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Erwan Taillanter

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Road network structure and traffic patterns

Réseaux complexes et trafic routier

Thèse de doctorat de l'Université Paris-Saclay

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Marc Barthelemy (directeur de recherche, Institut de Physique Théorique, CEA Saclay).

Thèse soutenue à Paris, le 11 septembre 2023, par

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Titre: Réseaux complexes et trafic routier

Mots clés: Réseaux urbains, embouteillages, diagramme fondamental macroscopique, percolation, hystérésis

Résumé en français: L'automobile personnelle a bouleversé le monde dans lequel nous vivons. En particulier, les infrastructures de transport ont été modifiées et agrandies à un rythme effréné au cours de ce siècle, pour répondre à la croissance du nombre d'automobilistes. Très vite, l'augmentation du nombre de véhicules a fait apparaître la nécessité d'établir un code de la route, pour fluidifier et sécuriser le trafic, mais également celle d'étudier avec une approche scientifique ce phénomène nouveau.

L'évolution du trafic routier sur un réseau urbain reste à ce jour un sujet mal compris. Les nombreuses intersections introduisent de fortes corrélations entre les différentes routes, et les outils disponibles sont rapidement limités lorsque l'on souhaite estimer l'impact d'une décision locale sur le réseau à une échelle globale. Le domaine de l'ingénierie du trafic se trouve ainsi dans une situation qui n'est pas sans rappeler celle rencontrée par la communauté physicienne lors de l'invention de la physique statistique. A l'évidence, une approche « microscopique » n'est pas appropriée à la problématique et il convient d'adopter un point de vue « macroscopique ». Cette nouvelle échelle est typiquement celle du réseau entier, composé de routes, en ignorant la réalité « microscopique » du comportement des automobilistes sur ces routes.

Dans une approche physicienne, nous essayons de trouver des quantités moyennes pertinentes et de décrire leur comportement. Nous nous concentrons en particulier sur trois aspects distincts du problème, ayant chacun donné lieu à une publication.

Le premier point concerne le réseau lui-même, et en particulier son évolution lorsque la demande en trafic augmenta. Nous montrons que la demande en trafic est le paramètre principal ayant mené à la construction des autoroutes urbaines et des périphériques dans les villes des Etats-Unis au cours des années 1960. Nous identifions empiriquement deux seuils de population au-delà desquelles les villes ont respectivement une

autoroute et/ou un périphérique et proposons une analyse coût-bénéfice permettant de comprendre cette observation.

Le deuxième point concerne la propagation des embouteillages sur le réseau urbain. Nous mettons en évidence le fait que la formation d'embouteillage a lieu simultanément sur l'intégralité du réseau aux heures de pointes. Plus précisément, nous mesurons pour chaque route son degré de saturation et observons la longueur de corrélation de cette mesure. Nous constatons que cette longueur de corrélation est faible la nuit (de l'ordre de quelques centaines de mètres), indiquant qu'une perturbation locale se dissipe en quelques rues à peine. Tout à l'inverse, aux heures de pointe, cette longueur de corrélation diverge, impliquant que toute perturbation sur le réseau sera ressentie par tous les usagers. Ceci suggère que la formation d'embouteillages sur le réseau est analogue à une transition de phase physique.

Enfin, le dernier point, certainement le plus intéressant au niveau de ses implications pour la planification urbaine, concerne le lien entre la structure du réseau et sa réponse au trafic. Daganzo et Geroliminis ont observé en 2008 que la densité de voitures moyenne et le trafic moyen sur le réseau sont liés entre eux par une relation proche de celle observée à l'échelle de la route, appelée dans ce cas Diagramme Fondamental Macroscopique (MFD). L'existence de cette relation est remarquable et pas encore tout à fait comprise. En particulier, son existence implique que le réseau a une capacité maximale (c'est-à-dire un flux maximal de voitures), atteint pour une certaine densité optimale. A l'aide d'une simulation, nous explorons l'impact des différents paramètres du réseau sur ces deux paramètres du MFD. Ceci marque un pas important en direction d'une meilleure planification des travaux de voirie. La situation idéale serait en effet de pouvoir évaluer à l'avance le plus finement possible l'impact de décisions locales sur l'efficacité globale du réseau.

Title: Road network structure and traffic pattern

Keywords: Urban networks, congestion, Macroscopic Fundamental Diagram, percolation, hysteresis

Abstract: The personal automobile has changed the world we live in. In particular, transportation infrastructure has been modified and expanded at a rapid pace during this century to accommodate the growing number of motorists. Very quickly, the increase in the number of vehicles made it necessary to establish a highway code, to make traffic flow more smoothly and safely, but also to study this new phenomenon with a scientific approach. The evolution of road traffic on an urban network remains to this day a poorly understood subject. The numerous intersections introduce strong correlations between the different roads, and the available tools are quickly limited when one wishes to estimate the impact of a local decision on the network at a global scale.

The field of traffic engineering is thus in a situation not unlike that encountered by the physics community when statistical physics was invented. Clearly, a "microscopic" approach is not appropriate to the problem and a "macroscopic" point of view must be adopted. This new scale is typically that of the entire network of roads, ignoring the "microscopic" reality of the behavior of motorists on these roads. In a physical approach, we try to find relevant average quantities and describe their behavior. We focus in particular on three distinct aspects of the problem, each of which led to a publication. The first point is about the network itself, and in particular its evolution when the traffic demand increases. We show that traffic demand is the main parameter that led to the construction of urban freeways and ring roads in US cities during the 1960s. We empirically identify two population thresholds above which cities have a freeway and a ring road respectively and pro-

pose a cost-benefit analysis to understand this observation. The second point relates to the propagation of traffic jams on the urban network. We show that traffic jams occur simultaneously on the entire network during peak hours. More precisely, we measure for each road its degree of saturation and observe the correlation length of this measure. We find that this correlation length is small at night (of the order of a few hundred meters), indicating that a local perturbation dissipates in just a few streets. Conversely, during peak hours, this correlation length diverges, implying that any disruption on the network will be felt by all users. This suggests that the formation of traffic jams on the network is analogous to a physical phase transition.

The final point, and certainly the most interesting in terms of implications for urban planning, concerns the link between the structure of the network and its response to traffic. Daganzo and Geroliminis observed in 2008 that the average car density and the average traffic on the network are related to each other by a relationship close to the one observed at the road scale, called in this case Macroscopic Fundamental Diagram (MFD). The existence of this relationship is remarkable and is not yet fully understood. In particular, its existence implies that the network has a maximum capacity (i.e., a maximum flow of cars), reached for a certain optimal density. Using a simulation, we explore the impact of different network parameters on these two MFD parameters. This is an important step towards better road planning. The ideal situation would obviously be to be able to assess in advance the impact of local decisions on the overall network efficiency as accurately as possible.

À Charlotte, pour sa présence à mes côtés et son soutien tout au long de ces trois ans

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Contents

General Introduction	13
I Cities, Urban networks and the need for a physical approach	15
1 Historical and political context	15
1.1 Cities	15
1.1.1 A universal concept ?	16
1.1.2 Heterogeneity in the spatial and socio-economical structure of cities .	17
1.2 Urban traffic : an eminently political issue	18
1.2.1 Modern cities are shaped by cars	18
1.2.2 Cost of congestion	19
1.2.3 Induced traffic and Braess's paradox	19
1.2.4 Some historical examples	21
2 Urban traffic through the lens of physics	26
2.1 Scientific context	26
2.1.1 Complex systems and statistical physics	26
2.1.2 Spatial networks	28
2.1.3 Measuring traffic in cities : Relevant parameters of the problem . . .	31
2.2 Traditional approaches and their limitations	33
2.2.1 Hydrodynamic perspective : derivation of the fundamental diagram .	33
2.2.2 Generalization attempts and the difficult scale-up to urban networks .	37
2.2.3 Introducing Branston's Bureau of Public Roads function	39
2.2.4 Agent-based Land Use and Transport Interaction models	40
2.3 Microscopic approach	41
2.4 Rise and promises of machine learning	41
2.5 Towards a macroscopic description of cities	42
Conclusion	43

II Interplay between population and road network structure and its impact on congestion

regimes	45
3 Introduction of some useful tools	46
3.1 Characterizing inhomogeneity	46
3.1.1 Measuring inhomogeneity: Lorenz curve and Gini coefficient	46
3.1.2 Measuring the spatial spread: the Venables distance	47
3.1.3 Some illustrations with real world measures on the Parisian network	48
3.2 Concept of centrality	50
3.2.1 Different centralities	50
3.2.2 Betweenness centrality as a proxy for traffic demand	51
3.2.3 Limits of the betweenness centrality	53
4 Network growth leads to abrupt changes in the congestion regimes	55
4.1 Symbiotic relationship between population growth and network growth	55
4.2 Network growth and changes in the congestion regimes	56
4.3 Adapting to changing congestion regimes, a challenge for cities	59
5 Emergence of urban highways and ring roads	61
5.1 Empirical study	61
5.1.1 Historical context	61
5.1.2 Data and method	62
5.1.3 Results	65
5.2 Cost-benefit analysis	66
5.2.1 Model	66
5.2.2 Two possible scenarios : urban highway or ring road ?	69
5.3 Discussion	74
5.3.1 Impact of the assumption $\mu = 1$	74
5.3.2 Limitations of our model	75
5.3.3 Influence on the congestion regimes	75
5.4 Externalities associated to urban highways	76
5.4.1 Identifying the critical social cost of urban highways	76
5.4.2 Assessing the cost of externalities is a difficult task	77
5.4.3 Perspectives	78
Conclusion	80

III Traffic dynamics on urban networks	81
6 State of the art	82
6.1 Existence of weak links for congestion	82
6.1.1 Identifying influential and weak links	82
6.1.2 Study of functional clusters, metastable states	82
6.2 Congestion propagation	84
6.2.1 Diffusion processes	84
6.2.2 Identifying clusters of congested roads, percolation of congestion	84
6.2.3 Epidemiological models and the limited influence of network structure	85
6.3 Alternatives to the macroscopic, physical approaches	87
7 Empirical evidence for a jamming transition in urban traffic	88
7.1 Divergence of the correlation function	88
7.1.1 Construction of the correlation function	88
7.1.2 Results	93
7.1.3 Singular role of ring-roads and urban highways	96
7.2 Study of the spatial distribution of congested links	97
7.2.1 Giant component	97
7.2.2 Core of congested links	98
7.2.3 A simple model for spatial contagion	100
7.3 Discussion and implications	102
7.3.1 Limitation of our dataset	102
7.3.2 Macroscopic effects	102
7.3.3 Consequence on the existence of street level fundamental diagrams	103
Conclusion	105
IV On the Macroscopic Fundamental Diagram	106
8 Introducing the Macroscopic Fundamental Diagram	107
8.1 History and empirical existence	107
8.1.1 Production MFD and outflow MFD	107
8.1.2 Critical parameters and traffic regimes	108
8.2 Theoretical framework	108
8.2.1 Choosing the right average	108
8.2.2 Conditions of existence and estimation methods	109
8.2.3 Reservoir based models and clustering methods	110

8.2.4	Understanding the multivaluedness of the MFD	112
8.2.5	Derivation of a functional form for the MFD	114
8.3	Applications	118
8.4	Open questions and outline of the chapter	118
9	Structure of road networks and the shape of the Macroscopic Fundamental Diagram	120
9.1	Method	120
9.1.1	The Chowdhury-Schadschneider model for car progression	120
9.1.2	Network model	121
9.1.3	Derivation of the MFD	123
9.2	Previous empirical results	125
9.3	Scaling laws for the MFD's critical parameters	126
9.3.1	Dimensional analysis	126
9.3.2	Influence of v_{max}	127
9.3.3	Simulation results for k^*	127
9.3.4	Simulation results for q^*	128
9.3.5	Influence of p and t_{tl} on n_c	129
9.4	Discussion	132
9.4.1	Comparison with empirical results	132
9.4.2	Advantage of a numerical approach over empirical studies	135
9.4.3	Limits of our model	136
9.5	Perspectives	136
9.5.1	Spectral analysis	136
9.5.2	Other sources of heterogeneity	137
10	Unifying the MFD with the spatio-temporal dynamics of congestion ?	139
10.1	Towards an Effective Macroscopic Fundamental Diagram ?	140
10.1.1	Quick introduction to effective medium theory	140
10.1.2	Building an effective medium theory for road networks	141
10.1.3	Challenges and perspectives	144
10.2	Relation between percolation and MFD	146
	Conclusion	148
V	Discussion and closing thoughts on urban mobility	149
11	Perspectives beyond the MFD	150
11.1	Taking traffic demand structure into account	150

11.2	Passenger-MFD and area-based efficiency	151
12	Enforcing cooperation and working towards a social optimum	152
12.1	Side-note : a brief introduction to mean-field games	152
12.2	Optimal traffic assignment	153
12.3	Encouraging a modal shift	155
	General conclusion	156
	Résumé détaillé en français	158
	Bibliography	166

List of Figures

I.1	Example of a network on which the addition of a road leads to an increase of travel time	21
I.2	Ratio between shortest path and euclidean distance	30
I.3	Spatial distribution constrained by a network	30
I.4	Some real world fundamental diagrams	36
I.5	Some simplified fundamental diagrams	36
II.1	Lorenz curve, definition of the Gini coefficient	47
II.2	Hotspots for traffic and congestion in Paris	49
II.3	Venables distance of traffic in Paris	49
II.4	Illustration of centrality measure	54
II.5	Grid-tree model	57
II.6	Analysis of congestion regimes for nine cities	58
II.7	Examples of cities with urban highways and ring-roads	62
II.8	Density-population relation for US urban areas	64
II.9	Study of the relationship between 1960 traffic demand and 2020 population	65
II.10	Fraction of cities with a urban highway	65
II.11	Fraction of cities with a ring-road	66
II.12	Variation of the cost functions with T	73
II.13	Variation of the cost functions with μ	74
III.1	Influence of the threshold for congestion on the number of congested links	90
III.2	Daily evolution of traffic measures in Paris	91
III.3	Relation between travel time and euclidean distance in the case of the Parisian network	92
III.4	Distribution of the distance between all pairs of links of the network	93
III.5	Correlation functions	94
III.6	Evolution of the correlation length	95
III.7	Average size of the Giant Component of congested links	98

III.8	Distribution among all links of the frequency of presence in the giant components . . .	98
III.9	Overlap between the cores of congested links from hour to hour	99
III.10	Numerical results for the proposed contagion process	101
III.11	Example of problematic fundamental diagrams	104
IV.1	Historical observation of Yokohoma's MFD	108
IV.2	MFD as a function of accumulation and its distribution	113
IV.3	Cut-based derivation of the MFD	115
IV.4	Illustration of our toy-network	122
IV.5	MFD obtained with the ChSch model	124
IV.6	Data collapse for the scaling of k^*, q^*	127
IV.7	Influence of v_{max} on k^* and q^*	127
IV.8	Influence of p and t_{tl} on k^*	128
IV.9	Influence of p and t_{tl} on q^*	129
IV.10	Data collapse confirming the exponential form of V^*	129
IV.11	Variations of n_c with p and t_{tl}	130
IV.12	Number of cars able to pass an intersection as a function of traffic light duration . . .	131
IV.13	Influence of ρ_r on k^*	132
IV.14	Distribution of average road lengths in cities and relationship with road density	133
IV.15	Relationship between q^* and k^*	133
IV.16	Capacity reduction as a function of bus production	135
IV.17	Scaling for roads of variable lengths	138
IV.18	Testing a real congestion distribution on the MFD	142
IV.19	Testing the impact of various road types on the MFD	143

List of Tables

II.1 Variables used in our cost-benefit analysis	68
IV.1 Variables describing the network and the MFD	122

General Introduction

Cities have played a major role in the economic, cultural and technological growth of countries around the globe since their inception, by facilitating trade and leading to the division of labor, which in turn allowed the appearance of experts in all kinds of fields. However, the concentration of people in densely inhabited areas poses problems in terms of transports, both to supply the city with food and goods and to allow the movement of people inside the city. From the very beginning, urban growth has thus been closely linked to the development of transportation networks. Remarkably, cities worldwide and their road network share a lot of characteristics. This suggests that the mechanisms which led to the observed road network structure are actually simple and universal, making them well suited to an approach based on statistical physics tools.

The emergence and generalization of cars in the course of the 20th century have dramatically reshaped cities, which needed to adapt their network to cater for the growing transportation demand. Today, traffic congestion is a central problem for cities worldwide, as it has negative repercussions on the economy, through the waste of time that it creates, as well as on health and the environment, because of the pollution that it generates. Once again, it appears that the patterns of traffic and congestion formation are shared by all cities.

The starting point of this thesis has been the study of the impact of the decision to close the “Voies sur Berges Georges Pompidou” in Paris in 2016. The decision to close this urban highway, running through Paris along the Seine river, epitomizes the tensions and the problems regarding urban traffic. Several points are particularly interesting. First, traditional approaches to road planning proved incapable of precisely predicting the impact of such a decision, which highlights the relevance of trying to tackle it with a different, physical, approach. Secondly, it shows how complex the decisions regarding road network planning are, as their evaluation greatly depends on the perspective taken. As an example, urban highways tend to be used mostly by suburbanites to travel to the city center, while being mostly detrimental to inhabitants of the zone in between.

While over the course of these three years, we tackled a variety of problematics not necessarily related to this initial question, the evaluation of the impact of the closure of the “Voies sur Berges”

will be some sort of *Leitmotiv* throughout this thesis. This is mostly due to the fact that, as far as empirical studies are concerned, we have worked with traffic measurements of the city of Paris, which included the period around 2016 and thus made it possible to study this decision, but also because it is, as we said, a relevant example of several of the problematics related to urban road networks and urban traffic.

This thesis will be organized as follows. In the first chapter, we will further introduce the problematic of cities and urban traffic. We will start by highlighting the relevance of a physical approach in the context of urban traffic, while also specifying our method and our objectives for this thesis. We will then introduce the traditional approach to traffic management, based on a hydrodynamical description, and show its limits in the urban context.

In the second chapter, we will investigate the evolution of the road network itself. We will on one hand show that it cannot be separated from the evolution of the city as a whole and in particular the population growth. On the other hand, we will show how the growth of the road network impacts the congestion patterns that are likely to emerge. In particular, this will lead to the realization that a switch from the historical decentralized evolution of the road network to a more centrally planned one is essential in large cities.

The third chapter will be devoted to the study of the traffic dynamics on the network. We will show that congestion is unevenly spread on the network, yet displays a remarkable stability over time, with patterns that repeat almost identically from day to day. We will show that two mechanisms seem to be at play, with weak links acting as the onset for congestion, from which congestion then percolates to the entire network. As we will see, this second mechanism of the congestion's propagation to the entire network remarkably seems to be almost independent from the details of the actual network structure.

Based on these results, the fourth chapter will introduce and investigate the most modern approach to urban traffic, namely the Macroscopic Fundamental Diagram (MFD). In this framework, details of the network are ignored, as only the relationship between average traffic and average car density at the network scale are studied, thus measuring the network's response to traffic demand. We will give an introduction to the recent and active literature on this subject, before proposing our own results. Using a simulation, we will investigate the influence of the network's structure on the MFD. We will then propose ideas to use results presented in the third chapter, regarding the spatio-temporal dynamics of traffic congestion, to deepen our understanding of the MFD in future research.

Finally, we will discuss limits of this framework and propose alternatives that could help to plan cities more efficiently in the future, highlighting in particular the importance of legislations to enforce socially optimal solutions in place of the current individualistic paradigm.

Chapter I

Cities, Urban networks and the need for a physical approach

1 Historical and political context

1.1 Cities

While there is no consensus on the exact date of the first emergence of cities, it is clear that human populations have tended to gather in larger settlements for at least 5,000 years. The existence of cities required a remarkable level of social organization, such as a division of labor, transports of goods from agricultural regions to the more densely populated urban areas or trade, but also enabled an increased level of human interactions and economies of scale, driving the exchange of ideas and the development of politics or religions. This, combined with the appearance of military forces made possible by the division of labor, led to the emergence of city-states and further to larger unified states. In parallel, cities drove the development of new technologies and the economical growth of states [1]. In this section, we will briefly describe the common denominators defining cities. We will then discuss the dynamics of urban growth, both in terms of population and in terms of spatial spread. Finally, we will discuss the evolution of the spatial structure of cities during the last century, and highlight the significant differences in scale, income or population distribution between cities.

Even though this section serves mostly as an historical introduction, we will present some recent results which highlight the potential of statistical physics in our understanding of objects which are traditionally most often associated with the fields of history, geography or economy.

1.1.1 A universal concept ?

Remarkably, cities have emerged all over the globe and share a significant amount of traits. Most obviously, cities are defined by an above average level of population density. Despite a lack of centralized urban planning, early cities developed following similar structures. Even before the emergence of cars, cities were characterized by dense road networks, allowing the movement of people and goods from one specialized district to another (e.g. from a residential area to the central market) by foot or by public transportation [2, 3]. While they resembled a minimal spanning tree, all pre-industrial road networks were characterized by a very sharp degree-distribution (most nodes of degree 2 or 3) and more importantly, by a vastly increased robustness to link removal when compared to purely tree-like graphs [4, 5].

As we have said, the very existence of a city relies on the import of food from surrounding agricultural regions. This implies that cities will typically have access to long-range transportation networks, historically often in the form of a river, but later also in the form of rail and road networks. The emergence [6, 7] of these inter-city connections and their role [8, 9, 10, 11] in the development of a hierarchy of cities at the scale of the countries has recently gained a lot of interest and display, once again, remarkable similarities between countries.

These similarities between cities, both in their structure and their interactions with other cities, indicate that, despite the apparent complexity of the problem, some simple, fundamental mechanisms are at play, making cities well suited to a study with tools from statistical physics. As an example, the mechanisms leading to a robust network, spanning the entire city in a unique connected component, achieved without central planning and with minimal investment, are actually very similar to the constraints faced by biological networks. Remarkably, strong similarities can indeed be found between the structure of cities and biological organisms [12, 13, 14], as we shall see in more details in the second chapter.

Following the industrial revolution, cities saw an extraordinary increase of their population all over the globe. For the first time in human history, more than half of the world's population lived in urban areas in 2008, a proportion that reaches 80% in western countries and should be generalized worldwide by the end of the century. In the last two centuries, the worlds largest cities saw their population increase exponentially [15].

While cities with larger population typically display higher population densities [16], the increase of population also goes hand in hand with a spatial spread of the cities [4, 17, 18, 19]. This in turn means that cities tend to merge into one another and form vast urban areas. The exact definition of urban areas depends on the country, but is typically a combination of a high population density (of

the order of 10^{2-4} inhabitants/km²), a non interrupted urban land use (gaps between buildings smaller than 100 – 500m) and a high proportion of commuters (typically working in the tertiary sector). In all the following, we will use cities or urban areas as synonyms as the difference is mostly administrative, while the most important “physical” factors for our studies are the high population density and the large number of commuters.

Despite their similarities, cities (or urban areas) describe a wide range of scales, with populations ranging from 1,000 to 20,000,000, densities ranging from 1,000 to 40,000 inhabitants/km² and areas ranging from 1 to 10,000 km².

1.1.2 Heterogeneity in the spatial and socio-economical structure of cities

While population is arguably the most important predictor for a city’s characteristics, highly correlated to numerous indicators such as a city’s GDP, innovation potential [1], or road network density [3], cities with a given population still display important socio-economical differences [1, 16].

Within a given city, neither the population nor the economic activity are homogeneously distributed. Economic activity can typically be gathered in one or several Central Business Districts (respectively mono-centric or poly-centric structure), or be evenly spread throughout the city [20, 21, 22]. Similarly, the population is not evenly spread in the city, with significant variations of the population density throughout cities [23]. The same is true for the populations income, with large disparities inside cities between “rich” and “poor” neighborhoods [24, 25]. Interestingly, these spatial distributions of economical activity and population density and income are not constant in time and can be modeled using “microscopic” household-based [24, 26, 27, 28] or “macroscopic” neighborhood-based models [29, 30, 31].

A common trait, especially in larger cities, is the distance between housing and workplaces for a large part of the population. This seems counter-intuitive and certainly is counter-productive : cities help to reduce the distance to facilities and work by increasing the population density, yet people tend to live far away from their workplaces and spend more time commuting when they live in urban areas than when they live in rural areas [12, 16, 32]. Multiple factors explain this observation : larger cities typically have a poly-centric or decentralized structure, meaning that different members of a household can work in different parts of the city. Additionally, quality of life in the surroundings and budget seem to be more important parameters than accessibility or distance to the workplace in the choice of living accommodations [30]. The suboptimal repartition of population with regard to their workplace dramatically increases the commuting distances and time, leading to additional congestion on road networks and in public transports [33, 34].

1.2 Urban traffic : an eminently political issue

1.2.1 Modern cities are shaped by cars

While the need for transport networks has always been closely tied to the existence of cities and has determined their structure throughout the centuries [2, 3, 35], the emergence and generalization of cars during the last century has dramatically reshaped cities worldwide. Their tree-like structure progressively changed to a more grid-like plan, with intersections of degree 4 becoming more common [3, 36, 37, 38]. Note that the history of cities around the globe remains visible in their modern infrastructure. In particular, the degree distribution of the intersections varies quite significantly between Europe and the US [38, 39], which does impact traffic slightly, as we shall see during this thesis. However, once again, cities around the globe display remarkable similarities which suggests that the process of urbanization displays some kind of universality and that the study of road networks should be well suited to a physical approach.

Contrary to the first evolutions of urban transportation networks, these changes were clearly planned on a global, centralized scale, moving away from the previous "self-organized" or "allometric"¹ type of expansion, in order to increase the robustness of the network against congestion [3, 12, 35, 40]. These transformation included widening the streets to accommodate the growing traffic, building new roads to harmonize their length, with lengths of the order of 200m [41, 42], transforming existing roads into one-way-streets in a strategically planned manner [39], and adapting the roads to the expected flows, depending on their position in the network [43, 44]. In particular, high-speed roads were added to the network in the hope to reduce traffic congestion, further modifying its structure and giving it small-world properties [45].

Overall, the small spatial efficiency of cars and the rapidly increasing number of car commuters meant that the space allocated to roads and parking spaces² increased sharply in only a few decades, which required to destroy a significant amount of buildings (and natural habitats) and dramatically reshaped the urban landscape [43, 48].

Car-centered networks are the backbone of modern cities, allowing the transport of goods and a vast amount of commuters, but also causing significant problems in terms of safety (especially to other road users such as pedestrians), air pollution or land devaluation. All problematics related to the urban networks have to be thought as part of a complex whole and are as such fundamentally political.

¹i.e. akin to biological systems displaying scale-free properties.

²It is estimated that cities in the USA have up to 8 times more parking spaces than cars. The cost [46] and environmental impact [47] associated to this wasted, asphalted space are increasingly criticized.

1.2.2 Cost of congestion

Despite all the efforts made to facilitate transportation in urban areas, congestion is very much a problem in cities all around the globe. Because of the poly-centric nature of large cities and the discrepancy between living and working places [49], travel distance does not decrease in large cities compared to smaller ones and, worse than that, travel time actually scales super-linearly with population because of increased congestion [12, 16, 32]. The time spent in urban traffic can reach as much as 300 hours per person and per year on average³, corresponding to a time wasted in congestion of as much as 160 hours per person and per year⁴. This has a significant cost for society, up to 10% of a city's GDP for the most extreme cases [32], as well as a detrimental impact on the environment through an excess of pollution [16].

Mitigating congestion (and car traffic in general) is therefore of paramount importance for the future of urban planning. Cities have the potential to contribute to the reduction of greenhouse emissions, as by essence, they bring people closer together, allowing huge economies of scale in the transport of goods to the customers. However, in its current form, urban development misses its goal, as the growing population in densely packed areas results in an increase rather than a decrease of CO₂ through inefficient urban transportation networks.

1.2.3 Induced traffic and Braess's paradox

For the best part of the 20th century, it was believed that congestion was simply the result of an insufficient traffic capacity with regards to a rapidly increasing number of cars. The solution was then systematically the construction of further and wider roads or highways. However, it is now well established that the construction of these new high-speed roads, because it increased travel speeds, participated to an exodus of the middle class from the city centers to the suburbs, made accessible by the newly built highways [50, 51]. This means that the travel demand increased and the new infrastructure quickly became saturated as well. More generally, it is clear that the increase of road capacity does not necessarily result in improved network performance, as it will automatically lead to an induced increase of traffic (see [51, 52] for empirical evidence and [53] for the theoretical aspects of induced traffic), but also because the construction of a poorly planned new road can paradoxically lead to an increase of the average travel time. We will detail this phenomenon, as it is a good illustration of the complexity of the problems related to urban planning and traffic congestion and especially the wildly non-linear relationship between mobility demand and actual route choice.

³Tomtom® traffic index

⁴Data from INRIX®

Wardrop's principles

In 1952, Wardrop introduced two “principles” dictating the route choice of agents on a network [54]. Those two principles are closely related to the well-known Nash equilibrium in game theory and read as follows:

1. **“User equilibrium” or “Selfish Wardrop equilibrium”**: Each road user chooses the shortest route to complete his trip, in the sense that his route couldn't be made shorter by a unilateral decision.
2. **“System optimal” or “Social Wardrop equilibrium”**: at equilibrium, the average journey time is at a minimum, implying that all users behave cooperatively in choosing their routes to ensure the most efficient use of the whole system.

The first principle seems very intuitive, but implicitly makes the assumption of rational agents with perfect (real-time) knowledge. Empirical validations indicate that this is in fact not exactly the case, but that the observed deviations between shortest path and actual route choice are small [55]. Moreover, the rapid spread in the use of GPS makes this principle likely to be more and more accurate. A direct consequence of this principle is that if two points A and B are connected by two equivalent roads, the traffic will be equally split on both, such that the increase of travel time due to congestion will also be equal.

The second principle, however, is not automatically fulfilled. In fact, the Braess paradox that we will describe is an example of a situation where the social Wardrop equilibrium is not met and where either a better road planning or a centralized road pricing scheme are required to overcome the “price of anarchy⁵” associated to the selfish Wardrop equilibrium.

Braess's paradox

We describe here an example of a situation first described by Braess in 1968 [56]. Consider a road network as shown in figure I.1, on which 2000 drivers wish to travel from the starting point to the ending point. We assume that A-B and C-D are high capacity roads, where the travel time will be independent of demand and equal to 30 minutes, while A-C and B-D are smaller roads, where the travel time will be a function of demand Q , for instance $\frac{Q_{AC}}{100}$ (in minutes). We compare the cases without and with a high speed high capacity junction B-C which takes 5 minutes to travel.

Without the junction and under Wardrop's first principle, the drivers will split evenly between A-B-D and A-C-D, which are equivalent. As a result, the travel time from A to D for the users will be $30 + \frac{1000}{100} = 40$ minutes.

⁵i.e. the difference in cost between the social optimum and the result of self-interested behaviors

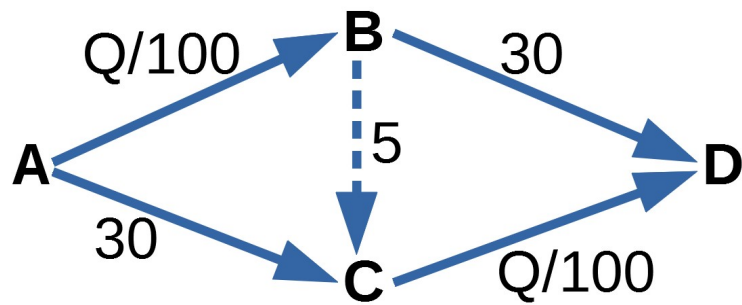


Figure 1.1: Diagram of a simplified network on which the addition of a road leads to an increase of travel time. Travel times between two points are indicated in minutes, Q is the traffic demand on the road.

If one adds the connection B-C, it is always faster to travel from A to C through B, with A-B-C now taking at most $\frac{2000}{100} + 5 = 25$ minutes, instead of 30 previously. This means that all users will take this new route, leading to a trip A-B-C taking indeed 25 minutes. However, the complete trip A-B-C-D now takes $2\frac{2000}{100} + 5 = 45$ minutes to be completed, as all users are now also concentrating on C-D. No user has an incentive to use A-B-D or A-C-D, as these routes now take 50 minutes, 5 minutes more than A-B-C-D. By cooperating and agreeing on not taking the new road, the users would benefit collectively and individually, yet they are stuck with the route A-B-C-D as any unilateral decision would only result in an increased travel time.

This situation might seem very artificial, but actually occurred on several occasions, such as in Stuttgart [57] (which led to the discovery of this phenomenon), Seoul or New York⁶. Interestingly, in those two latter examples, it was the closing of a road that led to an unexpected decrease in travel time.

1.2.4 Some historical examples

So far, we have shown that planning a urban road network is not a simple task, because of the complex interplay between urban development and road network. Furthermore, because it is now known that building new roads only has a limited impact on congestion, and will not necessarily improve greenhouse emissions because it will certainly result in increased traffic demand (as opposed to the construction of mass public transportations, see for instance [58]), it becomes clear that the decisions related to urban transport are fundamentally political. Building roads or public transports has a cost and choosing one rather than the other is always a political choice. For such important decisions, access to good quality information and prognosis is crucial, but as we shall illustrate on two historical examples, this has not always been the case and sometimes is not even possible.

⁶See New York Times article here

The Federal Highway Act

We have already briefly mentioned the fact that urban highways are a common infrastructure in large cities around the world, as they have been built abundantly in the second half of the 20th century. In this part, we would like to come back to the specific case of the United States and describe the political decisions at play during that time, as well as their consequences on today's cities. Sources for this paragraph are mostly [43, 48, 50, 59] (and references therein), which give a very good historical perspective on the topic⁷.

Starting in the 1920s, the USA were the first country to massively embrace individual car transportation, and congestion and car accidents soon became an issue. First steps to combat congestion and improve safety were the introduction of a set of standardized rules for cars and drivers taking part in urban traffic⁸, as well as traffic signals at intersections. Despite these efforts, car-use increased too rapidly for the network and congestion remained an issue. Engineers and planners soon realized that a limited access, grade-separated road could carry significantly larger volumes of traffic at much higher speeds and much more safely than the typical city street. Two visions of the modern freeway started to compete. Urban planners suggested to treat urban freeways as regular roads, simply with higher speed and higher capacity. This implied building a lot of short and relatively narrow freeway sections, connecting preexisting facilities in a densely packed high-speed network. Those visions often also included rail services, which would be directly integrated in the newly built grade separated network, and generally saw freeways as a tool in the broader picture of land use and city development. Engineers, however, argued that longer, wider and less connected highways would allow higher travel speeds and were clearly the best solution to increase traffic capacity. The proposed plans included fewer highways, which would typically cross the city and enable the fast transit from suburbs to a small number of exits in the city center. This kind of infrastructure did not adapt to the preexisting land-use scheme and the city's facilities and its construction would be very disruptive for the existing network and buildings.

While road planning and construction had historically been courtesy of the cities, building large infrastructures like freeways implied budgets on a whole other scale, which cities could not afford. As a result, the federal government introduced a gasoline tax in 1932 which would serve to finance a nationwide plan for the construction of highways. This meant that, with a few exceptions such as Los Angeles, which built a dense urban freeway network in 1939, the majority of urban highways were built as part of the Federal Highway Act which was introduced in 1956. This Highway Act combined two aspects in one, namely the inter-city mobility via highways and the decongestion of urban areas,

⁷Curious readers might also want to take a look at the Internet pages "*segregationbydesign*" and "*Strongtowns*", which are arguably more activist than scientific, but provide a lot of interesting pictures comparing cities before and after the highway construction

⁸Such as the 1926 International Convention relative to Motor Traffic or the 1949 Geneva Convention on Road Traffic

and using the same highways for both purposes was by far the easiest solution. Additionally, the construction of highways being financed almost exclusively by the gasoline tax, projects encouraging the automobile use were favored. In this context, projects were judged mostly on the promised increase of traffic capacity, rather than other criteria such as integration or multi-modality. Traffic engineers and automobile manufacturers lobbied successfully⁹ to advance their ideas among the general public and put pressure on officials to act. Starting in 1956, highways were built between and through all but the smallest US cities. In the US metropolitan areas, urban highway mileage increased from 161 to 16,716 between 1950 and 1990.

Because of the design of these highways, with a limited number of interfaces with the street network, city centers became easy to reach, but all the rest of the cities completely lost their attractiveness. The shorter travel times also meant that living in the suburbs became possible, causing the typical suburbanization phenomenon all across the USA [50]. Distance traveled, fuel consumption and car ownership sky-rocketed, as car and oil lobbies had been hoping, but the travel times didn't decrease as the newly built infrastructures soon were overwhelmed with the growing number of suburbanites. Even more concerning is the fact that the huge investments in road infrastructure were naturally made at the expense of public transportations, which even today remain critically under-developed in the majority of US cities.

Since the 1990s, we see a shift in the perception of highways in some US-cities. The rise of fuel prices has made it even clearer that a city built around cars is fundamentally segregating its poorest inhabitants, who are unable to afford a car and are thus unable to move. At a time where the infrastructure built under the Federal Highway Act reaches the end of its lifespan and will need considerable investments to be rebuilt, some cities decide to remove their freeways and promote more sustainable transportation modes such as mass public transport.

This historical example is striking, as one of the reasons why the "engineering vision" was preferred is that it was more mathematically robust and data driven. Calculating the capacity of a 6-lane highway is an easier task than to estimate the beneficial influence of a well integrated public transportation network. But urban environments are complex objects, and as everything that has to do with human lives, important decisions probably should not be taken on the base of a unique metric, no matter how well presented or robust it seems.

⁹See for instance this propaganda video produced by General Motors Corporation[®] to advocate for the Federal Highway act in 1954

Closure of the “Voies sur Berges” in Paris : a matter of perspective

Paris as well decided to build urban highways during the 1960s. The initial plan¹⁰, proposed and defended by Georges Pompidou, was to build several highways supplementing the street network and connecting Paris to its suburbs, at a time were the Parisian urban area was strongly growing. As such, the plan was some sort of hybrid vision between the “engineering” and the “planner” vision previously described and was supposed to “adapt Paris to the automobile”¹¹. However, strong opposition from the residents which would have been near to those new highways as well as from environmentalist associations delayed the project, until the first oil crisis led to its dismissal in 1973 [60]. Only one of the proposed highways was opened in 1967, namely the “Voie sur Berges Georges Pompidou” or short “Voie sur Berges”.

This highway offered rapid transit in the direction West-East along the Seine river, on 13km all the way through Paris. It was soon criticized, for the levels of noise and pollution that it introduced in the historical center, as well as for the symbol of the growing importance given to cars that it represented [60]. As early as in the 1990s, the idea of closing the “Voie sur Berges” emerged and gained in popularity until Anne Hidalgo, the mayor of Paris, finally decided to close the central section of the highway in 2016 and replace it by a cycle path and pedestrian zone, closed to any form of motorized traffic.

Several elements are of interest in this story. First, it should be noted that a majority of Parisians supported this decision, which comes as no surprise considering that less than half of the city’s population owns a car and that they were the most affected by the induced noise and pollution. On the other hand, residents of the suburbs of Paris, relying on the “Voie sur Berges” to commute, were very vocal against this decision. This highlights the fundamental issue of such highways, which really only benefits the city center and the suburbs, at the detriment of the zone in between, as can also be seen from the typical price of housing in cities which displays a minimum between the center and the suburbs [24, 25].

The other interesting point is the battle that both sides fought in the years preceding 2016. Proponents of the closure argued that the decision would reduce car traffic, by favoring cycling or pushing commuters towards public transportation. Additionally, the decision was defended by arguing that it would make the city center more attractive to tourists, which would be beneficial to the economy of the zone. Opponents claimed that the closure would lead to a transfer of traffic onto the nearest roads, increasing congestion and thus also pollution, while also costing the region money in the form of wasted time and by deterring locals to come and shop in the city center due to its inaccessibility. In particular, both sides could not agree on the share of commuters that would abandon the car commute in favor of another form of transportation. Both sides based their arguments on state-of-the art simulations,

¹⁰The so called *Plan Autoroutier pour Paris*

¹¹Georges Pompidou’s words at the inauguration of the “Voie sur Berges”

yet all outcomes appeared to be equally likely. This illustrates once again the difficulty to model and understand phenomena taking place on urban networks. We will come back in more details on the simulations typically used to model traffic and show that, in our opinion, a physical approach offers a good complement to the traditional approaches.

To conclude on the experience of the Voies sur Berges, it appears that there was some sort of truth in the arguments of both sides. In the short term, overall traffic did not decrease, but the increased demand on roads surrounding the Voies sur Berges actually led to a noticeable increase in travel time, a small increase of the pollution in the city center and a noticeable shift of the most polluted areas [61]. More importantly, the transfer of traffic was observed not only in the vicinity of the Voies sur Berges, but even in the suburbs of Paris, highlighting once again the complexity of the problem. However, now that a few years have passed, both overall traffic and average time spent in congestion start to decline¹². Unfortunately, a recent analysis of the impact on air pollution and noise levels has not been published and conclusions cannot be drawn on that front. However, this highlights the political nature of decisions regarding the traffic network. Paris continues to lead an ambitious politic to promote cycling and mass transports in place of car traffic. Opponents are quick to note short term negative impacts of these decisions¹³, but their full influence can only be measured after a few years, once the city has reached its new equilibrium¹⁴ in terms of spatial distribution of housing, jobs, facilities and commuting patterns.

¹²Data is available on the site opendataparis and we will discuss this extensively in a following section of this thesis

¹³See for instance the reactions to the closure of the rue de Rivoli

¹⁴Provided that a system as complex as a city can indeed be considered to be at some sort of equilibrium

2 Urban traffic through the lens of physics

The previous section introduced the many difficulties associated with the planning of road transportation in cities, mostly through historical examples. In this section, we will present in more detail the scientific context of complex systems such as cities and their road networks. This will allow us to further argue in favor of a physical description of urban road networks. We will then define the metrics used to describe car traffic and introduce traditional approaches of traffic engineering. In particular, we will spend some time describing tools that show great success on simple cases such as roads or highways and point-out the specificities of cities which make the scale up to a urban network difficult.

Finally, we will introduce more modern approaches to the problem of urban traffic. These include machine learning approaches, which are out of the scope of this thesis but which we will briefly touch on, as well as approaches rooted in statistical physics. We will give some successful examples of such methods, which will outline the following chapters of this thesis.

2.1 Scientific context

2.1.1 Complex systems and statistical physics

The difficulty to model urban systems in general and urban traffic in particular, comes from the multitude of actors composing it and whose behavior and interactions result in complex collective phenomena. We already introduced Wardrop's principle, which is a very basic example of game theory applied to understand the behavior of road users. Generally speaking, economic models tend to make strong assumptions about the agents, such as the fact that they are identical (or at least, can be categorized into a very small number of "types") or perfectly rational, which actually implies some sort of omniscience and foretelling. Further, it is often assumed that the sum of individual (selfish) optimizations will result in a global optimum for society [62], as if the system of human interactions necessarily was linear and agents never interacted with each other.

These assumptions are increasingly criticized in the economical field itself, but theories developed on their basis have enjoyed considerable success for decades. In the case of traffic, they very obviously do not work, as we have already mentioned about Wardrop's principles and illustrated using Braess's paradox and as one probably also would know from personal experience : on the road, selfish behaviors mostly lead to congestion or hazards.

The situation is then the following : a large number of agents with different interests and different characteristics interact strongly with each other. Yet, despite the numerous parameters defining the problem, the system seems to self-organize into a coherent behavior which, most importantly, is constant from day to day and from city to city.

This should sound familiar to statistical physicists. Indeed, statistical physics was invented in the late 1800s to explain observations made in the field of thermodynamics, when it became clear that a large number of interacting particles is chaotic and cannot be described exactly at the microscopic level. However, the knowledge of the basic microscopic rules of such a complex system would allow to successfully understand highly non-linear collective macroscopic phenomena, such as phase transitions. A particularly strong realization at that time was the consequence of the central limit theorem : when one averages over a large enough number of particles, the details of individual behaviors disappear and a new collective behavior defined by a very small number of parameters emerges.

This justifies the excursion of physicists into other fields which are *a priori* out of their expertise : any complex system where a multitude of details merge into a simpler collective behavior is well-suited to a study with the tools developed by statistical physicists over the past century. We will now give a few prescriptions as to what a good model should respect, which should again sound really familiar to physicists.

Universality

The core of the motivation to apply statistical physics tools to urban systems comes from the realization that road networks *look* and *behave* more or less identically all around the globe. In any given city, congestion patterns are very predictable and do not vary from day to day, despite the fact that individuals composing traffic do not execute the same actions identically every day. Clearly, a physical model of road traffic needs to respect this universality of the observations, meaning that it should not rely on hypotheses that are specific to any given city. This does not exclude the possibility to use parameters related to the cities structure (on the contrary), but the model should be transposable without any changes from one city to another. Fortunately, global GPS service providers now allow an access to high quality traffic data in a large number of cities worldwide, which facilitates the falsification of theories.

Parsimony principle

Another principle that is fundamental in physics is the limitation of the number of parameters of the network to the strict minimum¹⁵. Because parameters of the model need to be calibrated using real world data, it is obvious that the number of parameters should be small compared to the sample size, to avoid over-fitting. However, even beyond this consideration, every introduced parameter should have a physical interpretation and make sense in the context of the studied phenomenon.

¹⁵This principle is far from new, as it is a reformulation of "Occam's razor" : *Entia non sunt multiplicanda praeter necessitatem* (Entities must not be multiplied beyond necessity), attributed to 14th-century philosopher William of Ockham

This is motivated mainly by two reasons, which both relate to our strive for universality :

1. By using too much parameters, one actually tries to reproduce the *specificities* of each cities, rather than focusing on their *shared attributes*, thus failing to develop a universal model.
2. A model with too much parameters does not allow to study the individual influence of parameters. As a result, the model might *describe* reality adequately and even *predict* future evolutions, but it doesn't enable to *explain* and *understand* the phenomena.

In particular, Land Use and Transport Interaction (LUTI) models, which are widely used to plan changes to the urban infrastructure and which we will describe in more detail in a following section suffer from these problems. Guided by a desire to be as realistic as possible, they usually require a large amount of parameters and are thus inefficient to understand the mechanisms at play (e.g. hierarchize the influence of various competing mechanisms on the observed result). Worse, as we have described with the example of the closure of the Voies sur Berges, some parameters can't be measured precisely and must be chosen within a range. As a consequence, simulation results can vary strongly depending on the choice of parameter made, without one result being less "correct" than the other.

In a physical spirit, results presented throughout this work will thus try to reproduce empirical observations with the help of toy models, which rely on as little parameters as possible and we will try to systematically highlight their physical meaning.

