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Ilia Shilov

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**DE L'UNIVERSITÉ PSL**

Préparée à l'École Normale Supérieure

# Game-Theoretic Approaches for Peer-to-Peer Energy Trading

Soutenue par

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## Résumé

Les marchés de l'électricité subissent des transformations décentralisées importantes en raison de la numérisation croissante, de l'inclusion à grande échelle des ressources énergétiques distribuées (DER) et de l'autonomisation des consommateurs. Ces changements posent des défis qui nécessitent de formuler de nouveaux modèles de marché intégrant des structures décentralisées, des sources d'énergie renouvelables (RES) et le comportement stratégique des consommateurs. Dans de tels marchés, de multiples acteurs opèrent à différents niveaux hiérarchiques, induisant des problèmes d'équilibre et nécessitant de nouvelles approches pour le calcul des équilibres de marché. Cette thèse aborde plusieurs aspects complexes de la réalisation de l'équilibre sur les marchés décentralisés de l'électricité, notamment la coordination de divers acteurs, la gestion de la variabilité de l'offre et de la demande, la gestion des contraintes du réseau électrique et l'asymétrie de l'information entre les participants. Des approches théoriques des jeux sont appliquées, utilisant le concept d'équilibre de Nash généralisé (GNE) pour évaluer les actions des participants et les contraintes du réseau électrique.

Dans un premier temps, nous abordons les incertitudes dans les marchés de pair à pair (P2P) dans un contexte hétérogène d'aversion pour le risque. Nous proposons un nouveau cadre dans lequel les consommateurs peuvent couvrir les risques au moyen de contrats financiers. Le cadre utilise une approche de jeu de Stackelberg, impliquant une compagnie d'assurance et les consommateurs, avec deux conceptions du problème envisagées, chacune conduisant à un schéma de tarification de la couverture des risques. Le cadre atténue l'inefficacité du marché causée par les objectifs non alignés des consommateurs et de la compagnie d'assurance, en incluant les incitations de prix pour cette dernière, ce qui garantit l'existence d'une solution pour la formulation pessimiste. La méthode proposée simplifie l'analyse de l'équilibre du marché et permet de dériver l'équilibre du marché dans différents contextes.

En outre, l'interaction entre les niveaux physique et financier, gérés par le gestionnaire du réseau de distribution (GRD), est analysée. Deux modèles de marché financier pour les consommateurs sont comparés : un modèle centralisé et un modèle P2P entièrement distribué. L'étude démontre l'efficacité de Pareto de l'équilibre du marché et la façon dont la structure de prix proposée limite le comportement de resquillage au niveau financier. Les résultats numériques étudient l'influence de la flexibilité des agents et de l'inclusion de la production renouvelable sur le fonctionnement du marché proposé.

En outre, un cadre traitant de l'asymétrie de l'information est examiné lorsque les agents possèdent des informations privées. Cette question est formulée comme un jeu de communication non coopératif, permettant aux agents de déterminer leurs rapports aléatoires à partager avec les autres acteurs du marché. Des preuves de l'unicité de la solution d'équilibre variationnel du jeu sont dérivées et une expression en forme fermée du prix de la confidentialité est fournie.

Enfin, l'étude explore le couplage potentiel entre les marchés de prévision et les marchés P2P de l'électricité afin d'atténuer l'incertitude liée à l'imprécision des prévisions de production des SER, en permettant aux agents d'acquiescer une prévision de leur production basée sur les SER. Les conditions de l'efficacité du marché P2P sont identifiées, une condition clé étant la participation des prosommateurs au marché des prévisions. L'étude présente une preuve de la rationalité individuelle du modèle de marché couplé, montrant que les consommateurs sont toujours rationnellement incités à participer au marché des prévisions. En outre, les conditions sur les distributions de probabilité des prévisions garantissant cette propriété sont explorées. Les résultats numériques confirment les conclusions théoriques, illustrant la rentabilité du couplage des marchés tant pour les vendeurs de prévisions que pour les consommateurs.

Cette thèse conclut que les services complémentaires, la coordination et l'échange d'informations sont essentiels pour optimiser les opérations de marché dans des environnements où les agents agissent de manière stratégique, en explorant les complexités associées aux marchés P2P de l'électricité et les problèmes d'équilibre correspondants. La thèse se concentre sur la coordination de divers acteurs, la gestion de l'offre et de la demande, la couverture des risques, la prise en compte des contraintes du réseau électrique et l'asymétrie de l'information, en s'appuyant sur des approches de la théorie des jeux. Les résultats soulignent l'importance des services supplémentaires et de l'échange d'informations dans ces environnements stratégiques. En tant que telle, cette thèse présente des avancées clés pour améliorer l'état de l'art et la compréhension des complexités des marchés décentralisés de l'électricité, contribuant ainsi au débat actuel sur la restructuration du marché de l'électricité.

Les marchés de l'électricité sont de plus en plus décentralisés et intègrent davantage de ressources énergétiques distribuées (DER) et de modèles axés sur le consommateur. Cette transition présente de nouveaux défis et nécessite des modèles de marché actualisés. Cette thèse explore ces complexités, en se concentrant sur la coordination de divers acteurs, la gestion de l'offre et de la demande, les contraintes de distribution et l'asymétrie de l'information. Nous utilisons des approches théoriques des jeux, examinant des cadres pour les marchés de l'électricité de pair à pair avec des agents averses au risque et des prosommateurs couvrant les risques par le biais de contrats financiers. Nous étudions également l'interaction entre les niveaux physiques et financiers, la comparaison des modèles de marché centralisés et entièrement distribués, et aborde la question de l'asymétrie de l'information. Enfin, nous étudions le couplage des marchés prévisionnels avec le marché de l'électricité. Les résultats soulignent l'importance des services supplémentaires et de l'échange d'informations dans ces environnements stratégiques.

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**Mots clés :** La théorie des jeux, Smart grids, Algorithmes, Pair a pair, L'équilibre de Nash généralisé

## Abstract

Electricity markets are undergoing significant decentralization transformations due to the increasing digitalization, large-scale inclusion of Distributed Energy Resources (DERs), and broader consumer empowerment. These shifts present challenges requiring to formulate new market models incorporating decentralized structures, renewable energy sources (RES), and the strategic behavior of prosumers. In such markets, multiple actors operate at different hierarchical levels, inducing equilibrium problems and necessitating new approaches for market equilibria computation. This thesis addresses several complexities in achieving equilibrium in decentralized electricity markets, including coordinating diverse actors, managing supply and demand variability, handling power grid constraints, and information asymmetry among participants. Game-theoretic approaches are applied, using the Generalized Nash Equilibrium (GNE) concept to evaluate participants' actions and power grid constraints.

In the first phase of this research, we address uncertainties in peer-to-peer (P2P) markets with a heterogeneous risk-averse setting. A new framework is proposed where prosumers can hedge risks through financial contracts. The framework uses a Stackelberg game approach, involving an insurance company and the prosumers, with two problem designs considered, each leading to a risk-hedging pricing scheme. The framework mitigates market inefficiency caused by non-aligned objectives of the prosumers and the insurance company, by including the price incentives for the latter ensuring a solution exists for the pessimistic formulation. The proposed method simplifies market equilibrium analysis and allows the derivation of market equilibrium in various settings.

Further, the interaction between the physical and financial levels, managed by the Distribution System Operator (DSO), is analyzed. Two designs of the financial level prosumer market are compared: centralized and P2P fully distributed designs. The study demonstrates the Pareto efficiency of market equilibrium and how the proposed pricing structure limits free-riding behavior on the financial level. Numerical results investigate the influence of agents' flexibility and renewable generation inclusion on the proposed market operation.

Additionally, a framework addressing information asymmetry is examined where agents possess private information. This issue is framed as a non-cooperative communication game, allowing agents to determine their randomized reports to share with other market players. Proofs of the uniqueness of the Variational Equilibria solution of the game is derived and a closed-form expression of the privacy price is provided.

Lastly, the study explores the potential coupling between forecast and P2P electricity markets to mitigate the uncertainty from imprecise RES-generation forecasts, allowing agents to acquire a forecast of their RES-based generation. Conditions for the efficiency of the P2P market are identified, with a key

condition being prosumers' participation in the forecast market. The study presents proof of individual rationality of the coupled market model, showing that prosumers are always rationally incentivized to participate in the forecast market. Furthermore, conditions on the probability distributions of the forecasts ensuring this property are explored. Numerical results support theoretical findings, illustrating market coupling profitability for both the forecast sellers and the prosumers.

This research concludes that adjunct services, coordination, and information exchange are vital in optimizing market operations in environments where agents act strategically, exploring the complexities associated with P2P electricity markets and corresponding equilibrium problems. The research focuses on the coordination of diverse players, demand-supply management, risk-hedging, the inclusion of power grid constraints, and information asymmetry, relying on game-theoretic approaches. As such, this research presents key advancements to enhancing the state of the art and improving the understanding of the complexities of decentralized electricity markets, therefore contributing to the current debate on the electricity market restructuring.

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**Keywords :** Game theory, Smart grids, Algorithms, Peer-to-peer, Generalized Nash Equilibrium

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Writing this section is, on the one hand, the easiest and, on the other hand, the most difficult task I am undertaking as I finish my manuscript. Nothing is easier than being grateful to all those who have accompanied me on my journey, and nothing is more difficult than finding the right words to express my gratitude in the most comprehensive way. In an attempt to find those words, I have replayed in the recesses of my memory my entire path from the very beginning. And although my advisors Ana and H  l  ne have always taught me not to write everything in linear order (as well as to be less verbose), I will allow myself to depart from that rule in this case and write my remerciements in a nonorthodox chronological fashion, hoping for their favor.

*“Bard! by that God, whom thou didst not adore,  
I do beseech thee (that this ill and worse  
I may escape) to lead me, where thou saidst,  
That I Saint Peter’s gate may view, and those  
Who as thou tell’st, are in such dismal plight.*

*Onward he mov’d, I close his steps pursu’d.*

Dante Alighieri, *Divine Comedy*

Without my friend Daniel, I would not be where I am today, as I owe the beginning of my academic journey to him, who suggested I join the Master’s Program at Paris Sud after him. Studentship (while not Argo-like yet), game theory courses, an internship at the EDF where I met Saad and Adrien (which led to many wonderful evenings, especially in Russian restaurants), and, of course, the kind guidance of Clemence, without whose advice and recommendations I would not have met Ana and H  l  ne and the spark from which my thesis grew would never have ignited.

*”And maidens showed the way. And the axle, glowing in the socket -  
for it was urged round by the whirling wheels at each  
end - gave forth a sound as of a pipe, when the daughters of the  
Sun, hasting to convey me into the light, threw back their veils  
from off their faces and left the abode of Night.”*

Parmenides, *On Nature*

I cannot fully express the depth and greatness of my gratitude to Ana and H  l  ne - my permanent supervisors, to whom I am eternally indebted for their understanding, invaluable advice, belief in me, and support throughout my long and sometimes difficult journey. I could not have reached its end if they had not believed in me and the potential of my thesis at the very beginning. Their always right way to



motivate and guide me, long and frequent regular discussions of all types and often spontaneous, not only shaped and guided my work but also sharpened my thinking, setting new intellectual horizons before me and stretching my abilities to reach them. I also cannot but thank Anibal, who, joining us later, became an invaluable contributor to all the discussions and work, always ready to guide and help with all possible difficulties. I was once told that a Ph.D. student eventually becomes somewhat of a copy of his or her supervisors, and I can only hope that statement is true.

My only regret is that, separated by geographical constraints, we rarely had discussions together in person, limited mostly to remote calls. However, there are two sides to every coin, and without residing at Inria Paris I would not have met a beautiful Argo (formerly known as Dyogene) team.

*”On that day all the gods looked down from heaven upon the ship  
and the might of the heroes, half-divine, the bravest of men then sailing the sea;”*

Apollonius Rhodius, *Argonautica*

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who helped me a lot and who clearly will bring light to the team for years to come. And while the journey of Argo will continue without me, the real Golden Fleece is certainly the friends I made along the way.

*“If only because if it were not for them, I, both as a man and a writer, would amount to much less; in any case, I wouldn’t be standing here today.”*

Joseph Brodsky, *Nobel Prize Lecture*

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Me



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## Long abstract

Electricity markets are expected to undergo a significant transformation, evolving from centralized to decentralized structures, driven by digitalization, large-scale integration of renewable energy sources (RES) and distributed energy resources (DERs), and consumer-centric models that encourage active consumer engagement in electricity markets. This transition necessitates the development and investigation of novel decentralized market models, among which peer-to-peer (P2P) electricity markets constitute one of the most prominent forms. These markets, characterized by multiple decision-makers operating at different hierarchical levels with diverse objectives, motivate new approaches to study the market equilibria that will result in efficient market operation and to address the challenges associated with achieving the equilibria. These include the coordination of multiple actors with diverse objectives, the management of supply (inclusion of RES-based generation) and demand variability, the handling of distribution constraints, and the information asymmetry among participants. To address these challenges, we apply game-theoretic approaches, combining principles from game theory, optimization, and distributed algorithmic designs. To evaluate the impact of each participant's actions on the others' strategies, as well as the power grid constraints, we use the concept of the Generalized Nash Equilibrium (GNE), which, being more general than classical Nash Equilibrium, enables the incorporation of shared constraints depending on the strategies of all players.

In the initial phase of the research, this dissertation considers the effects of the inclusion of RES-based generation, more precisely, the framework of the P2P electricity markets involving heterogeneous risk-averse agents. When dealing with future uncertain losses, agents can have individual perceptions of uncertainties, i.e., different perceptions of risk (heterogeneous risk aversion framework), which might lead to market inefficiencies. To address this issue, a new framework is proposed, enabling the prosumers to hedge these risks through financial contracts that they can purchase from an insurance company or trade directly with their peers. The problem is formulated as a Stackelberg game, in which the insurance company is seen as the leader and the prosumers as followers. Subsequently, two formulations of the problem are considered. In the first model, only the insurance company acts as a source of risk-hedging contracts, while the second model supplements the first design by permitting inter-agent risk-hedging. A risk-hedging pricing scheme is derived in each design, and it is demonstrated that in the case when prosumers' objectives are not aligned with those of the insurance company (i.e., pessimistic formulation), the Stackelberg game may lack a solution, thus resulting in the market inefficiency. This technical difficulty is mitigated by the proposed equivalent game reformulation, i.e., as a parametrized GNEP, that simplifies the analysis of market equilibria characterization and its properties, yielding insights into the proper organization of insurance markets in a decentralized setting. It is proven that price incentives can



be designed by the insurance company, ensuring the existence of a solution to the pessimistic formulation, which is close to the optimistic one up to a preset precision parameter, thus allowing to achieve the market equilibria in both settings. Subsequently, economic properties of the Stackelberg equilibria, such as fairness, equity, and economic efficiency, are derived. Numerical simulations in support of the derived mathematical results quantify the effects of the proposed frameworks on the market organization and the resulting costs for the prosumers and the insurance company.

Second, as peer-to-peer financial transactions directly affect the grid operation, controlled by the distribution system operator (DSO) to guarantee grid security, our study develops a mechanism integrating both elements to address this issue by studying the interaction between the physical level, subject to the local and physical constraints of the distribution grid, managed by the DSO, and a financial market, subject to bilateral trading reciprocity coupling constraints. The focus has been placed on the effects of this interaction on the operation of the market, with the interaction being modeled as a GNEP. Two designs of the financial level prosumer market have been compared in this study : a centralized and a peer-to-peer fully distributed design. The study demonstrates that the Pareto efficiency of the market equilibrium can be guaranteed when the cost to trade is uniform across all participants while also illustrating that the proposed pricing structure of the noncooperative game limits exploitative behavior, utilizing price signals from the DSO. A case analysis is developed, focusing on the efficiency loss related to varying levels of agents' flexibility and the amount of renewable generation in the network. The impact of the prosumers' pricing on the social cost of the market has also been quantified.

Furthermore, a framework to address the information asymmetry challenge, in which agents possess private information they might be reluctant to share (e.g., demand or generation profile), is examined in this dissertation. This key problem is framed as a non-cooperative communication game, taking the form of a GNEP, which allows agents to determine their randomized reports to share with other market players while anticipating the form of the solution of another GNEP, representing the peer-to-peer market. In this non-cooperative game, the decision for each agent on the deterministic and random parts of the report is made, ensuring that (a) the distance between the deterministic part of the report and the truthful private information remains bounded, and (b) the expected value of the privacy loss random variable is also bounded. This structure facilitates a change in the privacy level for each agent. The equilibrium of the game is characterized, with the uniqueness of the Variational Equilibria proven and a closed-form expression of the privacy price provided. Furthermore, a closed-form expression has been developed to gauge the impact of privacy preservation, which is influenced by the inclusion of random noise and deterministic deviation from the agents' true values of the private information.

Finally, imprecise forecasts on RES-generation levels introduce additional uncertainty for the agents,

which impacts their trading in P2P markets, consequently affecting the market's efficiency. However, the advent of data markets, which facilitate the exchange of data to improve forecast accuracy, can mitigate this issue. Our study delves into the potential coupling between forecast markets to enhance the forecasts available to the agents, where agents can acquire a forecast of their RES-based generation, and the peer-to-peer electricity market. The electricity market is initially modeled as a two-stage peer-to-peer market, cast in the form of a GNEP. Further, this study identifies the conditions under which the efficiency of the electricity peer-to-peer market can be attained. A key one is the participation of prosumers in the forecast market. The study presents proof of individual rationality of the coupled market model, showing that prosumers are always rationally incentivized to participate in the forecast market. Furthermore, conditions on the probability distributions of the forecasts that assure this property are explored. The numerical results not only corroborate the analytical findings but also illustrate the profitability of the market coupling for both the forecast sellers and the agents in the peer-to-peer electricity market.

Our research results in several conclusions that help the understanding of P2P electricity markets, associated complexities, and resulting equilibrium problems, which are addressed using game-theoretic approaches. A novel risk-hedging framework to address uncertainties is proposed, with the expectation of enhancing the operational stability of these markets by resolving the difficulties associated with heterogeneous risk-aversion of the prosumers and arising bilevel optimization problems. The relationship between peer-to-peer transactions and grid security is investigated next, with the proposed mechanism resulting in stabilizing the operation of the market, consequently, increasing its efficiency. The issue of information asymmetry is addressed through a communication game, where a balance between information sharing and privacy is investigated, as well as providing insights into the strategic behavior of the agents within the framework with privacy constraints. The final focus is on exploring the coupling of data markets and P2P electricity markets to mitigate additional uncertainty brought by imprecise RES-generation forecasts. It is anticipated that the successful implementation of this approach will not only improve market efficiency but also drive profitability for all market participants, heralding a promising future for P2P electricity markets.



# Chapitre 1

## Introduction

As we witness an increasing reliance on digital technology and a shift towards renewable energy, the traditional electricity trading system is undergoing significant changes. This shift encouraging more active consumer participation in electricity markets, with consumers' active engagement being strongly supported by recent European Union (EU) policy recommendations.

In accordance with this transition, the central focus of this thesis leans towards the decentralization of the energy sector, acknowledging it as both an opportunity allowing to accentuate consumers' role and position them as proactive players in the energy landscape and a challenge, including technical difficulties created by the growth of local energy resources, the adoption of market models centered on consumers as well as decentralized markets, and the evolution of energy trading mechanisms.

The main goal of our research, therefore, is to propose innovative models that leverage game-theoretic methods to navigate these new and evolving markets effectively, by proposing game-theoretic models for centralized (acting as a benchmark) and decentralized electricity market designs; and to investigate their properties along with performance and their specific aspects taken into consideration, such as e.g. strategic behavior of the agents in a framework with privacy constraints, inclusion of intrinsic preferences and risk-aversion of the agents, information asymmetry or incomplete information, and locational properties - all in order to guarantee an efficient operation of the market.

**Motivation** The European Union (EU) has made a strong commitment to achieving net-zero greenhouse gas emissions by 2050. In addition to meeting environmental goals, this effort also presents opportunities for economic growth and technological innovation. To support this major change, the EU launched the Clean Energy Package in 2016. This law encourages the use of renewable energy, aiming for it to make up 42.5% of all energy in the EU by 2030 [45]. At the heart of this is a move to renewable energy (Figure 1.1), with constantly increasing share and a focused reduction in carbon emissions.

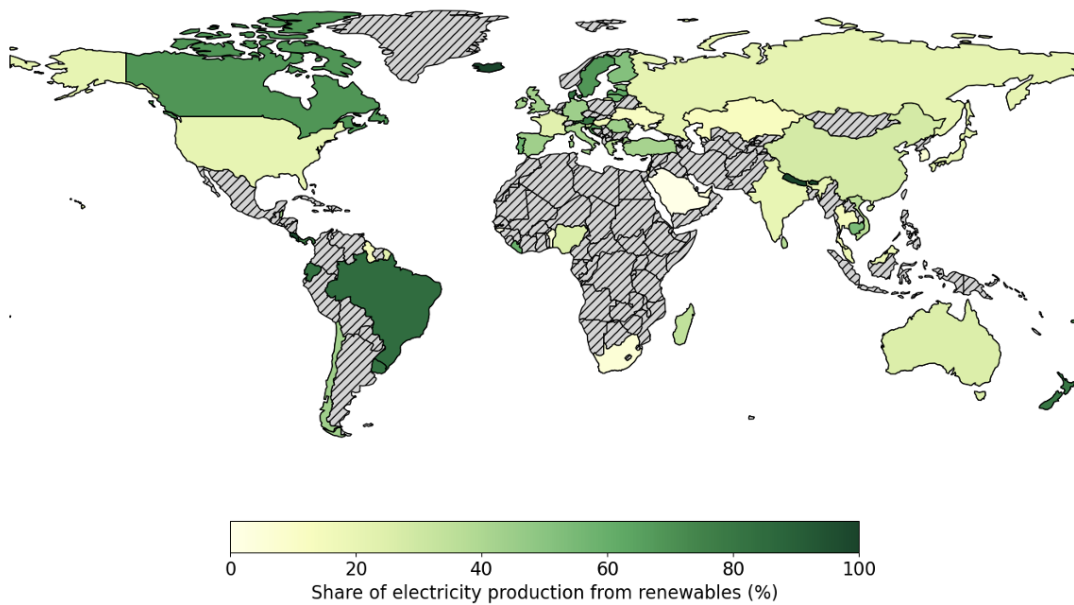


FIGURE 1.1 – Share of renewable electricity production in countries or regions that have committed to the net-zero emission target [43]

At the same time, the power system is benefiting from big technological advances. Digitalization and the falling costs of new technologies like electric vehicles (EVs), batteries, heat pumps, photovoltaic systems (PVs), as well as active installation of smart meters enable consumers to play an unprecedented proactive and interactive role. This emerging role, turning consumers into proactive prosumers (consumers who can both consume and produce energy), is expected to significantly impact the operations and organization of electricity markets, which were traditionally designed for large-scale dispatchable generation and largely inflexible loads. Indeed, energy systems are witnessing a clear move from large, centralized energy systems to smaller, decentralized ones, involving a wider range of participants. While these new players control relatively small-scale generation and consumption volumes, their influence and flexible energy use are growing. The combination of technological progress and new energy resources is changing how energy networks are managed. This shift aligns with the EU's goals to increase the use of renewable energy and make the market more consumer-focused.

This shift highlights a key trend : progression towards a more consumer-centric electricity market design. Consumers are now able and are planned to have a more active role, driving the development of new technologies and business models, as well as undertaking proactive participation in electricity markets (local and beyond). Taking this growing role of consumers into account in the design and operational analyses of electricity markets is essential for maintaining and improving their efficiency. At the same time, there is an increasing preference among consumers for local, renewable energy sources, with

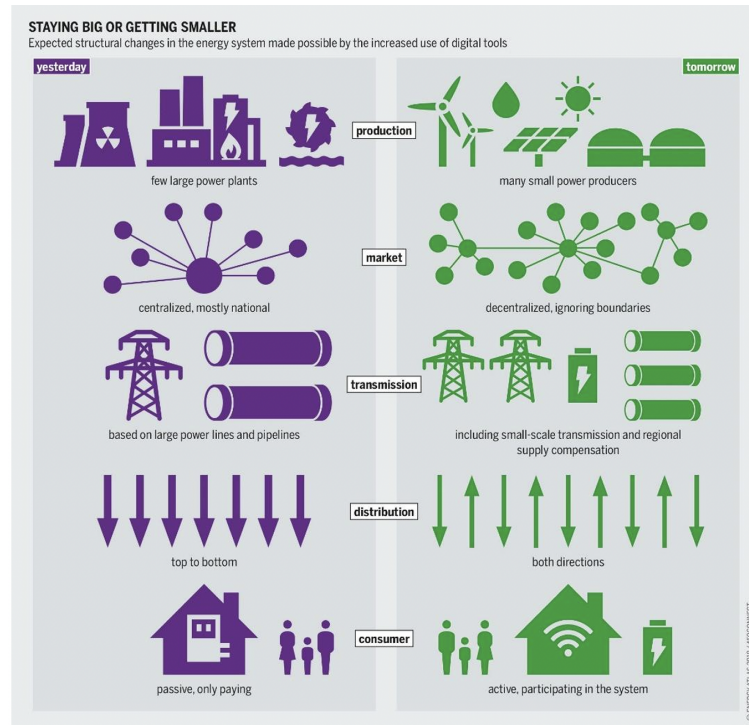


FIGURE 1.2 – Expected structural changes in the energy systems made possible by the increased use of digital tools, source : [60]

communities ready to jointly invest in and use renewable resources for improving their sustainability and efficiency. These large changes raise the need for rethinking the current market designs to avoid high costs and grid instability.

These evolving capabilities and assets of the agents in the energy sector open new opportunities for new market designs. In this respect, the research scope of this thesis is focused on one key emerging market design, namely, peer-to-peer (P2P) electricity markets. These markets allow consumers and producers or prosumers to interact directly, by trading energy among each other, i.e. in a peer-to-peer fashion. This direct interaction shifts the dynamics from a traditional centralized model, where a single entity regulates and facilitates the trading process, to a landscape where multiple selfish and independent agents act and make decisions based on their own interests. Such markets can lead to the rise of energy systems and local market organizations that are more suited to the needs and preferences of consumers, as they move away from the traditional centralized model.

**Equilibrium problems** In such settings, each participant’s optimal decision is intrinsically tied to the decisions of others in the network, which naturally gives rise to equilibrium problems. In other words, an individual’s plausible actions might depend on the actions of others. As every participant is trying to maximize their own benefit, it leads to a strategic game (as a game-theoretic concept, i.e. strategic

interactions among rational agents) where their individual choices collectively determine the equilibrium of the market.

Yet, finding this equilibrium state, where no participant can benefit from changing their strategy unilaterally, is a complex task. It is a point of balance that depends on many factors, such as the participants' consumption and generation capabilities, risk attitudes, information exchange, regulatory policies, or the uncertainty and variability inherent to renewable energy sources (which typically form a significant part of P2P energy trading). A change in any of these factors can cause a shift in the equilibrium, thus affecting the performance and stability of the market. Furthermore, achieving this equilibrium state poses several challenges. First, an effective mechanism is required to coordinate the actions of all participants, while respecting their autonomy and individual objectives. Secondly, commonly shared constraints implied by the grid usage and trading quantity reciprocity. Thirdly, due to the decentralized nature of P2P markets, this equilibrium must be achieved in a distributed manner, without a central authority to orchestrate the process.

As such, the strategic behaviors of agents and the associated equilibrium problems are crucial topics of exploration for understanding and improving the operation of decentralized P2P electricity markets using a game-theoretic approach.

The necessity to incorporate the specificity of these markets has led us to consider various equilibrium concepts that can better capture these dynamics. One such concept is the Generalized Nash Equilibrium (GNE). Unlike a traditional Nash Equilibrium, where each player optimizes their strategy, taking into account the dependence of its objective function (i.e., utility, or total cost) on the decision of others, a GNE takes into account that the actions of each player can influence not only the payoff but also the strategy space of the other players, or in other words, the feasible sets. This concept is particularly useful in decentralized electricity markets because it encapsulates how each participant's decision may affect the feasible strategies of others, through constraints such as physical distribution limits or bilateral trading reciprocity. It provides a framework to incorporate and analyze various aspects of these markets and to understand how strategic decisions in these markets may influence the policies.

In our research, we focused on the following factors that make achieving equilibrium in decentralized electricity markets a challenging task :

(i) **Coordination between multiple actors** : Decentralized markets involve various entities with different objectives, preferences, and constraints. Ensuring that their individual decisions result in a socially desirable outcome requires sophisticated coordination mechanisms.

(ii) **Variability in supply and demand** : Decentralized electricity markets are characterized by fluctuations in supply (e.g., intermittent renewable energy sources) and demand (e.g., time-varying consump-

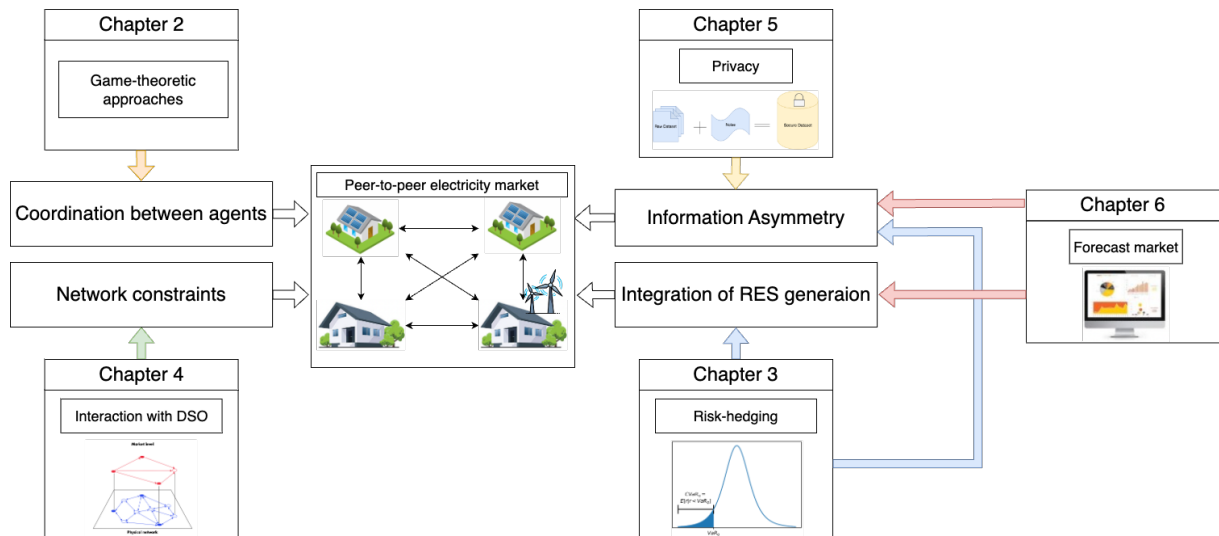


FIGURE 1.3 – A graphical description of the connection between the research questions and the chapters of the thesis.

tion patterns). This variability complicates the process of achieving and maintaining equilibria.

(iii) **Transmission and distribution constraints** : Physical constraints, such as limited transmission and distribution capacities, can restrict the flow of electricity and impact the attainment of equilibria in decentralized markets.

(iv) **Information asymmetry** : Participants in decentralized electricity markets may have access to different levels of information (due to e.g. privacy issues), which can lead to suboptimal decisions and hinder the achievement of equilibria.

Addressing these equilibrium problems in decentralized electricity markets is crucial for ensuring their efficiency, stability, and overall performance. This thesis employs game-theoretic mechanisms, distributed algorithms, and optimization techniques to address these challenges.

**Contributions and thesis structure** Chapter 3 addresses the challenge of facilitating coordination among diverse risk-averse actors in the P2P markets. Understanding and managing risk is crucial in any market, but even more so in a decentralized one, where individual entities may have different views on the risk (underlying uncertainty), thus deviating from an efficient market equilibrium. Using a model of a Stackelberg (that we interpret as a bilevel optimization problem) game, where insurance company plays a leading role anticipating the reaction of prosumers (i.e. followers), we derived a pricing scheme that promotes fairness and efficiency under various market structures, while also comparing and discussing potential insurance (mutual or not) scheme organizations. To overcome technical difficulties associated with the solution of bi-level problems, especially in a *pessimistic* setting (when the objective of the prosu-



mers is not aligned with the objective of the insurance company), we introduced an equivalent reformulation relying on parametrization of a set of the solutions of the Stackelberg game, with the values of the parameters corresponding to the solutions of modified generalized Nash equilibrium problems (GNEPs). This allowed us to study the equilibria characterization, solution existence, and market properties in a simpler setting, concluding the proper organization of the insurance market.

Subsequently, Chapter 4 introduces a *two-level* model, focused on the interaction between the financial and grid layers of the local electricity market, where the latter has a limited capacity for electricity distribution, a constraint that can severely impact the equilibrium formation and its efficiency in electricity markets. We model the interaction between the two levels as a GNEP, in which the distribution system operator (DSO) exchanges information with the prosumers using price signals, represented by the dual variables of its optimization problem. The developed framework, performed analyses, and derived results provide key insights into the intricacies of this interaction while showcasing that the proposed trading system effectively limits exploitative behavior (no free-lunch behavior) and guarantees Pareto efficiency when the cost to trade is uniform across all participants.

Chapter 5, focuses on the issue of 'information asymmetry', and in particular, the situation where some market participants possess information that they do not want to share with others. In this regard, we developed a novel model to predict how individuals might alter their behavior to maintain their privacy, constructing a communication game in the form of a GNEP in which the agents determine the amount of information they want to share with others to achieve equilibria in an original GNEP, representing the peer-to-peer electricity market. The developed framework enables predicting potential shifts in the behavior of the agents who might choose to withhold or manipulate their private information for competitive advantage. This model broadens our understanding of privacy dynamics in decentralized electricity markets and also provides us with a means to quantify the cost of privacy — a concept that helps to evaluate the trade-off between information sharing and the costs incurred at the equilibrium.

Finally, Chapter 6 addresses the handling of uncertainty that stems from the increasing integration of renewable energy sources into the energy mix, which introduces variability in power generation. In this regard, we propose an innovative coupling of electricity markets (modeled as a GNEP) with forecast markets of energy generation, aiming to mitigate this uncertainty and improve prosumers' decision-making. This combined market structure not only improves the quality and accuracy of forecasts but also encourages active participation from market players, mutually benefiting all parties involved. We show that it is possible to achieve maximized efficiency of the peer-to-peer electricity market if all the agents at least report their beliefs (forecasts) to the forecast market operator, which could be done if they participate in the forecast market. While this can be a strong condition, it is mitigated by the fact, which

we prove in our work, stating that the agents benefit from purchasing the forecasts, thus, they have an incentive to participate in the forecast market. In other words, we prove that Individual Rationality holds for the coupling of these two markets.

For addressing these problems, we have relied on fundamental principles from game theory, optimization, and algorithmic designs to introduce novel models and thorough solution mechanisms to address the different underlying technical challenges, allowing us to quantify the transformative potential of decentralized energy systems in improving the efficiency and consumer-centricity of emerging electricity systems and markets. As such, this work provides key contributions to advancing the state of the art and improving the understanding and navigating the complexities of decentralized electricity markets, rendering them more resilient and adaptable to future challenges.



# Chapitre 2

## Research Scope and Methodology

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## 2.1 Research Scope

### 2.1.1 Call for a Change in Electricity Markets

#### Shift towards renewable energy

The issue of climate change is a major concern due to its anticipated severe consequences, including increased frequency and intensity of extreme weather events, which could have negative impacts on productivity, infrastructure, health, biodiversity, food production, and political stability, directly impacting the sustainability, stability, and development of our societies. To address these risks, the European Union (EU) has proposed a long-term strategy aimed at achieving net-zero greenhouse gas emissions by 2050 [4], in accordance with the 2015 United Nations Climate Change Conference agreement [102]. This transition is expected to be both cost-efficient and socially fair, with a particular focus on eliminating emissions from the different energy sectors.

The EU has established regulations outlining the process for preparing long-term climate and energy plans. The EU and its Member States aim to lead the way toward climate neutrality, capitalizing on the opportunities for economic growth, new business models, job creation, and technological advancement that the implementation of the long-term plans will bring. As part of the EU's efforts to achieve climate neutrality and reduce greenhouse gas emissions, the European Commission introduced the Clean Energy Package in November 2016 which was later updated in 2019 [52], which consists of legislative proposals for various energy-related sectors. The Clean Energy Package suggests utilizing market-based approaches to incorporate Renewable Energy Sources while also promoting long-term policy consistency through a reliable investment framework. This strategy aims to reduce regulatory risks for investors and ensure a stable energy supply. The package sets an EU-wide goal of achieving 42.5% of gross final energy from RES by 2030 [52]. The EU prioritizes both RES integration and carbon emission reductions, with the RES target intended to incentivize Member States to continue providing subsidies for RES thus promoting the transition.

#### The power system is becoming more distributed

The power system's digitalization, facilitated by the implementation of information and communication technologies (ICTs), is occurring in parallel with - and supporting - the integration of renewable electricity resources. The decreasing costs of new technologies, such as smart meters, solar panels, small/medium wind farms, electric vehicles (EVs), and other distributed energy resources (DERs), have significantly impacted electricity markets [115]. These cost reductions have enabled the widespread adoption of DERs and promoted a transition from centralized energy systems to more decentralized ones.

In most power current systems, DERs remain minor players in the provision of electricity services; nonetheless, smart energy consumption and DER deployment are generally on the rise. The Clean Energy Package recognizes the potential of these technologies and aims to create a more integrated and consumer-centric energy market by promoting the participation of DERs, demand response, and energy storage. The expansion of DERs has allowed consumers to become active participants in the energy market, often as *prosumers* - the consumers who both generate and consume electricity. The combination of digitalization and the introduction of new energy resources allows for more active network management, potentially bringing an end to the passive network management paradigm, which relies on networks being designed to accommodate the aggregate peak demand of passive consumers. This will continue to transform electricity markets in support of the EU's efforts to include more renewables in the energy mix and make the market increasingly consumer-oriented.

**Consumer-Centric Markets** Factors listed above, such as energy sector liberalization, the rise of distributed energy resources (DERs), digitalization, and a focus on energy efficiency and demand-side management have facilitated the shift towards consumer-centric models. In these models end-users play a more active role in the energy system, driving new technologies, services, and business models.

As demonstrated in [46], an increasing number of prosumers desire the freedom to choose their energy sources or even achieve self-sufficiency. Neglecting to support this consumer preference could lead to economic inefficiencies within a microgrid, as evidenced by the authors. Another study [20] reveals a growing acceptance of renewable and local energy sources among consumers in recent years, with this trend expected to continue. Additionally, various local communities have demonstrated a willingness to share investment and use of renewable resources. For example, the Netherlands ranks highly in the number of EU initiatives in this respect [143]. The social aspect of cooperation encourages consumers to increasingly participate in energy community initiatives. Therefore, failing to capture a more consumer-centric approach within (and as a compliment to) electricity markets can run the risk of higher investment costs, reduced efficiency, and compromised reliability in power delivery [113]. This, as well as the shift towards renewable energy sources, motivates the need for consumer-centric market models as the next step of the progression of wholesale and retail electricity markets.

### 2.1.2 Current Organization of The Electricity Markets

To grasp how electricity markets operate, one must recognize two key physical properties of power systems. First, electricity is essentially a perishable commodity because we have limited capabilities to store power once it is generated. Therefore, it must be consumed as soon as it is produced, making it

necessary to maintain a balance between power supply and demand at all times and locations. Secondly, electricity must be transmitted from its point of generation to the node of consumption through electrical networks, subject to physical laws and constraints. These physical constraints, along with the power balance constraint, give rise to unique power values at specific locations and times within the distribution power grid.

Maintaining balance in the power system is a challenging task. To ensure that the system remains balanced at all times, the system operator (SO) must determine how much each power plant should increase or decrease its production, making use of centralized unit commitment in real-time electricity markets. The system operators must take into account all aspects of the network, including costs, ramp-rates, and plant locations, to make technically feasible and efficient decisions in real time. With the inclusion of RES-based generators and Distributed Energy Resources, it becomes evident that the complexity of managing the power grid escalates.

Current electrical energy markets provide long price histories on which to base future price projections, and the market fundamentals which drive prices are straightforward to model [119]. Traditional electricity markets are designed to accommodate the electricity generated in bulk, with low variability, and supply power to distributed consumers. This approach assumes that electricity that comes from controllable, large-scale generation sources (e.g., fossil-fired, hydro, and nuclear generation), flows downstream to the load buses. The SO acting on the balancing market stage (through procurement and activation of reserves) is accountable for maintaining a balance between generation and load by regularly adjusting the generators' output. While this system has provided some benefits in terms of competition and price stability, it requires adjustments to efficiently incorporate RES and proactive demand.

The Clean Energy Package proposes a power market where consumers can participate in demand response, self-consumption, or self-generation and engage in markets [118]. Decentralized energy resources, such as distributed generation, storage, and demand response, should be allowed to participate in wholesale electricity markets either directly or through aggregators, on an equal footing with traditional generators, provided they meet the necessary technical capabilities. This would require changes in the current design of retail and wholesale markets, as well as interactions and coordination between the TSOs and distribution system operators (DSOs).

This restructuring would involve a shift towards more flexible and dynamic market designs, which would enable the integration of distributed renewable energy sources into the grid. This introduces a direction both in the research and practice, dedicated to the models that would be based on a decentralized, digitalized system that enables the real-time trading of energy and provides incentives for the integration of renewable energy sources. These models would rely on new technologies and market designs to enable

the integration of a wide range of market participants, including prosumers, community energy projects, aggregators, etc. A more flexible, dynamic market design would also enable the integration of new technologies such as electric vehicles and energy storage, which are essential for the decarbonization of other sectors such as transport and industry. Additionally, it would enable greater market competition, which would result in lower prices for consumers [115]. In this respect, we focus on the local energy markets (LEMs) and propose novel models for P2P markets for efficient organization of the interactions among prosumers.

The shift towards LEMs necessitates the development of additional market clearing mechanisms that account for distributed systems, renewable energy sources (RES), and more active consumer participation. As the market incorporates consumer-centric approaches, distributed systems, and RES, institutions are developing common market rules, standards, and methodologies that facilitate the integration of these new resources and market actors, while addressing the equilibrium problems that may arise [52], [115] from the decentralization. This entails the development of additional market mechanisms that address emerging equilibrium problems and ensure efficient, reliable, and sustainable electricity markets empowering consumers in the face of new challenges.

## 2.2 Local Electricity Market Designs

**Local Electricity Markets** The concept of smart grids has been integral in managing the growing proliferation of DERs, especially at the low voltage level. Smart grids facilitate the incorporation of new elements into the power system, such as smart meters, DERs, and storage solutions. Given this, DSOs reevaluate their management approaches to accommodate these alterations at the low voltage level. Active distribution network management can significantly contribute to new systems by providing novel ways to govern the grid under diverse operating conditions, preferably in a preventive manner [161]. In this regard, the creation of local electricity markets (LEMs) and virtual power plants might be vital in implementing these methods and aiding DSOs in the management of the distribution network [107]. Microgrids, often understood as low voltage distribution grids containing DERs (i.e. structures with a technical focus on the local distribution network) that can operate in both islanding and grid-connected mode, have the potential to introduce LEMs (understood as social and economic organization of local electricity consumers/providers) where prosumers and consumers actively engage and exchange energy among themselves, fostering local energy consumption [107], [92]. They can enhance energy efficiency, reduce the vulnerability of larger grids, decrease blackouts, and support power for remote communities. Economically, they diminish greenhouse gas emissions, system losses, grid reinforcement expenses by



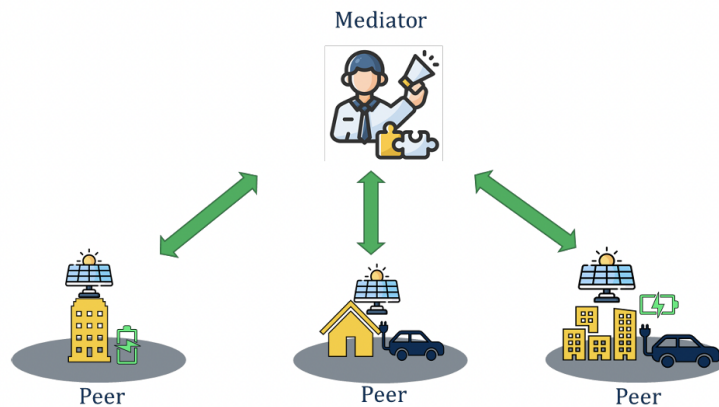


FIGURE 2.1 – Centralized Trading. Source : [101]

the DSO, and load-shedding costs to consumers, all while lowering fuel costs and ancillary services from a system-wide viewpoint. Environmentally, the widespread adoption of LEMs bolsters the usage of clean energy sources, thereby reducing pollutant and greenhouse gas emissions.

The advent of microgrids brings a new set of requirements and possibilities to the energy sector, necessitating the development of novel LEM designs. These new designs must not only accommodate the smaller scale and potentially isolated nature of microgrids, as well as the benefits from increased connectivity but also take into account the increased presence of DERs within these networks.

