0.077)	0.311*** (0.085)
Population (t-1)	−0.000 (0.000)	−0.000 (0.000)	−0.000 (0.000)	−0.000* (0.000)
Primary balance	0.226*** (0.055)	0.227*** (0.058)	0.230*** (0.058)	0.217*** (0.058)
Political stability	0.187 (0.659)	0.192 (0.662)	0.187 (0.661)	0.013 (0.682)
Current account	0.030 (0.054)	0.030 (0.055)	0.032 (0.055)	0.040 (0.059)
Constant	−6.289*** (1.974)	−6.299*** (2.003)	−6.169*** (2.029)	−5.823*** (2.039)
Observations	1379	1378	1369	1305
Countries	76	76	76	76

Notes: Regression results of unbalanced panel data. Robust, standard errors in parentheses. The dependent variable is the yearly real growth rate of GDP in percentage point. Growth is measured as the first difference of log GDP. The key explanatory variable is the deviation of temperature from its long run mean in 1900-1950. Temperature is measured in °C. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Source: Authors' estimates.

of country i in year t , $\tilde{T}_{i,t}$ is the deviation of temperature from its long run mean defined in equation (2.1), and $X_{i,t}$ is a vector of economic and political control variables. ω_i is a country fixed effect and $\varepsilon_{i,t}$ is an error term.

Equation (2.10) is a flexible way to account for potential autocorrelation in the growth process. As discussed in Section 2.4.3, one can have a time-varying debt limit if the growth rate is autocorrelated, that is, if the coefficient ρ_1 in (2.10) is statistically significant.

Table 2.8 presents estimation results of equation (2.10). We find a strong negative effect of temperature on the growth rate for all specifications: a degree Celsius increase of temperature relative to its long run mean decreases the current period growth rate by 0.437-0.489 percent on annual basis. This effect is statistically and economically significant. Interestingly, we note that the estimated value for the coefficient ρ_1 is also statistically significant for all specifications. Since growth is positively autocorrelated, temperature affects, not only the current period growth rate, but also future growth rates. Moreover, the fact that (log) GDP is I(1) implies that temperature has a permanent effect on the level GDP.

This finding confirms the existence of a debt limit channel of temperature through the growth rate: an increase in temperature has a negative effect on current and future growth rates, which lowers the country's debt limit. A decrease in the debt limit increases a country's probability of default and thus its CDS spread.

(Maximum) Surplus effects of temperature

We now focus on the effects of temperature anomalies on the maximum surplus. A main challenge of this exercise is that the maximum primary surplus is not observed in the data.¹⁹ However, although the maximum primary surplus is not observable, we have historical data on countries' primary surplus. We therefore use the primary surplus and relate this variable to temperature anomalies. The

¹⁹This is one reason why the debt limit is not observed in the data. However, even if one had empirical data on the maximum primary surplus, the debt limit remains an unobserved theoretical object. See [Diarra, Guillard and Kempf, 2022](#).

underlying assumption of this procedure is that the maximum primary surplus of a country is related to its historical fiscal behavior. This assumption is in line with the practice in the literature and in debt sustainability analyses by institutions like the IMF and the World Bank, which consists to use historical primary surpluses of countries to estimate the maximum primary surplus.²⁰

In line with the literature of debt sustainability and sovereign default (Bohn 1998; Davig, Leeper and Walker 2011; Bi 2012; Ghosh et al. 2013; Lorenzoni and Werning 2019), we consider a specification where the primary surplus reacts positively to past debt, augmented with temperature anomalies:

$$s_{i,t} = \delta_0 + \delta_1 \tilde{T}_{i,t} + \delta_2 b_{i,t-1} + \mathbf{\Lambda}' X_{i,t} + \omega_i + \eta_t + \varepsilon_{i,t}, \quad (2.11)$$

where $s_{i,t}$ is the primary surplus in percentage of GDP observed for country i in year t , $\tilde{T}_{i,t}$ is the deviation of temperature from its long run mean, and $b_{i,t-1}$ is the ratio of debt-to-GDP in the previous year. $X_{i,t}$ is a vector of economic and political control variables used in previous studies (see Bohn 1998; Ghosh et al. 2013). To account for potential non-linearity between the primary surplus and the debt-to-GDP ratio, as suggested by Ghosh et al. (2013), we include in the control vector $X_{i,t}$ the square and cubic terms of past debt, that is: $b_{i,t-1}^2$ and $b_{i,t-1}^3$. The other control variables are standard in the literature. Appendix 2.6 presents a detailed definition of these variables.

Table 2.9 reports estimation results of equation (2.11). We do not find any significant effect of temperature anomalies on the primary surplus. Since temperature anomalies do not affect the primary surplus, we conclude that the maximum primary surplus is not affected neither. As a result, temperature does not affect the debt limit through the maximum primary surplus. Thus the (maximum) primary surplus does not play any role in the transmission of the estimated effects of temperature to CDS spreads found in Section 2.4.1.

²⁰See, for example, Ghosh et al. (2013) and IMF (2011, 2018).

Table 2.9: Primary surplus and Temperature (1999-2017)

Dependent variable:	Primary surplus (% of GDP)		
	(1)	(2)	(3)
Climate			
Temperature	-0.12 (0.20)	-0.12 (0.20)	-0.12 (0.20)
Controls			
Debt-to-GDP (t-1)	0.05*** (0.01)	0.06*** (0.02)	0.08* (0.04)
Debt-to-GDP squared (t-1)		-0.00 (0.00)	-0.00 (0.00)
Debt-to-GDP cubic (t-1)			0.00 (0.000)
Output gap	0.19*** (0.04)	0.20*** (0.04)	0.20*** (0.04)
Expenditure (t-1)	-4.30*** (1.45)	-4.49*** (1.48)	-4.48*** (1.48)
Inflation	0.01 (0.03)	0.01 (0.03)	0.00 (0.03)
Governance effectiveness	1.47* (0.81)	1.58* (0.81)	1.54* (0.83)
Ratings	0.10	0.08	0.07
Trade openness	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)
Constant	118.44*** (42.62)	123.55*** (43.50)	123.51*** (43.54)
Observations	1289	1289	1289
Countries	76	76	76
Country fixed effects	Yes	Yes	Yes
Time fixed effects	Yes	Yes	Yes
R^2	0.465	0.466	0.466
R^2 -adjusted	0.419	0.420	0.419
AIC	6,836	6,836	6,838
BIC	7,363	7,368	7,375

Notes: Regression results of unbalanced panel data. Robust, country clustered standard errors in parentheses. The dependent variable is the primary surplus in percentage of GDP. The key explanatory variable is the deviation of temperature from its long run mean in 1900-1950. Temperature is measured in °C. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Source: Authors' estimates.

2.5 Conclusion

We estimate the effects of temperature anomalies— the deviation of temperature from its long run mean— on sovereign default risk, proxied by sovereign CDS spread. We use a large panel dataset covering 76 developing and advanced countries over the period 1999-2017. We consider CDS spreads at one, three, five and ten-year maturities and find that a higher temperature, relatively to the long run mean, increases sovereign CDS spreads. Moreover, the longer the maturity of the CDS spreads the larger the impact of temperature on spreads, suggesting that sovereign creditors price climate risk, when investing over both the short, medium and long horizons.

To better understand this finding, we build on an equilibrium pricing equation of sovereign debt present in most theoretical models of sovereign default to isolate the key transmission channels through which temperature can affect CDS spreads. We document the existence of a “debt limit channel” of temperature: a higher temperature has a negative effect on future growth rates, which lowers the country’s debt limit— the maximum debt-to-GDP ratio it can sustain without defaulting. As a result, the probability of default increases, leading to higher CDS spreads.

This debt limit channel can roughly explain all the estimated effect of temperature anomalies on sovereign CDS spreads. We show that the growth rate play a crucial role in this mechanism. In particular, the debt-to-GDP ratio and the primary surplus do not play any role in the transmission of the effects of temperature anomalies to spreads.

Our findings have interesting implications for the policy responses to climate change in the context of high public debt-to-GDP ratios and limited fiscal space available for countries. Our identification of the key mechanisms suggests that climate risk must be taken into account in the assessment of public debt sustainability.

2.6 Appendix: Data

Table 2.10: Definition of variables and data sources.

Variable	Definition	Source
Current account	Current account to GDP (% of GDP)	IMF (WEO, October 2020)
CDS spread	Credit default swap spread (basis point)	Macrobond
GDP	Gross Domestic Product constant (national currency)	World Bank (WDI)
GDP per capita	Gross Domestic Product per capita constant (national currency)	World Bank (WDI)
Debt-to-GDP ratio	General government gross debt to GDP ratio	IMF (WEO, October 2020)
Domestic credit t	Domestic credit provided by banking sector (% of GDP)	World Bank (WDI)
Foreign direct investment	Foreign direct investment, net inflows (% of GDP)	World Bank (WDI)
Gross capital formation	Gross capital formation (annual % growth)	World Bank (WDI)
Governance effectiveness		
Inflation rate	Average consumer prices (%)	IMF (WEO, October 2020)
Population	Total population of the country	World Bank (WDI)
Primary balance	General government primary fiscal balance (% of GDP)	IMF (WEO, October 2020)
Polity index	Political risk index	Polity IV (Center for Systemic Peace)
Ratings	Sovereign credit ratings	Fiscal space database (World Bank)
Temperature	Mean annual temperature deviation from mean over 1900-1950 (degree Celsius)	Matsuura and Willmott (2019) Terrestrial Air Temperature and Precipitation: Monthly and Annual Time Series (1900-2017) v.5.01
Trade openness	Trade openness (imports+exports in % of GDP)	World Bank (WDI)

