Essays on stock market returns' exposure to investor sentiment using time-series and panel switching regime models

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Essays on Stock Market Returns' Exposure to Investor Sentiment using Time-Series and Panel Switching Regime Models

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« The University of Paris Saclay and the Faculty of Economics and Management of Sfax do not approve nor disapprove the opinions expressed in a thesis: they must be considered as the *author's own* ».

I dedicate my thesis to my husband IMED and my aunt RAWYA

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Preface

In this thesis, the first chapter is theoretical, and its purpose is threefold. It recalls the informational efficiency concept, and provides an overview of the literature relating to puzzles in the stock markets and behavioral finance as a better way to present them. Moreover, it discusses the impact of investor sentiment on stock market returns.

The last two chapters of this thesis are written in the form of papers and they can be read independently of one another. Although they are closely linked, they answer the problem differently.

- Jawadi, F., Namouri, H. and Ftiti, Z. (2017), " An analysis of the effect of Investor Sentiment in a Heterogeneous Switching Transition Model for the G7 stock markets ", Journal of Economic Dynamics and Control. Under Review.

- Namouri, H., Jawadi, F., Ftiti, Z. and Hachicha, N. (2017), " Threshold effect in the relationship between investor sentiment and stock market returns: A PSTR specification ", Applied Economics. pp. 1-15.

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Introduction

"Dysfunctions that financial markets have recently encountered, particularly the crash of 19 October 1987, have reactivated in the economics community a series of debates - sometimes very elders- about market efficiency. At the heart of these debates lies the question of the relevance of prices that are formed there: do they satisfactorily express the constraints specific to production and exchange activities, or are they the products of a partially mass psychology, or even totally disconnected from these realities?"

Orléan (1989, p. 44)

While the hypothesis of market efficiency had been accepted in the 70s, the crash of 1987 was a catalyst of the movement that called into question the market efficiency. Even Grossman and Stiglitz (1980) have questioned since the 70s the market efficiency assumption. Thereafter, Shiller (1981) also confirmed market inefficiency by conducting an empirical study in the US market that showed an excess of stock price volatility with respect to dividends.

At the end of 1999, for the first time in history, the market capitalization of financial markets exceeded the world's gross domestic product. However, the financialization of the economies after the subprime crisis in 2008 ultimately led the world economy into the most severe recession since 1929.

After each stock market crash, severe criticism overwhelms the efficient market theory. An informed efficient market should evaluate the securities exchanged at their fundamental value. However, crashes reflect a general dysfunction rejecting this Efficient Market Hypothesis.

Investors have different information for traded assets and they analyze differently this information. Thus, the hypothesis of homogeneous rational agents is brought into question. On another side, the stylized facts observed in real markets such as volatility clustering, excess kurtosis, auto-correlation of returns, bubbles, and crashes cannot be reproduced by the random walk hypothesis implied by efficiency. Consequently, it is difficult to support the efficient market assumption as in the classical model. A new competing theory, behavioral finance, questions the rationality and efficiency hypotheses. According to behavioral finance,

psychological impulses or collective agreements can deviate asset prices from their fundamental values. From the early 1990s, the behavioral finance earned recognition while highlighting the role of psychology and experimental economics in the decision-making process. Furthermore, the prospect theory, proposed in 1979 by Kahneman and Tversky, was recognized as a model for the construction of a new financial theory that integrates more realistically individual behavior.

According to Tadjeddine (2013):"Behavioral finance led to more realistic assumptions for understanding financial behaviors. Each behavioral axe gives an explanation for a phenomenon observed on the markets or in the laboratory, which previously contradicted the results of the classical paradigm" (p. 24).

Behavioral finance suggests investors' psychological biases are important factors that help better understand stock price evolution. In this context, several researchers have tried to highlight the behavioral and emotional biases explaining investors' irrationality (eg. Barberis and Thaler, 2003; Shiller, 2000). In particular, investor sentiment, as part of emotional biases, has become a key concept in behavioral finance arousing the interest of many researchers to explain the sustainable misalignments of prices (Fisher and Statman, 2003; Brown and cliff, 2004, 2005; Chen, 2011; Lux, 2011; Ni et al., 2015, etc.).

1. Investor sentiment and stock market returns

By analyzing market behavior, many researchers noticed market anomalies as well as investor behaviors' biases.

To explain these anomalies, a new theory emerged in the United States in the early 1980s based on a behavioral approach to study investors' decisions and market movements. The first behavioral finance contributions appeared with the work of psychologists, in the forefront those of Kahneman and Tversky (1979), who confirmed, through the lottery, flaws in rationality during the decision-making process. The authors showed at the time of the search for a solution, the brain uses heuristics leading to irrational decision.

Given this reality of behaviors, which are more complex than that of conventional theory, behavioral finance seems destined to become the norm (Thaler, 1999). To become more operational, this new theory proposes tools to help understand stock market dynamics, the most important among them being the market sentiment.

Therefore, the general problem of this thesis focuses on the analysis of the relationship between investor sentiment and stock market dynamics.

Investors behave heterogeneously, thus the sentimental component affects the stock market returns. According to De long et al. (1990), the unpredictability of noise traders, based on their sentiment in decision making, accentuates price deviations from fundamentals and increases stock price volatility.

It turns out that investor sentiment creates inertia effects and complex price dynamics[1]. In this context, several researchers have tried to model the investor sentiment's role in the financial markets to explain the misalignment of prices. Shiller (2000) found that the bubble expectation index is correlated with past returns of the Dow Jones Industrial. Over the period 1989-1999, five of the six high points of the bubble expectation index correspond to the high points of returns over the past six months of the Dow Jones Industrial.

Still on the American market and over a period spanning from July 1981 to December 2008, Beer et al. (2012) showed that risk sentiment explains the profits of constructed portfolios that other risk factors do not. In addition, they found that profits increase as they introduce sensitive stocks to the portfolios. The authors conclude that fund managers should consider the risk sentiment in their valuations of financial assets.

It follows that investors and managers who take into account the dynamic behavior of asset prices and integrate the sentimental component into their financial decisions are more likely to have realistic expectations about future prices and the risks to which they are exposed.

One of the limitations of previous studies is that most empirical studies have only focused on the US market (Black, 1986; Daniel et al.,1998; Brown, 1999; Verma et al., 2008; Kumar and Lee, 2006; Lemmon and Portniaguina, 2006; Baker and Wurgler, 2006; Yu and Yuan, 2011; Chung et al., 2012; Stambaugh et al., 2012; Kim et al., 2014, etc.). Few studies have so far studied the link between investor sentiment and stock market returns across developed markets (Scmelling, 2009; Zouaoui et al., 2011; Corredor et al., 2013; Bathia and Bredin, 2013).

[1] As Shefrin (2005) documented: "In finance, sentiment is synonymous with error" (p. 213).

It is therefore relevant to study the market returns' exposure to sentiment risk in a competitive international context during crisis periods.

2. Overview on nonlinear models

The use of linear models is linked to restrictive assumptions (the assumption of the absence of transaction costs, the homogeneity of investors' anticipations, and information symmetry). This implies a rapid and symmetric adjustment of prices towards the fundamentals, with a constant speed of convergence. However, the presence of rigidities in the stock markets often prevents the prices from adjusting in a continuous and linear way. According to Frances and Van Dijk (2000), the asymmetry characterizing most market data makes the linear price adjustment unlikely.

In this context, several researches showed the persistence of asset price deviations (Boswijk et al., 2007; Manzan, 2003, 2007, etc.). These studies focused on the processes bringing these deviations to equilibrium in the presence of transaction costs and operators with heterogeneous expectations. They concluded the superiority of non-linear models toward linear models to take market frictions into account.

In the literature, many models have been explored to model nonlinearity. We can distinguish between two kind of models: nonlinear variance models and nonlinear mean models, including regime-switching models[2]. Models that allow for state-dependent or regime-switching behavior have been most popular over the last years.

All the above regime-switching models have allowed in the empirical literature to model asymmetries such as distinct dynamics in the ascending and descending phases by means of their different regimes.

We can distinguish between two main classes of models existing in time series: Markov switching models and thresholds models. In the context of Markov models, the transition mechanism is based on an unobservable state variable, which is supposed to follow a Markov chain. At each period, there is therefore a certain probability of belonging to a given regime.

In contrast, for threshold models, the transition mechanism is carried out using an observable transition variable, a threshold, and a transition function. A smooth transition

[2] For more details on regime-switching models, see Uctum (2007).

model has the advantage of containing a continuum of regimes. Then, belonging to a regime depends on the transition function and on the distance between the threshold and the transition variable. Some recent studies showed the superiority of the smooth transition autoregressive (STAR) models toward the Markov switching models (Phillips, 1991; Sarantis, 1999; Deschamps, 2008).

We thus use in this thesis the smooth transition regression models. In fact, Granger and Teräsvirta (1997) interpret the smooth transition as the abrupt regime shift that individuals make on different dates. The non-simultaneity of individual behaviors reflects that individual or institutional agents who better anticipate the government action can begin their transition before the regime shift. while, other agents can react late because of information costs. This is in line with Uctum (2007) who adds that "such interpretation can be extended to situations where individual reactions may be gradual to varying extents, reflecting behavioral inertia due to transaction costs, habits or uncertainty" (p. 457).

3. Research question and contribution of the thesis

In a financial market, investors are different from each other and might have heterogeneous expectations. This is due to information asymmetry and the heterogeneity of the analysis information. Moreover, the same information might be interpreted in a subjectively different way by each investor. The emotional state plays a very important role in decision making.

Investor sentiment is an intuitive tool affecting the stock price formation process proposed by behavioral finance. Within this framework arises the problem statement of our thesis, which deals with the study of the relationship between the investor sentiment and stock market returns. Thus, we propose to answer the following questions:

- Does investor sentiment influence the dynamics of stock market returns?
- Does the effect of investor sentiment vary with the phase of the business cycle?

The answers to the various questions related to our problem statement are interesting for investors and fund managers. They can help them in their decision-making process and foster a better allocation of financial assets. This contributes to a better integration of market sentiment in asset management. Moreover, the motivation of this topic is related to the realization that investor sentiment is a tool of behavioral finance that helps predict market

dynamics better. This allows us to understand the functioning of financial markets while enabling the various managers concerned to make relevant and appropriate decisions.

From an empirical point of view, we propose two econometrical specifications to study the nexus investor sentiment-stock market returns using the Smooth Transition Regression model, "STR model" developed by Granger and Teräsvirta (1993) and Panel Smooth Transition Regression model, "PSTR models" proposed by González et a1. (2005) in a nonlinear context.

The heterogeneity of agents' expectations is a source of nonlinearity. Investors have heterogeneous expectations and can change from one regime to another while involved in interpersonal interaction. In particular, the emotional component (sentiment) affects agents 'reaction following the arrival of new information. This causes adjustment delays in prices, which is difficult to represent by simple linear models.

The objective of this thesis is thus to consider the behavioral heterogeneity of investors induced by the sentimental component to model the nonlinear stock market returns' dynamics with threshold models.

Furthermore, we use panel data to capture the heterogeneity. Taking into account the individual and temporal dimensions brings out interesting characteristics that could not be distinguished in temporal series for lack of sufficient observations. By working simultaneously on several individuals instead of the aggregates, the risks of bias can be minimized.

In this context, the use of regime-switching models in panel data makes it possible both to combine the advantages of working on panel data and to simultaneously solve the problems of non-linearity, heterogeneity, and temporal instability of the relationship over time. More precisely, these models allow the existence of distinct individual dynamics that may evolve over time while taking asymmetries into account. In addition, to carry out our study, we use a database that integrates the major developed markets: G7.

4. Thesis plan

The present work is divided into three chapters, presenting theoretical concepts and empirical studies.

After recalling some different definitions (efficiency market hypothesis, puzzles and anomalies, behavioral finance, and investor sentiment), the first chapter will be largely

devoted to a study of the literature concerning investor rationality, since it is mainly on this point that behavioral finance, from which market sentiment arises, opposes the classical finance. The notions of behavioral heterogeneity and noise traders will allow us to address rationality in its collective dimension and to have a slightly different perspective on the phases of euphoria and pessimism.

Subsequently, we will define market sentiment by emphasizing the diverse approaches and issues posed by the use of the representative investor. To conclude, we will explain the link between investor sentiment and stock market returns.

In the second chapter, we will explore the exposure of market returns to sentiment risk in the G7 countries before and after the subprime crisis. For this purpose, we will propose a theoretical model inspired from Barberis et al. (1998). We postulate that stock market returns depend on the reaction of two kinds of investors: arbitrageurs and noise traders. Empirically, we will use the STR model. This methodology will allow us to capture the non-linearity and the smoothing effect between regimes. We identify threshold effects, different regimes, and different transition speeds. The detected regimes depend on the risk sentiment triggered by the noise traders.

Considered as an extension of the second chapter limited to a study of the relationship between investor sentiment and stock market returns within the context of time series, the third chapter aims to investigate the same problem with a nonlinear econometric modeling in Panel data. We will use the PSTR model that combines both the benefits of Panel data and nonlinear modeling. We found that when the study was carried out in time series, the thresholds effects have not emerged for all countries for lack of sufficient number of points in different regimes. This finding clearly illustrates one of the advantages of regime switching models in panel data.

Our threshold approach in this context has many advantages. First, the link between stock market returns and investor sentiment is allowed to change according to sentiment risk. Second, an ex-ante value of the threshold is not imposed but obtained thanks to the model. Third, the transition from one regime to another can be done smoothly, which allows the existence of a neutral zone in which the sentiment is inelastic to the risk sentiment. Besides, The panel study provides the opportunity to study individual behavior while observing the behavior of others. Also, it allows taking into account individual heterogeneity between different countries.

To the best of our knowledge, these econometric methods have never been used to examine the link between investor sentiment and stock market returns in the related literature. we therefore propose an original application on the subject, which constitutes the main contributions of this thesis.

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Chapter 1: Investor sentiment and stock market returns' dynamics

1. Introduction

Financial theory has been based for half a century on the Efficient-Market Hypothesis (EMH). According to this hypothesis, the price should be equal to the fundamental value (Samuelson, 1965), which means that the change in stock prices can only result from the change in fundamentals. However, the past few decades have been marked by frequent financial crises characterized by increased volatility and asset prices' correlation. In addition to these financial crises, we noted important stock price misalignments toward fundamentals.

Although fundamental variations may partially explain the increase in stock price deviations, fluctuations during the last decade have proved too important to be justified by changes in fundamentals. On the one hand, several empirical studies have detected puzzles and anomalies thus revealing significant deviations from equilibrium values. On the other hand, the assumption that all individuals are perfectly rational seems restrictive and unrealistic. This explains why the 1990s have witnessed the emergence and predominance of behavioral finance. This research stream challenges the assumption of a perfectly rational representative agent and opts for bounded rationality of agents.

As a result, the development of strategies that take advantage of both behavioral and emotional bias has become the main concern of many empirical studies (Shiller, 2000; Barberis and Thaler, 2003; Orléan, 2004 ; Akerlof and Shiller, 2010, etc.).

In this context, several researchers introduced more sophisticated models taking the market sentiment into account. In particular, De Long et al. (1990) highlighted the role of noise traders- investors whose decisions are primarily justified by their sentiments- on price formation. Moreover, noise traders might have a mimetic behavior and common emotions related to their social interactions (Kumar and Lee, 2006). Accordingly, the sentiment risk caused by noise traders can induce complex stock market return dynamics.

Designed to put into perspective the concept of market sentiment, this chapter will be structured as follows: the second section will be devoted to a definition of the main concepts

of our thesis. Section 3 will recall the informational efficiency theory. Finally, section 4 will focus on investor sentiment and its complex impact on stock market returns.

2. Concepts

2.1. A perfect market

A "perfect" market is supposed to give the most complete results. This means that there is no arbitrage opportunity. An alternative definition of such a market is therefore a market which is fully efficient. In other words, observed prices are reliable signals for resources allocation. Efficiency can thus be defined as the market's achievement of its function (Gillet, 2006).

2.1.1. Efficiency Market Hypothesis

Traditional financial theory is predominantly based on Efficient Market Hypothesis noted "EMH"[3]. This hypothesis primarily postulates that investors are rational, as they aim at maximizing their expected utility, by accounting for risk in the information they have. Accordingly, stock prices should reflect the information available to all investors.

Hence, the efficiency concept appears to integrate four types: the first one relates to the agents' rationality (Mandelbrot, 1966; Samuelson, 1965). The second type relates to the optimum allocation of resources. This implies that the market is able to direct funds to the most productive resources, thus allowing a satisfactory economic development. The third type corresponds to operational efficiency, which refers to stock-market transaction costs supported by buyers and sellers. The market is organized in such a way as to confront suppliers and capital seekers, considering that intermediaries perform their tasks by restricting themselves to a fair remuneration. In other words, the exchange process should lead to the price formation procedure at the lowest cost possible. The fourth type, informational efficiency, represents the stock prices' ability to reflect thoroughly and immediately all available information.

Even if these various efficiency types are independent, we will be interested in informational efficiency, the most studied type of efficiency in the literature, and the rationality of the agents' behaviors. This choice can be explained by the knowledge that, on

[3] See Gillet (2006) for more details on the Market Efficiency.

the one hand, informational efficiency is the essential pillar of financial theory and, on the other hand, rationality is a crucial concept in economic theory.

2.1.2. Rationality

Any agent acting in compliance with the expected utility maximization procedure is considered to be rational. In this regard, the Allais (1953) definition suggests that an agent is considered rational as long as his or her decisions turn out to be logical and appropriate for achieving his or her purposes.

As Allais (1953) indicated : "a man is considered rational when his purposes are consistent for each other and when he uses appropriate means to achieve his purpose"

(p.518).

The initial expected-utility-theory based model has been dubbed instrumental rationality. This form of rationality stresses that individuals seek to maximize their utility (under constraint) in a universe where information is predominantly free and unlimited. Hence, traditional models have been focused on judging preferences, which help assess a certain action and its possible yielded outcomes. Thus, the individual's behavior is exclusively based on preferences. According to Von Neumann and Morgenstern (1947), the preferences should satisfy three axioms. First, the continuity axiom stipulates the preference order is continuous in a uncertain situation. Second, the independence axiom posits autonomy between preferences and probabilities. Third, the transitivity axiom highlights that the individual is able to rank the entirety of actions and measures.

Hence, instrumental rationality helps reflect efficient actions as cognitive abilities that are usually assumed to be unlimited. In this context, Walliser (1982) took the difference between the real environment and the perceived one into consideration through introduction of the cognitive rationality concept. In other words, the representations drawn by agents in the real environment prove to depend highly on individual beliefs and differ from one individual to another.

In 1972, Simon introduced the bounded rationality assumption: "Rationality denotes a style of behavior that is appropriate to the achievement of given goals within the limits imposed by given conditions and constraints"(p.161).

While the constraints agents face in the instrumental rationality context are external, they become internal within the bounded rationality framework. Owing to his or her limited

capacity, the agent would seek to achieve a suitable satisfaction threshold rather than trying to find solution to maximize his or her satisfaction. Consequently, the selection process would stop as soon as the individual chooses an action he considers higher or equal to his satisfaction threshold.

Insofar as the market is efficient, in addition to the observation that agents prove to behave rationally, their expectations would, most often, turn out to be rational. That the price appears equal to the fundamental value should necessarily denote that the agents' expectations about dividends prove to be relatively rational.

According to Muth (1961): " that expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory. At the risk of confusing this purely descriptive hypothesis with a pronouncement as to what firms ought to do, we call such expectations "rational" "(p. 316).

Formally, the rational expectations hypothesis is defined as follows:

$$x_t^a = E(x_t|I_{t-1}) \quad (1\text{-}1)$$

Where x_t^a is the anticipation made at time t-1 for variable x_t

I_{t-1} is the set of information available at time t-1

E is the mathematical conditional expectation in the available information set.

This supposes a perfect coincidence tends to prevail between the model applied and the economic operational one. In other words, all agents would tend to use the same model, i.e. the traditional dividend discount model (DDM), to predict stock prices. In addition, all agents are supposed to enjoy freely unlimited information processing capabilities.

2.2. Puzzles in Stock markets

The efficiency theme lies at the core of financial theory. It has been the focus of interest of several research studies. Still, it has aroused noticeable controversy in the financial area. Thus, the efficiency subject has gained greater attention owing to the lack of result unanimity. The entirety of classical models and financial theories have predominantly based themselves in the study of the efficiency hypothesis.

Essentially, two major periods can be distinguished to mark financial market history. The first one concerns the 1970s, during which informational efficiency was recognized by

the major world markets. The second period starts at the 1980s and it has been characterized by controversies and puzzles striking the empirical literature incompatible with the EMH. It is the identification of these empirical anomalies that has led to a further reinforcement of the financial markets' inefficiency.

2.2.1. The excess volatility puzzle

According to the rational expectations hypothesis, the agents' expectations prove to heavily rely on the same equilibrium model. Given the importance of how anticipations have been formed and their impact on financial market efficiency, some studies tested rational expectations. More particularly, volatility tests have undertaken to underline the excessive volatility phenomenon persistent in stock prices.

Several authors have suggested we compare market price volatility to that derived from the equilibrium model. On applying the CAPM, the observed price p_t turns out to be a conditional expectation of ex-post price (rational), noted p_t^*, for any information available at time t : $p_t = E(p^*_t)$.

By definition, p_t is an optimum forecast of p_t^*. u_t is a forecast error. This error term is uncorrelated with p_t. We therefore obtain $u_t = p^*_t - p_t$ and $var(u_t) = var(p^*_t) - var(p_t)$. Given that a variance cannot be negative, it turns out that $var(p_t) \leq var(p^*_t)$.

This inequality appears to represent the variance bounds test (Shiller, 1981a), according to which the observed price volatility should be lower than that of the ex-post price. This inequality should be verified; it constitutes a test of the equilibrium model (LeRoy and Porter, 1981; Shiller, 1981).

The results obtained by Shiller (1981a) (in terms of standard deviation); for the S&P500 index (1871-1979): $\sigma(p) = 50,12$ and $\sigma(p^*) = 8,968$. For the Dow Jones index (1928-1979): $\sigma(p) = 355,9$ and $\sigma(p^*) = 26,80$.

According to these findings, it seems clear that the inequality marking the variances turns out to be strongly violated, which makes the observed price sound too volatile to be attributed to future fundamentals' change. Consequently, the CAPM model turns to be rejected.

Following the volatility test intended to assess the standard evaluation model CAPM's empirical validity, there are major second-class evaluation model relevant tests, along with their implications on the financial market. Stemming from applying the representative agent's

preference function, the second class figuring models brings together the entirety derived from Lucas' (1978) elaborated studies relating equilibrium models.

2.2.2. The equity premium puzzle

Lucas (1978) proposed a capital asset pricing model with consumption (C-CAPM; Consumption Based Capital Pricing Model), in which a new risk measure has been developed. It is given through the covariance binding the asset return and the consumption marginal utility. To test this model, Mehra and Prescott (1985), examined whether it is liable to reproduce the US stocks' different moments over the period 1889-1978.[4] They concluded that the Lucas (1978) model turns out to be incapable of reproducing a significant difference between the return of a risky asset and that of a risk-free asset, culminating in the excessive equity premium puzzle.

The equity premium can result from the risk aversion coefficient's fallacious level. The risk-averse investors usually tend to transfer the wealth of high growth marked periods towards the lower growth ones in a bid to homogenize the consumption growth over time. It is this high-risk aversion which often prompts investors to borrow for the sake of reducing the difference between the present consumption and the future consumption. This creates a high demand for capital, which naturally helps increase the interest rates. Still, as interest rates are low on the market, it is assumed that investors are overly patient so that future consumption would be as much appreciated as the current one. In other words, it is important to think that the investors' current preference rate would be zero and even negative. Such a state has been dubbed as the risk-free rate puzzle identified by Weil (1989).

Overall, the observed puzzles would imply that the standard model does not seem to reflect some of the fundamental features characterizing market functioning and operations.

2.2.3. The excess-volume puzzle

A major conundrum for researchers in the finance area is the excess-volume puzzle. In this context and according to the classical financial theory, investors are all rational and aware that the others are also rational. It is this assumption which keeps trading volumes predominantly low. Nevertheless, transaction volumes appear most often to be remarkably high with respect to most financial markets. In 2002, for instance, the number of the daily

[4] Using an additively separable utility function, the authors noted a highly remarkable risk premium as compared to the one the equilibrium model C-CAPM provided.

traded shares reached the threshold of 1.44 billion, and on February 2003, the NYSE annualized turnover roughly attained the level of 96%. Such figures contradict the efficiency assumption. Even by accounting for the diversity of personal situations, the trading volume puzzle cannot be explained (Glaser and Weber, 2007).

In this respect, behavioral finance provides an explanation for this conundrum through highlighting that some investors are prone to overconfidence[5]. In effect, Glaser and Weber (2007) tested the direct relationship binding the trading level concerning more than 215 on-line investors and individual overconfidence. The authors ensured that these investors should respond to a questionnaire purposefully designed to indicate their overconfidence level according to four measures; (incorrect calibration in probability estimates, the conviction of being better than average, the control illusion, and over-optimism). The authors studied their stock exchange transactions over the period ranging from January 1997 to April 2001. They found that level trading turns to be directly connected to the overconfidence measured through the "better than average effect."

Other empirical studies have examined the indirect link persisting between the trading volume and excess in individual and collective confidence. For instance, Statman et al. (2006) discovered that stock market trading volumes are positively correlated with price movements scored over the preceding months. The authors found that the stock trading volume is positively and significantly correlated with the previously recorded market returns as well as those relating to the stock itself. They also concluded that overconfidence turns out to be primarily due to the self-attribution bias whenever markets are bullish.

Strongly related to the investors' psychological patterns, these puzzles seem hardly compatible with the rationality assumption.

2.3. Behavioral finance and investor sentiment

Rationality is a common element in the efficient market hypothesis and in behavioral finance. For the proponents of the efficiency theory, rationality perfectly characterizes the behavior and expectations of the economic agents. The potential irrational agents are

[5] Several theoretical studies such as Daniel et al. (1998) and Odean (1998b) stressed that some investors' overconfidence helps greatly in maintaining a general equilibrium through excess in transaction volumes. Other studies, such as that conducted by Varian (1985), have concluded that the greater the opinions' heterogeneity, the higher the trading volume turns out to be.

eliminated quickly because of the losses their strategies generate. This is due to the intervention of the arbitrageurs who converge the price to its fundamental value. As for supporters of behavioral finance, any valid model or theory should take into account the irrational part of the behavior.

Often preceded by periods of sharp rise in stock markets, the various stock market crashes that have marked economic history suggest that prices can diverge strongly and permanently from the fundamental value before suddenly converging to it. Following the wave of financial crises, the rationality of investors has been questioned; behavioral finance has started to face the informational efficiency assumption.

2.3.1. Behavioral finance

Behavioral finance can be defined as the stream that appeals to the results of work in psychology and sociology and applies them to finance to develop more precise theories on investor behavior. Several definitions have been attributed to behavioral finance: Mangot (2004) stated this stream is the product of collaboration between psychology and finance since it takes into consideration the influence of the individuals' feelings about their decisions, thus allowing a better understanding of financial markets.

Ritter (2003) defined behavioral finance as the paradigm where financial markets are studied by using models, which are less narrow than those of the expected utility of Von Neumann Morgenstern and arbitrage theory. Barberis and Thaler (2003) indicated that behavioral finance is a new approach to financial markets that emerged in response to the difficulties faced by the traditional paradigm. In general terms, it emphasized that some financial phenomena can be better understood by using models where agents are not perfectly rational. Olsen (1998), in turn, showed that behavioral finance is based on the application of psychological principles to improve financial decision making.

According to Tadjeddine (2013) "The behavioral finance trend proposes to reconsider the behavioral hypotheses by abandoning the axioms of the rational decision and the hypothesis of the market efficiency" (p. 7).

Behavioral finance is based on two inseparable pillars: investor psychology and arbitrage limits (Shleifer, 2000). This research stream has led some to deduce that the arbitrage mechanisms are not always able to correct some efficiencies in the market, thus the notion of the limited arbitrage. On another side, numerous empirical and experimental studies

have highlighted the misinterpretations of information made by the various agents on the financial market.

Investors' decisions are often biased by their erroneous feelings and reasoning; they often make systematic errors that find their origin in the limited mental capacities of human beings. The different behaviors that create these anomalies are classified according to cognitive bias or emotional bias.

