



Joint design of control algorithms and communication protocols for Connected and Automated Vehicles

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Conception conjointe d'algorithmes de contrôle et de protocoles de communication pour les véhicules autonomes et connectés

*Joint design of control algorithms and communication
protocols for Connected and Automated Vehicles*

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Mots clés: Systèmes multi-agents, Communications 5G, Algorithme du Consensus, Contrôle de formation, Systèmes hybrides, Filtres de Kalman.

Résumé:

Dans cette thèse, nous nous adressons le problème du contrôle de systèmes multi-agents connectés via des modèles réalistes de systèmes de communication. Nous traitons principalement les systèmes de véhicules connectés et automatisés (CAVs) communiquant via des systèmes de communication 5G qui permettent deux types de communication : la communication directe entre les nœuds, connue sous le nom de communication véhicule-à-véhicule (V2V), et la communication à travers l'infrastructure réseau, qui est la manière traditionnelle de communiquer dans les réseaux cellulaires.

La thèse traite de trois problèmes : premièrement, nous analysons les propriétés de stabilité et de convergence de l'algorithme du consensus pour agents d'intégrateurs du premier ordre en utilisant un schéma d'accès multiple par répartition temporelle (TDMA) pour partager les ressources du réseau d'un canal de communication partagé. La stabilité exponentielle du système considéré est démontrée, et

une borne explicite dépendant des paramètres du système de communication est fournie pour estimer la vitesse de convergence. Ensuite, nous abordons le problème du contrôle de formation d'un groupe de véhicules connectés dans un contexte de communication 5G. Nous proposons un algorithme d'allocation de ressources pour sélectionner les utilisateurs émetteurs afin d'atteindre la formation souhaitée tout en respectant les contraintes imposées par la couche de communication. Enfin, nous étudions les propriétés de stabilité des filtres de Kalman pour les systèmes hybrides, précisément, des systèmes avec une dynamique en temps continu observée à travers des mesures en temps discret. La stabilité d'entrée-à-état (ISS) est démontrée pour de tels systèmes en utilisant une fonction de Lyapunov appropriée. Ce résultat peut être considéré comme une première étape dans l'analyse de la robustesse du système global, car il permet de prendre en compte les effets des erreurs de communication sur la stabilité du système contrôlé.

Title: Joint design of control algorithms and communication protocols for autonomous and connected vehicles

Keywords: Multi-agent systems, 5G communications, Consensus algorithm, Formation control, Hybrid systems, Kalman filters.

Abstract:

In this thesis, we address the problem of control of multi-agent systems connected over realistic models of communication systems. We mainly focus on systems of connected and automated vehicles (CAVs) that communicate through a 5G communication system, which allows two types of communication: direct communication between nodes, known as Vehicle-to-Vehicle (V2V) communications, and communication through the network infrastructure, which is the traditional way of communication in cellular networks.

The thesis discusses three problems: first, we analyze the stability and convergence properties of the consensus algorithm of first-order integrator agents using a time-division multiple access (TDMA) scheme to share the network resources of a broadcast shared communication channel. Exponential stability of the consid-

ered system is proved, and an explicit bound depending on the communication system parameters is provided to estimate the convergence rate. Second, we treat the problem of formation control of a float of connected vehicles in a 5G communication context. We propose a resource allocation algorithm to select the transmitting users to achieve the desired formation while satisfying the constraints imposed by the communication system. Finally, we study the stability properties of Kalman filters for hybrid systems, i.e., systems with continuous-time dynamics observed through discrete-time measurements. Input-to-state stability (ISS) is proved for such systems relying on an appropriate Lyapunov function. This result can be considered as a first step in the robustness analysis of the overall system since it allows to treat the effects of communication errors on the controlled system stability.

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APERÇU DE LA THÈSE

Cette thèse présente un résumé du travail accompli durant trois années sur le problème de la *conception conjointe d'algorithmes de contrôle et de protocoles de communication pour les véhicules connectés et automatisés* - connected and automated vehicles (CAVs). Les CAVs sont des véhicules qui intègrent des technologies de communication et des fonctions de conduite automatisée pour assister ou remplacer les humains dans la tâche de conduite grâce à des capteurs embarqués, des systèmes de positionnement géographiques et des systèmes de télécommunications [49]. Le cadre général de cette thèse est celui du contrôle des systèmes dynamiques composés de plusieurs véhicules qui échangent des informations via un canal de communication sans fil afin d'atteindre un objectif commun. Le concept de conception conjointe doit être interprété dans le sens où le système de communication est spécifiquement optimisé ou conçu pour atteindre un objectif de contrôle, plutôt que d'être optimisé uniquement par rapport aux performances liées à la communication. Cette thèse propose également des méthodologies pour analyser les performances des algorithmes de contrôle distribué en tenant compte de l'impact réel des protocoles de communication à partir d'une modélisation plus fine des systèmes de communications.

Dans les applications multi-véhicules, il existe deux problèmes principaux à résoudre : d'abord, la conception de contrôleurs locaux pour contrôler le comportement de chaque véhicule individuellement, et ensuite, la manière de gérer l'échange d'informations au sein du groupe pour atteindre l'objectif commun. Le problème de contrôle de tels systèmes se situe à la frontière de deux domaines scientifiques qui sont la *théorie du contrôle* et les *télécommunications*. Pour la communauté du contrôle, l'objectif est d'atteindre un niveau de performance souhaité compte tenu des caractéristiques du système à contrôler et des conditions de l'environnement dans lequel le système évolue. Pour la communauté des télécommunications, l'objectif est de garantir une transmission réussie des informations entre les nœuds connectés compte tenu des caractéristiques et capacités du système de communication. Le sujet de recherche de cette thèse se concentre sur la combinaison d'outils des deux domaines pour analyser et concevoir des systèmes dynamiques intégrant d'une manière conjointe le contrôle et les télécommunications.

Dans les applications liées à plusieurs véhicules, l'objectif du contrôle peut être d'atteindre une destination commune, connu sous le nom du problème du rendez-vous [25] [117], ou une vitesse commune, comme dans les problèmes de *platooning* [38] [120] [121] [122] [148], ou d'imposer aux véhicules de suivre une trajectoire tout en maintenant une configuration spatiale prédéfinie. Ce problème est connu dans la littérature sous le nom du problème du *contrôle de formation* [6] [8] [96] [110] [114]. Les problèmes cités ont suscité un intérêt considérable au cours des deux dernières décennies en raison de leur importance dans de nombreuses

applications d'ingénierie telles que celles liées aux véhicules autonomes [65] [10] [79], aux robots mobiles [82] [107], aux véhicules aériens sans pilote [26] [8], aux vaisseaux spatiaux [126], aux réseaux de capteurs [77], etc. Les mêmes techniques peuvent être appliquées à l'analyse du comportement social sur les réseaux sociaux [112] [113] ou dans les systèmes énergétiques distribués [90] [131], entre autres applications.

La conception de contrôle pour de tels systèmes repose sur les modèles de *systèmes multi-agents* - multi-agent systems (MASs) où le système étudié est composé de plusieurs sous-systèmes appelés *agents* [99] [101]. Ces agents sont représentés par des systèmes dynamiques qui échangent des informations les uns avec les autres à travers un système de communication. Les systèmes dynamiques, c'est-à-dire les systèmes évoluant dans le temps, sont généralement divisés en deux classes de systèmes : les systèmes *en temps continu* et les systèmes *en temps discret*. En effet, les circuits électroniques, les réactions chimiques ou les systèmes mécaniques sont naturellement décrits en temps continu selon les principes fondamentaux de la physique, tels que les lois du mouvement de Newton, les lois de la thermodynamique, les équations de Maxwell, etc. Cependant, les systèmes numériques tels que les microcontrôleurs, les réseaux numériques ou les algorithmes informatiques sont décrits en temps discret. Dans de nombreux systèmes récents tels que les systèmes cyber-physiques [21], on trouve les deux types de dynamique dans un même système. Par exemple, les actionneurs physiques tels que les moteurs électriques ou les vérins hydrauliques sont contrôlés par un système informatique. Dans les applications réelles impliquant des MASs, des robots mobiles, des drones ou des véhicules connectés, les systèmes sont contrôlés de la même manière par des contrôleurs distants envoyant des informations de contrôle via des canaux de communication sans fil. De plus, les systèmes modernes favorisent des architectures distribuées, où les contrôleurs sont implémentés à bord mais doivent être alimentés par des informations reçues des autres agents, ce qui ajoute un niveau de complexité supplémentaire.

L'analyse des systèmes multi-agents peut être divisée en trois niveaux : premièrement, le contrôle individuel, c'est-à-dire le contrôle du comportement de chaque agent individuellement et indépendamment des autres agents. À ce niveau, l'agent est contrôlé pour suivre une instruction donnée, comme suivre une trajectoire de référence ou atteindre une destination prédéfinie. Deuxièmement, le contrôle global ou distribué, c'est-à-dire le contrôle du comportement de l'ensemble des agents. À ce niveau, on considère le groupe d'agents comme un système unique où les différents composants interagissent les uns avec les autres via l'échange d'informations. L'échange d'informations influence le comportement du système global. Troisièmement, le niveau de communication qui englobe les stratégies et les techniques permettant aux agents de transmettre des informations à travers le réseau. La couche de contrôle individuel est, d'une part, affectée par les exigences de la couche de contrôle globale. Par exemple, un véhicule ne peut pas exécuter

certaines manœuvres pour éviter des collisions avec les autres véhicules du groupe. D'autre part, la couche de contrôle individuel affecte la couche de contrôle globale en imposant certaines contraintes sur le comportement ciblé du groupe qui dépend des performances et des caractéristiques de chaque véhicule. De même, les couches de contrôle globale et de communication s'influencent mutuellement en fonction des caractéristiques et des capacités du système de communication et des performances ciblées en matière de contrôle et d'échange d'informations. Par exemple, les performances de l'application de contrôle sont affectées par les capacités du système de communication telles que le débit, la latence, la bande passante, etc. Le système de communication, à son tour, peut adapter son fonctionnement en priorisant certains messages provenant de certains agents par rapport à d'autres pour optimiser l'échange d'informations entre les agents afin d'atteindre une performance de contrôle plus élevée.

Au niveau du contrôle individuel, on se concentre sur la dynamique des agents dont la complexité peut varier significativement allant de modèles simples tels que les systèmes d'intégrateur simple ou de double intégrateur [114] à des modèles plus complexes tels que les systèmes linéaires et non linéaires multidimensionnels [60]. De plus, ces dynamiques peuvent être exprimées en temps discret [115] ou en temps continu [99]. Ces deux types de formulations ont des propriétés différentes qui doivent être prises en compte, en particulier lors de la considérations des effets des deux autres niveaux, notamment le contrôle globale et la couche de communication.

Au niveau du contrôle global ou distribué, le point crucial est de déterminer quelles informations doivent être échangées entre les agents, quand elles sont transmises et comment elles sont utilisées pour atteindre l'objectif de contrôle commun. On distingue deux cas : celui des systèmes homogènes où tous les agents ont la même dynamique [137], et celui des systèmes hétérogènes où les agents ont des dynamiques différentes [103] [152]. La deuxième classe de MASs nécessite une analyse plus approfondie, car le niveau de contrôle individuel de chaque agent doit être étudié séparément.

Le niveau de la couche de communication spécifie le protocole de communication qui décrit la manière dont l'information est échangée entre les agents, ce qui est crucial pour déterminer le comportement du système global. Les informations et les données transmises d'un agent à un autre sont utilisées pour calculer et mettre à jour la décision de contrôle de l'agent, qui est influencée par les informations reçues. Par exemple, dans la navigation coopérative des véhicules, les informations échangées permettent aux véhicules d'adapter leurs manœuvres pour éviter les collisions ou réduire la consommation d'énergie. Cependant, lorsque les informations échangées sont erronées ou mal interprétées, cela peut conduire à un comportement indésirable du système, allant jusqu'à des problèmes de sécurité potentiels. Par conséquent, les imperfections du système de communication doivent être prises en compte lors de la conception du contrôleur au niveau de chaque agent.

Dans la plupart des résultats liés aux systèmes multi-agents présents dans la littérature, les chercheurs dans le domaine du contrôle font abstraction de la couche de communication en utilisant des modèles simples. Les modèles sélectionnés correspondent généralement aux conditions de communication parfaite. Alternativement, des hypothèses fortes, qui ne peuvent pas être satisfaites par de vrais systèmes de communication sans fil, sont formulées telles qu'une topologie du réseau fixe et invariante dans le temps, un flux continu d'informations ou des ressources de communication illimitées comme un débit de données infini. Certains des résultats présentés dans la littérature tiennent compte des imperfections de la couche de communication en utilisant différents cadres : des systèmes avec retard pour représenter le délai entre la transmission et la réception [99] [135], des systèmes de commutation pour capturer les changements dans la structure de la topologie du réseau [115], des systèmes à échantillonnage pour capturer les effets liés à la nature en temps discret du système de communication [151]. Dans de nombreux résultats, les erreurs dues au système de communication sont modélisées par un bruit de communication qui peut être stochastique [59] [78] ou déterministe [31], en fonction des hypothèses utilisées par les auteurs. Cependant, ces modèles restent insuffisants pour représenter fidèlement les imperfections des systèmes de communication, d'où la nécessité de faire appel à des modèles plus complexes tels que *les systèmes hybrides*.

Les systèmes comprenant des dynamiques en temps continu et en temps discret sont appelés systèmes hybrides. Goebel et al. présentent un cadre approprié pour étudier les systèmes dynamiques hybrides [48]. Plusieurs systèmes relèvent du cadre des dynamiques de systèmes hybrides, tels que les systèmes à échantillonnage [29], les systèmes de commutation [15], les systèmes déclenchés par des événements [53], les systèmes impulsifs [80], etc. L'idée novatrice principale de la modélisation des systèmes dynamiques hybrides dans le cadre des systèmes dynamiques hybrides [48] est d'étendre le domaine du temps à un domaine bidimensionnel, un domaine en temps continu, appelé le domaine flux (flow), et un domaine en temps discret, appelé le domaine saut (jump). Ainsi, l'état du système étudié dépend de deux variables temporelles incorporant les deux dynamiques.

Malgré les résultats intéressants liés à la coordination d'agents connectés sur des réseaux de communication imparfaits, les modèles utilisés dans la littérature sont insuffisants pour traiter les problèmes réels rencontrés lors de l'utilisation de systèmes de communication modernes tels que la technologie Wi-Fi, basée sur la famille de normes IEEE 802.11 [57], ou les systèmes de réseau cellulaire tels que ceux spécifiés par le 3rd Generation Partnership Project (3GPP), avec la technologie actuelle, la 5G [34]. Les principaux problèmes dans ces réseaux sont liés à la congestion du réseau, à la perte de paquets, aux interférences de signaux, à la mobilité des utilisateurs, à la perte de couverture, etc.

Les applications liées aux communications véhiculaires sont destinées à être utilisées sur de vastes zones, et doivent donc de préférence s'appuyer sur la tech-

nologie des réseaux cellulaires qui a été conçue dans cette optique. Le principe des *réseaux cellulaires* consiste à diviser la zone terrestre en régions appelées *cellules*. Chaque cellule est desservie par au moins un émetteur-récepteur fixe appelé la *station de base* - base station (BS). La station de base fournit des ressources radio qui peuvent être utilisées pour transmettre des informations, telles que des messages courts, de la voix ou des données. Typiquement, chaque cellule utilise un ensemble différent de fréquences par rapport aux cellules voisines, pour éviter les interférences et pour fournir une bande passante garantie. Lorsqu'elles sont réunies, ces cellules fournissent une couverture radio sur une vaste zone géographique permettant à plusieurs émetteurs-récepteurs, appelés *équipements utilisateur* - user equipment (UE), de communiquer entre eux en transmettant des messages via les stations de base. Les stations de base dans les réseaux cellulaires sont connectées à un réseau central qui permet de router l'information entre les UE connectés à différentes stations de base, ou vers un réseau externe, typiquement Internet. Les UE ont la capacité de se déplacer d'une cellule à une autre sans perdre leur connectivité, ce qui permet la mobilité des utilisateurs tels que les téléphones mobiles, les ordinateurs portables et les véhicules connectés. Les normes des réseaux cellulaires sont spécifiées par le 3GPP, qui est une organisation responsable du développement et de la maintenance des technologies de communication mobile. La technologie des réseaux cellulaires commerciaux a connu différentes évolutions. Jusqu'à présent, cinq générations de réseaux cellulaires ont été spécifiées et déployées, correspondant à une amélioration technologique significative, des premiers systèmes analogiques de la première génération aux systèmes de cinquième génération (5G) actuels. Comparée à sa prédécesseuse, chaque génération offre une plus grande capacité et un débit plus élevé, permettant de connecter davantage d'utilisateurs et de transmettre plus de données en moins de temps. Par conséquent, de nouveaux cas d'utilisation et applications deviennent concevables et réalisables. Comparée à la 4G, la 5G a été spécifiée pour fournir un débit de données 100 fois plus élevé, allant jusqu'à 10 gigabits par seconde, et une latence 250 fois plus faible allant jusqu'à 1 ms avec un support amélioré des mécanismes de qualité de service [30]. Ces améliorations ouvrent la voie à des applications à délai critique, notamment les communications véhiculaires en temps réel. Comme pour la 4G, la 5G propose un cadre de communication véhiculaire appelé Cellular Vehicle-to-Everything (C-V2X). En 4G, le composant C-V2X cible des cas d'utilisation liés à la sécurité en transmettant des messages de conscience coopérative - cooperative awareness messages (CAM) et des messages de sécurité de base - basic security messages (BSM) [34]. Outre la sécurité, le cadre 5G C-V2X se concentre sur l'assistance à la conduite avancée afin d'améliorer la fluidité de la circulation, de réduire la consommation d'énergie et d'augmenter la connectivité entre les utilisateurs routiers véhiculaires et non véhiculaires.

La principale contribution de cette thèse réside dans ce contexte à travers les trois aspects suivants. Premièrement, la prise en compte de modèles réalistes de

la couche de communication. Deuxièmement, l'amélioration de la modélisation du système en combinant les cadres de travail et les outils du contrôle et des communications. Enfin, la considération du système de contrôle-communication comme un système unique et global, où les paramètres liés au contrôle et à la communication sont optimisés conjointement pour atteindre l'objectif visé. Le paradigme traditionnel consiste à séparer la couche de communication de la couche applicative, qui est le contrôle des véhicules dans notre cas. Par exemple, dans le problème d'allocation de ressources ou le problème de contrôle de puissance, un problème d'optimisation est généralement posé où l'objectif est de trouver le schéma d'allocation de ressources optimal ou les niveaux de puissance optimaux pour atteindre le meilleur débit moyen dans le réseau. Du côté du contrôle, la couche de communication est considérée comme une contrainte donnée, et le contrôleur est conçu pour traiter le scénario le plus défavorable pour les applications liées à la sécurité ou pour atteindre un niveau de performance moyen, qui ne peut pas être garanti en permanence. Contrairement aux systèmes traditionnels, le paradigme de co-conception ou de conception conjointe consiste à considérer la couche de communication et la couche de contrôle comme deux composantes d'un système unique. Ces composantes interagissent entre elles grâce à des messages de rétroaction permettant d'atteindre une performance plus élevée. D'une part, le système contrôlé peut envoyer des demandes à la couche de communication afin de réserver les ressources nécessaires pour atteindre le niveau de performance souhaité. D'autre part, la couche de communication peut également informer le système contrôlé des conditions du réseau, telles que les niveaux de congestion dans le réseau, et exiger une adaptation de la performance visée.

L'idée de la co-conception ou de l'optimisation conjointe de l'algorithme de contrôle et du protocole de communication est exploitée dans certains résultats récents de la littérature. Cependant, ces résultats se limitent au problème du peloton - platooning. Par exemple, dans [89], Mei et al. proposent un schéma conjoint d'allocation de ressources radio et d'adaptation des paramètres de contrôle pour le peloton de véhicules. Un problème d'optimisation est formulé pour optimiser conjointement l'allocation de ressources radio pour la diffusion des messages CAM et les gains du régulateur de chaque véhicule afin de maintenir l'espacement inter-véhiculaire ciblé tout en garantissant la stabilité en chaîne du peloton de véhicules et les exigences de fiabilité des communications V2V. Dans [149], les auteurs présentent un cadre combiné de communication et de contrôle pour analyser la performance et la fiabilité de pelotons de véhicules connectés via des liaisons de communication sans fil. Ils proposent une contrainte sur les gains du régulateur pour garantir à la fois la stabilité du système et la stabilité en chaîne du peloton. Ils déterminent ainsi le délai de bout en bout maximum, comprenant les délais d'attente, de traitement et de transmission, qui peut être toléré sans perdre les propriétés de stabilité. De plus, ils proposent un problème d'optimisation pour sélectionner les paramètres du régulateur afin d'améliorer la fiabilité et la

performance du réseau de communication sans fil pour les systèmes de peloton. De même, dans [141], un problème de communication et de contrôle est étudié où les auteurs proposent une stratégie d'allocation de ressources permettant la coopération de pelotons à plusieurs voies et un algorithme de contrôle de puissance pour maximiser la taille du peloton et minimiser la consommation d'énergie. Ces exemples montrent l'importance de la modélisation des systèmes de contrôle et de communication conjointement pour fournir une solution efficace au problème étudié.

Dans cette thèse, nous adoptons le nouveau paradigme de co-conception ou de conception conjointe où les couches de contrôle et de communication sont considérées comme un système unique, et l'objectif est d'optimiser conjointement les paramètres des deux couches pour atteindre une meilleure performance en ce qui concerne l'application considérée, ici, le contrôle distribué de plusieurs véhicules. Par rapport aux publications citées, nous proposons un algorithme générique qui peut être utilisé dans plusieurs applications de contrôle véhiculaire. En effet, nos résultats ne se limitent pas aux pelotons, mais peuvent gérer toute configuration spatiale ciblée. De plus, nous utilisons les paramètres de la couche de communication comme des entrées de la couche du contrôle pour piloter la flotte de véhicules.

Ici, nous présentons la structure de la thèse et résumons brièvement les principaux résultats et contributions.

Le chapitre 1 est un chapitre préliminaire où nous introduisons brièvement les notions de base des systèmes de communication et de la théorie du contrôle utilisées dans cette thèse. Nous présentons le cadre technique dans lequel nos travaux ont lieu et expliquons la terminologie utilisée dans les chapitres suivants. La première section se concentre sur les systèmes de communication et les réseaux sans fil, et la deuxième partie se concentre sur le problème de consensus et la modélisation des réseaux pour les systèmes multi-agents. Dans la partie dédiée à la communication, nous nous concentrons principalement sur les techniques d'accès multiples utilisées dans les systèmes de communication pour gérer l'attribution des ressources aux utilisateurs partageant le même canal de communication. Cette notion est cruciale pour introduire les modèles de communication utilisés dans les chapitres 2 et 3. La partie contrôle présente les principales notions de la théorie des graphes utilisées pour étudier l'algorithme de consensus. Les différentes définitions sur la connectivité des graphes sont énoncées, et nous résumons les principaux résultats liés à la convergence de l'algorithme de consensus. Ensuite, nous présentons un aperçu du modèle de Hegselmann-Krause (HK) [54] que nous utilisons dans le chapitre 3. Une section spécifique est consacrée au résultat de Moreau [92] qui utilise une approche similaire à celle que nous utilisons dans le chapitre 2. Nous terminons ce chapitre en rappelant un théorème sur la stabilité exponentielle des systèmes non linéaires qui est discuté dans le chapitre 4.

Dans le chapitre 2, nous abordons le problème du consensus pour des systèmes multi-agents connectés via un canal de communication partagé. Dans les

applications réelles, les systèmes multi-agents utilisent un système de communication pour échanger des informations entre les agents. L'échange d'informations ne peut pas toujours être considéré comme parfait, comme c'est le cas dans la majorité des contributions liées au problème du consensus. La couche de communication a ses contraintes. En effet, pour communiquer entre eux, les agents ont besoin d'un support de communication généralement partagé entre eux, et l'accès à ce support est défini par certaines règles. Pour les réseaux de communication sans fil, les ressources du réseau en termes de temps et de fréquence ne peuvent pas être occupées par plus d'un utilisateur à la fois. Sinon, des interférences et des collisions de paquets peuvent se produire, et les informations transmises peuvent être perdues. Dans cette contribution, nous présentons une classe particulière d'algorithmes de consensus pour des systèmes multi-agents d'intégrateurs simples en temps continu communiquant via un canal de diffusion partagé. Pour traiter les problèmes d'interférences et de collisions de paquets, nous considérons un protocole d'accès multiple par répartition dans le temps - time-division multiple access (TDMA) représenté par une topologie de commutation où un seul agent peut transmettre à la fois. Une fonction de Lyapunov est proposée pour prouver la convergence exponentielle de cette classe d'algorithmes de consensus sous certaines hypothèses non restrictives. Nous déterminons également une borne explicite pour caractériser la convergence vers le consensus de tels systèmes. La borne donnée dépend des caractéristiques du protocole de communication, fournissant un outil puissant pour mieux concevoir le protocole de communication afin d'améliorer les performances de contrôle.

Dans le chapitre 3, nous abordons le problème du contrôle de formation pour un réseau de véhicules connectés communiquant via un système de communication 5G C-V2X avec des ressources réseau limitées. Le problème du contrôle de formation consiste à faire suivre à un groupe d'agents mobiles une trajectoire de référence tout en maintenant un motif ou une configuration spatiale prédéfinie. Pour résoudre ce problème, nous proposons un nouveau schéma combiné de commande de mouvement et d'allocation de ressources pour permettre aux véhicules d'atteindre la formation ciblée. Dans ce chapitre, nous nous concentrons d'abord sur la formulation du problème de commande de formation sous les contraintes imposées par le système de communication, à savoir, la ressource disponible limitée et la préservation de la connectivité. Ensuite, nous énonçons le problème d'allocation de ressources et présentons un algorithme d'optimisation pour sélectionner les agents émetteurs. L'algorithme proposé utilise le modèle du système pour prédire le comportement des véhicules et vérifier si les contraintes du système de communication sont satisfaites pour une configuration donnée d'agents émetteurs. L'objectif de cet algorithme est de sélectionner un sous-ensemble d'agents et de leur permettre de transmettre leurs états à leurs voisins pour converger vers la formation ciblée. Cette méthode utilise un schéma centralisé-distribué qui relève du Mode 1 d'allocation des ressources pour la communication V2V en 5G [34]. La

station de base attribue les ressources réseau aux agents sélectionnés de manière centralisée. Ensuite, l'algorithme du contrôle de formation est exploité de manière distribuée en s'appuyant sur une communication directe entre les agents. Une condition formelle sur la configuration spatiale initiale des véhicules est donnée pour assurer la convergence vers la formation souhaitée tout en préservant la connectivité de la formation et en utilisant efficacement les ressources du réseau.

Les systèmes contrôlés reposant sur des mesures de capteurs et une communication sont sujets à des erreurs dues aux imperfections du matériel de capteurs ou aux pertes de paquets dans le système de communication [41]. Ainsi, les performances des contrôleurs peuvent être fortement dégradées, entraînant des problèmes de stabilité voire de sécurité. Il est donc crucial de prendre en compte les imperfections des systèmes de communication qui peuvent être modélisées comme du bruit affectant le système aux instants de réception de l'information. Dans le chapitre 4, nous abordons le problème de la filtration du bruit de mesure pour les systèmes hybrides en utilisant les filtres de Kalman [73]. En effet, dans les applications du monde réel, les systèmes physiques évoluent en temps continu, et les informations externes telles que les mesures de capteurs, les observations ou les commandes de contrôle sont reçues en temps discret via un système de communication. Les deux modèles, en temps continu et en temps discret, présentent des lacunes. Le modèle en temps continu suppose que le flux d'observations est continu, et le modèle en temps discret ignore la dynamique du système pendant l'intervalle entre deux observations. Il est donc essentiel d'introduire une formulation hybride du filtre de Kalman qui combine la dynamique en temps continu du système et l'aspect en temps discret du flux d'observations reçues. Dans [124], Särkkä présente le principe de filtrage hybride, appelé par l'auteur filtrage continu-discret, et dérive les équations de la version hybride du filtre de Kalman à partir du cadre de l'inférence bayésienne sous l'hypothèse du bruit blanc gaussien d'espérance nulle. Dans ces travaux, nous étudions les propriétés de stabilité du filtre proposé dans le cadre déterministe. Nous proposons une caractérisation basée sur une analyse de stabilité en utilisant une fonction de Lyapunov pour prouver la stabilité exponentielle d'entrée-état - input-to-state stability (ISS) pour les systèmes hybrides. ISS est une notion de stabilité utilisée pour étudier la stabilité des systèmes dynamiques en présence d'entrées externes, telles que des perturbations provenant de l'environnement, des bruits de mesures ou des erreurs de modélisation. Cette notion est cruciale pour l'analyse de la robustesse des systèmes. Dans ce chapitre, une nouvelle formulation de l'algorithme du filtre de Kalman pour les systèmes hybrides est donnée en utilisant le cadre des systèmes dynamiques hybrides [48], où nous considérons la dynamique du système en temps continu et les observations en temps discret. En utilisant une analyse de stabilité basée sur une fonction de Lyapunov, nous prouvons la stabilité exponentielle d'entrée-état de l'erreur d'estimation du filtre hybride de Kalman sous des hypothèses légères sur le modèle du système. Le chapitre se termine par un résultat général sur la

stabilité exponentielle pour les systèmes non linéaires variant dans le temps.

Nous concluons cette thèse par un chapitre de conclusion où nous exposons certaines perspectives et directions de recherche qui peuvent être explorées en fonction des contributions et des résultats présentés dans cette thèse.

Les résultats de cette thèse sont publiés dans les articles suivants :

- Bechihi, Adel, Elena Panteley, and Arnaud Bouttier. "Exponential Convergence of the Consensus Algorithm over a Shared Broadcast Channel." 2021 60th IEEE Conference on Decision and Control (CDC). IEEE, 2021.
- Bechihi, Adel, Elena Panteley, Pierre Duhamel, and Arnaud Bouttier. "A Resource Allocation Algorithm for Formation Control of Connected Vehicles." IEEE Control Systems Letters 7 (2022): 307-312.
- Bechihi, Adel, Elena Panteley, and Arnaud Bouttier. "Input-to-state stability for hybrid Kalman filters." IFAC-PapersOnLine 55.20 (2022): 295-300.

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ABBREVIATIONS

2G	Second Generation
3G	Third Generation
3GPP	Third Generation Partnership Project
4G	Fourth Generation
5G	Fifth Generation
BS	Base Station
BSM	Basic Safety Message
C-V2X	Cellular Vehicle-to-Everything
CAM	Cooperative Awareness Message
CAV	Connected and Automated Vehicle
CDMA	Code-Division Multiple Access
CG	Configured Grant
CSMA	Carrier Sense Multiple Access
D2D	Device-to-Device
DG	Dynamic Grant
DL	Downlink
DLL	Data Link Layer
DSRC	Dedicated Short-Range Communication
FD	Frequency-Division
FDMA	Frequency-Division Multiple Access
FDX	Full-duplex
gNB	next Generation Node Base Station
GPS	Global Positioning System
GSM	Global System for Mobile Communications
HDX	Half-duplex
HK	Hegselmann-Krause
ISS	Input-to-State Stability
IEEE	Institute of Electrical and Electronics Engineer
LLC	Logical Link Control
LTE	Long Term Evolution
LTI	Linear Time-Invariant
LQR	Linear Quadratic Regulator
MAC	Medium Access Control
MAS	Multi-Agent System
MIMO	Multiple Input Multiple Output
NR	New Radio

OFDMA	Orthogonal Frequency-Division Multiple Access
OSI	Open Systems Interconnection
PER	Packet Error Rate
PID	Proportional–integral–derivative
QSC	Quasi-Strong Connectivity
RB	Resource Block
RSU	Road-Side Unit
SC	Strong Connectivity
SDMA	Space-Division Multiple Access
SL	Sidelink
SPS	Semi-Persistent Scheduling
TDMA	Time-Division Multiple Access
UE	User equipment
UL	Uplink
UQSC	Uniform Quasi-Strong Connectivity
V2I	Vehicle-to-Infrastructure
V2N	Vehicle-to-Network
V2P	Vehicle-to-Pedestrian
V2V	Vehicle-to-Vehicle
V2X	Vehicle-to-Everything
WAVE	Wireless Access in Vehicular Environment
Wi-Fi	Wireless Fidelity
WLAN	Wireless Local-Area-Network

INTRODUCTION

This thesis presents a summary of the work accomplished during three years on the problem of *co-design of control algorithms and communication protocols for connected and automated vehicles* (CAVs). CAVs integrate connectivity and automated driving technologies to assist or substitute humans in the task of driving using on-board sensors, positioning systems, and telecommunication systems [49]. The general framework of this thesis is the one of the controlling of dynamical systems composed of multiple vehicles that exchange information through a wireless communication channel in order to reach a common objective. The concept of co-design is to be understood here in the sense that the communication system is specifically optimized or designed to fulfil a control objective, instead of being optimized solely with respect to communication-only performance. This thesis also provides methodologies for analyzing distributed control algorithms performance accounting for the actual impact of communication protocols.

