Essays on labor market flows with endogenous job destruction and short-time work policies

Juho Peltonen

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Essais sur les flux du marché du travail avec destruction endogène d'emplois et politiques de chômage partiel

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THESE DE DOCTORAT

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ÉCOLE DOCTORALE
Droit, Économie, Management (DEM)

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HELSINGFORS UNIVERSITET
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Titre : Essais sur les flux du marché du travail avec destruction endogène d'emplois et politiques de chômage partiel

Mots clés : Marché du travail, Modèle d'appariement, Les séparations, Chômage partiel, Estimation bayésienne, Modèles d'équilibre général

Résumé : Cette thèse de doctorat est composée d'un chapitre introductif et de trois essais indépendants qui analysent différent aspects de la destruction d'emploi sur le marché du travail et ses conséquences macroéconomiques. En outre, cette thèse etudie les dispositifs de maintien dans l'emploi qui visent à accroître la stabilité du marché du travail.

Le premier essai s'intéresse à l'impact du facteur d'escompte subjectif dans les fluctuations du marché du travail dans un modèle d'équilibre général dynamique et stochastique avec appariements sur le marché du travail à la Krause and Lubik (2007). Une baisse du facteur d'escompte diminue les gains associés à la création d'emploi. En conséquence, les entreprises ouvrent moins de postes vacants et la valeur de chaque emploi diminue. Par ailleurs, lorsque les destructions d'emploi sont endogènes, les entreprises ont également la possibilité de supprimer des emplois à faible productivité, ce qui amplifie l'impact total du facteur d'actualisation sur le chômage. De plus, le modèle est estimé à l'aide de techniques bayésiennes sur des données américaines. Il en résulte que les variations du facteur d'actualisation sont la principale explication de la variance des variables du marché du travail, en particulier lorsque la période d'étude inclut la « grande récession ».

Le deuxième essai porte sur la politique du marché du travail dite de « chômage partiel » telle qu'elle a été mise en œuvre en Allemagne pendant la crise de la Covid-19. Le chômage partiel permet aux entreprises de réduire temporairement les heures de travail des salariés à temps plein afin de limiter la hausse du chômage en cas de choc important qui les pousserait autrement à se séparer de ces mêmes travailleurs. Pour cela, nous utilisons le modèle d'équilibre général avec chômage partiel de Balleer et al. (2016), à partir duquel nous réalisons une estimation bayésienne et une simulation d'un modèle contrefactuel sans chômage partiel. Nous trouvons ainsi que le taux de chômage allemand aurait été de 4,2 points de pourcentage plus élevé sans politique de chômage partiel pendant la Covid-19. Plus généralement, nous mettons en évidence que cette politique contribue à prévenir les destructions excessives d'emploi en période de ralentissement économique et à stabiliser les fluctuations du taux de chômage au cours des cycles économiques.

Dans le troisième essai, nous analysons l'optimalité sociale du marché du travail avec politique de chômage partiel dans un modèle d'équilibre général avec frictions de recherche d'emploi et cessations endogènes d'emploi. Lorsque les coûts liés au chômage partiel ne peuvent être internalisés dans les salaires, la création d'emploi s'avère systématiquement trop faible. Nous étudions ensuite la possibilité que certaines subventions restaurent l'efficacité sociale de l'économie décentralisée. Enfin, nous réalisons une illustration numérique calibrée sur l'économie allemande et trouvons que les transferts nécessaires à l'optimalité sociale sont de l'ordre de 1,9% de la production et réduisent le taux de chômage de 1,8 point de pourcentage par rapport au laissez-faire.

Title : Essays on labor market flows with endogenous job destruction and short-time work policies

Keywords : Labor markets, Search and matching, Endogenous separations, Short-time work policies, Bayesian estimation, General equilibrium models

Abstract : This doctoral dissertation consists of an introductory chapter and three independent essays which explore various aspects of job destruction on labor markets and their macroeconomic outcomes. Furthermore, this dissertation considers job retention policies aimed at preventing excess job destruction and at increasing labor market stability.

In the first essay, I analyze the role of discount factor shocks on labor market fluctuations within a dynamic stochastic general equilibrium search and matching labor market model à la Krause and Lubik (2007). Negative discount factor shocks decrease expected gains from job creation. As a result, firms are opening fewer job vacancies and the value of each job decreases. When job destruction is endogenous, firms also have the opportunity to cut the jobs with low productivity, which amplifies the impact of discount factor shocks on unemployment. Furthermore, the model is estimated with Bayesian techniques and a dataset from the United States. Discount factor shocks are found to be the main contributor in the variance of labor market variables, especially when the Great Recession is taken into account.

In the second essay, I examine the so-called short-time work (STW) labor market policy, as implemented in Germany in the Covid-19 period. STW enables firms to temporarily reduce the working hours of full-time workers in order to limit unemployment when a severe shock hits and would otherwise pressure them to separate from these workers. In this essay, I rely on the general equilibrium model with STW by \ Balleer et al. (2016), and a Bayesian estimation, as well as a simulation of a counterfactual model without STW. I find that the German unemployment rate would have been 4.2 percentage points higher without the policy during the Covid-19. In general, the essay shows that the STW program participates in preventing excess job destruction during economic downturns and stabilizes unemployment fluctuations over business cycles.

In the third essay, I study the social optimality of labor markets with short-time work policies (STW) in a general equilibrium model, with search frictions and endogenous job separations. When the costs related to STW cannot be internalized in wages, they create a systematic inefficiency through which job creation is found to be too low. I propose government transfers in the model to redistribute output in order to restore the social efficiency of the competitive economy. Furthermore, the essay includes a calibration exercise matching the German economy, which shows that transfers required for the social optimality are in the magnitude of 1.9% of output and achieve an unemployment rate which is 1.8 percentage points lower than it would be otherwise.

Abstract

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In the first essay, I analyze the role of discount factor shocks on labor market fluctuations within a dynamic stochastic general equilibrium search and matching labor market model à la Krause and Lubik (2007). Negative discount factor shocks decrease expected gains from job creation. As a result, firms are opening fewer job vacancies and the value of each job decreases. When job destruction is endogenous, firms also have the opportunity to cut the jobs with low productivity, which amplifies the impact of discount factor shocks on unemployment. Furthermore, the model is estimated with Bayesian techniques and a dataset from the United States. Discount factor shocks are found to be the main contributor in the variance of labor market variables, especially when the Great Recession is taken into account.

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Tiivistelmä

Tämä väitöskirja koostuu johdannosta ja kolmesta itsenäisestä esseestä, jotka käsittelevät työpaikkojen häviämistä ja tämän makrotaloudellisia seurauksia. Lisäksi väitöskirja tarkastelee työmarkkinapoliittisia järjestelmiä, joiden tavoitteena on estää tarpeetonta työpaikkojen häviämistä ja siten vakauttaa työmarkkinoita.

Ensimmäisessä esseessä analysoidaan diskonttokorkosokkien roolia työmarkkinoiden syklien selittäjänä käyttäen dynaamista ja stokastista yleisen tasapainon makrotalousmallia, jossa työmarkkinat on mallinnettu kuten artikkelissa Krause and Lubik (2007). Negatiiviset diskonttokorkosokit vähentävät uusien työpaikkojen luomisesta odotettuja tuottoja. Tämän seurauksena yritykset luovat vähemmän uusia työpaikkoja ja olemassa olevien työsuhteiden arvo laskee. Kun työpaikkojen häviäminen on endogeenistä, yrityksillä on myös mahdollisuus irtisanoa matalan tuottavuuden työntekijöitä, mikä vahvistaa diskonttokorkosokkien vaikutusta työttömyyden kasvuun. Lisäksi esseessä esitetty malli estimoidaan bayesiläisin menetelmin käyttäen Yhdysvaltojen aineistoa. Diskonttokorkosokkien havaitaan olevan tärkein epävarmuuden lähde työmarkkinamuuttujien syklisessä vaihtelussa erityisesti, kun otetaan huomioon vuosien 2007–2009 taantuma.

Toisessa esseessä tarkastellaan niin sanottua lyhennetyn työajan järjestelmää (short-time work, STW) ja analysoidaan Saksan talouden kehitystä Covid-19-pandemian aikana. Lyhennetyn työajan järjestelmä antaa yrityksille mahdollisuuden lyhentää tilapäisesti kokoaikaisten työntekijöiden työaikoja. Tämä järjestelmä rajoittaa työttömyyden nousua suurten negatiivisten sokkien seurauksena, sillä ilman sitä yritykset joutuisivat irtisanomaan työntekijöitä. Tämä essee hyödyntää Balleer et al. (2016) esittämää mallia, jossa yleisen tasapainon markotalousmalli sisältää lyhennetyn työajan järjestelmän. Esseessä estimoidaan tämä malli käyttäen bayesiläisiä menetelmiä ja simuloidaan kontrafaktuaalista tilannetta, jossa työajan lyhentäminen ei ole mahdollista. Tulosten perusteella Saksan työttömyysaste olisi ollut 4,2 prosenttiyksikköä korkeampi Covid-19-pandemian aikana ilman lyhennetyn työajan järjestelmää. Yleisemmin essee osoittaa, että kyseinen järjestelmä estää liiallisen työpaikkojen häviämisen taantumien aikana ja vakauttaa työttömyyden suhdannevaihteluita.

Kolmannessa esseessä tutkitaan työmarkkinoiden hyvinvointivaikutuksia, kun lyhen-

netyn työajan järjestelmä on käytössä. Tutkimuksessa analysoidaan yleisen tasapainon mallia, johon sisältyvät sellaiset etsintäteorian mukaiset työmarkkinat, joilla työpaikkojen häviäminen on endogeenistä. Koska lyhennetyn työajan järjestelmään liittyviä kustannuksia ei voida sisällyttää palkkoihin, ne luovat systemaattista tehottomuutta, jonka seurauksena työpaikkojen luominen on liian vähäistä. Mallissa tarkastellaan tulonsiirtoja, joilla tuotanto jaetaan uudelleen siten, että talouden optimi palautuu. Lisäksi tutkimuksessa esitetty malli kalibroidaan vastaamaan Saksan taloutta. Kalibroidussa mallissa sosiaalisen optimin edellyttämät tulonsiirrot ovat suuruusluokaltaan 1,9% tuotannosta, ja näiden tulonsiirtojen vaikutuksesta työttömyysaste on 1,8 prosenttiyksikköä matalampi.

Résumé

Cette thèse de doctorat est composée d'un chapitre introductif et de trois essais indépendants qui analysent diffèrent aspects de la destruction d'emploi sur le marché du travail et ses conséquences macroéconomiques. En outre, cette thèse etudie les dispositifs de maintien dans l'emploi qui visent à accroître la stabilité du marché du travail.

Le premier essai s'intéresse à l'impact du facteur d'escompte subjectif dans les fluctuations du marché du travail dans un modèle d'équilibre général dynamique et stochastique avec appariements sur le marché du travail à la Krause and Lubik (2007). Une baisse du facteur d'escompte diminue les gains associés à la création d'emploi. En conséquence, les entreprises ouvrent moins de postes vacants et la valeur de chaque emploi diminue. Par ailleurs, lorsque les destructions d'emploi sont endogènes, les entreprises ont également la possibilité de supprimer des emplois à faible productivité, ce qui amplifie l'impact total du facteur d'actualisation sur le chômage. De plus, le modèle est estimé à l'aide de techniques bayésiennes sur des données américaines. Il en résulte que les variations du facteur d'actualisation sont la principale explication de la variance des variables du marché du travail, en particulier lorsque la période d'étude inclut la ≪ grande récession ≫.

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Acknowledgments

Pursuing a PhD has never been a well-prepared plan for me, but more of a result of many coincidences. It was a long and demanding journey. An additional challenge was my decision to undertake the PhD at two universities in two countries, although this has also made my PhD journey even more unforgettable. The best part of it has been the chance to meet and to work with many extraordinary people. Without the support and contribution from several individuals, this dissertation would not have been possible. I would like to express my sincere gratitude at least to some of them explicitly here.

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Finally, I am grateful to my parents Marjo and Ari, who have always been my biggest supporters. My parents are not academics, but regardless they have never ceased to emphasize the importance of education. I would also like to thank my family and my friends for taking my mind out from the academic world from time to time, it has been essential to complete this dissertation.

Paris, September 2023
Juho Peltonen

Contents

Chapter 1

Introduction

This dissertation includes three independent essays, which all investigate aspects of job destruction on labor markets and further implied macroeconomic outcomes. In particular, I study the macro-labor fluctuations and job losses during the two recent economic crises, the Great Recession of 2007–2009 and the Covid-19 recession of 2020. Furthermore, this dissertation considers job retention policies which aim at preventing excess job destruction and at increasing labor market stability.

1.1 Background

In general, during an economic downturn, the unemployment rate tends to increase. Since wages and to a lesser extent working hours are rigid in the short run, firms are required to adjust their labor costs by decreasing hiring and increasing layoffs. While the former, i.e. job creation, has received considerable attention in the macro-labor literature, and its role in driving unemployment fluctuations is well understood, the latter, i.e. job destruction and its contribution in labor market volatility has been more debated.

Job destruction is a flow of workers from employment to unemployment, and job creation the opposite flow. However, there also exists constant flows between the labor force and outside it, which complicates the quantification of these flows. Indeed, measuring labor market flows is particularly challenging over long historical time series because of the lack of comprehensive micro datasets. Hence, the data on job openings, which can be collected from longer historical periods, has traditionally been used to measure job creation. Consequently, the role of new jobs in unemployment fluctuations had been thoroughly researched earlier in the economic literature.

Another challenge in observing job destruction is that a worker can move from employment to unemployment either from a voluntary quit or from an involuntary layoff.

The latter two move in opposite directions over business cycles, potentially offsetting each other. For instance, during a recession, quits are decreasing while layoffs are increasing, such that the flow from employment to unemployment seems to remain almost constant. Consequently, an easy interpretation is that unemployment fluctuations are the result of fluctuations in job creation instead of fluctuations in job destruction.

However, new micro data evidence, as well as developments in labor market theory, have provided new information about the importance of job destruction for labor market outcomes. In addition, large economic crises during recent decades have revealed that economic conditions, such as financial market disruptions, may increase the role of job destruction in business cycles. Literature reviews on job creation and destruction as drivers of unemployment fluctuations can be found for instance from Elsby et al. (2009), Fujita and Ramey (2009) and Portugal and Rua (2020).

The theoretical background of this dissertation is the search and matching theory, which originates from the seminal contributions by Diamond (1982), Mortensen (1982) and Pissarides (1985). Endogenous job destruction has been analyzed in this framework since the publication of another fundamental paper by Mortensen and Pissarides (1994). Considering macroeconomic fluctuations, Cooley and Quadrini (1999) integrated the theory of endogenous job destruction into a business cycle framework.

Further developments in the search and matching theory have increased interest in job destruction. In this regard, Den Haan et al. (2000) show that endogenous job destruction magnifies the effects of shocks. Furthermore, Mortensen and Nagypal (2007a), Mortensen and Nagypal (2007b), and Fujita and Ramey (2012) argue that models with endogenous separation generate larger volatilities than with constant job destruction, which improves the match of the model with the fluctuations observed in the data. Moreover, Elsby et al. (2010) and Portugal and Rua (2020) show that job destruction has played a larger role at the turning points of business cycles, i.e. the contribution of job losses to an increase in unemployment is significant at the beginning of recessions, while the diminishing job creation grows its importance later during economic downturns. The same result, specific to the Great Recession is provided by Clymo (2020).

1.1.1 Financial uncertainty in business cycles

To observe the relationship between financial uncertainty and labor market fluctuations, I depict the de-trended logs of the US employment rate and the Standard and Poor's 500 stock market index in Figure 1.1. The series exhibit visible correlation.

One potential rationale for the correlation in Figure 1.1 is as follows. The firms, which are hiring a new worker, compare the current recruitment costs with the expected future gains from an additional worker. Hence, the job creation is an investment, analogous with purchasing an equity share from the stock markets, and in both cases, the expected returns are discounted to present value. The increase in uncertainty decreases the discount factor, which further cuts job creation, and simultaneously depresses the stock prices. In addition, job destruction increases, because the expected values of existing jobs drop as a response to the uncertainty of future profits. Hall (2017) is the first to shows that the fluctuations in unemployment are the result of uncertainty, captured by the discount rates. He also argues that the discount rates can be estimated from the stock market data. In the first essay of this dissertation, I study the impact of time-varying discounting, more specifically discount factor shocks, in a dynamic model with endogenous job separations.

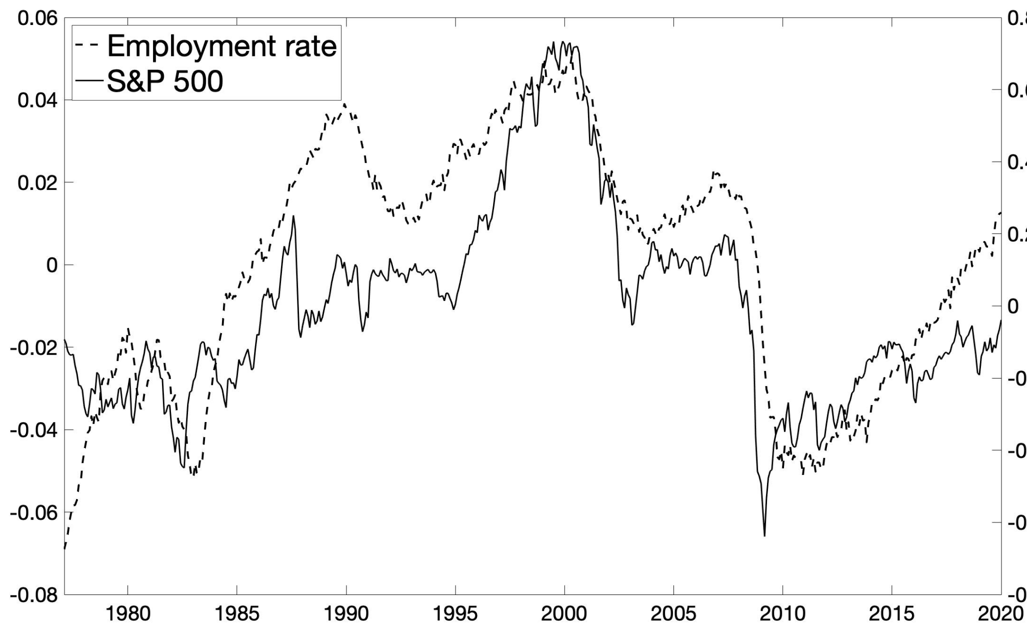


Figure 1.1: De-trended log of S&P 500 index (scale on the left axis) and de-trended log of employment rate (scale on the right axis) of All Persons for the United States aged 25-54. All series are in monthly frequency between 1977 and 2020. (Sources: datahub.io, spglobal.com, FRED, Federal Reserve Bank of St. Louis)

However, even if the stock market and the labor market cycles exhibit a correlation, the speed of recovery from recessions in these two markets is very different. As an illustration, Figure 1.2 shows the duration of the Standard and Poor's 500 stock market index and unemployment rate in the US to reach the pre-recession level, during the Great Recession and the Covid-19 pandemic. The unemployment rate clearly takes longer to reach its pre-crisis level. The same result holds for most of the recession periods in US history, even though the selection of the initial pre-recession value is not always trivial as the stock markets tend to drop earlier than the initial unemployment increase. The phenomenon of slow revival of unemployment after recessions is described as "jobless recoveries", as in Shimer (2012) for instance, who shows that also the gross domestic product, or the GDP growth, also returns to pre-recession level faster than unemployment does.

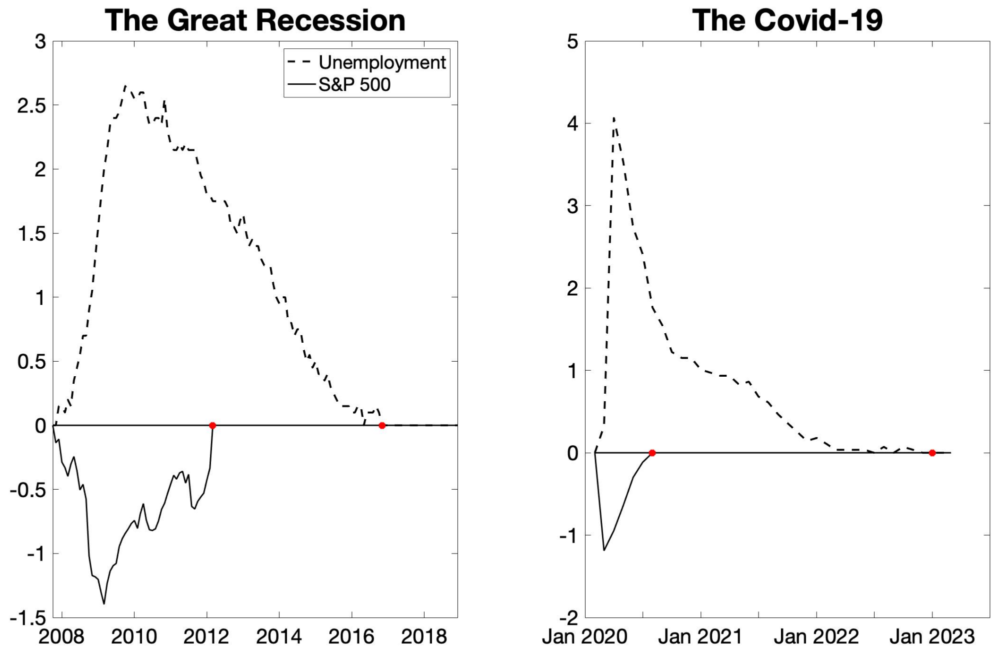


Figure 1.2: Recovery of unemployment rate (dashed line, above) and S&P 500 index (solid line, below), both series are normalized and set to start from zero. The red stars point the month when a series reaches back to zero. (Sources: datahub.io, spglobal.com, FRED, Federal Reserve Bank of St. Louis, NBER)

The economic theory of job creation and destruction offers one explanation for the sluggish recovery of labor markets from recessions. When firms are hit by negative shocks, they destroy the lowest productive jobs, and invest less into job creation. However, those jobs that are created have higher productivity. Consequently, mean productivity in the economy is increasing. As a result, achieving the same output as before can be produced with a lower number of workers, which leaves unemployment high even after the recession has passed.

Furthermore, the implied welfare consequences of jobless recoveries are a sum of two opposite effects. First, the productivity increase can be welfare enhancing, especially if the eventual outcome of an economic downturn is that workers move from "bad" jobs to "good" jobs. Second, if excess job destruction is followed by prolonged unemployment spells, the welfare losses, as well as fiscal burdens, can be considerable. Hence, the policy maker faces a tradeoff between these two effects. On the one hand, job retention policies, which incentivize firms to keep current workers employed, may reduce the welfare losses from job destruction and long unemployment recovery. On the other hand, these policies may suppress the allocative efficiency of labor markets, and consequently mitigate the productivity enhancements ignited by the economic downturns. Eventually, the essential question is the optimal level of policies, in terms of generosity and coverage of labor protection, for instance. In this regard, the second and the third essays of this dissertation study the job retention policy named short-time work, which was widely applied during the Covid-19 recession. Some key details of the Covid-19 period are discussed in the next section.

1.1.2 The Covid-19 crisis and job retention policies

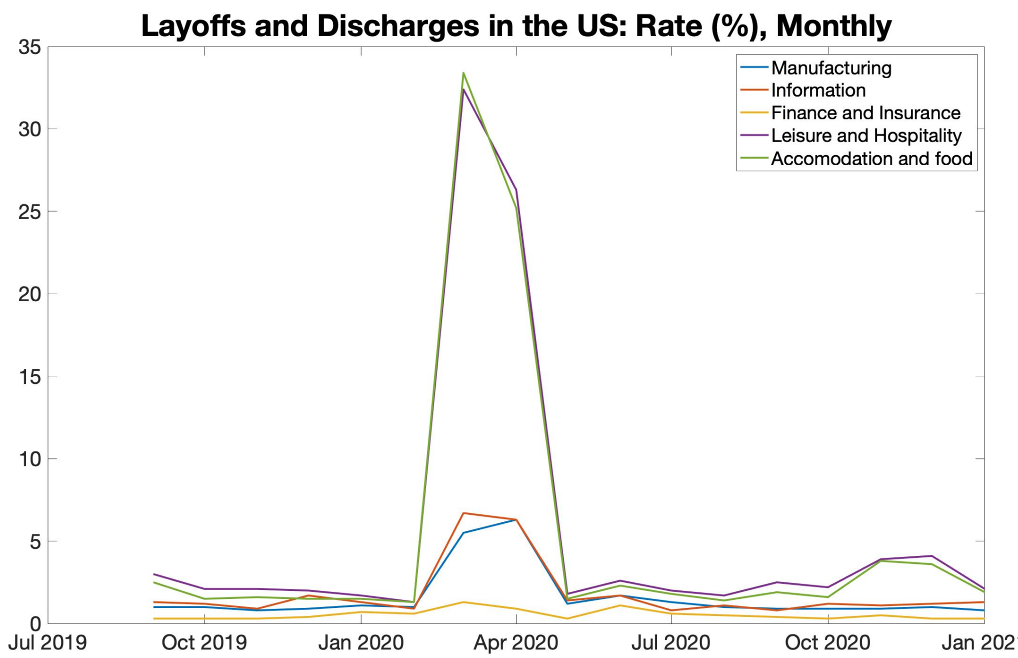


Figure 1.3: Some sector-specific job destruction rates in the US during the Covid-19 pandemic. Rate in percentages. (Source, FRED, Federal Reserve Bank of St. Louis)

The Covid-19 crisis of 2020 is an exceptional economic recession in the history. If one of the peculiarities of the Great Recession is that it started from the financial markets, the corresponding uniqueness of Covid-19 is that the cause of the recession is completely exogenous to the economy as a whole. More specifically, Covid-19 refers to the coronavirus disease 19, a respiratory virus infection, which resulted in a global pandemic, and a health crisis starting from Spring 2020. The economic consequences of the Covid-19 were a result of lockdown policies, by which the governments worldwide sought to prevent physical contacts between people in order to slow down the spread of the disease.[1]

From the perspective of labor markets, the lockdown policies were extreme since they prevented a significant proportion of employees in the economy from working. Consequently, firms employing these workers faced an unprecedented pressure to cut labor costs, for instance by destroying significant numbers of jobs. In addition, another peculiarity of the Covid-19 recession is that the impact on the economy was extremely asymmetric across business sectors. Figure 1.3 illustrates these two outcomes of the crisis, by showing some sector-specific rates of layoffs in the US during the Covid-19.

Two important aspects can be shown from Figure 1.3. First, the asymmetric impact of Covid-19 is striking, as the layoff rate reached over 30% of employed workers in some sectors, but did not increase at all in other sectors such as the Finance and Insurance sector. Second, the magnitude of this increase in layoff rates has been exceptionally large. As a comparison, the highest value of the total rate of layoffs in the US during the Great Recession was 2.0%, while the same value in the Covid-19 recession was 8.6% during March 2020. The figures for the most impacted industries in Figure 1.3 were multiple times larger.

From the perspective of job destruction, the increase in layoffs in the US were accompanied by an unprecedentedly sharp increase in unemployment. To be precise, the unemployment rate increased from 3.5% in February 2020 to a staggering value of 14.7% in just two months.[2] At the same time, job creation dropped sharply as well, since the vacancy rate decreased from 4.4% in February to 3.5% in April 2020. However, even though the pace at which the vacancy rate declined was unusual, the total magnitude of the decrease was less exceptional. For instance, during the Great Recession, the vacancy rate decreased by 1.5 percentage points in total. Hence, I think, it is safe to say that a considerable share of the unemployment fluctuations was driven by job destruction in the US, during the Covid-19 pandemic.

The stylized facts discussed so far this section were based on data from the United States. Yet, the US labor markets can be considered very flexible, meaning that the costs of job creation and destruction are relatively low, and the volatility in employment and unemployment are relatively high. Indeed, the sharp increase in unemployment was followed by a relatively rapid decrease, as the unemployment rate was 6.7% by the end of 2020. However, the jobless recovery described in the previous section is still present as the pre-Covid-19 level of 3.5% was reached only in the summer of 2022.

The second and the third essays of this dissertation will emphasize Europe, as they

[1] More about the lockdowns see Coibion et al. (2020)

[2] Source for all the US data in this section: FRED, Federal Reserve Bank of St. Louis

focus on a job retention policy, named short-time work (STW), which is traditionally a European policy program, even though it was widely implemented in developed economies all over the world as a response to the Covid-19 lockdowns. The aim of the short-time work policy is to preserve jobs by decreasing the working hours instead of increasing layoffs. Furthermore, the policy aims at lowering the unemployment fluctuations, and consequently the social costs of economic downturns.

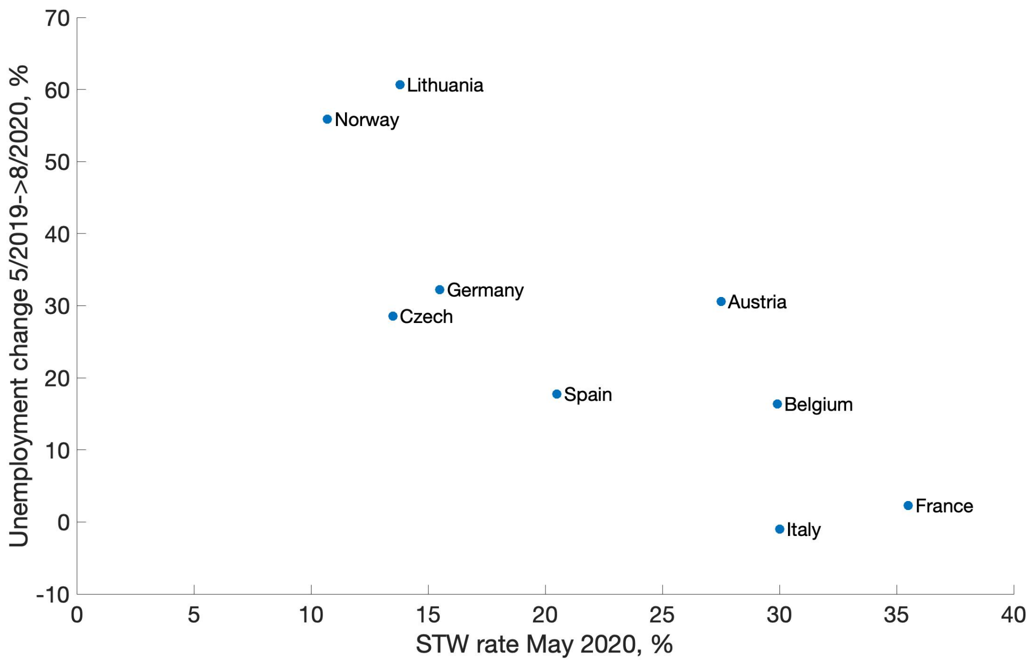


Figure 1.4: Relationship between a share of employed workers in short-time work program and a relative increase in unemployment rate in selected European countries, during Covid-19. (Source: OECD, Eurostat)

Figure 1.4 illustrates the relation between the short-time work (STW) policy and unemployment by depicting the relative unemployment rate change between May 2019 and August 2020 (y-axis), and the share of employed workers in STW in May 2020 (x-axis), which was the peak month of the STW participation in these selected countries. In general, there seems to be a relatively clear negative correlation, such that the higher the STW rate, the lower the increase in unemployment. In general, it is also worth mentioning the participation in the STW policy in Europe was at considerably high levels during the Covid-19. For instance in Germany, which is studied in detail in the second essay of this dissertation, the highest STW rate was 19% during the Covid-19 pandemic, while it was 4% during the Great Recession.[3]

[3]Source: Organisation for Economic Co-operation and Development (OECD) (2020)

To conclude, while in the US job destruction and unemployment increased significantly during Covid-19, the European countries responded to the crisis by allowing unprecedented levels of temporary working time reductions of normal full-time workers, in order to retain the job relationships. That said, there is no reason to assess that one or the other policy response is unarguably better. Eventually, the social welfare losses depend on the costs of job destruction and creation, compared with the costs of job retention policies. When the cost of hiring and firing is high, as in Europe with high labor market frictions, the job retention policies yield benefits in a social optimality perspective. On the contrary, when the same costs are low, as in the flexible labor markets in the US, the gains from preserving jobs are smaller, and the welfare improvements from productivity increase can compensate for the increased job destruction. Here again, the question of social optimality lies in the level of different labor market policies, to be further discussed in the third essay.

1.2 Summary of the essays

This section summarizes the main parts and the key results in the three essays of this dissertation. Each essay is self-contained and can be read separately.