### 2.2.1 Benchmark : Centralized Design

In the centralized market organization (Figure 2.1), analyzed in this study as a benchmark, a central entity – Market Operator (MO) – collects the information provided by the participants of the energy trading process and computes the pricing and allocation rules to optimize a certain objective function (maximizing social welfare, minimizing costs, etc.). Information provided by the consumers to the MO can differ in the level of detail as well as the level of complexity – the information reported could contain the intrinsic preferences of the consumers (eg. locality, type of energy), technological aspects of the equipment (solar panels, CHPs, boilers, etc.), target demand, RES-based generation profiles, etc. Constraints in the problem solved by the MO arise from different aspects of the energy trading such as physical properties of the transmission lines, DERs and RES, consumers' demand and consumption limits, etc. The advantages of the centralized design are that it relies on already well-established approaches for mathematical modeling of such systems and classical optimization techniques : for large-scale optimization, there exist decomposition techniques such as Bender's decomposition, Lagrangian relaxation, consensus Alternating Direction Method of Multipliers (ADMM), etc. that are extensively used in the power system economics and market literature. The full-information formulation implies the existence of a mechanism

that will motivate the participants to reveal their information and preferences to the central entity. The challenges in the centralized design may arise from large-scale data management scheme design, forecasting the generation from renewable energy sources, and from the need to provide a robust system that will be resistant to malicious behavior such as price information manipulation or attacks aimed at the privacy violation.

### 2.2.2 Hierarchical Design

Various types of complexity can be added depending on the considered degree of anticipation of the agents and the assumed information structure. When we assume that one (or even some of) the agents have some advantages with respect to the others regarding the order of decisions and his anticipation capability on the decisions of the others, a Stackelberg game which reflects hierarchy between certain agents, can be implemented. There is a leader who decides upon his strategy first, anticipating the reaction of the follower, governed by a so-called lower-level optimization problem. The follower can only react to the strategy of the leader – following the so-called rational reaction function [38]. A Stackelberg game with one leader can be represented by a bi-level optimization problem, i.e., an optimization problem that is constrained by a second optimization problem (the lower-level problem) [37]. Dedicated reformulations as mathematical programs with equilibrium constraints (MPECs) and reformulations as mixed integer linear programming (MILP) problems allow solving such problems with state-of-the-art commercial solvers (such as CPLEX and Gurobi). An MPEC can be used, e.g., to model the strategy of one profit-maximizing, price-making generator on a wholesale market. To study the interaction between multiple strategic agents (e.g., multiple price-making generators on a wholesale market or multiple local market operators in the local-global market framework), one may move to EPECs, i.e. a set of MPECs, parameterized in the decision variables of the other strategic agents, possibly with a common equilibrium problem in their constraints (in the local-global market framework, that would be the clearing of the global market by the global market operator) [79]. Solving such problems typically requires dedicated decomposition and solution techniques, such as diagonalization approaches.

### 2.2.3 Peer-to-peer Design

The European Commission Renewable Energy Directive refers to 'peer-to-peer trading' as to an exchange of renewable energy amongst market participants [44]. These exchanges are regulated by a contract having pre-defined conditions that control the automated procedure and settlement of the transaction. The transaction can take place directly amongst market participants or it can be mediated through a certified third-party market player like an aggregator. The directive ensures that the right to participate

in peer-to-peer trading does not infringe upon the rights and obligations of the involved parties, which may include end customers, producers, suppliers, or aggregators. In this context, peer-to-peer (P2P) energy trading can be seen as the reciprocal buying and selling of energy between two or more parties connected to the grid. P2P energy trading introduces a choice for consumers, allowing them to select their electricity providers and purchasers. It provides a mechanism for consumers to decide from whom they buy electricity and to whom they sell it, all facilitated by a secure platform. A brief overview is provided below, while a very comprehensive overview can be found in [101].

**Historical overview** P2P systems have been under rigorous study since the 1990s, predominantly in the computer science field for distributed computing and decentralized data sharing. The benefits, definitions, and architectures of this system have been explored over time [131]. A widely accepted definition of P2P encompasses systems and applications that utilize distributed resources to carry out tasks in a decentralized way instead of a centralized way [13], [131], [135], [71], [103], [2], [147], [112]. The concept of P2P systems is for agents (or peers) to directly offer their resources to other agents within the grid, bypassing the need for intermediary coordination. Fundamentally, P2P systems revolve around three main principles : resource sharing between peers, decentralization of the grid, and self-organization arising from decentralization.

P2P systems have been adapted across various application domains [112], mainly leveraging the principles of the collaborative and sharing economy. Codagnone et al. [31] describe the collaborative and sharing economy as a broad range of digital platforms, both commercial and non-profit, that facilitate exchanges among diverse players via a multitude of interaction modalities. Within this concept, peer collaboration often culminates in commercial platforms or platforms that generate common value, and therefore free/open source projects are created and available to all interested parties [55], [42].

The application of P2P and collaborative economy concepts in power systems is a relatively new development, largely evolving over the last decade. Aside from early exploratory work in the late 90s [152], [153], the first instance of proposing P2P concepts for power systems dates back to 2007 [13]. This study analyzed the P2P concept within the smart grid paradigm, where decentralized operation and control of grids, microgrids, and distributed energy resources are becoming a necessity.

The introduction of microgrids brings about infrastructures and technologies that allow bidirectional communication and self-metering in prosumers and end-users. Such technological progress aligns well with the basic information and structural needs of P2P markets, given that decentralized P2P systems necessitate IoT technologies to develop large-scale sharing communities with appropriate communication speed between participants. Therefore, microgrids can provide the physical infrastructure in the power

system that enables P2P LEMs to emerge and interact accordingly, allowing P2P markets to leverage LEMs characteristics to decentralize decision-making processes related to energy and service exchanges within the LEM context.

In light of these P2P traits, new designs for LEMs have been suggested, considering the relationship between producers, prosumers, and consumers, and based on the principles of a collaborative and sharing economy [21, 30, 68, 78, 84, 87, 93, 128, 148, 163]. Therefore, P2P markets seamlessly integrate within a microgrid. Mengelkamp et al. [93] proposed a LEM where prosumers and consumers could trade energy among themselves, thus retaining profits from energy trading within their community. The proposition is to enable prosumers and consumers to actively participate in the operation of the LEM, taking into account the environmental consciousness of consumers while saving money and fostering new investments in local renewable energy production.

Moreover, the challenges of applying this approach to the real-case scenario of the Brooklyn Microgrid are also demonstrated. Other research has designed the P2P market of a microgrid in simpler ways, like energy sharing based on energy profile matching among prosumers [163], and autonomous cooperative energy trading with protocols for sharing and matching energy schedules [87]. Alternatively, there is also research that focuses on schemes using a central entity to manage or oversee energy trades, in line with the community-based market described in the introductory section. Multi-agent systems for P2P markets are commonly developed to manage local energy transactions and generate appropriate price signals for LEM members [30]. In contrast, [84] proposed the use of an energy-sharing provider (also referred to as a community manager) that coordinates sharing activities among prosumers and consumers. Following an in-depth review of the literature, it's clear that microgrids are seen as a significant facilitator and incentive for P2P markets, and P2P markets, in turn, will greatly benefit from the technological advancements introduced by microgrids.

In this context, local energy markets have evolved, bearing the potential to neutralize local power imbalances, and thereby, enhance the flexibility of the current distribution grids, avoiding the need for new investments [93]. Hence, P2P systems have been seen as an attractive method to shape these budding local markets, considering all the distributed information required to be processed in the market. These models envisage the notion of collaborative markets where prosumers and consumers trade energy amongst themselves. This not only fosters cooperation and heightens social consciousness but also diminishes electricity bills and encourages reinvestment in local energy production. Importantly, by keeping the profits from local transactions within the community, they provide an additional economic incentive.

A crucial aspect of peer-to-peer (P2P) trading is the structure of its market design. This design lays out the specific guidelines that participants must adhere to in order to execute the trade. Within the

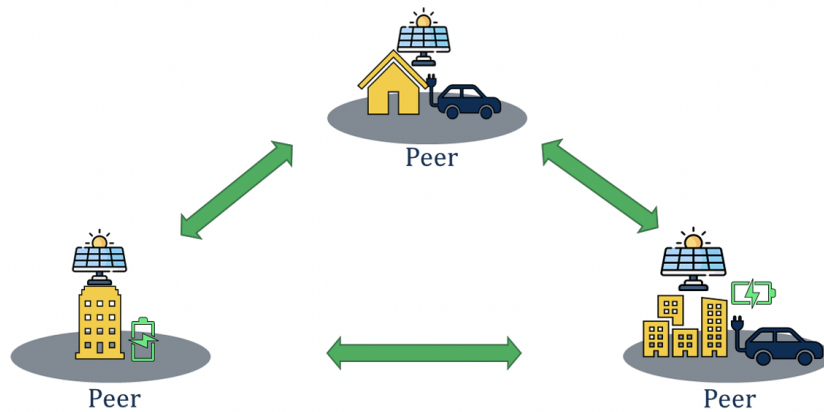


FIGURE 2.2 – Decentralized P2P Trading. Source : [101]

P2P framework, these guidelines encompass the necessary information to be submitted by the peers (for instance, the bid quantity and price), the protocols for bid matching, the adopted pricing model, and the mechanism for settling the market. Depending on how the trading process is conducted and how communication of information transpires among the participants, various market design strategies are employed for P2P trading.

### Fully distributed

The full P2P market (Figure 2.2) implies that each agent (i.e. producers, consumers and prosumers) directly interacts with the other agents without intermediary entities like a retailer or market operator. Two agents can agree on a transaction for a certain amount of energy and a price without centralized supervision. A trade between two agents in a local energy community supposes that these two have decided on a certain quantity to be sent from one side and received by the other side. Therefore, there must be an “agreement” or trade constraint between each pair of agents in a local community, which couples their respective decisions. In fact, a single P2P transaction between two agents constitutes a single *bilateral contract* for exchanging/sharing a product/service that may return a mutual benefit for both agents. A complete P2P market then translates to enabling simultaneous and numerous bilateral contracts among all agents at high temporal resolution and frequency. As a result, although the utility of a prosumer depends only on her own decisions, some of these decisions, such as the quantity she agrees to trade with all the other prosumers in her neighborhood, have an impact on the set of feasible actions of her neighbors ; in the same way, her feasible actions are determined by the actions of her neighbors - that introduces the notion of *coupling constraints*.

This approach constitutes a multi-agent system in which the decisions are made in a distributed way. While any distributed approach can be implemented at a centralized location for the purpose of being able

to parallelize computation and therefore improve computation speed, two situations in which physically distributed computations make sense include :

1. when the participating entities do not want to share all of their operational information with any other entity
2. when it is of importance to ensure that a failure of a single computational entity, i.e. the central coordinator, will not lead to an inability to control the system.

Indeed, the distributed optimization methods, such as ADMM make it possible to explicitly define individual problems for each agent, while guaranteeing their privacy : each agent only shares the information due to the set level of privacy : for example, in the case considered in [26], in centralized market design, all the private information is reported to the Market Operator, while in a peer-to-peer market design *local target demands* and *RES-based generations* are known only by an agent.

### **Coalition-based market design**

The second structure is the community-based market. This design relies on a community of members with a community manager to coordinate trading activities inside the community. It can readily be applied to microgrids. A group of neighbouring consumers with their own appliances, solar panels, or electric vehicles can form a community to share energy. In a smart city context, one may also think of a building, or block of buildings, where all its residents can form a community, which is a natural construct due to their location (i.e. being geographically close). In some cases, the community members also share the investment on DERs. More generally a community is to be based on members that share common interests and goals : for instance, a group of members that are willing to share green energy.

This market design is based on a distributed negotiation between the community manager and all its members. The community manager also serves as an interface between the community and the rest of the system.

*Coalition games*, including both coalition formation and a canonical coalition, can be implemented in the context of communities and has been already applied for designing services in the energy sector. For instance, demand response regulation in the EV domain has been implemented by using a coalition formation game in [75]. In [80], the authors demonstrate how to incentivize energy users with small-scale energy power production unit such as rooftop solar panels to directly energy trade with other users within a community instead of trading with the retailer. Further, the exploration of coalition game for regulation service can be found in [160], in which the authors design a coalition formation game to schedule the charging and discharging of EVs within a smart grid network such that the grid's stability is not compromised.

As for the previous design, the negotiation process could be solved in a centralized manner, but the best approach is to solve it in a distributed manner in order to respect the fundamentally decentralized nature of consumer-centric electricity markets. Naturally, this design relies on such a distributed structure, for which there is a central node (community manager) to manage the remaining ones (agents). Each agent solves its own problem and only shares the required information with the central node. In this way, privacy of all members of the community is guaranteed. To perform the comparison of the different market designs, performance indicators should be considered, reflecting various points of view on the system behavior (social welfare, efficiency loss, etc.)

### **Hybrid P2P market design**

Considering the advantages and drawbacks of the aforementioned models, and given the focus on energy communities in many prosumer-centric studies, a community or hybrid model for peer-to-peer (P2P) trade has emerged. Here, while trading is decentralized, communication among participating prosumers is centralized. A community manager, acting as a mediator, coordinates P2P energy trading among prosumers, indirectly encouraging participation through appropriate pricing signals. In this hybrid market model, prosumers share minimal information with the mediator, thereby maintaining a high level of privacy.

#### **2.2.4 The need for game-theoretic models in analyzing these markets**

Game theory is a mathematical framework that is often used to model strategic interactions between individuals or entities in complex systems. With the integration of advanced technologies and services in decentralized electricity markets, the relevance of game theory in this area has only grown. It's increasingly needed to develop innovative models and algorithms to tackle key challenges specific to these markets [129], [141]. Generally, the decentralized electricity market is composed of the nodes or players (agents) that can operate and interact with each other, aiming to reach an efficient equilibrium. Thus, the arising equilibrium problems necessitate the use of game-theoretic techniques to address the challenges encountered at various stages such as design, control, and implementation. Game theory provides a formal analytical framework equipped with mathematical tools that facilitate this study of agents' interactions.

Game theory is particularly beneficial in peer-to-peer (P2P) markets for several reasons. Firstly, the main objective in P2P is to motivate participants to trade energy with each other, forming a nearly autonomous energy community with minimal grid influence. In this context, traditional price signals from central power stations may not significantly impact P2P trading as they would in conventional systems.

Secondly, while existing energy trading systems in smart grids utilize various pricing schemes like real-time and time-of-use pricing, P2P requires the development of more innovative pricing strategies. For instance, a prosumer, as an independent decision-maker, might opt to sell surplus energy at variable rates to different network buyers, necessitating new pricing schemes. Lastly, the absence of centralized management places a heightened emphasis on secure energy trading transactions among P2P network participants.

In [141] authors highlight challenges associated to the P2P electricity markets as well as advantages and limitations of game-theory applications. We present them here, while referring to [141] for more details.

- The **challenges** include modeling user behavior, devising pricing schemes that facilitate user cooperation within the P2P network, managing the security and privacy of users, enabling strategy-proof transactions, maintaining trust between users in the absence of a centralized authority, reducing dependence on the central grid either partially or fully, managing network congestion when user numbers increase, stabilizing the system due to higher penetration of renewables, and integrating the central power station as part of P2P trading.
- Game theory offers a significant **advantage** in addressing these challenges. It can model user behavior and their interactive trading with one another, and seamlessly integrate pricing and incentive design as part of game framework development. It has the potential to establish trust between users within the network and motivate them to cooperate via its cooperative game framework. Furthermore, it can be combined with other signal processing techniques like fuzzy logic and machine learning.
- However, game theory has its **limitations**, with practical deployment of game theoretic models being somewhat restricted. It's challenging to implement when it involves human subjects directly in the optimization process. Notwithstanding, recent developments in game theory (such as auction game) have been deployed in pilot P2P projects like the Brooklyn microgrid.
- Game theory has found **similar applications** in sectors other than energy. These sectors include banking, where it's used in online financial transactions, IoT for device discovery and control, healthcare for peer-to-peer assistance with patients with chronic conditions, real estate for peer-to-peer lending, and finance for debt financing.

A comprehensive summary of game-theoretic approaches used in this dissertation is provided in the next Section.



## 2.3 Game theoretic approaches

**Brief historical overview** Even though the distinct concepts of game theory were known for several centuries, it emerged as a distinct field when John von Neumann published "On the Theory of Games of Strategy" in 1928 [145], introducing existence of solution for zero-sum games, utilizing Brouwer's fixed-point theorem. His 1944 book "Theory of Games and Economic Behavior," [146] co-authored with Oskar Morgenstern, marked the pinnacle of his work in game theory, defining strategies for two-person zero-sum games and revitalizing Daniel Bernoulli's theory of utility.

The central concept of game theory - Nash equilibrium, named after American mathematician John Forbes Nash Jr., was earlier used by Antoine Augustin Cournot in 1838 [34] to analyze oligopoly. In his theory, firms determine their optimal output based on others' outputs, resulting in a pure-strategy Nash equilibrium. The modern Nash equilibrium, however, considers mixed strategies, where players pick a probability distribution over possible strategies. Following findings of J. von Neumann and O. Morgenstern, Nash expanded the introduced solution concept in 1950/1951 [99], [100] to any game with finite actions, proving the existence of at least one mixed-strategy Nash equilibrium. Nash's equilibrium definition, stating each player's strategy is optimal against others', enabled him to use the Kakutani and Brouwer fixed-point theorems to prove equilibrium existence.

### 2.3.1 Basic Game Theory Concepts

Game theory, as a mathematical framework, can be broadly classified into two primary categories : cooperative and non-cooperative game theory. The latter is particularly valuable for investigating the strategic decision-making processes of multiple autonomous agents, or players, whose interests in the decision-making outcome may partially or fully conflict.

Fundamentally, **non-cooperative games** model a distributed decision-making process that enables the players to optimize their respective objective functions in the absence of coordination or communication, with these functions inherently interconnected through the players' actions. It's important to clarify that the term 'non-cooperative' does not necessarily signify a lack of cooperation among players. Instead, it indicates that any emerging cooperation must be self-sustaining without the need for strategic choice coordination or communication among the players. An emerging area of interest within this context is devising incentives to promote cooperation within a non-cooperative setting, as exemplified in some studies [141], [140].

Non-cooperative games often employ **strategic-form** representations. These representations primarily concentrate on individual strategic decisions and the selection of tactics each player would adopt to

achieve their objectives. A game in strategic (or normal) form is represented by a family of multi-variate functions  $\Pi_1, \dots, \Pi_N; N \geq 1$ . The index set of this family, which is denoted here by  $\mathcal{N} = \{1, \dots, N\}$ , is called the set of players and, for each  $n \in \mathcal{N}$ ,  $\Pi_n$  is commonly called the utility (or cost) function of player  $n$ . The strategic form assumes that  $\Pi_n$  can be any function of the following form :  $\Pi_n : \mathcal{X}_1 \times \dots \times \mathcal{X}_N \rightarrow R, (x_1, \dots, x_N) \mapsto \Pi_n(x)$  where  $\mathcal{X}_n$  is called the set of strategies of player  $n$ ,  $x_n$  is the strategy of player  $n$ ,  $x = (x_1, \dots, x_N) \in \mathcal{X}$  is the strategy profile, and  $\mathcal{X} = \mathcal{X}_1 \times \dots \times \mathcal{X}_N$ . A strategic-form game can be denoted by the compact triplet notation  $G = (\mathcal{N}, (\mathcal{X}_n)_{n \in \mathcal{N}}, (\Pi_n)_{n \in \mathcal{N}})$ . The notation  $x_{-n} = (x_1, \dots, x_{n-1}, x_{n+1}, \dots, x_N)$  is used to denote the strategies taken by all other players except player  $n$ . With a slight abuse of notation, the whole strategy profile is denoted by  $x = (x_n, x_{-n})$ . Thus, in generic form of a non-cooperative game, each player  $n \in \mathcal{N}$  solves the following optimization problem :

$$\begin{aligned} \min_{x_n} \quad & \Pi_n(x_n, x_{-n}), \\ \text{s.t.} \quad & x_n \in \mathcal{X}_n, \end{aligned} \tag{2.1}$$

**Note on cooperative games** When games permit players to communicate and engage in side payments (such as utility sharing), fully cooperative strategies may become appropriate. In such instances, the tools provided by **cooperative game theory** become highly relevant, as they help address an important question : "What is the outcome when players have the ability to communicate and choose to work cooperatively ?"

Cooperative games probe the potential of encouraging independent decision-makers to function collectively as an united entity to enhance their overall game positioning. An example can be seen in the realm of politics, where diverse parties may opt to merge or form a coalition, creating a cooperative group to improve their odds of gaining a share of power. Cooperative game theory comprises two main areas : Nash bargaining and coalition games. Nash bargaining involves scenarios where several players must agree upon the terms of their cooperation, while coalition game theory pertains to the establishment of cooperative groups or coalitions. Essentially, both aspects of cooperative game theory provide players with the necessary tools to determine potential partners for cooperation and the terms of such alliances, taking into consideration a variety of cooperation incentives and rules of fairness. A more in-depth discussion of cooperative game theory is available in [141].

### 2.3.2 Nash Equilibrium

The strategic-form representation can encompass a variety of situations in decentralized electricity markets or markets in general. For instance, players in a game can be : prosumers competing to improve

their performance in terms of incurred costs, electricity producers competing for market share etc.

Formally, the strategic form is characterized by two key features. Firstly, each player  $n$  can have its own objective, represented by a specific function  $\Pi_n(x)$ . Secondly, each player  $n$  has partial control over the optimization variables, as it can only control its strategy  $x_n \in \mathcal{X}_n$ . While the first feature resonates with multi-objective optimization, a difference lies in the control of optimization variables as in multi-objective optimization, one has full control over all variables. Moreover, multi-objective optimization problems often necessitate the definition of an aggregate objective.

The second feature aligns with the framework of distributed optimization, where a common objective function is usually considered, i.e.,  $\forall n, \Pi_n(x) = \Pi(x)$ . More crucially, the conventional assumption in distributed optimization is that the decision-making process is primarily driven by a single designer (or controller), who provides a set of strategies that the players strictly adhere to. While this scenario is possible and might be significant for some algorithmic aspects, game theory often allows players the freedom to choose their strategies.

A central question in this context is how to “solve” a strategic-form game. The notion of optimality is unclear since we have multiple objectives and the variables influencing the utility functions cannot be jointly controlled. Hence, the problem needs to be defined before it can be solved, leading to the necessity of introducing game-theoretic solution concepts.

The **Nash equilibrium (NE)** is a fundamental solution concept for a strategic-form game on top of which many other concepts are built. This section is mostly dedicated to the NE and discusses more briefly other solution concepts, which might also be considered. In [100], Nash proposed a simple but powerful solution concept, which is now known as an NE (or Nash point).

**Definition 1** *An NE of the game  $G = (\mathcal{N}, (S_n)_{n \in \mathcal{N}}, (\Pi_n)_{n \in \mathcal{N}})$  is a strategy profile  $x^* = (x_1^*, \dots, x_N^*) = (x_n^*, x_{-n}^*)$  such that :*

$$\forall n \in \mathcal{N}, \forall x_n \in \mathcal{X}_n, \Pi_n(x_n^*, x_{-n}^*) \geq \Pi_n(x_n, x_{-n}^*), \quad (2.2)$$

which describes a situation where an individual cannot improve their expected gains by altering their strategy, assuming that the strategies of all other participants remain constant, then such a selection of strategic choices is recognized as a Nash equilibrium. For this reason, an NE is said to be strategically stable to unilateral deviations.

### 2.3.3 Generalized Nash Equilibrium problems

It quickly became clear that there was a need to extend the Nash equilibrium problems (NEP), allowing players to also interact at the level of feasible sets. The term generalized Nash equilibrium problems

(GNEP) was first introduced by G. Debreu in 1952 [36], where he used the term 'social equilibrium'. This work was essentially a mathematical primer for the well-known paper by K. Arrow and G. Debreu in 1954 [7] about economic equilibria. In this paper, they called the GNEP an 'abstract economy', pointing out that "In a game, the pay-off to each player depends upon the strategies chosen by all, but the domain from which strategies are to be chosen is given to each player independently of the strategies chosen by other players. An abstract economy, then, may be characterized as a generalization of a game in which the choice of an action by one agent affects both the pay-off and the domain of actions of other agents". (p. 273, [7]).

K. Arrow and G. Debreu's 1954 paper and Debreu's subsequent 1959 book set the foundation for the development of mathematical economics. For a long time (until the early 90s), the GNEP was primarily the domain of economists and game theory experts. Some critics in this community have questioned the GNEP, saying that it is not a real game. For instance, Ichiishi, in his impactful 1983 book [67], stressed that an 'abstract economy' is not a game because a player needs to know others' strategies to identify his feasible strategy set, but others can't figure out their feasible strategies without knowing the player's strategy. Thus, he called an abstract economy a 'pseudo-game' and saw it only as a mathematical tool. However, further research showed the power of the GNEP model's ability to describe and explain phenomena; its potential for designing rules, protocols, and taxes to achieve specific goals; and the fact that different game paradigms can be used where it's conceivable, even in a non-cooperative setting, to have mechanisms that make it possible to satisfy the constraints [47].

**Generalized Nash Equilibrium** The main difference that distinguishes classical games from GNEP is that while solving GNEP, each player's strategy must belong to a set  $X_n(x_{-n}) \subseteq \mathbb{R}^{m_n}$  that depends on other players' strategies, i.e. the feasible set (or strategy space) of player  $n$  is defined by the value of  $x_{-n}$ . The goal of player  $n, \forall n \in \mathcal{N}$ , given the other players' strategies  $x_{-n}$ , is to choose a strategy  $x_n$  that solves the following minimization problem :

$$\begin{aligned} \min_{x_n} \quad & \Pi_n(x_n, x_{-n}), \\ \text{s.t.} \quad & x_n \in \mathcal{X}_n(x_{-n}). \end{aligned} \tag{2.3}$$

For any  $x_{-n}$ , the solution set of problem (2.3) is denoted by  $S_n(x_{-n})$ . The GNEP is the problem of finding a vector  $x$  such that

$$x_n \in S_n(x_{-n}), \quad \text{for all } n = 1, \dots, N.$$

Such a point  $x$  is called a solution of the GNEP or **generalized Nash equilibrium (GNE)**. As in the case of classical Nash equilibrium, a point  $x$  is therefore an equilibrium if no player can decrease his objective function by changing unilaterally  $x_n$  to any other feasible point :

**Definition 2** An GNE of the GNEP  $G = (\mathcal{N}, (S_n)_{n \in \mathcal{N}}, (\Pi_n)_{n \in \mathcal{N}})$  is a strategy profile  $x^* = (x_1^*, \dots, x_N^*) = (x_n^*, x_{-n}^*)$ , where  $\forall n \ x_n^* \in \mathcal{X}_n(x_{-n}^*)$ , such that :

$$\forall n \in \mathcal{N}, \forall x_n \in \mathcal{X}_n(x_{-n}), \Pi_n(x_n^*, x_{-n}^*) \geq \Pi_n(x_n, x_{-n}^*), \quad (2.4)$$

If the feasible sets  $X_n(x_{-n})$  do not depend on the rival players' strategies, i.e. we have  $X_n(x_{-n}) = X_n$  for all  $n = 1, \dots, N$ , the GNEP reduces to the standard game formulation.

**Example 3 (from [47])** Consider a game with two players, i.e.  $N = 2$ , each player controlling one variable. Assume that the players' problems are

$$\begin{array}{ll} \min_{x_1} (x_1 - 1)^2 & \min_{x_2} (x_2 - \frac{1}{2})^2 \\ \text{s.t. } x_1 + x_2 \leq 1, & \text{s.t. } x_1 + x_2 \leq 1. \end{array}$$

The optimal solution sets are then given by

$$S_1(x_2) = \begin{cases} 1, & \text{if } x_2 \leq 0, \\ 1 - x_2, & \text{if } x_2 \geq 0, \end{cases} \quad S_2(x_1) = \begin{cases} \frac{1}{2}, & \text{if } x_1 \leq \frac{1}{2}, \\ 1 - x_1, & \text{if } x_1 \geq \frac{1}{2}. \end{cases}$$

Then it is easy to check that the solutions of this problem are given by  $(\alpha, 1 - \alpha)$  for every  $\alpha \in [\frac{1}{2}, 1]$ . Note that the problem has infinitely many solutions.

In the example above the sets  $\mathcal{X}_n(x_{-n})$  are defined explicitly by inequality constraints. Defining the feasible set in GNEP through the **shared inequality constraints**, i.e. the constraints that depend on the decision variables of the other agents is a one way to define the GNEP. This is the case in our research and we will often use such an explicit representation. More precisely, in order to fix notation and terminology, first we say that the sets  $X_n(x_{-n})$  are given by

$$\mathcal{X}_n(x_{-n}) = \{x_n \in \mathbb{R}^{m_n} : g_n(x_n, x_{-n}) \leq 0\}, \quad (2.5)$$

where  $g_n(\cdot, x_{-n}) : \mathbb{R}^{m_n} \rightarrow \mathbb{R}^{k_n}$ , with equality constraints incorporated in a straightforward manner. Consider the following example that will be used as a first step for building the models of peer-to-peer

electricity markets used in our research.

**Example 4** Consider a (simplified) peer-to-peer electricity trading made of a set  $\mathcal{N}$  of  $N$  agents, each one of them being located in a node of a communication network, that is modeled as a graph  $G := (\mathcal{N}, E)$  where  $E \subseteq \mathcal{N} \times \mathcal{N}$  is the set of communication links between the players. Let  $\Gamma_n$  be the set of nodes, player  $n$  wants to trade electricity with.

Each agent  $n \in \mathcal{N}$  minimizes their costs  $\Pi_n(x_n, x_{-n})$ , associated with energy production / consumption / trading, while having a possibility to trade with the other peers in the network. Trading decisions are expressed as a decision variable  $(q_{nm})_{m \in \Gamma_n}$  with  $q_{nm} \in x_n$ , i.e. the vector of the quantities exchanged between  $n$  and  $m$  in the direction from  $n$  to  $m$ . We use the following convention : if  $q_{nm} \geq 0$ , then  $n$  buys  $q_{nm}$  from  $m$ , otherwise ( $q_{nm} < 0$ )  $n$  sells  $-q_{nm}$  to  $m$ .

The following condition on agent's trades called trading reciprocity constraint couples the decisions of two neighboring agents, ensuring for every node  $m \in \Omega_n$  that  $q_{nm} + q_{mn} = 0$ . Assume that this is the only **shared constraint** of the game. Thus, we can write each agent's optimisation problem as

$$\begin{aligned} \min_{x_n} \quad & \Pi_n(x_n, x_{-n}), \\ \text{s.t.} \quad & x_n \in \mathcal{X}_n^{\text{Individual}} \\ & q_{nm} + q_{mn} = 0 \quad \forall m \in \Gamma_n \quad (\zeta_{nm}), \end{aligned} \tag{2.6}$$

where  $\zeta_{nm}$  represents the dual variables, corresponding to the coupling constraint. Note that we clearly can express the **shared equality constraints** as the combination of two :  $q_{nm} + q_{mn} \geq 0$  and  $q_{nm} + q_{mn} \leq 0 \quad \forall m \in \Gamma_n$ .

Our next goal would be to introduce the basic assumptions and reformulations later used in our research.

### 2.3.4 Variational Equilibria

A first and important reformulation can be established under the following additional convexity assumption.

**Assumption 5** For every player  $n$  and every  $x_{-n}$ , the objective function  $\Pi_n(\cdot, x_{-n})$  is convex and the set  $X_n(x_{-n})$  is closed and convex.

This assumption is very common, especially in the economic applications, and is satisfied for the models considered in our research. For example, we assume that it holds for (2.6) of Example 4. Using this

assumption we can use the following reformulation of the GNEP, connecting it to the theory of (*quasi-*) *variational inequalities* :

**Theorem 6** ([47]) *Let a GNEP be given, satisfying the Convexity Assumption 5, and suppose further that the  $\Pi_n$  are  $C^1$  for all  $n$ . Then, a point  $x$  is a GNE if and only if it is a solution of the quasi-variational inequality  $QVI(\mathcal{X}(x), F(x))$ , where*

$$\mathcal{X}(x) := \bigcup_{n=1}^N X_n(x_{-n})$$

and

$$F(x) := (\nabla_{x_n} \Pi_n(x))_{n=1}^N,$$

where the quasi-variational inequality problem  $QVI(\mathcal{X}(x), F(x))$  consists in finding a vector  $x^* \in \mathcal{X}(x^*)$  such that  $(y - x^*)^T F(x^*) \geq 0$  for all  $y \in \mathcal{X}(x^*)$ .

Here we have to note that consideration of the GNE without consideration of its restrictions might lead to some undesirable results. Firstly, unfortunately, the theory for QVIs is less advanced than that for variational inequalities (VI) [47], where the variational inequality problem VI  $(X, F(x))$  consists in finding a vector  $x^* \in X$  such that

$$(y - x^*)^T F(x^*) \geq 0 \text{ for all } y \in X.$$

The second problem that we might encounter an infinite number of equilibria, as in Example 3, which is undesirable in general, but specifically in electricity markets, which we consider in our research. It is therefore of interest to see whether it is possible to reduce a GNEP to a special case, at least under some suitable conditions. In this respect it turns out that valuable results can be obtained for a special case and VI.

**Definition 7** *Let a GNEP be given, satisfying the Convexity Assumption 5. We say that this GNEP is **jointly convex** if for some closed convex  $X$  and all  $n = 1, \dots, N$ , we have*

$$X_n(x_{-n}) = \{x_n : (x_n, x_{-n}) \in X\} \tag{2.7}$$

Note that Example 3 and Example 4 (under convexity assumptions on the objective functions) are instances of jointly convex GNEPs. Also, when the sets  $X_n(x_{-n})$  are defined explicitly by a system of inequalities as in Example 3 or equalities as in Example 4, then it is easy to check that (2.7) is equivalent

to the requirement that  $g_1 = g_2 = \dots = g_N := g$  and that  $g(x)$  are (componentwise) convex with respect to all variables  $x$ ; furthermore, in this case, it obviously holds that  $\mathcal{X} = \{x : g(x) \leq 0\}$ .

This class of problems has been first studied in detail in a seminal paper by Rosen (1965) [127] and has been often identified with the whole class of GNEPs. Jointly convex GNEPs are also often termed as GNEPs with **coupled constraints**, which we will use later on, as it reflects the structure of our models. Finally, for this class of problems we can define a specific case of equilibria, shown to be extremely useful in the research dedicated to the GNEPs in peer-to-peer electricity markets :

**Definition 8** *Let a jointly convex GNEP be given with  $C^1$ -functions  $\Pi_n$ . We call a solution of the GNEP that is also a solution of VI  $(\mathcal{X}, F)$  a **variational equilibrium (VE)**.*

The first useful application of VE can be demonstrated by the following example :

**Example 9** ([47] continued) *In Example 3 the game has infinitely many solutions given by  $(\alpha, 1 - \alpha)$  for every  $\alpha \in [\frac{1}{2}, 1]$ . Consider now the VI  $(X, F)$  where*

$$X = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 + x_2 \leq 1\}, \quad F(x) = \begin{pmatrix} 2x_1 - 2 \\ 2x_2 - 1 \end{pmatrix}.$$

*$F$  is clearly strictly monotone and therefore this VI has a unique solution which is given by  $(\frac{3}{4}, \frac{1}{4})$  as can be checked by using the definition of VI. Note that, as expected, this is a solution of the original GNEP.*

This demonstrates that we can expect that the restriction of the set of considered GNEP might let us obtain unique equilibrium. Moreover, as we show next, VE has some valuable properties associated with desirable market properties in peer-to-peer setting. This is shown using KKT conditions.

### 2.3.5 KKT conditions

It is not difficult to derive primal-dual conditions for the GNEP. Assume, that the problem is defined as in (2.3) with the sets  $X_n(x^{-n})$  given by (2.5). With this structure in place, and assuming all functions involved are  $C^1$ , we can easily write down the KKT conditions for each player's problem; the concatenation of all these KKT conditions gives us what we can call the KKT conditions of the GNEP.

Suppose that  $x^*$  is a solution of the GNEP. Then, if for player  $n$  a suitable constraint qualification holds (for example, the Mangasarian-Fromovitz or the Slater constraint qualification), there is a vector  $\lambda_n^*$  of multipliers so that the classical Karush-Kuhn-Tucker (KKT) conditions

$$\nabla_{x_n} L_n(x_n, x_{-n}^*, \lambda_n) = 0,$$



$$0 \leq \lambda_n \perp -g_n(x_n, x_{-n}^*) \geq 0$$

are satisfied by  $(x_n^*, \lambda_n^*)$ , where  $L_n(x, \lambda_n) := \Pi_n(x) + g_n(x)^T \lambda_n$  is the Lagrangian associated with the  $n$ -th player's optimization problem. Concatenating these  $N$  KKT systems, we obtain that if  $x^*$  is a solution of the GNEP and if a suitable constraint qualification holds for all players, then a multiplier  $\lambda^*$  exists that together with  $x^*$  satisfies the system

$$\begin{aligned} L(x, \lambda) &= 0 \\ 0 \leq \lambda \perp -g(x) &\geq 0, \end{aligned} \tag{2.8}$$

where

$$\lambda := \begin{pmatrix} \lambda_1 \\ \cdot \\ \cdot \\ \cdot \\ \lambda_N \end{pmatrix}, \quad g(x) := \begin{pmatrix} g_1(x) \\ \cdot \\ \cdot \\ \cdot \\ g_N(x) \end{pmatrix}, \quad \text{and} \quad L(x, \lambda) := \begin{pmatrix} \nabla_{x_1} L_1(x, \lambda_1) \\ \cdot \\ \cdot \\ \cdot \\ \nabla_{x_N} L_N(x, \lambda_N) \end{pmatrix}.$$

**Theorem 10** ([47]) *Let a GNEP be given defined by (2.3) and (2.5) and assume that all functions involved are continuously differentiable.*

1. *Let  $x^*$  be an equilibrium of the GNEP at which all the player's sub-problems satisfy a constraint qualification. Then, a  $\lambda^*$  exists that together with  $x^*$  solves system (2.8).*
2. *Assume that  $(x^*, \lambda^*)$  solves the system (2.8) and that the GNEP satisfies the Convexity Assumption. Then  $x^*$  is an equilibrium point of the GNEP.*

**Remark 11** *The differentiability assumption on the problem functions involved can be relaxed by using some suitable notion of subdifferential.*

Next consider the case of a jointly convex GNEP with the feasible set  $X$  having the explicit representation  $\mathcal{X} = \{x : g(x) \leq 0\}$  for some (componentwise) convex function  $g$ . Hence the strategy space for player  $n$  is given by  $\mathcal{X}_n(x_{-n}) = \{x_n : g(x_n, x_{-n}) \leq 0\}$  for all  $n = 1, \dots, N$ . Similar to the previous discussion on general GNEPs, it follows that the KKT conditions of player  $n$ -th optimization problem is given by

$$\begin{aligned} \nabla_{x_n} \Pi_n(x_n, x_{-n}) + \nabla_{x_n} g(x_n, x_{-n}) \lambda_n &= 0 \\ 0 \leq \lambda_n \perp -g(x_n, x_{-n}) &\geq 0 \end{aligned} \tag{2.9}$$

for some multiplier  $\lambda_n$ . On the other hand, consider the corresponding VI  $(X, F)$ . The KKT conditions

of this VI (see [47]) are given by

$$\begin{aligned} F(x) + \nabla g(x)\lambda &= 0, \\ 0 &\leq \lambda \perp -g(x) \geq 0 \end{aligned} \tag{2.10}$$

for some multiplier  $\lambda$ . The precise relation between these two KKT conditions and a GNEP solution is given in the following result which, basically, says that (2.10) holds if and only if (2.9) is satisfied with the same multiplier for all players  $n$  or, in other words, that a solution of the GNEP is a variational equilibrium if and only if the shared constraints have the same multipliers for all the players.

**Theorem 12** [[47], [63]] *Consider the jointly convex GNEP with  $g, \Pi_n$  being  $C^1$ . Then the following statements hold :*

1. *Let  $x^*$  be a solution of the VI  $(\mathcal{X}, F)$  such that the KKT conditions (2.10) hold with some multiplier  $\lambda^*$ . Then  $x^*$  is a solution of the GNEP, and the corresponding KKT conditions (2.9) are satisfied with  $\lambda_1 := \dots := \lambda_N := \lambda^*$ .*
2. *Conversely, assume that  $x^*$  is a solution of the GNEP such that the KKT conditions (2.9) are satisfied with  $\lambda_1^* = \dots = \lambda_N^*$ . Then  $(x^*, \lambda^*)$  with  $\lambda^* := \lambda_1^*$  is a KKT point of VI  $(X, F)$ , and  $\bar{x}$  itself is a solution of VI  $(X, F)$ .*

This theorem has important consequences and interpretations for the models considered in this dissertation. Consider again the problem presented in Example 4. Following the definition of VE and Theorem 12, we can write that VE of the game defined by the maximization problems (2.6) with coupling constraints, is a GNE of this game such that, in addition, the Lagrangian multipliers associated to the coupling constraints are equal, i.e. :

$$\zeta_{nm} = \zeta_{mn}, \quad \forall n \in \mathcal{N}, \forall m \in \Gamma_n \tag{2.11}$$

By duality theory,  $\zeta_{nm}$  for  $n \in \mathcal{N}, \forall m \in \Gamma_n$  can be interpreted as bilateral energy trading prices [26]. In general,  $\zeta_{nm} \neq \zeta_{mn}$ , thus leading to non-symmetric energy trading prices between couple of agents. Relying on VE as solution concepts enforces a natural symmetry in the bilateral energy price evaluation between any couple of agents [26].

### 2.3.6 Algorithmic approaches

The problem of coordinating the agents to a Nash equilibrium is of great importance for control purposes and it has consequently been addressed by many authors in the last few years. In the field of GNEP, distributed (variational) GNE computation is an active research area. Generally, decomposition

algorithms are an intuitive choice when dealing with Generalized Nash Equilibrium Problems (GNEPs). One of the first proposed solutions is a best-response Gauss-Seidel algorithm [47, 48], applied to a Generalized Potential Game (GPG). Essentially, in a Nash game, each player tries to optimize their own objective function. Thus, an iterative algorithm, like a Gauss-Seidel method, suits well as an initial step in this direction. In this method, during each cycle, each player looks at the strategies the other players are using, then updates their own strategy by solving their respective optimization problem. The GPG is a special structure of the game that arises in various applications, e.g., demand side management in the smart grid [8, 81]. Briefly, this structure can be understood as that there is a single function  $P$  that, in some sense, reflects the changes in the players' objective functions. In [17] authors show that, with a correct preconditioning, the preconditioned proximal-point (PPP) method generates an efficient algorithm for GNE seeking in GPG with linearly coupled cost functions, in terms of convergence speed. Authors derive a single-layer, fixed-step, distributed algorithm for the computation of a GNE in network potential games with linear coupling constraints.

Another subclass of games is an aggregative game (AG), which involves a set of linked optimization problems related to noncooperative agents. In these games, each participant is affected by the collective impact of all participants' actions [73], [70]. Aggregative games appear in various types of problems. These include demand-side management in the smart grid [128], such as the charging and discharging of electric vehicles [57, 88, 89, 108]. They are also used for demand-response regulation in competitive markets [81], and for managing congestion in traffic and communication networks [11]. Authors in [105, 110], and [59] consider aggregative games with quadratic objective functions, adopting strong monotonicity and Lipschitz pseudo-gradient. In [109] authors consider average aggregated games, in which each agent's action is influenced by the average strategy of the entire population. Average aggregated games are also considered in [106], where authors focus on the relation between Nash and Wardrop equilibria in GNEPs and propose a decentralized two-level algorithm based on optimal response with an outer loop that updates a dual variable to achieve a Wardrop equilibrium, as well as propose a decentralized one-level asymmetric projection algorithm based on gradient step to achieve either a Nash or a Wardrop equilibrium. Semi-decentralized algorithm in aggregative games with coupling constraints and non-differentiable cost functions is considered in [16]. In this work, the authors first critically review the most relevant available algorithms by leveraging the framework of monotone operator theory and operator splitting. Following this review, authors design two novel schemes. The initial scheme is a single-layer, fixed-step algorithm with convergence guaranteed for general monotone aggregative games, even those not classified as co-coercive or strictly monotone. The subsequent scheme is a single-layer proximal-type algorithm that is specifically designed for a class of monotone aggregative games with

linearly coupled cost functions.

Operator-splitting approaches are widely used in the literature dedicated to GNE (VE) seeking; it allows the design of GNE algorithms that are guaranteed to globally converge with fixed step-sizes, with concise convergence proofs [111]. For games with affine coupling constraints, distributed and center-free GNE seeking is investigated via an operator approach in [156–158]: a forward-backward algorithm, convergent in strongly monotone games [156], [158], and preconditioned proximal algorithms for monotone games [157], while semi-decentralized algorithms, requiring a central node (coordinator) to broadcast the common multipliers and/or aggregative variables are presented in [15, 16, 58]. Another subclass of operator splitting approaches (Douglas–Rachford splitting) is a celebrated Alternating Direction Method of Multipliers (ADMM) in distributed optimization, which is a special implementation of the DR splitting method. In [130] authors develop an algorithm for GNE problems within the framework of inexact-ADMM. Players are only aware of their own cost functions (which are not in the form of aggregative but general game), problem data (which is related to a private coupled equality constraint for each player) and action set of all players (they are not aware of the others' actions). In [23] authors use a regularized version of ADMM and present a global convergence result for  $N \geq 2$  players under a partial strong monotonicity and a partial Lipschitz condition in Hilbert spaces, while Jacobi-type ADMM method for the similar problems is considered in [22]. ADMM methods are widely applied to LEMs as they align well with decentralized structure of these markets [64].