Table 2.11: Countries by region

<u>East Asia and Pacific</u>	United Kingdom
Australia	<u>Latin America and Caribbean</u>
China	Brazil
Indonesia	Chile
Japan	Colombia
Korea	Dominican Republic
Malaysia	Ecuador
New Zealand	Guatemala
Philippines	Jamaica
Vietnam	Mexico
<u>Europe and Central Asia</u>	Panama
Austria	Peru
Belarus	Uruguay
Belgium	<u>Middle East and North Africa</u>
Bulgaria	Cyprus
Croatia	Egypt
Czech Republic	Iraq
Denmark	Israel
Estonia	Jordan
Finland	Kuwait
France	Lebanon
Germany	Morocco
Greece	Oman
Hungary	Saudi Arabia
Iceland	Tunisia
Ireland	Turkey
Italy	United Arab Emirates
Kazakhstan	<u>North America</u>
Latvia	Canada
Lithuania	United States
Netherlands	<u>South Asia</u>
North Macedonia	India
Norway	Pakistan
Poland	<u>Sub-Saharan Africa</u>
Portugal	Angola
Romania	Ethiopia
Russia	Kenya
Serbia	Namibia
Slovenia	Nigeria
Spain	Rwanda
Sweden	South Africa
Ukraine	

Chapter 3

Sovereign Defaults in a World of Climatic Disasters: The Expectations Channel

3.1 Introduction.

The potential for large economic shocks in explaining assets prices and risk premia has received a great deal of attention since the seminal work of Barro (2006). This work sparked a large literature that emphasized the crucial role of the prospects of rare events, such as wars, major conflicts or economic crises, to explain several macro-finance puzzles.¹ In this regard, natural disasters may be equally important for risk premia, although their economic impacts are often less severe than major conflicts and crises considered by Barro (2006).²

This paper analyses the link between sovereign default risk and the prospects of natural disasters, especially climate-related ones. It focuses on the expectations channel linking sovereign default and the risk of climatic disasters.

This topic is important in at least three regards. First, *ex-post*, climatic disasters appear especially salient in light of some recent sovereign default and debt

¹See Gabaix (2008), Gourio (2012), and Wachter (2013).

²Barro (2006) define *rare disasters* as major political or economic events that cause at least 15 percent of economic collapse.

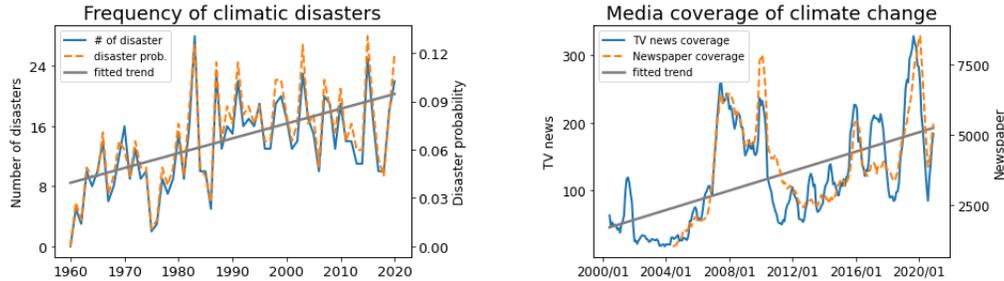


Figure 3.1: Note: The left panel shows the frequency of climatic disasters (left axis) and estimated disasters probabilities (right axis); The right panel shows monthly averages of media coverage of climate change by major TV stations and newspapers; Sources: EM-DAT, Media and Climate Change Observatory, and author’s computations.

restructuring episodes. Notable examples are Dominican Republic 1998, Grenada 2004, Antigua & Barbuda 2004 and 2009.³ Second, the frequency of climatic disasters has continuously increased over the past few decades, as illustrated in the left panel of Figure 3.1, and this trend is expected to accelerated with climate change.⁴ This may worsen fiscal space of countries, leading to an increase in the risk premium on their public debt. Third, climate change and related-disasters are receiving a growing attention of people, as reflected by the media coverage of this topic over the recent period (right panel of Figure 3.1). To the extent that media coverage reflects the perception of the general public, including creditors, one might expect climatic disasters to matter for their behavior and decisions.⁵

To address this issue, I develop a tractable model of sovereign default that allows for time-varying probability of climatic disasters. In the model, the probability of disaster is a deterministic function of time with a linear trend (Figure

³See [International Monetary Fund \(1999a\)](#) and [Asonuma et al. \(2018\)](#). Other default episodes related to climatic disasters are Moldova and Suriname which defaulted respectively in 1992 and 1998 following severe droughts ([International Monetary Fund, September 1999](#); [de Jong et al., 2000](#)). Ecuador defaulted in 1997 just a few months after floods caused major power shortages ([Sturzenegger and Zettelmeyer, 2006](#)).

⁴See [Intergovernmental Panel on Climate Change \(2007; 2021\)](#).

⁵Using a recent large scale survey for the United States, [Dietrich, Müller and Schoenle \(2022\)](#) confirms this intuition. The authors find that respondents to the survey do expect future increases in the probability of costly climate-related disasters, with a median probability of 5%. They also find that expected probability of disasters varies markedly with various individual characteristics of respondents, the degree of media consumption, and exposition to past disasters.

3.1).⁶ The model is purposely kept tractable, so that its main mechanisms can be characterized analytically.

Succinctly, I find the following: the default ratio– the maximum debt-to-GDP ratio that a country can be sustain without defaulting– is decreasing, nonlinear in the probability of disasters. Second, I show that different expectations of creditors on disaster risk can have very different effects on a country’s default ratio. Finally, I show that, in the presence of disaster risk, sovereign defaults can occur even in a very favorable environment with low real risk free below the growth rate.

The paper is organized as follows. Section 3.2 provides a brief review of the relevant literature. Section 3.3 presents the model. Section 3.4 characterizes the equilibrium default ratio. Section 3.5 conducts calibration and simulations to illustrate the role of creditors’ expectations on disasters. Section 3.6 extends the analysis to situations with low real risk free rate below the growth rate and uncertainty about the disaster probability. Section 3.7 concludes.

3.2 Related literature.

This paper combines the literature of sovereign default and a growing literature that considers the macroeconomic impacts of climate-related disasters. I briefly review important references that are relevant for my analysis.

Noy (2009), Lis and Nickel (2010), Loayza et al. (2012), and Botzen, Desch- enes and Sanders (2019) provide comprehensive reviews of the literature on the macroeconomic impacts of natural disasters, including those that are not directly related to climate change such as geophysical (earthquakes, mass movements, vol- canic activity) and biological (epidemics) disasters. This paper focuses on climatic disasters (floods, storms, droughts, landslides, wildfires and extreme temperature) whose frequency is projected to increase with climate change.

The effects of climatic disasters on sovereign default risk has received little attention so far. A few exceptions are Mallucci (2020) and Phan and Schwartzman

⁶In an extension, I also consider a situation where there is uncertainty about the disaster probability and creditors engage in Bayesian learning.

(2021) who introduce hurricane shocks in the strategic default framework à la Eaton and Gersovitz (1981).⁷ In their framework, the optimizing government can choose each period to repay its outstanding debt or to default, in which case it will face an exogenous cost. Surprisingly, Mallucci (2020) finds that the introduction of hurricane shocks in the strategic default model leads to lower debt levels. At the same time, interest rates spreads increase because the optimizing government chooses to default more often, despite the issuance of low debt levels. Phan and Schwartzman (2021) emphasize the role of disaster insurance and disaster-indexed bonds in the post-disaster recovery path and the government’s incentive to default.

In contrast to these papers, I abstract from strategic features and model sovereign defaults as “excusable events”, following Grossman and Huyck (1988). This type of default occurs only when the government is unable to get sufficient fiscal and debt issuance revenue to repay due debt.⁸ Second, in contrast to Mallucci (2020) and Phan and Schwartzman (2021) who assume a constant disaster probability, my framework allows for time-varying disaster probability and it emphasizes the role of creditors’s expectations of disaster risk, independently from the realizations of disaster shocks.

The expectation channel of climatic disasters has been less studied so far. An exception is a parallel paper by Dietrich, Müller and Schoenle (2022). The authors conduct a survey of a representative sample of U.S. households and find that respondents expect a high probability of costly disasters due to climate change. Incorporating this insight in a New Keynesian model, they find that disaster risk lowers the natural interest rate and contributes to business cyclical fluctuations. My paper extends the role of disaster expectations to the literature of sovereign default, contrasting different types of expectations about disaster risk.

This paper is also related to the literature of *rare disasters* that emerged following the seminal works of Rietz (1988) and Barro (2006). This literature emphasizes that the prospects of growth collapses following unlikely events such

⁷The empirical literature on this topic is also limited. Klomp (2015) is among the rare papers in this literature.

⁸Important references in this line are Bi (2012), Ghosh et al. (2013), Collard, Habib and Rochet (2015), Lorenzoni and Werning (2019), and Diarra, Guillard and Kempf (2022).

as wars and severe economic crises can explain several puzzles in macro-finance, in particular the high risk premia observed in stock markets. I extend this literature to the case of climate-related disasters and the linkage with countries' default risk premia.

3.3 The model.

The model builds on our previous work [Diarra, Guillard and Kempf \(2022\)](#). I expand this framework by introducing a time-varying probability of disaster in a tractable way so that the key mechanisms of the model can be analyzed analytically.

3.3.1 Environment.

Consider a small open economy with an infinite horizon. Time is discrete with periods $t = 0, 1, 2, \dots$ ⁹ The economy has access to international financial markets, which allow perfect coverage against risk, and investors behave as risk-neutral agents.

