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Electric vehicle charging scheduling problems : complexity and solution approaches

Imene Zaidi

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Imene ZAIDI

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*To my beloved Valentin.
In the memory of my beloved mother, Linda.*

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Abstract

It is widely recognized that greenhouse gas emissions are primarily responsible for climate crises. According to Our World Data, road transport was responsible for 11.9% of these emissions in 2020. Further, 60% of road transport emissions come from traveling by cars, motorcycles, and buses. Therefore, electrifying the road transport sector is a key solution to effectively reduce emissions and adopt cleaner energy, leading to the popularity of electric vehicles. Nowadays, owning an electric car has many benefits, like getting funds and grants from the government and driving and parking in restricted traffic zones. Moreover, with technological advancements, electric vehicles are becoming more attractive. They have improved range, wider model availability, and increased performance. For all these reasons, Electric vehicle sales keep breaking records year after year. According to the International Energy Agency, the number of electric vehicles attended 16.5 million in 2021, double the amount in 2019. However, charging an electric vehicle is time-consuming and requires considerable electric power. The large-scale adoption of electric vehicles will drastically increase the number of charging demands leading to the saturation of charging stations, long queues, and poor service quality. Moreover, the extra load created by the upcoming electric vehicle charging demands will have numerous detrimental electrical grid impacts. It is estimated that an uncontrolled large-scale adoption of electric vehicles in European countries would increase peak demand by 35% to 51%.

Scheduling the charging load becomes crucial to avoid these negative impacts without imposing expensive network reinforcements or upgrading the existing power grid. This thesis studies the electric vehicle scheduling problem in a single charging station. First, an overall background on electric vehicle technology and a review of studies on electric vehicle charging scheduling problems are provided. The latter are classified according to the objective functions, constraints, assumptions, and solving methods. Then, a variant of the electric vehicle charging scheduling problem with chargers delivering preemptive constant power rates is stated. Each charger is installed in a parking space and has one connector where a vehicle can be plugged in for charging. We are interested in finding the minimum chargers required to park a set of electric vehicles from their arrival to their departure times. Then, we tackle the problem of minimizing the power grid capacity needed to satisfy the charging demands of these vehicles. These two problems are investigated in the case of identical and non-identical chargers. Later, we fix the number of chargers and the grid capacity and change the objective to find the maximum charging demands we can satisfy. Afterward, we tackle the problem with constant and variable charging rates. For this problem, we maximize the delivered energy to each vehicle. For all these problems, assigning vehicles to a parking space with a suitable charger is a decision part of the schedule. For each studied problem, we provide complexity analysis, mathematical formulations, solving approaches, and extensive computational results to show the performance of the proposed algorithms on generated instances. Finally, key findings, a general conclusion, and clues for future research are summarized.

Keywords: Electric vehicles, Charging scheduling, Optimization, Heuristics, Metaheuristics, Complexity analysis.

Résumé

Il est incontestable que les émissions de gaz à effet de serre sont les principales responsables du réchauffement climatique. Selon Our World Data, le transport routier contribuait à 11,9% de ces émissions en 2020 et 60% de cette contribution proviennent des déplacements en voiture, moto et bus. Par conséquent, l'électrification du secteur du transport routier devient une solution clé pour réduire efficacement les émissions et favoriser l'adoption des énergies propres, d'où la popularité des véhicules électriques. Aujourd'hui, posséder un véhicule électrique présente de nombreux avantages écologiques et économiques. Plusieurs gouvernements ont mis en place différentes aides financières afin de favoriser l'acquisition des véhicules électriques. De plus, il n'existe pas de restrictions de circulation pour les véhicules électriques dans les zones à faibles émissions, qui d'ailleurs, deviennent de plus en plus nombreuses dans les agglomérations. En outre, avec les progrès technologiques, les véhicules électriques deviennent de plus en plus attractifs. Aujourd'hui, la diversité des modèles de véhicules électriques s'est largement densifiée laissant à l'acheteur un panel de sélection élargi. Ces modèles ont une meilleure autonomie et des performances accrues. Pour ces raisons, les ventes de véhicules électriques continuent de battre des records année après année. Selon l'Agence internationale de l'énergie (IAE), le nombre de véhicules électriques immatriculés a atteint 16,5 millions en 2021, soit deux fois plus qu'en 2019. Cependant, la recharge d'un véhicule électrique est chronophage et nécessite une immense quantité d'énergie électrique. Par conséquent, l'adoption de ces véhicules à grande échelle augmentera considérablement le nombre de demandes de recharge, ce qui entraînera la saturation des stations de recharge, de longues files d'attente et une mauvaise qualité de service. En outre, la charge supplémentaire créée par ces demandes aura de nombreux effets néfastes sur le réseau électrique. On estime qu'une recharge non pilotée à grande échelle dans les pays européens augmenterait le pic de consommation électrique de 35 à 51%. Pour éviter tous ces impacts négatifs, la planification et l'ordonnancement de la recharge devient cruciale. D'ailleurs, le but de cette thèse est de répondre à des problématiques liées à l'ordonnancement de la recharge de véhicules électriques dans une seule station de recharge. Tout d'abord, un aperçu sur la technologie des véhicules électriques et une revue des études sur les problèmes d'ordonnancement de la recharge de ces derniers sont fournis. Ces études sont classées en fonction des différentes fonctions objectives, des contraintes, des hypothèses et des méthodes de résolution. Ensuite, nous avons commencé par étudier le problème d'ordonnancement de la recharge des véhicules électriques avec des bornes fournissant des puissances constantes mais qui peuvent être préemptives. Chaque borne de recharge est installée dans un espace de stationnement et possède un seul connecteur où un véhicule peut être branché pour être rechargé. Nous nous intéressons à la recherche du nombre minimum de bornes de recharge nécessaires pour garer un ensemble de véhicules électriques depuis leurs arrivées jusqu'à leurs départs. Ensuite, nous abordons le problème de la minimisation de la capacité du réseau électrique nécessaire pour satisfaire les demandes de recharge de ces véhicules. Ces deux problèmes sont des problèmes tactiques où l'objectif est de trouver la quantité minimum de ressources (bornes de recharge, puissance, capacité de réseau) requises pour satisfaire un ensemble de demandes de recharge. Ils sont étudiés dans le cas où les bornes de recharge installées sont identiques ou non-identiques. Ensuite, nous passons à la version opérationnelle de ce problème où nous considérons que le nombre de bornes

ainsi que la capacité du réseau sont fixés. L'objectif est de trouver le nombre maximum de demandes de recharge que nous pouvons satisfaire. Par la suite, nous abordons le problème avec des puissances de recharge qui peuvent être soit constantes soit variables et nous relaxons la contrainte sur les arrivées des véhicules, i.e., un véhicule peut être branché à une heure ultérieure à celle prévue par le conducteur selon la disponibilité des bornes de recharge. Pour ce problème, la fonction objective est de maximiser l'énergie fournie à chaque véhicule. Pour tous ces problèmes, la décision concernant l'affectation des véhicules aux bornes appropriées relève aux algorithmes décisionnels. Pour chaque problème étudié, nous fournissons une analyse de complexité, des formulations mathématiques, des approches de résolution et des résultats détaillés pour démontrer la performance des algorithmes proposés sur les instances générées. Enfin, nous résumons les principaux résultats et citons les perspectives ouvertes par ces travaux de recherche.

Mots clés: Véhicules électriques, Ordonnancement de recharge, Optimisation, Heuristique, Métaheuristique, Analyse de complexité.

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General Introduction

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1.1 Context and Motivation

In recent years, the considerable development of the transportation sector has made it the main contributor to energy consumption and greenhouse gas emissions. As an example, transport accounts for 40% of CO_2 emissions in France in 2021¹, where 82% of passengers travel by car. In addition to the climate crisis, the surge in international fossil fuel prices and the advancement in electric vehicle technology accelerated the adoption of electric vehicles as a great alternative green technology. According to the International Energy Agency [IEA 2022], the number of electric vehicles attended 16.5 million in 2021, double the amount in 2019. Moreover, electric vehicle sales keep breaking records year after year. In 2021, almost 10% of vehicle sales were of electric models, four times the market share compared to 2019.

With this increase in sales, more investment is needed to support and extend charging infrastructures, which are critical to ensure the competitiveness of electric vehicles. Nowadays, electric vehicle owners mostly tend to charge their vehicles at home. However, they state that public charging is essential, mainly in workplaces. Moreover, the increase in the deployment of public chargers will facilitate longer trips, reduce range anxiety, and convince more consumers to purchase an electric vehicle, especially those who cannot access private charging. In 2021, more than 1.8 million public chargers were installed worldwide, an increase of 37% from the previous year [IEA 2022].

When electric vehicle adoption increases significantly, new challenges for electrical grid and charging infrastructure operators rise. On the one hand, the increasing power consumption due to charging will overload the grid and increase power losses and voltage deviation [Rahman *et al.* 2022]. On the other hand, charging infrastructure operators must meet the upcoming demands, maximize customer satisfaction, avoid long queuing, and minimize costs while respecting the power grid constraints. Therefore, it is crucial for these operators to include optimization strategies. Recently, there has been growing interest in developing electric vehicle

¹<https://www.climate-transparency.org/countries/europe/france>

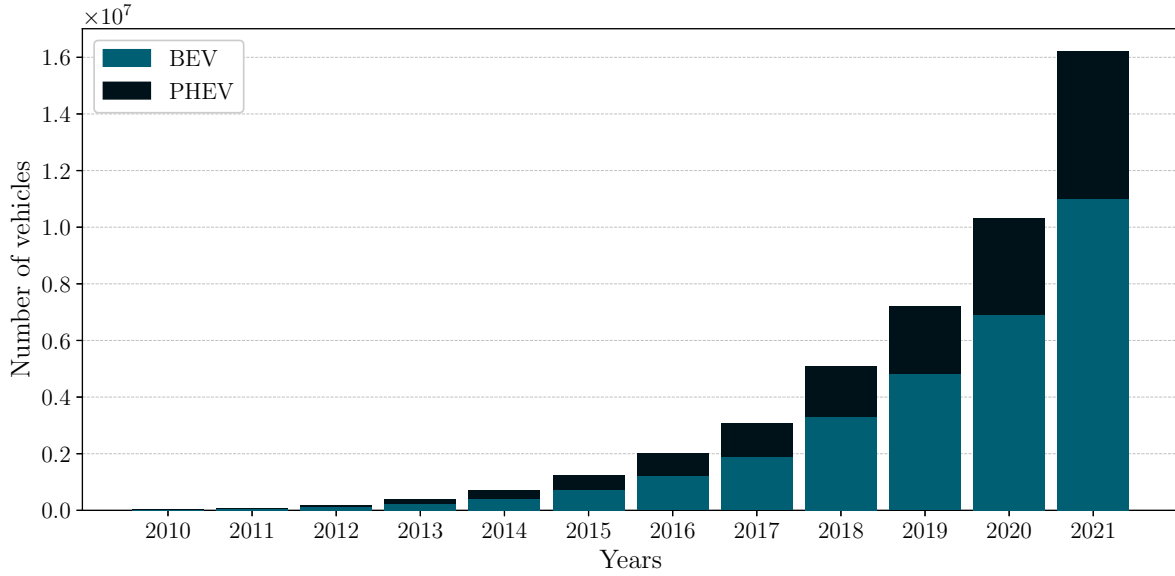


Figure 1.1: Electric vehicles stock, 2010-2021. BEV are battery electric vehicles. PHEV are plug-in hybrid electric vehicles. *Source: IEA 2022 [IEA 2022].*

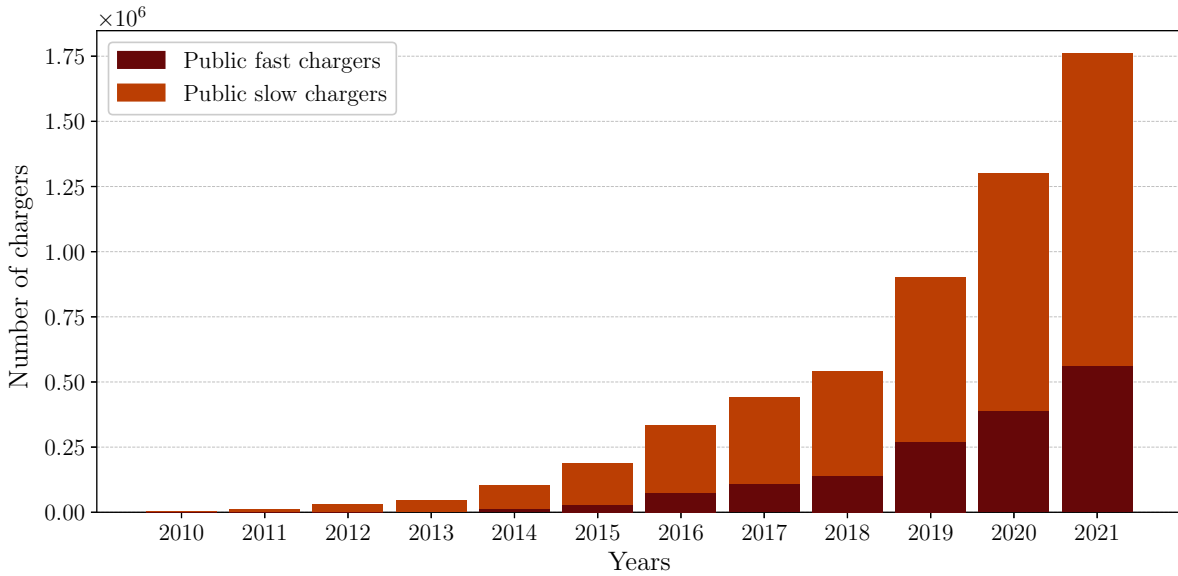


Figure 1.2: Total stock of electric vehicle chargers. Fast chargers have a charging power above 22 kW. Slow chargers have a charging power below 22 kW. *Source: IEA 2022 [IEA 2022].*

charging scheduling strategies that focus on economic objectives such as minimizing electricity costs or enhancing power grid reliability by minimizing power losses and voltage deviations. However, many studies assume that there is a sufficient number of chargers and focus on power allocation while neglecting the decision to assign electric vehicles to suitable chargers. Besides,

they assume an identical output power of chargers. Nevertheless, charging stations usually install chargers with different output power to satisfy more charging demands and improve service quality.

In this thesis, we consider optimizing the charging process in a public charging station. The charging station has a limited number of chargers and a maximum power capacity that the distribution-level transformer limits. A management system can remotely control chargers' activation or deactivation at any time. Each electric vehicle driver can submit a charging reservation demand before arriving to avoid queues.

1.2 Contributions

The general formulation of an instance of the electric vehicle charging scheduling problem can be defined as follows. There are a set of electric vehicle charging demands and a set of chargers. The total power delivered by all chargers must not exceed at any time. Each charging demand is characterized by its arrival time, departure time, and energy requirement. The required energy can be expressed directly in kWh or by providing the desired state-of-charge levels and the battery capacity. At each time, a charger can only charge one vehicle, and a vehicle can only be charged by one charger. The charging scheduling can either be preemptive or non-preemptive. Moreover, depending on the charger power rates, we can distinguish between two variants. The first one is called the constant power model, where chargers can deliver either their maximum output powers or zero. The second variant is the variable power model, where the charging rate of each charger varies over time from zero to its maximum output power.

We tackle different variants of the problem as follows. First, we assume that, when a charging demand is accepted, the vehicle must be plugged into a charger from arrival to departure. During this period, the required energy must be satisfied. We are first interested in preemptive charging scheduling with constant charging power rates. In this case, two tactical problems arise: the problem of finding the minimum number of chargers needed to plug all electric vehicles, and the problem of finding the minimum grid capacity required to charge all electric vehicles to their desired energy. We investigate these problems to carefully guide charging station operators to choose their subscribed maximum power. As the charging demands multiply, they will need to upgrade their capacity. In fact, power consumption peaks occur and cause high electricity bills. Generally, equipment such as power cutters and relays are installed at a small cost to avoid peaks, but they cause the system to shut down, which is not desirable. Consequently, it is essential to provide an overview of the minimum power limit depending on the installed charger types and charging demands.

After solving these problems, we move to operational problems. When scheduling all demands is impossible; thus we are interested in maximizing the number of satisfied charging demands. This problem is investigated in the case of identical and non-identical chargers.

Then, we investigate the problem where an electric vehicle is not necessarily charged to its desired energy. Moreover, to charge more vehicles, the plugging time of a vehicle can be later than the desired arrival time. Then, the vehicle will occupy a charger from the assigned plugging time until its departure time and cannot be plugged out during this period. We study the case where non-identical chargers are installed. The scheduling objective is to minimize the total

difference between the desired and the final state-of-charge levels. For this problem, we compare the constant power model and the variable power model.

For each variant of the problem, we study its complexity. If the problem is polynomial, we provide a polynomial solvable algorithm. Otherwise, we prove that it belongs to the complexity class \mathcal{NP} by reducing it to another \mathcal{NP} -hard problem. We propose different mathematical formulations. We propose heuristics and metaheuristics to obtain near-optimal solutions for the \mathcal{NP} -hard problems.

Finally, we study the electric vehicle charging scheduling problem in a private charging station where each vehicle has its own parking space as it is described in [Hernández-Arauzo *et al.* 2015]. The charging station is fed with a three-phase power supply. There are three lines, each carrying an alternating current of the same frequency and voltage amplitude from the source to the electrical outlets. Each line regroups a number of identical chargers that deliver power at constant rates. There is a limited number of chargers that can deliver power simultaneously due to charging station physical constraints. The first constraint defines the total power that can be drawn from each line to avoid overloading the system. The second constraint maintains the phase balance between the three lines. In fact, in a three-phase power system, the load should be distributed evenly between the three lines to minimize power losses and improve system reliability. In this problem, no reservation system is considered, and electric vehicles arrive at random unknown instants during the day with different request energies and departure times. However, vehicles are not allowed to leave before completing their charging, which may result in delays compared to requested departure times, called tardiness. Therefore, the objective is to build a real-time schedule that minimizes the total tardiness subject to the technical constraints of the charging station. We consider both preemptive and non-preemptive charging schedules. First, the problem is formulated as a mixed-integer linear programming (MILP) model. Then, a heuristic based on a priority rule is proposed to solve the online problem. Additionally, we implement a simple local search to improve the objective function value for preemptive charging.

1.3 Outline of the Thesis

The motivation and context of the thesis are outlined in this introductory chapter. In the next chapter, situate our study by reviewing the relevant literature and determining the research gap in the field. The first variant of the electric vehicle charging scheduling problem is presented and studied in Chapter 3. Chapter 4 is dedicated to the problem of maximizing the number of satisfied charging demands. Chapter 5 states the problem of maximizing the delivered energy and compares the constant and the variable power models. Chapter 6 is devoted to the online charging scheduling problem in an unbalanced three-phase power system. In the last chapter, we summarize the results of our works and outline the directions for future research.

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2.1 Introduction

This chapter provides the necessary background, terminology, and relevant related works to better understand the electric vehicle charging scheduling (EVCS) problems addressed in this thesis. The EVCS is an optimization problem closely related to the job scheduling problem with additional resources. Therefore, we provide a brief overview of the job scheduling problem to understand some of the complexity results and algorithms presented in this thesis. Then, we briefly review electric vehicle technologies, such as charging modes and the different entities involved. An essential part of this chapter is devoted to classifying and reviewing different works on electric vehicle charging scheduling problems. Then, we can position our problem and define the research gap that can be filled.

2.2 Optimization and Job Scheduling

Optimization problems involve finding the best solution that optimizes one or more objective functions while satisfying constraints. One of the well-studied optimization problems is the job scheduling problem. It involves allocating resources, such as machines, to perform jobs (also called activities or tasks) to optimize one or more objective functions. There are many

different variants of job scheduling problems, depending on the nature of jobs, the resources, the constraints on the schedule, and the objective function. In scheduling theory, the set of jobs, usually denoted \mathcal{J} , has characteristics that depend on the considered problem. For example, a job may have a processing time, a release date to design the earliest time at which the job can be processed, and a due date, which represents the time at which the processing of the job should be finished. Moreover, scheduling problems usually define a set \mathcal{M} of machines with characteristics that also depend on the definition of the problem. For example, we can find scheduling on single or identical parallel machines. Additional scheduling constraints can be defined, such as preemption, precedence constraints, and additional resources. Many objective functions exist in the literature, for example, minimizing the number of late jobs, minimizing the makespan, which corresponds to minimizing the time at which the last executed job ends, and so on.

Authors in [Graham *et al.* 1979b] introduced the $\alpha | \beta | \gamma$ notation to classify scheduling problems. The α field defines the machine environment. Job characteristics and scheduling constraints are specified by the β field, while the field γ defines the objective function. For example, $P | r_i | C_{max}$ designs parallel machines scheduling with release dates to minimize the makespan. Comprehensive book of classic scheduling problems with their most advanced and timely topics can be found in [Leung 2004].

It is worth mentioning that machines are not necessarily defined in all scheduling problems. An entire branch of scheduling theory called the Resource-Constrained Project Scheduling Problem (RCPSP) does not systematically use machines. The RCPSP is a well-known optimization problem where we are given a set of resources with specific capacities and a set of jobs. Each job has a duration and resource requirement per time unit. In addition, there are precedence constraints between these jobs. The principal objective of the problem is to schedule all jobs without exceeding resources' capacity so that the total duration of the project is minimized. This problem is proven to be \mathcal{NP} -hard [Blazewicz *et al.* 1983]. A comprehensive review of variants and extensions of the RCPSP can be found in [Hartmann & Briskorn 2010]. In recent decades, the RCPSP has attracted the interest of researchers because it embodies a wide range of scheduling problems, such as job shop and flow shop problems. One interesting special case of RCPSP is cumulative scheduling, in which a release and due date are added to each job, the precedence constraints are relaxed, and a single cumulative resource is considered at a time. The cumulative scheduling problem (CuSP) is \mathcal{NP} -complete in the strong sense [Baptiste & Le Pape 1997]. Recently, [Nattaf *et al.* 2015] considered a variant of cumulative scheduling with a cumulative, continuous, and renewable resource and presented a hybrid branch-and-bound method to solve it. Energetic reasoning, introduced by [Lopez 1991], is an efficient tool for dealing with the CuSP. Energetic reasoning is based on determining which jobs must be processed in any feasible schedule between two instants. Later, it was adapted by [Baptiste & Le Pape 1997, Nattaf *et al.* 2015] to develop a polynomial satisfiability test for their problems. When the resource consumption is uniform, and there are no constraints on its capacity, the problem turns into the interval scheduling problem, which is polynomially solvable.

This thesis focuses on the problem of electric vehicle charging scheduling (EVCS) at charging stations with a fixed number of chargers (charging points) and available power. This problem can be viewed as a resource-constraint scheduling problem, in which the jobs to be scheduled

are the charging demands and the resources are parking places, chargers, and electric energy.

2.3 Electric Vehicle Charging Technology

2.3.1 Electric Vehicles

According to their powertrains, electric vehicles can be classified into two main categories: hybrid electric vehicles (HEV) and all-electric vehicles (AEV). An HEV combines an internal combustion engine with an electric motor. It cannot be plugged in to charge its battery from external sources like the power grid. Instead, it is charged by the internal combustion engine or through an energy recovery mechanism called regenerative braking. This mechanism converts the vehicle's kinetic energy to electric energy to be stored in the battery. A plug-in hybrid electric vehicle (PHEV) is one type of HEV with a larger battery pack that can be recharged from the power grid. Meanwhile, an AEV, also called a pure-electric vehicle, is powered exclusively by electrical sources. We can classify AEVs into battery electric vehicles (BEV) and fuel cell electric vehicles (FCEV). A BEV relies on the power provided by the battery, which is recharged by plugging it into the power grid. An FCEV does not require an external charging system. Generally, it generates electricity using oxygen from the air and compressed hydrogen. In this study, we use the term "electric vehicle" to refer to both BEV and PHEV.

2.3.1.1 Electric vehicle battery

The battery is the core component of an electric vehicle, and it plays an essential role in developing and adopting electric vehicles since the driving range depends on it. The most commonly used rechargeable batteries are lithium-ion batteries since they have a higher energy density, a higher efficiency, and a longer cycle life than other types of batteries, like lead-acid batteries and nickel-metal hydride batteries [Andwari *et al.* 2017]. Several terms have been defined for batteries to characterize their performance. We summarize the most frequently used ones as follows.

- *Battery capacity*: it corresponds to the amount of electricity stored in the battery and can be released during the discharge. Usually, it is expressed in watt-hours (wh). A watt-hour is equivalent to the battery voltage multiplied by the current (in amperes) that can be drawn from the battery for an hour. The ampere-hours (Ah) measurement is the most commonly used since voltage is fixed for a battery type due to its internal chemistry.
- *Battery state of charge (SOC)*: it represents the ratio of the remaining battery capacity to its nominal capacity expressed in percentage.
- *Battery depth of Discharge (DOD)*: it represents the amount of utilized battery capacity in percentage.
- *Battery cycle life*: it represents the lifetime of a battery expressed as the number of charge-discharge cycles the battery can experience before its performance degrades significantly. The depth of discharge mainly affects the cycle life of lithium-ion batteries. For instance, a

battery discharged by 80% of its capacity has a much shorter cycle life than one discharged by only 20%. Battery manufacturers recommend maintaining the SOC between 20% and 80% to prolong battery life [Kostopoulos *et al.* 2020]. The cycle of life is an essential parameter for the widespread adoption of electric vehicles. In fact, when it is less than the vehicle's lifespan, the battery has to be changed after a few years, resulting in additional costs.

- *Energy density*: it refers to the energy stored in a battery per unit volume (wh/m³). Also, it can be defined as energy per unit mass (wh/kg). Energy density is essential since it specifies the battery size needed to reach a specific range.
- *Battery management systems (BMS)*: BMS is a set of electronic components and software algorithms designed to monitor battery state and ensure its safety. The BMS is essential for collecting information, such as the battery current, voltage, and temperatures. It processes those information to regulate the charging and discharging of the battery, update the SOC, ensure its optimum performance, and protect it from damages. In addition, the BMS ensures communication with the vehicle computer to provide information to the driver. The BMS may fulfill additional functions as well. [Andwari *et al.* 2017].

2.3.1.2 charging modes

The development of charging technology is crucial to the success of electric vehicles. Recently, charging is becoming more convenient and faster. Depending on the energy transfer mode, battery charging can be classified into three categories: conductive charging, inductive charging, and battery swapping. Each one of these methods requires different charging infrastructure and equipment.

- *Conductive charging*: in conductive charging, also called wired or plug-in charging, a physical connection is required to transform electrical power from the grid to the electric vehicle. There are two types of conductive chargers: on-board and off-board chargers. On-board chargers are built inside the vehicle and are generally used for slow charging. This topology allows the driver to charge his vehicle wherever an electric power outlet is available. In contrast, off-board chargers are located outside the vehicle and are mainly deployed for fast charging in parking lots or highways.
- *Inductive charging*: also known as wireless charging, it allows power to be transferred without wires or any physical connection. Generally, charging systems are installed under the road so vehicles can park or drive on them and efficiently charge simultaneously. There are three main modes of wireless charging: stationary, quasi-dynamic, and dynamic inductive charging [Jang 2018]. With stationary inductive charging, electric vehicles are charged when they are parked. In quasi-dynamic systems, charging can take a short time in a dynamic environment like bus stops and traffic lights. In dynamic inductive charging, electric vehicle drivers can charge their batteries while driving on the road, so they do not have to stop at a charging station. Therefore, wireless charging can be more convenient for electric vehicle drivers to overcome range anxiety and long waiting times. Some dynamic

charging systems are already commercialized, such as the On-Line Electric Vehicle (OLEV) deployed in 2009 at the Korea Advanced Institute of Science and Technology (KAIST). However, they are still in the early stages of development [Jang 2018].

- *Battery swapping*: battery swapping stations (BSS) allow electric vehicle drivers to replace their discharged batteries with fully charged ones. This strategy can effectively reduce the long charging time since exchanging batteries takes less than five minutes. Moreover, BSSs can sell the stored energy into electricity markets to maximize profit. However, the deployment of BSS is still challenging and expensive since it requires the availability of different battery packs and various standards for battery attachment. In addition to the increased battery degradation costs and higher infrastructure investment to support the vehicle-to-grid [Ahmad *et al.* 2020, Wang & Wang 2020].

2.3.1.3 Charging standards

Several organizations, such as the Society of Automotive Engineers (SAE), the International Electrotechnical Commission (IEC), and the Institute of Electrical and Electronics Engineers (IEEE), have developed different international standards for charging systems. These standards define general requirements for equipment, charging plugs, connectors, and inlets for conductive charging. It also defines requirements for inductive charging and battery swapping. SAE standards, like SAE J1772, are widely applied in the United States and Japan, whereas IEC standards are more widely adopted in Europe. The United States’ manufacturers also use IEEE standards. China has defined its Guobiao (GB/T) standard, similar to the IEC 62196-2 standard but with a different design. Table 2.1 shows a summary of the voltage and current levels of SAE J1772. The IEC 62196 standard has equivalent requirements but a different set of terms. Instead of levels, the term mode is used [Khan *et al.* 2018, Das *et al.* 2020]. The IEC 62196 standard defines four modes: mode 1 for slow charging from a household-type outlet, mode 2 for slow charging from a household-type outlet but with an in-cable protection device, mode 3 for slow or fast charging using a specific electric vehicle outlet with an installed control and protection function, and mode 4 for DC fast charging with off-board chargers. More discussions and details about charging standards can be found in [Das *et al.* 2020].

Table 2.1: Charging levels defined in SAE J1772 Standard for conductive charging [Chalia *et al.* 2021].

Power supply	Power level	Phase (AC)	Voltage (V)	Current (A)	Charging power (kW)
AC	level 1	single phase	120	12 or 16	1.44 or 1.92
AC	level 2	single phase	208-240	24-80	5.00 - 19.20
DC	level 1		200-450	80	36
DC	level 2		200-450	200	90
DC	level 3		200-600	400	240

2.3.2 Electricity Power Grid

The electric power grid is the infrastructure that allows the transportation of electrical energy from production facilities to consumers. It incorporates four main sectors: generation, transmission, distribution, and consumption. The increase in electric vehicle deployment will drastically affect those sectors, and thus a reinforcement of the existing grid will be required. Different studies have been conducted on the impact of electric vehicle integration on the different sectors, especially the distribution system. A recent review of those studies can be found in [Rahman *et al.* 2022].

The traditional power grid is evolving and will eventually be replaced by Smart Grids that enable the exchange of information between the utility grid and consumers. Also, Smart Grid provides advanced real-time management, metering, monitoring, and control through smart meters, sensors, and embedded appliances [Dileep 2020]. The smart grid concept has been introduced to monitor system stability efficiently, manage electricity load, incorporate distributed energy, balance energy demand response, and provide real-time electricity consumption measurements. The readers can refer to the survey for comprehensive details and information about Smart Grid's characteristics, evolution, and benefits [Dileep 2020]. Many researchers have focused on electric vehicle charging in the context of Smart Grids since smart grid technologies introduce additional communication features and enable data collection so that charging can be shifted to off-peak hours. Moreover, Smart Grids facilitate vehicle-to-grid integration and promote charging with renewable resources [Ahmadi *et al.* 2019].

2.3.3 Smart Charging Stakeholders

Many companies are concerned about the future deployment of electric vehicles and the adoption of smart charging. Smart charging strategies may differ from the point of view of these stakeholders. Therefore, it is essential to identify the different entities to position the reviewed studies and our work.

- *Power grid operators:* Power grid operators are the companies that manage the electrical grid. The main operators related to the electric vehicle charging optimization literature are Transmission System Operators (TSO), responsible for the high and ultra-high-voltage grids, and Distribution System Operators (DSO), responsible for lines that carry low and medium-voltage power from the transmission system to consumers. TSOs experiment and study the electricity system's resilience to future mass adoption of electric vehicles and its ability to sustain real-time demand-supply equilibrium. DSO will also encounter technical challenges with the future increasing charging demands, especially on transformers in low voltage grids. Moreover, some DSOs can be the owners of a percentage of charging points in their region.
- *Charging points operators:* Charging Point Operators (CPOs) are companies that own private or public charging stations. CPOs can control charging points to schedule charging operations. They can maximize their profits and limit the negative impacts on power grids by optimally scheduling electric vehicle loads. In addition, historical information can be

collected and used to predict future charging demands and implement robust optimization algorithms.

- *Aggregators*: Generally, aggregators are companies that act as intermediaries between electric power system operators and end consumers. They can play an essential role in future smart grids in managing charging loads by regrouping a certain number of electric vehicles and participating in electricity markets following the charging regulation requirements dictated by the DSO. Also, charging station operators act like aggregators.

2.4 Electric Vehicle Charging Scheduling Problem

Nowadays, electric vehicle drivers mostly tend to charge their vehicles directly when they arrive at home or at the nearest charging station [Azadfar *et al.* 2015]. The charging begins immediately when the vehicle is plugged in and continues until the battery is fully charged. This is called uncontrolled charging. Uncontrolled charging negatively impacts the power grid, charging infrastructure, and vehicle drivers. Consequently, smart charging approaches in large-scale electric vehicle deployment are crucial for proper energy utilization, grid stability, and consumer satisfaction. Different charging control and scheduling strategies were proposed. There are two types of control strategies: centralized and decentralized. In centralized strategies, charging decisions are taken by a high-level controller, such as aggregators. Oppositely, decentralized control strategies allow each EV to define its own charging plan. Both strategies have their advantages and limitations. Centralized approaches face long computation times and difficulty in collecting accurate charging information from many electric vehicles due to the uncertainties in their charging behavior [Kang *et al.* 2016]. Decentralized control strategies reduce computational requirements since the computation is distributed to individual agents. Also, they enhance customers' convenience and satisfaction.

In this section, we focus only on reviewing EVCS in a centralized manner. Also, this review does not consider papers dealing with vehicle-to-grid (V2G) technology, where electric vehicles can discharge energy back to the grid. Only unidirectional power flow models are considered. Also, papers considering renewable energy sources, such as solar-based energy and energy storage units, are not included.

In the literature, the formulation of the EVCS problem varies significantly from one study to another, depending on the characteristics of the charging infrastructure and the operational model considered. There is neither a generic model nor general benchmarks yet. This is due to the great variety of aspects to consider in the EVCS formulation, including charging infrastructures, electric vehicle drivers' behaviors, the electricity market, and the politics of charging services. Each aspect can be regarded in the model as an objective, a constraint, a parameter, or a decision variable. Hence, we first classify existing studies according to the different objectives considered. Then, we state the different constraints, assumptions, and considerations that would allow us to characterize and distinguish the various branches of the literature. Finally, we review the optimization methods used to tackle the EVCS problems. Figure 2.1 shows a summary of the classification of objectives, constraints, and optimization methods with non-exhaustive examples.

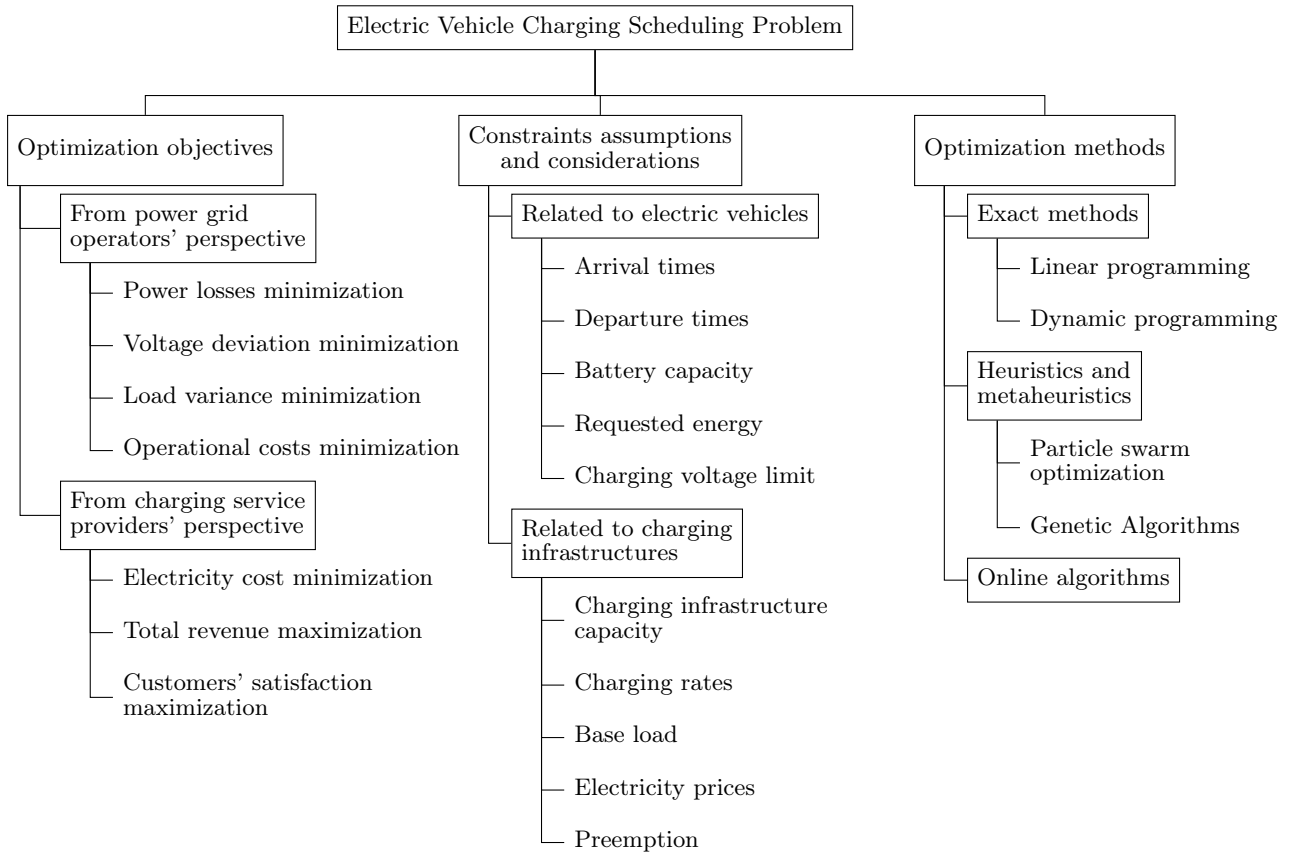


Figure 2.1: A classification of EVCS problem's objectives, constraints, and optimization approaches.

2.4.1 Objectives

A classification of problem objectives is required before classifying optimization methods used to solve them. We first classify EVCS problems into two classes from the stakeholders' perspective, namely, power grid operators and charging station operators. This classification is essential to distinguish the context of the study since, in the electricity sector, each unit is independent legally and functionally and has different objectives depending on its responsibility in the system. For example, the grid operators aim to maintain grid stability and reliability. On the other hand, the objectives of charging station operators involve maximizing profit and minimizing the electricity bills paid while respecting the grid constraints. Additionally, EVCS problems formulated from grid operators' perspectives will cover large and medium-scale distribution networks with multiple charging units (home charging, parking lots, commercial buildings). In contrast, the charging station operators will likely address the scheduling problems in the small and medium-scale charging infrastructures for which they are responsible. Despite the coupling between these two types of stockholders, they have been considered separately. Only a few papers consider both. Currently, it is more convenient for charging station operators to adopt smart charging strategies. Scheduling electric vehicle charging from grid operators' perspectives

will be more widely adopted in the context of the future Smart Grids, where electric vehicle aggregators manage geographically dispersed electric vehicles.

2.4.1.1 From power grid operators' perspectives

As mentioned before, the electric power system technically incorporates three operational sectors: generation, transmission, and distribution. Power grid operators are the entities in charge of managing these sectors. From the perspective of these operators, the increase in electricity demand raises concerns regarding the stability and reliability of the power grid. Moreover, it will result in more negative impacts, such as load peaks, significant voltage variance, power losses, lower energy efficiency, and an increase in voltage deviation. The extension and reinforcement of the power grid are typical approaches to address this problem. However, such a solution is not easy to achieve in the short term and requires expensive investments. A more attractive and economical alternative solution is implementing smart scheduling strategies. As a result, numerous works were proposed to minimize these issues and consider economic objectives, especially on distribution grids. Generally, the optimization problem is solved at the DSO level. We resume the further mentioned works from the power grid operators' perspectives in Table 2.2.

Table 2.2: Classification of some EVCS problems from the grid operators' perspectives.

Reference	Year	Objectives	Optimization approaches
[Clement-Nyngs <i>et al.</i> 2010]	2010	Minimize power losses, voltage deviations	Quadratic programming, dynamic programming
[Tan <i>et al.</i> 2013]	2013	Minimize voltage deviation, load variance, power losses	PSO
[Franco <i>et al.</i> 2014]	2014	Minimize power losses, the conventional loads, energy consumption and the energy that is not possible to provide to electric vehicles	a MINLP relaxed to a MILP
[Xu <i>et al.</i> 2014]	2014	Minimize the electricity purchase costs of all aggregators	Linear programming at DSO level, heuristics
[Lunci Hua <i>et al.</i> 2014]	2014	Minimizes the charging costs, optimizing power flow increments	Solving MILP models
[Hajforoosh <i>et al.</i> 2015]	2015	Minimize the active and reactive power loss, generation cost maximize delivered charging power	Online hybrid fuzzy discret PSO, hybrid Fuzzy GA
[Luo <i>et al.</i> 2016]	2016	Minimize load variance, the peak-to-valley load	Online algorithm
[Kang <i>et al.</i> 2016]	2016	Minimize charging cost, active power losses, voltage deviation	Improved PSO-GA
[Kang <i>et al.</i> 2017]	2017	Minimize load variation, revenue losses	Weight aggregation multi-objective PSO
[Suyono <i>et al.</i> 2019]	2019	Minimize charging cost, power losses, voltage deviation	Binary PSO, Binary Grey Wolves Optimization
[Liu & Zhou 2022]	2022	Minimize charging costs, peak-to-valley load	Heuristic for charging coordination

Cost minimization. The grid operators are more concerned about reducing the overall costs caused by electric vehicle charging, which can include energy generation costs [Hajforoosh *et al.* 2015], operational and electricity costs [Xu *et al.* 2014]. Authors in [Lunci Hua *et al.* 2014] minimize the overall daily charging and power loss costs in electrical distribution systems. In [Kang *et al.* 2017], the revenue losses are minimized. [Liu & Zhou 2022, Suyono *et al.* 2019] minimize the sum of charging cost of all electric vehicle drivers.

Power loss minimization. Power losses occur in distribution and transmission systems due to the Joule effect in power lines and transformers. Reducing these losses is essential for grid operators to enhance the power grid's efficiency and lower the operation cost. The introduction of electric vehicles increases power losses. For example, [Pieltain Fernandez *et al.* 2011] showed that power losses could increase by 40% in off-peak hours when most electric vehicles would be charged. There are two types of power loss: active power loss and reactive power loss. Studies can address the problem of minimizing one or both types as in [Hajforoosh *et al.* 2015, Franco *et al.* 2014]. The general formulation to calculate the active power loss used in [Tan *et al.* 2013, Kang *et al.* 2016, Clement-Nyns *et al.* 2010, Franco *et al.* 2014, Suyono *et al.* 2019] is defined as follows.

$$P_{loss} = \sum_{i=1}^N |I_i|^2 \times R_i \quad (2.1)$$

where N is total number of branches in the system, I_i is the magnitude of current flow in branch i and R_i is the resistance of branch i .

Voltage deviation minimization. Voltage deviation can be defined as the difference between the nominal voltage and the actual voltage in nodes. The smaller the deviation, the better the voltage condition of the system. These deviations may occur in distribution systems for different reasons, especially when the available reactive generation cannot meet the growing demand for reactive power on the customers' side. In other words, overloading the power system is the major contributor to the problem of voltage deviation. A *voltage deviation index* is defined as the sum of the squared value of the absolute voltage difference between the nominal voltage and the actual voltage for all nodes in the system.

$$V_{dev} = \sum_{i=1}^N |V_n - V_i|^2 \quad (2.2)$$

where N is the total network nodes, V_n is the nominal voltage, and V_i is the actual voltage at node i .

The voltage deviation cannot be below a given threshold. Thresholds are different from one country to another. According to the American national standard, the voltage deviation limit is $\pm 5\%$ of the nominal voltage, while it is limited to $\pm 10\%$ of the nominal voltage in Europe. [Kang *et al.* 2016, Tan *et al.* 2013, Suyono *et al.* 2019] includes the minimization of the voltage deviation in their objective function.

Load variance minimization. The load variance is generally defined as the expected value of the squared deviation from the mean of the load. Minimizing load variance will minimize the peak-to-valley load of the distribution system and improve power quality.

$$L_{dev} = \sqrt{\frac{1}{T} \sum_{t=1}^T (\bar{P} - P_t)^2} \quad (2.3)$$

where T denotes the number of the divided periods, \bar{P} is the average load of the distribution system or a specified district, and P_t is the scheduled load at time t .

[Tan *et al.* 2013] gave the relationship between minimizing the voltage deviation and minimizing the load variance. In [Kang *et al.* 2017], the electric vehicle load is scheduled to achieve grid stabilization. The problem is formulated as a multi-objective optimization problem. The first objective function is to minimize the load fluctuation caused by adding PHEVs to the grid. In [Luo *et al.* 2016], it is assumed that the electric vehicles charge directly from the distribution grid. The study combines the grid and transport system information for joint charging scheduling optimization and path planning. In charging scheduling, two strategies with two objectives are considered: minimizing the load variance in the distribution system as the fast charging load is added and the peak-to-valley load in a battery-swapping scenario.

[Luo *et al.* 2016, Kang *et al.* 2017, Liu & Zhou 2022] minimize the peak-to-valley load of the distribution system or a specified district defined as:

$$F = \max(P_t) - \min(P_t) \quad (2.4)$$

where $\max(P_t)$ and $\min(P_t)$ denote the values of the adjusted peak and valley loads, respectively.

2.4.1.2 From charging service providers perspective

Charging service providers (CSP) are companies that manage one or multiple charging stations installed in different locations like service stations, shopping centers, academic campuses, highway rest areas, or parking lots. The charging points installed can be private or public. Public charging points can be used by any electric vehicle and offer higher charging rates. These companies usually buy electricity from an energy supplier to meet the charging demands of their customers. Research works on EVCS problems from the perspective of CSPs focus on reducing electricity bills, maximizing their profits, and improving service quality while maintaining the physical constraints of the grid. We summarize the further mentioned works from the CSPs' perspectives in Table 2.3.

Cost minimization. Optimization problems with the objective of cost minimization are widely addressed. In EVCS problem formulations, cost minimization mainly involves reducing the CSP's electricity bills, considering the dynamic electricity pricing environment (see Section 2.4.2.13). Uncontrolled charging can lead to high electricity costs, which poses a significant obstacle to deploying public charging stations. This is particularly true for fast charg-

Table 2.3: Classification of some EVCS problems from the charging service providers' perspectives.

Reference	Year	Objectives	Optimization approaches
[Jin <i>et al.</i> 2013]	2013	Maximizing the aggregator's revenue, minimize the total charging cost of customers(as separate optimization problem)	Solving LP for static charging, a heuristic for the dynamic charging
[Tang <i>et al.</i> 2014]	2014	Minimize cost on electricity bill paid by the charging station	Online algorithm (OCHARD)
[Zheng & Shroff 2014]	2014	Maximize the total value of served vehicles, minimize the energy cost	Online algorithm (On-Arrival Allocation with Revocation)
[Jewell <i>et al.</i> 2014]	2014	minimizes the total makespan	Solving a MILP, heuristics, simulated annealing
[Zhu <i>et al.</i> 2014]	2014	Minimize the total charging time	Heuristic algorithms
[Bučar 2014]	2014	Maximize the total profit	MILP, local Ratio Technique, greedy heuristics, local search
[Ma <i>et al.</i> 2015]	2015	Minimize the charging cost to fully charge all electric vehicles	Offline: solving a binary LP, online: moving window optimization scheme
[Kuran <i>et al.</i> 2015]	2015	Maximizing parking lot revenues, maximizing the number of recharged electric vehicles	Solving a MILP, multi-layer system to implement day-ahead and real-time scheduling
[Tang & Zhang 2016]	2016	Minimize the total charging cost	Online algorithm, model predictive control based algorithm, a finite-horizon dynamic programming formulation
[Rahman <i>et al.</i> 2016]	2016	Maximizing the average SOC	Accelerated PSO
[?]	2017	Minimizing the preallocated energy	Heuristic using randomized algorithm approach
[Wu <i>et al.</i> 2018]	2018	Minimize the electricity cost	Fuzzy PSO algorithm
[Ki <i>et al.</i> 2018]	2018	Maximize total charging amount and earliness of charging completion time, minimize charging cost and time and dispersion of charging amounts	Solving a MILP, relaxation-based heuristic.
[García-Álvarez <i>et al.</i> 2018]	2018	Minimize the total tardiness	GRASP-like algorithm, memetic algorithm
[Niu <i>et al.</i> 2018]	2018	Maximize the charging service income, minimize customers' dissatisfaction, minimize power fluctuation degree	Hierarchical power control strategy, GA
[Zheng <i>et al.</i> 2018]	2018	Maximizing the level of satisfactions for electric vehicles charging	Online algorithm
[Yang 2019]	2019	Maximizing completion rate, and minimizing electricity bill.	Online algorithm: adaptive price-responsive early charging control
[Ayyadi <i>et al.</i> 2019]	2019	Minimize electric vehicles charging cost and the battery degradation cost	solving LP
[Liu <i>et al.</i> 2020]	2020	Minimize the total charging cost	Solving a bilevel programming model, LP, GA

ing stations, where electric vehicle charging can account for over 90% of the electricity costs [Garrett & Nelder 2016].

Authors in [Tang *et al.* 2014] minimize the total cost of the electricity bill paid by the charging station. In [Zheng & Shroff 2014], authors maximize the total value of served vehicles while minimizing the energy cost. [Ma *et al.* 2015] minimize the cost of fully charging all vehicles. [Tang & Zhang 2016] minimize the total charging cost, formulated as a strictly convex increasing function of the total load. [Wu *et al.* 2018] minimize the cost of electricity bought by the parking lot from the power grid, considering the varying electricity price at different time slots. Authors in [Ki *et al.* 2018] worked on the EVCS problem using M-to-N chargers adopted in actual charging stations in South Korea. Such a charger is designed to connect N electric vehicles, but only M electric vehicles will be charging simultaneously ($M < N$). The objective functions maximize the sum of four weighted terms, including the charging costs with a negative sign for minimization.

[Yang 2019] consider the charging scheduling problem in a residential parking station. They achieve cost minimization by incorporating a price preference index for each time interval. This index is dependent on the electricity price during that interval. The objective function is formulated as a maximization problem, with the index being higher when the electricity price is low and vice versa. In [Liu *et al.* 2020], the authors focus on minimizing the charging cost under Time-Of-Use electricity prices in a charging station with a limited number of chargers.

Total revenue maximization Authors in [Jin *et al.* 2013] addressed the problem of maximizing the aggregator’s revenue while imposing an upper bound to limit customers’ charging costs. The revenue of aggregators comes from the profit obtained by providing the regulation service to the power grid and the profit generated from the sale of energy to electric vehicles. Another revenue maximization problem was formulated in [Kuran *et al.* 2015] for smart parking lots. The revenue is determined as the result of subtracting the electricity buying price paid to the grid from the charging selling price for electric vehicles. [Bučar 2014] maximize the total profit generated from scheduling all charging demands. Further, [Niu *et al.* 2018] maximize the benefit of the fast charging station while considering the load fluctuation. When the load fluctuation is too important, the energy losses increases, eventually paid for by the charging station and electric vehicle drivers.

Customer satisfaction maximization Customer satisfaction represents an important factor in ensuring the success of a CSP business. Clearly, dissatisfied customers that experienced inconvenience during charging will not return to the charging station. Furthermore, poor quality service is detrimental to the CSP’s reputation. Thus, several papers propose to formulate the EVCS problem from the customer’s point of view or include customer satisfaction in their objective function. There are many ways for CSPs to improve their customers’ satisfaction; by focusing on maximizing the number of fulfilled charging demands [Kuran *et al.* 2015, Gerding *et al.* 2019], maximizing the energy delivered to each electric vehicle [Rahman *et al.* 2016, Ki *et al.* 2018, Yang 2019], by minimizing the customer’s charging cost [Jin *et al.* 2013], or minimizing the charging time [Zhu *et al.* 2014, Ki *et al.* 2018, García-Álvarez *et al.* 2018]. [Zhu *et al.* 2014] include the travel time to reach the charging point and the queuing time to the total charging

time to minimize. While in [García-Álvarez *et al.* 2018], the total tardiness is minimized, which occurs when the charging completion time exceeds the predefined departure time. Authors in [Zheng & Shroff 2014] maximize the total valuation of costumers. Further, authors in [?] quantify customer satisfaction by the percentage of the energy received. An electric vehicle driver is considered satisfied if this percentage is greater than a certain threshold that depends on the duration of plugging time. Then, the charging strategy is fulfilled when 95% of electric vehicle drivers are satisfied. Additionally, [Ki *et al.* 2018] maximize the charging balance among electric vehicles. [Zheng *et al.* 2018] minimize the supplying power while maximizing the level of satisfactions for electric vehicles charging. [Niu *et al.* 2018] balance between the costumers' satisfaction, the charging station incomes and load fluctuation.

2.4.2 Constraints, Assumptions and Considerations

2.4.2.1 Charging infrastructure capacity

Charging infrastructure capacity is a largely used constraint that defines the overall power limit of the charging infrastructure (electric distribution systems, charging stations, parking lots) expressed in kW. It limits electric vehicles' charging loads to avoid overloading other devices and transmission lines. Most of references [Jin *et al.* 2013, Xu *et al.* 2014, Lunci Hua *et al.* 2014, Zhang & Li 2015, Wu *et al.* 2018] defined it as the transformer capacity limit since the electrical power is provided by the nearest distribution transformer, as the distribution feeder subscribed power [Ayyadi *et al.* 2019], or as the maximum capacity of the distribution system [Suyono *et al.* 2019]. Authors in [Hajforoosh *et al.* 2015, Yao *et al.* 2017] limit the electric vehicles load demand. It can also be referred by charging station capacity [Kuran *et al.* 2015, Rahman *et al.* 2016, Kang *et al.* 2016]. Few references [Luo *et al.* 2016, García-Álvarez *et al.* 2018] defined it by the number of charging points that can deliver power simultaneously. Generally, the charging infrastructure capacity is defined as a time-constant power profile, especially when the base load (Section 2.4.2.2) is considered. Authors in [Gerding *et al.* 2019, Bučar 2014] consider a variable capacity. Authors in [Liu & Zhou 2022] limit the total charging power to the peak total load of coordinated charging.

2.4.2.2 Base load

In EVCS problems, authors refer to the load of all electricity consumption by appliances other than electric vehicles as the base load. Generally, it is used in problem formulations from power grid operators' perspectives where all daily power loads are considered to conduct the tests on a simulated electric power distribution grid. The base load information is used to schedule the charging load during off-peak hours when the base load is low and vice versa so that the total load is flattened. The base load does not constantly vary with time. Instead, it remains constant for a specific duration, usually in seconds or minutes, before it varies [Tang *et al.* 2014]. It can be estimated from historical data such as in [Jin *et al.* 2013] or based on day-ahead forecasts based on historical data [Xu *et al.* 2014], or known in advance [Ayyadi *et al.* 2019]. Authors in [Tang *et al.* 2014, Ma *et al.* 2015, Hajforoosh *et al.* 2015, Tang & Zhang 2016, Yang 2019] assumed that only the current base load is known and it is measured in real time.

2.4.2.3 Arrival times

Arrival time is the constraint that defines the availability of an electric vehicle for charging. It is always considered as a hard constraint since the electric vehicle cannot start charging before it the arrival time. With the scheduling problem notation, arrival time can be seen as the release date of a job. Most of the papers define the arrival time as the time when the electric vehicle is plugged in. This is usually the case for the problem from the power grid operators' perspective, where they consider the power distribution system that regroups home and parking lot chargers. From the charging service providers' perspective, arrival times correspond to plugging times since it is usually assumed that there are enough charging points or, in a few cases, reserved parking places for each vehicle [García-Álvarez *et al.* 2018]. Few papers consider that the arrival times are different from the plug-in times. An insufficient number of charging points is the major reason for queueing. Thus, they also focus on minimizing the waiting time in queue [Zhu *et al.* 2014], or consider a reservation system. The authors in [Luo *et al.* 2016] also assume a limited number of charging points in each station. The arrival time is estimated from the traveling time to reach the chosen station by an optimal traveling path planning optimization.

Another important consideration is the uncertainty in electric vehicles' arrival times. Some papers assume that the electric vehicle arrivals are known in advance [Jewell *et al.* 2014, Ayyadi *et al.* 2019], it can be provided by the electric vehicle owner [Jin *et al.* 2013], or predicted using historical data [Sundstrom & Binding 2012, Xu *et al.* 2014]. In many scenarios, future arrivals are unknown and depend on the random drivers' behaviors. Consequently, all other information about the future charging demands, such as the battery capacity, departure times, and requested energy, will also be unknown [Jin *et al.* 2013, Tang *et al.* 2014, Lunci Hua *et al.* 2014, García-Álvarez *et al.* 2018, Zheng & Shroff 2014, Ma *et al.* 2015, Tang & Zhang 2016, Yao *et al.* 2017, Zheng *et al.* 2018, Ki *et al.* 2018, Ki *et al.* 2018, Yang 2019, Gerding *et al.* 2019].

[Kuran *et al.* 2015] consider two cases: irregular vehicles whose arrival times are unknown and regular vehicles whose arrival times can be predicted using their historical behavior and then corrected in real-time. Additionally, authors in [Wu *et al.* 2018] consider charging vehicles with or without reservation. [Hajforoosh *et al.* 2015] do not consider arrival times. In [Franco *et al.* 2014], electric vehicle owners select their charging intervals. [Kang *et al.* 2017] define a home charging strategy to encourage customers to plug in their vehicles at a fixed period (e.g., from 21:00 to 6:00). If consumers join this plan, they will receive a discount on their electricity bill.

2.4.2.4 Departure times

Electric vehicle drivers will provide their departure time or parking duration in most problem formulations. In analogy to the scheduling problem notation, departure times can be seen as the due date of a job if it is a soft constraint where tardiness may occur [García-Álvarez *et al.* 2018]. Otherwise, it is a hard constraint, and it can be seen as the deadline of a job. In this case the electric vehicle will be unplugged at this time [Zheng & Shroff 2014, Kuran *et al.* 2015, Zhang & Li 2015, Tang & Zhang 2016, Yao *et al.* 2017, Zheng *et al.* 2018, Ki *et al.* 2018, Niu *et al.* 2018,

[Yang 2019, Gerding *et al.* 2019, Ayyadi *et al.* 2019, Liu *et al.* 2020, Liu & Zhou 2022]. In [Hajforoosh *et al.* 2016, Rahman *et al.* 2016, Pflaum *et al.* 2017, Wu *et al.* 2018], an electric vehicle is allowed to leave prior to its initially provided departure time. Further, [Yang 2019] the driver may provide the departure time when he plugs his vehicle; otherwise, it will be estimated based on the historical behavior of this vehicle. Authors in [Jewell *et al.* 2014] assume a common departure time for all electric vehicles.

2.4.2.5 Charging demands

Many researchers assume that electric vehicle drivers will provide their desired SOC when they plug their vehicles [Xu *et al.* 2014, Kumar *et al.* 2015, Hajforoosh *et al.* 2016, Jin *et al.* 2013, Hajforoosh *et al.* 2015, Zheng *et al.* 2018, Ki *et al.* 2018, Yang 2019, Liu & Zhou 2022]. In this case, the initial SOC and the battery capacity of the electric vehicle are either automatically detected by the charging point once the electric vehicle is connected [Xu *et al.* 2014, Hajforoosh *et al.* 2015, Zheng *et al.* 2018, Niu *et al.* 2018, Yang 2019] or also provided by drivers [Tang *et al.* 2014, Zheng & Shroff 2014, Tang & Zhang 2016, Wu *et al.* 2018, Gerding *et al.* 2019, Liu & Zhou 2022] assume that the electric vehicle owners will directly request the energy demanded expressed in kWh. [Vagropoulos *et al.* 2015, Niu *et al.* 2018] uses the battery State of Energy SOE instead of SOC. The SOE is defined as the ratio of the current stored energy to the rated energy capacity. The charging demand constraint can either be a hard constraint where the desired energy must be reached or a soft constraint where the scheduler tries to achieve acceptable energy by the departure time. In the first case, either the departure times will be a soft constraint [García-Álvarez *et al.* 2018], only feasible charging demands will be accepted [Tang *et al.* 2014, Zheng & Shroff 2014, Wu *et al.* 2018, Liu *et al.* 2020], no predefined departure time [Zhu *et al.* 2014, Hajforoosh *et al.* 2015, Suyono *et al.* 2019], study the feasibility of the charging to guarantee the satisfaction of each electric vehicle charging demand [Tang & Zhang 2016]. In the second case, the difference between the desired energy and the provided energy will be penalized in the objective function [Zhang & Li 2015, Franco *et al.* 2014, Zheng & Shroff 2014, Kuran *et al.* 2015, Zhang & Li 2015, Vagropoulos *et al.* 2015]. Further, in [Pflaum *et al.* 2017], the authors consider that a customer is satisfied if he has obtained more than a certain percentage of the energy required to fully charge the battery. This percentage depends on the connection time of the electric vehicle, since a customer who plugs in his vehicle for only a short time will not expect it to be fully charged. Also, in [Ki *et al.* 2018], if the available charging time is limited, then all the plugged electric vehicles cannot be charged to their desired levels. In this case, the charging balance among electric vehicles must be considered. Other papers consider charging electric vehicles to their rated battery capacity [Ahmad *et al.* 2014, Zhu *et al.* 2014, Zhang & Li 2015, Vagropoulos *et al.* 2015, Rahman *et al.* 2016, Yao *et al.* 2017, Pflaum *et al.* 2017, Niu *et al.* 2018]. Additionally, [Rahman *et al.* 2016, Hajforoosh *et al.* 2016, Ayyadi *et al.* 2019] restrict the charging demands by the recommended maximum SOC.

2.4.2.6 Charging power rates

The power charging rates can be classified into two groups:

- *Constant charging power rates*: also called fixed power rates, where a charger deliver a fixed value over time. In this case, we can distinguish between two cases: identical chargers, where all chargers in the station deliver the same output power [Jewell *et al.* 2014, Zhu *et al.* 2014, Lunci Hua *et al.* 2014, Yao *et al.* 2017], and non-identical chargers, where each charger (or vehicle) has a different power rate [Ma *et al.* 2015, Luo *et al.* 2016, Yang 2019, Suyono *et al.* 2019]. Further, [Bučar 2014] assume identical chargers where each charger can operate at different constant charging rates. [Liu & Zhou 2022] assume two constant charging power rates: fast and slow charging power.
- *Variable power rates*: also called continuous power rates, the charging rate can vary over time between a minimum value, usually zero, and a maximum value. In this case, the charging power rate will be a decision variable in the problem formulation. The charging power can be restricted by the rated power of the charging point [Xu *et al.* 2014, Vagropoulos *et al.* 2015, Kang *et al.* 2017, Ki *et al.* 2018, Wu *et al.* 2018, Niu *et al.* 2018, Liu *et al.* 2020], the charging demand of an electric vehicle [Jin *et al.* 2013, Niu *et al.* 2018, Kuran *et al.* 2015, Zhang & Li 2015, Ki *et al.* 2018, Niu *et al.* 2018], the maximum allowed charging power of an electric vehicle [Rahman *et al.* 2016, Tang *et al.* 2014, Zheng & Shroff 2014, Kuran *et al.* 2015, Yao *et al.* 2017, Ki *et al.* 2018, Niu *et al.* 2018, Gerding *et al.* 2019, Ayyadi *et al.* 2019], or the maximum charging infrastructure capacity [Rahman *et al.* 2016, Zhang & Li 2015, Kuran *et al.* 2015, Hajforoosh *et al.* 2015, Yang 2019, Gerding *et al.* 2019, Ayyadi *et al.* 2019]. In [Tang & Zhang 2016], charging power rates are not bounded, but since the optimal solution minimizes the total load variance, the charging capacity is also minimized.

