



Contribution to the development of new maintenance strategies integrated to control charts for a production process under service levels, operational, and quality constraints

Aminu Sahabi Abubakar

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Aminu Sahabi Abubakar. Contribution to the development of new maintenance strategies integrated to control charts for a production process under service levels, operational, and quality constraints. Automatic. Université de Lorraine, 2023. English. NNT : 2023LORR0055 . tel-04443235

HAL Id: tel-04443235

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Thèse

Présentée à
L'Université de Lorraine – Metz

Pour obtenir le titre de
Docteur (PhD) de

Spécialité Automatique, Traitement du Signal et des Images, Génie Informatique
UFR Mathématiques, Informatique, Mécanique
École doctorale IAEM-Lorraine

Par

Aminu Sahabi ABUBAKAR

Contribution to the development of new maintenance strategies
integrated to control charts for a production process under service
levels, operational, and quality constraints

Soutenue, le 23/05/ 2023 devant le jury composé de :

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Thesis

presented at

The University of Lorraine – Metz

For obtaining

PhD

Speciality, Automatic Signal, Image Processing, Computer, and Industrial Engineering

UFR Mathématiques, Informatique, Mécanique

École doctorale IAEM-Lorraine

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DEDICATION

*To the source of my life, my mother, Maimuna,
to the start of my strength, my father, Sahabi Abubakar,
to the houses of love and joy, Samira and Nafisa,
to the pillars of support, my family and friends,
to all the light sources, my dear teachers and mentors,
to all my lovely ones, including the dear readers.*

ACKNOWLEDGEMENT

This thesis is a product of support and encouragement from many to whom I would like to express my gratitude.

I am grateful to the thesis directors, Associate Professor (HDR) Zied HAJEJ and Dr Aime C. NYOUNGUE, for their valuable contributions and availability. I want to thank them for trusting me and admitting me to carry out this thesis. I also thank them for their kindness, rigour, and wisdom. Please know that this journey would not have been possible without you.

Thanks to Professor Nidhal REZG and Mrs Christel WIEMERT, Director and administrative manager of the LGIPM Laboratory. I would also like to thank Associate Professor Sadok TURKI, Dr Kammoun Mohamed Ali, Dr Nadia NDHAIEF, Associate Professor (HDR) Sofiene DELLAGI, and other senior members of the LGIPM for their warm welcome in the laboratory. Professors Marie SAWADOGO and Jérémie SCHUTZ, for being members of this thesis monitoring committee. Let me express my appreciation to my friends in the laboratory, Aisha SAAD, Ayoub TIGHAZOU, Olanrewaju Isaac ADENEKAN, and many others, for their kind support.

Thanks to the members of the jury who agreed to evaluate this thesis. I thank Professor Abderrahman ELMHAMED, and Dr (HDR) Mohamed Zied BABAI, for their honour of accepting to be rapporteurs. And, thanks to my invite Professor, Samaila UMARU. I also thank, Hamid ALLAOUI, Dr, Associate Professor Nadia HAMANI, Professors, Éric LEVRAT and Ali SIADAT for agreeing to examine this research work.

I am grateful to the Petroleum Technology Development Fund (PTDF) for providing me with the scholarship opportunity to pursue this program. Thanks to Dr Leila MATHIEU of the French agency for the promotion of higher education (CAMPUS-FRANCE). Let me also appreciate Mr Mahfoud Barhoumi and Dr Salawudeen A. Tijjani, an Alexander von Humboldt fellow of the institute for Automation of complex systems, RWTH-Germany for their academic collaborations and support.

I must admit that throughout this thesis, the support of my family and friends was never lacking, special thanks to Abubakar, Shafa'atu, Almustafa, Musa - Sahabi, then Alh. Bello Galadima, Alh. Mohd Kindi Zauo, Sadik Buhari Galadima, Nasmatu Dahiru Ambursa, Babangida B.B, Moh'd Abubakar, Nura Umar Galadima, Nasir Bello Gwadangaji, Dr Abubakar Shitu, Moh'd Baba, Idris Kaoje, Ibrahim Hamisu, and Mr Kevin Ifegbuike.

I wish to particularly acknowledge the love, joy, and understanding given by my spouses Samira Dahiru and Nafisa Aliyu. I will always owe immeasurable gratitude to my kind, encouraging and prayerful parents, Alh. Sahabi Abubakar and Haj. Maimuna Balaraba Bello.

Above all, thanks to the Almighty Allah for giving me all it takes to accomplish this project. I forever remain grateful.

Alhamdulillah.

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General introduction

The overall consequences of the stated global challenges are already affecting production planning, inventories, and delivery. These continue to change in customer behaviours, markets, product variety, cost, and quality. An increased requirement for the production of high-quality products implies the need to use modern approaches for modelling and analysing customer changing demands as well as contemporary computing tools for supporting decision-making (Jahan et al., 2016). About production planning, inventory management, maintenance strategies, and quality policies. Hence the increase in the requirement for advanced production control and decision support tools by company managers. Getting the best output from the production system and meeting the varied customer requirements (Quality, service, and cost) that become more important than ever before (Kölmel et al., 2014). Satisfying customers varied requirements in itself will require also, an integrated chain of actions (Pakurár et al., 2019). Beginning with production planning, maintenance strategies, and quality policies. Given the stated one can easily understand that the new products and markets challenges, exigent customer expectations, and product variety have already evolved. Therefore, the need for new and improved approaches to enable more effective collaboration and integration of production factors. Essentially, the technique we proposed to optimise maintenance and quality control of an integrated production system is a control chart tool based on statistical measurement and analysis of quality parameters. The control chart ensures the cost-effective and continuous production process reliability improvement, which reduces the non-quality products and consequently minimises the total production costs.

It has been established that production, maintenance, and quality are three principal factors of production system optimization, unfortunately, they have been often investigated separately in the literature (Xiao et al., 2019). Studies that considered these three concepts together mostly assumed only one assignable cause in the production process variation (Salmasnia et al., 2017). The assumption is a simplification and is unlikely to occur in actual production processes. This will result in poor performance criteria (financial and technical) when there are multiple assignable causes originating the shift and may differ differently from the control chart design. Nowadays, a customer can make a product demand, modify the demand or suddenly make a cancellation order with so much ease, via mobile phone applications or web platforms (B. Liu et al., 2020). Additionally, the present competition in the industrial environment today is not limited to increasing and dynamic demand but also the need for higher quality products at competitive prices (Hajej et al., 2021). Indeed, these stated challenges must-have consequences on operation planning, production processes, machine maintenance, and product quality. To address these challenges, production company managers must remain dynamic throughout the production horizon and the supply chain to remain competitive. Inventory management problems can also impact customer satisfaction (Mifdal et al., 2015). Machine breakdowns or delays in the delivery of products also have other consequences (Zied et al., 2014). To satisfy customers, several companies optimized their raw material inventories, production tools, energy, operation, or finished product inventory (Van Horenbeek et al., 2013). Recently, the maintenance

function has drawn much attention in the management of production or services companies. Strategic maintenance planning and optimization ensure production system reliability and availability by stabilizing the process also reducing costs by reducing the number of non-conformal items (Beheshti Fakher et al., 2017). The maintenance strategy allows the appropriate maintenance planning in collaboration with optimal production, inventory, and delivery plans, thereby achieving overall production performance efficiency, reducing total costs, and enhancing profitability (G. Q. Cheng et al., 2018). We find substantial work that deals with different problems in various ways so that the system can respond to the different constraints. However, many of these works have been restricted and have not considered quality, multi-warehouses, or multi-assignable causes. This justifies the need for solutions to optimize production, maintenance, and quality that will considerably improve both industry's (technical) and the market's (economic) requirements. This thesis exploits recent research gaps on production and maintenance systems integrating the service and quality constraints. We carried out integrated production planning, maintenance, and quality control problems for unreliable single-machine production systems responding to the random demand of a supply chain. The machine is subject to random failures resulting from the deterioration that disrupt production and impact the defective product rate, maintenance, and total production cost. This work is motivated by the ongoing research trends that integrate fundamental production functions based on the statistical process control and the control chart results to improve product quality, and quantity produced to minimize total production costs. In this perspective, we have developed and proposed new policies for an integrated model and organized the work as follows.

In the first chapter, we present the context of the study and highlight the problem of this thesis, and the objectives of our research. We then try to situate our study concerning the various works already carried out in production-integrated maintenance, we also cite some work carried out with quality control policies. The second chapter is devoted to modelling an integrated production, maintenance, and quality control model with a statistical process control chart of a supply chain production system. The aim is then to find the optimal production and maintenance plan which minimized the total costs, characterized by the optimal number of Preventive Maintenance actions according to production and inventory management plans throughout the production periods. The number of products to be programmed, their stock levels to allow the satisfaction of the customers at each of the multi-warehouse on each period and the number of preventive maintenance actions optimized the overall production process, reducing non-conformal products. To sum up, we formulated the model of the stochastic problem, the production policy, the production case, established equations, an analytical study, and resolutions. We carried out a study and analysis of the maintenance policy integrated with quality. Formulation of machine failure rate, maintenance cost, and quality. Numerical experimentation is conducted and conclusions are drawn. We determined the optimum production, delivery, and maintenance strategy with the optimal parameters related to the control chart. We achieved our objectives which minimized the total cost of inventory at different stores, the production, and delivery, and at the same time, the costs related to quality and maintenance strategy. This work is presented at MOSIM (Agadir-Morocco), & IEEM (Singapore, 2020), international

conferences, and published in the journal of applied science (MPDI - Appl. Sci. 2021, 11, 4192). In the third chapter, we studied an optimal production, maintenance, and quality forecasting Problem, with an improved statistical process chart of a supply chain under service and quality requirements. We designed and developed a more efficient, multi-layer process control chart of quality. A mathematical model that significantly reduces the total cost of production, maintenance, inventory and quality is developed. We established optimum production, inventory, delivery, and optimal parameters related to the improved control chart, with additional surveillance limits based on optimal decisions. The work is presented at the IEOM conference held in Istanbul-Turkey 2022 and IFAC-MIM conference, available online in the conference proceedings, its extension is also, submitted to the international journal of production research (IJPR). In the fourth chapter of the thesis, we develop integrated maintenance, production, and quality policy with a statistical process control chart in combination with the AFNOR rule, considering different assignable causes. We incorporated additional quality monitoring as a decision support technique for detecting some unusual trends in the production process, we seek to optimize the production planning of a three-echelon production system under uncertainties. We understand also, the impact of imperfect maintenance (IM) on the production system reliability, non-conformal items, and total production cost minimization. The results of this study are presented at international conferences on Trends & Innovation in Management Engineering Sciences and Humanities Dubai 2022, the 2023 20th International Multi-Conference on Systems, Signals & Devices (SSD) - Conference on Systems, Automation & Control, Mahdia Tunisia. And the CIGI Qualita MOSIM 2023 University of Quebec at Trois-Rivières.

The main contribution of this research is the study, review, and improvement in the design and application of a statistical process control chart. The development of mathematical models for an advanced decision support system involving the combination of different quality control techniques for maximum performance. New production control and optimization approaches are developing and being investigated. We also, implemented a sensitivity analysis on the developed algorithm to demonstrate its robust fullness. This deals with the challenging issue of the interaction between the different yet, related factors of production; production planning, inventory management, maintenance strategy, and quality policy. The factors were treated optimally and collaboratively, which improved system reliability and reduced the number of machine failures, false alarms, as well as the number of non-quality products, thereby minimizing the total cost. Conclusively, we ended the thesis report with a general conclusion commenting on the results presented generally and also, indicating the perspective for the continuity of this research.

Chapter 1:

State of the art: Production and Maintenance Strategies in industrial systems management.

This chapter presents the literature review of the thesis and discusses the importance of the study, thereby establishing the knowledge gap. The study begins with the presentation of the state of the art on production, production coupled with maintenance, or production and quality, then the production, maintenance, and quality. Given the leading works in the literature that are relevant to this research. We presented the different notions of production systems, including their classification and objectives. The resource and demand-based topology, supply chain and its management, life cycle, and waste management. The maintenance of production systems, main categories, and the law of failures. The state-of-the-art briefly highlighted different types of maintenance policies: perfect, minimal, and imperfect maintenance, and their significance in the management of the production system. A general overview of production and maintenance optimisation. And, we presented the description of the global thesis problem, also, highlighted the main goal and key objectives of the research. The study Significantly focused on quality, quality tools, and their applications in production and maintenance control. This study cantered on developing production and maintenance integrated into a control chart of quality policy. We propose new integrated maintenance strategies with an in-depth analysis of the continuous production process reliability improvements and treated under customer requirements (Service, quality, and cost) constraints. The technique we used to optimise the maintenance and quality control of an integrated production system is a control chart tool based on statistical measurement and analysis of quality parameters. We are interested in optimising maintenance that plays a critical role in customer satisfaction, sustainability, and the development of companies.

1.1 Introduction

The continuous changes and challenges in worldwide economic conditions request a robust and dynamic production industry capable of producing high-quality items at a competitive cost. The customer service requirements such as varying demand, and the increasing cost of production, inventory, and delivery have forced more constraints on producers to stay competitive. Nowadays, a customer can make a demand, modify the demand or suddenly make a cancellation order via mobile phone applications or web platforms. Additionally, the present competition in the industrial environment today is not limited to increasing and dynamic demand but also the need for higher quality products at lower prices. At the same time, production frameworks are getting more complex and prone to numerous instabilities, (Budai et al., 2008). Moreover, customers are becoming exigent, and their urgent and multiple needs (consciousness and demand) are requiring delivery with minimum time. (Sadok et al., 2012) decided on the ideal production, delivery, and maintenance accounting conveyance time, machine disappointment, and varying demand along with the right to withdraw, they write in their paper. The production systems must, after all, have a delivery time, quantity, and acceptable quality. The current industrial market is characterized by significant progress: a massive increase in high technology, the emergence of management and performance improvement techniques that contribute to the development of companies. Industrial managers and researchers are always trying to find the best solutions for the way companies operate. Production and Maintenance are the most critical aspects of the industrial system. Traditionally production and maintenance were treated separately, but with the development of the current industrial system, the emergence of exigent customers, competitive markets, varied products and random demands required the collaboration of these inter-related aspects to achieve overall objectives. For the determination of the appropriate Preventive maintenance policy and stock size, (Li, 2000) constructed a model of a serial production line consisting of several faulty machines to several buffers. (Kenné & Gharbi, 2004) created an integrated approach to production inventory control based on the age Preventive Maintenance strategy. The term "corrective maintenance" refers to an action taken to bring the machine back to its original condition (AGAN). While inventory management problems can impact customer satisfaction (consciousness and costs), machine breakdowns or delays in the delivery of products also, have other consequences. These challenges require good decisions making tools with multi-level controls; production planning, inventory management, and maintenance strategy, (Van Horenbeek et al., 2013). (Chen et al., 2015) implemented an integrated production scheduling and maintenance planning model in the context of imperfect preventive maintenance in which they use an immune clonal selection algorithm. A well-controlled production service contributes to the satisfaction of demands on time and ensures the company's sustainability. Recent work has focused more and more on the maintenance function and given it greater importance because of the critical role it plays in the industry. Maintenance is no longer traditional, it has become one of the strategic production functions which maximized reliability and minimized cost (Operation, Maintenance, and Repair of Land-Based Gas Turbines, 2021). Indeed, a sound, thought-out maintenance strategy allows the company to better manage its production,

inventory, and delivery according to demands also, ensure the adequate performance, reliability of the equipment, and reduction of losses used to meet total customer requirements. Integrated Production maintenance approaches have gained tremendous importance in the research community. To create modern industrial strategies of more structured and integrated production planning, and maintenance optimization tailored to satisfy customers' varied needs while the company makes profits and remained competitive (Mifdal et al., 2015). (Dellagi et al., 2017) discovered that production rate fluctuations between periods significantly impact production maintenance plans and total incurred costs, allowing for the optimal production and preventive maintenance plan to be determined while considering constraints related to the production system capacity. As a result, many companies are attempting to eliminate transportation delays such as delivery time, which is when it takes products to travel from a production store to a customer's warehouse and substantially impacts performance metrics. Recently, (G. Cheng & Li, 2020) proposed an appropriately integrated lot-sizing and feedback strategy for a serial-parallel multi-stage faulty system. (Rasay et al., 2022) Companies must cope with these random demand uncertainties, changing markets, and product variety requirements to remain competitive. Indeed, these must-have consequences on operation planning, and production machine maintenance. Production planning optimization coupled with the maintenance technique has continuously been an incredible challenge for industrial companies. In any case, the item or benefit diminished when it influences either the method that made it by expanding costs, time and imperatives or controlling the ultimate item or benefit utilized by the client, (Baklouti et al., 2020; Rasay et al., 2022, 2022).

On the other hand, among the first acts of a hierarchical production process decision-making action—the failure or dysfunction of the production system considered—are optimal maintenance and production plans that minimize overall cost, including production, inventory, delivery, and maintenance. As a result, several researchers have looked into integrating the $\bar{X} - \bar{bar}$ type control chart and the periodic preventive maintenance approach to stabilize the process and avoid the creation of non-conformity that plagues production systems. In their work effect of maintenance on the economic design of an \bar{x} -control chart, (Lee, 2005) investigated the impact of maintenance on cost; however, they ignored stock deficiency. Also, (Lee, 2005) developed a model for supporting investment strategies about inventory that considers the delivery time to the customer, and in an imperfect production system, preventive maintenance is essential. Various authors looked into multiple aspects of production and upkeep. The company's ideal production plan and maintenance strategy needed them to develop items that could meet changing demand over time. In this regard, in their paper, "Automatic transfer lines with buffer stock," (Buzacott, 1967) was one of the first authors to improve productivity. They studied the impact of buffer stocks in boosting system productivity to consider integrated maintenance to production plans. (Schutz et al. 2011) have chosen, in their study the development of a production (business plan) and maintenance plan, to integrate the notion of operating and environmental conditions of the system. The authors of this work then chose a system subject to random failures which performs different missions (M) in a finite time horizon. Each mission has well-defined characteristics that depend on the nature of the operating and environmental conditions. The

optimal production and maintenance plans were developed simultaneously (Zied et al., 2014), and the ideal delivery plan considered delivery time, machine breakdowns, varying customer demand, and right of withdrawal. Production systems must, after all, have a delivery time and a carried amount. As a result, many organizations are attempting to eliminate transportation delays such as delivery time, which is the time it takes for a product to travel from the main store to the customer purchase warehouse, and which usually has a significant impact on performance measures. (Si et al., 2018) suggested a framework of reliability and maintenance structure for a two-state process optimizing decision variables a mechanism for determining discrete timings in production system preventive maintenance. (G. Q. Cheng et al., 2018) established an integrated model for a flawed production system that dealt with lot sizing, quality, and joint and condition-based maintenance. (Darendeliler et al., 2020) pandered with an imperfect production system that used inferior rework processes and imperfect preventive maintenance to determine the ideal economic production quantity (EPQ), multiple inspections, and inspection times. The company's ideal production plan and maintenance strategy needed them to develop items that could meet changing demand over time. (Hadian et al., 2021) investigated the subject of integrated maintenance to production strategies by examining the function of buffer stocks in boosting system productivity in this setting. (Elattar et al., 2022) used a multi-objective optimization inventory model to establish the appropriate just-in-time buffer required for efficient operational management. The failure rate rises with time and the use of the machine, but the latter is unusual. Most researchers, once again, presume a flawless production system. For a multi-system, multi-period design with a growing hazard rate of degradation distribution, (Zhao et al., 2022) integrated mission and maintenance. The job results determine the best values for the mission plan and the Preventive Maintenance period, during which the machine is brought back into working order after failure. We found that most studies in the literature provided for the coordination of two essential capacities: production, maintenance, or quality. Production and maintenance integration, followed by whether it's quality maintenance or production with quality, and most recently, the complete trends are trying to address all three elements.

To lower the failure rate, and non-conformal products and increase impacts on performance measures, we may extend this approach to account for other critical manufacturing factors such as delivery quantity, cost, time, and quality control. Other researchers focused on upkeep and quality. (Aghezzaf et al., 2007) developed a methodology for joint optimization of maintenance planning, process quality, and production schedules but did not consider the effects of inventory control shortages. They discovered that production rate fluctuations between periods have a significant impact on production maintenance plans and total expenses incurred, enabling the determination of the best production and preventive maintenance plan while considering production system capacity restrictions. The work (Panda, 2007) allows for the ideal buffer stock size to be determined based on machine age and maintenance requirements. The work proposed an integrated preventive maintenance quality model for an imperfect manufacturing system that includes two decision variables: buffer stock quantity and the rate of non-conform units which must take preventive maintenance actions. The goal of (Amelian et al., 2015) was to detect process shifts and subject the production system to them —

production Process Simulation at the point of failure, prone to maintenance and quality control. And quality base maintenance, thereby resolving the issue. (G. Q. Cheng et al., 2018) developed an analytical model that integrates production sampling quality control and maintenance of a critical production system with an AOQL constraint. However, the model assumed that a single quality attribute deteriorated with ageing. Furthermore, products and processes are growing increasingly complex due to a variety of different causes, necessitating more quality checks and multiple inspections. (Al-Salamah, 2018) investigated the integration of Production Maintenance and Quality for the collective determination of Economic Production Quantity and the periodic Preventive Maintenance level for an imperfect system that switches to a control state after a random time interval. The study showed how Preventive Maintenance reduces non-quality. For the flawed production system, (uit het Broek et al., 2021) worked on combining production, quality control, and condition-based maintenance. They examine manufacturing with lot sizing and quality control based on inspection policies to determine the percentage of defectives. A supply chain management chart with integrated production, maintenance, and control under quality restrictions, (Hajej et al., 2021) was devised to reduce the total cost of conjugated production, delivery, and maintenance optimization. (Qu et al., 2022) created a model for optimizing production size and the time to perform Preventive Maintenance to reduce non-conform units when a process swings out of control. (Megoze Pongha et al., 2022) integrated the model of production quality. This research looked at a randomly failing manufacturing system supposed to serve random demand while maintaining a certain service level, hence overcoming production quality difficulties.

In this chapter, we present the context of the study and highlight the problem of this thesis. We then try to situate our study concerning the various works already carried out in production-integrated maintenance, we also cite some work carried out with quality control policies. In most cases, the quality control policies are modelled based on normal law (Hawkins* & Zamba, 2003). All aspects of production, inventory, delivery, maintenance, and customer requirements are affected by quality and quality management (cost and service satisfaction). Several process improvement strategies, such as the Six Sigma Method, have been researched in the literature to achieve resemblance of quality and fulfilment of customer needs in the manufacturing process, Analysis of the causes and implications of the system measurement. The control chart is based on the statistical process control (SPC) methodology, which is one of the fundamental tools used by Six Sigma to enhance the quality of goods and processes by minimizing process variance and the accompanying high defect rate (Sharikh et al., 2019), (Sinha & Singh, n.d.) and (Bahria et al., 2019). This research focuses on the Six Sigma approach, which can be used in production or service industries.

Our study, which consists of a production plan, inventory management, maintenance and quality strategy to minimize costs, was based on two sequential algorithms; the first algorithm is used for the determination of the optimal production plan and the second algorithm is used for the planning of the maintenance and quality strategy. Different types and combinations of maintenance strategies were employed and taken into consideration several assignable cause of process variation. For the

optimization of the maintenance and the control chart, the difficulty is in the modelling the modification of the indicator by the system failure (Dutoit et al., 2019a). While others use normal, or gamma laws, it is possible to model the reliability and maintenance model using the Weibull probability distribution with the shape and scale parameters.

The objective of this research is to model the impact of production rate on the production system degradation and link the failures on the quality indicators so as to determine the best maintenance and quality strategy (Statistical Process Control chart). And, establish a collaborative, integrated production control policy that minimised total production cost.

Figure 1.1 Illustrate the mean “shift” μ_0 to μ and the drift of the standard deviation of the quality indicator by an increase in dispersion.

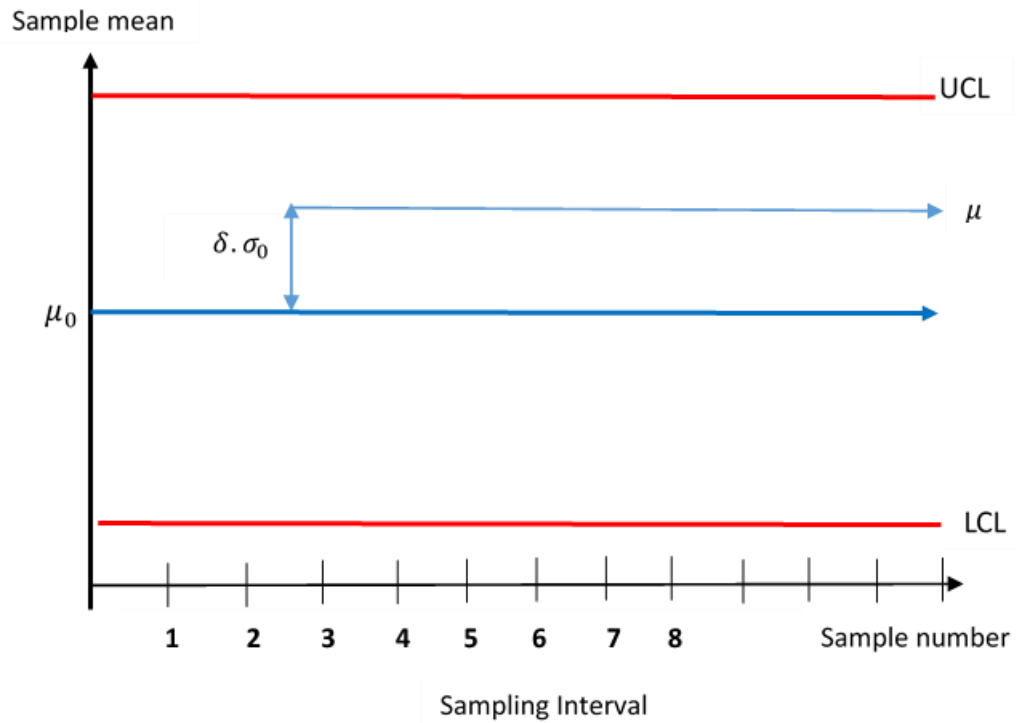


Figure 1. 1: The mean displacement and a drift of the standard deviation.

1.2 Production systems

Production systems are methods, procedures, or arrangements of elements that an enterprise uses to produce products or services to satisfy a given demand. In general, production includes all functions (processes) required to transform inputs of raw materials into the outputs of finished products or services aimed to satisfy the client's requirements, including delivery, quality, quantity,

price, and other conscious objectives. There are four main components of the production system as shown in the Figure 1.2 below.

1. Inputs which consist of raw materials, semi-finished products, machine parts, data, and other instructional resources.
2. Conversion Process which includes production planning, inspection, testing, storage, and transportation of products/Services. These involve all procedures and tactical operations (Industrial, chemical, or mechanical) aimed at transforming inputs into desired outputs.
3. Outputs could be in form of tangible or intangible goods, respectively products or services.
4. Control is the design and implementation of a well-organized function aimed at achieving overall production objectives.

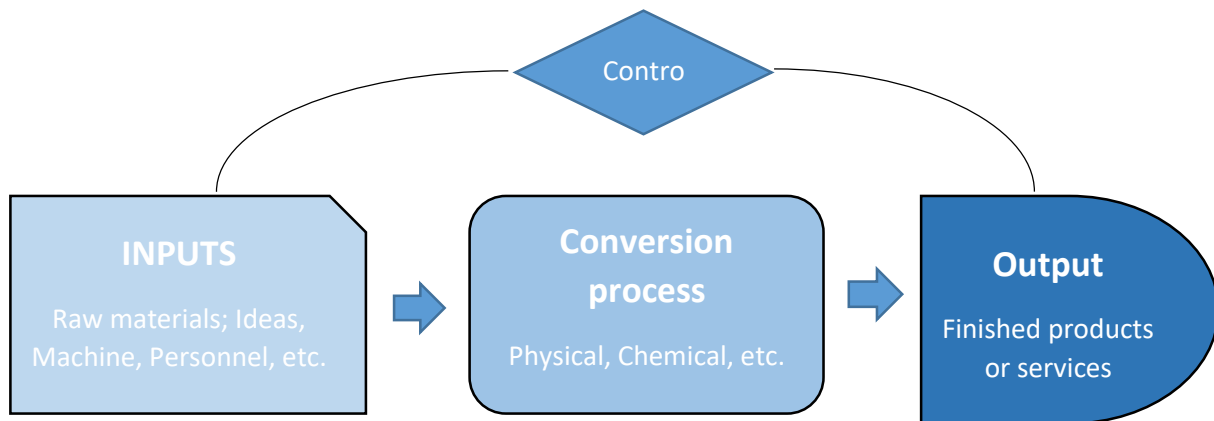


Figure 1. 2: Production System Components.

1.2.1 Objectives of production systems

In the context of increasingly fierce global competition, all production companies need to think about improving their production system through performance optimization techniques. In general, with the aid of resources; materials, funds, and all forms of infrastructure, the production system is designed to produce marketable outputs necessary to satisfy needs. This includes all functions (processes) required to transform inputs of raw materials into the outputs of tangible finished products or services aimed to satisfy the client's requirements, including delivery, quality, quantity, price, and other conscious specific objectives.

1.2.2 Classification of production systems

From the available literature, production systems can be classified into three categories:

1. Discrete state and discrete-time production system
2. Discrete state continuous-time production system
3. Mixed state and constant time production system

A discrete state production system is one in which discrete products are made. Also, the machines can be either in an operational or a failed state. A mixed-state model is characterized by a continuous buffer stock level and discrete-state machines. When a production system is a discrete-time, time is divided into intervals of equivalent length, and these time intervals must be sufficiently short of approximating a natural production system. Production line models can be homogeneous or inhomogeneous. The former means that the execution time of the machines is the same for all Machines, while the latter means that the execution time of the Machines is different for all Machines. (Gershwin 2002).

Another broad classification is a continuous and intermittent production system.

A- Continuous Production System is that which operates continuously with no irregularities or common halts. So, within the side, the non-stop production system, products are constantly produced according to the forecast call. The production does not longer rely on customers' demands. So, items may be made on a massive scale and be either stocked, sold, or both. The forms of non-stop production machines include: Mass manufacturing flows and Process production flows. In this system, the management mechanism isn't complicated as the intermittent system. In fact, in a non-stop system, massive portions of standardized products have produced the usage of the standardized production process. So, to recapitulate, the primary traits of a non-stop production system are indexed as follows:

1. The float of production is non-stop. It isn't intermittent
2. The products are standardised
3. The products are produced on predetermined quality standards
4. The products are made in anticipation of demand
5. Standardized routing sheets and schedules are prepared.

B - Intermittent Production System The adjective intermittent means something that begins off evolved and forestalls at irregular intervals (time gaps). In the periodic production system, the production relies upon customers' orders. These items are produced on a small scale. The float in output is intermittent (abnormal). Otherwise, the production float is disruptive, and numerous products may be made. Characteristics of products continuously change in step with the layout and size of the desired product from the customer. That is why this type of production system is characterized by using its flexibility. An intermittent production system may be used to produce the products where the essential nature of inputs modifications with the change within side the product's design, and the production method calls for non-stop adjustments. Ample storage among operations is required to perform those individual operations independently for similar usage of men and machines—non-stop adjustments.

Ample storage between processes is required to achieve those separate operations alone for comparable use of men and machines.

Examples of intermittent systems are machine shops, hospitals, standard offices, etc. To recapitulate, this form of the production system is generally characterized by:

1. The float of production isn't continuous.
2. It is intermittent
3. Wide types of products are produced
4. The quantity of production is small
5. General purpose machines are used. These machines may be used to produce unique kinds of products
6. The collection of operations is going on converting as consistent with the layout of the product
7. The product's quantity, size, shape, layout, etc., relies upon the customer's orders.

1.2.2.1 Resource-based typology

A Resource Topology encompasses the whole thing of resources (the resource set) with kinds and portions of types and their relationships of ways they may be connected. It is built based on dynamic capabilities to advance knowledge of how companies can manage disruptive external changes. The typology is based on three dimensions that align with the disaster life cycle, source of resources, and deployment of resources (Jiang et al., 2022).

1.2.2.2 Demand-related typology

The principal argument today is that the consumption system in the territorial organization should be more broadly taken into account in today's conceptual frameworks of economic geography or regional studies. The increasing importance of cultural industries, of the cultural dimension of industrial products in value creation (not only design and marketing but also for example aesthetics and image which are an integral part of the end product), give rise to highly complex consumption systems (e.g., specialized media, event organizers, legitimizing third parties, experts, all sorts of labelling and associated services) that mediate production-consumption processes and participate directly, or indirectly, in the creation of knowledge resources. The Production-demand relation has been addressed in different ways by literature. However, it is worth mentioning that the market is an aggregation of exchanges between quantities of supply and of demand for various goods that converge naturally and mechanically toward a general equilibrium mediated by prices. On the other, various critics have been made towards that former conception of markets and have argued that markets do not reveal a natural process of convergence toward equilibrium but rather a constant process of differentiation between different actors that are embedded in particular networks (White 2005), coordinated around particular conventions (Favereau et al., 2002).

1.3 The supply chain in the production system

Supply Chains are essential, especially with today's global situations loaded with uncertainties, we are witnessing an unprecedented breakdown of supply chains across the world today, as companies and businesses are struggling to keep their supply chains working. The supply chain in the production system is a competitive tool that has the capacities and capabilities of ensuring short terms as well as long terms advantages that foster end-to-end production system improvements, from raw material supply to finished product delivery. Supply Chain management improved overall production System performances, reduces costs, and ensures markets as well as Customer relation sustainability which are essentials for present-day global competition. Production Logistic System can be optimized with the support of a mathematical model in conjunction with a statistical process control chart. In this research, we consider a multi-warehouse chain network consisting of a principal manufacturing store (S) linked to the production system (M) on one hand, and the multi-retails warehouses (W_i) on the other side. Under disruptive (Random M Failures & Random Customer Demands) over a finite horizon. In this era of competition among supply chains, the success of a corporation is increasingly dependent on management's ability to integrate the company's networks of business relationships (*Surviving Supply Chain Integration*, 2000).

1.3.1 Supply chain

Supply chains are a set of industrial activities; planning, installation, operations, and control. It comprises monitoring, evaluations, and decisions about inventory, demand, and delivery linked to the distribution, retails/Suppliers, and Customers (Xu et al 2009). An effective supply chain is a competitive advantage for firms helping them to be capable of environmental turbulences. A supply chain is a network of suppliers, production, and distribution centres and channels between them configured to acquire raw materials, convert them to finished products, and distribute final products to customers. Supply chain network design is one of the most important strategic decisions in supply chain management. In general, network structure decisions contain setting the numbers, locations, and capacities of facilities and the quantity of flow between them (Hosseini et al., 2014).

1.3.2 Life cycle

This is an end-to-end study and analysis of a product, infrastructure, system, or service. Beginning with procurements, installation, operations, maintenance, and disposal. LCM is a business management approach that can be used by all types of businesses (and other organizations) to improve their sustainability performance. A method that can be used equally by both large and small firms, its purpose is to ensure more sustainable value chain management. LCM can be used to target, organize, analyse and manage product-related information and activities (Remmen et al, 2007) towards continuous improvement along the product life cycle. LCM is about making life cycle thinking and product sustainability operational for businesses that are aiming for continuous improvement. These

are businesses that are striving towards reducing their footprints and minimizing their environmental and socio-economic burdens while maximizing economic and social values.(Sonnemann & Editors, n.d.). Life Cycle Management is connecting various operational concepts and decision support tools. It aims to assist them to manage the growing demand and expectations in the field of Life Cycle Management.

1.3.3 Waste Management

Waste is more than just a symptom of an all-too-human demand for meaning or a merely technical problem for the likes of sanitary engineers and public health officials. From a production system point of view, any product, substance, or service that is no longer suited for its intended plan, schedules, or use fails to conform with the set standard and is said to be waste. An inevitable consequence of development and industrial progress is the generation of waste from an environmental point of view. Waste Management is devoted to the presentation and discussion of information on waste generation, characterization, minimization, collection, separation, treatment, and disposal, as well as manuscripts that address waste management policy, education, and economic and environmental assessments (Siddiqui et al., 2022).

1.3.4 Reverse logistics

Reverse logistics, a fairly new concept in logistics, has gained increasing importance as a profitable and sustainable business strategy. The process of planning, implementing, and controlling the efficient, cost-effective flow of raw materials, in-process inventory, finished goods, and related information from the point of consumption to the point of origin to recapture or create value or proper disposal (Tibben-Lembke & Rogers, 2002). The strategic factors consist of strategic costs, overall quality, customer service, environmental concerns, and legislative concerns. The operational factors consist of cost-benefit analysis, transportation, warehousing, supply management, remanufacturing and recycling, and packaging (Dowlathshahi, 2000). The reverse supply chain problem is one of the concepts in the supply chain which as well explore the waste management of the customer. In reverse supply chains, wastes are recovered and reproduced leading to the profit-making of new products which are produced after transferring by the costumer send to a production station, sorting station, and different manufacturing processes (melting, forging, clamping, painting ...) for reproduction. After the completion of several production processes, diverse products are resent to customers. In this study, we aimed to increase the reliability of the process thereby reducing the non-conformal products (wastes). Considering different production functions, and cost factors, the mathematical model of optimizing total production cost is developed. A solution is a useful tool in strategic decision-making for production control.

1.3.5 Reverse logistics processes

The reverse logistics process usually involves returns, recalls, repairs, repackaging for restocking or resale, recycling, and disposal. Reverse logistics comprises the sector of supply chains that process anything returning inwards through the supply chain or traveling 'backwards' through the supply chain. Traditional logistics involves direct order fulfilment, hub services, pick-and-pack services, and shipping. The reverse logistics process is a multi-stage logistics network including customer, collection, disassembly, refurbishment, and disposal centres. In the reverse flow, returned products are collected in collection centres and after inspection, the recoverable products are shipped to disassembly facilities, and scrapped products are shipped to disposal centres. With this methodology, intemperate transportation of returned items (particularly rejected items) is avoided and the returned items can be dispatched specifically to the fitting offices. After the refurbishment process, the refurbished parts are conveyed to clients as new parts. A predefined demand rate for each client zone is expected to result in return items and a predefined esteem is decided as a normal disposal rate. The normal disposal rate is related to the quality of returned items; since tall quality returns have a capability for recuperation handle (remanufacturing and de-manufacturing) and low-quality returns ought to be entered into a secure disposal process. Under the above situations, the remanufacturing company is interested in minimizing total remanufacturing cost so that eventually it can maximize total profit. To achieve the goal, while meeting part demands from manufacturing centres, the company determines how many returned products should be thrown into the remanufacturing process such as refurbishing and disassembling for 'as new condition. The other issues are to choose the location and determine the number of collection, disassembly, refurbishment, and disposal centres and determine the quantity of flow between network facilities. The following notation is used in the formulation of the proposed model.

In recent times, reverse logistics has become a key component of any successfully streamlined company's supply chain. In a warehouse environment, reverse logistics pertain to any of the following activities after the initial purchase:

- Returns
- Remanufacturing
- Refurbishing
- Unsold goods
- End-of-life
- Delivery failure
- Equipment rentals/leasing
- Equipment repairs/maintenance

1.3.6 Decision levels

Decision-making in production required to make a series of strategic and tactical decisions in the production functions; They plan, organize, direct, and control process activities from inputs to finished products or services. Production and operations management involves three main types of decisions, typically made at three different levels.

1. Production planning,
2. Production control, and
3. Production and operations improvements.

1.4 Maintenance in manufacturing and remanufacturing systems

The role of the maintenance function in a company (whatever its type and sector of activity) is, therefore, to guarantee the most significant possible availability of equipment at the best performance while respecting the allocated budget. (Alhouaij, 2010). The objective of maintenance in manufacturing and remanufacturing is essentially to keep the production system in its best operational condition, optimize overall performance, and minimized costs.

1.4.1 Maintenance: definition and history

According to the AFNOR, using NF X 60-010 standard, Maintenance is defined as "all the actions that make it possible to maintain or restore an asset in a specified state or in a position to provide a specified service". (NF X 60-010). A definition of Maintenance has been given by the standard NF EN 13306. It is presented as "all the technical, administrative and management actions during the life cycle of an asset, intended to maintain it or restore it to a state in which it can perform the required function". (NF EN 13306). It is a transversal activity since it is in service for all interrelated functions to ameliorate their sustainability. Therefore, maintenance engineering is nowadays a very important function in the industrial context and requires strategic planning actions for organizational process improvements as shown the figure 1. 3 below.

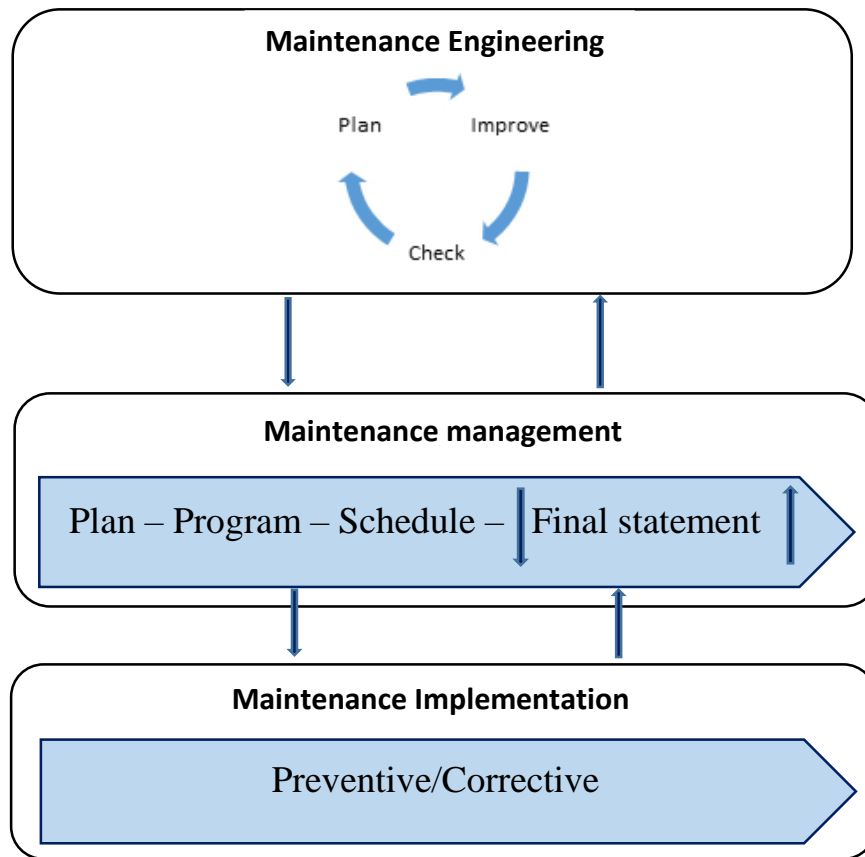


Figure 1. 3: Role of maintenance engineering in the maintenance organization process

1.4.2 Maintenance categories

Depending on the situation and conditions in which the equipment is to be maintained, we can distinguish two types of Maintenance:

Corrective Maintenance: is based on system failure which consists of carrying out maintenance tasks after a breakdown has been detected and is intended to restore a system to a state where it can perform its functions (Doyen & Gaudoin, 2006). Brown and Proschan (Brown & Proschan, 1983) stated that the application of corrective maintenance tasks returns the system either to the AGAN state 'as good as new (perfect repair) with a probability ' p ' or to a state where the system works again but retains an age that is equal to the age of the system at the time of the failure occurrence (imperfect repair) with a probability ' $q = 1 - p$ '.

Preventive Maintenance: This is a regularly scheduled practice that generally involves carrying out maintenance tasks before the occurrence of a failure to the system running smoothly and to reduce the

probability of equipment failure or degradation. (Smith & Hinchcliffe, 2003) describe the confusion that often arises between corrective and preventive Maintenance and shows that this is due to the old maintenance strategies still used by manufacturers, which consist of acting only when a breakdown occurs (corrective Maintenance). According to Smith and Hinchcliffe (Smith and Hinchcliffe, 2003), preventive Maintenance is then defined as the performance of previously planned maintenance tasks at well-defined time intervals (T) to maintain the proper functioning of an equipment or system.

However, it is important to mention that preventive maintenance actions at a given period T and corrective maintenance actions in case of failure can be *minimal*, *perfect*, or *imperfect*, depending on the maintenance activities carried out.

Maintenance actions

The state of the equipment after a maintenance action gives us an idea of the type of action performed as shown in Figure 1.4 below. In the literature related to maintenance reliability, three main types of maintenance action are distinguished. Perfect, imperfect, and minimal maintenance (or minimal repair). Based on literature studies, we detailed below these types of maintenance:

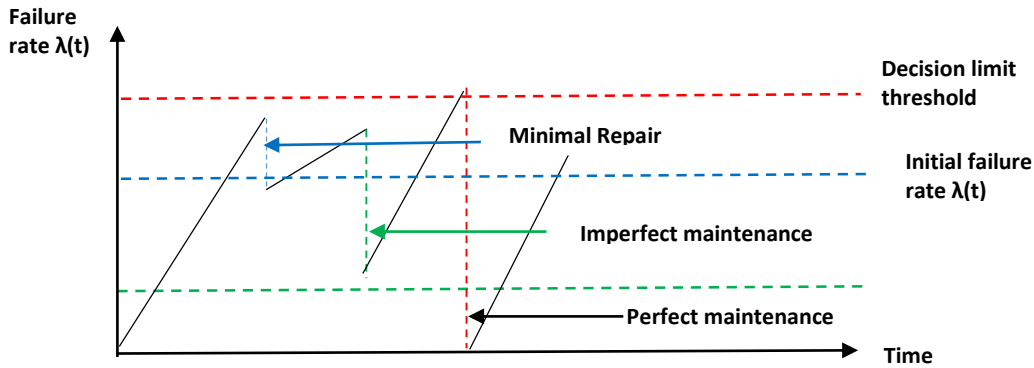


Figure 1. 4: Maintenance actions.

Perfect maintenance

It is expected that after a perfect maintenance action, the machine or system is returned to a state described as "As Good As New" (AGAN), a notation often used in the literature. In this case, the life distribution and failure rate function and the residual life are the same as those of a new system or this system at its first date of commissioning (Wang & Pham, 2011).

Minimal maintenance

This type of maintenance action sometimes referred to in some literature as "palliative maintenance", consists of restoring the system to an operational state, and more precisely, the state before the failure. In other words, we only repair what is necessary to get the system back into operation. This state is known in the literature as "As Bad As Old" (ABAO). It should be noted that

Barlow and Proscan, (Pohl, 2010) were the first to analyse this type of maintenance. In this type of maintenance, after minimal maintenance action, the failure rate remains unchanged. A good example of this type of maintenance is changing an electrical fuse of an electrical circuit or a fan belt for an engine can be considered as minimum repair actions.

Imperfect maintenance

This type of action is between "As good as new" and "As bad as old". The system is not restored to a new state but to an intermediate state. The tuning of an engine is a classic example of an imperfect maintenance action because it does not restore it to a perfect state but considerably improves its performance. Some causes may lead to this type of maintenance, such as replacing the wrong or an imperial part, partial repair of the defective part, partial or total repair of the damage to adjacent parts, or not acting on the real cause of failure, and finally on the choice of failure and finally on the timing of the intervention (Whitaker & Samaniego, 1989). There may also be the notion of the unsatisfactory quality of the maintenance technician or the unavailability of spare parts (Samet et al., 2010). In the literature, imperfect maintenance has been studied. In this study analytical methods have been used to mathematically model this type of maintenance.

1.4.3 Maintenance policies

To ensure the proper functioning of production equipment, it is essential to put in place a maintenance policy that guarantees the system's reliability while reducing costs and expenses. Maintenance policies have been the subject of many widely referenced reviews.

There are several classifications of maintenance policies found in the literature.

1.4.3.1 Age-type maintenance policy

Originally proposed by Barlow and Proshan (Enderlein, 1966) and known in the literature as the "Age Replacement Policy" (ARP). This type of maintenance is the most common maintenance practice. It replaces the production system after a predetermined preventive maintenance or as soon as a failure occurs. In initial studies, the existence of optimal age conditions for preventive maintenance was preventive maintenance, which is: a percentage of failures and repair costs higher than maintenance costs. Later, an age-type strategy was developed after accounting for very incomplete maintenance. Several modifications have been proposed, resulting in a significant number of interesting extensions. Now, this strategy consists of performing preventive maintenance at a given age (T) or a corrective procedure in case of failure. The age policy is more economical but has difficulties with planning.

1.4.3.2 Block-type maintenance policy

This was first proposed by (Nakagawa, 1979) and is known in the literature as the block replacement policy (BRP). In this strategy, units are replaced after each periodic failure, regardless of

the age or failure history of the production system. As with age-type strategies, planned replacements cost less than post-stop replacements. This policy is commonly used. An extension of this policy is known as the Minimum Repair Block Policy. This policy consists of replacing units at a specified frequency and performing minimal or palliative repairs in the event of failure, giving rise to the concepts of minimal repair and incomplete maintenance. (Barlow and Hunter, 1960). Several extensions have been discussed in the literature, and some modifications have been made to this strategy. For example, Nakagawa (1981) presented his three modifications of the block with minimal restoration. These changes were made to be as close to reality as possible. The Block policy is less economical than the Age policy but is simpler from an industrial scheduling point of view.

1.4.3.3 Failure rate based maintenance policy

This strategy assumes that preventive maintenance is performed only when the failure rate reaches a given limit and that in the event of a failure that does not reach the failure rate, repair actions will be carried out. Other studies have addressed the problem of maintenance strategies based on failure rate limits (Malik, 1979), (Pham and Wang, 1996), (Monga et al., 1997), (Sarkar et al., 2012), (Wang, 2012), and recently, (Volf, 2021).

1.4.3.4 Cost of maintenance action (duration) policy

A conservation strategy based on the cost (duration) of repairs is to make decisions according to the estimated cost of repairs. This policy was introduced in Gardens and Nonant (1963) and Drinkwater and Hastings (1967) as cited by (Sarkar et al., 2012). The downside of this policy is that the decision is repair-dependent. Therefore, if the repair cost per unit of time does not exceed a predefined limit, repair action will be taken, otherwise, the system will be replaced. (Nakagawa, 1979) studied maintenance strategies based on limited repair periods. If the duration of the repair action exceeds the limit, a replacement activity will be carried out. Otherwise, the system will be preserved. There are two types of cap policies: cost caps and duration caps.

1.4.3.5 Number of repair-type maintenance policy

According to (Morimura, n.d.), the number of repairs is an important criterion for this strategy. In their study, the decision variable is the value of k . In fact, after the first $(k-1)$ errors, minimal remedial actions fail and minimal remedial actions are taken. Then, after the k th failure, a replacement action is taken. In their study, the decision variable is the value of k . Several works have dealt with the problem of maintenance strategies based on limiting the number of repairs and several extensions have been proposed. Morimura, (1970) proposed an extension by adding a new variable T (reference time). According to this new policy, and like the basic strategy, the first $(k - 1)$ failures will be corrected with minimal repair action. If the k th failure occurred before the cumulative uptime T , it will be corrected with a minimum repair action and a replacement action will be performed after the next $((k - 1)^{th})$

failure. But if the k th failure occurs after the cumulative uptime T , a repair action will be performed. The decision variables for this strategy are k and T .

1.4.4 Joint Production-Maintenance planning

Integrated modelling is not simple, as it usually involves the application of multi-criteria to deal with the multi-objectives problem. It is essential to collaborate between production planning to meet customer requirements and the maintenance plan to avoid failures. However, the execution of these products coupled with maintenance tasks requires tactical planning of resources. Note that the development of production-integrated maintenance policies began in the 1990s, as can be noted in the literature. There are several works carried out in this direction, citing the work of (Ben-Daya & Makhdoum, 1998) investigates three different preventive maintenance policies, taking as optimizing the estimated total cost. In the study of developing a model that combines total production and the sum of estimated maintenance costs divided by the production cycle time. This study showed that the implementation of preventive maintenance results in a reduction of total control costs. (Weinstein & Chung, 1999) proposed a model to find a production schedule minimizing the total costs of setup, production, inventory, and maintenance. The production plan is obtained by minimizing the difference between the costs realized by the normal production system and the integrated production system. (Mohamed-Salah et al., 1999) presented a model for the simultaneous optimization of production flow and preventive maintenance activities, considering the interaction between production and maintenance services. The maintenance strategy of this model is based on the age of the machine producing a single product and the inventory of the production system to meet the constant demand. First, they determined the frequency of preventive maintenance on the machine, minimizing total maintenance costs. Second, they determined average inventory costs and unsatisfied demand. By combining these two costs, they were able to minimize the total cost function, which allowed them to find the optimal inventory and the optimal maintenance schedule. (Ait-El-Cadi et al., 2021) studied a repairable production unit subject to random breakdowns and operating with the "just-in-time" strategy. Preventive maintenance actions are conducted at times T , $2T$, $3T$..., then preventive and corrective maintenance with random durations. A buffer stock is set up to ensure the continuity of production at a rate β and compensate for the disruptions caused by breakdowns and planned maintenance. The strategy proposed by (Ait-El-Cadi et al., 2021) consists of building a buffer stock of capacity 'S' at the beginning of each cycle of preventive maintenance and covers the demands during the periods of maintenance when the unit is in stop. (Rezg et al., 2004), decided to study the case where a production line consists of n machines that do not contain buffer storage. The aim of their work then was to find an integrated maintenance strategy to prevent demand reduction based on cost minimization (storage costs, reduced demand costs, and maintenance costs). In the production strategy adopted by the authors, production occurs at the u_{max} rate until the last maximum inventory level Q is reached, and from that moment, production is made to order. In the event of a cable failure, the production line is interrupted and the requirements are fulfilled by the warehouse. We can also mention the work (Cassady & Kutanoglu, 2005), which aimed to minimize the weighted total delay of

all tasks performed by a single machine. Their work is summarized by optimizing the order of n jobs to be performed, optimizing the intervals of preventive maintenance to be performed, and finally integrating the production and maintenance to achieve the desired goal. The numerical results of this study show that the weighted total delay was reduced by an average of 30 percent. In another work, the authors (Sortrakul et al., 2005) integrated production planning and preventive maintenance to minimize the weighted completion time of all machine jobs. Indeed, their study compares the integrated solution with two solutions, production planning and preventive maintenance planning (independently). This study showed that the integrated solution has a 2% advantage (profit) over the other two solutions. The author considers this percentage, although small, high at the industrial level. (Aghezzaf et al., 2007), investigated the compromise between production strategy and maintenance policy during a finite time interval to minimize the total costs. The system is subject to random failures, and each maintenance task reduces its production capacity during the maintenance period. A minimum repair is employed under system breakdown, and a periodic replacement is performed every given fixed period. Based on an iterative solution and applying an algorithm to the problem at hand (using the CPLEX solver to solve the production plan problem), this work found an optimal production plan that minimizes production and maintenance costs. Also worth mentioning is the work done by (Kenne & Nkeungoue, 2008) who propose a stochastic optimization of connected manufacturing taking into account corrective and preventive maintenance. The authors proposed a method to optimally determine the periodicity and rate of preventive maintenance for identical machines in production lines. (Nakagawa, 2005) developed two maintenance strategies for an age and block-type production system over an infinite time horizon. (Nakagawa & Mizutani, 2009) reformulated the problem studied in (Nakagawa, 2005) considering the case of incomplete preventive maintenance initiatives. (Schutz et al., 2011) chose to integrate the concept of system operation and environmental conditions into production (business plan) and maintenance planning. The authors of this work then chose a system open to random failures, which performs different tasks (M) in a finite time horizon. Each task has well-defined characteristics that depend on the operation and environmental conditions. This study, which is about creating a business plan to maximize profits, was based on two genetic algorithms; the first algorithm is used to determine the optimal production plan and the second is to create a maintenance plan. (Nourelfath et al., 2012) studied set size problems in a parallel multi-machine system. The purpose of this work is to minimize the cost of preventive and corrective maintenance, installation, and production to meet the requirements of a limited period of operation. (Yalaoui et al., 2014) studied a production system with decreasing production capacity over the operating horizon and defined production and maintenance plans for that system. (Zhao et al. 2014) developed an iterative method that solves a single-machine system problem. (Cui et al., 2014) developed a model that jointly optimizes the production and maintenance schedules of a single-machine system using a heuristic approach based on three optimization steps. In the same context, (X. Liu et al., 2015) looked at a single machine system that produces several different types of goods and requires preventive maintenance planning to study their Economic Product Quantity (EPQ) model. During production, preventive maintenance measures are implemented between the two types of goods. (Hansen et al., 2016) also studied a case where a single machine system producing several different types of goods has random failures. Those

who do this work minimize the total cost of production and maintenance by considering inventory, customer requirements, equipment installation, and minor repairs in case of service constraints (customer satisfaction). In our work, we proposed a mathematical model that helps find an integrated strategy that will minimize the total production costs. The authors developed a model that jointly defines production location sizes and optimal maintenance policies. (Beheshti Fakher et al., 2017) integrated the multidimensionality problem into preventive maintenance planning, considering the quality aspect of the system, and using a genetic algorithm to solve the problem. (G. Q. Cheng et al., 2018) aim to address the issue of jointly optimizing production and maintenance planning considering production capacity and service level constraints. Recently, (Guo et al., 2022) studies the joint optimization of non-cyclical preventive maintenance and capacitated lot-sizing with lost sales. The objective is to determine the optimal lot size in each period, decide the optimal preventive maintenance for servicing machines over a planning horizon, and minimize the total cost related to service, operation, setup, production, inventory, and lost sales. The methods depend on three variables; the production planning involving the production rate U and the inventory quantity level Q_i at each of the multi-ware houses, then the determination of how many times N^* a preventive maintenance action is optimally carried out.

1.5 Quality in manufacturing systems

The unreliable production system is prone to random failure which directly impacts product quality. The variance in production rate, inventory management, and utilization affects the status of the production system and its maintenance evolution. Despite all preventive strategies available to eliminate the occurrence of deviations, additional reactive quality inspections are required. Therefore, companies now seek to take advantage of the constant technological development to explore new methods that are both cost-efficient and easy to improve Quality, (Bergman, 2021). A significant aspect of quality control is the establishment of well-defined controls.

Quality key points.

- Quality control is a process to ensure that product quality is maintained or improved.
- Quality control involves product testing, or inspection and gauging with standards.
- Quality control is dependent on the type of product or industry, and several tools of measurement.
- Quality control creates safe practices to ensure no quality products do not reach the customer as defined by ISO 9001-1996.

Quality is free, yes it is free but it's not a gift. It is the non-quality that costs (Phillips Crosby). Interestingly quality applies to all kinds of enterprises be it educational, engineering, construction, mining, manufacturing, or services. It must be applied to every enterprise.

1.5.1 Quality: definition and history

Quality can be defined as the standard of something or a measure of compliance to some given specifications. Product quality can be measured objectively by the number of defects, and non-conformal number (degree) while customer satisfaction is based on the subjective judgment regarding both service and overall product impression.

1.5.2 Quality Tools: 6 Sigma and SPC

Finding the right tools for the job significantly helps companies to solve problems and reap benefits such as improving process variation, product quality, and waste reduction. A Six Sigma control chart could be a simple, but effective device for assessing the soundness of a handle or operation over time. Making a control chart requires a chart that covers a period, a centreline that appears the comes about of a handle amid that time, and upper and lower control limits that demonstrate whether handle variety is to an acknowledged extent.

Six Sigma is a powerfully effective methodology for quality improvement, regardless of industry or vertical. Here are four of the most important of the Six Sigma statistical tools:

- Control charts,
- Histogram,
- Pareto Chart, and
- Failure Modes and Effects Analysis (FMEA).

While this is not an exhaustive list of Six Sigma tools, it does highlight some of the most critical ones' organizations may use to improve products, services, and processes as illustrated in the Figure 1.5 below.



Figure 1. 5: Six Sigma.

Control Charts

A control chart is a decision support document during the production process to record the results of the sampling control collected at the workstation. It allows us to visualize graphically the dimensional variations and to determine the right moments for an adjustment. The objective of the Control Chart allows the detection of any drifts (defects) in production, to intervene before producing non-conforming parts. Thus, it allows to increase in the quality level and therefore reduces the costs of non-quality.

