



HAL
open science

Essays in macroeconomics : green transition, macro-financial risk, and heterogeneity

Ghassane Benmir

► **To cite this version:**

Ghassane Benmir. Essays in macroeconomics : green transition, macro-financial risk, and heterogeneity. Economics and Finance. Université Paris sciences et lettres; Université de Tunis El Manar, 2023. English. NNT : 2023UPSLD017 . tel-04294227

HAL Id: tel-04294227

<https://theses.hal.science/tel-04294227>

Submitted on 19 Nov 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



THÈSE DE DOCTORAT

DE L'UNIVERSITÉ PSL

Préparée à l'Université Paris-Dauphine

**Essais en macroéconomie : transition verte, risque
macro-financier, et hétérogénéités**

Soutenue par

Ghassane BENMIR

Le 27 juin 2023

École doctorale n°ED 543

École doctorale SDOSE

Spécialité

Sciences Économiques

Composition du jury :

Patrick FÈVE
Professeur,
Toulouse School of Economics *Examineur*

Stephie FRIED
Senior Economist,
FED SF *Rapporteuse*

Benjamin MOLL
Professor,
London School of Economics *Examineur*

Katheline SCHUBERT
Professeure,
Paris School of Economics *Présidente*

Rick van der PLOEG
Professor,
University of Oxford *Rapporteur*

Gauthier VERMANDEL
Maître de conférence HDR,
PSL – Université Paris-Dauphine *Co-Directeur*

Bertrand VILLENEUVE
Professeur,
PSL – Université Paris-Dauphine *Co-Directeur*

Acknowledgments

First, I would like to express my gratitude to all my advisors, Dr. Gauthier Vermandel, Prof. Bertrand Villeneuve, Prof. Benjamin Moll, Prof. Luca Taschini, Prof. Simon Dietz, and Dr. Ivan Jaccard who guided me throughout this thesis. I am forever grateful to their continuous support during my Ph.D study and related research, to their patience, motivation, and immense knowledge. I could not have imagined having a better advisory board and mentors for my Ph.D journey. An immense thank you!

Second, I would like to thank the rest of my thesis committee: Prof. Patrick Fève, Dr. Stephie Fried, Prof. Katheline Schubert, and Professor Rick van der Ploeg, for the time they took to read my dissertation and for their insightful comments and encouragement, which incentivised me to widen my research from various perspectives.

Also, I would like to thank my colleagues at PSL Research – Paris Dauphine and at the LSE for all the time we shared together. A special thank you to Prof. Eve Caroli, Prof. Farid Toubal, Prof. Elise Huillery, Prof. Sidartha Gordon, and all the Leda members for their support.

I could not forget to mention my fantastic co-author Josselin (with whom we co-wrote different chapters of this thesis) for the stimulating exchanges, endless discussions, the sleepless nights working on debugging codes together, and all the fun we have had.

Last but not the least, I would like to especially thank my family and partner Tierra for their unconditional support. I would have never been able to do it without them.

Contents

Acknowledgments	i
General Introduction	1
Chapter 1: Policy Interaction and the Transition to Clean Technology	9
Chapter 2: The Distributional Costs of Net-Zero: A Heterogeneous Agent Perspective	105
Chapter 3: Endogenous Abatement Technology	210
General Conclusion	274

General Introduction

Climate change has shifted from a fringe issue to a worldwide emergency. Our understanding of the phenomenon and our willingness to act have developed significantly, in part paralleling the ways in which climate change is being experienced around the globe. It has become a hot topic where academics, industry, and lay people alike are finding common ground. As such, growing academic awareness is leading to important literature in the domain. The implementation of a strategy for the substantial reduction of greenhouse gases (GHG) at the global level has become a major priority. Since the Rio Conference in 1992, a debate has raged in academic and political circles over the growth-environmental trade-off. Discussions focus on the means by which economic activities could align with environmental concerns instead of being hindered by assumed mutual exclusivity.

The Intergovernmental Panel on Climate Change (IPCC), which is an international body of scientists established by the United Nations to assess the science related to climate change, periodically produces assessment reports that provide a comprehensive summary of the current state of scientific knowledge on climate change. The most recent IPCC report, released in 2021, provides projections of future climate change under different scenarios of greenhouse gas emissions. The report finds that human activities, particularly the burning of fossil fuels, are causing global temperatures to rise and that this is causing a wide range of impacts on natural systems and human societies. The report projects that under all emissions scenarios, global temperatures will continue to rise in the coming decades, leading to more frequent and severe heat waves, droughts, and extreme precipitation events.

Sea levels are also projected to rise, with potentially devastating impacts on coastal communities and infrastructure. The report finds that limiting global warming to 1.5 degrees Celsius above pre-industrial levels (a target agreed upon in the Paris Agreement) would require rapid and deep reductions in greenhouse gas emissions. Even if emissions were to reach net-zero by around 2050, some level of global warming is already locked in due to past emissions, and the impacts of climate change will continue to be felt for many decades to come. The IPCC projections highlight the urgent need for ambitious action to reduce greenhouse gas emissions and adapt to the impacts of climate change. Failure to do so will result in significant and potentially irreversible impacts on natural systems and human societies.

In practice, especially in the short and medium terms, however, financial and economic activity on one side, and environmental policy on the other, are in tension. A need for both medium/long and short-term policies aimed at bridging the gap between environmental sustainability and economic efficiency, as well as addressing financial stability and distributional impacts, are in dire need, in order to foster economic transition. Of special concern are climate actions that may strongly impact macroeconomic activity, given the potentially high added cost of GHG offsetting. With the substantial effects of climate actions on the overall economy, a growing body of research from the field of macroeconomics (including distributional macroeconomics) and macro-finance, among others, are now tackling these issues.

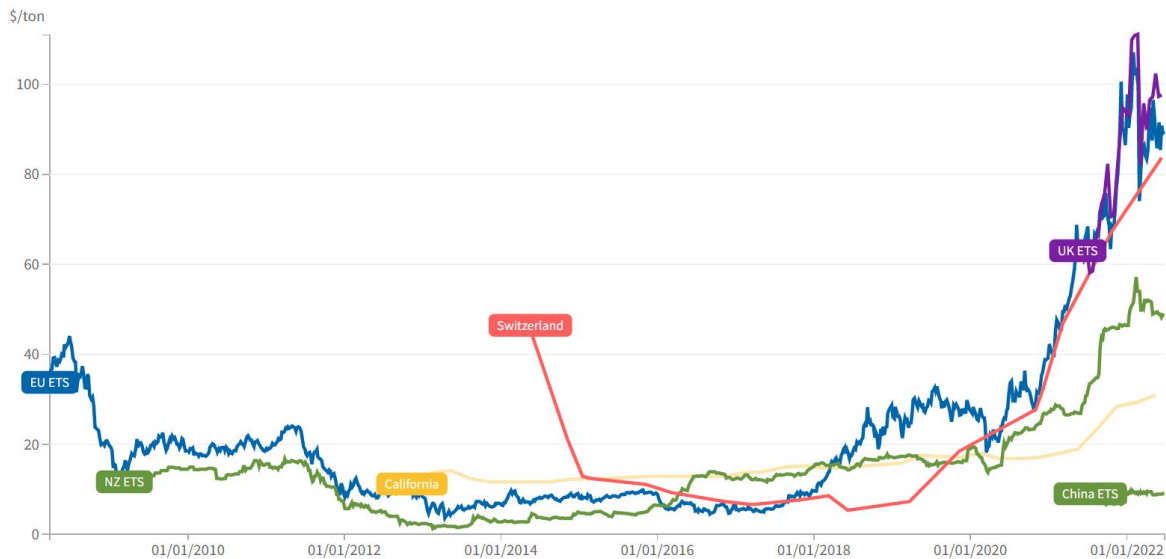
In recent years, financial policy makers have also become increasingly concerned by the challenges posed by climate change. As a step toward more actions, the ECB decided, as part of its financial stability and monetary policy strategy review, to monitor more closely climate risk and the consequences it could have on financial stability. For the time being, however, fiscal policy has been the main instrument to mitigate present and future damages from climate change.

As such, a successful green transition by 2050 would require substantial understanding of the social science (e.g. macroeconomic, financial, distributional) and natural science

(e.g. natural resources and energy availability) challenges that could be posed by both the environmental externality and policy. While the major focus of environmental and climate economists over the last decade as summarized in the literature review conducted by Schubert [2018] has been the pricing of the environmental externality and the global macroeconomic impacts of climate change, not much research investigating environmental externality and policy and their interlinks with the macro-finance and the inclusion of heterogeneity of agents have been conducted.

First, the potential linkages between i) environmental externality and policy and ii) financial markets could be of major concerns especially if shown that physical and mitigation climate related risks could cause significant distress on financial markets. Where carbon pricing could be used as mitigation risk for the climate physical risk, the design of carbon markets could entail potential market inefficiencies, which would hinder financial stability.

Second, as carbon prices reach historical heights (Figure 0.1), one of the major concerns with the 2050 net-zero emissions target lies in its potential distributional impacts. The example of the Yellow Vest Crisis (*Les Gilets Jaunes*) in France and Canada, to name only two instances of carbon-tax induced social upheaval, highlights the importance of accounting for distributional impacts when setting a carbon price, impacts of which may otherwise impede its implementation. The political economy of environmental policies could then become a major hurdle in achieving the net-zero policy and keeping temperatures below 1.5 or 2 degrees Celsius above pre-industrial levels. Thus, understanding the set of policies (e.g. finding ways to steer green innovation without solely relying on increasing carbon pricing), which could be used is crucial to avoid failing to achieve the 2050 net-zero objective.

FIGURE 0.1. Carbon Prices In the World's Major Cap-and-Trade Markets

Note: The figure presents the carbon prices in major cap-and-trade world markets and is constructed using data from the International Carbon Action Partnership: <https://icapcarbonaction.com>.

In the following paragraphs of the general introduction, we present the objectives of this thesis (where we investigate the three literature gaps identified in this introduction, namely: i) the interactions of environmental externality and policy with financial market, ii) the potential distributional impacts of net-zero policy, and iii) the set of policies which could be used to steer green innovation), the main results, a snapshot of the related literature (which is later developed within each chapter), as well as the main contributions of the thesis.

The objective of this thesis is to provide a framework to address the implications of the green transition that are aimed at combating physical and transition risks associated with climate change on the macroeconomy. To do so: i) in the first chapter, we explore the implications of setting a market of carbon permits (such as the European Union Emissions Trading Scheme) on macro-financial stability; then ii) we expand the framework in chapter two to study the implications of the net-zero policy (which aims for carbon neutrality by

2050) on household welfare by relying on recent advances in distributional macro theory; finally iii) in chapter three, we study the role that green innovation could play in the transition period and investigate which policies could be useful to steer green research and development (R&D).

The main results of the chapters of this thesis are as follows:

In the first chapter, we provide a macro-finance framework that includes financial intermediaries, heterogeneous firms, and the climate externality. We rely on this framework to investigate the impacts of market-based climate policy on financial stability and macroeconomic costs. We first show that as the cap policy diverges from the optimal social cost of carbon (SCC), the loss on welfare increases, whereas, in the short term, the ETS market design induces volatility in the carbon price, which distorts risk premia. We then propose tools to mitigate these inefficiencies. We find that: i) sectoral risk-weight (*i.e.* climate risk-weight) macroprudential policy is able to both reduce the wedge gap and mitigate financial stability risk related to climate change, and ii) short-term monetary policy instruments (*i.e.* QE rules) are able to restore the equilibrium in the financial markets.

In the second chapter, we focus on the distributional impacts related to the net-zero emissions target. We build on the recent distributional macroeconomic literature to which we cast a full-fledged climate dynamics framework and investigate the impacts of the green transition on household income and wealth joint distribution. We show that carbon price generates distributional impacts both in the short-run and the long-run. We also show that targeted redistribution of carbon revenues could be a major tool in government strategies to mitigate the distributional cost of the net-zero target. We note that, while fiscal transfers are able to offset some of the distributional impacts of carbon pricing, cheaper abatement technologies are paramount over the long-run.

In the third chapter, we investigate the role financial loans and fiscal policy could play in fostering green transition and steering green R&D. We start by empirically showing that both long-term loans and carbon price positively impact green innovation as measured by patent numbers. Then relying on a dynamic general equilibrium model we show that macro-financial policies could play a similar role to fiscal subsidies in steering green innovation and curving down the implicit carbon price.

The related literature (in a nutshell) of this thesis builds on both the macro-finance and macro-environmental economics literatures.

From macroeconomic business cycles models to distributional macroeconomics, passing through macro-finance: Since the first business cycle micro-founded framework of [Lucas \[1978\]](#) and [Kydland and Prescott \[1982\]](#), the last forty years witnessed major developments in the macro/business cycle literature. With an objective to better micro-found and estimate macro models using empirical data, the dynamic stochastic general equilibrium (DSGE) models became the state of the art framework used by central banks and policy institutions, especially with the canonical contributions of [Smets and Wouters \[2003\]](#) and [Christiano et al. \[2005\]](#), who established procedures to estimate large models using empirical macro data, thus allowing for many policy counterfactual analyses.

However, the 2007 financial crisis, showed some weaknesses and shortcomings of DSGE models, especially as these models were unable to make significant/robust predictions nor were they well suited to analyse the policy spectrum during recent financial distresses. These DSGE models lacked important financial components. Considerable research has since been conducted and models of financial intermediation were brought back to the forefront with their key mechanisms incorporated into state of the art DSGE models (e.g. [Gertler and Karadi \[2011\]](#)).

Recently, and building on the above advances of the literature and ground-breaking work in applied mathematics such as the field of “Mean Field Games” (MFG) introduced by [Lasry and Lions, 2007](#) (always with an objective to better micro-found the models),

[Achdou et al., 2022] among others, expanded the scope of DSGE models to new strand of literature referred to as distributional-macro. The main strength of this new literature is the inclusion of higher levels of heterogeneity within economic agents, which allows to increase the scope of economic and policy analysis.

From Integrated Assessment Models (IAMs) to environmental-macro E-DSGE: Since the early work of William Nordhaus, who aimed to incorporate geophysical and climate change dynamics within standard long-run macroeconomic models (where he developed IAM frameworks, e.g. Nordhaus [1992]), most of the research focused on the social cost of carbon, building on his framework and did not investigate the linkages with macro-finance and business cycles implications.

While IAMs offer a framework that features both climate dynamics and economic growth, it usually features a stylized representation of agent behaviors. In recent years, a body of research began incorporating climate dynamics within the more standard business cycle macro models (e.g. Heutel [2012], Golosov et al. [2014]), thus allowing for the birth of the new environmental-macro literature: E-DSGE models.

In the main body of this thesis, I build on all these recent macro-finance and distributional-macro literatures as well as environmental-macro literature¹.

The main contributions of the thesis with respect to the literature, can be summarized as follows:

i) Theoretically, this thesis offers four main contributions. First (and most generally), we propose a general equilibrium macro-finance and climate change frameworks that allows for studying the impacts of environmental policy on: a) financial markets, b) household income and wealth distributions, and c) green innovation. For the second theoretical contribution, the first chapter presents a framework that paves the way for a more comprehensive approach which allows for highlighting the role of the financial sector in the transition to

¹An extensive literature review is included in each chapter.

a low carbon economy and depicts the risks associated with climate transition policy and physical risk. For the third contribution, chapter two develops a novel and flexible heterogeneous climate macroeconomic framework that accounts for climate dynamics and allows for studying the distributional impacts along the transition to the net-zero emissions target and shows how accounting for heterogeneity is crucial for understanding climate policy implications in the long-run. Finally, the third chapter presents a new model of endogenous green growth that is able to account for the development of new sustainable technologies that lower abatement costs.

ii) Methodologically, we show how to cast non-stationary climate dynamics within the standard incomplete market model pioneered by [Imrohoroglu, 1989] and [Aiyagari, 1994], in continuous time following [Achdou et al., 2022] and retrieve stationary wealth distributions. The thesis proposes another methodological contribution, which involves computing pathways that are consistent with the transition to a low carbon economy. These pathways include both deterministic trends and stochastic processes, and they are derived using the extended path algorithm of Adjemian and Juillard [2013]. The resulting transition pathways exhibit some level of uncertainty at the business cycle frequency.

In the following, we present the three chapters of the thesis and then conclude.

Chapter 1

Policy Interaction and the Transition to Clean Technology

This chapter was presented at the Climate Risk Workshop of the Federal Reserve Bank of San Francisco, the Annual Meetings of the AEA, CEBRA and EAERE, the Annual Conference of the MIT Golub Center for Finance and Policy, the Annual Conference of the Money Macro and Finance Society, the 1st Sustainable Macro Conference, as well as the Paris-Dauphine Economics Ph.D. Workshop and Economic Department Seminar. It has also been published as a working paper at the Grantham Research Institute (LSE).

1.1 Introduction

In this paper, we study the implication of setting a market for carbon permits to meet the net-zero target (in the European Union (EU), this corresponds to an emission reduction objective of 55 percent by 2030 compared to the 1990 level). To de-carbonize the economy, the price of carbon is expected to rise sharply, as the welfare maximizing optimal policy is shown to not be sufficient ([Golosov et al., 2014] and [Hassler et al., 2020]). This could potentially lead to both welfare distortions in the long run and financial disruptions in the short run (depending on the market structure and price volatility). A framework seeking a better integration of macro-finance and environment would allow, on one hand, for a better understanding of carbon mitigation pricing policies as well as their impacts on different macro aggregates including consumer welfare, which is shown to be significantly impacted and differs depending on the carbon pricing policy market design in place ([Sager, 2019]). On the other hand, this framework would also allow for investigating the linkages and impacts of the climate externality on financial aggregates such as the natural rate of interest and the risk premium ([Benmir et al., 2020] and [Bauer and Rudebusch, 2021]). In our quantitative analysis, we take the EU net-zero policy as given and investigate how macro-financial policies could interact with it.

This paper is tightly linked to three strands of literature that address macro-environmental issues and the role of macro-financial authorities.

The first strand focuses on long-term analysis of the nexus between climate policies and the macroeconomy and can be traced back to the early work of [Nordhaus, 1991]. A wide range of literature of integrated assessment models (IAMs) extended the framework developed by Nordhaus to account for uncertainty in climate dynamics and damages (see [Stern, 2008], [Weitzman, 2012], and [Dietz and Stern, 2015], among others). [Golosov et al., 2014] use a dynamic stochastic general equilibrium (DSGE) model to show that the optimal carbon price is not impacted by future uncertainty. They also find that following the optimal

policy would not allow for global warming to be kept well below 2°C over a 50 years horizon. This is consistent with our simulations, which show that the price of carbon needs to rise well above its optimal counterpart to set the Euro Area (EA) on the net-zero path. While [Goloso et al., 2014] compute transition pathways resulting from the implementation of an optimal carbon price policy, we instead consider the carbon price resulting from the European Trading System (ETS) cap policy. In the same spirit of our work, [Hassler et al., 2020] investigate several sub-optimal policy scenarios using a multi-country IAM. These scenarios, however, are not designed to represent current carbon policies in the European Union (EU) and IAMs do not feature a role for the financial system. In a recent paper, [Van der Ploeg et al., 2020] study the financial consequences of climate risk with respect to portfolio choices. Although our article shares similar components with the latter, we differ by explicitly modeling financial intermediaries. [Carattini et al., 2021] and [Diluiso et al., 2021] also build environmental DSGE (E-DSGE) models with financial frictions, yet they do not account for trend growth and uncertainty around the level of TFP and carbon price in their long-term simulations, both of which are featured in our analysis. Furthermore, they both simulate transition pathways as a response to exogenous shocks, rather than using deterministic simulations. However, similar to [Carattini et al., 2021], we consider macroprudential policy as a long-term tool that can be used to shape banks' balance sheets in order to contain climate risk rather than a short-term tool to address financial shocks ([Diluiso et al., 2021]). With respect to the literature on long-term transition pathways, our simulations feature both deterministic trends and uncertainty on the level of TFP, as well as on the carbon price. While [Cai and Lontzek, 2019] also perform long-term transitions with uncertainty around the trend of TFP and climate damages, we focus on TFP and the price of carbon as we consider a shorter horizon. In addition, we use a Newton-based method to compute the solution where [Cai and Lontzek, 2019] use value function iteration. We also provide a dynamic analysis of welfare, which allows us to study the benefits of macroprudential policy along the transition to the net-zero target.

The second strand of literature relevant to our work focuses on business cycle impli-

cations of environmental policies. [Angelopoulos et al., 2010], [Fischer and Springborn, 2011], [Heutel, 2012], among others,² paved the way for business cycle analysis under an environmental externality. The main focus of these papers is to assess the efficiency of different environmental policies. In recent months, papers such as [Diluiso et al., 2021] or [Carattini et al., 2021] incorporated a financial sector in order to study the role of monetary and macroprudential policies in the fight against climate change. Our short-term analysis is tangentially related to these two papers. In our framework, however, the monetary authority intervenes to correct a distortion in risk premia stemming from carbon price volatility, which we estimate based on observed ETS futures price data. The role of the central bank thus arises endogenously from the transmission of carbon price shocks to financial variables through the marginal cost of firms, while [Diluiso et al., 2021] explore the benefits of both monetary and macroprudential policies in response to an exogenous shock to the quality of brown assets.

Finally, this paper is also linked to a strand of literature assessing central banks' large-scale asset purchases (LSAP) programs, and especially the so-called green quantitative easing (green QE). In the wake of the Great Financial Crisis, [Gertler and Karadi, 2011] provided a framework to study the impact of central banks' LSAP programs in response to a shock to the quality of capital. With respect to green QE, [Ferrari and Nispi Landi, 2021] investigate the impact of a series of positive unexpected shocks to the central bank's holdings of green bonds to simulate an assets purchase program. We differ by considering that LSAP programs are expected by agents, as central banks communicate about them beforehand. We also consider two types of green LSAP programs (transitory and permanent) and the interaction between them and pre-announced macroprudential policy.

Our modeling device borrows components from several macroeconomic types of models. We first build on the canonical versions of New Keynesian (NK) models such as [Woodford,

²E.g. [Bosetti et al., 2014], [Annicchiarico and Di Dio, 2015], and [Dissou and Karnizova, 2016]. For an extensive literature review distinguishing between the long-term and business cycle environmental macroeconomics, respectively, please refer to [Schubert, 2018].

2003], [Smets and Wouters, 2003] or [Christiano et al., 2005] to derive the core of our economy.³ Second, we add environmental components as in [Nordhaus, 2008], [Heutel, 2012], and [Dietz and Venmans, 2019], which allow for the analysis of the dynamics of the economy under the presence of the CO₂ externality. However, as opposed to [Heutel, 2012], we differentiate between green and brown firms instead of using one sole representation for firms, thus borrowing from the multi-sector literature ([Carvalho and Nechio, 2016] among others⁴). Finally, we include balance sheet constrained financial intermediaries as in [Gertler and Karadi, 2011]. Given that we introduce a macroprudential authority that can alter this constraint, we also draw on [Pietrunti, 2017].

As we will consider monetary policy, we only focus on the EA. We perform medium/long-term simulations both for transition pathways to meet the net-zero target and for LSAP programs along the transition to net-zero. As for business cycle simulations, we rely on second order impulse responses to analyze the impact of the ETS carbon price shock on macro-financial aggregates. The novelty of our approach is that our transition pathways feature both long-run deterministic growth rates (i.e. labor augmenting technology and carbon cap policy) and stochastic components around these trends. This allows us to compute confidence intervals for our variables of interest using Monte-Carlo simulations. Furthermore, we rely on the simulated method of moments (SMM) to estimate key structural parameters and match the EA macroeconomic, financial, and environmental empirical data.

Our main theoretical result highlights the inefficiencies stemming from the EU ETS design. In the long term we show that, as the cap policy diverges from the optimal social cost of carbon (SCC), the loss on welfare increases, whereas, in the short term the ETS market design induces volatility in the carbon price that distorts risk premia.

³Note that for simplicity we abstract from wages rigidities and labor disutility.

⁴We note that a substantial literature referred to as “directed technical progress” uses two sectors (green and dirty) to investigate the transition to a green economy and impacts of different environmental policies. See, for example, [Smulders and De Nooij, 2003], [Grimaud and Rouge, 2008], [Di Maria and Valente, 2008], [Acemoglu et al., 2012], [Aghion et al., 2016], [Acemoglu et al., 2019].

On applied grounds, our contribution is to propose tools to mitigate these inefficiencies. Using numerical simulations, we find that an instrument that deviates from the optimal policy (SCC), such as the ETS, is needed to meet the net-zero target. However, this induces a substantial cost in terms of welfare (3 percent consumption equivalent). To ease the welfare burden, we show that a sectoral risk-weight (*i.e.* climate risk-weight) macroprudential policy is able to reduce the wedge gap, without imposing infeasible regulatory weights on assets held by financial intermediaries and jeopardizing financial stability. In particular, a sectoral macroprudential policy favorable to the green sector boosts green capital and output, inducing a gain in welfare, compared to the sub-optimal policy economy without macroprudential policy, as the green sector is less sensitive to the rise in carbon price. With respect to the distortion on risk premia, we show that short-term monetary policy instruments (*i.e.* QE rules) are able to restore the equilibrium in the financial markets. Thus, macroprudential and monetary policies could play an important role in offsetting the negative effects stemming from the implementation of a market for carbon permits. Finally, we investigate the role of asset purchase programs over the net-zero transition and find that central banks would have an incentive to tilt their portfolio of assets toward the green sector when macroprudential policy takes into account climate risk. More generally, we show that QE rules could be used as a short-term countercyclical tool, while sectoral macroprudential policy could play a more structural role, allowing for a smooth transition toward net-zero emissions.

Our actual findings could be further reinforced if we were to see an increase in the share of the green sector, as illustrated in our simulated transition in [figure 1.2](#) and [figure 1.3](#), and as argued in the work of [[Acemoglu et al., 2016](#)], where the focus is on the long-term transition strategies.

This paper is organized as follows: section 2 presents the model, section 3 explains the solution method, section 4 discusses the results, and section 5 concludes.

1.2 The Model

Using the NK-DSGE framework as a foundation, the present paper investigates the potential role of fiscal policy, central bank unconventional monetary policy, and macroprudential policy, in mitigating climate change impacts on macroeconomic and financial aggregates. We first model our two-sector economy following [Carvalho and Nechio, 2016]. Then, we incorporate the environmental component following [Nordhaus, 2008], [Heutel, 2012], and [Dietz and Venmans, 2019], among others. Finally, we model financial intermediaries drawing on [Gertler and Karadi, 2011].

In a nutshell, the economy modeled is described using a discrete set up with time $t \in (0, 1, 2, \dots, \infty)$. The production sectors produce two goods (final and intermediate goods) using labor and capital. Households consume, offer labor services, and rent out capital to firms via financial intermediaries. Public authorities decide on the fiscal and environmental policy, the central bank decides on the monetary policy, and the financial authority sets the macroprudential policy.

1.2.1 The Household

At each period, the representative household supplies labor inelastically to the two sectors of our economy (i.e green and brown sectors denoted by $k \in \{g, b\}$ ⁵), while they also consume and save. Households can either lend their money to the government or to financial intermediaries, who will in turn leverage and finance firms. In each household there are bankers and workers. Each banker manages a financial intermediary and transfers profits to the household. Nevertheless, households cannot lend their money to a financial intermediary owned by one of their members. Household members who are workers supply labor and return their salaries to the household to which they belong.

Agents can switch between the two occupations over time. There is a fraction f of

⁵Where ‘g’ refers to the green sector and ‘b’ to the brown sector.

agents who are bankers and a probability θ_B that a banker remains a banker in the next period. Thus, $(1 - f)\theta_B$ bankers become workers every period and vice versa, which keeps the relative proportions constant. Exiting bankers give their retained earnings to households, which will use them as start-up funds for new bankers.

Households solve the following maximization problem:

$$\max_{\{C_t, B_{t+1}\}} E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} \right] \quad (1.1)$$

s.t.

$$C_t + B_{t+1} = \sum_k g(\varkappa) (W_{t,k} L_{t,k} + \Pi_{t,k}) + \Pi_t^T + T_t + R_t B_t, \quad (1.2)$$

where $\beta \in (0, 1)$ is the discount factor and σ shapes the utility function of the representative household associated with risk consumption C_t . The consumption index C_t is subject to external habits with degree $h \in [0; 1)$. Labor supply $L_{t,k}$ ⁶ in each sector is remunerated at nominal wage $W_{t,k}$. Note that the sector share for the green g is $g(\varkappa) = \varkappa$ and $(1 - \varkappa)$ for the brown sector b . $\Pi_{t,k}$ are profits from the ownership of firms, while Π_t^T are profits from the ownership of financial intermediaries and capital producing firms. T_t is lump sum taxes. As we assume that intermediaries deposits and government bonds are one period bonds, $R_t B_t$ is interest received on bonds held and B_{t+1} is bonds acquired.

Solving the first order conditions and denoting ϱ_t as the marginal utility of consumption, the consumption/saving equations are:

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \{ (C_{t+1} - hC_t)^{-\sigma} \}, \quad (1.3)$$

$$1 = \beta E_t \Lambda_{t,t+1} R_{t+1}, \quad (1.4)$$

⁶As the main focus of the paper is on medium run transitions, we impose inelastic labour supply to ease computational burden (without a loss of generality, as most of the effects will captured by the wages). We note that inelastic labor $L_{t,k} = \bar{L}_k$, where \bar{L}_k is the steady state level of labor in each sector.

with $\Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}$ the expected variation in the marginal utility of consumption.

1.2.2 The Firms

1.2.2.1 The Final Firms

Using the multi-sector framework from [Carvalho and Nechio, 2016], and under non-perfect competition, we assume that production comprises two sectors. Our representative final firms produce a final good $Y_{t,k}$ in these two competitive sectors. Using no more than capital and labor to produce the intermediate good Y_{jt} (where $j \in (0, 1)$ is the continuum of intermediate goods firms), intermediate firms supply the final sectors. In other words, the “bundling” of intermediate goods within the two sectors leads to a final good. The final economy good is a constant elasticity of substitution aggregate of the two sectors:

$$Y_t = \left(\varkappa^{\frac{1}{\theta}} Y_{t,g}^{1-\frac{1}{\theta}} + (1 - \varkappa)^{\frac{1}{\theta}} Y_{t,b}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}}, \quad (1.5)$$

with $\theta \in (1, \infty)$ the elasticity of substitution between the two sectors, and \varkappa the weight of each sector. The final firms in the model are looking for profit maximization (in nominal terms), at a given price P_t subject to the intermediate goods j in each of the two sectors k at prices $P_{jt,k}$:

$$\max_{Y_{jt}} \Pi_t^{\text{Final}} = P_t Y_t - \varkappa \int_0^1 P_{jt,g} Y_{jt,g} dj - (1 - \varkappa) \int_0^1 P_{jt,b} Y_{jt,b} dj, \quad (1.6)$$

where the aggregation of green and brown firms reads as:

$$Y_{t,k} = \int_0^1 \left(Y_{jt,k}^{1-\frac{1}{\theta_k}} \right)^{\frac{1}{1-\frac{1}{\theta_k}}}. \quad (1.7)$$

However, while we assume a constant elasticity of substitution between the final sectors, we consider a different elasticity of substitution θ_k between differentiated intermediate goods within each sector. As the goods of the two sectors entail different costs, a different

elasticity of substitution is considered. This assumption, which shapes the marginal cost structure, is based both on theoretical work of [Tucker, 2010] as well as on the empirical findings of [Chan et al., 2013] and [Chegut et al., 2019], where it is found that green projects entail higher marginal cost (7-13 percent higher costs for green projects in the construction industry compared to non green projects depending on the 'greenness' of the project, and 5-7 percent higher costs in the cement and iron & steel sectors, respectively).

The first order condition for the final firm profit maximization problem yields:

$$Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}} \right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t} \right)^{-\theta} Y_t. \quad (1.8)$$

Under perfect competition and free entry, the price of the final good is denoted P_t , while the price $P_{t,k}$ is the price index of sector- k intermediate goods. Finally, the price $P_{jt,k}$ is the price charged by firm j from sector k .

Prices of final aggregate goods and for each sector are given by:

$$P_t = (\varkappa P_{t,g}^{1-\theta} + (1 - \varkappa) P_{t,b}^{1-\theta})^{\frac{1}{1-\theta}}, \quad (1.9)$$

$$P_{t,k} = \left(\int_0^1 P_{jt,k}^{1-\theta_k} dj \right)^{\frac{1}{1-\theta_k}}. \quad (1.10)$$

1.2.2.2 The Intermediate Firms

Our economy is composed of two categories of firms: i) green firms, which are environmentally-friendly and ii) brown firms with a higher emission intensity. The representative firms j in each sector k of the modeled economy uses capital $K_{t,k}$ and labor $L_{t,k}$ to produce the intermediate good. In our framework, firms' productivity is subject to climate dynamics. As presented in [Goloso et al., 2014] real business cycle model, the environmental externality constrains the Cobb-Douglas production function of the firms, where the negative externality deteriorates the environment and alters production possibilities for firms. However, we differ from [Goloso et al., 2014] by incorporating damages

from the stock of emissions through the level of temperature as follows:

$$Y_{jt,k} = \varepsilon_t^{A_k} d(T_t^o) K_{jt,k}^\alpha (\Gamma_t L_{jt,k})^{1-\alpha}, \quad \alpha \in (0, 1), \quad (1.11)$$

where Γ_t is the economy growth trend and $d(T_t^o)$ a convex function relating the temperature level to a deterioration in output ($d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2} T_t^{o2}}$), with $(a, b) \in \mathbb{R}^2$, which is borrowed from [Nordhaus and Moffat, 2017]. As highlighted by [Benhabib et al., 1991], [Jaimovich and Rebelo, 2009], and [Queralto, 2020], the business cycle literature typically features preferences and/or production functions with $\Gamma_t = 1$ for all t . Within a business cycle framework, we usually assume no long-run growth. However, as we are also interested in the transition pathways, our economy features a growth trend Γ_t different than 1 in hours worked. Therefore, we introduce Γ_t^2 to the damage sensitivity parameter b , such that $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2} T_t^{o2}}$. The goal is to ensure the existence of a balanced growth path without a loss of generality, as over the studied period $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2} T_t^{o2}} \approx ae^{-bT_t^{o2}}$. In addition, the growth rate of Γ_t , which determines the growth rate of economy, is set exogenously to γ^Y where $\Gamma_t = \gamma^Y \Gamma_{t-1}$. Furthermore, α is the standard elasticity of output with respect to capital, and $\varepsilon_t^{A_k}$ is a sector-specific technology shock that follows an $AR(1)$ process: $\varepsilon_t^{A_k} = \rho_{A_k} \varepsilon_{t-1}^{A_k} + \sigma_{A_k} \eta_t^{A_k}$, with $\eta_t^{A_k} \sim \mathcal{N}(0, 1)$.

Global temperature T_t^o is linearly proportional to the level of the emission stock, which in turn is proportional to cumulative emissions as argued by [Dietz and Venmans, 2019]:^{7,8}

$$T_t^o = v_1^o (v_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o, \quad (1.12)$$

⁷To allow for convergence in the auto-regressive law of motion for the stock of emissions process (shown in equation (1.13)) we slightly depart from the transient climate response to cumulative carbon emissions theory and set $\gamma_d \neq 0$. However, we choose γ_d sufficiently low such that $X_t \approx X_0 + \sum_{i=0}^t (E_i + E_i^*)$.

⁸We note that while differences on climate dynamics and damages modeling over the long horizon (whether à la [Golosov et al., 2014], à la [Nordhaus, 2017], or à la [Dietz and Venmans, 2019], among others) induce consequent impacts on macroeconomic aggregate equilibriums, over the business cycle horizon (and under equivalent calibrations), these modeling specifications do not induce significant impacts on macroeconomic aggregate equilibriums.

with v_1^o and v_2^o chosen following [Dietz and Venmans, 2019].

Furthermore, the carbon emissions stock X_t follows a law of motion:

$$X_t = (1 - \gamma_d)X_{t-1} + E_t + E_t^*, \quad (1.13)$$

where $E_t = \sum_k g(\varkappa) \int_0^1 E_{jt,k} dj$ is the aggregate flow of emissions from both the green and brown firms at time t and γ_d is the decay rate. $E_t^* = E^* \Gamma_t$ represents the rest of the world emissions and is used to pin down the actual steady state level of the stock of emission in the atmosphere.⁹ We assume that the rest of the world's emissions grow at the same rate as the domestic GDP over the period studied.

The emissions level is shaped by a non-linear abatement technology $\mu_{jt,k}$ that allows firms to reduce their emissions inflows:

$$E_{jt,k} = (1 - \mu_{jt,k})\varphi_k Y_{jt,k}. \quad (1.14)$$

Emissions $E_{jt,k}$ at firm level are proportional to the production $Y_{jt,k}$ with φ_k the fraction of emissions to output in each sector.¹⁰ Also, emissions could be reduced at the firm level through an abatement effort $\mu_{jt,k}$. The firms are allowed to invest in an abatement technology, but it represents an extra cost.

We model the direct abatement effort costs as follows:

$$Z_{jt,k} = f(\mu_{jt,k})Y_{jt,k}, \quad (1.15)$$

where

$$f(\mu_{jt,k}) = \theta_{1,k} \mu_{jt,k}^{\theta_{2,k}}, \quad \theta_1 > 0, \theta_2 > 1, \quad (1.16)$$

with $\theta_{1,k}$ and $\theta_{2,k}$ the cost efficiency of abatement parameters for each sector.

⁹In order to retrieve a balanced growth path, we assume that the rest of the world's emissions grow at the rate of the Euro Zone.

¹⁰Contrary to [Cai and Lontzek, 2019], we consider $\varphi_{t,k} = \varphi_k$ constant overtime and calibrate it using Euro Area emissions to GDP data, as we focus on shorter time horizons (less than 50 years).

Thus, profits of our representative intermediate firms in each sector $\Pi_{jt,k}$ will be impacted by the presence of the environmental externality. Revenues are the value of intermediate goods $Y_{jt,k}$, while costs arise from: i) wages $W_{t,k}$ (paid to the labor force $L_{jt,k}$), ii) rents $R_{t,k}^K$ (on capital $K_{jt,k}$), iii) abatement investments $f(\mu_{jt,k})$, and iv) the cost of releasing carbon in the atmosphere $\tau_{et,k}E_{jt,k}$ (i.e. the carbon price paid to the government).

$$\begin{aligned}\Pi_{jt,k} &= \frac{P_{jt,k}}{P_t} Y_{jt,k} - W_{t,k} L_{jt,k} - R_{t,k}^K K_{jt,k} - \theta_{1,k} \mu_{jt,k}^{\theta_{2,k}} Y_{jt,k} - \tau_{et,k} E_{jt,k} \\ &= \left(\frac{P_{jt,k}}{P_t} - MC_{t,k} \right) Y_{jt,k},\end{aligned}\tag{1.17}$$

As firms are not free to update prices each period, they first choose inputs so as to minimize costs, given a price, subject to the demand constraint.

The cost-minimization problem yields the marginal cost, which can be expressed following the first-order conditions with respect to the firm's optimal choice of capital, labor, abatement, and production level, respectively:

$$R_{t,k}^K = \alpha \Psi_{jt,k} \frac{Y_{jt,k}}{K_{jt,k}},\tag{1.18}$$

$$W_{t,k}^K = (1 - \alpha) \Psi_{jt,k} \frac{Y_{jt,k}}{L_{jt,k}},\tag{1.19}$$

$$\tau_{et,k} = \frac{\theta_{1,k} \theta_{2,k}}{\varphi_k} \mu_{jt,k}^{\theta_{2,k}-1},\tag{1.20}$$

$$MC_{jt,k} = MC_{t,k} = \Psi_{t,k} + \theta_{1,k} \mu_{t,k}^{\theta_{2,k}} + \tau_{et,k} (1 - \mu_{t,k}) \varphi_k,\tag{1.21}$$

where $\Psi_{jt,k} = \Psi_{t,k}$ ¹¹ is the marginal cost component related to the same capital-labor ratio all firms from each sector choose. This marginal cost component is common to all intermediate firms, but differs across sectors.

Equation (1.20) is the optimal condition on abatement: abating CO₂ emissions is optimal when its marginal gain equals its marginal cost. This equation highlights the key

¹¹ $\Psi_{jt,k} = \Psi_{t,k} = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{1}{\varepsilon_t^{\alpha,k} d(T_t^\sigma)} (W_{t,k})^{1-\alpha} \left(R_{t,k}^K \right)^\alpha$.

role of the carbon price in shaping firms' decisions. In addition, abatement efforts $\mu_{t,k}$ are common to all firms of the same sector, as the environmental cost is also common to all firms of the same sector. Furthermore, as the impact of the environmental externality is not internalized by firms (i.e. they take X_t and T_t^o as given), the shadow value of the environmental externality is zero.

The total marginal cost captures both abatement and emissions costs as shown above in [equation \(1.21\)](#). Note that in the case of the laissez-faire scenario, $MC_{t,k} = \Psi_{t,k}$, as the firms are not subject to emissions and abatement constraints.

In addition, monopolistic firms engage in a price setting à la Rotemberg.¹² Price update is subject to an adjustment cost given by $\Delta_{jt,k}^P = \frac{\theta^P}{2} \left(\frac{P_{jt,k}}{P_{jt-1,k}} - 1 \right)^2$. Thus, profit maximization subject to the demand from final firms reads as follows:

$$\max_{P_{jt,k}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} (\Pi_{jt+i,k} - \Delta_{jt+i,k}^P Y_{t+i}) \quad (1.22)$$

$$\text{s.t. } Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}} \right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t} \right)^{-\theta} Y_t,$$

where $\beta^i \Lambda_{t,t+i} = \beta^i \frac{\varrho_{t+i}}{\varrho_t}$ is the real stochastic discount factor, or as commonly called in the macro-finance literature, the pricing kernel.

The NK Philips Curve pricing equation for each sector is as follows:

$$\theta^P \pi_{t,k} (\pi_{t,k} - 1) = \left(\frac{P_{t,k}}{P_t} \right)^{-\theta} \left(\frac{P_{t,k}}{P_t} (1 - \theta_k) + \theta_k MC_{t,k} \right) + E_t \left\{ M_{t,t+1} \frac{Y_{t+1}}{Y_t} \theta^P \pi_{t+1,k} (\pi_{t+1,k} - 1) \right\}, \quad (1.23)$$

with sectoral inflation $\pi_{t,k} = P_{t,k}/P_{t-1,k}$.

¹²As a robustness exercise we set price stickiness à la Calvo ([Appendix section 1.C.3](#)) and find similar results.

The aggregate inflation $\pi_t = \frac{P_t}{P_{t-1}}$ reads as:

$$\pi_t = \left(\varkappa^{\frac{1}{\theta}} \frac{P_{t-1,g}}{P_{t-1}} \pi_{t,g}^{1-\frac{1}{\theta}} + (1 - \varkappa)^{\frac{1}{\theta}} \frac{P_{t-1,b}}{P_{t-1}} \pi_{t,b}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}}. \quad (1.24)$$

In addition, please note that the j -index referring to our intermediate firms collapses as all firms for each sector, which are capable of setting their price optimally at t , will make the same decisions.

1.2.2.3 Capital Producing Firms

We assume that households own capital producing firms and receive profits. Capital producing firms buy specific types of capital from intermediate goods firms at the end of period t , repair depreciated capital, and create new capital. They then sell both the new and re-furbished capital. The relative price of a unit of capital is $Q_{t,g}$ for green and $Q_{t,b}$ for brown. We suppose that there are flow adjustment costs associated with producing new capital as in [Jermann, 1998]. Accordingly, capital producing firms face the following maximization problem:

$$\max_{\{I_{t,k}\}} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \{ (Q_{t+s,k} - 1) I_{t+s,k} - f_k(\cdot)(I_{t+s,k}) \} \quad (1.25)$$

$$\text{with } I_{t,k}^n = I_{t,k} - \delta K_{t,k}, \quad (1.26)$$

$$K_{t+1,k} = K_{t,k} + I_{t,k}^n, \quad (1.27)$$

$$\text{and } f_k(\cdot) = \frac{\eta_i}{2} \left(\frac{I_{t,k}}{I_{t-1,k}} - \theta^I \right)^2, \quad (1.28)$$

where $I_{t,k}^n$ and $I_{t,k}$ are net and gross capital created, respectively. $\delta K_{t,k}$ is the quantity of re-furbished capital, and η_i the inverse elasticity of net investment to the price of capital.¹³

¹³The term θ^I is set such that the over the balanced growth path ($f_k\left(\frac{i_{t,k}}{i_{t-1,k}}\right) = 0$), where $i_{t,k}$ is the de-trended net investment.

Thus, we get the following value for $Q_{t,k}$:

$$Q_{t,k} = 1 + f_k(\cdot) + f'_k(\cdot) \left(\frac{I_{t,k}}{I_{t-1,k}} \right) - \beta E_t \left\{ \Lambda_{t,t+1} f'_k(\cdot) \left(\frac{I_{t+1,k}}{I_{t,k}} \right)^2 \right\}. \quad (1.29)$$

1.2.3 Financial Intermediaries

We augment the setup of [Gertler and Karadi, 2011] to allow financial intermediaries to invest in both green and carbon-intensive firms. We also modify the incentive constraint to provide a realistic implementation of macroprudential policy through regulatory risk-weights on loans.

A representative bank's balance sheet can be depicted as:

$$Q_{t,g}S_{t,g} + Q_{t,b}S_{t,b} = N_t + B_t, \quad (1.30)$$

where $S_{t,g}$ and $S_{t,b}$ are financial claims on green and brown firms and $Q_{t,g}$ and $Q_{t,b}$ their respective relative price. Note that $S_{t,k} = K_{t,k}$, as firms from both sectors do not face frictions when requesting financing. On the liability side, N_t is the banks' net worth and B_t is debt to households. Over time, the banks' equity capital evolves as follows:

$$N_t = R_{t,g}Q_{t-1,g}S_{t-1,g} + R_{t,b}Q_{t-1,b}S_{t-1,b} - R_t B_{t-1}, \quad (1.31)$$

$$N_t = (R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,b} - R_t)Q_{t-1,b}S_{t-1,b} + R_t N_{t-1}, \quad (1.32)$$

where $R_{t,k} = \frac{R_{t,k}^K - (Q_{t,k} - \delta)}{Q_{t-1,k}}$ denotes the gross rate of return on a unit of the bank's assets from $t - 1$ to t for sector k .¹⁴

The goal of a financial intermediary is to maximize its equity over time. Thus, we can

¹⁴Note that the depreciated capital has a value of one as adjustment costs only apply to net investment.

write the following objective function:

$$V_t = E_t \left\{ \sum_{i=1}^{\infty} (\Delta\beta)^i \Lambda_{t,t+i} (1 - \theta_B) \theta_B^{i-1} N_{t+i} \right\}, \quad (1.33)$$

with $(1 - \theta_B)$ the exogenous probability of going out of business for a banker and Δ a parameter accounting for the subjective discount factor of bankers.¹⁵ We introduce a regulator in charge of the supervision of financial intermediaries. Drawing on [Pietrunti, 2017], we assume that the regulator requires that the discounted value of the bankers' net worth should be greater than or equal to the current value of assets, weighted by their relative risk:

$$V_t \geq \lambda(\lambda_g Q_{t,g} S_{t,g} + \lambda_b Q_{t,b} S_{t,b}), \quad (1.34)$$

with λ the risk-weight on loans and λ_g and λ_b sectoral specific weights that can be applied to loans for green and/or brown firms. The regulator can modify these weights, altering the constraint weighing on banks and thus the allocation of loans between sectors. In our baseline version of the model, however, we consider the case where λ_g and λ_b are both equal to one, and we calibrate λ and other banks-related parameters to match the capital ratio of banks in the Euro Area as well as risk premia levels. We guess that the value function is linear of the form $V_t = \Gamma_t^B N_t$ so we can rewrite V_t as:

$$V_t = \max_{S_{t,g}, S_{t,b}} E_t \{ \Delta\beta \Lambda_{t,t+1} \Omega_{t+1} N_{t+1} \}, \quad (1.35)$$

where $\Omega_t \equiv 1 - \theta_B + \theta_B \Gamma_t^B$. Maximization subject to the regulatory constraint (1.34) yields the following first order and slackness conditions:

$$\Delta\beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1,k} - R_{t+1}) \} = \nu_t \lambda_k \lambda, \quad (1.36)$$

$$\nu_t [\Gamma_t^B N_t - \lambda(\lambda_g Q_{t,g} S_{t,g} + \lambda_b Q_{t,b} S_{t,b})] = 0, \quad (1.37)$$

¹⁵This parameter allows us to perfectly match the steady state financial aggregates for the EA.

where ν_t is the multiplier for constraint (1.34). One interesting result is that we get:

$$N_t \geq \Xi_t(\lambda_g Q_{t,g} S_{t,g} + \lambda_b Q_{t,b} S_{t,b}), \quad (1.38)$$

where $\Xi_t = \lambda/\Gamma_t^B$ is the regulatory capital requirement for banks and λ_g and λ_b represent potential rewards or penalties on the weights required by the regulator on green and brown loans, respectively.¹⁶ Finally, we rewrite the value function to find Γ_t :

$$\begin{aligned} V_t &= \lambda \nu_t (\lambda_g Q_{t,g} S_{t,g} + \lambda_b Q_{t,b} S_{t,b}) + \Delta \beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} N_t \} \\ \Gamma_t^B N_t &= \nu_t \Gamma_t^B N_t + \Delta \beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} R_t N_t \} \\ \Gamma_t^B &= \frac{1}{1 - \nu_t} \Delta \beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \}. \end{aligned} \quad (1.39)$$

We close this part of the model with the aggregate law of motion for the net worth of bankers:

$$N_t = \theta_B [(R_{t,g} - R_t) Q_{t-1,g} S_{t-1,g} + (R_{t,b} - R_t) Q_{t-1,b} S_{t-1,b}] + (\theta_B R_t + \omega) N_{t-1}, \quad (1.40)$$

with $\omega \in [0; 1)$ the proportion of funds transferred to entering bankers.

1.2.4 Public Authorities

1.2.4.1 Central Bank

The central bank follows a simple [Taylor, 1993] rule to set the interest rate:

$$i_t - \bar{i} = \rho_c (i_{t-1} - \bar{i}) + (1 - \rho_c) [\phi_\pi (\pi_t - \bar{\pi}) + \phi_y (Y_t - Y_{t-1})], \quad (1.41)$$

¹⁶For instance, if $\lambda_g < \lambda_b$ banks will need to hold less capital for loans they grant to green firms compared to brown firms. Note that the actual capital ratio thus also depends on the risk-weights assigned to each asset, consistent with Basel III framework.

where \bar{i} is the steady state of the nominal rate i_t , $\rho_c \in [0, 1)$ is the smoothing coefficient, $\phi_\pi \geq 1$ is the inflation stance penalizing deviations of inflation from the steady state, ϕ_y is the output gap stance penalizing deviations of output from its previous period level Y_{t-1} . Moreover, the relationship between the nominal and the real interest is modeled through the Fisherian equation:

$$i_t = R_t E_t \{ \pi_{t+1} \}. \quad (1.42)$$

We match the observed level of nominal interest rate using the simulated method of moments with the German 10-year Bund as an observable.¹⁷ The estimation leads to a steady state value of about 1% annually over the sample period.

In addition to setting the nominal interest rate, the central bank conducts open market operations. Within our framework, it will be able to buy and sell assets that are otherwise held by financial intermediaries. We will explain in [section 1.2.7](#) how public financial intermediation (i.e. QE) works in this model.

1.2.4.2 Government

The government sets a budget constraint according to the following rule:

$$T_t + \tau_{et} E_t + RP_{t,g} \psi_{t,g} K_{t,g} + RP_{t,d} \psi_{t,b} K_{t,b} = G_t, \quad (1.43)$$

with public expenditure G_t finding its source from taxes T_t , revenues from the price of carbon $\tau_{et} E_t$ and from public financial intermediation on both green and brown firms $RP_{t,g} \psi_{t,g} K_{t,g}$ and $RP_{t,b} \psi_{t,d} K_{t,b}$ (with $RP_{t,k}$ the spread between each sector's risky rate and the riskless rate, also referred to as risk premia). Government spending is also assumed to be a fixed proportion of the GDP:

$$G_t = \frac{\bar{g}}{\bar{y}} Y_t. \quad (1.44)$$

¹⁷At the steady state, inflation is normalized to 1, so that $i_t = R_t$.

1.2.5 Normalization and Aggregation

Factors and goods markets clear as follows. First, the market-clearing conditions for aggregate capital and investment in the two sector economy read as: $K_t = \sum_k g(\varkappa) \int_0^1 K_{jt,k} dj$ and $I_t = \sum_k g(\varkappa) \int_0^1 I_{jt,k} dj$, respectively. Second, global aggregate emissions and aggregate emissions cost are two weighted sums of sectoral emissions $E_t = \sum_k g(\varkappa) \int_0^1 E_{jt,k} dj$, and sectoral emissions cost $Z_t = \sum_k g(\varkappa) \int_0^1 Z_{jt,k} dj$, respectively. Finally, the resource constraint of the economy features capital adjustment and abatement costs:

$$Y_t = C_t + G_t + I_t + \sum_k g(\varkappa) [f_k(\cdot)(I_{t,k})] + \sum_k g(\varkappa) \Delta_{t,k}^P Y_t + Z_t. \quad (1.45)$$

1.2.6 Climate Externality and Financial-Economics Inefficiencies

Retrieving the optimal allocation where the environmental cost is internalized by the central planner requires setting the carbon price in the decentralized equilibrium equals to the social cost of carbon found in the centralized problem. To keep the framework tractable and without a loss of generality, we solve the centralized problem for households and firms, given an allocation of investment, capital, financial intermediaries net worth and deposit as these do not enter the social cost of carbon derivation.¹⁸

1.2.6.1 Competitive Equilibrium

To pin down the optimal carbon policy, we solve for the Competitive Equilibrium (CE*). The CE* in this economy is defined as follows:

Definition 1.2.1 *A competitive equilibrium consists of an allocation $\{C_t, K_{t,k}, E_{t,k}, X_t, T_t^o\}$, a set of prices $\{P_t, P_{t,k}, R_t, R_{t,k}^k, W_{t,k}\}$ and a set of policies $\{\tau_{et,k}, T_t, B_{t+1}\}$ such that:*

- *the allocation solves the consumers' and firms' problems given prices and policies,*

¹⁸We can easily show that adding financial intermediaries as well as capital producing firms to the constraints of the centralized problem does not change the SCC derivation.

- the government budget constraint is satisfied in every period,
- temperature change satisfies the carbon cycle constraint in every period, and
- markets clear.

Result 1 *The optimal solution sets the carbon price policy $\tau_{et,k}$ as an optimal policy $\tau_{et,k}^*$, which maximizes total welfare in equation (1.1):¹⁹*

$$\tau_{et,k}^* = g(\varkappa)SCC_t. \quad (1.46)$$

with SCC_t the social cost of carbon:

$$SCC_t = \eta\beta\frac{\lambda_{t+1}}{\lambda_t}SCC_{t+1} + (v_1^o v_2^o)\beta\frac{\lambda_{t+1}}{\lambda_t}\S_{t+1}^T, \quad (1.47)$$

and with

$$\S_t^T = (1 - v_1^o)\beta\frac{\lambda_{t+1}}{\lambda_t}\S_{t+1}^T - \sum_k \Psi_{t,k}\varepsilon_t^{A,k}\frac{\partial d(T_t^o)}{\partial T_t^o}K_{t,k}^\alpha(\Gamma_t L_{t,k})^{1-\alpha}. \quad (1.48)$$

1.2.6.2 Departing from the Competitive Equilibrium to Meet Climate Goals

Definition 1.2.2 *Public authorities, however, do not optimally set the carbon price as highlighted in definition 2. In the EU, public authorities target a level of emissions that is consistent with their objective of a 55% emissions reduction by 2030. In practice, this means gradually increasing the cost of carbon through the reduction of emissions quotas distributed to firms within specific sectors. We model this situation by assuming that the cap set by the fiscal authority follows a decreasing trend, implying a growing price of carbon. The resulting carbon price can then be hit by exogenous shocks, to account in a ‘stylized’ way for price fluctuations on the ETS market:*

$$E_t = Cap_t \quad (1.49)$$

¹⁹The full derivation of the CE* can be found in the technical appendix

with $Cap_t = Cap/\Gamma_t^{Cap}$. Equivalently, a cap on emissions translates to a price of carbon such that:

$$\tau_{et,k} = Carbon\ Price_t, \quad (1.50)$$

where $Carbon\ Price_t = \varepsilon_t^\tau \Gamma_t^{Price}$ Carbon Price. In this case, Γ_t^{Price} is a trend on the carbon price that is proportional to the trend on the cap Γ_t^{Cap} and is consistent with the desired emissions reduction implemented through the cap policy. ε_t^τ represents the ETS price shock.²⁰

This stylized representation of the implementation of a permit market allows us to find theoretical fiscal pathways consistent with the EU climate objectives.

1.2.6.3 Welfare Distortion

Definition 1.2.3 *The welfare distortion arises when there is a difference between the optimal environmental policy and the targeted policy consistent with the EU objectives:*

$$\tau_{et,k}^* \neq \tau_{et,k} \quad (1.51)$$

When $\tau_{et,k}$ moves away from $\tau_{et,k}^*$, the loss in welfare grows.²¹

$$\Delta_{\{\tau-\tau^*\}} Welfare < 0 \quad (1.52)$$

where the welfare could be decomposed as follows:

$$\begin{aligned} Wedge_{C_k} \propto & (1-g)\varepsilon_t^{A,k}(\Gamma_t^{1-\alpha}\bar{L}^{1-\alpha})(d(T_t^o)K_{t,k}^\alpha - d(T_t^o)^*K_{t,k}^{\alpha*}) - (f(K_{t,k}) - f(K_{t,k})^*) \\ & - ((\Gamma_t^{1-\alpha}\bar{L}^{1-\alpha})(d(T_t^o)K_{t,k}^\alpha f(\mu_{t,k}) - d(T_t^o)^*K_{t,k}^{\alpha*} f(\mu_{t,k})^*) \end{aligned}$$

²⁰In our setup, carbon prices variations at the business cycle frequency are mainly driven by exogenous market forces. While sudden changes in abatement efficiency (i.e. the abatement cost) could in theory be a source of carbon price volatility, we abstract from considering this mechanism as there is a lack of empirical evidence and data availability (at the business cycle frequency) on abatement costs.

²¹A full decomposition of the welfare effect is presented in appendix [section 1.C.5](#).

Proposition 1.2.1 *Macroprudential climate risk-weights loosening the constraint on bank lending to the green sector can reduce the welfare loss on consumption, while addressing climate-related financial risk.²²*

Implementing a higher policy rate compared to an optimal policy clearly decreases damages from temperature to production $d(T_t^o) < d(T_t^o)^*$. However, abatement is costlier under the higher policy rate. This results in a loss of welfare, but prevents potential climate risks in the future that are not internalized by firms. The climate risk-weights macroprudential policy, which will lower (increase) the capital requirement for green (brown) assets, will in turn trigger a rise (decrease) in green (brown) firms' capital. As green firms are less subject to the carbon price, the increase in the relative size of the green sector in total output will lead to a welfare gain.

1.2.6.4 Risk Premium Wedge

Volatility in risk premia $RP_{t,k}$, defined as the difference between expected returns on risky assets $R_{t,k}$ and the return on the riskless asset R_t , could alter monetary policy transmission ([Doh et al., 2015]).

Definition 1.2.4 *When the carbon price is set through a market for carbon permits, it induces price uncertainty that is detrimental to firms. Ultimately, it affects the marginal cost of firms as well as the price of capital, and leads to movements in risk premia. In the case of a positive carbon price shock, the marginal cost of firms increases as they are now subject to higher CO_2 prices. This in turn could raise the risk premium.²³*

$$RP_{t,k} = R_{t,k} - R_t \tag{1.53}$$

$$= f(\Psi_{t,k}, Y_{t,k}, K_{t,k}, Q_{t,k}) - R_t \tag{1.54}$$

²²As detailed in [section 1.2.7](#) and shown in [figure 1.4](#), macroprudential policy arises as a tool to mitigate climate risk to the financial sector. While primarily intended to ensure financial stability, it also dampens the welfare effect of an increasing carbon price.

²³The impact is symmetric in the case of a negative carbon price shock. Furthermore, whether the shock is positive or negative, it implies higher volatility for the marginal cost and the risk premium.

Proposition 1.2.2 *Volatility in risk premia stemming from carbon price fluctuations could potentially distort the functioning of monetary policy operations. Short-term monetary policies (QE rules that react to changes in risk premia) can prevent this situation and ensure financial stability.*

The risky rate reacts to changes coming both from the firms' side and the financial side. In this case, the goal is to cut the link between the rise of the marginal cost (triggered by an increase in the carbon price) and the impact on the risk premium. One way to do so is to act on the financial side to compress the risk premium. Similar to models where a rise in risk premia comes from an exogenous shock on the quality of capital (e.g. crisis simulation in [Gertler and Karadi, 2011]), the central bank is able to offset this effect by intervening in the loan market.

1.2.7 Set of Policies

Environmental Policy

When acting optimally, the decentralized planner would set the environmental policy as shown in [result 1](#) ($\tau_{et,k}^*$ is set equal to the social cost of carbon $g(\varkappa)SCC_{t,k}$). However, as highlighted in the previous section, the EU authorities deviate from the optimal policy and set the environmental policy to be consistent with their net-zero emissions reduction objective ($\tau_{et,k} \neq \tau_{et,k}^*$).

Sectoral Macroprudential Weights

There is a macroprudential authority with the ability to alter the regulatory constraint weighing on banks ([equation \(1.34\)](#)) by modifying risk-weights on loans.

Environmental, social, and corporate governance (ESG) criteria are increasingly valued by both investors and authorities. As these criteria are also gaining importance in firms' credit ratings ([Escrig-Olmedo et al., 2019] and [Carbone et al., 2021]), it will likely im-

pact banks’ portfolio allocation. On the regulatory side, macroprudential authorities are starting to assess how they could consider climate risk within their frameworks. Recently, the [Basel Committee, 2021] issued a press release stating that “The Committee is taking a holistic approach to addressing climate-related financial risks to the global banking system. This includes the assessment and consideration of disclosure, supervisory and regulatory measures.” Within our framework, this would mean that firms with a low carbon intensity would carry a lower risk-weight in the RWA methodology, while carbon-intensive firms would carry a higher risk-weight. In our view, there are two means by which this could materialize. Either ESG criteria would become so important in standard credit ratings such that it could lead to environmentally friendly firms getting a higher rating, and thus a lower risk-weight in banks’ regulatory constraint. For instance, a green firm could see its rating upgraded from BBB+ to A-, implying a 25 percent drop in the risk-weight associated with this firm in banks’ regulatory capital constraint. On the other hand, a carbon-intensive firm could see its rating downgraded from BBB- to BB+, implying a 25 percent increase in the risk-weight associated with this firm.²⁴ In this case, this change in the importance of ESG criteria in credit ratings would endogenously transmit to macroprudential policy, and ultimately to banks’ portfolio allocation. Another possibility would be that macroprudential authorities apply an additional risk-weight related to the carbon intensity of firms. It could for instance multiply the risk-weight related to the credit rating of a firm by a climate risk-weight related to the environmental performance of a firm. In our setup, implementing climate risk-weights in the spirit of Basel III, would mean decreasing λ_g by 25 percent (i.e. $\lambda_g = 0.75$) and increasing λ_b by 25 percent (i.e. $\lambda_b = 1.25$).²⁵ This will loosen (tighten) the regulatory constraint on banks with respect to the green (brown) sector, triggering an increase (decrease) in loans to green (brown) firms. In addition to

²⁴Please refer to the high-level summary of Basel III reforms ([Basel Committee, 2017]) for a detailed description of the RWA methodology.

²⁵We consider this to be our baseline scenario, where both green and brown bonds held by financial intermediaries are mainly at the lower rank of investment grade bonds (i.e. BBB+ to BBB-). We also investigate other cases in our robustness exercises, where climate risk-weights applied are higher.

addressing climate-related financial risk, it would also support the transition to a greener economy.

Quantitative Easing

QE in this model can be both a short-term or a medium/long-term instrument. In the short term, the central bank can purchase or sell bonds as part of open market operations to ensure the smooth transmission of monetary policy. In this case, we model it as a QE rule, in the spirit of [Gertler and Karadi, 2011]. We will show quantitatively how QE rules targeting risk premia can offset the inefficiency stemming from the uncertainty over the carbon price. In the long term, the central bank can also implement LSAP programs, where it decides to buy a predefined portion of assets over a determined period of time. Much like the Corporate Sector Purchase Program in the EA, the central bank has the ability to finance non-financial firms in order to reduce corporate spread, steer private investment, and ultimately keep inflation within range of its target. In a complementary exercise, we will assess how green LSAP programs differ from conventional brown LSAP programs.

Then for each type of firm k we now have:

$$Q_{t,k}S_{t,k} = Q_{t,k}S_{pt,k} + Q_{t,k}S_{gt,k}, \quad (1.55)$$

with $Q_{t,k}S_{gt,k}$ the total real value of loans to firms of type k held by the central bank. $Q_{t,k}S_{pt,k}$ is the total real value of loans to firms of type k held by financial intermediaries, as defined in section 1.2.3. As in [Gertler and Karadi, 2011], we model this intervention by assuming that the central bank holds a portion $\psi_{t,k}$ of total loans to non-financial firms belonging to each sector:²⁶

$$Q_{t,k}S_{gt,k} = \psi_{t,k}Q_{t,k}S_{t,k}. \quad (1.56)$$

²⁶For simplicity, we abstract from monitoring costs.

To address the inefficiency stemming from carbon price uncertainty, we will assume that, for each sector, the central bank follows a counter-cyclical credit policy rule that reacts to the variations in the expected spread ($E_t\{\text{RP}_{t+1,k}\} = E_t\{R_{t+1,k} - R_{t+1}\}$) in order to decide the share of assets $\psi_{t,k}$ it holds. This rule is defined as follows:

$$\psi_{t,k} = \phi_k^s (E_t\{\text{RP}_{t+1,k}\} - \bar{\text{RP}}_k). \quad (1.57)$$

Note that in our baseline model $\psi_{t,k} = 0$ so that the central bank allows financial intermediaries to be the sole source of financing for firms.

1.3 Solution Method

1.3.1 Balanced Growth Path

In our economy, the labor-augmenting technology grows at rate Γ_t . As a number of variables (e.g. output, emissions, investment, ...) will not be stationary, we need to de-trend the model.²⁷ In the appendix [subsection 3.B.6](#) we present the de-trended economy, where all variables are stationary along an existing balanced growth path. The variables of our economy growing at the same rate Γ_t include: output per capita $Y_{t,k}$, investment per capita $I_{t,k}$, consumption per capita C_t , government spending G_t , lump sum taxes T_t , capital per capita $K_{t,k}$, emissions $E_{t,k}$, abatement costs $Z_{t,k}$, stock of emissions X_t , temperature T_t^o , debt to households B_t , net worth N_t , and the banks' value function V_t^B .

1.3.2 Model Solving and Methods

To solve for the medium/long-run pathway scenarios, we use the extended path algorithm, which allows us to integrate both deterministic trends and stochastic shocks. This approach maintains the ability of deterministic methods to provide accurate accounts of

²⁷This is also necessary to estimate our key structural parameters using the SMM.

non-linearities, while usual local approximation techniques do not perform as well under the presence of such non-linearities ([Adjemian and Juillard, 2013]). Furthermore, we account for uncertainty and compute confidence intervals along the net-zero transition pathways. We rely on the Monte Carlo method and simulate 2000 series for both stochastic shocks (i.e labor-augmenting technology and carbon price shocks) around their deterministic trends. As for addressing short-term business cycle implications of the ETS price volatility, we use second-order perturbation methods as they are usually performed in the macro-finance literature to retrieve impulse response functions.

1.3.3 Data and Fitting Strategy

As we will study the role of the central bank and macroprudential authority, we calibrate and estimate the model on the EA, even though the environmental ETS policy is set at the EU level. This is without a loss of generality, since all countries in the EA are members of the EU.

In order to best fit our model to real data,²⁸ we rely on the SMM ([Duffie and Singleton, 1993]) to estimate key structural parameters of our economy (table 1.4). In the spirit of [Jermann, 1998] we match the first and second moments of: output growth, investment growth, and consumption to output growth. As we are also interested in the financial and environmental sectors, we match the first moments of the real riskless and risky rates, the capital ratio of banks, the emission to output ratio, the global stock of carbon, and the ETS price level (at the beginning of 2021), as well as the difference between green and brown firms' marginal costs. We estimate the following key structural parameters: $\{\eta_{A_k}, \rho_{A_k}, \frac{\bar{g}}{\bar{y}}, \eta_i, \beta, \gamma_Y, h, \alpha, \delta, \theta_g, \theta_d, E^*, \varphi_k, \text{Carbon Price}, \lambda, \omega\}$, using the Metropolis–Hastings algorithm for the Markov Chain Monte Carlo over 5 chains of 2000 draws. The remaining parameters are calibrated and their values are reported in table 3.3, table 3.4, and table 3.5.

²⁸For macro-finance data, we match first and second moments using EA data between 2000 and 2020. All data sources are summarized in table 1.5.

1.3.3.1 Calibration

For parameters related to business cycle theory, their calibration is standard: the share of hours worked per day is set at one third in each sector and the coefficient of relative risk aversion σ in the CRRA utility function is set at 2, as argued by [Stern, 2008] and [Weitzman, 2007].