In the first case, the charging power is a parameter in the EVCS problem, and usually, a binary variable is defined to specify if a vehicle is charging or not. In the second case, charging power rates are continuous decision variables.

2.4.2.7 Preemption

In EVCS problems, preemption is the mechanism in which interruptions are allowed during the charging process to increase charging flexibility, avoid congestion and constraint violation, and reduce charging load. Moreover, even for home charging, electric vehicle drivers are not inconvenienced by preemptive charging if their demands are fulfilled [Huber *et al.* 2020]. Even though it is claimed that frequent preemptions may introduce additional deterioration for batteries, to the best of our knowledge, no general agreement or detailed study is explaining the negative impacts of interrupted charging methods on battery performance and lifespan [Guo *et al.* 2021]. Generally, preemption is implicitly used when the variable power rates have a lower bound equal to zero. In some papers, the charging scheduling is non-preemptive, especially with constant power rates [Zhu *et al.* 2014, García-Álvarez *et al.* 2018, Bučar 2014].

A comparison between non-preemptive and preemptive charging strategies can be found in [Ranjan *et al.* 2014, Han *et al.* 2017, Jahic *et al.* 2019]

2.4.2.8 Charging efficiency

The battery charging efficiency is the ratio between the effectively stored energy and input energy charging, which lies in $[0, 1]$. Efficiency values are declared by the manufacturers and generally range between 80% and 99% due to energy losses. Therefore, many studies [Hajforoosh *et al.* 2015, Vagropoulos *et al.* 2015, Yang 2019, Rahman *et al.* 2016, Yao *et al.* 2017, Ayyadi *et al.* 2019, Liu & Zhou 2022] calculate the energy retrieved by the vehicle as the product of charging power rate and a fixed efficiency value.

2.4.2.9 Calculation of charging time

The classic charging strategy is constant current–constant voltage (CCCV) [Hemavathi & Shinisha 2022]. In this scheme, the battery charging is accomplished in two stages: Constant Current (CC) mode and Constant Voltage (CV) mode. First, the battery is charged with CC mode, where a large constant current is used at varying voltages. This results in rapid charging until a certain voltage threshold is reached. Then, the charging process is switched to CV mode at constant voltage until the current decreases below the threshold resulting in slow charging. Other charging techniques exist, such as the pulse charging method, Constant temperature – constant voltage charging [Hemavathi & Shinisha 2022]. In the CC mode, generally to reach a SOC of 80%, the charging time can be approximated as linear function. Therefore, most of researchers Assuming linear charging times [Xu *et al.* 2014, Jin *et al.* 2013, Zhu *et al.* 2014, Ayyadi *et al.* 2019, Liu & Zhou 2022]. A more complex final SOC calculation can be found in [Rahman *et al.* 2016]. In addition, nonlinear battery charging is considered in [Sundstrom & Binding 2012]. The charging power rate and battery SOC relationship is described using a curve in [Niu *et al.* 2018].

2.4.2.10 Assignment of vehicles to chargers

The assignment of electric vehicles to charging outlets is less considered in the literature. Most papers consider that an electric vehicle will not enter full parking as in [Tang *et al.* 2014], or that there are always available charging points, and thus, electric vehicles do not need to queue or wait to be charged. Authors in [Bučar 2014] consider the assignment of electric vehicles to chargers as interval scheduling problem. In addition, [Jewell *et al.* 2014, Liu *et al.* 2020] consider scheduling with limited number of chargers. Few papers consider a reservation system. Others consider scheduling at different charging stations. In this case, many parameters must be considered, such as the distance, the remaining battery energy, and chargers' availability. [Zhu *et al.* 2014] assign vehicles to chargers such that the travel time to reach the chargers, the queuing time, and the charging time are minimized. [Luo *et al.* 2016] find the shortest routes in a weighted road network with Dijkstra's algorithm. In [Mkahl *et al.* 2017], the authors aim to choose a convenient charging station such that its battery SOC remains at its highest possible level at arrival.

2.4.2.11 Priority

Another aspect that can be considered in the EVCS problems is charging priority. [Rahman *et al.* 2016] express charging priority of a charging demand as a weighted term in the objective function. Authors in [Niu *et al.* 2018] propose a weighted dynamic priority ranking method that simultaneously considers the customers' and charging service providers' demands. [Vagropoulos *et al.* 2015] consider that priority should be given to the less charged vehicles closest to their departure time. Authors in [Yao *et al.* 2017] calculated the priority of charging for each electric vehicle at each time slot. Moreover, electric vehicle drivers with more membership fees have higher priority levels. In [Kumar *et al.* 2015], three priority criteria were compared: based on battery SOC, slack time available, and time and energy already allocated for the vehicle. In [Franco *et al.* 2014], electric vehicle drivers select their charging priority by choosing one of the pre-established charging intervals. In [Zheng *et al.* 2018], charging priority depends on the remaining charging energy, the remaining charging number of time slots, and the charging power rates. Authors in [Liu & Zhou 2022] calculate urgency factor for each vehicle. Then urgent charging demand are charged using fast charging while unurgent one is charged in slow charging mode.

2.4.2.12 Scheduling time horizon

One specific characteristic of scheduling problems is time representation. Usually, an electric vehicle charging schedule is built over a finite horizon in a discrete-time formulation, where a time horizon of one day is divided into a fixed number of time slots of equal duration. The lengths of time slots vary from one study to another. We find a time slot equals to 5 minutes in [Hajforoosh *et al.* 2016, Niu *et al.* 2018], 10 minutes in [Yao *et al.* 2017], 15 minutes in [Sundstrom & Binding 2012, Yao *et al.* 2017, Zheng *et al.* 2018, Ki *et al.* 2018], 30 minutes in [Kuran *et al.* 2015, Wu *et al.* 2018], and one hour in [He *et al.* 2012, Jin *et al.* 2013, Lunci Hua *et al.* 2014, Franco *et al.* 2014]. The problem in these models is that an electric vehicle may arrive or leave in the middle of a time slot, affecting the solution's quality and the calculation of charging energy and cost. Therefore, the smaller the time slot length, the better the accuracy of the schedule. However, this generally yields very large problem instances of intractable size.

Other papers consider event-driven models to reduce computational complexity, as in [Tang *et al.* 2014]. Then, the time interval is defined as the period between two consecutive events. These events can be the arrival of an electric vehicle, its departure, or a change in the base load. The advantage of using such a formulation is that the exact timing of events is accounted for within the scheduling horizon, making it more precise. However, when several events happen in a short period, it results in a more complex model than a time-indexed one. In addition, it may be more difficult to formulate some EVCS problems as event-based models..

2.4.2.13 Electricity market and prices

Electricity markets differ in their structure and design from one country or region to another. There are two main market elements: the day-ahead market and the real-time market [Cramton 2017]. The day-ahead market is based on the forecasted load for the next

day. Generally, the system operator publishes system conditions, forecasts, and other information for the next day. Then, customers can sell or buy energy for the next day in a closed auction. Therefore, electricity prices are fixed on the day before the day of delivery. In [Zhang & Li 2015, Yao *et al.* 2017, Yang 2019, Ayyadi *et al.* 2019], it is assumed that the electric vehicle charging is conducted in an environment with day-ahead electricity prices. This pricing is suitable for day-ahead schedules. The real-time market usually runs once every hour to account for real-time load changes that must be balanced with supply. The prices are time-varying and will increase as the load increases and vice versa [Ma *et al.* 2015, Hajforoosh *et al.* 2015].

We can also find EVCS under Time-of-Use pricing (TOU), as in [Xu *et al.* 2014, Kuran *et al.* 2015, Wu *et al.* 2018, Kang *et al.* 2017, Liu & Zhou 2022]. TOU prices are frequently used, where prices are set in advance and are higher during peak periods and lower during off-peak periods. This encourages consumers to shift discretionary consumption from peak demand periods to cheaper off-peak periods. TOU prices are determined several months in advance instead of at each hour. We can also distinguish between the wholesale and retail electricity markets. The wholesale market refers to the buying and selling of energy between the power grid and resellers. Resellers are the entities that have the intention to resell the electricity to end consumers. The purchase and sale of electricity to end consumers are made in the retail market. A CSP can buy energy from the retailer or wholesale market. If the CSP is separate from the retailer, the CSP has to aggregate a sufficient number of vehicles to meet the minimum bid volume on the wholesale market [Sundstrom & Binding 2012]. Authors in [Jin *et al.* 2013, Tang *et al.* 2014, Vagropoulos *et al.* 2015] assume that the CSP buys electricity from the grid at a wholesale price that varies on an hour-to-hour basis. Also in [Jin *et al.* 2013, Vagropoulos *et al.* 2015], it participates in regulation markets, while [Sundstrom & Binding 2012] assume that the aggregator buy electricity from a retailer. Studies on electric vehicle aggregators' participation in the electricity market are mostly done in the context of V2G. It allows an aggregator to gain an extra profit by selling energy by discharging into the grid if the real-time price exceeds the expected opportunity cost of recharging later.

2.4.2.14 Other constraints and considerations

There are other considerations and constraints defined in the literature. For example, authors in [Hajforoosh *et al.* 2015, Niu *et al.* 2018] set a charging voltage limit for each electric vehicle. Another aspect that is less considered is chargers' compatibility. In fact, not all vehicles can be charged from any charging point. It depends on the connector types and the supporting charging type. For example, many electric vehicles do not support fast charging.

2.4.3 Optimization Methods

This section reviews and classifies the optimization approaches used to solve the EVCS problem. As mentioned before, the EVCS problem varies from one study to another. Therefore, a comparison between these methods in terms of efficiency is irrelevant.

2.4.3.1 Exact methods and mathematical formulations

Several papers formulate the EVCS problem as a mathematical programming model. The general formulation of a mathematical model can be given as follows.

$$\begin{aligned} & \text{minimize} && f(x) \\ & \text{subject to} && g_i(x) \leq b_i \quad i = 1, \dots, m \end{aligned}$$

Where x_1, x_2, \dots, x_n are called the decision variables of the problem. Decision variables may take continuous or discrete values. The function f represents the objective function to be minimized. Functions g_i , $i = 1, \dots, m$ are the inequality constraints. A vector $x^* \in \mathbb{R}^n$ is called the optimal solution if it has the smallest value of f among all vectors that satisfy the constraints. Also, an optimization problem can be formulated as a maximization model.

Linear programming. Linear Programming (LP) involves minimizing a linear objective function subject to linear constraints. Linear models are convex ¹ and represent the simplest yet nontrivial optimization problems. Some powerful methods exist for such problems, such as the simplex algorithm and interior point methods. If variables x_1, \dots, x_n are constrained to take integer values, the problem is called an Integer Linear Program (ILP). Further, integer variables may be restricted to the values zero and one. In this case, they are called binary variables. Integer programming problems are often much more challenging to solve than LP with only continuous variables. A mixed integer linear programming (MILP) problem contains both integer and continuous variables. MILP models have the potential to be more difficult than their continuous LP and quadratics programming.

LP models are generally solved using commercial solvers, such as CPLEX, Gurobi, and MOPS. These solvers can solve many mathematical models effectively. However, depending on the problem, some problem instances are solvable in an acceptable amount of time, while others require days or weeks to find optimal solutions. Moreover, a problem can be expressed in many mathematically equivalent ways. In addition, each solver has its own set of default algorithmic parameter settings. Choosing from these various model expressions and algorithmic settings can profoundly influence solution time.

In [Jin *et al.* 2013], two problems were formulated as LP. One with the objective of maximizing the aggregator's revenue and the other to minimize the total electricity cost of electric vehicles owners. Authors in [Yang 2019], formulate the EVCS problem as a binary integer programming problem, which can be solved by proper algorithms such as the branch and cut. Another LP model was formulated in [Ayyadi *et al.* 2019] to minimize the charging cost and battery degradation cost. Authors in [Franco *et al.* 2014] formulate a mixed-integer nonlinear programming (MINLP) model to minimize power losses and maximize the total delivered energy. Then, a linearization is applied for the voltage drop calculation resulting in a MILP model to be solved using commercially available solvers. Authors in [Jewell *et al.* 2014] consider the EVCS problem as a parallel machine scheduling problem with availability constraints. A MILP model is given with the objective of minimizing the total makespan.

¹An optimization problem is convex functions f, g_1, \dots, g_m are convex. Solving a convex optimization problem is faster, simpler, and less computationally intensive than non-convex optimization

Nonlinear programming. the objective or constraint functions are not Nonlinear programming (NLP) involves solving a model where some of the constraints or the objective function are not linear. There are no effective methods for solving the general nonlinear programming problem. In a non-convex NLP, there may be more than one feasible region, and the best solution could be at any point in any of them.

One of the simplest forms of NLP is constrained quadratic programming (QP), in which the objective is a quadratic function and constraints are linear. QP is used in [Sundstrom & Binding 2012], the charging service provider buys the electricity from a retailer. The retailer computes the preferred charging curve, a time series of the preferred aggregated charging power while respecting the energy and power constraints. The objective of the optimization problem is to follow the preferred charging curve computed by the retailer as closely as possible. Thus the objective function is to minimize the sum of the quadratic differences.

2.4.3.2 Heuristic and meta-heuristic approaches

Many papers formulate the problem as a mathematical model but solve it with heuristics and metaheuristics. Due to the high complexity of most EVCS problems, solving a mathematical model is impracticable and time-consuming. Moreover, many EVCS problems are subject to stochastic events such as uncertain arrival times or electricity prices. As a result, many researchers have proposed heuristics and metaheuristics to find good solutions but not necessarily optimal ones in a reasonable time. In general, heuristics are problem-specific methods designed for a specific problem. In scheduling, most heuristics are priority rules. Metaheuristics algorithms are stochastic optimization algorithms that employ iterative processes to move towards better solutions. Unlike heuristics, they are not problem-specific and can be employed and adapted for a large optimization problem. There has been tremendous research in developing metaheuristics during the past two decades. Most of them are inspired by natural processes, including Simulated Annealing (SA), Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Ant Colony Optimization (ACO).

In [Jewell *et al.* 2014], heuristics based on machine scheduling algorithms are considered. In addition, a greedy local search and a SA algorithm with a pairwise exchange were proposed. The PSO algorithm was widely used to solve EVCS problems as we can see in Tables 2.2 and 2.3. PSO is a population-based metaheuristic inspired by the social behavior of organisms, such as a bee swarm, that moves in a group but benefits from the experience of all other members. In PSO, a number of searching agents, called particles, are placed in the search space. Each particle moves in the search place at each iteration based on its current personal and global best positions and random perturbations. In [Wu *et al.* 2018], a proportion-based assignment method is designed to determine the initial population for the PSO algorithm. The PSO algorithm is used in each optimization iteration to obtain better solutions. Then, a fuzzy system is deployed to determine the three crucial coefficients of PSO algorithms. Later, heuristics were used to improve the solutions when a particle is updated. [Kang *et al.* 2017] propose a weight aggregation (WA) strategy for multi-objective PSO to solve the EVCS problem. The proposed WA strategy assigns a weight factor to each objective function. Then, their value varies to obtain all combinations of objective functions so that the different relative importance between them is

shown. This strategy guides the PSO to update its particles and approach a Pareto front. WA could only reach the Pareto front in the case of a convex objective function. However, it can help reduce the computation time of the algorithm by avoiding a long evolution process. Also, new methods of updating elite particles are proposed to maintain the diversity of the final Pareto front in the implemented PSO. A binary PSO was used in [Liu & Zhou 2022]. [Niu *et al.* 2018] use a GA to solve the multi-objective optimization problem. Further, a heuristic based on a weighted dynamic priority ranking is proposed for the charging power control process. Authors in [Liu *et al.* 2020] formulate the problem as a bilevel programming model. Each vehicle's assigned charger and available charging duration are determined at the upper level. On the lower level, a GA is used to determine the charging power of each vehicle at each time slot. In [García-Álvarez *et al.* 2018], a memetic algorithm and a solving method inspired by the GRASP metaheuristic were used to minimize the total tardiness. The proposed metaheuristics are used for both static and dynamic version of the problem.

2.4.3.3 Online Algorithm

The EVCS problems are subject to uncertainty, such as the unknown electric vehicle arrival or departure times or changes in the base load, or electricity prices. In these cases, online scheduling algorithms are used. Online algorithms can only calculate and update the schedule with the currently available information. Information becomes available continuously with time. Therefore, decisions are made at each fixed time interval or when an event occurs. Moreover, online algorithms can involve predictions based on historical data. A standard metric to evaluate the performance of an online algorithm is the competitive ratio, which compares the performance of an online with an offline algorithm.

[Jin *et al.* 2013] solve an LP problem to schedule charging operations each time an electric vehicle arrives. Once the charging schedule for an electric vehicle is established, it cannot be changed. Further, the charging schedules are adjusted for the plugged vehicles when k more vehicles arrive. The smaller the value of k , usually, the better the performance. [Lunci Hua *et al.* 2014] propose an online adaptive EVCS framework to Minimize the total charging cost subject to battery demand and distribution grid constraints. A MILP is formulated to tackle this problem. The distribution grid constraints are taken into account using power flow optimization. The scheduling problem is solved at each time slot when a new vehicle arrives. Authors in [Tang *et al.* 2014] present an online algorithm, called ORCHARD, for the EVCS problem and prove that it is 2.39-competitive. The algorithm does not involve predictions about future information and decisions on the charging rates when a vehicle arrives or when the base load changes. The algorithm is based on the speed scaling problem, a power management technique that dynamically changes a processor's speed. Moreover, it does not consider the uncertainty in departure times, i.e., when an electric vehicle leaves before fulfilling its charging demand, which is not the case in real-life scenarios. In [Yao *et al.* 2017], the authors formulate the optimization problem as binary programming. Then, a convex relaxation method is used to maximize the number of chosen vehicles for charging at each time. [Zheng *et al.* 2018] a priority rule-based heuristic to achieve valley-filling is proposed. A capacity margin and charging priority indices are defined to construct the online algorithm. The first index is used to select the time

slot at which the power grid has enough surplus power for electric vehicle charging. The second index determines the charging priority of a vehicle at each time slot.

[Tang & Zhang 2016] consider minimizing charging cost as a strictly convex function. a Model Predictive Control (MPC) approach is applied to obtain a near-optimal solution. An MPC-based approach brings the stochastic problem to a deterministic one by replacing all uncertain future parameters with their predicted values. The performance gap between the near-optimal solution of the MPC-based approach and the optimal solution is evaluated using the Value of the Stochastic Solution, which represents the gap between the solution of the approximate approach and that of the dynamic programming problem. [Pflaum *et al.* 2017] propose a control strategy to determine an allowed day-ahead power consumption profile for an electric vehicle charging station to maximize customer satisfaction while minimizing power consumption. A heuristic is implemented in the real-time controller using the power profile obtained as an upper bound for the charging station's power consumption. In [Ki *et al.* 2018], a weighted-sum approach for solving the multi-objective optimization problem is proposed. The scheduling problem is solved iteratively whenever a new electric vehicle arrives. The sliding window scheme is adopted. When a new electric vehicle is connected to the charger, the control system verifies the charging status of plugged vehicles and the new electric vehicle. Then it defines a new sliding window for the scheduling as the time between the arrival of the new vehicle and the latest available charging time of the plugged ones.

Authors in [Wu *et al.* 2018] solve the problem at the beginning of each timeslot by determining a charging schedule for the immediate timeslot with the minimum electricity cost. Similarly, [Niu *et al.* 2018] calculates the schedule at the beginning of each time slot using a heuristic based on a priority rule. In [Yang 2019], a self-adaptive price-responsive early charging control is proposed to improve online scheduling.

Authors in [García-Álvarez *et al.* 2018] consider solving the dynamic problem as solving a sequence of instances of the static one. An instance is built at each time interval of a fixed length. It consists of the vehicles that arrived but have not started charging and those already charging. New schedules are calculated using the proposed metaheuristics.

2.5 Research positioning

Although the studies mentioned above have examined various aspects of the EVCS problem, they have mainly assumed that there is a sufficient number of chargers for all vehicles. Therefore, the scheduler does not decide which charger each vehicle is assigned. Few papers address the assignment of vehicles to chargers in the charging schedule. Authors in [Liu *et al.* 2020] considered a limited number of chargers to minimize the total charging cost under the time-of-use electricity price. However, the authors did not impose a total power, so the charging power of each charger is only limited by its maximum power. Moreover, few studies considered constant power charging rates. In addition, no comparison between considering identical or non-identical chargers was made.

Our study is most similar to the literature addressing the assignment of vehicles to chargers in the EVCS problem. However, the charging station operating model, the constraints, and the optimization objective are different. We outline the contributions of this thesis as follows.

- We consider a new charging station operating model where the charging station has limited total power and a limited number of chargers. Each charger is installed in a parking space.
- A reservation system is considered where electric vehicles submit their charging demands to avoid queuing. The scheduler allocates a suitable charger for each vehicle.
- Different objective functions were considered: minimizing the charging infrastructure capacity, maximizing the number of satisfied charging demands, maximizing the delivered energy, and minimizing total tardiness.
- We provide the \mathcal{NP} -hard hardness proof for each considered problem in detail.
- We propose heuristics and metaheuristics to jointly assign the electric vehicles to chargers and schedule the electric vehicle charging.
- Different comparison between various aspects of charging scheduling is given, namely:
 - between identical and non-identical constant power rates.
 - between the constant and variable charging power rates.
 - between the preemptive and non-preemptive schedules.
 - between time-indexed and event-based models.

2.6 Conclusion

In this chapter, we studied the state-of-the-art. First, we presented general scheduling problems; then, we defined different aspects related to electric vehicle technology. Also, we review existing works and classify them according to different objectives, constraints, and optimization methods used in the literature. Finally, we stated the research gap and positioned our work.

Preemptive EVCS problem to minimize grid capacity

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3.1 Introduction

In the previous chapter, we highlighted the wide variety of EVCS problems investigated in the literature. However, most of the existing works assume charging with variable power rates. In reality, even though the variable charging method is more flexible and promising to be commercialized in the future, only a few variable power chargers are currently available on the market. Besides, it is expected that constant power chargers will still co-exist with the variable ones since they are easier to deploy [Sun *et al.* 2016]. In fact, electric vehicle batteries draw approximately constant power when it charges from 20% to 80%. On the other hand, charging at constant power is more efficient since it can reduce the usable energy loss in the charging operation [Jeon *et al.* 2021].

Another important aspect is that very few studies consider the parking spaces and charging points (chargers) as limited resources in the charging station. Indeed, they assume that the charging station operators will not manage the parking space or the chargers, which is more suitable for charging stations without reservation where all chargers are identical. However, charging stations are more like parking lots with chargers that can deliver different power rates. Thus, it is necessary to assign each vehicle to a suitable charger to satisfy its demand before departure.

Motivated by the discussion above, this chapter studies the charging scheduling problem with a fixed number of chargers delivering power at constant rates. Precisely, we consider a charging station with a fixed number of parking spaces, and each is equipped with a charger with one connector on which only one vehicle can be plugged at a given time. The charging station has a limited grid power capacity to avoid overloading the power grid. Each electric vehicle driver has to submit a charging demand before arriving at the station to reserve a parking space. Given the limited number of chargers, the short range of electric vehicles, and the long time required to charge them, drivers of electric vehicles need to carefully plan their trips to ensure that they will have opportunities to recharge their batteries. As a result, it is preferable for them to confirm in advance that the charger they intend to use is available. Moreover, the already deployed Open Charge Point Protocol includes the reservation functionality of charging stations [OCP 2020]. A centralized management system controls and schedules the charging load to optimally utilize the parking spaces and the available power to satisfy charging demands without overloading the power grid.

This chapter mainly focuses on the following question. If the charging operators fix the goal of satisfying a certain number of charging demands, how many chargers and what is the limited power needed to achieve this?

Therefore, we address two problems. First, we study the problem of finding the minimum number of chargers needed to plug all vehicles. Then, we tackle the problem of minimizing the maximum power limit needed to satisfy all charging demands. We refer to this limit by the grid capacity. This problem is investigated to carefully guide charging stations to choose their subscribed maximum power. As the charging demands increase, they will need to upgrade their capacity. In fact, power consumption peaks occur and cause high electricity bills. Generally, equipment such as power cutters and relays are installed at a small cost to avoid peaks, but they cause the system to shut down, which is not desirable. Consequently, it is essential to provide an overview of the minimum power limit depending on the installed charger types and charging demands. We investigate both problems in the case of identical and non-identical chargers. For identical chargers, we prove that the two problems are polynomials in both cases. For grid capacity minimization, we prove that the problem is polynomial solvable in the case of identical chargers and \mathcal{NP} -hard in the case of non-identical chargers.

The remainder of this chapter is organized as follows. Section 3.2 describes in detail the investigated problem. Section 3.3 investigates the problem with identical charger, while Section 3.4 deals with non-identical chargers. For each case, the minimum number of chargers needed to plug all vehicles in the charging station and the minimum grid capacity problems are studied. Section 3.5 evaluates the performance of proposed methods and finally Section 3.6 concludes the chapter.

3.2 Problem Description

The EVCS problem can be stated as follows. We have a set $\mathcal{J} = \{1, \dots, n\}$ of electric vehicle charging demands to be scheduled on a set $\mathcal{M} = \{1, \dots, m\}$ of chargers. At each time, a charger can only charge one vehicle, and a vehicle can only be charged by one charger. Each charger i delivers a constant power w_i (kW). The total power that can be delivered by all chargers simultaneously must not exceed w_G (kW), which will further be denoted as the power grid capacity. The charging demand of each vehicle j is characterized by an arrival time r_j , a departure time d_j , and a required energy e_j (kWh). Each charging demand j has to be assigned to one of the m chargers, and its energy requirement must be fulfilled before the departure. Moreover, the vehicle j uninterruptedly occupies the charger, and the parking space, from its arrival time r_j to its departure time d_j and cannot be moved or unplugged during this time. However, the vehicle can charge preemptively, i.e., the charging of each vehicle j can be interrupted at any time and resumed later in the interval $[r_j, d_j]$. Even when the vehicle completes charging before d_j , it still occupies the charger until it departs. Unless otherwise mentioned, we divide the scheduling time horizon \mathcal{H} into T time slots of equal length τ . Without loss of the generality, we suppose that r_j and d_j are multiple of τ : $r_j \in \mathcal{H}$ and $d_j \in \mathcal{H}$. We assume linear charging times, meaning that the charging time p_{ij} of vehicle j on a charger i is equal to $\frac{e_j}{w_i}$ when the charging process is approximated with linear function. From now on, we shall assume, without loss of generality, that p_j is the number of time slots needed to satisfy j rounded to the nearest integer. Thus, $p_{ij} = \text{round}(\frac{e_j}{w_i \tau})$. The objective of this chapter is to find a feasible schedule with the minimum grid capacity w_G . We consider two cases of the problem: a charging station with identical chargers, where all chargers deliver the same charging power rate, and a charging station where chargers deliver different charging power rates are installed. In the next paragraph, we give an example of the optimal grid capacity values for a small instance with identical and non-identical chargers.

Example 3.2.1. Consider the charging demands of six vehicles. The arrival and departure times and the required power for these demands are listed in Table 3.1. We examine two cases. In the first case, we have a charging station with five chargers: the first charger delivers an output power of 30 kW, while the second delivers a power of 10 kW. The remaining chargers deliver 20 kW. In the second case, we consider a charging station with five identical chargers; each delivers 10 kW. We divided the scheduling horizon into time slots with a length of one hour. Figure 3.1(a) shows an optimal schedule of the charging demands in the first case, while Figure 3.1(b) depicts an optimal solution with identical chargers. In both solutions, all charging demands are scheduled, and their energy requests are satisfied. As we can see, each vehicle v_j occupies a charger from its arrival to its departure time. In the first case, the minimum grid capacity needed to schedule all charging needs is 30 kW. In the second case, it is equal to 40 kW since there are four chargers, and each one delivers 10 kW at time slot $t = 11$.

3.3 Identical Chargers

In this section, we consider an instance of the EVCS with m identical chargers, each delivering a constant power w (kW). In this case, each vehicle j has an identical charging time on all

Table 3.1: Instance for Example 3.2.1.

Vehicle	Arrival time r_j	Departure time d_j	Requested energy e_j
v_1	8:00	10:00	20 kWh
v_2	9:00	12:00	30 kWh
v_3	10:00	13:00	20 kWh
v_4	10:00	13:00	20 kWh
v_5	10:00	13:00	20 kWh
v_6	10:00	13:00	20 kWh

chargers, i.e., we have $p_{ij} = p_j$. Since we assumed that $p_j \leq d_j - r_j$, vehicle j can be assigned to any charger.

3.3.1 Minimum Number of Chargers

In this chapter, we aim to satisfy all charging demands with a minimum grid capacity. However, charging a vehicle j requires locating a charger that will be available during a period of uninterrupted length $[r_j, d_j)$. We say that a charger is available if no vehicle is plugged into it or parked in the parking space where it is installed. Without loss of generality, we assume that there is at least a charger i on which the energy demand of vehicle j can be fulfilled during the plugging time interval $[r_j, d_j)$, i.e., $\exists i, p_{ij} \leq d_j - r_j$. Otherwise, for a vehicle j for which this condition is not satisfied, or we have no charger available during the interval $[r_j, d_j)$, the vehicle can not be charged, which implies infeasibility. Hence, we first determine the minimum number of chargers needed to park and plug all vehicles. This problem is equivalent to finding the minimum number of machines that accommodate all the jobs in the fixed interval scheduling problem [Kovalyov *et al.* 2007]. Note that in this section, we only build the parking schedule, the charging schedule is not considered.

Theorem 3.3.1.1. *The minimum number of identical chargers needed to plug n vehicles can be established in $\mathcal{O}(n \log n)$ time.*

Proof. To prove Theorem 3.3.1.1, we present Algorithm 1 that runs in $\mathcal{O}(n \log n)$ time to calculate the minimum number of chargers \bar{m} . Then, we show that \bar{m} is optimal.

Feasibility and complexity. The resulting schedule of Algorithm 1 is feasible since, for each charger i , $i \in \{1, \dots, \bar{m}\}$, the plugging intervals of assigned vehicle to i are non-overlapping. Furthermore, sorting the set of charging demands in line 2 runs in $\mathcal{O}(n \log n)$ time and lines 3 to 9 can be implemented in $\mathcal{O}(n)$. Hence, Algorithm 1 runs in $\mathcal{O}(n \log n)$ time.

Optimality. Let \bar{m} be the number of chargers generated by Algorithm 1, and assume that it is not optimal. Let m^* be the optimal number of chargers, then, $m^* \leq \bar{m}$. Since $m^* \leq \bar{m}$, we have at least one charging demand j , that was assigned to the charger \bar{m} and could not be assigned to one of the m^* first chargers in the optimal solution. According to Algorithm 1, when vehicle j is available, all $\bar{m} - 1$ chargers are occupied with other charging demands. Then, at least $\bar{m} - 1$ demands conflict with demand j . This leads to $m^* \geq \bar{m}$. Therefore, we conclude that $\bar{m} = m^*$. \square

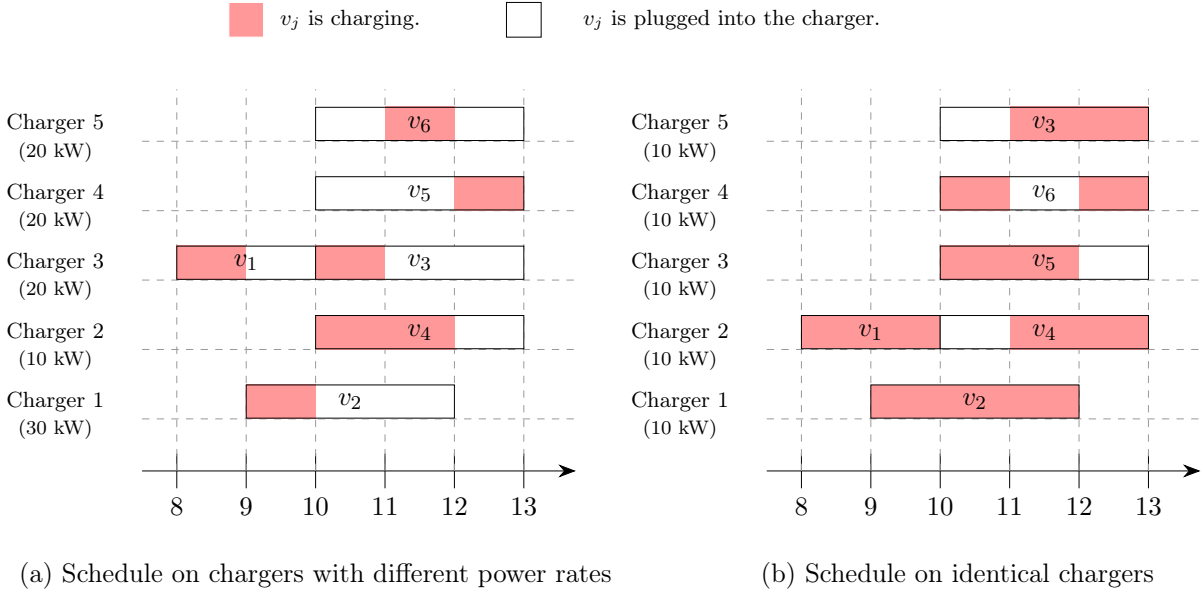


Figure 3.1: Optimal schedules of charging demands in Table 3.1. (a) depicts the optimal schedule using chargers with different power rates, and (b) depicts the optimal schedule where all chargers deliver 10 kW. Rectangles represent the vehicles' plugging intervals. We highlight charging intervals in red.

3.3.2 Minimum Grid Capacity

In this section, we assume that $m \geq \bar{m}$, where \bar{m} is the minimum number of chargers provided by Algorithm 1. We aim to minimize the grid capacity required to charge all vehicles to their desired energy.

3.3.2.1 Complexity

Let w_G^* denotes the optimal value of the grid capacity w_G . Clearly, in the optimal solution, w_G^* is a multiple of w and at most $m^* = \frac{w_G^*}{w}$ chargers can be activated at the same time. Therefore, minimizing the value of w_G is equivalent to minimizing $\tilde{m} = \frac{w_G}{w}$. To solve this optimization problem, we consider its decision problem, i.e.s, determining whether a feasible preemptive schedule exists that satisfies n charging demands by simultaneously activating at most \tilde{m} chargers. In the following, we show that this problem is polynomial solvable by reduction to the maximum flow problem [Ahuja *et al.* 1988].

Let \mathcal{L} be the set of L events that correspond to the distinct values of arrival and departure times, $\mathcal{L} = \{r_1, d_1, r_2, d_2, \dots, r_n, d_n\}$, sorted in non-decreasing order to obtain the sequence t_1, t_2, \dots, t_L with $L \leq 2n$. These L values divide the time horizon into $L - 1$ intervals $I_l = [t_l, t_{l+1})$, $l = 1, \dots, L - 1$. The corresponding network $N = (V, E)$ is constructed as follows. The set of vertices V consists of: (i) a source s , (ii) a vertex v_j for each charging demand $j \in \mathcal{J}$, (iii) a vertex I_l for each interval $[t_l, t_{l+1})$, $l = 1, \dots, L - 1$, and (iv) a sink p . The set of arcs E with restricted capacities consists of: (i) an arc from the source s to each charging demand vertex v_j

Algorithm 1: The calculation of the minimal number of identical chargers

Input : The set of charging demands \mathcal{J}

Output: The minimum number of chargers needed to plug all charging demands \bar{m}

```

1  $\bar{m} \leftarrow 0$ ;
2 Sort the set of charging demands  $\mathcal{J}$  in non-decreasing order of arrival times  $r_j$ ;
3 for  $j \in \mathcal{J}$  do
4   if there is a charger  $i$ ,  $i \in \{1, \dots, \bar{m}\}$ , available at  $r_j$  then
5     | Assign the demand  $j$  to charger  $i$ ;
6   else
7     | Assign the demand  $j$  to a new charger;
8     |  $\bar{m} \leftarrow \bar{m} + 1$ ;
9   end
10 end
11 return  $\bar{m}$ 

```

with capacity p_j , (ii) an arc from each vertex v_j to each interval vertex I_l if $[t_l, t_{l+1}) \subseteq [r_j, d_j)$ with a capacity equals to the length of the interval I_l , i.e, $t_{l+1} - t_l$, and (iii) an arc from each interval vertex I_l to the sink p with capacity $\tilde{m}(t_{l+1} - t_l)$ where $\tilde{m} = \lfloor \frac{w_G}{w} \rfloor$ corresponds to the number of chargers that can deliver power simultaneously. A feasible schedule of the charging problem exists if and only if from the source s to the sink p is equal to $\sum_{j=1}^n p_j$.

Theorem 3.3.2.1. *The minimum grid capacity problem with identical chargers can be solved in $\mathcal{O}(n^{2+o(1)} \log U \log n)$, where U is the maximum capacity of arcs in the corresponding network.*

Proof. Given an instance of the decision problem where parking all vehicles is feasible. Since the total capacity of arcs (s, v_j) , $j \in \mathcal{J}$, is equal to $C = \sum_{j=1}^n p_j$, then C is an upper bound on the maximum flow value in the network. Furthermore, if the optimal solution to the maximum flow problem equals C , then all arcs (s, v_j) are saturated, which means that all charging demands are satisfied. Extensive research on the maximum flow problem has been conducted over the past decades, leading to the proposal of several algorithms with different complexities. Authors [Chen *et al.* 2022] provided an algorithm to compute the maximum flow in $\mathcal{O}(|E|^{1+o(1)} \log U)$ time where $|E|$ is the number of arcs and U corresponds to the maximum capacity of arcs. We have $U = \max(\max_{j \in \mathcal{J}} p_j, \max_{l=1, \dots, L-1} \tilde{m}(t_{l+1} - t_l))$. In worst case we have, $|V| = 3n + 2$ and $|E| = n^2 + 3n$. Thus, the complexity of the feasibility charging problem is $\mathcal{O}(n^{2+o(1)} \log U)$. The optimal value of \tilde{m} can be found in $\mathcal{O}(\log n)$ using dichotomy search with an upper value of \tilde{m} equals to n . Therefore, the minimum grid capacity problem can be solved in $\mathcal{O}(n^{2+o(1)} \log U \log n)$. □

"

Example 3.3.1. Consider the charging demands given in Example 3.2.1 with five identical chargers with $w = 10$ kW. Obviously, the maximum possible value of w_G is 50 kW, which corresponds to the case where all chargers are activated at the same time slot. Therefore, the

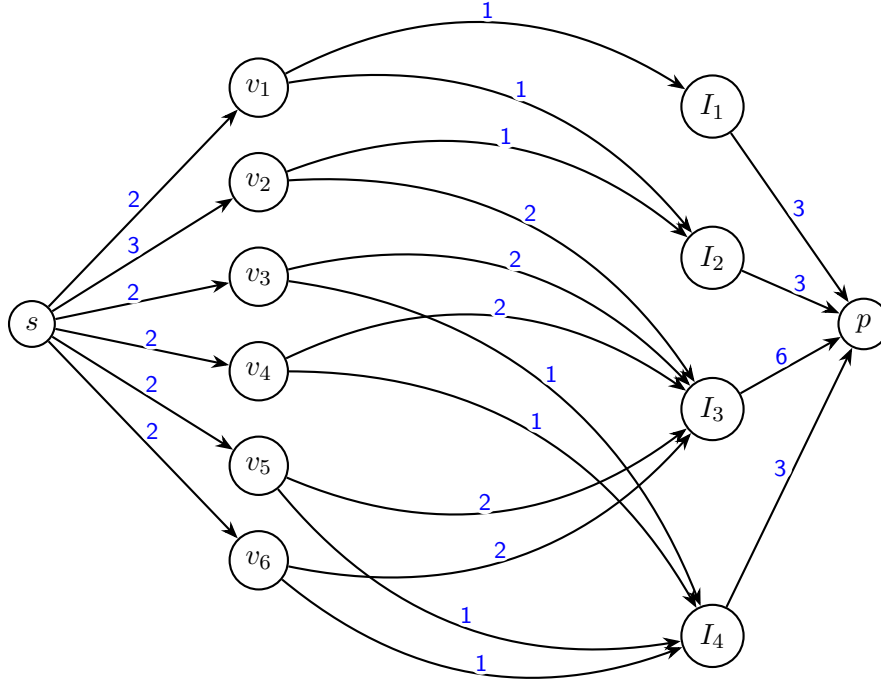


Figure 3.2: Flow network of the instance in Table 3.1 with five chargers, each delivers 10 kW, and a grid capacity equals to $w_G = 30$ kW.

set of possible values of \tilde{m} is $\{1, 2, 3, 4, 5\}$. The binary search algorithm determines the value of \tilde{m} to test, and the feasibility of the charging decision problem with this value is checked by constructing the equivalent network. The first value tested is the middle value in the set, $\tilde{m} = 3$. We construct the flow network considering $\tilde{m} = 3$ as depicted in Figure 3.2. Since each charger delivers 10 kW, the charging times are: $p_1 = 2$, $p_2 = 3$, $p_3 = 2$, $p_4 = 2$, $p_5 = 2$, and $p_6 = 2$, which corresponds to values on the arcs from the source s to v_j , $j = 1, \dots, 6$. we have $\sum_{j=1}^6 p_j = 13$. From Table 3.1, the set of distinct arrival and departure times is $\{8, 9, 10, 12, 13\}$. Thus, we have four vertices I_1 , I_2 , I_3 , and I_4 , which correspond to intervals $[8, 9)$, $[9, 10)$, $[10, 12)$, and $[12, 13)$, respectively. Since the vehicle v_1 is available for charging during its plugging interval $[8, 10)$, we have two arcs: one from v_1 to $I_1 = [8, 9)$ and another from v_1 to $I_2 = [9, 10)$. Both arcs have a capacity of 1, which is the length of each interval. Similarly, we add the remaining arcs from each demand vertex to each interval vertex. Finally, since we have $\bar{m} = 3$, and the intervals I_1 , I_2 , and I_4 have equal lengths of 1, each arc from the vertices corresponding to these intervals to the sink p has a capacity of 3. Since the length of the interval I_3 is 2, the capacity of the arc going from I_3 to the sink p is 6. Clearly, the maximum flow is 12, which is not equal to $\sum_{j=1}^6 p_j = 13$, and thus, charging all six vehicles is infeasible. Therefore, we move to the next value of \tilde{m} , that is $\tilde{m} = 4$. Similarly, we construct the equivalent network and calculate its maximum flow, which will equal 13. Consequently, the minimum grid capacity is 40 kW.

3.4 Distinct Types of Chargers

In this section, we consider the general case with m chargers, where each charger i , $i \in \mathcal{M}$, delivers a constant power of w_i (kW). Then, chargers are grouped into k types where chargers of the same type l , $l \in \mathcal{K} = \{1, \dots, k\}$ have the same charging power w_l (kW), i.e., there are m_l chargers of type l and $\sum_{l=1}^k m_l = m$. Obviously, if charger i is of type l , the charging time of demand j on that charger is $p_{jl} = p_{ij}$. When we have different types of chargers, it is possible that a charging demand cannot be satisfied on a specific type when its corresponding charging time exceeds the plugging interval. However, it can be satisfied on one of the chargers that deliver higher power. Without loss of generality, we assume that the k types of chargers are indexed in non-decreasing order of their power, i.e., $w_1 \leq w_2 \leq \dots \leq w_k$. Obviously, if a demand j is feasible on a charger of type l , it is also feasible on chargers of types $l+1, \dots, k$. Let l_j be the smallest index of charger type on which the demand j can be satisfied, i.e., $p_{jl_j} \leq (d_j - r_j)$ and $p_{j(l_j-1)} > (d_j - r_j)$.

3.4.1 Minimum Number of Chargers

This section deals with the case where there exists at least one charger on which each charging demand j can be satisfied, and thus, $l_j \leq k$, $j = 1, \dots, n$. As in the previous section, we seek the minimum number of chargers needed to plug all vehicles in \mathcal{J} .

Theorem 3.4.1.1. *The minimum number of non-identical chargers needed to plug n vehicles can be established in $\mathcal{O}(n \log n)$ time.*

Proof. To prove Theorem 3.4.1.1, we use the following Algorithm 2 to schedule all charging demands on a minimum number of chargers. Algorithm 2 has a time complexity of $\mathcal{O}(n \log n)$. The optimality proof of the algorithm is similar to the proof of Theorem 3.3.1.1.

□

Remark 3.4.1. Algorithm 2 provides the minimum number of chargers to plug all vehicles, but it does not provide the minimum number of chargers to satisfy these demands with the minimum possible grid capacity. To explain this, let us consider the instance in Example 3.2.1. All charging demands can be satisfied using a 10 kW charger; thus, Algorithm 2 will return $\bar{m} = 5$ by assigning all vehicles to chargers with 10 kW. However, as we can notice in Figure 3.1(1), the optimal value of w_G cannot be obtained with five chargers delivering each 10 kW. In fact, it depends on how we define the minimum grid capacity and whether we seek the minimum value of w_G with a fixed number of chargers or not. Nevertheless, Algorithm 2 is still helpful for checking the feasibility of the parking problem. Indeed, if $m < \bar{m}$, charging all vehicles is infeasible. Finally, if w_G^* is the minimum grid capacity required to satisfy all charging demands by considering the minimum number of chargers \bar{m} obtained by Algorithm 1, considering a number of identical chargers greater than \bar{m} will not reduce the value of w_G^* .

Algorithm 2: The calculation of the minimum number of non-identical chargers

Input : The set of charging demands \mathcal{J}
Output: The minimum number of chargers needed to satisfy all charging demands \bar{m}

```

1  $\bar{m} \leftarrow 0$ ;
2 Sort the set of charging demands  $\mathcal{J}$  in non-decreasing order of arrival times  $r_j$ ;
3 for  $j \in \mathcal{J}$  do
4   Let  $S_j$  be the set of chargers available at time  $r_j$  ordered in the non-decreasing order
   of their charging power;
5   if  $S_j = \emptyset$  then
6     Assign the demand  $j$  to a new charger of type  $l_j$ ;
7      $\bar{m} \leftarrow \bar{m} + 1$ ;
8   else
9     Let  $i$  be the first charger in  $S_j$  such that  $w_i \geq w_{l_j}$ ;
10    if  $i$  exists then
11      Assign the demand  $j$  to the charger  $i$ ;
12    else
13      Replace the last charger in  $S_j$  by a charger of type  $l_j$  and assign the demand
       $j$  to that charger;
14    end
15  end
16 end
17 return  $\bar{m}$ 

```

3.4.2 Minimum Grid Capacity

3.4.2.1 Complexity

In the following, we show that minimizing the grid capacity is \mathcal{NP} -hard even when there are only two types of chargers.

Theorem 3.4.2.1. *The problem of minimizing the grid capacity with at least two types of chargers is \mathcal{NP} -hard.*

Proof. We prove the \mathcal{NP} -hardness of the EVCS problem by reduction from the Partition problem, which is known to be \mathcal{NP} -hard [Garey & Johnson 1979]. The Partition problem can be stated as follows. Given a positive integer B and a set A of n positive integers $A = \{a_1, a_2, \dots, a_n\}$, where $\sum_{j \in A} a_j = 2B$. Can A be partitioned into two subsets A_1 and A_2 such that $\sum_{j \in A_1} a_j = \sum_{j \in A_2} a_j = B$?

Given an arbitrary instance of the Partition problem, we build an instance (I) of the EVCS problem as follows. Consider a set of two types of chargers where chargers of type 1 can deliver a power of $w_1 = 3$ and chargers of type 2 can deliver a power of $w_2 = 2$. There are $n + 2$ charging demands D_j , $j = 1, \dots, n + 2$. The arrival times r_j , the departure times d_j , and the energy requirements e_j of those demands ($j = 1, \dots, n + 2$) are given in Table 3.2. The charging time of each demand j , $j = 1, \dots, n + 1$, on each charger of type l , $l = 1, 2$, is given as $p_{jl} = \frac{e_j}{w_l}$.

Table 3.2: Charging instance I .

Charging demands	Arrival time r_j	Departure time d_j	Requested energy e_j
$D_j, j = 1, \dots, n$	0	9B	$6a_j$
D_{n+1}	0	6B	12B
D_{n+2}	2B	9B	21B

Now, we show that there is a feasible schedule for I with a grid capacity of $w_G = 5$ if and only if the Partition problem admits a solution.

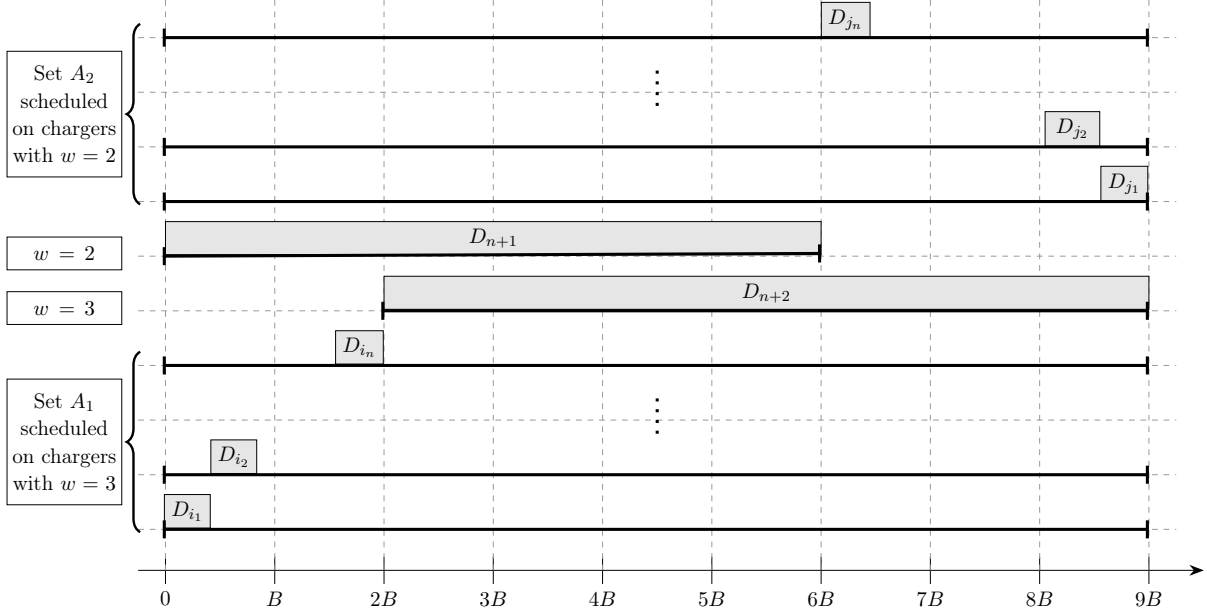
First, suppose the given instance of Partition has a solution. Let A_1 and A_2 be the required subsets of A where $\sum_{j \in A_1} a_j = \sum_{j \in A_2} a_j = B$. Then we can build the schedule of charging demands as follows:

- schedule the charging demands belonging to set A_1 on chargers delivering a power of $w_1 = 3$,
- schedule the charging demands belonging to set A_2 on chargers delivering a power of $w_2 = 2$,
- schedule the charging demand D_{n+1} on a charger delivering a power of $w_2 = 2$, and finally,
- schedule the charging demand D_{n+2} on a charger delivering a power of $w_1 = 3$.

The charging time of each charging demand j in set A_1 is equal to $2a_j$, while it is equal to $3a_j$ for each charging demand j in set A_2 . Charging demands in set A_1 are scheduled sequentially between 0 and $2B$, while charging demands in set A_2 are scheduled sequentially between $6B$ and $9B$. Charging demands D_{n+1} and D_{n+2} are scheduled in the intervals $[0, 6B)$ and $[2B, 9B)$, respectively. Figure 3.3 provides an illustration of this schedule. As we can notice, the grid capacity value at any time does not exceed 5.

Conversely, assume now that there exists a feasible schedule S of all charging demands in I without exceeding a grid capacity value of $w_G = 5$. Since charging demand D_{n+2} requires $21B$ of energy in the interval $[2B, 9B)$, it can only be scheduled on a charger with a power $w_1 = 3$. Thus, the charging time of D_{n+2} is equal to $7B$, and it starts charging at time $t = 2B$, and ends at time $t = 9B$. Now, suppose that D_{n+1} is scheduled on a charger delivering a power of $w_1 = 3$. Since D_{n+1} requests $12B$ amount of energy, its charging time on this charger is equal to $4B$. Therefore, when scheduling D_{n+1} in the interval $[0, 6B)$, its charging overlaps with the charging of D_{n+2} . This implies that the grid capacity limit exceeds 5 units of power. Consequently, D_{n+1} must be scheduled on a charger with a power $w_2 = 2$ and is charged for $6B$ starting at time $t = 0$ and ending at time $t = 6B$. Observe that by scheduling of D_{n+1} and D_{n+2} on chargers with a power of $w_2 = 2$ and $w_1 = 3$ as explained, there are five units of power of the grid capacity that are already consumed in the interval $[2B, 6B)$. Hence the remaining charging demands D_j , $j = 1, \dots, n$, cannot be charged in the interval $[2B, 6B)$.

Let A_1 and A_2 be the set of selected demands from the set of remaining demands D_j , $j = 1, \dots, 2n$, to be scheduled on chargers with power $w_1 = 3$ and $w_2 = 2$, respectively. The total charging time p_{A_1} and p_{A_2} of set A_1 and A_2 are $p_{A_1} = \sum_{j \in A_1} 2a_j$ and $p_{A_2} = \sum_{j \in A_2} 3a_j$,


 Figure 3.3: Schedule of instance I .

respectively. Since the grid capacity is limited to 5 units of power, and three units of power are already consumed by D_{n+2} in the interval $[2B, 9B)$, demands of a set A_1 can only be charged sequentially in the interval $[0, 2B)$. Thus,

$$\sum_{j \in A_1} a_j \leq B \quad (3.1)$$

In addition, with the remaining units of power after scheduling demands D_{n+1} and D_{n+2} , the total charging time of demands in sets A_1 and A_2 cannot exceed $5B$. Consequently, $\sum_{j \in A_1} 2a_j + \sum_{j \in A_2} 3a_j = 4B + \sum_{j \in A_2} a_j \leq 5B$, which implies that $\sum_{j \in A_2} a_j \leq B$ and

$$\sum_{j \in A_1} a_j \geq B \quad (3.2)$$

From inequalities (3.1) and (3.2), we have $\sum_{j \in A_2} a_j = B$ and $\sum_{j \in A_1} a_j = B$. Therefore, we form a solution for the Partition problem. \square

Remark 3.4.2. Observe that in the proof of Theorem 3.4.2.1, we know the charging power rates of the available types but the number of chargers of each type is not fixed beforehand. However, even if it was, the problem is still \mathcal{NP} -hard.

Remark 3.4.3. When we assign vehicles to chargers, the problem of determining the optimal charging schedule of these demands with a constant charging power rates so that we do not exceed a specific grid capacity value remains open.

3.4.2.2 Mathematical formulation

In contrast to identical chargers, the minimum number of chargers required to satisfy all charging demands in the case of different types of chargers with the minimum grid capacity cannot be determined before actually scheduling these demands (as explained in Remark 3.4.1). Therefore, we provide a mathematical formulation that only assigns each vehicle to a charger type, assuming that there are enough chargers of each type, i.e., the number of chargers m_l of each type l , $l \in \mathcal{K}$, is not fixed. Then, we extend the model to the problem of finding the minimum grid capacity with a fixed number of chargers of each type.

Consider the set of charger types $\mathcal{K} = \{1, \dots, k\}$, $k \geq 2$, where each charger of type l , $l \in \mathcal{K}$ delivers an output power of w_l (kW). Recall that the charging time needed to satisfy the demand of vehicle j on charger type l is equal to p_{jl} . The objective is minimizing the grid capacity. First, we restrict our attention to the case where there are enough chargers of each type. We define a binary decision variable x_{jt} , for each $j \in \mathcal{J}$, and $t \in \mathcal{H}$, that takes value one if vehicle j is charging at time slot t . In addition, we introduce the binary variable y_{jl} , for each $j \in \mathcal{J}$, and $l \in \mathcal{K}$, to indicate whether or not vehicle j is assigned to a charger of type l . Let w_G be a non-negative continuous variable that represents the grid capacity value to be minimized. This yields the following MILP model.

$$\min w_G \quad (3.3)$$

$$\sum_{t=1}^T x_{jt} = \sum_{l=1}^k y_{jl} p_{ij} \quad \forall j \in \mathcal{J} \quad (3.4)$$

$$\sum_{l=1}^k y_{jl} = 1 \quad \forall j \in \mathcal{J} \quad (3.5)$$

$$\sum_{j=1}^n \sum_{l=1}^k w_l y_{jl} x_{jt} \leq w_G \quad \forall t \in \mathcal{H} \quad (3.6)$$

Constraints (3.4) ensure that each charging demand j is satisfied. Constraints (3.5) imply that each vehicle j is only assigned to one type of charger l . The calculation of the grid capacity at any time slot t is represented in constraints (3.6). In addition, to restrict the charging of each vehicle j , $j \in \mathcal{J}$, to its plugging time interval $[r_j, d_j)$, binary variables x_{jt} are set to zero for all $t \in \mathcal{H}$ where $t < r_j$ and $t \geq d_j$.

Observe that constraints (3.6) contain a quadratic term involving products of binary variables y_{jl} and x_{jt} , which can be linearized by replacing each product $y_{jl}x_{jt}$ by an additional binary variable z_{jlt} . Then, for all $j \in \mathcal{J}$, $l \in \mathcal{K}$, and $t \in \mathcal{H}$, the binary variable z_{jlt} is set to one if and only if variables y_{jl} and x_{jt} are equal to one. Hence, constraints (3.6) can be expressed with the following constraints:

$$z_{jlt} \geq x_{jt} + y_{jl} - 1 \quad \forall j \in \mathcal{J}, l \in \mathcal{K}, t \in \mathcal{H} \quad (3.7)$$

$$z_{jlt} \leq x_{jt} \quad \forall j \in \mathcal{J}, l \in \mathcal{K}, t \in \mathcal{H} \quad (3.8)$$

$$z_{jlt} \leq y_{jl} \quad \forall j \in \mathcal{J}, l \in \mathcal{K}, t \in \mathcal{H} \quad (3.9)$$

$$\sum_{l=1}^k \sum_{j=1}^n w_l z_{jlt} \leq w_G \quad \forall t \in \mathcal{H} \quad (3.10)$$

Finally, When the number of chargers of each type is limited, i.e., there are only m_l available chargers of each type l , $l \in \mathcal{K}$, the following constraints are added to the MILP model.

$$\sum_{j=1}^n y_{jl} \leq m_l \quad \forall l \in \mathcal{K} \quad (3.11)$$

3.4.2.3 Solving methods

The problem of minimizing the grid capacity in the case of identical chargers is polynomial, as proven in Section 3.3.2.1. Therefore, it can be efficiently solved by either the flow network method presented in the proof in Section 3.3.2.1. As a result, this section only addresses the problem of minimizing the grid capacity with different types of chargers, which was shown to be \mathcal{NP} -hard in Section 3.4.2.1. Unless $\mathcal{P} = \mathcal{NP}$, there is no polynomial-time algorithm to solve this problem. Moreover, even for small problem instances (e.g., ten charging demands and three types of chargers), we find that solving the MILP model given in Section 3.4.2.2 is time-consuming and thus not practical. So, this section presents the developed heuristic and metaheuristic algorithms to obtain near-optimal schedules in a reasonable amount of time. Since it is more suitable to find the minimum grid capacity without fixing the number of chargers of each type, the presented algorithms only consider the assignment of vehicles to chargers types.

Solution representation. As previously stated, we only consider the assignment each vehicle j to a charger type l . Thus, the assignment solution can be presented with a vector $\sigma = (\sigma_1, \dots, \sigma_n)$ where σ_j , $j \in \mathcal{J}$, is the charger type selected to charge vehicle j . Then, we have to decide the charging schedule, i.e., choosing the appropriate time slots to charge each vehicle j according to σ_j . The charging schedule, also called the power allocation solution, is represented with a vector (T_1, \dots, T_n) where T_j is a vector with $(d_j - r_j)$ Boolean values, $T_j = (u_1, u_2, \dots, u_{r_j - d_j})$. The vehicle j is charging at time slot t if $u_{t-r_j+1} = 1$ in T_j . Otherwise, $u_{t-r_j+1} = 0$.

To simplify notation in the next subsections, we define the vector $(w_G^t)_{t \in \mathcal{H}}$ that stores the total power delivered to vehicles at each time slot t .

Consider Example 3.2.1. We group the five chargers into three types of chargers where chargers of type 1, 2, and 3 deliver 10 kW, 20 kW, and 30 kW, respectively. Then, the solution representation for the schedule of charging demands on different types of chargers given in Example 3.2.1 is shown in Figure 3.4.

Heuristic method The proposed heuristic, detailed in Algorithm 3, builds a charging schedule by considering vehicles in the non-decreasing order of their departure time d_j , and breaks ties first by non-increasing order of their energy request e_j , then by non-decreasing order of their arrival time r_j (line 1). First, the heuristic calculates a lower bound lb for the minimal grid capacity (line 4). Then, for each vehicle j (lines 5-15), it determines to which type of charger the vehicle is assigned (lines 7-12). Finally, the heuristic selects the time slots on which

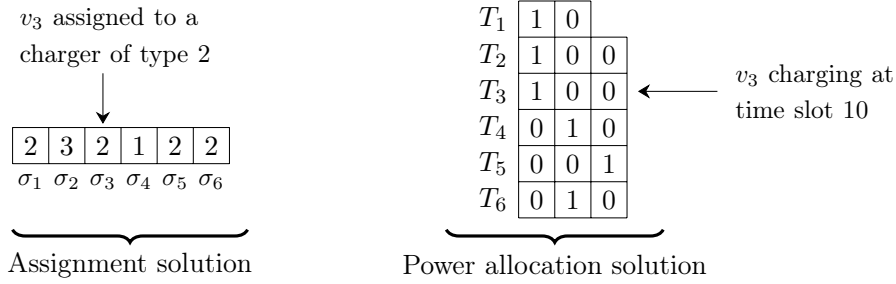


Figure 3.4: Solution representation of the schedule of Figure 3.1(1).

the vehicle is charged according to Algorithm 4. In the following, we provide details on each step of the heuristic.

Lower bound. A lower bound for the minimum grid capacity can be calculated by considering the total energy required by all vehicles divided by the total vehicle availability interval as follow:

$$lb = \max \left\{ \left[\frac{\sum_{j=1}^n e_j}{(\max_{j \in \mathcal{J}} d_j - \min_{j \in \mathcal{J}} r_j)} \right]; \left(w_l, l \in \mathcal{K} : w_{l-1} < \max_{j \in \mathcal{J}} \left[\frac{e_j}{d_j - r_j} \right] \leq w_l \right) \right\} \quad (3.12)$$

The first term in the max function of lb corresponds to the value of the minimum grid capacity when all demands are charged in the interval $[\min_{j \in \mathcal{J}} r_j, \max_{j \in \mathcal{J}} d_j)$, while the second term guarantee that the grid capacity is at least equal to the maximum power rate needed to satisfy a charging demand during its plugging interval. The time complexity of the lb calculation is $\mathcal{O}(n)$.

To illustrate the calculation of lb , consider the following example. We have four types of chargers, where chargers of type 1, 2, 3, and 4 deliver 10, 20, 40, and 50 kW, respectively. We have three charging demands: v_1 with $r_1 = 1$, $d_1 = 4$, and, $e_1 = 90$, v_2 with $r_2 = 4$, $d_2 = 8$, and, $e_2 = 20$, and v_3 with $r_3 = 4$, $d_3 = 8$, and, $e_3 = 30$. We have $\sum_{j=1}^n e_j = 140$, $\max_{j \in \mathcal{J}} d_j = 8$, and $\min_{j \in \mathcal{J}} r_j = 1$. Thus, The first lb term gives 20. We have $\left\lceil \frac{e_1}{d_1 - r_1} \right\rceil = 8$, $\left\lceil \frac{e_2}{d_2 - r_2} \right\rceil = 5$, and $\left\lceil \frac{e_3}{d_3 - r_3} \right\rceil = 30$. Then, the minimum type of chargers needed to satisfy the demand v_3 as expressed in the second term of lb is 40. Hence, $lb = 40$.