In multi-vehicle applications, there are two main problems to address: first, the design of local controllers to control the behaviour of each vehicle individually, and second, the way to handle the information exchange within the group to achieve the common objective. The problem of controlling such systems is placed on the edge between two scientific fields which are *control theory* and *telecommunication*. For the control community, the objective is to achieve a targeted performance level given the controlled system characteristics and the environment conditions. For the telecommunication community, the goal is to ensure successful information transmission between the connected nodes given the network features. The research topic of this thesis focuses on combining tools from the two fields to analyze and design dynamical systems integrating control and telecommunication.

In applications related to multiple vehicles, the control objective may be to reach a common destination, known as the rendezvous problem [25] [117], a common velocity, as in *platooning* problems [38] [120] [121] [122] [148], or to be set in a predefined spatial configuration, known as the *formation control* problem [6] [8] [96] [110] [114]. The cited problems have been of considerable interest in the last two decades due to the importance of these algorithms in many engineering applications such as autonomous vehicles [65] [10] [79], mobile robots [82] [107], unmanned air vehicles [26] [8], spacecrafts [126], networks of sensors [77], etc. The same techniques can be applied to the analysis of social behaviour on social networks [112] [113] or in distributed energy systems [90] [131], among other applications.

Control design for such systems rely on *multi-agent systems* (MAS) models where the studied system is composed of subsystems called *agents* [99] [101]. These agents are represented by dynamical systems that exchange information with each other using a communication system. Dynamical systems, i.e., systems

evolving in time, are generally divided into two classes of systems: *continuous-time* systems or *discrete-time* systems. Indeed, electronic circuits, chemistry reactions, or mechanical systems are naturally described in continuous time according to the basic principles of physics, such as Newton's laws of motion, thermodynamics laws, Maxwell's equations, etc. However, digital systems such as micro-controllers, digital networks, or computer-based algorithms are described in a discrete-time framework. In many recent systems like cyber-physical systems [21], we find both types of dynamics in a unique system. For example, physical actuators such as electrical motors or hydraulic cylinders are controlled by a computer system. In real-world applications involving MASs, mobile robots, drones, or connected vehicles are controlled, in the same way, by remote controllers sending control information through wireless communication channels. Moreover, modern systems foster distributed architectures, where controllers are implemented on board but require to be fed by information received from the other agents, which adds an additional level of complexity.

The analysis of multi-agent systems can be split into three levels: first, the individual control, i.e., the control of the behaviour of each agent independently. At this level, we control the agent to follow a given instruction, such as tracking a reference trajectory or reaching a destination. Second, the global or distributed control, i.e., the control of the agents behaviour altogether. At this level, we consider the group of agents as a unique system where the different components interact with each other through information exchange. The information exchange influences the behaviour of the global system. Third, the communication level which encompasses the strategies and techniques that allow agents to transmit information through the network. Figure 1 illustrates the interaction between the three levels. The individual control layer is, on the one hand, affected by the requirements of the global control layer. For example, a vehicle cannot execute some maneuvers to avoid collisions with the other vehicles within the group. On the other hand, the individual control layer affects the global control layer by imposing some constraints on the targeted behaviour of the group that depends on the performance and the characteristics of each vehicle. Similarly, the global control and communication layers are mutually influenced depending on the communication system features and capabilities and the control targeted performance and information exchange requirements. For example, the control application performance is affected by the communication system capabilities such as the throughput, the latency, the bandwidth, etc. The communication system, in turn, may adapt its scheduling by prioritizing some agents over others to optimize the information exchange between the agents in order to achieve a higher control performance.

For the individual control level, we focus on the dynamics of the agents and their complexity ranging from simple models such as single-integrator or double-integrator systems [114] to more complex models such as multi-dimensional linear and non-linear systems [60]. In addition, these dynamics can be expressed in

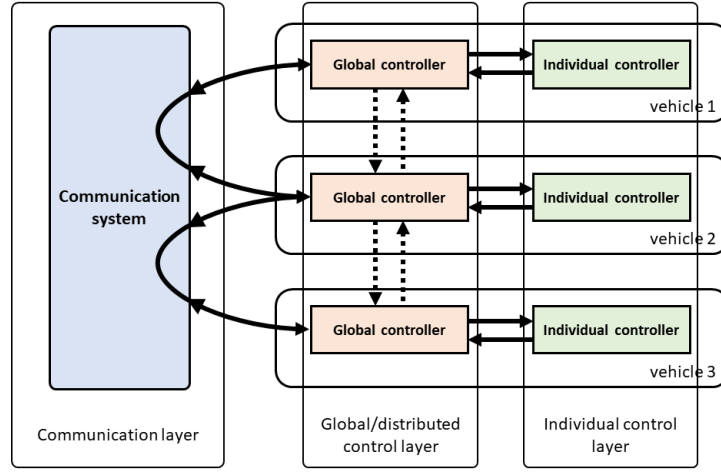


Figure 1: Three-level architecture: the individual control is adjusted according to the global control requirements, which is, in turn, influenced by the communication layer features. At the same time, the communication layer mechanisms are adjusted to satisfy the control targeted performance.

discrete-time [115] or continuous-time [99] frameworks which have slightly different properties that should be considered, in particular, when taking into account the effects of the two other levels.

At the global or distributed control level, the crucial point is to determine which information shall be exchanged between agents, when it is transmitted, and how it is used to achieve the common control goal. We distinguish the two cases of homogeneous systems where all the agents have the same dynamics [137], and heterogeneous systems where agents have different dynamics [103] [152]. Clearly, the second class of MASs requires a deeper analysis since the individual control level of each agent should be studied separately.

The communication layer level specifies the communication protocol that describes the way information is exchanged between agents, which is crucial to determine the behaviour of the global system. The information and data transmitted from one agent to another is used to compute and update the agent's control decision which is influenced by the received information. For example, in cooperative vehicular navigation, the exchanged information allows the vehicles to adapt their maneuvers to avoid collisions or reduce energy consumption. However, when the exchanged information is erroneous or misinterpreted, it may lead to a poor behaviour of the system inducing instabilities, up to possible safety issues. Therefore, the imperfections of the communication system must be taken into account when designing the controller at the level of each agent.

In the majority of results related to multi-agent systems, researchers make an abstraction of the communication layer by using simple models. Generally, the se-

lected models correspond to the perfect communication conditions. Alternatively, strong assumptions are made which cannot be satisfied by real wireless communication systems, such as fixed network topology, continuous flow of information, or unlimited network resources like infinite data rate. Some of the results presented in the literature consider the imperfections of the communication layer using different frameworks: time-delayed systems to capture the delay between the transmission and the reception [99] [135], switching systems to capture the changes in the network topology structure [115], sampled-data systems to capture the discrete-time nature of the communication system [151]. In many results, the errors due to the communication system are modelled by communication noise that can be stochastic [59] [78] or deterministic [31] depending on the assumptions used by the authors. However, these models are insufficient to represent finely the imperfections of communication systems. Thus, it is necessary to recall more complex models such as *hybrid systems*.

Systems including continuous-time and discrete-time dynamics are known as hybrid systems. Goebel et al. present a proper framework to study hybrid dynamical systems [48]. Several systems fall in the hybrid systems dynamics framework, such as sampled-data systems [29], switching systems [15], event-triggered systems [53], impulsive systems [80], etc. The main innovative idea of modelling hybrid dynamical systems in the framework of hybrid dynamical systems [48] is to extend the time domain to a two-dimensional domain, a continuous-time domain, known as the *flow* domain and a discrete-time domain, known as the *jump* domain. Thus, the state of the studied system depends on two time variables incorporating both dynamics.

Despite the interesting results related to the coordination of agents connected over imperfect communication networks, the models used in the existing literature are insufficient to address the problems encountered when using modern communication systems such as the well-known Wi-Fi technology based on the IEEE 802.11 family of standards [57] or cellular network systems such as those specified by the 3rd Generation Partnership Project (3GPP), with today's technology, the 5G [34]. The main problems in these networks are related to congestion in the network, packet losses, signal interference, mobility of users, loss of coverage, etc. Applications related to vehicular communications are meant to be applied over large areas, and thus shall preferably rely on cellular networks. The principle of *cellular networks* is to divide the land area into regions called *cells*. Each cell is served by at least one fixed-location transceiver called the *base station* (BS). The base station provides radio resources that can be used to transmit information, such as short messages, voice, or data. Typically, each cell uses a different set of frequencies from neighbouring cells, to avoid interference and provide a guaranteed bandwidth. When joined together, these cells provide radio coverage over a wide geographic area allowing multiple transceivers, called *user equipments* (UEs), to communicate with each other by transiting messages through base stations. BSs

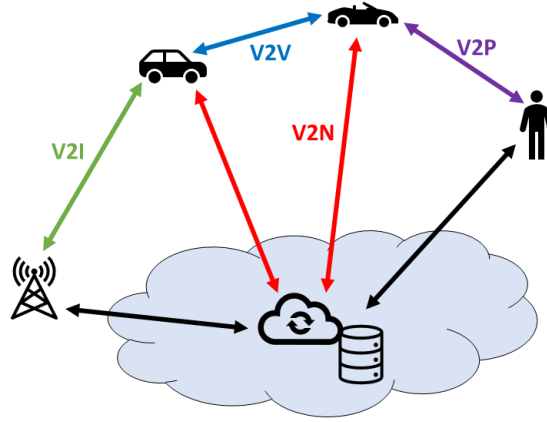


Figure 2: V2X communication types: Vehicle-to-Vehicle (V2V), Vehicle-to-Pedestrian (V2P), Vehicle-to-Infrastructure (V2I), Vehicle-to-Network (V2N).

in cellular networks are connected to a core network that allows to route information between UEs connected to different BSs, or to external network, typically internet. UEs have the ability to move from one cell to another without losing connectivity which enables the mobility of users such as mobile phones, laptops and connected vehicles. Cellular network standards are specified by the 3GPP, which is an organization responsible for the development and maintenance of mobile communication technologies. Commercial cellular networks technology has known different developments. Up to now, five generations of cellular networks have been specified and deployed corresponding to a significant technological improvement, from the first analog systems of the first generation to the current fifth generation (5G) systems. Compared to its predecessor, every generation offers more throughput and capacity, allowing to connect more users and transmit more data in a shorter time. Therefore, new use cases and applications become conceivable and achievable. Compared to 4G, 5G was specified to provide a 100 times higher data rate, up to 10 gigabits per second and 250 times lower latency going down to 1 ms with an enhanced support of Quality of Services mechanisms [30]. These enhancements open the door to delay-critical applications, notably real-time vehicular communications. As in 4G, 5G offers a vehicular communication framework known as Cellular Vehicle-to-Everything (C-V2X). In 4G, the C-V2X component targets safety-related use cases by transmitting Cooperative Awareness Messages (CAMs) and Basic Safety Messages (BSMs) [34]. Besides safety, the 5G C-V2X framework focuses on advanced driving assistance in order to improve traffic flow, reduce energy consumption, and increase connectivity between vehicular and non-vehicular road users.

Vehicle-to-Everything (V2X) is a general term that is used to refer to vehicles communication with other entities, including other vehicles: Vehicle-to-Vehicle (V2V), pedestrians: Vehicle-to-Pedestrian (V2P), roads infrastructure such as traffic lights and road signs: Vehicle-to-Infrastructure (V2I), network base stations: Vehicle-to-Network (V2N) [4] [61]. Wireless vehicular communication allows vehicles to be aware of their environment such as potential threats, traffic congestion, vehicles location, speed and direction, etc. The reception of such information allows the driver or the autonomous driving pilot to adjust its behaviour and adapt to the surrounding environment.

Dedicated short-range communication (DSRC) was the first V2X technology introduced in the IEEE 802.11p standard [76]. IEEE 802.11p introduced the communication protocol for wireless access in vehicular environments (WAVE), which is an enhancement of the Wi-Fi technology devoted to vehicular communications. It uses wireless local area network (WLAN) technology to establish DSRC channels so that the vehicles can communicate directly with other entities within short to medium ranges, typically 300 meters. DSRC technology presented a significant advance in the automotive industry because it allows data exchange between vehicles directly without the need to travel through a telecommunication infrastructure. After DSRC, another vehicular communication protocol was defined by the 3GPP in its release 14 [1], named Cellular V2X (C-V2X). C-V2X uses the cellular radio technology used by cell phones. The major difference that sets C-V2X apart from DSRC is that it allows both direct and indirect communication. In direct C-V2X, vehicles communicate directly with other vehicles (V2V) and roadside units (V2I) in the same way DSRC works. Under indirect C-V2X, vehicles communicate with other entities indirectly through the cellular network (V2N), which is not possible with DSRC. Indirect C-V2X is useful because the cellular network can collect data from many entities, and thus can be more effective at managing traffic on a larger scale. Initially designed in release 14 [1] [58] to use the Long Term Evolution (LTE) standard, 3GPP later added compatibility for 5G in releases 15 [2] and 16 [3] [52].

The main contribution of this thesis lies in this context through the following three aspects. First, considering realistic models of the communication layer. Second, enhancing the system modelling by combining the control and the communication frameworks. Finally, considering the control-communication system as a whole and unique system where control-related parameters and communication-related parameters are optimized jointly to achieve the targeted objective. The traditional paradigm consists in separating the communication layer from the application layer, which is the control of vehicles in our case. For example, in the resource allocation problem or power control problem, an optimization problem is generally posed where the objective is to find the optimal resources allocation scheme or the optimal power levels to achieve the best throughput on average in the network. From the control side, the communication layer is considered as a given constraint, and the controller is designed to handle the worst-case sce-

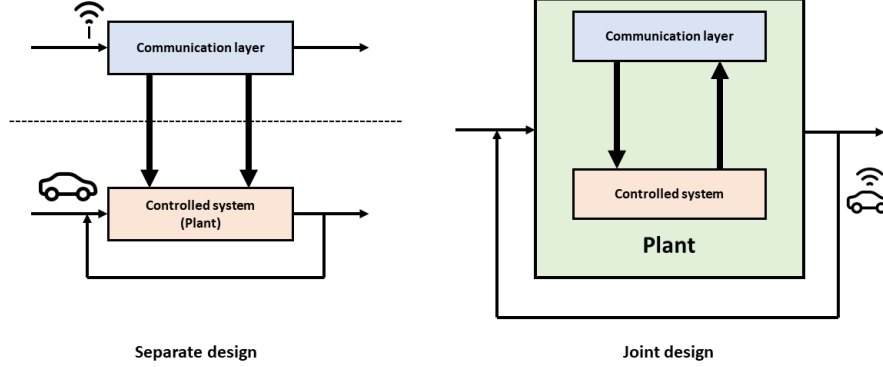


Figure 3: Separate-design paradigm vs. joint-design paradigm. In separate-design paradigm, the features of the communication layer is considered as a given constraint and the controlled system needs to adapt according to the provided performance levels. However, in joint-design paradigm, the controlled system and the communication layer interact with each other and adapt the adapt regarding the performance requirements.

nario for safety-related applications or to achieve an average performance level, which cannot be guaranteed all the time. Unlike traditional systems, the co-design or joint-design paradigm, illustrated in Figure 3, consists in considering the communication layer and the control layer as two components of a unique system. These components interact with each other through feedback messages allowing to achieve higher performance. On one hand, the controlled system is able to send requests to the communication layer in order to reserve the required resources to reach the targeted performance. On the other hand, the communication layer can also inform the controlled system about the network conditions, such as congestion levels, and require adaptation of the targeted performance.

The idea of joint-design or joint optimization of the control algorithm and the communication protocol is exploited in some recent results in the literature. However, these results are restricted to the platooning problem. For example, in [89], Mei et al. propose a joint scheme for radio resource allocation and control parameters adaptation for vehicle platooning. An optimization problem is formulated to jointly optimize the radio resource allocation for the CAM messages broadcasting and the controller gains of each vehicle in order to maintain the targeted inter-vehicle spacing while guaranteeing the string stability of the vehicles platoon and the reliability requirements of V2V communications. In [149], the authors present a combined communication-control framework for analyzing the performance and reliability of platoons of vehicles connected via wireless links. They propose a con-

straint on the controller gains to guarantee both plant and string stability. They derive the maximum wireless communication delay, including queuing, processing, and transmission delay, that can be tolerated without losing the stability properties. Moreover, they propose an optimization problem to select the controller parameters for improving the reliability and performance of the wireless communication network for platooning systems. Also, in [141], a communication-control problem is studied where the authors propose a resource allocation strategy that allows multi-lane platoons cooperation and a power control algorithm to maximize the platoon size as well as to minimize the power consumption. These examples show the importance of the system's modelling to provide a successful solution to the studied problem.

In this thesis, we adopt the new paradigm of co-design or joint-design where the control and the communication layers are considered as a unique system, and the objective is to optimize the two layers parameters jointly to achieve a better performance regarding the considered application, here, the distributed control of multiple vehicles. Compared to the cited publications, we propose a generic algorithm that can be used in several vehicular control applications. Indeed, our result is not restricted to platoons, but can handle any targeted spatial configuration. Moreover, we use the communication layer parameters as a control inputs to control the vehicles fleet.

Contributions of the thesis

Here we present the thesis's structure and briefly summarize the main results and contributions.

Chapter 1 is a preliminary chapter where we briefly introduce the basic notions of communication systems and control theory used in this thesis. We present the technical framework in which our works take place and explain the terminology used in the following chapters. The first section concentrates on communication systems and wireless networks, and the second one focuses on the consensus problem and network modelling for multi-agent systems. In the communication part, we mainly focus on the multiple access techniques used in communication systems to manage the resource attribution to users sharing the same network channel. This notion is crucial to introduce the communication models used in Chapters 2 and 3. The control part presents the main graph theory notions used to study the consensus algorithm. The different definitions of graph connectivity are stated, and we summarize the main results related to the convergence of the consensus algorithm. Then, we give an overview on the Hegselmann-Krause (HK) model[54] that we use in Chapter 3. A specific section is devoted to the result of Moreau [92] that uses an approach similar to the one that we use in Chapter 2. We close this chapter by recalling a theorem on exponential stability of non-linear systems that is discussed in Chapter 4.

In Chapter 2, we address the problem of consensus for multi-agent systems connected over a shared communication channel. In real-world applications, multi-agent systems use a communication system to exchange information between agents. The information exchange cannot always be assumed to be perfect, as it is done in the majority of the contributions related to the consensus problem. The communication layer has its constraints. In fact, to communicate with each other, agents need a communication medium that is generally shared between agents, and access to this medium is defined by some rules. For wireless communication networks, the network resources in time and frequency cannot be occupied by more than one user at a time. Otherwise, interference and packet collisions may occur, and the transmitted information may be lost. In this contribution, we present a particular class of consensus algorithms for multi-agent systems of continuous-time single integrators communicating over a shared broadcast channel. To deal with the interference and packet collisions problems, we consider a time-division multiple access (TDMA) protocol represented by a switching topology where only one agent can transmit at a time. A Lyapunov function is proposed to prove the exponential convergence of this class of consensus algorithms under mild assumptions. We also derive an explicit bound to characterize the convergence towards consensus of such systems. The given bound depends on the communication protocol features, which provides a powerful tool to better design the communication protocol in order to enhance the control performance.

Chapter 3 considers a formation control problem for a network of connected vehicles communicating over a 5G C-V2X communication system with limited network resources. The formation control problem consists in making a group of mobile agents follow a reference trajectory while maintaining a predefined spatial pattern or configuration (see Figure 4). To solve this problem, we propose a new combined motion control and resource allocation scheme to make the vehicles achieve the targeted formation. In this chapter, we first focus on formulating the formation control problem under the constraints imposed by the communication system, namely, the limited available resource and the connectivity preservation. Then, we state the resource allocation problem and present an optimization algorithm to select the transmitting agents. The proposed algorithm uses the system model to predict the behaviour of the vehicles and check whether the communication system's constraints are satisfied for a given configuration of transmitting agents. The goal of this algorithm is to select a subset of agents and allow them to transmit their state to their neighbours to converge toward the targeted formation. This method uses a centralized-distributed scheme which falls into Mode 1 of resources allocation for V2V communication in 5G [34]. The base station assigns the network resources to selected agents in a centralized way. Then, the formation control algorithm is operated in a distributed way relying on direct communication between the agents. A formal condition on the vehicles' initial spatial configuration is given to ensure the convergence towards the desired formation while preserving

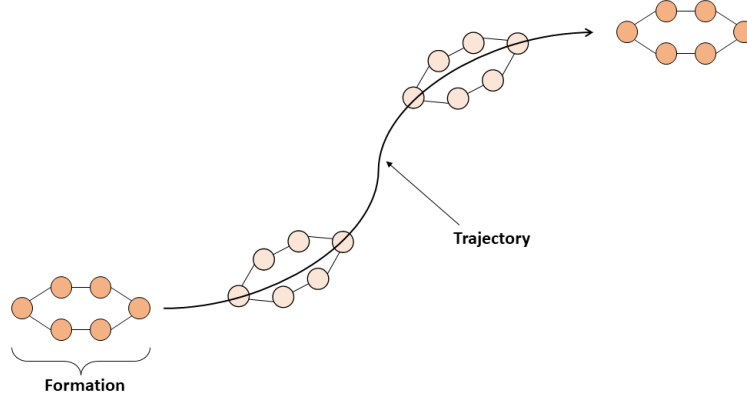


Figure 4: The formation control problem consists in making a group of agents follow a reference trajectory while maintaining a predefined spatial configuration that is described by their relative positions.

the formation connectivity and using the network resources efficiently.

Controlled systems relying on sensor measurements and communication are subject to errors due to imperfections of the sensor hardware or packet losses in the communication system [41]. Thus, the performance controllers can be highly degraded, leading to stability and even safety issues. Therefore, it is crucial to consider the imperfections of communication systems that can be modelled as noise affecting the system at the instants of information reception. In Chapter 4, we address the problem of filtering measurements noise for hybrid systems using Kalman filter [73]. Indeed, in real-world applications, physical systems evolve in continuous time, and external information such as sensor measurements, observations, or control commands are received in discrete time through a communication system. Both models, continuous-time and discrete-time, have their shortcomings. The continuous-time model supposes that the flow of observations is continuous, and the discrete-time model ignores the system dynamics during the interval between two observations. Thus, it is essential to introduce a hybrid formulation of the Kalman filter that combines the continuous-time dynamics of the system and the discrete-time aspect of the received flow of observations. In [124], Särkkä presents the hybrid filtering principle, called by the author continuous-discrete filtering, and derived the equations of the hybrid version of the Kalman filter starting from the Bayesian inference framework under the assumption of zero-mean white Gaussian noises. In our works, we study the stability properties of the proposed filter in the deterministic framework. We propose a Lyapunov-based characterization of exponential input-to-state stability (ISS) for hybrid systems. ISS is a stability notion

that is used to study the stability of dynamical systems in the presence of external inputs, such as disturbances, measurements noise or modelling errors. This notion is crucial for the systems robustness analysis. In this chapter, a new formulation of the Kalman filter algorithm for hybrid systems is given using the hybrid dynamical systems framework [48], where we consider continuous-time system dynamics and discrete-time observations. Using on a Lyapunov stability analysis, we prove exponential input-to-state stability on the estimation error of the hybrid Kalman filter under mild assumptions on the system's model. The chapter ends with a general result on exponential stability for non-linear time-varying systems.

We end this thesis with a conclusion chapter where we expose some perspectives and future directions of research that can be explored based on the contributions and results presented in this thesis.

The results of this thesis are published in the following papers:

- Bechihi, Adel, Elena Panteley, and Arnaud Bouttier. "Exponential Convergence of the Consensus Algorithm over a Shared Broadcast Channel." 2021 60th IEEE Conference on Decision and Control (CDC). IEEE, 2021.
- Bechihi, Adel, Elena Panteley, Pierre Duhamel, and Arnaud Bouttier. "A Resource Allocation Algorithm for Formation Control of Connected Vehicles." IEEE Control Systems Letters 7 (2022): 307-312.
- Bechihi, Adel, Elena Panteley, and Arnaud Bouttier. "Input-to-state stability for hybrid Kalman filters." IFAC-PapersOnLine 55.20 (2022): 295-300.

1 - PRELIMINARIES

The *Preliminaries* chapter introduces the basic notions and tools of communication systems and control theory used in this thesis. We present the technical framework in which our works take place and explain the terminology used in the following chapters. The first section concentrates on communication systems and wireless networks, and the second one focuses on the consensus problem and network modelling using graph theory for multi-agent systems.

1.1 . Communication systems: Wireless networks and multiple access techniques

A communication system consists of three main components: the transmitter, the channel, and the receiver [125]. The *transmitter* is the part that produces a message or a signal containing the information to be conveyed. It can be a computer in a data center, a satellite for TV broadcasting, a base station in a cellular network, or user equipments such as a cellular phone or a connected vehicle. The *channel* is the medium through which the transmitted message travels from the transmitter to the receiver. Electrical wires, optical fibers, and the free-space are all considered as channels. The *receiver* is the part that receives the message sent by the transmitter after traveling through the channel. It is responsible of decoding the received message and extract the conveyed information. Many equipments includes both transmission and reception functionalities such as mobile phones, computers, connected vehicles, etc.

The senders and receivers are called *nodes*, and the the interconnection between nodes composes a communication *network*. In a network, a node can be a sender sometimes and a receiver other times or both at the same time. The exchange of messages within the network is governed by the communication protocol that specifies how the channel is used by nodes.

The communication *protocol* covers the rules and conventions that allow nodes to exchange information over a communication medium [9]. This communication medium is usually shared between multiple users. One of the goals of the communication protocol is to manage the channel multiple access (see Subsection 1.1.3) and to allocate the network resources to users fairly and efficiently. By *fair*, we mean all users can use the network resources to transmit their information. It is not recommended that a few users occupy the resources for a long duration leaving no room for other users to transmit. By *efficient*, we mean that the resources are allocated to users depending on the network conditions. Communication protocols aim to guarantee a high rate of successful receptions of the transmitted data by reducing the impact of the channel noise, the interference levels, and the number

of packet collisions.

In this thesis, we focus on the control of multi-agent systems such as robots for industrial automation [16], or vehicles for road-traffic optimization [102]. For that purpose, the agents exchange information using wireless communications. In the following sections, we give a brief introduction on wireless communication systems and we mainly concentrate on vehicular networks using the fifth generation of cellular networks (5G) technology standardized by the 3GPP consortium [34].

1.1.1 . Wireless communication networks

Wireless communications consist in transmitting information remotely without the use of wires, cables, fibers or any of physical connection between the networked nodes. Wireless communications are widely used in several daily life applications such as television broadcasting, positioning applications (GPS, Galileo, etc.), mobile phones, headphones, etc. Compared to wired systems, wireless communication systems offer multiple advantages such as remote communication that can go from a few centimeters, e.g., from a cellphone to headphones, up to several kilometers, e.g. from a terrestrial gateway to satellites. Moreover, they provide flexibility and mobility [139]. Equipments connected through wireless communications can change their location while maintaining constant connectivity and join the network easily without the need of a physical wire [71]. This aspect is useful for cellular telephony and radio or TV broadcasting. Additionally, wireless networks are more cost-effective since they do not require an important physical infrastructure. For these reasons, among others, several applications are designed based on wireless communications. In this thesis, we focus on applications related to connected vehicles that use wireless communications to exchange information with nodes in their surrounding environment.

1.1.2 . Cellular communications and 5G C-V2X

Cellular networks are the most commonly used connectivity system for mobile phone calls and Internet access [17]. The communication between nodes is wireless and the system is designed to allow users to move from one cell to another without loss of connectivity using the handover process [37] [134]. Each cell is governed by one or multiple bases stations, and enable UEs to connect to the network. There exist two types of links between UEs and BSs: the downlink (DL) where the information flow goes from the BS to the UEs and the uplink (UL) where the information flow goes from the UEs to the BS. In addition to DL and UL, new generations of cellular network such as 4G and 5G support direct communication between users known in 5G as the sidelink (SL) (see Figure 1.1).

The access to the communication channel is governed by rules at the level of the data link layer (DLL), more specifically at the medium access control (MAC) [50]. MAC is a sublayer of the DLL layer in the open system interconnections (OSI) reference model for data transmission [153]. It is responsible for flow control and multiplexing in the transmission medium. It controls the transmission of data

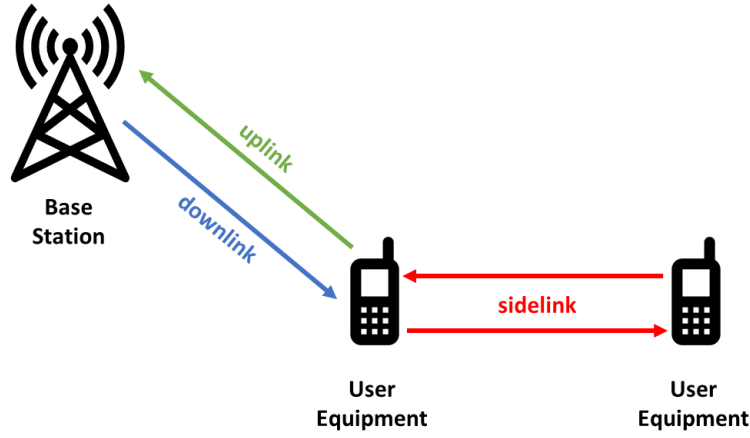


Figure 1.1: Types of link in cellular networks: Downlink (from BS to UE), Uplink (from UE to BS), Sidelink (from UE to UE).

packets via shared channels. The MAC layer provides an abstraction of the physical layer to the logical link control (LLC) sublayer and upper layers of the OSI model. It is responsible for encapsulating frames so that they are suitable for transmission via the physical medium. It resolves the addressing of source station as well as the destination station, or groups of destination stations [132]. Moreover, the MAC layer performs multiple access resolutions when more than one packet is to be transmitted. It determines the channel access methods for transmission. It also performs collision resolution and initiating the re-transmission process in case of collisions, and generates the frame check sequences and thus contributes to protection against transmission errors [127].

In our results, we study the behaviour of vehicular multi-agent systems connected over a 5G system, or similar systems. We focus on the way information is exchanged between the different agents. For this purpose, it is crucial to take into consideration the constraints related to the channel access and use. The channel use is governed by the multiple access techniques that we present in the following section.

1.1.3 . Multiple access techniques

Multiple access techniques deal with the division of the network resources to create multiple channels in frequency, time, space, and other forms [119]. The amount of available transmission resources is limited. Thus, we need to find methods to allow multiple users to share the available network resources efficiently by avoiding packet collisions. Shared access is used to implement a multiple access scheme when access to a channel by several users is required. Multiple access

schemes allow many users to share the radio spectrum. The resources sharing is based on the principle of orthogonality. The orthogonality allows the transmitted signals to be easily separated, and thus, a receiver can decode correctly the transmitted information. The variability of wireless channels presents both challenges and opportunities in designing multiple access communications systems. Multiple access strategies have an impact on the robustness and interference levels generated by the transmitted signals. Therefore, multiple access schemes are designed to maintain orthogonality and reduce interference effects. Multiple access schemes can be classified as reservation-based multiple access (e.g., FDMA, TDMA, CDMA) and random multiple access (e.g., ALOHA, CSMA) [5]. The family of reservation-based multiple access includes frequency-division multiple access (FDMA), time-division multiple access (TDMA), code-division multiple access (CDMA) and orthogonal frequency-division multiple access (OFDMA) that are presented hereafter.