Lastly, a fundamental problem in monotone game theory is the computation of a specific generalized Nash equilibrium (GNE) among all the available ones, e.g., the optimal equilibrium with respect to a system-level objective. In [19], authors address this open problem by leveraging results from fixed-point selection theory and derive distributed algorithms for the computation of an optimal GNE in monotone games. Authors also extend the technical results to the time-varying setting and propose an algorithm that tracks the sequence of optimal equilibria up to an asymptotic error, whose bound depends on the local computational capabilities of the agents.

Distributed computation of GNE and especially VE represents an active area of current research that hold immense potential for substantial improvements of the distributed systems, with peer-to-peer electricity markets in particular.



# Chapitre 3

## Risk-hedging

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# A Stackelberg Game Analysis of Risk-Hedging Strategies in Decentralized Electricity Markets

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## 3.1 Introduction

Motivated by the need for electricity market restructuring, this chapter quantitatively analyzes decentralized market designs formulated as equilibrium problems. In the last years, decentralization has been broadly seen as an upcoming trend in network economics [33, 49], and more specifically in the electricity market literature where it is perceived as an emerging topic [6, 26, 95], largely due to the liberalization of the energy sector, that has to account for the massive penetration of renewable energy sources (RES), and the more proactive role of prosumers.

Equilibrium problems used to analyze market designs rely heavily on the structure and the rules of the market, as well as on the way network constraints are handled [65]. As a first step, in order to account for the strategic behavior of consumers and the network constraints, we model the electricity market as a generalized Nash equilibrium problem (GNEP), i.e., a noncooperative game endogenizing shared coupling constraints within the agents' parametrized optimization problems. We employ generalized Nash equilibrium (GNE) as a solution concept [63, 74, 159], and a refinement of it, called variational equilibria (VE), assuming that the shadow variables of the shared coupling constraints are aligned among the agents [74, 127]. We focus on the design of decentralized electricity markets which rely on a network defining each agent's trading relationships, e.g., their neighbors. We focus on a financial level of a distribution network, but coupling between the market and the physical layer of a distribution network (seen e.g. as two inter-dependent layers [133]) constitutes an interesting direction for future work.

When dealing with future uncertain losses, agents can have individual perception of uncertainties or *risk perception*, that should be accounted for in the prosumers' optimization problems. Whenever agents have different perceptions of risk (*heterogeneous risk aversion* framework), it might lead to market inefficiencies [40, 54, 117, 121]. Additionally, the heterogeneous description of uncertainties makes the market incomplete for risk [96]. We allow financial contracts trading between agents to complete the market [123]. These contracts act as instruments to reduce the effect of heterogeneous risk attitudes on the outcome of the risk adjusted market. The questions that naturally arise from this inclusion are : **i)** how to define a mechanism with desirable market properties (e.g., economic efficiency, fairness) for risk hedging financial contracts ? **ii)** How to incentivize the prosumers to participate in this market ? And, **iii)** how to characterize the resulting equilibria ?

To answer these questions, we model financial contracts in the form of Arrow-Debreu securities. Several works have considered Arrow-Debreu securities for risk trading among prosumers in peer-to-peer electricity markets [54, 96, 144] in a decentralized one-level setting. Dealing with these securities, a first market design option requires the involvement in the market of an additional strategic agent, that can be interpreted as an insurance company. The insurance company acts as a seller of financial contracts with the goal to maximize its profit. In a second market design option, we allow the agents to trade Arrow-Debreu securities both with the insurance company and with the other agents, to hedge their risks. For both options, the presence of an insurance company that has to provide contract options to the prosumers calls for a Stackelberg formulation of the model, in which the insurance company acts as a *leader* and prosumers as *followers*.

Stackelberg games [138] have been extensively applied in various fields such as market design, financial hedging, security applications, etc. [27, 85, 132, 150, 155]. Stackelberg games can be casted as *bilevel optimization problems* where one problem (followers' or *lower-level*) is nested within another (leader's or *upper-level*). The structure of our problem naturally gives rise to a *one-leader, multi-follower* generalized Stackelberg game involving a GNEP at the lower level which might have multiple solutions [26], [144]. In that setting agents might either try to cooperate with the leader, or behave in an adversarial way, thus, either they choose the best solution with respect to the leader's objective (*optimistic bilevel problem*) or the worst one (*pessimistic bilevel problem*). We consider both optimistic (OBP) and pessimistic (PBP) formulations. PBP is usually considered to be more complicated to solve than OBP, due to the difficulties arising in the computation of its solution or even in the proofs of existence of solutions [18, 85, 86, 125]. To guarantee the existence of a solution of PBP, we include contract price based incentives for the prosumers, which allow us to characterize Stackelberg-Nash equilibrium of PBP, and to compare it to the solution of OBP.

In addition, the literature dedicated to the computation of PBP solutions often focuses on the computation of approximate equilibria [77, 85] or specific cases [12, 32, 149]. Using the structure of our model, we parametrize the response of the prosumers by introducing a *choice function*, which allows us to reformulate the initial bilevel problem as a set of parametrized GNEPs. This allows us to use results from the GNEP literature [139, 141, 159] to analyze the market equilibria. Finally, we compare the properties of our market models on data from Pecan Street, by computing the prosumers' individual costs, the social cost, analyzing fairness, and the impact of incomplete information on the insurance company's cost.



### 3.1.1 Related Work

Game-theoretic models have been widely employed to investigate agents' strategic behaviors in electricity markets [39, 65, 141, 149]. In [26], authors quantify the efficiency loss relying on the price of anarchy and capture the impact of incomplete information on the market equilibrium relying on GNE and VE. In the same vein, the economic dispatch in electricity trading with different structures of communication is analysed using consensus based approaches in [95, 96].

A large part of the literature focuses on the impact of risk on the agents' decisions in competitive settings [35, 53, 123] and in electricity markets in particular [1, 40, 54, 96, 117, 121, 144]. Among them, many papers explore equilibrium properties assuming that the market is not complete for risk [1, 35, 40, 117, 121, 123]. In [72], authors analyze the impact of heterogeneous risk preferences on the electricity market equilibrium. In [35], authors discuss incomplete risk trading and its impact on the long-term strategic investment decisions, and compare cases of complete and fully incomplete markets for risk. Risk trading alongside with the properties of complete market is explored in [123]. Financial hedging in a supply chain, modeled as a Stackelberg game, is considered in [27] and hedging inventory risk in [53], where authors show that risk hedging leads to a lower risk and a higher return on inventory investment.

Heterogeneous risk-adjusted decentralized electricity markets are considered in [96], [144]. In [96], a model for risk hedging via financial contracts is considered. It addresses the definition of fairness and the impact of risk in a one settlement two-stage market. In [144], a Nash equilibrium problem formulation is considered under different degrees of market completeness for risk. In [54], coherent risk measures are employed. In this paper, the authors analyse risk-adjusted markets and evaluate the impact of risk-hedging contracts on the market efficiency. The question of uniqueness and existence of risk-averse equilibria is addressed in [1, 54, 123], where one can find insights on some equivalences between social planner problems and equilibrium problems. The problem we address in our paper relies on a similar risk-averse setting. In [54, 96, 144], risk trading takes the form of Arrow-Debreu financial contracts, that prosumers can trade among themselves. We go further and supplement this one stage model with an additional layer operated by an insurance company. We thoroughly analyze the resulting Stackelberg game, considering both optimistic and pessimistic formulations and provide results on equilibria characterization, solution existence and market properties.

A wide range of problems, from security games [136] to general market design, are modeled as Stackelberg games. We provide only a few relevant examples and refer the reader to the literature for further information [38, 85, 136]. In [132], authors study Stackelberg-Nash-Cournot equilibria in a game with one leader and  $N$  followers and analyze its properties under mild economic assumptions. De Wolf and Smeers extend this result in [150] to a stochastic version, in which the decision of the leader is

taken while the market demand is uncertain, and provide a practical implementation of their model in the European gas market. Equilibrium problems with equilibrium constraints (EPECs) arising from the applications of Stackelberg game to the electricity markets are thoroughly analyzed in [65, 122, 155].

In the bilevel optimization literature, most chapters focus on the solution characterization and the development of computational approaches [12, 18, 32, 77, 85, 86, 136, 149]. In [77], authors focus on PBP, reformulating it in a standard form and then as a bilevel problem with a two player GNEP at the lower level, that later can be solved as a mathematical program with complementarity constraints (MPCC). They consider  $\varepsilon$  solution of the lower-level problem in order to overcome issues arising from PBP solution existence. We use the machinery from [77] to analyze PBP and its properties, but we focus on  $\varepsilon$  solution of the upper-level problem and introduce a parametrization of the reaction of the prosumers that allows us to use computational approaches suited for GNEPs.

### 3.1.2 Contributions and Chapter Organization

We provide a thorough analysis of equilibrium models for risk-averse market design taking into account uncertainties, the agents' strategic behaviors and network constraints. By comparison with the previous works that account for players with heterogeneous risk-aversion levels, in the context of local energy communities [96, 144], we focus on an imperfect competition setting in which the electricity market price is not enforced by an exogenous price setter, but is obtained as the result of the interactions between the prosumers. To that purpose, we first consider a noncooperative game model with coupled constraints capturing the energy trading reciprocity constraints between couples of agents, therefore leading to a GNEP framework. This allows us to include the connection graph structure, capturing the prosumers' trading preferences, in the prosumers' energy exchange model.

We first build a risk-averse model to capture risk-attitude heterogeneity among the agents and supplement it with an additional layer, operated by an insurance company. The insurance company can supply risk-hedging financial contracts for the agents as a sole supplier and as a competitor for an inter-agent financial contracts trading. We analyze the resulting Stackelberg game considering both optimistic and pessimistic formulations, and provide results on equilibria characterization, solution existence and market properties. We first prove the equivalence of the reformulation of the bilevel problem with a parameterized GNEP, by using a so-called *choice function*. Relying on this parametrization, we prove that the PBP formulation might not have a solution without including additional price incentives from the insurance company. Then, we prove that a slight decrease in the financial contracts' price leads to a solution of PBP that is  $\varepsilon$  close to the optimistic solution of OBP. We next discuss the situation where the insurance company has incomplete information about the prosumers' risk-aversion levels, and analyze

the resulting two-stage market equilibrium, proving that it is economically efficient and fair.

The organization of the rest of this chapter is as follows : after introducing the problem statement in section 3.2 and the agents in section 3.2.1, we analyse risk-averse market equilibria in an incomplete market setting, in section 3.2.2. We discuss completeness of the market in section 3.3, in which we build a two-stage market design involving an insurance company in section 3.3.1. In section 3.4, we provide a comprehensive analysis of the resulting Stackelberg game, considering both OBP and PBP. Numerical illustrations are provided in section 3.5.

## 3.2 Problem Description

We consider a single-settlement market for decentralized electricity trading modeled as a noncooperative game  $\mathcal{P}^G$  involving a set  $\mathcal{N}$  of  $N$  agents (prosumers). Each agent is located in a node of the network, which is modeled as an undirected connected graph  $G := (N, E)$  where  $E \subseteq N \times N$  is the set of links between the agents. Agent  $n$  can trade energy only with her neighbors in  $\mathcal{G}$ , denoted by  $\Gamma_n$ . The graph  $G$  does not necessarily reflect the distribution power network constraints. We do not consider the physical layer of the distribution network, but coupling it with the market (seen as two inter-dependent layers) constitutes an interesting direction for future work [133].

### 3.2.1 Agents

We assume that each agent  $n \in \mathcal{N}$  is equipped with RES-based self-generation which is denoted by  $\Delta g_n$ . To model randomness, we employ scenario based approach, which is widely used in the literature dedicated to the electricity markets [40, 54, 96, 144]. This approach allows to account for the stochasticity of the electricity market involving RES-based generation and risk hedging contracts. There are  $T$  possible scenarios :  $\mathcal{T} := (1, \dots, T)$ . Each agent's probability of being in a scenario  $t$  is given by  $p^t$  s.t.  $\sum_{t \in \mathcal{T}} p^t = 1$ . Probabilities  $p^t$  for all  $t \in \mathcal{T}$  are known by all the agents. Another approach to model individual and collective uncertainty is described in Appendix 3.7.3.

Each prosumer  $n$  chooses independently her *bilateral trades*  $\mathbf{q}_n^t$ , *flexible energy generation*  $g_n^t$  and *flexible demand*  $d_n^t$ , to minimize her cost function  $\Pi_n^t$ . The quantity exchanged between  $n$  and  $m$ , is denoted as  $q_{nm}^t$  for all  $m \in \Gamma_n \setminus \{n\}$ . If  $q_{nm}^t \geq 0$ , then  $n$  buys  $q_{nm}^t$  from  $m$ , otherwise ( $q_{nm}^t < 0$ )  $n$  sells  $-q_{nm}^t$  to  $m$ . We use subscript  $t$  to reflect the dependence of the decision variables on the scenario.

**Feasibility Set** In each node, we introduce  $\mathcal{D}_n := \{d_n^t \in \mathbb{R}_+ | \underline{D}_n \leq d_n^t \leq \overline{D}_n\}$  as agent  $n$ 's demand set and  $\mathcal{G}_n := \{g_n^t \in \mathbb{R}_+ | \underline{G}_n \leq g_n^t \leq \overline{G}_n\}$  as agent  $n$ 's generation set. Given a scenario  $t$ , we impose an

equality on the *trading reciprocity* :

$$q_{nm}^t + q_{mn}^t = 0, \quad \forall m \in \Gamma_n \quad (3.1)$$

which couples agents' bilateral trading decisions. It means that, in the case where  $q_{mn}^t > 0$ , the quantity that  $n$  buys from  $m$  should be equal to the quantity  $q_{nm}^t$  that  $m$  is willing to offer to  $n$ . Let  $\kappa_{nm} \in [0, +\infty)$  be the equivalent *trading capacity* between node  $n$  and node  $m$ , such that  $\kappa_{nm} = \kappa_{mn}$ . Then

$$q_{nm}^t \leq \kappa_{nm}, \quad \forall m \in \Gamma_n \quad (3.2)$$

Local *supply and demand balance* leads to the following equality in each node  $n$  in  $\mathcal{N}$ , with  $\Delta g_n^t$  denoting RES-generation :

$$d_n^t = g_n^t + \Delta g_n^t + \sum_{m \in \Gamma_n} q_{nm}^t \quad (3.3)$$

We denote the dual variable  $\xi_{nm}^t$  associated with the constraint (3.2),  $\zeta_n^t$  as the dual variable for (3.1) and  $\lambda_n^t$  for (3.3). Denote  $\mathbf{x}_n^t := (d_n^t, g_n^t, \mathbf{q}_n^t)$  to be the vector which contains the decision variables of prosumer  $n$ . We denote feasibility sets as  $\mathcal{K}_n(\mathbf{x}_{-n}^t) := \{\mathbf{x}_n^t | d_n^t \in \mathcal{D}_n, g_n^t \in \mathcal{G}_n, (3.1), (3.2), (3.3) \text{ hold } \forall t \in \mathcal{T}\}$ , where  $\mathbf{x}_{-n}^t$  is a vector which contains the decisions of all agents excluding agent  $n$ . Joint admissible set is written then as a  $\mathcal{K} := \prod_n \mathcal{K}_n(\mathbf{x}_{-n}^t)$ .

**Prosumer  $n$ 's Cost Function** We consider a *quadratic production cost* with  $a_n, b_n, c_n > 0$  :  $C_n(g_n^t) = \frac{1}{2}a_n g_n^{t^2} + b_n g_n^t + c_n$  for all  $t \in \mathcal{T}$ . We assume that the self-generation occurs at zero marginal cost with a quadratic form of the cost, that is seen as realistic for a large class of conventional generators [26]. The *usage benefit* perceived by agent  $n$  is modeled as a strictly concave function of agent  $n$ 's demand :  $U_n(d_n^t) = -\tilde{a}_n(d_n^t - \hat{d}_n^t)^2 + \tilde{b}_n$  for all  $t \in \mathcal{T}$ , where  $\hat{d}_n^t$  is a *target demand* defined exogenously for agent  $n$ .

We introduce price differentiation that characterizes both the locational aspects and the preferences of the prosumers. The preferences are modeled with (product) differentiation prices : each agent  $n$  has a price  $c_{nm}$  to trade with an agent  $m$  in her neighborhood  $\Gamma_n$ . The total trading cost of agent  $n$  is modeled by a linear function  $\tilde{C}_n(\mathbf{q}_n^t) = \sum_{m \in \Gamma_n, m \neq n} c_{nm} q_{nm}^t, \forall t \in \mathcal{T}$ , where parameters  $c_{nm} > 0$  can be interpreted as taxes for energy trading or agents' preferences regarding the trade characteristics. If  $q_{nm}^t > 0$  then  $n$  has to pay the cost  $c_{nm} q_{nm}^t > 0$ . Thus, the higher  $c_{nm}$  is, the less interesting it is for  $n$  to buy electricity from  $m$  but the more interesting it is for  $n$  to sell electricity to  $m$ . We write prosumer

$n$ 's cost function  $\forall t \in \mathcal{T}$  as follows :

$$\Pi_n^t = C_n(g_n^t) + \tilde{C}_n(\mathbf{q}_n^t) - U_n(d_n^t) \quad \forall t \in \mathcal{T} \quad (3.4)$$

**Local Market Operator (MO)** In the electricity market literature, the electricity trading problem is often considered to be solved in a centralized way, requiring the presence of a market operator (MO) to which all the private information is reported [144], [96]. In our work, the centralized electricity market design will be considered as the benchmark. The MO minimizes the social cost,  $SC := \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} \Pi_n^t$ , under constraints expressed by the joint feasible set  $\mathcal{K}$ .

### 3.2.2 Risk-Averse Electricity Market Design

In this chapter, we focus on a *risk-averse* design in which we consider a market with collective uncertainties that are common knowledge, but agents have different risk-aversion levels. On the contrary, in the *risk-neutral* formulation, prosumers optimize their costs with respect to the probabilities  $p^t$ , without taking into account the heterogeneity of the risk perception of the agents. A detailed description of the *risk-neutral* electricity market design can be found in the Appendix.

Under risk-averse market design, the prosumers act upon the set of risk attitudes  $\chi_n, n \in \mathcal{N}$ . Different risk attitudes imply different risk perception of the cost function (3.4). To account for the risk-averse behavior of the agents, we employ CVaR as a coherent risk measures in agents' objective functions. CVaR is known to have a lot of appealing properties, e.g., it is coherent, easy to integrate in an optimization problem, etc. [126].

By definition, CVaR is the average of all realizations larger than the VaR, where the latter is given by  $\eta_n = \min_{\eta_n} \{\eta_n \mid \mathbb{P}[\Pi_n^t \leq \eta_n] = \chi_n\}$ . Then, we write CVaR as follows :  $R[\Pi_n^t] = \eta_n + \frac{1}{(1-\chi_n)} \sum_{t \in \mathcal{T}} p^t [\Pi_n^t - \eta_n]^+$ . Note that  $R[\Pi_n^t]$  is convex in  $(d_n^t, g_n^t, \mathbf{q}_n^t, \eta_n)$  if  $\Pi_n^t$  is convex in  $(d_n^t, g_n^t, \mathbf{q}_n^t)$ , which is the case in our model. The non-differentiability of  $R[\Pi_n^t]$  can be overcome by leveraging the epigraph form [126] :  $R[\Pi_n^t] = \eta_n + \frac{1}{(1-\chi_n)} \sum_{t \in \mathcal{T}} p^t u_n^t$ , with  $u_n^t \geq 0$  and  $\Pi_n^t - \eta_n \leq u_n^t$  with dual variables  $\pi_n^t$  and  $\tau_n^t$  respectively. Define feasibility set  $\tilde{\mathcal{K}}_n$  as  $\tilde{\mathcal{K}}_n(\mathbf{x}_{-n}^t) := \{\mathbf{x}_n = (d_n^t, g_n^t, \mathbf{q}_n^t, u_n^t, \eta_n) \mid (d_n^t, g_n^t, \mathbf{q}_n^t) \in \mathcal{K}_n(\mathbf{x}_{-n}^t), u_n^t \geq 0, \Pi_n^t - \eta_n \leq u_n^t\}$  and denote  $\tilde{\mathcal{K}} := \prod \tilde{\mathcal{K}}_n(\mathbf{x}_{-n}^t)$  as a joint admissible set. We formulate agent  $n$ 's optimization problem as :

$$\min_{d_n^t, g_n^t, \mathbf{q}_n^t, u_n^t, \eta_n} \quad \eta_n + \frac{1}{(1-\chi_n)} \sum_{t \in \mathcal{T}} p^t u_n^t, \quad (3.5a)$$

$$s.t. \quad (d_n^t, g_n^t, \mathbf{q}_n^t, u_n^t, \eta_n) \in \tilde{\mathcal{K}}_n(\mathbf{x}_{-n}^t). \quad (3.5b)$$

To analyse the efficiency loss of decentralized electricity market designs, we apply solutions concepts of Generalized Nash Equilibria and Variational Equilibria, both of them exist under mild conditions [74], [159].

**Definition 13** A Generalized Nash Equilibrium (GNE) of the noncooperative game  $\mathcal{P}^G$  with coupling constraints is a vector  $\mathbf{x} := (x_n)_n$  that solves the maximization problems of the agents or, equivalently, a vector  $\mathbf{x}$  such that  $\mathbf{x}$  solve the KKT system for each  $n$ .

**Definition 14** A Variational Equilibrium (VE) is a GNE such that the Lagrangian multipliers of the coupling constraints (3.1), are equal, i.e. :

$$\zeta_{nm}^t = \zeta_{mn}^t, \quad \forall n \in \mathcal{N}, \forall m \in \Gamma_n \quad (3.6)$$

By duality theory,  $\zeta_{nm}^t$  for  $n \in \mathcal{N}, \forall m \in \Gamma_n$  can be interpreted as bilateral energy trading prices [26]. In general,  $\zeta_{nm}^t$  might not be aligned with  $\zeta_{mn}^t$ , thus leading to non-symmetric energy trading prices between couple of agents. Relying on VE as solution concepts enforces a natural symmetry in the bilateral energy price valuations [26]. However, this might require strong coordination between the agents.

### 3.3 Completeness of the Market

A market is said to be complete, whenever there exists an equilibrium price for every asset in every possible state of the world; the market is incomplete otherwise [10], [96]. To complete the market in the sense of this definition, we include financial contracts that are intended to hedge the risk of market participants. We discuss below two possible designs of the risk hedging market.

#### 3.3.1 Two Stage Design of the Risk-Hedging Market with an Insurance Company

We introduce an insurance company  $I$ , whose sole purpose is to sell the *state contingent claims* to the agents. We want to consider the situation in which financial contracts trading inside the community is not sufficient to satisfy all the demand, and agents still have the possibility to buy missing contracts from  $I$ . Below, we build a model of a risk hedging market, including first the insurance company and finally both the insurance company and inter-agent financial contracts trading. We model the insurance company as a distinct agent whose behavior is restricted purely to contract trading.  $I$  decides on the price  $\alpha_n^t$  and the maximum contract value  $\bar{J}^t$  for the contract  $J_n^t$ , which is paid if the outcome  $t$  is realized. In

this framework, the cost function of the insurance company is defined as follows :

$$\Pi_I = \sum_{n \in \mathcal{N}} \left[ \underbrace{- \sum_{t \in \mathcal{T}} \alpha_n^t J_n^t}_{\text{Received revenue}} + \underbrace{\sum_{t \in \mathcal{T}} p^t J_n^t}_{\text{Insurance payments}} \right] \quad (3.7)$$

With the presence of the sole insurance company at the upper-level and without inter-agent trading on the lower-level, the timeline of the risk-hedging market can be described as follows :

- (1) The insurance company  $I$  optimizes (anticipating the reaction of the prosumers) the contract price  $\alpha_n^t$  for prosumer  $n$  and the maximum amount  $\bar{J}^t$  (the same for all agents) for scenario  $t$ .
- (2) Each prosumer  $n \in \mathcal{N}$  determines the contracts  $J_n^t$  she wants to buy such that  $0 \leq J_n^t \leq \bar{J}^t$  and buys the contracts by paying the total price  $\sum_{t \in \mathcal{T}} \alpha_n^t J_n^t$  to receive  $J_n^t$  in scenario  $t$ .

Note that the price  $\alpha_n^t$  for the insurances is settled *per scenario, per agent* instead of *per scenario* in the inter-agent trading case. The motivation for this setting comes from the ability of the insurance company to evaluate the risks related to each agent as it is usually done in practice : the insurance company has means to assess these risks more accurately than the prosumers. Moreover, the insurance might propose contract prices that are discriminatory.

The sequence of decisions introduced in the timeline above made by insurance company  $I$  and prosumers have a hierarchical structure. It can be modelled as a Stackelberg one leader multi-follower game in which  $I$  acts as a leader and prosumers as followers. The leader anticipates the reaction of the followers when optimizing his strategy, while the followers react rationally to the actions of the leader by computing their best-response functions. Formally, the one leader multi-follower game can be written as follows :

$$\min_{(\alpha_n^t, \bar{J}^t)_{n \in \mathcal{N}}} \sum_{n \in \mathcal{N}} \left[ - \sum_{t \in \mathcal{T}} \alpha_n^t J_n^t + \sum_{t \in \mathcal{T}} p^t J_n^t \right] \quad (3.8a)$$

$$s.t. \quad 0 \leq \alpha_n^t \quad \forall n \in \mathcal{N} \quad (3.8b)$$

$$\forall n \in \mathcal{N} \quad J_n^t \in \arg \min_{J_n^t, \mathbf{x}_n^t} \overbrace{\sum_{t \in \mathcal{T}} \alpha_n^t J_n^t + \eta_n + \frac{1}{(1 - \chi_n)} \sum_{t \in \mathcal{T}} p^t u_n^t}^{\Pi_n} \quad (3.8c)$$

$$s.t. \quad \mathbf{x}_n^t \in \tilde{\mathcal{K}}_n(\mathbf{x}_{-n}^t) \quad \forall n \in \mathcal{N} \quad (3.8d)$$

$$0 \leq J_n^t \leq \bar{J}^t \quad \forall n \in \mathcal{N} \quad (3.8e)$$

where (3.8a) - (3.8b) constitute the *upper level* and (3.8c) - (3.8e), the *lower level* problems. Note that

the risk-adjusted costs of the agents are changing due to contract inclusion :

$$R[\Pi_n^t] = \sum_{t \in \mathcal{T}} \alpha_n^t J_n^t + \eta_n + \frac{1}{(1 - \chi_n)} \sum_{t \in \mathcal{T}} p^t \underbrace{[\Pi_n^t - J_n^t - \eta_n]^+}_{\leq u_n^t}. \quad (3.9)$$

for which we can employ the epigraph form accordingly.

### 3.3.2 Two-level Design of the Risk-Hedging Market with Insurance Company and Inter-Agent Trading

In this design we incorporate risk-hedging that includes both the insurance company and inter-agent financial contracts trading. We assume that the agents can not only acquire insurances  $J_n^t$  but also they can trade risk with each other using financial contracts, i.e., they pay a certain amount contingent on a given scenario occurring. We assume that agent  $n$  can trade risk with the whole community  $\mathcal{N}$ . The price for the contract corresponding to the scenario  $t \in \mathcal{T}$  is denoted  $\gamma^t$ . It is supposed to be homogeneous, e.g., the same price is proposed to all the agents in order to have non-discriminatory pricing on the prosumers' level. Note, that we impose no bound on the sign of  $W_n^t$ .

More precisely, we consider a modified formulation of (3.8) with prosumers' risk-adjusted costs at the lower level written as

$$R[\Pi_n^t] = \sum_{t \in \mathcal{T}} [\alpha_n^t J_n^t + \gamma^t W_n^t] + \eta_n + \frac{1}{(1 - \chi_n)} \sum_{t \in \mathcal{T}} p^t \underbrace{[\Pi_n^t - W_n^t - J_n^t - \eta_n]^+}_{\leq u_n^t}. \quad (3.10)$$

for which we can employ the epigraph form accordingly. It means that now in stage (2) of the timeline of the two-level game, each prosumer  $n \in \mathcal{N}$  decides on the contracts  $W_n^t, J_n^t$  she wants to buy after receiving price  $\alpha_n^t$ , s.t.  $0 \leq J_n^t \leq \bar{J}^t$  and buys the contracts by paying the total price  $\sum_{t \in \mathcal{T}} \alpha_n^t J_n^t + \sum_{t \in \mathcal{T}} \gamma^t W_n^t$  in order to obtain contingent payments  $J_n^t + W_n^t$  in scenario  $t$ .

## 3.4 Stackelberg Game Analysis

Section 3.4.2 starts with an introduction of the differences between OBP and PBP formulations for the two-level problem with  $I$  as the only seller of financial contracts. In section 3.4.2, we first discuss the prices of the financial contracts and existence of solution in PBP. We propose price incentives for the prosumers in section 3.4.2 and compare the resulting equilibria with the solution of OBP, investigated in 3.4.2. Further, in 3.4.2, we discuss an extension of the Stackelberg game, including imperfect information about risk attitudes of the prosumers. In section 3.4.3 we analyze the two-level formulation with both  $I$



and inter-agent trading of financial contracts. The also discuss the market properties of the resulting model.

### 3.4.1 Two-Stage Problem Preliminaries

First, we will need some standard bilevel optimization notations. We denote the insurance company's (at the upper level) variables as  $x_I = (\alpha_n^t, \bar{J}^t, u_{n,I}^t)_n$  and prosumer  $n$ 's (at the lower level) variables as  $x_n^L = (J_n^t, \mathbf{x}_n^t)$ . Let  $\phi(x_I)$  denote the value function of the lower level problem in (3.8) :  $\phi(x_I) := \min_{x_n} \{\Pi_n(x_I, x_n^L) | x_n \in \mathcal{K}_n^*(\mathbf{x}_{-n}^L, x_I)\}$  where  $\mathcal{K}_n^*(\mathbf{x}_{-n}^L, x_I)$  is the feasible set of the lower level problem for prosumer  $n$  and  $x_I$ , is the decision variables of  $I$ . Then, the dependent optimal point set of this problem can be written as  $S_n(x_I, \mathbf{x}_{-n}) := \{x_n \in \mathcal{K}_n^*(\mathbf{x}_{-n}^L, x_I) | \Pi_n(x_I, x_n^L) \leq \phi(x_I)\}$ .  $X_I$  denotes the feasible set of the upper-level optimization problem. Combining these definitions, we can write the bilevel optimization problem as

$$\begin{aligned} \text{'' min ''} & \quad \Pi_I(x_I, x_n^L) \\ & \quad s.t. \quad x_I \in X_I \\ & \quad \quad \quad x_n^L \in S_n(x_I, \mathbf{x}_{-n}^L) \quad \forall n \in \mathcal{N} \end{aligned} \quad (3.11)$$

and separately the individual problem for agent  $n$  on a lower-level problem in a compact form  $\forall n \in \mathcal{N}$  :

$$\begin{aligned} \min_{x_n^L} & \quad \Pi_n(x_I, x_n^L) \\ & \quad s.t. \quad x_n^L \in \mathcal{K}_n^*(\mathbf{x}_{-n}^L, x_I) \end{aligned} \quad (3.12)$$

For the KKT conditions, we use the following notation :  $g_{n,i}^L(\cdot) \leq 0$  ( $h_{n,j}^L(\cdot) = 0$ ) represents the generic inequality (equality) constraints of the lower-level problem for prosumer  $n$ , while  $\xi_n^L$  is the vector of dual variables for prosumer  $n$ 's problem. In the same manner, we use notations  $g_i^I(\cdot)$ ,  $h_j^I(\cdot)$ ,  $\xi_U$  for the upper-level problem. Let  $I^L$  ( $J^L$ ) denote the index set of the market level inequality (equality) constraints and  $I^U$  ( $J^U$ ) denote the index set for the upper level constraints. The optimal dual variable set of prosumer  $n$  for the lower-level problem is denoted as

$$\Xi_n(x_I, \mathbf{x}_{-n}^L, x_n^L) := \left\{ \xi_n^L \geq 0 : \xi_{n,i}^L g_{n,i}^L(x_I, \mathbf{x}_{-n}^L, x_n^L) = 0, \nabla_{x_n^L} \mathcal{L}(x_I, \mathbf{x}_{-n}^L, x_n^L, \xi_n^L) = 0 \right\} \quad (3.13)$$

**Definition 15 (Slater's condition)** We say that Slater's condition holds for prosumer  $n$ 's lower level problem (3.12) for a given  $x_I$ , if there exists  $x_n^L$  such that  $h_i^L(x_i, \mathbf{x}_{-n}^L, x_n^L) = 0$  and  $g_i^L(x_i, \mathbf{x}_{-n}^L, x_n^L) < 0$ .

**Proposition 16** 1. Slater's condition holds for the lower level problem for each  $n \in \mathcal{N}$  and for each  $x_I \in X_I$ .

2. The lower-level problem (3.12) is convex  $\forall n \in \mathcal{N}$  for each  $x_I \in X_I$ .

3.  $\Xi_n(x_I, \mathbf{x}_{-n}^L, x_n^L)$  is upper-semicontinuous.

**Proof.**

1. The first part of the proposition statement simply follows from the structure of the constraints of the lower-level problem : taking  $d_n^t = g_n^t > 0$ ,  $u_n^t > 0$  and  $q_{nm} = 0 \quad \forall n, m$  if  $\kappa_{nm} > 0$  and rewriting constraint (3.2) as an equality if  $\kappa_{nm} = 0$  we can guarantee its qualification.
2. Convexity of the lower level problem (3.12) follows from the fact that the matrices of the constraints are positive semi-definite.
3. Slater's condition implies MFCQ, then we use Theorem 3.1 from [38], Theorem 2.3 from [125]

■

Lastly, we recall the notion of equilibria we use in our analysis.

**Definition 17** A Stackelberg equilibrium of the game defined in (3.8) is a tuple  $(x_I^*, \mathbf{x}^{*,L})$  such that  $\mathbf{x}_n^{*,L} \in S_n(x_I^*, \mathbf{x}_{-n}^{*,L})$  and  $\Pi_I(x_I^*, \mathbf{x}^*) \leq \max_{x_n^L \in S_n(x_I, \mathbf{x}_{-n}^L) \forall n} \Pi_I(x_I, \mathbf{x}^L)$ .

**Efficiency of the Lower Level Equilibria** First market property that we prove is the economic efficiency of the Variational Equilibrium of the lower level problem (3.12), by considering its centralized formulation. A centralized problem is formulated by means of a local Market Operator (MO) who collects all the information of prosumers  $\in \mathcal{N}$  and then solves the problem (3.12) as a single optimization problem, reacting to the actions  $x_I$  of the leader (insurance company  $I$ ). It constitutes a single leader single follower game (SLSF), with the lower level problem written as

$$\begin{aligned} \min_{\mathbf{x}^L} \quad & \sum_{n \in \mathcal{N}} \Pi_n(x_I, x_n^L) \\ \text{s.t.} \quad & x_n^L \in \mathcal{K}_n^*(\mathbf{x}_{-n}^L, x_I) \quad \forall n \in \mathcal{N} \end{aligned} \tag{3.14}$$

Writing the KKT conditions for the problems (3.12) and (3.14), and using the property of the VE, we can establish the following result :

**Proposition 18** The set of Variational Equilibria of the GNEP given by (3.12) for all  $n \in \mathcal{N}$  coincides with the set of social welfare optima solutions of (3.14).

Besides the important property of efficiency, Proposition 18 allows us also to build theoretical analysis of both pessimistic and optimistic formulations of bilevel problem.

### 3.4.2 Optimistic Versus Pessimistic Formulations of the Game

In our analysis of the two-level insurance market, we focus on two formulations of the bilevel optimization problem that are classical in the literature : optimistic and pessimistic. These two formulations are needed to analyse the Stackelberg game, because the lower level problem in (3.8) does not have unique equilibrium. Thus, in order to optimize its problem,  $I$  has to choose an equilibrium among the set of the possible equilibria of the lower-level GNEP. Intuitively, we might see it as a situation in which there are some prosumers at the lower level who are indifferent between several outcomes of the game that result in the same cost, but the values of some decision variables of these prosumers have an impact on the cost of the leader. We illustrate this intuition on a simple example below.

**Example 19** *Assume that there are only two prosumers at the lower level of the Stackelberg game. Let us consider prosumer  $n = 1, 2$ . If at the equilibrium of the game the price for the contracts  $J_n^t$  established by insurance company equals  $\frac{p^n}{1-\chi_n}$  (which is a reasonable assumption as we show later) and  $u_n^t = \Pi_n^t - \eta_n - J_n^t > 0$ , then prosumer  $n$  is indifferent in her choice of insurance :  $J_n^t \in [0, \min\{\bar{J}_n^t, \Pi_n^t - \eta_n\}]$ . But this choice is crucial for the profits of  $I$ , as  $J_n^t = \min\{\bar{J}_n^t, \Pi_n^t - \eta_n\}$  is the best possible outcome for it and the worst one is  $J_n^t = 0$ , constituting two different outcomes of the game for  $I$ , while leading to the same result for prosumer  $n$ .*

Note that we consider only one of two prosumers in the example above. Indeed, prosumers might have different behavior on the risk trading market, depending on the values of their decision variables in the electricity peer-to-peer market equilibrium. We further partition the set  $N$  into groups and emphasize on the ones that are indifferent in the context of risk trading with  $I$ , i.e. the group of the agents for whom the objective function remains of the same value for all  $J_n^t \in [0, \min\{\bar{J}_n^t, \Pi_n^t - \eta_n\}]$ . We first formally define different formulations of the two-level interaction, depending on the response of the prosumers.

When  $I$  and prosumers  $n \in \mathcal{N}$  act in cooperative manner, that is prosumers seek not only to minimize their own costs, but also take into account the maximization of the profits of the insurance company, then  $I$  can choose to solve its own problem with respect to the best possible solution of the GNEP at the lower level (from its point of view). This leads us to the OBP formulation of (3.8) :

$$\begin{aligned}
 \min_{x_I, \mathbf{x}_n^L} \quad & \Pi_I(x_I, x_n^L) \\
 \text{s.t.} \quad & x_I \in X_I \\
 & x_n^L \in S_n(x_I, \mathbf{x}_{-n}^L) \quad \forall n \in \mathcal{N}
 \end{aligned} \tag{3.15}$$

Optimistic problems are widely studied in the literature [38], [136] and are considered to be more tractable as compared to the pessimistic position. The optimistic formulation is guaranteed to have an

optimal solutions under reasonable assumptions of regularity and compactness [136]. Indeed, it is easy to establish existence of solutions of problem (3.15) using Proposition 16 and [38]. On the other hand, the optimistic solution might not exactly correspond to the design of the risk hedging market, as there are no intrinsic incentives for the prosumers at the lower level to act in favor of the insurance company's profit maximization.

Under PBP setting, we assume that the insurance company and the prosumers do not act in the cooperative manner. It might be natural to assume that the insurance company  $I$  considers "worst case" with respect to the equilibrium of the lower-level GNEP. Indeed, if we refer to the Example 19 and consider indifferent prosumers, then it is natural to assume that given the choice of buying the insurance and not buying it with the same outcome, prosumers would choose the latter option. Then, we can rewrite bilevel problem (3.8) as

$$\begin{aligned} \min_{x_I} \max_{\mathbf{x}_n^L} \quad & \Pi_I(x_I, \mathbf{x}_n^L) \\ \text{s.t.} \quad & x_I \in X_I \\ & x_n^L \in S_n(x_I, \mathbf{x}_{-n}^L) \quad \forall n \in \mathcal{N} \end{aligned} \quad (3.16)$$

The scope of literature that investigates pessimistic formulations of bilevel problems is much smaller than that for the optimistic one, due to the fact that the pessimistic formulation is often more complicated than the optimistic one [38]. It is not always guaranteed that the solution of (3.16) exists even for very simple formulations [77], [86], and a lot of work is dedicated to the computation of approximate equilibria [77, 85] or focus on specific cases [12, 32, 85].

**Insurance Company's Information About the Prosumers' Parameters** Alongside with the different formulations of the bilevel optimization problem, we focus on the information structure of the Stackelberg game. For the insurance company  $I$ , it is crucial to have full information about the set of prosumers' electricity trading problem parameters : RES-based generation  $\Delta g_n^t$ , target demand  $\hat{d}_n^t$ , flexibility activation cost function  $C_n(\cdot)$  (more specifically parameters  $a_n, b_n, d_n^t$ ),  $u_n^t(\cdot)$  the usage benefit function (more specifically parameters  $\tilde{a}_n, \tilde{b}_n$ ), and bilateral trade cost function  $\tilde{C}_n(\cdot)$  (more specifically parameters  $(c_{nm})_{m \in \Gamma_n}$ ). Also it is crucial that  $I$  has full information about the risk-attitudes  $(\chi_n)_n$  of the prosumers to properly settle the prices  $(\alpha_n^t)_n$ .

We first discuss theoretical properties of the Stackelberg game assuming full information of  $I$  on the parameters listed above. We prove that the noncooperative game (3.16) has no solution, and propose a method to compute an approximate equilibrium that we compare to the equilibrium obtained as output of the optimistic formulation. Then, we discuss the game outcome in case of incomplete information, i.e. when the insurance company does not have an access to the true values of the listed parameters in

section 3.4.2. We start with a lemma about the insurance prices  $\alpha^t$  in case when there is only insurance company on the risk-hedging market and there is no inter-agent financial contracts trading.

**Lemma 20** *The price  $\alpha_n^t$  of the insurances  $J_n^t$  for agent  $n$  and scenario  $t$  does not exceed  $\frac{p^t}{1-\chi_n}$ .*

**Proof.** The objective function of the prosumer at the lower level takes the closed form

$$\Pi_n = \sum_{t \in \mathcal{T}} \left[ \eta_n + \alpha_n^t J_n^t + \frac{p^t}{1-\chi_n} u_n^t \right]$$

Note that prosumer  $n$  can belong to one of the following two groups, at the equilibrium, defined by the two cases below :

$$\text{case (i) } u_n^t = 0 \text{ if } \Pi_n^t - \eta_n - J_n^t \leq 0 \quad \text{case (ii) } u_n^t = \Pi_n^t - \eta_n - J_n^t > 0 \quad (3.17)$$

In case (i), we write the cost of prosumer  $n$  as  $\Pi_n = \sum_{t \in \mathcal{T}} [\eta_n + \alpha_n^t J_n^t]$  and in case (ii) as  $\Pi_n = \sum_{t \in \mathcal{T}} \left[ \eta_n \left(1 - \frac{p^t}{1-\chi_n}\right) + J_n^t \left(\alpha_n^t - \frac{p^t}{1-\chi_n}\right) + \frac{p^t}{1-\chi_n} \Pi_n^t \right]$ .

We aim to compute insurance company  $I$ 's strategy, e.g., the insurance prices  $(\alpha_n^t)_n$ . First, consider case (ii) : from the term  $J_n^t \left(\alpha_n^t - \frac{p^t}{1-\chi_n}\right)$ , it is clear that to have  $J_n^t \geq 0$ ,  $I$  needs to set  $\alpha_n^t \leq \frac{p^t}{1-\chi_n}$ . In case of strict inequality,  $J_n^t = \min\{\bar{J}^t, \Pi_n^t - \eta_n\}$ , in case of equality agent  $n$  is indifferent, so  $J_n^t \in [0, \min\{\bar{J}^t, \Pi_n^t - \eta_n\}]$ , and 0 otherwise.

For case (i), it is clear that  $J_n^t = 0$  if  $\Pi_n^t - \eta_n \leq 0$ , and  $J_n^t = \Pi_n^t - \eta_n$  otherwise. It means that the total price paid for the contract  $\alpha_n^t J_n^t$  should be smaller than the loss incurred without one :

$$\alpha_n^t J_n^t \leq \frac{p^t}{1-\chi_n} (\Pi_n^t - \eta_n) \Rightarrow \alpha_n^t \leq \frac{p^t}{1-\chi_n}$$

Thus, considering the response of the prosumers to the price settled by the insurance company, we obtain that in both cases  $\alpha_n^t \leq \frac{p^t}{1-\chi_n}$ . ■

**Choice function** In the text below we use the partition of the agents in groups. We denote agents with the value  $u_n^t = \Pi_n^t - \eta_n - J_n^t > 0$  at the GNE as  $\mathcal{N}' \subseteq \mathcal{N}$ . This group is later referred to as *indifferent prosumers* (see Example 19). Agents with the value  $\Pi_n^t - \eta_n - J_n^t \leq 0$  and  $\Pi_n^t - \eta_n \leq 0$  are denoted as  $\mathcal{N}'' \subseteq \mathcal{N}$  and with  $\Pi_n^t - \eta_n > 0$  as  $\mathcal{N}''' \subseteq \mathcal{N}$ .

Consider the group  $\mathcal{N}'$ . For this group of agents, we can describe *best-response* mapping of agent  $n$

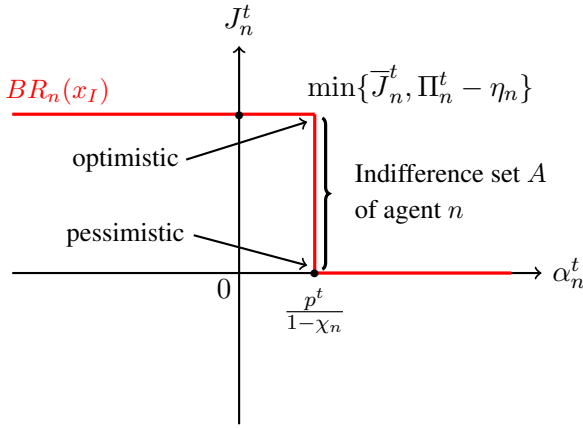
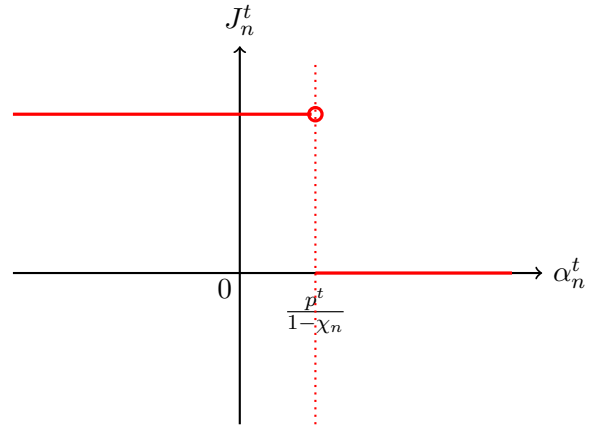
FIGURE 3.1 –  $BR_n(x_I)$  for  $n \in \mathcal{N}'$ .