Assignment to a charger type. For each vehicle j , the heuristic begins by seeking the greatest charging power rate allowed to charge j without exceeding the current grid capacity (lines 6 and 7). If a charger type delivering this power exists, the vehicle j is assigned to it (line 9). Otherwise, any selected charging power will increase the current grid capacity. Thus, the heuristic chooses a charger type with the smallest power that can satisfy the vehicle's charging demand (line 11). The time complexity for choosing a charger type is $\mathcal{O}(T)$ if we consider that the number of time slots T is larger than the number of charger types k .

Power allocation. Once the charging power w_{σ_j} is selected to charge the vehicle j , the power allocation heuristic displayed in Algorithm 4 is applied. The power allocation

heuristic starts charging vehicle j on time slots without exceeding the maximum between w_G and lower bound (lb) in chronological order (lines 2, 10-17), then, on time slots with the minimum w_G^t value (lines 3, 10-17). The time complexity of the power allocation is $\mathcal{O}(T \log T)$.

Finally, the overall time complexity of the heuristic is $\mathcal{O}(n \max(\log n, T \log T))$.

Algorithm 3: Heuristic to minimize the grid capacity with k types of chargers

Input : The set of charging demands \mathcal{J} , the set of chargers types \mathcal{K}

Output: The minimum required grid capacity w_G

- 1 Sort \mathcal{J} by non-decreasing order of departure times d_j . Break ties by the non-increasing order of energy demands e_j , then, by the non-decreasing order of arrival times r_j ;
 - 2 $(w_G^t) \leftarrow (0)_{t \in \mathcal{H}}$;
 - 3 $w_G \leftarrow 0$;
 - 4 $lb \leftarrow \max \left\{ \left\lceil \frac{\sum_{j=1}^n e_j}{(\max_{j \in \mathcal{J}} d_j - \min_{j \in \mathcal{J}} r_j)} \right\rceil ; \left(w_l, l \in \mathcal{K} : w_{l-1} < \max_{j \in \mathcal{J}} \left\lceil \frac{e_j}{d_j - r_j} \right\rceil \leq w_l \right) \right\}$;
 - 5 **for** $j \in \mathcal{J}$ **do**
 - 6 Let b be the number of time slots in the interval $[r_j, d_j]$ where $w_G^t < \max(lb, w_G)$;
 - 7 Let w_G^b be the maximum value of w_G^t where $w_G^t < \max(lb, w_G)$ and $t \in [r_j, d_j]$;
 - 8 **if** $\lceil \frac{e_j}{b} \rceil + w_G^b \leq \max(lb, w_G)$ **then**
 - 9 $\sigma_j \leftarrow$ the charger type with $w = \max_{l \in \mathcal{K}} w_l$ and $w \leq \frac{e_j}{b}$;
 - 10 **else**
 - 11 $\sigma_j \leftarrow$ the charger type with $w = \min_{l \in \mathcal{K}} w_l$ and $w \geq \frac{e_j}{(d_j - r_j)}$;
 - 12 **end**
 - 13 Schedule the charging of j on a charger that delivers w_{σ_j} on time slots according to Algorithm 4 ;
 - 14 Update the values of w_G and the vector $(w_G^t)_{t \in \mathcal{H}}$;
 - 15 **end**
 - 16 **return** w_G
-

Iterated Local Search (ILS). In this section, we adopt an Iterated Local Search (ILS) to solve the minimum power grid capacity problem. ILS is a metaheuristic which has been used to solve various optimization problems. A recent review of ILS and its various extensions and applications can be found in [Lourenço *et al.* 2019]. The essence of ILS can be given as follows. Starting from an initial solution as the current solution S' , the ILS iteratively perturbs S' , leading to a new solution S . Then, a local search procedure is applied to S . The new solution S is accepted or rejected at the end of each iteration according to an acceptance criterion.

Now, we detail the implemented ILS presented in Algorithm 5. The proposed ILS method requires an initial solution S_0 , and three parameters, namely, the initial perturbation level ($pert_0$), the maximum perturbation level ($pert_{\max}$), and the maximum number of consecutive non-improving iterations ($iter_{\max}$) for each perturbation level. The initial solution S_0 is set as the global best solution S^* as well as the current solution S' (line 2). At each iteration, a

Algorithm 4: Power allocation heuristic

Input : The charging demand j , the selected charging power w_{σ_j} , the vector $(w_G^t)_{t \in \mathcal{H}}$, the grid capacity w_G

Output: The minimum grid capacity w_G

- 1 Let p be the number of time slots required to charge j on a charger of type σ_j ;
- 2 $H_1 \leftarrow$ the set of time slots t where $t \in [r_j, d_j]$ and $\max(w_G, lb) \geq w_j + w_G^t$ sorted in chronological order;
- 3 $H_2 \leftarrow$ the set of time slots t where $t \in [r_j, d_j]$ and $t \notin H_1$ sorted in non decreasing order of w_G^t ;
- 4 **while** $p > 0$ **do**
- 5 **if** $H_1 \neq \emptyset$ **then**
- 6 $H_i \leftarrow H_1$;
- 7 **else**
- 8 $H_i \leftarrow H_2$;
- 9 **end**
- 10 Let t be the first time slot in H_i ;
- 11 Set u_{t-r_j+1} to 1 in T_j ;
- 12 $w_G^t \leftarrow w_G^t + w_{\sigma_j}$;
- 13 $p \leftarrow p - 1$;
- 14 $H_i \leftarrow H_i - \{t\}$;
- 15 **if** $w_G^t > w_G$ **then**
- 16 $w_G \leftarrow w_G^t$;
- 17 **end**
- 18 **end**
- 19 **return** w_G

new solution S is generated by applying p perturbations to the current solution S' (lines 5-6). The number of perturbations p is generated between the 1 and $pert$ (line 4), where $pert$ is the perturbation level initially set to a relatively small value $pert_0$. The perturbation level defines how much the perturbation changes the current solution. If it is too small, the ILS may not be able to escape the current local optimum, while if it is too large, the ILS may behave as a multi-start local search with randomly generated starting solutions. Therefore, we choose to randomize the perturbation level and adapt it at each iteration.

After perturbing the current solution, a local search procedure is applied, and the new solution S is updated (line 8). The objective value of S , denoted as $f(S)$, is then compared to that of the global best solution S^* . If S is better than S^* (lines 9-13), the new solution S replaces both S^* and S' and the number of iterations $iter$ and the perturbation level $pert$ are reset to 0 and $pert_0$, respectively. Otherwise (lines 14-19), a random number u is generated and the solution S may replace the current solution S' if the probability p_{iter} is less than u . p_{iter} decreases in a geometric way [Ogbu & Smith 1990] and is calculated as follows. $p_{iter} = p_0 \times r^{iter-1}$, where p_0 is the initial acceptance probability, r is the reducing factor ($0 < r < 1$), and $iter$ is the number of iterations. As a better solution become harder to find, the perturbation level $pert$ increases

after $iter_{\max}$ non-improving iterations (line 23). The increase of $pert$ allows the ILS to explore the search space away from the current solution. The search, i.e., lines 4-20, is resumed with the new perturbation level and a current solution set to the best solution found so far (line 24). The ILS loop is repeated until the maximum perturbation level ($pert_{\max}$) is met.

Algorithm 5: Iterated local search

Input : The initial solution S_0 ,
 ILS parameters: $pert_0, pert_{\max}, iter_{\max}, r$

Output: The best solution found S^*

```

1  $iter \leftarrow 0$ ;  $pert \leftarrow pert_0$ ;  $pert_{\max} \leftarrow pert_{\max} \times pert_0$ ;
2  $S \leftarrow S_0$ ;  $S' \leftarrow S_0$ ;  $S^* \leftarrow S_0$ ;
3 while  $pert < pert_{\max}$  do
4    $p \leftarrow$  choose a random number between 1 and  $pert$ ;
5   for  $s = 1$  to  $p$  do
6      $S \leftarrow$  Perturbation( $S'$ );
7   end
8    $S \leftarrow$  LocalSearch( $S$ );
9   if  $f(S) < f(S^*)$  then
10     $S' \leftarrow S$ ;
11     $S^* \leftarrow S$ ;
12     $iter \leftarrow 0$ ;
13     $pert \leftarrow pert_0$ ;
14  else
15    Generate a random number  $u \sim U(0, 1)$ ;
16    if  $u < p_0 \times r^{iter-1}$  then
17       $S' \leftarrow S$ ;
18       $iter \leftarrow iter + 1$ ;
19    end
20  end
21  if  $iter \geq iter_{\max}$  then
22     $iter \leftarrow 0$ ;
23     $pert \leftarrow pert + pert_0$ ;
24     $S' \leftarrow S^*$ ;
25  end
26 end
27 return  $S^*$ 

```

In what follows, we detail the four components to consider: the generation of the initial solution, the perturbation mechanism, and the local search procedure.

Initial solution The initial solution can be generated using one of these methods: (i) the solution found by the heuristic described in Section 3.4.2.3, (ii) by choosing a type l from \mathcal{K} , assigning

all vehicles to this type, then solving the the problem for identical chargers (Section 3.3.2).

We tried three strategies for the initial solution:

- Initialize with the heuristic solution only.
- Get the solutions generated by solving the problem with one type of charger and the heuristic, and then select the best one.
- Randomly choose between the heuristic and one-type charger solutions.

We observed that the last strategy was the better one. Initialization with the best solution between the four solutions did not result in the best results since the ILS would converge prematurely.

Perturbation mechanism In Algorithm 5, the current solution S' is perturbed (line 6) by modifying the assignment vector σ . More precisely, the perturbation consists of selecting a vehicle j , $j \in \mathcal{J}$ and changing its charger type σ_j to a new charger type l , $l \in \mathcal{K}, l \neq \sigma_j$ if it can satisfy j , i.e., $p_{jl} \leq d_j - r_j$. Three strategies are proposed to select a vehicle j :

- **A random selection:** select a random electric vehicle j and assigned it to a randomly selected type l .
- **Increase the charging power:** for each vehicle j , we calculate the value a_j as the number of time slots t where $u_{t-r_j+1} = 1$ in T_j and $w_G^t \leq w_G^{threshold}$, where $w_G^{threshold}$ is a parameter. Then, a roulette wheel selection [Lipowski & Lipowska 2012] is performed i.e., a vehicle j with a higher value a_j has a higher probability to be chosen. Then, the selected vehicle j is assigned to a type l with higher charging power, i.e., $w_l > w_{\sigma_j}$.
- **Decrease the charging power:** this selection is the similar to the previous one, except that it selects time slots t with $w_G^t > w_G^{threshold}$, and the selected vehicle is assigned to a charger type delivering a lower charging power.

The perturbation step is followed by Algorithm 4 to determine the instants of charging of vehicles. Finally, the objective value w_G is updated. Based on first experiments, $w_G^{threshold}$ is set to $\frac{1}{2}w_G$.

Local search procedure. As stated in Remark 3.4.3, the problem of finding the optimal grid capacity of an assignment solution remains open. So, instead of moving to another assignment solution, we have to explore its corresponding power allocation solutions. Hence, the local search procedure will try to minimize the grid capacity value without changing the assignment vector σ of vehicles to chargers. The performance of the ILS algorithm depends on the choice of the embedded local search. The better the local search, the better the corresponding ILS. Based on preliminary tests, using a local search algorithm that accepts only improving solutions, such as Hill Climbing, was less effective. Therefore, we choose the Simulated Annealing (SA) algorithm as the local search procedure for ILS. The SA algorithm will take the current solution of the ILS after perturbation as the initial solution. Then, it will iteratively improve this

solution by improving only its power allocation solution. The assignment solution remains unchanged. We first present the implemented SA algorithm's general framework and then present the neighborhood operators.

General framework of the simulated annealing algorithm. The simulated annealing (SA) algorithm, initially proposed by [Kirkpatrick *et al.* 1983], is a stochastic local search metaheuristic successfully adapted to address several scheduling problems. The detailed procedure of the implemented SA is presented in Algorithm 6. It starts by taking as input an initial solution (S_0), and five parameters: the maximum number of generated neighbors at each iteration ($MaxGenerated$), the acceptance ratio at each iteration ($AcceptanceRatio$), the final temperature (T_f), the maximum global number of generated solutions ($MaxTrials$), and the parameter for initializing the value of the temperature (μ). First (line 1), the initial solution S_0 is set as the current solution S and as the global best solution S_{best} . The temperature parameter T is initially set to a value proportional to the objective function value of the initial solution $T = \mu f(S_0)$. The maximum number of accepted solutions at each iteration ($MaxAccepted$) is initially set proportionally to the parameter ($MaxGenerated$) (line 2). At each iteration (lines 3-20), SA generates neighborhood solutions of the current solution S until reaching either the maximum generated neighbors ($MaxGenerated$) or the maximum number of accepted solutions ($MaxAccepted$). For each new solution S' , the global number of generated solutions ($trial$) and the number of generated neighbors of S' ($generated$) are incremented (lines 8-9). The objective function value of each solution, denoted as $f(S)$, represents the number of scheduled demands. The gap between the objective values of the new solution S' and the current solution S is calculated as $\Delta f = f(S') - f(S)$. The neighborhood solution S' is accepted and replaces the current solution based on the Metropolis criteria (lines 11-16); the new solution S' replaces the current solution if there is an improvement, i.e., $\Delta f < 0$. If S' improves the best solution found so far, it will become the new global best solution S_{best} . Otherwise, a random number u is generated following the uniform distribution $U[0, 1]$ and the neighborhood solution S' will become the current solution if $U(0, 1) \leq e^{-\Delta f/T}$ where T is the temperature parameter that controls the probability of accepting worse solutions. For each accepted solution, the parameter $accepted$ is incremented (line 13). Finally, a cooling scheme gradually decreases the temperature at each iteration (line 19). We consider the Lundy-Mees cooling scheme proposed by [Lundy & Mees 1986]. It updates the temperature T at each iteration l as $T_{l+1} = \frac{T_l}{a+bT_l}$. Connolly in [Connolly 1990] develops a variant of the Lundy-Mees scheme that set the parameter a to 1 and b as a function of the initial temperature T_0 , the final temperature T_f and the size of the neighborhood M as $b = \frac{T_0 - T_f}{MT_0T_f}$. Here, the number of iterations is not fixed directly. In fact, if we omit the condition on $MaxAccepted$, the number of iterations will be equal to $maxTrials$ divided by $MaxGenerated$. Thus, we set M to this value (line 1).

After updating the temperature, the number of generated neighbors ($generated$) and the number of accepted solutions ($accepted$) are reset to zero (line 5). The algorithm will stop if the number of generated solutions ($trial$) reaches its maximum ($MaxTrials$), or after generating ($MaxGenerated$) solutions that did not result in accepted solutions, i.e., $accepted = 0$ (line 20). When the stopping criterion is met, the algorithm terminates and returns the best solution S_{best}

Algorithm 6: Simulated annealing

```

input : The initial solution  $S_0$ ,
          Parameters:  $MaxGenerated$ ,  $AcceptanceRatio$ ,  $MaxTrials$ ,  $T_f$ ,  $\mu$ 
output: Best solution found  $S_{best}$ 
1  $S_{best} \leftarrow S_0$ ;  $S \leftarrow S_0$ ;  $T \leftarrow \mu f(S_0)$ ;  $M \leftarrow \frac{MaxTrials}{MaxGenerated}$ ;  $trial \leftarrow 0$ ;  $b \leftarrow \frac{T-T_f}{TMT_f}$ ;
2  $MaxAccepted \leftarrow AcceptanceRatio \times MaxGenerated$ ;
3 repeat
4    $accepted \leftarrow 0$ ;  $generated \leftarrow 0$ ;
5   while  $generated \leq MaxGenerated$  and  $accepted \leq MaxAccepted$  do
6      $S' \leftarrow Generate(S)$ ;
7      $\Delta f \leftarrow f(S') - f(S)$ ;
8      $generated \leftarrow generated + 1$ ;
9      $trial \leftarrow trial + 1$ ;
10    Generate a random number  $u \sim U(0, 1)$ ;
11    if  $f(S') < f(S)$  or  $u \leq e^{-\Delta f/T}$  then
12       $S \leftarrow S'$ ;
13       $accepted \leftarrow accepted + 1$ ;
14      if  $f(S) < f(S_{best})$  then
15         $S_{best} \leftarrow S$ 
16      end
17    end
18  end
19   $T \leftarrow \frac{T}{1+bT}$ ;
20 until  $trial \leq maxTrials$  and  $accepted > 0$ ;
21 return  $S_{best}$ 

```

found so far.

Local search neighborhood structure. A neighbor structure in the local search method moves the charging of a vehicle from peak to off-peak time slots. Let \mathcal{J}' be the set of charging demands where $d_j - r_j - p_{jl} > 0$, where p_{jl} is the charging time of vehicle j on its assigned charger type $l = \sigma_j$. First, the local search randomly selects an electric vehicle $j \in \mathcal{J}'$. Let H_1 be the set of time slots where the vehicle j is charging. That is, $H_1 = \{t | t \in [r_j, d_j] \text{ and } u_{t-r_j+1} = 1\}$. Let H_2 be the set of time slots where $H_2 = \{t | t \in [r_j, d_j] \text{ and } u_{t-r_j+1} = 0\}$. The local search will move the charging of j from t_1 to t_2 where $w_G^{t_1} = \max_{\{t \in H_1\}} w_G^t$, and $w_G^{t_2} = \min_{\{t \in H_2\}} w_G^t$. This procedure is repeated q times for the same vehicle, where q is randomly selected in $\{1, \dots, p_{jl}\}$. After each move, vectors w_G^t and T_j , and the objective function value w_G are updated.

Even though that introducing randomness in the choices made by the ILS and the SA was better for diversification, it was not the case for the choice of time slots. Indeed, we tried random choice combined with the one explained before. However, this led to worsening results.

3.5 Computational Results

In this section, we give our computational results. We first introduce the instances generated to evaluate the performance of the proposed methods. Next, we provide all the settings used for the different algorithms. We focus more on the problem of minimizing the grid capacity with non-identical chargers in since it is \mathcal{NP} -hard then we compare the results with identical chargers.

3.5.1 Instances Generation

In generated instances, we consider a charging station with three types of chargers where chargers of type 1 deliver a power of $w_1 = 11$ (kW), chargers of type 2 deliver a power of $w_2 = 22$ kW, and chargers of type 3 deliver a power of $w_3 = 43$ kW [LaMonaca & Ryan 2022]. We consider five groups of instances, where the number of charging demands n in groups 1, 2, 3, 4, and 5 is equal to 10, 20, 40, 50, and 100, respectively, and for each group, we generate ten different random instances as follows.

- The arrival times of vehicles are generated from the uniform distribution in the interval $[0, 0.2n]$ (in hours). This means that the time horizon length depends on the number of vehicles n .
- The required energy are generated from the uniform distribution $[5.5, 66]$ (in kWh).
- To generate the departure times of vehicles, we first calculate the charging times p_{1j} for each vehicle $j, j \in \mathcal{J}$, assuming that it is charged with type 1 chargers as $p_{1j} = \frac{e_j}{11}$. Then, the departure time of each vehicle j is calculated as $d_j = r_j + (1 + \alpha)p_{1j}$ where α is randomly chosen according to the value p_{1j} as follows:

p_{1j} (hours)	α
$[0.5, 1)$	$[0.1, 1]$
$[1, 2)$	$[0.1, 0.9]$
$[2, 3)$	$[0.1, 0.8]$
$[3, 4)$	$[0.1, 0.7]$
$[4, 5)$	$[0.1, 0.6]$
$[5, 6)$	$[0.1, 0.5]$

Table 3.3: Values of α depending on p_{1j} .

We generate another group of instances, denoted group 6, where the number of charging demands n equals 200. We generate ten different random instances as in previous instances groups, except for α values, where for half of the vehicles, α is fixed to 0.1, and for the other half, it is fixed to 0.2.

3.5.2 Computational and Parameters Settings

The proposed algorithms are implemented in C++ programming language and run on a desktop computer with an Intel Core i5 operating at 2.90 GHz and 8 GB RAM and running Linux OS (Ubuntu 20.04 LTS). The MILP models are solved using IBM CPLEX 12.8 with a time limit of 30 minutes for each instance. The length of time slots is set to $\tau = 0.1$ hour (6 minutes). For each instance, the number of time slots is set to $T = \max_{j \in \mathcal{J}} d_j$. Regarding the stochastic nature of the ILS algorithm and to obtain statistically significant results, 30 independent executions were done with a time limit of 30 minutes for each instance. Based on preliminary experiments, Table 3.4 provides the setting of ILS and SA parameters.

Table 3.4: ILS and SA parameters.

	Parameter	Value
ILS parameters	$pert_0$	3
	$pert_{\max}$	20
	$iter_{\max}$	20
	r	0.75
	p_0	0.5
SA parameters	μ	0.1
	$MaxGenerated$	50
	$MaxTrials$	500
	$AcceptanceRatio$	0.5
	T_f	0.001

3.5.3 Quality of the Lower Bound

In this section, we evaluate the quality of the lower bound (lb), given in Equation (3.12), compared to the optimal solution. Since it is hard to obtain the optimal solutions for instances with n greater than 20, we focused on the instances of group 1 where the number of charging demands n equals 10. We generated 50 instances with n equals 10 as described in Section 3.5.1. We use CPLEX without time limitation until the optimal solution is obtained and the results are reported in Table 3.5. For each instance, Table 3.5 reports the value of lower bound provided by equation (3.12) (column 2), the lower bound calculated by CPLEX (column 3), the value w_G of the optimal solution (column 4) and the percentage gap G_{lb} between the lower bound (lb) and the optimal solution where $G_{lb} = \frac{lb - w_G}{w_G}$. Detailed results for all instances are given in Appendix A. From Table 3.5, we observe that, on average, the lower bound of Equation (3.12) underestimates the optimal solution by about 10% with a standard deviation of 6.6. This observation allows us to evaluate the quality of solutions obtained by the different methods.

3.5.4 Simulation Results for Different Types of Chargers

In this section, we evaluate the performance of the methods proposed for the problem of minimizing the grid capacity value on the six groups of instances generated in Section 3.5.1. The

Table 3.5: Results for instances of group 1 ($n = 10$).

	lb (Eq. 3.12)	lb (CPLEX)	w_G (CPLEX)	G_{lb}
max	52.0	54	54	0.00
min	28.0	33	33	-25.58
median	40.0	43	43	-8.35
avg	40.3	44.9	45.02	-10.50
std. dev.	5.15	4.65	4.70	6.61

considered methods are: solving the MILP model (Section 3.4.2.2) using CPLEX, the heuristic (Section 3.4.2.3), and the ILS presented (Section 19). As mentioned before, we are only interested in knowing to which type of charger a vehicle is assigned, and we do not consider the assignment to a specific charger of this type. Hence, constraints (3.11) are not used in the solved MILP model. Recall that the computational times for CPLEX and ILS are limited to 30 minutes for each instance. Tables 3.6, 3.7, 3.8, 3.9, 3.10 and 3.11 represent the results of computational experiments when the number of electric vehicles n is 10, 20, 40, 50, 100 and 200, respectively. The first column displays the instance name. Column 2 reports the best calculated lower bound between the lower bound described by Equation (3.12) and CPLEX lower bound. Column 3 presents the best objective value (BS) found by the three methods (CPLEX, ILS and heuristic). The remaining columns report, for each method $M \in \{\text{CPLEX, Heuristic, ILS}\}$, the following indicators: $G_{lb}(M)$, $G_{BS}(M)$, and the average running *time* (in seconds); where $G_{lb}(M)$ represents the percentage gap between the lower bound (lb) and the solution generated by the method M , i.e., $G_{lb}(M) = \frac{w_G(M) - lb}{lb}$ and G_{BS} represents the percentage gap between the best solution (BS) and the solution generated by M , i.e., $G_{BS}(M) = \frac{w_G(M) - BS}{BS}$. If the method M can find the best solution then $G_{BS}(M) = 0\%$. Similarly, if the method M finds the optimal solution, then $G_{LB}(M) = 0\%$. Note that the grid capacity values obtained by ILS ($w_G(ILS)$) used in the calculation of the two indicators are the best value found over the 30 runs.

After 1800 seconds, CPLEX will stop and report the best solution so far unless it finds an optimal solution earlier. As we can see, the CPLEX computation time is only less than 1800 seconds for instances 1, 4, 6, and 9, which means that CPLEX could only find four optimal solutions out of 60. Even for small instances with only ten vehicles and three types of chargers, CPLEX struggled to solve the problem within 30 min.

As expected, the results found by CPLEX and ILS were better than those found by the heuristic for all instances. The results found by ILS were better than CPLEX in 35 instances by 4.64% on average. For these 35 instances, the gap between CPLEX and ILS lies between 0.48% and 11.79%, which is more significant for instances with $n \geq 100$. The ILS and CPLEX find solutions with the same value of w_G in 24 instances, mostly for instances with $n = 10$ and $n = 20$. We can observe that in instances with $n \geq 100$, CPLEX has found equal w_G values for two instances out of 20. CPLEX outperforms ILS in terms of minimizing w_G in one instance out of 60 (instance 13 in Table 3.7).

The gap between the lower bound and solutions found by CPLEX and ILS increases with the size of instances. For ILS, it lies between 0.0% and 33% for the first three groups of instances and between 8% and 38% for the last three groups. Moreover, in groups 2-6 of instances, we observe that the value of the lower bound (reported in the second column of each table) is

consistently given by the one calculated using Equation (3.12). As observed in Section 3.5.3, the lower bound of Equation (3.12) underestimates the optimal solution by at least 10%. We can therefore estimate the gap between the optimal solution and the solutions given by ILS for the large instances (groups 4-6) at less than 12%.

The running times of the heuristic and the ILS are relatively short compared to CPLEX; less than 0.1 ms are needed to solve large instances using the heuristic, and the ILS can find good solutions for instances with 200 vehicles in 20 seconds on average.

Table 3.6: Results for instances of group 1 ($n = 10$)

instance	LB	BS	CPLEX			Heuristic			ILS		
			$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)
1	43.00	43	0.00	0.00	63.56	2.33	2.33	4.89E-05	0.00	0.00	4.10
2	46.84	54	15.29	0.00	1801.33	17.42	1.85	5.85E-05	15.29	0.00	4.25
3	46.00	54	17.39	0.00	1800.39	19.57	1.85	6.11E-05	17.39	0.00	3.23
4	43.00	43	0.00	0.00	23.54	27.91	27.91	6.20E-05	0.00	0.00	4.23
5	44.00	54	22.73	0.00	1800.25	25.00	1.85	5.34E-05	22.73	0.00	3.92
6	43.00	43	0.00	0.00	16.69	27.91	27.91	7.05E-05	0.00	0.00	3.62
7	52.67	54	2.53	0.00	1800.06	25.32	22.22	6.67E-05	2.53	0.00	4.41
8	49.15	54	9.88	0.00	1800.30	34.29	22.22	7.07E-05	9.88	0.00	4.46
9	53.99	54	0.01	0.00	480.58	22.23	22.22	6.79E-05	0.01	0.00	3.86
10	50.03	54	7.93	0.00	1800.19	31.92	22.22	9.18E-05	7.93	0.00	4.31
average	47.17	50.7	7.58	0.00	1138.69	23.39	15.26	6.52E-05	7.58	0.00	4.04

Table 3.7: Results for instances of group 2 ($n = 20$)

instance	LB	BS	CPLEX			Heuristic			ILS		
			$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)
11	66.00	76	15.15	0.00	1800.60	31.82	14.47	9.89E-05	15.15	0.00	4.25
12	71.00	86	21.13	0.00	1800.20	54.93	27.91	1.48E-04	21.13	0.00	4.76
13	80.00	87	8.75	0.00	1800.10	37.50	26.44	1.24E-04	10.00	1.15	4.55
14	72.00	86	19.44	0.00	1800.12	37.50	15.12	1.40E-04	19.44	0.00	4.33
15	66.00	86	30.30	0.00	1800.48	50.00	15.12	1.37E-04	30.30	0.00	4.83
16	73.00	77	17.81	11.69	1800.89	35.62	28.57	8.71E-05	5.48	0.00	4.72
17	60.00	66	10.00	0.00	1864.95	28.33	16.67	9.09E-05	10.00	0.00	5.08
18	76.00	86	13.16	0.00	1800.62	30.26	15.12	1.43E-04	13.16	0.00	4.04
19	86.00	97	12.79	0.00	1800.21	27.91	13.40	1.27E-04	12.79	0.00	4.44
20	77.00	86	11.69	0.00	1800.11	42.86	27.91	1.34E-04	11.69	0.00	4.60
average	72.70	83.3	16.02	1.17	1806.83	37.67	20.07	1.23E-04	14.91	0.11	4.56

In previous tables, we reported the $G_{lb}(\text{ILS})$ and $G_{BS}(\text{ILS})$ based on the best values found by the ILS. Since ILS achieves all the best values (except one), we calculate the gap between the grid capacity value obtained by the ILS at each run and the best value obtained over the 30 runs for each instance. We give the five-number summary¹ for the calculated gap values in Figure 3.5. First, we remark that the lengths of the box plots are relatively short, suggesting that the calculated gap values are less spread out. Moreover, the gap values lie between 0% and

¹The five-number summary provides a concise summary of the distribution of a set of data: the minimum, the lower quartile, the median, the upper quartile, and the maximum.

Table 3.8: Results for instances of group 3 ($n = 40$)

instance	LB	BS	CPLEX			Heuristic			ILS		
			$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)
21	91.00	98	18.68	10.20	1801.58	57.14	32.41	1.62E-04	7.69	0.00	5.44
22	90.00	119	33.33	0.84	1800.26	58.89	19.17	1.83E-04	32.22	0.00	5.89
23	111.00	129	17.12	0.78	1802.51	58.56	35.38	2.02E-04	16.22	0.00	5.47
24	122.00	129	15.57	9.30	1800.16	35.25	17.02	2.11E-04	5.74	0.00	5.18
25	106.00	129	22.64	0.78	1800.14	75.47	43.08	2.23E-04	21.70	0.00	6.09
26	103.00	108	4.85	0.00	1800.13	49.51	42.59	1.91E-04	4.85	0.00	5.08
27	88.00	98	22.73	10.20	1800.13	62.50	32.41	1.72E-04	11.36	0.00	5.25
28	112.00	120	7.14	0.00	1827.46	27.68	19.17	1.90E-04	7.14	0.00	6.03
29	108.00	119	11.11	0.84	1801.16	32.41	19.17	2.61E-04	10.19	0.00	6.07
30	79.00	97	24.05	1.03	1803.36	67.09	34.69	1.69E-04	22.78	0.00	5.03
average	101.00	114.6	17.72	3.40	1803.69	52.45	29.51	1.96E-04	13.99	0.00	5.55

Table 3.9: Results for instances of group 4 ($n = 50$)

instance	LB	BS	CPLEX			Heuristic			ILS		
			$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)
31	119.00	129	17.65	8.53	1804.06	47.90	25.71	2.16E-04	8.40	0.00	6.53
32	106.00	120	14.15	0.83	1804.15	55.66	36.36	1.97E-04	13.21	0.00	6.32
33	114.00	141	24.56	0.71	1803.56	73.68	39.44	2.71E-04	23.68	0.00	7.22
34	96.00	129	34.38	0.00	1802.17	83.33	36.43	2.13E-04	34.38	0.00	5.95
35	93.00	120	39.78	8.33	1801.29	77.42	37.50	2.38E-04	29.03	0.00	6.32
36	108.00	129	19.44	0.00	1800.25	73.15	44.96	2.43E-04	19.44	0.00	5.74
37	103.00	129	26.21	0.78	1803.81	81.55	43.85	2.58E-04	25.24	0.00	5.28
38	123.00	140	15.45	1.43	1800.22	60.98	39.44	2.71E-04	13.82	0.00	6.44
39	116.00	140	22.41	1.43	1802.87	70.69	39.44	2.67E-04	20.69	0.00	5.73
40	104.00	120	15.38	0.00	1804.12	69.23	46.67	1.77E-04	15.38	0.00	6.27
average	108.20	129.7	22.94	2.20	1802.65	69.36	38.98	2.35E-04	20.33	0.00	6.18

Table 3.10: Results for instances of group 5 ($n = 100$)

instance	LB	BS	CPLEX			Heuristic			ILS		
			$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)
41	142.00	172	29.58	6.98	1800.54	85.92	43.48	4.74E-04	21.13	0.00	8.47
42	131.00	151	16.03	0.66	1800.48	51.15	30.26	3.74E-04	15.27	0.00	8.51
43	144.00	165	20.83	5.45	1800.59	52.78	26.44	3.68E-04	14.58	0.00	8.23
44	147.00	172	25.17	6.98	1800.59	72.11	37.50	4.02E-04	17.01	0.00	9.10
45	137.00	164	26.28	5.49	1800.57	68.61	33.53	3.97E-04	19.71	0.00	9.57
46	144.00	164	13.89	0.00	1800.51	37.50	20.73	3.61E-04	13.89	0.00	9.09
47	141.00	164	23.40	6.10	1800.51	56.03	26.44	3.75E-04	16.31	0.00	8.43
48	135.00	164	28.89	6.10	1800.53	62.96	26.44	3.60E-04	21.48	0.00	9.00
49	133.00	163	29.32	5.52	1800.55	48.87	15.12	3.73E-04	22.56	0.00	11.44
50	139.00	164	25.90	6.71	1800.51	66.19	32.00	3.76E-04	17.99	0.00	9.52
average	139.30	164.3	23.93	5.00	1800.54	60.21	29.19	3.86E-04	17.99	0.00	9.14

4% for the instances in the first four groups except for outliers values; for example, we have an outlier gap of 25.58% in group 1 instances. This can be explained by the fact that the w_G values are relatively small in these instances. Indeed, 25.58% corresponds to the difference between 54

Table 3.11: Results for instances of group 6 ($n = 200$)

instance	LB	BS	CPLEX			Heuristic			ILS		
			$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)	$G_{lb}(\%)$	$G_{BS}(\%)$	time (s)
51	158.00	172	18.35	8.72	1801.73	32.28	21.51	2.37E-04	8.86	0.00	15.11
52	157.00	205	31.85	0.98	1801.53	68.15	28.78	1.94E-04	30.57	0.00	17.20
53	148.00	172	31.76	13.37	1801.84	41.22	21.51	2.36E-04	16.22	0.00	14.53
54	162.00	198	34.57	10.10	1801.47	62.96	33.33	1.92E-04	22.22	0.00	19.87
55	160.00	206	29.38	0.49	1801.78	58.13	22.82	2.56E-04	28.75	0.00	17.26
56	161.00	194	27.95	6.19	1801.45	36.65	13.40	1.98E-04	20.50	0.00	15.28
57	149.00	165	16.78	5.45	1801.51	25.50	13.33	1.93E-04	10.74	0.00	17.45
58	160.00	198	23.75	0.00	1801.49	51.25	22.22	1.93E-04	23.75	0.00	15.90
59	168.00	187	17.86	5.88	1801.54	44.05	29.41	1.96E-04	11.31	0.00	19.01
60	156.00	215	39.74	1.40	1801.49	76.28	27.91	1.95E-04	37.82	0.00	16.00
average	157.90	191.20	27.20	5.26	1801.58	49.65	23.42	2.09E-04	21.07	0.00	16.76

kW and 43 kW. For the last two groups of instances, the gap lies between 0% and 10%. The median value, presented by an orange line, always lies between 0% and 3%. We can conclude that the proposed ILS is stable in terms of minimizing the grid capacity value since the difference between the w_G values at each run is small.

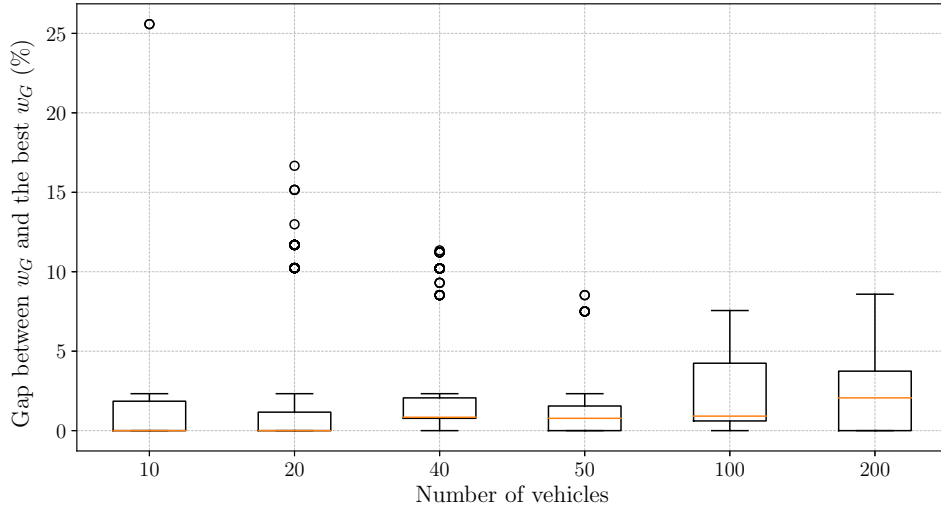


Figure 3.5: Distribution of the gap between the grid capacity value (w_G) and the best w_G value reached by the ILS for each instance. The results are grouped by instance group.

Further, in Figure 3.6, we display in the left box plot the five-number summary of the set of 300 w_G values for each group of instances (each group includes 10 instances and 30 runs are carried for each instance). This figure allows us to visualize the spread of the w_G values in each group of instances to compare the range of w_G value in function of the number of charging demands.

The results tables gave the average running time of ILS over 30 runs. However, it is important

to verify the stability of ILS in terms of execution time. Therefore, the right box plot in Figure 3.6 visualizes the five-number summary of 300 running time values for each group of instances. First, we notice that the worst execution time did not exceed one minute (outliers in box plot for instances with $n = 200$). We can remark that we have more outliers values in each group of instances. When examining these cases, we notice that the outliers values do not necessarily correspond to the best objective value. For example, the best w_G value for Instance 55 (206 kW) was found in 14.56 seconds, while a value of 208 kW was found in 38.37 seconds. Another example, for instance 55, the best w_G (198 kW) was found once in 45.7 seconds while in 11.20 seconds on average in ten other runs.

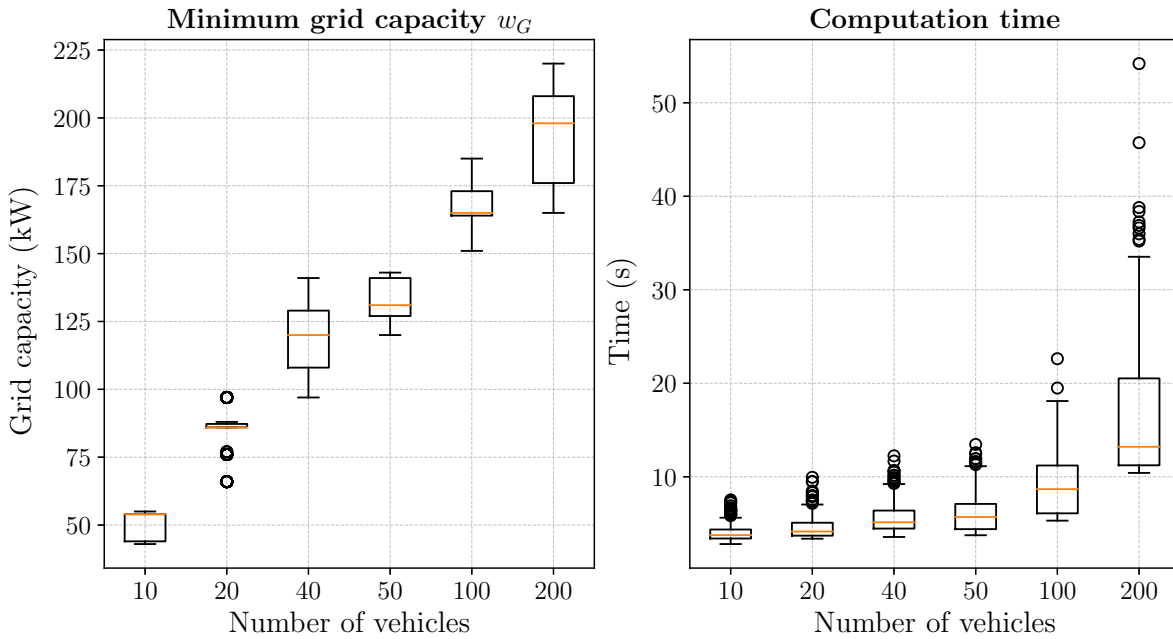


Figure 3.6: Distribution of the grid capacity value (w_G) and the computation time.

Figure 3.7 summarizes the results of tables 3.6 - 3.11. In addition to the best w_G value achieved by the ILS (ILS best), we plot the average w_G as "ILS (mean)". As we can see, the ILS mean w_G values are slightly higher than the best ones but still less than w_G values achieved by CPLEX for groups 3-6 and almost equal for groups 1 and 2. Considering the best values found by the ILS over 30 runs, the average grid capacity has increased by 64.5 % from 10 vehicles to 20, by 37.4% from 20 to 40, by 13.2 % from 40 to 50, by 26.7 % from 50 to 100 vehicles, and finally, by 16.37% from 100 to 200 vehicles.

Figure 3.8 shows five examples of the load profile. We choose one instance of each group to illustrate the power consumption in kW at each time slot.

To complete this section, for each group of instances, we calculate the percentage of electric

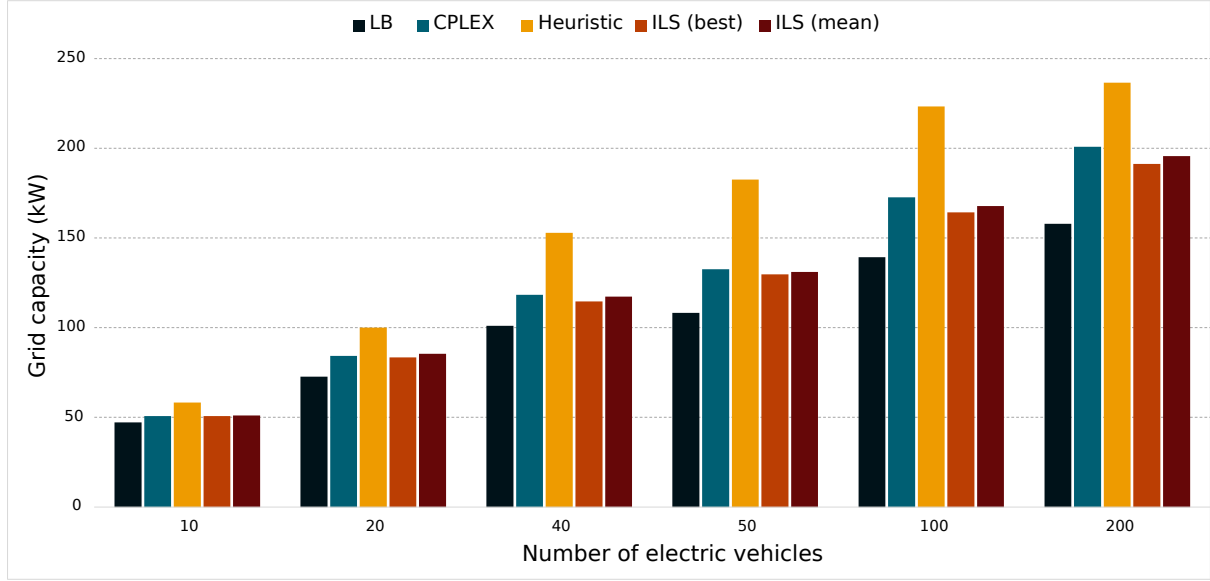


Figure 3.7: Comparison between the lower bound and the average grid capacity values achieved by CPLEX, Heuristic and ILS.

vehicles assigned to each type of chargers, using Heuristic, CPLEX and ILS. The result is reported in Table 3.12. We can see that the heuristic assigns most vehicles (86.43%) to chargers with power 11 kW while CPLEX and ILS assign most vehicles (64.30% and 69.30%, respectively) to chargers with power 43 kW.

Table 3.12: The average percentage of electric vehicles assigned to each type of chargers using Heuristic, CPLEX and ILS.

n	Heuristic			CPLEX			ILS		
	11 kW	22 kW	43 kW	11 kW	22 kW	43 kW	11 kW	22 kW	43 kW
10	83.00%	13.00%	4.00%	10.00%	9.00%	81.00%	12.00%	10.00%	78.00%
20	82.22%	17.22%	0.56%	8.33%	12.22%	79.44%	12.00%	19.68%	68.32%
40	86.75%	12.75%	0.50%	17.00%	25.00%	58.00%	16.50%	8.00%	75.50%
50	87.20%	11.60%	1.20%	11.60%	21.60%	66.80%	11.80%	9.80%	78.40%
100	88.30%	11.60%	0.10%	20.00%	24.00%	56.00%	24.60%	21.50%	53.90%
200	91.15%	8.75%	0.10%	14.60%	40.85%	44.55%	10.60%	27.85%	61.55%

3.5.5 Comparison between Considering One Type of Chargers and Multi-types of Chargers

In this section, we compare the best grid power value found using different types of chargers (results in Section 3.5.4) with the grid power value found using identical chargers. We conduct the simulation on the same instances by first considering a charging station with identical chargers delivering a power of 11 kW, then when all chargers deliver a power of 22 kW, and finally when

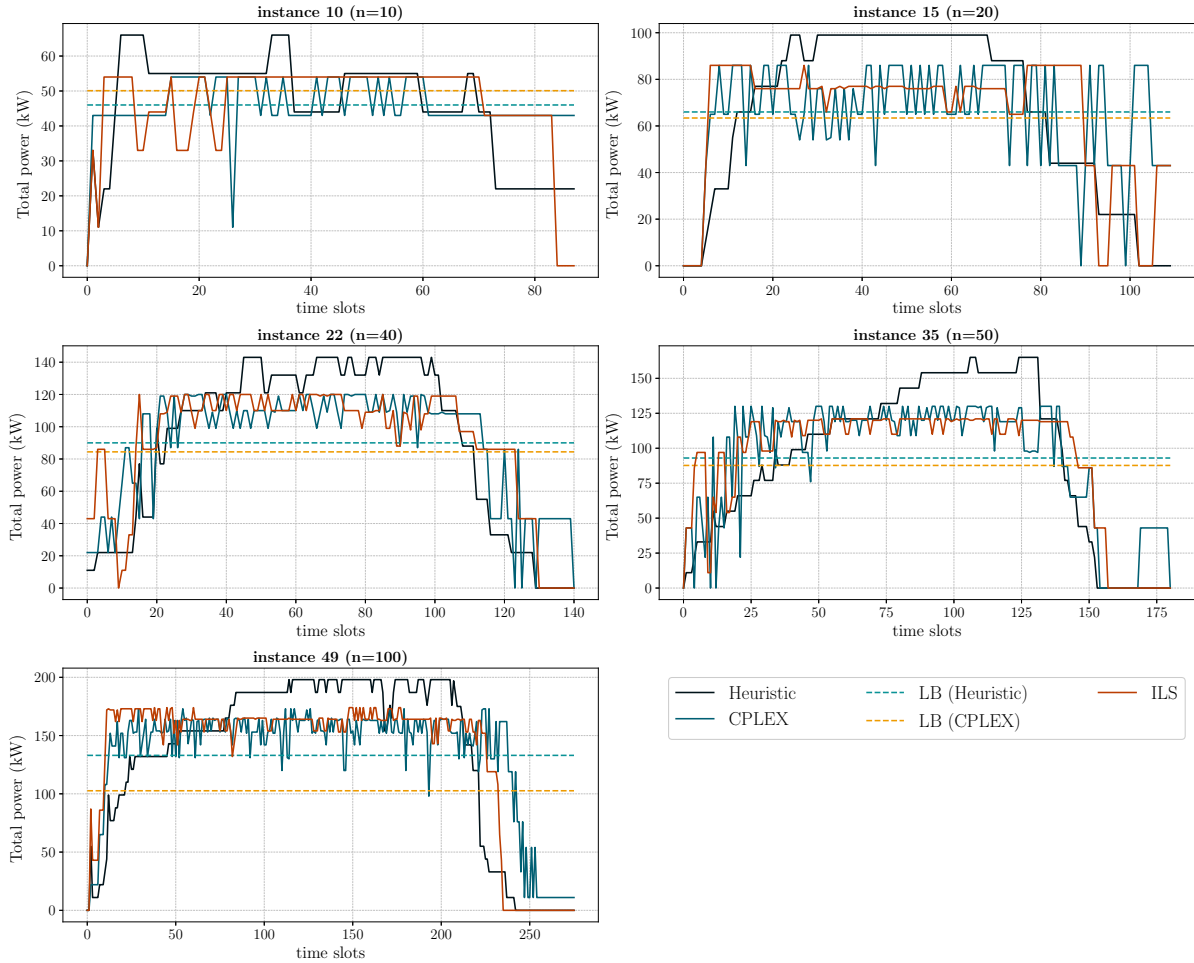


Figure 3.8: Example of load profiles for different instances.

all chargers deliver a power of 43 kW. This comparison allows the decision-maker to evaluate the contribution of installing different types of chargers in a station instead of identical ones and compare the costs of installing chargers according to the service that can be provided.

The results are shown in Table 3.13 which displays the average grid power value found when using: (i) different types of chargers (column w_G (diff. types)), (ii) chargers with power 11kW (column w_G (11 kW)), (iii) chargers with power 22 kW (column w_G (22 kW)), (iv) chargers with power 43kW (column w_G (43 kW)); and for each case, column (Nbr ch.) shows the average minimum number of chargers. We can see that the grid power value using chargers with power 11 kW is always worse than using different chargers for all instances with an average gap of 24.18 kW. The grid power value using chargers with 22 kW is worse than using different chargers over 59 instances out of 60 with an average gap of 10.43 kW. Finally, the grid power value found using chargers with 43 kW were worse in 41 instances with an average gap of 13.15 kW, and equal in 19 instances. As we can see, charging vehicles with different types of chargers is more advantageous. However, this advantage has to be compared with the installation costs of each

type of chargers.

It should be mentioned that the average running time to compute the minimum grid capacity for one type of charger was 0.8 ms (less than 0.01 ms for instances with 10 electric vehicles and less than 3.1 ms for instances with 100 vehicles).

Table 3.13: Comparison between considering one type of chargers and different types of chargers.

n	LB	diff. types of chargers		One type of charger			
		w_G	Nbr ch.	w_G (11 kW)	w_G (22 kW)	w_G (43 kW)	Nbr ch.
10	47.17	50.70	9.5	61.60	63.80	73.10	9.30
20	72.70	83.30	17.56	104.50	94.60	94.60	17.22
40	101.00	114.60	23.5	139.70	125.40	129.00	23.5
50	108.20	129.70	26.4	160.60	140.80	141.90	26.4
100	139.30	164.30	34.9	188.10	173.80	172.00	29.6
200	157.90	191.20	32.5	224.40	198.00	202.10	30.5

3.6 Conclusion

This chapter studied the EVCS problem in a charging station with identical and non-identical chargers. This chapter mainly indicates to charging station operators the complexity of sizing the charging station: finding the minimum grid capacity and the minimum number of chargers needed to satisfy a set of charging demands. We proved that the problem of minimizing the number of chargers required to plug a set of demands is polynomial in both cases. For the problem of minimizing the power grid, we proved that it is polynomial in the case of identical chargers. Even under ideal circumstances where vehicles are plugged into a charger for a fixed duration and considering linear charging times, the problem of finding an optimal grid capacity value is still \mathcal{NP} -hard for different types of chargers. We have developed a heuristic to solve the non-identical \mathcal{NP} -hard problem. An iterated local search (ILS) metaheuristic is further used to improve the heuristic results. Different scenarios were presented to evaluate the performance of the proposed algorithms. We have shown that using a MILP solver is not practicable for solving the minimum grid capacity problem with different types of chargers. The proposed ILS outperforms CPLEX in terms of minimizing the grid capacity and computational time. The ILS achieved better solutions in less than 20 seconds on average. Additional experiments revealed that installing chargers delivering different output power is more advantageous than installing identical chargers.

Preemptive EVCS problem to maximize the number of satisfied charging demands

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4.1 Introduction

This chapter aims to address the same EVCS problem in the previous one. However, the objective is different. The grid capacity and the number of chargers in the charging station are fixed and are considered as constraints in the underlying problem. Most charging station installations do not manage charging operations, so the number of chargers that can be installed is limited by the total power delivered by all chargers simultaneously. The main drawback of

such installation is that the number of chargers limits the number of vehicles parked in the charging station. Indeed, when a driver plugs his vehicle, the cable is locked to the vehicle's charging port, preventing random people from unplugging it. It can only be unlocked by the vehicle owner or the charging station owner. In this case, the vehicle has to be moved to another parking space so that other vehicles can charge. Installing more chargers and limiting the charging station capacity offers more flexibility and allows more vehicles to plug in and charge without overloading the power grid. We are interested in the following questions:

- Does a feasible schedule exist for all charging demands?
- If no feasible schedule exists, what is the maximum subset of charging demand that can be satisfied?

The first question is the decision version of the problem and it is related to the results of the previous chapter. The second question consists of maximizing the number of scheduled vehicles. In other words, we maximize the number of customers that can be served.

The rest of the chapter is constructed as follows. We present the studied problem and give an illustrative example in Section 4.2. In Section 4.3, we tackle the problem with identical chargers, while Section 4.4 investigates the problem with different types of chargers. Computational results are reported in Section 4.5, and finally, a conclusion is put forward in Section 4.6.

Table 4.1: Summary of notations

Notation	Description
\mathcal{J}	Set of n charging demands, indexed by j
\mathcal{M}	Set of m chargers, indexed by i
\mathcal{K}	Set of k types of chargers, indexed by l
\mathcal{H}	Set of T time-slots, indexed by t
r_j	Arrival time of vehicle j
d_j	Departure time of vehicle j
e_j	Energy request of vehicle j in kWh
p_{ij}	Charging time of vehicle j on charger i
p_{jl}	Charging time of vehicle j on charger type l
w_i	Charging power output of charger i in kW
w_G	Grid capacity constraint of the charging station in kW

4.2 Problem Description

We consider the same charging station operating model as in the previous chapter. We use similar notations, which are listed in Table 4.1. Before arriving, each vehicle submits a charging demand to reserve a charger. Charging demands can either be accepted or rejected. If the charging demand j is accepted, it is assigned to one of the m chargers, and its energy requirement must be fulfilled before the departure. Moreover, the vehicle j uninterruptedly occupies the charger,

and the parking space, from its arrival time r_j to its departure time d_j and cannot be moved or unplugged during this time. Again, the vehicle's charging operation can be preempted in the interval $[r_j, d_j)$. Even when the vehicle completes charging before d_j , it still occupies the charger until it departs.

The objective is to find a feasible schedule with the maximum number of satisfied charging demands. Again, we consider two variants of the problem: a charging station with identical chargers, where all chargers has the same charging power rate, and a charging station where chargers with different charging power rates are installed.

In the following paragraph, we give an example of optimal charging schedules for a small instance of the described problem.

Example 4.2.1. To illustrate the scheduling objective in this chapter, we consider the charging demands of six vehicles given in the previous chapter in Example 3.2.1. As a reminder, Table 3.1 gives the arrival and departure times and the requested energy of these demands. We consider the same two cases. In the first case, we have a charging station with five chargers: the first charger delivers an output power of 30 kW, while the second deliver a power of 10 kW. The remaining chargers deliver 20 kW. In the second case, we consider a charging station with five identical chargers, each delivers 10 kW. The scheduling horizon is divided into time slots, and each time slot is set to one hour. Compared to Example 3.2.1, the first difference is that the maximum charging station capacity w_G is fixed to 30 kW.

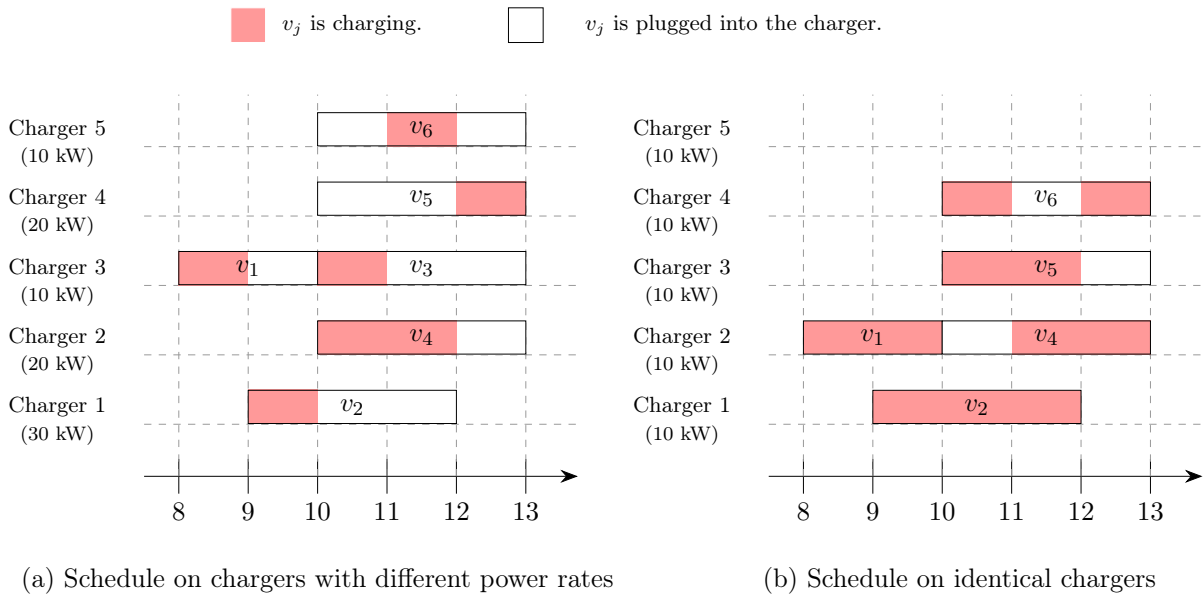


Figure 4.1: Optimal schedules of charging demands of Table 3.1. (a) depicts the optimal schedule using chargers with different power rates, and (b) depicts the optimal schedule where all chargers deliver 10 kW. Rectangles represent the vehicles' plugging intervals. We highlight charging intervals in red.

Figure 4.1(a) shows an optimal schedule of the charging demands in the first case, while Figure 4.1(b) depicts an optimal solution with identical chargers. In both solutions, the energy

requested by each accepted charging demands is satisfied. As we can see, each vehicle v_j is plugged into a charger from its arrival to its departure time. The total power delivered at each hour is always less than or equal to the fixed limit of 30 kW. As we can see, we can schedule all charging demands in the first case, whereas we can only schedule five charging demands in the second case.

4.3 Identical Chargers

In this section, we consider an instance of the electric vehicle charging scheduling problem with m identical chargers, each delivering a constant power w (kW). This implies that the charging time p_j of each demand j is independent of the choice of the charger, and it represents the number of time slots needed to satisfy j , i.e., $p_j = \frac{e_j}{w\tau}$. From now on, we shall assume without loss of generality that p_j is rounded to the nearest integer and that each charging demand can be satisfied during its plugging time interval, i.e., $p_j \leq d_j - r_j$. We first study the decision problem, that is, if it is possible to construct an effective algorithm to determine whether or not a feasible schedule for all charging demands exists. Then, we move to the optimization problem of finding the maximum size subset of satisfied charging demands.

4.3.1 Charging Scheduling Decision Problem

This section presents a polynomial test for the decision variant of the charging scheduling problem. The decision problem consists of determining whether a feasible schedule exists for n electric vehicle charging demands such that: each vehicle is parked and plugged into one of the m chargers from arrival to departure; during the plugging time interval, its requested energy is satisfied; and the instantaneously total power delivered by all chargers does not exceed w_G . Determining if a feasible schedule exists requires first checking if plugging all vehicles is feasible as explained in Section 3.3.1. As a reminder, this problem is equivalent to the fixed interval scheduling problem on identical machine [Bouzina & Emmons 1996]. It can be solved in $\mathcal{O}(n \max(\log(n), m))$ time by considering the set of charging demands \mathcal{J} in non-decreasing order of arrival times r_j , and then assigning each charging demand $j \in \mathcal{J}$ to the first available charger. If no charger is available, the schedule is infeasible. We can also calculate the minimum number of chargers \bar{m} required to plug all vehicles as in Algorithm 1. If $m < \bar{m}$, then the problem is infeasible. Now, consider that all vehicles can be parked at the charging station from its arrival to its departure. Determining the existence of a preemptive charging schedule that satisfies all charging demands without exceeding the grid capacity at any time can be determined in $\mathcal{O}(n^3)$ time by reduction to the maximum flow problem as presented in Section 3.3.2.1.

4.3.2 Complexity

This section addresses the optimization problem in which we maximize the number of satisfied charging demands.

Theorem 4.3.2.1. *The problem of maximizing the number of satisfied charging demands considering m identical chargers is \mathcal{NP} -hard.*

Proof. First, let us assume that parking all vehicles is feasible, and the charging scheduling with w_G is infeasible. Let \bar{m} be the maximum number of chargers that can be activated simultaneously. We have $\bar{m} = \lfloor \frac{w_G}{w} \rfloor$. Clearly, maximizing the number of satisfied charging demands is equivalent to minimizing the number of rejected charging demands, which is equivalent to the preemptive scheduling to minimize the number of late jobs on \bar{m} identical parallel machines with release and due dates, denoted as $P|pmtn; r_i | \sum U_i$ [Du et al. 1992]. The charging demands correspond to jobs. The arrival time is equivalent to the release date of the job, the departure time corresponds to its due date, and the charging time is the processing time of the job. The number of identical machines is equal to the maximum number of chargers that can be activated at the same time. A job is said to be late when it completes after its due date; otherwise, it is on time. The set of on-time jobs corresponds to the set of accepted demands in the problem of maximizing the number of satisfied charging demands. The set of rejected demands corresponds to the set of late jobs.

In scheduling problem $P|pmtn; r_i | \sum U_i$, a well-known property of the optimal schedules is that, for each machine, the on-time jobs precede all late jobs. To show the correctness of this property, suppose that a late job precedes an on-time job on a given machine. Moving this late job to the end of the schedule and shifting forward all the jobs currently succeeding it will not decrease the quality of the schedule. Hence, if a job is late, it is immaterial where it is scheduled. Therefore, minimizing the number of late jobs requires partitioning the set of jobs into a subset with maximum cardinality containing the on-time jobs to be scheduled on \bar{m} machines and one subset containing the late jobs. Moreover, the set of on-time jobs are scheduled in their interval $[r_j, d_j]$; in which no more than \bar{m} machines are used to schedule these jobs. Therefore, the optimal schedule for the $P|pmtn; r_i | \sum U_i$ problem is the optimal schedule for the problem of maximizing the number of satisfied charging demands described above.

In [Du et al. 1992] authors showed that the problem $P|pmtn; r_i | \sum U_i$ is \mathcal{NP} -hard even with two identical machines. As a result, the problem of maximizing the number of satisfied demands is \mathcal{NP} -hard. □

4.3.3 Mathematical Formulations

In the following, we develop integer linear programming (ILP) models of the charging scheduling problem. A commercial solver will be used to solve these models. Multiple formulations to the same problem are possible. We propose and compare four mathematical programming models. The first formulation is based on a time-indexed formulation, while the second is an event-based one. Then, relaxations are introduced for both models.