2.1.2 Spatial networks

Urban road networks are a particular subtype of networks. Their geographical embedding means that they are planar graphs, which strongly constrains their topology. As we already mentioned in the previous section, the degree distribution of the intersections is very narrow, with a majority of degree 3 or 4 nodes, in sharp contrast to other types of networks such as for instance social networks [63]. The study of road networks therefore needs to use tools suited to this geographical reality. The reader interested in a thorough description of spatial networks and events taking place on them might want to read [63] or [64]. We will simply highlight some salient properties that should be taken into account when studying these objects.

Describing the networks structure

We have in fact already started to use tools to describe the historical evolution of urban road networks. In an effort to use as little parameters as possible and to be able to describe all networks universally, one has to carefully choose global attributes of the network which still reflect its properties

at all scales. A good tool for this is the distribution of the centrality of the network's nodes¹⁶ [35, 36, 63, 65], or the study of the number and size of closed loops in the graph [66, 67], which give indications about the relative importance of intersections and subsequently, about the network's robustness and resilience against traffic jams. Note that road networks can be represented as their dual, in which nodes are roads, connected by links representing the intersections. The same type of analysis can then be made for this dual graph to study the importance of the network's links.

Other tools have been developed throughout the years to describe road networks. Studying the entropy of the orientation of the roads [38, 63, 66] or the shape of the cells formed by the street pattern [35, 36], one can estimate the similarities between networks of different cities or, more importantly, between a real network and a typical simplified graph model such as a tree, a lattice or a web. This is particularly useful, as numerous properties of these standard networks have already been studied and can often be treated analytically, giving precious insight into the expected behavior of real world networks. The simplification step exemplified here, using so-called "toy-models", is at the core of the physical method and will be used throughout this thesis.

Time dependent phenomena on spatial networks

Describing the road network is not the only difficulty when studying urban traffic. Indeed, traffic takes place on the network, which constrains the tools that can be used to describe it. Even though, at large scale, the travels seem to take place in the 2D-plane, the reality is a collection of 1D-roads, interacting together at numerous intersections. Once again, we will only briefly introduce some consequences of this fact, which will be a considerable simplification of [64].

The first consequence has to do with the distances traveled. The naive approximation one could be tempted to make, that distances between two points of the network should roughly be equal to the euclidean distance is false, as illustrated in I.2. Instead, one should use the shortest path along the underlying network, which poses a new problem : it is actually the travel time rather than the travel distance which is of interest. However, the shortest travel time can be hard to estimate and is time-dependent, as it is a function of traffic on the individual roads. This is particularly relevant when one wants to study the spatial correlation between points of the network.

This leads to the second difficulty related to traffic networks : the lack of spatial continuity. If one was to model particle density or movement in a 2D plane, one would assume some sort of continuity along both spatial direction x and y . But consider for instance the case of two parallel routes (which we will simply call 1 and 2) going from point A to point B. A continuous change of the congestion levels on the network could lead to an abrupt change of the shortest path, from route 1 to route 2.

¹⁶We will define and discuss centrality extensively in the following chapter of this thesis

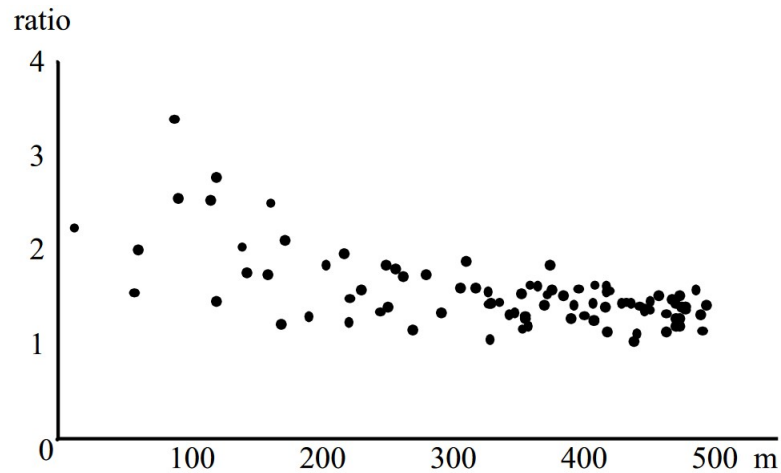


Figure I.2: Ratio of the shortest-path distance to its corresponding Euclidean distance for the street network in Kokuryo, a suburb in Tokyo. *Credits to Okabe and Sugihara [64]*

Finally, one needs to take particular care to treat all information as being constrained by the network. What we mean by this is in particular the discrepancy that exists between census data or information about origin and destinations, which is typically aggregated at the scale of cells in the euclidean plane, and the actual traffic that will be estimated using this data, which will move along the road network. Figure I.3 illustrates the issue : a phenomenon taking place on the network could be completely misinterpreted if one wrongly places it in the context of the euclidean plane. Fortunately, the modern era of mobile phones has given access to finely coarsed, high quality geo-localization data, making this problem less relevant than before.

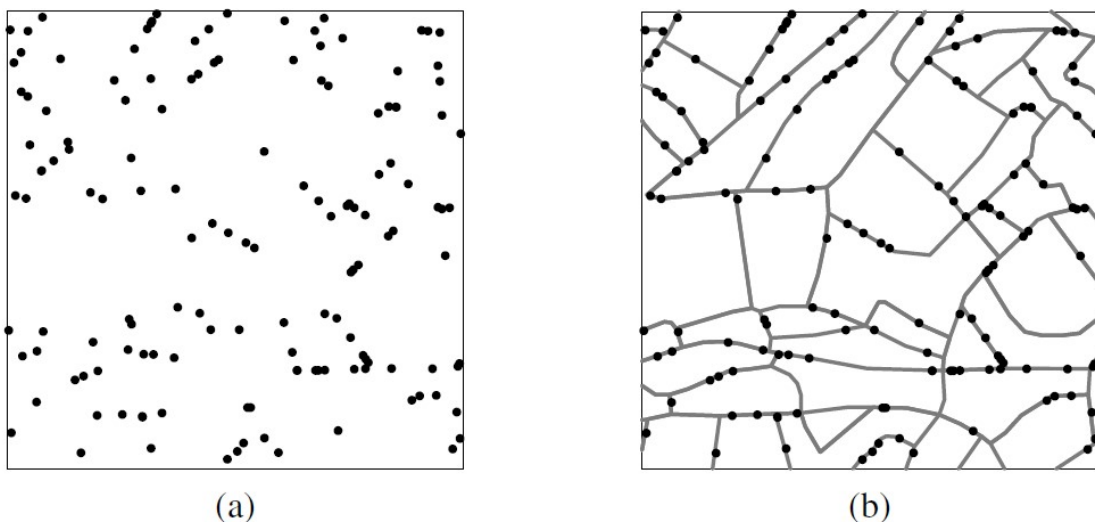


Figure I.3: The same point distributions: (a) non-randomly distributed on a bounded plane, (b) randomly distributed on a network. The network constrains the distribution, a low concentration of events in space could just be a consequence of the absence of roads. Typically, one cannot model a stochastic process on a network (e.g. road accidents) by applying stochastic methods designed for the plane *Credits to Okabe and Sugihara [64]*

2.1.3 Measuring traffic in cities : Relevant parameters of the problem

Having established that the exact behavior of drivers likely does not influence the collective dynamics of traffic and that we should strive to keep the number of parameters to a minimum, we shall now define the relevant parameters when one describes urban traffic. We will for now only define the parameters used in a classical description of traffic at the road level. Further chapters of this thesis will deal with descriptions of traffic at the scale of the network and will require their own parameters and notations.

In the most exact description of traffic, one would need to know each cars position and speed. Alternatively, from an exterior observer's perspective, one could measure the speed of passing cars and the time or distance between two cars. These measurements would appear like random variables following some given distribution. However, for obvious reasons of data storage, traffic measurements are always aggregated on a time-interval, which can range from a few minutes to an hour. As a result, the stochastic nature of traffic is almost completely hidden by virtue of the central limit theorem.

Quantities that can easily be measured and are thus typically accessible are the following:

- A flux (q)
- A measure of density (ρ) or occupation (o)
- An average speed (v) of passing cars

The flux is a straightforward quantity. A detector is placed on the roadside and counts cars passing during a given time interval. The measure of occupation and density requires a bit more details. The detector will typically be placed under the road and will count the fraction of time during which a car was above the detector, which is the measure of occupation $o = \frac{T_{occ}}{T_{tot}}$, where T_{occ} and T_{tot} are respectively the time where the detector was occupied and the total time interval. For moving traffic, one can then extrapolate the density of cars :

$$\rho = \frac{o}{L_c + L_d} \quad (1.1)$$

where cars have length L_c and the detectors is of size L_d .

The speed of cars can either be directly measured by the detector, which is extremely unusual (as it would require a significantly more involved detector than the usual loop-detectors used to measure traffic and occupation), or it can be extrapolated from q and ρ :

$$v = \frac{q}{\rho} \quad (1.2)$$

For the simplest case of a one-dimensional road along a direction x , the study of traffic is then the study of the two functions $q(x, t)$ and $\rho(x, t)$ (or alternatively $q(x, t)$ and $v(x, t)$). As we shall see in the following parts, this simple case already requires some simplifying assumptions to be analytically solvable, and gives rise to interesting non-linear phenomena such as shock-waves. Further, the scale-up to a 2D-network really is a challenging problem, that does not have any known solutions.

Loop Detector Data versus Floating Car Data

So far, we only described the case of loop detectors integrated in the streets (Loop detector data, LDD), which typically measure q and o over quite a long time-step. By definition, these detectors are fixed and therefore only give information about traffic and occupation in one point of the street. LDD thus provides an Eulerian observation of traffic.

Typically, a urban network will be equipped with one detector on each of its link. Conservation of flux guarantees that q is constant between two intersections¹⁷, but the density has no reason to be constant on the whole street, which is a clear limitation of such measuring methods. In particular, the extrapolated speed is not necessarily representative of the average speed on the link.

Another method exists, in which the speed and position of cars is directly measured via GPS (Floating Car Data or FCD). In theory, this would allow to give the measure of position and speed for cars on the network at every time step, in a Lagrangian style of observation. Again, for storage reasons, accessible data comes in the form of an average speed at a given point over a given time period $v(x, t)$. For the reasons described previously, this method of speed measurement does in general not give exactly the same value as the estimation using ρ and q .

However, a limitation of FCD is the measurement of a second quantity. Indeed, to measure flux or density on a street, one would need all of the cars to be equipped with a recording GPS. While some operators such as Google®, Waze® or TomTom® certainly come close to having this kind of data for some cities, they are typically not publicly available. Some extrapolation methods exist, but as for the speed estimation using LDD, they rely on some hypotheses and are not perfect.

In particular, the so-called “penetration rate” (i.e. the fraction of cars on the network which are actually equipped with sensors used in the study) is often space- and time-dependent and also depends on the origin and destinations of the cars. For instance, the likelihood of using a GPS is not the same for short trips as it is for longer trips, which skews the measurements of traffic and density made using FCD.

Discrepancies between both estimation methods are well known and an active research field. In particular, methods to increase the quality of the extrapolations when working with a single source of

¹⁷Ignoring the cars entering or exiting private properties, which is not necessarily a negligible quantity

data and to take advantage of both measurements when they are simultaneously available are currently investigated [68, 69, 70, 71, 72, 73].

Data access

Data access is essential to our method, either to give empirical evidence of a phenomenon, or to validate a model with real world measurements. We are fortunate to live in a time where data collection and spread is more generalized than ever. Loop Detector Data is generally property of cities or other public organizations and is thus mostly freely accessible. As we already mentioned, Floating Car Data, on the other hand, is generally owned by mobile phone or GPS companies, which make profit with it. Often, some aggregate index of traffic state are freely accessible¹⁸, but not the kind of detailed data we would need for our studies. While some results in the literature make use of FCD, most are based on LDD, as are the results presented in this thesis.

2.2 Traditional approaches and their limitations

2.2.1 Hydrodynamic perspective : derivation of the fundamental diagram

In the simplest case of a single-lane, non-intersecting road of length L , one can successfully apply a fluid mechanics analogy, considering vehicles as moving particles. The flow q of vehicles on this road can then be related to the car density ρ and the speed v by the relation $q = \rho v$.

The study of the dependence of q and v on ρ constitutes the so-called fundamental traffic diagram, empirically identified by Greenshield almost a century ago [74]. We first propose an intuitive interpretation of the existence of this fundamental diagram, before detailing more formally the equations describing road traffic. This will allow us to understand that the fundamental diagram is only an approximation of the evolution of traffic on a road, and to understand why scaling up to a network is a complex problem. Both [75, 76] have been excellent sources for this section, while [77, 78, 79] offer even more details and perspectives on the subject.

Intuitive origin of the fundamental diagram

The fundamental diagram has its origin in the behavior of motorists. For safety reasons, drivers tend to leave a distance d with the vehicle in front of them, which increases the greater their speed. From this it follows that the car density $\rho = \frac{1}{d}$ is a decreasing function of v and conversely, that v is a decreasing function of ρ . In this case, the function $v(\rho)$ can take different forms, but it always respects the following rules:

¹⁸e.g. the average time lost during commutes or the average speed in a city

1. $v(\rho)$ is a continuous, decreasing and positive function.
2. $v(0) = v_{max}$, which can be the maximum speed allowed on this road, or sometimes a little more (motorists don't always follow the rules).
3. $\exists \rho_{jam} > 0$ such that $v(\rho_{jam}) = 0$, where ρ_{jam} corresponds to the density in traffic jams, with an almost zero distance between vehicles. Taking the length of vehicles as $\ell = 6$ meters, we find $\rho_{jam} \approx 150$ vehicles/km, a value typically observed.

Most often, $v(\rho) = v_{max}$ over a range of small values of ρ , and it is only above a certain density that the velocity starts to decrease. Regardless of the exact form of $v(\rho)$, it follows from its properties and from the fact that $\rho \geq 0$ that the flow is only a function of ρ , $q(\rho) = \rho v(\rho)$ and has the following properties

1. $q(0) = 0$ and $q(\rho_{jam}) = 0$
2. $q(\rho)$ has positive values.
3. $q(\rho)$ is continuous and concave and so there exists $\rho_{crit} \in]0, \rho_{jam}[$ such that $q_{max} = q(\rho_{crit})$ is the maximum value of the flow on this route.
4. $q(\rho_{jam})$ is differentiable (at least by pieces, depending on the form given to $v(\rho)$).
5. $q'(0) = v_{max}$

The set of the two functions $v(\rho)$ and $q(\rho)$ constitutes the fundamental diagram of a road. It can be measured empirically and variations between roads can be observed, which can be explained by the properties of the road (straight, winding, wide, narrow...) but also of its environment (visibility...), through the distance d that motorists wish to leave between them. For instance, the fundamental diagram changes when it rains, with $v(\rho)$ and $q(\rho)$ taking lower values than in dry weather.

First order LWR approximation and shock-waves

This "hydrodynamic" description of traffic is based on the assumption of a continuous medium. Let us introduce the following notations: our problem is one-dimensional in space and depends on the position x on the road, as well as on the time t . We can then create the continuous quantities of vehicle density $\rho(x, t)$ and flow $q(x, t)$ from the count of vehicles $N(x, t)$ passing at x between t and $t + dt$,

$$\rho(x, t) = \frac{\partial N(x, t)}{\partial x} \quad \text{and} \quad q(x, t) = \rho(x, t) v(x, t) = \frac{\partial N(x, t)}{\partial t}$$

which verify the conservation of the number of vehicles:

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial q(x, t)}{\partial x} = 0 \quad (1.3)$$

Without additional assumptions, it is not possible to solve this differential equation. Lighthill, Witham and Richard [80, 81], using Greenshield's observations, made the hypothesis of the existence of a fundamental diagram, i.e. the existence for any value of ρ of a unique flow $q_{FD}(\rho)$ ¹⁹. The equation 1.3 is then simplified to:

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{dq_{FD}(\rho)}{d\rho} \frac{\partial \rho(x, t)}{\partial x} = 0 \quad (1.4)$$

The solution to this equation is of the form $\rho(x, t) = \rho(x - wt)$, with $w = \frac{dq_{FD}(\rho)}{d\rho}$ the velocity of shock-wave propagation in the medium. It follows that if we are interested in the case where a road is separated in two zones with distinct car densities $\rho(x, 0) = \rho_1$ for $x < x_w(0)$ and $\rho(x, t) = \rho_2$ for $x > x_w(0)$, the position $x_w(t)$ of the separation between the two density zones moves in space with :

$$x_w(t) = x_w(0) + \frac{q(\rho_2) - q(\rho_1)}{\rho_2 - \rho_1} t \quad (1.5)$$

It should be highlighted that the traditional approach described here still is an excellent tool to calibrate highways or to avoid congestion formation using dynamical speed limit control [82]. In particular, equation 1.5 gives prescriptions as to the maximal flow one can let enter into a congested road if one wants the jam to vanish rather than to grow.

The reasoning can also be made more complex and extended to the case of highways, with several lanes, different types of vehicles or even with entrances and exits. Still in the analogy with fluid mechanics, this requires to add a velocity profile (one speed for each lane) and a notion of viscosity (the interaction between drivers of different lanes due to the fact that a car needs time to reach a new speed). These models have been developed and used with great success over the past few decades [83, 84, 85], but are out of the scope of this thesis.

Empirical and experimental validation, form of the fundamental diagram

Since Greenshield's first observations, the fundamental diagram has been measured and described times and times again. Real-worlds fundamental diagrams do not follow a trivial mathematical function, such that several shapes and models have been proposed, as illustrated in figures 1.4 and 1.5

One might wonder if the proposed model of a fundamental diagram with a critical density above which congestion emerges is indeed inherent to road traffic, or if it is a consequence of imperfections

¹⁹Known as the LWR approximation from the name of its inventors

either in the drivers' behavior or in real world road infrastructures, where bottlenecks would trigger the emergence of congestion. This is one of the rare questions regarding traffic where, on top of empirical observations, experimental data exists. Experiments on closed circuits, without any bottlenecks, confirmed the existence of a critical density separating a free-flow phase from a congested phase with shock-waves [86, 87].

Additionally, a recent study has investigated the impact of self-driving cars and adaptive cruise-control on the existence and the shape of the fundamental diagram [88]. It appears that a diagram such as the one described here still exists when machines drive rather than humans, albeit with very different critical parameters, as the capacity on the road is increased, but at the cost of much faster traveling shock-waves (with upstream speeds of up to 100km/h , which could cause safety problems in case of a mix of human and non-human drivers on the road).

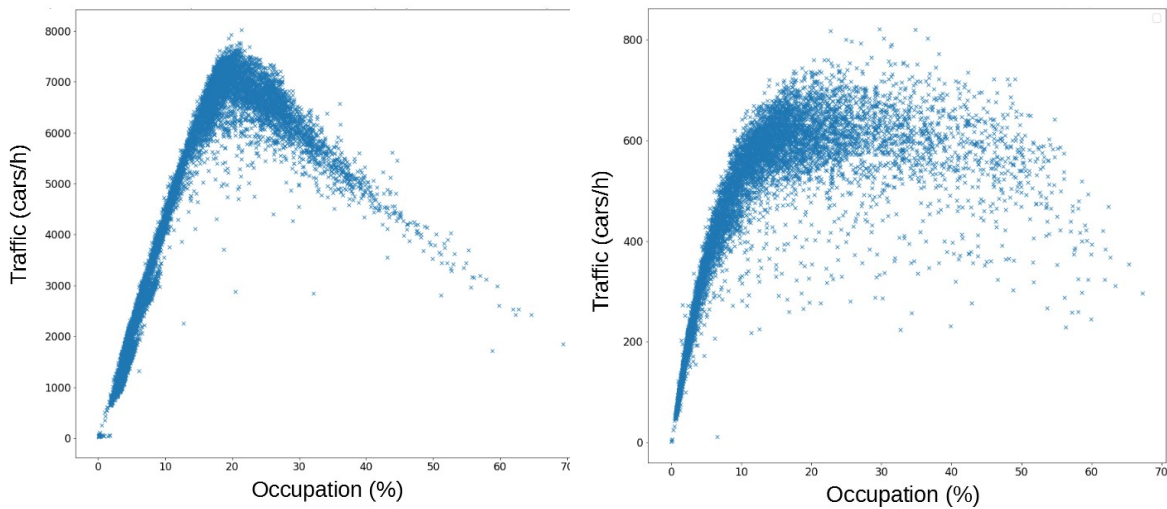


Figure I.4: Real-world observation of a fundamental diagram for the highway-like Boulevard Périphérique (left) and for the Avenue Jean Jaurès (right) in Paris, France

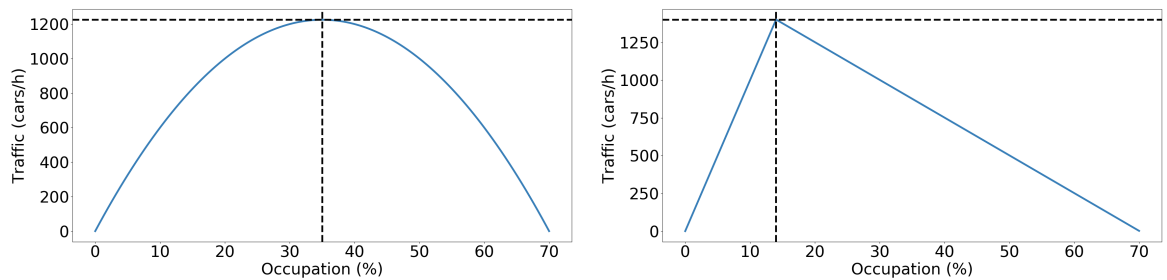


Figure I.5: Simplified models of fundamental diagrams, proposed by Greensfield (left, parabolic) and Newell-Daganzo (right, triangular)

2.2.2 Generalization attempts and the difficult scale-up to urban networks

Metastable congested states, generalized phase transition

Note that the reasoning we presented, originally proposed by Lighthill, Witham and Richard [80, 81], implicitly assumes the absence of driver anticipation. One could instead imagine a dependence of the speed on the density or speed variation encountered by the cars and thus a term in $\frac{\partial^2 \rho}{\partial x^2}$ or $\frac{\partial^2 v}{\partial x^2}$. The most general way to describe traffic is the Burgers equation for the speed of vehicles:

$$\frac{\partial v(x, t)}{\partial t} + v \frac{\partial v(x, t)}{\partial x} = \nu \frac{\partial^2 v}{\partial x^2} \quad (1.6)$$

Again, solving this equation 1.6 cannot be done without additional assumptions on the relationship between v , ρ and q . If, in empirical fundamental diagrams, the relation $q(\rho)$ is generally very well defined in the fluid regime $\rho < \rho_{crit}$, one finds that the measures are often more scattered in the congested regime $\rho > \rho_{crit}$, as shown in figure 1.4. New models have thus been proposed, satisfying both the Burgers equation 1.6 and the observation of metastable congested states in the fundamental diagram [89, 90]. Without going into details which would be out of the scope of this work, the most recent results in the field of traffic engineering agree on the following :

- The fundamental diagram is a measure of the average relationship between v , ρ and q .
- In the congested regime, we can observe deviations of the instantaneous measurements from the (average) fundamental diagram.
- More specifically, the free flowing regime is one stable regime, while the congested regime is actually a collection of metastable regimes [89, 90].
- The presence of a maximum q_{max} for a critical density ρ_{crit} remains preserved.
- There is **no** unique function $F(\rho, q, q_{max}, \rho_{crit}, \rho_{jam})$ which would allow to precisely describe all roads in a universal way [91].

It is important to keep this last point in mind. The fundamental diagram is a very practical description of traffic on a road scale, which accounts for the main phenomena (occurrence of congestion, shock waves). However, it can only be a simplified model of reality. We can therefore be satisfied with taking very simplified forms of the fundamental diagram in the framework of toy models, the simplest being the triangular model (Newell-Daganzo) and the parabolic model (Greenshield) (see figure 1.5).

Scale-up to a 2D-network

Equation 1.3 can easily be transformed for the case of a plane (x, y) :

$$\frac{\partial \rho(x, y, t)}{\partial t} + \nabla q(x, y, t) = 0 \quad (1.7)$$

This allows to successfully describe the movement of pedestrians, or vehicles at a very coarse scale. However, the specificity of road traffic is that it uses a network, which is only a sub part of the 2D-plane. Constructing a correspondence between the path in space and the path on the network is then already a difficult task in itself, but recent results [92] have proposed a method to find the vehicle flows on a network from the conservation equation 1.7. This model allows to anticipate the appearance of areas of high vehicle density on the network and is therefore promising. Nevertheless, this model ignores one of the essential components of an urban network, which is the interaction of vehicles at intersections.

Impact of signaled intersections on traffic

This hydrodynamic description of traffic faces an important limitation when one wants to take into account junctions and in particular signaled intersections²⁰. The addition of time-dependent boundary conditions for the flux on each of the links of the network further complicates the set of equations corresponding to the 2D-network and make it very poorly suited to the task.

One could in theory use the properties of the fundamental diagram to extract the characteristics corresponding to the formation and dissipation of queues at each traffic light, as proposed in [93]. However, the resulting system of equation soon becomes too large to be solved (as traffic jams can spill-back into upstream roads).

Further, it is important to note that the presence of traffic lights completely changes the hypotheses which led to the establishment of the Fundamental Diagram (e.g. the absence of exogenous factors and the absence of driver anticipation). In practice, streets of urban networks do not necessarily display a well defined fundamental diagram and do not respect the rules described in the previous section as visible in figure 1.4 and as we shall detail in section 7.3.3.

Let us give one fictional example to illustrate the issue. Imagine a low capacity road, reaching a signaled junction from which all exiting roads have a high capacity. This hypothesis ensures that, no matter the traffic exiting our road of interest, the traffic will flow freely when exiting the junction. This means that, at high density, the traffic on our road of interest becomes entirely determined by the capacity of the traffic light. The fundamental diagram does not display a decrease and the methods described previously cannot be applied. This illustrates the fact that for signaled intersections, methods

²⁰i.e. equipped with traffic lights

based on queuing theory or on cellular automata will be much more effective, as we shall explain in section 2.3.

2.2.3 Introducing Branston's Bureau of Public Roads function

Empirical study of travel times, origin of the BPR

Another empirical way to describe traffic at the road level was introduced by Branston for the US Bureau of Public Roads in 1976 [94]. He proposed a function, often referred to as the BPR function, relating travel time on a road to traffic demand.

More precisely, for a road of length L , the travel time τ behaves as:

$$\tau = \frac{L}{v_{max}} \left(1 + \alpha \left(\frac{Q}{q_{max}} \right)^\mu \right) \quad (1.8)$$

where v_{max} is the free-flow speed on the road, Q is the traffic demand and q_{max} is the maximum traffic that can be carried by this road. α and μ are coefficients that determine the sensitivity of the road to congestion and vary from road to road (just as the function $v(\rho)$ in the fundamental diagram may vary).

Note that traffic demand is not an easy quantity to measure. Indeed, we are talking about the number of people who want to use this road per unit of time, not the actual flow on the road. In the free-flow regime $Q < q_{max}$, we have $Q \approx q$, i.e. the traffic using the road is close to the demand. On the other hand, in the congested regime, we have $Q > q_{max} > q$ and we see the formation of traffic jams with an increase in the density of cars ρ on and upstream of the road.

Another source of information than the traffic measurements is then necessary to know the motorists' routes and to evaluate the traffic demand Q on this road to deduce the travel time. The BPR is thus particularly well suited to the case where the traffic demand is known and one wishes to calibrate a road accordingly. On the other hand, for the study of traffic behavior on an already built network, it is not the most adequate tool.

Relation with the fundamental diagram

To overcome this intrinsic limitation of the BPR, it has been shown in [76] that one can replace Q and q_{max} by the corresponding density (ρ and ρ_{crit}) and that one then obtains a new function of the same form as the BPR (with simply potentially different values of α and μ):

$$\tau = \frac{L}{v_{max}} \left(1 + \alpha \left(\frac{\rho}{\rho_{crit}} \right)^\mu \right) \quad (1.9)$$

In this form, the link between the BPR and the fundamental diagram becomes obvious. The inverse of the BPR gives us the velocity as a function of density:

$$v(\rho) = \frac{v_{max}}{1 + \alpha \left(\frac{\rho}{\rho_{crit}} \right)^\mu} \quad (I.10)$$

To satisfy the properties of the fundamental diagram, we must add the condition $v(\rho_{jam}) = 0$ (which, considering that $\mu > 1$ and usually $\rho_{jam} > 3 \rho_{crit}$ is a reasonable assumption to make) and the BPR then becomes a special case of fundamental diagram.

2.2.4 Agent-based Land Use and Transport Interaction models

Most decisions related to urbanism, including changes to the road network, are made using so called Land Use and Transport Interaction models (LUTI). The first such model was imagined by Lowry in 1964 [95], but it was from the 1980s onwards that they were considerably improved and that their use became generalized. They are based on the assumption of rational agents maximizing their utility. In particular, traffic assignment is typically done using some sort of variation of the Wardrop principle and the BPR is a very widely used tool to estimate the travel time under a given traffic demand [96, 97].

LUTI models aim at capturing the whole picture of human interactions in cities. Precisely, they were developed as a response to the unexpected suburbanization that followed the construction of urban highways, with the goal to evaluate all possible impacts a decision could have. First models included mainly land use²¹ and transportations networks, but recent softwares include an array of other aspects, such as water and energy supply or pollution [97, 98]. This ambitious program requires a large amount of initial data, that is then processed to give the most probable outcome [96]. In particular, this also requires to fit numerous parameters (e.g. the elasticity of utility with regards to every possible variable), which results in high uncertainties, as highlighted by the example of the closure of the Voies sur Berges.

LUTI models therefore fail to match the parsimony principle that we established earlier on. Perhaps more worryingly, the large number of parameters which need to be calibrated also means that they fail to be universal. In [98], the authors suggest that models working for the United States give wrong predictions when applied to European cities. All in all, there is no denying that these models are extremely useful to take very specific local decisions and anticipate all possible repercussions. Had they been used at the time of the Federal Highway Act, there is no doubt that the new highways would have been better integrated and that their negative impact could have been mitigated. However, LUTI models are not the correct tool if we want to progress in our understanding of the fundamental mechanisms governing urban traffic.

²¹Land use can cover several aspects such as the type of construction, population density, population income, etc...

2.3 Microscopic approach

As we have mentioned, the behavior of cars around traffic lights is well suited to models from queuing theory [99, 100]. However, because we are mostly interested in the spatial aspect of traffic (rather than just a number of cars waiting in a queue) and of the peculiar traffic dynamics which apply during braking and acceleration, cellular automata models soon gained interest to simulate traffic in general and in the urban context in particular.

Cellular automata models were introduced in the community of statistical physics around 1970. They are particularly suited to problems where moving particles cannot overlap or overtake each other and where space can easily be discretized. A very simple and famous example is the Asymmetric simple exclusion process (ASEP), introduced by Frank Spitzer in 1970 and which has since then become a standard toy model for the stochastic movement of particles with nearest-neighbor jumps [101, 102, 103]. Other examples include the Symmetric exclusion process (SEP) or any type of random walk [104].

Cellular automata models were soon applied to highway traffic [105] and urban traffic, first with a very simplified 2D-model which did include interactions of cars going in different directions, but no traffic lights [106, 107, 108, 109, 110], which showed the existence of a well defined critical density of cars on the network leading to a sharp jamming transition at a macroscopic level (through the phenomenon of gridlocked intersections). In further models, the effect of traffic light was investigated [111, 112, 113, 114, 115], which highlighted the importance of traffic light synchronization (also known as green waves) for the macroscopic efficiency of the network.

We should note that cellular automata are not the only microscopic models of urban traffic which have been used with success. Other models such as follow the leader [116, 117] or optimal velocity [117, 118] show good correlation with the observations. However, we decided to highlight the role of cellular automata, as we will be using it in the rest of this thesis.

All of these microscopic models share some very important traits, making them valuable tools: they rely on a very small number of parameters, which are easy to calibrate using real world data, and they capture the essential, universal properties of traffic. As such, they are very interesting toy models to gain insight into urban traffic, as it has for instance been the case for the generalization of traffic light synchronization strategies in the last decades.

2.4 Rise and promises of machine learning

The 21st century has seen an incredibly rapid increase in data collection and availability, as well as in computational power. Mobile phones, credit cards, smart traffic lights, etc, numerous objects are connected to the cloud, and record our movements and actions. This wealth of data can be used in a

new fashioned way, when compared to the traditional physical approach. Instead of using data to test a theory, one can explore patterns and correlations between a large array of parameters all at once, and identify the most important variables of the problem.

In the field of transportation, machine learning is mostly used to model and predict rail transport usage, as the smart cards used by train users in most countries allow for a very high quality data-set. Similarly, dock-less bikes, which are unlocked using a mobile phone application, are well suited to a statistical analysis of its mobility patterns. Machine Learning proves particularly useful to highlight the motives that emerge from the data, from a temporal and a spatial point of view [119, 120, 121], enabling a finer pricing strategy [122], or an efficient allocation of the trains [123, 124].

In the context of road traffic, Machine Learning sees its first applications in the dynamic management of traffic lights [125, 126, 127, 128, 129]. Further, it enables to reconstruct a more thorough picture of the state of traffic on the network from scarce real-time data [71, 130], which makes it a strong and complementary tool for traditional approaches²². It is hoped that it will allow to predict the formation of congestion in advance and help optimize real time traffic routing [122, 131, 132, 133, 134]. As for rail transportation, the growing data from car mobility patterns also enables the forecasting of long term trends in road usage or during occasional disruptive events [135, 136, 137]. Finally, methods from Machine Learning can be used to study a large number of networks and to compare their structure using new and not necessarily intuitive metrics [138], as well as the traffic dynamics patterns that emerge on them [139].

This overview of the applications of Machine Learning to urban traffic deserves a few comments. First, it is evident that it is a very young branch of sciences and that its full potential in the context of urban traffic has yet to be discovered and understood. Second, beyond its undeniable practical interest, Machine Learning can also be used to identify patterns and is in that sense a very powerful tool in the statistical physicist's empirical toolbox, as are probability distributions or dimensional analysis.

2.5 Towards a macroscopic description of cities

As made apparent from the examples listed throughout this section, the study of urban traffic needs its own methods and tools, distinct from those successfully used to describe traffic at the road level. In the same way that the hydrodynamic description of traffic ignored the "microscopic" reality of cars and driver behaviors and made use of "macroscopic" continuous quantities (flux, density and speed), a solution to the problem of urban traffic is to take an additional step back. New approaches have been developed over the two last decades, which can roughly be separated into two categories.

The first category ignores the heterogeneity of traffic inside of each street. Streets are then the

²²Recall that a limitation of both Loop Detector and Floating Car Data was their scarce nature

elementary constituents of the system and are defined by their (average) density, flux, speed or level of congestion. The work then concentrates on the interaction between streets inside the urban network and in particular the spread of congestion from one street to another.

The other category takes the macroscopic approach one step further, and simply considers the city as a whole (or more precisely, sub-parts of the city). The state of urban traffic is estimated using averages of density or flux on the entire network of chosen sub-parts of the city, typically referred to as “reservoirs”. The question is then to understand the dynamics of traffic from one such “reservoir” to its neighbors, which considerably simplifies the problem (as it reduces the number of interacting parts), or to study the differences between cities in terms of the properties of their average traffic.

We will make use of both approaches throughout this thesis. We do not give further details about both methods, as they will be introduced more thoroughly in the corresponding chapters.

Conclusion

Let us sum up this introductory chapter. As we have seen, cities and their road networks look remarkably similar all around the world, which suggests that the main mechanisms at play in their development are universal. Of course, some differences exist, as historical, geographical or cultural aspects vary from country to country and city to city, but they seem to be of a lower order of importance. Both the growth of the network and the dynamics of traffic on them are complex systems. They are the result of a myriad of individual decisions and details, which merge into complex collective behaviors. We aim at proposing a model for traffic on urban networks inspired by statistical physics. Such a model should be complex enough to reproduce the empirical observations made in cities. In particular, this implies that we find the most important parameters of the individual behaviors of automobilists, as well as their interactions. Examples of models ignoring the interactions, such as Wardrop’s second principle, are indeed unable to reproduce the complex non-linear phenomena observed empirically. Our model should also be as simple as possible, to guarantee its universality, i.e. its capacity to give accurate predictions at any time and in any geographic location. Parameters of our model should have a physical meaning in the context of the problem and should not be redundant.

In the second chapter of this thesis, we will study the growth of urban networks. We will compare models relating the network development to the population growth and show how this network growth can lead to abrupt transitions in the congestion patterns. Finally, we will present empirical and theoretical results suggesting that the construction of highways and ring-roads during the second half of the 20th century was mostly determined by the population increase.

The third chapter of this thesis will deal with traffic dynamics on the network. Using traffic mea-

surements averaged at the street level, we will investigate the propagation of congestion on the network. We will first give an overview of modern methods and then present empirical evidence suggesting that the network undergoes a phase transition during the formation of rush-hour traffic jams.

The fourth chapter embraces an even more macroscopic description of traffic. Recent work has shown that, much like the fundamental diagram for roads, a Macroscopic Fundamental Diagram (MFD) relates average traffic and average density at the scale of the network. We will first present the empirical and theoretical framework around this discovery, before exploring an important and missing part of our understanding of urban traffic, which is the interplay between network structure and its response to traffic demand, of which the MFD is a good measure. This will allow us to discuss future directions for the “physical” approaches to urban traffic. We will show that the existence of the MFD suggests that the problem of urban traffic certainly has some similarities with effective medium theories, which have interested physicists for some decades now, and that our knowledge of the MFD could certainly be refined by combining it with the knowledge about the spatio-temporal dynamics of traffic discussed in the third chapter.

Finally, we will discuss alternatives to the MFD to describe traffic in particular and urban transportation in general, highlighting the importance of collaboration to reach a socially optimal situation.

Chapter II

Interplay between population and road network structure and its impact on congestion regimes

The focus of this chapter is the study of the road network itself, which includes two complementary aspects. On one hand, we will highlight the fact that streets experience variable degrees of congestion and investigate the impact of the road network structure on the observed congestion regimes. On the other hand, as we shall see, the network structure is itself determined by the city's population growth.

We will begin by introducing tools used to measure and predict the heterogeneous repartition of traffic on the network. Using the concept of centrality, we will show that the growth of a city leads to abrupt transitions in the congestion regime that can form on its road network. After a first phase of self-organized growth of their road network, most large cities switched to a centralized urban planning to mitigate the congestion that inevitably starts to form.

We will in particular investigate the case of the emergence of high-speed infrastructure, using both an empirical approach and a cost-benefit analysis. We show that the appearance of urban highways and ring-roads in American cities in the second half of the 20th century only occurred for cities with populations of more than $\approx 300,000$ and $\approx 1,000,000$ respectively. This abrupt transition between city types suggests that simple mechanics drove the infrastructure decisions made in the 20th century. We thus propose a simple cost-benefit analysis, based on a comparison of the value of time lost to congestion and the value of infrastructure, and effectively recover these two transitions as traffic demand increases.

Finally, we will briefly discuss the prospects of this analysis if the externalities of an urban highway are taken into account, in contrast to the concerns of the funders and policymakers of the 1950s.

3 Introduction of some useful tools

It will come as a trivial fact to the reader that urban road networks are far from being homogeneous. They are composed of different types of streets of various widths, capacities and speed limits, meant for different purposes. The observed traffic and the level of congestion too are heterogeneous on the network. Everyone has the experience from individual streets or intersections particularly prone to congestion.

In this section, we will give some tools to characterize the heterogeneity of a given set of measures, initially developed in the context of economy, but also useful in this context. Then, we will introduce the concept of centrality and show how it can be used to predict the relative importance of roads in the network, which is a crucial step to tailor the network to the demand.

3.1 Characterizing inhomogeneity

3.1.1 Measuring inhomogeneity: Lorenz curve and Gini coefficient

Definitions

Corrado Gini initially proposed a coefficient to measure inequalities in the distribution of wealth or incomes amongst a population [140]. The best way to understand the Gini coefficient is graphically, based on the Lorenz curve as shown in Figure II.1. The Lorenz curve is obtained by sorting the population by increasing income, and then plotting the cumulative income as a function of the number of persons considered. The resulting curve is convex by construction and below the diagonal, which would correspond to the perfect equality of income amongst the population. The Gini coefficient G essentially measures the integrated distance between the real distribution and perfect equality and is normalized to vary from 0 (perfect equality) to 1 (one person owning everything). It can naturally also be used to characterize the inequality of traffic distribution on a network. For a set of n roads with traffics t_i , one has an explicit formula for the Gini coefficient:

$$G = \frac{2 \sum_{i=1}^n i * t_i}{n \sum_{i=1}^n t_i} - \frac{n+1}{n} \quad (II.1)$$

Defining hotspots

Obviously, the shape of the Lorenz curve contains more information than the Gini coefficient, which is an aggregated measure. Another relevant information one can get from the Lorenz curve is for example the number of “hotspots” in the distribution (i.e. individuals owning a majority of resources

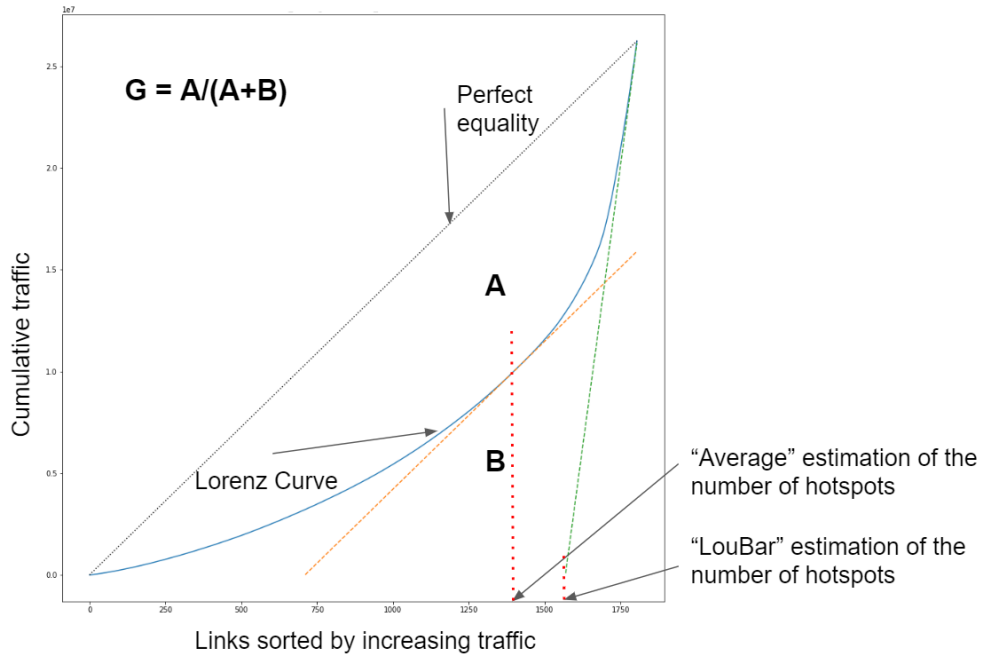


Figure II.1: Graphical definition of the Gini coefficient G , from the Lorenz curve of cumulative distributions and illustration of the estimation of the number of hotspots.

or in our case links concentrating most of the traffic or more traffic than the average). We can discuss two measures to do this, as illustrated in figure II.1. One can either look at the number of links having more than the average traffic (i.e. links above the part of the Lorenz curve which has the same slope as the diagonal), called “average estimation” on the figure.

Or one can use the intersection of the derivative in the last part of the Lorenz curve with the abscissa to account for the importance of the most used links of the graph. As for the average estimation, the hotspots will be the links with an index above the index of the intersection. This method “LouBar” was proposed by the authors Louail et al. in [21].

The Lorenz curve being convex, it is easy to demonstrate that the “LouBar” method systematically gives fewer hotspots than the average method.

3.1.2 Measuring the spatial spread: the Venables distance

Definitions

Having characterized the inhomogeneity of the repartition of traffic, we are now interested in answering the following question: do zones of high traffic or occupancy tend to be close to each other, or are they spread evenly on the graph?

To do so, we need an indicator of spatial spread, which we take from the literature [141]. The Venables distance is the average distance between pairs of links, when weighted by their traffic or

occupancy. It is defined in the following equation where $d(i, j)$ is the distance between links i and j , E is the set of edges in the graph and t_i is the traffic (or occupancy) of link i .

$$V = \frac{\sum_{i,j \in E} t_i t_j d(i, j)}{\sum_{k,l \in E} t_k t_l} \quad (11.2)$$

Understanding the Venables distance

Since we measure a distance, we need another distance as a reference. The natural choice is the unweighted average distance between pairs of links of the graph (i.e. taking $\forall i, t_i = 1$), which might change between dates, as the sensors available might vary with time. We will refer to this distance as V_g for Venables distance of the graph.

Another distance we will use as a comparison is the Venables distance one obtains for the null model where the traffic is randomly reshuffled over the links (and averaging over several permutations). We will refer to this distance as V_p for permutation. Indeed :

- if both the values taken by the traffic on a link, and the position of that link in the graph are random, one should get the same distance for each of those 3 measures. $V \approx V_p \approx V_g$
- if the traffic of each link is random, but all links with high traffic are concentrated in a specific place, one will have $V < V_p \approx V_g$. Conversely, if the links with high traffic are all on the borders of the network, one gets $V > V_p \approx V_g$
- if the traffic of each link is not random (e.g. one has two populations of links, one with traffic 1, the other with traffic 10), but their spatial distribution is random, then one will have: $V \approx V_p > V_g$

3.1.3 Some illustrations with real world measures on the Parisian network

We will rapidly illustrate the use of these metrics on the case of Paris. The Gini coefficient for Paris varies throughout the day, with lower value during the day than during the night. This result might seem surprising, but gives in fact a first argument as to why the Gini coefficient is not sufficient to capture the spatial distribution of traffic on the network : during the day, traffic is higher everywhere on the network, thus reducing the observed inequalities between roads. The Lorenz curve allows us to estimate the number of hotspots for traffic as being the $\approx 10\%$ most congested roads, i.e. 180 roads among the $\approx 1,800$ for which we have measures. Those roads with highest traffic and occupancy during the morning rush are displayed in figure 11.2

One can notice that the traffic is the highest on the Boulevard Périphérique and on the Voies sur Berges (which at that time were still open), i.e. on highway-like infrastructures, which comes as no

surprise. Hotspots for traffic also form continuous corridors. Hotspots for occupancy are however much more evenly spread and their position on the network, while somewhat correlated to the hotspots for traffic, is less intuitive.