Regarding environmental components, we calibrate the damage function according to [Nordhaus and Moffat, 2017].²⁹ The global temperature parameters v_1^o and v_2^o are set following [Dietz and Venmans, 2019] to pin down the ‘initial pulse-adjustment timescale’ of the climate system.³⁰ We use sectoral data made available by the Transition Pathway Initiative to set the share of the green sector \varkappa at 30 percent.³¹ Abatement parameters $\theta_{b,1}$, $\theta_{b,2}$, and $\theta_{g,2}$, which pin down the abatement costs for each sector, are set as in [Heutel, 2012]. We then proceed to set $\theta_{g,1}$ to match the drop in emissions induced by the introduction of the carbon price policy in the EA. More precisely, we retrieve the value of $\theta_{g,1}$ in such a way so as to be consistent with a reduction of emissions of 14.3 percent between 2009 and 2020,³² which is associated with an increase in the carbon price from 0 to 30 euro (the price of ETS at the end of 2020). In our model, this leads to a value of $\theta_{g,1}$ of 0.02, which means that the abatement technology is cheaper in the green sector. The decay rate of emissions δ_x is set at 0.21 percent as in [Heutel, 2012].

As for the financial parameters, we set the probability of remaining a banker θ_B at 0.98, meaning that 2 percent of bankers default every quarter, which is slightly less than in [Gertler and Karadi, 2011]. Δ is a parameter that introduces a different discount factor in the bankers’ objective function relative to households and is set to 0.99. This implies that bankers are slightly more impatient than households. Finally, the monetary rule

²⁹We perform a sensitivity analysis using values from [Dietz and Stern, 2015] and [Weitzman, 2012] in the next section.

³⁰We also perform a sensitivity analysis for v_2^o .

³¹What we consider green in our model is a sector with a carbon performance that allows for an emission target aligned with the Paris Agreement of 2 degrees Celsius or below.

³²We remove the first and last years of data.

parameters are set as in [Smets and Wouters, 2003].

Regarding the carbon price shock, we calibrate the standard deviation using ETS data (futures prices). We find a standard deviation of about 0.18 on a quarterly basis.

1.3.3.2 Estimation

Parameters estimated through the SMM are reported in table 1.4, while the empirical moments matched are reported in table 1.5. Although we only rely on a shock to the labor-augmenting technology, the model is able to match empirical moments for the EA.

More precisely, the depreciation rate of physical capital is estimated at 2.5 percent in quarterly terms, the government spending to GDP ratio at 28 percent, and the capital intensity in the production function α at 0.33. All these estimates are quite standard within the macroeconomic literature. The inverse elasticity of net investment to the price of capital η_i is estimated at 1.7354, in line with the value chosen by [Gertler and Karadi, 2011]. The parameter b , which allows us to pin down the discount factor, is set at 0.02. This ensures that we match the steady state real interest rate of about 1 percent (the mean rate of 10-year German Bund over the sampled period). Habits in consumption are found to be rather low (0.22) compared to the estimated value of [Smets and Wouters, 2003].

To replicate the global level of carbon stock in the atmosphere (i.e. 840 gigatons), the level of the rest of the world's emissions E^* is estimated at 3.37. Furthermore, as argued by [De Haas and Popov, 2019], CO₂ emissions intensity differs largely between sectors and industries. We use carbon intensity parameters φ_b and φ_g to match the observed ratio of emissions to output for the EA, which is at 21 percent.³³ Assuming that the carbon intensity in the green sector is approximately one third of what it is in the brown sector, we find that $\varphi_b = 0.29$ and $\varphi_g = 0.09$.

The value of θ_a , the brown firms' marginal cost parameter, is set as in [Smets and Wouters, 2003] to replicate the mean markup and marginal cost levels observed in the

³³We compute this value as the number of kCo2 per dollar of GDP using emissions data from the Global Carbon Project and GDP data from Eurostat.

economy. On the other hand, θ_g is estimated to match the green marginal cost, which is—as argued by [Chan et al., 2013] and [Chegut et al., 2019]—6 percent higher than the brown firms’ marginal cost.

The parameter shaping the leverage of banks $\bar{\lambda}$ is estimated at 0.0176 to generate a spread of 80 basis points between risky and riskless assets, consistent with [Fender et al., 2019]. The authors also find that the spread between green and brown bonds recently disappeared. Thus, we target the same steady state for R_g and R_d .³⁴ The proportional transfer to entering bankers ω is found to be around 0.006, allowing us to match a capital ratio of approximately 14.4 percent in the EA.

Finally, for the TFP shock, standard deviation and persistence are estimated at 0.006 and 0.78, which are both in line with previous estimates of [Smets and Wouters, 2003] for the EA.

1.4 Quantitative Analysis

In the EU, the carbon price resulting from the ETS cap policy is subject to high volatility. We use ETS futures weekly prices to retrieve the mean standard deviation over the period, before converting it to a quarterly level. We then set the standard deviation of the ETS carbon price σ_{ETS} to this value for all pathway simulations and exercises we conduct.

With respect to the long-term inefficiency (i.e. the welfare loss), we perform stochastic transition pathway simulations,³⁵ where we include stochastic shocks on both the price of carbon and the TFP around their respective deterministic growth rate. We perform 2000 Monte Carlo simulations to construct 95 percent confidence intervals around the deterministic trends for both the output and the carbon price needed to achieve the net-

³⁴This is also in line with recent findings of [Flammer, 2021] with respect to the so called “Greenium” puzzle (i.e. $R_g < R_d$). In this paper, she finds no evidence for the existence of a Greenium.

³⁵We compare two scenarios: a) the carbon policy is consistent with the net-zero objective and b) the carbon policy is consistent with the optimal social cost of carbon.

zero pledge. We then investigate the role that green macroprudential policy—which favors the green sector over the brown sector—could play in mitigating the welfare wedge, while ensuring financial stability.

Turning to the short-term inefficiency (i.e. risk premia distortion), we perform stochastic simulations to investigate the impulse responses to a shock to the price of carbon on risk premia and inflation, and highlight how the central bank could take into account this type of transition risk within its framework.

1.4.1 Fiscal Environmental Policy Scenario

The goal of this section is to present and analyze theoretical fiscal pathways consistent with the EU objective for 2030.³⁶ We first find the trajectory of the carbon price that leads to the desired reduction in emissions (i.e. a 55 percent emissions reduction relative to the 1990 level, which corresponds to a 33 percent reduction relative to the 2020 level). We then highlight the impact of sub-optimal carbon pricing policies on welfare.

1.4.1.1 Growth, carbon price, and the EU objectives

Figure 1.5 shows carbon price trajectories (according to two different growth scenarios) consistent with being on track for achieving the net-zero objective in the EU. The blue dashed line is the central scenario with a growth trend of 0.8 percent, corresponding to the average real growth rate per capita in the EA from 2000 to 2020. The orange dotted line is a scenario with a more optimistic growth trend of 1.2 percent. We also add stochastic components drawn from random disturbances to the TFP and the carbon price. The shaded blue and orange areas are 95 percent confidence intervals retrieved over the 2000 Monte Carlo draws. This allows us to account for uncertainty in output growth and the carbon pricing trajectory.³⁷ Depending on the growth scenario, reducing emissions by

³⁶In this section, as the main focus is long-term transition pathways, we do not consider nominal rigidities in prices.

³⁷Where trend growth in output and carbon prices are anticipated, but shocks can distort these deterministic processes in the short run.

55 percent compared to 1990 level would require a mean carbon price between 350€ and 375€ per ton of CO₂. Accounting for uncertainty, the price is found to fluctuate between 200€ and 500€, meaning that the target could be either undershot or overshoot. Note that this large confidence interval is computed assuming that future volatility can be inferred from past volatility. However, EU countries are considering measures to reduce price fluctuations in the ETS market,³⁸ which could lead to a lower standard deviation in the future. This exercise provides evidence that such measures are needed if the EU authorities want to improve their ability to meet their emission reduction objective. Furthermore, we also find that the price of carbon needs to follow the growth of output to be able to shrink the flow of emissions to the desired level. It is worth noting, however, that our model takes the abatement technology as given. With improvements in technology, the EU could reach the same target with a lower carbon price, but the mechanisms to trigger this improvement in the abatement technology are left for further research.

Figure 1.6 uses the central growth scenario (i.e. 0.8 percent growth rate) to compare the net-zero trajectory with a carbon market that exhibits uncertainty (blue solid line and shaded area) and a market that yields a completely deterministic carbon price (purple dotted line and shaded area). This is similar to comparing a cap policy with a tax policy. We find that a carbon tax like system, where volatility is controlled, would allow for reaching the net-zero objective with certainty. However, a cap and trade policy ensures that emissions reduction take place efficiently, as firms are able to trade permits while a tax system imposes a fixed reduction in emissions to all firms. In addition, [Karp and Traeger, 2018] show that, when considering a stock pollutant, a cap market guarantees efficiency gains (compared to a tax system) when the economy is subject to technology shocks that shift the marginal abatement cost curve and the social cost of carbon.

The ambitious net-zero goal would have several implications on output and consump-

³⁸A carbon price floor has been implemented in the Netherlands and is currently under consideration in Germany. The EU Market Stability Reserve was also introduced to regain some control over the carbon price.

tion alike. In [figure 1.7](#), we show that uncertainty in carbon pricing does not significantly alter consumption pathways and therefore does not alter the welfare, as shown in the case of the certainty equivalence in [[Golosov et al., 2014](#)]. Carbon price shocks do not propagate to the households as, on one hand, the stochastic discount factor—which is the central part in asset pricing and consumption smoothing mechanisms—is not directly impacted by the carbon pricing, and, on the other hand, the relative risk aversion is set different to 1 (the log utility case). In our setup, climate risk is not directly captured within the utility function, restraining the carbon price shock from propagating to consumption and welfare.³⁹ As such, we run deterministic transition pathway simulations instead of stochastic transition pathway simulations for the remaining welfare analysis.

1.4.1.2 Welfare implications

The first two plots in [figure 1.8](#) display the trajectory of the environmental policy consistent with the EU objective compared to the optimal environmental policy for both output and emissions. The optimal policy (i.e. setting the carbon price equals to the SCC) trajectory is not able to meet the net-zero pledge. The carbon price needed to achieve net-zero is found to be significantly higher than the SCC, thus altering the welfare pathway. Several key factors are in play. First, the fact that the environmental externality is a slow moving variable pushes the social planner to further its intervention at a late stage when the stock of carbon has significantly accumulated, and has become a major threat. Second, the absence of tipping points, which would force the social planner to account for uncertainty over the climate damages, would obligate the social planner to increase its actions by increasing the SCC ([[Dietz et al., 2021](#)]). Third, the household utility objective function does not capture the effects of climate change directly, which would impact the SCC ([[Barrage, 2020](#)] and [[Benmir et al., 2020](#)]).⁴⁰ Finally, in recent

³⁹While integrating climate risk as a dis-utility would allow for carbon price shocks to propagate to the welfare, we do not model it in this paper and leave it for future research.

⁴⁰[[Benmir et al., 2020](#)] show that the SCC increases when households account for the externality within their utility function ($u_{xc} \neq 0$).

work, [Cai and Lontzek, 2019], [Traeger, 2021], and [Van den Bremer and Van der Ploeg, 2021] both show that accounting for uncertainty in climate dynamics could increase the inherent level of the SCC. This increase in the carbon price, which would be welfare enhancing in our framework, is still, however, not sufficient to meet the net-zero emissions reduction goal. We show that the price difference between the optimal SCC and the net-zero ETS induced carbon price needed to reach the target (the “Extra Carbon Price”) is about 300€ higher by the end of 2030. While we do not explicitly model tipping points in the damage function, we perform a sensitivity analysis both on the climate damages specification and climate dynamics.

As reported in our sensitivity analysis (table 1.6), the optimal price of carbon depends on the specification of damages. We find carbon prices between 31.2€ to 144.1€ for different calibrations found within the literature. Furthermore, in the spirit of [Traeger, 2021], we perform a sensitivity analysis over the parameter v_2^o , which drives the climate dynamics for temperature. We show that for a higher value of v_2^o , temperature by 2030 could double, but the implied SCC (under both Nordhaus and Dietz damage specifications) would still be insufficient to obtain the desired emission reduction to be on track for net-zero by 2030. Under the Weitzman specification, we find that setting the carbon price equals to the SCC would lead to a 45 percent emissions reduction by 2030, which is higher than the EU objective. However, the carbon price that would be able to achieve such an objective is significantly high (846.65 €), thus suggesting major issues in terms of implementation. Therefore, for the remainder of the paper, we set the climate damage parameter “b” *à la* Nordhaus and v_2^o to the baseline value as in [Dietz and Venmans, 2019], as these are the closest to the ETS price at the start of January 2021 for all three estimates.

The two red plots in figure 1.8 show that the welfare loss increases over time as the extra carbon price continues to rise to about 300€. This deviation of the ETS carbon price from the SCC introduces a distortion with respect to the optimal allocation. By 2030, the household loses about 3 percent in consumption equivalent (CE) compared to the optimal case. We will see in the next section that this effect can be partially offset by

sectoral macroprudential risk-weights.

1.4.1.3 Introducing Macroprudential Policy

To reduce the welfare gap induced by the sub-optimal policy, we investigate the role macroprudential policy could play. We present transition pathway scenarios where the macroprudential authority varies regulatory risk-weights on loans granted to the green and the brown sectors by banks. While there is not yet such a policy in the EU, regulators are increasingly taking into account climate risk (see [section 1.2.7](#)).

In [figure 1.9](#), we present two net-zero emissions reduction scenarios: i) the scenario where macroprudential policy is neutral (i.e. $\lambda_g = 1$ and $\lambda_b = 1$) in blue, and ii) the scenario where a green macroprudential policy is implemented by the regulator in green (i.e. $\lambda_g \xrightarrow{t \rightarrow 2030} 0.75$ linearly, while $\lambda_b \xrightarrow{t \rightarrow 2030} 1.25$). We show that favoring the green sector over the brown sector in banks' regulatory constraint leads to an increase in the green capital (8.3 percent) and a decrease in the brown capital (4.8 percent) by the end of 2030, with respect to the scenario where risk-weights are left unchanged. The implementation of green macroprudential policy thus amplifies the rise (drop) in green (brown) capital induced by the rising carbon price along the transition. Compared to the neutral macroprudential policy case, increasing the capital stock in the green sector reduces the welfare loss (of about 1 percent CE). Intuitively, the increasing carbon price triggers a substitution between brown and green production, as the green sector is less emission intensive. Favoring the green sector in the RWA policy reinforces this substitution effect by tilting investments toward the green sector, leading to an increase in output.

In [figure 1.10](#), we investigate the case where the macroprudential authority favors the brown sector over the green sector to avoid a disorderly transition. The goal would be to attenuate the impact of the rising carbon price on the brown sector, as the current share of the brown sector is higher than the share of green sector (70 and 30 percent respectively). The brown macroprudential policy is displayed in brown (i.e. $\lambda_g \xrightarrow{t \rightarrow 2030} 1.25$ linearly, while

$\lambda_b \xrightarrow{t \rightarrow 2030} 0.75$). With sectoral shares held constant, this policy would lead to a lower welfare loss by the end of 2030 than in the case of the green macroprudential policy. The RWA policy reduces the substitution effect stemming from the environmental fiscal policy. At the aggregate level, the need for investment is lower, as the substitution effect is weaker than when macroprudential policy favors the green sector. Although output decreases relative to the green macroprudential policy scenario, welfare improves as investment spending is proportionally lower.

In [figure 1.11](#), we compare green and brown macroprudential policies, while assuming that the share of the green sector in the economy increases from 30 percent to 50 percent by the end of 2030.⁴¹ With an increasing share of the green sector,⁴² both types of macroprudential policies induce a substitution effect between the two sectors, which otherwise would not arise in the case of brown macroprudential policy (as shown previously in [figure 1.10](#)). In this case, green macroprudential policy is able to close the welfare wedge by the end of 2030. Two main factors are at play. First, as the share of the green sector grows, required investments in abatement decrease, thus increasing consumption. Second, green macroprudential policy induces lower investment costs in green capital, which at the aggregate level boosts consumption. Along the transition to a greener economy, favoring green firms in banks' capital requirements rules would ease the welfare burden on households, by lowering transition costs for firms. However, the main challenge would be to identify green firms in practice. As highlighted in [[Ehlers et al., 2020](#)], there is a need for a 'green label' at the firm-level for companies committed to the net-zero transition, as opposed to the current project-based green labels.

As a robustness exercise, we also report in [table 1.7](#) the steady state impacts of various macroprudential policy settings. We investigate several risk-weights combinations, where macroprudential policy is conducted as a one off. We consider a carbon price of about

⁴¹These results are further reinforced if the increase in the share of the green sector is greater than 50 percent.

⁴²In this setting, we exogenously change the share of the green sector over the 10 year transition period. One could endogenously model this shift in the share of the green sector. We leave this for future work.

300€ (the net-zero implied price by 2030). We then compare three scenarios: i) the model following the optimal policy ii) the model with a carbon price consistent with the net-zero target and no macroprudential policy iii) the model with a carbon price consistent with the net-zero target and various macroprudential policies. The robustness exercise shows that, the more the macroprudential authority decreases the risk-weight on green loans (while increasing the risk-weight on brown loans), the smaller the consumption loss is compared to the optimal. It would be possible to completely offset the consumption loss, but it would require drastic changes in risk-weight, which could threaten financial stability.

1.4.2 Risk Premia Stabilization

To offset the distortion of risk premia stemming from carbon price volatility, we assess the effectiveness of short-term QE rules set by the central bank.

The simulation reported in [figure 1.12](#) presents the responses of risk premia to a positive shock to the carbon price level. We first show how risk premia react to the volatility in the ETS market. As the EU decided to implement its environmental fiscal policy through carbon permits, there is an inherent variance in the price of carbon.⁴³ Estimating the standard deviation of the shock on the ETS series and simulating the model allow us to analyze how these unexpected variations in the carbon price could affect firms and banks. The blue line shows the reaction of risk premia in both the green and brown sectors following a positive shock on the carbon price. The shock leads to an increase in risk premia of about 10 basis points annually. This rise in risk premia could lead to financial instability and thus distortion in the transmission of monetary policy. To restore the equilibrium in risk premia, monetary policy could rely on quantitative easing rules (as a ‘fire-fighting’ tool), which would react to changes in the level of the risk premium. As such, the central bank would have the ability to substitute to financial intermediaries in financing either green or brown firms. This intervention will lead to a temporary increase

⁴³[Table 1.8](#) displays the moments of risk premia, marginal costs, and inflation for both sectors following a positive shock on carbon prices.

in the central bank balance sheet.

More specifically, we compare two scenarios: i) a model where the central bank does not implement QE rules, ii) a model where the central bank implements QE rules with various degrees of reaction. We show that the increase in spreads could be offset by an increase in asset purchases, where the intensity of the reaction of the central bank is represented by the parameter ϕ_k^s . For instance, asset purchases of about 0.23 percent (annually) of total assets within each sector (i.e. $\phi_k^s = 0.5$) are sufficient to almost completely offset the induced distortion in risk premia.⁴⁴ The mechanism at play here is the same as in the case of exogenous financial shocks on risk premia, except that the initial rise in risk premia is triggered by the shock on the carbon price and its subsequent effect on firms' marginal costs. Compared to the financial crisis simulation in [Gertler and Karadi, 2011], our carbon price shock triggers a reaction of risk premia that is smaller, but the magnitude of the intervention of the central bank is proportionally similar. By stepping in to directly lend to firms, the central bank is able to restore the equilibrium on the loans market and avoid potential negative effects coming from the rise of spreads. Table 1.8 confirms that the variance of risk premia is significantly reduced in the presence of QE rules. With respect to sectoral inflation, we find that central bank intervention increases inflation, though the magnitude is very small (less than 0.02 percent annually). Thus, a trade-off appears between financial stabilization and inflation control. However, in our framework, the benefits of mitigating the impact of the carbon price shock on risk premia seem to outweigh the inflationary consequences of asset purchases.

1.5 Asset Purchase Program Scenario – LSAP

To shed some light on the interest of tilting central banks portfolio toward green bonds, we simulate both transitory and permanent LSAP programs run by the central bank under

⁴⁴We also plot the case where $\phi_k^s = 5$ and $\phi_k^s = 0.05$. We show that when the central bank purchases about 0.27 percent of both green and brown assets annually, it is able to completely offset the rise in risk premia, while a purchase of about 0.15 percent annually reduces the impact on risk premia by about half.

two macroprudential policy scenarios. In the first case, the macroprudential authority implements climate-risk weights along the transition, while in the second case risk-weights are held constant.

1.5.1 Transitory LSAP

The first scenario studied is a transitory LSAP program where the central bank gradually increases the size of its balance sheet to hold around 8 percent of either green or brown total assets by 2028. Asset purchases are then reversed and holdings return to zero in approximately two years. As LSAP programs are announced by central banks before being implemented, we rely on perfect foresight simulations.

Figure 1.13 shows the impact of both green and brown transitory LSAP programs along the transition.⁴⁵ The main result is that there is no incentive for a central bank to purchase green rather than brown bonds as part of a LSAP program, since both programs lead to the exact same results. The reason is that green and brown bonds are seen as perfectly substitutable by banks. In this case, if the central bank favors one of the sectors in its asset purchases, the effect is completely offset by the reaction of financial intermediaries. An interesting point to note is that both green and brown transitory LSAP programs allow central banks to postpone the impact of the rising carbon price on brown capital and output by loosening the constraint on banks. If the transition to a low-carbon economy were to take place in a disorderly fashion, such LSAP programs could delay the potential negative impacts the transition might have on stranded assets.

Figure 1.14 shows how a transitory LSAP program focused on green bonds would interact with a sectoral macroprudential policy favoring the green sector. In this exercise, asset purchases are similar to those in the previous exercise, but the risk-weight on green loans is lowered along the transition, while the risk-weight on brown loans is gradually increased. Breaking the perfect substitution between green and brown assets allows to

⁴⁵As in the previous section, the carbon price is assumed to increase to reach the EU climate goals and trend growth is assumed to be 0.8 percent annually.

boost green sector capital and output compared to when macroprudential policy stays neutral over the period studied.⁴⁶ Overall, this leads to a positive effect on aggregate capital and output that disappears at the end of the simulation, as the central bank unwinds its asset purchases. Thus, a transitory green LSAP program coupled with a macroprudential policy favoring the green sector exacerbates the effect of the transition induced by the rise in the carbon price, which leads to a slightly better emission to output ratio.

1.5.2 Permanent LSAP

The second scenario studied is a permanent LSAP program where the central bank gradually increases the size of its balance sheet to hold around 8 percent of either green or brown total assets by 2028 and keeps this proportion constant from 2028 on.

Figure 1.15 displays the reaction of selected variables to both green and brown permanent LSAP programs along the transition. The results are quantitatively similar to the case of a transitory LSAP, except at the end of the simulation, where brown permanent LSAP seem to be more effective than transitory LSAP to mitigate the loss in brown capital and output associated with a decarbonization of the economy.

Figure 1.16 shows how a permanent LSAP program focused on green bonds would interact with a sectoral macroprudential policy favoring the green sector. The interaction of the two policies gives the best results in terms of accompanying the transition to a greener economy. Compared to the case where asset purchases were transitory, a permanent LSAP program yields an effect on capital, output, and emissions that is long-lasting. Overall, the emission to output ratio is lower, since green output rises sharply while brown output decreases over the period studied. It is also important to keep in mind that results presented in this section could be further reinforced if we were to witness an increase in the share of the green sector over the transition, as exemplified in the previous section.

⁴⁶Similarly, [Ferrari and Nispi Landi, 2021] break the perfect substitutability by introducing a quadratic cost related to the holding of green bonds by banks.

1.6 Conclusion

We develop a DSGE model with both endogenously-constrained financial intermediaries and heterogeneous firms. We then use the model to assess the implications of setting an environmental policy consistent with the net-zero target using a cap system.

We find that a price of about 350€ per ton of carbon is needed to be aligned with the net-zero target. However, the actual implementation of this price induces two inefficiencies. The first inefficiency is linked to the need of an increasingly higher price of carbon (compared to the optimal SCC) to meet the EU targets. This decoupling generates a growing welfare loss. To address this wedge, we show that a RWA policy favoring the green sector (i.e. green macroprudential policy) is efficient in partially offsetting the welfare loss while reaching the emissions target. Furthermore, green macroprudential would allow the regulator to address climate-related financial risk.

The second inefficiency is related to the market design of the environmental fiscal policy in the EU area. The present volatility in the ETS is shown to affect firms' marginal costs and thus to alter risk premia. We find that QE rules that react to changes in risk premia are able to completely offset movements in spread levels and volatility, allowing for a smooth transmission of monetary policy, while not significantly impacting inflation.

Turning to LSAP programs, we find that macroprudential policy is needed to provide an incentive to central banks to engage in both transitory and permanent green QE. However, permanent LSAP programs yields an effect on capital, output, and emissions that is long-lasting compared to transitory LSAP programs.

More generally, we show that QE rules could be used as a short-term countercyclical tool, while sectoral macroprudential policy could play a more structural role, allowing for a smooth transition toward net-zero.

In particular, we find that green macroprudential policy strengthen the substitution effect between the two sectors, which is triggered by the environmental fiscal policy. While

this result is obtained with a constant share of the green sector (\varkappa), increasing \varkappa along the transition reinforces our findings. Intuitively, making the green sector predominant (figure 1.2 and figure 1.3), would not only decrease substantially emissions, which in turn decreases the environmental policy cost (i.e. the carbon price), it would also help achieve the sought-after decoupling of emissions and output. The emissions to output ratio $E_Y = E/Y$ falls almost linearly with an increase in the green sector share and leads to lower level of carbon price.

Many extensions could be conducted using our framework. In particular, we think that further research could be devoted to the impact of non-linearities within the financial sector on the dynamics of the model and to the role that endogenous TFP could play in fostering the emergence of greener output growth. We also believe it could be fruitful to examine how to capture the environmental quality on the welfare of households in more direct ways than in existing models.

References

- D. Acemoglu, P. Aghion, L. Bursztyn, and D. Hemous. The environment and directed technical change. *American economic review*, 102(1):131–66, 2012.
- D. Acemoglu, U. Akcigit, D. Hanley, and W. Kerr. Transition to clean technology. *Journal of Political Economy*, 124(1):52–104, 2016.
- D. Acemoglu, D. Hemous, L. Barrage, P. Aghion, et al. Climate change, directed innovation, and energy transition: The long-run consequences of the shale gas revolution. In *2019 Meeting Papers*, number 1302. Society for Economic Dynamics, 2019.
- S. Adjemian and M. Juillard. Stochastic extended path approach. *Unpublished manuscript*, 2013.
- P. Aghion, A. Dechezleprêtre, D. Hemous, R. Martin, and J. Van Reenen. Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, 124(1):1–51, 2016.
- K. Angelopoulos, G. Economides, and A. Philippopoulos. What is the best environmental policy? taxes, permits and rules under economic and environmental uncertainty. *CESifo working paper series*, 2010.
- B. Annicchiarico and F. Di Dio. Environmental policy and macroeconomic dynamics in a new keynesian model. *Journal of Environmental Economics and Management*, 69:1–21, 2015.

- L. Barrage. Optimal dynamic carbon taxes in a climate–economy model with distortionary fiscal policy. *The Review of Economic Studies*, 87(1):1–39, 2020.
- Basel Committee. High-level summary of basel iii reforms. Bank for International Settlements, Basel Committee on Banking Supervision, 2017.
- Basel Committee. Basel committee consults on principles for the effective management and supervision of climate-related financial risks. Nov 2021. URL <https://www.bis.org/press/p211116.htm>.
- M. D. Bauer and G. D. Rudebusch. The rising cost of climate change: evidence from the bond market. *The Review of Economics and Statistics*, pages 1–45, 2021.
- J. Benhabib, R. Rogerson, and R. Wright. Homework in macroeconomics: Household production and aggregate fluctuations. *Journal of Political economy*, 99(6):1166–1187, 1991.
- G. Benmir, I. Jaccard, and G. Vermandel. Green asset pricing. *ECB Working Paper*, 2020.
- V. Bosetti, M. Maffezzoli, et al. Occasionally binding emission caps and real business cycles. *IGIER working paper*, 2014.
- Y. Cai and T. S. Lontzek. The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6):2684–2734, 2019.
- S. Carattini, G. Heutel, and G. Melkadze. Climate policy, financial frictions, and transition risk. *NBER Working Paper*, 2021.
- S. Carbone, M. Giuzio, S. Kapadia, J. S. Krämer, K. Nyholm, and K. Vozian. The low-carbon transition, climate commitments and firm credit risk. *ECB Working Paper*, 2021.

- C. Carvalho and F. Nechio. Factor specificity and real rigidities. *Review of Economic Dynamics*, 22:208–222, 2016.
- H. S. Chan, S. Li, and F. Zhang. Firm competitiveness and the european union emissions trading scheme. 2013.
- A. Chegut, P. Eichholtz, and N. Kok. The price of innovation: An analysis of the marginal cost of green buildings. *Journal of Environmental Economics and Management*, 98:102248, 2019.
- L. J. Christiano, M. Eichenbaum, and C. L. Evans. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113(1):1–45, February 2005.
- R. De Haas and A. A. Popov. Finance and carbon emissions. *ECB Working Paper*, 2019.
- C. Di Maria and S. Valente. Hicks meets hotelling: the direction of technical change in capital–resource economies. *Environment and Development Economics*, 13(6):691–717, 2008.
- S. Dietz and N. Stern. Endogenous growth, convexity of damage and climate risk: how nordhaus’ framework supports deep cuts in carbon emissions. *The Economic Journal*, 125(583):574–620, 2015.
- S. Dietz and F. Venmans. Cumulative carbon emissions and economic policy: in search of general principles. *Journal of Environmental Economics and Management*, 96:108–129, 2019.
- S. Dietz, J. Rising, T. Stoerk, and G. Wagner. Economic impacts of tipping points in the climate system. *Proceedings of the National Academy of Sciences*, 118(34), 2021.

- F. Diluiso, B. Annicchiarico, M. Kalkuhl, and J. C. Minx. Climate actions and macro-financial stability: The role of central banks. *Journal of Environmental Economics and Management*, 110:102548, 2021.
- Y. Dissou and L. Karnizova. Emissions cap or emissions tax? a multi-sector business cycle analysis. *Journal of Environmental Economics and Management*, 79:169–188, 2016.
- T. Doh, G. Cao, and D. Molling. Should monetary policy monitor risk premiums in financial markets? *Federal Reserve Bank of Kansas City: Economic Review*, 100(1):7–30, 2015.
- D. Duffie and K. J. Singleton. Simulated moments estimation of markov models of asset prices. *Econometrica: Journal of the Econometric Society*, pages 929–952, 1993.
- T. Ehlers, B. Mojon, and F. Packer. Green bonds and carbon emissions: exploring the case for a rating system at the firm level. *BIS Quarterly Review*, September, 2020.
- E. Escrig-Olmedo, M. Á. Fernández-Izquierdo, I. Ferrero-Ferrero, J. M. Rivera-Lirio, and M. J. Muñoz-Torres. Rating the raters: Evaluating how esg rating agencies integrate sustainability principles. *Sustainability*, 11(3):915, 2019.
- I. Fender, M. McMorrow, V. Sahakyan, and O. Zulaica. Green bonds: the reserve management perspective. *BIS Quarterly Review*, September 2019.
- A. Ferrari and V. Nispi Landi. Whatever it takes to save the planet? central banks and unconventional green policy. *Central Banks and Unconventional Green Policy (February 15, 2021)*. *Bank of Italy Temi di Discussione (Working Paper) No*, 1320, 2021.
- C. Fischer and M. Springborn. Emissions targets and the real business cycle: Intensity targets versus caps or taxes. *Journal of Environmental Economics and Management*, 62(3):352–366, 2011.
- C. Flammer. Corporate green bonds. *Journal of Financial Economics*, 2021.

- J. Gali and T. Monacelli. Optimal monetary and fiscal policy in a currency union. *Journal of international economics*, 76(1):116–132, 2008.
- M. Gertler and P. Karadi. A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34, January 2011.
- M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88, 2014.
- A. Grimaud and L. Rouge. Environment, directed technical change and economic policy. *Environmental and Resource Economics*, 41(4):439–463, 2008.
- J. Hassler, P. Krusell, C. Olovsson, and M. Reiter. On the effectiveness of climate policies. *IIES Working Paper*, 2020.
- G. Heutel. How should environmental policy respond to business cycles? optimal policy under persistent productivity shocks. *Review of Economic Dynamics*, 15(2):244–264, 2012.
- N. Jaimovich and S. Rebelo. Can news about the future drive the business cycle? *American Economic Review*, 99(4):1097–1118, 2009.
- U. J. Jermann. Asset pricing in production economies. *Journal of monetary Economics*, 41(2):257–275, 1998.
- L. S. Karp and C. P. Traeger. Prices versus quantities reassessed. *CESifo Working Paper*, 2018.
- F. E. Kydland and E. C. Prescott. Time to Build and Aggregate Fluctuations. *Econometrica*, 50(6):1345–1370, November 1982.
- R. E. Lucas. Asset prices in an exchange economy. *Econometrica: journal of the Econometric Society*, pages 1429–1445, 1978.

- W. Nordhaus. A question of balance: Weighing the options on global warming policies. *Yale University Press, New Haven, CT*, 2008.
- W. D. Nordhaus. To slow or not to slow: the economics of the greenhouse effect. *The economic journal*, 101(407):920–937, 1991.
- W. D. Nordhaus. The 'dice' model: Background and structure of a dynamic integrated climate-economy model of the economics of global warming. Technical report, 1992.
- W. D. Nordhaus. Revisiting the social cost of carbon. *Proceedings of the National Academy of Sciences*, 114(7):1518–1523, 2017.
- W. D. Nordhaus and A. Moffat. A survey of global impacts of climate change: replication, survey methods, and a statistical analysis. *NBER Working Paper*, 2017.
- M. Pietrunti. Financial frictions and the real economy. *ESRB Working Paper Series*, 2017.
- A. Queralto. A model of slow recoveries from financial crises. *Journal of Monetary Economics*, 114:1–25, 2020.
- L. Sager. Income inequality and carbon consumption: Evidence from environmental engel curves. *Energy Economics*, 84:104507, 2019.
- S. Schmitt-Grohé and M. Uribe. Optimal simple and implementable monetary and fiscal rules. *Journal of monetary Economics*, 54(6):1702–1725, 2007.
- K. Schubert. Macroeconomics and the environment. *Revue de l'OFCE*, (3):117–132, 2018.
- F. Smets and R. Wouters. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, 1(5):1123–1175, September 2003.
- S. Smulders and M. De Nooij. The impact of energy conservation on technology and economic growth. *Resource and Energy Economics*, 25(1):59–79, 2003.

- N. Stern. The economics of climate change. *American Economic Review*, 98(2):1–37, 2008.
- J. B. Taylor. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1):195–214, December 1993.
- C. P. Traeger. Uncertainty in the analytic climate economy. *CEPR Discussion Paper No. DP16065*, 2021.
- I. B. Tucker. *Microeconomics for today*. Cengage Learning, 2010.
- T. S. Van den Bremer and F. Van der Ploeg. The risk-adjusted carbon price. *American Economic Review*, 111(9):2782–2810, 2021.
- R. Van der Ploeg, C. Hambel, and H. Kraft. Asset pricing and decarbonization: Diversification versus climate action. *University of Oxford Working Paper*, 2020.
- M. L. Weitzman. A review of the stern review on the economics of climate change. *Journal of economic literature*, 45(3):703–724, 2007.
- M. L. Weitzman. Ghg targets as insurance against catastrophic climate damages. *Journal of Public Economic Theory*, 14(2):221–244, 2012.
- M. Woodford. *Interest and prices*. 2003.

Appendices

1.A Appendix: Tables

TABLE 1.1
Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
<u>Standard Macro Parameters</u>		
σ	Risk aversion	2
\varkappa	% of Green firms in the economy	30
θ	Price elasticity	5
ξ	Price stickiness (Calvo parameter)	2/3
θ^P	Price stickiness (Rotemberg parameter)	$\frac{(\theta-1)\xi}{(1-\xi)(1-\xi\beta)}$
\bar{L}	Labor supply	1/3

TABLE 1.2
Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
<u>Environmental Parameters</u>		
γ_d	CO ₂ natural abatement	0.0021
$\theta_{1,g}$	Abatement cost parameter for sector G	0.02
$\theta_{2,g}$	Abatement cost parameter for sector G	2.7
$\theta_{1,b}$	Abatement cost parameter for sector B	0.05
$\theta_{2,b}$	Abatement cost parameter for sector B	2.7
v_1^o	Temperature parameter	0.5
v_2^o	Temperature parameter	0.00125
a	Damage function parameter	1.004
b	Damage function parameter	0.02

TABLE 1.3
Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
<u>Banking Parameters</u>		
Δ	Parameter impacting the discount factor of bankers	0.99
θ_B	Probability of staying a banker	0.98
ρ_c	Smoothing monetary rule coefficient	0.8
ϕ_y	Output policy parameter	0.2
ϕ_Π	Inflation policy parameter	1.5

TABLE 1.4
Estimated Parameters

Parameters		Estimation	
		Mean	Standard Deviation
<u>Standard Macro Parameters</u>			
$\sigma_{A_{t,k}}$	Output shock standard deviation	0.0063361	7.2574e-06
$\rho_{A_{t,k}}$	Output shock persistence	0.76907	8.3156e-06
\bar{g}/\bar{y}	Public spending share in output	0.28503	1.9099e-05
η_i	Capital adjustment cost	1.7354	7.2439e-06
$1/(1 + b/100)$	Discount factor	0.027254	6.4961e-06
$1 + \gamma_Y/100$	Economy growth rate	0.21907	3.0773e-07
h	habits	0.22278	1.3859e-05
α	Capital intensity	0.34202	4.8802e-07
δ	Depreciation rate of capital	0.024995	1.5241e-07
θ_g	Price elasticity in sector G	11	6.1805e-06
θ_b	Price elasticity in sector B	7.0206	4.3802e-06
<u>Environmental Parameters</u>			
E^*	Rest of the world emissions	3.3666	3.0327e-06
φ_b	Emissions-to-output ratio in sector B	0.2849	1.5072e-06
Carbon Price	Carbon price level	0.0099078	4.5392e-06
<u>Banking Parameters</u>			
λ	Risk weight on loans	0.17618	5.9887e-06
ω	Proportional transfer to the entering bankers	0.006353	2.4101e-06

TABLE 1.5
Model moments compared to observed data (Euro Zone)

Target	Model	Data	Source
<u>Macro Aggregates:</u>			
Output Growth Volatility	0.0065	0.0066	Eurostat
Investment Growth Volatility	0.030	0.030	Eurostat
Consumption to output Growth Volatility	0.0047	0.0048	Eurostat
Mean Output Growth	0.0022	0.0023	Eurostat
Mean Investment Growth	0.0021	0.0023	Eurostat
Consumption to Output Ratio (%)	0.57	0.53	Eurostat
Government Spending to Output Ratio (%)	0.28	0.24	Eurostat
Marginal Cost of the Brown Sector (Normalized)	1	1	[Chegut et al., 2019]
Marginal Cost of the Green Sector (6% higher than 'B')	1.06	1.06	[Chegut et al., 2019]
<u>Financial Aggregates:</u>			
Risk-less Bond Mean Return (annualized)	1.07%	1.08%	ECB
Green Bonds Risk Premium (annualized)	0.80%	0.80%	[Fender et al., 2019]
Brown Bonds Risk Premium (annualized)	0.80%	0.80%	[Fender et al., 2019]
Banks' Capital Ratio (Equity as a % of RWA)	14.39%	14.40%	ECB
<u>Environmental Aggregates:</u>			
Global Level of Carbon Stock (GtC)	839	839	USDA
Emissions to Output Ratio (kCO ₂ per \$ of output)	0.21	0.21	Global Carbon Project/FRED
ETS Price (January 2021) in €	30	30	Bloomberg

TABLE 1.6
Sensitivity of the optimal carbon price to climate damages and dynamics

	Nordhaus		Dietz		Weitzman	
	$v_2^o = 0.00125$	$v_2^o = 0.0025$	$v_2^o = 0.00125$	$v_2^o = 0.0025$	$v_2^o = 0.00125$	$v_2^o = 0.0025$
Emissions Reduction (in%)	-	15%	5%	28%	15%	45%
Social Cost of Carbon (in €)	31.2	144.12	65.94	333.53	144.12	846.65
Temperature T^o (in Celsius)	1.06	2.07	1.05	2.04	1.03	2

Notes: The figures reported in the table show the sensitivity of the optimal price of carbon, temperature, and net-zero goal of 55 percent emissions reduction by 2030, to different levels of calibration of: i) the damage function (parameter “b”), and ii) the climate dynamics (parameter “ v_2^o ”). With respect to the damage function, $b = 0.01$ corresponds to [Nordhaus and Moffat, 2017], $b = 0.02$ corresponds to [Dietz and Stern, 2015], and $b = 0.04$ corresponds to [Weitzman, 2012]. For the climate dynamics, $v_2^o = 0.00125$ corresponds to baseline case with $T^o < 1.1C$ by 2030, and $v_2^o = 0.0025$ corresponds to case with $T^o < 2.1C$ by 2030.

TABLE 1.7
Steady state values

	Optimal Policy	ETS Policy	ETS and Macropru		
	(1)	(2)	$\lambda_g = 0.75$	$\lambda_g = 0.5$	$\lambda_g = 0.25$
			$\lambda_b = 1.25$	$\lambda_b = 1.5$	$\lambda_b = 1.75$
	(1)	(2)	(3)	(4)	(5)
Consumption	1.2419	1.2372	1.2387	1.2402	1.2418
Aggregate Output	2.1139	2.1029	2.1019	2.1013	2.1011
Green Output	1.0937	1.0937	1.1012	1.1111	1.1213
Brown Output	1.06	1.0515	1.0425	1.0337	1.0251
Emissions to Output	0.2183	0.1569	0.1569	0.1569	0.1569
Green Sector Emissions	0.1034	0.0747	0.0754	0.0760	0.0767
Brown Sector Emissions	0.2876	0.2049	0.2032	0.2014	0.1998
Green Capital Stock	11.4318	11.3383	11.6359	11.9468	12.2717
Brown Capital Stock	10.4235	10.1552	9.9001	9.6554	9.4207
Green Real Rate	1.0045	1.0045	1.004	1.0035	1.003
Brown Real Rate	1.0045	1.0045	1.005	1.0055	1.006
ETS Price (in euros)	31.2	300	303	304	306
Carbon Cost as % of GDP in Green Sector	0.3278	0.5122	0.5122	0.5122	0.5122
Carbon Cost as % of GDP in Brown Sector	0.7650	1.4580	1.4580	1.4580	1.4580

Notes: The first column is the economy subject to an optimal carbon price. The second column is the economy subject to a carbon price consistent with the EU climate goals for 2030 (i.e. ETS cap net-zero objective), and the three last columns feature both a carbon price consistent with the EU climate goals for 2030 and an intervention of the macroprudential authority. We show how the economy responds to different risk-weight requirements related to climate risk exposure of firms. For instance the baseline scenario presents the case where an upgrade in the rating of the green bonds of the asset class BBB+ to A- and the downgrade in the rating of the brown bonds of the asset class BBB+ to BBB- (i.e. $\lambda_g = 0.75$ and $\lambda_b = 1.25$). The two other cases: i) with $\lambda_g = 0.5$ and $\lambda_b = 1.5$, and ii) with $\lambda_g = 0.25$ and $\lambda_b = 1.75$, represent a higher cut in the risk-weight associated with climate risk exposure (i.e. a higher upgrade and downgrade in the ratings).

TABLE 1.8
Risk premia volatility under the carbon price shock

	Baseline Model		Model with QE Rules ($\phi_k^s=5$)	
	Mean	Standard Deviation	Mean	Standard Deviation
EP_g	0.1989	0.02	0.1989	0.0003
EP_b	0.1989	0.02	0.1989	0.0003
MC_g	0.9091	0.0001	0.9091	0.0003
MC_b	0.8571	0.0001	0.8571	0.0003
Q_g	1.0000	0.0002	1.0000	0.0001
Q_b	1.0000	0.0002	1.0000	0.0001
π_g	1.0000	0.0000	1.0000	0.0001
π_b	1.0000	0.0000	1.0000	0.0001

Notes: The figures reported in the table show the first and second moments of selected variables following a positive carbon price shock. The baseline model refers to the model with the ETS carbon price. The model with QE rules incorporates a reaction of the central bank to deviations in risk premia from their respective steady state.

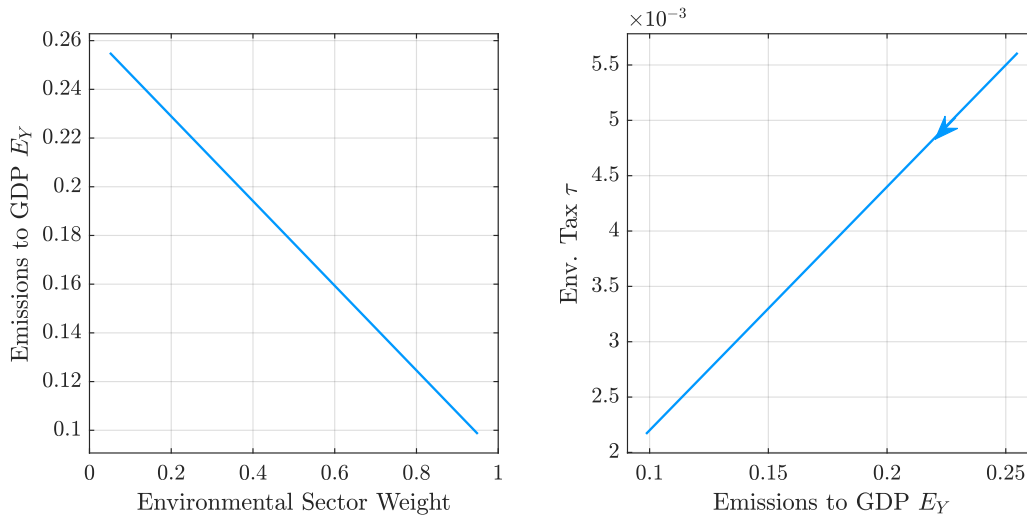
1.B Appendix: Figures

FIGURE 1.1. ETS Price in Euros per Ton of CO₂



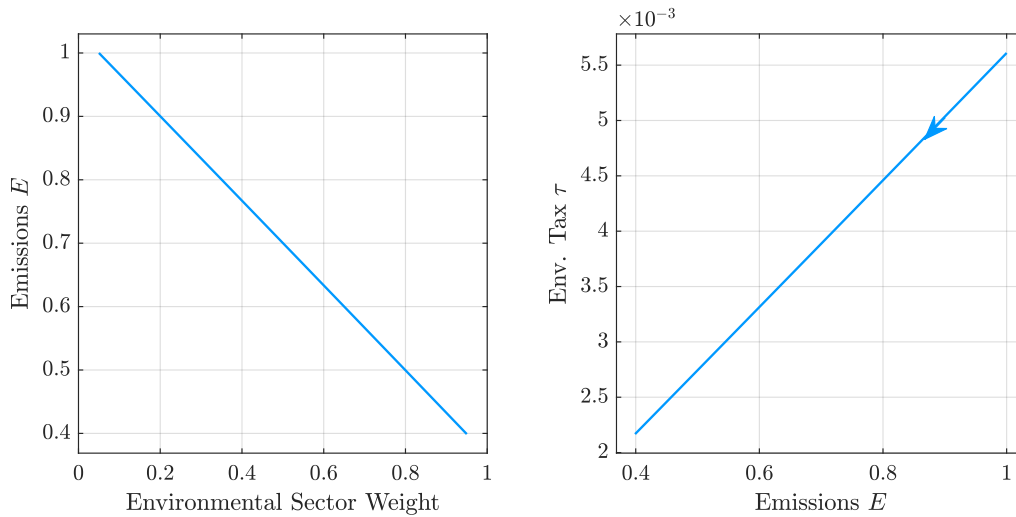
Notes: The figure displays the spot price of carbon permits traded within the ETS in euros per ton of CO₂. (Source: Bloomberg)

FIGURE 1.2. Share of the green sector, carbon intensity, and the environmental policy



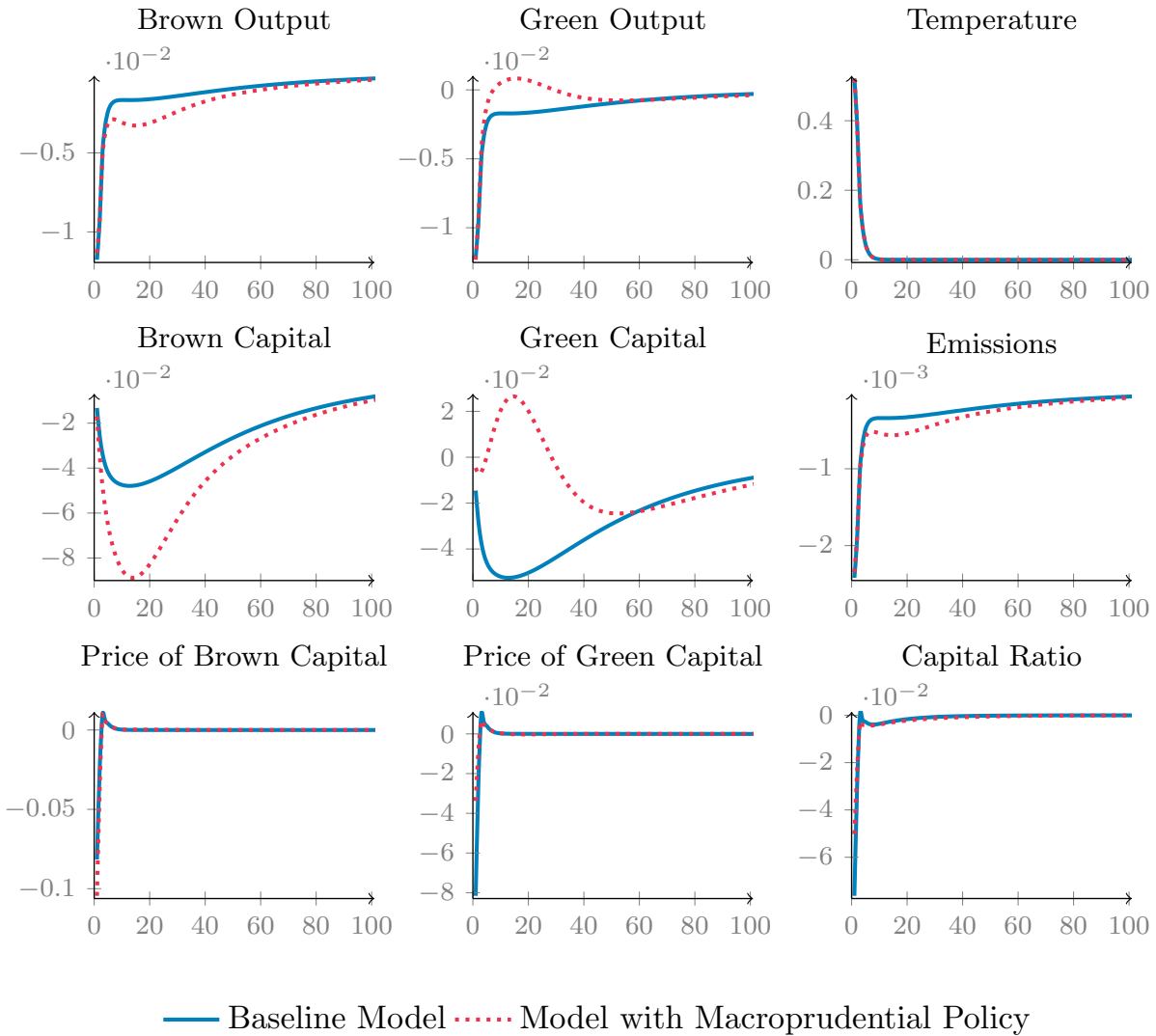
Notes: The graph on the left reports the interaction between emissions to output and the size of the green sector. The right graph reports how a change in the weight of the green sector drives the carbon price, through a decrease in the emissions to output ratio.

FIGURE 1.3. Share of the green sector, emission levels (normalized to one), and the environmental policy



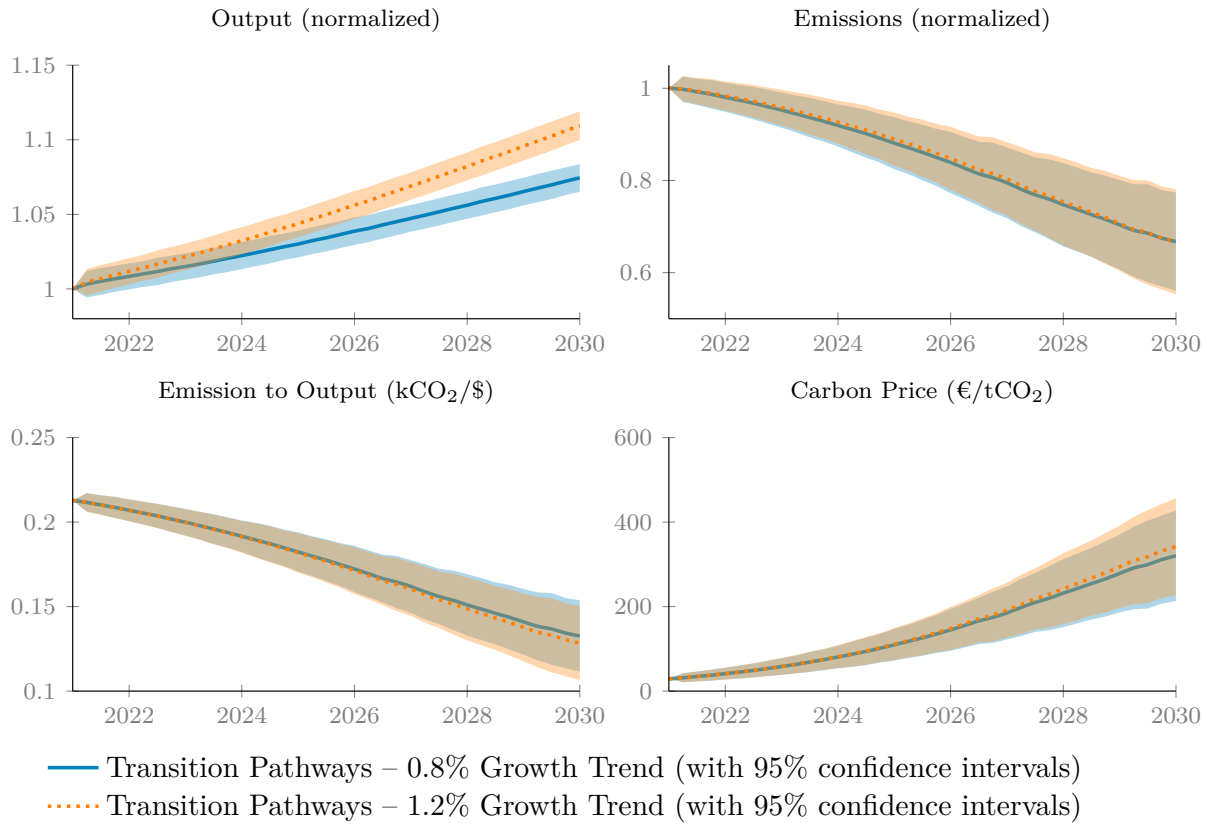
Notes: The graph on the left reports the interaction between emissions and the share of the green sector. The right graph reports how the share of the green sector shapes the carbon price.

FIGURE 1.4. Financial stability and climate risk



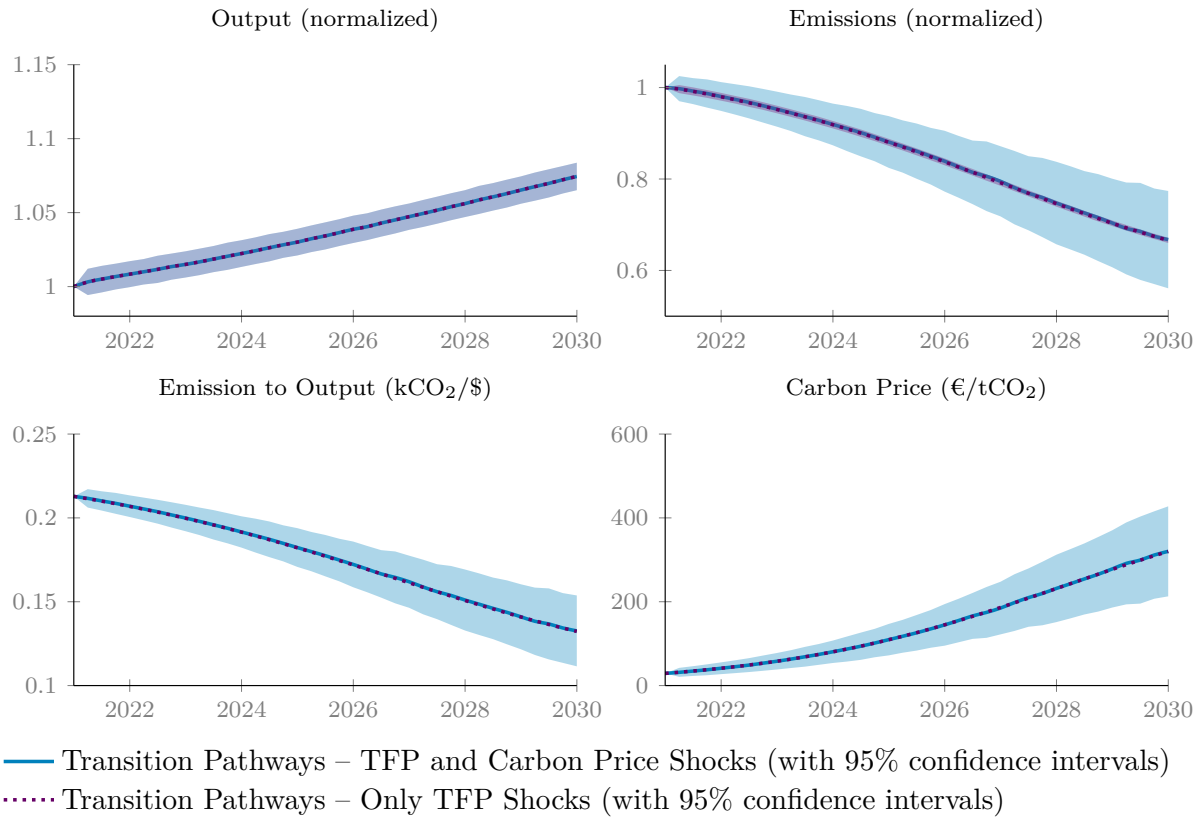
Notes: The figure shows the effect of a 0.5°C increase in the level of temperature, with and without macroprudential policy. In the baseline scenario, there is no sectoral macroprudential policy, which means $\lambda_b = \lambda_g = 1$. To illustrate the impact of green macroprudential policy on climate-related financial risk, we multiply/divide climate risk weights by a factor of 2, which means $\lambda_b = 2$ and $\lambda_g = 0.5$. Green macroprudential policy reduces the impact of a temperature increase on the global capital ratio by providing an incentive to banks to hold more green assets. The results are presented as percentage deviations from the steady state over quarterly periods.

FIGURE 1.5. Net-zero transition pathways with two different growth assumptions



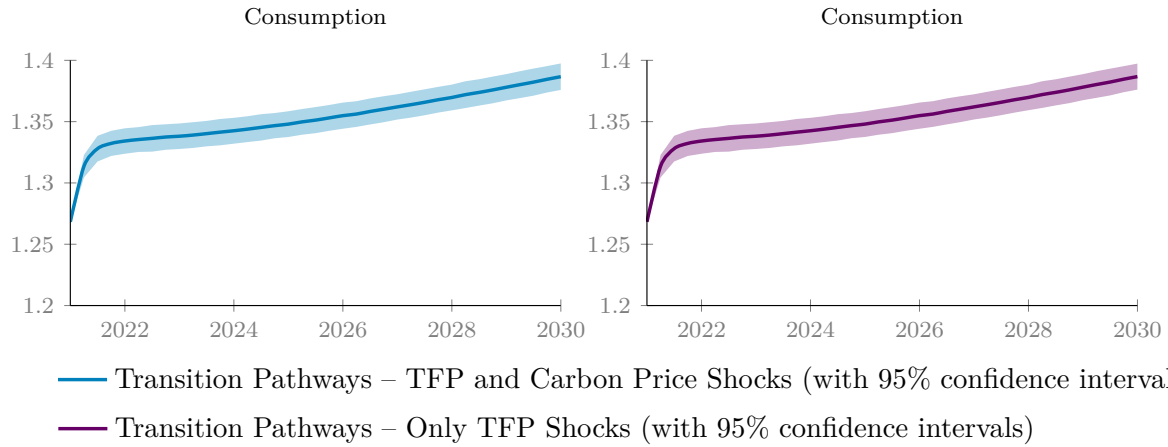
Notes: The figure reports the results of 2000 Monte Carlo simulation draws consistent with the net-zero target, according to two different growth scenarios. The blue line corresponds to the average per capita real growth over the last 20 years in the EZ (0.8%), while the orange dotted line corresponds to a more optimistic scenario in line with long term EZ trends (1.2%). The shaded blue and orange areas correspond to 95 percent confidence intervals for each scenario.

FIGURE 1.6. Net-zero transition pathways with and without carbon price uncertainty



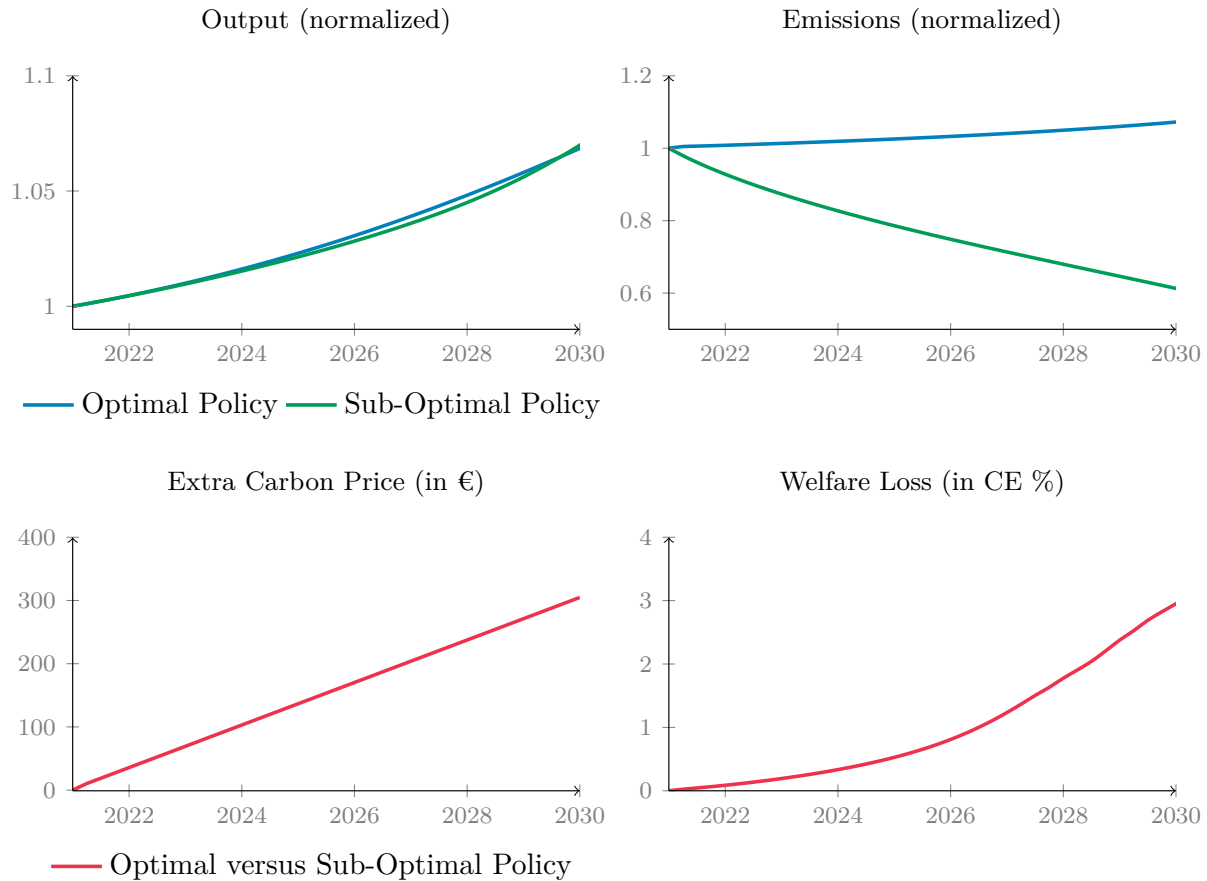
Notes: The figure reports the results of 2000 Monte Carlo simulation draws consistent with the net-zero target, according to the 0.8% growth scenario, where the carbon price is subject to carbon price volatility (i.e. carbon price shocks) and where the carbon price is not subject to carbon price volatility. The blue line corresponds to the average per capita real growth over the last 20 years in the EZ (0.8%) where the carbon price is subject to uncertainty, while the purple dotted line corresponds to the case where the carbon price is not subject to uncertainty. The shaded blue and purple areas correspond to the 95 percent confidence intervals for each scenario. Please note that for both scenarios output is subject to TFP shocks consistent with the past 20 years in the EZ.

FIGURE 1.7. Consumption pathways and carbon price uncertainty



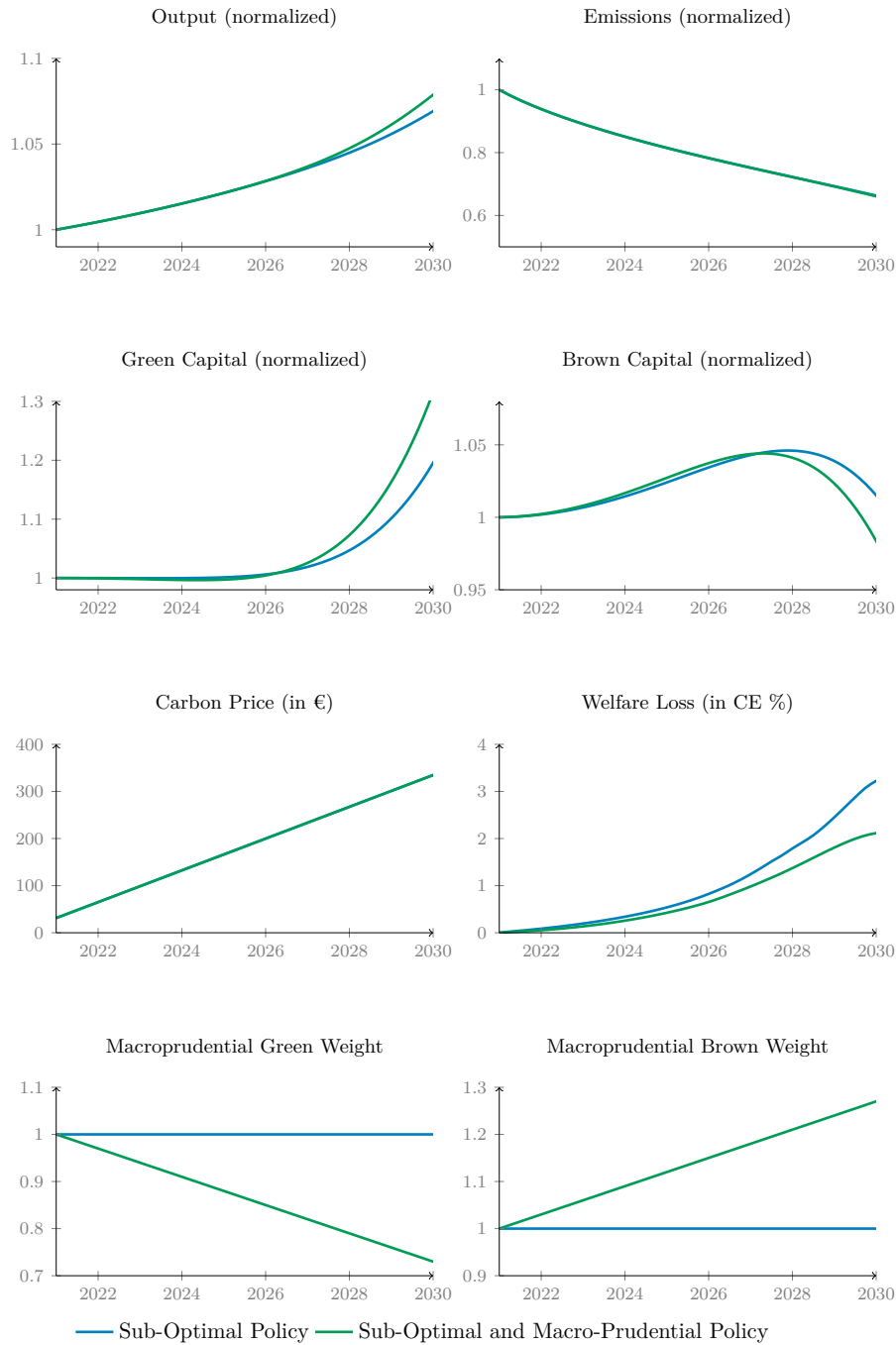
Notes: The figure reports the results of 2000 Monte Carlo simulation draws consistent with the net-zero target, according to the 0.8% growth scenario, where in one case the economy features carbon price volatility (i.e. carbon price shocks) and where in the other case the price of carbon is not subject to carbon price volatility. The blue line corresponds to the average per capita real growth over the last 20 years in the EZ (0.8%) where the carbon price is subject to uncertainty, while the purple line corresponds to the case where carbon price is not subject to uncertainty. The shaded blue and purple areas correspond to the 95 percent confidence intervals for each scenario. Please note that for both scenarios output is subject to TFP shocks consistent with the past 20 years in the EZ.