Let Y_t denote the country's GDP at date t and $\tilde{a}_t \equiv Y_t/Y_{t-1}$ the (gross) growth rate of GDP at this date. Each period the country may experience a climate-related disaster which occurs with probability p_t . When a disaster occurs in period t , the growth rate is negatively affected. [Assumption 3](#) formalizes the stochastic shocks in the economy.

Assumption 3.

1. \tilde{a}_t evolves randomly according to:

$$\tilde{a}_t = \begin{cases} a_t & \text{with probability } 1 - p_t \\ u a_t & \text{with probability } p_t, \end{cases} \quad (3.1)$$

⁹The discrete time framework is the standard framework in macroeconomic models.

where u is the severity of the disaster shock which is assumed to be constant and satisfies $0 < u < 1$; a_t is an *i.i.d.* random variable with a cumulative distribution function $G(a)$ and a density function $g(a)$, both defined on the interval $[0, +\infty)$, and $E(a) \equiv \bar{a} < \beta^{-1}$, where $\beta^{-1} = 1 + r$ is the risk-free real gross interest rate;

2. the density function $g(a)$ satisfies $\int_0^x ag(a) da > x^2g(x)$;
3. the probability of climatic disasters is a deterministic function of time:

$$p_t = \min(\alpha_0 + \alpha_1 t; 1), \quad (3.2)$$

where $\alpha_0 \geq 0$ and $\alpha_1 \geq 0$ are two parameters.

According to Assumption 3.1, the growth rate of output \tilde{a}_t is *i.i.d.* This implies that the severity of disaster u has a permanent effect on output.¹⁰ Assumption 3.2 is a regularity condition on the density function of a_t and it is introduced for analytical purpose only. This assumption essentially states that the density function of the growth rate decreases rapidly.¹¹ Assumption 3.3 is a tractable way to capture the increasing trend in the probability of climatic disasters, as illustrated in Figure 3.1. In Section 3.6, I consider a specification where the probability of disasters is random and creditors learn about it through Bayesian learning.

3.3.2 Creditors.

Creditors are risk-neutral¹² and have access to a risk-free asset that pays an interest rate r . They price government's debt taking into account the possibility of

¹⁰Barro (2006) also considers an i.i.d growth rate with constant disaster severity. In an earlier version of the paper, I consider the case where u is a random variable and the findings are qualitatively similar.

¹¹Natural candidates for this condition are probability distributions in the exponential family, such as the exponential distribution with density $g(a) = \lambda e^{-\lambda a}$. As we will see in Section 3.8.2, the widely used log-normal distribution also satisfies this condition under some parameter choices.

¹²The assumption of risk-neutral creditors is standard in theoretical models of sovereign default. One could of course assume risk-averse creditors. This would lower the discount factor

climatic disasters and government's defaults.

Let $\beta \equiv (1 + r)^{-1}$ be the discount factor of the economy. Assuming perfect international diversification of risks and rational expectations, the sovereign bond price at date t , denoted by q_t , writes as

$$q_t = \beta E_t h_{t+1}, \quad (3.3)$$

where h_{t+1} is the fraction of the end-of-period value that will be repaid in a given state of nature in period $t + 1$, with $h_{t+1} = 1$ if there is no default, and $h_{t+1} < 1$ in case of default.

3.3.3 Government.

The government has access to international financial markets where it can issue one-period maturity debt of facial value B_t , which is reimbursed at date $t + 1$. Let S_t denote the government's maximum primary surplus. It is assumed to evolve proportionally to output: $S_t = \hat{s}Y_t$, where \hat{s} is the maximum surplus-to-output ratio.¹³ Note however that even though \hat{s} is constant the primary surplus in level S_t is not, as this latter depends on output. Since output is affected by climatic disasters, so is S_t .¹⁴

The instantaneous government budget constraint writes:

$$q_t B_t = h_t B_{t-1} - \hat{s}Y_t, \quad (3.4)$$

with $h_t \in [0, 1]$. This parameter takes the value of 1 if there is no default in t and a lower value, given by a debt recovery rule, when the government is unable to

β , and thus amplify the expectations effects of disaster risk. Therefore, the results uncovered in the risk-neutral framework considered here can be seen as a lower bound with respect to a setting with risk-averse creditors.

¹³the constant character of \hat{s} is for simplicity only.

¹⁴One may wonder about the role of fiscal policy, in particular for the post-disaster recovery. I do not explore this mechanism here but instead focus on the expectations channel of disasters risk. Note however that since \hat{s} is, by definition, the maximum primary surplus that can be reached and this maximum is unlikely to increase following a disaster shock, my results can be seen as a lower bound with respect to the case where fiscal policy plays a role.

meet its financial obligations in t and thus defaults.

The government's budget constraint is subject to a no-Ponzi condition:

$$\lim_{T \rightarrow \infty} E_t \beta^T h_{t+T} B_{t+T-1} \leq 0. \quad (3.5)$$

3.3.3.1 Default and debt recovery rule.

Let Ω_t denote the maximum revenue that the Treasury can collect at date t to repay debt due at this date, B_{t-1} . It defines as the sum of the maximum primary surplus in period t and the maximum funding that the government can obtain in the same period by emitting new debt on financial markets.

Default occurs at date t if and only if $B_{t-1} > \Omega_t$. In this regard, we can refer to Ω_t as the period t default threshold, as in [Diarra, Guillard and Kempf \(2022\)](#). In case of default at t , the government reimburses a fraction $h_t < 1$ of its due debt. Therefore, the after-default (redeemed) debt level is $h_t B_{t-1}$. Following [Diarra, Guillard and Kempf \(2022\)](#), I assume that the fraction of repaid debt h_t is defined according to a default rule with the following specification:

$$h_t = \begin{cases} \mathbf{h} \cdot \Omega_t / B_{t-1} & \text{if } B_{t-1} > \Omega_t \\ 1 & \text{else,} \end{cases} \quad (3.6)$$

where $\mathbf{h} \in [0, 1]$ is a parameter. The parameter \mathbf{h} can be interpreted as the maximum recovery rate that creditors can expect in case of a sovereign default. Inversely, we can interpret $1 - \mathbf{h}$ as the minimum “haircut” on public debt, i.e the fraction of debt-to-GDP ratio that is written off in case of default. The higher \mathbf{h} the lower the haircut.

3.4 Equilibrium default ratio and expectations of disaster risk.

In this Section, I characterize the equilibrium default ratio and analyze how it is related to creditors' expectations about disaster risk. I consider three different

cases: a simple case with constant disaster probability, a naive expectation of disaster risk where creditors revised the probability of disaster but assume that it has no trend, and a third situation where creditors are forward-looking and assume an increasing trend in the disaster probability.

3.4.1 Debt valuation and the default ratio.

Let us first address the valuation of public debt in the presence of disaster risk. For convenience, it is useful to express all quantities in terms of fraction of output.

Define $b_t = B_t/Y_t$ as the debt-to-GDP ratio and $\omega_t = \Omega_t/Y_t$ as the default ratio. Using these definitions, equations (3.7) and (3.6) rewrite respectively:

$$q_t b_t = \frac{h_t b_{t-1}}{\tilde{a}_t} - \hat{s} \quad (3.7)$$

$$h_t = \begin{cases} \mathbf{h} \frac{\tilde{a}_t \omega_t}{b_{t-1}} & \text{if } b_{t-1} > \tilde{a}_t \omega_t \\ 1 & \text{else.} \end{cases} \quad (3.8)$$

Taking the sequence $\{\omega_t\}$ as given, equations (3.7) and (3.8), together with (3.3) and (3.5), are sufficient to analyze the valuation of public debt and the dynamics of emitted debt-to-output ratio b_t . Notice that the sequence of default ratios $\{\omega_t\}$ is endogenous and ultimately needs to be obtained. We will see below how this sequence obtains in equilibrium.

3.4.1.1 Market value of debt.

Take the sequence $\{\omega_{t+1}\}$ as given. Then using (3.1) and (3.8) the price of public debt defined in (3.3) rewrites

$$q_t = \beta (1 - E_t p_{t+1}) \left[1 - G \left(\frac{b_t}{\omega_{t+1}} \right) + \mathbf{h} \frac{\omega_{t+1}}{b_t} \int_0^{b_t/\omega_{t+1}} adG(a) \right] + \beta E_t p_{t+1} \left[1 - G \left(\frac{b_t}{u \omega_{t+1}} \right) + \mathbf{h} \frac{u \omega_{t+1}}{b_t} \int_0^{b_t/u \omega_{t+1}} adG(a) \right]. \quad (3.9)$$

It is a function of the ratio of public debt emitted on financial markets b_t , a risk premium linked to the probabilities of expected defaults in the future, based on the ratio ω_{t+1} , the probability law of a_t , the probability of disaster p_{t+1} , and the debt recovery rate to be applied in case of a sovereign default.

Let $v_t = q_t b_t$ denote the market value of debt, again as a fraction of GDP. From (3.9), it rewrites as follows:

$$v_t = \beta (1 - E_t p_{t+1}) \left[\left(1 - G \left(\frac{b_t}{\omega_{t+1}} \right) \right) b_t + \mathbf{h} \omega_{t+1} \int_0^{b_t/\omega_{t+1}} adG(a) \right] \\ + \beta E_t p_{t+1} \left[\left(1 - G \left(\frac{b_t}{u \omega_{t+1}} \right) \right) b_t + \mathbf{h} u \omega_{t+1} \int_0^{b_t/u \omega_{t+1}} adG(a) \right]. \quad (3.10)$$

The term on the first line of (3.12) is the market value of public debt if there is no disaster at date $t + 1$. The term on the second line is the market value of debt if there is a disaster at $t + 1$.