Figure II.3 shows the comparison of the Venables distance to the two reference distances introduced before. We only display the case of traffic measurements, but the situation is similar for occupancy measurements. We can see that, when one includes the Boulevard Périphérique, the Venables distance is higher than what it would be for a randomly distributed traffic. This indicates that the traffic concentrates on the outskirts of the network. Removing the Boulevard Périphérique from the study confirms this, as we see a drop of the Venables distance. In that case, the traffic is actually slightly concentrated towards the center of the network. This result highlights the fact that the Boulevard Périphérique plays a major role in the commutes around Paris.



Figure II.2: Hotspots for traffic and congestion in Paris during a morning commute in 2014. Positions along the axes are given in meters in the Lambert 93 conformal conic projection system (EPSG:2154)

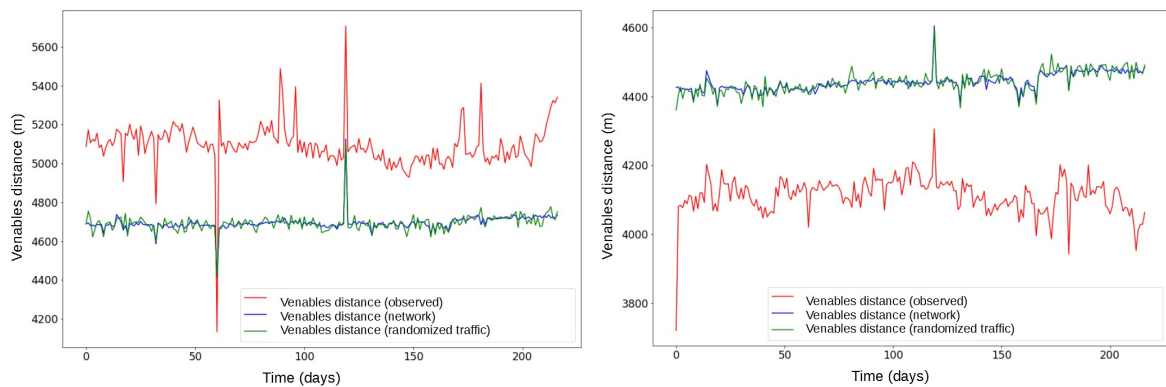


Figure II.3: Venables distance of the traffic distribution in Paris in 2014 with (left) and without (right) the Boulevard Périphérique

3.2 Concept of centrality

As soon as we are interested in irregular networks, it becomes obvious that the nodes and links of the network do not play the same role. It is then natural to define measures of "node importance", in order to rank them, or to identify possible weak points of the network, which we will call measures of centrality. The word "importance" is deliberately vague, since its interpretation depends strongly on the network under consideration and the question to be answered.

3.2.1 Different centralities

We will briefly illustrate a few types of centralities and their applications, before looking in more detail at the case of the *betweenness centrality* and its use in the case of road networks. Further details for each type of centrality, as well as a more comprehensive list of the (many) variants that exist can be found for instance in [142], while [143] studies the correlation between different centrality measures.

A very intuitive way of measuring the importance of a node is to compare its degree (i.e. its number of connections to other nodes) to the average degree on the network. This measure is called *degree centrality*, and is useful for instance to find popular individuals in a social network. Refined versions such as the *Katz centrality* or the *Eigenvector centrality*, also take into account the importance of the neighboring nodes when evaluating a node's importance.

We can also ask ourselves what influence an individual can have in the propagation of information or a virus. We are then interested in individuals who are close to a large number of individuals (i.e. separated by a small number of links from a significant part of the network). This can be measured using *closeness centrality*, which measures the inverse of the average distance of a node v to all other nodes of the network.

$$C(v) = \frac{\mathcal{N}}{\sum_{u \in \mathcal{V}} d(v, u)} \quad (11.3)$$

where \mathcal{V} denotes the ensemble of nodes of the network, $d(v, u)$ is the distance between nodes v, u and \mathcal{N} is a normalization term usually taken equal to the number of nodes in the network.

If we are now interested in a network carrying flows, such as a telecommunication or transportation network, we can be interested in two main questions: how to size the nodes of the network to support the demand, and how to make the network resilient to possible failures randomly affecting some nodes. The *Betweenness Centrality* (BC) is an appropriate measure for these questions. For each pair of points of the network, the shortest path is computed. Each node along this path sees its score increased accordingly. Nodes that appear in a large number of shortest paths, and which thus play an important

role in transport on this network will also have a high betweenness centrality. More specifically, the BC of a node v is defined as:

$$C(v) = \frac{1}{\mathcal{N}} \sum_{s,t|s \neq t \neq v} \frac{\sigma_{st}(v)}{\sigma_{st}} \quad (11.4)$$

where σ_{st} is the number of the shortest paths between s and t and $\sigma_{st}(v)$ is the number of the shortest paths between s and t that go through v . \mathcal{N} is a normalization term, that can for instance be taken equal to 1, or to the number of shortest paths that v could potentially be a part of (of order N^2 for a network of N nodes).

3.2.2 Betweenness centrality as a proxy for traffic demand

Computing shortest paths

The first ingredient to determine the betweenness centrality is to identify the shortest paths between any pair of points of the graph, using for instance Dijkstra's algorithm [144]. This implicitly requires that the network edges have some sort of length, and thus some sort of spatial embedding. Depending on the context, all edges can be taken of length one (e.g. communication or electrical network), randomly distributed or have a physical length for a spatial network.

In the case of a road network, the most relevant metric is actually the time it takes to travel a certain link or a certain path on the network, as it is typically what users try to minimize during their trips, according to Wardrop's first principle¹.

Origin-Destination Matrix, weighted betweenness centrality

In equation 11.4, σ_{st} represents the number of shortest paths going from s to t , from a strictly topological point of view. Indeed, for a perfectly regular grid lattice, multiple paths can be equivalent to travel from a node to another. The contribution of each path to the centrality of v is then weighted by the number of alternatives existing.

One can however imagine a case where the number of travels between two points s, t is a function of those two points. This is the case for road traffic in particular, as we know that people tend to live and work in similar places. One can then define the Origin-Destination Matrix T , which gives the number of commuters traveling between each pair of nodes s, t . The betweenness centrality 11.4 can then be adapted to:

¹Other metrics such as fuel consumption or total travel *price* (for networks with road tolls) can also be relevant

$$C(v) = \frac{1}{\mathcal{N}} \sum_{s,t|s \neq t \neq v} T_{st} \frac{\sigma_{st}(v)}{\sigma_{st}} \quad (11.5)$$

where the normalization term \mathcal{N} now is typically the number of travels on the network, i.e. $\sum_{s,t|s \neq t} T_{st}$.

Edge centrality, dual graphs

Note that for now, we discussed measures of the importance of nodes in a network. If we want to use centrality as a tool to estimate traffic, we would be more interested in the centrality of the links, rather than the nodes. Instead of using the physical graph of intersections connected by roads, one can represent the network by its dual, where roads are connected by intersections, and apply the same reasoning to this graph. Note that in the case of the BC, this is not an ideal solution, as it then ignores the length of the roads. Instead, one can slightly change the definition of the BC given in equation 11.4 to:

$$C((v, w)) = \frac{1}{\mathcal{N}} \sum_{s,t|s \neq t \neq v \neq w} \frac{\sigma_{st}((v, w))}{\sigma_{st}} \quad (11.6)$$

where (v, w) represents the edge from node v to node w .

Computing betweenness centrality

The naive approach to computing betweenness centrality, presented in the previous section, is very time consuming. Indeed, computing the shortest path between two points of the graph is of complexity $\mathcal{O}(N^2)$ (worst case scenario), and computing this for each pair of points yields a running time of $\mathcal{O}(N^4)$. However, Brandes [145] noticed that this naive method includes a lot of redundancy in the computed shortest paths and proposed a faster algorithm, running in at worst $\mathcal{O}(N^2 \log(N))^2$. This algorithm is extremely popular and is typically pre-implemented in *Python* libraries such as *networkx*.

Albeit faster, the $\mathcal{O}(N^2 \log(N))$ complexity still means that computing BC on large networks can be a challenge in terms of computing time.

²If we consider spatial graphs where the average degree is generally independent of N

3.2.3 Limits of the betweenness centrality

Empirical validation of the BC

Figure II.4 shows the values of closeness and betweenness centrality for the streets of Jinan city in China (in terms of distance traveled). One can see that closeness centrality typically highlights roads which have a central position in the graph, while betweenness centrality is maximal for links which play an important role in the transport of commuters from the suburb to the center and more generally larger, straighter, faster roads of the network. In that sense, the BC reasonably matches the observed flows of traffic on the network.

More thorough studies on the quality of the BC as a proxy for traffic demand show, however, that it does not perfectly match observed traffic. In particular, the authors of [55] show that Wardrop's first principle, which is implicitly assumed when using BC as a proxy for traffic demand, is not always correct. People do not always use the shortest path, but spread evenly across paths which are within a certain range of the shortest path, favoring other criteria such as fewer intersections or fewer sharp turns over shorter travel time in their route choice [147].

Further, the BC has a fundamental limitation, which is that it does not dynamically adapt to the changes in the network's congestion, thus missing potential changes to the shortest paths.

Alternatives or refinements exist but are harder to compute

To overcome this limitation, refinements of the betweenness centrality have been proposed. Authors of [148] proposed an approach based on an analogy with electrical circuits and considered the flow on each road as function of an imposed "potential" (which worked reasonably well for pedestrian flows), while authors of [149] proposed an approach mixing different measures of centrality into one metric to estimate traffic demand.

Alternatively, articles such as [150, 151] use a method where the network is loaded gradually. For each new trip, the shortest path given the current congestion levels is computed and the congestion and travel times are updated accordingly. These methods are much closer to real world observations, but also require significantly more computing time, as all shortest paths have to be computed independently (thus in $\mathcal{O}(N^4)$).

The BC should thus be seen as a good tool in the context of toy models, rather than a very precise one for real world traffic engineering. In particular, the BC has the advantage of giving a measure of which path people *would* follow if congestion didn't exist. This gives a tool to scale streets appropriately, while also giving an indication as to where congestion is likely to emerge in a network.

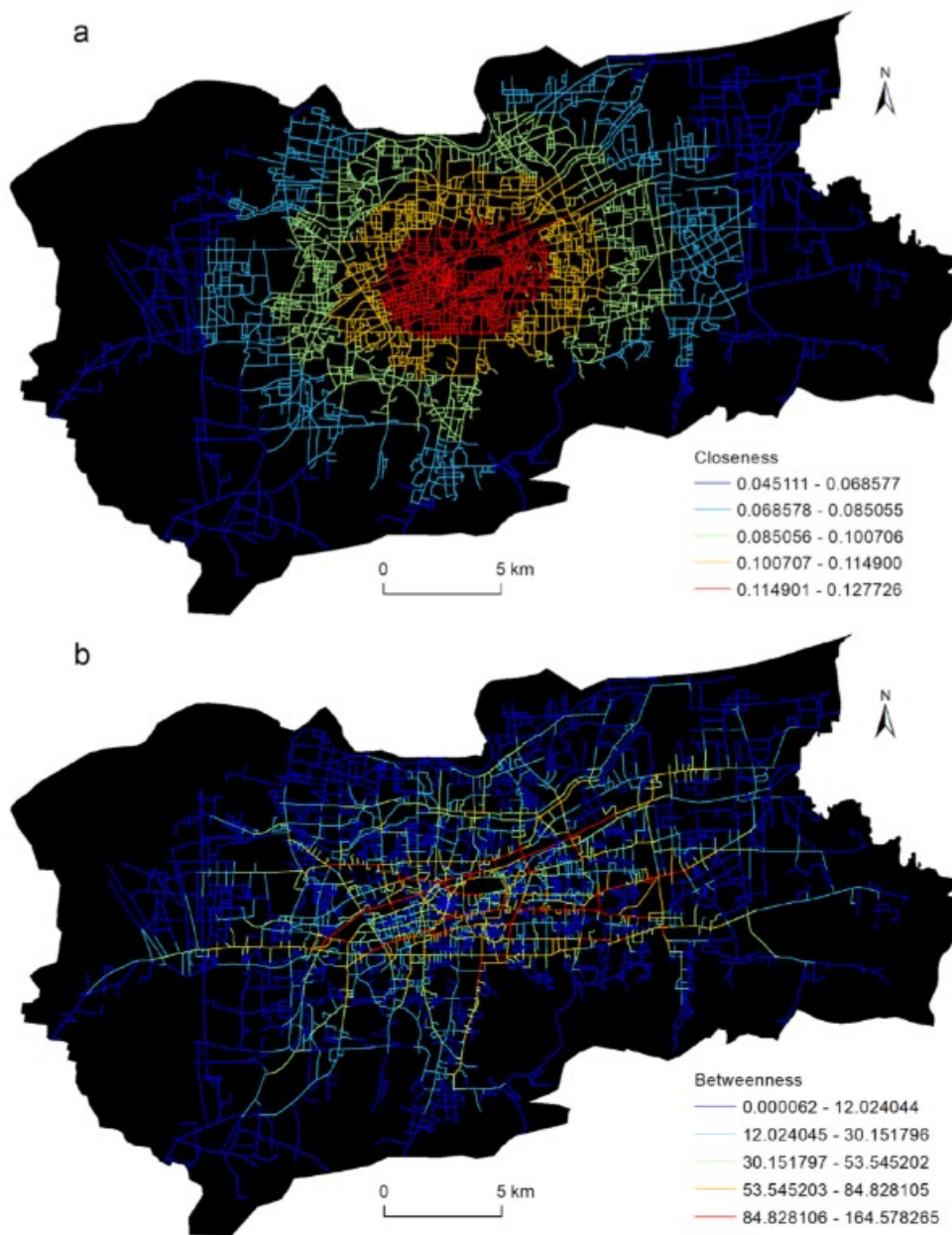


Figure II.4: (a) Closeness centrality and (b) Betweenness centrality (in terms of travel distance) measured for the road network of Jinan, China. Credits to *Yin et al.*[146]

4 Network growth leads to abrupt changes in the congestion regimes

Having established that congestion is *a priori* not spread evenly on the network, and having presented a tool which allows to estimate hotspots for traffic demand on the network, we will now study the impact of network structure on the position of the node with highest betweenness centrality. We will first focus on the dynamics of network growth, which as we will see is hard to disentangle from population and urban growth in general. Then, we will show that continuous changes in the structure of the network, by successive addition of links, can lead to abrupt changes in the congestion regimes experienced by the network. Results presented in this section are taken from the literature, but we believe that they are interesting to grasp the problematic of urban growth and urban traffic. Further, we will propose to combine different results and propose open questions on the topic which we believe present some novelty.

4.1 Symbiotic relationship between population growth and network growth

Historically, road networks emerged as a mean to connect different places, mainly for trade. Capacity was not an issue and the growth of the network was mainly dictated by the competition between travel-time and cost of construction of a new road. In particular, it appears that the decision to build new roads was mainly local (e.g. inhabitants of two localities agreeing on the fact that adding a shortcut between their two localities would be mutually beneficial) [3, 6, 35]. In that sense, the process governing the emergence of new roads and the formation of a road network are actually remarkably similar to those leading to the formation of biological networks [12, 13, 14, 7, 152]. This is of particular interest, as biological networks can easily be studied in a laboratory, giving experimental arguments to complement the empirical observations. Typically, very mono-centric biological networks (such as veins in the body) or networks with low flows and high construction costs will develop following a minimal spanning tree [12, 152], while decentralized biological networks with an incentive for resilience will form grids or honeycomb lattices [14].

Both in biological and in urban systems, it appears that the organism (resp. the city) and its transportation network influence each other [153, 154]. In the case of a city, the spread of the city will require the construction of new roads to connect new parts of the city. These new roads, however, make new parts of the network accessible and increase their attractiveness, thus influencing the dynamics of population growth and land use distribution and leading to the very common poly-centric structure of cities [30, 141]. Closeness centrality can be used as a reasonably good proxy for the attractiveness of nodes of the network, likely to gather population and/or economical activity [30, 146, 154].

4.2 Network growth and changes in the congestion regimes

As the population and the network grow, congestion starts to become an issue in some points of the network, typically of high betweenness centrality as we have discussed in the previous section. For organisms, the cost of congestion can become such that it has to adapt in order to survive. Adaptations can come in two forms, either a transition from a mono-centric to a poly-centric (or even completely decentralized) structure, which dilutes the demands for transportation more equally on the network, and/or the shift from a tree-like structure to a more redundant and resilient structure [153, 155]. In particular, this can lead to the formation of circular shortcuts around the center of the organism [153], also observed in public transportation networks [156] and obviously also in road networks, where, as we will discuss more thoroughly, ring-roads are quite a common infrastructure.

In this part, we will discuss in more detail a toy-model for such a transition in congestion regimes as a urban road network grows, proposed by Lampo et al. [157]. We will then discuss the importance of a top-down planning to combat the averse effects that a self-organized network can have on traffic congestion for larger cities.

A toy model for urban networks

Lampo et al. [157] proposed a simplified model for the structure of urban networks. The city center, typically consisting of strongly connected nodes, is approximated by a grid. On the contrary, the periphery consists mainly of weakly-connected arterial roads, which are in this model approximated by a tree-like structure. The resulting model of trees branched to a central grid, which the authors refer to as the grid-tree model, is illustrated in figure II.5. Without entering into all the details, the resulting network is defined by the following parameters :

1. The number of nodes in the grid $N_G = w^2$
2. The number of trees n_t
3. The number of nodes in each tree N_T , defined by the depth h (the number of ramifications) and the branching factor r (the number of branches at each ramification)

Additionally, different types of noise can be added to this model to account for local heterogeneities observed in real-world networks.

Authors assume that the origin destination matrix is flat, i.e. that people travel between each pair of nodes of the network with equal probability. However, one can note that this implies that the size of each tree is also in some way a measure of the population living in this periphery. They then concentrate

on the derivation of the betweenness centrality of the nodes which play a particular role in the network and could therefore be those with highest BC, namely the central node, the branching nodes between tree and grid, and the root node of each tree.

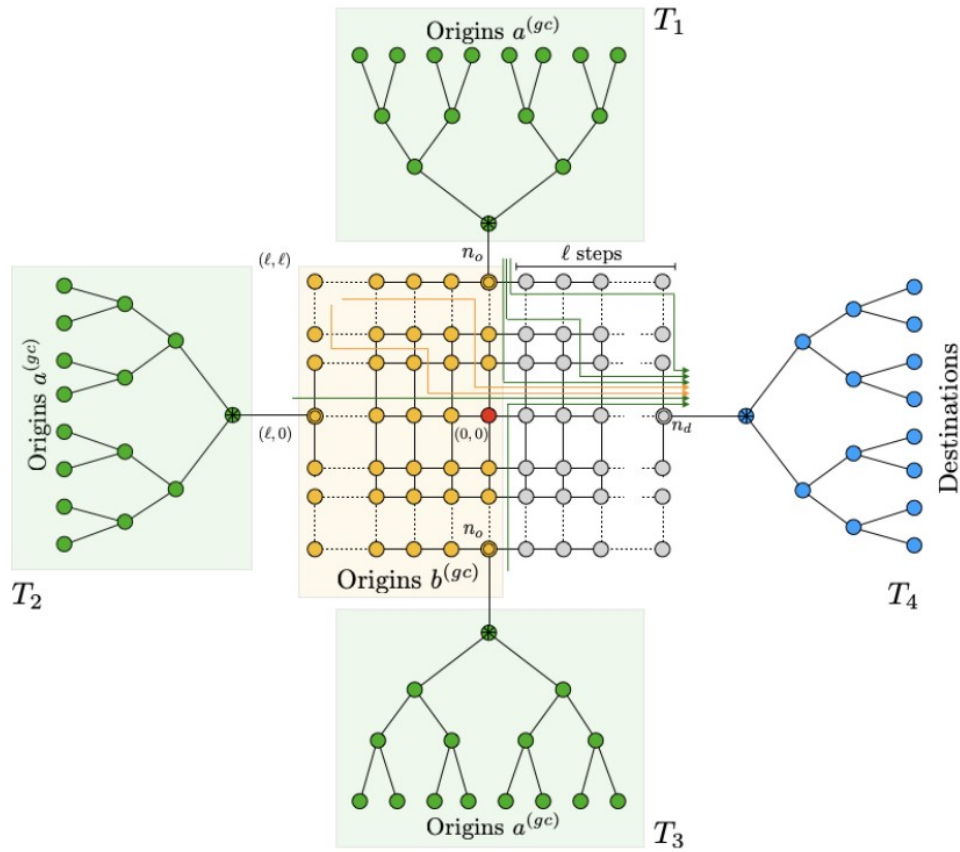


Figure 11.5: Grid-tree model for cities. n_t trees, defined by their branching factor r and the depth h , are connected to a central grid of size w . 3 types of points stand out in this network : the center of the grid $(0,0)$, in red, the interface between grid and trees (marked by a double circle), and the root of the trees (marked by a star on the image). Credits to *Lampo et al.*[157]

The aim of the study is to find which of those three points is the first to reach saturation, as a function of the topological features of the network w, r, h, n_t and in particular the ratio $\frac{n_t N_T}{N_G}$, which is expected to increase as a city grows. The main advantage of the method described in [157] is that the noiseless case can be solved exactly analytically.

Transition between congestion regimes and empirical validation

The authors find that, as the size of the trees increases, the onset of congestion moves from the city center, to the grid-tree interface and finally to the tree-roots. Even with addition of noise in the grid-tree structure, the position of the most congested links, measured by its distance to the city center, displays abrupt changes as the size of the trees increases.

They verify these analytical and numerical prediction made with the grid-tree model on data from real cities, as shown in figure II.6. For each city, they calculate the average position of the 15 nodes with highest betweenness centrality included in a circle of radius R_p from the city center. With the increase of R_p , this average position changes continuously inside a given regime and then brutally, as the network switches between two congestion regimes. Note however, that each node of the network sees its centrality change continuously as the network grows and that it is just the position of the most congested node that displays this brutal switch.

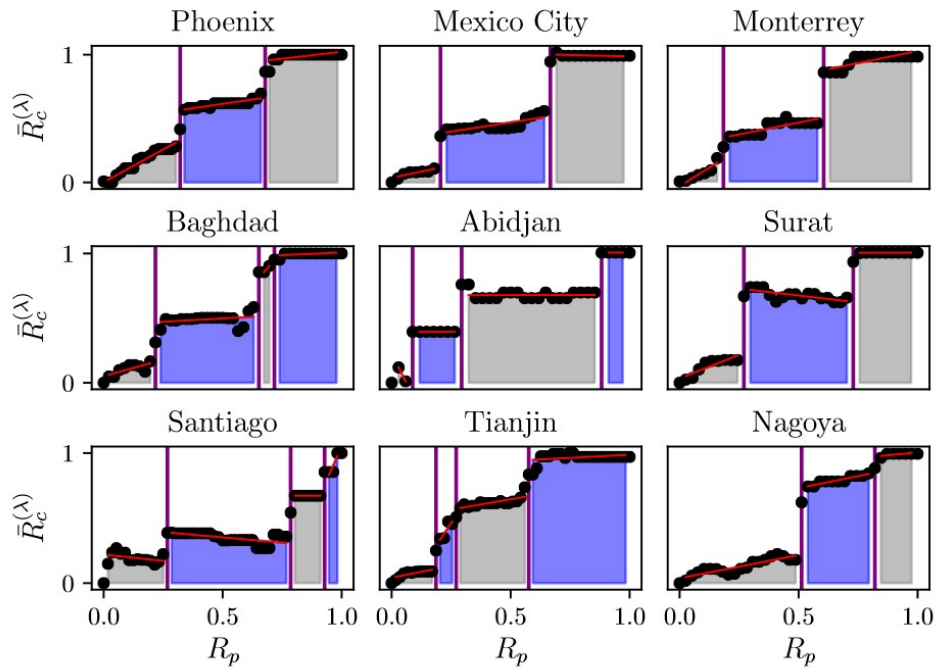


Figure II.6: Analysis of congestion regimes for nine cities. Points represent the average distance from city center of the $\lambda = 15$ nodes with largest betweenness. Vertical lines indicate the transitions between congestion regimes. Plots are normalized between 0 and 1 considering the radius of the different cities. Credits to *Lampo et al.*[157]

Discussion and perspectives

A limitation of the discussed article is the method used to empirically validate the results of the grid-tree model. Indeed, the authors looked at a given city and identified the most central nodes in an increasing radius. It would have been more powerful to look at the historical growth of the cities and to identify the described changes in congestion regimes as a function of network growth, rather than as a function of studied radius. However, as the authors highlight, cities grow mostly from their periphery, with the historical center remaining mostly constant, such that their method is reasonable.

This method can certainly be adapted and used to answer different questions. We did not have the time to investigate these questions, but present them for the curious reader.

1. What would be the most cost effective way to mitigate congestion in the grid-tree model ? In particular, when in the grid-tree-interface or the tree-root regime of congestion, could a few strategically placed links contribute to reduce congestion ?
2. In the context of a post-Covid world, where home-office is generalized, can we identify strategical routes to remove from the origin-destination matrix ? Indeed, one can imagine that removing a small number of well chosen origin and destination pairs (i.e. a small fraction of well chosen commuters) could lead to an abrupt change in the observed congestion regimes.

The relationship between network growth and congestion regimes is particularly interesting in the context of this thesis. It is indeed an illustration of the complex non-linear collective behaviors that can appear from seemingly simple “microscopic” elementary processes. In particular, the article [157] that we detailed highlights the interplay between population growth, network development and congestion regimes in cities and the need for careful urban planning. Indeed, as we mentioned in the introduction, it highlights the fact that local decisions can have an impact at the scale of the network. As we shall now see, the faculty of a city to adapt its road network to its growth, and in particular to adapt to those changes in the congestion regimes, is a determining factor of its economical success.

4.3 Adapting to changing congestion regimes, a challenge for cities

As we have seen, the free, self-organized (or decentralized) evolution of road networks reaches its limits for large cities. In more details, authors of [12, 158] have shown that cities which have grown following a decentralized evolution exhibit scale-free properties (e.g. in the number of road miles per unit area or the probability distribution of betweenness centrality on the network). Other cities, which have gone through a phase of centrally planned renovations of their urban network, do not exhibit these properties. They do however exhibit organization on larger scales (e.g. in the orientation of streets) when compared to self-organized cities where independent network structures coexist [35, 38]. Two main features distinguish centrally planned cities from self-organized ones :

1. As shown in [3, 35], cities densified their road network. Short, low BC roads were added to the network, as a mean to increase redundancy in the shortest paths and reduce the BC of the nodes with highest BC. In the context of the article by *Lampo et al.* [157], this would correspond to an increase of the size of the central grid with respect to the peripheral trees.
2. In order to cater for the increasing suburb to suburb (or activity center to activity center) transportation demand, cities built shortcuts in the form of ring-roads, to deviate the traffic from the city center [159].

The combination of these two effects can be seen in the results of [158]. Cities with a centralized planning exhibit a much narrower distribution of the BC : roads with high BC are avoided (by building alternatives such as ring-roads) and roads with low BC are avoided by densifying the network in the corresponding area. In particular, [42] shows that the length of dead ends scales sub-linearly with a city's size. This makes sense, as dead ends are effective as a mean to span a large area at a minimal construction cost, but are some sort of wasted opportunity if one wants to build a robust and resilient network, in which case it is preferable to connect the dead end to a second point of the network.

Implementing the recommended changes to the network as the city grows has however a cost. The authors of [159] show that there is a clear correlation between the way cities deal with suburb to suburb commuters and their economical situation. More precisely, they show that cities which segregate suburb to suburb traffic from the downtown traffic by building high-speed ring-roads, consistently show better results in terms of economical metrics such as productivity or GDP per capita. This is, however, some kind of chicken and egg situation: did these cities build ring-roads because they had the financial means to do so, or are they richer because they made the right decisions in terms of infrastructure planning ? The answer is like most of the time certainly a combination of both, but one way or another, it is clear from [158] and [159] that centrally planned networks are much more resilient against congestion and thus more efficient than self-organized ones.

5 Emergence of urban highways and ring roads

Building on the results made by [159] that ring-roads, and more generally speaking high speed connections are *a priori* positively correlated with a city's productivity, we will spend this section discussing the mechanisms that led to the appearance of urban highways in addition to the street network in US American cities. All results presented in this section are new contributions that were published in [160].

5.1 Empirical study

5.1.1 Historical context

The appearance of automobiles at the beginning of the 20th century very quickly led to the saturation of streets designed for pedestrians and horses, and traffic jams were a documented problem in the United States as early as the 1920s. It was then decided to introduce traffic rules (traffic regulations, signs, etc.), but also to modernize the network in order to adapt it to the growing demand. The first urban highway was built in New York City in 1923. The idea was that traffic jams occurred because of the chaotic nature of city traffic (interaction with pedestrians, frequent stops of cars for various reasons), and that a highway separating cars from all other users would be robust against the appearance of traffic jams and would allow a large number of motorists to travel at high speed. Under pressure from oil lobbies, the American government invested considerable sums of money in the 1950s and 1960s to equip major American cities with urban highways [48]. This decision, known as the Federal Highway Act, was at the time very positively received and considered as a way to affirm the position of the USA as the country of cars, symbol of individual freedom, in opposition to public transports, negatively connoted as socialist in those times of McCarthyism. However, the construction of these urban highways only accelerated the exodus of the middle class to the suburbs and thus considerably increased the number of trips made from the suburbs to the city center [48, 52, 161]. From 1960 to 1980, the number of automobiles in the United States doubled, while the driven-miles almost tripled³, more than the most "optimistic" projections of the oil companies backing the Federal Highway Act.

Inevitably, this high level of induced traffic quickly saturated the newly constructed highways. It was decided to continue to expand the highway system, both by adding more urban highways and by adding ring roads to facilitate suburb-to-suburb traffic and to divert through-traffic. With a few exceptions, most major U.S. cities were equipped with ring roads in the late 1960s or early 1970s. Again, history has shown that building new highways is only a temporary solution to congestion. In the case of the United States, the decisions taken at that time profoundly and durably defined the appearance of the cities. Indeed, the decision to finance very costly road infrastructure was made to the detriment of the

³According to the Bureau of Transportation statistics

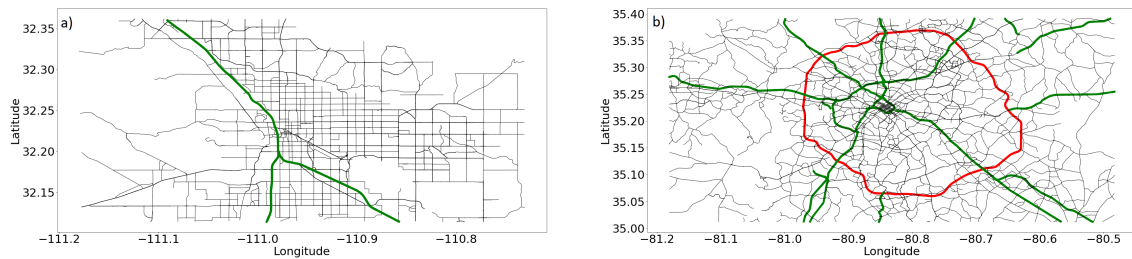


Figure II.7: a) Example of Tucson (AZ, USA, 540, 000 inhabitants in 2020), a urban area with two highways (in green), b) Example of Charlotte (NC, USA, 860,000 inhabitants in 2020), a urban area with a ring-road (in red) and several urban highways (in green)

development of public transport and was thus a driving force behind the social inequalities observed today [48].

Elsewhere in the world, similar contexts have led to the same results. For example, in the midst of the "Trentes Glorieuses"⁴, President Georges Pompidou supported a motorway plan for Paris, which eventually led to the construction of the "Voies sur Berges" in 1966, before the oil crisis of 1973 dampened the craze for a car-centered city. The Parisian ring-road "Boulevard Périphérique" was inaugurated in 1973 after 17 years of construction. In general, large European cities have fewer urban freeways than their American counterparts, but ring roads are a very common infrastructure. In countries with emerging economies or strong population and economic growth, the construction of urban highways and ring roads is on the rise, with Beijing and its now 7 concentric ring roads a striking example.

5.1.2 Data and method

A study on US American urban areas

Our work focuses on the emergence of these infrastructures in US cities. This choice is explained by the ease of access to a large sample of cities, as well as the homogeneity of the cities between them. Indeed, the American cities all have a recent and relatively similar history, whereas the European cities, for example, often have strong specificities linked to their medieval history. More importantly, we propose a cost-benefit analysis that takes into account the value of users' time and the cost of highway construction. These data are available in the literature for the United States and are quite homogeneous in the country. Moreover, as we have discussed in the previous section, cities in other countries might have benefited of building these infrastructures but simply couldn't afford them. Mixing different countries would add further complexity to the discussion.

The point here is to clarify what we mean by "city". We use data for the 926 "micropolitan" or "metropolitan" urban areas as defined by the US Census Bureau. In most cases, the micropolitan or

⁴Literally the "Glorious thirties", describing the time of strong economic growth from 1945 to 1975 in France

metropolitan area defined by the Census Bureau consists of a large city and the cities in its immediate vicinity. However, some urban areas are in fact a grouping of 2 to 4 cities of similar size, separated by a considerable distance ($> 10\text{km}$). In these cases, the population of the urban area has little correlation with the reality of the cities that comprise it, and we thus eliminate them from our analysis. Therefore, our analysis focuses on 888 U.S. urban areas with populations ranging from 13,948 (Ketchikan, AK) to 20,140,470 (New York City, NY) inhabitants in 2020. Throughout this chapter, we will use the terms "cities" and "urban areas" synonymously.

For each of these urban areas, we then manually checked using the *Google Maps "street view"* feature whether they include an urban highway. The choice to do this analysis manually is motivated by the specific criteria we chose to define an urban freeway, namely:

1. A multi-lane road, physically separated from the rest of the network, with interchanges and no signaled intersections.
2. To be "urban," the highway must separate two residential areas of the city and play a role in the transportation of commuters. This excludes in particular situations where a highway separates the city from its airport, for example, or where the city is clearly aligned in one direction (e.g., along a valley) and the highway is perpendicular to the city (and thus does not contribute to commuter travel).

The nomenclature of roads in the US is such that some road sections with intersections may be named after the highway they are an extension of, making automated analysis impossible. On the other hand, the study of ring-roads was easy thanks to the list of ring-roads found on *Wikipedia*. Finally, we cross-referenced this information with population surveys of the US Census Bureau [162].

Influence of population density

We wish to study the impact of traffic demand (i.e. the number of commuters using their cars) in the decision to build or not to build an urban highway in a city. One would expect that the population density in cities may also play a role in the decision to build or not a highway. However, the study of population density in the urban areas of our sample shows a relatively constant population density, independent of population, of about 10^3 inhabitants/ km^2 (see figure II.8). This result is different from what is known in the literature about the cities themselves or their centers, for which a correlation between population and density is observed instead (see for instance [19]). In any case, this justifies our choice to consider only traffic demand as the driving force behind the appearance of road infrastructure.

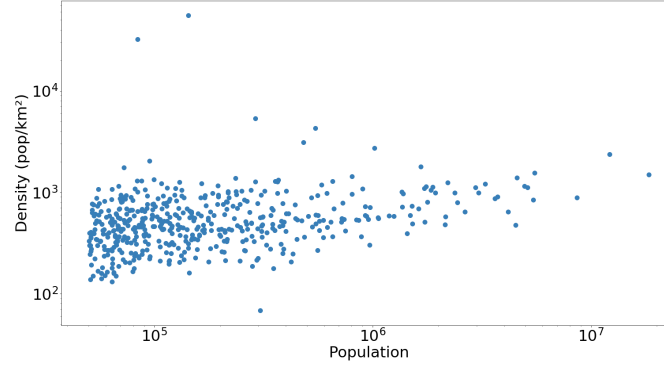


Figure II.8: For our data set of US American urban area, we observe that density is relatively independent of population

From the 2020 population to the traffic demand in 1960

As mentioned earlier, we study the emergence of highways and ring roads as a response to traffic demand. Ideally, we would need to know for each city the date of construction of the urban highway, as well as the number of commuters in that city at that date. Unfortunately, to the best of our knowledge, these data do not all exist. In particular, the population and number of commuters per year is only available for the 96 largest cities in our sample (population > 250,000 in 1960)[163]. We therefore make the following assumptions:

1. All highways were built at approximately the same time, in 1960.
2. The fraction of commuters among the population is the same from one city to another.
3. Cities have experienced the same population growth from 1960 to 2020.

We then use the 96 cities for which we have the number of commuters in 1960 to estimate the number of commuters for each city based on their population in 2020. Specifically, we introduce the 1960 and 2020 populations for a city i , $P_{1960}^{(i)}$ and $P_{2020}^{(i)}$ as well as the number of commuters $T_{1960}^{(i)}$ and $T_{2020}^{(i)}$. We then evaluate the distribution of the ratio $\frac{T_{1960}^{(i)}}{P_{2020}^{(i)}}$ for the 96 cities where this is possible. We find a log-normal distribution, with average $r \approx 0.14$ and standard deviation $\sigma \approx 0.06$. We then take for all cities $T_{1960}^{(i)} = rP_{1960}^{(i)}$, and estimate that the relative error on $T_{1960}^{(i)}$ is $\frac{3\sigma}{r}$. This pessimistic choice of 3σ seems to us to be justified by the fact that we have considered the traffic demand in 1960, while the actual date of construction varies from city to city.

We also illustrate the distribution of the ratio $\frac{T_{1960}^{(i)}}{P_{1960}^{(i)}}$ for the 96 cities where this is possible. This shows that assumption (2) is reasonable (at least among these 96 large urban areas), and that the uncertainty comes mainly from the distribution of the growth rate of cities from 1960 to 2020.

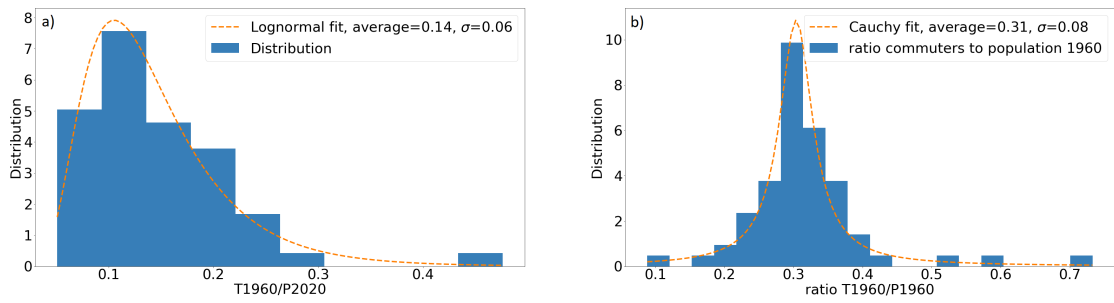


Figure II.9: Distribution of the ratios $\frac{T_{1960}^{(i)}}{P_{2020}^{(i)}}$ (a) and $\frac{T_{1960}^{(i)}}{P_{1960}^{(i)}}$ (b) for the 96 cities where this information is available

5.1.3 Results

Now that we have transformed our data to show the number of car commuters in 1960 for each urban area, we rank these urban areas by increasing traffic demand and group them into groups of $n = 40$ cities. Within each group, we calculate the fraction of cities with an urban highway (respectively a ring road). We evaluate the relative uncertainty on the average traffic demand in this group as $\frac{3\sigma}{r\sqrt{n}}$.

Emergence of a urban highway

There is a clear transition for a traffic demand $T_c \approx 10^4$, separating the smaller cities that do not have freeways from those that do. In terms of population in 2020, this translates into a critical population $P_c \approx 10^5$, and only three urban areas with more than 300,000 people are without freeways, the largest being Rockford, Illinois, with 350,000 people. Despite its cost of construction, this infrastructure is totally generalized in the medium and large cities of the United States.

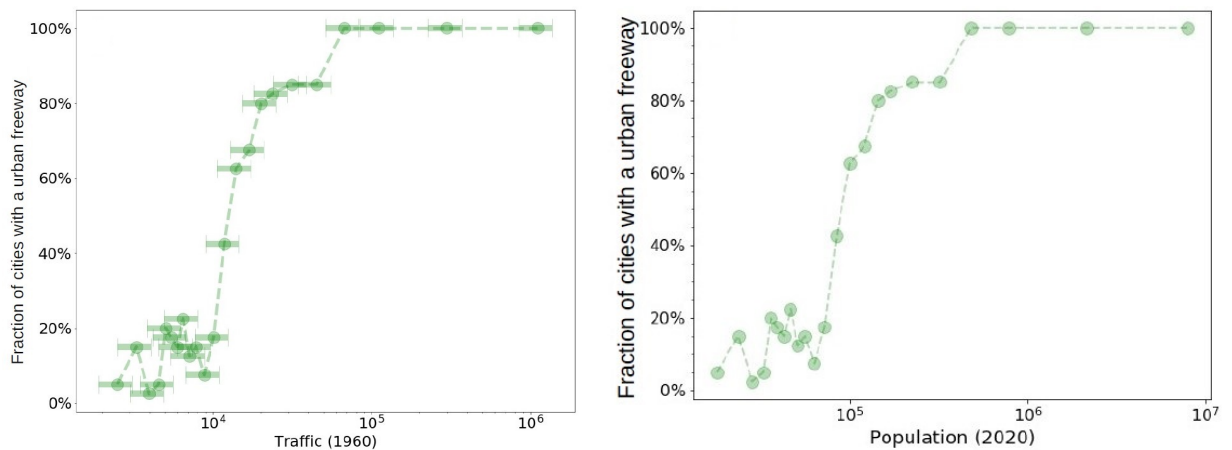


Figure II.10: Proportion of cities with a urban highway as a function of traffic demand in 1960 (a) and population in 2020 (b)

Emergence of a ring-road

Here we see that the transition is less marked and takes place in the range $[10^4 - 10^6]$ of traffic demand commuter. For a critical demand $T_c \approx 10^5$, 50% of cities have a ring road, which translates into a current population of $P_c \approx 10^6$. Unsurprisingly, all “megacities” with populations $> 10^7$ have one or more ring roads.

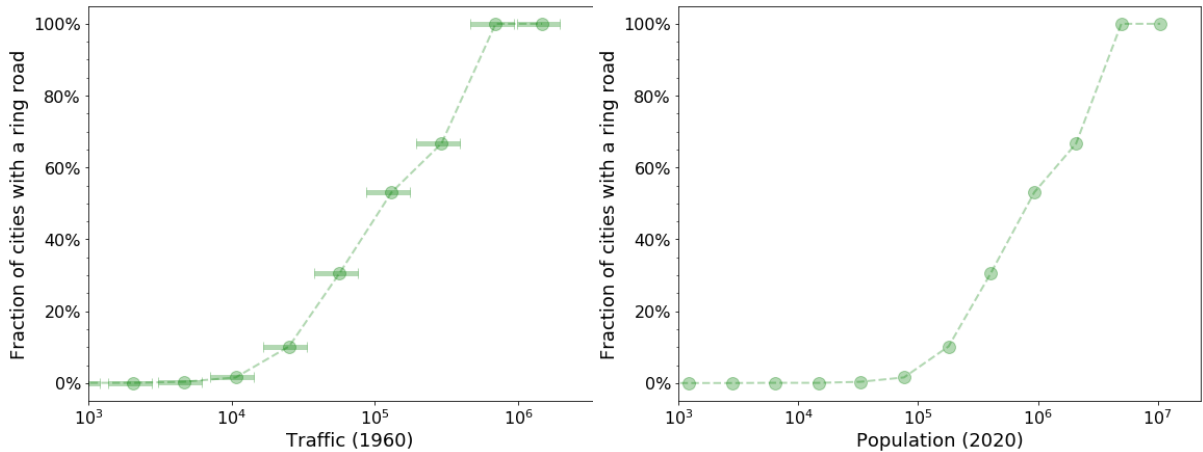


Figure II.11: Proportion of cities with a ring-road as a function of traffic demand in 1960 (a) and population in 2020 (b)

5.2 Cost-benefit analysis

We propose a toy model to understand these transitions. Our model is based on the comparison between the cost of building highways (respectively ring-roads) and the value of time lost by commuters in traffic jams. This model initially neglects all other costs associated with the presence of a highway. This strong assumption seems to us to be representative of the mindset and priorities when the decision was made to fund highways through the Federal Highway Act. Indeed, as we mentioned in the first chapter of this thesis, projects were judged first and foremost on the increase in traffic capacity.

5.2.1 Model

Simplified urban area

We consider an urban area with a number T of commuters taking their car every day. These commuters make their trip in the morning and evening during the peak hours, which we consider to be of duration $h = 2$ hours. We make the assumption that the commuters' trips are homogeneously distributed during these peak hours, i.e., at any time, $\frac{T}{h}$ commuters are on the road. We also make the simplifying assumption that these commuters live along a line from 0 to $2R$. This line represents

the corridor in which the urban highway is located, or the equivalent route via smaller roads. For each origin on this line, we consider that a fraction α of commuters want to go to the center of the city (located in R), while the rest $\beta = 1 - \alpha$ go to the opposite side of the city (in $2R$ for residents living in $< R$, 0 for others). We call τ_1 and τ_2 the time spent on the road by the entirety of the α and β commuters respectively.

The question arises whether to build a highway from L to $2R - L$. Our cost-benefit analysis is based on the study of the budget defined by:

$$Z = 2\epsilon(R - L) + 2NV(\Delta\tau_1 + \Delta\tau_2) \quad (11.7)$$

where the first term accounts for the cost of constructing and maintaining the urban highway (ϵ is the cost per unit length of the highway, $2(R - L)$ is the length of the highway) and the second term is the cost (or rather the benefit) associated with the time saved by the users (V is the value of the users' time, N is the number of days over which we are doing the analysis, $\Delta\tau_i = \tau'_i - \tau_i$ is the time saved by the users of type i thanks to the highway). The optimal size of the highway to build can then be determined by minimizing the cost function Z .

Adapted BPR function and derivation of travel times

To evaluate τ_1 et τ_2 , we use the Bureau of Public Roads function introduced by Branston in 1976 [94] (BPR function in the following). As mentioned in the introduction, this function links the travel time on a road to the traffic demand. For a road of length d and with maximal capacity Q_c , if the demand is Q , the corresponding travel time is given by :

$$\tau = \frac{d}{v} \left[1 + \left(\frac{Q}{Q_c} \right)^\mu \right] \quad (11.8)$$

where v is the free-flow speed on this road and μ is an exponent that accounts for the sensitivity of the road to congestion. Typically, μ varies from road to road and is in the range $[2, 4]$, but we will take $\mu = 1$ here to allow analytical calculations.