Time-indexed model (P_1). The first model is a basic discrete-time formulation containing three types of binary variables. First, binary variables s_j specify whether or not the charging demand of electric vehicle j is accepted. Second, the assignment binary variables x_{ijt} that specifies whether or not the electric vehicle j is plugged into the charger i at time slot t . Finally, we introduce the charging time binary variables y_{jt} to denote whether or not the electric vehicle j is charging at time slot t . The time-indexed formulation can be written as follows.

$$\max \sum_{j=1}^n s_j \quad (4.1)$$

$$\sum_{i=1}^m x_{ijt} \leq s_j \quad \forall j \in \mathcal{J}, t \in \mathcal{H} \quad (4.2)$$

$$\sum_{j=1}^n x_{ijt} \leq 1 \quad \forall i \in \mathcal{M}, t \in \mathcal{H} \quad (4.3)$$

$$\sum_{i=1}^m \sum_{t=r_j}^{d_j} x_{ijt} = s_j(d_j - r_j) \quad \forall j \in \mathcal{J} \quad (4.4)$$

$$x_{ijt} + \sum_{l=1, l \neq i}^m x_{lj(t+1)} \leq 1 \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, t \in \mathcal{H} \quad (4.5)$$

$$\sum_{t=r_j}^{d_j} y_{jt} = p_j s_j \quad \forall j \in \mathcal{J} \quad (4.6)$$

$$\sum_{j=1}^n w \times y_{jt} \leq w_G \quad \forall t \in \mathcal{H} \quad (4.7)$$

The objective function is given by (4.1) allows maximizing the total number of accepted charging demands. Constraints (4.2) ensure that when a demand j is accepted, it is assigned to at most one charger at each time slot t . Constraints (4.3) ensure that each charger i can only charge one vehicle j at each time slot t . Constraints (4.4) ensure that if a charging demand j is accepted, it will be plugged uninterruptedly from arrival r_j to departure d_j . Constraints (4.5) ensure that if a charging demand j is accepted then it will be plugged to only one charger i during its time interval $[r_j, d_j)$. Constraints (4.6) guarantee that the energy requirement of the vehicle j will be fulfilled if its charging demand is accepted. Constraints (4.7) are the power grid capacity restrictions, specifying that the total power delivered by all chargers simultaneously does not exceed w_G .

In addition, variables x_{ijt} and y_{jt} are set to zero for all j where $t \notin [r_j, d_j)$ to prevent vehicles from plugging or charging before their arrival time or after their departure time.

Event-based model (P_2). We aim to reduce the number of constraints to improve performance when solving the LP model. That can be achieved by reducing the number of time slots. Thus, instead of dividing the time horizon into T time slots of equal length, we divide it into $L - 1$ intervals as in Section 4.3.1. As previously defined in Section 4.3.1, the set of L events corresponds to the distinct values of arrival and departure times sorted in increasing order $t_1 < t_2 < \dots < t_L$ with $L \leq 2n$. Thus, we have $L - 1$ intervals $I_l = [t_l, t_{l+1})$. Let $\mathcal{L} = \{1, \dots, L - 1\}$ be the index set of these intervals. In addition to binary variables s_j , we define the assignment binary variables x_{ijl} that specifies whether or not the electric vehicle j is plugged into the charger i at event l , i.e., during the interval $[t_l, t_{l+1})$. Also, we introduce integer

variables p_{jl} that specify the charging duration of each vehicle j in the interval $[t_l, t_{l+1})$. We are led to the following formulation of the problem.

$$\max \quad \sum_{j=1}^n s_j \quad (4.8)$$

$$\sum_{i=1}^m x_{ijl} \leq s_j \quad \forall j \in \mathcal{J}, l \in \mathcal{L} \quad (4.9)$$

$$\sum_{j=1}^n x_{ijl} \leq 1 \quad \forall i \in \mathcal{M}, l \in \mathcal{L} \quad (4.10)$$

$$\sum_{i=1}^m \sum_{t_l=r_j}^{d_j} x_{ijl}(t_{l+1} - t_l) = s_j(d_j - r_j) \quad \forall j \in \mathcal{J} \quad (4.11)$$

$$x_{ijl} + \sum_{k=1, k \neq i}^m x_{kj(l+1)} \leq 1 \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, l \in \mathcal{L} \quad (4.12)$$

$$\sum_{t_l=t_1}^{t_L} p_{jl} = p_j s_j \quad \forall j \in \mathcal{J} \quad (4.13)$$

$$p_{jl} \leq t_{l+1} - t_l \quad \forall j \in \mathcal{J}, l \in \mathcal{L} \quad (4.14)$$

$$\sum_{j=1}^n p_{jl} \leq \frac{w_G}{w}(t_{l+1} - t_l) \quad \forall l \in \mathcal{L} \quad (4.15)$$

Constraints (4.9), (4.10), (4.11), (4.12) and (4.13) are similar to constraints (4.2), (4.3), (4.4), (4.5) and (4.6), respectively. Constraints (4.14) limit the charging duration of each demand j to the size of interval $[t_l, t_{l+1})$. Constraints (4.15) impose that, in each interval $[t_l, t_{l+1})$ the total power delivered by all chargers does not exceed w_G . In addition, variables x_{ijl} and p_{jl} are set to zero for all vehicles j , for l where $t_l < r_j$ and $t_l \geq d_j$.

In the optimal solution given by P_2 , the values of p_{jl} gives the charging duration of each accepted charging demand j in each interval $[t_l, t_{l+1})$. However, charging the vehicle j is not allowed to take place at any time in the interval $[t_l, t_{l+1})$; otherwise, the grid capacity constraint will not be respected. For example, consider a charging station with $m = 3$, $w = 10$ kW, and $w_G = 20$ kW. Let us assume that in the optimal solution given by P_3 , we have three vehicles, v_1 , v_2 , and v_3 ; each has to charge during two hours in the interval $[1, 4)$, i.e., $p_{j1} = 2$, $j = 1, 2, 3$. If we charge both v_1 and v_2 for two hours starting at the beginning of the interval (i.e., v_1 and v_2 are charging in the interval $[1, 3)$), v_3 cannot charge without exceeding the power grid capacity. The optimal solution is to charge v_1 in $[1, 3)$, v_2 in $[3, 4)$, and v_3 in $[1, 2)$ and in $[3, 4)$. Therefore, for each accepted charging demand j , we can define the charging time slots t from the values of variables p_{jl} as follows. Let m_t be the number of activated charger at t , $m_t \leq \bar{m} = \lfloor \frac{w_G}{w} \rfloor$. For each vehicle j , we schedule p_{jl} time slots in the interval $[t, t_{l+1})$ starting by the time slot t , $t \in [t_l, t_{l+1})$, with the minimum value m_t .

4.3.3.1 Relaxed LP formulations

The mathematical models P_1 and P_2 generate several constraints, in particular constraints (4.5) and (4.12), which may deteriorate the performance of a mathematical solver. In the following, we propose two new time-indexed and event-based relaxed models. In both models, we omit assignment variables and relax assignment constraints. In fact, as we deal with identical chargers, the assignment of a vehicle to a specific charger is immaterial. We only have to ensure that the number of chargers used at each time does not exceed m . In that way, we consider the m chargers as a supercharger with a capacity equal to m . Then we propose a method to repair the solutions of both models in order to make them feasible for the original one.

Relaxed time-indexed model (P_3). In this model, we use the same definitions of binary variables s_j and y_{jt} as in the previous model P_1 . We omit the variables x_{ijt} and relax the constraints (4.2), (4.3), (4.4) and (4.5). However, we ensure that at each time slot t , at most m vehicles are accepted. For this purpose, we define a parameter a_{jt} for each charging demand $j \in \mathcal{J}$, and time slot $t \in \mathcal{H}$, which is set to 1 if $t \in [r_j, d_j)$ and 0 otherwise. This yields the following relaxed time-indexed formulation.

$$\max \sum_{j=1}^n s_j \tag{4.16}$$

$$\sum_{j=1}^n a_{jt} s_j \leq m \quad \forall t \in \mathcal{H} \tag{4.17}$$

$$\sum_{t=r_j}^{d_j} y_{jt} = p_j s_j \quad \forall j \in \mathcal{J} \tag{4.18}$$

$$\sum_{j=1}^n w \times y_{jt} \leq w_G \quad \forall t \in \mathcal{H} \tag{4.19}$$

Constraints (4.17) guarantee that the number of plugged electric vehicles does not exceed the number of chargers m . Constraints (4.18) and (4.19) are identical to constraints (4.6) and (4.7), respectively. Besides, variables y_{jt} are set to zero for each j and each time slot t , $t \notin [r_j, d_j)$.

Relaxed event-based model (P_4). We apply the same relaxation above to the event-based model P_2 . In addition to binary variables s_j used in P_2 , we introduce integer variables p_{jl} that specify the charging duration of electric vehicle j in the interval $[t_l, t_{l+1})$. As in P_3 , we define the parameter a_{jl} for each charging demand $j \in \mathcal{J}$ and each event $l \in \mathcal{L}$ where $a_{jl} = 1$ if $[t_l, t_{l+1}) \subseteq [r_j, d_j)$, and $a_{jl} = 0$ otherwise.

$$\max \sum_{j=1}^n s_j \tag{4.20}$$

$$\sum_{j=1}^n a_{jl} s_j \leq m \quad \forall l \in \mathcal{L} \tag{4.21}$$

$$\sum_{t_l=t_1}^{t_L} p_{jl} = p_j s_j \quad \forall j \in \mathcal{J} \quad (4.22)$$

$$p_{jl} \leq t_{l+1} - t_l \quad \forall j \in \mathcal{J}, l \in \mathcal{L} \quad (4.23)$$

$$\sum_{j=1}^n p_{jl} \leq \frac{w_G}{w} (t_{l+1} - t_l) \quad \forall l \in \mathcal{L} \quad (4.24)$$

Constraints (4.21) ensure that, in each time interval $[t_l, t_{l+1})$, at most m vehicles are accepted. constraints (4.22), (4.23), and (4.24) are similar to constraints (4.13), (4.14), and (4.15), respectively. As in previous models, each vehicle j cannot be charged before its arrival or after its departure. Thus, variables p_{jl} are set to zero for all $j \in \mathcal{J}$ and all l where $[t_l, t_{l+1}) \not\subseteq [r_j, d_j)$.

4.3.3.2 Repairing the solution of relaxed models

In the optimal solution obtained by solving relaxed LP models, we do not have the assignment of vehicles to chargers. Since all chargers are identical, choosing a specific charger to charge a vehicle is irrelevant. Thus, we only have to guarantee that each accepted charging demand j is plugged during $[r_j, d_j)$ and that at most m vehicles are plugged at each time t , which is achieved by constraints (4.17) and (4.21) in P_3 and P_4 , respectively. Therefore, to repair the optimal solution, we only have to assign vehicles to chargers while the chosen charging time slots remain unchanged. The repairing solution procedure is shown in Algorithm 7.

Algorithm 7: Repairing the solution of relaxed models

Input : Optimal solution of relaxed problem P_3 (resp. P_4)

Output: Optimal solution to charging scheduling problem

- 1 Let S be the set of accepted demands obtained by solving one of relaxed models P_3 (resp. P_4), ordered in increasing order of their arrival times r_j . ;
 - 2 **while** $S \neq \emptyset$ **do**
 - 3 Let j be the first charging demand in the set S ;
 - 4 Assign j to the first available charger $i \in \mathcal{M}$;
 - 5 Delete j from S ;
 - 6 Charge each accepted demand according to the value of decision variables y_{jt} (resp. p_{jl}) of optimal solution of P_3 (resp. P_4)
-

Algorithm 7 can be implemented to run in $\mathcal{O}(n \max(\log n, \log m))$ time. Sorting the charging demands in ascending order of their release times takes $\mathcal{O}(n \log n)$ time and the inner loop (line 2-5) takes $\mathcal{O}(n \log m)$ time.

Proposition 4.3.3.1. *Algorithm 7 provides an optimal solution to the problem of maximizing the number of scheduled with identical chargers.*

Proof. In the following, we assume that P_3 is used to solve the relaxed charging scheduling problem. Let S be the set of accepted charging demands in the optimal solution of P_3 . Clearly, the number demands in the set S is an upper bound for the charging scheduling problem. Let S' be the subset of S that Algorithm 7 can schedule. We prove that $S = S'$.

Let $|S'|$ be the cardinality of the set S' and $|S|$ be the cardinality of the set S . Assume that $|S'| < |S|$. Let $j \in S$ be the first charging demand of S that cannot be scheduled by Algorithm 7. This means that at time r_j , all chargers are occupied by a subset $S^* \subseteq S$ of accepted demands at time $t = r_j$. Thus at time $t = r_j$, we have $\sum_{k \in S^*} s_k = m < \sum_{k \in S^*} s_k + s_j$, which violates constraints (4.21) of P_3 . Then at time r_j , at least one charger is available to schedule the demand j . By repeating this argument, we obtain $S' = S$. Finally, when all charging demands of S are assigned to chargers, the values of variables y_{jt} define the charging time slots of accepted demands. Similarly, we can prove that Algorithm 7 provides the optimal solution when fixing the optimal solution of P_4 . □

4.3.4 Heuristic Methods

In the previous sections, we showed that the problem of maximizing the number of satisfied demands with identical chargers is \mathcal{NP} -hard, and we provided a relaxed event-based LP model P_4 to solve it. Even though there are significantly fewer variables and constraints in model P_4 , a commercial optimizer can still spend too much time solving it, depending on the problem instances. Therefore, we develop a two-phase heuristic method to obtain near-optimal solutions within a reasonable computational time. The first phase selects a subset of charging demands based on one or more criteria, and the second phase uses the model P_4 to solve the problem considering only this subset. We present two methods for the first phase: selecting the vehicles based on interval scheduling and selecting the vehicles based on energetic reasoning. The proposed methods do not build the charging schedule; thus, we have no guarantee that the total charging power does not exceed the charging station capacity. Consequently, we use the model P_4 to solve the problem with only the selected charging demands.

4.3.4.1 Selection based on interval scheduling

In Algorithm 8, we determine a set of charging demands \bar{S} , selected from a set \mathcal{J} , to be assigned to chargers such that the plugging intervals of vehicles assigned to the same charger do not overlap. As stated before, this problem is similar to the interval scheduling problem on identical machines [Bouzina & Emmons 1996]. Algorithm 8 starts by sorting the set of charging demands S in non-decreasing order of their arrival times. Then for each demand j , $j \in S$, it will be assigned to the first charger i , $i \in \mathcal{M}$, on which j can be plugged from r_j to d_j , i.e., the charger i is available at r_j . Let j_1, j_2, \dots, j_m be the last vehicle assigned to chargers 1, 2, \dots , m . If all chargers are occupied by other vehicles (i.e. $d_{j_i} > r_j \forall i \in \mathcal{M}$), we have to choose to either reject j or one of vehicles j_1, j_2, \dots, j_m according to a criteria R (line 10).

It is possible to implement several criteria R in line (10) to choose the rejected vehicle when there is no available charger. We consider two criteria to select the rejected demand:

1. *Largest Departure Time* (LDT): Among the set $\{j_1, j_2, \dots, j_m\} \cup \{j\}$, select the charging demand k that has the largest departure time d_k . This criterion enables us to determine the maximum subset of charging demands that can be parked on m chargers which brings us to the interval scheduling algorithm in [Bouzina & Emmons 1996].

Algorithm 8: Charging demands selection

Input : The set of charging demands \mathcal{J} , the set of chargers \mathcal{M}
Output: The set of selected demands \bar{S}

- 1 $S \leftarrow \mathcal{J}$;
- 2 Sort S in non-decreasing order of their arrival times r_j ;
- 3 **while** $S \neq \emptyset$ **do**
- 4 Let j be the first demand of S ;
- 5 **if** there is a charger $i, i \in \mathcal{M}$ that is available at r_j **then**
- 6 Assign j on the first available charger ;
- 7 Add j to \bar{S} ;
- 8 **else**
- 9 Let j_i be the last charging demand assigned to the charger $i, i \in \mathcal{M}$;
- 10 Let k be the charging demand in $\{j_1, j_2, \dots, j_m\} \cup \{j\}$ that satisfies a criterion R ;
- 11 **if** $k \neq j$ **then**
- 12 Delete the demand k from \bar{S} ;
- 13 Add j to \bar{S} ;
- 14 **end**
- 15 **end**
- 16 Delete j from S ;
- 17 **end**

2. *Largest Idle Time* (LIT): Among the set $\{j_1, j_2, \dots, j_m\} \cup \{j\}$, select the charging demand k that has the largest idle time $d_k - r_k - p_k$.

It is easy to see that Algorithm 8 can be implemented to run in $\mathcal{O}(n \max(\log n, \log m))$ time. Using criterion LDT will lead to the optimal solution to the interval scheduling problem, i.e., the largest set $\bar{S} \subseteq \mathcal{J}$ containing vehicles that can be plugged into m chargers. If charging all vehicles in \bar{S} with the grid capacity w_G and the charging power rate w is feasible, \bar{S} is the optimal solution to our problem. Otherwise, finding the largest subset $\tilde{S} \subseteq \bar{S}$ that can be scheduled with w_G may not lead to the optimal solution to the problem. To see this, we construct the following illustrative example.

Example 4.3.1. We consider the charging demands of seven vehicles. Table 4.2 gives the arrival and departure times and the requested energy of these demands. We have a charging station with three identical chargers, and each delivers 10 kW. We set the grid capacity to 20 kW. Clearly, only two chargers can be activated at the same time. The scheduling horizon is divided into time slots, and each time slot is set to one hour. The assignment solutions obtained with Algorithm 8 with LDT and LIT are shown in Figure 4.2(a) and Figure 4.2(b), respectively. We observe that applying LDT criterion gives us a subset of six selected vehicles $\bar{S}_2 = \{v_1, v_2, v_3, v_5, v_6, v_7\}$ while applying LIT criterion gives us a subset of five selected vehicles $\bar{S}_2 = \{v_1, v_2, v_4, v_5, v_6\}$. However, when scheduling the charging of vehicles in subset \bar{S}_1 (using P_4 for example), we have to reject at least two vehicles to meet the grid capacity constraint. In contrast, all five charging demands in subset \bar{S}_2 can be scheduled without exceeding w_G .

Table 4.2: Charging demands data for Example 4.3.1

Vehicle	Arrival time	Departure time	Requested energy e_j
v_1	8:00	10:00	20 kWh
v_2	8:00	10:00	20 kWh
v_3	9:00	11:00	20 kWh
v_4	9:00	12:00	10 kWh
v_5	10:00	13:00	20 kWh
v_6	11:00	12:00	10 kWh
v_7	11:00	13:00	20 kWh

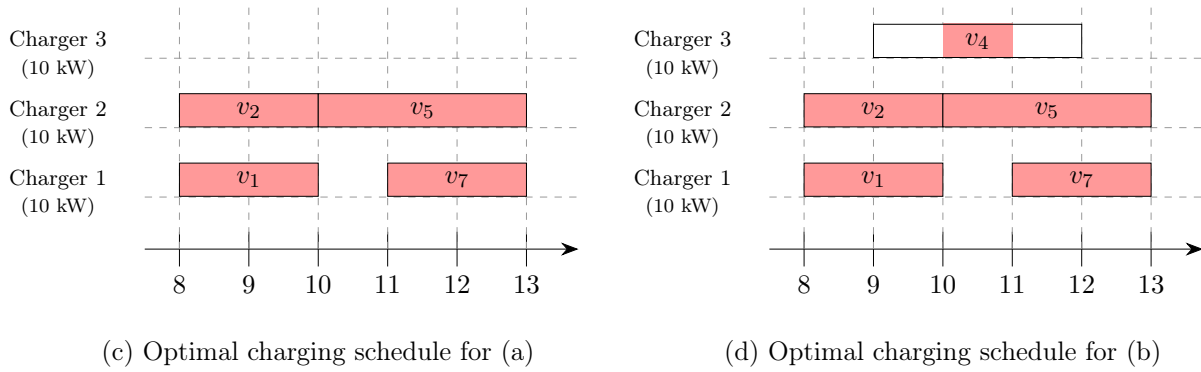
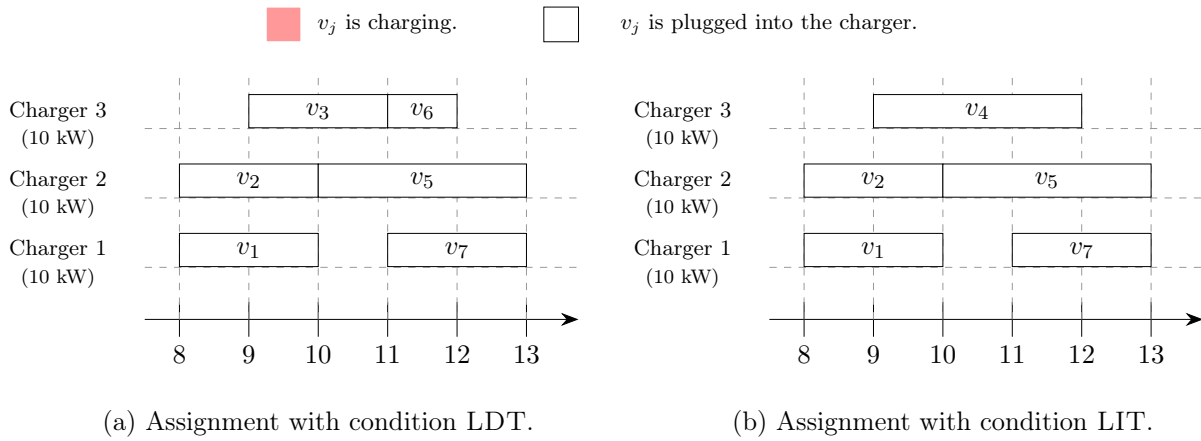


Figure 4.2: Example of schedules of the instance in Example 4.3.1. (c) and (d) are the optimal schedules for the assignment solutions (a) and (b) obtained using Algorithm 8 with condition LDT and LIT, respectively.

4.3.4.2 Selection based on energetic reasoning

The previous selection focuses more on the assignment problem and does not take into account the grid capacity constraint. To cope with that, we propose another selection that considers the grid capacity. Consider the charging scheduling problem with no constraints on the assignment of vehicles to chargers. This problem can be formulated, for example, using P_3 without constraints (4.17). Let $S \subseteq \mathcal{J}$ be the set of charging demands to be scheduled with w_G . The number of chargers that can be activated at the same time is $\bar{m} = \lfloor \frac{w_G}{w} \rfloor$. Let $w_G^*(S)$ be the minimum grid capacity required to charge all vehicles in the set S . Assume that $w_G^*(S) > w_G$. Let $m^*(S) = \lfloor \frac{w_G^*(S)}{w} \rfloor$ be the minimum number of chargers that can be activated in the same time to charge all vehicles in S . The main idea of the proposed Energetic Reasoning (ER) based selection is as follows. We calculate a lower bound (LB) on $m^*(S)$ using the energetic reasoning approach investigated by [Baptiste & Le Pape 1997] in the context of cumulative scheduling. Then, we try to delete charging demands from S until $LB(m^*(S))$ reaches \bar{m} .

Calculation of the lower bound on $m^*(S)$. Consider a relaxed problem where, instead of representing a vehicle's charging at time slot t by a binary variable y_{jt} , we have integer variable \tilde{y}_{jt} that represents the charging units of vehicle j at time t . Each vehicle j needs to charge for p_j units in the interval $[r_j, d_j]$ to be fulfilled. Moreover, for each time slot t , a vehicle charging units in $[r_j, t]$ is bounded by $(t - r_j)$ and by $(d_j - t)$ in the interval $[t, d_j]$. In addition, the total charging units at each time slot t must not exceed a given capacity C . The decision variant of the relaxed problem, as whether there exists a feasible schedule that satisfies these constraints, can be formulated as follows.

$$\sum_{t=r_j}^{d_j} \tilde{y}_{jt} = p_j \quad \forall j \in \mathcal{J} \quad (4.25)$$

$$\sum_{j=1}^n \tilde{y}_{jt} \leq C \quad \forall t \in \mathcal{H} \quad (4.26)$$

$$\sum_{u < t} \tilde{y}_{ju} \leq (t - r_j) \quad \forall j \in \mathcal{J}, t \in \mathcal{H} \quad (4.27)$$

$$\sum_{t \leq u} \tilde{y}_{ju} \leq (d_j - t) \quad \forall j \in \mathcal{J}, t \in \mathcal{H} \quad (4.28)$$

$$\tilde{y}_{jt} = 0 \quad \forall j \in \mathcal{J}, t \notin [r_j, d_j] \quad (4.29)$$

This problem is known as the partially elastic cumulative scheduling problem (PECuSP) investigated in [Baptiste & Le Pape 1997]. Now, consider the problem of minimizing the resource capacity C in the PECuSP. The optimal value for C , denoted as C^* , is a lower bound on $m^*(\mathcal{J})$, the minimum number of chargers needed to be activated at the same time to charge all charging demands in \mathcal{J} . Therefore, the calculation of the lower bound on $m^*(\mathcal{J})$ is based on the necessary and sufficient condition of existence for PECuSP, which can be stated as follows.

Let \mathcal{R} be the set of distinct arrival times, and \mathcal{D} be the set of distinct departure times. Let \mathcal{I} be the set of intervals $\tilde{I}_k = [t_1, t_2]$, $k \in \mathcal{K}$ where $t_1 \in \mathcal{R}$, $t_2 \in \mathcal{D}$, and $t_1 < t_2$. Notice that $K \leq n^2$. Let $|\tilde{I}_k| = t_2 - t_1$ be the length of the interval \tilde{I}_k .

For the PECUSP, the mandatory part of energy that the demand j must recover in the interval $\tilde{I}_k = [t_1, t_2]$ is defined as:

$$e(j, \tilde{I}_k) = \max(0, p_j - \max(0, t_1 - r_j) - \max(0, d_j - t_2)).$$

Then, the mandatory part of energy that must be recovered by all demands in the interval $\tilde{I}_k = [t_1, t_2]$ is:

$$E(\tilde{I}_k) = E(t_1, t_2) = \sum_{j \in \mathcal{J}} e(j, t_1, t_2).$$

Authors in [Baptiste & Le Pape 1997] proved that there is a feasible schedule for PECUSP for all demands with a limited resource capacity C if and only if

$$E(\tilde{I}_k) \leq C|\tilde{I}_k| \quad \tilde{I}_k \in \mathcal{I}$$

Therefore, the minimum resource capacity C^* is given as:

$$C^* = \max_{\tilde{I}_k \in \mathcal{I}} \left\lceil \frac{E(\tilde{I}_k)}{|\tilde{I}_k|} \right\rceil$$

The calculate of C^* , the lower bound on $m^*(\mathcal{J})$, can be done in $\mathcal{O}(n^2)$ time using the algorithm developed in [Baptiste & Le Pape 1997].

ER selection heuristic. We now describe the proposed ER-based selection heuristic. Consider a set of charging demands $S \subseteq \mathcal{J}$. The main idea of the heuristic is to seek the subset S' containing the minimum number of rejected demands such that the lower bound on $m^*(S - S')$ is less than or equal to \bar{m} . $m^*(S - S')$ denotes the minimum number of chargers that can be activated to charge all charging demands that are in S but not in S' . The overall heuristic is depicted in Algorithm 9. Consider the set of intervals \mathcal{I} as defined above. We associate to each interval $\tilde{I}_k = [t_1, t_2]$, $\tilde{I}_k \in \mathcal{I}$: **(i)** the length of the interval $|\tilde{I}_k| = t_2 - t_1$, **(ii)** the set $P(\tilde{I}_k)$ of charging demands $j \in S$ with $e(j, \tilde{I}_k) > 0$ sorted in non-increasing order of $e(j, \tilde{I}_k)$, and **(iii)** the mandatory part of energy that must be used by all demands in S in the interval $E(\tilde{I}_k)$.

To built the subset S' , we reject charging demands until $\frac{E(\tilde{I}_k)}{|\tilde{I}_k|}$ reaches \bar{m} for all intervals $\tilde{I}_k \in \mathcal{I}$ (lines 2-13). In order to iteratively reject the minimum number of charging demands, we start with the interval \tilde{I}_k that has the largest value $\frac{E(\tilde{I}_k)}{|\tilde{I}_k|}$ that exceeds \bar{m} , and we reject the first charging demands $j \in P(\tilde{I}_k)$ with the largest value of $e(j, \tilde{I}_k)$ until $E(\tilde{I}_k) \leq \bar{m}$. Before moving to the next interval, unscheduled demands are removed from all $P(\tilde{I}_l)$ and values $E(\tilde{I}_l)$ are updated for all $\tilde{I}_l \in \mathcal{I}$. Since this method does not guarantee that the set S' is minimal, we add an improvement phase by trying to reschedule some of unscheduled demands in S' (lines 14-24). We start by sorting the set of rejected charging demands S' in non-decreasing order of $\sum_{k=1}^K \frac{e(j, \tilde{I}_k)}{|\tilde{I}_k|}$. Then, for each demand $j \in S'$, we remove the charging demand j from the set of rejected demands S' if $\frac{E(\tilde{I}_k) + e(j, \tilde{I}_k)}{|\tilde{I}_k|} \leq \bar{m} \quad \forall \tilde{I}_k \in \mathcal{I}$.

Algorithm 9: Heuristic for selecting the vehicles to charge based on energetic reasoning

input : The set of intervals \mathcal{I} , The number of chargers that can be activated simultaneously \bar{m}

output: The set of rejected demands S'

- 1 $S' \leftarrow \emptyset, k \leftarrow 1;$
- 2 Let k be the index of interval $\tilde{I}_k \in \mathcal{I}$ with the largest value of $\frac{E(\tilde{I}_k)}{|\tilde{I}_k|};$
- 3 **repeat**
- 4 **while** $E(\tilde{I}_k) > \bar{m}|\tilde{I}_k|$ **do**
- 5 Let j be the first charging demand in $P(\tilde{I}_k);$
- 6 Add j to the set of rejected demands $S';$
- 7 **for** each $\tilde{I}_l \in \mathcal{I}$ **do**
- 8 Delete j from $P(\tilde{I}_l);$
- 9 $E(\tilde{I}_l) \leftarrow E(\tilde{I}_l) - e(j, \tilde{I}_l);$
- 10 **end**
- 11 **end**
- 12 Let k be the index of interval $\tilde{I}_k \in \mathcal{I}$ with the largest value of $\frac{E(\tilde{I}_k)}{|\tilde{I}_k|};$
- 13 **until** $E(\tilde{I}_k) > \bar{m}|\tilde{I}_k|;$
- 14 Sort the set of rejected charging demands S' in non-decreasing order of their values of $\sum_{k=1}^K \frac{e(j, \tilde{I}_k)}{|\tilde{I}_k|};$
- 15 **for** $j \in S'$ **do**
- 16 **if** $E(\tilde{I}_k) + e(j, \tilde{I}_k) \leq \bar{m}|\tilde{I}_k| \quad \forall \tilde{I}_k \in \mathcal{I}$ **then**
- 17 Delete j from $S';$
- 18 Add j to $P(\tilde{I}_l);$
- 19 **for** $\tilde{I}_l \in \mathcal{I}$ **do**
- 20 Add j to $P(\tilde{I}_l);$
- 21 $E(\tilde{I}_l) \leftarrow E(\tilde{I}_l) + e(j, \tilde{I}_l);$
- 22 **end**
- 23 **end**
- 24 **end**
- 25 **return** S'

The worst-case complexity of the Algorithm 9 is $\mathcal{O}(n^4)$. As mentioned, to obtain the final set of accepted charging demands, we use P_4 (or P_3) considering only the charging demands in the set $S - S'$ instead of the set \mathcal{J} .

Example 4.3.2. We consider the set $S = \{v_1, v_3, v_4, v_6\}$ of charging demands with their data given in Table 4.2. We have a charging station with three identical chargers, and each delivers 10 kW. We set the grid capacity to 10 kW. Clearly, only one charger can be activated at the same time ($\bar{m} = 1$). We set the length of each time slot to one hour. Notice that the optimal solution is to reject v_3 and schedule the set $\{v_1, v_4, v_6\}$. Now we select the charging demands to

reject by following Algorithm 9 step by step. First, we have $\mathcal{R} = \{8, 9, 11\}$ and $\mathcal{D} = \{10, 11, 12\}$. Therefore, we have seven intervals in \mathcal{I} . The values of $\frac{E(\tilde{I}_k)}{|\tilde{I}_k|}$ and $e(j, \tilde{I}_k)$ for each interval $\tilde{I}_k \in \mathcal{I}$ are given in Table 4.3.

Table 4.3: Calculation of the mandatory part of energy that must be recovered by all demands in S in each the interval \tilde{I}_k .

Interval \tilde{I}_k	t_1	t_2	Charging demand j	$e(j, \tilde{I}_k)$	$\frac{E(\tilde{I}_k)}{ \tilde{I}_k }$
\tilde{I}_1	8	10	$(v_1) j = 1$	2	1.5
			$(v_3) j = 3$	1	
\tilde{I}_2	8	11	$(v_1) j = 1$	2	1.33
			$(v_3) j = 3$	2	
\tilde{I}_3	8	12	$(v_1) j = 1$	2	1.5
			$(v_3) j = 3$	2	
			$(v_4) j = 4$	1	
			$(v_4) j = 4$	1	
\tilde{I}_4	9	10	$(v_1) j = 1$	1	2
			$(v_3) j = 3$	1	
\tilde{I}_5	9	11	$(v_1) j = 1$	1	1.5
			$(v_3) j = 3$	2	
\tilde{I}_6	9	12	$(v_1) j = 1$	1	1.67
			$(v_3) j = 3$	2	
			$(v_4) j = 4$	1	
			$(v_4) j = 4$	1	
\tilde{I}_7	11	12	$(v_3) j = 3$	1	1

The ER-based heuristic will start with the interval \tilde{I}_4 since it has the largest value $\frac{E(\tilde{I}_k)}{|\tilde{I}_k|}$. In this interval, we have two vehicles v_1 and v_3 with equal value $e(1, \tilde{I}_4) = e(3, \tilde{I}_4) = 1$. Thus, the heuristic adds one of these demands to the rejected set S' . Since we did not specify how ties are broken, the heuristic will reject the first demand, which is v_1 . The demand v_1 will be placed in S' and deleted from each \tilde{I}_k . Therefore, we have the values of $\frac{E(\tilde{I}_k)}{|\tilde{I}_k|}$ equal to 0.5, 0.67, 1, 1, 1, 1.33, and 1 for $\tilde{I}_1, \tilde{I}_2, \tilde{I}_3, \tilde{I}_4, \tilde{I}_5, \tilde{I}_6,$ and \tilde{I}_7 , respectively. In the next iteration, the interval with the largest value of $\frac{E(\tilde{I}_k)}{|\tilde{I}_k|}$ is \tilde{I}_4 , and in this interval, v_3 has the largest value $e(j, \tilde{I}_4)$. Thus, the heuristic removes charging demand v_3 and place it in S' . After that, we have $\frac{E(\tilde{I}_k)}{|\tilde{I}_k|} \leq \bar{m}, \forall \tilde{I}_k$. In the improvement phase (following lines 14-24), the values of $\sum_{k=1}^K \frac{e(j, \tilde{I}_k)}{|\tilde{I}_k|}$ for rejected demands $j = 1$ and $j = 3$ are 4 and 4.33, respectively. Hence, the heuristic reschedules the demand v_1 since it can be added without exceeding $\bar{m} = 1$. Consequently, the ER selection heuristic returns $S' = \{v_3\}$, which is exactly the set of charging demands to reject to obtain the optimal solution.

4.4 Distinct Types of Chargers

In this section, we consider the general case where each charger i , $i \in \mathcal{M}$, delivers a constant power of w_i (kW). Further, we can group the m chargers into k types where chargers of the same type l , $l \in \mathcal{K}$, have the same charging power w_l (kW). There are m_l chargers of type l and we have $\sum_{l=1}^k m_l = m$. The charging time of demand j on a charger type l is given as $\frac{e_j}{w_l \tau}$. Obviously, if charger i is of type l , the charging time of demand j on charger type l denoted as p_{jl} . In the next sections, we assume without loss of generality that the charging time p_{ij} (resp. p_{jl}) is rounded to the nearest integer. Each charging demand can be fulfilled during its plugging time interval, i.e., $p_{ij} \leq d_j - r_j$ (resp. $p_{jl} \leq d_j - r_j$). First, we revisit the decision variant of the problem. Then, we consider the problem of maximizing the number of the scheduled charging demands.

4.4.1 Charging Scheduling Decision Problem

Theorem 4.4.1.1. *The decision problem of the charging scheduling with at least two types of chargers is \mathcal{NP} -hard.*

Proof. The proof is given in the previous chapter in Section 3.4.2.1. \square

4.4.2 Complexity

Theorem 4.4.2.1. *The problem of maximizing the number of the scheduled charging demands with at least two types of chargers is \mathcal{NP} -hard.*

Proof. It is clear that the proof follows from the observation that scheduling with identical chargers is a special case of scheduling with k types of chargers. \square

4.4.3 Mathematical Formulations

In the following, we develop two linear programming models for the problem of maximizing the number of scheduled charging demands with different types of chargers. The difference between the two models concerns the assignment of vehicles. In the first formulation, we consider assigning each vehicle j to a charger i , while in the second formulation, we consider assigning each vehicle j to a charger type l .

4.4.3.1 LP formulation based on a vehicle-charger assignment (P_5).

For the first model, we define three groups of decision variables. Binary decision variables s_{ij} specify whether or not electric vehicle j is scheduled on charger i . Binary variables x_{ijt} decide whether or not the electric vehicle j is plugged into the charger i at time slot t . Binary variables y_{jt} determine whether or not vehicle j is charging at time slot t . Then, the mathematical formulation to maximize the number of scheduled charging demands can be expressed as follows.

$$\max \sum_{j=1}^n \sum_{i=1}^m s_{ij} \quad (4.30)$$

$$\sum_{i=1}^m s_{ij} \leq 1 \quad \forall j \in \mathcal{J} \quad (4.31)$$

$$\sum_{j=1}^n x_{ijt} \leq 1 \quad \forall i \in \mathcal{M}, t \in \mathcal{H} \quad (4.32)$$

$$\sum_{t=r_j}^{d_j} x_{ijt} = s_{ij}(d_j - r_j) \quad \forall i \in \mathcal{M}, j \in \mathcal{J} \quad (4.33)$$

$$\sum_{t=r_j}^{d_j} y_{jt} = \sum_{i=1}^m p_{ij}s_{ij} \quad \forall j \in \mathcal{J} \quad (4.34)$$

$$\sum_{j=1}^n \sum_{i=1}^m w_i \times s_{ij} \times y_{jt} \leq w_G \quad \forall t \in \mathcal{H} \quad (4.35)$$

The objective function is defined in (4.30). Constraints (4.31) ensure that each charging demand j is assigned to at most one charger. Constraints (4.32) and (4.33) guarantee that when a charging demand j is accepted to be scheduled on charger i , the vehicle j is plugged into this charger during the interval $[r_j, d_j]$. Constraints (4.34) ensure that when the charging demand of the vehicle j is accepted, its requested energy is fulfilled. The grid capacity constraint is determined by Constraints (4.35). In addition, for each charging demand j , $j \in \mathcal{J}$, variables x_{ijt} and y_{jt} are set to zero for all $t \in \mathcal{H}$ where $t < r_j$ and $t \geq d_j$.

Notice that constraints (4.35) contain a quadratic term involving only the multiplication of binary variables (y_{jt} and s_{ij}). Even though most state-of-the-art commercial optimizers can handle such constraints, a linearization approach can be more effective since the problem will be solved directly using MILP techniques. Given that the product of binary variables is itself a binary variable, the linearization can be done by introducing additional binary variables z_{ijt} to represent the binary conjunctions ($s_{ij} \times y_{jt}$) for all $i \in \mathcal{M}$, $j \in \mathcal{J}$, $t \in \mathcal{H}$. Basically, the variable z_{ijt} is set to 1 if and only if both variables y_{jt} and s_{ij} are equal to 1. Hence, we replace constraints (4.35) with the following constraints:

$$z_{ijt} \geq y_{jt} + s_{ij} - 1 \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, t \in \mathcal{H} \quad (4.36)$$

$$z_{ijt} \leq y_{jt} \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, t \in \mathcal{H} \quad (4.37)$$

$$z_{ijt} \leq s_{ij} \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, t \in \mathcal{H} \quad (4.38)$$

$$\sum_{i=1}^k \sum_{j=1}^n w_i z_{ijt} \leq w_G \quad t \in \mathcal{H} \quad (4.39)$$

4.4.3.2 LP formulation based on a vehicle-type assignment (P_6)

In this formulation, instead of considering the assignment of a vehicle to a specific charger i , $i \in \mathcal{M}$, we consider its assignment to a charger type l , $l \in \mathcal{K}$. As a result, we can omit the variables x_{ijt} and reduce the number of variables and constraints. Hence, the performance

of a commercial solver can be improved in terms of computation time and required memory, especially when the difference between l and m is significant. Even though it is acceptable for small instances to have $m = l$, it is not usually the case for larger instances (for example, with $m = 100$).

Thus, for this model, we redefine binary variables s_{jl} to specify whether or not charging demand j is scheduled on a charger of type l . We use the same definition for binary variables y_{jt} as in the previous model (P_5). Furthermore, we use parameters a_{jt} as defined in Section 4.3.3.1. Thus, the objective function and the set of constraints can be written as follows.

$$\max \quad \sum_{j=1}^n \sum_{l=1}^k s_{jl} \quad (4.40)$$

$$\sum_{l=1}^k s_{jl} \leq 1 \quad \forall j \in \mathcal{J} \quad (4.41)$$

$$\sum_{j=1}^n a_{jt} s_{jl} \leq m_l \quad \forall l \in \mathcal{K}, t \in \mathcal{H} \quad (4.42)$$

$$\sum_{t=r_j}^{d_j} y_{jt} = \sum_{l=1}^k p_{jl} s_{jl} \quad \forall j \in \mathcal{J}, l \in \mathcal{K} \quad (4.43)$$

$$\sum_{j=1}^n \sum_{l=1}^k w_l \times s_{jl} \times y_{jt} \leq w_G \quad \forall t \in \mathcal{H} \quad (4.44)$$

Constraints (4.42) limits the number of plugged vehicles into chargers of type l to m_l . Constraints (4.43) and (4.44) are equivalent to constraints (4.34) and (4.35), respectively.

Similarly as for constraints (4.35), we can linearize constraints (4.44) by defining binary variables z_{ljt} that are set to 1 if y_{jt} and s_{jl} are equal to 1. Then, Constraints (4.44) are rewritten using constraints (4.36), (4.37), ((4.38), and (4.39) in which we replace s_{ij} by s_{jl} and z_{ijt} by z_{ljt} .

4.4.3.3 Repairing the assignment solution of the model P_6

An optimal solution to the LP model P_6 only gives the type of charger on which a vehicle will charge. Therefore, a similar repairing procedure to the one used for P_3 (Section 4.3.3.2) can be applied to specify to which charger each vehicle is assigned. Let S_l be the set of vehicles assigned to type l and let M_l be the indexes of type l chargers, $M_l \subseteq \mathcal{M}$. A feasible assignment can be obtained by sorting the vehicles in S_l in non-decreasing order of their arrival time. Then, scheduling each vehicle $j \in S_l$ on the first available charger $i \in M_l$. this procedure is repeated for each type of charger l , $l \in \mathcal{K}$. This procedure can be implemented to run in $\mathcal{O}(n \max(\log n, \log(\max_{l \in \mathcal{K}} m_l)))$ time since the worst case is when all vehicles are assigned to one type of charger l with $m_l > m_{l'}, \forall l' \in \mathcal{K}$ and $l \neq l'$. We will omit the proof of the correctness of the repairing procedure, as it is similar to that of Proposition 4.3.3.1.

4.4.4 Simulated Annealing with Two-stage Local Search

The NP-hardness result for the charging scheduling problem suggests that finding an optimal solution will be challenging. Moreover, preliminary tests reveal that an optimizer could not solve the LP models to optimality within a reasonable time, especially for large-size instances. In addition, using a commercial LP solver may incur additional expenses for charging station operators. Hence, we propose a simulated annealing metaheuristic combined with a two-stage local search.

4.4.4.1 Solution representation

A solution to the charging scheduling problem consists of two parts: the assignment solution and the power allocation solution. In the assignment solution, electric vehicles are selected to be plugged into chargers so that there are no overlapping vehicles on the same charger. The power allocation solution defines the time slots for charging without exceeding the grid capacity. The choice of the solution representation is motivated by how we will explore the searching space. In other words, when a neighborhood operator is applied, it should be easy to check the feasibility and calculate the objective function value of the new solution. The assignment solution is represented by a vector $\Pi = (\pi_1, \dots, \pi_m)$ where π_i is the sequence of vehicles assigned to a charger i in non-decreasing order of their arrival times. Hence, checking if a vehicle can be assigned to a charger i can be done in $\mathcal{O}(\log n)$ time using binary search. Also, we keep a set of rejected demands \mathcal{L}_R .

The power allocation solution is represented with a vector (T_1, \dots, T_n) where T_j is $(d_j - r_j)$ -dimensional vector that stores Boolean values, $T_j = (u_1, \dots, u_{d_j - r_j}) \in \{0, 1\}^{(d_j - r_j)}$. If the vehicle j is charging at time slot t , the $t - r_j + 1$ component $u_{t - r_j + 1}$ is set to 1. Otherwise, it is set to 0. For convenience, we define a vector of real numbers $(w_G^t)_{t \in \mathcal{H}}$ that stores the delivered power by all chargers at each time slot t .

Notice that, unlike the assignment solution presentation, the power allocation solution representation is the same as for the problem of minimizing the grid capacity. Therefore, we can exploit some algorithms implemented for the previous chapter.

The solution representation for the schedule of charging demands on different types of chargers of Example 4.2.1 is shown in Figure 4.3.

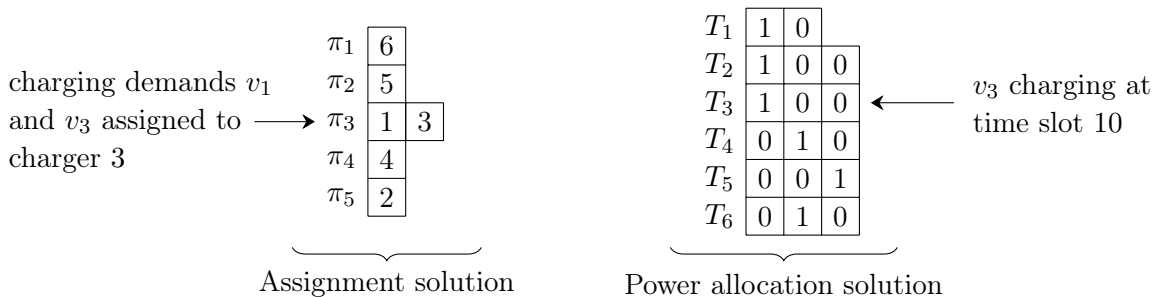


Figure 4.3: Solution representation of the schedule on chargers with different power rates in Figure 4.1(1).

4.4.4.2 Constructive heuristic method (CHM)

Exploring the neighborhood of a promising solution increases the chance of finding even better ones. As a result, a metaheuristic algorithm is greatly impacted by the quality of the initial solution since its neighborhood is more thoroughly explored. Given this, we propose a constructive heuristic method, denoted as CHM, to initialize the solution for the metaheuristic rather than using randomly generated ones. The heuristic is based on Algorithm 8, modified to include charging decisions for vehicles according to their energy requirement and the available power.

The proposed heuristic, described in Algorithm 10, builds a charging schedule by considering vehicles in non-decreasing order of arrival times r_j . Ties are broken using the non-decreasing order of departure times d_j . Further ties are broken using non-decreasing order of energy requirements e_j (line 1). The chargers are considered in non-decreasing order of charging power rates (line 2). w_G^t is initialized to 0 for each time slot t , $t \in \mathcal{H}$ (line 3). The CHM's basic idea is as follows. The CHM try to schedule vehicles in the order they appear in the set \mathcal{J} (lines 5-31). There are two situations where it would be infeasible to schedule a charging demand j : (i) when no charger is available, and (ii) when the power grid capacity is insufficient to charge this vehicle on an available charger. In both situations, we have to choose a vehicle to reject. Since vehicles are ordered in a non-decreasing order of arrival times, the CHM chooses either the vehicle j , or one of the last scheduled vehicles on the chargers (line 6).

We now describe in detail these decisions. For each vehicle j , if there are available chargers at r_j , the CHM begins by seeking an available charger with the smallest charging power rate j without exceeding the current grid capacity (lines 9-11). If such a charger exists, it is selected to charge vehicle j (line 12). Otherwise, the heuristic calculates the value $a(j', r_j, d_j)$ that represents the amount of energy allocated to each scheduled charging demand j' ($j' \neq j$) in the interval $[r_j, d_j]$. The charging demand with the largest value of $a(j', r_j, d_j)$ will be rejected if $a(j', r_j, d_j)$ is greater than the requested energy e_j (line 17). Otherwise, the vehicle j is rejected (line 21). When no charger is available at r_j (lines 23-30), the charging demand with the maximum departure time is rejected.

Note that when a charging demand other than j is rejected as in lines 18 and 26, we must repeat the procedure (lines 6-31) to reschedule j (lines 20 and 28).

The worst case complexity of the heuristic is $\mathcal{O}(\max(n \max(T, m), m \log n), m \log m)$

Example 4.4.1. Consider the instance in Example 3.2.1 with chargers of different types and $w_G = 30$ kW. We set the length of time slots to 30 minutes. Therefore, from 08:00 to 13:00, we have ten time slots. The schedule built by the heuristic is displayed in Figure 4.4. Following Algorithm 10, the heuristic will start by scheduling vehicles v_1, v_2, v_3, v_4 , and v_5 in this order. For vehicle v_6 , we have charger 1 available but not enough power to charge v_6 . Thus, the heuristic must reject either v_6 or one of the last scheduled vehicles (i.e., v_2, v_3, v_4 , and v_5) based on the values $a(j, 10, 13)$ and e_6 . Since we have $a(2, 10, 13) = 10$, $a(3, 10, 13) = 20$, $a(4, 10, 13) = 20$, $a(5, 10, 13) = 20$, and $e_6 = 20$, the vehicle v_6 is rejected. Similarly, vehicle v_7 will be rejected.

4.4.4.3 Simulated Annealing

We use a simulated annealing (SA) algorithm similar to Algorithm 6 but with different objective function $f(S)$ and different neighbor structure. The objective here is to maximize the number

Algorithm 10: Constructive heuristic method (CHM).

Input : The set of charging demands \mathcal{J} , the set of chargers \mathcal{M} , the grid capacity w_G

Output: The assignment of vehicles to chargers Π , a set of rejected demands \mathcal{L}_R , the power allocation vector (T_1, \dots, T_n) .

- 1 Sort \mathcal{J} in non-decreasing order of r_j , break ties by the non-decreasing order of d_j , then by in non-decreasing order of e_j ;
 - 2 Sort \mathcal{M} by non-decreasing order of charging power w_i ;
 - 3 $(w_G^t) \leftarrow (0)_{t \in \mathcal{H}}$;
 - 4 $\pi_i \leftarrow \emptyset$ for all $i \in \mathcal{M}$;
 - 5 **for** $j \in \mathcal{J}$ **do**
 - 6 Let j_1, j_2, \dots, j_m be the last vehicles scheduled in $\pi_1, \pi_2, \dots, \pi_m$;
 - 7 **if** there is an available charger at r_j **then**
 - 8 w_j^a be the first available charger in \mathcal{M} ;
 - 9 Let b be the number of time slots in $[r_j, d_j)$ where $w_G^t + w_j^a \leq w_G$;
 - 10 $E_j \leftarrow e_j / (b \times \tau)$;
 - 11 **if** j can be scheduled on an available charger i with a charging power $w_i \geq E_j$ without exceeding w_G **then**
 - 12 Add j to π_i ;
 - 13 Update T_j by charging vehicle j in the first p_{ij} time slots t where $t \in [r_j, d_j)$ and $w_G^t + w_i \leq w_G$;
 - 14 **else**
 - 15 Let $a(j', r_j, d_j)$ be the allocated energy to charging demand $j' \in \{j_1, j_2, \dots, j_m\}$ in the interval $[r_j, d_j)$;
 - 16 Let $k \in \{j_1, j_2, \dots, j_m\}$ be the charging demand satisfying $a(k, r_j, d_j) = \max_{j' \in \{j_1, j_2, \dots, j_m\}} a(j', r_j, d_j)$;
 - 17 **if** $a(k, r_j, d_j) > e_j$ **then**
 - 18 Add k to \mathcal{L}_R ;
 - 19 $T_k \leftarrow (0)_{t \in [0, d_k - r_k]}$;
 - 20 Reschedule j ;
 - 21 **else**
 - 22 Add j to \mathcal{L}_R ;
 - 23 **else**
 - 24 Let $k \in \{j_1, j_2, \dots, j_m\}$ be the charging demand satisfying $d_k = \max_{j' \in \{j_1, j_2, \dots, j_m\}} d_{j'}$;
 - 25 **if** $d_k > d_j$ **then**
 - 26 Add k to \mathcal{L}_R ;
 - 27 $T_k \leftarrow (0)_{t \in [0, d_k - r_k]}$;
 - 28 Reschedule j ;
 - 29 **else**
 - 30 Add j to \mathcal{L}_R ;
 - 31 Update w_G^t ;
-

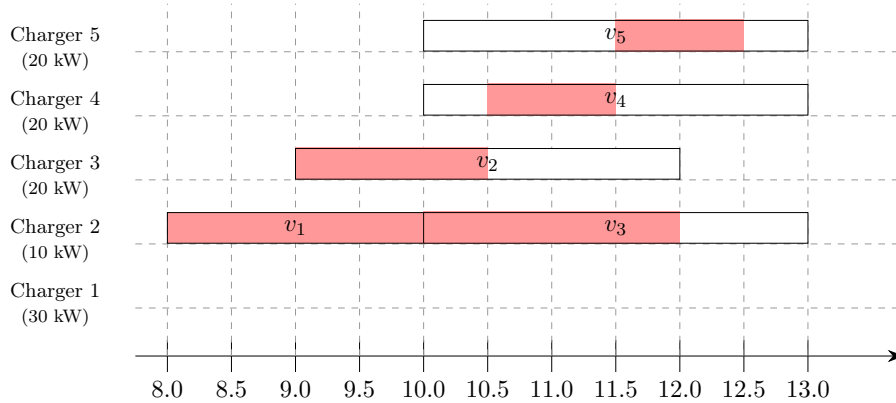


Figure 4.4: Charging schedule for charging demands in Example 4.2.1 on different types of chargers using the CHM. Rectangles represent the vehicles' plugging intervals. We highlight charging intervals in red.

of accepted charging demand. In addition, here is that the problem is a maximization problem. Therefore, we replace line 11 by $\Delta f > 0$ or $U(0, 1) \leq e^{\Delta f/T}$. Also, line 14 is replaced by $f(S) > f(S_{best})$.

4.4.4.4 Generation of new solution

When implementing a metaheuristic algorithm, one of the most critical decisions is defining the neighborhood operators used to explore the search space. In other words, how we generate new solutions highly influences the SA algorithm performance. Most neighborhood operators used for scheduling problems deal with assigning and sequencing jobs on different machines. Such neighborhood operators are efficient if calculating the objective function value from the assignment is simple. Unfortunately, in our case, each assignment solution can have one or more power allocation solutions. Furthermore, we cannot determine whether scheduling all charging demands in the assignment solution is feasible since it is an open problem (see Remark 3.4.3). In order to deal with this issue, a new solution is generated in Algorithm 6 line 6 by modifying the assignment part of the solution using the neighborhood operators (described Section 4.4.4.5). Then, a local search procedure (described in Section 4.4.4.6) is applied to get each generated solution's power allocation solution and objective function value.

4.4.4.5 Neighbor operators

The SA algorithm randomly chooses one of the following operators to generate a new assignment solution:

- **Change assignment:** this operator chooses a charging demand on a charger i_1 and moves it to another charger i_2 . The chargers and the charging demand are randomly selected. If a charging demand in charger i_2 overlaps with j , the move is discarded.

- **Assign Q_{assign} rejected charging demands:** this operator attempts to assign Q_{assign} charging demands from the rejected list as follows. At each attempt, a charging demand j is randomly selected to be inserted on a randomly selected charger i . The insertion is discarded if at least one charging demand in charger i overlaps with j . Q_{assign} is a parameter to be defined.
- **Reject a charging demand:** this operator moves a charging demand from a charger to the rejected list. The charger and the charging demand are randomly selected.
- **Reject the first q charging demands:** this operator selects a charger randomly, and then removes the first q charging demands from it and place them in the rejected list. The number q is selected randomly between 1 and the number of vehicles assigned to the selected charger. If the charger is empty, the move is discarded.
- **Reject the last q charging demands:** this operator is similar to the previous one, but instead of selecting the first q charging demands, the last q charging demands are removed.
- **Swap chargers:** this operator randomly selects two chargers and moves all charging demands in the first charger to the second one and vice versa.

Recall that the assignment of vehicles to chargers is represented by π_i , $i \in \mathcal{M}$, which is a sequence of non-overlapping plugging intervals sorted based on arrival times. As aforementioned, when assigning a vehicle j to charger i , we use binary search to check if the plugging interval $[r_j, d_j)$ overlaps with other vehicles' plugging intervals in π_i . If not, the binary search returns the position where to insert the vehicle. However, the insertion has a complexity of $\mathcal{O}(n)$ since the vehicle has to be deleted from either π_i or \mathcal{L}_R and inserted in $\pi_{i'}$. Still, the advantage of this operation is that we do not have to sort vehicles to determine whether or not the assignment solution is feasible, which has a time complexity of $\mathcal{O}(n \log n)$.

When a move is discarded, the SA algorithm randomly selects another operator. After each successful move, the SA algorithm applies a local search procedure to construct and improve the power allocation solution.

4.4.4.6 Two-stage local search procedure

Given an assignment solution, the two-stage local search (TsLS) procedure build the power allocation solution by selecting the maximum subset of scheduled demands from the assignment solution that can be satisfied without exceeding the grid capacity w_G . As mentioned before, the assignment solution may or may not be feasible, i.e., the grid capacity may not be sufficient to charge all assigned demands.

Let \mathcal{J}' be the set of assigned charging demands. Let \tilde{w}_G be the minimum grid capacity required to satisfy all charging demands in the set \mathcal{J}' . The basic idea is to obtain a charging schedule with the minimum value of \tilde{w}_G . When $\tilde{w}_G > w_G$, we insert and reject charging demands until \tilde{w}_G reaches w_G . Note that we can only insert the demands rejected by the TsLS procedure. Moreover, each demand can only be reinserted in their previously assigned charger, meaning that we cannot move a charging demand to another charger. Therefore, we keep a list \mathcal{L}_{LS} of rejected demands by the TsLS along with their previous chargers. At the end of the LS procedure,

charging demands in \mathcal{L}_{LS} are added to \mathcal{L}_R . The implemented TsLS algorithm (Algorithm 11) starts by building a power allocation solution for charging demands in the assignment solution S_0 using a heuristic described in Algorithm 4 (line 1). The current solution S is set to S_0 . At each iteration (lines 3-21), the TsLS procedure generates (*MaxGenerated*) neighbors of the current solution S (lines 4-9). For each generated neighbor, it applies a procedure to minimize \tilde{w}_G (line 6), which will be explained in more detail later. The best feasible solution S' in the neighborhood of S is selected (line 9). A solution is feasible if its grid capacity \tilde{w}_G is less than or equal to w_G . If the best neighborhood S' is better than the best solution found so far S^* , it will replace the current solution S and the best solution S^* . Otherwise, the number of non-improving iterations $iter$ is incremented (line 19). In this case, the current solution S is set to either the best solution in the neighborhood S' or to S^* as follows. The best solution in the neighborhood S' may replace the current solution S if a randomly generated number u is less than the probability p_{iter} (lines 15-17). p_{iter} decreases in a geometric way [Ogbu & Smith 1990] and is calculated as $p_{iter} = p_a \times r^{iter-1}$, where p_a is the initial acceptance probability, $r < 1$ is the reducing factor, and $iter$ is the number of iterations. When the number of non-improving iterations $iter$ exceeds *MaxNonImproving*, the search is considered as stagnating on a local optimum. If no feasible solution is found (line 23), a repair procedure is carried out on the current solution S .

Initial solution for power allocation Let \mathcal{J}' be the set of vehicles in the assignment solution. Let w_j be the charging power of each vehicle j and p_j the charging time of j , $j \in \mathcal{J}'$. The proposed heuristic, detailed in Algorithm 12, builds the power allocation solution for the set \mathcal{J}' by considering the assigned vehicles in non-decreasing order of their departure time d_j , and break ties first by non-increasing order of their energy request e_j , then by non-increasing order of their arrival time r_j (line 1). The grid capacity \tilde{w}_G and power allocation vectors are initialized to 0 (line 2). As mentioned before, we can exploit the algorithms defined in the previous chapter to get the power allocation solution. Hence, we charge a vehicle j according to Algorithm 4 (line 4). As a reminder, the power allocation heuristic starts by charging vehicle j at time slots without exceeding \tilde{w}_G in chronological order. Then, it charges the vehicle j on time slots with the minimum w_G^t value.

Local neighbor structure In the TsLS procedure, a neighbor N of the current solution S is generated in Algorithm 11 by one of the following operators:

- **Reject-LS** this operator is chosen if $\tilde{w}_G(S) > w_G$. It moves q charging demands from a charger to the rejected list \mathcal{L}_{LS} . The number q is proportional to the number of vehicles n . Based on preliminary tests, we observe that when q increases, the performance of the TsLS procedure worsens. Also, the TsLS procedure is more of an intensification phase in which we try to get as few rejected demands as possible from the assigned one. Thus, we fix q to $\left\lceil \frac{n}{10} \right\rceil$. We implements three methods to select the vehicle to reject. In the first one, a randomly chosen vehicle is selected. In the second method, we reject the vehicle that has the greatest contribution to the grid capacity constraint violation. Recall that the power allocation vector of the vehicle j is $T_j = (u_1, \dots, u_{t-r_j+1})$. Then, we reject the vehicle $j \in \mathcal{J}'$ that has largest value a_j where $a_j = \sum_{t \in H_1} u_{t-r_j+1} \times w_j$ and

Algorithm 11: Local Search Procedure**Input** : The assignment solution S_0 , $MaxNonImproving$, $MaxGenerated$ **Output:** Best feasible solution found S^*

```

1 Initialize the power allocation for  $S_0$  according to Algorithm 4;
2  $iter \leftarrow 0$ ;  $S \leftarrow S_0$ ;  $S^* \leftarrow$  empty solution;  $S' \leftarrow$  empty solution;
3 while  $iter < MaxNonImproving$  do
4   for  $k = 1$  to  $MaxGenerated$  do
5      $N \leftarrow$  Generate TsLS Neighbor( $S$ );
6     Apply minimizing grid capacity procedure on  $N$ ;
7     if  $\tilde{w}_G(N) \leq w_G$  and  $f(N) > f(S')$  then
8       if  $S^*$  is empty then  $S^* \leftarrow N$  ;
9        $S' \leftarrow N$ ;
10    if  $S^*$  is not empty then
11      if  $f(S') > f(S^*)$  then
12         $S \leftarrow S'$ ;
13         $S^* \leftarrow S'$ ;
14         $iter \leftarrow 0$ ;
15      else
16        Generate a random number  $u \sim U(0, 1)$ ;
17        if  $u < p_a \times r^{iter-1}$  then  $S \leftarrow S'$  ;
18        else  $S \leftarrow S^*$  ;
19         $iter \leftarrow iter + 1$ ;
20    else
21       $iter \leftarrow iter + 1$ ;
22 if  $S^*$  is empty then
23    $\leftarrow$  Apply repairing procedure to  $S$ ;
24 return  $S^*$ 

```

Algorithm 12: Power allocation heuristic**Input** : The set of charging demands \mathcal{J}' , the selected charging power w_j for each vehicle $j \in \mathcal{J}'$ **Output:** The power allocation solution $(T_j)_{j \in \mathcal{J}'}$, $(w_G^t)_{t \in \mathcal{H}}$, the grid capacity \tilde{w}_G

```

1 Sort  $\mathcal{J}'$  by non-decreasing order of  $d_j$ . Then, in non-increasing order of  $e_j$ . Then, in non-increasing order of  $r_j$  ;
2  $(w_G^t) \leftarrow (0)_{t \in \mathcal{H}}$  ;  $\tilde{w}_G \leftarrow 0$ ;  $(T_j) \leftarrow (0)_{j \in \mathcal{J}'}$  ;
3 for  $j \in \mathcal{J}'$  do
4    $\leftarrow$  Charge the vehicle  $j$  according to Algorithm 4;
5 return  $\tilde{w}_G$ ,  $(T_j)_{j \in \mathcal{J}'}$ ,  $(w_G^t)_{t \in \mathcal{H}}$ 

```

$H_1 = \{t | t \in [r_j, d_j) \text{ and } w_G^t > w_G\}$. The third method consists of using the roulette wheel selection [Lipowski & Lipowska 2012] to choose the vehicle to reject based on their values a_j . In other word, a vehicle j with a higher value a_j has a higher probability to be chosen. Note that after rejecting a vehicle j , T_j and w_G^t are updated.