We are going to focus on the emotional biases or what we called investor sentiment, since this latter is one of the main factors making the behavioral finance operational.

2.3.2. Investor sentiment

The investor sentiment concept serves mainly to depict how investors form their expectations and attempts to provide an explanation of their reactions. In this respect, accessible and intuitive as it might seem, this subject constitutes a complex concept that seems to be distinctively defined. In this respect, Broihanne et al. (2004) defined investor sentiment as being the set of behavioral phenomena likely to help explain how investors form beliefs on the basis of which they evaluate stocks.

Brown and Cliff (2004) confirmed that bullish (bearish) investor's hopes of return might prove to be higher (lower) than a given average. Zouaoui (2008) confirmed this definition and added that "the investor sentiment represents investor expectations that are not justified by the fundamentals. An optimistic investor (pessimistic) expects that the stock returns are higher (lower) to those which could be explained by fundamental indicators. In other words, defining the feeling means identifying an investor who is optimistic (pessimistic) without good (bad) economic reasons to make him so" (p. 21).

Regarding Baker and Wurgler (2007), they defined the sentiment as: " a belief about future cash flows and investment risks that is not justified by the facts at hand " (p.1).

According to Zhang (2008): "sentiment corresponds to erroneous beliefs that investors have against some kind of objective benchmark " (p.9).

Investor sentiment can be interpreted as being the variable that helps in generating investor evaluation errors along with their anticipation towards financial market, and which emanate from the subjective aspects involved in the stock evaluation. Although there should be only a single referential objective price for each asset, a certain subjectivity also appears to prevail on attempting to estimate such a price. Overall, investor sentiment can provide a

certain explanation as why prices may deviate from their fundamental value for quite long periods.

3. Informational efficiency framework

3.1. Informational efficiency: Principles and consequences

According to Fama (1965), a financial market is informationally efficient if all available information turns out to be correctly and fully reflected by the asset prices, thereby allowing an optimum allocation of financial resources at the lowest cost possible. Moreover, the efficiency assumption assumes the maintenance of the agents' atomicity. This implies that no investor is liable to influence the market.

The first definition was proposed by Fama (1965):

"an efficient market for securities, that is, a market where, given the available information, actual prices at every point in time represent very good estimates of intrinsic values.... Two factors that could possibly contribute toward establishing independence are (1) the existence of many sophisticated chart readers actively competing with each other to take advantage of any dependencies in series of price changes, and (2) the existence of sophisticated analysts, where sophistication implies an ability both to predict better the occurrence of economic and political events which have a bearing on prices and to evaluate the eventual effects of such events on prices." (p. 90).

The above quote stresses two major propositions; first, it highlights that the availability of a large number of operators helps minimize the asset prices' lasting deviations from their fundamentals. Second, it denotes that if the observed price reflects all available information, price fluctuations could not be due to unpredictable events, implying that price fluctuations are random.

In an informationally efficient market, the stock-market value should be equal to its fundamental value and any deviation reflecting uncertainty will be adjusted instantaneously. Thus, the observed market price would exclusively depend on expected future dividends.

In 1970, Fama proposed another definition that is more oriented towards the incorporation of information by the investors (p. 383):

"A market in which prices always fully reflect available information is called efficient".

Such a definition allows us to deduce that market efficiency could be guaranteed only when prices turn out to entirely and continuously reflect the available information. In other words, market efficiency happens once the observed price helps provide an unbiased estimation of its fundamental value. More particularly, information should be simultaneously detained by the all of the investors. This information will be incorporated instantaneously and completely into asset prices and it should also be freely available and accessible to all agents.

Since the price bears a reflection of all past and future events, price changes should stand an unpredictable phenomenon. In such a case, it is impossible to predict future returns from past ones and subsequently the return series would turn out to be independent, denoting the absence of any serial correlation. Consequently, prices would appear to follow a random walk.

Formally, the stock price follows a random walk if it satisfies the following equation:[6]

$$\ln(p_t) = \ln(p_{t-1}) + \varepsilon_t \quad (1\text{-}2)$$

Where p_t is the asset price at time t and ε_t is a white noise .

$$R_t \sim \frac{p_t - p_{t-1}}{p_{t-1}} \sim \ln\left(\frac{p_t}{p_{t-1}}\right) \quad (1\text{-}3)$$

Where R_t is the asset return at time t.

$$R_t \sim \frac{p_t - p_{t-1}}{p_{t-1}} = \varepsilon_t \quad (1\text{-}4)$$

Therefore, the observed price randomly fluctuates around its fundamental value and the returns to follow a white noise process. It turns out that the random walk model stands as a restrictive aspect. This led Samuelson (1965) to introduce the martingale model as an alternative to the random walk one. The stock price is called to follow a martingale:

$$E[P_{t+1}|I_t] = P_t \quad (1\text{-}5)$$

This equation suggests that the best forecast of the price that can be done in (t + 1), on the basis of the available information I, is the price at time t. Hence, unlike the random walk model, the martingale does not exclude the return's dependence. Samuelson (1965)

[6] The use of price logarithms rather than prices results from the observation that financial variables have non-stationary variances over time.

demonstrated that the martingale model includes the future returns' unpredictability and the equality between the observed price and its fundamental value at every moment. However, such a conclusion implies the impossibility to beat the market by reference to the available information.

Thus, Jensen (1978) proposes a new definition of efficiency:

"A market is efficient with respect to information set θ_t, if it is impossible to make economic profits by trading on the basis of information set θ_t"(p. 96).

Such a less restrictive definition states that returns might be dependent, though it is still impossible to speculate for achieving an excess return.

Empirically, the informational efficiency has several implications; the information is disseminated simultaneously and free of charge to all economic agents. In the presence of transaction costs or taxes, economic agents may be reluctant to act on the market fearing that these costs will cancel their potential realizable gains. The investor will not act in the market unless the expected gain is greater than the supported transaction costs. In reality, contrary to the theory of efficiency, transaction costs (bid-ask fees, commission fees, etc.) are not null. Furthermore, informational efficiency implies the existence of a total liquidity and investors' atomicity.

3.2. The paradox of informational efficiency

3.2.1. Questioning Rationality

The basic assumption on which market efficiency relies heavily is the investors' rationality. It assumes that individuals maximize their expected utility in uncertain situations. In other words, any investor is usually able to process the received information at any moment, revise his or her proper choices, and act in such way as to maximize his or her own satisfaction. However, people might tend to apply cognitive shortcuts to make a judgment. It is often this question of cognitive biases that guides the judgment in a predictable way, and sometimes it is a matter of heuristics that fixes it permanently.

a) Heuristics

Research on the subject dates back to the 1970s and was initially conducted by Tversky and Kahneman (1973, 1975) who identified three heuristics: the representativeness, the availability, and the anchoring. Let us start with the heuristics of representativeness. This

bias is about "to categorize a person, object or event in a category if these entities appear as representative of the category."[4] Individuals who mobilize this heuristic usually refer to the trend when it comes to making a judgment in a bid to generalize what is particular (Tversky and Kahneman, 1973). They usually rely on stereotypes for the sake of establishing general laws.

In this regard, Ritter (2003) suggests « As an example, when equity returns have been high for many years, many people begin to believe that high equity returns are normal » .

(P. 431)

So, investors tend to neglect the basic probabilities and follow the similarities observed with other typical or representative events. It is this notion that leads investors to frequently make quick irrational opinions (Shefrin, 2005). Individuals have a tendency to evaluate the occurrence of an uncertain future event through the degree to which it looks like a recent observed phenomenon. Thus, the representativeness bias helps greatly explain how prices over-react to past consistent information by suggesting to investors that they are in line with long term trends.

In this respect, Fisher and Statman (2002) analyzed several opinion surveys conducted over the period 1998-2001. Relying on the Gallup surveys conducted on behalf of the Paine Webber index of investors' optimism, they showed that during the last months of the Internet bubble, individual investors were aware that there was still enough time to make an investment decision. The authors concluded that investors were subject to over-optimism. Thus, investor optimism turns out to be strongly enhanced and powered by the representativeness bias prompting them to think the very high past returns bode well for high future returns. Hence, the representativeness bias helps indicate that individuals tend to make use of simple elements, easily accessible to memory, in a bid to achieve their estimates. Yet, these estimates turn out to be too biased due to such immediately available information. The availability heuristic documented by Tversky and Kahneman (1975) as the general principle by which individuals assess the probability associated with an event based on the ease with which examples of such an event come to mind.

In terms of financial choices, the availability heuristic may occur whenever the uninformed individual proves to use more analogical reasoning than logic reasoning to make

[4] This definition is provided by Lemaire (2006) in his book Psychology cognitive (p. 242).

judgments. Several empirical studies revealed that the availability heuristic helps greatly influence the financial analyst's behavior. In this context, Ganzach (2001) concluded that throughout the return assessment of thinly traded stocks, analysts tend to base their judgments on a global attitude. For instance, if stocks are perceived as good, they are judged to have a high return and low risk, whereas if they are perceived as bad, they are judged to have a low return and high risk. Primarily, Lee et al. (2008) discovered that analysts are relatively optimistic about their long-term forecasts of benefits once the economy is perceived to be expanding, and relatively pessimistic whenever the economy is noticed to be in a state of recession. According to the authors, such a result proves to be highly consistent with the availability heuristic, indicating that analysts tend to overweigh the current state of the economy when making forecasts of future profits.

Regarding the third heuristic, "the anchoring," Hirshleifer (2001) defined it as "the phenomenon that people tend to be unduly influenced in their assessment of some quantity by arbitrary quantities mentioned in the statement of the problem, even when the quantities are clearly uninformative" (p.1544).

In any assessment case, the interviewee has been asked to compare his estimates with another figure that can be either serious or totally random. The compared figure is called an anchor. This anchor has been discovered to remarkably affect the interviewee's choice. The anchoring lies in the observation that the comparison drawn between the interviewee's reached figure and the questionnaire's figure proves to influence the study responses.[7]

Kahneman and Tversky (1974) considered that the individuals were formulating their estimates by starting from an "initial value and by adjusting it to give their final answer". Hence, the newly obtained information can be insufficiently taken into account, as it may lead to errors in judgment and puts rationality into question[8]. Moreover, anchoring is considered as an individual over-confidence with regard to previous information.

[7] Kahneman and Tversky (1974) illustrated the anchoring bias through an experiment conducted among a set of individuals whose mission is to assess the number of United Nations member African countries having turned a wheel of fortune numbered according to a scale ranging from 0 to 100. The experiment reached results showed that the randomly obtained number significantly affect the respondents' estimates. Besides, on the financial markets, a variety of numbers may serve as anchors for the concerned agents.

[8] According to Bessière and Kaestner (2009), "the anchoring of a past value leads to underestimate the weight of new information for the anchor value's benefit. New beliefs are therefore insufficiently revised, which means, from an empirical point of view, an under-reaction" (p. 24).

Hence, it follows that the three above described heuristics indicate how individuals deal with available information. The responsive action seems biased, representing a primary deviation from the rationality hypothesis of individuals operating in the market. More recently, a better understanding of brain functioning has enabled us to have a new conception of the decision-making process. In this respect, neuroeconomics undertakes to confirm that the human brain functions by making various shortcuts. This can be explained through the process in which several choices are made reflexively.

b) Neuroeconomics

Neuroeconomics can be defined as the science that studies decision making and economic activities using experimental methods to make a direct observation of brain activity. It helps in providing a better understanding of rationality and the preferences guiding the economic agents' decisions. The neuroeconomics' major achievement lies in highlighting the emotions' crucial role in economic decision making[9] (Bechara and Damasio, 2005).

It is generally accepted that the decision-making process involves both controlled and automatic processes (Schneider and Shiffrin, 1977; Sloman, 1996). According to Camerer et al. (2005), there exist three outstanding brain characteristics that serve as the background process for generating automatic processes. In the first place, there exists the brain's power to process information simultaneously and quickly. In the second place, the neurons are divided into several specialized neurons able to communicate with each other. Third, there is a certain coordination among the various brain's areas to execute a particular task. Once the brain is familiarized with that particular task, the latter is directly dispatched and directed toward the specialist area.[10]The observation that the brain operates with a set of specialized systems proves to contradict the idea of Homo Oeconomicus that seeks to maximize its utility through preferences prioritized by a single set of rules.

In addition to the controlled and automatic processes, Camerer et al. (2005) distinguished between cognitive and affective processes. According to the authors, cognitive processes cannot generate an action without the affective processes' contribution. They also

[9] By locating brain activity while making specific decisions and bringing it closer to the localized neuro-anatomical functions, the brain's functional imagery serves to appropriately visualize the neural networks responsible for emotions during the decision-making process.

[10] Whenever the brain decides to execute several simultaneous operations, the neural system's intermingling connections might result in the predominance of certain illogical propositions.

confirmed that most behaviors result from the interaction among both processes. It turns out that neuroeconomics undertakes to describe an individual rationality that is distant from the assumptions on which the economy and conventional finance rest.

Furthermore, Akerlof and Shiller (2010) provide an explanation of the economy relating emotional reasoning through the animal spirit's elements. They thus contribute in providing a clearer understanding of the limits of rationality. In fact, they defended that individuals pursue their economic interests but they also do it for non-economic reasons. The authors concluded that the individuals' presumed rationality does not explain the current economic and financial crisis context. Hence, investors do not stand as perfectly rational, their emotions represent an integral part of their decisions. In some situations, emotions can guide judgments or even determine them.

3.2.2. Price predictability relating anomalies

Several empirical studies have predominantly revealed that future returns are discovered to be partly predictable from past returns. Trend movements have characterized both aggregate markets and individual stocks. Hence, two anomalies have been identified; "Momentum" and "reversal," suggesting investors tend to predominantly under-react to the present information while over-reacting to past information. Culter et al. (1991) found monthly autocorrelation to be significantly positive in the short-term regarding 13 stock markets studied over the period 1960-1988. This result reflects a momentum effect, indicating that performance is continuing in the short run.

Jegadeesh and Titman (1993) also showed a momentum effect regarding US stocks over three to twelve-month periods during 1965-1989. The authors showed that 25% of the excess return enjoyed by past winners appears to occur during the quarterly earnings announcement periods. Furthermore, the momentum effect is more important for securities with extreme performance.

On comparing the NYSE listed stock returns over the period 1926-1982, De Bondt and Thaler (1985) showed that, in the long-term, winning stocks recorded a potential decline. They also found that the correction of past over-reaction proves to be rather sensitive for losers and that this phenomenon would often occur especially during the month of January. So, Jegadeesh and Titman (1993) and DeBondt and Thaler (1985) suggested there is primarily a persistence of abnormal stock performance, subsequently followed by a reversal of this past trend.

In the same context, Lee and Swaminathan (2000) concluded that regarding the period 1965-1995, the winning stocks over six months continued their outperformance for almost one to three years before underperforming. The reversal of long-term performances indicates that the momentum effect, reflecting the investors' under-reaction, would eventually lead to their over-reaction.

More recently, on analyzing the profitability of momentum and reversal strategies over the period 1964-2008, Alwathainani (2012) constructed the winners' and losers' portfolios as follows: While the first set involves stocks having achieved the best monthly performance, the second consists of stocks that have recorded poor performance over the past two to four months. The pursued strategy consists of buying winning stocks and selling losing ones, while maintaining this position over a holding period of one to five years, whose performance is measured by the yearly average monthly return. Such a strategy is likely to generate positive abnormal returns over a one-year holding period and negative returns over the remaining four years. Such findings confirm the short-term momentum effect as well as the long-term reversal one.

It turns out that stock prices do not really follow a random walk, given the observation that returns seem to be positively correlated in the short term and negatively correlated in the long run. Strongly related to the investors' psychological patterns, this cyclical development seems hardly compatible with the rationality assumption, also called into question by the heuristics and neuroeconomics. In compliance with market conditions, investors usually undergo various emotional states. Thus, emotions turn out to play a particularly important role in the decision-making process whenever basic crucial information seems to be lacking. It thus appears necessary to quantify the investors' emotions. This is the challenge of behavioral finance and market sentiment. In the fourth section we will highlight the impact of investor sentiment on expectations' and returns' dynamics. We will end up by analyzing the complexity of the relationship binding investor sentiment and stock returns.

4. Investor sentiment and stock return dynamics

Traditional finance related models have often assumed the validity of the traditional assumption of homogeneity. They tend to reduce the investor behavior to that of a representative agent. Nevertheless, investors are very heterogeneous. This results in the development of the investor sentiment concept and the relevant models helping study its impact on stock market returns.

4.1. Emotions, moods, and decisions

Unlike the classical theory, several psychological studies have shown the individual is far from being placid. The investors' decisions are often biased due to their predominating emotions and false reasoning. They often make systematic errors stemming from their limited mental capacity as human beings.

a) Investor emotion and financial decisions

Emotions have an important place in the reasoning process. Individuals are able to make judgments and decisions on the basis of mental images with which they associate positive or negative feelings. Finucane et al. (2000) describe reasoning based on emotions as "affect heuristic." They conducted an experiment to see how the risk or benefits related to information manipulation has an influence on perception, despite the complete absence of any logical connection between the provided information and the second variable. The heuristic model's predictions have been confirmed with respect to the entirety of situations. New information has been discovered to change the perception of non-manipulated variables towards a direction opposite to the manipulated variable's one. Such a finding supports the idea that risk judgments are partially and jointly determined by a global effective evaluation.

Consequently, the shares of company enjoying a positive image (a glamour field) are more likely to be bought than those of the negatively perceived ones. It is this overall positive feeling arousing among investors that makes them underestimate the risk and overestimate the expected return.

The emotional evaluation might take precedence over cognitive assessment if the investor lacks experience. Experienced investors are more cognitively able to control their behaviors.

Emotions are dependent on the investor's successes and failures along with his or her role in such situations. For every emotion, there is a reflex response allowing the investor to respond quickly to each situation, while setting up the goal for improving such a situation.

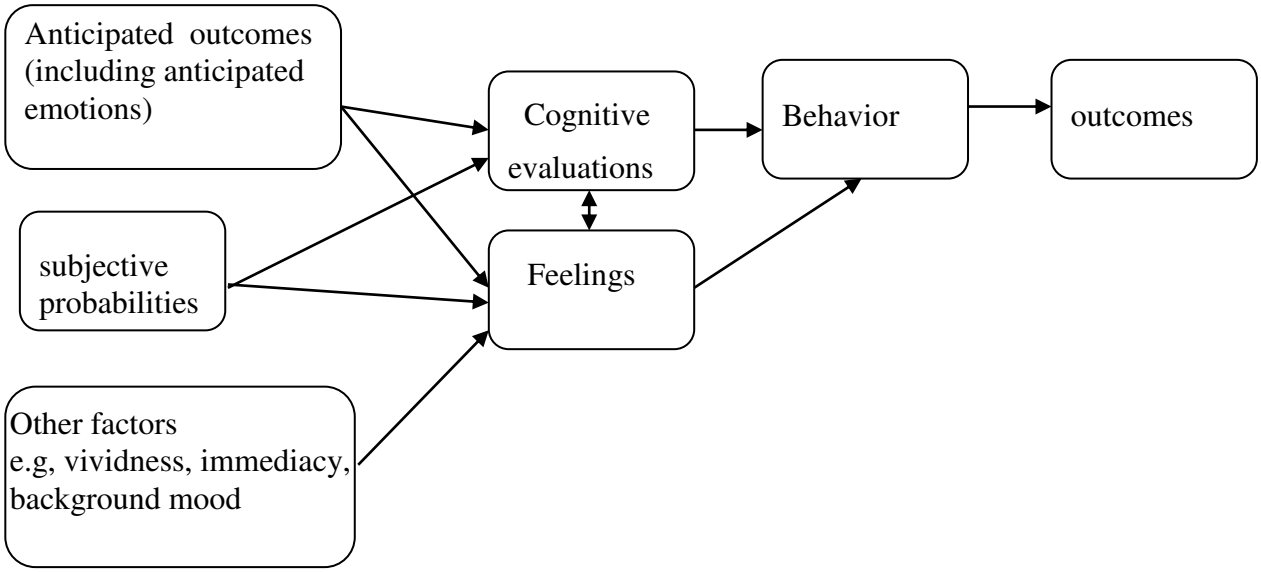
Loewensetein et al. (2001) suggested that cognitive assessment of risk and the perceived risk are divergent. The cognitive assessment process is most often based on probabilities relevant to different scenarios, as well as on the results emanating from each scenario. Affective perception of risk is mainly focused on results since they can be imagined more easily. The more extremely possible outcomes seem for the individual, the more

automatic his or her responses will be, regardless of the actual probabilities attached to such extreme situations.

Loewenstein (2000) revealed that emotions experienced during the decision-making process tend to remarkably influence investor behavior in the sense that they are different from those dictated by cost constraints or long-term benefits. It seems therefore reasonable to assume that the investor's emotions may influence the stock price setting process.

In this context, the "risk-as-feeling" model has been developed by Loewenstein et al. (2001) as a descriptive and prescriptive model of human behavior in terms of both cognitive properties and their interactions with emotions. The developed model is based on a number of premises highlighting that every aspect of the decision-making process is influenced by the investor sentiment. This process is illustrated in the figure below.

Figure 1. 1 Risk as feeling perspective



Source : Loewenstein et al. (2001), p. 270

In a context of uncertainty and risk, Loewenstein et al. (2001) relied on three factors to confirm the conclusion that investors' decisions are influenced by their feelings:

- Cognitive evaluations provoke emotional reactions:

In this respect, Zajonc (1980) showed emotions are regarded by most contemporary theories as cognitive post reactions likely to occur following the completion of considerable cognitive operations.

- Emotions help influence cognitive assessments:

The idea highlighting the emotion's noticeable influence on cognitive assessments is established by researchers in psychology. They recommend that people who are in a good mood usually tend to make optimistic judgments, unlike people who are in a bad mood who rather tend to make pessimistic ones.

- Emotions can remarkably influence individual behavior:

In this context, Damasio (1994) elucidated the emotion's vital role in the decision-making process in a context of uncertainty and risk through an experiment carried out on people with emotional difficulties. This experiment resulted in the conclusion that emotions appear to play a significant role in human performance in terms of decision making, problem solving, and memory, etc.

According to Dowling and Lucey (2005), two areas have proved to highlight the impact of sentiments on investors' decisions. The first one covers the "mood-misattribution effect." This research undertakes to study the impact of such environmental factors as weather, human body biorhythms, and social factors on the financial assets' returns. This area is based on psychological research works advancing that people decisions are partly guided by their sentiments. This implies that sentiments induced by factors such as weather influence even complex decisions including risks and uncertainties. Concerning the second research area, it stresses the image impact on the decision-making process; the argument put forward by this research trend concludes that the image of a stock affects investors emotions and, therefore, their behaviors.

b) Moods and seasonality in returns

As general emotional states, moods influence financial decisions by skewing the anticipations. In general, a good mood is discovered to yield better results since it allows a better understanding of information and a better resolution of problems. A positive mood is likely to induce the individual's optimism about the future. As a result, good humor would enhance the fact of acting as a buyer and reducing risks. Accordingly, moods would kindle the effect of changing expectations as regard return and risk.

It is widely documented that good weather usually helps people enjoy a good mood. So, if moods help in encouraging investors to turn into buyers, sunny days should then be distinguished by higher returns. This has been confirmed by Hirshleifer and Shumway (2003),

who, on investigating 26 stock markets, concluded that the better the weather is, the better the returns will be. As Rick and Loewenstein (2008) have documented "bad weather should lead to negative emotions which should in turn lead to negative price movements since negative emotions can exert conflicting effects on risk-taking" (p. 153).

Another more intuitive effect is that emanating from weekends and their ending. It has been maintained that the more the weekend approaches, the more pleasurable the mood will be. Conversely, Monday marks the work week return and consequently leads to the prevalence of a negative mood. This Monday effect can be explained by the observation that returns are abnormally low on Monday as compared to other days of the week.[11]

Alongside the weekends, days off also make investors in good mood, thus helping increase market returns. Several empirical studies confirmed that markets outperform historical averages during the days preceding vacations (Fields, 1934; Lakonishok and Smidt, 1988; Ariel, 1990; Cervera and Keim, 2000; Frieder and Subrahmanyam, 2004). This is dubbed the holidays effect. The returns observed in respect to the American markets revealed that this effect is rather remarkable on Christmas Eve.

Other researchers such as Tetlock (2007) investigated the mass media's linguistic and found that the media pessimism predict the price decrease. Tetlock et al. (2008) have further showed that earnings and stock returns are predicted by the negative words' fraction used in news stories.

Another non-economic phenomenon affecting investor decisions is the sports scores. In this context, Edmans et al. (2007) used the football results as mood variable to investigate the impact of an international football match on 39 national indexes during the day after the game. The empirical study showed that there exist a positive "abnormal return" of 5 basis points for the winner countries along with a significantly higher loss of 8.4 basis points for the losing countries.

[11] On studying the stock returns over the period 1953-1977, French (1980) noted that stock returns prove to be generally negative on Monday and positive over the remaining days of the week. Venezia and Shapira (2007) confirmed this result by pointing out that the returns are lower during the days following the weekends than during the other days of the week. The authors also stressed the weekend effect which differs according to the investors' type. The daily transaction analysis revealed that individual investors increase their buying and selling activities following the weekends, and that their propensity to sell highly exceeds their purchasing propensity. Still, professionals tend to reduce transactions at the beginning of the week with their buying and selling propensity, tending almost towards a proportionate decrease.

4.2. Behavioral heterogeneity in stock prices and sentiment risk

In this subsection, we define the noise trader risk also known as the "sentiment risk". In addition, the behavioral model as considered by De Long, Shleifer, Summers, and Waldman (1990) or the "DSSW model", introducing the "noise trader risk" will be examined.

4.2.1. The role of noise traders

Most of the abnormal situations intervening within the efficiency hypothesis have been attributed to the noticeable presence of non-informed investors (Barberis et al., 2005; Froot and Dabora, 1999; Rosenthal and Young, 1990; West, 1988). The latter's activities stand as the origin of the increasing deviation between the actual stock prices and the theoretical ones. This is mainly due to the special signs they apply whenever they tend to get into the market, along with their strategic irrational rules. Worth citing in this context is Black (1986), who defines noise trading as follows: "Noise trading is trading on noise as if it were information. People are willing to trade even though from an objective point view they would be better off not trading. Perhaps, they think the noise they are trading is information. Or perhaps, they just like to trade"(p. 531).

Hence, noise traders don't withhold or make use of all relevant and available information likely to generate potential benefits. According to Black (1986), noise is likely to result in imperfect observations. Thus, should there be no noise in the markets, only few transactions would take place. Yet, high daily trade volumes are frequently noticed to pervade the markets. For any exchange to take place, participants should detain relevant information that seems quite disparate. Such an intense activity arises predominantly from the persistence of noise traders who make frequent use of noisy information.

In this respect, Grossman and Stiglitz (1980) developed a special model involving both informed and uninformed investors. It helps in studying how information flows from the first type of investors to the second. The authors explain that in the absence of noise, prices would accurately reflect the entirety of relevant information. In case of prevalence of noise traders, the market is less informative.

Efficiency supporters tend to defend the idea that noise traders should be discarded rather rapidly. For instance, Fama (1965) argued that noise traders have to be excluded mainly because they act wrongly with arbitrageurs who are most often right since they are well informed and have the right information. Hence, these arbitrageurs would eliminate the uninformed investors. Still, this reasoning will not hold unless the noise traders perform a small part of the transactions and their action is not overriding.

Similarly, De long et al. (1990) argued that arbitrageurs are unable to take sufficient risks to rule out the noise traders. Their risk-aversion makes removing these noise traders take longer. They added that arbitrageurs are hesitant to sell or buy whenever noise traders buy or sell in a bullish or bearish market, as they may not achieve the maximum gain by buying or selling too early. Furthermore, arbitrageurs often seem unable to predict the noise traders' reactions since noise traders do not often rely on effectively relevant information. In other words, the noise traders can lead prices to retain their bullish or bearish nature over longer periods. But, it is just as likely their opinion would suddenly veer according to their sentiments. The sentiment's disruptive nature is likely to produce an additional risk relevant to their exchanged assets. The noise trader risk or sentiment risk makes riskier assets. Accordingly, arbitrageurs would certainly require a higher risk premium to invest in assets that noise traders made too risky.