Let us define

$$\delta_t \equiv \frac{b_t}{\omega_{t+1}}. \quad (3.11)$$

From the default rule (3.8) and, again, taking ω_{t+1} as given, the ratio δ_t can be interpreted as the minimum growth rate necessary to avoid default at $t + 1$. Since the ratio ω_{t+1} is subject to the prospect of future disasters so is δ_t . The higher δ_t the higher the probability of default.

Using definition (3.11), equation (3.12) can be rewritten as

$$v_t = \beta \chi(\delta_t; E_t p_{t+1}) \cdot \omega_{t+1}, \quad (3.12)$$

where

$$\chi(\delta_t; E_t p_{t+1}) \equiv (1 - E_t p_{t+1}) \left[(1 - G(\delta_t)) \delta_t + \mathbf{h} \int_0^{\delta_t} adG(a) \right] \\ + E_t p_{t+1} \left[(1 - G(\delta_t/u)) \delta_t + \mathbf{h} u \int_0^{\delta_t/u} adG(a) \right]. \quad (3.13)$$

According to (3.12), the market value of public debt is linearly increasing in

the next period default ratio ω_{t+1} . Notice that $\chi(\delta_t; E_t p_{t+1})$ is a potentially non monotone function. We will see in the next subsection how this function can be used to derive the equilibrium default ratio.

3.4.1.2 Equilibrium default ratio.

We can now characterize the default ratio ω_t . By definition it is the sum of the maximum primary surplus (as a fraction of GDP) and the maximum borrowing proceeds obtained on financial markets, that is, from (3.12) and (3.13):

$$\omega_t = \max_{\delta_t} \beta \chi(\delta_t; E_t p_{t+1}) \cdot \omega_{t+1} + \hat{s}. \quad (3.14)$$

Let $\Phi(\delta_t; E_t p_{t+1}) \equiv \chi(\delta_t; E_t p_{t+1}) / \partial \delta_t$ denote the first derivative of $\chi(\delta_t; E_t p_{t+1})$ with respect to δ_t . Using this notation, the first order condition to problem (3.14) defines

$$\begin{aligned} \Phi(\delta_t; E_t p_{t+1}) &\equiv (1 - E_t p_{t+1}) \left[1 - (1 - \mathbf{h}) \delta_t g(\delta_t) - G(\delta_t) \right] \\ &\quad + E_t p_{t+1} \left[1 - (1 - \mathbf{h}) \frac{\delta_t}{u} g(\delta_t/u) - G(\delta_t/u) \right] \\ &= 0. \end{aligned} \quad (3.15)$$

Let δ_t^* denote the value of δ_t that solves (3.15). Note that δ_t^* depends on the expected probability of disaster, that is

$$\delta_t^* \equiv \delta^*(E_t p_{t+1}). \quad (3.16)$$

Inserting (3.16) into (3.13), equation (3.14) becomes

$$\omega_t = \hat{s} + \beta \chi(E_t p_{t+1}) \cdot \omega_{t+1}. \quad (3.17)$$

Equation (3.17) is a forward looking dynamic equation. How much funding the government can obtain in the current period depends on how much it can obtain in the next period as the latter determines the opportunities for public

funding. This equation also makes it clear that the default ratio ω_t depends on the prospect of future disasters. I turn to the investigation of this relationship.

3.4.2 Constant expectations of disaster risk.

Let us first consider the situation where the probability of disaster is constant over time, that is $p_t = p \forall t$. Under this assumption, equation (3.17) rewrites

$$\omega_t = \hat{s} + \beta\chi(p) \cdot \omega_{t+1}. \quad (3.18)$$

Iterating (3.18) forward, we obtain a stationary solution if $\beta\chi(p) < 1$, and an infinite solution otherwise. The following proposition formalizes the existence of a unique stationary solution:

Proposition 6. *Under Assumption 3, if $p_t = p$, the function $\chi(p)$ satisfies $\beta\chi(p) < 1$. The equilibrium default ratio defined by (3.18) is locally unique and equal to:*

$$\omega_t = \frac{\hat{s}}{1 - \beta\chi(p)} \equiv \omega_p^*, \forall t. \quad (3.19)$$

The ratio ω_p^ is strictly increasing in \hat{s} and strictly decreasing in p .*

Proof. See Appendix 3.8.1. □

Despite the stochastic nature of this environment, the default ratio is a constant, $\omega_t = \omega_p^* \forall t$, independent from the history of shocks, in particular the realizations of disaster shocks. Notice however that it is affected by the prospect of these latter shocks or, loosely speaking, by their “frequency”, which is assumed to be constant here.

The constant property of the default ratio is related to the fact that here we assume a constant disaster probability, $p_t = p \forall t$.¹⁵ We will see below that this is no longer the case once we allow the probability of disaster to vary over time.

¹⁵Collard, Habib and Rochet (2015) and Diarra, Guillard and Kempf (2022) obtain a similar result for the default ratio, but they abstract from disaster risk, that is $p = 0$ in their framework. I generalize their findings to the case with disaster risk.

3.4.3 Naive expectations of disaster risk.

Suppose that creditors have “naive” expectations about disaster risk in the following sense: they assume that there is no particular trend in the probability of disaster but each period they revise the probability of disaster, disregarding any possible changes in the future. That is, creditors’ expectations about future disaster probabilities are such that

$$\tilde{E}_t p_{t+1} = p_t, \forall t, \quad (3.20)$$

where I use the “tilde” notation to distinguish naive expectations from rational expectations, denoted by E_t .

The probability of disaster p_t in (3.20) can be interpreted as the frequency of disasters observed in period t . Given the relatively low number of disasters that can be observed in single period, an elaborate way to model p_t would be to consider this later as a random variable. I will consider this possibility in Section 3.6. Here I simply assume that naive creditors set p_t to the deterministic (and time-varying) mean frequency of disasters, based on (3.2), while ignoring the time-varying nature of this mean.

Inserting (3.20) into (3.15), equation (3.16) redefines

$$\tilde{\delta}_t^* \equiv \tilde{\delta}^*(p_t). \quad (3.21)$$

Then, using (3.13) and (3.21), equation (3.17) becomes

$$\omega_t = \hat{s} + \beta\chi(p_t) \cdot \omega_{t+1}. \quad (3.22)$$

Iterating on (3.22) as before, we obtain the following solution

$$\omega_t = \frac{\hat{s}}{1 - \beta\chi(p_t)} \equiv \omega_t^{\text{naive}}. \quad (3.23)$$

As for the case of constant disaster probability in Section 3.4.2, the ratio ω_t^{naive}

is a strictly decreasing function of the probability of disaster.

Let \hat{b}_t^{naive} denote the maximum quantity of debt (-to-GDP) that can be emitted under naive expectations of disaster risk. From (3.21) and (3.23), recalling the definition in (3.11), it defines as

$$\hat{b}_t^{\text{naive}} = \tilde{\delta}_t^* \omega_t^{\text{naive}}. \quad (3.24)$$

3.4.4 Forward looking expectations of disaster risk.

Let us now consider the case with forward looking expectations of disaster risk where, as in the naive case, the probability of disaster varies over time, but creditors now anticipate the increasing trend in disaster probability, as specified in (3.2).

Using (3.2), (3.13), and (3.16), equation (3.17) now becomes

$$\begin{aligned} \omega_t &= \hat{s} + \beta \chi(E_t p_{t+1}) \cdot \omega_{t+1} \\ &= \hat{s} + \beta \chi(t+1) \cdot \omega_{t+1}. \end{aligned} \quad (3.25)$$

As it is clear from (3.25), $\chi(\cdot)$ is now a deterministic function of time.

The definition of the disaster probability in (3.2) implies that there exists a finite date $T < \infty$ such that $p_t = 1$ for any date $t \geq T$. Using this property, we can solve equation (3.25) by backward induction, starting from date T .¹⁶

After date T , the probability of disaster is constant and equal to one: $p_T = p_{T+1} = p = 1$. Therefore, the default ratio is defined by (3.19), that is: $\omega_T = \omega_{T+1} = \omega_1^*$. Then, from (3.25) and setting $\omega_T = \omega_1^*$, we can compute the default ratio at date $T - 1$:

$$\omega_{T-1} = \hat{s} + \beta \chi(T) \cdot \omega_1^* \equiv \omega_{T-1}^{\text{forward}}. \quad (3.26)$$

Proceeding with this backward iteration, we obtain the equilibrium default

¹⁶One could also iterate and solve (3.25) forward. This alternative approach would lead to the same path for ω_t , but it is slightly more complicated than the backward induction method pursued here.

ratio under forward looking expectations of disaster risk for all periods $t \leq T - 2$, denoted by $\omega_t^{\text{forward}}$.

Denoting $\hat{b}_t^{\text{forward}}$ the corresponding maximum quantity of debt that can be emitted on financial markets, recalling the definition in (3.11), we have

$$\hat{b}_t^{\text{forward}} = \delta_t^* \omega_t^{\text{forward}}, \quad (3.27)$$

where δ_t^* is defined by (3.2) and (3.16).

3.5 Numerical illustration.

In this Section, I calibrate the model and simulated the model illustrate the link between the default ratio and creditors' expectations about disaster risk. Building on the historical trend in the frequency of climatic disasters, I assess the potential effects of a gradual increase in the probability of disaster on the default ratio.

3.5.1 Estimating α_0 and α_1 .

A starting of point of the numerical analysis is the estimation of the parameters of disaster probability α_0 and α_1 . A natural strategy to estimate these parameters is to use the historical frequency of climatic disasters. For this purpose, I use the Emergency Events Database (EM-DAT).¹⁷ EM-DAT is a global database that records the occurrence of natural events, their monetary damage, the number of deaths they cause and other information such as the location. The database covers all countries in the world and goes back up to 1900. It includes both climate-related events (eg., storms, floods, droughts, landslides, wildfires, extreme temperature) and those that are not directly related to climate (eg., earthquakes, biological and technological events). I consider the period 1960-2019 to avoid recording issues in earlier periods and focus on events that are directly related to climate change since their frequency is predicted to increase over the next decades.