- **Re-assign-LS** this operator is chosen if $\tilde{w}_G(S) \leq w_G$. It chooses q vehicles from \mathcal{L}_{LS} to be assigned back to their chargers. q is fixed to $\left\lceil \frac{|\mathcal{L}_{LS}|}{10} \right\rceil$ where $|\mathcal{L}_{LS}|$ is size of the set \mathcal{L}_{LS} . We implement two methods to determine which vehicle to re-assign. The first randomly chooses a vehicle from \mathcal{L}_{LS} . The second method re-assigns the vehicle that is expected to have the smallest impact on overloading the power grid. More precisely, we choose the vehicle that has the largest value \bar{a}_j where \bar{a}_j is the number of time slots $t \in [r_j, d_j)$ and $w_G^t + w_j \leq w_G$. The power allocation vector T_j for the re-assigned vehicle j is obtained by applying Algorithm 4.

Minimizing grid capacity procedure (MINWG-LS) We used a heuristic to choose the time slots for charging the vehicles to satisfy their energy requirements regardless of the grid capacity limit. The resulting charging schedule is not guaranteed to be optimal, meaning the obtained \tilde{w}_G is not necessarily the lowest possible value. For this reason, it is necessary to establish a method to improve the charging schedule and attempt to minimize \tilde{w}_G so it remains within the grid's capacity limit w_G . One way to achieve this is by moving these vehicles' charging operations to different time slots. If \tilde{w}_G reaches w_G , the generated neighbor is feasible. This problem is addressed in Chapter 3 in Section 27. For this reason, we use the same SA in Algorithm 6 with the same objective function $f(S)$ (minimize \tilde{w}_G) but a slightly different neighbor structure. A neighbor structure in the minimizing grid capacity local search method (MINWG-LS) moves q charging operations of a vehicle j from a time slot t_1 where $u_{t_1-r_j+1} = 1$ in T_j to another time slot t_2 where $u_{t_2-r_j+1} = 0$. The number of moves q is randomly chosen between 1 and a maximum value $\left\lceil \frac{n}{10} \right\rceil$. Let H_1 be the set of time slots where the vehicle j is charging. That is, $H_1 = \{t | t \in [r_j, d_j) \text{ and } u_{t-r_j+1} = 1 \text{ in } T_j\}$. Let H_2 be the set of time slots where $H_2 = \{t | t \in [r_j, d_j) \text{ and } u_{t-r_j+1} = 0 \text{ in } T_j\}$.

Let $\mathcal{J}'' \subseteq \mathcal{J}$ be the set of scheduled charging demands where $d_j - r_j - p_j > 0$. First, we randomly select an electric vehicle $j \in \mathcal{J}''$, then the charging of the vehicle j is modified by one of the following methods:

- **Peak shifting:** this method moves a charging operation from peak to off-peak time slots. Basically, a charging operation is moved from the first time slot t_1 with $w_G^{t_1} = \max_{\{t \in H_1\}} w_G^t$ to the first time slot t_2 with $w_G^{t_2} = \min_{\{t \in H_2\}} w_G^t$.
- **Filling:** the basic idea is to charge the vehicles assigned to the chargers with lower power simultaneously to free up time to charge the vehicles assigned to the chargers delivering higher power. Basically, a charging operation is moved from the first time slot t_1 with $w_G^{t_1} = \min_{\{t \in H_1\}} w_G^t$ to the first time slot t_2 with $w_G^{t_2} = \max_{\{t \in H_2\}} w_G^t$ and $w_G^{t_2} + w_j \leq w_G$.

After each move, $u_{t_1-r_j+1}$ is set to 0 and $u_{t_2-r_j+1}$ is set to 1. The vector w_G^t is updated as well as the objective value \tilde{w}_G .

An illustration of these operators is given in Figure 4.5. In the initial schedule, we have $w_G^1 = 20$, $w_G^2 = 10$, and $w_G^3 = 40$. Thus, $\tilde{w}_G = 40$. Clearly, rescheduling the charging of v_1 at time slot $t = 2$ will not change the value of $\tilde{w}_G = 40$. Notice that for v_3 , $\min w_G^t = w_G^2 = 10$ to $\max w_G^t = w_G^1 = 20$. If we choose v_3 and apply the operator "Filling", it will move the charging operation from $t = 2$ to $t = 1$. As a result, we will have $w_G^1 = 30$, $w_G^2 = 0$, and $w_G^3 = 40$. When we choose v_1 and we apply the "peak shifting" operator, it will move the charging operation of v_1 from $t = 3$ to $t = 2$. Finally, we will have $w_G^1 = 30$, $w_G^2 = 30$, and $w_G^3 = 10$. Therefore, $\tilde{w}_G = 30$.

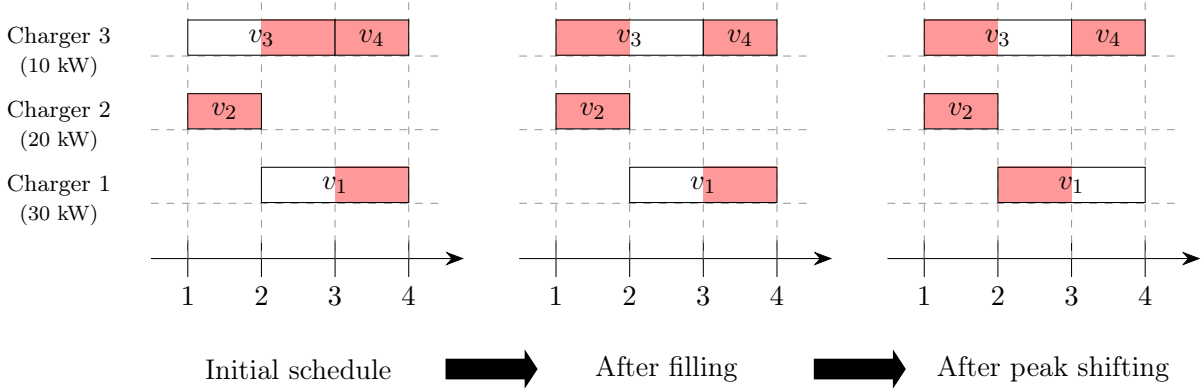


Figure 4.5: Example of application of "Filling" and "Peak Shifting" operators. Rectangles represent the vehicles' plugging intervals. We highlight charging intervals in red.

After moving q charging operations, if $\tilde{w}_G > w_G$, we apply a procedure similar to the "peak-shifting" operator as follows. For each time slot $t \in \mathcal{H}$ where $w_G^t > w_G$, for each vehicle j scheduled on this time slot ($u_{t-r_j+1} = 1$ in T_j), we try to move the charging operation to another t' where $u_{t'-r_j+1} = 1 = 0$ and $w_G^{t'} + w_j > w_G$. This procedure is time-consuming for large instances. The SA is also susceptible to get stuck in a local optimum. Thus, it is applied to the generated neighbor in the MINWG-LS procedure with a probability of 25%.

Repair procedure The global SA algorithm relies on the TsLS procedure to provide a feasible solution to the problem. If the search fails to yield a feasible solution, the TsLS method applies the following steps to construct one. The TsLS will reject a vehicle chosen using the second method of the Reject-LS operator. Then, MINWG-LS is applied. This instructions are repeated until $\tilde{w}_G \leq w_G$.

TsLS improvement phase At the end of the TsLS, an improvement phase is performed to reschedule some rejected charging demands in the best solution found by the TsLS. The first step involves calculating \bar{a}_j values, as defined in the "Re-assign-LS" operator, for each charging demand j in \mathcal{L}_{LS} . Let j' denote the charging demand with the maximum value $\bar{a}_j - p_j$. If $\bar{a}_{j'} - p_{j'} \geq 0$, we reschedule j' , and then MINWG-LS is applied. This procedure is repeated until $\max_{j \in \mathcal{L}_{LS}} \bar{a}_j - p_j < 0$.

4.5 Computational Results

In this section, we report and discuss the results of the proposed methods in this chapter. We start by describing the generated instances and algorithms' parameters. Then, we perform a computational comparison for the proposed methods for one type of charger and then for different types of chargers.

4.5.1 Instances

Again, we consider three types of chargers where chargers of type 1, 2, and 3 deliver an output power of $w_1 = 11$ (kW), $w_2 = 22$ (kW), and $w_3 = 43$ (kW), respectively. We consider two types of instances. The type 1 instances regroup the first five groups of instances generated in Section 3.5.1. In type 2 instances, we generate six groups of new instances. The number of charging demands n in groups 1 and 4 is equal to 100, it is equal to 200 for groups 2 and 5, and it is equal to 400 for groups 3 and 6. For each group, we generate 10 different random instances as in type 1 instances, except for α values. For groups 1, 2, and 3 for half of the vehicles, α is fixed to 0.1, and for the other half, it is fixed to 0.2. For groups 4, 5, and 6 for 75% of vehicles, α is fixed to 0.1, and for the other half, it is fixed to 0.2. A summary of those values are shown in Table 4.4. Type 1 instances represent typical scenarios, whereas type 2 instances represent more extreme situations to demonstrate the difficulty of solving LP models. In type 2 instances, each vehicle plugged into type one chargers must spend at least 80% of its parked time charging.

Table 4.4: Values of α depending on charging time p_{1j} for type 2 instances.

instances group	instances number	n	$\alpha = 0.1p_{1j}$	$\alpha = 0.2p_{1j}$
1	1 to 10	100	50%	50%
2	11 to 20	200	50%	50%
3	21 to 30	400	50%	50%
4	31 to 40	100	75%	25%
5	41 to 50	200	75%	25%
6	51 to 60	400	75%	25%

4.5.2 Settings and Parameters

Again, all algorithms are implemented in C++ programming language and run on a desktop computer with an Intel Core i5 operating at 2.90 GHz and 8 GB RAM and running Linux OS (Ubuntu 20.04 LTS). For this chapter, the LP models are solved using IBM CPLEX 12.8 running on an Intel Xeon Silver 4216 CPU operating at 2.10 GHz and 64 GB RAM and running Linux OS (Ubuntu 20.04 LTS). For precision, we set the length of each time slot to 6 minutes. Parameter values for the proposed SA with two-stage LS were selected after several preliminary tests and summarized in Table 4.5.

For instances with $n > 100$, the MINWG-LS parameters *MaxGenerated*, *MaxTrials*, and *AcceptanceRatio* are set to half of their values to reduce the computational time. In contrast

Table 4.5: SA and LS parameters.

	Parameter	Value
Global SA parameters	μ	0.1
	<i>MaxGenerated</i>	100
	<i>MaxTrials</i>	50
	<i>AcceptanceRatio</i>	0.1
	T_f	0.001
TsLS parameters	<i>MaxNonImproving</i>	6
	<i>MaxGenerated</i>	5
MINWG-LS (SA) parameters	μ	0.2
	<i>MaxGenerated</i>	20
	<i>MaxTrials</i>	100
	<i>AcceptanceRatio</i>	0.5
	T_f	0.001

to instances with $n \leq 100$, the SA's performance in maximizing the objective function was not highly affected by the changes in those parameters.

4.5.3 Comparison Results for One Charger Type

We evaluate the performance of the method proposed in Section 4.3. All simulations are done with chargers delivering a power w of 11 kW.

4.5.3.1 Comparison between the LP models P_1 , P_2 , P_3 , and P_4

We compare the performance of the four LP models P_1 , P_2 , P_3 , and P_4 on type one instances. The stopping criteria for CPLEX is set to a maximum time of one hour or when the optimal solution is achieved. The results are shown in Tables 4.6 and 4.7. In Table 4.6, the first column displays the instance group. Also, we display the number of vehicles n , the number of chargers m , and the grid capacity w_G for each instance group. Remark that we set w_G to 50% $\sum_{i=1}^m w_i$, which is equal to $0.5mw$. Column *avg.T* depicts the average number of time slots considering a length of 6 minutes. The number of time slots differs from one instance to another since we consider that $T = \max_{j \in \mathcal{J}} d_j$. Column *avg.L* gives the average number of events. The values n , m , and T (or L) gives the instances sizes. In the remaining columns, we report the average time CPLEX took to solve each model in seconds. It should be noted that this time includes the construction and repair solutions procedures.

The first noticeable result is that the computation time of P_1 and P_2 is significantly higher than P_3 and P_4 , especially for $n > 20$. The relaxation is more effective in reducing the computation time than considering event-based. The event-based formulation was more effective for P_1 than for P_3 since there are significantly fewer variables and constraints in P_3 , making it less sensitive. We observe that CPLEX reached the time limit of one hour for 29 instances when solving P_1 and 14 instances when solving P_2 . The worst running times to solve P_3 and P_4 were

3.79 seconds and 0.26 seconds, respectively.

Table 4.6: The average computation time for instances type 1

instances group	n	m	w_G	$avg.T$	$avg.L$	P_1 time(s)	P_2 time(s)	P_3 time(s)	P_4 time(s)
1	10	10	55	84.80	17.10	1.81	0.15	0.03	0.01
2	20	10	55	103.80	32.90	1073.89	110.97	0.15	0.05
3	40	20	110	140.10	58.90	3448.51	1247.15	0.18	0.06
4	50	20	110	166.40	71.60	3519.49	3142.80	0.27	0.06
5	100	30	165	267.70	135.90	3616.35	2278.48	1.39	0.20
avg.				152.56	63.28	2332.01	1355.91	0.40	0.08

More details on the objective function values (i.e. the number of accepted charging demands) found by the four models are given in Table 4.7. For each model, column $avg\ obj$ reports the average objective function value found, column $\#OPT$ gives the number of optimal solutions found in each group, and column $avg\ G_{OPT}$ provides the average gap between the objective function value and the optimal solution value in percentage, and it is calculated as follows. Let s_{P_1} be the objective function value found by CPLEX by solving model P_1 within a time limit of one hour, and let s_{OPT} be the optimal objective value. The gap between s_{P_1} and s_{OPT} is calculated as $G_{OPT} = \frac{s_{P_1} - s_{OPT}}{s_{P_1}}$. Similarly, we calculate the gap between P_2 , P_3 , and P_4 and the optimal solution. We observe that P_3 and P_4 achieved all optimal solutions. P_1 and P_2 found all optimal solutions for instances in groups 1 and 2, despite exceeding the time limit of one hour when solving P_1 in two instances in group 2. P_1 has worse performance when solving instances with $n > 10$. In fact, when taking a closer look, for instances where the $G_{OPT} < 0$ (26 instances), the gap lies between -96% and -75% except for one instance (instance 40 with $G_{OPT} = -2.22\%$). P_2 reaches a gap below 0.00% in 11 instances: for nine instances, it lies between -58% and -2.2% while it is above -80% in the remaining two. Therefore, we can confirm that P_2 outperforms P_1 , demonstrating the importance of considering event-based formulation.

Table 4.7: Results for instances type 1

instances group	P_1			P_2			P_3			P_4		
	$avg\ obj$	$\#OPT$	$G_{OPT}(\%)$	$avg\ obj$	$\#OPT$	$avg\ G_{OPT}(\%)$	$avg\ obj$	$\#OPT$	$avg\ G_{OPT}(\%)$	$avg\ obj$	$\#OPT$	$avg\ G_{OPT}(\%)$
1	9.40	10	0.00	9.40	10	0.00	9.40	10	0.00	9.40	10	0.00
2	13.00	10	0.00	13.00	10	0.00	13.00	10	0.00	13.00	10	0.00
3	15.60	3	-57.35	35.50	10	-0.32	35.60	10	0.00	35.60	10	0.00
4	14.20	1	-66.89	40.80	4	-3.71	42.40	10	0.00	42.40	10	0.00
5	13.40	0	-86.03	60.50	4	-37.22	96.20	10	0.00	96.20	10	0.00
avg	13.12	4.80	-42.05	31.84	7.60	-8.25	39.32	10	0.00	39.32	10	0.00

4.5.3.2 Performance of P_4 on type 2 instances

We showed that P_4 efficiently solves the problem with $n \leq 100$ and $w_G = 0.5mw$ within less than 0.26 seconds. However, we showed that the problem is \mathcal{NP} -hard. Therefore, we want to investigate the performance of P_4 on more challenging instances (instances type 2) with different values of w_G and m . For each instance, we set the execution time limit to 1800 seconds, and we run P_4 for $m \in \{15, 20, 25\}$ and $w_G \in \{0.25mw, 0.5mw, 0.75mw\}$.

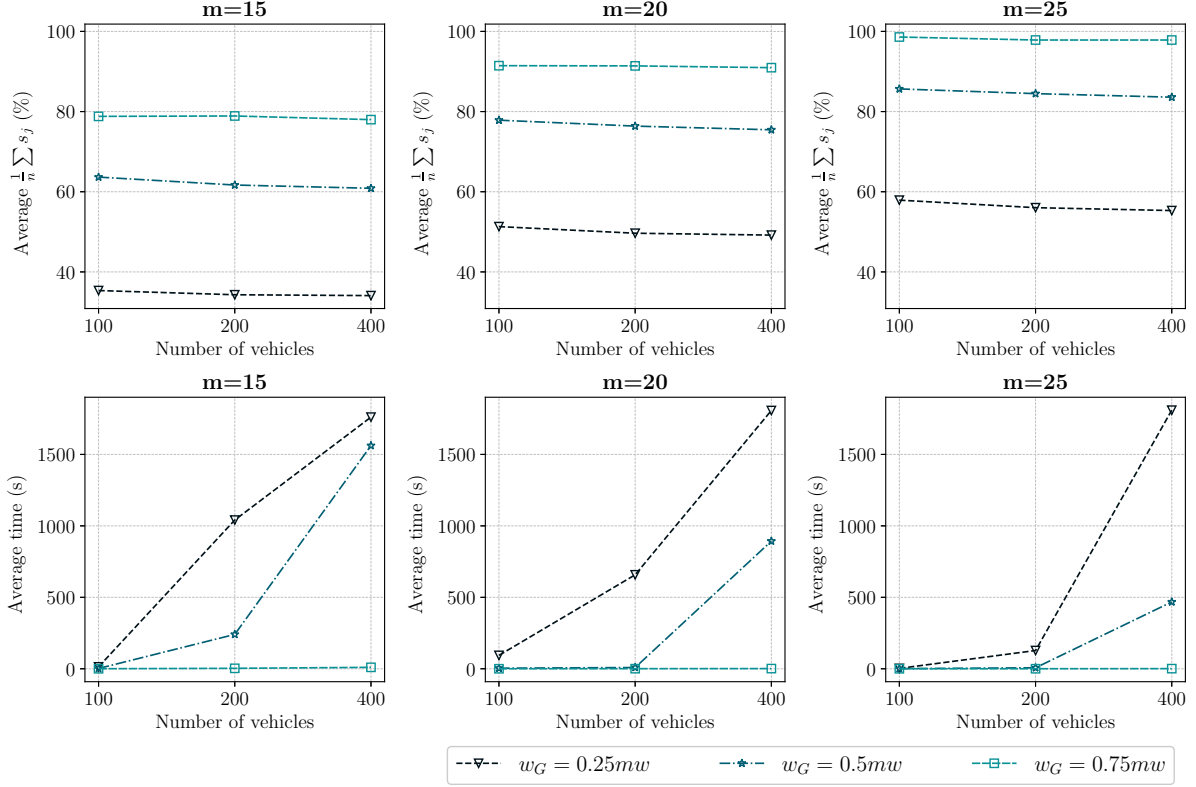


Figure 4.6: The effect of m and w_G values on the performance of P_4 .

We are interested in the average number of scheduled charging demands, i.e., $\frac{1}{n} \sum_{j=1}^n s_j$. Recall that $s_j = 1$ if the vehicle j is scheduled. Otherwise, $s_j = 0$. The average number of scheduled vehicles (in percentage) and the average running times are given in Figure 4.6.

Overall, we remark that the model's performance is more impacted by the value of the grid capacity w_G than by the number of chargers m . Increasing the grid capacity value improves the objective value and the running time of P_4 . The impact of the number of chargers m on the average running time is more important with $n \geq 200$ and w_G set to $25\%wm$ and $50\%mw$. When w_G is set to 75% of mw , the average running time was 1.95 seconds with a maximum of 41.13. Also, instances with $n = 100$ are always solved to the optimal within less than one second for all values of m and w_G . For $w_G \in \{0.25mw, 0.5mw\}$, CPLEX found 39 optimal solutions out of 120 for $n = 400$ and 106 optimal solutions out of 120 for $n = 200$. We observed that instances in group 6 were harder to solve than in group 3. Further, remark that the percentage of scheduled vehicles in each group is approximately similar considering the same values of w_G and m .

4.5.3.3 Comparison between P_4 and the heuristics

Consider the selections proposed in Section 4.3.4. Five methods can be driven by combining these selections. We can perform an interval scheduling (IS) based selection (LDT or LIT)

followed by the energy reasoning (ER) based selection to obtain a subset of charging demands. Then P_4 is solved considering only this subset. These two methods are denoted LDT+ER+ P_4 and LIT+ER+ P_4 . We can also choose only one selection (LDT, LIT, or ER) and then solve the model P_4 , which yields three methods: LDT+ P_4 , LIT+ P_4 , and ER+ P_4 . Remark that if we choose an interval scheduling-based selection, P_4 is solved without the assignment constraints (4.21), and therefore the repairing solution procedure (Algorithm 7) is not applied. Also, P_4 is limited to 30 minutes for each method. We test the performance of these methods on type 2 instances with $m = 15$ and $w_G = 83$ kW. In Figure 4.7, we plot the average number of scheduled demands achieved by the five methods and P_4 in percentage. In addition, we report the average gap, expressed as a percentage, between the objective value of each method s_M and the best objective value s_{best} , calculated as $G_{best}(M) = \frac{s_M - s_{best}}{s_{best}}$ in percentage. The average execution times for each method are also displayed.

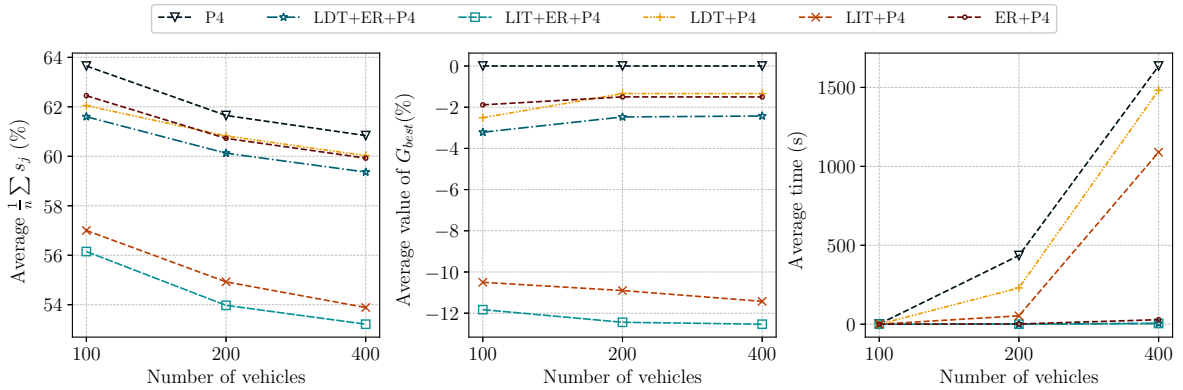


Figure 4.7: Comparing the heuristics with the model P_4 on type 2 instances.

First, we notice that P_4 achieved all the best solutions since its $G_{best} = 0$. The next best methods for maximizing the objective function are ER+ P_4 and LDT+ P_4 . Taking a closer look, the objective function values achieved by ER+ P_4 were better than LDT+ P_4 in 20 instances, equal in 23 instances, and worse in 16 instances. Then, the best method after these two is LDT+ER+ P_4 . The worse methods are the ones involving the LDT selection. In terms of running time, we can see that methods without the ER based selection have the worse performance, especially for instances with $n = 400$ where the time limit of 30 minutes is reached in most instances.

Overall, the ER+ P_4 method shows the best performance; the average gap is low (averagely -1.63%), corresponding to an average difference of 2.23 vehicles. Furthermore, the average running time is 0.13, 1.60, and 28.67 for $n = 100$, $n = 200$, and $n = 400$, respectively. In conclusion, developing a heuristic that considers the power allocation solution is better than the one dealing with the assignment solution.

4.5.4 Comparison Results for Distinct Charger Types

This section compares the results of the proposed methods for k types of chargers. Note that the values chosen for the grid capacity w_G and the number of chargers of each type m_l , $l \in \mathcal{K}$

are based on the results of the previous chapter. Specifically, those values are set so that it is impossible to schedule all vehicles in most instances to make the problem more challenging.

4.5.4.1 Impact of linearization on the performance of P_5 and P_6

To confirm the efficiency of the linearization proposed in Section 4.4.3, we run CPLEX to solve the LP models P_5 and P_6 with and without it on group 1 instances of type 1 ($n = 10$). For each instance, the grid capacity is limited to 50 kW, and the number of charges of each type m_l is set of 5. Once again, CPLEX running time is limited to 30 minutes. The results are shown in Table 4.8. For each model, we report the number of scheduled demands (column obj) and the execution time in seconds (column time). First, P_6 and P_5 with linearization achieved all optimal solutions. However, P_5 with linearization reached the time limit in four instances out of 10. Meanwhile, P_5 without linearization scheduled at most half of the optimum number of vehicles within 30 minutes. Also, P_6 without linearization scheduled at most half the number of vehicles in 8 instances. Even though we only tested ten instances, it is expected that the larger the instances, the worsen the models' performance. Therefore, in the rest of the section, we consider P_5 and P_6 with linearization.

Table 4.8: Results for type one instances of group 1 ($n = 10$) with $m_l = 5$, and $w_G = 50$ (kW)

instance	without linearization				with linearization			
	P_5		P_6		P_5		P_6	
	obj	time (s)	obj	time (s)	obj	time (s)	obj	time (s)
1	5	1802.57	5	1800.09	10	6.07	10	0.50
2	3	1800.22	3	1800.22	9	1800.66	9	18.74
3	4	1800.24	8	1800.18	9	1800.38	9	27.77
4	4	1800.22	3	1800.20	10	3.98	10	3.17
5	3	1800.21	6	1800.24	9	537.35	9	62.00
6	2	1800.24	4	1800.14	10	18.17	10	3.16
7	3	1800.26	2	1800.10	9	1800.48	9	17.70
8	3	1800.18	3	1800.07	9	742.27	9	21.76
9	1	1800.13	3	1800.13	9	46.03	9	8.29
10	4	1800.45	4	1800.31	9	1800.66	9	35.67
avg.	3.20	1800.47	4.10	1800.17	9.30	855.60	9.30	19.88

4.5.4.2 Comparison between P_5 , P_6 and the SA metaheuristic on type one instances

We test the LP models P_5 and P_6 , the heuristic CHM, and the proposed SA on type one instances. We set the number of chargers of each type m_l and the grid capacity value w_G to different values depending on the instance group. Each instance is run within a limit of 30 minutes. Tables 4.9, 4.10, 4.11, 4.12, and 4.14 compare the results of these methods on instance groups 1, 2, 3, 4, and 5, respectively. The comparison is based on:

- the objective function value reported in columns obj for P_5 , P_6 , and CHM. Due to the stochastic nature of the SA, it is run 30 times for each instance. The best, mean, and

worst objective values over these 30 runs are reported in columns best, mean, and worst, respectively. The column std reports the standard deviation to reflect the variation in the objective values;

- the computation time in seconds given in columns time. For the SA, the average running time is reported.

Group 1 instances were optimally solved by P_6 , P_5 , and the SA, while the heuristic found one optimal solution. We remark that the computation time varies significantly for CPLEX when solving the LP models. For example, P_6 took 0.5 seconds to solve instance one while it took 62 seconds to solve instance 5. This variation is more apparent when solving the model P_5 ; for instance 4, it took 4 seconds while it reached the time limit of 30 minutes in 4 instances. On the other hand, the running times of the SA and the heuristic tend to be relatively stable.

Table 4.9: Results for instances of group 1 ($n = 10$) with $m_l = 5$ and $w_G = 50$ kW.

instance	P_5		P_6		CHM		SA				
	obj	time (s)	obj	time (s)	obj	time (s)	best	avg	worst	std	time (s)
1	10	6.07	10	0.50	7	5.45E-05	10	9.90	9	0.31	2.38
2	9	1800.66	9	18.74	7	3.89E-05	9	9.00	9	0.00	5.22
3	9	1800.38	9	27.77	7	2.67E-05	9	9.00	9	0.00	4.94
4	10	3.98	10	3.17	7	8.70E-05	10	9.03	9	0.18	4.76
5	9	537.35	9	62.00	9	1.96E-05	9	9.00	9	0.00	4.95
6	10	18.17	10	3.16	6	4.74E-05	10	9.23	9	0.43	4.52
7	9	1800.48	9	17.70	7	3.17E-05	9	8.27	7	0.52	5.20
8	9	742.27	9	21.76	7	3.50E-05	9	8.33	8	0.48	4.86
9	9	46.03	9	8.29	6	5.14E-05	9	7.67	7	0.55	5.01
10	9	1800.66	9	35.67	7	3.21E-05	9	8.80	7	0.48	5.24
Average	9.30	855.60	9.30	19.88	7.00	4.24E-05	9.30	8.82	8.30	0.30	4.71

For group 2 instances, P_6 achieved all best solutions, where 8 of them are optimal. The SA achieved four of the best solutions, where three of them are optimal. In the remaining instances (six out of ten), the SA scheduled one less vehicle compared to the solutions found by P_6 . P_5 found one of the best solutions while the heuristic did not find any. The running time of P_6 is always higher than 800 seconds, while the SA tooks 8.87 seconds, at least 100 times less than CPLEX. We notice that P_5 reached the time limit for all instances.

In groups 3 and 4, P_6 found all the best solutions, while the SA found nine out of 20. Again, there is a difference of one vehicle between the SA best solutions and P_6 solutions, except for instance 39, where we have a difference of two. The solutions found by P_5 and the heuristic are worse than P_6 by five vehicles on average in group 3 instances. Then, the heuristic finds better solutions than P_5 in group 4 instances. We notice that P_6 reached its time limit for all instances in groups 3 and 4, whereas the SA has an average running time of 20.91 seconds.

To investigate why the SA scheduled fewer vehicles than P_6 , we replaced the local search (TSLS) with an LP model. Recall that the local search builds and improves an assignment's power allocation solution. The LP model is similar to P_3 (Section 4.3.3), in which we omit constraints (4.17). In constraints (4.19), w is replaced by the charging power of each vehicle according to the assignment solution. For each instance in groups 2, 3, and 4, the SA combined

Table 4.10: Results for instances of group 2 ($n = 20$) with $m_l = 7$ and $w_G = 50$ kW.

instance	P_5		P_6		CHM		SA				
	obj	time (s)	obj	time (s)	obj	time (s)	best	avg	worst	std	time (s)
11	15	1812.11	16	986.11	13	4.50E-05	15	14.97	14	0.18	8.29
12	12	1801.04	13	1755.78	10	1.49E-04	13	12.00	11	0.59	9.56
13	13	1807.87	14	1800.42	10	5.79E-05	13	12.37	11	0.61	8.62
14	13	1810.42	14	1051.17	10	8.86E-05	14	12.83	12	0.46	8.46
15	13	1800.70	14	1046.63	10	7.89E-05	13	12.10	11	0.55	8.86
16	13	1804.40	14	824.74	10	8.07E-05	13	12.60	12	0.50	9.13
17	16	1801.07	16	1800.61	14	4.01E-05	16	14.93	14	0.37	8.75
18	13	1810.07	14	845.37	10	5.97E-05	13	12.33	11	0.55	8.90
19	10	1800.94	13	1036.67	9	2.48E-04	12	11.47	11	0.51	9.03
20	12	1804.69	13	919.92	10	8.33E-05	13	12.33	12	0.48	9.07
Average	13.00	1805.33	14.10	1206.74	10.60	9.32E-05	13.50	12.79	11.90	0.48	8.87

Table 4.11: Results for instances of group 3 ($n = 40$) with $m_l = 8$ and $w_G = 75$ kW.

instance	P_5		P_6		CHM		SA				
	obj	time (s)	obj	time (s)	obj	time (s)	best	avg	worst	std	time (s)
21	26	1802.38	31	1806.91	27	1.37E-04	31	29.16	28	0.55	15.65
22	26	1802.27	31	1816.23	25	2.19E-04	31	29.23	28	0.68	17.26
23	11	1801.35	28	1807.00	23	2.08E-04	27	25.63	25	0.61	16.71
24	23	1802.33	26	1806.98	19	4.86E-04	25	23.27	22	0.58	15.15
25	25	1801.64	28	1806.13	21	2.17E-04	27	25.07	23	0.78	25.92
26	26	1802.17	30	1811.43	25	1.34E-04	30	28.93	28	0.45	17.90
27	26	1801.81	32	1817.55	26	1.02E-04	32	30.47	30	0.57	18.44
28	24	1801.51	29	1806.98	23	1.66E-04	28	27.27	26	0.52	16.53
29	25	1801.89	28	1812.80	23	1.38E-04	28	27.27	27	0.45	15.51
30	30	1802.85	34	1813.90	30	9.83E-05	33	32.07	31	0.45	16.48
Average	24.20	1802.02	29.70	1810.59	24.20	1.91E-04	29.20	27.84	26.80	0.56	17.56

Table 4.12: Results for instances of group 4 ($n = 50$) with $m_l = 9$ and $w_G = 100$ kW.

instance	P_5		P_6		CHM		SA				
	obj	time (s)	obj	time (s)	obj	time (s)	best	avg	worst	std	time (s)
31	21	1802.28	43	1807.80	36	1.88E-04	42	40.87	40	0.68	29.37
32	31	1802.03	45	1824.74	37	1.16E-04	45	43.33	43	0.55	22.50
33	16	1801.76	42	1800.62	35	1.06E-04	41	39.90	39	0.71	22.34
34	34	1802.02	45	1821.81	38	1.45E-04	44	43.43	43	0.50	25.09
35	22	1802.04	46	1811.01	41	1.35E-04	46	44.77	44	0.50	31.81
36	24	1801.96	44	1809.33	38	1.69E-04	43	41.83	41	0.53	24.14
37	23	1801.92	44	1808.52	37	1.99E-04	43	41.23	40	0.68	21.38
38	31	1801.90	40	1806.74	34	4.25E-04	40	38.27	37	0.74	22.14
39	33	1801.90	40	1807.43	34	1.53E-04	38	36.77	36	0.63	21.17
40	15	1802.46	46	1811.08	40	1.71E-04	45	44.10	42	0.61	22.72
Average	25.00	1802.03	43.50	1810.91	37.00	1.81E-04	42.70	41.45	40.50	0.61	24.27

with the LP (SA+LP) is run ten times. The best objective value and the average running time are reported in columns best and time in Table 4.13, respectively. We highlight in blue the cases where $s_{P_6} = s_{SA+LP} > s_{SA+TsLS}$, in red the cases where $s_{P_6} = s_{SA+LP} = s_{SA+TsLS}$, and in

orange the cases where $s_{P_6} > s_{SA+LP} = s_{SA+TsLS}$. This can help us identify if the issue is caused by the assignment solution or by the power allocation solution. As we can notice, in nine instances, both SA+TsLS and SA+LP failed to find the P_6 solutions. The SA+TsLS failed to find SA+LP solutions in eight instances. Consequently, it is necessary to enhance the global SA even if we improve power allocation solution exploration.

For group 5 instances with $n = 100$, the SA best solutions were better than P_6 in six instances, equal in 3, and worse in one instance (instance 41). The average running time of the SA has increased to 82.83 seconds. P_5 has the worst performance where it scheduled 83.81% less vehicles than P_6 .

Overall, the standard deviation (std) of the SA solutions is relatively small. It is always less than one, except for instance 48. It suggests that the objective values found by the SA over 30 runs are similar and less variable. We notice that the difference between the best solution value and the average increases with the size of instances but always remains below 2.5. The SA also keeps a small margin between the best and worst solutions (at most, five vehicles are unscheduled in the worst solutions). Hence it is more stable in terms of solution quality.

Table 4.13: Comparative results of P_6 , SA+LP, and SA+TsLS on group 2, 3, and 4 instances.

instance	P_6		SA+LP		SA+TsLS	
	obj	time (s)	best	time (s)	best	time (s)
11	16	986.11	16	11.00	15	8.29
12	13	1755.78	13	13.44	13	9.56
13	14	1800.42	14	10.38	13	8.62
14	14	1051.17	14	11.17	14	8.46
15	14	1046.63	13	12.23	13	8.86
16	14	824.74	14	9.83	13	9.13
17	16	1800.61	16	13.41	16	8.75
18	14	845.37	14	10.72	13	8.90
19	13	1036.67	12	556.89	12	9.03
20	13	919.92	13	13.47	13	9.07
avg.	14.1	1206.74	13.9	66.25	13.5	8.87
21	31	1806.91	32	75.88	31	15.65
22	31	1816.23	31	48.65	31	17.26
23	28	1807.00	27	55.02	27	16.71
24	26	1806.98	26	62.05	25	15.15
25	28	1806.13	27	55.44	27	25.92
26	30	1811.43	30	79.87	30	17.90
27	32	1817.55	32	71.06	32	18.44
28	29	1806.98	29	64.78	28	16.53
29	28	1812.80	28	72.37	28	15.51
30	34	1813.90	34	50.45	33	16.48
avg.	29.70	1810.59	29.60	63.56	29.2	17.56
31	43	1807.80	42	54.08	42	29.37
32	45	1824.74	45	63.12	45	22.50
33	42	1800.62	42	46.14	41	22.34
34	45	1821.81	44	50.40	44	25.09
35	46	1811.01	46	54.84	46	31.81
36	44	1809.33	43	72.98	43	24.14
37	44	1808.52	44	62.94	43	21.38
38	40	1806.74	40	64.79	40	22.14
39	40	1807.43	39	67.03	38	21.17
40	46	1811.08	45	40.75	45	22.72
avg.	43.50	1810.91	43	57.71	42.70	24.27

Table 4.14: Results for instances of group 5 ($n = 100$) with $m_l = 10$ and $w_G = 125$ kW.

instance	P_5		P_6		CHM		SA				
	obj	time (s)	obj	time (s)	obj	time (s)	best	avg	worst	std	time (s)
41	18	1806.49	79	1801.71	76	6.08E-04	81	78.57	77	0.90	76.69
42	11	1806.12	90	1834.36	81	2.39E-03	89	87.77	87	0.50	71.09
43	13	1805.92	82	1801.22	76	2.32E-04	84	82.33	81	0.76	75.62
44	14	1806.03	83	1801.06	75	3.54E-04	83	81.00	79	0.98	82.14
45	14	1806.01	85	1800.83	75	6.24E-04	86	84.30	83	0.70	104.83
46	13	1805.99	85	1801.46	77	2.65E-04	85	83.27	82	0.69	77.56
47	13	1805.57	83	1801.95	78	2.43E-04	86	84.23	83	0.77	71.34
48	14	1806.07	84	1833.32	73	6.58E-04	85	82.87	81	1.04	89.38
49	12	1806.21	86	1830.84	74	3.89E-04	86	84.27	81	0.94	78.99
50	14	1806.14	83	1801.24	77	3.88E-04	85	82.83	82	0.70	100.69
Average	13.60	1806.06	84.00	1810.80	76.20	6.15E-04	85.00	83.14	81.60	0.80	82.83

Table 4.15: Results for type 2 instances with $n = 100$, $m_l = 10$, and $w_G = 125$ kW.

instance	P_6		CHM		SA				
	obj	time (s)	obj	time (s)	best	avg	worst	std	time (s)
1	90	1807.48	79	1.49E-04	89	87.33	86	0.84	88.62
2	79	1801.08	73	1.59E-04	80	79.20	78	0.61	93.59
3	85	1816.11	74	1.35E-04	84	82.70	82	0.53	98.25
4	82	1826.77	74	1.43E-04	84	81.57	80	1.01	109.79
5	84	1809.77	76	1.44E-04	85	83.80	82	0.71	92.52
6	91	1833.77	80	1.20E-04	90	89.10	88	0.48	90.50
7	83	1814.54	73	1.40E-04	82	80.63	79	0.67	117.68
8	88	1810.21	80	1.22E-04	88	87.00	85	0.83	105.24
9	85	1819.91	73	1.37E-04	85	83.63	83	0.61	113.75
10	84	1809.07	76	1.40E-04	85	83.63	82	0.67	116.96
31	97	1821.13	88	1.08E-04	95	93.55	93	0.60	65.08
32	87	1811.14	79	1.28E-04	86	84.40	83	0.75	96.75
33	85	1819.42	76	1.34E-04	85	84.15	83	0.49	69.34
34	80	1801.33	72	1.62E-04	81	78.90	77	0.85	76.93
35	80	1808.20	74	1.51E-04	81	80.45	79	0.60	67.80
36	80	1817.84	71	1.62E-04	82	80.55	80	0.60	117.01
37	85	1826.97	75	1.46E-04	85	83.45	83	0.60	85.26
38	83	1806.21	76	1.53E-04	82	81.05	79	0.83	96.55
39	88	1803.81	78	1.33E-04	87	85.70	85	0.57	79.05
40	87	1815.47	77	1.30E-04	86	84.85	84	0.49	82.80
Average	85.15	1814.01	76.2	1.40E-04	85.1	83.78	82.55	0.67	93.17

4.5.4.3 Comparison between P_6 and the SA metaheuristic on type two instances

This section compares the SA with P_6 on type 2 instances. We did not run P_5 since it is expected to perform poorly when $n \geq 100$. For all instances, w_G is set to 125 kW and m_l is set to 10. For the SA, 20 runs are performed per instance.

For instances with $n = 100$, the best solutions found by the SA are better than P_6 in 7 instances, equal in 4 instances, and worse in 8 instances. Again, the SA scheduled only one

vehicle less than P_6 , except for instance 31, where it scheduled two fewer vehicles. The average running time of the SA is always less than 2 minutes per instance, while P_6 reached 30 minutes time limit. Although the heuristic took 0.14 milliseconds on average, it scheduled 10.5 % fewer vehicles than the SA.

Table 4.16: Results for type 2 instances with $n = 200$, $m_l = 10$, and $w_G = 125$ kW.

instance	P_6		CHM		SA				
	obj	time (s)	obj	time (s)	best	avg	worst	std	time (s)
11	155	1802.97	149	2.82E-04	165	162.55	160	1.19	113.71
12	133	1802.85	147	3.29E-04	160	158.25	155	1.16	124.07
13	142	1802.87	152	3.02E-04	168	165.85	164	0.99	118.70
14	143	1802.71	142	3.10E-04	156	154.20	153	0.95	126.31
15	150	1802.80	145	3.06E-04	160	156.80	152	1.61	102.73
16	147	1802.72	145	3.11E-04	160	158.30	156	0.98	124.39
17	164	1803.68	152	2.73E-04	169	167.20	165	1.11	117.31
18	136	1803.09	140	2.91E-04	157	155.45	154	0.89	126.89
19	145	1827.14	137	3.29E-04	154	151.55	150	1.00	115.15
20	138	1802.84	144	3.02E-04	160	158.35	157	0.81	121.97
41	152	1802.61	149	2.93E-04	162	160.50	159	0.83	112.02
42	150	1802.43	145	3.15E-04	161	158.65	157	1.14	114.86
43	153	1826.44	142	3.06E-04	160	158.35	157	0.75	117.58
44	149	1802.78	146	3.19E-04	159	156.95	156	1.00	112.83
45	156	1802.35	152	2.55E-04	168	165.10	163	1.29	111.86
46	158	1803.79	151	2.86E-04	167	164.65	162	1.23	121.39
47	150	1802.39	152	2.97E-04	163	160.50	159	1.00	126.11
48	147	1802.68	142	3.00E-04	160	157.25	154	1.21	119.95
49	140	1803.43	147	2.98E-04	162	160.60	160	0.68	125.29
50	151	1802.73	147	2.88E-04	165	163.25	161	0.91	112.88
Average	147.95	1805.27	146.3	3.00E-04	161.8	159.72	157.7	1.04	118.30

The best solutions found by the SA are better than P_6 in all instances with $n = 200$, with an average gap of 8.57%. The difference between the SA and P_6 in the number of scheduled charging demands varies from 1 to 22 vehicles. Concerning the average running time of the SA, it is always less than 130 seconds per instance. It is noticeable that from this point onward, the problem seems more challenging for CPLEX. Indeed, the average gap between the SA and P_6 increased to 27.11% in instances with $n = 400$ (an average of 78.7 vehicles). As we can notice, the difference between the SA and P_6 in the number of scheduled charging demands varies from 16 to 187 vehicles. Even the heuristic outperforms P_6 in 18 instances out of 20. The execution time of the SA has increased but remained less than 5 minutes per instance. The standard deviation values are higher in instances with $n = 400$ with a maximum value of 2.81 and an average value of 1.56. Also, the difference between the worst and the best solutions found by the SA increased to 6.05 on average. However, all worst solutions are better than P_6 solutions.

4.5.4.4 Additional results

Additional statistics are provided to explore further the solutions found by the three methods. Figures 4.8 and 4.9 compares the total energy demand of all vehicles with the total scheduled

Table 4.17: Results for type 2 instances with $n = 400$, $m_l = 10$, and $w_G = 125$ kW.

instance	P_6		CHM		SA				
	obj	time (s)	obj	time (s)	best	avg	worst	std	time (s)
21	196	1808.51	287	5.60E-04	310	307.95	305	1.36	259.96
22	264	1819.51	295	5.10E-04	319	316.30	314	1.63	265.95
23	291	1810.17	304	5.15E-04	322	319.80	317	1.44	257.01
24	184	1807.83	290	5.51E-04	313	309.25	306	1.77	255.84
25	266	1810.56	285	5.54E-04	308	305.10	302	1.52	274.76
26	264	1817.08	290	5.33E-04	312	308.20	305	1.51	264.96
27	246	1809.12	287	5.86E-04	305	303.00	300	1.41	275.92
28	201	1808.88	289	5.74E-04	313	310.50	307	1.54	250.01
29	256	1809.86	288	5.50E-04	311	309.10	307	1.12	251.44
30	178	1808.89	288	5.73E-04	311	307.70	306	1.53	267.93
51	225	1808.38	291	6.71E-04	318	314.55	312	1.47	262.70
52	279	1808.06	301	6.40E-04	323	319.00	310	2.81	255.16
53	189	1808.49	295	6.40E-04	323	319.85	317	1.66	288.95
54	297	1809.10	287	6.62E-04	317	314.05	310	1.82	276.73
55	166	1807.32	285	6.95E-04	312	308.00	306	1.69	292.16
56	165	1807.88	288	7.08E-04	312	308.70	306	1.38	305.07
57	235	1809.78	283	7.30E-04	306	302.25	300	1.48	299.94
58	296	1808.08	292	6.63E-04	317	314.15	312	1.23	298.44
59	112	1807.82	282	7.47E-04	306	302.25	299	1.68	310.98
60	267	1808.14	291	6.78E-04	314	311.70	310	1.26	290.25
Average	228.85	1809.67	289.9	6.17E-04	313.6	310.57	307.55	1.56	275.21

energy for type 1 and type 2 instances, respectively. In type 1 instances The SA scheduled 90.19%, 54.90%, 59.01%, 77.75%, and 77.15% of the total energy in instance groups 1, 2, 3, 4, and 5, respectively. In type 2 instances The SA scheduled 76.75%, 70.53%, and 67.64% of the total energy in instances with $n = 100$, $n = 200$, and $n = 400$, respectively.

To further detail the assignment of vehicles to charger types, we summarize the percentage of vehicles assigned to each type in Figure 4.10 and Figure 4.11. We recall that there are three types of chargers: 11 kW, 22 kW, and 43 kW. The plotted percentages represent the number of vehicles assigned to each type compared to the number of charger demands. The first thing to notice is that each method assigns vehicles approximately the same way. The percentage of assigned vehicles to each type slightly varies from one instance to another. This is interesting because it can help guide the search for future experiments. For example, we know that 30% of scheduled vehicles should be assigned to 22 kW chargers to obtain good solutions.

In type 1 instances, P_6 assigned fewer vehicles to 11 kW chargers (17.56%), and the remaining vehicles were distributed approximately equally between 22 kW chargers (33.43 %) and 43 kW chargers (30.76%). On the other hand, the SA assigned fewer vehicles to 43 kW chargers (17.98%), and the remaining vehicles were distributed approximately equally between 11 kW chargers (32.67%) and 22 kW chargers (30.13%). The heuristic prioritized chargers with 11 kW (45.53%) while assigning 19.79% of vehicles to 22 kW chargers and only 1.42% to 43 kW chargers. This is expected because the heuristic is designed to choose the first available charger with the lowest power. Since the heuristic had the worst solutions, we can confirm that installing faster chargers helps maximize the number of served vehicles even with the same limited power.

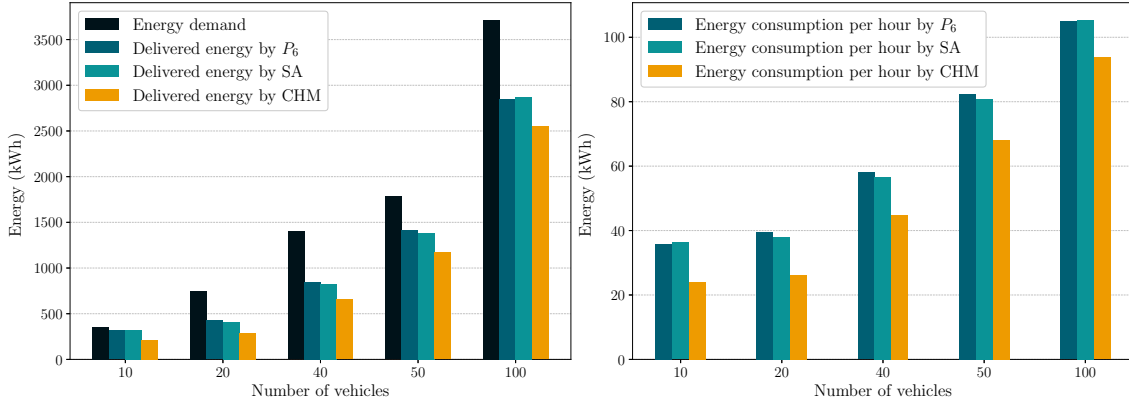


Figure 4.8: The average total energy demand of type 1 instances compared to the average total energy delivered to vehicles in schedules with P_6 , SA, and CHM.

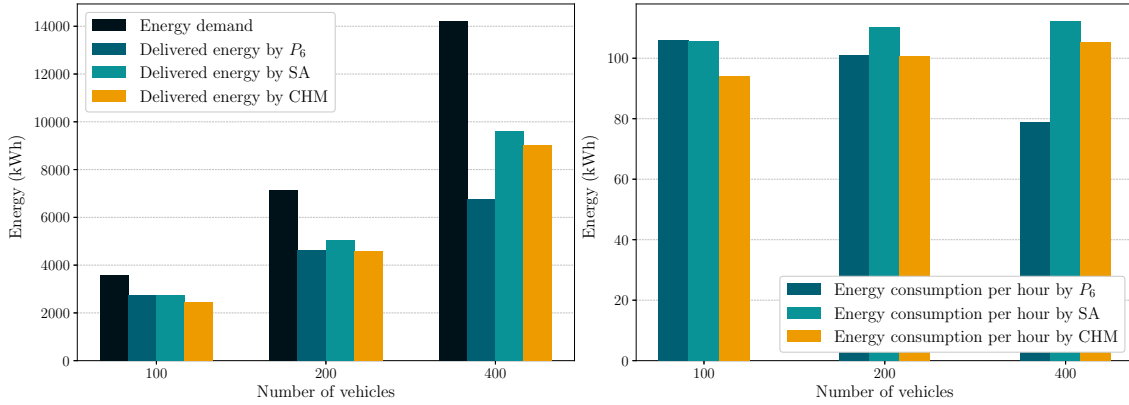


Figure 4.9: The average total energy demand of type 2 instances compared to the average total energy delivered to vehicles in schedules with P_6 , SA, and CHM.

The assignment in type 2 instances is very similar. P_6 assigned 9.81%, 26.99%, and 35.31% to type 1, 2, and 3 chargers, respectively. The SA scheduled 37.29% of charging demands on 11 kW chargers, 27.87% on 22 kW chargers, and 16.36% on 43 kW chargers. For the heuristic, we have 47.65% of vehicles assigned to 11 kW while 25.18% set to 22 kW and only 1.12% to 43 kW chargers.

To close this section, we select one instance from each group and plot the allocated power at each time slot to compare the three methods (P_6 , SA, and CHM). We also plot the value of the grid capacity w_G . Obviously, the allocated power never exceeds w_G . There is a difference between w_G and the maximum peak power, as may be observed. For example, w_G is set to 50 kW in instance 11, while the peak power is 44 kW, which is expected since we consider constant power and, therefore, the peak power depends on the maximum power delivered simultaneously. Also, we observe a significant variation in P_6 load curves, implying more frequent preemptions. Also, in instance 51, it is mainly caused by the fact that P_6 scheduled fewer vehicles.

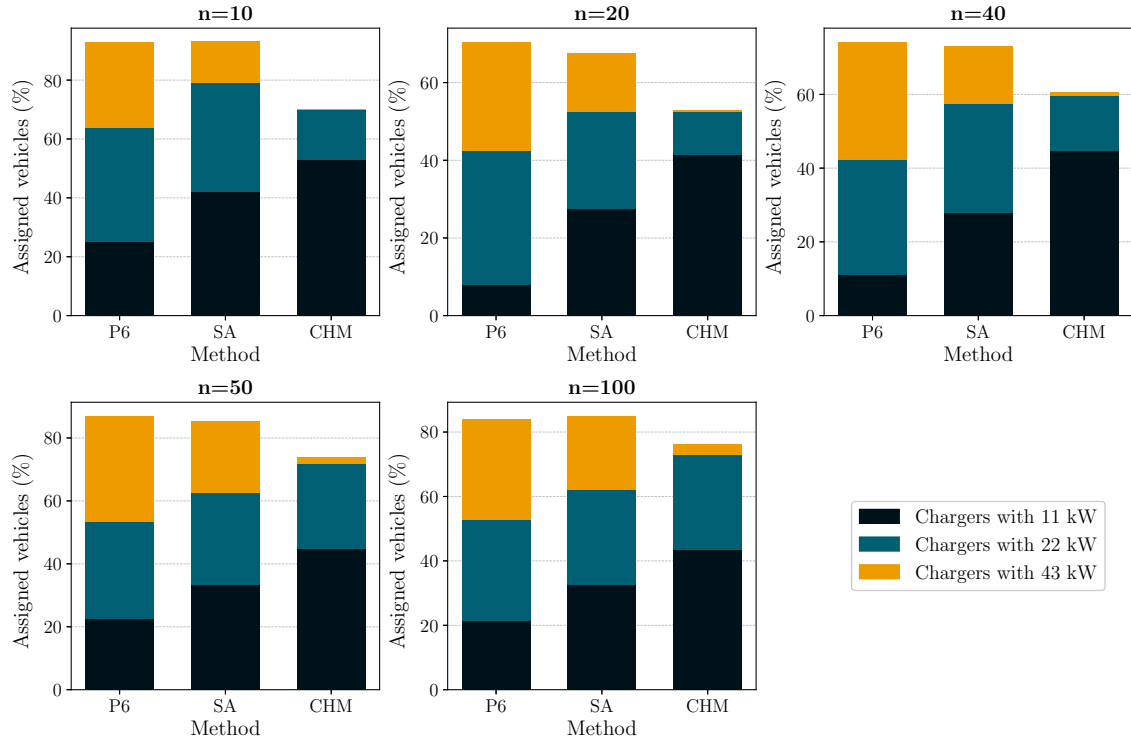


Figure 4.10: The average number of vehicles assigned to each type of charger in schedules for type 1 instances with P_6 , SA, and CHM.

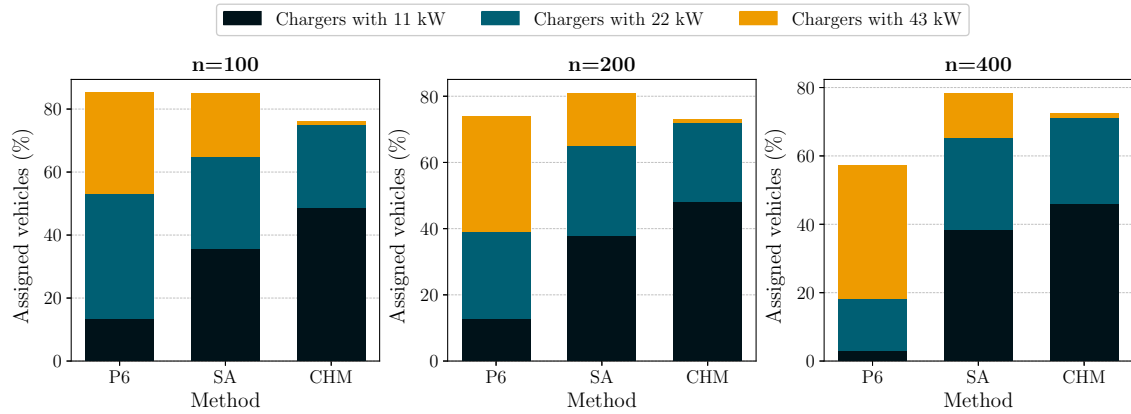


Figure 4.11: The average number of vehicles assigned to each type of charger in schedules for type 2 instances with P_6 , SA, and CHM.

4.6 Conclusion

In this chapter, we presented the EVCS problem to maximize the number of satisfied charging demands and proved it is \mathcal{NP} -hard. First, we tackled the problem with identical chargers, in

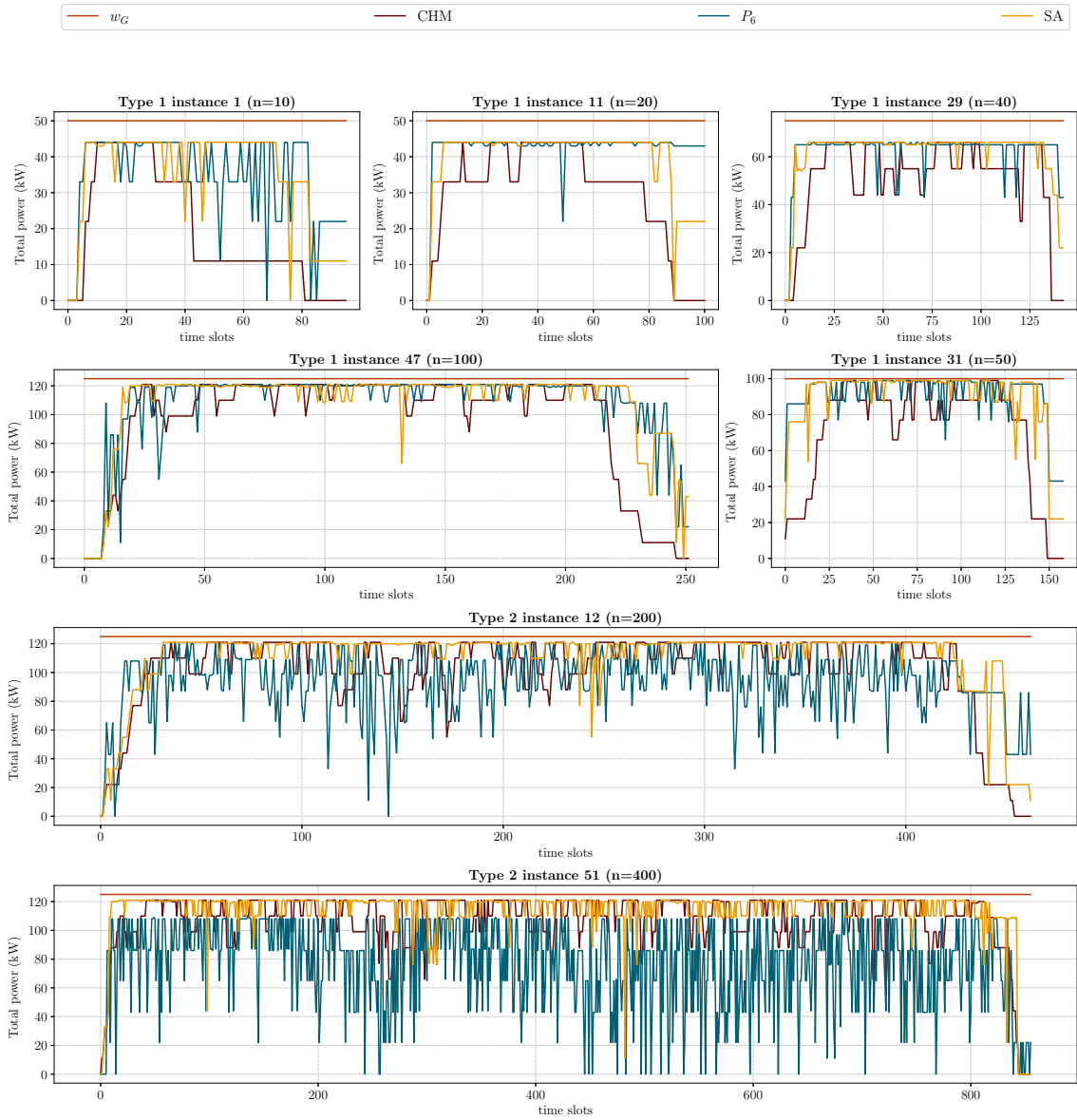


Figure 4.12: Example of load profiles for five instances obtained by P_6 , SA, and CHM.

which we provided different LP models and heuristics. We provided a comprehensive comparison between these methods. The results show that the relaxed event-based model outperforms the others and can efficiently solve large instances with 400 vehicles in less than 45 seconds. More challenging instances were generated to demonstrate its limitations. We found that its performance depends not only on the number of vehicles and chargers but also on the grid capacity. Furthermore, comparing heuristics demonstrates that it is more efficient to select the vehicles to schedule based on the power allocation solution rather than the assignment one.

Later, the problem with different chargers is studied, which is more challenging than the

previous one. We provide two LP formulations. Then, we developed a SA combined with a two-stage local search. From computational results, it is observed that the proposed SA finds good solutions in a small amount of time. Besides, the SA outperforms the LP model mainly in instances with more than 200 vehicles. Additional results confirm that installing different types of chargers is more beneficial in maximizing the number of charging demands that can be satisfied.

Preemptive EVCS to maximize the delivered energy

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5.1 Introduction

In previous chapters, we studied the problem with constant charging power rates. However, in the literature review in Chapter 2, we pointed out that two charging power rates can be considered: variable rates and constant rates. This chapter examines and compares two variants of the EVCS problem. In the first variant, called the constant power model, chargers can deliver either their maximum output powers or zero. The second variant is the variable power model, where the charging rate of each charger varies over time from zero to its maximum output power.

The charging station operating model is basically similar, where the charging station has limited total power and a limited number of chargers. Electric vehicles submit their charging demands, and the scheduler allocates a suitable charger for each vehicle. However, in this chapter, the scheduler can also decide the plugging time of each vehicle, which can be later than the requested arrival time. To serve more vehicles, the objective of the scheduler is to maximize the final state of charges.

The remainder of this chapter is organized as follows. Section 5.2 describes in detail the investigated problem. Section 5.3 formulates it as a mixed-integer linear programming (MILP). We examine the computational complexity of these problems in Section 5.4. Section 5.5 proposes optimization methods to tackle charging scheduling problem. Section 5.6 evaluates the performance of proposed methods and finally Section 5.7 concludes the chapter.