This function must be adapted for use in our model. Indeed, we consider a case where the demand Q varies with the position and the flow is therefore not uniform. Let us first consider the case of the interval $[0, R]$ and admit that commuters are distributed according to a function $\rho(x)$, and all go to R . The traffic at a point x is then given by the number of commuters living upstream of x :

$$Q(x) = \int_0^x \rho(y)dy \quad (11.9)$$

Variable	Description	Unit and value (if fixed)
Population and traffic		
P	Population of the urban area	
T	Number of car commuters	
h	Duration of the rush hours	$h = 2$ hours
N	Number of rush hour periods for or cost-benefit analysis	$N = 730$
BPR function		
τ	Travel time on a road segment	hours
Q	Traffic on a road (flux)	cars/h
Q_c	Critical traffic, capacity of the road	$Q_c = 10^3$ cars/h
μ	Critical exponent of the BPR	We suppose $\mu = 1$
Geometry		
R	Radius of the urban area	km
$L, (2R - L)$	Start and end of the freeway	km
$\ell, (2R - \ell)$	Start and end of the ring road	km
g	Geometric factor for the length of ring road	$g = \frac{\pi}{2}$
v_0	Free-flow speed on urban streets	$v_0 = 20$ km/h
v	Free-flow speed on freeways	$v = 100$ km/h
α	Part of commuters traveling to the city center	Typically $\alpha = 50\%$
$\beta = 1 - \alpha$	Part of commuters traveling to other suburbs	Typically $\beta = 50\%$
Cost analysis		
$\Delta\tau$	Travel time variation between scenarios	hours
C	Cost associated to the travel time	\$
ΔC	Cost variation between scenarios	\$
ϵ	Cost of construction of a freeway	$\epsilon = 10^7$ \$/km
V	Value of time	$V = 20$ \$/hour
T_c	Critical number of commuters leading to a change of infrastructure	
Compound variables		
$x = \frac{L}{R}$	Dimensionless position of the freeway	
$y = \frac{\ell}{R}$	Dimensionless position of the ring road	
$\eta = \frac{v_0}{v}$	Ratio between free-flow speeds	$\eta = 0.2$
$c = \frac{T}{P}$	Fraction of the population who commutes by car	$c = 40\%$

Table II.1: Variables used in our cost-benefit analysis

According to the BPR function, traveling the infinitesimal segment $[x, x + dx]$, then takes the infinitesimal time:

$$d\tau(x) = \frac{dx}{v_0} \left[1 + \left(\frac{Q(x)}{Q_c} \right)^\mu \right] \quad (II.10)$$

and the travel time from point a to point b is given by :

$$\tau_0(a, b) = \int_a^b \frac{dx}{v_0} \left[1 + \left(\frac{Q(x)}{Q_c} \right)^\mu \right] \quad (II.11)$$

Let us consider for the following that $\mu = 1$ and that the commuters are homogeneously distributed on the network: $\rho(x) = \frac{T}{2R}$ and $Q(x) = \frac{Tx}{2Rh}$. We can then express the total travel time of commuters living between 0 and R and going to R and $2R$ (where we introduced $\tilde{T} = \frac{T}{2hQ_c}$):

$$\begin{cases} \tau_1 = \frac{\alpha T}{2Rv_0} \int_0^R dx \int_x^R dx' \left(1 + \tilde{T} \frac{x'}{R} \right) \\ \tau_2 = \frac{\beta T}{2Rv_0} \int_0^R dx \left[\int_x^R dx' \left(1 + \tilde{T} \frac{x'}{R} \right) + \int_R^{2R} dx' (1 + \beta \tilde{T}) \right] \end{cases} \quad (II.12)$$

and thus the initial cost associated with the trips of commuters of type α and β :

$$C^0 = NV(\tau_1 + \tau_2) = \frac{NVT R}{2v_0} \left[1 + \frac{2}{3} \tilde{T} + 2\beta(1 + \beta \tilde{T}) \right] \quad (II.13)$$

5.2.2 Two possible scenarios : urban highway or ring road ?

Case of the urban highway

In the presence of an urban highway running from L to $2R - L$, with speed $v > v_0$ and the same capacity Q_c as the urban network, the total travel time from 0 to R (resp. $2R$) $\tau'_{1(2)}$ is now

$$\begin{aligned} \tau'_1 &= \frac{\alpha T R}{4v_0} \left[y^2 + \eta(1 - y^2) + \frac{2}{3} \tilde{T} (y^3(1 - \eta) + \eta) \right] \\ \tau'_2 &= \frac{\beta T R}{4v_0} \left[y^2 + \eta(1 - y^2) + \frac{2}{3} \tilde{T} (y^3(1 - \eta) + \eta) + 2(1 + \beta \tilde{T})(y + \eta(1 - y)) \right] \end{aligned} \quad (II.14)$$

where we introduced $y = \frac{L}{R}$, $\tilde{T} = \frac{T}{2hQ_c}$ and $\eta = \frac{v_0}{v}$.

The cost for this situation is then $C^{FW} = 2(R - L)\epsilon + 2NV(\tau'_1 + \tau'_2)$, which we compare to the

initial cost:

$$\begin{aligned}\Delta C^{FW} &= C^{FW} - C_0 \\ &= 2R(1-y)\epsilon - \frac{NVT R}{2v_0}(1-\eta) \left[1 - y^2 + \frac{2}{3}\tilde{T}(1-y^3) \right. \\ &\quad \left. + 2\beta(1+\beta\tilde{T})(1-y) \right]\end{aligned}\quad (II.15)$$

Let's reformulate this expression in terms of the variable $\Delta = 1 - y$, which gives

$\Delta C^{FW} = A_1\Delta + A_2\Delta^2 + A_3\Delta^3$, with:

$$\begin{cases} A_1 = 2R\epsilon - \frac{NVT R}{v_0}(1-\eta)(1+\beta + \frac{T}{2hQ_c}(1+\beta^2)) \\ A_2 = \frac{NVT R}{2v_0}(1-\eta)(1 + \frac{T}{hQ_c}) \\ A_3 = -\frac{NVT R}{2v_0} \frac{(1-\eta)T}{3hQ_c} \end{cases}\quad (II.16)$$

For the construction of a highway to be profitable, there must exist a negative value for ΔC^{FW} in the interval $y \in]0, 1[$. Since $A_2 > 0$ and $A_3 < 0$, this is only possible if $A_1 < 0$.

Let us make the assumption - which we will check later - that $T \gg Q_c$. We can then neglect all first-order terms in T in the expression of A_1 , and thus find the critical demand for which building a highway becomes profitable, given by :

$$T_c^{FW} \approx \sqrt{\frac{4}{(1+\beta^2)(1-\eta)}} \sqrt{\frac{\epsilon h v_0 Q_c}{NV}}\quad (II.17)$$

For speed values corresponding to an urban environment ($v_0 = 20\text{km/h}$, $v = 100\text{km/h}$), for $V = 20\text{\$/h}$, $h = 2$ hours, $\epsilon = 10^7\text{\$/km}$, $Q_c = 10^3\text{cars/h}$, $\beta = 0.5$ and considering the morning and evening peak hours over a full year ($N = 730$), we find a numerical value $T_c^{FW} \approx 10^4$.

On one hand, we have $T_c^{FW} \gg Q_c$, which justifies a posteriori our hypothesis, and on the other hand, we note that this value is in good agreement with our empirical observations.

Case of the ring-road

If instead of an urban highway, we imagine that the city builds a ring road, again ranging from L to $2R - L$, we can again estimate the cost associated with this infrastructure:

$$\Delta C_2 = 2R(1-y)g\epsilon + 2NV(\Delta\tau_1 + \Delta\tau_2)\quad (II.18)$$

where again, $y = \frac{L}{R}$ and g is a geometric factor, equal to $\frac{\pi}{2}$ for a circular ring road, which takes into account the longer trip around the city. The calculation is similar to the previous case, except that the

α and β flows now separate in L and not in R , and that the path along the ring road is affected by this coefficient g . We thus find:

$$\begin{aligned}\Delta C_2 = & 2R(1-y)g\epsilon + \frac{NVTR}{2v_0} \left[2\beta y(y-1) \right. \\ & + \frac{2}{3}\tilde{T} \left[(1-y^3)(\beta^2 - 2\beta) + \beta^2(1-y)^3 \right] \\ & \left. + 2\beta(1+\beta\tilde{T})(1-y)(2\eta g - 1) \right]\end{aligned}\quad (II.19)$$

As previously, we introduce the variable $\Delta = 1 - y$ and write $\Delta C^2 = A_1\Delta + A_2\Delta^2 + A_3\Delta^3$, where:

$$\begin{cases} A_1 = 2Rg\epsilon + \frac{NVTR}{2v_0} \left[2\beta\eta g - 2 + 2\beta(\beta\eta g - 1)\frac{T}{hQ_c} \right] \\ A_2 = \frac{NVTR}{2v_0} (2\beta + (1-\alpha^2)\frac{T}{hQ_c}) \\ A_3 = -\frac{NVTR}{2v_0} \left(\frac{T}{hQ_c} + 1 \right) \end{cases}\quad (II.20)$$

If the condition $\beta\eta g < 1$ is fulfilled (which is the case for realistic values of these three parameters), we find here again a critical demand for which the appearance of a ring road becomes profitable:

$$T_{c2} = \sqrt{\frac{2g}{\beta(1-\beta\eta g)}} \sqrt{\frac{\epsilon hv_0 Q_c}{NV}}\quad (II.21)$$

We can check the behavior of this expression for the limiting cases of β . As expected, for $\beta \rightarrow 0$, $T_{c2} \rightarrow \infty$ since the users all go to the center and thus do not use the ring road. If $\beta = 1$, the condition now concerns η and g : the higher speed on the ring road must compensate for the longer journey for the ring road to be an interesting option. Here again, we find $T_{c2} \approx 10^4 \gg Q_c$.

A urban highway is more profitable than a ring-road

The natural question is then to know in which order these two infrastructures appear, i.e. to compare the two critical demands T_c^{FW} and T_{c2} computed previously. They share the term $\sqrt{\frac{\epsilon hv_0 Q_c}{NV}}$, but differ in their prefactor. Let us note about these expressions that the ratio between the parameters could have been found by dimensional analysis, but that the square root behavior is not trivial.

We therefore calculate the sign of the difference of the prefactors $\frac{2g}{(1-\beta\eta g)} - \frac{4}{(1+\beta^2)(1-\eta)}$. We can easily convince ourselves that the sign of this expression is the same as that of the second order polynomial Ψ :

$$\Psi(\beta) = g(1-\eta) - 2\beta + \beta^2 g(1+\eta)\quad (II.22)$$

Ψ reaches its minimum for $\beta_0 = \frac{1}{g(1+\eta)}$ and is then equal to:

$$\Psi_{min} = g(1 - \eta) - \frac{1}{g(1 + \eta)} \quad (II.23)$$

$$= \frac{g^2(1 - \eta^2) - 1}{g(1 + \eta)} \quad (II.24)$$

For a realistic choice of values, i.e. for $g \approx \frac{\pi}{2}$ (more or less circular ring road) and $\eta < \frac{1}{3}$ (significantly higher free-flow speed on a freeway than in the city), we find $\Psi_{min} > 0$. Thus, whatever the value of β , the construction of a highway is always profitable before the construction of a ring road.

This result perfectly reproduces the historical order in which these infrastructures were built: first one (or more) urban highways, then a ring road.

Emergence of a ring-road on top of a urban highway

We therefore repeat our previous calculation, this time considering a city that already has a highway between L and $2R - L$ and for which the question of building a ring road from ℓ to $2R - \ell$ arises.

Before the construction of the ring road, the travel times are (previously established result):

$$\tau_1 = \frac{\alpha TR}{4v_0} \left[y^2(1 - \eta) + \eta + \frac{2}{3} \tilde{T}(y^3(1 - \eta) + \eta) \right]$$

$$\tau_2 = \frac{\beta TR}{4v_0} \left[y^2(1 - \eta) + \eta + \frac{2}{3} \tilde{T}(y^3(1 - \eta) + \eta) + 2(1 + \beta \tilde{T})(y + \eta(1 - y)) \right]$$

where as previously $y = \frac{L}{R}$, $\tilde{T} = \frac{T}{2hQ_c}$. Introducing $x = \frac{\ell}{R}$, we find for the ring road case:

$$\tau'_1 = \frac{\alpha TR}{4v_0} \left\{ y^2(1 - \eta) + \eta + \frac{2}{3} \tilde{T} [y^3(1 - \eta) + \eta(x^3(1 - \alpha) + \alpha)] \right\} \quad (II.25)$$

$$\tau'_2 = \frac{\beta TR}{4v_0} \left\{ y^2(1 - \eta) + \eta x^2 + \eta(1 - x)^2 + \frac{2}{3} \tilde{T} [(y^3(1 - \eta) + x^3\eta + \eta\beta(1 - x)^3] \right. \\ \left. + 2(1 + \beta \tilde{T}) [y(1 - \eta) + \eta(x + 2g(1 - x))] \right\} \quad (II.26)$$

Again, we look at the cost variation between both situations, $\Delta C^{RR} = 2(R - \ell)g\epsilon + 2NV(\Delta\tau_1 + \Delta\tau_2)$, where, using the previous results :

$$\Delta\tau_1 = -\frac{\alpha\beta\eta TR}{6v_0} \tilde{T}(1 - x^3) \quad (II.27)$$

$$\Delta\tau_2 = \frac{\beta TR\eta}{4v_0} \left\{ 2x(1 - x) + \frac{2}{3} \tilde{T}(x^3 - 1 + \beta(1 - x)^3) + 2(1 + \beta \tilde{T})(1 - x)(2g - 1) \right\} \quad (II.28)$$

Introducing $\Delta = 1 - x$, we get $\Delta \tilde{C}^{RR} = A_1\Delta + A_2\Delta^2 + A_3\Delta^3$ where:

$$\begin{cases} A_1 = 2Rg\epsilon + \frac{RNV\eta\beta}{v_0} \left[2gT + \frac{T^2}{2hQ_c}(1 - \beta g) \right] \\ A_2 = -\frac{RNV\eta\beta}{v_0} \left[\frac{T}{h} + \frac{(2-\beta)T^2}{2hQ_c} \right] \\ A_3 = -\frac{RNV\eta\beta}{v_0} \frac{T^2}{3hQ_c} \end{cases} \quad (II.29)$$

Again, the condition for a ring road to be profitable is to have $A_1 < 0$. This condition requires to have $\beta > 0$ and $\beta < \frac{1}{g}$. We then find a critical demand for which the ring road becomes profitable:

$$T_c^{RR} = \sqrt{\frac{4g}{\eta\beta(1-\beta g)}} \sqrt{\frac{\epsilon hv_0 C}{NV}} \quad (II.30)$$

Note that, in this case, the condition $\beta < \frac{1}{g}$ is not necessarily satisfied and, rather counter-intuitively, for a large number of suburb-to-suburb commuters, the construction of the ring road is not profitable. Indeed, the city already has a highway for rapid transit of commuters, and the construction of the ring road is only useful if it separates the two types of commuters α and β on two separate routes. Thus, only if a significant share of α commuters can benefit from the reduction of traffic in the center is the construction of the ring road advantageous. Similarly, if $\beta = 0$, adding a ring road to the network is unnecessary and $T_c^{RR} \rightarrow \infty$.

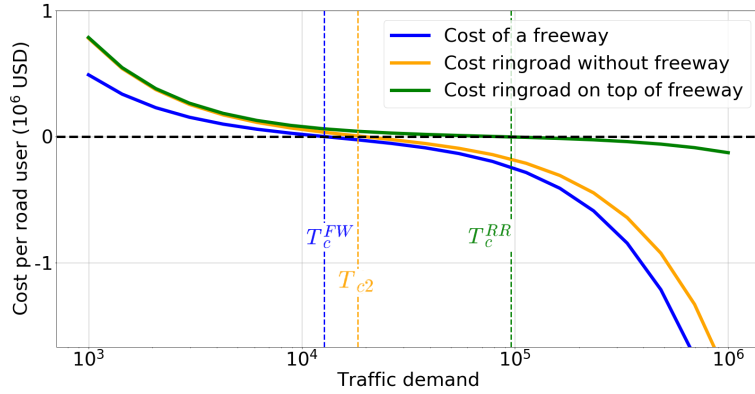


Figure II.12: Cost associated to the 3 presented scenarios, as function of traffic demand in the case $\mu = 1$ and derivation of the critical demand

For reasonable choices of parameters (i.e. as before, $v_0 = 20km/h$, $v = 100km/h$, $V = 20\$/h$, $h = 2$ hours, $\epsilon = 10^7\$/km$, $Q_c = 10^3$ cars/h, $\beta = 0.5$ and $N = 730$), we find $T_c^{RR} \approx 10^5$, again in good agreement with our empirical results. Figure II.12 shows the dependence of ΔC^{FW} , ΔC_2 and ΔC^{RR} on the traffic demand. We can see that the transition for ΔC^{FW} is sharp, for $T_c^{FW} \approx 10^4$, while the transition for ΔC^{RR} is much smoother and covers the whole interval $[10^4 - 10^6]$, again in

very good agreement with the observations.

5.3 Discussion

5.3.1 Impact of the assumption $\mu = 1$

We have in all this part made the assumption $\mu = 1$ in the BPR function $d\tau(x) = \frac{dx}{v_0} \left[1 + \left(\frac{Q(x)}{C} \right)^\mu \right]$, instead of a more realistic value in the interval $[2, 4]$. This allowed us to carry out the calculations analytically, but it is a strong assumption that we need to verify. To do this, we numerically compute the dependence of ΔC^{FW} on the demand T for different values of μ (see figure II.13). We find that the order of magnitude of T_c^{FW} is the same whatever the choice of μ in the interval $[1, 4]$, with $T_c^{FW} \approx 5 \cdot 10^3$ for $\mu = 4$, against $T_c^{FW} \approx 10^4$ for $\mu = 1$.

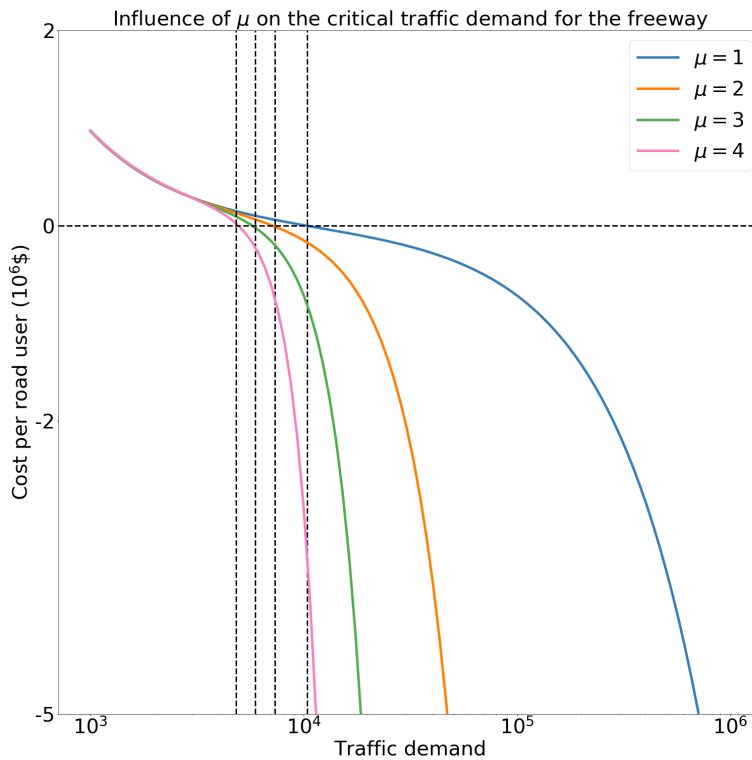


Figure II.13: Illustration of the impact of the choice of μ on the evaluation of the critical demand for the construction of a urban highway

Another way to test the realism of our results in spite of our assumptions is to compute the average travel time among commuters $\frac{\tau_1 + \tau_2}{T}$, for example in the simplest case with no freeway or ring road. We find that our 1D city model with $\mu = 1$ behaves well in the demand interval $10^4 - 10^5$, which

corresponds to average commuting times between 0.2 and 2 hours, which are realistic values for urban commuting. Fortunately, the transitions we observe are in this $10^4 - 10^5$ commuter range. Our model would be quite inappropriate for describing larger cities with 10^6 or 10^7 commuters.

5.3.2 Limitations of our model

Beside the choice of $\mu = 1$ in the BPR-function, our model relies on several strong simplifications, which naturally limit its validity and its use as a tool in real-life decisions. We would like to use this paragraph to acknowledge and discuss rapidly the different simplifications that were made.

First, in terms of our empirical analysis, we used a strong assumption when deriving the traffic demand in 1960. Secondly, in terms of our cost-benefit analysis, our model of a 1D-city is very simplified. In particular, it excludes two important factors from the possible discussion. On one hand, we cannot treat the case of commuters using the ring road to travel for instance from North to East rather than around the whole city. Similarly, we did not treat the possibility for a city to have several urban highways before the construction of its ring road.

The other point that our model ignores is the possibility for the urban street network to grow in parallel to the construction of the highway. Indeed, we treated the capacity C as fixed and equal for the street network and the urban highway. This is a very strong simplification, as it is very likely that different cities have in general a different ratio between their street network capacity and their highway capacity. This is particularly relevant when one recalls that the 1960s were, in general, very car-centric, meaning that the whole network was likely growing, beyond the simple addition of highways on top of a fixed urban network.

5.3.3 Influence on the congestion regimes

Adding a urban highway or a ring-road to the urban network obviously has a significant impact on the distribution of traffic and congestion on the network. As we mentioned in the previous section, suburb-to-suburb commutes make up for the majority of the traffic in cities and proposing high-speed alternatives to a trip through the city center is beneficial to a city's productivity. However, building urban highways and ring-roads also means that the pre-existing street network will be put under a lot of stress at the interfaces with the (few) interchanges with the highways. As we mentioned in the introductory chapter of this thesis, the construction of urban highways in the USA led to a further increase of congestion in the city centers, combined with a report of the traffic from the rest of the city to its suburbs [50]. This means that in most cities, the newly built high-speed infrastructure is not only responsible for most of the city's traffic, but actually also turns into the city's new most congested roads, as visible on the example of Paris, described in 3.1.3. Opponents of urban highways thus accuse

them of having displaced or worse exacerbated the problem of urban traffic instead of combating it [59, 164].

Some very recent work by Chondrogiannis and Grossniklaus [165] focuses on the question of the efficacy of a highway system in terms of its topological properties. In particular, they show that not all highway networks have been created equal, and confirm that strategically chosen interfaces with the street network are essential to the effectiveness of the highway system.

5.4 Externalities associated to urban highways

So far, we have only considered the cost of building the freeways, ignoring their negative impact on people and the environment. This is a strong assumption, which we believe is close to the concerns that existed when the decision to build these highways was made. However, in the years to come, many of these infrastructures will have to be thoroughly renovated if not completely rebuilt. Indeed, we know today that the life span of a highway is typically 50 to 70 years.

The question now is whether the decision taken in the past is still the right one. We know today that urban freeways are not the miracle solution to congestion that they were promised to be, while having a strong impact on the city. Among the negative impacts, we can obviously think of the air pollution to which a large number of inhabitants are exposed. We also think of the very negative impact that highways have in terms of urban planning. They constitute wide strips that cannot be crossed by pedestrians, which segregate the city into different districts and make the city unsuitable for soft mobility (a particularly striking problem in the United States). Lastly, the land taken up by highways is large and constitutes an important waste in a context where housing is lacking or is becoming increasingly expensive.

Some cities, already confronted with the aging of their highways, have chosen either to rebuild them identically, or to bury them (Boston's "*Big Dig*" being the most famous example) in order to combat most of the negative points mentioned above [59, 164]. Nevertheless, this decision remains a notable investment in favor of the car and is necessarily to the detriment of public transport. Other cities have decided to simply destroy their urban highways without replacing them and to use the space thus recovered for other purposes. Paris comes to mind, of course, with the closure of the Voies sur Berges, while Seoul is another example [59].

5.4.1 Identifying the critical social cost of urban highways

Our model can be adapted to discuss this new problem. We have seen that the urban highway and the ring road have a certain redundancy. Just as the addition of a ring road is less beneficial if the city already has a freeway, one can ask whether the presence of a freeway is always justified in a city with a

ring road. We propose to modify the previous discussion by introducing a "social cost" ϵ' related to the presence of a highway, which is added to the construction cost ϵ . We can then find for each city the critical value of this cost beyond which the removal of the urban highway should be considered instead of its renovation. The cost of the "renovation" scenario is now:

$$C = [2(R - L) + 2(R - \ell)g]\epsilon + NV(\tau_1 + \tau_2) + 2(R - \ell)\epsilon' \quad (\text{II.31})$$

while the cost without the highway reads:

$$C' = [2(\ell - L) + 2(R - \ell)g]\epsilon + NV(\tau'_1 + \tau'_2) \quad (\text{II.32})$$

The difference between both scenarios is:

$$\Delta C^{SC} = -2(R - \ell)(\epsilon' + \epsilon) + 2NV(\Delta\tau_1 + \Delta\tau_2) \quad (\text{II.33})$$

where :

$$\begin{cases} \Delta\tau_1 = \frac{\alpha TR}{4v_0}(1 - \eta) \left[1 - x^2 + \frac{2}{3}\tilde{T}\alpha(1 - x^3) \right] \\ \Delta\tau_2 = \frac{\beta TR}{4v_0}(1 - \eta) \left[(1 - x)^2 + \frac{2}{3}\tilde{T}\beta(1 - x)^3 \right] \end{cases} \quad (\text{II.34})$$

We look for the value of the social cost ϵ'_c beyond which removing the highway becomes the most cost-effective option, i.e. $\Delta C^{SC} < 0$. We find:

$$\begin{aligned} \epsilon'_c &= (1 - \eta) \frac{NV\alpha^2}{4hv_0Q_c} T^2 - \epsilon \\ &= (1 - \eta) \frac{NV\alpha^2}{4hv_0Q_c} c^2 P^2 - \epsilon \end{aligned} \quad (\text{II.35})$$

where the ratio $c = \frac{T}{P}$ depends *a priori* of the year and of the urban area considered. Nevertheless, as discussed earlier, there is little variation in this ratio between urban areas, so we can take the number of commuters and the population at the national level. In 2020, we find $c \approx \frac{140.10^6}{330.10^6} \approx 40\%$.

5.4.2 Assessing the cost of externalities is a difficult task

It then remains to assess the cost of the externalities associated with the presence of a highway, to compare it with ϵ'_c , and to make a decision. As we shall see, this is in practice quite difficult to do.

As mentioned earlier, the externalities associated with the presence of a freeway are numerous and their cost is often difficult to assess, constituting a subject for research in its own right.

First, air pollution is responsible for chronic respiratory diseases, impacting the lives of people living near highways and reducing their life expectancy [166, 167, 168, 169]. It is questionable to what extent replacing a highway with urban streets will change this finding, if the number of vehicles remains constant. On one hand, it has been shown that pollution levels are particularly high in the vicinity of major roads, before the wind is able to disperse them. Diluting the traffic on several roads would therefore make it possible to reduce the areas exposed to a very high level of pollution. On the other hand, it has been shown that vegetation has a very positive impact on air quality. In this sense, replacing a freeway with a boulevard with rows of trees represents considerable progress [170, 171, 172]. Finally, and this is a point that our model ignores completely, the removal of a freeway is generally accompanied by a reduction in total traffic (the same mechanism as for induced traffic described in the first chapter) and therefore in pollution. It should thus be noted that it is not simply a question of removing the urban freeway, but of proposing an alternative urban development, ideally in the form of a green boulevard, or even a green corridor, favoring soft mobility and public transport to encourage a modal shift of the former freeway users. The examples of Paris and Seoul show that this strategy does lead to a significant reduction in total traffic in the city.

The second major cost associated with a highway is real estate. Not only does the highway occupy a large area that could be used for buildings, but even the area around the highway is greatly devalued by its presence. In Seoul, the removal of the urban expressway and its replacement by a park has led to a significant increase in the attractiveness of the neighborhood and a sharp rise in the price per square meter [59].

Note that in this discussion we have implicitly assumed that a ring-road is less detrimental than a urban highway. This of course depends on the situation. If the ring-road is indeed in a less densely inhabited zone of the urban area and serves mainly to divert the through-traffic from the city center, we argue that its impact on the local population will be minimal compared to a urban highway. If, however, the ring-road has a small radius and is therefore completely integrated in the urban area, the same external costs that we described for highways are also associated to its presence. In that case, there is no easy solution, as one will necessarily have to choose between traffic performance and public health and safety.

5.4.3 Perspectives

Alternative strategies to mitigate the impact of urban highways

We should note that other strategies can be employed to reduce the negative impact of highways without completely replacing them, and in particular to optimize the highways throughput. One such

strategy is the allocation of one or more lanes of the highway to specific groups of vehicles. Typically, this could be bus lanes, offering a cheap but relatively efficient substitute to rail transport to rapidly bring suburbanites to the city center. Alternatively, some urban areas reserve the left lane of their urban highways to cars carrying 2 or more passengers, in order to promote car-sharing and reduce the overall number of cars on the road, thus the congestion and the pollution⁵, or electric vehicles. These policies are already implemented in some urban areas, for instance in most of California, USA, and are discussed in others, such as Paris for 2024.

A tool among others to help political decisions

We have been deliberately vague in describing the cost of the externalities associated with an urban freeway. The true "social cost" associated with the freeway is the sum of a multitude of factors, and depends strongly on the city (attractiveness, population density around the freeway, etc.). Moreover, as mentioned above, our model does not take into account other forms of mobility and therefore ignores the modal shift of commuters.

For such an important subject, it is neither possible nor desirable to propose a single model, prescribing the removal or reconstruction of a highway. Rather, this work should be seen as one tool among many that can be used by cities that find themselves considering the future of their urban highways. As stated in the first chapter of this thesis, however, the final decision remains an uncertain and political choice.

⁵This method has a large potential, as the average number of commuters per car is merely of 1.5 [173], resulting in an important waste of resources.

Conclusion

In this chapter, we introduced the notion that the traffic demand and the roads composing the urban network are widely heterogeneous. Using the betweenness centrality as a proxy for traffic and different results taken from the literature, we investigated the relationships between population growth and network growth and between network structure and congestion regimes. We showed that the urban road network cannot be thought without taking into account the population and land use dynamics of the city it is embedded in.

Road networks initially developed in a decentralized, self-organized manner, displaying strong similarities with biological networks. In particular, they display scale-free properties, especially in the distribution of the betweenness centrality among their roads. In the context of road networks, this is far from optimal, as it means that some road are used below their capacity, while others are saturated.

We further detailed the shift in congestion regimes that occurs when the road network grows, following the grid-tree model proposed by *Lampo et al.* [157]. From there, we showed that cities benefit from switching from this kind of self-organized structure to a more globally planned infrastructure, narrowing the distribution of the betweenness centrality. In particular, cities which could afford it built ring-roads to divert the suburb-to-suburb traffic from the city centers.

We then presented our own results regarding the emergence of urban highways and ring-roads during the 1960s in the USA. We found empirically that the decision to build these infrastructures seems to be explained mainly by the size of the city and proposed a simple cost-benefit analysis to explain this observation. Finally, we discussed the potential of this tool to assess whether to rebuild or to destroy these aging infrastructures in the upcoming decades.

Having established the interplay between population, network structure and links prone to be congested, we will now concentrate on the spatio-temporal dynamics of congestion on a given network.

Chapter III

Traffic dynamics on urban networks

In this chapter, we deal with the issue of the formation and propagation of traffic jams on a network. These phenomena are probably those that come most naturally to the reader's mind when thinking about the problematic of urban traffic.

First, we will review the state of the art of the physical approaches that have been successfully applied today to describe the spatial distribution of traffic jams on the network and their propagation. In particular, these studies confirm the intuition - introduced in the previous chapter - that traffic jams generally originate from a few "weak" links from which they then propagate to the rest of the network.

We will then show on the example of the city of Paris that the appearance of traffic jams actually occurs simultaneously on the entire network during peak hours. More precisely, we will compute the correlation length of the traffic on the network and show that it diverges during peak hours. From a small length at off-peak hours, indicating that traffic jams propagate only locally on the network, the correlation length increases at peak hours to a value of the order of the size of the network, indicating that any disruption affects the entire network. This phenomenon is characteristic of a phase transition.

Finally, we will discuss the implications of these results on how traffic behaves in the city, and in particular the notable differences between the fundamental diagrams of streets, compared to roads, as introduced in the first chapter.

6 State of the art

Albeit relatively new, the macroscopic approach to urban congestion has already given birth to different ideas and various interesting results. In particular, it appears that congestion formation and propagation stem from relatively simple elementary processes, but lead to complex collective behaviors such as phase transitions.

6.1 Existence of weak links for congestion

6.1.1 Identifying influential and weak links

As we have mentioned in the previous chapter, all roads do not play the same role in the network. The traffic demand can vary significantly based on the position of the link in the network. Links of high importance that are not properly dimensioned can then become bottlenecks for the traffic which, as we will see, have a detrimental influence on traffic at a macroscopic level.

Using the Betweenness centrality, as we have done previously, gives a good overview of links susceptible to be hotspots for congestion. However, they are also quite often dimensioned accordingly, such that actual congestion does not necessarily occur first on those links. An empirical study of the links on which congestion forms is therefore required.

By studying the correlation of traffic between pairs of links, [174] identified the most influential roads of the network. Changes in the congestion level of these roads were particularly correlated with the overall congestion of the region. More simply, [175, 176] identified weak links as roads that are consistently more congested than other roads of the network. Interestingly, [175] also showed that the duration for which a link remains congested scales linearly with its level of congestion and showed that there is a clear hierarchy of congestion levels among the links of a city.

More precisely, they found that the fraction F of links of the network experiencing a certain level of occupation o scales as $F(o) = o^{-\beta}$, with $\beta \approx 1.2$. This study was conducted in Beijing and Shenzhen, and small differences in the exponent between both cities were identified. It would be very interesting to have results of the same type for other cities. Indeed, this type of hierarchy between congestion levels is not trivial and one might wonder how the heterogeneity of congestion (captured in the exponent β) affects the efficiency of the network, and how it is correlated to the network structure of the city.

6.1.2 Study of functional clusters, metastable states

Because travel time is significantly increased on a congested link, one can make the assumption that drivers will choose a route that ignores these roads as long as it is possible. One can then measure the resilience of a network to the appearance of congestion on its weak links by measuring the increase

in length of the shortest path (measured as a travel time) when congested links are removed [150]. Interestingly, it appears that the resilience of the network varies significantly between cities, but that the structure of traffic demand plays a much more important role in this variability than network structure. This result is particularly promising, as it gives indications on the necessity to reroute traffic at a macroscopic scale to increase the network's resilience, but requires further studies to be fully understood.

An alternative way to look at the problem is to consider that the city is composed of functional clusters (of roads functioning normally), separated by dysfunctional links. Obviously, for a same given number of dysfunctional links, the resulting structure of functional clusters can vary greatly, with a significant impact on the macroscopic performance of the network. In [177], the authors find that for a given fraction of congested roads, the network empirically oscillates between essentially two metastable states. There can either be a giant component of functional links which spreads across the network, meaning that the macroscopic effect of individual congested links is weak, or a multitude of small functional clusters. In this second case, drivers will spend a significant amount of their travel time stuck in the few congested links to go from one functional cluster to the other.

The reality is even more complex than this, as hysteresis is observed in the transition between the two metastable states. As traffic demand increases and the network is loaded, it stays in a generalized functional state before collapsing into small functional clusters. At the end of the rush hour, however, the network tends to remain in its small cluster state. Further research in this direction is therefore needed, but this is clearly a very promising result : because the transition between the states is abrupt, one can imagine that small changes to the network could be enough to vastly improve the network's efficiency.

In [151], the authors combine both approaches. They gradually load the network with routes between randomly chosen origins and destinations. Each route follows the shortest path (in travel time) on the network. As the network is loaded, congested links are removed from the possible options for subsequent routes. They show that the network typically goes through two phases as the network is loaded and links are removed. First, the shortest path evolves such that its distance increases, but the travel time does only marginally increase, as the traffic is rerouted on alternative routes. Then, the network brutally collapses into small disconnected clusters and it becomes impossible to perform long-range routes on the network without being stuck in congestion. Their work also highlights the importance of planning networks under load. Indeed, they show that the congestion of roads with high centrality, which were planned to cater for high traffic but nonetheless experience important congestion, leads to a situation where smaller roads start to play an important role in the congested network, as they have a high BC once the travel times are updated according to the observed congestion.

6.2 Congestion propagation

6.2.1 Diffusion processes

Having identified the weakest links of the network, from which congestion is likely to spread, it is also interesting to look at the propagation of congestion from a link to its neighbors. Both [175, 178] proved that the probability of congestion decays with the distance to a congestion core as a power-law. This observation does not give informations about the temporal characteristics of the phenomenon, contrary to [174], which observed that the state of two links on the network is correlated, but shifted in time by a delay that depends on the distance between the links.

These results very much suggest that the congestion spreads from a link to another following some sort of diffusion process. Bellocchi et al. [179] proposed such a reaction-diffusion process for urban congestion propagation which successfully reproduces observations made during rush hours. For low demand, low congestion hours, however, this model proved less efficient.

This suggests that two mechanisms are at play and should be studied together. Some links are intrinsically prone to congestion due to a discrepancy between traffic demand and capacity, while on other links, the appearance of congestion is only a consequence of the diffusion of congestion from the initial “seeds” of congestion. Traffic spill-back and gridlocked intersections likely play an important role in the propagation of congestion.

Again, this study was made on a single city and it would be interesting to study the effects that network topology or methods for the active prevention of gridlocked intersections (e.g. through dynamic traffic light timing) could have on the propagation speed of congestion.

6.2.2 Identifying clusters of congested roads, percolation of congestion

A consequence of this reaction-diffusion like process for congestion propagation is that clusters of congested links tend to appear on the network. Just as for the functional clusters, attention has been given to the study of these congested clusters.

A first result that should be noted is that the number of congested links increases non-linearly when traffic demand increases [151, 176]. More precisely, the number of congested links is equal or close to zero for a large range of traffic densities, before brutally increasing and reaching a significant fraction of the total number of links. This result, characteristic of a phase transition, suggests that the congestion percolates through the network, which is confirmed by the study of the size [151, 176] and the spatial spread [180] of the giant component of congested links. Like individual roads, congested clusters also display scale-free properties, in their size [181] and recovery time [181, 182].

In more details, the probability $P(S)$ of observing a cluster of size S (defined by the number of links involved in the history of that cluster) scales as $P(S) = S^{-\alpha}$, the probability $\Phi(T)$ for a random cluster to dissipate in time T scales as $\Phi(T) = T^{-\beta}$ and the average recovery time T scales with the cluster size as $T = S^\gamma$. Not only is it remarkable that such scaling laws exist, but the authors of [181] included two cities (Beijing and Shenzhen) as well as data on highways in their study. The results are very surprising, as for both cities, the exponents were equal (respectively $\alpha = 2.3, \beta = 3.1$ and $\gamma = 0.65$), but also very different from the values observed for highways. Furthermore, they included different dates into their study and could confirm the invariance of α, β, γ from day to day.

These results are very powerful and suggest that only a few very basic elements of urban traffic are responsible for most of the observed phenomena regarding urban traffic congestion. Additionally, the scale-free property of congestion, materialized by these power-law behaviors, suggests a possible relationship with the framework of percolation. The next chapter of the thesis will deal with this promising perspective in more detail.

Note that a framework to study and understand scale-free properties of traffic at the *road level* from a theoretical point of view is currently investigated [183]. Results suggest that some sort of scale-free behavior naturally emerges close to criticality (i.e. at the switch between the free-flow and congested regimes, or in other words, at maximum flow). This also suggests that strategies focused on optimizing the flux by keeping the system close to criticality are naturally chaotic, and that the highest efficiency *on average* comes at the expense of a large standard deviation. We should however note that this is an active research topic, currently only focused on single roads close to criticality, and thus that its consequences for the urban traffic dynamics described here are not trivial.

6.2.3 Epidemiological models and the limited influence of network structure

One might note that the entire previous discussion about the percolation of congestion on the network and the distribution of congested cluster sizes suggests that the structure of the network is, in fact, not particularly relevant. The reaction-diffusion process proposed by [179] limited the propagation of congestion from one congested link to its neighbors. In [184], the authors go further and propose a simple contagion process based on the susceptible-infected-recovered (SIR) model. Links of the network are treated equally, without taking their spatial location or their interactions into considerations and the number of congested links simply is a macroscopic network variable.

Let us be a bit more precise and introduce the differential equations governing the SIR-process, as their derivation and understanding is intuitive. The authors use the following notation, $f(t)$ is the fraction of links which are in free-flow at time t , $c(t)$ denotes the fraction of congested links at time t , and $r(t)$ is the fraction of recovered links.

In the context of disease spreads, the latter term is justified by the fact that patients that just recovered from a disease usually have their anti-body levels high, such that they are relatively immune to the disease for a certain amount of time following their recovery. In the case of road congestion, the authors of [179] base this assumption on the observation that links usually stay congested until congestion dissipates for good, and that it is rather unusual to see links oscillate rapidly between free-flow and congested state. Like in the context of epidemics, the introduction of this term allows the SIR model to describe congestion propagation much more realistically than the simpler susceptible-infected or susceptible-infected-susceptible models.

The equations for the SIR process are then :

$$\frac{dr(t)}{dt} = \mu c(t) \quad (\text{III.1})$$

$$\frac{df(t)}{dt} = -\gamma c(t)(1 - r(t) - c(t)) \quad (\text{III.2})$$

$$\frac{dc(t)}{dt} = -\mu c(t) + \gamma c(t)(1 - r(t) - c(t)) \quad (\text{III.3})$$

These equations are intuitive to understand : the congested roads recover at a rate μ , the free-flowing links turn into congested links at a rate γ and thus the number of congested links can then be derived by the competition between congestion and recovery. In fact, the authors rewrite the transmission rate as $\gamma = \beta k$, where k is the average number of neighbors of each link. This is the only parameter which takes some sort of information about the network's topology into account. μ and β are then parameters that can be tuned to fit $c(t)$ to real-world traffic measurements.

The authors find a remarkable agreement between their results and empirical data. In more details, they find that their SIR model manages to reproduce the temporal evolution of congestion on the network, but that the parameters μ and β of the model have to be adjusted slightly for each city. Interestingly, two groups of cities tend to emerge. North-American and Australian cities (Montreal, Chicago, Melbourne) require similar parameters, as do Paris and London for Europe. These small differences between the studied cities suggest that this kind of simplification might be slightly too radical and that the network structure, beyond simply the average degree k , does in fact play some role (which would be what one intuitively expects). It would thus be interesting to investigate the correlation between macroscopic properties of the network (e.g. degree-distribution rather than average degree, BC-distribution...) and the macroscopic dynamics of congestion captured in the parameters of the SIR-model.

In any case, this result shows that the "microscopic" details of the city (i.e. the precise arrangement of streets and intersections) do not play a major role in the propagation of congestion. Interestingly, exactly as for the reaction-diffusion model, this model gives insight into the propagation of congestion,

but not on the first appearance of congested links. Indeed, one needs to give the SIR model at least one congested link from which congestion can then spread. In their article, the authors used real world data to determine the initial conditions of the simulation and then verified that the SIR model evolved similarly to the observations in subsequent times. As stated previously, the identification of weak links from which the congestion will start is therefore an independent and equally relevant problem. Moreover, as we have explained with [150, 151], while the number of congested links might be almost independent of the network structure, the consequences of this congestion on travel time can vary greatly from one network to another.

6.3 Alternatives to the macroscopic, physical approaches

In this section, we mostly described the state-of-the-art in terms of physical approaches to the problem of urban traffic dynamics. As we have said in the first chapter of this document, other tools exist and are used in practice to model urban networks and take informed decisions regarding the infrastructure. Typically, the two scales of the problem have to be treated separately. First, the time-dependent Origin-Destination matrix is determined. Then, using approximate capacities for each street, the network is loaded gradually, in order to determine the flux and car-density on every road of the network. Finally, given these informations, specific streets or intersections of the network can be simulated microscopically in order to optimize their capacity (e.g. by tuning the traffic light timing).

Note however, that some promising alternatives are rapidly emerging. In particular, Machine Learning models can be useful to anticipate the impact of changes in the network structure or the Origin-Destination matrix on the traffic dynamics. It appears that the most effective Machine Learning models are those that combine data analysis with some sort of physical constraints such as described in this section. In particular, Machine Learning models respecting the observed SIR-equations are currently investigated [139], while authors of [185] have shown that Learning-Based Control strategies should be trained using free-flow data, as the chaotic nature of the congested regime reduces the effectiveness of control methods trained on congested regimes.

This shows another aspect of the interest of a physical understanding of urban traffic : even if the observations are not sufficient to give a complete model for urban traffic dynamics, the discoveries can be used to constrain or to refine existing models used for practical applications.

7 Empirical evidence for a jamming transition in urban traffic

As we have seen, studying the propagation of congestion from street to street allows for powerful results. It appears that the emergence of network-wide congestion is a phase transition, somewhat related to percolation and is universally shared by all cities. In this section, we present our own approach to this problem, complementary to the previously presented articles. Using data from the city of Paris covering the period 2014-2018, we study the correlation between congestion levels of the streets. We build the correlation function corresponding to the network and identify a typical correlation length that diverges during rush hours, giving an additional proof of the existence of a jamming transition in urban traffic. We also discuss the spatial structure of congestion as we identify a core of congested links that participate in most traffic jams and show that the spatial structure of congestion is consistent with the reaction-diffusion picture proposed previously. Finally, we discuss the implications of such strong levels of correlation on traffic at the macroscopic and microscopic level, as well as the singular role of ring-roads observed in our data. Results presented in this section were published in the article [186].

7.1 Divergence of the correlation function

7.1.1 Construction of the correlation function

Description of the data and the Parisian network

We use traffic measurements provided by the city of Paris¹. The data consists of Loop Detector Data (flux and occupation), aggregated over 1h time windows, and cover the period 2014-2018². It concerns here the city of Paris ('Paris Intra-muros') which is surrounded by a ring road (the 'Boulevard Périphérique') which plays an important role for commuters going from one part of the suburb to another one. Paris is roughly circular with a diameter of order 10 km. For most inhabitants of the urban area that comprises Paris, it is common to live and work in different places. The average commute to work is of order 6.6 km for Parisians and of order 14.6 km for individuals living in the suburbs [187, 188]. This means that the typical trip spans a significant part of the network.