1.2.1 Discount factor shocks and labor market outcomes in the Great Recession

In this essay, I investigate the role of discount factor shocks on labor market fluctuations, by considering a dynamic stochastic general equilibrium search and matching labor market model à la Krause and Lubik (2007), supplemented with discount factor shocks. A negative discount factor shock makes households more impatient and thus value present costs more as compared to future expected gains. Therefore, the firms that they own open fewer job vacancies when those job openings imply a direct cost. This result is in line with Hall (2017) who shows in a state-contingent partial equilibrium model, that unemployment fluctuations are a result of time-varying discounting. At a macroeconomic level, I find that, endogenous job destruction amplifies the response of unemployment to discount factor shocks. As firms put more weight on the present wage bill as compared to future output, the value of each occupied job vacancy decreases, resulting in more job separations when the model allows for these endogenous flows.

The model is estimated using Bayesian techniques and a dataset from the US that covers the Great Recession. Discount factor shocks are shown to be the main contributor in the variance of labor market variables. The historical variance decomposition shows that the discount factor shock is the main contributor in the variance of vacancies (85%), job filling rate (93%), labor market tightness (73%), unemployment (44%), the wage bill (57%) and the idiosyncratic productivity cutoff (60%).

In order to investigate the importance of discount factor shocks in normal times, I also consider the same empirical exercise without the Great Recession. The discount factor shocks are still the main contributor in the variance of vacancies (76%), job filling rate (90%) and labor market tightness (53%). In the other variables, such as unemployment, wage bill and idiosyncratic productivity cutoff, the variance is explained by the matching efficiency shock (65%, 43% and 62% respectively), which captures the time-variance in the matching frictions. These results are consistent with Michaillat (2012), in that matching frictions are responsible for unemployment fluctuations during expansions but much less in recessions.

1.2.2 Short-time work in search and matching models: Evidence from Germany during the Covid-19 crisis

This essay evaluates the extent to which a short-time work (STW) labor-market policy can prevent unemployment fluctuations, in the case of Germany during the Covid-19 crisis. STW is a policy program that aims to preserve jobs by allowing and incentivizing employers to reduce working hours of full-time workers instead of laying them off. A key element of the policy is unemployment benefits that workers are paid from hours not worked, while their job contracts are preserved. In Germany the policy, namely Kurzarbeit, has a long tradition, since it had been established already in the 1920s.

In order to investigate the impact of the policy in the Covid-19 recession, I first estimate a general equilibrium model with the STW policy by Balleer et al. (2016) with state-of-the-art Bayesian techniques. This allows the model parameters to be quantified, as well as to yield an estimate of shock processes which are driving the labor market fluctuations over the sample period, including the Covid-19 crisis. In addition, the same model but without STW is simulated using the estimated shock series, resulting in counterfactual labor market outcomes without the STW policy. Comparing these two sets of results the materialized 1.4 percentage points increase in the unemployment rate would have been 5.6 percentage points without STW in Germany between February and April 2020.

This essay makes two quantitative contributions. First, the model with STW is estimated using Bayesian techniques. Second, the counterfactual model without STW is simulated in order quantify the impact of STW policy on unemployment stabilization. To the best of my knowledge, Bayesian techniques have not previously been used for this purpose. This quantitative method answers to some of the challenges in the existing research of STW, such as the cross-country comparisons, which are hampered by the fact that countries are different, or the utilization of microdata on a firm or an establishment level which cannot capture general equilibrium effects on employment or consumption from the demand side. The method of estimation and simulation allows for the construction of a counterfactual of an identical economy without STW in a general equilibrium framework.

1.2.3 On the efficiency of labor markets with short-time work policies

In this essay, I build on the decentralized general equilibrium search and matching labor market model with a short-time work policy (STW) which is close to the one by Balleer et al. (2016). I solve a constraint-efficient social-planner solution of the model, and compared it to a decentralized solution, in order to evaluate the social optimality of labor markets with STW. In addition, I introduce two government subsidies and evaluate their capacity to restore the decentralized efficiency.

In the model, a job-specific productivity shock may result in some workers generating profit losses. In this case, a firm can either participate in STW and decrease the working hours of low productive workers, or endogenously separate from these workers. Both choices come with a cost. Consequently, if the net value of a job occupied by a worker in STW is positive, the firm participates in STW and keeps the worker employed. However, the costs related to STW and separation have an impact on job creation. Indeed, when a firm considers opening a vacancy, it considers the expected STW and separation costs. Therefore, the firm opens fewer vacancies in the presence of these costs. Moreover, because the firm cannot internalize these costs in the collective wage bargaining, they create a systematic inefficiency in a competitive economy. This inefficiency has two consequences. First, competitive firms are creating less than the socially optimal number of new jobs. Second, a competitive level of STW is lower than the socially optimal level.

This essay shows that the social optimality can be restored under three sufficient conditions, which are the so-called Hosios rule (Hosios, 1990), and the sizes of two government transfers which redistribute output to correct the inefficiency from the separation and STW costs. As a final step, a calibration exercise matching the German economy over the period 2000-2021 suggests that transfers required for the social optimality are in the magnitude of 1.9% of output. In addition, the unemployment rate is found to be 1.8 percentage points lower in the presence of optimal transfers.

Chapter 2

Discount factor shocks and labor market outcomes in the Great Recession

Abstract

This paper investigates the role of discount factor shocks on labor market fluctuations. In a dynamic stochastic general equilibrium search and matching labor market model, negative discount factor shocks decrease expected gains from job creation. As a result, firms are opening fewer job vacancies and the value of each job decreases. When job destruction is endogenous, firms also have the opportunity to cut low productive jobs. This amplifies the impact of discount factor shocks to unemployment. The model is estimated using Bayesian techniques and a dataset from the US that covers the Great Recession. Discount factor shocks are shown to be the main contributor in the variance of labor market variables, especially when the Great Recession is taken into account.

Keywords: Search and matching, discount factor shock, endogenous separations, Bayesian estimation.

JEL: E24, E32, J63, J64.

2.1 Introduction

During the Great Recession in the US, unemployment has doubled, from 5% in December 2007 to 10% in May 2009, both due to a drop in job creation and an increase in job destruction. More specifically, the job vacancy rate decreased from 3.2% in December 2007 to 1.7% in April 2009, while the rate of layoffs and discharges increased from 1.4% in

December 2007 to 2.0% in April 2009. Hall (2017) suggests that labor market fluctuations are explained by discount factor shocks. When creating new jobs, employers compare the cost of hiring an additional worker to the discounted expected returns on the new job. Time-varying discount factors capture changes in the rate of preferences for the future, for instance due to macroeconomic or financial uncertainty (Basu and Bundick, 2017).

In addition to job creation, firms can adjust job separation over the business cycle. In such a context, Hall (2017) shows that changes in the discount factor increase the volatility of unemployment.[1] In order to investigate the interaction of discount factor shocks with time-varying job destruction, this paper develops a search and matching labor market model, in which endogenous job destruction is modeled in the spirit of Mortensen and Pissarides (1994) and Den Haan et al. (2000). The labor market model is embedded in a dynamic stochastic general equilibrium framework à la Krause and Lubik (2007). The endogenous job destruction magnifies the impact of discount factor shocks on unemployment, and the empirical evaluation confirms the relevance of these shocks on all labor market outcomes.

A negative discount factor shock makes households more impatient and thus value more present costs as compared to future expected gains. Therefore, the firms that they own open less job vacancies when those job openings imply a direct cost. In addition, the firms put more weight on the present wage bill over the future output. This lowers the value of each occupied job vacancy and increases incentives for the firms to cut less productive ones. Therefore, endogenous job destruction amplifies the response of unemployment to the discount factor shock.

To assess the quantitative importance of discount factor shocks, the model is estimated using Bayesian techniques and a quarterly dataset from the US between 2001-Q1 and 2019-Q3. The results show the significance of the discount factor shock in explaining labor market volatility. The historical variance decomposition shows that the discount factor shock is the main contributor in the variance of vacancies (85%), job filling rate (93%), labor market tightness (73%), unemployment (44%), the wage bill (57%) and the idiosyncratic productivity cutoff (60%). The variance of the job separation rate is explained by an exogenous separation shock and the discount factor shock in approximately equal shares (37%). Moreover, the results show that almost all responses of labor market variables are larger when job separation is endogenous. Only the responses of labor market tightness are close, as both the vacancies and unemployment change in the same proportion.

[1] As an extension of his model, Hall (2017) makes the job destruction rate depend on the discount rate and the growth rate of productivity. However, the general equilibrium effect has not been developed further.

In order to investigate the importance of discount factor shocks in normal times, I also consider the same empirical exercise without the Great Recession. The discount factor shocks are still the main contributor in the variance of vacancies (76%), job filling rate (90%) and labor market tightness (53%). In the other variables, the discount factor shock is no longer the main contributor. The largest share of the variance in unemployment, wage bill and idiosyncratic productivity cutoff is then explained by the matching efficiency shock (65%, 43% and 62% respectively). The matching efficiency shock captures the time-variance in the matching frictions. These results are consistent with Michaillat (2012), who argues that search and matching frictions explain unemployment fluctuations during expansions but not as much during recessions.

The rest of the paper is organized as follows. Section 2.2 discusses the related literature. Section 2.3 describes the model. Section 2.4 presents the Bayesian estimation, results and analysis. Section 2.5 concludes.

2.2 Literature

This paper is related to two strands of literature. The first one is the literature about discount factor shocks. Shocks to discount factors have a long tradition in the macroeconomic literature starting from at least Smets and Wouters (2003). Smets and Wouters (2003) and Smets and Wouters (2007) have discount factor shocks as reduced-form demand shocks which affect households' intertemporal substitution. The former is estimated using European and the latter with the US data. Both show the significance of discount factor shocks in explaining output and employment fluctuations. Since the Great Recession, macroeconomic papers often feature shocks to discount factors. For instance, in Christiano et al. (2011a) the discount factor shock increases household's propensity to save, which makes the zero lower bound binding.

Beraja et al. (2019) interpret discount factor shocks as aggregate demand shocks and find that these shocks explain a major share of the US cross-regional variation in unemployment during the Great Recession. Other interpretations include disaster risks and their effect on investment returns (Gourio, 2012), which was in play during the Great Recession.[2]

Basu and Bundick (2017) associate the time-variance in discount factors with uncertainty. They study how an increase in uncertainty can contract output. In their model, households' discount factor is subject to an exogenous shock process. Moreover, the

[2]Gourio (2012) argues that the probability of disaster, not the realization of it, is sufficient to affect the economy through discounting. Further, Isoré and Szczerbowicz (2017) show how the disaster probability generates simultaneously procyclical consumption and wages but countercyclical risk premia.

volatility of this process itself is subject to an innovation. This second-moment shock is called an uncertainty shock, since an increase in volatility increases uncertainty about the changes in household's demand.

Hall (2017) argues how in a search and matching labor market model, discount factor shocks are driving unemployment fluctuations. In his baseline model, the increase in unemployment is a result of diminishing vacancy postings while job destructions are constant. As an extension, Hall (2017) makes the job destruction rate a state-contingent variable and concludes that this considerably increases the unemployment volatility. In this paper, the model is in a general equilibrium framework and the job destruction is an endogenous choice based on the whole state of the economy. The amplification of discount factor shocks on unemployment is confirmed.

Both Basu and Bundick (2017) and Hall (2017) suggest that the volatility of discount factors can be estimated from stock market data. Following the same idea, Clymo (2020) builds a search and matching model where capital and labor are discounted with different time-varying discount factors. He estimates these discount rates from the US stock markets. He finds that 52% of the rise in unemployment during the Great Recession can be attributed to the two discount factor shocks. In Clymo (2020) the two discount factor shocks affect hirings and capital investments, while job destruction is constant, or subject to exogenous shocks as a robustness check. My estimation results of a model without capital, but with endogenous job destruction show comparable magnitude as the discount factor shocks account for 44% of the variance in unemployment.

The closest works to mine in this strand are Albertini and Poirier (2014) and Leduc and Liu (2020). Both papers do the Bayesian estimations of search and matching models with discount factor shocks. Albertini and Poirier (2014) have estimation with two shocks, i.e. TFP and discount factor shocks, and two observables, unemployment and real GDP from the US over a period of 1948-Q1–2014-Q2. They find that the bulk of variations in vacancies and unemployment is mainly explained by the discount factor shock. Leduc and Liu (2020) estimate a search and matching model with time-varying search and recruiting intensity. They find that the discount factor shock accounts about 60% of labor market fluctuations. Yet, neither of the two papers has endogenous job destruction. Albertini and Poirier (2014) have a constant job separation rate, while in Leduc and Liu (2020) separation is a result of an exogenous shock process. However, as shown in this paper, the endogenous job destruction are important in amplifying the discount factor shock consequences on the labor market.

Second, this paper is related to the literature about endogenous job destruction. A seminal contribution by Mortensen and Pissarides (1994) developed a search and matching model with endogenous job creation and destruction processes. In another fundamental paper, Cooley and Quadrini (1999) integrated the theory of endogenous job destruction to a business cycle framework, and Den Haan et al. (2000) argued that endogenous job destruction magnifies the output effects of shocks. The model in this paper is mostly based on Krause and Lubik (2007), who themselves build on the abovementioned papers.

Mortensen and Nagypal (2007b), Mortensen and Nagypal (2007a) and Fujita and Ramey (2012) are discussing the role of job destruction in generating plausible fluctuations in labor market variables over business cycles. They argue that models with endogenous separation generate larger volatilities than the models with exogenous job destruction. This improves the model match with the fluctuations observed in the data and suggests a possible solution to the so-called "Shimer puzzle" (Shimer, 2005). While the Shimer puzzle is related to productivity shocks, I find that also the impact of discount factor shocks on unemployment is amplified when the model has endogenous separation. The Shimer puzzle is an essential part of a large debate about whether unemployment fluctuations result from volatility in job creation or in job destruction. This paper does not take a stance on this question but acknowledges that endogenous separation is an intrinsic part of this literature. Summaries and discussions about the topic can be found for instance from Elsby et al. (2009), Fujita and Ramey (2009) and Portugal and Rua (2020). In this strand, a closely related paper is Näf et al. (2022), which has a model containing endogenous separation with discount factor shocks. The firing costs are shown to limit the impact of shocks on job destruction. A calibrated model in Näf et al. (2022) demonstrates that discount factor shocks generate more unemployment volatility than aggregate productivity shocks. The model in this paper has no firing cost, and the endogenous and exogenous separation are compared. In addition, I do a Bayesian estimation with five different shocks that shows discount factor shocks to be the main contributor in the variance of unemployment.

This paper is also related to Christiano et al. (2011b) and Wesselbaum (2015) in that they do a Bayesian estimation of search and matching models with endogenous job destruction. Christiano et al. (2011b) have a monetary DSGE-model, which is extended to a small open economy setup and estimated using Swedish data. They find that financial shocks are pivotal to explaining business cycle fluctuations and that foreign shocks explain approximately a third of the variation in unemployment. Neither Christiano et al. (2011b) nor Wesselbaum (2015) focuses on discount factor shocks as in this paper.

2.3 Model

The model presented here is a search and matching model with endogenous job separation à la Krause and Lubik (2007). The economy encompasses a representative household, a

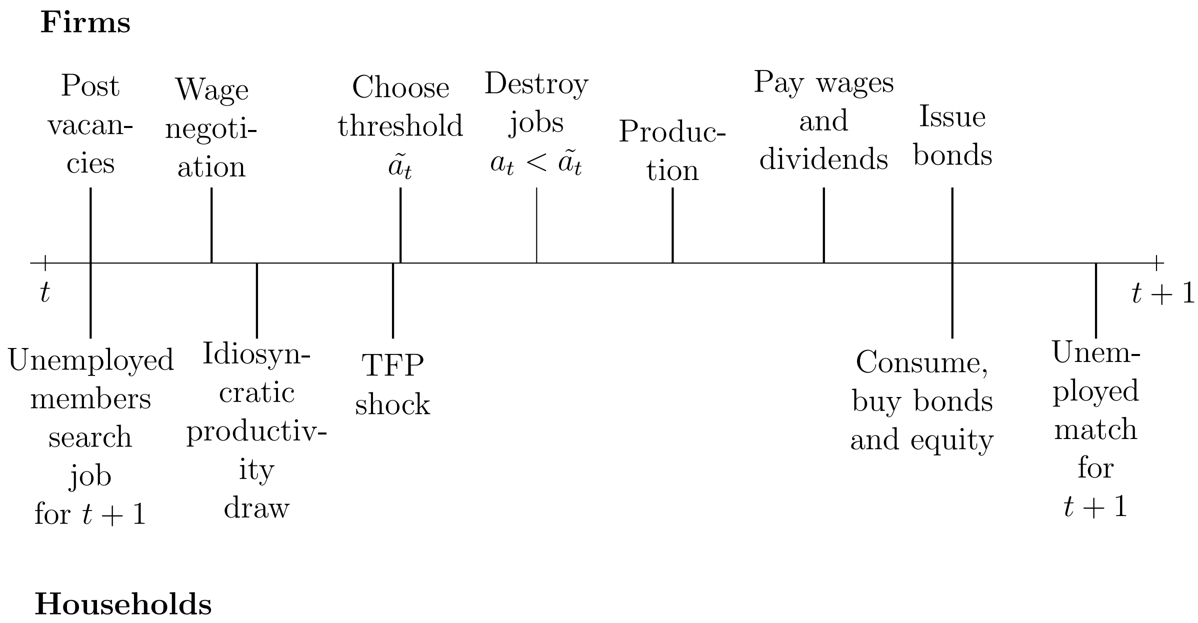


Figure 2.1: Timeline of events

representative firm and a government. The firm produces a homogeneous consumption good using labor as the only productive input. The household provides labor, consumes and saves to bonds and equity of the firm. Household members are either employed or unemployed. Following Merz (1995) and Andolfatto (1996), the household is large, providing its members with a perfect consumption insurance against the variations of individual income. The government collects lump sum tax to finance an unemployment benefit to unemployed workers.[3]

The timeline of the model is depicted in Figure 2.1. The order of events is the following. Household members who are employed in the productive jobs at period $t-1$ remain matched with the firm to the next period. The new workers who are matched with the firm during period $t-1$ are ready to enter the production during period t. To create new jobs, the firm posts vacancies to match with unemployed workers. The vacancy posting is forward looking on the expectation of period $t+1$ and does not depend on the state of the economy on current period, hence it can be considered to happen at the beginning of period t and the matches take place during the period. Next, the firm and the worker negotiate about the wage. The negotiated wage is conditional on the productivity of the job. Then, the idiosyncratic productivity is drawn and the total productivity is observed. Once the aggregate state of the economy is learned, the firm endogenously chooses a level

[3]The original model in Krause and Lubik (2007) has a continuum of firms producing differentiated goods with pricing power. The government issues money and bonds. Here firms produce an identical good, absent of any nominal rigidity, while public expenditures are limited to unemployment benefits.

of productivity $\tilde{a}_t$, such that jobs below this threshold ($a < \tilde{a}_t$,) are unprofitable and the firm destroys them. Both, the old existing jobs and the new jobs, created during the previous period, are destroyed if they turn out to be unprofitable. Before the production commences, the unprofitable jobs are cut. Next the production takes place. From the output, the firm pays wages, dividends, vacancy posting cost and repays the bonds. After the household has received the total income, it decides between consumption and savings, in the form of corporate equities and corporate bonds. Main details of the model are presented below and more detailed derivations can be found in the Appendix.

2.3.1 Household

The representative household lives infinitely and maximizes utility given by

$$E_t \sum_{j=0}^{\infty} \prod_{k=0}^{j} \beta_{t+k} \left[\frac{C_{t+j}^{1-\sigma} - 1}{1-\sigma} \right], \tag{1}$$

in which C_t is consumption, σ is a risk aversion parameter and β_t a subjective discount factor. The discount factor is subject to an exogenous shock and follows an $AR(1)$-process

$$\beta_t = \rho^\beta \beta_{t-1} + (1-\rho^\beta)\bar{\beta} + \epsilon_t^\beta, \tag{2}$$

in which $\rho^\beta \in (0,1)$ is the persistence, $\bar{\beta}$ the steady state value of β, σ^β the volatility and $\epsilon_t^\beta \sim \mathcal{N}(0, \sigma^\beta)$ an independently and identically distributed shock with standard deviation σ^β. This discount factor shock is one of the shocks in the Bayesian estimation later.

Each period, there is a unit mass of household members in the economy either employed n_t or unemployed u_t. However, they pool all of their income together before deciding on consumption and saving. Hence, the maximization of the household is subject to a single budget constraint,

$$C_t + P_t S_{t+1} + \frac{B_{t+1}}{R_t} = w_t n_t + b u_t + B_t + S_t(P_t + D_t) - T_t, \tag{3}$$

in which $w_t n_t$ are the wages that the employed household members earn, b is a constant unemployment benefit, B_t bond holdings, R_t a periodical gross return on bonds, S_t equity share holdings , P_t the price of a share, D_t a dividend from the firm and T_t a lump sum tax. Each period, the employed household members supply one unit of labor. The unemployed members earn constant unemployment benefit which the government finances by collecting a lump sum tax. The household saves by investing in bonds and shares of the representative firm. The corporate bonds are added to introduce a risk free asset to the model without a central bank. The corporate bonds are safe since the representative firm cannot default on its debt. Nothing prevents bonds B_t to be negative, thus technically the household can borrow from the firm.

The household chooses consumption C_t, bond purchases B_{t+1} and equity purchases S_{t+1} to maximize equation (1) subject to the budget constraint (3). This results in a consumption Euler equation

$$\frac{1}{R_t} = \beta_t E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}, \tag{4}$$

and to an equity pricing condition [4]

$$P_t = E_t \sum_{j=1}^{\infty} m_{t,t+j} D_{t+j}, \tag{5}$$

in which the $m_{t,t+j}$ is the household's stochastic discount factor. The stochastic discount factor between periods t and $t+1$, and a simplified expression for one-period ahead discount factor are respectively given by

$$m_{t,t+j} \equiv E_t \prod_{k=0}^{j} \beta_{t+k} \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} \quad \text{and} \quad m_{t+1} \equiv \beta_t E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}. \tag{6}$$

2.3.2 Firm

The representative firm posts job vacancies v_t with a constant vacancy posting cost c, to attract unemployed workers. The number of job matches taking place each period is given by a matching function $M(u_t, v_t)$, which is assumed to be increasing in both arguments, concave and homogeneous of degree 1. From a firm's perspective, the labor market tightness θ_t is defined as a ratio of vacancies to unemployment, i.e. $\theta_t = \frac{v_t}{u_t}$. The probability of vacancy being filled, i.e. the job filling rate is $q(\theta_t) = \frac{M(u_t,v_t)}{v_t}$.

The representative firm has amount of n_t filled jobs. Each job includes an idiosyncratic productivity component a_t. Every period, before the production commences, the job-specific productivity is drawn from a time-invariant distribution with a cumulative distribution function $F(a)$ and density $f(a)$. The endogenous separation rate is $F(\tilde{a}_t)$, in which $\tilde{a}_t$ is a productivity threshold. The threshold $\tilde{a}_t$ is endogenously chosen by the firm. In addition, a fraction ρ_t^X of jobs is assumed to separate exogenously each period.

[4]The equity shares and the pricing equation do not exist in Krause and Lubik (2007), in which firms simply transfer their profits to households. The pricing equation clarifies the definition of the stochastic discount factor, which is subject to a shock here.

The exogenous separation rate is subject to a shock and follows an $AR(1)$-process,

$$\rho_t^X = \rho^\rho \rho_{t-1}^X + (1-\rho^\rho)\bar{\rho^X} + \epsilon_t^\rho, \tag{7}$$

in which $\rho^\rho \in (0,1)$ is the persistence, $\bar{\rho^X}$ the steady state value of exogenous separation rate and $\epsilon_t^\rho \sim \mathcal{N}(0,\sigma^\rho)$ an independently and identically distributed shock with standard deviation σ^ρ. [5] The separation shock is one of the shocks in the Bayesian estimation. The definition of the separation shock follows Zanetti (2019). Overall, the total separation rate ρ_t is thus a sum of the exogenous and the endogenous part, given by

$$\rho_t \equiv \begin{cases} \rho_t^X + (1-\rho_t^X)F(\tilde{a}_t) & \text{if endogenous separation} \\ \rho_t^X + (1-\rho_t^X)\rho^e & \text{if constant separation.}^6 \end{cases} \tag{8}$$

The representative firm takes the idiosyncratic productivity distribution as given and chooses an optimal productivity threshold $\tilde{a}_t$. The conditional mean of idiosyncratic productivity, $H_t(\tilde{a}_t)$ is an aggregator over individual jobs surviving the cutoff $\tilde{a}_t$ as

$$H_t(\tilde{a}_t) = \int_{\tilde{a}_t}^{\infty} a \frac{f(a)}{1-F(\tilde{a}_t)} da. \tag{9}$$

In case of exogenous separation, the idiosyncratic productivity becomes constant such that the threshold $\tilde{a}_t = F^{-1}(\rho^e)$, $\forall t$, and the firm does not choose the threshold as part of the maximization problem defined below.

The firm produces output Y_t with a constant return-to-scale technology as

$$Y_t = A_t n_t H_t(\tilde{a}_t), \tag{10}$$

in which n_t the labor supply from the household and A_t is an aggregate productivity, which follows an $AR(1)$-process as

$$A_t = \rho^A A_{t-1} + (1-\rho^A)\bar{A} + \epsilon_t^A, \tag{11}$$

in which $\rho^A \in (0,1)$ is the persistence, $\bar{A}$ the steady-state aggregate productivity, σ^A the volatility and $\epsilon_t^A \sim \mathcal{N}(0,\sigma^A)$ an independently and identically distributed shock with

[5] The exogenous separation part ensures that the steady state flow from employment to unemployment is high enough. During booms the firm may not want to destroy any jobs endogenously.

[6] This definition allows a comparison of constant and endogenous separation by setting $\rho^e = F(\tilde{a}_{ss})$, where $\tilde{a}_{ss}$ is the steady-state threshold. In general, the choice of ρ^e results the threshold from the inverse CDF as $\tilde{a} = F^{-1}(\rho^e)$. This constant separation rate is a way to consider exogenous separation without changing the structure of the model.

standard deviation σ^A.

The profit flow of the firm at time t is D_t as

$$D_t = Y_t - W_t - cv_t - B_t + \frac{B_{t+1}}{R_t}, \tag{12}$$

in which W_t is the wage bill, c constant cost per vacancy v_t, B_t the bonds the firm issues and R_t the gross interest rate of the bonds. The wage bill is an aggregate of individual wages

$$W_t = n_t \int_{\tilde{a}_t}^{\infty} w_t(a) \frac{f(a)}{1 - F(\tilde{a}_t)} da. \tag{13}$$

The individual wages depend on the idiosyncratic productivity of the job, hence $w_t(a)$ is the wage function for an individual job in (13).

At time t, the firm chooses the number of employed workers n_t, the number of open vacancies v_t, the productivity cutoff $\tilde{a}_t$ and the quantity of bonds it issues B_{t+1} so as to maximize the expected value of real term profits

$$\max_{n_t, v_t, \tilde{a}_t, B_{t+1}} E_t \sum_{j=0}^{\infty} \left[m_{t+j} \Pi_{t+j} \right], \tag{14}$$

subject to the employment evolution equation

$$n_{t+1} = (1 - \rho_{t+1})(n_t + q(\theta_t) v_t), \tag{15}$$

Both existing and new matches are subject to an idiosyncratic productivity draw and hence possible separation. The definition that the new matches enter production only the next period makes the job creation completely forward-looking. The firm may end up destroying the new matches before they eventually produce, since the productivity shocks, idiosyncratic and total, are unknown at the moment of the match. [7]

Since all the workers are either employed or unemployed, the unemployment develops symmetrically to the employment as $u_t = 1 - n_t$. This means also that u_t and n_t are the rates of unemployment and employment, respectively.

The maximization of (14) subject to (15) yields two optimality conditions. (Details of the derivation are in the appendix.) First, a job creation condition obtained from the

[7] The firm does not know ex-ante what kind of jobs it creates, which simplifies the implementation of endogenous separation. This definition follows Krause and Lubik (2007), while for instance in Mortensen and Pissarides (1994) new jobs are created with the highest possible productivity, which ensures that a new job is profitable.

first-order conditions on employment n_t and vacancies v_t, is

$$\frac{c}{q(\theta_t)} = E_t m_{t+1}\left[(1-\rho_{t+1})\left(A_{t+1}H(\tilde{a}_{t+1}) - \frac{W_{t+1}}{n_{t+1}} + \frac{c}{q(\theta_{t+1})}\right)\right]. \tag{16}$$

The job creation condition equates the marginal cost of hiring with the expected gain from an additional worker. The expected gain is the output net of wage, plus the value of the match. The value of the match is the expected cost of hiring a worker, which the firm avoids paying the next period if it is matched with a worker already. The separation rate is taken into account, since the expected gain materializes only if the worker is not separated before production.

Second, a job destruction condition, obtained from the first-order conditions on employment n_t, vacancies v_t and productivity cutoff $\tilde{a}_t$, is

$$A_t\tilde{a}_t - w_t(\tilde{a}_t) + \frac{c}{q(\theta_t)} = 0. \tag{17}$$

The job destruction condition states that the firm chooses a productivity cutoff $\tilde{a}_t$ such that a surplus from a job with productivity $\tilde{a}_t$ is zero. The jobs with productivity below the threshold, $a_t < \tilde{a}_t$, are not profitable such that the firm destroys these jobs and the worker and the firm separate. After the wage function is derived below, an explicit value for the $\tilde{a}_t$ can be derived from (17).

2.3.3 Wage Setting

A wage is the outcome of a Nash-bargaining process. The firm and the worker bargain about the sharing of the joint surplus. The worker has a relative bargaining power $\eta_t \in (0,1)$. The bargaining power is subject to an exogenous shock and follows an $AR(1)$-process

$$\eta_t = \rho^\eta \eta_{t-1} + (1-\rho^\eta)\bar{\eta} + \epsilon_t^\eta, \tag{18}$$

in which $\rho^\eta \in (0,1)$ is the persistence, $\bar{\eta}$ the steady-state value of bargaining power and $\epsilon_t^\eta \sim \mathcal{N}(0,\sigma^\eta)$ an independently and identically distributed shock with standard deviation σ^η. The bargaining power shock is one of the shocks in the Bayesian estimation.

The bargaining problem is

$$w_t(a_t) = \arg\max(\mathcal{W}_t(a_t) - \mathcal{U}_t)^{\eta_t}\mathcal{J}_t(a_t)^{1-\eta_t} \tag{19}$$

in which $\mathcal{W}_t(a_t)$ and $\mathcal{U}_t$ are the value functions of employed and unemployed worker respectively, and $\mathcal{J}_t(a_t)$ is the value function of firm. (Value functions are presented in

Appendixes 2.A.2(a) and 2.A.2(b).) The bargaining takes place every period before the idiosyncratic productivity draw and production. Both parties are willing to maintain the match as long as it preserves a positive surplus to them. The worker's surplus is the difference between the value of being employed with a wage $w_t(a_t)$, i.e. $\mathcal{W}_t(a_t)$ and the value of being unemployed and earning the unemployment benefit b, i.e. $\mathcal{U}_t$. The continuation value for the worker is the probability of remaining employed or becoming unemployed. The firm does not have an outside option because it is assumed that the job has zero value without a matched worker. Hence, the firm's surplus $\mathcal{J}_t(a_t)$ is the output from a job with productivity a_t minus the wage to the worker, plus the continuation value of the match.

Following Krause and Lubik (2007), the worker's wage is a weighted average of the competitive Nash bargaining wage and the previous period's wage. The wage is an exogenously given parameter γ and the mean wage becomes

$$w_t(a_t) = \gamma w_t^{flex}(a_t) + (1-\gamma)w_{t-1}, \tag{20}$$

in which $w_t^{flex}(a_t)$ is the flexible, Nash-bargained wage.

By inserting the functional forms of the value functions to the Nash bargaining solution (19) the flexible wage is solved as (See Appendix 2.A.2 for the details)

$$w_t^{flex}(a_t) = \eta_t(A_t a_t + \theta_t c) + (1-\eta_t)b. \tag{21}$$

The first part of the flexible wage equation (21) is the surplus from the job match with a productivity a_t and the latter part is the worker's outside option b. If the workers have a maximum bargaining power, i.e. $\eta_t = 1$, they obtain all the surplus. On the contrary, if the workers do not have any bargaining power i.e. $\eta_t = 0$, the firm pays each job only a wage that equals the unemployment benefit.

2.3.4 Market-Clearing and Competitive Equilibrium

This economy is composed of a bond market, an equity market and a consumption good market. On the bond market, the representative firm cannot default on its debt. Hence, the market clears when the firm is borrowing with interest rate R_t from the household, according to the Euler equation (4).