The control chart is a graph used to study how a process changes over time. Comparing current data to historical control limits leads to conclusions about whether the process variation is consistent (in control) or unpredictable (out of control, affected by special causes of variation). Data are plotted in

time order. A control chart always has a central line for the average, an upper line for the upper control limit, and a lower line for the lower control limit. These lines are determined from historical data. One of the fundamental tools of *Six Sigma's process* improvement methodology is the *control chart*. It is an instrument or technique to support and improve the activities of quality management and improvements. It helps to identify causes understand the processes, collect, organized, and analyse data to generate ideas, keep records and keep projects on track. Control charts have two primary uses in an improvement project. The first and most common application is to monitor process stability and control. The second, and not quite as expected, is using it as an analysis tool. The *control chart* is responsible for monitoring the variation of critical characteristics and providing diagnostic data about process capability. This data will then be used to analyse variation in-process data to uncover any process defects. Control charts help production managers make informed decisions for all continuous improvement activities. Once *statistical formulas* have been applied to the data gathered from the process, the control chart's upper and lower control limits are calculated. Data points that fall outside this range represent the process variations caused by irregularities and can be identified and eventually eliminated. These charts are highly effective in preventing future defects, and they help ensure process consistency. Using this tool, an organization can monitor, control, and improve processes over time precisely, which lowers costs and improves efficiencies.

Statistical monitoring is based on the interpretation of the results recorded at the workstation by the operator. It consists of taking a representative sample from a batch of products (population and checking certain characteristics of all the individuals in the sample. The result of the control, for each sample, is plotted on a graph which is called a control chart as shown below in Figure 1. 6

Statistical monitoring should allow:

- intervene in the process before non-quality is produced.
- measure the "capability" of a process: the process is capable of producing good parts.
- Act on variations to ensure the stability of the process over time.

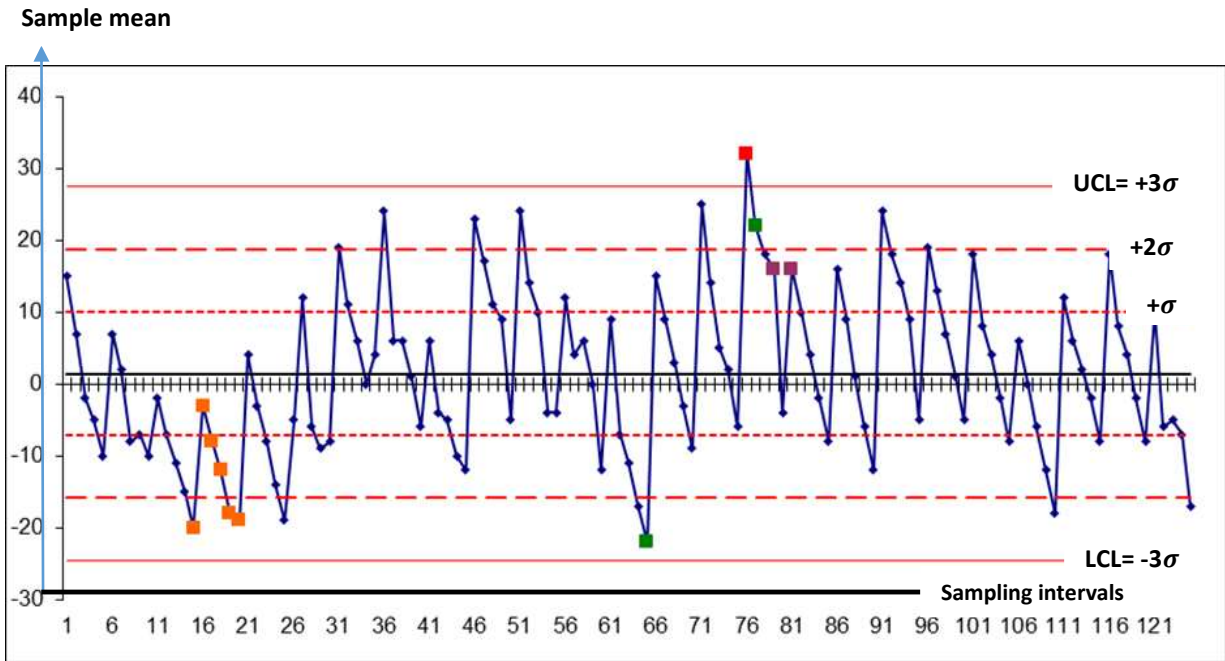


Figure 1. 6: Control Chart_ Out-of-Control Signal

1.5.3 Production and Quality

Providing products with consistent quality is at the forefront of any industrial company's priorities. And it's no surprise: consequences when facing a quality drop are multiple, ranging from an increase in production costs, pressure on supply chain management, an increase in waste generation, and reduced sales down to fatalities and serious injuries. Overall, as quality drops alone can sustainably damage customer trust and company brand reputation, they demand tight surveillance. The link between quality improvement and productivity is well-established (Montgomery et al. 1985). Production systems are presently faced with an increasingly demanding customer in terms of product quality, low prices, and delivery times as well as technological progress that forces them to follow new trends and makes old production techniques obsolete (Tersine and Wacker 2000), (Lau et al. 2002), (Ho et al. 2005). There is a need for industrial companies to make effective, more flexible, and more efficient processes to produce higher-quality products at reduced costs. In this context, it becomes a must-do to shift from traditional ways to joint optimal solutions. Production coupled with Quality Control, allows factory managers, production engineers, quality engineers, and maintenance teams to quickly integrate the forecast of joint models in the surveillance of key production processes. Several production techniques have been developed in recent years that allow production lines to be configured according to customer and company needs (Hajej et al., 2018). Quality is becoming a business strategy leading to success, growth, and enhanced competitive position (Beheshti Fakher et al., 2017). Organizations with successful quality improvement programs can enjoy significant competitive advantages. With increasing automation and mechanization, production processes are shifting from workers to machines (Xiao et al., 2019). Consequently, the role of equipment maintenance in controlling quantity, quality,

and costs is more evident and important than ever (Rasay et al., 2022). To succeed in this new environment, equipment must be maintained in ideal operating conditions and must run effectively.

1.5.4 Maintenance and Quality

The link between maintenance and quality, although not completely missing, is not adequately addressed in the literature. The focus of our work is production system management and improvement through machine maintenance and product quality control methods. Quality improvement means the elimination of waste such as scrap and rework, which increases productivity and often leads to cost reductions (G. Q. Cheng et al., 2018). Final control inspection is currently being moved to the process level through adequate process monitoring and control techniques (Dutoit et al., 2019b). Consequently, defects and variations are eliminated or greatly minimized at their source, and in the process, in particular, machine performance problems are identified early on. There is a lot of evidence showing that the quality of products and services needs to improve in almost every industry. Hardly any manufacturing or service organizations create their products and services without some sort of QMS in place to ensure the customers get a quality product at a reasonable price. There is an interrelation between the production process condition and the product output quality. The maintenance activities are to ensure the reliability of the process and deliver a quality product in uptime at a reasonable price. Quality control (QC) is activities designed to ensure that desired quality levels are achieved by the process. It takes both types of activities to reach desirable quality goals. Well-designed maintenance integrated into quality control will enable industrial companies to do that. While the importance and benefits of Quality management are clear in maintenance, it is in the implementation that often where many companies get hung up (Hadian et al., 2021). Although it is clear that maintenance affects quality, this relationship needs to be modelled and measured, analysed and improved. Models relating maintenance to product quality have not been adequately developed in the literature. Excessive maintenance results in unnecessary costs. On the other hand, if the equipment is not well maintained this will lead to breakdowns and results in defective products (Ben-Daya & Duffuaa, 1995). We are interested in models that determine the optimal maintenance schedule which minimizes the total production cost and ensures a high-quality product.

1.5.5 Integrated maintenance strategies and quality

Increasing emphasis on sustainable production requires preserving the efficiency of degrading resources over time. Additionally, maintenance planning must consider the interactions with quality control and production planning, since they are fundamental functions for economic success in production companies. Several works have considered different and sometimes simplified production systems to solve production planning problems. The earliest works in the literature were concerned with one-machine systems (S.E. Elmaghraby 1968), (H. Emmons 1969), (Held and Karp 1962), (Lawler, 1963), (Srinivasan 1971), (Baker and Martin 1974), (Fisher 1976). Other works have been interested in systems closer to reality composed of several machines mounted either in series or parallel. Brandolese

et al (1996), (Ouali et al (2002), Kenne and Gharbi (2004), Kenne and Nkeungoue (2008). Recently, the integration of production, sampling inspection, and age-based maintenance planning for an unreliable production system subject to gradual deterioration(Rivera-Gómez et al., 2021) The optimal joint production, maintenance, and product quality control policies for a continuously deteriorating manufacturing system investigated by (Megoze Pongha et al., 2022). The production planning field has developed methods for reducing work in progress (WIP) while meeting desired production rate levels. The Statistical Quality Control (SQC) field has introduced optimized tools for monitoring the behaviour of processes to achieve the desired product quality (Dutoit et al., 2019b). The Maintenance Management field has developed policies for preserving the efficiency of degrading resources over time by offering proactive and tactical capabilities (Megoze Pongha et al., 2022). Therefore, optimization and good planning of maintenance and quality become a necessity in optimal production decision-making.

1.6 Description of the problem

In this thesis, we focus on an industrial issue that revolves around the development of different integrated optimization of production, maintenance planning, and quality control. A supply chain comprises a production system with a single machine that produces a single type of product and a primary production store S and multi-purchases warehouse (w_0, w_1, \dots, w_L), where the customer demand (items) is met. Over a finite horizon, each warehouse strives to serve many random requests within a set quality specification limit H and with a delivery time τ_i ($i = 0, \dots, L$) For each demand, as indicated in Figure 1.7 below.

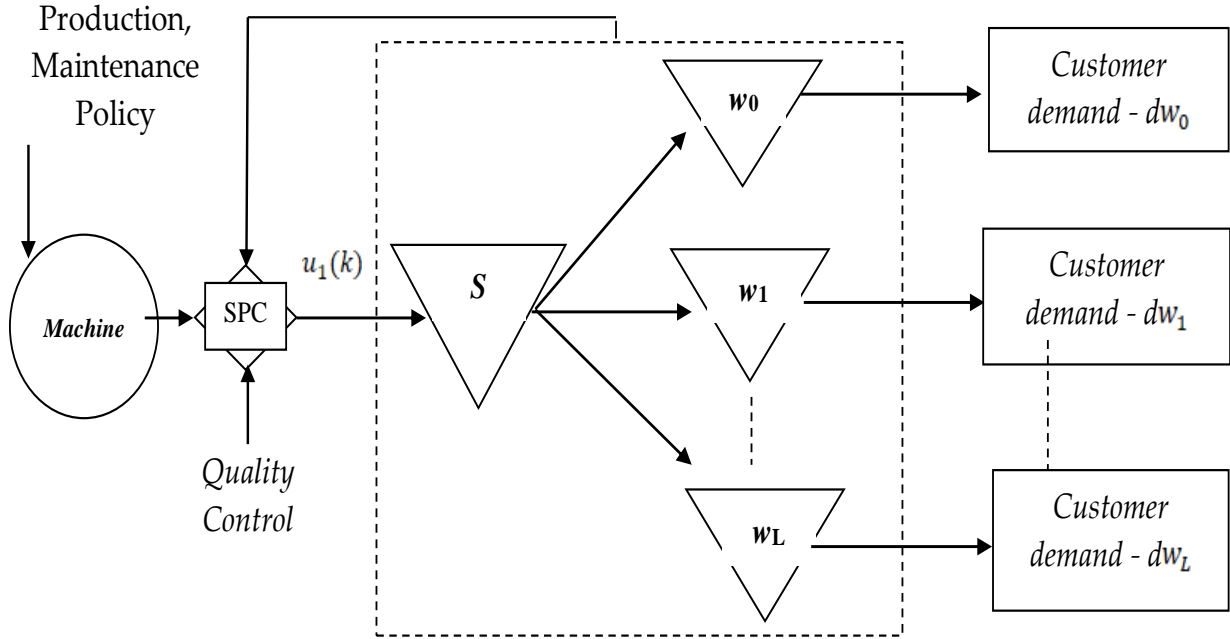


Figure 1. 7: Production System with control chart integrated into maintenance and quality.

The production machine M is prone to failure at any time. The rate of production has an impact on the machine's degradation. As a result, the failure rate $\lambda(t)$ increases with the age of the machine and the rate of production $u(k)$. It also impacts the production process's reliability and leads to the production of non-conforming products. For the sample X_t , we assumed a quantitative quality statistic X with n measurements, controlling the machine every h time unit during the production period Δt . Individual metrics are assumed to be between the UCL upper control limits and the LCL lower control limits. The measurement findings are kept on a measurement card and the initial samples were sacrificed. Assuming that the law of X_t is a Gaussian constant with known or well-estimated mean μ_0 and standard deviation σ_0 , the production process is stable and under control when all statistics fall between the lower and the upper control limits (LCL & UCL). To reduce the degradation and failure rate that depends on the machine's production rate variation, different maintenance strategies are collaborated with quality policy to achieve the research objectives. Our quality policy integrated into the maintenance strategy is driven by a control chart. When a production system is subject to failures, maintenance intervention can restore the system back to normal or a better status. From the literature review different failure impact models, we study how production machines failure affects the quality of the products. In this research we defined a new failure rates for different model problems and used to build numerical methods.

1.6.1 Motivation/issue

The increase in the requirement for advanced production decision support tools (SPC), to get the best output from the production system that can meet the present-day global instabilities, highly competitive markets, and dynamic customer requirements. And, the increasing attention paid to sustainable development around the world which become important more than ever before has compelled companies to find strategies and means to guarantee their production system sustainability (Reliability/Availability). Ameliorating the stated industrial challenges certainly requires simultaneous process and product improvements that minimized cost and maximized customer satisfaction. This thesis contributes to the development of a new integrated model of production, maintenance, and quality control problems linked to supply chain management under customer service and quality constraints to achieve the set objectives. Good quality products can affect demand and increase profits by minimizing losses, and vice versa. In fact, the quality improvement of production products with a statistical process control can save number of inspection, size of products to be inspected, inspection intervals, and other difficult trouble shooting.

1.6.2 Problem of the thesis

The problem raised in this thesis considers a production system composed of an unreliable, single machines production system to satisfy the random demand of customers during a finite production horizon, under service and quality constraints. The system produces only one type of product with a variable production rate and it is subject to degradation due to production and stock

management which can lead to failures over time. To do this, the production system must operate with a variable production rate and is linked to a supply chain management with a multi-warehouse to satisfy customer service level and minimize total production costs.

The model is described in Figure 1.8. The production process is evaluated by measuring quality indicators from the processed products with the aid of a statistical process control chart (SPC) of quality. The aims are to define, measure, analyse, and continuously improve the process reliability, and products quality, thereby reducing the non-conformal products and minimising the total production cost. In real industrial practice, the production process cannot be in a stable condition at all times, as it may begin in good condition and later shifts to instability. To have a production control, it is therefore necessary to monitor and evaluate the process states when the process is stable (in-control) producing quality products and when it becomes unstable (out-of-control) producing non-quality products.

The objective of this thesis is therefore to determine optimal production control parameters. Beginning with the determination of optimal production rates and inventory management plans throughout the finite production horizon, the product quantities to be produced and stock at each warehouse from one period to another as well as the delivery considerations. Then the second objective is to formulate the optimal maintenance and quality strategies that minimize the total costs related to production, inventory, delivery, delay penalty, and total maintenance and quality actions. The tool we used is the statistical control chart integrated into maintenance and quality. The maintenance and quality model is described as a successive optimization of two sub-models, with the first sub-model as a constraint to optimize the second sub-model. The goal is to control quality and machine failure rates by designing and developing new integrated maintenance, production, inventory, and delivery strategies under quality constraints. To optimize production and inventory while considering the impacts on maintenance, quality, maintenance, and the total expected cost. Next, it is important to define the main notations used throughout this thesis.

1.6.3 Notations

The following parameters are utilized in the mathematical formulation of the model's problem in Chapters 2, 3, and 4.

Chapters 2 and 3:

Productions parameters

$u(k)$: machine production quantity u in period k , ($k = 0, 1, \dots, H - 1$)

$S(k)$: level of S inventory at the end of period k , ($k = 0, 1, \dots, H - 1$)

S : principal store S (Manufacturing Stock).

$w_i(k)$: inventory level of w_i , ($i = 0, \dots, L$) at the end of period k , ($k = 0, 1, \dots, H - 1$) for each warehouse

τ_i : delivery duration for warehouse w_i

$Q_i(k)$: delivery quantity at period k , ($k = 0, 1, \dots, H - 1$) for the warehouses

L : ware-houses number.

Δt : production period length
 $d_i(k)$: average demand during period k , ($k: 0, 1, \dots, H - 1$) for each customer
 $V_{di}(k)$: demand variance in period k , ($k: 0, 1, \dots, H$) for each customer
 H : the production planning horizon
 $H \cdot \Delta t$: finite production horizon
 Q_v : delivery vehicle capacity.
 c_p : machine unit production cost
 c_{hs} : cost of holding inventory product units during one period at the primary store.
 c_{hwi} : cost of inventory holding unit product during one period at the warehouse w_i , ($i = 0, \dots, L$)
 c_d : a shortfall cost of one product unit during one timeframe.
 c_l : the unit delivery cost
 u_{max} : machine maximum production capacity.
 u_{min} : machine minimum production capacity.
 θ_i : probability index represented to each customer I service level degree of pleasures.

Maintenance parameters – Chapter 2

$\lambda_k(t)$: Failure rate at period k , ($k = 1, \dots, T$)
 N_1^* : The number of Preventive Maintenance actions in the control state (Scenario 1).
 N_2^* : Number of Preventive Maintenance actions in – out-of-control state (Scenario 2).
 t_p : Perfect Maintenance action duration
 mu : monetary unit.
 $P(S_1)$: the probability of being in the control state Scenario 1
 $P(S_2)$: the probability of being in the out - of - control state Scenario 2
 C_{pm1} : preventive maintenance cost for each detected cause assignable
 C_{pm2} : preventive maintenance cost during the in-control state
 C_{cm} : corrective maintenance cost
 C_{MT} : total maintenance cost

Quality parameters – Chapter 2

j : the average number of samples required to detect an 'out of control condition.
 h : interval of sampling.
 δ_j : the degree of the movement from the centreline to the state of out-of-control.
 n_{pc} : number of assignable causes detected by the control chart
 k_p : Number of standard deviations between the centreline of the control chart and the control limits.
 c_{si} : cost of sampling inspection
 c_r : the unit cost of one defective item
 c_{NC} : cost of non-conformal items
 δ_j : the degree of the movement from the centreline to the state of out-of-control.
 ACQ : average cost of quality

Maintenance parameters – Chapter 3

$\lambda_k(t)$: Failure rate at period k , ($k = 1, \dots, H$)
 N_{pef} : Number of perfect maintenance actions during the out-of-control state (Scenario 3).
 N_{imp} : Number of imperfect maintenance actions during the surveillance state (Scenario 2).

t_p	: Duration of perfect maintenance
t_{imp}	: Duration of imperfect maintenance
C_{cm}	: Corrective maintenance cost
C_{Im}	: Imperfect maintenance cost
C_{pm}	: Perfect maintenance cost
C_{MT}	: Total maintenance cost

Quality parameters – Chapter 3

h	: The sampling interval.
α_2	: The probability of non-identification of the surveillance state
α_3	: The probability of non-identification of the critical out-of-control state
ψ_1	: The parameter showing if the surveillance limits were exceeded before the control limits
j_p	: Average number of samples to detect the “out of control
j_{imp}	: Average number of samples to detect the surveillance state
δ_p	: Magnitude of a shift in between SL to the Critical state
δ_{imp}	: Degree of a shift in been the centreline to the SL
k_p	: Control chart coefficient samples at a critical stage.
k_{imp}	: Control chart coefficient of samples at surveillance zone
ARC	: The average duration of a restoration cycle
c_{si}	: The unit sampling inspection cost
c_r	: The unit cost of one defective product
c_{NC}	: The cost of the non-conformal product
ACQ	: Average total cost of quality

Chapter 4: notations

Quality Parameters:

C_s	: Sampling inspection cost of one unit
C_r	: Rejection cost of one non-quality product
C_F	: False alarm cost.
UCL	: Upper control limit
LCL	: Lower control limit
δ_c	: The magnitude of the shift to a critical state compared to the centre line
k_c	: Standard deviations of the sample distribution between mean -CL and the control limit of the control chart
σ	: Standard deviation
n	: Samples size
I_s	: Sampling interval
TCQ	: Total cost of quality

Maintenance Parameters:

C_{M1}	: maintenance cost of the in-control state (scenario 1)
α_1	: The probability of the process being in a control state (Scenario 1)
C_{ipm}	: Unit cost of imperfect maintenance in the in-control state
N_{imp1}	: Number of imperfect preventive maintenance actions during the in-control (scenario1)

T_{imp1}	: Time interval between 2 successive imperfect preventive maintenance during the in-control (scenario1)
$C_{M2.1}$: maintenance cost of only the presence of a point outside the main limits (scenario 2)
α_2	: The probability of detection of the shift to the control limits.
C_{pm2}	: Unit cost of perfect maintenance action performed when a point is detected outside main limits
$N_{imp2,1},$ $T_{imp2,1}$: Number and interval of imperfect preventive maintenance before detecting points outside main limits
$N_{imp2,2},$ $T_{imp2,2}$: Number and interval of imperfect preventive maintenance after detecting points outside main limits
RCD_2	: Average restoration cycle duration for scenario 2
$C_{M2.2}$: maintenance cost of the only presence of seven points in one direction (scenario 3)
α_3	: The probability of detection of seven points in one direction (scenario 3)
C_{pm3}	: Unit cost of perfect maintenance action performed when detecting seven points in one direction
$N_{imp3,1},$ $T_{imp3,1}$: Number and interval of imperfect preventive maintenance before detecting seven points in one direction
$N_{imp3,2},$ $T_{imp3,2}$: Number and interval of imperfect preventive maintenance after detecting seven points in one direction
RCD_3	: Average restoration cycle duration for scenario 3
$C_{M2.3}$: Maintenance cost of only the presence of 7 consecutive points falling on the same side (scenario 4)
α_4	: The probability of detection of seven consecutive points falling on the same side (scenario 4)
C_{pm4}	: Unit cost of perfect maintenance performed for seven consecutive points falling on the same side
$N_{imp4,1},$ $T_{imp4,1}$: Number and interval of imperfect preventive maintenance before detecting seven consecutive points falling on the same side
$N_{imp4,2},$ $T_{imp4,2}$: Number and interval of imperfect preventive maintenance after detecting seven consecutive points falling on the same side.
RCD_4	: Average restoration cycle duration for scenario 4
-----	<i>Conditional Out-of-Control Scenarios (2,4, X, X)</i>
$C_{M2.4.1.1}$: Maintenance cost of when scenarios 2 and 3 are present at the same time with scenario 2 before 3
$\alpha_{2,3}$: Probability of occurrence scenario 2 before 3
C_{pm23}	: Unit cost of perfect maintenance action performed when scenarios 2 and 3 are present at the same time as scenario 2 before 3

$N_{imp23,1}$: Number and interval of imperfect preventive maintenance before detecting points outside main limits
$T_{imp23,1}$	
$N_{imp23,2}$: Number and interval of imperfect preventive maintenance between detecting of point outside main limits and of seven points in one direction
$T_{imp23,2}$	
$N_{imp23,3}$: Number and interval of imperfect preventive maintenance after detecting seven points in one direction
$T_{imp23,3}$	
$C_{M2.4.1.2}$: Maintenance cost of when scenarios 2 and 3 are present at the same time with scenario 3 before 2
$\alpha_{3,2}$: the probability of occurrence of scenario 3 before 2
C_{pm32}	: Unit cost of perfect maintenance action performed when scenarios 2 and 3 are present at the same time as scenario 3 before 2
$N_{imp32,1}$: number and interval of imperfect preventive maintenance before detecting seven points in one direction
$T_{imp32,1}$	
$N_{imp32,2}$: number and interval of imperfect preventive maintenance between detecting seven points in one direction and of points outside main limits
$T_{imp32,2}$	
$N_{imp32,3}$: number and interval of imperfect preventive maintenance after detecting points outside main limits
$T_{imp32,3}$	

$C_{M2.4.2.1}$: Maintenance cost of when scenarios 2 and 4 are present at the same time with scenario 2 before 4
$\alpha_{2,4}$: The probability of occurrence of scenario 2 before 4
C_{pm24}	: The unit cost of perfect maintenance action performed when scenarios 2 and 4 are present at the same time as scenario 2 before 4
$N_{imp24,1}$: Number and interval of imperfect preventive maintenance before detecting of point outside the main limit
$T_{imp24,1}$	
$N_{imp42,2}$: Number and interval of imperfect preventive maintenance between detecting of point outside main limits and of seven consecutive points falling on the same side
$T_{imp42,2}$	
$N_{imp42,3}$: Number and interval of imperfect preventive maintenance after detecting seven consecutive points lying on the same side
$T_{imp42,3}$	
$C_{M2.4.2.2}$: Maintenance cost of when scenarios 2 and 4 are present at the same time with scenario 4 before 2
α_{42}	: The probability of occurrence of scenario 4 before 2
C_{pm42}	: The unit cost of perfect maintenance action performed when scenarios 2 and 4 are present at the same time as scenario 4 before 2
$N_{imp42,1}$: Number and interval of imperfect preventive maintenance before detecting Seven consecutive points falling on the same side
$T_{imp42,1}$	
$N_{imp42,2}$: Number and interval of imperfect preventive maintenance between detecting seven consecutive points falling on the same side and of point outside main limits
$T_{imp42,2}$	
$N_{imp42,3}$: Number and interval of imperfect preventive maintenance after detecting points outside main limits
$T_{imp42,3}$	

$C_{M2.4.3.1}$: Maintenance cost when scenarios 3 and 4 are present at the same time as scenario 3 before 4
α_{34}	: The probability of occurrence of scenario 3 before 4
C_{pm34}	: The unit cost of perfect maintenance action performed when scenarios 3 and 4 are present at the same time as scenario 3 before 4
$N_{imp34,1}$, $T_{imp34,1}$: Number and interval of imperfect preventive maintenance before detecting seven points in one direction
$N_{imp34,2}$, $T_{imp34,2}$: Number and interval of imperfect preventive maintenance between detecting seven points in one direction and of seven consecutive points falling on the same side
$N_{imp34,3}$, $T_{imp34,3}$: Number and interval of imperfect preventive maintenance after detecting seven consecutive points falling on the same side
$C_{M2.4.3.2}$: Maintenance cost when scenarios 3 and 4 are present at the same time as scenario 4 before 3
α_{43}	: Probability of occurrence scenario 4 before 3
C_{pm43}	: The unit cost of perfect maintenance action performed when scenarios 3 and 4 are present at the same time as scenario 4 before 3
$N_{imp43,1}$, $T_{imp43,1}$: Number and interval of imperfect preventive maintenance before detecting seven consecutive points falling on the same side
$N_{imp43,2}$, $T_{imp43,2}$: Number and interval of imperfect preventive maintenance between detecting seven consecutive points falling on the same side and seven points in one direction and
$N_{imp43,3}$, $T_{imp43,3}$: Number and interval of imperfect preventive maintenance after detecting seven points in one direction and
<i>Conditional Out-of-Control Scenario (2,5, X, X)</i>	
$C_{M2.5.1.1}$: Maintenance cost when scenarios 2, 3, and 4 are present at the same time as scenario 2 before scenarios 3 and scenario 3 before 4.
α_{234}	: The probability of occurrence of scenario 2 before 3 and 3 before 4
C_{pm234}	: The unit cost of perfect maintenance action performed when scenarios 2, 3, and 4 are present at the same time as scenario 2 before scenario 3 and scenario 3 before scenario 4.
$N_{imp234,1}$, $T_{imp234,1}$: Number and the interval of imperfect preventive maintenance before detecting points outside main limits
$N_{imp234,2}$, $T_{imp234,2}$: number and interval of imperfect preventive maintenance between detecting points outside main limits and of seven points in one direction
$N_{imp234,3}$, $T_{imp234,3}$: Number and interval of imperfect preventive maintenance between detecting seven points in one direction and of seven consecutive points falling on the same side
$N_{imp234,4}$, $T_{imp234,4}$: Number and interval of imperfect preventive maintenance after detecting seven consecutive points falling on the same side
$C_{M2.5.1.2}$: Maintenance cost of when scenarios 2, 3, and 4 are present at the same time with

	scenario 2 before scenario 4 and scenario 4 before 3
α_{243}	: The probability of occurrence of scenario 2 before scenario 4, and scenario 4 before 3
C_{pm243}	: The unit cost of perfect maintenance action performed when scenarios 2, 3, and 4 are present at the same time as scenario 2 before scenario 4 and scenario 4 before scenario 3.
$N_{imp243,1}$ $T_{imp243,1}$: Number and interval of imperfect preventive maintenance before detecting points outside main limits
$N_{imp243,2}$ $T_{imp243,2}$: Number and interval of imperfect preventive maintenance between detecting of point outside main limits and of seven consecutive points falling on the same side
$N_{imp243,3}$ $T_{imp243,3}$: Number and interval of imperfect preventive maintenance between detecting seven consecutive points falling on the same side and seven points in one direction
$N_{imp243,4}$ $T_{imp243,4}$: Number and interval of imperfect preventive maintenance after detecting seven points in one direction and

$C_{M2.5.2.1}$: Maintenance cost of when scenarios 2, 3, and 4 are present at the same time with scenario 3 before scenario 2, and scenario 2 before 4.
α_{324}	: Probability of occurrence scenario 3 before 2 and 2 before 4
C_{pm324}	: The unit cost of perfect maintenance action performed when scenarios 2, 3, and 4 are present at the same time as scenario 3 before scenario 2 and scenario 2 before scenario 4.
$N_{imp324,1}$ $T_{imp324,1}$: Number and interval of imperfect preventive maintenance before detecting seven points in one direction
$N_{imp324,2}$ $T_{imp324,2}$: Number and interval of imperfect preventive maintenance between detecting seven points in one direction and of points outside main limits
$N_{imp324,3}$ $T_{imp324,3}$: Number and interval of imperfect preventive maintenance between detecting points outside main limits and of seven consecutive points falling on the same side
$N_{imp324,4}$ $T_{imp324,4}$: Number and interval of imperfect preventive maintenance after detecting seven consecutive points falling on the same
$C_{M2.5.2.2}$: Maintenance cost of when scenarios 2, 3, and 4 are present at the same time with scenario 3 before scenario 4, and scenario 4 before 2.
α_{342}	: The probability for the occurrence of scenario 3 before 4 and 4 before 2
C_{pm342}	: The unit cost of perfect maintenance action performed when scenarios 2, 3, and 4 are present at the same time as scenario 3 before scenario 4 and scenario 4 before scenario 2.
$N_{imp342,1}$ $T_{imp342,1}$: Number and interval of imperfect preventive maintenance before detecting seven points in one direction
$N_{imp342,2}$ $T_{imp342,2}$: number and interval of imperfect preventive maintenance between detecting seven points in one direction and of seven consecutive points falling on the same side
$N_{imp342,3}$ $T_{imp342,3}$: Number and interval of imperfect preventive maintenance between detecting seven consecutive points falling on the same side and of point outside main limits
$N_{imp342,4}$: Number and interval of imperfect preventive maintenance after detecting points

$T_{imp342,4}$: outside main limits

$C_{M2.5.3.1}$: Maintenance cost of when scenarios 2, 3, and 4 are present at the same time with scenario 4 before scenario 2, and scenario 3 before 3

α_{423} : The probability for the occurrence of scenario 4 before 2 and 2 before 3

C_{pm423} : The unit cost of perfect maintenance action performed when scenarios 2, 3, and 4 are present at the same time as scenario 4 before scenario 2 and scenario 2 before scenario 3.

$N_{imp423,1}, T_{imp423,1}$: Number and interval of imperfect preventive maintenance before detecting seven consecutive points falling on the same side

$N_{imp423,2}, T_{imp423,2}$: Number and interval of imperfect preventive maintenance between detecting seven consecutive points falling on the same side and of points outside main limits

$N_{imp423,3}, T_{imp423,3}$: Number and interval of imperfect preventive maintenance between detecting points outside main limits and of seven points in one direction.

$N_{imp423,4}, T_{imp423,4}$: Number and interval of imperfect preventive maintenance after detecting seven points in one direction

$C_{M2.5.3.2}$: Maintenance cost of when scenarios 2, 3, and 4 are present at the same time with scenario 4 before scenario 3, and scenario 3 before 2

α_{432} : The probability for the occurrence of scenario 4 before 3 and 3 before 2

C_{pm432} : The unit cost of perfect maintenance action performed when scenarios 2, 3, and 4 are present at the same time as scenario 4 before scenario 3 and scenario 3 before scenario 2.

$N_{imp432,1}, T_{imp432,1}$: Number and interval of imperfect preventive maintenance before detecting seven consecutive points falling on the same side

$N_{imp432,2}, T_{imp432,2}$: Number and interval of imperfect preventive maintenance between detecting seven consecutive points falling on the same side and seven points in one direction

$N_{imp432,3}, T_{imp432,3}$: Number and interval of imperfect preventive maintenance between detecting seven points in one direction and of seven consecutive points falling on the same side

$N_{imp432,4}, T_{imp432,4}$: Number and interval of imperfect preventive maintenance after detecting points outside main limits.

CM_{Total} : Total maintenance cost

μ_p : Perfect maintenance duration

Production Parameters:

$u(k)$: Production rate $u = \{u(0), u(1), \dots, u(H - 1)\}$

$S(k)$: Stocks inventory at the period end ($k = 0, \dots, H$)

$w_i(k)$: Inventory level of $w_i, (i = 0, \dots, l)$ at the end of period $k, (k = 0, 1, \dots, H - 1)$ for each warehouse

τ_i : Delivery time for warehouse w_i

$Q_i(k)$: Delivery rate during period k , ($k = 0, 1, \dots, H - 1$) for each warehouse
l	: Number of warehouses
Δt	: Length of a production period
$d_i(k)$: Average demand during period k , ($k: 0, 1, \dots, H - 1$) for each customer
$V_{di}(k)$: Variance of demand during period k , ($k: 0, 1, \dots, H$) for each customer
H	: Number of production periods in the planning horizon
$H \cdot \Delta t$: Length of the finite planning horizon
Q_v	: Delivery vehicle capacity
c_p	: Unit production cost of the machine
c_h	: Inventory holding cost of one product unit during one period at the Principal Store S
c_{hi}	: Inventory holding cost of the unit product during one period at the warehouse S_i , ($i = 0, \dots, l$)
c_d	: Shortage cost one product unit during one period
c_l	: Delivery cost
μ	: Monetary unit
u_{max}	: Maximal production rate of machine
u_{min}	: Minimal production rate of machine
θ_i	: Probability index is related to each customer's (i) service level satisfaction degree.

1.7 Conclusion

After reviewing different production and maintenance optimization strategies, it is established that the joint optimization of production and maintenance plays a significant role in the development, operations, growth, and sustenance of all production and service enterprises. In this first chapter, we have presented the relevant and different notions of production systems, including their classification and objectives. The resource and demand-based topology, supply chain and its management, life cycle, and waste management. The maintenance of production systems, main categories, and the law of failures. The state-of-the-art briefly highlighted the types of maintenance policies; perfect, minimal, and imperfect maintenance, and their significance in the management of the production system. The study also presented quality, quality tools, and their applications in production and maintenance control. Indeed, the literature indicates that some tools (Control charts) are more effective than others, depending also on the failure models. We choose the best control chart suitable for detecting production process variation. We found out that statistical process chart (SPC) was good for process shift but lacks capability in detecting small drifts. Therefore, we proposed an improved statistical process chart (ISPC) is found to be robust in detecting problems (Shifts & Small drifts). Additionally, we used the developed improved control chart in combination with the AFNOR principles for some special kind of production process variation. We also reviewed different works which joins production and maintenance, quality and production, then maintenance strategies and quality. Also, we have observed during the literature review that most of the proposed approaches either consider one or two main production factors, a

single cause of process variation, or horizons of infinite time, for which the system is considered in a constant operating mode over time. The remaining chapter presented the thesis problem, and motivational issues, and described the industrial problem, then situated our work with the existing literature. We integrated maintenance strategy with quality, considering the variation in the production rate and inventory management from one period to another (dynamically) throughout the finite production horizon. The performance of the production process is evaluated, and monitored statistically to ensure product conformity with the help of Statistical process control (SPC), and maintenance actions employed. We define the parameters utilized in the mathematical formulation of the model's problem in the remaining chapters.

Chapter 2:

An integrated production, maintenance, and quality control model with a statistical process control chart of a supply chain.

*This chapter investigates integrated maintenance, production, and product quality control policy for a supply chain consisting of a single machine producing one type of product. The production system aims to satisfy varying demands under service and quality levels during a finite horizon, linked to a primary storage, and multi-warehouses. The variation of the production rate and its use over time affect the production system's **reliability** status. Hence, the system is subject to a random breakdown that directly impacts the quality of the products. The tool we used to optimised the maintenance and the quality control of an integrated production system is a control chart tool based on statistical measurement and analysis of quality parameters. This work aims to establish an optimal production planning with inventory management considering the production, holding, delivery, and delay costs, and then the appropriate maintenance strategy. The study also provides a quality control policy to mitigate the proportion of non-compliant products with a statistical process chart, which indicates the process variations. Numerical experimentation is to show the effectiveness of the suggested model.*

2.1 Introduction

Recently, the global economic challenges such as the demand for some high-quality varied products at a low price have imposed more constraints on companies to remain competitive. Moreover, customers are becoming more exigent, while production systems are getting more complex and subject to multiple uncertainties (Ben-Daya & Rahim, 2000). Therefore, developing new industrial strategies, including production, maintenance, and quality, has become crucial to reducing costs and satisfying customers' multiple requirements. Production planning optimization and maintenance strategy have always been a tremendous challenge for industrial companies. The new role of maintenance function to better support value creation by contributing to the economic dimension and considering the environmental and social aspects are the trends (Jasiulewicz-Kaczmarek et al., 2021). However, the quality of the product or service decreases when the process is affected by time, total cost, and constraints or influences the quality of the customer's final product or service. This chapter will use the six-sigma method to improve product and process quality, guaranteeing that products meet customer requirements. The main objective is to devise a control that indicates the production process variation and the related high deficiency rate by using, for example, the control chart based on the "statistical process control" (SPC) methodology. The six-sigma method will employ the Define, Measure, Analyse, Improve, and Control (DMAIC) methodologies (Sharikh et al., 2019).

A decision-support system based on a quality follow-up by the control chart. The control chart identifies the control state of the process, to minimize the losses by controlling the variability. To meet customers' multiple requirements, as well as ensure sustainability. The optimum production plans, maintenance, and quality policy procedures, including inventory management to meet customer service level requirements. A production system malfunction is among the causes of non-compliant products and losses. The statistical process quality control, coordinated maintenance strategy, and optimal production planning will be a solution to minimize costs and increases production system reliability. Most of the works reported in the literature allowed the studies of either a single factor or the coordination of no more than two of the fundamental functions: production, maintenance, and quality. Beginning with the integration of production and maintenance, then production with quality or maintenance with quality. Recently, the most comprehensive trends deal with integrating all three factors. According to the production bounds and the service level requirement, this work's originality is in the determination of the ideal combination of production, maintenance, and delivery amounts based on the control chart. That identifies the state of the process (stable state or critical state) and the correlation between production and maintenance.

The works of literature in the direction of production and maintenance problems. (Buzacott, 1967) were amongst the earliest authors to improve production in their work "Automatic transfer lines with buffer stock". They studied the problem of integrated production to maintenance planning by considering the buffer stocks in supporting the productivity of the process. In their work "Impact of delivery time on optimal production, delivery, and maintenance planning," conducted by (Sadok et al.,

2012) studied the ideal production, delivery, and maintenance accounting for the delivery time, machine failure, and random demand, in addition to the withdrawal right. (Chen et al., 2015) dealt with an unreliable production system using partial rework processes and maintenance (imperfect), thereby finding the economic production period, inspection times, and preventive maintenance interval. The works of (Kenné & Gharbi, 2004), also by (Tlili et al., 2015) consider corrective maintenance at each failure of the production system between preventive maintenance actions, thereby optimizing the preventive maintenance of the buffer stock. (Nahas, 2017) developed a stochastic model of an unreliable system to determine the optimum stock size and preventive maintenance strategy. (G. Q. Cheng et al., 2018) actualized the joint model of production plan and imperfect maintenance, in which the immune clonal selection algorithm is used. Few works consider the impact of production variation on system degradation in their maintenance strategy. (Hajej et al., 2018) found a significant influence of production rate fluctuations between periods on the production maintenance plans and total incurred cost. Thus they determine the optimal production and preventive maintenance plan considering constraints related to the production system capacity. The present work studies this aspect by considering the increase of the production process deterioration according to both time and use. After that, we discuss the quality and production/maintenance strategies by presenting the different literature on the integrated maintenance strategy to quality.

A cost-benefit model for investments in inventory and preventive maintenance, developed by (Lee, 2005), considers the delivery time to the customer as a constraint. (X. Liu et al., 2015) developed joined production and inventory control methods based on age type of preventive maintenance plan, and where corrective maintenance actions are to restore the machine to as good as new state. Their work allows to them decide the optimum buffer stock size concerning machine age and relating to maintenance. (Holgado et al., 2016), the study investigates maintenance and its effects on production operations, the study did not consider the quality.

Other researchers dealt with maintenance and quality. (Radhoui et al., 2010) introduce integrated preventive maintenance and quality model consisting of a buffer stock size and the rate of non-conforming units as decision variables for which preventive maintenance (PM) actions are taken. (Pandey et al., 2011) studied a joint optimization method for production planning, maintenance planning, and process quality but ignored the effects of inventory control shortages. Nevertheless, the model assumed specific quality attributes deteriorate according to age. (Suliman & Jawad, 2012) developed a model to optimize the production and preventive maintenance period to minimize non-conforming products when the process shifts out of control. (Bouslah et al., 2016) developed an optimally combined lot-sizing and feedback plan for a failing system using a sampling plan and corrective maintenance at failures to improve quality.

Recently, the integration of production, maintenance, and quality as studied by different researchers. For example, (Antosz & Stadnicka, 2018), use the six-sigma methodology to collect and analyse data concerning the maintenance process and waste identification element of the lean concept but are limited to the maintenance service improvements. (Ben-Daya & Rahim, 2000), presented joint

maintenance and the economic approach of an x-control chart for a deteriorating process, where the in-control state is characterized by a probability distribution. The study proposed a preventive maintenance strategy to reduce the out-of-control state considering the increasing hazard rate and to illustrate the impact of maintenance level on the costs of quality control. (Nourelfath et al., 2012), proposed a maintenance and reliability arrangement for a two-state process optimizing the decision variable of a manufacturing system to determine discrete times of preventive maintenance activities. Production processes are becoming complex, therefore require improved quality checks and multiple inspections. (Xiao et al., 2019) carried out integrated production, quality, and condition-based maintenance for the unreliable manufacturing system. They consider production with lot sizing and quality control done by entirely utilizing the strategy of inspection to obtain the defective proportion. (Lesage & Dehombreux, n.d.) aimed to identify the shifts in the production process that are subject to maintenance and quality control at failure, dealing with the problem with a simulation approach. (Bouslah et al., 2016) developed a combined production, control quality by sampling, and a deteriorating production system with an “Average Outgoing Quality” (AOQL) constraint. (Beheshti Fakher et al., 2017) have integrated production, maintenance, and quality concerning a multi-production, multi-period system characterized by an increasing hazard rate of deterioration distribution. The outcomes of the works determine the optimum values of production planning and the preventive maintenance interval, where the system is returned to an in-control state after producing non-conforming items. Concerning the joint production, maintenance, and six-sigma methodology, (Salmasnia et al., 2017), developed a joint model of production, maintenance policy, and control chart by considering various assignable causes. (Bahria et al., 2019) have developed an integrated method for joint production, quality, and maintenance control for batch production systems subject to degradation. They proposed a preventive and corrective maintenance strategy depending on the control of lots of produced quality performed using a control chart and minimizing the total cost of setup, inventory, maintenance, and quality. An advanced control chart is a means of “statistical process control (SPC) “using pattern recognition. (Addeh et al., 2018) used the patterns of the control chart corresponding to the manufacturing process specific factors to improve the “control chart pattern recognition” (CCPR) accuracy. A perfect and automatic CCPR is designed with various complex features, increasing the statistical process control.

A decision-support system based on a quality follow-up control chart. The control chart identifies the control state of the process, to minimize losses by controlling the variability. (Hajej et al., 2021), studied an integrated model of production, maintenance, and quality issues. The work examined a randomly deteriorating manufacturing system that should satisfy stochastic demand under a given service requirement based on dynamic inspection of quality.

Methodologies coordinating the SPC and stock issue generally pointed toward a control chart planning boundary. To limit stock and quality costs without considering the different issues that can impact the process stability and the products’ quality. In the literature, the integrated maintenance strategy with statistical process control based on the increasing failure rate with time and the machine’s use is not adequately studied.

We can observe that works involving quality aspects and or an analytic study for the production, maintenance, and quality correlation are still new in the existing pieces of literature. The present research proposes an original idea combining the fundamental functions by integrating a statistical control process tool. Our study considered that the machine degradation is impacted by the production rate variation, and it's used over time. This study contributes to the literature by investigating an analytical relationship between production and the failure rate analytical relation. The study demonstrated the progression of process failure rate with its use, regarding the process reliability continuity from one production period to another. The study considered the evolution of machine degradation impacted by both production and time as the source of the non-quality.

Unlike numerous literary works, the present study deals with an original and realistic idea integrating all functions: production, maintenance, and quality. This work aimed to determine the economic production planning, optimum delivery planning for various warehouses considering the cost/time of delivery, the delay penalty even at a random demand, and the optimal maintenance and quality strategy. A developed mathematical model will help determine the decision variables, which will optimize the objective function. Section 2 presents the control chart parameters, and a description of the production, maintenance, and quality problem. Section 3 develops the mathematical expressions of production and delivery strategies. The robustness of the solution is demonstrated using a numerical experiment in Section 4. Conclusively, the conclusion is drawn in Section 5.

2.2 Description of Industrial problem

This work contributes to the appropriate maintenance integrated into the production planning, and quality level requirements. For this purpose, we use a statistical process monitoring mechanism of a control chart of averages (X-bar chat tool). The idea is to monitor the machine failure rate by developing new integrated production, maintenance, and quality strategy, to increase the reliability of the process and minimize the total production cost. We consider the effects of production, inventory, maintenance, and quality, thereby optimizing overall production performance. The considered supply chain comprises a single machine producing one type of product, serially linked to a primary store and multi- warehouses (w_1, w_2, \dots, w_L). With a delivery time, $\tau_i, (i = 0, \dots, L)$, from which the client receives their demand items, under certain service level and quality requirements, and throughout a finite horizon H (Figure 2.1). The machine of the process is subject to random failures and repairs. The deterioration of the process machine is impacted both by time and production. The produced items are delivered from the primary stock to the warehouses (w_1, w_2, \dots, w_L), by a conveyance vehicle with a capacity (Q_v).

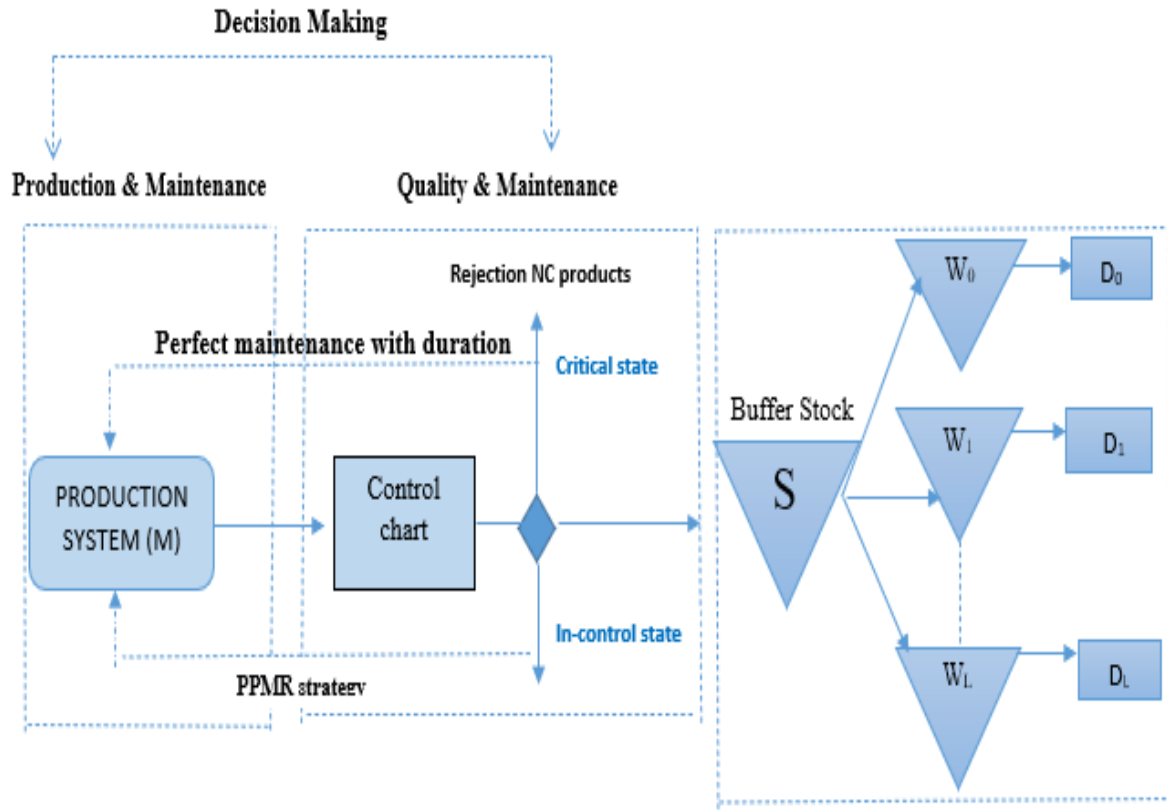


Figure 2. 1: Supply chain management

As a result, the production system capability and reliability are affected by the production rate $u(k)$, which causes the production of non-conforming units. The failure rate of the process machine $\lambda(t)$, increases with both time and use. Supposedly the quality control of the production system is inspected every h time unit, production interval (Δt), by a quantifiable quality statistic (Xt), taking n size sample at each inspection (t). The specific statistical measures (average and standard deviation) for each sampling must all be between the “upper control limit” (UCL) and the “lower control limit” (LCL). It is assumed that the samples follow a Gaussian distribution, and the production process is stable if the Gaussian law (Xt), is constant with a mean (μ_0), and standard deviation (σ_0), which are known or well estimated. When all statistics are within control limits, the process is considered under control, since the average sample of the quality indicator measurements is at its target value between the “lower control limit” (LCL) and the “upper control limit” (UCL). To reduce the chances of being out of control and the stability/capability of the production process resulting from degradation and the failure rate, a maintenance policy characterised by a preventive maintenance activity with minimal repair (PPMR) is performed through the finite production horizon. Perfect preventive maintenance activities are periodically scheduled at some defined intervals $T = a \cdot \Delta t = a \cdot b \cdot h$ with $a = \text{integer}$, $\Delta t = \text{production length}$, and during each failure between two successive preventive maintenance activities,

corrective maintenance with a minimal repair is performed. Therefore, we consider two scenarios as follows (Figure 2.2).

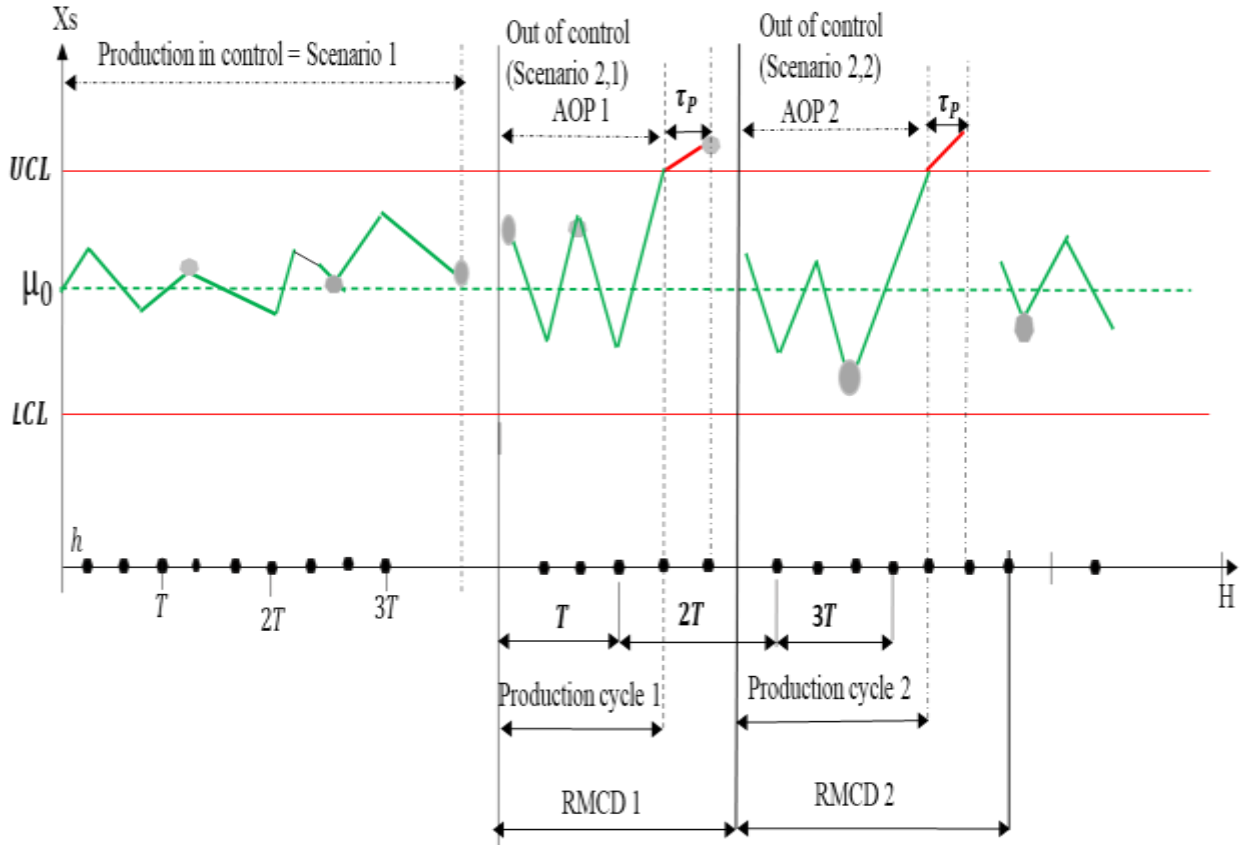


Figure 2. 2: Control chart of quality.

2.2.1 Principles of control chart

The “statistical process control” (SPC) is a tool of the six-sigma method that is employed to reduce the quality variability, support analysis- and decision-making to help determine if a process is stable and predictable. An ‘x-bar’ control chart with a mean and standard deviation tool is utilized to monitor the quality characteristics and alert when the production process shifts to an out-of-control situation. Statistical monitoring is based on the interpretation of the results recorded at the workstation by the operator or quality inspector. It consists of taking sized (n) samples from a batch of products and inspecting certain characteristics of all the individual samples. The result of the control, for each sample, is plotted on a graph called a control chart.

Statistical monitoring should allow:

- intervene in the process when the product quality changes to non-conformal.
- measure the "capability" of a process.
- Act on variations to ensure the process over time.

2.2.1.1 Statistical Monitoring of Production.

The control chart is used to monitor the condition of the process. Statistical monitoring is based on the interpretation of the results collected at the workstation by the operator. It consists of taking a representative sample from a batch of products (sample size) and checking certain characteristics of all the individuals in the sample. The result of the control, for each sample, is reported by plotting samples of the process output collected over time on a graph.

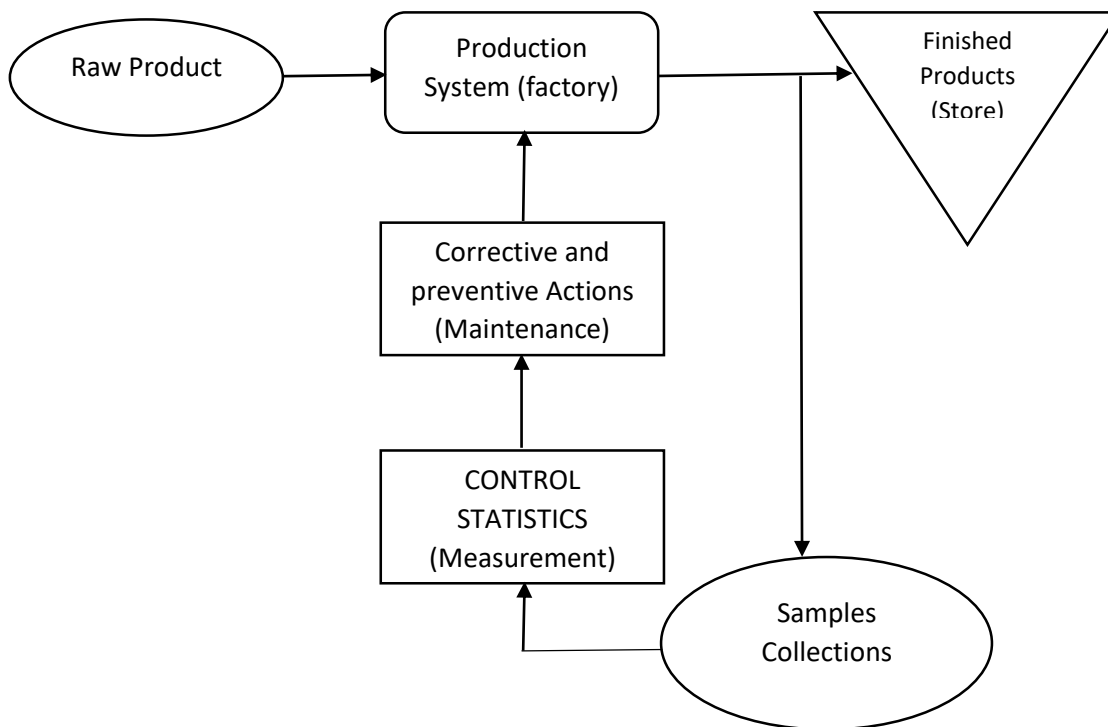


Figure 2. 3: Statistical Monitoring of Production.

The basic principles of statistical techniques as presented in Figure 2. 3 use charts to present a picture of what is happening to and in a process as a service is produced. The production monitoring aims to prevent the occurrence of defects, when properly used, control charts will indicate the followings:

1. When you are doing something you shouldn't.
2. When you are not doing something you should.

3. When you are doing things right.

In short, control charts will indicate how consistent your job or process is. Statistical monitoring should allow:

- to intervene in the process before non-quality is produced.
- measure the "capability" of a process: the process is capable of producing good parts.
- act on variations to ensure the stability of the process over time.

2.2.1.2 The Control Chart.

A control chart is an in-process decision support document to record the results of sample checks collected at the workstation. It allows us to visualize graphically the dimensional variations and to determine and to determine the right moments for an adjustment.

a - Objective:

- The Control Chart allows the detection of any drifts (defects) in production, to intervene before the production of non-conforming parts. Thus, it allows to increase the quality level and thus reduce the costs of non-quality.

b - The different zones on a Control Chart:

- Zone 1: Identification (machine, frequency, part, operation...).
- Zone 2: Table of values of the characteristics measured on the parts.
- Area 3: Calculation of the average (\bar{X}) and the range R for each sample.
- Zone 4: Calculation of the average of the means ($\bar{\bar{X}}$) and the average of the ranges (\bar{R}).

Area 5: 2 charts with decision limits:

- The range map (R): Plot all points representing the range for each sample.
- The mean map (\bar{X}): Plot all points representing the mean for each sample.

c - Limits for the mean and range (Field 6)

- L.C.L: Lower Control Limit.
- U.C.L.: Upper Control Limit.
- L.S.L: Lower Surveillance Limit.
- L.S.L: Upper Surveillance Limit.