FIGURE 1.8. Transition pathways: optimal versus net-zero



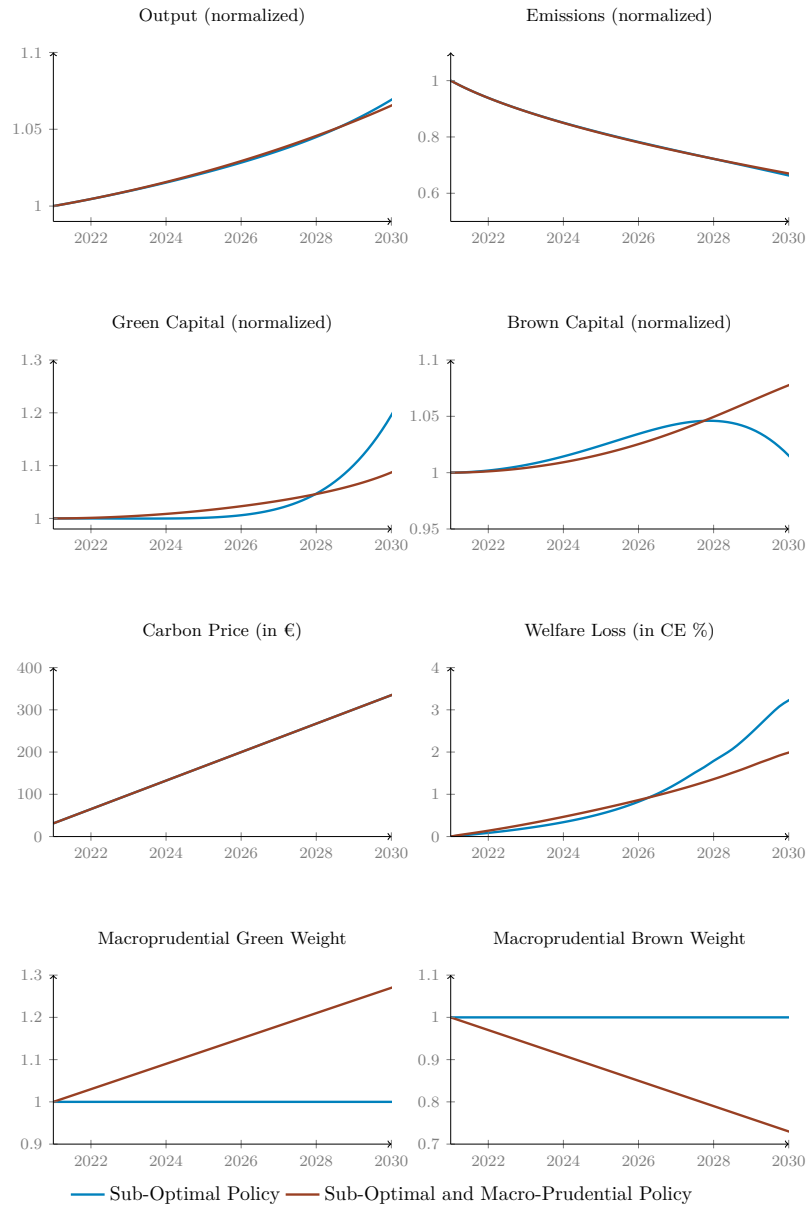
Notes: The figure compares the pathway consistent with the optimal carbon price (the social cost of carbon) to the net-zero ETS cap policy pathway. The blue line corresponds to the social planner choice, while the green dotted line corresponds to a pathway consistent with a reduction of emissions of 33 percent by 2030 (55 percent compared to 1990 level). The red lines show both the difference in carbon price and the welfare loss, between the optimal and sub-optimal policy (ETS inherent price). More specifically, the red graph on the left shows the trajectory of the extra carbon price, which is the carbon price consistent with the net-zero ETS cap policy minus the optimal price of the social planner. The graph on the right shows the welfare loss in consumption equivalent (CE), which is the difference between the welfare implied by the pathway of the social planner and the welfare implied by the pathway consistent with the net-zero objective.

FIGURE 1.9. Transition pathways (net-zero) with and without green macroprudential policy



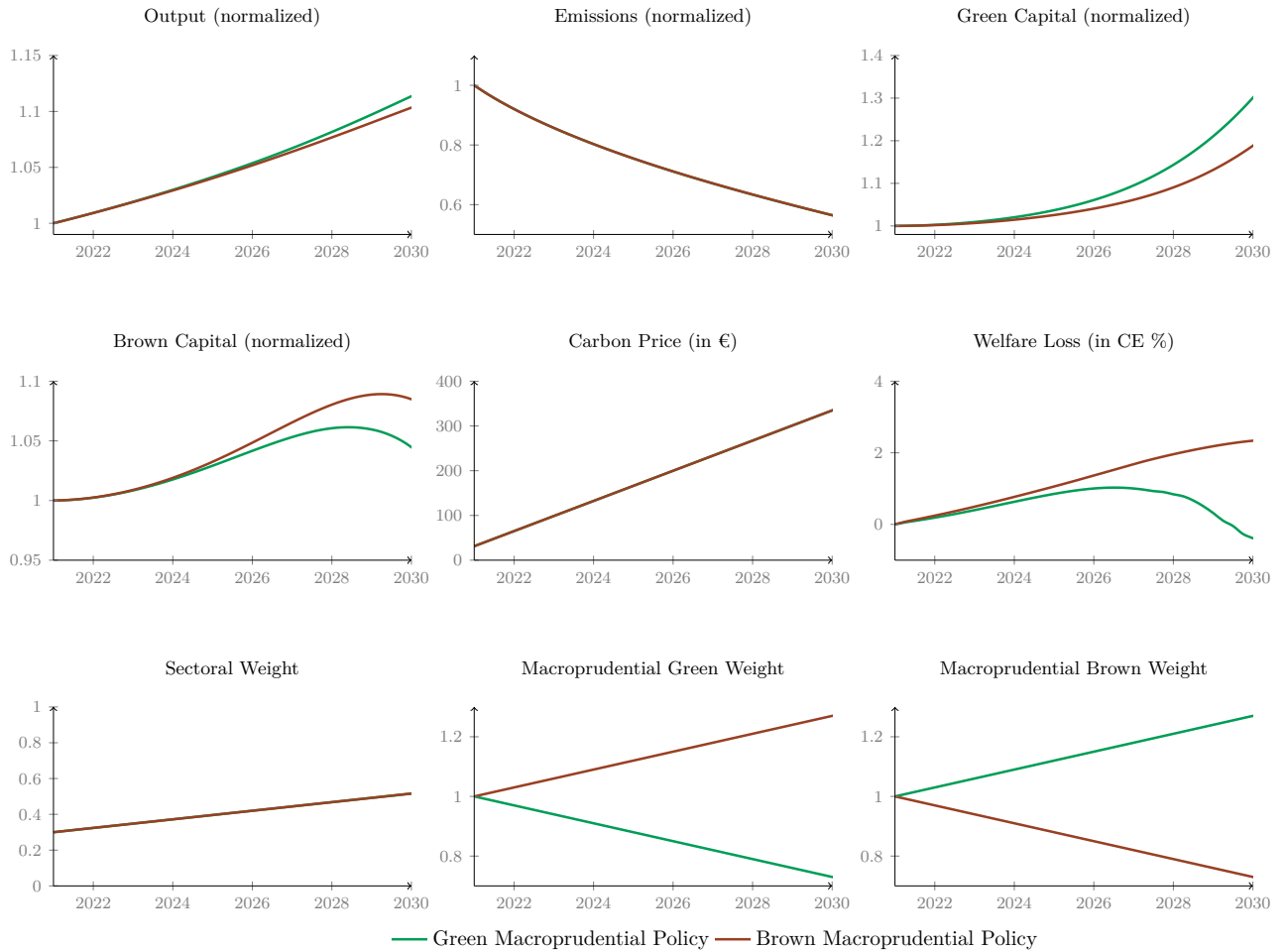
Notes: The figure compares a pathway consistent with the net-zero objective where a macroprudential policy takes into account climate risk and where it does not. The blue line corresponds to the case where no climate risk is considered ($\lambda_g = 1$ and $\lambda_b = 1$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \rightarrow 0.75$ and $\lambda_b \rightarrow 1.25$).

FIGURE 1.10. Transition pathways (net-zero) with and without brown macroprudential policy



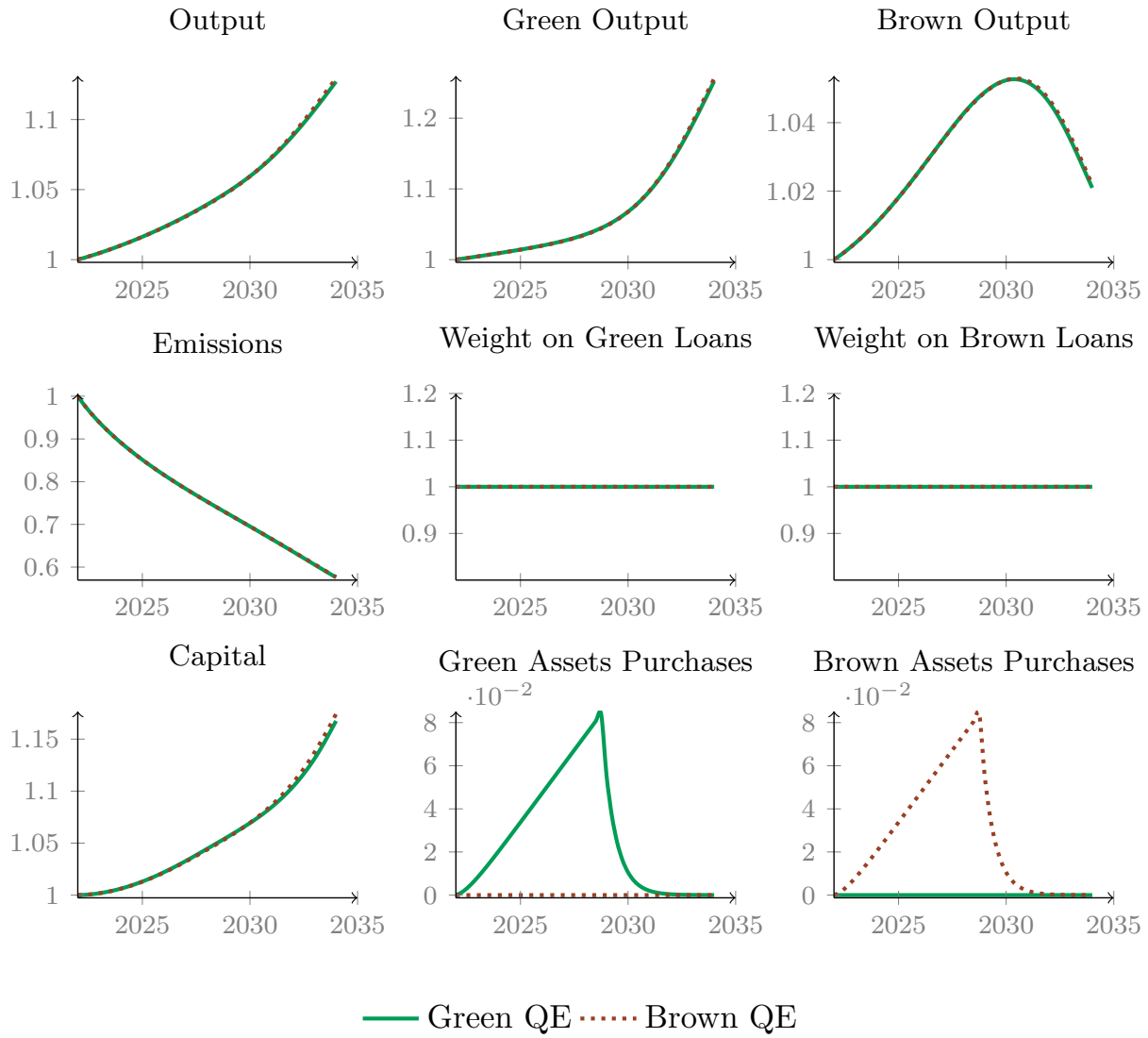
Notes: As a robustness exercise, we compare a pathway consistent with the net-zero objective where a macroprudential policy favors the brown sector over the green and where it stays neutral. The blue line corresponds to the neutral case ($\lambda_g = 1$ and $\lambda_b = 1$) and the brown line corresponds to the case where the macroprudential authority favors the brown sector ($\lambda_g \rightarrow 1.25$ and $\lambda_b \rightarrow 0.75$).

FIGURE 1.11. Transition pathways (net-zero) with macroprudential policy and an increase in the green sector share



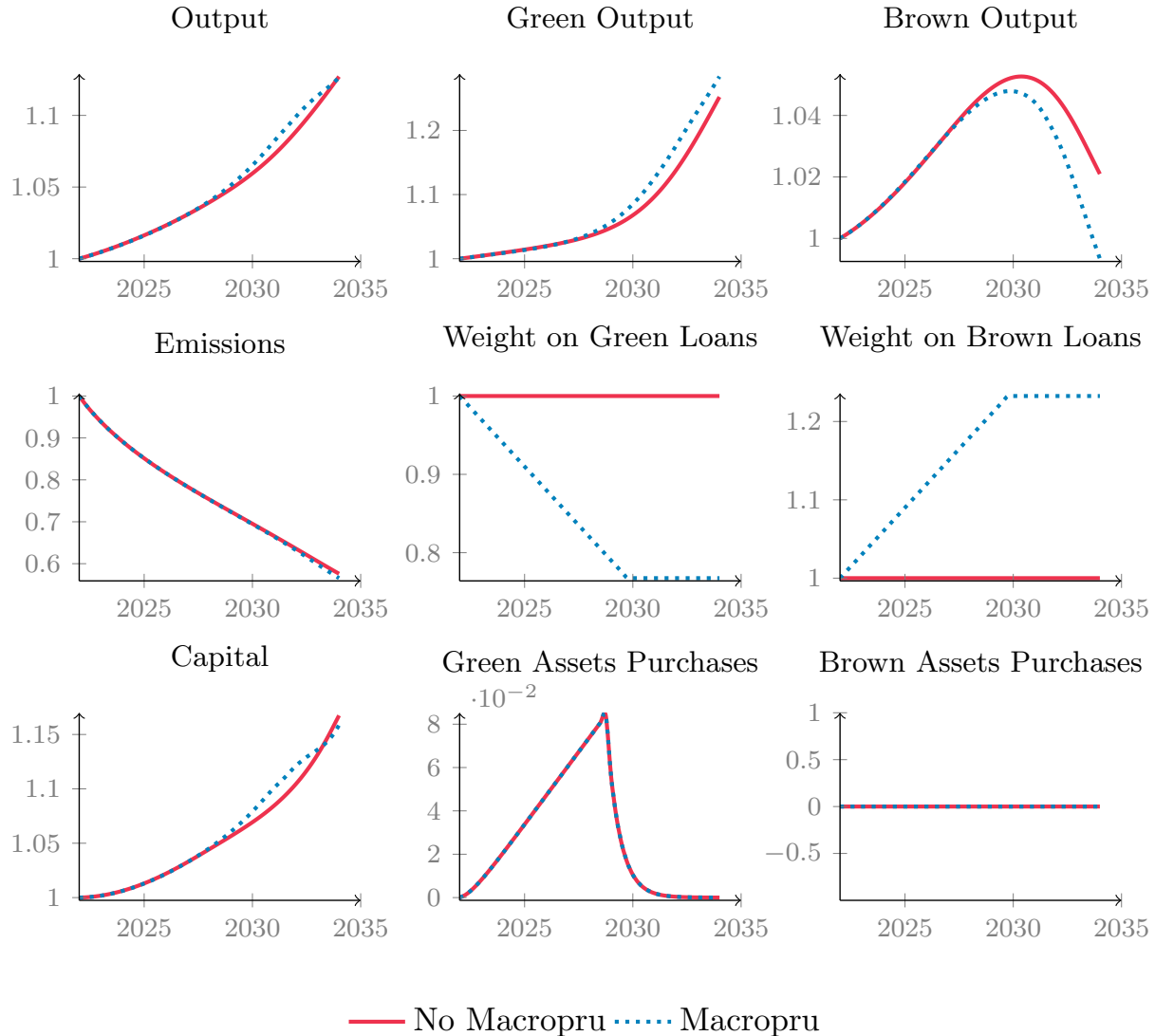
Notes: The figure compares a pathway consistent with the net-zero objective where the share of the green sector increases overtime ($\alpha \rightarrow 50\%$) and where a macroprudential policy: i) takes into account climate risk, and ii) favors the brown sector over the green. The brown line corresponds to the case where the brown sector is favored over the green ($\lambda_g = 1.25$ and $\lambda_b = 0.75$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \rightarrow 0.75$ and $\lambda_b \rightarrow 1.25$).

FIGURE 1.13. Effect of transitory green and brown asset purchase programs



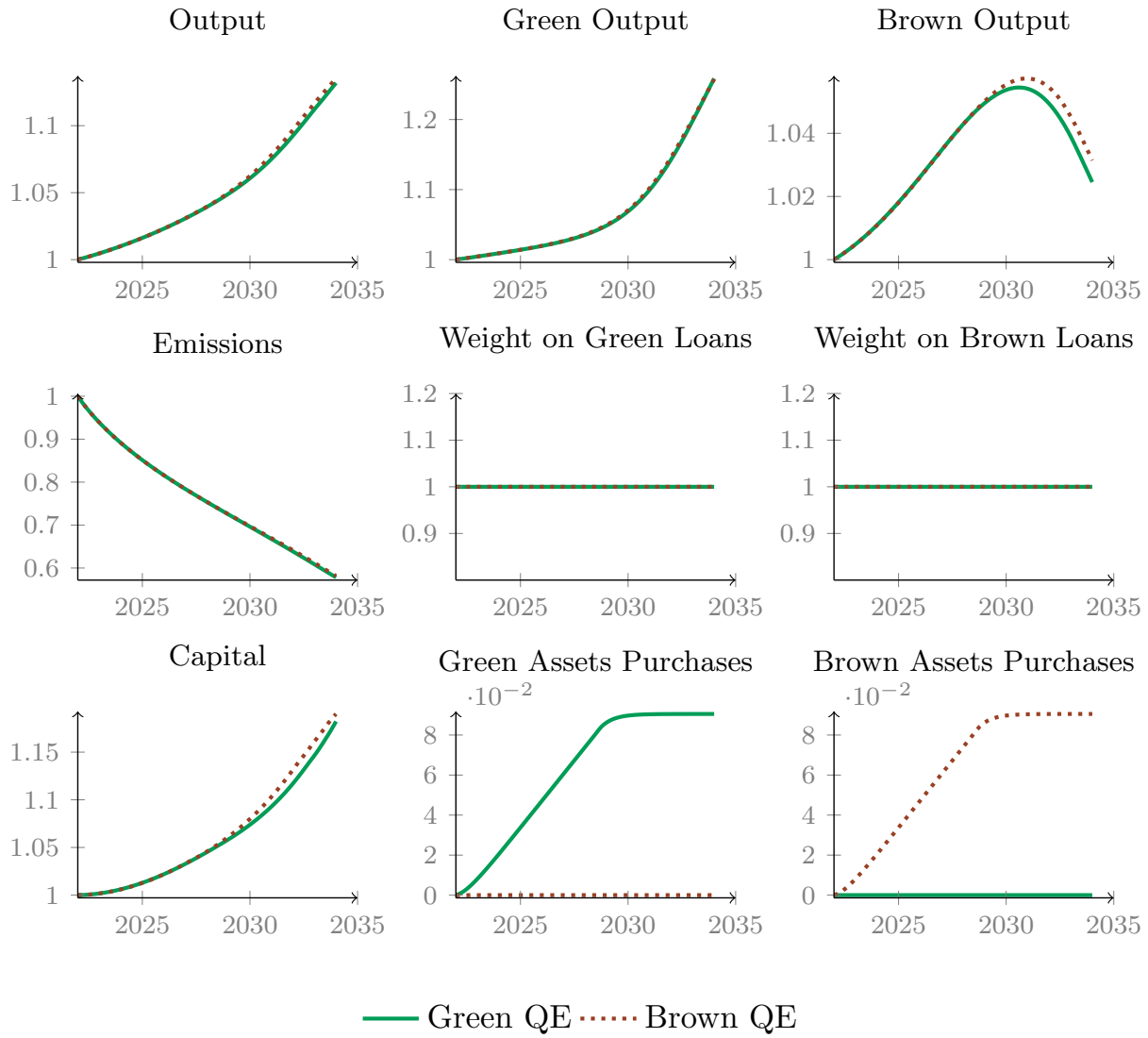
Notes: The figure shows the effect of transitory green and brown asset purchase programs (of about 9% of total asset in the economy) on a selection of variables, where the central bank stops purchasing bonds by 2028.

FIGURE 1.14. Effect of a transitory green asset purchase program with and without green macroprudential policy



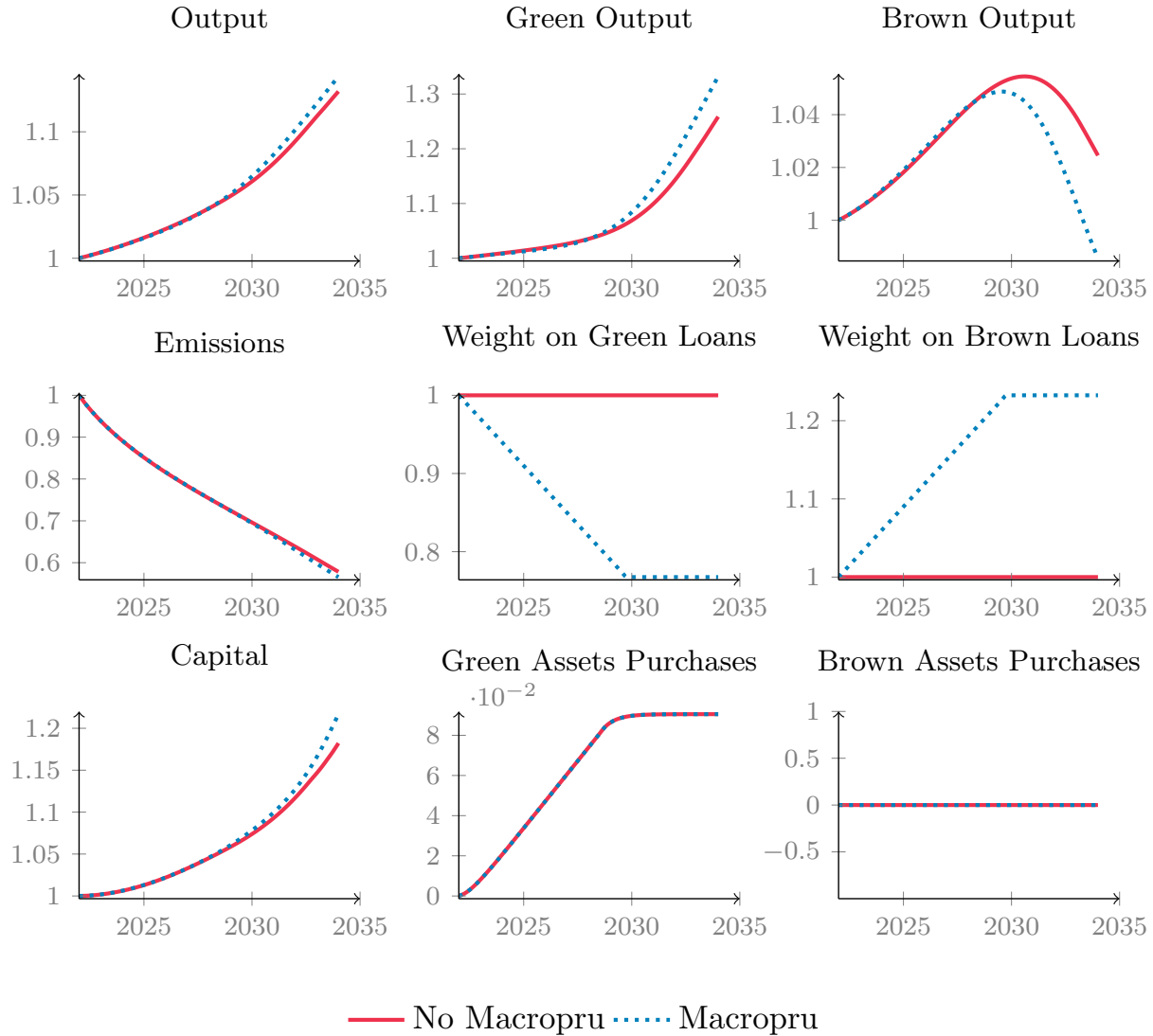
Notes: The figure shows the effect of transitory green asset purchase program (of about 9% of total asset in the economy) on a selection of variables, where the central bank stops purchasing bonds by 2028. In blue, the macroprudential authority sets a green macroprudential policy as presented in the previous section, while in red, it remains neutral.

FIGURE 1.15. Effect of permanent green and brown asset purchase programs



Notes: The figure shows the effect of permanent (where the central bank keeps the share of asset constant at about 9% of total assets in the economy) green and brown asset purchase programs on a selection of variables.

FIGURE 1.16. Effect of a permanent green asset purchase program with and without green macroprudential policy



Notes: The figure shows the effect of a permanent (where the central bank keeps the share of asset constant at about 9% of total assets in the economy) green asset purchase program on a selection of variables. In blue, the macroprudential authority sets a green macroprudential policy as presented in the previous section, while in red, it remains neutral.

 (Online Appendix)

1.C Appendix: Climate Externality and Inefficiencies

1.C.1 The Social Planner Equilibrium: Centralized Economy

The benevolent social planner optimal allocation and optimal plan would choose to maximize welfare by choosing a sequence of allocations, for given initial conditions for the endogenous state variables, that satisfies the economy constraints.⁴⁷

The planners' social problem for the households reads as follows:⁴⁸

$$\begin{aligned}
 & \max E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} \right. \\
 & + \lambda_t \left(\sum_k \left(g(\varkappa) W_{t,k} L_{t,k} + \Pi_{t,k} \right) + \Pi_t^T + T_t + R_t B_t - C_t - B_{t+1} \right) \\
 & + \lambda_t \sum_k q_{t,k} \left(\frac{P_{t,k}}{P_t} Y_{t,k} - W_{t,k} L_{t,k} - R_{t,k}^K K_{t,k} - f(\mu_{t,k}) Y_{t,k} - \Pi_{t,k} \right) \\
 & + \lambda_t \sum_k \Psi_{t,k} (\varepsilon_t^{A,k} d(T_t^o) K_{t,k}^\alpha (\Gamma_t L_{t,k})^{1-\alpha} - Y_{t,k}) \\
 & + \lambda_t \varrho_t (E_t - \sum_k g(\varkappa) E_{t,k}) \\
 & + \lambda_t \xi_t^X (X_t - \eta X_{t-1} - E_t) \\
 & + \lambda_t \xi_t^T (T_t^o - v_1^o (v_2^o X_{t-1} - T_{t-1}^o) - T_{t-1}^o) \\
 & \left. + \lambda_t \sum_k \xi_{t,k}^E (E_{t,k} - (1 - \mu_{t,k}) \varphi_k Y_{t,k}) \right)
 \end{aligned}$$

where as we will show below the Social Cost of Carbon SCC_t is the shadow value with

⁴⁷This equilibrium will provide a benchmark solution, which we use to compare with the allocation obtained in the decentralized economy for the carbon policy.

⁴⁸The social planner optimizes in an economy without price/financial frictions. This frictionless economy is the bare-bone model. In the following section, we present the decentralized economy, where we include financial and price frictions.

respect to the temperature damages ξ_t^t . $\Psi_{t,k}$ is the marginal cost component related to the firm's choice of labour and capital.

The first order conditions determining the SCC_t are the ones with respect to T_t^o, X_t , while the FOCs with respect to $E_{t,k}, \mu_{t,k}$ and $\Pi_{t,k}$ determine the level of abatement needed:

$$\lambda_t \xi_t^T = E_t \beta (1 - v_1^o) \lambda_{t+1} \xi_{t+1}^T - \lambda_t \sum_k \Psi_{t,k} \varepsilon_t^{A,k} \frac{\partial d(T_t^o)}{\partial T_t^o} K_{t,k}^\alpha (\Gamma_t L_{t,k})^{1-\alpha} \quad (1.58)$$

$$\lambda_t \xi_t^X = E_t \beta (v_1^o v_2^o) \lambda_{t+1} \xi_{t+1}^T + E_t \beta \eta \lambda_{t+1} \xi_{t+1}^X \quad (1.59)$$

$$\lambda_t \xi_{t,k}^E = g(\boldsymbol{\varkappa}) \lambda_t \xi_t^X \quad (1.60)$$

$$\lambda_t q_{t,k} f'(\mu_{t,k}) = \varphi_k \lambda_t \xi_{t,k}^E \quad (1.61)$$

$$\lambda_t = \lambda_t q_{t,k}. \quad (1.62)$$

Rearranging these FOCs we obtain the following SCC_t and abatement level:

$$\xi_t^T = E_t (1 - v_1^o) \Lambda_{t,t+1} \xi_{t+1}^T - \sum_k \Psi_{t,k} \varepsilon_t^{A,k} \frac{\partial d(T_t^o)}{\partial T_t^o} K_{t,k}^\alpha (\Gamma_t L_{t,k})^{1-\alpha} \quad (1.63)$$

$$\xi_t^X = E_t (v_1^o v_2^o) \Lambda_{t,t+1} \xi_{t+1}^T + E_t \eta \Lambda_{t,t+1} \xi_{t+1}^X \quad (1.64)$$

$$\xi_{t,k}^E = g(\boldsymbol{\varkappa}) \xi_t^X \quad (1.65)$$

$$f'(\mu_{t,k}) = \varphi_k \xi_{t,k}^E \quad (1.66)$$

1.C.2 The Decentralized Economy

The competitive equilibrium problem for the firms reads as follows:

$$\begin{aligned} \max E_t \sum_{i=0}^{\infty} & \left(\left(\frac{P_{t,k}}{P_t} Y_{t,k} - W_{t,k} L_{t,k} - R_{t,k}^K K_{t,k} - f(\mu_{t,k}) Y_{t,k} - \tau_{et,k} E_{t,k} - \Pi_{t,k} \right) \right. \\ & + \lambda_t \Psi_{t,k} (\varepsilon_t^{A,k} d(T_t^o) K_{t,k}^\alpha (\Gamma_t L_{t,k})^{1-\alpha} - Y_{t,k}) \\ & \left. + \lambda_t \Psi_{t,k}^E (E_{t,k} - (1 - \mu_{t,k}) \varphi_k Y_{t,k}) \right) \end{aligned}$$

The first order conditions determining the environmental policy $\tau_{et,k}$ are the ones with respect to $E_{t,k}$ and $\mu_{t,k}$:

$$\Psi_t^E = \tau_{et,k} \quad (1.67)$$

$$f'(\mu_{t,k}) = \Psi_t^E \varphi_k \quad (1.68)$$

Thus, from both the household and firm FOCs, we get⁴⁹:

$$\Psi_{t,k}^E = \tau_{et,k} \quad (1.69)$$

$$\Psi_{t,k}^E = \xi_{t,k}^E \quad (1.70)$$

$$f'(\mu_{t,k}) = \xi_{t,k}^E \varphi_k \quad (1.71)$$

$$\xi_t^T = (1 - v_1^o) \Lambda_{t,t+1} \xi_{t+1}^T - \sum_k \Psi_{t,k} \varepsilon_t^{A,k} \frac{\partial d(T_t^o)}{\partial T_t^o} K_{t,k}^\alpha (\Gamma_t L_{t,k})^{1-\alpha} \quad (1.72)$$

$$\xi_t^X = (v_1^o v_2^o) \Lambda_{t,t+1} \xi_{t+1}^T + \eta \Lambda_{t,t+1} \xi_{t+1}^X \quad (1.73)$$

$$\xi_{t,k}^E = g(\varkappa) \xi_t^X \quad (1.74)$$

The competitive equilibrium problem for the capital producing firms and financial intermediaries remains the same as the one presented in the financial intermediaries section.

⁴⁹Since $q_{t,k} = 1$ (as showed above), we retrieve that the input shadow cost $\Psi_{t,k}^E$ in the firms optimization problem is equal to $\xi_{t,k}^E$.

In the next section we present the Calvo problem for price frictions.⁵⁰

1.C.3 The New Keynesian Phillips Curve à la Calvo

When monopolistic firms engage in infrequent price setting à la Calvo, we assume that intermediate goods producers for each sector re-optimize their prices $P_{jt,k}$ only when a price change signal is received. The probability (density) of receiving such a signal h periods from today is assumed to be independent from the last time the firm received the signal. A number of firms ξ will receive the price-change signal per unit of time. All other firms keep their old prices. Thus, the profit maximization of our intermediate firms reads as follows:

$$\max_{P_{jt,k}} \mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \Pi_{jt+i,k} \quad (1.75)$$

$$\text{s.t. } Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}} \right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t} \right)^{-\theta} Y_t,$$

$$\text{and, } Y_{jt,k} = d(T_t^o) \varepsilon_t^{A_k} K_{jt,k}^\alpha L_{jt,k}^{1-\alpha}.$$

where $\beta^i \Lambda_{t,t+i} = \beta^i \frac{\varrho_{t+i}}{\varrho_t}$ is the real stochastic discount factor as in the Rotemberg case.

The NK Philips Curve pricing equations are as follows:

$$p_{t,k}^* = \frac{P_{t,k}^*}{P_t} = \frac{\theta_k}{\theta_k - 1} \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \text{MC}_{t+i,k} \mathfrak{S}_{t+i,k}}{\mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \mathfrak{S}_{t+i,k}}, \quad (1.76)$$

⁵⁰The Rotemberg case is presented in the core text.

where

$$\begin{aligned}\mathfrak{S}_{t+i,k} &= \left(\frac{1}{P_{t+i,k}}\right)^{-\theta_k} \left(\frac{P_{t+i,k}}{P_{t+i}}\right)^{-\theta} P_t^\theta Y_{t+i} \\ &= P_{t+i,k}^{\theta_k - \theta} \left(\frac{P_{t+i}}{P_t}\right)^\theta Y_{t+i},\end{aligned}\tag{1.77}$$

or equivalently:

$$p_{t,k}^* = \frac{P_{t,k}^*}{P_t} = \frac{\theta_k}{\theta_k - 1} \frac{S_{t,k} + \Upsilon_{t,k}}{\Theta_{t,k}},\tag{1.78}$$

$$\text{with: } S_{t,k} = P_{t,k}^{\theta_k - \theta} \Psi_{t,k} Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi_{t+1}^\theta S_{t+1,k},$$

$$\text{and: } \Theta_{t,k} = P_{t,k}^{\theta_k - \theta} Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi_{t+1}^{\theta-1} \Theta_{t+1,k},$$

$$\text{and: } \Upsilon_{t,k} = P_{t,k}^{\theta_k - \theta} \left[\theta_{1,k} \mu_{t,k}^{\theta_{2,k}} + \tau_{et,k} (1 - \mu_{t,k}) \varphi_k \right] Y_t + \frac{\varrho_{t+1}}{\varrho_k} \xi \beta \mathbb{E}_t \pi_{t+1}^\theta \Upsilon_{t+1,k},$$

with inflation $\pi_t = P_t/P_{t-1}$.

The optimal pricing condition p^* is obtained by equating the dynamic marginal revenues to the dynamic marginal costs. As in each period a fraction ξ of the intermediate firms of each sector choose their optimal price P_k^* , we can rewrite the final firms goods price P_k as a weighted average of the last period's price level and the price set by firms adjusting in the current period: $P_{t,k} = (\xi P_{t-1,k}^{1-\theta_k} + (1-\xi) P_{t,k}^{*1-\theta_k})^{\frac{1}{1-\theta_k}}$. In addition, please note that the j -index referring to our intermediate firms collapses as all firms for each sector, which are capable of setting their price optimally at t , will make the same decisions.

As presented in [Gali and Monacelli, 2008], the Calvo price dispersion $D_{pt,k}$ is essentially a measure of distortion introduced by dispersion in relative prices. Price dispersion is bounded below at 1, where 1 would be the value in the case of flexible prices. Price dispersion in our two-sector economy reads as:

$$\int_0^1 Y_{jt,k} dj = \int_0^1 \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_{t,k} dj = D_{pt,k} Y_{t,k},\tag{1.79}$$

with $D_{pt,k}$ the aggregate loss of efficiency induced by price dispersion of the intermediate goods. In other words, it also reads as $D_{pt,k} = (1 - \xi) \left(\frac{P_{t,k}}{P_t}\right)^{(\theta_k - \theta)} (p_{t,k}^*)^{-\theta_k} + \xi \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} \pi_{t,k}^{\theta_k} D_{pt-1,k}$.

Furthermore, as outlined in [Annicchiarico and Di Dio, 2015], our two-sector environmental components are impacted by the price dispersion as following:⁵¹

$$E_{t,k} = (1 - \mu_{t,k}) \varphi_k D_{pt,k} Y_{t,k}, \quad (1.80)$$

$$Z_{t,k} = \theta_{1,k} \mu_{t,k}^{\theta_{2,k}} D_{pt,k} Y_{t,k}. \quad (1.81)$$

1.C.4 The Non-Stationnary Equilibrium Conditions

The following equations represent the model equilibrium conditions.

Households:

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \{ (C_{t+1} - hC_t)^{-\sigma} \}, \quad (1.82)$$

$$1 = \beta E_t \Lambda_{t,t+1} R_{t+1}, \quad (1.83)$$

Final firms:

$$Y_t = \left(\varkappa^{\frac{1}{\theta}} Y_{t,g}^{1-\frac{1}{\theta}} + (1 - \varkappa)^{\frac{1}{\theta}} Y_{t,b}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}}, \quad (1.84)$$

⁵¹Note that, as in the canonical NK models, production and profits are also affected by the price dispersion $Y_{t,k} = d(T_t^o) \varepsilon_t^{A_k} K_{t,k}^\alpha L_t^{1-\alpha} D_{pt,k}^{-1}$ and $\Pi_{t,k} = (1 - MC_{t,k} D_{pt,k}) Y_{t,k}$.

Intermediate firms:

$$Y_{t,k} = d(T_t^o) \varepsilon_t^{A_k} K_{t,k}^\alpha L_t^{1-\alpha} D_{pt,k}^{-1}, \quad (1.85)$$

$$T_t^o = v_1^o(v_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o, \quad (1.86)$$

$$X_t = (1 - \gamma_d) X_{t-1} + E_t + E_t^*, \quad (1.87)$$

$$E_{t,k} = (1 - \mu_{t,k}) \varphi_k D_{pt,k} Y_{t,k}, \quad (1.88)$$

$$Z_{t,k} = \theta_{1,k} \mu_{t,k}^{\theta_{2,k}} D_{pt,k} Y_{t,k}, \quad (1.89)$$

$$R_{t,k}^K = \alpha \Psi_{t,k} \frac{Y_{t,k}}{K_{t,k}}, \quad (1.90)$$

$$W_{t,k}^K = (1 - \alpha) \Psi_{t,k} \frac{Y_{t,k}}{L_{t,k}}, \quad (1.91)$$

$$\tau_{et,k} = \frac{\theta_{1,k} \theta_{2,k}}{\varphi_k} \mu_{jt,k}^{\theta_{2,k}-1}, \quad (1.92)$$

$$MC_{t,k} = \Psi_{t,k} + \theta_{1,k} \mu_{t,k}^{\theta_{2,k}} + \tau_{et,k} (1 - \mu_{t,k}) \varphi_k, \quad (1.93)$$

New Phillips Curve equation (the Rotemberg case):

$$\theta^P \pi_{t,k} (\pi_{t,k} - 1) = \left(\frac{P_{t,k}}{P_t} \right)^{-\theta} \left(\frac{P_{t,k}}{P_t} (1 - \theta_k) + \theta_k MC_{t,k} \right) + E_t \left\{ \beta \Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} \theta^P \pi_{t+1,k} (\pi_{t+1,k} - 1) \right\} \quad (1.94)$$

New Phillips Curve equations (the Calvo case):

$$p_{t,k}^* = \frac{P_{t,k}^*}{P_t} = \frac{\theta_k}{\theta_k - 1} \frac{S_{t,k} + \Upsilon_{t,k}}{\Theta_{t,k}}, \quad (1.95)$$

$$S_{t,k} = P_{t,k}^{\theta_k - \theta} \Psi_{t,k} Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi_{t+1}^\theta S_{t+1,k}, \quad (1.96)$$

$$\Theta_{t,k} = P_{t,k}^{\theta_k - \theta} Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi_{t+1}^{\theta-1} \Theta_{t+1,k}, \quad (1.97)$$

$$\Upsilon_{t,k} = P_{t,k}^{\theta_k - \theta} \left[\theta_{1,k} \mu_{t,k}^{\theta_{2,k}} + \tau_{et,k} (1 - \mu_{t,k}) \varphi_k \right] Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi_{t+1}^\theta \Upsilon_{t+1,k}, \quad (1.98)$$

$$D_{pt,k} = (1 - \xi) \left(\frac{P_{t,k}}{P_t} \right)^{(\theta_k - \theta)} (p_{t,k}^*)^{-\theta_k} + \xi \left(\frac{P_{t,k}}{P_t} \right)^{-\theta} \pi_{t,k}^{\theta_k} D_{pt-1,k}, \quad (1.99)$$

$$P_{t,k} = (\xi P_{t-1,k}^{1-\theta_k} + (1 - \xi) P_{t,k}^{*1-\theta_k})^{\frac{1}{1-\theta_k}}, \quad (1.100)$$

Other NK equations:

$$\pi_t = P_t/P_{t-1}, \quad (1.101)$$

$$\pi_{t,g} = P_{t,g}/P_{t-1,g}, \quad (1.102)$$

$$\pi_{t,b} = P_{t,b}/P_{t-1,b}, \quad (1.103)$$

$$P_t = (\varkappa P_{t,g}^{1-\theta} + (1-\varkappa)P_{t,b}^{1-\theta})^{\frac{1}{1-\theta}}, \quad (1.104)$$

$$\pi_t = \left(\varkappa \frac{P_{t,g}}{P_t} \pi_{t,g}^{\frac{\theta-1}{\theta}} + (1-\varkappa) \frac{P_{t,b}}{P_t} \pi_{t,b}^{\frac{\theta-1}{\theta}} \right), \quad (1.105)$$

Capital producing firms:

$$I_{t,k}^n = I_{t,k} - \delta K_{t,k}, \quad (1.106)$$

$$K_{t+1,k} = K_{t,k} + I_{t,k}^n, \quad (1.107)$$

$$f_k(\cdot) = \frac{\eta_i}{2} \left(\frac{I_{t,k}}{I_{t-1,k}} - \theta^I \right)^2, \quad (1.108)$$

$$Q_{t,k} = 1 + f_k(\cdot) + f'_k(\cdot) \left(\frac{I_{t,k}}{I_{t-1,k}} \right) - \beta E_t \left\{ \Lambda_{t,t+1} f'_k(\cdot) \left(\frac{I_{t+1,k}}{I_{t,k}} \right)^2 \right\}, \quad (1.109)$$

Financial Intermediaries:

$$Q_{t,g}S_{t,g} + Q_{t,b}S_{t,b} = N_t + B_t, \quad (1.110)$$

$$N_t = \theta_B [(R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,b} - R_t)Q_{t-1,b}S_{t-1,b}] + (\theta_B R_t + \omega)N_{t-1}, \quad (1.111)$$

$$V_t = \lambda \nu_t (\lambda_g Q_{t,g}S_{t,g} + \lambda_b Q_{t,b}S_{t,b}) + \Delta \beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} N_t \}, \quad (1.112)$$

$$\Gamma_t^B N_t = \nu_t \Gamma_t^B N_t + \Delta \beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} R_t N_t \}, \quad (1.113)$$

$$\Gamma_t^B = \frac{1}{1-\nu_t} \Delta \beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \}, \quad (1.114)$$

$$\nu_t \lambda_k \lambda = \Delta \beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1,k} - R_{t+1}) \}, \quad (1.115)$$

$$0 = \nu_t [\Gamma_t^B N_t - \lambda (\lambda_g Q_{t,g}S_{t,g} + \lambda_b Q_{t,b}S_{t,b})], \quad (1.116)$$

$$RP_{t,k} = R_{t,k} - R_t, \quad (1.117)$$

Central Bank:⁵²

$$i_t - \bar{i} = \rho_c (i_{t-1} - \bar{i}) + (1 - \rho_c) [\phi_\pi (\pi_t - \bar{\pi}) + \phi_y (y_t - y_{t-1})], \quad (1.118)$$

$$i_t = R_t E_t \{ \pi_{t+1} \}, \quad (1.119)$$

Government:

$$G_t = T_t + \tau_{et} E_t + RP_{t,g} \psi_{t,g} K_{t,g} + RP_{t,d} \psi_{t,b} K_{t,b}, \quad (1.120)$$

$$G_t = \frac{\bar{g}}{\bar{y}} Y_t, \quad (1.121)$$

Environmental Policy (when the policy is sub-optimal ($E_t = \text{cap}$)):

$$E_t = \text{Cap}_t, \quad (1.122)$$

$$\text{Cap}_t = \text{Cap} / \Gamma_t^{\text{Cap}}, \quad (1.123)$$

Environmental Policy (when the policy is optimal ($\tau_{et,k} = \text{social cost of carbon}$)):

$$\tau_{et,k} = g(\varkappa) \text{SCC}_t, \quad (1.124)$$

$$\text{SCC}_t = \eta \beta \frac{\lambda_{t+1}}{\lambda_t} \text{SCC}_{t+1} + (v_1^o v_2^o) \beta \frac{\lambda_{t+1}}{\lambda_t} \xi_{t+1}^T, \quad (1.125)$$

$$\xi_t^T = (1 - v_1^o) \beta \frac{\lambda_{t+1}}{\lambda_t} \xi_{t+1}^T - \sum_k \Psi_{t,k} \varepsilon_t^{A,k} \frac{\partial d(T_t^o)}{\partial T_t^o} K_{t,k}^\alpha (\Gamma_t L_{t,k})^{1-\alpha}, \quad (1.126)$$

⁵²To ensure stationarity over the BGP, the central bank sets its interest rates following the Taylor rule in the spirit of [Smets and Wouters, 2003].

Aggregate variables:

$$E_t = \sum_k g(\varkappa) E_{t,k}, \quad (1.127)$$

$$K_t = \sum_k g(\varkappa) K_{t,k}, \quad (1.128)$$

$$I_t = \sum_k g(\varkappa) I_{t,k}, \quad (1.129)$$

$$Z_t = \sum_k g(\varkappa) Z_{t,k}, \quad (1.130)$$

Aggregate resource constraint (price stickiness à la Rotemberg):

$$Y_t = C_t + G_t + I_t + \sum_k g(\varkappa) [f_k(\cdot) I_{t,k}] + \sum_k g(\varkappa) \Delta_{t,k}^P Y_t + Z_t. \quad (1.131)$$

Aggregate resource constraint (price stickiness à la Calvo):

$$Y_t = C_t + G_t + I_t + \sum_k g(\varkappa) [f_k(\cdot) I_{t,k}] + Z_t. \quad (1.132)$$

1.C.5 Welfare Distortion

When $\tau_{et,k}$ moves away from $\tau_{et,k}^*$, losses in household lifetime consumption and welfare grow:

$$\Delta_{\{\tau-\tau^*\}} \text{Welfare} < 0,$$

As in [Schmitt-Grohé and Uribe, 2007], we define welfare under the optimal policy, conditional on the state of the economy in period $i = 0$ being the non-stochastic steady state associated with that regime and remaining under that regime forever, as $\text{Welfare}_t^{\tau^*}$.

Similarly, Welfare_t^τ represents welfare under the sub-optimal policy:

$$\text{Welfare}_t^{\tau^*} = E_t \sum_{i=0}^{\infty} \beta^i U(C_t^{\tau^*}) \quad (1.133)$$

$$\text{Welfare}_t^\tau = E_t \sum_{i=0}^{\infty} \beta^i U(C_t^\tau) \quad (1.134)$$

where $C_t^{\tau^*}$ and C_t^τ denote the particular plans for consumption under the optimal regime and sub-optimal regime, respectively.

Now, let λ_W denote welfare costs associated with the sub-optimal fiscal policy in terms of consumption. It is defined as the fraction of the optimal consumption process that a household would be willing to give up to be as well off under the sub-optimal policy (τ) as under the optimal policy (τ^*).

$$\text{Welfare}_t^\tau = E_t \sum_{i=0}^{\infty} \beta^i U((1 - \lambda_W)C_t^{\tau^*}) \quad (1.135)$$

As the utility function is a CRRA, no closed form solution exists to characterize the loss in welfare denoted λ_W . We perform a numerical exercise⁵³ to compute the unconditional λ_W .

We can reduce the problem to the following expression:⁵⁴

$$\begin{aligned} \text{Wedge}_C &= \left(\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{(C_{t+i}^* - hC_{t+i-1}^*)^{1-\sigma}}{1-\sigma} \right) \propto \Delta C_t \\ &\propto \Delta Y_t - \Delta I_t - \Delta G_t - \Delta Z_t \\ &\propto \Delta(1-g)Y_t - \Delta I_t - \Delta Z_t \end{aligned}$$

⁵³Where we use policy functions approximated to the second order.

⁵⁴First by using the fact that the utility function is strictly increasing. Then by using the economy budget constraint (and abstracting from adding-without a loss of generality—the investment adjustment costs as well as the price stickiness adjustment costs): $Y_t = C_t + I_t + G_t + Z_t$, and that $G_t = gY_t$, and $Z_t = f(\cdot)Y_t$.

Thus, the total effect on consumption reads as follows:

$$\text{Wedge}_C \propto (1 - g)(Y_t - Y_t^*) - (I_t - I_t^*) - (Z_t - Z_t^*)$$

As argued above, and without a loss of generality, we can focus on one sector and draw the same conclusion for the model with both sectors:

$$\begin{aligned} \text{Wedge}_{C_k} \propto & (1 - g)(\varepsilon_t^{A,k} \Gamma_t^{1-\alpha} \bar{L}^{1-\alpha})(d(T_t^o) K_{t,k}^\alpha - d(T_t^o)^* K_{t,k}^{\alpha*}) \\ & - (f(K_{t,k}) - f(K_{t,k})^*) \\ & - ((\varepsilon_t^{A,k} \Gamma_t^{1-\alpha} \bar{L}^{1-\alpha})(d(T_t^o) K_{t,k}^\alpha f(\mu_{t,k}) - d(T_t^o)^* K_{t,k}^{\alpha*} f(\mu_{t,k})^*)) \end{aligned}$$

Comparing now the impact of a higher carbon price to the optimal, we can first clearly see that the damages from higher temperature will be lower under the higher carbon price than under the optimal one $d(T_t^o) < d(T_t^o)^*$, as temperature is lower since emissions are reduced at a higher rate. Similarly, abatement is higher under the higher carbon price. As such, we propose a sectoral-maropprudential policy, which will loosen the regulatory constraint on loans to the green sector. This policy will boost the relative share of the green sector in total output, which will partially offset the welfare loss, as the green sector is less carbon intensive⁵⁵.

Similarly, the sectoral-maropprudential policy will decrease the wedge on the labor component of welfare.

⁵⁵Thus, abatement costs are less impacted by the rise in the carbon price in this sector.

1.C.6 Premium Distortion

Risk premia are defined as:

$$\begin{aligned} EP_{t,k} &= R_{t,k} - R_t \\ &= \frac{\alpha L_{t,k}^{1-\alpha} \epsilon_t^{t,k} \Psi_{t,k} d(T_t^o) K_t^{\alpha-1} - (Q_{t,k} - \delta)}{Q_{t-1,k}} - R_t \end{aligned}$$

At the steady state, as we chose $L_k = \bar{L}$ to match hours worked in the economy, the previous expression simplifies to:

$$\begin{aligned} EP_{t,k} &= \frac{\alpha \bar{L}^{1-\alpha} \epsilon_t^{t,k} \Psi_{t,k} d(T_t^o) K_t^{\alpha-1} - (Q_{t,k} - \delta)}{Q_{t-1,k}} - R_t \\ &= \frac{\alpha \Psi_{t,k} \frac{Y_{t,k}}{K_{t,k}} - (Q_{t,k} - \delta)}{Q_{t-1,k}} - R_t \end{aligned}$$

Thus, relying on a market-based instrument such as the ETS implies sudden changes and volatility in the carbon price. This uncertainty will generate fluctuations in the marginal cost components and in the price of capital, that will translate to volatility in risk premia. In the case of an increase in the carbon price:

- $MC_{t,k}$, which represents the marginal cost of firms would increase as a result of higher abatement costs ($MC_{t,k} = \Psi_{t,k} + \theta_{1,k} \mu_{t,k}^{\theta_{2,k}} + \tau_{t,k} (1 - \mu_{t,k}) \varphi_k$).
- Thus, firms' investment decreases, leading to a lower price of capital $Q_{t,k}$.

While a positive shock would trigger volatility in risk premia (as the price increase impacts all component in $R_{t,k}$), the direction of the change depends on the calibration. As such two cases arise:

1. $\alpha \Psi_{t,k} \frac{Y_{t,k}}{K_{t,k}} - (Q_{t,k} - \delta) > 0$.

In this case, risk premia would increase following a positive shock on the carbon price. Intuitively, the decrease in the price of capital is proportionally higher than the impact the shock would have on output, capital, and capital/labor input cost.

$$2. \alpha \Psi_{t,k} \frac{Y_{t,k}}{K_{t,k}} - (Q_{t,k} - \delta) < 0.$$

In this case, risk premia would decrease following a positive shock on the carbon price. Intuitively, the decrease in the price of capital cost is proportionally smaller than the impact the shock would have on output, capital, and capital/labor input cost.

In either case, it is possible to offset the level and volatility effect by acting on $Q_{t,k}$. From the macro-finance literature, we know that QE rules reacting to deviations in risk premia from their steady states are able to eliminate risk premia distortions. In our case, the distortion arises from a shock to the carbon price and not to the quality of capital.

1.C.7 Balanced Growth Path Equilibrium

1.C.7.1 The Firms

In order to perform our structural parameters estimation through the simulated method of moments, we first need to specify the de-trended economy over its balanced growth path.

The growth rate of Γ_t determines the growth rate of the economy along the balanced growth path.⁵⁶ This growth rate is denoted by γ^Y , where:

$$\Gamma_t = \gamma^Y \Gamma_{t-1} \tag{1.136}$$

Stationary variables are denoted by lower case letters, whereas variables that are growing are denoted by capital letters. For example, in the growing economy, output in each sector is denoted by $Y_{t,k}$. De-trended output is thus obtained by dividing output in the growing economy by the level of growth progress:

⁵⁶In our setup both sectors grow at the same rate Γ_t .

$$y_{t,k} = \frac{Y_{t,k}}{\Gamma_t} \quad (1.137)$$

Sectoral emissions, which we denote by $E_{t,k}$, in the growing economy are given as follows:

$$E_{t,k} = (1 - \mu_{t,k})\varphi_k Y_{t,k} D_{tp,t,k} \quad (1.138)$$

Thus, in the de-trended economy, per sector emissions law of motion reads as follows:

$$e_{t,k} = (1 - \mu_{t,k})\varphi_k y_{t,k} D_{tp,t,k} \quad (1.139)$$

where:

$$e_{t,k} = \frac{E_{t,k}}{\Gamma_t} \quad (1.140)$$

and the price dispersion $D_{tp,k}$ is a stationary variable⁵⁷.

Therefore, the total flow of emissions reads as:

$$e_t = \frac{E_t}{\Gamma_t} \quad (1.141)$$

The abatement cost in the growing economy is:

$$Z_{t,k} = f(\mu_{t,k}) Y_{t,k} D_{tp,t,k} \quad (1.142)$$

Thus, in the de-trended economy, the abatement cost reads as follows:⁵⁸

$$z_{t,k} = f(\mu_{t,k}) y_{t,k} D_{tp,t,k} \quad (1.143)$$

⁵⁷In the baseline case (i.e. the Rotemberg case), the term $D_{tp,k}$ collapses (i.e. $D_{tp,k} = 1$). Only when relying on Calvo pricing that the dispersion appears.

⁵⁸Please note that $\mu_{t,k}$ is stationary.

The stock of emissions in the atmosphere is denoted by X_t , while the temperature is called T_t^o in the growing economy:

$$X_t = (1 - \gamma_d)X_{t-1} + E_t + E_t^* \quad (1.144)$$

$$T_t^o = v_1^o(v_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o, \quad (1.145)$$

The de-trended X_t and T_t^o read as follows:

$$x_t = \frac{(1 - \gamma_d)}{\gamma^Y} x_{t-1} + e_t + E^* \quad (1.146)$$

$$\gamma^Y t_t^o = v_1^o(v_2^o x_{t-1} - t_{t-1}^o) + t_{t-1}^o \quad (1.147)$$

where:

$$x_t = \frac{X_t}{\Gamma_t} \quad (1.148)$$

$$t_t^o = \frac{T_t^o}{\Gamma_t} \quad (1.149)$$

In the growing economy, with the above growth progress, the production function is as follows:

$$Y_{t,k} = \varepsilon_t^A d(T_t^o) K_{t,k}^\alpha (\Gamma_t L_{t,k})^{1-\alpha} D_{pt}, k \quad (1.150)$$

where per sector labor $L_{t,k}$ and the technology shock $\varepsilon_t^{A_k}$ are stationary variables. Furthermore, the climate damage function captures the growth rate Γ_t such that $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2} T_t^{o2}}$. Capturing the growth rate of the economy within the damage function allows us to simplify the de-trended form of the damage function without a loss of generality as over the studied period (a 10-15 year horizon) $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2} T_t^{o2}} \approx ae^{-bT_t^{o2}}$.

De-trending the production function gives the following:

$$y_{t,k} = \varepsilon_t^A d(t_t^o) k_{t,k}^\alpha L_{t,k}^{1-\alpha} D_{pt}, k^{-1} \quad (1.151)$$

As for aggregate emissions, the de-trended aggregate output reads as:

$$y_t = \frac{Y_t}{\Gamma_t} \quad (1.152)$$

The capital-accumulation equation for both the green and brown sectors in the growing economy read as:

$$K_{t,k} = (1 - \delta)K_{t-1,k} + I_{t-1,k} \quad (1.153)$$

In the de-trended economy, we thus have:

$$k_{t,k} = \gamma^{Y-1} [(1 - \delta)k_{t-1,k} + i_{t-1,k}] \quad (1.154)$$

with both capital and investment de-trended variables reading as: $k_{t,k} = \frac{K_{t,k}}{\Gamma_t}$ and $i_{t,k} = \frac{I_{t,k}}{\Gamma_t}$, respectively.⁵⁹

1.C.7.2 The Economy Constraint (Rotemberg case)

The economy budget constraint reads as:

$$Y_t = C_t + I_t + G_t + Z_t + \sum_k g(\varkappa)[f_k(\cdot)I_{t,k}] + \sum_k g(\varkappa)[\Delta_{t,k}^P]Y_t \quad (1.155)$$

Thus,

$$y_t = c_t + i_t + g_t + z_t + \sum_k g(\varkappa)[f_k(\cdot)i_{t,k}] + \sum_k g(\varkappa)[\Delta_{t,k}^P]y_t \quad (1.156)$$

where: $c_t = \frac{C_t}{\Gamma_t}$.

⁵⁹We note that both the return on capital $R_{t,k}^k$ and wage $W_{t,k}$ are stationary. This can be easily seen by looking at the intermediate firms FOC.

The calvo case reads as:

$$y_t = c_t + i_t + g_t + z_t + \sum_k g(\varkappa)[f_k(\cdot)i_{t,k}] \quad (1.157)$$

1.C.7.3 Households

Under the presence of a labor-augmenting technology Γ_t , the utility function reads as:

$$U(C_t) = \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma}.$$

Thus, the de-trended utility reads as:

$$\sum_{t=0}^{\infty} \beta^t U(c_t) = \sum_{t=0}^{\infty} \tilde{\beta}^t \left(\frac{(c_t - (\gamma^Y)^{-1} h c_{t-1})^{1-\sigma}}{1-\sigma} \right) \quad (1.158)$$

where $\tilde{\beta} = \beta\gamma^{1-\sigma}$.

Turning to households, the equilibrium de-trended conditions read as:

$$\varrho_t = (c_t - h(\gamma^Y)^{-1} c_{t-1})^{-\sigma} - \tilde{\beta}(\gamma^Y)^{-1} h E_t \left\{ (c_{t+1} - h(\gamma^Y)^{-1} c_t)^{-\sigma} \right\} \quad (1.159)$$

$$1 = \tilde{\beta} E_t \Lambda_{t,t+1} R_{t+1} \quad (1.160)$$

with $\Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}$ the expected variation in the marginal utility of consumption.

1.C.7.4 The Firms Monetary Aggregates (NK related variables)

The presence of trend growth in output will impact the NK variables. Hence, the stationarized New Phillips Curve reads as:

$$\theta^P \pi_{t,k} (\pi_{t,k} - 1) = \left(\frac{P_{t,k}}{P_t} \right)^{-\theta} \left(\frac{P_{t,k}}{P_t} (1 - \theta_k) + \theta_k MC_{t,k} \right) + E_t \left\{ \gamma^Y \tilde{\beta} \Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} \theta^P \pi_{t+1,k} (\pi_{t+1,k} - 1) \right\} \quad (1.161)$$

Turning now to the Calvo case, we stationarize $S_{t,k}$, $\Upsilon_{t,k}$, and $\Theta_{t,k}$, dividing these

variables by the trend Γ_t . The NK Philips Curve stationary equations are as follows:

$$s_{t,k} = P_{t,k}^{\theta_k - \theta} \Psi_{t,k} y_t + \gamma^Y \frac{\varrho_{t+1}}{\varrho_t} \xi \tilde{\beta} \mathbb{E}_t \pi_{t+1}^\theta s_{t+1,k}, \quad (1.162)$$

$$\theta_{t,k} = P_{t,k}^{\theta_k - \theta} y_t + \gamma^Y \frac{\varrho_{t+1}}{\varrho_t} \xi \tilde{\beta} \mathbb{E}_t \pi_{t+1}^{\theta-1} \theta_{t+1,k}, \quad (1.163)$$

$$v_{t,k} = P_{t,k}^{\theta_k - \theta} \left[\theta_{1,k} \mu_{t,k}^{\theta_{2,k}} + \tau_{et,k} (1 - \mu_{t,k}) \varphi_k \right] y_t + \gamma^Y \frac{\varrho_{t+1}}{\varrho_t} \xi \tilde{\beta} \mathbb{E}_t \pi_{t+1}^\theta v_{t+1,k}. \quad (1.164)$$

1.C.7.5 Government

The lump sum taxes T_t and government spending G_t grow at the growth rate of the economy Γ_t :

$$g_t = t_t + \tau_t e_t, \quad (1.165)$$

with $T_t = t_t \Gamma_t$.

1.C.7.6 Capital Producing Firms

The de-trended tobin Q reads as:

$$Q_{t,k} = 1 + f_k(\cdot) + f'_k(\cdot) \left(\gamma^Y \frac{i_{t,k}}{i_{t-1,k}} \right) - \tilde{\beta} \mathbb{E}_t \left\{ \Lambda_{t,t+1} f'_k(\cdot) \left(\gamma^Y \frac{i_{t+1,k}}{i_{t,k}} \right)^2 \right\}. \quad (1.166)$$

1.C.7.7 Financial Intermediaries

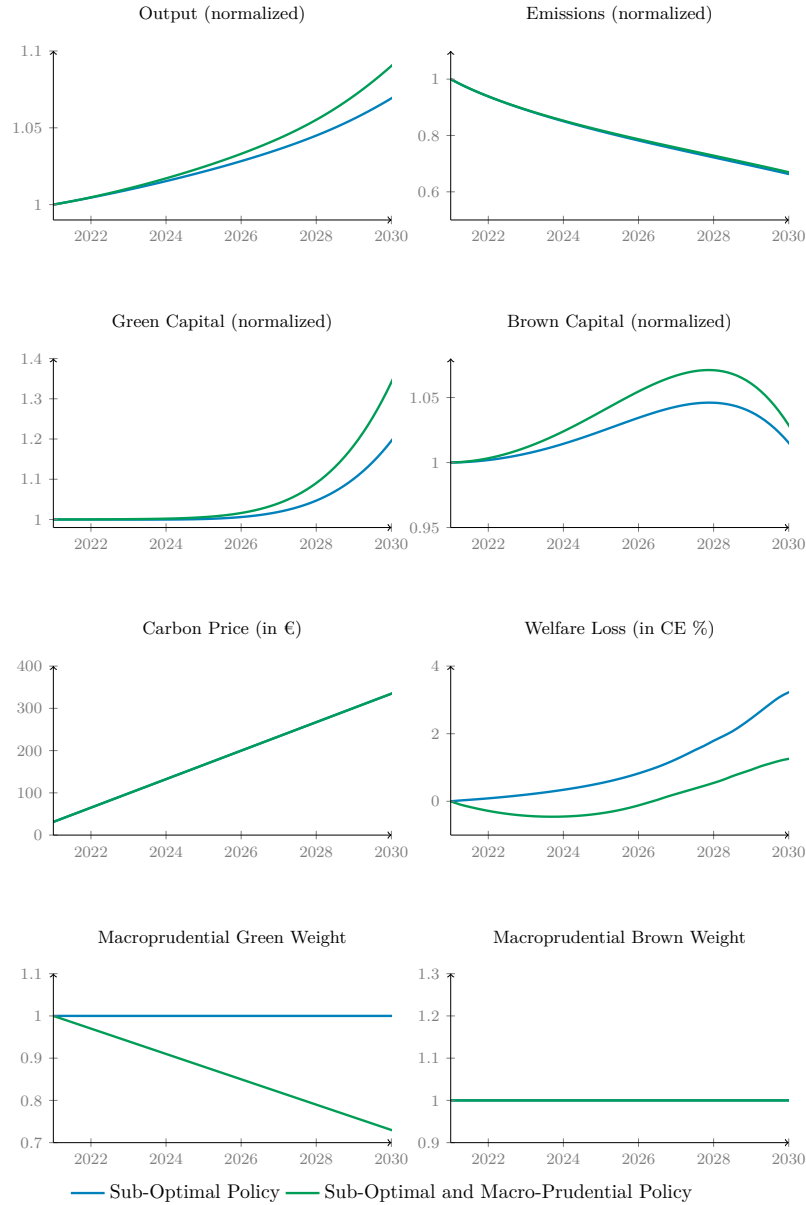
All financial intermediary variables are made stationary by dividing aggregate variables by the trend Γ_t . The only equation that needs to be adjusted is the net worth of bankers. Therefore, the stationary net worth of bankers reads as:

$$N_t = \gamma^{Y-1} (\theta_B [(R_{t,g} - R_t) Q_{t-1,g} S_{t-1,g} + (R_{t,d} - R_t) Q_{t-1,d} S_{t-1,d}] + (\theta_B R_t + \omega) N_{t-1}). \quad (1.167)$$

Chapter 1: Policy Interaction and the Transition to Clean Technology

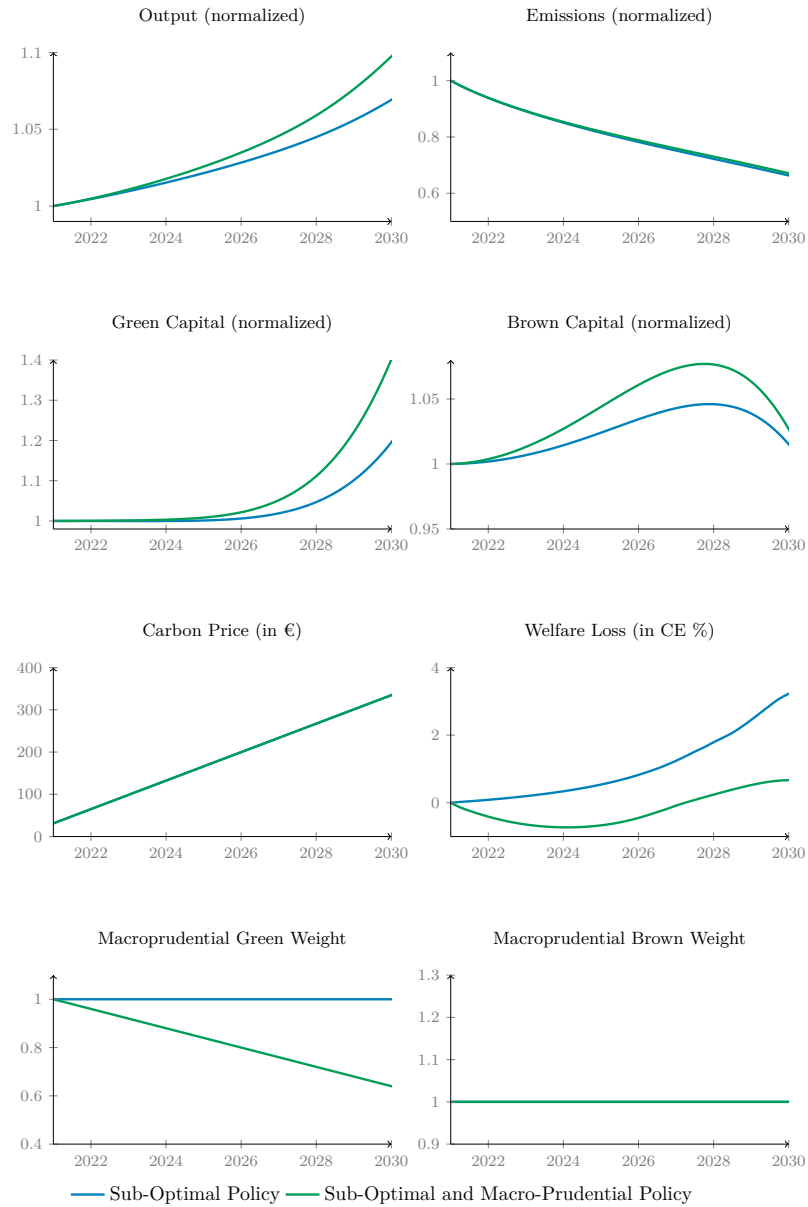
1.D Appendix: Additional Figures

FIGURE 1.17. Implications of transition pathways (Net-Zero) Without and With Macroprudential Policy



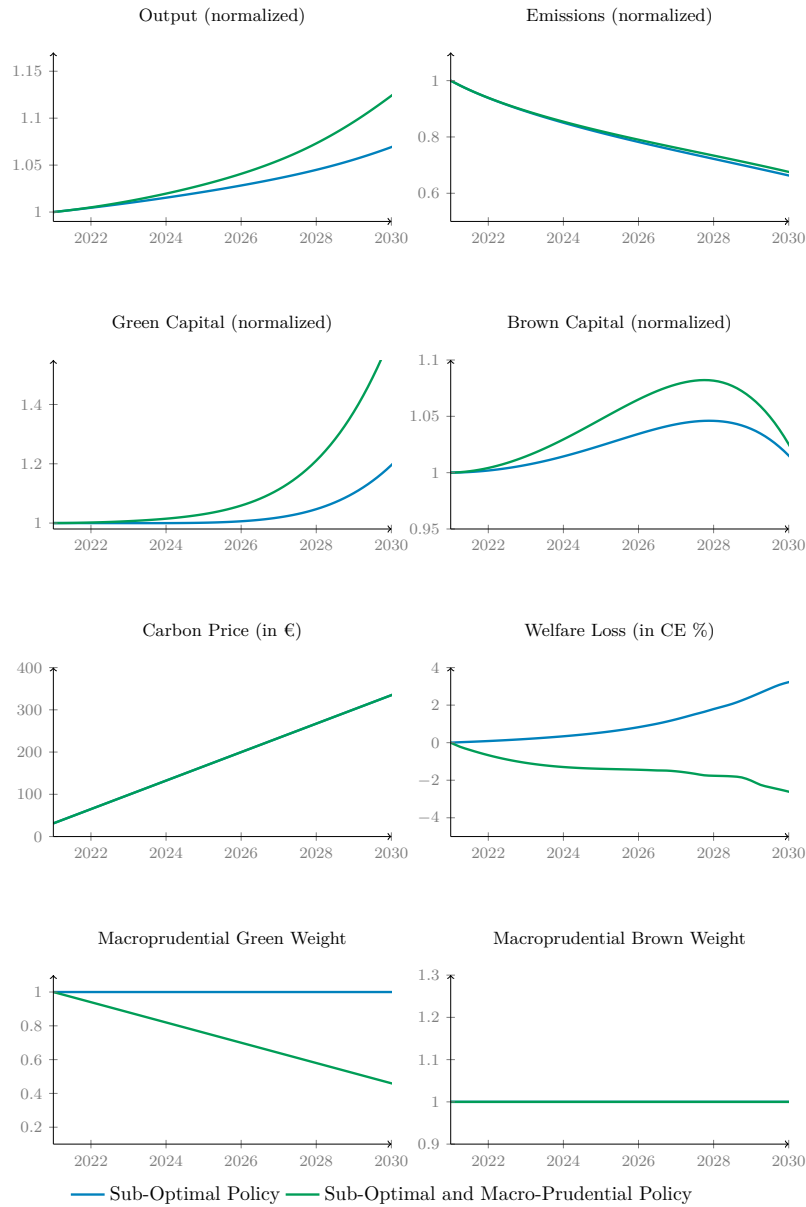
Notes: As a robustness exercise, we compare a pathway consistent with the net-zero objective where a macroprudential policy takes into account climate risk and where it does not. The blue line corresponds to the case where no climate risk is considered ($\lambda_g = 1$ and $\lambda_b = 1$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \rightarrow 0.75$ and $\lambda_b = 1$).