Lee et al. (1991) added that: "If different noise traders traded randomly across assets, the risk their sentiment would create would be diversifiable, just as the idiosyncratic fundamental risk is diversifiable in conventional pricing models. However, if fluctuations in the same noise trader sentiment affect many assets and are correlated across noise traders, then the risk that these fluctuations create cannot be diversified. Like fundamental risk, noise trader risk will be priced in equilibrium " (p. 81).

According to Daniel et al. (2001), all investors could have the same information but use it differently; this is mainly due to the over confidence of the informed investors. "Uninformed individuals can infer all the signals perfectly from market prices. The uninformed end up with the same information as the informed traders but use it differently as they are not overconfident about these signals" (p. 932).

Kogan et al. (2006) examined the link between long-run survival of the noise traders and their influence on asset prices. They noted that noise traders can significantly impact asset prices even when their health goes to zero. These results are similar to that documented by Mendel and Shleifer (2012), who developed a model composed by rational but uninformed traders and sophisticated investors. They also demonstrated that noise traders have an effect on equilibrium price regardless of their size in the market.

Consequently, the noise trading approach seems to highlight that ignorant strategies help in influencing markets and are not canceled out automatically. Noise traders might dominate the market in matters of transaction volume, relying heavily on the illusion that they

are acting rationally. Thus, noise traders could generate higher returns than those achieved by rational investors (De Long et al., 1990). Correspondingly, arbitrage might be unable to absorb all the demand-related shocks, in so far as the noise traders' sentiment unpredictability might restrict the shares of arbitrageurs to take action against the noise traders' devised strategies. As a result, stock prices might diverge permanently from the fundamental value. As Zhang (2008) indicated:" the noise trader model posits that if there are limits to arbitrage and investor beliefs are correlated, then noise unrelated to fundamentals, such as sentiment, may lead asset prices to deviate from what is expected from the benchmark of market efficiency" (p. 4).

4.2.2. A noise-model relevant behavior

De Long et al. (1990) developed a model whereby the noise traders' integration within the entirety of acting investors could be maintained. The model involves two types of investors. On the one hand, it includes noise traders (denoted by n) and, on the other, arbitrageurs (denoted by i) or sophisticated investors who are highly aware of the stocks' fundamental value.

Two major periods lie explicitly in the De Long et al. (1990) consideration. In the first one, investors construct their portfolios with an initial endowment exogenously determined by the model. Their exclusive decision consists in choosing the portfolio's nature. In the second period, however, they undertake to settle their positions, along with measuring their strategic profitability. Agents pertaining to the same group are usually identical in terms of strategy. Noise traders represent a μ proportion of investors, while the others represent a 1-μ proportion.

In the market, two major asset types are available. A risk-free asset (denoted as s) offering a certain dividend r stands for the risk-free rate. The asset attached offer is perfectly elastic and De Long et al.(1990) set its relevant price to 1. The other asset is the risky asset (denoted u) whose supply is not elastic. It offers a return equal to r + ε where ε is a random variable distributed according to a normal distribution, with zero esperance and constant variance. The price of u in period t is denoted p_t.

Initially, De Long et al.(1990) presented a model bearing no fundamental risk (ε = 0), before studying the implications of such a risk. The model will be immediately exposed in our context as incorporating fundamental risk.

Falsely perceived by the noise trader, the risky asset price is described by a random variable as assuming to follow a normal distribution i.i.d: $\rho_t \rightsquigarrow N(\rho^*, \sigma_\rho^2)$.

The mean misperception $\rho*$ is a measure of the average noise traders' related "bullishness". σ_ρ^2 stands for the noise traders' variance of the expected return misperceptions. The agents are striving to maximize their expected utility as standing under the return normality assumption, an expected utility function expressed in terms of mean-variance, including their relevant risk aversion degree:

$$u = -e^{-(2\gamma)w} \tag{1-6}$$

Where γ denotes the absolute risk aversion. Maximizing expected utility amounts to maximizing the following equation:

$$\overline{w} - \gamma\sigma_w^2 \tag{1-7}$$

Where $\overline{w}$ is the expected final wealth, and σ_w^2 is the one-period-ahead variance of w. The arbitrageur expresses a demand λ_t^i for risky assets, which helps maximize the following expected utility function in respect of the risky-asset amounts requested:

$$E(U) = \overline{w} - \gamma\sigma_w^2 = c_0 + \lambda_t^i\left[r + {}_tp_{t+1} - p_t(1+r)\right] - \gamma(\lambda_t^i)^2({}_t\sigma_{pt+1}^2 + \sigma_\varepsilon^2) \tag{1-8}$$

Where c_0 represents the labor income as derived from the first period. The lower index $_t$ preceding the letters denotes the date on which the anticipation was made.

$${}_t\sigma_{pt+1}^2 = E_t\{(p_{t+1} - E_t(p_{t+1}))^2\} \tag{1-9}$$

Similarly, the noise trader would undertake to maximize the following:

$$E(U) = \overline{w} - \gamma\sigma_w^2 = c_0 + \lambda_t^n\left[r + {}_tp_{t+1} - p_t(1+r)\right] - \gamma(\lambda_t^n)^2({}_t\sigma_{pt+1}^2 + \sigma_\varepsilon^2) + \lambda_t^n\rho_t \tag{1-10}$$

Where λ_t^n represents the risky-asset amount requested at time t by the noise traders. ρ_t denotes the error committed by the noise trader at time t.

Thus, expected utility functions relevant maximization procedure helps determine the asset quantities relevant to each type of market agent. By solving the equations (1-8) and (1-10), we obtain:

$$\lambda_t^i = \frac{\left[r + {}_tp_{t+1} - p_t(1+r)\right]}{2\,\gamma({}_t\sigma_{pt+1}^2 + \sigma_\varepsilon^2)} \tag{1-11}$$

$$\lambda_t^n = \frac{\left[r + {}_tp_{t+1} - p_t(1+r)\right]}{2\,\gamma({}_t\sigma_{pt+1}^2 + \sigma_\varepsilon^2)} + \frac{\rho_t}{2\,\gamma({}_t\sigma_{pt+1}^2 + \sigma_\varepsilon^2)} \tag{1-12}$$

It turns out that the noise traders' required amounts sound to be identical to those requested by informed investors except for the term marking the noise traders error committed in the expected return rate estimation. The fundamental risk attached to the asset financial and economic environment along with the noise traders' induced risk allow the market to persist while remaining unfixed. When noise traders overestimate the expected returns, they tend to ask for greater risky asset amounts than that requested by sophisticated investors and whenever they prove underestimate the returns, they consider asking for less quantities. The risk caused by noise traders affects not only their own strategy but also the strategy established by the other investors.

The equilibrium price related equation looks as follows:[12]

$$p_t = \frac{1}{1+r}\left[r + {}_tp_{t+1} - 2\,\gamma\left({}_t\sigma_{pt+1}^2 + \sigma_\varepsilon^2\right) + \mu\rho_t\right] \tag{1-13}$$

At time t+1, on accounting for the conditional expectation and assuming that the variance of the price is a constant, it can be formulated as follows:

$${}_tp_{t+1} = 1 - \frac{2\,\gamma\left(\sigma_p^2 + \sigma_\varepsilon^2\right) - \mu\rho^*}{r} \tag{1-14}$$

Concerning the noise traders' presence resultant variance, it can be constructed in terms of the investors' error committed features:

$$\sigma_{pt+1}^2 = \frac{\mu^2\sigma_\rho^2}{(1+r)^2} \tag{1-15}$$

[12] By resuming the equations (1-11) and (1-12) adding them up to determine the total demand, and by placing $(1-\mu)\,\lambda^i + \mu\,\lambda^n = 1$, this relation is the result of the assumption presuming that investors could sell at (t) all their accumulated assets at (t + 1).

By substituting ${}_tp_{t+1}$in the equation (1-13), De Long et al. (1990) defined the risky assets' price evolution on the base of data allowing the measurement of the noise traders' impact on the market:

$$p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{2\gamma}{r}\left(\frac{\mu^2\sigma_\rho^2}{(1+r)^2} + \sigma_\varepsilon^2\right) \quad (1\text{-}16)^{13}$$

The last term of the equation (1-16) is considered by De Long et al. (1990) as the most important element in this equation. It presents noise trader risk consisting primarily in the impossibility to predict the noise traders' potential response. For instance, if they keep pushing the price to rise frequently, no evidence would appear ensuring what they would potentially pursue with such a procedure or that they would in such a way as to influence the price decline. So, this term helps highlight that, in addition to the fundamental risk, an additional risk proves to persist.

De Long et al. (1990) demonstrated that noise traders could acquire higher returns than the sophisticated investors could. They indicated that through their stochastic biased beliefs, noise traders have not only maintained their persistence in the market but have also secured greater gain achievements than those achieved by sophisticated investors.

As a result, such a situation stands as a major cause of the large price deviations toward fundamental values likely to prevail if the noise traders' errors are correlated. The presence of noise traders who are unpredictable, along with the perfectly informed investors who help maintain the fundamental value, generate the special price fluctuations. Thus, driven by an excessive optimism or pessimism, noise traders contribute in generating a bullish or bearish market from which they could derive remarkable benefits. The arbitrageurs' strategy no longer consists in diversifying their overall portfolio, but also in acting contrary to the noise traders' reactions (market timing strategy).

The DSSW model revealed that the noise traders' behaviors predominantly based on their sentiment has an impact on stock returns while limiting the arbitrageurs' actions. It is

[13] The second term of the equation (1-16), accounts for the role of the noise traders' assessment error shifts on the risky asset price. For instance, should the noise traders appear to believe that the price should rather increase, they would push the prices up, and all the stronger that their presence in the market (μ) would turn out to be important. Concerning the third term of the equation (1-16), it captures the price deviation with respect to the fundamental value given the observation that the noise traders' average error is not null. Once optimistic ($\rho^*>0$), these traders would usually prompt the price to be overvalued with respect to its normal value.

now possible to model such behaviors while integrating them it into the market movements' general review. This suggests a growing interest placed on the investor sentiment related area and the relevant works undertaken to investigate the stock market dynamics field.

4.3. Investor sentiment measures

Several proxies proposed in the literature to measure investor sentiment. Eventually, these studies have identified two categories of measures: the direct measures, as arising from investor surveys, and the indirect measures, as based on financial and economic variables in correlation with investor sentiment.

The indirect measures of investor sentiment are multiple: the closed-end fund discounts (Lee et al.,1991; De Long et al.,1990; Swaminathan, 1996; Neal and Wheatley, 1998; Gemmill and Thomas, 2002; Bruch et al., 2003), the dividend premium (Baker and Wurgler, 2004a,b), the composite sentiment indicator (Baker and Wurgler, 2006, 2007; Baker et al., 2012; Chang et al., 2012; Brown and Cliff, 2004; Ho and Hung, 2009), the trading volume (Baker and Stein, 2004; Scheinkman and Xiong, 2003), the number of IPO's (Ljungqvist et al., 2006), the Arms index (Arms 1988), the implied volatility (Whaley, 2000), the FEARS index (Da et al., 2015).

The indirect measures of investor sentiment are recognized as indirect because they are objective measures indirectly related to investor sentiment. This denotes that the entirety of the subjective parts could be neutralized. Yet, the implicated data are frequently used owing to their diversities as well as their highly frequent availability.

The direct measures of investor sentiment do not entail applying any theoretic frame. They are primarily based on investor surveys and business reviews. Thus, they allow for a direct measuring of the respondents' expectations. Polls are increasingly used for a twofold purpose: understanding the customers' motivations and identifying the voters' expectations. Should several indicators appear to be proposed, methodologies would turn out to differ chiefly according to whether it is consumers or investors who are being interrogated.

To determine the individual investors' attitude towards risk, several institutions have devised questionnaires through which a score could be computed in terms of the points accumulated with respect to each response. The score achieved would indicate either an upward trend (optimism) or a downward one (pessimism) allotted to various investors. In this context, the direct measures proponents are the US-oriented research works, such as the index of the American Association of Individual Investors (AAII) and the Investors Intelligence (II)

(Brown and Cliff, 2004). The American Association of Individual Investors helps defend the small investors' interests. The AAII proposes to calculate a sentiment index as drawn from an extensive survey of its members. The weekly index serves to measure the interviewers' sentiment about market returns likely to be recorded over the six upcoming months.

Similarly, the Investors Intelligence is a company providing technical analyses involving strong behavioral dimensions. It has monitored the publication of over one hundred editors of financial newsletters and has accumulated a historical report of their respective opinions as gathered since 1963. Each financial newsletter's editor would fall into one of the following categories: optimistic, pessimistic, or neutral.

Shiller (2000) suggested a different methodology relevant to measuring other investor sentiment related features. He proposed a questionnaire and accumulated the relevant responses to reach three distinct indicators: bubble expectation, negative bubble expectation, and investor confidence. The poll applied a questionnaire sent every six months starting from 1989 to institutional investors based in the United States and Japan. It was also sent to individual investors in the United States.

Moreover, Shiller (1999) developed the Yale School of Management Stock Market Confidence indexes by collecting the relevant data since the end of 1980, with a biannual frequency. These indexes serve to effectively measure the respondents' perception with respect to four different variables: the one-year confidence index, buy on dips index,[14] the crash confidence, and the valuation confidence index.

Additionally, the Sentix index seeks to measure investor sentiment, at both private and institutional levels. The applied indicators are based on the stock markets both in their entirety and by activity sector, as long with the bond markets (American federal bond and futures contracts on the bund), currencies, and commodities (oil, gold). To deal with the usual optimism / pessimism issues regarding the evolution of over-specific markets, Sentix publishes different opinion surveys concerning such finance related items as the relative popularity of a particular management style or the attractiveness of different maturities on the European income markets.

[14] Buy on dips index measures the investors' proportion expecting a rebound after a drop in the Dow by 3% in each day.

In addition, several researchers undertake to apply the consumer confidence index as a direct measure of investor sentiment, highlighting its robustness in relevance to other measures (Lemmon and Portniaguina, 2006). This index tries to measure the sentiment about the risk of a possible stock market collapse rather than the general confidence and expectations of an overall market growth. For the sake of constructing such a proxy, the information is collected on a monthly survey basis, including key issues related to the surveyed people's expectations.

In this context, Brown and Cliff (2004) showed that both methods, direct and indirect, appear to collect almost the same type of information. A particular method's ability to measure one or another component of investor sentiment appears to help greatly in anticipating the market movements in terms of returns.

4.4. Investor sentiment impact on stock market returns

Several studies on behavioral finance studied the relationship binding investor sentiment and stock market returns. Implicitly, they also tend to highlight the investors' rationality limits.

First, the pioneering work elaborated by De Long et al. (1990) shed light on the covariance between market sentiment and stock returns, stressing the noise traders' impact on price behavior. However, Solt and Statman (1988) as well as Clarke and Statman (1998) discovered no significant relationship binding the sentiment measured by the Investors Intelligence (II) index and returns reached at different temporal periods of 4, 26, or 52 weeks observed on the Dow Jones industrial or SP 500.

Similarly, Fisher and Statman (2000) demonstrated that a significant relationship persists between individual investor sentiment as measured by the AAII index and the monthly returns of large capitalization over the period extending between July 1987 and July 1998. Nonetheless, no significant relationship have been discerned between investor sentiment measured by the II index and the monthly returns of large capitalization.

Through a study conducted regarding several specialized markets for agricultural products, Wang (2001) revealed that the commercial traders' sentiment forecast price continuations while the non-commercial traders' sentiment represents a contrarian indicator of price movement. However, the small trader sentiment appearing to have no remarkable forecasting of future market movements.

Simon and Wiggins (2001) found a significant investor sentiment's impact on the returns of S&P 500 futures contracts. Their study revealed that the highly noticeable pessimistic or optimistic periods provide greater opportunities to purchase or sell.

Lee et al. (2002) studied the link between the noise trader risk, conditional volatility, and expected return. They employed a GARCH framework containing contemporaneous shifts in investor sentiment in the mean equation and lagged shifts in sentiment in the conditional volatility equation. They used the Investors Intelligence measure to investigate the impact of shifts in investor sentiment on the volatilities along with the excess returns of the DJIA, S&P 500, and NASDAQ indexes, from January 1973 to October 1995. They concluded that changes in sentiment are contemporaneously positively correlated with excess returns, while they are negatively correlated with market volatility.

Brown and Cliff's (2004) composite model demonstrated a noticeable correlation persists between the composite indicator and market movements. However, the investor sentiment does not undertake to anticipate any returns. The authors have also noted the strongest persistent link is that binding large capitalization and the institutional investor sentiment.

While studying the market sentiment's long-term impact, Brown and Cliff (2005) documented that the Investors Intelligence index has a noticeable impact on returns over a one to three-year horizon. In addition, they concluded that the index represents a relevant measure of market sentiment and has to be included in the asset valuation.

Hirose et al. (2009) applied margin trading to investigate the Japanese market by studying the link binding sentiments and returns. They found that variations in the margin buying volume are positively correlated with the previous period's registered returns. Moreover, they concluded that the margin purchases' volume increases whenever the market expanded in the recent past and the concerned company's shares registered a poor performance.

Qiang and Shue (2009) revised the DDSW model in the Chinese market over May 1998 - December 2006. Using a generalized autoregressive conditional heteroskedasticity in mean model (GARCH-M), they found a significant investor sentiment impact on stock prices. They also noted that the positive impact of change in investor sentiment on stock prices proves to be stronger than that of a negative one.

Sheu et al. (2011) studied the causal link between daily sentiments and returns in the Taiwan stock market over the period 2003- 2006. They used a threshold model to account for different market states. Their results further confirmed that noise traders affect market behavior. The authors found that sentiment, measured by the ARMS index, is an important indicator once the market is more bearish. Yet, they noted that when sentiment is measured by the put-call trading volume and option volatility index, its effect is noticed once the market more bullish.

More recently, Daszyńska-Żygadło et al. (2014) studied the nexus between investor moods and excess market returns in eight emerging markets. They noted a positive contemporaneous relationship between excess returns and investor sentiment only in Brazil and China. They also found that the impact of the change in investor sentiment on excess returns is stronger during periods of negative moods than that of positive moods in Brazil, China, India, and Mexico.

5. Conclusion

This chapter has been predominantly dedicated to present the main concepts on which our thesis is based. In a first step, the literature review highlights anomalies and puzzles in the context of classical finance. This cast noticeable doubt on market efficiency based on the perfect rationality. In a second step, the literature review showed that behavioral finance while adopting a psychological approach permits a better explanation the reality of stock markets.

Durable price deviations from fundamentals followed by crashes along with heuristics are factors that have prompted several researchers to introduce more sophisticated models taking investor sentiment into account (Barberis et al., 1998; Black, 1986; De Long et al., 1990, etc.). Such models confirm agents' heterogeneity as eluding the traditional representative-agent based approach.

Indeed, the market has a sentiment. More specifically, it has thoughts, beliefs, moods, and emotions. In this context, several studies showed the significant impact of investor sentiment on the decision-making process, and thus on the return generating processes (Dowling and Lucey, 2005; Finuance et al., 2000; Loewenstein et al., 2001). It follows that sentiment risk is a real tool promoting understanding of stock prices' dynamics since it offers hope to quantify investors' psychology and behavior.

The next two chapters of this thesis will have an empirical character. We will use two econometric specifications to model the stock market returns' exposure to investor sentiment.

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Chapter 2: An Analysis of the Effect of Investor Sentiment in a Heterogeneous Switching Transition Model for G7 Stock Markets

ABSTRACT

In line with Chiarella and He (2000b) and Boswijk et al. (2007), this study examines the stock market dynamics in a behavioural heterogeneity framework. In particular, we analyse the stock market dynamics in G7 countries over recent decades while considering the effect of heterogeneous investor sentiment on stock returns. We develop an empirical and nonlinear heterogeneous stock price specification allowing for the presence of two types of investors (arbitrageurs and noise traders) and two regimes. With reference to the heterogeneous smooth transition regression model, we enable the price to switch smoothly between regimes after accounting for an endogenous change in investor sentiment. Our findings do not reject the hypothesis of the nonlinear investor sentiment effect on stock returns. Further, we show that these two regimes characterize the stock price dynamics for which the price is closely governed by fundamentals in the first one, while emotions and sentiment drive the price in the second. Our model, therefore, captures the main stylized facts observed in the market and shows good in- and out-of-sample forecasting power.

Keywords: Stock Return Dynamics; Investor Sentiment, Nonlinearity; Smooth Transition Regression, Forecasting.

JEL classification codes: C20; G10

1. Introduction

Modern financial theory is based on the efficient market hypothesis and paradigm of rational and homogeneous expectations and representative agents. This theoretical work has, however, been unable to explain some of the stylized facts observed in financial markets such as high trading volume, excess volatility, skewness, excess kurtosis and fat tails, long-range dependency, and volatility clustering. Accordingly, alternative frameworks have been developed to better explain stock market dynamics. These frameworks include the model with chartists and fundamentalists (Chiarella, 1992), model with imitating agents (Kirman,1992, 1993), adaptive belief system (Brock and Hommes, 1997, 1998), and model with heterogeneous investors (Chiarella and He, 2000b). These studies have opened the avenue to the development of heterogeneous agent models (HAMs) in which investors with different sentiments, levels of risk aversion, and expectations concerning price dynamics might interact and adapt their beliefs.

Mean reversion in stock prices and the efficient market hypothesis do not seem to hold naturally. Indeed, the price becomes the result of the interaction of at least two reaction functions. The first reaction function is that of arbitrageurs who rely more on fundamentals, do not follow emotions, and show evidence of rationality (Fama, 1965; Samuelson, 1965). The second reaction function corresponds to less rational investors, also called noise traders, who do not trust fundamentals but rather follow their emotions, sentiment, and cognitive bias. Against this background, the market is expected to alternate between at least two major states. The first state is predominated by fundamentalists whose actions will mean-revert the price, while the second state is governed by noise traders who will mislead the price and generate price trends, thereby preventing the price from converging to its fundamental value.

Over recent years, most stock markets have alternated between different levels, showed excess volatility, and experienced different crashes and crisis, such as the 1987 crash, Internet bubble, subprime crisis, and global financial crisis (Jawadi and Prat, 2012; Shiller, 1989, 2000), making the identification of these states necessary to better explain and forecast stock market dynamics. Resolving this issue would be possible by identifying these two groups of investors and analysing their actions. However, as these actors are not easily observed, an indirect approach, based on behavioural finance theory and the analysis of

investor sentiment, has been proposed to analyse investors' actions and their effect on stock market dynamics. This approach is based on psychological and emotional aspects, which are identified to explain decision-making processes, movement in financial markets, and agents' strategies. Therefore, the use of investor sentiment might play a central role in asset price formation. Indeed, while classical theory supports that asset returns are mean-reverting and fundamentals drive the equity price, behavioural theory suggests that equity prices exhibit misalignment and can be explained by emotional bias.

From a theoretical point of view, one strand of the literature developed since the beginning of the 1990s investigates the role of investor sentiment in determining stock prices (Barberis et al., 1998; Campbell and Kyle, 1993; De Long et al., 1990; Lakonishok et al., 1994; Shleifer and Summers, 1990). Such studies have argued that noise traders are likely to affect the stock price based on their unpredictable changes in sentiment. In this context, a large body of the literature has discussed the extent to which investor sentiment affects the decision-making process (Finuance et al., 2000; Loewenstein, 2001).

From an empirical point of view, several studies have analysed the role of investor sentiment in stock price formation (Fisher and Statman, 2000; Simon and Wiggins, 2001). In line with the theoretical framework, the consensus is that noise traders are able to affect the stock price. However, both theoretical and empirical studies are inconclusive in terms of the dynamics of asset price formation through arbitrageurs and noise traders. One strand of the literature employs a linear framework to investigate the sentiment index (Baker and Wurgler, 2006; Singer et al., 2013; Verma et al., 2008; Verma and Soydemir, 2009; Zouaoui et al., 2011). Another strand has supported the asymmetric dynamics of investor sentiment in stock price formation, as symmetric models are inadequate at producing stock market return dynamics (Anderson, 1997; Boswijk et al., 2007; Jawadi and Prat, 2012)[15].These studies note that investor sentiment is one of the main causes of asymmetry in stock returns (e.g. Ding et al., 2004; McMillan, 2003; Zhang and Semmler, 2009). Most studies analysing the asymmetric effect of the sentiment index on stock prices have employed a Markov-switching model based on the state of the economy (Chen, 2011; Chung et al., 2012), with others

[15] From an economics point of view, linearity has been questioned by the presence of transaction costs (Anderson, 1997), mimetism (Orléan, 1990, 1992), the behavioural heterogeneity of market participants (De Grauwe and Grimaldi, 2005), and information asymmetry (Artus, 1995).

analysing the asymmetric effect based on the bear and bullish states of the market by considering a representative investor (Barberis et al., 1998; Hong et al., 2004).

Our study is related to this second strand of the literature investigating the nonlinear effect of the sentiment index in asset price formation. However, we differ from theoretical and empirical previous studies in two main ways. First, we relax the assumption of investor homogeneity and propose a heterogeneous agent framework, thus extending the theoretical model of Barberis et al.(1998) developed with a representative investor. Second, in line with Chiarella and He (2000b), we propose a two-agent HAM and show that the interaction of arbitrageurs and noise traders might generate nonlinear dynamics. The consideration of this nonlinearity is useful for explaining the dynamics of the market and its stylized facts.

In particular, we propose an asset stock price dynamic tested empirically based on the smooth transition regression (STR) model developed by Granger and Teräsvirta (1993). The transition between regimes in this model is endogenously governed by changes in investor sentiment. When applying this model to G7 countries, our results show three main findings. First, the alternation of two regimes (arbitrageurs and noise traders) validates the heterogeneous agent hypothesis, rejects the efficiency hypothesis, and yields significant findings to better explain market dynamics. Second, the model provides interesting results in terms of in- and out-of-sample forecasting with regard to the linear benchmark model. Finally, investor sentiment has a significant effect on the stock return dynamics that enter asymmetrically and nonlinearly.

The remainder of this paper is structured as follows. Section 2 presents the theoretical framework and summarizes the related literature. The econometric specification is presented in Section 3. Section 4 describes the data and descriptive statistics. Section 5 outlines the empirical results. Finally, the last section concludes.

2. Theory and Related Literature

2.1. Theoretical Background

The relationship between investor sentiment and asset price dynamics is justified by the dynamic interaction between investors (e.g. noise traders, arbitrageurs) who might behave differently and have distinct expectations. With regard to the informational efficiency framework of Fama (1965), the actions of rational investors lead to the disappearance of noise traders from markets. Therefore, the sentiment factor is unimportant, as rational investors

have rational and homogeneous beliefs that might be correctly captured by a representative agent.

Shiller (1984) and Campbell and Kyle (1987) suggested that noise traders can affect asset price formation in the long term. Trading by noise traders compared with that by arbitrageurs can serve as an additional source of risk, as noise traders' expectations are not mean-reverting in the long term. For example, pessimistic (optimistic) noise traders' behaviour drives down (up) the price of an asset. Further, if noise traders remain pessimistic (optimistic) for long horizons, the price movement will be extreme and it will be driven down (up) further. On the contrary, an arbitrageur, holding this asset, at the moment of a position's liquidation, might assume losses.

A framework suitable for modelling an investor's interaction has been developed based on behavioural finance theory, with particular reference to two important phenomena: conservatism and representativeness. One example of conservatism is when an investor appears slow to change his/her beliefs in the face of new announcements (Edwards, 1968). In finance, Barberis et al. (1998) associated conservatism with an investor's underreaction to good news, which can lead to mean reversion in stock prices. Representativeness, which has been discussed in psychology by Tversky and Kahneman (1974), means the similarity of an uncertain event with a past event. For example, the representativeness heuristic can be seen when a stock market experiences an increase in returns for several years. In this setting, investors might consider past history to be representative of potential future return growth. This behaviour implies an investor's overreaction and yields a non-mean-reverting price process. These two phenomena also justify the emergence of Heterogeneous Agent Models (HAMs (Chiarella and He, 2000b)) with two types of investors and models explicitly including investor sentiment (Barberis et al., 1998).

Next, in line with Barberis et al. (1998), we investigate the properties of a theoretical relationship to assess the extent to which investor sentiment affects stock returns while reconciling the two arguments above, namely the tendency of arbitrageurs to drive asset price formation and the presence of noise traders who might affect stock price formation[16].