2.2.1.3 Interpretation of a Control Chart.

We will see below examples of cases of control chart of the average, to deduce an interpretation for each case.

2.2.2 The production process scenarios.

We use an " \bar{X} - bar" control chart to monitor the process and control the quality level of products by analyzing the sampled results.

2.2.2.1 The scenarios

The SPC works based on control chart parameters and shows when and what is an appropriate action to take. We used the normal law to model the random variation of the process. The normal law is widely used in statistical process control, especially in its reduced-cantered form. Before using the control chart, it must verify the sample's normality during process control. The machine status can be assessed by measuring the quality indicator X_t with the product produced. From an initial population with a mean (μ_0), and a standard deviation (σ_0), The evolution of this indicator follows a normal law with a mean (\bar{X}_s), and a standard deviation (σ_s). Every unit of time (h), a sample size of (n) products are taken and examined to study the evolution of the quality indicator X_t .

Scenario 1: When the process is in a stable state (under control), and PMMR planning with negligible duration is applied. For each conducted preventive and corrective maintenance action in this situation.

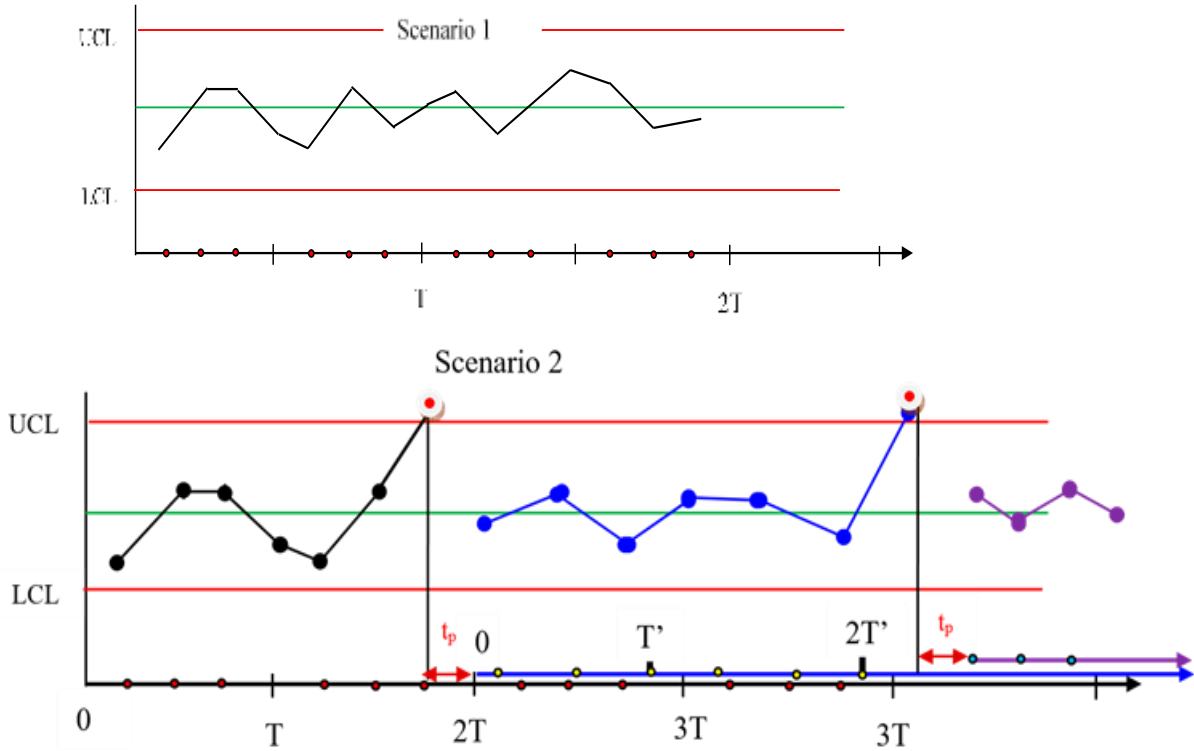


Figure 2. 4: The two scenarios of quality control and evolution of maintenance.

Scenario 2: When the process moves out of control between j^{th} and $(j + 1)^{th}$ sampling, due to an assignable cause (changes the mean value). Consequently, affecting the product quality and the indicator measurements of the sample are traced outside the control standards. Under this situation, the production process is suspended and perfect maintenance activity with non-negligible duration (t_p) is conducted to restore the machine to as good a new state. As shown in Figure 2. 4, then continue the planning according to the cumulative failure rate, a new periodical preventive maintenance planning (PMMR).

2.2.2.2 The Control Chart parameters

The quality indicator \bar{X}_s follows a normal distribution $\bar{X}_s \sim \left(\mu, \frac{\sigma_0}{\sqrt{n}}\right)$ with μ is the mean and σ_0 is the standard deviation of the initial population. The average value \bar{X}_s is recorded in 'X - bar'. For all S sample products, X_s , the average value shown in figure 2.5. The control chart is defined as follows:

$$\mu = \bar{\bar{X}}_s \approx \mu_0 = \frac{1}{m} \times \sum_{i=1}^m X_i \quad (2.1)$$

m is the number of sampling.

And, for all S samples, we define the related standard deviation:

$$\sigma_s = \frac{\sigma_0}{\sqrt{n}} \quad (2.2)$$

Control charts have the following attributes determined by the data:

An average or centreline for the data: It's the sum of all the input data divided by the total number of data points.

An upper control limit (UCL): It's typically three process standard deviations above the average.

A lower control limit (LCL): It's typically three process standard deviations below the average.

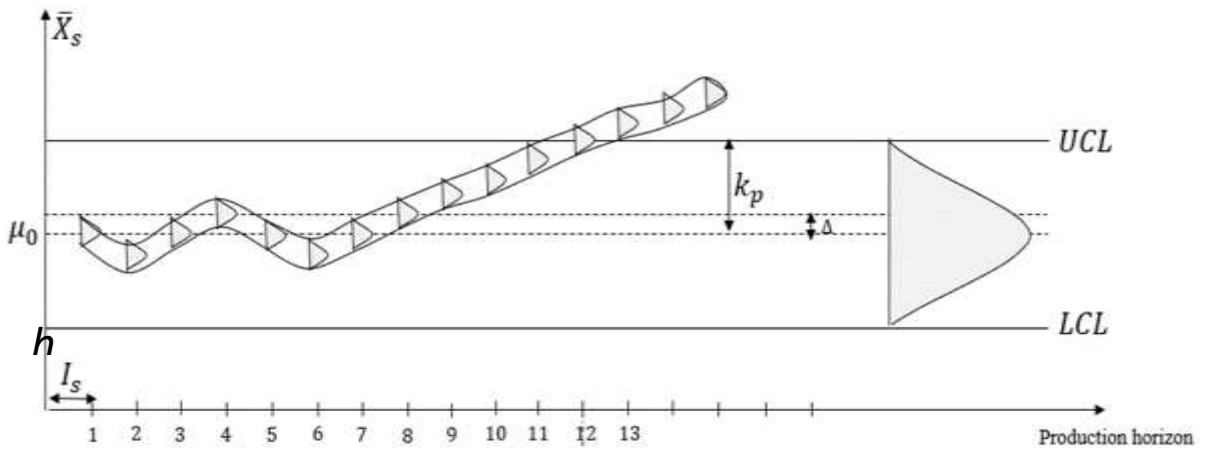


Figure 2. 5: Control chart.

We, therefore, define the control limits as follows:

The upper limits of the control chart:

$$UCL = \mu_0 + \frac{k_p}{\sqrt{n}} \times \sigma_0 \quad (2.3)$$

The lower limits of the control chart:

$$LCL = \mu_0 - \frac{k_p}{\sqrt{n}} \times \sigma_0 \quad (2.4)$$

Taking after a sample inspection of an n size, two scenarios are conceivable depending on whether the limits of control are surpassed or not.

Scenario 1: Under control process.

When the process measurements of an average quality indicator are situated between the control limits $LCL \leq \bar{X}_s \leq UCL$, the production process is considered stable (under control), and a maintenance strategy is described by preventive maintenance with minimal repair (PMMR) maintenance activities is scheduled. And, the production process continues as normal without disruptions.

we let p_1 be the probability of the first scenario:

$$p_1 = Pr(LCL < \bar{X}_s < UCL)$$

$$p_1 = F\left(\mu_0 + k_p \times \frac{\sigma_0}{\sqrt{n}}\right) - F\left(\mu_0 - k_p \times \frac{\sigma_0}{\sqrt{n}}\right) \quad (2.5)$$

With F : distribution function of the reduced centred normal law.

Scenario 2: Out of control process.

When the process measurements' average of the quality indicator is located outside the control limits ($(UCL < \bar{X}_s)$ or $(\bar{X}_s < LCL)$), then the production process is considered in critical condition. In this case, the production process is stopped and a perfect preventive maintenance action with a mean duration (t_p) is employed.

we let p_2 be the probability of the second scenario:

$$p_2 = Pr(UCL < \bar{X}_s) + Pr(\bar{X}_s < LCL)$$

$$p_2 = 1 - F\left(\mu_0 + k_p \times \frac{\sigma_0}{\sqrt{n}}\right) + F\left(\mu_0 - k_p \times \frac{\sigma_0}{\sqrt{n}}\right) \quad (2.6)$$

With F : distribution function of the reduced centred normal law.

Therefore, the parameters of the control chart consist of the number of inspections (m), the sample size (n), the sampling time interval (h), and the number of standard deviations associated with

the sample distribution between the centreline and the limits (k_p) of the control chart. Therefore, the control chart decision variables are m, n, h, k_p .

2.2.3 The Sequential Optimization Strategy

The optimization of production and maintenance strategy can be achieved in two ways: the sequential strategy and the simultaneous strategy. The choice is independent of the results to be obtained but rather dependent on the method to solve the problem. While integrated simultaneous optimization is based on a simultaneous resolution of the production and maintenance plans. Sequential optimization on the other hand is a strategy of two activities, which mostly considers the optimal solution found in the production planning as a constraint to determine the optimal maintenance strategy. The selection of the sequential optimization strategy in our work is by the fact that the failure rate of each period depends directly on the production rate and the duration of use associated with that period (k). Consequently, and given that the maintenance plan is strongly dependent on the system failure rate in question, the determination of the strategy to employ requires first, the determination of the production plan. Figure 2.6 below demonstrates an overview of the sequential optimization strategy.

In this work, we use a sequential scheme by proposing two sub-models (production, inventory, and delay-penalty model), then the (maintenance and quality model), described as a successive optimization of two sub-models, with the first sub-model as a constraint to optimize the second sub-model.

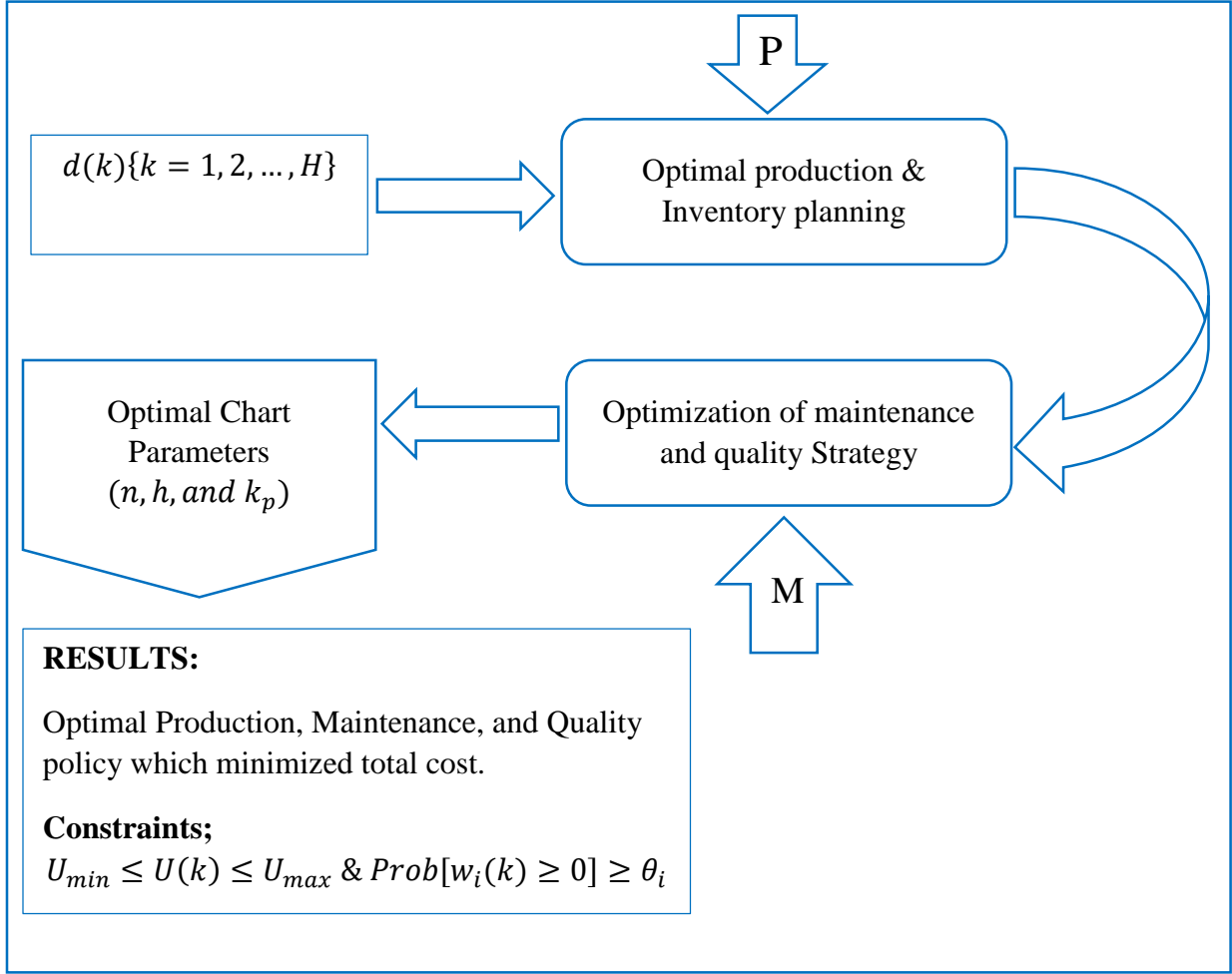


Figure 2. 6: The Description of the sequential strategy.

2.3 Formulation of the stochastic problem

To establish an optimal production, maintenance, and quality integrated strategy, we define a stochastic model that minimizes the total costs over a finite horizon. Our goal is to establish, the optimum production rates, the optimal inventory/delivery quantities, as well as the optimal maintenance strategy corresponding to the quality control chart's optimal parameters, which minimizes the total production cost. The total expected cost includes the costs of production, inventory holding, delay penalties, and quality/maintenance.

Formally, the problem is defined as follows:

$$\text{Min } F = PC + HC + DC + DPC + TMC \quad (2.7.0)$$

Under the following

$$S(k) = S(k-1) + u(k) \cdot \Delta t - \sum_{i=1}^L Q_i(k) \quad \text{with } k = \{0, 1, \dots, H\} \quad (2.7.1)$$

$$Z_S(k) = \text{Max}\{S(k-1), 0\} \cdot \Delta t + \frac{1}{2} \cdot u(k) \cdot \Delta t^2 \quad (2.7.2)$$

$$w_i(k) = w_i(k-1) + Q_i(k - \tau_i / \Delta t) - d_i(k), \text{ where } k, (k = 1, 2, \dots, H) \text{ and } \tau_i \geq 1 \quad (2.7.3)$$

$$Z_{w_i}(k) = \text{Max}\{w_i(k-1), 0\} \cdot \Delta t + \frac{1}{2} Q_i(k - \tau_i / \Delta t) \cdot \Delta t^2, \text{ where } k, (k = 1, 2, \dots, H) \text{ and } \tau_i \geq 1 \quad (2.7.4)$$

$$\text{Prob}[w_i(k) \geq 0] \geq \theta_i \quad (2.7.5)$$

$$u_{\min} \leq u(k) \leq u_{\max} \quad (2.7.6)$$

Where F represents the total production costs function; production, holding, maintenance, and quality. PC denotes the production cost, HC is the total inventory holding cost at both the principal store and the warehouses i . The delivery and delay penalty costs are respectively represented with DC & DPC . The study set defines the inventory balance equation for each period (k) taking into account the dynamic production, the supply quantities to the principal store S as well as the multi-warehouses w_i . The $S(k)$ states the principal stock quantity at any given period k , which accounts for quantity Q taken to any of the warehouses and dynamically throughout the production horizon H . The demand d is the product quantity taken out from the retail warehouses w_i . The τ_i is the transportation duration factor accounting the for times it takes to supply products from the principal store to all the warehouses. The equations (2.7.1, 2.7.6) define constraints imposed by the service level requirement for each period and the production capacity limitation by the lower and upper production bounds. The probabilistic constraint of inventory is taken as a chance constraint to ensure that the inventory level is greater than zero with a conditional probability of at least θ at each period k .

The optimization problem is described by a random parameter of our problem (forecasting demand and production, the stochastic constraint of production capacity limitations, and service level requirements) and the decision variables. Production plan according to the model, it is important to make assumptions to be adopted; in our case, we adopt a dynamic modelling convention (Hajej et al, 2011). The main objective is total cost minimization. We begin by planning the optimal rate of production ($u(k)$ and inventory level ($Q_i(k)$) to satisfy customers' demands at all the warehouses and for all periods. Secondly, using the production rates and stock quantities as constraints to find the optimal number of preventive maintenance actions (N)* as well as the optimal parametric combination of the key decision variables (n, h, m, k_p), with the aid of a statistical control chart of a quality tool. The production horizon (H) is equally divided into periods of length equal to Δt . For each production period k , we assume that production takes place at the beginning of the period, and at the end of the period, the unsatisfied demands are penalized. Also, we assumed that the demand fluctuation follows a normal process with mean and variance.

2.3.1 Stochastic model of problem

To develop the model, the following hypotheses are considered based on some industrial reality and constraints. This strategy of maintenance planning is characterized by the optimal number of preventive maintenance actions and the intervals between two successive preventive maintenances, we have an idea to plan the necessary personnel, tools, and all other maintenance resources required, thereby reducing the intervention durations. Therefore, periodic preventive maintenance with minimal repair (PPMMR) is proposed during the in-control situation and for each failure respectively. This does not require significant maintenance time as it is assumed that the necessary logistics (tools, technicians, finance, and other resources) are available. On the other hand, a non-negligible time (t_p) for each action of perfect maintenance corresponding to each out-of-control event is considered.

This work aims to determine for all periods, H , the optimal production, $u(k)^*$, to meet the varying demand of the customer simultaneously with the optimal inventory quantities, Q_i^* , at all the warehouses, under service and quality requirements. Then, according to the optimal plan of production, establish the best maintenance strategy based on the optimum number of preventive maintenance actions (N)*. Also, the optimal parameters of the control chart; the quality inspection number (m), the sample size (n), the control interval (h), and the number of standard deviations between the centreline and the control limits of the control chart (k_p). The study's main aim is to minimize the total cost associated with production, inventory/holding, maintenance, and quality, comprising sampling, false alarm, and rejection costs over the finite horizon time, H . In this study, it is assumed that the production horizon is proportionally divided into k production periods with a Δt length. Additionally, we assume that demand variation is followed by a normal distribution, with mean \hat{d}_k and variance V_{dk} .

To resolve the joint optimization problem, we use a sequential scheme by proposing two sub-models (production, inventory, and delivery model, and maintenance and quality model), defined as a successive optimization of two sub-models. The first sub-model is a constraint to optimize the second. The optimization problem is described by a random parameter of our problem (forecasting demand and production, the stochastic constraint of service level, etc.) and the decision variables' dependence.

Assumptions

1. The random customer demand follows a normal law, and every unsatisfied demand in a period causes a delayed penalty.
2. Each delivery time, τ_i , from S to each warehouse, w_i , is constant and is a multiple of Δt .
3. The maximum and minimal production rates are known and constant.
4. The repair time is negligible for the maintenance strategy performed under control.
5. perfect preventive maintenance activities are performed under control

6. For each out-control, the time of perfect preventive maintenance performed is t_p^* , and a new maintenance cycle is continued afterward the non-conformal products are not returned to production.
7. Non-conformal products are due to the degradation of the machine.
8. All unit costs are known and constant.
9. All required human and material resources to carry out maintenance actions are always available.
10. The preventive maintenance activities are perfect and restore the equipment to “as good as new” condition.

2.4 Analytical study of production policy

2.4.1 Production policy

This section aims to optimize the total cost of production and inventory stock in the primary store as well as all warehouses linked in the supply chain. It is important first to equate the different costs illustrated in the previous subsection: In general, with the aid of resources; materials, funds, and all forms of infrastructures, the production system is designed to produce marketable outputs necessary to satisfy needs. This includes all functions (process) required to transform inputs of raw materials into the outputs of tangible finished products or services aimed to satisfy the client's requirements, including delivery, quality, quantity, price, and other conscious specific objectives. Recall that our formulated problem in Subsection 2.3.

2.4.2 Production case

The production system studied is composed of a single machine M which produces a single product at a production rate $u(k)$, to supply a principal manufacturing store $S(k)$, linked to multi warehouse's $w_i(k)$ to allow the satisfaction of a client random demand throughout the production horizon H . The law is characterized by a Normal distribution with mean and standard deviation parameters respectively denoted by \hat{d} and σd 2. This research aims first to find the best production plan to meet the varying demand of customers at a specified service and quality level. Second, the optimal control chart parameters, which are the sample size (n), the interval of control (h), and the control limits coefficients (k_p), as well as the best preventive maintenance plan, which is defined by the best number of preventive maintenance tasks N are determined by the economic plan of production. During the finite time horizon, the goal is to keep overall costs as minimum as possible regarding the production, inventory, maintenance, and quality, including sampling inspection, false alarm, and rejection costs of non-conformal items. The horizon is assumed to be divided equally into H periods with lengths equal to Δt . Furthermore, we assume that demand fluctuation is a normal process with mean $d_i(k)$ and variance $V_{di}(k)$ provided by and that all demands are met after each period

2.4.3 Analytical study of Production policy

This section focused on transforming the total cost into some analytical but deterministic expressions that would be friendly. Based on the Stochastic model given in the previous subsection of the problem, the production costs can be calculated as follows:

- **Production Cost**

The optimal production control policy will largely depend on production rate $u(k)$ as a fundamental control parameter. It is central to the production bounds, that is between the minimum and maximum production rates ($u_{min} \leq u(k) \leq u_{max}$). It is also, worthy of note that the production rate is adapted based on the random demand $d(k)$. Hence, the cost of production at a period of k

$$PC = C_p \times \sum_{k=1}^H u(k) \cdot \Delta t \quad (2.8)$$

- **Holding cost at the primary store (S)**

To avoid shortages, as well as unnecessary product holding costs, there is a need for effective inventory management. The inventory balance equation defining the progress of the primary inventory for all periods is expressed by:

$$S(k) = S(k-1) + u(k) \cdot \Delta t - \sum_{i=1}^L Q_i(k), \quad \text{with } k, = \{0, 1, \dots, H\} \quad (2.9)$$

$Z_s(k)$ is the area generated by the evolution of the inventory quantity in period k , ($k = 1, 2, \dots, H$)

$$Z_s(k) = \text{Max}(S(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot u(k) \cdot \Delta t^2 \quad (2.10)$$

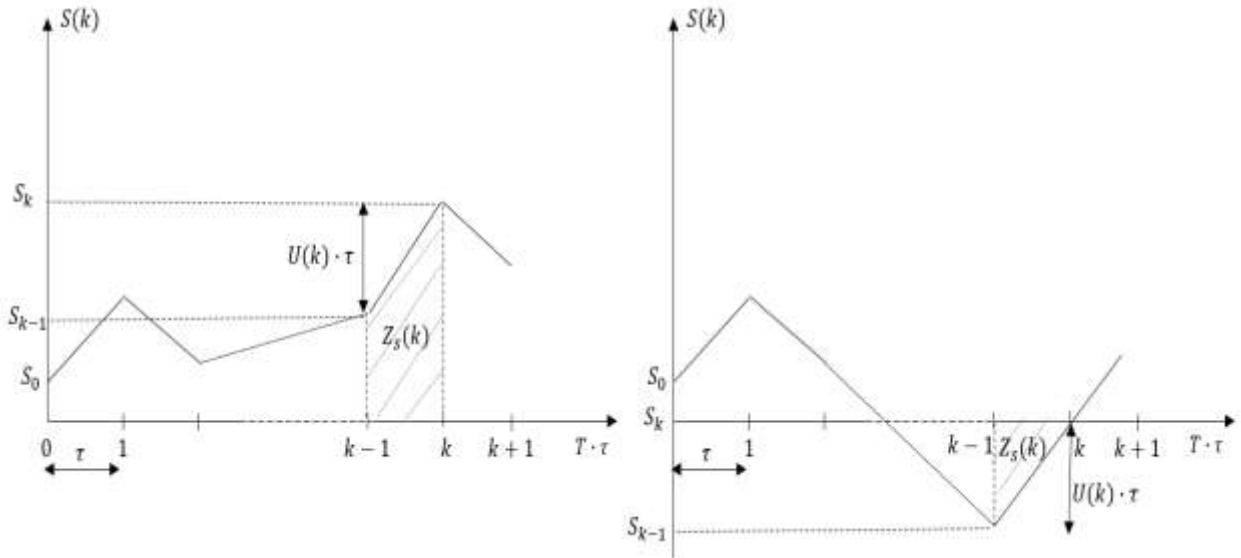


Figure 2. 7: The inventory evolution.

As a result, the holding cost (primary store) is calculated as follows:

$$HC_S = \sum_{k=1}^H C_{hs} \times Z_S(k) \quad (2.11)$$

Where c_{hs} = The unit cost of holding inventory.

- **Holding cost at the retail warehouses (w_i)**

The inventory quantity entering each warehouse w_i at any given period k equals the inventory quantity that leaves S at the period. $(k - \tau_i/\Delta t) \cdot \tau_i/\Delta t$, the latter defines the times it takes products to be transported from S to any of the w retail stores i .

The inventory level of each warehouse inventory w_i at the period k is given by the following equation:

$$w_i(k) = w_i(k-1) + Q_i(k - \tau_i/\Delta t) - d_i(k), \text{ where } k, (k = 1, 2, \dots, H) \text{ and } \tau_i \geq 1 \quad (2.12)$$

$Z_S(k)$ is the area generated by the inventory level evolution during the period $k, (k = 1, 2, \dots, H)$

The inventory level of each warehouse w_i at any given period k equals the inventory level of w_i at period $k-1$, plus the number of products that arrives at w_i (i.e. $Q_i(k - \tau_i/\Delta t)$) minus the customer demand d_i at period k .

The generated area of the inventory level evolution for each warehouse (w_i) during any given period k is given as follows:

$$Z_{w_i}(k) = \text{Max}(w_i(k-1), 0) \cdot \Delta t + \frac{1}{2} Q_i(k - \tau_i/\Delta t) \cdot \Delta t^2, \text{ where } k, (k = 1, 2, \dots, H) \text{ and } \tau_i \geq 1 \quad (2.13)$$

The inventory holding cost for all warehouses is given by the following expression;

$$HC_W = \sum_{k=1}^H \sum_{i=1}^L C_{hwi} \times Z_W(k) \quad (2.14)$$

Consequently,

- **Total Inventory Holding cost (HC)**

This consists of the inventory holding cost at the main production store (S), as well as all inventory costs from numerous retail warehouses ($w_i \{i = 1, \dots, L\}$) expressed as follows:

$$HC = HC_S + HC_W$$

$$HC = \sum_{k=1}^H \left(C_{hs} \times \left(\text{Max}(S(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot u(k) \cdot \Delta t^2 \right) + \sum_{i=1}^L C_{hwi} \times \left(\text{Max}(w_i(k-1), 0) \cdot \Delta t + \frac{1}{2} Q_i(k - \tau_i/\Delta t) \cdot \Delta t^2 \right) \right) \quad (2.15)$$

- **Delivery cost**

The delivery cost to all warehouses at period k is:

$$DC(Q(k)) = c_l \times \sum_{i=1}^L \left(\frac{Q_i(k)}{Q_v} \right) \quad (2.16)$$

- **Delay Penalties**

Having produced the appropriate quantity of products, and also planned optimal inventory levels, without effective delivery, the plan can create a severe problem for the supply chain. In this study, we consider some delay penalties. Whether a scenario of delay after period k results in a shortage being restored in period $(k + 1)$ as shown in Figure 2. 7.

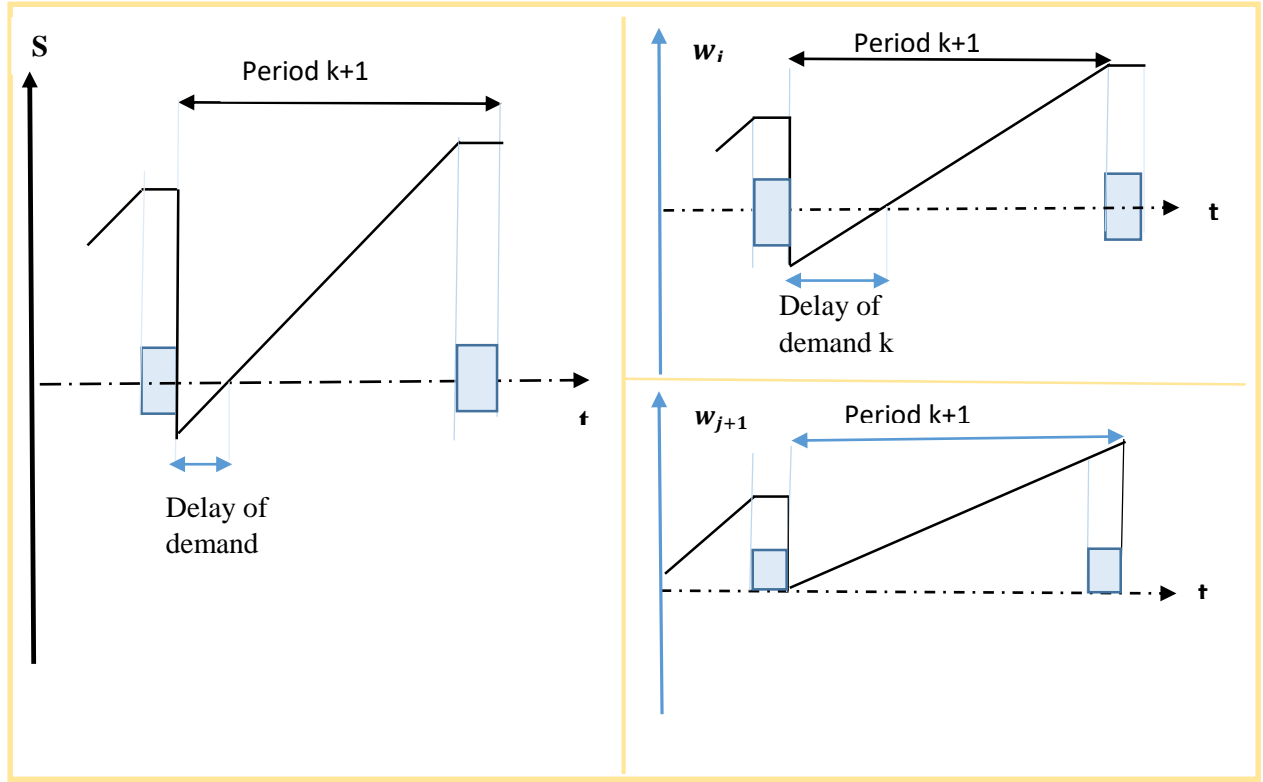


Figure 2. 8: Shortage of inventory level

The penalties are determined by the length of time required $dw(.)$ to generate the lost quantity that was lost at the end of each period, as determined by:

$$PC = C_d \times \left(\sum_{k=1}^H \left(\sum_{i=1}^L dw_i \right) \right) \quad (2.17)$$

$$dw_i = \frac{|\min(w_i(k), 0)|}{Q(k+1 - t_i / \Delta t)} \quad (2.18)$$

The followings are the constraints of the model as expressed below;

- ***Production bounds***

These are the production capacity limitations during each of the production periods k , expressed below;

$$u_{min} \leq u(k) \leq u_{max} \quad (2.19)$$

- ***Service level constraints***

These are the service level requirements for each warehouse during all the production periods k , as represented by the following:

$$Prob[w_i(k) \geq 0] \geq \theta_i \quad (2.20)$$

The model given is a stochastic model that includes the production bounds, and the service level requirements described by constraint in equations (2.19), and (2.20) respectively. The uncertainty around the variation in demand gives the randomness to service level constraints. Solving such a sequential problem under constraints is generally difficult due to the stochastic nature of the problem. Hence we begin by transforming the stochastic problem into an equivalent deterministic problem which will then be easier to solve to facilitate its solution.

To transform the probabilistic problem into a deterministic equivalent, it is necessary to first determine the change in the variance of inventory over the planning horizon. we use the "certainty-equivalent" approach, which consists of associating the variables with their average values and considering that the variation of the random demand follows a Gaussian distribution. Since demand is a random variable, it affects the value of the stock level $S(k)$, which becomes a stochastic variable, whereas the production rate $u(k)$ and the number of stock in the warehouses $w_i(k)$ are deterministic variables. To cast the service level constraint into a deterministic form, also specifying certain minimum and maximum production quantities that depend on the service level requirements.

Lemma:

For $k = 1, 2, \dots, H$ and $i = 0, 1, \dots, L$ we have;

$$Prob[w_i(k) \geq 0] \geq \theta_i \Rightarrow Q_i(k - \tau_i / \Delta t) \geq \sqrt{V_{di}(k)} \times \varphi^{-1}(\theta_i) - w_i(k - 1) + \hat{d}_i(k) \quad (2.20.1)$$

With

φ : Cumulative Gaussian distribution function with mean $\hat{d}_i(k)$ and finite variance $V_{d_i}(k)$

And

$U_{\theta_i}(\cdot)$: Minimum production quantity at delivery

φ^{-1} : inverse distribution function.

Proof:

We have $w_i(k) = w_i(k-1) + Q_i(k - \tau_i/\Delta t) - d_i(k)$ and according to the equation (2.20) we have

$Prob[w_i(k) \geq 0] \geq \theta_i$, then the service level requirement constraint is given by:

$$Prob[w_i(k-1) + Q_i(k - \tau_i/\Delta t) - d_i(k) \geq 0] \geq \theta_i$$

$$Prob[w_i(k-1) + Q_i(k - \tau_i/\Delta t) \geq d_i(k)] \geq \theta_i$$

$$Prob[w_i(k-1) + Q_i(k - \tau_i/\Delta t) - \hat{d}_i(k) \geq d_i(k) - \hat{d}_i(k)] \geq \theta_i \quad (2.20.2)$$

We divide the expression by $\sqrt{V_{di}(k)}$, then we have:

$$Prob\left[\frac{w_i(k-1) + Q_i(k - \tau_i/\Delta t) - \hat{d}_i(k)}{\sqrt{V_{di}(k)}} \geq \frac{d_i(k) - \hat{d}_i(k)}{\sqrt{V_{di}(k)}}\right] \geq \theta_i \quad (2.20.3)$$

Note that:

$X = \frac{d_i(k) - \hat{d}_i(k)}{\sqrt{V_{di}(k)}}$ is a variable follows the reduced centred Gaussian distribution $N(0, 1)$

The cumulative Gaussian distribution function is denoted by φ .

$$\varphi\left[\frac{w_i(k-1) + Q_i(k - \tau_i/\Delta t) - \hat{d}_i(k)}{\sqrt{V_{di}(k)}}\right] \geq \theta_i \quad (2.20.4)$$

Since $\lim_{-\infty} \varphi \rightarrow 0$ and $\lim_{-\infty} \varphi \rightarrow 1$ we conclude that φ is strictly increasing. We note that φ is indefinitely differentiable, so we conclude that φ is invertible.

$$\frac{w_i(k-1) + Q_i(k - \tau_i/\Delta t) - \hat{d}_i(k)}{\sqrt{V_{di}(k)}} \geq \varphi^{-1}(\theta_i) \quad (2.20.5)$$

Then

$$w_i(k-1) + Q_i(k - \tau_i/\Delta t) - \hat{d}_i(k) \geq \sqrt{V_{di}(k)} \times \varphi^{-1}(\theta_i)$$

$$Q_i(k - \tau_i/\Delta t) \geq \sqrt{V_{di}(k)} \times \varphi^{-1}(\theta_i) - w_i(k-1) + \hat{d}_i(k) \quad (2.20.6)$$

Thus

$$Prob[w_i(k) \geq 0] \geq \theta_i \Rightarrow Q_i(k - \tau_i/\Delta t) \geq \sqrt{V_{di}(k)} \times \varphi^{-1}(\theta_i) - w_i(k-1) + \hat{d}_i(k) \quad (2.20.7)$$

End proof

2.4.4 Resolution to solve the production problem

There are many optimization approaches that can be used to solve a stochastic problem. The optimal policy, which determines the optimal production plan, is based on the production bound and the service level requirements. The solution to this policy is obtained by dynamic programming. Formally, these techniques are used to implement a real optimal policy that provides a feasible solution that is easy to compute and execute. The optimization model is composed of multi-objective functions which minimize the total inventory, delivery, and production cost, CT , on the finite horizon, $H \cdot \Delta t$, as presented in Figure 2.9 below.

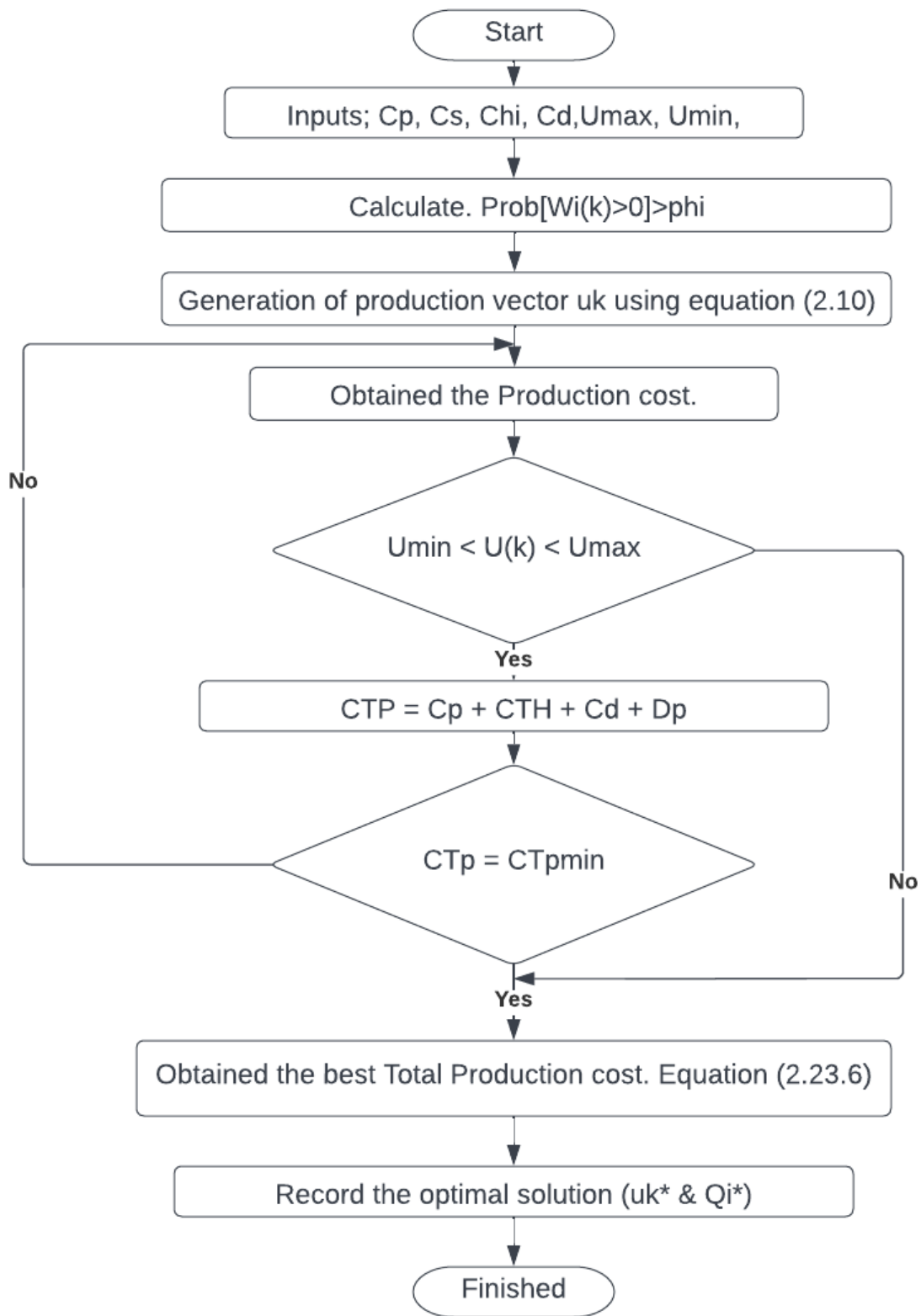


Figure 2. 9: Numerical procedure for determining optimal production rates

2.5. Study and analysis of quality integrated maintenance policy

The unreliable production system is prone to random failure which has a direct impact on product quality. The variance in production rate, and utilization affects the status of the production system and its maintenance evolution. Despite all preventive strategies available to eliminate the occurrence of deviations, additional reactive quality inspections are required. Therefore, companies now seek to take advantage of the constant technological development to explore new methods that are both cost-efficient and easy to improve Quality, (Bergman, 2022).

2.5.1. Description of a quality integrated maintenance policy

We use the Control Chart as a decision support tool to allow the detection of any drift (defects) in the production and to intervene by carrying the appropriate maintenance strategy commensurate to the optimal production and inventory plan. Thus, it allows the increase in process reliability, and product quality level and thus decreases the costs of non-quality.

The maintenance strategy is preventive maintenance actions with minimal repair (PMMR) during the process in-control state, planned on the finite horizon $H \cdot \Delta t$. when the process is in the in-control state, the production periods is divided into N equal parts. The preventive activities are performed at each $i \cdot T$ ($i = 1, 2, \dots, N$), to restore the machine state to as good as new condition. However, if a failure occurs between two planned successive preventive actions, corrective maintenance is therefore employed with minimal repair (as Bad as Old) as illustrated in Figure 2 .10. It is assumed that the minimal repair and replacement durations are negligible when the process is in the “in-control” state. The maintenance plan is developed based on the optimal production plan, which minimizes the failure rate and costs of maintenance.

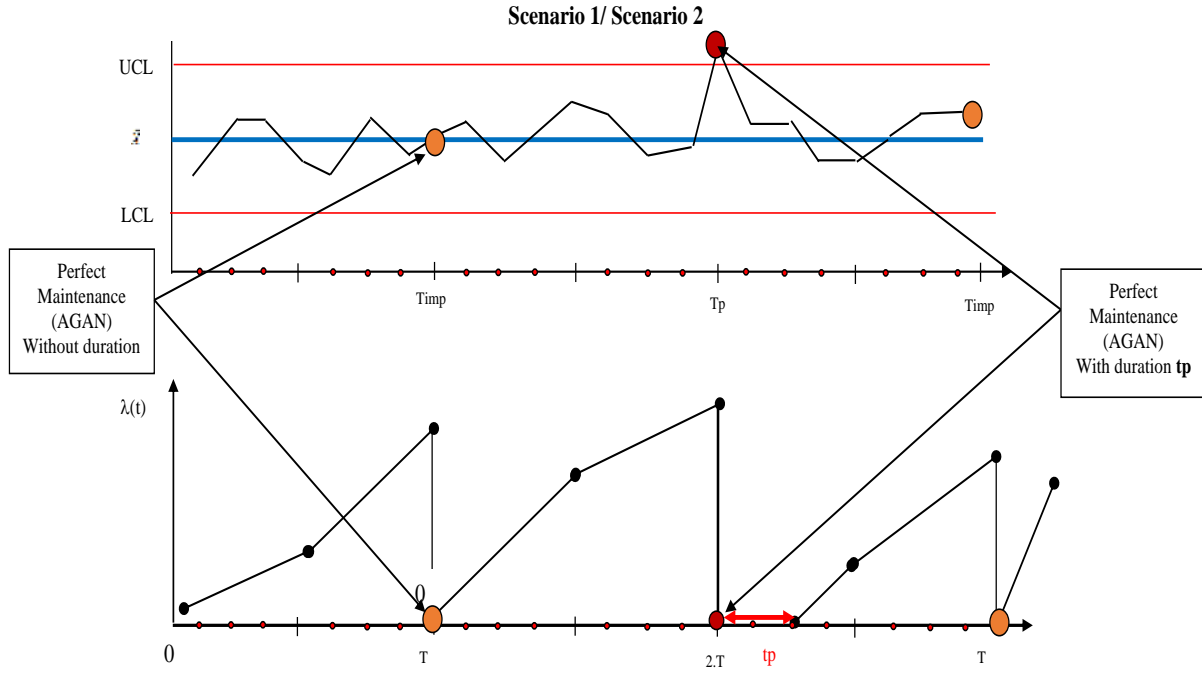


Figure 2. 10: Integrated maintenance and control chart decision.

The Effectiveness of the Control Chart and the Average Restoration Maintenance Cycle

The effectiveness of the control charts can be gauged by evaluating the risk of not detecting a change while using an n sample size. This efficiency of the chart is inversely proportional to the probability. Assume that at some point there is an adjustment that only affects the average. After the adjustment, the new value of the central tendency of the production process is:

$$\mu = \mu_0 + \Delta \quad (2.21)$$

where Δ is the amplitude of the adjustment. Considering that the adjustment is positive, we have $\Delta = \mu - \mu_0$.

Let β_1 be the probability that the point \bar{X}_s falls within the control limits, i.e., β_1 is the probability of non-detecting the adjustment Δ with a one-sample set taken after the adjustment. So, β_1 is the non-detection probability associated with the critical state. Hence, the average quality indicator does not surpass the control limits ($LCL \leq \bar{X}_s \leq UCL$) as long as the production process is in a critical state and has perfect maintenance activity with time.

$$\beta_{1j} = \{P(LCL \leq \bar{X}_s \leq UCL) | \mu = \mu_0 + \Delta\} \quad (2.22)$$

After adjustment, the law of the quality characteristic followed is:

$$\bar{X}_s \sim N(\mu, \sigma_0)$$

The mean sample:

$$\bar{X}_s \sim N\left(\mu, \frac{\sigma_0}{\sqrt{n}}\right)$$

Expressing the adjustment as a function of the standard deviation of X ,

$$\Delta = \delta_{pj} \times \sigma_0$$

With: the magnitude of the move to the “out of control” for assignable cause j with $j = \{1, \dots, n_{pc}\}$.

Equation (22) is written as:

$$\beta_1 = \left\{ P\left(\frac{LCL-\mu}{\frac{\sigma_0}{\sqrt{n}}} \leq \bar{X}_s \leq \frac{UCL-\mu}{\frac{\sigma_0}{\sqrt{n}}}\right) \right\} \quad (2.23)$$

$$\beta_{1j} = F\left(\frac{UCL-\mu}{\frac{\sigma_0}{\sqrt{n}}}\right) - F\left(\frac{LCL-\mu}{\frac{\sigma_0}{\sqrt{n}}}\right) \quad (2.24)$$

Therefore:

$$\beta_{1j} = F(k_p - \delta_{pj} \times \sqrt{n}) - F(-k_p - \delta_{pj} \times \sqrt{n}) \quad (2.25)$$

$$j = \{1, \dots, n_{pc}\}$$

Where F: is the reduced cantered normal distribution function.

So, the average operational period (AOP_1) defined the average number of consecutive samples leading to detecting the first out-of-control (critical state). It is expressed, as follows, for each j :

$$AOP_{1,j} = \frac{1}{1-\beta_{1j}} \quad (2.26)$$

So by replacing β_{1j} with $F(k_p - \delta_{pj} \times \sqrt{n}) - F(-k_p - \delta_{pj} \times \sqrt{n})$ from (2.29) we obtained

$$AOP_{1,j} = \frac{1}{1-F(k_p - \delta_{pj} \times \sqrt{n}) - F(-k_p - \delta_{pj} \times \sqrt{n})}$$

Proof:

$$\text{This is to demonstrate that } \sum_{i=1}^{+\infty} i \times P(N=i) = \frac{1}{1-P(\delta_j)}$$

$$\text{with } P(N=i) = P(\delta_j)^{i-1} \times (1-P(\delta_j))$$

$$\text{When } 0 < q < 1, \text{ we have } \sum_{i=0}^{+\infty} q^i = \frac{1}{1-q} \text{ and } \sum_{i=0}^{+\infty} i \times q^{i-1} = \frac{1}{(1-q)^2}$$

This last result is obtained by derivative with respect to q the two members of $\sum_{i=0}^k q^i = \frac{1-q^{k+1}}{1-q}$

and making tender k to $+\infty$.

Thus by replacing q by $P(\delta_j)$ with $(0 < P(c) < 1)$, we have :

$$\sum_{i=1}^{+\infty} i \times (P(\delta_j))^{i-1} = \frac{1}{(1-P(\delta_j))^2}$$

We deduce that:

$$\sum_{i=1}^{+\infty} i \times P(N=i) = \frac{1-P(\delta_j)}{(1-P(\delta_j))^2} = \frac{1}{1-P(\delta_j)}$$

End proof

The Average Restoration Maintenance Cycle Duration

Average restoration of maintenance cycle duration after perfect preventive maintenance due to the shift to the out-of-control state $RMCD_j$, with $j = \{1, \dots, n_{pc}\}$, is given by:

$$RMCD_{1,j} = AOP_{1,j} \times h + t_{pj} \quad (2.27)$$

2.5.1.1 Machine failure rate

A failure rate is simply a count of failures over time. It is usually first a frequency observation of how often the machine has failed over some previous period. A failure rate can also be a prediction of the number of failures to be expected in a given future period.

Failure Rate $\lambda(t)$

The failure rate is the probability that a failure occurs per unit time interval Δt . The failure rate is the conditional probability, which can be expressed as

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{\{F(t+\Delta t) - F(t)\} / \Delta t}{R(t)} = \frac{f(t)}{R(t)} \quad (2.28)$$

The traditional bathtub curve can describe the variance of failure rate shown in Figure 2.11

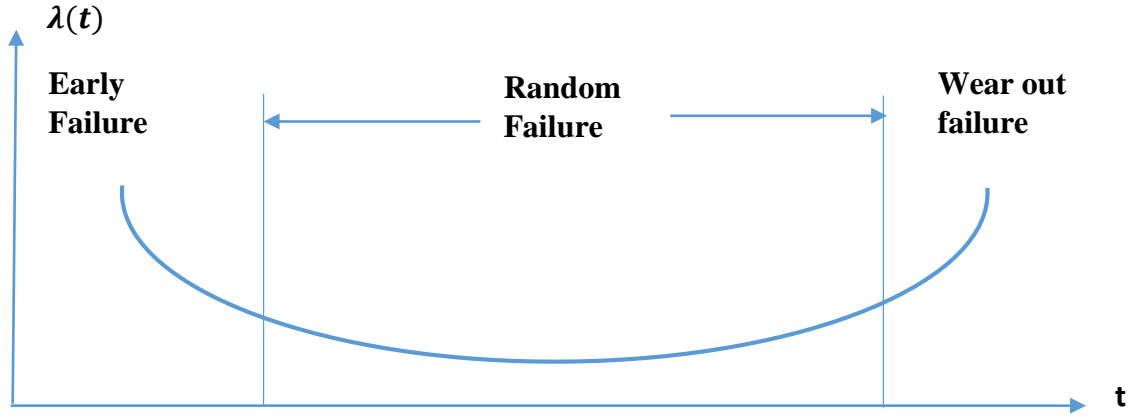


Figure 2. 11: bathtub curve failure rate.

In this work, we were inspired by the failure rate model developed in the work of Hajej et al. (2011) and which gives a thorough analytical study of the relationship between the production rate and machine failure rate. We have adapted it to our system to give rise to the failure rate expression,

$$\lambda_k(t) = \lambda_{k-1}(\Delta t) + \frac{u(k)}{U_{\max}} \cdot \lambda_n(t) \quad \forall t \in [0, \Delta t]$$

Where k is the production period, Δt is the length of production, $u(k)$ is a production rate at any period k , u_{\max} is the maximum production rate and (λ_n) is the nominal function of machine failure rate, i.e the failure rate corresponding to the maximum production rate of the machine throughout the production horizon H . Δt .

The failure rate of the process machine, $\lambda(t)$, increases with both time and use as shown in Figure 2.12 below.

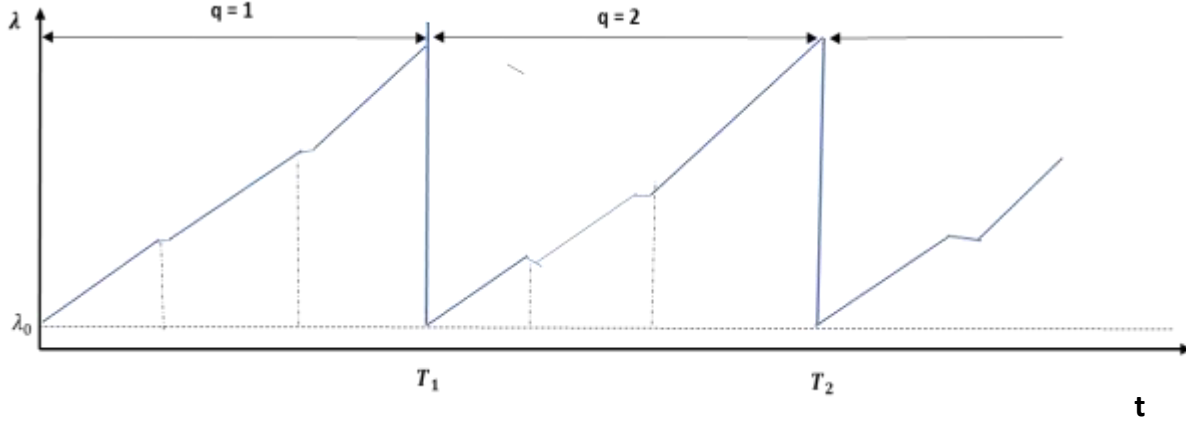


Figure 2. 12: evolution of the failure rate as a function of time

2.5.1.2 Formulation of maintenance cost and quality

Total Cost of Maintenance

The total cost of maintenance consists of the preventive and corrective maintenance cost components of maintenance (1) C_{M1} and maintenance (2) C_{M2} , corresponding to the two scenarios (1 and 2) that are bound to happen in the production process (Figure 2.13). We take into cognizant the comprehensive maintenance strategy to ensure both the cost-efficiency and the reliability of the production process.

Maintenance planning will optimize costs related to maintenance corrective and preventive maintenance.

The proposed maintenance policy is characterized by the optimum number, N^* , of preventive maintenance activities, and the best adequate interval between them is noted as T^* which is obtained by the following equation:

$$C_{MT} = C_{M1} + C_{M2} \quad (2.29)$$

With:

$$C_{M1} = p_1 \times \left(C_{pm} \times \left\lfloor \frac{H}{T/\Delta t} \right\rfloor + C_{cm} \times \varphi_M(U, N) \right) \quad (2.30)$$

Where $\varphi(U, N)$ is the average number of failures, $U = \{u(k), k: \text{from } 1 \dots, H\}$.

It is corresponding to the obtained production planning specified by the vector and the number of preventive maintenance actions N . Assuming that the preventive maintenance action is consistently applied at the end of the production period, the average number of failures is given by the following relation:

$$\varphi_M(U, T) = \sum_{j=0}^{N-1} \left[\sum_{i=\left\lfloor \frac{j \times T}{\Delta t} \right\rfloor + 1}^{\left\lfloor (j+1) \times \frac{T}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=(N \times T / \Delta t)}^H \int_0^{\Delta t} \lambda_i(t) dt \quad (2.31)$$

$\lfloor \cdot \rfloor$: the integer part, and $T = \left\lfloor \frac{H \cdot \Delta t}{N} \right\rfloor$,

$$C_{M2} = p_2 \times \left(C_{pm1} \times n_{pc} + \sum_{j=0}^{n_{pc}-1} \left(C_{pm2} \times \left\lfloor \frac{\left(\frac{AOP_{1,j+1} \times h - RMCD_{1,j}}{b \times h} \right)}{\frac{T_j}{\Delta t}} \right\rfloor + C_{cm} \times \varphi_j(U, N_j) \right) \right. \\ \left. + C_{pm2} \times \left\lfloor \frac{H - \frac{RMCD_{1,npc}}{b \times h}}{\frac{T_{npc}}{\Delta t}} \right\rfloor + C_{cm} \times \varphi_{npc}(U, N_{npc}) \right) \quad (2.32)$$

$$AOP_{1,0} = 0, \quad t_{p0} = 0$$

$$RMCD_{1,j} = AOP_{1,j} \times h + t_{pj}$$

$$T = a \cdot \Delta t = a \cdot b \cdot h \quad \text{with } a = \text{integer}, \Delta t = \text{production length}$$

With n_{pc} as the number of shifts, N_j is the number of preventive maintenance activities between two shifts. The N_{npc} are numbers of preventive maintenance activities between the last shift and the end of the production horizon (H). The probability of being in scenario 1 and 2 (p_1 & p_2) are defined in equation 2.5 and equation 2.6 respectively.

$$Q_{Mj}(U, N_j) = \sum_{Z=0}^{N_j} \left[\sum_{i=\left\lfloor \frac{AOP_{1,j}}{b} \right\rfloor + tp_j + \left\lfloor \frac{(Z+1) \times \frac{T_j}{\Delta t}}{\Delta t} \right\rfloor}^{\left\lfloor \frac{AOP_{1,j}}{b} \right\rfloor + tp_j + \left\lfloor \frac{Z \times \frac{T_j}{\Delta t}}{\Delta t} \right\rfloor + 1} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=(N_j \times \frac{T_j}{\Delta t})}^{AOP_{1,j+1}/b} \int_0^{\Delta t} \lambda_i(t) dt, \quad (2.33)$$

Where;

$$N_j = \left\lfloor \frac{\left(\frac{AOP_{1,j+1} \times h - RMCD_{1,j}}{b \times h} \right)}{\frac{T_j}{\Delta t}} \right\rfloor$$

And;

$$Q_{npc}(U, N_{npc}) = \sum_{Z=0}^{N_{npc}} \left[\sum_{i=\left\lfloor \frac{AOP_{1,npc}}{b} \right\rfloor + tp_{npc} + \left\lfloor \frac{(Z+1) \times \frac{T_{npc}}{\Delta t}}{\Delta t} \right\rfloor}^{\left\lfloor \frac{AOP_{1,npc}}{b} \right\rfloor + tp_{npc} + \left\lfloor \frac{Z \times \frac{T_{npc}}{\Delta t}}{\Delta t} \right\rfloor + 1} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=(N_{npc} \times \frac{T_{npc}}{\Delta t})}^H \int_0^{\Delta t} \lambda_i(t) dt \quad (2.34)$$

$$N_{npc} = \left\lfloor \frac{H - \left(\frac{AOP_{1,npc}}{b} \right) - tp_{npc}}{T_{npc}} \right\rfloor$$

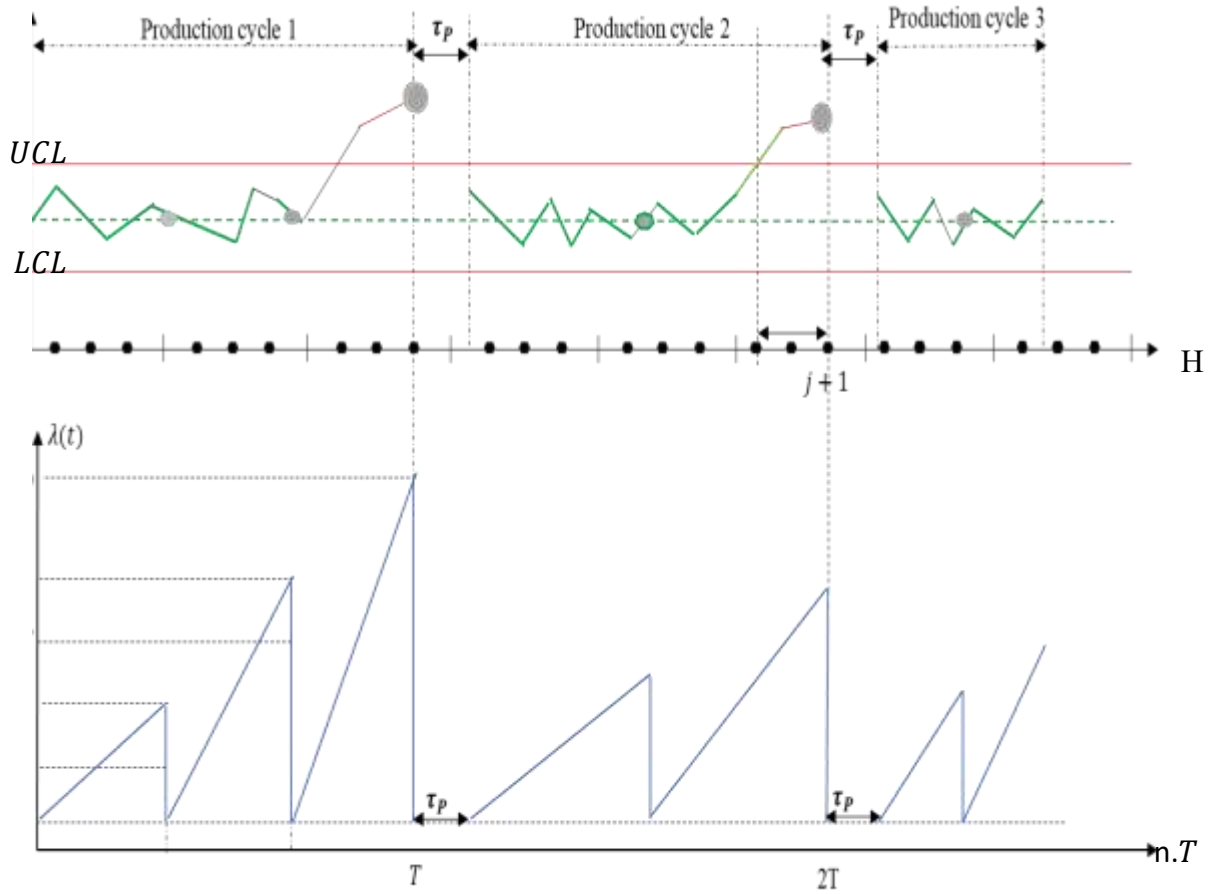


Figure 2. 13: evolution of the failure rate

Total Cost of Quality

The total cost of quality is the summation of the cost of false alarms, C_{FA} , the cost of sampling inspection (C_{Si}), and the cost of non-conformal items (CNC).