5.2 Problem Description

We consider a charging station with a set $\mathcal{M} = \{1, \dots, m\}$ of chargers. A management system can remotely control chargers' activation or deactivation at any time. When a charger i , $i \in \mathcal{M}$ is activated, it delivers a constant power output w_i (kW). Otherwise, no power is delivered even if a vehicle is plugged into this charger. We also consider the variable power model, where the output power of each charger i can vary over time from 0 to w_i . The charging station has a maximum power supply of w_G (kW), which is generally insufficient to sustain simultaneous activation of all chargers. Specifically, the sum of the delivered output power of all chargers cannot exceed w_G (kW) at any time.

The scheduling time horizon of one day \mathcal{H} is divided into T time slots of length τ . Let $\mathcal{J} = \{1, \dots, n\}$ be the set of electric vehicles that need charging throughout the time horizon. Each electric vehicle j , $j \in \mathcal{J}$, submits a charging demand by providing the following information: the desired arrival time to the station r_j , the estimated initial state-of-charge at the arrival e_j^0 , the desired state-of-charge at the departure e_j^d , the battery capacity B_j (kWh), and the departure time d_j . Without loss of the generality, we suppose that r_j and d_j are the indices of the arrival and departure times of vehicle j , respectively: $r_j \in \mathcal{H}$ and $d_j \in \mathcal{H}$. The scheduler collects all charging demands and determines a day-ahead charging schedule by assigning electric vehicles to chargers. Due to the limited number of chargers, a vehicle may be required to arrive later than its intended arrival. Let st_j be the plugging time of vehicle j , where $st_j \geq r_j$. An electric vehicle will occupy a charger from the assigned plug-in time st_j until its departure time d_j and cannot be plugged out during this period. The preemption of charging operation is allowed. Ideally, all vehicles should be charged to their desired state-of-charge levels. However, this may not be possible, given the limited number of chargers and the limited power capacity of the charging station. Thus, the scheduling objective is to minimize the total difference between the desired and the final state-of-charge levels. An illustrative example is given in the following paragraph.

Example 5.2.1. We consider a charging station with three chargers. The first charger delivers an output power of 20 kW, while the second and the third chargers deliver a power of 10 kW. We set the maximum grid capacity to 30 kW. We consider the charging of five demands. Table 5.1 gives the arrival and departure times and the requested energy of these demands. The scheduling horizon is divided into time slots, and each time slot is set to one hour.

Table 5.1: Illustrative example instance.

Vehicle	Arrival time r_j	Departure time d_j	Initial SOC e_j^0	Desired SOC e_j^d	Battery capacity B_j
v_1	8:00	10:00	0.20	0.70	40 kWh
v_2	8:00	11:00	0.20	0.70	40 kWh
v_3	9:00	11:00	0.25	0.50	80 kWh
v_4	9:00	12:00	0.20	0.70	40 kWh
v_5	10:00	12:00	0.20	0.70	40 kWh

Figure 5.1(a) shows a feasible schedule of charging demands with a constant power charging, while Figure 5.1(b) depicts a feasible solution with variable power of chargers. In both solutions, the amount of energy requested by vehicles is satisfied. As we can see, each vehicle v_j is plugged into a charger to its departure time. The total power delivered at each hour is always less than or equal to the fixed limit of 30 kW.

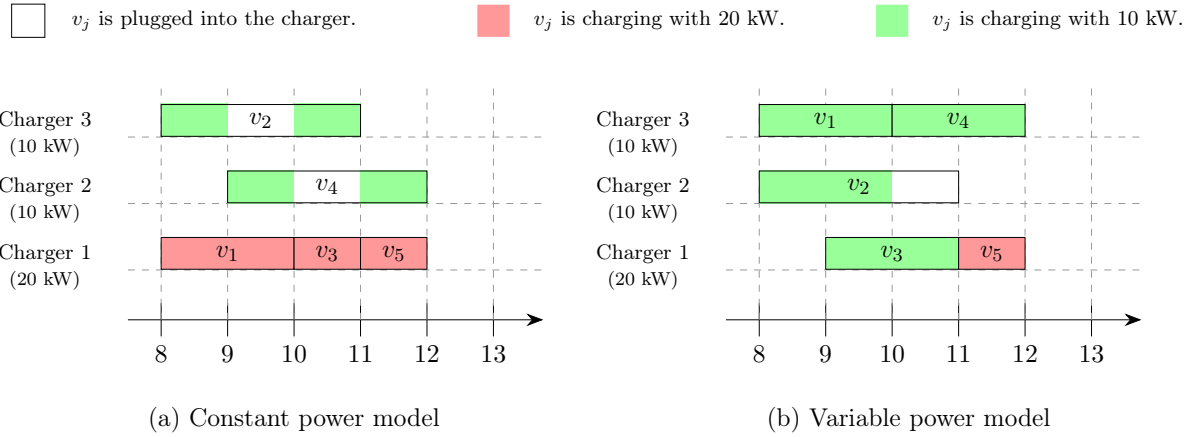


Figure 5.1: Feasible schedules of charging demands of Table 5.1. (a) depicts the schedule using chargers with constant power, and (b) depicts the schedule with variable power charging. Rectangles represent the plug-in times of the vehicles. We highlight charging intervals with 10kW in red, whereas charging intervals with 20 kW are colored in green.

5.3 MILP Formulations

In this section, we propose three mixed-integer linear programming (MILP) formulations for the EVCS problem described above. The first one is a time-indexed formulation considering the constant power variant. The variable power variant is presented using a time-indexed and an event-based formulation in the second and the third MILPs, respectively.

5.3.1 Time-indexed Constant Power Model

In the case of a constant power model, we use the following decision variables: binary variables x_{ijt} to specify if the electric vehicle j is charged by the charger i during the time slot t and

continuous variables e_j^f to denote the final state-of-charge of vehicle j at its departure time d_j .

$$\min \quad \sum_{j=1}^n (e_j^d - e_j^f)$$

$$\sum_{i=1}^m x_{ijt} \leq 1 \quad \forall j \in \mathcal{J}, t \in \mathcal{H} \quad (5.1)$$

$$\sum_{j=1}^n x_{ijt} \leq 1 \quad \forall i \in \mathcal{M}, t \in \mathcal{H} \quad (5.2)$$

$$x_{i't't'} \leq 1 - x_{ijt} \quad \forall i, i' \in \mathcal{M}, i' \neq i, j \in \mathcal{J}, t, t' \in \mathcal{H}, t' < t \quad (5.3)$$

$$e_j^0 \leq e_j^f \leq e_j^d \quad \forall j \in \mathcal{J} \quad (5.4)$$

$$e_j^f = e_j^0 + \frac{\tau \sum_{t=r_j}^{d_j} w_i \times x_{ijt}}{B_j} \quad \forall j \in \mathcal{J} \quad (5.5)$$

$$x_{ijt} = 0 \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, t \in \mathcal{H}, t < r_j, t \geq d_j \quad (5.6)$$

$$x_{ijt} + x_{ij't'} \leq 1 \quad \forall i \in \mathcal{M}, j, j' \in \mathcal{J}, j' \neq j, t \in \mathcal{H}, t' \in [t, d_j] \quad (5.7)$$

$$\sum_{i=1}^m \sum_{j=1}^n w_i \times x_{ijt} \leq w_G \quad \forall t \in \mathcal{H} \quad (5.8)$$

The objective function (5.13) minimize the difference between the state-of-charge at the departure e_j^f and the desired state-of-charge e_j^d . Constraints (5.1) ensure that, at each time slot t , each vehicle j is assigned to at most one charger. Constraints (5.2) ensure that, at each time slot t , each charger i charges one vehicle. Constraints (5.3) ensure that each vehicle j is charged by one charger i , i.e., when a vehicle is assigned to a charger, it cannot be moved to another. Constraints (5.4) and (5.5) calculate the final state-of-charge of each vehicle j . Constraints (5.6) ensure that vehicle j can only be charged between its desired arrival time r_j and its departure time d_j . Constraints (5.7) ensure that the charger i is allocated to the vehicle j from the time it begins to charge (it represents its plugging time st_j) to its departure time d_j . Constraints (5.8) ensure that, at each time slot, the total output power does not exceed the grid capacity.

5.3.2 Time-indexed Variable Power Model

The following MILP represents the EVCS problem where each charger can deliver a time-varying output power. In addition to the previously defined decision variables, we introduce continuous variables p_{ijt} to denote the power delivered by the charger i to the vehicle j at time slot t . Then, Constraints (5.5) and (5.8) are replaced by the following constraints:

$$e_j^f = e_j^0 + \frac{\tau \sum_{i=1}^m \sum_{t=r_j}^{d_j} p_{ijt}}{B_j} \quad \forall j \in \mathcal{J} \quad (5.9)$$

$$\sum_{i=1}^m \sum_{j=1}^n p_{ijt} \leq w_G \quad \forall t \in \mathcal{H} \quad (5.10)$$

We also add the following constraints to guarantee that a charger does not deliver more than its maximum output power.

$$w_i \times x_{ijt} \geq p_{ijt} \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, t \in \mathcal{H} \quad (5.11)$$

5.3.3 Event-based Variable Power Model

In contrast to the previous formulation using variables indexed by time, we propose a model in which variables are indexed by events. Event-based formulations are often used for problems with large-size scheduling horizons since it involves fewer variables. In the studied problem, events correspond to vehicle plug-in and departure times. Therefore, we have at most $2n$ events. Let $\mathcal{V} = \{1, \dots, V\}$ be the index set of these events, $V \leq 2n$. We define continuous variables t_v , $v \in \mathcal{V}$, which represent the time of the event v . Also, we introduce binary variables x_{ijv} to define whether or not t_v is the plug-in time of vehicle j into charger i . In addition, binary variables y_{ijv} equal one if t_v is the event when an electric vehicle j is unplugged from charger i . We also define decision variables p_{ij}^v representing the energy received by vehicle j from charger i during interval $[t_v, t_{v+1})$. Finally, continuous variables e_j^v represent the state-of-charge of vehicle j at the end of interval $[t_v, t_{v+1})$. Let $D_{max} = \max_{j \in \mathcal{J}} d_j$ and M a large enough constant.

Using these notations, the variable power model can be formulated as follows.

$$\min \quad \sum_{j=1}^n (e_j^d - e_j^f) \quad (5.12)$$

$$t_v \leq t_{v+1} \quad \forall v \in \mathcal{V} \quad (5.13)$$

$$\sum_{v=1}^V \sum_{i=1}^m x_{ijv} \leq 1 \quad \forall j \in \mathcal{J} \quad (5.14)$$

$$x_{ij}^v \times r_j \leq t_v \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, v \in \mathcal{V} \quad (5.15)$$

$$t_v \leq x_{ijv} d_j + (1 - x_{ijv}) D_{max} \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, v \in \mathcal{V} \quad (5.16)$$

$$y_{ijv} \times d_j \leq t_v \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, v \in \mathcal{V} \quad (5.17)$$

$$t_v \leq y_{ijv} d_j + (1 - y_{ijv}) D_{max} \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, v \in \mathcal{V} \quad (5.18)$$

$$\sum_{v=1}^V x_{ijv} - \sum_{v=1}^V y_{ijv} = 0 \quad \forall i \in \mathcal{M}, j \in \mathcal{J} \quad (5.19)$$

$$\sum_{u=v}^d x_{iku} \leq M(1 - (x_{ijv} + y_{ijd} - 1)) \quad \forall i \in \mathcal{M}, j, k \in \mathcal{J}, v, d \in \mathcal{V}, k, k \neq j, d > v \quad (5.20)$$

$$p_{ijv} \leq w_i(t_{v+1} - t_v) \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, v \in \mathcal{V} \quad (5.21)$$

$$p_{ijv} \leq (e_j^d - e_j^0)B_j \sum_{u=1}^V x_{iju} \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, v \in \mathcal{V} \quad (5.22)$$

$$p_{ijv} \leq M \left(\sum_{u=1}^v x_{iju} - \sum_{u=1}^v y_{iju} \right) \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, v \in \mathcal{V} \quad (5.23)$$

$$\sum_{i=1}^v \sum_{j=1}^n p_{ijv} \leq w_G(t_{v+1} - t_v) \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, v \in \mathcal{V} \quad (5.24)$$

$$e_j^v \leq \frac{\sum_{j=1}^n p_{ijv}}{B_j} \quad \forall j \in \mathcal{J}, v \in \mathcal{V} \quad (5.25)$$

$$e_j^0 \leq e_j^f \leq e_j^d \quad \forall j \in \mathcal{J} \quad (5.26)$$

$$e_j^f = e_j^0 + \sum_{v=1}^V e_j^v \quad \forall j \in \mathcal{J} \quad (5.27)$$

Constraints (5.14) ensure that each vehicle j is plugged into at most one charger and at most one event v . Constraints (5.15) ensure that each vehicle j will not be plugged before the arrival time r_j . Constraints (5.16) ensure that each vehicle j will not be plugged after the departure time d_j . Constraints (5.17) (5.18) and ensure that each vehicle j will be unplugged at the departure time d_j . Constraints (5.19) ensure that if the vehicle j is plugged to charger i , it will be unplugged from the same charger. Constraints (5.20) ensure that the charger i will be reserved to vehicle j during plug-in time $[t_v, t_d]$ (no other vehicle will be plugged). Constraints (5.21) ensure that the energy delivered by charger i to vehicle j will not exceed its output energy. Constraints (5.22) ensure that the energy delivered by charger i to vehicle j will not exceed its demand. Constraints (5.8) ensure that the vehicle j will not be charging before plug-in time and after plug-out time. Constraints (5.24) ensure that the power delivered by all chargers will not exceed the total grid capacity. Constraints (5.25), (5.26) and (5.27) calculate the final state-of-charge of each vehicle j .

Remark 5.3.1. We did not propose an event based formulation for constant power model since we consider different charging power rates. As stated in Remark 3.4.3, power allocation problem with different constant power rates is an open problem. Therefore, in an event interval $[t_v, t_{v+1})$, we do not have an algorithm to schedule the charging without exceeding the grid capacity.

5.4 Complexity

In this section, we prove that the charging problem is strongly \mathcal{NP} -hard for both constant and variable power models.

Theorem 5.4.0.1. *The EVCS problem to maximize the delivered energy is strongly \mathcal{NP} -hard considering constant and variable charging powers.*

Proof. To prove the NP-hardness of the charging problem, we use the reduction from 3-Partition, which is known to be \mathcal{NP} -hard [Garey & Johnson 1979]. The 3-Partition problem is stated as follows. Given a positive integer B and a set $N = \{a_1, \dots, a_{3n}\}$ of $3n$ positive integers with $B/4 < a_j < B/2$ and $\sum_{j \in N} a_j = nB$, is there a partition of N into n mutually disjoint subsets N_1, \dots, N_n , such that each subset N_i contains 3 elements of N and $\sum_{j \in N_i} a_j = B$?

Given an arbitrary instance of the 3-Partition problem, we build an instance (I) of the considered EVCS problem, consisting of a set of $3n$ chargers where each charger can deliver a constant charging power $w = 1$, and a capacity of the grid value of n . There are $3n$ charging requests D_j , $j = 1, \dots, 3n$. Arrival time r_j and departure time d_j of all demands are equal to 0 and B , respectively. The energy requirement e_j of demand j is equal to a_j , $j = 1, \dots, 3n$.

In what follows, we show that there exists a feasible schedule of charging demands with the objective function equal to zero if and only if the 3-Partition admits a solution.

First, assume that the 3-Partition problem has a solution and let N_1, \dots, N_n be the required subsets of N . Each subset N_i is then composed of 3 elements and $\sum_{j \in N_i} a_j = B$. Let a_{i1}, a_{i2} and a_{i3} be the elements of a set N_i . Since all chargers deliver a constant power equal to 1, then the maximum charging time of demand j is equal to a_j , $j = 1, \dots, 3n$. Thus the desired schedule of charging requests is constructed as follows: for each set N_i we charge a_{i1} on the charger $3(i-1) + 1$ in the time interval $[0, a_{i1}]$, then we charge a_{i2} on the charger $3(i-1) + 2$ in the time interval $[a_{i1}, a_{i1} + a_{i2}]$, and finally we charge a_{i3} in the interval $[a_{i1} + a_{i2}, a_{i1} + a_{i2} + a_{i3}]$. We note that the charging schedule respects the capacity of the grid, and each request is satisfied with the requested amount of energy. Conversely, assume now that there exists a feasible schedule S of all charging demands in which all vehicles are charged to their requested amount of energy. Since all chargers deliver a constant power equals to 1, then the charging time of each request j is equal to a_j . Furthermore, as all demands have the same arrival and departure times, then all chargers are occupied between instants 0 and B . Also, we know that the total power that can be delivered by all chargers is limited to n , then at each time t , $t \in [0, B]$, at most n chargers can be activated in the same time. Let T be the total charging time spent to charge vehicles when exactly n chargers are activated at the same time in the interval $[0, B]$, then $T \leq nB$. Since all demands are satisfied and $\sum_{j \in N} a_j = nB$, then we deduce that at each time t , $t \in [0, B]$, exactly n chargers are activated in the same time. Let i_1, \dots, i_n be the first n demands that start charging at time $t = 0$. Since at each time $t \in [0, B]$ exactly n chargers are activated, then at each time t on which one of the demands i_1, \dots, i_n ends another demands starts charging. Let i'_1, \dots, i'_n be the demands that start charging when demands i_1, \dots, i_n ends, respectively. Again, at each time t on which one of the demands i'_1, \dots, i'_n ends another demands starts charging. Let i''_1, \dots, i''_n be the demands that start charging when demands i'_1, \dots, i'_n ends, respectively. Since all demands are charged to their requested amount of energy, then $a_{i_u} + a_{i'_u} + a_{i''_u} \leq B$, $u = 1, \dots, n$. We have $\sum_{u=1}^n (a_{i_u} + a_{i'_u} + a_{i''_u}) = nB$ then $a_{i_u} + a_{i'_u} + a_{i''_u} = B$, $u = 1, \dots, n$. Thus, the sets $N_u = \{i_u, i'_u, i''_u\}$, $u = 1, \dots, n$ are a solution to the 3-Partition problem. Therefore, the scheduling problem is strongly \mathcal{NP} -hard.

In the case of variable charging power, it is sufficient to consider that charger can deliver variable power with maximum power equal to 1, and the proof remains valid. \square

5.5 Solution Methods

5.5.1 Solution Representation and Evaluation

In the constant charging model, a solution to the charging scheduling problem consists of determining the charging demands assigned to chargers and then choosing the appropriate time slots for charging. In the variable power model, an additional decision on the charging power at each time slot must be made. Therefore, we propose a two-step decomposition approach to solve the charging scheduling problem. The first step deals with assigning charging demands to chargers and the second step determines the charging schedule of vehicles. A solution is evaluated by solving the power allocation to determine the objective function.

5.5.1.1 Assignment solution

We represent the assignment of charging demands to chargers as a vector $\Pi = (\pi_1, \dots, \pi_m)$ where π_i is the sequence of charging demands assigned to a charger i . Once we have a solution to the assignment problem, we solve the power allocation by determining the amount of power delivered by each charger to each demand at each time slot. The unassigned demands will be placed on the list of rejected demands.

To get plug-in times st_j from the assignment vector Π , we simply schedule all demands sequentially without idles times while respecting their arrival times. For each demand j in the sequence π_i , we select the earliest possible plug-in time $st_j = \max(r_j, d_{j'})$ where j' is the demand scheduled before j in π_i . In this case, demand j will be plugged from st_j to its departure time d_j . If $st_j > d_j$, the charging demand j will be rejected and added to the set of rejected demands \mathcal{L}_R . The details of generating assignment solutions in the proposed heuristics and metaheuristics are given further in Sections Sections 5.5.2 and 5.5.3.

5.5.1.2 Power allocation solution

After determining the assignment of demands to the chargers, the next step is to decide the amount of electric power delivered by each charger at each time slot. We formulate the problem for both constant and variable power variants as an MILP model.

Complexity of the power allocation problem. Before presenting the power allocation MILP models, we should present the complexity of the power allocation problem. As mentioned in Chapter 3, the power allocation considering constant power model with different types of chargers is an open problem (see Remark 3.4.3). Remark that when the assignment is decided, the power allocation is exactly the same one as in the previous chapter. However, for the variable power model, the problem is polynomial using the reduction from maximum flow problem [Ahuja *et al.* 1988]. The equivalent network $N = (V, E)$, illustrated in Figure 5.2, is constructed as follows. The set of vertices V consists of: (i) a source s , (ii) a vertex for each time slot t , $t \in \mathcal{H}$ (iii) a vertex v_j fore each charging demand $j \in \mathcal{J}$, and (iv) a sink p . The set of arcs E with restricted capacities consists of: (i) an arc from the source s to each time slot vertex t with capacity w_G , (ii) an arc from each vertex t to each charging demand vertex v_j if $t \in [r_j, d_j]$ with a capacity equals to w_j , where w_j is the maximum output power of the charger where j is

assigned, and (iii) an arc from each vehicle vertex v_j to the sink p with capacity $E_j = (e_j^d - e_j^0)B_j$. A feasible charging schedule is determined by solving the flow network.

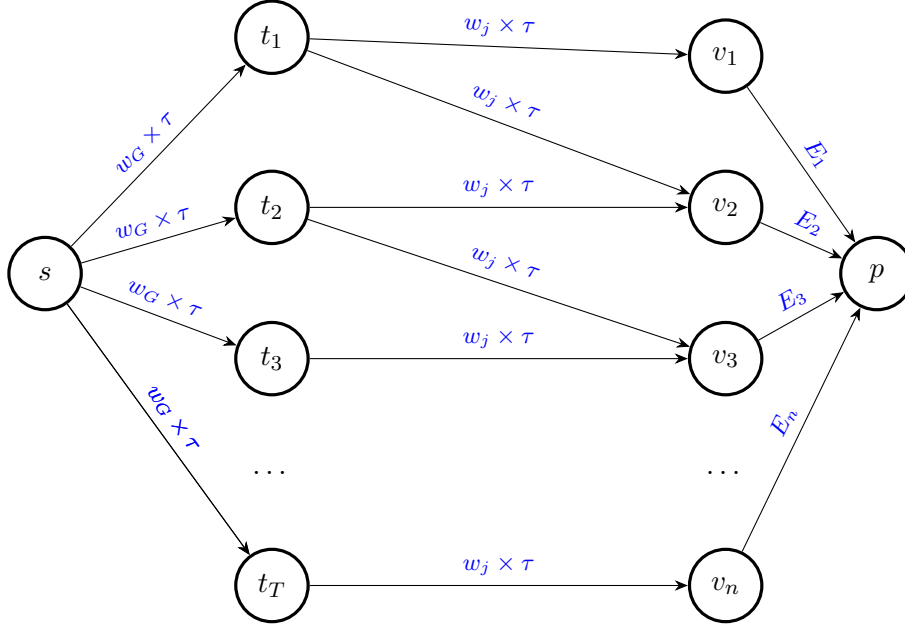


Figure 5.2: Flow network for solving the power allocation solution with variable power model.

LP for constant power model. We define two parameters: Boolean parameters a_{ij} that specifies if the demand j is assigned to charger i , and parameters st_j to specify the plug-in time of the demand j . Both parameters are obtained in the assignment phase of demands to chargers. To get the plug-in time st_j , we simply schedule all electric vehicles assigned to the same charger sequentially without idle times while respecting their arrival times. We define the binary decision variable y_{jt} that specifies whether or not the electric vehicle j is charging at time slot t . We define a parameter $p_{jt} = \sum_{i=1}^n a_{ij} \times w_i$ if $st_j \leq t < d_j$ and $p_{jt} = 0$ otherwise. Variables y_{jt} are set to zero for all t where $t < st_j$ and $t > d_j$.

$$\min \sum_{j=1}^n (e_j^d - e_j^f)$$

$$e_j^0 \leq e_j^f \leq e_j^d \quad \forall j \in \mathcal{J} \quad (5.28)$$

$$e_j^f = e_j^0 + \frac{\sum_{t=st_j}^{d_j} y_{jt} \times p_{jt} \times \tau}{B_j} \quad \forall j \in \mathcal{J} \quad (5.29)$$

$$\sum_{j=1}^n y_{jt} \times p_{jt} \leq w_G \quad \forall t \in \mathcal{H} \quad (5.30)$$

Constraints (5.28) and (5.29) calculate the final state-of-charge of each electric vehicle j . Constraints (5.30) ensure that at each time slot t , the total output power does not exceed the charging station limit.

LP for variable power model. Even though the problem is polynomial by solving its equivalent maximum flow problem, it was more convenient and faster to solve an LP model using a commercial solver. For the variable power model, event-based decision variables are used instead of time-indexed variables. Let t_l , $l \in \mathcal{L} = \{1, \dots, L\}$, be distinct plug-in times st_j and departure times d_j ordered in non decreasing order, where $L \leq 2n$. We define parameter $p_{jl} = \sum_{i=1}^n a_{ij} \times w_i$ for each l where $st_j \leq t_l < d_j$, and $p_{jl} = 0$, otherwise. We define the continuous decision variables e_j^l which represents the energy delivered to the electric vehicle j in the time interval $[t_l, t_{l+1})$.

$$\begin{aligned} \min \quad & \sum_{j=1}^n (e_j^d - e_j^f) \\ & e_j^0 \leq e_j^f \leq e_j^d \quad \forall j \in \mathcal{J} \end{aligned} \quad (5.31)$$

$$e_j^f = e_j^0 + \frac{\sum_{l=1}^L e_j^l}{B_j} \quad \forall j \in \mathcal{J} \quad (5.32)$$

$$0 \leq e_j^l \leq p_{jl}(t_{l+1} - t_l) \quad \forall j \in \mathcal{J}, l \in \mathcal{L} \quad (5.33)$$

$$\sum_{j=1}^n e_j^l \leq w_G(t_{l+1} - t_l) \quad \forall l \in \mathcal{L} \quad (5.34)$$

Constraints (5.31) and (5.32) calculate the final state-of-charge of each electric vehicle j . Constraints (5.33) ensure that, at each time t , the delivered power by each charger i does not exceed its maximum rated power w_i . Constraints (5.34) ensure that the total output power does not exceed the charging station limit.

5.5.2 Heuristic Methods

In the following, we define two greedy rules for the assignment of charging demands to chargers. The power charging schedule of assigned demands is determined by solving the MILP presented in Section 5.5.1.2.

5.5.2.1 First Come First Served (FCFS) Heuristic.

The First-come-first-served (FCFS) rule is a popular approach to scheduling electric vehicle charging [Jin *et al.* 2013, Yao *et al.* 2017, Yang 2019]. It assigns the vehicle with the earliest arrival time to the first available chargers. The detail of the heuristic is given in Algorithm 13.

The time complexity of the Heuristic FCFS is $\mathcal{O}(nm)$.

5.5.2.2 Interval Graph Coloring based Heuristic (IGCH).

In this heuristic, we use a graph coloring approach to select the set of demands to be assigned to chargers. Let consider the interval graph $G = (V, E)$ where each vertex $v \in V$ represents a charging demand. There is an edge $e \in E$ between two vertices if and only if their associated intervals overlap, i.e., $(j, j') \in E$, $j' \neq j$, if $[r_j, d_j] \cap [r_{j'}, d_{j'}] \neq \emptyset$. Assigning a set of demands to a given charger is equivalent to the k-coloring problem of the graph G . The k-coloring problem

Algorithm 13: Heuristic FCFS

```

input : The set of charging demands  $\mathcal{J}$ , the set of chargers  $\mathcal{M}$ 
output: Assignment of vehicles to chargers  $\Pi$ , list of rejected demands  $\mathcal{L}_R$ 
1 Sort the set of charging demands  $\mathcal{J}$  in non-decreasing order of their arrival times  $r_j$ ,
  breaking ties according to the non-decreasing order of their departure times  $d_j$ . ;
2 for  $j \in \mathcal{J}$  do
3   if there is a charger  $i$  available at time  $r_j$  then
4     | Add  $j$  to  $\pi_i$ ;
5   else
6     | Let  $t$  be the earliest time when a charger  $k$  is available, where  $t > r_j$ 
7     | if  $t < d_j$  then
8     | | Add  $j$  to  $\pi_k$ ;
9     | else
10    | | Add  $j$  to the list of rejected demands;
11    | end
12  end
13 end

```

assigns a color $c \in \{1, \dots, k\}$ to each vertex of G so that no adjacent vertices have the same color. The set of vertices colored with the same color corresponds to the set of demands assigned to the same charger, which is called a color class. Since an interval graph is a chordal graph, a greedy coloring algorithm provides an optimal coloring on a chordal graph following the perfect elimination ordering [Gilmore & Hoffman 1964]. A perfect elimination ordering in a graph is an ordering of the vertices so that each vertex v and the neighbors of v that occur after v in the order form a clique. We use the lexicographic breadth-first (LEXBFS) search proposed by [Rose *et al.* 1976] to find the perfect elimination ordering in linear time. Generally, an interval graph admits more than one optimal coloring solution since we can generate different perfect elimination ordering. However, enumerating all perfect sequences might be exponential in the size of the graph in general [Chandran *et al.* 2003].

In our case, we add randomness to the algorithm to generate different perfect elimination orders by introducing a random weight w to each vertex. Therefore, when two vertices have the same label, we choose the vertex with maximum weight w . Algorithm 14 shows the pseudo-code of randomized LEXBFS ordering.

The overall procedure of the IGCH algorithm is depicted in Algorithm 15. After solving the graph coloring problem (lines 1-3), we obtain the chromatic number k and the set of color classes C_1, \dots, C_k . First, the set of chargers is ordered in decreasing order of their output powers (line 4). Then, the set of color classes is ordered in decreasing order of their cardinalities (line 5), i.e., it begins with the class color with the highest number of demands and ends with the class with the smallest number of demands. The demands with the same color class are ordered in increasing order of their arrival times (line 6). Next, we assign the demands in the first m color classes to the first m chargers (line 7). Consequently, demands in the color class with the highest cardinality are assigned to the charger with the highest charging output power. When

Algorithm 14: Randomized LEXBFS

```

input : Interval graph  $G = (V, E)$ 
output: A lexicographic ordering  $\sigma = (v_1, \dots, v_n)$ 
1 for  $v \in V$  do
2   | label( $v$ )  $\leftarrow$   $\square$ ;
3   |  $w(v) \leftarrow$  random();
4 end
5 for  $i = |V|$  down to 1 do
6   | Choose a vertex  $v \in V$  with lexicographical maximal label, breaking ties according
   | to the decreasing order of  $w(v)$ ;
7   |  $\sigma(i) \leftarrow v$ ;
8   | for  $u \in Neighborhood(v)$  do
9     | label( $u$ )  $\leftarrow$  label( $u$ ).concatenate( $i$ );
10  | end
11 end
12 return  $\sigma$ 

```

Algorithm 15: Heuristic IGCH

```

input : Scenario of  $n$  electric vehicles and  $m$  chargers
output: Assignment of the electric vehicles to the chargers, list of rejected demands
1 Construct the interval graph  $G$  of the scenario;
2 Get a lexicographic ordering  $\sigma$  of  $G$  using Algorithm 14;
3 Color the vertices with the smallest color that is not already used by one of its
  neighbors in the reverse of the order of  $\sigma$ ;
4 Sort chargers in decreasing order of their output power;
5 Sort the color classes in decreasing order of their cardinality;
6 Sort electric vehicles in the same class color in increasing order of their arrival times;
7 Assign the electric vehicles of the  $k$  first classes to the  $k$  first chargers;
8 if  $k > m$  then
9   | for each vehicle  $j$  in the non assigned classes do
10  |   | Choose the charger  $i$  that has the largest sub-interval  $[t_1, t_2) \subseteq [r_j, d_j)$  where the
   |   | charger is free (no vehicle is plugged in) ;
11  |   | Assign the vehicle  $j$  to the charger  $i$  at the position of  $[t_1, t_2)$ ;
12  |   | if no sub-interval available then
13  |   |   | Add  $j$  to the list of rejected demands;
14  |   | end
15  | end
16 end

```

the chromatic number k is greater than m , each remaining non assigned demand j in classes C_{m+1}, \dots, C_k will be assigned to a charger i as follows. Let $[t_1, t_2)$ be the largest time period

between r_j and d_j where a charger i is available, and let j' be the demand assigned to this charger i where $d_j = t_1$, then demand j is assigned to the charger i after j' (lines 10,11). If no charger is available between $[r_j, d_j)$, then demand j is rejected (lines 12-14).

Example 5.5.1. We give an illustrative example of the IGCH. Consider a charging station with three chargers. The first two chargers deliver an output power of 20 kW, while the third delivers an output power of 10 kW. The maximum grid capacity is set to 30 kW. We consider the charging of six demands. Data related to these demands are given in Table 5.2 and its interval graph is giving in Figure 5.3. The chromatic number of this graph is $k = 4$, which is greater than the number of chargers $m = 3$. Thus, we have four color classes C_l , $l = 1, \dots, 4$.

Table 5.2: IGCH illustrative example instance.

Vehicle	Arrival time r_j	Departure time d_j	Initial SOC e_j^0	Desired SOC e_j^d	Battery capacity B
v_1	8:00	10:00	0.2	0.7	40 kWh
v_2	8:00	11:00	0.2	0.7	40 kWh
v_3	8:00	12:00	0.2	0.7	40 kWh
v_4	9:00	12:00	0.2	0.7	40 kWh
v_5	11:00	14:00	0.2	0.7	40 kWh
v_6	11:00	13:00	0.2	0.7	40 kWh

Figures 5.4 and 5.5 show two different assignment solutions according to two different coloration. The first possible coloration (Figure 5.4) is $C_1 = \{v_1, v_5\}$, $C_2 = \{v_3\}$, $C_3 = \{v_4\}$, and $C_4 = \{v_2, v_6\}$. According to lines (4-7) in Algorithm 15, the heuristic starts by assigning demands in classes C_1 and C_4 to charger 1 and 2, respectively, then C_2 to charger 3. After that, the heuristic assigns the demand of $C_3 = \{v_4\}$ by applying the rule in lines (8-15). More precisely, following lines (8-15) there are two interval times $[10, 11)$ on charger 1 and $[12, 13)$. Since the two intervals have the same length, and chargers are already sorted in decreasing order of their charging outputs, the heuristic assigns v_4 to charger 1, and v_4 is then plugged after v_1 . Also, v_4 is plugged to its departure $d_4 = 13$ and thus v_6 is plugged at time $t = 13$ (following the rule described in Section 5.5.1.1). An interval graph has one optimal value for the chromatic number k but it does not generally have one optimal coloration. A second possible coloration of the graph in Figure 5.3 can be $C_1 = \{v_1, v_6\}$, $C_2 = \{v_2, v_5\}$, $C_3 = \{v_4\}$, and $C_4 = \{v_3\}$. The assignment of demands following this coloration by the IGCH is depicted in Figure 5.5.

5.5.3 Simulated Annealing with LP models

We use the simulated annealing (SA) algorithm described in Chapter 3 in Algorithm 6. Clearly, the objective function $f(S)$ and the neighbor structure are different. The new solution is generated (Algorithm 6 line 6) by a modifying the assignment of vehicles to chargers as described in Section 5.5.3.1. The MILP of power allocation, described in Section 5.5.1.2, is solved to get the objective function value of each generated solution. Also, we consider both the Lundy-Mees and the original cooling schemes in line 19. In the original SA paper [Kirkpatrick *et al.* 1983], authors propose the following decreasing geometric cooling scheme:

$$T_{l+1} = \alpha T_l$$

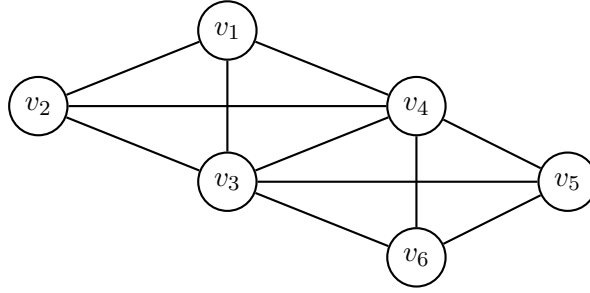


Figure 5.3: Interval Graph corresponding to the example of Table 5.2. The vertices v_j , $j = 1, \dots, 6$ represent the charging demands. There is an edge between two vertices if corresponding intervals overlap.

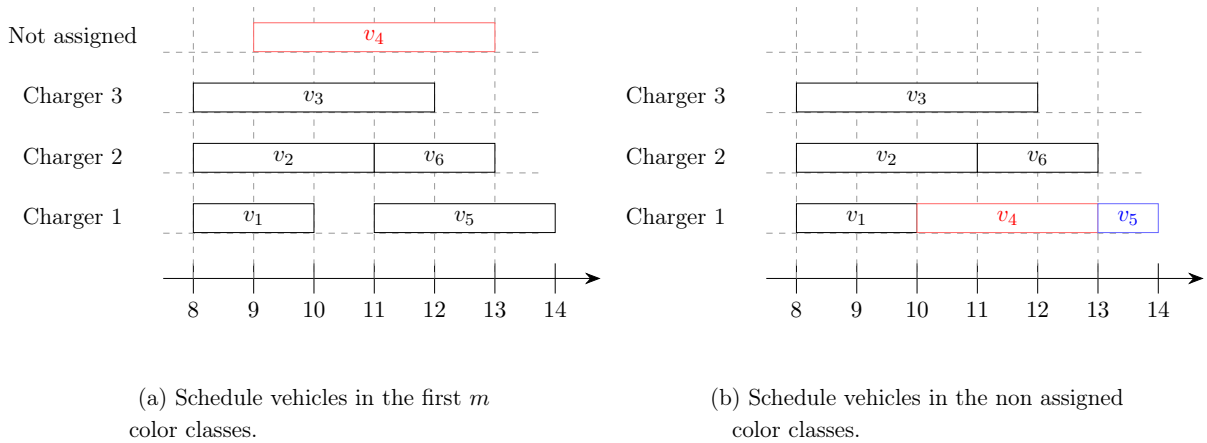


Figure 5.4: IGCH assignment according to coloration 1.

5.5.3.1 Neighborhood generation

A neighborhood solution of the current solution is obtained through a perturbation of the assignment of charging demands to chargers using one of the following moves:

- *Swap on the same charger:* a neighborhood solution is generated by randomly choosing a charger and exchanging the position of two charging demands scheduled on this charger. The positions of charging demands are randomly selected.
- *Swap between two different chargers:* a neighborhood solution is generated by swapping two charging demands between two chargers. The chargers and the charging demands are randomly selected.
- *Insert:* a neighborhood solution is generated by randomly choosing a charging demand on a charger and moving it to another position on another charger. The chargers and the position of a charging demand are randomly selected.
- *Shift left:* a neighborhood solution is generated by moving a charging demand from a position o_1 to o_2 on the same charger where $o_2 < o_1$. Positions o_1 and o_2 are randomly

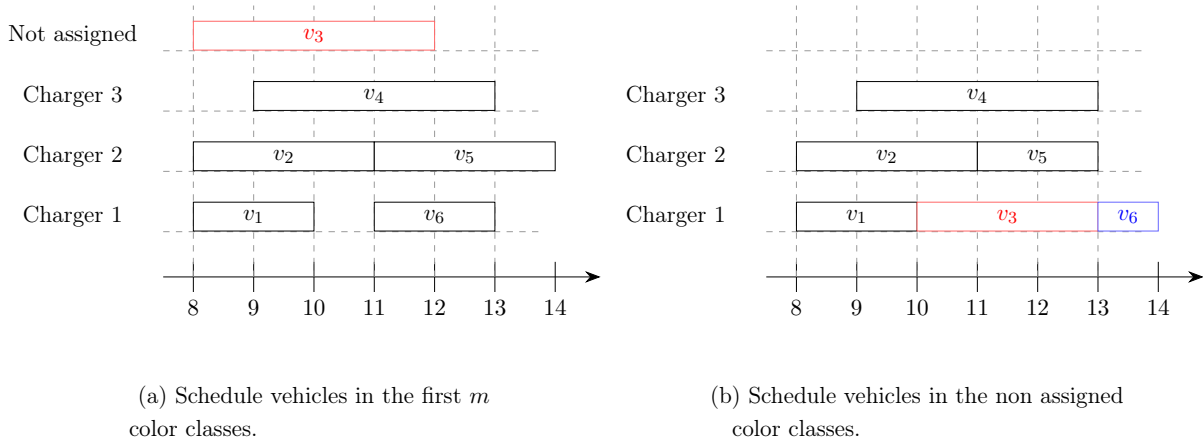


Figure 5.5: IGCH assignment according to coloration 2.

selected.

- *Shift right*: a neighborhood solution is generated by moving a charging demand from a position o_1 to a position o_2 on the same charger, where $o_2 > o_1$. Positions o_1 and o_2 are randomly selected.
- *Insert from the list of rejected demands*: a neighborhood solution is generated by choosing a charging demand from the rejected list of demands and inserting it on a charger at position o . The charger and the charging demand in the rejected list are randomly selected.
- *Swap from the list of rejected demands*: a neighborhood solution is generated by moving a charging demand from position o_1 to the rejected list and replacing it with a rejected demand at position o_2 . The charger and the position of charging demands are randomly selected.
- *Reject a demand*: a neighborhood solution is generated by moving a charging demand from position o to the rejected list. The charger and the position of charging demand are randomly selected.

At each neighbor generation, the move operator is selected randomly from the list of moves. Then, the power allocation MILP described in Section 5.5.1.2 is solved. Note that none of these moves can generate an unfeasible solution since we apply the procedure explained in Section 5.5.1.1 to get the plug-in times from the assignment vector. For example, if a demand j is sequenced after a demand j' where $d_j \leq d_{j'}$, then j will be rejected. Therefore, certain moves may reject some charging demands instead of changing their assignment, but it will not generate an infeasible assignment.

5.6 Computational Results

In this section, we present the results of the experimental study. We start by describing generated scenarios. Then we perform parameter tuning to choose the best values for SA parameters. Finally, we compare the results obtained by the proposed optimization methods.

All algorithms are implemented in C++ programming language and run on a personal computer with CPU Intel Core i5-4570S operating at 2.90 GHz and 8 GB RAM and running Linux OS (Ubuntu 20.04 LTS). We use CPLEX 12.7 as a solver for the MILP models in our heuristics and SA. The full MILP models, presented in Section 5.3, were solved by CPLEX running on an Intel Xeon Silver 4216 CPU operating at 2.10 GHz and 64 GB RAM and running Linux OS (Ubuntu 20.04 LTS). We could only get results for scenarios with 10 charging demands and 5 chargers. CPLEX could not solve scenarios with more than 10 demands due to out-of-memory errors, even when we limited the computing time to a few minutes.

5.6.1 Instances Generation

Since this study was done before the preceding chapters, the generation of instances differs from the one before. The following instances were less challenging for grid capacity minimization. Therefore, these instances were not used in the previous chapters.

Regarding the charging station, we consider four classes of instances with different number of chargers $m = \{5, 10, 20, 40\}$. In the first class, the output power delivered by the first three chargers is 3.7 kW, 22 kW, and 43 kW, respectively. The last two chargers deliver 11 kW. For the remaining classes, we set 30% of chargers to deliver 3.7 kW, 30% to deliver 11 kW, 30% to deliver 22 kW and 10% to deliver 43 kW. This charging rates are chosen from the the standard IEC 61851 [Std 2017] that defines the classification of the different charging modes. The charging station maximum capacity w_G is set to 70% of $\sum_{i=1}^n w_i$. The scheduling time horizon is one day, and the time slot τ is set to 6 minutes. To test our algorithms, we must model the stochastic electric vehicle charging demands. Electric vehicle arrivals are randomly occurring and independent events. Therefore, the arrival time is modeled using a non-homogeneous Poisson Process with an arrival rate $\lambda(h)$ that varies at each hour $h = \{1, \dots, 24\}$. The arrivals are high in the morning and low in the afternoon. The parking duration pr_j follows an exponential distribution with a mean parking duration that also varies over time. There is no correlation between the arrival time and the parking duration, so the two variables can be generated independently [?]. The departure time d_j of each electric vehicle can be directly obtained with the formula $d_j = r_j + pr_j$. The initial state-of-charge e_j^0 at the arrival r_j is considered uniformly distributed in the range of $[0.2, 0.7]$ of the capacity of the vehicle's battery. The desired state-of-charge e_j^d of each electric vehicle j is uniformly chosen from $[e_j^0, 1]$. The battery capacities are randomly chosen from the current real-world electric vehicle battery capacities [EVDB 2020]. We generate 15 instances for each class. In the first 10 instances of each class, the chromatic number k of the interval graph corresponding to the electric vehicle charging demand instance is greater than the number of chargers. In comparison, it is less than or equal to the number of chargers in the last five instances.

The first class of instances with $m = 5$ and $n = 10$ are used, as small-sized instances, to obtain the optimal objective values by solving the full MILPs and comparing it with the ones

obtained using the proposed algorithms. These instances are denoted from 1 to 15 with the prefix "s". As an example, s-3 denotes the third small instance with 10 vehicles. The remaining instances (classes 2, 3, and 4) are named using only their sequence number from 1 to 45.

All scenario data is public and accessible on the web https://github.com/imyzz/EVCSP_RA_instances.

5.6.2 Parameter Tuning of SA Algorithm

The tuning of optimization algorithms is essential to obtain good results. We use IRACE (Iterated Racing for Automatic Algorithm Configuration) package [López-Ibáñez *et al.* 2016] that implements a general iterated racing procedure to find optimal parameter settings. From classes 2 to 4, we select two instances as the training instances for IRACE. Table 5.3 presents the parameter settings for the SA algorithm tuned by IRACE. The first column reports the selected parameters for tuning. The second column reports, for each parameter, the range of values tested. Parameters μ , α , and "final temperature" are numerical, meaning each takes real values in its corresponding interval. At the same time, "max neighbors", "cooling technique", and "max accepted" are categorical parameters that take discrete values in their corresponding set. The third column reports the best parameter settings after tuning, which are further used in all experiments. Here, a dash "-" means that the best value is unavailable. Indeed, The value of α only needs to be set if the "Cooling technique" parameter is "Geometric", and since the best cooling scheme is "LandyMees", IRACE did not report the value of α . Thus, no best value for α was reported in Table 5.3.

Table 5.3: Parameter settings for SA algorithm tuned by IRACE.

Parameter	Value range	Best
μ	[0.01, 0.9]	0.12
Max neighbors	{20, 50, 100}	100
Cooling technique	{LandyMees, Geometric}	LandyMees
α	[0.01,0.9]	-
Final temperature	[0.001,0.1]	0.001
Acceptance ratio	{0.1, 0.2, 0.3, 0.4}	0.1

5.6.3 Evaluation of Algorithms on Small Instances

Here, we use the set of small-size instances to compare the proposed algorithms with the result of solving the full MILP models using CPLEX. We set the maximum computation time of CPLEX to 30 minutes. Results are reported in Table 5.4. The first column denotes the instance name. The second column " k " depicts the chromatic number of each instance's interval graph. For CPLEX, column "obj" reports the objective value found by solving the full MILP, column "LB" gives the lower bound calculated by CPLEX, and column "time" displays the total running time in seconds. For the FCFS heuristic, the objective function value and the total running time in seconds are displayed in the columns "obj" and "time", respectively. Due to the stochastic nature

Table 5.4: Comparison of results with $n = 10$ and $m = 5$.

scenario	k	CPLEX			FCFS		IGCH				SA			
		obj	LB	time(s)	obj	time(s)	mean	best	std	time(s)	mean	best	std	time (s)
Model with constant power														
s-1	9	0.75	0.00	1807.92	1.86	1.17	0.49	3.96E-03	1.56	6.11E-03	0.12	0.08	0.04	5.57
s-2	8	0.15	0.00	1808.16	0.25	0.17	0.09	5.99E-03	0.42	4.99E-03	0.13	0.08	0.06	5.20
s-3	8	0.17	0.01	1807.16	0.72	0.65	0.07	4.56E-03	0.43	6.00E-03	0.10	0.07	0.02	5.35
s-4	8	0.17	0.01	1806.69	1.67	1.16	0.19	6.15E-03	1.06	5.39E-03	0.21	0.14	0.05	5.23
s-5	9	0.94	0.00	1806.63	1.65	1.10	0.18	3.67E-03	0.87	4.15E-03	0.10	0.06	0.04	5.11
s-6	8	0.13	0.02	1808.59	1.03	0.74	0.15	6.90E-03	0.25	5.27E-03	0.11	0.09	0.01	4.78
s-7	6	0.12	0.00	1812.29	0.39	0.16	0.25	5.46E-03	0.41	6.13E-03	0.11	0.09	0.02	3.60
s-8	9	0.79	0.00	1807.03	1.82	1.57	0.13	4.68E-03	0.68	5.38E-03	0.25	0.16	0.11	5.62
s-9	10	1.46	0.00	1807.60	1.86	1.43	0.16	3.91E-03	1.00	5.81E-03	0.34	0.20	0.07	6.22
s-10	8	0.07	0.02	1811.51	0.34	0.13	0.16	4.94E-03	0.37	4.81E-03	0.08	0.07	0.01	4.88
s-11	4	0.08	0.07	1844.80	0.33	0.24	0.03	5.94E-03	0.32	3.81E-03	0.06	0.05	0.01	3.74
s-12	5	0.09	0.08	1813.99	0.88	0.55	0.13	5.93E-03	0.65	6.30E-03	0.18	0.09	0.12	5.47
s-13	4	0.06	0.05	1812.98	0.33	0.19	0.04	6.19E-03	0.99	5.77E-03	0.12	0.08	0.04	4.25
s-14	5	0.06	0.01	1811.90	0.56	0.16	0.16	4.68E-03	0.44	4.28E-03	0.07	0.06	0.02	4.24
s-15	4	0.15	0.05	1811.17	0.33	0.27	0.08	9.68E-03	1.46	7.46E-03	0.26	0.15	0.10	4.26
Model with variable power														
s-1	9	0.00	0.00	114.61	1.58	0.92	0.37	7.40E-04	1.42	8.96E-04	0.00	0.00	0.00	0.44
s-2	8	0.00	0.00	682.86	0.17	0.00	0.20	7.26E-04	0.30	7.67E-04	0.01	0.00	0.03	0.25
s-3	8	0.00	0.00	60.95	0.58	0.48	0.05	8.24E-04	0.35	7.45E-04	0.00	0.00	0.02	0.25
s-4	8	0.00	0.00	76.02	1.48	1.06	0.21	7.41E-04	0.90	6.68E-04	0.01	0.00	0.04	0.48
s-5	9	0.00	0.00	71.31	1.44	0.89	0.26	6.85E-04	0.75	1.78E-03	0.00	0.00	0.00	0.39
s-6	8	0.00	0.00	94.16	0.86	0.47	0.17	6.43E-04	0.12	9.29E-04	0.00	0.00	0.00	0.12
s-7	6	0.00	0.00	75.16	0.10	0.00	0.18	6.77E-04	0.21	7.08E-04	0.00	0.00	0.00	0.14
s-8	9	0.00	0.00	1732.09	1.68	1.48	0.11	6.49E-04	0.54	1.55E-03	0.06	0.00	0.05	1.08
s-9	10	0.08	0.00	1809.56	1.79	1.39	0.14	6.94E-04	0.85	7.87E-04	0.16	0.05	0.09	1.02
s-10	8	0.00	0.00	85.45	0.16	0.00	0.17	7.33E-04	0.24	7.30E-04	0.00	0.00	0.00	0.09
s-11	4	0.00	0.00	43.61	0.13	0.11	0.04	7.23E-04	0.23	7.26E-04	0.00	0.00	0.00	0.25
s-12	5	0.00	0.00	44.68	0.62	0.33	0.10	8.38E-04	0.54	6.96E-04	0.06	0.00	0.08	0.94
s-13	4	0.00	0.00	30.99	0.14	0.14	0.00	7.26E-04	0.87	7.81E-04	0.00	0.00	0.02	0.49
s-14	5	0.00	0.00	28.34	0.39	0.07	0.12	8.58E-04	0.32	6.81E-04	0.00	0.00	0.00	0.17
s-15	4	0.00	0.00	46.37	0.12	0.00	0.11	1.18E-03	1.30	6.94E-04	0.03	0.00	0.06	0.78

of the IGCH and the SA algorithm, 30 independent executions were done for each instance. We report the average and the best objective function value over the 30 runs in columns "mean" and "best", respectively. Also, we report the standard deviation of the mean objective function value in column "std" and the average running time in column "time". The best objective values are shown in bold.

First, we remark that the SA algorithm achieved all the best objective values for both constant and variable power models, and the average standard deviation was 0.04, which suggests that the SA algorithm is stable.

For the constant power model, CPLEX found 6 of the best solutions out of 15, mainly when $k \leq m$. All instances were hard to solve for CPLEX within 30 minutes. The average gap between the best objective values found by SA, CPLEX, FCFS, and IGCH and the lower bound (LB) calculated by CPLEX were 0.08, 0.34, 0.66, and 0.57, respectively.

For the variable power model, we notice that the SA algorithm achieved 14 optimal solutions

out of 15 in less than 1.08 seconds. CPLEX also achieved 14 optimal solutions. However, it took an average time of 227.61 seconds. One instance, "s-9", was hard to solve with CPLEX within 30 minutes. For the heuristics, IGCH found three optimal solutions out of 15. The average gap between the best objective values found by FCFS and IGCH and the lower bound (LB) were 0.56 and 0.35, respectively.

For both constant and variable models, IGCH outperforms FCFS. However, the average objective values found by IGCH were worse than FCFS heuristic. For this reason, we chose solutions found by the FCFS heuristic as the initial solutions for SA Algorithm. We established this choice by the initial experiments in which SA Algorithm achieved the best results when starting from the FCFS heuristic solutions, even when running IGCH several times and selecting the best solution as the initial solution for SA Algorithm.

Now, we compare the three proposed methods in terms of running time. As expected, heuristics are faster than the SA algorithm. For the constant power model, FCFS and IGCH took an average running time of 5.4 and 5.5 milliseconds, whereas the SA algorithm took an average running time of 4.96 seconds. For the variable power model, the average running time for FCFS and IGCH were 0.8 and 0.9 milliseconds, respectively, whereas the SA algorithm took an average running time of 0.5 seconds.

In summary, the SA algorithm outperformed CPLEX in significantly less time. Also, considering the variable power model rather than the constant power model is more efficient for satisfying more charging demands and running in less computation time.

5.6.4 Evaluation of Algorithms on Classes 2-4 of Instances

In this section, we focus on the evaluation of algorithms on instances of classes 2-4. Note that no instances could be solved by CPLEX (both time-indexed and event-based models) due to out-of-memory errors, even though we used a machine with 64 GB RAM. Using similar column headings as in Table 5.4, Tables 5.5, 5.6 and 5.7 provide comparison of results obtained with a number of charger m equals to 10, 20 and 40, respectively. From tables Tables 5.5, 5.6 and 5.7, we made the following observations:

- In the variable power model, the objective function values obtained were better for all instances, regardless of the optimization method. Indeed, the average objective values obtained by the SA algorithm were reduced by 95.62%, 54.34%, and 73.00% for m equal to 10, 20, and 40, respectively.
- For all instances, the performance of the SA algorithm is significantly better than the heuristics IGCH and FCFS in both best and mean objective function values. For example, the SA algorithm improves best the objective function value by 85.37% and by 65.59% compared to the FCFS heuristic and IGCH, respectively.
- the objective function values obtained by the variable power model is better for all instances, regardless of the optimization method. Indeed, the average objective values obtained by the SA algorithm is reduced by 95.62%, 54.34%, and 73.00% for m equal to 10, 20, and 40, respectively.

Table 5.5: Comparison of results with $m = 10$.

scenario	n	k	FCFS		IGCH				SA			
			obj	time (s)	mean	best	std	time (s)	mean	best	std	time (s)
Model with constant power												
1	20	12	0.52	0.01	0.87	0.31	0.33	0.01	0.18	0.13	0.05	9.18
2	29	18	3.77	0.01	3.41	2.05	0.61	0.02	0.73	0.42	0.19	11.68
3	24	16	2.22	0.01	1.80	1.02	0.35	0.01	0.31	0.22	0.06	10.51
4	25	17	1.64	0.01	1.77	1.02	0.49	0.02	0.35	0.24	0.07	19.58
5	30	16	4.81	0.01	3.07	2.16	0.63	0.02	1.05	0.64	0.26	12.37
6	27	16	2.80	0.01	1.95	1.11	0.58	0.01	0.57	0.36	0.12	9.49
7	29	14	2.10	0.01	1.49	0.72	0.40	0.02	0.53	0.36	0.13	15.84
8	26	16	2.48	0.01	1.51	0.56	0.52	0.01	0.32	0.19	0.09	11.08
9	22	14	2.48	0.01	1.81	0.77	0.38	0.01	0.33	0.24	0.07	9.14
10	22	15	0.73	0.01	0.84	0.33	0.26	0.01	0.22	0.13	0.06	9.32
11	27	9	2.21	0.01	1.66	0.98	0.36	0.01	0.65	0.42	0.12	8.88
12	25	10	1.87	0.01	0.69	0.43	0.21	0.01	0.35	0.25	0.06	7.69
13	20	8	2.03	0.01	0.55	0.39	0.11	0.01	0.29	0.16	0.10	6.44
14	23	9	2.21	0.01	0.96	0.52	0.28	0.01	0.45	0.31	0.13	7.33
15	22	9	1.47	0.01	0.65	0.34	0.27	0.01	0.35	0.27	0.05	6.34
Model with variable power												
1	20	12	0.27	0.00	0.52	0.02	0.27	0.00	0.00	0.00	0.00	1.34
2	29	18	3.36	0.00	3.12	1.54	0.72	0.00	0.27	0.01	0.21	1.97
3	24	16	1.96	0.00	1.41	0.57	0.40	0.00	0.04	0.00	0.07	1.65
4	25	17	1.26	0.00	1.45	0.75	0.36	0.00	0.06	0.00	0.06	1.74
5	30	16	4.53	0.00	2.81	1.32	0.52	0.00	0.54	0.05	0.26	1.84
6	27	16	2.46	0.00	1.53	0.71	0.43	0.00	0.23	0.05	0.10	1.89
7	29	14	1.90	0.00	1.15	0.34	0.46	0.00	0.12	0.00	0.11	2.10
8	26	16	2.23	0.00	1.03	0.24	0.45	0.00	0.04	0.00	0.05	1.84
9	22	14	2.13	0.00	1.45	0.52	0.44	0.00	0.04	0.00	0.06	1.45
10	22	15	0.48	0.00	0.54	0.04	0.26	0.00	0.01	0.00	0.02	1.49
11	27	9	1.89	0.00	1.05	0.08	0.42	0.00	0.19	0.08	0.12	2.04
12	25	10	1.64	0.00	0.42	0.08	0.23	0.00	0.06	0.00	0.11	1.72
13	20	8	1.73	0.00	0.24	0.02	0.13	0.00	0.06	0.00	0.09	1.39
14	23	9	2.04	0.00	0.57	0.03	0.25	0.00	0.16	0.00	0.13	1.58
15	22	9	1.14	0.00	0.25	0.00	0.21	0.00	0.03	0.00	0.05	1.50

- For instances in classes 2-4, we observe that IGCH outperforms the FCFS heuristic for both the mean and the best objective function values. The best objective function values of the IGCH is 57.63% less than those of the FCFS heuristic.
- In the case of the variable power model, the SA algorithm found 14 optimal solutions with an objective value equal to 0 while the IGCH found two optimal solutions.
- We perform the Mann-Whitney U test [Mann & Whitney 1947] to compare the results

Table 5.6: Comparison of results with $m = 20$.

scenario	n	k	FCFS		IGCH				SA			
			obj	time (s)	mean	best	std	time (s)	mean	best	std	time (s)
Model with constant power												
16	48	22	6.04	0.02	2.75	1.89	0.49	0.03	1.71	1.12	0.27	13.54
17	50	22	6.22	0.02	3.30	2.16	0.48	0.02	2.24	1.49	0.36	12.32
18	59	30	4.95	0.02	3.72	2.39	0.87	0.03	1.18	0.90	0.20	19.44
19	66	28	7.40	0.02	6.13	4.62	0.76	0.04	2.37	1.75	0.32	22.46
20	51	23	4.73	0.02	2.85	1.86	0.52	0.02	1.63	1.22	0.22	13.04
21	58	33	6.63	0.02	7.55	4.64	1.15	0.03	2.64	1.75	0.67	18.80
22	53	26	3.76	0.02	3.21	1.81	0.60	0.03	1.3	0.72	0.24	18.89
23	68	37	9.24	0.02	8.73	6.60	0.88	0.04	3.65	2.73	0.41	21.40
24	52	27	4.21	0.02	3.39	1.80	0.70	0.04	1.25	0.75	0.29	15.91
25	54	28	5.89	0.02	6.19	4.26	0.90	0.03	2.76	1.91	0.47	19.49
26	40	20	4.11	0.01	1.93	1.37	0.34	0.02	0.78	0.51	0.13	12.43
27	40	18	4.05	0.01	1.86	1.41	0.26	0.02	0.90	0.61	0.15	13.04
28	37	20	3.52	0.01	1.63	0.88	0.33	0.02	0.63	0.45	0.11	20.50
29	33	15	2.88	0.01	0.72	0.50	0.15	0.01	0.38	0.27	0.09	15.17
30	39	18	3.48	0.02	1.85	1.19	0.39	0.02	0.58	0.42	0.11	11.43
Model with variable power												
16	48	22	5.39	0.01	2.03	0.94	0.46	0.01	1.08	0.43	0.34	4.86
17	50	22	5.79	0.01	2.59	1.34	0.61	0.01	1.60	1.06	0.31	4.85
18	59	30	4.20	0.01	2.85	0.94	1.02	0.02	0.6	0.17	0.32	5.44
19	66	28	6.74	0.01	5.09	3.68	0.81	0.02	1.50	0.75	0.39	7.08
20	51	23	4.24	0.01	2.06	1.2	0.51	0.01	0.93	0.67	0.21	5.09
21	58	33	6.06	0.01	7.17	5.23	1.15	0.02	1.80	1.02	0.52	5.80
22	53	26	3.18	0.01	2.79	1.72	0.46	0.01	0.59	0.31	0.21	5.85
23	68	37	8.75	0.01	8.18	6.46	0.72	0.02	2.63	1.80	0.58	7.32
24	52	27	3.75	0.01	2.73	1.34	0.72	0.01	0.57	0.30	0.26	5.16
25	54	28	5.44	0.01	5.25	3.24	0.78	0.01	1.99	1.00	0.37	4.79
26	40	20	3.61	0.00	1.20	0.49	0.36	0.01	0.16	0.02	0.10	3.31
27	40	18	3.75	0.00	1.08	0.61	0.28	0.01	0.31	0.05	0.16	3.51
28	37	20	3.22	0.00	0.94	0.39	0.23	0.01	0.10	0.00	0.09	3.42
29	33	15	2.60	0.00	0.20	0.00	0.16	0.01	0.04	0.00	0.08	2.66
30	39	18	3.16	0.00	1.09	0.26	0.36	0.01	0.15	0.00	0.14	3.57

between the IGCH and the SA algorithm in constant and variable power models. The Mann-Whitney U test is a non-parametric statistical test to determine whether two independent samples were drawn from a population with the same distribution. We compare the p-value to a significance level of 0.05. The p-value found is 0.00001, 0.0002 and 0.00001 for results with $m = 10$, $m = 20$, and $m = 40$, respectively. This suggests a significant difference between the results of SA and IGCH algorithms.