FIGURE 3.2 – Best-response function in PBP.

to the insurance company w.r.t the decision  $x_I = (\alpha_n^t, \bar{J}^t)$ :

$$J_n^t = BR_n(x_I) = \begin{cases} 0 & \text{if } \alpha_n^t > \frac{p_n^t}{q - \chi_n} \\ \min\{\bar{J}_n^t, \Pi_n^t - \eta_n\} & \text{if } \alpha_n^t < \frac{p_n^t}{q - \chi_n} \\ Ch_n^t(A, \omega) & \text{if } \alpha_n^t = \frac{p_n^t}{q - \chi_n} \end{cases}$$

where  $A := [0, \min\{\bar{J}_n^t, \Pi_n^t - \eta_n\}]$  and  $Ch_n^t(A, \omega)$  is a choice function  $Ch_n^t(A, \omega) : \{A\} \times \Omega \rightarrow A$ . We refer to Figure 3.1 as an illustration of  $BR_n(x_I)$ . More precisely, for each agent  $n \in \mathcal{N}'$ , for each scenario  $t$  this function takes as input the interval  $A$  and parameter  $\omega \in \Omega$  and returns a single value  $J_n^t$ , corresponding to the insurance bought by agent  $n$ :  $Ch_n^t(A, \omega) := \omega[\Pi_n^t - \eta_n]$ . Parameter  $\omega \in \Omega := [0, 1]$  controls the optimality of the choice of the prosumer for the insurance company  $I$ . Using this function, we write  $Ch_n^t(A, \omega)$  instead of  $J_n^t$  in upper-level optimization problem (3.8a). We denote this formulation of (3.8) as  $\mathcal{G}_{ch}(\omega)$ .

### Pessimistic Formulation Analysis

**Connection between pessimistic formulation and  $\mathcal{G}_{ch}(\omega)$**  In the next lemmas, we show the link between pessimistic formulation (3.16) and  $\mathcal{G}_{ch}(\omega)$ . First, we follow the path established in [77] and reformulate problem (3.16) as a one leader, multi-follower game, where the lower level is modeled as a GNEP with  $2N$  players. Intuitively, at the lower level, each prosumer cares not only about minimization of her own cost function, but also about maximization of the cost of insurance company  $I$ . To formalize the

setting, we introduce an auxiliary agent who takes care of this maximization task :

$$\begin{aligned} \min_{x_I, (\mathbf{x}_n^L, \mathbf{z}_n^L)_n} \quad & \Pi_I(x_I, \mathbf{x}_n^L) \\ \text{s.t.} \quad & x_I \in X_I \\ & (x_n^L, z_n^L) \in E(x_I) \quad \forall n \in \mathcal{N} \end{aligned} \quad (3.18)$$

where  $E(x_I)$  is the equilibrium set of the following GNEP :

$$\begin{aligned} \min_{x_n^L} \quad & -\Pi_I(x_I, \mathbf{x}_n^L) & \min_{z_n^L} \quad & \Pi_n(x_I, \mathbf{x}_n^L) \\ \text{s.t.} \quad & x_n^L \in \mathcal{K}_n^*(\mathbf{x}_{-n}^L, x_I) & \text{s.t.} \quad & z_n^L \in \mathcal{K}_n^*(\mathbf{x}_{-n}^L, x_I) \\ & \Pi_n(x_I, \mathbf{x}_n^L) \leq \Pi_n(x_I, \mathbf{z}_n^L) \end{aligned} \quad (3.19)$$

**Lemma 21** *If solution  $(\hat{x}_I, \hat{\mathbf{x}}_n^L)$  of (3.16) exists and is a local optimal point of this problem, then for any  $\hat{z}_n^L \in S_n(\hat{x}_I, \hat{\mathbf{x}}_{-n}^L)$ , the tuple  $(\hat{x}_I, \hat{\mathbf{x}}_n^L, \hat{z}_n^L)$  is a local optimal point of (3.18).*

**Proof.** Denote the optimal value function  $\psi(x_I) := \max_{x_n^L \in S_n(x_I, \mathbf{x}_{-n}^L) \forall n \in \mathcal{N}} \Pi_I(x_I, \mathbf{x}_n^L)$ . Suppose by contradiction that  $(\hat{x}_I, \hat{\mathbf{x}}_n^L, \hat{z}_n^L)$  is not a local optimal point for (3.18), i.e., there exists a sequence  $(x_I^k, \mathbf{x}_n^{L^k}, z_n^{L^k})$  with  $x_I^k \in X_I$  and  $(x_n^{L^k}, z_n^{L^k}) \in E(x_I^k)$  for all  $n \in \mathcal{N}$  such that  $(x_I^k, \mathbf{x}_n^{L^k}, z_n^{L^k}) \rightarrow (\hat{x}_I, \hat{\mathbf{x}}_n^L, \hat{z}_n^L)$  and  $\psi(x_I^k) = \Pi_I(x_I^k, \mathbf{x}_n^{L^k}) < \Pi_I(\hat{x}_I, \hat{\mathbf{x}}_n^L) = \psi(\hat{x}_I)$ . This contradicts the optimality of  $(\hat{x}_I, \hat{\mathbf{x}}_n^L)$ . ■

**Lemma 22** *Stackelberg equilibria of (3.18) with VE at the lower level belong to the set of equilibria  $E_{\mathcal{G}_{ch}}$  of  $\mathcal{G}_{ch}(0)$ .*

**Proof.** First, note that from Proposition 18, VE at the right part of (3.19) is efficient. From which it follows that at the left part of (3.19), instead of the last inequality we have an equality. Fix some  $\alpha_n^t$  as a solution of the upper-level problem. Then, we can rewrite (3.19) as

$$\begin{aligned} \min_{x_n^L} \quad & \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} [\alpha_n^t - p^t] J_n^t \\ \text{s.t.} \quad & x_n^L \in VE \end{aligned} \quad (3.20)$$

where  $\alpha_n^t - p^t > 0$ . Thus, each agent chooses minimal possible  $J_n^t$  while satisfying the KKT conditions of (3.19). For prosumers  $n \in \mathcal{N}'' \cup \mathcal{N}'''$ , the choice of best response  $J_n^t$  is fixed, so it follows that we should consider prosumers  $n \in \mathcal{N}'$ . The response of this group is fixed unless  $\alpha_n^t = \frac{p^t}{1-\chi_n}$ , for which, in order to minimize the profits of the insurance company, each agent in  $\mathcal{N}'$  chooses  $J_n^t = 0$ . ■

**No solution** Finally, we return to our initial statement that might be seen as the situation where the absence of the additional price incentives from the insurance company  $I$  for the group of indifferent agents  $n \in \mathcal{N}'$  leads to a non-existence of solution of (3.16).

**Proposition 23** *In a pessimistic framework, the problem (3.16) admits no solution.*

**Proof.** Suppose by contradiction that the solution of problem (3.16) exists. We first state that at optimum of  $\mathcal{G}_{ch}(0)$ , problem  $\alpha_n^t$  can not be strictly less than  $\frac{p^t}{1-\chi_n}$ . Suppose, by contradiction, that at the optimum,  $\alpha_n^t < \frac{p^t}{1-\chi_n}$ . Then, the insurance company  $I$  can always increase its profit by adding sufficiently small  $\varepsilon : \alpha_n^t + \varepsilon < \frac{p^t}{1-\chi_n}$ , and taking the limit  $\varepsilon \rightarrow 0$ , we obtain a contradiction wrt the optimality of  $\alpha_n^t$ . Then, from Lemma 20, it follows that  $\alpha_n^t = \frac{p^t}{1-\chi_n}$ . In addition, from the definition of problem  $\mathcal{G}_{ch}(0)$ , it follows that the values of contracts  $J_n^t$  acquired by agents  $n \in \mathcal{N}'$  are equal to 0. Thus, decreasing  $\alpha_n^t$  by small  $\varepsilon > 0$  (see Figure 3.2), insurance company can increase its profits. Thus, we obtain a contradiction which concludes the proof. ■

**Price incentives** Although the non-existence of a solution of the pessimistic bilevel problem is not a rare case [86], it is not desirable from the market point of view. Several works deal with the question of overcoming this issue by computing  $\varepsilon$ -optimal solution [77, 85] of the lower level problem. From Proposition 23, it is natural to consider an approximate solution of the upper level as a way to incentivize the indifferent prosumers  $n \in \mathcal{N}'$  to act in favor of buying insurances thus, increasing the profit of the insurance company. Indeed, consider again Example 19 and assume that the insurance company chooses to set  $\alpha_n^t = \frac{p^t}{1-\chi_n} - \varepsilon$  for some now fixed parameter  $\varepsilon > 0$ . Then, the amount bought by the prosumer becomes  $\min\{\bar{J}_n^t, \Pi_n^t - \eta_n\}$  as now it is profitable for her to acquire the insurance. In that case the profit of insurance company  $I$  will become  $[\frac{p^t}{1-\chi_n} - \varepsilon - p^t] \min\{\bar{J}_n^t, \Pi_n^t - \eta_n\} > 0$ . We formalize this in the following proposition :

**Proposition 24** *For any given  $\varepsilon$ , if insurance company sets the prices  $\alpha_n^t = \frac{p^t}{1-\chi_n} - \varepsilon$  for prosumers  $n \in \mathcal{N}'$ , then the problem (3.16) has a solution.*

**Proof.** The proof follows directly from the reformulation of PBP as a Stackelberg game with  $2N$  agents at the lower level and the proof of Lemma 20. ■

Moreover, these considerations allow us to evaluate how much the insurance company fails to receive when the agents are reluctant to cooperate by comparison with the optimistic solution. More precisely, we show that the value of the objective function  $\Pi_I^P$  in this formulation is at most  $\varepsilon \sum_{n \in \mathcal{N}'} \min\{\bar{J}_n^t, \Pi_n^t - \eta_n\}$  less than the value of  $\Pi_I^O$  at the equilibrium of the optimistic problem (3.15).



**Optimistic Formulation Analysis** Optimistic formulation can be described by means of the choice function if we set parameter  $\omega = 1$ . That means that those agents who are indifferent in their choice of  $J_n^t$  choose the best possible option for  $I : Ch_n^t(A, 1) = \Pi_n^t - \eta_n$ . First, note that if the type of the two-level game is optimistic, then we can set the price  $\alpha_n^t = \frac{p_n^t}{1-\chi_n}$  for all agents  $n \in \mathcal{N}$ . Indeed, we follow the proof of Lemma 20 and extend it by considering slightly lower prices. But in the optimistic framework there is no need to provide incentives to the indifferent prosumers, so the insurance company can always increase price for them up to  $\alpha_n^t = \frac{p_n^t}{1-\chi_n}$ . In the next proposition we show the connection between (3.15) and  $\mathcal{G}_{ch}(\omega)$  :

**Proposition 25** *Stackelberg equilibria of (3.15) with VE at the lower level coincides with the set of equilibria  $E_{\mathcal{G}_{ch}}$  of  $\mathcal{G}_{ch}(1)$ .*

**Proof.** (i) Assume by contradiction that a solution  $(x_I^{\mathcal{G}}, \mathbf{x}^{L^{\mathcal{G}}})$  of  $\mathcal{G}_{ch}(1)$  is not a solution of (3.15). It means that there exists a solution  $(\hat{x}_I, \hat{\mathbf{x}}^L)$  of (3.15) such that  $\Pi_I(\hat{x}_I, \hat{\mathbf{x}}^L) \leq \Pi_I(x_I^{\mathcal{G}}, \mathbf{x}^{L^{\mathcal{G}}})$ . Equivalently :

$$\sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} \hat{J}_n^t [p^t - \hat{\alpha}_n^t] \leq \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} [\Pi_n^t - \eta_n]^+ [p^t - \alpha_n^t]$$

We showed that  $\hat{\alpha}_n^t = \alpha_n^t = \frac{p_n^t}{1-\chi_n}$ . Then, dividing by the term  $p^t - \frac{p_n^t}{1-\chi_n}$ , we obtain

$$\sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} \hat{J}_n^t \geq \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} [\Pi_n^t - \eta_n]^+$$

The set  $\mathcal{N}$  can be split into three groups :  $\mathcal{N}'$ ,  $\mathcal{N}''$ ,  $\mathcal{N}'''$ , that are defined as follows : (1)  $\hat{J} = \hat{\Pi}_n^t - \hat{\eta}_n > 0$ , (2)  $\hat{J} = 0$  and (3)  $\hat{J} \in [0, \hat{\Pi}_n^t - \hat{\eta}_n]$ ,  $\hat{\Pi}_n^t - \hat{\eta}_n > 0$ , thus

$$\sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} [\hat{\Pi}_n^t - \hat{\eta}_n]^+ \geq \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} [\Pi_n^t - \eta_n]^+$$

which contradicts Proposition 18.

(ii) We use the fact that  $\alpha_n^t = \frac{p_n^t}{1-\chi_n}$  and write the closed form of the objective function in (3.15) :

$$\min_{x_I, \mathbf{x}_n^L} \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} J_n^t \left[ p^t - \frac{p_n^t}{1-\chi_n} \right]$$

from which it follows that each agent  $n$  maximizes  $J_n^t$ , while satisfying the KKT conditions. From Proposition 18, it follows that  $J_n^t = Ch_n^t(1)$  for all agents  $n \in \mathcal{N}'$  which gives us exactly a solution of  $\mathcal{G}_{ch}(1)$ . ■

In view of the above results, we can directly establish the following proposition :

**Proposition 26** *The value of the objective function  $\Pi_I^P$  in PBP is at most  $\varepsilon \sum_{n \in \mathcal{N}'} \min\{\bar{J}_n^t, \Pi_n^t - \eta_n\}$  less than the value of  $\Pi_I^O$  at the equilibrium of OBP (3.15) :*

$$\Pi_I^O - \Pi_I^P = \varepsilon \sum_{n \in \mathcal{N}'} \min\{\bar{J}_n^t, \Pi_n^t - \eta_n\} \quad (3.21)$$

**Incomplete Information About the Risk Attitudes** Up to this section, we assumed that the insurance company  $I$  can correctly assess the risk attitudes  $(\chi_n)_n$  of the prosumers and compute the prices accordingly, alongside with the parameters  $(a_n, b_n, d_n^t, \tilde{a}_n, \tilde{b}_n, \Delta g_n^t, \hat{D}_n, c_{nm})$  of the electricity trading problem. Nevertheless, in practice the insurance company does not have an access to the agents' perception of the risk, thus, the only information insurance company  $I$  has access to is some a priori belief about  $(\chi_n)_n$ , expressed by means of some distribution  $(\mathbb{X}_n)_n$ . We also assume that the insurance company has an access to good estimations of the electricity trading problem parameters in the sense that the difference in the resulting assessments and the true values bring negligible difference to our model. We leave the discussion about the ways to achieve this out of the scope of the chapter. It follows that the insurance company solves the problem (3.8) by taking the expectation of (3.8c), where the expectation is taken with respect to some distribution  $\chi_n \sim \mathbb{X}_n, \forall n$ . Following the same path as in the proof of Lemma 20, we establish the following result :

**Proposition 27** *When the only information the insurance company has access to is a distribution  $\chi_n \sim \mathbb{X}_n$ , then the price for the contract for agent  $n$  is given by  $\alpha_n^t = \mathbb{E}\left[\frac{p^t}{1-\chi_n}\right]$ .*

It is straightforward to determine which agent acquires the contracts, depending on the relation between  $\alpha_n^t = \mathbb{E}\left[\frac{p^t}{1-\chi_n}\right]$  and  $\alpha_n^t = \frac{p^t}{1-\chi_n}$  and the partition into groups  $\mathcal{N}', \mathcal{N}'', \mathcal{N}'''$ . The only interesting situation appears when  $\alpha_n^t = \mathbb{E}\left[\frac{p^t}{1-\chi_n}\right] = \frac{p^t}{1-\chi_n}$  for some agent  $n \in \mathcal{N}'$  (e.g., when  $\mathbb{X}_n$  is discrete). Then, we again have to consider optimistic and pessimistic formulations and use the machinery established in section 3.4.2.

### 3.4.3 Analysis of the Two-Level Formulation with Inter-Agent Trading

The most comprehensive formulation proposed in section 3.3.2 poses a lot of questions regarding the market organization. First, we provide an illustration, why the prices  $\alpha_n^t$  settled as in the previous sections may generate market imperfections in the framework with inter-agent trading.

**Example 28** *Consider decentralized electricity market with two agents at the lower level with risk attitudes  $\chi_1 < \chi_2$ . Then  $\alpha_1^t < \alpha_2^t$  and if the price  $\gamma^t$  is less than  $\frac{p^t}{1-\chi_2}$ , then agent 1 can buy the insurances*

from 1 and resell them to agent 2. This behavior clearly reflects a market imperfection that must be addressed.

We again consider two formulations, optimistic (OBP) and pessimistic (PBP) and analyse the market prices and its properties at equilibrium. We use the results established in the previous section to address the problems encountered in the full formulation of the decentralized electricity market with risk hedging.

We consider OBP first. We first show that the price of the contracts at the lower level is settled as  $\gamma^t = \frac{p^t}{1-\chi_n}$ .

**Proposition 29** *For a inter-agent financial contracts trading, the risk-adjusted probabilities are aligned across market participants. Furthermore, the risk-adjusted probabilities coincide with those of the least risk averse agent and are equal to the prices of financial contracts, i.e.,  $\gamma^t = \frac{p^t}{1-\min_n \chi_n}$*

**Proof.**

From the KKT conditions, we get that  $\frac{p^t}{1-\chi_n} = \tau_n^t + \pi_n^t$ . From the complementarity constraints, we see that the set of risk adjusted probabilities in the modified problem with risk-hedging contracts  $W_n^t$  implies zero probability on the scenarios with  $\Pi_n^t - W_n^t \leq \eta_n$  and  $\tau_n^t = \frac{p^t}{1-\chi_n}$  for scenarios with  $\Pi_n^t - W_n^t \geq \eta_n$ .

From the KKT conditions, we infer that the  $\tau_n^t$  are aligned across agents :  $\tau_n^t = \tau_m^t = \tau^t \forall n, m \in \mathcal{N}$ . To show that they coincide with those of the least risk averse agent, assume that the price  $\gamma^t$  if fixed. Then, for those agent  $n' \in \mathcal{N}$  for whom  $\gamma^t \geq \frac{p^t}{1-\chi_{n'}}$ , it is profitable to sell the contracts ( $n' \in S$ ) :  $W_{n'}^t \leq 0$ . The opposite holds for agent  $n''$  with  $\gamma^t \leq \frac{p^t}{1-\chi_{n''}}$ , thus making her the buyer of the financial contracts ( $n'' \in B$ ).

Consider  $n'' \in B \subseteq \mathcal{N}$ . If  $u_{n''}^t \geq 0$ , then  $u_{n''}^t = \Pi_{n''}^t - \eta_{n''} - W_{n''}^t$ , and the term representing financial contract trading becomes  $W_{n''}^t(\gamma^t - \frac{p^t}{1-\chi_{n''}}) \leq 0$ , which implies that increasing  $W_{n''}^t$  up to  $\Pi_{n''}^t - \eta_{n''}$  leads to the decrease in the cost of the agent. In other words, taking the sub-dervative  $\partial R_{n''}[\Pi_{n''}^t]$  w.r.t.  $W_{n''}^t$ , it is straightforward to establish that  $0 \in \partial_{W_{n''}^t} R_{n''}[\Pi_{n''}^t](\Pi_{n''}^t - \eta_{n''})$ .

On the other hand, for  $n' \in S \subseteq \mathcal{N}$  we can similarly establish that  $0 \notin \partial_{W_{n'}^t} R_{n'}[\Pi_{n'}^t](x)$  for  $x \in \mathbb{R}^-$ , which means that it is always profitable to sell any amount of financial contracts for agents in  $S \subseteq \mathcal{N}$ . Hence, agent  $n^*$  with the lowest risk aversion can sell contracts  $W_{n^*}^t$  at the lowest prices  $\gamma^t$  equal (by taking infinitesimal  $\varepsilon$ , and setting  $\varepsilon \rightarrow 0$ ) to  $\frac{p^t}{1-\chi_{n^*}} = \frac{p^t}{1-\min_n \chi_n}$ . ■

It follows that the agent with the minimal risk aversion can supply risk-hedging demand for the agents at the lower level. In such setting, optimistic formulation again is expressed through the choice function

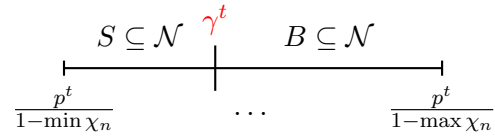


FIGURE 3.3 –  $\gamma^t$  on the risk-aversion scale

$Ch_n^t(A, 1)$  and by using choice function reformulation, we establish that the prices for the insurances provided by  $I$  are equal to  $\alpha_n^t = \frac{p^t}{1 - \min \chi_n}$ ,  $\forall n \in \mathcal{N}$ .

In PBP, the prosumers prefer to trade contracts directly with their peers than with  $I$ . More precisely, we follow the proofs of Lemmas 21 and 22 to derive Proposition 23. It leads to the question of how to design price incentives described in section 3.4.2, and similarly to Proposition 24. We show that the prices of the insurances with price incentives are equal to  $\alpha_n^t = \frac{p^t}{1 - \min \chi_n} - \varepsilon$ ,  $\forall n \in \mathcal{N}$ .

**Remark 30** *In the two-level model the insurance prices do not depend on their own risk aversion, but solely on the risk aversion of the least risk averse agent. Therefore, it is more appropriate to speak about equity than about any other kind of fairness, as the prices are now aligned across the agents.*

**Fairness** Investigating the impact of risk preferences on cost allocation in decentralized electricity markets becomes fundamental in order to design mechanisms that grant fairness among prosumers. We provide a definition for fairness of cost allocation in risk-adjusted market with financial contracts. Intuitively, in our framework involving the presence of an insurance company, fairness should relate the insurances' price  $(\alpha_n^t)_n$  to the prosumers' risk aversion levels. More precisely :

**Definition 31** *We say that the risk-hedging market with an insurance company is fair if the insurances' price  $\alpha_n^t$  is lower for less risk-averse agents, that is  $\alpha_{n'}^t \leq \alpha_{n''}^t$  if  $\chi_{n'} \leq \chi_{n''}$ .*

From the Propositions 24 and 25, it is straightforward that the risk-hedging market is fair :

**Proposition 32** *The risk-hedging market described by the two-level game (3.11) is fair in the sense of Definition 31.*

## 3.5 Numerical Results

We compare the performance of the various electricity and financial contracts trading market designs proposed in this article, and analyze the impact of heterogeneous risk aversion on the prosumers' and  $I$ 's costs as well as social cost, by solving the noncooperative games from section 3.3.

**Data** We use residential data provided by Pecan Street [69] for Austin, Texas. The data consists of 15-minutes intervals specifying renewable generation, load and facilities energy consumption for 25 individual homes. We sample the distribution of scenarios for RES-based generation and demand from the generation data and aggregated consumption respectively. Histograms representing 100 scenarios of the RES-based generation and demand of three agents are given in Figures 3.4 and 3.5. To run the

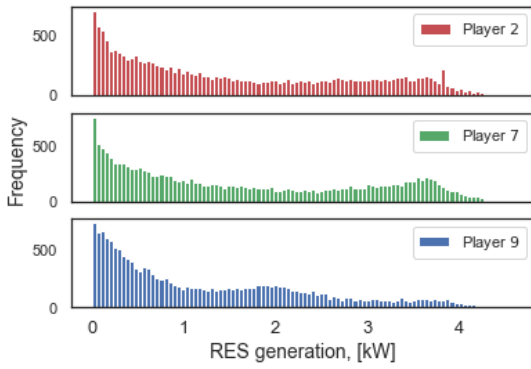


FIGURE 3.4 – Histograms of RES-generation.

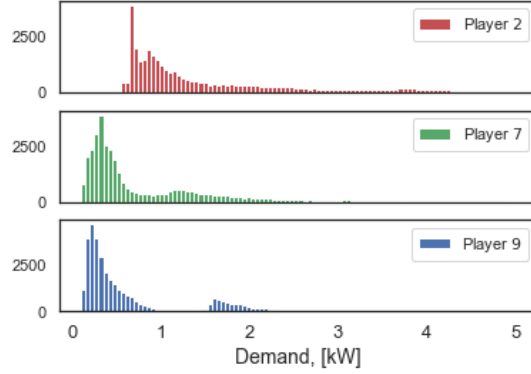


FIGURE 3.5 – Histograms of demand.

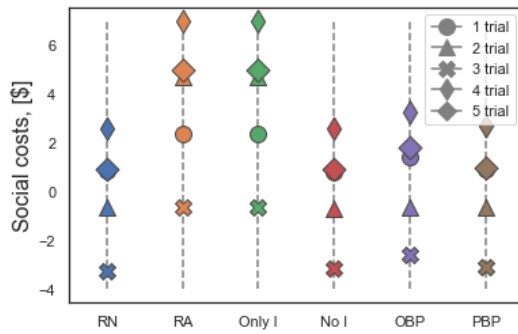
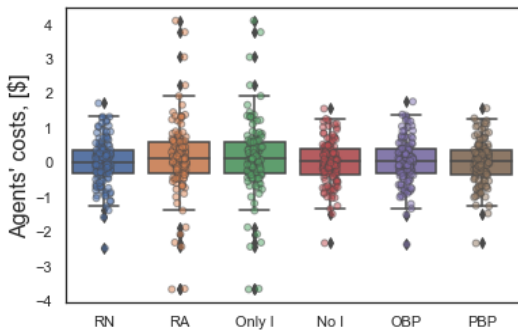


FIGURE 3.7 – Spread of the prosumers’ costs and social cost in each market design for 5 trials. (RN) - risk-neutral, (RA) - risk-averse, (Only I) - two level with only  $I$ , (No I) - one level with risk-hedging, (OBP) - two-level optimistic, (PBP) - two-level pessimistic.

experiments, we use the same probabilities to generate the scenarios for all the agents. Extensions of the model that account for different distributions across the agents is described in the Appendix. We solve the problem using a radial connection graph depicted in Figure 3.6. Here, edges represent the neighborhood of each prosumer. Edges were generated randomly, such that the radial structure of the graph is preserved.

**Comparison of different market designs** In order to compare the market design outcomes, we run a set of experiments (trials) with different values of the parameters  $(\tilde{a}_n, \tilde{b}_n, a_n, b_n, \hat{a}_n^t, \chi_n)_n$  sampled from a uniform distribution  $U[0, 1]$ . Then, for each set of parameters and 100 sampled scenarios for RES-based generation and target demand we solve the corresponding model and compute the prosumers’ costs, the social cost and the insurance company  $I$ ’s profit where applicable. Prosumers’ costs and social costs for 5 trials in the different market designs are shown in Figure 3.7; where a different mark is used for each trial. The costs are the lowest in the risk-neutral framework (RN).

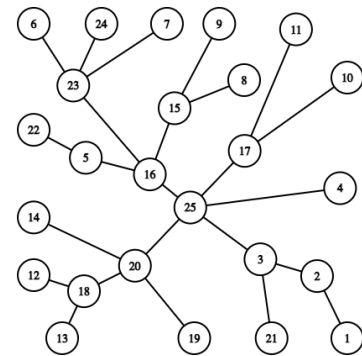


FIGURE 3.6 – Connection graph.

They increase in the risk-averse framework (RA), as one could expect, due to increased risk-aversion of the agents, which motivates them to make more conservative decisions such that the volatility of their overall costs reduces [144].

The participation of an insurance company in the two-level (optimistic) framework (*Only I*) allows the agents to hedge their risk towards uncertainties, but numerical tests show that the prosumers adjust their electricity generations, demands and tradings to belong to the set of indifferent prosumers.

	RN	RA	Only I	No I	OBP	PBP
SC [\$]	0.101	3.686	3.686	0.186	0.192	0.162
I's cost [\$]	-	-	-1.41	-	-0.437	-0.018
Fairness		-	+	-	-	-
Equity		-	-	+	+	+

This means that by buying insurances they do not decrease their costs, but due to the optimistic formulation, the purchases of the insurance company enable it to increase its profits. Pessimistic formulation in the *Only I* setting slightly decreases both the profits of *I* and the costs of the prosumers, but the difference is

TABLE 3.1 – Prosumers' social cost, *I*'s cost, market properties for different market designs computed on a 15 min interval.

minor. The introduction of financial contracts in the one-level setting (*No I*) [96] sets more profitable financial contracts conditions for the prosumers, because now the contract prices are uniform and are set to be  $\frac{p^t}{1-\min \chi_n}$ .

It allows agents to decrease their costs significantly by comparison with the risk-averse (RA) and two-level with the sole *I* (*Only I*) settings. The possibility for *I* to propose insurances in the optimistic (OBP) formulation, allows prosumers to reduce their costs by comparison with the risk-averse setting – in the same manner as the one-level formulation – while bringing more profits to *I*. The pessimistic formulation entails slightly lower costs. That is an important consequence of the *price incentives* that *I* uses in PBP market : the relative reluctance of the agents to buy insurances from *I* motivates the company to slightly decrease its contract prices, making them cheaper to the agents. On the other hand, it leads to a decrease in *I*'s profits.

From the experiments below, one can conclude that the most profitable framework for insurance company *I* is the *Only I*, in which it is the only provider of risk-hedging contracts. On the other hand, allowing competition between inter-agent contracts and *I*'s acting as a contract provider allows prosumers to decrease their costs significantly, at the expense of the insurance company.

**Impact of incomplete information** In sections 3.4.2 and 3.4.2, we discussed the effects of incomplete information of *I* about the prosumers' parameters. We focus here more specifically on *I*'s incomplete

information about  $(\chi_n)_n$  in PBP, as introduced in section 3.4.2. We test several distributions  $\mathbb{X}_n$  available to  $I$  as the beliefs about  $\chi_n$  and compare  $I$ 's resulting profits induced by these distributions, in Table 3.2. The *True value* is a vector of risk attitudes for the 25 prosumers sampled from the *beta distribution*  $\beta(1, 1)$ . Distributions of  $\mathbb{X}_n$  are taken from the same family of beta distributions with different parameter values for the distribution. The best outcome is obtained when the parameters of the distribution are guessed correctly by  $I$ .

	True value	$\beta(1, 1)$	$\beta(0.5, 1)$	$\beta(1, 0.5)$	$\beta(2, 2)$
$I$ 's cost [\$]	-0.0316	-0.0082	-0.0015	-0.0062	-0.0036

TABLE 3.2 –  $I$ 's cost differences under different distributions of the prosumers' risk attitudes in PBP.

However, Table 3.2 shows that the insurance company's profit even in the best outcome is still 3.85 times lower than with complete information. The same holds for other two-level formulations. Clearly, this highlights the fact that the insurance company has incentives to learn the distribution of the risk-aversion levels of the prosumers. Further research should be done in order to understand how to choose suitable distribution to model the prosumers' risk aversion, and design learning mechanism for  $I$  that also enables it to maximize its profit. A more detailed analysis of the agents' parameters impact on the results of the model, is presented in the Appendix.

### 3.6 Concluding Remarks

In this work, we investigated two-level risk-hedging market designs of a decentralized electricity market, and provided a comprehensive analysis of the underlying equilibrium problems. An insurance company is included in the two-level market first as the only insurance supplier and then as a competitor with the inter-agent financial contract trading. We showed that the structure of the two-level design might lead to the nonexistence of a solution, but that problem can be overcome by designing price-based incentives which aim to incentivize the prosumers to buy insurances instead of trading contract with their peers. To that purpose, we reformulated the resulting Stackelberg game as a parametrized GNEP. The price incentives only slightly decrease the profits of the insurance company, but also allow prosumers to decrease their costs, as we illustrate in the numerical experiments.

The discussion around incomplete information on the prosumers' risk-aversion levels poses several questions for future research, e.g., how can the insurance company optimize the electricity trading parameters while learning the risk-aversion levels of the agents? One way to achieve that could be to build a dynamic incentive-compatible mechanism such that the agents report their private information to

the insurance company. Another important extension of the proposed market design would be to allow competition among several insurance companies at the upper level of the Stackelberg game, leading to a *multi-leader, multi-follower* framework. Finally, another interesting branch of future research would be developing more efficient distributed algorithms to compute market equilibria.

## 3.7 Appendix

### 3.7.1 Risk-Neutral Framework

In a *risk-neutral* framework, prosumers optimize their costs with respect to the probabilities  $p^t$ , without taking differences in risk perception of the agents into account. This corresponds to the classical economic dispatch model, in which prosumers account for the uncertainty of their generation and inflexible demand when optimizing their strategy, with a common view on the collective uncertainty.

**Centralized case** The first formulation that is considered in this chapter, and will be used as a benchmark, is formulated in a centralized manner, where a global Market Operator minimizes the social cost for the risk-neutral community. We can write the formulation as follows :

$$\begin{aligned} \min_{\mathbf{D}^t, \mathbf{G}^t, \mathbf{q}^t} \quad & \mathbb{E}[SC] \\ \text{s.t.} \quad & \mathbf{x} := (\mathbf{D}^t, \mathbf{G}^t, \mathbf{q}^t) \in \mathcal{K}. \end{aligned}$$

The Social Cost function  $SC(\cdot)$  is convex as the sum of convex functions defined on a convex feasibility set. Indeed, the feasibility set is obtained as Cartesian product of convex sets. Thus, the optimization problem can be solved using standard convex optimization algorithms.

**Decentralized case** We propose different decentralized market designs, in which each prosumer  $n \in \mathcal{N}$  selfishly optimizes her demand ( $d_n^t$ ), energy generation ( $g_n^t$ ) and bilateral trades ( $\mathbf{q}_n^t$ ) with other prosumers in her neighborhood under constraints on demand, generation and trading capacity so as to minimize her expected costs. Formally, each prosumer in node  $n \in \mathcal{N}$  solves the following optimization problem :

$$\min_{d_n^t, g_n^t, \mathbf{q}_n^t} \quad \mathbb{E}[\Pi_n^t], \quad (3.22a)$$

$$\text{s.t.} \quad x_n := (d_n^t, g_n^t, \mathbf{q}_n^t) \in \mathcal{K}_n(\mathbf{x}_{-n}), \quad (3.22b)$$



where expectation is given by

$$\mathbb{E}[\Pi_n^t] = \sum_t p^t \underbrace{\left[ C_n(g_n^t) + \tilde{C}_n(\mathbf{q}_n^t) - U_n^t(d_n^t) \right]}_{\Pi_n^t} \quad (3.23)$$

This formulation can be viewed as a decomposition of the centralized problem which accounts for the strategic behavior of all the prosumers. We first show the efficiency of the equilibria of the game (3.22) :

**Proposition 33** *The KKT conditions of the centralized market design coincide with the KKT conditions at any variational equilibrium (VE) of the decentralized market design. It follows that the set of VEs obtained as outcome of the decentralized market design contains economically efficient outcomes.*

**Proof.** The proof follows from the KKT conditions and the definition of VE that impose that  $\zeta_{nm}^t = \zeta_{mn}^t, \forall m \in \mathcal{N}, \forall m \in \Gamma_n$ . ■

**Lemma 34** *At equilibrium,  $\Pi_n^t$  is uniquely defined,  $\forall n \in \mathcal{N}$ . Moreover, if the values  $p^t(c_{nm} - c_{mn})_{nm}$  are not equal for any couple  $(n, m) \in \mathcal{N} \times \Gamma_n$  and corresponding scenarios, then prosumer  $n$ 's strategy  $x_n$  at VE is unique.*

**Proof.** We start the proof by decomposing the problem into quadratic and linear parts. First, let  $Q_n = \sum_{m \in \Gamma_n} q_{nm}$  be prosumer  $n$ 's net import, and note that  $Q_n^t \in \mathcal{Q}_n$  where due to (3.2)  $\mathcal{Q}_n$  is closed and bounded set. We consider the following problem  $\forall (Q_n^t)_n$  :

$$\begin{aligned} \min_{\mathbf{D}^t, \mathbf{G}^t} \quad & \sum_{n \in \mathcal{N}} \sum_t p^t \left[ C_n(g_n^t) - u_n^t(d_n^t) \right] \\ \text{s.t.} \quad & (d_n^t, g_n^t) \in \mathcal{D}_n \times \mathcal{G}_n, \quad \forall n \in \mathcal{N} \\ & d_n^t = g_n^t + \Delta g_n^t + Q_n^t. \end{aligned} \quad (3.24)$$

Problem (3.24) has unique solution  $(\mathbf{D}^t, \mathbf{G}^t)$  for each  $Q_n^t \in \mathcal{Q}_n$  because it is strictly convex in  $\mathbf{D}^t, \mathbf{G}^t$ . To prove the statement of the lemma, we have to consider the linear subproblem, which is formulated as follows :

$$\begin{aligned} \min_{\mathbf{q}^t} \quad & \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} p^t \tilde{C}_n(\mathbf{q}_n^t) \\ \text{s.t.} \quad & q_{nm}^t \leq \kappa_{nm}, \quad \forall m \in \Gamma_n, \forall n \in \mathcal{N} \\ & q_{nm}^t + q_{mn}^t = 0 \quad \forall m \in \Gamma_n, \forall m \in \mathcal{N} \\ & Q_n^t = \sum_{m \in \Gamma_n} q_{nm}^t. \end{aligned} \quad (3.25)$$

Using (3.1) we can rewrite the objective function of (3.25) as

$$\sum_{(n,m) \in \mathcal{N} \times \Gamma_n} \sum_{t \in \mathcal{T}} p^t \cdot q_{nm}^t (c_{nm} - c_{mn}) \quad (3.26)$$

For convenience, we index all possible combinations  $(n, m, t) \in \mathcal{N} \times \Gamma_n \times T$  of trades between agents and denote them as  $k_1, \dots, k_M$ , where  $M = |E| \cdot |T|$ . Coefficients for  $q_{k_i}$  appearing in (3.26) are denoted as  $c_{k_i}$ . Then, the linear subproblem can be written as follows :

$$\begin{aligned} \min_{\mathbf{q}^t} \quad & \sum_{k_i, i=1, \dots, M} c_{k_i} q_{k_i} \\ \text{s.t.} \quad & -\kappa_{k_i} \leq q_{k_i} \leq \kappa_{k_i} \\ & Q_j = \sum_{k_i \in J_n} q_{k_i}, \end{aligned} \quad (3.27)$$

where  $J_n$  is a subset of indices  $k_1, \dots, k_M$  representing the trades of agent  $n$ . By Theorem 1 from [91], a solution  $\bar{x}$  of the linear problem  $\{\min_x c^t x \mid Ax = b, Cx \geq d\}$  is unique if and only if it remains a solution to all linear programs obtained by arbitrary but sufficiently small perturbation of its cost vector  $c$ , or equivalently, for each  $b$  in  $\mathbb{R}^n$ , there exists a real positive number  $\varepsilon$  such that  $\bar{x}$  remains a solution of the perturbed linear program  $\{\min_x (c + \varepsilon b)^T x \mid Ax = b, Cx \geq d\}$ . Thus, to finish the proof, we order the coefficients  $c_{k_i}$  such that  $c_{k_1} \leq c_{k_2} \leq \dots \leq c_{k_M}$ , and consider two cases : (i)  $\nexists i, j : c_{k_i} = c_{k_j}$  or, equivalently,  $c_{k_i} < c_{k_{i+1}} \leq \dots \leq c_{k_M}$ . Then, it is clear that solution  $\bar{\mathbf{q}}^t$  of the original LP is a solution of the perturbed LP for any vector  $b$ , because the order of coefficients  $c_{k_i}$  can be preserved by choosing sufficiently small  $\varepsilon$ . In case (ii) in which the ordering of the coefficients is not strict, i.e.  $\exists i, j : c_{k_i} = c_{k_j}$ , perturbing the cost by vector  $b = (\dots, b_{k_i}, \dots, b_{k_j}, \dots)$  with  $b_{k_i} \neq b_{k_j}$  restricts us from preserving the order of coefficients thus leading to a non unique solution. It remains to conclude that equal coefficients  $c_{k_i}$  among agents lead to the same trading costs, thus leading to the unique values of  $\Pi_n^t$ . ■

**Remark 35** *The implication of the above result is that even if the solution  $x_n$  at VE is not unique, it still bears the same individual costs for all the prosumers. Moreover, the condition for having a unique VE in practice is pretty mild, e.g., it reduces to the agents having non-symmetric coefficients  $c_{nm}$ . If the condition to have symmetric coefficients is crucial, e.g., when they represent taxes, it is possible to achieve uniqueness of VE by adding a regularization quadratic term  $\phi q_{nm}^2$  that accounts for transaction costs [144]. This track is discussed later in section 3.7.2.*

### 3.7.2 Regularized lower level problem

In this section, we modify the risk-adjusted costs of the agents in order to ensure the strict convexity of the lower-level problem :

$$R[\Pi_n^t] = \sum_{t \in \mathcal{T}} [\alpha_n^t J_n^t + \beta_1 J_n^{t^2}] + \eta_n + \frac{1}{(1 - \chi_n)} \sum_{t \in \mathcal{T}} p^t [\Pi_n^t - J_n^t - \eta_n]^+. \quad (3.28)$$

with the costs  $\Pi_n^t$  redefined as  $\Pi_n^{tR} := \Pi_n^t + \frac{1}{2}\beta_2 \sum_{m \in \Gamma_n} c_{nm} q_{nm}^2 + \beta_1 J_n^{t^2}$  for some small  $\beta_1, \beta_2 > 0$  and  $\Pi_n^R := \sum_{t \in \mathcal{T}} \Pi_n^{tR}$ . The regularization formulation is common in the literature; for example, in [144], authors interpret the regularizer  $\beta_2 \sum_{m \in \Gamma_n} c_{nm} q_{nm}^2$  as a transaction cost arising from trades. In [96] the regularizer  $\beta_1 J_n^{t^2}$  is introduced as a transaction cost for financial contracts. These terms allow us to obtain a unique solution  $(J_n^t, x_n^t)$  for all  $n \in \mathcal{N}$ , for all values of  $x_I$ . With this modification, we can write the KKT conditions of the lower-level problem (3.12) with modified  $\Pi_n^t$ , in which the only changes appear for the optimality condition w.r.t.  $q$  :

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial q_{mn}^t} = 0 &\Leftrightarrow \zeta_{nm}^t + \xi_{nm}^t + \tau_n^t c_{nm} (1 + \beta_2 q_{nm}) - \lambda_n^t = 0 \\ \frac{\partial \mathcal{L}}{\partial J_n^t} = 0 &\Leftrightarrow \alpha_n^t + \beta_1 J_n^t - \tau_n^t - \underline{\sigma}_n^t + \bar{\sigma}_n^t = 0 \end{aligned}$$

**Proposition 36** *For any  $\varepsilon > 0$ , there exists  $\beta_1, \beta_2 > 0$  s.t.  $\Pi_n^R \leq \Pi_n + \varepsilon$ , i.e., we can approximate any  $\varepsilon$ -GNE of the lower level using a regularized formulation. Moreover, there exists a sequence  $\beta_1^k, \beta_2^k$  s.t.  $x^k \rightarrow \hat{x} \in E_{LL}$  for some  $\hat{x}$ .*

**Proof.** We first observe that  $\frac{1}{2}\beta_2 \sum_{m \in \Gamma_n} c_{nm} q_{nm}^2 + \beta_1 J_n^{t^2}$  is non-negative, and that  $|q_{nm}| \leq \kappa_{nm} \forall n, m \in \mathcal{N}$ , from which it follows that the difference between the objective functions for agent  $n$  can be bounded by  $\beta_2 \sum_{m \in \Gamma_n} c_{nm} \kappa_{nm}^2$ . Fixing other decision variables at the equilibrium and taking  $\beta_2 = \frac{\varepsilon}{\sum_{m \in \Gamma_n} c_{nm} \kappa_{nm}^2}$ , we obtain the first statement.