Cost of False Alarm

The notion of false alarm is introduced when the limits of the card are exceeded, even if the process is under control. In this case, another cost called the false alarm cost is built into the model. The probability β_2 presents the false alarm where the control chart detects a point that exceeds the control limits while the process is under control. So, the central value of the card remains constant, $\Delta = \delta_p \times \sigma_0 = 0$:

$$\mu_1 = \mu_0 + \delta_p \times \sigma_0 = \mu_0 \quad (2.33)$$

The probability of achieving a false alarm is provided by:

$$\beta_2 = \{Prob(\bar{X}_s \leq LCL \mid \mu_1 = \mu_0) + Prob(\bar{X}_s \geq UCL \mid \mu_1 = \mu_0)\} \quad (2.34)$$

$$\beta_2 = F\left(\frac{LCL - \mu_1}{\frac{\sigma_0}{\sqrt{n}}}\right) + 1 - F\left(\frac{UCL - \mu_1}{\frac{\sigma_0}{\sqrt{n}}}\right) \quad (2.35)$$

$$\beta_2 = 1 + F\left(\frac{\mu_1 - k_p \times \frac{\sigma}{\sqrt{n}} - \mu_1}{\frac{\sigma_0}{\sqrt{n}}}\right) - F\left(\frac{\mu_1 + k_p \times \frac{\sigma}{\sqrt{n}} - \mu_1}{\frac{\sigma_0}{\sqrt{n}}}\right) = 1 + F(-k_p) + 1 - F(k_p) = 2 \times F(-k_p) \quad (2.36)$$

Therefore, the average number of inspections (of successive samples) that must be recorded before obtaining a false alarm (AOP_2) is given by:

$$AOP_2 = \frac{1}{\beta_2} \quad (2.37)$$

Consider S to be a discrete random variable indicating the samples' number when the production process is under control. S take values between 0 and m . So, the probability distribution related to the random variable S is expressed by:

$$P(S = m) = p_1^m \times (1 - p_1) \quad (2.38)$$

Hence, the average number of samples when the production process is under control can be expressed by:

The cost of false alarm is given by:

$$C_{FA} = C_F \times \frac{E(S)}{AOP_2}, \quad C_{FA} = 2 \times C_F \times E(S) \times F(-k_p) \quad (2.39)$$

Cost of Sampling inspection

The inspection cost is defined by the total cost of sampling inspections (C_{si}), which is a product of the number of samples taken (n) during inspection, the cost of sampling one unit product, and the number of times the sample is drawn expressed as;

$$C_{si} = c_i \times n \times m \quad (2.40)$$

Cost of Non-Conforming Item

The proportion of defective units at the period when the process is in the out-of-control state, also known as non-quality rejected products is defined by:

$$C_{NC} = C_r \times \left(\sum_{j=1}^{n_{pc}} \left(u \left(\left\lfloor \frac{AOP_j}{b} \right\rfloor \right) \times \left(\frac{AOP_j}{b} - h \right) \right) \right) \quad (2.41)$$

2.5.2 Optimization procedure

An iterative numerical optimization procedure is developed and coded using MATHEMATICA software, as shown in the algorithm flow diagram (Figure 2.14) below. The

procedure integrated the following user inputs (initials) data; $u_k, u_{min}, u_{max}, \alpha, \beta, \mu, \sigma, \delta, \Delta t, u, H, C_{pm1}, C_{pm2}, C_{cm}, C_i, C_r, C_f$

- ✓ The multi-decision variables (n, m, h, k_p) are initialized with small values by the user.
- ✓ The decision parameters are varied with some increments $(\Delta n, \Delta m, \Delta h, \Delta k_p)$ to some defined maximum limits $(n_{max}, m_{max}, h_{max}, k_{p_{max}})$
- ✓ The average operational period $(AOP_{1,j})$ defined for each “j” is calculated by (2.26)
- ✓ The average restoration of maintenance cycle duration $(RMCD_j)$ is calculated by equation (2.27)
- ✓ The total cost of maintenance is calculated using the equation (2.29)
- ✓ The average number of checks (of successive samples) to have a false alarm (AOP_2) is calculated using equation (2.37)

The accuracy or quality of the solution (optimal parameters) obtained is a function of the increments step-length operated by the user.

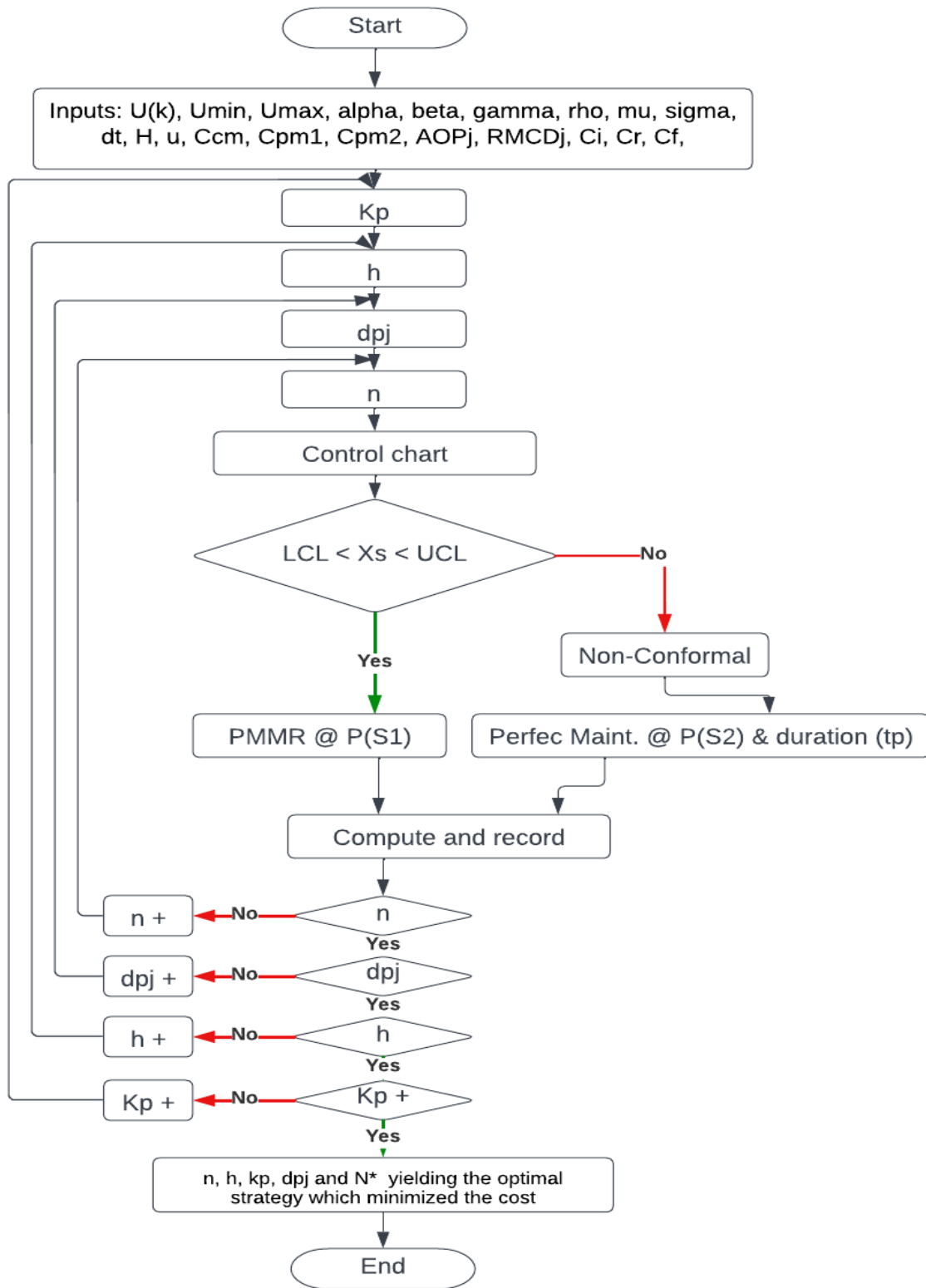


Figure 2. 14: The diagram of the numerical optimization procedure

2.6 Numerical example

In this study, our first objective was to minimize the total cost of production and inventory by determining the economic planning of production to meet the varying demands. Secondly, to minimize the total maintenance and quality cost for different process scenarios. To resolve the joint model of production, maintenance, and quality, we used Mathematica software with its integrated algorithms due to the complexity of the problem. The optimal results are represented by the best production plan and a numerical procedure for the maintenance and quality problem to determine the optimal value of sample size, sampling interval, control limits, and the optimal number of perfect preventive maintenance actions corresponding to integrated policy.

2.6.1. Production Problem

To validate the robustness and effectiveness of our work, we consider three different cases. Considering a supply chain composed of a production system that produces one type of product, one primary stock, and multi-warehouses that will satisfy random demands over a finite planning horizon $H = 12$ months each of period length $\Delta t = 1$ month. Assumed that the standard deviation of each demand of a product is the same for all periods, $\sigma d_i = 1.2$, ($\{i: 1, 2\}$), and it is assumed that the initial level of inventory $S(0) = 100$. The average demand for warehouse 1 and warehouse 2 is $\hat{d}_1(k) = \hat{d}_2(k) = 300 \{k: 0, \dots, H - 1\}$.

Lower and upper limits of production capacities: $U_{min} = 0$ and $U_{max} = 500$ and $c_p = 2 \text{ mu}$, $c_h = 0.2 \text{ mu}$, $c_{si} = 0.2 \text{ mu}/k$, $\{i: 1, \dots, L = 2\}$, $w_i(0) = 200$ with $\{i: 1, 2\}$. The consumer satisfaction level is equal to 90% $\theta_i = 0.9 (i = 1, 2)$, and (i) is the number of warehouses.

Example 1:

Tables 2.3, Tables 2.4, and Figure 2.12 show some of the results; the optimum production planning and the lowest total cost for different values of the delivery time τ . Meanwhile, Figure 2.14 illustrates the total cost (production, delivery, inventory) as a delivery time function τ . So, the lowest total cost value corresponds to $\tau^* = 2$, with a minimal cost equal to 13,772 mu . Firstly, the optimal results represent the best production plan to satisfy the random demand presented in Tables and 2.1, Tables 2.2, and secondly to be used as constraints in determining the most appropriate number of PM actions to minimize total cost. We used Mathematica software with an embedded algorithm. And a numerical procedure for the maintenance and quality problem finds the optimal values of the sample size, sampling interval, and the control limits coefficient corresponding to the integrated policy.

$d_1(1)$	$d_1(2)$	$d_1(3)$	$d_1(4)$	$d_1(5)$	$d_1(6)$
150	170	150	150	150	140
$d_1(7)$	$d_1(8)$	$d_1(9)$	$d_1(10)$	$d_1(11)$	$d_1(12)$
160	140	160	150	150	15

Table 2. 1: Average demand 1.

$d_1(1)$	$d_1(2)$	$d_1(3)$	$d_1(4)$	$d_1(5)$	$d_1(6)$
160	130	150	150	140	160
$d_1(7)$	$d_1(8)$	$d_1(9)$	$d_1(10)$	$d_1(11)$	$d_1(12)$
160	160	140	150	150	140

Table 2. 2: Average demand 2.

$u^*(1)$	$u^*(2)$	$u^*(3)$	$u^*(4)$	$u^*(5)$	$u^*(6)$
350	290	250	320	320	420
$u^*(7)$	$u^*(8)$	$u^*(9)$	$u^*(10)$	$u^*(11)$	$u^*(12)$
350	350	350	230	150	150

Table 2. 3: The optimal production plan.

$Q1^*(1)$	$Q1^*(2)$	$Q1^*(3)$	$Q1^*(4)$	$Q1^*(5)$	$Q1^*(6)$
160	180	125	160	170	200
$Q1^*(7)$	$Q1^*(8)$	$Q1^*(9)$	$Q1^*(10)$	$Q2^*(11)$	$Q2^*(12)$
210	200	180	170	100	-

Table 2. 4: The optimal delivery plan for warehouse 1 ($\tau^* = 2$).

$Q2^*(1)$	$Q2^*(2)$	$Q2^*(3)$	$Q2^*(4)$	$Q2^*(5)$	$Q2^*(6)$
180	110	125	160	150	220
$Q2^*(7)$	$Q2^*(8)$	$Q2^*(9)$	$Q2^*(10)$	$Q2^*(11)$	$Q2^*(12)$
200	220	180	60	50	-

Table 2. 5: The optimal delivery plan for warehouse 1 ($\tau^* = 2$).

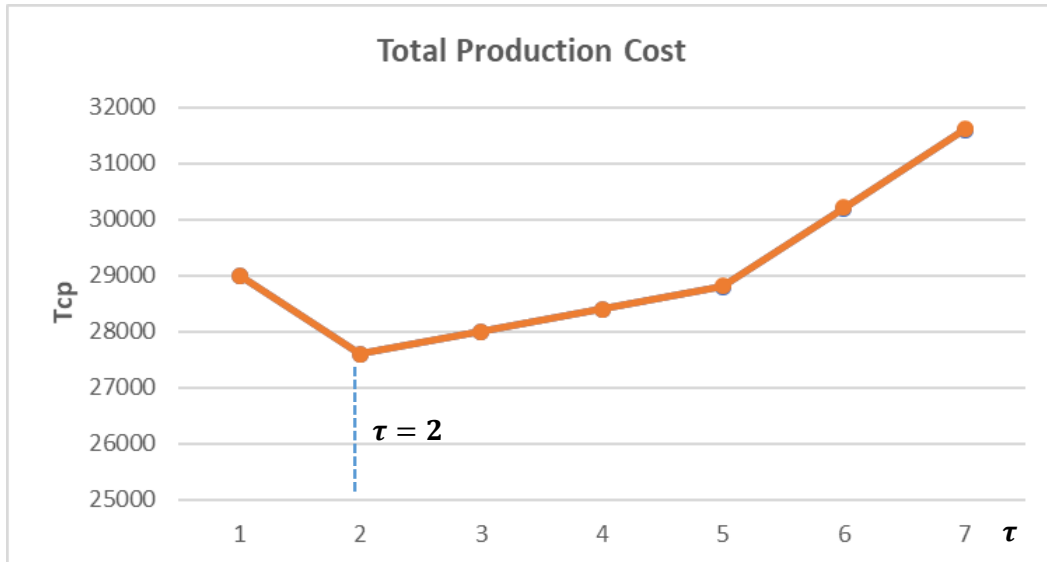


Figure 2. 15: The optimal delivery plan for warehouse 2 ($\tau^* = 2$).

Example 2: (Higher demand):

We consider a second example by increasing the average of the demands (Tables 2.6, Tables 2.7) to verify the effectiveness of our proposed model. Table 2.8, Table 2.9, and Table 2.10 show the optimum production and delivery planning and the lowest total cost of different values of the delivery time.

$d_1(1)$	$d_1(2)$	$d_1(3)$	$d_1(4)$	$d_1(5)$	$d_1(6)$
300	340	300	300	300	280
$d_1(7)$	$d_1(8)$	$d_1(9)$	$d_1(10)$	$d_1(11)$	$d_1(12)$
320	280	320	300	300	300

Table 2. 6: Average demand of customers at warehouse 1.

$d_2(1)$	$d_2(2)$	$d_2(3)$	$d_2(4)$	$d_2(5)$	$d_2(6)$
320	260	300	300	280	320
$d_2(7)$	$d_2(8)$	$d_2(9)$	$d_2(10)$	$d_2(11)$	$d_2(12)$
320	320	280	300	300	280

Table 2. 7: Average demand of customers at warehouse 2.

$u^*(1)$	$u^*(2)$	$u^*(3)$	$u^*(4)$	$u^*(5)$	$u^*(6)$
440	290	250	320	320	420
$u^*(7)$	$u^*(8)$	$u^*(9)$	$u^*(10)$	$u^*(11)$	$u^*(12)$
410	420	360	230	240	440

Table 2. 8: The economic production plan.

$Q_I^*(1)$	$Q_I^*(2)$	$Q_I^*(3)$	$Q_I^*(4)$	$Q_I^*(5)$	$Q_I^*(6)$
250	250	250	250	2200	90
$Q_I^*(7)$	$Q_I^*(8)$	$Q_I^*(9)$	$Q_I^*(10)$	$Q_I^*(11)$	$Q_I^*(12)$
160	210	70	-	-	-

Table 2. 9: The economic delivery plan for warehouse 1 ($\tau^* = 1$).

$Q_2^*(1)$	$Q_2^*(2)$	$Q_2^*(3)$	$Q_2^*(4)$	$Q_2^*(5)$	$Q_2^*(6)$
160	120	150	160	250	170
$Q_2^*(7)$	$Q_2^*(8)$	$Q_2^*(9)$	$Q_2^*(10)$	$Q_2^*(11)$	$Q_2^*(12)$
100	250	70	-	-	-

Table 2. 10: The optimal delivery plan for warehouse 2 ($\tau^* = 1$).

For the optimal planning of production, delivery quantities varied according to the delivery time, then impacted the total cost. Consequently, through the analysis of the variability of delivery

time, we obtained the proper delivery time value that minimizes the total cost, which relates to $\tau = 1$, with minimal cost equal to 26,500 mu in our numerical Example 2. Moreover, compared with the first example, the higher value of the total cost relates to the higher value of the average demand can note that, as average demands increase, the process produces more. Then, we need to take more of the products in the process store and the purchase warehouses to satisfy the demand with a maximum service level. Consequently, the transported amount also increases with a faster delivery time.

Example 3: (Varying standard deviation):

In this case, we increase the standard deviation to $\sigma d_i = 5$, ($\{i: 1,2\}$) and use the same average demands presented in Example 2, and we find the economic planning given in Tables 2.1.1 to 2.1.3.

$u^*(1)$	$u^*(2)$	$u^*(3)$	$u^*(4)$	$u^*(5)$	$u^*(6)$
380	114	112	8	293	366
$u^*(7)$	$u^*(8)$	$u^*(9)$	$u^*(10)$	$u^*(11)$	$u^*(12)$
433	40	216	402	255	203

Table 2. 11: Economic production plan.

$Q_1^*(1)$	$Q_1^*(2)$	$Q_1^*(3)$	$Q_1^*(4)$	$Q_1^*(5)$	$Q_1^*(6)$
201	160	243	239	79	291
$Q_1^*(7)$	$Q_1^*(8)$	$Q_1^*(9)$	$Q_1^*(10)$	$Q_1^*(11)$	$Q_1^*(12)$
278	353	115	346	135	309

Table 2. 12: The economic delivery plan for warehouse 1 ($\tau^* = 2$).

$Q_2^*(1)$	$Q_2^*(2)$	$Q_2^*(3)$	$Q_2^*(4)$	$Q_2^*(5)$	$Q_2^*(6)$
395	304	38	210	336	218
$Q_2^*(7)$	$Q_2^*(8)$	$Q_2^*(9)$	$Q_2^*(10)$	$Q_2^*(11)$	$Q_2^*(12)$
25	289	117	0	292	200

Table 2. 13: The optimal delivery plan for warehouse 2 ($\tau^* = 2$).

By increasing the standard deviation, the proper delivery time value is determined, also minimizing the total cost, which relates to $\tau = 2$, with minimal cost equal to 35,310 *mu* from Example 3. Moreover, in comparison with example 2, the higher total cost corresponds to the higher value of the standard deviation. We can observe the increase in standard deviation from Figure 2.16, and the process produces more. Then more products in the primary store and the warehouses to satisfy the higher demand. However, increasing the standard deviation will require more of the production system, the transported amount also increases with a faster delivery time, and consequently, these will impact the degradation of the process as well as the corresponding strategy of maintenance quality.

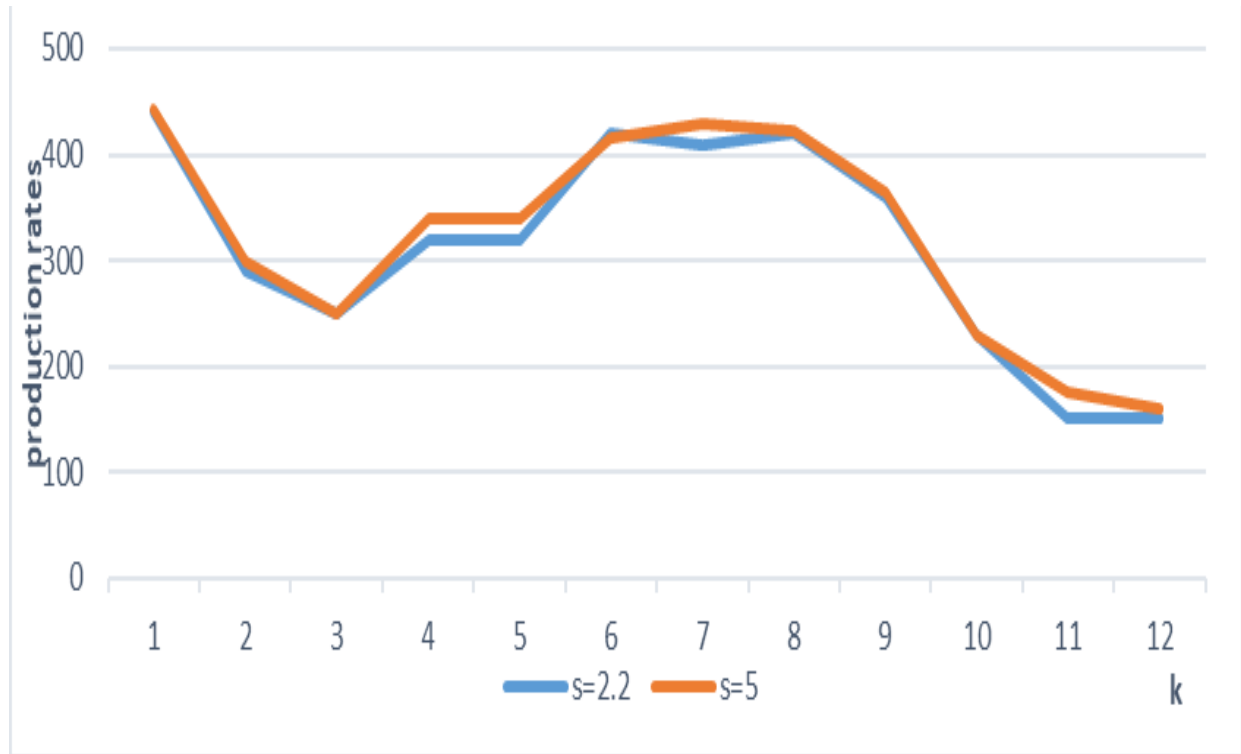


Table 2. 14: Production rates as a function of standard deviation.

2.6.2. Maintenance and Quality Problem

We developed the following numerical optimization procedure based on the obtained optimal production plan and the following different parameters arbitrarily chosen:

The average duration of each preventive maintenance action, at each out-of-control state, is $\Delta t/28$ as shown in Figure 2.13 below. Weibull distribution is used for the evolution of the production system failure rate in its wear-out life, where failure rate increases quickly with both time and use, and which requires preventive maintenance actions unlike the other phases (early life, useful life) which are

not needed. The degradation law of the production system is, therefore, characterized by a Weibull distribution with scale and shape parameters respectively $\beta_1 = 100$ and $\alpha_1 = 2$.

$C_{CM} = 5500 \text{ } \mu\text{u}$, $C_{PM1} = 300 \text{ } \mu\text{u}$, $C_{PM2} = 700 \text{ } \mu\text{u}$, $c_r = 70 \text{ } \mu/\text{unit lost}$, $c_F = 120 \text{ } \mu/\text{false alarm}$, $c_i = 15 \text{ } \mu/\text{product}$,

$\mu_0 = 5$, $\sigma_0 = 1.5$,

$\Delta n = 2$; $\Delta h = 1$; $\Delta k_p = 0.05$; $n_{max} = 50$; $h_{max} = 10$; $k_{p-max} = 4.00$

Using the economic plan of production obtained in Table 2.3, Using a numerical procedure, for the first scenario where the process is stable and in control during all horizons, the optimum values of m, n, h, kp , and numbers of preventive actions (in-control & in the out-of-control scenarios) are presented in Table 2.14, 2.15 below. Thus, the best strategy consists in taking a sample of size 42 every 48h. In addition, for the design of the control chart, the ideal number of standard deviations between the centre line and the control chart and limits is found to be $k_p = 3.50$ as indicated in Figure 2.16. Therefore, during the in-control state, an average of 4 preventive maintenance actions would be carried out with a maintenance interval of $48h = 2 \text{ days}$ as shown in Figure 2.17. The total cost average corresponding to the integrated policy is 45,000 μu .

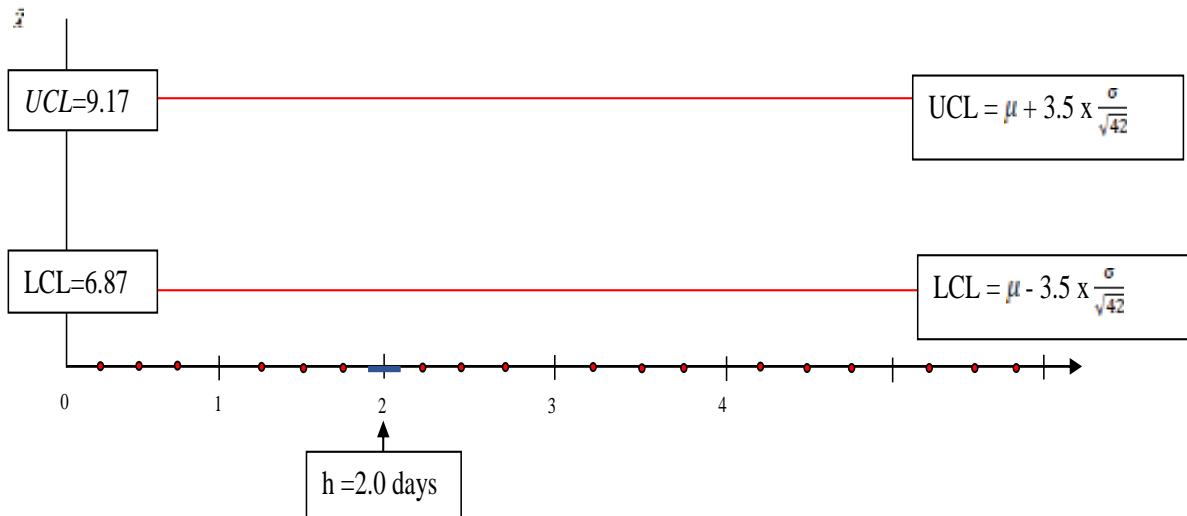


Figure 2. 16: PM duration according to Control Chart.

Regarding the second scenario where the control chart can detect an out-of-control, the best strategy is for the control chart to detect one point out-of-control with the average number of samples

to have a false alarm corresponding to $AOP_2 = 120$ h. The average number of samples to indicate when the control limits are exceeded is equal to $AOP1 = 1554h = 6. \Delta t$. Hence, the control card is found to be efficient as it's tolerable to false alarm after a relatively more number of samples and reacts to the exceeding limits quickly. From Table 2.8, the total best number of preventive maintenance activities between the out of control equals 3. Concerning the optimal number of preventive maintenance activities between the out-of-control point, maintenance cycle 1 ($N_{2,1} = 2$) and maintenance cycle 2 ($N_{2,2} = 1$), as shown in Table 2.15.

The first maintenance cycle is characterized by the production period before the out-of-control point, where the $AOP1 = 84$ h $= 6. \Delta t$, and we applied two preventive maintenance actions without duration according to the average number of failures. The second cycle is characterized by the production periods from $AOP1 + tp = 7. \Delta t + \Delta t/20$ to $H. \Delta t$, where we applied one preventive maintenance action.

Cycle 1 of maintenance	Cycle 2 of maintenance
$[0, AOP1] = [0, 6. \Delta t]$	$[AOP1, H] = [6. \Delta t + \Delta t/20, 8. \Delta t]$
$N_1 = 2$	$N_2 = 1$

Table 2. 14: An optimal number of preventive maintenance activities

2.6.3. Sensitivity Analysis for Maintenance/Quality Strategy

To analyse and validate the system's behaviour, we have studied the different possible causes of variations in model parameters, namely: the cost of inspection (c_i), the cost of rejecting defective items (c_r), perfect maintenance duration (t_p), and the magnitude of the transition to the critical "out-of-control" state relative to the centreline (δ). Using the economical production plan presented in table 2.8 (higher demand) For example, 1, the effects of variations in these parameters on the optimum solutions are given in Table 2.15 and Figure 2.17.

Parameter	Variation	n^*	h^*	k_p^*	AOP	N_1^* (scenario 1)	$N_{2,1}^{**}$ (scenario 2- Cycle 1)	$N_{2,2}^{**}$ (scenario 2- Cycle 2)	Total Production Cost (1)	Total Production Cost (2)
Base		42	2	3.50	1554	3	2	1	19426	15884
c_i	10	48	2	3.65	2544	3	1	0	22083	12323
	15	42	2	3.70	1544	2	2	0	22552	12353
	20	36	2	3.60	1162	3	3	0	22114	13528
	50	48	2	3.65	2544	3	2	0	22506	12353
c_r	70	46	2	3.60	2162	2	2	0	21706	13418
	90	42	2	3.70	2520	2	1	0	22796	12437
	0.5	30	1.5	3.45	2544	3	3	0	22552	12362
t_p	0	42	2	3.60	2162	3	2	0	21706	13418
	1.5	48	3	3.75	1848	2	1	1	20613	14966
	0.2	32	2	3.70	2368	1	1	0	23021	12938
δ	0.4	38	2	3.65	2438	2	1	1	22464	12640
	0.6	48	3	3.50	960	3	2	2	17125	18101
	200	38	2	3.65	1544	3	2	0	22623	12353
Cpm ₁	300	42	2	3.50	1628	3	2	1	19305	15449
	400	50	3	3.65	2544	2	1	0	22552	12353
	500	32	2	3.50	2368	3	1	0	23021	12938
Cpm ₂	700	48	2	3.65	2544	3	2	1	22624	12353
	900	48	2	3.75	2544	2	3	0	23146	12353
	4000	48	3	3.50	1628	2	3	2	18312	14773
Ccm	5500	40	2	3.55	1848	3	2	1	20175	14975
	7000	32	1	3.65	2544	4	1	0	24142	12439

Table 2. 15: Sensitivity analysis of different parameters. (*denotes the optimal value).

From the sensitivity analysis (Table 2.15), we can notice that:

- ***Variation of the unit inspection cost:*** The increase of the unit inspection cost causes the reduction of the AOP and also the decrease of the inspection number of items n^* to minimize the total cost of sampling. In addition, a slight variation in the other decision variables such as the k_p^* was noted where the control limits move away and the severity of the control chart is lower. The number of preventive maintenance actions N_2 is increased to reduce the degradation of the process.
- ***Variation of the cost of a defective unit:*** The cost of rejection has a slight influence on the AOP and the number of inspections, m^* . The increase of the rejection cost causes the decrease of the sampling size n^* and the increase in the control chart limit (k_p) in order to reduce the proportion of non-conforming units. However, it mainly acts on the proportion of non-conforming units and subsequently on the control limits.
- ***Variation of PM duration at out-of-control:*** The increase of preventive maintenance duration at each out-of-control point generates the increase of the optimal number of preventive maintenance decreases when the process is under control reducing the frequency to undertake the preventive maintenance action with duration. If the preventive maintenance time increases, the system reacts to ensure excellent process quality while increasing the number of inspections and the sampling interval. Moreover, the number of standards deviation between the centre line and the control limits k_p increases.
- ***Variation of the magnitude of the shift to the "out-of-control" state:*** The increasing of the parameter δ implies that the average sample of the quality indicator will move away from the line of the control chart. Subsequently, the process is more degraded. Hence, the control chart becomes tighter to cope with the increasing process degradation, allowing potentially more points to fall close to the control limits. Consequently, the preventive maintenance actions with duration increases as well as the number of preventive maintenance actions. Furthermore, the sample size and the sampling frequency increase improving the process quality.
- ***Variation of Cpm1:*** When the cost of preventive maintenance action C_{pm1} for each shift increases, the number of preventive maintenance actions during the in-control decreases reducing the frequency to undertake preventive maintenance actions for out-of-control and increasing the average period before the shift to the out-of-control state. As the severity of the control chart increases the sample size and the sampling frequency, yielding high-quality requirements.
- ***Variation of Cpm2:*** When the cost of preventive maintenance action increases, the number of preventive maintenance action during the in-control state decrease reducing the frequency to undertake preventive maintenance actions. Furthermore, the control limits move away, which illustrates the that preventive maintenance actions at each out-of-control are performed more frequently due to the low-quality requirements and the severity of the control chart increases the sample size and the sampling frequency.

- *Variation of Ccm*: When the cost of corrective maintenance action increases, the number of preventive maintenance action during the in-control state increase reducing the average number of failure and increasing the reliability of process as well as the high-quality. Furthermore, the average period before the shift to the out-of-control state increases and fewer preventive maintenance actions with duration is performed. Because the severity of the control chart is lower, the sample size and the sampling rate are reduced.

2.7 Conclusion

This work studied an integrated model of production policy, maintenance strategy, and quality control, considering the impact of the production variation and control chart parameters on the optimum maintenance plan. The optimization model finds the optimal economical plan of production and delivery and the best configuration of the control chart, consequently, the optimal preventive maintenance plan while minimizing the total cost. Also, we provided a quality control policy to reduce the non-compliant proportion of products by using the statistical process control chart, where the best configuration allowed us to decide the optimal maintenance strategy. A numerical illustration and sensitivity analysis have been studied to show the variation influence of different parameters on the system behaviour.

The approach considers the best combination between the production, maintenance, and quality policies and the information obtained by the control chart based on established analytical relationships. This is an efficient and robust model, that will be ready to apply in real practical cases, due to the stochastic characteristics of most production systems. Our proposed strategy was compared with several research works, that have used the control chart in the quality control policy and assumed that the out-of-control state is due to the degradation of the production system such as Bahria et al. [28] and Ben-Daya et al. [26], but these studies did not consider the degradation source and its relationship with the production part. Our work has defined and studied the correlation between production and maintenance by establishing an analytical relationship between the non-conformal items, failure rate, the average number of failures, the production rate, and its impact on the production system. Additionally, in their work, the plan as well as the number of maintenance are defined as data and not considered as decision variables. While, in our work, we consider that the maintenance planning characterized by the optimal number of preventive maintenance actions during the in-control state is dynamic and depends on the control chart parameters that are considered as decision variables.

The subsequent models take into account different assignable causes. In the next chapter (3) An optimal production, maintenance, and quality forecasting Problem, with an improved statistical process chart of a supply chain under service and quality requirements is investigated. The product quality control is done with a multi-level statistical process chart (SPC) with additional surveillance. The SPC indicates when the process is respectively in control, surveillance, and critical stages. While

the process is in a control state, producing conforming items is subjected only to minimal corrective maintenance. But when the operation fails (out of the control) state until the process shifts to a surveillance stage (SL-CL), it requires imperfect preventive maintenance with duration (t_{imp}) to reduce the failure rate. And when the operation moved to a critical “out of control” state, then a perfect maintenance action with significant time (t_p) is employed to restore the process to (AGAN). The decision variables are the sample size n , the control interval h , and the control charts coefficients k_{imp} and k_p .

Chapter 3:

An optimal production, maintenance, and quality forecasting Problem, with an improved statistical process chart of a supply chain under service and quality requirements.

This chapter investigated optimal production, maintenance, and quality forecasting Problem, with an improved statistical process chart of a supply chain under service and quality requirements. The product quality control is done with a multi-level statistical process chart (SPC) with additional surveillance. The SPC indicates when the process is respectively in control, on surveillance, and under critical stages. While the process is in a control state, producing conforming items is subjected only to minimal corrective maintenance. But when the operation falls to a surveillance stage (SL-CL), it requires imperfect preventive maintenance with duration (t_{imp}) to reduce the failure rate. And when the operation moved to a critical state, then a perfect maintenance action with significant time (t_p) is employed to restore the process to (AGAN). The decision variables are the sample size n , the control interval h , and the control charts surveillance limits coefficients k_{imp} and the critical limits k_p . The multi-ware houses are to satisfy random customer demands during the finite production horizon with service, quality, and production bounds constraints. We use a combination of stochastic mathematical formulation, simulation, and optimization to determine the optimal chart parameters, which minimizes the total expected cost of the production. To highlight the interesting aspects in the interactions between the production, quality, and maintenance, a numerical examples and sensitivity analysis are presented. The work established a collaborative production planning decision support system which increase the reliability of the process, reduces the non-conformal losses and subsequently, achieved the total cost minimization.

3.1 Introduction.

There is a need for industrial companies to reduce their costs, satisfy multiple customer requirements, and at the same time make profits while maintaining market competitiveness. Production, maintenance, and quality are the most critical aspects of an industrial system. These factors are interrelated with each other. Production planning reduces the work in progress and ensures the satisfaction of the demand. Maintenance increases availability with a reduction of failures at increased reliability. While quality control will guarantee the desired product quality. Therefore, the integration of these fundamental interdependent factors will result in achieving improved production system efficiency as well as significant total production cost minimization. However, the planning and implementation of these factors jointly and optimally represent a challenge to production companies. And traditionally, these factors were not treated altogether or optimally, but with the development of the current industrial system, the emergence of exigent customers, competitive markets, new products, and random demands required the collaboration of these interrelated factors to achieve overall objectives. There are several works in the literature conducted by different authors and considering one or two of these factors, until recently, the integration of all three factors. According to (Abubakar et al.,2020), customers are becoming more exigent, while production systems are getting more complex and subject to multiple uncertainties, and demand also becomes random. Hence the need to develop integrated industrial strategies (maintenance, production, and quality). Unfortunately, production planning, maintenance, and quality control policies are rarely studied together in the literature. For industrial organizations, determining the best production planning and maintenance strategy has always been difficult. As a hierarchical decision-making process, the best maintenance and production plans minimize the entire cost, including production, inventory, and maintenance. As a result, we combine a statistical quality control process with extra surveillance measures to make more interactive decisions about new combined maintenance strategies. Optimal production and delivery planning will be an ideal solution for lowering total production costs by boosting the production system's reliability and, as a result, reducing non-conformance production. (Buzacott, 1967) is one of the earliest authors to accomplish the improvement of production. In his work "Automatic transfer lines with buffer stock," he studied the role of buffer stocks in boosting system productivity to solve the challenge of integrating maintenance into production plans. (Nourelfath et al.,2016) studied a multi-period multi-product capacitated lot-sizing context, integrating production, maintenance, and quality for an imperfect process. In their model, during each period, the machine is inspected, and imperfect preventive maintenance activities are simultaneously performed. The outcomes of the works determine the optimal values of the production plan and the preventive maintenance period where the machine is restored to in control state after producing non-conformal items. (Guo et al.,2018) consider lot sizing, quality, and maintenance for an imperfect production system. (Rivera-Gomez et al.,2020) proposed an integrated production maintenance and quality control policy for an unreliable single-product manufacturing system subject to degradation. Depending on the defective proportion determined during the inspection, the inspector can decide on the appropriate preventive maintenance. A minimal repair is

carried out at failure to restore the production system to its previous status (ABAO). (Boushla et al.,2016) investigated production, preventive maintenance, and quality control using a sampling method. The production lot size, the sampling plan, the safety stock, and overhaul planning are under quality constraints. The latter determined the optimal buffer stock size of the production system subjected to periodic preventive maintenance (PM) action, thereby reducing the probability of failure and the machine age, proportional to the preventative maintenance. (Nahas,2017) created a stochastic model of unstable equipment, which focused on joint optimization of economic production and aged-based preventive maintenance policy for deteriorating production systems using proportional hazard (PHM). (Ben-Daya M.,2002) in their paper effect of maintenance on the economic design of the x-control chart, and its expansion (Zang and Zang,2017) suggested a reliability and maintenance structure for a two-state process optimizing decision variables in industrial systems to identify discrete timeframes for Preventive Maintenance tasks. Nevertheless, they also neglected inventory scarcity. (Salmasnia et al.,2017) use a particle mass optimization algorithm. A collaborative design of production run length, maintenance policy, and control chart is built using numerous assignable reasons. The best values are determined by the result of the effort of the production plan and the preventive maintenance period where the machine is restored to in controlled state after producing non-conformal products. They likewise explored the effects of imperfect production on the optimal production cycle time. Production and quality issues were attended to by Hajej et al., in their work which Integrated the model of production, maintenance, and quality. The work studied a randomly failing manufacturing system that has to satisfy random demand during a finite production horizon and under a given service level. (Selcuk, 2017) worked out a suggestion for the implementation of production quality. Other researchers who dealt with maintenance and quality are (El cadi et al.,2021), who worked on integrated optimization of Production planning, maintenance, and quality control policy without considering the effects of inventory control shortages.

To the best of our knowledge, all the works on integrated strategies that study the use of control charts consider that CM actions are initiated following the exceeding of the control limits, while PM is planned independently of the control chart. Besides, those works do not consider the evolution of the degradation degree as well as the failure rate according to both time and production during preventive maintenance and/or corrective maintenance actions.

Considering these two major limitations, we propose in this chapter to have both preventive (perfect and imperfect) and corrective maintenance actions triggered by the process control with the control chart on one hand, and the evolution of degradation degree of the process according to production variation during maintenance actions on the other hand. Besides the control limits on the control chart, we establish surveillance limits (lower than the control limits) that trigger preventive maintenance actions. Hence, given the measurement results on the control chart, one can decide whether to undertake or not preventive or corrective maintenance actions at the end of each sampling interval following the production and the degradation of the process.

This reinforces the integration of maintenance and quality control compared to existing strategies in the literature for which PM actions are not triggered directly by the process control. Moreover, the proposed approach will also simultaneously take into account production through the degradation degree of the process.

The originality of this work is in its collaborative determination of production and inventory quantities under varying customer requests and throughout the production horizon, then gauging the variable production rate impacts in the finding of the appropriate maintenance decisions (*CM*, *Imperfect* & *Perfect*). The new maintenance approach is determined by the multi-layer quality control chart parameters, taking into account the different control chart coefficients (*kimp*, *kp*). A mathematical model that significantly reduces the total cost of production, maintenance, inventory, and quality is developed. The remainder of the paper tests the solution's effectiveness using a numerical example from which results are obtained and a conclusion drawn.

A mathematical model is developed to determine the optimal values of the sample size, the sampling interval, the surveillance, and the control limits of the control chart, which minimize the average total cost including production cost, inventory costs, the cost of unused products, maintenance costs, and quality costs. The remainder of the chapter is organized as follows: Section 3.2 described the problem of the study, motivation, and targeted contributions. Section 3.3 present the stochastic model of production, maintenance, and quality problem. Section 3.4 give the production, maintenance, and quality strategy. Section 3.5 deal with the sequential optimization procedure employed in this model. Section 3.6 reported the numerical experiment conducted, in which the input data, results and discussion of the results. In section 3.7 the sensitivity analysis is presented. Section 3.8 presented the drawn conclusion of the study.

3.2 Problem Description

The production system is based on a single machine that produces only one sort of product, linked to a supply chain. The supply chain is made up of a principal manufacturing store S , connected to multi-purchases warehouses (w_1, w_2, \dots, w_L) with a delivery volume capacity Q_v and time τ_i ($i: 1, \dots, L$) where the customers receive their demand products. The system operates at a given service level θ_i , production capacity limits: $u_{min} \leq u_k \leq u_{max}$, over a finite horizon H , as shown in Figure 3.1 below. Machine M is unreliable, hence are subjects to faults and repairs at any time. The deterioration of the machine is influenced by both time and production cadences. Consequently, the failure rate $\lambda(t)$ increases with time, and the production rate $u(k)$ affects the reliability and the capability of the production process responsible for non-conforming units. The integrated strategy presented a summary of our proposal in Figure 3.1, which illustrates the control chart's three process states and is based on which decisions are made to keep and reject defective products.

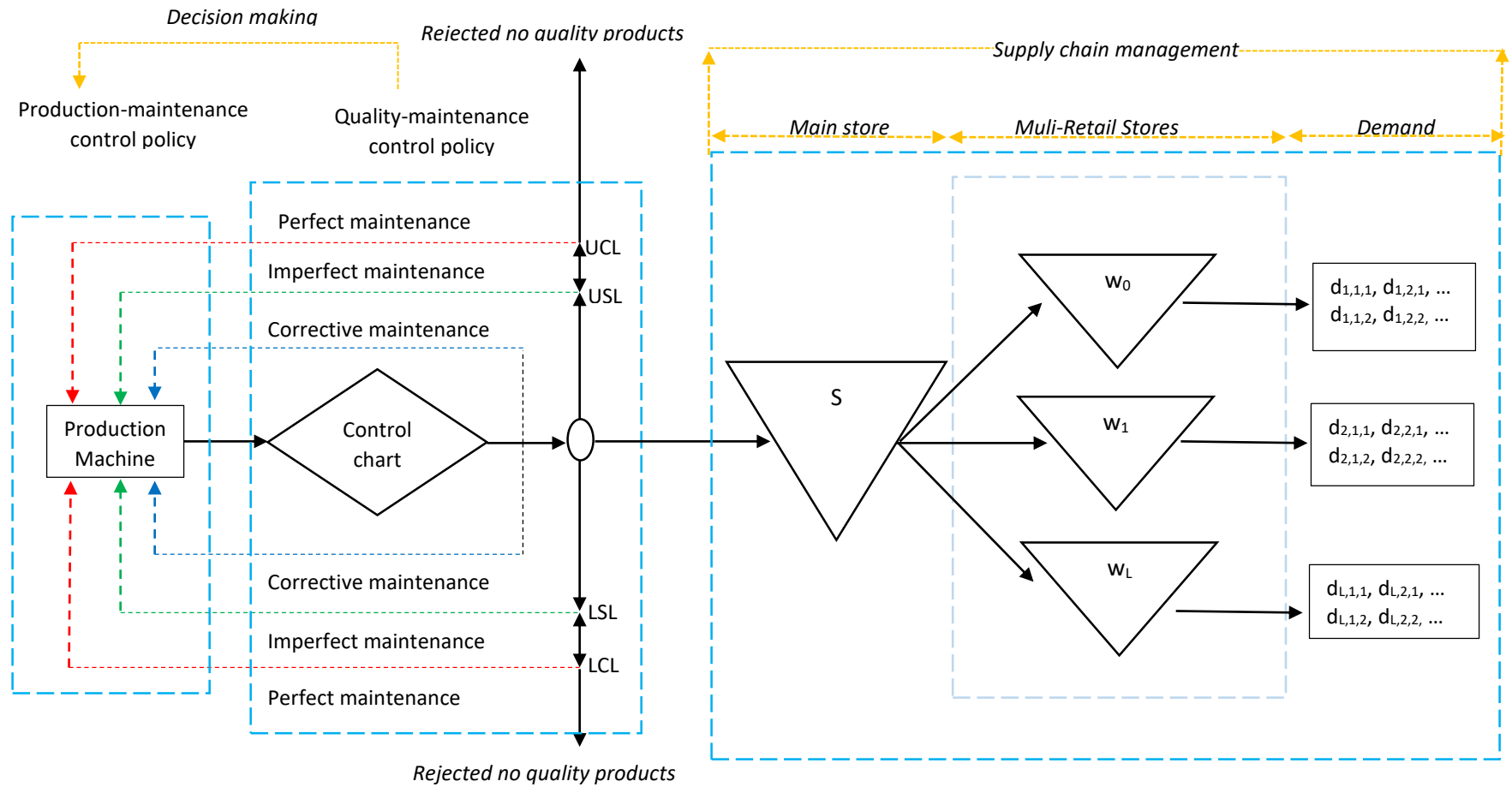


Figure 3. 1: Production Maintenance and Quality System link to supply chain management

Let's assume a joint production to maintenance and quality control problem of an imperfect manufacturing system to satisfy random demands of a multi-retail house inventory arrangement at a given service and capacity constraints. While the process is in a control state, producing conforming items is subjected only to minimal corrective maintenance. But when the operation fails (out of the control) state until the process shifts to a surveillance stage (SL-CL), it requires imperfect preventive maintenance with duration (t_{imp}) to reduce the failure rate. And when the operation moved to a critical "out of control" state, then a perfect maintenance action with significant time (t_p) is employed to restore the process to (AGAN). The decision variables are the economical production plan, the optimal maintenance plan and sample size n , the control interval h , and the control charts coefficients k_{imp} and k_p .

3.2.1 Improved control chart

Although the normal control chart is widely used and regarded as an effective tool in statistical quality control. It is worth notifying that in today's world, the pursuit of high-quality products is asking for high precision quality control in the present and future industrial applications to improve production performance. In our previous work, we evaluated the performance of normal control charts in the previous chapter for the detection of a process variation (out-of-control) condition for only two scenarios. The results suggested the need for improvement. This work presented recent development in the design and application of an improved statistical Process control chart (ISPC).

3.2.2 Principles and characteristics of the control chart

The main purpose of the control chart is to give a signal when a control process deviates from the standard, the secondary purpose is to give a signal when item-to-item variability has increased. Find below Figure 3.2 illustrating the schematic usage of the control chart in a production line.

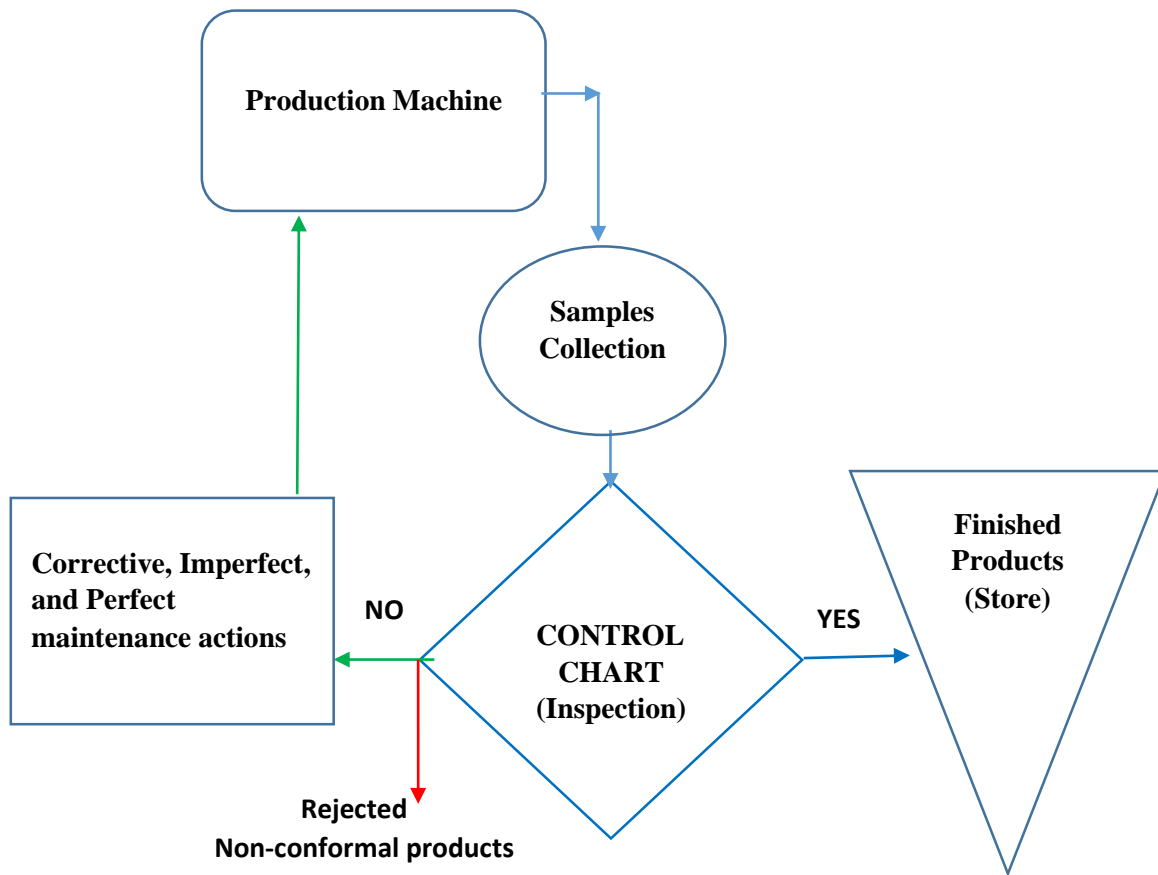


Figure 3. 2: Control Chart Monitoring of Production

The ISPC is a more powerful tool with additional limits to monitor processes in different scenarios to signal multiple maintenance actions. This will guarantee high-quality precision, system reliability, and maintainability. Consider Figure 3.3 below, if the sample means are found within the approved limits, before the surveillance limits (USL/LSL) as zone 1, the process is on target (in-control), where corrective maintenance with minimal repairs is employed. If the mean value lies outside the control limits (UCL/LCL) referred to as zone 2 the process is off target and in a critical state (out of control), hence the machine is stopped and a perfect maintenance action is employed with time (t_p). If it is found in between the surveillance limits and the control limits referred to as zone 3 the process is on surveillance, and maybe in or off target, in this case, imperfect maintenance with time (t_p) is carried out.

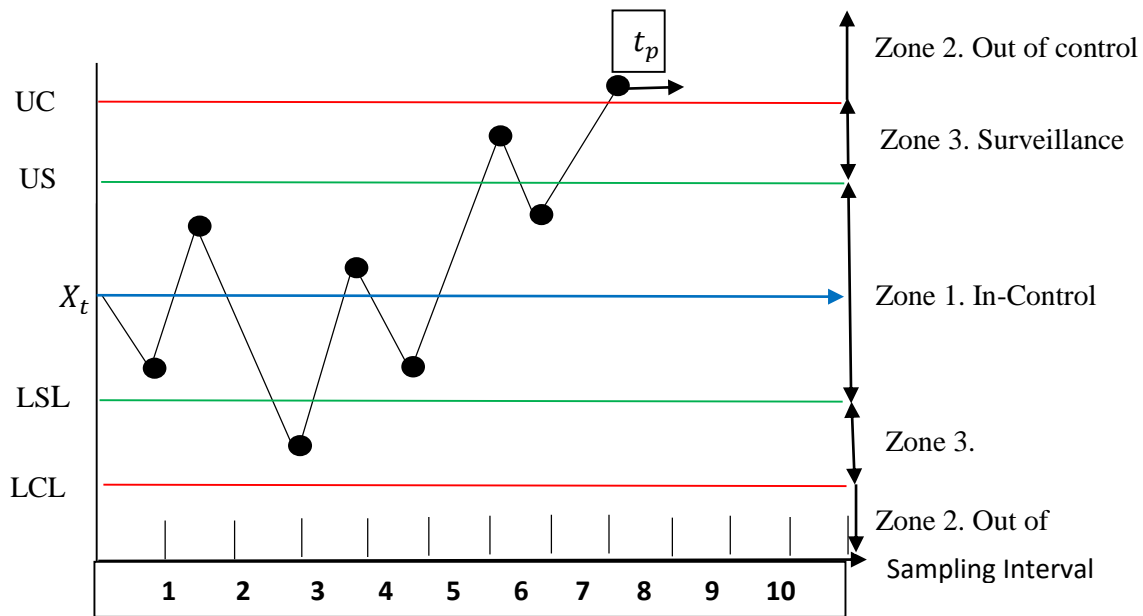


Figure 3. 3: Control chart with surveillance

3.2.3 The difference with the normal control cart

By improvements in monitoring and controlling a process, we can assure that it operates at its fullest potential. A stable process is a process, in which monitored statistics are kept within control limits LCL-UCL. Whereas the improved control charts with additional warning limits are designed to achieve high-precision products. Also to alert small drifts or ascertain some uncertainties in-between the target and off-target are needed for optimal decision support. There are several points to note about this chart.

- ISPC extension of normal SPC
- The detection ability of the improved chart is enhanced.
- Central limits, and multi-action limits (imperfect & perfect) maintenance signals
- Allow decision support related to multiple scenarios
- Better variability control, the “statistical process control” (ISPC) with surveillance limits (USL/LSL) is needed for higher quality products, and also is required for an optimal maintenance decision support system.
- Suitable for processes involving many assignable causes of variation.
- Applicable to a single machine producing multiple products with different quality standards.

Whichever means the chosen charts are constructed almost in the same way. They depend on an estimate of standard deviation and also assume that the data are normally distributed. Control charts may also be applied for measuring the propagation of defective or non-conforming products.

The aim is to improve process control. the modification can be applied to detect variance shifts, and these shifts prove to detect an increase in σ better than the decrease and are useful to monitor both the process mean and the process variance shift.

In this case, three scenarios are possible: In control, small drifts, and critical shifts. The statistical monitoring for the process is designed based on statistical analysis as presented in the mathematical formulation section. In general, the surveillance limits are set based on the mean target value, standard deviation, sample number, and sampling interval. The best measure of variability is the standard deviation. However, the sample range is simpler to use as a measure of variability in statistical process control.

This, however, is not deemed to be enough to keep production under control. The pattern made by the points on the chart needs to be interpreted. Corrective actions are taken to keep production under control and bring the points back in between the control limits for the product to be at a tolerable distance to the specified nominal values. The performance of a control chart is precise, even if it is the result of small sample sizes in production for tests done every day.

The case studies presented herein showcase that control charts result in higher production efficiency and are as such used widely in industry. This work has equally highlighted performed research in the area of control charts. Indeed, research on control charts is done on a global basis, and from the findings discussed in this work, statistical methods and techniques are further empowered by the use of computer technology and, in particular, dynamic software packages and artificial neural networks, to name a few. Given the above, it may be stated that Statistics may further assist its users by refined and selected methods to improve quality in a modern way besides control charts.