On the equity markets, the representative firm does not issue new equity shares. Thus, the quantity of shares can be normalized to one and the gross return on equity can be defined as

$$R_{t+1}^s = (P_{t+1} + D_{t+1})/P_t, \tag{22}$$

in which the price of equity P_t follows from household's optimality condition (5). Since there is no risk of bankruptcy of the representative firm, the return on equity equals the return on bonds. The equity market clears such that the representative household holds the shares. The government finances unemployment benefits by collecting a lump-sum tax T_t. The government budget constraint is then $bu_t = T_t$.

The model is closed by combining the market clearing conditions and the budget constraints of the firm, the household and the government. This results in an aggregate resource constraint of the economy (Detailed calculations are in the appendix.)

$$C_t = Y_t - cv_t, \tag{23}$$

such that the output is either consumed or invested in vacancy posting each period.

The competitive equilibrium consists of output Y_t, consumption, C_t, vacancy posting, v_t, employment n_t, unemployment u_t, labor market tightness θ_t, an idiosyncratic productivity cutoff $\tilde{a}_t$, a separation rate ρ_t, a mean productivity $H_t(\tilde{a}_t)$, job filling rate $q(\theta_t)$, wages $w_t(a_t)$, a wage bill W_t, a lump-sum tax T_t, returns on bonds and equity, R_t and R_t^s, equity price P_t, dividends D_t and a household's stochastic discount factor m_t, such that v_t satisfies the job creation condition (16), $\tilde{a}_t$ is defined by the cutoff equation (26), the separation rate is defined by (8), employment evolves according to (15), unemployment is $u_t = 1 - n_t$, labor market tightness is defined as a ratio of vacancies to unemployment, mean productivity H_t is an aggregate over the productivity distribution in (9), the job filling rate is defined by the matching function $q(\theta_t) = M(\theta_t^{-1}, 1)$, $w_t(a_t)$ is defined by the wage equation (20), the wage bill is employment times the mean wage $W_t = n_t w_t$, C_t and R_t satisfy household's Euler equation (4), price of equity P_t follows equation (5), the stochastic discount factor follows equation (6), the firm pays profits as dividends, i.e. the profit flow D_t equals dividends, return on equity R_t^s satisfies equation (22), T_t balances government budget and the aggregate resource constraint is satisfied.

2.4 Estimation

The model is estimated using Bayesian techniques. First, the model is log-linearized around a deterministic steady state. For a given set of parameters, the solution takes the form of a state-space representation. The model thus describes a data-generating process, that is used to compute a likelihood function. I choose the prior distributions for the estimated parameters and use the Metropolis-Hastings algorithm to estimate their posterior distributions. The estimation and numerical solution of the model is done using Dynare software (Adjemian et al., 2022).

2.4.1 Data

The estimation uses data from the US. Observed variables are output Y_t, the unemployment rate u_t, the vacancy rate v_t, the separation rate ρ_t and the mean real wage w_t. The output and the wage data are from the National Income and Product Accounts by the US Bureau of Economic Analysis. The labor market series are from The Job Openings and Labor Turnover Survey (JOLTS) by the US Bureau of Labor Statistics. All the data is retrieved from FRED, the Federal Reserve Bank of St. Louis.

The data set is in quarterly frequency and covers a sample period of 2001-Q1–2019-Q3. The output and the wage series are converted to per-capita terms by scaling with the labor force. Logarithms of all the data series are passed through the Hodrick-Prescott-filter with a smoothing parameter 1600.

2.4.2 Calibration and Prior Distributions

First, the the matching function $M(u_t, v_t)$ is specified to be in the Cobb-Douglass form:

$$M(u_t, v_t) = \zeta_t u_t^{\mu} v_t^{1-\mu}, \tag{24}$$

in which ζ_t is a matching efficiency, μ a matching elasticity with respect to unemployment. The matching efficiency follows an $AR(1)$-process

$$\zeta_t = \rho^{\zeta}\zeta_{t-1} + (1-\rho^{\zeta})\bar{\zeta} + \epsilon_t^{\zeta}, \tag{25}$$

in which in which $\rho^{\zeta} \in (0,1)$ is the persistence, $\bar{\zeta}$ the steady state value of matching efficiency and $\epsilon_t^{\zeta} \sim \mathcal{N}(0, \sigma^{\zeta})$ an independently and identically distributed shock with standard deviation σ^{ζ}. The matching shock is one of the shocks in the Bayesian estimation.

Table 2.1 summarizes the calibration. An annual risk free interest rate over the sample period is 1.6 % implying the steady state discount factor $\bar{\beta} = 0.996$. I set the relative risk aversion parameter $\sigma = 2$ following Krause and Lubik (2007). The idiosyncratic productivity is chosen to follow a log-normal distribution. The parameters of the distribution are chosen such that the mean μ_F is normalized to zero and the standard deviation σ_F is set to match the standard deviation of the separation rate over the sample period which is 0.17. I set the total matching efficiency in steady state $\zeta = 0.7$. This is approximately the same value which the calibration used by Krause and Lubik (2007) implies.

Table 2.1: Calibrated parameters

Parameter	Description	Value	Source
$\bar{A}$	Steady-state TFP	1	Standard in the literature
$\bar{\beta}$	Discount factor	0.996	During sample period the annual risk-free interest rate is 1.6 %
σ	Relative risk aversion	2	Krause and Lubik (2007)
μ_F	Mean of idiosyncratic productivity distribution	0	Normalized s.t $\mu_F = E[ln(a)] = 0$
σ_F	STD of idiosyncratic productivity distribution	0.17	Chosen to match volatility of separations (Krause and Lubik (2007)) . Standard deviation 0.17 over the sample period
$\bar{\zeta}$	Matching efficiency	0.7	From Krause and Lubik (2007)

The prior and posterior values for the parameters are summarized in Table 2.2. A Beta distribution prior is assigned to those parameters that can get values on the unit interval. A Gamma distribution is used for a positive real valued parameter ρ^X.

Table 2.2: Results from Metropolis-Hastings (parameters)

		Prior			Posterior			
	Parameter	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
γ	Wage rigidity	beta	0.200	0.1000	0.283	0.0445	0.2126	0.3480
η	Bargaining power	beta	0.300	0.1000	0.172	0.0236	0.1334	0.2105
μ	Match elasticity	beta	0.500	0.1000	0.750	0.0449	0.6751	0.8231
ρ^X	Exog. separations	gamm	0.060	0.0200	0.108	0.0002	0.1074	0.1082

I set the prior mean for the real wage rigidity parameter $\gamma = 0.2$. This implies that 80% of the wages are renegotiated annually, since the model is in quarterly frequency. Wages are found to be rigid in previous studies, e.g. by Gertler et al. (2008), so I set the standard deviation to 0.1, allowing relatively narrow prior in low values of γ.

In the search and matching literature the firm and the worker are often attributed symmetric bargaining power 0.5 like in Krause and Lubik (2007). However, the estimation mostly results in lower values e.g. Petrosky-Nadeau et al. (2018) set the parameter to 0.04. Hence, to target values between 0 and 0.5, I choose the prior mean for the bargaining power $\eta = 0.3$, with a standard deviation of 0.1.

The prior mean for the match elasticity μ is set to 0.5 with a standard deviation of 0.1. This prior is in line with the survey findings by Petrongolo and Pissarides (2001), who found the elasticity to be between 0.4 and 0.7.

The prior for the endogenous separation parameter is chosen to follow Den Haan et al. (2000). They define that the endogenous separation leads to the destruction of the job. Exogenous separation is considered as voluntary quits, or the mutually agreed terminations of work contracts. These jobs are not destroyed and the firm opens a new vacancy to replace the separated worker. Using this definition, the mean of exogenous separation is 6% during the sample period. Hence, the prior mean is set as 0.06 with a larger standard deviation of 0.02.

The priors and posteriors for the structural shocks are summarized in Table 2.3. The persistence parameters are set to follow a Beta distribution, due to their range between zero and one. The standard deviations of the shocks follow inverse Gamma distribution, which is the standard practice in the Bayesian estimation.

Table 2.3: Results from Metropolis-Hastings (structural shocks)

		Prior			Posterior			
		Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ρ^A	TFP persistence	beta	0.800	0.1000	0.768	0.0695	0.6553	0.8848
σ^A	TFP std	invg	0.004	0.0010	0.005	0.0004	0.0041	0.0053
ρ^β	β persistence	beta	0.800	0.1000	0.722	0.0454	0.6457	0.7947
σ^β	β std	invg	0.200	Inf	0.035	0.0041	0.0279	0.0410
ρ^ρ	ρ^X persistence	beta	0.500	0.1000	0.428	0.0753	0.3042	0.5513
σ^ρ	ρ^X std	invg	0.002	Inf	0.025	0.0021	0.0214	0.0283
ρ^η	η persistence	beta	0.500	0.1000	0.553	0.0730	0.4312	0.6715
σ^η	η std	invg	0.010	Inf	0.073	0.0092	0.0580	0.0873
ρ^ζ	ζ persistence	beta	0.500	0.1000	0.609	0.0703	0.4949	0.7267
σ^ζ	ζ std	invg	0.040	Inf	0.053	0.0047	0.0457	0.0610

I set the persistence of both the productivity shock and the discount factor shock to 0.8, with a standard deviation of 0.1 to point towards a strong autocorrelation of the series (see Table 2.5). The separation, the bargaining power and the matching efficiency shocks are closely related to the separation, the wage and the vacancy data series, respectively. These series exhibit moderate autocorrelation. Hence, I choose weakly informative priors for the persistence of these shocks with a mean 0.5 and a standard deviation 0.1.

The priors of the volatilities of the shocks are set to a relatively small mean and infinite standard deviation. The priors are based on the maximum likelihood estimation of the shocks. The exception from the others is the separation shock, which has the standard deviation prior set to 0.0002. Since the exogenous part of the separation cannot exceed the total separation rate, too large values are prevented by a small standard deviation.

Table 2.4: Parameter values implied by the prior and the posterior mean of estimated parameters in Table (2.2)

	Parameter	With prior mean	With posterior mean
b	Unemployment benefit	0.36	0.59
c	Vacancy cost	0.18	0.028

Two parameters of the model, the unemployment benefit b and the vacancy posting cost c are solved according to the steady-state parameter values. The values of b and c which are implied by the prior mean and the posterior mean values are presented in Table 2.4.

There is an ongoing debate in the literature about the calibration of the parameter b, see e.g. Petrosky-Nadeau et al. (2018) for a discussion. On the one hand, b can be considered as an unemployment insurance benefit only. Using this definition Shimer (2005) pins down a value of 0.4. On the other hand, b may encompass not only the unemployment insurance, but also home production, self-employment and utility of leisure. Hagedorn and Manovskii (2008) end up in a value as high as 0.955. The priors that I have chosen for other parameters imply $b = 0.36$ which is acceptable since it is close to Shimer (2005).

The vacancy cost c can also take a large range of values. In the literature, the vacancy cost consists of at the least labor costs of recruiting, but it can also include advertisement costs, agency fees, travel costs and other indirectly related costs. Michaillat (2012) estimates the labor cost as 0.098 of a worker's wage and the flow cost of recruiting as high as 0.54 of worker's wage. Petrosky-Nadeau et al. (2018) calibrate the total vacancy cost as 2.6% of quarterly wages. Using these definitions, the values for parameter c here would vary between 0.025 and 0.51. Based on this, the value of 0.18, implied by the prior means, is appropriate.

2.4.3 Posterior distributions

Posterior means, standard deviations and the 5th and 95th percentiles of the estimated parameters are reported in Table 2.2 and the shocks in Table 2.3. The posterior distributions have shifted enough away from the priors, implying a good identification.

The posterior mean for the wage rigidity parameter γ is estimated as 0.28 which is slightly higher than the prior mean. The value implies that the wages are renegotiated approximately once per year. This is in line with the survey data evidence in Gottschalk (2005). The posterior is relatively tight with 95% confidence interval being from 0.22 to 0.31, which implies a good identification.

The worker's bargaining power η is estimated to be low. The posterior mean is 0.17 with a standard deviation of 0.024. This means that the firm is able to keep most of the surplus to itself. The value suggests that in calibrations in the literature very commonly used value 0.5 may be overweighting the worker's bargaining power.

The elasticity of matching with respect to the unemployment, i.e. parameter μ estimates slightly higher than the prior. The posterior mean is 0.75 which is approximately on the upper end of the findings by Petrongolo and Pissarides (2001). The posterior is relatively tight with a standard deviation of 0.031 and 95% confidence interval from 0.62 to 0.72

The posterior estimate for the exogenous separation rate parameter $\rho^X = 0.108$ has moved away considerably from the prior. The posterior distribution is also highly concentrated with a standard error 0.0002 and 95% confidence interval from 0.1077 to 0.1082. The concentrated posterior suggest that the data are informative. The posterior estimates mean that around 98% of the separations in the steady state are due to the exogenous part of the total separations.

The posterior estimate of the persistence of discount factor shock is 0.722. The value is moderately less persistent than the prior mean of 0.8. The persistence of the discount factor shock has a concentrated posterior with a confidence interval between 0.65 and 0.79. The standard deviation of discount factor shock σ^β is 0.035.

The persistence of productivity shock estimated as 0.768. It is also moderately less persistent than the prior 0.8. The persistence of productivity shock has a wide posterior with the 95% confidence interval covering the values from 0.66 to 0.88. The standard deviations of the shock is 0.005. The estimated values suggest that the innovation of the discount factor shock is almost four times bigger than the productivity shock innovation.

The posterior mean of the persistence of the separation shock ρ^{ρ} is 0.43 and the standard deviation $\sigma^{\rho} = 0.002$. The latter value is small, since the estimate of the exogenous separation rate ρ^{X} is close to the steady state value of total separation rate, and hence there is relative little room for the innovation to increase the exogenous share of separations. In addition, the estimated posterior distribution of σ^{ρ} is very concentrated.

The parameters of the bargaining power shock are $\rho^{\eta} = 0.553$ and $\sigma^{\eta} = 0.078$ and the matching efficiency shock $\rho^{\zeta} = 0.609$ and $\sigma^{\zeta} = 0.053$. All of these estimates are close to their prior means with concentrated posterior distributions. Both of these shocks show less persistence than the productivity shock but larger standard deviations.

The parameter values for the unemployment benefit b and the vacancy posting cost c which are implied by the posterior means are reported in Table 2.4. Both values change from the prior, but are within the plausible value ranges from the literature.

2.4.4 Model Fit

Table 2.5 presents the mean, the standard deviation and the first-order autocorrelation of the observed variables from the data and the theoretical moments from the model. The theoretical moments are computed by simulating the model with the estimated posterior mean calibration.

Table 2.5: Mean, standard deviation and autocorrelation from the data and the model

	Data			Model		
Variable	Mean.	σ	Autocorr	Mean	σ	Autocorr
$\log Y$ (Output)		0.010	0.861		0.010	0.784
$\log w$ (Wage)		0.015	0.780		0.016	0.678
ρ (Separation rate)	0.111	0.056	0.616	0.110	0.048	0.597
u (Unemployment rate)	0.060	0.115	0.922	0.060	0.088	0.816
v (Vacancy rate)	0.094	0.108	0.876	0.327	0.093	0.614

The model steady-state with prior means is targeted to match separation and unemployment rates in the data. The mean of vacancy rate from the model is quite considerably off when compared with the data. This is not surprising, since in the model, the total vacancy posting cost is determined by the aggregate resource constraint as the output minus consumption (see equation (23)). The posterior mean steady state ratio of the vacancy cost to output $\frac{c}{\bar{Y}} = 0.0279$ and the total vacancy costs are almost 10% of output. This is much higher than the most common value in the literature 1% of GDP, see e.g. in Andolfatto (1996). Also the ratio of consumption to output $\frac{\bar{C}}{\bar{Y}} = 0.902$ is very high. In the data the average over the sample period is 0.675. The consumption-output ratio is larger than in the data partly because the model does not feature public consumption and partly because the investments are done to the vacancy posting. The latter also explains the high level of vacancy costs.[8]

The model captures the volatilities reasonably well. The standard deviations of output and wage are almost identical between the data and the model. The volatilities of the separation rate, the unemployment rate and the vacancy rate are slightly lower than in the data.

Furthermore, the model exhibits the negative correlation between vacancies and unemployment, i.e. the Beveridge curve. This correlation is -0.78 in the simulated model in comparison to a value of -0.91 in the data used in the estimation. This is an interesting difference with Krause and Lubik (2007) in which the model does not succeed in generating this correlation. The same applies in the correlation between job destruction and creation which is positive in the model in Krause and Lubik (2007) unlike in the data. Here, the simulated correlation between vacancies and separation is -0.45 while the same correlation in the data is -0.57.

In order to investigate the ability of discount factor shocks in capturing changes in uncertainty, the estimated discount factor shock innovations are compared with the so-called JLN-index, that is an uncertainty measure from Jurado et al. (2015).[9] Over the whole sample period, the correlation between the discount factor shock series and the JLN uncertainty index is 0.42. This relatively low correlation is mainly due to higher volatility in the shock innovations during the expansion periods. In comparison, in a subsample around the Great Recession, i.e. 2006–2011, the correlation is 0.78 suggesting that discount factor shocks indeed capture increase in uncertainty during big economic

[8]Consumption data: U.S. Bureau of Economic Analysis, Personal Consumption Expenditures and Gross Domestic Product, retrieved from FRED, Federal Reserve Bank of St. Louis

[9]Index value of JLN 3-Month Ahead Macroeconomic Uncertainty by Jurado et al. (2015) from FRED, Federal Reserve Bank of St. Louis, series JLNUM3M.

downturns.

2.4.5 Impulse Responses

In order to study mechanisms behind the labor market outcomes in the Great Recession, Figure 2.2 presents the impulse response functions to a negative discount factor shock. The shock impacts the household's subjective discount factor causing the household to become more impatient. As a result, the stochastic discount factor decreases, consumption jumps and savings drop. The impulse response of the return on bonds increases due to a higher return rate required by the household.

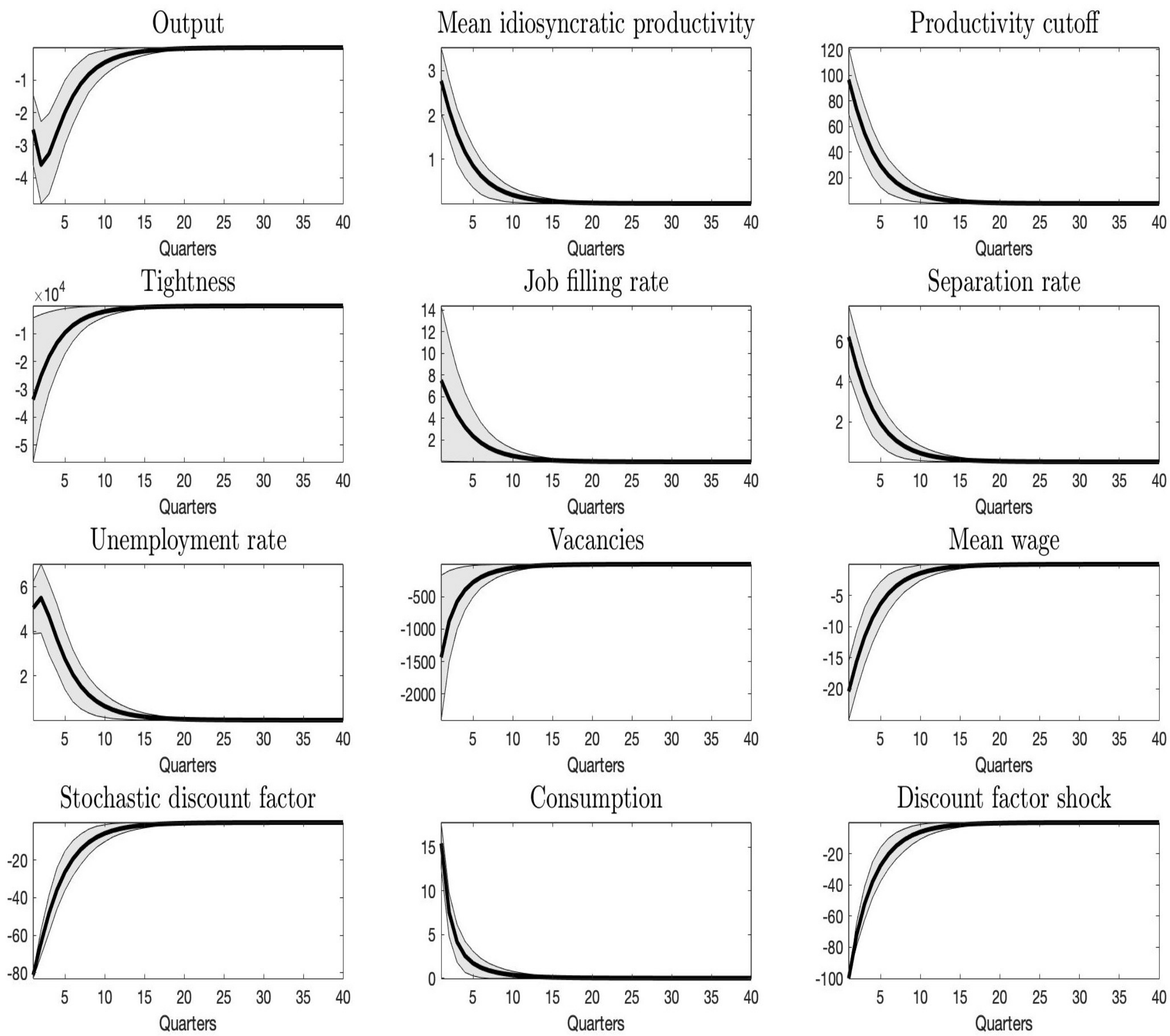


Figure 2.2: Impulse response functions to a negative one-standard-deviation discount factor shock. The gray area is defined by the 5th and 95th percentile Bayesian confidence band. The variable responses are normalized by the shock size.

The drop in savings affects the firms by cutting the funds available for investing in job vacancies. The firm creates fewer jobs resulting in a negative response of vacancies. In addition, the investment and vacancy drop drives the output to recession regardless of the positive response of consumption. The vacancy drop has a negative impact on the labor market tightness. The job filling rate increases since there are less vacancies open to be filled. The drop in the labor market tightness and the increase in the job filling rate makes the filled job less valuable to the firm. As a consequence of the decreased value of a filled job, the firm has higher incentive to cut low profitable jobs and it chooses a higher productivity cutoff. This leads to an increase in the mean idiosyncratic productivity and in the separation rate. Due to the higher amount of separation and the lower amount of new vacancies, the unemployment increases and employment decreases.

From the theoretical perspective, the most surprising result is the impact of the discount factor shock to wages. However, it is consistent with the data. Even if the idiosyncratic productivity and hence the surplus of each job increases, the mean wage still decreases. The wage rigidity and especially the workers' bargaining power are estimated to be considerably low. Hence, the firm can obtain the most of the increased job specific surplus.

The negative effect of the drop in the value of a filled job dominates the positive effect of the increase in the job specific surplus which the worker obtains. As a result, the total impact of the negative discount factor shock is negative to the mean wage. The negative impact on the wage bill is further enhanced by the drop in employment.

Figure 2.3 presents the Bayesian impulse response functions of a negative productivity shock, which is more standard in macroeconomic models. A negative productivity shock directly decreases total output and the individual output of all the existing jobs within a firm. This has two direct consequences. The first, workers obtain a share of additional output based on the wage bargaining outcome. As a result, the mean wage decreases. Second, the value of each filled job decreases, hence the higher idiosyncratic productivity is needed for profitability and the firm chooses a higher productivity threshold leading to an increase in the mean idiosyncratic productivity. In addition, the increase in the idiosyncratic productivity threshold leads to a positive response in the separation rate.

The household has less income due to the wage decrease, but also from the profits of the firm. The decrease in income leads to a decrease in consumption. The consumption decrease has negative impact on the stochastic discount factor implying consumption smoothing. Hence, savings decreases and the rate of return on corporate bonds increases.

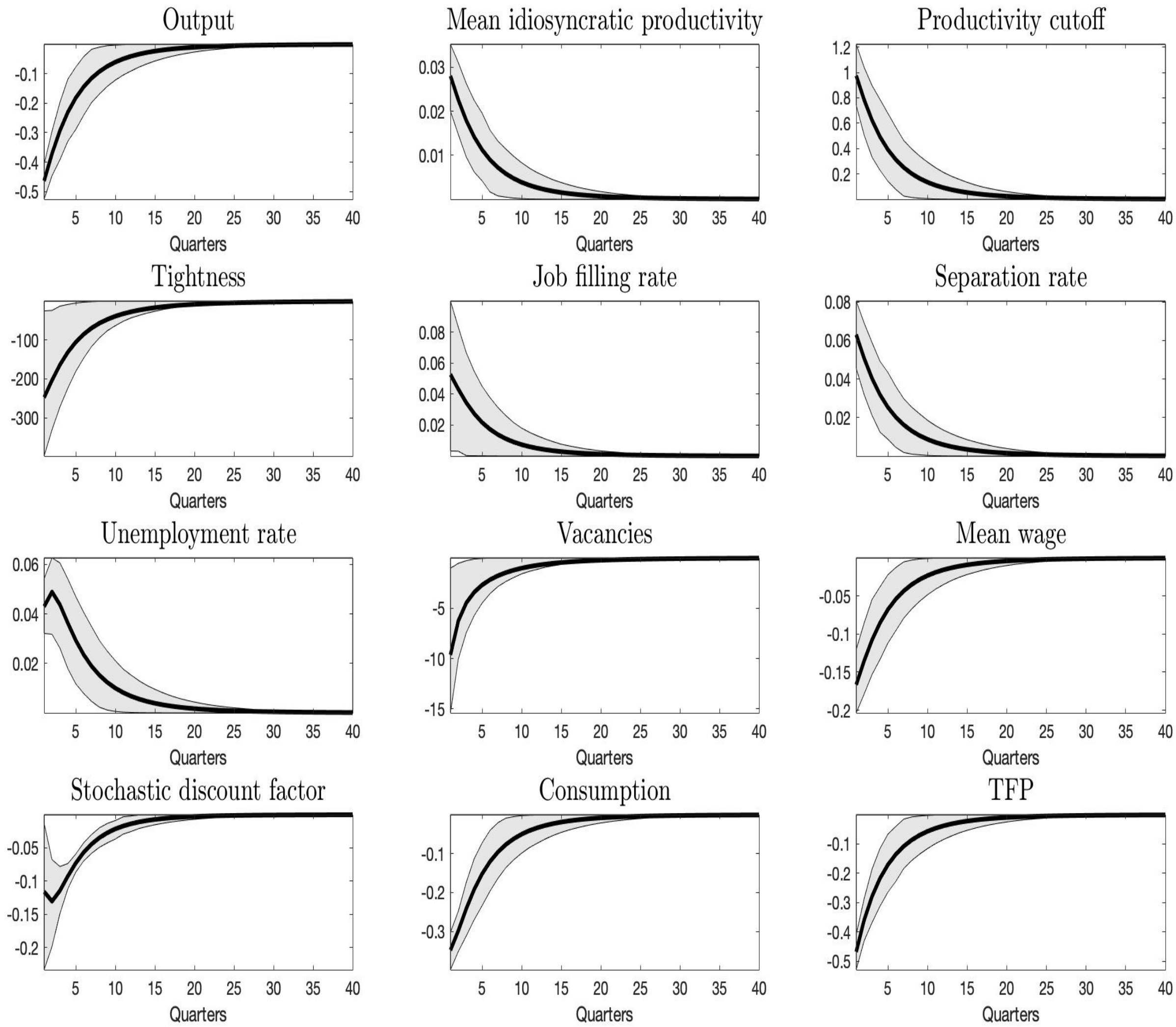


Figure 2.3: Impulse response functions to a negative one-standard-deviation productivity shock. The gray area is defined by the 5th and 95th percentile Bayesian confidence band. The variable responses are normalized by the shock size.

The decrease in savings means that the firm has less funds to invest in job creation. As a result, the vacancy posting decreases. Simultaneously, the positive response of separation rate raises the inflow of workers into unemployment. As a result unemployment increases and employment decreases. The labor market tightness, defined as vacancies over unemployed decreases due to the fall in vacancy posting and the raise in unemployment. The easier labor market results in a positive response in the job filling rate. Finally, the wage bill response is negative because of both the lower mean wage and employment.

2.4.6 The effect of endogenous job destruction

This section discusses about endogenous job destruction in more detail. The job destruction condition (17) was derived in the model section. The firm decides to destroy jobs which are less productive than the cutoff $\tilde{a}_t$. The productivity cutoff is positively related to wages and negatively to the value of a filled job. The first one, since a higher wage requires higher productivity to remain profitable. The latter one, because if filled jobs become increasingly valuable, the firm hoards labor by preserving less and less productive jobs. However, on a detailed level, the effect is more ambiguous. The wages are determined by bargaining over a joint surplus which itself contains the value of a job match. Thereby, an increase in match value, increases wages.

The filled job has a value because it is costly to post vacancies. If the labor market is tight, more open vacancies are required to match with a new worker. Hence, an increasing labor market tightness results in higher match values and further to higher wages. In order to examine how job destruction depends on the labor market tightness, the functional form of cutoff $\tilde{a}_t$ is derived by inserting the wage equations (20) and (21) into the job destruction condition (17). This yields a threshold equation as (see Appendix 2.A.2 for details)

$$\begin{aligned}\tilde{a}_t = \frac{1}{A_t}\Bigg[&\frac{\eta_t}{1-\gamma\eta_t}\theta_t c + \frac{1-\eta_t}{1-\gamma\eta_t} b + \frac{1-\gamma}{1-\gamma\eta_t} w_{t-1}(\tilde{a}_{t-1}) \\ &- \frac{1}{1-\gamma\eta_t}\frac{c}{q(\theta_t)}\Bigg].\end{aligned} \tag{26}$$

Here, the labor market tightness θ_t appears in the first and the last term on the right-hand side of equation. The first term is positive and increasing in θ_t, while the second one is negative and decreasing. The first conclusion is clear. Since, by definition, η_t and $q(\theta_t)$ vary between 0 and 1, the negative term is larger than the positive one when $\theta_t \leq 1$. However, the labor market tightness is not restricted to below one values. The positive term is linear in θ_t while the negative term is convex. With sufficiently large values of θ_t, its impact on the job destruction turns from negative to positive. In this case the value of a match dictates wages. Then an increase in labor market tightness causes an increase in wages which outweighs the decrease in the match value.

In order to quantify the level of labor market tightness which starts to have positive impact on the job destruction threshold, the posterior mean calibration is used in equation (26). The comparative statics shows that θ_t starts to have positive impact on the threshold $\tilde{a}_t$ when the value is approximately 1500 or larger. This is an unrealistically high number for labor market tightness. Hence, it is safe to say that in the model of this paper, the job destruction decreases when the labor market tightness and as a result, the value of a filled job increases.

The labor market tightness is the channel through which the discount factor shocks affect job destruction. The discount factor shock directly affects vacancy postings through the job creation condition (16). Since the labor market tightness θ_t is defined as vacancies over unemployed, a decrease in vacancies decreases labor market tightness. As presented above, the decrease in tightness increases endogenous separation.

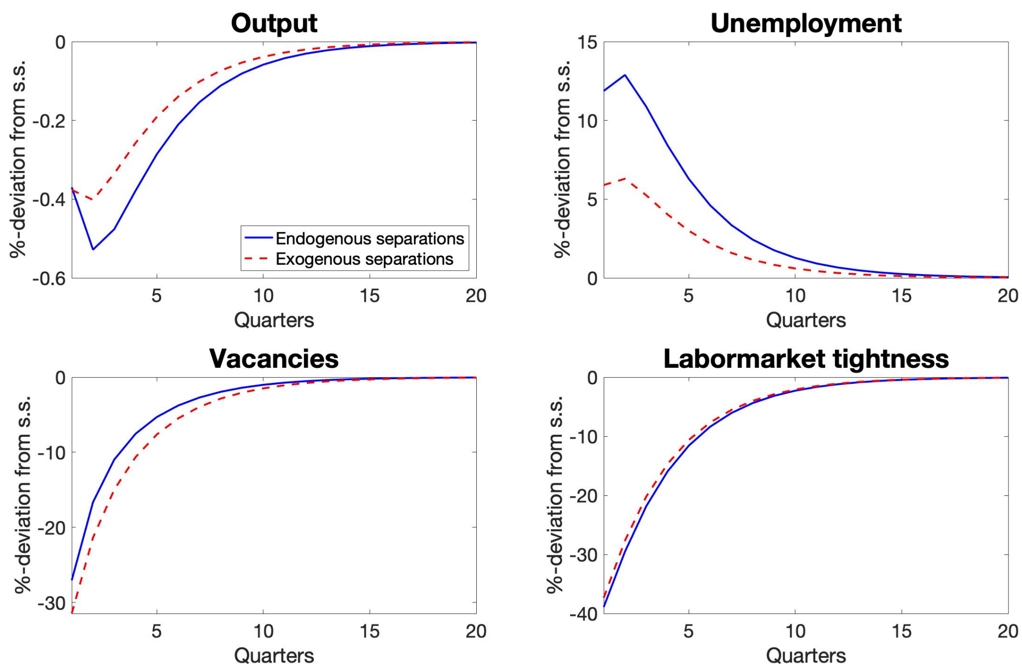


Figure 2.4: The impulse response functions of output, unemployment, vacancies and labor market tightness to a one standard deviation size negative discount factor shock. The solid line is a response from the full estimated model with endogenous separation. The dashed line is build from the full model by fixing the separation rate to its steady state value.