FIGURE 1.18. Implications of transition pathways (Net-Zero) Without and With Macroprudential Policy



Notes: As a robustness exercise, we compare a pathway consistent with the net-zero objective where a macroprudential policy takes into account climate risk and where it does not. The blue line corresponds to the case where no climate risk is considered ($\lambda_g = 1$ and $\lambda_b = 1$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \rightarrow 0.65$ and $\lambda_b = 1$).

FIGURE 1.19. Implications of transition pathways (Net-Zero) Without and With Macroprudential Policy



Notes: As a robustness exercise, we compare a pathway consistent with the net-zero objective where a macroprudential policy takes into account climate risk and where it does not. The blue line corresponds to the case where no climate risk is considered ($\lambda_g = 1$ and $\lambda_b = 1$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \rightarrow 0.45$ and $\lambda_b = 1$).

Chapter 2

The Distributional Costs of Net-Zero: A Heterogeneous Agent Perspective

This chapter is part of my Job Market Paper and was presented at the Board of the Federal Reserve, the University of California Berkeley, Paris School of Economics, King's College, LSE Environment week, Leipzig University, IE University, Amsterdam School of Economics, as well as the LSE Environmental Economics and Paris-Dauphine Economics Ph.D. Workshop and Economic Department Seminar.

2.1 Introduction

While a number of macroeconomic climate policy analyses include heterogeneity in the production sector or in climate damages, the literature mostly relies on a representative household sector and suffers from an absence of frameworks that include full household heterogeneity. Little is known about the properties of consumption and saving behavior in the presence of both: i) climate dynamics, and ii) income and wealth heterogeneity; and even less is known about such behavior under the presence of borrowing constraints.

In this paper we investigate the distributional impacts of setting a net-zero carbon policy by 2050 in the U.S. and elucidate the roles fiscal redistribution, inflation dynamics, and green innovation play over the transition to net-zero. To this end, we develop a heterogeneous agent macroeconomic model that accounts for climate dynamics and allows for studying the distributional impacts of carbon net-zero policy. We first contrast carbon-MIT shock simulations with empirical findings on the California cap-and-trade market to ensure that the model responses are consistent with empirical findings. Then we compute transition pathways toward the net-zero 2050 emissions target and investigate the impacts of the environmental policy on the joint distribution of income and wealth.

Our main finding is that the net-zero emissions policy leads to contrasting short/medium-run and long-run outcomes. In particular, we show that the net-zero policy is ultimately welfare-enhancing over the long-run, while it increases distributional costs by asymmetrically decreasing households consumption (compared to the *laissez-faire* scenario) over the transition period (i.e. 2022-2050). These welfare results are mainly driven by wealth distribution dynamics shifting toward the borrowing constraint. We begin by showing the important role climate dynamics play over the transition and how they shape the joint distribution dynamics over the long run. We find that accounting for climate dynamics, in a *laissez-faire* scenario, reduces asset holdings over the long run for all income and wealth quantiles. In contrast, implementing a carbon policy that aims to achieve the

net-zero target by 2050—which would allow for temperature to stay below 2°C over the long run—destroys wealth over the transition (i.e. between 2022 and 2050). Households engage in precautionary savings⁶⁰ as they expect carbon prices to significantly increase over the transition, which mechanically raises capital holdings in the first few years. However, as carbon prices increase, more households join the borrowing constraint, ultimately increasing by 6-10 percent the total number of households financially constrained when compared to the laissez-faire scenario. Over the long run, the joint distribution of income and wealth shifts to the right compared to the laissez-faire case (i.e. more households move away from the borrowing constraint and hold higher levels of capital) as temperature damages are now offset. Therefore, our framework uncovers (and quantifies) a clear trade-off between the long-run welfare benefits of carbon reduction policies and the short-run welfare/distributional costs of the net-zero transition, which ought to be taken into consideration when designing these policies. Quantitatively, we find that achieving net-zero target by 2050 (compared to the laissez-faire scenario) implies a 0.54 percent welfare gain (in terms of consumption equivalent (CE) variation) in the long run, while it induces up to a 20 percent consumption loss for the poorest households in the short/medium run given a medium-range abatement cost.

In the following, we highlight the four main results, which are related to: i) the impacts of carbon prices on consumption, ii) the role of fiscal redistribution, iii) the interactions between the net-zero transition and inflation, and iv) the role of green innovation.

Our first result is that carbon pricing impacts consumption via wages and rates of return. To achieve this result, we decompose the effect on consumption into direct and indirect effects. (In our framework we only have indirect effects.) In particular, we show how asset returns, wages, profits, and transfers interact to determine the consumption shift following a carbon price shock. While transfers contribute positively to consumption, in contrast, wages and asset returns, which are the main drivers, decrease consumption

⁶⁰This isn't precautionary saving in the strict sense of the word, which is often linked to aggregate risk. In our framework households increase their savings to be able to face the expected rise in carbon prices.

at the aggregate level. As firms face carbon costs, they reallocate resources (capital and labor costs), whereby the shadow input cost⁶¹ decreases, driving both capital returns and labor wages down. We then subject the model results to the case of the California carbon market and find that the model is able to reproduce the same empirical findings. More specifically, in the case of the cap-and-trade market in California, we show that the carbon price shock diffuses through the economy via the energy sector aggregates and then impacts consumers via a drop in wages and asset returns. Using U.S. climate sentiment data provided by [Ardia et al., 2020], we construct a climate news instrument. Our high-frequency instrument allows us to capture a wide range of events (e.g. regulatory, disaster, and green technological innovations). We then use the climate news instrument to identify carbon shocks, before using the carbon shocks series in an instrumental variable structural vector auto-regressive framework (IV-SVAR) ([Gertler and Karadi, 2015], [Montiel Olea et al., 2021], and [Känzig, 2021]) to investigate the aforementioned impacts of carbon pricing on household wages and asset returns.

Our second result addresses the importance of fiscal redistribution in smoothing the distributional impacts both in the short run and over the transition to net-zero by 2050. To this end, we decompose households into different wealth quantiles and income levels. We find that transfers play a major part in smoothing the impact of the carbon price. For instance, we find that an income-based tax rebate best smooths the carbon price shock, as households with a low income level and within the bottom 25 percent wealth quantile—who otherwise most suffer from the carbon tax shock—are able to keep the same level of pre-carbon-shock consumption. Similar to the first exercise, we take the model to the data once again and show how California carbon pricing asymmetrically impacts households, depending on their level of income, and do so by using quarterly consumer survey data. Of particular interest, we find that positive carbon price shocks within the California cap-and-trade market tend to increase the price of energy, which in turn decreases net energy consumption, resulting in lower wages and asset returns. The results are robust

⁶¹The marginal cost related to firms' choice of capital return and labor wages.

to both weak IV bootstrapping and Cholesky decomposition. In the case of California, the bottom 50 percent income households see their consumption fall, while consumption tends to temporarily increase for the top 50 percent, suggesting unequal consequences of carbon pricing. We conduct a series of sensitivity checks, which indicate that the results are robust along a number of dimensions including the selection of news, the estimation technique, the model specification, and the sample period.

Our third result is that nominal rigidities are an important feature of the net-zero transition. We highlight the linkages between inflation and carbon pricing by considering the case of sticky prices. We show that carbon pricing induces lower inflation over the transition period, while inflationary pressure manifests over the long run, which could represent a challenge for monetary policy conduction in a world with high inflation. This is largely due to the increasing total marginal cost that is driven by higher carbon prices. Although firms decrease their shadow input cost by reallocating capital and labour resources, the rising carbon prices and increasing abatement costs outweigh the decrease in the shadow input cost and thus increase the total marginal cost. As such, inflationary pressures could further deepen the distributional impacts discussed above.

Finally, our fourth result is that green innovation (represented by abatement cost in our framework) plays a major role over the transition and could make the net-zero 2050 emissions target difficult to achieve if cheaper technologies are not developed rapidly. To this extent, we investigate the case of abatement learning and show how fiscal redistribution and green innovation decreases carbon prices and boosts consumption over the transition. With green innovation decreasing abatement investment costs, the impacts on the joint distribution of income and wealth are less pronounced and the net-zero transition is less costly for households and firms alike.

As with the empirical component of our work, we perform a comprehensive series of sensitivity checks, including different calibrations for abatement costs, climate dynamics, climate damages, and policy trajectories.

In addition, we provide a methodological contribution, under which, climate dynamics

are cast within the standard incomplete market model pioneered by [Imrohoroglu, 1989] and [Aiyagari, 1994], in continuous time following [Achdou et al., 2022]. Our methodological contributions are twofold. First, we develop a novel and flexible heterogeneous climate macroeconomic framework that accounts for climate dynamics and allows for studying the distributional impacts along the transition to the net-zero emissions target. To this end, we make use of heterogeneous agents models to expand the scope of literature on climate macroeconomics by including household heterogeneity to contribute to the climate mitigation debate. One of our contributions is to integrate the heterogeneous agents climate models into the framework provided by [Achdou et al., 2022] and more broadly into the field of “Mean Field Games” (MFG) introduced by [Lasry and Lions, 2007]. Whereas, a number of climate-macroeconomic empirical studies (e.g. [Dell et al., 2012], [Burke et al., 2015], [Colacito et al., 2018]) and theoretical work such as [Rudik et al., 2021] focus on heterogeneity of climate damages, we, on the other hand, focus on household heterogeneity in terms of income/wealth, and are able to contrast the distributional impacts of the net-zero transition between the short/medium run and the long run, impacts that are not due to environmental preferences or heterogeneous climate modeling choices. Second, we show how the long-run steady states of the economy are solved under the presence of climate dynamics, as well as how transition dynamics are computed following the seminal work of [Achdou et al., 2022]. Moreover, we highlight that under the presence of nominal rigidities, relying on the system of equations method to solve the transition dynamics for the marginal cost is necessary, as the updating iterative algorithm rule does not allow for convergence when used to clear the New Phillips Curve.

Literature Review. Where an extensive part of the literature focuses on the optimal price of carbon, also referred to as the social cost of carbon ‘SCC’ ([Nordhaus, 1991], [Stern, 2008], [Weitzman, 2012], and [Dietz and Stern, 2015], among many others), the macroeconomic impacts of reaching net-zero emissions have received far less attention. These papers focus on the level of the optimal cost of carbon in a representative agent model, where the goal is to characterize the price level needed to offset carbon emissions. The uncertainty,

however, around the optimal price ([Cai and Lontzek, 2019], [Van der Ploeg et al., 2020], [Barnett et al., 2020], and [Traeger, 2021]) suggests difficulties regarding its implementation. In a recent paper, [Benmir and Roman, 2020] investigate the consequences of net-zero emissions targets in the context of the EU. They show that following an optimal policy is not sufficient, and, therefore, there is a need for sub-optimal policy (such as the European Trading System (ETS)) to reach the net-zero target. This sub-optimal price level induces welfare losses at the aggregate level and could dissimulate disparities at the household level, suggesting potential negative impacts on the distribution. However, none of these papers clearly identify the transition dynamics and its impacts over the distribution using a fully heterogeneous agent model.

Another major part of the the literature focuses on the role of technological change and innovation in climate change mitigation (e.g. [Smulders and De Nooij, 2003], [Grimaud and Rouge, 2008], [Di Maria and Valente, 2008], [Acemoglu et al., 2012], [Aghion et al., 2016], and [Acemoglu et al., 2019]), where household heterogeneity is often overlooked. While these papers shed light on the role of technology over the transition, they do not capture the potential trade-off between: i) using carbon pricing revenue to steer green innovation, and ii) smoothing the potential distributional costs linked to setting a carbon price.

However, recently, building on [Bosetti and Maffezzoli, 2013], who were the first to show the importance of accounting for heterogeneity (in household income) when investigating climate policy interaction with macroeconomic aggregates, [Cavalcanti et al., 2021] study the distributional effects of climate change mitigation policies within and across countries. Similarly, [Malafry and Brinca, 2022] assess how household heterogeneity implies different levels of carbon price preferences. They, however, do not investigate the transition dynamics with a joint household income/wealth distribution and endogenous energy sector where energy prices are subject to demand and supply markets. Furthermore, [Fried et al., 2018] and [Fried et al., 2021] use a heterogeneous life-cycle model to investigate the impact of carbon taxes on future generations, while [Goulder et al., 2019] use a computable general equilibrium model to assess the carbon tax's negative distributional impacts. In addition,

[Bakkensen and Barrage, 2021] investigate the impact of belief heterogeneity on coastal housing markets, using a dynamic model. We contribute to this literature by providing a framework: i) that allows for transition pathways, where energy is an endogenous input to other sectors of economy; ii) that encompasses full climate dynamics; and iii) that captures full household heterogeneity (in income and wealth); all of which, we argue are essential components for understanding the full scope of the impacts of the net-zero emissions target.

These largely theoretical studies contrast with a number of empirical findings by, for example, [Metcalf, 2019], [Shapiro and Metcalf, 2021], and [Bernard and Kichian, 2021], who find no significant effect of carbon policy on macroeconomic aggregates. Our work bolsters the findings of [Känzig, 2021], who in contrast to the aforementioned empirical papers, find a significant and negative impact of carbon pricing on macroeconomic aggregates. Similar to [Känzig, 2021], others, such as [Mansanet-Bataller and Pardo, 2009], and [Bushnell et al., 2013], also use event study methodology to investigate the impacts of regulatory carbon and energy news on prices. We contribute to this growing literature by employing the Sentometrics index developed by [Ardia et al., 2020] in our study of the California cap-and-trade carbon market.

While the heterogeneous macroeconomic literature proposes a set of methods (e.g. [Ahn et al., 2018] and [Auclert, 2019]) to solve dynamic systems, we follow [Achdou et al., 2022] and use the finite difference method developed by the authors for solving our heterogeneous agent model and for computing the Hamilton-Jacobi-Bellman (HJB) as well as for the Kolmogorov Forward equations. As the main focus of our paper is the net-zero distributional impacts, we rely on MIT shocks and do not focus on aggregate risk ([Den Haan, 1997], [Krusell and Smith, 1998], [Reiter, 2009], [Boppart et al., 2018], and [Auclert et al., 2021], among others) in this paper.

For practical purposes, we will first present the empirical findings and then move to the theoretical results, which are the core of the paper. Our empirical results are to be considered in light of the theoretical model's numerical exercises. The empirical exercises serve to ensure that the results of the theoretical model are consistent with the carbon

pricing propagation channels for the case of California, which is an imperfect but available representation of what could happen at the U.S. level and is the only large carbon market in the country.

Section 2 presents our empirical findings, while section 3 outlines our continuous-time climate macroeconomic model of income and wealth distribution. Section 4 describes our computational algorithm for both stationary and time-varying equilibria. Section 5 delineates our net-zero transition quantitative results. Section 6 highlights the impacts of net-zero on inflation. Section 7 presents the case of learning by doing. Section 8 concludes.

2.2 Empirical Analysis

As the main objective of our paper is to investigate the net-zero distributional impacts on households, understanding the channels through which carbon pricing propagates in the economy is paramount. Our empirical study on the California cap-and-trade market sheds some light on the ways by which carbon pricing impacts aggregate prices and different consumers. We then use the empirical findings to discipline our theoretical framework and ensure consistency of the channels through which carbon pricing impacts the economy at large and households more specifically.

While our main study looks at U.S. net-zero distributional impacts, our choice of the California carbon market is due to the absence of a generalized carbon market in the U.S. (as is the case for the European Union). We, therefore, use California as a proxy for the U.S. in terms of potential propagation channels when setting a carbon price.

To conduct our empirical analysis, we make use of the event studies found in the monetary literature (e.g. [Kuttner, 2001], [Gertler and Karadi, 2015], and [Nakamura and Steinsson, 2018]) that use news shock strategies to identify structural shock instruments, which we then couple with a climate “Sentometric” index ([Ardia et al., 2020]) that summarizes the climate sentiment (i.e. whether media report positive or negative news about climate change) at a daily frequency in the U.S.

2.2.1 The California Market at a Glance

The California carbon cap-and-trade program is considered to be one of the largest⁶² multi-sectoral emissions trading systems in the world, along with the EU ETS.

The program aims at a reduction of emissions by 40 percent below 1990 levels by 2030, and has a goal of reaching carbon neutrality by 2045, which is a far more ambitious goal than the U.S. net-zero recent pledges (carbon neutrality by 2050). California's program covers GHG sources responsible for approximately 85 percent of the state's CO₂ emissions. It relies on two types of compliance instruments: i) allowances and ii) offsets, which are traded on secondary markets (spot and futures markets).

Revenue from carbon pricing, which the regulator has amassed, comes to 5 billion dollars of total revenue since the beginning of the program. The total revenue is used, on one hand, for a Greenhouse Gas Reduction Fund (65 percent) to help implement programs aiming at further reducing CO₂ emissions, and, on the other hand, as a redistribution tool for environmentally disadvantaged and low-income communities (35 percent).

In the following section, we investigate the linkages between the California cap-and-trade system and different macroeconomic prices and aggregates.

2.2.2 The Carbon Policy Instrument

Building on the event study literature, we use the comprehensive Sentometric index⁶³ by [Ardia et al., 2020], which lists all daily news on climate sentiment in the U.S. from 2003 to 2018. We then take the mean over the period of interest (2012 to 2018) and only consider a news shock to be the days where a higher level of climate news was observed compared to the mean. This reflects a movement in the sentiment and/or the regulatory constraints, which we use as an event news shock to the California carbon price. As the

⁶²It is the fourth largest in the world, following the cap-and-trade programs of China, the European Union, and the Republic of Korea.

⁶³All index data are publicly shared by the authors: <https://sentometrics-research.com/download/mccc/>.

selection of events is a fundamental factor in event studies, we run a sensitivity analysis with different thresholds to control for possible confounding noise in the data.

Sentometric index data are provided daily, which allows us to perform a high-frequency analysis when constructing the carbon policy surprise series. Following [Gertler and Karadi, 2015], we construct the carbon surprise series (τ_t^{Shock}) as the change of carbon prices (τ_t^C) between the event day⁶⁴ and the previous day as follows:

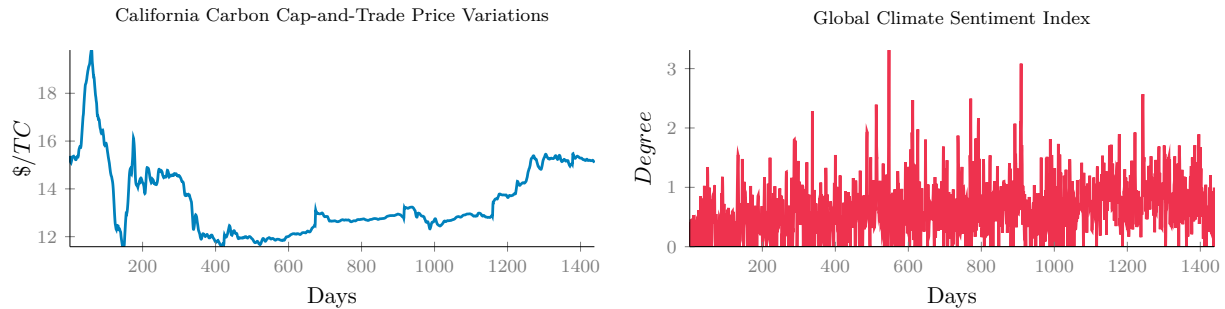
$$\tau_t^{\text{Shock}} = \begin{cases} \tau_t^C - \tau_{t-1}^C & \text{If } \text{day}_t(\text{Carbon Index}) \geq \frac{1}{T} \sum_{i=1}^T \text{Carbon Index}_i, \\ 0 & \text{otherwise.} \end{cases} \quad (2.1)$$

A question that is usually of concern is the reverse causality. In our framework, we are less subject to it as our measure of the price change is at a sufficiently high frequency (daily news), which allows us to isolate the impact of the news sentiment confidently.

Furthermore, although we construct our carbon surprise series at a daily frequency, we aggregate all data to a monthly level in order to fit with the other macro-aggregate data, which are only available at a monthly frequency. In order to study the macroeconomic impact of carbon policy we rely on four aggregates, namely: i) energy composite price, ii) energy net generation, iii) weekly wages of the energy and utilities sector, iv) returns on equity index, which are all taken at a monthly frequency (or aggregated to monthly for the returns on equity index and energy prices), all for the state of California.⁶⁵ The sample spans the period from April 2012 to April 2018, a period for which we have available data on climate sentiment as well as for all the other variables.

⁶⁴Where we use front contract on carbon allowances futures.

⁶⁵For details on data sources, please refer to Appendix [section 2.A](#).

FIGURE 2.1. Carbon Prices and Climate Index

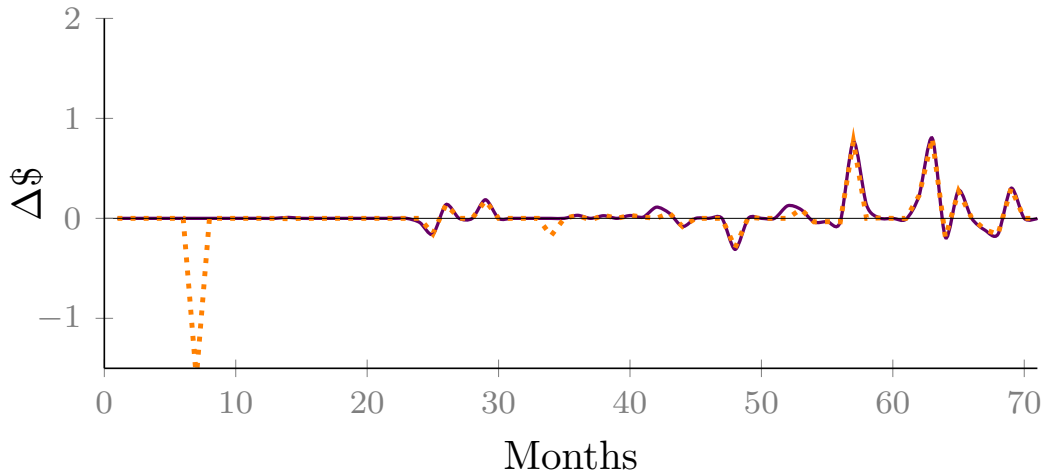
Note: The left figure displays the CO₂ future prices for the California cap-and-trade market between the 1st of May 2011 and the 27th of March 2018 at a daily frequency. The figure on the right, shows the climate sentiment index for the same period.

To illustrate the data used to extract the policy instrument in [figure 2.2](#), we present the carbon price and climate sentiment index in [figure 2.1](#). Relying then on the strategy outlined above, we show the carbon policy surprise series for two cases: i) where we include all days with zero news on climate, and ii) where we exclude all days with zero news on climate. A ‘zero’ news day means that either we have no information or that sentiment over climate change is positive.⁶⁶ Excluding these zero news days shifts the mean of the sentiment around climate change, which shifts movements over the policy price shock.

We can see that excluding days with no or zero news adds more variation (orange dotted line) compared to our baseline case where we consider that the days with zero news are days with positive sentiment over climate change.⁶⁷

⁶⁶What is meant by positive, is a lack of negative news on climate change.

⁶⁷In [figure 2.22](#) and [figure 2.21](#) we show that our results remain strong to this hypothesis.

FIGURE 2.2. Carbon Price Policy Instrument

Note: The figure presents the shock to futures contract carbon prices (price difference for relevant days) for the California carbon market used as an external instrument in our study. We use the climate index daily data to extract the shock from the carbon prices, which then we aggregate to monthly levels. Data are presented at a monthly frequency for the period between the 1st of May 2011 and the 27th of March 2018. In dotted orange we present the case where the aggregation of climate news events excludes all ‘zeros’ (i.e. days without any climate news). Whereas, the purple solid line presents the aggregation of climate news events events where we includes all zero news days.

2.2.3 Validity of the Carbon Policy Instrument

Following both [Ramey, 2016] and [Montiel Olea et al., 2021], we first investigate the auto-correlation function and verify that our policy instrument is not explained by our macroeconomic aggregate series. We do this by conducting a Granger causality test. We find no auto-correlation (p-value of Q-stat for H_0 is 0.99) and no significance of the Granger causality test.

2.2.4 Impacts of Carbon Price Policy on Aggregate Macro-Variables: IV-SVAR Model

While in our framework, we could use the constructed carbon policy surprise series as a direct measure of our shock of interest,⁶⁸ we allow for some errors in our policy surprise series, and thus use it as an instrument instead of a direct policy shock.

We use an SVAR to derive the impulse responses of the variables of interest following our carbon policy shock. To set our VAR, we follow [Montiel Olea et al., 2021].

Let Y_t be a 4 x 1 vector of observables (energy prices, net energy generation, wages, equity index returns). We assume that the dynamics of the observables are described by a system of linear simultaneous equations:

$$Y_t = \sum_{j=1}^p A_j Y_{t-j} + \eta_t, \quad (2.2)$$

where η_t is a vector of reduced-form VAR innovations. We can re-write the reduced form innovations as a vector of structural shocks ϵ_t :

$$\eta_t = \Gamma \epsilon_t, \quad (2.3)$$

where Γ is a non-singular 4 x 4 matrix.

The method relies on two main assumptions: i) the invertability of the structural VAR (i.e. VAR forecast errors at date t are a non-singular transformation of the structural errors at date t) and ii) the structural shocks are assumed to be serially and mutually uncorrelated:

$$E(\epsilon_t) = 0, \quad (2.4)$$

$$E(\epsilon_t \epsilon_t') = \text{diag}(\sigma_1^2, \dots, \sigma_n^2). \quad (2.5)$$

⁶⁸We do that as robustness check, and find similar results. Please refer to Appendix [section 2.A](#).

Therefore, the covariance matrix for the reduced form innovations reads as:

$$E(\eta_t \eta_t') = \Sigma = \Gamma \text{diag}(\sigma_1^2, \dots, \sigma_n^2) \Gamma', \quad (2.6)$$

In our research question, we are interested in the causal impact of the carbon policy shock on the set of observables. In other words, we are interested in the structural impulse response coefficient. In our framework this is the response of our observables to a one unit change in the policy shock, which we denote as $\epsilon_{1,t}$:

$$\frac{\partial Y_{i,t+k}}{\partial \epsilon_{1,t}} = e_1' C_k(A) \Gamma e_1, \quad (2.7)$$

where C_k corresponds to the Wold decomposition of the VAR and emphasizes the dependence of the MA coefficients on the AR structure coefficients in A , and e_1 is the first column of the identity matrix.

Since we use the carbon policy as an instrument—which we denote as z_t —instead of a direct measure,⁶⁹), we require both the relevance and the exogeneity conditions to hold:

$$E(z_t \epsilon_{1,t}) = \alpha \neq 0, \quad (2.8)$$

$$E(z_t \epsilon_{j,t}) = 0 \text{ for } j \neq 1. \quad (2.9)$$

Having outlined the instrumental variable SVAR framework, we estimate the impulse responses function coefficients for the VAR in levels ([Sims et al., 1990]) where all variables are taken in logs, using a Two-Stage Least Squares (2SLS) methodology. We conduct the 2SLS using our instrument z_t and regressing $\hat{\eta}_t$ on $\hat{\eta}_{1,t}$. We rely on the bootstrap residual moving block as in [Montiel Olea et al., 2021].

Furthermore, we use eight lags in our SVAR as suggested by the AIC criterion. We also allow for twelve lags on the Newey-West standard errors in order to capture possible

⁶⁹In the Appendix [section 2.A](#) we show the impulse responses when the policy shock is used as a direct measure in the SVAR.

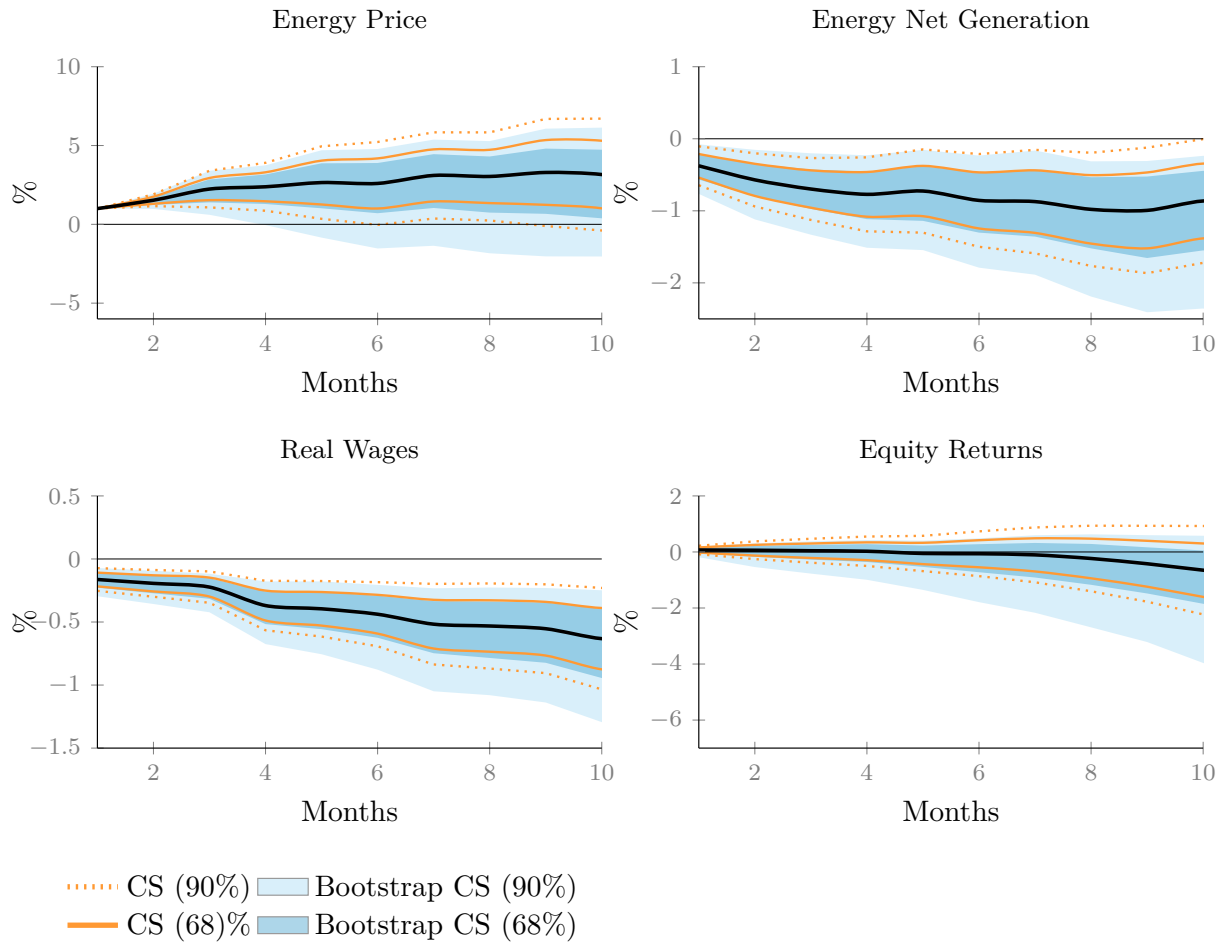
auto-correlation within our monthly data.

Finally, we conduct both standard inference and weak IV inference as suggested by [Montiel Olea et al., 2021], since the heteroskedasticity robust F-statistic in the first stage of the IV-SVAR is less than the critical value of ten but higher than four.

2.2.5 Impacts of Carbon Price Policy on Aggregate Macro-Variables: Results

Turning to the results of our IV-SVAR model, [Figure 2.3](#) presents the standard inference results. More precisely, it shows the impulse responses (IRFs) to the identified carbon policy shock, normalized to increase the energy price by one percent on impact. The solid black lines represent the estimated paths. The shaded blue areas are the 68 and 90 percent confidence bands, while the orange solid and dotted lines are the 68 and 90 percent confidence bands computed using the bootstrapping procedure.

FIGURE 2.3. IV-SVAR



Note: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this exercise, the carbon shock is constructed excluding including all days with zero or no news.

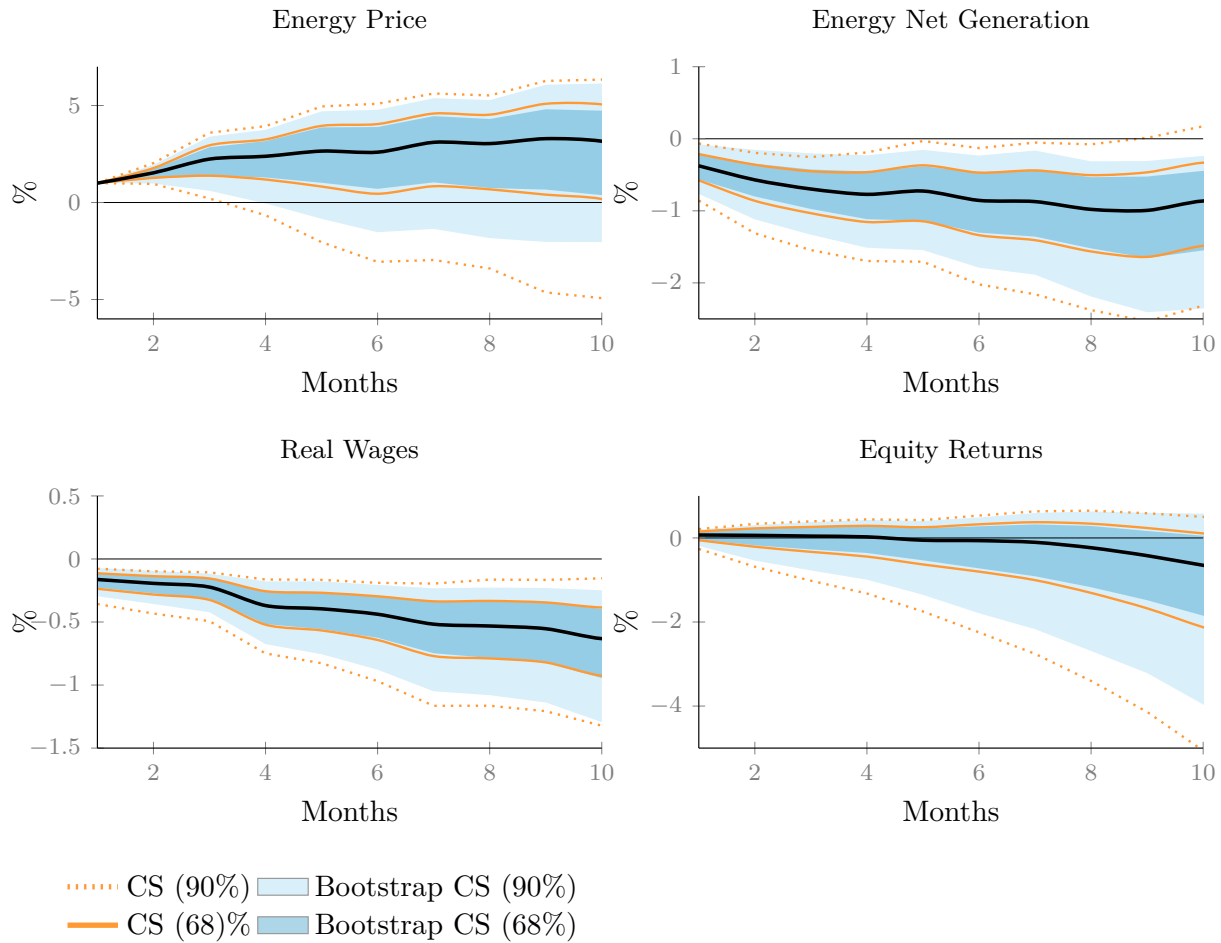
Results show that a carbon policy shock leads to a persistent increase in energy prices, which triggers a persistent decrease in net energy generation. This rise in energy price and fall in energy generation induce a cost to firms and consumers. As firms input costs increase with higher carbon prices, they engage in resource reallocation, which leads to a persistent decrease in wages. With respect to equity returns, the fall does not manifest immediately, but is observed seven periods following the shock.

From both a statistical and economic perspective, the results are significant. As shown by the confidence intervals, the directions of the effects are clearly identified. In terms of magnitudes, the results are also economically significant. A carbon policy shock increasing energy prices by 1 percent leads to a 1 percent decrease in net energy generation and to about 0.6 percent decrease in wages paid to employees of the energy and utilities sector, whereas returns on equity fall by about 2 percent by the end of the 15 months period.

When relying on the weak IV inference, the results turn out to be robust and similar in terms of magnitude, direction, and statistical significance, as shown in [Figure 2.4](#). Finally, the results from both: i) the instrument where we exclude zero day news and ii) the standard Cholesky SVAR (where we use the external instrument as a direct internal variable),⁷⁰ turn out to be very similar, which supports our overall results.

⁷⁰Refer to appendix [section 2.A](#) for more details.

FIGURE 2.4. Weak IV-SVAR



Note: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this exercise, the carbon shock is constructed excluding including all days with zero or no news. The inference is conducted using weak IV robust bootstrapping procedure.

2.2.6 The Impacts of Energy Prices on Consumption Quantiles: SVAR

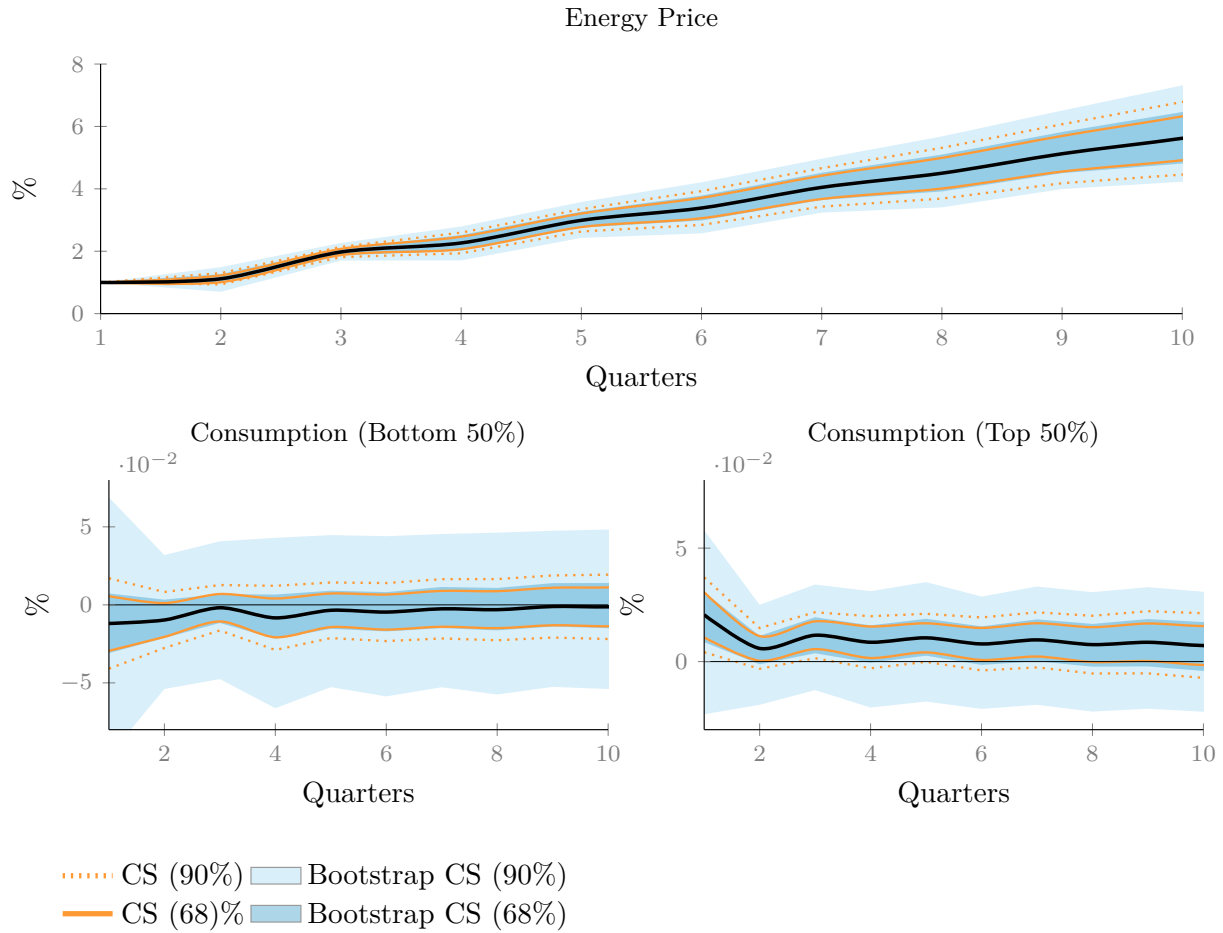
As the main focus of this paper is to investigate the heterogeneous impacts of carbon pricing on households, we use the quarterly Consumer Expenditure Surveys (CES), which

provide detailed data on household consumption baskets and income levels. The CES provide data on locations of participants, so we focus on California (as it is the main carbon market in the U.S.) and expand the data to five years prior to the first future carbon contracts in order to have sufficient data points to conduct our inferences (Q1-2006 to Q4-2019). The instrument values are set to zero for all quarters prior to 2012 (the time at which futures carbon contracts are available) as argued by [Känzig, 2021]. We follow the same methodology as in the previous section to construct our carbon instrument, with the only difference being that we aggregate Sentometric climate news over quarters and not months for this exercise.

Figure 2.5 presents the standard inference results. More precisely, it shows the impulse responses (IRFs) to the identified carbon policy shock, normalized to increase the energy price by one percent on impact. The solid black lines represent the estimated paths. The shaded blue areas are the 68 and 90 percent confidence bands, while the orange solid and dotted lines are the 68 and 90 percent confidence bands using bootstrapping procedures as was the case for the first IV-SVAR model presented above. The standard errors are computed with Newey-West four lags to account for potential auto-correlation within quarters. We also include two lags in the VAR.⁷¹

⁷¹The results are robust even when we include 4 lags.

FIGURE 2.5. IV-SVAR Consumption Qunatiles



Note: The figure presents the cumulative impulse responses of bottom 50 percent income household versus top 50 percent income household located in California to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this exercise, the carbon shock is constructed excluding including all days with zero or no news. The inference is conducted using robust bootstrapping procedure. We conduct to IV-SVAR separate regressions for each income quantile.

Results show that a carbon policy shock leads to a persistent increase in energy prices, which triggers an asymmetric consumption reaction. The bottom 50 percent income households see their consumption fall, while the top 50 percent income households experience a rise in their consumption before it falls back to its steady state level.

Both from a statistical and economic perspective, the results are significant (at 68

percent). As shown by the confidence intervals, the directions of the effects are clearly identified. Two main reasons could explain the small magnitude of the results. First, California recycles its revenues from carbon pricing and redistributes a part of it (35 percent) to low income households, which could smooth the carbon price shocks transmitted via the energy price increase. However, as we do not have access to such fiscal data, we cannot control for the impacts of the redistribution. Second, to conduct our inference, we included a period of five years where there was no carbon pricing market in place. As mentioned above, the instrument values are set to zero for all quarters prior to 2012. Under such a framework, the carbon price instrument contains multiple zeros, which could result in biasing estimates downward.

2.3 The Theoretical Model

Building on [Golosov et al., 2014], [Dietz and Venmans, 2019] and [Achdou et al., 2022], we develop a heterogeneous agent climate model with two production sectors. Accordingly, where [McKay et al., 2016] and [Kaplan et al., 2018], among others, rely on *MIT shocks* to analyze the responses of the economy to a monetary shock, we use the same methodology to investigate a carbon price shock as environmental authorities plan far-ahead changes to their tax regulations and/or emission cap system and leave little space for uncertainty.

The modeled economy is characterized by continuous-time and an infinite horizon and is comprised of two types of *firms* (*energy producers and non-energy producers*),⁷² *heterogeneous households*, and a *government*. In this setup, production by firms induces an environmental externality through CO₂ emissions. A damage function relates rising emissions generated by firms' production to a deterioration in global productivity.

We first present the climate dynamics of our economy, and then present the energy firms followed by an explication of the non-energy firms' intermediate and final goods. We

⁷²Both type of firms are infinitely lived and of measure one.

thereafter present the household problem, and the government policy framework.

2.3.1 Climate Dynamics

As highlighted in the standard integrated assessment models (IAMs) (see [Nordhaus, 1991]), a large part of the accumulation of CO₂ and other Greenhouse Gases (GHGs) in the atmosphere results from the human activity of economic production. Following recent work by [Dietz and Venmans, 2019], we describe the concentration process of Carbon Dioxide X_t in the atmosphere as follows:

$$\dot{X}_t = E_t + E_t^{Row}, \quad (2.10)$$

where $X_{2020} = 840$ is the initial value of emissions stock in GTons of CO₂ and $E_t \geq 0$ is the inflow of Greenhouse Gases at time t , and E_t^{Row} is the inflow of the rest of the world's emissions.⁷³

The total level of emissions flow is a sum of all emissions of its j firms of its s sectors:

$$E_t = \sum_s E_t^s = \sum_s \int_0^1 e_{j,t}^s dj, \quad (2.11)$$

with $e_{j,t}^s$ being the emissions flow of firm j of sector s . In our framework, the total emissions flow reads as:

$$E_t = E_t^y + E_t^e = \int_0^1 e_{j,t}^y dj + \int_0^1 e_{j,t}^e dj, \quad (2.12)$$

where $e_{j,t}^y$ are emissions from non-energy firms and $e_{j,t}^e$ are emissions from energy firms.

In addition, we define the relationship between the temperature vector T_t° and the stock

⁷³ E_t^{Row} is assumed to evolve similarly to domestic emissions. This assumption implies international cooperation and is important to achieve the climate target of staying below 2C degrees.

of emissions in the atmosphere X_t as follows:⁷⁴

$$\dot{T}_t^o = \phi_1(\phi_2 X_t - T_t^o), \quad (2.13)$$

with ϕ_1 and ϕ_2 representing the climate sensitivity parameters.

The impact of global warming on the economy is reflected by a convex damage function of temperature in the atmosphere. This is a standard feature of the IAMs family:

$$d(T_t^o) = ae^{-b(T_t^o)^2}, \quad (2.14)$$

with a and b the two parameters shaping climate damages.

2.3.2 Non-Energy Firms

The non-energy production sector is comprised of final and intermediate firms. We first present the final before turning to the intermediate firms.

2.3.2.1 The Final Firms

Our representative final firms produce a final good Y_t in a competitive sector, which is an aggregate of intermediate firms output $y_{j,t}$ (where $j \in (0, 1)$ is the continuum of intermediate firms):

$$Y_t = \int_0^1 \left(y_{j,t}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}}, \quad (2.15)$$

where $\theta \in (1, \infty)$ is the elasticity of substitution between the intermediate goods. Final firms in the model are looking for profit maximization (in nominal terms), at a given price P_t , subject to the intermediate goods j prices $p_{j,t}$. The first order condition for the final

⁷⁴In our setup T_t^o represents the atmospheric temperature level. As a robustness exercise we model the climate following the three boxes framework as in [Cai and Lontzek, 2019] (please refer to Appendix section 2.B for more details). The results remain similar to the [Dietz and Venmans, 2019] specification.

firm profit maximization problem yields:

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t. \quad (2.16)$$

Under perfect competition and free entry, the price of the final good denoted as P_t is expressed with respect to the intermediate firm price $p_{j,t}$:

$$P_t = \left(\int_0^1 p_{j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (2.17)$$

2.3.2.2 Intermediate Firms and Environmental Externality

Our intermediate representative firm j produces goods using a standard Cobb-Douglas function with climate damages. It seeks profit maximization by making a trade-off between, on one hand, the desired level of capital, labor, and energy, subject to climate damages, and on the other hand, the price of energy paid to energy producers, capital and abatement investment, as well as the cost implied by the environmental policy paid to the regulator. The production function reads as:

$$y_{j,t} = A_t d(T_t^o) (k_{j,t}^y)^{\alpha_1} (e_{j,t}^n)^{\alpha_2} (l_{j,t}^y)^{1-\alpha_1-\alpha_2}, \quad (2.18)$$

where α_1 and α_2 are the elasticities of output with respect to capital $k_{j,t}^y$ and energy $e_{j,t}^n$, A_t is the TFP,⁷⁵ $k_{j,t}^y$ the capital used by intermediate firms, $e_{j,t}^n$ the level of energy demand, and $l_{j,t}^y$ the effective units of labour input. In our framework, firms' productivity is subject to climate dynamics. As in the real business cycle model presented in [Goloso et al., 2014], the environmental externality constrains the Cobb-Douglas production function of the firms, where the emissions feedback deteriorates the environment and alters the production possibilities for firms. However, we differ from [Goloso et al., 2014] by

⁷⁵In the context of the net-zero quantitative simulations, $A_t = \gamma A_{t-1}$ where γ is the exogenous growth rate of the economy.

incorporating damages from the stock of emissions through the level of temperature as outlined in [subsection 2.3.1](#).

Economic production results in emission flows of CO₂, which is modeled as follows:

$$e_{j,t}^y = (1 - \mu_{j,t}^y) \varphi_t^y y_{j,t}, \quad (2.19)$$

where $e_{j,t}^y$ represents the emissions flow generated by firm j , and $0 \leq \mu_{j,t}^y \leq 1$ the fraction of emissions abated by firms.

This functional form for emissions allows us to take into account the high-frequency variations in CO₂ emissions. The term φ_t^y denotes the total inflow of emissions resulting from production, prior to abatement. In this expression, $\varphi_t^y = \bar{\varphi}^y \Omega_t$ with $\bar{\varphi}^y$ being the carbon-intensity parameter that pins down the steady-state ratio of emissions-to-output, while Ω_t represents a trend in the emissions intensity, which captures the decoupling of emissions to output that results from technological improvements.

Furthermore, intermediate firm j incurs a cost $F(\mu_{j,t})$ for every emission unit abated, where $\mu_{j,t}$ is the abatement level. Following [[Nordhaus, 2008](#)], abatement costs read as follows:

$$F(\mu_{j,t}^y) = f(\mu_{j,t}^y) y_{j,t}, \quad (2.20)$$

where

$$f(\mu_{j,t}^y) = \theta_1 (\mu_{j,t}^y)^{\theta_2}, \quad \theta_1 > 0, \theta_2 > 1, \quad (2.21)$$

with θ_1 and θ_2 shaping the cost of abatement for the non-energy sector.

The profits of the representative intermediate firm $\Pi_{j,t}^F$ will thus be impacted by the presence of the environmental externality. As the firm do not internalize its impacts, the regulator then imposes an environmental policy, which forces the firm to engage in abatement efforts, as otherwise it would pay a carbon price to the regulator with respect to its emissions level. The revenue is the real value of intermediate goods $y_{j,t}$, while the cost arises from the following: energy needed $e_{j,t}^n$ in the production cycle, the capital investment

level $i_{j,t}^y$, wages w_t^y paid to the labor force $l_{j,t}^y$, abatement effort $\mu_{j,t}^y$, and the environmental carbon price τ_t^y . The profit equation reads as:

$$\begin{aligned}\Pi_{j,t}^F &= \frac{p_{j,t}}{P_t} y_{j,t} - w_t^y l_{j,t}^y - i_{j,t}^y - p_t^e e_{j,t}^n - f(\mu_{j,t}^y) y_{j,t} - \tau_t^y e_{j,t}^y \\ &= \left(\frac{p_{j,t}}{p_t} - mc_{j,t} \right) y_{j,t},\end{aligned}\tag{2.22}$$

subject to,

$$\dot{k}_{j,t}^y = i_{j,t}^y - \delta k_{j,t}^y,\tag{2.23}$$

$$y_{j,t} = A_t d(T_t^o) (k_{j,t}^y)^{\alpha_1} (e_{j,t}^n)^{\alpha_2} (l_{j,t}^y)^{1-\alpha_1-\alpha_2},\tag{2.24}$$

with p_t^e the price paid to the energy firms for energy supplied.

Given a price, and subject to the demand constraint, the cost-minimization problem yields the real marginal cost, which can be expressed following the first-order conditions with respect to the firm's optimal choice of energy, capital investment, labour, and abatement investment, respectively:

$$p_t^e = \varrho_t^y \alpha_2 \frac{y_t}{e_t^n},\tag{2.25}$$

$$r_t^y = \varrho_t^y \alpha_1 \frac{y_t}{k_t^y} - \delta,\tag{2.26}$$

$$w_t^y = \varrho_t^y (1 - \alpha_1 - \alpha_2) \frac{y_t}{l_t^y},\tag{2.27}$$

$$\tau_t^y = \frac{f(\mu_t^y)'}{\varphi_t^y},\tag{2.28}$$

where $\varrho_{j,t}^y = \varrho_t^y$ is the marginal cost component related to the same capital demand all firms choose. This price component is common to all intermediate firms as they are identical.

Equation (2.28) is the optimal condition on abatement: abating CO₂ emissions is optimal when its marginal gain equals its marginal cost. As in [Benmir and Roman, 2020], this equation highlights the key role of the carbon price in shaping firms' decisions. In addition,

abatement efforts μ_t^y are common to all firms of the same sector, as the environmental cost is also common to all firms of the same sector. Furthermore, as the impact of the environmental externality is not internalized by firms (i.e. they take X_t and T_t^o as given), the shadow value of the environmental externality is zero under the laissez-faire. This means firms will have no incentive to engage in abatement effort and emission reduction.

In addition, we can express the total marginal cost as the sum of input cost, abatement cost, and net abatement carbon price:

$$mc_{j,t} = mc_t = \varrho_t^y + f(\mu_t^y) + \varphi_t^y \tau_t^y (1 - \mu_t^y), \quad (2.29)$$

When prices are flexible (i.e monetary neutrality),⁷⁶ the only distortion in our framework arises from monopolistic competition. Using [equation \(2.22\)](#) as well as [equation \(2.16\)](#), we can write the marginal cost and firm's profit as follows:⁷⁷

$$mc_t = \frac{\theta - 1}{\theta}, \quad (2.30)$$

$$\Pi_t^F = (1 - mc_t)Y_t. \quad (2.31)$$

2.3.3 The Energy Producers

Energy producers provide energy resources $e_{j,t}^n$ to the intermediate firms $y_{j,y}$ by relying on physical capital $k_{j,t}^n$, and labour $l_{j,t}^n$. They produce energy using a Cobb-Douglas production function:

$$e_{j,t}^n = A_t^n (k_{j,t}^n)^{\alpha_n} (l_{j,t}^n)^{1-\alpha_n}, \quad (2.32)$$

with α_n the elasticity of energy production to capital $k_{j,t}^n$, and $l_{j,t}^n$ the fraction of labour used by the energy sector.

Producing energy generates a level of emissions, which if not abated (i.e. made greener),

⁷⁶In [section 2.6](#) we investigate the case where prices are sticky (i.e. the New Keynesian Heterogeneous Agent framework–HANK).

⁷⁷Refer to appendix [section 2.C.2](#) for full derivations.

is costly for the energy producers. The emissions level is modeled by a nonlinear technology similar to the one used by the intermediate non-energy firms that allows for reducing the inflow of emissions. The emission flow of CO₂ from energy producers ($e_{j,t}^e$) reads as:

$$e_{j,t}^e = (1 - \mu_{j,t}^n) \varphi_t^n e_{j,t}^n. \quad (2.33)$$

As is the case for the intermediate firms, $0 \leq \mu_t^n \leq 1$ is the fraction of emissions abated by energy firms. The energy firm will face an abatement investment technology similar to the non-energy firms $F(\mu_{j,t}^n) = \theta_1(\mu_{j,t}^n)^{\theta_2}$ for every emission unit abated,⁷⁸ where $\mu_{j,t}^n$ is the abatement level of the energy firm. φ_t^n is the carbon-intensity function for the energy sector and follows a similar law of motion as the non-energy firms. Similarly, the trend in the carbon-intensity process of non-energy firms allows us to match the empirical sectoral decoupling in the U.S.

Again, similar to non-energy firms, the profits of the representative energy firm $\Pi_{j,t}^E$ will be impacted by the presence of the environmental policy. In this case, revenues are the real value of energy production $e_{j,t}^n$, while the costs arise from investment $i_{j,t}^n$ in physical capital $k_{j,t}^n$, wages w_t^n paid to the labor force $l_{j,t}^n$, and the abatement investment $\mu_{j,t}^n$, as well as the environmental carbon price τ_t^n . The profit equation reads as:

$$\Pi_{j,t}^E = p_t^e e_{j,t}^n - w_t^n l_{j,t}^n - i_{j,t}^n - f(\mu_{j,t}^n) e_{j,t}^n - \tau_t^n e_{j,t}^n, \quad (2.34)$$

where

$$\dot{k}_{j,t}^n = i_{j,t}^n - \delta k_{j,t}^n, \quad (2.35)$$

$$e_{j,t}^n = A_t^n (k_{j,t}^n)^{\alpha_n} (l_{j,t}^n)^{1-\alpha_n}. \quad (2.36)$$

⁷⁸We assume both sectors use the same abatement technology (i.e. the same abatement cost function). While abatement cost functions are assumed to be the same across sectors, the total abatement cost will be different across the two sectors.

Subject to the demand constraint, the cost-minimization problem yields the real marginal cost, which can be expressed following the first-order conditions with respect to the energy firm's optimal choice of energy prices, capital, abatement investments, and the environmental policy cost, as well as the energy firm's optimal choice of labour, respectively:

$$p_t^e = \varrho_t^e + f(\mu_t^n) + \varphi_t^n \tau_t^n (1 - \mu_t^n), \quad (2.37)$$

$$r_t^e = \varrho_t^e \alpha_n \frac{e_t^n}{k_t^n} - \delta, \quad (2.38)$$

$$\tau_t^n = \frac{f(\mu_{j,t}^n)'}{\varphi_t^n}, \quad (2.39)$$

$$w_t^n = \varrho_t^e (1 - \alpha_n) \frac{e_t^n}{l_t^n}, \quad (2.40)$$

where $\varrho_{j,t}^e = \varrho_t^e$ is the marginal cost component related to the same capital demand of all energy firms.

Equation (2.39) is the optimal condition for abatement in the energy sector: abating CO₂ emissions is optimal when marginal gain equals marginal cost. In addition, abatement effort μ_t^n is common to all energy firms as highlighted in the previous section.

2.3.4 Households

The household problem is approached using a CRRA utility function,⁷⁹ whereby households that are heterogeneous in their wealth a and income y , choose consumption expenditures c_t .

$$\max_{\{c_t\}} E_0 \int_0^\infty e^{-\rho t} u(c_t) dt, \quad (2.41)$$

where $\rho \in [0, 1]$ is the time discount factor.

The household budget constraint reads:

$$\dot{a}_t = r_t^a a_t + w_t^y z_t^y + w_t^n z_t^n + \frac{z_t^y}{\bar{z}} \Pi_t^F + T_t - c_t, \quad (2.42)$$

⁷⁹ $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$.

where a_t is the households' choice of asset with r_t^a the interest rate. z_t^y is the vector of labour productivity for households working for non-energy firms, while z_t^n is the corresponding vector of labour productivity for household working for energy firms. For simplicity, income is assumed to have two states $z_t^s \in \{z_1, z_2\}$ for each sector $s \in (y, n)$ and to follow similar Poisson processes with intensities $\lambda(jj')$,⁸⁰ while w_t^y and w_t^n are wages for both non-energy and energy labour (l_t^y and l_t^n), which is supplied inelastically by households. Finally, profits from monopolistic intermediate firms are transferred to households proportional to their income productivity levels z_t^y as in [Ahn et al., 2018], where \bar{z} is the average productivity.⁸¹

Individuals also face a borrowing constraint:

$$a_t \geq \underline{a}, \quad (2.43)$$

where $-\infty < \underline{a} < 0$.

Given this model setup, individual consumption–saving decisions and the evolution of the joint distribution of their income and wealth can be summarized with two differential equations: a HJB equation and a Kolmogorov Forward (or Fokker–Planck) equation:

$$\begin{aligned} \rho v(a, z_j^y, z_j^n, t) = & \max_c u(c) + \partial_a v(a, z_j^y, z_j^n, t)(r(t)^a a + w(t)^n z_j^n + w(t)^y z_j^y + \frac{z_j^y}{\bar{z}} \Pi_t^F + T - c) \\ & + \sum_{j'} \lambda_{jj'} v(a, z_{j'}^y, z_{j'}^n, t) + \partial_t v(a, z_j^y, z_j^n, t), \end{aligned} \quad (2.44)$$

and

$$\partial_t g(a, z_j^y, z_j^n, t) = -\partial_a [s(a, z_j^y, z_j^n, t)g(a, z_j^y, z_j^n, t)] + \sum_{j'} \lambda_{j'j} g(a, z_{j'}^y, z_{j'}^n, t). \quad (2.45)$$

with the first order condition yielding the optimal consumption sequence $c(a, z_j^y, z_j^n, t) = u'^{-1}(\partial_a v(a, z_j^y, z_j^n, t))$.

⁸⁰In our setup $j = 1, 2$. As in [Ahn et al., 2018] we adopt the convention that $j = 1$ and $j' = 2$.

⁸¹This is meant to minimize the redistribution implied by cyclical fluctuations in profits.

2.3.5 Public Authorities

The public authority could set a carbon price (equation (2.46)) to meet the net-zero objectives as follows:

$$\tau_t^s = \text{Carbon Price}_t^s, \quad (2.46)$$

where Carbon Price_t^s is the price level for the energy and non-energy sectors $s \in \{y, n\}$ that the public authority sets.

Or alternatively/equivalently,⁸² the public authority could chose to follow an emission cap system, where it sets an emission cap as follows:

$$E_t^s = \text{Carbon Cap}_t^s, \quad (2.47)$$

with $\text{Carbon Cap}_{t=0}^s$ the actual emission level and $\text{Carbon Cap}_{t=2050}^s$ the net-zero objective (i.e. $\text{Carbon Cap}_{t=2050}^s = 0$). This cap then implies a price for carbon, depending on the level of production, the abatement cost, and carbon intensity.

The government uses the environmental policy revenues $\tau_t^s E_t^s$ to finance exogenous expenditures G_t and transfers to households T_t . The public authority budget constraint reads as:

$$G_t + T_t = \sum_s \tau_t^s E_t^s, \quad (2.48)$$

with $\sum_s \tau_t^s E_t^s = \int_0^1 (\tau_t^e e_{j,t}^y + \tau_t^n e_{j,t}^e) dj$.

2.3.6 No Arbitrage

Households hold all assets in the economy and thus are subject to a unique asset return r_t^a . Both the no-arbitrage condition and the share of capital between sectors yield

⁸²Under an equivalent calibration.

the capital level invested in each sector (i.e. the energy and non-energy sectors):

$$r_t^a = r_t^y = r_t^e, \quad (2.49)$$

and

$$K_t = K_t^y + K_t^n, \quad (2.50)$$

where K_t^y and K_t^n are the aggregate capital stock in each sector.

2.3.7 Equilibrium and Market Clearing

An equilibrium in this framework is defined as pathways for individual household and firm decisions $\{a_t, c_t, l_t^y, l_t^n, e_t^n, k_t\}_{t \geq 0}$, input prices $\{w_t^y, w_t^n, p_t^e\}_{t \geq 0}$, returns on assets $\{r_t^a\}_{t \geq 0}$, fiscal variables $\{T_t, G_t, \tau_t\}_{t \geq 0}$, measures $\{\nu_t\}_{t \geq 0}$, and aggregate quantities such that, at every time t : (i) households and both types of firms maximize their objective functions taking as given equilibrium prices, taxes, and transfers; (ii) the sequence of distributions satisfies aggregate consistency conditions; (iii) the government budget constraint holds; and (iv) all markets clear. There are three markets in our economy: the market for capital of energy and non-energy firms (that can be glossed as a single asset), the labor market, and the goods market.

The asset market clears when physical capital K_t equals households' holdings of assets $A_t = \int a d\nu_t$,

$$K_t = A_t. \quad (2.51)$$

The labor market clears as follows:

$$L_t^s = \int z^s l_t^s(a, z^y, z^n) d\nu_t, \quad (2.52)$$

where s represents our two sector (i.e. energy and non-energy).

The goods market clearing condition reads as:

$$Y_t = C_t + I_t + G_t + \sum_s F(\mu_t^s), \quad (2.53)$$

where Y_t is the aggregate output, C_t is total consumption expenditure, $I_t = I_t^y + I_t^n$ aggregate investment in total capital K_t . $F(\cdot) = \int_0^1 f(\cdot) dj$ is the aggregate abatement cost for each sector s .

2.4 Solution Method

In this section, we describe the general solution framework necessary to solve our model. We then detail the custom MATLAB algorithm we developed to address specific issues related to having climate dynamics in the model.

2.4.1 Method

To solve our heterogeneous-agent model, the first step is to find a stationary equilibrium. The consumer's problem is solved on a grid using finite differences à la [Achdou et al., 2022]. We discretize time in addition to wealth and income. The income process follows a two state Poisson and we construct a linearly-spaced asset grid with 201 points. The dynamic programming problem is then solved by evaluating the value function using an upwind scheme finite difference method.⁸³

Stationary equilibrium:

A stationary recursive competitive equilibrium is defined as:

1. Value and policy functions: $v(a, z^y, z^n)$, $c(a, z^y, z^n)$, and $s(a, z^y, z^n)$

⁸³For further details about the method, please refer to [Achdou et al., 2022].