3.2.4 Production process scenario with the improved control chart

The process is monitored by taking samples of size when the process is running well and calculating the standard deviation using the mathematical formula expressed in equation 3.2 below. A large number of samples are taken from the production line at regular intervals and the number of non – conforming products are counted.

The SPC works based on control chart parameters and shows when and what is an appropriate action to take. We used the normal law to model the random variation of the process. The normal law is widely used in statistical process control, especially in its reduced-cantered form. Before using the control chart, it must verify the sample's normality during process control. From an initial population with a mean (μ_0), and a standard deviation (σ_0), the machine status can be assessed by measuring the

quality indicator X_s with the product produced. The evolution of this indicator follows a normal law with a mean (μ), and a standard deviation (σ_s). Every unit of time (h), a sample size of (n) products are taken and examined to study the evolution of the quality indicator X_s .

Measuring the product's quality indicator X_s can determine the state of the process. Sampling results are recorded using an "x-bar" control chart. The normal law of mean μ_0 and standard deviation σ_0 governs the evolution of this quality indicator X_s to track the change of the indicator X_s . A sample of size n is taken and verified every h unit of time. An "X-bar" control chart is used to record the sample data. For each of all samples, the average value shown in the control chart is defined as follows:

$$\mu = \overline{\overline{X_s}} \approx \mu_0 = \frac{1}{m} \times \sum_{i=1}^m X_i \quad (3.1)$$

The associated standard deviation is given by:

$$\sigma_s = \frac{\sigma_0}{\sqrt{n}} \quad (3.2)$$

So, we define the control limits: the upper and lower control limits (UCL and LCL) and surveillance limits (USL and LSL) for the "X-bar" control chart as follows:

we define the control limits as follows:

Upper control chart limits

$$UCL = \mu_0 + \frac{k_p}{\sqrt{n}} \times \sigma_0 \quad (3.3)$$

Lower control limit

$$LCL = \mu_0 - \frac{k_p}{\sqrt{n}} \times \sigma_0 \quad (3.4)$$

Upper Surveillance limit

$$USL = \mu_0 + \frac{k_{imp}}{\sqrt{n}} \times \sigma_0 \quad (3.5)$$

Lower Surveillance limit

$$LSL = \mu_0 - \frac{k_{imp}}{\sqrt{n}} \times \sigma_0 \quad (3.6)$$

Hence, the parameters of the chart are; the sampling interval (h in time units), the sample size (n), the number of standard deviations of the sample distribution between the control limits of the control chart and the surveillance limits (k_p) and (k_{imp}) the number of standard deviations of the sample distribution between the Centre line of the control chart and the control limits. Decision variables related to the control chart are n , h , k_p , and k_{imp} .

In this paper, we consider situations in which process property degradation may be related to problems related to machine degradation states (e.g., Bouslah, Gharbi, and Pellerin (2016), Radhoui, Rezg, and Chelbi (2010)). Therefore, depending on the evolution of the quality index (X_s), it is decided whether and what kind of maintenance action should be performed. Following an inspection of sample

size n , depending on the control limits, three scenarios are possible as shown in Figure 3.4, Figure 3.5, and Figure 3.6 below; Corrective maintenance without duration in the control state, imperfect maintenance in a surveillance state, and Perfect maintenance with duration (t_p) in the critical state.

Scenario I: As shown in Figure 3.4, when the production system is under control, the average of the measurements of the quality indicator \bar{X}_s is located between the surveillance limits $LOL \leq \bar{X}_s \leq UOL$. The process is considered to be under control, and Corrective Maintenance actions with minimal repair are executed without any duration.

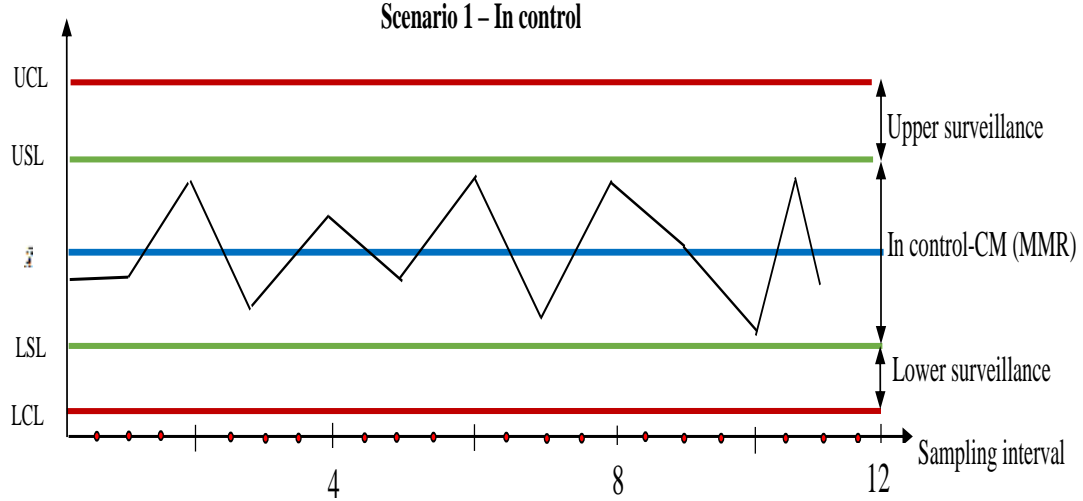


Figure 3. 4: Control chart at scenario 1

Scenario II: As shown in Figure 3.5, When the production system is shifted away from the central limit to the surveillance state, the average of the measurements of the quality indicator \bar{X}_s is located between the surveillance and the control limits ($LCL \leq \bar{X}_s \leq LSL$ or $USL \leq \bar{X}_s \leq UCL$) for a sample j_{imp} . In this case, the process is considered to have entered an "out of control" state called the "monitoring" state. Hence, the machine is stopped and an imperfect preventive maintenance action with an average duration (t_{imp}) is carried out to mitigate the process and reduce the failure rate of the process. In this case, a proportion of non-confirming units is detected and rejected. Assume that the number of non-quality is proportional to the production volume between the $(j_p - h)^{th}$ and $(j_{imp})^{th}$ sampling interval. So the measure of inspections m is therefore equal to j_{imp} .

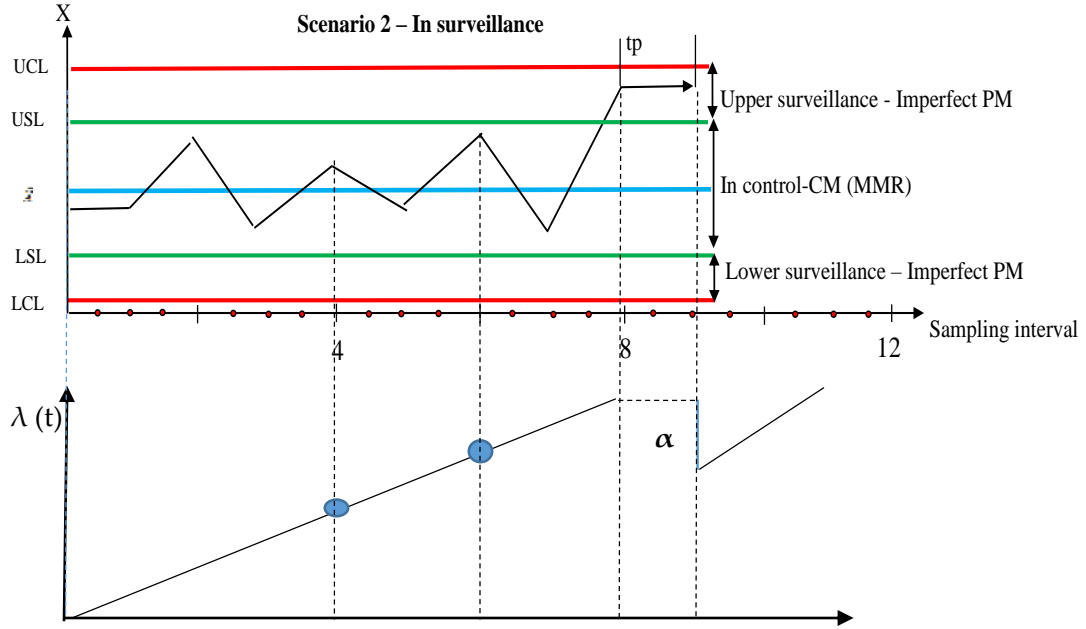


Figure 3. 5: Case of an imperfect PM action triggered by the control chart

Scenario III: When the average of the quality indicator measurements is located beyond the control limits ($LCL \geq \bar{X}_s$ or $\bar{X}_s \geq UCL$) as illustrated in Figure 3.6 below, the machine is considered in an 'out-of-control' state called 'critical' state, and the production unit is considered in critical condition. In this case, the production is stopped and a Perfect maintenance action with a mean duration of t_p is performed on it. All items produced between sampling intervals $((j_p - h)^{th})$ and $(j_p)^{th}$ are rejected. Therefore, the number of inspections m is equal to j_p .

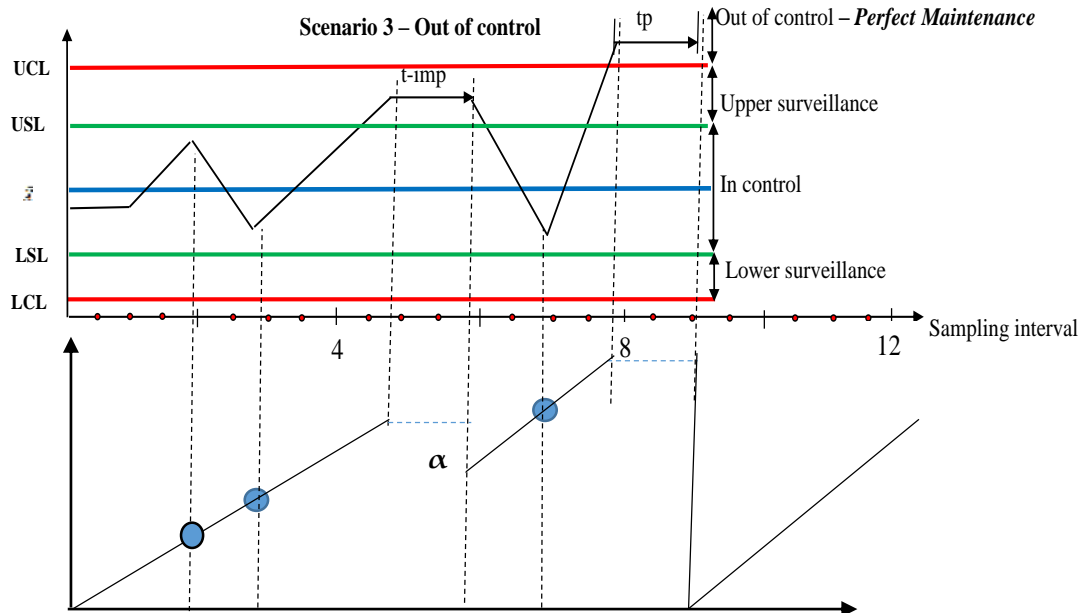


Figure 3. 6: Case of an imperfect and perfect maintenance action triggered by the control chart

3.2.5 Assumption

The study of this chapter is based on the following assumptions:

Assumptions

- The failure rate of a production unit is depending both time and production
- The demands that are satisfied at the end of each period and which cannot be satisfied are lost.
- The non-conformity of the products is due only to the degradation of the production unit.
- Maintenance actions depend on the type of maintenance (perfect, imperfect, and with minimal repair)
- The resources needed to perform maintenance actions are available.
- The unit costs related to production, inventory, maintenance, and quality are known and constant.

3.3 Model of the stochastic production maintenance quality problem

To establish an optimal production, maintenance, and quality integrated strategy, we define a stochastic model that minimizes the total costs over a finite horizon. Our goal is to establish, the optimum production rates, the optimal inventory/delivery quantities, as well as the optimal maintenance strategy corresponding to the quality control chart's optimal parameters, which minimizes the total production cost. The total expected cost includes the costs of production, inventory holding, delay penalties, and quality/maintenance.

Formally, the problem is defined as follows:

The optimization model is composed of the objective functions that minimize the expected total costs (production, inventory, delivery, delay penalty, quality, and sum of maintenance) over the finite horizon $H \cdot \Delta t$.

$$\text{Min } F = PC + HC + DC + DPC + TMC \quad (3.7)$$

Under the following constraints:

$$S(k) = S(k-1) + u(k) \cdot \Delta t - \sum_{i=1}^L Q_i(k) \text{ with } k = \{0, 1, \dots, H\} \quad (3.8)$$

$$w_i(k) = w_i(k-1) + Q_i(k - \tau_i / \Delta t) - d_i(k), \text{ where } k, (k = 1, 2, \dots, H) \text{ and } \tau_i \geq 1 \quad (3.9)$$

The service level, for $(k = 1, \dots, H-1)$ and $(i = 1, \dots, L)$

$$Prob[w_i(k) \geq 0] \geq \theta_i \quad (3.10)$$

The upper and lower bounds of production, during each period

$$u_{min} \leq u(k) \leq u_{max} \quad (3.11)$$

Where F represents the total production costs function; production, holding, maintenance, and quality. PC denotes the production cost, HC is the total inventory holding cost at both the principal store and the warehouses i . The delivery and delay penalty costs are respectively represented with DC & DPC . The study set defines the inventory balance equation for each period (k) taking into account the dynamic production, the supply quantities to the principal store S as well as the multi-warehouses w_i . The $S(k)$ states the principal stock quantity at any given period k , which accounts for quantity Q taken to any of the warehouses and dynamically throughout the production horizon H . The demand d is the product quantity taken out from the retail warehouses w_i . The τ_i is the transportation duration factor accounting the for times it takes to supply products from the principal store to all the warehouses. The equations (3.10, 3.11) define constraints imposed by the service level requirement for each period and the production capacity limitation by the lower and upper production bounds. The probabilistic constraint of inventory is taken as a chance constraint to ensure that the inventory level is greater than zero with a conditional probability of at least Q_i at each period k .

The optimization problem is described by a random parameter of our problem (forecasting demand and production, the stochastic constraint of production capacity limitations, and service level requirements) and the decision variables. Production plan according to the model, it is important to make assumptions to be adopted; in our case, we adopt a dynamic modelling convention (Hajej et al, 2011). The main objective is total cost minimization. We begin by planning the optimal rate of production ($u(k)$ and inventory level ($Q_i(k)$) to satisfy customers' demands at all the warehouses and for all periods. Secondly, using the production rates and stock quantities as constraints to find the optimal number of preventive maintenance actions (N)* as well as the optimal parametric combination of the key decision variables (n, h, m, k_{imp}, k_p), with the aid of a statistical control chart of a quality tool. The production horizon (H) is equally divided into periods of length equal to Δt . For each production period k , we assume that production takes place at the beginning of the period, and at the end of the period, the unsatisfied demands are penalized. Also, we assumed that the demand fluctuation follows a normal process with mean and variance."

The objective is to establish optimum production, inventory, delivery, and optimal parameters related to the improved control chart, with additional surveillance limits (n, h, m, k_{imp}, k_p). The maintenance strategy is corrective maintenance in the control state, then PM (imperfect & perfect) with the most appropriate durations (t_{imp} & t_p), and numbers (N_{imp} & N_{pef}) when the process shifted. To achieve this conjugated production, delivery, quality, and maintenance optimization, we minimize the sum of maintenance costs (C_{cm}, C_{Im} & C_{pm}) and total inventory holding costs, as well as total costs related to the quality (c_{si} & c_r).

3.4 Production, maintenance, and quality strategy

Equipment Availability-Reliability-Maintenance, Product Quality, and System Productivity are strongly interrelated to each other. The present industrial realities have compelled companies to find strategies and means to guarantee their production system sustainability (Reliability/cost) through optimal integrated policies which collaborate between production planning to meet customer requirements and the maintenance plan to avoid failures. Production, Maintenance, and Quality are the most critical aspects of the industrial system but were traditionally treated separately. With the development of the current industrial system, the emergence of exigent customers, competitive markets, new products, and random demands, therefore, requires the collaboration of these interrelated factors to achieve overall objectives. However, the execution of these strategies requires tactical planning of integrated strategies.

3.4.1. Production policy

Recall the production problem as presented in the previous chapter. In this section, we aimed to minimize the total production and inventory cost by determining the optimal and economical production planning to meet the random demands of the customers at multi-warehouses and throughout the production period. The production horizon is divided into H periods of equal length Δt , each delivery time τ_i from S to warehouses w_i is multiple of Δt . Furthermore, client demand follows the regular legislation and any demand that is not met within specific period results in a penalty (Delay penalty cost), the unit's costs are also known and constant.

To solve the proposed model, and due to the complexity of the problem, we develop a numerical procedure to determine an approximate possible solution for the optimal production plan $u(k)^*$, we use the random exploration method to randomly generate a production vector during the finite production horizon (H) and under the service level constraints ($Prob[w_i(k) \geq 0] \geq \theta_i$) and production bounds ($u_{min} \leq u_k \leq u_{max}$). From this vector, we calculate the production cost in each period. The calculation of the number of Products being produced to satisfy the service level constraint determined the optimal quantity (Q_i^*) to be taken to warehouses and for all periods. Finally, when all the conditions are verified and satisfied, the minimum total production cost (TCP_{min}) is saved. The optimal production plan $u(k)^*$ as well as optimal delivery quantities are generated using our developed multi-objectives function algorithm. The detailed algorithm flowchart diagram is shown in chapter 2.

Recall that the total production Cost:

$$TCP = Production\ cost + Holding\ cost + Delivery\ cost + Penalties\ cost \quad (3.12)$$

With:

- Production Cost

$$PC = C_p \times \sum_{k=1}^H u(k) \cdot \Delta t \quad (3.13)$$

- Holding cost at the Manufacturing Store (S)

$$HC_S = \sum_{k=1}^H C_{hs} \times \left(\text{Max}(S(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot u(k) \cdot \Delta t^2 \right) \quad (3.14)$$

- Holding costs at the retail warehouses (w_i)

$$HC_W = \sum_{k=1}^H \sum_{i=1}^L C_{hwi} \times \left(\text{Max}(w_i(k-1), 0) \cdot \Delta t + \frac{1}{2} Q_i \left(k - \frac{\tau_i}{\Delta t} \right) \cdot \Delta^2 \right) \quad (3.15)$$

- Total Inventory Holding cost (HC)

$$HC = HC_S + HC_W$$

$$HC = \sum_{k=1}^H \left(C_{hs} \times \left(\text{Max}(S(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot u(k) \cdot \Delta t^2 \right) + \sum_{i=1}^L C_{hwi} \times \left(\text{Max}(w_i(k-1), 0) \cdot \Delta t + \frac{1}{2} Q_i \left(k - \frac{\tau_i}{\Delta t} \right) \cdot \Delta^2 \right) \right) \quad (3.16)$$

- Delivery cost

$$DLC(Q(k)) = C_t + C_o \times \sum_{i=1}^L \left(\frac{Q_i(k)}{Q_v} \right) \quad (3.17)$$

- Delay Penalties cost

$$DPC = C_d \times \left(\sum_{k=1}^H \left(\sum_{i=1}^L dw_i \right) \right), \quad dw_i = \frac{|\min(w_i(k), 0)|}{Q(k+1 - \frac{\tau_i}{\Delta t})} \quad (3.18)$$

Hence, the inventory level of each warehouse inventory w_i given by the equation (3.9) is characterized by a random variable due to the random demand, that follows a Gaussian distribution defined by:

$$w_i(k) = w_i(k) + \varepsilon_k \times \sqrt{\text{Var}(w_i(k))} \Leftrightarrow w_i(k) = w_i(k) + \omega_k \times \sqrt{k} \times \sigma_d \quad (3.19)$$

With w_k is the standard deviation of the reduced centered Gaussian law $N(0, 1)$

Therefore, the deterministic service level is expressed as follows:

$$Q_i(k - \tau_i) \geq \varphi^{-1}(\theta) \times \sqrt{k+1} \times \sigma_d - w_i(k-1) + d(k) \quad \text{with } k \in \{0, 1, \dots, H-1\} \quad (3.20)$$

So, the problem formulation is expressed as follows:

$$\text{Min } U\{u(k), Q(k), k: 1, \dots, H\}$$

$$\left[(C_p \times \sum_{k=1}^H u(k) \cdot \Delta t + C_h \times \sum_{k=1}^H \left\{ \begin{array}{l} \max(S(k-1), 0) \Delta t + \frac{1}{2} \cdot u(k) \cdot \Delta t^2 \\ + \sum_{i=1}^L \max(w_i(k-1), 0) \Delta t + \frac{1}{2} \cdot Q_i \left(k - \frac{\tau_i}{\Delta t} \right) \cdot \Delta t^2 + C_l \times C_o \times \end{array} \right. \right. \\ \left. \left. \sum_{i=1}^L \left(\frac{Q_i(k)}{Q_v} \right) \right) \right] \quad (3.21)$$

3.4.2. Maintenance and quality policy

The maintenance cost consists of preventive and repair costs and depends on the three scenarios. The optimization of maintenance strategy consists to minimize the costs related to perfect (critical state) and imperfect (surveillance state) preventive maintenance actions and corrective maintenance actions (in-control state). In practice, it is not always possible to restore the systems after each maintenance action. We cannot do a simple repair either. We do what we call a partial renewal or partial overhaul (only a few system components are replaced or restored) to reduce the failure rate. Hence, the integration of imperfect maintenance with switching to perfect maintenance depending on the state of the process is important and practical, economically speaking since manufacturing systems are looking for reducing maintenance costs with respecting the reliability of the process. Assuming that when the process is in a controlled state, we apply corrective maintenance with minimal repair. In this part, we aimed to minimize the total maintenance and Quality cost by finding the optimal control chart parameters (n, h, m, k_{imp}, k_p) to optimize the total maintenance and quality cost. The total maintenance cost is composed of the sum of two costs: the cost of preventive maintenance actions (imperfect and perfect) (CM_1) and the cost of corrective maintenance (CM_2). The cost of CM_1 depends on the number of imperfect (N_{imp}) and the number of perfect preventive maintenance (N_{perf}), and the unit price of each action according to the state of the process. While the CM_2 will depends on the cost of corrective maintenance depending on the average number of failures between two preventive maintenance, as well as the duration of imperfect and perfect maintenance (t_{imp}, t_p). Hence, the average number of failures depends on the evolution of the failure rate that depends on the economical production plan obtained by the production policy.

Failure rate modelling

According to the production plan, we determined and developed the new failure rate which depends on the average number of samples to detect the “out of control” and the average number of samples to detect the surveillance state (j_p, j_{imp}) as well as the durations (t_{imp}, t_p).

The expression for the failure rate depends on the imperfect and perfect maintenance actions and the production rates as expressed below:

$$\begin{aligned}
\lambda_k(t) = & \left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_{imp}} \right\rfloor + 1\right) \times j_{imp}} \right\rfloor\right) \times \left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_p} \right\rfloor + 1\right) \times j_p}\right\rfloor\right) \times \left(\lambda_{k-1}(\Delta t) + \frac{u(k)}{U_{max}} \lambda_n(t)\right) + \left(1 - \right. \\
& \left. \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_p} \right\rfloor + 1\right) \times j_p}\right\rfloor\right) \times \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_{imp}} \right\rfloor + 1\right) \times j_{imp}} \right\rfloor \times \lambda_{\left(k - \left\lfloor \frac{j_{imp}}{\Delta t} \right\rfloor\right)} \left(\left(\left\lfloor \frac{k \times \Delta t}{j_{imp}} \right\rfloor - 1\right) \times j_{imp}\right) \times e^\alpha + \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_p} \right\rfloor + 1\right) \times j_p}\right\rfloor \times \\
& \frac{u(k)}{U_{max}} \lambda_n(t) \quad \forall t \in [0, \Delta t]
\end{aligned} \tag{3.21}$$

With $\lfloor \cdot \rfloor$: the minimum integer part

This failure rate function mainly consists of four essential parts:

- $A = \lambda_{k-1}(\Delta t) + \frac{u(k)}{U_{max}} \lambda_n(t)$ which describes the cumulative failure rate according to production rate and previous failure rates when there is not a preventive maintenance action.
- The term $\left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_{imp}} \right\rfloor + 1\right) \times j_{imp}} \right\rfloor\right) \left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_p} \right\rfloor + 1\right) \times j_p}\right\rfloor\right)$ is equal to zero to annul the failure rate A to zero when there is a perfect or imperfect preventive maintenance action, i.e., at times j_{imp} or j_p , otherwise equals to 1.
- The $B = \lambda_{\left(k - \left\lfloor \frac{j_{imp}}{\Delta t} \right\rfloor\right)} \left(\left(\left\lfloor \frac{k \times \Delta t}{j_{imp}} \right\rfloor - 1\right) \times j_{imp}\right) \times e^\alpha$ accumulates the failure rate when imperfect maintenance is applied, basing on the partial renewal at times j_{imp} . If there are some observed states before the critical state, at each interval $I_k = [(k-1), j_{imp}, k, j_{imp}]$, which that the failure rate will be equal to the failure rate of the previous interval I_{k-1} multiplied by a degradation factor e^α with $\alpha \in [0,1]$.
- The expression $\left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_p} \right\rfloor + 1\right) \times j_p}\right\rfloor\right) \times \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_{imp}} \right\rfloor + 1\right) \times j_{imp}} \right\rfloor$ equal to zero to annul the failure rate B when there is perfect preventive maintenance (failure rate C different to zero), i.e., at each j_p , otherwise equals to 1.
- As the term $C = \lambda_n(t)$ accumulates the failure rate when perfect maintenance is applied and restores the failure to zero, based on the production rate on the machine with $\lambda_n(t)$ presented the failure rate of the process under nominal conditions.
- $\left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{j_p} \right\rfloor + 1\right) \times j_p}\right\rfloor$ equals zero to annul the failure rate C when there is imperfect preventive maintenance (failure rate B different from zero), i.e., at each j_{imp} , otherwise equals to 1.

- As the term $C = \frac{u(k)}{U_{max}} \lambda_n(t)$ accumulates the failure rate when perfect maintenance is applied and restores the failure to zero, based on the production rate on the machine with $\lambda_n(t)$ presented the failure rate of the process under nominal conditions.
- $\left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{J_p} \right\rfloor + 1\right) \times J_p} \right\rfloor$ equals zero to annul the failure rate C when there is imperfect preventive maintenance (failure rate B different from zero), i.e., at each J_{imp} , otherwise equals to 1.

The average total maintenance cost

Considering that either PM (perfect, imperfect) or CM actions can be performed in each cycle, the total average maintenance cost can be expressed as:

$$C_{TM}(j_p, j_{imp}) = CM_{perf} * (1 - \psi_1) + CM_{imp} * \psi_1 + C_{cm} * \varphi(j_p, j_{imp}) \quad (3.22)$$

With the average number of failures $\varphi(j_p, j_{imp})$ is expressed as follows:

$$\varphi(j_p, j_{imp}) = \sum_{k=1}^{\left\lfloor \frac{ADRC - (t_{imp} \times \psi_1 + t_p \times (1 - \psi_1))}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_k(t) dt + \sum_{k=\left\lfloor \frac{ADRC}{\Delta t} \right\rfloor}^H \int_0^{\Delta t} \lambda_k(t) dt$$

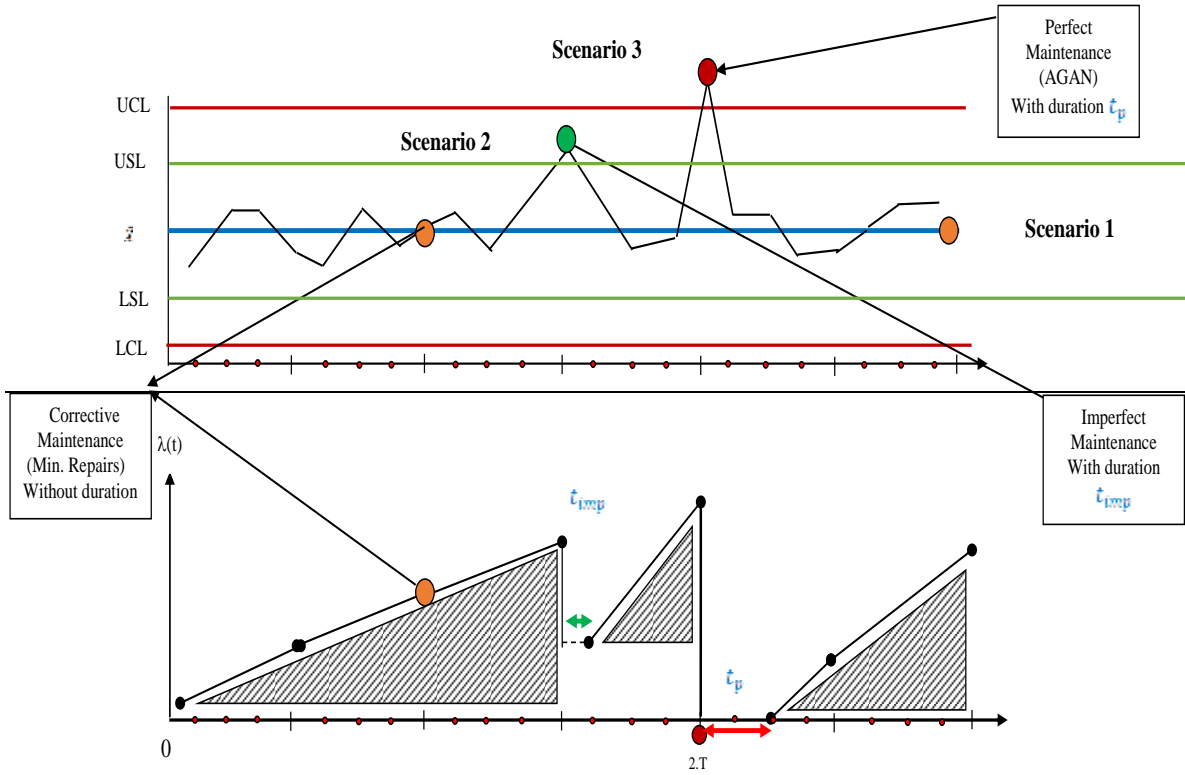


Figure 3. 7: Integrated maintenance and control chart decisions

Average Duration of a Restoration Cycle

Consider j_{imp} to be the average number of samples to signal the process moves into the surveillance state.

From (Montgomery 2004) it follows that;

$$j_{imp} = \frac{1}{1-\alpha_2} \quad (3.23)$$

α_2 is the probability of non-identification of the surveillance state, for this situation, the quality marker is within ($LSL \leq \bar{X}_s \leq USL$), while the production process is in a surveillance state, and the machine should be submitted to an imperfect PM activity with duration (t_{imp}).

$$\alpha_2 = F\left(\frac{USL - \mu_1}{\sigma/\sqrt{n}}\right) - F\left(\frac{LSL - \mu_1}{\sigma/\sqrt{n}}\right) \quad (3.24)$$

Hence:

$$\alpha_2 = F(k_{imp} - \delta_{imp} \times \sqrt{n}) - F(-k_{imp} - \delta_{imp} \times \sqrt{n}) \quad (3.25)$$

Using Equation (3.23), we obtain:

$$j_{imp} = \frac{1}{1 - F(k_{imp} - \delta_{imp} \times \sqrt{n}) + F(-k_{imp} - \delta_{imp} \times \sqrt{n})} \quad (3.26)$$

Similarly;

Consider j_p to be the average number of samples to signal the shift to the critical out-of-control state. It also follows that;

$$j_p = \frac{1}{1-\alpha_3} \quad (3.27)$$

α_3 is the probability of non-identification of the critical out-of-control state, for this situation, the average of the quality indicator is within ($LCL \leq \bar{X}_s \leq UCL$), while the production process is in a critical state, and the machine should be submitted to perfect maintenance (PM) activity with duration (t_p).

$$\alpha_3 = \text{Prob}(LCL \leq \bar{X}_s \leq UCL / \mu_2 = \mu + \delta_p \times \sigma_s) \quad (3.28)$$

$$\alpha_3 = F\left(\frac{UCL - \mu_2}{\sigma/\sqrt{n}}\right) - F\left(\frac{LCL - \mu_2}{\sigma/\sqrt{n}}\right)$$

$$\alpha_3 = F(k_p - \delta_p \times \sqrt{n}) - F(-k_p - \delta_p \times \sqrt{n}) \quad (3.29)$$

And from Equation (3.27), we have:

$$j_p = \frac{1}{1 - F(k_p - \delta_p \times \sqrt{n}) + F(-k_p - \delta_p \times \sqrt{n})} \quad (3.30)$$

The average duration of a restoration cycle.

$$ADRC = (j_{imp} \times \psi_1 + j_p \times (1 - \psi_1)) \times h + \mu_{imp} \times \psi_1 + \mu_p \times (1 - \psi_1) \quad (3.31)$$

ψ_1 is the parameter showing if the surveillance limits were exceeded before the control limits. This indicator is defined by the following expression;

$$\psi_1 = \text{mark}(j_{imp} < j_p) = \begin{cases} 1, & \text{If } j_{imp} < j_p \\ 0, & \text{Otherwise} \end{cases} \quad (3.32)$$

The average overall cost of quality

The average total quality is consisting of the sampling inspection cost (C_i) which depends on the sample size (n) and the number of times the sample is taken (m). And, the cost of non-conformal products (C_{NC}) depends on the production rate, the average number of samples to detect the surveillance state (j_{imp}), and the “out of control” critical state (j_p), the production length (Δt), and the control interval (h).

The average total cost of sampling inspection is expressed by;

$$C_{si} = C_i \times n \times m \quad (3.33)$$

The proportion of non-conforming units was rejected.

$$C_{NC} = C_r \times \left(\psi_1 \times \left(u \left(\left\lfloor \frac{j_p * h}{\Delta t} \right\rfloor \right) \times \left(\frac{j_p * h}{\Delta t} - h \right) \right) + (1 - \psi_1) \times \left(u \left(\left\lfloor \frac{j_{imp} * h}{\Delta t} \right\rfloor \right) \times \left(\frac{j_{imp} * h}{\Delta t} - h \right) \right) \right) \quad (3.34)$$

3.5 Optimization model

To solve the proposed model, the sequential optimization procedure is employed. Solving the first sub-problem of production and considering its output, characterized by economical production and inventory plans, as input to the integrated maintenance quality sub-problem. And due to the complexity of the second sub-problem, we proposed a numerical procedure to determine the optimal values for sample size, sampling interval, monitoring, and control limits according to the integration policy. Using developed optimization algorithms, the best strategies for inventory, quality, and maintenance are achieved. Finally, we perform a sensitivity analysis to validate the restriction and robustness of the optimal policy.

An iterative numerical optimization procedure is developed and coded using MATHEMATICA software, as shown in the algorithm flow diagram (Figure 3.8) below. The procedure integrated the following user inputs (initials) data;

$$u_k, u_{min}, u_{max}, \alpha, \beta, \mu_o, \sigma_o, \delta, \Delta t, u, H, C_{pm1}, C_{pm2}, C_{cm}, C_i, C_r, C_f$$

- ✓ The multi-decision variables (n, m, h, k_p, k_{imp}) are initialized with small values by the user.
- ✓ The decision parameters are varied with some increments ($\Delta n, \Delta m, \Delta h, \Delta k_p, \Delta k_{imp}$) to some defined maximum limits ($n_{max}, m_{max}, h_{max}, k_{p_{max}}, k_{imp_{max}}$)
- ✓ The average number of checks (of successive samples) to be in or out of control (j_{imp}, j_p) is calculated by using equations (3.23 & 3.27)
- ✓ The average duration of a restoration cycle (ADRC) is calculated by equation (3.31)
- ✓ The total cost of maintenance is calculated using the equation (3.22)

The accuracy or quality of the solution (optimal parameters) obtained is a function of the increments step-length operated by the user.

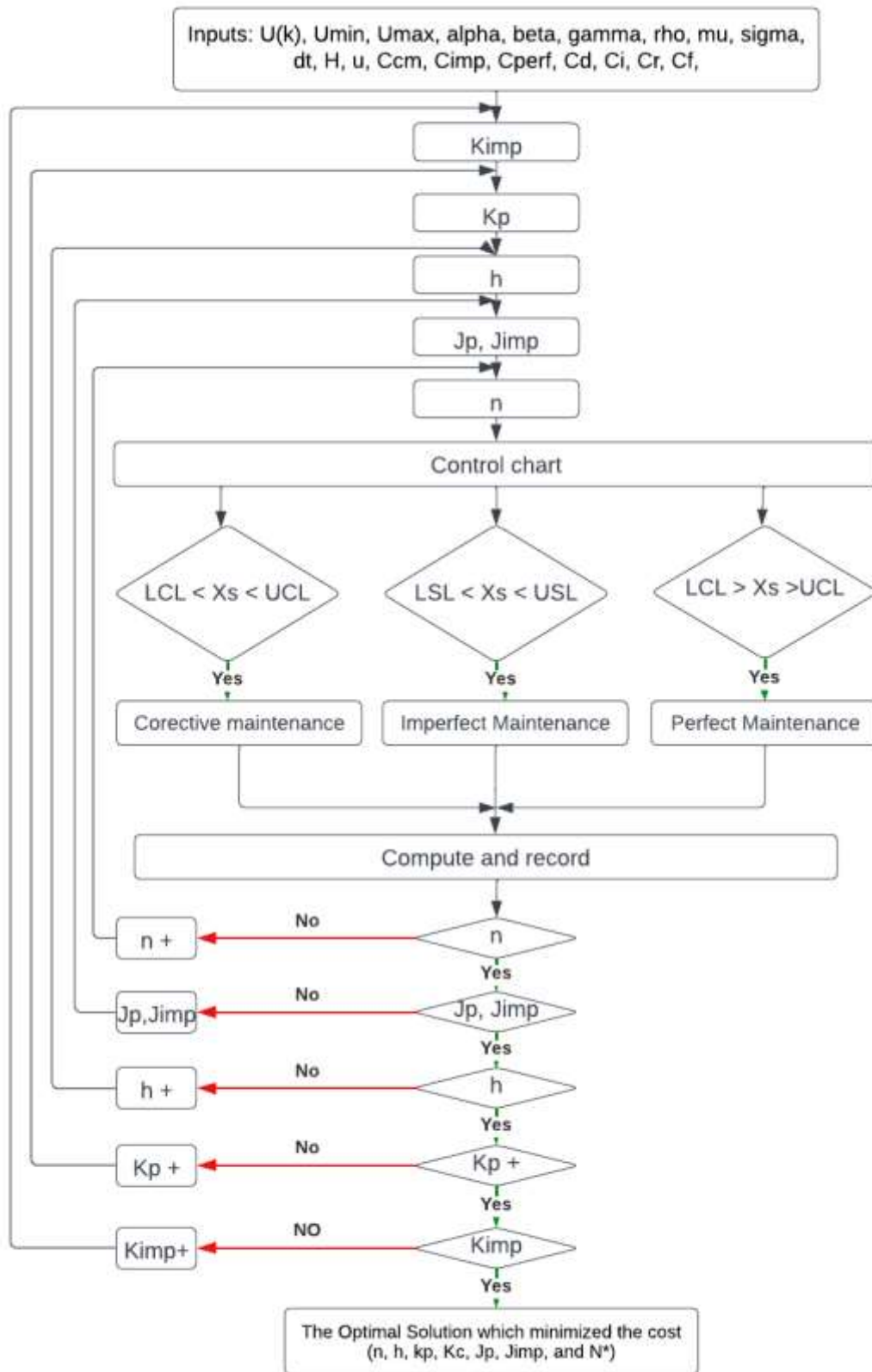


Figure 3. 8: Schematic diagram of the Maintenance & Quality algorithm

3.6. Numerical experiment

A Supply Chain consists of a system producing a single sort of product to meet the delivery of two retail houses ($L = 2$) that will satisfy random demands within twelve months ($H = 12$), with periods of duration $\Delta t = 1$ month. We assume that the standard deviation of each product demand is the same for all periods and each demand $\sigma d_i(\{i: 1,2\}) = 1.2$. We also assume that the initial stock level is $S(0) = 0$. Lower and upper boundaries of production, as well as other production, maintenance, and quality parameters, are presented below:

The following input data were used for experiments with our model as an approach to finding the best strategy.

Input data.

PRODUCTION		MAINTENANCE		QUALITY	
PARAMETER	VALUE	PARAMETER	VALUE	PARAMETER	VALUE
u_{max}	5000	β_1	100	Ci	50
u_{min}	0	α_1	2	Cr	70
$S(0)$	0	CM_{imp}	500	μ_0	5
L	2.0	CM_{perf}	1000	σ_0	1.5
Δt	1.0	C_{cm}	70000	δ_{imp}	0.8
$d_i(k)$	1500	t_p	8 hours	δp	1.0
H	12	t_{imp}	4 hours		
Q_v	3000				
c_p	50				
c_{hs}	2.2				
c_{hwi}	2.2				
c_d	4.0				
τ_i	1.0				
θ_i	0.9				

Table 3. 1: Initial input data

d(1)	d(2)	d(3)	d(4)	d(5)	d(6)	d(7)	d(8)	d(9)	d(10)	d(11)	d(12)
4500	3700	4500	3500	5300	4400	3560	4450	4510	1540	2500	3400

Table 3. 2: Average customer demand

Results.

The optimal production quantity of products $u(k)^*$ to meet customer demands respecting the given service level requirements is presented in Table 3.3. The production plans are according to the probabilistic function defined by equation (3.10). This strategy used the inventory balance equation, and also, considers both the cost of inventories as well as costs of penalties. The optimal planning allowed us to minimize the total cost of production TC_p^* to 2502 *m. u.*

$u^*(1)$	$u^*(2)$	$u^*(3)$	$u^*(4)$	$u^*(5)$	$u^*(6)$
4540	2940	2530	3220	3210	4200
$u^*(7)$	$u^*(8)$	$u^*(9)$	$u^*(10)$	$u^*(11)$	$u^*(12)$
4110	4220	3630	2430	2540	4643

Table 3. 3: Optimal Production Plan

$Q1^*(1)$	$Q1^*(2)$	$Q1^*(3)$	$Q1^*(4)$	$Q1^*(5)$	$Q1^*(6)$
2250	2150	2050	2500	1250	2040
$Q1^*(7)$	$Q1^*(8)$	$Q1^*(9)$	$Q1^*(10)$	$Q1^*(11)$	-
1340	1280	2150	1080	1010	-

Table 3. 4: The optimal delivery plan for retail warehouse 1

$Q2^*(1)$	$Q2^*(2)$	$Q2^*(3)$	$Q2^*(4)$	$Q2^*(5)$	$Q2^*(6)$
2150	640	2430	2520	1310	1500
$Q2^*(7)$	$Q2^*(8)$	$Q2^*(9)$	$Q2^*(10)$	$Q2^*(11)$	-
470	320	2130	1130	2500	-

Table 3. 5: The optimal delivery plan for retail warehouse 2

We used the outputs of the first part findings ($u(k)^*$ & Q_i^*) of our production optimization procedure as some constraints (input) to the second part (maintenance and quality) procedure in sequential order. The optimal combination of the multi-level control chart tool is determined using the developed algorithm. The detailed algorithm flowchart diagram is shown above in (Figure 3.9).

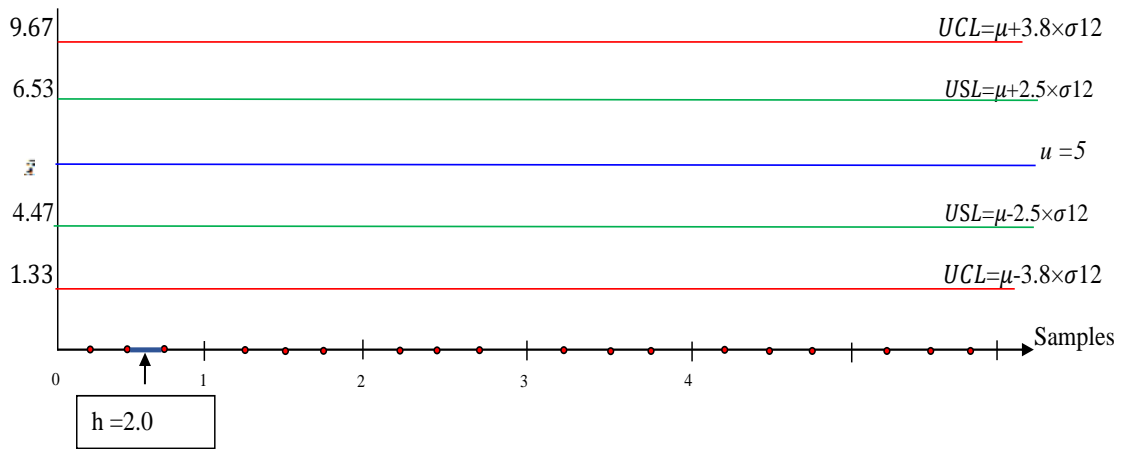


Figure 3. 9: Optimal design of the \bar{X} - control chart.

h^*	n^*	m^*	k_{imp}^*	k_p^*	$J_p T^*$	$J_{imp} T$	N_{imp}
2.0	48	57	2.50	3.85	2928	54	1

Table 3. 6: Optimal decision parameters

Control chart effects on maintenance cost

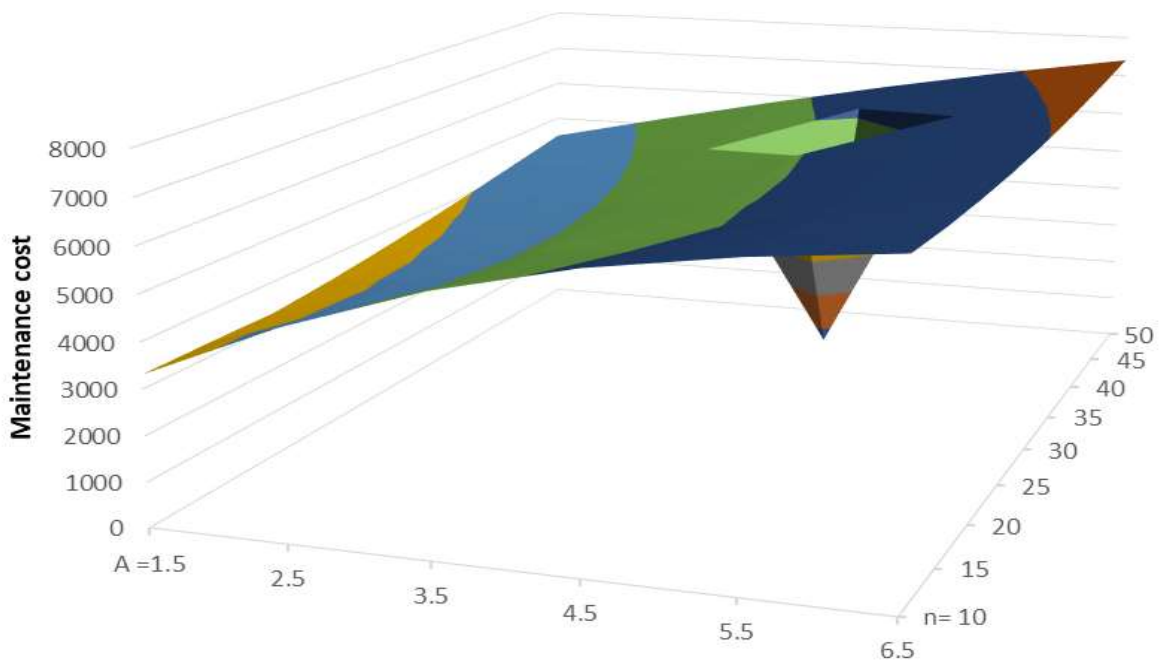


Figure 3. 10: The total cost of maintenance against control chart combinations

Table 3.1 presented the optimal planned production quantities. The optimal delivery quantities at the multi-retail store are presented in Tables 3.2 and 3.3 which guarantee the customer service level requirement. The best maintenance and quality strategy consist in considering one sample of size 48 every h , with $h = 2$ days. Furthermore, concerning the scheme of the control chart, the ideal number of standard deviations between the Centreline, the surveillance limits, and the control limits are respectively 2.50 and 3.85. The shift to the surveillance state and out-of-control state would occur on average after 68h and 3774h respectively. The optimal parameters of the control chart are shown in table 3.4 which minimizes the total cost of maintenance and quality.

Whenever a sample is considered, the percentage that the system is in the 'in-control' state is found to be equal to 74%, the probability to be in the surveillance state (performing an imperfect PM action) is 18%, and the probability to be in the critical state and perform a perfect maintenance action is 8%.

3.7 Sensitivity Analysis for Maintenance/Quality Strategy

In this subsection, we investigate the influence of changing some key study parameters. To analyse and validate the system's behaviour, we have studied the different possible causes of variations in model parameters, namely: the cost of inspection (c_i), the cost of rejecting defective items (c_r), imperfect maintenance duration (t_{imp}), and the magnitude of the transition to the surveillance state and the critical "out-of-control" state relative to the centreline (δ_{imp}). Using the forecasted demand data to established the economical production plan presented in table 3.2. For example, 1, the effects of variations in these parameters on the optimum solutions are given in Table 3.5 and Figure 3.12. Additionally, we have studied and analysed the impacts of the costs of maintenance imperfect (CM_{imp}).

Parameter	Variation	k_p^*	k_{imp}^*	h^*	n^*	m^*	$J_p T^*$	$J_{imp} T^*$	N_{imp}^*	N_{perf}^*
Base		3.85	2.5	2	48	57	2928	54	1	1
c_i	-5.00	3.85	2.5	2	48	57	2928	57	1	0
	0	3.95	2.5	2	48	57	3784	68	1	0
	+5.00	3.85	2.5	2	42	65	2928	74	1	0
c_r	-20.0	3.85	2.5	2	48	57	2928	54	1	0
	0	3.90	2.5	2	46	60	2928	54	1	0
	+20.0	3.95	2.5	2	42	65	3276	74	1	1
t_{imp}	-0.50	3.95	2.5	2	38	72	2964	78	1	1
	0	3.95	2.5	2	36	75	2964	78	1	1
	+0.50	3.85	2.5	2	48	54	2928	54	1	0
δ_{imp}	-0.1	3.95	2.5	2	46	58	3588	32	1	1
	0	3.95	2.5	2	48	57	3714	68	1	1
	+0.1	3.85	2.5	2	46	58	2806	32	1	0
CM_{imp}	-100	3.95	2.5	2	48	57	3744	68	1	1
	0	3.95	2.5	2	44	62	3432	72	1	1
	+100	3.95	2.5	2	44	62	3432	72	1	1

Table 3. 7: Sensitivity analysis of different parameters. (*denotes the optimal value)

From the sensitivity analysis (Table 3.7), we can deduce the following:

- **Variation of the unit inspection cost (C_i):** The increase in unit inspection cost causes a decrease in the number of inspection items size (n) to minimize the total cost of sampling, then a slight increase several times to take the sampling (m), to the total sampling expenditures. In addition, the sampling interval h^* increases to perform sampling less frequently. This makes both

surveillance and control limits coefficients k_p^* , and k_c^* increase, yielding lower quality requirements.

- **Variation of the cost of a defective unit (C_r):** The increase in the rejection cost causes a decrease in the sampling size (n), the increase in the times the sampling inspection is conducted (m), And the increase in the control chart limit (k_p) to reduce the proportion of non-conforming units. However, the cost of rejection affects the average number of samples to detect the surveillance state duration ($J_{imp}T^*$) without significance on the average number of samples to detect the “out of control” (J_pT^*). The surveillance and the control limits get closer, when the unit rejection cost C_r take a tall, permitting possibly more focuses to drop near to them. Moreover, the optimal interval of sampling diminishes and the optimal sampling size increases in order to quicker detect small process shifts and reduce the proportion and the cost of non-conforming items. In like manner, the average numbers of samples (j_p and j_c) controlled before the shift to the surveillance or critical state diminishes, which illustrates the truth that maintenance activities are performed greater..
- **Variation of Imperfect Maintenance duration at out-of-control (t_{imp}):** As the preventive maintenance (imperfect) duration μ_{imp} gain, the coefficients of the control chart boost, the surveillance and control limits move further apart, and PM actions are done less frequently. After that, the average number of samples taken (J_{imp}) before going into the surveillance or critical state increase. Control charts with lower severity have higher sampling interval and lower sample size. Subsequently, the quantity of non-conforming items hike, resulting to an increase in the costs of rejection.
- **Variation of the magnitude of the shift to the surveillance state (δ_{imp}):** The increasing of the parameter (δ_{imp}) implies that the average sample of the quality indicator to detect the surveillance state ($J_{imp}T$) will move away from the center line of the control chart. Hence, the control chart becomes more relaxed to cope with the increasing process degradation, allowing potentially more points to fall within the control limits. Consequently, the perfect maintenance actions with duration decreases as well as the average number of samples to detect the surveillance state ($J_{imp}T$). Furthermore, the sample size (n), decreases, and the sampling frequency increase thereby improving the process quality. As the parameter δ_p increases, the process deteriorates further and the new mean of the process moves away from the target mean μ . As a result, the monitoring limits have shorter coefficients and cramped charts. The result is a decrease in the average number of samples exceeding the monitoring limit and hold the frequency of maintenance activities. Other points are very close to the surveillance limits and between the surveillance and the control limits. A process may not run properly and the need

to quickly retrieve process-related information. Therefore, the sample size is increased the sampling interval decreased, and the quality of the process is improved

- **Variation of Cost of Preventive Maintenance imperfect (CM_{imper}):** The CM_{imper} which increase the cost of a IPM action, leads to the less frequent execution of IPM actions (higher value of J_{imp}). In this case, the surveillance and the control limits are moving apart. So potentially more points fall within the limits of the monitor, and fewer points are very close to those limits. This accounts for the fact that the optimal sample size decreases and the sampling interval increases.

3.8 Conclusion

This chapter studied an integrated model of production plan, maintenance strategy, and quality control policy, considering the impact of the production variation and control chart parameters on the optimum maintenance plan. We have developed an integrated production model subjected to a random failure to satisfy different customer demands under a given service and quality requirements for all the production periods. An optimal combination of machine degradation which depends on production plans, and control chart parameters are developed. A mathematical model that significantly reduces the total cost of production, maintenance, inventory, and quality is built based on a proposed integrated optimization of production, maintenance, and quality and. Unlike most of the scientific work in the literature which uses the traditional production process control tools, they also, focused on either one or two of the production factors. While this work designed an improved statistical process control tool. The tool is used to investigate an integrated model of production, maintenance, and quality control policy, considering the impact of the production variation and control chart parameters on the optimum maintenance plan. We have developed an integrated production model subjected to a random failure to satisfy different customer demands under a given service and quality requirements for all the production periods. An optimal combination of machine degradation depends on production plans, and control chart parameters are developed. A mathematical model that significantly reduces the total cost of production, maintenance, inventory, and quality is built based on a proposed integrated optimization of production, maintenance, and quality. The new maintenance approach considered the deployment of different types of maintenance with appropriate durations commensurate to process scenarios from the evolution of the control chart quality indicator. The approach provided the information obtained by the control chart based on established analytical relationships and significantly increases the process reliability, reduces the generation of non-conformal items, and minimized total cost. This is found to be a robust model, that will apply to industrial cases in need of high precision principles required for the production of high-quality products.

The subsequent models take into account different assignable causes. In the next chapter (4) we proposed integrated maintenance production, and quality policy considering different assignable causes. A quality management technique in conjunction with a process monitoring control chart is

employed. The new joint control policy considers the interactions between production quality and maintenance strategies. We developed the policy for randomly failing production systems and in a dynamic way throughout a finite production horizon. We employed the Rule of AFNOR (Seven), as involved in quality management in conjunction with a control chart tool, to define multiple process scenarios as decision levels.

Chapter 4:

An Integrated Maintenance, Production, and Quality Policy with a Statistical Process Control Chart considering different assignable causes.

In the past decades, studies of production and maintenance planning have been focusing on age-dependent machine failure. The integration and optimal coordination of the three major factors of production (production, maintenance, and quality) represents a challenge for industrial companies and are more challenging with multi-assignable causes in the production process. To address this problem, this chapter employed the AFNOR Rule as involved in quality management, jointly with the control chart tool. To monitor, analyze and direct appropriate maintenance actions along the different multiple process scenarios as decision levels. We use the interactions between the production process and product quality for strategies of maintenance that minimized costs, maintain accuracy, and improve reliability. We developed the policy for randomly failing production systems in a dynamic way throughout a finite production horizon. The optimization of maintenance strategy consists to minimize the costs related to preventive imperfect, corrective (minimal repairs) in the several cases of four different possible scenarios. Based on the production rate variation and its impact on machine degradation the number of maintenance, times, and intervals needed for prudent decision-making is determined by the probabilities of different cases. In this context, this chapter presents the derivation of all the probabilities for the process to be in or out of control, the average run lengths, and the restoration cycle durations. Based on the degradation factor, a new failure rate equation is formulated, then the average failure rate for each case, we formulated different maintenance costs. The proposed approach is useful for a precise calculation and minimization of the total maintenance cost which optimizes the total production cost in consideration of all possible process statuses due to multiple causes

4.1 Introduction

Maintenance has a significant role in the current industrial revolution (Industry 4.0) focusing on product quality and production process reliability. Imperfect maintenance (IM) is a new kind of maintenance approach, which has evolved in the last decades, as an alternative in-between the classical PM and CM. Production systems linked to the supply chain models usually use traditional maintenance and inventory management policies that result in higher total production costs due to either shortages or redundancies. Improved methods can drive dynamic and integrated decision models that conduct production, maintenance, and inventory management actions more efficiently and in a collaborative way while reducing total costs. In this work, we propose a new framework that employed combined and continuous quality monitoring techniques involving statistical process control in conjunction with the AFNOR principles (rule of seven). To jointly optimize maintenance policy according to planned production and inventory decisions that satisfy a randomly failing production system's customer service and quality requirements. We formulate our problem as a stochastic program that considers the interaction between the process reliability state and product quality. To determine the optimal number of imperfect maintenance be planned periodically before, during, and after the process variation. The framework presented herein applies to real-world industrial problems with uncertainties in terms of random customer demand, high quality, lower cost, and production system reliability requirements. With the rapid change in global economic conditions, there is a need for implementing effective maintenance policies to ensure the satisfaction of customers' demand for high-quality products at competitive prices. Few works consider the impact of production variation on system degradation in their maintenance strategy. Among these works, Hajej et al. [1] found the significant influence of production rate fluctuations between periods on the production maintenance plans and total cost. In a real production setup, machines are subject to multiple uncertainties; random failures, and non-accurate maintenance strategies, resulting in a quality output decline which will have consequences on the overall performance of such companies. Therefore, optimizing production and maintenance strategies are key to mitigating the effects of such random disturbances. Additionally, product quality has become a critical third factor in recent years and has been gradually incorporated into the field of production systems. Quality control is a process that is employed to guarantee that product quality is sustained or improved. The use of a new role of maintenance function to better support value creation, by contributing to the economic dimension is the new trend according to Jasiulewicz et al. [2]. Various works in the literature considered integrating two or more of these essential functions. Consequently, the integrated maintenance strategy with quality management techniques in combination with SPC (statistical process control) based on the increasing failure rate with time and the machine's use is rarely considered in the literature. The control chart is based on the "statistical process control" (SPC) methodology, the six-sigma method will employ the Define, Measure, Analyse, Improve, and Control (DMAIC) methodologies [3]. Therefore, the combination of statistical process control of quality, maintenance strategy, and optimal production planning will be an unfailing solution that minimizes costs and maximizes production system

reliability. Different authors studied various issues of production and maintenance. Buzacott et al. [4] were able to improve production in their work titled “Automatic transfer lines with buffer stock”. The study considered the problem of integrated production to maintenance methods by analysing the inventory stocks in improving production. Chen [5] studied an unreliable system of production considering partial rework processes with preventive maintenance imperfect, thereby finding the economic production period, best controls, inspection times, and preventive maintenance interval. Chelbi and Ait-Khadi [6] advanced the works of Salameh and Ghattas [7] by considering the option of corrective maintenance at each failure of the manufacturing system between preventive maintenance actions, thereby optimizing the preventive maintenance and the buffer stock. Nahas [8] developed a stochastic model of an unreliable machine to determine the optimum stock size and preventive maintenance strategy. Recently, Chen et al. [9] actualized the joint model of production plan and maintenance schedule under imperfect preventive maintenance, in which they use the immune clonal selection algorithm. In the work of Holgado et al. [10], although this study investigates maintenance management and its impacts on manufacturing operations by relating the economic, environmental, and social benefits of maintenance function to plant operation, alas, the study did not consider the correlation of maintenance with quality. Other researchers dealt with maintenance and quality, for example, Pandey et al. [11] worked on a joint optimization approach for production planning, maintenance schedule, and process quality, but did not consider the impacts of inventory control shortages. Lesage and Dehombreux [12] aimed to identify the shifts in the production process that are subject to maintenance and quality control at failure, dealing with the problem with a simulation approach. Bouslah et al. [13] developed a combined production, control quality by sampling, and a deteriorating production system with an “Average Outgoing Quality” (AOQL) constraint. Nevertheless, the model assumed specific quality attributes deteriorate according to age. Suliman and Jawad [14] developed a model for optimizing the production and preventive maintenance action period to reduce the non-conforming items when the process moves out of control. The joint integration of production, maintenance, and quality was conducted by Ben-Daya and Rahim [15]. They presented joint maintenance and the economic approach of an x-control chart for a deteriorating process, where the in-control state is characterized by a probability distribution. They proposed a preventive maintenance strategy to reduce the out-of-control state considering the increasing hazard rate and to illustrate the impact of maintenance level on the costs of quality control. Its extension by Zang and Zang [16] suggested a maintenance and reliability structure for a two-state process optimizing the decision variable of a manufacturing system to determine discrete times of preventive maintenance activities. Recently, Bahria et al. [17] have developed an integrated method for joint production, quality, and maintenance control for batch production systems subject to degradation. They proposed a preventive and corrective maintenance strategy depending on the control of lots of produced quality performed by using a control chart and minimizing the total cost. Presently, products and processes are becoming more complex, with several assignable causes, and require more quality checks and improved monitoring tools. The novelty of our work would be the product of improved quality monitoring methods to trigger appropriate maintenance actions. The use of SPC in conjunction with the rule of seven quality management techniques. In practice, it is not always possible to restore

the systems to a new state after each maintenance action, neither we can rely on an imperfect or minimal repair that returns the system to as bad as old. However, more accurate planning of the number and time of these different maintenance actions for several cases is needed in the economic planning of a maintenance program. The number of maintenance, times, and intervals needed for prudent decision-making cannot be estimated unless the probability of different cases possible in both the in-control and out-of-control scenarios is calculated. In this context, we present the formulation of all the probabilities for the process to be in or out of control, the average run lengths, and the restoration cycle durations depending on the control chart parameters. And, also, based on the production rate variation and its impact on the machine degradation throughout the finite horizon. The statistical distribution function of the reduced cantered normal law in conjunction with quality management methodologies. We based on the degradation factor, a new failure rate equation is formulated to derive the characteristic function, then the average failure rate for each case, we formulated different maintenance costs. The study is aimed at a precise and comprehensive calculation and minimization of the total maintenance and quality cost, which optimizes the total production cost, considering all possible process cases under different scenarios due to multiple causes.