Frequency-Division Multiple Access (FDMA)

The FDMA is the simplest scheme used to provide multiple access. It separates different users by assigning them a different carrier frequency (see Figure 1.2). In FDMA, signals from various users are assigned different frequencies, and frequency guard bands are provided between contiguous signal spectra to minimize cross-talk between contiguous channels. The main advantages of FDMA are the easiness of implementation and the possibility to increase the capacity by reducing the information bit rate. Moreover, hardware design is simple, because multiple users are isolated by employing basic band-pass filters. However, FDMA presents some disadvantages such as the fixed bit-rate per channel, leading to inefficient use and waste of network resources. Indeed, if a channel is not in use by its assigned user, it remains idle and cannot be used by other users to enhance the system capacity.

Time-Division Multiple Access (TDMA)

In the TDMA scheme, each user occupies the whole channel bandwidth for a fraction of time, which is the opposite of the FDMA scheme where a single user occupies a fraction of the channel bandwidth for the entire duration (see Figure 1.2). In a TDMA system, time is divided into equal time intervals, called slots, and users data is transmitted in the slots. Several slots make up a frame. Guard times are used between each user transmission to minimize cross-talk between channels. In a TDMA system, the user can use multiple slots to support a wide range of bit rates by selecting the lowest multiplexing rate or multiples of it. Depending on the data rate used and the number of slots per frame, a TDMA system can use the entire bandwidth of the system or can employ a frequency-division (FD) scheme. The resultant multiplexing is a mixture of frequency division and time division. When used exclusively and not with FD, TDMA schemes use the bandwidth more efficiently because no frequency guard band is required between channels. However, it requires a substantial amount of signal processing for matched filtering and

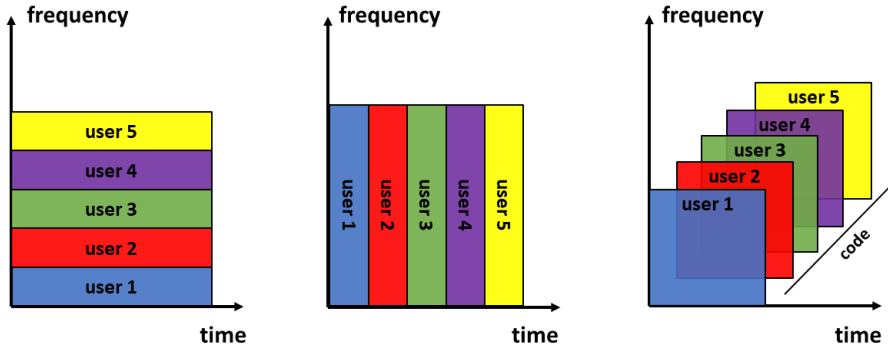


Figure 1.2: Examples of reservation-based multiple access schemes: FDMA, TDMA, CDMA. In FDMA, each user occupies a fraction of the channel bandwidth for the entire duration. In TDMA, each user occupies the whole channel bandwidth for a fraction of time. In CDMA, users can transmit data over the entire frequency all the time using unique code sequences.

correlation detection for synchronizing with a time slot. Moreover, TDMA requires synchronization among users. If the time slot synchronization is lost, the channels may collide with each other.

Code-Division Multiple Access (CDMA)

In CDMA, all the users can transmit data simultaneously over multiple frequencies. The CDMA scheme allows each user to transmit data over the entire frequency range all the time (see Figure 1.2). Simultaneous transmissions are separated by orthogonal code sequences called spreading codes. A spreading code is a pseudo-random code used to modulate the data signal in order to generate a faster signal, i.e., with a larger bandwidth than the data signal bandwidth [111]. For each user, a unique code sequence is assigned. Users combine their signals with the spreading codes to generate new signals that are transmitted simultaneously through the channel. The receiver receives the sum of the transmitted signals. Using the orthogonality property of the spreading codes, he can decode the received signal to extract the transmitted information. The receiver must know the spreading code used to encode the transmitted signal to reconstruct it correctly. CDMA allows high bit-rates and does not require synchronization. However, as transmissions on the same frequency with different codes are still received, they appear as noise for the receiver after performing the decoding. This means the greater the number of users, the higher the noise level on the system, which can highly affect the channel capacity [45].

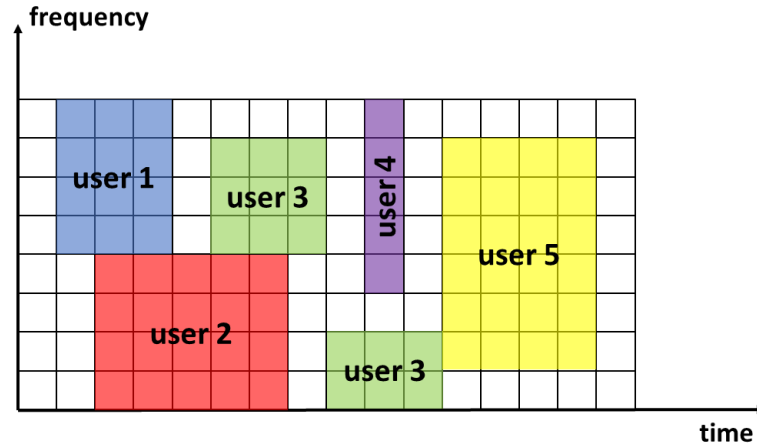


Figure 1.3: OFDMA is a mixture between frequency-division multiplexing and time-division multiplexing where the radio resources are divided over time and frequency. A number of time slots and a number of contiguous or non-contiguous sub-carriers are assigned to each user.

Orthogonal Frequency-Division Multiple Access (OFDMA)

Orthogonal frequency-division multiple access (OFDMA) is a mixture between frequency-division multiplexing and time-division multiplexing where transmissions occur over overlapping but orthogonal sub-carriers. Compared to FDMA, the sub-carrier frequency in OFDMA are chosen in a way that such that sub-carriers are orthogonal to each other. Thus, no inter-carrier guard bands are required, which increases the spectral efficiency [147]. In OFDMA, the radio resources are divided over time and frequency (see Figure 1.3). For each user, a number of time slots called OFDM symbols and a number of contiguous or non-contiguous sub-carriers are assigned. OFDMA provides several advantages regarding the network efficiency which make it the reference multiple access technique in current technologies such as in cellular communications and Wi-Fi. It offers high diversity and flexibility as network resources are divided in small time-frequency blocks and are assigned to users depending on the channel quality. It also gives better coverage over networks and reduces the interference levels between cells. However, because of its diversity, the resource attribution may become very complex. Moreover, it has more sensitivity to frequency offsets and time synchronization [93].

There exist other methods of multiplexing related to antenna diversity, such as Multiple Input Multiple Output (MIMO) communications [106], and signal processing methods, such as beamforming [138], which allow to generate directional signals for transmission and reception in order to achieve spatial diversity. This technique is known as spatial multiplexing space-division multiple access (SDMA).

1.2 . Control theory: Graph theory and the consensus problem

Multi-agent systems represent a class of systems where the considered system is composed of subsystems interacting with each others. These subsystems are called agents. In order to control multi-agent systems, we need to model the system at two levels. First, at every agent's level, we need to provide the dynamics of the considered agent which are typically given by differential equations for continuous-time models or difference equations for discrete-time models. Some system models may include a combination of differential and difference equations, or more complex mathematical models. These dynamics describe the behaviour of the agents over time. Second, we need to model the communication network over which the agents can exchange information. The interconnections between agents represent the *network topology*. The combination of the two modelling layers, individual dynamics and network topology, will give the model of the multi-agent system as a whole system where the behaviour of one agent depends on the behaviour of the other agents.

1.2.1 . Graph theory for network topology modelling

A *graph* consists of a set of elements called *nodes* connected via links called *edges*. We consider $\mathcal{V} = \{v_i \mid i \in \mathcal{I}_N\}$ a set of N elements, where \mathcal{I}_N denotes the set of integers between 1 and N , i.e., $\mathcal{I}_N = \{1, 2, \dots, N\}$. We introduce the *graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of order N , where \mathcal{V} is the set of nodes and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of edges [46]. An edge is denoted by $e_{i,j} = (v_i, v_j) \in \mathcal{V} \times \mathcal{V}$. Saying that the edge $e_{i,j}$ belongs to \mathcal{E} means that a link exists from the node v_i to the node v_j . In the sequel, we assume that self-loops are not allowed, i.e., $e_{i,i} \notin \mathcal{E}$ for all $i \in \mathcal{I}_N$.

To represent the structure of a graph \mathcal{G} , we introduce the *adjacency matrix* $\mathcal{A} = [a_{i,j}]$ of order N , defined as follows

$$\begin{cases} a_{i,j} = 1, & \text{if } e_{j,i} \in \mathcal{E} \\ a_{i,j} = 0, & \text{otherwise} \end{cases}, \quad \forall i, j \in \mathcal{I}_N. \quad (1.1)$$

We also introduce the Laplacian matrix $\mathcal{L} = [l_{i,j}]$ associated to the graph \mathcal{G} which is defined as follows

$$\begin{cases} l_{i,j} = -a_{i,j}, & \text{if } i \neq j \\ l_{i,i} = \sum_{j=1}^N a_{i,j} \end{cases}, \quad \forall i, j \in \mathcal{I}_N. \quad (1.2)$$

Laplacian matrices play an important role in the analysis of the consensus algorithm (see Subsection 1.2.2).

In graph theory, we distinguish two classes of graphs: *undirected* graphs and *directed* graphs, also known as *digraphs*. In undirected graphs, the links between agents are bidirectional, which means that if there is an edge from v_i to v_j , there

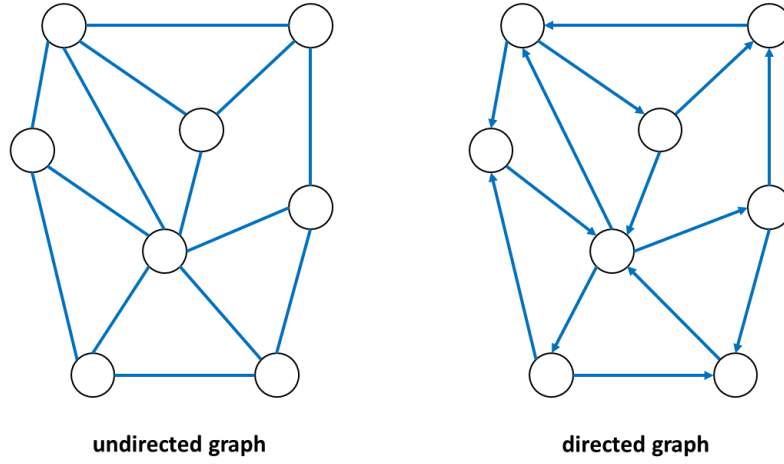


Figure 1.4: Undirected graph vs. directed graph: in undirected graphs, the links between nodes are bi-directional, i.e., if there exists an information flow going from node i to node j , then there exists an information flow from node j to node i . However, in directed graphs, the links are uni-directional, i.e., the existence of an information flow from node i to node j does not imply the existence of an information flow from node j to node i .

is also an edge from v_j to v_i ($e_{j,i} \in \mathcal{E} \iff e_{i,j} \in \mathcal{E}$). These links are also called symmetric, and therefore undirected graphs are also called symmetric graphs.

A directed path from v_{i_1} to v_{i_n} in the graph \mathcal{G} is a sequence of edges $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \dots, (v_{i_{n-1}}, v_{i_n})$ such that $(v_{i_j}, v_{i_{j+1}}) \in \mathcal{E}$ for all $j \in \{1, \dots, n-1\}$. We say that the graph \mathcal{G} has a *spanning tree*, if there exists $k \in \mathcal{I}_N$ such that, for any $j \neq k$, there exists a directed path from the node v_k to the node v_j . The node v_k is called the root of the directed spanning tree.

We present the different definitions of graph connectivity.

Definition 1.1 (Connectivity)

For undirected graphs, we say that a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is connected if for any $v_i, v_j \in \mathcal{V}$ there exists a path from v_i to v_j . The existence of a path from v_j to v_i is trivial because of the symmetry of the links.

The following definitions are related to directed graphs.

Definition 1.2 (Strong connectivity)

We say that a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is strongly connected (SC) if for any $v_i, v_j \in \mathcal{V}$ there exists a direct path from v_i to v_j .

Definition 1.3 (Quasi-strong connectivity)

We say that a directed graph \mathcal{G} is quasi-strongly connected (QSC) if \mathcal{G} has a spanning tree.

If the network topology changes over time, the objects introduced in the previous paragraph are time-dependent. Assuming that the nodes set is constant over time, i.e., $\mathcal{V}(t) = \mathcal{V}$, we denote by $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ the instantaneous directed graph that represents the network topology at time t . The instantaneous adjacency matrix $\mathcal{A}(t)$ and instantaneous Laplacian matrix $\mathcal{L}(t)$ are defined for $\mathcal{G}(t)$ as above. Given a time-varying directed graph $\mathcal{G}(t)$, we define the union graph over an interval $[t_1, t_2]$ as follows

$$\mathcal{G}([t_1, t_2]) = \left(\mathcal{V}, \bigcup_{t \in [t_1, t_2]} \mathcal{E}(t) \right). \quad (1.3)$$

Graph $\mathcal{G}([t_1, t_2])$ is a directed graph that has \mathcal{V} as a nodes set and its edges set contains all the edges of graphs $\mathcal{G}(t)$ for $t \in [t_1, t_2]$.

Note that, in the definition of union graphs (1.3), we considered closed multi-agent systems (in opposition to open systems [43]) where the set of agents is supposed to be fixed over time, i.e., no agent is entering or leaving the global system.

We introduce a weaker connectivity definition for time-varying graphs.

Definition 1.4 (Uniform quasi-strong connectivity [81])

We say that a directed graph is uniformly quasi-strongly connected (UQSC) if there exists a constant $T > 0$ such that for any $t \geq t_0$, the union graph $\mathcal{G}([t, t + T])$ has a spanning tree (quasi-strongly connected). We say that a graph is T -UQSC if it is UQSC with the time constant T .

From the previous definitions, note that strong connectivity (SC) implies quasi-strong connectivity (QSC) which implies uniform quasi-strong connectivity (UQSC) for time-varying graphs. For undirected graphs SC and QSC are equivalent to simple connectivity.

1.2.2 . Consensus algorithm

The consensus algorithm is a well-studied class of algorithms for multi-agent systems. Consensus algorithms aim to make the agents achieve a common value. This property has several applications in data fusion [97], sensing [146], tracking [74], etc. Similar to [99], we consider the following consensus problem for a multi-agent system composed of N continuous-time single-integrators x_i interconnected over a graph \mathcal{G}

$$\begin{cases} \dot{x}_i = u_i \\ u_i = \sum_{j=1}^N a_{i,j}(t)(x_j - x_i) \end{cases} \quad ; \quad i \in \mathcal{I}_N, \quad (1.4)$$

where $x_i \in \mathbb{R}$ is the state of the i -th agent, $u_i \in \mathbb{R}$ represents its input command and $a_{i,j}(t)$ are the corresponding coefficients of the adjacency matrix associated to a time-varying graph $\mathcal{G}(t)$ representing the network topology, i.e., the links

between agents. We assume that the functions $a_{i,j}(t)$ are piece-wise constant functions which represent the switching behaviour of the network topology.

For a network of single integrators interconnected over a switching graph $\mathcal{G}(t)$, the consensus algorithm can be written as

$$\dot{x} = -\mathcal{L}(t)x, \quad (1.5)$$

where $x = (x_1, \dots, x_N)^T \in \mathbb{R}^N$ is the state vector of the network composed of N agents and $\mathcal{L}(t)$ is a Laplacian matrix associated to graph $\mathcal{G}(t)$.

For system (1.5) we introduce the following characterization of convergence of the network trajectories to consensus.

Definition 1.5 (Asymptotic consensus)

We say that the system (1.5) achieves consensus asymptotically if

$$\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| = 0; \quad \forall i, j \in \mathcal{I}_N,$$

or equivalently,

$$\lim_{t \rightarrow \infty} \|x_i(t) - \bar{x}(t)\| = 0; \quad \forall i \in \mathcal{I}_N,$$

where $\bar{x}(t) = \frac{1}{N} \sum_{i=1}^N x_i(t)$ and $\|\cdot\|$ is the Euclidean norm.

Definition 1.6 (Asymptotic consensus)

We consider system (1.5) and we define the disagreement errors $\delta_i(t) = x_i(t) - \bar{x}(t)$ and the disagreement vector $\delta = (\delta_1, \dots, \delta_N)^T$. We say that system (1.5) achieves consensus asymptotically if

$$\lim_{t \rightarrow \infty} \|\delta(t)\| = 0. \quad (1.6)$$

Asymptotic consensus ensures convergence to consensus asymptotically without giving any addition information on the required time to achieve a certain region around the consensus set. In some applications, a stronger convergence property is required to inform on the time needed to achieve consensus. Thus, we introduce the uniform exponential convergence definition.

Definition 1.7 (Uniform Exponential consensus)

We consider system (1.5) and disagreement vector $\delta = (\delta_1, \dots, \delta_N)^T$. We say that system (1.5) achieves consensus exponentially uniformly (in time) if there exist two constants $K > 0$ and $\lambda > 0$ such that for any initial states $x_0 \in \mathbb{R}^N$ and any initial time $t_0 \geq 0$ we have

$$\|\delta(t)\| \leq K e^{-\lambda(t-t_0)} \|\delta(t_0)\|; \quad \forall t \geq t_0. \quad (1.7)$$

The graph topology may be dependent on time or on the state of the agent itself. For example, in applications related to mobile robots or vehicles, the communication links between agents depend on their positions. We present, in the next subsection, the Hegselmann–Krause model that uses a state-dependant network topology [19].

1.2.3 . Hegselmann–Krause model

The Hegselmann–Krause (HK) model, also known as the Krause model, proposes an enhanced version of the consensus algorithm presented in [99] to take into account the state-dependence of the network topology [54] [133]. The Krause model is generally presented in discrete time where, at each step, each agent updates its state based on the states of its close neighbours, i.e., the agents belonging to some space around the considered agent. At each time step, the agent state is updated as the average of its state and its neighbours states. This rule implies a dynamic topology of the graph connecting the agent, which leads to complex behaviours of the whole system that should be investigated. A relevant problem to analyze regarding the HK model is the convergence of the system to a steady state that may be either a consensus, i.e., when a common value is reached, or to multiple groups called clusters [18].

Formally, the HK model is presented as follows. Let $\rho > 0$ be a positive constant that represents the connectivity threshold. Two agents i and j with states $x_i, x_j \in \mathbb{R}$, respectively, are said to be connected if $\|x_j - x_i\| \leq \rho$. In this case, agent i is called a neighbour of agent j and vice versa. We define the set of neighbours \mathcal{N}_i for a given agent $i \in \mathcal{I}_N$ as $\mathcal{N}_i = \{j; \|x_j - x_i\| \leq \rho\}$. Note that $i \in \mathcal{N}_i$.

In the standard HK model, the dynamics of agent i are given by the following equation [19]:

$$x_i(t+1) = \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} x_j(t). \quad (1.8)$$

At each time step, the state of agent i is updated by averaging the states of its neighbours. As $i \in \mathcal{N}_i$, the state of agent i is counted in the average state by giving the same weight to agent i and the other neighbouring agents.

The following lemma presents a summary of the relevant properties regarding the convergence of the HK model system to consensus.

Lemma 1.1

The main properties of the standard HK model are the following:

- *The order of states is preserved.*
- *If the connectivity is lost between agents i and j at time t_0 , then this split remains for all $t \geq t_0$.*
- *If the graph remains connected, then consensus is achieved.*
- *If agents achieve consensus, then it is achieved in finite time.*

The proof of these results are given in [18], [91]. They mainly rely on the notion of order in the real numbers domain \mathbb{R} . Most of these properties become invalid for the higher dimensional case as it will be shown in Section 1.3. For the scalar continuous-time HK model, the property of convergence to consensus in finite time is no more valid as presented in Lemma 1.2.

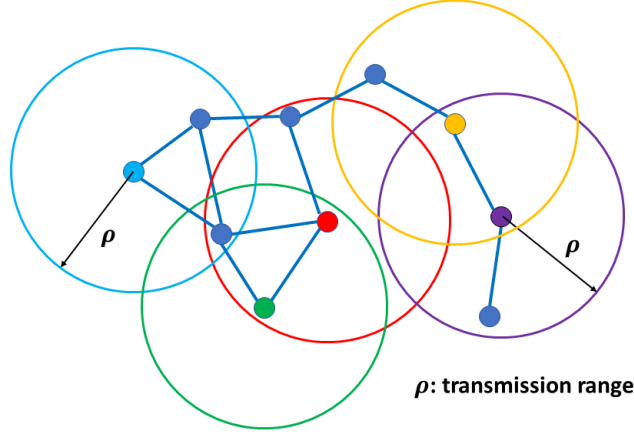


Figure 1.5: Network topology of the HK model: the network topology is defined with respect to the nodes positions. When the nodes positions change, the network topology may change and thus it is called *state dependent*.

There exists a modified version of the discrete-time HK model where the state update is performed using an averaging with unequal weights [39]. This version of HK model, also known as DeGroot model [36], gives a larger weight to the state of agent i compared to its neighbours states. The purpose of this unbalanced modification is to represent the notion of self-confidence. Formally, the system is written as follows

$$x_i(t+1) = \alpha_i x_i(t) + (1 - \alpha_i) \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t)). \quad (1.9)$$

where $\alpha_i \in (0, 1)$ is gain factor. The properties of system (1.9) are very similar to system (1.8) (see Lemma 1.1). However the convergence to consensus in finite time is lost. In the next paragraph, we focus on the continuous-time HK model.

The continuous-time HK model

We consider a multi-agent system composed of N agents where each agent $i \in \mathcal{I}_N = \{1, \dots, N\}$ is represented by its continuous-time state $x_i \in \mathbb{R}$. As presented in Subsection 1.2.2, the agents form a graph where the connectivity coupling function is defined by

$$w(x_i, x_j) = \begin{cases} 1; & \text{if } \|x_i - x_j\| \leq \rho \\ 0; & \text{otherwise} \end{cases},$$

where ρ is called the connectivity maximum distance or the transmitting range.

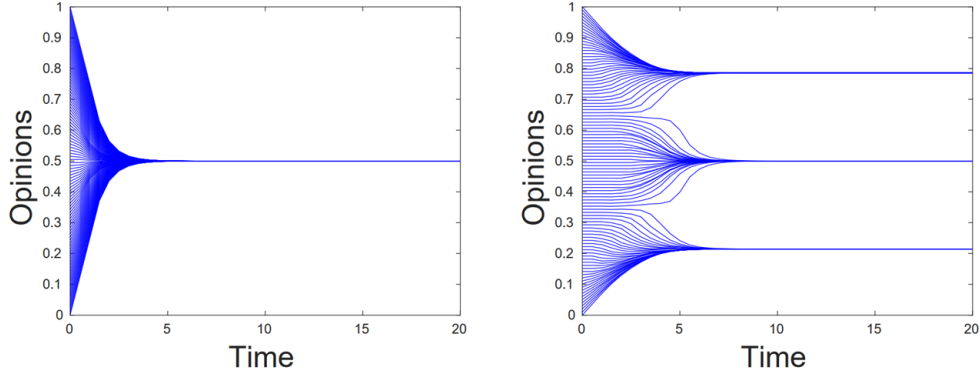


Figure 1.6: Consensus vs. Clustering [56]: in the steady state, the agents states may converge to a common value, which corresponds to the consensus case. However, the clustering case occurs when the agents states converge to different values.

For any $i \in \mathcal{I}_N$, we define the set of neighbours $\mathcal{N}_i = \{j; \|x_i - x_j\| \leq \rho\}$. The associated consensus problem is defined by the following dynamics

$$\dot{x}_i = \sum_{j \in \mathcal{N}_i} (x_j - x_i) = \sum_{j=1}^N w(x_i, x_j)(x_j - x_i); \quad \forall i \in \mathcal{I}_N. \quad (1.10)$$

Lemma 1.2 ([27])

The main properties of the continuous-time HK model are the following:

- *The order of states is preserved.*
- *If the connectivity is lost between agents i and j at time t_0 , then this split remains for all $t \geq t_0$.*
- *If the graph remains connected, then consensus is achieved.*

As for system (1.9), the property of convergence to consensus in finite time is lost for the the continuous-time HK model (1.10).

The high-dimensional HK model

The HK model can be applied for high-dimensional systems where the agents states are represented by vectors $x_i \in \mathbb{R}^n$ [95]. As in the scalar case, the control law is written as follows in discrete time

$$x_i(t+1) = \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} x_j(t), \quad (1.11)$$

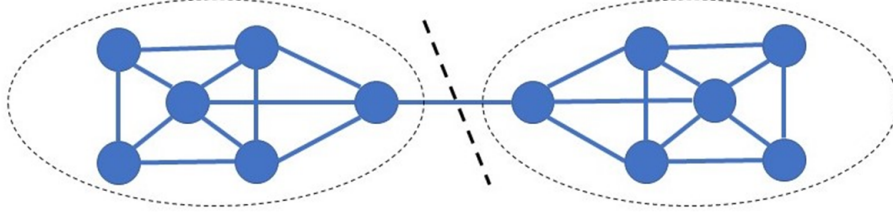


Figure 1.7: Creation of clusters: Although the system is connected at the initial time, connectivity may be lost if the initial link between two or more groups is weak.

and in continuous time

$$\dot{x}_i = \sum_{j \in \mathcal{N}_i} (x_j - x_i). \quad (1.12)$$

The sets \mathcal{N}_i are defined with respect to the agents states as in the scalar case, but with the euclidean norm.

The high-dimensional HK model presents more challenges and most of the properties that are valid for the scalar case become invalid. Indeed, we cannot use the line of analysis applicable to the scalar case since the notion of order used for scalar agents state cannot be applied in higher dimensions. Stability and finite time convergence for multi-dimensional model are proved by Lorenz in [83, Theorem 3.3.1], where the analysis relies on non-negative matrix theory. As in the scalar case, multi-dimensional HK may converge to consensus or may lead to the creation of clusters. The creation of clusters depends on the initial conditions. It is obvious that starting in a clustering condition will lead to clustering convergence. However, when starting from a unique connected group, convergence to consensus is not guaranteed (see Figure 1.7).

Lemma 1.3

The high-dimensional HK model satisfy the following property:

- *If the graph remains connected, then consensus is achieved.*

Note that most of the properties in the scalar case are lost for high dimensional systems. In Chapter 3, we provide a stronger condition on the initial states to ensure convergence to consensus and prevent the split of the vehicles formation, i.e., the creation of clusters (see Figure 1.7).

1.2.4 . Results related to the consensus algorithm

The convergence of the consensus algorithm depends on the agents dynamics and the network topology. In this thesis, we consider only multi-agent systems composed of first-order integrators ($\dot{x}_i = u_i$) where the agents states may be scalars $x_i \in \mathbb{R}$ or vectors $x_i \in \mathbb{R}^n$. We focus mainly on the properties of the network topology and its impact on the convergence to consensus. We present here some fundamental results regarding the network topology and its properties required to achieve consensus.

Up to our knowledge, the consensus algorithm was formally introduced in the case of discrete-time agents for the first time in [62], inspired by the Vicsek model [140]. The authors provide convergence results using Laplacian matrices properties. In [99], Olfati-Saber et al. give the convergence conditions for consensus achievement under fixed and switching directed graphs. For undirected graphs, the convergence of the consensus protocol depends on the second smallest eigenvalue of the Laplacian matrix, known also as the algebraic connectivity [145]. In [150], the authors present an overview of convergence results regarding the consensus problem under directed graphs. Quadratic Lyapunov functions are proposed for different types of graphs depending on their connectivity properties: strongly connected, balanced and undirected graphs.

In this thesis, we focus on continuous-time systems and will use the squared norm of the disagreement vector introduced in Definition 1.7 as a candidate Lyapunov function $V(\delta) = \frac{1}{2}\|\delta\|^2 = \frac{1}{2}\delta^T\delta$. Note that the disagreement vector can be written as

$$\delta = Gx, \quad (1.13)$$

where $G = \mathbf{I} - \frac{1}{N}\mathbf{1}\mathbf{1}^T$.

Under this notation, system (1.5) can be written using the disagreement vector as

$$\dot{\delta} = -G\mathcal{L}(t)\delta. \quad (1.14)$$

Equation (1.14) is obtained from the property $\mathcal{L}\mathbf{1} = \mathbf{0}$. The derivative of V is given by

$$\dot{V}(\delta) = -\frac{1}{2}\delta^T (G\mathcal{L} + \mathcal{L}^T G) \delta = -\frac{1}{2}\delta^T (\mathcal{L} + \mathcal{L}^T) \delta. \quad (1.15)$$

The analysis of the properties of matrix $\hat{\mathcal{L}} = \frac{1}{2}(\mathcal{L} + \mathcal{L}^T)$ leads to the convergence properties of the system. For example, for undirected graphs, we have $\mathcal{L}^T = \mathcal{L}$. Then, $\hat{\mathcal{L}}$ is a Laplacian matrix which guarantees that V is a decreasing function. Moreover, if the associated graph is connected, we have asymptotic convergence to consensus. For directed graphs, similar results can be shown for balanced graphs [99], strongly connected graphs [150] and graphs containing a spanning tree [104]. However, these analysis are insufficient for a general connected

graph because of the sign of matrix $\hat{\mathcal{L}}$. Thus, we will use a different approach (a different Lyapunov function) to analyze the consensus problem presented in the next chapter Chapter 2.

1.2.5 . Convergence of the consensus algorithm under time-varying network topology

In [92], Moreau proposes a new Lyapunov function to prove exponential convergence of the consensus algorithm under mild assumptions on the graph connectivity.

Theorem 1.1 (Theorem 1 [92])

Consider the system

$$\dot{x} = -\mathcal{L}(t)x. \quad (1.16)$$

Assuming that \mathcal{L} is a bounded and piecewise continuous function of time. If there exist an index $k \in \mathcal{I}_N$, a threshold $\delta > 0$ and an interval length $T > 0$ such that for all $t \in \mathbb{R}$ the digraph associated to

$$\int_t^{t+T} \mathcal{L}(\tau) d\tau \quad (1.17)$$

has the property that all nodes may be reached from the node k and all non-zero coefficients of the associated adjacency matrix are greater than δ , then the equilibrium set of consensus states is uniformly exponentially stable.

The connectivity condition given in Theorem 1.1 states that on each interval of time $[t, t + T)$ the union graph has a spanning tree. Moreover, the root of the spanning tree remains the same over time.

The second particularity of this paper result is the use of a Lyapunov function of the form

$$W(x) = \max_{i \in \mathcal{I}_N} x_i - \min_{i \in \mathcal{I}_N} x_i, \quad (1.18)$$

where $x = (x_1, \dots, x_N)^T$.

In our result in Chapter 2, we use a similar Lyapunov function to derive the convergence properties of the consensus algorithm for a special class of graphs under a weaker connectivity condition.

1.2.6 . Asymptotic stability of non-linear time-varying systems using non-strict inequalities

In this section, we recall a result on asymptotic stability for non-linear systems that involves a non-strict inequality on the derivative of the Lyapunov function. Most of the results regarding asymptotic stability use a strict inequality to guarantee the convergence of the system state to the origin. However, this condition may be relaxed when other properties are satisfied, as in the following theorem.

Theorem 1.2 (Theorem 4.5 [72])

Let $D = \{x \in \mathbb{R}^n \mid \|x\| < r\}$ and suppose that $f(t, x)$ is piecewise continuous in t and locally Lipschitz in x on $[0, \infty) \times D$. Let $x = 0$ be an equilibrium point for $\dot{x} = f(t, x)$ at $t = 0$. Let $V : [0, \infty) \times D \rightarrow \mathbb{R}$ be a continuously differentiable function such that

$$\begin{aligned} W_1(x) &\leq V(t, x) \leq W_2(x), \\ \dot{V}(t, x) &= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) \leq 0, \\ \int_t^{t+\delta} \dot{V}(\tau, \phi(\tau, t, x)) &\leq -\lambda V(t, x), \end{aligned}$$

$\forall t \geq 0, \forall x \in D$, for some $\delta > 0$, where $W_1(x)$ and $W_2(x)$ are continuous positive definite functions on D and $\phi(\tau, t, x)$ is the solution of the system that starts at (t, x) .