Second, we note that from (3.7.2), boundeness of  $|q_{nm}|$  and Proposition 16.3, that for each  $\beta_2^k$  there exists a set of dual variables s.t.

$$|\zeta_{nm}^{t^k} + \xi_{nm}^{t^k} + \tau_n^{t^k} c_{nm} - \lambda_n^{t^k}| \leq \beta_2^k \kappa_{nm} \xrightarrow{\beta_2^k \rightarrow 0} 0,$$

which approaches exactly the set of solutions described by KKT for original problem. We note that from the reformulation (3.19), the set of equilibria solutions of the lower-level problem in PBP is a subset of equilibria solutions of the lower level of OBP. Thus, the bound is proved for both formulations. ■

### 3.7.3 Numerical results supplement

	RN	RA	No I	OBP	PBP
CS	0.7	1.21	1.2	1.28	1.28
DS	~1200	~1600	~1600	~1600	~1600

TABLE 3.3 – Average computational time in seconds

**Computational approaches** In general, computing a GNE can be a challenging task. Many algorithms have been proposed, especially in recent years, but the conditions that guarantee their convergence, such as strong monotonicity of the pseudo-gradient of the game, aggregative structure, potential structure, etc. [48], [104], [139], [159], might be too strong to

justify in practice. Computing solutions of bilevel problems, especially in the pessimistic framework, can be even more challenging [136], [12], [85]. Reformulation as a centralized optimization problem might lead to the inefficiency of the solution [123] and also, due to computational and communication limitations, it is not always possible to solve a large-scale optimization problem, and it is preferable to decompose the problem so that it can be solved by a distributed algorithmic approach. Using reformulation  $\mathcal{G}_{ch}$  with choice function, we can implement both centralized and distributed approaches to solve the two-level problem. To solve the problem in a distributed fashion, we use gradient-descent method discussed for e.g. in [159] implemented using PyTorch and for the centralized solution we use Gurobi Optimizer 9.5. The comparison of the computational time for centralized (CS) and decentralized (DS) solutions of different models with 100 scenarios are given in Table 3.3.

**Parameters' impact** We want to assess the agents' parameters' impact on their costs in different frameworks. To that purpose, we use linear regression on a set of 2000 parameters sampled from the uniform distribution and extract the weights corresponding to the parameters, summing them over all agents. The result is depicted in Figure 3.8. The  $R^2$  scores obtained are  $> 0.75$ . It might seem surprising that the main weights are put on  $\tilde{b}_n$  and  $d_n^t$ , which are the constant terms in agent's demand and

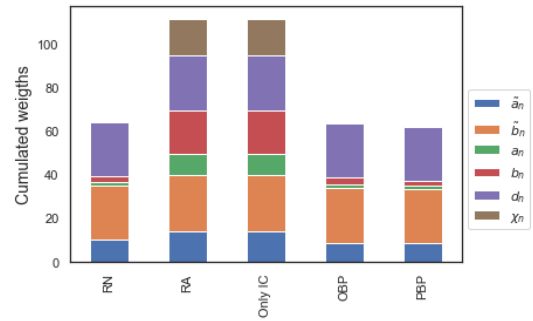


FIGURE 3.8 – Parameters' weights

generation costs. This is due to the fact that these terms are not affected by the decisions of the prosumers, and, while prosumers minimize the terms in their costs that depend on  $d_n^t, g_n^t$ , these constants remain unchanged. Coefficients  $a_n, b_n$  of generation cost  $C_n(g_n^t)$  as well as risk aversion  $\chi_n$  affect a lot *RA* and *Only I* while not having a significant impact in *OBP* and *PBP* frameworks. This is due to the *equity* property of the latter, i.e., the price for the financial contracts being the same (and minimal

possible) for all the prosumers.

**Uncertainties** Scenario approach considered in the chapter can be supplemented with a distinction between the correlated and independent random variables reflecting prosumers' generation, demand etc. It is possible to adapt the notion of the general types of individual risk and collective risk investigated in [28], where authors accounted for both of them and investigated the effects of the combination of both Arrow-Debreu and Malinvaud's models of insurances on this type of uncertainty. In our work we can employ the former ones, while considering the same type of uncertainty division. Thus, each agent faces two sorts of uncertainty : individual uncertainty and collective uncertainty. It allows to speak about the independence of the random variables we focus on.

For each agent, there are possible  $S_n$  individual states  $(1, \dots, S_n)$  and  $T$  possible collective states  $(1, \dots, T)$ . Each agent correctly believes that her probability of being in a joint state  $(s, t)$  is given by  $p_n(s, t) > 0$  s.t.  $\sum_{(s,t)} p_n(s, t) = 1$ . We denote the corresponding random variables as  $\mathcal{S}_n$  and  $\mathcal{T}$ . Agents view  $\mathcal{T}$  as a possible state of nature (e.g. weather conditions) which are common knowledge for everyone.  $\mathcal{S}_n$ , on the other hand, reflects individual uncertainties conditioned on the state of nature (e.g. the demand of agent  $n$ ). It is natural to assume that after the state of nature  $t$  is observed by the agents, their individual r.v.  $\mathcal{S}_n$  are independent i.e.  $\mathcal{S}_n$  are conditionally independent given  $\mathcal{T}$  and the conditional probabilities are given by

$$p_n^{(s|t)} = \frac{p_n(s, t)}{\sum_{s'} p_n(s', t)} \quad (3.29)$$

All the results in the chapter can be proven for this modified scenario approach. Additional constraints are introduced due to the trading in the electricity market. For pairs of agents  $n, m \in N, n \neq m$ , in order to align their trading decisions, we have to consider pairs of individual scenarios  $(s^i, s^j) \in \mathcal{S}_n \times \mathcal{S}_m$ , given collective state  $t$ . Under the assumption of conditional independence of individual scenarios, we can write the joint probabilities of individual scenarios conditionally to the state of nature, as  $p(s^i, s^j | t) = p_n^{(s^i|t)} \cdot p_m^{(s^j|t)}$ . Given a scenario  $(s^i, s^j, t)$ , we impose an equality on the *trading reciprocity* :

$$q_{nm}^{(s^i, s^j, t)} + q_{mn}^{(s^i, s^j, t)} = 0, \quad \forall m \in \Gamma_n, \quad (3.30)$$

which couples the agents' bilateral trading decisions. It means that in the case where  $q_{nm}^{(s^i, s^j, t)} > 0$ , the quantity that  $n$  buys from  $m$  should be equal to the quantity  $q_{mn}^{(s^i, s^j, t)}$  that  $m$  is willing to offer to  $n$ . Individual uncertainties sets  $\mathcal{S}_n$  are unknown by other agents in the network, thus it follows that the

trades of agent  $n$  decided for scenario  $s^i$  should be equal for all the scenarios  $s^j, s^k$  of the agent  $m \in \Gamma_n$  :

$$q_{nm}^{(s^i, s^j, t)} = q_{nm}^{(s^i, s^k, t)}, \quad \forall s^j, s^k \in \mathcal{S}_m. \quad (3.31)$$



# Chapitre 4

## Interaction with a DSO

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# A Generalized Nash Equilibrium analysis of the interaction between a peer-to-peer financial market and the distribution grid

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## 4.1 Introduction

Within peer-to-peer electricity market, agents (prosumers) negotiate with their peers their energy procurement seeking to minimize their costs with respect to both individual and trading reciprocity coupling constraints taking into account trading cost preferences. Many studies focus on the financial modeling of peer-to-peer energy trading market. Game theoretic approaches integrating the prosumers' strategic behaviors in the peer-to-peer trading are considered in [26]. The economic dispatch in energy communities under different structures of communications is analysed in e.g. [96] using optimization approaches. An increasing amount of attention is brought to the determination of suitable pricing mechanisms in the peer-to-peer market that reflect the contribution of the prosumers to the state of the distribution grid. Several studies investigate distribution locational marginal pricing (DLMP), e.g. [26, 61, 62, 82] and provide an insight on impact on the market (efficiency, individual rationality, incentive compatibility), and regulatory properties (transparency, fairness, etc.), and computational cost.

Practical problem for peer-to-peer implementation is related to the feasibility of the power flows corresponding to the bilateral trades negotiated on the financial market, regarding distribution grid network constraints. In case of infeasibility, some trades might be curtailed and the resulting loss allocated to the agents. Another important aspect which justifies the need for financial and physical level decoupling is the information sharing between prosumers and the DSO, as the latter might be reluctant to share the sensitive power grid related information with the former. This research topic has been addressed by several studies, e.g. [61] proposes a methodology to assess the impact of peer-to-peer transactions on the physical network and ensures that the physical network constraints are not violated. [137], [162] investigate multi-agent simulation framework and a consensus-based approach for peer-to-peer electricity trading in a microgrid respectively. Cooperative or noncooperative Stackelberg games are studied in [90, 142], with an assumption that the distribution system operator (DSO) acts as the leader and prosumers, as followers.

In our model we focus on the interaction between (i) the financial level, in which the agents minimize the sum of their *generation flexibility* cost and *bilateral trading* costs minus their *usage benefit*, and (ii) the physical level, in which the DSO minimizes the total generation flexibility cost taking into account the

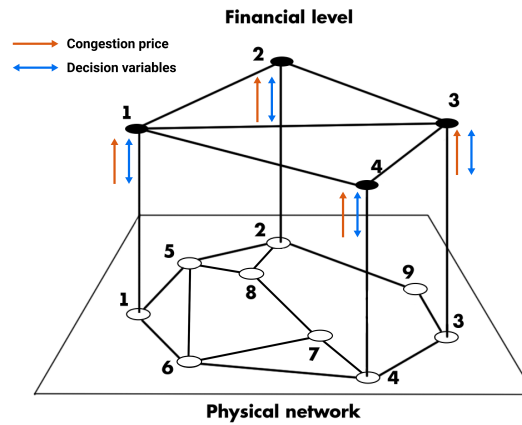


FIGURE 4.1 – Simplified example of the two-level interaction

physics of the distribution network, which we model through a linear DC power-flow approximation. The interactions between the two levels are modeled as generalized Nash equilibrium problems (GNEP), i.e., noncooperative games which endogenized shared coupling constraints within the agents' parametrized optimization problems. We analyze generalized Nash equilibrium (GNE) [74, 159], and a refinement of it, called variational equilibria (VE)[74], assuming that the shadow variables associated with the shared coupling constraints are aligned among the agents.

This chapter key contributions can be summarized as follows : (1) We formally define the optimization problems for financial market and DSO, and formulate the interaction between the financial and physical levels as a GNEP (see Fig. 4.1, inspiration from [85]). We consider a two-player GNEP, in which the financial level is operated in a centralized fashion by a Market Operator (MO). We compare the two-player GNEP outcome to a N+1 GNEP outcome, in which the financial level made of N prosumers is operated in a fully distributed peer-to-peer fashion. We show that the resulting GNEs are Pareto efficient under certain conditions. (2) We characterize the solution of the GNEPs and discuss the effects of the prosumers' pricing mechanism, which captures the interaction between the financial and physical levels. We provide an illustrative example that demonstrates that in case the two levels are uncoupled, there might be a free-lunch behaviors, i.e. agents increasing their trades up to infinity to minimize their costs. On the contrary, we prove that there is no such possibility in our model. (3) To illustrate our results, we show that our problem is a generalized potential game (GPG) and implement the Gauss-Seidel best response algorithm, which converges to a solution of the two-level game. In addition, we investigate the impact of RES-based generation on the market social cost and DSO's activation cost under the different levels of the generation flexibility and flexible demand available. We also consider the impact of functional dependence of the congestion cost term in the agents' trading costs functions, on the market social cost.

### 4.1.1 Notations

Bold symbol  $\mathbf{x}$  denotes a vector and capital italic symbol  $\mathcal{X}$  denotes a set. Exclusion of a set is denoted by  $\mathcal{X} \setminus \mathcal{A} := \{x | x \in \mathcal{X}, x \notin \mathcal{A}\}$ .  $x \perp y$ , means  $x \geq 0, y \geq 0$  and  $xy = 0$ .  $\Pi_n$  denotes a cost function of agent  $n$ . To make a presentation concise, we sometimes omit the dependence of  $\Pi_n$  on the decision variables.

## 4.2 Description

We consider a single-settlement market for energy trading made of a set  $\mathcal{N}$  of  $N$  agents (prosumers) – each one of them being located in a node of the distribution grid. On top of the physical level network, the agents form a trading network which is modeled as a connected undirected graph  $G^M := (\mathcal{N}, E^M)$  where  $E^M \subseteq \mathcal{N} \times \mathcal{N}$  is the set of trading links between the players, which reflects the financial level network structure. We denote  $\Gamma_n$  to be the set of neighbors of  $n$  in this trading network, that reflects the agents she wants to trade with. In this financial level agents make the decisions about their *demand*  $D_n$ , *generation flexibility*  $G_n$  and bilateral *financial trades*  $q_{nm} \forall m \in \Gamma_n \setminus \{n\}$ . If  $q_{nm} \geq 0$ , then  $n$  buys  $q_{nm}$  from  $m$ , otherwise ( $q_{nm} < 0$ )  $n$  sells  $-q_{nm}$  to  $m$ . Inequality means that we allow for the surplus during the electricity trading. The surplus is handled by an aggregator, who can sell it on the wholesale market. The modelling of the aggregator's optimization problem is out of the scope of the current work.

On the physical level, we consider a distribution grid, which is represented by an undirected graph  $G^{DSO} := (\mathcal{N}, E^{DSO})$ , where  $E^{DSO} \subseteq \mathcal{N} \times \mathcal{N}$  is a set of the distribution lines between agents. Let  $\Omega_n$  be the set of the agents with whom agent  $n$  is connected in the distribution grid (note that  $\Omega_n$  does not necessary coincide with  $\Gamma_n$ ). DSO makes a decision about *power flows*  $F_{nm}$ , *voltage angles*  $\theta_n$  and coefficient  $\rho_n$  for the *fraction of the generation flexibility* to be used.

To model the interaction between the two levels, we assume that the decision variables  $D_n$  and  $G_n$  of the agents act as the parameters in the DSO optimization problem. DSO's decision variable  $\rho_n$  and the Lagrangian multiplier  $\gamma_n$ , which can be interpreted as the *congestion price*, are used as parameters in the agent  $n$ 's optimization problem. This interaction model implies that each agent  $n$  chooses the level of the generation flexibility she is willing to utilize, while the DSO chooses the share of this generation flexibility to use. Supply-demand balance constraint should hold both on the financial and physical levels. Moreover, the coupling between the two levels appears explicitly through the congestion price, a function of which is a component of the bilateral trading costs of the agents.

### 4.2.1 Financial level

**Feasibility sets** For each agent  $n \in \mathcal{N}$ , we introduce  $\mathcal{D}_n := \{D_n \in \mathbb{R}_+ | \underline{D}_n \leq D_n \leq \overline{D}_n\}$  as agent  $n$ 's demand set, with  $\underline{D}_n$  and  $\overline{D}_n$  being the lower and upper-bounds on demand capacity and  $\mathcal{G}_n := \{G_n \in \mathbb{R}_+ | \underline{G}_n \leq G_n \leq \overline{G}_n\}$  be agent  $n$ 's generation flexibility set, where  $\underline{G}_n$  and  $\overline{G}_n$  are the lower and upper-bounds on activation capacity. Let  $\Delta G_n$  denote the *RES-based generation* at node  $n$ .

We impose an inequality on the *trading reciprocity* :

$$q_{nm} + q_{mn} \leq 0, \quad (4.1)$$

which couples agents' bilateral trading decisions<sup>1</sup>. We denote  $\zeta_{nm}$  the corresponding dual variable. It means that, in the case where  $q_{nm} > 0$ , the quantity that  $n$  buys from  $m$  can not be larger than the quantity  $q_{mn}$  that  $m$  is willing to offer to  $n$ .

*Local supply and demand balance* leads to the following equality in each node  $n$  in  $\mathcal{N}$  :

$$D_n = G_n + \Delta G_n + \sum_{m \in \Gamma_n} q_{mn} = G_n + \Delta G_n + Q_n, \quad (4.2)$$

where  $Q_n$  is defined as the net import at node  $n$ . Corresponding dual variable is denoted as  $\lambda_n$

**Objective function** In each node  $n$  we model the *generation flexibility cost* as a quadratic function of local activated flexibility, using three positive parameters  $a_n, b_n$  and  $d_n$  :

$$C_n^G(\rho_n G_n) = \frac{1}{2} a_n (\rho_n G_n)^2 + b_n \rho_n G_n + d_n, \quad (4.3)$$

where  $\rho_n \in [0, 1]$  is a decision variable of the DSO, which represents the *the fraction of the flexibility* offered by agent  $n$  that is activated by the DSO.

The *usage benefit* perceived by agent  $n$  is modeled as a strictly concave function of node  $n$  demand, using two positive parameters  $\tilde{a}_n, \tilde{b}_n$  and a *target demand*  $D_n^*$ , defined exogenously for agent  $n$  :

$$U_n(D_n) = -\tilde{a}_n (D_n - D_n^*)^2 + \tilde{b}_n \quad (4.4)$$

The total *trading cost* function of agent  $n$  is denoted by :

$$\tilde{C}_n(\mathbf{q}_n) = \sum_{m \in \Gamma_n, m \neq n} q_{nm} (c_{nm} + f(\gamma_n)), \quad (4.5)$$

---

1. Inequality means that there might be an energy surplus in the system. Surplus might be handled by a third party player (aggregator) who is part of the game and who would compensate the consumers for the energy surpluses.

where parameters  $c_{mn} > 0$  can model taxes for energy trading or agents' preferences regarding trade characteristics [26]. In real systems, DSO does not reveal  $\gamma_n$  explicitly, but some function  $f(\gamma_n)$  as a function of *congestion price*, computed by the DSO.

Then, we write prosumer  $n$ 's cost function as follows :

$$\Pi_n = C_n^G(\rho_n G_n) + \tilde{C}_n(\mathbf{q}_n) - U_n(D_n) \quad (4.6)$$

### 4.2.2 Physical level

On the physical, the DSO solves the Optimal Power Flow (OPF) problem. The original power flow equations for AC systems are non-linear equations of complex numbers, having a quadratic relationship between power and voltage, bringing non-convexity to the problem. We use the DC-OPF linearization of the original problem, which is classical in the OPF literature. DC-OPF formulation is used to represent distribution grids when it is important to obtain the analytical results and interpretation of the dual variables corresponding to the different prices in the electricity market [62], [82].

**Constraints** In DC-OPF approximation, *power flow* on the line  $nm$  can be expressed as

$$F_{nm} = \frac{1}{x_{nm}}(\theta_n - \theta_m), \quad (4.7)$$

with the dual variable  $\tau_{nm}$  associated to it, where  $x_{nm}$  is the line reactance. We include the upper and lower bounds  $\underline{F}_{nm} \leq F_{nm} \leq \bar{F}_{nm}$  for which we use dual variables  $\underline{\phi}_{nm}, \bar{\phi}_{nm}$  correspondingly. In order to approximate the angles, we impose limits on the angle difference between connected buses :

$$-\frac{\pi}{3} \leq \theta_n - \theta_m \leq \frac{\pi}{3}. \quad (4.8)$$

with the corresponding dual variables  $\underline{\alpha}_n, \bar{\alpha}_n$ . For each node, DSO ensures that local supply and demand balance holds :

$$D_n = \rho_n G_n + \Delta G_n + \sum_{m \in \Omega_n} F_{nm} \quad (4.9)$$

with  $\gamma_n$  as a dual variable.

**Optimization problem** Denote the joint strategy vector for the market level decision variables as  $\mathbf{s}_{MO} := (\mathbf{s}_n)_1^N$ . We denote feasibility set for a DSO operator as  $\mathcal{S}_{DSO}(\mathbf{s}_{MO}) := \{\mathbf{s}_{DSO} = (\rho_n, \theta_n, \mathbf{F}_n)_{1, \dots, N} | \underline{F}_{nm} \leq F_{nm} \leq \bar{F}_{nm}, (4.7), (4.8), (4.9) \text{ hold } \forall n \in \mathcal{N}\}$ . The DSO takes *demand*  $D_n$  as the parameter and minimizes the sum of *generation flexibility costs*  $C_n^G(\rho_n G_n)$  subject to *power flow* equations (4.7), *node*

balance (4.9) and upper and lower bounds on angles and power flows constraints :

$$\min_{\rho_n \in [0,1], \theta_n, F_{nm}} \quad \Pi_{DSO} := \sum_n C_n^G(\rho_n G_n) \quad (4.10a)$$

$$s.t. \quad \mathbf{s}_{DSO} \in \mathcal{S}_{DSO}(\mathbf{s}_{MO}) \quad (4.10b)$$

## 4.3 Market designs

### 4.3.1 Centralized financial market operation

We consider two designs of the financial market level, a centralized and a decentralized (peer-to-peer) ones. Under *centralized* market design, in which the global Market Operator minimizes social cost of the agents on the financial level. We denote the feasibility set of agent  $n$  as  $\mathcal{S}_n(\mathbf{s}_{-n}) := \{\mathbf{s}_n = (D_n, G_n, \mathbf{q}_n) | D_n \in \mathcal{D}_n, G_n \in \mathcal{G}_n, (4.1), (4.2) \text{ hold } \forall n \in \mathcal{N}\}$ , where  $\mathbf{s}_{-n}$  denotes is a vector that contains the concatenation of all the agents' actions excluding agent  $n$ .

$$\min_{G_n, D_n, \mathbf{q}_{nm}} \quad \Pi_{MO} := \sum_n \Pi_n \quad (4.11a)$$

$$s.t. \quad \mathbf{s}_{MO} \in \mathcal{S}_{MO} := \prod_n \mathcal{S}_{-n}(\mathbf{s}_{-n}) \quad (4.11b)$$

We formulate the interaction between DSO and a local MO as a two-player generalized Nash equilibrium game :  $\mathcal{G} := \{I, (S_i)_{i \in I}, (\Pi)_{i \in I}\}$ , where  $I$  is the set of agents, which in this framework is defined as  $I := \{MO, DSO\}$ , for each  $i \in I$ ,  $S_i$  is the strategy set and  $\Pi_i$  is the cost function. We denote the systems of the KKT conditions for the financial and physical levels as  $KKT_{MO}$ ,  $KKT_{DSO}$  respectively.

### 4.3.2 Peer-to-peer financial level market design

In this section we consider peer-to-peer setting on the financial level, in which each agent  $n \in \mathcal{N}$  selfishly optimizes her demand ( $D_n$ ), energy generation ( $G_n$ ) and bilateral trades ( $\mathbf{q}_n$ ) with other agents in her neighborhood under constraints on demand, generation and trading capacity so as to minimize her costs. Formally, each agent in node  $n \in \mathcal{N}$  solves :

$$\min_{D_n, G_n, \mathbf{q}_n} \quad \Pi_n, \quad (4.12a)$$

$$s.t. \quad \mathbf{s}_n = (D_n, G_n, \mathbf{q}_n) \in \mathcal{S}_n(\mathbf{s}_{-n}) \quad (4.12b)$$

We formulate the interaction between the DSO and the agents as an  $N + 1$ -player generalized Nash equilibrium game :  $\mathcal{G}_p := \{I, (S_i)_{i \in I}, (\Pi)_{i \in I}\}$ , where  $I$  is the set of agents, which in this framework is defined as  $I := \mathcal{N} \cup \{DSO\}$ , for each  $i \in I$ ,  $S_i$  is the strategy set and  $\Pi_i$  is the cost function of agent  $i$ . We consider the KKT conditions of the game as a system given by  $KKT_p$  which is the concatenation of  $KKT_{DSO}$  and  $KKT_n$  for all  $n = 1 \dots N$ , where  $KKT_n$  denotes the KKT conditions for (4.12).

#### 4.4 Equilibrium analysis

In our analysis we rely on Generalized Nash Equilibria and Variational Equilibria; both of them exist under mild conditions [74], [159].

**Definition 37** ([74]) *A Generalized Nash Equilibrium (GNE) of the game  $\mathcal{G}(\mathcal{G}_p)$  with coupling constraints, is a vector  $(x_{MO}, x_{DSO})$  that solves the system given by  $KKT_{MO} \vee KKT_{DSO}$  ( $KKT_p$ ).*

**Definition 38** *We say that GNE of  $\mathcal{G}_p$  is induced by the Variational Equilibrium (VE) of the financial level problem if it is a GNE of  $\mathcal{G}_p$  s.t.*

$$\zeta_{nm} = \zeta_{mn}, \quad \forall n \in \mathcal{N}, \forall m \in \Gamma_n \quad (4.13)$$

*We denote such equilibria as  $GNE_{VE}$ .*

As it was stated above,  $\zeta_{nm}$  for  $n \in \mathcal{N}, \forall m \in \Gamma_n$  can be interpreted as bilateral energy trading prices [26]. In general,  $\zeta_{nm} \neq \zeta_{mn}$ , thus leading to non-symmetric energy trading prices between couple of agents. Relying on VE as solution concepts enforces a natural symmetry in the bilateral energy price evaluation between any couple of agents [26].

We note that  $(s_1, \dots, s_n)$  that solve  $\bigvee_{n=1}^N KKT_n$  are defined by exactly the same KKT system as the social cost minimizer of the market level problem -  $KKT_{MO}$ . Therefore, we obtain the following result :

**Proposition 39** *GNE given in two-player game  $\mathcal{G}$  coincides with  $GNE_{VE}$  of  $\mathcal{G}_p$ .*

**Pareto-efficiency of GNE** A strategy is a *Pareto efficient* outcome if no joint strategy is both a weakly better outcome for all players and a strictly better outcome for some player. Formally, if  $S_{GNE}$  denotes the set of the joint equilibrium strategies, then  $s \in S_{GNE}$  is Pareto efficient if  $\nexists s' \in S_{GNE}$  s.t.

$$\forall i \in I : \Pi_i(s') \leq \Pi_i(s), \quad \exists i \in I : \Pi_i(s') < \Pi_i(s)$$

**Proposition 40** *If the coefficients  $c_{nm}$  in the trading costs  $\tilde{C}_n(\mathbf{q}_n)$  of the agents are homogeneous, i.e.  $c_{nm} = c_{n'm'} \forall n, m, n', m'$ , then GNE of  $\mathcal{G}$  and  $GNE_{VE}$  of  $\mathcal{G}_p$  are Pareto-efficient.*

**Proof.** Denote  $c := c_{nm} \forall n, m$ . Then, we can rewrite the trading cost of the agents using the supply-demand balance equality :

$$\tilde{C}_n(\mathbf{q}_n) = cQ_n = c(D_n - G_n - \Delta G_n), \quad (4.14)$$

thus the objective function  $\Pi_n$  doesn't depend on  $\mathbf{q}_n$ . Note that  $\Pi_n$  is strictly convex in  $D_n, G_n$  and the feasibility set of the financial level optimization problem is convex. Then, solution of the financial level problem given by the  $KKT_{MO}$  (or the  $\forall_1^N KKT_n$  s.t.  $\zeta_{nm}$  are equal) is unique w.r.t.  $D_n, G_n$ . Now we consider the strategies of the DSO. Objective of the DSO doesn't depend on  $\theta_n$  and  $F_{nm}$  is strictly convex in  $\rho_n$ . The feasible set is convex. Thus, we have a unique  $\rho_n$  that solves the KKT conditions. It follows that there's no other GNE that can decrease the costs of the agents. ■

**Pricing** In this section we focus on the Lagrangian multipliers that can be interpreted a market prices. From  $KKT_{DSO}$  conditions, we obtain the expression for the  $\gamma_n$  :

$$\gamma_n = \bar{\phi}_{nm} - \underline{\phi}_{nm} + (\bar{\alpha}_n - \underline{\alpha}_n)x_{nm} \quad (4.15)$$

Note that the angles  $\theta_n, \theta_m$  unambiguously define  $F_{nm}$ , thus, for the pair of the agents  $n, m$  there can be only one active constraint out of the power flow bounds  $\underline{F}_{nm} \leq F_{nm} \leq \bar{F}_{nm}$  and the angles bounds (4.8). It follows that the dual variable  $\gamma_n$  represents the *congestion price* on the physical level. From the  $KKT_{MO}$  conditions for the financial level we have that the nodal price  $\lambda_n$  associated with the supply and demand balancing constraint in node  $n$  can be expressed as the sum of the bilateral trade price  $\zeta_n$  offered by  $n$  to  $m$  associated with the trading reciprocity constraint (4.1), the coefficient  $c_{nm}$  and the function  $f(\gamma_n)$  of congestion price  $\gamma_n$  on the physical level :

$$\lambda_n = c_{nm} + f(\gamma_n) + \zeta_{nm} \quad (4.16)$$

From the balance equations (4.9) and (4.2), we have that

$$G_n(1 - \rho_n) = \sum_{m \in \Gamma_n} F_{nm} - \sum_{m \in \Omega_n} q_{nm}$$

Since  $\rho_n \in [0, 1]$ ,  $1 - \rho_n \leq 0$  we consider three cases :

1.  $G_n \geq 0$ , then  $\sum_{m \in \Gamma_n} q_{nm} \leq \sum_{m \in \Omega_n} F_{nm}$



2.  $G_n \leq 0$ , then  $\sum_{m \in \Gamma_n} q_{nm} \geq \sum_{m \in \Omega_n} F_{nm}$
3.  $G_n = 0$  or  $\rho_n = 1$ , then  $\sum_{m \in \Gamma_n} q_{nm} = \sum_{m \in \Omega_n} F_{nm}$

If node  $n$  is injecting power in the grid, i.e.  $G_n \geq 0$ , then the trading cost allocated to this agent is less than the total congestion cost  $f(\gamma_n) \sum_{m \in \Omega_n} F_{nm}$  caused by this agent on the physical level and vice versa.

**No free lunch behavior** Term  $\sum_{m \in \Gamma_n} f(\gamma_n) q_{nm}$  is crucial in the trading costs of the agents. In the absence of  $f(\gamma_n)$  in the trading costs, we might observe the *free lunch behavior* - situation in which the financial trades are increasing up to infinity in order to decrease the trading costs  $\tilde{C}_n(\mathbf{q}_n)$ . We illustrate this in the following example :

*Example* : Consider 3-node network, which is represented by a complete graph. Assume that the trading costs of the agents are given by  $\tilde{C}_n(\mathbf{q}_n) = \sum_{m \in \Gamma_n} c_{nm} q_{nm}$  and the coefficients  $c_{nm}$  are  $(c_{12}, c_{13}) = (1, 1)$ ,  $(c_{21}, c_{23}) = (1, 3)$ ,  $(c_{31}, c_{32}) = (2, 1)$ . Let  $(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$  be a feasible vector of the trading decisions s.t.  $q_{13}, q_{21}, q_{32} < 0$  and  $q_{31}, q_{12}, q_{23} > 0$ . Then, w.l.o.g. pick node 3 and assume that it increases amount of energy she sells to node 1 :  $q'_{31} = q_{31} - \varepsilon$ . Then, it also has to buy the same additional amount from node 2 :  $q'_{32} = q_{32} + \varepsilon$ . Then, new trading cost  $\tilde{C}_3(\mathbf{q}'_3) = 2 * (q_{31} - \varepsilon) + 1 * (q_{32} + \varepsilon) = \tilde{C}_3(\mathbf{q}_3) - \varepsilon$ . Similarly, for node 2,  $\tilde{C}_2(\mathbf{q}'_2) = \tilde{C}_2(\mathbf{q}_2) - 2\varepsilon$  and for node 1 :  $\tilde{C}_1(\mathbf{q}'_1) = \tilde{C}_1(\mathbf{q}_1)$ . Note, that all the bilateral trade constraints (4.1) remain feasible and that  $Q_n$  do not change. Thus, by increasing  $\varepsilon$ , agents are able to decrease their costs without violating any constraint.

The following result states, that adding the term  $f(\gamma_n)Q_n$  in the trading costs of the agents prohibits the *free-lunch behavior*.

**Proposition 41** *Free lunch behavior is not possible in the  $GN E_{VE}$  if the trading costs of the agents are given by (4.5), i.e.  $\nexists \mathbf{s}_n$  s.t.  $\mathbf{s}_n \in S_n$  and  $\tilde{C}_n \rightarrow -\infty$ .*

**Proof.** To consider free-lunch behavior, it is sufficient to investigate the cycles in graph  $G$ . Consider a cycle of length  $k$  :  $G_{C_k} := ((n_1, \dots, n_k), E_{C_k})$ . We consider a part of the trading cost function of agent  $i$  is that corresponds to the trades made inside this cycle :

$$\begin{aligned} \tilde{C}_i^{G_{C_k}}(\mathbf{q}_i) &= c_{i,i-1}q_{i,i-1} + c_{i,i+1}q_{i,i+1} + f(\gamma_i)(q_{i,i-1} + q_{i,i+1}) \\ &= \lambda_i(q_{i,i-1} + q_{i,i+1}) - \zeta_{i,i-1}q_{i,i-1} - \zeta_{i,i+1}q_{i,i+1} \end{aligned}$$

Assume that one agent changes her trades by adding  $\varepsilon$  to the amount she buys and subtracting  $\varepsilon$  from the amount she sells. In order to have free-lunch behavior, it is necessary that all the changes in the trading costs of the agents in  $G_{C_k}$  induced by this change are non-positive with one agent having strictly

decreased cost. Thus, the change in the sum of the total costs of all agents in  $G_{C_k}$  should be negative. Note, that the first term  $\lambda_i(q_{i,i-1} + q_{i,i+1})$  does not change, so it is sufficient to consider the last. Taking the sum over all the nodes in  $G_{C_k}$

$$\begin{aligned} & - \sum_{i \in G_{C_k}} [\zeta_{i,i-1} q_{i,i-1} + \zeta_{i,i+1} q_{i,i+1}] = \\ & = - [\zeta_{2,1} q_{2,1} + \zeta_{1,2} q_{1,2} + \dots + \zeta_{1,k} q_{1,k} + \zeta_{k,1} q_{k,1}] \\ & = - [\zeta_{1,2} (q_{2,1} + q_{1,2}) + \dots + \zeta_{1,k} (q_{1,k} + q_{k,1})] = 0 \end{aligned}$$

Where the equivalence to zero follows from the complementarity conditions for the bilateral trading constraints (4.1). We finish the proof by noting that it contradicts the necessary condition for the free-lunch behavior. ■

## 4.5 Numerical results

We consider the 18-node distribution network [25], for which all the parameters and the scheme are provided in [134]. Each node is a consumer with  $\bar{D}_n > 0$  and some nodes are generators (RES or conventional), therefore producing energy that can be consumed locally to meet demand  $D_n$  and exported to the other nodes to meet the unsatisfied demand.

We are interested in two aspects of the interaction between financial and physical levels : (i) how does the penetration of RES generation affect the efficiency and performance of the two-level noncooperative game, (ii) how does the form of the trading cost affect the efficiency of the market, more precisely, what is the effect of the different functions  $f(\cdot)$  that we apply to the congestion price  $\gamma_n$ .

**Potential form of the game** Computing GNE in the general case might be a challenging task. For a specific type of GNEP, which is called Generalized Potential Games (GPG) there are established approaches in the literature that compute both GNE [48] and VE [14]. A GNEP is a Generalized Exact Potential Game if (i) the feasible set of the game is non empty, (ii) there exists a continuous function  $P(\mathbf{x}) : \mathbb{R}^N \rightarrow \mathbb{R}$  such that for all  $n$ , for all  $\mathbf{s}_{-n}$  (such that  $\mathcal{S}_n(\mathbf{s}_{-n})$  is not empty), and for all  $\mathbf{s}_n, \mathbf{z}_n \in \mathcal{S}_n(\mathbf{s}_{-n})$

$$\Pi_n(\mathbf{s}_n, \mathbf{s}_{-n}) - \Pi_n(\mathbf{z}_n, \mathbf{s}_{-n}) = P(\mathbf{s}_n, \mathbf{s}_{-n}) - P(\mathbf{z}_n, \mathbf{s}_{-n}) \quad (4.17)$$

**Proposition 42** *Problem  $\mathcal{G}_p$  is a Generalized Exact Potential Game (GPG) with a potential function*

given by

$$P = \sum_n \left[ C_n^G(\rho_n G_n) + \tilde{C}_n(\mathbf{q}_n) - U_n(D_n) \right] \quad (4.18)$$

**Proof.** Indeed, the feasible set of the problem is nonempty and we can check directly that (4.17) holds.

■

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**Algorithm 1** Regularized Gauss-Seidel BR algorithm [48]

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- 1: Choose a feasible starting point  $x^0 = (x_1^0, \dots, x_N^0)$ ,  
a positive regularization parameter  $\tau > 0$  and set  $k := 0$ .
- 2: If  $x^k$  satisfies a suitable termination criterion : STOP.
- 3: For  $n = 1, \dots, N$ , compute a solution  $x_n^{k+1}$  of :

$$\begin{aligned} \min_{\mathbf{x}^n} \quad & \Pi_n(x_1^{k+1}, \dots, x_{n-1}^{k+1}, x_n, x_{n+1}, \dots, x_N^k) + \\ & + \tau \|x_n - x_n^k\|^2, \\ \text{s.t.} \quad & \mathbf{x}_n \in \mathcal{S}_n(x_1^{k+1}, \dots, x_{n-1}^{k+1}, x_{n+1}^k, \dots, x_N^k). \end{aligned}$$

- 4: Set  $x^{k+1} := (x_1^{k+1}, \dots, x_N^{k+1})$ ,  $k \leftarrow k + 1$  and go to (2).
- 

First, we investigate the efficiency loss caused by the peer-to-peer equilibrium computation with respect to centralized solution. Formally, we compute the ratio of the total cost of the system computed at an optimum and the total cost of the system at a GNE :

$$R_{cost} := \frac{Cost_{(\mathbf{s}_1, \dots, \mathbf{s}_N, \mathbf{s}_{DSO}) \in SC}(\mathbf{s}_1, \dots, \mathbf{s}_N, \mathbf{s}_{DSO})}{Cost_{(\mathbf{s}_1, \dots, \mathbf{s}_N, \mathbf{s}_{DSO}) \in GNE}(\mathbf{s}_1, \dots, \mathbf{s}_N, \mathbf{s}_{DSO})}$$

**RES penetration and flexibility** Define the percentage of the RES-based generation penetration in the system as

$$R_{res} = \frac{\sum_{n \in \mathcal{N}} \Delta G}{\sum_{n \in \mathcal{N}} \bar{G}_n}$$

There are two important points to consider when we increase the amount of RES in the system : (i) how much does it increase the efficiency loss of the system, (ii) what is the maximum amount of RES that we can inject so that the problem remains feasible.

Figure 4.2 illustrates how increasing RES generation percentage in the system we can approach the social cost optimum in the different settings of the *generation flexibility* availability. *Low GF* and *High GF* correspond to the low and high values of the  $\bar{G}_n \forall n \in \mathcal{N}$  respectively. On the horizontal axis we put the percentage  $R_{res}$  of the RES-based generation in the network, which varies in the intervals that ensure that the problem is feasible in all settings considered. For the *Low GF* framework, the problem becomes

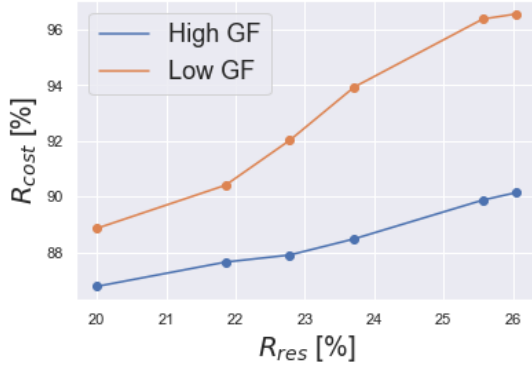


FIGURE 4.2 – Flexible generation

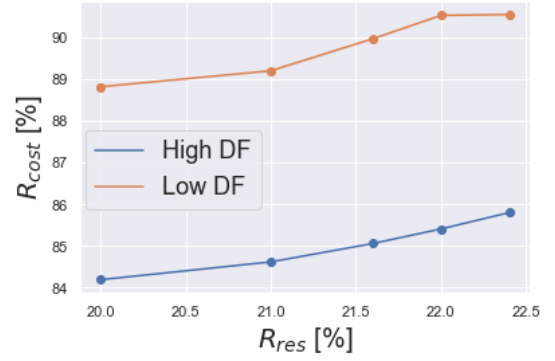


FIGURE 4.3 – Flexible demand

infeasible for  $R_{res} > 23\%$  (or  $R_{res} > 27\%$ ) as there is not enough flexible resource in the nodes to avoid congestion. First, note, that the efficiency of the GNE in the *High GF* is lower, which is caused by the bigger feasible sets, thus, the increased ability of the agents to act selfishly and deviate more from the SC optimum. Increasing the amount of RES generation available affects the *Low GF* the most : as soon as the RES generation of the agents increases, agents obtain more freedom and are able to better adjust their generation/trades, which leads to the increase in the efficiency with a high growth rate. *High GF* efficiency growth is much less affected by the RES injection, because the agents already have had a lot of freedom for their actions even on the low values of  $R_{res}$ .

Similar behavior is observed when we investigate the effects of the *demand flexibility* on the efficiency of the system. Figure 4.3 shows the changes in the efficiency for the *High DF* and *Low DF* settings w.r.t.  $R_{res}$ . Again, more flexibility in the decisions of the agents leads to the lowest values of the efficiency of the system and lower level of the flexibility induces the highest efficiency growth rate w.r.t.  $R_{res}$ .

**RES penetration and pricing functions** We investigate how the form of the function  $f(\cdot)$  affects the total cost of the agents. For our experiments we choose  $f(\cdot)$  to be monotonically increasing, continuous function : we take  $f(x) = \sqrt{x}$  and  $f(x) = x$ . Figures 4.4 and 4.5 illustrate how the total costs of all agents and DSO change when we increase the RES penetration. On the y-axis we put the ratio between the costs at the given value of  $R_{res}$ , divided by the first entry, i.e. the cost at a value  $R_{res} = 20\%$ .

The decrease of the agents' and DSO costs shows the same tendency for both of the functions  $f(x) = \sqrt{x}$  and  $f(x) = x$  on Figures 4.4 and 4.5. While the decrease rate of the DSO cost is higher for  $f(x) = \sqrt{x}$  it is the opposite for the agents' cost. In this example  $f(x) = \sqrt{x}$  seems to be a good choice for the DSO, while bringing some disadvantages for the agents. As it is illustrated in this example, it is important to design a suitable  $f(\cdot)$  which would benefit both layers of the model, thus calling for a further research.

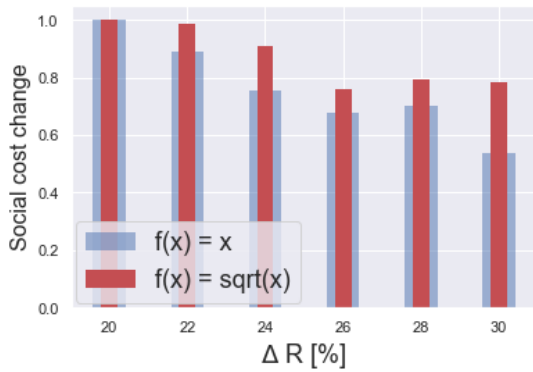


FIGURE 4.4 – Agents' cost changes

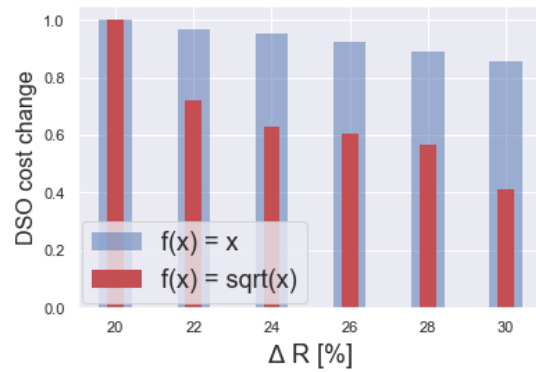


FIGURE 4.5 – DSO cost changes

## 4.6 Conclusion

We formulate a generalized Nash equilibrium problem which models the interaction between the financial peer-to-peer electricity market level and the physical level (distribution grid) operated by the DSO. We provide characterization of the GNE under different designs of the financial level prosumers' market. We discuss the effects of the trading cost form on the equilibria, focusing on properties such as Pareto efficiency, no free-lunch behavior and Lagrangian multipliers pricing interpretation.

As a future research direction, we will investigate formally the impact of the functions used to compute the modified nodal prices that the DSO uses to charge the prosumers. Another interesting direction is to consider the different physical layer models, e.g. second cone order programming (SOCP). It is interesting to provide the bound on the Price of Anarchy to evaluate the efficiency loss caused by the decentralization of the decisions in the financial level.

# Chapitre 5

## Privacy Impact

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# Privacy Impact on Generalized Nash Equilibrium in Peer-to-Peer Electricity Market

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## 5.1 Introduction

Within the peer-to-peer electricity market, agents negotiate their energy procurement seeking to minimize their costs with respect to both individual and coupling constraints, while preserving a certain level of privacy [26]. In this chapter the problem is modeled as a generalized Nash equilibrium problem (GNEP), parametrized in the privacy level, chosen by the agents.

Information sharing in the peer-to-peer market can improve agents' performance, but also may violate their privacy, leading to the disclosure of agent's private information [154]. This calls for the design of new communication mechanisms that capture the agents' ability to define the information they want to share (their report) with the other market participants, while preserving their privacy [51]. In many applications, this problem is usually addressed by including noise to the reports that the agents subsequently use to compute the market equilibrium [26]. However, this approach does not include the ability of the agents to act strategically on the values of their report. Moreover, the question of the optimal noise distribution is crucial in such a framework [97].

To analyse the market in presence of shared coupling constraints, we employ Generalized Nash Equilibrium (GNE) as solution concept [74], and a refinement of it, called Variational Equilibria (VE), assuming the shadow variables associated with the shared coupling constraints are aligned among the agents. In our proposed framework, agents compute GNE with respect to the constraints that bound (a) the distance between the deterministic deviation from the true values of the private information and (b) the Kullback-Leibler divergence, that measures the effect of the additive random noise included in the reports.

Game theoretic approaches integrating the prosumers' strategic behaviors in the peer-to-peer trading are considered in [26], [14]. The economic dispatch in energy communities under different structures of communications is analysed in [95], [96]. The impact of privacy on an energy community was analyzed in the literature, e.g. in [26], where the sensitive information and the noise added to agents' reports were considered as exogenous parameters. Using a prediction model, Fioretto et al. provide a privacy-preserving mechanism, to protect the information exchanged between the different market operators while guaranteeing their coordination [51].