4.2 Industrial Issues

In this chapter, we focus on an industrial issue that revolves around the development of integrated production optimization. we seek to optimize the production planning of a three-echelon production system under uncertainties. We aim to understand also, the impact of (IM) on the production system reliability, non-conformal items, and total production cost minimization as further explained in the next section.

4.2.1 Problem description

This model as illustrated in figure 4.1, integrated economic production quantity linked to multi warehouses, and statistical process monitoring in conjunction with the rule of seven to coordinate maintenance in the presence of several assignable causes. An improved production process monitoring management techniques to monitor quality characteristics, and trends of assignable causes of variation, and notify when the process is either stable or shifted to an out-of-control state. Unlike the traditional SPC which indicates common causes of variation, our new method will also consider uncommon trends involving special and multi-assignable causes of variation. In the real production process, a production system may start in a controlled state operation producing quality and acceptable products and after a while may shift to a state of out-of control and will begin the generation of no conformal products. The unhealthy process variation is due to so many factors such as machine degradation, operator skills, fatigue, material complexities, quality standards, etc.

This research considered a randomly failing production system of a single machine linked to a supply chain consisting of a single product aimed to satisfy random customer demand. The process

begins in-control state, producing conforming products with the capacity of having a variable production rate u_k and sometimes goes out of control under four possible scenarios; 1, 2, 3, and 4. The four distinct scenarios are defined by the control chart in combination with the rule of seven techniques of quality management. As involved in quality management, the Rule of Seven says, “A run of seven or more consecutive points in a control chart, either above the mean, or below the mean, or continuously increasing or decreasing, may indicate the process to be out-of-control”.

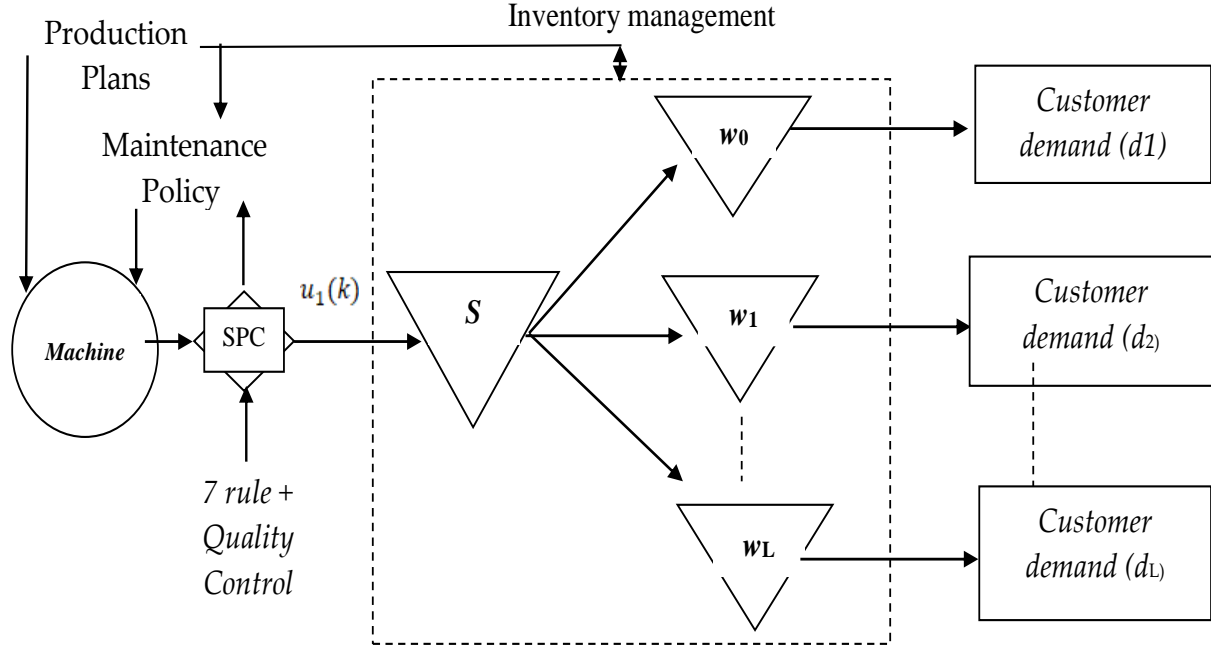


Figure 4. 1: Production and Supply chain management

The production system is subject to random failures due to the degradation degree of the production machine influenced by the production rates $u(k)$ and consequently, the failure rate $\lambda(t)$ increases with time. This will reduce the production process's reliability and generate the production of non-conforming units. To sustain the overall production objectives, the process is monitored using the rule of seven quality management methods in conjunction with the SPC. The control chart parameters are the sample size (n), the sampling intervals (I_s), and the number of standard deviations of the sample distribution between the center line of the control chart and the control limits (k_c). The quality parameters x_s respect the normal distribution factor (F) with mean and standard deviation (μ & σ). The results comprise the control chart measurements and observations used to decide appropriate maintenance actions throughout the control horizon ($H \cdot \Delta t$). The process is in control when all statistics are within control limits. The average sample of the quality indicator measurements \bar{x}_s is at its target value between the lower control limit LCL , and the upper control limit UCL .

1. The probability of the occurrence of a new i^{th} assignable cause equaled after each restoration cycle.
2. The resources needed to perform maintenance actions are available.
3. The unit costs related to inventory, maintenance, and quality are known and constant.

4.2.2 The quality control management

Control charts are decision-making tools that provide information for timely decisions concerning recently produced products. The rule of seven states that when seven or more consecutive measurement points fall on one side of the mean, an assignable cause must be investigated. Control charts can be used to identify sources of variation, both common and special causes. Common cause variation is the variation inherent in the process. A process with only common cause variation is highly predictable. A process that has a significant inherent common cause variation may not be capable of producing standard products that meet predetermined quality specifications. While special cause variation is that which is not inherent to the process. A process with special cause variation is highly unpredictable.

A process is referred to be in control when the control chart does not indicate any out-of-control condition and contains only common causes of variation. Control charts are used for continuous process monitoring activities when the common cause variation happened to be small. But, if the common cause of variation is too large, the process will need to be improved or modified to reduce the amount of the inherent variation to an acceptable level. When a control chart indicates an out-of-control condition (a point outside the control limits or matching one or more of the criteria in the rules below), the assignable causes of variation must be identified and eliminated.

4.2.2.1 The rule of seven

Key Issues

- The rule of seven is also referred to as a thumb or heuristic.
- All seven points may be within the control limits.
- Such a situation is considered abnormal, and hence warrants some attention.
- When seven consecutive measurement points fall on the same side of the control chart central line. (mean) either below or above, the process is referred to as out of control and needs adjustments.
- It does not imply the need for corrective action, but the need to be investigated (to find an assignable cause).
- Also, it does not imply the frequency at which a process becomes out of control is acceptable or otherwise.

The figure 4.2 below illustrates an example of seven points below, and above the control chart mean.

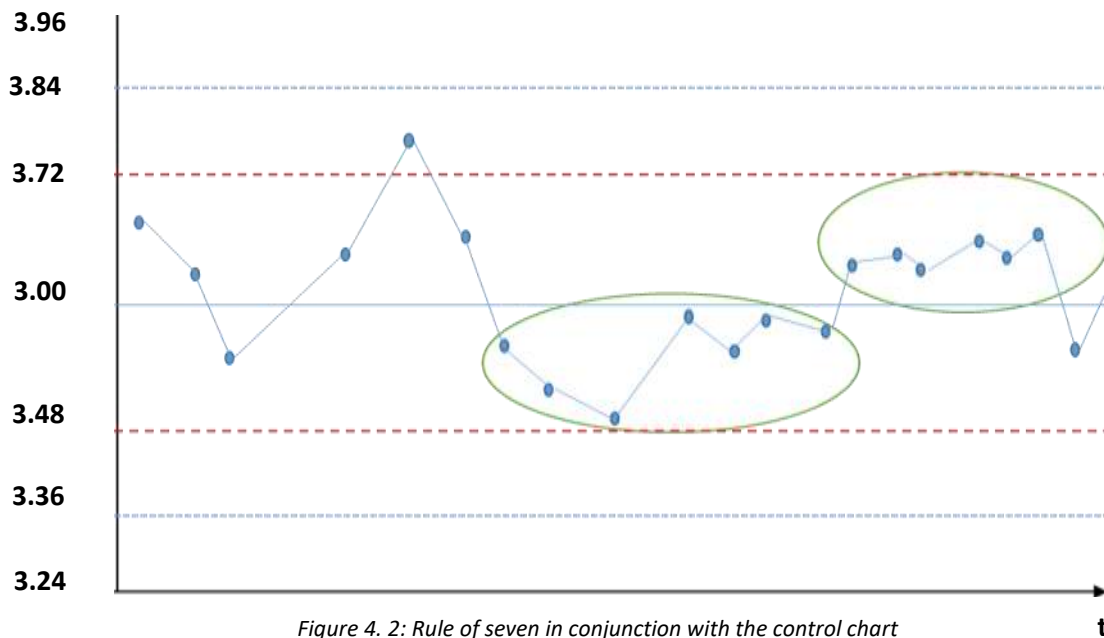


Figure 4. 2: Rule of seven in conjunction with the control chart

We use only four of the following rules (1, 4, 5, and 7), however, all can be used to properly interpret control charts:

Rule 1 – One point beyond the 3σ control limit

Rule 2 – Eight or more points on one side of the centreline without crossing

Rule 3 – Four out of five points in a row between 1σ to 2σ or beyond

Rule 4 – Six points or more in a row steadily increasing or decreasing

Rule 5 – Two out of three points in between 2σ to 3σ

Rule 6 – 14 points in a row alternating up and down

Rule 7 – Any noticeable/predictable pattern, cycle, or trend

4.2.2.2 Control chart

Different control chart configurations are considered when the system is out-of-control and should also adopt another maintenance strategy to prevent possible out-of-control, non-compliant products in the in-control state.

The control chart parameters are the sample size (n), the sampling interval h , and the number of standard deviations of the sample distribution k_c , which is between the center line and the control limits of the control chart. In this study, the decision variables related to the control chart are n , I_s , and k_c .

Also, the control chart average value for each sample s is;

$$\bar{x}_s = \frac{1}{n} \times \sum_{i=1}^n x_i \quad (4.1)$$

We define the associated standard deviation is given by:

$$\sigma_s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x}_s)^2}{n-1}} \quad (4.2)$$

We calculate the mean of the means (central value) and the average of the standard deviations of the indicator x for all m samples.

$$\mu = \frac{1}{m} \times \sum_{s=1}^m \bar{x}_s \quad (4.3)$$

$$\sigma = \frac{1}{m} \times \sum_{s=1}^m \sigma_s \quad (4.4)$$

The upper and lower control limits are defined by the following equation.

$$UCL = \mu + \frac{k_c}{\sqrt{n}} \times \sigma \quad (4.5)$$

$$LCL = \mu - \frac{k_c}{\sqrt{n}} \times \sigma \quad (4.6)$$

4.2.3 Different scenarios and associated maintenance actions

To reduce the probability of shifting to out-of-control, quality monitoring of products is essential to employ an appropriate maintenance strategy commensurate to the production process degradation due to the production rate variations. Imperfect maintenance actions with negligible durations are applied in the “in control” state, with minimal repairs at the ($T_{\text{imp}} = \alpha \cdot \Delta t = \alpha \cdot \beta \cdot h_s$, where α and β are integers) time interval, where Δt is a production period length and h is the sampling interval. A minimum repair is applied for each failure between two successive imperfect preventive maintenances (Scenario I). To help determine whether or not the variation within the data set is due to the random variation typical of the performance of that process, or due to non-random attributable change in the process, there are different rules to run a chart. In the case where the production process is “out of control” between the j^{th} and $(j+1)^{\text{th}}$ sample due to the occurrence of any of the assignable causes as well as to machine degradation, from μ to $\mu + \delta \times \sigma$ (Scenario II). Hence, production is interrupted and a PPM (perfect preventive maintenance) action of average duration μ_p must be employed. On the other hand, a control chart can indicate the presence of a special cause if a point goes beyond the limits of control or when the plots of the data show non-random behaviour. This behaviour shows the existence of non-random conditions in the process using graphs such as Ford rules or AFNOR rules. In this study, we deal with the examples from ford rules where the process instability can be generated during the presence of a series of more than 7 consecutive points below or above the centre line (Scenario III) or ascending or descending series of more than 7 points (Scenario IV).

The following are the four possible scenarios of the unreliable process due to real situations that likely occur in a production run.

Scenario I: The sample average of the quality indicator is between the control limits ($LCL < \bar{x}_s < UCL$) for all samples. The production process always remains in the in-control state from the start to the end of the production cycle, in other words, the total time of being out of control is zero. In this situation, the maintenance strategy is to carry out periodic imperfect maintenance throughout the horizon ($H, \Delta t$) to guarantee the production system's reliability and sustainability, as shown in figure 4.3 below.

- (a) Periodic IMPERFECT preventive maintenance following degradation of the machine.
- (b) Minimal repair is implemented at each failure between two successive preventive maintenance actions

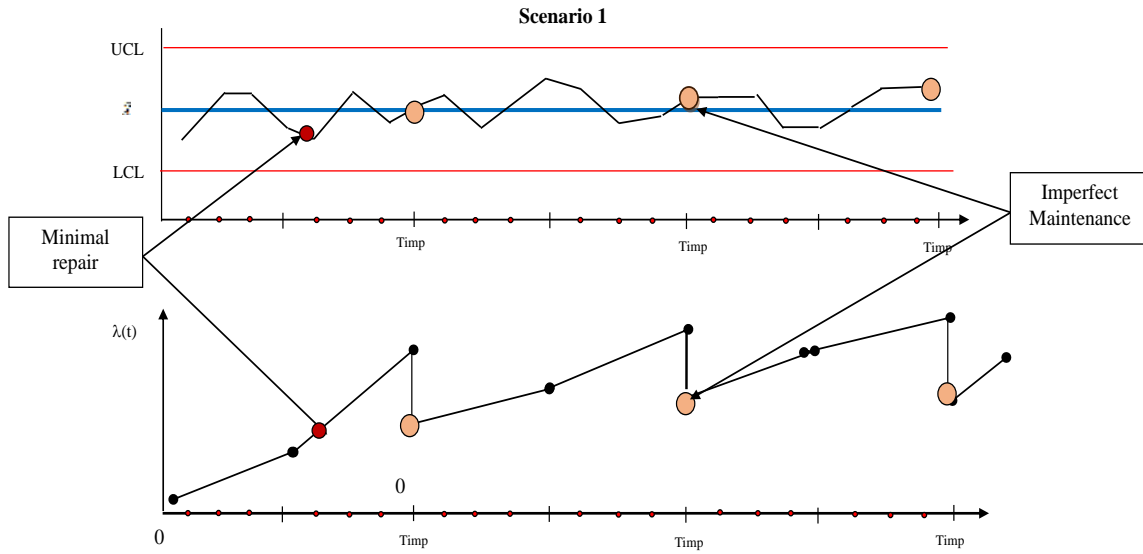
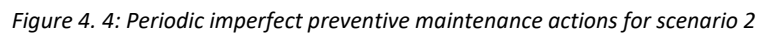


Figure 4. 3: Periodic imperfect preventive maintenance actions for scenario 1

Scenario II: As shown in figure 4.4, the production process starts in the in-control state and is monitored by taking n sample sizes, at times $I, (1, 2, i, 3, i, \dots)$ to check the status of the mean process. The process shifts to the out-of-control state between the j^{th} and $(j + 1)^{th}$ sampling due to the occurrence of the i^{th} assignable cause A_i ($i = 1, 2, \dots, S$, and consequently, the mean value of the quality characteristic changes from μ to $\mu + \delta_i \cdot \mu$. In that event, operators seek to detect the assignable cause and perform imperfect maintenance with the average duration of μ_p . Then, the process is restored to an as-good-as-new state, and periodically reschedule the maintenance plan according to the production rate. Items produced during the last sampling interval transition to the 'out of control' state (between the samples $(j - 1)$ and j) are all rejected.



The diagram illustrates the AGAN framework. The top part shows a process with two phases: before and after AGAN. The bottom part shows the AGAN model's output, which is a sequence of points (red, yellow, blue, orange) that are clustered into two groups (red and blue) and then mapped to the process output. The AGAN model is shown as a box labeled 'AGAN'.

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Scenario IV: The mean charts are very good at detecting either significant biases or enormously increased random fluctuations. Used in this way, they are not very good at finding small shifts or slow drifts. For this reason, one sometimes adds additional rules such as more than seven consecutive points situated on the same side of the CL (Figure 4.6) Hence, the process starts in the in-control state, and the process instability is generated during the presence of a series of more than 7 points on the same side of the CL at time j^{th} to $(j + 7)^{th}$ sampling. Consequently, the process becomes unstable and will eventually manage the non-product quality. The operators seek to detect the assignable cause and conduct the imperfect preventive maintenance with an average duration of μ_p . Then, the process is restored to an as-good-as-new situation and periodically reschedules the maintenance plan according to the production rate—all items produced during the last sampling interval $(j + 7)^{th}$ are rejected.

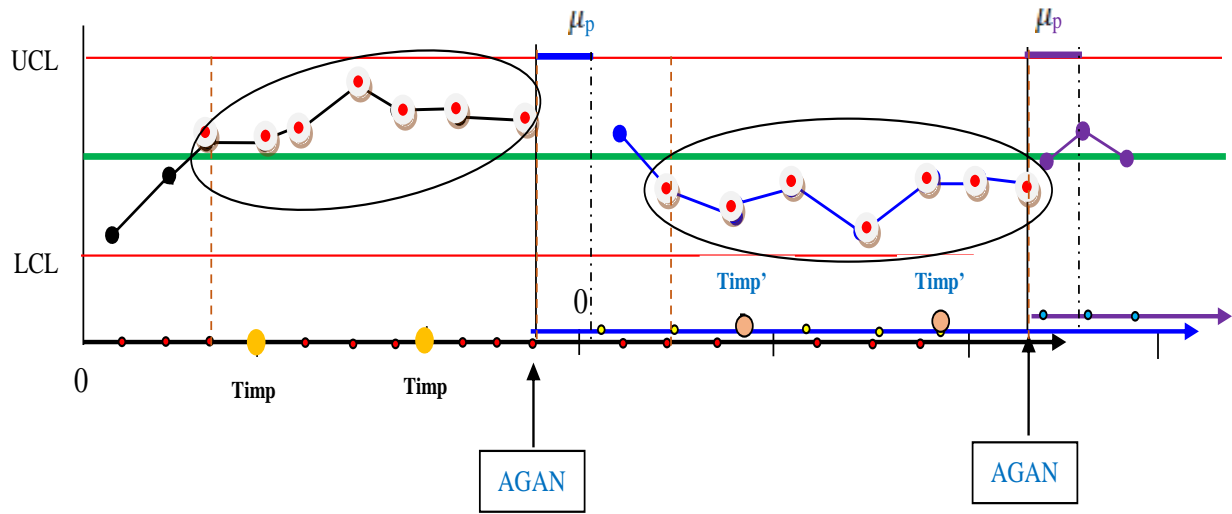


Figure 4. 6: Periodic imperfect maintenance actions for scenario 4

4.2.4 Assumptions.

The considered assumptions in the proposed model are as follows:

1. The demands are characterized by a normal distribution with mean and variance given respectively by \hat{d}_i , and V_{d_i} , and the demands are satisfied at the end of each period.
2. The sample size (n) taken at each step is known and constant.
3. Non-conforming items are not integrated into the production process.
4. The non-conformity of the products is due to the degradation of the machine, and the assignable causes.
5. Perfect maintenance actions restore the production unit to an as good as new state
6. Imperfect maintenance actions reduce the failure rate between as-bad as-old and as-good as-new states.
7. minimal repairs are applied when the production unit fails between two imperfect maintenance actions.

8. The quality characteristic is normally distributed. The occurrence of i^{th} assignable cause leads to a mean shift from μ_0 to either $\mu_0 + \delta_i \times \sigma$ or $\mu_0 - \delta_i \times \sigma$. Since the probability of detecting positive and negative shifts is the same.
9. The probability of the occurrence of new i^{th} assignable cause is equal after each restoration cycle
10. Each assignable cause is observed one time during the finite production horizon
11. The resources needed to perform maintenance actions are available.
12. The unit costs related to inventory, maintenance and quality are known and constant.

4.2.5 Notation

This section provides definitions; production, quality, decision variables (SPM), maintenance, and costs required to mathematically develop the proposed model. Find the notation used specifically for this chapter as shown in Appendix VI, and it turns out that the notation is divided into three subdivisions: quality, maintenance, and production parameters.

4.3 Stochastic problem model

The production policy characterized by a stochastic production problem produces a rate $u(k)$ with $k: 1, \dots, H$ during the finite production horizon $H \cdot \Delta t$. We assume that the horizon is portioned equally into H periods with a length equal to Δt . At the beginning of each production period, a stock is created to ensure random warehouse discrete satisfaction $Q(k)$, and each warehouse is created to ensure a random demand $d(k)$ under a given service level. The objective is to determine the economic production plan satisfying the forecasting demand and respecting a given service level while minimizing production and inventory costs. This study aims to determine the economic production plan to satisfy customers' demands under a given service and quality level constraints. Secondly, according to the obtained economic production plan and considering the influence of production rates on the degradation degree and the different scenarios, we determine the optimal maintenance plan and the control chart's quality management methodologies. The cost model of production and maintenance policies should consider the expected costs related to production, inventory, maintenance, and quality, including sampling and rejection costs during the finite time horizon. Similar to the previous production model.

4.3.1 The Mathematical model

The goal of the production policy in this study is to determine the economic plan of production that satisfies the random customer's demand under a given service and quality level constraints. Secondly, according to the obtained economic plan of production and the influence of the production rates on the degradation degree, and depending on the different scenarios, we determine the optimal

number and time of imperfect preventive maintenance, according to the optimal parameters of the control chart. The cost model of production and maintenance policies considered the expected total costs related to production, inventory, maintenance, and quality including sampling, false alarm, and rejection costs from the no-conformal products throughout the finite production horizon.

4.3.2 The Production model

Recall that the production policy characterized by a stochastic production problem, consists in producing a rate $u(k)^*$, with $k: 1, \dots, H$ during the finite production horizon $H \cdot \Delta t$. Assumed that the horizon is portioned equally into H periods with a length equal to Δt , at beginning of each production period, a stock is created to ensure the discrete satisfaction of random warehouses $Q_i(k)$ and each warehouse (i) to ensure the satisfaction of a random demand $d(k)$ under a given service level θ_i . The objective is to determine the economic plan of production to satisfy a forecasting demand respecting a given service level by minimizing the total cost of production, and inventory. To solve the proposed model, we develop a numerical procedure to determine an approximate possible solution for the optimal production plan $u(k)^*$, we use the random exploration method to randomly generate a production vector during the finite production horizon (H) and under the service level constraints ($Prob[w_i(k) \geq 0] \geq \theta_i$) and production bounds ($u_{min} \leq u_k \leq u_{max}$). From this vector, we calculate the production cost in each period. The calculation of the number of Products being produced to satisfy the service level constraint determined the optimal quantity (Q_i^*) to be taken to warehouses and for all periods.

Finally, when all the conditions are verified and satisfied, the minimum total production cost (TCP_{min}) is saved. The optimal production plan $u(k)^*$ as well as optimal delivery quantities, are generated using our developed multi-objective function algorithm. The detailed algorithm flowchart diagram is shown in the appendix.

- Production Cost

$$PC = C_p \times \sum_{k=1}^H u(k) \cdot \Delta t \quad (4.7)$$

- Holding cost at the Main Store (S)

$$HC_S = \sum_{k=1}^H C_h \times Z_S(k) \quad (4.8)$$

- Holding costs at the retail warehouses (w_i)

$$HC_W = \sum_{k=1}^H \sum_{i=1}^L C_h \times Z_{wi}(k) \quad (4.9)$$

- Total Inventory Holding cost (HC)

$$HC_T = C_{hi} \times \sum_{k=1}^H \left\{ \begin{array}{l} \max(S(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot u(k) \cdot \Delta t^2 \\ + \sum_{i=1}^L [\max(w_i(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot Q_i(k - \tau_i) \cdot \Delta t^2 \end{array} \right. \quad (4.10)$$

- Delivery cost

$$DLC(Q(k)) = C_l + C_o \times \sum_{i=1}^L \left(\frac{Q_i(k)}{Q_v} \right) \quad (4.11)$$

- Delay Penalties

$$DPC = C_d \times \left(\sum_{k=1}^H \left(\sum_{i=1}^L dw_i \right) \right), \quad dw_i = \frac{|\min(w_i(k), 0)|}{Q(k+1 - \tau_i)} \quad (4.12)$$

The minimization of the different cost components mentioned above is carried out considering the model constraints such as the service level, inventory balance, and the production bounds for each period k .

$$Min CTP = \underset{U=\{u(k), k=1, \dots, H\}}{Min} \left\{ C_p \times \sum_{k=1}^H u(k) \cdot \Delta t + C_h \times \sum_{k=1}^H \left(\begin{array}{l} \max(S(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot u(k) \cdot \Delta t^2 \\ + \max(w_i(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot Q_i(k - \tau_i) \cdot \Delta t^2 \end{array} \right) + C_d \times \left(\sum_{k=1}^H \left(\sum_{i=1}^L \frac{|\min(w_i(k), 0)|}{Q(k+1 - \tau_i)} \right) \right) \right\}$$

Under the following:

$$S(k) = S(k-1) + u(k) \cdot \Delta t - \sum_{i=1}^L Q_i(k), \quad k = 0, 1, \dots, H-1, \text{ and } i = 1, \dots, L \quad (4.13)$$

$$w_i(k) = \begin{cases} w_i(k-1) + Q_i(k - \tau_i) - d_i(k) & \text{If } k \geq \tau_i \\ w_i(k-1) - d_i(k) & \text{otherwise} \end{cases} \quad (4.14)$$

$$Prob[w_i(k) \geq 0] \geq \theta_i \quad (k = 0, 1, \dots, H-1, \text{ and } i = 1, \dots, L) \quad (4.14.1)$$

$$U_{min} \leq u(k) \leq U_{max}, \quad k = 1, \dots, H \quad (4.15)$$

4.3.3 The maintenance and quality policy

In this part, we aimed to minimize the total maintenance and quality cost by finding the conditional probabilities defining cases in different scenarios. And in conjunction with the optimal control chart parameters (n, h, k_c) to optimize the number of maintenance, thereby minimizing the total maintenance and quality cost.

4.3.3.1 The maintenance policy

The cost model of maintenance should take into account all possible scenarios presented above. Four scenarios are possible depending on the production plan obtained by the production policy and

its influence on the degradation degree of the processes. The total cost of maintenance depends on the cost of unit costs of imperfect (all cases/scenarios), preventive maintenance, and the Average Number of Failures (ANF in all cases/scenarios).

The Probabilities:

Scenarios (2, 3, and 4) from one of the unions.

$Pr(S_2 \cup S_3 \cup S_4)$ probability to have a scenario from the three scenarios 2,3 and 4

Scenarios (2, 3, and 4) are one and the other in the union of other scenarios.

$Pr((S_2 \cap S_3) \cup (S_2 \cap S_4) \cup (S_3 \cap S_4))$ probability to have two scenarios at the same time from scenarios 2, 3, and 4.

Scenarios (2, 3, and 4).

$Pr(S_2 \cap S_3 \cap S_4)$ probability to have three scenarios at the same time

The Scenarios:

Scenario 1.

The first scenario is characterized by the stable state of the process during the all finite horizon H.Δt. In this situation, no variation of the average of the quality indicator outside the control limits, and a periodic maintenance strategy is applied, during the finite horizon, which depends only on the degradation of the process. Imperfect preventive maintenance actions are planned to reduce the failure rate of the process. The probability that the process is in the 'in-control' state is formulated as follows:

$$\alpha_1 = Pr(LCL < \bar{x}_s < UCL), \quad (4.16)$$

$$\alpha_1 = F\left(\mu + k_c \times \frac{\sigma}{\sqrt{n}}\right) - F\left(\mu - k_c \times \frac{\sigma}{\sqrt{n}}\right) \quad (4.16.1)$$

With F: distribution function of the reduced cantered normal law.

Let ARL₁ be the average run length characterized by the average number of successive samples where the process is stable during the finite horizon H. Δt (scenario 1). It is expressed as follows.

$$ARL_1 = \frac{1}{\alpha_1} \quad (4.17)$$

Scenario 2.

For scenario 2, when the process is unstable, the average can vary and it takes as value μ_1 . For a sample j, We note δ_j the expression of the adjustment of the mean in several standard deviations:

$$\delta = \frac{\mu_1 - \mu_0}{\sigma_0} \quad (4.18)$$

Let ARL₂ be the average run length characterized by the average number of successive samples to detect the shift to the control limits (scenario 2). It is expressed as follows (Montgomery 2004).

$$ARL_2(\delta) = \frac{1}{1-P_N(\delta)} = \frac{1}{\alpha_2} \quad (4.19)$$

$P_N(\delta)$ is the probability of non-detection of the shift to the out-of-control limit.

α_2 is being the probability of detection of the shift to the control limits.

Proof of (4.19).

This is to demonstrate that $\sum_{i=1}^{+\infty} i \times P(N=i) = \frac{1}{1-P(\delta_j)}$ with $P(N=i) = P(\delta_j)^{i-1} \times (1-P(\delta_j))$

$$\text{When we have } \sum_{i=0}^{+\infty} q^i = \frac{1}{1-q} \text{ and } \sum_{i=0}^{+\infty} i \times q^{i-1} = \frac{1}{(1-q)^2} \quad (4.19.1)$$

This last result is obtained by derivative concerning q the two members of $\sum_{i=0}^k q^i = \frac{1-q^{k+1}}{1-q}$

and making tender k to $+\infty$.

Thus by replacing q by $P(\delta_j)$ with ($0 < P(c) < 1$), we have : $\sum_{i=1}^{+\infty} i \times (P(\delta_j))^{i-1} = \frac{1}{(1-P(\delta_j))^2}$

$$\text{We deduce that: } \sum_{i=1}^{+\infty} i \times P(N=i) = \frac{1-P(\delta_j)}{(1-P(\delta_j))^2} = \frac{1}{1-P(\delta_j)} \quad (4.19.2)$$

End proof;

In this situation, the average quality indicator does not exceed the control limits $LCL < \bar{x}_s < UCL$ while the process is in out-of-control, and perfect preventive maintenance action is needed to restore the state of the process to AGAN state.

$$P_N(\delta) = \{\text{Prob}(LCL \leq \bar{x}_s \leq UCL \mid \mu_1 = \mu + \delta \cdot \sigma)\} \quad (4.20)$$

$$P_N(\delta) = F\left(\frac{UCL - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right) - F\left(\frac{LCL - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right) \quad (4.21)$$

$$P_N(\delta) = F(k_c - \delta \times \sqrt{n}) - F(-k_c - \delta \times \sqrt{n}) \quad (4.22)$$

With F: distribution function of the reduced cantered normal law.

The average restoration cycle duration for scenario 2, RCD_2 , is given by

$$RCD_2 = ARL_2(\delta) \times h + \mu_p \quad (4.23)$$

Scenario 3.

The probability that the points continuously increase or decrease. So, the probability of seven consecutive points continuously ascending or descending, when the process is in stable status i.e.

$$(LCL \leq \bar{x}_s \leq UCL).$$

$$\alpha_3 = (P(7 \text{ ascending points})|LCL < \bar{x}_s < UCL) + (P(7 \text{ descending points})|LCL < \bar{x}_s < UCL) \quad (4.24)$$

Let ARL_3 be the average run length presented the average as several successive samples to detect seven consecutive points either ascending or descending or while the process is in control (scenario 3).

$$ARL_3 = \frac{1}{\alpha_3} \quad (4.25)$$

The average restoration cycle duration for scenario 3, RCD_3 , is given by

$$RCD_3 = ARL_3 \times h + \mu_P \quad (4.26)$$

Scenario 4.

Concerning scenario 4, note that the probability of any one point falling above (or below) the centreline is $\frac{1}{2}$ when the process is in control. Then, from the Multiplicative Law of probability for independent events, the probability of seven consecutive points falling above the centreline is equal to $(1/2)^7 = (1/128)$. Likewise, the probability of seven consecutive points falling below the centreline is $(1/2)^7 = (1/128)$. Therefore, the probability of seven consecutive points falling on the same side of the centreline is, by the Additive Law of Probability.

$$\begin{aligned} \alpha_4 &= (P(7 \text{ points on the same side of the center line})|LCL < \bar{x}_s < UCL) \\ &= (P(7 \text{ points above center line})|LCL < \bar{x}_s < UCL) + \end{aligned}$$

$$(P(7 \text{ points below the center line})|LCL < \bar{x}_s < UCL) \quad (4.27)$$

Let ARL_4 be the average run length presented as the average number of successive samples to detect seven consecutive points on the same side of the centreline while the process is in control (scenario 4).

$$ARL_4 = \frac{1}{\alpha_4} \quad (4.28)$$

The average restoration cycle duration for scenario 4, RCD_4 , is expressed as;

$$RCD_4 = ARL_4 \times h + \mu_P \quad (4.29)$$

4.3.3.2 Total maintenance cost

Maintenance costs consist of preventive and repair costs and depend on the four scenarios. The optimization of maintenance strategy consists to minimize the costs related to imperfect preventive, corrective (minimal repairs) maintenance, and perfect maintenance actions. In practice, restoring the systems to a new state is not always possible after each maintenance action, and we can rely on a simple repair. It is possible to carry out a partial renewal or partial overhaul (Restoration or replacement of a few system components). Hence, the integration of imperfect maintenance is important in real industrial situations. Economically speaking production companies are seeking for minimizing maintenance costs while respecting the reliability of the process. The objective of the maintenance strategy is to determine the optimal parameters of the control chart which are: the sample size n , the sampling interval h , the control limits coefficients k_c , as well as the optimal number N_{imp}^* preventive maintenance actions and the most adequate time between them, noted T_{imp}^* in the 'in-control' state and for each new cycle after the perfect maintenance action with non-negligible duration (μ_p) applied due to assignable causes. To perform a partial renewal maintenance policy, we need first to define the failure rate equation and from it, we can determine the average number of failures. Concerning the partial renewal, Assumed that we do a partial renewal at the instants $T_{imp}, 2T_{imp}, 3T_{imp}, \dots$, at each interval $h = [(k-1)T_{imp}, kT_{imp}]$, the failure rate will be equal to the failure rate over the previous interval $h-1$ multiplied by a 'degradation factor' e^α with α a strictly positive real.

The following figure 4.7 represented the evolution of the failure rate the imperfect preventive maintenance actions in each production period.

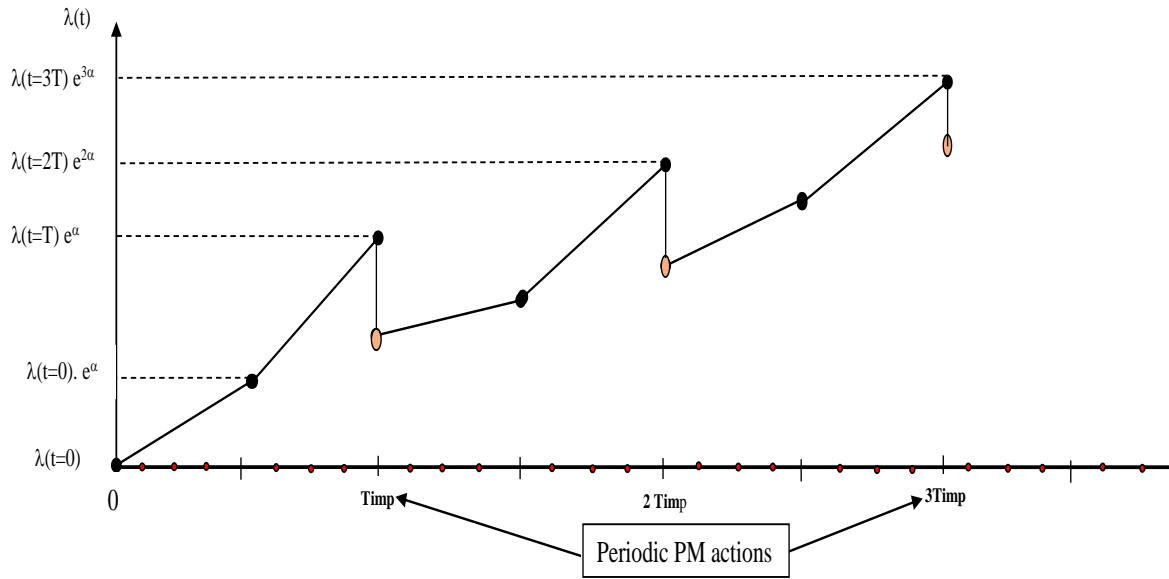


Figure 4. 7: Evolution of the failure rate.

Formally, the expression of failure rate in the case of imperfect maintenance actions is expressed as follows:

$$\lambda_k(t) = \left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T_{imp}} \right\rfloor + 1 \right) \times T_{imp}} \right\rfloor \right) \underbrace{\left(\lambda_{k-1}(\Delta t) + \frac{u(k)}{U_{max}} \lambda_n(t) \right)}_A + \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T_{imp}} \right\rfloor + 1 \right) T_{imp}} \right\rfloor \times \underbrace{\lambda_{\left(k - \left\lfloor \frac{T_{imp}}{\Delta t} \right\rfloor \right)} \left(\left(\left\lfloor \frac{k \times \Delta t}{T_{imp}} \right\rfloor - 1 \right) \times T_{imp} \right) \times e^\alpha}_{B} \quad \forall t \in [0, \Delta t] \quad (4.30)$$

$$T_{imp} = \text{Int} \left\lfloor \frac{H}{N_{imp}} \right\rfloor \times \Delta t$$

With $\lfloor \cdot \rfloor$: the minimum integer part

This failure rate function mainly consists of four essential parts:

- $A = \lambda_{k-1}(\Delta t) + \frac{u(k)}{U_{max}} \lambda_n(t)$ which describes the cumulative failure rate according to production rate and previous failure rates when there is no preventive maintenance action.
- The term $\left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T_{imp}} \right\rfloor + 1 \right) \times T_{imp}} \right\rfloor \right)$ is equal to zero to annul the failure rate A to zero when there is an imperfect preventive maintenance action, i.e., at times $q \times T_{imp}$ with $(q = 1, \dots, N_{imp})$, otherwise equals to 1.
- The $B = \lambda_{\left(k - \left\lfloor \frac{T_{imp}}{\Delta t} \right\rfloor \right)} \left(\left(\left\lfloor \frac{k \times \Delta t}{T_{imp}} \right\rfloor - 1 \right) \times T_{imp} \right) \times e^\alpha$ accumulates the failure rate when imperfect maintenance is applied, basing on the partial renewal at times $q \times T_{imp}$ with $(q = 1, \dots, N_{imp})$. At each interval $h_k = [(k-1) T_{imp}, k T_{imp}]$, in which the failure rate will be equal to the failure rate of the previous interval h_{k-1} multiplied by a degradation factor e^α with $\alpha \in [0,1]$.

The average number of failures is presented in each different case:

Case 1: (in-control)

In this scenario, a maintenance strategy is characterized by imperfect preventive maintenance planning with minimal repair for each 'in-control' state of the process. Imperfect preventive maintenance actions should be scheduled over the finite horizon H which is divided equally into N_{imp} parts of duration T_{imp} . Assume that performing imperfect preventive actions corresponds to the times $i.T_{imp}$ that the machine will be younger but not as good as new. However, between successive imperfect preventive interventions, the minimal repair is performed. It is assumed that the repair and overhaul durations μ_p related to the maintenance planning correlated to the degradation of the process are negligible.

The average number of failures is expressed as follows:

$$\varphi_{S1}(U, N_{imp1}) = \sum_{j=1}^{N_{imp1}} \left(\sum_{k=(j-1) \times T_{imp1}+1}^{(j \times T_{imp1})} \int_0^{\Delta t} \lambda_k(t) dt \right) + \sum_{i=(N_{imp1} \times T_{imp1})+1}^H \int_0^{\Delta t} \lambda_k(t) dt \quad (4.31)$$

$$T_{imp1} = \left\lfloor \frac{H}{N_{imp1}} \right\rfloor \times \Delta t$$

So the cost of maintenance in scenario 1, is given by the expression:

$$C_{M1} = (C_{imp} \times (N_{imp1} - 1) + C_{cm} \times \varphi_{S1}(U, N_{imp1})) \quad (4.32)$$

Case 2: (out-control)

In the case of an 'out-control' state, imperfect preventive maintenance with duration is applied to restore the process with a new cycle due to the presence of point outside main limits or a series of more than 7 points in one direction, up (ascending) or down (descending) or the presence of seven consecutive points falling on the same side of the centreline or either or several scenarios at the same time. In each new production cycle, a new arrangement of imperfect preventive maintenance actions is performed. In this case, the average number of failures between these two phenomes as well as the maintenance cost is expressed according to different cases:

Case 2.1: the presence of a point outside main limits (scenario 2)

$$C_{M2.1} = \alpha_2 \times (C_{pm2} + C_{imp} \times (N_{imp2,1} - 1) + C_{imp} \times (N_{imp2,2} - 1) + C_{cm} \times \varphi_{S2}(U, N_{imp2,1}, N_{imp2,2})) \quad (4.33)$$

With $\varphi_{S2}(U, N_{imp2,1}, N_{imp2,2})$: the average number of failures

$$\begin{aligned} \varphi_{S2}(U, N_{imp2,1}, N_{imp2,2}) = & \sum_{Z=0}^{N_{imp2,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp2,1}}{\Delta t} \right\rfloor+1}^{\left\lfloor (Z+1) \times \frac{T_{imp2,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=N_{imp2,1} \times \frac{T_{imp2,1}}{\Delta t}}^{\left(\frac{(RCD_2 - \mu_p)}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \\ & \sum_{Z=0}^{N_{imp2,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_2)}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp2,2}}{\Delta t} \right\rfloor+1}^{\left\lfloor (Z+1) \times \frac{T_{imp2,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=N_{imp2,2} \times \frac{T_{imp2,2}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \end{aligned} \quad (4.34)$$

$$T_{imp2,1} = \left\lfloor \frac{\left(\frac{(RCD_2 - \mu_p)}{\beta h} \right)}{N_{imp2,1}} \right\rfloor, \text{ and } T_{imp2,2} = \left\lfloor \frac{H - \left(\frac{(RCD_2)}{\beta h} \right)}{N_{imp2,2}} \right\rfloor$$

Case 2.2: the presence of seven points in one direction, (scenario 3)

$$C_{M2.2} = \alpha_3 \times (C_{pm3} + C_{imp} \times (N_{imp3,1} - 1) + C_{imp} \times (N_{imp3,2} - 1) + C_{cm} \times \varphi_{S3}(U, N_{imp3,1}, N_{imp3,2})) \quad (4.35)$$

$$\varphi_{S3}(U, N_{imp3,1}, N_{imp3,2}) = \sum_{Z=0}^{N_{imp3,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp3,1}}{\Delta t} \right\rfloor+1}^{\left\lfloor (Z+1) \times \frac{T_{imp3,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=N_{imp3,1} \times \frac{T_{imp3,1}}{\Delta t}}^{\left(\frac{(RCD_3 - \mu_p)}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt +$$

$$\sum_{Z=0}^{N_{imp3,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_3)-\mu_P}{\beta h} \right) \right\rfloor}^{\left\lfloor (Z+1) \times \frac{T_{imp3,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=N_{imp3,2} \times \frac{T_{imp3,2}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \quad (4.36)$$

$$T_{imp3,1} = \left\lfloor \frac{\left(\frac{(RCD_3)-\mu_P}{\beta h} \right)}{N_{imp3,1}} \right\rfloor, \text{ and } T_{imp3,2} = \left\lfloor \frac{H - \left(\frac{(RCD_3)}{\beta h} \right)}{N_{imp3,2}} \right\rfloor$$

Case 2.3: the presence of seven consecutive points falling on the same side (scenario 4)

$$C_{M2.3} = \alpha_4 \times \left(C_{pm4} + C_{imp} \times (N_{imp4,1} - 1) + C_{imp} \times (N_{imp4,1} - 1) + C_{cm} \times \varphi_{S4}(U, N_{imp4,1}, N_{imp4,2}) \right) \quad (4.37)$$

$$\begin{aligned} \varphi_{S4}(U, N_{imp4,1}, N_{imp4,2}) &= \sum_{Z=0}^{N_{imp4,1}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_4)-\mu_P}{\beta h} \right) \right\rfloor}^{\left\lfloor (Z+1) \times \frac{T_{imp4,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=N_{imp4,1} \times \frac{T_{imp4,1}}{\Delta t}}^{\left(\frac{(RCD_4)-\mu_P}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \\ &\sum_{Z=0}^{N_{imp4,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_4)}{\beta h} \right) \right\rfloor}^{\left\lfloor (Z+1) \times \frac{T_{imp4,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=N_{imp4,2} \times \frac{T_{imp4,2}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \\ T_{imp4,1} &= \left\lfloor \frac{\left(\frac{(RCD_4)-\mu_P}{\beta h} \right)}{N_{imp4,1}} \right\rfloor, \text{ and } T_{imp4,2} = \left\lfloor \frac{H - \left(\frac{(RCD_4)}{\beta h} \right)}{N_{imp4,2}} \right\rfloor \end{aligned} \quad (4.38)$$

Case 2.4: the occurrence of two scenarios at the same time

▪ **Case 2.4.1: the presence of scenarios 2 and 3**

- Case 2.4.1.1: the presence of S2 before S3

$$C_{M2.4.1.1} = \alpha_{23} \times \left(C_{pm2} + C_{pm3} + C_{imp} \times (N_{imp23,1} - 1) + C_{imp} \times (N_{imp23,2} - 1) + C_{imp} \times (N_{imp23,3} - 1) + C_{cm} \times \varphi_{S23}(U, N_{imp23,1}, N_{imp23,2}, N_{imp23,3}) \right) \quad (4.39)$$

$$\begin{aligned} \varphi_{S23}(U, N_{imp23,1}, N_{imp23,2}, N_{imp23,3}) &= \sum_{Z=0}^{N_{imp23,1}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_2)-\mu_P}{\beta h} \right) \right\rfloor}^{\left\lfloor (Z+1) \times \frac{T_{imp23,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp23,1} \times \frac{T_{imp23,1}}{\Delta t}}^{\left(\frac{(RCD_2)-\mu_P}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp23,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_2)}{\beta h} \right) \right\rfloor}^{\left\lfloor (Z+1) \times \frac{T_{imp23,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp23,2} \times \frac{T_{imp23,2}}{\Delta t}}^{\left(\frac{(RCD_3)-\mu_P}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp23,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_3)}{\beta h} \right) \right\rfloor}^{\left\lfloor (Z+1) \times \frac{T_{imp23,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp23,3} \times \frac{T_{imp23,3}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \end{aligned} \quad (4.40)$$

$$T_{imp23,1} = \left\lfloor \frac{\left(\frac{RCD_2 - \mu p}{\beta h}\right)}{N_{imp23,1}} \right\rfloor, T_{imp23,2} = \left\lfloor \frac{\left(\frac{RCD_3 - \mu p}{\beta h}\right) - \left(\frac{RCD_2}{\beta h}\right)}{N_{imp23,2}} \right\rfloor, \text{ and } T_{imp23,3} = \left\lfloor \frac{H - \left(\frac{RCD_3}{\beta h}\right)}{N_{imp23,3}} \right\rfloor$$

- Case 2.4.1.2: the presence of S3 before S2

$$C_{M2.4.1.2} = \alpha_{32} \times \left(C_{pm2} + C_{pm3} + C_{imp} \times (N_{imp32,1} - 1) + C_{imp} \times (N_{imp32,2} - 1) + C_{imp} \times (N_{imp32,3} - 1) + C_{cm} \times \varphi_{S32}(U, N_{imp32,1}, N_{imp32,2}, N_{imp32,3}) \right) \quad (4.41)$$

$$\begin{aligned} \varphi_{S32}(U, N_{imp32,1}, N_{imp32,2}, N_{imp32,3}) &= \sum_{Z=0}^{N_{imp32,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp32,1}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp32,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp32,1} \times \frac{T_{imp32,1}}{\Delta t}}^{\left(\frac{RCD_3 - \mu p}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp32,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_3}{\beta h}\right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp32,2}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp32,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp32,2} \times \frac{T_{imp32,2}}{\Delta t}}^{\left(\frac{RCD_2 - \mu p}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp32,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_2}{\beta h}\right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp32,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp32,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp32,3} \times \frac{T_{imp32,3}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \end{aligned} \quad (4.42)$$

$$T_{imp32,1} = \left\lfloor \frac{\left(\frac{RCD_3 - \mu p}{\beta h}\right)}{N_{imp32,1}} \right\rfloor, T_{imp32,2} = \left\lfloor \frac{\left(\frac{RCD_2 - \mu p}{\beta h}\right) - \left(\frac{RCD_3}{\beta h}\right)}{N_{imp32,2}} \right\rfloor, \text{ and } T_{imp32,3} = \left\lfloor \frac{H - \left(\frac{RCD_2}{\beta h}\right)}{N_{imp32,3}} \right\rfloor$$

▪ Case 2.4.2: scenario 2 and 4

- Case 2.4.2.1: the presence of S2 before S4

$$C_{M2.4.2.1} = \alpha_{24} \times \left(C_{pm2} + C_{pm4} + C_{imp} \times (N_{imp24,1} - 1) + C_{imp} \times (N_{imp24,2} - 1) + C_{imp} \times (N_{imp24,3} - 1) + C_{cm} \times \varphi_{S24}(U, N_{imp24,1}, N_{imp24,2}, N_{imp24,3}) \right) \quad (4.43)$$

$$\begin{aligned} \varphi_{S24}(U, N_{imp24,1}, N_{imp24,2}, N_{imp24,3}) &= \sum_{Z=0}^{N_{imp24,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp24,1}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp24,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp24,1} \times \frac{T_{imp24,1}}{\Delta t}}^{\left(\frac{RCD_2 - \mu p}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp24,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_2}{\beta h}\right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp24,2}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp24,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp24,2} \times \frac{T_{imp24,2}}{\Delta t}}^{\left(\frac{RCD_4 - \mu p}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp24,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_4}{\beta h}\right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp24,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp24,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp24,3} \times \frac{T_{imp24,3}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \end{aligned} \quad (4.44)$$

$$T_{imp24,1} = \left\lfloor \frac{\left(\frac{RCD_2 - \mu p}{\beta h}\right)}{N_{imp24,1}} \right\rfloor, T_{imp24,2} = \left\lfloor \frac{\left(\frac{RCD_4 - \mu p}{\beta h}\right) - \left(\frac{RCD_2}{\beta h}\right)}{N_{imp24,2}} \right\rfloor, \text{ and } T_{imp24,3} = \left\lfloor \frac{H - \left(\frac{RCD_4}{\beta h}\right)}{N_{imp24,3}} \right\rfloor$$

- Case 2.4.2.2: the presence of S4 before S2

$$C_{M2.4.2.2} = \alpha_{42} \times \left(C_{pm2} + C_{pm4} + C_{imp} \times (N_{imp42,1} - 1) + C_{imp} \times (N_{imp42,2} - 1) + C_{imp} \times (N_{imp42,3} - 1) + C_{cm} \times \varphi_{S42}(U, N_{imp42,1}, N_{imp42,2}, N_{imp42,3}) \right) \quad (4.45)$$

$$\begin{aligned} \varphi_{S42}(U, N_{imp42,1}, N_{imp42,2}, N_{imp42,3}) &= \sum_{Z=0}^{N_{imp42,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp42,1}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp42,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp42,1} \times \frac{T_{imp42,1}}{\Delta t}}^{\left(\frac{(RCD_4 - \mu p)}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp42,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_4)}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp42,2}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp42,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp42,2} \times \frac{T_{imp42,2}}{\Delta t}}^{\left(\frac{(RCD_2 - \mu p)}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp42,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_2)}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp42,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp42,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp42,3} \times \frac{T_{imp42,3}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \end{aligned} \quad (4.46)$$

$$T_{imp42,1} = \left\lfloor \frac{\left(\frac{(RCD_4 - \mu p)}{\beta h} \right)}{N_{imp42,1}} \right\rfloor, T_{imp42,2} = \left\lfloor \frac{\left(\frac{(RCD_2 - \mu p)}{\beta h} \right) - \left(\frac{(RCD_4)}{\beta h} \right)}{N_{imp42,2}} \right\rfloor, \text{ and } T_{imp42,3} = \left\lfloor \frac{H - \left(\frac{(RCD_2)}{\beta h} \right)}{N_{imp42,3}} \right\rfloor.$$

▪ Case 2.4.3: scenario 3 and 4

- Case 2.4.3.1: the presence of S3 before S4

$$C_{M2.4.3.1} = \alpha_{34} \times \left(C_{pm3} + C_{pm4} + C_{imp} \times (N_{imp34,1} - 1) + C_{imp} \times (N_{imp34,2} - 1) + C_{imp} \times (N_{imp34,3} - 1) + C_{cm} \times \varphi_{S34}(U, N_{imp34,1}, N_{imp34,2}, N_{imp34,3}) \right) \quad (4.47)$$

$$\begin{aligned} \varphi_{S34}(U, N_{imp34,1}, N_{imp34,2}, N_{imp34,3}) &= \sum_{Z=0}^{N_{imp34,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp34,1}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp34,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp34,1} \times \frac{T_{imp34,1}}{\Delta t}}^{\left(\frac{(RCD_3 - \mu p)}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp34,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_3)}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp34,2}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp34,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp34,2} \times \frac{T_{imp34,2}}{\Delta t}}^{\left(\frac{(RCD_4 - \mu p)}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp34,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_4)}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp34,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp34,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp34,3} \times \frac{T_{imp34,3}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \end{aligned} \quad (4.48)$$

$$T_{imp34,1} = \left\lfloor \frac{\left(\frac{(RCD_3 - \mu p)}{\beta h} \right)}{N_{imp34,1}} \right\rfloor, T_{imp34,2} = \left\lfloor \frac{\left(\frac{(RCD_4 - \mu p)}{\beta h} \right) - \left(\frac{(RCD_3)}{\beta h} \right)}{N_{imp34,2}} \right\rfloor, \text{ and } T_{imp34,3} = \left\lfloor \frac{H - \left(\frac{(RCD_4)}{\beta h} \right)}{N_{imp34,3}} \right\rfloor$$

- Case 2.4.3.2: the presence of S4 before S3

$$C_{M2.4.3.2} = \alpha_{43} \times \left(C_{pm3} + C_{pm4} + C_{imp} \times (N_{imp43,1} - 1) + C_{imp} \times (N_{imp43,2} - 1) + C_{imp} \times (N_{imp43,3} - 1) + C_{cm} \times \varphi_{S43}(U, N_{imp43,1}, N_{imp43,2}, N_{imp43,3}) \right)$$

$$1) + C_{cm} \times \varphi_{S43}(U, N_{imp43,1}, N_{imp43,2}, N_{imp43,3})) \quad (4.49)$$

$$\begin{aligned} \varphi_{S43}(U, N_{imp43,1}, N_{imp43,2}, N_{imp43,3}) = & \sum_{Z=0}^{N_{imp43,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp43,1}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp43,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ & \sum_{i=N_{imp43,1} \times \frac{T_{imp43,1}}{\Delta t}}^{\left(\frac{(RCD_4 - \mu p)}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp43,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_4)}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp43,2}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp43,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ & \sum_{i=N_{imp43,2} \times \frac{T_{imp43,2}}{\Delta t}}^{\left(\frac{(RCD_3 - \mu p)}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp43,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_3)}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp43,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp43,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ & \sum_{i=N_{imp43,3} \times \frac{T_{imp43,3}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \end{aligned} \quad (4.50)$$

$$T_{imp43,1} = \left\lfloor \frac{\left(\frac{(RCD_4 - \mu p)}{\beta h} \right)}{N_{imp43,1}} \right\rfloor, \quad T_{imp43,2} = \left\lfloor \frac{\left(\frac{(RCD_3 - \mu p)}{\beta h} \right) - \left(\frac{(RCD_4)}{\beta h} \right)}{N_{imp43,2}} \right\rfloor, \text{ and } T_{imp43,3} = \left\lfloor \frac{H - \left(\frac{(RCD_3)}{\beta h} \right)}{N_{imp43,3}} \right\rfloor$$