Then, the origin $x = 0$ is uniformly asymptotically stable. If all assumptions hold globally and $W_1(x)$ is radially unbounded, then the origin is globally uniformly asymptotically stable. If

$$\begin{aligned} W_1(x) &\geq k_1 \|x\|^c, \\ W_2(x) &\geq k_2 \|x\|^c, \end{aligned}$$

$k_1 > 0, k_2 > 0, c > 0$, then the origin is exponentially stable.

In Chapter 4, we present a similar result for exponential stability where the Lyapunov function may increase during some intervals, but additional conditions guarantee its vanishing over time. The convergence rate is given in terms of the conditions constants.

2 - CONSENSUS OVER A SHARED BROADCAST-CAST CHANNEL

In real-world applications, multi-agent systems use a communication system to exchange information between agents. The information exchange cannot be assumed to be perfect and the effects of the communication layer cannot always be simplified or abstracted as done in the majority of the contributions related to the consensus problem (see Chapter INTRODUCTION). The communication layer has its own constraints. In fact, to communicate with each other, agents need a communication medium that is generally shared between agents and the access to this medium is defined by some rules. For wireless communication networks, the network resources (time, frequency) cannot be occupied by more than one user at a time. Otherwise, interference and packet collisions may occur, and the transmitted information may be lost. To avoid such situations, different approaches, known as multiple access schemes, were proposed to allow multiple users to share the network resources efficiently (see Chapter 1). In this chapter, we will focus on the scenario of a multi-agent system of first-order integrators communicating over a shared wireless broadcast channel. The word *broadcast* means that when any one node transmits a frame, each of the other nodes in the radio coverage receives a copy of the transmitted frame when the communication conditions are sufficiently good. This model is suitable for networks of connected vehicles where the communication medium corresponds to the electromagnetic spectrum, and cooperative awareness messages (CAMs) are broadcasted periodically by vehicles to inform the surrounding users of the vehicle's state [44] [42]. To avoid packets collision and coordinate the agents access to the shared channel, we use a time-division multiple access (TDMA) scheme. TDMA is one of the first methods used in digital communication to allow multiple users to share the network resources such as in 2G cellular systems. It also allows, by design, to support half-duplex communications which is suitable for 5G sidelink communications for connected vehicles [7]. In TDMA, time is divided into consecutive intervals called time slots. During each time slot, the communication channel is entirely used by only one user, then it is released, and another user can use it for transmission. This technique ensures an interference-free communication, however, it brings more complexity at the level of the system modelling and the stability analysis.

Compared to the classical results in the literature on multi-agent systems and consensus problems, the network topology under TDMA scheme cannot be modelled by an undirected graph or a fixed graph, i.e., constant over time. The network topology plays an important role in achieving consensus. In the majority of results on consensus of MAS are provided for networks with fixed and undirected topologies. However, in real-world applications, the network topology is directed and not

fixed but rather time-varying or switching due to different reasons such as failure or creation of links between nodes [105], formation reconfiguration [75], state-dependence of the links [95], unreliable communications and packet-losses [11]... Convergence results on the consensus problem under time-varying or switching topology utilize different types of assumptions on the interconnections. For example, in the continuous-time setting, classical results [63], [99] for networks with switched directed graphs impose connectivity conditions that have to be satisfied for all graphs. In [115], [92] these results were extended by imposing connectivity conditions that should be satisfied on average over bounded time intervals, see also [100], [118] for surveys. Alternatively, the cut-balance assumption on the interactions was considered, for example, in [55], [87] for the interactions that satisfy some form of reciprocity. In [88] it was shown that consensus is achieved if the interconnections satisfy the cut-balance condition on average on a sequence of contiguous intervals. For a broadcast channel using TDMA scheme, the network topology is directed and switching. Moreover, we cannot assume, as in [115], that the graph contains a spanning at any time. In fact, in our model, we need milder assumptions on the graph connectivity that do not concern the instantaneous graph but the behaviour of the graphs over a period of time. Thus, in this chapter, we pay attention to the problem formulation and the connectivity assumption. In addition to the proof of the main results, these two aspects represent the main contribution of this chapter. In the sequel, we consider the consensus problem for switching multi-agent systems where communications between the agents resemble those using the TDMA scheme. Specifically, we study the consensus problem under directed switching topology where only one agent can transmit at a time. We present conditions that ensure exponential consensus for a network of first-order integrators interconnected using such a scheme and, at the same time, provide an estimate of the convergence rate in such a network.

2.1 . Model description

We start the analysis by giving the reformulation of the consensus problem (see Chapter 1) where we consider the effects of the communication protocol. First, we define the communication protocol that we use and briefly discuss its properties, then we present the model after taking into account the effects of communication using such a protocol.

2.1.1 . Communication protocol modelling

The proposed communication protocol relies on a TDMA scheme. Time is divided into sequential intervals of the form $\{[t_k, t_{k+1})\}_{k \in \mathbb{N}}$. The time moments t_k are called the switching instants. During each time interval, only one agent can use the entire channel to transmit its information. To every time slot $\{[t_k, t_{k+1})\}$ we associate an integer $i_k \in \mathcal{I}_N$ to represent the index of the transmitter agent, and

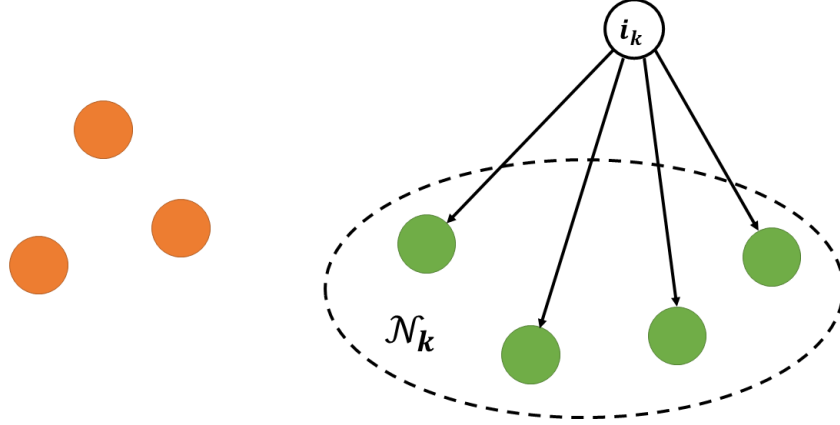


Figure 2.1: Network topology under a TDMA protocol: the network topology is represented by a directed graph composed of directed links going from the transmitter nodes to the receivers.

a set $\mathcal{N}_k \subset \mathcal{I}_N$ for the set of receivers indexes. Note that for a given transmitter node, the set of receivers can vary over various steps. Indeed, the set can be empty for some steps $k \in \mathbb{N}$, which allows us to deal with packet losses or the absence of transmission. Then, the triplets sequence $(t_k, i_k, \mathcal{N}_k)$ completely defines the communication protocol of the system. The network topology and the time-division scheme are illustrated, respectively, in Figure 2.1 and Figure 2.2.

The sequence of switching instants and the order of transmitters are supposed to be known by all agents. Thus, every agent knows when to update its control command and when to transmit. In practice, a preliminary step is needed to allocate the shared network resources to a specific node at a given time. This step is known as the scheduling step which is not discussed in this chapter. It is assumed that an external entity is able to attribute the network resources for the chosen user during a specific duration.

2.1.2 . Consensus problem description under a TDMA communication protocol

We suppose that the agents have the continuous-time dynamics (1.4) that we recall here

$$\begin{cases} \dot{x}_i = u_i \\ u_i = \sum_{j=1}^N a_{i,j}(t)(x_j - x_i) \end{cases} \quad ; \quad i \in \mathcal{I}_N. \quad (2.1)$$

We consider the following scenario. Let $\{t_k\}_{k \in \mathbb{N}}$ be a sequence of switching instants defined by the protocol. During each interval $[t_k, t_{k+1})$, the transmitting agent broadcasts its state $x_{i_k}(t_k)$ observed/measured at time t_k . This information



Figure 2.2: Time-division scheme: the communication protocol is entirely characterized by the triplets sequence $(t_k, i_k, \mathcal{N}_k)$ defining the transmitter node i_k , the set of receivers \mathcal{N}_k and the topology switching time t_k .

is used by the receivers to update their control input, while the non-receivers, i.e., the transmitting node and those that are not connected to the transmitting node, keep their state constant. In this case, the consensus algorithm (1.4) is transformed as follows.

For an agent $j \in \mathcal{I}_N$,

$$\begin{cases} \dot{x}_j(t) = x_{i_k}(t) - x_j(t), & \text{if } j \in \mathcal{N}_k; \\ \dot{x}_j(t) = 0, & \text{if } j \notin \mathcal{N}_k; \end{cases} \quad \forall t \in [t_k, t_{k+1}), \quad (2.2)$$

where i_k is the index of the transmitter agent and \mathcal{N}_k is the set of the receivers in the k -th step. Note that, since $i_k \notin \mathcal{N}_k$, we have $x_{i_k}(t) = x_{i_k}(t_k)$ for any $t \in [t_k, t_{k+1})$ which allows us to omit the time parameter t in (2.2).

This system is a particular case of the consensus problem under switching topology where the set of possible network topologies is restricted to the class of directed graphs presented in Figure 2.1.

In order to analyse the stability properties of the considered system (2.2), we start by using the traditional framework of consensus problem analysis by studying the property of the Laplacian matrices modelling the network topology.

2.2 . Properties of Laplacian matrices of one-sender graphs

To study the behaviour of system (2.2), we start by writing it in the following matrix form:

$$\dot{\delta} = -G\mathcal{L}_k\delta; \quad \forall t \in [t_k, t_{k+1}), \quad (2.3)$$

where the δ vector is defined as in (1.13). $\mathcal{L}_k = \mathbf{I}_{\mathcal{N}_k} - \mathbf{1}_{\mathcal{N}_k}\mathbf{1}_{i_k}^T$. $\mathbf{I}_{\mathcal{N}_k}$ and $\mathbf{1}_{\mathcal{N}_k}$ are, respectively, a diagonal matrix and a vector where j -th element is equal to 1 if $j \in \mathcal{N}_k$ and 0 if $j \notin \mathcal{N}_k$. $\mathbf{1}_{i_k}$ is a vector with 1 at the i_k -th position and 0 otherwise. Note that $\mathbf{I}_{\mathcal{N}_k} = \text{diag}(\mathbf{1}_{\mathcal{N}_k})$.

Example:

We consider the case where $N = 5$, $i_k = 2$ and $\mathcal{N}_k = \{1, 4, 5\}$. The corresponding Laplacian matrix can be written as follows

$$\mathcal{L}_k = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{pmatrix}.$$

We define $\hat{\mathcal{L}}_k$ as

$$\hat{\mathcal{L}}_k = \frac{1}{2} (\mathcal{L}_k + \mathcal{L}_k^T) = \mathbf{I}_{\mathcal{N}_k} - \frac{1}{2}\mathbf{1}_{\mathcal{N}_k}\mathbf{1}_{i_k}^T - \frac{1}{2}\mathbf{1}_{i_k}\mathbf{1}_{\mathcal{N}_k}^T. \quad (2.4)$$

We denote by n_k the cardinality of \mathcal{N}_k , i.e., $n_k = \text{card}(\mathcal{N}_k)$. Matrix $\hat{\mathcal{L}}_k$ is crucial in the analysis of convergence of MASs to consensus. A discussion on such matrices is presented in Subsection 2.2.1.

Proposition 2.1

Assuming that $n_k > 0$, the eigenvalues of $\hat{\mathcal{L}}_k$ are:

- $\lambda_- = \frac{1-\sqrt{n_k+1}}{2}$ with a multiplicity of 1, and the associated eigenspace \mathcal{S}_{λ_-} is $\text{span}(v_{\lambda_-})$, where $v_{\lambda_-} = \mathbf{1}_{\mathcal{N}_k} + \lambda_+\mathbf{1}_{i_k}$.
- $\lambda_+ = \frac{1+\sqrt{n_k+1}}{2}$ with a multiplicity of 1, and the associated eigenspace \mathcal{S}_{λ_+} is $\text{span}(v_{\lambda_+})$, where $v_{\lambda_+} = \mathbf{1}_{\mathcal{N}_k} + \lambda_-\mathbf{1}_{i_k}$.
- $\lambda_1 = 1$ with a multiplicity of $n_k - 1$, and the associated eigenspace \mathcal{S}_{λ_1} is the orthogonal complement of $\text{span}(v_{\lambda_2}, v_{\lambda_3})$ in $\text{span}(\{\mathbf{1}_j | j \in \mathcal{N}_k \cup \{i_k\}\})$.
- $\lambda_0 = 0$ with a multiplicity of $N - n_k - 1$, and the associated eigenspace \mathcal{S}_{λ_0} is the orthogonal complement of $\text{span}(\{\mathbf{1}_j | j \in \mathcal{N}_k \cup \{i_k\}\})$, i.e., $\mathcal{S}_{\lambda_0} = \text{span}(\{\mathbf{1}_j | j \in \{\mathcal{N}_k \cup \{i_k\}\}^c\})$.

Remark

From the result of proposition 2.1, we notice that:

- If the node i_k is connected to all the other nodes ($n_k = N - 1$), then 0 is not an eigenvalue of $\hat{\mathcal{L}}_k$.
- $\hat{\mathcal{L}}_k$ has one and only one strictly negative eigenvalue $\lambda_- = \frac{1 - \sqrt{n_k + 1}}{2}$.

2.2.1 . Discussion on the disagreement function

We consider a disagreement vector with the property $\dot{\delta} = -G\mathcal{L}\delta$. Let V be the candidate Lyapunov function defined as follows

$$V = \frac{1}{2} \delta^T \delta. \quad (2.5)$$

Then, we have

$$\dot{V} = -\frac{1}{2} \delta^T (\mathcal{L} + \mathcal{L}^T) \delta. \quad (2.6)$$

We consider the decomposition of δ in the eigenspaces of $\hat{\mathcal{L}} = \frac{1}{2} (\mathcal{L} + \mathcal{L}^T)$ (see Section 2.1)

$$\delta = \delta_0 + \delta_1 + \delta_+ + \delta_-. \quad (2.7)$$

Then, we have

$$V = \frac{1}{2} (\delta_0^T \delta_0 + \delta_1^T \delta_1 + \delta_+^T \delta_+ + \delta_-^T \delta_-),$$

and

$$\begin{aligned} \dot{V} &= -\delta_1^T \delta_1 - \lambda_+ \delta_+^T \delta_+ - \lambda_- \delta_-^T \delta_- \\ &= -2V + (1 - \lambda_+) \delta_+^T \delta_+ + (1 - \lambda_-) \delta_-^T \delta_- + \delta_0^T \delta_0 \\ &= -2V + \lambda_- \delta_+^T \delta_+ + \lambda_+ \delta_-^T \delta_- + \delta_0^T \delta_0. \end{aligned}$$

The sign of the derivative of function V cannot be determined from the previous decomposition. Thus, function V is not necessarily decreasing. The standard quadratic Lyapunov function, used for undirected or balanced graphs, fails to give a straightforward conclusion on the convergence of the proposed class of graphs. Therefore, we need a different approach to analyse the convergence properties of system (2.2).

2.3 . Exponential convergence of the consensus algorithm under a TDMA communication protocol

In this section, we present a convergence result for system (2.2) that states that under mild assumptions on the communication protocol and the interconnections

graph, a consensus is achieved uniformly in time, exponentially fast, and we give an explicit bound for the convergence rate.

We assume that the following assumptions are satisfied.

Assumption A1 (dwell time): There exists a constant $\underline{\tau} > 0$ such that for any $k \in \mathbb{N}$, $t_{k+1} - t_k \geq \underline{\tau}$.

Assumption A2 (bounded intervals): There exists a constant $\bar{\tau} > 0$ such that for any $k \in \mathbb{N}$, $t_{k+1} - t_k \leq \bar{\tau}$.

Assumption A3 (T-UQSC): There exists a constant $T > 0$ such that the graph $\mathcal{G}([t, t+T])$ is T-UQSC.

Theorem 2.1 ([12])

Consider system (2.2) and let Assumptions **A1-A3** be satisfied. Then the system achieves consensus exponentially uniformly (in time), and the following bound on the norm displacement vector is satisfied for any initial states $x_0 \in \mathbb{R}^N$ and any $t_0 \geq 0$

$$\|\delta(t)\| \leq K e^{-\lambda(t-t_0)} \|\delta(t_0)\|; \quad \forall t \geq t_0,$$

where

$$\begin{cases} K &= \sqrt{\frac{N}{2}} \frac{1}{1-\beta e^{-\alpha}} \\ \lambda &= -\frac{\ln(1-\beta e^{-\alpha})}{\alpha} \end{cases},$$

with $\alpha = \gamma(N)(T + \bar{\tau})$, $\beta = (e^{\underline{\tau}} - 1)^{N-1}$ and $\gamma(N) = \frac{(N-1)(N-2)}{2}$. The parameters T , $\underline{\tau}$ and $\bar{\tau}$ are defined in Assumptions **A1-A3**.

The proof of Theorem 2.1 is based on the same max-min Lyapunov function as in [92] but using different arguments to estimate the convergence rate and avoid assumption on the rooted spanning tree with the same root for every $\mathcal{G}([t, t+T])$. While detailed proof of the theorem is postponed to Section 2.3.1, we present some notations and technical lemmas that are crucial for the proof of Theorem 2.1.

First, for the system (2.2), we introduce the following auxiliary variables

$$\begin{cases} x_M(t) &= \max_{j \in \mathcal{I}_N} x_j(t) \\ x_m(t) &= \min_{j \in \mathcal{I}_N} x_j(t) \end{cases}; \quad \forall t \geq t_0. \quad (2.8)$$

and the Lyapunov function

$$W(t) = W(x(t)) = x_M(t) - x_m(t); \quad \forall t \geq t_0 \quad (2.9)$$

The following result allows us to link the norm of the displacement vector δ and Lyapunov function W introduced above.

Lemma 2.1

Let $\{x_i \mid i \in \mathcal{I}_N\}$ be a set of N real numbers. Let $x_M = \max_{i \in \mathcal{I}_N} x_i$, $x_m = \min_{i \in \mathcal{I}_N} x_i$, $\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$, $\delta_i = x_i - \bar{x}$ and $\delta = (\delta_1, \dots, \delta_N)^\top$. Then, we have

$$\frac{1}{2}(x_M - x_m)^2 \leq \|\delta\|^2 \leq \frac{N}{4}(x_M - x_m)^2.$$

The result of Lemma 2.1 follows from the Popoviciu's inequality [108] and the Von Szokefalvi Nagy's inequality [94].

Lemma 2.2 states that the Lyapunov function $W(t)$ is non-increasing along solutions of the system.

Lemma 2.2

Consider the system (2.2) and Lyapunov function $W(t)$ defined in (2.9). Then, for any $x_0 \in \mathbb{R}^N$ and $t_0 \geq 0$ the function $W(x(t))$ is not increasing for all $t \geq t_0$, that is

$$W(t) \leq W(t_0).$$

The following lemma gives a bound of the max-min function over a subset of \mathcal{I}_N .

Lemma 2.3

Let $\mathcal{S} \subset \mathcal{I}_N$ be a non-empty subset of \mathcal{I}_N . We consider the function $W_{\mathcal{S}}(t) = \max_{i \in \mathcal{S}}(x_i(t)) - \min_{i \in \mathcal{S}}(x_i(t))$. Then, for any $t \geq t_0$, we have

$$W_{\mathcal{S}}(t) \leq \left(1 - e^{-(t-t_0)}\right) W(t_0) + e^{-(t-t_0)} W_{\mathcal{S}}(t_0).$$

These results are straightforward from the definition of function W in (2.9) and the system dynamics described in (2.2).

2.3.1 . Proof of Theorem 2.1

In this section, we give a detailed proof of Theorem 2.1 by analysing the variation of the function W defined in (2.9).

Based on the sequence of switching instants $\{t_k\}_{k \in \mathbb{N}}$, we introduce the subsequence $\{t_{k_j}\}_{j \in \mathbb{N}}$ defined as follows

$$\begin{aligned} t_{k_0} &= t_0, \\ t_{k_1} &= \min_{s \in \mathbb{N}} \{t_s | t_s \geq t_{k_0} + T\}, \\ &\vdots \\ t_{k_{j+1}} &= \min_{s \in \mathbb{N}} \{t_s | t_s \geq t_{k_j} + T\}. \end{aligned}$$

The purpose of the proof is to estimate the decrease rate of the function W over a time interval of the form $[t_{k_j}, t_{k_j + \gamma(N)}]$ where $\gamma(N) = \frac{(N-1)(N-2)}{2}$.

We introduce the collection of sets $\mathcal{H}_p \subset \mathcal{I}_N$ for $1 \leq p \leq N$ such that the cardinality of \mathcal{H}_p is equal to p . These sets are time-dependant and are constructed recursively as follows. We know that $\mathcal{H}_N = \mathcal{I}_N$. For a fixed $t \geq t_0$, the set \mathcal{H}_p is deduced for \mathcal{H}_{p+1} by removing the maximum-valued agent or the minimum-valued agent in \mathcal{H}_{p+1} such that the $W_{\mathcal{H}_p}(t) = \max_{i \in \mathcal{H}_p}(x_i(t)) - \min_{i \in \mathcal{H}_p}(x_i(t))$ is maximal.

We start by proving the following claim.

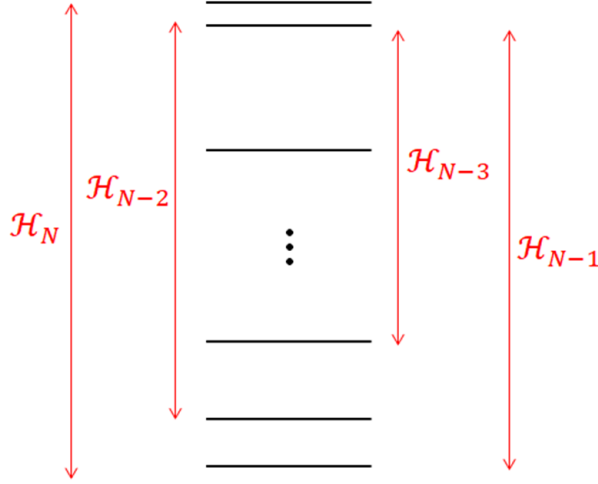


Figure 2.3: Construction of sets \mathcal{H}_p : the set \mathcal{H}_p is deduced for \mathcal{H}_{p+1} by removing the maximum-valued agent or the minimum-valued agent in \mathcal{H}_{p+1} .

Claim 2.1

Let $1 < p \leq N$. If Assumption **A3** holds, then for any $j \in \mathbb{N}$, there exists an integer $l_p^* \in \llbracket k_j, k_{j+N-p+1} - 1 \rrbracket$ such that

$$W_{\mathcal{H}_p}(t_{l_p^*+1}) \leq e^{-(t_{l_p^*+1}-t_{l_p^*})} W(t_{l_p^*}) + \left(1 - e^{-(t_{l_p^*+1}-t_{l_p^*})}\right) W_{\mathcal{H}_{p-1}}(t_{l_p^*}). \quad (2.10)$$

Note that this inequality is different from the result in Lemma 2.3 since it involves the sets \mathcal{H}_p and \mathcal{H}_{p-1} .

Similar to (2.8) let $x_M(t)^{\mathcal{H}_p}$ and $x_m(t)^{\mathcal{H}_p}$ be respectively the maximum and minimum states for agents belonging to \mathcal{H}_p . We next estimate the evolution of the function $W_{\mathcal{H}_p} = x_M^{\mathcal{H}_p} - x_m^{\mathcal{H}_p}$. Note that, when Assumption **A3** is satisfied, for any $j \in \mathbb{N}$, the union graphs $\mathcal{G}([t_{k_j}, t_{k_{j+1}}))$ are quasi-strongly connected.

Under Assumption **A3**, for any $j \in \mathbb{N}$, there exists an integer $l_p^* \in \llbracket k_j, k_{j+1} - 1 \rrbracket$ such that at least one of the extreme-valued agents belonging to \mathcal{H}_p receives information at time $t_{l_p^*}$, i.e., $x_M^{\mathcal{H}_p}(t_{l_p^*+1}) \neq x_M^{\mathcal{H}_p}(t_{l_p^*})$ or $x_m^{\mathcal{H}_p}(t_{l_p^*+1}) \neq x_m^{\mathcal{H}_p}(t_{l_p^*})$. Otherwise, the two agents do not receive information during an interval larger than T which is contradictory with Assumption **A3**.

We first address the case where only one of the extreme-valued agents in \mathcal{H}_p can receive information. Without loss of generality, we suppose that it is a maximum. Therefore there exists an integer l_p^* such that the one of the maximum-valued agents in \mathcal{H}_p belongs to $\mathcal{N}_{l_p^*}$.

If $x_{i_{l_p^*}}(t_{l_p^*}) \leq x_M^{\mathcal{H}_p}(t_{l_p^*})$, then we always have $\dot{x}_M^{\mathcal{H}_p}(t) \leq x_M^{\mathcal{H}_{p-1}}(t_{l_p^*}) - x_M^{\mathcal{H}_p}(t)$ since \mathcal{H}_{p-1} is constructed from \mathcal{H}_p by removing the receiving agent having the value

$x_M^{\mathcal{H}_p}$. Therefore, using Gronwall's lemma [51], we conclude from the inequality $\dot{W}_{\mathcal{H}_p}(t) \leq -\left(W_{\mathcal{H}_p}(t) - W_{\mathcal{H}_{p-1}}(t_{l_p}^*)\right)$ the following result:

$$\begin{aligned} W_{\mathcal{H}_p}(t_{l_p^*+1}^*) &\leq e^{-(t_{l_p^*+1}^*-t_{l_p}^*)} W_{\mathcal{H}_p}(t_{l_p}^*) \\ &\quad + \left(1 - e^{-(t_{l_p^*+1}^*-t_{l_p}^*)}\right) W_{\mathcal{H}_{p-1}}(t_{l_p}^*) \\ &\leq e^{-(t_{l_p^*+1}^*-t_{l_p}^*)} W(t_{l_p}^*) \\ &\quad + \left(1 - e^{-(t_{l_p^*+1}^*-t_{l_p}^*)}\right) W_{\mathcal{H}_{p-1}}(t_{l_p}^*). \end{aligned}$$

This inequality shows that $W_{\mathcal{H}_p}$ decreases between $t_{l_p}^*$ and $t_{l_p^*+1}^*$ when the transmitting agent's value is lower than the maximum value over \mathcal{H}_p at time $t_{l_p}^*$.

Let $0 \leq s \leq N - p$ and let $l_s \in \llbracket k_{j+s}, k_{j+s+1} - 1 \rrbracket$ be an integer such that the maximum valued agent belonging to \mathcal{H}_p receives information at time t_{l_s} . We suppose that for any $s \in \llbracket 0, p - 1 \rrbracket$ we have $x_{i_l}(t_l) > x_M^{\mathcal{H}_p}(t_l)$. We define the sets

$$\begin{aligned} \mathcal{S}_p^+(t) &= \{i \mid x_i(t) > x_M^{\mathcal{H}_p}(t)\}, \\ \mathcal{S}_p^-(t) &= \{i \mid x_i(t) \leq x_M^{\mathcal{H}_p}(t)\}. \end{aligned}$$

If Assumption **A3** holds, then for any $0 \leq s \leq N - p$, we know that the union graph $\mathcal{G}([t_{k_{j+s}}, t_{k_{j+s+1}}])$ has a spanning tree. Therefore, there exists $l'_s \in \llbracket k_{j+s}, k_{j+s+1} - 1 \rrbracket$ such that the transmitting agent belongs to $\mathcal{S}_{p-1}^-(t_{l'_s})$ and one of the receivers belongs to $\mathcal{S}_p^+(t_{l'_s})$. Since $\text{card}(\mathcal{S}_p^+) \leq N - p$ then, after $N - p$ intervals of the form $[t_{k_j}, t_{k_{j+1}})$ the maximum value is necessarily different because, at worse, the $N - p$ agents have the same value. Then, $x_M(t_{k_j}) < x_M(t_{k_{j+N-p+1}})$ which implies that $W(t_{k_j}) < W(t_{k_{j+N-p+1}})$. Let $l_p^* \in \llbracket k_j, k_{j+N-p+1} - 1 \rrbracket$ such that the maximum value x_M is non constant. Then,

$$\begin{aligned} W_{\mathcal{H}_p}(t_{l_p^*+1}^*) &\leq W(t_{l_p^*+1}^*) \\ &\leq e^{-(t_{l_p^*+1}^*-t_{l_p}^*)} W(t_{l_p}^*) + \left(1 - e^{-(t_{l_p^*+1}^*-t_{l_p}^*)}\right) W_{\mathcal{H}_{p-1}}(t_{l_p}^*). \end{aligned}$$

When both of the extreme-valued agents in \mathcal{H}_p can receive information, we can easily show that $\dot{W}_{\mathcal{H}_p} = -W_{\mathcal{H}_p}$ during the subintervals when two extreme-valued nodes are in the receiving set. Otherwise, due the symmetry of the problem, we return to the previous case. In both cases the inequality (2.10) is satisfied.

This achieves the proof of Claim 2.1.

Based on this claim we can obtain the following statement that gives an estimate of the decrease rate of the function $W_{\mathcal{H}_p}$ over the interval $[t_{k_j}, t_{k_{j+N-p+1}})$

Claim 2.2

Let $1 < p \leq N$. If Assumption **A3** holds, then for any $j \in \mathbb{N}$, we have

$$\begin{aligned} W_{\mathcal{H}_p}(t_{k_{j+N-p+1}}) &\leq e^{-(t_{k_{j+N-p+1}}-t_{k_j})} \left(e^{(t_{l_p^*+1}^*-t_{l_p}^*)} - 1 \right) W_{\mathcal{H}_{p-1}}(t_{k_j}) \\ &\quad + \left(1 - e^{-(t_{k_{j+N-p+1}}-t_{k_j})} \left(e^{(t_{l_p^*+1}^*-t_{l_p}^*)} - 1 \right) \right) W(t_{k_j}). \end{aligned}$$

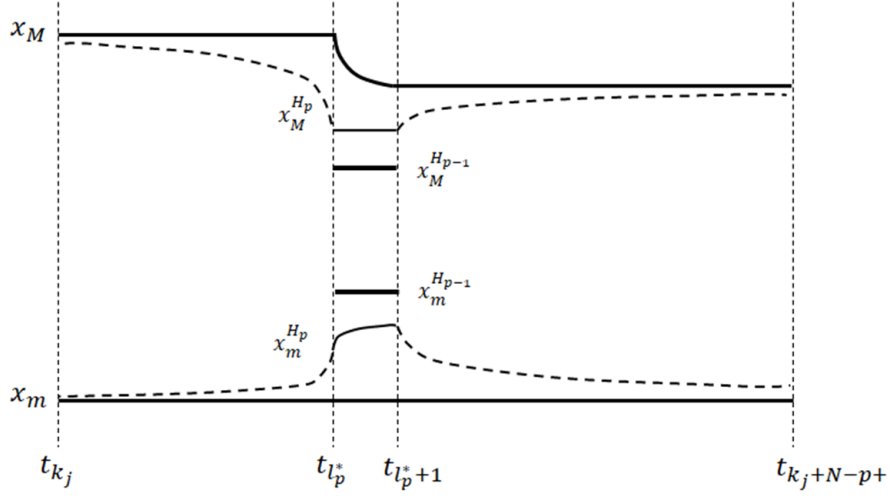


Figure 2.4: Illustration of the result of Claim 2.1.

To prove the inequality in Claim 2.2, we use the result of Claim 2.1, for the interval $\llbracket k_j, k_{j+N-p+1} - 1 \rrbracket$. We apply Lemma 2.3 for \mathcal{H}_{p-1} between t_{k_j} and $t_{l_p}^*$, and for \mathcal{H}_p between $t_{l_p}^*+1$ and $t_{k_j+N-p+1}$. By combining the three obtained equations and subtracting $W(t_{k_j})$ from both sides and use the property $W(t_{l_p}^*) \leq W(t_{l_p}^*+1) \leq W(t_{k_j})$ to get the result of Claim 2.2.