The anticipation of the actions of the agents in our model is represented by the common knowledge of the *form of the solution*. This anticipation will be used in strategic behavior framework to compute the prosumers' optimal deviation in their private information reports.

Various definitions of privacy have been introduced in the data science literature [56]. Several information metrics : e.g., mutual information, entropy, Kullback-Leibler divergence, and Fisher information are used to quantify information release [50], [97]. Differential privacy (DP) was recently successfully applied to multi-energy market operations [51] and dynamical systems [97]. DP relies on adding noise to the reports from predetermined distributions. In our model we also use the additive noise, but we relax the assumptions of DP mechanism and focus on the prosumers' ability to determine their noise distribution. It is done by bounding the the expectation of the *privacy loss random variable* [9], which constitutes exactly the Kullback-Leibler divergence for the introduced privacy-preserving randomized mechanism.

Similarly to local DP, in our model we assume that additive noise is added locally and independently by each agent, but we relax the assumptions of DP mechanism and assume that the prosumers have the ability to determine their noise distribution and act strategically on the values of their reports. Similarly to Eilat et al. [41], we bound the expectation of the *privacy loss random variable* [9], which constitutes the Kullback-Leibler divergence for the introduced privacy-preserving randomized mechanism.

Allowing the agents to control their privacy level raises questions about the trade-off between privacy preservation and utility maximization both at the agents and at the system (market) levels. This research track has been widely analyzed in the literature [83]. In [5], Alvim et al. derive bounds both on the information leakage and utility of the agents under differential privacy. In [24], Boursier and Perchet address this question by the means of the Optimal Transport while including the Kullback-Leibler divergence term in the utility of the agents.

### 5.1.1 Contributions

We relate the notion of privacy preservation resulting from the non-disclosure of the nominal demands and RES-based generations of the prosumers in [26], to the privacy mechanism with the additive Gaussian noise, that allows each agent to control her privacy level. It is done, firstly, by choosing the deterministic value to report to other agents; secondly, by using the random noise added to that value. We quantify the impact of privacy on the prosumers' costs and provide an analytical expression of the market equilibria. In addition, we allow each agent to change her level of privacy and show the existence of the incentives for the prosumers to deviate from their true sensitive parameter values. We rely on the notion of strong monotonicity to prove the existence and uniqueness of the solution to our problem. Using Kullback-Leibler divergence, we measure the cost of privacy, caused by inclusion of the random



noise. All the theoretical results are illustrated on the 14-bus IEEE network.

The organization of the rest of this chapter is as follows : in section 5.2 we first describe the peer-to-peer electricity trading problem in Subsections 5.2.1 and 5.2.2, which constitutes a basis for our communication game, that will be defined in Subsection 5.2.4. In section 5.3 we provide the analytical expression of the GNE, prove the uniqueness of the VE of the game and show that under mild conditions, the communication game has a potential form. We illustrate our theoretical results on the 14-bus IEEE network and discuss privacy-utility relation in section 5.4.

## 5.2 Statement of the problem

### 5.2.1 Preliminaries

In distributed control systems there is a usual trade-off between privacy and cost : to obtain a better solution, each agent relies on the information of the other agents in the system, which they might not have incentives to provide.

Consider a single-settlement market for peer-to-peer electricity trading made of a set  $\mathcal{N}$  of  $N$  agents, each one of them being located in a node of a communication network, that is modeled as a graph  $G := (N, E)$  where  $E \subseteq N \times N$  is the set of communication links between the players. Let  $\Omega_n$  be the set of nodes, player  $n$  wants to trade electricity with. Being the interface node between the local electricity market and at the distribution level and the transmission power network, node 0 can communicate with any other nodes in  $\Omega_0 := \mathcal{N} \setminus 0$ . The graph  $G$  does not necessarily reflect the distribution power network constraints.

In this chapter we focus on the privacy issues that arise after solving the peer-to-peer electricity trading problem [26].

Each agent  $n$  chooses independently her bilateral trades  $\mathbf{q}_n$  with agents she wants to trade electricity with, self-generation  $G_n$  and flexible demand  $D_n$ , in order to minimize her cost function  $\Pi_n$  :

$$\begin{aligned} \Pi_n(D_n, G_n, \mathbf{q}_n) := & \underbrace{1/2 \cdot a_n G_n^2 + b_n G_n + d_n}_{C_n(G_n)} + \\ & + \underbrace{\tilde{a}_n (D_n - D_n^*)^2 - \tilde{b}_n}_{U_n(D_n)} + \underbrace{\sum_{m \in \Omega_n, m \neq n} c_{nm} q_{mn}}_{\tilde{C}_n(\mathbf{q})}, \end{aligned} \quad (5.1)$$

where  $a_n, b_n, d_n, \tilde{a}_n, \tilde{b}_n > 0$  and  $D_n^*$  denotes the *nominal demand* of agent  $n$  [26]. Thus, the vector of agent  $n$ 's decision variables is  $(D_n, G_n, \mathbf{q}_n)$ , where  $\mathbf{q}_n := (q_{mn})_{m \in \Omega_n}$  is the vector of the quantities

exchanged between  $n$  and  $m$  in the direction from  $m$  to  $n$ ,  $q_{mn}$ , for all  $m \in \Omega_n \setminus \{n\}$ . We use the following convention : if  $q_{mn} \geq 0$ , then  $n$  buys  $q_{mn}$  from  $m$ , otherwise ( $q_{mn} < 0$ )  $n$  sells  $-q_{mn}$  to  $m$ . We let  $Q_n$  denote the *net import* of agent  $n$  :  $Q_n := \sum_{m \in \Omega_n} q_{mn}$ .

Each agent computes trading cost  $\tilde{C}_n(\mathbf{q})$  using  $c_{nm}$  which might represent preferences measured through product differentiation prices [26], [137] on the possible trades with the neighbors, or taxes. The following condition on agent's trades called *trading reciprocity constraint* couples the decisions of two neighboring agents, ensuring for every node  $m \in \Omega_n$  that  $q_{mn} + q_{nm} = 0$ . Note, that this formulation of the coupling constraints differs from the one presented in [26], as we use equality constraint in our model, instead of the inequality. That means that the energy surplus is not allowed in the electricity trading model. Let  $\kappa_{nm} \in [0, +\infty)$  be the equivalent trading capacity between node  $n$  and node  $m$ , such that  $\kappa_{nm} = \kappa_{mn}$  and  $\forall m \in \Omega_n$ . This equivalent trading capacity is used to bound the trading flows such that  $q_{mn} \leq \kappa_{mn}$ .

Local supply and demand should satisfy the following balance equality in each node  $n$  in  $\mathcal{N}$  :  $D_n = G_n + \Delta G_n + \sum_{m \in \Omega_n} q_{mn}$ , where  $\Delta G_n$  is the *renewable energy sources (RES)-based generation* at node  $n$ , assumed to be non-flexible.

### 5.2.2 Electricity trading problem

As it was discussed in the introduction, each agent holds some private information that takes the form of nominal demand  $D_n^*$  and RES-based generation  $\Delta G_n$ , which she does not desire to reveal to the other agents in the system. In the further analysis, we assume  $y_n := D_n^* - \Delta G_n$  to be the private information of agent  $n$ . We assume that the agents desire to solve the electricity trading problem endowed with the set of coupled constraints while not allowing the other agents to infer their values of  $y_n$ . We denote  $\mathbf{x}_n := (D_n, G_n, \mathbf{q}_n)$  to be the vector that contains agent  $n$ 's decision variables and  $\mathbf{x}_{-n}$  is the vector of the other agents' actions. We recall the optimization problem formulated in [26] for the clearing of the peer-to-peer electricity market.

**Peer-to-peer market design** In the peer-to-peer setting the problem of the electricity trading takes the form of generalized Nash equilibrium problem i.e., a game where the feasible sets of the players depend on the other players' actions. With the notation introduced above, it means that each agent solves the

following optimization problem :

$$\min_{\mathbf{x}_n} \quad \Pi_n(\mathbf{x}_n), \quad (5.2a)$$

$$s.t. \quad \underline{G}_n \leq G_n \leq \overline{G}_n \quad (\underline{\mu}_n, \overline{\mu}_n) \quad (5.2b)$$

$$\underline{D}_n \leq D_n \leq \overline{D}_n \quad (\underline{\nu}_n, \overline{\nu}_n) \quad (5.2c)$$

$$q_{mn} + q_{nm} = 0 \quad (\zeta_{nm}) \quad (5.2d)$$

$$q_{mn} \leq \kappa_{mn} \quad (\xi_{nm}) \quad (5.2e)$$

$$D_n = G_n + \Delta G_n + \sum_{m \in \Omega_n} q_{mn} \quad (\lambda_n), \quad (5.2f)$$

where the corresponding dual variables are placed in blue at the right of each constraint. Note that in (5.2) the feasible set of the agent  $n$  can be rewritten in a more compact form  $\mathcal{C}_n(\mathbf{x}_{-n}) = \{\mathbf{x}_n | (5.2b) - (5.2f) \text{ hold}\}$ . This notation will be used later in the chapter.

We introduce the following assumption to guarantee that the interface trading capacities are big enough to supply trading needs of all the agents and that differentiation prices are symmetric for trading with the root node.

**Assumption 43** *We assume that there are large trading capacities from and to node 0 – that is  $\xi_{0n} = \xi_{n0} = 0 \forall n \in \mathcal{N}$  and  $c_{n0} = c_{0n}$  for all  $n \in \mathcal{N}$ .*

The following subsection describes the computation of  $\tilde{C}_n(\mathbf{q})$  in the different setting for the differentiation prices.

### 5.2.3 Computation of the trading cost

Under the conditions of Assumption 43, Proposition 8 in [26] states that :

**Proposition 44** *For any couple of nodes  $n \in \mathcal{N}, m \in \Omega_n, m \neq n$  with asymmetric preferences (such as  $c_{mn} > c_{nm}$  or  $c_{mn} < c_{nm}$ ) imply that the node with the smaller preference for the other saturates the line.*

We focus on two opposite instances :

**1. All  $c_{nm}$  are homogeneous** That means that  $c_{nm} = c$  for all  $n, m \in \mathcal{N}$ . This case reflects the interpretation of  $c_{nm}$  as the taxes for energy trading, that should be naturally non-discriminating among agents. In this case bilateral trade cost is given by :

$$\tilde{C}_n(\mathbf{q}_n) = c \cdot Q_n \quad (5.3)$$

**2. All  $c_{nm}$  for  $m, n \neq 0$  are heterogeneous** This framework represents the case, when all  $c_{nm}, m, n \neq 0$  are drawn from some continuous distribution (e.g. uniform). Under the Assumption 43 we are able to obtain the expressions for  $\tilde{C}_n(\mathbf{q}_n)$  in this framework for agent  $n$ . Again using Proposition 44 we have that  $q_{n0} = Q_n - \sum_{m \in \Omega_n, m \neq 0} \kappa_{nm} \text{sgn}(c_{mn} - c_{nm})$ , where  $Q_n$  is obtained by combining (5.2f) and expressions for  $D_n, G_n$ . Thus, we are able to obtain the cost expressions for each agents  $n$  directly :

**Proposition 45** *Bilateral trade costs for any agent  $n \in \mathcal{N}$  in the network except root node 0 are given by*

$$\begin{aligned} \tilde{C}_n(\mathbf{q}_n) &= c_{0n} \left[ Q_n - \sum_{m \in \Omega_n, m \neq 0} \kappa_{nm} \text{sgn}(c_{mn} - c_{nm}) \right] \\ &+ \sum_{k \in \Omega_n, k \neq 0} c_{nk} \kappa_{nk} \text{sgn}(c_{kn} - c_{nk}). \end{aligned} \quad (5.4)$$

*Bilateral costs for node 0 are expressed as*

$$\tilde{C}_0(\mathbf{q}_0) = \sum_{n \in \Omega_0} c_{0n} \left[ \sum_{m \in \Omega_n, m \neq 0} \kappa_{nm} \text{sgn}(c_{mn} - c_{nm}) - Q_n \right] \quad (5.5)$$

**Remark 46** *Note, that we do not impose any condition on the ratio between the values of the coefficients  $c_{nm}$  : that is, choosing  $c_{0n} < c_{mn}, \forall m, n \in \mathcal{N}$ , we can ensure the preference for the local trades.*

**3. Intermediate case** To demonstrate the difficulties arising in the general case for computing bilateral trades, we consider the intermediate case, in which there exists **one** additional symmetric relation  $c_{n'm'} = c_{m'n'}$  for  $m', n' \neq 0$ . Thus, for this pair of nodes we have that

$$\begin{aligned} Q_{n'} &= q_{0n'} + q_{m'n'} + \sum_{k \neq m' \in \Omega_{n'}} \kappa_{n'k} \text{sgn}(c_{kn'} - c_{n'k}) \\ Q_{m'} &= q_{0m'} + q_{n'm'} + \sum_{k \neq n' \in \Omega_{m'}} \kappa_{m'k} \text{sgn}(c_{km'} - c_{m'k}), \end{aligned}$$

where  $q_{m'n'} = -q_{n'm'}$ , which gives us a system of two equations with three unknown variables  $q_{0n'}, q_{0m'}, q_{m'n'}$ . Writing the similar equation for every node  $k \neq m', n', 0$ , we get  $N - 3$  equations with  $N - 3$  unknowns and adding the expression for  $Q_0$  we obtain linear system with  $N$  independent equations and  $N$  unknown variables. It follows that adding even one symmetric relation leads to the system of  $N$  equations with  $N + 1$  unknowns.

**Link between electricity trading and communication game** It is shown in [26], that at the VE, agent  $n$ 's decision variables  $\mathbf{x}_n^*$  depend on the dual variable  $\lambda_n$ , which, under the Assumption 43 is aligned across agents :  $\lambda_n = \lambda_0, \forall n \in \mathcal{N}$ , where  $\lambda_0$  is the *uniform market clearing price*. The equilibrium

expressions, provided in [26], also hold for our model with equality constraint (5.2d).  $\lambda_0$  depends on the private information  $y_n$  of the agents. Formally,  $\lambda_0$  is given by :

$$\lambda_0 = \frac{\sum_n y_n + \sum_n \frac{b_n}{a_n}}{\sum_n \left( \frac{1}{2\bar{a}_n} + \frac{1}{a_n} \right)} \quad (5.6)$$

and the decision variables  $D_n$  and  $G_n$  are given at the equilibrium by the following expressions :  $D_n(\mathbf{y}) = D_n^* - \frac{1}{2\bar{a}_n} \lambda_0$ ,  $G_n(\mathbf{y}) = -\frac{b_n}{a_n} + \frac{1}{a_n} \lambda_0$ . The expression for  $Q_n$  is obtained from the supply demand equality condition (5.2f) :  $Q_n(\mathbf{y}) = D_n^* + \frac{b_n}{a_n} - \left( \frac{1}{a_n} + \frac{1}{2\bar{a}_n} \right) \lambda_0 - \Delta G_n$ .

Thus, to solve (5.2) each agent needs to compute the uniform market clearing price  $\lambda_0$ , which requires a knowledge of all the  $(y_n)_n$  in the system. It leads to a question for each agent  $n$  of how to determine the report of her private information, so that it has the minimal impact on her cost, while guaranteeing that the certain level of privacy is met. That is, each agent  $n$  anticipates the form of the solution of the electricity trading problem at the equilibrium and determines the report  $\tilde{y}_n$  of her private information, that she submits to the other agents in the system.

In order to do so, each agent  $n$  minimizes the difference between the cost of the problem with the modified values and the optimal solution of the problem (5.2) with the truthful reports  $\Pi_n^*$  :

$$\begin{aligned} \min_{\tilde{y}_n} \quad & \mathbb{E} \left[ \Pi_n(\tilde{y}_n, \tilde{\mathbf{y}}_{-n}) - \Pi_n^* \right], \\ \text{s.t.} \quad & \mathbf{x}_n^*(\tilde{\mathbf{y}}) \in \mathcal{C}_n(\mathbf{x}_{-n}^*(\tilde{\mathbf{y}})), \end{aligned} \quad (5.7)$$

where the expectation is taken in order to account for both randomized and deterministic cases. Note that  $\mathbf{x}_n^*$  depends on  $\tilde{\mathbf{y}}$  because in the expressions for the decision variables  $D_n(\cdot)$ ,  $G_n(\cdot)$  and  $q_n(\cdot)$  we use the reports  $\tilde{\mathbf{y}}$  instead of the true values  $\mathbf{y}$  as the input. Also note that  $\Pi_n^*$  is a constant as it is calculated using true values of  $\mathbf{y}$ , thus it can be omitted from the objective function.

**Remark 47** In (5.7) we assume that the form of the electricity trading problem is known by all the agents in the system. It enables each agent to anticipate the **form of the solution**  $x_n^*(\cdot)$ , for all  $n \in \mathcal{N}$  and thus, based on this form to decide on the optimal information  $\tilde{y}_n, \forall n \in \mathcal{N}$  to report to the other agents **before** they actually obtain the solution of the electricity trading problem. Note that it differs from [51], as we take the form of the solution  $\mathbf{x}^*$  of the GNEP as given.

#### 5.2.4 Communication game

The report of the agent  $n$  takes the form  $\tilde{y}_n = \hat{y}_n + \varepsilon_n$ . The first part of the report captures the ability of agent  $n$  to act strategically on her report by determining the deterministic part  $\hat{y}_n$  that solves the cost

minimization problem. In the second part of the report, each agent implements a randomized mechanism  $M(\cdot)$  by choosing the noise  $\varepsilon_n$  to add to  $\hat{y}_n$  in order to preserve a certain level of privacy.

**Privacy loss definition** First, we define an upper bounded distance as a symmetric *adjacency relation*  $y_n \simeq \hat{y}_n$  for agent  $n$  :  $y_n \simeq \hat{y}_n \iff d(y_n, \hat{y}_n) \leq \alpha_n$ , where  $\alpha_n$  is chosen beforehand and reflects the amount of information agent  $n$  desires to preserve [29].

**Definition 48 (Privacy loss)** Given a randomized mechanism  $M$ , let  $p_{M(y_n)}(z)$  denote the density of the random variable  $Z = M(y_n)$ . The privacy loss function of  $M(\cdot)$  on a pair of  $y_n \simeq \hat{y}_n$  is defined as  $l_{M, y_n, \hat{y}_n}(z) = \log(p_{M(y_n)}(z)/p_{M(\hat{y}_n)}(z))$ . The privacy loss random variable  $L_{M, y_n, \hat{y}_n} := l_{M, y_n, \hat{y}_n}(Z)$  is the transformation of the output random variable  $Z = M(y_n)$  by the function  $l_{M, y_n, \hat{y}_n}$ .

We assume that each agent samples a Gaussian noise  $\varepsilon_n \sim \mathcal{N}(0, \sigma_n^2)$ , thus obtaining the report  $\tilde{y}_n \sim \mathcal{N}(\hat{y}_n, \sigma_n^2)$ .

When the Gaussian isotropic random noise is added to the deterministic value of the input, it is well-known that the privacy loss random variable is also Gaussian :

**Lemma 49 ([9])** The privacy loss  $L_{M, y_n, \hat{y}_n}$  of a Gaussian output perturbation mechanism follows a distribution  $\mathcal{N}(\eta, 2\eta)$ , with  $\eta = D^2/2\sigma^2$ , where  $D = \|y_n - \hat{y}_n\|$ .

**A randomized mechanism for information reporting** We aim to allow agents to be able to decide on the optimal noise added to their private information, by choosing the optimal variance  $V_n$ . For simplicity of notations, we denote  $V_n := \sigma_n^2$ .

First, each agent chooses the neighboring input  $\hat{y}_n \simeq y_n$ , on which she later implements  $M(\cdot)$ . It is reflected in the constraint (5.8d). In the constraint (5.8e), the expectation of the privacy loss random variable measures the expected privacy loss of the mechanism  $M(y_n)$  on the fixed private information  $y_n, \hat{y}_n$ . In other words, it shows, how much information can be extracted from the report  $\tilde{y}_n$ . Note, that it is exactly the Kullback-Leibler divergence (or the relative entropy) between  $M$ 's output distributions on  $y_n$  and  $\hat{y}_n$ .

Thus, to decide on the optimal value of the report  $\tilde{y}_n$ , each agent needs to solve the following opti-

mization problem :

$$\min_{\hat{y}_n, V_n} \mathbb{E}_{\varepsilon_n \sim \mathcal{N}(0, V_n)} [\Pi_n(\hat{\mathbf{y}}, \varepsilon)] \quad (5.8a)$$

$$s.t. \quad \underline{G}'_n \leq \mathbb{E}_{\varepsilon_n \sim \mathcal{N}(0, V_n)} [G_n(\tilde{\mathbf{y}})] \leq \overline{G}'_n \quad (\underline{\mu}_n, \overline{\mu}_n) \quad (5.8b)$$

$$\underline{D}'_n \leq \mathbb{E}_{\varepsilon_n \sim \mathcal{N}(0, V_n)} [D_n(\tilde{\mathbf{y}})] \leq \overline{D}'_n \quad (\underline{\nu}_n, \overline{\nu}_n) \quad (5.8c)$$

$$(\hat{y}_n - y_n)^2 \leq \alpha_n^2 \quad (\underline{\gamma}_n, \overline{\gamma}_n) \quad (5.8d)$$

$$\mathbb{E}[L_{M, y_n, \hat{y}_n}] \leq A_n \quad (\underline{\beta}_n, \overline{\beta}_n) \quad (5.8e)$$

where  $\underline{G}'_n = \underline{G}_n + \omega_{G_n}$ ,  $\overline{G}'_n = \overline{G}_n - \omega_{G_n}$  and  $\underline{D}'_n = \underline{D}_n + \omega_{D_n}$ ,  $\overline{D}'_n = \overline{D}_n - \omega_{D_n}$ , in which  $\omega_{D_n}, \omega_{G_n} > 0$  are introduced in order to account for the strictly feasible solutions of problem (5.2). In the numerical experiments we set  $\omega_{D_n}, \omega_{G_n}$  to be a small, e.g.  $10^{-3}$ .

As it is shown below, the only term depending on the variance in the utility function of the agent  $n$  is  $\frac{B_n}{B} \sum_m V_m$ , where we denote  $B_n := \frac{1}{a_n} + \frac{1}{2\hat{a}_n}$  and  $B := \sum_n B_n$ . In the special case  $y_n = \hat{y}_n$  for some  $n$ , where the constraint (5.8e)  $\frac{(\hat{y}_n - y_n)^2}{2V_n} \leq A_n$  holds for any  $0 < V_n < \infty$ . The possible convention could be to exclude this constraint from the consideration, when  $y_n = \hat{y}_n$  and set  $V_n = 0$ .

The condition for the uniform market clearing price  $\lambda_0$  to have a form given in (5.6) is to have zero total net import, i.e.  $\sum_n Q_n = 0$ . In the case a *fully coordinated mechanism* is implemented, i.e the local Market Operator has an access to all the constraints and parameters of the agents and solves the problem in a centralized way, it is possible to oblige agents to align their reports  $\tilde{\mathbf{y}}$  such that  $\mathbb{E}[\sum_n Q_n(\tilde{\mathbf{y}})] = \sum_n (y_n - \hat{y}_n) = 0$ . It follows that  $\sum_n \hat{y}_n = \sum_n y_n$ . So, when we compute  $\lambda_0$  using  $\tilde{\mathbf{y}}$  instead of  $\mathbf{y}$ , we obtain  $\mathbb{E}[\lambda_0(\tilde{\mathbf{y}})] = \frac{1}{B} (\sum_n \hat{y}_n + \sum_n \frac{b_n}{a_n}) = \frac{1}{B} (\sum_n y_n + \sum_n \frac{b_n}{a_n})$ . Thus, the final market clearing price does not depend on the reports of the agents, which is formalized in the following statement :

**Proposition 50** *When the prosumers align their reports  $\tilde{\mathbf{y}}$  so that the condition  $\mathbb{E}[\sum_n Q_n(\tilde{\mathbf{y}}_n)] = 0$  is met, then the uniform market clearing price  $\lambda_0$  depends only on the true values of their initial parameters  $\mathbf{y}$ .*

In the case a peer-to-peer communication mechanism is implemented, the sum of the net imports at each node might not be equal to zero. Indeed, agents might have incentives to violate this condition in order to decrease their costs. Thus, the condition  $\sum_n Q_n = 0$  might not hold.

On the market level it is necessary for the condition of zero total net import to hold such that supply and demand balance each other in problem (5.2) [96] in order to preserve the balance of the market. Also, note that non zero total net import  $\sum_n Q_n \neq 0$  implies that there exists at least one pair of agents  $(n, m)$  with  $q_{nm} + q_{mn} \neq 0$ . Besides, this might cause the violation of the capacity condition  $q_{nm} \leq \kappa_{nm}$ .

It means that the local MO has to compensate the difference  $\mathbb{E}[\sum_n Q_n(\tilde{\mathbf{y}}_n)]$  caused by the lack of coordination in the agents' reports. In the case  $\mathbb{E}[\sum_n Q_n(\tilde{\mathbf{y}}_n)] \leq 0$ , there is an energy surplus in the system, which can be sold by the MO (by the intermediate of an aggregator) to the wholesale market at price  $p^0$ . If  $\mathbb{E}[\sum_n Q_n(\tilde{\mathbf{y}}_n)] \geq 0$ , then the MO (by the intermediate of an aggregator) has to buy the energy on the wholesale market at price  $p^0$ , which depends on the wholesale market price, in order to supply the system demand.

When the constraints and the private information of the agents are not shared, the MO only knows the aggregate deviation  $\sum_n (y_n - \hat{y}_n)$  thus penalties imposed on the agents depend on it and not on the personal deviation  $y_n - \tilde{y}_n$  of the agent  $n$ .

**Remark 51** For prosumers, imports/exports of energy from/to the community manager are possible at prices  $p^-/p^+$  respectively such that  $p^+ \leq p^0 \leq p^-$ . To avoid non-differentiability in the utility function, we let  $p^+ = p^0 = p^-$ .

To compensate for the cost of buying the lack of energy at the local market level from the wholesale market, the MO imposes penalty to each prosumer that takes the form  $P(\tilde{\mathbf{y}}) = \frac{p^0}{N} \sum_n (y_n - \tilde{y}_n)$ . Note that in case of the excess of the production on the local market level, the prosumers will be equally reimbursed based on the surplus produced. The division by  $N$  is introduced in order to equally split the burden of the non zero total net import and mitigate the possible volatility of the price  $p^0$ .

**Assumption 52** A local MO ensures the compensation for the nonzero total net import. This implies that the formula for  $\lambda_0$  in (5.6) is used by all the prosumers to compute their decision variables.

**Proposition 53** Dual variables  $\underline{\beta}_n, \bar{\beta}_n$  for the constraint (5.8e) can be interpreted as the privacy price for agent  $n$  and are computed by the formula

$$(\bar{\beta}_n + \underline{\beta}_n)^2 = \frac{B_n^2 (\hat{y}_n - y_n)^2}{4B^4}$$

**Proof.** Constraint (5.8e) can be rewritten as follows, when we consider  $V_n \neq 0$  for all  $n \in \mathcal{N}$  :  $\mathbb{E}[L_{M,y_n,\hat{y}_n}] \leq A_n \iff (\hat{y}_n - y_n)^2 \leq 2V_n A_n$ . In the following analysis, we denote  $B_n := \frac{1}{a_n} + \frac{1}{2\hat{a}_n}$  and  $B := \sum_n B_n$ . The objective function (5.8a) of the agent  $n$  depends linearly on the  $V_n$ , thus attaining the minimum with respect to this decision variable on the lower boundary of the feasible region. The lower boundary is given by the constraint (5.8e), from which we can conclude that  $V_n = \frac{(\hat{y}_n - y_n)^2}{2A_n}$  for any given value of the decision variable  $\hat{y}_n$  for any agent  $n$ . From the KKT conditions we have that  $V_n = \frac{2B^4}{A_n B_n^2} (\bar{\beta}_n + \underline{\beta}_n)^2$ , from which we obtain the expression for  $(\bar{\beta}_n + \underline{\beta}_n)^2$ .



First, from the complementarity conditions we know that either of  $\underline{\beta}_n, \bar{\beta}_n$  equals 0. Clearly, it is non-negative term that appears in the utility of the agent  $n$  at the equilibrium. Thus, we can view  $\underline{\beta}_n, \bar{\beta}_n$  as a *privacy price*. ■

**Remark 54** *The privacy price increases with respect to the distance between the truthful ( $y_n$ ) and biased ( $\hat{y}_n$ ) values of agent  $n$ 's private information.*

## 5.3 Equilibrium problems

### 5.3.1 Aggregate game formulation

Note that as  $\lambda_0$  depends on the sum of  $\sum_n (D_n^* - \Delta G_n)$ , the objective function in (5.8) has an *aggregate game* structure, i.e. it depends on player  $n$ 's decision  $\hat{y}_n$  and on the aggregate of the other agents' decisions.

Below we provide the computations of the objective function of agents  $\Pi_n(\tilde{y}_n, \hat{y}_{-n})$  both in (i) the fully coordinated mechanism and (ii) the peer-to-peer coordination mechanism.

To arrive to this closed form expression, we observe that  $\tilde{y}_n \sim \mathcal{N}(\hat{y}_n, V_n)$ . The sum of normal variables is a normal variable itself :  $\sum_n \tilde{y}_n \sim \mathcal{N}(\sum_n \hat{y}_n, \sum_n V_n)$ , from where it follows that  $(\sum_n \tilde{y}_n + \sum_n \frac{b_n}{a_n}) \sim \mathcal{N}(\sum_n \hat{y}_n + \sum_n \frac{b_n}{a_n}, \sum_n V_n)$ . Using a formula for the second moment of the normal distribution, expression (5.3) and Proposition 45, we obtain the expression for the utility of the agents in cases (i) and (ii).

In the homogeneous differentiation price case  $c_{nm} = c$ , the cost function of agent  $n$  is given by

$$\begin{aligned} \mathbb{E}[\Pi_n(\tilde{\mathbf{y}})] &= \frac{B_n}{2B^2} \left[ \left( \sum_m \hat{y}_m + \sum_m \frac{b_m}{a_m} \right)^2 + \sum_m V_m \right] \\ &+ c \left[ D_n^* - \Delta G_n + \frac{b_n}{a_n} - \frac{B_n}{B} \left( \sum_m \hat{y}_m + \sum_m \frac{b_m}{a_m} \right) \right] \\ &+ \frac{p^0}{N} \sum_m (y_m - \hat{y}_m) - \frac{b_n^2}{2a_n} + d_n - \tilde{b}_n. \end{aligned}$$

Expression for the utility in the case when  $c_{nm}$  are heterogeneous is similar, except that for  $\tilde{C}_n(\mathbf{q}_n)$ , we use the expressions from Proposition 45.

### 5.3.2 GNE computation

**$c_{nm}$  are homogeneous** From the computations of the KKT conditions, we obtain that  $\frac{B_n}{B^2} \sum_{m \in \mathcal{N}} \hat{y}_m + M'_n = 0, \quad \forall n \in \mathcal{N}$ , where  $M'_n := \frac{B_n}{B^2} \sum_m \frac{b_m}{a_m} - \frac{p^0}{N} - \frac{cB_n}{B} + \frac{1}{a_n B} (\bar{\mu}_n - \underline{\mu}_n) + \frac{1}{2\bar{a}_n B} (\underline{\nu}_n - \bar{\nu}_n) + \bar{\gamma}_n - \underline{\gamma}_n + \bar{\beta}_n - \underline{\beta}_n$ .

$c_{nm}$  are heterogeneous for  $m \neq 0$  Analogously, first order stationarity conditions for agents  $n \in \mathcal{N}$  are given by  $\frac{B_n}{B^2} \sum_{m \in \mathcal{N}} \hat{y}_m + M_n'' = 0$ , with  $M_n'' := \frac{B_n}{B^2} \sum_n \frac{b_n}{a_n} - c_{0n} \frac{B_n}{B} - \frac{p^0}{N} + \frac{1}{a_n B} (\bar{\mu}_n - \underline{\mu}_n) + \frac{1}{2\bar{a}_n B} (\underline{\nu}_n - \bar{\nu}_n) + \bar{\gamma}_n - \underline{\gamma}_n + \bar{\beta}_n - \underline{\beta}_n$  and  $M_0'' = \frac{B_0}{B^2} \sum_n \frac{b_n}{a_n} + \frac{1}{a_0 B} (\bar{\mu}_0 - \underline{\mu}_0) + \sum_{n \neq 0} c_{0n} \frac{B_n}{B} - \frac{p^0}{N} + \frac{1}{2\bar{a}_0 B} (\underline{\nu}_0 - \bar{\nu}_0) + \bar{\gamma}_0 - \underline{\gamma}_0 + \bar{\beta}_0 - \underline{\beta}_0$ .

### 5.3.3 Uniqueness of the Variational Equilibrium

**Definition 55** An operator  $F : K \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^n$  is strongly monotone on the set  $\hat{K} \subseteq K$  with monotonicity constant  $\alpha > 0$  if  $(F(x) - F(y))^\top (x - y) \geq \alpha \|x - y\|^2$ ,  $\forall x, y \in \hat{K}$ . The operator is monotone if  $\alpha = 0$ .

In order to show the uniqueness of the VE of the problem (5.8), we check if the operator

$$F(\hat{\mathbf{y}}, \mathbf{V}) := [\nabla_n \mathbb{E}(\Pi_n(\hat{\mathbf{y}}, \mathbf{V}))]_{n=1}^N \quad (5.9)$$

is strongly monotone. For homogeneous differentiation price  $c_{nm} = c$ ,  $\mathbf{F}(\hat{\mathbf{y}}, \mathbf{V})$  writes as follows :

$$\mathbf{F}(\hat{\mathbf{y}}, \mathbf{V}) = \text{col} \left\{ \left( \frac{B_i}{B^2} \left( \sum_m \hat{y}_m + \sum_m \frac{b_m}{a_m} \right) - c \frac{B_i}{B} - \frac{p^0}{N}, \frac{V_i B_i}{B^2} \right)_{i=0}^{N-1} \right\} \quad (5.10)$$

When differentiation prices  $c_{nm}$  are heterogeneous for  $m \neq 0$ , operator  $\mathbf{F}(\hat{\mathbf{y}}, \mathbf{V})$  is obtained similarly, but for the expressions of  $\tilde{C}_n(\mathbf{q}_n)$ , we take expressions from Proposition 45. To do so, we use the following lemma :

**Lemma 56** ([104]) A continuously differentiable operator  $F : K \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^n$  is  $\alpha$ -strongly monotone with monotonicity constant  $\alpha$  (resp. monotone) if and only if  $\nabla_x F(x) \succeq \alpha I$  (resp.  $\nabla_x F(x) \succeq 0$ ) for all  $x \in K$ . Moreover, if  $K$  is compact, then there exists  $\alpha > 0$  such that  $\nabla_x F(x) \succeq \alpha I$  for all  $x \in K$  if and only if  $\nabla_x F(x) \succ 0$  for all  $x \in K$ .

**Lemma 57** Operator  $\mathbf{F}(\hat{\mathbf{y}}, \mathbf{V})$  defined in (5.10) is strongly monotone.

**Proof.** First, note that for heterogeneous  $c_{nm}$ , operator  $\mathbf{F}(\hat{\mathbf{y}}, \mathbf{V})$  writes as follows for nodes  $i \neq 0$  :

$$\mathbf{F}(\hat{\mathbf{y}}, \mathbf{V}) = \text{col} \left\{ \left( \frac{B_i}{B^2} \left( \sum_m \hat{y}_m + \sum_m \frac{b_m}{a_m} \right) - c_{i0} \frac{B_i}{B} - \frac{p^0}{N}, \frac{V_i B_i}{B^2} \right)_{i=1}^{N-1} \right\}, \quad (5.11)$$

and for node 0 we can write it as follows :

$$\mathbf{F}(\hat{\mathbf{y}}, \mathbf{V}) = \left( \frac{B_0}{B^2} \left( \sum_m \hat{y}_m + \sum_m \frac{b_m}{a_m} \right) + \sum_{m \neq 0} c_{0m} \frac{B_m}{B} - \frac{p^0}{N}, \frac{V_0 B_0}{B^2} \right). \quad (5.12)$$

We want to prove that the operator  $F(\hat{\mathbf{y}}, \mathbf{V})$  defined in (5.10) or in (5.11) and (5.12) is strongly monotone.

We denote vector  $z$  to be  $z := (\hat{y}_0, V_0, \dots, \hat{y}_{N-1}, V_{N-1})$ . We need to investigate whether  $\nabla_z F(\hat{\mathbf{y}}, \mathbf{V})$  is positive-definite. Denote  $F(\hat{\mathbf{y}}, \mathbf{V})_{i1} := \frac{B_i}{B^2} \left( \sum_n \hat{y}_m + \sum_n \frac{b_n}{a_n} \right) - c \frac{B_i}{B} - \frac{p^0}{N}$  for the homogeneous  $c_{nm}$  case, and  $F(\hat{\mathbf{y}}, \mathbf{V})_{i2} = \frac{V_i B_i}{B^2}$ . Similarly, for the heterogenous  $c_{nm}$  case, denote  $F(\hat{\mathbf{y}}, \mathbf{V})_{i1} := \frac{B_i}{B^2} \left( \sum_n \hat{y}_m + \sum_n \frac{b_n}{a_n} \right) - c_{i0} \frac{B_i}{B} - \frac{p^0}{N}$  for  $i \neq 0$  and  $F(\hat{\mathbf{y}}, \mathbf{V})_{01} := \frac{B_i}{B^2} \left( \sum_n \hat{y}_m + \sum_n \frac{b_n}{a_n} \right) + \sum_{m \neq 0} c_{0m} \frac{B_m}{B} - \frac{p^0}{N}$ . Thus we have that  $\frac{\partial F(\hat{\mathbf{y}}, \mathbf{V})_{i1}}{\partial \hat{y}_j} = \frac{B_i}{B^2}$ ,  $\forall j \in \mathcal{N}$  and  $\frac{\partial F(\hat{\mathbf{y}}, \mathbf{V})_{i2}}{\partial V_i} = \frac{B_i}{B^2}$ . All other partial derivatives are 0. Thus  $\nabla_z F(\hat{\mathbf{y}}, \mathbf{V})$  is a matrix defined with its entries to be

$$\nabla_z F(\hat{\mathbf{y}}, \mathbf{V})_{ij} = \begin{cases} \frac{B_{i+2}}{B^2} & \text{if } i, j \text{ are even} \\ \frac{B_{i+1}}{B^2} & \text{if } i, j \text{ are odd and } i = j \\ 0 & \text{otherwise} \end{cases}$$

Symmetric matrix  $A$  is positive definite on compact if its quadratic form is positive :  $x^\top A x > 0$ ,  $\forall x \in \mathbb{R}^n \setminus 0$ . Note, that non-symmetric matrix  $A$  is positive definite iff symmetric matrix  $\frac{1}{2}(A + A^\top)$  is. In our case the quadratic form is given by the following expression :

$$\begin{aligned} \frac{1}{2} z^\top \left( \nabla_z F(\hat{\mathbf{y}}, \mathbf{V}) + \nabla_z F(\hat{\mathbf{y}}, \mathbf{V})^\top \right) z &= \\ &= \frac{1}{B^2} \sum_{i=1}^N B_i \hat{y}_i^2 + \frac{1}{2B^2} \sum_{i=1}^N \sum_{j \neq i} (B_i + B_j) \hat{y}_i \hat{y}_j + \frac{1}{B^2} \sum_{i=1}^N B_i V_i \\ &\geq \frac{1}{B^2} \sum_{i=1}^N B_i \hat{y}_i^2 + \frac{1}{B^2} \sum_{i=1}^N \sum_{j \neq i} \sqrt{B_i B_j} \hat{y}_i \hat{y}_j + \frac{1}{B^2} \sum_{i=1}^N B_i V_i \\ &= \frac{1}{B^2} \left( \sum_{i=1}^N \sqrt{B_i} \hat{y}_i \right)^2 + \frac{1}{B^2} \sum_{i=1}^N B_i V_i, \end{aligned}$$

which is positive for all  $(\hat{y}_i, V_i)_i$  in the feasible region. ■

**Proposition 58** *By the strong monotonicity of  $F(\hat{\mathbf{y}}, \mathbf{V})$ , VE of the game (5.8) is unique [74].*

### 5.3.4 Generalized Potential Game extension

**Assumption 59** *Assume that  $\forall i, j \in \mathcal{N} : \frac{B_i}{B} \simeq \frac{B_j}{B}$ , i.e. each agent  $n$ 's contribution  $B_n$  to the sum  $B$  is relatively small. Denote  $H := \frac{B_n}{B} \forall n \in \mathcal{N}$ .*

**Proposition 60** *Under Assumption 59, the game (5.8) is a Generalized Potential Game.*

**Proof.** The notion of Generalized Potential Game is widely used in the literature, see e.g. [48]. To verify that our game is indeed the potential game, consider function  $\mathcal{P}(\hat{\mathbf{y}})$ , defined as follows :

$$\begin{aligned} \mathcal{P}(\hat{\mathbf{y}}, \mathbf{V}) = & \sum_{i=1}^N \left[ \frac{H\hat{y}_i}{B} \left( \frac{1}{2} \sum_n \hat{y}_n + \sum_n \frac{b_n}{a_n} \right) - \frac{p^0 \hat{y}_i}{N} - \frac{cH\hat{y}_i}{B} \right. \\ & \left. + \frac{H}{B} V_i \right] \end{aligned}$$

We can check that it is a potential function. Indeed, for all  $\hat{\mathbf{y}}_{-n}$ , and for all admissible  $x_n, z_n, x'_n, z'_n$  :

$$\begin{aligned} \Pi_n(x_n, \hat{\mathbf{y}}_{-n}, x'_n, \mathbf{V}_{-n}) - \Pi_n(z_n, \hat{\mathbf{y}}_{-n}, z'_n, \mathbf{V}_{-n}) = & \\ = \frac{H}{2B} \left[ (x_n - z_n)(x_n + z_n + 2 \sum_{m \neq n} \hat{y}_m + 2 \sum_{k \in \mathcal{N}} \frac{b_k}{a_k}) \right] & \\ - \frac{(x_n - z_n)cH}{B} - \frac{p^0}{N}(x_n - z_n) + \frac{H}{B}(x'_n - z'_n) = & \\ = P(x_n, \hat{\mathbf{y}}_{-n}, x'_n, \mathbf{V}_{-n}) - P(z_n, \hat{\mathbf{y}}_{-n}, z'_n, \mathbf{V}_{-n}) & \quad (5.13) \end{aligned}$$

■

Generalized Potential Games constitute a subclass of games for which the convergence of the BR algorithms is established [48] in the deterministic case. Taking into account that the BR scheme is suited for our private framework, an interesting direction of the research would be to establish the convergence of the BR algorithm for the stochastic NE of the GPG.

## 5.4 Numerical Results

### 5.4.1 Algorithm description

In [160], authors employ the penalized individual cost functions to deal with coupled constraints and provide three stochastic gradient strategies with constant step-sizes in order to approach the Nash Equilibrium. In order to establish their results, authors consider the model with the operator  $F(\hat{\mathbf{y}}, \mathbf{V})$  to be *strongly-monotone* and *Lipschitz continuous*, which holds for our case. We consider the scheme, called by the authors as *Diffusion Adapt-then-Penalize* :

$$\begin{cases} \psi_\nu^k = \hat{y}_\nu^{k-1} - \mu \nabla_{\hat{y}_\nu} \Pi_\nu(\hat{y}_\nu, \tilde{\mathbf{y}}_{-\nu}) \\ \hat{y}_\nu^k = \psi_\nu^k - \mu R \nabla_{\hat{y}_\nu} (\theta_\nu(\psi_\nu^k)), \end{cases}$$

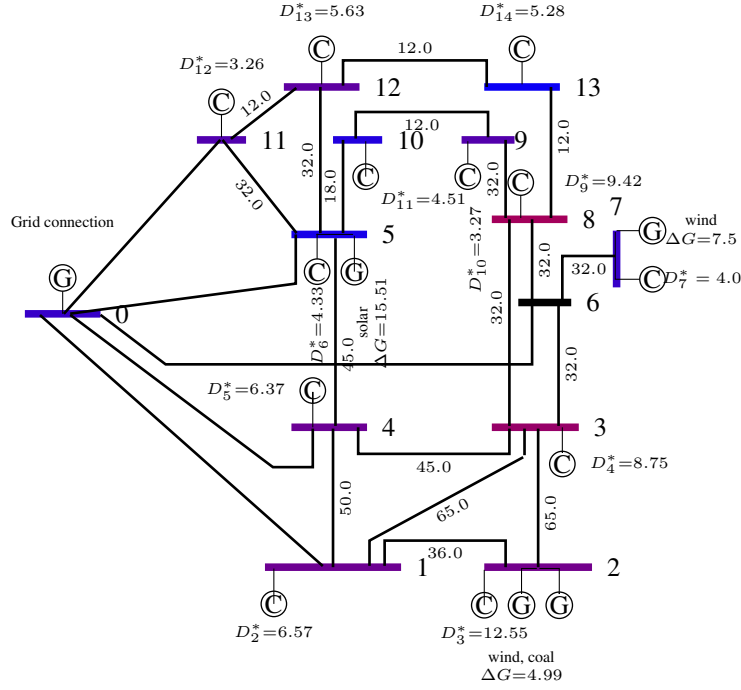


FIGURE 5.1 – IEEE 14-bus network system

where  $\mu$  denotes the step-size,  $R$  - penalty parameter and  $\theta_\nu(\cdot)$  - penalty function for the coupling constraints, which we choose to be the sum over all the constraints of the form  $x \leq 0$ , of all functions  $p_\nu(x)$  such that  $p_\nu(x) = \sum \mathbb{I}_{x \geq 0} \cdot x^2/2$ .