[16] Barberis et al. (1998) supposed that an earning asset is modelled by a Markov process with two states depending on the state of the economy. They further supposed that the transition probabilities from one regime to another are fixed by a representative investor.

Specifically, the asset price might be driven by these two types of investors, and thus two regimes can be identified. In the first regime, arbitrageurs follow fundamentals, show conservatism, and yield the mean-reverting price. In the second regime, noise traders follow their emotions, show representativeness, and generate price trends and non-mean-reverting prices through their actions. The combination of these two states enables us to capture the complexity and further nonlinearity generated by an investor's interaction. Such nonlinearity might, for example, be captured by an exponential function, as its central regime would capture the actions of arbitrageurs, while its extreme regime could reproduce the behaviour of noise traders.

Unlike Barberis et al. (1998), we suppose that two investors form the market: Arbitrageurs (AR) ad noise traders (NT). Further, we suppose that some investors of each type vary from one period to another. We note by (λ) and $(1-\lambda)$ the percentage of arbitrageurs and noise traders in the market, respectively. Accordingly, conservatism is shown by an arbitrageur's action, while a noise trader's behaviour is influenced by representativeness. The market price is the consequence of the interaction of these two aspects and the dominance of the investor type. Indeed, at time (t), depending on available information at $(t-1)$, if conservatism behaviour dominates representativeness, the price will be mean-reverting and vice versa.

Three states can thus be identified: (i) all investors are arbitrageurs, (ii) all investors are noise traders, and (iii) both types of investors are present in the market at time t: (λ) arbitrageurs and $(1-\lambda)$ noise traders. However, given that, as suggested by De Long et al. (1990), noise traders are not observed in the financial market, $(1-\lambda)$ is not observed and this remains undetermined. Therefore, in line with the above theoretical background based on emotional bias characterizing noise traders, their presence in the market might be determined by the analysis of investor sentiment. Specifically, we suppose that noise traders only appear in the market above a certain level of investor sentiment. Based on these assumptions, investors react to asset returns in two ways. The first state is with only arbitrageurs and the second state with only noise traders. Therefore, we suggest that the transition from the first state to the second state is governed by the change in the investor sentiment index. This threshold is determined endogenously.

Based on the above assumptions, asset price dynamics are the sum of arbitrageurs and noise traders' reaction functions, that is the sum of the linear function (mean-reverting process

of arbitrageurs) and a nonlinear part (the non-mean-reverting process of noise traders). Therefore, our model proposes that return dynamics are modelled by two functions covering two states. However, price dynamics can be in the first state (Fama,1965) or the second state (Shiller, 1984)). Interestingly, while Barberis et al. (1998) assumed an abrupt transition between the under-reaction and overreaction regimes, we propose a more general framework that enables a smooth transition to capture behavioural heterogeneity among investors.

2.2. Related Literature

The concepts of market efficiency, rational expectations, and representative agents are insufficient to appropriately explain the real function of stock markets, yielding new explanations in the literature on HAMs (Brock and Hommes, 1998; Chiarella and He, 2000; Hommes, 2006; LeBaron, 2006) and behavioural heterogeneity in stock markets (Boswijk et al., 2007)[17]. This framework has the advantage of introducing more flexibility and versatility into modelling stock price dynamics, as it takes account of heterogeneity among investors and therefore captures their differences in risk tolerance, risk attitude, and expectations about the evolution of prices.

According to He et al. (2009), HAMs might reproduce the most observed stylized facts in stock markets (e.g. skewness, excess kurtosis and fat tails, volatility clustering); as such, they have started to be widely applied in financial markets. For example, although the discrete-time HAM model (Chiarella et al., 2006) has helped scholars understand cyclical market behaviour, He et al. (2009) extended this model to a continuous framework to capture the Hopf bifurcation that characterizes the market. In the same context, Chiarella et al. (2011) developed a direct approach with two agents (chartists and fundamentalists) and showed the important effect of noise for explaining market dynamics in HAMs. Similarly, the continuous-time HAM including fundamental, contrarian, and momentum traders developed by He and Li (2014) can also reproduce market dynamics. More recently, He and Zheng (2016) proposed a framework including uncertainty about the fundamental value to describe investor heterogeneity and showed that their model supplants the usual HAM to reproduce bubbles, crashes, and mean reversion in stock prices.

[17] See Hommes (2006), Hommes and Wagener (2009), and Chiarella et al. (2009) for a survey of HAMs.

Regarding the effect of behavioural heterogeneity on stock markets, He and Li (2008) developed a new stochastic market fraction asset pricing model and showed that asset prices might be driven by news and market psychology. Moreover, He and Shi (2012) investigated the effect of behavioural heterogeneity on the market equilibrium and risk premium and showed that both optimistic and pessimistic sentiment might affect the risk premium. Indeed, heterogeneity in beliefs (optimism, pessimism, doubt, overconfidence) may not only affect financial market dynamics (Bohm and Chiarella, 2005; Chiarella et al., 2006, 2009), but also generate complex nonlinear dynamics and switching regimes (Boswijk et al., 2007, Jawadi and Sahut, 2009).

Chiarella et al. (2007) extended related works (Brock and Hommes, 1998; Chiarella and He, 2001, 2003; Lux, 1998) by developing a model with two agent groups (fundamentalists and trend chasers), highlighting that the investor's interaction might generate complex dynamics. Chiarella and He (2010) showed that a model with chartists and fundamentalists can generate multiple steady states or regimes. These regimes appear to be unstable (stable) when chartists carry out strong (weak) extrapolations. Further, the interaction between these two types of agents might generate different switching regimes among these states. Chiarella et al. (2011) showed that rational HAMs in financial markets might generate nonlinear dynamics. Indeed, they produced both a stable equilibrium corresponding to the well-known framework of the rational paradigm and instable states that reflected noise traders' actions, suggesting that these models are suitable for capturing the most important market features.

In addition, several recent theoretical and empirical studies have focused on the effect of behavioural heterogeneity through another direct channel associated with investor sentiment. This literature highlights not only that investor sentiment might affect stock market dynamics but also that investor sentiment changes according to the investors who dominate the market (arbitrageurs or noise traders), which might affect the market in different ways. De Long et al.(1990) were the first to offer a model introducing 'sentiment risk' that considered both arbitrageurs, who anticipate assets by using the available information, and noise traders, whose decisions lead to the over- or undervaluation of financial assets. The authors showed that arbitrageurs are unable to predict noise traders' reactions that depend on their sentiment. This may cause systematic risk to affect stock prices and yield deviations from fundamentals,

even in the absence of fundamental risk. Thus, sentiment risk becomes a price factor of financial markets.

Barberis et al. (1998) developed a theoretical model to analyse the effect of investor sentiment on asset price formation by considering a representative investor. They assumed investor homogeneity and investigated under-reaction and over-reaction by investors (representing conservatism and representativeness, respectively). By using a Markov-switching model, the authors showed that their model can reproduce stock price dynamics. Verma et al. (2008) proposed a theoretical model to analyse the asymmetry between rational and irrational sentiment, distinguishing between individual and institutional investor sentiment. Their model was tested based on a VAR model[18].They concluded that the effect of rational sentiment on stock markets is greater than that of irrational sentiment. Further, they showed that individual and institutional investors are driven by both rational and irrational factors.

In addition to theoretical works, the effect of investor sentiment on asset price formation has been investigated empirically[19]. Baker and Wurgler (2006) explored the US market between 1962 and 2000 and showed that when the level of sentiment is high (low) at the start of the period, returns are low (high) at the end of the period. They justified this finding by stating that the reaction of uninformed investors limits arbitrage possibilities. Baker and Wurgler (2006, 2007) also showed that the impact of psychological factors is greater on the performance of securities with high costs and high arbitrage risk. Verma and Soydemir (2009) employed a similar model to analyse the short-term effects of investor sentiment and market price risk. By using a VAR model, they suggested that when rational investors are optimistic (pessimistic), noise traders are pessimistic (optimistic), thus favouring contrarian strategies, and that the market reaction is particularly affected by the change in investor sentiment.

Schmeling (2009) investigated the effect of investor sentiment on expected stock returns for 18 industrialized countries. He estimated a predictive regression linking stock market returns and the investor sentiment measure through both time series and panel data

[18] Empirically, they investigated the short-term impact of rational and irrational investors on Dow Jones Industrial Average and S&P 500 index returns, using monthly data between October 1988 and April 2004.

[19] We intentionally mention only recent related studies.

approaches. The author showed a significant sentiment effect, but one that depends on the level of institutional quality and cultural factors specific to each country. Indeed, the effect of investor sentiment is more pronounced for countries that have less market integrity and are culturally more prone to herd-like behaviour and overreaction.

Corredor et al. (2013) analysed the short-term effect of investor sentiment on four European stock markets (France, Germany, Spain, United Kingdom) from 1990 to 2007 based on a VAR approach. The authors did not reject the sentiment effect hypothesis but found that the forecasting power of sentiment varies by country. Singer et al. (2013) also applied a VAR model to investigate the relationship between weekly stock returns and professional analysts' sentiment for the case of the German stock market. They constructed a sentiment index based on the published stock recommendations in print and online media and showed that past returns predict current investor sentiment.

Further, some studies provide evidence of asymmetry in the sentiment effect on stock returns (Ding et al., 2004; McMillan, 2003; Zhang and Semmler, 2009). This asymmetry has its foundation in behavioural finance theory. Indeed, prospect theory (Kahneman and Tversky, 1979) claims that investors have a reference point when they evaluate possible gains and losses. Owing to loss aversion[20], they are more interested in market downturns than upturns. Accordingly, negative sentiment might have a larger impact on returns than does positive sentiment.

To explain this complex relationship between stock returns and investor sentiment, some studies model the asymmetry inherent in this relationship. McMillan (2003) showed that stock returns are characterized by nonlinear dynamics resulting from the interaction of noise traders and arbitrageurs. Dridi and Germain (2004) found a nonlinear effect of investor sentiment on stock prices. Chen (2011) studied the link between the lack of consumer confidence and stock market behaviour by using monthly returns on the S&P 500 index price for 1978–2009. The author employed a Markov-switching framework to model the fluctuations in the stock market between two regimes (bear and bull markets) to investigate the effect of confidence loss (negative sentiment) on the stock market. He showed that pessimism has an asymmetric influence during market fluctuations as negative sentiment has a larger effect on stock returns in bear periods than in bull periods. In addition, the results

[20] This refers to an investor's tendency to strongly prefer avoiding losses to acquiring gains.

showed that a lack of consumer confidence induces a higher probability of switching to a bear market regime. Chung et al. (2012) focused on the US market between 1966 and 2007 and tested the asymmetry in the impact of investor sentiment on the cross-section of stock returns across expansion and recession states. By using a Markov multivariate model, they concluded that investor sentiment has a significant impact on stock returns only during periods of economic expansion.

Overall, the hypothesis of investor sentiment effect is not rejected and the sentiment–stock return relationship seems to exhibit nonlinearity and asymmetry. To extend this related literature, we propose a more general empirical nonlinear framework inspired by the theoretical framework of Barberis et al. (1998) to investigate the reaction of stock returns after a shock affecting investor sentiment. We also propose the in- and out-of-sample forecasting of our model to test the forecasting power of investor sentiment. Finally, in contrast to most previous studies that focus on the US market, we extend examine G7 countries.

3. Threshold Empirical Specification

Abandoning the representative agent assumption (and therefore the rational expectation hypothesis) and considering HAMs would enable us to better characterize the dynamics of stock markets. In particular, considering two groups of agents (i.e. arbitrageurs and noise traders) and investigating their interactions would be useful for better identifying market states and reproducing the most stylized facts. Further, the consideration of the linear and nonlinear effects of investor sentiment is helpful to better capture the actions of both arbitrageurs and noise traders on asset prices. More specifically, as in Barberis et al. (1998), a switching regime model provides an appropriate framework with which to better assess behavioural heterogeneity in stock prices as well as measure the effect of investor sentiment on stock prices.

The advantage of this framework is twofold. First, it offers sufficient flexibility to enable investor sentiment to enter asymmetrically and nonlinearly in order to take account of the different forms of cognitive and emotional biases (excess optimism / pessimism, fads, mimetic behaviour) and the expectations of agent (arbitrageurs, chartists, noise traders). Second, it enables the sentiment–stock return relationship to be sufficiently flexible to account for the time-varying sentiment effect with regard to the market state (expansion, recession).

Recall that the above-described theoretical framework for stock returns derived in line with Barberis et al. (1998) stipulates the combination of two dynamics. A first refers to stock return dynamics when arbitrageurs significantly dominate the market and for which the sentiment effect does not hold. The second (nonlinear) dynamics enable investors to follow noise traders' cognitive thoughts and sentiment. This combination yields a more appropriate specification that can characterize the fluctuations and states of stock markets, while this nonlinearity offers more flexibility for our model.

Next, from an empirical view, we propose a two-regime Heterogeneous empirical specification based on the smooth transition regression (STR) model of Granger and Teräsvirta (1993), to approximate the above theoretical model and assess for the effect of investor sentiment in a nonlinear framework. Our empirical STR model reproduces two extreme regimes (arbitrageurs and noise traders), thereby enabling the transition between these regimes to occur nonlinearly and smoothly. When we enable switching to be endogenously governed by changes and shocks affecting investor sentiment, as we do through the consideration of sentiment as an endogenous threshold variable, it becomes possible to enable the model to identify regimes that distinctly capture the differences among heterogeneous market agents and expectations.

Formally, our empirical model is written as follows:

$$y_t = \alpha' W_{1t} + \theta' W_{2t} F(s_t, \gamma, c) + u_t \qquad (2\text{-}1)$$

Where (y_t) denotes stock returns, the first part $[\alpha' W_{1t}]$ captures the reaction of arbitrageurs in the first regime, and the second term $[\theta' W_{2t} F(s_t, \gamma, c)]$ reproduces the reaction of noise traders in the second regime. We suppose that $u_t \rightarrow \text{iid}(0, \sigma^2)$. $\gamma > 0$, $W_{1t} = (1, y_{t-1,} \dots y_{t-d}, x_{1t}, \dots, x_{1,t-d}, x_{2t}, \dots., x_{2,t-d}, x_{3t}, \dots., x_{3,t-d}, x_{4,t}, \dots x_{4,t-d})'$ denotes the explanatory variables of the linear part and $W_{2t} = (1, x_{5,t}, \dots., x_{5,t-d})'$ presents the explanatory variables ofthe nonlinear part, while y_{t-1} , y_{t-2} , $\dots y_{t-d}$ are the lagged dependent variables.

The variables x_1 to x_5 refer to the main determinants of the fundamental value of asset prices as follows: x_1: dividend yield (DY), x_2: price earnings ratio (PER), x_3: growth in industrial production (IP), $x_{4:}$ term structure of the interest rate (ST), and $x_{5:}$ investor sentiment (Sent). $\alpha = (\alpha_0, \alpha_1, \alpha_2, \dots \dots \alpha_j)'$ denotes the coefficients of the first regime and $\theta =$

$(\theta_0, \theta_1, \theta_2, \dots \dots \theta_p)'$ refers to the coefficients of the second regime. s_t is the transition variable.

F is the transition function, which is continuous and bounded between 0 and 1. This enables model (2-1) to be linear if F reaches zero or to capture the extreme regime if F equals unity but also to capture an important number of intermediate states for each values between 0 and 1. According to Granger and Teräsvirta (1993), F may be an exponential function (Equation (2-2)) or a logistic function (Equation (2-3)). Accordingly, systems (2-1)–(2-2) yield an exponential STR (ESTR) model, while systems (2-1)–(2-3) define a logistic STR (LSTR) model[21]:

$$F(s_t, \gamma, c) = 1 - \exp\{-\gamma(s_t - c)^2\} \quad , \gamma > 0 \tag{2-2}$$

$$F(s_t, \gamma, c) = \{1 + \exp(-\gamma(s_t - c))\}^{-1} \quad , \gamma > 0 \tag{2-3}$$

With regard to our theoretical model and to check whether the transition between regimes occurs after a given change in investor sentiment, we suppose hereafter that the transition variable s_t corresponds to one-period lagged investor sentiment[22]. γ is the transition speed and c denotes the threshold parameter.

In line with the modelling strategy proposed by Granger and Teräsvirta (1993) and Teräsvirta (1994), we model the nonlinear model in four steps. First, we specify the linear model, check the linearity hypothesis, and select the appropriate transition function. Second, we estimate the model by adopting the nonlinear least squares method. Third, we run a battery of misspecification tests to check the robustness of our estimation. Finally, we check the forecasting performance of our model compared with a linear benchmark model.

[21] See Jawadi and Koubbaa (2007) for a more complete discussion on the properties of these functions.

[22] This hypothesis will, however, be statistically tested when applying the linearity tests. As in Teräsvirta (1994), the optimal threshold variable is the one that maximizes the rejection of linearity.

4. Data and Preliminary Results

4.1. Data

Our data include the closing stock market prices and investor sentiment indexes of G7 countries (France, Germany, Italy, the United States, the United Kingdom, Canada, Japan) as well as other related variables (used as control variables): dividend yield, price earnings ratio, growth in industrial production, and term structure of the interest rate. These data are collected from three databases: DataStream, International Financial Statistics, and the OECD. The proxy used for investor sentiment corresponds to the consumer confidence index.

The selection of the G7 sample is justified by the fact that it includes most developed stock markets and reflects sufficient heterogeneous investment and trading strategies to assess the effect of investor sentiment on stock market dynamics. This choice also enables us to propose an interesting comparison among countries. Direct and indirect measures are developed to measure investor sentiment. However, sentiment data are much more developed for the US market and, to our knowledge, the consumer confidence index is the only proxy for investor sentiment available for all G7 countries.

Monthly data are collected for all seven countries. Table 2.1 provides more details on the data and Table 2.2 explains the sample period for each country. The highest (lowest) number of observations is for the United States (Japan). However, the number of observations is sufficient to carry out nonlinear modelling for all countries. Further, overall, our samples include different downturn and crisis periods (1987 crash, Internet bubble, subprime crisis, global financial crisis), which enables to assess the reaction of stock markets to further changes in investor sentiment during calm and crisis periods.

Table 2. 1 Data description

Variable		Proxy	Source
Y	Stock returns	Log (P_t/P_{t-1})	DATASTREAM
$\mathbf{X_1}$	Dividend yield (**DY**)	Dividend /price	DATASTREAM
$\mathbf{X_2}$	Price Earnings Ratio (**PER**)	Price / profit after tax	DATASTREAM
$\mathbf{X_3}$	Growth in industrial production (**PI**)	Log (PI_t / PI_{t-1})	International Financial Statistics
$\mathbf{X_4}$	Term structure of the interest rate (**ST**)	Difference between long and short rates	International Financial Statistics
$\mathbf{X_5}$	Investor sentiment (**Sent**)	Confidence consumer index	Main Economic Indicators, OECD

Table 2. 2 Sample period by country

Country	Period of analysis	Number of observations
France	09/07/1973- 09/02/2014	488
United States	09/07/1973- 09/04/2014	490
United Kingdom	09/04/1974 - 09/03/2014	480
Germany	09/07/1975 - 09/02/2014	464
Italy	09/01/1986 - 09 /02/2014	338
Canada	09/04/1980 - 09/03/2014	407
Japan	09/06/1987 - 09/02/2014	321

4.2. Investor Sentiment Measure

As mentioned above, several proxies have been proposed to measure investor sentiment. These measures can be grouped into two classes: direct measures based on investor surveys and indirect measures based on financial and economic variables correlated with investor sentiment. While direct measures are preferred to indirect measures, their limitation concerns whether the interviewee sample represents the whole investor population.

In our framework, we also refer to the first strand of direct measures, since when using indirect measures, sentiment might endogenously vary with changes in economic fundamentals that overlook investors' subjectivity and preferences. The direct measure of investor sentiment adopted in our analysis is the consumer confidence index, which is available for all G7 countries and for reasonable periods. This index assesses consumers' confidence and trustiness, making it the most appropriate measure available for our sample[23]. However, consumer confidence does contain an irrational component in that the investor is hardly affected by the news when responding to economic reports.

To construct this index, information is collected through a monthly survey, including a set of standardized questions. Lemmon and Portniaguina (2006) and Ho and Hung (2009) confirmed the positive correlation between consumer confidence and household participation

[23] Most surveys in developed countries have adopted standardized questions to ensure the international comparability of this index. These questions are similar to those asked in the survey by the University of Michigan and are usually structured around three themes: (i) past and future financial situation, (ii) past and future economic situation, and (iii) major purchases of durable goods.

in the stock market. In other words, when investors are optimistic about the economy, they are also optimistic about the stock market. Schmeling (2009) also confirmed that consumer confidence is a reasonable proxy for individual sentiment.

4.3. Preliminary Analysis

We first checked the stationary hypothesis for all variables, which is a necessary condition for STR modelling. To this end, three unit root tests were applied: the Augmented Dickey–Fuller (ADF) test, Philips–Perron (PP) test, and Kwiatkowiski–Phillips–Schmidt–Shin (KPSS) test. Further, as our sample includes some downturns, we applied the test of Zivot and Andrews (1992) that is robust for further structural breaks. Table 2.3 summarizes the results of these tests.

Table 2. 3 Results of the unit root tests

Country(ies)	Stationary variables in levels	Stationary variables in first differences
US and Italy	$Y, X3$	$DX1, DX2, DX4, DX5$
Japan	$Y, X1, X3, X4$	$DX2, DX5$
France	$Y, X1, X2, X3, X4, X5$	
Canada	$Y, X1, X2, X3, X4$	$DX5$
UK	$Y, X1, X2, X3$	$DX4, DX5$
Germany	$Y, X2, X3$	$DX1, DX4, DX5$

Table 2.4 reports the main descriptive statistics of the stock returns series. Except for the United Kingdom, all series exhibited asymmetry. The negative sign for skewness indicated the nonlinearity of the data. Their distribution also showed further evidence of excess leptokurtic and fat tails except for Germany and Japan. Further, the normality assumption was rejected for all stock market return series. Finally, the null hypothesis of the ARCH test was rejected for all countries except France and Japan.

Table 2. 4 Descriptive statistics of the stock market returns series

Countries	Mean	Variance	Skewness	Kurtosis	J-B	ARCH
France	0.005	0.004	-0.960	4.190	432.102	1.515
US	0.006	0.002	-0.943	6.109	834.754	3.467***
UK	0.006	0.358	0.005	11.316	2571.458	9.924***
Germany	0.003	0.002	-0.398	2.078	95.796	3.147**
Italy	0.007	0.005	-0.674	3.662	281.813	6.595***
Canada	0.005	0.002	-0.824	4.937	459.429	4.135***
Japan	-0.001	0.003	-0.272	1.512	34.544	1.527

Note: J-B refers to the statistic of the Jarque–Bera test. ARCH denotes the statistic of the ARCH test of Engle (1982).(***) denotes statistical significance at the 1% level.

5. Empirical Analysis

5.1. Specification of the Heterogeneous STR Model

When assessing the extent to which investor sentiment affects the dynamics of stock markets, a misspecified linear model might wrongly reject nonlinearity; further, a linear model constitutes the benchmark of nonlinear modelling. Moreover, the specification of the nonlinearity form might affect the forecasting performance of the model. In practice, as mentioned earlier, this specification step includes three steps: (i) specification of the linear model, (ii) linearity tests, and (iii) selection of the transition function.

5.1.1. Linear Heterogeneous Model

We started with an initial linear model that contains four lags for each explanatory variable and followed a general-to-specific procedure that consists of running regressions with four lags for each explanatory variable and subsequently eliminating the non significant delays. This linear model should minimize the information criteria, maximize the adjusted coefficient of determination, and show the best statistical properties for the error term. We carried out this procedure for all the indexes under consideration and retained for each index the most appropriate specification including only the significant explanatory variables. Overall, investor sentiment seems to significantly affect the dynamics of stock returns for all G7 countries, suggesting further evidence of market inefficiency and implying that the

hypothesis of heterogeneous agents who react to their sentiment and cognitive bias does hold[24]. To better characterize this sentiment effect, we next investigated the relationship between investor sentiment and stock returns in a nonlinear framework.

5.1.2. Linearity Tests

The application of linearity tests is an important step to check whether the dynamics of stock returns exhibit further nonlinearity either in the mean or in the variance[25]. To check for the presence of nonlinearity in the data, we applied several linearity tests to test the null hypothesis of linearity (H_0) against the alternative hypothesis of nonlinearity (H_1). The null hypothesis of linearity might be specified as follows: (H_0): $\gamma = 0$. However, this test suffers from nuisance parameters because some parameters are unidentified under the null hypothesis and accordingly the standard asymptotic distribution results do not apply. To resolve this problem, Luukkonen et al.(1988) proposed a linearization solution that consists of replacing the transition function with its Taylor approximation. This makes it possible to test linearity and the standard statistical inference by using the Lagrange multiplier (LM) test, which follows a standard asymptotic Chi-squared distribution under the null hypothesis of linearity. The LM tests test the null hypothesis of linearity against the alternative test of the nonlinearity of the STR type (ESTR or LSTR).

To briefly recall the principle of these tests[26], we consider the following LSTR model:

$$y_t = \alpha' W_{1t} + \theta' W_{2t} \left(1 + \exp\left(-\gamma(s_t - c)\right)\right)^{-1} + u_t \quad (2\text{-}4)$$

Following Luukkonen et al. (1988), when approximating the logistic function by using a third-order Taylor approximation around $\gamma = 0$, we got the following auxiliary regression:

$$y_t = \mathrm{M}_0 + \alpha_1' \breve{w}_{1t} + \breve{w}_{2t}\mathrm{B'}_0 + \breve{w}_{2t}\mathrm{B'}_1\, \mathrm{s_t} + \breve{w}_{2t}\mathrm{B'}_2\, \mathrm{s_t^2} + \breve{w}_{2t}\mathrm{B'}_3\, \mathrm{s_t^3} + \mathrm{e_t} \quad (2\text{-}5)$$

[24] To save space, we do not report the detailed results of the linear specifications, but they are available upon request.

[25] In our study, as in Jawadi and Koubbaa (2007), we focus on testing and modelling nonlinearity in the first moment, as we are interested in modelling stock return dynamics.

[26] For more details and discussion on the LM tests of linearity against nonlinearity for the STR type, see Van Dijk et al. (2002) and Jawadi and Koubbaa (2007).

Where $\breve{w}_{1t} = (y_{t-1}, \ldots . y_{t-d}, x_{1t}, \ldots x_{1,t-d}, x_{2t}, \ldots x_{2,t-d}, x_{3t}, .. x_{3,t-d}, x_{4,t}, .. x_{4,t-d})'$ and $\breve{w}_{2t} = (x_{5,t}, .. x_{5,t-d})'$, $e_t = u_t$ + the remainder term from the Taylor expansion. B'_1, B'_2, B'_3 are function of the STR parameters. That is, the null hypothesis corresponds to $H'_0: B_1 = B_2 = B_3 = 0$

As shown in Table 2.5, our findings retained one-period lagged investor sentiment as the more suitable transition variable for which linearity is strongly rejected for all countries except Germany. This result suggests that stock return dynamics for the G7 are rather nonlinear and that their dynamics exhibit a switching regime governed by shocks or changes affecting investor sentiment. While this finding suggests that the hypotheses of rational expectation and informational efficiency do not hold, this a priori conclusion is in line with He et al. (2009) and Chiarella et al. (2011) for which the interactions of different agents, including noise traders who rely on their cognitive bias, might affect the formation of financial asset prices.

Table 2. 5 Results of the standard linearity test (p-values)

Transition variable	**France**	**US**	**Italy**	**Japan**	**UK**	**Germany**	**Canada**
One-period lagged investor sentiment	0.050*	0.004***	0.000***	0.006***	0.000***	0.218	0.001***

Note: (*), (**), and (***) denote the rejection of linearity at 10%, 5%, and 1%, respectively.

Nevertheless, recall that linearity might be improperly rejected and that the result of the standard test of linearity could be biased if the data include further outliers. To account for this bias, Van Dijk et al. (1999) developed a linearity test that has a similar distribution and principle to the standard LM test, but that is robust to outliers. To check the robustness of the results of the standard linearity tests, we therefore applied the test of Van Dijk et al. (1999). Table 2.6 reports the results.

Table 2. 6 Results of the robust linearity test to outliers (p-values)

Transition variable	US	France	Italy	Japan	UK	Canada
One-period lagged investor sentiment	0.001***	0.000***	0.000***	0.038**	0.046**	0.000***

Note: (*), (**), and (***) denote the rejection of linearity at 10%, 5%, and 1%, respectively.