From Claim 2.2, we deduce that, under Assumption **A1** and Assumption **A2**, for $j \in \mathbb{N}$ and any $1 < p \leq N$,

$$\begin{aligned} & W_{\mathcal{H}_p}(t_{k_j+N-p+1}) - W(t_{k_j}) \\ & \leq e^{-(N-p)(T+\bar{\tau})} (e^{\bar{\tau}} - 1) [W_{\mathcal{H}_{p-1}}(t_{k_j}) - W(t_{k_j})] / \end{aligned}$$

We apply the last result for $1 < p \leq N$. Then, since $\mathcal{H}_N = \mathcal{I}_N$ and $W_{\mathcal{H}_1}(t_{k_j}) = 0$, we get

$$\begin{aligned} & W\left(t_{k_j + \frac{(N-1)(N-2)}{2}}\right) - W(t_{k_j}) \\ & \leq -e^{-\frac{(N-1)(N-2)}{2}(T+\bar{\tau})} (e^{\bar{\tau}} - 1)^{N-1} W(t_{k_j}). \end{aligned}$$

Therefore, for any $q \in \mathbb{N}$,

$$W(t_{k_{q\gamma(N)}}) \leq \left[1 - e^{-\gamma(N)(T+\bar{\tau})} (e^{\bar{\tau}} - 1)^{N-1}\right]^q W(t_0)$$

, where $\gamma(N) = \frac{(N-1)(N-2)}{2}$.

Let $t \geq t_0$ and $q^* = \operatorname{argmax}_{q \in \mathbb{N}} \{t_{k_{q\gamma(N)}} \leq t\}$. Then, using Lemma 2.2 and the inequality $\frac{t-t_0}{\gamma(N)(T+\bar{\tau})} - 1 \geq q^*$, we get

$$W(t) \leq K' e^{-\lambda'(t-t_0)} W(t_0),$$

where

$$K' = \frac{1}{1 - e^{-\gamma(N)(T+\bar{\tau})} (e^{\bar{\tau}} - 1)^{N-1}},$$

and

$$\lambda' = -\frac{\ln \left(1 - e^{-\gamma(N)(T+\bar{\tau})} (e^{\bar{\tau}} - 1)^{N-1} \right)}{\gamma(N)(T + \bar{\tau})}.$$

Using Lemma 2.1, we deduce that

$$\begin{aligned} \|\delta(t)\| &\leq \frac{\sqrt{N}}{2} W(t) \leq \frac{\sqrt{N}}{2} K' e^{-\lambda'(t-t_0)} W(t_0) \\ &\leq \sqrt{\frac{N}{2}} K' e^{-\lambda'(t-t_0)} \|\delta(t_0)\|, \end{aligned}$$

which concludes the proof with $K = \sqrt{\frac{N}{2}} K'$ and $\lambda = \lambda'$.

2.3.2 . Simulation of the consensus algorithm using standard TDMA protocols

We consider a multi-agent system composed of $N = 5$ agents. Each agent is represented by its state. The initial states at $t_0 = 0$ are evenly distributed over the interval $[0, 1]$, i.e., for any $i \in \mathcal{I}_5$, $x_i(0) = \frac{(i-1)}{4}$. We compare 4 scenarios of the consensus algorithm over a shared broadcast channel.

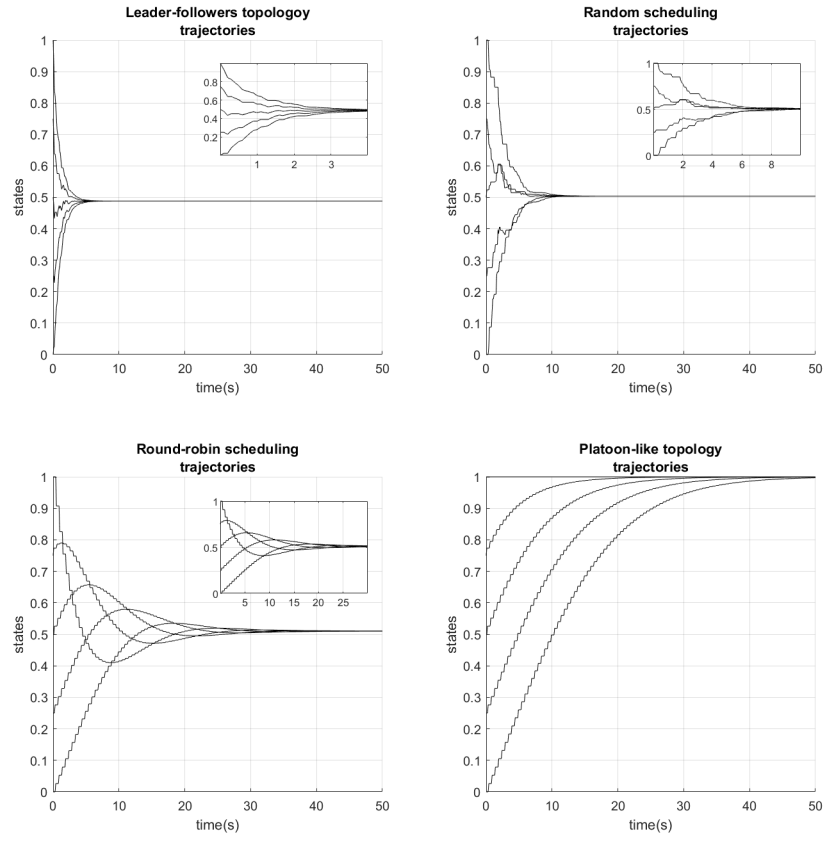
Scenario 1: (Leader-followers topology) for each time step of length $\tau = 0.1s$, the transmitting agent i_k is selected randomly. All the remaining agents receive the transmitted information \mathcal{I}_5 , i.e., $i_k \rightarrow \mathcal{I}_5$.

Scenario 2: (Random scheduling) for each time step of length $\tau = 0.1s$, the transmitting agent i_k is selected randomly and the set of receiving agents is selected randomly $\mathcal{N}_k \subset \mathcal{I}_5$, i.e., $i_k \rightarrow \mathcal{N}_k$.

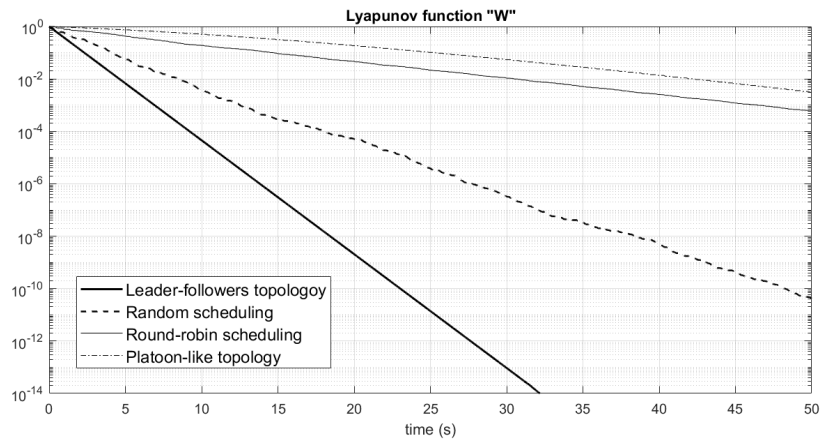
Scenario 3: (Round-robin scheduling) The network topology is a ring and the scheduling is periodic with a period $T = 0.5s$. Every period is divided in 5 steps of length $\tau = 0.1s$: $i_1 \rightarrow i_2$, $i_2 \rightarrow i_3$, $i_3 \rightarrow i_4$, $i_4 \rightarrow i_5$ and $i_5 \rightarrow i_1$.

Scenario 4: (Platoon-like topology) The network topology is a chain and the scheduling is periodic with a period $T = 0.5s$. Every period is divided in 5 steps of length $\tau = 0.1s$: $i_1 \rightarrow i_2$, $i_2 \rightarrow i_3$, $i_3 \rightarrow i_4$, $i_4 \rightarrow i_5$ and there is no transmission in the last step.

Figure 2.5a shows the trajectories of the agents states in the four proposed scenarios. And, in Figure 2.5b, we compare the Lyapunov function W of the four proposed scenarios in *logarithmic scale*. As proven in Theorem 2.1, there is an exponential bound of W that depends on the protocol parameters. This explains why the curves in Figure 2.5b, are almost linear.



(a) States trajectories for the four proposed scenarios



(b) Evolution of the function W over time for the four proposed scenarios in logarithmic scale

Figure 2.5: Comparison of four scenarios of the proposed system. Consensus is achieved under an exponential rate.

2.4 . Conclusion

In this chapter, we proposed a new model for the consensus algorithm for a multi-agent system of continuous-time single integrators under a TDMA communication protocol. The TDMA method allows the interconnected nodes to share the network resources efficiently and to avoid packet collisions. We proved uniform exponential convergence of the proposed system under mild assumptions regarding the network topology connectivity and the switching times. We provided an explicit exponential bound on the norm of the displacement errors vector. The proposed exponential bound depends on the parameters of the communication protocol. This bound allows to design the communication protocol to guarantee the achievement of control performance targets.

3 - CO-DESIGN OF FORMATION CONTROL AND NETWORK RESOURCE ALLOCATION

In the control theory literature, the stability properties of consensus algorithms are studied under strong assumptions on the communication system. Most of the time, the network topology is assumed to be undirected and/or fixed in time [63] [99]. However, in real-world applications, the network topology changes over time and depends on the states of the agents. For example, when dealing with connected vehicles, the network topology depends on their positions since a vehicle can communicate only within a specific range. The Hegselmann-Krause (HK) model proposes an enhanced version of the consensus algorithm presented in [99] to take into account the state-dependence of the network topology [54] [133]. The HK model is generally presented in discrete-time where, at each step, each agent updates its state based on the states of its close neighbours, i.e., agents that belong to some vicinity of the considered agent. In both discrete and continuous time, the HK model provides interesting properties on the network topology that allow to achieve consensus [18] [91]. However, it is assumed that all agents can always communicate with their neighbours. In real communication systems, communication links can be interrupted at some moments due to packet losses. Moreover, the network resources are limited. Therefore, agents can transmit information only if there are available resources allocated to them during a specific period.

The fifth generation of cellular networks (5G) standards provide a framework for vehicular communications known as cellular vehicle-to-everything (C-V2X) [34] [44]. 5G wireless technology is meant to deliver higher peak data rates, lower latency, improved reliability, massive network capacity, increased availability, and a more uniform user experience. Higher performance and improved efficiency empower new user experiences and pave the way to more vertical applications. C-V2X uses cellular connectivity to send and receive signals from a vehicle to other vehicles, pedestrians, or fixed stations such as traffic lights in its surroundings. It offers several tools that enable, for example, road traffic optimization, safety level improvement and reduction of energy consumption [85] [28].

Compared to previous cellular standards like 4G LTE, the 5G technology allows more flexibility in the design of application-oriented communication systems by providing optimized technologies to implement C-V2X communications. For example, for road-traffic optimization applications, it is crucial that vehicles can communicate with each other in order to agree on a common target such as the vehicles speed or the inter-vehicular distance. In this context, the consensus algorithm represents an efficient tool to make the inter-connected vehicles agree on a common value. The concept of designing jointly the communication and control

is explored in [149] and [89] for vehicles platooning. In this work, we present a method to tune the communication system configuration to optimize a criterion related to the control of an arbitrary formation of vehicles, which includes platoons. We propose a resource allocation algorithm that guarantees convergence to the desired formation by preventing the split of the formation that may occur because of the loss of connectivity.

The chapter is organized as follows. First, we propose a formulation similar to the continuous-time HK model for the formation control problem in a 5G C-V2X communication context. We focus on the communication system's model and the constraint of limited network resources. In the second part of this chapter, a resource allocation algorithm is presented along with a characterization of the initial conditions ensuring convergence. In Section 3.3, a simulation example is given to assert the relevance of the proposal. Finally, we conclude the chapter and present a discussion on the robustness of the proposed algorithm regarding the communication errors.

3.1 . Problem formulation

Consider a network of connected and automated vehicles communicating over a 5G C-V2X communication network. Vehicles are connected to a base station (gNB) and, as described in Section 3.1.2, they can communicate directly with each other through the C-V2X sidelink. Each vehicle is identified by its position in a 2-dimensional space $p_i \in \mathbb{R}^2$ for $i \in \mathcal{I}_N = \{1, \dots, N\}$. The network structure depends on the vehicles' positions. Information is exchanged between vehicles through radio signals. A vehicle can successfully receive the transmitted signals only if it is within the radio coverage of the transmitting vehicles. The radio coverage, also called transmission range or distance, depends on the power of the transmitted signal and the radio conditions. This distance is assumed identical for all vehicles and is denoted as ρ . The communication links are determined by the connectivity function w , defined by

$$w(p_i, p_j) = \begin{cases} 1; & \text{if } \|p_i - p_j\| \leq \rho \\ 0; & \text{otherwise} \end{cases} . \quad (3.1)$$

Function $w(\cdot, \cdot)$ is an indicator of the presence of a communication link between two agents. This link is symmetric, i.e., if an agent i can receive from agent j , then agent j can also receive from agent i . In this case, i and j are called neighbours. Note that transmission errors are not considered in this model. The main cause of communication errors in modern communication systems is packet losses. 5G systems guarantee a very low packet error rate (PER) of around 10^{-3} [109].

3.1.1 . Formation control problem for connected vehicles

There are many applications in which the vehicles are required to follow a predefined trajectory while maintaining a specific formation [6]. For example, in the case of platooning, the vehicles need to travel at a specific speed while forming a line with equal inter-vehicle distances. We suppose that the agents goal is to track a specific trajectory generated by the dynamics $\dot{p}_i(t) = u_{ref}(t)$ while maintaining a formation defined by the offsets with respect to the center of formation. The reference trajectory is assumed to be given by an external controller and known by all agents. The center of the formation is a virtual point that is used as a reference to express the shape of the formation in terms of fixed vectors called offsets $d_i \in \mathbb{R}^2$. It can be associated to the average position $\bar{p} = \frac{1}{N} \sum_{i=1}^N p_i$ or to the position of a specific agent, for example agent 1, which implies $d_1 = 0$. The goal of the formation control algorithm is to guarantee

$$\lim_{t \rightarrow \infty} \|(p_i(t) - p_j(t)) - (d_i - d_j)\| = 0; \quad \forall i, j \in \mathcal{I}_N. \quad (3.2)$$

This condition states that the positions of every couple of agents $(i, j) \in \mathcal{I}_N^2$ tends to the desired separation $(d_i - d_j)$ in the targeted formation [114]. Note that the relevant information in formation control problems is the separation vectors rather than the offset vectors.

In order to solve the formation control problem, we propose the following algorithm

$$\begin{cases} \dot{p}_i &= u_i, \\ u_i &= u_{ref} + \sum_{j=1}^N w(p_i, p_j) [(p_j - d_j) - (p_i - d_i)] \end{cases}, \quad (3.3)$$

where $u_i \in \mathbb{R}^2$ represents the control command of the i -th agent. The command u_i is composed of two control parts: the first part is the reference control input that indicates the nominal behaviour of the vehicle, and the second part is a consensus-like control input that aims to maintain the shape of the formation. The position of a vehicle is updated according to the positions of its neighbours.

We introduce the relative positions $x_i = p_i - d_i$ for $i \in \mathcal{I}_N$ as the state of i -th agent. The state variables $x_i(t)$ tend to the center of formation when time tends to infinity. Using the new notations, the proposed algorithm is written as follows.

For any $i \in \mathcal{I}_N$,

$$\dot{x}_i = u_{ref} + \sum_{j=1}^N w(p_i, p_j)(x_j - x_i). \quad (3.4)$$

Equation (3.4), shows that the dynamics of x_i depends on both the actual positions and the relative positions. This is an important difference between our model and the standard formulation of the HK model [54]. Note that we can substitute $w(p_i, p_j)$ by $w(x_i + d_i, x_j + d_j)$ in order to have a more homogeneous expression.

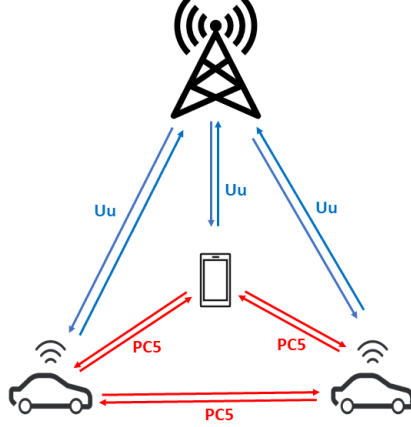


Figure 3.1: Mode 1 of resources allocation: the resource allocation is performed by the BS over the Uu link. The communication is performed directly between UEs over the PC5 interface.

Then, the convergence (consensus) condition (3.2) can be written as

$$\lim_{t \rightarrow \infty} \|x_i - x_j\| = 0; \quad \forall i, j \in \mathcal{I}_N.$$

3.1.2 . Formation control problem under a 5G communication framework

In addition to the traditional downlink (DL) and uplink (UL) transmissions between the base station (gNB) and the user equipments (UEs), the 3GPP 4G and 5G cellular systems also enable device-to-device (D2D) communications through the so-called sidelink (SL). We focus here on transmissions between vehicular agents (V2V) over the 5G sidelink that supports two modes of resources allocations, namely Mode 1 and Mode 2 [52].

We focus here on Mode 1 where the gNB assigns and manages the SL radio resources for V2V communications using the DL/UL interfaces. Mode 1 uses dynamic grant (DG) scheduling or configured grant (CG) scheduling that extends the semi-persistent scheduling (SPS) of LTE V2X Mode 4. For clarity purposes, we will use the terminology of SPS in its general meaning to refer to the scheduling mechanism where resource allocation is performed periodically and the same resources are maintained by users for a specific duration.

While the 3GPP standard defines the principle of Mode 1 scheduling and the signaling required to establish and maintain communication links, the method used to select which communication resources to allocate to each UE remains implementation-specific. One of the main purposes of the scheduling is to allocate the less noisy frequencies to UEs in order to optimize the channel usage while

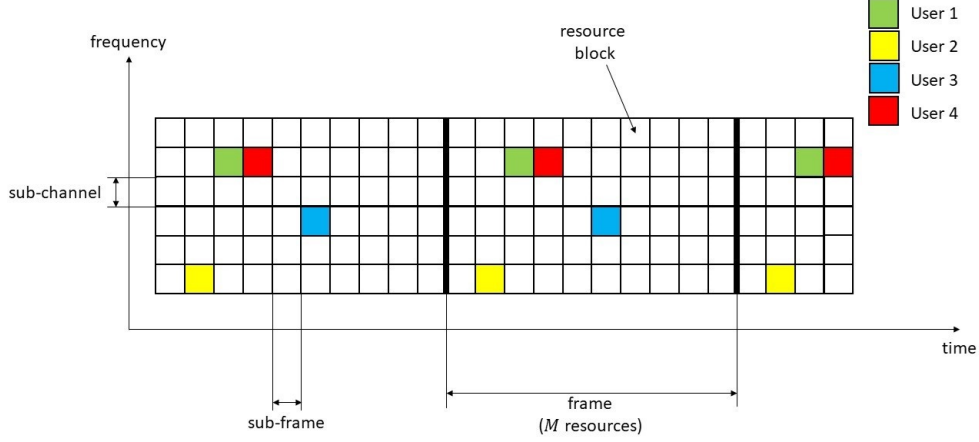


Figure 3.2: Structure of a frame: a frame is divided in time and frequency corresponding, respectively, to sub-frames and sub-channels. The resulting intersection is called a resource block (RB).

ensuring to all UEs a fair access to resources. However, the scheduling can be configured in order to optimize various criteria related to the application performance. In our work, we propose a scheduling strategy that aims to accelerate the convergence of the vehicles system to the desired formation.

The time-continuity assumption

We assume here that all agents operate in Mode 1 with SPS scheduling. It is further assumed that all agents transmit with the same periodicity over synchronous periods of time called hereafter frames. In this chapter, we use the word "frame" to refer to a logical frame repeating periodically over time, which is different from the physical frame of duration 10ms specified in the 5G standards [44].

The information transmitted by agents is encoded and mapped in a resource block (RB) as presented in Figure 3.3. Thus, the flow of transmitted information is not continuous over time, but transmissions occur at specific instants. Indeed, when a base station allocates an RB to a user i , then user i transmits periodically its state x_i at the corresponding subframes. As illustrated in Figure 3.3, the duration between two consecutive transmissions corresponds to the frame duration T_f and the number of transmissions corresponds to the number q of frames per super-frame. A super-frame is composed of q consecutive frames, and its duration is given by $T_p = qT_f$.

To simplify the system model, assume that the number of frames per super-frame is sufficiently large $q \gg 1$. In doing so, we can assume that the information transmission is continuous over time. Note that, despite the continuous-time

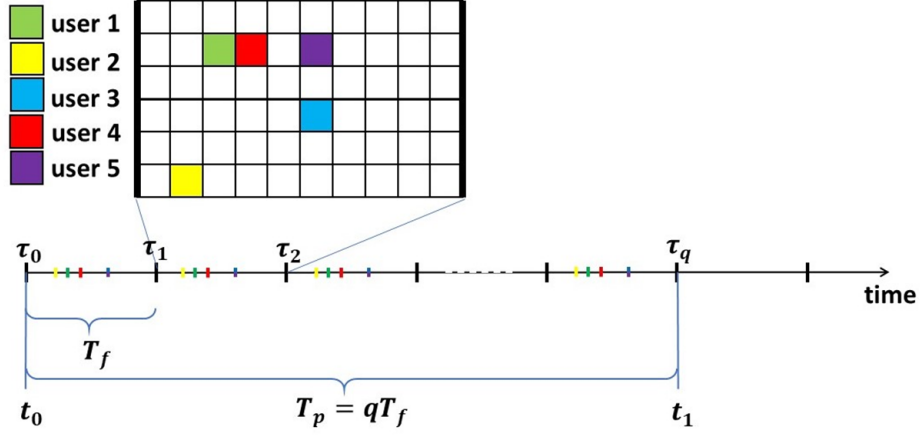


Figure 3.3: Structure of a super-frame: the transmission bandwidth is divided in a grid of time and frequency resource blocks (RBs). A block is associated to a user for a given period T_p of time corresponding to q frames of duration T_f .

transmission, the communication link between a transmitter and a receiver can be interrupted during a super-frame if the receiver leaves the transmission range.

Let $\{t_k\}_{k \in \mathbb{N}}$ be the sequence of starting times of the super-frames such that t_0 corresponds to the initial time, and $t_{k+1} - t_k = T_p$ for any $k \in \mathbb{N}$.

For any $i \in \mathcal{I}_N$ and $t \in [t_k, t_{k+1})$,

$$\dot{x}_i = u_{ref} + \sum_{j=1}^N h_{k,j} w(p_i, p_j) (x_j - x_i), \quad (3.5)$$

where $h_{k,j} \in \{0, 1\}$ for any $j \in \mathcal{I}_N$ and $k \in \mathbb{N}$ such that $h_{k,j} = 1$ if agent j is transmitting at step k and $h_{k,j} = 0$ otherwise. Index j represents the agent number and index k corresponds to the index of the super-frame which is incremented at the beginning of every super-frame.

Note that the value of parameters $h_{k,j}$ in (3.5) is fixed during the interval $[t_k, t_{k+1})$. Thus, the new system model (3.5) includes hybrid-dynamics, i.e., continuous-time dynamics and discrete-time dynamics [47]. The control law of $h_{k,j}$ is given by the resource allocation algorithm proposed in Section 3.2. This decision is made at the level of the communication system (the base station gNB) and transmitted to the users, at the beginning of every super-frame (t_k), to parameterize their controllers u_i . This aspect justifies the terminology of joint design of control and communication systems.

The limited resources constraint

In the case of the sidelink or D2D communication, if two close enough UEs use the same resource, interference may occur leading to packet losses. On the other hand, two UEs can use the same resource if they are far away from each other. This possibility is used in cellular networks to re-use radio resources. Assume that all UEs have the same coverage, i.e., the maximum distance at which a receiver can successfully receive the transmitted information. When a UE needs to transmit a packet, it competes with all other UEs within its coverage. If there are fewer resources per frame than the number of UEs in the coverage region, one has to select which UEs can transmit.

The proposed algorithm aims to assign available resources to users that are allowed to transmit. Let M be the number of available resources per frame. Locally, the number of transmitting users cannot exceed M . However, when two transmitting users are far away from each other, their signals cannot interfere due to power loss. Then, they are allowed to use the same resource if it is guaranteed that there is no receiver in the common range of these two transmitting users. In this context, we define a resource reuse distance R as shown in Figure 3.4. For example, user U_j belongs simultaneously to the set of neighbours of U_i and U_k . If the same resource is associated to users U_i and U_k then, an interference will occur at the level of U_j . Therefore, in order to ensure an interference-free communication, the resources reuse distance should respect the following condition

$$R > 2\rho.$$

For each agent $i \in \mathcal{I}_N$, define the set of potentially interfering users $\varphi_i = \{j \in \mathcal{I}_N; \|p_j - p_i\| \leq R\}$.

The limited resources constraint can be formulated by the following inequality

$$\sum_{j \in \varphi_i} h_{k,j} \leq M; \quad \forall t \in [t_k, t_{k+1}), \forall i \in \mathcal{I}_N. \quad (3.6)$$

This constraint ensures that in a circle of radius R around a given agent, the number of transmitting users cannot exceed the number of resources per frame. Thus, the communication system is guaranteed to be interference-free. Note that φ_i is a time-varying set that depends on the users positions p_i . Since the positions vary over time, then the content of sets φ_i varies over time too.

The transmission indicators change over time and are set by the base station gNB at the beginning of each super-frame. In the next section, we present the algorithm that allows selecting the transmitting agents during each super-frame.

3.2 . Resource allocation algorithm

The proposed algorithm aims to allocate available network resources per super-frame to users in order to ensure the convergence to consensus and accelerate the

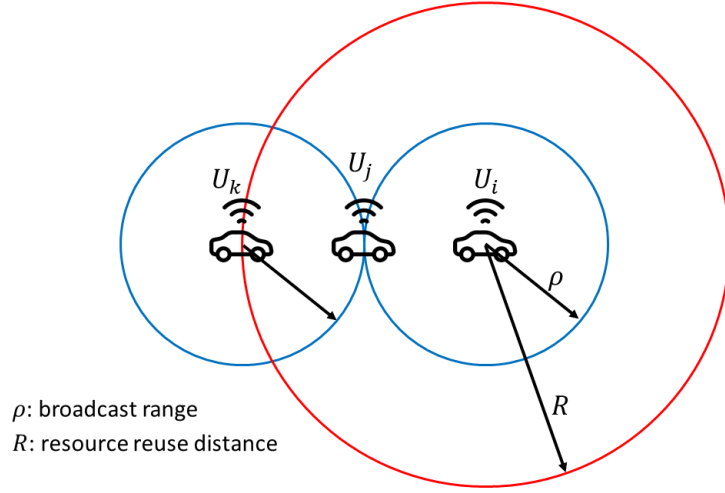


Figure 3.4: Illustration of the resource reuse distance. Users belonging to a circle of radius R cannot be associated to the same resource because of the potential existence of a common receiver.

convergence rate. The algorithm is performed by the base station at the beginning of each super-frame. It makes it possible to select the transmitting users while ensuring that the limited resources constraint is respected. Since the number of transmitting users is lower than the number of available resources, the 5G system will be able to attribute the resources to users and ensure an interference-free communication during the convergence to the desired formation.

To characterize the distance to the desired formation, i.e., achieving consensus, we use the cost function

$$V(x) = \frac{1}{2} \sum_{i,j=1}^N \|x_i - x_j\|^2. \quad (3.7)$$

This function is positive definite on the consensus set, i.e.,

$$V(x) = 0 \iff x_1 = \dots = x_N.$$

In the next section, we present an iterative algorithm that minimizes the cost function V at the end of each super-frame under the agents' dynamics and the communication system constraints. Let us first recall some notations about graph theory.

Let $p = (p_1, \dots, p_N)^T \in \mathbb{R}^{2N}$ be the vector containing the vehicles positions. We denote by $\mathcal{G}(p) = (\mathcal{V}(p), \mathcal{E}(p))$ the graph composed of vertices in positions p_i . An edge $e_{i,j} \in \mathcal{E}(p)$ if $\|p_i - p_j\| \leq \rho$, or equivalently $w(p_i, p_j) = 1$, where $w(\cdot, \cdot)$ is the connectivity function defined in (3.1).

Note that, in formation control problems, it is implicitly assumed that the graph formed by vehicles is connected. Indeed, when the graph is unconnected, i.e., the graph is split into clusters that do not have shared communication links, these clusters cannot be attracted to each other without the need of external dynamics.

Therefore, we impose the following condition to ensure that convergence to the desired formation is possible

$$\mathcal{G}(p(t)) \text{ is connected; } \forall t \in [t_k, t_{k+1}). \quad (3.8)$$

The controller that we propose for system (3.5) has two levels. During each super-frame, the control input is defined by (3.3) while the transmission indicators $h_{k,l}$ during this super-frame are defined via the algorithm described in the next section.

3.2.1 . Transmission protocol design

For the k -th super-frame, at time t_k , the base station uses its knowledge of the system dynamics to predict the system's behaviour over the interval $[t_k, t_{k+1})$ and sends the transmission instructions to users after selecting the optimal transmission configuration $h_k^* = (h_{k,1}^*, \dots, h_{k,N}^*)^T \in \{0, 1\}^N$ that minimizes the value of the cost function at the end-time of the super-frame $V(t_{k+1})$. The optimization problem is stated as follows [13]

$$\begin{aligned} h_k^* &= \underset{h_k \in \{0,1\}^N}{\operatorname{argmin}} V(x(t_{k+1})) & (3.9) \\ \text{subject to } \dot{x}_i &= u_{ref} + \sum_{j=1}^N h_{k,j} w(p_i, p_j)(x_j - x_i); & \forall t \in [t_k, t_{k+1}), \forall i \in \mathcal{I}_N, \\ \sum_{j \in \varphi_i} h_{k,j} &\leq M; & \forall t \in [t_k, t_{k+1}), \forall i \in \mathcal{I}_N, \\ \mathcal{G}(p(t)) &\text{ is connected; } & \forall t \in [t_k, t_{k+1}). \end{aligned}$$

The first two constraints correspond to the dynamics of the agents given in (3.5) and the limited resources constraint (3.6). The third constraint ensures that the connectivity is preserved during the whole super-frame. Similar to the standard Krause model, it can be shown for model (3.5) that if the connectivity is lost at some moment t , it is lost for all future time. For this reason, we need to check the graph connectivity only at the end-time of the super-frame t_{k+1} . This property allows us to replace the third constraint by " $\mathcal{G}(p(t_{k+1}))$ is connected" which is equivalent to checking that the second smallest eigenvalue of the Laplacian matrix associated to graph $\mathcal{G}(p(t_{k+1}))$ is strictly positive. This modification allows reducing the algorithm complexity.

The formulated optimization problem fits in the class of Boolean programming problems. Various methods, such as the Branch-and-Bound method [20], are proposed in the literature to handle such problems and can be used to solve (3.9).

Rather than finding the best method for solving the optimization problem (3.9), we focus in this chapter on the convergence properties of the closed loop system (3.5)-(3.9).

3.2.2 . Existence of a solution for the transmission protocol

Even though the proposed optimization problem guarantees the existence of at least one feasible solution $h_k = (0, \dots, 0)^T$, this solution is not satisfactory since it means that there is no information transmission between users. In this case, only the reference dynamics u_{ref} would contribute to the vehicles control and thus the desired formation will not be achieved.

To guarantee the existence of a non-zero feasible solution, we impose assumptions on the graph connectivity at initial time instant $t_0 = 0$ and on the graph of the desired formation. The latter is defined as follows. Let $d = (d_1, \dots, d_N)^T \in \mathbb{R}^{2N}$ be the vector of the offsets. Similar to graph $\mathcal{G}(p)$ we define the graph of the desired formation $\mathcal{G}(d) = (\mathcal{V}(d), \mathcal{E}(d))$ which is composed of vertices in positions d_i s and edges given by $e_{i,j} \in \mathcal{E}(d)$ if $w(d_i, d_j) = 1 \iff \|d_i - d_j\| \leq \rho$. To ensure an information flow between all agents in the desired formation we introduce the following assumption.