2. Factor demands: K and L^s
3. Distribution of household wealth: $g(a, z^y, z^n)$
4. Prices: r^a , p^e , w^y , and w^n

such that:

1. Given a set of prices r^a , w^y , and w^n , the value function $v(a, z^y, z^n)$ solves the household problem, thus satisfying the HJB equation:

$$\rho v(a, z_j^y, z_j^n) = \max_c u(c) + \partial_a v(a, z_j^y, z_j^n)(r(t)^a a + w^n z_j^n + w^y z_j^y + \frac{z_j^y}{\bar{z}} \Pi^F + T - c) + \sum_{j'} \lambda_{jj'} v(a, z_{j'}^n, z_{j'}^y)$$

on (\underline{a}, ∞) and for $j \in (1,2)$, which implies policy and saving functions:

$$c(a, z^y, z^n) = (u')^{-1}(\partial_a v(a, z^y, z^n)) \text{ and } s(a, z^y, z^n) = (1 + r^a)a + w^y z^y + w^n z^n + T + \frac{z^y}{\bar{z}} \Pi^F - c(a, z^y, z^n)$$

2. Given the prices r^a , p^e , w^y , and w^n , the factor demands K and L^s , solve the intermediate and energy firms first order conditions,
3. Given the saving policy function $s(a, z^y, z^n)$, the distribution $g(a, z^y, z^n)$ satisfies the stationary Kolmogorov Forward equation:

$$0 = -\partial_a [s(a, z_j^y, z_j^n) g(a, z_j^y, z_j^n)] + \sum_{j'} \lambda_{j'j} g(a, z_j^y, z_j^n)$$

on (\underline{a}, ∞) and for $j \in (1,2)$,

4. Given the distribution $g(a, z, z^n)$, the markets for capital and labor clear:

$$\sum_j \int_{\underline{a}}^{\infty} a g(a, z_j^y, z_j^n) da = K \text{ and } \sum_j z_j^s f_j^s = L^s.$$

Transition dynamics:

Turning now to the transition dynamics, we define the time-dependent recursive competitive equilibrium as:

1. Value and policy functions: $v(a, z^y, z^n, t)$, $c(a, z^y, z^n, t)$, and $s(a, z^y, z^n, t)$
2. Factor demands: $K(t)$ and $L(t)^s$
3. Distribution of household wealth: $g(a, z^y, z^n, t)$
4. Prices: $r^a(t)$, $p(t)^e$, $w(t)^y$, and $w(t)^n$

such that:

1. Given a set of prices $r(t)^a$, $w(t)^y$, and $w(t)^n$, as well as a terminal condition for the value function $v_\infty(a, z_j^y, z_j^n)$, the value function $v(a, z^y, z^n, t)$ solves the dynamic household problem, and satisfies the HJB equation:

$$\begin{aligned} \rho v(a, z_j^y, z_j^n, t) = & \max_c u(c) + \partial_a v(a, z_j^y, z_j^n, t)(r(t)^a a + w(t)^n z_j^n + w(t)^y z_j^y + \frac{z_j^y}{z} \Pi_t^F + T_t - c) \\ & + \sum_{j'} \lambda_{jj'} v(a, z_{j'}^y, z_{j'}^n, t) + \partial_t v(a, z_j^y, z_j^n, t) \end{aligned}$$

with the terminal condition $\lim_{T \rightarrow \infty} v(a, z_j^y, z_j^n, T) = v_\infty(a, z_j^y, z_j^n)$

2. Given the prices $r(t)^a$, $p(t)^e$, $w(t)^y$, and $w(t)^n$, the factor demands $K(t)$ and $L(t)^s$ solve the intermediate and energy firms first order conditions,
3. Given the saving policy function $s(a, z^y, z^n, t)$ and the initial distribution $g_0(a, z_j^y, z_j^n)$, the distribution $g(a, z^y, z^n, t)$ satisfies the dynamic Kolmogorov Forward equation:

$$\partial_t g(a, z_j^y, z_j^n, t) = -\partial_a [s(a, z_j^y, z_j^n, t) g(a, z_j^y, z_j^n, t)] + \sum_{j'} \lambda_{j'j} g(a, z_{j'}^y, z_{j'}^n, t)$$

with an initial condition on the distribution $g(a, z_j^y, z_j^n, t) = g_0(a, z_j^y, z_j^n)$,

4. Given the distribution $g(a, z^y, z^n, t)$, the markets for capital and labor clear:

$$\sum_j \int_{\underline{a}}^\infty a g(a, z_j^y, z_j^n, t) da = K(t) \text{ and } \sum_j z_j^s f_j^s = L(t)^s.$$

2.4.2 Solution Algorithm under Climate Dynamics

Contrary to standard models with idiosyncratic income risk, climate dynamics in our model imply different methods for finding the initial and final steady states. With the initial and final steady states in hand, we proceed to compute transition pathways following MIT shocks. In what follows, we rely on [Achdou et al., 2022] for solving the HJB and Kolmogorov Forward equations and adapt their method to our [Aiyagari, 1994] framework with two production sectors and an environmental externality.

Initial state

For the initial steady state, the procedure is fairly standard, as emissions and temperature are fixed at the current level. Compared to the [Aiyagari, 1994] framework, however, our model features two types of capital. While looping over values for aggregate capital, we exploit the no-arbitrage condition and build an inner loop where we guess a share of aggregate capital going to the energy sector. We then use firms' first order conditions to ensure that returns on capital in both sectors are equal (i.e. the share of capital guessed clear the no-arbitrage condition), before aggregating household wealth and checking that our market clearing conditions hold.

Final state

For the final state, the presence of climate dynamics complicates the search for a fixed point (i.e. the final steady state level of temperature and stock). To understand why, consider [equation \(2.13\)](#) evaluated at the steady state:⁸⁴

$$\bar{T}^o = \phi_2 \bar{X}. \tag{2.54}$$

⁸⁴ \bar{T}^o and \bar{X} represent the steady state values.

While the parameter ϕ_1 does not appear in the steady state equation, it plays an important role in temperature dynamics over the transition. It is also not possible to know the terminal value of X without knowing the path of emissions over the period studied. To address these issues, we compute a synthetic path for emissions consistent with the Representative Concentration Pathway (RCP) 8.5 scenario,⁸⁵ which allows us to get the terminal value of X and T . With the value of temperature at the final state, we are then able to compute the remaining terminal values within the inner loop used to find the level of capital in each sector.

Transition dynamics

For transition dynamics, we rely on the same method developed for finding the final state of the economy. The only difference is that we now need to find the full path of all the endogenous variables. To do so, we use a vector of synthetic emissions fitted to the studied scenario to retrieve the complete path of temperature. We then derive the vector of output subject to climate damages. The remaining part of the procedure is standard.

2.4.3 Calibration

The model is calibrated on U.S. data. While we do not have two assets (liquid and illiquid as in [Kaplan et al., 2018]), which would otherwise allow for a refined representation of U.S. households portfolios, we calibrate income shocks to retrieve a realistic distribution of wealth.⁸⁶ The wide range of assets found in the economy is represented in our model as a generic productive asset that households hold and are allowed to borrow. We set the borrowing constraint \underline{a} to a value corresponding to roughly one year of average wages. For simplicity, the income process within each sector follows a two-state Poisson, representing

⁸⁵In RCP 8.5 emissions continue to rise throughout the 21st century.

⁸⁶The scope of our paper being the transition to net-zero, we are more interested in the dynamics of the distribution rather than the initial steady state.

high and low income realizations. The productivity of high earners compared to low earners is proportional across sectors.

For parameters related to standard macroeconomic theory, their calibration is in line with the literature: the share of hours worked is set at one third in each sector and the coefficient of relative risk aversion σ in the CRRA utility function is set at 2. Discount rate ρ is set at 5 percent to target an interest rate of about 4 percent annually. The depreciation rate of capital δ is calibrated at 5 percent annually. Turning to the production sector, the elasticity of substitution θ is set at 6, leading to a markup of around 17 percent. The non-energy sector relies on three inputs. We set α_1 and α_2 to target an energy production to total output ratio of 4-5 percent. The share of labour in production for non-energy firms is set at 0.66, while the share of capital α_1 is set at 0.19, and the share of energy α_2 at 0.15. We use sectoral data on the U.S. to set the share of the energy sector α_n at two thirds, which allows us to recover the share of wages from the energy sector with respect to the non-energy sector, and to account for large investment needed in this sector. These calibrations lead to an average labor share of 57 percent and an average capital and profit share of 26 percent.

Regarding environmental components, we calibrate the damage function according to [Dietz and Stern, 2015]. The global temperature parameters ϕ_1^o and ϕ_2^o are set following [Dietz and Venmans, 2019] to pin down the ‘initial pulse-adjustment timescale’ of the climate system.⁸⁷ Abatement parameters θ_1 and θ_2 , which represent the abatement costs for each sector, are borrowed from [Nordhaus, 2008].⁸⁸ To match the U.S. level share of emissions from each sector (25 percent of total emissions generated by the energy sector), we calibrate the emission-to-sectoral-production ratio $\bar{\varphi}^y$ and $\bar{\varphi}^n$ to 2 and 0.3 respectively. Finally, the decoupling rate of emissions is calibrated to 1 percent to match U.S. Energy

⁸⁷We perform a sensitivity analysis on the damage function using values from [Nordhaus and Moffat, 2017] and [Weitzman, 2012], in the next section. We also perform a robustness analysis on climate sensitivity using various values of ϕ_2^o .

⁸⁸We assume that firms from both sectors have access to the same abatement technology. We also perform sensitivity analysis on the efficiency of abatement technology in the next section.

Information Administration (EIA) data.

TABLE 2.1
Model Matching Moments

Target	Model	Data	Source
<u>Macro Aggregates:</u>			
Labor Share	0.567	0.597	FRED (2019)
Capital Share	0.260	0.311	BEA (2020)
<u>Environmental Aggregates:</u>			
Global Level of Carbon Stock (GtC)	840	840	USDA (2020)
Temperature °C (in excess to pre-industrial level)	1.15	1.19	NOAA (2020)
Share of Emissions from Energy	0.25	0.25	EIA (2020)
Share of Emissions from Non-Energy	0.75	0.75	EIA (2020)
Emissions Decoupling Rate	0.01	0.01	EIA

2.5 Net-Zero Transition Results

2.5.1 Understanding the Impact of Carbon Pricing under Heterogeneous Agents

In this section we will investigate the impact of putting a price on carbon in an economy with idiosyncratic income risk. Using our model, we compute the transition following an MIT shock under three different scenarios that all trigger a 25-percent reduction in total emissions.⁸⁹ Our main scenario relies on carbon taxation on both the energy and the non-energy firms. We also assess how solely taxing either energy firms or non-energy firms

⁸⁹We conduct this exercise by setting the carbon price $\tau_t^s = \varepsilon_t^s$ following a bounded Ornstein-Uhlenbeck process: $d\varepsilon_t^s = \theta^s(\hat{\varepsilon}^s - \varepsilon_t^s)dt + \sigma^s dB_t$. B_t is a F_t -adapted idiosyncratic Brownian motion and θ^s , $\hat{\varepsilon}^s$, and σ^s are positive constants. $s \in \{n, y\}$ represents the energy and non-energy sectors.

would change the outcome of the policy. We then disentangle theoretically and compute numerically how pricing carbon on the firm side ultimately affects household consumption according to level of income and wealth.

2.5.1.1 Energy Sector, Carbon Pricing, and Macroeconomic Drivers

We first focus on how carbon price shocks, when set at the energy sector level, propagate through the economy and impact macroeconomic prices and aggregates. When the regulator sets a carbon price, energy firms are forced to engage in abatement efforts and to pay a carbon price. In doing so, the demand for energy decreases, which increases energy prices and decreases wages and returns. This result holds as long as the drop in energy generation is higher than the total environmental cost. Otherwise, energy prices fall on impact:

$$\underbrace{p_t^e}_{\text{Energy Price}} = \left(\underbrace{mc_t}_{\text{Total Marginal Cost}} - \underbrace{\underbrace{f(\mu_t^y)}_{\text{Abatement Investment}} - \underbrace{\tau_t^y(e_t^y/y_t)}_{\text{Emission Intensity Carbon Price}}}_{\text{Total Environmental Costs}} \right) \alpha_2 \frac{y_t}{e_t^n} \quad (2.55)$$

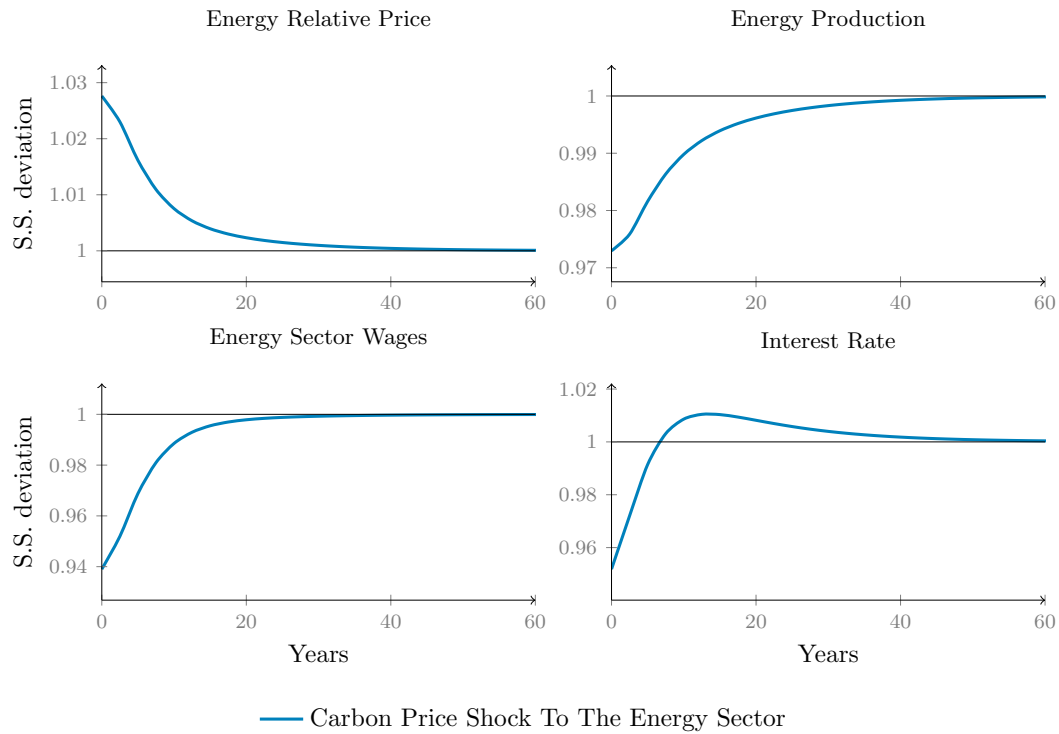
$$r_t^e = \left(\underbrace{p_t^e}_{\text{Energy Price}} - \underbrace{\underbrace{f(\mu_t^n)}_{\text{Abatement Investment}} - \underbrace{\tau_t^n(e_t^e/e_t^n)}_{\text{Emission Intensity Carbon Price}}}_{\text{Total Environmental Costs}} \right) \alpha_n \frac{e_t^n}{k_t^n} - \delta \quad (2.56)$$

$$w_t^n = \left(\underbrace{p_t^e}_{\text{Energy Price}} - \underbrace{\underbrace{f(\mu_t^n)}_{\text{Abatement Investment}} - \underbrace{\tau_t^n(e_t^e/e_t^n)}_{\text{Emission Intensity Carbon Price}}}_{\text{Total Environmental Costs}} \right) (1 - \alpha_n) \frac{e_t^n}{l_t^n} \quad (2.57)$$

As shown in [figure 2.6](#), following a carbon price shock that aims at reducing emissions by 25 percent, energy prices increase, whereas energy generation, as well as wages and returns, decrease. These results are consistent with our empirical findings for the case of

California outlined in the previous empirical section. Energy firms decide to reallocate resources by lowering their capital investment level and decreasing labor wages. Firms then increase energy prices to recover profit loss. This increase in energy prices decreases the intermediate firms' demand for energy and thus decreases the total energy generation level.

FIGURE 2.6. Energy Sector Carbon Pricing and Macroeconomic Aggregates



Note: The figure plots the impulse responses to a carbon shock leading to an initial 25% reduction in total emissions in the energy sector as deviation from a normalized steady state.

In the next exercise, we explore three scenarios where the public authority sets carbon pricing within: i) the energy sector only, ii) the non-energy sector only, and iii) both the energy and non-energy sectors at the same time. These scenarios allow us to understand the implications of different policy design.

2.5.1.2 Carbon Price and Macroeconomic Dynamics

Figure 2.25 displays the reaction of the economy to an introduction of a carbon price that engenders a 25 percent emissions reduction, under our three scenarios. In the case where the price on carbon is only applied to the non-energy sector (dotted red line), this implies cutting emissions in this sector by approximately one third. Whereas, when the price on carbon is only applied to the energy sector (dashed blue line), this represents a reduction of almost all emissions in the energy sector. Since both sectors rely on the same abatement technology, the difference in response is due to the specific role each sector plays in the economy. The energy sector being a small part of the economy, taxing it does not greatly impact the dynamics of the interest rate nor the capital stock, compared to other scenarios. However, as energy producers provide an input for the non-energy sector, implementing a carbon pricing scheme targeted to energy firms still leads to a gradual decline in the capital stock and output, since firms have to bear a higher input cost for energy.

When taxing energy firms as opposed to taxing non-energy firms (and vice versa), the impacts on energy prices are found to be widely different. Our simulations show that when taxing energy firms, on one hand, the increasing cost related to carbon triggers an immediate drop in energy production, implying in turn an increase in the relative price of energy. On the other hand, taxing only non-energy firms reduces the demand for energy and its relative price. Energy producers thus gradually lower their production and the price returns to its initial steady state. The situation on the market for labor also depends on the type of policy implemented. Although both non-energy and energy sector wages fall regardless of the policy implemented, the effect is comparable across scenarios in the energy sector but different in the non-energy sector. Concretely, this means that taxing non-energy firms' carbon emissions transmits to both sectors wages, when a policy implemented only in the energy sector minimally affects wages in the non-energy sector. Overall, the analysis of aggregate variables suggests that implementing a carbon fiscal policy on energy firms

before targeting other firms (as is the case in Californian and European cap-and-trade schemes) is efficient, since it is less costly to first abate emissions from energy production.

2.5.1.3 Carbon Price Transmission Mechanism

When a regulator plans to implement carbon pricing, it is important to understand beforehand how it is going to impact household consumption according to income and wealth level. To uncover the heterogeneous effects of taxing firms' emissions on household consumption, we start by detailing direct and indirect drivers of consumption. This allow us to later tie these drivers of consumption to the empirical findings in [section 2.2](#).

We first decompose the response at time zero of consumption with respect to its main components:

$$C_t(\{\Gamma_t\}_{t \geq 0}) = \int c_t(a, z^y, z^n; \{\Gamma_t\}_{t \geq 0}) d\nu_t \quad (2.58)$$

Here $c_t(a, z^y, z^n; \{\Gamma_t\}_{t \geq 0})$ is the household consumption policy function and $\nu_t(da, dz; \{\Gamma_t\}_{t \geq 0})$ is the joint distribution of illiquid assets and idiosyncratic income.

Following [[Kaplan et al., 2018](#)],⁹⁰ by total differentiation, we can decompose the consumption response at $t = 0$ as:

$$dC0 = \int_0^\infty \left(\frac{\partial C0}{\partial r_t^a} dr_t^a + \frac{\partial C0}{\partial w_t^n} dw_t^n + \frac{\partial C0}{\partial w_t^y} dw_t^y + \frac{\partial C0}{\partial \Pi_t^F} d\Pi_t^F + \frac{\partial C0}{\partial T_t} dT_t \right) dt \quad (2.59)$$

In our framework, and opposite to [[Kaplan et al., 2018](#)], we only have indirect effects of carbon price shocks to consumption, as the carbon pricing scheme studied in this paper only affects firms directly. The implementation of a carbon price will impact consumers through five channels: asset returns, both types of wages, profits, and transfers. Note that we do not consider redistributing proceeds from the carbon tax until [section 2.5.3](#), which means that at present transfers will not impact consumption. Intuitively, and consistent with our empirical findings in [section 2.2](#), a positive shock to the price of carbon should lead

⁹⁰A similar exercise can be found in [[Auclert, 2019](#)].

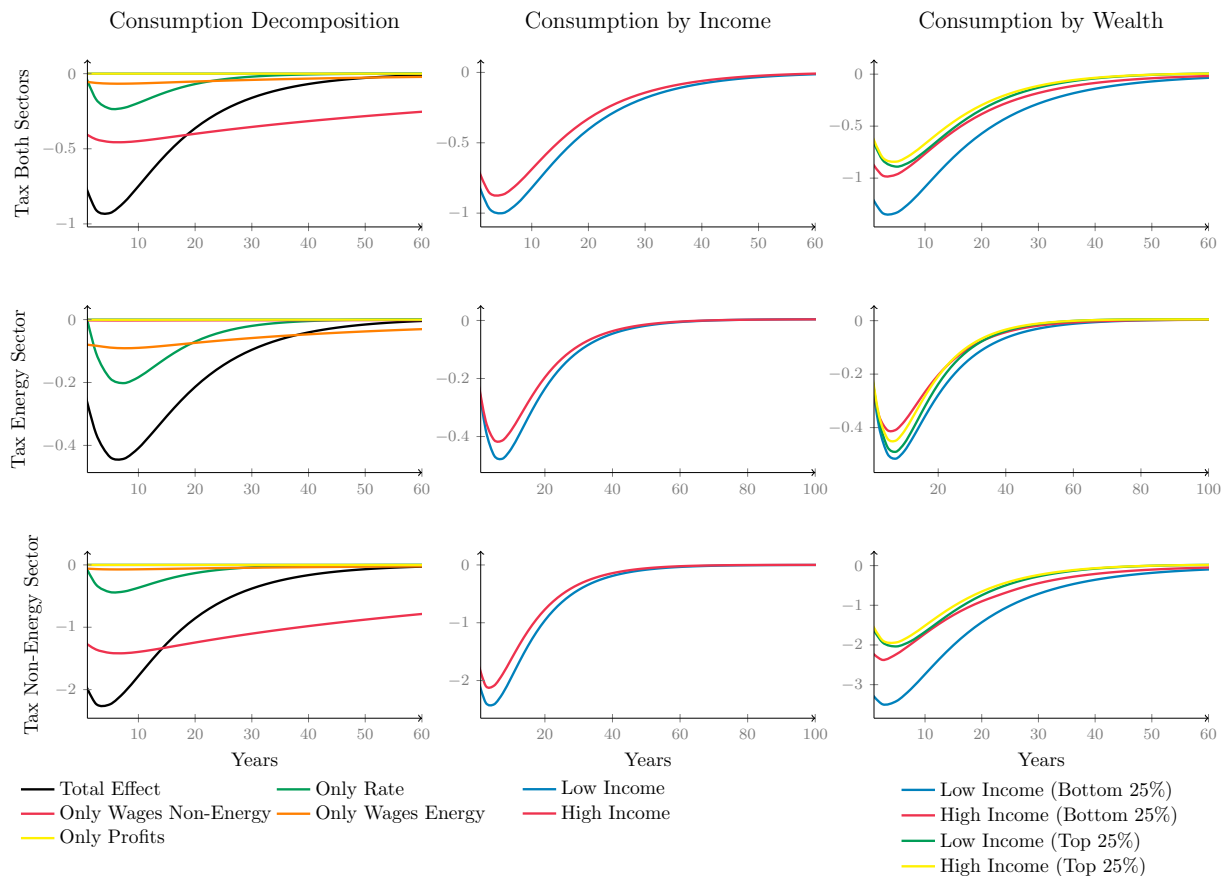
to lower asset returns, wages, and profits, as putting a higher price on carbon implies an additional input cost for firms. These effects should transmit to households and ultimately reduce consumption.

2.5.1.4 Distributional Effects of Carbon Pricing

Figure 2.7 shows the impact on consumption decomposed into various indirect effects, by income, and by wealth for our three carbon pricing scenarios. To decompose the effect, we mute all but the component of interest by setting them to their respective steady state values over the transition. Consistent with our findings on aggregate variables, the first column shows that taxing the energy sector only is the less costly policy in terms of aggregated consumption. The reason is that most of the effect goes through wages in the sector concerned by the price on carbon. Since wages from the energy sector only account for a small part of total wages, their reduction is less detrimental to consumption than a reduction of non-energy wages. As expected, changes in the interest rates put pressure on household revenues from capital, which also induces lower consumption. In the same spirit, [Malafry and Brinca, 2022] use a two-period heterogeneous agent model to disentangle the effect of setting a carbon price on household aggregate welfare.⁹¹ In their setup, however, implementing a tax on carbon always benefits consumption, even without redistributing carbon revenues.

⁹¹The three channels they use are consumption, redistribution, and risk.

FIGURE 2.7. Carbon Price Shock and Consumption Responses



Note: The figure plots the reaction of consumption according to three different scenarios leading to an initial 25% reduction in emissions. The first row corresponds to the case where the tax is implemented in both sectors. The second row corresponds to the case where only the energy sector is taxed. And the last row corresponds to the case where only the non-energy sector is taxed. The first column plots the reaction of consumption as well as its four components. The second column plots the reaction of consumption according to the realization of income. And the last column plots the reaction of consumption according to the realization of income and the level of wealth.

Columns two and three display the distributional impact for our three scenarios. We find that taxing the energy sector only generates less distributional costs than other policies. Not only the aggregate impact on consumption is smaller, but the consumption reaction for low/high income and low/high wealth is quite homogeneous compared to the other two scenarios. In the case where only the non-energy sector is subject to the carbon price, the loss in consumption for low income households that are also at the lower end

of the wealth distribution is twice the loss households at the upper end of the wealth distribution experience. This suggests that policy makers should pay particular attention to distributional effects throughout the transition to net-zero. It is especially true for countries that plan to move from taxing only emissions generated in the energy sector to taxing emissions generated in all sectors.

2.5.2 Net-Zero Transition and Wealth Dynamics

Turning now to net-zero transition dynamics, we present and analyze various scenarios. We first highlight how incorporating climate dynamics and accounting for climate damages have a significant impact on the long-term equilibrium of the model. We also investigate the role that climate sensitivity, damage uncertainty, and abatement efficiency play on laissez-faire and net-zero emissions transitions, respectively. Finally, we show that the speed at which carbon policy is implemented matters for transition dynamics.

The baseline scenario features a trend growth rate of 2 percent annually over the period 2022 to 2100. The growth process is then stopped and we let the model converge to the new steady state. Although we use an average calibration (i.e. consistent with intermediate values found in the literature) for parameters related to climate sensitivity, climate damages, and abatement efficiency, we also provide in the appendix a detailed sensitivity analysis for each of the exercises we perform.

2.5.2.1 Why climate dynamics matters

In this section, we investigate how climate feedback shapes long-term dynamics, both at the aggregate level and at the household level. We compare the simulations of our model to a counterfactual model where we remove the link between temperature and production. The objective is to assess whether ignoring climate dynamics leads to an erroneous view of what might happen to the economy and wealth distribution in the future if no action is taken.

Climate damages and laissez-faire transition pathways

Figure 2.26 and figure 2.27 display transition pathways from our baseline model (with climate damages – green solid line) and from the counterfactual model (without climate damages – brown dashed line). As argued by [Cai and Lontzek, 2019], [Traeger, 2021], and [Van den Bremer and Van der Ploeg, 2021] among others, uncertainty over climate dynamics plays a significant role in shaping macroeconomic dynamic responses. As such, we provide transition dynamics taking into account a range of values for ϕ_2 , which corresponds to the uncertainty over climate dynamics in our framework.

In both scenarios, economic activity increases the flow and stock of emissions (as firms do not internalize the climate externality), yielding a temperature level $T_{2100}^o \in (2.8^\circ\text{C} - 3.5^\circ\text{C})$ by 2100.⁹² The simulations in figure 2.26 show that, when taking into account the effect of climate change on productivity, output and capital start to decline rapidly once the growth process is over. Thus, failing to account for climate change leads to overestimating GDP and consumption in the long run. As output decreases compared to the case where temperature does not impact productivity, energy demand falls and wages in both sectors are reduced (figure 2.27). Interestingly, the energy relative price is also lower in this case, since demand for energy plummets. In addition, as households expect sustained long-run economic growth, they increase their consumption, substituting away from capital savings in the first few periods, which increase the return on capital firms have to pay. As the growth process stops in 2100, households anticipate and start smoothing their consumption, bringing the interest rate back to a level close to its initial steady state.

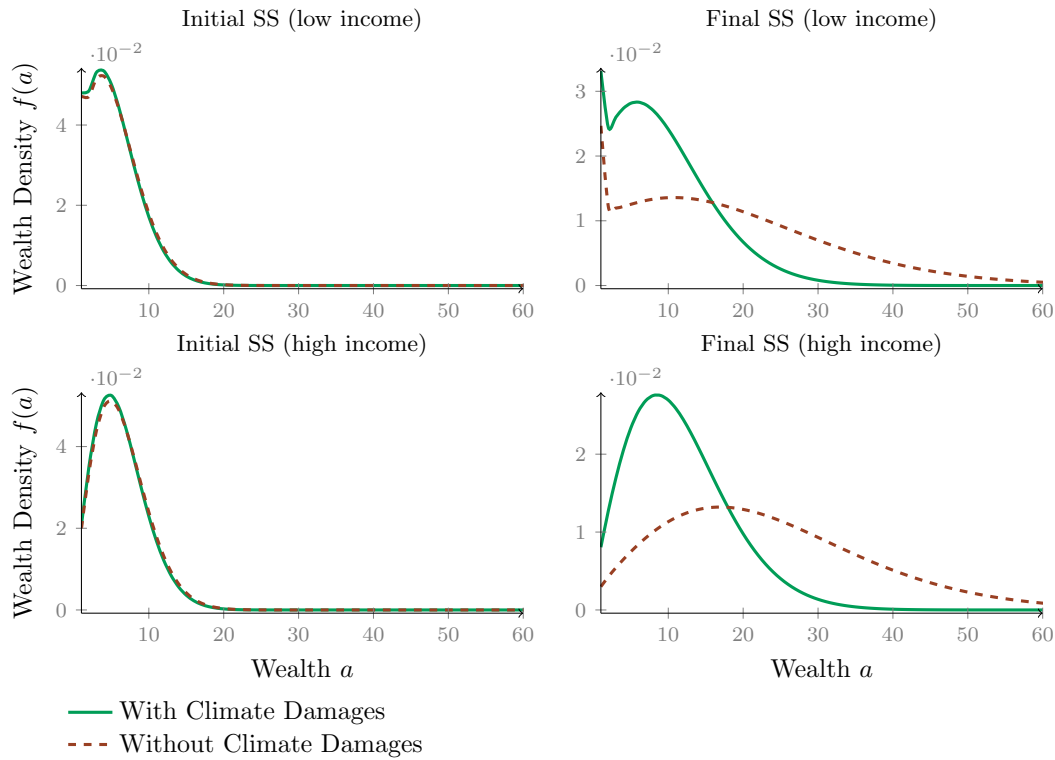
Similarly, uncertainty over climate damages (figure 2.28 and figure 2.29) plays an important role over the transition. While there appears to be less uncertainty about damages compared to climate sensitivity, the range of economic losses remains large enough to motivate aggressive mitigation policies.

⁹²We choose the range of ϕ_2 to match the latest IPCC RCP scenarios.

Overall, the sensitivity analysis on climate and damages shows that accounting for climate dynamics and uncertainty is crucial to derive credible long-term scenarios. Therefore, models that do not include this type of mechanism are likely to yield biased results and lead to myopic policy recommendations. An interesting additional question concerns the consequences of ignoring climate dynamics on the study of the distribution over time.

Implications for the distribution of wealth

Figure 2.8 displays initial and final stationary distributions from our baseline model (with climate damages) and our counterfactual model (without climate damages). One can see that what was true for aggregate variables is even more relevant for distributional costs. When ignoring the negative feedback from temperature to productivity, the distribution of wealth flattens and drastically shifts to the right, which means that the average household becomes significantly wealthier. However, correcting for the impact of climate paints a completely different picture. In this more realistic case, the decrease in distributional costs is marginal, despite 80 years of sustained economic growth. In other words, when global warming goes unchecked, it has the ability to destroy gains from increased productivity. This fact, along with the other findings in this section, motivates our choice to include climate damages and take uncertainty into account when studying the distributional impacts of carbon policy during the transition to net-zero.

FIGURE 2.8. Distribution Impacts With and Without Climate Damages

Note: The figure compares initial and final stationary distributions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line).

2.5.2.2 Meeting the Net-Zero Target

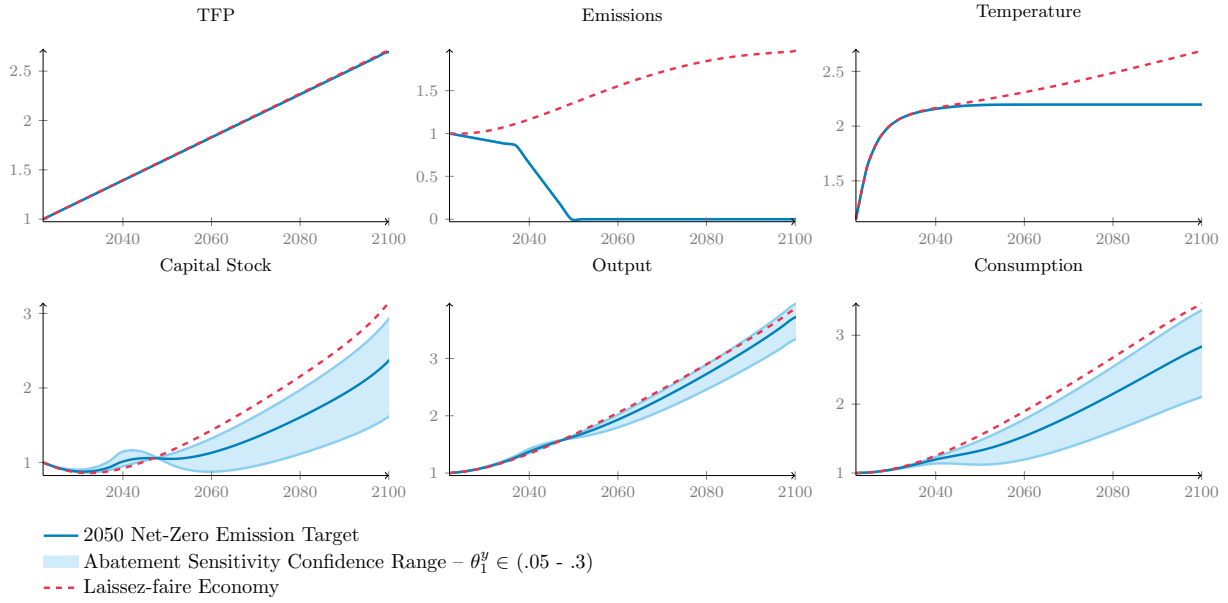
We now investigate the transition pathways over the net-zero emission target scenario. We first start by showing how abatement technologies play a pivotal role in shaping the transition pathways as well as the wealth distribution along the transition. More precisely, we analyze how different levels of abatement costs for firms lead to more or less significantly different macroeconomic responses and severe distributional impacts from a rising carbon price.

Net-zero objective transition pathways

A large part of the literature focuses on the optimal path of carbon pricing (e.g. [Golosov et al., 2014], [Dietz and Venmans, 2019], [Cai and Lontzek, 2019], among many others). The main question then is whether the optimal carbon price is able to achieve net-zero emissions by 2050. When accounting for different levels of uncertainty (e.g. climate damages, climate sensitivity, and abatement technology efficiency, among others), achieving such a target is severely hindered. [Benmir and Roman, 2020] show how optimal policy is not sufficient and investigate, using a representative agent (RA) macro model, the implication of gradually setting the carbon cap to meet a net-zero objective by 2050. However, [Benmir and Roman, 2020] do not specifically model energy sectors and focus on green and brown sectors. Including energy producers allows for investigating sequential policy setting and the implications on the macroeconomy.

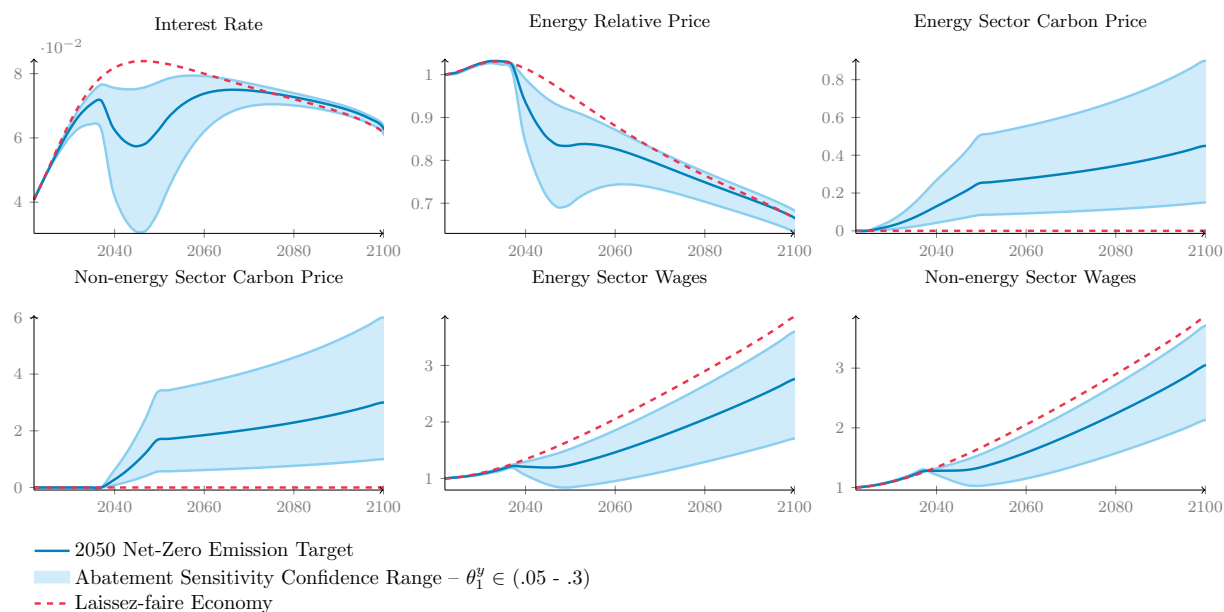
In figure 2.9 we compare the laissez-faire scenario to a cap policy leading to net-zero emissions by 2050. We show how under a 2 percent growth rate, a laissez-faire scenario clearly overshoots the Paris Agreement objective of keeping temperature below 2°C with temperature rising to a level above 2.5°C . In contrast, a net-zero strategy where emissions are reduced linearly and gradually across sectors as is the case for most cap-and-trade regimes (in our case, the cap first targets the energy sector before spanning all non-energy sectors 15 years later), allows for maintaining a temperature below 2.2°C . Furthermore, the cap policy induces a loss in the capital used to produce energy, leading to both a consumption and output loss for the net-zero case. During the transition this also means greater distributional and welfare (as it will be highlighted in the next section) costs. However, this effect does not hold in the long run. As the effects of global warming start materializing, output deteriorates in the laissez-faire case and the gains from not transitioning to net-zero are quickly reversed.

FIGURE 2.9. Net-Zero Emission Target and Laissez-faire Economy – Macro Aggregates



Note: This figure compares the net-zero scenario to a laissez-faire scenario under a 2 percent growth rate over the period 2022 to 2100. We perform a sensitivity analysis (blue shaded area) over abatement efficiency (i.e. abatement cost parameter θ_1).

Turning now to prices as shown in figure 2.10, our HA model under the net-zero policy induces a rise in carbon prices, energy price, and interest rate over the first cap period where only the energy sector is subject to the environmental policy. When the regulator generalizes the cap policy to all other sectors, the interest rate, the energy price, and wages in all sectors decrease to levels significantly lower than in the laissez-faire scenario. Intuitively, there are two trusts in play. First, growth expectations trigger higher levels of consumption within households as they expect higher income in the near future, which reduces investment levels (i.e. substitution effect is higher than income effect in this case). Second, future carbon policy expectations cool down the heat generated by the growth expectations, as when the cap hits all non-energy sectors, the continuously higher levels of carbon prices reduce profits and capital demanded, which in turn decreases wages and other aforementioned factor prices.

FIGURE 2.10. Net-Zero Emission Target and Laissez-faire Economy – Prices

Note: This figure compares the net-zero scenario to a laissez-faire scenario under a 2 percent growth rate over the period 2022 to 2100. We perform a sensitivity analysis (blue shaded area) over abatement efficiency (i.e. abatement cost parameter θ_1).

Welfare Costs: Laissez-faire versus Net-zero

We summarize the welfare effects of 2050 net-zero policy in [table 2.2](#). We use a standard consumption-equivalent welfare metric (CE), that is, the percentage change in consumption one would have to give individuals in the laissez-faire equilibrium each year to make them as well off as under the net-zero policy (see [section 2.C.4](#) for details).

Our first finding, is that implementing a net-zero policy by 2050 is welfare enhancing compared to the case of the laissez-faire (as shown in [table 2.2](#)) where no environmental policy is enforced by the regulator (whether we account or not for heterogeneity in income and wealth). However, accounting for heterogeneity in households' income and wealth appears to be highly important in the analysis of welfare costs of climate and environmental policy. When ignoring heterogeneity in households income and wealth, the RA framework is found to over estimate the welfare gains from the 2050 net-zero policy by a factor of

1/4 (i.e. 25 percent in the baseline case)⁹³ compared to the Heterogeneous Agent (HA) Climate analogous framework, as the agents saving and investment behaviors change the level of aggregate capital holding, which in turn have significant impacts on the welfare cost. In particular, the net-zero emission target allows for temperature to remain below 2.2°C in the long-run. Keeping temperature below 2.2°C reduces the damages related to increasing temperatures, which in the case of the laissez-faire have increasing impacts on productivity and capital holdings, forcing firms to relocate resources across the economy, therefore driving wages and returns down.

In quantitative terms, we find that the net-zero policy (for the baseline case), increases the combined welfare of households by 0.54 percent and 0.72 percent, respectively for the HA and RA climate frameworks in consumption-equivalent units. That is, the welfare increase from this policy is equivalent to the effect of a 0.54 percent increase in consumption in every period for every agent in the economy in the case of the HA model. Our results are similar in terms of magnitude to those of the optimal labour policies studied in [Itskhoki and Moll, 2019], and are at least one order of magnitude larger than those from eliminating the business cycle, which are typically on the order of 0.01 percent (see, e.g., [Lucas Jr, 2003]).

TABLE 2.2
Welfare: Net-zero versus Laissez-faire

	HA-Climate	RA-Climate
Low Abatement Cost	0.84%	1.05%
Welfare gains (in CE) Moderate Abatement Cost (baseline case)	0.54%	0.72%
High Abatement Cost	0.13%	0.26%

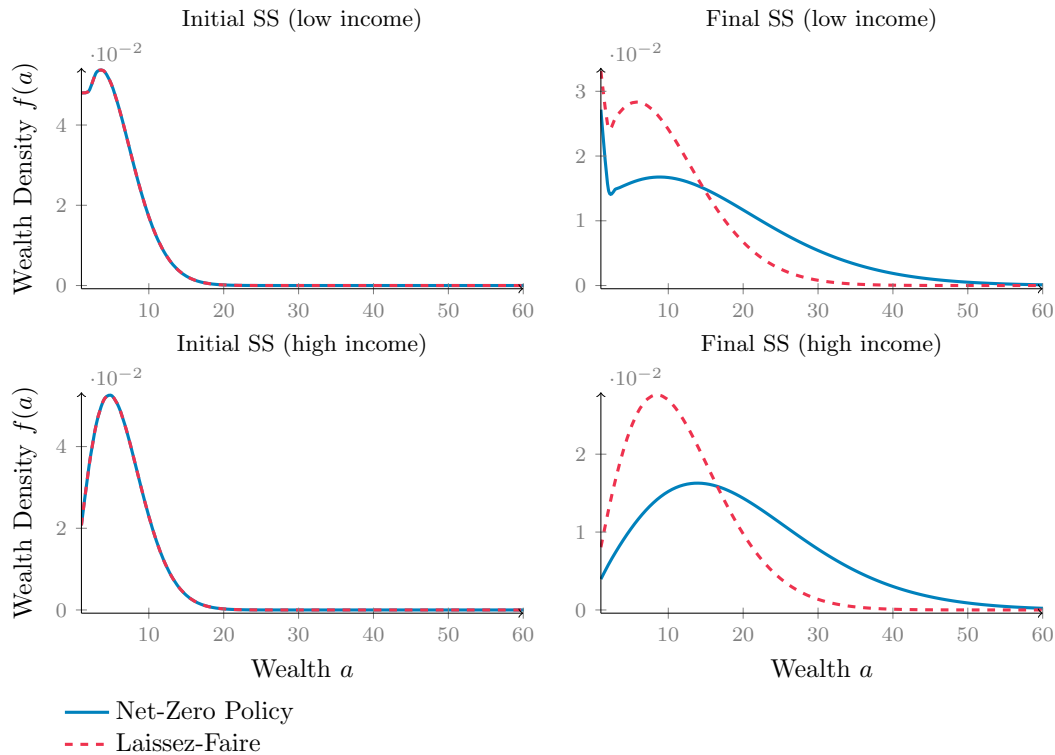
Note: This table compares the welfare gains in consumption equivalent terms (CE) from a 2050 net-zero scenario to a laissez-faire scenario under a 2 percent growth rate over the period 2022 to 2100 from our HA-Climate Model to the RA-Climate analogous model.

⁹³The factor goes up to 1/2 in the case of high abatement cost.

Net-zero long-run distributional impacts

One main strength and advantage of our framework is its ability to investigate the social impacts of public policies such as a net-zero climate policy over the transition, which is not possible with RA models and not investigated with the climate carbon pricing HA models developed up to date ([Bosetti and Maffezzoli, 2013], [Fried et al., 2018], [Goulder et al., 2019], [Cavalcanti et al., 2021], [Fried et al., 2021], [Känzig, 2021], and [Malafry and Brinca, 2022]). These models do not feature distributional long-run transition pathways and mainly focus on steady state analysis or impulse responses.

When comparing the initial and final steady state value of the stationary distributions of wealth for high and low income households shown in [figure 2.11](#), engaging in a net-zero path reduces the distributional costs at the end of the transition in the economy as the distribution of wealth for both low and high earners shifts to the right (i.e. all household quantiles become wealthier compared to those in the laissez-faire scenario). This is largely due to the net-zero policy keeping temperature at levels below 2.2°C, which ensures that temperature rise induced damages are not increasing overtime, which is otherwise the case under the laissez-faire scenario. Climate damages under the laissez-faire scenario rise to high levels following significant increases in temperature, which thus destroys capital and output, and in turn lowers the future realization of labour income and decreases consumption, leading to a higher level of distributional and welfare costs than with the net-zero case.

FIGURE 2.11. Distributional Impacts of the Net-Zero Compared to the Laissez-faire

Note: This figure compares the net-zero scenario to a laissez-faire scenario at the initial and final steady state stationary wealth distributions for both low and high income households.

Net-zero short and medium-run distributional impacts

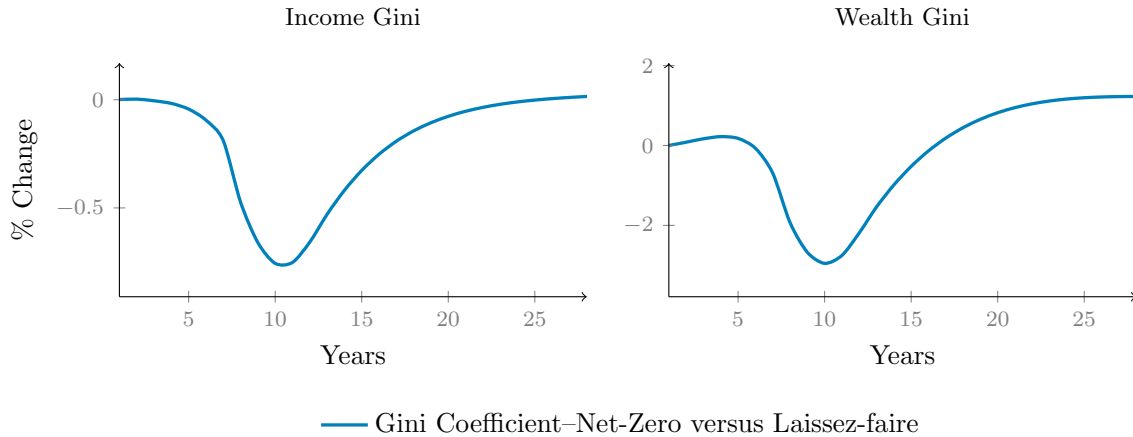
Focusing on a steady state analysis, however, is problematic when carbon prices are expected to rise over the transition to reach the net-zero objective. The political economy aspect of carbon pricing should not be underestimated when formulating public policy aimed at addressing the climate externality, as seen for example during the social upheaval in France with the *Gilets Jaunes*, whose extended protests were initially a reaction to a change in the carbon tax policy.

Looking at the Gini coefficients for both income and wealth⁹⁴ over the transition (2022-2050) as shown in [figure 2.12](#), we find that inequalities decrease in the first 15 years of the

⁹⁴As the Gini coefficient is often used as standard measure of inequalities.

transition by almost 1% and 2% for income and wealth, respectively, and that inequalities continue to be lower than 0 for the remaining 13 years of the transition (i.e. net-zero is equality enhancing compared to the laissez-faire scenario).

FIGURE 2.12. Income and Wealth Gini Coefficient Overtime: Net-Zero versus Laissez-faire



Note: This figure compares the net-zero scenario to a laissez-faire scenario for both the income and wealth Gini over the transition period.

One could be tempted to conclude that net-zero policy is able to improve the inequalities in income and wealth. However, when we turn to the household joint distributions of income and wealth, the picture is much more nuanced.

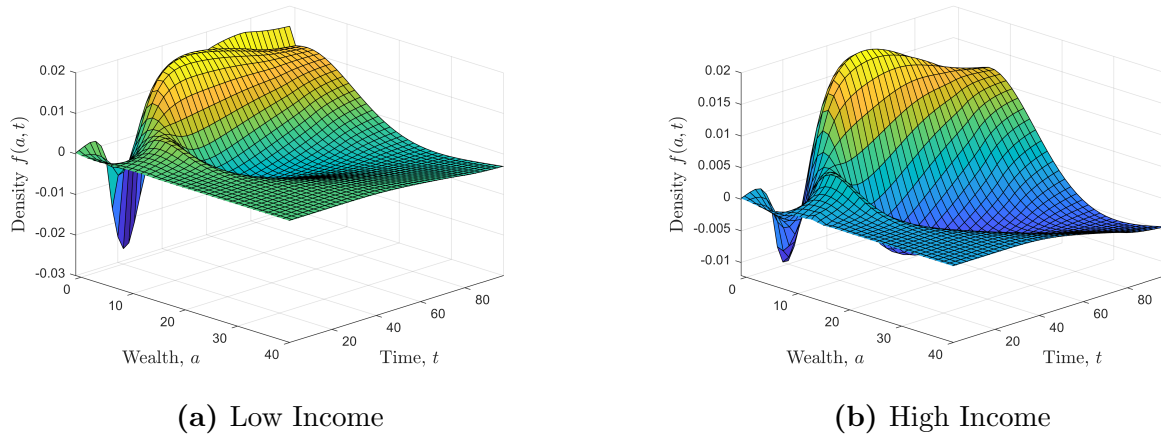
In figure 2.13 we present the wealth distribution transition pathways over the transition period 2022-2100. The left figure (a) shows how the distribution of wealth for low income is impacted over the transition to net-zero compared to the laissez-faire scenario, while figure (b) displays the results for high income earners.⁹⁵ At the beginning of the transition households expect higher output due to the announced 2 percent growth rate but also expect higher carbon prices as the government initializes the cap policy. As mentioned above, the growth expectation leads to an increase in consumption as household expect higher income in the future. Furthermore, environmental policy sectoral targeting

⁹⁵We take the difference of the wealth distribution pathways between the net-zero and the laissez-faire

(starting with the energy sector) allows for a decrease capital losses during the period in which only the energy sector is emission capped. Intuitively, as the energy sector is smaller than the non-energy sectors in our economy, the growth expectation dynamics are stronger than the impacts of the rising carbon price expectation for the first 15-20 years. However, as soon the second phase of the cap policy is launched (in 2037), the percentage of households financially constraint wealth starts to rise (a considerable spike of about 5 percent and 3 percent in the case of moderate abatement costs in a period of less than 20 years is noted within low and high income earners, respectively)⁹⁶ as consumption is now directly impacted by the high carbon price that spans all economic sectors. Where the level impact of the carbon net-zero cap by 2050 is comparable between low and high income households with respect to the laissez-faire scenario (about 2 percent increase), the low income households remain the most impacted on aggregate. By substantially substituting toward higher consumption levels at the beginning of the transition, low income earners compared to high income earners, are of particular carbon prices rise concern.

For the remaining analysis, we will mainly focus on the joint distributions of income and wealth, rather than on the Gini coefficients, as the Gini suggests an improvement in inequalities, while the joint distributions show that all households are getting poorer (less wealthy) over the transition.

⁹⁶In [figure 2.33](#), [figure 2.34](#), and [figure 2.35](#), we present different transition of the wealth distribution for three levels of abatement efficiency cost. In the case of inefficient abatement scenario, the percentage of households financially constrained increase by about 10 percent when economic sectors are subject to emission cap.

FIGURE 2.13. Net-Zero versus Laissez-faire with Moderate Abatement

Note: This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution. Figure (a) show the household wealth pathway between 2022 and 2100 for low income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

2.5.2.3 Net-Zero Transition Speed

Net-zero speed and transition pathways

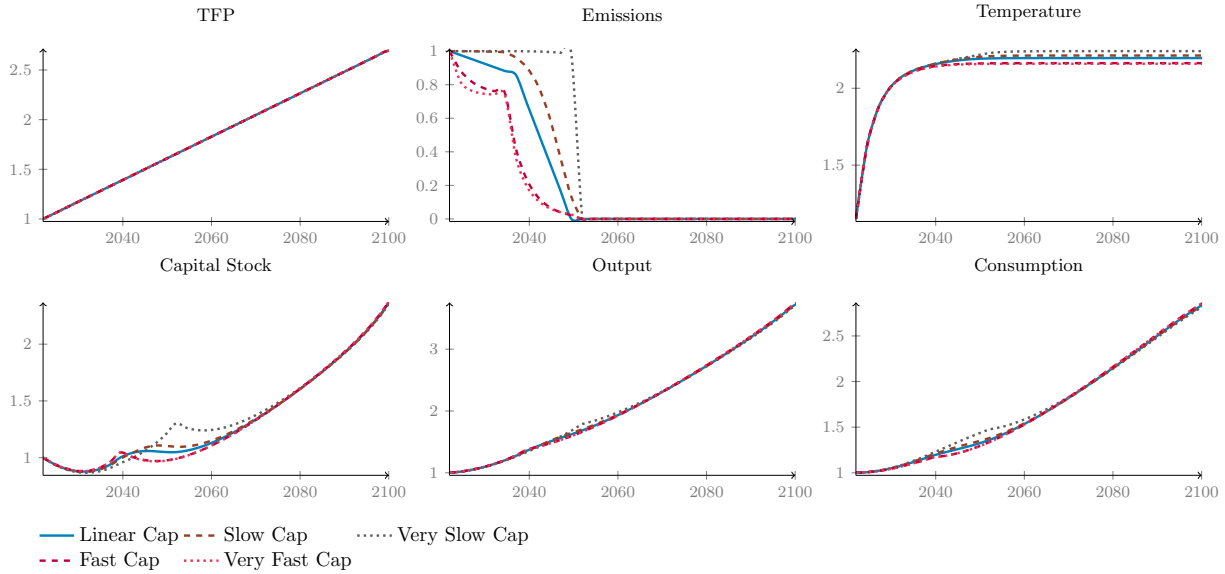
As discussed above, the lion's share of the climate literature focuses on the drivers and the level of social cost of carbon, which, in a decentralized equilibrium, corresponds to the optimal carbon price level. In an HA framework, defining the optimal carbon pricing (i.e. the social cost of carbon) is not straightforward, as it falls within the sphere of normativity. The level of optimal carbon pricing in an HA model will depend on the weight applied to the different agents' utilities. There is therefore a real need to identify a normative framework to be able to begin to answer the important question: what is the optimal carbon price in an HA framework?⁹⁷

While identifying the optimal social cost of carbon requires first an agreement over the normative framework to be used, we investigate four different trajectories (concave

⁹⁷Adrien Auclert (2022) discussion at the FED of New York.
<https://libertystreeteconomics.newyorkfed.org/2022/01/the-effect-of-inequality-on-the-transmission-of-monetary-and-fiscal-policy/>.

and convex) in addition to our baseline linear carbon cap scenario. The four additional trajectories, which we refer to as fast, very fast, slow, and very slow allow us to attain a wide range of possible transition scenarios to net-zero.

FIGURE 2.14. Net-Zero Emission Target Trajectories – Macro Aggregates

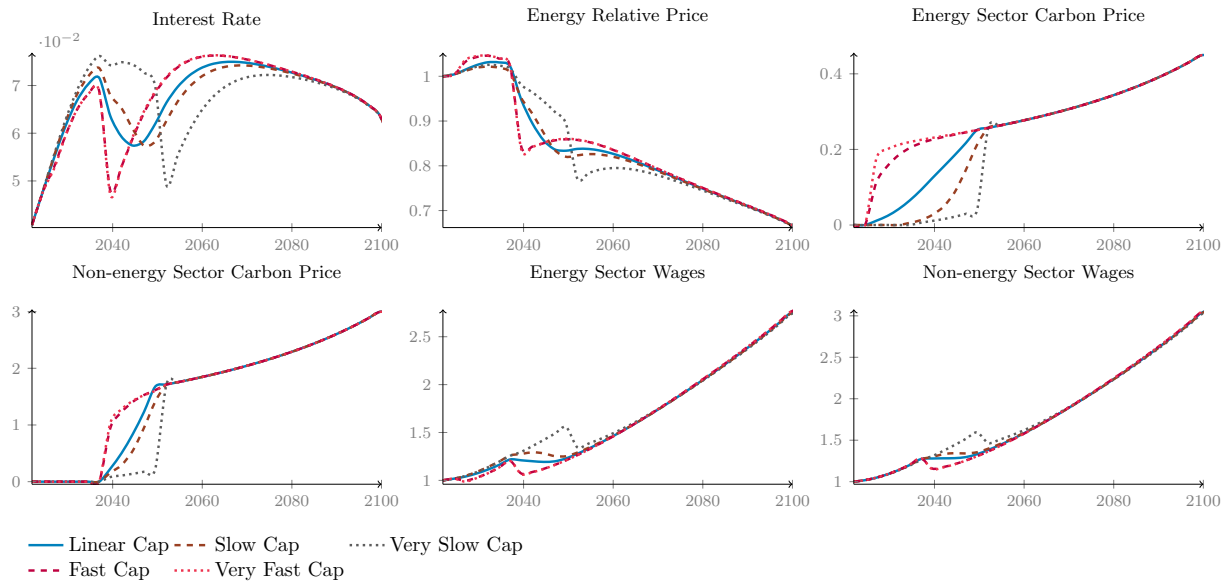


Note: This figure compares five different net-zero trajectories: i) linear (baseline case), ii) fast cap, iii) very fast cap, iv) slow cap, and v) very slow cap.

Figure 2.14 presents the transition pathways for our economy’s macro-aggregates. Acting promptly with aggressive environmental policies or, in contrast, delaying the intervention until the last minute, has little impact on the output over the transition period (2022-2100). However, acting today to reduce emissions to zero as opposed to acting 28 years from now would have consequences on the temperature level. While over the transition, the temperature variation is of a small magnitude, the inertia over the long-run would mean a small deviation today would lead to significant difference over the long-run (as we demonstrate and discuss above figure 2.26). The impact on consumption during the transition follows the pathway of capital movements. When the regulator decides to act fast, agents expect a sharp decrease in emissions, which requires major investment by firms. This triggers capital investment relocation toward higher investment in abatement

costs. This substitution toward abatement investment is costly and thus leads to a lower level of capital (in the case of a fast cap compared to a slow cap), which in turn leads to a lower level of consumption and a rise in wealth losses (as shown in [figure 2.44](#), [figure 2.45](#), [figure 2.41](#), and [figure 2.42](#)).

FIGURE 2.15. Net-Zero Emission Target Trajectories – Prices



Note: This figure compares five different net-zero trajectories: i) linear (baseline case), ii) fast cap, iii) very fast cap, iv) slow cap, and v) very slow cap.

With respect to prices, [figure 2.15](#) shows how the interest rate, energy price, and carbon prices, as well as wages are impacted following our four plus one (linear) policy speeds. Policy speed is shown to have a significant impact on the interest rate and the energy price, where the dips at the start of the environmental policy are rather strong compared to the case where the environmental cap is set linearly. This suggests potential volatility issues within financial markets that could lead to further consumption drops. Conducting a slow versus a fast cap has a significant impact on labor income (about 25-30 percent decrease when policy is conducted following a fast cap versus a slow cap). This is due to the sudden needs of abatement investment and reallocation of factors of production.

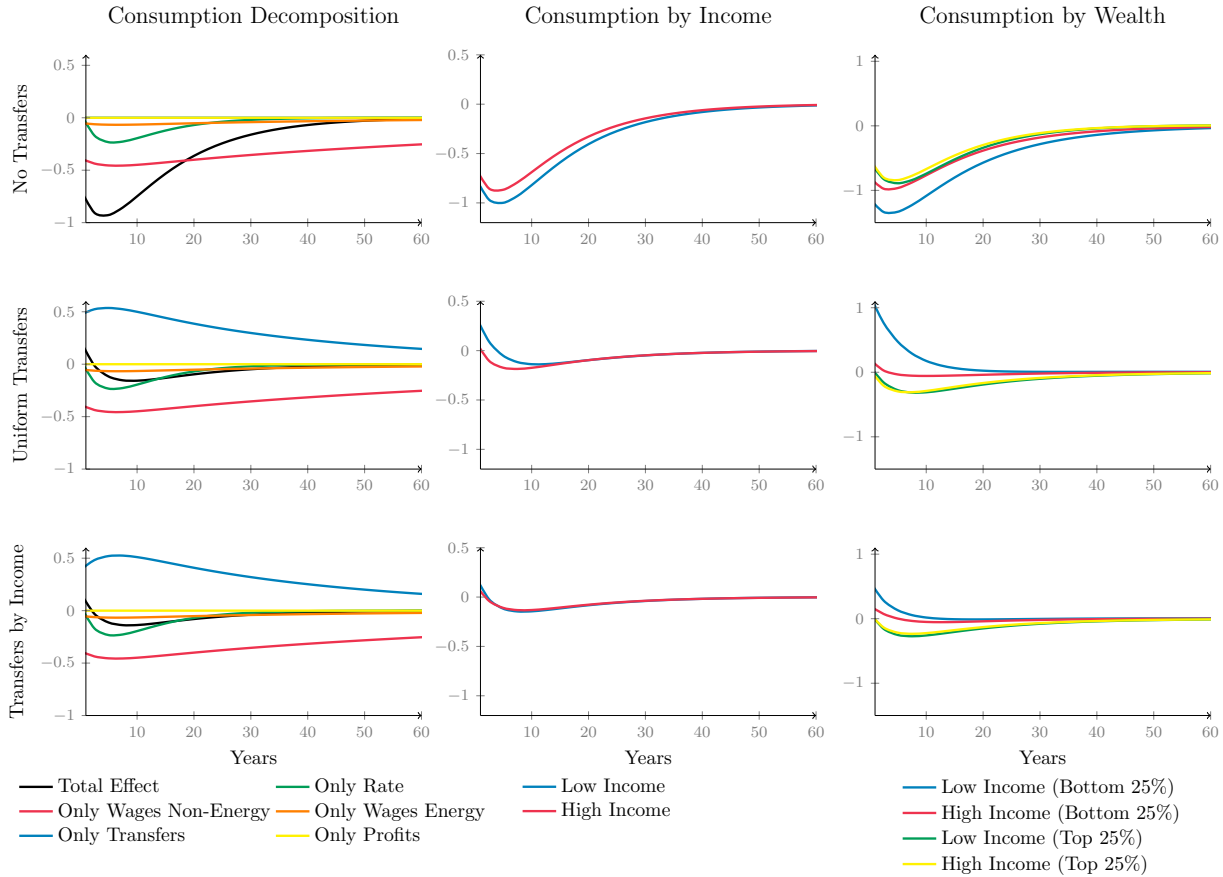
2.5.3 Redistribution of Carbon Revenues

As argued above, implementing carbon pricing consistent with the net-zero target is not a free lunch and leads to a rise in financially constrained and poor households over the transition period. In the following section we show how redistributing the carbon fiscal revenues could help smooth the net-zero transition and offset some of the negative effect.

2.5.3.1 Carbon Policy and Transfers

Figure 2.16 shows the impact on consumption decomposed into various indirect effects, by income and by wealth (as in section 2.5.1.4), according to the use made of carbon revenues. In the case where no transfer scheme is implemented by the government (first row in figure 2.16), the proceeds from carbon taxation are used for unproductive government spending and this scenario corresponds to the first row in figure 2.7.

FIGURE 2.16. Fiscal Transfers and Consumption Drivers



Note: The figure plots the reaction of consumption according to three different fiscal transfer scenarios. The first row corresponds to the case with no fiscal transfers. The second row corresponds to the case with uniform fiscal transfers. And the last row corresponds to the case with per income fiscal transfers. The first column plots the reaction of consumption as well as its four components. The second column plots the reaction of consumption according to the realization of income. And the last column plots the reaction of consumption according to the realization of income and the level of wealth.

When the government decides to redistribute revenues uniformly (second row), it is able to completely offset the negative impact on consumption, for both low-income and high-income households. Moreover, uniform redistribution particularly benefits low-income households with little wealth. The reason is that these households do not earn much return on capital and/or profits, which implies that transfers represent a high share of their disposable income compared to other types of households. Therefore, low-income

households with low wealth actually increase their consumption when the carbon price shock is combined with uniform redistribution of revenues. This result is consistent with [Goulder et al., 2019], who show that recycling carbon proceeds can benefit lower income households and induce a progressive effect overall. Although this result may seem very promising, one should keep in mind that distorting the trade-off between consumption and savings may reduce the potential for future growth. If this type of redistribution policy prevents households from accumulating wealth, the long-run impact could ultimately be regressive. In that sense, income-based redistribution of carbon revenues is an attractive alternative, as it generates less volatility in consumption across income/wealth groups of households, while still offsetting the negative effect of carbon pricing exhibited in [figure 2.7](#).

Analysis of various recycling policies following an MIT carbon price shock shows that revenue redistributions, whether uniform or income-based, can offset most of the negative impact on consumption, and thus on welfare. These findings are in line with [Malafry and Brinca, 2022], who show that the optimal price of carbon is higher when the government engages in revenue redistribution. Keeping a long-run perspective in mind, however, this exercise implies that recycling carbon revenues by income would be less distortionary. To confirm this intuition, we now turn to the analysis of the distribution during the transition to net-zero.

2.5.3.2 Net-Zero Distributional Impacts and Transfers

[Figure 2.17](#) and [figure 2.18](#) compare the net-zero scenario distributed fiscal transfers (uniformly and by income) with the net-zero scenario without fiscal transfers, over the transition for the wealth distribution. When the density value is positive, this means that we have a negative impact on the wealth distribution and vice versa. In other words, when the density function is positive at a given point, this means that the distribution of households shifted toward the right.

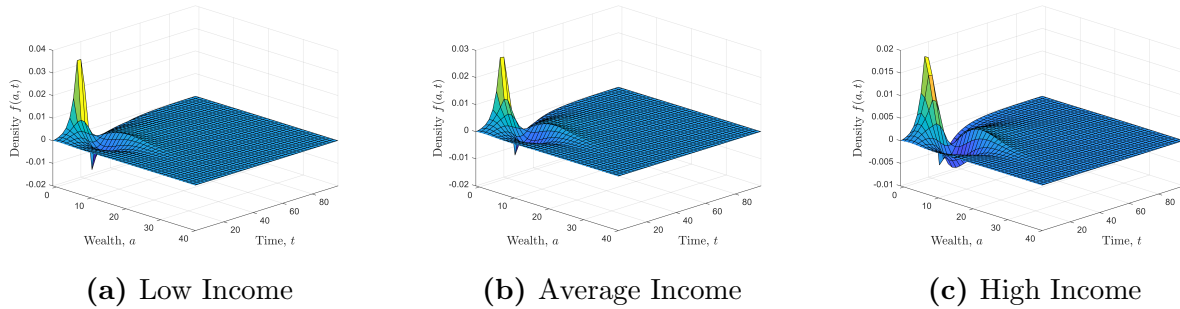
Over the net-zero transition, redistributing carbon fiscal revenues to households, both

uniformly and by income, allows for decreasing disparities between different household and over 2043 and 2050 (that is, when the second phase of the cap policy is engaged, which includes all other non-energy sectors). Focusing, however, on the first 20 years of the transition period, and as highlighted in the case of distributional impacts over the transition with no fiscal transfers (i.e. [figure 2.13](#) results), both uniform and per income fiscal transfers allow household to engage in less precautionary savings to face the future impacts of the rising carbon costs, and thus achieve a higher consumption level. In other words, fiscal redistribution acts as a smoothing mechanism that reduces household saving incentives during the first 20 years, and boosts their consumption, which as shown allows for reducing the impacts on the wealth distribution (and welfare) compared to the case with no fiscal transfers between 2043 and 2050.

Between uniform and per income fiscal transfer, it appears (as it is also the case in the previous section) that per income transfers allow for the fewest spikes and the least change overtime.

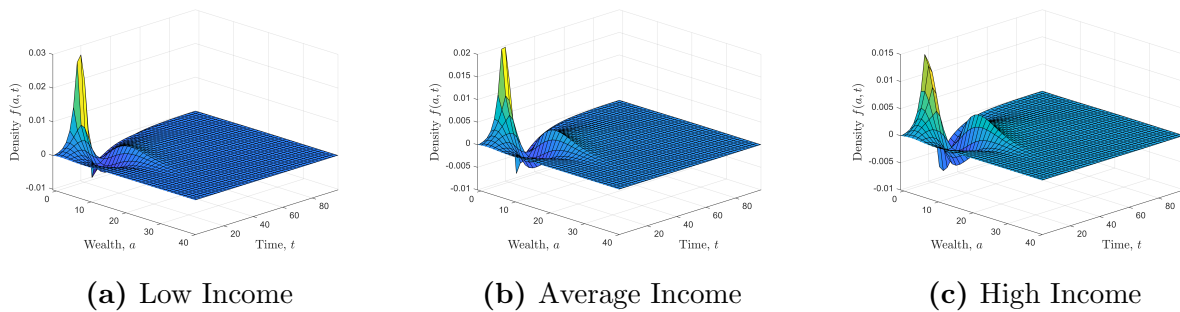
We note that, although, fiscal transfers are able to offset some of the unequal impacts of carbon pricing, the medium/long run effect cannot be addressed solely by redistributing carbon revenues. Carbon proceeds after 2050 are equal to zero ($\tau_t E_t = 0$), and firms still engage in abatement investments to maintain emissions at zero. The distribution density function in [figure 2.17](#) and [figure 2.18](#) which is between 40 and 80 (and corresponds to 2060 and 2102) is almost equal to zero (flat plane). This means that there is no difference between net-zero with or without fiscal transfers. In contrast, for example in [figure 2.13](#) (which represents the net-zero scenario with no fiscal transfers compared to the laissez-faire scenario), we see that for the same period overall poverty (i.e. losses in capital holdings) rises. Thus, under fiscal transfers, distributional costs still rises. This last result is of special importance and suggests the need to investigate the ways by which abatement costs can be made cheaper.