The endogenous and exogenous job separation are compared by equating the steady-state separation rates as presented in Section 2.3.2, i.e. by setting $\rho^e = F(\tilde{a_{ss}})$. In the model with exogenous separation the firm is unable to affect the idiosyncratic productivity distribution. The impulse responses of the key variables are presented in Figure 2.4. The negative discount factor shock decreases vacancy postings on impact. This increases unemployment and decreases output and labor market tightness. When job separation is endogenous, the firm also cuts low productive jobs, by choosing higher idiosyncratic productivity cutoff. As a result, all the remaining jobs are more productive. This increase in the mean idiosyncratic productivity cannot happen when the separation is exogenous. Hence, the drop in output on the impact of the shock is the same in both models, but it is accompanied by a twice as large increase in unemployment when job destruction is endogenous.

The drop in the vacancy postings is smaller in the model with endogenous separation. The difference in the drop is approximately in the same proportion as the increase in unemployment. As a result, the decrease in labor market tightness is almost identical in both models. However, the increase in job destruction in the endogenous separation model have a consequence on the recovery from the shock. Because of the larger unemployment increase, more jobs need to be created in order to restore the steady state. Since a part of output is used to cover the vacancy costs, the output recovery is slower in the model with endogenous job destruction.

2.4.7 Variance decomposition

Table 2.6 presents shock variance decomposition. The discount factor shock ϵ^{β} stands out considerably when it comes to labor market variables. It explains 85% of the variance in vacancies, 93% in the job filling rate, 60% in the productivity cutoff, 57% in the wage bill and 44% in unemployment. In addition the discount factor shock is almost the sole driver of fluctuations in the stochastic discount factor and hence the bonds rate.

The discount factor shock affects directly on the vacancy posting through the job creation condition (16). The shock alters the expected gain from a new job. The change in vacancy posting has direct impact on the labor market tightness, which changes the job filling rate which is a function of the tightness. Hence, the large share of variance comes from the discount factor shock.

The value of a filled job depends on the labor market tightness, which affects the surplus of each job and hence impact wages. This shows in the variance decomposition of both the mean wage and the wage bill. The impact of the discount factor shock to the productivity cutoff also comes through the value of a filled job. The discount factor is not directly present in the job destruction condition (17) and the derived productivity cutoff equation (26). The impact of discount factor shocks on the job destruction, i.e. on the separation rate is indirect. Even though the discount factor shock is the most important driver of the variance in productivity cutoff, it is still not dominant in the separation rate volatility. The separation shock ϵ^{ρ} and the discount factor shock explain approximately equal shares of the variance of separation rate.

Table 2.6: Variance Decomposition (in percent)

	ϵ^A TFP	ϵ^ρ Separation	ϵ^η Bargaining	ϵ^ζ Matching	ϵ^β Discount factor
Y (Output)	69.29	3.05	0.17	21.10	6.41
H (Mean id. prod.)	5.86	0.31	12.89	20.89	60.06
$\bar{a}$ (Prod. cutoff)	5.86	0.31	12.89	20.89	60.06
θ (Tightness)	3.60	0.37	0.67	22.05	73.30
q (Job filling rate)	4.58	0.48	0.86	0.79	93.29
ρ (Separation)	3.69	37.13	8.13	13.17	37.87
u (Unemployment)	3.22	9.67	2.96	39.77	44.38
n (Employment)	3.22	9.67	2.96	39.77	44.38
v (Vacancies)	3.06	3.44	0.12	8.38	85.00
W (Wage bill)	3.83	1.62	13.06	24.20	57.30
w (Mean wage)	3.46	0.27	26.49	16.47	53.31
m (Discount factor)	0.38	0.31	0.06	0.61	98.64
R (Bonds rate)	0.38	0.31	0.06	0.61	98.64
C(Consumption)	39.67	11.11	0.47	7.57	41.18

Output volatility is dominated by the total productivity shock ϵ^A. The discount factor shock has a very small impact. On the other hand, the variance in consumption is explained in approximately equal shares by the productivity shock and the discount factor shock. The output which is not invested in vacancy posting is consumed by the aggregate resource constraint (23) which explains the importance of the productivity shock in the volatility of consumption.

The analogous results about the importance of the discount factor shock can be found in Albertini and Poirier (2014) and Leduc and Liu (2020). Like here, in both of these papers the productivity shock accounts for most of the volatility in output while the discount factor shock explains the volatility of labor market variables. For instance, Leduc and Liu (2020) find that the discount factor shock accounts for about 67% and 59% of variance in unemployment and vacancies respectively.

The other important source of volatility in the labor market variables is the matching efficiency shock ϵ^ζ. It explains almost 40% of the variance in unemployment, 24% in wage bill and 21% in output. The matching efficiency shock affects both job creation and destruction. If the matching efficiency increases, hiring new workers becomes cheaper for the firm and it increases vacancy posting. On the other hand, the firm destroys an increasing number of low productive jobs, because their value decreases when the job filling rate increases. Hence, the job destruction increases. Since, the matching efficiency affects both the creation and destruction of jobs, it is not surprising that a large share of the volatility in unemployment is explained by the matching efficiency shock.

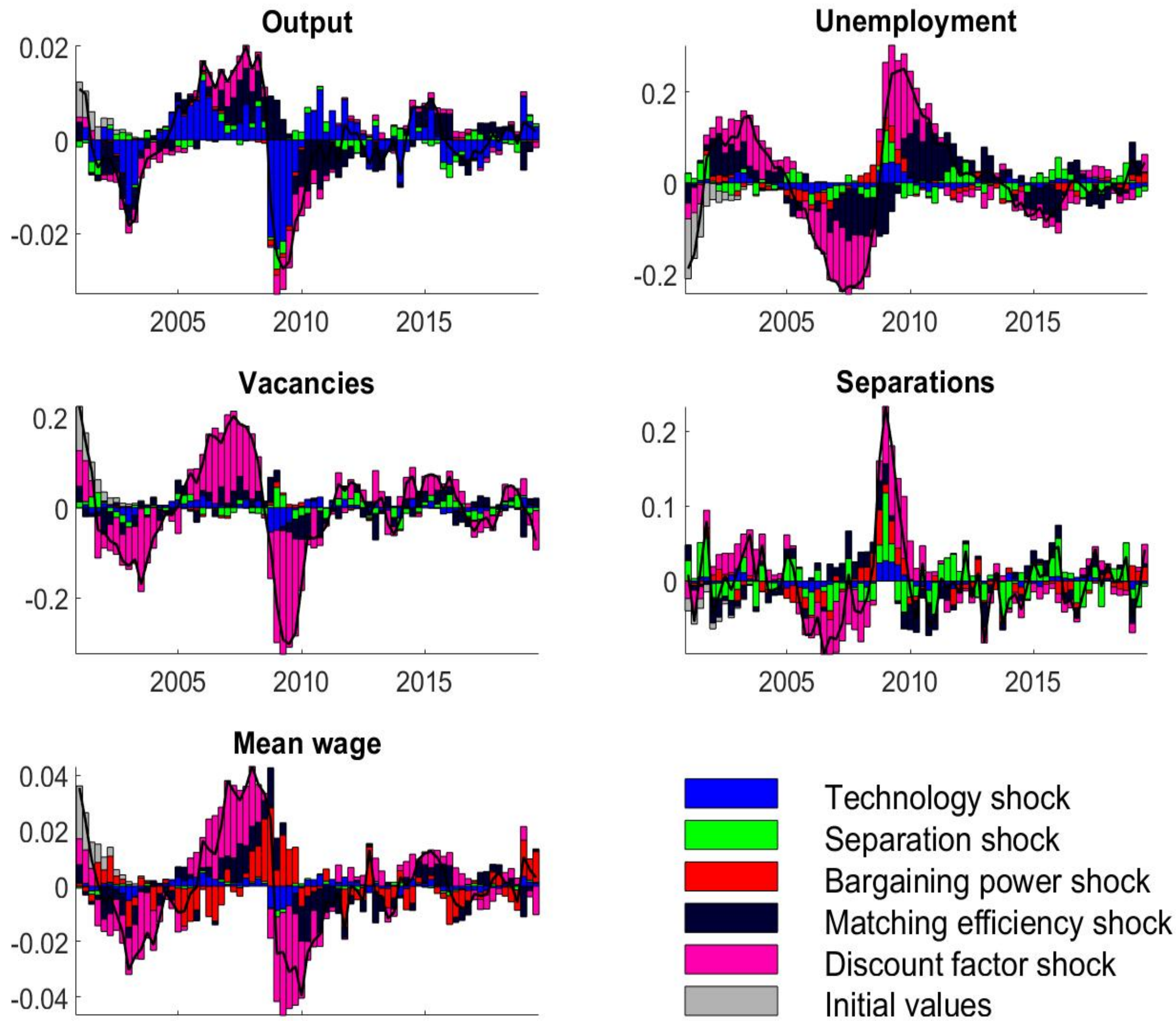


Figure 2.5: Historical shock decompositions of output, the unemployment rate, vacancies, the separation rate and the mean wage.

A similar importance of the matching efficiency shock can be found in the literature e.g. by Lubik (2009) or Sedláček (2014). Using a maximum likelihood estimation and a long dataset covering 1948–2011, Sedláček (2014) finds that the matching efficiency explains 23% of the increase in unemployment during the recessions in the US.

Lubik (2009) estimates a search and matching labor market model with an exogenous separation rate, flexible wages and firms which produce differentiated goods. He finds that 92% of the variation in unemployment and 38% in vacancies are due to the matching efficiency shock. Neither of two studies has a discount factor shock and wage rigidity similar than in this paper. Hence, the results are different in magnitudes, but qualitatively they are in line, by showing the importance of the matching efficiency shock, especially in explaining unemployment volatility. This effect is stronger during "normal times", as I will show in Section 2.4.8.

Figure 2.5 depicts the historical shock decompositions of output, the unemployment rate, the separation rate, vacancies, and the mean real wage. The time period is from 2001-Q1 to 2019-Q3.

The variance in output is mainly driven by the productivity shock through the whole sample period. A drop during the Great Recession of 2008–2009 is clearly visible and it is largely explained by the productivity shock.

However, the discount factor shock explains most of the variance in unemployment over the sample period. This also applies to the fluctuations during the Great Recession. The productivity shock explains a very small portion of the increase in unemployment. This finding closely coincides with the results by Albertini and Poirier (2014). The importance of the matching efficiency shock also stands out. Sedláček (2014) explains the changes in the matching efficiency as changes in the hiring standards of the firms. In Figure 2.5, the impact of the matching efficiency shock to unemployment seems to be stronger around the turning points of business cycles.

The historical decomposition of the vacancy rate clearly illustrates the mechanism whereby the discount factor shock impacts the labor markets, namely through the job creation. The graphical representation also illustrates the mechanism, that uncertainty affects firms' decision on job cuts less than on vacancy posting.

Contrary to the vacancies, the separation rate is explained by the combination of the shocks over the whole sample period. While the discount factor shock is important, it is not similarly dominant in separation than in vacancies. The separation shock seems to capture a large share of the variance in the separation rate from approximately 2010 onwards. During this period, the volatility of the separation rate is relatively small. However, the separation shock is not the main driver of the larger fluctuations especially between 2005 and 2010. The other shocks affect on separation through the endogenous channel. This points out the importance of the endogenous separation as a mechanism which explains larger fluctuations in job destruction. This further suggests that implementing endogenous separation in a search and matching model can improve the fit with the data that includes a large economic crisis with a considerable increase in job losses.

The separation shock is important in explaining fluctuations in the separation rate, but less significant for the volatility of other variables. Another similarly specific shock is the bargaining power shock ϵ^{η}, which explains 26,5% of the variance in the mean real wage. This is the second highest share after the discount factor shock. The effect on the other variables is limited at least partly due to the wage rigidity.

The historical decomposition of the mean real wage in Figure 2.5 depicts that the discount factor shock and the bargaining power shock explain a considerable share of the variance in wages. The decomposition graph shows some level of asymmetry in the large fluctuation around the Great Recession. The effect of the bargaining power shock is mostly positive and the drop in the mean wage during the Great Recession is explained mostly by the discount factor shock.

2.4.8 Sensitivity analysis, the role of the Great Recession

In order to study the effect that the Great Recession has on the estimation results, the estimation is redone using a shorter sample without the Great Recession. The recession peak and through dates are the fourth quarter of 2007 and the second quarter of 2009[10], but since the labor market variables show slower recovery than the output, the sample period of the observed variables is cut to begin from the first quarter of 2010. The downside is that the sample becomes relatively short, but this way the sample is clearly more stable with smaller volatility making it considerably different from the original sample. The shorter sample is estimated using the same priors and calibration than the original sample.

The posterior estimates of the parameter values move away from the priors implying that the new shorter sample is informative in identifying the parameters. The posterior mean values are still close to the ones estimated using the full sample. The estimated exogenous separation parameter mean is exactly the same in both estimations. The wage rigidity estimates are close, with 0.26 on the shorter sample and 0.28 on the full sample. Workers' bargaining power estimates higher with 0.23 with the short sample and 0.17 with the full sample. The match elasticity parameter estimate decreases to 0.69 with the short sample from 0.75 with the full sample. The standard deviations of all posterior estimates are very small, but slightly larger than with the full sample estimation.

The posterior estimates of the shock persistences differ more than the parameter values between the two samples. Unsurprisingly, all the shocks are estimated less persistent,

[10]Source NBER business cycle dating https://www.nber.org/research/business-cycle-dating

since the sample contains less large fluctuations. However, the standard deviations of the shocks remain almost unchanged with the only exception of the bargaining power shock with the standard deviation of 0.055 while it was 0.073 with the full sample estimation.

Table 2.7 summarizes the variance decompositions without the Great Recession in the sample. The notable difference is that the contribution of the discount factor shock decreases. The most significant is the decrease of the discount factor shock contribution to the job destruction related variables. For instance, the variance in the separation rate, the discount factor shock explains, is only 6,77% which is the second lowest share amongst the five shocks. In the sample without the Great Recession, the separation shock explains the largest share, over 59% of the variance in the separation rate.

Table 2.7: Variance Decomposition (in percent), without the Great Recession

	ϵ^A TFP	ϵ^ρ Separation	ϵ^η Bargaining	ϵ^ζ Matching	ϵ^β Discount factor
Y (Output)	56.53	4.37	0.14	34.98	3.98
H (Mean id. prod.)	2.61	0.19	18.11	62.52	16.57
$\bar{a}$ (Prod. cutoff)	2.61	0.19	18.11	62.52	16.57
θ (Tightness)	2.79	0.62	0.59	42.41	53.60
q (Job filling rate)	4.71	1.05	1.00	2.52	90.71
ρ (Separation)	1.07	59.20	7.40	25.56	6.77
u (Unemployment)	1.52	12.97	2.27	65.02	18.21
n (Employment)	1.52	12.97	2.27	65.02	18.21
v (Vacancies)	3.00	4.96	0.11	15.68	76.25
W (Wage bill)	2.74	2.17	14.45	43.31	37.33
w (Mean wage)	2.76	0.45	26.54	31.39	38.86
m (Discount factor)	0.55	0.38	0.04	0.89	98.14
R (Bonds rate)	0.55	0.38	0.04	0.89	98.14
C(Consumption)	33.92	15.90	0.40	13.49	36.28

On the contrary to job destruction, the volatility in job creation is still mostly explained by the discount factor shock. It contributes 76% to the variance in vacancies and over 90% to the job filling rate. These are only slightly smaller shares than with the full sample. This result suggests, that even outside of recessions, the uncertainty about the future returns of new jobs, captured by the discount factor shock, dictates job creation. While the indirect effect that discounting has on the job cuts diminishes outside the Great Recession, the direct impact on job creation still prevails.

The other interesting result is that the importance of the matching efficiency shock increases with the shorter sample. The matching efficiency shock explains the largest share of variance in unemployment, the productivity cutoff and the wage bill and the second largest share in output, the labor market tightness, the separation rate and the mean wage. The change in the decomposition of unemployment variance is interesting. With the full sample, the discount factor shock is the most important driver of fluctuations in unemployment explaining about 44% and matching efficiency shock the second with about 40% share. With the shorter sample the impact of the discount factor shock drops to about 18% while the matching efficiency shock increases to 65%.

The change in the shock decomposition of unemployment between the sample with and without the Great Recession is consistent with the arguments by Michaillat (2012). He shows that the search and matching frictions are explaining the unemployment during expansion periods, but not during recessions. During recessions, unemployment is the result of the shortage of jobs even absent the matching frictions. In my estimations, the time-varying matching frictions are captured by the matching efficiency shock, which explains a larger share of unemployment volatility outside the Great Recession estimations.

Figure 2.6 presents the historical shock decomposition graphs of output, unemployment rate, vacancies, separation rate and mean wage. All the graphs, with the exception of separation rate have some bigger volatility at the beginning of the sample period. This suggests that at the beginning of the sample, the economy is recovering fast from the Great Recession towards the sample trend.

The output volatility is largely explained by the technology shock, but also the matching efficiency shock stands out, especially at the beginning of the sample period. The separation shock explains the largest share of the volatility in unemployment, throughout the whole period. The discount factor shock is still clearly dominant in explaining the volatility in vacancies, but also has impact on the mean wage. The variance in wages is the combination of contributions by the discount factor, the matching efficiency and the bargaining power shocks as seen also from Table 2.7. However, during most of the sample, the matching efficiency and the discount factor shocks have opposite effect on the wages than the bargaining power shock. The separation rate is largely explained by the separation shock over the whole sample.

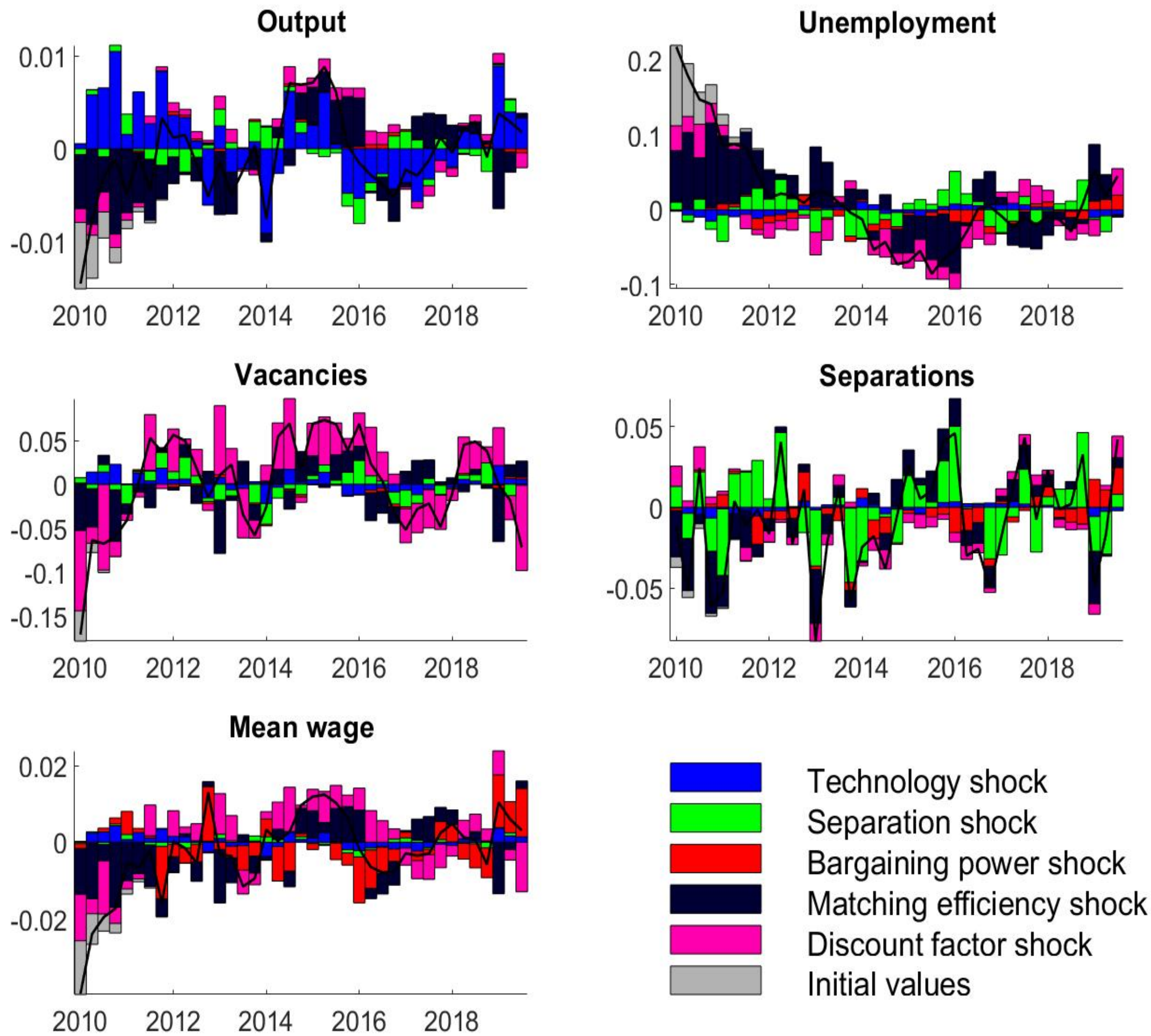


Figure 2.6: Historical shock decompositions of output, unemployment rate, vacancies, separation rate and mean wage.

2.5 Conclusion

The Bayesian estimation of a search and matching labor market model with endogenous separation show that shocks to the discount factor are the most significant drivers of fluctuations in labor market variables. The discount factor shock changes the expected profits from a new job and affects the incentives to open new vacancies. The impact of the shock is magnified, when the firm has an option to cut low productive jobs. The frictions in labor markets are important in explaining the importance of the discount factor shock.

If the labor is adjusted without a cost, the job creation is not an investment for the firm and hence it is not directly subject to discounting.

The discount factor shock changes household's intertemporal substitution. Since output is used to either in consumption or invested in job creation, the discount factor shock has small effect on output in the absence of price rigidities. Hence, one addition to the model here could be pricing. Price rigidities could potentially amplify the impact of discount factor shocks on output and further on labor markets.

The model in this paper does not include capital. This means that the firm is investing only in job creation, i.e. vacancy posting. Adding capital would create another channel for the discount factor shock to impact the labor markets. For instance, if the discount factor shock decreases capital investments and hence the capital stock, the productivity of each job decreases. The lower productivity makes more jobs unprofitable for the firm leading to larger job cuts. Also, the substitution between labor and capital would potentially impact on job creation and destruction. Thus, adding capital in the model could amplify the impact of the discount factor shock even further.

The addition of capital would also enable the implementation of financial frictions in the form of a collateral constraint. Then capital is a factor of production but also holds collateral value. In this setting, the impact of the discount factor shock can be propagated by a financial accelerator effect. The collateral constraint also enables the model to exhibit financial shocks. A comparison of financial shocks and discount factor shocks can explore whether the fluctuations during the Great Recession are attributed to a tightening of financial markets or to an increase in uncertainty. This question is especially interesting for job destruction, which in this paper is not directly subject to discounting. I leave these topics for future research.

Appendix

2.A Model derivation

2.A.1 Household's problem

The representative household optimizes intertemporal utility subject to budget constraint. The optimization problem is

$$\max_{C_t, S_{t+1}, B_{t+1}} E_t \sum_{j=0}^{\infty} \prod_{k=0}^{j} \beta_{t+k} \left[\frac{C_{t+j}^{1-\sigma} - 1}{1-\sigma} \right], \tag{A.1}$$

$$\text{subject to } \; C_t + P_t S_{t+1} + \frac{B_{t+1}}{R_t} = W_t + S_t(P_t + D_t) + B_t + bu_t - T_t,$$

where C_t is consumption, P_t is the price of the equity share of representative firm, S_t is the quantity of equity shares, D_t is the dividend the firm pays, B_t is the quantity of representative firm's bonds, R_t is the rate of return on bond, W_t is the wages earned by the employed household members, b is the unemployment benefit to the unemployed household members, u_t is unemployment, T_t is a lump sum tax, β_t is the household's subjective discount factor and σ is the household's risk aversion parameter.

Let us denote λ_t as the Lagrange multiplier of the budget constraint. FOCs with respect to consumption, equity shares and bonds are

$$\frac{\partial \mathcal{L}}{\partial C_t} : \lambda_t = \frac{1}{C_t^{\sigma}} \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial B_t} : \frac{1}{R_t} = E_t \beta_t \frac{\lambda_{t+1}}{\lambda_t} \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial S_{t+1}} : \lambda_t P_t = E_t \beta_t \lambda_{t+1}(P_{t+1} + D_{t+1}) \tag{A.4}$$

Taking equation (A.2) one period ahead yields

$$\lambda_{t+1} = \frac{1}{C_{t+1}^{\sigma}}, \tag{A.5}$$

which is inserted into (A.2) to derive the consumption Euler equation as

$$\frac{1}{R_t} = E_t \beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \tag{A.6}$$

and to (A.4) to derive the equity Euler equation as

$$1 = \beta_t E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_{t+1} + D_{t+1}}{P_t} \right] \tag{A.7}$$

Iterating (A.7) forward results in

$$P_t = E_t \sum_{j=1}^{\infty} \left(\frac{\beta_{t+j}^{j} C_{t+j}}{C_t} \right)^{-\sigma} D_{t+j} \tag{A.8}$$

which determines the stochastic discount factor as

$$m_{t,t+j} \equiv E_t \prod_{k=0}^{j} \beta_{t+k} \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma}, \tag{A.9}$$

and for the sake of brevity, let us define the one period ahead stochastic discounter as

$$m_{t+1} \equiv \beta_t E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}. \tag{A.10}$$

Let us define the gross return of equity on period $t+1$ as R^s_{t+1}. From the household's budget constraint we obtain that the periodical return per share is the dividend plus the change in price of the share. Hence, the gross return can be expressed relative to the price on period t as

$$\begin{gathered} (S_{t+1} P_t) R^s_{t+1} = S_{t+1}(P_{t+1} + D_{t+1}) \\ \Leftrightarrow \\ R^s_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}, \end{gathered} \tag{A.11}$$

which simplifies the equation (A.7) to

$$1 = E_t m_{t+1} R^s_{t+1}. \tag{A.12}$$

2.A.2 Firm's problem

The representative firm maximizes profits by choosing employment n_t, vacancies v_t, a productivity cut-off $\tilde{a}_t$ and borrowing B_{t+1}. The constraint is the employment evolution. Let us denote ξ_t as the Lagrange multiplier associated with the constraint.

The Lagrange equation for the firms problem is

$$\begin{aligned} \mathcal{L} = E_t \sum_{s=0}^{\infty} m_{t+s} \Big[& A_t n_t H(\tilde{a}_t) - W_t - c v_t + B_{t+1} - R_t B_t \\ & + \xi_t [(1-\rho_t)(n_{t-1} + v_{t-1} q(\theta_{t-1})) - n_t] \Big], \end{aligned} \tag{A.13}$$

where A_t is the TFP, $H(\tilde{a}_t)$ is an aggregate idiosyncratic productivity, c is a vacancy cost, ρ_t is a separation rate, θ_t is a labor market tightness and $q(\theta_t)$ is a vacancy filling rate. The stochastic discount factor m_t is derived in the household problem.

The FOCs w.r.t. employment, vacancies, a productivity cutoff and borrowing are

$$\frac{\partial \mathcal{L}}{\partial n_t} : \xi_t = E_t \xi_{t+1} m_{t+1}(1-\rho_{t+1}) - \frac{\partial W_t}{\partial n_t} + A_t H(\tilde{a}_t) \tag{A.14}$$

$$\frac{\partial \mathcal{L}}{\partial v_t} : \frac{c}{q(\theta_t)} = E_t \xi_{t+1} m_{t+1}(1-\rho_{t+1}) \tag{A.15}$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{a}_t} : \xi_t \frac{\partial \rho_t}{\partial \tilde{a}_t}(n_{t-1} + v_{t-1} q(\theta_{t-1})) = A_t n_t \frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} - \frac{\partial W_t}{\partial \tilde{a}_t} \tag{A.16}$$

$$\frac{\partial \mathcal{L}}{\partial B_t} : \frac{1}{R_t} = E_t m_{t+1} \tag{A.17}$$

Deriving the job creation condition

First, let us insert the expected future value of a job solved in (A.15) into the first order condition of vacancies (A.14), which yields

$$\xi_t = A_t H(\tilde{a}_t) - \frac{\partial W_t}{\partial n_t} + \frac{c}{q(\theta_t)}. \tag{A.18}$$

Taking this equation (A.18) one period ahead gives the expectation of the shadow value of a vacancy as

$$E_t \xi_{t+1} = E_t \left(A_{t+1} H(\tilde{a_{t+1}}) - \frac{\partial W_{t+1}}{\partial n_{t+1}} + \frac{c}{q(\theta_{t+1})} \right), \tag{A.19}$$

which can then be inserted back to the equation (A.15) resulting in the job creation condition as

$$\frac{c}{q(\theta_t)} = E_t m_{t+1}(1-\rho_{t+1}) \left[A_{t+1} H(\tilde{a_{t+1}}) - \frac{\partial W_{t+1}}{\partial n_{t+1}} + \frac{c}{q(\theta_{t+1})} \right]. \tag{A.20}$$

The wage bill W_t is an aggregate of individual wages given by

$$W_t = n_t \int_{\tilde{a}_t}^{\infty} w(a) \frac{f(a)}{1-F(\tilde{a}_t)} da, \tag{A.21}$$

and hence the partial derivative of the wage bill with respect to employment is

$$\frac{\partial W_t}{\partial n_t} = \int_{\tilde{a}_t}^{\infty} w(a) \frac{f(a)}{1-F(\tilde{a}_t)} da \qquad \left(= \frac{W_t}{n_t} \right). \tag{A.22}$$

The exact form of the job creation condition is derived by inserting the partial derivative of the wage bill (A.22) into the job creation condition (A.20), resulting in

$$\frac{c}{q(\theta_t)} = E_t m_{t+1}(1-\rho_{t+1})\left[A_{t+1}H(\tilde{a_{t+1}}) - \int_{\tilde{a}_t}^{\infty} w(a)\frac{f(a)}{1-F(\tilde{a}_t)}da + \frac{c}{q(\theta_{t+1})}\right] \tag{A.23}$$

Deriving the job destruction condition

The derivation of job destruction conditions is started by combining the Lagrange multiplier ξ_t, from the equation (A.18), with the FOC of the idiosyncratic productivity cutoff, i.e. equation (A.16). This results in

$$\left[A_t H(\tilde{a}_t) - \frac{\partial W_t}{\partial n_t} + \frac{c}{q(\theta_t)}\right]\frac{\partial \rho_t}{\partial \tilde{a}_t}(n_{t-1} + v_{t-1}q(\theta_{t-1})) = A_t n_t \frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} - \frac{\partial W_t}{\partial \tilde{a}_t}. \tag{A.24}$$

Let us first notice that the unemployment evolution equation can be expressed as

$$\frac{n_t}{1-\rho_t} = (n_{t-1} + v_{t-1}q(\theta_{t-1})). \tag{A.25}$$

Then we can replace part of the left-hand side of equation (A.24) with this employment evolution equation (A.25), which yields

$$\left[A_t H(\tilde{a}_t) - \frac{\partial W_t}{\partial n_t} + \frac{c}{q(\theta_t)}\right]\frac{\partial \rho_t}{\partial \tilde{a}_t}\frac{n_t}{1-\rho_t} = A_t n_t \frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} - \frac{\partial W_t}{\partial \tilde{a}_t}. \tag{A.26}$$

Now, there are four partial derivatives in equation (A.26). The first is the partial of wage bill W_t w.r.t employment derived earlier in equation(A.22). The second and third are the wage bill, W_t, and the idiosyncratic productivity aggregate, $H(\tilde{a}_t)$, derived w.r.t the idiosyncratic productivity cut-off $\tilde{a}_t$. These are solved using Leibniz integral rules. The fourth partial is the separation rate ρ_t w.r.t. the idiosyncratic productivity cut-off $\tilde{a}_t$, and it is calculated using the definition of probability density function and cumulative density function, i.e. $F'(\tilde{a}_t) = f(\tilde{a}_t)$. To sum up, these partial derivatives are

$$\frac{\partial W_t}{\partial \tilde{a}_t} = \frac{f(\tilde{a}_t)}{1-F(\tilde{a}_t)}(W_t - n_t w(\tilde{a}_t)), \tag{A.27}$$

$$\frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} = \frac{f(\tilde{a}_t)}{1-F(\tilde{a}_t)}(H(\tilde{a}_t) - \tilde{a}_t) \tag{A.28}$$

and

$$\frac{\partial \rho_t}{\partial \tilde{a}_t} = (1-\rho^X)f(\tilde{a}_t) \tag{A.29}$$

Inserting all the partial derivatives into the equation (A.26) yields

$$\begin{aligned}&\left[A_t H(\tilde{a}_t) - \frac{W_t}{n_t} + \frac{c}{q(\theta_t)}\right](1-\rho^X)f(\tilde{a}_t)\frac{n_t}{1-\rho_t} \\ &= A_t n_t \frac{f(\tilde{a}_t)}{1-F(\tilde{a}_t)}(H(\tilde{a}_t) - \tilde{a}_t) - \frac{f(\tilde{a}_t)}{1-F(\tilde{a}_t)}(W_t - n_t w(\tilde{a}_t)).\end{aligned} \tag{A.30}$$

In equation (A.30), $f(\tilde{a}_t)$ can be eliminated from the both sides of the equation. Then, a part of the LHS of the equation (A.30) can be simplified by using the definition of total separations ρ_t. This simplification is the following

$$\begin{aligned}(1-\rho^X)\frac{n_t}{1-\rho_t} &= (1-\rho^X)\frac{n_t}{1-(\rho^X+(1-\rho^X)F(\tilde{a}_t))} \\ &= (1-\rho^X)\frac{n_t}{(1-\rho^X)(1-F(\tilde{a}_t))} = \frac{n_t}{1-F(\tilde{a}_t)}.\end{aligned} \tag{A.31}$$

After inserting this result into equation (A.30), we can derive the job destruction condition by removing corresponding terms from both sides of the equation as

$$\begin{aligned} & \left[A_t H(\tilde{a}_t) - \frac{W_t}{n_t} + \frac{c}{q(\theta_t)}\right]\frac{n_t}{1-F(\tilde{a}_t)} \\ &= A_t n_t \frac{1}{1-F(\tilde{a}_t)}(H(\tilde{a}_t)-\tilde{a}_t) - \frac{1}{1-F(\tilde{a}_t)}(W_t - n_t w(\tilde{a}_t)) \\ & \Leftrightarrow \\ & A_t n_t H(\tilde{a}_t) - W_t + \frac{c}{q(\theta_t)} n_t \\ &= A_t n_t H(\tilde{a}_t) - A_t n_t \tilde{a}_t - W_t + n_t w(\tilde{a}_t) \\ & \Leftrightarrow \\ & A_t \tilde{a}_t - w(\tilde{a}_t) + \frac{c}{q(\theta_t)} = 0, \end{aligned} \tag{A.32}$$

in which the last equation is the job destruction condition in the main text.