¹⁷<https://www.emdat.be/>

An issue with EM-DAT is that the majority of recorded events cannot be considered as “disasters” as they have negligible economic impacts. A standard approach in the empirical literature on the impacts of natural disasters on economic activity is to set a threshold of monetary damage or a number of deaths above which an event is classified as a disaster. Here I consider a threshold of 95th percentile and classify as a disaster any climatic event that causes a monetary damage or a number of deaths (per population) that is above this threshold. That is, an event is defined as a disaster if it is among the top 5% extremes. The blue curve in Figure 3.1 (left axis) presents the yearly number of disasters observed according to this definition.

Next, I compute the probability of disaster based on the number of disasters observed in each year. Specifically, for a given year, I compute the probability of disaster as the fraction of countries that experience a disaster in that year. The dashed orange curve in Figure 3.1 plots the obtained disaster probabilities.

Finally, I estimate the parameters α_0 and α_1 by fitting a linear trend to probabilities of disaster obtained in the previous step. The associated estimates of α_0 and α_1 are 0.035 and 0.002, respectively. Note that these estimates are common to all countries. Given the relatively small frequency of disaster events, reliable country estimations of the parameters α_0 and α_1 cannot be conducted.

3.5.2 Calibration.

To illustrate the role of disaster expectations, I calibrate the model using data on advanced economies that have experienced climatic disasters in 1960-2019.¹⁸ Appendix 3.8.2 extends the analysis to emerging countries and simulates country specific debt ratios according to the naive and forward looking expectations of disasters.

I consider a log-normal distribution for the (potential) growth rate a_t :

$$\ln a \sim N(\mu, \sigma^2).$$

¹⁸See Table 3.4 in Appendix 3.8.2 for the full list of advanced countries considered.

The last column of Table 3.1 presents the parameter values for the numerical exercises in the next subsection.

Table 3.1: Baseline parameter values (annual basis).

		Emerging	Advanced
Risk-free rate	r	0.0293 ^a	0.0293 ^a
Debt recovery parameter	\mathbf{h}	0.70 ^b	0.93 ^b
Maximum primary surplus	\hat{s}	0.02 ^c	0.04 ^c
Mean of growth rate	μ	0.0367 ^d	0.0256 ^d
Volatility of growth rate	σ	0.0313 ^d	0.0252 ^d
Disaster impact	u	0.98 ^e	0.98 ^e
Disaster probability	α_0	0.035 ^f	0.035 ^f
Disaster probability	α_1	0.002 ^f	0.002 ^f
Length of one period	$t, t + 1$	4 years ^g	4 years ^g

Notes : a : Annualized rate on 4-year-maturity German bonds (1980-2019)

b : Diarra, Guillard and Kempf (2022), c : IMF(2018), d : Historical (1980-2019), e : Hsiang and Jina (2014)

f : Estimates based on historical frequency of large climatic disasters,

g : US debt duration (2010).

The length of one period in the model is set to 4 years.¹⁹ The risk-free rate is set to the average annualized real yield on the 4-year maturity German bonds in 1980-2019. The debt recovery parameter \mathbf{h} is calibrated following Diarra, Guillard and Kempf (2022); the maximum primary surplus \hat{s} is calibrated according to estimates by IMF(2018). The disaster shock u is calibrated in accordance with estimates found in the empirical literature on the impacts of disasters on growth.²⁰ The parameters of disaster probability are calibrated to their estimated values obtained previously. The mean and volatility of the log growth rate are sample averages in 1980-2019.

¹⁹Although the length of one period in the model is 4 years, I will report all results of the numerical exercises on annual basis.

²⁰See, for example, Hsiang and Jina (2014), and Felbermayr and Groschl (2014).

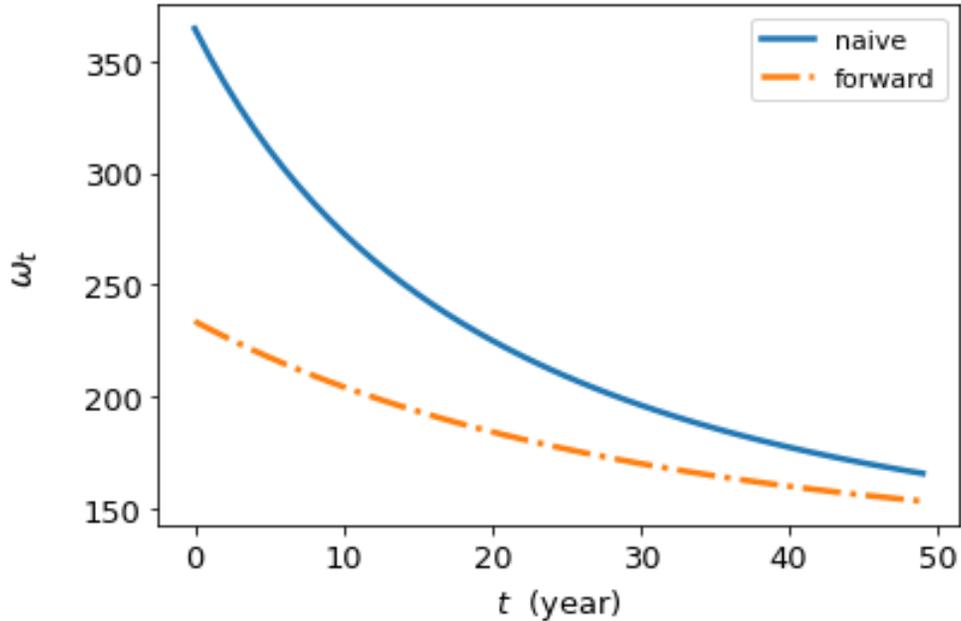


Figure 3.2: Simulated paths of the default ratio (% of GDP).

3.5.3 Simulation results.

I simulate the model over 50 years period. Figure 3.2 illustrates the evolution of the ratios $\omega_t^{\text{forward}}$ and ω_t^{naive} over 50 periods.

We notice that both ratios are decreasing in the probability of disaster. However, $\omega_t^{\text{forward}}$ is much lower than ω_t^{naive} , especially in the early periods. This is because, in contrast to the case of naive expectations of disaster risk, when creditors have a forward looking view they anticipate future increases in disaster probability. Higher probabilities of disasters imply lower expected growth rates, a higher risk premium on public debt and thus a lower default ratio. In contrast, under naive expectations the negative feedback of disaster risk to the default ratio is weaker as creditors ignore future increases in the probability of disasters. Nonetheless, the two ratios converge overtime as creditors with naive expectations become more “aware” of the increasing frequency of disasters, although they do not anticipate this trend.

The figure also highlights the sensitivity of the default ratio to specifications that underlies creditors’ expectations about disaster risk. A sudden change in

creditors expectations of disaster risk, from naive to forward looking ones, can cause a sharp fall in the default ratio, which can lead to a rapid increase in the default risk.

3.6 Extensions.

3.6.1 Sovereign default and disaster risk when $r < g$.

There has been a large debate recently on the sustainability of public debt and the possibility of government default in an environment where the real risk free rate is below the rate of growth, that is when the differential $r - g$ is negative.²¹ This situation, while it is not new (Sergeyev and Mehrotra 2020, Mauro and Zhou 2020), has led some researchers to conclude that the sustainability of public debt is not a real concern, in particular in the near future (see Blanchard 2019).

In this section, I relax the assumption $\bar{a} < \beta^{-1} = 1 + r$ and analyze how the introduction of disaster risk may contribute to the emergence of sovereign default risk, further emphasizing the crucial role of creditors' expectations about disaster risk.

To set the intuition, let us consider the simple case with constant expectations of disaster risk of Section 3.4.2, that is, $p_t = p \forall t$. In this case, the default ratio is defined by (3.19), and in particular the term $\beta\chi(p)$ of this equation, which depends on the distribution of the growth rate of output (see equation 3.13). Consider a situation with a low value of the disaster probability p . Therefore, from equation (3.22), the default ratio is finite if only if the risk free rate and the (mean) growth rate are such that $\beta\chi(p) < 1$, and infinite otherwise. However, since $\chi(p)$ is a decreasing function of p according to Proposition 7, for sufficiently high values of p a finite default ratio may exist even if β is close to one, i.e., if the risk free rate is close to zero and/or the growth rate is very high. The following proposition formalizes this finding:

²¹Here g refers to the (net) real growth rate of output, and r is the real risk-free rate as before.

Proposition 7. *Suppose that the probability of disasters is constant, $p_t = p$. Therefore, the default ratio defined by (3.19) is infinite if only if p is sufficiently low and the risk free rate and the mean growth rate are such that $\beta\chi(p) \geq 1$.*

Proof. The proof of Proposition 7 is immediate from (3.17) and Proposition 7. \square

According to Proposition 7, the possibility of sovereign defaults cannot be always ruled out in the presence of disaster risk, even if the risk free rate remains below the growth rate, that is $r < g$.

Following the same reasoning, this finding generalizes to the case with naive expectations of disaster risk.²² We also obtain a similar result for the case with forward looking expectations of disaster risk by inspecting (3.26). From this equation, we notice that the ratio $\omega_t^{\text{forward}}$ is finite if and only if $\omega_T^{\text{forward}} = \omega_1^* < \infty$. This inequality is verified if only if the risk free and growth rates are such that the function $\chi(p_t)$ satisfies $\beta\chi(1) < 1$ for any $t \geq T$.

Figure 3.3 illustrates this result, considering the cases of naive and forward looking expectations of disaster risk as in Section 3.4. To construct Figure 3.3, I set $r = 1\%$, in contrast to 2.9% in Figure 3.2, while keeping the other parameters the same as before.