Case 2.5: The occurrence of three scenarios at the same time

▪ Case 2.5.1: S2 before S3 and S4

- Case 2.5.1.1: the presence of S2 before S3 before S4

$$\begin{aligned} C_{M2.5.1.1} = & \alpha_{234} \times \left(C_{pm2} + C_{pm3} + C_{pm4} + C_{imp} \times (N_{imp234,1} - 1) + C_{imp} \times (N_{imp234,2} - 1) + C_{imp} \times \right. \\ & \left. (N_{imp234,3} - 1) + C_{imp} \times (N_{imp234,4} - 1) + C_{cm} \times \varphi_{S234}(U, N_{imp234,1}, N_{imp234,2}, N_{imp234,3}, N_{imp234,4}) \right) \end{aligned} \quad (4.51)$$

$$\begin{aligned} \varphi_{S234}(U, N_{imp234,1}, N_{imp234,2}, N_{imp234,3}, N_{imp234,4}) = & \sum_{Z=0}^{N_{imp234,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp234,1}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp234,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ & \sum_{i=N_{imp234,1} \times \frac{T_{imp234,1}}{\Delta t}}^{\left(\frac{(RCD_2 - \mu p)}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp234,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_2)}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp234,2}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp234,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ & \sum_{i=N_{imp234,2} \times \frac{T_{imp234,2}}{\Delta t}}^{\left(\frac{(RCD_3 - \mu p)}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp234,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_3)}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp234,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp234,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ & \sum_{i=N_{imp234,3} \times \frac{T_{imp234,3}}{\Delta t}}^{\left(\frac{(RCD_4 - \mu p)}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp234,4}-1} \left[\sum_{i=\left\lfloor \left(\frac{(RCD_4)}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp234,4}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp234,4}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ & \sum_{i=N_{imp234,4} \times \frac{T_{imp234,4}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \end{aligned} \quad (4.52)$$

$$T_{imp234,1} = \left\lfloor \frac{\left(\frac{(RCD_2 - \mu p)}{\beta h} \right)}{N_{imp234,1}} \right\rfloor, \quad T_{imp234,2} = \left\lfloor \frac{\left(\frac{(RCD_3 - \mu p)}{\beta h} \right) - \left(\frac{(RCD_2)}{\beta h} \right)}{N_{imp234,2}} \right\rfloor, \quad T_{imp234,3} = \left\lfloor \frac{\left(\frac{(RCD_4 - \mu p)}{\beta h} \right) - \left(\frac{(RCD_3)}{\beta h} \right)}{N_{imp234,3}} \right\rfloor,$$

$$\text{and } T_{imp234,4} = \left\lfloor \frac{H - \left(\frac{RCD_4}{\beta h}\right)}{N_{imp234,4}} \right\rfloor$$

- Case 2.5.1.2: the presence of S2 before S4 before S3

$$C_{M2.5.1.2} = \alpha_{243} \times \left(C_{pm2} + C_{pm3} + C_{pm4} + C_{imp} \times (N_{imp243,1} - 1) + C_{imp} \times (N_{imp243,2} - 1) + C_{imp} \times (N_{imp243,3} - 1) + C_{imp} \times (N_{imp243,4} - 1) + C_{cm} \times \varphi_{S243}(U, N_{imp243,1}, N_{imp243,2}, N_{imp243,3}, N_{imp243,4}) \right) \quad (4.53)$$

$$\begin{aligned} \varphi_{S243}(U, N_{imp243,1}, N_{imp243,2}, N_{imp243,3}, N_{imp243,4}) &= \sum_{Z=0}^{N_{imp243,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp243,1}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp243,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp243,1} \times \frac{T_{imp243,1}}{\Delta t}}^{\left(\frac{RCD_2 - \mu p}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp243,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_2}{\beta h}\right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp243,2}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp243,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp243,2} \times \frac{T_{imp243,2}}{\Delta t}}^{\left(\frac{RCD_4 - \mu p}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp243,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_4}{\beta h}\right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp243,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp243,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp243,3} \times \frac{T_{imp243,3}}{\Delta t}}^{\left(\frac{RCD_3 - \mu p}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp243,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_3}{\beta h}\right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp243,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp243,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp243,3} \times \frac{T_{imp243,3}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \end{aligned} \quad (4.54)$$

$$T_{imp243,1} = \left\lfloor \frac{\left(\frac{RCD_2 - \mu p}{\beta h}\right)}{N_{imp243,1}} \right\rfloor, T_{imp243,2} = \left\lfloor \frac{\left(\frac{RCD_4 - \mu p}{\beta h}\right) - \left(\frac{RCD_2}{\beta h}\right)}{N_{imp243,2}} \right\rfloor, T_{imp243,3} = \left\lfloor \frac{\left(\frac{RCD_3 - \mu p}{\beta h}\right) - \left(\frac{RCD_4}{\beta h}\right)}{N_{imp243,3}} \right\rfloor,$$

$$\text{and } T_{imp243,4} = \left\lfloor \frac{H - \left(\frac{RCD_4}{\beta h}\right)}{N_{imp243,4}} \right\rfloor$$

▪ Case 2.5.2: S3 before S2 and S4

- Case 2.5.2.1: the presence of S3 before S2 before S4

$$C_{M2.5.2.1} = \alpha_{324} \times \left(C_{pm2} + C_{pm3} + C_{pm4} + C_{imp} \times (N_{imp324,1} - 1) + C_{imp} \times (N_{imp324,2} - 1) + C_{imp} \times (N_{imp324,3} - 1) + C_{imp} \times (N_{imp324,4} - 1) + C_{cm} \times \varphi_{S324}(U, N_{imp324,1}, N_{imp324,2}, N_{imp324,3}, N_{imp324,4}) \right) \quad (4.55)$$

$$\begin{aligned} \varphi_{S324}(U, N_{imp324,1}, N_{imp324,2}, N_{imp324,3}, N_{imp324,4}) &= \sum_{Z=0}^{N_{imp324,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp324,1}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp324,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp324,1} \times \frac{T_{imp324,1}}{\Delta t}}^{\left(\frac{RCD_3 - \mu p}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp324,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_3}{\beta h}\right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp324,2}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp324,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\ &\sum_{i=N_{imp324,2} \times \frac{T_{imp324,2}}{\Delta t}}^{\left(\frac{RCD_2 - \mu p}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp324,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_2}{\beta h}\right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp324,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp324,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \end{aligned}$$

$$\sum_{i=N_{imp324,3} \times \frac{T_{imp324,3}}{\Delta t}}^{\left(\frac{(RCD_4-\mu p)}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp324,3}-1} \left[\sum_{i=\left\lfloor \frac{(RCD_4)}{\beta h} \right\rfloor + \left\lfloor Z \times \frac{T_{imp324,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp324,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=N_{imp324,3} \times \frac{T_{imp324,3}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \quad (4.56)$$

$$T_{imp324,1} = \left\lfloor \frac{\left(\frac{(RCD_3-\mu p)}{\beta h}\right)}{N_{imp324,1}} \right\rfloor, T_{imp324,2} = \left\lfloor \frac{\left(\frac{(RCD_2-\mu p)}{\beta h}\right) - \left(\frac{(RCD_3)}{\beta h}\right)}{N_{imp324,2}} \right\rfloor, T_{imp324,3} = \left\lfloor \frac{\left(\frac{(RCD_4-\mu p)}{\beta h}\right) - \left(\frac{(RCD_2)}{\beta h}\right)}{N_{imp324,3}} \right\rfloor,$$

and $T_{imp324,4} = \left\lfloor \frac{H - \left(\frac{(RCD_4)}{\beta h}\right)}{N_{imp324,4}} \right\rfloor$

- Case 2.5.2.2: the presence of S3 before S4 before S2

$$C_{M2.5.2.2} = \alpha_{342} \times \left(C_{pm2} + C_{pm3} + C_{pm4} + C_{imp} \times (N_{imp342,1} - 1) + C_{imp} \times (N_{imp342,2} - 1) + C_{imp} \times (N_{imp342,3} - 1) + C_{imp} \times (N_{imp342,4} - 1) + C_{cm} \times \varphi_{S342}(U, N_{imp342,1}, N_{imp342,2}, N_{imp342,3}, N_{imp342,4}) \right) \quad (4.57)$$

$$\varphi_{S342}(U, N_{imp342,1}, N_{imp342,2}, N_{imp342,3}, N_{imp342,4}) = \sum_{Z=0}^{N_{imp342,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp342,1}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp342,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=N_{imp342,1} \times \frac{T_{imp342,1}}{\Delta t}}^{\left(\frac{(RCD_3-\mu p)}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp342,2}-1} \left[\sum_{i=\left\lfloor \frac{(RCD_3)}{\beta h} \right\rfloor + \left\lfloor Z \times \frac{T_{imp342,2}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp342,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=N_{imp342,2} \times \frac{T_{imp342,2}}{\Delta t}}^{\left(\frac{(RCD_4-\mu p)}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp342,3}-1} \left[\sum_{i=\left\lfloor \frac{(RCD_4)}{\beta h} \right\rfloor + \left\lfloor Z \times \frac{T_{imp342,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp342,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=N_{imp342,3} \times \frac{T_{imp342,3}}{\Delta t}}^{\left(\frac{(RCD_2-\mu p)}{\beta h}\right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp342,3}-1} \left[\sum_{i=\left\lfloor \frac{(RCD_2)}{\beta h} \right\rfloor + \left\lfloor Z \times \frac{T_{imp342,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp342,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \sum_{i=N_{imp342,3} \times \frac{T_{imp342,3}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt \quad (4.58)$$

$$T_{imp342,1} = \left\lfloor \frac{\left(\frac{(RCD_3-\mu p)}{\beta h}\right)}{N_{imp342,1}} \right\rfloor, T_{imp342,2} = \left\lfloor \frac{\left(\frac{(RCD_4-\mu p)}{\beta h}\right) - \left(\frac{(RCD_3)}{\beta h}\right)}{N_{imp342,2}} \right\rfloor, T_{imp342,3} = \left\lfloor \frac{\left(\frac{(RCD_2-\mu p)}{\beta h}\right) - \left(\frac{(RCD_4)}{\beta h}\right)}{N_{imp342,3}} \right\rfloor,$$

And $T_{imp342,4} = \left\lfloor \frac{H - \left(\frac{(RCD_2)}{\beta h}\right)}{N_{imp342,4}} \right\rfloor$

▪ Case 2.5.3: S4 before S3 and S2

- Case 2.5.3.1: the presence of S4 before S3 before S2

$$C_{M2.5.3.1} = \alpha_{432} \times \left(C_{pm2} + C_{pm3} + C_{pm4} + C_{imp} \times (N_{imp432,1} - 1) + C_{imp} \times (N_{imp432,2} - 1) + C_{imp} \times (N_{imp432,3} - 1) + C_{imp} \times (N_{imp432,4} - 1) + C_{cm} \times \varphi_{S432}(U, N_{imp432,1}, N_{imp432,2}, N_{imp432,3}, N_{imp432,4}) \right) \quad (4.59)$$

$$\begin{aligned}
\varphi_{S432}(U, N_{imp432,1}, N_{imp432,2}, N_{imp432,3}, N_{imp432,4}) = & \sum_{Z=0}^{N_{imp432,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp432,1}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp432,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\
& \sum_{i=N_{imp432,1} \times \frac{T_{imp432,1}}{\Delta t}}^{\left(\frac{RCD_4 - \mu p}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp432,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_4}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp432,2}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp432,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\
& \sum_{i=N_{imp432,2} \times \frac{T_{imp432,2}}{\Delta t}}^{\left(\frac{RCD_3 - \mu p}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp432,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_3}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp432,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp432,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\
& \sum_{i=N_{imp432,3} \times \frac{T_{imp432,3}}{\Delta t}}^{\left(\frac{RCD_2 - \mu p}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp432,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_2}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp432,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp432,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\
& \sum_{i=N_{imp432,3} \times \frac{T_{imp432,3}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt
\end{aligned} \tag{4.60}$$

$$\begin{aligned}
T_{imp432,1} &= \left\lfloor \frac{\left(\frac{RCD_4 - \mu p}{\beta h} \right)}{N_{imp432,1}} \right\rfloor, T_{imp432,2} = \left\lfloor \frac{\left(\frac{RCD_3 - \mu p}{\beta h} \right) - \left(\frac{RCD_4}{\beta h} \right)}{N_{imp432,2}} \right\rfloor, T_{imp432,2} = \left\lfloor \frac{\left(\frac{RCD_2 - \mu p}{\beta h} \right) - \left(\frac{RCD_3}{\beta h} \right)}{N_{imp432,3}} \right\rfloor, \\
\text{and } T_{imp432,4} &= \left\lfloor \frac{H - \left(\frac{RCD_2}{\beta h} \right)}{N_{imp432,4}} \right\rfloor
\end{aligned}$$

- Case 2.5.3.2: the presence of S4 before S2 before S3

$$\begin{aligned}
C_{M2.5.3.2} = & \alpha_{423} \times \left(C_{pm2} + C_{pm3} + C_{pm4} + C_{imp} \times (N_{imp423,1} - 1) + C_{imp} \times (N_{imp423,2} - 1) + C_{imp} \times \right. \\
& \left. (N_{imp423,3} - 1) + C_{imp} \times (N_{imp423,4} - 1) + C_{cm} \times \varphi_{S423}(U, N_{imp423,1}, N_{imp423,2}, N_{imp423,3}, N_{imp423,4}) \right)
\end{aligned} \tag{4.61}$$

$$\begin{aligned}
\varphi_{S423}(U, N_{imp423,1}, N_{imp423,2}, N_{imp423,3}, N_{imp423,4}) = & \sum_{Z=0}^{N_{imp423,1}-1} \left[\sum_{i=\left\lfloor Z \times \frac{T_{imp423,1}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp423,1}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\
& \sum_{i=N_{imp423,1} \times \frac{T_{imp423,1}}{\Delta t}}^{\left(\frac{RCD_4 - \mu p}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp423,2}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_4}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp423,2}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp423,2}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\
& \sum_{i=N_{imp423,2} \times \frac{T_{imp423,2}}{\Delta t}}^{\left(\frac{RCD_2 - \mu p}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp423,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_2}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp423,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp423,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\
& \sum_{i=N_{imp423,3} \times \frac{T_{imp423,3}}{\Delta t}}^{\left(\frac{RCD_3 - \mu p}{\beta h} \right)} \int_0^{\Delta t} \lambda_i(t) dt + \sum_{Z=0}^{N_{imp423,3}-1} \left[\sum_{i=\left\lfloor \left(\frac{RCD_3}{\beta h} \right) \right\rfloor + \left\lfloor Z \times \frac{T_{imp423,3}}{\Delta t} \right\rfloor + 1}^{\left\lfloor (Z+1) \times \frac{T_{imp423,3}}{\Delta t} \right\rfloor} \int_0^{\Delta t} \lambda_i(t) dt \right] + \\
& \sum_{i=N_{imp423,3} \times \frac{T_{imp423,3}}{\Delta t}}^H \int_0^{\Delta t} \lambda_i(t) dt
\end{aligned} \tag{4.62}$$

$$\begin{aligned}
T_{imp423,1} &= \left\lfloor \frac{\left(\frac{RCD_4 - \mu p}{\beta h} \right)}{N_{imp423,1}} \right\rfloor, T_{imp423,2} = \left\lfloor \frac{\left(\frac{RCD_2 - \mu p}{\beta h} \right) - \left(\frac{RCD_4}{\beta h} \right)}{N_{imp423,2}} \right\rfloor, T_{imp423,2} = \left\lfloor \frac{\left(\frac{RCD_3 - \mu p}{\beta h} \right) - \left(\frac{RCD_2}{\beta h} \right)}{N_{imp423,3}} \right\rfloor, \\
\text{and } T_{imp423,4} &= \left\lfloor \frac{H - \left(\frac{RCD_3}{\beta h} \right)}{N_{imp423,4}} \right\rfloor
\end{aligned}$$

Hence, the maintenance cost of the production process is formulated as follows:

$$C_{MT} = \alpha_1 \times C_{M1} + (1 - \alpha_1) \times \left(Pr(S_2 \cup S_3 \cup S_4) \times (C_{M2.1} + C_{M2.2} + C_{M2.3}) + (1 - Pr(S_2 \cup S_3 \cup S_4)) \times \left((Pr((S_2 \cap S_3) \cup (S_2 \cap S_4) \cup (S_3 \cap S_4))) \times (C_{M2.4.1.1} + C_{M2.4.1.2} + C_{M2.4.2.1} + C_{M2.4.2.2} + C_{M2.4.3.1} + C_{M2.4.3.2}) + Pr(S_2 \cap S_3 \cap S_4) \times (C_{M2.5.1.1} + C_{M2.5.1.2} + C_{M2.5.2.1} + C_{M2.5.2.2} + C_{M2.5.3.1} + C_{M2.5.3.2})) \right) \right) \quad (4.63)$$

4.3.4 Total cost of quality

The total cost of quality is characterized by the cost of false alarms, the cost of sampling, and the cost of the non-conforming products for each scenario.

4.3.4.1 Cost of false alarms

A false alarm is a situation where the control limits on the control chart are exceeded while the process is in fact "in control".

Since $\alpha_2 = P_j(\delta_j)$ the probability of non-detection of the shift to the control limits, if $\delta_j = 0$, there is no deregulation on the average, so the central value of the control chart remains constant:

$$\mu_1 = \mu + \delta_j \cdot \sigma = \mu \quad (4.64)$$

In this case, the probability (1- P (0)) is, so, the probability of whether the average of sampling taken is above UCL or below LCL.

So it's the probability to trigger a false alarm can be expressed as follows:

$$\alpha_5 = \{\text{Prob}(\bar{x}_s \leq LCL \mid \mu_1 = \mu) + \text{Prob}(\bar{x}_s \geq UCL \mid \mu_1 = \mu)\} \quad (4.65)$$

Thus,

$$\begin{aligned} \alpha_5 &= F\left(\frac{LCL - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right) + 1 - F\left(\frac{UCL - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right), \\ \alpha_5 &= 1 + F\left(\frac{\mu_1 - k_p \times \frac{\sigma}{\sqrt{n}} - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right) - F\left(\frac{\mu_1 + k_p \times \frac{\sigma}{\sqrt{n}} - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right) \\ \alpha_5 &= 1 + F(-k_p) + 1 - F(k_p), \quad \alpha_5 = 2 \times F(-k_p), \quad \alpha_5 = 1 - \alpha_2(0) \end{aligned}$$

Thus, the average number of samples $E(S)$ taken between two false alarms is given by:

$$E(S) = \sum_{i=0}^{H \cdot \beta} \{i \times \alpha_1^i \times (1 - \alpha_1)\} \quad (4.66)$$

With $H \cdot \beta$: number of inspections

Thereafter:

The average number of false alarms $E(f)$ during the finite horizon $H \cdot \Delta t$ (Ben Daya and Rahim, 2000), can be expressed as follows:

$$E(f) = \alpha_5 \times E(S) \quad (4.67)$$

So, the average cost of false alarms is given by:

$$C_{FA} = C_F \times E(f) = 2 \times C_F \times F(-k_p) \times \sum_{i=0}^{H \cdot \beta} \{i \times \alpha_1^i \times (1 - \alpha_1)\} \quad (4.68)$$

4.3.4.2 cost of sampling

The average total cost of sampling is expressed by:

$$C_{sampling} = C_s \times n \times m \quad (4.69)$$

4.3.4.3 Cost of non-confirming (defective) products

The quantity of non-confirming products rejected due to the shift to the critical state is equal to the product quantity during the last sampling interval. So, for scenario 2, the number of defective items rejected at the shift to the control state is equal to the quantity produced during the production period of the last sampling interval before the shift to the upper or lower control chart limits. For scenarios 3 and 4, the number of defective items rejected is equal to the quantity produced during the last sampling interval (j+7)th.

Thus, the total cost of non-confirming or defectives products depends on the different scenarios and is expressed as follows:

$$\begin{aligned} C_{NC} = C_r \times & \left[(1 - \alpha_1) \times \left(Pr(S_2 \cup S_3 \cup S_4) \times \left(\alpha_2 \times \left(u\left(\left\lfloor \frac{RCD_2}{\beta h} \right\rfloor\right) \times \left(\frac{RCD_2 - I_s - \mu_P}{\beta h}\right)\right) + \alpha_3 \times \left(u\left(\left\lfloor \frac{RCD_3}{\beta h} \right\rfloor\right) \times \right. \right. \right. \\ & \left. \left. \left(\frac{RCD_3 - I_s - \mu_P}{\beta h}\right)\right) + \alpha_4 \times \left(u\left(\left\lfloor \frac{RCD_4}{\beta h} \right\rfloor\right) \times \left(\frac{RCD_4 - I_s - \mu_P}{\beta h}\right)\right) \right) \right) + (1 - Pr(S_2 \cup S_3 \cup S_4)) \times \left(\left(Pr((S_2 \cap S_3) \cup \right. \right. \\ & (S_2 \cap S_4) \cup (S_3 \cap S_4)) \times \left((\alpha_{23} + \alpha_{32}) \times \left(\left(u\left(\left\lfloor \frac{RCD_2}{\beta h} \right\rfloor\right) \times \left(\frac{RCD_2 - I_s - \mu_P}{\beta h}\right)\right) + \left(u\left(\left\lfloor \frac{RCD_3}{\beta h} \right\rfloor\right) \times \left(\frac{RCD_3 - I_s - \mu_P}{\beta h}\right)\right) \right) \right) + \right. \\ & (\alpha_{24} + \alpha_{42}) \times \left(\left(u\left(\left\lfloor \frac{RCD_2}{\beta h} \right\rfloor\right) \times \left(\frac{RCD_2 - I_s - \mu_P}{\beta h}\right)\right) + \left(u\left(\left\lfloor \frac{RCD_4}{\beta h} \right\rfloor\right) \times \left(\frac{RCD_4 - I_s - \mu_P}{\beta h}\right)\right) \right) + (\alpha_{34} + \alpha_{43}) \times \\ & \left. \left(\left(u\left(\left\lfloor \frac{RCD_3}{\beta h} \right\rfloor\right) \times \left(\frac{RCD_3 - I_s - \mu_P}{\beta h}\right)\right) + \left(u\left(\left\lfloor \frac{RCD_4}{\beta h} \right\rfloor\right) \times \left(\frac{RCD_4 - I_s - \mu_P}{\beta h}\right)\right) \right) \right) + Pr(S_2 \cap S_3 \cap S_4) \times \left((\alpha_{234} + \alpha_{243} + \right. \\ & \alpha_{324} + \alpha_{342} + \alpha_{423} + \alpha_{432}) \times \left(\left(u\left(\left\lfloor \frac{RCD_3}{\beta h} \right\rfloor\right) \times \left(\frac{RCD_3 - I_s - \mu_P}{\beta h}\right)\right) + \left(u\left(\left\lfloor \frac{RCD_4}{\beta h} \right\rfloor\right) \times \left(\frac{RCD_4 - I_s - \mu_P}{\beta h}\right)\right) + \right. \\ & \left. \left. \left(u\left(\left\lfloor \frac{RCD_2}{\beta h} \right\rfloor\right) \times \left(\frac{RCD_2 - I_s - \mu_P}{\beta h}\right)\right) \right) \right) \right) \right) \right] \quad (4.70) \end{aligned}$$

Therefore,

4.3.4.4 The total cost of quality

The total cost of quality consists of the cost of sampling inspection, the non-conformal cost, and the cost of false alarms. The total cost of quality is given by:

$$TCQ = C_{FA} + C_{NC} + C_{sampling} \quad (4.71)$$

So, the total cost of maintenance and quality is given as follows,

$$C_{tot_{M/Q}} = C_{MT} + TCQ \quad (4.72)$$

4.4 The Optimization approach

Taking into consideration all the possible scenarios, cases, and sub-cases associated with the problem, the specific failure rates, the respective probabilities of occurrence as well as the order of happenings. An iterative numerical procedure has been developed to minimize the total costs given by equation (4.72). Based on the determination of the optimal control chart parameters ($n^* h^* k_c^*$), and the corresponding number of preventive maintenance imperfect to be carried out under every distinct case of all the production system scenarios. The Figure 4.8 below demonstrates the algorithm diagram-solving procedure in which an iterative numerical optimization procedure is developed and coded. The obtained results are presented and discussed in section 5 with arbitrarily chosen input data. A stochastic dynamic programming approach is used to develop the optimization solution.

- ✓ The multi-decision variables (n, h, k_p) are initialized with small values by the user.
- ✓ The decision parameters are variated with some increments ($\Delta n, \Delta h, \Delta k_p$) to some defined maximum limits ($n_{max}, h_{max}, k_{p_{max}}$)
- ✓ The probability that the process in scenarios 1, 2, 3, & 4 ($\alpha_1, \alpha_2, \alpha_3, \& \alpha_4$) are defined by equations 4.16, 4.18, 4.24, & 4.27 respectively.
- ✓ The average run length ($ARL_1, ARL_2, ARL_3, \& ARL_4$) defined for each run is calculated by equations 4.17, 4.19, 4.25, & 4.28 respectively.
- ✓ The average restoration of cycle duration (RCD_j) is calculated by equations 4.23, 4.26, & 4.29.
- ✓ The total maintenance cost of the production process is formulated by equation (4.63)
- ✓ The total cost of maintenance and quality is calculated using the equation (4.72)

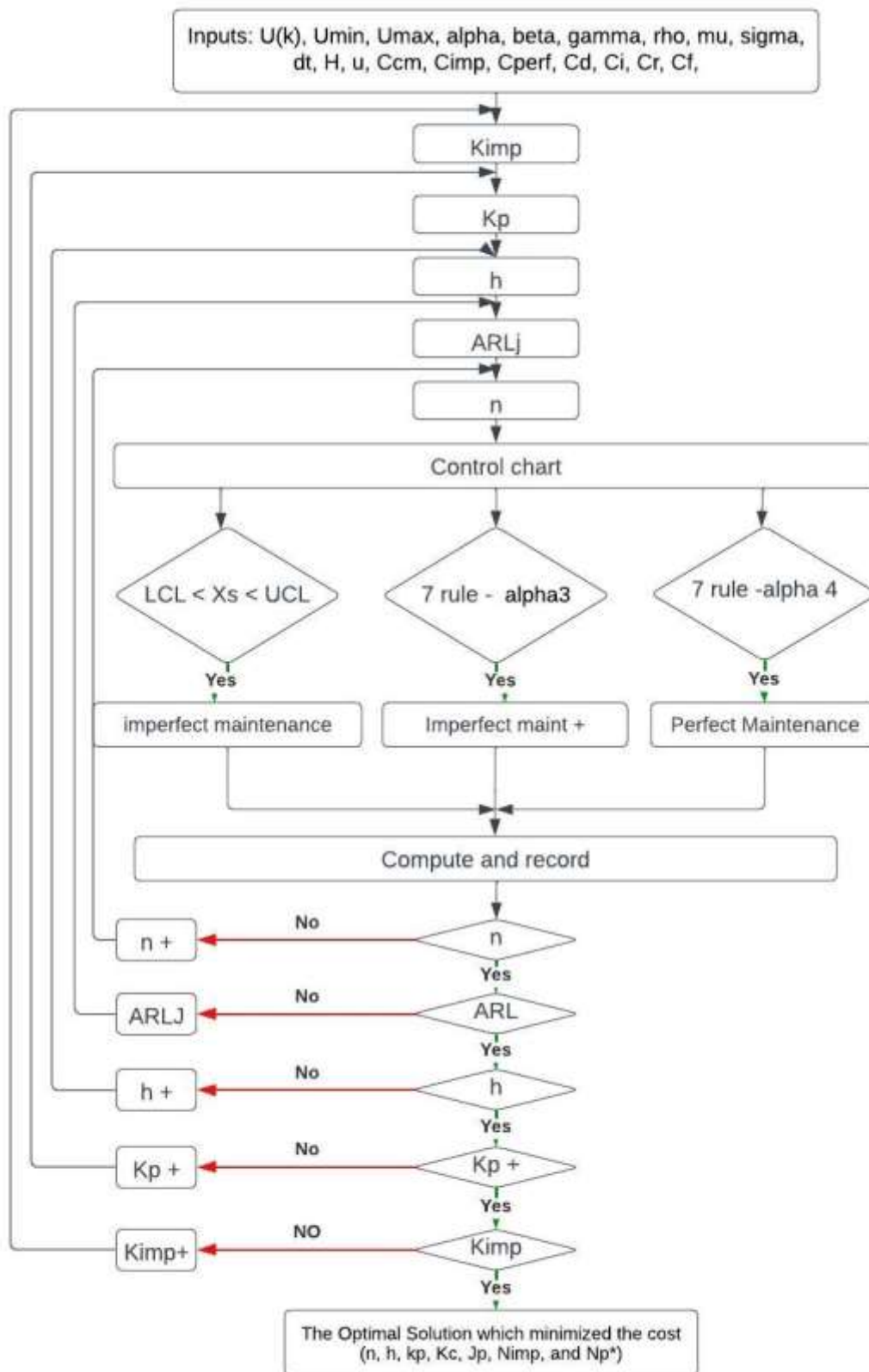


Figure 4. 8: The diagram of the numerical optimization procedure

4.5 Numerical examples

To illustrate this work, we have simulated a numerical example of integrated production, maintenance, and quality model. In this study, our first objective is to minimize the total cost of production and inventory by determining the production's economic plan to satisfy the random demands. Secondly is to reduce the total cost of maintenance and quality for different process scenarios. To solve the production and maintenance/quality model, we used the developed algorithm0 and a numerical procedure for the maintenance and quality problem to determine the optimal value of the time and number of imperfect preventive maintenance actions corresponding to the integrated policy.

4.5.1 Production problem

We consider a supply chain problem composed of a production system, one main stock, and three warehouses ($L = 3$) to satisfy random demand. The production system is composed of one machine and produces one type of product over a finite planning horizon $H = 24 \text{ months}$ in which the period length $\Delta t = 1 \text{ month}$. Assuming the standard deviation of each demand of a product is the same for all periods and each demand $\sigma_{di}(\{i: 1, 2, \dots\}) = 20$ and the initial inventory level, we assume that $S(0) = 2050$. The average demand for customers of warehouse 1, warehouse 2, and warehouse 3: $d_1(k) = d_2(k) = d_3(k) = 1050 \{k: 0, \dots, H - 1\}$. Lower and upper boundaries of production, as well as other production, maintenance, and quality parameters, are presented below:

The following input data were used for experiments in our model as an approach to finding the best strategy.

Input data.

PRODUCTION		MAINTENANCE		QUALITY	
PARAMETER	VALUE	PARAMETER	VALUE	PARAMETER	VALUE
$S(0)$	2050	α_1	1.00	Ci	25
τ_i	1.0	$\alpha_{2.2}$	0.556	Cr	55
L	3.0	$\alpha_{2.3}$	0.333	C_f	125
Q_v	3000	$\alpha_{2.4}$	0.111	μ_1	5.72
c_p	50	CM_{imp-1}	1000	LCL	1.5
c_{hs}	20.2	CM_{imp-2}	2000	UCL	8.5
c_{hwi}	20.2	CM_{imp-3}	5000	$E(S)$	2

c_d	45.0	$CM_{minimal}$	500		
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Table 4. 1: Initial input data

d(1)	d(2)	d(3)	d(4)	d(5)	d(6)	d(7)	d(8)	d(9)	d(10)	d(11)	d(12)
1500	1700	1500	1500	1300	1400	1560	1450	1510	1540	1500	1400
d(13)	d(14)	d(15)	d(16)	d(17)	d(18)	d(19)	d(20)	d(21)	d(22)	d(23)	d(24)
1560	1450	1510	1540	1500	1400	1700	1500	1500	1300	1400	1560

Table 4. 2: Average customer demand

Results.

The optimal production quantity of products $u(k)^*$ to meet customer demands respecting the given service level requirements is presented in Table 2. The production plans are according to the probabilistic function defined by equation (4.14). Tables 3, 4, and 5 presented the optimal inventory management from one period to another throughout the production horizon. This strategy used the inventory balance equation, and also, considers both the cost of inventories as well as costs of penalties. The optimal planning allowed us to reduce the total cost of production TC_p^* to 2080428 $m.u.$

$u(1)^*$	$u(2)^*$	$u(3)^*$	$u(4)^*$	$u(5)^*$	$u(6)^*$	$u(7)^*$	$u(8)^*$	$u(9)^*$	$u(10)^*$	$u(11)^*$	$u(12)^*$
1756	1168	1436	1166	1200	1123	1042	1243	1564	1423	1045	1254
$u(13)^*$	$u(14)^*$	$u(15)^*$	$u(16)^*$	$u(17)^*$	$u(18)^*$	$u(19)^*$	$u(20)^*$	$u(21)^*$	$u(22)^*$	$u(23)^*$	$u(24)^*$
1042	1243	1564	1423	1045	1254	1168	1436	1166	1200	1123	1042

Table 4. 3: Optimal Production Plan

$Q1(1)^*$	$Q1(2)^*$	$Q1(3)^*$	$Q1(4)^*$	$Q1(5)^*$	$Q1(6)^*$	$Q1(7)^*$	$Q1(8)^*$	$Q1(9)^*$	$Q1(10)^*$	$Q1(11)^*$	$Q1(12)^*$
676	298	436	486	360	223	342	243	464	342	345	245
$Q1(13)^*$	$Q1(14)^*$	$Q1(15)^*$	$Q1(16)^*$	$Q1(17)^*$	$Q1(18)^*$	$Q1(19)^*$	$Q1(20)^*$	$Q1(21)^*$	$Q1(22)^*$	$Q1(23)^*$	$Q1(24)^*$
351	443	548	423	404	354	361	422	1166	1200	1123	1042

Table 4. 4: The optimal delivery plan warehouse 1.

$Q2(1)^*$	$Q2(2)^*$	$Q2(3)^*$	$Q2(4)^*$	$Q2(5)^*$	$Q2(6)^*$	$Q2(7)^*$	$Q2(8)^*$	$Q2(9)^*$	$Q2(10)^*$	$Q2(11)^*$	$Q2(12)^*$
685	468	536	284	440	511	412	382	536	614	465	425
$Q2(13)^*$	$Q2(14)^*$	$Q2(15)^*$	$Q2(16)^*$	$Q2(17)^*$	$Q2(18)^*$	$Q2(19)^*$	$Q2(20)^*$	$Q2(21)^*$	$Q2(22)^*$	$Q2(23)^*$	$Q2(24)^*$
442	367	564	514	385	434	439	546	1166	1200	1123	1042

Table 4. 5: The optimal delivery plan warehouse 2.

$Q3(1)^*$	$Q3(2)^*$	$Q3(3)^*$	$Q3(4)^*$	$Q3(5)^*$	$Q3(6)^*$	$Q3(7)^*$	$Q3(8)^*$	$Q3(9)^*$	$Q3(10)^*$	$Q3(11)^*$	$Q3(12)^*$
396	406	468	366	380	323	302	443	641	423	214	554
$Q3(13)^*$	$Q3(14)^*$	$Q3(15)^*$	$Q3(16)^*$	$Q3(17)^*$	$Q3(18)^*$	$Q3(19)^*$	$Q3(20)^*$	$Q3(21)^*$	$Q3(22)^*$	$Q3(23)^*$	$Q3(24)^*$
262	543	480	492	245	354	368	436	1166	1200	1123	1042

Table 4. 6: The optimal delivery plan warehouse 3.

4.5.2 Maintenance and quality problem

To resolve the maintenance and quality problem, we developed and used the control chart tools, the optimal production plan obtained by the production policy, to model maintenance, and quality formulations the following different parameters were nominated considering realistic settings.

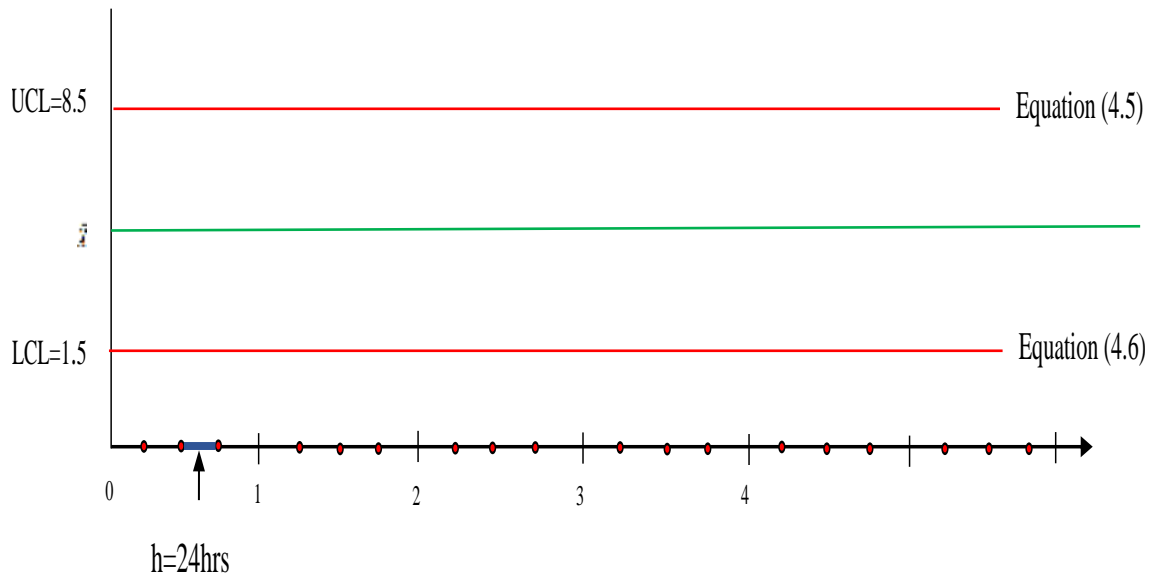


Figure 4. 9: Control Chart limits and deviations

Numerical results

Parameter	n^*	k_c^*	h^*	σ	μ	α_1	N_{imp1}	C_{M1}	C_{TQ1}	C_{TP1}
Value	35	3.85	3.0	1.2	5.0	1.0	8	188730	840050	2097273

Table 4. 7: The in-control state – scenario 1 – CASE 1.

CASES	α_X	FAILURE	$N_{imp\ 2-1}$	$N_{imp\ 2-2}$	$N_{imp\ 2-3}$	C_{TM}	C_{TQ}	C_{TP}
CASE 2.1	0.556	$\varphi_{S2} : 0.36$	5	2	1	538800	645000	2255805
CASE 2.2	0.333	$\varphi_{S3} : 0.37$	8	0	0	125800	840050	2238555
CASE 2.3	0.111	$\varphi_{S4} : 0.36$	0	7	3	672900	215275	2081899

Table 4. 8: The out-of-control state scenarios 2, 3, and 4.

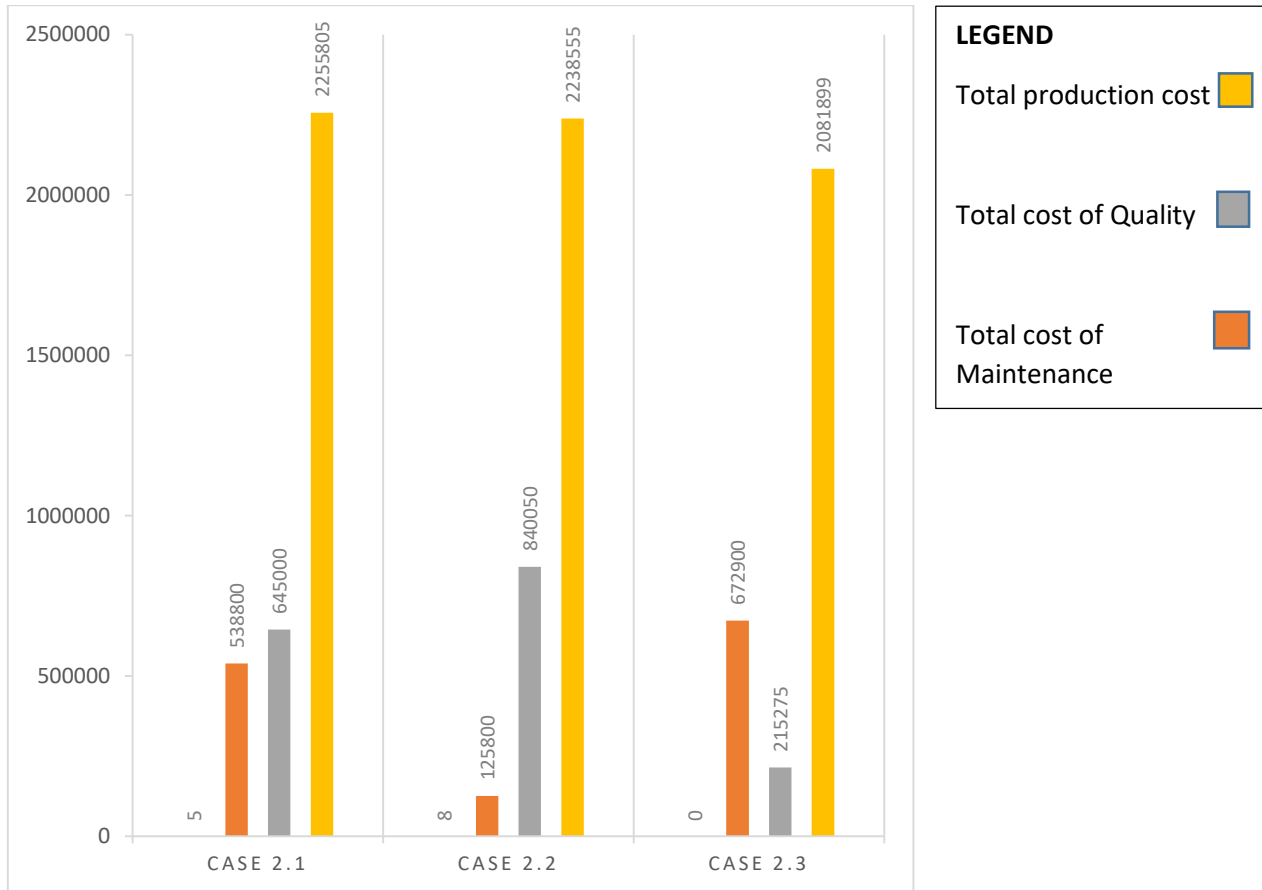


Figure 4. 10: Number of PM for out-of-control scenarios

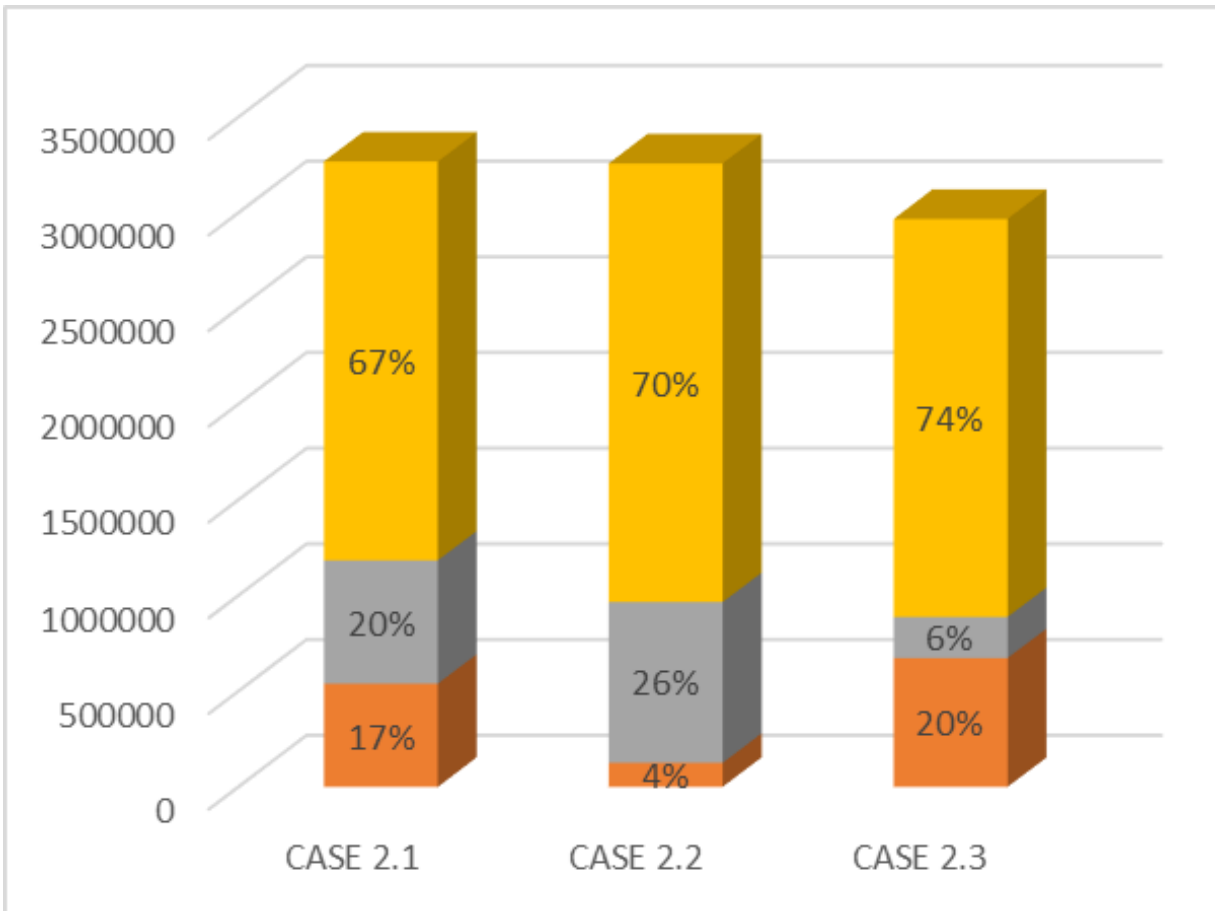
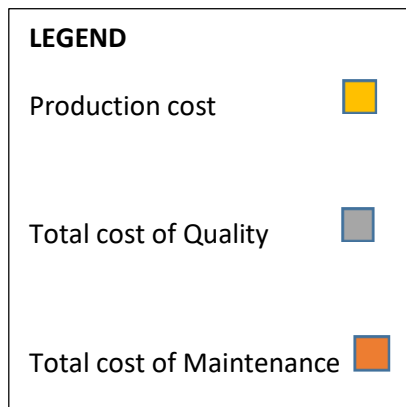


Figure 4. 11: Total Cost of Production for out-of-control scenarios



CASES	α	FAILURE	$N_{imp\ 2.4-1}$	$N_{imp\ 2.4-2}$	$N_{imp\ 2-3}$	$C_{TM2.4}$	$C_{TQ2.4}$	$C_{TP2.4}$
CASE 2.4.1.1	0.167	$\varphi_{S2411}:0.363$	8	0	0	2028	8400	2080428
CASE 2.4.1.2	0.166	$\varphi_{S2412}:0.366$	8	0	0	4407	21025	2080729
CASE 2.4.2.1	0.169	$\varphi_{S2421}:0.359$	3	5	1	2259	8400	2133114
CASE 2.4.2.2	0.166	$\varphi_{S2422}:0.356$	0	8	1	2848	8400	2092378
CASE 2.4.3.1	0.167	$\varphi_{S2431}:0.361$	0	8	1	3102	8400	2096238
CASE 2.4.3.2	0.165	$\varphi_{S2432}:0.360$	1	6	2	7448	244150	2243628

Table 4. 9: CASES 2.4, the occurrence of two scenarios at the same time ($Kc=3.85$)

CASES	α	FAILURE	$N_{imp\ 2.4-1}$	$N_{imp\ 2.4-2}$	$N_{imp\ 2-3}$	$C_{TM2.4}$	$C_{TQ2.4}$	$C_{TP2.4}$
CASE 2.4.1.1	0.167	$\varphi_{S2411}:0.363$	8	0	0	2028	8400	2080428
CASE 2.4.1.2	0.166	$\varphi_{S2412}:0.366$	8	0	0	2329	8400	2080729
CASE 2.4.2.1	0.169	$\varphi_{S2421}:0.359$	0	7	1	5900	51865	1693549
CASE 2.4.2.2	0.166	$\varphi_{S2422}:0.356$	2	5	2	8508	24690	2325408
CASE 2.4.3.1	0.167	$\varphi_{S2431}:0.361$	8	0	0	3102	8400	2081503
CASE 2.4.3.2	0.165	$\varphi_{S2432}:0.360$	1	6	2	10979	109650	2190629

Table 4. 10: CASES 2.4, the occurrence of two scenarios at the same time ($Kc=3.5$)

CASES	α	FAILURE	$N_{imp\ 2.4-1}$	$N_{imp\ 2.4-2}$	$N_{imp\ 2-3}$	$C_{TM2.4}$	$C_{TQ2.4}$	$C_{TP2.4}$
CASE 2.4.1.1	0.167	$\varphi_{S2411}:0.363$	8	0	0	20280	84500	2080428
CASE 2.4.1.2	0.166	$\varphi_{S2412}:0.366$	8	0	1	46950	225775	2308470
CASE 2.4.2.1	0.169	$\varphi_{S2421}:0.359$	4	4	1	52150	160250	2235465
CASE 2.4.2.2	0.166	$\varphi_{S2422}:0.356$	6	1	1	55030	219525	2295028
CASE 2.4.3.1	0.167	$\varphi_{S2431}:0.361$	8	0	0	31030	84300	2081503
CASE 2.4.3.2	0.165	$\varphi_{S2432}:0.360$	6	1	1	70200	191650	2268670

Table 4. 11: CASES 2.4, the occurrence of two scenarios at the same time ($Kc=4.25$)

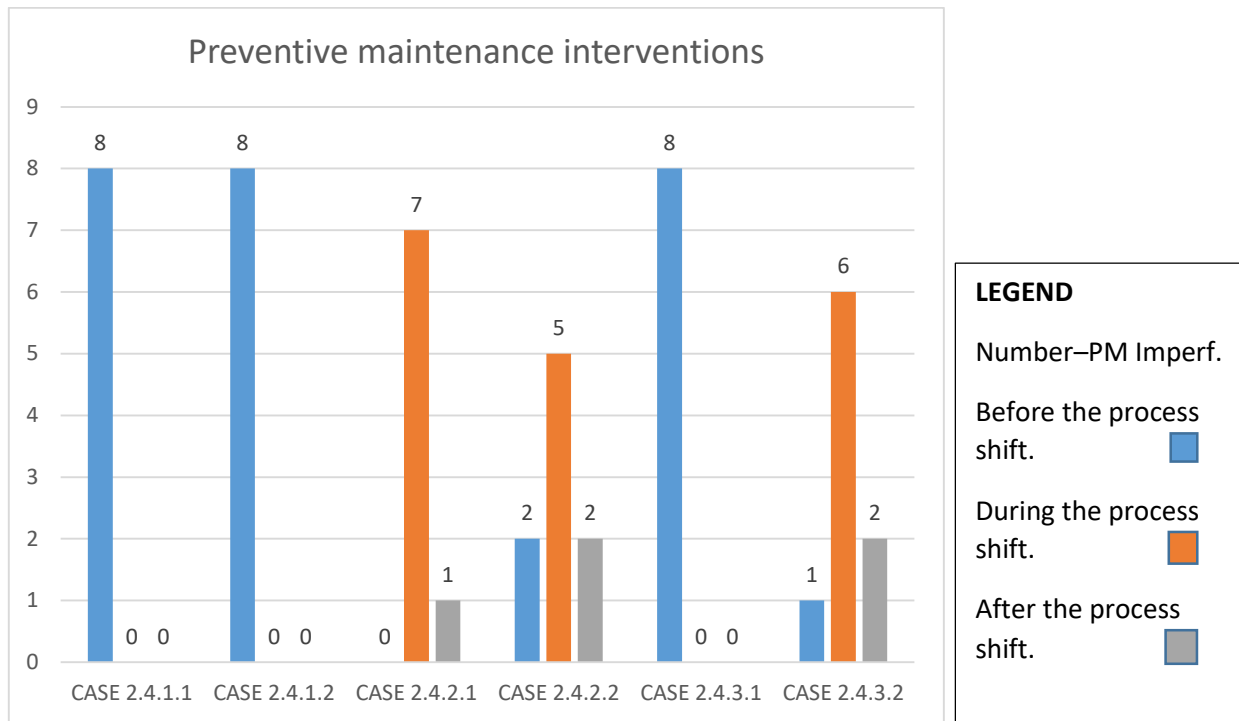


Figure 4. 12: The occurrence of the 2-point out-of-control scenario (CASES 2. 4 - $K_c=3.85$)

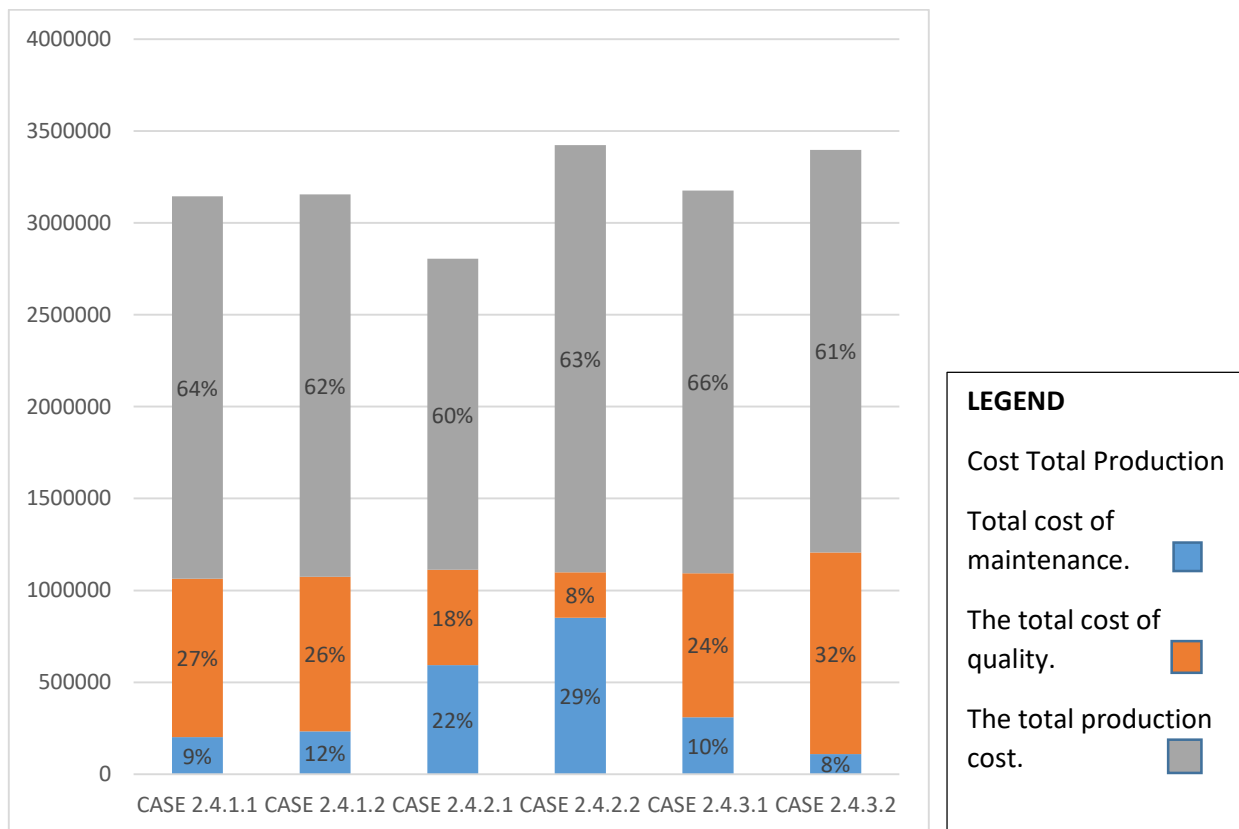


Figure 4. 13: Total production cost implication on CASE's 2.4 ($K_c=3.5$)

CASES	α	FAILURE	$N_{imp\ 2.5-1}$	$N_{imp\ 2.5-2}$	$N_{imp\ 2-3}$	$C_{TM2.5}$	$C_{TQ2.5}$	$C_{TP2.5}$
CASE 2.5.1.1	0.169	$\varphi_{S2511}: 0.364$	8	0	1	850510	220400	2298951
CASE 2.5.1.2	0.166	$\varphi_{S2512}: 0.365$	1	7	1	930104	481036	2127404
CASE 2.5.2.1	0.167	$\varphi_{S2521}: 0.354$	7	1	1	106015	249125	2329730
CASE 2.5.2.2	0.165	$\varphi_{S2522}: 0.365$	0	8	1	119704	114600	2096574
CASE 2.5.3.1	0.167	$\varphi_{S2531}: 0.362$	7	1	1	136325	228650	2312285
CASE 2.5.3.2	0.166	$\varphi_{S2532}: 0.365$	1	7	2	233559	139625	2232948

Table 4. 142: CASES 2.5, the occurrence of three scenarios at the same time ($Kc=3.85$)

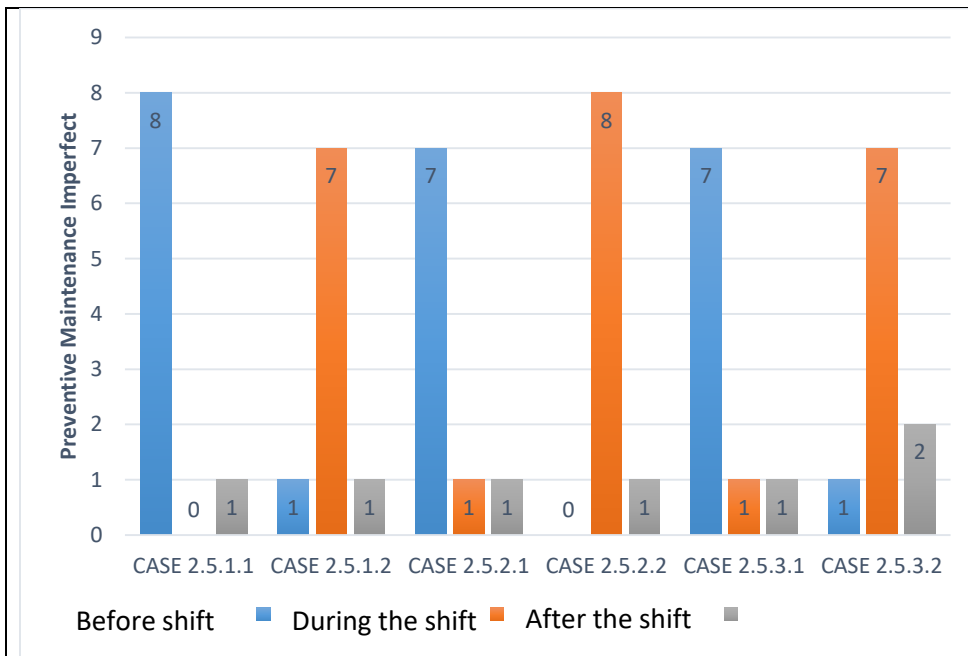


Figure 4.14. The occurrence of the 3-point out-of-control scenario (CASES 2.5)

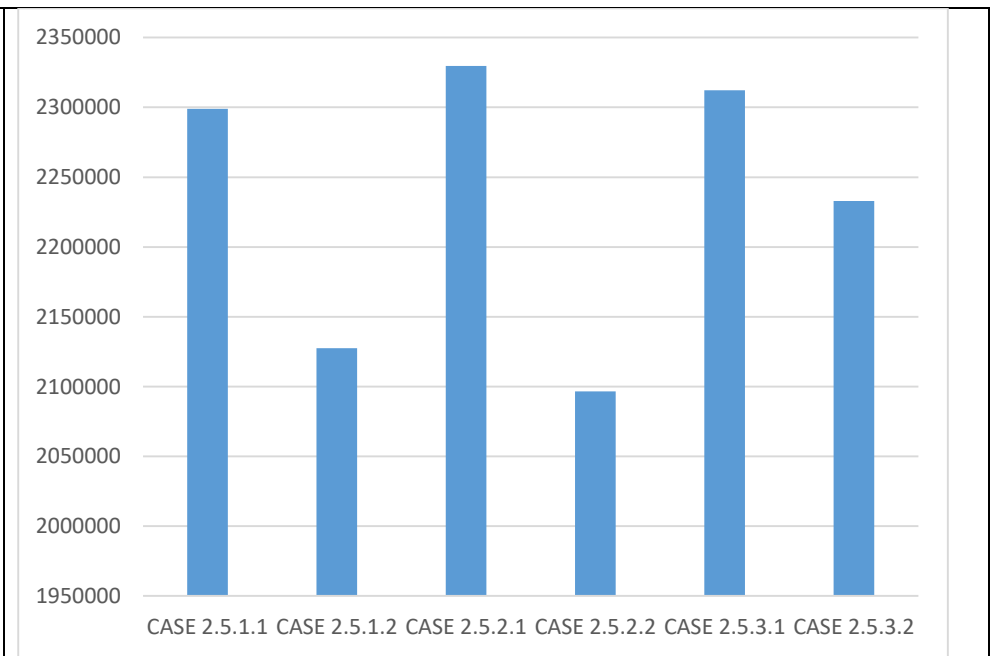


Figure 4.15 Total production cost implication on CASE's 2.5

4.5.3 Discussion of the results

Hence, according to the production policy's economic plans shown in Table 4.3 based on the forecasted demand and its impact on the degradation degree of processes, also, the use of the product maintained by the inventory levels as contained in Tables 4.4, 4.5, and Tables 4.6. We can consider that the best production, maintenance, and quality control strategy consists of taking one sample of size 35 every 24 hours = 3.0 days, followed by 3 ARL_2 , 5 ARL_3 , and 8 ARL_4 . Furthermore, regarding the control chart's design, the optimal number of standard deviations between the Centreline and the control limits is 3.85 and conducting 8 imperfect maintenance during the in-control state process as presented in Table 4.7. In the case of the out-of-control process (Scenarios 2, 3, and 4), the results are presented in Table 4.8. For scenario 2, 6 imperfect maintenance activities are conducted before the process shift, 1 imperfect maintenance during the shift, and then 1 maintenance after the operator detects the assignable cause. In scenario 3, only 8 imperfect maintenance activities are conducted before the process shift. While for scenario 4, 7 imperfects maintenance before the shift of the process, no action during the shift, and then 1 maintenance after the operator detects the assignable cause. Further digestion as presented by Tables 4.9 and 4.10 respectively, for the cases of occurrences of two and three points at the same time consisting of the number of imperfect maintenance before, during, and after the shift. According to the optimal production control policies, process scenarios-cases, and specific failure rates, which minimized the total production cost consisted of total maintenance and total quality cost.