FIGURE 2.17. Net-Zero with Uniform Fiscal Transfers versus without Transfers – Medium Abatement Efficiency



Note: This figure compares the net-zero scenario with uniformly distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution. Figure (a) show the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with uniformly distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

FIGURE 2.18. Net-Zero with Fiscal Transfers (by Income) versus without Transfers – Medium Abatement Efficiency



Note: This figure compares the net-zero scenario with per income distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution. Figure (a) show the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with per income distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

2.6 What About Inflation And Carbon Pricing?

2.6.1 Case of Sticky Prices (HANK): Model Changes

Firms

In the case of sticky prices, monopolistic non-energy firms engage in a price setting à la [Rotemberg, 1982]. Price update is subject to a quadratic adjustment in the rate of price change and is expressed as a fraction of aggregate output:

$$\Delta_{j,t}^P = \frac{\theta^P}{2} \left(\frac{\dot{p}_{j,t}}{p_{j,t}} \right)^2 Y_t. \quad (2.60)$$

For the ease of reading and as firms are identical, we suppress notational dependence on j . Thus, profit maximization subject to the demand from final firms yields the New Philips Curve⁹⁸:

$$\left(r_t^a - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\theta}{\theta^P} (mc_t - mc^*) + \dot{\pi}_t \quad (2.61)$$

where, $mc^* = \frac{\theta-1}{\theta}$, and π_t is the inflation rate.

The flow profits before price adjustment is similar to the RBC case, as such the flow profits in the case of sticky prices will include the price adjustment costs:

$$\Pi_t^F = (1 - mc_t)Y_t - \frac{\theta^P}{2} \pi_t^2 Y_t. \quad (2.62)$$

The Monetary Authority

Under the presence of price stickiness (i.e. the non-neutrality of monetary policy), the

⁹⁸The full derivation can be found in the appendix [section 2.C.2](#).

central bank follows a simple [Taylor, 1993] rule to set the nominal interest rates i_t^i :

$$i_t^i = \bar{r}^a + \phi_\pi \pi_t + \phi_Y (Y_t - \bar{Y}) \quad (2.63)$$

where \bar{r}^a is the steady state of real rate and $\phi_\pi \geq 1$ is the inflation stance. \bar{Y} is the steady state level of output, while ϕ_Y is the central bank reaction to output gap.

In addition, the relationship between the nominal and the real interest is modeled through the Fisherian equation:

$$i_t^i = r_t^a + \pi_t \quad (2.64)$$

2.6.2 Solution Method

In the presence of price stickiness, the updating iterative algorithm rule does not allow for convergence when used to clear the New Phillips Curve. We instead rely on the system of equations method to solve the transition dynamics for the marginal cost:

$$\text{MC}(k_1^*, \dots, k_N^*) = 0 \quad (2.65)$$

where $\text{MC}: \mathbb{R}^N \rightarrow \mathbb{R}^N$ denotes the N-period excess marginal cost function.

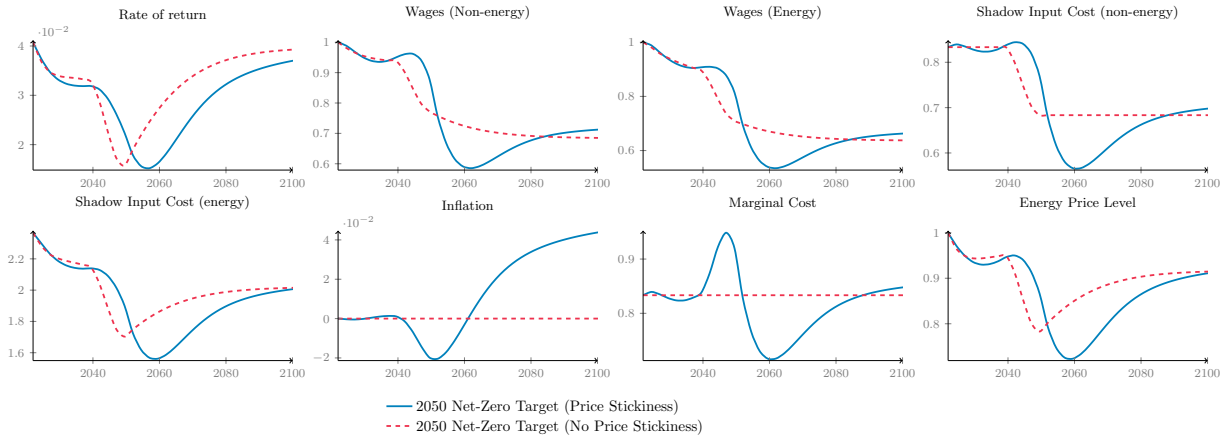
2.6.3 Results

Under the presence of price rigidities, the marginal cost for firms is subject to fluctuations. To understand the implications of the net-zero emissions target and its interaction with inflation, we simulate a transition pathway consistent with the net-zero emissions target under a linear cap, and with by income fiscal transfers and no TFP growth. [Figure 2.19](#) shows the cases both of sticky prices (in blue) and flexible prices (in red) where inflation has no role.

Over the net-zero transition (i.e. 2022-2050), the high cost of offsetting carbon emissions pushes firms to decrease wages, which in turn pushes the input shadow costs down-

ward, thus decreasing inflationary pressures. This, however, is not the case in the first few years (i.e. until 2038), whereby households perfectly foresee the high cost of the environmental transition and engage in precautionary savings. This response allows for the level of capital to remain close to that of the case of flexible prices, and ultimately keeps inflation stable, as the shadow input costs and marginal costs remain stable. Thereafter (once the transition to net-zero has been accomplished, i.e. after 2050), inflationary pressures kick in, as both wages and rates of return rise given that the tax revenue becomes zero, which increases the shadow input costs. While inflation could be less of a concern over the transition, the long-run consequences could see inflation rise to over 5 percent by 2080. This could be of major concern to the conduction of monetary policy. However, we recognize that modeling choices are paramount to these results, and further research should investigate the inflationary pressures over the transition, using a full two asset modeling framework à la [Kaplan et al., 2018].

FIGURE 2.19. Inflation and Net-Zero Target Interactions



Note: The figure plots the reaction of relevant macro-aggregates and prices according to two modeling choices: i) in blue the presence of price stickiness, and ii) in red under the assumption of flexible prices. In both cases, we plot the net-zero trajectory under no TFP growth.

2.7 Learning By Doing and Abatement Efficiency

2.7.1 Model Changes

In this section we highlight the role of green innovation. The cost function of abatement is now steered by endogenous green innovations:

$$f(\mu_t^s) = \left(\int_0^{A_t^g} f(\mu_{j,t}^s)^{\frac{1}{\theta_3}} dj \right)^{\theta_3}, \quad (2.66)$$

Thus,

$$f(\mu_t^s) = \theta_1 (\mu_t^s)^{\theta_2} (A_t^g)^{-\theta_3} \quad (2.67)$$

where $\theta_3 > 0$ is the elasticity of green innovations⁹⁹ and s the two sectors in our economy (i.e energy and non-energy sectors).

Where one could model green innovations A_t^g with an endogenous growth process as in [Benmir and Roman, 2021], we use abatement level μ_t as a learning indicator. This reduced form allow for capturing the learning by doing, without necessarily worrying about the sources of green innovation funding. As such, the abatement cost function reads as:

$$f(\mu_t^s) = \theta_1 (\mu_t^s)^{\theta_2 - \theta_3} \quad (2.68)$$

2.7.2 Results

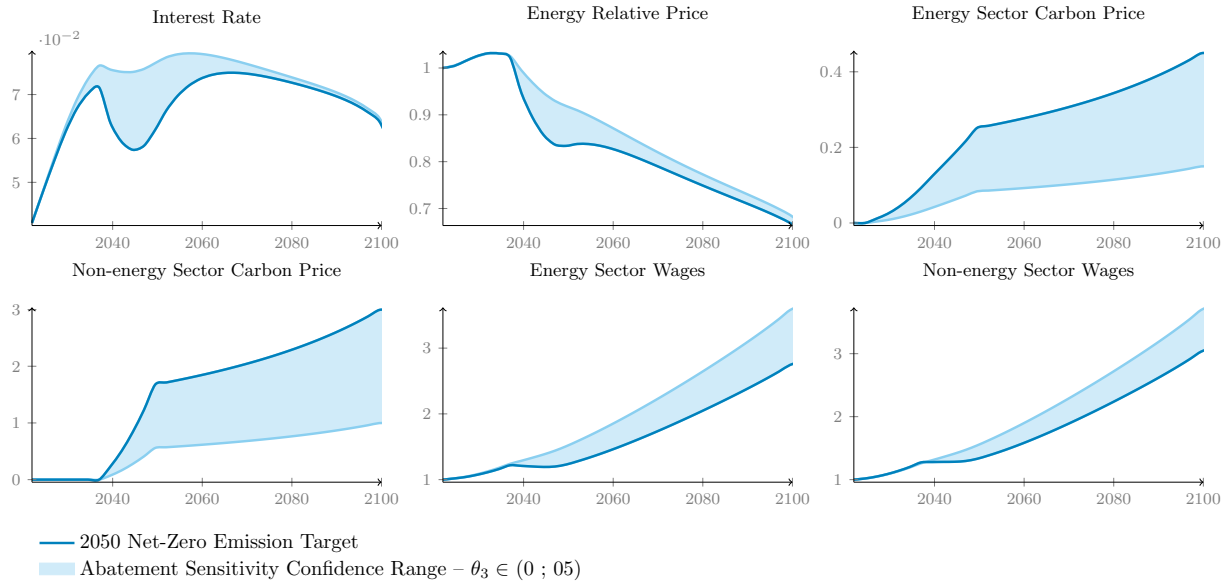
Figure 2.19 shows how accounting for learning by doing within abatement costs, allows for a smoother transition as carbon price costs decrease over the transition, thus allowing for higher wages and rates of returns. This in turn stimulates the economy and would decrease distributional impacts stemming from the net-zero transition.

Intuitively, with green innovation decreasing abatement investment costs, firms do not

⁹⁹We conduct sensitivity analysis over different values of θ_3 .

need to engage in costly resource reallocation, where they decrease both their capital holdings and labor wages. Instead, firms are able to make cheap investments in abatement technologies as the cost is low, which ultimately maintains the shadow input cost levels close to the laissez-faire scenario. In such a case, the impacts on the distribution are less pronounced and the net-zero transition is less costly for households and firms alike, as both capital holdings and wages remain high in the economy.

FIGURE 2.20. Abatement Learning By Doing and Macro Prices



Note: The figure plots the relevant macro-prices according to two modeling choices: i) in tick blue the baseline abatement intensity, and ii) in shaded blue a range of values for learning intensity. In both cases, we plot the net-zero trajectory under 2 percent TFP growth.

2.8 Conclusion

In this paper, we provide a framework to study the effects of the transition to a low-carbon economy on household income and wealth distribution.

We first conduct an empirical analysis of the California carbon cap-and-trade market to investigate the propagation channels and impacts of carbon price shocks on Californian households, and do so using U.S. climate Sentometric data. We show how California

carbon pricing shocks increase energy prices and decrease net-energy generation, which decreases wages and momentarily increases equity returns before the latter decreases over time. Furthermore, when focusing on household bottom and top income quantiles, the carbon price shock is found to impact households asymmetrically depending on their level of income. In particular, we find that the bottom 50 percent income level households see their consumption fall, while a positive shock on the price of carbon tends to momentarily increase consumption for the top 50 percent income level households. We conduct a series of sensitivity checks, which indicate that the results are robust along a number of dimensions including the selection of news, the estimation technique, the model specification, and the sample period.

We then develop a heterogeneous household model with two production sectors: i) an energy sector and ii) a non-energy sector. We first use the model to decompose the effect of a carbon price shock on households, before assessing the impact of the net-zero target on aggregate variables and the distribution. Much of the transmission of a shock on the price of carbon goes through wages and the interest rate. As such, implementing carbon taxation in the energy sector or in the non-energy sector leads to different outcomes. We find that it is overall less costly to first abate emissions in the energy sector, consistent with policies implemented in the EU and in California. Furthermore, putting a price on carbon in the non-energy sector has higher impacts in terms of distributional costs on consumption and wealth. These findings are confirmed by the study of the transition dynamics to net-zero. Although we show that acting to lower emissions is required to avoid major economic losses on a long-run horizon, distributional and welfare costs are expected to rise in the short run. To mitigate the rapid changes in the distribution of wealth over the transition, we investigate the role of transfers. Income-based redistribution of carbon revenues proves to be the most effective in smoothing household consumption and savings decisions during the uncertain emissions reduction period.

Overall, the findings of this paper suggest that while the transition to net-zero is a necessary step toward a long-run sustainable economy, it induces changes in the distribu-

tion of income and wealth that could potentially lead to social unrest. Public authorities need to anticipate and monitor the impact of large-scale environmental policies on different types of households (especially financially-constrained households) if the transition is to be successful. In this perspective, targeted redistribution of carbon revenues could be a major tool in government strategies. We note that, although fiscal transfers are able to offset some of the unequal impacts of carbon pricing, the medium/long run effect cannot be addressed solely by redistributing carbon revenues. Additionally, the need for cheaper abatement technologies is paramount. To this extent, we consider the case of abatement learning and show how fiscal redistribution and green innovation decreases carbon prices and boosts consumption over the transition. Finally, turning to the linkages between inflation and carbon pricing, we show that net-zero carbon pricing costs induce inflationary pressure over the long run, thus suggesting a potential challenge for monetary policy in so far as keeping inflation under the desired target.

References

- D. Acemoglu, P. Aghion, L. Bursztyn, and D. Hemous. The environment and directed technical change. *American economic review*, 102(1):131–66, 2012.
- D. Acemoglu, D. Hemous, L. Barrage, P. Aghion, et al. Climate change, directed innovation, and energy transition: The long-run consequences of the shale gas revolution. In *2019 Meeting Papers*, number 1302. Society for Economic Dynamics, 2019.
- Y. Achdou, J. Han, J.-M. Lasry, P.-L. Lions, and B. Moll. Income and wealth distribution in macroeconomics: A continuous-time approach. *The review of economic studies*, 89(1):45–86, 2022.
- P. Aghion, A. Dechezleprêtre, D. Hemous, R. Martin, and J. Van Reenen. Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, 124(1):1–51, 2016.
- S. Ahn, G. Kaplan, B. Moll, T. Winberry, and C. Wolf. When inequality matters for macro and macro matters for inequality. *NBER macroeconomics annual*, 32(1):1–75, 2018.
- S. R. Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684, 1994.
- D. Ardia, K. Bluteau, K. Boudt, and K. Inghelbrecht. Climate change concerns and the performance of green versus brown stocks. *National Bank of Belgium, Working Paper Research*, 2(1):229–260, 2020.

- A. Auclert. Monetary policy and the redistribution channel. *American Economic Review*, 109(6):2333–67, 2019.
- A. Auclert, B. Bardóczy, M. Rognlie, and L. Straub. Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5):2375–2408, 2021.
- L. A. Bakkensen and L. Barrage. Going underwater? flood risk belief heterogeneity and coastal home price dynamics. *The Review of Financial Studies*, 2021.
- M. Barnett, W. Brock, and L. P. Hansen. Pricing uncertainty induced by climate change. *The Review of Financial Studies*, 33(3):1024–1066, 2020.
- G. Benmir and J. Roman. *Policy interactions and the transition to clean technology*. Grantham Research Institute on Climate Change and the Environment, 2020.
- G. Benmir and J. Roman. Endogenous abatement technology. 2021.
- J.-T. Bernard and M. Kichian. The impact of a revenue-neutral carbon tax on gdp dynamics: The case of british columbia. *The Energy Journal*, 42(3), 2021.
- T. Boppart, P. Krusell, and K. Mitman. Exploiting mit shocks in heterogeneous-agent economies: the impulse response as a numerical derivative. *Journal of Economic Dynamics and Control*, 89:68–92, 2018.
- V. Bosetti and M. Maffezzoli. Taxing carbon under market incompleteness. 2013.
- M. Burke, S. M. Hsiang, and E. Miguel. Global non-linear effect of temperature on economic production. *Nature*, 527(7577):235–239, 2015.
- J. B. Bushnell, H. Chong, and E. T. Mansur. Profiting from regulation: Evidence from the european carbon market. *American Economic Journal: Economic Policy*, 5(4):78–106, 2013.

- Y. Cai and T. S. Lontzek. The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6):2684–2734, 2019.
- T. Cavalcanti, Z. Hasna, and C. Santos. Climate change mitigation policies: Aggregate and distributional effects. 2021.
- R. Colacito, B. Hoffmann, and T. Phan. Temperatures and growth: A panel analysis of the united states. *Journal of Money, Credit, and Banking*, 51(2-3):2019, 2018.
- M. Dell, B. F. Jones, and B. A. Olken. Temperature shocks and economic growth: Evidence from the last half century. *American Economic Journal: Macroeconomics*, 4(3):66–95, 2012.
- W. J. Den Haan. Solving dynamic models with aggregate shocks and heterogeneous agents. *Macroeconomic dynamics*, 1(2):355–386, 1997.
- C. Di Maria and S. Valente. Hicks meets hotelling: the direction of technical change in capital–resource economies. *Environment and Development Economics*, 13(6):691–717, 2008.
- S. Dietz and N. Stern. Endogenous growth, convexity of damage and climate risk: how nordhaus’ framework supports deep cuts in carbon emissions. *The Economic Journal*, 125(583):574–620, 2015.
- S. Dietz and F. Venmans. Cumulative carbon emissions and economic policy: in search of general principles. *Journal of Environmental Economics and Management*, 96:108–129, 2019.
- S. Fried, K. Novan, and W. B. Peterman. The distributional effects of a carbon tax on current and future generations. *Review of Economic Dynamics*, 30:30–46, 2018.
- S. Fried, K. M. Novan, and W. Peterman. Recycling carbon tax revenue to maximize welfare. 2021.

- M. Gertler and P. Karadi. Monetary policy surprises, credit costs, and economic activity. *American Economic Journal: Macroeconomics*, 7(1):44–76, 2015.
- M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88, 2014.
- L. H. Goulder, M. A. Hafstead, G. Kim, and X. Long. Impacts of a carbon tax across us household income groups: What are the equity-efficiency trade-offs? *Journal of Public Economics*, 175:44–64, 2019.
- A. Grimaud and L. Rouge. Environment, directed technical change and economic policy. *Environmental and Resource Economics*, 41(4):439–463, 2008.
- A. Imrohoroğlu. Cost of business cycles with indivisibilities and liquidity constraints. *Journal of Political economy*, 97(6):1364–1383, 1989.
- O. Itskhoki and B. Moll. Optimal development policies with financial frictions. *Econometrica*, 87(1):139–173, 2019.
- D. R. Känzig. The economic consequences of putting a price on carbon. *Available at SSRN 3786030*, 2021.
- G. Kaplan, B. Moll, and G. L. Violante. Monetary policy according to hank. *American Economic Review*, 108(3):697–743, 2018.
- P. Krusell and A. A. Smith, Jr. Income and wealth heterogeneity in the macroeconomy. *Journal of political Economy*, 106(5):867–896, 1998.
- K. N. Kuttner. Monetary policy surprises and interest rates: Evidence from the fed funds futures market. *Journal of monetary economics*, 47(3):523–544, 2001.
- J.-M. Lasry and P.-L. Lions. Mean field games. *Japanese journal of mathematics*, 2(1):229–260, 2007.

- R. E. Lucas Jr. Macroeconomic priorities. *American economic review*, 93(1):1–14, 2003.
- L. Malafry and P. Brinca. Climate policy in an unequal world: Assessing the cost of risk on vulnerable households. *Ecological Economics*, 194:107309, 2022.
- M. Mansanet-Bataller and A. Pardo. Impacts of regulatory announcements on co2 prices. *The Journal of Energy Markets*, 2(2):1–33, 2009.
- A. McKay, E. Nakamura, and J. Steinsson. The power of forward guidance revisited. *American Economic Review*, 106(10):3133–58, 2016.
- G. E. Metcalf. On the economics of a carbon tax for the united states. *Brookings Papers on Economic Activity*, 2019(1):405–484, 2019.
- J. L. Montiel Olea, J. H. Stock, M. W. Watson, et al. Inference in structural vector autoregressions identified with an external instrument. *Journal of Econometrics*, 225(1):74–87, 2021.
- E. Nakamura and J. Steinsson. High-frequency identification of monetary non-neutrality: the information effect. *The Quarterly Journal of Economics*, 133(3):1283–1330, 2018.
- W. Nordhaus. A question of balance: Weighing the options on global warming policies. *Yale University Press, New Haven, CT*, 2008.
- W. D. Nordhaus. To slow or not to slow: the economics of the greenhouse effect. *The economic journal*, 101(407):920–937, 1991.
- W. D. Nordhaus and A. Moffat. A survey of global impacts of climate change: replication, survey methods, and a statistical analysis. *NBER Working Paper*, 2017.
- V. A. Ramey. Macroeconomic shocks and their propagation. *Handbook of macroeconomics*, 2:71–162, 2016.

- M. Reiter. Solving heterogeneous-agent models by projection and perturbation. *Journal of Economic Dynamics and Control*, 33(3):649–665, 2009.
- J. J. Rotemberg. Sticky Prices in the United States. *Journal of Political Economy*, 90(6): 1187–1211, 1982.
- I. Rudik, G. Lyn, W. Tan, A. Ortiz-Bobea, et al. Heterogeneity and market adaptation to climate change in dynamic-spatial equilibrium. Technical report, Iowa State University, Department of Economics, 2021.
- A. F. Shapiro and G. E. Metcalf. The macroeconomic effects of a carbon tax to meet the us paris agreement target: The role of firm creation and technology adoption. Technical report, National Bureau of Economic Research, 2021.
- C. A. Sims, J. H. Stock, and M. W. Watson. Inference in linear time series models with some unit roots. *Econometrica: Journal of the Econometric Society*, pages 113–144, 1990.
- S. Smulders and M. De Nooij. The impact of energy conservation on technology and economic growth. *Resource and Energy Economics*, 25(1):59–79, 2003.
- N. Stern. The economics of climate change. *American Economic Review*, 98(2):1–37, 2008.
- J. B. Taylor. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1):195–214, December 1993.
- C. P. Traeger. Uncertainty in the analytic climate economy. *CEPR Discussion Paper No. DP16065*, 2021.
- T. S. Van den Bremer and F. Van der Ploeg. The risk-adjusted carbon price. *American Economic Review*, 111(9):2782–2810, 2021.
- R. Van der Ploeg, C. Hambel, and H. Kraft. Asset pricing and decarbonization: Diversification versus climate action. *University of Oxford Working Paper*, 2020.

M. L. Weitzman. Ghg targets as insurance against catastrophic climate damages. *Journal of Public Economic Theory*, 14(2):221–244, 2012.

Appendices

2.A Appendix: A

2.A.1 Data

The data used¹⁰⁰ in this section were obtained from following sources:

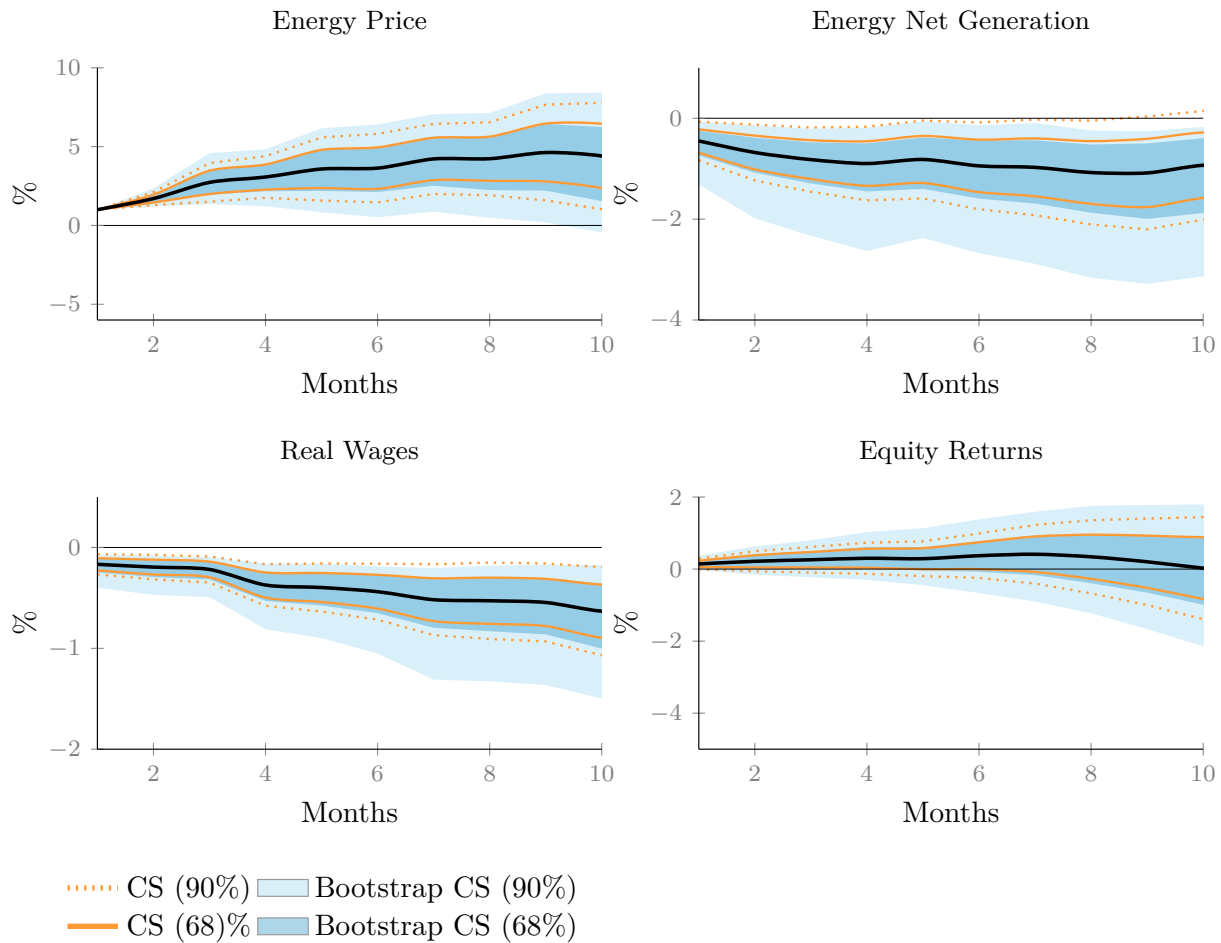
- U.S. climate sentiment data were extracted from the Sentometric data source, ([Ardia et al., 2020]),
- California carbon futures prices data are obtained from Climate Policy Initiative database,
- California daily energy prices are taken from California Independent System Operator (California ISO) database,
- California net energy generation monthly data are taken from U.S. Energy Information Administration (EIA) database,
- California monthly data wages are obtained from the U.S. Bureau of Labor Statistics (BLS) database,
- California monthly equity returns index is received from Bloomberg,
- California quarterly consumption data by income quartile are constructed using CES collected by BLS,
- All other U.S. macro data (mainly used for model Calibration purposes) are obtained through Fred database.

¹⁰⁰All data used were either for the heterogenous impact of carbon pricing on households (i.e. second IV-SVAR on consumption qunatiles) were extracted directly on a quarterly basis (CES data) or transformed from a daily frequency to a quarterly frequency (california energy composite prices and sentometric data). Similarly for all the other empirical regression, data were extracted on a monthly basis except data on energy prices and sentometric data.

2.A.2 IV-SVAR Robustness

Figure 2.21 presents the results when we exclude all days with no news. The results remain robust to including no climate news days (i.e. days with zero or unknown news about climate).

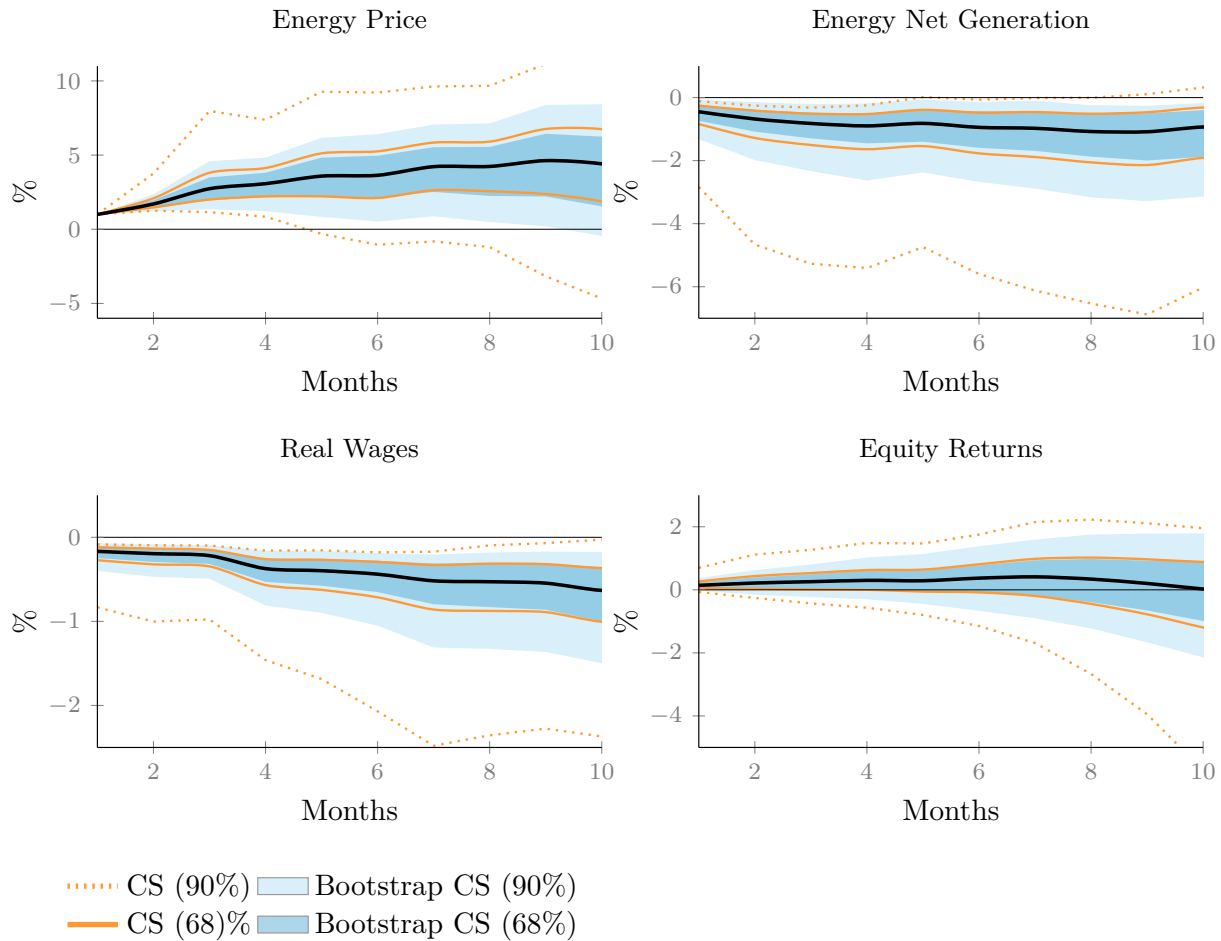
FIGURE 2.21. IV-SVAR



Note: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this robustness exercise, the carbon shock is constructed excluding all days with zero or no news.

Figure 2.22 presents the results when we exclude all days with no news under weak IV robust inference specification. The results remain robust to including no climate news days (i.e. days with zero or unknown news about climate).

FIGURE 2.22. Weak IV-SVAR



Note: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this robustness exercise, the carbon shock is constructed excluding all days with zero or no news. The inference is conducted using weak IV robust bootstrapping.

2.A.3 SVAR Model

In this section we present the structural vector auto-regressive model (SVAR), where the policy shock is used as a direct measure. The estimated SVAR reads as:

$$AY_t = \sum_{s=1}^4 B_s Y_{t-s} + C\epsilon_t \quad (2.69)$$

where variables are ranked in the following order and the following imposed restriction on the structural matrix A:

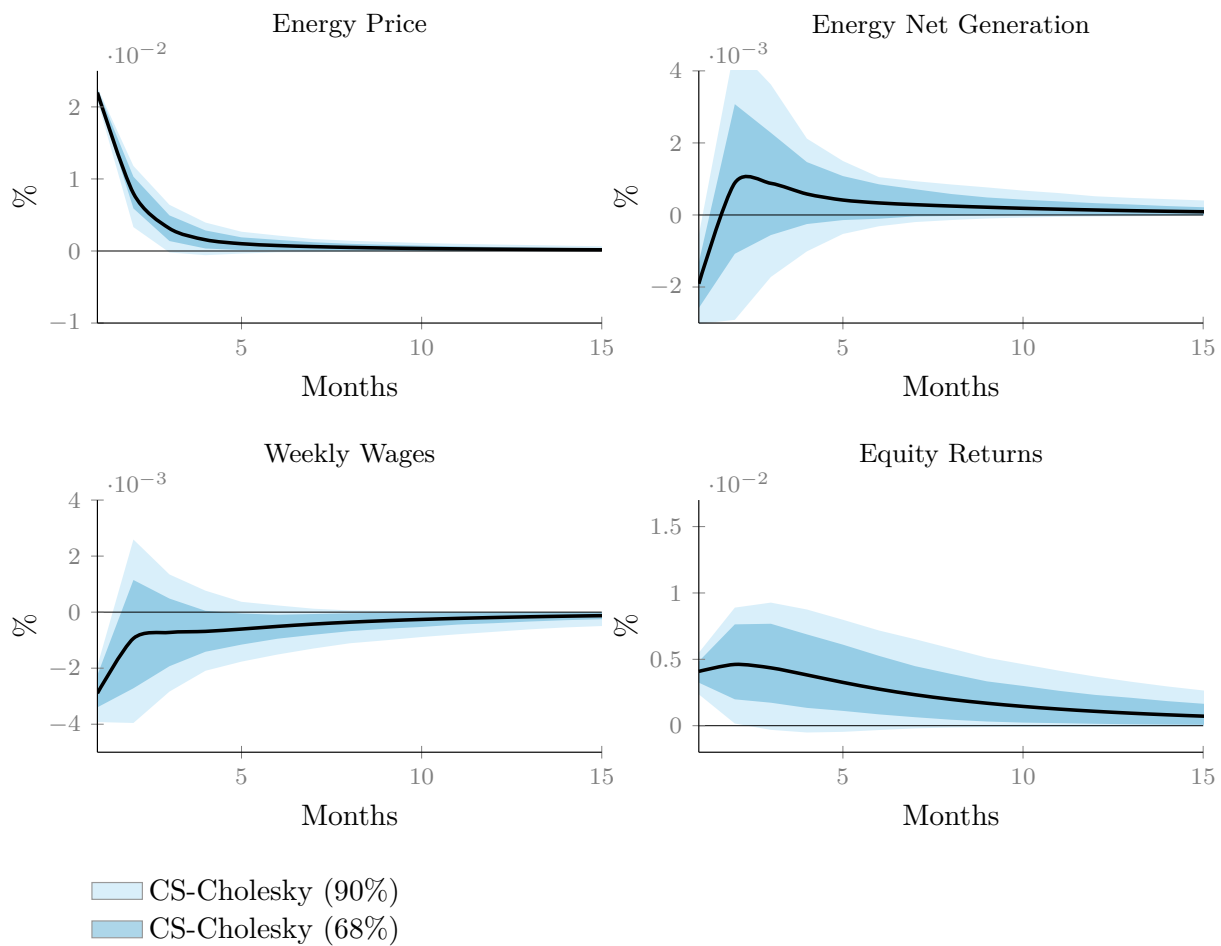
$$Y_t = \begin{pmatrix} \tau_t^C \\ P_t^{en} \\ E_t^{en} \\ W_t \\ R_t \end{pmatrix} = \begin{pmatrix} \text{Carbon Price Shock} \\ \text{Energy Prices} \\ \text{Energy Cons} \\ \text{Wages} \\ \text{Equity Return} \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix}$$

2.A.4 SVAR Results

Similarly, to the IV-SVAR results presented in the main empirical section of the paper, on impact energy prices increase, which lead to a decrease in energy net-generation, and thereafter a fall in wages and an increase in equity return. The Cholesky IRF results are aligned to the IV-SVAR results.

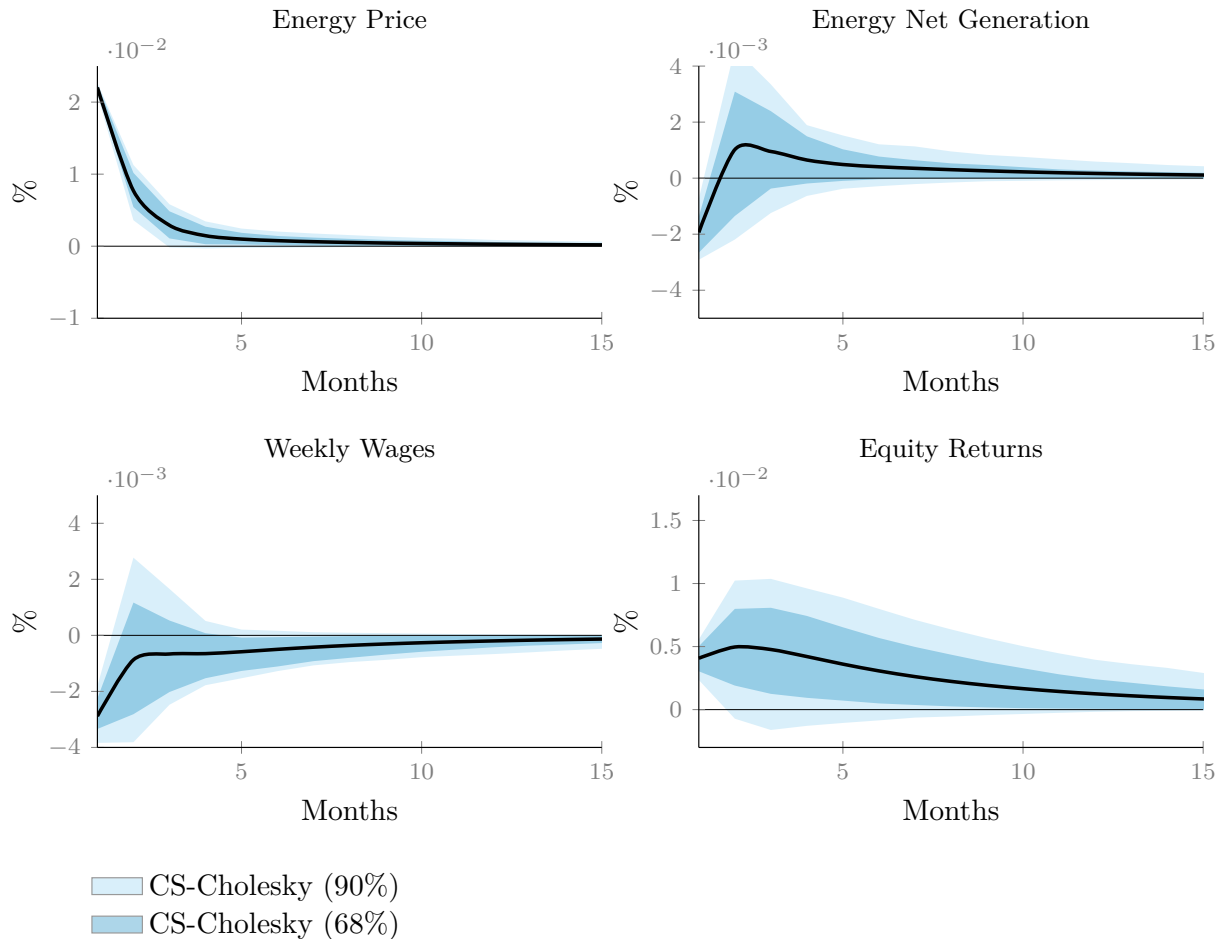
FIGURE 2.23. SVAR with Cholesky Decomposition



Note: The figure presents a 1 lagged SVAR where the carbon policy instrument (with all zero news days are included) is used as an internal instrument. We rely on the Cholesky decomposition to compute the impulse responses at both 90 percent and 68 percent confidence intervals.

Figure 2.24 presents the results of the Cholesky IRFs where we exclude days of zero news. The results remain robust to the main specification (i.e. where days with no news are included in the sample).

FIGURE 2.24. SVAR with Cholesky Decomposition



Note: The figure presents a 1 lagged SVAR where the carbon policy instrument (with all zero news days are excluded) is used as an internal instrument. We rely on the Cholesky decomposition to compute the impulse responses at both 90percent and 68 percent confidence intervals.

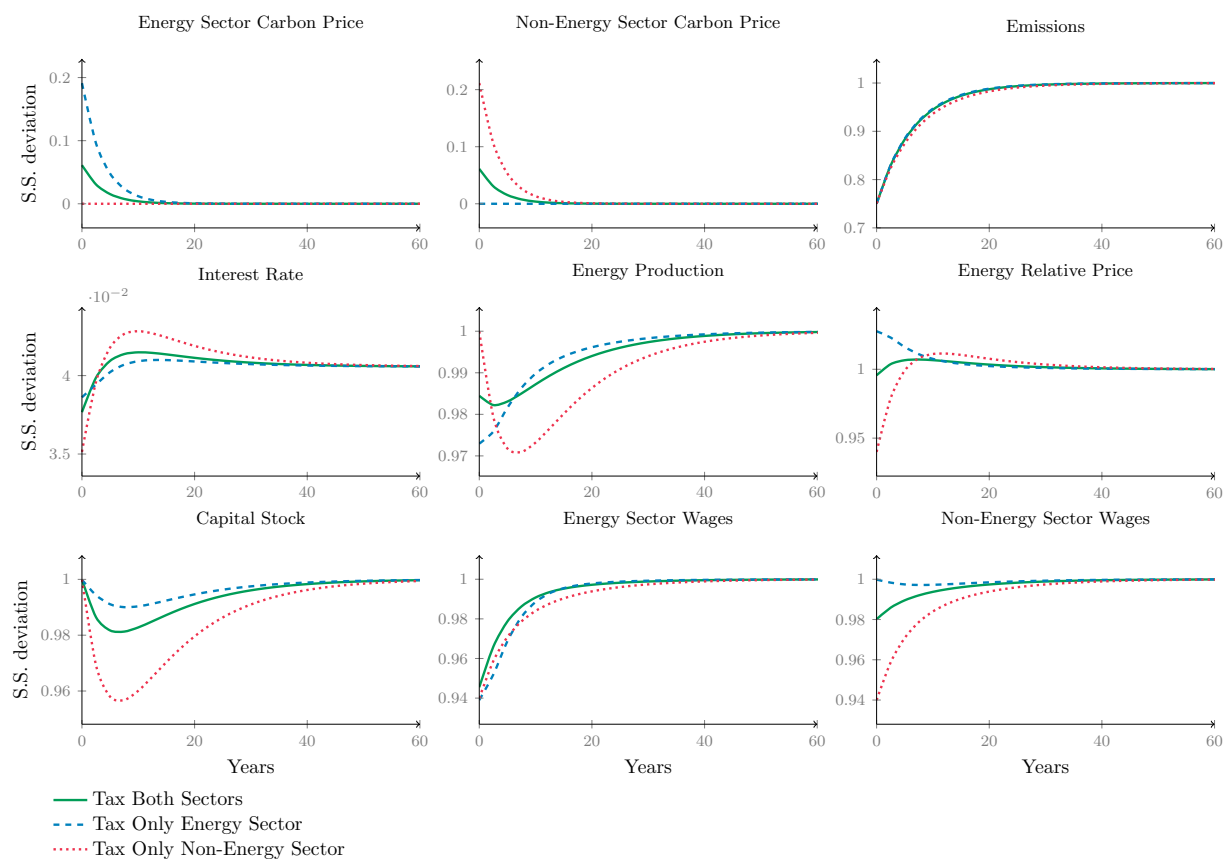
2.B Appendix: B

2.B.1 Appendix: Calibration

TABLE 2.3
Calibrated parameter values (annually)

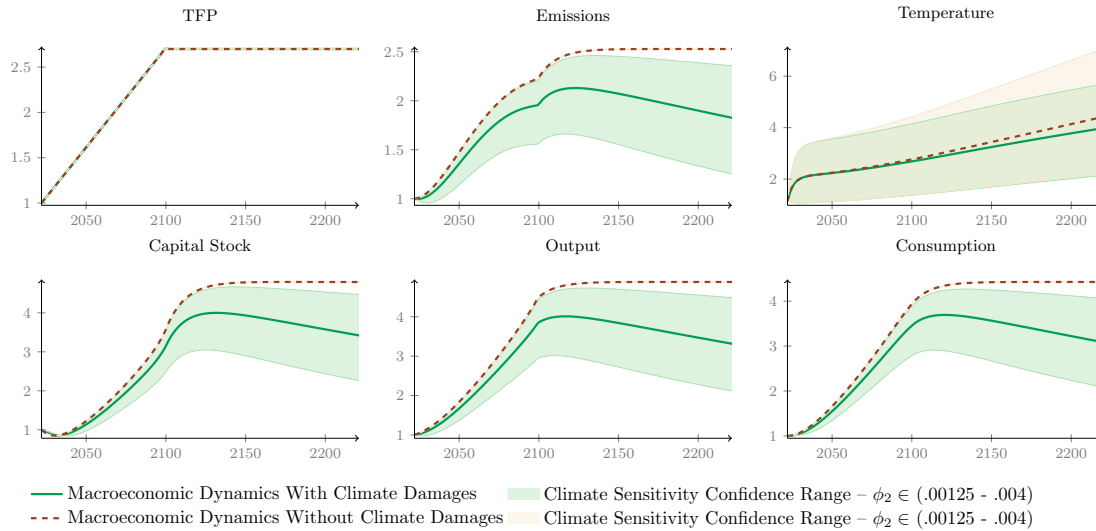
	Calibrated parameters	Values
<u>Standard Macro Parameters</u>		
α^1	Capital intensity for non-energy firms	0.19
α^2	Elasticity of energy to non-energy production	0.15
α^n	Capital intensity for energy firms	2/3
δ	Depreciation rate of capital	0.05
σ	Risk aversion	2
ρ	Discount rate	5%
θ	Price elasticity	6
\bar{L}	Labor supply	1/3
<u>Environmental Parameters</u>		
$\bar{e}^n/\bar{e}^e = \varphi^n$	Emissions-to-output ratio in energy sectors	0.3
$\bar{e}^y/\bar{y} = \varphi^y$	Emissions-to-output ratio in non-energy sectors	2
θ_1	Abatement cost parameter	0.1
θ_2	Abatement cost parameter	2.7
θ_3	Abatement learning elasticity	$\in (0,1)$
ϕ_1^o	Temperature parameter	0.5
ϕ_2^o	Temperature parameter	0.00125
a	Damage function parameter	1.004
b	Damage function parameter	0.02
<u>NK Parameters</u>		
θ^P	Rotemberg quadratic cost parameter	100
ϕ^π	Inflation stance	1.5
ϕ_Y	Output gap reaction parameter	0.1

FIGURE 2.25. Carbon Pricing and Macroeconomic Aggregates



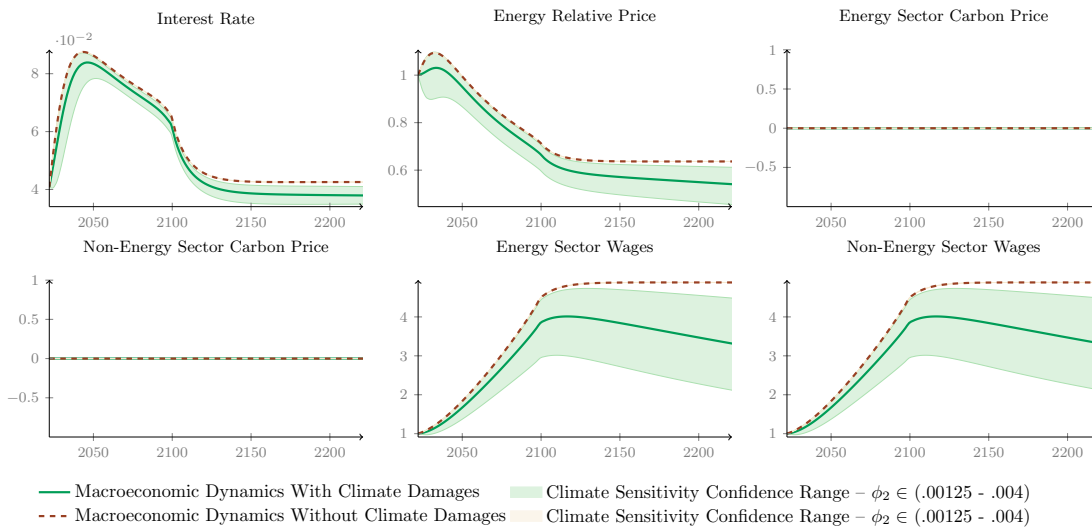
Note: The figure plots three different scenarios leading to an initial 25% reduction in total emissions. The dotted red line corresponds to the case where only the non-energy sector is taxed. The dashed blue line corresponds to the case where only the energy sector is taxed. The solid green line corresponds to the case where the tax is implemented in both sectors.

FIGURE 2.26. Climate Uncertainty and Macroeconomic Dynamics – Macro Aggregates



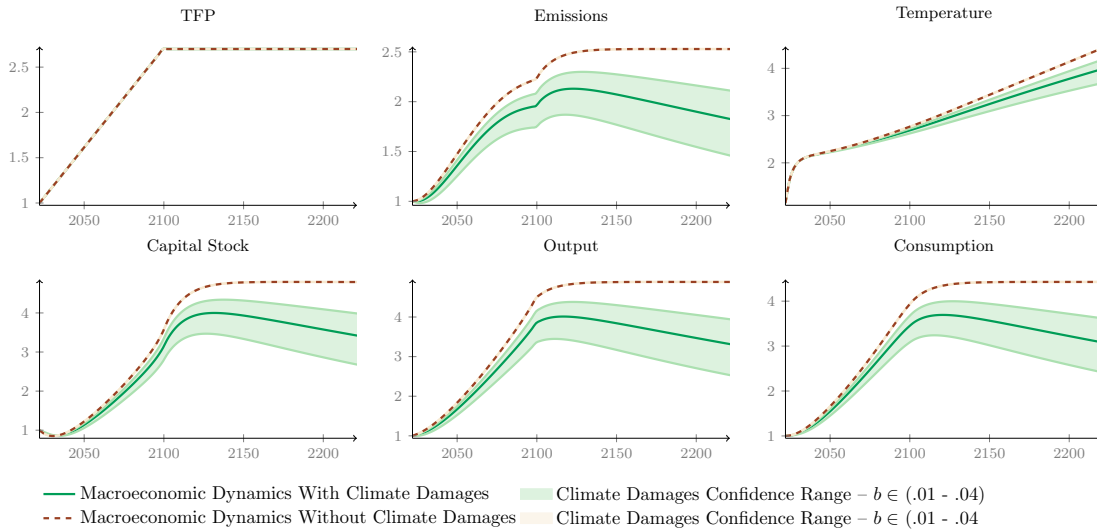
Note: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of ϕ_2 in line with IPCC scenarios.

FIGURE 2.27. Climate Uncertainty and Macroeconomic Dynamics – Prices



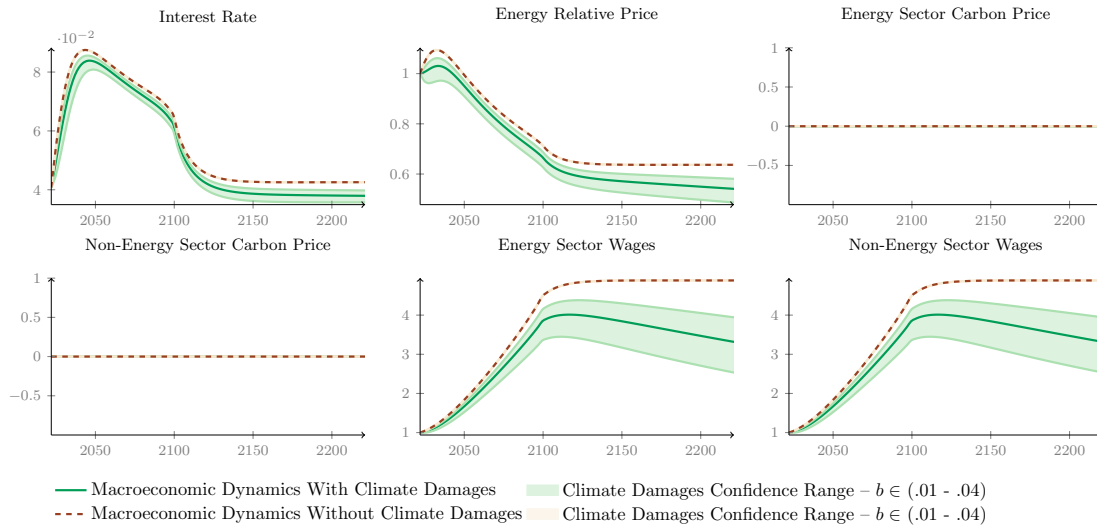
Note: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of ϕ_2 in line with IPCC scenarios.

FIGURE 2.28. Sensitivity To Climate Damages – Macro Aggregates



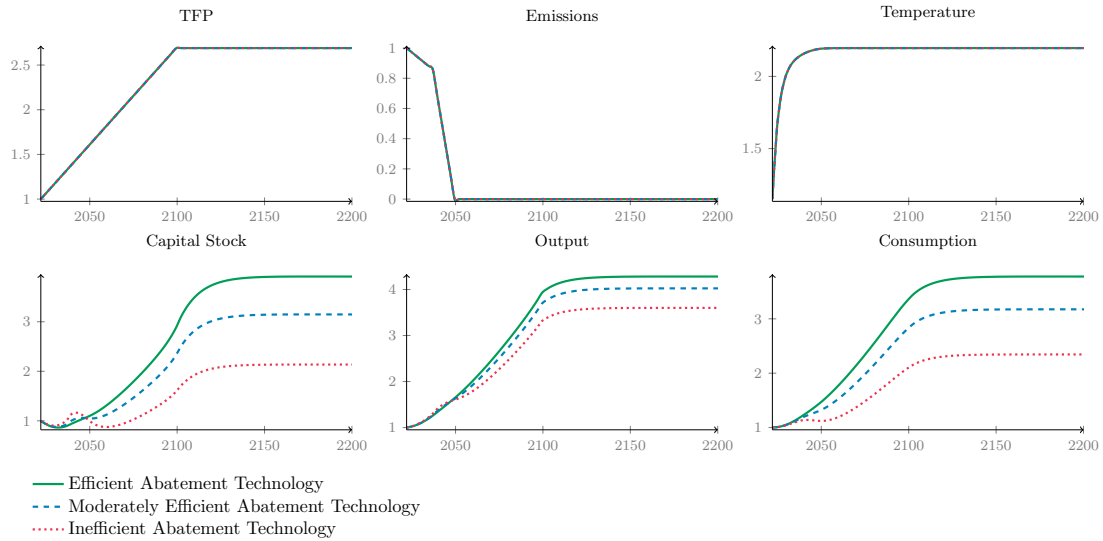
Note: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of climate damages parameter b as argues by Nordhaus, Dietz, and Weitzman.

FIGURE 2.29. Sensitivity To Climate Damages – Prices



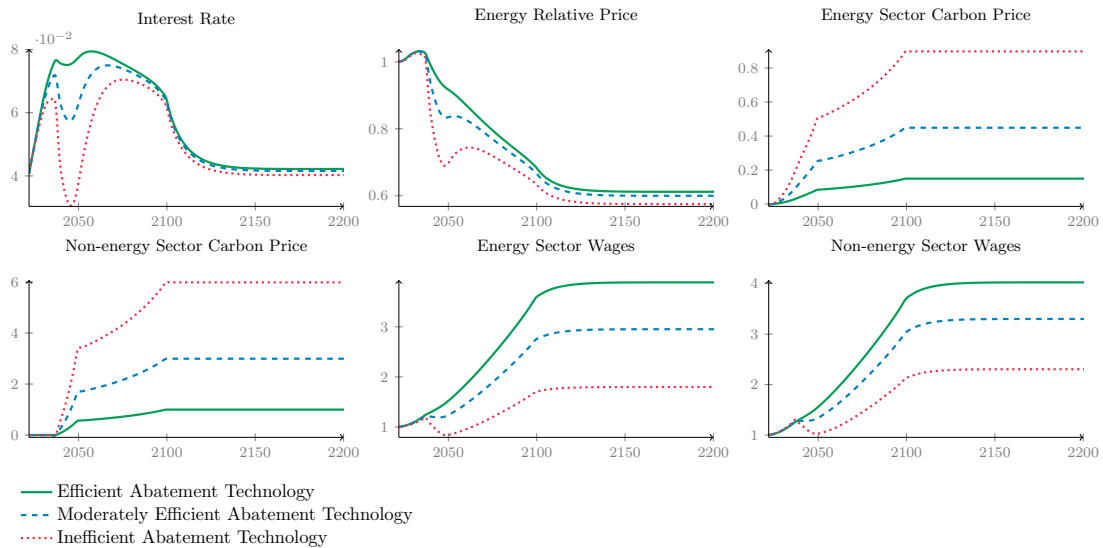
Note: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of climate damages parameter b as argues by Nordhaus, Dietz, and Weitzman.

FIGURE 2.30. Net-Zero Emission Target and Abatement Efficiency – Macro Aggregates



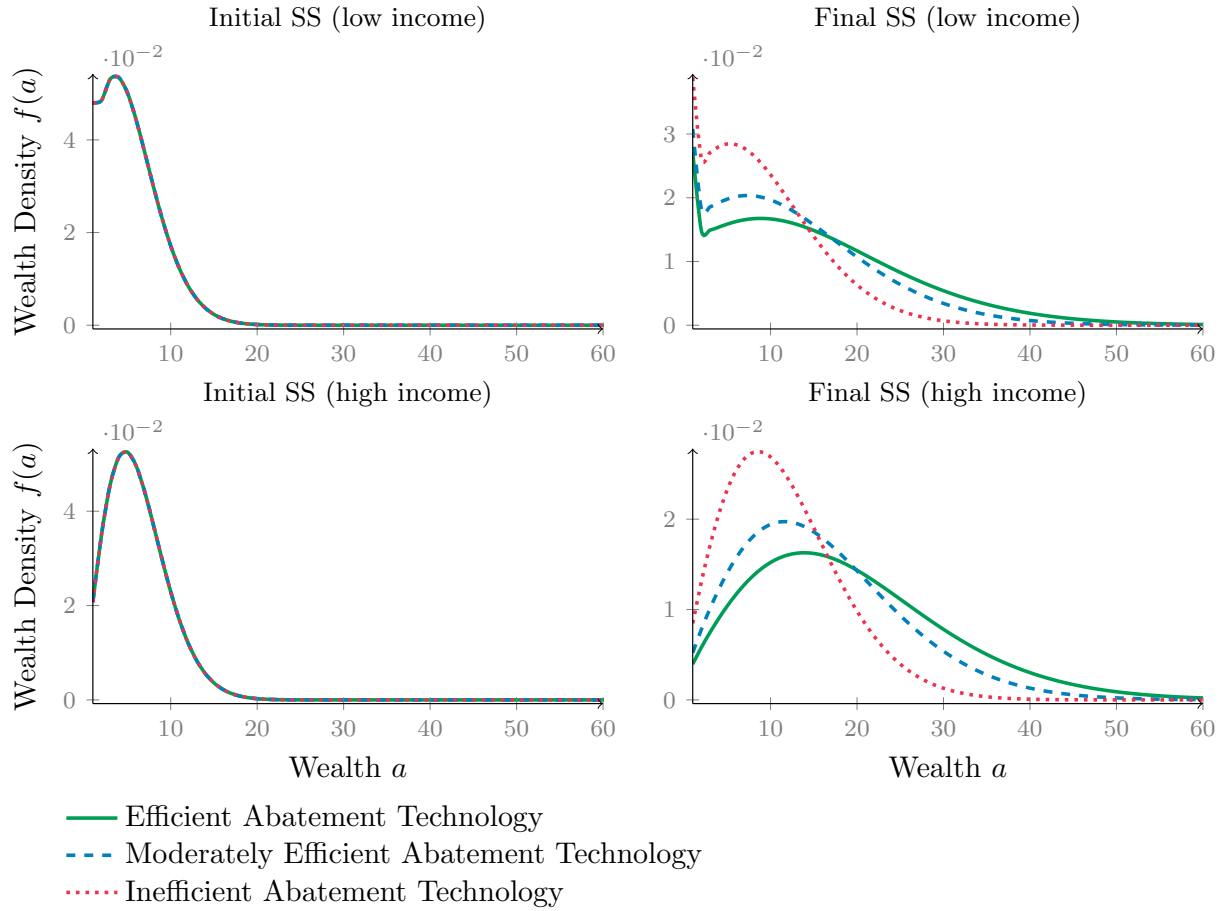
Note: The figure compares transitions under 2 percent growth rate computed using three different abatement cost efficiency levels: i) efficient abatement in green, ii) moderate abatement cost in dashed blue, and iii) inefficient abatement technology with high cost in dotted red.

FIGURE 2.31. Net-Zero Emission Target and Abatement Efficiency – Price



Note: The figure compares transitions under 2 percent growth rate computed using three different abatement cost efficiency levels: i) efficient abatement in green, ii) moderate abatement cost in dashed blue, and iii) inefficient abatement technology with high cost in dotted red.

FIGURE 2.32. Distributional Impacts of the Net-Zero For Different Abatement Efficiencies



Note: The figure compares initial and final stationary distributions computed using a model with three different abatement cost efficiency levels: i) efficient abatement in green, ii) moderate abatement cost in dashed blue, and iii) inefficient abatement technology with high cost in dotted red.