The vertical line on the figure indicates the date after which the default ratio under naive expectations of disaster risk becomes finite, that is: $\omega_t^{\text{naive}} < \infty$. After this date, a public default cannot be ruled out. Before this date, the ratio ω_t^{naive} is infinite as the probability of disaster is not high enough to have $\beta\chi(p_t) < 1$. In this case, there is no concerns about sovereign default.

The picture is however different in the case with forward looking expectations of disaster risk. In this case the default ratio $\omega_t^{\text{forward}}$ is always finite and, for sufficiently high initial public debt-to-GDP ratio, a sovereign default can occur at any date t . Compared to Figure 3.2, the ratio $\omega_t^{\text{forward}}$ is now markedly lower than ω_t^{naive} . This finding shows that creditors' expectations about disaster risk plays a crucial role even in an environment with low interest rates.

²²See equations (3.22) and (3.23).

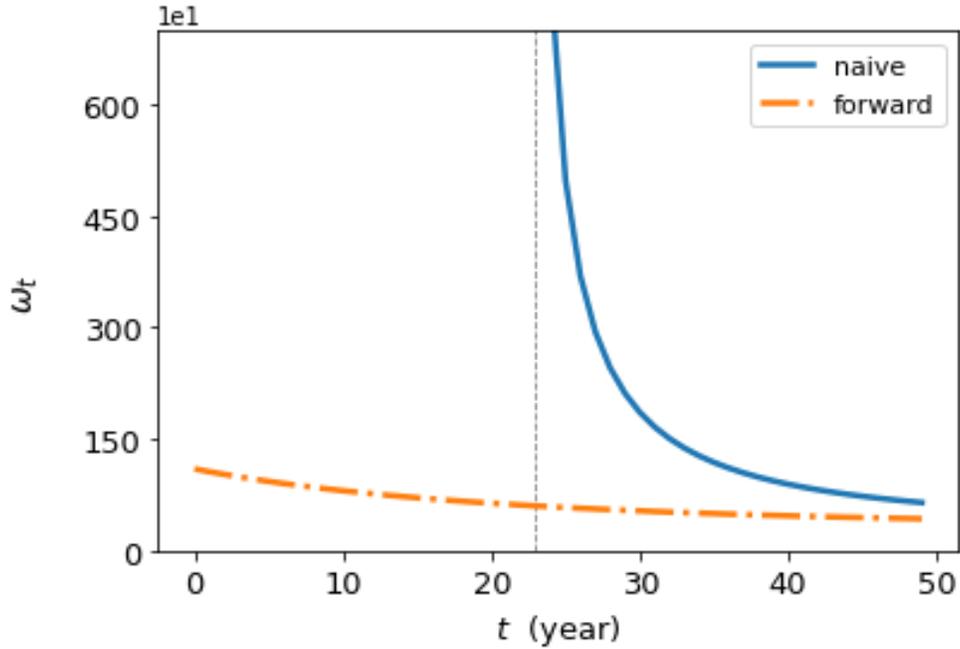


Figure 3.3: Simulated paths of the default ratio (% of GDP) when $r < g$.

3.6.2 Learning from disasters.

The analysis in the previous sections assumed that the probability of disaster is deterministic and observed by creditors. In this section, I relax this assumption and consider a situation where there is uncertainty about the disaster probability and creditors engage in Bayesian learning: they do not observe the probability of disaster p_{t+1} but each period they form a prior belief on the distribution of p_{t+1} and update their belief based on the history of disaster realizations up to the current period.

Let z_t denote a disaster indicator that equal to one if a disaster occurs at t , and zero otherwise. Then creditors' posterior expectations of p_{t+1} , given the observed sequence of disaster realizations, is $E_t(p_{t+1}|z^t)$, where z^t is the disaster history up to date t . Using this notation in (3.13), (3.15) and (3.16), equation

(3.17) becomes

$$\omega_t = \hat{s} + \beta\chi \left(E_t \left(p_{t+1} | z^t \right) \right) \cdot \omega_{t+1}. \quad (3.28)$$

To solve (3.28), I consider the case where the sequence $\{z_t\}$ are i.i.d. random variables,²³ each following a Bernoulli distribution with the same unknown, random parameter²⁴

$$p_{t+1} = \tilde{p} \equiv \Pr(z_{t+1} = 1) \quad \text{and} \quad 1 - \tilde{p} \equiv \Pr(z_{t+1} = 0), \quad (3.29)$$

where I use the tilde to highlight the random feature of the disaster probability.

Given the distribution that underlies the occurrence of disasters, the next step is to decide on creditors' prior belief on the distribution of \tilde{p} , that is, the distribution that they have in mind before observing any disaster (i.e., in the initial period $t = 0$). I assume that creditors' prior belief on the distribution of \tilde{p} at the initial date $t = 0$ is a beta distribution with shape parameters $\alpha > 0$ and $\gamma > 0$.²⁵ Then for each period $t = 1, 2, \dots$, creditors observe whether a disaster occurs (i.e., $z_t = 1$) or not (i.e., $z_t = 0$) and update their prior distribution of \tilde{p} . Therefore, using Bayes's rule, it can be shown that creditors' posterior distribution of \tilde{p} given the disaster history z^t is also a beta distribution with parameters $\alpha + n$ and $\gamma + t - n$, where n is the total number of periods where a disaster occurs up to date t , that is: $n = \sum_{s=1}^t z_s$.²⁶ Symmetrically, $t - n$ is the number of periods with no disaster occurrence.

Creditors' posterior expectations of \tilde{p} is

$$E_t \left(\tilde{p} | z^t \right) = \frac{\alpha + n}{\alpha + \gamma + t}. \quad (3.30)$$

²³The assumption of i.i.d. disaster occurrences is made for simplicity. One could consider a more elaborated, non i.i.d. process but at the cost of cumbersome complexities in the model.

²⁴The Bernoulli distribution is a standard probability distribution used for random variables with a binary outcome, as it is the case here.

²⁵The beta distribution is a commonly used distribution for a continuous random variable that can only take on values on the interval $[0, 1]$, as it is the case for \tilde{p} .

²⁶Inversely, the term $t - n$ represents the number of period where no disaster occurs.

Notice that, given α and γ , it is increasing in the number of disaster occurrences n and a decreasing function of time or, more precisely, of the number of non-disaster periods $t - n$. Intuitively, as the number of disaster occurrences increases, creditors will expect a higher probability of disaster in the future. On the contrary, if the number of non-disaster periods increases, creditors will expect a lower probability of disaster.

Inserting (3.30) in (3.28), we obtain

$$\omega_t = \hat{s} + \beta\chi(t, n; \alpha, \gamma) \cdot \omega_{t+1}, \quad (3.31)$$

where $\chi(\cdot)$ is a decreasing function of n and an increasing of t . Iterating (3.31) forward, we obtain the equilibrium default ratio under Bayesian learning:

$$\omega_t = \frac{\hat{s}}{1 - \beta\chi(t, n; \alpha, \gamma)} \equiv \omega_t^{\text{bayesian}}. \quad (3.32)$$

From (3.16) and (3.32), recalling the definition in (3.11), we deduce the maximum quantity of debt-to-GDP under Bayesian learning:

$$\hat{b}_t^{\text{bayesian}} = \delta_t^* \omega_t^{\text{bayesian}}, \quad (3.33)$$

where δ_t^* is defined by (3.16) and (3.30).

Numerical illustration. Here is a numerical example of this Bayesian perception of disaster risk. Suppose that the truth probability is 0.035.²⁷ Suppose also that in period in period $t = 0$ creditors' prior distribution of \tilde{p} is a beta distribution with parameters $\alpha = 1$ and $\gamma = 27.57$, so that the prior expectation of \tilde{p} , defined as $\alpha/(\alpha + \gamma)$, equal to the truth probability.²⁸ The other parameter values are set as in Figure 3.2.²⁹

²⁷This corresponds to the historical estimated value for α_0 , the intercept of (3.2), which is present in Table (3.1).

²⁸Alternatively, one could use any values of α and γ that put most of the prior probability at small values of \tilde{p} . It does not matter very much which values one chooses, the resulting posteriors given the data would be very similar.

²⁹See Table 3.1.

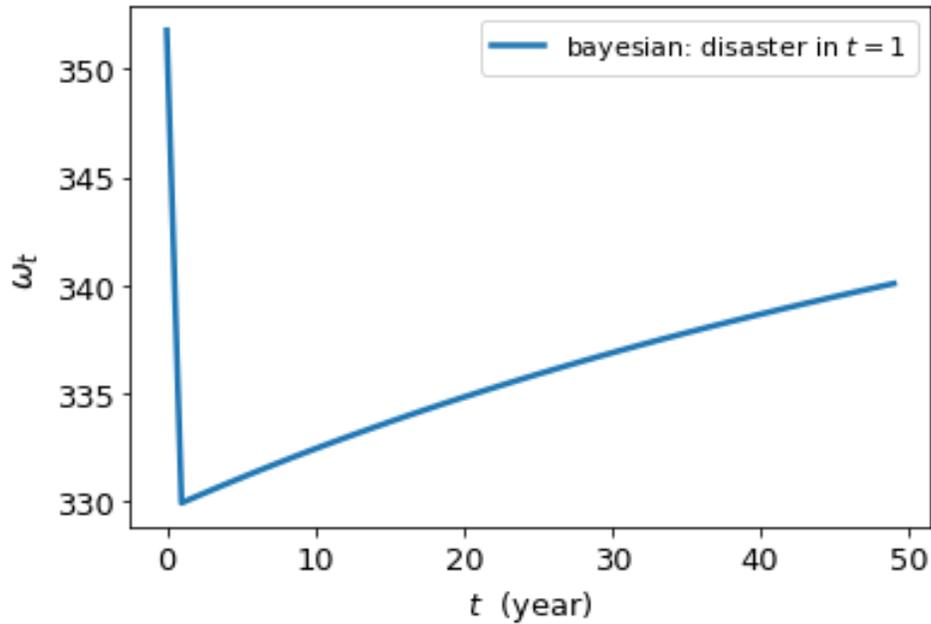


Figure 3.4: Simulated paths of the default ratio (% of GDP) with Bayesian learning.