- From the obtained results, we can deduce that being in a particular production process state is not a guarantee for lower total production cost, it depends on the maintenance strategy, quality policy, the occurrence, and characteristics of the out-of-control scenarios.
- **From Table 4.8:** The out-of-control state – scenarios 2, 3, and 4 as demonstrated graphically in Figures 4.10 and 4.11, the **case 2.3** (out of control state – Scenario 3) having the lowest total production cost, the 8.4% reduction is due to the higher number of imperfect maintenance (10) as compared with other cases. We also noted the significance of the order in performing IM before, during, and after the shift, by comparing **cases 2.1 and 2.2** more IM before the shift is better than during the shift which results in higher maintenance and quality costs. Although **case 2.2** is not characterized by the lowest total maintenance cost, the maintenance strategy resulted in the lowest total quality cost from the optimal maintenance cost.
- **From Table 4.9: CASES 2.4**, the occurrence of two scenarios at the same time, we observed that the case with the highest total production cost in this category is **case 2.4.1.2** due to the higher number of imperfect maintenance, also resulted to higher quality cost. As demonstrated in Figure 4.13, a cost reduction of 11% is achievable by simply limiting the maintenance action to only before the shift.

- Take **case 2.4.1.1** and compare it with **case 2.4.1.2**, it will be observed that the higher total production cost of the latter case is due to lower IM action resulting in higher non-conformal cost. However, the lower IM action in the latter case did not reduce the total maintenance cost of this case.
- **From Table 4.10: CASES 2.5**, the occurrence of three scenarios at the same time is graphically presented in Figures 4.14, and 4.15. We observed that the lowest total production cost **case 2.5.2.2** is not attributed to any PM action before the shift. While the case with the highest total production cost is **case 2.5.2.1** is having PM interventions all through.
 - And compare **case 2.5.1.1** with a similar **case 2.5.2.1** both having the same total number of Maintenance interventions, but with different order of IM before, during, and after the shift, which causes a significant TCP variation of 17%.

4.6 Conclusion

The optimal maintenance strategy is achieved using a combined approach involving product inspection and continuous process monitoring to maintain, improve, and control overall production system performance. In this work, we incorporated additional quality monitoring as a decision support technique for detecting some unusual trends in the production process, we seek to optimize the production planning of a three-echelon production system under uncertainties. We considered a production, maintenance, and quality control policy, for unreliable single-product production systems linked to a multi-warehouse supply chain. The stochastic model uses forecasting production based on random demand to satisfy certain service levels and quality requirements. A maintenance and quality strategy based on the production process scenarios due to several possible cases represents an additional uncertain information structure. We introduce a new stochastic mathematical formulation that relies on different scenarios probabilities and specific failure rates of the scenarios. We then developed some specialized optimization techniques to solve each of the production, maintenance, and quality-constrained problems with a dynamic programming algorithm built based on the model-stated decomposition. Our numerical results show that the proposed solution approach can provide precise optimal solutions for varied instances with reasonable computational accuracy.

The main contribution of this study is the development of the mathematical model of an advanced decision system and the design of improved quality control techniques. This deals with the challenging issue of the interaction between the different yet, related factors of production; production planning, inventory management, maintenance strategy, and quality policy. The factors were treated optimally and collaboratively, which improved system reliability and reduced the number of machine failures, and false alarms, as well as the number of non-conformal products, thereby, minimizing the total cost. Numerical examples were conducted with an exhaustive procedure and based on the results of the integrated model which considers multiple assignable causes of production process variation.

We have significantly minimized the total cost by a range of 2.5% to 32% depending on the wide choices of decision-level parameters. We understand also, the impact of imperfect maintenance on the production system reliability, non-conformal items, and total production cost minimization. The production cost minimization justifies the need for research in this field, and also support the future studies that we recommend to consider cases of multi-product manufacturing system, multi-stage quality inspection, or remanufacturing integration.

GENERAL CONCLUSION

This research work reviewed and improved the design, development, and industrial application of novel decision support tools for integrated production systems. We used the improved statistical process chart for the control and optimisation of an integrated model of production problems. The optimal production planning, inventory management, and maintenance strategies are performed in collaboration with quality control policies to direct appropriate maintenance policies for an unreliable production system under certain customer service constraints. In this thesis work, we are interested in the development of new maintenance strategies integrated into production for a single-product production system linked to a multi-warehouse under operational and quality constraints.

The main contribution of this study is the development of the mathematical model of an advanced decision system and the design of improved quality control techniques. The technique we used to optimise maintenance and quality control of an integrated production system is a control chart tool based on statistical measurement and analysis of quality parameters. Also, it has been established, that production, maintenance, and quality are three principal factors of production system optimisation. This deals with the challenging issue of the interaction between the different yet, related factors of production; production planning, inventory management, maintenance strategy, and quality policy. These factors were treated optimally and collaboratively, which improved system reliability and reduced the number of machine failures, and false alarms, as well as the number of non-quality products, thereby, minimising the total production cost.

In the first chapter, we have constituted a bibliographical study of the research works concerning the production system. Beginning with simple and integrated production planning with maintenance or quality control policies. This allowed us to situate our thesis problem about the existing literature works. From the literature review, we realised the existence of knowledge gaps. Indeed, the study of non-continuous, finite production horizons for the integrated production system is rarely studied. Very few works address the problem of variable production rates from one production period to the other over time due to random demand. Additionally, there is a need for work to address the influence of variation in product production as well as operational and quality factors on the failure rate of the production system and consequently on the optimal maintenance plan to apply. In addition, there is a lack of work that proposes new integrated maintenance strategies with an in-depth study of the continuous production process reliability improvements and treated under customer requirements (Service, quality, and cost) constraints. Based on these aspects, which are not well studied and investigated adequately in the literature, we have set out in detail the problems as well as the objectives and the methods of resolution of our research outcome.

In the second chapter, we defined and model our first production and product quality control policy for a supply chain problem. The production system consists of a single machine (unreliable) producing one type of product and trying to satisfy a random demand of multiple customers, over a finite horizon, under service and quality constraints. The impacts of the production rate variation and its use over time on the production system's reliability are investigated. We employed a sequential

optimisation strategy which determined the optimal production rate and inventory quantities from one production period to another throughout the finite production horizon. with the first sub-model as a constraint to optimise the second sub-model of maintenance integrated into the control chart of quality. We established, the optimum production rates, the optimal inventory/delivery quantities, and consequently, the optimal periodic maintenance strategy with minimal repairs (PPMMR) corresponding to the best configuration of the control chart's optimal parameters, which minimises the total production cost.

In the third chapter, we developed our second model production problem. We evaluated the performance of normal control charts in the previous chapter for the detections of a process variation (out-of-control) condition for only two scenarios and three control chart decision parameters. The results suggested the need for improvement. This work presented recent development in the design and application of an improved statistical Process control chart (ISPC). In this chapter, we studied and developed a more efficient, multi-layer process control chart of quality with additional surveillance, and more control limit coefficients needed for higher precision products and optimal production. The work established a collaborative production planning and decision support system integrated into maintenance and quality that increases the reliability of the process, reduces the non-conformal losses, and subsequently, achieved total cost minimization. This is found to be a robust model that will apply to industrial cases in need of high precision principles required for the production of high-quality products.

Subsequently, in the fourth chapter, we modelled a production problem accounting for different assignable causes of process variation. We incorporated additional quality monitoring as a decision support technique for detecting unusual trends in production processes. We seek to optimise the production planning of a three-echelon production system under uncertainties. In this work, we employed a combination of quality management techniques. The rule of Seven (AFNOR) is involved in quality management in conjunction with a control chart tool. The new joint control policy considers the interactions between production quality and maintenance strategies. To monitor, analyse and direct appropriate maintenance actions along the different multiple process scenarios as decision levels. The optimisation of maintenance strategy consists to minimise the costs related to preventive maintenance imperfect, corrective (minimal repairs) in several cases of four different possible scenarios, and other multiple cases of singles or combinational possibilities. In this context, this chapter presents the derivation of all the probabilities for the process to be in or out of control, the average run lengths, and the restoration cycle durations. Based on the degradation factor, new failure rate equations were formulated, and then the average failure rate for each case, we formulate different maintenance costs. The numerical results show that our proposed approach is useful for a precise calculation with reasonable computational accuracy, and optimised the production planning of a three-echelon production system under uncertainties.

We understand also, the impact of imperfect maintenance on the production system reliability, non-conformal items, and total production cost minimization in consideration of all possible process statuses due to multiple causes. The production cost minimization justifies the need for research in this field, and also support the future studies that we recommend to consider cases of multi-product manufacturing system, multi-stage quality inspection, or remanufacturing integration.

BIBLIOGRAPHY

References

- Addeh, A., Khormali, A., & Golilarz, N. A. (2018). Control chart pattern recognition using RBF neural network with new training algorithm and practical features. *ISA Transactions*, 79, 202–216. <https://doi.org/10.1016/j.isatra.2018.04.020>
- Aghezzaf, E. H., Jamali, M. A., & Ait-Kadi, D. (2007). An integrated production and preventive maintenance planning model. *European Journal of Operational Research*, 181(2), 679–685. <https://doi.org/10.1016/j.ejor.2006.06.032>
- Ait-El-Cadi, A., Gharbi, A., Dhoub, K., & Artiba, A. (2021). Integrated production, maintenance and quality control policy for unreliable manufacturing systems under dynamic inspection. *International Journal of Production Economics*, 236, 108140. <https://doi.org/10.1016/j.ijpe.2021.108140>
- Al-Salamah, M. (2018). Economic production quantity with the presence of imperfect quality and random machine breakdown and repair based on the artificial bee colony heuristic. *Applied Mathematical Modelling*, 63, 68–83. <https://doi.org/10.1016/j.apm.2018.06.034>
- Antosz, K., & Stadnicka, D. (2018). Possibilities of Maintenance Service Process Analyses and Improvement Through Six Sigma, Lean and Industry 4.0 Implementation. In P. Chiabert, A. Bouras, F. Noël, & J. Ríos (Eds.), *Product Lifecycle Management to Support Industry 4.0* (Vol. 540, pp. 465–475). Springer International Publishing. https://doi.org/10.1007/978-3-030-01614-2_43
- Bahria, N., Chelbi, A., Bouchriha, H., & Dridi, I. H. (2019). Integrated production, statistical process control, and maintenance policy for unreliable manufacturing systems. *International Journal of Production Research*, 57(8), 2548–2570. <https://doi.org/10.1080/00207543.2018.1530472>
- Baklouti, A., Mifdal, L., Dellagi, S., & Chelbi, A. (2020). An Optimal Preventive Maintenance Policy for a Solar Photovoltaic System. *Sustainability*, 12(10), 4266. <https://doi.org/10.3390/su12104266>

- Beheshti Fakher, H., Nourelfath, M., & Gendreau, M. (2017). A cost minimisation model for joint production and maintenance planning under quality constraints. *International Journal of Production Research*, 55(8), 2163–2176. <https://doi.org/10.1080/00207543.2016.1201605>
- Ben-Daya, M., & Duffuaa, S. O. (1995). Maintenance and quality: The missing link. *Journal of Quality in Maintenance Engineering*, 1(1), 20–26. <https://doi.org/10.1108/13552519510083110>
- Ben-Daya, M., & Makhdoum, M. (1998). Integrated production and quality model under various preventive maintenance policies. *Journal of the Operational Research Society*, 49(8), 840–853. <https://doi.org/10.1057/palgrave.jors.2600586>
- Ben-Daya, M., & Rahim, M. A. (2000). Effect of maintenance on the economic design of \bar{x} -control chart. *European Journal of Operational Research*, 120(1), 131–143. [https://doi.org/10.1016/S0377-2217\(98\)00379-8](https://doi.org/10.1016/S0377-2217(98)00379-8)
- Bergman, M. W. (2021). *Implementing AI vision for Quality Inspection within a Manufacturing Environment A study to Explore the Functionality of Detecting Deviations in Manual Assembly*. <https://doi.org/10.13140/RG.2.2.30797.92649>
- Bouslah, B., Gharbi, A., & Pellerin, R. (2016). Integrated production, sampling quality control and maintenance of deteriorating production systems with AOQL constraint. *Omega*, 61, 110–126. <https://doi.org/10.1016/j.omega.2015.07.012>
- Brown, M., & Proschan, F. (1983). Imperfect repair. *Journal of Applied Probability*, 20(4), 851–859. <https://doi.org/10.2307/3213596>
- Budai, G., Dekker, R., & Nicolai, R. P. (2008). Maintenance and Production: A Review of Planning Models. In *Complex System Maintenance Handbook* (pp. 321–344). Springer London. https://doi.org/10.1007/978-1-84800-011-7_13
- Buzacott, J. A. (1967). AUTOMATIC TRANSFER LINES WITH BUFFER STOCKS. *International Journal of Production Research*, 5(3), 183–200. <https://doi.org/10.1080/00207546708929751>

- Cassady, C. R., & Kutanoglu, E. (2005). Integrating Preventive Maintenance Planning and Production Scheduling for a Single Machine. *IEEE Transactions on Reliability*, 54(2), 304–309. <https://doi.org/10.1109/TR.2005.845967>
- Chen, X., Xiao, L., Zhang, X., Xiao, W., & Li, J. (2015). An integrated model of production scheduling and maintenance planning under imperfect preventive maintenance. *Eksploracja i Niezawodność - Maintenance and Reliability*, 17(1), 70–79. <https://doi.org/10.17531/ein.2015.1.10>
- Cheng, G., & Li, L. (2020). Joint optimization of production, quality control and maintenance for serial-parallel multistage production systems. *Reliability Engineering & System Safety*, 204, 107146. <https://doi.org/10.1016/j.ress.2020.107146>
- Cheng, G. Q., Zhou, B. H., & Li, L. (2018). Integrated production, quality control and condition-based maintenance for imperfect production systems. *Reliability Engineering & System Safety*, 175, 251–264. <https://doi.org/10.1016/j.ress.2018.03.025>
- Cui, Y., Zong, H., Yan, H., Li, N., & Zhang, Y. (2014). Degarelix versus Goserelin plus Bicalutamide Therapy for Lower Urinary Tract Symptom Relief, Prostate Volume Reduction and Quality of Life Improvement in Men with Prostate Cancer: A Systematic Review and Meta-Analysis. *Urologia Internationalis*, 93(2), 152–159. <https://doi.org/10.1159/000356272>
- Darendeliler, A., Claeys, D., Khatab, A., & Aghezzaf, E.-H. (2020). Joint Optimization of Dynamic Lot-sizing and Condition-based Maintenance: *Proceedings of the 9th International Conference on Operations Research and Enterprise Systems*, 151–158. <https://doi.org/10.5220/0008941601510158>
- Dellagi, S., Chelbi, A., & Trabelsi, W. (2017). Joint integrated production-maintenance policy with production plan smoothing through production rate control. *Journal of Manufacturing Systems*, 42, 262–270. <https://doi.org/10.1016/j.jmsy.2016.12.013>
- Dowlatshahi, S. (2000). Developing a Theory of Reverse Logistics. *Interfaces*, 30(3), 143–155. <https://doi.org/10.1287/inte.30.3.143.11670>

- Doyen, L., & Gaudoin, O. (2006). Imperfect maintenance in a generalized competing risks framework. *Journal of Applied Probability*, 43(03), 825–839. <https://doi.org/10.1017/S0021900200002138>
- Dutoit, C., Dehombreux, P., Lorphèvre, E. R., & Equeter, L. (2019a). Statistical Process Control and Maintenance Policies for Continuous Production Systems Subjected to Different Failure Impact Models: Literature Review. *Procedia CIRP*, 86, 55–60. <https://doi.org/10.1016/j.procir.2020.01.050>
- Dutoit, C., Dehombreux, P., Lorphèvre, E. R., & Equeter, L. (2019b). Statistical Process Control and Maintenance Policies for Continuous Production Systems Subjected to Different Failure Impact Models: Literature Review. *Procedia CIRP*, 86, 55–60. <https://doi.org/10.1016/j.procir.2020.01.050>
- Elattar, S., Mohamed, H. G., & Hussien, S. A. (2022). A Multi-Objective Optimization of Secure Pull Manufacturing Systems. *Applied Sciences*, 12(12), 5937. <https://doi.org/10.3390/app12125937>
- Enderlein, G. (1966). Barlow, E. E., F. Proschan and L. C. Hunter: Mathematical Theory of Reliability. Wiley, New York-London-Sydney 1965. 256 S., Preis 83 s. *Biometrische Zeitschrift*, 8(4), 278–278. <https://doi.org/10.1002/bimj.19660080409>
- Guo, Y., Hrouga, M., Liu, J., Afifi, S., Allaoui, H., & Zhang, Y. (2022). JOINT OPTIMIZATION OF CAPACITATED LOT-SIZING WITH LOST SALES AND NON-CYCLICAL PREVENTIVE MAINTENANCE. *국제산업공학저널*, 29(3). <https://doi.org/10.23055/IJITAP.2022.29.3.8013>
- Hadian, S. M., Farughi, H., & Rasay, H. (2021). Joint planning of maintenance, buffer stock and quality control for unreliable, imperfect manufacturing systems. *Computers & Industrial Engineering*, 157, 107304. <https://doi.org/10.1016/j.cie.2021.107304>
- Hajej, Z., Nyoungue, A. C., Abubakar, A. S., & Mohamed Ali, K. (2021). An Integrated Model of Production, Maintenance, and Quality Control with Statistical Process Control Chart of a Supply Chain. *Applied Sciences*, 11(9), 4192. <https://doi.org/10.3390/app11094192>

- Hajej, Z., Rezg, N., & Gharbi, A. (2018). Quality issue in forecasting problem of production and maintenance policy for production unit. *International Journal of Production Research*, 56(18), 6147–6163. <https://doi.org/10.1080/00207543.2018.1478150>
- Hansen, J., Sato, M., Hearty, P., Ruedy, R., Kelley, M., Masson-Delmotte, V., Russell, G., Tselioudis, G., Cao, J., Rignot, E., Velicogna, I., Tormey, B., Donovan, B., Kandiano, E., von Schuckmann, K., Kharecha, P., Legrande, A. N., Bauer, M., & Lo, K.-W. (2016). Ice melt, sea level rise and superstorms: Evidence from paleoclimate data, climate modeling, and modern observations that 2 °C global warming could be dangerous. *Atmospheric Chemistry and Physics*, 16(6), 3761–3812. <https://doi.org/10.5194/acp-16-3761-2016>
- Hawkins*, D. M., & Zamba, K. D. (2003). On Small Shifts in Quality Control. *Quality Engineering*, 16(1), 143–149. <https://doi.org/10.1081/QEN-120020780>
- Holgado, M., Macchi, M., & Fumagalli, L. (2016). Value-in-use of e-maintenance in service provision: Survey analysis and future research agenda. *IFAC-PapersOnLine*, 49(28), 138–143. <https://doi.org/10.1016/j.ifacol.2016.11.024>
- Hosseini, S. M. H., Kramer, J. H., & Kesler, S. R. (2014). Neural correlates of cognitive intervention in persons at risk of developing Alzheimer’s disease. *Frontiers in Aging Neuroscience*, 6. <https://doi.org/10.3389/fnagi.2014.00231>
- Jahan, A., Edwards, K. L., & Bahraminasab, M. (2016). Materials selection in the context of design problem-solving. In *Multi-criteria Decision Analysis for Supporting the Selection of Engineering Materials in Product Design* (pp. 25–40). Elsevier. <https://doi.org/10.1016/B978-0-08-100536-1.00002-3>
- Jasiulewicz-Kaczmarek, M., Żywica, P., & Gola, A. (2021). Fuzzy set theory driven maintenance sustainability performance assessment model: A multiple criteria approach. *Journal of Intelligent Manufacturing*, 32(5), 1497–1515. <https://doi.org/10.1007/s10845-020-01734-3>

- Jiang, Y., Ritchie, B. W., & Verreyne, M.-L. (2022). A Resource-Based Typology of Dynamic Capability: Managing Tourism in a Turbulent Environment. *Journal of Travel Research*, 61(5), 1006–1023. <https://doi.org/10.1177/00472875211014960>
- Kenné, J. P., & Gharbi, A. (2004). Stochastic optimal production control problem with corrective maintenance. *Computers & Industrial Engineering*, 46(4), 865–875. <https://doi.org/10.1016/j.cie.2004.05.024>
- Kenne, J. P., & Nkeungoue, L. J. (2008). Simultaneous control of production, preventive and corrective maintenance rates of a failure-prone manufacturing system. *Applied Numerical Mathematics*, 58(2), 180–194. <https://doi.org/10.1016/j.apnum.2006.11.010>
- Kölmel, A., Sauer, A., & Lanza, G. (2014). Quality-oriented Production Planning of Battery Assembly Systems for Electric Mobility. *Procedia CIRP*, 23, 149–154. <https://doi.org/10.1016/j.procir.2014.10.075>
- Lee, H.-H. (2005). A cost/benefit model for investments in inventory and preventive maintenance in an imperfect production system. *Computers & Industrial Engineering*, 48(1), 55–68. <https://doi.org/10.1016/j.cie.2004.07.008>
- Lesage, A., & Dehombreux, P. (n.d.). *Optimisation conjointe des stratégies de maintenance et de contrôle qualité: Modèle intégrant la maintenance imparfaite*. 9.
- Li, J. (2000). Production variability in manufacturing systems: Bernoulli reliability case. *Annals of Operations Research*, 93(1/4), 299–324. <https://doi.org/10.1023/A:1018928007956>
- Liu, B., He, K., & Xie, M. (2020). Integrated production and maintenance planning for a deteriorating system under uncertain demands. *IFAC-PapersOnLine*, 53(3), 222–226. <https://doi.org/10.1016/j.ifacol.2020.11.036>

- Liu, X., Wang, W., & Peng, R. (2015). An integrated production, inventory and preventive maintenance model for a multi-product production system. *Reliability Engineering & System Safety*, 137, 76–86. <https://doi.org/10.1016/j.ress.2015.01.002>
- Megoze Pongha, P., Kenné, J.-P., De Jesus Garcia, E., & Hof, L. A. (2022). Optimal joint production, maintenance and product quality control policies for a continuously deteriorating manufacturing system. *International Journal of Modelling and Simulation*, 1–18. <https://doi.org/10.1080/02286203.2022.2056799>
- Mifdal, L., Hajej, Z., & Dellagi, S. (2015). Joint Optimization Approach of Maintenance and Production Planning for a Multiple-Product Manufacturing System. *Mathematical Problems in Engineering*, 2015, 1–17. <https://doi.org/10.1155/2015/769723>
- Mohamed-Salah, O., Daoud, A.-K., & Ali, G. (1999). A simulation model for opportunistic maintenance strategies. *1999 7th IEEE International Conference on Emerging Technologies and Factory Automation. Proceedings ETFA '99 (Cat. No.99TH8467)*, 1, 703–708. <https://doi.org/10.1109/ETFA.1999.815422>
- Monga, A., Zuo, M. J., & Toogood, R. W. (1997). Reliability-Based Design of Systems Considering Preventive Maintenance and Minimal Repair. *International Journal of Reliability, Quality and Safety Engineering*, 04(01), 55–71. <https://doi.org/10.1142/S0218539397000059>
- Morimura, H. (n.d.). *ON SOME PREVENTIVE MAINTENANCE POLICIES FOR IFR*. 31.
- Nahas, N. (2017). Buffer allocation and preventive maintenance optimization in unreliable production lines. *Journal of Intelligent Manufacturing*, 28(1), 85–93. <https://doi.org/10.1007/s10845-014-0963-y>
- Nakagawa, T. (1979). A summary of block replacement policies. *RAIRO - Operations Research*, 13(4), 351–361. <https://doi.org/10.1051/ro/1979130403511>

- Nakagawa, T., & Mizutani, S. (2009). A summary of maintenance policies for a finite interval. *Reliability Engineering & System Safety*, 94(1), 89–96. <https://doi.org/10.1016/j.ress.2007.04.004>
- Nourelfath, M., Châtelet, E., & Nahas, N. (2012). Joint redundancy and imperfect preventive maintenance optimization for series–parallel multi-state degraded systems. *Reliability Engineering & System Safety*, 103, 51–60. <https://doi.org/10.1016/j.ress.2012.03.004>
- Pakurár, M., Haddad, H., Nagy, J., Popp, J., & Oláh, J. (2019). The Service Quality Dimensions that Affect Customer Satisfaction in the Jordanian Banking Sector. *Sustainability*, 11(4), 1113. <https://doi.org/10.3390/su11041113>
- Panda, S. (2007). Optimal JIT safety stock and buffer inventory for minimal repair and regular preventive maintenance. *International Journal of Operational Research*, 2(4), 440. <https://doi.org/10.1504/IJOR.2007.014173>
- Pandey, D., Kulkarni, M. S., & Vrat, P. (2011). A methodology for joint optimization for maintenance planning, process quality and production scheduling. *Computers & Industrial Engineering*, 61(4), 1098–1106. <https://doi.org/10.1016/j.cie.2011.06.023>
- Pohl, E. (2010). System Reliability. In G. S. Parnell, P. J. Driscoll, & D. L. Henderson (Eds.), *Decision Making in Systems Engineering and Management* (pp. 227–272). John Wiley & Sons, Inc. <https://doi.org/10.1002/9780470926963.ch8>
- Qu, L., Liao, J., Gao, K., & Yang, L. (2022). Joint Optimization of Production Lot Sizing and Preventive Maintenance Threshold Based on Nonlinear Degradation. *Applied Sciences*, 12(17), 8638. <https://doi.org/10.3390/app12178638>
- Radhoui, M., Rezg, N., & Chelbi, A. (2010). Joint quality control and preventive maintenance strategy for imperfect production processes. *Journal of Intelligent Manufacturing*, 21(2), 205–212. <https://doi.org/10.1007/s10845-008-0198-x>

- Rasay, H., Naderkhani, F., & Azizi, F. (2022). Opportunistic maintenance integrated model for a two-stage manufacturing process. *The International Journal of Advanced Manufacturing Technology*.
<https://doi.org/10.1007/s00170-021-08571-5>
- Rezg, N., Xie, X., & Mati, Y. (2004). Joint optimization of preventive maintenance and inventory control in a production line using simulation. *International Journal of Production Research*, 42(10), 2029–2046. <https://doi.org/10.1080/00207540310001638235>
- Rivera-Gómez, H., Gharbi, A., Kenné, J.-P., Ortiz-Zarco, R., & Corona-Armenta, J. R. (2021). Joint production, inspection and maintenance control policies for deteriorating system under quality constraint. *Journal of Manufacturing Systems*, 60, 585–607. <https://doi.org/10.1016/j.jmsy.2021.07.018>
- Sadok, T., Zied, H., & Nidhal, R. (2012). Impact of delivery time on optimal production/delivery/maintenance planning. *2012 IEEE International Conference on Automation Science and Engineering (CASE)*, 335–340. <https://doi.org/10.1109/CoASE.2012.6386395>
- Salmasnia, A., Abdzadeh, B., & Namdar, M. (2017). A joint design of production run length, maintenance policy and control chart with multiple assignable causes. *Journal of Manufacturing Systems*, 42, 44–56. <https://doi.org/10.1016/j.jmsy.2016.11.003>
- Samet, S., Chelbi, A., & Ben Hmida, F. (2010). Optimal availability of failure-prone systems under imperfect maintenance actions. *Journal of Quality in Maintenance Engineering*, 16(4), 395–412.
<https://doi.org/10.1108/13552511011084544>
- Sarkar, A., Behera, D. K., & Kumar, S. (2012). Maintenance Policies of Single and Multi-unit Systems in the Past and Present. *International Journal of Current Engineering and Technology*, 10.
- Schutz, J., Rezg, N., & Léger, J.-B. (2011). Periodic and sequential preventive maintenance policies over a finite planning horizon with a dynamic failure law. *Journal of Intelligent Manufacturing*, 22(4), 523–532. <https://doi.org/10.1007/s10845-009-0313-7>

- Sharikh, N. M. A., El Farra, M. M., Abuzerr, S., & Madi, K. (2019). The Impact of Lean Six Sigma Practices on Physiotherapy Services Quality at UNRWA Healthcare Centers, Gaza Strip, Palestine. *Journal of Biosciences and Medicines*, 07(12), 79–94. <https://doi.org/10.4236/jbm.2019.712008>
- Si, W., Love, E., & Yang, Q. (2018). Two-state optimal maintenance planning of repairable systems with covariate effects. *Computers & Operations Research*, 92, 17–25. <https://doi.org/10.1016/j.cor.2017.11.007>
- Siddiqui, S. A., Ristow, B., Rahayu, T., Putra, N. S., Widya Yuwono, N., Nisa', K., Mategeko, B., Smetana, S., Saki, M., Nawaz, A., & Nagdalian, A. (2022). Black soldier fly larvae (BSFL) and their affinity for organic waste processing. *Waste Management*, 140, 1–13. <https://doi.org/10.1016/j.wasman.2021.12.044>
- Sinha, P. K., & Singh, N. N. (n.d.). *THE ADVANCEMENT & EFFECT OF SIX SIGMA APPROACH IN A MODERN INDUSTRY AND CURRENT BUSINESS ENTERPRISE*. 4(3), 15.
- Smith, A. M., & Hinchcliffe, G. R. (2003). *RCM—Gateway to World Class Maintenance*. Elsevier Science. <https://books.google.fr/books?id=BnQN2ODPHNAC>
- Sonnemann, G., & Editors, M. M. (n.d.). *Life Cycle Management*. 353.
- Sortrakul, N., Nachtmann, H. L., & Cassady, C. R. (2005). Genetic algorithms for integrated preventive maintenance planning and production scheduling for a single machine. *Computers in Industry*, 56(2), 161–168. <https://doi.org/10.1016/j.compind.2004.06.005>
- Suliman, S. M. A., & Jawad, S. H. (2012). Optimization of preventive maintenance schedule and production lot size. *International Journal of Production Economics*, 137(1), 19–28. <https://doi.org/10.1016/j.ijpe.2012.01.006>
- Surviving Supply Chain Integration: Strategies for Small Manufacturers* (p. 6369). (2000). National Academies Press. <https://doi.org/10.17226/6369>

- Tibben-Lembke, R. S., & Rogers, D. S. (2002). Differences between forward and reverse logistics in a retail environment. *Supply Chain Management: An International Journal*, 7(5), 271–282.
<https://doi.org/10.1108/13598540210447719>
- Tlili, L., Radhoui, M., & Chelbi, A. (2015). Condition-Based Maintenance Strategy for Production Systems Generating Environmental Damage. *Mathematical Problems in Engineering*, 2015, 1–12.
<https://doi.org/10.1155/2015/494162>
- uit het Broek, M. A. J., Teunter, R. H., de Jonge, B., & Veldman, J. (2021). Joint condition-based maintenance and condition-based production optimization. *Reliability Engineering & System Safety*, 214, 107743. <https://doi.org/10.1016/j.ress.2021.107743>
- Van Horenbeek, A., Buré, J., Cattrysse, D., Pintelon, L., & Vansteenwegen, P. (2013). Joint maintenance and inventory optimization systems: A review. *International Journal of Production Economics*, 143(2), 499–508. <https://doi.org/10.1016/j.ijpe.2012.04.001>
- Volf, P. (2021). Application of Stochastic Optimization to Optimal Preventive Maintenance Problem. *Journal of Applied Mathematics and Physics*, 09(10), 2461–2475.
<https://doi.org/10.4236/jamp.2021.910157>
- Wang, Y., & Pham, H. (2011). Imperfect preventive maintenance policies for two-process cumulative damage model of degradation and random shocks. *International Journal of System Assurance Engineering and Management*, 2(1), 66–77. <https://doi.org/10.1007/s13198-011-0055-8>
- Weinstein, L., & Chung, C.-H. (1999). Integrating maintenance and production decisions in a hierarchical production planning environment. *Computers & Operations Research*, 26(10–11), 1059–1074.
[https://doi.org/10.1016/S0305-0548\(99\)00022-2](https://doi.org/10.1016/S0305-0548(99)00022-2)
- Whitaker, L. R., & Samaniego, F. J. (1989). Estimating the Reliability of Systems Subject to Imperfect Repair. *Journal of the American Statistical Association*, 10.

- Xiao, S., Chen, Z., & Sarker, B. R. (2019). Integrated maintenance and production decision for k -out-of- n system equipment with attenuation of product quality. *International Journal of Quality & Reliability Management*, 36(5), 735–751. <https://doi.org/10.1108/IJQRM-07-2017-0138>
- Yalaoui, A., Chaabi, K., & Yalaoui, F. (2014). Integrated production planning and preventive maintenance in deteriorating production systems. *Information Sciences*, 278, 841–861. <https://doi.org/10.1016/j.ins.2014.03.097>
- Zhao, X., Li, R., Cao, S., & Qiu, Q. (2022). Joint modeling of loading and mission abort policies for systems operating in dynamic environments. *Reliability Engineering & System Safety*, 108948. <https://doi.org/10.1016/j.ress.2022.108948>
- Zied, H., Sadok, T., & Nidhal, R. (2014). *A Jointly Optimization of Production, Delivery and Maintenance planning for multi-warehouse/multi-delivery problem*. 11.

LIST OF PUBLICATIONS

- i. Hajej, Z., Nyoungue, A. C., **Abubakar, A. S.**, & Ali, K. M. (2021). An Integrated Model of Production, Maintenance, and Quality Control with Statistical Process Control Chart of a Supply Chain. *Applied Sciences*, MDPI, 11(9), 4192. <https://doi.org/10.3390/app11094192>
- ii. **Aminu Sahabi Abubakar**, Hajej Zied, Aime C. Nyoungue (2023). Strategic imperfect maintenance optimization with collaborative quality control methods for an integrated production problem, considering assignable causes. *Journal of Risk and Reliability* (Under revision).
- iii. **Abubakar, A. S.**, Nyoungue, A., & Hajej, Z. (2020). Stratégie intégrée de production, maintenance et de carte de contrôle d'une chaîne logistique sous contrainte de qualité. *International Conference on Modelling Optimization and Simulation* (MOSIM 2020, Agadir Morocco.) <https://hal.archives-ouvertes.fr/hal-03178050>
- iv. **Abubakar, A. S.**, Nyoungue, A., & Hajej, Z. (2020). Integrated Production, Maintenance, and Control Chart of Supply Chain Management Under Quality Constraints. *2020 IEEE International Conference on Industrial Engineering and Engineering Management* (IEEM. Singapore) (pp. 1215-1219). <https://doi.org/10.1109/IEEM45057.2020>
- v. **Aminu Sahabi Abubakar**, Hajej Zied, Aime C. Nyoungue. Collaboration of Production, Maintenance, and Quality for a Multi-product, Multi-warehouse Problem, with a Multi-Level Statistical Process Chart of a Supply Chain under Service and Quality requirements. *12th Annual International Conference on Industrial Engineering and Operations Management* (IEOM 2022). <http://ieomsociety.org/istanbul2022/proceedings>
- vi. **Aminu Sahabi Abubakar**, Hajej Zied, Aime C. Nyoungue. An optimal production, maintenance, and quality forecasting Problem, with an improved statistical process chart of a supply chain under service and quality requirements. *The 10th IFAC triennial conference* (MIM 2022 Nantes, France) www.mim2022.com. Available online at <http://www.sciencedirect.com>
- vii. **Aminu Sahabi Abubakar**, Hajej Zied, Aime C. Nyoungue. Stochastic Production, Maintenance, and Quality Policy for Unreliable Production Systems under Service Level Constraint. *International Conference on Trends and Innovations in Management, Engineering, Sciences, and Humanities* (ICTIMESH 2022 - Dubai, UAE) Published by *International Journal of Latest Trends in Engineering and Technology* (IJLTET) ISSN: 2278-621X
- viii. **Aminu Sahabi Abubakar**, Hajej Zied, Aime C. Nyoungue. Integrated Production, Maintenance, and Quality Strategy for Production Systems subject to Several Assignable

Causes of Process Variation. The 20th IEEE International Multi-Conference on Systems, Signals & Devices 2023 Mahdia, Tunisia.

- ix. **Aminu Sahabi Abubakar**, Hajej Zied, Aime C. Nyoungue. A Statistical Process Monitoring Control for an Integrated Production, Maintenance, and Quality Policy of a Forecasting Production Problem. The CIGI Qualita MOSIM congress 2023 at the University of Quebec at Trois-Rivières, Canada. (Accepted)

ABSTRACT

The current production industry is characterising by significant progress: a massive increase in high technology, the emergence of exigent customers, competitive markets, varied product quality, and random demands required the collaborations of the inter-related aspects of production. The production, maintenance, and quality are the most critical aspects of the industrial system. We are interested in optimising maintenance that plays a critical role in customer satisfaction, sustainability, and the development of companies. Our study is centred on developing production and maintenance integrated into a control chart of quality policy. We propose new integrated maintenance strategies with an in-depth analysis of the continuous production process reliability improvements and treated under customer requirements (Service, quality, and cost) constraints. The integration and optimal coordination of these factors of production is not easy and represent a challenge for industrial companies, and are more challenging with multi-assignable causes of production process variation. To address this problem, this research work employed the use of a control chart tool in combination with the AFNOR principles known as Rule of seven as involved in quality management. To monitor, analyses and direct appropriate maintenance actions for continuous system reliability improvements along the different multiple process scenarios as decision levels. We use the interactions between the production process and product quality for strategies of maintenance which reduce the failure of the production system by improving process reliability and reducing the no conformal products. The study Significantly focused on quality, quality tools, and their applications in production and maintenance control. The technique we used to optimise the maintenance and quality control of an integrated production system is a control chart tool based on statistical measurement and analysis of quality parameters. We model different production problems and develop a control policy for randomly failing production systems that must satisfy customer requirements dynamically throughout the finite production horizon. We introduced a mathematical model to minimise the total costs of production, inventory, maintenance, and quality control. The optimisation of the maintenance strategy was integrated into a control chart tool information. Based on the production rate variation and its impact on machine degradation the number of maintenance, times, and intervals needed for prudent decision-making is determined by the developed algorithm. In this context, this research presents the derivation for different cases of all the probabilities for the process to be in or out of control, the average run lengths, and the restoration cycle durations. Based on the degradation factor, new failure rate equations are formulated, and then the average failure rate for each case, we formulated different maintenance cost optimisation models. The proposed approach is useful for precise calculation and minimisation of the total maintenance cost. Which optimises total production cost considering all possible production process statuses due to multiple causes. Our work contributes to the emergence of quality management and performance improvement techniques that will contribute to the development of production companies.

Keywords: Production, Maintenance, Quality, Statistical Process control (SPC) chart, Improved Statistical Process control chart (ISPC), AFNOR Principles, Total production cost optimisation.

RÉSUMÉ

De nos jours, l'industrie de la production se caractérise par des progrès significatifs en termes de : augmentation massive de la haute technologie, émergence de clients exigeants, marchés compétitifs, qualité de produit variée et demandes aléatoires. Cette révolution d'exigences implique interdépendants de tous les processus de l'entreprise, plus ou moins liés à la production. La production, la maintenance et la qualité sont les aspects les plus critiques du système industriel. Dans cette thèse, nous nous intéressons à l'optimisation de la maintenance qui joue un rôle essentiel dans la satisfaction du client, la durabilité et le développement des entreprises. Notre étude est centrée sur le développement des politiques intégrant conjointement la production, la maintenance et la qualité au travers d'une carte de contrôle moyenne/écart type. Nous proposons de nouvelles stratégies de maintenance intégrée avec une analyse approfondie de l'amélioration continue de la fiabilité du processus de production. Cette stratégie tient compte des contraintes et des exigences du client (taux de service, qualité, coût...). L'intégration et la coordination optimale de des facteurs de production n'est pas facile. Celle-ci représente un énorme défi pour les entreprises industrielles. De plus, elles sont plus difficiles avec les causes multi-assignables de la variation du processus de production. Pour répondre à l'ensemble de ces problèmes, nous avons dans ce travail de recherche, utilisé l'outil carte de contrôle en combinaison avec les principes, connus sous le nom de Règle des sept l'AFNOR, impliqués dans la gestion de la qualité.

Pour définir la maintenance appropriée permettant l'amélioration continue de la fiabilité du système pour les différents scénarios de processus, nous utilisons dans cette étude, une politique intégrée de production, qualité du produit et de maintenance. Cette des stratégies nous amène à réduisent la défaillance du système de production en améliorant la fiabilité du processus et en réduisant les produits non conformes. Notre étude a porté de manière significative sur l'utilisation des outils de qualité et leurs applications dans le contrôle de la production et de la maintenance. Nous avons utilisé outil de carte de contrôle pour optimiser la maintenance et le contrôle de la qualité d'un système de production. C'est un outil basé sur la mesure et l'analyse statistique des paramètres de qualité de la production. Nous modélisons différents problèmes de production et développons une politique de contrôle pour des systèmes de production à défaillance aléatoire. Ces systèmes doivent, en toute circonstance, satisfaire les exigences des clients de manière dynamique tout au long de l'horizon de production fini. Nous avons introduit un modèle mathématique permettant de minimiser les coûts totaux de production, d'inventaire, de maintenance et de contrôle de la qualité.

L'optimisation de la stratégie de maintenance a été intégrée dans un outil d'information sur les cartes de contrôle. Sur la base de la variation du taux de production et de son impact sur la dégradation de la machine, le nombre de maintenance, les temps et les intervalles nécessaires à une prise de décision prudente sont déterminés par l'algorithme développé. Dans ce contexte, cette recherche présente pour différents scénarios de toutes les probabilités pour un processus sous contrôle ou hors de contrôle, les durées moyennes d'exécution, et les durées de cycle de restauration. En se basant sur facteur de dégradation, de nouvelles équations de taux de défaillance ont été formulées, puis le taux

de défaillance moyen pour chaque cas. Nous avons formulé différents modèles d'optimisation des coûts de maintenance. L'approche proposée est utile pour le calcul précis et la minimisation du coût total de maintenance. Cette approche optimise le coût total de production en considérant tous les états possibles du processus de production dus à des causes multiples.

Notre travail contribue à l'émergence de techniques de gestion de la qualité et d'amélioration des performances qui contribueront au développement des entreprises de production.

Mots clés : Production, Maintenance, Qualité, Carte de contrôle statistique des processus (SPC), Carte améliorée de contrôle du processus statistique (ISPC), Principes AFNOR, Optimisation du coût total de production.

RÉSUMÉ GÉNÉRAL DE LA THÈSE

Aujourd'hui, les entreprises industrielles sont constamment à la recherche de solutions pour améliorer l'efficacité de la production, minimiser les coûts et maximiser les profits tout en satisfaisant les diverses exigences des clients. Ces exigences sont liées à la qualité des produits, aux délais de livraison ainsi qu'aux niveaux de service.

Notre analyse de ce sujet commence par la présentation de l'état de l'art sur la production, la production couplée à la maintenance, la production et la qualité, puis l'intégration de la production, la maintenance et la qualité. Compte tenu des principaux travaux de la littérature qui sont pertinents pour cette recherche. Nous avons présenté les différentes notions de systèmes de production, y compris leur classification et leurs objectifs. La topologie basée sur la disponibilité des ressources et la demande, la chaîne d'approvisionnement et sa gestion, le cycle de vie et la gestion des déchets. La maintenance des systèmes de production, les principales catégories et la loi de défaillance.

L'état de l'art a brièvement mis en lumière les différents types de politiques de maintenance : maintenance parfaite, minimale et imparfaite, et leur importance dans la gestion des systèmes de production, un aperçu général de l'optimisation de la production et de la maintenance. Enfin, nous avons présenté la description du problème global de la thèse, ainsi que le but principal et les objectifs clés de notre sujet de recherche. L'étude s'est concentrée de manière significative sur la qualité, les outils de la qualité et leurs applications dans le contrôle de la production et de la maintenance. Cette étude est centrée sur le développement de la production et de la maintenance intégrées dans une carte de contrôle de la politique de qualité. Nous proposons de nouvelles stratégies de maintenance intégrée avec une analyse approfondie des améliorations continues de la fiabilité du processus de production. La technique que nous avons utilisée pour optimiser la maintenance et le contrôle de la qualité d'un système de production intégré est un outil de carte de contrôle basé sur la mesure statistique et l'analyse des paramètres de qualité. Nous nous sommes intéressés à l'optimisation de la maintenance qui joue un rôle essentiel dans la satisfaction des clients, la durabilité et le développement des entreprises.

Une exigence accrue de production de produits de haute qualité implique la nécessité d'utiliser des approches modernes pour modéliser et analyser les demandes changeantes des clients, ainsi que des outils informatiques pour soutenir la prise de décision (Jahan et al., 2016). Il est plus important que jamais d'obtenir le meilleur rendement du système de production et de répondre aux diverses exigences des clients (qualité, service et coût) (Kölmel et al., 2014). La satisfaction de diverses exigences des clients nécessite en soi une chaîne d'actions intégrée (Pakurár et al., 2019). À commencer par la planification de la production, les stratégies de maintenance et les politiques de qualité. Compte tenu de ce qui précède, on peut facilement comprendre que les nouveaux produits et les défis des marchés, les attentes exigeantes des clients et la variété des produits ont déjà évolué. Il est donc nécessaire d'adopter des approches nouvelles et améliorées pour permettre une collaboration et une intégration plus efficaces des facteurs de production. Il a été établi que la production, la maintenance et la qualité sont trois facteurs principaux de l'optimisation des systèmes de production, malheureusement, ils ont souvent été étudiés séparément dans la littérature (Xiao et al., 2019). Les études qui ont pris en compte

ces trois concepts ensemble, ont généralement supposé une seule cause assignable dans la variation du processus de production (Salmasnia et al., 2017). Cette hypothèse est une simplification et il est peu probable qu'elle se produise dans les processus de production réels. Il en résultera des critères de performance médiocres (financiers et techniques). Lorsqu'il existe de multiples causes assignables à l'origine d'un changement qui peut différer de la conception de la carte de contrôle. De nos jours, un client peut facilement demander un produit, le modifier ou l'annuler soudainement par le biais d'applications de téléphonie mobile ou de plateformes web (B. Liu et al., 2020). La concurrence actuelle dans l'environnement industriel ne se limite pas à une demande croissante et dynamique, mais aussi à la nécessité d'obtenir des produits de meilleure qualité à des prix compétitifs (Hajej et al., 2021). En effet, ces défis doivent avoir des conséquences sur la planification des opérations, les processus de production, la maintenance des machines et la qualité des produits. Pour relever ces défis, les responsables des entreprises de production doivent rester dynamiques tout au long de l'horizon de production et de la chaîne d'approvisionnement afin de rester compétitifs. Les problèmes de gestion des stocks peuvent également avoir une incidence sur la satisfaction des clients (Mifdal et al., 2015). Les pannes de machines ou les retards dans la livraison des produits ont également d'autres conséquences (Zied et al., 2014). Pour satisfaire les clients, plusieurs entreprises ont optimisé leurs stocks de matières premières, leurs outils de production, leur énergie, leurs opérations ou leurs stocks de produits finis (Van Horenbeek et al., 2013). Récemment, la fonction de maintenance a attiré beaucoup d'attention dans la gestion des entreprises de production ou de services. La planification et l'optimisation stratégiques de la maintenance garantissent la fiabilité et la disponibilité du système de production en stabilisant le processus et en réduisant les coûts grâce à la diminution du nombre d'éléments non conformes (Beheshti Fakher et al., 2017). La stratégie de maintenance permet une planification appropriée de la maintenance en collaboration avec des plans optimaux de production, d'inventaire et de livraison, ce qui permet d'atteindre une efficacité globale des performances de production, de réduire les coûts totaux et d'améliorer la rentabilité (G. Q. Cheng et al., 2018). Nous trouvons des travaux substantiels qui traitent différents problèmes de diverses manières afin que le système puisse répondre à différentes contraintes. Cependant, beaucoup de ces travaux ont été restreints et n'ont pas pris en compte la qualité, les entrepôts multiples ou les différentes causes qui leurs être imputables. Cela justifie le besoin de solutions pour optimiser la production, la maintenance et la qualité qui amélioreront considérablement les exigences de l'industrie (technique) et du marché (économique).

Dans cette thèse, nous nous concentrons sur un problème industriel qui tourne autour du développement de différentes optimisations intégrées de la production, de la planification de la maintenance et du contrôle de la qualité. Une chaîne d'approvisionnement comprend un système de production avec une seule machine qui produit un seul type de produit et un magasin de production primaire S et un entrepôt à achats multiples (w_0, w_1, \dots, w_L) , où la demande du client (articles) est satisfaite. Sur un horizon fini, chaque entrepôt s'efforce de répondre à de nombreuses demandes aléatoires dans une limite de qualité définie H et avec un délai de livraison τ_i ($i = 0, \dots, L$) pour chaque demande.

La machine de production est susceptible de tomber en panne à tout moment. Le taux de production a un impact sur la dégradation de la machine. Par conséquent, le taux de

défaillance $\lambda(t)$ augmente avec l'âge de la machine et le taux de production $u(k)$. Il a également un impact sur la fiabilité du processus de production et conduit à la production de produits non conformes. Pour l'échantillon Xt , nous avons supposé une statistique quantitative de qualité X avec n mesures, contrôlant la machine toutes les h unités de temps pendant la période de production Δt . Les mesures individuelles sont supposées se situer entre les limites supérieures de contrôle UCL et les limites inférieures de contrôle LCL . Dans cette perspective, nous avons développé et proposé de nouvelles politiques pour un modèle intégré et organisé le travail comme suit.

Dans le premier chapitre, nous présentons le contexte de l'étude, mettons en évidence la problématique et les objectifs de notre recherche. Nous essayons ensuite de situer notre étude par rapport aux différents travaux déjà réalisés dans le domaine de la maintenance intégrée à la production. Nous citons également quelques travaux réalisés dans le cadre des politiques de contrôle de la qualité. L'objectif de cette recherche est de modéliser l'impact de la cadence de production sur la dégradation du système de production et de relier les défaillances sur les indicateurs de qualité afin de déterminer la meilleure stratégie de maintenance et de qualité (diagramme de Maîtrise Statistique des Procédés). Et d'établir une politique de contrôle de la production intégrée et collaborative qui minimise le coût total de production.

Le deuxième chapitre est consacré à la modélisation d'un modèle intégré de production, de maintenance et de contrôle de la qualité à l'aide d'un diagramme de contrôle du processus statistique d'un système de production à chaîne d'approvisionnement. L'objectif est alors de trouver le plan de production et de maintenance optimal qui minimise les coûts totaux, caractérisé par le nombre optimal d'actions de maintenance préventive en fonction des plans de production et de gestion des stocks tout au long des périodes de production. Le nombre de produits à programmer, le niveau du stock dans chacun des entrepôts sur chaque durée pour permettre la satisfaction des clients et le nombre optimal d'actions de maintenance préventive sur le processus de production global, en réduisant les produits non-conformes. En résumé, nous avons formulé le modèle du problème stochastique, la politique de production, le cas de production, les équations établies, une étude analytique et les résolutions. Nous utilisons la carte de contrôle comme outil d'aide à la décision pour permettre la détection de toute dérive (défauts) dans la production et pour intervenir en appliquant la stratégie de maintenance appropriée en fonction du plan de production et d'inventaire optimal. Ainsi, elle permet d'augmenter la fiabilité du processus et le niveau de qualité du produit et donc de diminuer les coûts de non-qualité.

La stratégie de maintenance consiste en des actions de maintenance préventive avec réparation minimale (PMMR) pendant l'état de contrôle du processus, planifiées sur l'horizon fini $H \cdot \Delta t$. Lorsque le processus est dans l'état de contrôle, les périodes de production sont divisées en N parties égales. Les activités préventives sont exécutées à chaque $i \cdot T$ ($i = 1, 2, \dots, N$), afin de restaurer l'état de la machine dans un état aussi bon que neuf. Toutefois, si une défaillance survient entre deux actions préventives successives planifiées, la maintenance corrective est donc employée avec une réparation minimale (aussi mauvaise que l'ancienne). On suppose que les durées minimales de réparation et de remplacement sont négligeables lorsque le processus est dans l'état "sous contrôle". Le plan de maintenance est élaboré sur la base du plan de production optimal, qui minimise le taux de défaillance et les coûts de maintenance.

Nous avons réalisé l'étude et l'analyse de la politique de maintenance intégrée à la qualité. Formulation du taux de défaillance des machines, du coût de la maintenance et de la qualité. Expérimentation numérique et conclusions. Nous avons déterminé la stratégie optimale de production, de livraison et de maintenance avec les paramètres optimaux liés à la carte de contrôle. Nous avons atteint nos objectifs en minimisant le coût total de l'inventaire dans les différents magasins, la production et la livraison, et en même temps, le coût lié à la qualité et à la stratégie de maintenance. Ce travail a été présenté aux conférences internationales MOSIM (Agadir-Maroc) et IEEM (Singapour, 2020), et publié dans la revue des sciences appliquées (MPDI - Appl. Sci. 2021, 11, 4192).

Dans le troisième chapitre, nous avons étudié les problèmes de production optimale, de maintenance et de prévision de la qualité, avec un diagramme de processus statistique amélioré d'une chaîne d'approvisionnement soumise à des exigences de service et de qualité. Nous avons conçu et développé un diagramme de contrôle de la qualité plus efficace, à plusieurs niveaux, comme le montre la figure 4 ci-dessous. Nous avons mis au point un modèle mathématique qui réduit considérablement les coûts de production, de maintenance, d'inventaire et de qualité. Nous avons établi une production optimale, un inventaire, une livraison et des paramètres optimaux liés à la carte de contrôle améliorée, avec des limites de surveillance supplémentaires basées sur des décisions optimales. Le travail a été présenté à la conférence IEOM qui s'est tenue à Istanbul-Turquie en 2022 et aux conférences IFAC-MIM, disponible en ligne dans les actes de la conférence, et son extension a été soumise au journal of risk and reliability (JRR). Le système de production est basé sur une seule machine qui ne produit qu'une seule sorte de produit, liée à une chaîne d'approvisionnement. La chaîne d'approvisionnement est composée d'un magasin principal de fabrication S , connecté à des entrepôts multi-achats (w_1, w_2, \dots, w_L) avec une capacité de volume de livraison Q_v et un temps τ_i ($i: 1, \dots, L$), où les clients reçoivent les produits qu'ils ont demandés. Le système fonctionne à un niveau de service donné θ_i , avec des limites de capacité de production: $u_{min} \leq u_k \leq u_{max}$, sur un horizon fini H . La machine M n'est pas fiable, elle est donc sujette à des pannes et à des réparations à tout moment. La détérioration de la machine est influencée par le temps et les cadences de production. Par conséquent, le taux de défaillance $\lambda(t)$ augmente avec le temps, et le taux de production affecte la fiabilité et la capacité du processus de production $u(k)$ responsable des unités non conformes. La stratégie intégrée présente un résumé de notre proposition les trois états du processus de la carte de contrôle et sur la base desquels sont prises les décisions de conserver et de rejeter les produits défectueux. Nous supposons qu'il s'agit d'un problème conjoint de production, de maintenance et de contrôle de la qualité d'un système de fabrication imparfait visant à satisfaire les demandes aléatoires d'un établissement de vente au détail multiple, avec un service donné et des contraintes de capacité. Lorsque le processus est dans un état de contrôle, la production d'articles conformes n'est soumise qu'à une maintenance corrective minimale. Mais lorsque l'opération échoue (hors du contrôle) jusqu'à ce que le processus passe à un stade de surveillance (SL-CL), elle nécessite une maintenance préventive imparfaite d'une durée (t_{imp}) pour réduire le taux de défaillance. Et lorsque l'opération est passée à un état critique "hors contrôle", une action de maintenance parfaite d'une durée significative (t_p) est employée pour rétablir le processus (AGAN). Les variables de décision sont le plan de production économique, le plan de maintenance optimal et la taille de l'échantillon n , l'intervalle de contrôle h et les coefficients des cartes de contrôle k_{imp} and k_p .

Dans le quatrième chapitre de la thèse, nous développons une politique intégrée de maintenance, de production et de qualité à l'aide d'une carte de contrôle du processus statistique en combinaison avec la règle AFNOR, en tenant compte de différentes causes assignables. Nous avons incorporé une surveillance supplémentaire de la qualité en tant que technique d'aide à la décision pour détecter certaines tendances inhabituelles dans le processus de production, nous cherchons à optimiser la planification de la production d'un système de production à trois échelons dans des conditions d'incertitude. Le processus commence par un état de contrôle, produisant des produits conformes avec la capacité d'avoir un taux de production variable u_k et échappe parfois au contrôle dans quatre scénarios possibles : 1, 2, 3 et 4. Les quatre scénarios distincts sont définis par la carte de contrôle en combinaison avec la règle des sept techniques de gestion de la qualité. Dans le cadre de la gestion de la qualité, la règle des sept stipule que "une série de sept points consécutifs ou plus dans une carte de contrôle, soit au-dessus de la moyenne, soit au-dessous de la moyenne, soit en augmentation ou en diminution continue, peut indiquer que le processus n'est pas maîtrisé".

À la fin de l'étude, nous comprenons l'impact de la maintenance imparfaite (IM) sur la fiabilité du système de production, les articles non conformes et le coût total de production. Les résultats ont été présentés aux conférences internationales Trends and Innovation in Management Engineering Sciences and Humanities Dubai 2022, the 20th International Multi-Conference on Systems, Signals & Devices (SSD 2023) - Conference on Systems, Automation & Control, Mahdia Tunisia, et the 50th International Conference on Computers and Industrial Engineering CIE50, 2023.

Ce travail de recherche a examiné et amélioré la conception, le développement et l'application industrielle de nouveaux outils d'aide à la décision pour les systèmes de production intégrés. Nous avons utilisé le diagramme de processus statistique amélioré pour le contrôle et l'optimisation d'un modèle intégré de problèmes de production. Les stratégies optimales de planification de la production, de gestion des stocks et de maintenance sont réalisées en collaboration avec les politiques de contrôle de la qualité afin d'orienter les politiques de maintenance appropriées pour un système de production peu fiable soumis à certaines contraintes de service à la clientèle. La principale contribution de cette recherche est l'étude, l'examen et l'amélioration de la conception et de l'application d'un diagramme de contrôle du processus statistique. Le développement de modèles mathématiques pour des systèmes avancés d'aide à la décision. Cela implique la combinaison de différentes techniques de contrôle de la qualité pour une performance maximale. De nouvelles approches de contrôle et d'optimisation de la production sont développées et étudiées. Nous avons également mis en œuvre une analyse de sensibilité sur l'algorithme développé afin de démontrer sa robustesse. Nous avons également mis en œuvre une analyse de sensibilité sur l'algorithme développé afin de démontrer sa robustesse et sa complétude. Ces facteurs ont été traités de manière optimale et en collaboration, ce qui a permis d'améliorer la fiabilité du système et de réduire le nombre de pannes de machines, de fausses alarmes et de produits de mauvaise qualité, minimisant ainsi le coût total.