Another interesting feature of the network is the impact of the Seine river crossing the city. As in most cities with a river, there are roads along it on each bank. In addition in the case of Paris until 2016, there was a urban highway (the 'Voie sur berge Georges Pompidou') going from West to East through Paris, while there were only regular roads (smaller and with traffic lights) for the direction from East to West. As a result of the closure of the Voies sur Berges in the fall of 2016, the dataset studied here effectively contains two quite different networks and traffic structures depending on the

¹And accessible on the website [opendataparis](https://opendataparis.com/)

²Measurements started in 2014 and are updated once per year, for the year $n-1$. In theory, we could therefore give results for the period 2014-2022, but we choose to give here only the results featured in our article

period of focus. Our results will be displayed comparing years 2014 to 2018, with 2016 being split into its beginning and its end.

Measuring congestion

For each road, we build the fundamental diagram using the traffic measurements, and extract the free flow speed u_i^{max} from the slope at the origin. Because some roads changed throughout the studied period, we do this step for every year of our dataset. The free flow velocity varies strongly from a link to another and we find that for Paris, it is log-normally distributed among the links. It is natural to normalize the velocity on each link by its free flow value and to define $v_i(t) = \frac{u_i(t)}{u_i^{max}}$, which is the standard quantity used for instance in [174]. Here, we want to define a measure of the macroscopic state of the network, i.e. an aggregation of the states of the links, and the natural quantity is then the relative increase of travel time (or relative delay) $d_i(t)$ on each link. For a link of length L_i , it is given by :

$$\begin{aligned} d_i(t) &= \frac{L_i/u_i(t) - L_i/u_i^{max}}{L_i/u_i^{max}} \\ &= \frac{v_i(t) - 1}{v_i(t)} \end{aligned} \quad (III.4)$$

Using this delay, we can compute the average delay over the whole network

$$d_{network}(t) = \frac{\sum_{l_i} d_i(t)}{N_{links}} \quad (III.5)$$

or other quantities such as the time lost on an average commute.

Another important quantity is the length of the links experiencing delays which, for a same average delay, will vary between a high local congestion or a mild but generalized congestion. To do so, we define a congested link as a link on which the travel time has been increased by more than 10%. This choice is somewhat arbitrary and we will discuss it briefly here. First, we note that such a choice has to be made for any of the similar studies that we discussed previously (e.g. [179, 184]). We consider that when the travel time is increased (and the velocity therefore decreased) we are in the congested regime. We could therefore in principle choose any criterion on the travel time increase as long as it is strictly positive. We choose a 10% increase of travel time as something that would “noticeably” impact the road users, but this is obviously a matter of choice. In particular, we choose not to include links where the increase of travel time is close to 0%, as we might then include links which are in fact in free flow state. What we mean by this are the points that are below the average Fundamental Diagram but at an occupancy lower than the critical occupancy. More importantly, we note that the dependence on

this threshold is very weak as shown in Fig. III.1: the fraction of congested links is roughly constant up to approximately 10%. Any threshold taken in this range will then not affect our results.

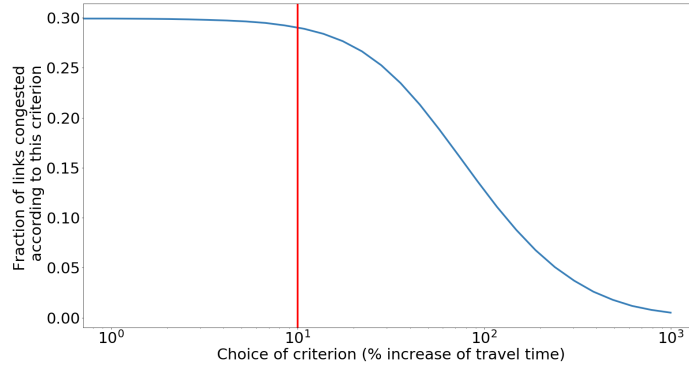


Figure III.1: Dependence of the fraction of links in congested state on the criterion used to define congestion (year 2017, hour 14). In red, our definition of congestion (links are congested if the travel time is increased by over 10%)

The focus of this work is the formation of congestion and we will therefore consider working days and remove weekends and the month of August when the traffic demand is (much) lower. Unless specified otherwise, all the figures displaying average quantities are obtained by averaging over all working days in a year at a given hour. For example, we show in Fig. III.2 the daily evolution of relevant measures of traffic in Paris for the studied period. Interestingly, we note that the traffic level is high for most of the day without large differences between rush hours and mid-day. In contrast, the occupancy rate and the fraction of network with delays display more pronounced peaks during rush hours (in the morning around 8am and in the evening around 7pm). In addition, we note that even during rush hours, only 30 to 40% of the total number of roads of the network experience significant delays, indicating a concentration of delays in a few 'hotspots', in agreement with the results in [151, 174] or others.

Correlation function

The goal in this section is to find an intrinsic marker of a phase transition in the data and to show that congestion in a complex road network during rush hours can be viewed as a sort of network-wide jamming transition. A typical marker of a phase transition in classical statistical physics is the divergence of the correlation length close to the critical value of the control parameter (see for instance [189]). We will therefore analyze the correlation between the delays on all roads of the network, and identify the correlation length and its variation. We will show that it 'diverges' (i.e. it becomes comparable to the size of the network) during rush hours suggesting the existence of a transition. Note that this approach is similar to a previous work by Petri et al. [178] with a few major differences that we will discuss.

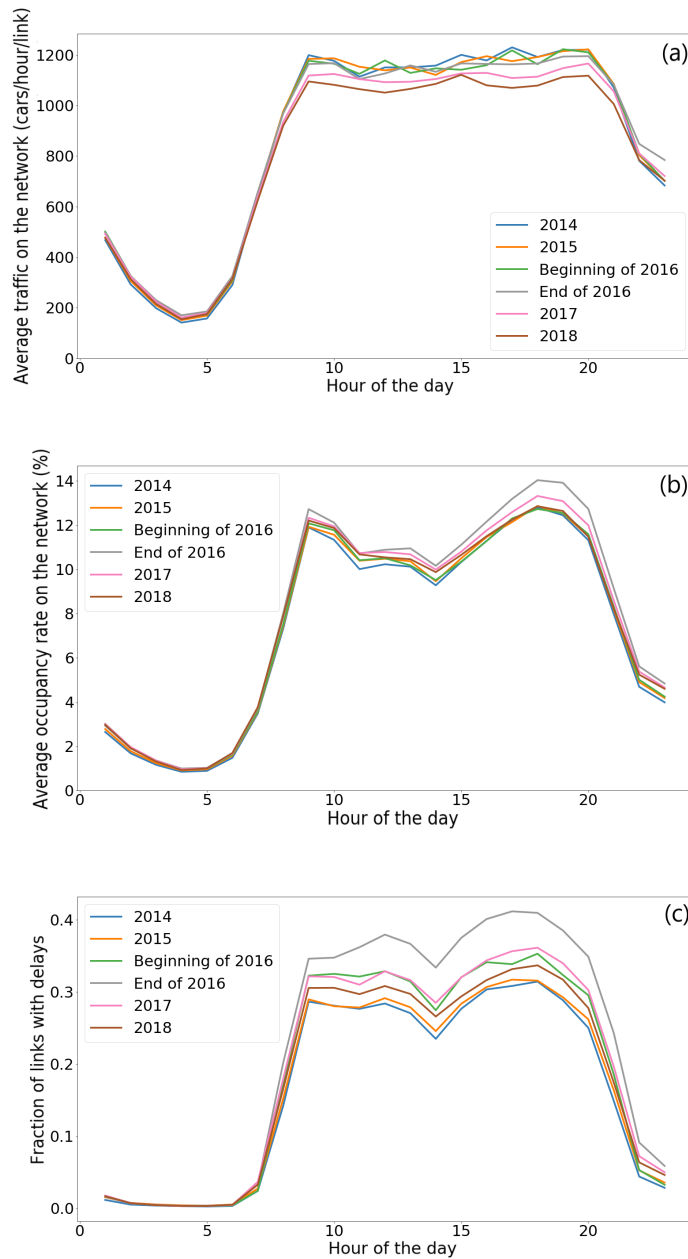


Figure III.2: Daily evolution for years 2014 to 2018 of (a) the average traffic, (b) the average occupancy rate, and (c) the average fraction of links of the network experiencing delays. The traffic level is high during the whole day and rush hours can be observed on the occupancy rate and delays at about 8am and 7pm.

The authors of [178] computed the correlation between traffic fluctuations in London (i.e. for each time step, the difference between measured traffic and expected traffic on each road). There are however some limits to using the values of traffic for measuring the state of the congestion state of the network. Indeed, imagine that we have two links A and B on which the congestion level is growing in a correlated way. Depending on the fundamental diagrams of A and B (and on the initial state of congestion), one could have an increase of traffic on A and a decrease of traffic on B, despite both

being more congested. As a result, the correlation of traffic fluctuations between A and B is not an ideal measure of the real correlation of the traffic state between A and B.

As mentioned above, we compare roads based on the relative delay experienced by users. For each year and each hour, we thus have the delay $d_i(t)$ experienced on link i and we introduce the quantity $T_i(t)$ which denotes the average travel time for that year and hour on this link. The relative delay is then given by $\tau_i(t) = d_i(t)/T_i(t)$ and indicates the importance of congestion on this link. We consider the correlation function of this quantity and measure it for the year y and hour t

$$C_{i,j}(y, t) = \langle \tau_i(t)\tau_j(t) \rangle - \langle \tau_i(t) \rangle \langle \tau_j(t) \rangle \quad (\text{III.6})$$

The averages (denoted by the brackets $\langle \cdot \rangle$) are performed over all working days of a given year y and at a given hour t . From this definition, we construct a distance dependent correlation function by sorting the pairs i, j according to their distance. Since we are considering a phenomenon that takes place on a network, some caution is needed for defining this distance.

Defining the distances

As discussed in [64], the naive choice of the euclidean distance is not adequate as all physical processes discussed here occur on the network itself which somehow distorts distances. Instead, one should consider in general that each pair of roads is separated by the distance along the shortest path on the network connecting them. Here we will use the shortest travel time between roads (we use the path which, on average, is the shortest for a given year and hour).

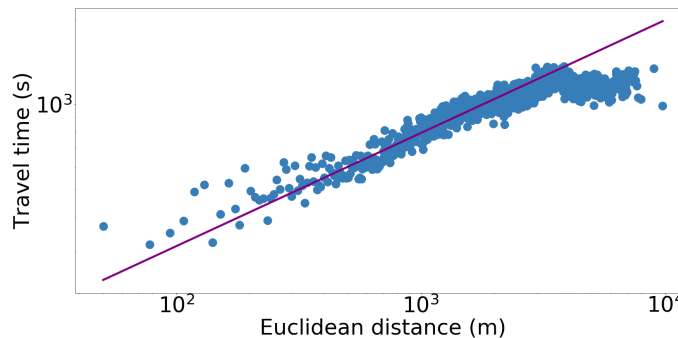


Figure III.3: Relation between travel time and euclidean distance in the case of the Parisian network. The straight line corresponds to the power law fit with exponent $\beta \approx 0.4$.

For most distances, the travel time scales as d^β with $\beta \approx 0.4$ (Fig. III.3), while at larger distances, the travel time becomes independent of the distance. This effect is an artifact of the geometry of our network as consider Paris and its ring road only. The points furthest away from each other are

diametrically opposed on the ring road, and the travel between them can be made at high speeds. Points slightly closer from each other will most likely need a fair bit of travel inside the urban network, explaining why it doesn't translate into shorter travel times. Had we considered a larger network, including roads outside of the 'Boulevard Périphérique', we would most likely not see this effect.

7.1.2 Results

We find in our dataset that the correlation function displays a typical behavior of the form

$$C(r, y, t) = \frac{1}{r^\eta} \exp\left(-\frac{r}{\xi(y, t)}\right) \quad (\text{III.7})$$

where r is the distance between two roads and where $\xi(y, t)$ is the correlation length, depending on the hour of the day t and calculated for each year y . As mentioned earlier, distances, including the correlation length, are converted in the time needed to travel them. As in [178], we find that very short range correlations (i.e. between one link and its immediate neighbors) are strongly affected by the exact configuration of the network and extrinsic parameters such as the timing of traffic lights. For this reason, we perform our fits for distances greater than 60 seconds of travel time, which corresponds roughly to the roads separated by more than just one traffic junction. Only a very small number of roads is distant of more than 3000 seconds of travel time (see figure III.4).

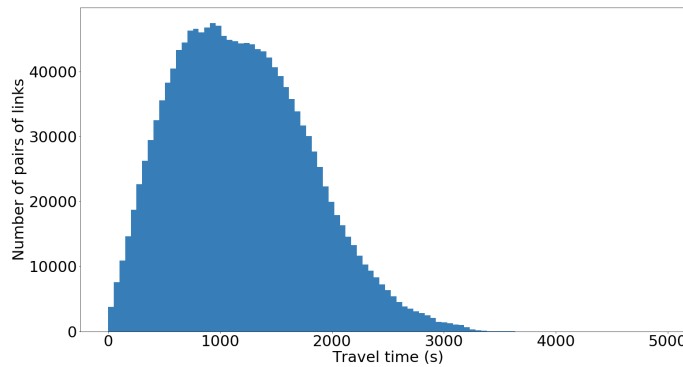


Figure III.4: Distribution of the distance between all pairs of links of the network

In practice, we observe that the correlation function at short-medium distances ($< 3,000s$) is relatively stable between years or between consecutive hours. Above 3,000s, we observe large fluctuations from a year to another, confirming that we observe individual events rather than an average trend. We thus perform our fit on the range $[60, 3,000]$ seconds of travel time and we find an exponent $\eta \approx 0.4$.

More precisely, we apply the following procedure to extract the parameters. We first perform a 3 parameters fit (η , ξ , and C), which gives a good approximation of η : around 0.4 for most hours and

years. We then enforce $\eta = 0.4$ and perform a new fit with two parameters (ξ and C), hoping to be able to fix C and to fit only for ξ . It appears that for most days C is rather constant from one year to another but displays small fluctuations during the day. We thus perform a new two parameters fit (over C and ξ) for which the variation of C is constrained to the small interval observed empirically. This procedure provides us with robust values of η , C , and ξ . Some example of correlation functions are displayed in figure III.5.

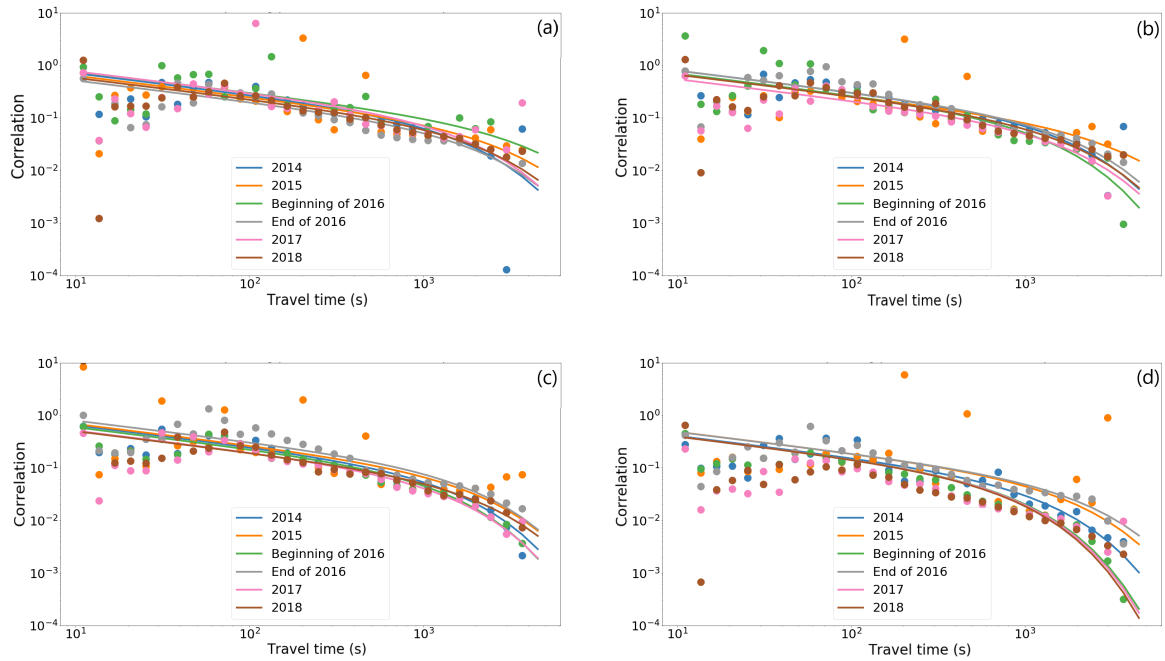


Figure III.5: Illustration of the correlation functions and the corresponding fit for hours 6pm (a), 7pm (b), 8pm (c), 9pm (d). The distance is expressed in terms of travel time along the network. The exponent of the power law decay is $\eta \approx 0.4$

This correlation function depends a priori on the year and the hour of the day. Performing the fits for all these cases, we are able to extract the correlation length and to examine its variation shown in Fig. III.6(a). First, we obtain consistently a value of $\eta \simeq 0.4$, which is higher than the value (≈ 0.26) found in [178]. Another major difference is of course the presence in our fit of an exponential cutoff for the correlation. In addition to the fact that we don't consider the same quantity for computing the correlation, the choice of euclidean distance in [178] might also affect the results. Indeed, two roads far away in space could be either separated by a highway and a short travel time apart (case (1)), or separated by the urban network and a long travel time (case (2)). We argue that these two cases translate in different correlations, with case (1) being more correlated than (2). In [178], pairs of roads in both cases (1) and (2) are averaged over, resulting in larger values of correlation at long distances, which might explain why they don't observe an exponential cutoff in their results. The non-

linear relation between travel-time and euclidean distance (figure III.3) might also explain in part the difference between the values of the exponent η .

We also observe a clear difference between night and day hours. During night hours, the correlation length is very small, typically of the order of the time needed to travel from a link to two or three links further: delays appearing on a link during the night propagate to the vicinity of this link only. In sharp contrast, we observe during day hours a dramatic increase of the correlation length to values close to 1,500 seconds and during rush hours, the correlation length displays a peak well above this value. We recall here that the typical trip of a road user on the network is known to be around 1.500 seconds long and to span a significant part of the city, thus having $\xi \approx 1,500s$ indicates a correlation at the scale of the whole system. This divergence is the sign of a transition that occurs during rush hours with congestion expanding over the whole network.

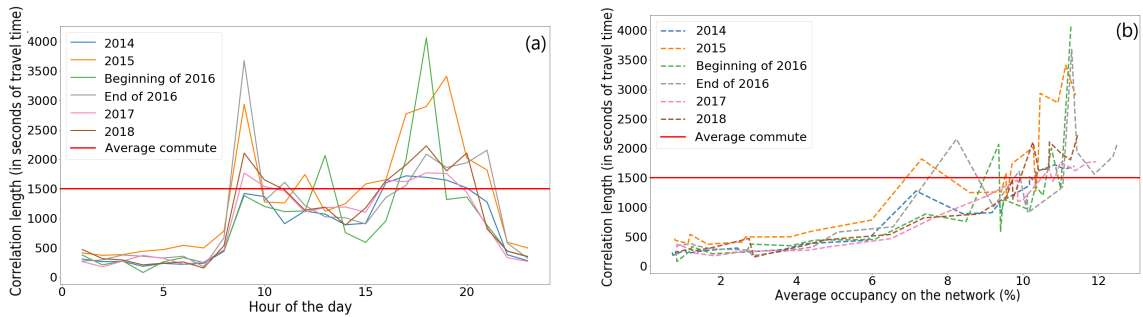


Figure III.6: Correlation length ξ for the delays on the links of the network, as function of time (a) and of the average occupancy rate (b). The horizontal line indicates the duration of a typical commute (1500 seconds), which geographically approximately corresponds to the size of the system

These results therefore suggest the existence of a transition between a state where congestion is localized to another state where congestion occurs over the whole network during rush hours. The control parameter which varies with time is the traffic demand and it is then natural to consider the average density (or the average occupancy rate) on the network as the control parameter here. This leads us to investigate the relation between the correlation length and the average occupancy rate on the network (see Fig. III.6(b)). As expected, the correlation length increases with the occupancy rate, and grows very rapidly for occupancy rates above 10%. In principle, one would like to study this divergence in more detail. From other examples in statistical physics, one expects to find:

$$\xi \approx \left| \frac{o - o_c}{o_c} \right|^\nu \quad (III.8)$$

Where o_c is the critical value of average occupancy on the network and ν is the critical exponent.

7.1.3 Singular role of ring-roads and urban highways

As we discussed in the previous chapter, most large cities built a ring road in order to reduce traffic in the urban center by proposing an alternative route around the city. It is therefore important to understand its role in the spread of congestion. In the case of Paris, the 'Boulevard Périphérique' is encircling the central Paris area and has a length of about 35km . We compute the correlation function for the links belonging to this ring road and the results are very different from those for the entire network. More precisely, the Boulevard Périphérique displays two distinct regimes. During the night, as in the rest of the network, the correlation is small and local, but during the day the correlation is almost independent from the distance. It is maximal at short distances, but then remains large regardless of the distance. The Boulevard Périphérique being an urban highway, we can apply standard methods of traffic theory to understand this result (see for example [190]). In particular, as we introduced in the first chapter, a traffic jam propagates on a highway like a wave with a speed given by :

$$u_w = \frac{q_d - q_u}{k_d - k_u} \quad (\text{III.9})$$

where $q_{d(u)}$ and $k_{d(u)}$ are the downstream (upstream) flow and density. If we consider rush hours during which the Boulevard Périphérique is congested both upstream and downstream, using the Fundamental Diagram of the Périphérique (introduced in the first chapter as figure I.4) we find that the congestion wave moves against the direction of traffic with a speed of order 50km/h . Considering the fact that this ring road is about 35 km long and that the peak for transport demand during the morning or the evening rush is over 2 hours long, it is clear that any unexpected incident on the Boulevard Périphérique has the time to spread over the whole ring, explaining the high correlation values observed in our data. If the ring road is not congested, an incident will simply vanish locally and the correlation is small.

We expect that this phenomenon happens for urban highway (ring-road or not) used above capacity during the day. This almost binary switch between small, short range correlations and large correlation independent from the distance on the ring road has an impact on the entire network. Indeed, roads that are on opposite sides of the city, but both close to the Boulevard Périphérique will see their congestion level correlated, especially when the Périphérique is strongly congested. In other words, the boundary conditions of the system change during the day and induces correlations between locations far apart from each other. During low demand hours, the Périphérique acts as an open boundary (no correlation), while during rush hours, it can virtually be reduced to a single point. In other words, it acts during rush hours as a propagator of congestion at the boundary of the whole city.

To our knowledge, Zeng et al. [191] are the only other authors who investigated the peculiar role

of urban highways in congestion propagation, even though their approach is somewhat different. They found that the critical exponent for the percolation varied with the hour of the day and in particular with the level of congestion of highways. During rush-hours, when the whole network (including the highway) is congested, the network behaves approximately as a 2D-lattice, while outside of rush hours, highways give the network some properties of a small world, which translate into a different exponent for the cluster size distribution.

However, both their and our conclusion is that traffic management should be designed in priority as to ensure free-flow on those high speed arterials and facilitate long-range connections. One can understand this by going back to the discussion about the metastable states of traffic proposed by [177]. Keeping the long-range connections effective would most certainly help to keep the network in the “efficient” state of a giant functioning component spanning the entire network, while the collapse of these few long range connections makes it far more likely for the network to be separated into small disconnected functional clusters.

7.2 Study of the spatial distribution of congested links

In this section, we investigate the spatial structure of congestion in Paris. In particular, we will focus on clusters of delayed roads (as done for example in [181]) in order to analyze the spreading process of the congestion on the network.

7.2.1 Giant component

For each hour, we define congested roads as those on which the travel time is increased by more than 10% compared to the free flow situation. These roads form clusters of congestion on the network. Due to the limited spatial resolution of the data-set, we merged neighboring congested clusters based on the following rule: (i) if two clusters of congestion are separated by a path shorter than 350 meters (which corresponds to the typical length of a link) and (ii) if along this path, roads are not equipped with sensors (making it impossible to evaluate the delay), we consider that the two clusters are connected. For each date, we then look at the largest connected component of congested links. We find that the congestion percolates during rush hours and the dependence of the average giant component size on the average occupancy rate displays a typical shape of percolation in a finite size system, as displayed in figure III.7(b).

Zeng et al. [177] have shown that during rush hours the network oscillates between multiple metastable states: for a given fraction of congested links, they found that there could either be a giant functional cluster or several small functional clusters. We try to reproduce these results for our case of the cluster of congestion rather than functional clusters. However, we do not find these meta-stable

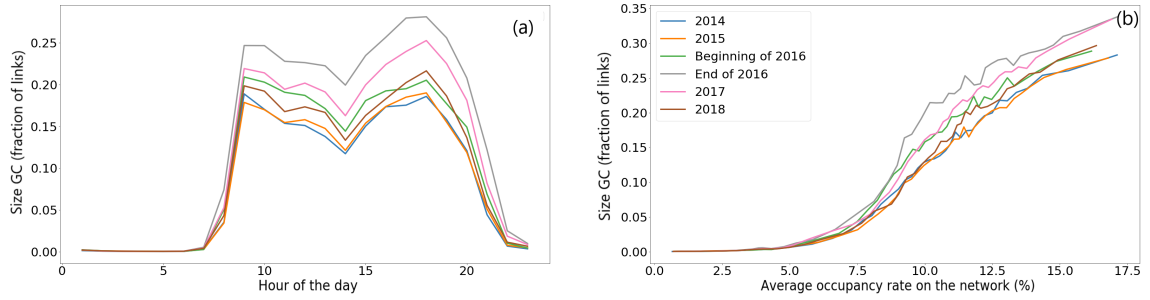


Figure III.7: Average size of the Giant Component (fraction of links), depending on the hour of the day (a) and on the average occupancy rate (b). Averages performed over all working days of a given year at a given hour.

states and each measure of the size of the giant component as a function of occupancy is close to the average relation displayed in Fig. III.7(b). The most likely explanation for this discrepancy is that our measures are averaged over an hour, which is a long period of time compared to the typical stability duration of the metastable states found in [177], which is of order 10 minutes. This suggests that our measure thus only reflects the average of the systems oscillations between the two states.

7.2.2 Core of congested links

Once we have defined the giant component of congested links for each day and hour, it is natural to look at its stability in time. We thus identify each link that contributed to the giant component throughout the year and plot the distribution of its appearance frequency in the giant component (Fig. III.8). This result allows us to identify “core” links that are present in the giant component for

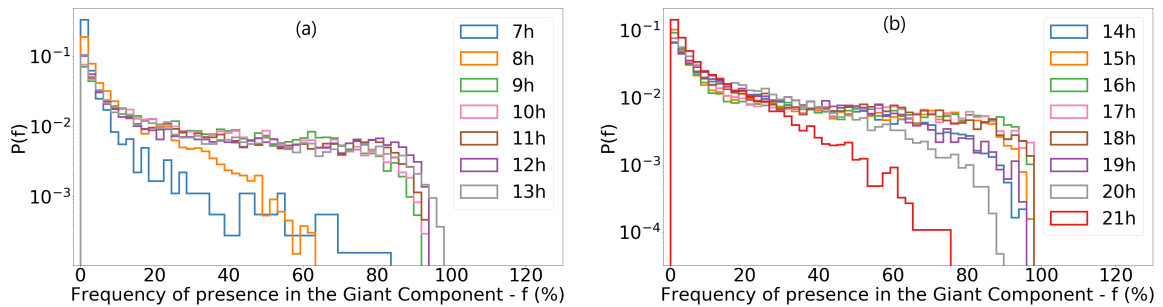


Figure III.8: Distribution among all links of the frequency of presence of the link in the giant components of congestion of the year 2018 for morning (a) and afternoon (b) hours respectively. Note the stability between hours.

most days and other links that appear only occasionally in the giant component and we can make some observations. First, there isn't a core of congested links during the night as few roads are congested and vary from a day to another. During rush hours, the core of congested links consists mostly of the

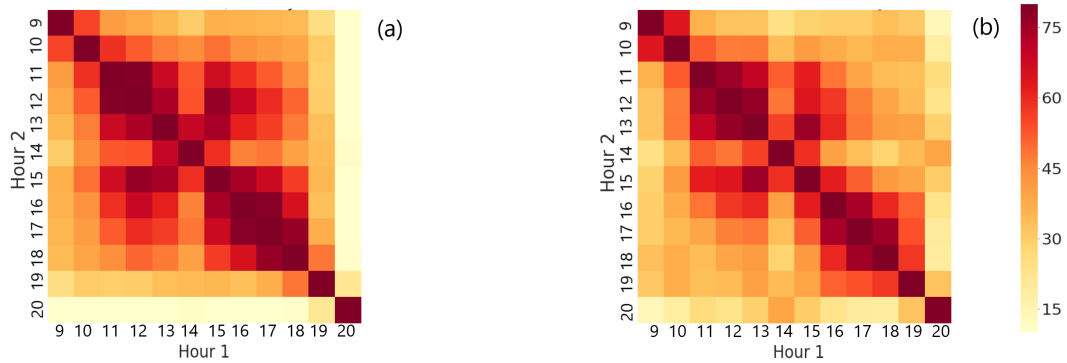


Figure III.9: Overlap between the cores of congested links from hours to hour, for 2014 (a) and 2018 (b) respectively. The color codes the percentage of overlap between two hours. One can clearly identify a continuity in the congested links during the morning and the evening rush, but only a small overlap between the two periods.

Boulevard Périphérique and other roads on the outer part of Paris, while during the other hours of the day, the core of congested links is smaller and concentrated in the center of Paris. These results are consistent with the expected flows of commuters, coming from the suburbs to the center of Paris via the Périphérique, but the high level of congestion of the city center throughout the day is more surprising.

We can further investigate the temporal evolution of the spatial distribution of traffic. Of particular interest is the evolution of the cores from hour to hour, in particular between the morning and evening rush, as well as the evolution from year to year. Fig. III.9 displays the overlap between the cores of congested links, defined as links present in the giant component of congested links for more than 50% of the days. The overlap is then calculated as the size of the intersection between the two cores, divided by the size of their union.

These figures highlight the stability of the core during the morning or the evening rush hour. In addition, in 2014 there is an important overlap between congestion cores in the morning and in the afternoon. Even if there are differences between the morning and evening cores, this result points to the existence of 'weak' links that are sensitive to demand and become easily congested, independently from the actual origins and destinations of the commuters. We note that the overlap between cores is weaker in 2018. Since we know that the average congestion on the network and the average size of the giant component have remained constant between 2014 and 2018 (figures III.2, III.7), this indicates that the congestion must be more evenly spread on the network.

In particular, it seems that the weak links, from which the giant component of congestion starts its progression, are not as clearly identified as in 2014. This is not necessarily good news, as it means that there is more randomness in the source of the giant component of congestion (any part of the network

could potentially become congested randomly) and that there is no clear indication on where to put the focus for future road planning in order to mitigate the emergence of congestion. An interpretation for this difference between 2014 and 2018 could be the following. As we mentioned, an important highway along the Seine was closed in 2016. In 2014, most users traveling from West to East used this ‘Voie sur Berges’, which carried high levels of traffic and therefore was prone to congestion. In 2018, the former users of this highway have been distributed on several alternative routes across the network. The city center in particular now operates at higher traffic density, which leads to more roads susceptible to trigger congestion.

7.2.3 A simple model for spatial contagion

Fig. III.8 yields other results than simply the study of the core of congested links. Indeed, we observe that the frequency at which links appear in the giant component follows a peculiar distribution that is essentially constant for the range 20% – 80% (except for hours in the range 8pm-7am where we observe a strong decay). This is very different from what we would observe if the giant component consisted essentially of the same links every day. In this case we would find a distribution with a high peak at large frequency and a smaller peak at low frequency. Here, in contrast we observe a whole range of frequency and we discuss here a simple toy model that allows us to understand this behavior. This model is based on the intuitive idea that congestion spreads from a link to its neighbor in a diffusive way.

Rather than building a reaction-diffusion process (like has been done by Bellocchi and Geroliminis [179]), we follow the results of [175, 180]) to build a much simpler model. In particular, it has been shown in [180] that the probability for two links to belong to the same functional cluster decays exponentially with distance, and that the probability of a link to become congested decays linearly with the distance to a congestion ‘seed’ [175]. We thus propose the following model: we assume that a single given link of the network can act as the seed for the giant component of congestion, and a link i at distance d_i from the seed link has a probability to belong to the giant component of congestion which reads as

$$p_i = \max(p_s(1 - e^{-d_i/L}), p_r) \quad (\text{III.10})$$

where p_s is the probability for congestion to emerge on the seed, and p_r is a small residual probability, accounting for the fact that any link might be subject to a random incident causing its congestion. The probability to find a link (among a total of N_L links) in k of N realizations of the congested network

is then :

$$\begin{aligned}
 P(k, N) &= \frac{1}{N_L} \sum_i \binom{N}{k} p_i^k (1 - p_i)^{N-k} \\
 &= \frac{\sum_d n(d) \binom{N}{k} p(d)^k (1 - p(d))^{N-k}}{\sum_d n(d)}
 \end{aligned} \tag{III.11}$$

where in the second line we replace the sum over links by a sum over the distance d from the seed, weighted by the number $n(d)$ of links at distance d from the seed. For a regular lattice in dimension D , $n(d)$ scales as d^{D-1} . However, for a real network the dependence between n and d might be different, for instance if the seed is placed in the periphery of the network. Moreover, in the case of emerging congestion on the network, it is likely that several links will act simultaneously as seeds for congestion. For these reasons, we expect in general to have $n(d) = d^\alpha$, where $\alpha \leq 1$ and can vary with the hour of the day. In addition, we expect α to be smaller for hours where the number of seeds in the network is larger such as during rush hours where the formation of congestion might happen on several links. In fact, the core of links of the giant component should be seen as the set of weakest links of the network on which jams are likely to appear spontaneously.

We compute numerically the expression Eq. III.11 and show the result in Fig. III.10 for different values of α . Comparing this result to the empirical measure shown in Fig. III.8, we find that for hours in the range 9am-5pm, we have $\alpha \approx 0$ which is consistent with the many congestion seeds observed during the day. For hours in the range 6pm-8pm, we find a value closer to $\alpha \approx 1$ consistent with the fact that the congested core is much smaller in the evening (during night hours, the traffic demand is too low to lead to observe a growth of congestion from links acting as seeds).

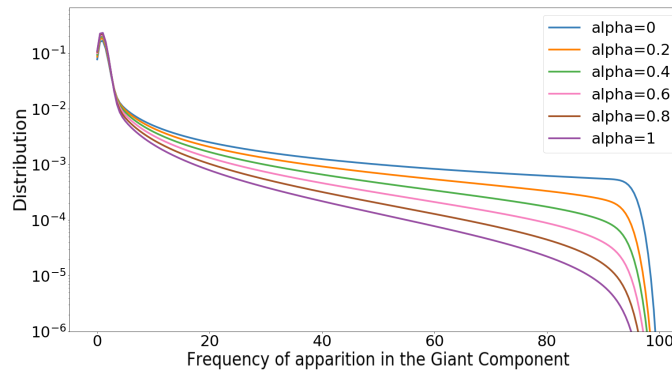


Figure III.10: Numerical results for equation III.11 for different distributions of links on the network $n(d) = d^\alpha$ (with $L = 100m$, $p_s = 0.95$ and $p_r = 0.01$).

7.3 Discussion and implications

7.3.1 Limitation of our dataset

In general, this study would have benefited from a higher quality dataset, be it in terms of spatial or temporal resolution. We did not have access to all roads of the network, which introduces a bias in our results (as we effectively just study the largest streets of the network).

Further, as we said, a lot of phenomena relative to traffic jam emergence take place on much shorter time scales than an hour, such that we describe here an average situation, while missing out on higher frequency phenomena, essential to fully understand the dynamics of traffic congestion on the network. In particular, the resolution of our data doesn't allow us to fit the correlation length close to criticality with enough precision. Indeed, our data is aggregated over one hour, while the transition certainly takes place over a much shorter time period, but determining the critical exponent for this transition and thus the universality class of the system constitutes an interesting objective for future studies.

7.3.2 Macroscopic effects

Let us take a moment to reflect on the implications of our results. From a practical point of view, the existence of long range correlation during rush hours means that the entirety of road users will be affected by any incident happening on the network. We need to remember that our data is aggregated over an hour and that we therefore cannot give details about the spatio-temporal spread of congestion. However, from our results, it appears that if you travel in the South of Paris and an accident happened in the North in the preceding hour, you will most likely feel an increased level of congestion because of it, which is a remarkable phenomenon.

From a urban planning point of view, it highlights how difficult the task is to anticipate the effect of a local decision. One cannot simply estimate the changes to traffic in the vicinity of the road of interest, but should in theory take the whole network into account. This also begs the question as to the real impact of the closure of the Voies sur Berges. How can one estimate the long term impact of this decision, if every other changes made to the network in the years that follow also influenced traffic in the center of Paris ? In particular, the north-east of Paris saw a sharp increase of its population over the past few years [188]. The commutes from and to the highway A3 most certainly affect commuters traveling along the Seine.

Finally, from a scientific point of view, we can link this result to the existing articles described in the previous section. In particular, this result further highlights that the problem of traffic in cities really should be treated macroscopically. We already mentioned that [181, 184] suggest that the dynamics of congestion on the network actually is not the most relevant problem and that more attention should be

given to studies like [150, 151] to increase the network efficiency at a given average congestion level. This is precisely the type of questions that the Macroscopic Fundamental Diagram tries to answer, as we will discuss in the following chapter. Another particularly interesting result linking the traffic dynamics to the network wide response to traffic demand has recently been proposed by J.A. Laval [192]. He showed that the co-existence in the network of congested and uncongested roads could be mapped to the coexistence curve of a simple chemical component at the critical point between the fluid and gas phases, opening an abundance of new perspectives to treat traffic dynamics at the network scale using tools developed for physical phase transitions. We will, once again, come back to this result in the following chapter.

7.3.3 Consequence on the existence of street level fundamental diagrams

We have already mentioned the fact that urban fundamental diagrams do not necessarily respect all the rules defined by Greenshield, with in particular situations where the flux becomes independent of density under the influence of traffic lights (see section 2.2.1). Regardless, it would be a valuable information for urban traffic management to be able to identify a unique fundamental diagram, i.e. a unique relation $q(k)$ for each road of the network. Identifying this relation and its critical parameters q_{max} , ρ_{crit} and ρ_{jam} would indeed, in theory, open the possibility of a dynamic management keeping the roads in in their optimal regime ($\rho \approx \rho_{crit}$) and avert gridlock by enforcing $\rho < \rho_{jam}$. The authors of [91] propose a systematic study of urban fundamental diagrams, in the search of a unique functional form that would allow to extract said parameters from any given urban traffic measurements. They face however obstacles of different nature, which we believe deserve some discussion, in the light of the results that we just presented. We would like to stress that this discussion is at this point mainly speculative, and should serve as a source for future reflexion on the subject.

First, the distribution of traffic on the network is such that some roads never experience congestion. Traffic measurements only contain points in the free-flow state, which obviously makes it impossible to properly fit a fundamental diagram. Second, the observed fundamental diagrams display a variety of non trivial shapes. This is the motivation of the study [91], which aims at finding the functional form which would be the best compromise between this variety of shapes, but we believe that this could be at the expense of some information hidden in the fundamental diagram's shape. Third, the authors face a considerable amount of traffic measurements which they consider as corrupted and reject for their study. While some are clearly the result of malfunctioning loop detectors (e.g. large number of points with non-zero density and zero flow), they also reject measurements which exhibit more ambiguous patterns such as the appearance of multiple branches. We illustrate two example of such ambiguous traffic measurements in figure III.11.

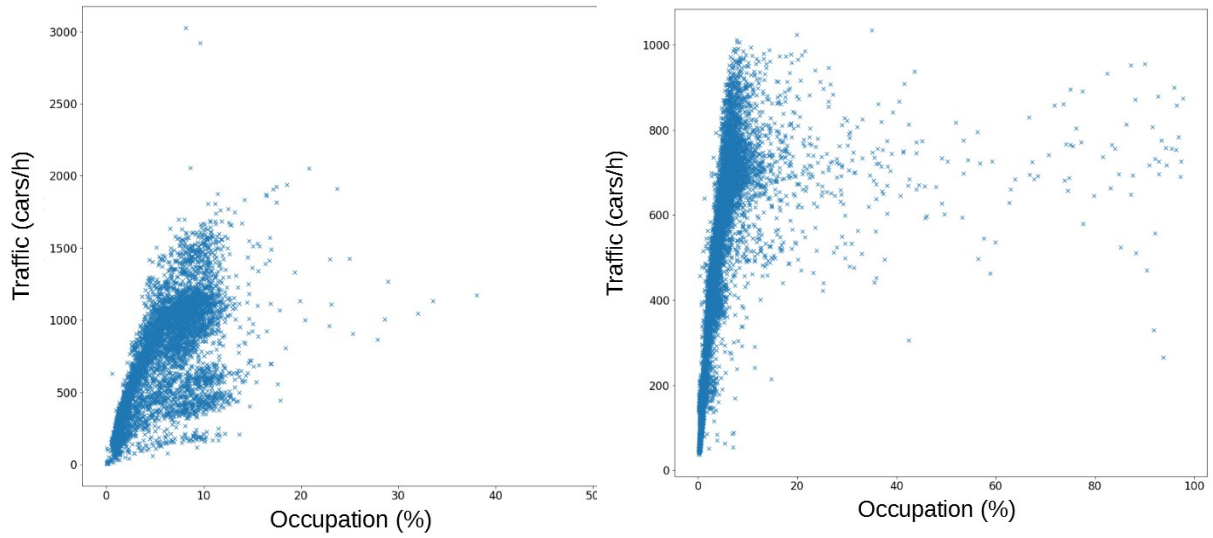


Figure III.11: Example of urban traffic measurements which do not exhibit a clear fundamental diagram. Left: Avenue des Champs Élysées, right: Boulevard Batignolles, Paris, France

Let us take a few steps back. A fundamental diagram is the assumption that for a given road, there is unique relation $q(k)$, defined by unique values q_{max} , ρ_{crit} and ρ_{jam} . As we have mentioned, the uniqueness of these critical parameters is not true even in the case of a rural, 1D road, where q_{max} and ρ_{crit} typically are functions of the visibility conditions. It seems unlikely to us that, in a urban network where roads are highly correlated to one another, the critical parameters q_{max} , ρ_{crit} and ρ_{jam} of a road are constant and independent of the traffic state of neighboring roads. In particular, figure III.11 (left) suggests that a given road can have different fundamental diagrams depending on the congestion state of the network.

Let us present our thoughts on this subject slightly more formally. Consider that road i has a fundamental diagram $q^i(k^i)$ defined by q_{max}^i , ρ_{crit}^i and ρ_{jam}^i . We believe that contrary to the implicit assumption made by [91], those critical parameters are not unique and are in fact functions of q^j, k^j for roads j that are at least the immediate downstream neighbors of i .

In the case where k^i and the different k^j can vary independently, we get situations like displayed in figure III.11, with different branches appearing in the traffic measurements. In the case where k^i is highly correlated with the k^j , the resulting traffic measurements still is not a fundamental diagram in the sense imagined by Greenshield, as it not only a measure of the road i 's response to traffic demand, but of the entire system of i , its neighbors and the signaled intersection between them. In our opinion, this explains the vast collection of shapes encountered in street diagrams. Further, it means that the use of such diagrams for dynamic traffic management would not be as simple as for the case of highways, as each local decision might change the shape of the neighboring roads fundamental diagrams and thus their traffic state.

Conclusion

In this chapter, we have established that urban networks are a system composed of highly correlated streets. The emergence of congestion on the network likely has to do with the existence of bottlenecks for traffic on the network, which are a result of both the local structure of the network and the structure of traffic demand, as we have discussed in the previous chapter. However, it appears that the propagation of congestion from these weak links is not particularly dependent of the network structure. Urban road networks display remarkable similarities in the temporal evolution of the number of congested links as well as the size of the clusters that they form.

Intriguingly, as congestion spreads to the entire network, the system seems to oscillate between metastable states. The congestion or decongestion of a small number of links is sufficient to move from a state of generalized, network-wide congestion and a state where several small size congestion clusters are separated by functioning roads. The further study of this phenomenon, in particular its hysteresis patterns and the peculiar role that urban highways and ring-roads seem to play, appears as a promising solution to leverage the impact of congestion on the travel time of road users.

Finally, these results show that traffic congestion in cities and schemes to combat it can only be imagined macroscopically. In particular, the concept of fundamental diagram, as useful as it might be in traditional problems such as highways, does not resist the scale up to a highly correlated network. Much like the fundamental diagram can lead to powerful results by ignoring the “microscopic” reality of the cars and driver behaviors, we now need to take one more step back and ignore the reality of individual streets. The next chapter will thus deal with modern approaches that consider the problematic of urban traffic and congestion directly at the scale of the network.

Chapter IV

On the Macroscopic Fundamental Diagram

In the same way that the fundamental diagram links density to flow at the scale of a road by a simple function, it appears empirically that such a relationship exists at the scale of the network. We will first present the empirical evidence for the existence of this Macroscopic Fundamental Diagram (MFD), the conditions necessary for its observation, as well as the theoretical framework existing to date. We will also illustrate the practical applications linked to its existence in terms of traffic management in cities. Nevertheless, its understanding remains for the moment incomplete. In particular, two aspects for a thorough understanding of road traffic in cities are missing today.

On one hand, the MFD can currently only be measured from traffic surveys. To make informed decisions about changes in the urban network, a theory linking the MFD to the structure of the urban network would be a valuable aid. Here, using numerical modeling and dimensional arguments, we will propose scaling laws for the MFD's critical parameters in terms of the road density, the intersection density, the average car size and the maximum velocity. This framework is able to explain the scaling observed empirically for several cities in the world, and opens the way to a better understanding of the traffic on a road network at a large urban scale.

On the other hand, the current theoretical framework of the MFD ignores the spatial distribution of traffic on the network. While it is known both empirically and theoretically that the inhomogeneity of vehicle distribution on the network is detrimental to network's capacity, a theory unifying the models of traffic jam nucleation and propagation with the MFD theory is missing today. The end of this chapter will thus be dedicated to the perspectives for future research in the field of the MFD.