Figure 3.4 presents a simulated default ratio $\omega_t^{\text{bayesian}}$ over 50 periods, considering a situation where a disaster occurs at $t = 1$ and no disaster occurs afterwards.

We notice a sharp fall in the default ratio after the realization of disaster at date $t = 1$. This is because, after observing the disaster, there is an upward shift in creditors' posterior expectations of disaster risk, leading to a sudden fall of the default ratio. However, if no disaster occurs in the subsequent periods, the default ratio recovers gradually towards its pre-disaster level as creditors' posterior expectations about disaster risk fade over time.

3.7 Conclusion.

I have developed a tractable stochastic model to analyze the potential effects of a gradual increase in the probability of climatic disasters on public debt sustainability and sovereign default risk. In the baseline model, the probability of disaster is modeled as a deterministic function with a linear trend. I show that when a

country moves from an environment with low disaster risk to one with high disaster risk it will face a sharp fall in the maximum debt-to-GDP ratio that can be sustained sustain without defaulting.

The model emphasizes the crucial role of creditors' expectations about disaster risk. Different expectations of disaster risk have very different implications for debt sustainability and sovereign default risk. Calibration and simulation exercises based on the historical frequency of climatic disasters show that a gradual increase in the probability of disasters over the coming years, if anticipated by creditors, can contribute to the reemergence of sovereign defaults, even when the risk free rate is lower than the growth rate of output. I show that this result also applies when there is uncertainty about the probability of disasters and creditors learn this probability through Bayesian learning. From a policy perspective, my findings call for more attention and caution about public debt sustainability in the context of climate change and the related increasing probability of extreme events.

3.8 Appendix

3.8.1 Proof of Proposition 2.

Proposition 2: *Under Assumption 3, with $p_t = p$, the equilibrium default ratio is locally unique and equal to:*

$$\omega_t = \frac{\hat{s}}{1 - \beta\chi(p)} \equiv \omega_p^* \forall t. \quad (3.8.1)$$

The ratio ω_p^* is strictly increasing in \hat{s} and strictly decreasing in p .

Proof. First, assume that $\beta\chi(p) < 1$. We will see below this inequality is verified under Assumption 3. Then, iterating (3.17) forward we obtain the stationary solution (3.8.1).

It remains to shown that $\partial\chi(p)/\partial p < 0$. Using (3.13), setting $p_{t+1} = p$ and differentiating the function $\chi(\cdot)$ with respect to the p , we have

$$\frac{\partial\chi(p)}{\partial p} = \delta [G(\delta) - G(\delta/u)] + \mathbf{h} \left[\int_0^\delta adG(a) + u \int_0^{\delta/u} adG(a) \right]. \quad (3.8.2)$$

Recalling that $0 < u < 1$, we deduce from (3.8.2):

$$\delta \left[G(\delta) - G(\delta/u) \right] \leq 0.$$

Then, the left-hand side of (3.8.2) is negative if

$$u \int_0^{\delta/u} adG(a) < \int_0^\delta adG(a). \quad (3.8.3)$$

Let us define

$$\psi(\delta) \equiv \int_0^\delta ag(a) da. \quad (3.8.4)$$

Using this definition, we have

$$\begin{aligned} \frac{d}{d\delta} \left(\frac{\psi(\delta)}{\delta} \right) &= \frac{\delta\psi'(\delta) - \psi(\delta)}{\delta^2} \\ &= \frac{\delta^2 g(\delta) - \psi(\delta)}{\delta^2} < 0, \end{aligned} \tag{3.8.5}$$

where the last inequality is implied by Assumption 3.2.

From (3.8.5), and recalling $u < 1$, we have

$$\frac{\psi(\delta/u)}{\delta/u} < \frac{\psi(\delta)}{\delta} \Leftrightarrow u\psi(\delta/u) < \psi(\delta). \tag{3.8.6}$$

Inserting the definition (3.8.4) in (3.8.6), we obtain

$$u \int_0^{\delta/u} ag(a) da < \int_0^{\delta} ag(a) da, \tag{3.8.7}$$

which is a sufficient condition to have $\partial\chi(p)/p < 0$.

□

3.8.2 Country analysis.

This section further investigates the link between the default ratio and creditors' expectations about disaster risk by resorting to country calibration and simulations.

I calibrate and simulated the model to a sample of 12 emerging and 22 advanced countries that have experienced at least one climatic disaster in 1960-2019.³⁰ Table present parameter values for the risk-free rate r , the debt recovery parameter \mathbf{h} , the maximum primary surplus \hat{s} , and the disaster parameters u , α_0 and α_1 . The mean μ and volatility σ of the (log) gross growth rate are set to

³⁰I follow the definition of the IMF to classify the countries in the Advanced and Emerging groups. In line with the assumption of access to international financial markets in the model and Following [Diarra, Guillard and Kempf \(2022\)](#), I require the selected countries to have at least ten consecutive years of observations of sovereign bond yield. Sovereign yields used for this selection are from Reuters.

their country specific values.³¹

Tables 3.2 and 3.3 report simulated maximum debt ratios, \hat{b}_t , for advanced and emerging countries, respectively.³² In each table, I report simulation results for both the naive and forward looking expectations of disaster risk over three time horizons: 2019 (the last year in my sample), 2030 (a decade ahead), and 2050 (mid century). For comparison, I also report results for the standard case with no disaster risk, along with the actual debt-to-GDP ratio of each country in 2019 so as to have a measure of their fiscal space.³³

Consider the group of advanced countries (Table 3.2). Assuming no disaster risk, as in Diarra, Guillard and Kempf (2022), the maximum debt ratio in advanced countries amounts to 375% of GDP on average (Column 2). This value is well above the average debt-to-GDP ratio of 95% observed for advanced countries in 2019 (Column 1). There is however some heterogeneity across countries, reflecting differences in their growth rates. For instance, some countries such as Hong Kong and Lithuania present fairly large maximum debt ratios evaluated at 3050% and 1090% of GDP, respectively.³⁴ In contrast, other countries such as Italy and Greece present debt ratios amounting to 150% and 125% of GDP, respectively. Comparing maximum debt ratios to actual debt-to-GDP ratios observed in each country in 2019 to have a measure of their fiscal space, one would conclude that sovereign default is not an immediate concern in advanced countries, except for Italy and Greece.³⁵

Assuming naive expectations of disaster risk, the maximum debt ratios in 2019 (Column 3) of Hong Kong and Lithuania are respectively reduced by 63%, 35% of GDP relative to the case with no disaster risk (Column 2). The corresponding

³¹See Appendix ?? Table 3.4.

³²I focus on the ratio \hat{b}_t , instead of the default ratio ω_t , as the former is directly comparable to actual debt-to-GDP ratios.

³³The fiscal space is defined as the difference between the maximum debt ratio that a country can sustain and its actual debt ratio, that is: $\hat{b}_t - b_t$. See Ghosh et al. (2013) and Diarra, Guillard and Kempf (2022).

³⁴The relatively large maximum debt ratios of these countries is partly due to their high growth rates.

³⁵The fiscal space given by the difference ($\hat{b}_{\text{nodisas}} - b_{2019}$) indicates that Greece should be in default in 2019, while Italy is on the verge of default. This situation is not very surprising given the recent debt sustainability issues experienced by these countries.

Table 3.2: Simulated maximum debt-to-GDP ratio, \hat{b}_t (%): Advanced economies.

Country			Naive expectations			Forward expectations		
	b_{2019}	\hat{b}_{nodisas}	\hat{b}_{2019}	\hat{b}_{2030}	\hat{b}_{2050}	\hat{b}_{2019}	\hat{b}_{2030}	\hat{b}_{2050}
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Australia	47.47	489.80	357.07	260.47	188.18	257.27	215.60	175.51
Austria	70.51	220.87	189.17	158.09	128.05	164.12	144.19	123.05
Belgium	98.06	216.80	185.89	155.53	126.19	161.63	142.05	121.37
Czech Republic	94.04	189.41	170.99	150.65	128.16	156.25	141.52	123.90
France	30.25	206.92	178.22	149.79	122.15	155.97	137.32	117.49
Germany	8.44	181.18	160.72	139.09	116.16	144.90	129.74	112.37
Greece	98.07	125.58	116.94	106.69	94.34	111.13	102.84	92.40
Hong Kong*	59.64	3050.66	1126.82	603.60	362.59	481.81	393.01	304.26
Iceland	184.90	456.37	361.66	281.17	212.71	276.96	238.38	196.65
Israel	0.27	728.30	478.48	325.55	223.50	308.54	255.45	204.10
Italy	68.33	150.96	136.31	120.24	102.45	125.64	113.70	99.68
Latvia	59.98	705.96	525.06	387.67	280.95	366.97	312.23	253.56
Lithuania	134.56	1090.47	703.07	471.72	319.65	421.22	352.24	280.26
Luxembourg	36.97	693.89	490.27	349.33	247.30	329.18	277.20	223.26
Netherlands	35.91	219.49	189.67	159.84	130.10	165.71	146.35	125.00
New Zealand	21.99	291.77	241.68	195.59	153.30	200.11	173.72	145.63
Portugal	47.60	193.86	172.49	149.69	125.37	155.39	139.50	120.98
Slovak Republic	32.06	774.95	529.47	368.87	257.07	343.15	287.74	230.62
Spain	116.84	230.19	199.17	168.01	136.76	173.64	153.59	131.11
Switzerland	58.12	199.18	173.39	147.24	121.01	153.28	135.76	116.67
United Kingdom	48.46	221.28	191.56	161.70	131.75	167.51	148.12	126.53
United States	65.60	297.45	245.45	197.96	154.69	202.32	175.39	146.84
Sample average	95.51	375.46	285.56	219.28	166.66	216.23	186.79	155.57

Notes: *: Countries excluded from the computation of the sample mean in Columns 2 to 8. For each country, the mean μ and volatility σ of the growth rate are calibrated to their historical values. The other parameters (r , \hat{s} , \mathbf{h} , u , α_0 , and α_1) are set to their baseline values in Table 3.1.