- The objective function values are better minimized in instances where k is less than or

Table 5.7: Comparison of results with $m = 40$.

scenario	n	k	FCFS		IGCH				SA			
			obj	time (s)	mean	best	std	time (s)	mean	best	std	time (s)
Model with constant power												
31	93	45	6.27	0.04	5.14	3.57	0.95	0.08	1.87	1.38	0.26	38.93
32	99	53	7.27	0.03	7.60	5.91	1.00	0.08	2.57	1.95	0.33	51.29
33	79	41	6.64	0.03	3.07	1.81	0.85	0.06	1.66	1.11	0.21	28.74
34	102	52	9.59	0.03	7.61	5.20	1.06	0.09	2.76	2.09	0.37	28.84
35	93	52	7.77	0.03	8.20	6.77	0.70	0.07	2.34	1.40	0.33	41.19
36	96	50	4.30	0.03	5.86	4.01	0.78	0.07	1.79	1.38	0.21	54.29
37	96	52	9.47	0.03	6.92	4.53	1.43	0.07	2.67	2.18	0.36	30.28
38	112	58	9.82	0.04	9.19	7.24	0.98	0.09	3.30	2.72	0.41	37.45
39	95	43	7.04	0.03	3.94	2.68	0.71	0.06	2.54	1.94	0.29	27.32
40	78	38	6.56	0.03	2.94	2.15	0.37	0.04	1.77	1.35	0.25	27.10
41	85	37	6.40	0.03	3.80	1.95	0.76	0.05	1.82	1.30	0.28	22.99
42	82	39	5.47	0.03	3.32	2.15	0.45	0.05	1.45	1.23	0.16	26.55
43	88	40	6.36	0.03	4.76	3.68	0.58	0.06	2.11	1.58	0.28	27.38
44	91	40	6.13	0.03	3.54	2.24	0.60	0.06	1.76	1.26	0.26	22.73
45	79	39	6.81	0.02	3.49	2.45	0.41	0.05	1.96	1.24	0.34	25.21
Model with variable power												
31	93	45	5.32	0.01	3.31	2.13	0.97	0.04	0.78	0.17	0.25	13.39
32	99	53	6.30	0.01	5.85	3.05	1.00	0.05	1.48	0.81	0.38	15.54
33	79	41	5.79	0.01	1.73	0.80	0.67	0.03	0.77	0.46	0.20	10.36
34	102	52	8.16	0.02	6.16	3.83	0.91	0.06	1.38	0.53	0.45	14.82
35	93	52	6.69	0.01	6.82	4.21	1.12	0.05	1.20	0.39	0.38	12.87
36	96	50	3.18	0.01	4.29	2.40	0.80	0.05	0.70	0.29	0.25	14.76
37	96	52	8.31	0.01	5.39	3.24	1.18	0.05	1.49	0.90	0.36	14.05
38	112	58	8.57	0.02	7.40	4.67	1.19	0.07	1.75	1.05	0.34	18.14
39	95	43	5.79	0.02	1.95	0.58	0.72	0.04	1.02	0.26	0.26	14.25
40	78	38	5.61	0.01	1.35	0.59	0.38	0.03	0.64	0.23	0.25	9.63
41	85	37	5.56	0.01	2.37	1.27	0.69	0.03	0.85	0.35	0.32	11.14
42	82	39	4.70	0.01	1.80	0.82	0.43	0.03	0.48	0.19	0.21	10.44
43	88	40	5.33	0.01	3.07	1.86	0.67	0.04	0.92	0.34	0.33	13.26
44	91	40	5.11	0.01	1.99	0.67	0.49	0.04	0.68	0.24	0.29	12.79
45	79	39	5.98	0.01	2.09	1.23	0.48	0.03	0.91	0.30	0.24	9.91

equal to m (the last five instances for each case). This is explained by the fact that there are enough charges to park all vehicles. Also, the IGCH heuristic performs better in these instances since it is designed to take advantage of interval graph properties.

- The average running time for FCFS, IGCH, and the SA algorithm are 0.01, 0.02, and 13.18 seconds, respectively. As expected, the average time taken by the SA algorithm is greater in instances with more charging demands, while it is still less than 0.09 seconds for the heuristics FCFS and IGCH. This is due to the fact that SA algorithm solves multiple

MILP with CPLEX at each neighbor generation while one MILP is solved for each IGCH or FCFS run. It can also be noted that solving the variable power model took less time than solving the constant power model showing the efficiency of using an event-based formulation than a time-indexed one.

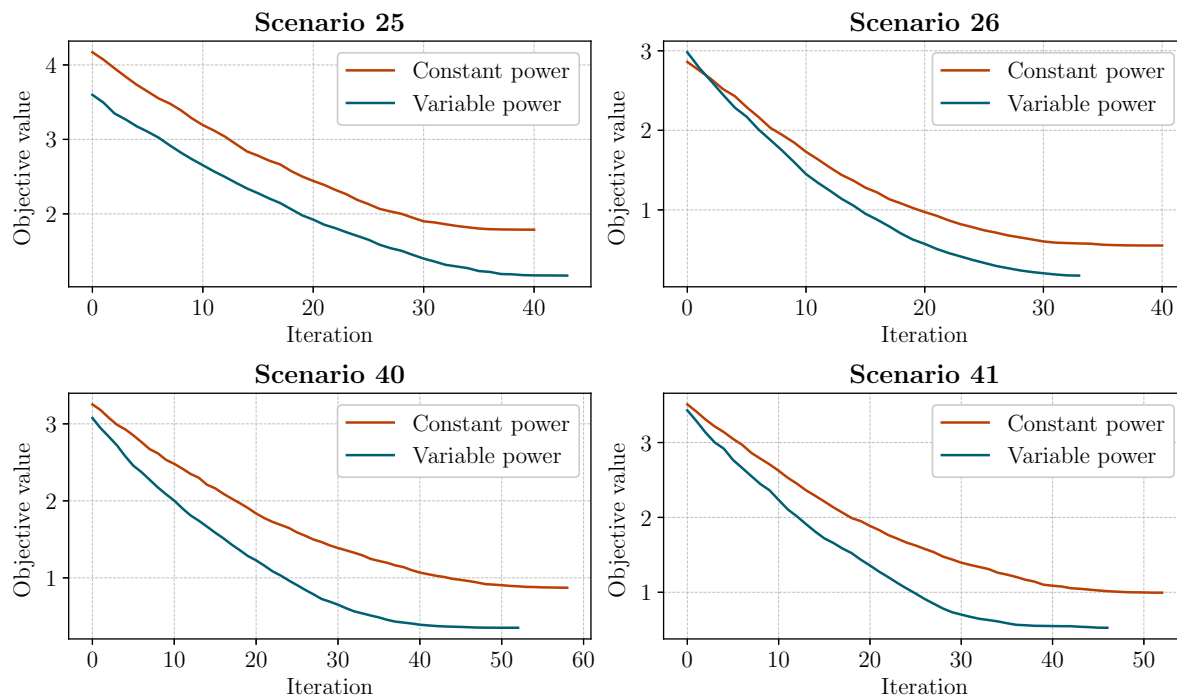


Figure 5.6: The convergence curve of the best objective value obtained by the SA algorithm for scenarios 25, 26, 40, and 41.

5.6.5 Convergence of the SA Algorithm

Another aspect worth to be studied is the convergence of the SA algorithm. To do that, we choose four instances from classes 3 and 4, namely instances 25, 26, 40, and 41, and plot the convergence curves of the SA algorithms in Figure 5.6. For each instance, we plot the average of the best objective function values found at each iteration over 30 runs in both variable and constant models. We observe that the SA algorithm converges better toward a good quality of solutions in the variable power model. As we can see, the number of iterations is different for each instance and run since it is determined by the evolution of the best objective value.

5.6.6 Additional Results

In this chapter, we consider the objective of minimizing the total gap between the requested and the final state-of-charges, which provides overall customer satisfaction. However, optimization algorithms may charge fewer vehicles with more energy rather than more vehicles with less

energy. A rejected charging demand is still penalized in the objective function since the gap between its initial and requested state-of-charge is added to the objective function value. However, when this gap is too small, rejecting a demand in order to charge another one may be more efficient. Therefore, it is worthwhile to evaluate the quality of solutions using an additional measure. To assess how each algorithm performs in terms of customer satisfaction, we track for each instance the percentage of rejected demands and the percentage of satisfied demands. Mainly, we are interested in the percentage of demands that are satisfied to at least 75%, i.e., the demands $j \in \mathcal{J}$ where $e_j^f - e_j^0 \geq 0.75(e_j^d - e_j^0)$. Since the SA algorithm and the IGCH are non-deterministic, the presented results are the average values obtained over the 30 runs. Results of CPLEX are not available for classes 2-4 of instances and are replaced by a dash "-" in tables.

Table 5.8: Comparison between the average percentages of rejected demands and partially satisfied demands.

		Rejected demands				Satisfied demands to (75%)			
		CPLEX	FCFS	IGCH	SA	CPLEX	FCFS	IGCH	SA
Model with constant power									
Class 1	instances with $k > m$	0.00%	1.00%	9.15%	0.60%	86.00%	79.00%	69.73%	96.43%
	instances with $k \leq m$	0.00%	0.00%	0.00%	0.13%	100.00%	78.00%	86.53%	93.13%
Class 2	instances with $k > m$	-	2.72%	6.63%	1.23%	-	75.47%	79.53%	96.96%
	instances with $k \leq m$	-	0.00%	0.00%	0.91%	-	76.68%	89.78%	97.30%
class 3	instances with $k > m$	-	0.50%	7.00%	2.39%	-	67.22%	76.67%	89.22%
	instances with $k \leq m$	-	0.00%	0.00%	0.80%	-	68.49%	86.62%	97.47%
class 4	instances with $k > m$	-	0.00%	5.08%	0.89%	-	76.07%	83.24%	93.28%
	instances with $k \leq m$	-	0.00%	0.00%	0.89%	-	78.97%	88.50%	94.42%
Model with variable power									
Class 1	instances with $k > m$	0.00%	1.00%	8.95%	0.23%	100.00%	83.00%	74.10%	98.63%
	instances with $k \leq m$	0.00%	0.00%	0.00%	0.00%	100.00%	82.00%	92.87%	93.87%
Class 2	instances with $k > m$	-	2.72%	6.21%	0.83%	-	77.52%	83.45%	98.30%
	instances with $k \leq m$	-	0.00%	0.00%	0.89%	-	77.59%	92.78%	98.64%
class 3	instances with $k > m$	-	1.36%	1.55%	0.44%	-	39.25%	84.10%	95.53%
	instances with $k \leq m$	-	0.00%	0.00%	0.73%	-	69.49%	91.93%	98.02%
class 4	instances with $k > m$	-	0.00%	5.07%	0.84%	-	77.41%	86.31%	94.77%
	instances with $k \leq m$	-	0.00%	0.00%	0.88%	-	80.41%	91.76%	95.65%

Table 5.8 shows the average percentage of rejected and satisfied demands in solutions found by the proposed methods. The percentage of satisfied demands is higher for instances with $k \leq m$. In addition, no charging demand was rejected for these instances (except for the SA algorithm in the constant power model). CPLEX never rejected charging demands and achieved a percentage of 96.5% of partially satisfied demands. As expected, the FCFS heuristic never

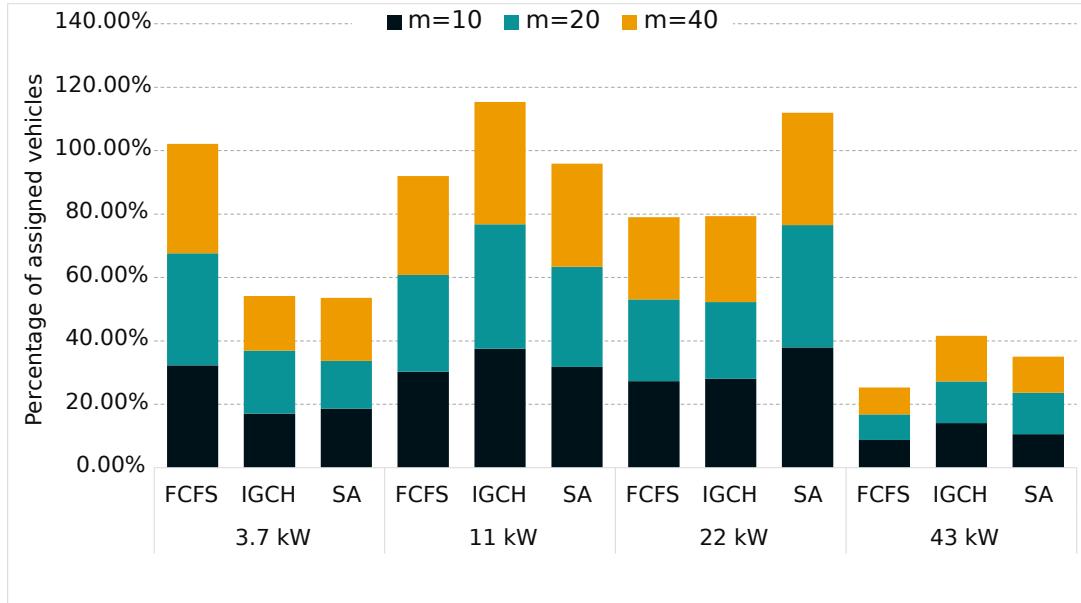
rejected demands when $k \leq m$, since there are enough chargers to park all vehicles, and the heuristic schedules each vehicle on the first available charger. The IGCH also has a 0% rejection in instances with $k \leq m$ since it constructs an interval graph and assigns the vehicle in each color class to a charger. The IGCH achieves the highest rejection percentage in instances with $k > m$ (averagely 6.20%). Even though the FCFS rule rejects less than 3% of vehicles, it satisfies fewer demands compared to other methods. For the SA algorithm, the percentage of satisfied demands is always above 93% except for class 3 instances with constant power charging mode and $k > m$, which dropped to 89%. Moreover, the rejection percentage stays under 2.5%.

Since we assumed that vehicles could be plugged-in after their desired arrival time at the charging station, it is interesting to show the average gap between the desired arrival times and the decided plug-in times in solutions found by each method when demands are accepted. Table 5.9 shows results of the experiment. We can notice that CPLEX and the SA algorithm achieved the highest average gap for instances with $n = 10$ and $k \leq m$. More precisely, 9 minutes gap for CPLEX and about 7 minutes gap for the SA considering the variable power model. In the remaining instances, the proposed methods did not exceed 5 minutes delay. As CPLEX and the SA algorithm achieved the best performance, we can conclude that relaxing the arrival time constraints can be more advantageous in satisfying more charging demands with an average delay of a few minutes.

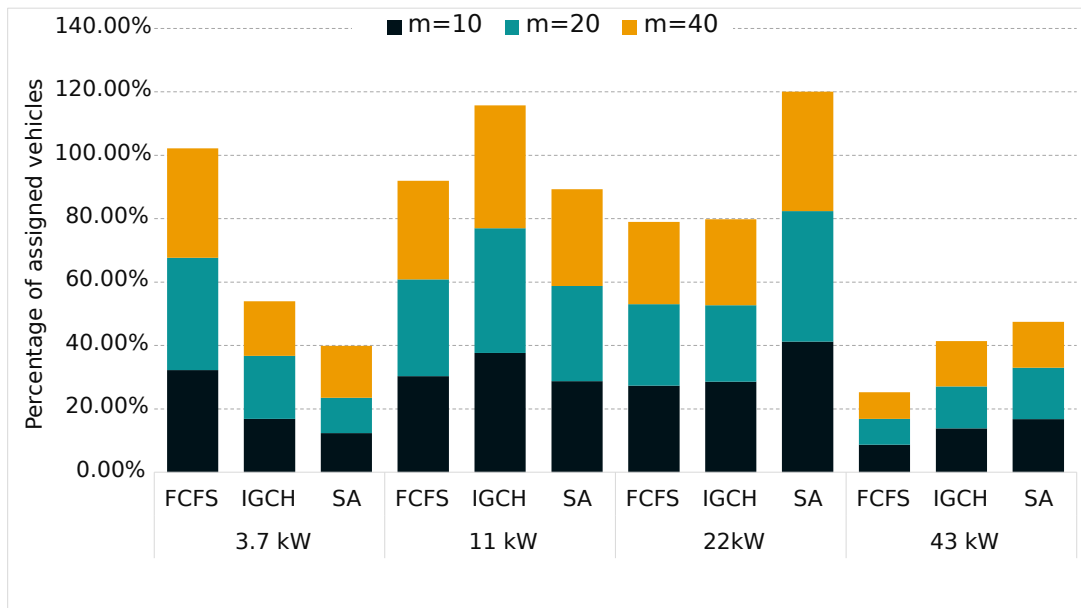
Table 5.9: Comparison of the average gap between the desired arrival times and the actual plug-in times in hours.

		CPLEX	FCFS	IGCH	SA
Model with constant power					
Class 1	instances with $k > m$	0.11	0.07	0.05	0.11
	instances with $k \leq m$	0.03	0.00	0.00	0.03
Class 2	instances with $k > m$	-	0.02	0.03	0.05
	instances with $k \leq m$	-	0.00	0.00	0.02
Class 3	instances with $k > m$	-	0.00	0.00	0.01
	instances with $k \leq m$	-	0.00	0.00	0.01
Class 4	instances with $k > m$	-	0.00	0.00	0.01
	instances with $k \leq m$	-	0.00	0.00	0.00
Model with variable power					
Class 1	instances with $k > m$	0.15	0.07	0.05	0.12
	instances with $k \leq m$	0.06	0.00	0.00	0.04
Class 2	instances with $k > m$	-	0.02	0.03	0.06
	instances with $k \leq m$	-	0.00	0.00	0.02
Class 3	instances with $k > m$	-	0.00	0.00	0.01
	instances with $k \leq m$	-	0.00	0.00	0.01
Class 4	instances with $k > m$	-	0.00	0.00	0.01
	instances with $k \leq m$	-	0.00	0.00	0.01

To further detail the assignment of vehicles to chargers, in Fig. 5.7, we summarize the percentage of vehicles assigned to each type of charge. We recall that there are four types of chargers: 3.7 kW, 11 kW, 22 kW, and 43 kW. The first observation we can make is that the three methods assign vehicles to a type of charger nearly the same way in the constant power model



(a) Constant power model



(b) Variable power model

Figure 5.7: The average percentage of electric vehicles assigned to each type of charger using FCFS, IGCH, and SA considering (a) constant power model and (b) variable power model.

as in the variable power model. We can spot that the SA algorithm prioritizes chargers with 22 kW, where 40% of vehicles are assigned. The IGCH assigned more vehicles to 11 kW chargers. However, 60% of vehicles are assigned to 3.7 kW and 11 kW chargers by the FCFS heuristic showing why it performs worse than the other methods. Even though it may seem that the SA algorithm assigns fewer vehicles to fast chargers with 43 kW, we recall that only 10% of chargers deliver 43 kW. Furthermore, We observe that the SA algorithm assigns 13.3% of vehicles to the 30% chargers delivering 3.7 kW while it assigns 15.82% of vehicles to the 10% chargers delivering 3.7 kW. However, we cannot conclude that using only one fast type of charger is better. More tests should be conducted, and one must consider the installation costs, chargers compatibility, and impact of fast charging on batteries.

5.7 Conclusion

This chapter addressed the electric vehicle charging scheduling problem in a charging station with different charging types and limited overall power. We have assumed that the data related to vehicle charging demands, such as arrival time, departure time, and state-of-charge, are known in advance. This assumption is realistic since many charging service operators require a reservation in advance to avoid queues. For each vehicle's charging demand, the scheduler has to determine the allocation of chargers to vehicles and the charging planning to maximize customer satisfaction. Arrival times were considered as soft constraints where vehicles could be plugged later than the desired arrival time.

We considered constant charging rates as well as variable charging rates. We proved that both problems are \mathcal{NP} -hard. Further, we formulate the scheduling problems as a mixed-integer linear programming (MILP) model. Even for small instances, it was hard to solve the MILP models. Therefore, we designed a heuristic based on an interval graph coloring algorithm (IGCH) and a simulated annealing (SA) algorithm combined with linear programming. Different instances were generated to evaluate the performance of the proposed methods. The results show that the variable charging model is better for satisfying charging demands. We show that the SA algorithm performs better in minimizing the objective function in less than a minute. Additional results on the scheduling solutions quality were conducted. The SA algorithm scheduling solutions charge at least 93% of charging demands to more than 75% of their desired energy. On the other hand, the percentage of rejected demands is under 2.5%.

Online EVCS in unbalanced three-phase power system

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6.1 Introduction

Electric vehicles can be charged with either single-phase or three-phase chargers. In single-phase charging, the power flows through a single wire or conductor while it flows through 3 conductors in three-phase charging. The main difference between single-phase and three-phase charging is the amount of power that can be delivered to the vehicle. The charging power of three-phase chargers is higher and used for fast charging. In contrast, single-phase chargers use the standard 230V with a maximum current of 32A. In this case, the electric vehicle can be charged with a maximum power of 7.4kW. However, single-phase chargers are less expensive and easy to install. Moreover, not all vehicles are compatible with three-phase charging.

This chapter studies the EVCS problem in a charging station fed with a three-phase power supply. Electric vehicles are connected to one of the three phases as single-phase loads. In this installation, the phase imbalance can be aggravated when more vehicles are charging on one phase, especially since electric vehicles are heavy load and their charging depends on the random drivers' behavior. Such imbalance has a negative impact on the power system, causing waste of network capacity, additional energy losses, and extra costs.

Another challenge in this study is that vehicles arrive at the charging station randomly during the day with different charging demands and departure times. Each vehicle has its own parking space. The objective is to build a real-time schedule that minimizes the total tardiness

subject to the technical constraints of the charging station. We consider preemptive as well as non-preemptive EV charging. A mixed-integer linear programming (MILP) model is formulated for the offline problem. We propose heuristics based on the priority rule to solve the online problem. Further, a local search is implemented to improve the objective value of preemptive EV charging. Computational results show that the proposed solving approaches outperform the existing heuristics developed in the literature. Moreover, we show that total tardiness is significantly reduced when preemption is exploited.

6.2 Problem Description

6.2.1 Charging Station Model

Our study concerns the EVCS problem in a charging station designed to be installed as a large public parking or a collective garage as described in [Sedano Franco *et al.* 2013]. This station is fed with a three-phase current power source. Thus, there are three conductors, each carrying an alternating current of the same frequency and voltage amplitude from the source to the electrical outlets. These conductors are called lines, and each line regroups a number of power outlets, or chargers, into which electric vehicles can be plugged for charging. In other words, each connected electric vehicle is considered as a single-phase load. However, two constraints limit the number of chargers delivering power simultaneously. The first constraint is related to the maximum power that can be drawn from each line so that system overload can be avoided. The second constraint maintains the load balance between any two lines. In fact, in a three-phase power system, the load should be distributed equally between the three lines to minimize power losses and improve the system's efficiency.

Figure 6.1 illustrates the design of such architecture. Each secondary machine commands the switching on or off of two power outlets. It also records the arrival time of the electric vehicle and communicates it to the corresponding primary machine. The primary machines have a user interface where the electric vehicle drivers enter their departure time and the desired energy demand. All these data are communicated to the central server, where a scheduler of the electric vehicle charging demands is implemented.

To simplify the operating model of the charging station, we make the same assumption as in [Hernández Arauzo *et al.* 2013, Hernández-Arauzo *et al.* 2015], where each electric vehicle driver has a dedicated parking place so he can plug his vehicle into the charger at any time. He also provides the parking duration and charging demand through the user interface. Therefore, there is no queue, and the driver does not have to wait before plugging his electric vehicle. However, the electric vehicle may not start charging immediately. Once it starts charging, it cannot be unplugged from the charger before satisfying its charging demand. Furthermore, all chargers are identical and deliver the same power at a constant rate.

6.2.2 Charging Scheduling Problem

Chargers switching on and off can be controlled according to the defined charging schedule. This schedule is built so that each line's total charging load does not exceed its capacity. Besides, the maximum imbalanced load between any two lines must be maintained. Since the completion

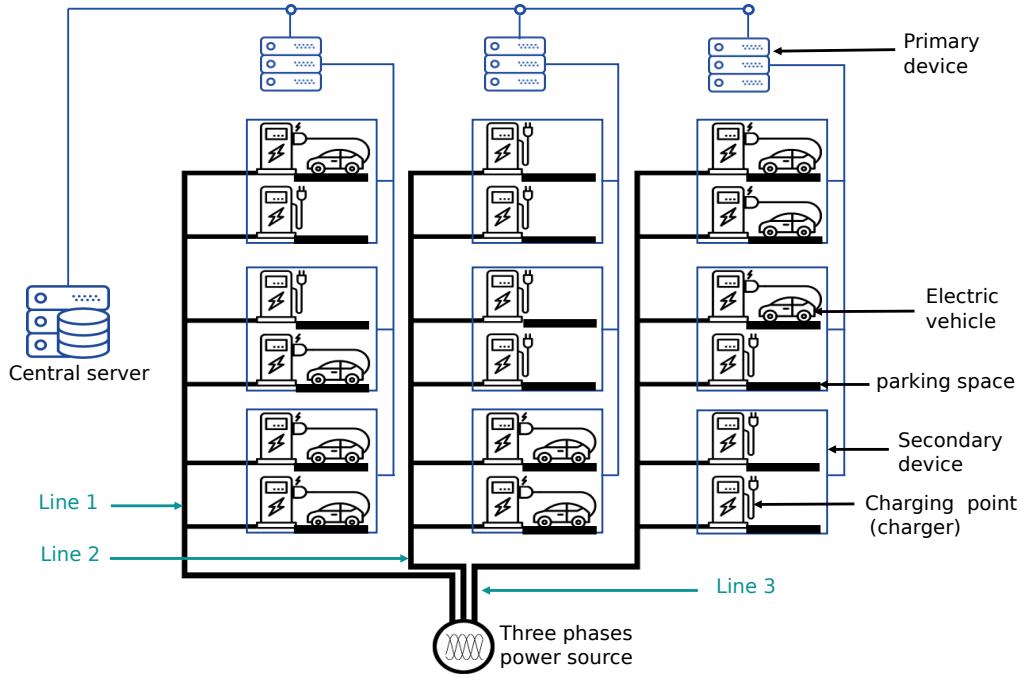


Figure 6.1: Scheme of the considered charging station.

time of each charging operation can exceed the requested departure time, the objective function will be to minimize the total tardiness.

The charging scheduling horizon \mathcal{H} runs from 00:00h to 23:59h and is divided into equal time slots of length τ , which is set to 6 min in our case. Each electric vehicle has its own parking space and arrives at the station at random instants. This implies that arrival times, charging times, and departure times are unknown until they actually arrive. As a result, the schedule must be built iteratively. In literature, there are two main strategies to handle this uncertainty: solving the scheduling problem whenever a new electric vehicle is connected or at each time slot. We adopt the second strategy since the time slot we defined is relatively small, and it prevents the system from collapsing when a large number of electric vehicles arrive at once.

First, we consider the non-preemptive scheduling as in [Hernández-Arauzo *et al.* 2015, Hernández Arauzo *et al.* 2013], where the charging of an electric vehicle cannot be interrupted until it completes charging. In other words, once a charger is switched on to deliver power to the electric vehicle, it cannot be switched off before the completion of the charging. Thus, the scheduling problem consists of assigning a starting time of charging for each arrived electric vehicle. Then, we investigate the case where preemption is allowed. In particular, the charging of an electric vehicle can be interrupted to charge another instead. The amount of energy for a preempted electric vehicle charging is not lost. When preempted charging is afterward resumed, it only needs power for its remaining charging time. Such a recharging strategy is highly recommended in smart charging [Kara *et al.* 2015]. Moreover, the open charge point protocol (OCPP) currently integrates these operations without human intervention [Vaidya & Mouftah 2018].

6.3 Mathematical Formulation

We formulate the problem as an integer linear programming (ILP) model. Indices, sets, and parameters are listed in Table 6.1.

Table 6.1: Summary of notations

Notation	Description
L_j	Line j , $j = 1, 2, 3$
N	Total number of chargers
\mathcal{N}	Set of chargers indexed by i
n_1	Number of chargers in line L_1 indexed $1, \dots, n_1$
n_2	Number of chargers in line L_2 indexed $n_1 + 1, \dots, n_1 + n_2$
n_3	Number of chargers in line L_3 indexed $n_1 + n_2 + 1, \dots, n_1 + n_2 + n_3 = N$
\tilde{N}	The maximum of power outlets that can deliver power simultaneously in any line at any time
Δ	Parameter that establishes the balance between any two lines, $\Delta \in [0, 1]$
\mathcal{H}	Set of T time slots indexed by t
τ	Length of time slot t
r_i	Arrival time of vehicle i
d_i	Departure time of vehicle i
p_i	Charging time of electric vehicle i

The further decision variables used in this formulation are given as follows:

- Binary variables $x_{i,t}$ to specify if electric vehicle i is charging at the time slot t .
- Integer variables N_j^t to denote the number of electric vehicle being charged in line L_j at time slot t .
- Integer variables T_i to represent the tardiness of electric vehicle i and it is calculated as $T_i = \max(0, C_i - d_i)$, where C_i represents the completion time of charging of vehicle i .

Then, the formulation of the EVCS problem as an LP model is as follows.

$$\sum_{i=1}^{n_1+n_2+n_3} T_i \quad (6.1)$$

$$\sum_{t=r_i}^H \tau x_{i,t} \geq p_i \quad \forall i \in \mathcal{N} \quad (6.2)$$

$$(t+1) \times \tau \times x_{i,t} - d_i \leq T_i \quad \forall i \in \mathcal{N}, t \in \mathcal{H} \quad (6.3)$$

$$\tau p_i(x_{i,t} - x_{i,t-1}) \leq \sum_{l=t}^{t+p_i/\tau} \tau x_{i,l} \quad \forall i \in \mathcal{N}, t \in \mathcal{H} \quad (6.4)$$

$$N_1^t = \sum_{i=1}^{n_1} x_{i,t} \quad \forall t \in \mathcal{H} \quad (6.5)$$

$$N_2^t = \sum_{i=n_1+1}^{n_1+n_2} x_{i,t} \quad \forall t \in \mathcal{H} \quad (6.6)$$

$$N_3^t = \sum_{i=n_1+n_2+1}^{n_1+n_2+n_3} x_{i,t} \quad \forall t \in \mathcal{H} \quad (6.7)$$

$$N_j^t \leq \tilde{N} \quad \forall t \in \mathcal{H}, j \in \{1, 2, 3\} \quad (6.8)$$

$$N_j^t - N_k^t \leq \Delta \tilde{N} \quad \forall t \in \mathcal{H}, k, j \in \{1, 2, 3\} j \neq k \quad (6.9)$$

$$-N_j^t + N_k^t \leq \Delta \tilde{N} \quad \forall t \in \mathcal{H}, j \neq k, k, j \in \{1, 2, 3\} \quad (6.10)$$

The objective function (6.1) minimizes the total tardiness. Constraints (6.2) ensure that electric vehicle i charges to its desired charging time p_i . Constraints (6.3) calculate the tardiness of charging electric vehicle i . Constraints (6.4) ensure the non-preemption of charging in case of non-preemptive scheduling. Constraints (6.5), (6.6) and (6.7) calculate the number of electric vehicle that are charging at the same time at time slot t in lines 1, 2 and 3 respectively. Constraints (6.8) define to the maximum power that can be drawn from each line. \tilde{N} restricts the number of electric vehicles that can be charged simultaneously in each line. Since each outlet delivers power at the same constant rate, the power delivered by each line can be expressed by the number of active outlets. Constraints (6.9) and (6.10) establish the maximum imbalance between any two lines, such as the difference between the numbers of electric vehicles in any two lines does not exceed $\Delta \tilde{N}$ with $\Delta \in [0, 1]$.

6.3.1 Online Optimization Algorithms

Although the LP model has been developed, finding the optimal schedule with an exact method cannot be done in polynomial time. In the simple case where we have only one line, the problem is equivalent to scheduling jobs on parallel machine $P|r_i|\sum_i T_i$ following the $\alpha|\beta|\gamma$ notation [Graham *et al.* 1979a] which is \mathcal{NP} -hard. Furthermore, the EVCS problem, as defined in this chapter, is a dynamic and real-time optimization problem that requires an online fast optimization method. Hence, we propose a heuristic to solve the non-preemptive charging scheduling problem. The heuristic is based on the *PRTT* (Priority Rule for Total Tardiness criterion) dispatching rule used in [Chu & Portmann 1992]. The same dispatching rule is used for the preemptive problem, and then we try to improve its results by applying a local search procedure.

6.3.1.1 Non-preemptive Scheduling Algorithm

Heuristics based on priority dispatching rules have been widely used in the literature to find near-optimal solutions for \mathcal{NP} -hard scheduling problems since they are simple methods and require less computation time than sophisticated meta-heuristics. This makes them adequate for real-time and dynamic problems. We adopt the *PRTT* rule proposed in [Chu & Portmann 1992] to the electric vehicle scheduling problem presented in the previous section. The *PRTT* of a charging operation i at time t is defined as:

$$prtt_i^t = \max(t, r_i) + \max(d_i, \max(t, r_i) + p_i) \quad (6.11)$$

Recall that the schedule is built at each time slot considering only the arrived electric vehicles, (i.e., $r_i \leq t$), with no knowledge of upcoming vehicles. As results, the term $\max(r_i, t)$ will always equal t . Then, the *PRTT* value of electric vehicle i is calculated as follows:

$$prtt_i^t = \max(d_i, t + p_i) \quad (6.12)$$

The charging operations are afterward scheduled at each step in ascending order of their *PRTT* values. Once an electric vehicle starts charging, no interruption is allowed until completion. Thus, the objective is to assign a starting time st_i for each electric vehicle without breaking the constraints (6.8), (6.9) and (6.10). The pseudo-code of the non-preemptive charging scheduling is shown in Algorithm 16. At each time slot, for each line, the *PRTT* of ready and not assigned electric vehicles are calculated using the equation (6.12), and they are ordered in increasing order. Then, we calculate N' , the number of electric vehicles that can be added to the schedule at time t in line j without breaking the constraints (6.8), (6.9) and (6.10). The value of N' can be obtained by:

$$N' = \min\left(\min_{k=1, \dots, L, k \neq j} (\Delta \tilde{N} - N_j^t + N_k^t), \tilde{N} - N_j^t\right) \quad (6.13)$$

The starting times of the first N' electric vehicles that will not break the constraints (6.8), (6.9) and (6.10) in the next time slots are set to t . We improve the assignments of electric vehicles at the end of each time slot in case an assignment of a vehicle breaks a previous imbalance constraint. This happens when the assignment of an electric vehicle i in line j to a starting time t verifies that the assignment of another electric vehicle i' in line k will not break the imbalance constraints (6.9) and (6.10) at time t .

$$\min\{|N_k^t + 1 - N_j^t| - \Delta \tilde{N}; k \in \{1, 2, 3\}, k \neq j\} \leq 0 \quad (6.14)$$

In this case, we redo the scheduling for the current time slot t (lines 2-21) to assign this vehicle and eventually others. An illustration of such situation is given in Example 6.3.1.

Example 6.3.1. Consider the charging demands of six vehicles. Table 6.2 gives the arrival, departure, and charging times for these demands. We set $\tilde{N} = 4$ and $\Delta = 0.5$, which means that the difference between any two lines should not exceed two vehicles. We start by scheduling vehicles line by line. Figure 6.2(a) shows the schedule of the charging demands at time $t = 8$. As we can see the vehicle v_3 cannot be scheduled when scheduling v_1 and v_2 since due to imbalance constraints (6.9) and (6.10). After that, we schedule v_4 and v_5 in line 2. Also, v_6 is scheduled on line 3; therefore, the imbalance constraints that prevent v_3 from being scheduled are broken. Hence, we redo its assignment as in Figure 6.2(b).

Algorithm 16: *PRTT* based pseudo-algorithm for non preemptive charging scheduling

input : r_i, p_i, d_i for each vehicle i , Δ, \tilde{N}
output: Total tardiness $\sum T_i$

```

1 for each time slot  $t$  do
2   for each line  $L_j, j = 1, \dots, 3$  do
3     Get the set  $I'$  of ready electric vehicles in the line  $j$  ( vehicles that have arrived
       during the interval  $[0, t]$  and not assigned);
4     if  $I'$  is not empty then
5       for each arrived vehicle do
6         Calculate its PRTT then we added to the set  $I_{prtt}$ ;
7       end
8       Sort the set  $I_{prtt}$  in increasing order of PRTT values ;
9       Calculate  $N'$  the number of electric vehicles that can be added to the
       schedule at time  $t$  in line  $j$  without breaking the constraints (6.8), (6.9), and
       (6.10);
10      for each vehicle  $i$  in the first  $N'$  vehicles in  $I_{prtt}$  do
11        if  $|N_j^{t'} + 1 - N_k^{t'}| \leq \Delta N$  for  $t'$  from  $t$  to  $t + p_i, k = 1, 2, 3k \neq j$  then
12          Set the starting time of the electric vehicle to  $t$ ;
13          Update  $N_j^{t'}$  for the next time slots  $t'$  starting from  $t$  to  $t + p_i$  ;
14          Calculate the tardiness of the assigned vehicle  $T_i = \max(0, t + p_i - d_i)$  ;
15          if the assignment breaks a previous balance constraint then
16            redo the time slot  $t$ ;
17          end
18        end
19      end
20    end
21  end
22 end
23 return  $\sum_{i=1}^N T_i$ 

```

Table 6.2: Instance for Example 6.3.1.

Vehicle	Line	Arrival time r_j	Departure time d_j	Charging time p_j
v_1	L_1	8:00	10:00	2 hours
v_2	L_1	8:00	10:00	2 hours
v_3	L_1	9:00	12:00	3 hours
v_4	L_2	8:00	10:00	2 hours
v_5	L_2	8:00	10:00	2 hours
v_6	L_3	9:00	13:00	4 hours

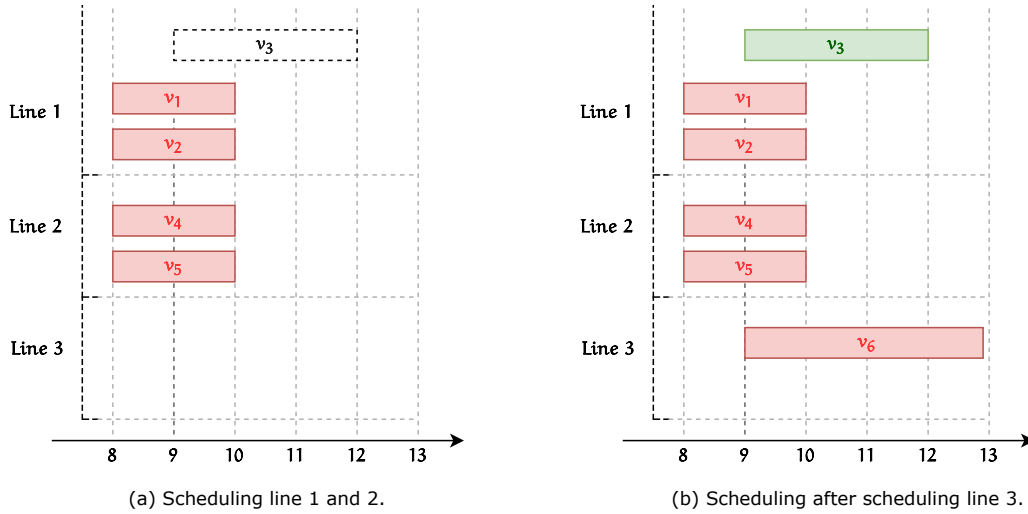


Figure 6.2: Example of improving the assignment in case of breaking a previous imbalance constraint.

6.3.2 Preemptive Scheduling Algorithms

Allowing preemption in scheduling problems is a commonly used relaxation to make the problem easier to solve, especially in our case where the schedule is built iteratively. In particular, the decision to start charging a vehicle at a given time is unchangeable, even if another electric vehicle with higher priority to charge arrives. In our case, a high-priority electric vehicle has the smallest $PRTT$ value at a given time. Furthermore, preemptive charging is allowed with the new generation of chargers that use the OCPP protocol [OCP 2020].

6.3.2.1 Heuristic

Unlike the non-preemptive scheduling heuristic proposed, we schedule the charging of the arrived electric vehicle at the current time slot only. No charging schedule is built for the next time slots. We calculate the $PRTT$ of already and unfinished charging operations at the beginning of each time slot. Then, we schedule the charging operations with smaller $PRTT$ values that verify the constraints (6.8), (6.9) and (6.10) for this time slot only.

We modify the $PRTT$ function to take into account only the remaining charging time of an electric vehicle instead of the whole charging time. The pseudo-code of the preemptive charging scheduling is given in Algorithm 17.

Algorithm 17: *PRTT* based pseudo-algorithm for preemptive online charging scheduling

input : r_i, p_i, d_i for each vehicle, Δ, N
output: Total tardiness $\sum T_i$

- 1 Let p'_i be the remaining charging time for each electric vehicle;
- 2 $p'_i \leftarrow p_i$ for each vehicle;
- 3 **for** each time slot t **do**
- 4 **for** each line $j = 1, \dots, 3$ **do**
- 5 Get the set I' of ready electric vehicles in the line j (vehicles that have arrived during the interval $[0, t]$ and did not complete charging);
- 6 **end**
- 7 **if** I' is not empty **then**
- 8 **for** each arrived vehicle **do**
- 9 Calculate its *PRTT* then we added to the set I_{prtt} ;
- 10 **end**
- 11 Sort the set I_{prtt} in increasing order;
- 12 Calculate N' the number of electric vehicle that can be added to the schedule in line j at time t without breaking the constraints (6.8), (6.9) and (6.10);
- 13 **for** the first N' electric vehicle in I_{prtt} **do**
- 14 $N_j^t \leftarrow N_j^t + 1$;
- 15 $p'_i \leftarrow p'_i - 1$;
- 16 **if** $p'_i = 0$ **then**
- 17 Calculate the tardiness of the assigned job $T_i = \max(0, t - d_i)$;
- 18 **end**
- 19 **if** the assignment breaks a previous balance constraint **then**
- 20 redo the time slot t ;
- 21 **end**
- 22 **end**
- 23 **end**
- 24 **end**
- 25 **return** $\sum T_i$

6.3.2.2 Local Search

In the previous section, a heuristic based on the *PRTT* dispatching rule is used to obtain the preemptive charging scheduling. To explore solutions with better tardiness, we propose a local search algorithm described in Algorithm 18.

At each time slot t , a partial schedule is built using the *PRTT* based heuristic. At this time, we schedule the arrived electric vehicle in the time horizon \mathcal{H} allowing preemption. Then, a hill-climbing strategy is used to improve the partial schedule. Although this procedure can improve the current partial schedule at the considered time t , it does not necessarily improve the whole final schedule. Given the real-time setting, we can only change the scheduling of electric vehicles in the next time slots. Changes cannot be made for previous time slots t' such $t' < t$.

The fundamental part of any local search method is the neighborhood structure. A neighbor is generated by moving a charging operation from one time slot to another. We consider moves that only can improve the current partial schedule. Since the problem is constrained, we must maintain the schedule's feasibility each time we need to move a charging operation.

For the description of the neighbor generation moves, we call a time slot t a "source" if we can remove a charging operation from it without breaking the balance constraints (6.9) and (6.10). A time slot t is called a "target" if adding a charging operation at t does not exceed its capacity \tilde{N} and does not break constraints (6.9) and (6.10).

- **Shift right:** a neighbor in shift right is generated by moving a random a charging operation of an electric vehicle i in a selected line j from a random source time slot t_1 to a random target time t_2 , $t_2 > t_1$. Since we consider only moves that can improve a partial schedule, t_2 should not be greater than its departure time $t_2 \leq d_i$. We guide the search by first shifting the charging operations that have a high $PRTT$, so other charging operations with lower $PRTT$ can replace it.
- **Shift left:** a neighbor in shift left is generated by moving a charging operation of an electric vehicle i in a selected line j from a random source time slot t_1 to a random target time $t_2 < t_1$ such that $t_2 \geq r_i$. A variant of shift left is implemented considering the tardy electric vehicle ($T_j > 0$). Thus, we shift left the charging operation at its completion time slot.
- **Shift right on three lines:** a neighbor in shift right on the three lines is generated by moving three random charging operations i_1, i_2, i_3 from each line $j = 1, 2, 3$ from a random time slot t_1 to a random time t_2 with $t_2 > t_1$ and $N_j^{t_2} + 1 \leq N$. By moving a charging operation per line, we make sure that the balance constraints (6.9) and (6.10) are maintained.

Shift left on three lines: a neighbor in shift left on 3 lines is generated by moving three random charging operations i_1, i_2, i_3 of each line $j = 1, 2, 3$ at a random time slot t_1 to a random time t_2 with $t_2 < t_1$, $N_j^{t_2} + 1 \leq N$ and $t_2 \leq \min(r_{i_1}, r_{i_2}, r_{i_3})$.

A local search algorithm starts by initializing an empty solution. A solution S is a feasible partial schedule that describes the set of charging operations scheduled in any line at each time slot t , $t \in \mathcal{H}$. Then, at each time slot t , and for each line j , a partial solution S is built by scheduling the charging operation of arrived vehicles preemptively according to their $PRTT$ values in the time slots t' such $t' \geq t$. A shift right move is considered when we have a charging operation of an electric vehicle i that has a lower $PRTT$ value than an already scheduled charging operation at the time slot t' . After scheduling the charging operations in each line, we start exploring the neighborhood of the solution S . A neighbor solution S' is generated using one of the moves described below. S' will replace S if the total tardiness of S' is less or equal to the total tardiness of S ($f(S)$). Since we are in a real-time setting, the stopping criterion will be either a given limited time that does not exceed τ the length of a time slot or that we cannot generate new solutions using one of the moves.

Algorithm 18: Hill-Climbing Pseudo-Algorithm for preemptive online charging scheduling

input : r_i, p_i, d_i for each vehicle, Δ, N
output: Total tardiness $\sum T_i$

- 1 Initialize the solution S ;
- 2 **for** each time slot t **do**
- 3 **for** each line $j = 1, \dots, L$ **do**
- 4 Get the set I' of ready electric vehicles in the line j (vehicles that have arrived during the interval $[0, t]$ and did not complete charging);
- 5 **end**
- 6 **if** I' is not empty **then**
- 7 **for** each ready vehicle **do**
- 8 Calculate its $PRTT$ then we added to the set I_{prtt} ;
- 9 **end**
- 10 Sort the set I_{prtt} in increasing order ;
- 11 **for** each electric vehicle i in I_{prtt} **do**
- 12 $t' \leftarrow t$;
- 13 $p'_i \leftarrow p_i$;
- 14 **while** $t' \leq T$ and $p'_i > 0$ **do**
- 15 **if** a vehicle can be added to the schedule at time t' in line j without breaking the constraints (6.8), (6.9), and (6.10) **then**
- 16 Add the electric vehicle i to the list S_j^t ;
- 17 Decrements p'_i ;
- 18 **else**
- 19 Shift right a job i' with $prtt(i') > prtt(i)$;
- 20 **end**
- 21 $t' \leftarrow t' + 1$;
- 22 **end**
- 23 **end**
- 24 **end**
- 25 **while** not stagnation or time limit not reached **do**
- 26 $S' \leftarrow$ generate a neighbor of S ;
- 27 **if** $f(S') = f(S)$ **then**
- 28 stagnation \leftarrow True;
- 29 **else**
- 30 $S \leftarrow S'$;
- 31 **end**
- 32 **end**
- 33 **end**
- 34 **return** $\sum T_i$

6.4 Computational Results

Table 6.3: Comparison of results of the first scenario.

\tilde{N}	Δ	EV	$PRTT$	$PRTT$ pmtn	HC pmtn
Type 1 Instances					
20	0.2	8386.30	10.53%	-35.07%	-39.69%
20	0.4	4120.40	-3.53%	-26.46%	-31.53%
20	0.6	3670.60	-3.59%	-27.09%	-31.65%
20	0.8	3590.90	-3.79%	-27.01%	-31.42%
30	0.2	1959.30	6.57%	-56.56%	-60.92%
30	0.4	421.20	-2.52%	-68.02%	-82.03%
30	0.6	347.90	8.30%	-67.63%	-84.94%
30	0.8	347.60	8.24%	-67.61%	-84.95%
40	0.2	735.00	-3.01%	-68.03%	-69.77%
40	0.4	14.00	0.71%	-85.71%	-85.71%
40	0.6	3.40	15.00%	-100.00%	-100.00%
40	0.8	3.40	15.00%	-100.00%	-100.00%
Type 2 Instances					
20	0.2	128185.00	0.84%	-6.17%	-5.82%
20	0.4	46319.30	-0.46%	-5.64%	-5.08%
20	0.6	22966.80	-0.22%	-6.06%	-5.91%
20	0.8	14573.10	0.79%	-6.30%	-7.28%
30	0.2	72860.80	-0.31%	-6.07%	-5.53%
30	0.4	21479.90	-0.34%	-6.66%	-6.46%
30	0.6	8088.90	0.94%	-8.57%	-9.52%
30	0.8	4486.30	0.72%	-14.10%	-16.93%
40	0.2	46135.40	-0.48%	-5.85%	-5.27%
40	0.4	10869.30	1.20%	-7.11%	-7.99%
40	0.6	3599.10	2.15%	-14.45%	-18.11%
40	0.8	1635.50	3.86%	-26.54%	-42.75%

In this section, we evaluate the performance of the proposed methods. We consider the benchmarks proposed in [Hernández Arauzo *et al.* 2013]. There are a total number of 180 power outlets in the charging station, in which 60 chargers are connected to each line. The arrival, charging demands, and departure times are generated following normal distributions with different means and deviations that model the electric vehicle charging patterns. Three different scenarios are considered, where 60 instances are generated for each scenario. In each instance, 180 electric vehicles arrive at the station on a time horizon of 24 hours. There are two types of instances according to how the electric vehicles are distributed between the three lines. In the first type instances, the electric vehicles are distributed uniformly: 60 electric vehicles arrive at each line throughout the day. In the second type instances, 60%, 30%, and 10% of electric vehicles arrive at the first, second, and third lines, respectively. This makes the imbalance con-

straints harder to solve. The instances of the first scenario are generated to represent a normal weekday. The instances of the second scenario are obtained by increasing the arrival rate of electric vehicles in a short period of time with different charging and departure times. This makes the charging demands exceed the charging station capacity for these periods. In the third scenario, the instances are generated as in the second scenario but with tighter departure times, making it the most challenging scenario to solve.

Note that the algorithms are implemented in Python 3.7 and run on an Intel Core i5-7440HQ (4 CPUs) operating at 2.8 GHz and 8 GB RAM.

Table 6.4: Comparison of results of the second scenario.

\tilde{N}	Δ	EV	<i>PRTT</i>	<i>PRTT</i> pmtn	HC pmtn
Type 1 Instances					
20	0.2	16886.00	5.30%	-19.21%	-20.27%
20	0.4	14131.20	-0.68%	-17.41%	-18.29%
20	0.6	13648.30	-0.52%	-17.35%	-18.06%
20	0.8	13547.90	-0.48%	-17.34%	-18.07%
30	0.2	6394.70	-0.71%	-31.71%	-42.00%
30	0.4	4824.30	0.75%	-33.92%	-49.33%
30	0.6	4668.60	0.41%	-34.51%	-51.19%
30	0.8	4672.60	0.45%	-34.57%	-51.28%
40	0.2	1951.40	2.48%	-50.95%	-61.59%
40	0.4	1212.70	7.72%	-59.98%	-86.77%
40	0.6	1210.70	7.17%	-60.52%	-89.48%
40	0.8	1210.60	7.18%	-60.52%	-89.51%
Type 2 Instances					
20	0.2	143883.00	1.57%	-4.60%	-6.10%
20	0.4	62246.20	-0.38%	-4.42%	-3.62%
20	0.6	39869.40	-0.45%	-5.31%	-4.12%
20	0.8	30887.20	-0.57%	-6.90%	-5.23%
30	0.2	86392.00	0.45%	-4.79%	-4.25%
30	0.4	34475.40	-0.50%	-5.87%	-4.97%
30	0.6	19355.70	-0.35%	-8.92%	-8.05%
30	0.8	14375.10	-0.18%	-12.22%	-11.89%
40	0.2	59775.00	0.15%	-4.90%	-4.15%
40	0.4	22254.00	-0.25%	-7.80%	-6.96%
40	0.6	10991.30	-0.30%	-12.17%	-12.22%
40	0.8	7260.50	0.28%	-18.22%	-19.13%
40	0.4	1212.70	7.72%	-59.98%	-86.77%
40	0.6	1210.70	7.17%	-60.52%	-89.48%
40	0.8	1210.60	7.18%	-60.52%	-89.51%

Table 6.3 , Table 6.4 and Table 6.5 shows the comparison of results obtained for scenarios 1, 2 and 3 respectively with those obtained in [Hernández-Arauzo *et al.* 2015] by the heuristic

"EV". We refer to the proposed heuristic for non-preemptive charging scheduling by "*PRTT*", by "*PRTT* pmtn" for preemptive one, and by "HC pmtn" for the hill-climbing preemptive scheduling. We have 30 instances for each group (Scenario, type). For charging station parameters, \tilde{N} varies between 20, 30 and 40 and Δ varies between 0.2, 0.4, 0.6 and 0.8. The tardiness is calculated for each instance and then summed up for each group. Then, the decrease (or increase) in the total tardiness of the proposed methods compared to the total tardiness in [Hernández-Arauzo *et al.* 2015] is calculated and reported in percentage. The percentage will be negative if there is a decrease in total tardiness, i.e., the solutions are better and positive otherwise.

Table 6.5: Comparison of results of the third scenario.

\tilde{N}	Δ	EV	<i>PRTT</i>	<i>PRTT</i> pmtn	HC pmtn
Type 1 Instances					
20	0.2	20704.7	7.20%	-10.36%	-8.54%
20	0.4	18001.7	-0.68%	-8.57%	-4.87%
20	0.6	17528.2	-0.61%	-8.58%	-4.08%
20	0.8	17463.3	-0.57%	-8.75%	-3.97%
30	0.2	9051.1	-0.30%	-15.33%	-15.81%
30	0.4	7347.3	-0.76%	-12.13%	-11.82%
30	0.6	7150.4	-0.17%	-11.90%	-11.01%
30	0.8	7144.5	-0.17%	-11.87%	-11.01%
40	0.2	3478.1	0.61%	-23.76%	-31.48%
40	0.4	2373.1	-0.63%	-14.74%	-29.56%
40	0.6	2276.7	0.39%	-12.36%	-28.51%
40	0.8	2276.7	0.39%	-12.36%	-28.51%
Type 1 Instances					
20	0.2	138064.00	1.82%	-5.97%	-7.80%
20	0.4	62988.80	-1.27%	-4.56%	-3.75%
20	0.6	42528.30	-0.59%	-4.69%	-2.76%
20	0.8	33998.20	-0.44%	-5.33%	-2.21%
30	0.2	83169.50	0.93%	-5.78%	-5.29%
30	0.4	34799.70	-0.17%	-5.32%	-4.01%
30	0.6	21321.10	-0.66%	-5.70%	-3.71%
30	0.8	16972.00	-0.73%	-6.80%	-4.06%
40	0.2	58306.30	0.09%	-5.62%	-4.95%
40	0.4	22988.70	-0.60%	-5.93%	-4.28%
40	0.6	12220.30	-0.47%	-6.24%	-4.91%
40	0.8	9127.30	-0.59%	-8.05%	-6.29%

For the first scenario, the *PRTT* outperforms the heuristic EV in 10 groups out of 24 groups, whereas it outperforms the heuristic EV in 12 groups out of 24 in the second scenario and 18 groups out of 24 in the third scenario. This shows that our heuristic has better objective values when scheduling electric vehicles that arrive simultaneously with tighter departure times.

For all scenarios, the *PRTT* pmtn and the HC pmtn consistently outperform the heuristic EV since scheduling the preemption will allow an electric vehicle with high priorities with tighter departure times and shorter charging times to charge instead. Thus, Relaxing the preemption constraints is not redundant, and it will have a great advantage in minimizing total tardiness. Comparing the results between the two types of instances, we notice that the decrease in total tardiness in type 1 instances is more significant than in type 2 instances. For *PRTT* pmtn, The total tardiness was averagely lower by 36.61% in type 1 instances, while it was averagely lower by 7.76% in type 2 instances. For HC pmtn, The total tardiness was averagely lower by 44.70% in type 1 instances, while it was averagely lower by 7.83% in type 2 instances.

About the comparison between the *PRTT* pmtn and the HC pmtn, the tardiness of HC pmtn is better in 39 groups and worse in 33. We notice that the local search is better in type one instances, especially in the first and second scenarios, and slightly worse in the third. This is because charging operations in the third scenario have tighter departure times, so it is harder to find moves that improve the whole schedule's total tardiness. However, the overall tardiness was reduced by 26.26% using HC pmtn on average, whereas it was reduced by 22.2% using *PRTT* pmtn.

6.5 Conclusion

In this chapter, we proposed heuristics and a local search procedure to solve the online charging scheduling problem while maintaining the balance in the three phases system. We first formulate the offline problem as LP model. To solve the problem in a real-time setting, we proposed a new heuristic based on *PRTT* dispatching rule. Computational results show that our heuristic outperforms other heuristics in most instances. Moreover, the computation time is relatively negligible, which is suitable for real-time scheduling. Also, the effectiveness of preemptive scheduling in reducing total tardiness is shown through the computational results.

General conclusion and perspectives

In order to decarbonize road transport and promote the utilization of green energy, research on electric vehicle technologies has gained considerable attention. Over the years, the market for electric vehicles has been expanding remarkably. Nonetheless, more smart charging infrastructures are required to meet the increase in electricity demands in the next decade. Implementing smart charging strategies to schedule the electric load is critical to avoid negative impacts on the power grid. Hence, this thesis addressed the electric vehicle charging scheduling problem.

In the state-of-the-art, we briefly revisited the classic job scheduling problem. Then, we gave a brief overview of electric vehicle technologies. Also, we pointed out the large variety in the formulation of the electric vehicle charging scheduling problem depending on charging models, objectives, considerations, and constraints.

Then, we addressed the electric vehicle charging problem in a charging station with limited grid capacity and a limited number of chargers delivering power at constant rates. All vehicles must submit a charging demand reservation. Initially, we studied the problem of charging all vehicles with the minimum grid capacity in the case of identical and non-identical chargers. We proved that the problem is polynomial solvable in the first case, while it was \mathcal{NP} -hard in the second one. Mathematical formulation and solving methods were provided and compared. Simulation results suggest that installing chargers with different charging power rates is more suitable. When the grid capacity is not sufficient to meet all charging demands, we aim to maximize the number of accepted ones. This problem is proved to be \mathcal{NP} -hard for both identical and non-identical chargers. Different mathematical formulations, heuristics, and metaheuristics were proposed to tackle it.

Next, the charging problem with variable power rates is studied and compared to the one with constant power rates. The charging station operating model was slightly different. A relaxation on arrival time was considered, where vehicles can be plugged later than the desired arrival. The objective was to maximize the state of charge at departure. We proved that the problem was \mathcal{NP} -hard for both constant and variable power models. We proposed a mixed-integer linear programming models, heuristics, and metaheuristics to solve it. Simulation results revealed that solving the linear programming models with a commercial solver was almost impossible, even for small instances. Moreover, using variable charging power rates was more efficient than constant ones in delivering more energy with the same grid capacity.

All the mentioned charging problems were preemptive. We compared preemptive and non-preemptive charging scheduling in an online setting in an unbalanced three-power system previously stated in the literature. We provided fast heuristics and a simple local search. We showed how preemption is more efficient in optimizing the schedule even with simple methods.

Table 7.1 summarizes objectives, constraints, complexity results, and solving methods for all electric vehicle charging scheduling problems addressed in this thesis.

Table 7.1: Summary of EVCS problem addressed in this thesis.

	Chapter 3		Chapter 4		Chapter 5	Chapter 6	
Chargers	identical	non-identical	identical	non-identical	non-identical	identical	
Assignment to chargers	Yes					No	
Preemption	Yes					Yes	No
Departure times	Hard constraints					Soft constraints	
Charging rates	Constant				Constant	Variable	Constant
Arrival times	Hard constraints				Soft constraints		Hard constraints
Energy demands	Hard constraints				Soft constraints		Hard constraints
Objective	Minimize the grid capacity		Maximize the number of accepted demands		Maximize the delivered energy		Minimize the total tardiness
Complexity class	\mathcal{P}		\mathcal{NP}				
Solving Methods	Max Flow	Heuristics MILP, ILS+SA	ILP, heuristics	ILP, SA+TsLS, heuristics	SA+MILP heuristics	Online heuristics and LS	Online heuristics

Before ending this dissertation, we state the following perspectives:

- The presented work could be taken as a starting point for developing a more complex optimization model by including more constraints related to electric vehicle technology, such as nonlinear charging times, charging efficiency, and charger compatibility. This will provide more precise models that reflect the various requirements of electric vehicles.
- The proposed system can be extended to handle charging demands with or without reservation. In addition, it is interesting to handle cases where the vehicle makes a reservation but never shows up and also where the vehicle might leave later or earlier than planned. In this case, online scheduling algorithms can be developed and compared to the results of the offline ones proposed in this thesis.
- In this thesis, we assumed preemptive charging. Even though it is more flexible and beneficial for handling constraints and optimizing the objective function, as we observed in Chapter 6, frequent preemptions may introduce extra deterioration for the batteries. In particular, manufacturers of electric vehicles recommend that a charging phase last at least 15 minutes to avoid undesirable chemical reactions in lithium-ion batteries. In our model, the problem can be avoided by just setting the time slot to 15 minutes, but still, we lose accuracy. Therefore, more research should be done to tackle this problem.
- The model could be extended to consider a network of charging stations. In this case, choosing a charging station will depend on the distance, the energy required to reach the station, the availability of chargers, and the remaining energy.
- Considering other optimization methods, namely population-based metaheuristics, could be interesting. Metaheuristics based on a single solution were more convenient for finding good solutions in a reasonable amount of time, especially since evaluating solutions after perturbation was complex and time-consuming. In this case, using parallelisms and multi-threaded algorithms could be helpful.
- Another idea would consist of considering multi-objective by adding the minimization of the charging electricity bills the charging station pays or maximizing its profit. In

particular, under Time-of-Use pricing, charging vehicles when electricity costs are low should be privileged.

- When the assignment of vehicles to chargers is known, the class complexity of the power allocation problem with different constant power rates is not yet determined.
- For the problem with an unbalanced power system, it would be more interesting to consider chargers with different power rates. Since single-phase charging is slow charging, we can only consider chargers up to 7.4 kW. Moreover, the imbalance constraint cannot be expressed only with the number of vehicles. More complex measures such as VUF (Voltage Unbalance Factor) can be used. In this case, the voltage of each phase (line) should be measured.
- It could be relevant to model the charging station on a simulator and implement the optimization algorithms to validate an conduct more realistic tests.

Appendices

Results For Lower Bound on w_G

Table A.1: Results for instances of group 1 ($n = 10$)

Instances	n	lb (eq. (3.12))	lb(Cplex)	w_G (Cplex)	G_{lb}
1	10.00	35.00	43.00	43.00	-18.60
2	10.00	43.00	43.00	43.00	0.00
3	10.00	36.00	43.00	43.00	-16.28
4	10.00	46.00	52.67	54.00	-14.81
5	10.00	42.00	53.99	54.00	-22.22
6	10.00	40.00	43.00	43.00	-6.98
7	10.00	42.00	43.00	43.00	-2.33
8	10.00	52.00	53.99	54.00	-3.70
9	10.00	50.00	53.84	54.00	-7.41
10	10.00	42.00	43.00	43.00	-2.33
11	10.00	41.00	43.00	43.00	-4.65
12	10.00	35.00	43.00	43.00	-18.60
13	10.00	39.00	43.00	43.00	-9.30
14	10.00	35.00	43.00	43.00	-18.60
15	10.00	42.00	43.95	44.00	-4.55
16	10.00	38.00	43.00	43.00	-11.63
17	10.00	36.00	43.00	43.00	-16.28
18	10.00	28.00	33.00	33.00	-15.15
19	10.00	42.00	43.00	43.00	-2.33
20	10.00	32.00	43.00	43.00	-25.58
21	10.00	50.00	54.00	54.00	-7.41
22	10.00	43.00	43.00	43.00	0.00
23	10.00	36.00	43.00	43.00	-16.28
24	10.00	36.00	43.00	43.00	-16.28
25	10.00	41.00	43.00	43.00	-4.65
26	10.00	41.00	43.00	43.00	-4.65
27	10.00	44.00	54.00	54.00	-18.52
28	10.00	32.00	43.00	43.00	-25.58
29	10.00	41.00	43.00	43.00	-4.65
30	10.00	40.00	43.00	43.00	-6.98
31	10.00	50.00	53.99	54.00	-7.41
32	10.00	41.00	43.00	43.00	-4.65
33	10.00	50.00	54.00	54.00	-7.41
34	10.00	47.00	53.99	54.00	-12.96
35	10.00	35.00	43.00	43.00	-18.60
36	10.00	38.00	43.00	43.00	-11.63
37	10.00	36.00	43.00	43.00	-16.28
38	10.00	40.00	43.00	43.00	-6.98
39	10.00	38.00	43.00	43.00	-11.63
40	10.00	41.00	43.00	43.00	-4.65
41	10.00	39.00	43.00	43.00	-9.30
42	10.00	34.00	43.00	43.00	-20.93
43	10.00	41.00	43.00	43.00	-4.65
44	10.00	51.00	54.00	54.00	-5.56
45	10.00	36.00	43.00	43.00	-16.28
46	10.00	40.00	43.00	43.00	-6.98
47	10.00	41.00	43.00	43.00	-4.65
48	10.00	39.00	43.00	43.00	-9.30
49	10.00	40.00	43.00	43.00	-6.98
50	10.00	38.00	43.00	43.00	-11.63

Résumé étendu en français

B.1 Introduction Générale

B.1.1 Contexte et motivation

Le transport est le secteur qui émet le plus de gaz à effet de serre. En 2021, les transports représentent 40% des émissions de CO_2 en France, où 82% de ces émissions proviennent du transport routier ¹. Ainsi, l'adoption des véhicules électriques comme technologie alternative aux véhicules thermiques apparaît comme une des solutions majeures non seulement face à la crise climatique, mais aussi aux problèmes de hausse des prix des énergies fossiles. Selon l'Agence internationale de l'énergie (IEA) [IEA 2022], le nombre de véhicules électriques a atteint 16,5 millions en 2021, soit deux fois plus qu'en 2019. Les ventes de véhicules électriques continuent de battre des records année après année. En 2021, près de 10% des ventes de véhicules étaient des modèles électriques, soit une part de marché quatre fois supérieure à celle de 2019.