For our numerical simulations, we choose the following parameters of the algorithm :  $\mu = 0.003$  and  $R = 700$ .

### 5.4.2 Numerical Results

We consider the IEEE 14-bus network system, which is depicted in Figure 5.1, where each bus (node) of the network corresponds to a prosumer in our model. We consider the system, which consists of the agents with the non-zero self-generation and demand parameters, thus we exclude one interim node 6, which sole purpose in the initial 14-bus system is to connect the flows. Thus, in our model there are 13 buses (nodes).

We first focus on the homogeneous differentiation price case  $c_{nm} = 1.0$  [\$/MWh], for all  $n, m \in \mathcal{N}$ . The cost  $p^0$ , used by local MO to trade with the wholesale market, is set to be higher than  $c$  and equals 5.0 [\$/MWh]. The natural assumption is the homogeneity of the self-generation parameters of the prosumers, which we set to be  $a_n = 0.5, b_n = 6.0$  for all  $n \in \mathcal{N}$ . Also, there are three nodes (2, 5, 7) that are additionally equipped with a RES-based generation. Values on the links between the nodes on Figure 5.1 specify the trading capacity parameters  $\kappa_{nm}$ . Recall from the Assumption 43 that there are

large trading capacities to and from node 0, thus we do not specify them on the scheme. The nominal demands and RES-based generations in Figure 5.1 are given in [GWh]. All the parameters that are used to calibrate the agents' utility functions are specified in Table 5.1.

TABLE 5.1 – Agents' utility function parameters.

Node	$\tilde{a}$	$\tilde{b}$	$\mathbf{d}$	$\overline{D}$	$\overline{G}$
<b>0</b>	1.5	0.0	9	25.0	100.0
<b>1</b>	1.18	5.09	15	26.7	100.0
<b>2</b>	1.0	3.78	14	99.2	80.0
<b>3</b>	0.57	4.36	0.0	52.8	20.0
<b>4</b>	1.24	5.03	0.0	12.6	20.0
<b>5</b>	1.62	3.04	2.0	16.2	20.0

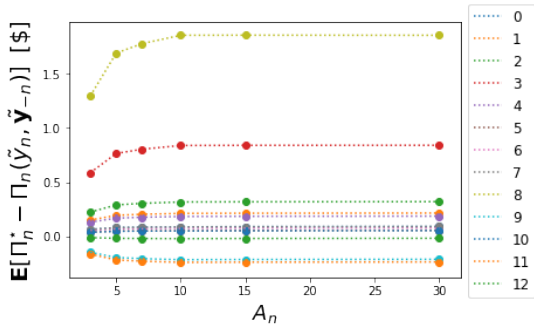
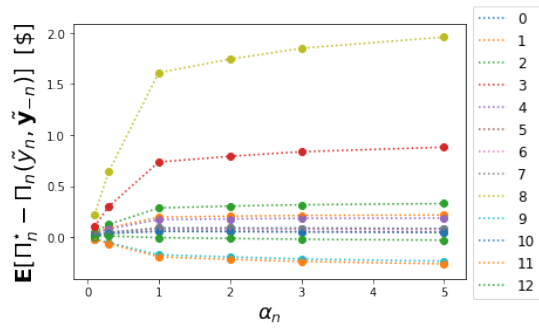
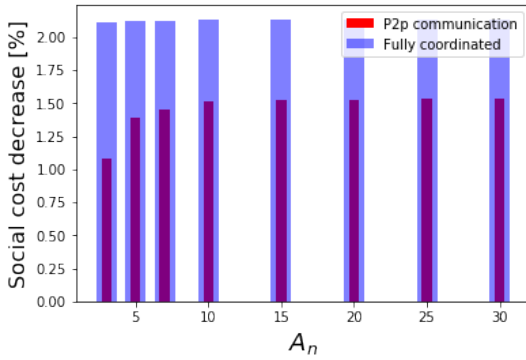
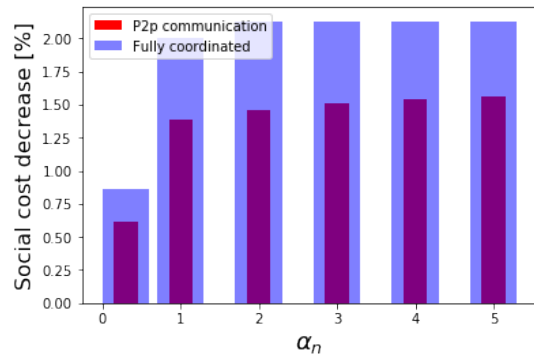
Node	$\tilde{a}$	$\tilde{b}$	$\mathbf{d}$	$\overline{D}$	$\overline{G}$
<b>6</b>	1.54	4.29	0.0	19.9	20.0
<b>7</b>	1.5	0.0	11.0	25.0	50.0
<b>8</b>	0.31	2.75	0.0	34.5	20.0
<b>9</b>	4.36	4.67	0.0	14.0	20.0
<b>10</b>	1.63	3.32	0.0	8.5	20.0
<b>11</b>	5.16	5.5	0.0	11.1	20.0
<b>12</b>	1.96	6.21	0.0	18.5	20.0

To provide insights on the privacy-utility relation under strategic behavior of the agents in our model, we measure the impact of our mechanism on the cost of the agents, quantified through the agent's utility gap  $\mathbb{E} \left[ \Pi_n^* - \Pi_n(\tilde{y}_n, \tilde{\mathbf{g}}_{-n}) \right]$  for each agent  $n$ . In Figure 5.2, we plot the utility gap as a function of  $A_n$  for all the agents. We observe that the nodes 3 and 8 decrease their costs the most among all the prosumers and nodes 9 and 11 have, on the contrary, increasing costs. From Table 5.1 it can be seen that node 3 and 8 have the minimal flexible demand coefficients :  $\tilde{a}_3 = 0.57$  and  $\tilde{a}_8 = 0.31$ . Similarly, nodes 9 and 11 have the biggest flexible demand coefficients :  $\tilde{a}_9 = 4.36$  and  $\tilde{a}_{11} = 5.16$ . The cost of the demand flexibility affects the utility of the agents, i.e. small cost allows them to adjust their demand such that they can decrease their costs, while deviating from their true values.

For the graphs shown below, we set  $\alpha_n = 3.0$  when we plot the dependance w.r.t.  $A_n$ , and  $A_n = 10.0$  when we plot the dependance w.r.t.  $\alpha_n$ . For this choice of parameters, the color of the nodes in Figure 5.1 shows the *privacy price*  $\underline{\beta}_n, \overline{\beta}_n$  [\$/MWh] from Proposition 53 in each  $n \in \mathcal{N}$ . Light blue denotes the lowest privacy price ( $1.129 \cdot 10^{-3}$  [\$/MWh]) and dark violet denotes the highest ( $2.827 \cdot 10^{-2}$  [\$/MWh]).

Figure 5.3 represents the dependence of the plot of the utility gap on the parameter  $\alpha_n$  of the agents. It is shown, that when the maximal bound on the distance is low, the agents expectedly deviate from their costs  $\Pi_n^*$ . As soon as  $\alpha_n$  increases, thus providing more possibility to deviate, agents tend to show the similar behavior as on the plot with respect to  $A_n$  : nodes 3 and 8 gain the most and nodes 9 and 11 have the increasing costs.

Figures 5.4 and 5.5 depict the dependance of the social cost of the market w.r.t.  $A_n$  and  $\alpha_n$  respectively. We compare three instances : peer-to-peer communication mechanism, fully coordinated communication mechanism and the social cost evaluated in the truthful reports. Note, that the latter one provides the same cost when Proposition 50 holds.

FIGURE 5.2 – Utility gap wrt.  $A_n$ FIGURE 5.3 – Utility gap wrt.  $\alpha_n$ FIGURE 5.4 – Social cost decrease wrt.  $A_n$ FIGURE 5.5 – Social cost decrease wrt.  $\alpha_n$ 

It can be seen that increase in  $A_n$  affects the peer-to-peer communication the most, which is caused by the decrease of the privacy price induced by the noise in the agents' reports (recall, that the variance is given by  $V_n = \frac{2B^4}{A_n B_n^2} (\bar{\beta}_n + \underline{\beta}_n)^2$ ), thus allowing them to compute their decision variables more precisely. Clearly, it affects the centralized communication mechanism less. On the other hand, increase of the  $\alpha_n$  affects the centralized communication mechanism the most, as it allows the local MO to find an optimal solution for each agent in the system, thus leading to the biggest decrease in the costs.

The results above are shown in the homogeneous  $A_n$ ,  $\alpha_n$  and  $c_{nm}$  case. Heterogeneity in the distance parameters  $\alpha_n$  and  $A_n$  does not affect the behavior of the agents in the system described above. Nevertheless, setting parameters  $\alpha_n$  to be small for those who deviate the most (e.g. nodes 3 and 8) can bound their influence on the sum  $\sum_n \tilde{y}_n$ , thus, bounding the deviation from the  $\sum_n y_n$ . In the case of heterogeneous differentiation prices  $c_{nm}$  for  $n, m \neq 0$ , we compute the trading costs of the agents, using the expressions given in Proposition 45. Numerical experiments show similar behaviors for all the agents in the system, while distinguishing the node 0 : in this setting it decreases its cost the most.

Our numerical results suggest that demand flexibility allows the agents to mitigate increase of the costs induced by privacy, i.e. we observe that the agents with low demand flexibility costs (resp. high demand flexibility costs) are able to benefit (resp. lose) the most from the proposed privacy mechanism. It

is remarkable that this advantage holds regardless of the heterogeneity/homogeneity of market parameters such as privacy levels  $(A_n, \alpha_n)$  or preferences/taxes  $(c_{nm})$ .

## 5.5 Conclusion

In this chapter, we considered a peer-to-peer electricity market, in which agents have private information. The problem is modeled as a noncooperative communication game, which takes the form of a GNEP, where the agents determine their randomized reports to share with the other market players, while anticipating the form of the peer-to-peer market equilibrium. Agents decide on the deterministic and random parts of the report, such that the (a) the distance between the deterministic part of the report and the truthful private information is bounded (b) expectation of the privacy loss random variable is bounded. This allows them to act strategically on the values of the deterministic part and to choose the random noise included in their reports. We characterized the equilibrium of the problem and proved the uniqueness of the Variational Equilibria. We provided a closed form expression for the privacy price. The theoretical results are illustrated on the 14-bus IEEE network, using the stochastic gradient descent algorithm. We show the impact of the privacy preservation caused by inclusion of random noise and deterministic deviation from agents' true values.

Since our problem has a potential form under mild assumptions, as next step, we will focus on the development of a distributed learning algorithm to compute a stochastic NE solution of the Generalized Potential Game. Bayesian games constitute an additional direction of research enabling the study of more sophisticated information structures.





# Chapitre 6

## Coupling with Forecast Market

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# Towards Forecast Markets For Enhanced Peer-to-Peer Electricity Trading

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## 6.1 Introduction

There is a significant amount of literature addressing uncertainty management in electricity markets through methods such as multistage stochastic bidding strategies, flexibility activation, and forward contracts [98], [117]. In traditional market structures, uncertainties faced by end-users are handled by bigger entities such as retailers and DSOs who can benefit from reduced uncertainty by managing a large portfolio of users. In decentralized electricity markets without such entities, one of the main challenges is how the agents can accommodate the uncertainty in their generation profiles emanating from variable renewable energy resources, without an access to its real distribution. When faced with future uncertain outcomes, agents make decisions based on their own beliefs (forecasts). Therefore, in these markets, decision-makers need accurate information about the probability distributions for improved decision-making.

We model the electricity market as a two-stage model, consisting of the day-ahead (DA) forward market and the real-time (RT) market. An agent may purchase (sell) electricity in both markets to guarantee an adequate supply to meet their demand. In the literature, these markets are commonly modeled as a variation of stochastic inventory models [98], [96]. Such models typically require full knowledge of the probability distribution, which can be difficult to attain due to limited historical data or subjective forecasting techniques. The decision maker must select a probability distribution to use as an input, either relying on their forecasts or those provided by another source, which will affect the computed order quantity [98].

Therefore, forecasting is crucial for making informed decisions and planning, and it has been widely studied by researchers and practitioners. In [116] authors have written a comprehensive review of the field and its advancements. In the literature dedicated to stochastic inventory models, a data-driven approach was investigated in [66] while in [114] authors investigated the effects of partial information about the random variable distribution (e.g., mean, variance, symmetry, unimodality) available to the decision maker and how updated information affects agent's costs.

Prediction (forecast) markets have generated a lot of interest in the research, see e.g. [151] for a detailed overview. They can be used to aggregate and disperse information into efficient forecasts of uncertain future events. Such a forecast improvement by combining or utilizing more data from various

sources was shown in multiple studies, see [94] the example of energy applications. Different models for the data/information/forecast markets were proposed in the literature, associated with different properties and prediction tasks (see e.g. [3], [124], [76]). In our application, we focus on the forecast market for the improvement of buyers' forecasts in a probabilistic format.

The possibility of obtaining an improved forecast could improve the outcome on the electricity market side, but raises important research questions such as : (i) how to evaluate the impact of the forecast purchase on the market outcome, (ii) how to reward the accuracy of the forecasters relative to each other and their contribution to the client's utility, (iii) what constitutes a "better" forecast, and (iv) what are the properties of this connection and under what conditions is there an incentive for prosumers to buy the forecasts.

In this study, we make the first step in the connection between the forecast market and the peer-to-peer electricity market. To the best of our knowledge, this is the first work in this direction. To account for consumer behavior and network restrictions, we model the electricity market as a generalized Nash equilibrium problem (GNEP), a non-cooperative game that incorporates shared coupling constraints in the agents' optimization problems. We use the generalized Nash equilibrium (GNE) as the solution [63, 74] and a refined version called variational equilibria (VE), which assumes alignment of shadow variables among agents [74, 127]. Our focus is on designing decentralized electricity markets that are based on a network defining each agent's trading relationships, for example, with their neighbors. In this work, we concentrate on the financial aspect of the market connection, excluding the physical distribution network from consideration.

For the forecast market, we employ the model introduced in [120], in which authors describe platforms for sharing predictive information with compensation for the cost of data collection, processing, etc., without exposing private data. In that work, the authors generalize the wagering mechanism described in [76] and design a mechanism that considers both the forecast skill of the players and the utility of the forecasts for a decision-maker (prosumer in a peer-to-peer electricity market in our study). The proposed payoff function for the mechanism fairly rewards forecasters based on the accuracy of the forecasters and their contribution to the improvement of the client's utility. This suits our goals, answering question number (ii) from our research questions. Nevertheless, the obstacle in the direct connection of the proposed forecast market and the peer-to-peer electricity market lies in the evaluation of the forecast impact on the outcome of the latter. As in [120] the utility of the client is assumed to be exogenous to the model, the challenge in linking this proposed forecast market to the peer-to-peer electricity market is to evaluate the impact of the forecast on the market outcome. Unlike previous research, our study takes a step forward by endogenizing the change in the prosumers' utility resulting from the acquisition of

the forecast. Our model inherits all the desirable economic properties of the forecast market properties demonstrated in [120].

**Contributions** The main contributions of the work concern (i) the characterization of the equilibria of the two-stage peer-to-peer electricity market in the presence of random generation and (ii) the impact of the connection of electricity peer-to-peer market and the forecast market, which allows the prosumers to update their forecast. We first provide an important result that shows that the impact of the forecast update prosumer's outcome on the electricity market can be evaluated independently of the other prosumers' forecasts. Thus, it allows us to endogenize the utility of prosumers brought by the forecast update, which has been traditionally assumed as an exogenous factor in the literature on forecast models [120], [76].

Furthermore, our analytical results allow us to demonstrate that the efficiency of the VE of the electricity peer-to-peer market can be achieved if the prosumers participate in the forecast market. We prove the conditions on VE existence and then show that to achieve social optima it is required that the centralized welfare optimizer has access to prosumers' forecasts. Although this is a strong requirement, we show that there are incentives for the prosumers to participate in the forecast market, in other words, individual rationality of the coupled market model. We show the conditions on the distributions under which we can guarantee this property and discuss related limitations. These results are significant as they shed light on the connection between the peer-to-peer electricity market and the forecast market, and provide insights into how such frameworks can be utilized to enhance the electricity market operation.

Finally, we provide numerical illustration using a dataset specifying renewable generation, load, and facilities energy consumption for 25 individual homes in Austin, Texas available through Pecan Street [69] to illustrate the effects and properties of the proposed market connection. We demonstrate that the introduction of the forecast market connection to the electricity market is beneficial both for the prosumers and the forecast sellers if the quality of their forecasts allows prosumers to better predict RES-based generation.

## 6.2 Coupled Markets Model

### 6.2.1 Agents and Markets

In this section, we introduce two models used for the peer-to-peer electricity market and the forecast market and discuss the framework for the coupling. We focus on a two-stage (day-ahead and real-time) model for the electricity market, in which agents make their decisions on the day-ahead market while anticipating the outcome of their renewable energy-based generation based on their forecasts. These

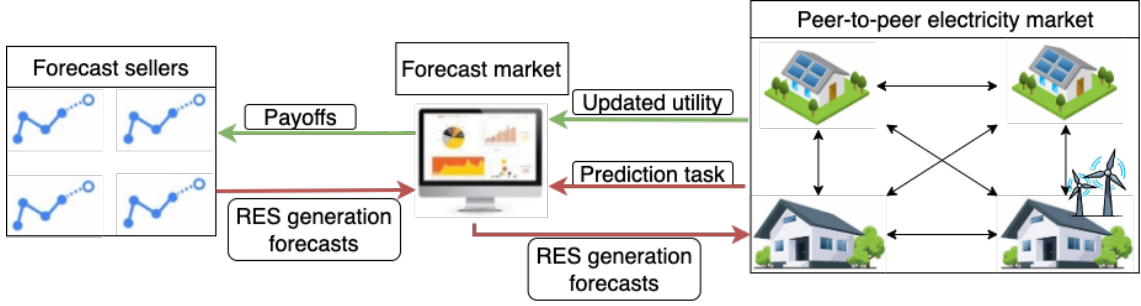


FIGURE 6.1 – High-level presentation of the proposed framework. Red arrows indicate actions before electricity market clearing, green arrows indicate post-clearing actions.

decisions encompass electricity trading with backup retailers and other prosumers in the network. We consider the set  $\mathcal{N}$  of  $N$  agents (prosumers). Each agent is located in a node of the network, which is modeled as an undirected connected graph  $\mathcal{G} := (\mathcal{N}, E)$  where  $E \subseteq \mathcal{N} \times \mathcal{N}$  is the set of links between the agents. Our focus is on designing decentralized electricity markets that are based on a network defining each agent's trading relationships. In this work, we concentrate on the financial aspect of the market connections, excluding the physical distribution network from consideration. In the real-time market, agents observe the actual generation and adjust the remaining unsettled quantities. This model represents a variant of stochastic inventory model, in which prosumers minimize their costs on both stages, complicated by inclusion of the peer-to-peer trading. The quality of agents' decisions in such models, and subsequently, their costs, depend on the quality of their forecasts.

In its simplest form, connection between the forecast market and stochastic inventory models can be represented by the following motivating example, in which we consider the classical variant of the newsvendor problem :

$$\max_q \mathbb{E}_i[\text{profit}] := \mathbb{E}_{F_r}[p \min(q, D)] - cq$$

where  $D$  is a random variable with distribution with a cumulative distribution function (CDF)  $F_r$  representing demand, each unit is sold for price  $p$  and purchased for price  $c$ ,  $q$  is the number of units stocked. The solution to the optimal stocking quantity is  $q = F_i^{-1}(\frac{p-c}{p})$ , where  $F_i^{-1}$  denotes the generalized inverse cumulative distribution function of agent  $i$ 's belief  $F_i(\cdot)$  about distribution  $F_r$  of  $D$ . If newsvendor has an access to the forecast market, they can choose to make a purchase of  $\hat{F}_i$  (before taking decision about  $q$ ), thus, leading to solution  $q = \hat{F}_i^{-1}(\frac{p-c}{p})$  and the payment can be made based on the relative improvement to the base forecast  $F_i$  of the newsvendor.

In our framework, each agent has the option to purchase a forecast before trading on the electricity market. To do so, agents report their base forecast to the forecast market operator (MO) and receive the aggregation of the forecasts, provided by the sellers on the forecast market. The set of the sellers

is denoted as  $I$ . We note that it does not necessarily coincide with the set  $\mathcal{N}$ , but we do not put any restriction on that. After the electricity market clearing, part of the utility obtained by the agent in  $\mathcal{N}$  from the forecast purchase is distributed between forecast sellers in  $I$  by MO, who computes payoff allocation according to the contribution of each forecast seller. Proposed framework is illustrated on Figure 6.1. Below we describe model for electricity and forecast markets in more details.

### 6.2.2 Peer-to-Peer Electricity Market

We employ a two-settlement electricity market design consisting of day-ahead and balancing (real-time) markets. We assume the presence of a backup retailer from whom the community can purchase energy both in day-ahead (hereafter, referred to as first stage) and in real-time (hereafter, referred to as second stage). Therefore, we fix the buying (b) and selling (s) prices for first (or day-ahead  $p_{da}$ ) and second (or real time  $p_{rt}$ ) stages, such that  $p^{rt,b} > p^{da,b} > p^{da,s} > p^{rt,s}$ . The community is seen as a price-taker in the electricity market, hence making the prices exogenous to this problem.

Let  $\Gamma_i \subseteq \mathcal{N}$  denote the set of neighbors of agent  $i$ , which reflects the agents with whom she wants to trade. We denote the trade between agent  $i$  and  $j \in \Gamma_i$  as  $q_{ij}$  (limited with upper-limit  $\kappa_{ij}$ ), where  $q_{ij}$  is the amount of power  $i$  purchases from (or sells to)  $j$  if  $q_{ij} \geq 0$  ( $q_{ij} \leq 0$ ) and impose a bilateral trading reciprocity constraint  $q_{ij} + q_{ji} = 0$ . Trading cost term is presented in the cost function as  $\sum_{j \in \Gamma_i} c_{ij} q_{ij}$ , where parameters  $c_{ij} > 0$  represent (product) differentiation prices and reflect agent  $i$ 's preferences for energy trading. Denote  $d_i$  as agent  $i$ 's demand and  $\Delta g_i$  as agent  $i$ 's renewable energy generation (wind, solar, etc.) which we assume to be a random variable with a CDF,  $F_r \in [0, \infty)$ . In this work, we do not account for a correlation between agents' random variables while it constitutes an important step for a further research. Then, each agent has to make a trading decision in the first stage (day-ahead market) about acquiring ( $q_i^{da,b}$ ) or selling ( $q_i^{da,s}$ ) energy at prices  $p^{da,b}$ ,  $p^{da,s}$  respectively. At the second stage (real-time market), agents settle imbalances after observing the realization of  $\Delta g_i$  for the prices  $p^{rt,b}$  (buying) and  $p^{rt,s}$  (selling).

**Remark 61** *For the sake of simplicity, demand  $d_i$  is assumed to be deterministic, as  $d_i$  is prosumer  $i$ 's own demand, and hence may be controlled by  $i$ . This reflects the practical case where prosumers can have relatively precise control over their demand (e.g. using flexible demand and/or storage in an energy management system). In the more general case, demand can also be a random variable, as it is subject to factors which can be out of the control of the prosumer (e.g., weather conditions, occupancy, etc). Our results can be extended to this case by considering the forecast of the net load  $d_i - \Delta g_i$  (instead of simply  $\Delta g_i$ ), which introduces an additional term in the proof of Theorem 62 (net load random variables can have negative values).*

$$\min_{q_i^{da}, q_i^{rt}, \mathbf{q}_i} \overbrace{p^{da,b} q_i^{da,b} - p^{da,s} q_i^{da,s} + \sum_{j \in \Gamma_i} c_{ij} q_{ij}}^{1^{st} \text{ stage costs}} \quad (6.1a)$$

$$+ \mathbb{E} \left[ \overbrace{p^{rt,b} q_i^{rt,b} - p^{rt,s} q_i^{rt,s}}^{2^{nd} \text{ stage costs}} \right]$$

$$\text{s.t. } q_{ij} + q_{ji} = 0, \quad \forall j \in \Gamma_i \quad (6.1b)$$

$$d_i = \Delta g_i + \sum_{j \in \Gamma_i} q_{ij} + q_i^{da,b} - q_i^{da,s} + q_i^{rt,b} - q_i^{rt,s} \quad (6.1c)$$

$$q_{ij} \leq \kappa_{ij}, \quad \forall j \in \Gamma_i \quad (6.1d)$$

$$q_i^{da,b} \geq 0, q_i^{da,s} \geq 0, q_i^{rt,b} \geq 0, q_i^{rt,s} \geq 0 \quad (6.1e)$$

Note, that the expectation of the second stage costs is taken with respect to a distribution with CDF,  $F_r$ , which represents a real distribution of  $\Delta g_i$ . Nevertheless, without full knowledge of this distribution, agent  $i$  has access to a forecast (belief) with CDF  $F_i$  about the distribution of  $\Delta g_i$ , which she uses for computing the solution of the problem (6.1). In the next section, we provide a description of the forecast market, which allows agent  $i$  to update her belief to be subsequently used in (6.1).

### 6.2.3 Forecast Market

We adopt a model for the forecast market from [120], [76] illustrated in Figure 6.2. Let prosumer  $i$  be a buyer of the forecast who is interested in improving a forecast (e.g., forecasting algorithm, weather forecast or a generation forecast for their renewable energy asset). For this purpose, the prosumer enters the market by posting a forecasting task for a realization of  $\Delta g_i$ , their own forecast report  $F_i$  as a reference for improvement and an offer of a coefficient  $\phi$  for an improvement. Each seller  $j \in I$  reports their forecast  $f_j$  along with a wager  $m_j > 0$  defined by themselves, which expresses their confidence on their forecast. We note that the client is also allowed to enter the market as a player with their own forecast report and wager. The client can compete for a relative forecasting skill reward and also influence the resulting forecast. Finally, the forecast market operator aggregates all the forecasts provided by the players, considering their wagers, and delivers the resulting report  $\hat{F}_i(\mathbf{m}, \mathbf{f})$  to the client.

After the occurrence of the event, i.e., the time interval for which the forecast is being elicited, the market operator observes the true outcome  $\Delta \bar{g}_i$  and evaluates the score  $s(f_j, \Delta \bar{g}_i)$  of each seller  $j \in I$ , which shows how “good” was the forecast reported by seller  $j$ . Furthermore, the operator also evaluates the utility allocated by the client for the forecast improvement  $U(F_i, \hat{F}_i, \Delta \bar{g}_i, \phi) := \phi(\hat{\Pi}_i(\hat{F}_i) - \Pi_i(F_i))$



in monetary terms, which then has to be distributed among the players that have contributed to the improvement.

The design of this market model requires three main components : (i) an aggregation operator (to combine forecasts), (ii) a scoring rule, and (iii) a payoff allocation mechanism.

- *Aggregation operator* : For each forecast seller  $j \in I$ , let  $f_j$  be the forecast report in terms of probability distribution function (PDF) and  $F_j$  be the corresponding cumulative distribution function (CDF). Then, the average quantile (QA) forecast is given by  $\hat{F}_i = \sum_j \hat{m}_j F_j^{-1}$ , where  $\hat{m}_j := \frac{m_j}{\sum_{k \in I} m_k}$ .
- *Scoring rule* : For an event of interest  $x$ , let the PDF reported by a player  $j$  be  $f_j$ , and let  $\Delta \bar{g}_i$  be the event that actually occurred. Let  $F_j$  denote the cumulative distribution. Then, the continuous ranked probability score is defined as

$$CRPS(F, \Delta \bar{g}_i) := \int_{-\infty}^{\infty} [F_j(x) - F_{\Delta \bar{g}_i}(x)]^2 dx,$$

where  $F_{\Delta \bar{g}_i}(x) = 0$  if  $x < \Delta \bar{g}_i$  and  $F_{\Delta \bar{g}_i}(x) = 1$  otherwise. To obtain a score  $s$ , renewable energy production can be normalized to obtain a continuous random variable with the values in  $[0, 1]$ . Then, we can re-orient the scoring function by defining  $s(f_j, \Delta \bar{g}_i) = 1 - CRPS$ .

- *Payoff allocation mechanism* : Payoff function is divided in two parts, one representing the allocation from the wager pool and another from the prosumer's allocated utility. The former evaluates the relative forecasting skill of a player, and the latter compensates for their contribution to an improvement of the client's utility  $U$ . Let the wager payoff of a forecast seller  $j$  be

$$W_j(\mathbf{f}, \mathbf{m}, \Delta \bar{g}_i) := m_j \left( 1 + s(f_j, \Delta \bar{g}_i) - \frac{\sum_k s(f_k, \Delta \bar{g}_i) m_k}{\sum_k m_k} \right).$$

An overall payoff is given as

$$\tilde{W}_j := W_j + \mathbb{1}_{U > 0} \left( \frac{\tilde{s}(r_j, \Delta \bar{g}_i) m_j}{\sum_k \tilde{s}(f_j, \Delta \bar{g}_i) m_k} U \right),$$

where  $\tilde{s}(f_j, \Delta \bar{g}_i) := \mathbb{1}_{s(f_j, \Delta \bar{g}_i) > s(f_i, \Delta \bar{g}_i)} s(f_i, \Delta \bar{g}_i)$ ,

in which  $s(f_i, \Delta \bar{g}_i)$  is the score of prosumer's initial forecast. As authors show in [120], this forecast market mechanism enjoys some desirable properties, adapted from [76]. We provide their brief descriptions below, while referring to [120], [76] for more details.

- *Budget-balance* : A mechanism is budget-balanced if the market generates no profit and creates no loss.

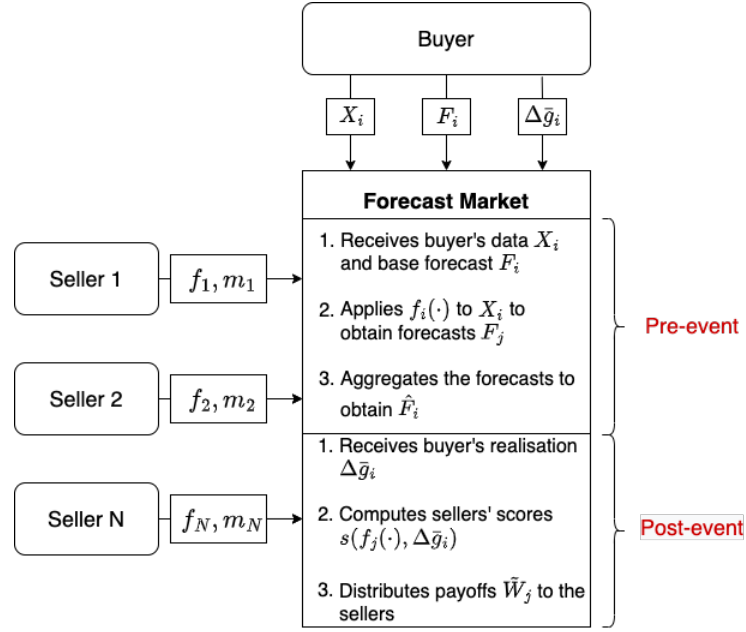


FIGURE 6.2 – Proposed data market scheme

- *Anonymity* : A mechanism satisfies anonymity if the payoff received by a player does not depend on their identity ; rather it depends only on the forecast reports and the realization of an uncertain event.
- *Individually rational for the forecast sellers* : expected profits from participation in the forecast market are non-negative for the seller.
- *Sybilproofness* : A truthful mechanism is sybilproof if the players cannot improve their payoff by creating fake identities and copies of their identities.
- *Truthful for the client* : A mechanism is truthful for a client, in terms of reported prediction, if the client's expected payment is minimized by reporting their true belief as their own forecast.

### 6.3 Coupled Markets Analysis

In this section we first derive the optimal procurement quantities for the agents, by solving problem (6.1) as a variant of stochastic inventory management problem complicated by the peer-to-peer trading. Then we focus on two important properties of our connection between the forecast and peer-to-peer market : efficiency and individual rationality. We show condition for both properties to hold and discuss the limitations of these assumptions.

Firstly, total cost of agent  $i$  can be expressed as a sum of first-stage cost, second-stage cost observed after realization of random variable  $\Delta g_i$  and trading cost. First, note that the second stage decision is

completely defined by the first-stage decision through the supply-demand balance constraint :

$$q_i^{rt,b} - q_i^{rt,s} = \overbrace{d_i - \Delta g_i - q_i^{da,b} + q_i^{da,s}}^{s_i} - \sum_{j \in \Gamma_i} q_{ij}$$

Note that  $q_i^{rt,b} \cdot q_i^{rt,s} = 0$ , thus we can rewrite the equation above as

$$q_i^{rt,b} = s_i \quad \text{if} \quad s_i \geq 0 \quad \text{else} \quad q_i^{rt,s} = -s_i$$

Then, we can insert it into the expected cost function, where  $F_i$  denotes a cdf of a forecast of agent  $i$  (with pdf  $f_i$ ) :

$$\Pi_i := p^{da,b} q_i^{da,b} - p^{da,s} q_i^{da,s} + \mathbb{E}_{F_i} [p^{rt,b} s_i \mathbb{I}_{s_i \geq 0} + p^{rt,s} s_i \mathbb{I}_{s_i \leq 0}] + \sum_{j \in \Gamma_i} c_{ij} q_{ij} \quad (6.2)$$

Denote residual after first-stage decisions as  $r_i := d_i - q_i^{da,b} + q_i^{da,s} - \sum_{j \in \Gamma_i} q_{ij}$  and note that it is obviously non-negative. We first prove the following important result about optimal procurement strategy in the presence of day-ahead and real-time contracts in a market with random renewable generation :

**Theorem 62** *The residual  $r_i$  of agent  $i$  after the day-ahead market is given by*

$$\begin{aligned} q_i^{da,b} - q_i^{da,s} + \sum_{j \in \Gamma_i} q_{ij} &= d_i - F_i^{-1} \left( \frac{p^{da,b} - p^{rt,s} - \mu_i^{da,b}}{p^{rt,b} - p^{rt,s}} \right) \\ &= d_i - F_i^{-1} \left( \frac{p^{da,s} - p^{rt,s} + \mu_i^{da,s}}{p^{rt,b} - p^{rt,s}} \right) = d_i - F_i^{-1} \left( \frac{c_{ij} - p^{rt,s} + \zeta_{ij} + \xi_{ij}}{p^{rt,b} - p^{rt,s}} \right). \end{aligned} \quad (6.3)$$

More precisely,

$$r_i = F_i^{-1} \left( \frac{p^{da,b} - p^{rt,s}}{p^{rt,b} - p^{rt,s}} \right) \quad (6.4)$$

if agent  $i$  purchases electricity on the first stage

$$r_i = F_i^{-1} \left( \frac{p^{da,s} - p^{rt,s}}{p^{rt,b} - p^{rt,s}} \right) \quad (6.5)$$

if agent  $i$  sells electricity on the first stage

**Proof.** First, we expand the second-stage costs :

$$\begin{aligned}
\Pi_i^{second} &= \mathbb{E}_{F_i} [p^{rt,b}[d_i - \Delta g_i - q_i^{da,b} + q_i^{da,s} - \sum_{j \in \Gamma_i} q_{ij}] \mathbb{I}_{s_i \geq 0} \\
&\quad + p^{rt,s}[d_i - \Delta g_i - q_i^{da,b} + q_i^{da,s} - \sum_{j \in \Gamma_i} q_{ij}] \mathbb{I}_{s_i \leq 0}] \\
&= -p^{rt,b} \int_0^\infty f_i(\Delta g_i) \Delta g_i \mathbb{I}_{s_i \geq 0} d\Delta g_i + p^{rt,b} \int_0^\infty f_i(\Delta g_i) r_i \mathbb{I}_{s_i \geq 0} d\Delta g_i \\
&\quad - p^{rt,s} \int_0^\infty f_i(\Delta g_i) \Delta g_i \mathbb{I}_{s_i \leq 0} d\Delta g_i + p^{rt,s} \int_0^\infty f_i(\Delta g_i) r_i \mathbb{I}_{s_i \leq 0} d\Delta g_i \\
&= -p^{rt,b} \int_0^{r_i} \Delta g_i f_i(\Delta g_i) d\Delta g_i + p^{rt,b}(r_i) \mathbb{P}_i(\Delta g_i \leq r_i) \\
&\quad - p^{rt,s} \int_{d_i - q_i^{da,b} + q_i^{da,s} - \sum_{j \in \Gamma_i} q_{ij}}^\infty \Delta g_i f_i(\Delta g_i) d\Delta g_i + p^{rt,s}(r_i) \mathbb{P}_i(\Delta g_i \geq r_i)
\end{aligned}$$

Then, second stage costs can be rewritten as

$$\begin{aligned}
\Pi_i^{second} &= p^{rt,b} r_i F_i(r_i) + p^{rt,s} r_i (1 - F_i(r_i)) \\
&\quad - p^{rt,b} \int_0^{r_i} \Delta g_i f_i(\Delta g_i) d\Delta g_i - p^{rt,s} \int_{r_i}^\infty \Delta g_i f_i(\Delta g_i) d\Delta g_i
\end{aligned} \tag{6.6}$$

Then, taking a derivative of the second stage costs w.r.t  $r_i$ , we obtain :

$$\frac{\partial}{\partial r_i} \Pi_i^{second} = p^{rt,b} F(r_i) + p^{rt,s} - p^{rt,s} F(r_i)$$

Next we write Lagrangian for (6.1) in which we omit supply-demand balance constraint as it is already satisfied in (6.6).

$$\begin{aligned}
\mathcal{L}_i &= p^{da,b} q_i^{da,b} - p^{da,s} q_i^{da,s} + \Pi_i^{second} + \sum_{j \in \Gamma_i} c_{ij} q_{ij} + \sum_{j \in \Gamma_i} \zeta_{ij} (q_{ij} + q_{ji}) \\
&\quad + \sum_{j \in \Gamma_i} \xi_{ij} (q_{ij} - \kappa_{ij}) - \mu_i^{da,b} q_i^{da,b} - \mu_i^{da,s} q_i^{da,s} - \mu_i^{rt,b} q_i^{rt,b} - \mu_i^{rt,s} q_i^{rt,s}
\end{aligned}$$

Then, first order stationarity conditions give :

$$\frac{\partial \mathcal{L}_i}{\partial q_i^{da,b}} = p^{da,b} - (p^{rt,b} - p^{rt,s}) F_i(r_i) - p^{rt,s} - \mu_i^{da,b} = 0 \tag{6.7}$$

$$\frac{\partial \mathcal{L}_i}{\partial q_i^{da,s}} = -p^{da,s} + (p^{rt,b} - p^{rt,s}) F_i(r_i) + p^{rt,s} - \mu_i^{da,s} = 0 \tag{6.8}$$

$$\frac{\partial \mathcal{L}_i}{\partial q_{ij}} = \zeta_{ij} + \xi_{ij} + c_{ij} - (p^{rt,b} - p^{rt,s}) F_i(r_i) - p^{rt,s} = 0 \tag{6.9}$$

From first two equations we get that  $p^{da,b} - p^{da,s} = \mu_i^{da,b} + \mu_i^{da,s} > 0$ , which (from complementarity conditions) means that agent  $i$  can not simultaneously buy and sell energy from the backup retailer (the same holds for a real-time market). Then, we obtain exactly equation (6.3). Note that as  $q_i^{da,b} q_i^{da,s} = 0$ , we can consider these cases separately. First, assume that agent  $i$  is buying energy from the backup retailer, i.e.  $q_i^{da,b} > 0$ . It means that  $\mu_i^{da,b} = 0$  and  $\mu_i^{da,s} = p^{da,b} - p^{da,s} > 0$ . Thus,

$$q_i^{da,b} + \sum_{j \in \Gamma_i} q_{ij} = d_i - F_i^{-1} \left( \frac{p^{da,b} - p^{rt,s}}{p^{rt,b} - p^{rt,s}} \right) \quad (6.10)$$

Similarly, if agent  $i$  is selling energy on the day-ahead market to the backup retailer, then

$$-q_i^{da,s} + \sum_{j \in \Gamma_i} q_{ij} = d_i - F_i^{-1} \left( \frac{p^{da,s} - p^{rt,s}}{p^{rt,b} - p^{rt,s}} \right) \quad (6.11)$$

■

The result above expresses the agents' decision on the day-ahead market in terms of residuals  $r_i$ , i.e. the quantities representing the additional purchases that each agent needs to make to balance the uncertainty of the supply after observing the realization of the renewable generation  $\Delta g_i$ . While not providing us the explicit expressions for the decision variables  $q_i^{da,b}, q_i^{s,da}, q_{ij}$ , it allows us to obtain important results about electricity market properties.

### 6.3.1 Efficiency of the Peer-to-Peer Market

When designing market rules, it is important to choose an equilibrium with desirable properties from a set of equilibria (possibly infinite). In our analysis we rely on a notion of Generalized Nash Equilibria and its refinement, Variational Equilibria (VE) as discussed in [74].

**Definition 63** A Generalized Nash Equilibrium (GNE) of the game defined by the maximization problems (6.1) with coupling constraints, is a vector  $x_i := (q_i^{da}, q_i^{rt}, \mathbf{q}_i)$  that solves the maximization problems (6.1) or, equivalently, a vector  $x_i := (q_i^{da}, q_i^{rt}, \mathbf{q}_i)$  such that  $x_i := (q_i^{da}, q_i^{rt}, \mathbf{q}_i)$  solve the system  $KKT_i$  for each  $i$ .

**Definition 64** A Variational Equilibrium (VE) of the game defined by the maximization problems (6.1) with coupling constraints, is a GNE of this game such that, in addition, the Lagrangian multipliers  $\zeta_{ij}$  associated to the coupling constraints  $q_{ij} + q_{ji} = 0$  are equal, i.e. :

$$\zeta_{ij} = \zeta_{ji}, \quad \forall i \in \mathcal{N}, \forall j \in \Gamma_i \quad (6.12)$$

By duality theory,  $\zeta_{ij}$  for  $i \in \mathcal{N}, \forall j \in \Gamma_i$  can be interpreted as bilateral energy trading prices [26]. In general,  $\zeta_{ij}$  might not be aligned with  $\zeta_{ji}$ , thus leading to non-symmetric energy trading prices between couple of agents. Relying on VE as solution concepts enforces a natural symmetry in the bilateral energy price valuations [26]. To give a condition on VE existence, we need the following lemma :

**Lemma 65** *For any pair of agents  $(i, j)$  such that  $j \in \Gamma_i$ , dual variables  $\xi_{ij}$  associated with capacity constraints  $q_{ij} \leq \kappa_{ij}$  are equal to zero,  $\xi_{ij} = \xi_{ji} = 0$ .*

**Proof.** From complementarity conditions for (6.1), we have that  $(q_{ij} - \kappa_{ij})\xi_{ij} = 0$  and  $(q_{ji} - \kappa_{ji})\xi_{ji} = 0$ . Using the fact that  $q_{ij} + q_{ji} = 0$  and the fact that  $\kappa_{ij} = \kappa_{ji}$ , we rewrite the second equality as  $(q_{ij} + \kappa_{ij})\xi_{ji} = 0$ . Taking the sum with the first equality, we obtain that  $q_{ij}(\xi_{ij} + \xi_{ji}) - \kappa_{ij}(\xi_{ij} - \xi_{ji}) = 0$ . Next we note that  $\xi_{ij} \neq 0$  only when  $q_{ij} = \kappa_{ij}$ , thus,  $\kappa_{ij}(2\xi_{ji}) = 0$ , from which it follows that  $\xi_{ji} = 0$ . Symmetric reasoning for  $q_{ji}$  concludes the proof. ■

Lemma 65 allows us to use (6.9) in the following way : consider (6.9) for agent  $i \in \mathcal{N}$  and agent  $j$  for  $j \in \Gamma_i$ . Recap that  $r_i := d_i - q_i^{da,b} + q_i^{da,s} - \sum_{j \in \Gamma_i} q_{ij}$  and assume that agent  $i$  buys energy on the day-ahead market and agent  $j$  sells it on the day-ahead market (the opposite case is considered similarly). Then, at the Variational Equilibrium  $\zeta_{ij} = \zeta_{ji}$ , thus

$$c_{ij} - c_{ji} = (p^{rt,b} - p^{rt,s}) \left[ F_i(r_i) - F_j(r_j) \right] = p^{da,b} - p^{da,s},$$

which might raise a question, whether VE exists only if the condition  $c_{ij} - c_{ji} = p^{da,b} - p^{da,s}$  holds. It is indeed the case as we show in the next proposition :

**Proposition 66** *VE of the problem (6.1) exists if  $c_{ij} - c_{ji} = p^{da,b} - p^{da,s}$  when agent  $i$  ( $j$ ) purchases (sells) electricity on the day-ahead market and if  $c_{ij} - c_{ji} = p^{da,s} - p^{da,b}$  in the opposite case.*

**Proof.** Denote  $x_i := (q_i^{b,da}, q_i^{s,da}, q_i^{b,rt}, q_i^{rt}, \mathbf{q}_i)$  the vector of agent  $i$ 's decision variables. Following Definition 1.2 in [74], Variational Equilibrium exists, if there exists  $x \in \mathbb{C}$ , where  $x := (x_1, \dots, x_N)$  - tuple of all the decision variables of all agents and  $\mathbb{C}$  - shared constraint set, such that  $x$  solve the following variational inequality :

$$F(x)^T(y - x) \geq 0, \quad \forall y \in \mathbb{C}, \quad (6.13)$$

where  $F(x) := (\nabla_{x_1} \Pi_1(x)^T, \dots, \nabla_{x_N} \Pi_N(x)^T)$ . Assume that such  $x$  exists. Denoting  $q_{ij}^*, q_{ji}^*$  the values of bilateral trades in the VE as well as  $q_i^{b,da,*}, q_j^{s,da,*}$  the values of the day-ahead trades in the VE. Let  $y \in \mathbb{C}$  be equal to  $x$  except modification of the above quantities : take sufficiently small  $\varepsilon$ , such that  $q_i^{b,da} = q_i^{b,da,*} + \varepsilon, q_j^{s,da} = q_j^{s,da,*} + \varepsilon$  and  $q_{ij}^* = q_{ij} + \varepsilon, q_{ji}^* = q_{ji} - \varepsilon$ . Then, writing (6.13) for a pair of

agents  $i$  and  $j \in \Gamma_i$  we obtain (after taking the sum) :

$$\begin{aligned} & p^{da,b}(q_i^{b,da,*} + \varepsilon) - p^{da,b}q_i^{b,da} - p^{da,s}(q_j^{s,da,*} + \varepsilon) + p^{da,s}q_j^{b,da} \\ & + c_{ij}(q_{ij}^* - \varepsilon) + c_{ji}(q_{ji}^* + \varepsilon) - c_{ij}q_{ij}^* - c_{ji}q_{ji}^* \geq 0, \end{aligned}$$

which reduces to

$$\varepsilon(p^{da,b} - p^{da,s}) + \varepsilon(c_{ji} - c_{ij}) \geq 0.$$

As this inequality should hold for any values of  $\varepsilon$ , it follows that  $c_{ij} - c_{ji} = p^{da,b} - p^{da,s}$ . The opposite case is considered similarly. ■ This result gives a necessary condition for VE to exist, which allows us to further prove the efficiency property for our peer-to-peer market coupled with the forecast market. Indeed, solving problem (6.1) in a centralized way leads to redundancy of the bilateral trading conditions  $q_{ij} + q_{ji} = 0$  for a pair of agents  $(i, j)$ . Thus, considering VE we can guarantee that the solutions at VE are defined by exactly the same KKT system as the social welfare maximizer, if the distributions used for the solutions of these problems are the same.