Assumption A1: The graph $\mathcal{G}(d)$ associated to the desired (targeted) formation is connected.

Theorem 3.1 ([13])

Let $\mathcal{G}(d)$ be the graph associated to the desired formation and let ρ be the transmission range. Let assumption **A1** be satisfied.

If initial positions $p(t_0)$ are such that $\|p_i(t_0) - p_j(t_0)\| \leq \rho$ holds for all pairs $(v_i, v_j) \in \mathcal{E}(d)$, then at each step, the optimization problem in (3.9) has a non-zero solution. Moreover, solutions of system (3.5) converge to consensus when the optimal transmission indicators h_k^ defined by (3.5) are applied.*

Proof.

The first step of our proof is to show the existence of a feasible solution, and the second step is to study convergence to the desired formation. Lyapunov function $V(x)$ defined in (3.7) is used in both steps of the proof.

To prove the existence of a feasible solution to problem (3.9), it is enough to show that there is at least one configuration that gives a feasible solution. Let the assumptions of the theorem be satisfied at t_k . Then, to prove the existence of a feasible solution, it is enough to prove the existence of parameters $h_{k,l}$ such that $\dot{V} \leq 0$ over the interval $[t_k, t_{k+1})$ and the connectivity constraint is satisfied at the moment t_{k+1} .

Taking the derivative of function V along solutions of (3.5) over the interval

$[t_k, t_{k+1})$, we obtain

$$\begin{aligned}\dot{V} &= \sum_{i,j=1}^N (x_i - x_j)^T (\dot{x}_i - \dot{x}_j) \\ &= \sum_{i,j,l=1}^N h_{k,l} w(p_l, p_i) (x_i - x_j)^T (x_l - x_i) \\ &\quad - \sum_{i,j,l=1}^N h_{k,l} w(p_l, p_j) (x_i - x_j)^T (x_l - x_j) \\ &= 2 \sum_{i,j,l=1}^N h_{k,l} w(p_l, p_i) (x_i - x_j)^T (x_l - x_i).\end{aligned}$$

The last equality is obtained from the dynamics equation (3.5) and the interchange of variables i and j in the second term. Next, using equalities $w(p_l, p_j) + [1 - w(p_l, p_j)] = 1$ and $2ab = (a + b)^2 - a^2 - b^2$, we get

$$\begin{aligned}\dot{V} &= - \sum_{i,j,l=1}^N h_{k,l} w(p_l, p_i) w(p_l, p_j) \|x_i - x_j\|^2 \\ &\quad - \sum_{i,j,l=1}^N h_{k,l} w(p_l, p_i) [1 - w(p_l, p_j)] \|x_i - x_l\|^2 \\ &\quad - \sum_{i,j,l=1}^N h_{k,l} w(p_l, p_i) [1 - w(p_l, p_j)] \|x_j - x_l\|^2 \\ &\quad + \sum_{i,j,l=1}^N h_{k,l} w(p_l, p_i) [1 - w(p_l, p_j)] \|x_j - x_l\|^2.\end{aligned}$$

Since in the expression above only the last term is positive, then to guarantee that $\dot{V} \leq 0$ during the super-frame k , it is enough to find a configuration h_k that allows us to dominate the last term by the other terms.

Assume that at super-frame k , there is only one transmitting agent¹ l_{0_k} and therefore $h_{k,l_{0_k}} = 1$ and $h_{k,l} = 0$ for $l \neq l_{0_k}$. Note that, in this case, \dot{V} verifies the following inequality

$$\begin{aligned}\dot{V} &\leq - \sum_{i,j=1}^N h_{k,l_{0_k}} w(p_{l_{0_k}}, p_i) [1 - w(p_{l_{0_k}}, p_j)] \|x_j - x_i\|^2 \\ &\quad + \sum_{i,j=1}^N h_{k,l_{0_k}} w(p_{l_{0_k}}, p_i) [1 - w(p_{l_{0_k}}, p_j)] \|x_j - x_{l_{0_k}}\|^2.\end{aligned}$$

By choosing the transmitting agent as $l_{0_k} = \operatorname{argmin}_{l \in \mathcal{I}_N} \sum_{j=1}^N [1 - w(p_l, p_j)] \|x_j(t_k) - x_l(t_k)\|^2$ we guarantee that the first term in the \dot{V} inequality dominates the second term. Therefore, we get $\dot{V} \leq 0$. The condition $\dot{V} = 0$ is achieved only if $x_i = x_j$ for $i, j \in \mathcal{I}_N$.

At the same time, from the equation (3.5), for any $t \in [t_k, t_{k+1})$, we get

$$\begin{aligned}p_i(t) - p_{l_{0_k}}(t) &= e^{-(t-t_k)} (p_i(t_k) - p_{l_{0_k}}(t_k)) \\ &\quad + (1 - e^{-(t-t_k)}) (d_i - d_{l_{0_k}}).\end{aligned}$$

¹Note that such a choice guarantees that the constraint of limited network resources is satisfied, since $M \geq 1$.

Then for any $(v_i, v_j) \in \mathcal{E}(d)$, such that $\|d_i - d_j\| \leq \rho$ and $\|p_i(t_k) - p_j(t_k)\| \leq \rho$, using the triangular inequality we obtain that $\|p_i(t) - p_j(t)\| \leq \rho$ for any $t \in [t_k, t_{k+1})$. This property guarantees that the graph $\mathcal{G}(p(t))$ remains connected during this interval and since k is arbitrary, the connectivity property of $\mathcal{G}(p(t))$ is preserved for all $t \geq 0$. Thus we showed the existence of a non-zero feasible solution.

The optimization algorithm (3.9) provides an optimal selection of transmitters that minimizes the value of the cost function $V(x)$ at time t_{k+1} while satisfying the constraints of connectivity preservation and limited network resources. The existence of such an optimal solution is guaranteed from the existence of a single-transmitting-agent feasible solution. Let such an optimal selection of transmitters given by (3.9) be applied to the system (3.5) and denote by $V_{opt}(x(t))$ the value of the function $V(x)$ along trajectories based on this selection. Then at instants t_k , values of function $V_{opt}(x(t))$ are upper bounded by a strictly decreasing function $V(x(t))$ that corresponds to the single-transmitting-agent solution. Therefore, the vehicles converge to the desired formation.

□

The connectivity condition given in Theorem 3.1 can be interpreted as follows. If two agents are supposed to be in each other's transmission range in the desired formation, they must be in each other's transmission range at the initial time. In practical applications, this constraint is easily satisfied since vehicles positions in the formation are chosen such that a minimal maneuver is required.

Remark

The problem of assigning network resources to users can be solved using a graph coloring algorithm [64], namely, the Welsh-Powell algorithm [142]. Graph coloring algorithms allow to attribute labels, or colors, to nodes in a graph under certain constraints. In the context of the studied problem, we aim to attribute colors to vehicles that are designated to transmit. Colors represent network resources and a fixed number of colors is available. We need to satisfy the constraint that two vehicles belonging to a set φ_i , defined in (3.6), must be of different colors (see Figure 3.5).

3.3 . Simulation results

In this section, we present a numerical example to illustrate the behaviour of the algorithm that we propose to solve the formation control problem for connected vehicles. We consider a vehicular network composed of $N = 15$ vehicles. The desired configuration is given in the top-right plot in Figure 3.6. This configuration is defined by the offsets $d_1 = (0, 0)^T, d_2 = (-2, 2)^T, d_3 = (-2, -2)^T, d_4 = (-4, 0)^T, \dots, d_{14} = (-18, 2)^T, d_{15} = (-18, -2)^T$. At time t_0 , the vehicles are distributed randomly in a 2-dimensional space such that their distribution satisfies

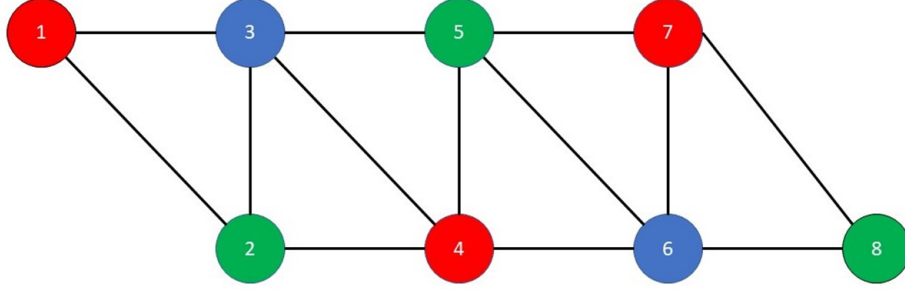


Figure 3.5: Graph coloring for resources attribution: adjacent nodes are associated to different colors.

the condition given in Theorem 3.1. The initial configuration is given in the top-left plot in Figure 3.6.

The frame duration is equal to $T_f = 10ms$ and the resources are used by the same users for $q = 100$ consecutive frames. Thus, the duration of the super-frame is equal to $T_p = 1s$. We suppose that the number of available resources is $M = 4$. The transmitting range is given by $\rho = 4$. The reference trajectory is $\dot{p}_i(t) = u_{ref}$, with $u_{ref} = (30, 0)^T$.

At instants $t_k = 0s, 1s, 2s, \dots$ the proposed algorithm is executed at the base station and the resources are assigned to the transmitting agents selected as the optimal solution given by (3.9). In this example, problem (3.9) is solved using the non-linear Branch-and-Bound algorithm. The simulation results are plotted in Figure 3.6. The bottom-left plot shows the vehicles trajectories in the plan (2-dimensional space). The markers on the trajectories correspond to the positions of the transmitting agents at the beginning of each step, where different markers correspond to different steps. The bottom-right plot shows the number of transmitting users at each period.

Figure 3.6 shows that the vehicles achieve the desired formation. Moreover, the network resources are used efficiently. Indeed, while agents are tending to the desired formation, the number of transmitting agents is higher than the number of available resources $M = 4$. Thus, the proposed algorithm benefits from the vehicles positions to allow multiple users to use the same resource while guaranteeing an interference-free communication.

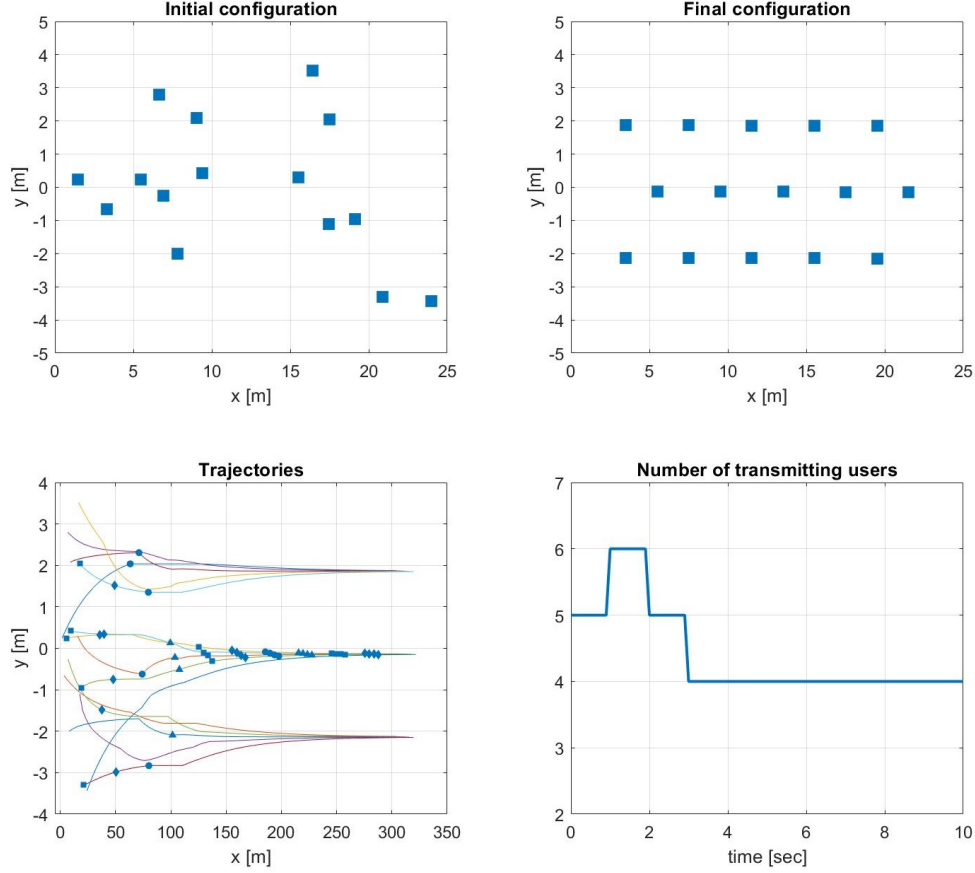


Figure 3.6: Vehicles positions at (top-left) the initial time and (top-right) the final time. (bottom-left) Trajectories of the vehicles and transmitting agents at each step. (bottom-right) the number of transmitting users during the simulation. The vehicles converge to the desired formation.

3.4 . Conclusion

This chapter proposes a joint design method for formation control and communication resources allocation strategy for connected vehicles over a 5G network. The proposed scheme includes a consensus-type algorithm for formation control, where agents communication is constrained by the network resources availability. For the communication strategy, we proposed an algorithm relying on an optimization problem that selects the transmitting agents in order to accelerate the convergence rate towards the desired formation while guaranteeing an efficient use of the network resources. We characterize the domain of initial condition (vehicles positions) from which it is guaranteed to converge to the desired formation. This domain is related to the targeted formation.

The proposed algorithm relies on the knowledge of the true positions of the vehicles. Indeed, the correct position of each vehicle is assumed to be known by

the vehicle itself at each time step, and is transmitted to the base station at the beginning of each super frame. Moreover, the communication between vehicles and the base station are assumed to be synchronized. Therefore, the network resources attribution is performed according to the perfect information on the vehicles positions. In real world applications, this assumption does not hold. The vehicles positions are provided through embedded sensors at the level of vehicles such as GPS positioning or positioning through Road-Side Units (RSUs). These positions given by sensors are not the exact positions and provide a noisy estimation of the actual positions. Moreover, the synchronization of positions acquirement and the communication is not generally satisfied which increase the inaccuracy of the received positions by the base station. Due to these imperfections in real-world applications, the inaccuracies on the vehicles positions need be taken into account, and their effect on the algorithm performances should to be evaluated.

In the next chapter, we propose a Lyapunov-based method to evaluate the performance of the well-known filtering algorithm, the Kalman filter. We propose to formulate a hybrid dynamical system model where observations occur at discrete-time in the same way as communication between vehicles and the base station, or from vehicle to vehicle. However, the vehicles dynamics are modelled in continuous time. We evaluate the accuracy of the positions estimation regarding the amplitude of the errors related to communication and sensing such as communication delays, mis-synchronization, packet loss, sensors imperfections and errors related to vehicles modelling such as modelling uncertainties, external disturbances, etc. The proposed result gives an explicit bound of the state estimation error in terms of the maximum amplitude of the observation and process errors.

4 - HYBRID KALMAN FILTER

Kalman filters are highly used in industry and engineering systems for tracking, estimation, filtering and prediction applications thanks to their low computation complexity and their high performance. Kalman filter are the optimal Bayesian filter for systems with linear dynamical models and linear state-to-observation mapping under additive zero-mean Gaussian noise [124]. Kalman filters are also the solution of the quadratic optimal control problem [70]. In this chapter, we consider the Kalman filter as the solution of the quadratic optimal control problem. We consider the assumption of bounded errors and we will evaluate the Kalman filter performance regarding the errors bounds.

In this chapter, we present a theoretical result based on an input-to-state stability analysis [130] to characterize the behaviour of Kalman filters applied on hybrid dynamical systems [48] including continuous-time dynamics and discrete-time dynamics. Systems combining the control of continuous-time physical systems such as mobile robots, air-crafts and autonomous vehicles, and communication systems that operate in discrete time. The system presented in Chapter 3 falls in this class of system. This motivated us to investigate the behaviour of Kalman filters on such systems in order to extend the result in Theorem 3.1 to be robust against communication errors and dynamics modelling uncertainties.

The Kalman filter is one of the most used algorithms for state estimation and filtering problems. The principle of state estimation consists of reconstructing the system's state based on observations, also known as the system's outputs or measurements. The standard Kalman filter is the optimal discrete-time linear filter that guarantees the minimum variance of the state estimation error for a linear system with white Gaussian disturbances of known variances. In [73], Kalman proposed the first formulation for Kalman filters as a solution to the Wiener problem. Since then, many enhanced versions of the Kalman filter have been proposed to deal with non-linear systems, such as the extended Kalman filter [32] and the unscented Kalman filter [68]. The continuous-time version of the Kalman filter, known as the Kalman-Bucy filter, was proposed in [23], where the system state and the observations are represented by continuous-time variables.

Both models, continuous-time and discrete-time, have their shortcomings. The continuous-time model assumes that the flow of observations is continuous in time. The discrete-time model ignores the system dynamics during the interval between two observations. In real-world applications, physical systems evolve in continuous time, and external information such as sensors measurements, observations or control commands are received in discrete time through a communication system. Thus, it is essential to introduce a hybrid formulation of the Kalman filter that combines the continuous-time dynamics of the system and the discrete-time aspect of the received flow of observations. In [124], the author presents the hybrid

filtering principle, designated as continuous-discrete filtering. He derives the equations of the hybrid version of the Kalman filter starting from the Bayesian inference framework under the assumption of zero-mean white Gaussian noises.

The assumption of Gaussian noises is unsuitable for some systems because of the unboundedness of their distributions. In [70] and [143], the authors present, respectively, a deterministic formulation of the discrete-time and the continuous-time Kalman filter algorithm using the principle of least squares estimation. The deterministic formulation allows to use the notion of input-to-state stability (ISS) [130] in order to quantify the state estimation error with respect to the amplitude of the external perturbations.

In our works, we focus on the deterministic formulation of the Kalman filter algorithm for hybrid systems. We propose a new formulation of the Kalman filter algorithm under the hybrid systems framework where the system dynamics are modelled with a continuous-time variable and the observations are represented by a discrete-time variable. We prove exponential input-to-state stability for the proposed model which includes exponential stability [72] in the absence of noise and boundedness of the state depending on the process and observations noises amplitude. For this purpose, a Lyapunov function is proposed to prove exponential ISS for the hybrid Kalman filter under mild assumptions on the system model.

The chapter is organized as follows. In Section 4.1, we give a brief presentation of the hybrid systems notations and present the definition and a characterization of exponential ISS. Then, in Section 4.2, we present the hybrid Kalman filter model and prove the exponential ISS property of this model. A numerical example is presented in Section 4.3 to illustrate the main result. A discussion on the stability the Kalman filter and the stability of the observed system is given in Section 4.4. Finally, we state a theoretical result on exponential stability for non-linear time-varying systems in Section 4.5 before closing the chapter with summarizing conclusion.

4.1 . Input-to-state stability for hybrid systems

4.1.1 . Hybrid systems framework

The hybrid systems framework [48] allows to model systems that combine continuous-time and discrete-time dynamics such as sampled-data systems [29], switching systems [15], event-triggered systems [53], impulsive systems [80], etc. Investigating the properties of these classes of systems is crucial to study multiple real-world systems, particularly those including communication between their different components or with external units. The hybrid systems framework provides a common set of tools to analyse the behaviour of such systems. We briefly recall, hereafter, the basic notions of this framework that we used to present our result.

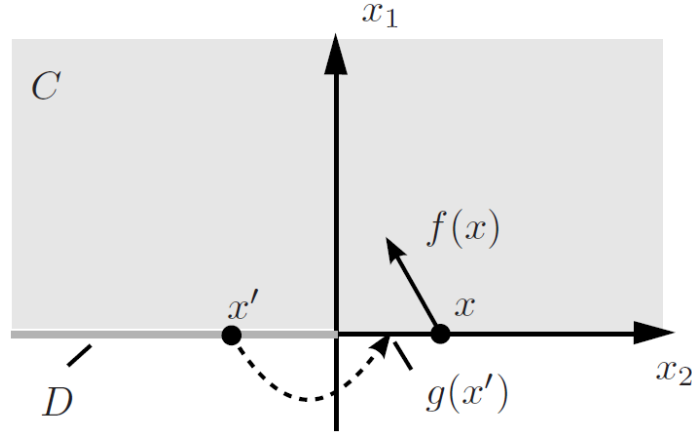


Figure 4.1: Flow and jump domains [48].

A hybrid system is formulated as follows [48].

$$\begin{cases} \dot{x} = f(x, u); & (x, u) \in \mathcal{C}, \\ x^+ = g(x, u); & (x, u) \in \mathcal{D} \end{cases} \quad (4.1)$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^p$ correspond to the state and the input of the system respectively.

This system representation (4.1) states that the state of the hybrid system, represented by x , evolves according to the differential equation $\dot{x} = f(x)$ while in the set \mathcal{C} , and it evolves according to the equation $x^+ = g(x)$ while in the set \mathcal{D} . The notation x^+ represents the value of the state after an instantaneous change.

The first equation in (4.1) is called the flow map and \mathcal{C} the flow domain. The second equation is called the jump map and \mathcal{D} the jump domain. A solution of the system (4.1) is denoted by $x(t, k)$ where the first parameter t represents the flow dynamics (continuous-time) and the second parameter k represents the jump dynamics (discrete-time). Unlike the classical formulation of hybrid systems framework, here the time is defined on \mathcal{E} not the state itself (See Figure 4.2).

4.1.2 . Exponential input-to-state stability for hybrid systems

The notion of input-to-state stability (ISS), introduced initially in [129] for continuous-time systems, appears to be very useful to characterize the behaviour of systems with external or uncontrolled inputs such as disturbances or noise. , a system is ISS if it is asymptotically stable in the absence of external inputs, and its trajectories, after a sufficiently large time, are bounded by a function that depends on the amplitude of the external inputs. It has been widely studied for continuous-time and discrete-time systems, and Lyapunov-based characterizations for ISS are given in [129] [66].

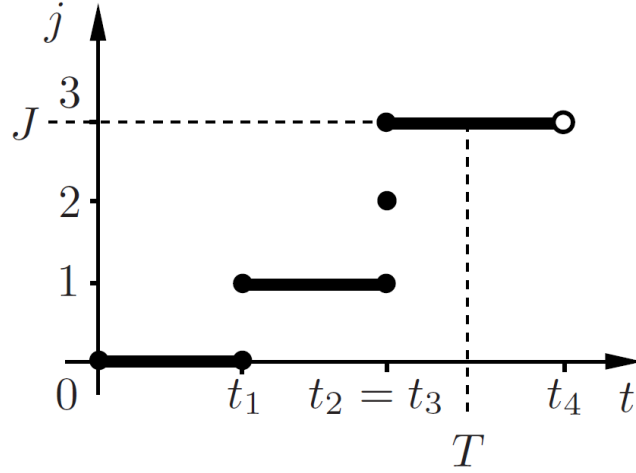


Figure 4.2: Hybrid time domain [48]: time is defined by two variables, a continuous variable evolving when the system state is in the flow domain, and a discrete variable evolving when the system state is in the jump domain.

For the considered hybrid system, we suppose that the jumps occur at specific instants called switching instants. Thus, unlike (4.1), the flow and jump domains depend only on the sequence of switching instants, not on the system's state or the inputs. Let $\{t_k\}_{k \in \mathbb{N}}$ be an increasing and unbounded sequence of real scalars that represent the sequence of switching instants. We propose a simpler formulation of the hybrid system.

As in [48], we define the hybrid domain $\mathcal{E} = \bigcup_{k=1}^{\infty} ([t_{k-1}, t_k] \times \{k\})$, the jump domain given by

$$\mathcal{D} = \bigcup_{k=1}^{\infty} (\{t_k\} \times \{k\}),$$

and the flow domain

$$\mathcal{C} = \bigcup_{k=1}^{\infty} ([t_{k-1}, t_k] \times \{k\}).$$

We consider the hybrid system defined by

$$\begin{cases} \dot{x}(t, k) = f(x(t, k), q(t)); & \forall (t, k) \in \mathcal{C}, \\ x(t_k, k+1) = g(x(t_k, k), r_k); & \forall (t_k, k) \in \mathcal{D}, \end{cases} \quad (4.2)$$

where $q(t) \in \mathbb{R}^{m_q}$ and $r_k \in \mathbb{R}^{m_r}$ are external inputs.

We introduce the definition of exponential ISS systems and give a Lyapunov-based characterization for such systems.

Definition 4.1

The system (4.2) is said to be exponentially input-to-state (ISS) stable if there exist constants $K, \lambda_1, \lambda_2, \gamma_1, \gamma_2 > 0$ such that, for any $(t, k) \in \mathcal{E}$, we have

$$\|x(t, k)\| \leq \max \left\{ K e^{-(\lambda_1 t + \lambda_2 k)} \|x_0\|; \gamma_1 \sup_{\tau \geq t_0} \|q(\tau)\|; \gamma_2 \sup_{l \in \mathbb{N}^*} \|r_l\| \right\}, \quad (4.3)$$

where $x_0 = x(t_0, 1)$ is the initial state.

The previous definition states that for an exponential ISS system, the norm of the state decreases exponentially during the transitory phase. Then, after a sufficiently large time, the effect of the initial state vanishes, and the error becomes bounded by a function that depends on the maximum amplitude of the external inputs and the system constants.

A similar definition is given for asymptotic ISS and exponential ISS in [24] and [35] respectively. Here we emphasize the difference between the disturbances in the flow dynamics and the jump dynamics.

Definition 4.2

A function $V : (\mathbb{R}^n, \mathbb{R}, \mathbb{N}^*) \rightarrow \mathbb{R}_{\geq 0}$ is an exponential ISS-Lyapunov function for system (4.2) if there exist constants $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \mu_1, \mu_2 > 0$ such that

$$\begin{aligned} \alpha_1 \|x(t, k)\|^2 &\leq V(x(t, k), t, k) \leq \alpha_2 \|x(t, k)\|^2; \quad \forall (t, k) \in \mathcal{E}, \\ \frac{\partial V}{\partial t} + \nabla V.f(x(t, k), q(t)) &\leq -\alpha_3 \|x(t, k)\|^2; \\ \forall (t, k) \in \mathcal{C} \text{ with } \|x(t, k)\| &\geq \mu_1 \|q(t)\|, \\ V(x(t_k, k+1), t_k, k+1) - V(x(t_k, k), t_k, k) &\leq -\alpha_4 \|x(t_k, k)\|^2; \\ \forall (t_k, k) \in \mathcal{D} \text{ with } \|x(t_k, k)\| &\geq \mu_2 \|r_k\|. \end{aligned}$$

The definition of the exponential ISS Lyapunov function claims that the function V is decreasing exponentially during flow steps and jump steps when the state of the system is sufficiently large. The threshold depends on the amplitude of the external inputs.

Lemma 4.1

A function $V : (\mathbb{R}^n, \mathbb{R}, \mathbb{N}^*) \rightarrow \mathbb{R}_{\geq 0}$ is an exponential ISS-Lyapunov function for system (4.2) if and only if there exist constants $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4, \hat{\gamma}_1, \hat{\gamma}_2 > 0$ such that

$$\begin{aligned} \hat{\alpha}_1 \|x(t, k)\|^2 &\leq V(x(t, k), t, k) \leq \hat{\alpha}_2 \|x(t, k)\|^2; \quad \forall (t, k) \in \mathcal{E} \\ \frac{\partial V}{\partial t} + \nabla V.f(x(t, k), q(t)) &\leq -\hat{\alpha}_3 \|x(t, k)\|^2 + \hat{\gamma}_1 \|q(t)\|^2; \quad \forall (t, k) \in \mathcal{C} \\ V(x(t_k, k+1), t_k, k+1) - V(x(t_k, k), t_k, k) &\leq -\hat{\alpha}_4 \|x(t_k, k)\|^2 + \hat{\gamma}_2 \|r_k\|^2; \\ \forall (t_k, k) \in \mathcal{D}. \end{aligned}$$

Theorem 4.1

If the system (4.2) has an exponential ISS-Lyapunov function, then it is exponentially ISS.

As in [24], Theorem 4.1 gives a Lyapunov-based characterization of exponential ISS for hybrid systems. The proof of this result uses the Gronwall's inequality [51] to integrate the second inequality in Lemma 4.1. A bound for $\|x(t, k)\|$ is then derived using a recursive formula.

The main contribution of this chapter is to prove exponential ISS for the hybrid Kalman filter using the result of Lemma 4.1.

4.2 . Hybrid Kalman filter stability

In this section, we present the equations of the Hybrid Kalman filter and apply the result of Theorem 4.1 using a suitable exponential ISS Lyapunov function.

4.2.1 . Hybrid Kalman filter algorithm

We consider a linear time-invariant (LTI) continuous-time system which state is partially observed through measurements that are received at arbitrary discrete instants $\{t_k\}_{k \in \mathbb{N}^*}$. The system's state $x(t)$ is a continuous-time function that describes the system's behaviour over time. However, the observations are represented by the sequence $\{y_k\}_{k \in \mathbb{N}^*}$ and are available only at specific instants $\{t_k\}_{k \in \mathbb{N}^*}$.

The described system can be written in the following state-space form

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + q(t), \\ y_k &= Cx(t_k) + r_k \end{cases}, \quad (4.4)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y_k \in \mathbb{R}^p$ represent, respectively, the state of the system, the control input and the measurements vector. $q(t) \in \mathbb{R}^{d_1}$ and $r_k \in \mathbb{R}^{d_2}$ are unknown deterministic disturbances that represent, respectively, the process noise and the measurements noise. The process noise is used to represent the disturbances, the uncontrolled inputs and the unmodelled components of the system. The measurements noise models the errors due to the sensors accuracy and the communication network.

We assume that the disturbances/noises are **bounded** and we present the following state estimator. The proposed algorithm is called the continuous-discrete Kalman filter in [124, Algorithm 3.15].

We denote by \hat{x} the state estimate given by the hybrid Kalman filter algorithm. For the jump domain $\mathcal{D} = \bigcup_{k=1}^{\infty} (\{t_k\} \times \{k\})$, we define the jump map:

For all $(t_k, k) \in \mathcal{D}$

$$\begin{aligned} \hat{x}(t_k, k+1) &= (I - K_k C) \hat{x}(t_k, k) + K_k y_k, \\ P(t_k, k+1) &= (I - K_k C) P(t_k, k), \end{aligned}$$

where

$$\begin{aligned} K_k &= P(t_k, k)C^T S_k^{-1}, \\ S_k &= CP(t_k, k)C^T + R_k. \end{aligned} \quad (4.5)$$

For the flow domain $\mathcal{C} = \bigcup_{k=1}^{\infty} ([t_{k-1}, t_k] \times \{k\})$, we define the flow map:
For all $(t, k) \in \mathcal{C}$

$$\begin{aligned} \dot{\hat{x}}(t, k) &= A\hat{x}(t, k) + Bu(t, k), \\ \dot{P}(t, k) &= AP(t, k) + P(t, k)A^T + Q(t), \end{aligned}$$

where the initial conditions are given by the previous jump map. Matrices $Q(t) \in$ and $R_k \in$ are arbitrary weighting positive-definite matrices.

The purpose of this work is to study the state estimation error with respect to the process and measurements noises. Let $\tilde{x}(t, k) = \hat{x}(t, k) - x(t)$ be the state's estimate error. Its dynamics are given by the following equations:

$$\begin{cases} \dot{\tilde{x}}(t, k) = A\tilde{x}(t, k) - q(t); & \forall (t, k) \in \mathcal{C}, \\ \tilde{x}(t_k, k+1) = (I - K_k C)\tilde{x}(t_k, k) + K_k r_k; & \forall (t_k, k) \in \mathcal{D} \end{cases} \quad (4.6)$$

The dynamics of the matrix P are given by

$$\begin{cases} \dot{P}(t, k) = AP(t, k) + P(t, k)A^T + Q(t); & \forall (t, k) \in \mathcal{C}, \\ P(t_k, k+1) = (I - K_k C)P(t_k, k); & \forall (t_k, k) \in \mathcal{D} \end{cases} \quad (4.7)$$

Remark

*In the stochastic formulation of the Kalman filter, the matrices $Q(t)$ and R_k represent the covariance matrices of the process and the observations noises, which are assumed to be known. In the deterministic formulation, these matrices are symmetric and positive definite matrices, that can be **arbitrary** chosen by the designer. The same principle is used in linear quadratic regulators (LQR) [70].*

4.2.2 . Exponential ISS for the hybrid Kalman filter

To state our main result in Theorem 4.2, we suppose that the following assumptions hold true.