8 Introducing the Macroscopic Fundamental Diagram

We will start by giving a historical review of the Macroscopic Fundamental Diagram (MFD), from the first intuition of a relation between traffic measures at the network scale to its empirical observation. We will then give an overview of the work that has been done in the past 15 years to understand the origin and the functional form of the MFD, before giving an insight in its real-world usage. The review [193] has been a very precious support for this section and constitutes a valuable resource for readers who wish to explore the subject further. Throughout this section, we will highlight questions that remain open regarding the MFD, which we recall is still a very new and dynamical field of research. Those open questions will then guide the structure of the remaining parts of the chapter.

8.1 History and empirical existence

Although the concept of a network-wide relationship between average accumulation and average speed on the network was intuited since the 1960s [194, 195], it is only since the generalization of high-quality, network-wide traffic measurements that its existence has been proven. First empirical proofs of such a Macroscopic Fundamental Diagram can be traced back to the end of the 20th century [196], but it is truly the work of Daganzo and Geroliminis that made the subject popular among traffic engineers and physicists [197, 198]. They were the first to measure a well-defined relationship between average accumulation and average flow during a field experiment in Yokohama, akin to the fundamental diagram at the road level. Their historical result is displayed in figure IV.1.

In particular, they demonstrated that, much like the fundamental diagram, this relationship truly is a property of the network, as it is insensitive to the Origin-Destination matrix. Indeed, it remains identical at different hours of the day or during the weekend, even though the travel patterns change. Since then, the existence of such a Macroscopic Fundamental Diagram has been confirmed in numerous cities worldwide. Further work has concentrated on justifying the existence of such a relationship, finding parameters affecting its shape, as well as on real-world application of the MFD.

8.1.1 Production MFD and outflow MFD

We should note that, as already intuited by Daganzo and Geroliminis [198], two types of MFD exist. Indeed, instead of the so-called *production MFD* that we presented here, one can also look at the number of cars exiting a given perimeter as a function of the car density inside the perimeter. This also gives rise to a well-defined MFD, called in this case *outflow MFD*.

Both functions are equal in the case of a stationary traffic flow on all streets. However, real life traffic has a much more stochastic nature, such that the ratio of inbound to outbound vehicles in the

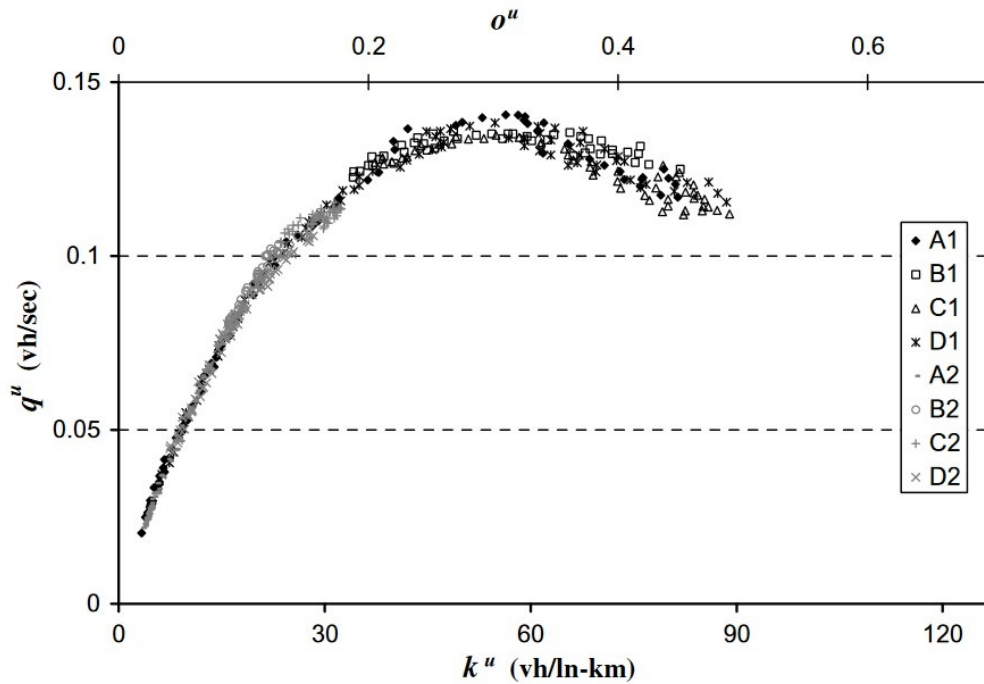


Figure IV.1: Historical result by Daganzo and Geroliminis. Empirically observed Macroscopic Fundamental Diagram of Yokohama, Japan. Each point is a measure of density (x-axis) and traffic (y-axis) averaged over all links of the network. The different types of markers represent different time periods (morning, evening, weekday, weekend), but remarkably all fall on the same well defined relationship, proving the insensitivity to the Origin-Destination Matrix. *Credits to Geroliminis and Daganzo [198]*

perimeter usually fluctuates with time. As a result, both MFDs are in general not completely equivalent. In all the following, we will implicitly refer to the production MFD when we mention the term MFD.

8.1.2 Critical parameters and traffic regimes

Just like the fundamental diagram, the MFD is concave. This means that the network has a maximal capacity, reached for a unique and well defined critical average density. These critical parameters, which we will call q^* and k^* respectively, separate the MFD into two regimes, free-flow for average densities below k^* and generalized congestion for densities above k^* .

8.2 Theoretical framework

8.2.1 Choosing the right average

So far, we have only alluded to *average* values of density and flux. However, as the reader might have noticed, there are several ways one can define those averages.

Let us consider a network composed of N links denoted by the subscript i . Each link has a length ℓ_i , a traffic q_i and a density k_i . Assume that the urban area corresponding to the network has an area \mathcal{A} . One can then think of at least three ways to define the MFD:

- **Unweighted average** : One simply performs an average over the number of links, i.e.

$$k^u = \frac{\sum_i k_i}{N}, q^u = \frac{\sum_i q_i}{N}$$

- **Weighted average** : The average is weighted by the length of the links, i.e.

$$k^w = \frac{\sum_i k_i \ell_i}{\sum_i \ell_i}, q^w = \frac{\sum_i q_i \ell_i}{\sum_i \ell_i}$$

- **Surface average** : One can also look at the average density and traffic per unit surface,

$$k^s = \frac{\sum_i k_i}{\mathcal{A}}, q^s = \frac{\sum_i q_i}{\mathcal{A}}$$

Note that one could also opt for a description without normalization, where one would simply express the sum of traffics and of densities on the network. While this might be useful for a given city to track the evolution of traffic production, normalizing is useful if one wants to compare cities with one another.

All those descriptions lead to the existence of an MFD, as they are fundamentally equivalent. However, their use-case and interpretation varies. It is important to keep the existence of these various definitions in mind, as there is not yet a consensus on which one to choose when speaking about the MFD. In order to compare results from article to article or from city to city, an harmonization is thus sometimes necessary.

8.2.2 Conditions of existence and estimation methods

Data acquisition

As MFDs were studied in different cities around the world, it rapidly appeared that the relationship between average traffic and density could sometimes be quite scattered. Buisson and Ladier. [199] showed that the traffic measures used to derive the MFD has to be homogeneous for the MFD to be meaningful. They focused on the case of Loop Detector Data (see chapter 1 for more details), and showed that the detectors should be placed similarly on each road with regard to the intersections. More recent research has shown that the influence of heterogeneity in the LDD placement is hard to anticipate, as it strongly depends on other properties of the network [73].

Later, Leclercq et al. [200, 201] showed that both Loop Detector Data and Floating Car Data used alone led to significant errors in the observed MFD, mainly for the reasons described in the first chapter : LDD is likely to skew density and speed measures, while the extrapolation of FCD data to the whole network requires strong assumptions on the penetration rate of the cars into the network.

Recent research on the subject agrees on the fact that a mixture of both LDD and FCD leads to the most precise observation of the MFD [68].

Influence of the traffic demand structure

Additionally, Geroliminis and Sun [202] showed that the claim of a network wide MFD might be a bit too ambitious : for the MFD to be well defined, one should ideally use a sub-part of the urban network on which the levels of accumulation are relatively homogeneous. In particular, heterogeneity in the density can lead to the emergence of hysteresis loops in the MFD, meaning that the relationship between density and flux is not well defined anymore. Gayah, Daganzo and Gonzales [203, 204] proposed a mechanism for the formation of hysteresis loops based on the asymmetry between the loading and unloading speed of respectively uncongested and congested roads. Subsequently, [205, 206] showed that the observed hysteresis depends on the way the network is loaded, which is sensitive to the Origin-Destination matrix.

It should thus be noted that two different types of objects are described by the same term MFD. On one hand, the response of the network to traffic in *steady-state* gives an Origin-Destination independent relation. This relation is often referred to as the MFD, but is in reality an upper-bound for the *dynamical response* of the network to a load, which can be time dependent and sensitive to the OD-matrix. We will start by describing some methods that have been proposed to estimate the “steady-state” MFD, but throughout this chapter, we will also come back to the impact of heterogeneity of traffic density on the MFD and the interpretations of the observed hysteresis.

Additionally, it has recently been shown that the specific use of intersections by the road users can greatly affect the MFD. In particular, intersections with high turning rates have a negative impact, noticeable at a macroscopic scale [185].

8.2.3 Reservoir based models and clustering methods

Single-reservoir model

Daganzo [197] first introduced the model of a single-reservoir, even before empirically observing network-wide MFDs. In a nutshell, the single-reservoir model is some sort of generalization of the fundamental diagram to the case where cars can enter and exit a road-segment exogenously (i.e. coming from the previous street), but also endogenously (i.e. starting or stopping their trip inside the road). This refinement is necessary for the urban context, as trips starting and stopping inside of a road rather than at an intersection are usual. Under the assumption that, at the scale of the reservoir, variations of density with time are slow enough that density can be considered homogeneous inside the

reservoir at any time¹, Daganzo proposed the response of the reservoir to a mixture of endogenous and exogenous demand. The name for this relationship is quite ambiguous : it is not what one would typically call a fundamental diagram, as it takes two inputs (traffic demand and current accumulation) and gives two outputs (traffic flow and updated accumulation), nor is it a macroscopic fundamental diagram. Because of its nature, we will in the following refer to it as the transfer function associated to the reservoir, but note that this is not the conventional denomination.

Daganzo then notices that the notion of reservoir can actually be extended beyond a single-road : provided that the homogeneity of density is respected, multiple roads can be assembled into one larger reservoir and will still display some sort of transfer function. The geometry of the network inside the reservoir plays a significant role. In particular, Daganzo shows that “directional” reservoirs (i.e. reservoirs with an entry, an exit and a clear direction for the traffic flow) are stable systems : if congestion emerges because of exogenous parameters such as a traffic jam downstream, it will resolve naturally when the jam disappears. Closed loops, however, exhibit a problematic behavior. Indeed, once this reservoir reaches a saturated state, it does not automatically go back to free-flow.

Daganzo then notices that such configurations are in fact extremely common in urban networks. For instance, four intersections around a square have the potential to be completely gridlocked precisely because they form a closed loop. Fortunately, he proposes a solution to this problem : by controlling the access to the reservoir, based on its current state, one can make sure that the reservoir never reaches the tipping-point for gridlock. We will come back to this strategy in the paragraph about the applications of the MFD.

Single-reservoir and MFD

Let us clarify the relationship between the single-reservoir model and the MFD. First, as we have said, Daganzo showed that a reservoir can be larger than a street, provided that the traffic density is homogeneous inside the reservoir. The reservoir then is defined by some sort of transfer function, that can in theory be derived from the fundamental diagrams of the streets composing the reservoir and their geometry. We say in theory, because as already evident in the seminal paper [197], this is actually only possible for simple geometries.

Because this transfer function, among other things, relates accumulation to traffic flow, it is clear that it is related to the notion of macroscopic fundamental diagram. Different methods can be used to derive the MFD of a reservoir, knowing its transfer function. We will not detail these methods here and redirect the reader towards the article by Mariotte et al. [207], which proposes an analytical and

¹The main idea behind this assumption is that one should not simultaneously have congested and uncongested segments in the reservoir

numerical comparison of different estimation methods, and the more recent work by J.A. Laval [208]. Note, however, that this step also requires some kind of knowledge about the network and the drivers, such as the temporal profile of traffic demand or the average trip-length.

Clustering methods

The reservoir based model has its merit as the first theoretical framework to understand the existence of the MFD. In particular, it justifies the need to cluster the network into sub-graphs of similarly congested roads, on which the MFD will be well-defined. As the criterion to form clusters is based on the level of car density, which should be similar for roads within each cluster, the optimal clustering is *a priori* varying in time, as the spatial distribution of car density on the network is time-dependent. Efforts have thus been made to develop dynamical clustering methods [209, 210] which take congestion propagation into account.

8.2.4 Understanding the multivaluedness of the MFD

Impact of density heterogeneity on the MFD

As we have discussed, the inclusion of roads with vastly different degrees of congestion in the same calculation of average density and traffic can lead to multivaluedness and hysteresis phenomena in the MFD. A simple explanation for this observation has been provided by Knoop et al. [211]. Consider a collection of links sharing the same triangular fundamental diagram defined by a free-flow speed u_f , a wave-speed w and a jamming density κ , such that for each link:

$$q_i(k_i) = \min(u_f k_i, (\kappa - k_i)w)$$

Assume now that the average density on the network (which the authors call accumulation) is given by A and that the link densities k_i follow a distribution² of average A and standard deviation σ . They then look at the average traffic on the network (which they call production and denote P) as a function of both A , which would be the traditional MFD and σ . The resulting function, which they call Generalized Macroscopic Fundamental Diagram (GMFD) is reproduced in figure IV.2. It shows that, for a given average accumulation, the heterogeneity among links negatively affects the network production. The derivation of this GMFD and discussions about its future refinements will be provided in the section 10.

²In their article [211], the authors use a uniform distribution to simplify calculations, but any distribution would give similar results

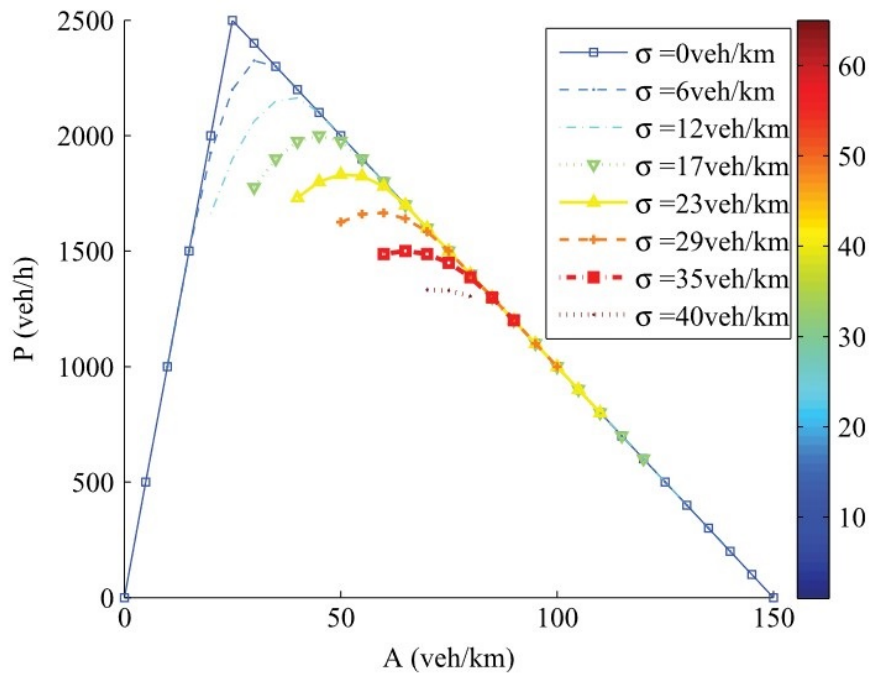


Figure IV.2: Production as a function of average accumulation and its standard deviation in the toy model proposed by [211]. For a given average accumulation, the heterogeneity among links negatively affects the network production. *Credits to Knoop et al. [211]*

Asymmetry in the loading and unloading speed, hysteresis

Having established that the MFD can indeed *a priori* be multivalued if we take inhomogeneous links into account, the question remains as to whether this situation actually happens in real life. Indeed, one could imagine two possibilities :

1. That all links of the network naturally tend to equilibrate their densities, such that the dispersion on the network is small. Of course, we know from the previous chapter that this is not the case.
2. That the observed heterogeneity is constant over time, i.e. that for a given value of average density, the whole distribution of densities on the network is fixed. Considering that, as we have seen in the previous chapter, the patterns for congestion seem to be relatively stable in time and to be reproduced nearly identically from day to day, this hypothesis would actually be quite plausible.

Gayah and Daganzo [203, 204] looked at the second hypothesis in the case of a toy model. They considered a two-reservoir model, i.e. an extension of the single-reservoir model presented earlier, where the two reservoirs can exchange vehicles. We will redirect the reader to the original paper [203] for the calculation and only present the essence of their result. Solving the differential equations for the evolution of traffic and density (q_i, k_i) in each reservoir, they found two important results :

1. The perfect repartition ($k_1 = k_2$) is an unstable equilibrium, thus in real life heterogeneity naturally arises in the network
2. The dynamics of the system and in particular the ratio $\frac{k_1}{k_2}$ evolve differently during the loading and the unloading of the network and display hysteresis.

In other words, a simple toy model based on two reservoirs is enough to display hysteresis in the congestion distribution, which then translates into hysteresis in the MFD. The intuitive interpretation of this phenomenon in real world networks is explained for instance in [203, 211]. As the network is loaded, congestion starts to appear in some weak links of the network, but the increase of density is generalized on the network. The densities of all links are strongly correlated and the standard deviation of density is relatively small. However, once the congestion is formed in these specific links, it will last until the network is almost empty : as the network is unloaded, roads that are not too congested see their density decrease, while congested links remain congested, with high density. Thus, during the unloading, the standard deviation of density is much higher and the network is less efficient for a given average accumulation.

Time dependent traffic demand leads to hysteresis in single reservoirs

Other hypotheses to explain the observed hysteresis of the MFD are currently investigated. In particular, Yuan and Knoop [212] recently suggested that the hysteresis of the MFD could also be traced back to the choice of the hour of departure of the road users. In a nutshell, road users will tend to avoid leaving during the worst part of the rush hour, when congestion is highest.

Interestingly, the authors consider a single reservoir, which is interesting, because it means that the actual details of the network do not play a role in the appearance and shape of hysteresis patterns in the MFD, while the demand profile does very clearly influence the shape and magnitude of the hysteresis loops. The article also explains why in empirical MFDs, the congested branch is never observed : people prefer to delay their departure rather than experience really high and generalized levels of congestion.

8.2.5 Derivation of a functional form for the MFD

Simple fluid model

Recall from the previous chapter that we introduced a recent article by J.A. Laval [192], in which the author proposed an analogy between the coexistence of congested and free-flowing links in the network and the coexistence of a liquid and a gaseous phase at the phase transition of a simple fluid.

If we denote by θ and θ_c the temperature and the critical temperature, the proportion of matter ρ in one of the phases (liquid or gas), is known to vary as:

$$|\rho - \rho_c| \propto \left(1 - \frac{\theta}{\theta_c}\right)^\beta \quad (\text{IV.1})$$

where β is an exponent which defines the *universality class* of the phenomenon. Typically, for simple fluids, one observes consistently $1/\beta \approx 3$.

The promising results of [192] suggest that the MFD might be the result of the coexistence between two phases of link-states (free-flow/congested), thus of the form :

$$q(k) = C \left[\frac{\beta}{2} \left(1 - |1 - 2k|^{1/\beta}\right) \right] \quad (\text{IV.2})$$

where C is a prefactor that depends on the network. Further studies are needed to confirm this result and find the value of β more precisely.

However, the first results indicate that $1/\beta \approx 1.715$ is plausible, which would mean that the evolution of congestion on an urban network belongs to the same universality class as directed percolation phenomena. This result is coherent with results introduced in the previous chapter, but also very exciting and promising.

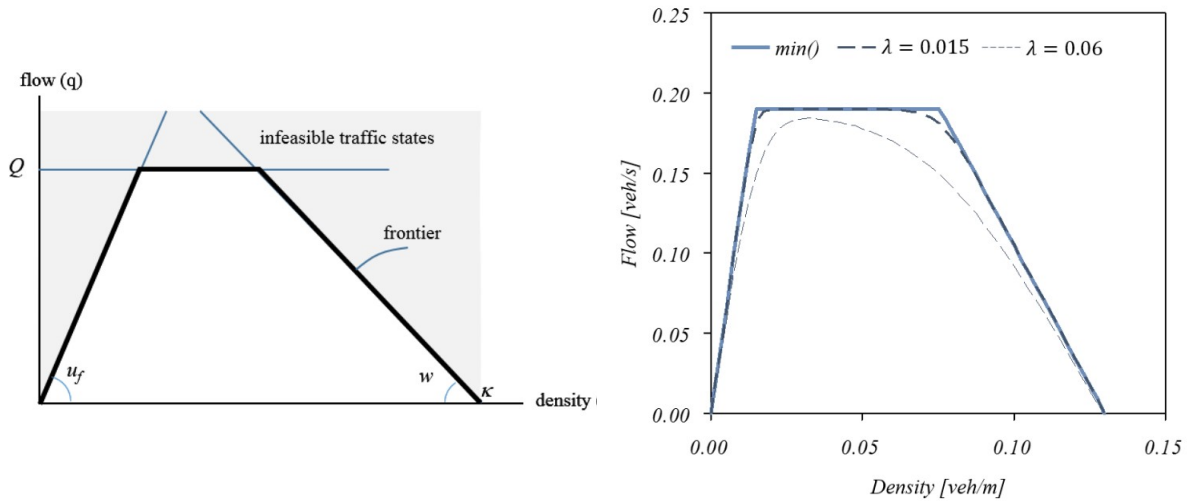


Figure IV.3: Left : Three “cuts” (in blue) defining the upper bound for the MFD, as introduced by [213]. Right : Functional form of the MFD proposed by [214] based on the cut-method. *Credits to Ambuhl et al. [214]*

Tight cuts based methods

As we have said, single-reservoir models give an insight into the existence of a macroscopic relationship between accumulation and traffic flow. However, even under the assumption of a homogeneous car density, the derivation of the MFD of a given network is not a simple task. Indeed, the MFD will be at least constrained by the roads fundamental diagrams, the network design and the signaled intersections. A method proposed by Daganzo and Geroliminis [213] was then to use the following assumptions to give upper bounds to the MFD :

- As the road level diagrams are concave, the MFD should be concave (which also matches empirical observations)
- Free-flow speed at the macroscopic scale should be the same as at the microscopic scale. Indeed, at low average density, all links are in free-flow.
- Conversely, at high densities, the MFD should be constrained by the congested branches of the individual fundamental diagrams
- The maximal flow on the network is constrained by the throughput of the intersections, which is determined by the traffic lights and can be calculated

From these assumptions, they can give a family of 3 “cuts” which will be upper bounds to the concave MFD³. More precisely, if the individual links of the network have an identical triangular fundamental diagram defined by a free-flow speed u_f , a wave-speed w and a jamming density κ , and if the maximal flow supported by each intersection is Q , then the MFD is bound by the upper-envelope (illustrated in figure IV.3) :

$$q(k) = \min(u_f k, Q, (\kappa - k)w) \quad (\text{IV.3})$$

A natural way to fit a function knowing its value and derivative in 3 points is to use a polynomial, which is for instance proposed in [215]. However, this poses the issue of the physical sense of the 3 to 6 parameters that one introduces (depending on how rigidly one wants to fit the cuts). Ambühl et al. therefore proposed a functional form for the MFD based on this cut method and using only one additional free parameter λ . This smoothing parameter measures how close the actual MFD is to the upper-bound (which would be the optimal performance for the network), as illustrated in figure IV.3.

³The paper by Daganzo and Geroliminis [213] actually proposed more involved arguments to find, in theory, as much tight cuts as one wants. In practice however, the 3 cuts described here are the easiest to determine.

The functional form then writes⁴ :

$$q(k) = -\lambda \ln \left(\exp \left(-\frac{u_f k}{\lambda} \right) + \exp \left(-\frac{Q}{\lambda} \right) + \exp \left(-\frac{(\kappa - k)w}{\lambda} \right) \right) \quad (\text{IV.4})$$

This functional form satisfyingly reproduces real-world MFDs of different cities (with a few exceptions), and is thus a promising tool for real-world applications, as we should detail in the following paragraph. It is also of high value for our theoretical understanding of the MFD, because it shows that the elementary constituents of the networks, namely the streets and intersections, determine most of the MFD. However, it poses a few questions that we shall keep in mind for the rest of this chapter, of which we give a short list :

- Real-world MFDs are well defined by this functional form. However, as we know, the diagrams of the streets are not triangular and certainly not identical. How do the macroscopic parameters u_f , w , κ relate to the distribution of their link-level counterparts ?
- If λ is some sort of measure of the efficiency of the network, what are the parameters that define it ? Is it the dispersion of the parameters of the individual links, the network's topology, the structure of the traffic demand ?
- Can this description be completed to describe the multivaluedness and the hysteresis observed in real-world MFDs ?

Note that the article [214] uses arguments related to the “microscopic” structure of the network to derive the functional form of the MFD, but that the actual extraction of the MFD is made with empirical data. More recent work has focused on semi-analytical methods which try to extrapolate the MFD from the knowledge of some fundamental diagrams, based on the tight cut method [216]. In particular, it confirms that the MFD can not solely be anticipated based on the diagrams of the streets, as the traffic demand patterns also play an important role.

Taking congestion dispersion into account in a Generalized MFD (GMFD)

As we have said, the standard deviation of congestion also seems to play a role in the value of the average network traffic. Different models have been proposed, but the literature is relatively scarce. What is clear is that, at fixed average density, average traffic is a decreasing function of density dispersion. Authors of [215] proposed a GMFD of the form :

$$q(k) = \Psi(k) \exp \left(-\frac{\sigma_k}{\sigma_0} \right) \quad (\text{IV.5})$$

⁴Note that the limit of equation IV.4 as $\lambda \rightarrow 0$ is equation IV.3

Where Ψ is a polynomial dependence of traffic to density and σ_k is the standard deviation of the density on the links.

However, as evident from figure IV.2, the influence of σ_k on the traffic q depends on the average density k in a non-linear and non-trivial manner (i.e., in equation IV.5, the normalization term σ_0 should be a function of k). An elegant functional form relating q , k , σ_k has not yet been proposed, but recent empirical results [210] give new elements to work towards a satisfying model.

8.3 Applications

As we have said, at the scale of a single-reservoir, the presence of closed loops can mean that the reservoir can be stuck in a stable state of congestion (gridlock) [213]. To avoid this, strategies of perimeter control have been developed. The idea is to dynamically limit the number of cars entering a given perimeter, in order to ensure that the reservoir always stays in a free-flow state [217, 218, 219, 220, 221]. Some cities like London have already implemented this kind of strategies to decongest their centers with the use of a toll to access the city-center during working days.

One can even think of more general and ambitious strategies for traffic control. Having established that the dispersion of the congestion levels is detrimental to the macroscopic efficiency of the network, one can imagine traffic assignment methods that would minimize this dispersion and/or segment the city into multiple reservoirs and try to maintain each reservoir at a high efficiency level [222, 223, 224].

For each of these application, a fine real-time estimation of the state of traffic is necessary, as one would want to have each reservoir operating close to its maximal capacity, but without reaching its congested state. This highlights the need for reliable estimation methods which we discussed before.

Finally, a last application of the MFD can be the validation of new simulation methods. Indeed, as we have mentioned in the introduction chapter, simulations of traffic often struggle to be realistic both at a local and at a macroscopic scale. In particular, one can tune the parameters of the simulation as to match observation in some streets, but using all real-world traffic measurements of all the streets would lead to an over-fit of the simulation. Using the MFD as a validation of the simulation can thus be a practical tool : the simulation can now be calibrated to match a few local observations as well as an aggregated macroscopic one [225].

8.4 Open questions and outline of the chapter

Let us conclude this section by summing up open questions related to the MFD that we have presented so far.

An important question that we have only touched upon is the relationship between network structure

and shape of the MFD. At the reservoir level, Daganzo et al. gave some arguments to show that the MFD existed. However, the relationship between network structure and MFD remained unclear until first empirical results by Loder et al. [226]. The next section will thus deal with this problematic, as we will highlight relationships between the macroscopic network properties and the critical parameters of the MFD.

Additionally, we saw that congestion emerges on some weak links of the network, before spreading to the rest of the network, where the first mechanism is certainly influenced by the local structure of the network. We thus will discuss other metrics that could be used to better describe the network at all scales.

Another important point is how to unify the MFD with the spatio-temporal dynamics of traffic on the network that we presented extensively in the previous chapter. In particular, as we have seen, the MFD is sensitive to the inhomogeneity of traffic congestion. As it happens, we have given several results on that matter in the previous chapter and one might wonder how they can be applied in the framework of the MFD. We will present perspectives for future work in this direction in section 10.

9 Structure of road networks and the shape of the Macroscopic Fundamental Diagram

This section will concentrate on the first question that we identified regarding the MFD, which is the relationship between network structure and shape of the MFD. More precisely, using a cellular automaton simulation and a toy model for urban networks, we will identify the impact of network parameters on the critical parameters of the MFD. Results presented in this section have been published in [227].

9.1 Method

9.1.1 The Chowdhury-Schadschneider model for car progression

We simulate the traffic with the ChSch model [112, 228] which was introduced as an extension to urban road networks of the NaSch model [105] for highway traffic. It allows to study the interplay between vehicle dynamics and interactions induced by intersections controlled by traffic lights [112, 115].

The model

We consider a regular 2D-lattice of roads and for the sake of simplicity, we consider cars moving from West to East and from South to North only. On each road, the cars move by following the rules of the ChSch model that we shall describe briefly⁵.

The road is divided into discrete cells and time is also discretized. Each cell can be either empty or occupied by a vehicle with a velocity v that can take integer values in $[0, v_{max}]$ where v_{max} is the maximum speed allowed on the road.

At each time step τ , the speed v_i of each car is updated in parallel following, if the traffic light in front is green:

$$v_i(t + \tau) = \max(v_{max}, v_i(t) + 1, d_{i,i+1}(t) - 1)$$

where v_i is the speed of the car i , $v_{max} = 5$ cells/timestep and $d_{i,i+1}$ is the number of cells separating the car i from its predecessor.

If the traffic light in front of car i is red, this rule changes to:

$$v_i(t + \tau) = \max(v_{max}, v_i(t) + 1, d_{i,i+1}(t) - 1, d_{i,tl}(t))$$

where $d_{i,tl}(t)$ is the distance to the traffic light.

⁵for more details on this models, see for example [228]

Notice in particular the following:

- The speed of the car i only depends on the position of car $i + 1$, not on its speed.
- Cars accelerate at most by 1 cell/ τ^2 , but can decelerate by up to 5 cells/ τ^2 if needed.
- Contrary to the original paper, we do not introduce a probability for cars to randomly decelerate.

Once all the speeds are determined, the cars progress together by a discrete number of simulation cells, the new positions of car i being:

$$d_i(t + \tau) = d_i(t) + v_i(t + \tau)$$

Historical results of the ChSch model

The ChSch model and the NaSch model of which it is a derivation are both microscopic approaches to traffic progression. We have described the attributes and some benefits of this approach in the first chapter of the thesis, but we will briefly come back to some interesting results found using these models. The historical NaSch model [105] was introduced to model highway traffic and reproduced its main attributes, mainly the transition between a free-flow state and a congested regime with shock-waves. The model was adapted to study the impact of traffic lights in the urban context [112, 115]. This gave a theoretical framework to the development and generalization of synchronized traffic lights in cities. Additionally, refinements were introduced to study the dynamics of merging traffic [111, 229] and to further characterize the phase transitions which the system undergoes as congestion emerges at the road level. In particular, it was shown that the congested regime is actually composed of different phases as far as the microscopic nature of the congestion is concerned, such as the distribution of distances between cars [113, 229], or the probability for cars to be distributed in a given manner on the street [230, 231].

9.1.2 Network model

We have described the original ChSch model which considered a regular 2D-lattice. Real world road networks, however, display some level of irregularity as we have discussed in the first chapter of this thesis. In particular, there is quite an important difference in the average degree of intersections between cities. To account for this parameter of urban road networks, we introduce a probability p for each link of the lattice to be removed. Starting from a regular 2D-lattice, we can transform the network into a less regular network, with intersections of degree 3 and 4 (see figure IV.4). Additionally, in contrast with the original model [112], at each intersection, every car picks a new direction (east or north) at random and continues its progression.

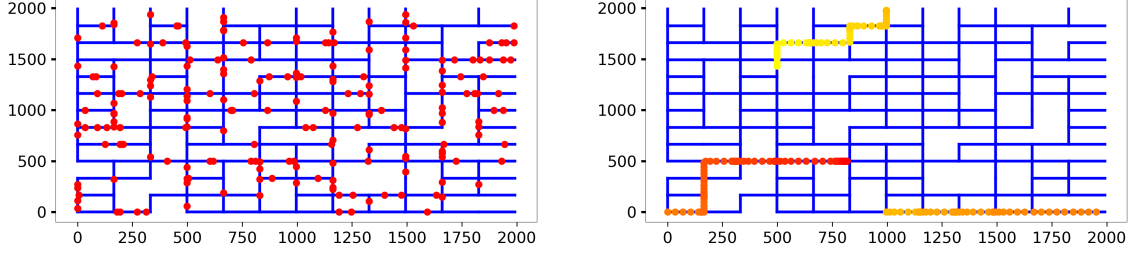


Figure IV.4: Illustration of our simulation for a network with $p = 0.2$, with 13×13 nodes and roads of length $166m$ (distances are indicated in meters). We use periodic boundary conditions. (Left) Initial time: $N = 200$ cars located at random in the network. (Right) Trajectory of a car during 100 consecutive time-steps (time goes from yellow to red)

Variable	Description	Unit	Default value
q^*	Network capacity	$km/h/km^2$	
k^*	Critical accumulation	$cars/km^2$	
ρ_r	Density of roads in space	$lane \cdot km/km^2$	
ρ_i	Density of intersections in space	km^{-2}	
p	Fraction of links missing compared to regular lattice	Number $\in [0; 1]$	0
L_{car}	Length of our simulation cell	km	$7m$
τ	Discrete timestep of the simulation	h	2 seconds
v_{max}	Maximal speed on our simulated network	h^{-1} (cells/timestep)	$5 \text{ cells}/\tau$
t_{tl}	Traffic light timing (duration of a green or red light)	h	30 seconds
V_s	Average speed of the cars in our simulation	h^{-1} (cells/timestep)	
V_{lim}	Average speed on the empty network, combination of L_{car} , v_{max} and t_{tl}	km/h	

Table IV.1: Definition of the variables used in this section

Varying the network parameters independently

We use the network parameters described in Table IV.1. In real networks, the density of roads ρ_r and of intersections ρ_i can vary independently⁶, thanks to multi-lane roads, the sinuosity of roads, as well as the existence of bridges and tunnels (i.e. roads might physically cross without intersecting). In order to reproduce this in our simulation, we introduce a geometrical factor g linking the actual length of the roads L_{road} to the geometrical distance between intersections L_{grid} , such that $L_{road} = gL_{grid}$. Indeed, if we only considered a 2D lattice, we would have only one parameter L_{grid} determining $\rho_r = \frac{2}{L_{grid}}$ and $\rho_i = \frac{1}{L_{grid}^2}$.

Instead in our model we then have $\rho_r = \frac{2g(1-p)}{L_{grid}}$ and $\rho_i = \frac{1}{L_{grid}^2}$, for the density of roads and intersection, respectively, which we can indeed vary independently.

Note that the influence of p is twofold : there is an implicit dependence in ρ_r and an explicit one that we can explore at fixed ρ_r .

⁶Even though they are, of course, strongly correlated

Note on the relation between car length and simulation speed

By design, cars travel an integer number of cells per timestep. This means that, when we vary the length of the cars, we also vary their speed. More precisely:

$$V = 2V_s L_{car}$$

with V the real speed (m/s), as used throughout this section, and V_s the simulation speed (cells/timestep). In all the following, we plot the data collapse $q^* L_{car} / \rho_r$ versus $\rho_r / L_{car} \rho_i$. To achieve this in practice, we actually plot q^* / ρ_r versus $\rho_r / L_{car} \rho_i$.

9.1.3 Derivation of the MFD

Each point of the MFD is obtained by averaging the density of cars and the flux on the network over a given time span. The choice of this time span is important in real-world network. It should be longer than the typical duration of green light cycles, as we are interested in the average state of traffic rather than in its high frequency variations. On the other hand, averaging over too long a period means that some cars will enter or exit the network, resulting in a biased estimation of the state of the traffic. This problem is also well-known for the road fundamental diagrams, where the measured diagram is always smaller than the "real" diagram.

The way we obtain the MFD is the following:

1. We pick a network corresponding to the desired values of the parameters
2. We pick a number of cars and add them randomly to the network. This stage determines the car density k (ranging from 1% to 50% of the maximal number of cars one could fit on the network, i.e. values of density respectively far smaller and larger than the MFDs maximum).
3. We let the network evolve during 500 time steps towards its equilibrium
4. We let the network evolve during another 500 times steps and estimate the average flow q during this period
5. We repeat steps 3 and 4 for the same network but with an increasing number of cars.

Hence, for each estimation of the flux, the density k is fixed (unlike for real life measurements). This allows us to estimate the flow over a longer period of time than what would typically be done on real networks.

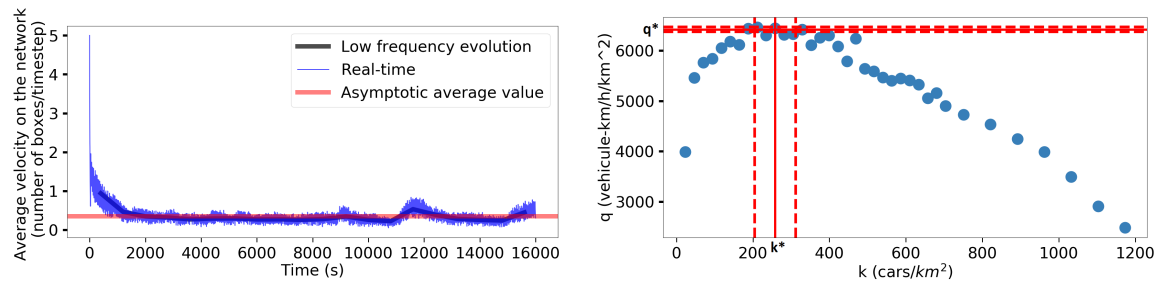


Figure IV.5: Left : Evolution of the average velocity on the network with time. The density on the network is close to k^* . Starting from a random configuration, the average speed decreases as traffic jams emerge during the first 1000 seconds. The network then remains in a steady state with relatively constant average velocity. Right : Corresponding MFD obtained over 500 timesteps. We show the values of k^* and q^* with the corresponding confidence intervals (red dashed lines). The shape of the MFD implies in general that the estimate of k^* is noisier than the one for q^* .

Step 3 is necessary to have a realistic MFD. Indeed, on real networks, cars are never randomly distributed. We determine the time needed to reach the equilibrium using the average velocity. Starting from our random initial configuration, we measure the average velocity on the network as a function of time. We observe a decrease of the velocity as cars regroup in traffic jams in specific points of the network. The choice of 500 time steps (or 1,000 seconds) corresponds to a duration which guarantees that the network has reached a steady state (average velocity is constant, see figure IV.5).

Step 4 also needs some clarification. The average flow can be estimated with two methods, which are subject to debate amongst traffic engineers. Remembering that the average flow is expressed as a number of traveled kilometers during a period of time and per unit surface, one can either sum the speeds of the cars at each time step, average this sum over all time steps and finally divide by the network surface.

Alternatively, one can measure the number of cars exiting a road at each time step and consider that this car covered the length of the road during that time step. Summing these distances over all time steps, dividing by the duration of the measurement (here 500 time steps correspond to 1,000 seconds) and dividing by the network surface, one gets another estimation of the flux.

The former method corresponds to real life data provided by on-board speed measurements (Floating Car Data, FCD), while the latter corresponds to measurements obtained with fixed loop detectors on the road (Loop Detector Data, LDD). As we discussed in the previous section, whilst one might intuitively think that both methods are equivalent, they are not and the resulting MFD depends on the chosen method. The choice of a method, or a mix between both of them is not settled among the traffic engineer community. In our case, because our network is in steady-state (as the number of cars is fixed), both methods are indeed equivalent and in practice, we chose to emulate LDD results.

9.2 Previous empirical results

A few articles have addressed the question of the influence of road layout on the macroscopic fundamental diagram. Indeed, as the MFD started to be studied in more and more cities, it became clear that its shape and its critical parameters vary between cities. Knoop et al. [232] first proposed simulated networks to study their MFD. This confirmed that the MFD does depend on network structure and also confirmed that, much like the fundamental diagram, there is no unique functional form capturing the shape of every possible MFD. They didn't systematically explore the impact of network parameters, but concentrated on the influence of arterial roads⁷. They found that the addition of such roads surprisingly decreases the capacity of the network, while also introducing multivaluedness in the congested branch of the MFD. Their hypothesis to explain these results are a combination of Braess's paradox and the impact of heterogeneity on the MFD discussed previously. The addition of an arterial road leads to a socially suboptimal solution where some roads are underused and others are saturated. This heterogeneous distribution of traffic reduces the total flux, while introducing hysteresis patterns in the MFD during the loading and unloading of the network.

More recently, Loder et al. [226, 233], Aghamohammadi and Laval [234] and Wong et al. [235] used real-world data on a large set of cities to find correlations between network topology and critical parameters of the MFD. Using the notations that we just introduced, the empirical results can be summed up as follows:

- Positive but sub-linear relationship between ρ_r and k^* [226]
- Some sort of negative influence of ρ_i on k^* [226] and v^* [235], but the exact scaling is not clear
- Some influence of the network's topology, studied in terms of average betweenness centrality [226], average degree of the intersections or "integration⁸" [235]. The influence of these quantities on k^* , q^* is unclear, but more redundant network (i.e. high average betweenness or closeness centrality) surprisingly seem to have slightly lower values of k^* , q^*
- An apparent proportionality between k^* and q^* [226]
- A linear decrease of q^* when the proportion of buses amongst the traffic is increased [226]

We will now describe the results obtained using our simulation. As we will see, we reproduce some empirical results, contradict others and more generally are able to refine over these empirical studies. We will end this section by discussing how our results compare to the empirical work presented here and why we believe that our approach offers, in this context, important advantages.

⁷i.e. not highways, but still high capacity roads crossing cities in a straight line

⁸Which in its spirit is closely related to average closeness centrality

9.3 Scaling laws for the MFD's critical parameters

9.3.1 Dimensional analysis

In the spirit of Loder et al. [226], we define all our variables as averages per unit area: k^* is in veh/km², and q^* in cars·km/h/km². The network is defined by the fraction p of missing links, the density ρ_r of roads in space (with unit km/km²) and the density ρ_i of intersections in space (with unit km⁻²). The progression of cars using the ChSch model is defined by 4 parameters: the length L_{car} (km) of the discrete cells on which cars evolve, the time step on which we update the speeds and positions (taken equal to 2 seconds), the maximal speed of the cars v_{max} (expressed in our model as a number of steps per time step) and the duration of each green or red light phase, t_{tl} (in seconds).

Despite its simplicity, this model is complex enough to display a MFD, from which we extract q^* and k^* (see Fig. IV.5). We compute those critical parameters for various values of ρ_r , ρ_i , p , t_{tl} and L_{car} , running 100 simulations for each set of values to estimate the error. We will use these numerical simulations to identify scaling laws for the parameters k^* and q^* . We start with the most general form for these quantities that are

$$k^* = F(\rho_i, \rho_r, L_{car}, v_{max}, p, \dots) \quad (IV.6)$$

$$q^* = G(\rho_i, \rho_r, L_{car}, v_{max}, p, \dots) \quad (IV.7)$$

where F and G are unknown functions at this point. The critical accumulation k^* has the unit of ρ_r/L_{car} while the q^* has the unit of k^* times a velocity. We first define the adimensional quantity $n = \rho_r/L_{car}\rho_i$ which is the average number of cars which can be fitted on a road between two intersections of the network. Keeping the number of relevant parameters minimal, the natural scaling for k^* and q^* can then be written as

$$k^* = \frac{\rho_r}{L_{car}} \Phi(n, p, t_{tl}, v_{max}) \quad (IV.8)$$

$$q^* = \frac{\rho_r}{L_{car}} V^*(n, p, t_{tl}, v_{max}) \quad (IV.9)$$

where the functions Φ and V^* are at this stage also unknown. Φ has no dimension, while V^* is the average speed on the network at the critical point. We tested numerically these scaling forms Eqs. (IV.8), (IV.9) by plotting k^*L_{car}/ρ_r and q^*L_{car}/ρ_r versus $n = \rho_r/L_{car}\rho_i$ (at fixed p and t_{tl}). If they are correct, we should obtain a data collapse for various values of ρ_r , ρ_i and L_{car} . We show in Fig. IV.6 this data collapse (for $p = 0$) which confirms that our scaling assumptions are correct.

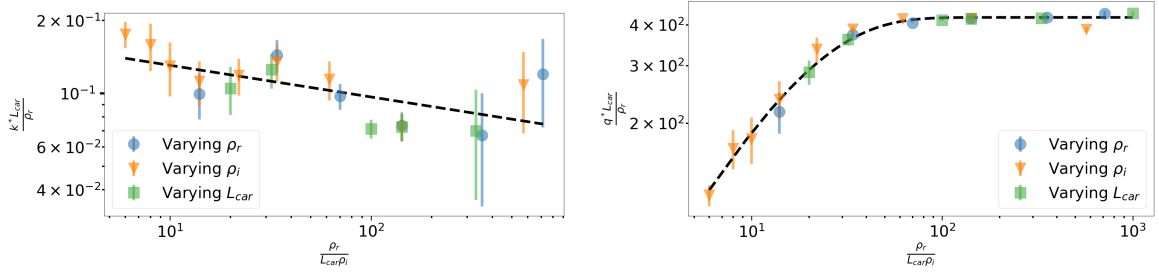


Figure IV.6: Test of the scaling *ansätze* Eqs. (V.3,IV.9). We plot the rescaled variables $k^* L_{car}/\rho_r$ and q^*/ρ_r versus the adimensional variable $n = \rho_r/(L_{car}\rho_i)$. In both cases we observe a good data collapse. (Left) For k^* , we observe a roughly constant function (the dashed line is a power law fit with exponent ≈ -0.1 , $R^2 \approx 0.5$). (Right) For q^* , we observe an increasing function followed by a plateau, which we fit with an exponential function ($R^2 = 0.98$). All figures were obtained using 100 random configurations of a network of 13x13 nodes, with $p = 0$.