Wages

Wages are defined as a Nash bargaining problem given by

$$\arg\max_{w_t} (\mathcal{W}_t(a_t) - \mathcal{U}_t)^{\eta} \mathcal{J}_t(a_t)^{1-\eta}, \tag{A.33}$$

in which $\mathcal{W}_t(a_t)$ is the worker's value when employed with productivity a_t, $\mathcal{U}_t$ is the worker's value when unemployed, $\mathcal{J}_t(a_t)$ is the firm's value from a job with productivity a_t, and the parameter η is the bargaining power of workers. In order to derive the wage, the following sections define the value functions of the problem.

Firm's value function

A job with the threshold productivity $\tilde{a}_t$ has zero value for the firm, i.e. $\mathcal{J}(\tilde{a}_t) = 0$. Using the job destruction condition, i.e. the equation (A.32), the value function at the threshold can be written as

$$\mathcal{J}_t(\tilde{a}_t) = A_t \tilde{a}_t - w(\tilde{a}_t) + \frac{c}{q(\theta_t)}, \tag{A.34}$$

which generalizes to a value function with any idiosyncratic productivity as

$$\mathcal{J}_t(a_t) = A_t a_t - w(a_t) + \frac{c}{q(\theta_t)}. \tag{A.35}$$

Now, for the clarity, we can notice first that

$$\begin{aligned}\int_{\tilde{a_t}}^{T} \frac{c}{q(\theta_t)} \frac{f(a)}{1-F(\tilde{a_t})} da &= \frac{1}{1-F(\tilde{a_t})} \frac{c}{q(\theta_t)} \int_{\tilde{a_t}}^{T} f(a) da \\ &= \frac{1}{1-F(\tilde{a_t})} \frac{c}{q(\theta_t)} \Big(F(T) - F(\tilde{a_t} \Big)\end{aligned} \tag{A.36}$$

and since, by the definition of cumulative density function $F(a)$

$$\lim_{T\to\infty} F(T) = 1, \tag{A.37}$$

the last term from the equation (A.35) can be written as

$$\frac{c}{q(\theta_t)} = \int_{\tilde{a_t}}^{\infty} \frac{c}{q(\theta_t)} \frac{f(a)}{1-F(\tilde{a_t})} da, \tag{A.38}$$

The job creation condition from equation (A.23) is repeated here for clarity

$$\frac{c}{q(\theta_t)} = E_t m_{t+1}(1-\rho_{t+1}) \Big[A_{t+1} H(\tilde{a_{t+1}}) - \int_{\tilde{a_t}}^{\infty} w(a) \frac{f(a)}{1-F(\tilde{a_t})} da + \frac{c}{q(\theta_{t+1})} \Big]. \tag{A.39}$$

Now, inserting the definition of $H(\tilde{a_{t+1}})$ and the hiring cost term from (A.38) for the period $t+1$, the job creation condition (A.39) becomes

$$\begin{aligned}\frac{c}{q(\theta_t)} &= E_t m_{t+1}(1-\rho_{t+1}) \Big[A_{t+1} \int_{\bar{a_{t+1}}}^{\infty} a \frac{f(a)}{1-F(\tilde{a_{t+1}})} da - \int_{\tilde{a_t}}^{\infty} w(a) \frac{f(a)}{1-F(\tilde{a_t})} da \\ &\quad + \int_{\tilde{a_{t+1}}}^{\infty} \frac{c}{q(\theta_{t+1})} \frac{f(a)}{1-F(\tilde{a_{t+1}})} da \Big] \\ &\Leftrightarrow \\ \frac{c}{q(\theta_t)} &= E_t m_{t+1}(1-\rho_{t+1}) \Big[\int_{\bar{a_{t+1}}}^{\infty} \Big(A_{t+1} a - w(a) + \frac{c}{q(\theta_{t+1})} \Big) \frac{f(a)}{1-F(\tilde{a_{t+1}})} da \Big] \\ &\Leftrightarrow \\ \frac{c}{q(\theta_t)} &= E_t m_{t+1}(1-\rho_{t+1}) \Big[\int_{\bar{a_{t+1}}}^{\infty} \mathcal{J}_{t+1}(a) \frac{f(a)}{1-F(\tilde{a_{t+1}})} da \Big].\end{aligned} \tag{A.40}$$

Above, the last step uses the definition of value function (A.35). By inserting this result into the value function equation (A.35), the final form of the firm's value is given by

$$\mathcal{J}_t(a_t) = A_t a_t - w(a_t) + E_t m_{t+1}(1-\rho_{t+1}) \Big[\int_{\bar{a_{t+1}}}^{\infty} \mathcal{J}_{t+1}(a) \frac{f(a)}{1-F(\tilde{a_{t+1}})} da \Big]. \tag{A.41}$$

Worker's value functions

The worker's value function, when she is matched with a job that has idiosyncratic productivity a_t consists of a present wage and the future expectations of remaining employed or becoming unemployed. When the worker remains employed, the expected value depends on the new idiosyncratic draw. The value function is given by

$$\begin{aligned}\mathcal{W}_t(a_t) = w_t(a_t) + E_t m_{t+1}(1-\rho_{t+1}) \int_{\tilde{a_{t+1}}}^{\infty} \mathcal{W}_{t+1}(a) \frac{f(a)}{1-F(\tilde{a_{t+1}})} da \\ + E_t m_{t+1} \rho_{t+1} \mathcal{U}_{t+1}.\end{aligned} \tag{A.42}$$

The unemployed worker earns an unemployment benefit b, and can remain unemployed or find a job with probability $\theta_t q(\theta_t)$. The value of unemployed worker is given by

$$\begin{aligned}\mathcal{U}_t = b + E_t m_{t+1} \theta_t q(\theta_t)(1-\rho_{t+1}) \int_{\tilde{a_{t+1}}}^{\infty} \mathcal{W}_{t+1}(a) \frac{f(a)}{1-F(\tilde{a_{t+1}})} da \\ + E_t m_{t+1}(1-\theta_t q(\theta_t)(1-\rho_{t+1}))\mathcal{U}_{t+1}.\end{aligned} \tag{A.43}$$

The next section derives the exact wage.

Deriving the wage equation

The Nash bargaining problem is presented in the equation (A.33). The solution of the problem is given by

$$\mathcal{W}_t(a_t) - \mathcal{U}_t = \frac{\eta}{1-\eta} \mathcal{J}_t(a_t). \tag{A.44}$$

For the sake of brevity, let us first take a look at the LHS of the bargaining solution (A.44). Inserting the worker's value functions and reorganizing the LHS result in

$$w_t(a_t) + E_t m_{t+1}(1-\rho_{t+1}) \int_{a_{\bar{t+1}}}^{\infty} \mathcal{W}_{t+1}(a) \frac{f(a)}{1-F(a_{\tilde{t+1}})} da + E_t m_{t+1} \rho_{t+1} \mathcal{U}_{t+1}$$

$$-$$

$$b + E_t m_{t+1} \theta_t q(\theta_t)(1-\rho_{t+1}) \int_{a_{\bar{t+1}}}^{\infty} \mathcal{W}_{t+1}(a) \frac{f(a)}{1-F(a_{\tilde{t+1}})} da + E_t m_{t+1}(1-\theta_t q(\theta_t)(1-\rho_{t+1}))\mathcal{U}_{t+1}$$

$$= \frac{\eta}{1-\eta} \mathcal{J}_t(a_t)$$

$$\Leftrightarrow$$

$$w_t(a_t) - b + E_t m_{t+1}\Bigg[(1-\theta_t q(\theta_t))(1-\rho_{t+1}) \int_{a_{\bar{t+1}}}^{\infty} \mathcal{W}_{t+1}(a) \frac{f(a)}{1-F(a_{\tilde{t+1}})} da$$

$$+\rho_{t+1}\mathcal{U}_{t+1} - (1-\theta_t q(\theta_t))\mathcal{U}_{t+1} - \theta_t q(\theta_t) \rho_{t+1} \mathcal{U}_{t+1}\Bigg]$$

$$= \frac{\eta}{1-\eta} \mathcal{J}_t(a_t)$$

$$\Leftrightarrow$$

$$w_t(a_t) - b + E_t \beta_{t+1}\Bigg[(1-\theta_t q(\theta_t))(1-\rho_{t+1}) \Bigg(\int_{a_{\bar{t+1}}}^{\infty} \mathcal{W}_{t+1}(a) \frac{f(a)}{1-F(a_{\tilde{t+1}})} da - \mathcal{U}_{t+1}\Bigg)\Bigg]$$

$$= \frac{\eta}{1-\eta} \mathcal{J}_t(a_t)$$

$$\Leftrightarrow$$

$$w_t(a_t) - b + E_t m_{t+1}\Bigg[(1-\theta_t q(\theta_t))(1-\rho_{t+1}) \int_{a_{\bar{t+1}}}^{\infty} \Bigg(\mathcal{W}_{t+1}(a) - \mathcal{U}_{t+1}\Bigg) \frac{f(a)}{1-F(a_{\tilde{t+1}})} da\Bigg]$$

$$= \frac{\eta}{1-\eta} \mathcal{J}_t(a_t)$$

$$\Leftrightarrow$$

$$w_t(a_t) - b + E_t m_{t+1}\Bigg[(1-\theta_t q(\theta_t))(1-\rho_{t+1}) \int_{a_{\bar{t+1}}}^{\infty} \Bigg(\frac{\eta}{1-\eta}\mathcal{J}_{t+1}(a)\Bigg) \frac{f(a)}{1-F(a_{\tilde{t+1}})} da\Bigg] = \frac{\eta}{1-\eta} \mathcal{J}_t(a_t)$$

$$\Leftrightarrow$$

$$w_t(a_t) - b + \frac{\eta}{1-\eta}(1-\theta_t q(\theta_t)) E_t m_{t+1}\Bigg[(1-\rho_{t+1}) \int_{a_{\bar{t+1}}}^{\infty} \mathcal{J}_{t+1}(a) \frac{f(a)}{1-F(a_{\tilde{t+1}})} da\Bigg] = \frac{\eta}{1-\eta} \mathcal{J}_t(a_t) \tag{A.45}$$

where the second to last step follows from the equality in the Nash rule (A.44). Now, the term in square brackets equals to the hiring cost term as derived in the equation (A.40). Inserting that and the firm's value function from (A.35), the previous equation (A.45) becomes

$$
\begin{aligned}
w_t(a_t) - b + \frac{\eta}{1-\eta}(1-\theta_t q(\theta_t))\frac{c}{q(\theta_t)} &= \frac{\eta}{1-\eta}\left(A_t a_t - w_t(a_t) + \frac{c}{q(\theta_t)}\right) \\
&\Leftrightarrow \\
(1-\eta)(w_t(a_t)-b) + \eta\frac{c}{q(\theta_t)} - \eta\theta_t c &= \eta A_t a_t - \eta w_t(a_t) + \eta\frac{c}{q(\theta_t)} \\
&\Leftrightarrow \\
w_t(a_t) &= \eta A_t a_t + \eta\theta_t c + (1-\eta)b \\
&\Leftrightarrow \\
w_t(a_t) &= \frac{\eta}{1+\eta}(A_t a_t + \theta_t c) + \frac{1-\eta}{1+\eta}b.
\end{aligned}
\tag{A.46}
$$

The aggregate real wage, i.e. the mean of individual wages, is given by

$$
w_t = \int_{\tilde{a}_t}^{\infty} w_t(a)\frac{f(a)}{1-F(\tilde{a}_t)}da, \tag{A.47}
$$

which can be inserted into the wage equation (A.46) to derive the exact wage as

$$
\begin{aligned}
w_t &= \int_{\tilde{a}_t}^{\infty}\left(\frac{\eta}{1+\eta}(A_t a + \theta_t c) + \frac{1-\eta}{1+\eta}b\right)\frac{f(a)}{1-F(\tilde{a}_t)}da \\
&\Leftrightarrow \\
w_t &= \frac{\eta}{1+\eta}A_t\int_{\tilde{a}_t}^{\infty} a\frac{f(a)}{1-F(\tilde{a}_t)}da + \frac{\eta}{1+\eta}\theta_t c + \frac{1-\eta}{1+\eta}b
\end{aligned}
\tag{A.48}
$$

Deriving the job productivity threshold

The job destruction condition, i.e. equation (A.32), is repeated here from the previous chapter as

$$
A_t\tilde{a}_t - w(\tilde{a}_t) + \frac{c}{q(\theta_t)} = 0. \tag{A.49}
$$

Now, the exact wage is derived in equation (A.46). Inserting this to the job destruction condition yields

$$\begin{gathered}
A_t\tilde{a}_t - \left[\frac{\eta}{1+\eta}(A_t\tilde{a}_t + \theta_t c) + \frac{1-\eta}{1+\eta}b\right] + \frac{c}{q(\theta_t)} = 0 \\
\Leftrightarrow \\
A_t\left(1 - \frac{\eta}{1+\eta}\right)\tilde{a}_t = \frac{\eta}{1+\eta}\theta_t c + \frac{(1-\eta)}{1+\eta}b - \frac{c}{q(\theta_t)} \\
\Leftrightarrow \\
A_t(1-\eta)\tilde{a}_t = \eta\theta_t c + (1-\eta)b - (1+\eta)\frac{c}{q(\theta_t)} \\
\Leftrightarrow \\
\tilde{a}_t = \frac{1}{A_t}\left[\left(\frac{\eta}{1-\eta}\theta_t c + b\right) - \frac{1+\eta}{1-\eta}\frac{c}{q(\theta_t)}\right],
\end{gathered} \tag{A.50}$$

in which the last equation gives the job destruction threshold $\tilde{a}_t$.

2.A.3 Real wage rigidity

The real wage rigidity is defined such that an exogenously determined fraction γ of the job specific wage is given by the efficient Nash bargaining wage defined in the previous section. The remaining fraction $(1-\gamma)$ of the wage is the same as in the previous period. The wage from the job with productivity a_t is then given by

$$w_t(a_t) = \gamma w_t^n(a_t) + (1-\gamma)w_{t-1}(a_{t-1}), \tag{A.51}$$

where $w_t^n(a_t)$ is the wage from the Nash bargaining as

$$w_t^n(a_t) = \frac{\eta}{1+\eta}(A_t a_t + \theta_t c) + \frac{1-\eta}{1+\eta}b. \tag{A.52}$$

Job productivity threshold with the real wage rigidity

As before the job destruction conditions is

$$A_t\tilde{a}_t - w(\tilde{a}_t) + \frac{c}{q(\theta_t)} = 0. \tag{A.53}$$

Inserting the rigid wage equations results in

$$A_t\tilde{a}_t - \left[\gamma w_t^n(a_t) + (1-\gamma)w_{t-1}(a_{t-1})\right] + \frac{c}{q(\theta_t)} = 0 \tag{A.54}$$

and further

$$A_t\tilde{a}_t - \left[\gamma\left(\frac{\eta}{1+\eta}(A_t\tilde{a}_t + \theta_t c) + \frac{1-\eta}{1+\eta}b\right) + (1-\gamma)w_{t-1}(a_{t-1})\right] + \frac{c}{q(\theta_t)} = 0$$

$$\Leftrightarrow$$

$$\left(1 - \frac{\gamma\eta}{1+\eta}\right)\tilde{a}_t = \frac{1}{A_t}\left[\left(\frac{\gamma\eta}{1+\eta}\theta_t c + \frac{\gamma(1-\eta)}{1+\eta}b + (1-\gamma)w_{t-1}(a_{t-1})\right) - \frac{c}{q(\theta_t)}\right]$$

$$\Leftrightarrow$$

$$\left(\frac{1+\eta[(1-\gamma)-\gamma]}{1+\eta}\right)\tilde{a}_t = \frac{1}{A_t}\left[\left(\frac{\gamma\eta}{1+\eta}\theta_t c + \frac{\gamma(1-\eta)}{1+\eta}b + (1-\gamma)w_{t-1}(a_{t-1})\right) - \frac{c}{q(\theta_t)}\right] \quad \text{(A.55)}$$

$$\Leftrightarrow$$

$$\tilde{a}_t =$$

$$\frac{1}{(1+\eta[(1-\gamma)-\gamma])A_t}\left[\left(\gamma\eta\theta_t c + \gamma(1-\eta)b + (1+\eta)(1-\gamma)w_{t-1}(a_{t-1})\right)\right.$$

$$\left. -(1+\eta)\frac{c}{q(\theta_t)}\right],$$

in which the last equation gives the job destruction threshold with real wage rigidity.

2.A.4 Aggregate resource constraint

The aggregate resource constraint is derived by combining the constraints of the household, the firm and the government.

The household budget constraint is

$$C_t + P_t S_{t+1} + \frac{B_{t+1}}{R_t} = W_t + S_t(P_t + D_t) + B_t + bu_t - T_t. \quad \text{(A.56)}$$

The government simply covers the unemployment benefit, i.e. $T_t = bu_t$. Since the household and the firm are representative, the household owns all the shares. It is assumed that the firm does not issue new shares, so the shares can be normalized to 1. Taking this into account and combining the government budget constraint to (A.56) yields

$$C_t + \frac{B_{t+1}}{R_t} = W_t + D_t + B_t. \quad \text{(A.57)}$$

The firms budget constraint is

$$\Pi_t = Y_t - W_t - cv_t - B_t + \frac{B_{t+1}}{R_t}. \quad \text{(A.58)}$$

The firm pays all the profits out as dividends, i.e. $D_t = \Pi_t$. Adding this to the intermediate result (A.57) yields the aggregate resource constraint

$$C_t + \frac{B_{t+1}}{R_t} = W_t + Y_t - W_t - cv_t - B_t + \frac{B_{t+1}}{R_t} + B_t$$
$$\Leftrightarrow \tag{A.59}$$
$$C_t = Y_t - cv_t$$

2.A.5 Equilibrium set

Endogenous variables

$Y_t, C_t, H_t, \tilde{a}_t, \theta_t, q(\theta_t), \rho_t, u_t, n_t, v_t, W_t, w_t, m_t, R_t, R_t^s, D_t, P_t, T_t$

Equilibrium equations

Production function:

$$Y_t = A_t n_t H_t \tag{A.1}$$

Dividend:

$$D_t = Y_t - W_t - cv_t - B_t + \frac{B_{t+1}}{R_t} \tag{A.2}$$

Price of equity:

$$P_t = E_t \sum_{j=1}^{\infty} m_{t+j} D_{t+j} \tag{A.3}$$

Gross return on equity:

$$R_{t+1}^s = \frac{(P_{t+1} + D_{t+1})}{P_t} \tag{A.4}$$

Expectation of idiosyncratic productivity conditional on surviving cutoff $\tilde{a}_t$:

$$H_t = \int_{\tilde{a}_t}^{\infty} a \frac{f(a)}{1 - F(\tilde{a}_t)} da \tag{A.5}$$

Productivity threshold:

$$\tilde{a}_t = \frac{1}{A_t} \left[\frac{\eta_t}{1 - \gamma\eta_t} \theta_t c + \frac{1 - \eta_t}{1 - \gamma\eta_t} b + \frac{1 - \gamma}{1 - \gamma\eta_t} w_{t-1}(\tilde{a_{t-1}}) - \frac{1}{1 - \gamma\eta_t} \frac{c}{q(\theta_t)} \right] \tag{A.6}$$

Labour market tightness:

$$\theta_t = \frac{v_t}{u_t} \tag{A.7}$$

Matching probability:

$$q(\theta_t) = \zeta_t \theta_t^{-\mu}. \tag{A.8}$$

Separation rate:

$$\rho_t = (\rho_t^X) + (1 - \rho_t^X) F(\tilde{a}_t). \tag{A.9}$$

Unemployment:

$$u_t = 1 - n_t. \tag{A.10}$$

Employment:

$$n_{t+1} = (1 - \rho_{t+1})(n_t + v_t q_t). \tag{A.11}$$

Job creation condition:

$$\frac{c}{q_t} = E_t m_{t+1} \left[(1 - \rho_{t+1}) \left(A_{t+1} H_{t+1} - w_{t+1} + \frac{c}{q_{t+1}} \right) \right]. \tag{A.12}$$

Bonds Euler equation:

$$\frac{1}{R_t} = \beta_t E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}. \tag{A.13}$$

Equity pricing:

$$1 = \beta_t E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} R_{t+1}^s \right]. \tag{A.14}$$

Discount factor:

$$m_{t+1} = \beta_t E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \tag{A.15}$$

Wage setting rule:

$$w_t = \gamma \left(\eta_t (A_t a_t + \theta_t c) + (1 - \eta_t) b \right) + (1 - \gamma) w_{t-1} \tag{A.16}$$

Wage bill:

$$W_t = n_t w_t. \tag{A.17}$$

Aggregate budget constraint:

$$Y_t = C_t + c v_t \tag{A.18}$$

Exogenous shock processes

$$A_t = \rho_A A_{t-1} + (1-\rho_A)\bar{A} + \sigma^A \epsilon^A \tag{A.19}$$

$$\beta_t = \rho^\beta \beta_{t-1} + (1-\rho^\beta)\bar{\beta} + \sigma^\beta \epsilon_t^\beta \tag{A.20}$$

$$\rho_t^X = \rho^\rho \rho_{t-1}^X + (1-\rho^\rho)\bar{\rho^X} + \epsilon_t^\rho, \tag{A.21}$$

$$\eta_t = \rho^\eta \eta_{t-1} + (1-\rho^\eta)\bar{\eta} + \epsilon_t^\eta, \tag{A.22}$$

$$\zeta_t = \rho^\zeta \zeta_{t-1} + (1-\rho^\zeta)\bar{\zeta} + \epsilon_t^\zeta, \tag{A.23}$$

with innovations

$$\epsilon^A \sim \mathcal{N}(0,1) \tag{A.24}$$

$$\epsilon^\beta \sim \mathcal{N}(0,1) \tag{A.25}$$

$$\epsilon_t^\rho \sim \mathcal{N}(0,\sigma^\rho) \tag{A.26}$$

$$\epsilon_t^\eta \sim \mathcal{N}(0,\sigma^\eta) \tag{A.27}$$

$$\epsilon_t^\zeta \sim \mathcal{N}(0,\sigma^\zeta) \tag{A.28}$$

2.B Estimation

2.B.1 Data

This section describes the data sources and the construction of the observables used in estimation.

Output. U.S. Bureau of Economic Analysis, Real Gross Domestic Product [GDPC1], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/GDPC1

Unemployment rate. U.S. Bureau of Labor Statistics, Unemployment Rate [UNRATE], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/UNRATE

Vacancy rate. U.S. Bureau of Labor Statistics, Job Openings: Total Nonfarm [JTSJOR], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/JTSJOR

Layoffs rate. U.S. Bureau of Labor Statistics, Layoffs and Discharges: Total Nonfarm [JTSLDR], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/JTSLDR

Wages. U.S. Bureau of Economic Analysis, Compensation of Employees: Wages and Salary Accruals [WASCUR], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/WASCUR

Consumption. U.S. Bureau of Economic Analysis, Real Personal Consumption Expenditures [PCECC96], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/PCECC96

Total separation rate. U.S. Bureau of Labor Statistics, Total Separations: Total Nonfarm [JTSTSR], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/JTSTSR

Labor force. U.S. Bureau of Labor Statistics, Population Level [CNP16OV], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/CNP16OV

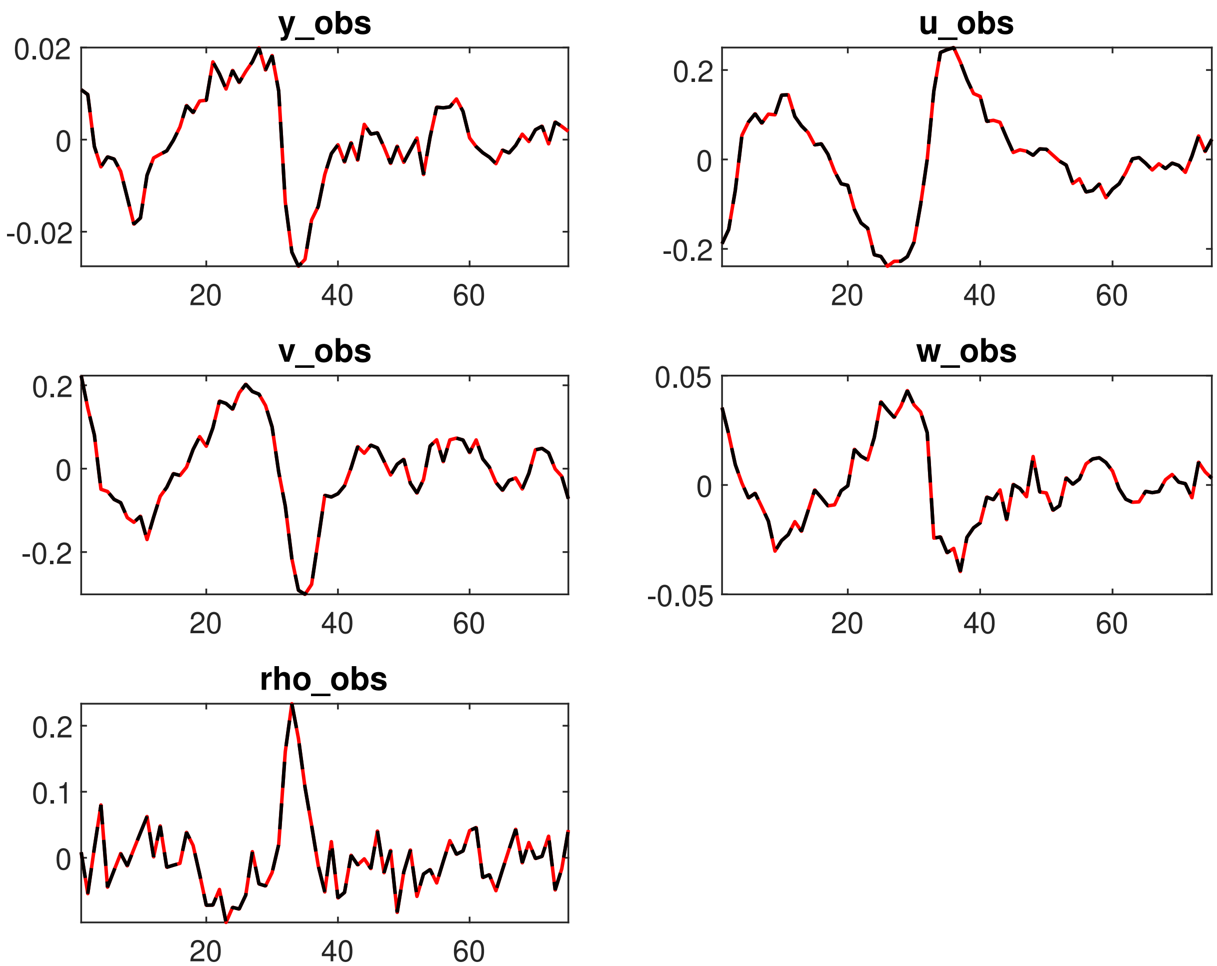
GDP deflator. U.S. Bureau of Economic Analysis, Gross Domestic Product: Implicit Price Deflator [GDPDEF], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/GDPDEF

The baseline estimation uses five series output, unemployment rate, vacancy rate, firings rate and wages. The robustness check estimation uses also consumption. The monthly series (unemployment, vacancy, layoffs, total separations, labor force) are aggregated into a quarterly frequency. I convert output, consumption and wage to per-capita terms by scaling with labor force, and divide wage series by the GDP-deflator to capture a real wage. For estimation, all series are passed through the Hodrick-Prescott filter with smoothing parameter 1,600 to obtain the cyclical component.

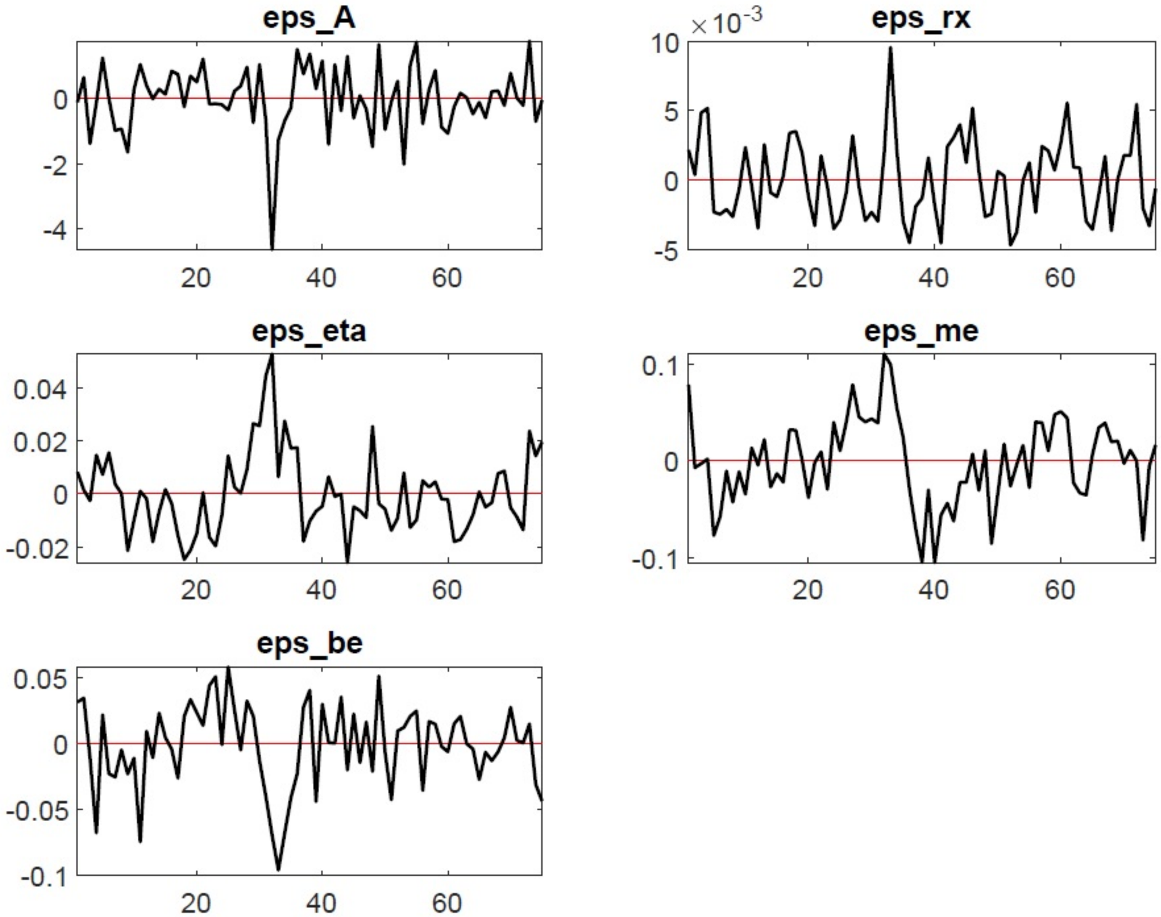
2.B.2 Estimation graphs

This section presents the graphical output of the estimation process.