Assumptions:

A1. There exist $\underline{\tau}, \bar{\tau} > 0$ such that for any $k \in \mathbb{N}$,

$$\underline{\tau} \leq t_{k+1} - t_k \leq \bar{\tau}$$

A2. There exist constants $\lambda_{Q1}, \lambda_{Q2}, \lambda_{R1}, \lambda_{R2} > 0$ such that for any $t \geq t_0$ and $k \in \mathbb{N}^*$

$$\begin{aligned} \lambda_{Q1}I &\preceq Q(t) \preceq \lambda_{Q2}I \\ \lambda_{R1}I &\preceq R_k \preceq \lambda_{R2}I \end{aligned}$$

A3. There exist $m \in \mathbb{N}^*$ and $\lambda_{Ob} > 0$ such that for any $k \in \mathbb{N}$,

$$\sum_{i=0}^{m-1} \Phi^T(t_{k+m-i}, t_{k+m}) C^T C \Phi(t_{k+m-i}, t_{k+m}) \succeq \lambda_{Ob} \mathbf{I}$$

Assumption **A1** states that the duration of the intervals between two consecutive observations is upper and lower bounded. This assumption is necessary to guarantee that an update of the state's estimate will occur after a specific duration and to avoid the Zeno effect [48]. Moreover **A1** gives more flexibility for the system's model and take into consideration the effect of jitter and delays [144]. Assumption **A2** states that the weighting matrices are positive definite and bounded. Finally, assumption **A3**, known as the uniform observability assumption, ensures that the system state can be reconstructed based on the observations, i.e., the outputs of the system.

Theorem 4.2 ([14])

Under assumptions **A1-A3**, the system (4.6)-(4.7) is exponentially ISS. The corresponding exponential ISS Lyapunov function has the form

$$V(\tilde{x}(t, k), t, k) = \tilde{x}^T(t, k) P^{-1}(t, k) \tilde{x}(t, k) \quad (4.8)$$

Moreover, for any $(t, k) \in \mathcal{E}$ where $k \geq m + 2$, we have

$$\|x(t, k)\| \leq \max \left\{ K e^{-(\lambda_1 t + \lambda_2 k)} \|x(t_0, 1)\|; \gamma_1 \sup_{\tau \geq t_0} \|q(\tau)\|; \gamma_2 \sup_{l \in \mathbb{N}^*} \|r_l\| \right\}$$

with $K = 3\sqrt{\frac{\lambda_{P_2}}{\lambda_{P_1}\beta_1}}$, $\lambda_1 = \frac{\hat{\alpha}_3}{2\hat{\alpha}_2}$, $\lambda_2 = -\frac{1}{2} \ln(\beta_1)$, $\gamma_1 = 3\sqrt{\frac{1}{\lambda_{P_1}} \left(\frac{1+\beta_1(1-\beta_2)}{1-\beta_1\beta_2} \right) \beta_3 \frac{\hat{\alpha}_2}{\hat{\alpha}_3} \hat{\gamma}_1}$,
 $\gamma_2 = 3\sqrt{\frac{1}{\lambda_{P_1}} \frac{1}{1-\beta_1\beta_2} \hat{\gamma}_2}$, $\hat{\alpha}_1 = \frac{1}{\lambda_{P_2}}$, $\hat{\alpha}_2 = \frac{1}{\lambda_{P_1}}$, $\beta_1 = \left(1 - \frac{\hat{\alpha}_4}{\hat{\alpha}_2}\right)$, $\beta_2 = e^{-\frac{\hat{\alpha}_3}{\hat{\alpha}_2} \tau}$,
 $\beta_3 = \left(1 - e^{-\frac{\hat{\alpha}_3}{\hat{\alpha}_2} \tau}\right)$ and $\hat{\alpha}_3, \hat{\alpha}_4, \hat{\gamma}_1, \hat{\gamma}_2$ are defined in the inequalities (4.10)-(4.11).

Before presenting the proof of the main result, we first state the following lemma, which is essential for further development. The proof of this lemma is given in Appendix A.

Proposition 4.1

If assumptions **A1-A3** hold, then, there exist constants $\lambda_{P_1}, \lambda_{P_2} > 0$ such that for any $(t, k) \in \mathcal{E}$ where $k \geq m + 2$. The matrix $P(t, k)$, defined in (4.7), satisfies the following bounds

$$\lambda_{P_1} I \preceq P(t, k) \preceq \lambda_{P_2} I \quad (4.9)$$

where $m \in \mathbb{N}^*$ is defined in assumption **A3**.

The proof of Proposition 4.1 is given in Appendix A.2.

4.2.3 . Proof of Theorem 4.2.

The result of Proposition 4.1 allows us to get the first inequality in Lemma 4.1. We prove that the function (4.8) verifies the second and third inequality in Lemma 4.1 (See Appendix A.3).

For $(t, k) \in \mathcal{C}$,

$$\dot{V}(t, k) \leq -\frac{\theta_1 \lambda_{Q_1}}{\lambda_{P_2}^2} \|\tilde{x}(t, k)\|^2 + \frac{1}{\lambda_{Q_1}(1 - \theta_1)} \|q(t)\|^2 \quad (4.10)$$

where $0 < \theta_1 < 1$ is an arbitrary constant.

For $(t_k, k) \in \mathcal{D}$,

$$\begin{aligned} V(t_k, k+1) - V(t_k, k) &\leq -\frac{\theta_2 \xi}{\lambda_{P_2} \lambda_C + \lambda_{R_2}} \|\tilde{x}(t_k, k)\|^2 \\ &\quad + \left(\frac{\lambda_{P_2} \lambda_C}{\lambda_{R_1}^2} + \frac{\lambda_{P_2} \lambda_C + \lambda_{R_2}}{\lambda_{R_1}^2 (1 - \theta_2)} \right) \|r_k\|^2 \end{aligned} \quad (4.11)$$

where $0 < \theta_2 < 1$ is an arbitrary constant.

Equations (4.10) and (4.11) allow us to satisfy the second and third equations in Lemma 4.1. Thus, using the result of Proposition 4.1, we deduce recursively the following inequality.

For any $t \geq t_{m+1}$, let $k^* = \arg\max_k \{t_k \leq t\}$. Then, we get

$$\begin{aligned} V(t, k^* + 1) &\leq e^{-\frac{\hat{\alpha}_3}{\hat{\alpha}_2}(t-t_0) - \ln\left[\left(1 - \frac{\hat{\alpha}_4}{\hat{\alpha}_2}\right)^{-1}\right]k^*} V(t_0, 1) \\ &\quad + \frac{1 + \left(1 - \frac{\hat{\alpha}_4}{\hat{\alpha}_2}\right) \left(1 - e^{-\frac{\hat{\alpha}_3}{\hat{\alpha}_2} \tau}\right)}{1 - \left(1 - \frac{\hat{\alpha}_4}{\hat{\alpha}_2}\right) e^{-\frac{\hat{\alpha}_3}{\hat{\alpha}_2} \tau}} \left(1 - e^{-\frac{\hat{\alpha}_3}{\hat{\alpha}_2} \tau}\right) \frac{\hat{\alpha}_2}{\hat{\alpha}_3} \hat{\gamma}_1 \|q\|_\infty^2 \\ &\quad + \frac{1}{1 - \left(1 - \frac{\hat{\alpha}_4}{\hat{\alpha}_2}\right) e^{-\frac{\hat{\alpha}_3}{\hat{\alpha}_2} \tau}} \hat{\gamma}_2 \|r\|_\infty^2 \end{aligned}$$

where $\|q\|_\infty^2 = \sup_{t \geq t_0} \|q(t)\|^2$ and $\|r\|_\infty^2 = \sup_{k \in \mathbb{N}^*} \|r_k\|^2$.

Finally, using the first inequality in Lemma 4.1 and the properties of the square root function and the max function we deduce the bound for the norm of the state estimation error $\|\tilde{x}(t, k)\|$ in Theorem 4.2.

4.3 . Numerical example

To illustrate the result of Theorem 4.2, we consider the system

$$\begin{cases} \begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix} \\ y_k = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1(t_k) \\ x_2(t_k) \end{pmatrix} + r_k \end{cases}$$

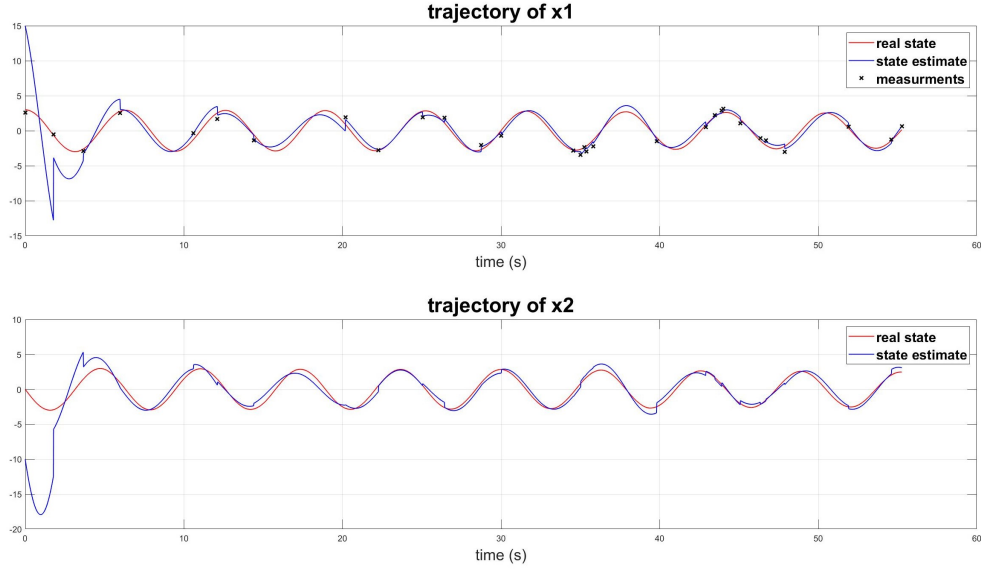


Figure 4.3: Trajectories of the state components x_1 and x_2 , the state estimate components \hat{x}_1 and \hat{x}_2 and observations y_k .

where $x = (x_1, x_2)^T$ represents the system's state. $q_1(t)$, $q_2(t)$ and r_k are bounded signals that represent the process and observation noises. We set $\sup_{t \geq t_0} \|q(t)\| = 0.1$ and $\sup_{k \in \mathbb{N}^*} \|r_k\| = 1$. The observations y_k are received at random instants. The initial state is $x_0 = (1, 0)^T$ and the initial state estimate is $\hat{x}_0 = (5, -2)^T$.

In Figure 4.3., we plot the trajectories of the state components x_1 and x_2 , the estimate components \hat{x}_1 and \hat{x}_2 and observation y_k . The state estimator given by the hybrid Kalman filter algorithm provides an accurate estimate for the system's state. The norm of the state estimation error is depicted in Figure 4.4. It decreases exponentially during the transitory phase during $[0, 15s]$ where the effect of the initial estimation error vanishes, and it remains bounded afterwards.

4.4 . Application of hybrid Kalman filter on perturbed linear unstable system

For linear time-invariant (LTI) continuous-time systems, a necessary and sufficient condition for asymptotic or exponential stability is to have a Hurwitz state-transition matrix, i.e., its eigenvalues have strictly negative real parts [69]. In this section, we aim to show that it is possible to estimate the state of an unstable *continuous-time* system using *discrete-time* observations.

4.4.1 . Motivating Example

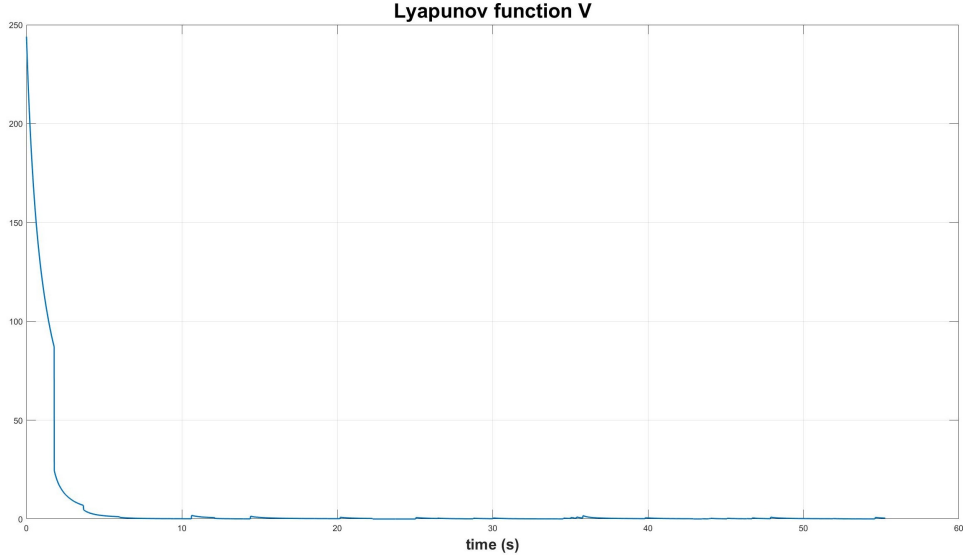


Figure 4.4: Norm of the state estimation error $\tilde{x}(t, k)$.

We recall the state space model of the system to track (4.4).

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + q(t) \\ y_k &= Cx(t_k) + r_k \end{cases} \quad (4.12)$$

We consider the case where $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $C = (1 \quad 0)$. The matrix $P(t, k)$ is scalar and the Kalman gain is given by $K_k = \frac{P(t, k)}{P(t, k) + R_k}$. Thus, as introduced above in (4.6)-(4.7), the Kalman filter equations are given by:

$$\begin{cases} \dot{\tilde{x}}(t, k) = \tilde{x}(t, k) - q(t), & \forall(t, k) \in \mathcal{C} \\ \tilde{x}(t_k, k+1) = \frac{R_k}{P(t, k) + R_k} \tilde{x}(t_k, k) + \frac{P(t, k)}{P(t, k) + R_k} r_k, & \forall(t_k, k) \in \mathcal{D} \end{cases} \quad (4.13)$$

and

$$\begin{cases} \dot{P}(t, k) = 2P(t, k) + Q(t), & \forall(t, k) \in \mathcal{C} \\ P(t_k, k+1) = \frac{P(t, k)R_k}{P(t, k) + R_k}, & \forall(t_k, k) \in \mathcal{D} \end{cases} \quad (4.14)$$

Remark

Note that, as above in Subsection 4.2.1, in this analysis we deal with two different dynamical systems (See Figure 4.5).

1. The system represented by the state $x(t)$ which is the system to estimate or track.
2. The system represented by the state $\tilde{x}(t, k)$ which is the estimation error resulting from the application of the hybrid Kalman filter introduced in Subsection 4.2.1.

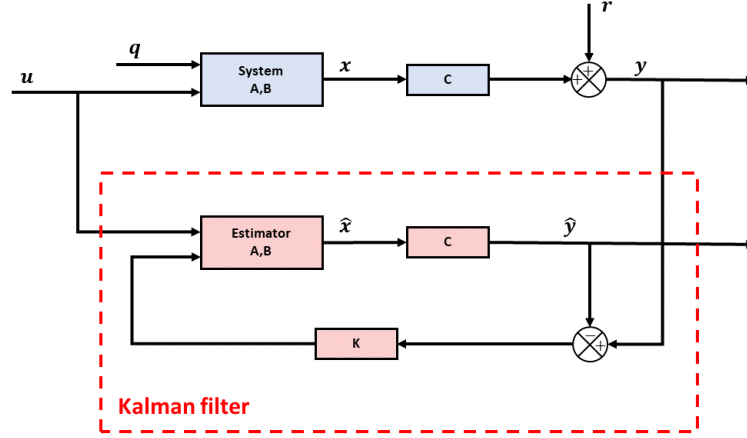


Figure 4.5: Kalman filter block diagram: Kalman filter uses noisy measurements to estimates the unknown state of the system by comparing the expected and the observed measurements.

The goal of this section is to emphasize the deference between the stability of the system to estimate and the systems representing the estimation error. We aim to show that the Kalman filter algorithm can estimate and track the state of the system represented by $x(t)$, even if the later is not stable in the sense of Lyapunov stability.

We consider the noise-free case, i.e., $q(t) = 0$ and $r_k = 0$, and we assume that $Q(t) = Q$ and $R_k = R$.

Let $V(t, k) = \frac{\tilde{x}^2(t, k)}{P(t, k)}$ and $W(t, k) = \tilde{x}^2(t, k)$. Then,
For all $(t, k) \in \mathcal{C}$

$$\begin{aligned}
 \dot{V}(t, k) &= \frac{2\tilde{x}(t, k)\dot{\tilde{x}}(t, k)P(t, k) - \tilde{x}^2(t, k)\dot{P}(t, k)}{P^2(t, k)} \\
 &= \frac{2\tilde{x}^2(t, k)P(t, k) - 2\tilde{x}^2(t, k)P(t, k) - \tilde{x}^2Q}{P^2(t, k)} \\
 &= -\frac{Q\tilde{x}^2(t, k)}{P^2(t, k)} \\
 &= -\frac{Q}{P(t, k)}V(t, k)
 \end{aligned}$$

and

$$\dot{W}(t, k) = 2\tilde{x}(t, k)\dot{\tilde{x}}(t, k) = 2\tilde{x}^2(t, k) = 2W(t, k)$$

Moreover, for all $(t, k) \in \mathcal{D}$

$$\begin{aligned}
V(t_k, k+1) - V(t_k, k) &= \frac{\tilde{x}^2(t_k, k+1)}{P(t_k, k+1)} - \frac{\tilde{x}^2(t_k, k)}{P(t_k, k)} \\
&= \frac{R}{P(t_k, k) + R} \frac{\tilde{x}^2(t_k, k)}{P(t_k, k)} - \frac{\tilde{x}^2(t_k, k)}{P(t_k, k)} \\
&= -\frac{\tilde{x}^2(t_k, k)}{P(t_k, k) + R} \\
&= -\frac{P(t_k, k)}{P(t_k, k) + R} V(t_k, k)
\end{aligned}$$

and

$$\begin{aligned}
W(t_k, k+1) - W(t_k, k) &= \tilde{x}^2(t_k, k+1) - \tilde{x}^2(t_k, k) \\
&= \left[\left(\frac{R}{P(t_k, k) + R} \right)^2 - 1 \right] \tilde{x}^2(t_k, k) \\
&= -\frac{P^2(t_k, k) + 2P(t_k, k)R}{(P(t_k, k) + R)^2} \tilde{x}^2(t_k, k)
\end{aligned}$$

Similarly,

$$\begin{aligned}
W(t_k, k+1) &= \left(\frac{R}{P(t_k, k) + R} \right)^2 W(t_k, k+1) \\
&= \left(\frac{R}{P(t_k, k) + R} \right)^2 e^{2(t_k - t_{k-1})} W(t_{k-1}, k) \\
&= \left(\frac{1}{1 + \frac{P(t_k, k)}{R}} \right)^2 e^{2(t_k - t_{k-1})} W(t_{k-1}, k) \\
&= \left(\frac{e^{(t_k - t_{k-1})}}{1 + \frac{e^{2(t_k - t_{k-1})} (P(t_{k-1}, k) + \frac{1}{2}Q) - \frac{1}{2}Q}{R}} \right)^2 W(t_{k-1}, k) \\
&< W(t_{k-1}, k)
\end{aligned}$$

Remark

If $t_k - t_{k-1} \gg 1$, then the estimation uncertainty is high which gives more reliability to the observation. Thus, in the noise-free case, not receiving information enhance the estimation accuracy.

The integrated form of the system's equations is given by

$$\begin{cases} \tilde{x}(t, k) = e^{(t - t_{k-1})} \tilde{x}(t_{k-1}, k) \\ P(t, k) = e^{2(t - t_{k-1})} (P(t_{k-1}, k) + \frac{1}{2}Q) - \frac{1}{2}Q \end{cases}, \quad \forall (t, k) \in \mathcal{C} \quad (4.15)$$

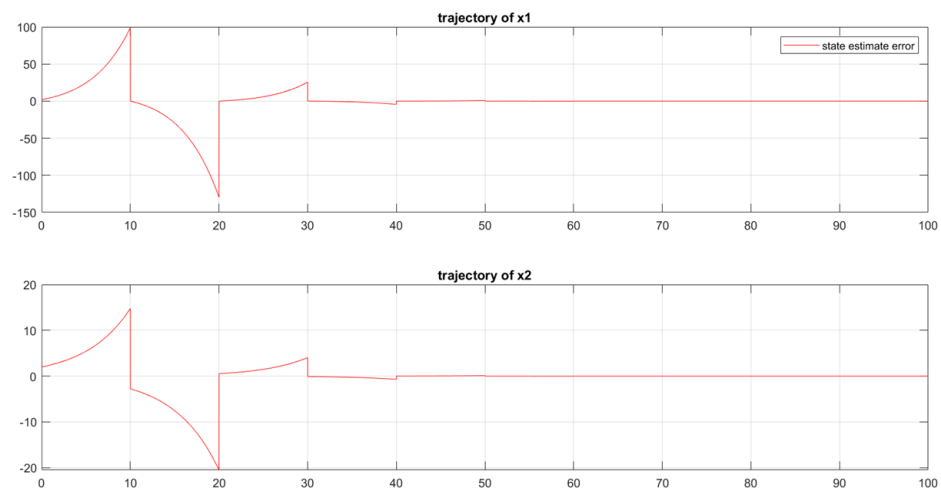


Figure 4.6: Trajectories of the filtered system over time.

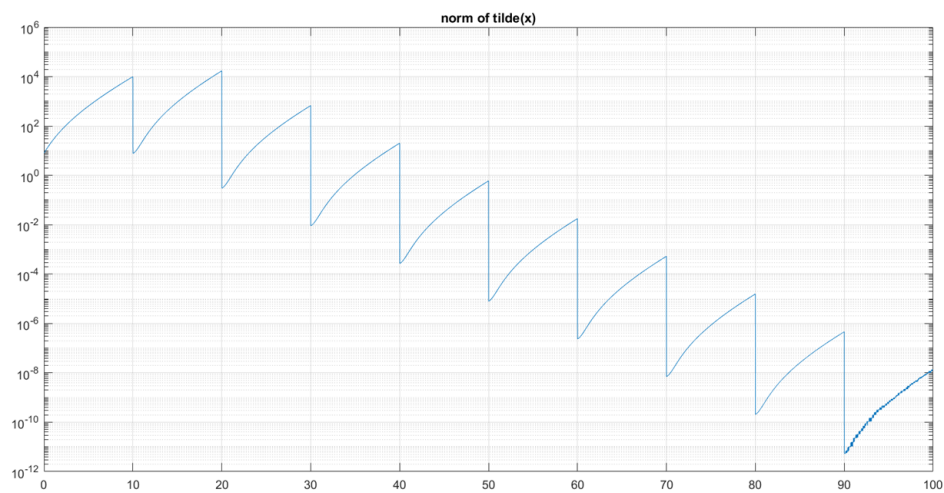


Figure 4.7: Evolution of function $W = \tilde{x}^T \tilde{x}$ over time.

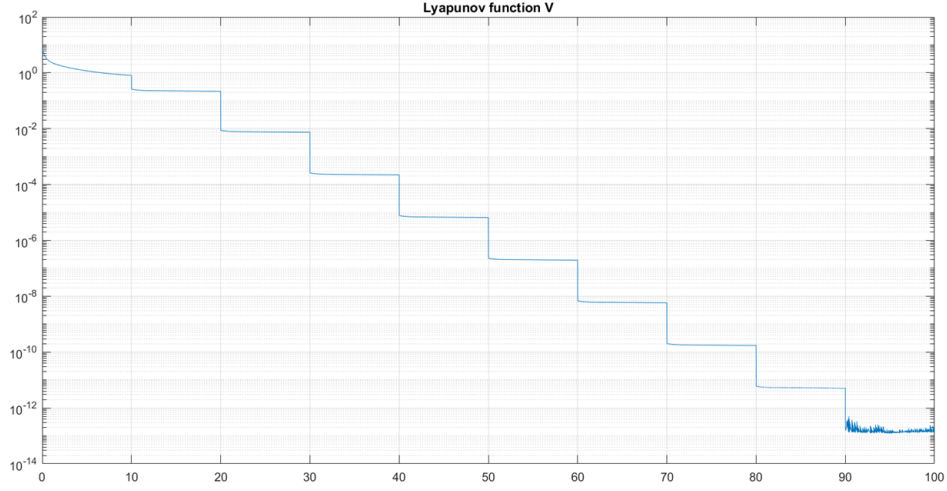


Figure 4.8: Evolution of function $V = \tilde{x}^T P^{-1} \tilde{x}$ over time.

and

$$\begin{cases} \tilde{x}(t_k, k+1) = \frac{R_k}{P(t_k, k) + R_k} \tilde{x}(t_k, k) \\ P(t_k, k+1) = \frac{R_k}{P(t_k, k) + R_k} P(t, k) \end{cases}, \forall (t_k, k) \in \mathcal{D} \quad (4.16)$$

Therefore,

$$\tilde{x}(t_k, k+1) = \frac{R_k}{P(t, k) + R_k} e^{(t_k - t_{k-1})} \tilde{x}(t_{k-1}, k) \quad (4.17)$$

$$= \frac{e^{(t_k - t_{k-1})} R_k}{e^{2(t_k - t_{k-1})} (P(t_{k-1}, k) + \frac{1}{2}Q) - \frac{1}{2}Q + R_k} \tilde{x}(t_{k-1}, k) \quad (4.18)$$

4.5 . New relaxed conditions for exponential stability of non-linear time-varying systems

In this section, we present a theoretical result on exponential stability of non-linear time-varying systems under relaxed assumptions. This result is an extension of Theorem 4.5 in [72] introduced in 1.2.

Theorem 4.3

We consider the system

$$\dot{x} = f(t, x) \quad (4.19)$$

where $f : [t_0, \infty) \times D \rightarrow \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x on $[t_0, \infty) \times D$, and $D \subset \mathbb{R}^n$ is a domain that contains the origin $x = 0$. Suppose that $x = 0$ is an equilibrium point for (4.19).

Let $V : [t_0, \infty) \times D \rightarrow \mathbb{R}$ be a continuously differentiable function such that

$$0 < \alpha_1 \|x\|^p \leq V(t, x) \leq \alpha_2 \|x\|^p \quad (4.20)$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) \leq \lambda \|x\|^p \quad (4.21)$$

$\forall t \geq 0$ and $\forall x \in D$, where $\lambda \geq 0$ and α_1, α_2 and p are strictly positive.

Let $\{t_k\}_{k \in \mathbb{N}}$ be a strictly increasing sequence such that $\lim_{k \rightarrow \infty} t_k = \infty$ and $\forall k \in \mathbb{N}, t_{k+1} - t_k \leq T$. Let $0 < \mu < \alpha_2$ such that

$$V(t_{k+1}, x(t_{k+1})) - V(t_k, x(t_k)) \leq -\mu \|x(t_k)\|^p \quad (4.22)$$

$\forall k \in \mathbb{N}$ and $x(t_k) \in D$. Then $x = 0$ is exponentially stable. If the assumptions hold globally, then $x = 0$ is globally exponentially stable.

Proof.

From (4.20) and (4.21), we have the inequality

$$\dot{V}(t, x) \leq \frac{\lambda}{\alpha_1} V(t, x), \quad \forall (t, x) \in [t_0, \infty) \times D \quad (4.23)$$

then, by Grönwall's inequality, for all $t^* \in [0, \infty)$, we have

$$V(t, x(t)) \leq V(t^*, x(t^*)) e^{\frac{\lambda}{\alpha_1}(t-t^*)}, \quad \forall t \geq t^* \quad (4.24)$$

From (4.20) and (4.22), we have the inequality

$$V(t_{k+1}, x(t_{k+1})) \leq \left(1 - \frac{\mu}{\alpha_2}\right) V(t_k, x(t_k)), \quad \forall (k, x(t_k)) \in \mathbb{N} \times D \quad (4.25)$$

then, $\forall k \in \mathbb{N}$

$$V(t_k, x(t_k)) \leq \left(1 - \frac{\mu}{\alpha_2}\right)^k V(t_0, x(t_0)) \quad (4.26)$$

$$= V(t_0, x(t_0)) e^{-\gamma k} \quad (4.27)$$

where $\gamma = -\ln\left(1 - \frac{\mu}{\alpha_2}\right) > 0$. γ is well defined because $V(t_{k+1}, x(t_{k+1})) \leq (\alpha_2 - \mu) \|x(t_k)\|^p$ (using (4.22)).

For all $t \in [0, \infty)$, there exists $k^* \in \mathbb{N}$ such that $t \in [t_{k^*}, t_{k^*+1})$. Then, we have

$$\begin{aligned} V(t, x(t)) &\leq V(t_{k^*}, x(t_{k^*})) e^{\frac{\lambda}{\alpha_1}(t-t_{k^*})} \\ &\leq V(t_0, x(t_0)) e^{-\gamma k^*} e^{\frac{\lambda}{\alpha_1}(t-t_{k^*})} \end{aligned}$$

The first inequality is obtained from (4.24) between t and t_{k^*} . The second inequality is obtained from (4.26) for k^* .

From the boundedness assumption on the sequence $\{t_{k+1} - t_k\}_{k \in \mathbb{N}}$ (i.e., $t_{k+1} - t_k \leq T$), we deduce that $\forall k \in \mathbb{N}, t_k - t_0 \leq kT$. Thus

$$\begin{aligned} V(t, x(t)) &\leq V(t_0, x(t_0)) \exp\left(-\frac{\gamma}{T}(t_{k^*} - t_0)\right) \exp\left(\frac{\lambda}{\alpha_1}(t - t_{k^*})\right) \\ &= V(t_0, x(t_0)) \exp\left(-\frac{\gamma}{T}(t - t_0)\right) \exp\left(\left(\frac{\gamma}{T} + \frac{\lambda}{\alpha_1}\right)(t - t_{k^*})\right) \\ &\leq V(t_0, x(t_0)) \exp\left(\gamma + \frac{\lambda T}{\alpha_1}\right) \exp\left(-\frac{\gamma}{T}(t - t_0)\right) \end{aligned}$$

The first inequality is obtained by "replacing" (upper-bound) k^* by $\frac{t_{k^*} - t_0}{T}$. The last inequality is deduced using the property $t - t_{k^*} \leq T$. In the second equality, we add and subtract t is the first exponential term. The last inequality uses the fact that $t - t_{k^*} \leq T$ since $t \in [t_{k^*}, t_{k^*+1}]$.

Using (A.1) and (4.20), we can conclude that

$$\begin{aligned} \|x(t)\| &\leq \left[\frac{V(t, x(t))}{\alpha_1} \right]^{\frac{1}{p}} \\ &\leq \left[\frac{V(t_0, x(t_0)) \exp\left(\gamma + \frac{\lambda T}{\alpha_1}\right) \exp\left(-\frac{\gamma}{T}(t - t_0)\right)}{\alpha_1} \right]^{\frac{1}{p}} \\ &\leq \left[\frac{\alpha_2 \|x(t_0)\|^p \exp\left(\gamma + \frac{\lambda T}{\alpha_1}\right) \exp\left(-\frac{\gamma}{T}(t - t_0)\right)}{\alpha_1} \right]^{\frac{1}{p}} \\ &\leq \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{p}} \exp\left(\frac{\gamma \alpha_1 + \lambda T}{p \alpha_1}\right) \exp\left(-\frac{\gamma}{pT}(t - t_0)\right) \|x(t_0)\| \end{aligned}$$

which shows that $x = 0$ is exponentially stable. If the assumptions hold globally then $x = 0$ is globally exponentially stable. □

Remark

Compared 1.2, the result of 4.3 does not request the uniformity of Lyapunov function decrease over intervals of the form $[t, t + \delta)$ which is hard assumption to verify an important class of systems. The alternative assumption that we propose require the existence of a sequence of time instants such that the Lyapunov function decreases "enough" during the interval between two consecutive elements of the time sequence $[t_k, t_{k+1}]$. Moreover, the Lyapunov function can increase during such intervals, at a controlled rate determined by λ in (4.21). However, condition (4.22) ensures that the Lyapunov function value at the end of the interval is necessarily lower than its value at the beginning of the interval of the form $[t_k, t_{k+1}]$.