FIGURE 2.33. Net-Zero versus Laissez-faire with Efficient Abatement

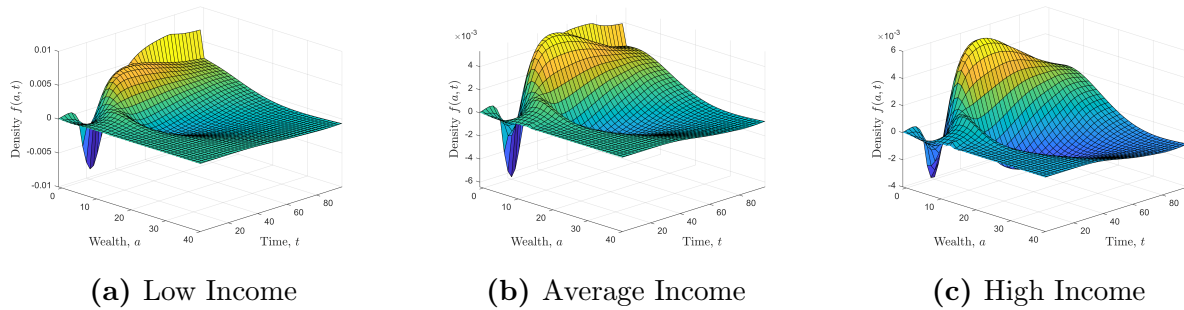


FIGURE 2.34. Net-Zero versus Laissez-faire with Medium Abatement Efficiency

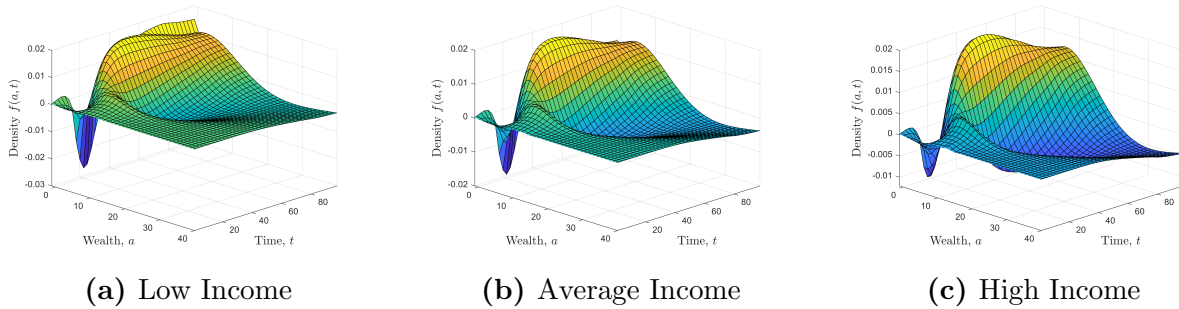
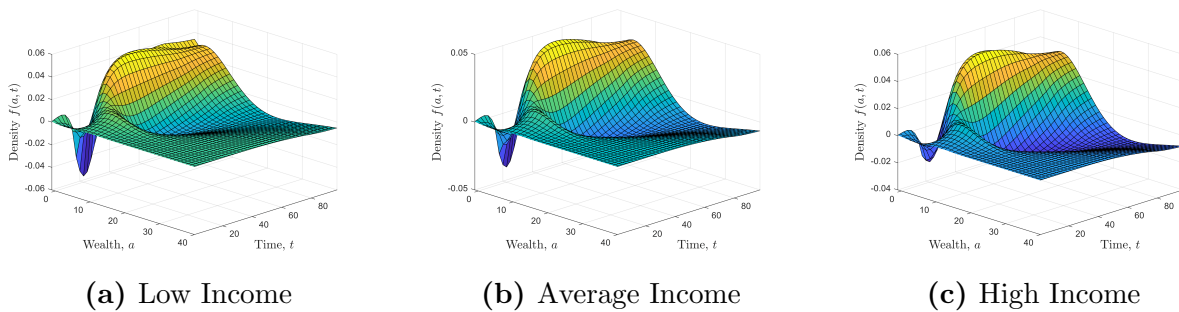


FIGURE 2.35. Net-Zero versus Laissez-faire with Inefficient Abatement



Note: This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution for three different abatement efficiency levels. For example the first row displays the results for efficient abatement costs where figure (a) show the household wealth pathway between 2022 and 2100 for low income households, figure (b) displays the results for average income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

FIGURE 2.36. Net-Zero with Uniform Fiscal Transfers versus without Transfers – Efficient Abatement

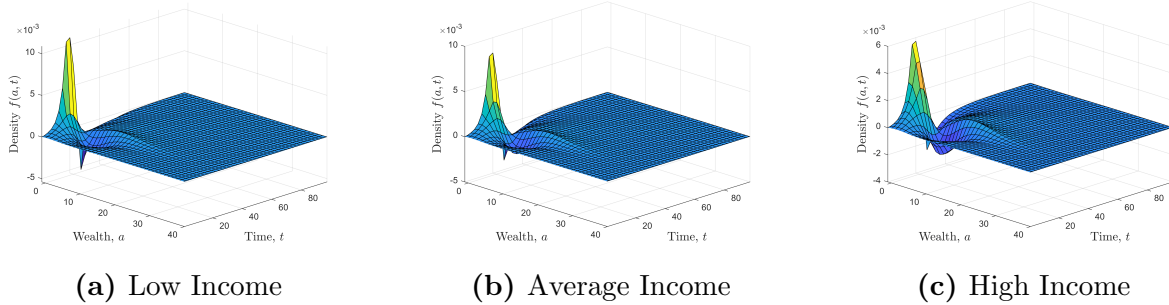
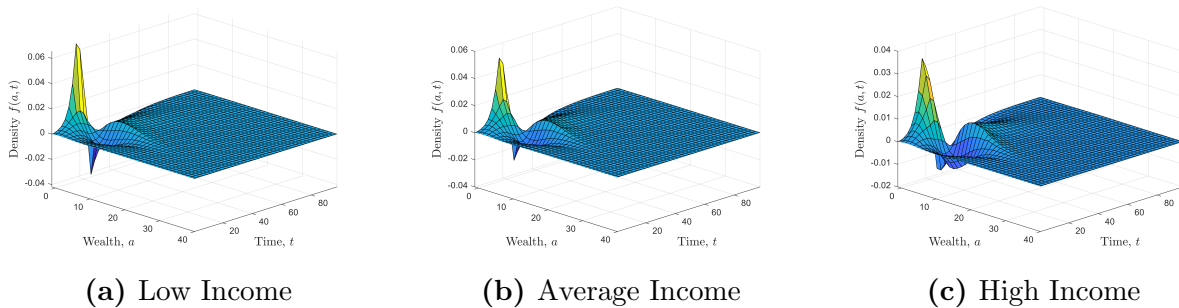


FIGURE 2.37. Net-Zero with Uniform Fiscal Transfers versus without Transfers – Inefficient Abatement



Note: This figure compares the net-zero scenario with uniformly distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution for two different abatement efficiency. For example, in the first row, figure (a) shows the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with per income distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

FIGURE 2.38. Net-Zero with Fiscal Transfers (by Income) versus without Transfers – Efficient Abatement

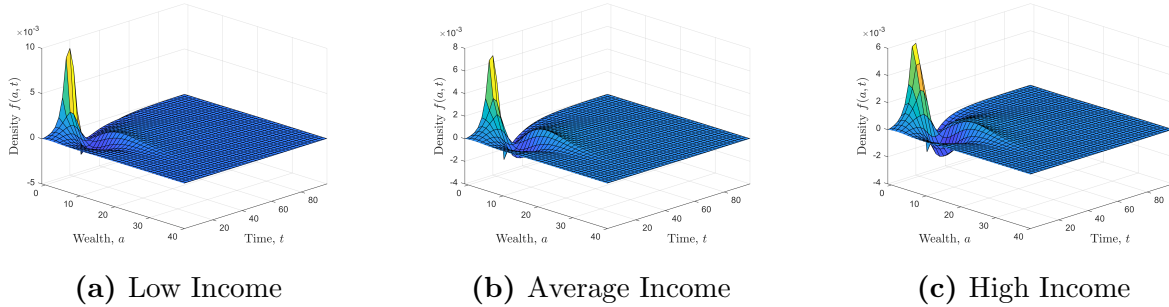
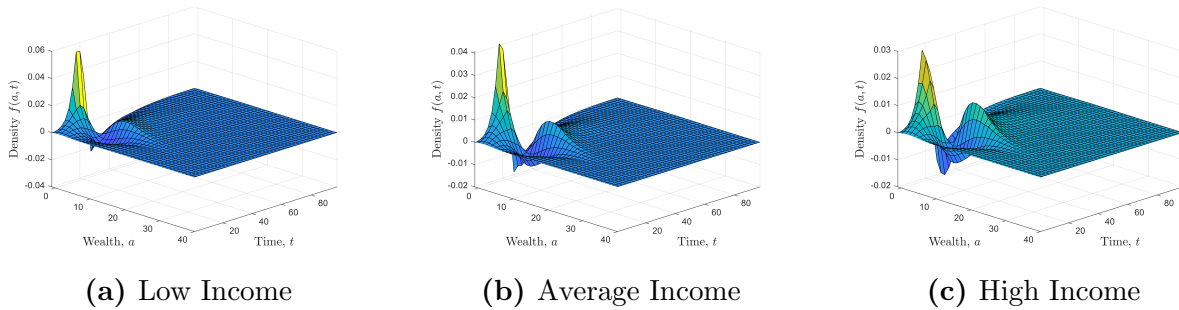


FIGURE 2.39. Net-Zero with Fiscal Transfers (by Income) versus without Transfers – Inefficient Abatement



Note: This figure compares the net-zero scenario with per income distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution for two different abatement efficiency. For example, in the first row, figure (a) shows the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with per income distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

FIGURE 2.40. Net-Zero versus Laissez-faire with A Linear Trajectory

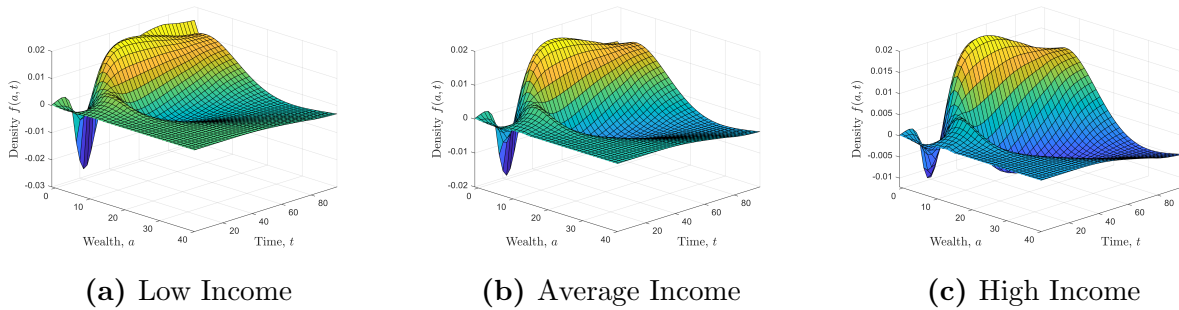


FIGURE 2.41. Net-Zero versus Laissez-faire with A Slow Trajectory

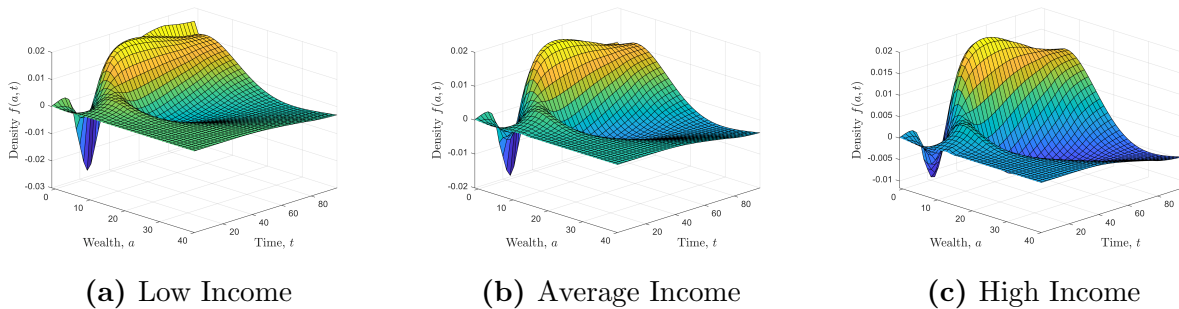
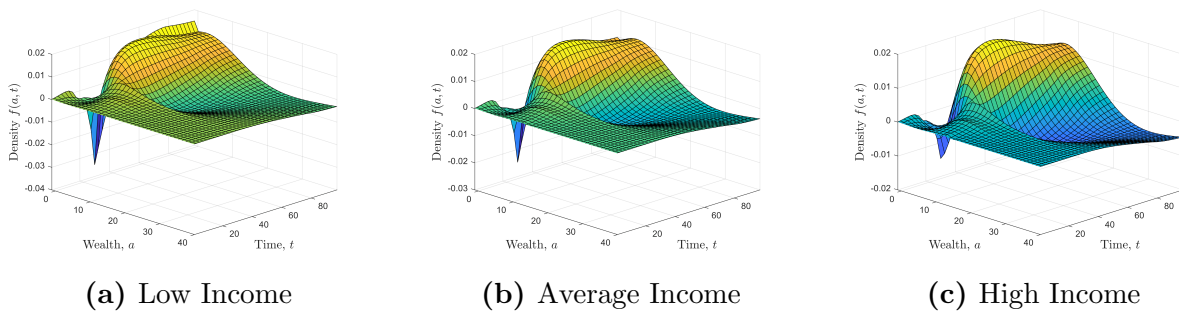


FIGURE 2.42. Net-Zero versus Laissez-faire with A Very Slow Trajectory



Note: This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution for three different emission cap trajectories (linear, slow, and very slow). For example the first row displays the results for a linear cap trajectory costs where figure (a) show the household wealth pathway between 2022 and 2100 for low income households, figure (b) displays the results for average income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

FIGURE 2.43. Net-Zero versus Laissez-faire with A Linear Trajectory

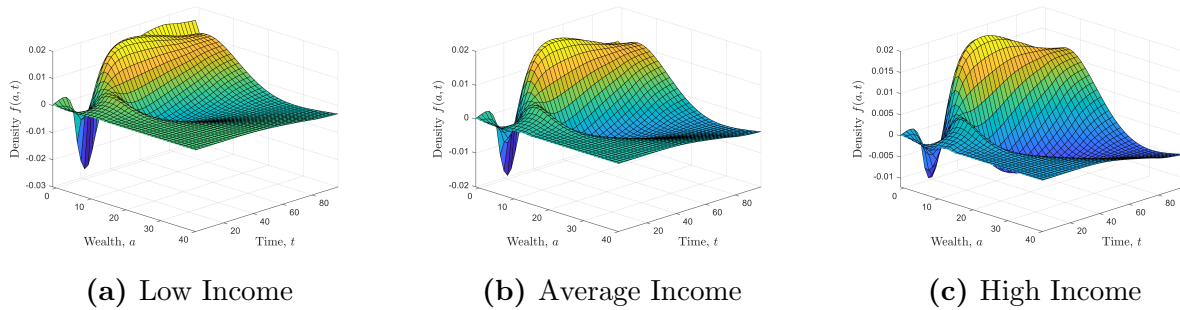


FIGURE 2.44. Net-Zero versus Laissez-faire with A Fast Trajectory

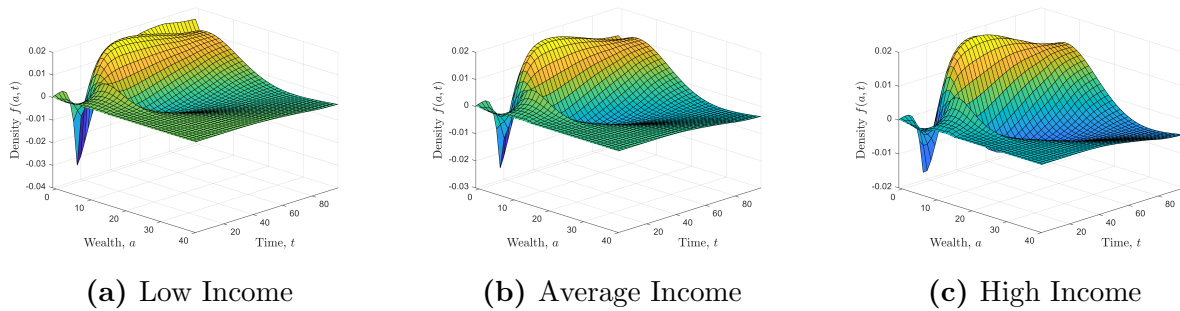
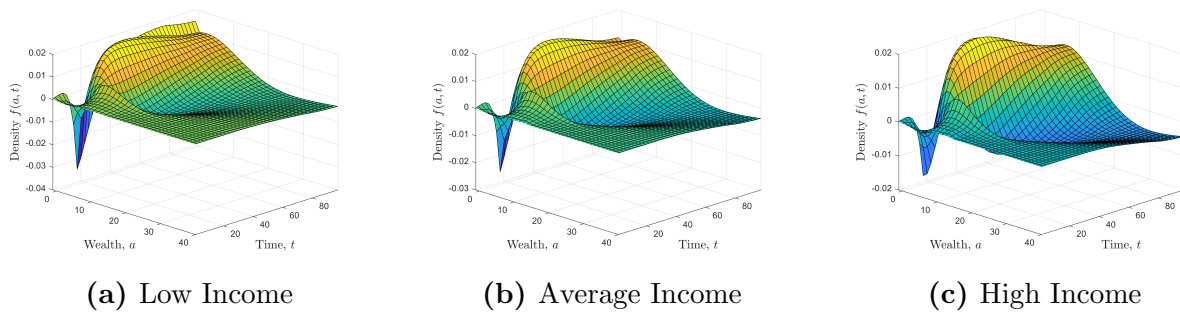


FIGURE 2.45. Net-Zero versus Laissez-faire with A Very Fast Trajectory



Note: This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution for three different emission cap trajectories (linear, fast, and very fast). For example the first row displays the results for a linear cap trajectory costs where figure (a) show the household wealth pathway between 2022 and 2100 for low income households, figure (b) displays the results for average income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

2.C Appendix: C

2.C.1 The Three Box Climate Model

The three box climate dynamics is modeled following [Cai and Lontzek, 2019] specification. First, the carbon emissions stock M_t law of motion reads:

$$\dot{M}_t = (\Phi_M - I)M_t + b_1 E_t \quad (2.70)$$

with $M_t = (M_t^{AT}, M_t^{UO}, M_t^{LO})^T$ the three-dimensional vector describing the masses of carbon concentrations in the atmosphere, and upper and lower levels of the ocean. $E_t = \sum_i \int_0^1 e_{i,j,t} dj$ is the total current concentration of carbon dioxide in the atmosphere with $e_{i,j,t}$ the intermediate firm emissions j and sector i and $b_1 = (1, 0, 0)^T$. The matrix Φ_M summarizes the relationship between the actual stocks of emissions and the pre-industrial equilibrium states of the carbon cycle system.

In addition, we define the relationship (as seen in the DICE model) between the temperature vector T_t^o (i.e. both the atmosphere and ocean temperatures) and the stock of emissions in the atmosphere M_t^{AT} as following:

$$\dot{T}_t^o = (\Phi_T - I)T_t^o + b_2 \text{RF}(M_t^{AT}) \quad (2.71)$$

with $T_t^o = (T_t^{oAT}, T_t^{oOC})^T$ and the matrix Φ_T represents the heat diffusion process between ocean and air. $b_2 = (\xi_T, 0)^T$ with ξ_T the climate sensitivity parameter. Furthermore, atmospheric temperature is affected by radiative forcing, $\text{RF}(\cdot)$, which is the interaction between radiation and atmospheric CO_2 as following:

$$\text{RF}(M_t^{AT}) = \eta_F \log_2 \left(\frac{M_t^{AT}}{\bar{M}^{AT}} \right) + \text{RF}_t^{Exo} \quad (2.72)$$

where RF_t^{Exo} represents the exogenous radiative forcing dynamic and reads as:

$$\text{RF}_t^{Exo} = \begin{cases} -0.06 + 0.0036t, & \text{for } t < 100 \\ 0.3 & \text{otherwise} \end{cases} \quad (2.73)$$

The impact of global warming on the economy is reflected by the same convex damage function of temperature in the atmosphere presented in the paper:

$$d(T_t^{oAT}) = ae^{-b(T_t^{oAT})^2} \quad (2.74)$$

2.C.2 The Non-Energy Firm Problem

The non-energy intermediate firm seeks profit maximization:

$$v(k^y, t) = \max_{p, y, i^y, l^y, \mu^y, e^n} \int_t^\infty e^{-\int_t^s r_u^y du} \Pi^F \quad (2.75)$$

subject to

$$\dot{k}_{j,t}^y = i_{j,t}^y - \delta k_{j,t}^y, \quad (2.76)$$

$$y_{j,t} = A_t d(T_t^o) k_{j,t}^y{}^{\alpha_1} e_{j,t}^n{}^{\alpha_2} l_{j,t}^y{}^{1-\alpha_1-\alpha_2}, \quad (2.77)$$

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t. \quad (2.78)$$

with profits:

$$\Pi_{j,t}^F = \frac{p_{j,t}}{P_t} y_{j,t} - w_t^y l_{j,t}^y - i_{j,t}^y - p_t^e e_{j,t}^n - f(\mu_{j,t}^y) y_{j,t} - \tau_t^y (1 - \mu_t^y) \varphi_t^y y_{j,t}$$

To solve the problem above, first we solve the cost minimization problem of choosing production inputs to minimize total cost subject to producing at least $y_{j,t}$:

$$\min_{i^y, l^y, \mu^y, e^n} \int_t^\infty e^{-\int_t^s r_u^y du} \text{Cost}^F \quad (2.79)$$

subject to

$$\dot{k}_{j,t}^y = i_{j,t}^y - \delta k_{j,t}^y, \quad (2.80)$$

$$A_t d(T_t^o) k_{j,t}^y \alpha_1 e_{j,t}^n \alpha_2 l_{j,t}^y 1^{-\alpha_1 - \alpha_2} \geq y_{j,t}. \quad (2.81)$$

where,

$$\text{Cost}^F = w_t^y l_{j,t}^y + i_{j,t}^y + p_t^e e_{j,t}^n + f(\mu_{j,t}^y) y_{j,t} + \tau_t^y (1 - \mu_t^y) \varphi_t^y y_{j,t}. \quad (2.82)$$

The optimality conditions are:

$$\lambda_t^y = 1 \quad (2.83)$$

$$r_t^y \lambda_t^y - \dot{\lambda}_t^y = \alpha_1 \frac{y_t}{k_t^y} \varrho_t^y - \delta \lambda_t^y, \quad (2.84)$$

$$p_t^e = \varrho_t^y \alpha_2 \frac{y_t}{e_t^n}, \quad (2.85)$$

$$w_t = \varrho_t^y (1 - \alpha_1 - \alpha_2) \frac{y_t}{l_t^y}, \quad (2.86)$$

$$\tau_t^y = \frac{f(\mu_t^y)'}{\varphi_t^y}, \quad (2.87)$$

where λ_t^y is the co-state, while the ϱ_t^y is the shadow value of input costs. In addition, the transversality condition reads:

$$\lim_{t \rightarrow \infty} k_{j,t}^y \lambda_t^y e^{-\int_0^t r_u^y du} \leq 0 \quad (2.88)$$

Using these first order conditions and the expression of profits ($\Pi^F = \left(\frac{p_{j,t}}{P_t} - mc_t\right) y_{j,t}$) we can then retrieve the expression of the total marginal cost $mc_t = \varrho_t^y + f(\mu_t^y) + \tau_t^y \varphi_t^y (1 - \mu_t^y)$.

Furthermore, using [equation \(2.22\)](#) as well as [equation \(2.16\)](#), we can derive the marginal cost and profit of the firms by solving the firms maximization problem:

Case of flexible prices (i.e. Real Business Cycles)

$$v(p_j, t) = \max_{p_j} \int_t^{\infty} e^{-\int_t^s r_u^y du} \Pi^F \quad (2.89)$$

s.t.

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t. \quad (2.90)$$

where,

$$\Pi^F = \left(\frac{p_{j,t}}{P_t} - mc_t \right) y_{j,t}. \quad (2.91)$$

The first order condition yields the price level p_t as firms are all identical (i.e. $p_{j,t} = p_t$)

:

$$\frac{p_t}{P_t} = \frac{\theta}{\theta - 1} mc_t \quad (2.92)$$

Using the symmetric equilibrium condition where $P_t = p_t$, we can rewrite the marginal cost and profits as follows:

$$mc_t = \frac{\theta - 1}{\theta} \quad (2.93)$$

$$\Pi_t^F = (1 - mc_t) Y_t \quad (2.94)$$

Case of sticky prices (i.e. New-Keynesian)

$$v(p_j, t) = \max_{p_j} \int_t^{\infty} e^{-\int_t^s r_u^y du} \left(\tilde{\Pi}^F - \Delta^P \right) \quad (2.95)$$

where,

$$\tilde{\Pi}_{j,t}^F = \left(\frac{p_{j,t}}{P_t} - mc_t \right) \left(\frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t, \quad (2.96)$$

$$\Delta_{j,t}^P = \frac{\theta^P}{2} \left(\frac{\dot{p}_{j,t}}{p_{j,t}} \right)^2 Y_t. \quad (2.97)$$

The Hamiltonian of this problem (where we drop j for ease for writing as all firms are subject to same input costs) reads as:

$$H(p, \dot{p}, \lambda^p, t) = \tilde{\Pi}_t^F - \Delta_t^P + \lambda_t^p \dot{p}_t \quad (2.98)$$

The first order condition yields:

$$\lambda_t^p = \theta^P \frac{\dot{p}_t}{p_t} \frac{P_t}{p_t} Y_t, \quad (2.99)$$

$$\dot{\lambda}_t^p = r_t^y - \left((1 - \theta) \frac{p_t}{P_t} Y_t + \theta \frac{mc_t}{p_t} \left(\frac{p_t}{P_t} \right)^{-\theta} Y_t + \theta^P \left(\frac{\dot{p}_t}{p_t} \right)^2 \frac{P_t}{p_t} Y_t \right). \quad (2.100)$$

where λ_t^p is the co-state.

Using the symmetric equilibrium condition once again ($P_t = p_t$) and setting inflation $\pi_t = \frac{\dot{p}_t}{p_t}$, we can rewrite the optimality conditions as follows:

$$\lambda_t^p = \theta^P \pi_t Y_t, \quad (2.101)$$

$$\dot{\lambda}_t^p = r_t^y \lambda_t^p - \left((1 - \theta) Y_t + \theta \frac{mc_t}{P_t} Y_t + \theta^P \pi_t^2 Y_t \right). \quad (2.102)$$

Differentiating the first optimality condition with respect to time, we get:

$$\theta \dot{\pi}_t Y_t + \theta \pi_t \dot{Y}_t = \dot{\lambda}_t^p, \quad (2.103)$$

Finally we substitute this last equation into the equation for co-state and rearrange to get:

$$\left(r_t^a - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\theta}{\theta^P} (mc_t - mc^*) + \dot{\pi}_t \quad (2.104)$$

where, $mc^* = \frac{\theta-1}{\theta}$.

Finally, firms profit after price adjustment costs read as follows:

$$\Pi_t^F = (1 - mc_t)Y_t - \frac{\theta^P}{2} \pi_t^2 Y_t. \quad (2.105)$$

2.C.3 The Energy Firm Problem

Similar to the non-energy intermediate firms, the energy firms problem reads as:

$$v(k^n, t) = \max_{i^n, k^n, l^n, \mu^n} \int_t^\infty e^{-\int_t^s r_u^e du} \Pi^E \quad (2.106)$$

subject to

$$\dot{k}_{j,t}^n = i_{j,t}^n - \delta k_{j,t}^n \quad (2.107)$$

and where:

$$\begin{aligned} \Pi_{j,t}^E = & p_t^e A_t^n k_{j,t}^{\alpha_n} l_{j,t}^{1-\alpha_n} - w_t^n l_{j,t}^n - i_{j,t}^n \\ & - f(\mu_{j,t}^n) A_t^n k_{j,t}^{\alpha_n} l_{j,t}^{1-\alpha_n} - \tau_t^n \varphi_t^n (1 - \mu_t^n) A_t^n k_{j,t}^{\alpha_n} l_{j,t}^{1-\alpha_n} \end{aligned} \quad (2.108)$$

The Hamiltonian of this problem reads as:

$$H(\cdot, \lambda^n, t) = \Pi^E + \lambda_t^n (i_t^n - \delta k_t^n) \quad (2.109)$$

The optimality conditions are:

$$\lambda_t^n = 1 \tag{2.110}$$

$$r_t^e \lambda_t^n - \dot{\lambda}_t^n = \alpha_n \frac{e_t^n}{k_t^n} (p_t^e - f(\mu_t^n) - \tau_t^n \varphi_t^n (1 - \mu_t^n)) - \delta \lambda_t^n, \tag{2.111}$$

$$w_t^n = (1 - \alpha_n) \frac{e_t^n}{l_t^n} (p_t^e - f(\mu_t^n) - \tau_t^n \varphi_t^n (1 - \mu_t^n)), \tag{2.112}$$

$$\tau_t^n = \frac{f(\mu_{j,t}^n)'}{\varphi_t^n}, \tag{2.113}$$

and the transversality condition:

$$\lim_{t \rightarrow \infty} k_{j,t}^n \lambda_t e^{-\int_0^t r_u^e du} \leq 0 \tag{2.114}$$

We can then note that $q_t^e = p_t^e - f(\mu_t^n) - \tau_t^n \varphi_t^n (1 - \mu_t^n)$ is the energy production input cost.

2.C.4 Welfare Analysis

We measure the welfare gain of the net-zero policy compared to the the laissez-faire equilibrium, using a standard consumption-equivalent welfare metric, which we denote by Δ . Denoting the equilibrium allocation under laissez-faire with ‘LF’ and Net-Zero by ‘NZ’, Δ solves:

$$\mathbb{E}_0 \left(\int_0^\infty e^{-\rho t} u((1 + \Delta) c_t^{LF}) dt | (a_0, z_0^y, z_0^n) = (a, z^y, z^n) \right) dg_0^{LF}(a, z^y, z^n) \tag{2.115}$$

$$= \mathbb{E}_0 \left(\int_0^\infty e^{-\rho t} u(c_t^{NZ}) dt | (a_0, z_0^y, z_0^n) = (a, z^y, z^n) \right) dg_0^{NZ}(a, z^y, z^n) \tag{2.116}$$

with

$$v_0^{NZ}(a, z_j^y, z_j^n, t) = \mathbb{E}_0 \left(\int_0^\infty e^{-\rho t} u(c_t^{NZ}) dt | (a_0, z_0^y, z_0^n) = (a, z^y, z^n) \right) dg_0^{NZ}(a, z^y, z^n) \quad (2.117)$$

Using the functional form of the utility function $u((1 + \Delta)c_t^{LF}) = (1 + \Delta)^{1-\sigma} u(c_t^{LF})$ we can simplify the above equation as follows:

$$(1 + \Delta)^{1-\sigma} v_0^{LF} = v_0^{NZ} \quad (2.118)$$

$$\Delta = \left(\frac{v_0^{NZ}}{v_0^{LF}} \right)^{\frac{1}{1-\sigma}} - 1 \quad (2.119)$$

Chapter 3

Endogenous Abatement Technology

This chapter was presented in the poster session of the Annual AEA Meeting.

3.1 Introduction

While carbon pricing is the major tool used in climate mitigation policies nowadays, this policy is not a free lunch as it induces unintended effects. In Europe, Canada, and California (US), as well as elsewhere, governments have opted for a market cap-and-trade system instead of a targeted price, where carbon permits are traded, which facilitates the attainment of desired emission level reductions. As, this market design is not set optimally from a welfare perspective and is subject to market volatility and business cycle fluctuations, a number of inefficiencies arise (e.g. financial transition risk, welfare losses, net-zero distributional costs, and risk premium distortions as highlighted in [Benmir and Roman, 2020] and [Benmir and Roman, 2022]). In order to address the inefficiencies induced by such a carbon market design, finding ways to steer green innovation without solely relying on increasing carbon pricing becomes a major priority.

The goal of this paper is explore the role of financial institutions and macro-financial policy in steering green innovation. First, we seek to empirically investigate the different linkages between green innovation, fiscal and financial carbon policies. Second, we show on how fiscal and/or financial policies could help steer some of the main drivers that contribute to the next zero carbon emissions transition. To do so, we build a quantitative model to address the evidence and provide a framework that allows for analyzing the role of various green innovation policies in the transition to a low carbon economy.

With respect to the first part, we rely on empirical data on the Euro Zone (EZ), the US, and a panel of the 19 EZ countries. We find that, macro-financial factors (e.g. long-term loans) contribute to higher levels of green innovation as it is the case for carbon pricing.

Regarding the second part of the paper, the model introduces two modifications to the standard real business cycle economy: i) it explicitly accounts for the process of endogenous green innovation by lowering the cost of abatement; ii) it includes an agency friction in financial markets that may disrupt the financing of investments in innovation à la

[Queralto, 2020]). Endogenous green innovation financed by the banking sector allow for the substantial emissions reduction by triggering higher levels of abatement, and without subsequently relying on increasingly higher levels of carbon pricing.

In the spirit of [Romer, 1990], [Acemoglu et al., 2012], and [Anzoategui et al., 2019], we introduce sustained growth in green R&D arising from an endogenously expanding variety of green technologies. Green entrepreneurs invest in projects that could lead to an improvement of the green technology, but lacks the funds to finance the necessary expenditures. When it is successful, the green technology allows firms to abate at a cheaper cost, which in turn lower emissions. To obtain funds, our green firms borrow from banks. The outcome from green innovation efforts consists of novel varieties of intermediate goods, which are then used in final abatement efforts.

The main quantitative application of our model is to explore the EZ net-zero transition pathways, under the presence of green innovation boosting policies (i.e. fiscal and financial). Three main reasons are behind the focus on the EZ. First, the ETS (European Trading System) carbon pricing market is the most advanced environmental fiscal policy in the world. Second, the European Union (EU) global strategy in emissions reduction is moving toward finding ways in which green innovation could be steered more efficiently. Finally, the availability of data allows for running both empirical exercises and counterfactual scenarios.

Using a macro-financial framework as a foundation, the present paper builds on [Heutel, 2012], [Fischer and Springborn, 2011], and [Golosov et al., 2014], among others, to account for the effect of the environmental externality on the economy, while also following [Gertler and Karadi, 2011] to model financial intermediaries. The novelty of the model is that we introduce green innovators in the spirit of [Romer, 1990], [Comin and Gertler, 2006], and [Acemoglu et al., 2012]. The main divergences of our paper with this literature are that: i) endogenous growth in green R&D directly impacts the abatement technology by making it cheaper, thus triggering higher abatement levels, ii) green innovators need to obtain funds from financial intermediaries to set up projects as in [Anzoategui et al., 2019] and

[Queralto, 2020], and iii) we estimate the model trends and endogenous growth structural parameters using data on R&D and green innovation patents expenditure.

The paper is divided into three main sections: i) an empirical analysis on the linkages between green R&D, and macro-financial factors; and ii) an analysis of output and green innovation trends as well as net-zero pathways, using a full fledged estimated model with both financial intermediaries and an endogenously-determined abatement technology.

3.2 Motivational Evidence: Emission, Carbon Pricing, and Green Innovation

3.2.1 Data

Data used¹⁰¹ in this section were obtained from the ECB Statistical Data Warehouse, Eurostat database, FRED, and European Patent Office (EPO) database.¹⁰²

The data set includes series from all 19 EZ countries with data spanning from the first quarter (Q1) of year 2000 to the last quarter (Q4) of year 2019.

Table 3.2 presents the descriptive statistics for the data set we use in our empirical analysis (i.e. the Panel OLS on the EZ 19 countries). We use green patents, ETS futures prices, long-term loans granted by the financial sector to domestic non-financial corporations, and macro aggregates (e.g. output), which are end of the date quarterly, and in millions of currency. We focus on the 19 EZ countries. We add lags (4, 8, and 12) to the ETS carbon price and to the long-term loans, as this represents the time for both the fiscal and financial policies to impact green innovation.

¹⁰¹All data were either extracted directly on a quarterly basis or transformed from a monthly frequency to a quarterly frequency.

¹⁰²For a detailed list of data used and treatment, please refer to the appendix, [subsection 3.A.1](#).

3.2.2 Green Innovation: EZ Panel OLS Analysis

To the assessment of the impacts of both fiscal and macro-financial variables on green innovation (i.e. green patents), we use a pool of panel data from the 19 EZ countries. The focus of our analysis is on the fiscal (ETS carbon pricing) and financial (long-term credits to non-financial firms) impacts on green innovation. Unfortunately, due to scarcity of data on green subsidies for the EZ, we are unable to clearly show the impact of such policies on green patenting. However, different studies (e.g. [Bai et al., 2019]) show the positive and significant impact of such fiscal tools in facilitating green innovation.

Previous papers—such as [Acemoglu et al., 2012] and [Aghion et al., 2016], which (using panel data) assess the impacts of carbon policies (via subsidies or taxes) on fuel prices and clean innovation, or such as [Acemoglu et al., 2019], which rely on diff-in-diff between the US and the EU to assess shell gas discovery and its impact on patents and green innovation—fail to capture the impacts of macro-financial variables on R&D. As such, we focus on investigating both fiscal and macro-financial drivers of green innovation.

Understanding the role macro-financial variables could play in steering green innovation could become a major tool in mitigating climate change and efficiently reducing emissions. In this empirical assessment, we conduct a panel regression analysis to investigate the role financial loans could play in boosting green patents. We start our analysis from Q1 of 2008 to Q4 of 2019 in order to have a balanced panel sample for all the EZ 19 countries, as data on the ETS carbon pricing are only available from 2008. Then to assess the drivers of green innovations, which we proxy through green patents, we regress the green patents for each of the EZ countries on both the ETS prices and macro-financial indicators, namely the long-term loans, as well as on a number of macro controls, time fixed and country fixed effects:

$$GreenPatent_{i,t} = \beta_1 ETS_{i,t} + \beta_2 FI_{i,t} + \sum_i \beta_i X_{i,t} + T_t + State_i + \epsilon_{i,t} \quad (3.1)$$

Table 3.1 results suggest both a significant and positive role of the ETS price system as well as the long term credit trends for boosting green innovation. The results are consistently significant as we run robustness checks with different timing lags for both the ETS carbon price and for long-term loans, and they underscore the importance of conducting more R&D.

Output is found to play an important role, suggesting that the stronger the economic growth, the higher the levels of green innovation. This is inline with the finding of [Song et al., 2015], where green innovation benefits from the positive spillovers of economic growth.

TABLE 3.1
Green Innovation Drivers: Panel OLS Regression

Green R&D	(1)	(2)	(3)
ETS Price Level (1 year lag)	22.65* (12.92)		
Long-term Loan (1 year lag)	0.0801*** (0.0149)		
ETS Price Level (2 years lag)		7.882* (4.167)	
Long-term Loan (2 years lag)		0.0990*** (0.0140)	
ETS Price Level (3 years lag)			7.761** (3.724)
Long-term Loan (3 years lag)			0.112*** (0.0140)
GDP per capita	1.502*** (0.290)	1.474*** (0.350)	1.442*** (0.422)
Constant	-772.8** (339.0)	-392.9*** (119.8)	-389.4*** (119.9)
Observations	772	700	628
R-squared	0.969	0.970	0.968
Time fixed effect	Y	Y	Y
Country fixed effect	Y	Y	Y

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notes: The regression features both time and countries fixed effects that are not reported for simplicity.

3.3 General Framework

In this section we present a standard endogenous growth model enhanced with an environmental externality à la [Dietz and Venmans, 2019] and the possibility of emission abatement for firms. We assume that this abatement technology can be improved exogenously. The goal is to check whether the model is able to replicate the empirical finding presented above, and perform a forecast simulation for the EZ. In the next section, we will show how it is possible to endogenize the cost and efficiency of the abatement technology.

In a nutshell, the economy modeled is described using a discrete set up with time $t \in (0, 1, 2, \dots, \infty)$. The production sector produces two goods (final and intermediate goods) using labor and capital. Households consume, offer labor services, and rent out capital to firms. Public authorities decide on the fiscal and environmental policy.

3.3.1 The Household

The economy is populated by a continuum of measure one of households, and each household has a unit measure of members. Households make decisions on consumption, labor supply, investment in physical capital and saving through a risk-free one-period international bond. A fraction of the workers are specialized workers L_s who supply labor inelastically to entrepreneurs. Regular workers in a household are monopolistic suppliers of a differentiated specific labor type, used to produce intermediate goods. Both types of labor return wages to the family. Profits made by firms are paid to entrepreneurs for their innovative ideas.

We note that, as it is highlighted by [Benhabib et al., 1991], [Jaimovich and Rebelo, 2009], and [Queralto, 2020], the business cycle literature typically features preferences with $\Gamma_t = 1$ for all t . These business cycle frameworks assume no long-run growth. However, as we are also interested in the transition pathways, Γ_t cannot be considered as constant for the long-run simulations. Thus, it is important to consider trend growth in hours worked.

In addition, the presence of $\Gamma_t^{1-\sigma}$ ¹⁰³ ensures a balanced growth path with constant hours. Furthermore, as long as the volatility of the growth rate $(\gamma^Y)^{1-\sigma}$ is small, fluctuations in $\Gamma_t^{1-\sigma}$ will have a small impact on labor supply at medium frequencies, consistent with the usual formulation of GHH preferences.

The household maximization problem reads:

$$\max_{\{C_t, I_t, K_{t+1}, L_t, B_{t+1}\}} E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} \Gamma_t^{1-\sigma} L_{t+i}^{1+\varphi} \right], \quad (3.2)$$

s.t.

$$C_t + B_{t+1} + I_t + f(K_t, I_t) = W_t L_t + W_{s_{t,s}} \bar{L} s^s + T_t + R_t B_t + R_t^K K_t \quad (3.3)$$

$$K_{t+1} = (1 - \delta) K_t + I_t \quad (3.4)$$

where $\beta \in (0, 1)$ is the discount factor, parameters $\sigma, \varphi > 0$ shape the utility function of the representative household associated with risk consumption C_t , and labor L_t . The consumption index C_t is subject to external habits with degree $h \in [0; 1)$ while $\chi > 0$ is a shift parameter allowing us to pin down the steady state amount of hours worked. Labor supply L_t is remunerated at real wage W_t . As we assume that government bonds are one period bonds, $R_t B_t$ is interest received on bonds held and B_{t+1} is bonds acquired. Households also choose the level of investment I_t and lend capital K_t at a return rate R_t^K . Adjustment costs $f(K_t, I_t) = \frac{\gamma I}{2} (\frac{I_t}{K_t} - \delta)^2 I_t$ allow for capital building time, as in [Christiano et al., 2005]. $\bar{L} s^s$ is the inelastic labor supply to the R&D sector remunerated at real wage $W_{s_{t,s}}$. Note that firms do not reverse profits back to households. These profits will instead be revenues for entrepreneurs, as shown in the next section.

The first order conditions read¹⁰⁴:

¹⁰³We adjust the growth rate with $1 - \sigma$ as we consider a separable dis-utility of labour.

¹⁰⁴We note ϱ_t^C and ϱ_t^K the Lagrange multipliers associated with budget and capital constraints, respectively.

$$\varrho_t^C = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \{ (C_{t+1} - hC_t)^{-\sigma} \}, \quad (3.5)$$

$$\varrho_t^C = \chi \frac{\Gamma_t^{1-\sigma} L_t^\varphi}{W_t}, \quad (3.6)$$

$$1 = \beta E_t \{ \Lambda_{t,t+1} R_{t+1} \}, \quad (3.7)$$

$$\varrho_t^C = \frac{\varrho_t^K}{1 + f_I(\cdot)}, \quad (3.8)$$

$$\varrho_t^K = \beta E_t \{ (1 - \delta) \varrho_{t+1}^K + \varrho_{t+1}^C (R_{t+1}^K - f_K(\cdot)) \}, \quad (3.9)$$

where the stochastic discount factor (i.e. the expected variation in marginal utility of consumption) reads as follows $\Lambda_{t-1,t} = \frac{\varrho_t^C}{\varrho_{t-1}^C}$.

3.3.2 R&D Entrepreneurs

As in [Comin and Gertler, 2006] entrepreneurs are an unbounded mass of prospective innovators with the ability to introduce new varieties of intermediates in each period. Each entrepreneur use resources to create a new project $RD_{t,s}$. Both new projects $RD_{t,s}$ and existing varieties $A_{t,s}$ face the risk of an exogenous exit shock $(1 - \phi_{RD,s})$. This process is meant to capture in a simple way the life-cycle dynamics of firms. Note that we also consider that entrepreneurs are not using energy heavy output, thus emitting zero CO₂ emissions. The evolution of the aggregate stock of innovations $A_{t,s}$ reads as follows:

$$A_{t+1,s} = \phi_{RD,s} (A_{t,s} + RD_{t,s}), \quad (3.10)$$

Entrepreneurs are able to produce new varieties by employing materials and skilled workers as inputs, according to the following production function:

$$RD_{t,s} = N_{t,s}^{\eta_s} (A_{t,s} L_{S,t,s})^{1-\eta_s}, \quad \eta_s \in (0, 1), \quad (3.11)$$

where $N_{t,s}$ is the amount of materials used (in units of final output) and $L_{s,t,s}$ is the number of skilled workers hired. Once the variety created, entrepreneurs lend it to monopolist firms in exchange for patent exclusivity. The monopolists then manufacture the new good and reverse profits Π_t (as shown in ??) back to the entrepreneurs. Furthermore, as in [Romer, 1990], in order to generate endogenous growth, the entrepreneurs production function captures the externality of the aggregate level of knowledge $A_{t,s}$.

The entrepreneurs problem will read as follows:

$$\max_{\{RD_{t,s}, N_{t,s}, L_{s,t,s}\}} E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} [\Pi_t RD_{t+i,s} - (N_{t+i,s} + W_{s,t+i,s} L_{s,t+i,s})] \quad (3.12)$$

s.t.

$$RD_{t+i,s} = N_{t+i,s}^{\eta_s} (A_{t+i,s} L_{s,t+i,s})^{1-\eta_s}. \quad (3.13)$$

The first order conditions read:

$$1 = MC_t^{RD,s} \eta_s N_{t,s}^{\eta_s - 1} (A_{t,s} L_{s,t,s})^{1-\eta_s}, \quad (3.14)$$

$$W_{s,t,s} = MC_t^{RD,s} (1 - \eta_s) A_{t,s} N_{t,s}^{\eta_s} (A_{t,s} L_{s,t,s})^{-\eta_s}, \quad (3.15)$$

$$\Pi_t = MC_t^{RD,s}, \quad (3.16)$$

where $MC_t^{RD,s}$ the Lagrange multiplier associated to the production constraint. Entrepreneurs equalize their marginal cost to the profit they receive from the firms and are subject to the inelastic supply of skilled labor ($L_{s,t,s} = \bar{L}^s$).

3.3.3 The Firms

3.3.3.1 Energy Producers

$$E_{n,t} = A_{n,t}L_{n,t} \quad (3.17)$$

The emissions level is modeled by a nonlinear technology (i.e. abatement technology μ) that allows for reducing the inflow of emissions:

$$E_t = (1 - \mu_t)\vartheta E_{n,t}. \quad (3.18)$$

The emissions E_t at firm level are proportional to the production Y_{jt} with ϑ the carbon intensity parameter. Contrary to [Lontzek et al., 2015], we consider $\vartheta_t = \vartheta$ constant overtime and calibrate it using Euro Area emission to energy output levels, as in our model, we capture the effects of green R&D directly through the abatement cost.

Furthermore, we allow the energy producer to reduce emissions through an abatement effort μ_t . This can be thought as the cost to transition from carbon-intensive to clean energy sources. When energy producers decide on abatement efforts, they incur a technology cost:

$$Z_t = f(\mu_t)E_{n,t}, \quad (3.19)$$

where

$$f(\mu_t) = g(\theta_t^1)\mu_t^{\theta_2}, \quad \theta_2 > 1, \quad (3.20)$$

and

$$g(\theta_t^1) = \frac{\theta_1}{\Gamma_t^{\theta_1}\epsilon_t^{\theta_1}}, \quad \theta_1 > 0, \quad (3.21)$$

with θ_1 and θ_2 representing the cost efficiency of abatement parameters. In this section, we assume that the cost function of abatement $g(\theta_t^1)$ follows an exogenous trend $\Gamma_t^{\theta_1}$ and

can be hit by a random shock $\epsilon_t^{\theta_1}$.¹⁰⁵ The goal is to capture exogenously the impact of improvements in green technology that we will concretely model in the next section. This will result in a decrease in abatement costs that will allow for substantially higher levels of abatement μ_{jt} .

A decrease in $g(\theta_t^1)$ triggers a drop in the marginal cost of abatement, which we define as:

$$MC_\mu = \frac{f(\mu_t)'}{\mu_t} \quad (3.22)$$

Thus, the profits of our representative energy producers $\Pi_{n,t}$ will be affected by the presence of the environmental externality. The revenues are the total value of energy production $P_{n,t}E_{n,t}$, while the costs arise from wages $W_{n,t}$ (paid to the labor force $L_{n,t}$), abatement μ_t , and the price of emissions E_t associated with the environmental policy τ_{et} :

$$\Pi_{n,t} = P_{n,t}E_{n,t} - W_{n,t}L_{n,t} - g(\theta_t^1)\mu_t^{\theta_2}E_{n,t} - \tau_{et}E_t \quad (3.23)$$

The cost-minimization problem yields the input costs, which can be expressed following the first-order conditions with respect to the firm's optimal choice of labor and abatement, respectively:

$$W_{n,t} = A_{n,t} (P_{n,t} - g(\theta_t^1)\mu_t^{\theta_2}) - \tau_{et}(1 - \mu_t)\vartheta, \quad (3.24)$$

$$\tau_{et} = \frac{\theta_2 g(\theta_t^1) \mu_t^{\theta_2-1}}{\vartheta E_{n,t}}, \quad (3.25)$$

capturing the CO₂ externality firms face an additional trade-off (equation (2.28)) between paying the environmental policy τ_t or incurring abatement cost related to the abatement levels they chose μ_t .¹⁰⁶ This last optimality condition highlights the key role of the carbon price dynamics in shaping the abatement level of firms.

¹⁰⁵ $\epsilon_t^{\theta_1}$ follows an $AR(1)$ shock process: $\log(\epsilon_t^{\theta_1}) = \rho_{\theta_1} \log(\epsilon_{t-1}^{\theta_1}) + \sigma_{\theta_1} \eta_t^{\theta_1}$, with $\eta_t^{\theta_1} \sim \mathcal{N}(0, 1)$.

¹⁰⁶In addition, both the environmental policy τ_t and abatement effort μ_t are common to all firms, as the environmental cost, which firms are subject to, is constant.

3.3.3.2 The Final Firms

The final good is produced by a competitive sector, which uses the different varieties of intermediates produced by entrepreneurs as inputs, yielding the following production function:

$$Y_t = \int_0^{A_{t,s}} \left(Y_{jt}^{1-\frac{1}{\theta}} dj \right)^{\frac{1}{1-\frac{1}{\theta}}}. \quad (3.26)$$

Final firms are looking for profit maximization at a given price P_t , subject to the intermediate goods j with prices P_{jt} :

$$P_t = \left(\int_0^{A_{t,s}} P_{jt}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (3.27)$$

The first order condition for the final firm profit maximization problem yields:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} Y_t. \quad (3.28)$$

3.3.3.3 Intermediate Firms

Contrary to the standard RBC framework, representative firms (indexed by j) of the modeled economy face a trade-off between the desired level of abatement level and the environmental policy level, in addition to the usual capital and labor trade-off.

As the environmental externality is a global phenomena, firms do not internalize its impacts, thus, they incur the externality costs as the social planner or government imposes an environmental policy in order to fix the market failure. Setting an environmental policy then pushes firms to optimally choose a level of abatement to maximize their profit. Following [Heutel, 2012], the environmental externality enters the Cobb-Douglas production function of the firms, through a damage function linked to the level of temperature à la [Nordhaus and Moffat, 2017] as follows:

$$Y_{jt} = \varepsilon_t^A d(T_t^o) K_{jt}^{\alpha_1} E_{n,jt}^{\alpha_2} L_{jt}^{1-\alpha_1-\alpha_2}, \quad \alpha_1 + \alpha_2 \in (0, 1), \quad (3.29)$$

where $E_{n,jt}$ is the energy composite used in production by firm j and $d(T_t^o)$ is a convex polynomial function of order 2 displaying the temperature level ($d(T_t^o) = ae^{-\left(\frac{b}{\Gamma_t^2}T_t^{o2}\right)}$), with $(a,b) \in \mathbb{R}^2$, which is borrowed from [Nordhaus and Moffat, 2017]. As in the case of the disutility of labour, we introduce Γ_t^2 to the damage sensitivity parameter b , such that $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2}T_t^{o2}}$. The goal is to allow for the existence of the balance growth path without a loss of generality, as over the business cycle or for a period of less than 30 years $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2}T_t^{o2}} \approx ae^{-bT_t^{o2}}$ ¹⁰⁷. ε_t^A is an exogenous technology shock that follows an AR(1) shock process: $\log(\varepsilon_t^A) = \rho_A \log(\varepsilon_{t-1}^A) + \sigma_A \eta_t^A$, with $\eta_t^A \sim \mathcal{N}(0, 1)$.

As argued by [Dietz and Venmans, 2019], global temperature $d(T_t^o)$ is assumed to be linearly proportional to the level of cumulative emissions:

$$T_t^o = v_1^o(v_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o. \quad (3.30)$$

Furthermore, the carbon emissions stock X_t follows a law of motion:

$$X_t = (1 - \gamma_d)X_{t-1} + E_t + E_t^*, \quad (3.31)$$

where E_t is the aggregate flow of emissions at time t and γ_d is the decay rate. E_t^* represents the rest of the world emissions and is used to pin down the actual steady state level of the stock of emission in the atmosphere.¹⁰⁸

The cost-minimization problem yields the real marginal cost, which can be expressed following the first-order conditions with respect to the firm's optimal choice of capital,

¹⁰⁷This point is further discussed in the Balanced Growth Path section.

¹⁰⁸For simplicity we assume that the rest of the world emissions follow the same growth rate of our closed economy: $E_t^* = E^* \Gamma_t$.

labor, as well as the energy consumption, respectively:

$$R_t^K = \alpha_1 \Psi_{jt} \frac{Y_{jt}}{K_{jt}}, \quad (3.32)$$

$$W_t = (1 - \alpha_1 - \alpha_2) \Psi_{jt} \frac{Y_{jt}}{L_{jt}}, \quad (3.33)$$

$$P_{n,t} = \alpha_2 \Psi_{jt} \frac{Y_{jt}}{E_{n,jt}}. \quad (3.34)$$

The first two equation equation Equation (3.32) and (3.33) are the standard optimal choice of capital and labor. When using energy as an input, firms face an additional cost (3.34). $\Psi_{jt} = \Psi_t = MC_t^f$ ¹⁰⁹ is the marginal cost of the representative firms, as they all choose the same level of capital, labor, and energy inputs.

The aggregate production function of the intermediate firms will now features the measure A_t . Using both the Cobb-Douglas production form (3.29) and the final firms production equation (3.26), we can rewrite the production function as following:

$$Y_t = A_{t,s}^{\frac{1}{\theta-1}} d(T_t^o) K_t^\alpha L_t^{1-\alpha}. \quad (3.35)$$

The firm profit maximization with respect to output and prices, yields the following pricing rule:¹¹⁰

$$MC_t^f = \frac{P_{jt} \theta - 1}{P_t \theta} \quad (3.36)$$

Each intermediate producer sets its price equals to a constant markup over the marginal cost. Finally, the profits equation will also capture the measure $A_{t,s}$ and can be presented as following:¹¹¹

$$\Pi_t = \frac{1}{\theta} \frac{Y_t}{A_{t,s}}. \quad (3.37)$$

¹⁰⁹We can rewrite the firm problem as follows: $\Pi_{jt} = \left(\frac{P_{jt}}{P_t} - MC_t^f \right) Y_{jt}$, where $MC_{jt}^f = MC_t^f = \Psi_t$.

¹¹⁰With $\frac{P_{jt}}{P_t} = 1$, as we abstract from price stickiness.

¹¹¹For the full mathematical derivations please refer to the appendix.

3.3.4 Government

Government levies a lump sum tax and sets an environmental policy to finance its spending as following:

$$T_t + \tau_{et}E_t = G_t, \quad (3.38)$$

with the public expenditure G_t , taxes T_t , and revenue from emissions tax $\tau_{et}E_t$. The government spending is also assumed to be a fixed proportion of the GDP:

$$G_t = \frac{\bar{g}}{\bar{y}}Y_t. \quad (3.39)$$

3.3.5 The environmental policy

Competitive Equilibrium

To pin down the optimal policy,¹¹² we solve for the Competitive Equilibrium (“CE”). The CE in this economy is defined as follows:

Definition 3.3.1 *A competitive equilibrium consists of an allocation*

$\{C_t, L_t, K_{t+1}, E_t, X_t, T_t^o\}$, a set of prices $\{P_t, R_t, R_t^K, W_t\}$ and a set of policies $\{\tau_t, T_t, B_{t+1}\}$ such that

- *the allocations solve the consumers’, firms’ problems given prices and policies,*
- *the government budget constraint is satisfied in every period,*
- *temperature change satisfies the carbon cycle constraint in every period, and*
- *markets clear.*

Definition 3.3.2 *The optimal solution sets the carbon price τ_t as an optimal policy τ_t^* ,*

¹¹²As we consider a closed economy, we assume that cooperation takes place in such a way to avoid free-riding and potential carbon leakages. This is achieved by setting E^* to a constant.

which maximizes the total welfare in [equation \(3.2\)](#):

$$\tau_t^* = SCC_t. \quad (3.40)$$

with SCC_t the social cost of carbon:

$$SCC_t = \eta\beta \frac{\lambda_{t+1}}{\lambda_t} SCC_{t+1} + (v_1^o v_2^o)\beta \frac{\lambda_{t+1}}{\lambda_t} \mathfrak{S}_{t+1}^T, \quad (3.41)$$

and with,

$$\mathfrak{S}_t^T = (1 - v_1^o)\beta \frac{\lambda_{t+1}}{\lambda_t} \mathfrak{S}_{t+1}^T - \sum_k \Psi_t \varepsilon_t^A \frac{\partial d(T_t^o)}{\partial T_t^o} K_{t-1}^\alpha L_t^{1-\alpha} \quad (3.42)$$

Departing from the Competitive Equilibrium to Meet Climate Goals

Definition 3.3.3 *The public authorities, however, do not always optimally set the carbon policy. For instance, in the EU area, public authorities target an emissions level that is consistent with their objective of a 55% emissions reduction by 2030. As in [[Benmir and Roman, 2020](#)] we model this situation by assuming that the cap on emissions implies a specific carbon price that can be hit by exogenous shocks and which also incorporates an endogenous trend:*

$$\tau_t = \Gamma_t^\tau \text{Carbon Price.}^{113} \quad (3.43)$$

where $\Gamma_t^\tau = \gamma^\tau \varepsilon_t^\tau \Gamma_{t-1}^\tau$ is the stochastic growth rate of the tax which allows to reduce emissions to be aligned with the cap policy, and where ε_t^τ the stochastic AR(1) shock on tax that represents the market volatility of the ETS system.

This stylized representation of the implementation of a permit market allows us to find theoretical fiscal pathways consistent with the EU climate objectives. That said, the targeted CO₂ level/price is assumed to be constant at the business cycle frequency.

¹¹³Although the policy used in the EU is $E_t = \text{Cap Policy}$, it is analogous to set $\tau_t = \text{Carbon Price}$ that would allow for decreasing emissions to match the cap.

3.3.6 Normalization and Aggregation

In equilibrium, factors and goods markets clear as shown below. First, the market-clearing conditions for aggregate capital, investment, labor, and wages, read as: $A_t K_t = \int_0^1 K_{jt} dj$, $I_t = \int_0^1 I_{jt} dj$, $A_t L_t = \int_0^1 L_{jt} dj$, and $W_t = \int_0^1 W_{jt} dj$. Similarly, global aggregate emissions and aggregate emissions cost reads as: $E_t = \int_0^1 E_{jt} dj$, and emissions cost $Z_t = \int_0^1 Z_{jt} dj$, respectively. Finally, the resource constraint of the economy reads as follows:

$$Y_t = C_t + G_t + I_t + N_{t,s} + f(\cdot)I_t + Z_t. \quad (3.44)$$

3.3.7 Transition Pathways with Exogenous Abatement Technology

3.3.7.1 Calibration

Calibrated parameters for the standard endogenous growth model are reported in [Table 3.3](#) and [Table 3.4](#). For parameters related to business cycle theory, their calibration is standard: the depreciation rate of physical capital is set at 2.5 percent in quarterly terms, the government spending to GDP ratio at 40 percent,¹¹⁴ the share of hours worked per day at 0.33 for firms and 0.15 for entrepreneurs, and the capital share in the production function α at 0.3. The inverse elasticity of net investment to the price of capital γ_I is set at 1.728 as in [\[Gertler and Karadi, 2011\]](#) and the coefficient of relative risk aversion σ in the utility function at 2, as argued by [\[Stern, 2008\]](#) and [\[Weitzman, 2007\]](#). We set the discount factor at 0.9975 to get a steady state real interest rate of 1 percent. This choice is motivated by the low interest rate environment witnessed in recent years.

Regarding the environmental part, we calibrate the damage function according to [\[Dietz and Stern, 2015\]](#). The global temperature parameters v_1^o and v_2^o are set following [\[Dietz and Venmans, 2019\]](#) to pin down the ‘initial pulse-adjustment timescale’ of the climate

¹¹⁴We match the level of the Euro Area.

system. The level of the remainder of the world's emissions E^* is set at 1.59 in order to replicate the global level of carbon in the atmosphere of 840 gigatons. We use the carbon intensity parameter ϑ to match the observed ratio of emissions to output for the Euro Area (EA) at 21%.¹¹⁵ The abatement parameters θ_1 and θ_2 are taken from [Heutel, 2012]. The decay rate of emissions δ_x is set at 0.21 percent. Finally, the firms' marginal cost parameter θ is set to 11.

3.3.7.2 Transition Pathways Simulations

In order to solve for the medium/long-run pathways scenarios, we use the extended path algorithm ([Adjemian and Juillard, 2013]), which allows for both integrating deterministic trends and stochastic shocks, as it is shown in [Benmir and Roman, 2020].

The goal of this section is to find and analyze a theoretical pathway consistent with the objective of the EU for 2030 under the presence of i) a targeted carbon price policy, ii) an exogenously growing green technology, and iii) an optimal policy coupled with an exogenously growing green technology.

We thus find the trajectory of the output, the marginal cost of abatement, and the carbon price, that leads to a desired reduction in emissions (55 percent relative to the level of 1990). We then highlight the main differences between relying solely on a carbon policy or solely on an abatement technology, versus using an optimal policy which maximizes the welfare (but would alone fails to attain the 55 percent emissions reduction desired) coupled with an abatement technology that is increasing over time.

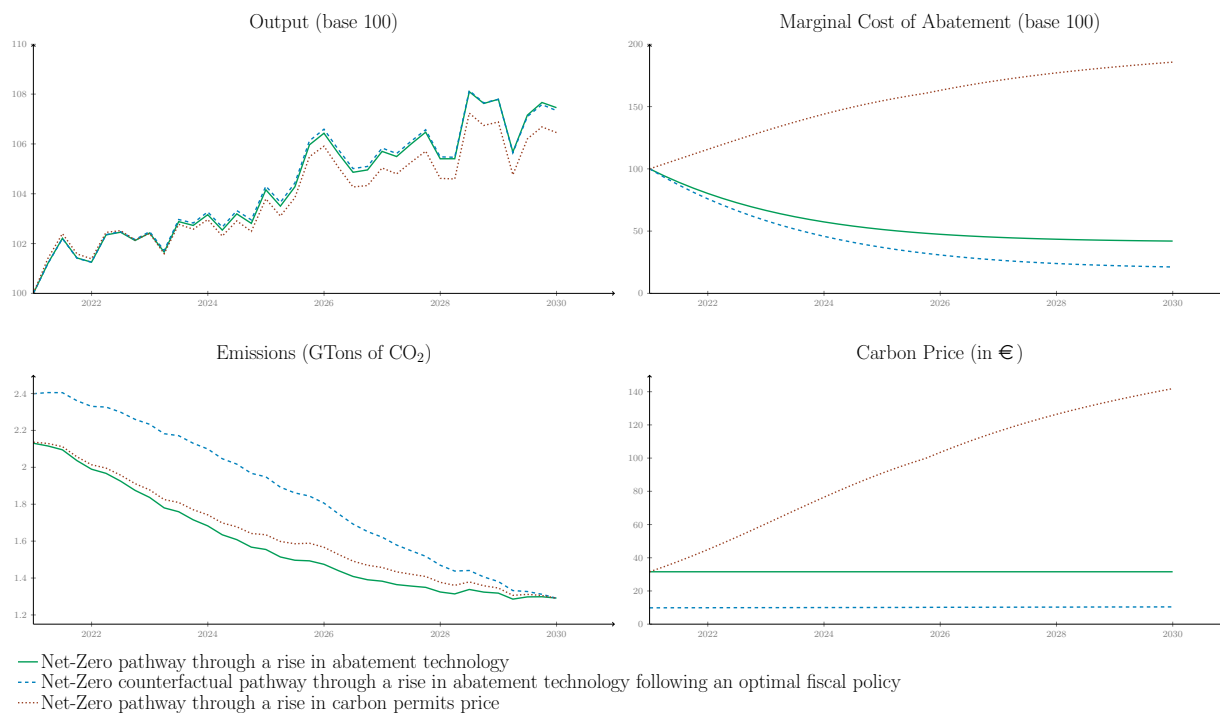
Figure 3.1 shows what carbon price and/or reduction in abatement costs trajectories would be needed to be on track for achieving the net-zero target in the EZ, assuming a growth trend of 0.8 percent.¹¹⁶ We also add a stochastic shock process to TFP, that we calibrate according to the estimation in [Smets and Wouters, 2003]. This allows us to

¹¹⁵We compute the emissions to output ratio as the number of kCo2 per dollar of GDP using emissions data from the Global Carbon Project and GDP data from FRED.

¹¹⁶The average real growth rate per capita in the EZ area from 2000 to 2020

simulate a realistic transition scenario, where the trend in growth is anticipated, but shocks can distort this deterministic process in the short run. The blue dashed line is a scenario where we build a counterfactual highlighting the pathway if an optimal policy is set and coupled with decreasing marginal abatement costs. The green solid line is a scenario where green technology—coupled with a fixed tax rate—is the only long-run driver of emissions reduction. Finally the dotted red line corresponds to the scenario where the targeted environmental policy (e.g. EU ETS cap system) is the only instrument used to mitigate the climate externality and keeps the economy on track for achieving the desired level of emissions reduction. Relying on a targeted tax alone, requires high levels of carbon price to be on target for net-zero by 2050, and induces a higher output loss than both other scenarios where green innovation is boosted to allow for lower marginal cost of abatement, which in turn triggers higher abatement levels. We find that either fixing the environmental policy at a targeted level and allowing for green innovation to boost abatement levels, or using an optimal fiscal policy coupled with green innovation are more efficient in keeping higher levels of output than just relying on a carbon fiscal policy alone. It suggests that an optimal policy with green innovation boosting is the optimal choice from a welfare perspective.

This results comforts our empirical finding where both a fiscal environmental policy and green innovation growth which we characterized in the empirical section via increasing numbers of green patents (i.e. lower costs of abatement as new technology are efficient and thus allow for abating at lower costs) are both major contributors to significant emissions reduction. In addition, higher fiscal carbon prices are also shown to negatively impacts the costs of abatement where green patents tends to decrease following spikes in carbon prices.

FIGURE 3.1. Net-Zero Transition Pathways - 2030

3.4 Introducing Endogenous Green Technology

In this section, we introduce green entrepreneurs who produce innovations in the abatement technology. An improvement in green technologies will, in turn, reduce the cost of abatement for firms. However, green innovators will need to rely on loans from banks to start new projects. Thus, we also show how financial intermediaries are modeled. Finally, we propose a set of policies that could help fostering green innovations. The goal is to show how public policies could ultimately impact the abatement efficiency.

3.4.1 Household

In this new setup, households are populated of both workers and bankers, with measures $1 - f$ and f respectively. In addition, a new specialized green R&D worker force Ls_g join the global R&D skilled labour Ls_s as well as the unspecialized labour force L_t are now

either specialized in who supply labor inelastically to entrepreneurs. Bankers manage a financial intermediary that uses borrowed funds to make loans to green innovators. There is perfect consumption insurance among family members.

There is random turnover between bankers and workers: a banker becomes a worker with probability $1 - \theta_B$ at which time he or she transfers accumulated earnings to the family. Workers become bankers with probability $(1 - \theta_B) \frac{f}{1-f}$ so there is a measure $(1 - \theta_B)f$ of new bankers each period. This allows for offsetting the number that exit. The household transfers a small amount of resources to new bankers so they are able to start operations. Banker exit is introduced as a device to ensure that the financial imperfection will remain relevant—otherwise banks might reach a point where internal resources are enough to finance all desired lending.

The budget constraint of households is modified to display wages for skilled labor employed by green entrepreneurs, as well as profits from the ownership of financial intermediaries.

$$C_t + B_{t+1} + I_t + f(K_t, I_t) = W_t L_t + W_{s_t, g} \bar{L} s^g + W_{s_t, s} \bar{L} s^s + \Pi_t^{FI} + T_t + R_t^K K_t + R_t B_t \quad (3.45)$$

where $\bar{L} s^g$ is the inelastic labor supply to green entrepreneurs associated with wage $W_{s_t, g}$, and Π_t^{FI} are the profit from the financial intermediaries.¹¹⁷

3.4.2 Green Innovators

Similarly to the R&D entrepreneurs presented in [subsection 3.3.2](#), we follow [[Comin and Gertler, 2006](#)] and introduce an unbounded mass of prospective green innovators with the ability to improve the abatement technology. However, we differ from their set up insofar as we consider that the innovators are green R&D creators that allows for improving the abatement efficiency via a reduction in abatement costs ($g(\theta_t^1)$). Each green innovator

¹¹⁷These changes to the household budget constrain do not have any impact on the first order conditions presented in [subsection 3.3.1](#).

use resources to create a new project $RD_{t,g}$. Both new projects $RD_{t,g}$ and existing technologies $A_{t,g}$ face the risk of an exogenous exit shock $(1 - \phi_{RD,g})$. Similarly to the R&D entrepreneurs, we assume that green innovators do not emit CO_2 while developing new technologies.

Our innovators or research and development centers need to obtain funding from banks to finance entry. Here the idea is that financial intermediaries are the economic entities with the expertise and knowledge when evaluating and monitoring green entrepreneurial projects.

The total number of green technologies in operation at any given time t is denoted by $A_{t,g}$, while the green projects $RD_{t,g}$ are the number of new technologies in process in period t . Accordingly, the evolution of the aggregate stock of green innovations, $A_{t,g}$, is given by:

$$A_{t+1,g} = \phi_{RD,g}(A_{t,g} + RD_{t,g}), \quad (3.46)$$

To be more specific, each green innovator can produce a new potential technology by employing materials and skilled workers as inputs, according to the following production function:

$$RD_{t,g} = N_{t,g}^{\eta_g}(A_{t,g}L_{t,g})^{1-\eta_g}, \quad \eta_g \in (0, 1), \quad (3.47)$$

where $N_{t,g}$ is the amount of materials used (in units of final output) and $L_{t,g}$ is the number of skilled workers hired. $A_{t,g}$ denotes the aggregate green technological level of the economy, which as explained below is equal to the total number of technologies in operation. Similarly to the R&D entrepreneurs, the innovators production function captures the externality of the aggregate level of knowledge $A_{t,g}$, which allows for generating endogenous growth.¹¹⁸

Once the technology created, entrepreneurs lend it to monopolist firms in exchange for

¹¹⁸For simplicity, we consider that spillovers on the green innovation only originate from the green technological level $A_{t,g}$.

patent exclusivity. The monopolists then use these technologies to lower their abatement cost and pay a rent Z_t corresponding to abatement costs to the green innovators.

As in [Queralto, 2020], we assume that green entrepreneurs can borrow to face the entry cost without any friction. More specifically, when seeking funding, our innovators can emit a financial intermediaries security which is perfectly contingent on the success of the green project. However, as in [Gertler and Karadi, 2011], banks do face frictions relative to their leverage ratio, as we will show in the next section. As long as the innovation does not become obsolete, the underlying securities pay in each future period. If the innovation becomes obsolete, then the payoff is zero. We denote the price of one unit of these securities $Q_{t,e}$.

The green innovators optimize over the revenues from selling securities subject to the inherent costs of developing the innovation by using materials N_t and paying wages W_{S_t} to the skilled labor L_{S_t} . The maximization problem reads as follows:

$$\max_{\{RD_{t,g}, N_{t,g}, L_{S_{t,g}}\}} E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} [Q_{t+i,e} RD_{t+i,g} - (N_{t+i,g} + W_{S_{t+i,g}} L_{S_{t+i,g}})] \quad (3.48)$$

s.t.

$$RD_{t+i,g} = N_{t+i,g}^{\eta_g} (A_{t+i,g} L_{S_{t+i,g}})^{1-\eta_g} \quad (3.49)$$

The first order condition reads (denoting $MC_t^{RD,g}$ the Lagrange multiplier associated to the production constraint):

$$1 = MC_t^{RD,g} \eta_g N_{t,g}^{\eta_g-1} (A_{t,g} L_{S_{t,g}})^{1-\eta_g}, \quad (3.50)$$

$$W_{S_{t,g}} = MC_t^{RD,g} (1 - \eta_g) A_{t,g} N_{t,g}^{\eta_g} (A_{t,g} L_{S_{t,g}})^{-\eta_g}, \quad (3.51)$$

$$Q_{t,e} = MC_t^{RD,g}. \quad (3.52)$$

Using these first order conditions¹¹⁹ and equation (3.47), we can rewrite the price of the inherent security $Q_{t,e}$ in terms of the marginal cost components as following:¹²⁰

$$Q_{t,e} = MC_t^{RD,g} = \frac{1}{\eta_g} \left(\frac{1}{\bar{L}S^g} \right)^{\frac{1-\eta_g}{\eta_g}} \left(\frac{RD_{t,g}^N}{A_{t,g}} \right)^{\frac{1-\eta_g}{\eta_g}}, \quad (3.53)$$

Contrary to the previous section, where the cost of abatement was driven by an exogenous process, the cost function of abatement is now steered by endogenous green technological changes. Thus, green innovators projects will ultimately lead to higher abatement and lower emissions. The equation (3.21) now reads:

$$f(\mu_t) = \left(\int_0^{A_{t,g}} f(\mu_{jt})^{\frac{1}{\theta_3}} dj \right)^{\theta_3} \quad (3.54)$$

Thus,

$$g(\theta_t^1) = \theta_1 A_{t,g}^{-\theta_3}, \quad \theta_1 > 0 \text{ and } \theta_3 > 0, \quad (3.55)$$

where θ_3 is now the elasticity of the cost of abatement with respect to the green technology.

3.4.3 Financial Intermediaries

A representative financial intermediary make use of deposits from households as well as its own net worth to leverage and invest in green entrepreneurs. We model this part following [Gertler and Karadi, 2011]. We can write the representative bank's balance sheet as:

$$Q_{t,e} S_{t,e} = N_t + B_t, \quad (3.56)$$

¹¹⁹With $W_{S_{t,g}} = \frac{1-\eta_g}{\eta_g} \frac{N_{t,g}}{A_{t,g} L_{S_{t,g}}}$

¹²⁰We also use the market clearing condition for skilled labor: $L_{S_{t,g}} = \bar{L}S^g$.

where $S_{t,e}$ are financial claims on green innovators and $Q_{t,e}$ their relative price. Note that market clearing implies that $S_{t,e} = A_{t,g} + RD_{t,g}$, as assets held by banks must match the total number of existing green technologies. On the liability side, N_t is the banks' net worth and B_t is debt to households. Over time, banks' retained earnings evolve as follows:

$$N_t = R_{t,e}Q_{t-1,e}S_{t-1,e} - R_t B_{t-1}, \quad (3.57)$$

$$N_t = (R_{t,e} - R_t)Q_{t-1,e}S_{t-1,e} + R_t N_{t-1}, \quad (3.58)$$

where $R_{t,e}$ denotes the gross rate of return on a unit of the bank's claims on green innovators:

$$R_{e,t} = \frac{\phi_{RDg}(Z_t + Q_{t,e})}{Q_{t-1,e}}. \quad (3.59)$$

Financial intermediaries will maximize equity on an infinite horizon, yielding the following objective function:

$$V_t^B = E_t \left\{ \sum_{i=1}^{\infty} \beta^i \Lambda_{t,t+i} (1 - \theta_B) \theta_B^{i-1} N_{t+1+i} \right\}, \quad (3.60)$$

where θ_B is the probability of a bank exiting the market. The constraint on banks arise from the existence of a supervisory regulator. Drawing on [Pietrunti, 2017], we assume that this regulator requires that the discounted value of the bankers' net worth should be greater than or equal to the current value of assets, weighted by their relative risk:

$$V_t^B \geq \lambda Q_{t,e} S_{t,e}. \quad (3.61)$$

In this simplified setup, banks only hold one asset, so the regulator will set a value for λ in order to target a specific capital ratio for banks. By modifying this parameter, the financial authority will be able to tighten or relax the constraint on banks, which will impact the number of entrepreneurial projects the financial sector can fund. In our baseline model,

we will calibrate λ to match the capital ratio of European banks at the steady state. We guess that the value function is linear of the form $V_t = \Gamma_t^B N_t$ so we can rewrite V_t^B as:

$$V_t^B = \max_{S_{t,e}} E_t \{ \beta \Lambda_{t,t+1} \Omega_{t+1} N_{t+1} \}, \quad (3.62)$$

where $\Omega_t \equiv 1 - \theta_B + \theta_B \Gamma_t^B$. Maximization subject to constraint (3.61) yields the following first order and slackness conditions:

$$\beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1,e} - R_{t+1}) \} = \nu_t \lambda, \quad (3.63)$$

$$\nu_t [\Gamma_t^B N_t - \lambda Q_{t,e} S_{t,e}] = 0, \quad (3.64)$$

where ν_t is the multiplier for constraint (3.61). We can thus write the capital ratio as $\Xi_t = \lambda / \Gamma_t^B$. Finally, we rewrite the value function to find Γ_t^B :

$$\begin{aligned} V_t^B &= \nu_t \lambda Q_{t,e} S_{t,e} + \beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} N_t \} \\ \Gamma_t^B N_t &= \nu_t \Gamma_t^B N_t + \beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} R_t N_t \} \\ \Gamma_t^B &= \frac{1}{1 - \nu_t} \beta E_t \{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \}. \end{aligned} \quad (3.65)$$

We close this part of the model with the aggregate law of motion for the net worth of bankers:

$$N_t = \theta_B (R_{t,e} - R_t) Q_{t-1,e} S_{t-1,e} + (\theta_B R_t + \omega) N_{t-1}, \quad (3.66)$$

with $\omega \in [0; 1)$ the proportion of funds transferred to entering bankers.