As shown in Table 2.6, one-period lagged investor sentiment still appears to be the most suitable transition variable. Further, linearity is more strongly rejected for all G7 countries, particularly at the 5% level for the United Kingdom and Japan and at 1% for Canada, Italy, France, and the United States. These results again point to nonlinearity in the data, suggesting further evidence of the nonlinear effect of investor sentiment on the stock returns of G7 countries.

In the same context, recall that standard linearity tests might be affected if the data are characterized by excess heteroscedasticity, which does seem to be the case given the ARCH effect reported for most series in Table 2.4. Accordingly, we must apply the linearity tests robust to heteroscedasticity developed by Granger and Teräsvirta (1993). The results in Table 2.7 confirm the choice of the transition variable, but show that the linearity hypothesis is rejected only for the United States, the United Kingdom, Japan, and Canada at the 1% level. This finding suggests that taking further outliers and heteroscedasticity into account when testing linearity shows that only the dynamics of Canadian, Japanese, US, and UK stock returns series seem to be nonlinearly dependent on investor sentiment.

Table 2. 7 Results of the robust linearity test to heteroscedasticity (p-values)

Transition variable	US	France	Italy	Japan	UK	Canada
One-period lagged investor sentiment	0.000***	0.230	0.541	0.000***	0.000***	0.006***

Note: (*), (**), and (***) denote the rejection of linearity at 10%, 5%, and 1%, respectively.

5.1.3. Selecting the transition function

The final step associated with the specification of the heterogeneous STR models consists of selecting the appropriate transition function to identify the regimes and therefore characterize the sentiment effect for each one. Here, we used a sequence of Fisher tests developed by Teräsvirta (1994) based on the comparison of the auxiliary regressions for the linearity tests against the LSTR and ESTR models[27]. Table 2.8 reports the results. Further, we applied standard and robust tests to check the robustness of the transition function choice. We showed that for Canada, Japan, the United Kingdom, and the United States, the exponential function is preferred to the logistic function. This specification enables us to identify a central regime and an extreme regime that have distinct dynamics but for which the transition from the outer regime to the upper regime is similar to that from the upper to the outer regime. This result is in line with Jawadi and Koubbaa (2007) and Jawadi and Prat (2012), who also found that the exponential function fits transition regimes better when considering stock returns.

To further ensure robustness, we also estimated both the LSTR and the ESTR models and compared the results, again finding the superiority of the ESTR model.

[27] See Jawadi and Koubbaa (2007) for more details on these tests.

Table 2. 8 Results of the transition function tests (p-values)

Standard selection tests			
Series	**H_{03}**	**H_{02}**	**H_{01}**
US	0.624	0.000	0.000
Japan	0.572	0.001	0.000
UK	0.617	0.000	0.000
Canada	0.350	0.000	0.021
Robust selection tests to outliers			
Series	**H_{03}**	**H_{02}**	**H_{01}**
US	0.504	0.075	0.000
Japan	0.535	0.014	0.157
UK	0.086	0.595	0.037
Canada	0.062	0.002	0.005
Robust selection tests to heteroscedasticity			
Series	**H_{03}**	**H_{02}**	**H_{01}**
US	0.022	0.000	0.001
Japan	0.351	0.000	0.000
US	0.000	0.000	0.000
Canada	0.227	0.044	0.231

Note: The rejection of H_{03} and acceptance of H_{01} and H_{02}imply that the model is an LSTR. However, if H_{02} is rejected more strongly than H_{03} and H_{01}, the true model is an ESTR. In other cases, where H_{03} and H_{01} are more strongly rejected than H_{02}, an ESTR model should be selected.

5.2 Estimation of the Heterogeneous STR Model

Next, the ESTR model was estimated for the four countries (Canada, Japan, the United Kingdom, and the United States) for which linearity was rejected (Table 2.9). Our findings point to different findings. First, the estimated values for the transition speed is rather low (with highest value for Japan: $\gamma = 4.733$), suggesting that the transition between regimes is higher for the United States and Japan than for Canada and the United Kingdom. As for the threshold parameter, the US market shows the lowest threshold value (c= -0.0039), implying that US investors might react rapidly after a small change in their sentiment and emotions.

For the United Kingdom and Canada, the estimated threshold parameters are positive (c = 0.3416 and c = 0.0183, respectively), indicating that a more important shock or change in investor sentiment (particularly for the United Kingdom) is required to push investors to switch regimes and follow their emotions. This finding might also suggest that British and Canadian investors are less sensitive to their emotions and cognitive bias than are US investors.

Table 2. 9 Estimation results of the ESTR model

US			**Japan**			**UK**			**Canada**		
$\bar{R}^2$		0.93	$\bar{R}^2$		0.85	$\bar{R}^2$		0.74	$\bar{R}^2$		0.84
Regression-F		6432.90 (0.0000)	Regression-F		131.20 (0.0000)	Regression-F		1378.52 (0.0000)	Regression-F		2222.77 (0.0000)
D-W Statistic		2.00	D-W Statistic		1.99	D-W Statistic		1.95	D-W Statistic		1.95
γ		3.313	γ		4.733	γ		0.265	γ		0.877
c		-0.003	c		-0.011	c		0.341	c		0.018
α_0		-0.000 (0.804)	α_0		0.020** (0.049)	α_0		-0.032* (0.089)	α_0		0.030*** (0.001)
Y_{t-3}	α_1	0.182*** (0.000)	Y_{t-1}	α_1	0.271*** (0.000)	Y_{t-1}	α_1	-0.101** (0.012)	Y_{t-2}	α_1	0.058** (0.027)
Y_{t-4}	α_2	0.075*** (0.005)	Y_{t-2}	α_2	0.038* (0.081)	Y_{t-2}	α_2	-0.036 (0.135)			
DY_t	α_3	-0.170*** (0.000)	DY_t	α_3	-0.403*** (0.000)	DY_t	α_3	-0.117*** (0.000)	DY_t	α_2	-0.238*** (0.000)
DY_{t-3}	α_4	0.019** (0.030)	DY_{t-1}	α_4	0.550*** (0.000)	DY_{t-1}	α_4	0.105*** (0.000)	DY_{t-1}	α_3	0.243*** (0.000)
DY_{t-4}	α_5	0.018** (0.021)	DY_{t-2}	α_5	-0.150*** (0.000)	DY_{t-2}	α_5	0.016* (0.059)	DY_{t-3}	α_4	-0.009* (0.088)
PER_t	α_6	0.025*** (0.000)	PER_t	α_6	0.009*** (0.000)	PER_t	α_6	0.033*** (0.000)	PER_t	α_5	0.008*** (0.000)
PER_{t-3}	α_7	-0.001** (0.010)									
PER_{t-4}	α_8	-0.008*** (0.000)	PER_{t-1}	α_7	-0.002*** (0.000)	PER_{t-1}	α_7	-0.032*** (0.000)	PER_{t-1}	α_6	-0.008*** (0.000)
PI_t	α_9	0.049 (0.605)	PI_t	α_8	0.047 (0.510)	PI_t	α_8	0.142 (0.319)	PI_t	α_7	-0.035 (0.731)
PI_{t-2}	α_{10}	0.360*** (0.000)							PI_{t-2}	α_8	0.187* (0.060)
									PI_{t-3}	α_9	0.229** (0.021)
PI_{t-3}	α_{11}	0.320*** (0.000)							PI_{t-4}	α_{10}	0.310*** (0.001)
ST_t	α_{12}	-0.000 (0.585)	ST_t	α_9	-0.003 (0.312)	ST_t	α_9	-0.008** (0.046)	ST_t	α_{11}	-0.001* (0.063)
						ST_{t-3}	α_{10}	-0.010** (0.015)			
θ_0		0.004 (0.109)	θ_0		-0.010 (0.118)	θ_0		0.012* (0.066)	θ_0		-0.008** (0.019)
$SENT_t$	θ_1	0.010*** (0.000)	$SENT_t$	θ_1	-0.023 (0.107)	$SENT_t$	θ_1	0.024*** (0.009)	$SENT_t$	θ_1	-0.007 (0.290)
			$SENT_{t-1}$	θ_2	0.028* (0.061)				$SENT_{t-1}$	θ_2	0.012* (0.073)

Note: The number in brackets denotes the p-values. DW refers to the statistics of the Durbin–Watson test. (*), (**), and (***) denote significance at the statistical levels of 10%, 5%, and 1%, respectively.

This result and the observation of the weak estimated transition speed in the case of the United Kingdom and Canada suggest that British and Canadian investors would stay longer in the central regime than US and Japanese investors. This finding indicates that while the hypotheses of HAMs and nonlinear sentiment–stock return relationship are not rejected for these four countries, the activation of the investor sentiment effect and switching from the central regime (regime dominated by arbitrageurs) to the extreme regime (regime of irrationality and emotional bias dominated by noise traders) may occur more rapidly for the United States and Japan than for Canada and the United Kingdom.

Further, to date the switching regime and identify the regime (arbitrageurs, noise traders), we examined the value of the threshold parameters and searched for the corresponding date. Our analysis shows that for the US stock market, regime switching occurred in December 2002. This finding suggests that the investor sentiment effect arose in the aftermath of the collapse of the Internet bubble, leading to the bankruptcies of WorldCom and Global Crossing in 2002. Indeed, the NASDAQ index lost 78% of its value in the last quarter of 2002 before rising between October and January 2013.

Regarding the Japanese index, the estimated threshold value corresponds to September 2012, suggesting that the regime shift might be explained by political effects in Japan[28]. Indeed, before the Japanese political change in 2012[29], deflation characterized the Japanese economy for almost two decades. However, since the announcement of the stimulus package under Abenomics, consumer confidence has been restored. This positive shock has contributed, in addition to other new measures, to recovering economic agents' confidence. In fact, the Abe government launched an economic program focusing on boosting economic activity through a change in the monetary policy regime as well as the introduction of new structural reforms and a fiscal policy that yielded positive inflation in Japan by the end of 2013.

For the United Kingdom, the switching regime was carried out in August 2009, suggesting that the activation of the sentiment effect occurred in the aftermath of the global financial crisis, no surprise given the strong economic and financial linkages between the

[28] Political events might affect investor sentiment and therefore stock prices (Drakos, 2010; Li and Born, 2006; Niederhoffer, 1971; Schneider and Troeger, 2006).

[29] Shinzo Abe became the president of the Democratic Liberal Party on 26 September, 2012.

United Kingdom and the United States (source of the subprime crisis). In particular, this coincides with the implementation of the quantitative easing policy in the first quarter of 2009 and the decision by the Bank of England to increase its asset purchase program by £50 billion to £175 billion on 6 August, 2009. These measures influenced the dynamics of inflation and economic activity through several channels, notably investor behaviour.

Concerning Canada, the switching date was March 2006. Indeed, the Canadian stock market witnessed strong growth in 2006 and the S&P/TSX Canadian rose by 9% in the first quarter of 2006.This performance might be attributed to various sectors including the energy sector.

Second, looking at the direct effect of investor sentiment on the dynamics of stock returns, we do not reject the nonlinear effect hypothesis. Indeed, investor sentiment has a positive and significant effect (at 1% for the United Kingdom and the United States and 10% for Japan and Canada) either with one lag (for Canada and Japan, suggesting further evidence of persistence and long memory) or only a contemporaneous effect for the United States and the United Kingdom. The size of the effect is higher for the United Kingdom, reflecting the importance of UK market capitalization and behavioural heterogeneity among investors. This finding is particularly interesting as it validates the heterogeneous behaviour sentiment effect on the dynamics of stock returns and is in line with the conclusions of He et al. (2009) and Chiarella et al. (2011). In other words, the market efficiency hypothesis and rational expectation paradigm cannot explain stock price variations; rather, the heterogeneous expectations framework better fits the data to reproduce the stylized facts of the financial market.

Third, the analysis of the effects of the control variables in the first regime shows that most are significant and have the expected sign for all countries. In particular, industrial production seems to stimulate the stock market cycle, while the negative effect of the interest rate points to the competitive or arbitrage character between investing in the monetary market (less risky financial assets) and in stock markets (risky assets). This finding is line with those of Chen et al. (1986) and Jawadi and Prat (2012)[30]. The price earnings ratio also has the

[30] This finding suggests that when the difference between the long and short interest rates increases, stock market returns decrease. Indeed, when interest rates increase, bonds will be more profitable, which would lead investors to buy debt securities.

expected positive effect, while the dividend yield effect is negative. Further, the autoregressive terms have significant effects, suggesting further evidence of persistence and the memory effect in stock return dynamics.

Table 2. 10 Descriptive statistics of the stock returns for each regime

series	Mean		Variance		Skewness		Kurtosis		J-B	
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
US	0.006	0.005	0.002	0.003	-0.532	-1.616	2.744	10.817	127.876	727.571
Japan	-0.003	0.025	0.003	0.004	-0.269	-0.599	1.737	-0.218	41.929	1.112
UK	0.006	0.008	0.005	0.004	0.394	0.427	11.314	3.921	2277.922	37.593
Canada	0.006	0.001	0.001	0.002	-0.693	-1.004	4.393	5.462	275.941	135.497

Overall, these results suggest that the first regime for all markets is significantly governed by fundamentals and that arbitrageurs might find these variables to be informative when forecasting stock market dynamics. Indeed, financial ratios might significantly affect stock returns, particularly in the central regime, while the dynamics of stock returns might be predicted by using the information provided by the above financial variables, which is line with Campbell and Shiller (1989, 1998) and Goyal and Welch (2008).

Our main contribution, however, is showing that this mechanism and the dependency of stock returns on fundamentals is activated only in the central regime (i.e. the first regime); by contrast, when the threshold variable (investor sentiment) exceeds the threshold estimated by our model endogenously, the market evolves by following investor sentiment and emotions. In other words, the complexity of stock market dynamics might be relativized when considering an on/off formation price model that incorporates two regimes. The first regime governed by fundamentals and dominated by arbitrageurs yields a price close to its fundamentals, whereas the second regime for which irrational investors and noise traders dominate, emotions and opinions imply a misleading price. Interestingly, our model enables this sentiment effect to enter nonlinearity and has the advantage of detecting endogenously the threshold distinguishing the two regimes. Further, it enables the transition between these

regimes to occur smoothly so as to capture the large behavioural heterogeneity among investors in stock markets. To better characterize these regimes, we computed the main descriptive statistics of the stock returns for each regime (see Table 2.10), showing that the mean, variance, skewness, and kurtosis significantly vary for each regime for all indexes.

Finally, to better illustrate these different regimes and show the interaction between investor sentiment and stock returns, Figures 2.1–2.4 illustrate the estimated transition function (on the left), the temporal dynamics of the transition function (on the right), and the dynamics of stock returns, sentiment change, and transition function (at the bottom). These figures highlight three interesting results. First, the estimated transition function, for all indexes, always reaches unity and persists in the outer regime, reflecting the importance of the noise traders' regime and the effect of investor sentiment. As mentioned above, the duration of the central regime is highest for the United Kingdom. Second, the continuous interaction between stock returns and investor sentiment changes suggest that the sentiment effect has always been active. Further, the excess volatility inherent in sentiment change as well as that in the temporal dynamics of the transition function highlights the importance of the actions of noise traders, suggesting an active arbitrage between the first regime (driven by fundamentals) and the second regime (governed by investor sentiment). Interestingly, the demonstration of investor sentiment and actions of noise traders are in line with the conclusions of the eminent works of Carl Chiarella and Robert Shiller as well as the literature on HAMs.

Figure 2. 1 Estimated dynamics of the transition for the United States

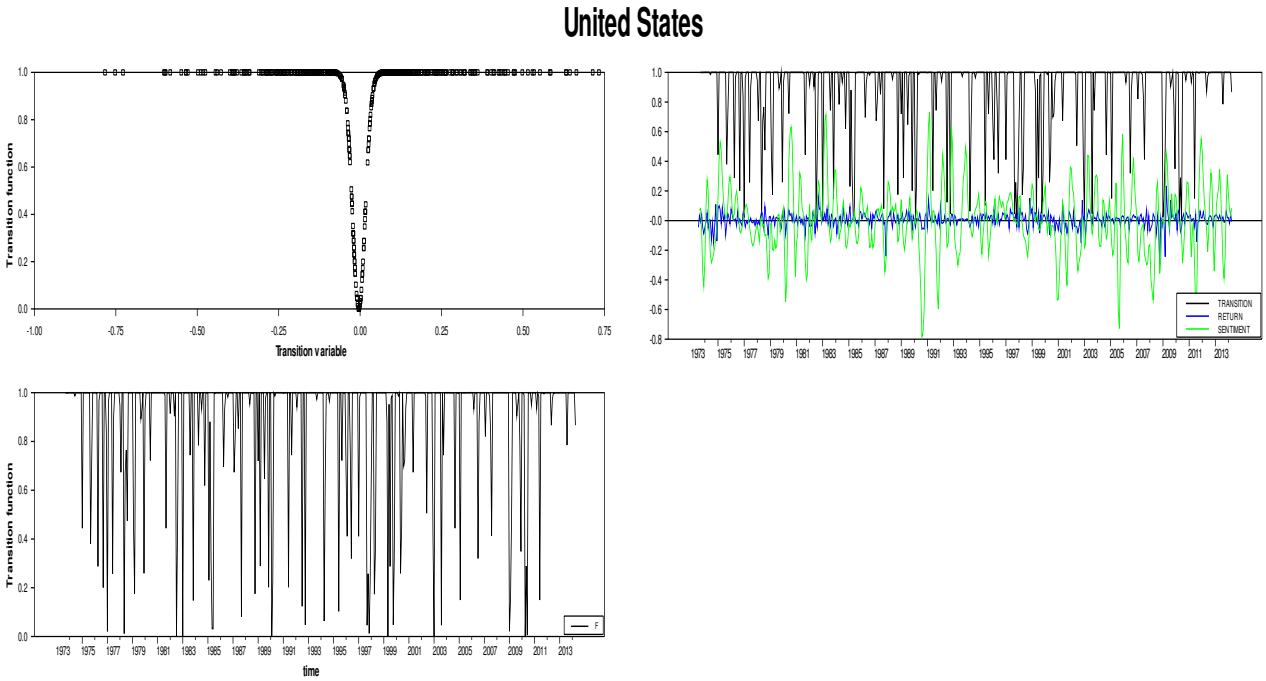


Figure 2. 2 Estimated dynamics of the transition for Japan

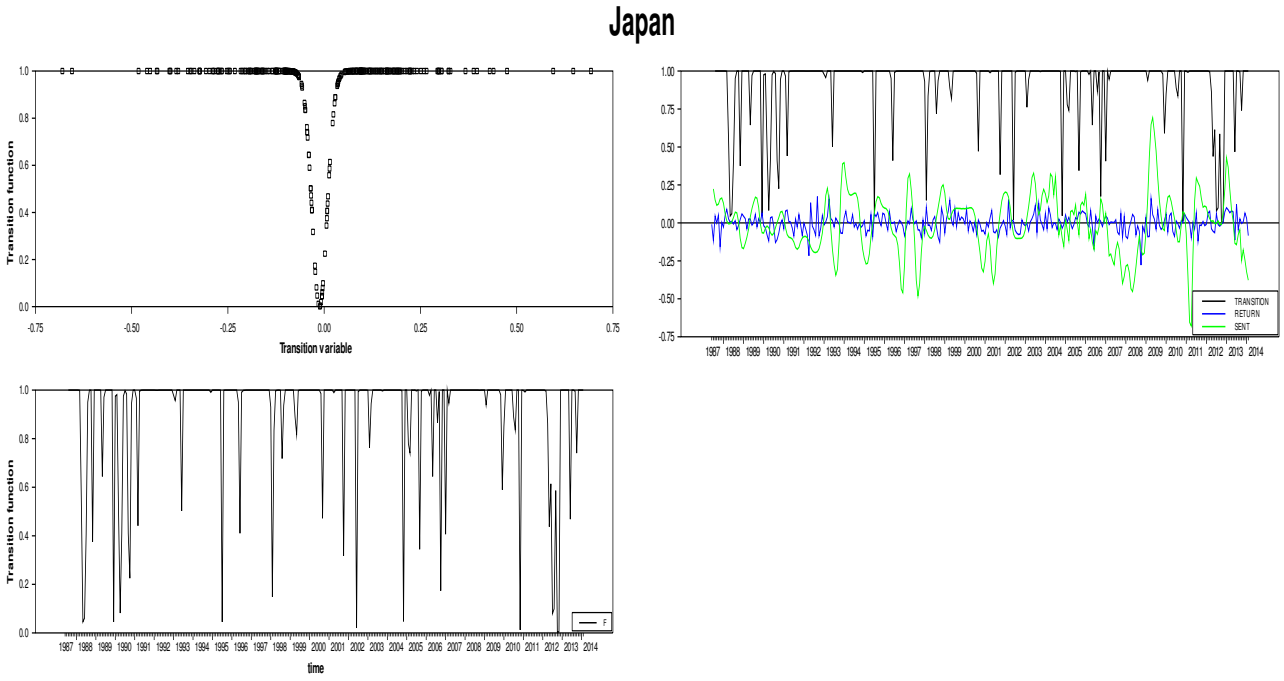


Figure 2. 3 Estimated dynamics of the transition for the United Kingdom

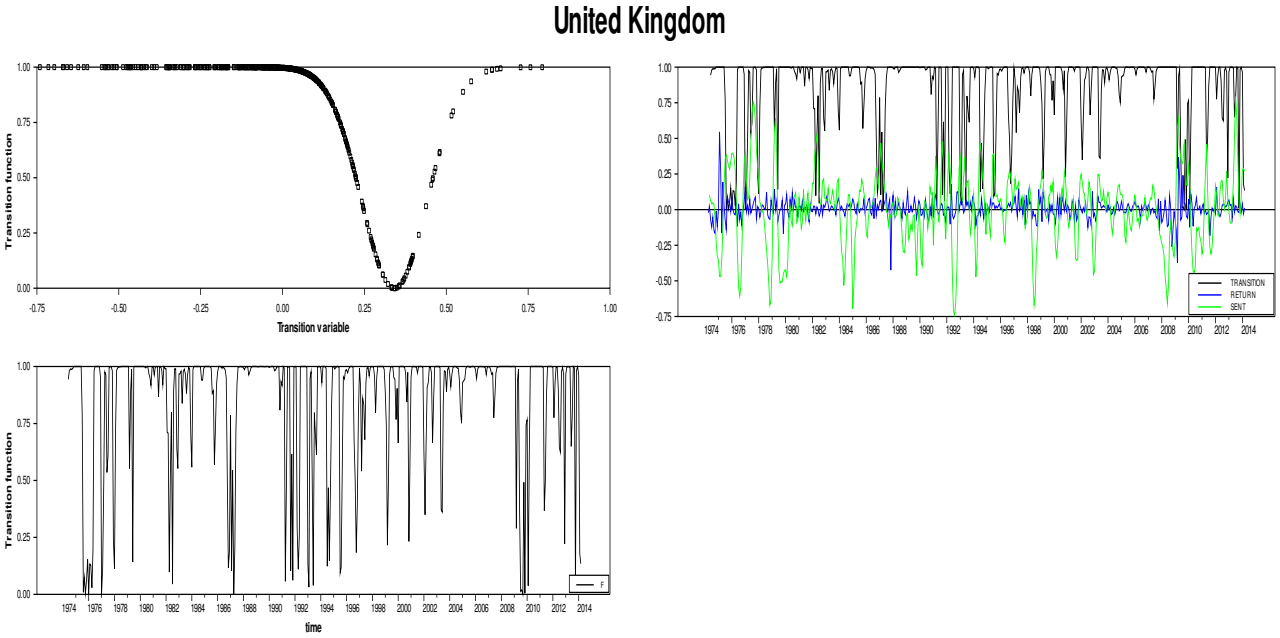
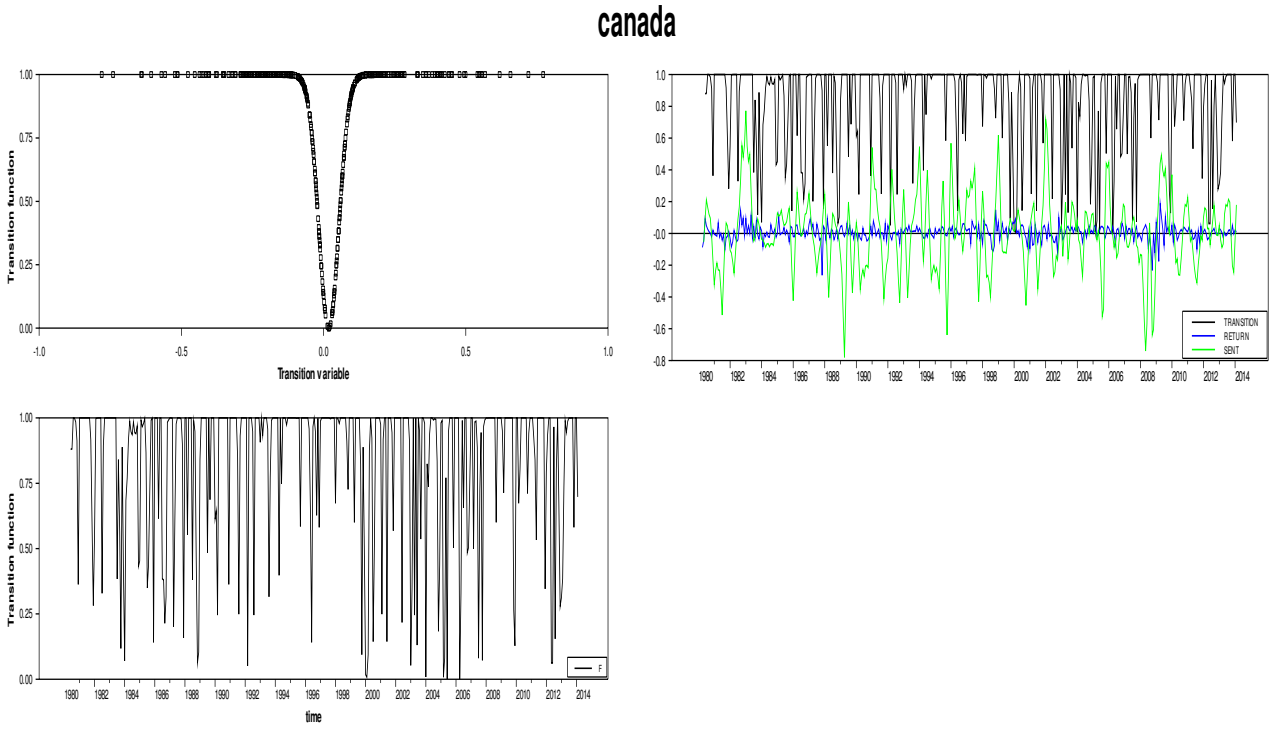


Figure 2. 4 Estimated dynamics of the transition for Canada



5.3. Robustness Tests

The last step in the STR modelling process consists of checking the robustness of our specification and estimation. To this end, we applied the three misspecification tests introduced by Eitrheim and Teräsvirta (1996) to the estimated residuals of the nonlinear tests. First, we checked whether the residuals of the STR models are autocorrelated (Table 2.11). We found that the residuals are non-correlated, suggesting that the estimators are efficient for Canada, while the errors are correlated for the United Kingdom. As for the United States and Japan, the autocorrelation is rejected for a small order of dependency.

Table 2. 11 Residual autocorrelation test: Chi-Squared values

Order of serial dependence	US	Japan	UK	Canada
$q = 2$	2.940 (0.229)	1.688 (0.429)	8.526 (0.014)	0.626 (0.731)
$q = 4$	7.485 (0.112)	2.324 (0.676)	16.091 (0.002)	2.431 (0.656)
$q = 8$	22.710 (0.003)	17.927 (0.021)	24.325 (0.002)	8.671 (0.370)
$q = 12$	22.919 (0.028)	19.620 (0.074)	28.002 (0.005)	13.769 (0.315)

Note: The number in brackets denotes the p-values of the LM test.

Second, we applied a test of no remaining nonlinearity that checks whether the nonlinearity in the data has been absorbed. In other words, this test checks whether our specification and the number of regimes are sufficient to capture the nonlinear effect of investor sentiment on stock returns. Table 2.12 shows the correctness of our specification, as the hypothesis of no remaining nonlinearity is not rejected for the United States, the United Kingdom, or Canada. This finding suggests that the two-regime STR specification fits the data appropriately, except for Japan.