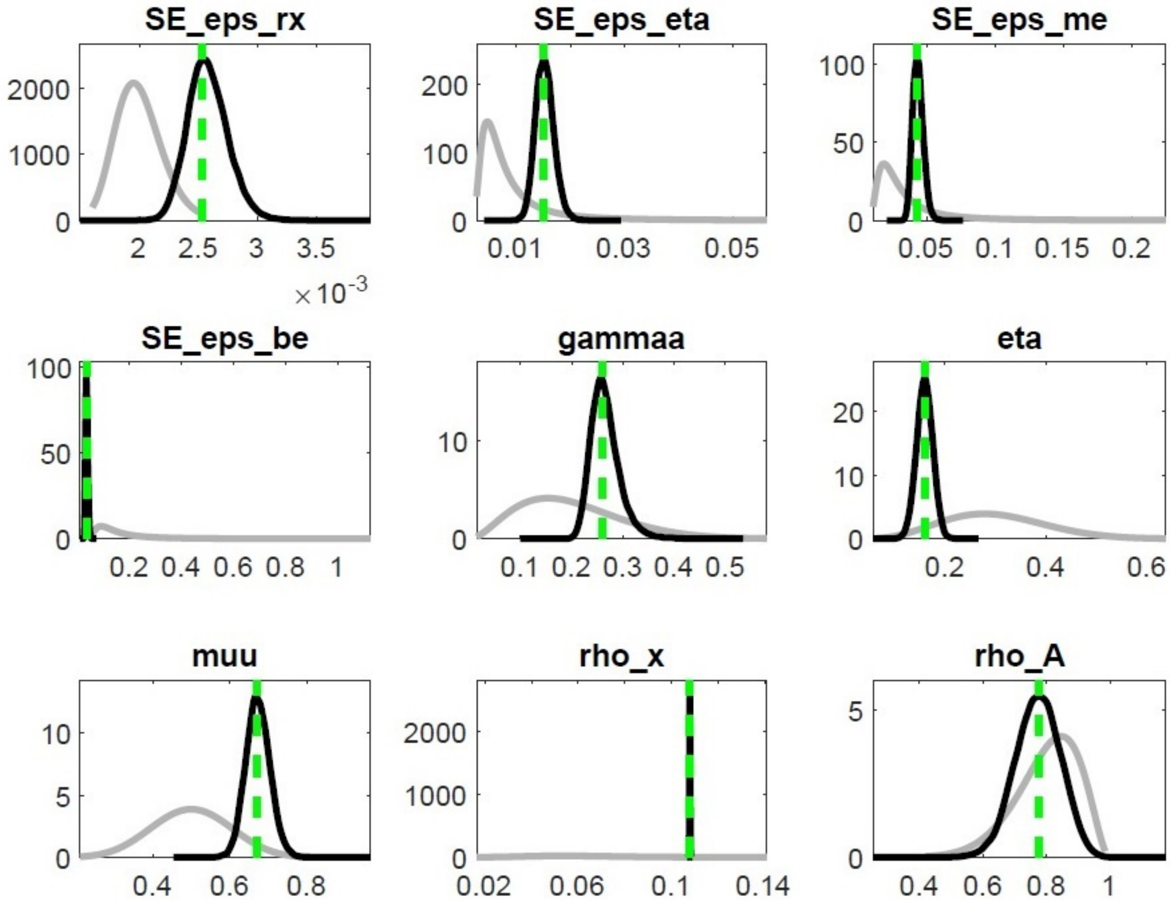
Historical and smoothed variables

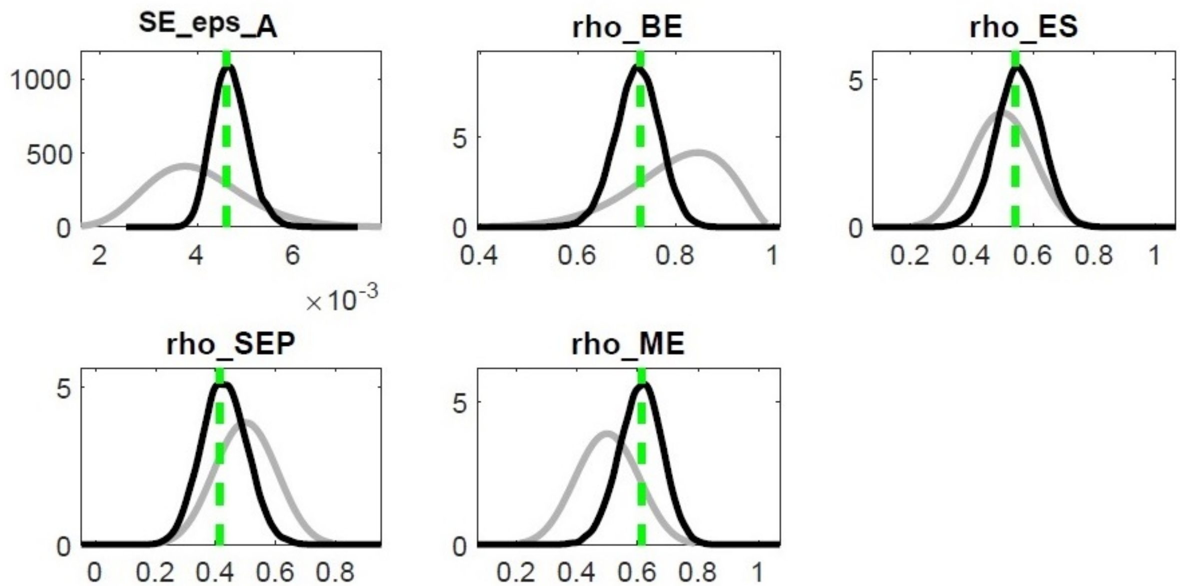


Smoothed shocks

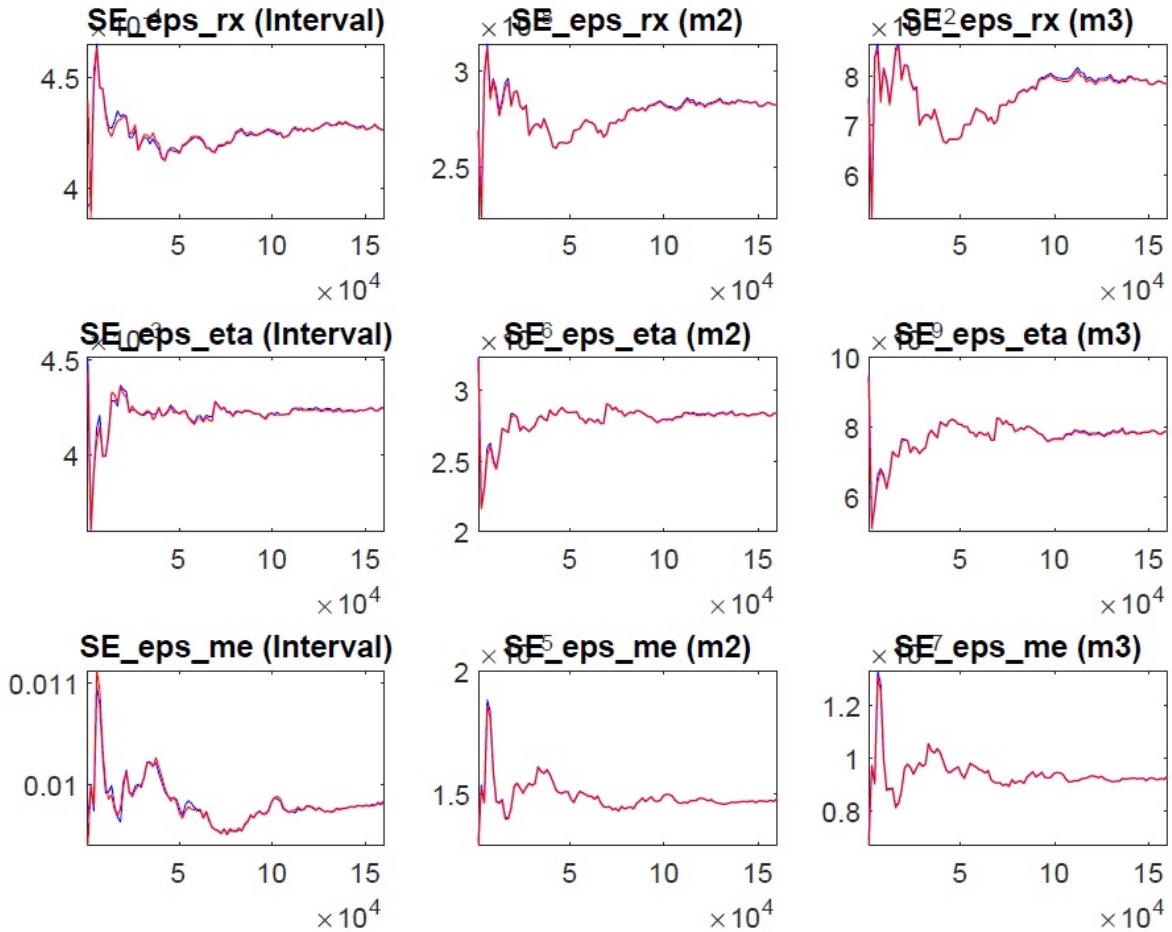


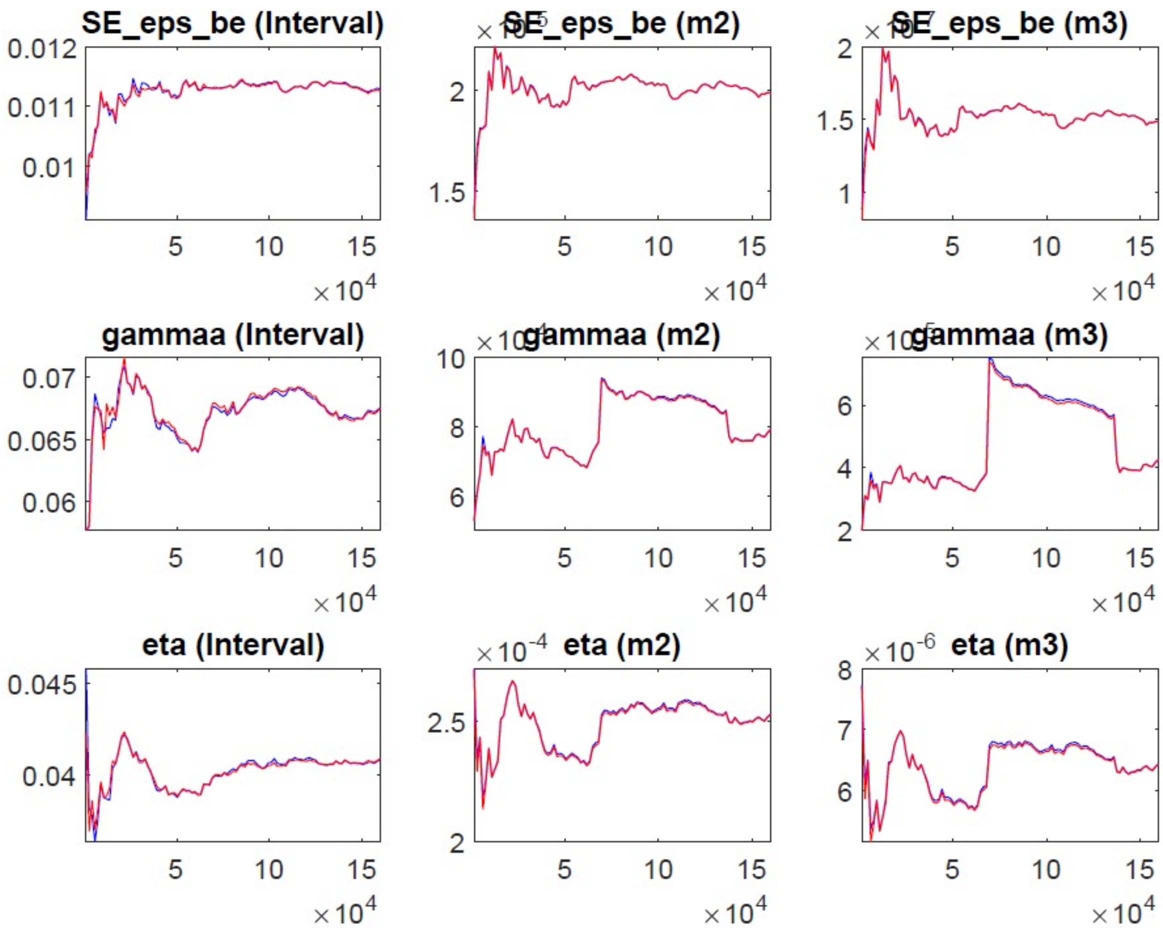
Posteriors and priors

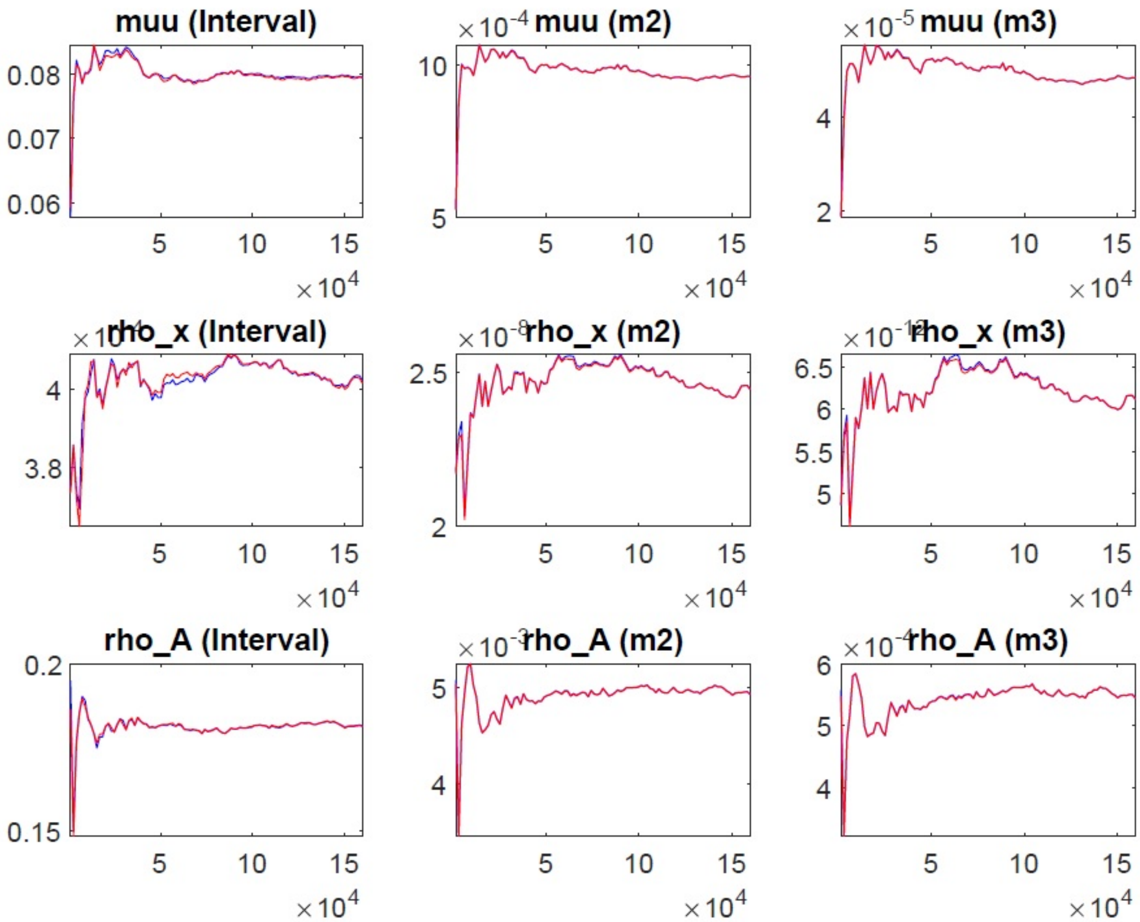


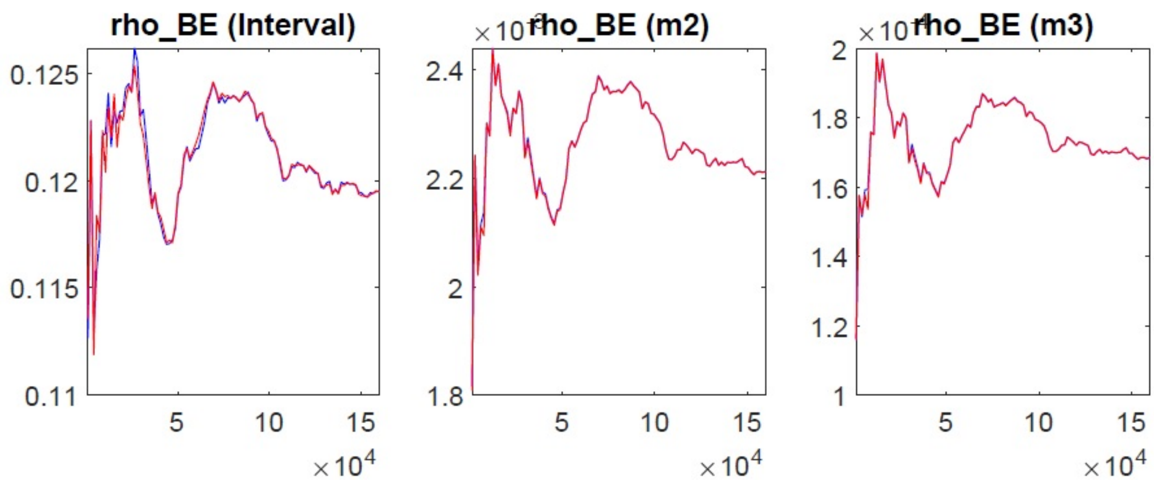

SE_eps_A
1000
500
0
2
4
6
×10^-3
rho_BE
5
0
0.4
0.6
0.8
1
rho_ES
5
0
0.2
0.4
0.6
0.8
1
rho_SEP
5
0
0
0.2
0.4
0.6
0.8
rho_ME
5
0
0.2
0.4
0.6
0.8
1

Univariate diagnostics

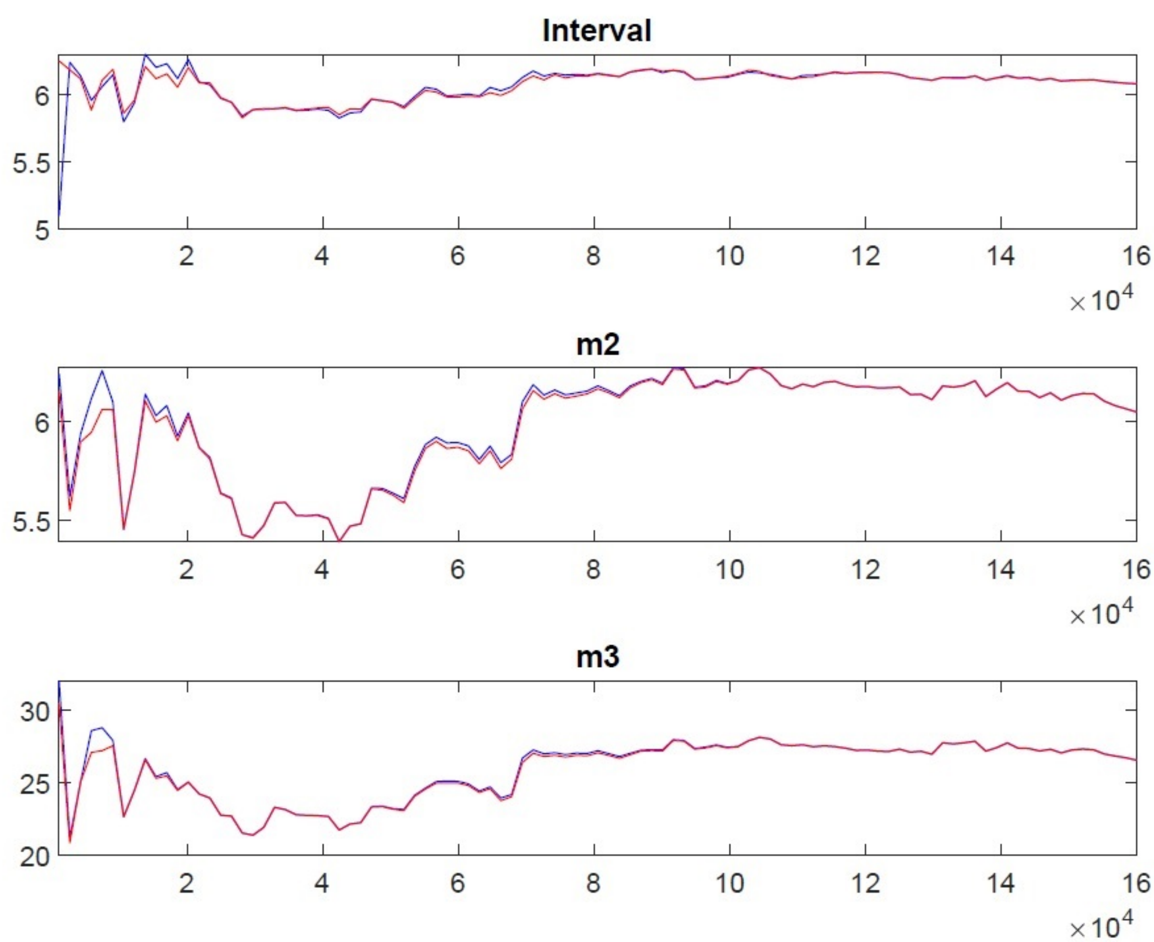



SE_eps_be (Interval)
0.012
0.011
0.01
5
10
15
×10^4
SE_eps_be (m2)
2
1.5
SE_eps_be (m3)
2
1.5
1
gammaa (Interval)
0.07
0.065
0.06
gammaa (m2)
10
8
6
gammaa (m3)
6
4
2
eta (Interval)
0.045
0.04
eta (m2)
×10^-4
2.5
2
eta (m3)
×10^-6
8
7
6


muu (Interval)
muu (m2)
muu (m3)
rho_x (Interval)
rho_x (m2)
rho_x (m3)
rho_A (Interval)
rho_A (m2)
rho_A (m3)
×10^-4
×10^-5
×10^-3
×10^-4
×10^4



Multivariate convergence diagnostics



2.C Robustness

2.C.1 IRFs to a matching efficiency shock

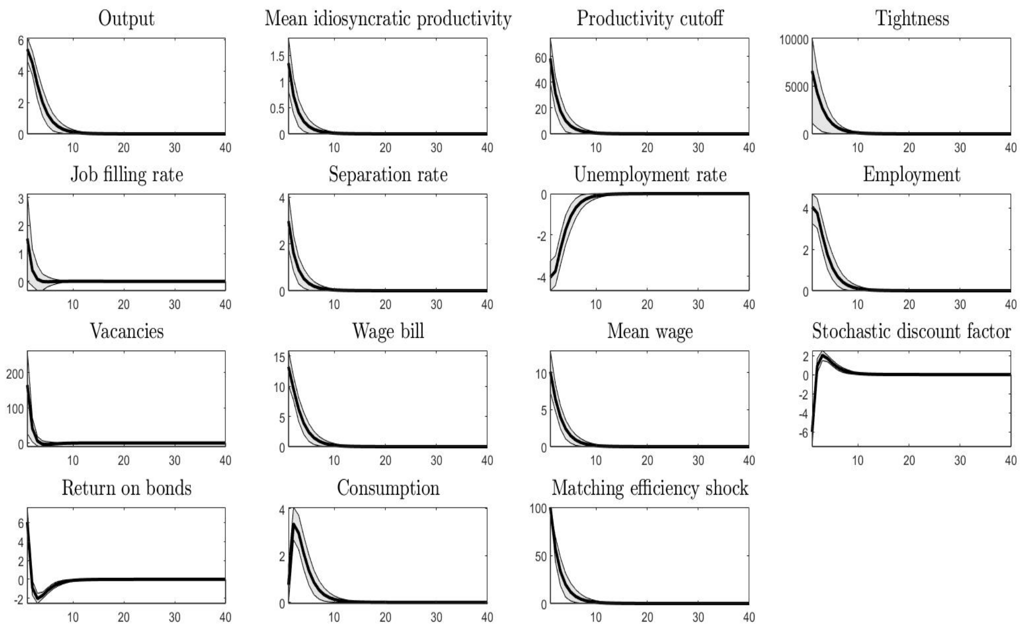


Figure 2.7: Positive orthogonalized shock to the matching efficiency

An estimation using a shorter sample without the Great Recession is presented in the sensitivity analysis section of the paper. The estimation results show that during the extension period after the Great Recession, the matching efficiency shock is the most important driver of several labor market variables. Figure 2.7 presents the impulse response functions of the matching efficiency shock.

2.C.2 Robustness checks

This section studies the robustness of the estimation results. First, the outcome that the discount factor shock drives the labor market variables considerably, is expected considering the results from the existing literature. On the contrary, the result that the discount factor shock explains very small share of output volatility but still somewhat large share of variance in consumption is less clear. To test the robustness of this finding, I remove the output from observed variables and add consumption series instead.

The aggregate budget constraint of the model states that the output is used in consumption or investment in job creation. Job creation is observed from the data. The main difference in using consumption instead of output is that consumption appears in the stochastic discount factor and hence is directly connected to the discount factor shock. In addition, consumption is smoother that output in the data.[11]

The alternative estimation setup using consumption data instead of output does not change the qualitative result from the baseline estimation. However, the quantitative impact of the technology shock becomes more significant. The share of volatility in output which the discount factor shock explains drops to only 1% compared with the baseline estimation of 6.4% while the technology shock explains over 90% of the variance. Hence, the result that the output volatility is mostly explained by the productivity shock is robust to the use of output or consumption as observable. The increased share of the technology shock also applies to the variance decomposition of consumption, in which the share of technology shock increases from approximately 40% to 77%. The discount factor shock explains now only 12% of the variance in consumption compared with about 40/% when using output as an observable.

The variance decompositions of the labor market variables react to the change in estimation setup the same way as output and consumption. The share of variances that technology shock explains increases and the share of discount factor shock decreases. The discount factor shock remains the most important in explaining variance in the job creation related variables and wages. However, in variance decomposition of unemployment, the share of discount factor shock decreases from 44% to 21% and the matching efficiency shock becomes the most important contributor with approximately 43% share.

[11]The robustness could also be tested using both output and consumption as observables, but the estimation does not identify well when all the variables in the aggregate budget constraint are observables.

Chapter 3

Short-time work in search and matching models: Evidence from Germany during the Covid-19 crisis

Abstract

This paper estimates the extent to which unemployment in Germany would have been increased during the Covid-19 recession without a short-time work (STW) labor-market policy which enables employers to reduce temporarily the working hours of full-time workers. A Bayesian estimation of a general equilibrium model with a STW policy, and a simulation of a counterfactual model without STW, show that the German unemployment rate would have been 4.2 percentage points higher without the policy. These results indicate that the STW participates in preventing excess job destruction during economic downturns, and in stabilizing unemployment fluctuations over business cycles.

Keywords: Search and matching, short-time work, Bayesian estimation.

JEL: E24, E32, J63.

3.1 Introduction

Broadly defined, short-time work (STW) is a job retention policy, which aims at preserving employed workers during recessions, and at preventing unemployment increases. This policy saves jobs by allowing and incentivizing employers to reduce working hours of full-time workers instead of laying them off. A key element of the policy is that a government pays workers in STW benefits from hours not worked. In Germany, this policy, named Kurzarbeit, has a long tradition, since it was first established already in the 1920s (Müller and Schulten, 2020). Furthermore, during the Great Recession of 2007–2009, the German labor markets experienced a deep recession period with a very moderate unemployment increase compared to that the US.

During the Covid-19 recession of 2020, the German government supported unprecedented participation in the STW program. Up to 19% of employed workers were in STW in Spring 2020, which is a larger share than ever before. As a comparison, the same figure was 4% during the Great Recession (Organisation for Economic Co-operation and Development (OECD), 2020) and the average is 0.78% over the past 20 years.[1] In addition, the working time of workers in STW was reduced on average by almost 50% during the Covid-19, while the average reduction is approximately 30%, and furthermore the reduction during the Great Recession was not higher than the average (Herzog-Stein et al., 2021).

This paper estimates what the level of the unemployment rate Germany would have experienced during the Covid-19 crisis without the short-time work program. To answer this question, I estimate Balleer et al. (2016)'s labor-market model with STW using Bayesian techniques. Estimation quantifies the model parameters, and yields series of shock innovations. These estimated shock processes account for fluctuations in labor market variables over the sample period, including the Covid-19 pandemic. Then, I consider the same model without STW and simulate it using the estimated shock series, in order to obtain counterfactual labor market outcomes without the STW policy. Finally, the two sets of results are compared, which concludes, for instance, that the materialized 1.4 percentage points increase in the unemployment rate would have been 5.6 percentage points without STW.

The contribution of this paper is quantitative and twofold. First, I estimate a structural general equilibrium model with STW, using Bayesian techniques. Second, I simulate a counterfactual model without STW, in order to evaluate the impact of STW policy on unemployment stabilization. To the best of my knowledge, Bayesian techniques have not been used for this purpose so far. STW is generally quantified either from cross-country comparisons, for instance in Hijzen and Venn (2011) and Boeri and Bruecker (2011), or by exploiting microdata, for instance in Cahuc et al. (2021) and Kopp and Siegenthaler (2021). However, both approaches face challenges. First, the cross-country comparisons are hampered by the fact that countries are different. Even if the STW programs are comparable, many other dimensions such as institutions, administrative practices, or cultural traits differ which complicates the identification of STW from the other country-specific characteristics. Second, utilizing microdata at a firm or an establishment level within a

[1] Source: Bundesbank (https://www.bundesbank.de/en/statistics) and author's calculations.

single country cannot capture general equilibrium effects on employment or consumption from the demand side. The method of estimation and simulation in this paper has the additional benefit that it allows for the construction of a counterfactual of an identical economy without STW in a general equilibrium framework.

In general, using a novel method, this paper complements the existing literature about the effectiveness of STW in stabilizing unemployment, and preventing job destruction. Indeed, the simulation results over the last two decades of the German economy show that the variance in unemployment is 2.3 times higher, and the variance in job separations is 2.1 times higher without STW. In turn, this paper also detects some of the trade-offs of the policy. For instance, when firms are subsidized in keeping workers employed in STW, the low productive jobs are preserved. The simulations in this paper result in up to approximately 30% higher firm-level productivities during recessions in the economy without STW. Furthermore, another issue of the policy is detected from the parameter estimates of the model. They suggest that the number of workers in STW has been higher than the number of jobs which are saved from separation, which in turn implies deadweight costs for the society. This result is affected by the generous STW policy during the large recessions in the sample, but it also shows that the STW program is vulnerable to moral hazard issues, and further suggest the need for research about the optimal level of the policy.

Finally, this paper provides support for the decisions of governments that incentivized exceptionally high levels of STW during Covid-19. Without these policy extensions, the firms would have needed to adjust the labor costs through costly job destruction and re-creation, resulting in high volatility in unemployment. Moreover, as argued by Näf et al. (2022), job destruction and creation are especially costly in Continental European countries, where the role of STW has indeed been essential during the Covid-19. However, since this paper also detects the trade-offs of STW, it suggests that the policy expansions, should be limited in duration, for instance in coverage, eligibility and generosity during economic downturns.

The rest of the paper is organized as follows. Section 3.2 discusses the related literature. Section 3.3 describes the model. Section 3.4 presents the estimation and simulation procedure. Section 3.5 contains the results and analysis. Section 3.6 concludes.