3.4.4 Carbon Policy and Green Innovation

As argued in the section above on the model equilibrium, many economies rely on a permit-market-based instrument instead of an optimal carbon price (e.g. the ETS in the EU and the carbon permit markets in Canada in California (US)). Thus, in order

to reach the Paris Agreement objective of the net-zero emissions by 2050, such carbon pricing strategy requires carbon prices to constantly increase, which in turn incentivizes firms to engage in continuously higher abatement efforts. However, investing in abatement technologies is costly and has a number of consequences such as welfare losses as shown in [Benmir and Roman, 2020]. Steering green innovation via other tools besides carbon pricing would be less welfare distortionary. Incentivizing green innovation that lowers the cost of abatement, however, might prove difficult if the price of carbon increases substantially and in places where no green abatement technology is yet available.

Definition 3.4.1 *A government, when relying on a carbon permit market solely to tackle the climate externality, sets a carbon cap:*

$$E_t = Cap_t \quad (3.67)$$

which inherently determines a carbon price level τ_{et} :

$$\tau_{et} = Carbon\ Price_t. \quad (3.68)$$

where Cap_t is the path of the cap on emissions consistent with the net-zero objective, and $Carbon\ Price_t$ the inherent carbon price associated with this objective. To reach the net-zero target, the price is expected to steadily increase in order to match the expected decrease in the cap.

However, under the presence of endogenous green innovation that contributes to lowering the cost of abatement, the social planner is not limited anymore in terms of tools it could use, and is able to rely on both a carbon price τ_{et} and the green technologies $A_{t,g}$:

Definition 3.4.2 *To decrease emissions, firms engage in higher abatement efforts:*

$$\mu_t = 1 - \frac{Cap_t}{vY_t} \quad (3.69)$$

with $\Delta\left(\frac{Cap_t}{vY_t}\right) < 0$. Otherwise, the optimal social cost of carbon presented in the initial exogenous framework would be able to achieve the target. Therefore, the carbon price, as defined in [equation \(3.25\)](#), is driven by two instruments, namely, i) the environmental cap Cap_t and ii) the green technologies $A_{t,g}$:

$$Carbon\ Price_t = \theta_1\theta_2 \frac{\left(1 - \frac{Cap_t}{vY_t}\right)^{\theta_2-1}}{v} A_{t,g}^{-\theta_3} \quad (3.70)$$

Effectively, when $Cap_t = vY_t = \bar{E}_t$ ¹²¹ (i.e. a laissez-faire economy)

$$\min(Carbon\ Price_t) = 0 \quad (3.71)$$

And when $Cap_t = 0 \Rightarrow \mu_t = 1$ (i.e. a net-zero objective)

$$\max(Carbon\ Price_t) = \theta_1\theta_2 \frac{1}{v} A_{t,g}^{-\theta_3} \quad (3.72)$$

Corollary 3.4.1 $\Delta Cap_t < 0 \Rightarrow \Delta Z_t > 0 \Rightarrow \Delta R_{t,e} > 0 \Rightarrow \Delta RD_{t,g} > 0$

An increase in the carbon price (i.e. a decrease in the cap), triggers more abatement, which in turn increases the cost of abatement $Z_t = f(\mu_t)Y_t$, as firms would equate their marginal benefit from investing in abatement to the carbon price. This increase in Z_t imply a higher rate of return on entrepreneurs equity $R_{e,t}$ as entrepreneurs' profits are reversed to banks. The higher the profitibility of entrepreneurs, the more banks would direct investment toward green projects, which would spur green innovation $A_{t,g}$.

Proposition 3.4.1 To ensure we meet the net-zero target with a deacreasing cap on emissions, while trying to mitigate the effect on welfare of a rising carbon price, we investigate three macro-financial tools that could foster green innovation: i) the fiscal authority uses revenues from carbon pricing policy to subsidize green innovators; ii) the macroprudential authority adapt its capital requirement to give an incentive to financial intermediaries to

¹²¹ \bar{E}_t the steady state level of emissions at each period t

invest in green entrepreneurs' equity, thus generating a greater number of successful green technologies; and iii) the central bank engages in an asset purchase program aiming to ease funding conditions for the green innovation sector.

i) Fiscal Policy

As presented in the model section, the government finances its government spending as follows:

$$T_t + (1 - \bar{s})\tau_t E_t = G_t, \quad (3.73)$$

with the public expenditure G_t finding its source from taxes T_t and revenues from the carbon tax $\tau_t E_t$.

In this setting we will consider the possibility for the government to divert part \bar{s} or all of the environmental policy revenues back to the green innovators (if $\bar{s} = 0$ no subsidy is diverted to the green innovators). In this case, subsidies would raise profits of green entrepreneurs and ultimately be reversed to banks as interest:

$$R_{e,t} = \frac{\phi_{RD}(Z_t + Q_{t,e} + \bar{s}\tau_t E_t)}{Q_{t-1,e}}. \quad (3.74)$$

ii) Financial Policy

As detailed in section 3.4.3, the financial authority imposes a capital constraint on banks modeled through the parameter λ that pins down the steady state capital ratio. In a more sophisticated model, claims on green entrepreneurs could be one of several assets held by banks. In this case, different weights could be applied to different assets, and the regulator could favor a specific sector.¹²² Our setup is without loss of generality, since modifying λ in our model is similar to modifying the weight on loans to entrepreneurs in a model with several assets, keeping all other weights constant.

¹²²See [Benmir and Roman, 2020].

3.4.5 Normalization and Aggregation

When introducing green innovators, the resource constraint of the economy is modified as follows:

$$Y_t = C_t + G_t + I_t + N_{t,g} + N_{t,s} + f(\cdot)I_t. \quad (3.75)$$

3.5 The Balanced Growth Path

From the empirical data on global patents, green patents, and output, both green investment $N_{t,g}$ and global R&D investments $N_{t,s}$ are found to have higher trend growth than output. This empirical finding requires us to balance the growth rates of the green and global R&D investments on the supply side of the resource constraint of our economy to ensure balanced growth. Thus, to allow for a balanced growth path, we introduce investment-specific trends à la [Greenwood et al., 1997] that we denote as $V_{t,g} = \gamma_g^V V_{t-1,g}$ and $V_{t,s} = \gamma_s^V V_{t-1,s}$, where γ_g^V and γ_s^V are constant growth rates. These investment goods $N_{t,g}$ and $N_{t,s}$ are produced from final goods by means of a linear technology, whereby $\frac{1}{V_{t,g}}$ and $\frac{1}{V_{t,s}}$ units of final goods yield one unit of investment goods, respectively.¹²³

Furthermore, the non-linear climate damages within the production function does not allow for a balanced growth path when considered as the following: $d(T_t^o) = ae^{-bT_t^{o2}}$. To allow for a balanced growth path trajectory, we show that over the period horizon we consider for our estimation (2000-2020), the low growth rate Γ_t had a small to no effect on the damage function dynamics $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2}T_t^{o2}} \approx ae^{-bT_t^{o2}}$. Capturing the growth rate of the economy within the damage function allows for simplifying the de-trended form of the damage function without a loss of generality, given that over the period sample of our estimation, climate damages that are corrected for the economy growth rate Γ_t are not significantly different from climate damages that are not corrected for the economy growth rate. In addition, given that both climate is defined as the average change over the past

¹²³The slope of this investment-specific trend crucially appears in the measurement equation of the model and is estimated.

30 years, and that the stock of emissions is a slow moving variable, our 20 year sample period allows us to consider the damage function as a de-trended equation, which allows for reconciling the balanced growth path.

Our economy presents three sources of permanent growth: i) an endogenous source of growth $A_{t,s}$, ii) two exogenous sources of growth $V_{t,s}$ and $V_{t,g}$, and iii) a fourth endogenous source of green innovation growth $A_{t,g}$ which impacts the efficiency of abatement. Having these different sources of growth requires that we de-trend our model as a number of variables (e.g. output, emissions, investment, ...) will not be stationary. In the appendix section [subsection 3.B.6](#) we present the de-trended economy. The aggregate variables of our economy,¹²⁴ include: output per capita Y_t , investment per capita I_t , consumption per capita C_t , government spending G_t , lump sum taxes T_t , capital per capita K_{t-1} , emissions E_t , abatement costs $Z_t/V_{t,g}$, green investment expenditures $N_{t,g}/V_{t,g}$, global R&D investment expenditures $N_{t,s}/V_{t,s}$, stock of emissions X_t , Temperature T_t^o , R&D varieties per capita $RD_{t,s}$, and green innovation varieties per capita $RD_{t,g}$, wages W_t , skilled labour wages $W_{t,s}$, relative price of financial claims $Q_{t,e}$, debt to households B_t , net worth N_t , and the banks value function V_t^B , and all grow at the same rate Γ_t , which reads as the following:

$$\Gamma_t = A_{t,s}^{\frac{1}{(\theta-1)(1-\alpha)}} \quad (3.76)$$

where $\Gamma_t = \gamma_t^Y \Gamma_{t-1}$, the stock growth of R&D $A_{t,s}$ is $\gamma_t^{A_s} = \frac{A_{t,s}}{A_{t-1,s}}$, and the stock growth of green innovation $A_{t,g}$ is $\gamma_t^{A_g} = \frac{A_{t,g}}{A_{t-1,g}}$.

¹²⁴Along the balanced growth path.

3.6 Quantitative Analysis

3.6.1 Calibration and Estimation

3.6.1.1 Data and Measurement Equations

The model is estimated using Bayesian methods and EZ quarterly data over the sample time period 2000Q1 to 2019Q4. Data are taken from both Eurostat and the European Patent Office. We focus on the period between 2000 and 2019, as the decoupling between emissions and output started to be more significant in the 2000s. Furthermore, empirical data also support this strategy, since investment in decarbonized technologies started to exhibit a trend at the same time.

In order to estimate the key shocks and parameters of our model, we start by making our four series (output, emissions, R&D and green innovation expenditures, which we proxy via patents numbers) stationary. We first divide the sample by the working age population. Second, data are taken in logs and we then use a first difference filtering to obtain growth rates. Finally, we use the GDP price index to deflate all nominal variables.

To measure the empirical contribution of endogenous growth in green and standard technologies, we follow [Vermandel, 2019] and use a cost-based approach. As there is no data available for quarterly investment in both green technologies and global R&D, we use the number of patents filed to proxy expenditures.

Measurement equations are given by:

$$\begin{bmatrix} \text{Real Per Capita Output Growth} \\ \text{Per Capita } CO_2 \text{ Emissions Growth} \\ \text{Real Per Capita R\&D Expenditure Growth} \\ \text{Real Per Capita Green Innovation Expenditure Growth} \end{bmatrix} = \begin{bmatrix} \log \gamma_t^Y + \Delta \log(\tilde{y}_t) \\ \log \gamma_t^Y + \Delta \log(\tilde{e}_t) \\ \log(\gamma_t^Y / \gamma_s^V) + \Delta \log(\tilde{n}_{t,s}) \\ \log(\gamma_t^Y / \gamma_g^V) + \Delta \log(\tilde{n}_{t,g}) \end{bmatrix}, \quad (3.77)$$

where tilde denote de-trended variables.¹²⁵

3.6.1.2 Calibration and Prior Distribution

As the main objective of our paper is to assess trends in R&D and green innovation growth, all standard macro-finance and environmental parameters are calibrated from the literature. The calibration values for the standard macro block and the environmental components are reported in [table 3.3](#) and [table 3.4](#). [Table 3.5](#) reports the calibration of financial parameters related to the full model. We set the probability of remaining a banker θ_B at 0.972 as in [[Gertler and Karadi, 2011](#)]. We find the values of the proportional transfer to the entering banker ω and the regulatory parameter λ to approximately match both the debt to equity ratio¹²⁶ and the capital ratio in the EA. Because we only model loans to entrepreneurs, that are seen to carry a high level of risk, we assume that the regulator applies a 150% weight¹²⁷ to such assets before multiplying it by the theoretical capital requirement for banks of 10.5%. This yields an effective capital ratio of 15.75% in our baseline model.

For the remaining set of parameters and shocks, we rely on Bayesian methods. In a nutshell, a Bayesian approach can be followed by combining the likelihood function with prior distributions for the parameters of the model to form the posterior density function. The posterior distributions are drawn through the Metropolis-Hastings sampling method (MCMC). In the following fit exercise, we solve the model using a linear approximation to the policy function, and employ the Kalman filter to form the likelihood function. [Table 3.6](#) summarizes the prior—as well as the posterior—distributions of the structural parameters for the U.S. economy. As in [[Smets and Wouters, 2003](#)] the persistence of shocks follows a beta distribution with a mean of 0.5 and a standard deviation of 0.2, while the standard deviation of shocks follow an inverse gamma distribution with mean 0.001 and standard

¹²⁵The balanced growth path of the model can be found in the appendix.

¹²⁶We compute the debt to equity ratio by taking the sum of the debt to equity ratios of the 19 EZ countries, weighted by their relative shares in total banks assets, using data from Eurostat and the ECB.

¹²⁷Corresponding to the highest weight possible for corporate loans according to Basel III regulation.

deviation of 0.005.

The output growth rate γ_y and green innovation growth rate γ_{A_g} are estimated using a prior standard deviation of a gamma distribution with mean 0.05 and 0.01, respectively, while we use a beta distribution with mean 0.125 and 0.15 for the investment share in R&D η_s and green innovations η_g . Finally, the exogenous R&D and green innovation investment growth rates γ_{V_s} and γ_{V_g} are estimated using a normal distribution with means 1 and standard deviations of 0.2.

3.6.1.3 Posterior Distribution

In addition to prior distributions, [table 3.6](#) reports the means and the 5th and 95th percentiles of the posterior distributions drawn from four parallel MCMC chains of 20,000 iterations each. The sampler employed to draw the posterior distributions is the Metropolis-Hasting algorithm with a jump scale factor so as to match an average acceptance rate close to 25-30 percent for each chain.

Results of the posterior distributions for each estimated parameter are listed in [table 3.6](#). It is clear from [table 3.6](#) that the data were informative, as the shape of the posterior distributions differs from the priors. Results for structural shocks parameters that are common with [\[Smets and Wouters, 2003\]](#) are in line with the values they find. Regarding investment elasticities η_k with $k \in \{s, g\}$, our values are close to [\[Queralto, 2020\]](#). As for the endogenous and exogenous trends, our estimates are consistent with the observed empirical output and green innovation investment growth rates.

3.6.2 Endogenous Trends

In this section, we first discuss the results of our estimation of endogenous growth trends in output and green innovation. We then perform a counterfactual exercise to assess the relevance of policies aiming at boosting the growth trend in green innovation.

3.6.2.1 Estimated Trends

Figure 3.2 and figure 3.3 display the estimated trends in output and green technology, respectively. Those two trends are highly correlated,¹²⁸ but the trend on green innovation is approximately twice as high as the trend on output. This can explain the decoupling between emissions and output witnessed over the studied period. The trend on green innovation also exhibits more volatility at the business cycle frequency, which is consistent with the fact that the green technology sector is less mature than standard R&D.

3.6.3 Transition Pathways with Endogenous Abatement Technology

In this section, we characterize the dynamics of the economy when considering the net-zero pathway consistent with the objective of the EU for 2050 ($E_t = Cap_t$) under the presence of i) a fiscal subsidy scheme where 70% of the environmental revenues are reversed to the financial intermediaries to incentive higher investments in green technologies, and ii) a permanent financial policy, which lowers the capital constraint on financial intermediaries by 30%, thus allowing them to increase investments in green entrepreneurs. We use the estimated values of the structural parameters to replicate the growth rates in productivity and green technologies of the EZ economy. Furthermore, as we are unable to estimate the elasticity θ_3 of abatement costs $f(\mu_t)$ to green technology $A_{t,g}$ due to data unavailability, we consider three different cases that corresponds to three different values of $\theta_3 \in (0, 1)$.

Figure 3.4 shows the dynamics of our key variables (output, emissions, carbon price, marginal abatement cost, green technology, and global R&D) under a net-zero scenario. The carbon price is significantly driven by the elasticity θ_3 . The scenario where $\theta_3 = 1$ (the blue line) is the most optimal in terms of welfare, as the price of carbon is constantly decreasing, which is not the case when $\theta_3 = .7$ and $\theta_3 = .3$. With a higher theta, the output

¹²⁸This is not surprising, since the model features a spillover effect from the global technology to the green technology.

growth rate is also higher as profits are less impacted negatively by the carbon price. This impact on profits in turn lowers the global R&D investments and level. Turning to innovation in green technologies, a higher elasticity lowers the marginal cost of abatement, which leads to a lower carbon price to meet the emissions reduction goal. We note that a scenario where $\theta_3 = 1$ is highly unlikely as carbon prices are increasing nowadays, suggesting that $\theta_3 < 1$.¹²⁹

Figure 3.5, figure 3.6, and figure 3.7 display the counterfactual exercises where the public authorities implement either a fiscal or financial policy. Since the level of θ_3 is highly uncertain, we show the transition paths for the 3 values considered above. Focusing, however, on the the case where $\theta_3 = .3$ (the most conservative case), a financial fiscal subsidy, which reverses 70% of the carbon policy revenues to green innovators, is found to be the most effective in steering both growth in green technologies as well as global R&D. The financial policy act as carbon price stabilizers (a lower increase in the first half of the 30 years than the subsidy case). In all scenarios, the carbon price increases in the first 15 years, until the technology is mature enough to trigger higher abatement without having to raise the price on carbon as explained in section 3.4.4.

3.7 Conclusion

In this article, we first conduct an empirical analysis to assess the impacts of fiscal environmental policies and long-term bank lending on green innovation using a panel set on the EZ. We find that both the environmental policy and the availability of funds play an important and significant role in boosting green innovation.

Second, we develop a dynamic general equilibrium model based on the empirical evidence to assess the role fiscal and macro-financial policies can play both in the long-run.

We use a reduced form model to get the long-run transition pathways toward the net-

¹²⁹Further research could be done to investigate the elasticity of θ_3 of abatement cost to green technologies to better characterize the economy dynamics.

zero transition and find that making abatement technology available and cheap coupled with an optimal environmental policy is the most efficient tool (from a welfare perspective) in achieving climate goals. Relying solely on a carbon price could reach the same target, but comes with higher welfare costs.

Finally, we use a full fledged model incorporating both endogenous green innovation growth and financial intermediaries to quantitatively estimate trends on output and green innovation. We then assess the role fiscal subsidies and financial policy could play in boosting green innovation. We show that these two policies differently affect the path of the trend growth in green innovation. In addition, we show that fiscal subsidies are more effective than financial policy in reaching the net-zero while ensuring a lower carbon price over time. This leads us to conclude that policy makers could optimally foster growth in projects that enable cheaper and more effective abatement by giving incentives to financial intermediaries and entrepreneurs. In the context of the fight against climate change, and keeping in mind the ambitious goals that it requires, these findings represent both a glimmer of hope and a call for more action.

References

- D. Acemoglu, P. Aghion, L. Bursztyn, and D. Hemous. The environment and directed technical change. *American economic review*, 102(1):131–66, 2012.
- D. Acemoglu, D. Hemous, L. Barrage, P. Aghion, et al. Climate change, directed innovation, and energy transition: The long-run consequences of the shale gas revolution. In *2019 Meeting Papers*, number 1302. Society for Economic Dynamics, 2019.
- S. Adjemian and M. Juillard. Stochastic extended path approach. *Unpublished manuscript*, 2013.
- P. Aghion, A. Dechezleprêtre, D. Hemous, R. Martin, and J. Van Reenen. Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, 124(1):1–51, 2016.
- D. Anzoategui, D. Comin, M. Gertler, and J. Martinez. Endogenous technology adoption and r&d as sources of business cycle persistence. *American Economic Journal: Macroeconomics*, 11(3):67–110, 2019.
- Y. Bai, S. Song, J. Jiao, and R. Yang. The impacts of government r&d subsidies on green innovation: Evidence from chinese energy-intensive firms. *Journal of cleaner production*, 233:819–829, 2019.
- J. Benhabib, R. Rogerson, and R. Wright. Homework in macroeconomics: Household

- production and aggregate fluctuations. *Journal of Political economy*, 99(6):1166–1187, 1991.
- G. Benmir and J. Roman. Policy interactions and the transition to clean technology. *Grantham Research Institute on Climate Change and the Environment*, 2020.
- G. Benmir and J. Roman. The distributional costs of net-zero: A heterogeneous agent perspective. 2022.
- L. J. Christiano, M. Eichenbaum, and C. L. Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1):1–45, 2005.
- D. Comin and M. Gertler. Medium-term business cycles. *American Economic Review*, 96(3):523–551, 2006.
- S. Dietz and N. Stern. Endogenous growth, convexity of damage and climate risk: how nordhaus’ framework supports deep cuts in carbon emissions. *The Economic Journal*, 125(583):574–620, 2015.
- S. Dietz and F. Venmans. Cumulative carbon emissions and economic policy: in search of general principles. *Journal of Environmental Economics and Management*, 96:108–129, 2019.
- C. Fischer and M. Springborn. Emissions targets and the real business cycle: Intensity targets versus caps or taxes. *Journal of Environmental Economics and Management*, 62(3):352–366, 2011.
- M. Gertler and P. Karadi. A model of unconventional monetary policy. *Journal of monetary Economics*, 58(1):17–34, 2011.
- M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88, 2014.

- J. Greenwood, Z. Hercowitz, and P. Krusell. Long-run implications of investment-specific technological change. *The American economic review*, pages 342–362, 1997.
- G. Heutel. How should environmental policy respond to business cycles? optimal policy under persistent productivity shocks. *Review of Economic Dynamics*, 15(2):244–264, 2012.
- N. Jaimovich and S. Rebelo. Can news about the future drive the business cycle? *American Economic Review*, 99(4):1097–1118, 2009.
- T. S. Lontzek, Y. Cai, K. L. Judd, and T. M. Lenton. Stochastic integrated assessment of climate tipping points indicates the need for strict climate policy. *Nature Climate Change*, 5(5):441–444, 2015.
- W. D. Nordhaus and A. Moffat. A survey of global impacts of climate change: replication, survey methods, and a statistical analysis. Technical report, National Bureau of Economic Research, 2017.
- M. Pietrunti. Financial frictions and the real economy. Technical report, ESRB Working Paper Series, 2017.
- A. Queralto. A model of slow recoveries from financial crises. *Journal of Monetary Economics*, 114:1–25, 2020.
- P. M. Romer. Endogenous technological change. *Journal of political Economy*, 98(5, Part 2):S71–S102, 1990.
- F. Smets and R. Wouters. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, 1(5):1123–1175, September 2003.
- M. Song, J. Tao, and S. Wang. Fdi, technology spillovers and green innovation in china:

analysis based on data envelopment analysis. *Annals of Operations Research*, 228(1): 47–64, 2015.

N. Stern. The economics of climate change. *American Economic Review*, 98(2):1–37, 2008.

G. Vermandel. Endogenous trends. *Unpublished manuscript*, 2019.

M. L. Weitzman. A review of the stern review on the economics of climate change. *Journal of economic literature*, 45(3):703–724, 2007.

Appendices

3.A Appendix: Empirical Part

3.A.1 Data Sources

The data used¹³⁰ in this section were obtained from following sources:

- “Long-term loans granted by the financial sector to domestic non-financial corporation” were extracted from the ECB Statistical Data Warehouse.
- All EZ macro data (e.g. output) were obtained from the Eurostat database.
- EZ area and countries deflators are extracted from Fred database.
- Quarterly population for all samples are obtained from the OECD database.
- ‘Green Patent’ data are extracted from the European Patent Office (EPO) database.¹³¹
- ETS carbon price data are obtained from the European Environment Agency.

¹³⁰All data used were either extracted directly on a quarterly basis or transformed from a monthly frequency to a quarterly frequency.

¹³¹Data on green patents are selected through the new search filter introduced by the EPO: “cpc = y02”, which allows for identifying patents with green applicability.

3.A.2 Empirical Results

TABLE 3.2
Descriptive Statistics EZ aggregate.

Variable	Obs	Mean	Std. Dev.	Min	Max
The aggregate EZ					
GDP in (Million of Currency)	864	140894.1	207878.3	1509.7	872335
Population in (Million)	856	19 233 600	24 911 430	486 000	83 145 000
ETS price in (Euro)	846	11.63263	6.868454	3.8696	27.13354
Deflator in (100 scale)	864	98.48927	5.351341	80.69107	115.0133
Green Patents in (number)	864	202.1134	485.9407	0	2672
Gross capital formation	864	29631.05	43870.17	80	189979
Long-term loans	862	196488.1	254660.6	2612.26	920094

3.B Appendix: Model Part

3.B.1 Calibration

TABLE 3.3
Standard parameter values (quarterly basis)

	Calibrated parameters	Values
β	Discount factor	0.9975
α	Capital share	0.33
δ	Depreciation rate of capital	0.025
h	Habits formation parameter	0.8
σ	Risk aversion	2
φ	Disutility of labor	1
θ	Price elasticity	11
\bar{L}	Labor supply	0.33
\bar{L}_s	Labor supply	0.15
\bar{g}/\bar{y}	Public spending share in output	0.4

TABLE 3.4
Environmental and Entrepreneurs parameter values (quarterly basis)

	Calibrated parameters	Values
η	Material share	.125
a	Damage function parameter	1.004
b	Damage function parameter	0.02
v_1^o	Temperature parameter	0.5
v_2^o	Temperature parameter	0.00125
E^*	Emissions from the rest of the world	1.59
ϑ	Carbon intensity	0.287
δ_x	CO ₂ natural abatement	0.0021
θ_1	Abatement cost parameter	0.05
θ_2	Abatement cost parameter	2.7
θ_3	Abatement cost parameter	-0.6

TABLE 3.5
Financial parameter values (quarterly basis)

	Calibrated parameters	Values
γ_I	Capital adjustment cost	1.728
ω	Proportional transfer to the entering bankers	0.008
λ	Steady state risk weight on loans	0.43
θ_B	Probability of staying a banker	0.98

TABLE 3.6
Prior and Posterior distributions of structural parameters

		Prior distributions			Posterior distributions
		Shape	Mean	Std.	Mean $[0.050;0.950]$
<u>Shock processes:</u>					
Std. productivity	σ_A	\mathcal{IG}_1	0.001	0.005	0.0061 $[0.0050 ; 0.0071]$
Std. emission	σ_E	\mathcal{IG}_1	0.001	0.005	0.0082 $[0.0070 ; 0.0093]$
Std. R&D	σ_{A_s}	\mathcal{IG}_1	0.001	0.005	0.0352 $[0.0307 ; 0.0401]$
Std. green innovation	σ_{A_g}	\mathcal{IG}_1	0.001	0.005	0.0451 $[0.0392 ; 0.0512]$
AR(1) productivity	ρ_A	\mathcal{B}	0.50	0.20	0.9641 $[0.9349 ; 0.9934]$
AR(1) emission	ρ_E	\mathcal{B}	0.50	0.20	0.9796 $[0.9636 ; 0.9983]$
AR(1) R&D	ρ_{A_s}	\mathcal{B}	0.50	0.20	0.5456 $[0.3704 ; 0.7129]$
AR(1) green innovation	ρ_{A_g}	\mathcal{B}	0.50	0.20	0.9237 $[0.8509 ; 0.9832]$
<u>Endogenous growth parameters:</u>					
Trend slope	$\gamma_y - 1$	\mathcal{G}	0.005	0.001	0.0043 $[0.0029 ; 0.0058]$
Green innovation trend slope	$\gamma_{A_g} - 1$	\mathcal{G}	0.01	0.002	0.0100 $[0.0067 ; 0.0132]$
R&D investment exogenous trend	γ_{V_s}	\mathcal{N}	1	0.20	1.0020 $[1.0011 ; 1.0027]$
Green investment exogenous trend	γ_{V_g}	\mathcal{N}	1	0.20	1.0097 $[0.9951 ; 1.0276]$
R&D investment elasticity	η_g	\mathcal{B}	0.15	0.20	0.0721 $[0.0001 ; 0.1501]$
Green investment elasticity	η_s	\mathcal{B}	0.125	0.20	0.1088 $[0.0001 ; 0.2170]$
Log-marginal data density					666.668864

Notes: \mathcal{B} denotes the Beta, \mathcal{IG}_1 the Inverse Gamma (type 1), \mathcal{N} the Normal, and \mathcal{G} the Gamma distribution.

TABLE 3.7
Steady state values

	Baseline	Financial Policy	Fiscal Subsidies
Output	0.8318	0.8330	0.8401
Consumption	0.3776	0.3781	0.3813
Emissions	0.1749	0.1749	0.1750
Emissions to Output	0.2102	0.2100	0.2083
Overall Technology	1	1.0102	1.0720
Green Projects	0.1055	0.1065	0.1130
Abatement Cost	0.0536	0.0535	0.0523
Abatement Share	0.2675	0.2685	0.2742
Tax in Euros	28.50	28.50	28.50
Entrepreneurs' Profits	0.0756	0.0750	0.0756
Entrepreneurs' Risk Premium	0.0029	0.0020	0.0029
Banks' Capital Ratio	0.1581	0.1107	0.1581

3.B.2 Trend Figures

FIGURE 3.2. The Economy Trend Growth Rate (in %).

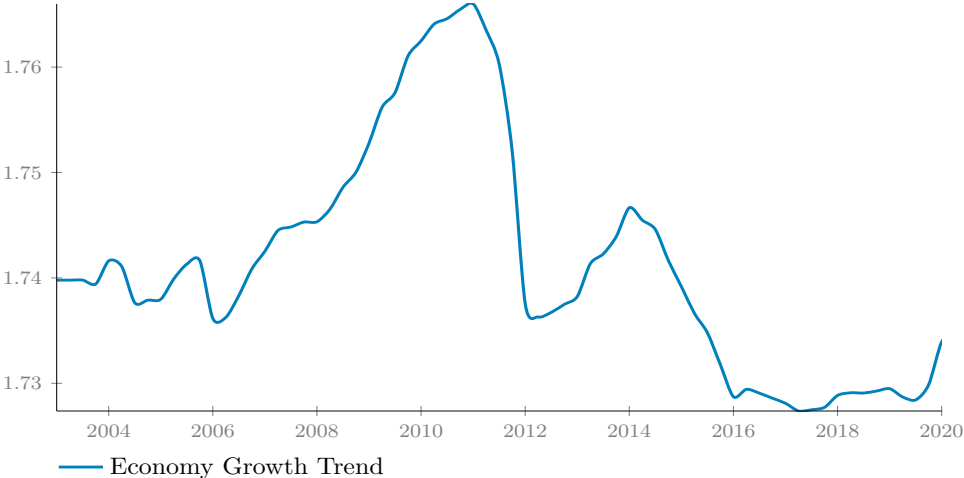


FIGURE 3.3. The Green Innovation Trend Growth Rate (in %).



3.B.3 Transition Pathways Figures

FIGURE 3.4. The Net-Zero Transition Pathway Under Different Abatement to Green Technology Elasticities θ_3 .

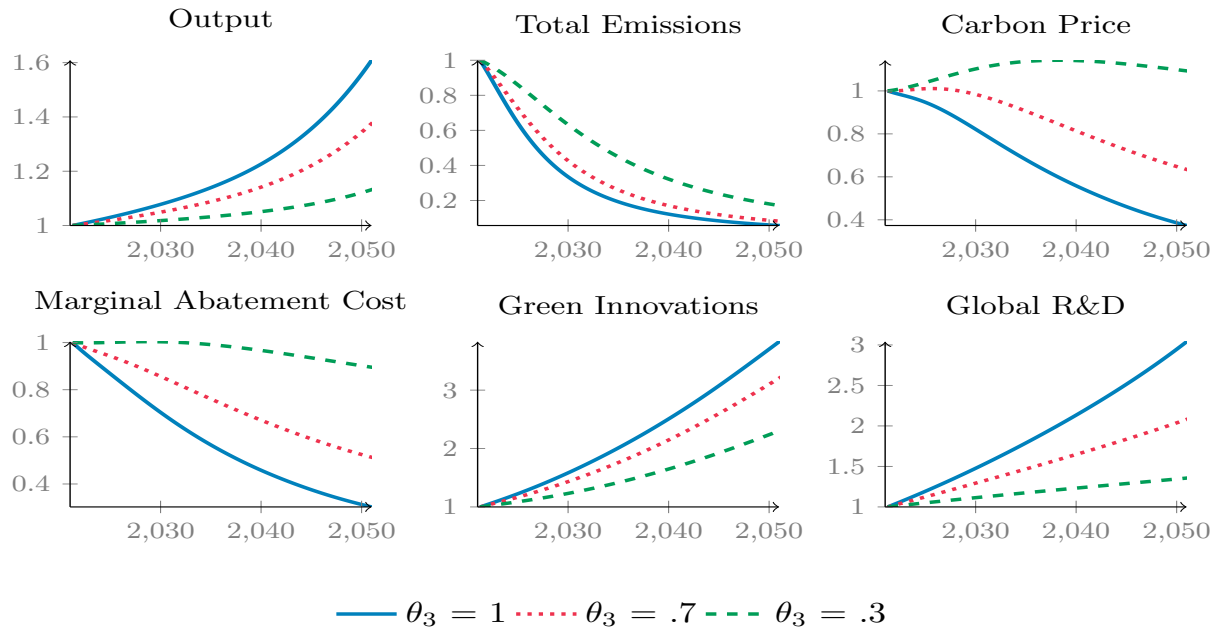


FIGURE 3.5. The Net-Zero Transition Pathway Under The Three Macro-Financial Policies (with $\theta_3 = 1$).

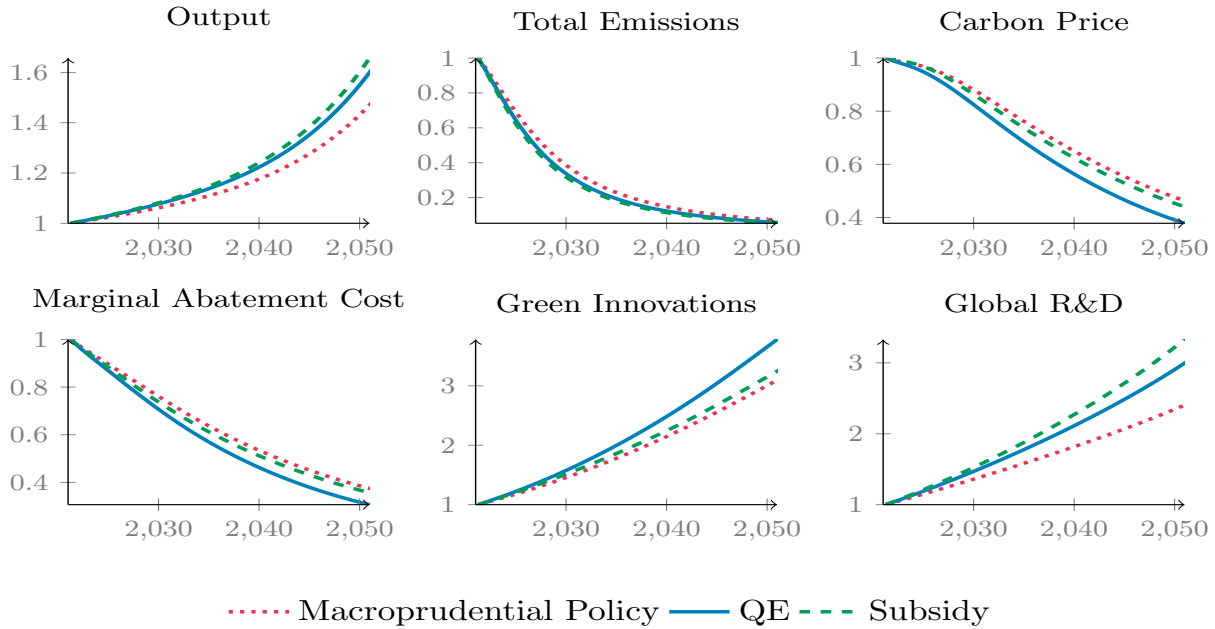


FIGURE 3.6. The Net-Zero Transition Pathway Under The Three Macro-Financial Policies (with $\theta_3 = .7$).

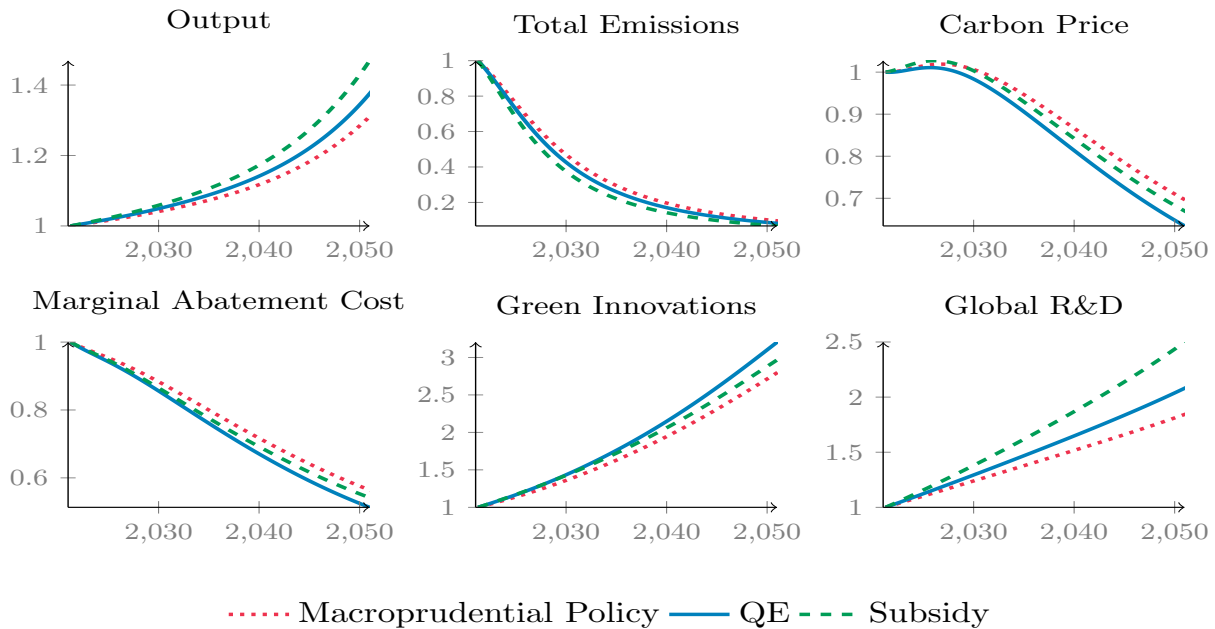
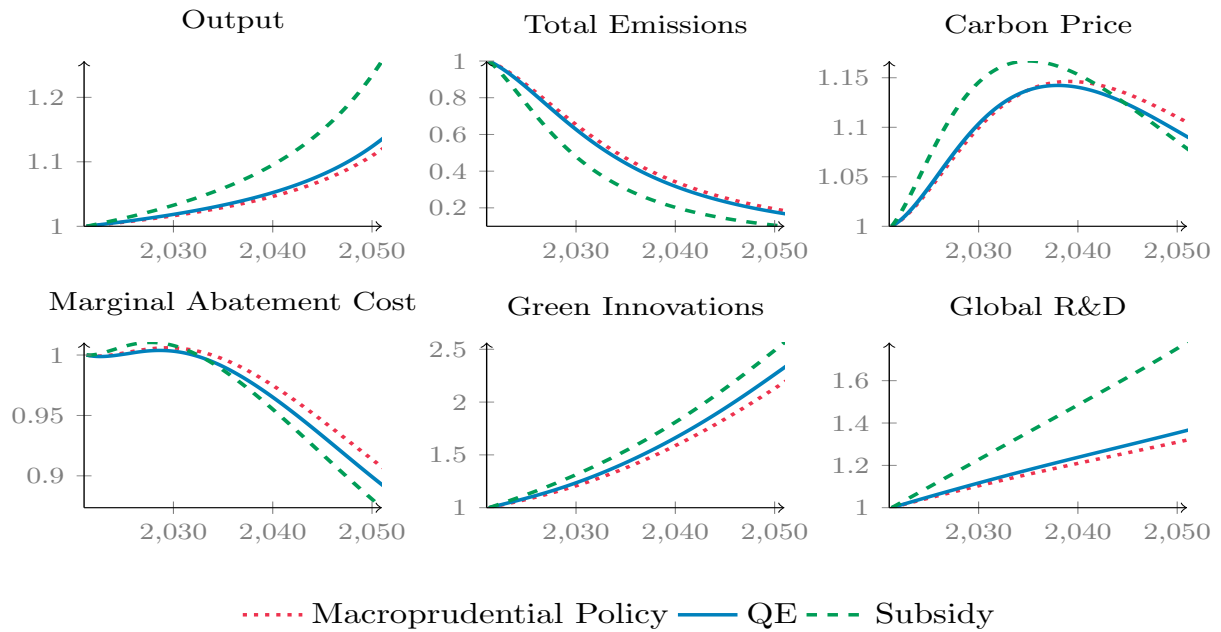


FIGURE 3.7. The Net-Zero Transition Pathway Under The Three Macro-Financial Policies (with $\theta_3 = .3$).



3.B.4 Model Equilibrium

3.B.4.1 The Social Planner Solution

The planners social problem for the households reads as following¹³²:

$$\begin{aligned}
\max E_t \sum_{i=0}^{\infty} \beta^i & \left(\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} \Gamma_t^{1-\sigma} L_{t+i}^{1+\varphi} \right. \\
& + \lambda_t (W_t L_t + W_t^s \bar{L}^s + W_t^g \bar{L}^g + R_t^K K_t + \Pi_t + T_t + R_t B_t - C_t - I_t - B_{t+1}) \\
& + \lambda_t \varrho_t^C ((1-\delta)K_t + I_t - K_{t+1}) \\
& + \lambda_t q_t (Y_t - W_t L_t - R_t^K K_t - f(\mu_t)Y_t - \Pi_t) \\
& + \lambda_t \Psi_t (d(T_t^o)K_t^\alpha L_t^{1-\alpha} - Y_t) \\
& + \lambda_t \xi_t^X (X_t - \eta X_{t-1} - E_t - E^*) \\
& + \lambda_t \xi_t^T (T_t^o - v_1^o (v_2^o X_{t-1} - T_{t-1}^o) - T_{t-1}^o) \\
& \left. + \lambda_t \xi_t^E (E_t - (1-\mu_t)\varphi_t Y_t) \right),
\end{aligned}$$

where the Social Cost of Carbon SCC_t is ξ_t^X , and Ψ_t the marginal cost component related to the firms problem.

The first order conditions determining the SCC_t are the ones with respect to T_t^o , X_t , E_t , μ_t and Π_t :

¹³²Please note that the social planner problem is not impacted by the financial intermediaries nor by the R&D entrepreneurs or the green innovators.

$$\lambda_t \xi_t^T = \beta(1 - v_1^o) \lambda_{t+1} \xi_{t+1}^T - \lambda_t \Psi_t \varepsilon_t^A \frac{\partial d(T_t^o)}{\partial T_t^o} K_{t-1}^\alpha L_t^{1-\alpha} \quad (3.78)$$

$$\lambda_t \xi_t^X = \beta(v_1^o v_2^o) \lambda_{t+1} \xi_{t+1}^T + \beta \eta \lambda_{t+1} \xi_{t+1}^X \quad (3.79)$$

$$\lambda_t \xi_{t,k}^E = g(\varkappa) \lambda_t \xi_t^X \quad (3.80)$$

$$\lambda_t q_{t,k} f'(\mu_{t,k}) = \varphi_{t,k} \lambda_t \xi_{t,k}^E \quad (3.81)$$

$$\lambda_t = \lambda_t q_{t,k}. \quad (3.82)$$

Rearranging these FOCs we obtain the following SCC_t :

$$\xi_t^T = (1 - v_1^o) \Lambda_{t,t+1} \xi_{t+1}^T - \sum_k \Psi_{t,k} \frac{\partial d(T_t^o)}{\partial T_t^o} K_t^\alpha L_t^{1-\alpha} \quad (3.83)$$

$$\xi_t^X = (v_1^o v_2^o) \Lambda_{t,t+1} \xi_{t+1}^T + \eta \Lambda_{t,t+1} \xi_{t+1}^X \quad (3.84)$$

$$\xi_t^E = g(\varkappa) \xi_t^X \quad (3.85)$$

$$f'(\mu_t) = \varphi_t \xi_t^E \quad (3.86)$$

The competitive equilibrium problem for the firms reads as following:

$$\begin{aligned} \max E_t \sum_{i=0}^{\infty} & \left(\left(\frac{P_{jt}}{P_t} Y_t - W_t L_t - R_t^K K_t - f(\mu_t) Y_t - \tau_t E_t - \Pi_t \right) \right. \\ & + \lambda_t \Psi_t (d(T_t^o) K_{t-1}^\alpha L_t^{1-\alpha} - Y_t) \\ & \left. + \lambda_t \xi_t^E (E_t - (1 - \mu_t) \varphi_t Y_t) \right) \end{aligned}$$

The first order conditions determining the tax rate τ_t are the ones with respect to E_t and

μ_t :

$$\xi_t^F = \tau_t \quad (3.87)$$

$$f'(\mu_t) = \xi_t^F \varphi_t \quad (3.88)$$

Thus, from both the household and firm FOCs, we get:

$$\xi_t^F = \tau_t \quad (3.89)$$

$$\xi_t^F = \xi_t^E \quad (3.90)$$

$$f'(\mu_t) = \xi_t^E \varphi_t \quad (3.91)$$

$$\xi_t^T = (1 - v_1^o) \Lambda_{t,t+1} \xi_{t+1}^T - \Psi_t \frac{\partial d(T_t^o)}{\partial T_t^o} K_{t-1}^\alpha L_t^{1-\alpha} \quad (3.92)$$

$$\xi_t^X = (v_1^o v_2^o) \Lambda_{t,t+1} \xi_{t+1}^T + \eta \Lambda_{t,t+1} \xi_{t+1}^X \quad (3.93)$$

$$\xi_t^E = \xi_t^X \quad (3.94)$$

3.B.4.2 The Firms

The firm maximization of profits reads:

$$\Pi_{jt} = \max_{P_{jt}, Y_{jt}} \left(\frac{P_{jt}}{P_t} - MC_t^f \right) Y_{jt}, \quad (3.95)$$

s.t.

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} Y_t. \quad (3.96)$$

The first order condition yields:

$$\frac{P_{jt}}{P_t} = \frac{\theta}{\theta - 1} MC_t^f \quad (3.97)$$

Now using the pricing equation $P_t = (\int_0^{A_{t,s}} P_{jt}^{1-\theta} dj)^{\frac{1}{1-\theta}}$ we get:

$$\frac{P_{jt}}{P_t} = A_{t,s}^{\frac{1}{\theta-1}} \quad (3.98)$$

Thus, we can rewrite the first order condition as:

$$\frac{\theta}{\theta-1} MC_t^f = A_{t,s}^{\frac{1}{\theta-1}}. \quad (3.99)$$

Therefore,

$$\Pi_{jt} = \left(\frac{P_{jt}}{P_t} - MC_t^f \right) Y_{jt}, \quad (3.100)$$

$$= \frac{1}{\theta} \frac{Y_t}{A_{t,s}} \quad (3.101)$$

Turning now to the Cobb-Douglas production function, we use the inputs market-clearing conditions $\int_0^{A_{t,s}} L_{jt} dj = A_{t,s} L_t$ and $\int_0^{A_{t,s}} K_{jt} dj = A_{t,s} K_t$ to retrieve the final form of the production function:

$$Y_t = A_{t,s}^{\frac{1}{\theta-1}} d(T_t^o) K_t^\alpha L_t^{1-\alpha}. \quad (3.102)$$

The rest of the first order condition remains similar to the ones presented in the reduced form model.

3.B.4.3 The Households, Innovators, and Financial intermediaries

For the household, the entrepreneurs, and the banking sector, all equilibrium equations are presented in the core text.

3.B.5 Carbon Cap and Green Innovation

By substituting the environmental cap policy equation ($E_t = Cap_t$) into the emissions flow equation ($E_t = (1 - \mu_t)vY_t$), we get:

$$\mu_t = 1 - \frac{Cap_t}{vY_t} \quad (3.103)$$

Using the FOC on abatement [Equation 2.28](#):

$$\text{Carbon Price}_t = \theta_1 \theta_2 \frac{\left(1 - \frac{Cap_t}{vY_t}\right)^{\theta_2 - 1}}{v} A_{t,g}^{-\theta_3} \quad (3.104)$$

We see that the carbon price could be steered by either Cap_t and/or $A_{t,g}$ ¹³³. It is then clear that when:

$$\Delta A_{t,g}^{\theta_3} > \Delta \left(1 - \frac{Cap_t}{vY_t}\right)^{\theta_2 - 1} \Rightarrow \text{Carbon Price}_t \text{ decrease.}$$

While when:

$$\Delta A_{t,g}^{\theta_3} < \Delta \left(1 - \frac{Cap_t}{vY_t}\right)^{\theta_2 - 1} \Rightarrow \text{Carbon Price}_t \text{ increase.}$$

Turning now to the abatement cost, we have:

$$f(\mu_t) = \theta_1 \left(1 - \frac{Cap_t}{vY_t}\right)^{\theta_2} A_{t,g}^{-\theta_3} \quad (3.105)$$

Likewise, when:

$$\Delta A_{t,g}^{\theta_3} > \Delta \left(1 - \frac{Cap_t}{vY_t}\right)^{\theta_2} \Rightarrow \text{the per unit abatement cost decrease.}$$

While when:

$$\Delta A_{t,g}^{\theta_3} < \Delta \left(1 - \frac{Cap_t}{vY_t}\right)^{\theta_2} \Rightarrow \text{the per unit abatement cost increase.}$$

As the total abatement cost $Z_t = f(\mu_t)Y_t$ enters the banks returns equation $R_{t,e} = \frac{\phi_{RDg}(Z_t + Q_{t,e})}{Q_{t-1,e}}$, a drop in Z_t would reduce the returns $R_{e,t}$. In turn, the decrease in $R_{e,t}$ gives less incentives to financial intermediaries to finance green equity innovators, which end up

¹³³The changes on Y_t being very small over the business cycle with respect to climate damages, we don't focus on their effects on carbon prices.

decreasing their overall number of innovations $A_{t,g}$.

3.B.6 Balanced Growth Path Equilibrium

The growth rate of Γ_t determines the growth rate of the economy along the balanced growth path. This growth rate is denoted by γ_t^Y , where:

$$\Gamma_t = \gamma_t^Y \Gamma_{t-1} \quad (3.106)$$

Stationary variables are denoted by lower case letters, whereas variables that are growing are denoted by capital letters. For example, in the growing economy output is denoted by Y_t . De-trended output is thus obtained by dividing output in the growing economy by the level of growth progress:

$$y_t = \frac{Y_t}{\Gamma_t} \quad (3.107)$$

Emissions, which we denote by E_t , in the growing economy are given as follows:

$$E_t = (1 - \mu_t)vY_t \quad (3.108)$$

where v the elasticity of emissions to output.

Thus, in the de-trended economy, emissions law of motion reads as following:

$$e_t = (1 - \mu_t)vy_t \quad (3.109)$$

where:

$$e_t = \frac{E_t}{\Gamma_t} \quad (3.110)$$

The stock of emissions in the atmosphere is denoted by X_t , while the temperature is

called T_t^o in the growing economy:

$$X_t = (1 - \gamma_d)X_{t-1} + E_t \quad (3.111)$$

$$T_t^o = v_1^o(v_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o, \quad (3.112)$$

where $(1 - \gamma_d)$ the decay rate.

The de-trended X_t and T_t^o read as following:

$$x_t = \frac{(1 - \gamma_d)}{\gamma_t^Y} x_{t-1} + e_t \quad (3.113)$$

$$\gamma_t^Y t_t^o = v_1^o(v_2^o x_{t-1} - t_{t-1}^o) + t_{t-1}^o \quad (3.114)$$

where:

$$x_t = \frac{X_t}{\Gamma_t} \quad (3.115)$$

$$t_t^o = \frac{T_t^o}{\Gamma_t} \quad (3.116)$$

$$(3.117)$$

In the growing economy, with the above growth progress, the production function is as follows:

$$Y_t = \varepsilon_t^A A_{t,s}^{\frac{1}{\theta-1}} d(T_t^o) K_t^\alpha L_t^{1-\alpha} \quad (3.118)$$

where labour L_t and the technology shock ε_t^A are stationary variables. Furthermore, the climate damage function captures the growth rate Γ_t such that $d(T_t^o) = a e^{-\frac{b}{\Gamma_t^2} T_t^o^2}$. Capturing the growth rate of the economy within the damage function allows us to simplify the de-trended form of the damage function without a loss of generality.

De-trending the production function, gives the following:

$$\Gamma_t y_t = \varepsilon_t^A A_{t,s}^{\frac{1}{\theta-1}} d(t_t^o) \Gamma_t^\alpha k_t^\alpha L_t^{1-\alpha} \quad (3.119)$$

Thus, the growth rate of the economy will satisfy:

$$\Gamma_t = A_{t,s}^{\frac{1}{\theta-1}} \Gamma_t^\alpha, \quad (3.120)$$

with the de-trended production function:

$$y_t = \varepsilon_t^A d(t_t^o) k_t^\alpha L_t^{1-\alpha} \quad (3.121)$$

Rewriting the [Equation 3.120](#), we retrieve the growth rate of the economy:

$$\Gamma_t = A_{t,s}^{\frac{1}{(\theta-1)(1-\alpha)}} \quad (3.122)$$

The capital-accumulation equation in the growing economy is:

$$K_t = (1 - \delta)K_{t-1} + I_{t-1} \quad (3.123)$$

In the de-trended economy, we thus have:

$$k_t = \gamma_t^{Y-1} [(1 - \delta)k_{t-1} + i_{t-1}] \quad (3.124)$$

with both capital and investment de-trended variables reading as: $k_t = \frac{K_t}{\Gamma_t}$ and $i_t = \frac{I_t}{\Gamma_t}$, respectively. The wage as shown in the model section reads as following:

$$W_t = (1 - \alpha) \Psi_t \frac{Y_t}{L_t} \quad (3.125)$$

The de-trended wages¹³⁴ will therefore read as:

$$w_t = (1 - \alpha)\Psi_t \frac{y_t}{L_t} \quad (3.126)$$

with the de-trended wage w_t reads as $w_t = \frac{W_t}{\Gamma_t}$.

Moving to the endogenous growth components of our economy, both $A_{t,s}$ and $RD_{t,s}$, as well as $A_{t,g}$ and $RD_{t,g}$ grow at similar rates $A_{t,s}$ and $A_{t,g}$, respectively. The law of motion for both the adjusted global R&D entrepreneurs ($\gamma_{A_{t,g}} = \frac{A_{t,g}}{A_{t-1,g}}$ and $\tilde{R}D_{t,g} = \frac{RD_{t,g}}{A_{t,g}}$) and green innovators ($\gamma_{A_{t,s}} = \frac{A_{t,s}}{A_{t-1,s}}$ and $\tilde{R}D_{t,s} = \frac{RD_{t,s}}{A_{t,s}}$) reads as:

$$\gamma_{A_{t,s}} = \phi_{RD,s}(1 + \tilde{R}D_{t-1,s}), \quad (3.127)$$

$$\gamma_{A_{t,g}} = \phi_{RD,g}(1 + \tilde{R}D_{t-1,g}), \quad (3.128)$$

With these new forms of technology growth rates, we can derive the de-trended expression for initial investment $N_{t,s} = \eta RD_{t,s} MC_t^{RD,s}$ and skilled labour wages $W_{t,s} = (1 - \eta) MC_t^{RD,s} \frac{RD_{t,s}}{L_s^s}$:

$$n_{t,s} = \eta_s \tilde{R}D_{t,s} MC_t^{RD,s} \quad (3.129)$$

$$w_{t,s} = (1 - \eta_s) MC_t^{RD,s} \frac{\tilde{R}D_{t,s}}{L_s^s} \quad (3.130)$$

where $N_{t,s} = n_{t,s} A_{t,s}$ and $W_{t,s} = w_{t,s} A_{t,s}$.

To insure a balanced growth path within the economy, we added the two exogenous growth rates $V_{t,g}$ and $V_{t,s}$, which will impact the specific investment on green and R&D expenditures, respectively, in order to ensure stationnarity. As explained in the balanced growth path section in the core of this paper, the growth rates of green and global R&D expenditures have been increasing faster than output. As such, to capture this trend in the

¹³⁴We note, that Ψ_t the labour/capital share is a stationnary variable. The same can be noticed for the returns on capital R_t^k , the total marginal cost MC_t , abatement μ_t , and the environmental policy τ_t .

expenditure side of GDP and still satisfy the supply side ratios in output, we introduce the common investment-specific trends $V_{t,s}$ and $V_{t,g}$ ¹³⁵, which grow at gross rates $\gamma_s^V = \frac{V_{t,s}}{V_{t-1,s}}$ and $\gamma_g^V = \frac{V_{t,g}}{V_{t-1,g}}$. Therefore, the economy's resource constraint reads as:

$$y_t = c_t + i_t + g_t + V_{t,g}n_{t,g} + V_{t,s}n_{t,s} \quad (3.131)$$

where the de-trended variables read as: consumption $c_t = \frac{C_t}{\Gamma_t}$, investment $i_t = \frac{I_t}{\Gamma_t}$, government spending $g_t = \frac{G_t}{\Gamma_t}$, and both $n_{t,s} = \frac{N_{t,s}}{A_{t,s}}$ the initial investment overall technologies and $n_{t,g} = \frac{N_{t,g}}{A_{t,g}}$.

Now that we have the expression for investment specific expenditures for the global R&D sector, we can easily derive the de-trended expression for the aggregate firms' profits Π_t , which will be subject to the same exogenous growth rate defined above:

$$MC_t^{RD,s} = \Pi_t \quad (3.132)$$

$$\Pi_t = \frac{1}{\theta} \frac{Y_t}{A_{t,s}} \quad (3.133)$$

As the marginal cost for R&D is stationary, the profits would be as well. However, the output over endogenous growth grows at a slightly different growth than the BGP growth rate Γ_t . We thus add the same exogenous growth rate $V_{t,s}$ as follows:

$$\Pi_t V_{t,s} = \frac{1}{\theta} y_t \quad (3.134)$$

¹³⁵Using the fact that $A_{t,s}/V_{t,s} = \Gamma_t$ and $A_{t,g}/V_{t,g} = \Gamma_t$ we get the following economy investment specific growth rate which satisfies the BGP $V_{t,s} = A_{t,s}^{1 - \frac{1}{(\theta_P - 1)(1 - \alpha)}}$ and $V_{t,g} = A_{t,g} A_{t,s}^{-\frac{1}{(\theta_P - 1)(1 - \alpha)}}$.

Similarly, the green innovation sector de-trending reads as the following:

$$n_{t,g} = \eta_g \tilde{RD}_{t,g} MC_t^{RD,g} \quad (3.135)$$

$$w_{t,g} = (1 - \eta_g) MC_t^{RD,g} \frac{\tilde{RD}_{t,g}}{L_s^g} \quad (3.136)$$

$$MC_t^{RD,s} = Q_{t,e} \quad (3.137)$$

with, $N_{t,g} = n_{t,g} A_{t,g}$ and $W_{t,g} = w_{t,g} A_{t,g}$.

The abatement cost Z_t , which is impacted by the level of green innovation in the economy $A_{t,g}$ reads as:

$$Z_t = \theta_1 A_{t,g}^{-\theta_3} \mu_t^{\theta_2} Y_t \quad (3.138)$$

When de-trended, it reads as:

$$Z_t V_{t,z} = \theta_1 \mu_t^{\theta_2} y_t \quad (3.139)$$

where $V_{t,z} = A_{t,g}^{\theta_3} / \Gamma_t$ is the exogenous growth rate which acts to correct for the balanced growth path such that $\gamma_z^V = V_{t,z} / V_{t-1,z}$. Notice that for $\theta_3 = 1$, we retrieve the same BGP exogenous correcting growth rate $V_{t,g}$.

The lump sum taxes T_t grow at the growth rate of the economy Γ_t :

$$g_t = t_t + \tau_t e_t \quad (3.140)$$

with $T_t = t_t \Gamma_t$.

Moving now to the household's maximization of utility problem, the de-trended lifetime welfare maximization reads:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(\Gamma_{t+i} c_{t+i} - h \Gamma_{t+i-1} c_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} \Gamma_{t+i}^{1-\sigma} L_{t+i}^{1+\varphi} \right] \quad (3.141)$$

Thus, rewriting the above equation by denoting $\tilde{\beta} = \beta(\gamma_t^Y)^{1-\sigma}$ and $h_t = h(\gamma_t^Y)^{-1}$ we get:

$$E_t \sum_{i=0}^{\infty} \tilde{\beta}^i \left[\frac{(c_{t+i} - h_t c_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \right] \quad (3.142)$$

Moving now to the financial intermediaries, the de-trended balance sheet reads as:

$$Q_{t,e} \tilde{S}_{t,e} = n_t + b_t \quad (3.143)$$

where the net worth $n_t = \frac{N_t}{A_{t,g}}$ grows at Γ_t , the debt to households $b_t = \frac{B_t}{A_{t,g}}$ grows at $A_{t,g}$, and assets held by the bank ($S_{t,e} = A_{t,g} + RD_{t,g}$) grow at $A_{t,g}$ and satisfy $\tilde{S}_{t,e} = S_{t,e}/A_{t,g} = 1 + \tilde{RD}_{t,g}$.

The de-trended retained earnings reads as:

$$n_t = (\gamma_{t,g}^V)^{-1} (R_{t,e} Q_{t-1,e} \tilde{S}_{t-1,e} - R_t b_{t-1}) \quad (3.144)$$

The value function de-trended value $V_t^B = v_t^B A_{t,g}$ reads as¹³⁶:

$$v_t^B = \max E_t \left\{ \sum_{i=1}^{\infty} \beta^i \Lambda_{t,t+i} (1 - \theta_B) \theta_B^{i-1} n_{t+1+i} \right\} \quad (3.145)$$

¹³⁶We note that Γ_t^B is both stationnary as $V_t^B = \Gamma_t^B N_t$. In addition, as Γ_t^B is stationnary, Ω_t , and ν_t are also stationnary.

General Conclusion

In this thesis, we examine the macro-financial and distributional impacts of net-zero environmental policies that aim for carbon neutrality by 2050.

In the first chapter, we assess the macro-financial implications of setting an environmental policy consistent with the net-zero target using a cap system. We incorporate a climate dynamics framework into the business cycle macro-finance framework. The model includes both endogenously-constrained financial intermediaries, heterogeneous firms, and climate dynamics. Using this framework, we examine the implications of setting a carbon cap (i.e. a sub-optimal policy) with respect to the social cost of carbon (i.e. the first best). We find that a price of about 350 euro per ton of carbon is needed to be aligned with the net-zero target. However, such implicit carbon price induces two inefficiencies. The first inefficiency is linked to the need for an increasingly higher price of carbon (compared to the optimal price) in order to meet EU targets. This decoupling generates a growing welfare loss. The second inefficiency is related to climate-related financial risk, which can be addressed through a green macroprudential policy. To address these inefficiencies, we propose a green macroprudential policy that favors the green sector. This policy is efficient in partially offsetting the welfare loss while reaching emission targets and allows regulators to address climate-related financial risk.

In the second chapter, we expand our investigation and look further at the welfare and potential distributional impacts that households could be subjected to during the transition to a net-zero economy. To do so, we provide a macro framework with heterogeneous

agents and climate dynamics in order to study the effects of the transition to a low-carbon economy on household income and wealth distributions. We first show that carbon price shocks impact households asymmetrically depending on their level of income. Specifically, the bottom 50 percent income level households see their consumption fall, while a positive shock on the price of carbon tends to momentarily increase consumption for the top 50 percent income level households. The main finding of this paper suggest that while the transition to net-zero is a necessary step toward a long-run sustainable economy, it induces changes in the distribution of income and wealth that could potentially lead to social unrest. Public authorities need to monitor the impact of large-scale environmental policies on different types of households (especially financially-constrained households), if the transition is to be successful. With this perspective, targeted redistribution of carbon revenues could be a major tool in government strategies. We note that, although fiscal transfers are able to offset some of the unequal impacts of carbon pricing, the medium/long run effect cannot be addressed solely by redistributing carbon revenues. Additionally, the need for cheaper abatement technologies remains key to a low (social) cost transtion.

In the third chapter, we study the impacts of fiscal environmental policies and long-term bank lending on green innovation. After empirically establishing that both policies play an important and significant role in boosting green innovation, we then develop a dynamic general equilibrium model to assess the role fiscal and macro-financial policies can play in the long-run. This model incorporates both endogenous green innovation growth and financial intermediaries to quantitatively estimate trends on output and green innovation. Using this framework, we estimate the macro trends on output and green innovation, before turning to assessing the role fiscal subsidies and financial policy could play in boosting green innovation. We show that these two policies positively contribute to trend growth in green innovation. We show that financial policy aimed at boosting green loans could achieve the same objective as fiscal subsidies, thus providing a more flexibly to fiscal authorities in their revenue management (e.g. redistributing carbon revenues to households).

RÉSUMÉ

Dans cette thèse, nous fournissons des cadres théoriques pour étudier les implications de la transition verte sur la macroéconomie et la stabilité financière, les distributions de revenus/richeesse, et l'innovation verte. Nous utilisons ces cadres novateurs pour évaluer comment différentes politiques (par exemple fiscales, macro-financières) pourraient être utilisées dans les stratégies de mitigation du changement climatique.

Nous étudions d'abord les inefficacités du marché des permis carbone où i) le niveau du prix du carbone requis pour atteindre les objectifs climatiques implique une perte de bien-être qui peut être mitigée par la mise en place de pondérations sectorielles dans les exigences de fonds propres des banques favorables aux secteurs à moins forte intensité de carbone, ce qui atténue également le risque climatique physique dans l'économie, et ii) la conception d'une politique de plafonnement et d'échange implique une incertitude sur le prix du carbone qui entraîne une volatilité des primes de risque qui peut être compensée suivant des règles d'assouplissement quantitatif. Deuxièmement, nous étudions les impacts distributifs de la mise en œuvre de l'objectif de zéro émission aux États-Unis à l'horizon 2050. Nous modélisons une économie de ménages hétérogène et montrons que la politique du net-zéro 2050 améliore le bien-être à long terme, mais induit des coûts de distribution à court/moyen terme. Nous montrons ensuite comment la distribution des revenus de la politique carbone pourrait partiellement compenser les pertes de consommation et faciliter la transition vers le zéro carbone. Troisièmement, nous fournissons une motivation empirique sur le rôle important que jouent le prix du carbone et les prêts financiers dans le pilotage de l'innovation verte. Nous montrons ensuite, à l'aide d'un modèle de croissance endogène, comment une politique macrofinancière visant à augmenter les prêts verts à long terme peut jouer un rôle similaire dans la stimulation de l'innovation verte (comme le font les subventions fiscales) et la baisse des prix du carbone, allégeant ainsi le fardeau des entreprises lié à la hausse du prix du carbone et permettant en même temps plus de flexibilité aux autorités fiscales dans leur gestion des revenus (par exemple, en redistribuant les revenus du carbone aux ménages).

MOTS CLÉS

Politiques Macro-financières, Politique Environnementale, Macro-Climat et Distribution, Innovation Verte.

ABSTRACT

In this dissertation, we provide frameworks to investigate the implications of the green transition on the macroeconomy and financial stability, income/wealth distributions, and green innovation. We use these novel frameworks to assess how different policies (e.g. fiscal, macro-financial) could be utilized in climate change mitigation strategies.

We first study the inefficiencies of a market for carbon permits where i) the level of carbon price required to achieve climate goals imply a loss in welfare that can be mitigated through the implementation of sector-specific weights in the capital requirements of banks favorable to less carbon intensive sectors, which also mitigates the climate physical risk in the economy, and ii) the design of a cap and trade policy implies uncertainty on the price of carbon that leads to volatility in risk premia that can be offset by means of quantitative easing rules. Second, we investigate the distributional impacts of implementing the net-zero emissions target in the U.S. for the 2050 horizon. We model a heterogeneous household economy and show that 2050 net-zero policy is welfare enhancing in the long run, but induces short/medium-run distributional costs. We then show how distributing revenue from the carbon policy could partially offset consumption losses and smooth the net-zero transition. Third, we provide empirical motivation on the significant role carbon price and financial loans play in steering green innovation. We then show using an endogenous growth model how macro-financial policy aiming to increase long-term green loans can play a similar role in boosting green innovation (as do fiscal subsidies) and lowering carbon prices, thus easing the burden on firms related to the rise in the carbon price and providing more flexibly to fiscal authorities in their revenue management (e.g. redistributing carbon revenues to households).

KEYWORDS

Macro-financial Policies, Environmental Policy, Distributional Macro-Climate, Green Innovation.