9.3.2 Influence of v_{max}

We start by investigating the influence of v_{max} on k^* and q^* . We find no significant differences, with v_{max} ranging from 1 to 20 cells per time step (see figure IV.7). This is not surprising when one considers figure IV.5 showing that the average velocity V^* of cars on the network when close to the maximum capacity is of order 0.5 cells/time step.

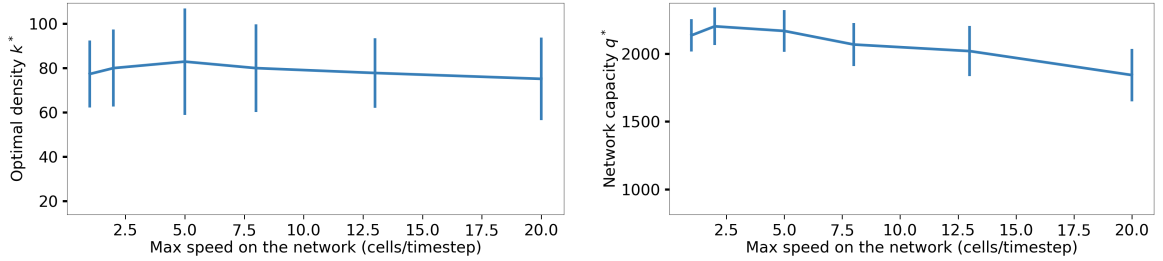


Figure IV.7: Influence of v_{max} on k^* and q^* . No significant impact on either quantity is observed.

9.3.3 Simulation results for k^*

We also test the dependence of k^* on the other parameters p , v_{max} and t_{tl} . In particular, there is no explicit dependence on p (only via ρ_r) and no dependence on v_{max} and t_{tl} . This leads us finally to the following scaling form for k^*

$$k^* \approx \frac{\rho_r}{L_{car}} \left(\frac{\rho_r}{L_{car}\rho_i} \right)^\beta \quad (\text{IV.10})$$

with $\beta \approx -0.1 \pm 0.1$ ($R^2 \approx 0.5$). The prefactor in this expression has a natural interpretation as it is the number of cars per unit area that can be fitted in the network.

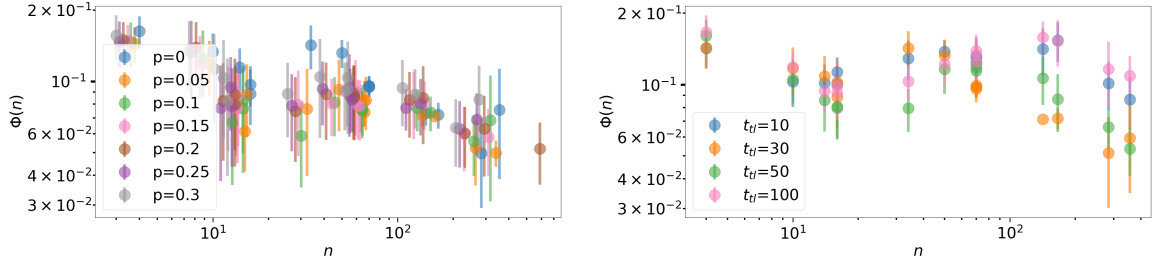


Figure IV.8: Data collapse for k^* for different values of p (at fixed $t_{tl} = 30$ seconds) and t_{tl} (at fixed $p = 0$). Neither p nor t_{tl} have a significant influence on Φ

9.3.4 Simulation results for q^*

First, we recall that in our simulation, the cars move by an integer number of cells per time step. Thus the physical speed is the product of the simulation speed v_s and the length of the cars, $V = L_{car}v_s$. We thus have:

$$q^* = \rho_r V^* \left(\frac{\rho_r}{L_{car}\rho_i}, p, t_{tl} \right) \quad (IV.11)$$

Fig. IV.6(b) shows that the scaling of q^* is more complicated and displays essentially two regimes. We use a simple *Ansatz* suggested by Fig. IV.6 of the form $V^*(n, p, t_{tl}) = V_{lim} (1 - e^{-n/n_c(p, t_{tl})})$ where $n_c(p, t_{tl})$ is a crossover value that depends *a priori* on p and t_{tl} . We verify that the general exponential form of V^* and the limit V_{lim} at large n are independent of p and t_{tl} by plotting the data collapse at fixed p and t_{tl} for various p and t_{tl} (Fig. IV.9):

$$\frac{q^*}{\rho_r} \approx V^* \left(\frac{\rho_r}{L_{car}\rho_i} \right) \quad (IV.12)$$

We will also plot $e^{-n/n_c(p, t_{tl})}$ as a function of $1 - \frac{V^*}{V_{lim}}$ for various p and t_{tl} and verify that the graphs collapse into one (Fig. IV.10).

Based on our results, we confirm the following fit for V^* :

$$V^*(n, p, t_{tl}) = V_{lim} \left(1 - e^{-n/n_c(p, t_{tl})} \right) \quad (IV.13)$$

where we recall that the quantity $n = \frac{\rho_r}{\rho_i L_{car}}$ corresponds to the number of cars that can fit on each road. n_c is thus a critical size of the roads and V_{lim} is a constant.

The shape of the function V^* deserves some discussion. The quantity n_c , that characterizes the

maximum number of cars that can be fitted on the network, separates two regimes for the function V^* . For $n > n_c$, V^* is essentially constant, meaning that q^* becomes proportional to ρ_r and does not depend on ρ_i . If the roads of the network are long enough, the MFD will then be governed by the roads themselves and the impact of intersections will be negligible. This also gives an interpretation for V_{lim} which is the average speed of a car traveling on the empty network but respecting all traffic lights. On the opposite side of the spectrum, if roads are short ($n < n_c$), the MFD will be mostly determined by intersections. Two mechanisms are at play here. On one hand, cars will spend a larger fraction of their time accelerating or decelerating, which is suboptimal, and on the other, short roads increase the likelihood of a spill-back of congestion into an intersection, leading to a gridlocked network leading to a decrease of q^* .

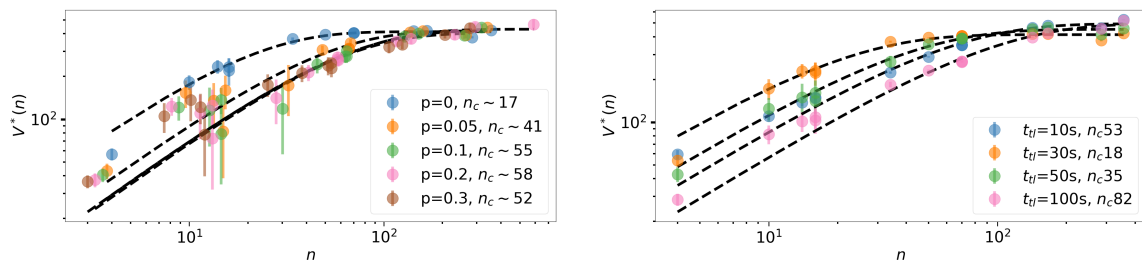


Figure IV.9: Data collapse for q^* for different values of p (at fixed $t_{tl} = 30$ seconds) and t_{tl} (at fixed $p = 0$) and corresponding value of n_c . All fits have $R^2 > 0.98$

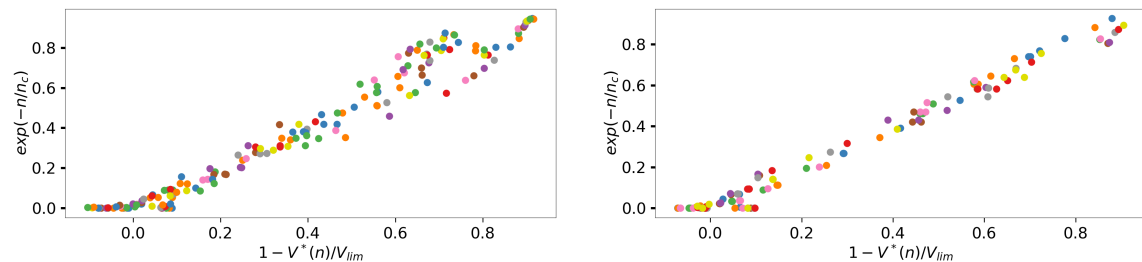


Figure IV.10: Collapse of the dependence of V^* to p (at fixed $t_{tl} = 30$ seconds) and t_{tl} (at fixed $p = 0$). Each color corresponds to a value of p (resp. t_{tl})

9.3.5 Influence of p and t_{tl} on n_c

For $p = 0$, the network is completely regular and, under our assumption of completely random route choice, there is no reason for the traffic to accumulate at a given place. The probability of a gridlocked intersection is small and even relatively short roads are sufficient to prevent spill-backs⁹. However, if one removes some links ($p > 0$), breaking this regularity, one introduces congestion hotspots into the

⁹This is some sort of reformulation of the results presented in [234], where the *critical block length*, i.e. the minimal length one should have for any road of the network, was identified as an important parameter of the MFD

network. More specifically, the absence of some links creates intersections where two roads merge into one. In order to be resilient against spill-backs, roads of the resulting network have to be at least twice as long, which is consistent with the observed increase of n_c from ≈ 20 to $n_c \approx 50$ for $p > 0$. We notice here that the transition occurs in the range $p \in [0; 0.1]$. For larger values of p , n_c remains constant, which is consistent with the proposed mechanism. Note that this discussion implies that the global efficiency of the network, of which q^* is a measure, is ultimately defined by the few (local) weakest points of the network. Creating only few hotspots of congestion is all it takes to reduce the macroscopic capacity of the network by up to a factor 3.

The variations of n_c with t_{tl} , displayed in Fig. IV.11 are consistent with previous results such as those obtained in the original study by Chowdhury and Schadschneider [112]. The traffic flow on the network is maximized for a correctly chosen value of the traffic light duration. With our variables, this translates into a minimum for $n_c(t_{tl})$, reached for $t_{tl} \approx 30$ seconds. The reason for this optimum is a competition between two optimization problems. On one hand, cars need to accelerate from a standstill. Increasing t_{tl} increases the average speed of cars passing the intersection during the green phase, thus increasing the flow. If t_{tl} is too large, one reaches a point where few cars pass the intersection and a large number of cars wait at the red light of the intersecting road. Our optimal value of $t_{tl} \approx 30$ seconds is in good agreement with green light duration chosen in typical urban networks.

This discussion is well known and can be found in more details in traffic engineering handbooks such as [190], but we will illustrate it here in the context of our toy model.

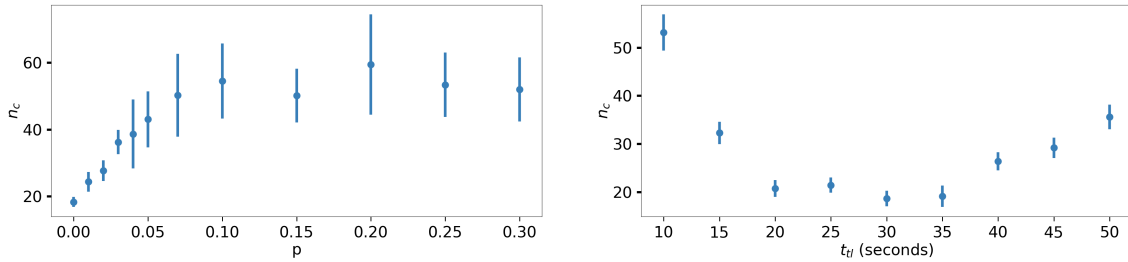


Figure IV.11: Variation of n_c (left) with p (at fixed $t_{tl} = 30$ seconds) and (right) with t_{tl} (at fixed $p = 0$).

Explicit calculation of the number of cars exiting an intersection

Consider the case where cars wait at a red light (all cars have a null speed), densely packed to each other (no empty cell in between). Assume that the roads exiting the intersection are completely empty (no influence on the exiting cars). Recall that the cars follow the ChSch rules, which means that their speed evolves at each time step as $v_i(t + \tau) = \max(v_{max}, v_i(t) + 1, d_{i,i+1}(t) - 1)$,

where v_i is the speed of the car i , $v_{max} = 5$ cells/timestep and $d_{i,i+1}$ is the number of cells

separating the car i from its predecessor. Notice in particular that the speed of the car i only depends on the position of car $i + 1$, not on its speed.

This means that in our queue of cars, when the light turns green at $t = 0$, the car i will only start to move at time step i . It is easy to note that, under our assumption of empty roads, the distance to the preceding car will not play a role in v_i and that the speed of each car will simply go from 0 to 5 in 5 time steps. Thus, the i -th car will reach speed $v = 1$ at time step i , and $v = 5$ at time step $i + 4$. After time step $i + 3$, it will be at distance $i - 10$ of the intersection (having covered $1+2+3+4$ cells in the 4 preceding time steps). Knowing that each time step is 2 seconds long, we can then find the number of cars $N(t_{tl})$ able to pass the green light during t_{tl} . If $t_{tl} \geq 10$ seconds, it satisfies:

$$\lfloor \frac{t_{tl}}{2} \rfloor = \frac{N - 10}{5} + N + 3$$

$$N(t_{tl}) = \lfloor \frac{5}{6} \left(\lfloor \frac{t_{tl}}{2} \rfloor - 1 \right) \rfloor \quad (\text{IV.14})$$

If $t_{tl} < 10s$, the situation is a bit more complex, as cars will pass the intersection with different speeds depending on t_{tl} . More precisely, car i will pass the intersection if and only if $\sum_1^{\lfloor \frac{t_{tl}}{2} \rfloor + 1 - i} j \geq i$.

By definition, $N(t_{tl})$ is the largest i satisfying this inequality, which gives: $N(2) = 1, N(4) = 1, N(6) = 2, N(8) = 3$ (and $N(10) = 3$, as does equation IV.14).

We represent equation IV.14, as well as the rate at which cars exit the intersection $N(t_{tl})/t_{tl}$ in figure IV.12. We observe that for values larger than ≈ 25 seconds, t_{tl} only has a marginal influence on the rate of cars leaving the road. This is coherent with an optimal value of t_{tl} close to 30 seconds.

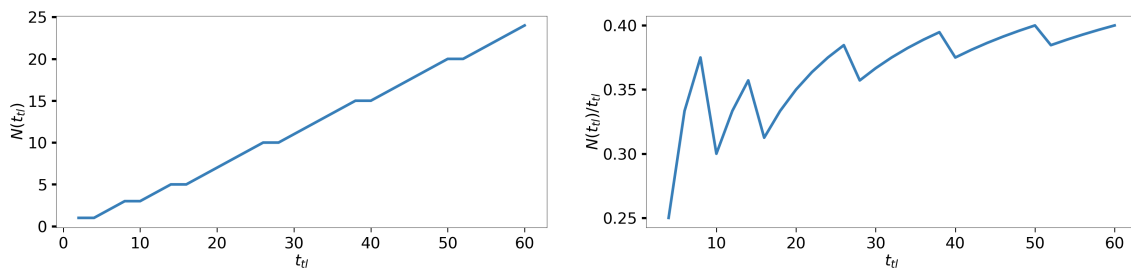


Figure IV.12: Number of cars able to pass an intersection as a function of traffic light duration (equation IV.14) and ratio $N(t_{tl})/t_{tl}$

Note that in equation IV.14, we can be more general and get the influence of v_{max} . If we are in the described situation where cars start from a standstill and the queue is densely packed, the number of cars passing the intersection varies as $\frac{v_{max}}{v_{max}+1}$ (instead of $\frac{5}{6}$), showing the limited impact of this parameter (as observed in figure IV.7).

9.4 Discussion

9.4.1 Comparison with empirical results

Let us come back to the empirical results [226, 234, 235] that we presented earlier and see how they compare to our proposed scalings.

Influence of intersection density

Interestingly, we find no explicit influence of the intersection density ρ_i , which only intervenes through the distance between intersections, or in our discrete model the number of cars n fitting on each road. Note that this result is actually in good agreement with [235], which looked at different definitions to measure intersection density. The linear density of intersections (i.e. the inverse of the distance between intersections) appeared to be more correlated with k^* than area based ones (such as ρ_i). As we already mentioned, our discussion about the critical size of the roads is also in good agreement with [234].

Influence of the road density

In their study [226], Loder et al. found a power law of the form $k^* \approx \rho_r^\alpha$ with exponent $\alpha = 0.85$ and rejected the assumption of a linear relation between these variables. Our scaling form Eq. IV.10 implies that

$$k^* \sim \rho_r^{1+\beta} \quad (\text{IV.15})$$

where $1 + \beta \approx 0.9 \pm 0.2$ in agreement with the results obtained in [226]. In Fig. IV.13 we plot the numerical result for k^* versus ρ_r confirming our scaling result.

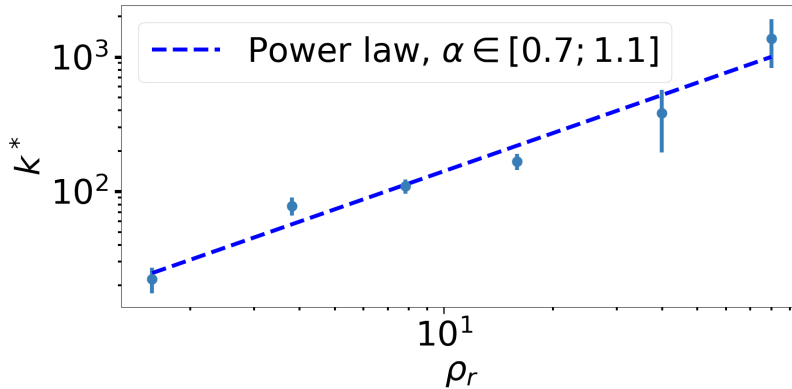


Figure IV.13: Influence of ρ_r on k^* . A power law fit gives an exponent in $[0.7, 1.1]$ in agreement with the scaling form result $k^* \sim \rho_r^{1+\beta}$ (network of 13×13 nodes with $p = 0$)

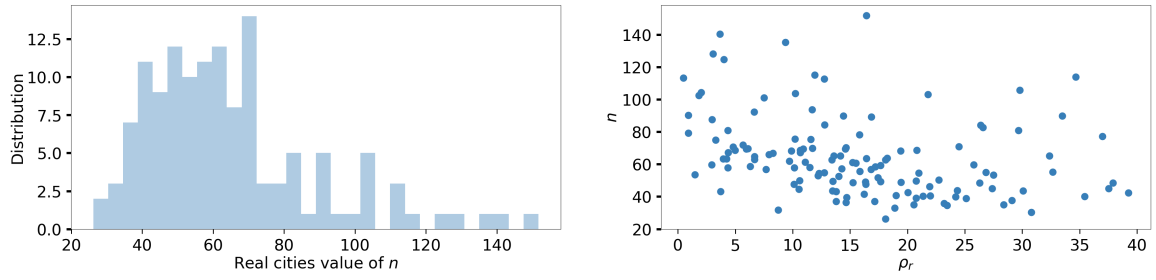


Figure IV.14: Left: Distribution of the value of n in real cities, varying by not more than a ratio 5. Right: Influence of ρ_r on the value of n for real cities, showing close to no correlation ($r = -0.32$)

Relationship between capacity and critical accumulation

Loder et al. [226] also found empirically a linear relationship between k^* and q^* . Our scalings show that this linear relationship is not true *a priori*. In fact, using these scaling forms, the ratio q^*/k^* can be rewritten as :

$$\frac{q^*}{k^*} = V_{Lim} n^{-\beta} \left(1 - e^{-n/n_c(p, t_{tt})} \right) \quad (\text{IV.16})$$

where we recall that $n = \rho_r / (L_{car} \rho_i)$. For real networks, the variations of ρ_r, ρ_i, L_{car} are such that n spans no more than a factor 6 and is usually in the range [25; 100] (see figure IV.14(a)). This leads to V^* varying by a factor ± 2 , while the variations of $n^{-\beta}$ (with $\beta \approx -0.1$) are negligible. V_{Lim} is also known to display small variations among cities, typically by a factor 1.5 at most (see for instance [226]), as it represents some sort of speed limit. Additionally, figure IV.14(b) shows that the average length of roads does surprisingly not seem to be a function of road density in real cities. These empirical considerations thus show that both quantities k^* and q^* can vary independently between cities by up to a factor ± 3 .

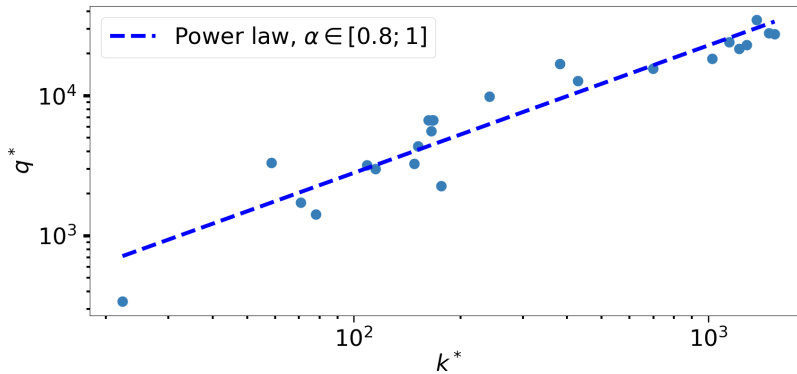


Figure IV.15: q^* as a function of k^* ($R^2 = 0.9$)

However, as shown in [226] as well as in our results displayed in figure IV.15, both quantities vary

by more than a factor 10 mainly under the common influence of ρ_r . This common ρ_r term makes them seem proportional, even though it is not strictly speaking the case. We also note that a simple approximation for computing the MFD, as used in [211] and described in section 10, further suggests that k^* and q^* have no reason to be proportional.

Influence of the bus production

Another important result found empirically is that the capacity of the network is reduced when the bus production increases. More precisely, if we denote by $q^*(P_b)$ the maximum capacity when the bus flow is P_b ($q^*(0)$ is then the capacity for the network when no bus is present), the capacity reduction of the network is defined by

$$\delta = \frac{q^*(0) - q^*(P_b)}{q^*(0)} \quad (\text{IV.17})$$

The authors of [226] express the bus production as a flux per unit surface, i.e. in the same units as the capacity. It is therefore natural to denote by θ_b the fraction of vehicles that are buses and we can then write $\theta_b = P_b/q^*(P_b)$. Using our proposed scaling, the main dependence of q^* on the vehicle length is $q^* \sim 1/\overline{L_{veh}}$. If we consider that the length of a bus is $L_b \approx 3L_{cars}$, the average length of the vehicles in the network is $\overline{L_{veh}} = L_{car}(2\theta_b + 1)$, and we then obtain

$$\begin{aligned} q^*(P_b) &= \frac{q^*(0)}{1 + 2\theta_b} \\ &\approx q^*(0) - 2P_b \end{aligned} \quad (\text{IV.18})$$

which leads to a capacity reduction $\delta = 2P_b/q^*(0)$. We thus expect a linear relationship between the capacity reduction and the bus production, with a slope $2/q^*(0) \approx 10^{-4} \text{ km.h}$ (considering the observed values for $q^* \approx 10^4 \text{ veh.km/h/km}^2$). This is in excellent agreement with the empirical results observed in [226] (see figure IV.16), and also explains the large dispersion observed in their results : for each value of P_b , the observed capacity reduction can vary significantly based on the capacity $q^*(0)$ of that particular city.

Note that this result is very simplistic. More involved studies such as proposed by Geroliminis et al. [236] showed that, if one tries to count buses as “equivalent” to a given number of cars (which they call Bus-Car-Unit BCU), this BCU changes as congestion levels increase. This means that, when one studies the influence of buses on the entire MFD (i.e. the whole range of densities), one has to consider car density and bus density independently, in what they coin as a “3D-MFD”. However, both Loder et al. and ourselves look at a very small subpart of the MFD, corresponding to its maximal capacity. In this specific range, the approximation of a unique BCU likely is correct, as suggested by our results.

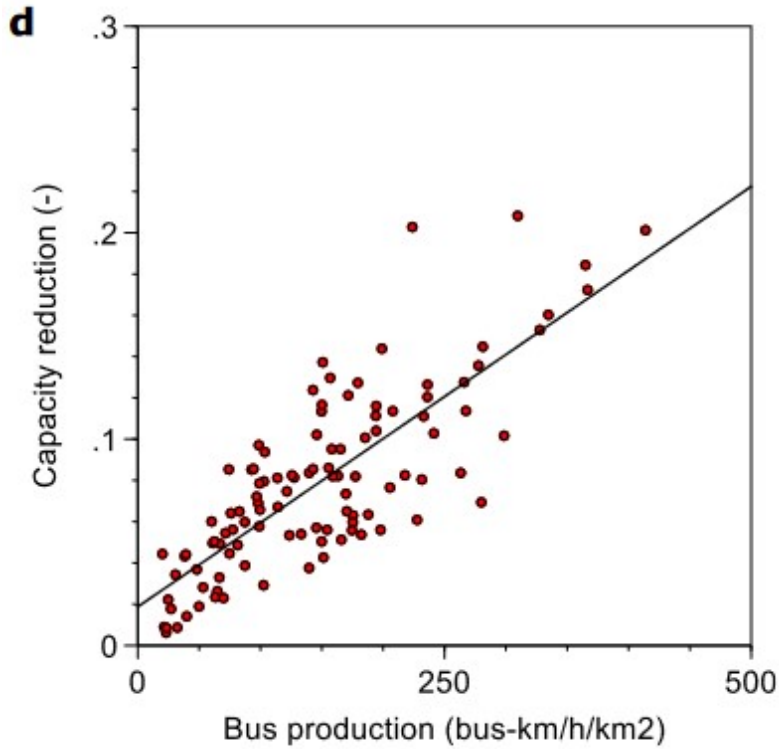


Figure IV.16: Figure by *Loder et al.* [226]: Empirically observed capacity reduction as a function of bus production. The slope is $\approx 4 \cdot 10^{-4} \text{ km} \cdot \text{h}$

9.4.2 Advantage of a numerical approach over empirical studies

In our opinion, our method offers two advantages over previous results. First, we used dimensional arguments to constrain the form of the scaling laws, rather than only testing blindly the influence of each parameter. In particular, this introduced the need to consider the car length as a relevant parameter of the problem, on top of parameters of the network, and allowed us to study the influence of bus production with a different perspective. It also proved helpful to construct the dimensionless quantity n and find the relatively complicated scaling of q^* .

Secondly and more importantly, our simulation allowed us to vary the parameters independently, which is not the case in real world networks. In particular, ρ_r, ρ_i are strongly correlated, with, at first order, $\rho_i \sim \rho_r^2$. In empirical studies, distinguishing the influence of ρ_i from the influence of ρ_r is difficult. The same applies to measures of network redundancy (be it betweenness centrality or closeness centrality) which are strongly correlated to intersection density and thus again to ρ_r .

In particular, the scaling for q^* is far from trivial, as it is not only a power-law. Trying to fit directly the relationship between q^* and ρ_r , for instance, one would completely miss the V^* term and only see the first order term in ρ_r .

Finally, as we have discussed, the position of the detectors on the roads, and the choice of roads

equipped with detectors vastly influences the resulting empirical MFD [234]. A numerical study allows to get rid of this bias and to compare different network with the same conditions of traffic measurement.

9.4.3 Limits of our model

We tried to present a model that is as simple as possible, mainly motivated by the idea of proposing a dimensional analysis of the problem. Introducing too many parameters would have made this difficult if not impossible. However, it is clear that our model is not realistic. The main ingredients lacking in order to reproduce the attributes of real-world MFDs are the possibility to tune the Origin-Destination patterns, and in particular to have the cars choose their trajectory following the shortest path. Indeed, as we mentioned, bottlenecks seem to play an important role in the MFD, thus it is really the interplay between network structure and OD-matrix that is relevant. However, reducing this to a simple quantity that one could use in a formal expression for the MFD will require some creativity and additional work.

Another important aspect is that we only looked at steady-state MFDs. While, as we mentioned earlier, this has advantages to extract a “clean” MFD and allow to study the influence of network parameters, it should be noted that a real-world network should be efficient first and foremost in the transient regimes of loading and unloading, which are far more frequent than an ideal steady-state regime. Again, studying the network’s response during the loading and unloading phases would be of interest, but would require additional parameters, not immediately compatible with the framework introduced in this section.

9.5 Perspectives

9.5.1 Spectral analysis

We would like to redirect the reader to the figure IV.5, showing the evolution of the average velocity in our simulation with time. As we mentioned, the velocity decreases rapidly as traffic jams form, before reaching what is approximately a steady-state. However, as can be seen in the figure, the velocity displays some non-negligible fluctuations.

The high frequency fluctuations are likely related to the traffic light timing, but other frequencies seem to be present in the time series, which we believe would be worth to investigate. Interesting questions include :

- Are these fluctuations an artifact of our simulation, or are they observed in real-world networks ?
- Are the fluctuations of the average velocity random or somewhat periodic ?
- Can the probability or period of the fluctuations be related to the network parameters ?

- Is the probability or period of the fluctuations related to the average steady-state velocity ? In other words, if one has the choice, is it better to design a network which does display fluctuations or not ?
- We recall that these fluctuations are observed at fixed k , implying that the MFD of our simulation is multivalued. Can we progress in our understanding of the relationship between *average* and *instantaneous* MFD and the hysteresis patterns observed in real-world networks ?

We believe that this kind of spectral analysis is different from the current approaches to urban traffic and could give complementary insights into the mechanisms at play.

9.5.2 Other sources of heterogeneity

Both our study and the previously existing empirical results focused mainly on average measures of the network structure. We merely introduced the parameter p to account for the heterogeneity in the intersection's degree distribution. As suggested by our results and as could already be inferred from the influence of density heterogeneity on the MFD discussed in the previous section, we believe that a lot of interesting behaviors are still to be discovered from the study of the relationship between network heterogeneity and critical parameters of the MFD, both empirically and using simulations as we proposed. Here, we suggest some ideas of such sources of heterogeneity. A challenge will also be to find relevant metrics to capture this heterogeneity in a unique but meaningful parameter which would allow (1) to extend our scaling laws and (2) to compare cities with one another.

Behavior when the roads have different lengths

A natural source of heterogeneity is to introduce different types of roads. We started to investigate the situation where all roads are the same and when roads have variable lengths.

Figure IV.17 shows the data collapse $\Gamma(n)$ in the case of networks with roads of variable lengths. The shape of $\Gamma(n)$ as well as the parameters C , n_c are the same as previously, indicating that the quantities ρ_r , ρ_i , etc, remain relevant in this case. However, we see an increase of the deviation around this average behavior (especially for q^*), indicating that the introduction of disorder certainly requires the introduction of a new network parameter describing the spatial distribution of roads to fully understand the relationship between network structure and MFD. One might for instance think of the Gini coefficient introduced earlier, but some kind of distance measuring the spatial correlation of the road lengths would probably also be needed. This is only a preliminary result and should be taken with a pinch of salt, but would be coherent with the discussion about the Generalized MFD presented in the previous section.

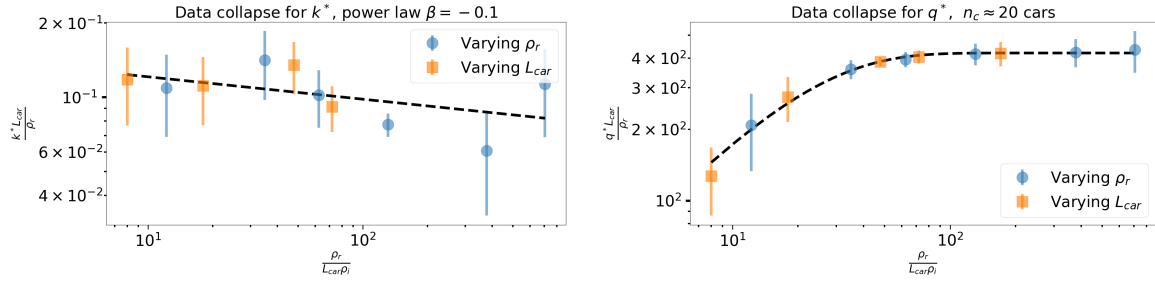


Figure IV.17: Data collapse for k^* (left) and q^* (right) in the case of a network with variable road lengths, with $p = 0$. The average behavior is the same as for the regular network described in the article, but the error-bars are larger, indicating that the position of the roads becomes relevant.

Other types of grids with other degree distributions

Similarly, one could try to test various types of grids. Here, we started from a lattice and removed some links, which led to some intersections of degree 3 and others of degree 4. One could for instance test the influence of some very high degree nodes (e.g. the central node in a web graph). Much like for the roads, our proposed mechanism for the influence of p suggests that it is not only the degree distribution that matters, but also the position in the network of these peculiar intersections. Again, it would be interesting to find simple metrics capturing the combination of intersection-type and position in the network and to study its impact on the MFD.

Testing heterogeneous Origin-Destination matrices

In our study, cars moved randomly, without a given origin or destination. Essentially, this is equivalent to a flat origin-destination matrix (all routes are equally likely). Again, we could test the influence of different matrices on the MFD. In particular, we could study the interplay between the structure of traffic demand (captured in the OD matrix) and network structure. Indeed, as we have explained in the previous chapter, it is likely the combination of those two factors that leads to the appearance of congestion on a set of weak links, from which it then spreads to the rest of the network.

It is therefore likely that a metric measuring the coherence of the network structure given the traffic demand would be strongly correlated with the capacity of the network q^* . Authors of [226] studied the impact of the average betweenness centrality of a network on k^* , with limited success. We believe that the study should focus on the weakest links of the network, thus either on the maximal BC, the Gini coefficient of the BC or on other measures such as the cumulative BC introduced in [151].

This is at this stage speculative, but we shall use the following section to further explore the idea of combining the results of the previous chapter (namely congestion emergence and spread at the street level) with the MFD described here.

10 Unifying the MFD with the spatio-temporal dynamics of congestion ?

The MFD is a very powerful tool for traffic control, as it allows to estimate the traffic state at a broad scale and thus considerably reduces the complexity of the control scheme. However, the relationship between the “microscopic” reality of the city (i.e. the state of individual streets) and its “macroscopic” state remains unclear. Remember in particular from the first chapter that we would like to find a tool which allows to predict the global, “macroscopic” influence of local decisions.

The previous section gave elements in that direction, by linking the critical parameters of the MFD to aggregated measures of the network’s structure. However, a theory unifying the numerous results regarding the dynamics of congestion at the scale of streets (introduced in chapter 3) with the MFD would be a major step forward. Important questions include the reasons for the existence of a MFD, even in cities far from Daganzo’s idealized *reservoir* (different types of streets, complicated geometries...), the interplay between network structure and shape of the MFD (even beyond the critical parameters, the shape of the MFD can vary significantly) or the relationship between observed traffic dynamics and in particular hysteresis in the congestion patterns and the observation of hysteresis in the MFD.

This section will be slightly different from the rest of the thesis. We will try to highlight relationships that might exist between different articles of the literature on urban traffic, and propose new approaches to the problem, but this section will remain mostly speculative. We will try to organize our thoughts as best as possible, as we hope that this might inspire future work in the field of the MFD. We would like to highlight two possible approaches to the problem of the scale-up from the street level to the network level, and their related challenges.

The first is based on the realization that a central question related to the MFD is to know how a network composed of many fundamental diagrams will react to traffic demand. In that sense, it is closely related to Effective Medium Theories, that have been developed and investigated quite thoroughly in the physics community over the last decades. We will start by introducing this field, before trying to construct an Effective Macroscopic Fundamental Diagram, from a given collection of Fundamental Diagrams. As we will see, some central hypotheses that allow simplifications in most Effective Medium Theories are not verified in road networks as, in particular, roads are not randomly distributed in space.

The second approach is based on the idea, introduced in the previous chapter, that congestion somehow percolates through the network during rush hours. We will try to present recent research and our thoughts on how to take advantage of some empirical observations introduced in the previous chapter regarding the spatio-temporal dynamics of congestion on the network to work towards a more general form and a better understanding of the MFD.

10.1 Towards an Effective Macroscopic Fundamental Diagram ?

In essence, the problem of the MFD is really to know how a network composed of several roads, which each have a given “microscopic” response to traffic, will macroscopically respond to traffic demand (assuming that streets have indeed a reasonably well defined fundamental diagram). The problem is thus analog to effective medium theories developed to model for instance the electromagnetic properties of heterogeneous materials. We will start by introducing quickly the concept of effective medium theory, and the hypothesis that the material has to respect for it to be applicable. We will then try to apply the same concept to urban network and show the significant hurdles that one encounters in doing so.

10.1.1 Quick introduction to effective medium theory

Effective medium theory has mainly been introduced in the context of the study of electrical properties of composite materials. The typical case is that of a mostly pure conductor, in which some insulating impurities have been trapped. The question is then to know how these impurities affect the overall conductivity of the material. One would for instance like to be able to express its “effective” dielectric susceptibility. Another example is the case of a network of resistors. In the simplest case, one can assume that there are two types of resistors randomly distributed on the network and one would like to know the “effective” resistance of the network. As previously in this thesis, we would like to highlight important results, which are of relevance in the context of our work, without entering into too much details. More formally, effective medium theory is the search of a function F such that:

$$\epsilon_e = F(\epsilon_1, \epsilon_2, p) \quad (\text{IV.19})$$

Where ϵ_e is the effective dielectric susceptibility of the composite made of a fraction p of material of susceptibility ϵ_1 and fraction $1 - p$ material of susceptibility ϵ_2 .

Important results on this simple case are the following :

- The naive approximation by the average is in general false, $\epsilon_e \neq p \epsilon_1 + (1 - p) \epsilon_2$
- The shape, size and spatial distribution of the impurities plays a role, the sole parameter p is not sufficient.
- The resolution of the effective medium equations heavily relies on knowledge on electromagnetic properties of conductors and insulators, and in particular Maxwell's equations in case of a continuous medium, or Kirchhoff's laws in the case of a network of resistors. As a result, the transposition to other problems is not trivial.

All detailed calculations and more involved cases can be found for instance in [237, 238, 239, 240], while increasingly complex cases are studied up to this day and have allowed the development of meta-materials with finely tuned electro-magnetic properties [241, 242, 243].

10.1.2 Building an effective medium theory for road networks

Back to the impact of congestion heterogeneity on the MFD

We shall give more details on the approach used by Knoop et al. [211] that we introduced at the beginning of this chapter. As we will see, it is a first step towards an effective macroscopic fundamental diagram for traffic.

Assume that all links of the network have a triangular fundamental diagram of parameters q_{max}, k_c, w

$$q(k) = \begin{cases} q_{max} \frac{k}{k_c} & \text{for } k < k_c \\ w(k_{jam} - k) & \text{for } k > k_c \end{cases}$$

In the presence of disorder and due to the dynamics, the density of vehicles is not uniform and we observe a distribution around a mean A with variance σ . These different values k_1, k_2, \dots, k_N for the N links in the network, are in general correlated random variables. We expect indeed a spatial correlation between neighbors due to the jam nucleation and propagation. However, the simple approximation made in [211] is to consider these values as uncorrelated and following the same distribution $P(k)$. In this approximation, the MFD $Q = Q(k)$ is computed as the average traffic :

$$Q = \frac{1}{L} \sum_i q(k_i) \approx \int P(k)q(k) \quad (\text{IV.20})$$

where L is the total network length. If one knows the distribution of congestion $P(k)$, one can then compute this average. For the sake of simplicity, Knoop et al. use a uniform distribution in the range $[A - \sigma; A + \sigma]$.

One has then three different cases to treat :

1. Case where $A + \sigma < k_c$. All links are in the free flow regime
2. Case where $A - \sigma > k_c$. All links are in the congested regime
3. Case where we have a mixture of both congested and non congested links $A - \sigma < k_c < A + \sigma$

The first two cases are trivial. Because all links are in a regime where the fundamental diagram is linear, the average traffic is the flux corresponding to the average density, $Q = q(A)$ and the MFD has thus the same shape as the local fundamental diagram.

The third case is more involved and more interesting. Q is now given by :

$$Q = \frac{1}{2\sigma} \left(\int_{A-\sigma}^{k_c} dk q_{max} \frac{k}{k_c} + \int_{k_c}^{A+\sigma} dk w(k_{jam} - k) \right) \quad (IV.21)$$

$$= \frac{w}{4\sigma} \left(\frac{k_{jam} - k_c}{k_c} (k_c^2 - (A - \sigma)^2) + 2k_{jam}(A + \sigma - k_c) - ((A + \sigma)^2 - k_c^2) \right)$$

If one is interested in the critical parameters of the MFD, one can notice that equation IV.21 is a 2nd order polynomial in A and derive its maximum (k^*, Q^*) :

$$k^* = k_c + \sigma \left(1 - 2 \frac{k_c}{k_{jam}} \right) \quad (IV.22)$$

$$Q^* = q_{max} \left(1 - \frac{\sigma}{k_{jam}} \right) \quad (IV.23)$$

We observe that Q^* is always reduced compared to the initial triangular fundamental diagram. For k^* , this depends on the ratio between k_c and k_{jam} . However, in real life, $k_{jam} > 2k_c$, and thus $k^* > k_c$, which corresponds to the results by Knoop et al. presented in figure IV.2.

Adding empirical knowledge about the congestion distribution

We have here presented the results of Knoop et al., using a uniform probability distribution for congestion. One could refine this model using some real-world measurements of this distribution and see the effect on the MFD. Remember from chapter 3 that Jiang et al. [175] measured this probability distribution for Beijing and Shenzhen, finding that the probability to find a link with occupation level o scaled approximately as $o^{-\beta}$, with $\beta \approx 1.2$.

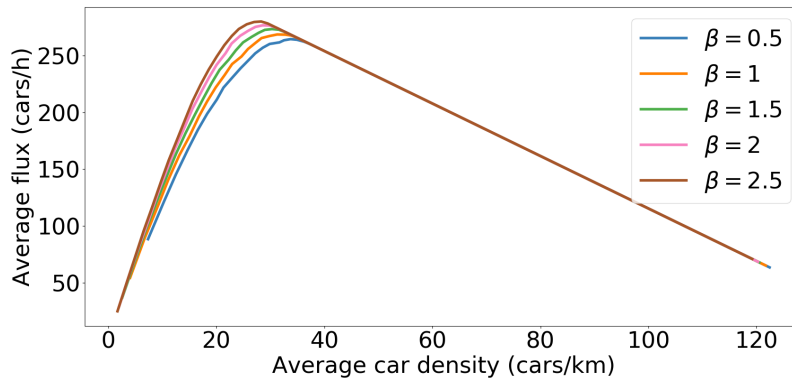


Figure IV.18: Building the average MFD based on the empirically observed power-law distribution of congestion $P(o) = o^{-\beta}$.

One can for instance look at the impact of this factor β on the MFD (see figure IV.18). Unsurprisingly, we find that larger values of β , which correspond to a narrower distribution of congestion on the

links, perform better in terms of network capacity. Recall from chapter 2 that narrower distributions of the betweenness centrality of the roads was also associated with higher efficiency. Generally, it seems that an efficient city should be very symmetrical and very homogeneous both in its network structure and in the traffic demand.

Introducing different types of roads

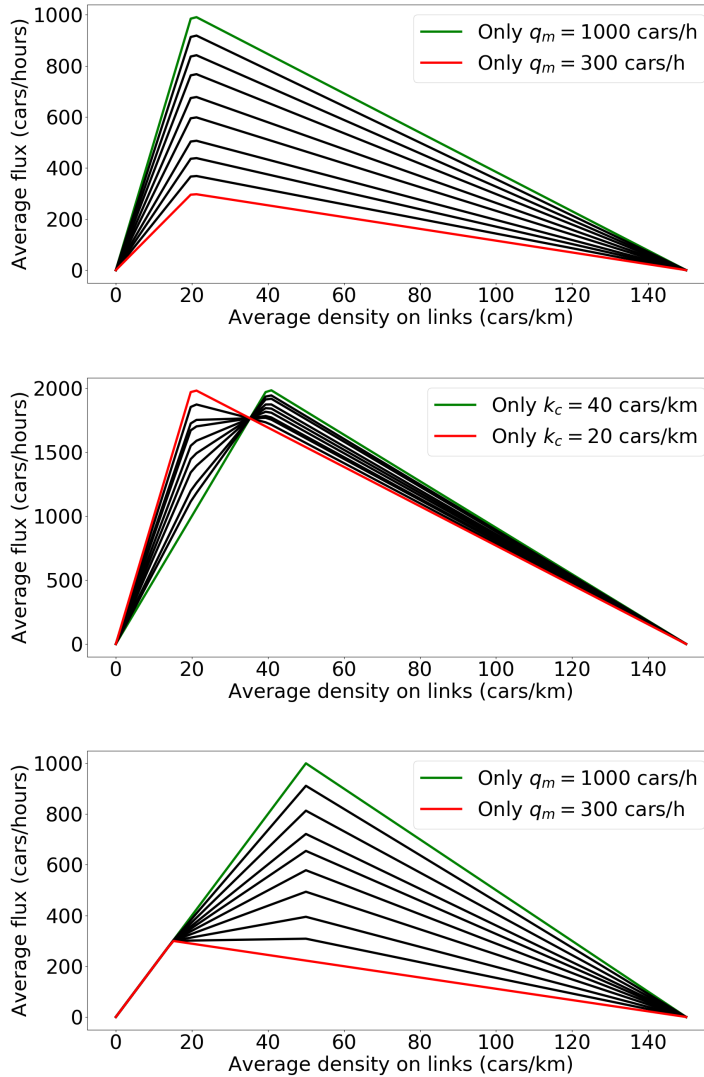


Figure IV.19: Building the average MFD for a network with two road types in varying proportions. Top : two different values of q_{max} at fixed k_c , Middle : two different values of k_c at fixed q_{max} , Bottom : two different (k_c, q_{max}) with fixed ratio. As one can see, the evolution of k^*, q^* with the proportion of roads is not trivial.

One can imagine the same reasoning where, instead of having a distribution of the car density on the roads, we have a distribution of the parameters of the fundamental diagrams. The network could be composed of a collection of randomly chosen fundamental diagrams $q_{max}^{(i)}, k_c^{(i)}, w^{(i)}$. Keeping the

model relatively simple, we only consider one-lane roads. Then k_{jam} is approximately constant and the parameters are $q_{max}^{(i)}$ and $k_c^{(i)}$. One can imagine four different ways to vary these parameters :

1. Vary q_{max} at fixed k_c
2. Vary k_c at fixed q_{max}
3. Vary both at fixed $v_0 = \frac{q_{max}}{k_c}$
4. Vary both independently

We only present here numerical results for the simplest case where there are two types of roads and we vary the proportion of those two roads in the network, while the car density on all roads is equal. The first case (different q_m at fixed k_c) is not very interesting : one can intuitively imagine that we will get k^* equal to the links' critical density and that q^* will be the average of the links' capacities. The two cases where different critical densities are mixed, however, are more interesting and less intuitive : as one can see in figure IV.19, this is enough to display non-trivial behaviors in the evolution of the shape and critical parameters of the MFD, while also highlighting once again that k^* and q^* have a priori no reason to be related by a simple relationship.