3.2 Related literature

This paper investigates the German STW program, named Kurzarbeit. In Germany, Kurzarbeit has a long history, and consequently, the German STW is probably the most researched policy program in the economic literature. Three closely related papers are

Cooper et al. (2017), Gehrke et al. (2019) and Aiyar and Dao (2021). Cooper et al. (2017) estimate the parameters of a structural heterogeneous-agents search-model using a simulated method of moments and confidential plant-level micro data. Further, the paper imposes a negative productivity shock generating a recession in the model. Finally, the authors compare the impact of the shock with and without the STW policy. The same shock increases unemployment by 4 percentage points without STW, while the increase is 0.5 percentage points when the policy is active. As a comparison, this paper uses a Bayesian estimation which allows for an assessment of the full series of shocks accountable for the Covid-19 fluctuations, and further, for these series to be applied on the counterfactual simulations. However, the stabilization of 4.2 percentage points in unemployment during the Covid-19 recession in this paper is close to the results found by Cooper et al. (2017).

Gehrke et al. (2019) are estimating the impact of STW and other institutional factors on the German labor markets during the Great Recession. They are estimating a model that builds on Balleer et al. (2016) with Bayesian techniques, which makes their paper the closest to this one. However, there are important differences, most notably, this paper focuses on the Covid-19, which is not part of the data in Gehrke et al. (2019). The dataset and structural shock also diverge as Gehrke et al. (2019) have four observables, i.e. the number of short-time workers, the unemployment rate, GDP and government spending, and four shocks, i.e. TFP, matching efficiency, government spending and STW shocks. My paper has the two former series and two former shocks in common, but the third observable is the vacancy rate and the third shock is the unemployment benefit shock. Other differences are, that the data are detrended with an HP-filter and the model is log-linearized in Gehrke et al. (2019), while in this paper a linear trend is removed and the model is linearized as all the observables are rates. Finally, the counterfactual exercises also differ.

Gehrke et al. (2019) estimate that STW had a small stabilizing effect on unemployment during the Great Recession, as they find that the STW program only prevented 0.3 percentage points increase in the unemployment rate. The simulation exercise in this paper finds a more than four times larger impact, since the increase in unemployment would have been 1.3 percentage points higher without STW, during the Great Recession. At least one of the differences is likely to be important in explaining this divergence in the results. Gehrke et al. (2019) implement an STW shock in the model. As the authors show, this shock, that changes the policy criterion for participation in STW, has almost no impact on unemployment, but explains a significant fraction of the increase in the level of STW. In this paper, the exogenous policy criterion is fixed, and hence the volatility of STW is explained by the other shocks, which are also responsible for unemployment fluctuation. In addition, this paper matches the vacancy rate to data. Consequently, the volatility in job creation is restricted to observations, and it cannot prevent excessive fraction of unemployment increase. However, Gehrke et al. (2019) do not report the responses of vacancies, so the comparison of this effect is not possible. In turn, there are also equivalent results. For instance, both papers find that unemployment fluctuations are mostly driven by the TFP and matching efficiency shocks. In addition, both papers argue that STW may generate deadweight costs, since the number of workers in STW can be larger than the number workers that are actually preserved from layoffs.

Aiyar and Dao (2021) investigate the contribution of the STW expansion during the Covid-19 in preventing the unemployment increase. They are exploiting high-frequency regional data from Germany, and OLS-regression to detect that, during the second quarter of 2020, the average unemployment rate would have been 2.9 percentage points higher without the expansion of the program. Furthermore, the regional variation is considerable, for instance the increase would have been almost 4 percentage points in Hamburg. These estimates are in line with this paper, since the 4.2 percentage points higher unemployment found here is the difference between actual and simulated peak monthly values. Moreover, the counterfactual in this paper is an economy without STW, while Aiyar and Dao (2021) compare the effect of the policy coverage expansion with the STW coverage outside the Covid-19 pandemic.

Another interesting aspect of Aiyar and Dao (2021) is that they choose the strategy of analyzing regional data from a single country in order to solve the challenges related to cross-country panels, or country-level micro data. In this paper, these same challenges are solved by estimating and simulating a general equilibrium model. The benefits of the method by Aiyar and Dao (2021) are that it enables the authors to empirically identify the impact of the changes in the STW policy during the Covid-19. In turn, the method here allows this paper to investigate a fully counterfactual case where STW does not exist at all.

The German STW is also researched by Balleer et al. (2016) and Niedermayer and Tilly (2016) related to the Great Recession, Herzog-Stein et al. (2021) who compare the Great Recession and the Covid-19, and Teichgräber et al. (2022) who apply a mechanism design approach with a calibration exercise matching the pre-Great Recession Germany. Related to unemployment stabilization, Balleer et al. (2016) find that STW lowers the unemployment fluctuations by 21% and Niedermayer and Tilly (2016) that one job was retained for every four workers on STW. The stabilization during Covid-19 is larger, which is expected as the participation in STW was higher. This fact is documented in detail by Herzog-Stein et al. (2021) who show that the STW program was more expanded, and it also accounted alone for the working hour reductions during Covid-19, whereas during the Great Recession all the working-time instruments contributed to the intensive margin adjustments. The calibration exercise by Teichgräber et al. (2022) finds that the job separation rate decreases by 1.2 – 2.4 percentage points with STW, which is a qualitatively corresponding result with this paper, but difficult to compare exactly, since the Teichgräber et al. (2022)'s calibration targets an outside of recession period.

The model in this paper is from Balleer et al. (2016). They calibrate the model to match the German economy and show that STW stabilizes unemployment, and that a discretionary change in STW policy does not have an impact on unemployment. The model is recently extended by Dengler and Gehrke (2021) to include the precautionary savings of households, which are shown to amplify the stabilization effectiveness of STW.

More generally, this paper contributes to the empirical research about STW, which investigates whether the STW policy can prevent job losses and stabilize unemployment fluctuations. Related papers using cross-country analysis include Abraham and Houseman (1994) comparing the Belgian, French, and German economies with the US, and Hijzen and Venn (2011), and Boeri and Bruecker (2011) investigating OECD and developed economies respectively. Furthermore, related country-specific research includes Cahuc et al. (2021), Benghalem et al. (2023) and Albertini et al. (2022) who analyze France, Kopp and Siegenthaler (2021) and Hijzen and Salvatori (2022) Switzerland, and Osuna and Pérez (2021) Spain. In general all this literature supports the contribution of STW in stabilizing unemployment over business cycles. However, to the best of my knowledge, this paper is the first to use a Bayesian estimation and simulation method to quantify the impact of STW policy during the Covid-19 pandemic.

3.3 Model

The model is the search and matching model by Balleer et al. (2016), who extend Mortensen and Pissarides (1994) with a STW policy. The model is composed of households, firms and a government. The households supply labor, consume and save. Furthermore, all household members are either employed or unemployed, and the latter ones are searching for a job. The firms are producing homogeneous consumption good using labor as the input of production. In order to hire workers, the firms are posting vacancies with a fixed cost. Unemployed workers and open vacancies are matched according to a matching function as in canonical search and matching models. The government collects a lump-sum tax to finance unemployment benefits. As an addition in this model, these unemployment benefits are also paid to the workers in STW for hours not worked.

Workers in the model are ex-ante identical. In turn, job-specific productivity is determined by an idiosyncratic shock, drawn each period for all the existing job matches. If the job-specific productivity is low, the worker may generate profit losses. In this case, the firm has two choices. One, it can separate with this worker as in Mortensen and Pissarides (1994). Two, it can participate in STW and reduce working hours.

The timing of the events is the following. First, unemployed workers search for jobs and firms post vacancies. These two are matched according to a matching function. Second, an exogenous proportion of matches dissolve without decisions from any of the agents, i.e. exogenous separation occurs. Third, wages are negotiated, resulting in a surplus sharing rule, which is dependent on the aggregate state of the economy. Fourth, exogenous total productivity is realized, and all employed workers draw an idiosyncratic shock from a time-invariant distribution. Fifth, firms make a decision about endogenous separations and STW. These decisions depend on the aggregate and idiosyncratic states, as well as the wage rule. Finally, production is done and surplus is shared.

The following section presents the essential ingredients of the model, details of the full model and derivation are in Appendix 3.A.

3.3.1 The firm

An idiosyncratic shock ε_t, is drawn from a time-invariant distribution with PDF $g(\varepsilon)$. This shock is transitory and drawn again each period. The value of a job with the idiosyncratic productivity ε_t is given by

$$J_t(\varepsilon_t) = a_t - \varepsilon_t - w_t - c_f + \beta E_t J_{t+1}(\varepsilon_{t+1}), \tag{1}$$

where a_t is a TFP, w_t is a wage, c_f is a fixed cost of production and β is the household's subjective discount factor.

First, the firm chooses a condition for STW. However, it cannot benefit unconditionally from the policy, hence the government sets a criterion under which the firm can participate in STW. Formally, this criterion is given by

$$J_t(\varepsilon_t) < \bar{J}, \tag{2}$$

in which $\bar{J}$ an exogenous threshold parameter. More specifically, if the value of a job is smaller than $\bar{J}$, the working hours of worker in job $J_t(\varepsilon_t)$ can be reduced in STW.

Criterion (2) also implies a threshold value of an idiosyncratic shock. On the threshold, the condition (2) holds in equality, i.e. the threshold value identifies the lowest productive job, which is not applicable for STW. The threshold value of shock is named v_t^k and solved as

$$J_t(v_t^k) = \bar{J}$$
$$\Leftrightarrow \tag{3}$$
$$v_t^k = a_t - w_t - c_f - \bar{J} + \beta E_t J_{t+1}(\varepsilon_{t+1}).$$

Noticing that the idiosyncratic shock is subtracted from the TFP in equation (1), i.e. it has a negative impact on the job value, the interpretation of the threshold v_t^k is that all the jobs with $\varepsilon_t > v_t^k$ are applicable to STW, while the jobs with $\varepsilon_t \leq v_t^k$ are full-time jobs.

After the firm has chosen the STW threshold, it decides about the optimal working-hours of workers participating in the program. The full working time is normalized to one, and the firm decides a fraction, which is reduced from the full-time. The working-time condition is a result of a maximization problem given by

$$\max_{K(\varepsilon_t)} \ (a_t - \varepsilon_t - w_t)(1 - K(\varepsilon_t)) - c_f - C(K(\varepsilon_t)), \tag{4}$$

where $K(\varepsilon_t) \in [0, 1]$ is the share of hours not worked, and $C(K(\varepsilon_t))$ is a convex cost of hour reduction. The convex cost ensures an interior solution, which enables the situation in the German labor markets to be captured, where for instance during the Covid-19 the hour cut was approximately 50% of full working time according to Herzog-Stein et al. (2021). As a comparison, a linear cost would result in a corner solution, in which the firm cuts either all the working hours or none in STW.

The optimization problem in equation (4) is solved by assuming a quadratic from for cost function $C(K(\varepsilon_t))$, which results in an optimal hour reduction as

$$K(\varepsilon_t)^* = -\frac{a - \varepsilon_t - w_t}{c_K}, \tag{5}$$

in which c_K is a fixed cost of hour cut.

Next, the firm makes a decision about endogenous separation. A difference from the canonical search and matching model is that the firm is now aware of the optimal conditions under which it participates in STW. Consequently, the optimal STW outcome is considered in a separation decision. More specifically, the firm compares the value of a job with optimal working time in STW, i.e. $(1 - K_t(\varepsilon_t)^*)$, with the cost of separation. If it is more costly to separate, the firm retains the job in STW. The separation condition is given by

$$J_t(\varepsilon_t, K(\varepsilon_t)^*) < -f, \tag{6}$$

where $J_t(\varepsilon_t, K_t(\varepsilon_t))$ is the value of STW job with optimal working time reduction K_t^* and

$-f$ is a fixed separation cost. The intuition of separation condition (6) is that, if the worker with optimal working time in STW is still generating more profit losses than the separation cost $-f$, it is beneficial for the firm to separate.

Analogically with the STW threshold in equation (3), the separation threshold for the idiosyncratic productivity is named v_t^f and derived from condition (6) as

$$J_t(v_t^f, K_t(v_t^f)^*) = -f$$
$$\Leftrightarrow \tag{7}$$
$$v_t^f = a_t - w_t - c_f + \frac{1}{(1 - K(v_t^f)^*)}\Big[f + \beta E_t J_{t+1}(\varepsilon_{t+1}) - C(K(v_t^f)^*)\Big],$$

where $K(v_t^f)^*$ is the optimal working hour reduction of a job with the threshold idiosyncratic productivity v_t^f, and $C(K(v_t^f)^*)$ the cost of the corresponding hour cut. The interpretation of threshold is also analogical with the STW threshold, such that the jobs with idiosyncratic productivity $\varepsilon_t > v_t^f$ are separated, and the jobs with $\varepsilon_t \leq v_t^f$ are preserved in STW. Figure 3.1 illustrates how the firm's choice of STW and separation thresholds divides workers into three groups.

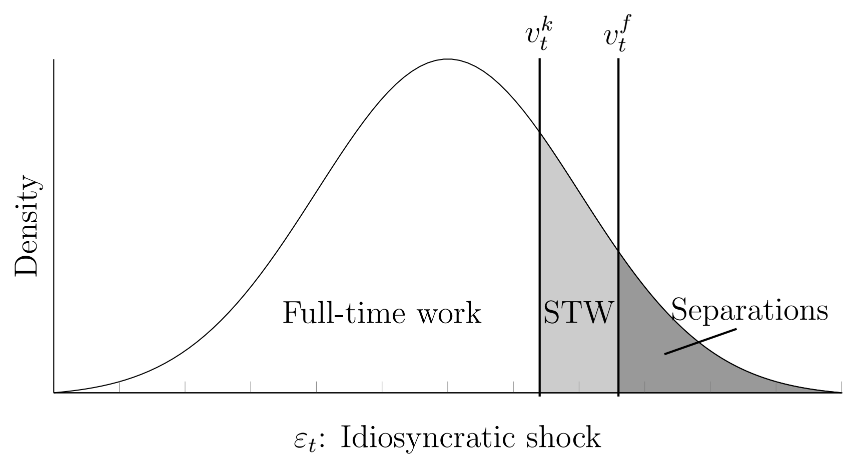


Figure 3.1: The idiosyncratic productivity is drawn from a time-invariant distribution. The firm chooses two thresholds for the idiosyncratic shock: STW threshold v_t^f and separation threshold v_t^k. These thresholds divide workers in three groups: separations, STW, and full-time work.

The STW and separation conditions above are dictated by exogenous parameters $\bar{J}$ and $-f$. In order to investigate further the relationship between these two parameters, let us first consider a case without STW, i.e. the case where hours are not reduced. More specifically, this means $K(v_t^f)^* = 0$ and $C(K(v_t^f)^*) = 0$ in the separation threshold equation (7). In this case, the separation threshold (7) would collapse to correspond the

STW threshold in equation (3), with the difference that the former has f and the latter $-\bar{J}$ on the RHS. Consequently, setting $\bar{J} = -f$ would imply $v_t^k = v_t^f$. Next, if we keep the assumption that $\bar{J} = -f$, and consider the case where the firm cuts hours in STW, i.e. $K(\varepsilon_t)^* > 0$ the two thresholds diverge such that $v_t^k < v_t^f$. Furthermore, this implies that the jobs are preserved from separation, only if the firm benefits from hour reductions in STW. In other words, if $\bar{J} = -f$, STW is saving jobs which would be destroyed without the policy.

The productivity distribution $g(\varepsilon)$ and the thresholds v_t^k and v_t^f define the share of workers in STW, named χ_t, and given by

$$\chi_t = \int_{v_t^k}^{v_t^f} g(\varepsilon) d\varepsilon, \tag{8}$$

and the rate of endogenous separations, named ρ_t^e, and given by

$$\rho_t^e = \int_{v_t^f}^{\infty} g(\varepsilon) d\varepsilon. \tag{9}$$

Finally, the total separation rate is a sum of exogenous and endogenous shares, following Den Haan et al. (2000), as

$$\rho_t = \rho^X + \left(1 - \rho^X\right) \rho_t^e, \tag{10}$$

where ρ_t is the total separation rate and ρ^X is an exogenous separation rate.

Job creation is more standard and follows canonical search and matching models. More specifically, new workers are not hired directly to STW, since the idiosyncratic productivity is not known at the moment of vacancy posting. The firm is posting vacancies with a fixed cost to hire unemployed workers. The free entry condition of vacancies is assumed to hold, i.e. an open vacancy does not have any value. As a result, the job creation condition equates the cost of hiring with the expected value of a filled job, and is given by

$$\frac{\kappa}{q(\theta_t)} = \beta E_t J_{t+1}(\varepsilon_{t+1}), \tag{11}$$

where κ is a fixed vacancy posting cost, θ_t is labor market tightness and $q(\theta_t)$ is a vacancy filling rate.

The employment evolution is also standard

$$n_{t+1} = (1 - \rho_{t+1})(n_t + q(\theta_t) v_t), \tag{12}$$

in which n_t is employment and v_t is a number of vacancies. As is common in the literature the labor force is normalized to one, yielding unemployment, u_t, as $u_t = 1 - n_t$.

Wages are bargained collectively, meaning that the labor union is bargaining the same wage for all workers. In addition, the outside option in the bargaining process is a strike. During a strike production does not occur, and wages are not paid, but the workers remain employed, and the firm holds the value of job matches. In addition, it is assumed that the workers receive a strike allowance, which is equivalent to an unemployment benefit. In this setting, the wage becomes dependent only on the aggregate productivity and the reservation wage. Due to the assumption of the strike allowance, the latter equals the unemployment benefit as is standard in the wage bargaining models. The wage bargaining result is then given by

$$w_t = \gamma a_t + (1-\gamma) b_t, \tag{13}$$

where γ is the bargaining power of workers, and b_t is an unemployment benefit, which is subject to an exogenous shock, and hence time varying. This shock is added for the estimation purposes and specified later.

Finally, the aggregate output is a combination of outputs by full-time and STW workers net of fixed costs, as

$$\begin{aligned} Y_t = & \underbrace{\frac{n_t}{1-\rho_t^e}}_{\text{Employment}} \underbrace{\int_{-\infty}^{v_t^k} (a_t - \varepsilon_t) g(\varepsilon_t) d\varepsilon_t}_{\text{Full-time workers' output}} \\ & + \frac{n_t}{1-\rho_t^e} \underbrace{\int_{v_t^k}^{v_t^f} [(a_t - \varepsilon_t)(1 - K^*(\varepsilon_t)) - C(K^*(\varepsilon_t))] g(\varepsilon_t) d\varepsilon_t}_{\text{STW workers' output}} \\ & \underbrace{- n_t c_f - \frac{n_t}{1-\rho_t^e} \rho_t^e f - v_t \kappa}_{\text{Fixed costs of production, separations, vacancies}} . \end{aligned} \tag{14}$$

The stock of employees in equation (14) is derived from employment evolution equation (12) and the definition of total separation rate (10).

3.3.2 The household, the government and closing the model

The household in the model supplies labor, consumes and saves to risk-free government bonds. The members of the household are either employed or unemployed. The household's income consists of wages and unemployment benefits. As usual, the unemployed household members earn only unemployment benefits and the employed workers, who are working full working hours, earn full wages. In turn, as a difference from canonical labor market models, the STW workers earn wages from hours worked, and unemployment benefits from hours not worked. Hence, the household budget constraint is

$$
\begin{aligned}
C_t + B_{t+1} = & \underbrace{w_t \frac{n_t}{1-\rho_t^e} \int_{-\infty}^{v_t^k} g(\varepsilon_t) d\varepsilon_t}_{\text{Wages of full-time workers}} + \underbrace{w_t \frac{n_t}{1-\rho_t^e} \int_{v_t^k}^{v_t^f} (1 - K^*(\varepsilon_t)) g(\varepsilon_t) d\varepsilon_t}_{\text{Wages of STW workers}} \\
& + \underbrace{b_t \frac{n_t}{1-\rho_t^e} \int_{v_t^k}^{v_t^f} K^*(\varepsilon_t) g(\varepsilon_t) d\varepsilon_t}_{\text{Unemployment benefits of STW workers}} + b_t u_t + R_t B_t + \Pi_t - T_t,
\end{aligned}
\tag{15}
$$

in which C_t is consumption, B_t a government risk-free bond, R_t a gross return of the bond, Π_t the profits from firms which the household owns, and T_t a lump sum tax.

The household derives utility from consumption. The utility function is named $U(C_t)$, which the household maximizes over time, by choosing consumption C_t and the government bond purchases B_{t+1} subject to a budget constraint in equation (15). The first order conditions for consumption and bonds yield to standard consumption Euler equation as

$$
\frac{1}{R_{t+1}} = \beta E_t \frac{U'(C_{t+1})}{U'(C_t)}, \tag{16}
$$

where U' is the derivative of utility function w.r.t consumption.

Respectively, the government pays unemployment benefits not only to unemployed workers, but also for hours not worked to workers in STW. The benefits are financed by lump sum tax and borrowing. The government runs a balanced budget, with budget constraint given by

$$
b_t \frac{n_t}{1-\rho_t^e} \int_{v_t^k}^{v_t^f} K^*(\varepsilon_t) g(\varepsilon_t) d\varepsilon_t + b_t u_t + R_t B_t = T_t + B_{t+1}. \tag{17}
$$

All output which is left after the frictional costs, is consumed. The frictional costs are subtracted in equation (14). Hence, the aggregate budget constraint is simply

$$
Y_t = C_t. \tag{18}
$$

3.3.3 Shocks and functional forms

This section defines the exogenous shock processes which are estimated later in the paper. In addition, the functional forms which are not presented before in the model section are specified here.

The matching function is defined to take Cobb-Douglas form, given by

$$m_t = \mu_t u_t^{\alpha} v_t^{1-\alpha}, \tag{19}$$

where m_t is the number of matches, μ_t a matching efficiency, and α the elasticity of matches w.r.t. to unemployment. The matching efficiency is subject to an exogenous shock. Moreover, the matching function implies a vacancy filling rate as

$$q(\theta_t) = \mu_t \theta_t^{-\alpha}. \tag{20}$$

The second function which is specified, is the household's utility function. It is assumed to take CRRA form as

$$U(C_t) = \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} \right), \tag{21}$$

in which σ is a risk-aversion parameter.

There are three exogenous shocks in the model, productivity shock, workers' outside option shock, and matching efficiency shock. These shocks are selected such that they can be identified, since they generate distinct responses of the variables of the model. Furthermore, all of the shocks are assumed to take $AR(1)$ form.

Productivity shock is imposed on the common productivity component, for instance in output equation (14), and given by

$$a_t = (1-\rho^a)\bar{a} + \rho^a a_{t-1} + \epsilon_t^a, \qquad \epsilon_t^a \sim \mathcal{N}(0, \sigma^a), \tag{22}$$

where ρ^a is the persistence parameter, $\bar{a}$ a steady-state value of the common productivity a_t, and ϵ_t^a an independently and identically distributed shock with standard deviation σ^a.

Workers' outside option, i.e. the unemployment benefit b_t in the wage rule (13), is subject to the workers' outside option shock as

$$b_t = (1-\rho^b)\bar{b} + \rho^b b_{t-1} + \epsilon_t^b, \qquad \epsilon_t^b \sim \mathcal{N}(0, \sigma^b), \tag{23}$$

in which ρ^b is the persistence parameter, $\bar{b}$ a steady-state value of the unemployment benefit b_t, and ϵ_t^b an independently and identically distributed shock with standard deviation σ^b.

Finally, the matching efficiency shock is imposed on the matching efficiency μ_t in the matching function (19), and given by

$$\mu_t = (1-\rho^{\mu})\bar{\mu} + \rho^{\mu} \mu_{t-1} + \epsilon_t^{\mu} \qquad \epsilon_t^{\mu} \sim \mathcal{N}(0, \sigma^{\mu}), \tag{24}$$

where ρ^μ is the persistence parameter, $\bar{\mu}$ a steady-state value of the matching efficiency, and ϵ_t^μ an independently and identically distributed shock with standard deviation σ^μ.

3.4 Estimation and simulation

The model is estimated using Bayesian techniques. The model is linearized around a deterministic steady state. I choose the prior distributions for the estimated parameters and use the Metropolis-Hastings algorithm to estimate their posterior distributions. Furthermore, the estimation yields shock processes which are accountable for the fluctuations in the model. These shock series are used to simulate a counterfactual model without the STW policy. The estimation, numerical solution of the model, and the simulation of counterfactual model are done using Dynare software version 5.0 (Adjemian et al., 2022).

3.4.1 Data

The estimation uses monthly data from Germany between January 2000 and November 2021. The observed variables are the unemployment rate u_t, the vacancy rate v_t, and the STW rate χ_t. All the data is retrieved from the Bundesbank statistics database, and provided there by the Federal Employment Agency. The unemployment rate is available as such, but the vacancy rate and STW rate are calculated from the levels, by using an additional data series of employed workers. For vacancies, we should note, that the Federal Employment Agency has data about the official vacancies opened through the public employment agencies, and hence lacks vacancies which are only available through private search channels (Merkl and Sauerbier, 2023). (Details in appendix 3.B.1).

A peculiarity of the data is, that the German unemployment series has a strong declining trend between mid-2005 and mid-2019. This trend is attributed to a series of labor market reforms, known as the Hartz reforms (see Krause and Uhlig (2012), Krebs and Scheffel (2013), Launov and Wälde (2013) and Hochmuth et al. (2021) for the details of the reform). Since this paper studies cyclical fluctuations, with the primary time-period of interest being the Covid-19, the trend is removed in a following way. First, the linear trend between the beginning of the sample and the beginning of the Covid-19 in February 2020 is removed. Then, the Covid-19 period is attached at the end of the detrended series. Finally, the new observation series is demeaned. The described approach to process the unemployment data is used, in order to capture precise fluctuations during the Covid-19, which is at the very end of the sample. For instance, HP-filter results in a significant boom before a bust of the Covid-19, which is counterfactual, as the German unemployment rate remains completely stable between 4.9–5% for 16 months preceding the Covid-19 pandemic. (See also appendix 3.B.2).

In addition, the Hartz reforms have resulted in an increasing trend in vacancy rate series. The vacancy rate trend is less considerable, and also more stable throughout the whole sample as compared with the unemployment rate data. Hence, the linear trend is removed from the whole series. Furthermore, the vacancy rate series is also demeaned. The STW series is not detrended or demeaned, since the participation in the policy is very low outside recessions. More precisely, clearly less than 1% of employed workers are in STW during expansions, and the average STW rate over the whole sample, including the Great Recession and the Covid-19 is only 0.78%. In turn, the increases in STW participation during the two big recession periods are significant, as approximately up to 4% and 14% of employed workers were in STW during the Great Recession and the Covid-19 respectively. This is clearly visible in Figure 3.2, which depicts the data series used in estimation.

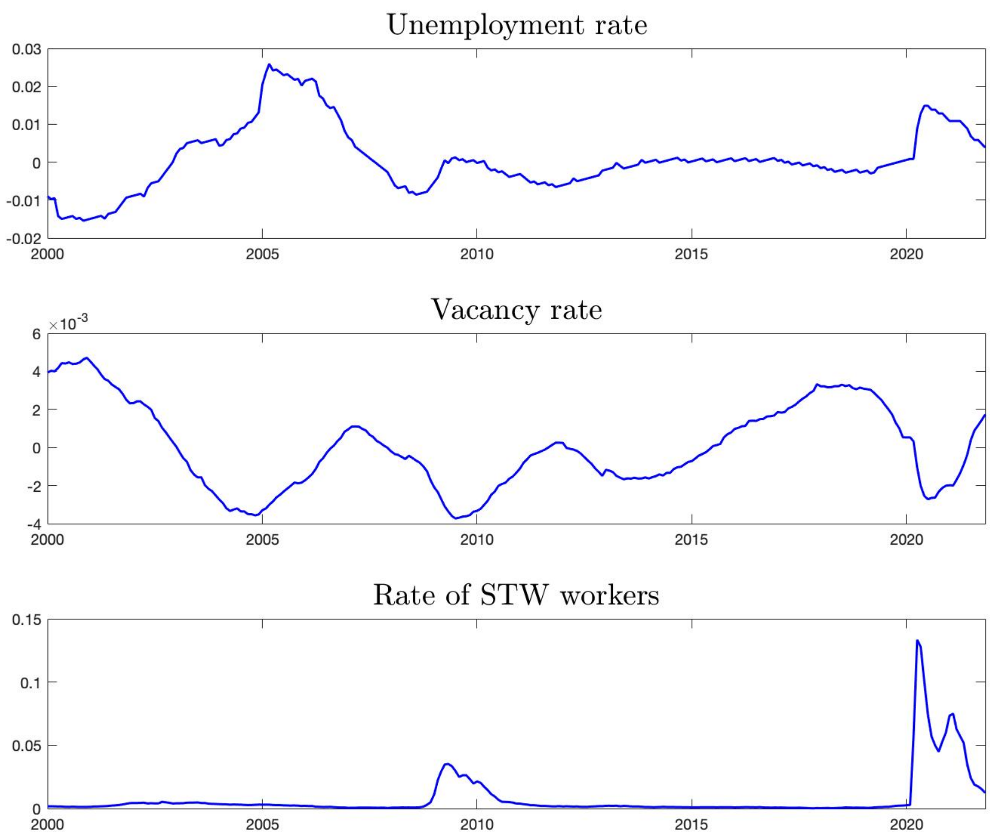


Figure 3.2: Observed variables in estimation. The unemployment and vacancy rate detrended and demeaned cyclical components.

The shift in trend which follows the Hartz reforms can be dated to begin around year 2005. The choice to remove the linear trend from the beginning of the sample, which is earlier than the shift can then result in overemphasizing the fluctuations in early 2000s. On one hand, this may lead to parameter estimates which exaggerate the tightness of criterion to participate in STW policy, since there is no significant volatility in the STW participation in the early 2000s. On the other hand, the higher volatility in the sample improves identification. Furthermore, the lower volatility at the beginning of the sample could increase the relative magnitudes of estimated shocks affecting the Great Recession and the Covid-19 fluctuations, which would then lead to an even larger increase in unemployment in counterfactual simulations. Hence, this filtering method is not likely to result in an overestimation of the stabilization effect of STW during the large recessions.

3.4.2 Calibration

This section presents the calibrated parameter values. As the number of observables in the estimation is limited, and these observables are related to labor markets, there is insufficient information to identify all parameters in the Bayesian estimation. Hence, a number of parameters are calibrated. In addition, the steady-state targets for variables are chosen.

Table 3.1 summarizes the calibration, which closely follows the original article by Balleer et al. (2016). The subjective discount factor is chosen to match annual risk-free interest rate of 4.5%. I follow Balleer et al. (2016) in setting the fixed cost of production, as there is not enough information in the data to estimate this cost. The same applies to the bargaining power of workers, and to the household's risk aversion parameters. The former is set to a neutral value of 0.5, which equalizes the surplus sharing between workers and firms. The latter is set to a non-informative value of 1.

The monthly vacancy rate in the data is 0.0111, which further implies the steady-state separation rate. Following the arguments in Den Haan et al. (2000), approximately two-thirds of the total separations are assumed exogenous and one third endogenous, i.e. here 0.01 and 0.006 respectively. Moreover, the distribution of idiosyncratic productivity is assumed to be a logistic distribution, which is the definition in Balleer et al. (2016). The benefit of the logistic distribution is that it is close to normal distribution with thicker tails but allows for analytical solutions. The unconditional mean of the distribution is set to zero, and the standard deviation, also named a scale parameter in a logistic distribution case, is set to 1.029 following Balleer et al. (2016). Moreover, the chosen calibration results in profit losses from the workers in STW, while the full-time jobs are creating positive profits.

Parameter	Description	Value	Source
β	Discount factor	0.996	Annual risk-free rate 4.5%
c_f	Fixed cost of production	0.225	Balleer et al. (2016)
γ	Workers' bargaining power	0.5	Balleer et al. (2016)
σ	Risk aversion	1	Balleer et al. (2016)
ρ^X	Exogenous separations	0.01	Monthly vacancy rate 0.0111
s	STD of idiosyncratic distribution	1.029	Losses in STW and profits in FT
μ^g	Mean of idiosyncratic distribution	0	Losses in STW and profits in FT
g	Idiosyncratic distribution	Logistic	Balleer et al. (2016)
Steady-state target			
$\bar{\mu}$	Matching efficiency	0.433	Monthly $q = 0.70$, $v = 0.0111$
$\bar{u}$	Unemployment rate	0.078	Average 2000-2021
$\bar{\rho^e}$	Endogenous separations	0.006	Monthly vacancy rate 0.0111
$\bar{\chi}$	Rate of STW	0.0078	Average 2000-2021
$\bar{K}^*$	Working time reduction	0.33	Balleer et al. (2016)
$\bar{A}$	TFP	1	Standard in the literature
$\bar{b}$	Unemployment benefit	0.65	Replacement rate 65%

Table 3.1: Calibration

The steady-state target of unemployment rate is set to 7.8% which is the mean in the data. Jointly, the aforementioned vacancy rate and unemployment rate imply a steady-state labor market tightness. Further, the matching efficiency in the steady state is set to 0.433, in order to target a vacancy filling probability around 0.7 and job finding probability around 0.2. We should note here that the matching efficiency is subject to an exogenous shock which is estimated later, lowering the significance of the steady-state calibration.