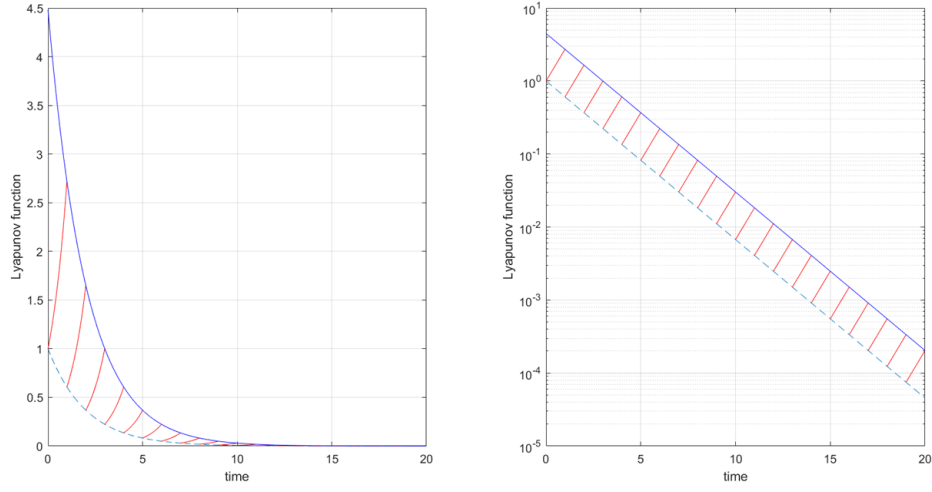


Figure 4.9: Illustration of the evolution of the Lyapunov function in Theorem 4.3 over time: (left-hand side) linear scale, (right-hand side) logarithmic scale. The dashed blue curve represents an interpolation of the Lyapunov function bound at times t_k . The red curves represent the Lyapunov function bound during intervals $[t_k, t_{k+1})$. The blue curve represents the Lyapunov function bound $t \geq t_0$.

4.6 . Conclusion

In this chapter, we proposed a Lyapunov-based proof of exponential ISS for the estimation error of the hybrid Kalman filter algorithm. The main result states that the state estimation error decreases exponentially during the transitory phase and is bounded after a sufficiently large time. This bound depends on the bounds of the process and observations noises. The proposed analysis allows us to study the Hybrid Kalman filter behaviour for bounded deterministic errors. This analysis can be extended in the stochastic framework for bounded distributions instead of Gaussian distributions. The development of this result was motivated by the extension of the result in Chapter 3 to the case where packet losses may occur.

Moreover, we presented a theoretical result on exponential stability of non-linear time varying systems. The presented result is suitable for hybrid dynamical systems including continuous-time dynamics and discrete-time dynamics such as event-triggered systems, sampled-data system, switching systems, etc. Such a framework is a powerful tool to analyze combining control of physical systems with continuous-time dynamics, and communications that occur in discrete time.

CONCLUSION

Multi-agent systems control, high-speed and reliable communications, and distributed and cooperative control have been trending research topics in the last few years and are likely to further attract researchers in the upcoming years. Multi-agent systems are effective for modelling complex networks of interacting systems. MAS modelling appears in numerous applications ranging from economic, biological and social systems to various engineering systems. Designing distributed algorithms based on the multi-agent approach has several advantages related to optimizing energy consumption, data privacy, and scalability. However, it presents technical challenges that systems designers have to overcome regarding information exchange, stability of the global systems, and robustness against attacks and information errors. MAS became very popular thanks to the large spectrum of applications that can be addressed using distributed algorithms. Compared to centralized algorithms, distributed algorithms are harder to design but they offer several advantages regarding flexibility, security, scalability, etc. They are used in several areas such as smart grids for the production and distribution of energy, distributed computing like distributed databases management or distributed information processing and in intelligent transport systems for cooperative control of connected vehicles via wireless communication networks. In this thesis, we focused on vehicular systems, but the results can be applied to other fields after some modifications to capture the specificity of the considered application.

In Chapter 2, we studied the problem of multi-agent systems communicating over a shared broadcast channel. We proposed a new model for the consensus algorithm for a multi-agent system of continuous-time first-order integrators under a TDMA communication protocol. The TDMA scheme is the simplest and the first technique for multiple access to the communication channel. The principle of TDMA consists in assigning the whole network resources to a single user for a fixed interval of time, and then they are released to be assigned to another user. This method allows the interconnected nodes to share the network resources efficiently and avoid signal interference and packet collisions.

We proved uniform exponential convergence of the proposed system under mild assumptions on connectivity, and we provided an explicit exponential bound on the norm of the displacement errors vector. The proposed exponential bound depends on the parameters of the communication protocol, namely, the duration of time slots, the period of topology switching, and the number of users.

In Chapter 3, we considered the problem of formation control of a group of connected vehicles communicating over a 5G communication system. The first challenge we encountered was to properly formulate the problem to include the communication system's feature in the vehicle control law. The first contribution of this chapter is the system modelling since we considered the general case of

the formation control problem for a two-dimensional vehicular movement rather than the one-dimensional used generally for the case of platooning. Through this example, we highlighted the notion of controlling the system using communications. Indeed, a good choice of transmitting users leads to faster convergence to the desired formation and a more robust stability.

To solve this problem, we proposed a joint design method for formation control and communication strategy for connected vehicles over a 5G network. The proposed scheme includes a consensus-type algorithm for formation control, where the agents communication is constrained by the network resources availability. For the communication strategy, we proposed a resource allocation algorithm that selects the transmitting agents in order to accelerate the convergence rate toward the desired formation while guaranteeing efficient use of the network resources.

We characterize the domain of initial condition on the vehicles positions from which it is guaranteed to converge to the desired formation. This domain is related to the targeted formation.

In Chapter 4, we analyzed the stability properties of the hybrid Kalman filter algorithm introduced in [124]. We proposed a Lyapunov-based proof of exponential ISS for the estimation error of the hybrid Kalman filter algorithm. The main result states that the state estimation error decreases exponentially during the transitory phase and is bounded after a sufficiently large time. This bound depends on the bounds of the process and observations noises. The proposed analysis allows us to study the Hybrid Kalman filter behaviour for bounded deterministic errors. This analysis can be extended in the stochastic framework for bounded distributions instead of Gaussian distributions.

Futures directions

The thesis falls on the border between telecommunication and control theory fields. The most challenging task we encountered was to use jointly tools from both fields to study the behaviour of multi-agent systems connected over a wireless communication channel. In Chapter 2 and Chapter 3, we focused on application problems related to the control of connected vehicles over a realistic communication channel. First, we presented a result on the stability analysis of the consensus algorithm when a TDMA scheme is used for channel multiple access. Second, we proposed a resource allocation algorithm for vehicles connected over a 5G communication channel. In Chapter 4, we proposed a theoretical result to characterize the state estimate error of a physical system observed through discrete-time observations.

The problems that we addressed in this thesis can be considered as a first step in the direction of joint of control and communication for engineering systems. Multiple extensions and improvements are still to be considered in order to study the behaviour of multi-agent systems connected over real-world communication

systems such as 5G cellular networks or Wi-Fi networks. The approach that we proposed can, not only be applied to connected and autonomous vehicles, but also to robots in automated factories, drones, aircraft, and any engineering systems of interconnected systems communicating through a wireless communication network.

Below, we develop some specific perspectives and future directions of research:

Joint design of control and communication systems

As mentioned in the introduction chapter, the paradigm of joint design of control and communication systems should be explored further in the future. In this context, one of the possible approaches is to pose the overall control-communication design problem as a multiple-objective optimization problem that aims to optimize communication and control parameters simultaneously. In [89], the authors propose an algorithm that solves an optimization problem with respect to the network resources allocation scheme and the control parameters of each vehicle belonging to the platoon. Similarly, the algorithm that we propose in Chapter 3 can be extended to support more complex vehicle models with parameterized controllers such as proportional-integral-derivative (PID) controllers [67]. PID controllers are widely used in industry to control complex systems thanks to their performances and their simplicity of implementation [123]. It will be interesting to consider optimizing simultaneously the communication transmitting indicators and the PID coefficients in the context of formation control of connected vehicles, and compare its performance with the Algorithm 3.9 when applied for vehicles with pre-tuned PID coefficients that were optimized separately.

Presence of communication errors

Regarding Chapter 4, the result presented on input-to-state stability of the Kalman filter for hybrid dynamical systems can be applied in the context of MAS to measure the effect of communication noise on the steady state of the agents. As in [98], the Kalman filter algorithm is used for distributed estimation relying on a consensus-like algorithm to improve the agents sensing quality. Using the principle of consensus-based estimation, the result of Theorem 4.2 in Chapter 4 can be applied for distributed estimation of continuous-time systems observed through discrete-time measurements with noise. The Kalman filter algorithm provides an estimate of the state and a reliability matrix measuring the quality of the estimation for each agent. An agent uses these matrices as a weight of the contribution of his neighbours to improve its estimation accuracy.

In this context, the resource allocation algorithm developed in Chapter 3 can be improved by considering the effect of communication errors. Indeed, the proposed algorithm problem (3.9) solves an optimization problem having as an input the real positions of the vehicles at a given time while ensuring that the constraints of interference-free communication and connectivity preserving are satisfied. When

errors occur at the level of the system dynamics due to disturbances and modelling uncertainty, or at the communication system level because of delays or packet losses, the algorithm may become inefficient or even unsafe. Thus it is essential to study the robustness of the proposed scheme when the given inputs are polluted by noise.

More realistic vehicle models

Another direction for future research is to extend the presented results by considering more realistic models of vehicles dynamics. Indeed, in Chapters 2 and 3, we considered a simple model for vehicles dynamics which is the first-order integrator. However, in real-world applications, vehicles have more complex dynamics that adds additional complexity to control design and stability analysis of the controlled system. There exist famous models used in the literature. Among such models, we can mention double integrators [116], non-holonomic unicycle model [86], differential-drive robots [84], or rigid body model with input constraints [22]. Combining realistic dynamics models with realistic communication systems models is even more challenging. At the same time, it can lead to design robust algorithms for real-world applications.

The basic models used in chapters 2 and 3 lead to the effect that changes of state of the transmitting nodes are not taken into account during the transmitting duration. In realistic settings, this is not true. Hence, a more careful analysis, using sampled-data systems is required to address this issue. As well as robustness with respect to the different types of disturbances and unmodelled dynamics of the vehicles.

Communication and data fusion

Regarding the size of multi-agent systems studied in this thesis, the presented results are valid for large-size systems. However, the convergence time and the complexity of the proposed algorithms may significantly increase when the network size increases. Another interesting aspect to study is the management of agents joining or leaving the formation. This problem requires establishing a network topology model that supports the varying number of participant agents, such as considering open systems models. In [136], the problem of open systems under the Krause model is considered. The authors presented some properties of open Krause systems and compared them to the standard Krause model. Convergence properties and connectivity constraints become stronger to ensure the desired behaviour of the system. It is interesting to check how the entrance of new agents can reestablish connectivity after a topology split. Moreover, the communication system should be designed to support a varying density of users (number of agents). In this direction, different strategies for resources allocation can be considered depending on the network density to provide the required quality of service.

Distributed resource allocation

In Chapter 3, we proposed a resource allocation operating in Mode 1. Mode 1 implies that resource allocation is decided at the level of the base station, but communication between agents is operating in D2D, i.e., communication is done directly between agents. The second resource allocation mode, called Mode 2 [34], consists in selecting available resources autonomously by users. In Mode 2, agents operate a first step of sensing to discover available resources. Then, the second step consists in transmitting information over these resources for a given amount of time. After that, resources are released and can be occupied by other users.

Mode 2 of resource allocation has the advantage of being operational in out-of-coverage situations where users cannot be connected to a base station. However, the rate of packet collision increases since two users sensing at the same time can select the same resources because they do not see each other. Moreover, the transmitting time intervals may become asynchronous, which increases the complexity of the control algorithm.

A - APPENDIX

A.1 . Proof of Proposition 2.1

From the definition of matrix $\hat{\mathcal{L}}_k$ (2.4), we have

$$\hat{\mathcal{L}}_k \mathbf{1}_j = \begin{cases} -\frac{1}{2} \mathbf{1}_{\mathcal{N}_k}, & \text{if } j = i_k \\ \mathbf{1}_j - \frac{1}{2} \mathbf{1}_{i_k}, & \text{if } j \in \mathcal{N}_k \\ \mathbf{0}, & \text{if } j \in \{\mathcal{N}_k \cup \{i_k\}\}^c \end{cases}$$

The vectors $\mathbf{1}_j$ with $j \in \{\mathcal{N}_k \cup \{i_k\}\}^c$ are eigenvectors of $\hat{\mathcal{L}}_k$ associated to the eigenvalue 0. The eigenvalue 0 has at least a multiplicity of $\text{card}(\{\mathcal{N}_k \cup \{i_k\}\}^c) = N - n_k - 1$.

Let λ be an eigenvalue of $\hat{\mathcal{L}}_k$, and v_λ the eigenvector of $\hat{\mathcal{L}}_k$ associated to λ .

The vector v_λ belongs in the orthogonal complement of $\text{span}(\{\mathbf{1}_j | j \in \{\mathcal{N}_k \cup \{i_k\}\}^c\})$ which is equivalent to $\text{span}(\{\mathbf{1}_j | j \in \{\mathcal{N}_k \cup \{i_k\}\}\})$, then it can be written as:

$$v_\lambda = \sum_{j \in \mathcal{N}_k \cup \{i_k\}} \alpha_j \mathbf{1}_j \quad (\text{A.1})$$

We have

$$\begin{aligned} \hat{\mathcal{L}}_k v_\lambda = \lambda v_\lambda &\iff \sum_{j \in \mathcal{N}_k \cup \{i_k\}} \alpha_j \hat{\mathcal{L}}_k \mathbf{1}_j = \lambda v_\lambda \\ &\iff \sum_{j \in \mathcal{N}_k} \alpha_j \left(\mathbf{1}_j - \frac{1}{2} \mathbf{1}_{i_k} \right) - \frac{\alpha_{i_k}}{2} \mathbf{1}_{\mathcal{N}_k} = \lambda v_\lambda \\ &\iff \sum_{j \in \mathcal{N}_k} \alpha_j \left(\mathbf{1}_j - \frac{1}{2} \mathbf{1}_{i_k} \right) - \frac{\alpha_{i_k}}{2} \sum_{j \in \mathcal{N}_k} \mathbf{1}_j = \lambda v_\lambda \\ &\iff \sum_{j \in \mathcal{N}_k} \left(\alpha_j - \frac{\alpha_{i_k}}{2} \right) \mathbf{1}_j - \frac{1}{2} \left(\sum_{j \in \mathcal{N}_k} \alpha_j \right) \mathbf{1}_{i_k} = \sum_{j \in \mathcal{N}_k} \lambda \alpha_j \mathbf{1}_j + \lambda \alpha_{i_k} \mathbf{1}_{i_k} \end{aligned}$$

Then,

$$\begin{cases} \alpha_j - \frac{\alpha_{i_k}}{2} = \lambda \alpha_j \\ \frac{1}{2} \sum_{j \in \mathcal{N}_k} \alpha_j = -\lambda \alpha_{i_k} \end{cases} \iff \begin{cases} (\lambda - 1) \alpha_j = -\frac{\alpha_{i_k}}{2} \\ \frac{1}{2} \sum_{j \in \mathcal{N}_k} \alpha_j = -\lambda \alpha_{i_k} \end{cases} \quad (\text{A.2})$$

If $\lambda = 1$, then $\alpha_{i_k} = 0$, and thus $\sum_{j \in \mathcal{N}_k} \alpha_j = 0$ which define the equation of a hyperplane in \mathbb{R}^{n_k} . Therefore, we can find $n_k - 1$ eigenvector of $\hat{\mathcal{L}}_k$ associated to the eigenvalue 1.

If $\lambda \neq 1$, then

$$\begin{aligned}
\begin{cases} (\lambda - 1)\alpha_j = -\frac{\alpha_i}{2} \\ \frac{1}{2} \sum_{j \in \mathcal{N}_k} \alpha_j = -\lambda\alpha_i \end{cases} &\iff \begin{cases} \alpha_j = -\frac{\alpha_i}{2(\lambda-1)} \\ \frac{1}{2} \sum_{j \in \mathcal{N}_k} \alpha_j = -\lambda\alpha_i \end{cases} \\
&\iff -\frac{n_k}{4} \frac{\alpha_i}{(\lambda-1)} = -\lambda\alpha_i \\
&\iff n_k\alpha_i = 4\lambda(\lambda-1)\alpha_i \\
&\iff n_k\alpha_i = 4 \left[\left(\lambda - \frac{1}{2} \right)^2 - \frac{1}{4} \right] \alpha_i \\
&\iff \frac{n_k+1}{4} \alpha_i = \left(\lambda - \frac{1}{2} \right)^2 \alpha_i
\end{aligned}$$

Since $\alpha_i \neq 0$, because if $\alpha_i = 0$ then $\alpha_j = 0$ for all $j \in \mathcal{N}_k$, we have

$$\lambda_{\pm} = \frac{1 \pm \sqrt{n_k + 1}}{2} \quad (\text{A.3})$$

From (A.2), we can deduce the normalized eigenvectors of λ_+ and λ_- respectively:

$$v_{\lambda_+} = \frac{1}{\sqrt{n_k + 4\lambda_-^2}} \sum_{j \in \mathcal{N}_k} \mathbf{1}_j + 2\lambda_- \mathbf{1}_i \quad (\text{A.4})$$

$$v_{\lambda_-} = \frac{1}{\sqrt{n_k + 4\lambda_+^2}} \sum_{j \in \mathcal{N}_k} \mathbf{1}_j + 2\lambda_+ \mathbf{1}_i \quad (\text{A.5})$$

A.2 . Proof of Proposition 4.1

We prove the result of Proposition 4.1 under assumptions **A1-A3**.

Lemma A.1

Let $\Phi^T(t, t_0) = e^{A(t-t_0)}$, then, for any $t \geq t_0$, we have

$$e^{-2\|A\|(t-t_0)} \mathbf{I} \preceq \Phi(t, t_0) \Phi^T(t, t_0) \preceq e^{2\|A\|(t-t_0)} \mathbf{I}$$

Proof.

For any $x \in \mathbb{R}^N$,

$$\begin{aligned}
x^T \Phi(t, t_0) \Phi^T(t, t_0) x &= \|\Phi^T(t, t_0) x\|^2 \\
&\leq \|\Phi^T(t, t_0)\|^2 x^T x
\end{aligned}$$

Since $\Phi^T(t, t_0) = e^{A^T(t-t_0)}$, then $\|\Phi^T(t, t_0)\| \leq e^{\|A\|(t-t_0)}$. Therefore,

$$x^T \Phi(t, t_0) \Phi^T(t, t_0) x \leq e^{2\|A\|(t-t_0)} x^T x$$

On the other hand,

$$\begin{aligned}
x^T x &= \|x\|^2 = \|\Phi^{-T}(t, t_0)\Phi^T(t, t_0)x\|^2 \\
&\leq \|\Phi^{-T}(t, t_0)\|^2 \|\Phi^T(t, t_0)x\|^2 \\
&= \|\Phi^{-T}(t, t_0)\|^2 x^T \Phi(t, t_0) \Phi^T(t, t_0) x \\
&\leq e^{2\|A\|(t-t_0)} x^T \Phi(t, t_0) \Phi^T(t, t_0) x
\end{aligned}$$

Therefore,

$$e^{-2\|A\|(t-t_0)} x^T x \leq x^T \Phi(t, t_0) \Phi^T(t, t_0) x$$

□

Lemma A.2

For $k \in \mathbb{N}^*$

$$\gamma \mathbf{I} \preceq P(t_k, k+1)$$

$$\text{where } \gamma = \left[\frac{2\|A\|}{\lambda_{Q_1}(1-e^{-2\|A\|\mathcal{T}})} + \frac{\lambda_C}{\lambda_{R_1}} \right]^{-1}$$

Proof.

We start by integrating the first equation is (4.7). Thus, for any $k \in \mathbb{N}^*$, we have

$$\begin{aligned}
P(t_k, k) &= \Phi(t_k, t_{k-1})P(t_{k-1}, k)\Phi^T(t_k, t_{k-1}) \\
&\quad + \int_{t_{k-1}}^{t_k} \Phi(t_k, \tau)Q(\tau)\Phi^T(t_k, \tau)d\tau
\end{aligned}$$

Using the property $P^{-1}(t_k, k+1) = P^{-1}(t_k, k) + C^T R_k^{-1} C$, we get

$$\begin{aligned}
P^{-1}(t_k, k+1) &\preceq \left(\int_{t_{k-1}}^{t_k} \Phi(t_k, \tau)Q(\tau)\Phi^T(t_k, \tau)d\tau \right)^{-1} + C^T R_k^{-1} C \\
&\preceq \left[\frac{1}{\lambda_{Q_1}} \left(\int_{t_{k-1}}^{t_k} e^{-2\|A\|(t_k-\tau)} d\tau \right)^{-1} + \frac{\lambda_C}{\lambda_{R_1}} \right] \mathbf{I} \\
&= \left[\frac{1}{\lambda_{Q_1}} \left(\frac{1 - e^{-2\|A\|(t_k-t_{k-1})}}{2\|A\|} \right)^{-1} + \frac{\lambda_C}{\lambda_{R_1}} \right] \mathbf{I} \\
&\preceq \left[\frac{2\|A\|}{\lambda_{Q_1}(1 - e^{-2\|A\|\mathcal{T}})} + \frac{\lambda_C}{\lambda_{R_1}} \right] \mathbf{I}
\end{aligned}$$

where λ_C is greatest eigenvalue of $C^T C$. The first inequality uses the fact that $\Phi(t_k, t_{k-1})P(t_{k-1}, k)\Phi^T(t_k, t_{k-1})$ is a positive matrix. The second inequality is deduced from assumption **A1** and Lemma A.1. The last inequality is deduced from assumption **A2**.

□

Corollary A.1

For $k \in \mathbb{N}^*$ and $t \in [t_k, t_{k+1}]$

$$\lambda_{P_1} \mathbf{I} \preceq P(t, k+1) \quad (\text{A.6})$$

where $\lambda_{P_1} = \left[\frac{2\|A\|}{\lambda_{Q_1}(1-e^{-2\|A\|\bar{\tau}})} + \frac{\lambda_C}{\lambda_{R_1}} \right]^{-1} e^{-2\|A\|\bar{\tau}}.$

Proof.

This result is deduced from assumption **A2** and the integrated form of $P(t, k+1)$. □

Lemma A.3

For any $k \geq 2$

$$P(t_k, k) \preceq (1 + \beta) \Phi(t_k, t_{k-1}) P(t_{k-1}, k) \Phi(t_k, t_{k-1})$$

where $\beta = \frac{\lambda_{Q_2}(e^{2\|A\|\bar{\tau}} - 1)}{2\|A\|} \left[\frac{2\|A\|}{\lambda_{Q_1}(1-e^{-2\|A\|\bar{\tau}})} + \frac{\lambda_C}{\lambda_{R_1}} \right].$

Proof.

From equation (4.7) we get

$$P(t_k, k) = \Phi(t_k, t_{k-1}) (P(t_{k-1}, k) + \Gamma_k) \Phi^T(t_k, t_{k-1})$$

where $\Gamma_k = \int_{t_{k-1}}^{t_k} \Phi(t_{k-1}, \tau) Q(\tau) \Phi^T(t_{k-1}, \tau) d\tau.$

Using the assumptions **A2** and the result of lemma A.1, we deduce that

$$\begin{aligned} \Gamma_k &\preceq \lambda_{Q_2} \int_{t_{k-1}}^{t_k} \Phi(t_{k-1}, \tau) \Phi^T(t_{k-1}, \tau) d\tau \\ &\preceq \lambda_{Q_2} \left(\int_{t_{k-1}}^{t_k} e^{2\|A\|(\tau-t_{k-1})} d\tau \right) \mathbf{I} \\ &\preceq \frac{\lambda_{Q_2}(e^{2\|A\|\bar{\tau}} - 1)}{2\|A\|} \mathbf{I} \end{aligned}$$

Finally, Using the result of Lemma A.2, we get

$$\Gamma_k \preceq \frac{\lambda_{Q_2}(e^{2\|A\|\bar{\tau}} - 1)}{2\|A\|} \gamma^{-1} P(t_{k-1}, k)$$

□

Lemma A.4

Under assumption **A3** of uniform observability, we have, for any $k \geq 2$

$$P(t_{k+m}, k+m+1) \preceq \frac{\lambda_{R_2}}{\lambda_{Ob}} (1 + \beta)^{1-m} \mathbf{I}$$

Proof.

Using the result of Lemma A.3, we deduce that, for any $k \geq 2$

$$\begin{aligned} P^{-1}(t_k, k+1) &= P^{-1}(t_k, k) + C^T R_k^{-1} C \\ &\succeq \hat{\beta} \Phi^T(t_{k-1}, t_k) P^{-1}(t_{k-1}, k) \Phi(t_{k-1}, t_k) + C^T R_k^{-1} C \end{aligned}$$

where $\hat{\beta} = (1 + \beta)^{-1}$.

By applying the previous inequality recursively, we get

$$\begin{aligned} P^{-1}(t_{k+m}, k+m+1) &\succeq \hat{\beta}^m \Phi^T(t_k, t_{k+m}) P^{-1}(t_k, k+1) \Phi(t_k, t_{k+m}) \\ &\quad + \sum_{i=0}^{m-1} \hat{\beta}^i \Phi^T(t_{k+m-i}, t_{k+m}) C^T R_{k+m-i}^{-1} C \Phi(t_{k+m-i}, t_{k+m}) \end{aligned}$$

Since $\Phi^T(t_k, t_{k+m}) P^{-1}(t_k, k+1) \Phi(t_k, t_{k+m})$ is a positive matrix and $\beta > 0$, then under assumption **A3**, we get

$$\begin{aligned} P^{-1}(t_{k+m}, k+m+1) &\succeq \frac{1}{\lambda_{R_2}} \hat{\beta}^{m-1} \sum_{i=0}^{m-1} \Phi^T(t_{k+m-i}, t_{k+m}) C^T C \Phi(t_{k+m-i}, t_{k+m}) \\ &\succeq \frac{\lambda_{Ob}}{\lambda_{R_2}} \hat{\beta}^{m-1} \mathbf{I} \end{aligned}$$

□

Corollary A.2

For $k \geq m+2$ and $t \in [t_k, t_{k+1}]$

$$P(t, k+1) \preceq \lambda_{P_2} \mathbf{I} \quad (\text{A.7})$$

where $\lambda_{P_2} = \frac{\lambda_{R_2}}{\lambda_{Ob}} (1 + \beta)^{m-1} e^{2\|A\|\bar{\tau}} + \frac{\lambda_{Q_2}}{2\|A\|} (e^{2\|A\|\bar{\tau}} - 1)$

Proof.

From the integrated form (4.7) and assumption **A2**, we get

$$\begin{aligned} P(t, k+1) &\preceq \frac{\lambda_{R_2}}{\lambda_{Ob}} (1 + \beta)^{1-m} e^{2\|A\|\bar{\tau}} \mathbf{I} \\ &\quad + \frac{\lambda_{Q_2}}{2\|A\|} (e^{2\|A\|\bar{\tau}} - 1) \mathbf{I} \end{aligned}$$

□

A.3 . Proof of equations (4.10)-(4.11)

In this section, we give more details to show how we obtained the equations (4.10) and (4.11).

A.3.1 . Proof of equation (4.10)

Using the dynamics of \tilde{x} and P during the flow steps (4.6)-(4.7), Proposition 4.1, and the property of the matrix inverse derivative $\dot{P}^{-1} = -P^{-1}\dot{P}P^{-1}$, we prove that under assumption **A2**

$$\begin{aligned}\dot{V}(t, k) &= -\tilde{x}^T(t, k)P^{-1}(t, k)Q(t)P^{-1}(t, k)\tilde{x}(t, k) \\ &\quad - 2\tilde{x}^T(t, k)P^{-1}(t, k)q(t) \\ &\leq -\frac{\lambda_{Q_1}}{\lambda_{P_2}^2}\|\tilde{x}(t, k)\|^2 - 2\tilde{x}^T(t, k)P^{-1}(t, k)q(t)\end{aligned}$$

Then, using the property $2ab \leq a^2 + b^2$, we get

$$\begin{aligned}\dot{V}(t, k) &\leq -\frac{\lambda_{Q_1}}{\lambda_{P_2}^2}\|\tilde{x}(t, k)\|^2 + \frac{2}{\lambda_{P_2}}\|\tilde{x}(t, k)\|\|q(t)\| \\ &= -\frac{\lambda_{Q_1}}{\lambda_{P_2}^2}\|\tilde{x}(t, k)\|^2 \\ &\quad + 2\sqrt{\frac{\lambda_{Q_1}(1-\theta_1)}{\lambda_{P_2}^2}}\|\tilde{x}(t, k)\|\sqrt{\frac{1}{\lambda_{Q_1}(1-\theta_1)}}\|q(t)\| \\ &\leq -\frac{\theta_1\lambda_{Q_1}}{\lambda_{P_2}^2}\|\tilde{x}(t, k)\|^2 + \frac{1}{\lambda_{Q_1}(1-\theta_1)}\|q(t)\|^2\end{aligned}$$

where $0 < \theta_1 < 1$ is an arbitrary constant.

A.3.2 . Proof of equation (4.11)

Using the dynamics of \tilde{x} and P during the jump steps (4.6)-(4.7), and the K_k and S_k formula, we get

$$\begin{aligned}V(t_k, k+1) - V(t_k, k) &= -\tilde{x}^T(t_k, k)C^T S_k^{-1}C\tilde{x}(t_k, k) + r_k^T(R_k^{-1} - S_k^{-1})r_k \\ &\quad + 2\tilde{x}^T(t_k, k)C^T S_k^{-1}r_k\end{aligned}$$

Then, under assumptions **A2-A3**, we have

$$\begin{aligned}
& V(t_k, k+1) - V(t_k, k) \\
& \leq -\frac{\xi}{\lambda_{P_2}\lambda_C + \lambda_{R_2}} \|\tilde{x}(t_k, k)\|^2 + \frac{\lambda_{P_2}\lambda_C}{\lambda_{R_1}^2} \|r_k\|^2 \\
& \quad + 2\frac{\sqrt{\xi}}{\lambda_{R_1}} \|\tilde{x}(t_k, k)\| \|r_k\| \\
& = -\frac{\xi}{\lambda_{P_2}\lambda_C + \lambda_{R_2}} \|\tilde{x}(t_k, k)\|^2 + \frac{\lambda_{P_2}\lambda_C}{\lambda_{R_1}^2} \|r_k\|^2 \\
& \quad + 2\sqrt{\frac{\xi(1-\theta_2)}{\lambda_{P_2}\lambda_C + \lambda_{R_2}}} \|\tilde{x}(t_k, k)\| \sqrt{\frac{\lambda_{P_2}\lambda_C + \lambda_{R_2}}{\lambda_{R_1}^2(1-\theta_2)}} \|r_k\| \\
& \leq -\frac{\theta_2\xi}{\lambda_{P_2}\lambda_C + \lambda_{R_2}} \|\tilde{x}(t_k, k)\|^2 \\
& \quad + \left(\frac{\lambda_{P_2}\lambda_C}{\lambda_{R_1}^2} + \frac{\lambda_{P_2}\lambda_C + \lambda_{R_2}}{\lambda_{R_1}^2(1-\theta_2)} \right) \|r_k\|^2
\end{aligned}$$

where $\xi = \frac{\lambda_{Ob}e^{-2(m-1)\|A\|^\tau}}{m}$ and $0 < \theta_2 < 1$ is an arbitrary constant.

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