First, we prove an interesting result, that shows that there is no impact of the agents' forecasts on the other agents. Note that the final trading costs of agent  $i$  consist of two terms :  $\sum_{j \in \Gamma_i} c_{ij}q_{ij}$ , representing preferences and  $\sum_{j \in \Gamma_i} \zeta_{ij}q_{ij}$ , representing the bilateral payment for the trade. Then, from (6.9) we obtain that

$$\begin{aligned} \Pi_i^q &= \sum_{j \in \Gamma_i} (\zeta_{ij} + c_{ij})q_{ij} = \sum_{j \in \Gamma_i} \left[ (p^{rt,b} - p^{rt,s})F_i(r_i) + p^{rt,s} \right] q_{ij} \\ &= \left[ (p^{rt,b} - p^{rt,s})F_i(r_i) + p^{rt,s} \right] \sum_{j \in \Gamma_i} q_{ij}, \end{aligned} \tag{6.14}$$

which, with the fact that  $\sum_{j \in \Gamma_i} q_{ij}$  and residual  $r_i$  depend solely on agent  $i$ 's parameters (expressions (6.4) and (6.5)) gives us the following result :

**Theorem 67** *Total cost of agent  $i$  depends only on the parameters of agent  $i$ . It means that forecast market operator can compute utility change of agent  $i$  without information from the other agents.*

**Proof.** Follows directly from equations (6.4), (6.5) and (6.14). ■

This result is not only interesting by itself, it is also crucial for the coupling of the forecast market and the peer-to-peer market, as it allows to endogenize the utility of the prosumer introduced by the forecast update without collecting information from the other agents in the system. The seller's scores and payoff distribution then occur as in [120].

Nonetheless, this result does not immediately lead to a market efficiency. Indeed, in order to guarantee it, we have to assume that all the agents report their forecasts to the market operator (not necessarily purchase the forecast).

**Theorem 68** *If all the agents report their forecasts to the Market Operator (participate in the forecast market), then the VE of (6.1) coincides with the set of social welfare optima.*

**Proof.** The proof follows from Lemma 65, Proposition 66 and KKT conditions written for peer-to-peer electricity market and centralized formulation of (6.1). ■

While being a strong assumption, it is mitigated by the fact that we establish in the next section, more precisely, individual rationality of the coupling between forecast market and peer-to-peer electricity market. We show, that under mild conditions, agents benefit from purchasing the forecasts, thus, they have incentive for participation in the forecast market.

### 6.3.2 Individual Rationality

From (6.3) we obtain that  $r_i = F_i^{-1}\left(\frac{\zeta_{ij} + c_{ij} - p^{rt,s}}{p^{rt,b} - p^{rt,s}}\right)$ , thus,  $\Pi_i^q = (\zeta_{ij} + c_{ij}) \sum_{j \in \Gamma_i} q_{ij}$ . Note, that from (6.9) we have that  $\zeta_{ij} + c_{ij} = c_i$  for each  $j \in \Gamma_i$ , where  $c_i$  is some constant specific for each agent with  $p^{da,s} \leq c_i \leq p^{da,b}$ . Then, from (6.3) we obtain that either  $c_i$  is equal to  $p^{da,b}$ , if agent  $i$  buys energy from the backup retailer, or to  $p^{da,s}$  otherwise. It allows us to finally write expressions for the total cost imposed on the agent  $i$ . First, consider the case when  $i$  buys energy from backup retailer on the day-ahead market

$$\begin{aligned} \Pi_i^{total} &= p^{da,b} q_i^{da,b} + p^{da,b} \sum_{j \in \Gamma_i} q_{ij} + \Pi_i^{second} \\ &= p^{da,b} \left[ d_i - F_i^{-1}\left(\frac{p^{da,b} - p^{rt,s}}{p^{rt,b} - p^{rt,s}}\right) \right] + \Pi_i^{second}, \end{aligned} \quad (6.15)$$

where  $\Pi_i^{second}$  is given by

$$\begin{aligned} \Pi_i^{second} &= \overbrace{p^{rt,b} r_i F_r(r_i) + p^{rt,s} r_i (1 - F_r(r_i))}^1 \\ &\quad - \overbrace{p^{rt,b} \int_0^{r_i} \Delta g_i f_r(\Delta g_i) d\Delta g_i - p^{rt,s} \int_{r_i}^{\infty} \Delta g_i f_r(\Delta g_i) d\Delta g_i}^2, \end{aligned}$$

where  $F_r$  ( $f_r$ ) denotes CDF (PDF) of a real distribution of  $\Delta g_i$ . It means that  $\Pi_i^{total}$  gives an expected cost of agent  $i$  who takes  $r_i$  as a first stage decision ( $r_i$  denotes residual after the first stage). Considering the first part of the expression :

$$p^{rt,b} r_i F_r(r_i) + p^{rt,s} r_i (1 - F_r(r_i)) = r_i (p^{rt,b} - p^{rt,s}) F_r(r_i) + p^{rt,s} r_i$$



The second part can be expressed as follows, where the expectation with respect to the real distribution is denoted as  $\mathbb{E}_r[\cdot]$  :

$$\begin{aligned}
& p^{rt,b} \int_0^{r_i} \Delta g_i f_r(\Delta g_i) d\Delta g_i + p^{rt,s} \int_{r_i}^{\infty} \Delta g_i f_r(\Delta g_i) d\Delta g_i \\
&= p^{rt,b} \mathbb{E}_r[\Delta g_i | \Delta g_i \leq r_i] \mathbb{P}_r(\Delta g_i \leq r_i) + p^{rt,s} \mathbb{E}_r[\Delta g_i | \Delta g_i \geq r_i] \mathbb{P}_r(\Delta g_i \geq r_i) \\
&= p^{rt,b} \mathbb{E}_r[\Delta g_i | \Delta g_i \leq r_i] F_r(r_i) + p^{rt,s} \mathbb{E}_r[\Delta g_i | \Delta g_i \geq r_i] (1 - F_r(r_i)) \\
&= p^{rt,b} \mathbb{E}_r[\Delta g_i] - (p^{rt,b} - p^{rt,s}) \mathbb{E}_r[\Delta g_i | \Delta g_i \geq r_i] (1 - F_r(r_i))
\end{aligned}$$

Summing it all together :

$$\begin{aligned}
\Pi_i^{total} &= p^{da,b} [d_i - r_i] - (p^{rt,b} - p^{rt,s}) F_r(r_i) + p^{rt,s} r_i - p^{rt,b} \mathbb{E}_r[\Delta g_i] \\
&\quad + (p^{rt,b} - p^{rt,s}) \mathbb{E}_r[\Delta g_i | \Delta g_i \geq r_i] (1 - F_r(r_i)),
\end{aligned} \tag{6.16}$$

which gives us expected cost of an agent  $i$ .

Expression (6.16) allows us to consider the effects of the forecast market on the peer-to-peer market in an expectation with respect to the real distribution of  $\Delta g_i$ . First, note that CDF  $F_r(x)$  and its PDF  $f_r(x)$  are defined on  $x \in [0, \infty)$ . The first question to answer (and to show the expected rewards of the sellers on the forecast market) is the definition of order between distributions. Naturally, we would like to show that  $\mathbb{E}_r[\hat{\Pi}_i^{total}] \leq \mathbb{E}_r[\Pi_i^{total}]$  if distribution  $\hat{F}_i$  is "better" than  $F_i$ . Intuitively, for one shot game it should compare two distributions by the amount of probability mass concentrated around the realization of a random variable  $\Delta g_i$ . It provide us a hint that the comparison should be made by conditioning the distance between distributions. The question on how to choose the metrics is non-trivial as shown below.

Consider an agent  $i$  who has an initial forecast about the distribution of  $\Delta g_i$  with CDF  $F_i$  and a 'better' forecast with PDF  $\hat{F}_i$ . Then, we want to show that

$$\mathbb{E}_r[\hat{\Pi}_i^{total}] \leq \mathbb{E}_r[\Pi_i^{total}] \tag{6.17}$$

Now, fix prices  $p^{rt,b} > p^{da,b} > p^{da,s} > p^{rt,s}$  and denote  $\rho := \frac{p^{da,b} - p^{rt,s}}{p^{rt,b} - p^{rt,s}}$ . Assume that agent  $i$  buys energy on the day-ahead market if she uses  $F_i$  or  $\hat{F}_i$  or  $F_r$  (this can be expressed as  $F_i^{-1}(\rho)$ ,  $\hat{F}_i^{-1}(\rho)$ ,  $F_r^{-1}(\rho) \leq d_i$ ). Situation in which  $i$  sells energy on the day-ahead market is considered similarly. Moreover, denote  $r_i^r := F_r^{-1}(\rho)$  and  $\hat{r}_i := \hat{F}_i^{-1}(\rho)$ .

Denote as  $\Pi_r^{total}$  the cost obtained by the agent  $i$  associated with the decision  $r_i = F_r^{-1}(\rho)$ , taken when she knows the real distribution  $F_r$ . Subtracting it from both sides of (6.17), and using (6.16) we

write for the right side of the inequality

$$\begin{aligned} S_r &:= p^{da,b}[r_i^r - r_i] + p^{rt,s}[r_i - r_i^r] + (p^{rt,b} - p^{rt,s})[r_i F_r(r_i) - r_i^r F_r(r_i^r)] \\ &\quad + (p^{rt,b} - p^{rt,s}) \left[ \int_{r_i}^{\infty} x f_r(x) dx - \int_{r_i^r}^{\infty} x f_r(x) dx \right] \end{aligned}$$

with the left side ( $S_l$ ) written in the same way but with  $\hat{r}_i$  instead of  $r_i$ . Now, dividing both sides by  $(p^{rt,b} - p^{rt,s})$ , we can write  $S_r$  (or  $S_l$  if we use  $\hat{F}_i$ ) as

$$\begin{aligned} S_r &= \rho[F_r^{-1}(\rho) - F_i^{-1}(\rho)] + [F_i^{-1}(\rho)F_r(F_i^{-1}(\rho)) - F_r^{-1}(\rho)F_r(F_r^{-1}(\rho))] \\ &\quad + \left[ \int_{F_i^{-1}(\rho)}^{\infty} x f_r(x) dx - \int_{F_r^{-1}(\rho)}^{\infty} x f_r(x) dx \right] \\ &= F_i^{-1}(\rho)[F_r(F_i^{-1}(\rho)) - \rho] + \int_{F_i^{-1}(\rho)}^{F_r^{-1}(\rho)} x f_r(x) dx \end{aligned}$$

Integrating by parts gives

$$\begin{aligned} S_r &= F_i^{-1}(\rho)[F_r(F_i^{-1}(\rho)) - \rho] + F_r^{-1}(\rho)F_r(F_r^{-1}(\rho)) \\ &\quad - F_i^{-1}(\rho)F_r(F_i^{-1}(\rho)) - \int_{F_i^{-1}(\rho)}^{F_r^{-1}(\rho)} F_r(x) dx \\ &= \int_{F_i^{-1}(\rho)}^{F_r^{-1}(\rho)} (\rho - F_r(x)) dx \end{aligned}$$

Thus, we want to prove that

$$\int_{\hat{F}_i^{-1}(\rho)}^{F_r^{-1}(\rho)} (\rho - F_r(x)) dx \leq \int_{F_i^{-1}(\rho)}^{F_r^{-1}(\rho)} (\rho - F_r(x)) dx \quad (6.18)$$

**Remark 69** From that one can expect that the metric would require a distance between quantiles, e.g. assume that 1-Wasserstein metric  $W_1(\hat{F}_i, F_r)$  is lower than  $W_1(F_i, F_r)$  :

$$\int_0^1 |\hat{F}_i^{-1}(p) - F_r^{-1}(p)| dp \leq \int_0^1 |F_i^{-1}(p) - F_r^{-1}(p)| dp$$

This does not suffice as can be seen by the following example : assume that  $W_1(\hat{F}_i, F_r) \leq W_1(F_i, F_r)$  but  $F_r^{-1}$  and  $F_i^{-1}$  intersect at  $\rho$  :  $F_r^{-1}(\rho) = F_i^{-1}(\rho)$ . Then  $S_r = 0$  and if the condition  $\hat{F}_i^{-1}(\rho) \leq F_r^{-1}(\rho)$  holds, we obtain a contradiction. Thus, a stronger condition is required on the distance between distributions.

Assume now that for given  $\rho$ ,

$$|\hat{F}_i^{-1}(\rho) - F_r^{-1}(\rho)| \leq |F_i^{-1}(\rho) - F_r^{-1}(\rho)| \quad (6.19)$$

which, while being strong, doesn't immediately guarantee that (6.18) holds without additional assumptions on  $F_r$  in the neighbourhood of  $F_r^{-1}(\rho)$ . Next we discuss the conditions on the distributions and  $\rho$  such that (6.17) holds. First, note that  $\rho = F_r(F_r^{-1}(\rho))$ , thus, inequality clearly holds when  $F_i^{-1}(\rho) \leq \hat{F}_i^{-1}(\rho) \leq F_r^{-1}(\rho)$  or when  $F_i^{-1}(\rho) \geq \hat{F}_i^{-1}(\rho) \geq F_r^{-1}(\rho)$ . We next assume that  $\hat{F}_i^{-1}(\rho) \leq F_r^{-1}(\rho) \leq F_i^{-1}(\rho)$ , while the opposite case can be considered similarly. In this case with the change of variables we can rewrite (6.18) as

$$\int_{F_r(\hat{F}_i^{-1}(\rho))}^{\rho} [F_r^{-1}(x) - \hat{F}_i^{-1}(\rho)] dx \leq \int_{\rho}^{F_r(F_i^{-1}(\rho))} [F_i^{-1}(\rho) - F_r^{-1}(x)] dx \quad (6.20)$$

in which the left part is upper-bounded by  $\int_{F_r(\hat{F}_i^{-1}(\rho))}^{\rho} [F_r^{-1}(\rho) - \hat{F}_i^{-1}(\rho)]$  and the left part is lower-bounded by  $\int_{\rho}^{F_r(F_i^{-1}(\rho))} [F_i^{-1}(\rho) - F_r^{-1}(\rho)]$ . Note that without additional assumptions on the distributions the bounds are tight :

**Example 70** Let  $F_r$ ,  $F_i$  and  $\hat{F}_i$  represent CDFs of Gaussian distributions with means  $\mu_r, \mu_i$  and  $\hat{\mu}_i$  respectively, where  $\hat{\mu}_i \leq \mu_r \leq \mu_i$  and  $\mu_r - \hat{\mu}_i \leq \mu_i - \mu_r$ . Assume that the variance is the same for all the distributions. Then assumption (6.18) holds  $\forall \rho \in (0, 1)$  and  $F_r^{-1}(x) - \hat{F}_i^{-1}(\rho) = F_r^{-1}(\rho) - \hat{F}_i^{-1}(\rho) \leq F_i^{-1}(\rho) - F_r^{-1}(x) = F_i^{-1}(\rho) - F_r^{-1}(\rho)$  for all  $x$ .

The reasoning above gives us the following result :

**Theorem 71** Forecast's update from  $F_i$  to  $\hat{F}_i$  decreases agent  $i$ 's costs (i.e. inequality (6.17) holds) if

1.  $|\hat{F}_i^{-1}(\rho) - F_r^{-1}(\rho)| \leq |F_i^{-1}(\rho) - F_r^{-1}(\rho)|$
2.  $\frac{b-a}{c-a} \int_a^c f_r(x) dx \leq \int_b^c f_r(x) dx$ ,

where  $a := \hat{F}_i^{-1}(\rho)$ ,  $b := F_r^{-1}(\rho)$ ,  $c := F_i^{-1}(\rho)$  and  $F_r$  denotes real CDF of  $\Delta g_i$ .

**Proof.** Proof follows from the derivations above. Using the bounds in (6.20) and denoting  $a := \hat{F}_i^{-1}(\rho)$ ,  $b := F_r^{-1}(\rho)$ ,  $c := F_i^{-1}(\rho)$  we can write it as

$$(b - a)[F_r(b) - F_r(a)] \leq (c - b)[F_r(c) - F_r(b)] \quad (6.21)$$

Which can then be transformed into

$$\frac{b - a}{c - a} \int_a^c f_r(x) dx \leq \int_b^c f_r(x) dx \quad (6.22)$$

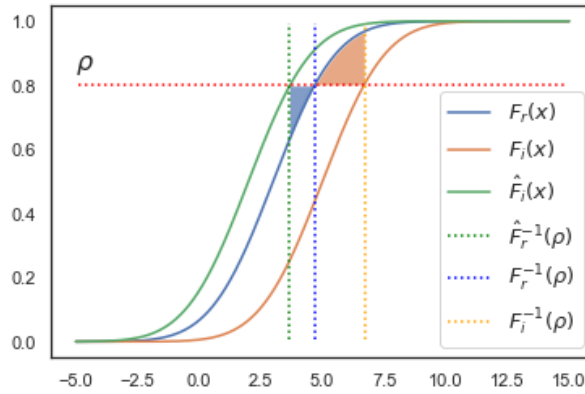


FIGURE 6.3 – Illustration for condition (6.22). Blue area should be smaller than the orange one.

where coefficient  $K := \frac{b-a}{c-a} \in (0, \frac{1}{2}]$ . This, combined with (6.19) gives exactly the conditions of the theorem. Note that the case with  $F_i^{-1}(\rho) \leq F_r^{-1}(\rho) \leq \hat{F}_i^{-1}(\rho)$  is considered similarly. ■

Taking the worst case with  $K = \frac{1}{2}$ , we obtain that the condition for (6.22) holds if  $F_r''(x) \geq 0$  for  $x \in [a, c]$  which represents a sufficient condition for (6.17) to hold. In the general case, condition (6.22) defines the relationship between the quantiles of the forecasts and CDF of a real distribution of  $\Delta g_i$ . This condition is illustrated in Figure 6.3 : blue area should be less than the orange one.

To illustrate the implications of our result we consider the distributions from Example 70 :

**Example 72** For shifted Gaussian distributions, condition (6.22) reduces to

$$\rho \leq G(\rho) := \frac{\mu_i - \mu_r}{\mu_i - \hat{\mu}_i} \Phi\left(\frac{\mu_i - \mu_r}{\sigma} + \Phi^{-1}(\rho)\right) - \frac{\hat{\mu}_i - \mu_r}{\mu_i - \hat{\mu}_i} \Phi\left(\frac{\hat{\mu}_i - \mu_r}{\sigma} + \Phi^{-1}(\rho)\right),$$

which can be easily evaluated numerically. Figure 6.4 demonstrates the values of  $G(\rho) - \rho$  for different  $\mu_r$  while  $\mu_i$  and  $\hat{\mu}_i$  are fixed and are equal to 5 and 2 respectively. The closer  $\hat{\mu}_i$  to  $\mu_r$  comparing to  $\mu_i - \mu_r$ , the bigger admissible values of  $\rho$  are. For example, when  $\mu_r = 3$  (as in Figure 6.3), condition (6.22) is satisfied with  $\rho \lesssim 0.97453$ .

## 6.4 Numerical Experiments

In this section, we provide several numerical examples to provide an illustration on the proposed market model coupling and to numerically demonstrate the properties of the proposed model on the real-world data example. The code for the experiments and the values of all the parameters are provided in [134].

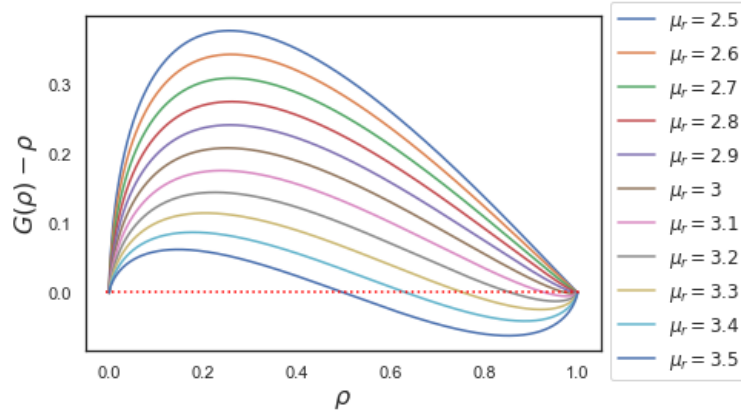


FIGURE 6.4 –  $G(\rho) - \rho$  for different distances between distributions. The closer 'better' forecast to the real distribution, the higher the value of  $\rho$  for which we can guarantee Individual Rationality.

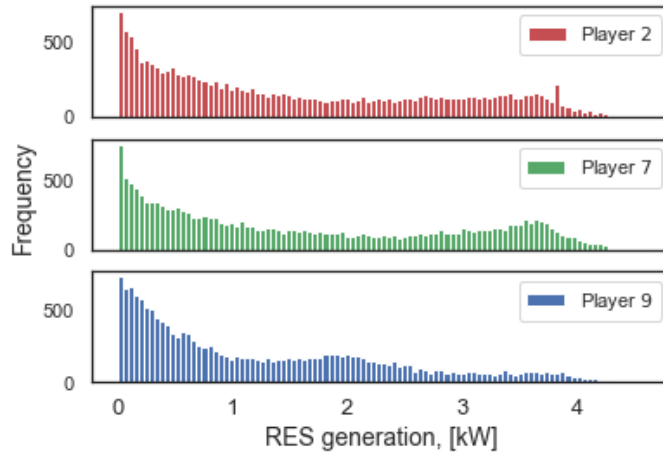


FIGURE 6.5 – Histograms of RES-generation.

**Experimental setup** We use residential data provided by Pecan Street [69] for Austin, Texas. The data consists of 15-minutes intervals specifying renewable generation, load and facilities energy consumption for 25 individual homes. We sample the distribution of scenarios for RES-based generation and demand from the generation data and aggregated consumption respectively. Histograms representing 100 scenarios of the RES-based generation of three agents in the network are given in Figure 6.5. The electricity market prices are chosen arbitrarily to be  $p^{da,b} = 0.3$  [\$/kWh],  $p^{da,s} = 0.2$  [\$/kWh],  $p^{rt,b} = 0.4$  [\$/kWh] and  $p^{rt,s} = 0.1$  [\$/kWh]. These prices were selected for demonstration purposes only, and they can be set to any value as long as they maintain the assumed order  $p^{rt,b} > p^{da,b} > p^{da,s} > p^{rt,s}$ .

For the purpose of our experiment, we assume that there are 25 sellers in the forecast selling market. This allows us to precisely assess the impact of forecasts on electricity trading, with each seller in the forecast market having the optimal forecast for their counterpart in the electricity market. This does not detract from the generality of our results, while it is important to note that in real-life setting the number

of sellers on the forecast or electricity market is arbitrary.

Each agent on the peer-to-peer electricity market can arrive at the wagering based forecasting market, described in section 6.2.3 ([120] for detailed description), as a client. The client submits the task of forecasting the next 15-minutes of renewable energy generation  $\Delta g_i$ . As a historical data  $X_i$ , agent  $i$  provides observations of RES-generation (e.g. Figure 6.5), from which the sellers build distributions, subsequently used in a forecast  $\hat{F}_i$ . It follows that the forecast sellers utilize same data but different models (forecasting skill) to construct the forecasts.

In order to obtain an aggregated forecast  $\hat{F}_i$  we use quantile averaging (QA) described in section 6.2.3. In order to showcase the differences in the forecast only, equal weight values of  $m_n = 1$  are assigned for all  $n$  in  $\{1, \dots, 25\}$ . The methodology for selecting wagers and their effect are thoroughly discussed in [120]. After the task and the forecasts are submitted to the Data Trading Platform, the forecast market operator evaluates the scores of submitted forecasts compensates accordingly.

Each buyer of the forecast (an agent in the peer-to-peer market) is equipped with a base forecast in a form of truncated normal distribution with a mean equal to a sample mean, computed from historical observations. Variance is computed using the scaling factor of 2 and lies in  $[1.455, 2.452]$ .

We test two types of the forecasts available to the sellers on the forecast market :

1. Firstly, we employ truncated normal distribution with the variance smaller than those in the base forecasts of the buyers (prosumers). The means remain the same, while the variance for the 'updated' forecasts now lies in a range  $[0.364, 0.852]$ .
2. Secondly, we test empirical distributions built from historical data as the 'real' forecasts : each seller has access to a distribution  $i \in \{1, \dots, 25\}$ , from which we then draw  $\Delta g_i$  as a realization of a random variable, corresponding to RES-based generation of agent  $i$ .

Finally, we examine a scenario where only one (or two) prosumers purchase the forecast on the forecast market. Due to Theorem 67 we do it without any loss of generality, as the results can be easily extended to any number of buyers. This allows us to concentrate on the impact of the forecast market and the individual rationality property on each agent's level.

**Results** We first illustrate the conditions of Theorem 71 on the real data. Figure 6.6 illustrates the distributions used for agent 1. 'Aggregated' distribution is composed of empirical distributions held by forecast sellers (this corresponds to the setting number 2 above). 'Updated' forecast represents setting 1, while 'Base' represents the naive forecast available to agent 1 initially. Note that the distance between quantiles of these distributions and the empirical distribution calculated at  $\rho = \frac{p^{da,b} - p^{rt,s}}{p^{rt,b} - p^{rt,s}}$  is the lowest for the aggregated forecast, while the distance between the updated forecast is lower than that for the

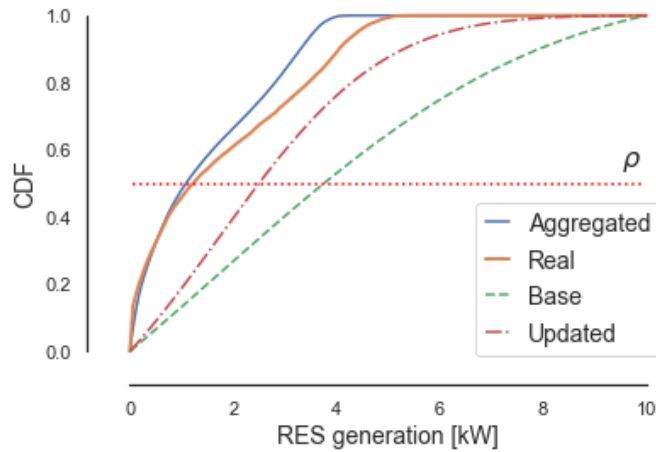


FIGURE 6.6 – CDF of different distributions used for agent 1. (i) 'Aggregated' - aggregated forecast  $\hat{F}_i$  using empirical distributions, (ii) 'Real' - empirical distribution of  $\Delta g_i$ , (iii) 'Base' - truncated normal distribution, (iv) 'Updated' - truncated normal distribution with reduced variance.

base one.

To represent the costs associated with different distributions used as the forecast of an agent, we a sliding mean. Figure 6.7 illustrates the costs of agent 1 on the electricity market associated with different distributions used by the agent to compute the values of her decision variables. We can observe that while the costs associated with a base forecast are the largest, the costs associated with a 'real' forecast and 'updated' one are very close, with the latter rarely exceeding the former. We also observe that the interquartile range (IQR) associated with the distributions decreases as the distribution is closer (in a sense of inequality (6.19), as illustrated on Figure 6.6) to the real one. This is an interesting observation, as it means that even if in some time intervals the 'best' forecast might result in higher costs for the agent, the volatility in the costs of the agent is reduced, which indicates reduced volatility in the electricity market.

On can intuitively guess that better forecast does not guarantee individual rationality in the one-shot game and the purchase of the forecast on a forecast market can even increase the costs of the agent in a short-term. Figure 6.8 demonstrates that even the 'best' forecast leads to the increase in the costs of agent 1 in the beginning, within  $\sim 50$  of 15-minutes intervals. Nevertheless, the total effect of the forecast purchase tends to affect the outcome for the agents positively. We observed that it decreased real costs for the agents purchasing the forecast, while the costs of the other agents remained the same, as illustrated by the red dotted line on Figure 6.8. Note that the cumulative costs associated with the 'real' forecasts are lower than those associated with the 'updated' one, which, with Figure 6.6 provides an illustration of the effect of the 'closeness' of the distributions which we discuss in section 6.3.2.

To demonstrate the outcome of the proposed coupling for the the sellers on the forecast market,

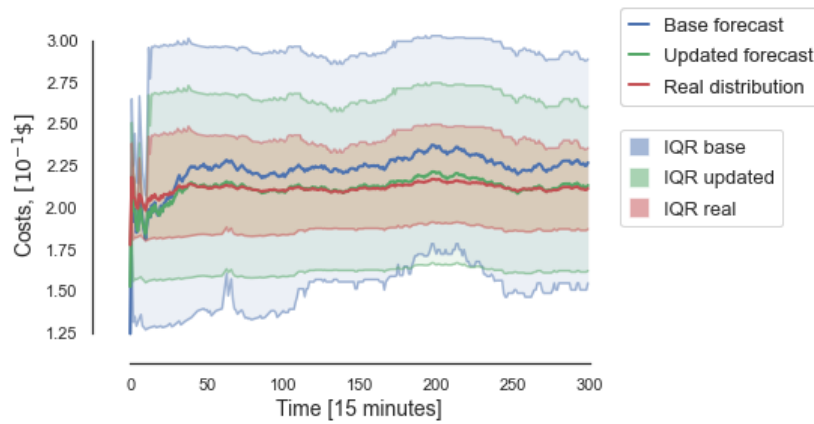


FIGURE 6.7 – Sliding mean costs of agent 1 in electricity market with different distributions used to estimate expected costs. Interquartile range (IQR) for each distribution is represented by the shaded area on the graph.

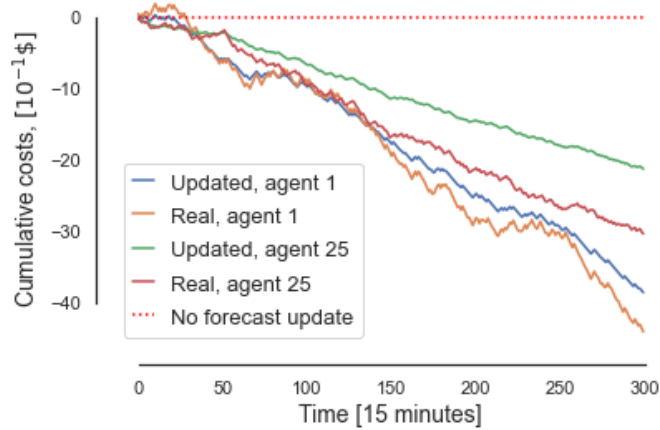


FIGURE 6.8 – Cumulative changes in the costs of agents 1 and 25 due to the improvements in their forecasts.

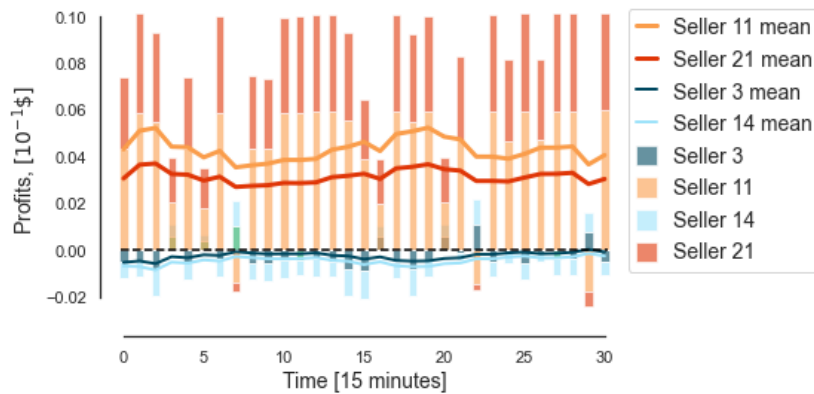


FIGURE 6.9 – Profits of the top two and bottom two scoring sellers in the forecast market. Bars represent obtained profit (loss) in each 15-minutes interval, while sliding means are depicted as the lines of a corresponding color.



we provide an illustration of the profits (losses) entailed by the forecast purchase by agent 1 in Figure 6.9. For the sake of presentation, we illustrate only the profits (losses) of the top (bottom) two scoring sellers in the forecast market. We measure the utility obtained by agent 1 from purchasing the forecast by measuring the difference between the agent's cost associated with a 'base' forecast and the 'updated' one in this case. In our proposed coupling mechanism, this is carried out (as well as scoring allocation for the forecast providers) by the forecast market operator. In average, seller 11 and 21 performed better than the others. Nevertheless, as it is shown by the bar plot, in some instances this agents suffer some losses, caused by misprediction. The opposite holds for sellers 3 and 14.

## 6.5 Conclusion

In this work we presented a framework for coupling electricity peer-to-peer and forecast markets. We addressed the main research questions arising from this coupling : how to evaluate the impact of the forecast update on the agent's utility, how does it improve the operability of the electricity market and what are the incentives for the agents in the electricity market to purchase the forecasts. We proved that i) the impact of the forecast purchase on the prosumers' utility can be computed without information from the other prosumers ; ii) if all the prosumers participate in the forecast market, then the electricity market economic efficiency is achieved ; iii) for the agents there exist incentives to participate in the forecast market, i.e., the purchase of the forecast leads to decreased costs, given condition on the 'distance' between the distributions. Thus, the importance of this result lies not only in the fact that it is profitable for the agent to purchase a forecast, but also in that it allows for achieving electricity market efficiency.

This chapter serves as a starting point in exploring the connection between electricity and forecast markets, highlighting various critical issues that need to be addressed. One of these issues is determining a metric for comparing probabilistic forecasts that ensures individual rationality in the electricity market. Another promising avenue for future research would be to expand the modeling of the electricity market by incorporating factors such as exploring peer-to-peer trading in real-time markets, and adding a stage to account for long-term contracts. Continuing in these directions, we can move towards the most efficient way for the forecast markets to interact with electricity markets with renewable generation.

# Chapitre 7

## Conclusions and Future Work

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### 7.1 Concluding discussion

This dissertation primarily addresses the challenges posed by the development of decentralized peer-to-peer electricity markets, a result of the increasing digitalization, shift towards renewable energy, and emergence of consumer-centric models. The main goal of the work has been to propose game-theoretic solutions to a number of key challenges arising as equilibrium problems in these new market landscapes. Thus, this thesis contributes to the ongoing research on the transformation of electricity markets, and related equilibrium problems, supporting the adoption of new market models for electricity market decentralization.

To better capture the market dynamics related to a more proactive role of consumers in decentralized electricity markets, this dissertation has considered various equilibrium concepts, primarily the Generalized Nash Equilibrium (GNE), which allowed us to take into account how each participant's actions can influence the strategy space of others. This framework enabled us to understand how strategic decisions in these markets might influence the operation of the markets, as well as providing a set of decision-making rules and methods to guide the decentralized electricity market evolution.

Mainly, our research spotlighted several factors that make achieving equilibrium in decentralized electricity markets challenging. These include the coordination between multiple actors with diverse objectives, uncertainty in generation and demand, distribution network constraints, and information asymmetry among participants. To address these issues, which are vital for the efficiency and overall performance of these markets, we have developed several game-theoretic models, incorporating several key

aspects to the studied problems, which are described next.

## Contributions

The first step of the dissertation has focused on capturing the presence and effects of heterogeneous risk-aversion agents in peer-to-peer electricity markets. In this regard, Chapter 3 of this dissertation has highlighted the importance of managing risk in such markets, where diverse risk-averse actors may have different perceptions of risk. We used a Stackelberg bi-level game model to derive a pricing scheme that promotes fairness and efficiency. Our proposed model's reformulation allowed a simpler analysis of equilibria characterization and market properties, thus contributing to the understanding of the proper organization of insurance markets in a peer-to-peer setting.

To connect the financial level of electricity markets with the physical system captured by the electric distribution grid, we have introduced in Chapter 4 a model that addresses the interaction between the financial and grid layers of local electricity markets, focusing on constraints in electricity distribution. We showed how the proposed trading system could limit exploitative behavior by leveraging price signals from the distributed system operator in the form of dual variables generated from a formulated DSO-level optimization problem. The derived results show that we can guarantee Pareto efficiency of the market equilibrium when the cost to trade is uniform across all participants.

In Chapter 5, we tackled the challenge of 'information asymmetry', aiming at capturing the fact that decision-makers might possess information they did not want to share with others. In this regard, we have developed a novel model that predicted strategic behavior of the agents while maintaining privacy within a GNEP context. This model has offered insights into privacy dynamics in decentralized electricity markets and provided a means to quantify the cost of privacy, evaluating the trade-off between information sharing and equilibrium costs.

Finally, in Chapter 6, we addressed the uncertainty arising from the integration of renewable energy sources into the energy mix. We proposed an innovative combination of electricity markets with forecast markets to improve decision-making and reduce uncertainty. Our results showed the possibility of achieving efficiency in the peer-to-peer electricity market when all agents report their forecasts, thus encouraging their active participation.

Overall, our research used fundamental principles from game theory, optimization, and algorithmic designs to develop novel models and mechanisms, capable of addressing technical challenges, quantifying the potential of decentralized energy systems in enhancing the efficiency and consumer-centricity of emerging electricity systems and markets. In addition to the advancements to the state of the art described above, as well as in the detailed conclusions given in each chapter, several generalizations can be

drawn from this dissertation :

- Implementation of adjunct services plays a pivotal role in optimizing the functionality of decentralized electricity markets and is particularly beneficial in an environment where multiple agents act strategically in their interests. These services function as corrective mechanisms, offering tailored solutions to address the specific, distinctive issues raised by the strategic behavior of individual agents, facilitating the attainment of efficient market equilibrium. These issues reflect numerous facets of decentralized markets, such as, e.g., decision-makers possessing information they did not want to share with others, heterogeneous inherent properties (e.g., risk-aversion), or the quality of the prior belief about the probability distribution of the uncertainty, introduced by renewables integration. Introduction of targeted services for each of these problems mitigates the negative impacts of potential disparities and conflicts, thus resulting in better market operation.
- Information exchange is crucial in decentralized electricity markets, essential not only among prosumers (e.g., private information exchange) but also between prosumers and other market participants such as distributed system operators (e.g., dual variables, price-signals, etc.). However, the implementation of such information exchanges should be carefully calibrated. In the absence of control measures, there is a risk that certain agents strategically manipulate the information exchange system for personal gain, thus exploiting the system to their advantage, thereby inducing free-rider behavior. As such, the design of an information exchange system should balance the need for transparency and mutual benefit with safeguards to prevent exploitation and promote fair and efficient market organization.

## 7.2 Future work

The modeling and experimental contributions of this dissertation paves the way for a number of future research directions. Motivated by the results and discussion in this thesis, future works could be advanced through the following recommendations.

**Refined network representation** The models introduced in this work predominantly dismiss the grid modeling or, when considering grid-related aspects, rely on linearized power flows models (known as the DC power flow representation, as in Chapter 4), which notably simplifies the non-linear and, importantly, non-convex nature of the alternating current (AC) power flows. This assumption is frequently made in both short-term and long-term planning issues. However, recent progress in formulating and solving models based on AC power flows, which are computationally efficient, could enhance the resolution of solutions derived from the proposed models. Incorporating AC constraints into the proposed

models will enable accounting for reactive power flows and nodal voltages, which may lead to more accurate representation of the grid, and the impact the P2P trading can have on the grid operation. This also allows assessing the technical and financial impacts of fluctuating renewable power generation on ancillary reactive power and voltage control services. With anticipated increases in renewable energy usage, ensuring AC feasibility becomes crucial. These model enhancements further allow us to evaluate the financial worth of the delivery of grid services by the peers. These models enhancement also allow exploring the potential of peer-to-peer markets to deliver grid services.

**Refined modelling approaches** In addition to the models presented in this work, there is considerable scope for enhancing these models by integrating them with other game-theoretic methods to enhance operations of peer-to-peer electricity markets. This integration would enable us to express different valuation types (e.g., various auction models) in a clear and extractable way. Another possibility would be to incorporate more intricate methods for updating information, akin to what is seen in Bayesian games. Moreover, in the data-driven analysis, the question of the restricted size of available datasets or noisy data comes into play. In this case, the next step should be to employ equilibrium refinements to address the challenges tied to achieving an equilibrium in games with noisy observations.

**Refined algorithmic approaches** While the algorithmic approaches utilized in this study have shown considerable effectiveness in addressing the challenges of GNEP in decentralized electricity markets, there is substantial potential for further improvements. The complexities of GNEPs necessitate the use of algorithms that are not only efficient but also suitable for the complexity of the optimization problems inherent to decentralized systems. Advanced algorithmic designs, particularly those capable of better handling the intricacies of GNEP, could significantly enhance the practical implementation of the proposed models.

**Refined information assessment** Designing effective market mechanisms is traditionally based on the assumption of the rationality of the agents, as well as the objective perception of their environment, underlying uncertainties, and decision-making processes. However, deviations from rationality and noise in available data are well-known problems, especially in markets shifting towards a decentralized structure. Thus, capturing subjective preferences, perceptions, and valuations (e.g., private data evaluation) of decision-makers, as well as the underlying uncertainty, is crucial in multi-player distributed decision-making processes. This approach leads to a more accurate reflection of the decisions made by the agents while reacting to price signals (e.g., by acquiring/releasing data), forming coalitions (e.g., collective investment agreements), or participating in novel markets. This direction closely aligns with the evolution

of market structures (e.g., data markets), which requires innovative mechanism designs (using auction, bi-level problems, etc.), enabling decision-makers to reach more efficient outcomes by accurately leveraging their information and preferences.



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## RÉSUMÉ

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Les marchés de l'électricité sont de plus en plus décentralisés et intègrent davantage de ressources énergétiques distribuées (DER) et de modèles axés sur le consommateur. Cette transition présente de nouveaux défis et nécessite des modèles de marché actualisés. Cette thèse explore ces complexités, en se concentrant sur la coordination de divers acteurs, la gestion de l'offre et de la demande, les contraintes de distribution et l'asymétrie de l'information. Nous utilisons des approches théoriques des jeux, examinant des cadres pour les marchés de l'électricité de pair à pair avec des agents averses au risque et des prosommateurs couvrant les risques par le biais de contrats financiers. Nous étudions également l'interaction entre les niveaux physiques et financiers, la comparaison des modèles de marché centralisés et entièrement distribués, et aborde la question de l'asymétrie de l'information. Enfin, nous étudions le couplage des marchés prévisionnels avec le marché de l'électricité. Les résultats soulignent l'importance des services supplémentaires et de l'échange d'informations dans ces environnements stratégiques.

## MOTS CLÉS

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La théorie des jeux, Smart grids, Algorithmes, Pair a pair, L'équilibre de Nash généralisé

## ABSTRACT

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Electricity markets are increasingly being decentralized, integrating more Distributed Energy Resources (DERs) and consumer-focused models. This transition presents new challenges and requires updated market models. This thesis explores these complexities, focusing on the coordination of diverse players, demand-supply management, distribution constraints, and information asymmetry. It utilizes game-theoretic approaches, examining frameworks for peer-to-peer electricity markets with risk-averse agents and prosumers hedging risks through financial contracts. It also investigates the interplay between physical and financial levels, comparison of centralized and fully distributed market designs, and addresses the issue of information asymmetry. Lastly, it explores coupling forecast markets with the electricity market. The findings emphasize the importance of additional services and information exchange in these strategic environments.

## KEYWORDS

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Game theory, Smart grids, Algorithms, Peer-to-peer, Generalized Nash Equilibrium