Table 2. 12 Test of no remaining nonlinearity: Chi-Squared values

US	Japan	UK	Canada
2.948 (0.815)	21.408 (0.001)	9.492 (0.147)	1.326 (0.970)

Note: The number in brackets denotes the p-values of the LM test.

Third, we applied a test of parameter constancy to check whether the STR is constant as we assumed when we estimated the two-regime STR model. As shown in Table 2.13, the

p-values of the three parameter constancy tests show that the parameters do not vary over time. These results also confirm the appropriateness of our two-regime STR model with constant parameters.

Table 2. 13 Test of parameter constancy: Chi-Squared values

US	Japan	UK	Canada
LM_1 = 0.5012 (0.9186)	LM_1 = 1.726 (0.631)	LM_1 = 7.714 (0.052)	LM_1 = 0.115 (0.989)
LM_2 = 7.1857 (0.3040)	LM_2 = 6.779 (0.341)	LM_2 = 10.242 (0.114)	LM_2 = 0.691 (0.994)
LM_3 = 7.2064 (0.6156)	LM_3 = 6.914 (0.646)	LM_3 = 14.524 (0.104)	LM_3 = 3.263 (0.952)

Note: The number in brackets denotes the p-values of the LM test.

5.4. Forecasting with the Heterogeneous STR Model

With regard to the linear benchmark model, the two-regime STR model stipulates the presence of two distinct regimes and supposes that the sentiment effect enters nonlinearity. Examining the forecasting power of the STR model aims to check whether with these hypotheses the nonlinear model supplants the benchmark model when forecasting the future dynamics of stock returns. To this end, we re-estimated the nonlinear model and computed in- and out-of-sample forecasts. In particular, we computed two loss functions for both the linear and the nonlinear models: the mean absolute error (MAE) and mean squared error (MSE). Next, we compare these statistics for both models.

Formally, let y_t be an observed stock return series and $\hat{y}_{t+h/t}$ the (linear or nonlinear) prediction of y_{t+h} at $(t+h)$.This forecasting corresponds to:

$$\hat{y}_{t+h/t} = E(y_{t+h/t}) \tag{2-6}$$

Where It is the available information at time$t$, $E(.)$ is the operator of the conditional expectation, and h is the forecasting horizon.

The forecasting error corresponds to

$$e_{t+h/t} = y_{t+h} - \hat{y}_{t+h/t} \tag{2-7}$$

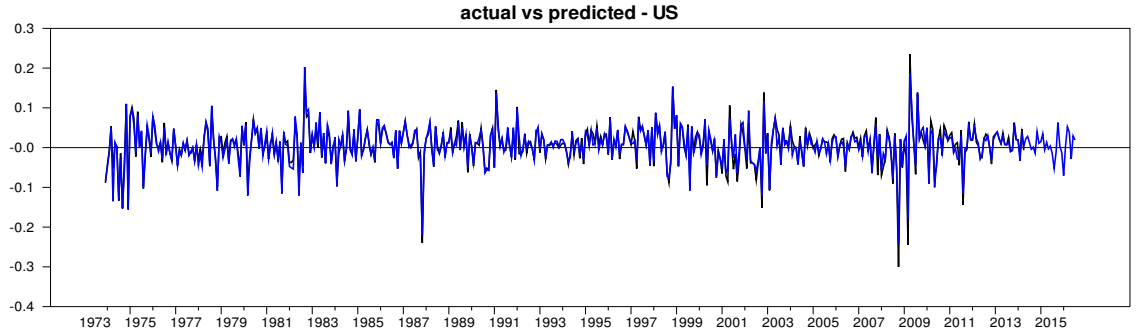
By using this forecasting error, we can compute the MSE and MAE as[31]:

$$MSE = \frac{1}{m}\sum_{j=1}^{m}(\hat{y}_{T+h+j/T+j} - y_{T+h+i})^2 \quad (2\text{-}8)$$

$$MAE = \frac{1}{m}\sum_{j=1}^{m}\left|\hat{y}_{T+h+j/T+j} - y_{T+h+i}\right| \quad (2\text{-}9)$$

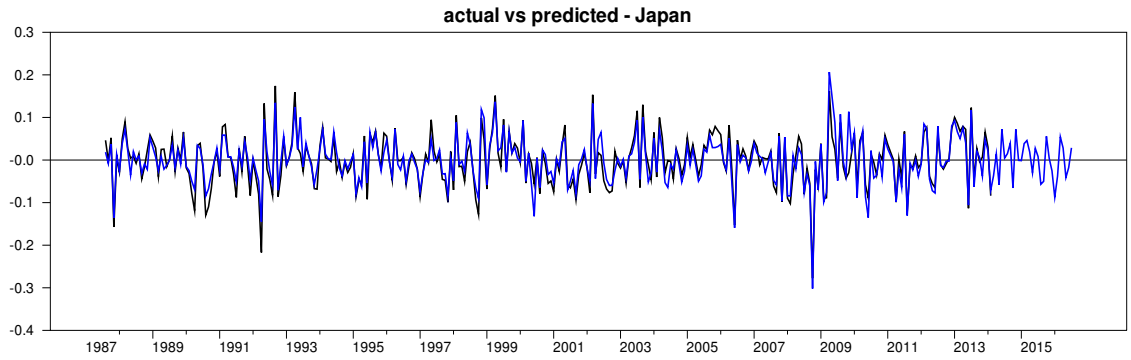
A given model has superior forecasting performance if it minimizes the forecasting measurement error and therefore the MSE and/or MAE. We first calculated the linear and nonlinear in-sample forecasting. To do this, both models were re-estimated for the United States, the United Kingdom, Japan, and Canada for 1973:07 to 2014:04, 1974:04 to 2014:03, 1987:06 to 2014:02, and 1980:04 to 2014:03, respectively. Figures 2.5–2.8 show the actual and predicted returns $(\hat{y}_{t+h/t})$, highlighting that the two-regime STR always reproduces the dynamics of stock returns appropriately.

Figure 2. 5 Actual and STR-predicted returns for the United States



Note: The black line denotes the actual return, while the blue one refers to the predicted return according to the STR model.

Figure 2. 6 Actual and STR-predicted returns for Japan



[31] m denotes the number of predictions.

Figure 2. 7 Actual and STR-predicted returns for the United Kingdom

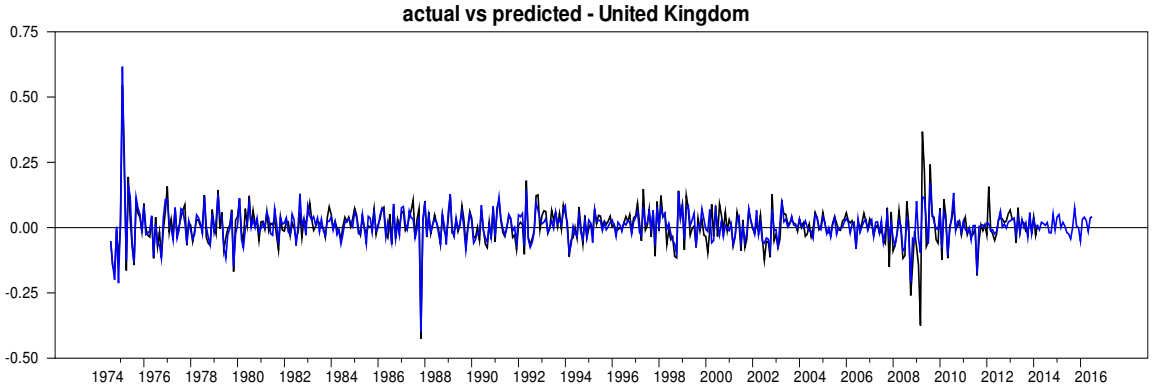
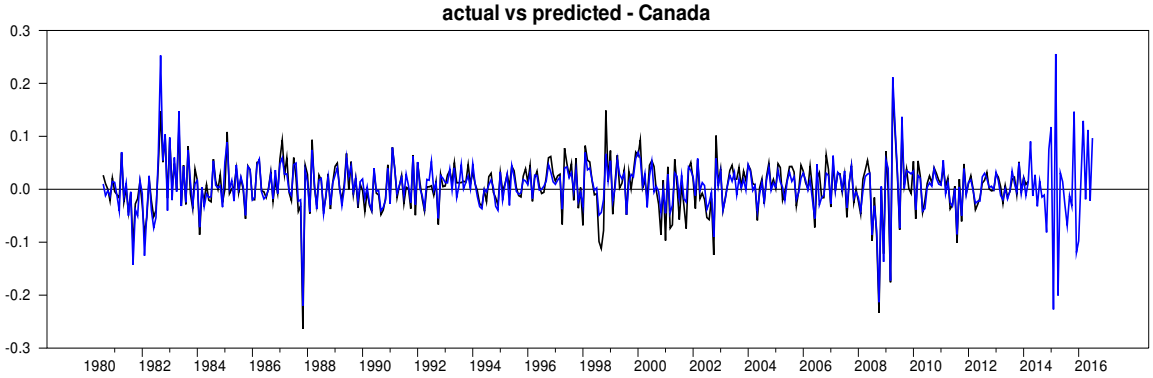


Figure 2. 8 Actual and STR-predicted returns for Canada



Second, the out-of-sample forecasting are computed for the United Kingdom and Canada over the period 2014:03 to 2016:07, for Japan over the period 2014:02 to 2016:07, and for the United States over the period 2014:04 to 2016:07. We calculated the out-of-sample forecasting in the short and medium terms, retaining the three forecasting horizons: $h = 1, 2, 3$ months.

To compare the linear and nonlinear forecasting results, we again computed the MSE and MAE, which provided similar findings. Thus, we focused on the results of the MAE. Finally, we tested the significance of these forecasting statistics by using the Harvey et al. (1997) test. The null hypothesis of this test is that the linear and STR models present the same predictive performance, while the nonlinear model supplants the linear model according to the alternative hypothesis.

Harvey et al.(1997) showed that, in small samples, the classic statistic of Dibeold and Mariano (1995) tends to have an empirical size much greater than the nominal size. The authors used an unbiased estimator of variance $var^*_{est}\ (\bar{d})$ and proposed a modified DM statistic. First, they considered the exact long-run variance of the empirical mean of the loss differentials VAR (d) and not the asymptotic variance of this empirical mean. Then, they estimated this variance by using the estimators of the auto-covariances $\hat{\gamma}_d$ obtained from a small sample correction. This correction can reduce the erroneous rejection under the null hypothesis of the equality of the two models.

Harvey et al. (1997) proposed the following modified DM statistic:

$$DM^* = \frac{\bar{d}}{\sqrt{var^*_{est}\ (\bar{d})}} \backsimeq \left[\frac{m+1-2h}{m} + \frac{h(h-1)}{m^2}\right]^{\frac{1}{2}} DM \tag{2-10}$$

where:

$$var^*_{est}(\bar{d}) = \frac{1}{m}\left[\hat{\gamma}^*_d(0) + \frac{2}{m}\sum_{j=1}^{h-1}(m-j)\hat{\gamma}^*_d(j)\right] \tag{2-11}$$

$$\hat{\gamma}^*_d(j) = (m-j)^{-1}\sum_{i=j+1}^{m}(d_i - \bar{d})(d_{i-j} - \bar{d}) \tag{2-12}$$

Table 2.14 shows that the STR model is relatively more appropriate than the linear model for in-sample forecasting returns for Canada and the United States. However, concerning out-of-sample forecasting, the STR model is more suitable than the linear model for Canada (for all forecasting horizons) and the United Kingdom (for h = 1). Finally, for Japan and the United States, the STR model does not supplant the linear model.

Table 2. 14 Results of the Harvey et al. (1997) test (p-values)

Country	**In-sample forecasting**	**Out-of-sample forecasting**		
		$\boldsymbol{h=1}$	$\boldsymbol{h=2}$	$\boldsymbol{h=3}$
Canada	0.045**	0.000***	0.000***	0.000***
US	0.085*	0.994	0.994	0.995
Japan	0.294	0.448	0.887	0.943
UK	0.629	0.003***	0.127	0.239

Note: (*), (**), and (***) denote the rejection of the equality of the null hypothesis at 10%, 5%, and 1%, respectively.

6. Conclusion

This study contributes to the growing literature on behavioural finance and HAMs related to modelling and forecasting stock return dynamics. In particular, in line with Chiarella and He (2000b), it proposes a new way in which to model the relationship between the stock market dynamics in G7 countries and investor sentiment taking account of the effect of heterogeneous investor sentiment on stock returns. To this end, we developed a new nonlinear behavioural stock price specification that allows for the presence of two types of investors (arbitrageurs and noise traders) and two regimes. We refer to the heterogeneous STR model of Granger and Teräsvirta (1993) to construct our modelling procedure, while enabling the stock price to switch smoothly between regimes after accounting for an endogenous change in investor sentiment.

Our results show two interesting findings. First, we do not reject the hypothesis of the nonlinear investor sentiment effect on stock returns that is activated for each regime. Second, we show that the two distinct regimes characterize the stock price dynamics. Indeed, the price is closely governed by fundamentals in the first regime and is mean-reverting, while emotions and sentiment drive the price in the second one yielding price trends and a non-mean-reverting process. Further, our model captures the main stylized facts observed in the market and shows good in- and out-of-sample forecasting power.

This study can be extended by introducing other economic factors or considering the ARCH effect using an ESTR-GARCH specification. Further, future research might consider testing the linearity in a nonlinear-time panel data framework to take account of the heterogeneity observed between countries (e.g. estimating a panel STR model).

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Chapter 3: Threshold effect in the relationship between investor sentiment and stock market returns: A PSTR specification

ABSTRACT

This article verifies whether the hypothesis of heterogeneous agent modelling and the behavioural heterogeneity framework can reproduce recent stylized facts regarding stock markets (e.g. the 1987 crash, internet bubble, and subprime crisis). To this end, we investigate the relationship between investor sentiment and stock market returns for the G7 countries from June 1987 to February 2014. We propose an empirical non-linear panel data specification based on the panel switching transition model to capture the investor sentiment-stock return relationship, while enabling investor sentiment to act asymmetrically, non-linearly, and time varyingly according to the market state and investor attitude towards risk. Our findings are twofold. First, we show that the hypotheses of efficiency, rationality, and representative agent do not hold in reproducing stock market dynamics. Second, investor sentiment affects stock returns significantly and nonlinearly, but its effects vary with the market conditions. Indeed, the market appears predominated by fundamental investors in the first regime. In the second regime, investor sentiment effect is positively activated, increasing stock returns; however, when their overconfidence sentiment exceeds some threshold, this effect becomes inverse in the third regime for a high threshold level of market confidence and investor over-optimism.

Keywords: Dynamic non-linear stock returns; investor sentiment; threshold effect; panel smooth threshold regression.

JEL: C20; G10

1. Introduction

For several decades, financial theory has relied on the efficient market hypothesis (EMH), the paradigm of the representative agent, and the rational expectation hypothesis. These theoretical backgrounds postulate that agents are rational and that all available information is considered in the decision-making process. Accordingly, even with new information arriving in the market, rational investors are expected to react instantaneously, and asset prices tend to converge to their fundamental values, which is consistent with Fama's (1965) and Samuelson's (1965) findings. Additionally, the numerous empirical studies conducted in the 1970s confirmed these assumptions, showing that irrational investors might be automatically eliminated from the market through arbitrage forces.

However, during the past two decades, stock markets have experienced different episodes of crashes, crises, bubbles (e.g. the crash of 1987, the internet bubble of 2000, the subprime crisis of 2007, and the recent global financial crisis of 2008–2009), marked by several anomalies and abnormal stylized facts. These facts have both challenged and placed doubt on market efficiency and investor's rationality. Indeed, the hypotheses of homogeneity and rationality of agents have failed to reproduce these stylized facts and explain anomalies[32].

Accordingly, an alternative framework has recently emerged, yielding a growing body of research on behavioural finance. Unlike the classical framework of financial theory, behavioural finance shows more flexibility in the hypotheses of rationality and representative agents. It implies that investors deviate from rationality depending on their psychology during decision-making, which means they might have heterogeneous expectations about future events and market dynamics. Interestingly, studies regarding heterogeneous agent models (HAM) were conducted to justify market inefficiency and reproduce the dynamics and states of stock prices associated with investor and market opinion (optimism, pessimism, confidence, nervousness, irrationality, etc.). Specifically, the investor sentiment paradigm may have a central role in explaining asset price misalignments and stock market dynamics (Black 1986; DeLong et al. 1990; Baker and Wurgler 2007; Zouaoui, Nouyrigat, and Beer

[32] Shleifer (2000) highlights that financial market anomalies might prevent rational investors from benefitting from price misalignments, as transaction costs distort arbitrage opportunities.

2011; Lux 2011; Chung, Hung, and Yeh 2012; Chen, Chen, and Lee 2013; Li, Guo, and Park 2017). For example, Shleifer (2000) highlights that both arbitrage's limited capacity to reduce stock prices to their fundamental values and market sentiment constitute the main pillars of behavioural finance in terms of the reasons for price misalignments. Brown and Cliff (2005) and Baker and Wurgler (2006) confirm this suggestion, in that uniform demand shocks and arbitrage's limited capacity are the main reasons for stock market anomalies. Particularly, Brown and Cliff (2005) show that uniform demand shocks drive investor sentiment. Investor sentiment is a broad concept, as it includes any misperception that can cause mispricing. It comprises investor beliefs, moods, and emotions. Positive (negative) sentiment leads investors to be more (less) confident about their abilities to evaluate situations and more willing (fearing) to take risks. Hence, when investors are optimistic (pessimist), they tend to overvalue (undervalue) price stocks (Kuhnen and Knutson 2005).

Literature based on behavioural finance theory relaxes the classical assumption of representative agents in favour of models that consider heterogeneous agents. For instance, DeLong et al. (1990) state that economy is characterized by two types of investors: professional investors or arbitrageurs, who anticipate asset prices based on the available information, and noise traders, whose decisions lead to periods of over- or undervaluation of financial assets. The latter investors react irrationally and noisily based on their emotions and sentiments, since they do not have access to information. Consequently, asset prices can significantly diverge from fundamental values and 'sentiment risk', also called 'noise traders' risk', is considered a price determinant in financial markets.

Interestingly, evidence of heterogeneous investors in financial markets and the effect of sentiment on financial price determination have increased, as illustrated by several recent empirical studies (Barber et al. 2009). Baker and Wurgler (2007) defined investor sentiment as the beliefs about future cash flows and investment risks that are not justified by the available information. They found that sentiment has the greatest impact on stocks that are difficult to arbitrage and showed the negative impact of the contemporaneous optimism on stock returns. Schmeling (2009) also studied the causality between investor sentiment and stock returns for 18 industrialized countries and found that an increase in investor sentiment yields a decrease in future market returns and vice versa. Bathia and Bredin (2013) explored the investor sentiment-stock returns relationship in G7 markets over the period January 1995 to December 2007, using several investor sentiment proxies. Their results confirmed the negative relationship between investor sentiment and subsequent future returns. Dergiades

(2012) examined the causality between American stock market returns and investor sentiment by employing non-linear causality test. They used monthly time series data over the period 1965–2007 and showed that investor sentiment might significantly predict stock returns. Focusing on the German stock market, Lux (2011) found that investor sentiment might forecast returns; however, the impact of sentiment decreases during large market movement periods. Using a panel approach for 16 international stock markets over the period April 1995 to June 2009, Zouaoui, Nouyrigat, and Beer (2011) showed that investor sentiment measured by the confidence index has a significant impact on the occurrence probability of financial crises since crisis periods are always preceded by rising investor euphoria. Therefore, the more optimistic the investors, the more overvalued the prices, and greater the occurrence of stock market crises. Akhtar et al. (2011) explored the link between the announcement of investor sentiment and the Australian stock market's reaction. They studied whether positive and negative sentiment announcements have the same impact on stock prices. Using monthly data from June 1992 to December 2009, the authors highlighted that the Australian stock market reacts negatively to the announcement of negative consumer sentiment. In contrast, a positive consumer announcement does not lead to a market reaction, suggesting further asymmetry in the sentiment-stock return relationship. Considering the various states of the US economy between January 1966 and December 2007, Chung, Hung, and Yeh (2012) also studied the asymmetry in the predictive power of investor sentiment in the cross-section of stock returns during economic expansions and recessions. They showed, using a Markov multivariate model, that investor sentiment has a significant forecasting power only during periods of economic expansion. Li, Guo, and Park (2017) also investigated the causality between investor sentiment and stock returns in the American market by using two direct proxies. Contrary to Chung, Hung, and Yeh (2012), who only considered two states (expansion and recession), the authors used various regimes reflecting different economic states using a quantile causality approach. Their results showed that investor sentiment possesses a predictive power for the stock market only in a normal or a recession state, rather than in an expansion one. The authors attribute this finding to the asymmetry in agent reactions regarding losses and gains.

Overall, the opinions regarding the effect of sentiment on stock market dynamics is not unanimous and varies based on the data sample and methodology used.

Further, although this field of literature appears to favour non-linear models when considering the sentiment effect[33], most previous studies have analysed the sentiment effect for individual cases based on time series models. However, as the emotion or heuristic affect originates from psychological factors, such as social factors, habits, and culture (Dowling and Lucey 2005; Shefrin 2010), the risk sentiment might vary across countries. Therefore, analysing this issue for individual cases might mask some individual behaviour. Consequently, it is more appropriate to investigate it on a broader level to consider the differences across countries and markets as well as under the non-linearity hypothesis.

In practice, studies analysing the relationship between stock prices and investor sentiment for a large panel market in a non-linear framework are rather scarce. In particular, in related literature, the investigation of the sentiment effect has used either linear panel data modelling or non-linear individual time series modelling, but never to our knowledge in a non-linear context with panel data. For example, asymmetric modelling for panel data uses different empirical frameworks. Ni, Wang, and Xue (2015) employed a panel quantile regression model for the Chinese stock market, from January 2005 to September 2013. They found a significant asymmetric impact of investor sentiment on stock returns lasting from 1 to 24 months. Chen, Chen, and Lee (2013) analysed the asymmetric relationship between industry returns and investor sentiment based on Hansen's (1999) threshold model and found abrupt regime changes for 11 Asian countries from 1996 to 2010. Using volume turnover as a proxy for investor sentiment, industry returns are dependent on investor sentiment changes, with a positive relationship between global sentiment and industry returns.

Our study is related to this literature regarding the effect of investor sentiment[34] on stock price dynamics and aims to fill this gap through the simultaneous consideration of non-linearity and panel data frameworks. In particular, we add to existing literature regarding the relationship between asset returns and investor sentiment by considering the heterogeneity of both the investor and sentiment across countries. Econometrically, we propose a new non-linear panel data specification based on the panel smooth transition regression model (PSTR) developed by González, Teräsvirta, and Van Dijk (2005). The choice of this approach has two

[33] Specifically, previous studies highlight that investor sentiment is the main source of non-linearity in asset price dynamics (Lee, Jiang, and Indro 2002; among others).

[34] It is broadly defined as 'investors' belief about future cash flows and risk not justified by the facts at hand' (Baker and Wurgler 2007, p. 129). Hereafter, we employ this definition to investigate the link between investor sentiment and stock market returns.

main advantages. First, the PSTR model helps to capture investors' behavioural heterogeneity in the data more extensively. Second, it allows for determining the time-varying threshold level[35] endogenously and smoothly[36] to distinguish between investors' regimes, contrary to the abrupt changes adopted by Chen, Chen, and Lee (2013), for example. To the best of our knowledge, this model has not been used earlier to analyse the impact of investor sentiment on stock returns, thereby providing an original approach to the field of behavioural finance[37].

Our study contributes to literature on asymmetric relationships between stock prices and the sentiment index as follows. First, we propose a new empirical threshold panel data specification, based on the PSTR model, to model the effect of heterogeneous investor sentiment. We also include several macroeconomic and financial variables (dividend yield, price earnings ratio, industrial production growth, term structure of interest rates, world stock market returns) as control variables into the model to improve the predictive power of our model. Second, the panel data option allows us to consider two dimensions – individual and temporal – thus considering both dynamic behaviour and further heterogeneity, resulting in more consistent estimators. Third, while previous studies predominantly use U.S. data, our analysis extends this literature by considering a more original sample: the G7 countries.

The results highlight two main findings. First, we do not accept the hypotheses of efficiency, rationality, and the representative agent model. Second, we show that investor sentiment has a significant and per regime effect on the dynamics of stock returns. We identify three distinct regimes. In the first regime, sentiment does not affect stock returns and markets seem to be predominated by rational investors and fundamentals. However, its effect

[35] In line with previous literature (e.g. Ivanov, Tkalec, and Vizek 2011; Chang and Su 2010; Chang and Lee 2012; Chen, Chiang, and So 2003; Easaw and Ghoshray 2008), showing the evidence of threshold effects in finance, our model is the most suitable to consider this feature.

[36] The motivation behind the smooth transition behaviour is justified in that we assume that investors change their market' sentiment progressively. This assumption allows the number of investors for each regime to be time varying.

[37] Obviously another option would be to conduct this research question with only a non-linear time-series modelling of the investor sentiment-stock return relationship, rather than focusing on panel non-linear data analysis, as in the current study. We carried out this time series analysis, tested linearity, and estimated a smooth regression model (for countries whose linearity is rejected). For brevity, we do not report these results in this paper but they are available upon request. Accordingly, even for time series analysis, it appears that the non-linear time-variation of sentiment's effect on stock returns is supported and that a switching regime hypothesis can improve the modelling of stock return dynamics. Our preference for the panel non-linear investigation is doubly justified by the fact that it provides more consistent and efficient estimators and that considering these double dimensions (time and countries) and non-linearity provides us with richer information and a more flexible framework to assess the heterogeneous effects of investor sentiment on stock returns. We thank the referee who suggests us to explain at this stage the methodology choice made in this paper.

becomes significant and alternates from positive to negative according to the regime and level of market confidence and optimism.

The remainder of this article is organized as follows: Section II presents the PSTR methodology. Section III presents the data and preliminary tests, while Section IV presents the empirical results. Finally, Section V concludes the study.

2. Econometric Methodology

The relationship between investor sentiment and stock market returns is analysed based on the PSTR model of González, Teräsvirta, and Van Dijk (2005). Unlike other econometric tools used in previous studies to investigate this relationship, this new approach has different advantages. First, it allows us to consider both investor heterogeneity and their asymmetric reactions. Second, it considers both time series and panel data dimensions, thereby providing more consistent estimators.

Third, PSTR modelling enables a smooth rather than an abrupt transition between extreme regimes, which is a more flexible and reliable framework. This smoothness assumption is particularly justified by the fact that investor sentiment depends on several psychological factors for which any further change cannot be brutal. Fourth, the PSTR framework offers the possibility of considering more than two regimes, which helps to capture the different market states induced by different investors' feelings and positions. Formally, a two-regime PSTR model corresponds to

$$y_{it} = \alpha_i + \beta_0' x_{it} + \beta_1' x_{it} g(q_{it}; \gamma, c) + \varepsilon_{it} \quad (3\text{-}1)$$

Where $t = 1 \ldots \ldots T$, $i = 1 \ldots \ldots \ldots N$, T and N representing the panel temporal and individual dimensions, respectively. y_{it} denotes the endogenous variable and x_{it} refers to a vector of exogenous variables, excluding delayed endogenous variables. α_i is the vector of individual fixed effects; ε_{it} denotes the error term; $g(q_{it}; \gamma, c)$ is a transition function associated with a transition variable q_{it}, threshold parameter c, and smoothing parameter γ. The transition function $g(q_{it}; \gamma, c)$ is continuous and bounded between 0 and 1 according to the value it takes.

González, Teräsvirta, and van Dijk (2005) consider two different interpretations of the PSTR model. First, PSTR can be seen as a linear and heterogeneous panel data model, where the coefficients vary between individuals and over time. Second, it can be considered as a

non-linear homogeneous panel, where the transition between two linear and homogeneous extreme regimes might occur gradually. As for the transition function, González, Teräsvirta, and Van Dijk (2005) suggest retraining a logistic transition function of order m:

$$g(q_{it};\gamma,c) = \left(1 + \exp\left(-\gamma \prod_{j=1}^{m}(q_{it} - c_j)\right)\right)^{-1} \quad (3\text{-}2)$$

Where $\gamma > 0, c_1 \leq \cdots \leq c_m$ and c is a m-dimensionnel vector of threshold parameters.