In particular, this simple example echoes with the discussion we had on the functional form for the MFD proposed by Ambühl et al in [214]. It would be interesting to investigate on this simple "average" effective MFD how the four parameters of their model relate to the distribution of the road level fundamental diagrams.

10.1.3 Challenges and perspectives

The approach proposed by Knoop et al. is close in its spirit to effective medium theory, but is in fact not quite the answer to the problem we try to solve. Indeed, we would like to know the response of the network to traffic demand. However, both the distribution of density on the network and the resulting MFD are part of the response of the network to traffic. The second approach we investigated, where we mix different types of roads and look at the resulting average diagram is closer to an effective medium theory, but also poses problems. Indeed, there is no reason to expect that the car density will be homogeneously distributed on the resulting network, and we would need to know it in order to estimate the MFD. Additionally, one would expect that, once we have introduced different types of roads, their spatial distribution plays a role as well. The results presented here correspond indeed to the naive average, which is known to be wrong for the case of the electric properties of composite materials or networks and has no reason to be true in the case of roads. We would like to highlight once more what makes the problem of urban networks particularly complex when compared to electric networks.

Building an analogy to Kirchoff's laws ?

For electric networks, one can use Kirchoff's laws¹⁰. Each edge of the network is defined by two state variables which are its current I_i and its potential V_i . Each of these quantities is then constrained by one of Kirchoff's laws, resulting in a system of equations. Adding the knowledge of the relationship between I_i and V_i for each link (e.g. Ohm's law for resistors), one can then in theory solve the system of equations and find the values I_i and V_i on each link. This last step is in practice difficult, which is the reason why effective medium theories rely on simplifications to find the macroscopic properties of the network, but as we shall see, even building the analogy between electric networks and road networks is not trivial.

Indeed, in our model, the equivalent of the potential would be the traffic demand, i.e. the number of cars which want to travel on the link, while $q^{(i)}$ and $k^{(i)}$ are both results of this demand. A simplification we can make is to assume that traffic will flow freely to where there is space available. This is in fact not too much of a stretch, if one considers Wardrop's first principle, which implies that traffic will naturally tend to equilibrate on equivalent roads. Ignoring the reality of traffic demand and considering only $k^{(i)}$, $q^{(i)}$ as the state variables of the roads on our network, we too can devise two laws relating these quantities on the network, which both stem from the conservation of the number of cars. At the scale of each link, we have :

$$\frac{dk^{(i)}}{dt} = (q_{in}^{(i)} - q_{out}^{(i)}) \frac{L_{car}}{L} \quad (\text{IV.24})$$

where $q_{in}^{(i)}$ is the flow entering the link, $q_{out}^{(i)}$ is the flow exiting the link, L_{car} is the length of a car and L is the length of the link. Of course, for each link, we also have the fundamental diagram giving the relationship $q_{out}^{(i)}(k^{(i)})$.

At each intersection, we have :

$$\sum q_{in} = \sum q_{out} \quad (\text{IV.25})$$

Possible simplifications ?

Using this knowledge, one can again in theory find the evolution of $k^{(i)}$, $q^{(i)}$ on each roads, which is typically what is done in traffic simulations. However, we would like to find a scheme to simplify this system of equations. We propose here perspectives that could be explored. A first step could be to consider the equilibrium state of the network, i.e. ignoring the loading and unloading phenomena and assume for all roads $\frac{dk^{(i)}}{dt} = 0$, and solve the equations in that case. One could then consider a regular network of roads, identical except for one parameter (e.g. random distribution of q_{max} , k_c , $L...$).

¹⁰As a reminder, the sum of currents is null at each node and the sum of potentials is null on each closed loop of the network

Note however that one would then solve the equations for the flow of cars q_i . Because the fundamental diagram is not bijective, one could not completely solve for all k_i, q_i without taking into account the temporal aspect of traffic.

Another possible simplification is therefore to consider a different shape for the fundamental diagram, and to consider that congested roads do not contribute to traffic and can be removed from the network. The natural analogy for the study of traffic would then be the study of fracture phenomena in heterogeneous mediums, and is closely linked to the framework of percolation. We presented results based on this percolation framework in the third chapter, but we shall now present some perspectives to reconcile these results with the observation and understanding of the MFD.

10.2 Relation between percolation and MFD

A recent article by Ambühl et al. [244] showed an intriguing relationship between percolation of congestion on the network and MFD. The authors showed that the percolation of the congested links at the network scale coincides with the network-wide switch between free-flow and congestion, captured in the critical parameter k^* of the MFD. In this framework, congestion percolates on the roads of the network. Thus, one would expect to find properties of the universality class of standard percolation.

However, in the framework proposed by Aghamogammadi and Laval that we introduced previously [234], which we recall showed that the MFD can be fitted with a coexistence function, the emerging critical exponent seems to link the MFD to the universality class of directed percolation. More generally, the point of view proposed by Laval et al. is that congestion is a state of strong correlation between cars, and that the system is in its critical state once the interaction between cars percolates the network. With this point of view in mind, of cars traveling on the network being the medium for percolation, the relationship with directed percolation becomes easier to understand. The reality is probably a mix of those two frameworks, but the question is not settled yet.

Because the literature regarding the evolution of congestion at the street level, and in particular the study of the evolution of congestion clusters, is quite abundant, there is certainly a lot of information about the MFD to be gained from this approach. We think in particular of two aspects that could be interesting for future research.

First, the study of the percolation of congestion on the network could be refined to find important parameters of the network's structure that influence, for instance, the critical demand at which congestion percolates. These studies could certainly benefit from the vast knowledge existing about percolation in general, and could then be transposed into valuable information for traffic planners through its correspondence with the MFD shown in [244].

Secondly, questions related to the existence and the shape of the MFD could certainly be answered

by using empirical knowledge regarding the evolution of the number and spatial distribution of congested links (for instance [151, 176, 181, 184]). Recall for instance that the distribution of the size of congested clusters displays scale free properties. Empirically, the exponent for this distribution is found to be close to 2.3, which is relatively close to the expected exponent for both directed (≈ 2.1) and standard percolation (≈ 2.05).

Conclusion

In this chapter, we have presented the most recent and promising approach to urban traffic. The empirical discovery of a network wide relationship between car density and traffic opened new perspectives both from a theoretical point of view and in terms of practical applications. However, as we discussed, our understanding of this MFD is only partial and significant efforts are still to be made.

In particular, we presented our results for an important aspect of future urban planning which is the relationship between network structure and MFD. Using a simulation on a toy-model for urban traffic and dimensional arguments, we showed in particular that the density of roads in space is the most important predictor of the MFD, while intersection density or maximal speed on the network only play a limited role. Moreover, our results confirmed the detrimental impact of weak links in the network on traffic capacity. It thus appears that, on top of the macroscopic properties of the network, its microscopic details do indeed play a significant role on urban traffic. The impact of network disorder on the MFD remains to be studied more systematically, which will require the use of appropriate tools to describe the network structure at all scales.

Besides, the influence of heterogeneously distributed car density on the network, while clearly identified as detrimental for the MFD, remains to be studied. Indeed, a difficulty comes from the fact that both the congestion distribution and the MFD are results of the interplay between the network's structure and the (time-dependent) traffic demand, and that one can therefore not simply treat congestion distribution as an independent parameter of the problem. In particular, it is not certain that the framework that we proposed, based on dimensional analysis, can be extended to take these other aspects into account. We reach here a manifestation of the trade-off between our strive for realism and our desire for parsimony.

We thus presented in the last section other possibilities that are currently being explored to reconcile the knowledge of traffic dynamics at the street level, discussed in the third chapter, with the existence and the shape of the MFD. This section aimed at showing how other methods and results traditionally associated with physics, such as effective medium theory and percolation theory can certainly play an important role in our understanding of urban traffic, which we hope will gather interest from the physics community.

Chapter V

Discussion and closing thoughts on urban mobility

In this chapter, we would like to give some perspectives regarding the future of urban planning. So far, we have discussed urban planning under the angle of traffic efficiency. In the first two chapters, we have mentioned that decisions regarding the road infrastructure have impacts on all aspects of a city, leading to shifts in the distribution of population, jobs or wealth and thus contributing to enhance or reduce inequalities. Apart from that, however, the tools and studies that we presented were mostly interested in maximizing the road network efficiency, i.e. ensure that the network could carry as many cars as possible as quickly as possible.

The MFD is a prime example of this paradigm, as it measures the efficiency of the network through the prism of the number of kilometers traveled by cars per unit time. We would now like to propose alternative metrics, which also take into account the structure of the city. Indeed, it might be easier to change the spatial distribution of housing and jobs as to reduce the number of traveled kilometers, rather than changing the network in order to accommodate for a wasteful traffic demand.

Another angle we would like to explore is to consider not only the efficiency of the road network, but rather what one could call the spatial efficiency of the city, i.e. the number of commuters that successfully arrive at destination per unit time in a given city, using all possible sorts of transportation. The central question for cities is indeed to maximize this efficiency at minimal cost, and to achieve this, to find the optimal mix between transportation modes.

Finally, we would also like to highlight the potential of enforcing cooperation between road users rather than selfish behaviors. As shown in the beginning of this thesis with Braess's paradox, the sum of individual interests does not lead to a social optimum. Using the results of this thesis, we will show

that some collaborative strategies could be imagined to improve traffic in particular and mobility in general. Because of the complex nature of cities and human interactions, tools derived from statistical physics such as Mean-Field Games are, once again, well suited to the study of such strategies. The generalization of the use of the GPS and maybe soon of autonomous cars opens new perspectives to implement such strategies in real-life.

The key takeaway message from this chapter is that we need to change our habits, and that centrally planned and thus fundamentally political decisions are mandatory. In particular, legislating to dilute the opening hours of business could significantly decrease congestion. In parallel, a combination of good public transportation and strategically planned perimeter control based on dynamical pricing would further reduce traffic congestion.

11 Perspectives beyond the MFD

11.1 Taking traffic demand structure into account

As we have said, the MFD simply measures the number of kilometers driven on the network per unit time, as a function of car density. However, this metric is not necessarily correlated to the travel time experienced by commuters of that city. Imagine for instance two cities A and B with the same population such that :

- City A has a low population density, congestion is not really an issue, but people commute over large distances to their workplace.
- City B has a high population density. As a result, congestion is higher, but people only need to commute over a few kilometers.

In terms of the MFD, city A will appear to be more efficient than city B. However, in terms of travel time to work, city B might be more efficient. Because, as we have explained in the second chapter, the road infrastructure does not only influence the traffic flow, but also the patterns of traffic demand (by increasing the attractiveness of some parts of the city which are easier to reach), the average length of the commute should in our opinion also be a parameter used to measure the efficiency of a city's road network. Authors of [34] investigated the potential of optimally distributing jobs and housing in Paris in terms of distance traveled. They found that, if everyone worked in the business district closest to their house, total distance traveled for commutes could be reduced by up to 56%. This would either lead to significantly reduced travel times, or facilitate a shift towards soft-mobilities such as bicycles, or both. As we alluded to in the second chapter, this also means that generalizing home-office, in particular for some strategically chosen commuters, could potentially have a significant positive impact on traffic

congestion. The lockdowns that occurred during the Covid pandemic allowed for empirical studies on this subject [245], but no clear results emerge yet.

In the same vein, authors of [246] showed that, knowing the MFD of a city, one could achieve significant improvements in the travel time of commuters by diluting the rush hours for their departures. Spreading the bulk of the commutes over a two hour rush-hour period compared to one hour could potentially reduce average travel time by up to a factor 3. Once again, this parameter is not directly linked to the road network structure, but proves to be a much more powerful lever to mitigate congestion than any road planning measures could be.

It follows from what we just said that metrics such as average travel time and rate of arrival at the destination are actually much more relevant to measure the network efficiency such as experienced by the road users. Ideally, one wants a network which minimizes travel time and maximizes arrival rate.

11.2 Passenger-MFD and area-based efficiency

A first step one can take to generalize the MFD is to count not only the distance traveled by cars, but the distance traveled by actual commuters. Authors of [236, 247] proposed a framework to include a given fraction of buses into the mix of vehicles, in what they called a “3D-MFD”, which was then empirically measured by Loder et al [248]. As we have discussed in the previous section, the impact on the overall MFD is actually limited, but the superior number of seats of a bus greatly enhances the efficiency of the network in terms of passenger transport. Molkenhain et al. [249] looked at the optimal number of buses to optimize passenger transportation from a more theoretical point-of-view, in particular in relationship with the network’s topology. From all those articles, it emerges that there exists an optimum for the number of buses on the network which maximizes the overall flux of passengers on the network. Similarly, [246] studied the socially optimal balance between road and rail transportation in cities and found that trains play a significant role in strategies to mitigate car congestion.

Note that this framework of the MFD could actually be extended to any form of transportation, such as bicycles [250], pedestrians [251], trains or trams. Those specific fields and the combination of different transportation modes in one single framework is a concept in its infancy. In particular, it is unlikely that such a multi-modal MFD exists, in the sense that there is no reason for a clear relationship between commuter density and flux to exist once all transportation modes are aggregated. However, the idea of estimating the efficiency of the entire transportation network of a city, in terms of the flux per unit surface that it can support, the flux per dollar invested in the transportation network or the flux per ton of emitted pollutant, surely is an interesting alternative to keep in mind.

12 Enforcing cooperation and working towards a social optimum

As shown with the example of Braess's paradox, individuals optimizing their travel can lead to socially sub-optimal situations. We believe that a lot of efforts can still be made to model and understand the influence of individual decisions on the macroscopic traffic patterns that emerge in cities using game theory. The goal is indeed to find the best levers to bring people to change their behavior, mitigate traffic congestion and reach a socially optimal situation.

12.1 Side-note : a brief introduction to mean-field games

Considering that cities are a system of a very large number of agents and that the agents' "strategies" are often complex, the framework of mean-field games (introduced by Lasry and Lions in [252]) is particularly suited to model them. By "complex", we mean that a strategy could be a continuous function of time and of the system state, rather than a binary decision such as "cooperate" or "betray" in the simplest game theory example that is the prisoner's dilemma. Mean-field games are out of the scope of this thesis and we will therefore not introduce them rigorously but rather present their interest and some results which are interesting in the context of this discussion.

Let us however explain rapidly the difference between mean-field games and traditional game theory. It relies on the hypothesis of a large number of identical agents, such that each agent is influenced by the average decisions of other agents (rather than by the detail of these individual decisions), but cannot change this average by a unilateral decision. Further, mean-field games consider the state and the strategy of each player as a continuous function of time, which is a major difference with classical game theory, which typically uses discrete time. This means that the evolution of the system can be described by a set of differential equations, making the resolution of the problem much easier than the equivalent description in game theory when a large number of agents is considered. Moreover, Mean-field games are well-suited to cases where the system state can be influenced by random exogenous factors. They are thus another illustration of the use of statistical physics to describe complex systems *a priori* out of the realm of physics.

This framework has very powerful applications in economics and in particular to model financial markets (see for instance [253]), to model pedestrians and prevent crowd movements [254, 255] and has recently been used in the context of road traffic. Authors of [256], for instance, have used mean-field games in combination with the MFD to derive the time-of-departure distribution among commuters in Lyon, France.

12.2 Optimal traffic assignment

Ideally, one would like to devise a traffic assignment scheme which optimizes the trip-completion rate, while minimizing other metrics such as time wasted in congestion or pollution. As we have mentioned, the MFD is a very useful tool to achieve this goal, as it allows to separate the network into perimeters in which the density of vehicles should be kept close to an optimal value. Different approaches to achieve this are possible and currently explored.

Intelligent traffic lights

A first strategy is to dynamically manage the traffic lights such that cars are physically impeded to enter a given road or a given perimeter. This method has been used on highway entrances (in which case it is called ramp metering) for a few decades, but is far from being generalized. Considering the discussion we had on the peculiar role of urban highways in the propagation of congestion, we believe that a more systematic use of ramp metering could be useful. In the more general urban context, methods start to be developed to optimize the in-flow into critical parts of the network using machine learning to pilot traffic lights at intersections [125, 127, 129, 257].

Dynamic pricing schemes

A second strategy to deter road users from entering congested zones is to add tolls to the road. Different pricings can be imagined, such as hotspot pricing [217] where the travel through a congested intersection is taxed, or more involved schemes [258, 259, 260, 261, 262, 263]. In particular, [258] also explores the possibility of taxing or subsidizing employers depending on their schedule for work hours, in order to enforce a better repartition of the commutes in time.

The most radical method, used by some cities, is to purely forbid the use of private cars for some parts of the population in the hope to reduce congestion. However, as shown in [264], this strategy is non-local and is thus not as efficient to reduce congestion as strategies that would target congested zones of the network in real-time.

Perspectives offered by self-driving cars and generalized GPS-use

Note that the strategies based on tolls and even more on driving restrictions can potentially be negatively received by the population. Instead, one might wonder if it could be possible to “positively” bring road users to cooperate. In particular, the fact that GPS is now widely used and further, that autonomous cars are likely to enter the market in at most a few decades, begs the question of a possible switch from selfish routing strategies to socially optimal, altruistic strategies.

Let us first start by describing briefly the current strategy employed by GPS services. For each customer, the shortest path given the current traffic state is computed (using typically Dijkstra's algorithm). This means that, even when everyone uses a GPS, suboptimal situations such as the Braess paradox arise [265]. In the case of Waze[®], which has arguably become one of the most prominent GPS providers worldwide, the used strategy is more involved. Even though the exact algorithm is kept secret¹, it clearly uses historical knowledge about the expected congestion and certainly also about the measured traffic evolution to give the best route not only at the present time, but over the entire travel time. Recent strategies have been proposed to refine this selfish route choice, by adding a form of resilience to stochastic incidents on the route [132]. The best route is then the shortest on average, considering everything that could go wrong, and thus often the route which offers the easiest switch to alternatives.

However, one could imagine a change of paradigm in favor of a centralized routing scheme, in the same way that the evolution of the road network changed from a decentralized growth to a centrally planned one during the 20th century. Authors of [133, 222] proposed such frameworks which dispatch each new vehicle on a route such as to reach a balance between minimizing its individual travel time and minimizing the overall impact on the network efficiency. Even though a specific road user could be under the impression of "sacrificing" a few minutes, the example of Braess's paradox shows that everyone could actually benefit from such a cooperative scheme.

The main issue with such a strategy is that it requires every road user (or at least a significant proportion) to be part of the centrally planned routing scheme. By definition, this would thus require a GPS provider to dominate the market, or to convince the different operators to collaborate², which is not obvious in a market economy. However, it is not certain that this is so unlikely. Even today, the impact of the use of Waze[®] is very noticeable in cities around the world, as the diversion of traffic on small residential roads during peak hours becomes a problem. If, conversely, all Waze[®] users were routed following a collaborative scheme, the overall impact on congestion would most certainly already be noticeable as well.

The solution becomes even more likely when one considers the emergence of autonomous cars. Indeed, as shown by [266], non-cooperative autonomous cars are very inefficient even in their "microscopic" interactions (e.g. giving way at intersections). If they were to be generalized, they would likely require some sort of cooperation and thus some sort of central planning. Adding a centrally planned routing scheme would then be a relatively natural step.

¹Some information about the algorithm can be found on the website

²Even though the incentive to "betray" would be high, as one could still individually benefit from a selfish behavior

12.3 Encouraging a modal shift

As discussed in the paragraph on area-based efficiency, cars are very inefficient means of transportation, both in terms of wasted space and in terms of energy consumption. So far, we have discussed methods to optimize traffic flow on the network and reduce commuting times for road users. However, the detrimental impact of personal transportation on the environment is of such scale that a fundamental change in our travel habits is necessary. The move towards a social optimum requires that we collectively move away from cars and promote the use of soft mobilities or public transportations. Simply banning cars from cities is a radical solution, but is very negatively perceived by the population and it is of paramount importance to first develop credible alternatives to the car use. Authors of [58] have shown that access to rapid public transportation is by far the most important factor explaining the *spontaneous* modal choice of the population. Other articles have focused on the best incentive to pull commuters away from their cars, as for instance the price of public transportation [262, 267, 268], which is arguably the second most important factor.

A vast open question is now to adapt the entire network (not just the road network) to the traffic demand in cities. As shown in [22], the majority of commutes in large cities are actually not to central business districts but rather from everywhere to everywhere. Current urban rail networks, for instance, do not have the ideal structure to cater for this demand, which results in significant time losses for commuters [156, 269] and makes the use of the car more attractive than it should be. Besides, car traffic represents the majority of commutes in a large number of cities, and a brutal switch from cars to public transportations would completely saturate the existing infrastructure, further illustrating the need to invest in this sector.

This discussion should be seen in perspective with the discussion of the second chapter. The combination of growing awareness about the detrimental effect of cars on the environment and our health, with the fact that a large part of the road infrastructure will require significant investment in the upcoming years to be kept viable, makes the next decades a pivotal moment in the way we think of cities and urban mobility. Fortunately, in a large number of cities we see a trend in favor of public transportation. In Paris, for instance, the decision to close the Voies sur Berges and further roads of the network is accompanied by efforts to promote cycling and, at the scale of the urban area, an ambitious plan to modernize public transportation and adapt it to the new reality of a decentralized city with a large number of suburb-to-suburb commutes, known as the *Grand Paris Express*.

General conclusion

The aim of this thesis was to offer a different perspective on urban road traffic, applying methods derived from statistical physics. The motivation for this approach stemmed from the observation that both the structure of road networks and the traffic patterns that emerge on them are actually very similar between cities from different countries, and of vastly different sizes. We started by defining our approach, focused on two main aspects that are universality, i.e. the capacity to reproduce results observed all over the world, and parsimony, i.e. the strive to propose models that are as simple as possible, in order to identify the main mechanisms at play in a phenomenon.

The first important aspect that we studied is the evolution of urban road networks. We showed that urban networks are closely linked to the more general urban evolution. As the city grows, new roads are built to accommodate growing traffic demand, which in turn influences future urban development by facilitating the access to some parts of the city. While this process historically took place spontaneously, meaning that alterations of the road network were decided locally, we showed that the appearance of cars and the associated issue of traffic congestion required a shift to a centrally planned urban road network. In particular, cities built around car traffic are characterized by structures closer to a grid, in contrast to the historical tree-like structure of self-organized cities, and generally by an increased resilience of their network to disruptions.

We then studied in more detail the appearance of urban highways and ring roads during the 20th century. Empirical results show a well-defined population threshold separating cities with or without these infrastructures, which suggests that a simple criteria dictated the decision to build them. We thus proposed a simple cost-benefit analysis based on the balance between cost of construction and value of time, explaining our observations.

Then, we focused on the dynamics of traffic on a given network. More specifically, having established since the first chapter that the traffic evolution at the scale of a network is too complex to be described exactly, we used an approach at the level of the street, and studied the propagation of congestion from one street to another. We showed different arguments pointing towards a combination of two phenomena, namely the initial appearance of congestion on a few weak points of the network, followed

by the percolation of congestion throughout the network. In particular, we presented our own results giving one such argument, by showing that the correlation length of traffic congestion is low when traffic demand is low, but diverges during rush hours, indicating that the system does indeed go through some sort of phase transition.

The fourth chapter presented the intriguing relationship that emerges between average car density and car traffic at the scale of the network, called the MFD. We summarized the literature on this relatively new field, insisting on the fact that the existence of such a relationship is both remarkable from a theoretical point of view and very useful in terms of traffic management. We also highlighted the many open questions related to the existence of the MFD and the relationship between network structure, traffic dynamics and the shape of the MFD. We presented our results based on dimensional analysis and a simple simulation, showing that the most important parameters defining the MFD are the density of roads in space, the distance between intersections, as well as the regularity of the network. Further studies are needed to confirm this latter point but we found that, starting from a regular 2D-lattice, removing a small number of links resulted in a significant reduction of traffic capacity.

We then proposed two different approaches to enhance our understanding of the MFD, both deeply rooted in statistical physics and which, we believe, could appeal to physicists. Those two approaches are effective medium theories on one hand, and percolation theory on the other. In both cases, we highlighted the promises, but also the many challenges that have yet to be solved to offer a comprehensive understanding of the interplay between network structure and traffic dynamics at the network scale. While a lot still has to be discovered, it appears that heterogeneity is the main enemy of an efficient road network. This includes both the heterogeneity of types of roads on the network and the heterogeneity of traffic demand on the network, which are both likely to lead to local bottlenecks with a detrimental effect on traffic at the network scale.

Finally, and we believe that this has been a theme throughout this thesis, we stressed how important it is to consider the question of urban traffic and urban road networks in the broader picture of urban mobility and urban life-quality in general. The example of the Federal Highway Act, which led to the construction of urban highways in a large number of US American cities, shows how much impact the urban infrastructure has on subsequent urban, social and economical development. As we highlighted in the last chapter of this thesis, road transport is only one aspect of an efficient urban mobility and the development of new tools for traffic management, such as the MFD, should never lose sight of this fact as real-world decisions are ultimately multi-factorial and political.

Nonetheless, the study of urban traffic is a fascinating field, including for statistical physicists, in which a lot has yet to be discovered.

Résumé détaillé en français

Introduction

La généralisation de l'automobile au cours du siècle dernier a drastiquement remodelé les villes de part le monde. Il convient cependant de noter les remarquables similitudes que l'on retrouve entre ces villes, indépendamment de leur taille, de leur histoire ou de leur emplacement géographique. Bien sûr, des différences subsistent, mais toutes les villes se distinguent par exemple par des distances entre intersections, des densités de route par surface ou encore le degré moyen des intersections relativement proches. De la même façon, le comportement du trafic routier sur ce réseau, et en particulier les phénomènes collectifs tels que l'apparition d'embouteillages, sont eux aussi remarquablement similaires d'une ville à l'autre.

Ce constat laisse penser que les réseaux routiers urbains peuvent avec succès être décrits à l'aide d'outils développés pour l'étude des systèmes complexes, branche de la physique statistique. L'objet de cette thèse est donc l'étude de différents aspects de la problématique du transport routier en ville, l'étude de l'évolution de l'infrastructure, d'une part, et du trafic routier sur ce réseau d'autre part. Le cœur de notre démarche sera toujours ancré dans la physique statistique, dans le sens où nous nous efforcerons de proposer des modèles respectant **l'universalité** des observations discutée précédemment, ainsi que le **principe de parcimonie**. Ce principe dicte que le nombre de paramètres d'un modèle doit être réduit à son minimum, ce qui interdit les paramètres redondants et oblige à justifier le sens physique de chacun des paramètres. Nous pensons qu'une telle approche, complémentaire des approches "traditionnelles" telles qu'utilisées dans le domaine de l'ingénierie du trafic routier par exemple, qui visent elles à reproduire dans le plus de détails possibles la réalité, permet au contraire de saisir les principaux mécanismes à l'œuvre dans un phénomène et plus généralement d'affiner notre connaissance théorique de ce domaine.

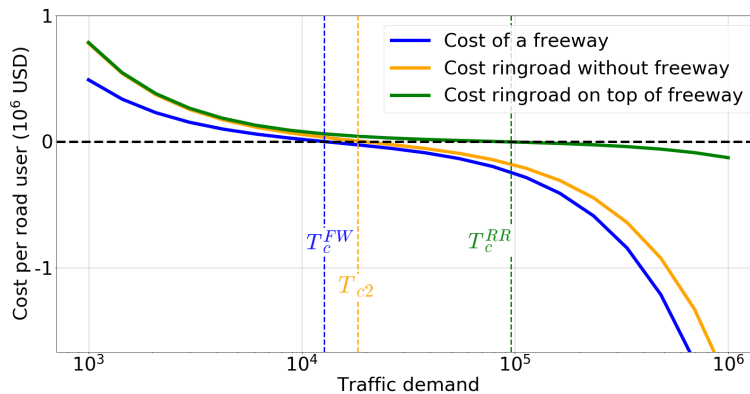
L'apparition d'autoroutes urbaines et de périphériques au XX^e siècle, exemple de la relation entre population et infrastructures

Nous l'avons dit, la croissance du réseau routier s'est faite en réponse à la demande croissante du trafic routier. Plusieurs stratégies furent adoptées pour contrer la problématique naissante des embouteillages. Outre la densification du réseau et l'introduction de signalisations et de feux tricolores pour organiser le flux de véhicules, il apparût dès les années 1920 qu'une route destinée uniquement aux voitures, à sens unique et sans intersections, serait bien plus efficace pour transporter un grand nombre de véhicules. La première autoroute fût ainsi construite à New York en 1923, avant que le Federal Highway Act, voté en 1956, ne conduise à la construction de plusieurs dizaines de milliers de kilomètres d'autoroutes de part les Etats Unis, et en particulier à travers les centre-villes des grandes villes américaines. Peu après, constatant que ces nouvelles infrastructures, loin de leurs promesses, étaient elles aussi rapidement saturées par le trafic croissant, de nombreuses villes décidèrent de construire des autoroutes périphériques, permettant aux véhicules de contourner les centres-villes à grande vitesse. Notons que nous employons ici le terme "ville", mais que, selon la situation, l'objet d'étude est bien souvent l'aire urbaine.

De façon surprenante, bien que la décision de construire ces autoroutes et périphériques ait été prise au cas par cas, il apparaît que c'est finalement la population qui permet d'expliquer si une ville dispose ou non d'une autoroute ou d'un périphérique. Une étude empirique menée sur les villes étatsuniennes permet en effet d'identifier deux populations seuil $P^{FW} \approx 10^5$ et $P^{RR} \approx 10^6$ au-delà desquelles les villes ont respectivement une autoroute urbaine et un périphérique. Ces seuils bien définis laissent penser qu'un mécanisme simple suffit à expliquer la décision de construire ces infrastructures. Nous proposons donc une analyse coût-bénéfice basée sur la comparaison entre le coût de construction d'une telle infrastructure, et l'estimation du coût associé au temps perdu dans les embouteillages par les habitants d'une ville. Nos travaux, publiés dans [160], montrent que cette analyse simple permet de retrouver les valeurs numériques pour les seuils de population observés empiriquement.

Sans rentrer dans le détail de notre méthode, nous présentons le résultat principal en figure A. Nous exprimons l'analyse coût-bénéfice non pas en terme de population de la ville, mais en terme de navetteurs utilisant leur voiture et bénéficiant donc de l'infrastructure. Empiriquement, nous trouvons que ce nombre de navetteurs en 1960 (au moment de la décision de construire l'infrastructure), est environ proportionnel à la population actuelle des villes, avec $T^{1960} \approx 0.14 P^{2020}$. Nous retrouvons les valeurs numériques observées empiriquement, et en particulier le fait que les autoroutes urbaines ont été construites avant les périphériques

Notons que notre analyse n'inclue aucune externalité négative associée à la présence d'une autoroute.



A. Coût associé à différents scénarios d'infrastructures en fonction du nombre de navetteurs utilisant leur voiture dans la ville. On retrouve les valeurs numériques observées empiriquement, et en particulier le fait que les autoroutes urbaines ont été construites avant les périphériques

Nous pensons que ce point de vue reflète la mentalité des décideurs politiques dans les années 1950 et 1960, mais qu'il n'est plus en phase avec nos connaissances actuelles, l'impact négatif des autoroutes sur la santé des riverains, ainsi que sur les phénomènes de ségrégation étant désormais bien documentés [166, 167, 168, 169]. Alors que ces infrastructures atteignent aujourd'hui leur fin de vie, de nombreuses villes vont être confrontées à la question de leur devenir. L'évaluation de la balance coût-bénéfice devra alors intégrer ces nouveaux éléments, conduisant probablement à la destruction d'un certain nombre d'autoroutes urbaines, comme ce fût par exemple déjà le cas des Voies sur Berges à Paris.

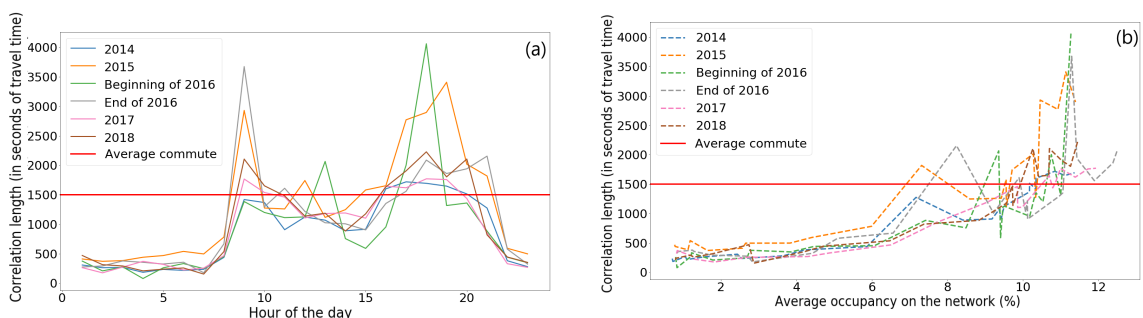
Un autre point important lorsque l'on évoque l'infrastructure routière, est son impact sur l'évolution de la ville. Pour l'heure, nous avons discuté de l'impact de la population sur l'infrastructure routière. Mais cette dernière influence également la façon dont la densité de population évolue. Ainsi, la construction des autoroutes urbaines, en permettant un accès rapide des banlieues aux centre-villes, a contribué à un exode massif des classes moyennes américaines vers les *suburbs*, façonnant profondément et durablement la société. En particulier, cet exode a conduit à une forte hausse du nombre de kilomètres parcourus (phénomène connu sous le nom de trafic induit), avec un effet néfaste sur l'environnement et contribuant également à annuler le bénéfice de l'autoroute sur les temps de trajets des utilisateurs.

Evolution spatiale et temporelle du trafic à l'échelle urbaine

Un second axe de cette thèse est l'étude du trafic routier sur le réseau urbain. Si, pour des routes unidimensionnelles et sans intersections, des méthodes inspirées de la mécanique des fluides permettent de décrire le trafic à une échelle locale, comme une fonction continue du temps et de l'espace, le passage à l'échelle d'un réseau urbain complique sensiblement la tâche. En effet, les intersections

introduisent de fortes corrélations entre les différentes routes, tandis que les feux tricolores ou encore les trajectoires diverses des usagers introduisent une dimension stochastique, rendant vain l'espoir de pouvoir calculer le trafic en tout point du réseau et à tout instant. Ainsi, pour étudier le trafic routier en ville, on s'intéresse plutôt à des mesures prises à l'échelle de chaque route et à la façon dont les rues interagissent entre elles. Les principaux résultats permis par cette approche sont qu'il existe deux phénomènes principaux à l'oeuvre lors de l'apparition d'embouteillages en ville. D'une part, certaines rues du réseau sont sensibles aux embouteillages, a priori en raison d'une disparité entre la demande et la capacité de l'infrastructure [175, 176]. Ce phénomène dépend donc de la structure du réseau. Ensuite, les embouteillages se propagent depuis ces "sources" au reste du réseau. De façon remarquable, cette deuxième phase semble, elle, indépendante de la structure du réseau [179].

Le résultat de ces deux phénomènes est que, lorsque la demande en trafic augmente, les embouteillages percolent à travers le réseau, comme le montre par exemple l'étude la taille des zones embouteillées [177]. Durant cette thèse, nous avons proposé un argument supplémentaire en faveur de cette idée de percolation des embouteillages [186]. L'étude de la corrélation entre les niveaux de congestion des différentes routes du réseau parisien montre que les rues sont corrélées sur une distance typique qui dépend de l'heure de la journée et de la demande en trafic. En particulier, la longueur de corrélation est très faible la nuit (de l'ordre d'une centaine de mètre), indiquant qu'une perturbation locale n'aura qu'un impact de faible portée, mais atteint des valeurs bien plus grandes aux heures de pointes, de l'ordre de la taille du réseau entier. Ce phénomène, caractéristique d'une transition de phase, est cohérent avec l'idée d'une percolation des embouteillages. En outre, cela implique aussi qu'aux heures de pointe, toute perturbation locale aura *in fine* un impact global sur l'intégralité des usagers du réseau.



B. Longueur de corrélation ξ de la mesure du niveau de congestion des rues du réseau parisien en fonction du temps (a) et de l'occupation moyenne du réseau (b). La ligne horizontale indique la durée moyenne d'un trajet de navetteur, correspondant à une distance de l'ordre de la taille du réseau

Approche à l'échelle de la ville : le MFD

Il apparaît de ce résultat que l'échelle de la rue n'est pas forcément la plus pertinente si l'on souhaite étudier le trafic routier en ville. En effet, l'étude du trafic à l'échelle de la ville (ou plutôt de sous-parties bien définies de la ville) a récemment gagné en intérêt et donné lieu à des résultats puissants et intrigants. Daganzo et Geroliminis [197, 198] ont ainsi trouvé empiriquement que la densité de voitures moyen k et le trafic moyen q dans une zone donnée exhibent une relation bien définie et semble-t-il indépendante de la matrice origines-destinations des usagers. En particulier, la fonction $q(k)$ est concave et admet un maximum q^* pour une densité optimale k^* .

Cette description du trafic, connue sous le nom de diagramme fondamental macroscopique (MFD en anglais), par opposition au diagramme fondamental connu pour une route qui peut être dérivé avec des arguments hydrodynamiques, reste pour l'heure mal comprise. Ceci n'a pas empêché l'apparition et la mise en pratique de stratégies de contrôle du trafic à l'échelle de la ville reposant sur la connaissance du MFD. En particulier, des stratégies de contrôle de périmètre par péage urbain ont été mises en place dans des villes comme Londres : la tarification variable permet de contrôler le nombre de véhicules présents dans le centre-ville, et de garantir que le système est toujours le plus proche possible de son optimum (k^*, q^*) [217, 218, 219, 220, 221].

L'une des questions ouvertes importantes concernant le MFD est la relation entre sa forme (conditionnée principalement par ses paramètres critiques k^* et q^*) et la structure du réseau sous-jacent. Quelques articles récents ont étudié cette question avec une approche empirique [226, 233, 235]. Durant cette thèse, nous avons proposé une approche basée sur des arguments dimensionnels et une simulation numérique pour affiner notre connaissance du MFD [227]. Notre modèle repose sur l'automate cellulaire ChSch proposé par Chowdury et Schadschneider [112, 228]. Le temps et l'espace sont discrétisés et les voitures se déplacent à chaque pas de temps d'un nombre entier de "cellules". Le réseau est quant à lui une matrice 2D (toutes les routes ont la même longueur), dont nous pouvons enlever quelques routes. Les intersections sont équipées de feux, conformément au modèle ChSch et les voitures choisissent aléatoirement une nouvelle direction à chaque intersection. Notre modèle repose sur les paramètres suivants :

La forme la plus générale pour k^* , q^* est alors :

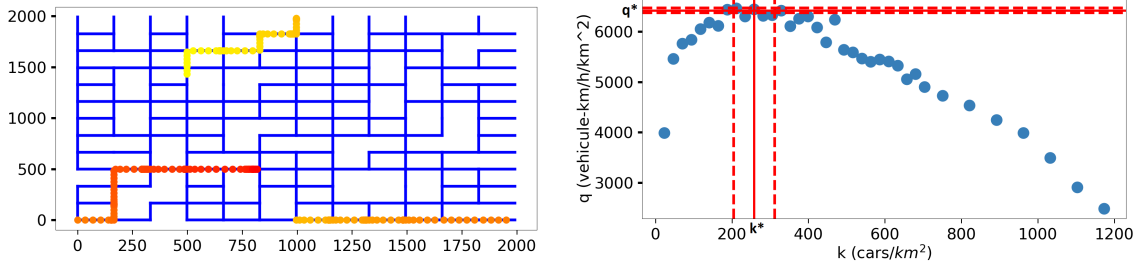
$$k^* = F(\rho_i, \rho_r, L_{car}, v_{max}, p, \dots) \quad (V.1)$$

$$q^* = G(\rho_i, \rho_r, L_{car}, v_{max}, p, \dots) \quad (V.2)$$

où F, G sont à ce stade des fonctions inconnues. L'analyse dimensionnelle permet de réécrire ces

Variable	Description	Unit	Default value
q^*	Network capacity	$km/h/km^2$	
k^*	Critical accumulation	$cars/km^2$	
ρ_r	Density of roads in space	$lane \cdot km/km^2$	
ρ_i	Density of intersections in space	km^{-2}	
p	Fraction of links missing compared to regular lattice	Number $\in [0; 1]$	0
L_{car}	Length of our simulation cell	km	$7m$
τ	Discrete timestep of the simulation	h	2 seconds
v_{max}	Maximal speed on our simulated network	h^{-1} (cells/timestep)	5 cells/ τ
t_{tl}	Traffic light timing (duration of a green or red light)	h	30 seconds
V_s	Average speed of the cars in our simulation	h^{-1} (cells/timestep)	
V_{lim}	Average speed on the empty network, combination of L_{car} , v_{max} and t_{tl}	km/h	

C. Définition des variables utilisées dans notre modèle



D. *Gauche* : Illustration de la trajectoire d'un véhicule sur notre simulation durant 100 pas de temps consécutifs (du jaune au rouge). *Droite* : illustration du MFD correspondant à cette simulation. Nous utilisons ici un réseau de 13×13 noeuds, avec des routes de longueur $166m$ et $p = 0.2$. Nous utilisons des conditions aux limites périodiques. Notons que, de part la forme du MFD, l'estimation de k^* est naturellement plus bruitée que celle de q^* .

fonctions en faisant apparaître la quantité adimensionnée $n = \rho_r/L_{car}\rho_i$, qui correspond au nombre de voitures pouvant être présente sur une route du réseau. On a alors :

$$k^* = \frac{\rho_r}{L_{car}} \Phi(n, p, t_{tl}, v_{max}) \quad (V.3)$$

$$q^* = \frac{\rho_r}{L_{car}} V^*(n, p, t_{tl}, v_{max}) \quad (V.4)$$

A l'aide de notre simulation numérique, nous testons alors l'influence de chaque paramètre du réseau indépendamment sur k^* et q^* . Cette capacité à tester les paramètres indépendamment représente une plus-value importante vis-à-vis d'une analyse empirique, les paramètres du réseau étant a priori fortement corrélés. Nous aboutissons alors aux formes fonctionnelles suivantes :

$$k^* \approx \frac{\rho_r}{L_{car}} \left(\frac{\rho_r}{L_{car}\rho_i} \right)^\beta \quad (V.5)$$

où $\beta \approx -0.1 \pm 0.1$ ($R^2 \approx 0.5$), et

$$q^* = \rho_r V_{lim} \left(1 - e^{-n/n_c(p, t_{tl})} \right) \quad (V.6)$$

où V_{lim} est la vitesse limite des véhicules se déplaçant sur le réseau vide en respectant les feux tricolores, n est le nombre moyen de voitures par route et n_c est une taille critique des routes qui dépend de l'irrégularité du réseau (capturée dans le paramètre p) et du durée du cycle des feux (paramètre t_{tl}).

Introduire des intersections de degré 3 en place d'intersections de degré 4 (c'est à dire augmenter la valeur de p) a ainsi un impact négatif sur le trafic : n_c augmente rapidement avec p , impliquant que, toutes choses égales par ailleurs, la capacité du réseau q^* a une valeur plus faible.

Concernant l'impact de t_{tl} , il apparaît qu'il existe une plage de valeurs optimales (autour de 30 secondes) minimisant n_c et maximisant donc q^* . Ce résultat est bien connu en ingénierie du trafic et peut être compris par des arguments de probabilité des files d'attentes.

Ces résultats viennent compléter l'approche empirique déjà existante, avec pour intérêt principal la possibilité de tester l'influence de chaque paramètre indépendamment, permettant ainsi des résultats plus fins et une meilleure compréhension de l'impact de la structure du réseau sur le MFD, qui pourra être bénéfique pour planifier au mieux de futures modifications apportées à la voirie.

De nombreuses questions restent cependant ouvertes, notamment concernant le rôle de l'hétérogénéité (par exemple en introduisant différents types de rues dans le réseau) sur le MFD.

Conclusion

L'objectif de cette thèse était d'offrir une perspective différente sur le trafic routier urbain, en appliquant des méthodes dérivées de la physique statistique. Cette approche a été motivée par l'observation que la structure des réseaux routiers et les modèles de trafic qui y apparaissent sont en fait très similaires entre les villes de différents pays et de tailles très différentes.

Dans un premier temps, nous avons ensuite étudié plus en détail l'apparition des autoroutes urbaines et des périphériques au cours du XX^e siècle. Les résultats empiriques montrent un seuil de population bien défini séparant les villes avec ou sans ces infrastructures, ce qui suggère qu'un critère simple a dicté la décision de les construire. Nous avons donc proposé une analyse coût-bénéfice simple basée sur l'équilibre entre le coût de construction et la valeur du temps, expliquant nos observations.

Nous nous sommes ensuite intéressés à la dynamique du trafic sur un réseau donné. En particulier, nous avons présenté nos propres résultats, montrant que la longueur de la corrélation de la congestion du trafic est faible lorsque la demande de trafic est faible, mais qu'elle diverge pendant les heures de pointe, ce qui indique que le système subit une sorte de transition de phase lors de l'apparition d'embouteillages.

Nous avons enfin présenté la relation intrigante qui émerge entre la densité moyenne de voitures et le trafic automobile à l'échelle du réseau, appelée MFD. Nous avons présenté nos résultats basés sur une analyse dimensionnelle et une simulation simple, montrant que les paramètres les plus importants définissant le MFD sont la densité des routes dans l'espace, la distance entre les intersections, ainsi que la régularité du réseau. D'autres études sont nécessaires pour confirmer ce dernier point, mais nous avons constaté que, à partir d'un réseau 2D régulier, la suppression d'un petit nombre de liens entraîne une réduction significative de la capacité de trafic.

Pour finir, nous souhaitons souligner l'importance de considérer la question du trafic urbain et des réseaux routiers urbains dans le cadre plus large de la mobilité urbaine et de la qualité de vie urbaine en général. L'exemple du Federal Highway Act, qui a conduit à la construction d'autoroutes urbaines dans un grand nombre de villes américaines, montre à quel point l'infrastructure urbaine a un impact sur le développement urbain, social et économique ultérieur. Le transport routier n'est qu'un aspect d'une mobilité urbaine efficace et le développement de nouveaux outils de gestion du trafic, tels que le MFD, ne doit jamais perdre de vue ce fait. Les décisions concernant la mobilité ou la voirie sont en fin de compte multifactorielles et politiques.

Néanmoins, l'étude du trafic urbain est un domaine fascinant, y compris pour les physiciens, dans lequel beaucoup reste à découvrir.

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