Table 3.3: Simulated maximum debt-to-GDP ratio, \hat{b}_t (%): Emerging economies.

Country			Naive expectations			Forward expectations		
	b_{2019}	\hat{b}_{nodisas}	b_{2019}	\hat{b}_{2030}	\hat{b}_{2050}	b_{2019}	\hat{b}_{2030}	\hat{b}_{2050}
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Brazil	89.47	62.73	55.42	48.49	42.69	51.81	46.73	42.07
Chile	27.91	104.62	88.14	73.60	61.66	77.93	68.66	59.90
China*	52.63	∞	∞	∞	∞	13273.28	8723.88	5902.37
Colombia	52.29	137.55	97.94	75.13	64.84	81.79	70.56	63.60
Hungary	66.34	66.33	57.36	49.43	43.58	53.14	47.59	42.99
Malaysia	57.24	508.96	260.12	163.76	116.94	159.83	131.42	108.17
Mexico	53.75	64.43	56.72	49.47	43.47	52.88	47.62	42.83
Pakistan	85.56	683.00	228.53	136.56	107.99	142.45	117.87	103.09
Philippines	36.97	106.14	87.58	72.21	60.67	76.88	67.43	59.10
Poland	45.97	126.92	97.58	77.12	64.71	82.74	71.62	63.12
Russia	13.92	32.02	30.48	28.61	26.38	29.91	28.22	26.19
South Africa	62.15	73.68	61.11	51.31	45.61	55.71	49.42	45.07
Sample average	53.68	178.76	101.91	75.06	61.69	78.64	67.92	59.65

Notes: ∞ : Cases where the mean growth is larger than the risk free rate and $\hat{b}_t = \infty$. *: Countries excluded from the computation of the sample mean in Columns 2 to 8. For each country, the mean μ and volatility σ of the growth rate are calibrated to their historical values. The other parameters (r , \hat{s} , \mathbf{h} , u , α_0 , and α_1) are set to their baseline values in Table 3.1.

figures for Italy and Greece are 10% , and 7% of GDP, respectively. Overall, maximum debt ratios in advanced countries fall on average by 24% of GDP in 2019, 42% in 2030 (Column 4) and 56%in 2050 (Column 5) relative to the case with no disaster risk. At debt-to-GDP ratios observed in 2019, Italy and Greece will be in default by 2030, and Portugal will be on the verge of default by 2050.

With forward looking expectations of disaster risk, the decreasing pattern in maximum debt ratios is even more accelerated. Maximum debt ratios in advanced countries now fall on average by 42% of GDP in 2019, 50% by the 2030 and 58% by 2050.

Turning to emerging countries (Table 3.3), we observe a pattern similar to advanced countries. In the absence of disaster risk (Column 2), China presents an infinite maximum debt ratio,³⁶ followed by Pakistan with a maximum debt ratio of 683% of GDP. Brazil and Russia present the lowest maximum debt ratios evaluated to 62% and 32% of GDP, respectively. Comparing maximum debt ratios to actual debt-to-GDP ratios observed in each country in 2019, we notice that Brazil and Hungary have exhausted their fiscal spaces in 2019 and thus should be already in default.

Assuming naive expectations of disaster risk, the maximum debt ratio of emerging countries in 2019 (Column 3) is reduced by 42% on average relative to the case with no disaster risk (Column 2). At debt-to-GDP ratios observed in 2019, four of the twelve emerging countries (Brazil, Hungary, Mexico and South Africa) would be in default by 2030. This pattern is further accelerated if creditors have a forward looking view about disaster risk and anticipate future increases in this risk. In this case, Brazil, Hungary, Mexico and South Africa would be in already default in 2019.

This simple calibration exercise further illustrates the role of creditors expectations about disasters risk. It shows how a gradual increase in the probability of disasters, due to climate change, may affect public debt sustainability and (potentially) lead to more sovereign defaults across countries.

³⁶This is because, in this case, the growth rate is higher than the risk-free rate and thus the default ratio, as well as the maximum debt ratio \hat{b}_t , tend to infinity

Table 3.4: Mean and volatility of the growth rate (1980-2019, %)

Advanced countries	μ	σ	Emerging countries	μ	σ
Australia	3.06	1.44	Brazil	2.35	3.23
Austria	1.96	1.45	Chile	4.13	3.93
Belgium	1.90	1.40	China	8.99	2.50
Czech Republic	1.99	3.77	Colombia	3.39	2.03
France	1.78	1.32	Hungary	2.23	2.75
Germany	1.68	1.89	Malaysia	5.59	3.41
Greece	0.82	3.49	Mexico	2.42	3.20
Hong Kong	4.37	3.67	Pakistan	4.70	2.00
Iceland	3.43	3.48	Philippines	3.80	3.31
Israel	3.48	1.74	Poland	3.69	2.62
Italy	1.19	1.82	Russia	0.66	6.44
Latvia	3.82	5.62	South Africa	2.18	2.21
Lithuania	4.11	5.08	Sample	3.68	3.14
Luxembourg	3.74	3.01			
Netherlands	2.06	1.77			
New Zealand	2.57	1.81			
Portugal	1.97	2.55			
Slovak Republic	3.82	3.00			
Spain	2.24	2.11			
Switzerland	1.78	1.54			
United Kingdom	2.11	1.89			
United States	2.59	1.79			
Sample	2.57	2.53			

Conclusion

In this doctoral thesis, I have studied three different issues related to the topic of public debt sustainability, sovereign default, and their interplay with climate change and the risk of climate-related disasters. To address these issues, I combine theoretical modeling with econometric and simulation methods. Throughout the thesis, I adopt a perspective that considers sovereign defaults as “excusable events” in the sense of [Grossman and Huyck \(1988\)](#). This type of defaults are associated with identifiable bad states of nature. They occur only when, following a negative shock, the government is unable to get sufficient fiscal and debt issuance proceeds to repay due debt. This definition of default is in contrast with the one used in the literature of strategic default *à la* [Eaton and Gersovitz \(1981\)](#), which emphasizes the willingness of the government to repay due debt.

The thesis is structured around three chapters. Chapter 1 revisits the old issue of public debt sustainability in stochastic environment. We have challenged the standard assumption zero debt recovery in the literature and developed a stochastic model that allows for partial and, potentially, repetitive defaults. This model incorporates a debt recovery rule that depends on a unique parameter, which is grossly equal to one minus the “haircut”—the fraction of debt-to-GDP ratio lost by creditors following a sovereign default. It can take any value from 0 to 1. The case with 0 corresponds to a full repudiation of the defaulted debt, while the case with 1 is equivalent to full repayment of public debt. We have solved the model explicitly and have clarified the notions of public debt sustainability, sovereign default, and solvency, which are quite often overlooked by economists and in public debate. We have provided more precise definition to these concepts,

discussed the importance to distinguish them, and provided operational measure to each of them. A key message of this chapter is that the assessment of the sustainability of public debt, in particular the estimation of country fiscal spaces, depends crucially on the value of the debt recovery parameter. A small change in this parameter can have substantial effects on the dynamics of public debt and, therefore, lead to very different conclusions in terms of debt sustainability analysis. We illustrate the role of this parameter through calibrations, simulations and estimations of the model using historical data on a well defined sample of emerging and advanced countries.

Chapter 2 has embarked on the growing literature of climate change and the link with sovereign default risk. Indeed, although climate change is a hot topic for academics, policy makers, and investors, there is little evidence on the link, and the nature of the link, between climate change and sovereign risk, and whether financial markets effectively price climate-related risks. To address this issue, we have estimated the effects of temperature anomalies – temperature’s deviation from its long-run mean – on sovereign default risk and explore the transmission channels. As a proxy for sovereign default risk, we have used sovereign credit default swap (CDS) spread. We considered sovereign CDS spread at several maturities– one, three, five and ten-year maturities. Econometrically, we addressed this issue using a cross-country panel data covering 76 developing and advanced countries with sufficient data over the period 1999-2017. Using a standard two-way fixed effects estimation method, we have documented a strong positive impact of temperature on sovereign CDS spread. Interestingly, we found that the longer the maturity of the CDS spreads the larger the impact of temperature on spreads. An implication of this finding is that sovereign creditors price climate risk, not only when investing over a short horizon but also the medium and long ones.

Regarding the transmission channels of the effects of temperature to CDS spread, we have used the equilibrium bond pricing equation found in theoretical sovereign default models to isolate the key transmission mechanisms. We have shown that the effect of temperature on CDS spread is mostly driven by the neg-

ative effect of the former on the growth rate of GDP. Interestingly, we find that the debt-to- GDP ratio and the primary balance, which are two macroeconomic determinants present in the basic pricing equation, do not play any role in the transmission of the effect of temperature to CDS spread. These findings emphasized the need to take climate risk into account in the assessment of public debt sustainability. They have important implications for the policy responses to climate change in the context of high public debt-to-GDP ratios and limited fiscal space available for countries.

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