The order of the transition function impacts the transition between extreme regimes. González, Teräsvirta, and Van Dijk (2005) support that a first ($m = 1$) or second ($m = 2$) order of transition function is sufficient to ensure the necessary variations of slope coefficients suitable to most non-linear cases. Further, González, Teräsvirta, and Van Dijk (2005) develop a generalized PSTR model by proposing an addictive model for several transition functions:

$$y_{it} = \alpha_i + \beta_0' x_{it} + \sum_{j=1}^{r} \beta_j' x_{it} g_j\left(q_{it}^{j};\gamma_j,c_j\right) + \varepsilon_{it} \quad (3\text{-}3)$$

Where $g_j\left(q_{it}^{j};\gamma_j,c_j\right)$ is the j^{th} transition function with $j = 1,\ldots,r$.

To estimate the PSTR model, the authors adopt a three-steps strategy: i) model specification and linearity tests, ii) estimation, and iii) misspecification and validation tests. Hereafter, we recall these tests.

2.1. Linearity test

This test, known as the homogeneity test, tests linearity hypothesis against the alternative hypothesis of the PSTR model and corresponds, regarding equation (3-1), to:

$$\begin{cases} H_0 : \beta_1 = 0 \\ H_1 : \beta_1 \neq 0 \end{cases} \quad \text{or} \quad \begin{cases} H_0 : \gamma = 0 \\ H_1 : \gamma \neq 0 \end{cases}$$

However, as for the time-series STR models, this test suffers from a nuisance parameters, suggesting the presence of non-identified parameters under the null hypothesis. To solve the non-identification problem under H_0, González, Teräsvirta, and Van Dijk (2005) adopt the same approach as Luukkonen et al. (1988) proposed in the Smooth Autoregressive Transition Autoregressive, noted STAR, model context. Indeed, they suggest replacing the transition function $g(q_{it};\gamma,c)$ by its first-order Taylor approximation around $\gamma = 0$ that corresponds to:

$$y_{it} = \alpha_i + \beta_0^{'*} x_{it} + \beta_1^{'*} x_{it} q_{it} + \cdots \ldots \ldots + \beta_m^{'*} x_{it} q_{it}^{m} + \varepsilon_{it}^{*} \quad (3\text{-}4)$$

Where the parameter vectors $\beta_1^*, \ldots, \beta_m^*$ are multiples of γ, $\varepsilon_{it}^{*} = \varepsilon_{it} + R_m \beta_1^{'} x_{it}$, and R_m is the residual of Taylor approximation.

With this parameterization, the null hypothesis becomes:

$$H_0^{*}: \beta_1^{'*} = \cdots = \beta_m^{'*} = 0 \quad (3\text{-}5)$$

This auxiliary equation can be tested with the following standard Lagrange Multiplier test (LM):

$$LM = T * N \frac{(\mathrm{SCR}_0 - \mathrm{SCR}_1)}{\mathrm{SCR}_0} \quad (3\text{-}6)$$

Where SCR_0 is the sum of squared residuals of a linear panel model with individual effects and SCR_1 is the sum of squared residuals of auxiliary equation (3-4). Under the null hypothesis, the LM statistic has an asymptotic χ^2 distribution with $(m * k)$ degrees of freedom, where k is the number of explanatory variables.

González, Teräsvirta, and Van Dijk (2005) also propose another linearity test when the sample size is small. The statistic of this test corresponds to:

$$LM_F = \frac{(\mathrm{SCR}_0 - \mathrm{SCR}_1)/mk}{\mathrm{SCR}_0/(TN - N - mk)} \quad (3\text{-}7)$$

Under the null hypothesis, $LM_F \rightsquigarrow \mathrm{F}(mk, TN - N - mk)$.

2.2. Estimation of the PSTR parameters

The estimation strategy is similar to that used in a time series for a STAR or STR model, except for the elimination phase of individual fixed effects. The estimation procedure of the PSTR model with a single transition function (r = 1) comprises, first, removing individual fixed effects and, second, estimating the slope coefficients, thresholds, and smoothing parameters using the non-linear least-squares method. We start extracting the individual average of the endogenous variable and the residuals, computing $\tilde{y}_{it} = y_{it} - \bar{y}_{it}$ with $\bar{y}_{it} = \frac{\sum_{t=1}^{T} y_{it}}{T}$ and $\tilde{\varepsilon}_{it} = \varepsilon_{it} - \bar{\varepsilon}_{it}$ with $\bar{\varepsilon}_{it} = \frac{\sum_{t=1}^{T} \varepsilon_{it}}{T}$.

For the explanatory variables of the first regime, we also extract individual averages $\tilde{x}_{it}=x_{it} - \bar{x}_{it}$ with $\bar{x}_{it} = \frac{\sum_{t=1}^{T} x_{it}}{T}$. However, the explanatory variables of the second regime depend on parameters γ and c and, thus, we make the following transformation:

$$\widetilde{W}_{it}(\gamma, c) = x_{it}g(q_{it};\gamma,c) - \overline{W}_{it}(\gamma, c) \tag{3-8}$$

$$\overline{W}_{it}(\gamma, c) = \frac{\sum_{t=1}^{T} x_{it}g(q_{it};\gamma,c)}{T} \tag{3-9}$$

Therefore, the new matrix of explanatory variables becomes $x_{it}^{*}(\gamma, c) = \left[\tilde{x}_{it}{}'; \widetilde{W}_{it}{}'(\gamma, c)\right]'$.

After eliminating the individual effects, for a given pair (c, γ) it is possible to estimate the slope coefficients of extreme regimes through the ordinary least squares method as follows:

$$\hat{\beta}(\gamma, c) = \left[\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}^{*}(\gamma, c)x_{it}^{*}(\gamma, c)'\right]^{-1}\left[\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}^{*}(\gamma, c)\tilde{y}_{it}\right] \tag{3-10}$$

In the second step and conditionally on $\hat{\beta}(\gamma, c)$, we estimate γ and c using the non-linear least square method, and the optimal values for these estimators should correspond to:

$$(\hat{\gamma}, \hat{c}) = ArgMin \sum_{i=1}^{N}\sum_{t=1}^{T}\left[\tilde{y}_{it} - \hat{\beta}'(\gamma, c)x_{it}^{*}(\gamma, c)\right]^2 \tag{3-11}$$

In this last stage, we can then re-estimate the slope coefficients from the estimators of γ and c. However, as for the time series STR model, the convergence of the estimation procedure depends on the choice of initial values for γ and c. Colletaz and Hurlin (2006) propose creating a grid of search on these parameters by choosing several initial values of γ and c. Subsequently, it is sufficient to select the couple minimizing the residual sum of squares.

2.3. Validation tests

After estimating the PSTR model, it is recommended to apply at least two main misspecifications to verify the specification and estimation robustness of the PSTR model.

2.3.1. Constancy Parameter Test

As the parameter constancy test has been more successful in time series than in panel data, González, Teräsvirta, and Van Dijk (2005) explain this by the fact that the time dimension in panel data is shorter than in time series. Therefore, the more the temporal dimension is important, the more useful this test is.

It tests the null hypothesis of parameter's constancy against the alternative hypothesis of slope coefficients' smooth variation over time. Under the alternative hypothesis, the PSTR model becomes a time-varying (TV)-PSTR as follows:

$$y_{it} = \alpha_i + \left(\beta_{10}' x_{it} + \beta_{11}' x_{it} g_j(q_{it}; \gamma_1, c_1)\right) + f(t; \gamma_2, c_2)\left(\beta_{20}' x_{it} + \beta_{21}' x_{it} g_j(q_{it}; \gamma_1, c_1)\right) + \varepsilon_{it} \quad (3\text{-}12)$$

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This test tests the null hypothesis of a parameter's constancy against the alternative hypothesis of slope coefficients' smooth variation over time. Under the alternative hypothesis, the PSTR model becomes a time-varying (TV)-PSTR as follows:

$$y_{it} = \alpha_i + \left(\beta_{10}' x_{it} + \beta_{11}' x_{it} g_j(q_{it}; \gamma_1, c_1)\right) + f(t; \gamma_2, c_2)\left(\beta_{20}' x_{it} + \beta_{21}' x_{it} g_j(q_{it}; \gamma_1, c_1)\right) + \varepsilon_{it} \quad (3\text{-}12)$$

Where $f(t; \gamma_2, c_2)\ is$ a h order logistical transition function that has a transition variable the time t. Under the null hypothesis, the same identification problem arises. It is

therefore necessary to replace the function $f(t;\gamma_2,c_2)$ by its first-order Taylor approximation around $\gamma_2 = 0$:

$$y_{it} = \alpha_i + \beta_{10}'^{*}x_{it} + \beta_1'^{*}x_{it}\,t + \beta_2'^{*}x_{it}t^2 + \cdots \ldots \ldots + \beta_h'^{*}x_{it}t^h + (\beta_{20}'^{*}x_{it} + \beta_{h+1}'^{*}x_{it}\,t + \cdots \ldots + \beta_{2h}'^{*}x_{it}t^h\ )g_j(q_{it};\gamma_1,c_1) + {\varepsilon_{it}}^{*} \quad (3\text{-}13)$$

Where ${\varepsilon_{it}}^{*} = \varepsilon_{it} + R(t;\gamma_2,c_2)$ and $R(t;\gamma_2,c_2)$ is the residual term and the coefficients $\beta_j^{'*}$ for $j = 1,2,\ldots,h,h+1,\ldots,2h$ are multiples of γ_2. The null hypothesis becomes $H_0^*: \beta_j^* = 0$. This test is also performed based on the following Lagrange test (equation (3-14)) or the Fisher test (equation (3-15)), depending on sample size.

$$LM = T * N\frac{(\mathrm{SCR}_0 - \mathrm{SCR}_1)}{\mathrm{SCR}_0} \rightsquigarrow \chi^2\,(2hk) \quad (3\text{-}14)$$

Where SCR_0 is the sum of squared residuals of the PSTR model (3-1) and SCR_1 is the sum of squared residuals of auxiliary equation (3-13).

$$LM_F = \frac{(\mathrm{SCR}_0 - \mathrm{SCR}_1)/2hk}{\mathrm{SCR}_0/(TN - N - 2hk)} \rightsquigarrow F\left(2hk,(TN - N - 2hk)\right) \quad (3\text{-}15)$$

2.3.3. No remaining non-linearity test

The no remaining heterogeneity test verifies whether the number of regimes has captured all non-linearity inherent to the data, and determines the optimal number of transition functions (r). Specifically, this test comprises testing the residual non-linearity null hypothesis of PSTR with a single transition function $(r = 1)$ against the alternative with two transition functions $(r = 2)$

Let us consider the following three-regime PSTR model:

$$y_{it} = \alpha_i + \beta_0'x_{it} + \beta_1'x_{it}g_1\left({q_{it}}^{(1)};\gamma_1,c_1\right) + \beta_2'x_{it}g_2\left({q_{it}}^{(2)};\gamma_2,c_2\right) + \varepsilon_{it} \quad (3\text{-}16)$$

The null hypothesis of the non-heterogeneity test becomes $H_0: \gamma_2 = 0$. This test also suffers from the identification problem, which can be resolved when replacing the transition function $\left({q_{it}}^{(2)};\gamma_2,c_2\right)$ by its following first Taylor approximation around $\gamma_2 = 0$:

$$y_{it} = \alpha_i + \beta_0^{*'} x_{it} + \beta_1' x_{it} g_1\left(q_{it}^{(1)}; \widehat{\gamma_1}, \widehat{c_1}\right) + \beta_{21}'^{*} x_{it} q_{it}^{(2)} + \cdots + \beta_{2m}'^{*} x_{it} q_{it}^{(2)m} + \varepsilon_{it}^{*} \quad (3\text{-}17)$$

Where parameters $\widehat{\gamma_1}, \widehat{c_1}$ are estimates under the null hypothesis, and the coefficients β_{2j}^* for $j = 1, \ldots, m$ are multiples of γ_2.

The hypothesis of non-linearity can be formulated as $H_0^* : \beta_{21}^* = \cdots = \beta_{2m}^* = 0$. To test this hypothesis, it is possible to also use the following Lagrange or Fisher's statistic based on sample size.

$$LM = T * N \frac{(\mathrm{SCR}_0 - \mathrm{SCR}_1)}{\mathrm{SCR}_0} \rightsquigarrow \chi^2(mk) \quad (3\text{-}18)$$

Where SCR_0 is the sum of squared residuals of PSTR model (3-1) and SCR_1 is the sum of squared residuals of auxiliary equation (3-17).

The Fisher version of this test corresponds to:

$$LM_F = \frac{(\mathrm{SCR}_0 - \mathrm{SCR}_1)/mk}{\mathrm{SCR}_0/(TN - N - mk)} \rightsquigarrow F(mk, (TN - N - 2mk)) \quad (3\text{-}19)$$

Where m is the order of the transition function.

3. Data and Preliminary Analysis

We use a monthly data panel of seven developed countries: France, the United States, the United Kingdom, Germany, Italy, Canada, and Japan from June 9, 1987 to February 9, 2014. Table 3.1 summarizes our variables, their measures, as well as sources. We use a direct measure of investor sentiment given by the consumer confidence index.

Table 3. 1 Data description: Measures and sources

Variables		Measure	Source
Endogenous variable	Return	Log (P_t/P_{t-1})	Datastream
Explanatory variables	Investor sentiment	Confidence consumer index	Main Economic Indicators, OECD
	Dividend yield	Dividend /price	Datastream
	Price Earnings Ratio	Price / profit after tax	Datastream
	Growth of industrial production	Log (PI_t / PI_{t-1})	International Financial Statistics
	Term structure of interest rate	Difference between long rate and short rate	International Financial Statistics
	World market return	Log (P_t/P_{t-1})	Datastream

Next, we investigate data statistical properties.

3.3. Preliminary analysis

3.2.1. Panel unit root tests

Econometric literature on panel unit root tests has further developed since the work of Levin and Lin (1992). First, most tests model heterogeneity at an average level, while preserving the homogeneity hypothesis of the autoregressive root. To overcome this limitation, Im, Pesaran, and Shin (1997) and Maddala and Wu (1999) presented the initial first generation of unit root tests for heterogeneous panels, allowing for the heterogeneity of the autoregressive root. However, these tests might wrongly reject the unit root null hypothesis, as they do not consider the correlation between individuals. Subsequently, second generation unit root tests have been developed, using the independence hypothesis between individuals (Bai and Ng 2001; Choi 2002; Pesaran 2007; etc.).

We applied three unit root tests: the Im, Pesaran, and Shin (2003); Maddala and Wu (1999); and Pesaran (2007) test.[38] The various results of these tests are reported in Table 3. 2. First, all tests reject the null hypothesis of non-stationarity for stock market return, price-earnings ratio, growth of industrial production, investor sentiment, and word stock market return.

Table 3. 2 Results of panel unit root tests

Tests	**First generation**				**Second generation**	
Variables	**Maddala and Wu (1999)**		**Im, Pesaran and Shin (2003)**		**Pesaran (2007)**	
	Without drift	With drift	Without drift	With drift	Without drift	With drift
Stock market return	136.864 (0.000)	110.198 (0.000)	-13.233 (0.000)	-13.061 (0.000)	-9.915 (0.000)	-9.593 (0.000)
Dividend yield	21.914 (0.080)	20.600 (0.112)	-1.547 (0.061)	-1.324 (0.093)	0.366 (0.643)	1.318 (0.906)
Price earnings ratio	31.862 (0.004)	22.937 (0.061)	-2.392 (0.008)	-3.787 0.000)	-2.867 (0.002)	-3.401 (0.000)
Growth of industrial production	135.216 (0.000)	111.010 (0.000)	-12.122 (0.000)	-11.820 (0.000)	-6.926 (0.000)	-6.472 (0.000)
Term structure of interest rate	45.206 (0.000)	52.001 (0.000)	-0.889 (0.187)	-2.199 (0.014)	-2.807 (0.002)	-1.839 (0.033)
Investor sentiment	45.768 (0.000)	34.302 (0.002)	-3.886 (0.000)	-3.568 (0.000)	-1.811 (0.035)	-1.682 (0.046)
World stock market return	166.959 (0.000)	134.018 (0.000)	-22.462 (0.000)	-22.728 (0.000)	-12.530 (0.000)	-12.417 (0.000)

Notes: The numbers between parentheses are the p-values. For all tests, the null hypothesis is non-stationarity. For Im, Pesaran, and Shin's (2003) test, reported values are the values of the W_{tbar} statistic. The number of delays introduced in the tests have been selected with the criteria of Akaike (AIC) and Schwarz (BIC).

[38] The retained tests are summarized in the appendix.

Then, both first-generation tests lead to mixed results on presence of unit root in the dividend yield. The Im, Pesaran, and Shin (2003) test rejects the non-stationarity hypothesis for the model with or without trend. Based on the test of Maddala and Wu (1999), the results are slightly different and lead to accepting the null hypothesis of non-stationarity in a model without deterministic trend. However, this result is not confirmed when considering the correlation between individuals. The test statistic of Pesaran (2007) rejects the unit root null hypothesis for the dividend yield.

Finally, regarding the term structure of interest rates, the Im, Pesaran, and Shin (2003) test leads to rejecting the unit root null hypothesis for the model with drift, contrary to the model without drift. However, the implementation of the second-generation test shows the term structure of interest rate is stationary regardless of the hypothesis on the deterministic component, making these results desirable. Additionally, applying wrongly first-generation tests with inter-individual dependencies leads to size distortions. Accordingly, we can conclude that these unit root tests generally indicate dividend yield non-stationarity.

Subsequently, all modelling steps focus on variables stationary in levels (price earnings ratio, growth of industrial production, term structure of interest rate, world stock market return), as well as the first difference of non-stationary variables (dividend yield).

Table 3.3 reports the main descriptive statistics for the variables above, that is, mean, variance, minimum, and maximum. As per Table 3.3, we have 2,247 observations.

Table 3. 3 Descriptive statistics

Variables	Observations	Mean	Variance	Min	Max
Stock market return	2247	0. 002	0.003	-0.424	0.367
Dividend yield	2247	0.0007	0.045	-2.91	3.38
Price earnings ratio	2247	20.145	136.645	5	85
Growth of industrial production	2247	0.0009	0.0001	-0.158	0.065
Term structure of interest rate	2247	3.494	34.194	-4.01	26.106
Investor sentiment	2247	99.965	1.792	95.722	103.025
World stock market return	2247	0.004	0.002	-0.401	0.105

From Table 3.3, investor sentiment varies significantly over time and shows high volatility, while the stock returns range between significant negative and positives values.

3.2.2. Baseline linear framework

Under the hypothesis of linearity, to asses for the effect of investor sentiment on the dynamics of stock returns, we estimate the following linear specification:

$$y_{it} = \alpha_i + \sum_{j=0}^{p_i} \beta_j S_{i,t-j} + \sum_{j=0}^{p_i} \theta_j Z_{i,t-j} + \varepsilon_{i,t} \quad (3\text{-}20)$$

Where y_{it} is the stock market return for the i^{th} country at time t, $S_{i,t-j}$ is the investor sentiment for the i^{th} country at time $t-j$, $Z_{i,t-j}$ is a vector of control variables for the i^{th} country at time $t-j$, α_i is an individual fixed effect, and $\varepsilon_{i,t}$ is assumed to be i. i. d. $\mathrm{N}(0, \sigma^2)$, $i = 1, \ldots, N;\ t = 1, \ldots, T;\ N = 7;\ T = 321;\ j = 0, \ldots 4.$

The initial linear model in equation (3-20) is a linear fixed effect panel regression model, with four delays for each explanatory variable, since we used the general-to-specific method that consists in running regression with several lags p_j ($j = 4$, in this case) and

subsequently eliminate the insignificant delays. This allows obtaining the linear model that minimizes the information criteria and maximizes the adjusted coefficient of determination.

We obtained, next, the following linear fixed effect panel regression;

$$y_{it} = \alpha_i + \beta_0 s_{i,t} + \theta_{01} z_{1,i,t} + \theta_{11} z_{1,i,t-1} + \theta_{02} z_{2i,t} + \theta_{12} z_{2i,t-1} + \theta_{22} z_{2i,t-2} + \theta_{03} z_{3i,t} + \theta_{43} z_{3i,t-4} + \theta_{04} z_{4i,t} + \theta_{34} z_{4i,t-3} + \theta_{44} z_{4i,t-4} + \theta_{05} z_{5i,t} + \theta_{15} z_{5i,t-1} + \varepsilon_{i,t} \quad (3\text{-}21)$$

Where y is the stock market return, s denotes the investor sentiment, $z_1$refers to the dividend yield, z_2 is the price earnings ratio, z_3 is the growth of industrial production, z_4 is the term structure of interest rate, z_5 is world stock market return..

Table 3. 4 Estimation results of linear fixed effects panel model

Explanatory variables		**Parameters**
Investor sentiment	$s_{i,t}$	$\beta_0 = 0.010$*** (0.004)
Dividend yield	$z_{1,i,t}$	θ_{01}= -0.134*** (0.000)
	$z_{1,i,t-1}$	θ_{11}= 0.013*** (0.000)
Price Earnings Ratio (PER)	$z_{2i,t}$	θ_{02}= 0.013*** (0.000)
	$z_{2i,t-1}$	θ_{12}= -0.014*** (0.000)
	$z_{2i,t-2}$	θ_{22}= 0.001*** (0.006)
Growth of industrial production	$z_{3i,t}$	θ_{03}= 0.073

		(0.224)
	$z_{3i,t-4}$	θ_{43}= 0.120** (0.042)
Term structure of interest rate	$z_{4i,t}$	θ_{04}= 0.0008 (0.539)
	$z_{4i,t-3}$	θ_{34}= -0.006*** (0.0212)
	$z_{4i,t-4}$	θ_{44}= 0.005** (0.028)
World stock market return	$z_{5i,t}$	θ_{05} = 0.161*** (0.000)
	$z_{5i,t-1}$	θ_{15} = 0.175*** (0.0000)

The numbers between parentheses denote the p-values. (***) and (**) denote statistical significance at the 1% and 5% statistical levels, respectively.

This linear model corresponds to our baseline specification. The fixed-effect estimates of the linear regression are reported in Table 3.4. Accordingly, the coefficient of the investor sentiment appears to be positive and significant at 1%. Our findings confirm the assumption of behavioural finance that when the investor sentiment increases (decreases), the stock market return decreases (increases). However, this specification supposes linearity and this conclusion regarding the sentiment effect needs to be verified using a more robust and appropriate specification, allowing for further non-linearity in the sentiment stock return relationship.

4. Empirical Analysis

Next, we assess the interaction between stock return dynamics and investor sentiment in a more general framework: the PSTR model. The relationship can thus be asymmetrical

and nonlinear, and can capture further data heterogeneity in the data. Next, we analyse the PSTR estimates and discuss the effect of investor sentiment according to the market state.

4.1. PSTR model specifications

To specify the relationship between stock returns and investor sentiment for the G7 countries, we first use the following PSTR model with two extreme regimes and a single transition function:

$$\begin{aligned} y_{it} = {} & \alpha_i + \beta_0 s_{it} + \theta_{01} z_{1,i,t} + \theta_{11} z_{1,i,t-1} + \theta_{02} z_{2i,t} + \theta_{12} z_{2i,t-1} + \theta_{22} z_{2i,t-2} \\ & + \theta_{03} z_{3i,t} + \theta_{43} z_{3i,t-4} + \theta_{04} z_{4i,t} + \theta_{34} z_{4i,t-3} \\ & + \theta_{44} z_{4i,t-4} + \theta_{05} z_{5i,t} + \theta_{15} z_{5i,t-1} + \beta_1 s_{it} g(q_{it}; \gamma, c) + \varepsilon_{i,t} \end{aligned} \tag{3-22}$$

Where s_{it} denotes the investor sentiment at t; β_0 notes the coefficient of the investor sentiment in the first regime, β_1 denotes the coefficient of the investor sentiment in the second regime. The transition function $g(q_{it}; \gamma, c)$ is continuous and depends on the threshold variable. q_{it} is an exogenous variable for one-period lagged investor sentiment; $c = (c_1, \dots \dots . c_m)$ is a vector of parameters; and parameter γ measures the slope of the transition function.

Focusing on the investor sentiment-stock market return relationship, we can show that sentiment elasticity is obtained as a weighted average of β_0 and β_1:

$$\frac{e_{it}^{PSTR}}{s_{i,t}} = \frac{\partial y_{it}}{\partial s_{i,t}} = \beta_0 + \beta_1 g(q_{it}; \gamma, c) \tag{3-23}$$

Our PSTR model can be generalized to $r + 1$ extreme regimes as follows:

$$\begin{aligned} y_{it} = {} & \alpha_i + \beta_0 s_{it} + \theta_{01} z_{1,i,t} + \theta_{11} z_{1,i,t-1} + \theta_{02} z_{2i,t} + \theta_{12} z_{2i,t-1} + \\ & \theta_{22} z_{2i,t-2} + \theta_{03} z_{3i,t} + \theta_{43} z_{3i,t-4} + \theta_{04} z_{4i,t} + \theta_{34} z_{4i,t-3} + \\ & \theta_{44} z_{4i,t-4} + \theta_{05} z_{5i,t} + \theta_{15} z_{5i,t-1} + \sum_{j=1}^{r} \beta_j S_{it} g_j\left(q_{it}^{j}; \gamma_j, c_j\right) + \varepsilon_{it} \end{aligned} \tag{3-24}$$

Then, the sentiment elasticity for the i^{th}country at time t is given by the weighted average of the $r + 1$ elasticities, β_j being obtained in the $r + 1$ extreme regimes presented as follows:

$$\frac{e_{it}^{PSTR}}{s_{i,t}} = \frac{\partial y_{it}}{\partial s_{i,t}} = \beta_0 + \sum_{j=1}^{r} \beta_j \, g_j\left(q_{it}^{j}; \gamma_j, c_j\right) \tag{3-25}$$

Consequently, when examining vectors β_0 and β_j, we can only interpret parameter signs rather than their values. In fact, the sum of these parameters corresponds to the elasticity only if the transition function tends toward 1. Therefore, it is necessary to compute the elasticity to quantify the relative coefficient increases or decreases.

Further, for robustness, it is recommended to also consider two specifications with two transition functions, such as a transition function of order 1 ($m = 1$) and another of order 2 ($m = 2$). For each specification (each value of m), the first step is to test a linear model versus a threshold effect model (or a linearity test). If linearity is rejected, we then determine the number of transition functions needed to absorb possible non-linearity (using the no remaining non-linearity test).

As such, we compute the statistics LM and LM_F for linearity tests ($H_0: r = 0 \; versus \; H_1: r = 1$) and for residual non-linearity tests ($H_0: r = a \; versus \; H_1: r = a + 1$), where a is the number of transition function[39].

Table 3. 5 Linearity and no remaining non-linearity tests

	m = 1	**m = 2**
$\boldsymbol{H_0: r = 0 \; versus \; H_1: r = 1}$	2.422 (0.008)	2.607 (0.034)
$\boldsymbol{H_0: r = 1 \; versus \; H_1: r = 2}$	4.817 (0.008)	2.230 (0.107)
$\boldsymbol{H_0: r = 2 \; versus \; H_1: r = 3}$	0.793 (0.452)	-

Notes: The LMF statistic follows an asymptotic distribution $F\left(mk, TN - N - (r+1)\,mk\right)$, where m is the number of threshold parameters and k the number of explanatory variables. The values between parentheses indicate the p-values.

[39] We only report LMF tests as the LMF tests have main properties in small samples (see Van Dijk et al. (2002) for more details).

Table 3. 5 presents the results of linearity and no remaining non-linearity tests. We show that the null hypothesis of linearity is rejected, suggesting that the relationship between stock market returns and investor sentiment in G7 exhibits further non-linearity. For $m = 2$, the result shows that the model has only two regimes, as the remaining non-linearity results indicate that the null hypothesis cannot be rejected for $r = 2$. For $m = 1$, the remaining non-linearity test shows that two regimes are not sufficient to capture all non-linearity in the model. This might suggest that the optimal number of transition functions is equal to two when $m = 1$. We verify this through the choice of transition function and report the main results in Table 3.6.