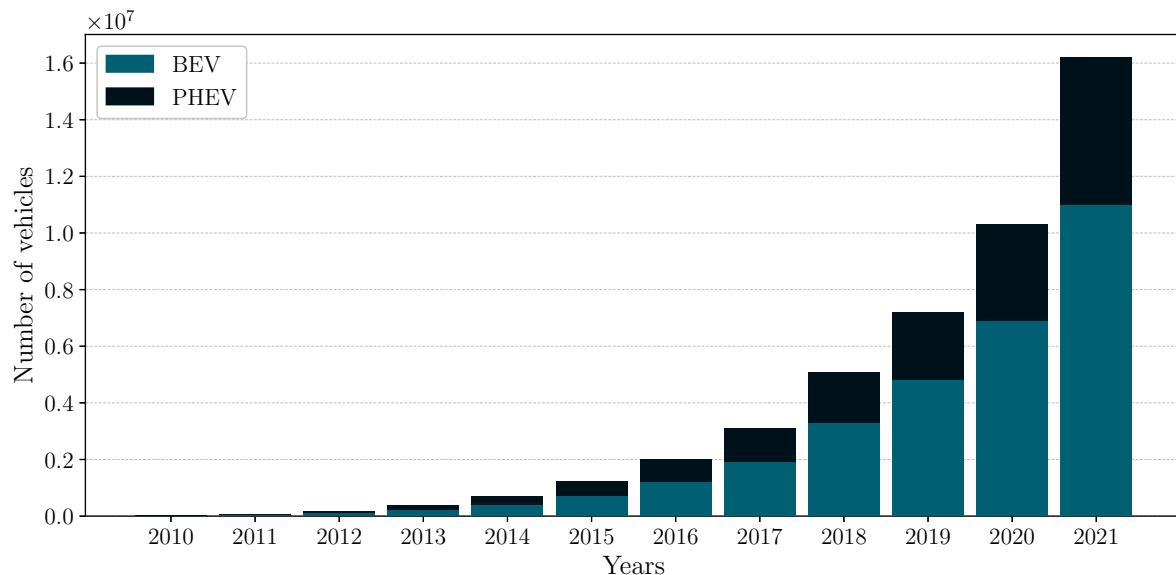


Figure B.1: Nombre de véhicules électriques dans le monde, 2010-2021. Les BEV représente les véhicules électriques purs qui embarquent une batterie rechargeable. Les PHEV sont les véhicules électriques hybrides rechargeables. *Source*: IEA 2022 [IEA 2022].

¹<https://www.climate-transparency.org/countries/europe/france>

Avec les ventes des véhicules électrique en hausse, il est nécessaire d'investir davantage pour déployer vigoureusement des infrastructures de recharge, qui sont essentielles pour garantir la compétitivité des véhicules électriques. De nos jours, la plupart des propriétaires de véhicules électriques rechargent leur véhicule à la maison. Toutefois, ils considèrent que la recharge publique est indispensable, notamment sur les lieux de travail. De plus, l'augmentation du déploiement des bornes de recharge publiques facilitera les trajets plus longs, réduira l'anxiété liée à l'autonomie et convaincra davantage de consommateurs d'acheter un véhicule électrique, en particulier ceux qui n'ont pas accès à une recharge privée. En 2021, plus de 1,8 million de bornes de recharge publiques ont été installées dans le monde, ce qui représente une augmentation de 37% par rapport à l'année précédente [IEA 2022].

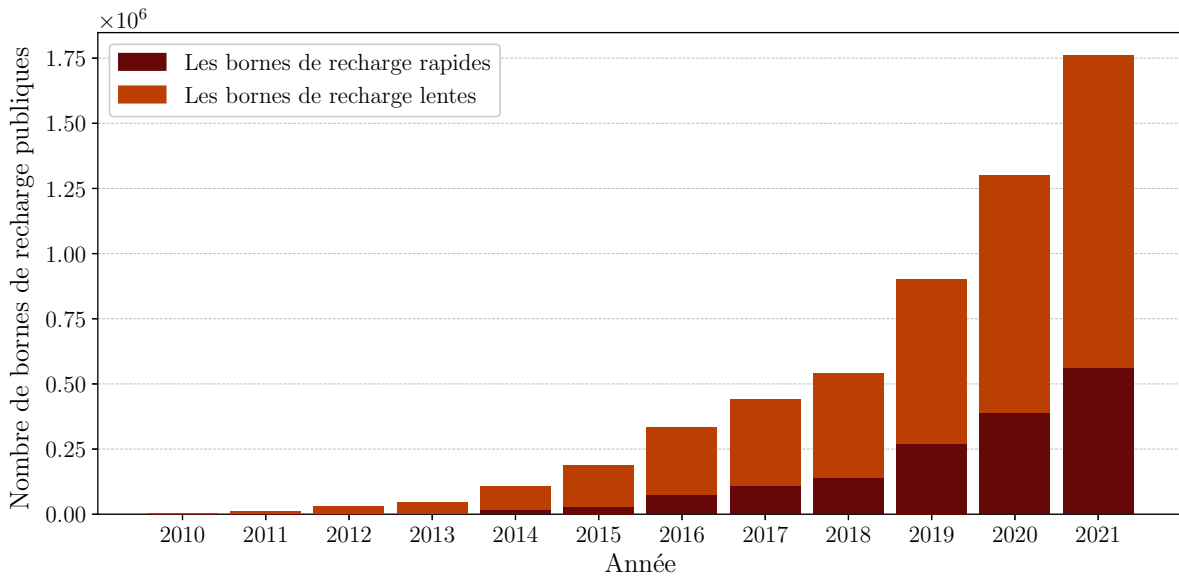


Figure B.2: Le nombre total de bornes de recharge installées. Les bornes de recharge rapide ont une puissance de charge supérieure à 22 kW. Les bornes de recharge lente ont une puissance de charge inférieure à 22 kW. *Source:* IEA 2022 [IEA 2022].

L'adoption à grande échelle des véhicules électriques présente de nouveaux défis pour les gestionnaires du réseau électrique et les opérateurs des stations de recharge. D'une part, la consommation importante d'énergie liée aux demandes de recharge impactera négativement le réseau électrique et augmentera les pics de consommation. D'autre part, les opérateurs des infrastructures de recharge doivent répondre aux demandes à venir, maximiser la satisfaction des clients, éviter les longues files d'attente et minimiser les coûts, tout en respectant les contraintes liées au réseau électrique. En d'autres termes, ces opérateurs doivent minimiser la consommation d'énergie en satisfaisant de multiples demandes de recharge. Il est donc crucial pour ces opérateurs d'inclure des stratégies d'optimisation. Récemment, plusieurs études sur le développement de stratégies de planification de la recharge des véhicules électriques ont émergé. On peut ainsi classer ces études en deux principales catégories. La première regroupe les études menées du point de vue des opérateurs du réseau électrique, dont l'objectif est d'améliorer et d'assurer la

fiabilité du réseau électrique. Dans la seconde catégorie d'études, on trouve celles menées du point de vue des opérateurs de stations de recharge, qui se concentrent plutôt sur des objectifs économiques tels que la minimisation des coûts d'électricité et la maximisation de la satisfaction des clients.

Dans le cadre de cette thèse, nous abordons exclusivement la planification de la recharge des véhicules électriques du point de vue des opérateurs des stations de recharge. Plus précisément, nous considérons une station de recharge disposant d'un nombre limité de bornes de recharge, pouvant être identiques (c'est-à-dire pouvant délivrer la même puissance) ou non identiques. Cette station dispose également d'une puissance maximale allouée à l'ensemble des bornes pour éviter la surcharge du réseau électrique. Les demandes de recharge des véhicules électriques sont caractérisées par leur heure d'arrivée, leur heure de départ et leur demande en énergie. L'objectif est de planifier la recharge de ces véhicules en respectant les différentes contraintes, qu'elles soient liées aux demandes (temps d'arrivée, temps de départ, quantité d'énergie, etc.) ou aux ressources de la station de recharge (nombre de bornes, disponibilité des bornes, puissances des bornes, puissance maximale, etc.).

Les travaux présentés dans la littérature proposent des stratégies efficaces pour optimiser la planification de la recharge des véhicules électriques. Cependant, les décisions relatives à l'affectation des véhicules aux bornes de recharge appropriées semblent être négligées. De plus, ces travaux supposent que les bornes de recharge ont une puissance de sortie maximale identique, mais qui peut être variable dans le temps. Néanmoins, les stations de recharge installent généralement des bornes de recharge avec des puissances différentes pour satisfaire davantage de demandes de recharge et améliorer la qualité de service. De plus, les stations de recharge proposent des différents services selon leurs modes opérationnels. On peut trouver des stations publiques sans ou avec réservations, disposant de bornes identiques ou non, ayant une puissance constante ou variable, etc. La variation du mode de fonctionnement affecte le problème d'optimisation et sa complexité. Il est donc nécessaire d'étudier le problème en considérant les différentes configurations.

B.1.2 Contributions

La formulation générale d'une instance du problème de planification de la charge des véhicules électriques peut être définie comme suit. Considérons un ensemble de demandes de recharge de véhicules électriques et un ensemble de bornes de recharge. La puissance totale délivrée par toutes les bornes de recharge ne doit pas dépasser un certain seuil à tout moment, que nous appelons la capacité du réseau. Chaque demande de recharge est caractérisée par son heure d'arrivée, son heure de départ et ses besoins en énergie. La quantité d'énergie demandée peut être exprimée directement en kWh ou en précisant l'état de charge (State-of-Charge - SOC) souhaité ainsi la capacité de la batterie. À chaque instant, une borne de recharge ne peut charger qu'un seul véhicule et un véhicule ne peut être chargé que par une seule borne de recharge. La recharge des véhicules peut être soit préemptive, où la recharge d'un véhicule peut être arrêter à tout moment et reprise plus tard, soit non préemptive ou l'interruption de la recharge d'un véhicule est interdite. De plus, en fonction des taux de puissance des bornes de recharge, il est possible de distinguer deux cas. Le premier, que nous noterons "la recharge à

puissance constante", est caractérisé par des bornes de recharge qui délivrent soit leur puissance maximale, soit zéro. La seconde cas, "la recharge à puissance variable", permet quant à elle au taux de charge de chaque borne de recharge de varier dans le temps, de zéro à sa puissance maximale.

Nous commençons par nous intéresser à la recharge à puissance constante et préemptive. Nous considérons une station de recharge équipée d'un système de réservation. Un client souhaitant recharger son véhicule doit faire une demande de réservation en précisant l'heure d'arrivée, l'heure de départ ainsi que la quantité d'énergie requise. Dans un premier temps, nous supposons que lorsqu'une demande de recharge est acceptée, le véhicule doit être connecté à une borne de recharge du début à la fin de la période de recharge, et sa demande d'énergie doit être satisfaite. Le système de gestion de la station de recharge collecte toutes les demandes et doit les planifier en fonction des ressources disponibles de la station (nombre de bornes, puissances disponibles et la capacité du réseau). Ainsi, deux questions se posent :

1. Quel est le nombre minimum de bornes de recharge nécessaires pour brancher tous les véhicules électriques de leurs arrivées à leurs départ?
2. Quelle est la capacité minimale de réseau requise pour charger tous les véhicules électriques jusqu'à leur niveau d'énergie souhaité ?

Après avoir résolu ces problèmes, nous abordons les problèmes opérationnels. Lorsque la planification de toutes les demandes est impossible, nous cherchons à maximiser le nombre de demandes de recharge satisfaites. Ces problèmes sont étudiés pour les bornes de recharge identiques et non identiques.

Ensuite, nous étudions une autre variante du problème avec un mode opérationnel différent. Pour charger plus de véhicules, l'énergie demandée n'est pas nécessairement satisfaite dès le départ, mais est maximisée. De plus, la contrainte qui impose de brancher le véhicule dès son arrivée est relaxée. Le système choisira l'heure à laquelle le véhicule sera branché, qui peut être postérieure à l'heure d'arrivée souhaitée, et le véhicule occupera la borne de recharge jusqu'à son heure de départ. Dans ce cas, l'objectif de la planification consiste à réduire autant que possible la différence entre les états de charge (SOC) souhaités et les états de charge réels au moment du départ. De plus, nous comparons les deux modèles de recharge, à savoir la recharge à puissance constante et celle à puissance variable.

B.1.3 Contenu de la thèse

La motivation et le contexte de la thèse sont présentés dans le chapitre introductif, Chapitre 1. Dans le deuxième chapitre (Chapitre 2), nous présentons un état de l'art essentiel sur les problèmes de planification de la recharge des véhicules électriques. Avant cela, nous résumons les concepts d'ordonnancement et de planification en général, ainsi que la technologie des véhicules électriques, afin d'établir les bases nécessaires à la compréhension des problématiques abordées dans le cadre de cette thèse.

La première variante du problème de planification de la recharge des véhicules électriques est présenté et étudié dans le Chapitre 3. Le Chapitre 4 est consacré au problème de maximisation du nombre de demandes de recharge satisfaites. Le Chapitre 5 aborde le problème de maximisation

de l'énergie fournie et compare les modèles de puissance constante et de puissance variable. Le Chapitre 6 est consacré au problème de planification de la recharge en ligne dans un système de puissance triphasé déséquilibré. Enfin, dans le dernier chapitre, nous résumons les résultats de notre travail et décrivons les perspectives pour les prochaines études.

B.2 État de l'art

Cette section fournit le contexte, la terminologie et les travaux pertinents nécessaires pour une meilleure compréhension des problèmes de planification de la recharge des véhicules électriques (EVCS) abordés dans cette thèse. L'EVCS est un problème d'optimisation étroitement lié au problème d'ordonnancement de tâches sous contraintes de ressources. Par conséquent, nous présentons un bref aperçu de ce dernier pour comprendre certaines preuves de complexité et algorithmes présentés dans cette thèse. Ensuite, nous passons brièvement en revue les technologies des véhicules électriques, telles que les modes de recharge et les différentes entités impliquées. Une partie essentielle de cette section est consacrée à la classification et à la revue de différents travaux sur les problèmes de planification de la recharge des véhicules électriques. Ainsi, nous pouvons identifier notre problème et déterminer les domaines de recherche qui nécessitent d'être explorés pour combler les lacunes.

B.2.1 Optimisation et problèmes d'ordonnancement de tâches

Un problème d'optimisation consiste à trouver la meilleure solution parmi un ensemble de solutions possibles, qui optimise une ou plusieurs fonctions objectives tout en respectant des contraintes. Les problèmes d'ordonnancement, qui forment une classe de ces problèmes d'optimisation, impliquent l'organisation de tâches (également appelées activités) avec des contraintes de temps et de ressources, afin d'optimiser une ou plusieurs fonctions objectives. Il existe de nombreuses variantes de problèmes d'ordonnancement, selon la nature des tâches, des ressources, des contraintes et de la fonction objectif. Dans la théorie de l'ordonnancement, les tâches présentent des caractéristiques qui varient selon le problème considéré. Une tâche peut être définie par sa date de début, sa date de fin et sa durée d'exécution. Selon la nature du problème, certaines activités doivent être exécutées sans interruption, ce qui est appelé une activité non préemptive. Néanmoins, si les activités peuvent être interrompues et exécutées en plusieurs parties, on parle alors d'une activité préemptive. Les activités peuvent nécessiter une ou plusieurs ressources durant leur exécution, et il est nécessaire de modéliser l'allocation de chaque ressource à chaque activité. Les ressources utilisées par les activités peuvent être de nature diverse, telles que des ressources renouvelables (qui peuvent être réutilisées dès qu'elles sont libérées) ou des ressources consommables (dont la consommation globale est limitée dans le temps). Parmi les ressources renouvelables, on trouve les machines. Dans ce cas, on parle de problèmes d'atelier. On parle de problème à une machine lorsque toutes les tâches doivent être exécutées sur une seule machine. Toutefois, si plusieurs machines sont disponibles pour l'exécution des tâches, on parle de problèmes à machines parallèles. Des contraintes supplémentaires peuvent être définies en matière d'ordonnancement, telles que les contraintes de précedence, les contraintes de disponibilité et d'échéance, les contraintes de partage de ressources, etc. De nombreux objectifs différents

sont également proposés dans la littérature, tels que la minimisation du nombre de tâches en retard, la minimisation du makespan qui correspond à la durée totale de l'exécution de toutes les tâches, la maximisation de la capacité de production, etc.

Les auteurs de [Graham *et al.* 1979b] ont introduit la notation $\alpha | \beta | \gamma$ pour classer les problèmes d'ordonnancement. Le champ α définit l'environnement de la machine. Les caractéristiques de la tâche et les contraintes d'ordonnancement sont spécifiées par le champ β , tandis que le champ γ définit la fonction objectif. Par exemple, $P | r_i | C_{max}$ concerne l'ordonnancement sur machines parallèles avec des tâches ayant des dates de début pour minimiser le makespan. Un livre complet sur les problèmes d'ordonnancement classiques avec plus de détails peut être trouvé dans [Leung 2004].

Il convient de mentionner que l'utilisation des machines pour l'exécution des tâches n'est pas nécessairement considérée pour tous les problèmes d'ordonnancement. Une branche entière de la théorie de l'ordonnancement appelée problème d'ordonnancement de projet à contraintes de ressources (RCPSP) n'utilise pas systématiquement des machines. Le RCPSP peut être défini comme suit. On dispose d'un ensemble de tâches à ordonnancer et d'un ensemble de ressources renouvelables ayant des capacités qui définissent leurs quantités disponibles tout au long du projet. Chaque tâche a une durée et une quantité de ressources à consommer par unité de temps. De plus, il existe des contraintes de précédence entre ces tâches. L'objectif principal du problème est de planifier toutes les tâches sans dépasser la capacité des ressources afin que la durée totale du projet soit minimisée. Ce problème est prouvé être \mathcal{NP} -difficile [Blazewicz *et al.* 1983]. Un état de l'art des variantes et des extensions du problème RCPSP peut être trouvé dans [Hartmann & Briskorn 2010]. Au cours des dernières décennies, le RCPSP a suscité l'intérêt des chercheurs car il englobe une large classe de problèmes d'ordonnancement, tels que les problèmes de job shop et de flow shop. Un cas spécial intéressant du RCPSP est l'ordonnancement cumulatif, dans lequel une date de début et une date d'échéance sont ajoutées pour chaque tâche, les contraintes de précédence sont relâchées et une seule ressource cumulée est considérée à la fois. Le problème d'ordonnancement cumulatif (CuSP) est \mathcal{NP} -complet [Baptiste & Le Pape 1997]. Récemment, les auteurs de [Nattaf *et al.* 2015] ont considéré une variante de l'ordonnancement cumulatif avec une ressource cumulée, continue et renouvelable, et ont présenté une méthode hybride de branch-and-bound pour le résoudre.

Le raisonnement énergétique, introduit par [Lopez 1991], est un outil efficace pour traiter le CuSP. Le raisonnement énergétique repose sur la comparaison entre l'énergie que doit consommer un ensemble de tâches entre deux instants et les ressources disponibles. Plus tard, il a été adapté par [Baptiste & Le Pape 1997, Nattaf *et al.* 2015] pour développer un test de satisfiabilité polynomiale pour leurs problèmes. Lorsque la consommation des ressources est uniforme et qu'il n'y a aucune contrainte sur leur capacité, le problème se transforme en un problème d'ordonnancement par intervalle, qui peut être résolu en temps polynomial.

Cette thèse se concentre sur le problème de l'ordonnancement de la recharge des véhicules électriques (EVCS) dans des stations de recharge avec un nombre fixe de bornes de recharge (points de recharge) et une puissance disponible. Ce problème peut être considéré comme un problème d'ordonnancement avec contraintes de ressources, dans lequel les travaux à ordonnancer sont les demandes de recharge et les ressources sont les places de stationnement, les bornes de recharge et l'énergie électrique disponible.

B.2.2 Problèmes d'ordonnement de la recharge des véhicules électriques

Les véhicules électriques peuvent être classés en deux catégories principales: les véhicules électriques hybrides (HEV) et les véhicules entièrement électriques (AEV). Un HEV combine un moteur à combustion interne avec un moteur électrique. Il ne peut pas être branché pour recharger sa batterie à partir de sources externes telles que le réseau électrique. Il est rechargé par le moteur à combustion interne ou grâce à un mécanisme de récupération d'énergie appelé freinage régénératif. Ce mécanisme convertit l'énergie cinétique du véhicule en énergie électrique qui est stockée dans la batterie. Un véhicule électrique hybride rechargeable (PHEV) est un type de HEV avec une batterie plus grande qui peut être rechargée depuis le réseau électrique.

D'autre part, un AEV, également appelé véhicule électrique pur, est alimenté exclusivement par des sources électriques. Nous pouvons classer les AEV en véhicules électriques à batterie (BEV) et en véhicules électriques à pile à combustible (FCEV). Un BEV dépend de l'énergie fournie par la batterie, qui est rechargée en la branchant sur le réseau électrique. Un FCEV ne nécessite pas de système de charge externe. Généralement, il génère de l'électricité en utilisant l'oxygène de l'air et de l'hydrogène comprimé.

Dans cette thèse, nous utilisons le terme "véhicule électrique" pour désigner à la fois les BEV et les PHEV.

De nos jours, les conducteurs de véhicules électriques ont principalement tendance à charger leurs véhicules directement lorsqu'ils arrivent chez eux [Azadfar *et al.* 2015] ou à la station de recharge la plus proche [Azadfar *et al.* 2015]. La recharge commence immédiatement lorsque le véhicule est branché et se poursuit jusqu'à ce que la batterie soit complètement chargée. On appelle cela une recharge non contrôlée. La recharge non contrôlée a un impact négatif sur le réseau électrique, l'infrastructure de recharge et les conducteurs de véhicules. Par conséquent, des approches de recharge intelligente dans le déploiement à grande échelle de véhicules électriques sont cruciales pour une utilisation appropriée de l'énergie, la stabilité du réseau et la satisfaction des consommateurs. Différentes stratégies de contrôle et de planification de la recharge ont été proposées. Il existe deux types de stratégies de contrôle : centralisées et décentralisées. Dans les stratégies centralisées, les décisions de recharge sont prises par un contrôleur de haut niveau, tel que des agrégateurs. À l'opposé, les stratégies de contrôle décentralisées permettent à chaque véhicule électrique de définir son propre plan de recharge. Les deux stratégies ont leurs avantages et leurs limites. Les approches centralisées nécessitent des temps de calcul longs et rencontrent des difficultés pour collecter des informations de recharge précises auprès de nombreux véhicules électriques en raison des incertitudes liées à leur comportement [Kang *et al.* 2016]. Les stratégies de contrôle décentralisées réduisent les exigences de calcul car le calcul est réparti entre les agents individuels. De plus, elles améliorent la commodité et la satisfaction des clients. Cependant, les stratégies décentralisées peuvent entraîner une utilisation inefficace des ressources disponibles et des conflits de charge. De plus, ils peuvent rencontrer des limitations en termes d'évolutivité lorsque le nombre de véhicules électriques augmente. La coordination et la gestion des plans de recharge individuels deviennent plus complexes à mesure que le système s'agrandit, ce qui peut affecter la scalabilité du système de recharge.

Dans cette section, nous nous concentrons uniquement sur les problèmes liés à la planification de la recharge des véhicules électriques de manière centralisée. De plus, nous n'incluons pas les articles traitant de la technologie de véhicule-réseau (V2G), où les véhicules électriques peuvent

restituer de l'énergie au réseau. Seuls les modèles de flux d'énergie unidirectionnels sont pris en compte. De plus, nous n'incluons pas les articles traitant des sources d'énergie renouvelable, telles que l'énergie solaire et les unités de stockage d'énergie.

Dans la littérature, la formulation du problème de planification de la recharge des véhicules électriques (EVCS) varie considérablement d'une étude à l'autre, en fonction des caractéristiques de l'infrastructure de recharge et du modèle opérationnel considéré. Il n'existe pas encore de modèle générique ni de références générales. Cela est dû à la grande variété d'aspects à prendre en compte dans la formulation de l'EVCS, notamment les infrastructures de recharge, les comportements des conducteurs de véhicules électriques, le marché de l'électricité et les politiques des services de recharge. Chaque aspect peut être considéré dans le modèle en tant qu'objectif, contrainte, paramètre ou variable de décision. Ainsi, nous classifions d'abord les études existantes en fonction des différents objectifs considérés. Ensuite, nous exposons les différentes contraintes, hypothèses et considérations qui nous permettraient de caractériser et de distinguer les différentes branches de la littérature. Enfin, nous passons en revue les méthodes d'optimisation utilisées pour aborder les problèmes de l'EVCS. La figure B.3 présente un résumé de la classification des objectifs, contraintes et méthodes d'optimisation avec des exemples non exhaustifs.

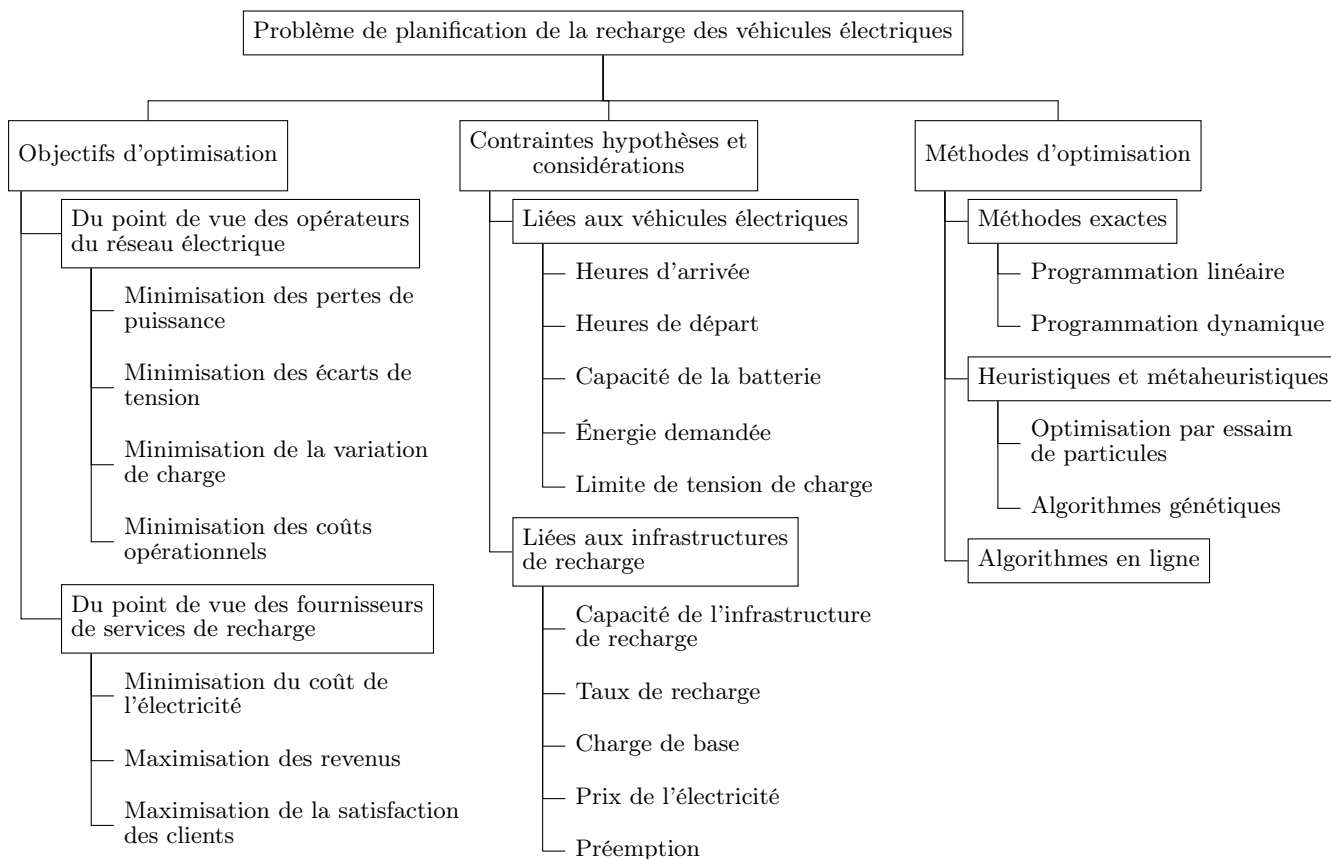


Figure B.3: Une classification des objectifs, contraintes et approches d'optimisation du problème de planification de la recharge des véhicules électriques (EVCS).

Une classification des objectifs du problème est nécessaire avant de classer les méthodes d'optimisation utilisées pour les résoudre. Nous classifions d'abord les problèmes de planification de la recharge des véhicules électriques (EVCS) en deux catégories selon la perspective des parties prenantes, à savoir les opérateurs de réseau électrique et les opérateurs de stations de recharge. Cette classification est essentielle pour distinguer le contexte de l'étude, car dans le secteur de l'électricité, chaque entité est indépendante légalement et fonctionnellement et a des objectifs différents en fonction de sa responsabilité dans le système. Par exemple, les opérateurs de réseau visent à maintenir la stabilité et la fiabilité du réseau. En revanche, les objectifs des opérateurs de stations de recharge consistent à maximiser les profits et à minimiser les factures d'électricité tout en respectant les contraintes du réseau. De plus, les problèmes de planification de la recharge formulés du point de vue des opérateurs de réseau couvriront des réseaux de distribution à grande et moyenne échelle avec plusieurs unités de recharge (recharge à domicile, parkings, bâtiments commerciaux).

En revanche, les opérateurs de stations de recharge aborderont probablement les problèmes de planification dans les infrastructures de recharge de petite et moyenne taille pour lesquelles ils sont responsables. Malgré le couplage entre ces deux types d'acteurs, ils ont été considérés séparément. Seuls quelques articles les prennent en compte tous les deux. Actuellement, il est plus pratique pour les opérateurs de stations de recharge d'adopter des stratégies de recharge intelligentes. La planification de la recharge des véhicules électriques du point de vue des opérateurs de réseau sera plus largement adoptée dans le contexte des futurs réseaux intelligents, où les agrégateurs de véhicules électriques gèrent des véhicules électriques répartis géographiquement.

Dans ce qui suit, nous nous concentrons sur les problèmes formulés du point de vue des opérateurs de stations de recharge, car les problèmes traités dans cette thèse relèvent de cette catégorie.

Minimisation des coûts. Les problèmes d'optimisation visant à minimiser les coûts sont les plus abordés dans la littérature. Cet objectif implique principalement la réduction des factures d'électricité payées par les opérateurs de stations de recharge, en tenant compte des offres d'électricité à tarification dynamique (voir la section 2.4.2.13). Une recharge non contrôlée entraînera des coûts d'électricité élevés, ce qui constitue un obstacle important au déploiement des stations de recharge publiques. Surtout pour les stations de recharge rapide où la recharge des véhicules électriques peut représenter plus de 90% des coûts d'électricité [Garrett & Nelder 2016].

Les auteurs de [Tang *et al.* 2014] minimisent le montant de la facture d'électricité payée par la station de recharge. Dans [Zheng & Shroff 2014], les auteurs maximisent le nombre totale des véhicules servis tout en minimisant le coût de l'énergie. [Ma *et al.* 2015] optimisent le coût de recharge en assurant une recharge complète pour tous les véhicules. Dans l'article de [Tang & Zhang 2016], les auteurs formulent le coût total de recharge comme une fonction strictement convexe et croissante. Dans leur étude publiée dans [Wu *et al.* 2018], les auteurs cherchent à réduire le coût de l'électricité achetée par un parking auprès du réseau électrique. Ils prennent en considération la variation des prix de l'électricité à différents intervalles de temps. Dans leur publication [Ki *et al.* 2018], les auteurs se sont penchés sur le problème des systèmes de charge pour véhicules électriques (EVCS) en utilisant des chargeurs de type M-to-N qui sont réellement utilisés dans les stations de recharge en Corée du Sud. Ces chargeurs sont conçus pour permettre

la connexion de N véhicules électriques, mais seuls M véhicules électriques peuvent se recharger simultanément ($M < N$). La fonction objectif a pour but de maximiser la somme de quatre termes pondérés, incluant les coûts de recharge avec un signe négatif pour leur minimisation.

Dans leur étude mentionnée dans [Yang 2019], les auteurs considèrent sur la planification de la recharge dans un parking résidentiel. Ils parviennent à minimiser les coûts en intégrant un indice de préférence de prix pour chaque intervalle de temps, qui dépend du prix de l'électricité à chaque intervalle. La fonction objectif est formulée sous forme de maximisation, donc l'indice est plus élevé lorsque le prix de l'électricité est bas, et vice versa. Les auteurs de l'article [Liu *et al.* 2020] se concentrent sur la minimisation des coûts de recharge en fonction des tarifs d'électricité en vigueur dans une station de recharge qui dispose d'un nombre limité de chargeurs.

Maximisation des revenus Cet objectif est étroitement lié à l'objectif précédent. En effet, pour maximiser les revenus d'une station de recharge, il est essentiel de trouver un équilibre entre les coûts d'approvisionnement en électricité et les prix de vente de la recharge. En minimisant les coûts de l'électricité, la station peut augmenter sa marge bénéficiaire en réduisant les dépenses liées à l'achat d'électricité auprès du réseau.

Les auteurs de l'article [Jin *et al.* 2013] maximisent des revenus d'un agrégateur, tout en établissant une limite supérieure pour contrôler les coûts de recharge des clients. Les agrégateurs tirent leurs revenus à la fois du bénéfice généré par la fourniture de services de régulation au réseau électrique et des bénéfices résultant de la vente d'énergie aux véhicules électriques. Dans le contexte des parkings intelligents, l'article [Kuran *et al.* 2015] aborde un problème similaire de maximisation des revenus. Les revenus sont déterminés en soustrayant le coût d'achat de l'électricité du prix de vente de la recharge des véhicules électriques. L'article [Bučar 2014] se concentre sur la maximisation du profit total en planifiant toutes les demandes de recharge. On trouve aussi l'article [Niu *et al.* 2018] qui cherche à maximiser le bénéfice d'une station de recharge rapide tout en tenant compte des fluctuations de charge. Lorsque ces fluctuations sont importantes, cela entraîne une augmentation des pertes d'énergie, qui finissent par être supportées par la station de recharge et les conducteurs de véhicules électriques.

Maximisation de la satisfaction des clients La satisfaction des clients représente un facteur important pour garantir le succès d'une entreprise gérant les stations de recharge. De toute évidence, les clients mécontents qui ont connu des désagréments lors de la recharge ne reviendront pas à la station de recharge. De plus, un service de mauvaise qualité nuit à la réputation du CSP. Ainsi, plusieurs articles proposent de formuler le problème EVCS du point de vue du client ou d'inclure la satisfaction des clients dans leur fonction objectif. Il existe de nombreuses façons pour les opérateurs de stations de recharge d'améliorer la satisfaction de leurs clients : en se concentrant sur la maximisation du nombre de demandes de recharge satisfaites [Kuran *et al.* 2015, Gerding *et al.* 2019], en maximisant l'énergie fournie à chaque véhicule électrique [Rahman *et al.* 2016, Ki *et al.* 2018, Yang 2019], en minimisant les coûts de recharge des clients [Jin *et al.* 2013], ou en minimisant le temps de recharge [Zhu *et al.* 2014, Ki *et al.* 2018, García-Álvarez *et al.* 2018].

L'article [Zhu *et al.* 2014] considère l'inclusion du temps de déplacement jusqu'au point de recharge et du temps d'attente dans le temps total de recharge à minimiser. En paral-

lèle, l'article [García-Álvarez *et al.* 2018] se concentre sur la minimisation du retard total, qui survient lorsque l'heure de fin de recharge dépasse l'heure de départ prédéfinie. Dans l'article [Zheng & Shroff 2014], les auteurs se focalisent sur la maximisation de la valorisation totale des clients. De plus, dans l'article [?], la satisfaction des clients est quantifiée en termes de pourcentage d'énergie reçue. La satisfaction d'un conducteur de véhicule électrique est atteinte lorsque ce pourcentage dépasse un seuil spécifique, dépendant de la durée du temps de branchement. La stratégie de recharge est considérée comme satisfaisante lorsque 95% des conducteurs de véhicules électriques sont satisfaits. Également, dans l'article [Ki *et al.* 2018], l'accent est mis sur la maximisation de l'équilibre de charge entre les véhicules électriques. Les auteurs de l'article [Zheng *et al.* 2018] cherchent à minimiser la puissance fournie tout en maximisant le niveau de satisfaction pour la recharge des véhicules électriques. Enfin, l'article [Niu *et al.* 2018] vise à équilibrer la satisfaction des clients, les revenus de la station de recharge et la fluctuation de la charge.

B.2.3 Le positionnement de la recherche

Bien que les études précédemment mentionnées aient abordé divers aspects du problème de planification de la recharge des véhicules électriques, elles ont principalement supposé l'existence d'un nombre suffisant de bornes de recharge pour tous les véhicules. Par conséquent, l'accent était mis sur l'ordonnancement des recharges sans prendre en compte l'attribution spécifique de chaque véhicule à une borne de recharge. Peu d'articles ont réellement exploré la question de l'affectation des véhicules aux bornes de recharge dans le cadre de l'élaboration d'un calendrier de recharge. Dans l'étude menée par les auteurs [Liu *et al.* 2020], un nombre limité de bornes de recharge a été pris en considération afin de minimiser le coût total de la recharge en fonction du tarif d'électricité selon les heures d'utilisation. Toutefois, cette approche n'imposait pas de restriction sur la puissance totale allouée, ce qui signifie que la puissance de recharge de chaque borne était uniquement limitée par sa capacité maximale. De plus, peu d'études ont examiné la recharge à puissance constante, et aucune comparaison n'a été faite pour déterminer les avantages de l'utilisation de bornes de recharge identiques ou non identiques.

Notre étude est proche de la littérature portant sur la planification de la recharge avec l'attribution des véhicules aux bornes de recharge. Toutefois, nous nous distinguons par notre modèle opérationnel des stations de recharge, nos contraintes et nos objectifs d'optimisation. Les contributions de cette thèse peuvent être exposées de la manière suivante :

- Nous proposons un nouveau modèle opérationnel pour les stations de recharge qui prend en compte le nombre limité de chargeurs disponibles, chacun étant installé dans une place de parking. De plus, nous considérons une contrainte de puissance totale limitée pour la station de recharge. En fixant une limite de puissance totale pour la station, on s'assure de ne pas dépasser la capacité du réseau et d'éviter les pics de consommation et les surcharges qui pourraient entraîner des problèmes de stabilité du réseau, des coupures d'électricité et des coûts supplémentaires.
- Nous introduisons un système de réservation, permettant aux véhicules électriques de soumettre leurs demandes de recharge afin d'éviter les files d'attente. L'ordonnanceur attribue ensuite un chargeur adapté à chaque véhicule.

- Différentes fonctions objectifs sont prises en compte, incluant la minimisation de la capacité requise pour l'infrastructure de recharge, la maximisation du nombre de demandes de recharge satisfaites, la maximisation de l'énergie délivrée et la minimisation des retards.
- Nous fournissons une preuve détaillée de la difficulté \mathcal{NP} -complète pour chacun des problèmes envisagés.
- Nous proposons des heuristiques et des métaheuristiques visant à optimiser à la fois l'affectation des véhicules aux chargeurs et l'ordonnancement de la recharge des véhicules électriques.
- Nous effectuons des comparaisons approfondies entre différents aspects de la planification de la recharge, notamment :
 - Entre des taux de charge constants identiques et non identiques.
 - Entre des taux de charge constants et variables.
 - Entre des ordonnancements préemptifs et non préemptifs.
 - Entre des modèles indexés sur le temps et des modèles basés sur les événements.

En résumé, notre étude apporte des contributions significatives en matière d'attribution des véhicules aux chargeurs dans le contexte des stations de recharge pour véhicules électriques, en proposant un nouveau modèle opérationnel, des contraintes spécifiques, des objectifs d'optimisation variés, ainsi que des preuves de difficulté et des approches heuristiques novatrices.

B.3 Problème de planification préemptive de la recharge des véhicules électriques pour minimiser la capacité du réseau

Nous avons souligné la diversité des problèmes de planification de la recharge des véhicules électriques (EVCS) étudiés dans la littérature. Cependant, la plupart des travaux existants supposent une recharge avec des taux de puissance variables. Malgré le potentiel commercial prometteur de cette méthode de recharge, seuls quelques chargeurs à puissance variable sont actuellement disponibles sur le marché. En outre, il est prévu que les chargeurs à puissance constante continueront de coexister avec ceux à puissance variable, en raison de leur facilité de déploiement [Sun *et al.* 2016]. Cette coexistence est motivée par le fait que les batteries des véhicules électriques consomment une puissance relativement constante lorsqu'elles se rechargent dans la plage de 20% à 80%. Par ailleurs, la recharge à puissance constante est considérée comme plus efficace, car elle réduit les pertes d'énergie utilisable lors de l'opération de recharge [Jeon *et al.* 2021].

Un autre aspect crucial est que très peu d'études prennent en compte les places de parking et les points de recharge (chargeurs) en tant que ressources limitées à gérer dans les stations de recharge. En effet, ces études supposent généralement que les opérateurs de stations de recharge ne gèreront ni les places de parking ni les chargeurs et que le conducteur choisira son emplacement à son arrivée, ce qui convient davantage aux stations de recharge sans réservation où tous les chargeurs sont identiques. Cependant, les stations de recharge ressemblent davantage

à des parkings équipés de chargeurs capables de délivrer des taux de puissance différents. Par conséquent, il est essentiel d'attribuer à chaque véhicule un chargeur approprié pour répondre à sa demande avant le départ.

Basé sur la discussion précédente, le problème présenté dans cette section (et dans le Chapitre 3) se concentre sur l'étude du problème de planification de la recharge en considérant un nombre fixe de chargeurs délivrant une puissance constante. Plus spécifiquement, la station de recharge comprend un nombre déterminé de places de parking, chacune étant équipée d'un chargeur avec un seul connecteur permettant le branchement d'un véhicule à la fois. Elle dispose d'une capacité d'alimentation électrique limitée afin d'éviter toute surcharge du réseau. Avant de se rendre à la station, chaque conducteur de véhicule électrique doit soumettre une demande de recharge pour réserver une place de parking.

En raison du nombre limité de chargeurs, de l'autonomie restreinte des véhicules électriques et des longs temps de recharge, les conducteurs doivent soigneusement planifier leurs déplacements pour s'assurer d'avoir l'occasion de recharger leurs batteries. Il est donc préférable pour eux de vérifier à l'avance la disponibilité du chargeur qu'ils ont l'intention d'utiliser. De plus, le protocole de point de charge ouvert (Open Charge Point Protocol) déjà déployé intègre la fonctionnalité de réservation des stations de recharge [OCP 2020]. Un système de gestion centralisé contrôle et planifie la charge afin d'optimiser l'utilisation des places de parking et de la puissance disponible pour répondre aux demandes de recharge sans surcharger le réseau électrique.

Nous nous focalisons principalement sur la question suivante : si les opérateurs de stations de recharge fixent comme objectif de satisfaire un certain nombre défini de demandes de recharge, combien de chargeurs sont nécessaires et quelle est la puissance limitée nécessaire pour atteindre cet objectif ?

Nous abordons donc deux problèmes. Tout d'abord, nous étudions le problème de déterminer le nombre minimum de chargeurs requis pour brancher tous les véhicules. Ensuite, nous nous attaquons au problème de minimisation de la limite de puissance maximale nécessaire pour satisfaire toutes les demandes de recharge. Nous désignons cette limite par la capacité du réseau. Cette problématique est étudiée dans le but d'orienter de manière précise les stations de recharge dans le choix de leur puissance maximale souscrite. La limitation de la puissance totale peut également avoir un impact financier. En réduisant la puissance totale allouée, on peut réduire les coûts d'investissement dans l'infrastructure électrique nécessaire à la station de recharge. Au fur et à mesure de l'augmentation des demandes de recharge, il est impératif d'adapter leur capacité en conséquence. En effet, les pics de consommation d'énergie entraînent des factures d'électricité élevées. Généralement, des équipements tels que des dispositifs de coupure de puissance et des relais sont installés à faible coût pour éviter ces pics, mais ils entraînent l'arrêt du système, ce qui est indésirable. Par conséquent, il est essentiel de fournir un aperçu de la limite de puissance minimale en fonction des types de chargeurs installés et des demandes de recharge. Nous examinons ces deux problèmes dans le cas de chargeurs identiques et non identiques. Pour les chargeurs identiques, nous démontrons que les deux problèmes sont polynomiaux dans les deux cas. En ce qui concerne la minimisation de la capacité du réseau, nous prouvons que le problème est polynomiale dans le cas de chargeurs identiques et \mathcal{NP} -difficile dans le cas de chargeurs non identiques.

Nous avons développé une heuristique pour résoudre le problème \mathcal{NP} -difficile avec des

chargeurs non identiques. Ensuite, nous avons utilisé une métaheuristique de recherche locale itérée (ILS) pour améliorer les résultats de cette heuristique. Nous avons présenté différents scénarios afin d'évaluer les performances des algorithmes, proposés. Nous avons démontré que l'utilisation d'un solveur MILP, notamment CPLEX, n'est pas pratique pour résoudre le problème avec des chargeurs de différents types. L'ILS que nous avons proposée dépasse les performances de CPLEX en termes de minimisation de la capacité du réseau et de temps de calcul. En moyenne, l'ILS a obtenu de meilleures solutions en moins de 20 secondes. Des expériences supplémentaires ont révélé que l'installation de chargeurs fournissant une puissance de sortie différente est plus avantageuse que l'installation de chargeurs identiques.

Ainsi, Le chapitre 3 qui traite ces problèmes est organisée comme suit. La section 3.2 décrit en détail le problème étudié. La section 3.3 examine le problème avec des chargeurs identiques, tandis que la section 3.4 traite des chargeurs non identiques. Pour chaque cas, nous étudions le nombre minimum de chargeurs requis pour brancher tous les véhicules dans la station de recharge ainsi que les problèmes de capacité minimale du réseau. La section 3.5 évalue les performances des méthodes proposées, et enfin, la section 3.6 conclut le chapitre.

B.4 Problème de planification préemptive de la recharge des véhicules électriques pour maximiser le nombre de demandes satisfaites

Ce problème, présenté dans le chapitre 4, est similaire à celui décrit précédemment. Cependant, la fonction objectif est différente. La capacité du réseau et le nombre de chargeurs dans la station de recharge sont fixés et sont considérés comme des contraintes dans ce problème. La plupart de stations de recharge limite le nombre de chargeurs installés en fonction de la puissance totale souscrite. Dans ce cas, tout les chargeurs peuvent délivrer jusqu'à leurs puissances maximales en même temps. Un inconvénient majeur de cette configuration est que le nombre de chargeurs limite le nombre de véhicules pouvant stationner dans la station de recharge. Lorsqu'un conducteur branche son véhicule, le câble est verrouillé au port de recharge du véhicule, empêchant ainsi toute personne non autorisée de le débrancher. Seul le propriétaire du véhicule ou le propriétaire de la station de recharge peut le déverrouiller. Dans ce cas, le véhicule doit être déplacé vers un autre emplacement de stationnement afin de permettre à d'autres véhicules de se charger. L'ajout de plus de chargeurs et la limitation de la capacité de la station de recharge offrent une plus grande flexibilité et permettent à un plus grand nombre de véhicules de se brancher et de se charger sans surcharger le réseau électrique. Nous nous intéressons aux questions suivantes :

- Existe-t-il un ordonnancement faisable pour toutes les demandes de recharge ?
- Si aucun ordonnancement faisable n'existe, quel est le sous-ensemble maximal de demandes de recharge pouvant être satisfaites ?

La première question est la version décisionnelle du problème et elle est liée aux résultats du chapitre précédent (chapitre 4). La deuxième question vise à maximiser le nombre de véhicules chargés. En d'autres termes, nous cherchons à maximiser le nombre de clients pouvant être servis. Nous avons démontré que ce problème est \mathcal{NP} -difficile. Nous avons abordé en premier lieu le cas

des chargeurs identiques, pour lequel nous avons proposé différents modèles de programmation linéaire ainsi que des heuristiques. Une comparaison détaillée entre ces méthodes a été réalisée. Les résultats mettent en évidence que le modèle événementiel relaxé surpasse les autres approches et peut résoudre efficacement des instances de grande taille comportant 400 véhicules en moins de 45 secondes. Des instances plus complexes ont été générées afin de démontrer les limites du modèle. Nous avons constaté que ses performances dépendent non seulement du nombre de véhicules et de chargeurs, mais également de la capacité du réseau. En outre, la comparaison des heuristiques révèle que la sélection des véhicules à planifier en se basant sur la solution d'allocation de puissance s'avère plus efficace que celle basée sur la solution d'affectation. Par la suite, nous nous penchons sur le problème avec des chargeurs différents, qui s'avère plus complexe que le précédent. Nous proposons deux formulations de programmation linéaire pour aborder ce cas. Ensuite, nous développons une méthode de recuit simulé (SA) combinée à une recherche locale à deux étapes. Les résultats révèlent que le SA proposé parvient à trouver de bonnes solutions en un laps de temps court. De plus, le SA surpasse le modèle linéaire, principalement dans les instances comprenant plus de 200 véhicules. Des résultats supplémentaires confirment que l'installation de différents types de chargeurs est plus bénéfique pour maximiser le nombre de demandes de recharge qui peuvent être satisfaites.

Ainsi, le chapitre 4 est structuré comme suit. Nous présentons le problème étudié et donnons un exemple illustratif dans la section 4.2. Dans la section 4.3, nous abordons le problème avec des chargeurs identiques, tandis que la section 4.4 examine le problème avec différents types de chargeurs. Les résultats de simulation sont présentés dans la section 4.5, et enfin, une conclusion est proposée dans la section 4.6.

B.5 Problème de planification préemptive de la recharge des véhicules électriques pour maximiser l'énergie délivrée

Dans les chapitres 3 et 4, nous avons étudié le problème de planification de la recharge des véhicules électriques avec des taux de puissance de recharge constants. Cependant, dans la revue de littérature présentée au chapitre 2, nous avons mis en évidence la possibilité de considérer deux types de taux de puissance : les taux variables et les taux constants. Ce chapitre se penche sur l'examen et la comparaison de deux variantes du problème de planification de la recharge. Dans la première variante, appelée modèle de puissance constante, les chargeurs ont la capacité de délivrer soit leur puissance maximale, soit aucune puissance du tout. La seconde variante est le modèle de puissance variable, où le taux de charge de chaque chargeur peut varier dans le temps, allant de zéro à sa puissance maximale.

Le modèle de fonctionnement de la station de recharge reste essentiellement le même, avec une puissance totale limitée et un nombre restreint de chargeurs. Les véhicules électriques soumettent leurs demandes de recharge, et c'est à l'ordonnanceur d'attribuer un chargeur approprié à chaque véhicule. Toutefois, l'ordonnanceur dispose également de la possibilité de décider de l'heure de branchement de chaque véhicule, qui peut être postérieure à l'heure d'arrivée demandée. Dans le but de servir un maximum de véhicules, l'objectif de l'ordonnanceur est de maximiser l'état final des charges.

Nous avons prouvé que ce problème est \mathcal{NP} -difficile. De plus, nous avons formulé le problème d'ordonnancement sous la forme d'un modèle de programmation linéaire en nombres entiers (MILP). Même pour de petites instances, il était difficile de résoudre les modèles MILP. Par conséquent, nous avons conçu une heuristique basée sur un algorithme de coloration de graphe d'intervalles et un algorithme de recuit simulé (SA) combiné à la programmation linéaire. Différentes instances ont été générées pour évaluer les performances des méthodes proposées. Les résultats montrent que le modèle de puissance variable est plus efficace pour satisfaire les demandes de recharge. Nous démontrons que l'algorithme SA obtient de meilleurs résultats en minimisant la fonction objectif en moins d'une minute. Des résultats supplémentaires sur la qualité des solutions d'ordonnancement ont été présentés. Les solutions d'ordonnancement de l'algorithme SA chargent au moins 93% des demandes de recharge à plus de 75% de leur énergie souhaitée. D'autre part, le pourcentage de demandes rejetées est inférieur à 2,5%.

Ainsi, le chapitre 5 traitant ce problème est organisée comme suit. La section 5.2 décrit en détail le problème étudié. La section 5.3 le formule comme un problème de programmation linéaire mixte en nombres entiers (MILP). Nous examinons la complexité dans la section 5.4. La section 5.5 propose des méthodes d'optimisation. La section 5.6 évalue les performances des méthodes proposées, et enfin la section 5.7 conclut le chapitre.

B.6 Conclusion et perspectives

La transition vers des transports routiers décarbonés et l'encouragement à l'utilisation des énergies vertes ont suscité un vif intérêt pour la recherche sur les technologies des véhicules électriques. Au fil des années, le marché des véhicules électriques a connu une expansion remarquable, ce qui a entraîné une demande croissante en infrastructures de recharge. Dans la prochaine décennie, il sera nécessaire de mettre en place davantage d'infrastructures de recharge intelligentes pour répondre à cette demande croissante en électricité.

L'un des défis clés liés à l'augmentation du nombre de véhicules électriques réside dans la gestion efficace de la charge électrique. Il est essentiel d'adopter des stratégies de recharge intelligente afin d'optimiser l'utilisation de l'énergie électrique tout en évitant les impacts négatifs sur le réseau électrique. La planification de la recharge des véhicules électriques joue un rôle crucial dans ce domaine.

La présente thèse s'est donc penchée sur le problème de planification de la recharge des véhicules électriques. L'objectif était de développer des méthodes et des stratégies permettant de gérer de manière optimale la demande de recharge tout en prenant en compte les contraintes techniques et les capacités du réseau électrique. En examinant les différents aspects de ce problème complexe, des solutions ont été proposées pour optimiser l'utilisation des infrastructures de recharge et minimiser les coûts énergétiques.

Les résultats de cette thèse ont contribué à mieux comprendre les défis et les opportunités liés à la recharge des véhicules électriques. Les approches développées ont montré leur efficacité dans la gestion de la demande de recharge, ce qui permet de minimiser les pics de consommation et d'améliorer l'efficacité énergétique globale. Ces avancées sont essentielles pour promouvoir l'adoption des véhicules électriques et garantir une transition réussie vers une mobilité plus durable.

Dans l'état de l'art, nous avons brièvement revisité le problème classique d'ordonnement des tâches. Ensuite, nous avons donné un aperçu des technologies des véhicules électriques, soulignant la grande variété dans la formulation du problème de planification de la recharge des véhicules électriques en fonction des modèles de recharge, des objectifs, des considérations et des contraintes.

Ensuite, nous nous sommes intéressés au problème de recharge des véhicules électriques dans une station de recharge avec une capacité de réseau limitée et un nombre limité de chargeurs délivrant de l'énergie à des taux constants. Tous les véhicules doivent soumettre une demande de réservation de recharge. Dans un premier temps, nous avons étudié le problème de la recharge de tous les véhicules avec une capacité minimale du réseau, dans le cas de chargeurs identiques et non identiques. Nous avons prouvé que le problème est polynomiale dans le premier cas, tandis qu'il est \mathcal{NP} -difficile dans le second. Des formulations mathématiques et des méthodes de résolution ont été proposées et comparées. Les résultats des simulations suggèrent qu'il est préférable d'installer des chargeurs avec des taux de recharge différents. Lorsque la capacité du réseau n'est pas suffisante pour satisfaire toutes les demandes de recharge, notre objectif est de maximiser le nombre de demandes acceptées. Ce problème s'est avéré être \mathcal{NP} -difficile, que les chargeurs soient identiques ou non identiques. Différentes formulations mathématiques, heuristiques et métaheuristiques ont été proposées pour le résoudre.

Ensuite, nous avons étudié le problème de la recharge avec des taux de puissance variables et l'avons comparé à celui avec des taux de puissance constants. Le modèle de la station de recharge était légèrement différent. Nous avons pris en compte une relaxation sur l'heure d'arrivée, où les véhicules peuvent être branchés plus tard que l'heure d'arrivée souhaitée. L'objectif était de maximiser l'état de charge au moment du départ. Nous avons prouvé que le problème était \mathcal{NP} -difficile, que les puissances de recharge soient constants ou variables. Nous avons proposé des modèles de programmation linéaire en nombres entiers, des heuristiques et des métaheuristiques pour le résoudre. Les résultats des simulations ont révélé que résoudre les modèles de programmation linéaire avec un solveur commercial était presque impossible, même pour de petites instances. De plus, l'utilisation de taux de recharge variables était plus efficace que des taux constants pour fournir plus d'énergie avec la même capacité du réseau.

Tous les problèmes de recharge mentionnés étaient de nature préemptive. Nous avons comparé la planification de la recharge préemptive et non préemptive dans un contexte en ligne, dans un système triphasé déséquilibré précédemment décrit dans la littérature. Nous avons proposé des heuristiques rapides et une recherche locale simple. Nous avons montré comment la préemption était plus efficace pour optimiser l'ordonnement, même avec des méthodes simples.

Le tableau B.1 résume les objectifs, les contraintes, les résultats de complexité et les méthodes de résolution pour tous les problèmes de planification de la recharge des véhicules électriques abordés dans cette thèse.

Avant de conclure cette dissertation, nous présentons les perspectives suivantes :

- Le travail présenté peut servir de point de départ pour développer un modèle d'optimisation plus complexe en incluant davantage de contraintes liées à la technologie des véhicules électriques, telles que les temps de charge non linéaires, l'efficacité de charge et la compatibilité

Table B.1: Résumé des problèmes de planification de la recharge des véhicules électriques abordés dans cette thèse.

	Chapitre 3		Chapitre 4		Chapitre 5		Chapitre 6	
Chargeurs	identiques	non identiques	identiques	non identiques	non identiques		identiques	
Affectation aux chargeurs	Oui						Non	
Préemption	Oui						Oui	Non
Temps de départ	Contraintes strictes						Contraintes souples	
Taux de charge	Constant				Constant	Variable	Constant	
Temps d'arrivée	Contraintes strictes				Contraintes souples		Contraintes strictes	
Demandes d'énergie	Contraintes strictes				Contraintes souples		Contraintes strictes	
Objectif	Minimiser la capacité du réseau		Maximiser le nombre de demandes acceptées		Maximiser l'énergie fournie		Minimiser le retard total	
Classe de complexité	\mathcal{P}		\mathcal{NP}					
Résolution	Max Flow	Heuristiques	MILP,	MILP, SA+TsLS,	SA+MILP	Heuristiques en ligne	Heuristiques en ligne	
Méthodes		MILP, ILS+SA	heuristiques	heuristiques	heuristiques	et LS		

des chargeurs. Cela permettra d'obtenir des modèles plus précis qui reflètent les différentes exigences des véhicules électriques.

- Le système proposé peut être étendu pour gérer des demandes de charge avec ou sans réservation. De plus, il est intéressant de traiter les cas où le véhicule fait une réservation mais ne se présente jamais, ainsi que les cas où le véhicule peut partir plus tard ou plus tôt que prévu. Dans ce cas, des algorithmes de planification en ligne peuvent être développés et comparés aux résultats des algorithmes hors ligne proposés dans cette thèse.
- Dans cette thèse, nous avons supposé une recharge préemptive. Bien qu'elle soit plus flexible et bénéfique pour gérer les contraintes et optimiser la fonction objective, comme nous l'avons observé au chapitre 6, des préemptions fréquentes peuvent entraîner une détérioration supplémentaire des batteries. En particulier, les fabricants de véhicules électriques recommandent qu'une phase de charge dure au moins 15 minutes pour éviter des réactions chimiques indésirables dans les batteries lithium-ion. Dans notre modèle, le problème peut être évité en définissant simplement la plage horaire à 15 minutes, mais nous perdons tout de même en précision. Par conséquent, des recherches supplémentaires doivent être menées pour résoudre ce problème.
- Le modèle pourrait être étendu pour prendre en compte un réseau de stations de recharge. Dans ce cas, le choix d'une station de recharge dépendra de la distance, de l'énergie nécessaire pour atteindre la station, de la disponibilité des chargeurs et de l'énergie restante.
- Il serait intéressant de considérer d'autres méthodes d'optimisation, notamment des métaheuristiques basées sur une population. Les métaheuristiques basées sur une seule solution étaient plus pratiques pour trouver de bonnes solutions dans un délai raisonnable, notamment étant donné que l'évaluation des solutions après perturbation était complexe et chronophage. Dans ce cas, l'utilisation de parallélisme et d'algorithmes multithread pourrait être utile.
- Une autre idée consisterait à considérer des objectifs multiples en ajoutant la minimisation

des factures d'électricité de charge que la station de recharge paie ou en maximisant son profit. En particulier, avec une tarification selon les heures creuses, il conviendrait de privilégier la charge des véhicules lorsque les coûts de l'électricité sont bas.

- Lorsque l'affectation des véhicules aux chargeurs est connue, c'est-à-dire que nous savons quel véhicule est connecté à quel chargeur, une question intéressante se pose : quelle est la complexité de classe du problème d'allocation de puissance lorsque les chargeurs ont des taux de puissance différents ? Des études futures pourraient apporter des éclaircissements sur cette question et contribuer à une meilleure compréhension de ce problème d'optimisation.
- Pour le problème avec un système électrique déséquilibré, il serait plus intéressant de considérer des chargeurs avec des taux de puissance différents. Comme la charge monophasée est une charge lente, nous ne pouvons considérer que des chargeurs jusqu'à 7,4 kW. De plus, la contrainte de déséquilibre ne peut pas être exprimée uniquement par le nombre de véhicules. Des mesures plus complexes telles que le facteur de déséquilibre de tension (VUF - Voltage Unbalance Factor) peuvent être utilisées. Dans ce cas, la tension de chaque phase (ligne) doit être mesurée.
- Il pourrait être pertinent de modéliser la station de recharge sur un simulateur et de mettre en œuvre les algorithmes d'optimisation pour valider et réaliser des tests plus réalistes. Enfin, la mise en œuvre pratique de ces méthodes dans les stations de recharge réelles et leur intégration dans les systèmes de gestion existants nécessitent une étude plus approfondie et une collaboration entre les acteurs de l'industrie, les chercheurs et les décideurs.

En conclusion, cette thèse a jeté les bases pour une meilleure compréhension et une optimisation de la recharge des véhicules électriques. Elle ouvre la voie à de nouvelles recherches et à des développements futurs visant à promouvoir l'adoption des véhicules électriques et à favoriser la transition vers une mobilité plus durable.

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