We run regressions with $m = 1$ and $m = 2$, and the most suitable model is the one that maximizes the adjusted determination coefficient ($\bar{R}^2$) and minimizes the sum of squared residuals. From Table 3.6, results are relatively close, but the PSTR model with $m = 1$ is preferred. Moreover, the rejection of linearity is stronger for $m = 1$.

To check the adequacy of our model through the parameter constancy tests reported in Table 3.7, we show that parameters do not vary over time. Hence, the following PSTR specification with constant parameters seems to be adequate to assess the non-linear effect of investor sentiment on stock return dynamic

$$
\begin{aligned}
y_{it} = {} & \alpha_i + \beta_0 s_{it} + \beta_1 s_{it}\left(1 + \exp\left(-\gamma_1(q_{\mathrm{it}} - c_1)\right)\right)^{-1} \\
& + \beta_2 s_{it}\left(1 + \exp\left(-\gamma_2(q_{\mathrm{it}} - c_2)\right)\right)^{-1} + \theta_{01} z_{1,i,t} + \theta_{11} z_{1,i,t-1} \\
& + \theta_{02} z_{2i,t} + \theta_{12} z_{2i,t-1} + \theta_{22} z_{2i,t-2} \\
& + \theta_{03} z_{3i,t} + \theta_{43} z_{3i,t-4} + \theta_{04} z_{4i,t} + \theta_{34} z_{4i,t-3} \\
& + \theta_{44} z_{4i,t-4} + \theta_{05} z_{5i,t} + \theta_{15} z_{5i,t-1} + \varepsilon_{i,t}
\end{aligned}
\tag{3-26}
$$

Table 3. 6 Choice of transition function order (m)

Transition function order	**m=1**	**m=2**
($\bar{R}^2$)	0.681	0.680
Sum of Squared Residuals	2.596	2.600

Table 3. 7 Test of parameter constancy (p- values)

LM_1 (p-value)	LM_2 (p-value)
0.111	0.443

Note: LM denotes the statistics of parameter constancy tests.

4.2. PSTR model estimation results

We estimated the three-regime PSTR model by the non-linear least square method and report the main results in Table 3.8. First, most control variables show signs in line with financial theory and have a significant effect on stock return dynamics. Indeed, the dividend yield, price earnings ratio, term structure of interest rate, and world stock market return have significant impact on stock market returns, while the effect of the growth of industrial production is not significant.

Table 3. 8 Parameter estimates for the final PSTR model $(\mathbf{m = 1, r = 2})$

Regimes	**Variables**	**Parameters**	
Control variables	Dividend yield	$z_{1i,t}$	θ_{01}= -0.135*** (0.000)
		$z_{1i,t-1}$	θ_{11}= 0.012*** (0.001)
	Price Earnings Ratio	$z_{2i,t}$	θ_{02}= 0.013*** (0.000)
		$z_{2i,t-1}$	θ_{12}= -0.014*** (0.000)
		$z_{2i,t-2}$	θ_{22}= 0.001*** (0.004)
	Growth of industrial production	$z_{3i,t}$	θ_{03}= 0.057

			(0.346)
		$z_{3i,t-4}$	θ_{43}= 0.097 (0.106)
	Term structure of interest rate	$z_{4i,t}$	θ_{04}= 0.0005 (0.698)
		$z_{4i,t-3}$	θ_{34}= -0.005** (0.0454)
		$z_{4i,t-4}$	θ_{44}= 0.004** (0.048)
	World stock market return	$z_{5i,t}$	θ_{05}= 0.161*** (0.000)
		$z_{5i,t-1}$	θ_{15}= 0.176*** (0.000)
First regime	Investor sentiment	$s_{i,t}$	β_0= 0.012 (0.121)
Second regime	Investor sentiment	$s_{i,t}$	β_1= 0.044*** (0.000)
	γ_1= 1609.440 c_1= 99.208		
Third regime	Investor sentiment	$s_{i,t}$	β_2= −0.098*** (0.000)
	γ_2= 0.603 c_2= 101.265		

Note: The numbers between parentheses denote the p-values. The PSTR parameters cannot be directly interpreted as elasticity.

Second, we show that the effect of investor sentiment varies by regime and over time. In the first regime, the investor sentiment does not seem to affect stock returns, suggesting that market regime is predominated by rational investors who would rather rely on fundamentals than follow their sentiments.

In the second regime, investor sentiment significantly and positively influences stock return dynamics. This regime refers to the actions of investors who rely more on their beliefs than on information provided by fundamentals. Interestingly, the transition from the first (rational investors or fundamentalists) to the second regime occurs abruptly based on the estimated value of the first parameter slope (γ_1= 1609.4403), which is also illustrated in Figure 3.1. This transition is guided by changes in the investor sentiment variable and might suggest that the border between rationality and irrationality is derived from sentiment, and the sentiment effect is activated when investor sentiment exceeds the level of c_1= 99.208. Further, it appears that, in this regime, sentiment first affects stock return dynamics positively, suggesting that an optimistic attitude would increase stock returns.

The third regime refers to the presence of investors who still follow their sentiment, but whose sentiment also affects stock returns significantly but negatively. In this regime, the effect of sentiment becomes negative. Further, the transition speed of the second transition function is rather weak (γ_2= 0.6033)compared to the first transition function, suggesting that transition between the second and the third regime occurs smoothly rather than abruptly, compared to the transition between the first two regimes (Figure 3.2). This suggests that the switching regime is more sensitive to investor sentiment only when moving from rational to irrational behaviour. However, the transition between optimistic and over-optimistic market states is less elastic or linked to investor sentiment.

Figure 3. 1 Dynamics of the first transition function

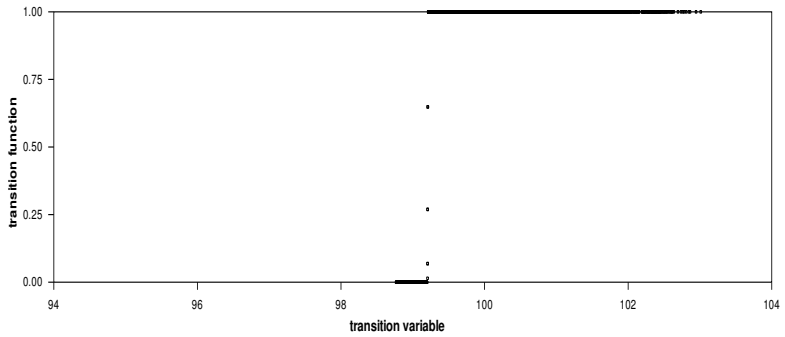
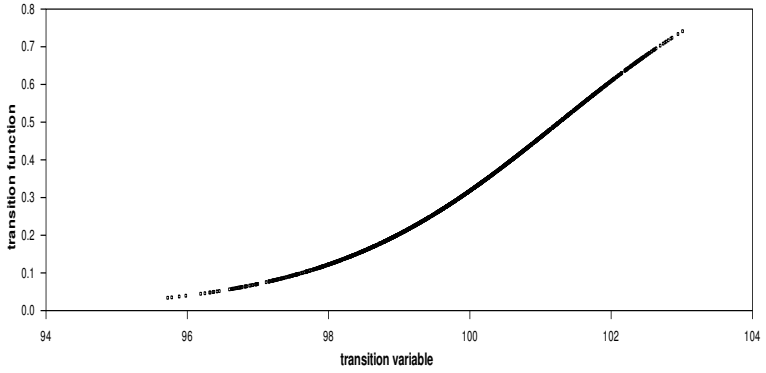


Figure 3. 2 Dynamics of the second transition function



Our results show (Figure 3.1) that, interestingly, the first threshold value (c_1) corresponds to the sentiment of December 1994. December 1994. Economically, this regime change might be explained through the euphoria preceding the internet bubble. Indeed, the first symptoms of the technology bubble appeared at the beginning of 1995. In fact, the investor frenzy for Netscape's initial public offering tripled the share price of the young company in one day. At the end of its first trading day, its market capitalization had grown to USD 2 billion. For nearly five years, the gains promised by companies in the information and communications technology sector sharpened investors' appetite, which resulted in large volumes of share issues, loans, as well as bank loans. The stock market value of the companies in that sector increased regardless of their actual sales or profits.

Additionally, following the popularization of the internet in 1994, the NASDAQ electronic market index of 1,000 in early 1995 multiplied five times over five years. As such, during the entire second regime, investor sentiment and the excess of optimism stimulated the increase of stock returns in the G7 countries.

The second threshold's value (c_2) corresponds to the investor sentiment value of August 2008, referring to the global financial crisis that began in 2007.

The bursting price bubbles, including the US housing price bubble of the 2000s, along with the subprime crisis, caused a significant fall in the stock market followed by the collapse of several financial institutions. Additionally, this downturn might justify the fact that the effect of investor sentiment became negative in the third regime. As previously mentioned, we can only interpret the signs of estimated parameters β j. For a better illustration of the impact

of the sentiment level on returns, Figure 3 depicts the estimated elasticity defined by equation (3-25) against the actual sentiment, $e_{it}^{PSTR}/s_{i,t}$.

Overall, in terms of sentiment effect, we can interpret the three above regimes as the neutral (first), optimistic (second), and over-optimistic regimes (third). For the first regime, containing only rational investors, β_0 is not statistically significant, which indicates that the investor sentiment effect does not old in this regime. Therefore, investors do not pay specific attention to their emotions and their decisions are based only on fundamentals. Therefore, this first regime is relatively stable, since psychology is not yet involved in the decision-making process. This finding confirms the initial hypothesis that noise traders do not affect the market during stable periods, which is also in line with the efficiency and rationality hypotheses.

In the second regime, noise traders dominate in the market through their overconfidence and optimistic behaviour. Accordingly, the investor sentiment effect, illustrated by β_1 affects the stock returns positively and significantly at 1%. Note that the signs of these regression coefficients are consistent with empirical literature on behavioural finance, such as West (1988), who showed that the volatility excess of stock prices might be justified by the actions of noise traders.

The third regime is a regime of noisy investors who are extremely overconfident, as observed during 2008, and investor over-optimism negatively affects stock market returns as illustrated by the coefficient associated with the current sentiment β_2. This result can be explained by the fact that investors were more confident and optimistic than they should have been and that the market would support their overestimated beliefs and market ability, which led to inverse effects. Indeed, in this regime, the excess of confidence might have misled investors and pushed them to follow unrealistic beliefs, increase their transactions, support higher transaction costs, which can induce market corrections.

Overall, we find that investor sentiment is a crucial determinant of stock market returns. However, its effect varies with market state and the level of change in investor beliefs. In particular, regarding the values and signs of St, the effect is about 10.2%, in the first regime but insignificant; 4.4% in the second regime while being significant, and around −9.8% in the third regime. Thus, the magnitude of the sentiment effect differs per regime, with a highest absolute value in the third regime. A possible explanation for the time-varying sentiment effect lies in the learning sentiment and confidence theory of Gervais and Odean (2001), which states that the sentiment of overconfidence might have different effects on

market returns. Regarding our results, when the market is benchmarked by fundamentals, as in the first regime, the sentiment effect does not significantly work, however the less market returns are governed by fundamentals, the more the sentiment effect will affect the market returns positively and significantly. In other words, when investor sentiment or overconfidence exceeds a given threshold, this excess might result in 'miscalibration' or 'overprecision' of information because given that spirit state investors might trade excessively and yield a negative effect on stock returns, implying losses for investors as shown in the third regime Interestingly, according to our results, the negative sentiment effect in the third regime is higher that two-times its positive effect in the second regime, suggesting that overall the sentiment effect is more destabilizing and reveals further asymmetry in the sentiment-stock return relationship.

This result is consistent with Gervais, Heaton andOdean's (2011) work as well as the eminent papers of Terrence Odean. These authors proved that investor overconfidence might negatively affect their profits and stock return dynamics. When more confident, investors tend to overestimate their beliefs and knowledge. This feeling pushes them to be more active in the market, buying and selling more shares than a normal or a median investor because they believe they can beat the market. The direct consequences of increasing their actions will be an increase in transaction costs, taxes etc., which would negatively affect their benefits and therefore stock returns. Our modelling shows that the consideration of non-linearity, panel data dimension and switching regime hypotheses would not only capture this feature, but also endogenously identify the threshold that separates normal overconfidence (second regime) from an irrational excess of overconfidence (third regime).

The coexistence of these different regimes is a well-established illustration of the Gervais and Odean's (2001) conclusion, and multi-period market model in which the investor learns to be overconfident through his/her failures and successes and his/ her ability also evolves with the time and the experience. With less experience, an investor's overconfidence would be higher and would place him in the third regime, but with more experience, he/she would be able to recognize his own ability and therefore switch to the other regimes.

5. Conclusion

Our article explored the threshold effects on the return-sentiment relationship for the G7 countries. We proposed an empirical specification, based on the PSTR model developed by González, Teräsvirta, and Van Dijk (2005), to better capture the effects of investor

sentiment on stock return dynamics for the G7 countries over the analysed period. Our main findings point to two interesting results. First, our results do not support the hypotheses of market efficiency, rationality, and representative agents, showing that heterogeneity among investors cannot be ignored, and that it may yield some nonlinearity, time-variation, and threshold effects in stock return dynamics. Second, we show that investor sentiment has a significant and per regime effect on stock returns. Particularly, we identify three different regimes. In the first neutral regime, rational investors or fundamentalists predominate and are governed by fundamentals rather than information from market opinion and investor sentiment. The second regime is predominated bynoise traders, with investor sentiment having a positive and significant effect on the stock returns. The third regime, with over-optimism and an excessive overconfidence, investor sentiment yields inverse (negative) effects on stock returns. Finally, while the sentiment does seem to imply a rapid transition between the first (rational) and second (irrational) regimes, its effect is smoother for the transition between the second and third regimes. This confirms the important role played by investor sentiment, but also suggests further ambiguity and complexity in assessing its effect. However, it is important to note that this conclusion depends on the proxy used for investor sentiment. In future research, it would be worthwhile considering other proxies to verify these results' robustness. Further, it would be interesting to investigate the multidimensional relationship between investor sentiment and stock market returns in a non-linear framework, such as the two-dimensional smooth transition vector autoregressive (STVAR) developed by Camacho (2004).

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Appendices of chapter 3

Presentation of the panel data unit root tests

This appendix shows the unit root tests used in this study. There are three tests: Im, Pesaran, and Shin (2003); Maddala and Wu (1999); and Pesaran (2007).

Im, Pesaran, and Shin (2003) test

Im, Pesaran, and Shin's (2003) was the first test developed for panel data unit root releasing the homogeneity assumption of the autoregressive root imposed by Levin and Lin (2002). Generally, Im, Pesaran, and Shin's (2003) test proposes an augmented Dickey-Fuller test (ADF) for each individual $i = 1, \ldots . N$ of a panel:

$$\Delta y_{i,t} = \propto_i + \rho_i y_{i,t-1} + \sum_{j=1}^{p_i} \beta_{i,j} \Delta y_{i,t-j} + \varepsilon_{i,t},$$

where $\propto_i = -\rho_i \gamma_i$, with $\gamma_i \in \mathbb{R}$ and $\varepsilon_{i,t}$ is N.i.d $(0, \sigma_i^2)$.

$H_0: \rho_i = 0, \forall\, i = 1, \ldots .. N,$

$H_1 : \rho_i < 0, \forall\, i = 1, \ldots .. N_1,$

$\rho_i = 0, \forall\, i = N_1 + 1, N_2 + 2, \ldots N.$

To perform this test, Im, Pesaran, and Shin (2003) offer first a statistic $Z_{tbar}(p, \beta)$ that is centred upon the expectation of the asymptotic distribution of individual statistic ADF and reduced by the variance of that same distribution:

$$Z_{tbar}(p, \beta) = \frac{\sqrt{N}\left[t_{\_bar_{NT}} - E(t_{iT})\right]}{\sqrt{\mathrm{Var}(t_{iT})}},$$

with $t_{\_bar_{NT}} = \frac{1}{N} \sum_{i=1}^{N} t_{iT},$

Where the moments $E(t_{iT})$ and $Var(t_{iT})$ correspond to the moments of expectation and variance of the asymptotic distribution (when T→∞) of an ADF statistic under the unit root null hypothesis $\rho_i = 0$ in a model with constant. The statistic $Z_{tbar}(p, \beta)$ follows a standard normal distribution when T and N tend to infinity. However, this distribution may be problematic in small sized panels T. To overcome this problem, Im, Pesaran, and Shin (2003) offer a second statistic $W_{tbar}(p, \beta)$:

$$W_{tbar}(p,\beta) = \frac{\sqrt{N}\left[t\_bar_{NT} - N^{-1}\sum_{i=1}^{N} E[t_{iT}(\rho_i,0)/\rho_i=0]\right]}{\sqrt{N^{-1}\sum_{i=1}^{N} Var[t_{iT}(\rho_i,0)/\rho_i=0]}},$$

Where the values of $E[t_{iT}(\rho_i,0)/\rho_i=0]$ and $Var[t_{iT}(\rho_i,0)/\rho_i=0]$ were tabulated for various levels of delays p_i and sizes of T by the authors.

Maddala and Wu (1999) test

The test of Maddala and Wu (1999) is similar to Im, Pesaran, and Shin's (2003) test, since it does not retain the homogeneity hypothesis of the autoregressive root. This is the nonparametric test of Fisher (1993), based on the set of $p-values$ of N independent individual unit root tests. Generally, Maddala and Wu (1999) retain the following statistic:

$$P_{MW} = -2\sum_{i=1}^{N}\ln(p_i),$$

with $p_i = F_{Ti}(G_i)$ being the p-value of the test statistic G_i of the unit root null hypothesis for an individual i, where F_{Ti} refers to the distribution function associated with the individual statistic G_i for a sample of size T_i.

Under the assumption of the absence of inter-correlation, the statistic P_{MW} follows a χ^2 (2N).

Pesaran (2007) test

Pesaran (2007) enriches the Im, Pesaran, and Shin (2003) test while considering the correlation between individuals. In fact, this test studies the unadjusted series $y_{i,t}$ corrected by individual average of $y_{i,t-1}$ and the first differences $\Delta y_{i,t-1}$. Pesaran (2007) then obtained a cross-sectional ADF (CADF) model. For this test, the null hypothesis H_0 is that each individual time series contains a unit root. The alternative hypothesis is that part of the series are stationary.

$$\Delta y_{i,t} = a_i + \rho_i\, y_{i,t-1} + u_{i,t}$$

where $u_{i,t} = \gamma_i\theta_t + \varepsilon_{i,t}$, $a_i = -\rho_i\gamma_i$, $\gamma_i \in \mathbb{R}$, θ_t is the common factor between individuals and the individual effect and is i.i.d (0,1).

The CADF model is written:

$$\Delta y_{i,t} = a_i + b_i y_{i,t-1} + c_i\bar{y}_{t-1} + d_i\Delta\bar{y}_t + v_{i,t}$$

with $\bar{y}_t = \frac{1}{N}\sum_{j=1}^{N} y_{jt}$

The Pesaran (2007) statistic of the cross-sectional augmented Im, Pesaran, and Shin (2003) test is an average of individual statistics noted $t_i(N,T)$:

$$\mathrm{CIPS(N,T)} = \frac{1}{N}\sum_{i=1}^{N} t_i(N,T).$$

Where $t_i(N,T)$ is the statistic of the cross-sectionally augmented Dickey–Fuller for the i[th] cross-section unit given by the t-ratio of the coefficient of $y_{i,t-1}$ in the CADF model.

Since this distribution is nonstandard when N tends to infinity, Pesaran (2007) tabulated critical values for different T and N.

Conclusion

This thesis answers the questions posed in the introduction. Primarily, it questions the impact of investor sentiment on stock market returns in the G7 markets during periods of crisis and calm. More specifically, it seeks to explain the nonlinearity and the threshold effects in the relationship between investor sentiment and stock market returns. Another question is whether the nonlinearity would be interpreted in the same way in panel and in time series.

The growing interest of financial researchers in the concept of investor sentiment arises in an environment that is marked by a dichotomy between arbitrageurs and noise traders. These noise traders open up an alternative path in the study of investors in the financial markets. The imperfect nature of their behavior allows researchers to move significantly closer to the realities observed in financial markets across the world.

Risk sentiment is derived from behavioral finance, and can be seen as an attempt to aggregate into a single variable the complete behavior of noise traders. In this context, several studies, such as that of Barberis et a1.(1998), highlight the fact that investor sentiment can best describe the reality of stock markets.

The objective of this thesis is to examine the impact of investor sentiment on the stock market dynamics of the G7 countries before and after the crisis periods by using two smooth threshold models. We take the nonlinearity induced by risk sentiment into account. Moreover, the explicit consideration in our modeling of behavioral heterogeneity leads to a more realistic presentation. Our results allow us to refine the study of stock market dynamics through the detection of regimes with smooth transitions. This finding seems to be interesting because market returns series are often characterized by dynamics with abrupt changes. We highlight the impact on stock returns of the change in investor sentiment resulting from regime shift.

From an operational point of view, our results are interesting and useful for investors and financial asset managers. They might help financial analysts and stock market supervisors, to offer them a clearer view of the return–risk couple in terms of recommendations. It is advisable to sell securities when investors are optimistic. Indeed, investor sentiment can offer investors excellent market timing signals. It is a more accurate

indicator than other fundamentals and economic factors when judging the overvaluation of the stock market.

The thesis is composed of three complementary and synchronized chapters.

The first chapter sets out the theoretical framework for our research topic. After the recapitulation of the basic concepts, the other two chapters present the empirical work, and on this basis we provide answers to all the questions asked at the beginning of the thesis. The periods covered are calm periods as well as crisis periods. They are determined according to the availability of the data.

At the end of this work, we can, on the one hand, draw various conclusions concerning the different paths we have followed and, on the other hand, suggest several paths of research that should be explored in the future.

We analyze the impact of investor sentiment before and after the subprime crisis in time series, through the use of the Smooth Transition Regression (STR) model developed by Granger and Teräsvirta (1993). Our results show that smooth transition threshold models can provide a relevant alternative to linearity and better explain the exposure of stock market returns to investor sentiment in the United States, Japan, Canada, and the United Kingdom. Moreover, we find that the sentiment effect depends on the country as well as on the regime under consideration.

The STR model has been used by researchers to model hedge funds, daily cycles of urban traffic, exchange rates, the forecasting of stock returns by volume, interest rates, and inflation (Jawadi and Khanniche, 2012; Kamarianakis et al., 2010; Akram et al., 2006; McMillan, 2007, 2009; Chen and Maringer, 2011). However, to our knowledge, no study has applied the STR model to examine the nexus between investor sentiment and stock market returns.

In order to refine our analysis of stock market dynamics, in the third chapter of this thesis we use a model that unites the advantages of panel data and those of nonlinear models. We employ the Panel Smooth Transition Regression (PSTR) model proposed by Gonzales et al. (2005). This regime-switching model with smooth transition in panel data allows the nonlinear behavior of the price adjustment process with respect to the equilibrium value to be modeled, while nonlinearity and heterogeneity are taken into account.

The use of a PSTR model allows us to propose an original and unified framework to study the impact of investor sentiment on stock market returns. This study is carried out while taking into account the inter-individual heterogeneity and the temporal variability of the coefficient of investor sentiment. Our results suggest that the PSTR model is a robust and appropriate specification for our G7 panel. Our results also allow us to highlight, in the same equation, the nonlinearity that was not completely detected in the second chapter. We find, on the one hand, that the nonlinearity can be generated by three regimes and, on the other hand, that the impact of investor sentiment depends on the regime being considered. Indeed, we confirm the result obtained in the second empirical chapter that suggests the existence of a normal regime where investor sentiment is not significant. However, we emphasize two other regimes (optimism and over-optimism) that are reduced to a single regime during our time series analysis.

The PSTR model was first used by Colletaz and Hurlin (2006) to study the nonlinearity of Public Capital Productivity. Bessec and Fouquau (2008) showed that the nonlinear link between electricity consumption and temperature can be described using the PSTR model. Others researchers, such as Seleteng et al. , 2013; Eggoh , 2012; Thanh, 2015 have employed the PSTR model to examine the effects of inflation on growth. To our knowledge, the PSTR model has not yet been used to study the link between investor sentiment and stock market returns.

Finally, this thesis underlines the interest in following a threshold approach in time series as well as in panel data. This is a promising path for research. In fact, the use of regime-switching modeling makes it possible to take nonlinearity and the heterogeneity of expectations into account. This work aims to enrich previous empirical work and tries to obtain new results by relying on a new approach that has not before been explored in the framework for modeling the impact of investor sentiment on stock market dynamics.

Nevertheless, as for any research study, there are some limitations to our work. First, it should be noted that our conclusions depend to some extent upon the measure of investor sentiment. Indeed, the absence of available data for other measures of investor sentiment for all the G7 countries condemned us to work with the consumer confidence index alone.

At the end of this analysis, new pathways can be followed in order to extend the themes evoked in this thesis. Thus, we propose two possible extensions that are part of the current research into threshold models. First, we propose that the simultaneous presence of

nonlinearity in the mean and the variance should be taken into account through the estimation of ESTR-GARCH or ESTR-STGARCH models. A second potential axis of research consists of examining the link between investor sentiment and stock market returns in a multi-varied context by using smooth transition vector autoregressive (STVAR) developed by Camacho (2004).

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ÉCOLE DOCTORALE
Sciences de l'homme et de la société (SHS)

Title: Essays on Stock Market Returns' Exposure to Investor Sentiment using Time-Series and Panel Switching Regime Models.

key words: investor sentiment, nonlinear stock market returns' dynamics, threshold effects.

Abstract: This thesis presents a contribution to the study of the impact of investor sentiment upon stock market dynamics in the developed markets of the G7 countries. For this purpose, two econometric methods are proposed for modeling stock market dynamics that take the behavioral heterogeneity of investors into account. This thesis includes three chapters. The first chapter is theoretical. After recalling the hypothesis of informational efficiency, it introduces the basic concepts (rationality, behavioral finance, investor sentiment, etc.). The following two chapters offer two empirical tests for the impact of investor sentiment on the dynamics of stock market returns. In the first, the Smooth Transition Regression "STR" model is used to provide a better reproduction of the different regimes for market returns triggered by investor sentiment. The second empirical chapter aims to take the nonlinearity and the heterogeneity into account simultaneously, through the dual use of temporal and individual dimensions and by applying the Panel Smooth Transition Regression "PSTR" model. Our empirical results imply a rejection of the efficiency hypothesis and a rejection of the hypothesis of a representative agent, suggesting the nonlinearity of stock market returns. We also show that there are different transition speeds between different regimes, suggesting that the exposure of stock market returns to risk sentiment varies nonlinearly over time and by regime.

Titre: Essais de modélisation de l'exposition des rentabilités boursières au sentiment de l'investisseur: Application des modèles à changement de régimes en séries temporelles et en données de panel.

Mots clés: sentiment de l'investisseur , dynamique non linéaire des rentabilités boursières, effets de seuil.

Résumé: Cette thèse présente une contribution relative à l'étude de l'impact du sentiment de l'investisseur sur la dynamique boursière dans les marchés développés de G7. A cette fin, deux spécifications économétriques ont été proposées pour modéliser la dynamique des marchés boursiers tenant compte de l'hétérogénéité comportementale des investisseurs. Cette thèse est structurée autour de trois chapitres. Le premier chapitre est d'ordre théorique. Après avoir rappelé l'hypothèse de l'efficience informationnelle, ce chapitre introduit les concepts de base (rationalité, finance comportementale, sentiment de l'investisseur, etc.). Les deux autres chapitres proposent deux essais empiriques sur l'effet du sentiment de l'investisseur sur la dynamique des rentabilités boursières. Dans le premier chapitre empirique, le modèle Smooth Transition Regression "STR" est utilisé afin de mieux reproduire les différents régimes des rentabilités boursières activés par l'effet du sentiment de l'investisseur. Le second chapitre empirique vise à tenir en compte simultanément la non linéarité et l'hétérogénéité comportementale à travers le double usage des dimensions temporelle et individuelle en employant le modèle Panel Smooth Transition Regression "PSTR". Nos résultats empiriques impliquent le rejet de l'hypothèse d'efficience ainsi que celle d'agent représentatif, en suggérant que la dynamique des rentabilités boursières exhibe de la non-linéarité. Nous montrons également l'existence d'effets de seuil significatifs permettant de distinguer différents régimes, suggérant que l'exposition des rentabilités au risque sentiment varie non-linéairement dans le temps et par régime.