The steady-state rate of workers in STW is set to 0.78% of employment, which is the average in the data. This is 0.1 percentage points higher than the estimate in Balleer et al. (2016) due to the impact of the Covid-19 recession. However, the working hour reduction is set to one-third of full working time as in Balleer et al. (2016). One third, is approximately the long-term average that applies also during the Great Recession, even if the hour reduction is momentarily higher during the Covid-19 (Herzog-Stein et al., 2021).

Finally, the steady state of common productivity component $\bar{A}$ is set to one, which is standard in the macro-literature. The steady-state target of unemployment benefit is set to 0.65 which implies approximately 65% replacement rate. Both these variables are subject to an exogenous shock processes, which are estimated later.

3.4.3 Priors

The parameters for estimation are the separation cost f, the STW policy criterion $\bar{J}$, the elasticity of matching function α, the vacancy posting cost κ, and hour reduction cost c_k. The costs are estimated as they cannot be directly observed from the data. Especially, the parameters of interest are the separation cost f and the STW criterion $\bar{J}$, since these two dictate the choices between job destruction and STW.

	Prior			Posterior			
Parameter	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
f	gamma	2.400	0.5000	2.681	0.0671	2.5816	2.7899
$\bar{J}$	normal	-2.000	0.2000	-0.874	0.0722	-0.9945	-0.7582
α	beta	0.600	0.1000	0.968	0.0076	0.9552	0.9781
κ	gamma	1.200	0.1000	1.383	0.0161	1.3570	1.4066
c_k	normal	22.000	0.1000	21.984	0.0975	21.8321	22.1433

Table 3.2: Results from Metropolis-Hastings, parameter priors and posteriors

Table 3.2 contains the prior distributions and the posterior results from the Metropolis-Hastings algorithm. Parameters f and κ are costs, so they can only get a positive real value, hence the prior is set to follow a gamma distribution. I choose the prior means of the separation cost f and the vacancy cost κ, based on the calibration in Balleer et al. (2016), as values 2.4 and 1.2 respectively. The standard deviation of the separation cost is set to 0.5, which is relatively loose allow for potentially large moves due the big recessions in the sample, especially the Covid-19 recession. I set the standard deviation of vacancy cost κ to 0.1.

A beta distribution prior is assigned to the matching elasticity parameter α which can only get values on the unit interval. The prior mean is set to 0.6, which is supported by the survey findings by Petrongolo and Pissarides (2001) for large European economies, and in addition is the calibration value in Balleer et al. (2016). A relatively loose standard deviation of 0.1 allows the values from the whole unit interval.

The value of STW criterion $\bar{J}$ can, at least in theoretical sense, be either positive or negative, hence its prior is set to follow normal distribution. As discussed in the model section of this paper, the relationship between the separation cost f and the STW criterion $\bar{J}$ is particularly interesting. More specifically, the situation, in which the STW participation is an efficient option for separations requires that $\bar{J} = -f$. Since, the policy makers have an incentive to change the STW criterion to more generous during the economic downturns, I choose the prior mean of $\bar{J}$ higher than $-f$, and set the value as -2.0. In addition, the standard deviation is set to 0.2, resulting in a wider distribution, in order to allow a significantly more generous criterion, which can be assumed to be a result of the Covid-19 period in the sample.

Finally, the most peculiar parameter for the estimation is the hour reduction cost c_k. Correspondingly with the other costs, it is assumed to be positive. However, the calibration in Balleer et al. (2016) uses rather large value of 22.0 of this parameter. Hence, I choose a normal distribution with a mean of 22.0 and a standard deviation of 0.1. This prior yields to a symmetric distribution between lower and higher values, but simultaneously the negative values of the parameter are extremely unlikely.

	Prior			Posterior			
Parameter	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ρ^a	beta	0.800	0.1000	0.977	0.0048	0.9697	0.9853
ρ^μ	beta	0.800	0.1000	0.995	0.0018	0.9930	0.9987
ρ^b	beta	0.500	0.1000	0.173	0.0252	0.1332	0.2133
σ^a	invg	0.100	Inf	0.017	0.0015	0.0140	0.0187
σ^μ	invg	0.100	Inf	0.042	0.0019	0.0387	0.0446
σ^b	invg	0.100	Inf	0.834	0.0610	0.7438	0.9370

Table 3.3: Results from Metropolis-Hastings, shocks

Table 3.3 contains the prior distributions and the posterior results of the shock processes. Priors are chosen following the tradition in the literature about the Bayesian estimation of structural macro models. More specifically, the persistence parameters are set to follow a beta distribution as they can get values on a unit interval, and the standard deviations follow an inverse gamma distribution. Furthermore, the latter parameters have identical priors as the means are set to 0.1 and standard deviations to infinity. For the productivity shock persistence ρ^a, and the matching efficiency shock persistence ρ^μ, the prior means are set to 0.8 and standard deviations to 0.1. In order to allow the estimation of only the shock processes without structural parameters for the simulation purposes later, the third shock, i.e. the workers' outside option shock has a different prior with a mean of 0.5 and a standard deviation of 0.1. This specification of priors avoids problems of symmetricality when estimating only the shocks. In addition, the productivity and matching efficiency shocks are present in the literature with dynamic labor market models, in which they are usually assumed to be persistent. For instance, Lubik (2009) sets a prior persistence to a high value of 0.9 for both shocks in a related model. In turn, the shock on unemployment benefit is less common, which supports a weakly-informative prior.

3.4.4 Posteriors

The posterior estimates of means, standard deviations, and 5th and 95th percentiles are presented with the priors in Tables 3.2 and 3.3. Posterior estimates for the separation cost f, the STW criterion $\bar{J}$, the matching elasticity α, the vacancy cost κ, and all the parameters of structural shocks are not overlapping with the prior implying a good identification of these parameters. In turn, the hour reduction cost c_K seems to be converging to a value following the prior, which suggest that there is insufficient information in the observation series for identification of this parameter value. However, the other estimates seem to be robust, whether the c_K parameter is part of the estimation set or not, hence it is not likely to impact the outcome of the estimation, and simulation exercise later.

The posterior estimate of separation cost f is slightly higher than the prior, more specifically, approximately 2.7 against 2.4 respectively. In turn, the posterior mean of STW criterion $\bar{J}$ is considerably higher than the prior, i.e. approximately -0.9 versus -2.0. The values of both of these posteriors are most likely impacted by the large recession periods in the sample, during which the STW has been specifically incentivized by the government. When the benefits of participation in STW are increased, the relative cost of separation has also increased, which is shown in the estimate of parameter f.

The matching efficiency parameter α has a posterior which is significantly moved from the prior value. The posterior value of 0.97 is very high, suggesting that the increase in unemployment affects the new matches in almost one-to-one relation. This may be a result of the existence of STW in the model, since the policy stabilizes unemployment fluctuations. Since the changes in unemployment are smaller due to STW, the job matches become more reactive to these changes leading to a high elasticity estimate.

Finally, the two remaining parameters, the vacancy cost κ and the hour reduction cost c_K are estimated as being close to their prior values. The vacancy cost increases from a prior value of 1.2 to approximately 1.4 as posterior mean, with a narrow confidence interval, suggesting a good identification. In turn, as mentioned above, the hour reduction cost is not well identified, which is implied by the posterior values that follow closely the prior.

The shock parameters are all well identified. The productivity shock A_t and the matching efficiency shock μ_t have high persistences of 0.977 and 0.995 respectively, and low standard deviations of 0.017 and 0.042 respectively. The third shock, the shock on unemployment benefit b_t, is more peculiar as it is very transient with persistence of 0.173, but with a considerably high standard deviation of 0.834. In other words, it seems that the workers have occasionally experienced relatively strong, but very short lived shifts in their outside option. These types of temporary changes in unemployment benefits may be the result of policies which have been implemented during the large recessions, such as the Great Recession or the Covid-19 recession.

3.4.5 Simulation

The purpose of the simulation exercise, is to compare the economy with STW with a counterfactual economy without the policy. The strategy is the following. The previous section described the estimation process. As one of the outcomes, the estimation yields the time-series of the shocks which are accountable for the fluctuations in macro-variables over the sample period. Next, these shock series are imposed on a counterfactual model which does not have STW but is otherwise equivalent with the estimated model. Finally, the responses of the counterfactual model on the estimated shocks are compared with the baseline model that has the STW policy.

The counterfactual model is constructed by removing the STW policy from the estimated model. This has an impact on the steady state of the counterfactual model. Since, the firms no longer have access to STW, job destruction increases i.e., the separation rate shifts from 2.9% to 3.7%. However, simultaneously also job creation is higher, i.e. the vacancy rate increases from 4.1% to 4.8%. Jointly, these two effects change the steady-state unemployment rate from 8.5% in the estimated model to 11.0% in the counterfactual model. The removal of STW has the largest steady-state impact on these three variables. On the contrary, output, the job value, labor market tightness and the vacancy filling rate remain close between the two models, with less than 10% difference in relative magnitudes.

For the simulation procedure, the shock processes are estimated independently from the baseline model with STW. More specifically, the structural parameters which are part of the estimation in previous section, are mostly calibrated to their prior means. The reason for this is that the parameter estimates are induced by the existence of STW. Hence, it is probable that the calibration with posterior means is more accurate for the model with STW but would be less accurate for the counterfactual model. As such, the calibration would be unfavorable to a model without STW, and could overemphasize the role of the policy in stabilizing labor market fluctuations.

Furthermore, as the main period of interest in this paper is the Covid-19 pandemic, the calibration of STW criterion becomes essential. In this regard, the German government was strongly incentivizing firms to participate in STW during Covid-19. Consequently, it can be assumed that the STW criterion was less stringent at the time. Hence, for the shock estimation, I set the STW criterion parameter $\bar{J}$ to a value of -1.4, which is a higher value than the prior mean of -2.4, but lower than the posterior mean of -0.9. The chosen value is a trade-off between two opposite effects. First, the low values, such as the prior of -2.4, would result in unfeasibly large shocks during the recessions in the sample, most importantly the Covid-19 recession. Second, the high values, for instance the posterior of -0.9, would distort the steady-state values far from the long-term averages of observed variables. The chosen level of the parameter $\bar{J}$ results in correlation 0.28 between the wage and output, and 0.24 between productivity and output in the estimated model. These values are more feasible for the Covid-19 period, since the drop in GDP was 4.2% in 2020, while the gross-wages droped 1.3% in 2021. Over the longer period, the wage and output growth are closely correlated.[2]

The final part of the simulation is a choice of the initial-state from which the simulation begins. In general, a Bayesian estimation rarely results in an initial-state of the model which equals the calibrated steady-state. Hence, the initial-state of counterfactual simulation is set to have the same relative deviation from the steady-state as the estimated initial-state of the STW model. In addition, in order to obtain simulated shock decompositions, the simulation process is repeated with one shock series at a time. These simulations are set to begin from the steady-state instead of the initial-state, in order to make them comparable.

3.5 Results and analysis

This section discusses the results of the estimation outcomes and the simulation exercises described in the previous section. The main results of this paper are related to the simulation results, so those are analyzed first. However, the structural parameter estimates have interesting policy implications in their own right, which are considered at the end of the section.

3.5.1 Stabilization of labor markets

Figure 3.3 shows an illustration of the unemployment rate increase during Covid-19. In order to compare more easily the two economies, the counterfactual unemployment rate has been scaled to 5% in January 2020. From this starting point, the unemployment rate would have increased to over 10% without STW during the spring 2020. The right panel of Figure 3.3 shows the sharp increase in the STW level, which emphasizes the difference between the two economies. In the counterfactual economy, adjustment happens through job destruction, while in the estimated model the working hours are adjusted with STW.

[2]Source: https://www.wsi.de/de/loehne-und-gehaelter-14576-entwicklung-der-bruttostundenloehne-in-deutschland-ab-1970-26336.htm

Figure 3.3: An illustration of unemployment increase during the Covid-19 is presented in the left panel. The blue line (below) is the observed unemployment rate, and the red line (above) the simulated counterfactual rate. For the comparison, the counterfactual unemployment rate has been scaled to 5% in January 2020. The right panel depicts the observed increase in STW level.

Figure 3.4 shows the comparison of the observed variables in the estimation with the counterfactual simulation outcomes of the same variables, over the whole sample period. In brief, the maximum increase in unemployment rate during the Covid-19 recession is 5.6 percentage points without STW, instead of 1.4 percentage points with the STW policy. Likewise, the stabilization can be detected during the Great Recession when the increase in unemployment rate is 2.3 percentage points without STW, instead of 1 percentage points in the data.

Considering the whole sample period, the contribution of STW in stabilizing labor markets is investigated by comparing the variances of the observed and simulated series. In this comparison, the variance of unemployment is 2.3 times higher in the economy without STW, and the variance of separation rate is 2.1 times higher. These results confirm that the STW policy effectively participates in preventing job losses, and mitigation of unemployment fluctuations.

In addition, the volatility of the separation rate is asymmetrically lower in the economy with STW, i.e., the increases during recessions are lower. This has important welfare implications. Hairault et al. (2010) show that the separation rate is asymmetric over business cycles in the US economy. More specifically, the separation rate decreases more during expansions than it increases during recessions. Consequently, the higher volatility of separations leads to lower average unemployment and lower business cycle costs. Here, the lower volatility due to STW increases this asymmetry by lowering the increases in recessions, which has an analogical effect in lowering business cycle costs and improving welfare.

Figure 3.4: A comparison of the observed variables in estimation, and their simulated counterparts in the counterfactual model without STW. The graphs show the absolute deviations from the steady states of the corresponding models.

An interesting feature shown in Figure 3.4 is that it reveals a known challenge related to vacancy creation in the search and matching models with endogenous separations. The counterfactual model, i.e. the red line in the graph, first has a drop in vacancy rate at the beginning of recessions, which is followed by a significant jump almost immediately after the initial drop. The reason is that the increase in separations, for instance in Figure

3.5, sharply increases unemployment and decreases labor market tightness, which in turn incentivizes vacancy postings. This effect is visible in the simulated series even when the shocks are estimated from the data with a smooth decline in the vacancy rate during recessions.

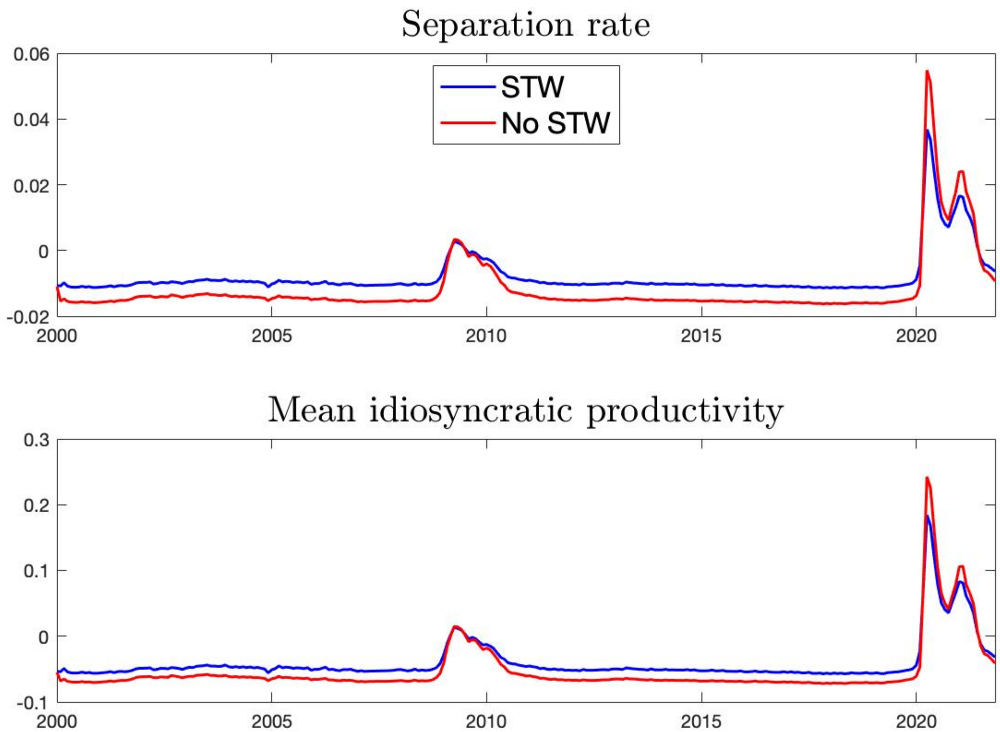


Figure 3.5: A comparison of separation rate and the firm level productivity between the models with and without STW. The graphs show the absolute deviations from the steady-states.

Figure 3.5 shows the comparison of separation rate and firm level productivity in the economies with STW and without the STW policy. It presents one of the trade-offs of STW. Job destruction is increasing more when the STW policy does not exist. Correspondingly, the firms are increasing the firm specific productivity more, as a response to negative shocks, when the labor hoarding is not subsidized with STW. The productivity increase is a result of firms destroying a larger number of low productive jobs, and simultaneously creating a larger number of jobs with higher productivity. To be exact on the size of this effect, the mean idiosyncratic productivity increases up to 30% more during the Covid-19 recession in the economy without STW.

3.5.2 Shock decompositions

The second simulation exercise imposes the three shock series on the counterfactual model one shock at a time. The outcome of these simulations is compared with the corresponding historical shock decompositions from the estimated model. Figure 3.6 shows the shock decompositions of the unemployment rate and the separation rate from the two models. Out of the three shocks, the TFP and matching efficiency accounts for the largest share of the volatility in unemployment. In turn, the matching efficiency shock does not contribute to the fluctuations in the separation rate. In addition, the contribution of each shock on both models follows the same patterns. However, the magnitudes are systematically larger in the model without STW.

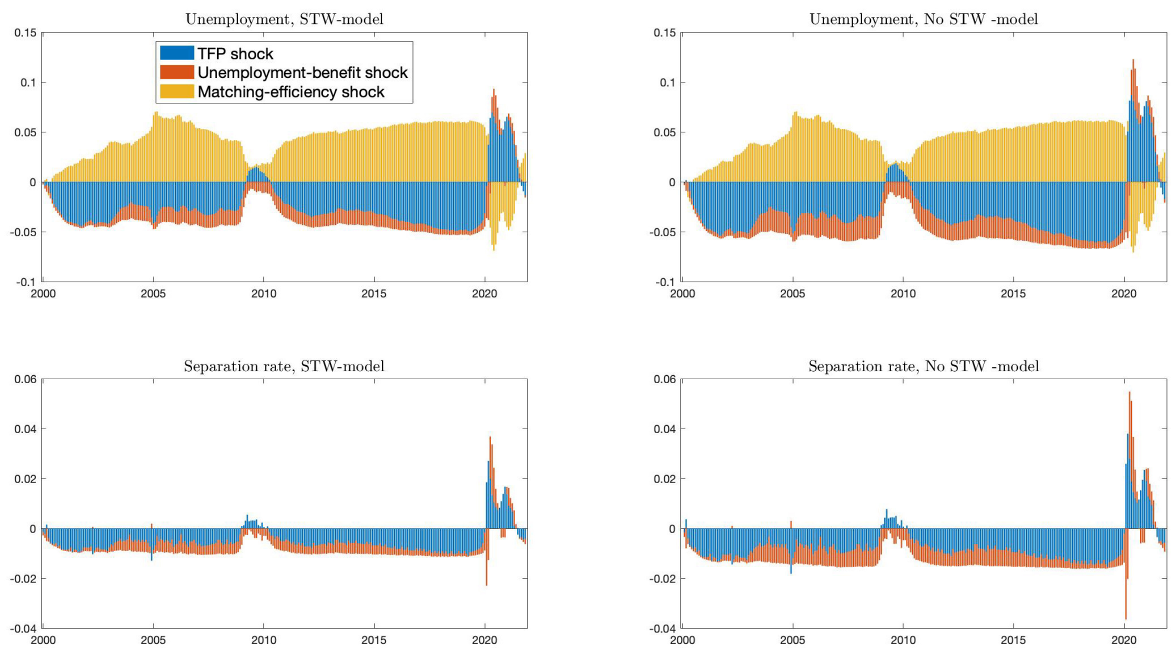


Figure 3.6: The shock decompositions of unemployment rate and separation rate in the two models, with and without STW. The scales are the absolute deviations from the steady states.

In order to observe more specifically the difference in magnitudes of each shock in the two models, Figure 3.7 presents the contributions of the three shocks on unemployment. As discussed in the previous section the total differences in unemployment rates between the model with and without STW are significant, especially during the Covid-19 crisis, but also over the whole sample period. The shock decomposition shows that the difference between the two models arises from the productivity and the workers' outside option shocks. In turn, the impact of the matching efficiency shock on unemployment is identical in both models. In other words, the STW policy does not increase the propagation of the changes in the labor market matching on unemployment. The matching efficiency either increases or decreases the job finding probability for unemployed workers, and hence the relative effect is equivalent if the labor market tightness is the same in both economies. This is the case if the number of vacancies and the rate of unemployment are systematically higher in the economy without STW, compared with the economy with the policy.

The shock decompositions also indirectly confirm that the economies in estimation and simulation are identical, notwithstanding the STW policy. The above discussed matching efficiency shock has the same effect on unemployment, but the other decompositions also point out that it does not have an impact on the STW rate, and more surprisingly on the vacancy filling rate. The reason is that the matching efficiency shock makes vacancies and unemployment move with the same proportions, perfectly compensating each other. Moreover, as the shock has no impact on the STW rate, or working hours in STW, it seems clear that it makes no difference between the two economies, in which the only difference is the STW policy. (See Appendix 3.C.1 for more details.)

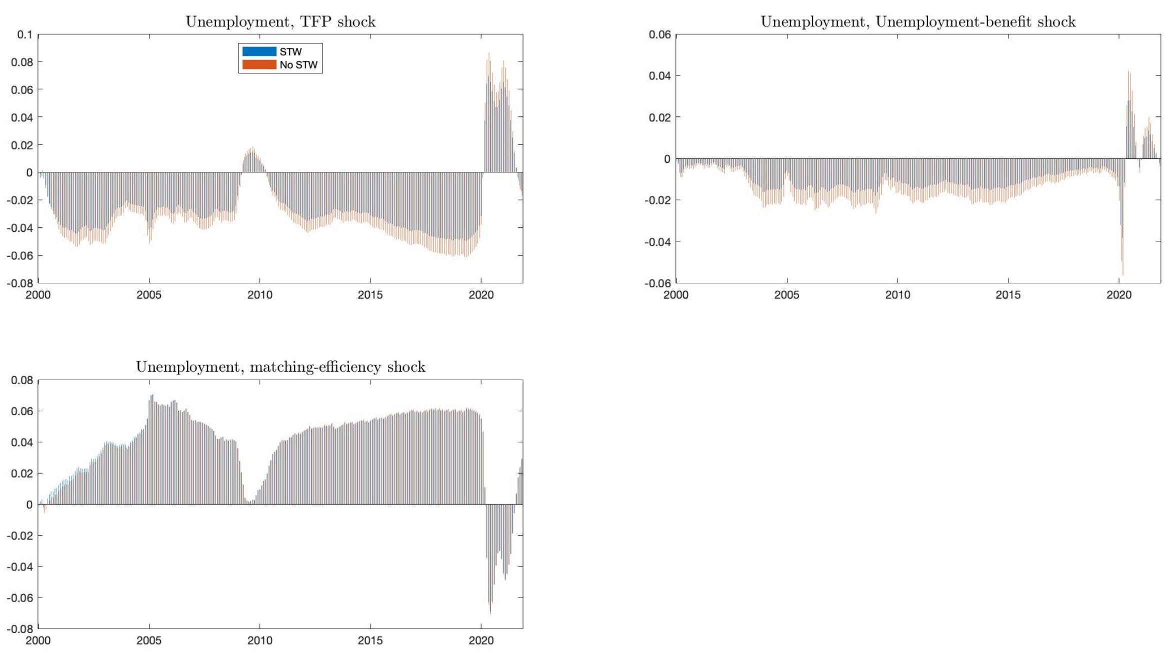


Figure 3.7: The impact of different shocks on the unemployment rate in the two models, with and without STW. Monthly impact on each model is depicted side by side, in order to compare the magnitudes. The scales are the absolute deviations from the steady-states.

3.5.3 Parameter estimates

This sections discusses two key parameter estimates namely the STW criterion $\bar{J}$ and the separation cost f. The priors and posteriors are presented in Table 3.2. The estimates of these two parameters are particularly interesting, since they capture if the STW criterion allows a larger number of workers in STW than the number of jobs which are saved from separations. This indeed seems to be the case.

As argued in the model section, if the parameter values are set such that $\bar{J} = -f$, the choice of STW is a true alternative to separation. However, the posterior mean estimates are approximately $\bar{J} = -0.9$ and $f = 2.7$, i.e. the STW criterion is higher than the cost of separation. This allows the situation in which the value of the job is low enough for STW, but too high for separation. More specifically, let us consider a job with value J_t such that $-f < J_t < \bar{J}$, which is possible given the estimated parameter values. Now, this job meets the STW condition in equation (2), meaning that the firm sends this worker to STW. Simultaneously, the same job generates less profit losses than the cost of separation, i.e., it does not meet the separation condition in equation (6) even with the full working hours. Hence, the firm has no incentive to separate with the worker, but is still reducing the working hours of this job in STW, because the policy criterion allows this.

The estimated values of the STW criterion and separation costs suggest that the policy is too slack. Hence, the policy does not prevent job losses, but allows firms to participate opportunistically in the STW program, in order to increase profits. However, this result is heavily influenced by the two large recession periods in the sample, especially the Covid-19 recession, when the German government deliberately incentivized higher participation in the program. Consequently, these results propose that if the STW policy is expanded during the economic downturns, the duration of this expansion should be carefully limited to avoid the fiscal deadweight costs.

3.6 Conclusion

The short-time work policy (STW) stabilized unemployment and prevented excess job destruction in Germany during the Covid-19 recession. This paper investigates the effectiveness of STW during Covid-19 by estimation of a general equilibrium labor market model with STW using Bayesian techniques, in order to obtain a series of shocks accountable for the labor market fluctuations. These shock series are then imposed on a counterfactual model without STW to detect that the unemployment rate in Germany would have been 4.2 percentage points higher during the Covid-19 in the absence of the STW policy.

This paper confirms results reported earlier in the economic literature about the contribution of STW programs in stabilizing unemployment. In addition, I find some trade-offs of the policy. For instance, the firms may not increase their performance sufficiently by creating jobs with high productivity if the government is subsidizing the working time reductions of current workers with STW. Furthermore, if the STW policy is overly generous, the firms may increase their profits by participating in the program, and as such cutting the labor costs with subsidized working time reductions, even when the jobs in STW are profitable and not in danger of layoffs. Hence, the results in this paper indicate the need for a further investigation into the optimal level of the policy. I leave these considerations for future research.

Appendix

3.A Model

This appendix presents the details of the model derivation. The model is from Balleer et al. (2016), and detailed information about the model is available in the original article and its Appendix.

3.A.1 Firms and labor markets

Value of a job or worker with idiosyncratic realization ε_t is J_t as

$$J_t = a_t - w_t - \varepsilon_t - c_f + \beta E_t J_{t+1} \tag{A.1}$$

in which a_t is TFP, w_t a wage, c_f fixed cost of production and β a subjective discount factor of households. ε_t is drawn each period to each job surviving exogenous separations from a time-invariant distribution with CDF $G(\varepsilon)$ and PDF $g(\varepsilon)$.

The government imposes a criteria of eligibility for workers in STW. These criteria is presented by an exogenously given parameter $\bar{J}$. Workers with value less than $\bar{J}$ can be sent to STW, yielding a following condition

$$a_t - w_t - \varepsilon_t - c_f + \beta E_t J_{t+1} < \bar{J} \tag{A.2}$$

A threshold value v_t^k for the idiosyncratic shock ε_t is solved as

$$v_t^k = a_t - w_t - c_f + \beta E_t J_{t+1} - \bar{J} \tag{A.3}$$

The firm chooses between STW and full-time work such that, if $\varepsilon_t > v_t^k \Leftrightarrow J_t < \bar{J}$, the worker is sent to STW , and if $\varepsilon_t < v_t^k \Rightarrow J_t > \bar{J}$ the worker is regular full time worker.

A firm chooses the optimal level of working-time reduction by maximizing the contemporaneous profit of a worker in STW. Maximization problem is given by

$$\max_{K(\varepsilon_t)} \ \mathit{\Pi}_t = (a_t - w_t - \varepsilon_t)(1 - K(\varepsilon_t)) - c_f - C(K(\varepsilon_t)), \tag{A.4}$$

in which $K(\varepsilon_t)$ is the share of working hours reduced from full working time and $C(K(\varepsilon_t))$ is the cost of reducing $K(\varepsilon_t)$ units of working time.

Defining a quadratic cost function as

$$C(K(\varepsilon_t)) = c_k \frac{1}{2} K(\varepsilon_t)^2, \tag{A.5}$$

in which c_k is a fixed unit cost of work time reduction, the optimal hour reduction choice is given by

$$K^*(\varepsilon_t) = -\frac{a_t - w_t - \varepsilon_t}{c_k}. \tag{A.6}$$

The least profitable workers are in STW, which reduces the losses they generate. If these losses, albeit the hour cuts, are larger than the cost of layoff, the worker and the firm separate, and the job is destroyed. Assuming a separation cost f, the condition of layoff becomes

$$(a_t - w_t - \varepsilon_t)(1 - K^*(\varepsilon_t)) - C(K^*(\varepsilon_t)) - c_f + \beta E_t J_{t+1} < -f, \tag{A.7}$$

from which a separation threshold of idiosyncratic component named v_t^f can be solved as

$$v_t^f = a_t - w_t - c_f + \frac{1}{(1 - K^*(v_t^f))}(f + \beta E_t J_{t+1} - C(K^*(v_t^f))). \tag{A.8}$$

If $\varepsilon_t > v_t^f \Leftrightarrow J_t < -f$, the worker and the firm separate.

The clarify the choices between STW and firing, let us assume $\bar{J} = -f > J_t$, the firm first chooses an optimal hour cut K_t^*. The value of the job after hour reduction K_t^* is named J_t'. If $J_t' > -f$ the worker is sent to STW with $(1 - K_t^*)$ working hours. In turn, if $J_t' < -f$ the worker and the firm separate.

The idiosyncratic shock follows a distribution with PDF $g(\varepsilon)$. Hence, the share of workers in STW is given by

$$\chi_t = \int_{v_t^k}^{v_t^f} g(\varepsilon) d\varepsilon. \tag{A.9}$$

Analogically, the rate of endogenous separation is

$$\phi_t^e = \int_{v_t^f}^{\infty} g(\varepsilon) d\varepsilon, \tag{A.10}$$

Total separation rate is defined as a combination of exogenous rate ϕ^X, and the endogenous rate. Total separation rate is named ϕ_t, and defined as

$$\phi_t = \phi^X + \left(1 - \phi^X\right) \phi_t^e. \tag{A.11}$$

Finally, we can define the expected value of a job before the idiosyncratic shock is realized, which is J_t as

$$J_t = (1-\phi^X)\underbrace{\int_{-\infty}^{v_t^k}(a_t - w_t - \varepsilon_t)g(\varepsilon_t)d\varepsilon_t}_{\text{Full time work}}$$

$$+(1-\phi^X)\underbrace{\int_{v_t^k}^{v_t^f}[(a_t - w_t - \varepsilon_t)(1-K^*(\varepsilon_t)) - C(K^*(\varepsilon_t))]g(\varepsilon_t)d\varepsilon_t}_{\text{STW work}} \tag{A.12}$$

$$\underbrace{-(1-\phi_t)c_f - (1-\phi^X)\phi_t^e f}_{\text{Fixed cost of production and separatoin costs}} \quad +(1-\phi_t)\beta E_t J_{t+1},$$

On the labor markets, the employment evolution follows the canonical search and matching models, and is defined as

$$n_t = (1-\phi_t)(n_{t-1} + m_{t-1}) \tag{A.13}$$

in which n_t is employment and m_t is a number of new matches. The new matches are resulted by a matching function, defined in Cobb-Douglass form, and given by

$$m_t = \mu_t u_t^\alpha v_t^{1-\alpha}, \tag{A.14}$$

in which v is vacancies, μ_t is an exogenous matching efficiency and α is an elasticity of matches with respect to unemployment.

As is typical in this type of labor market models, the labor force is normalized to one and all workers are either employed or unemployed, equation employment and unemployment as

$$u_t = 1 - n_t. \tag{A.15}$$

Labor market tightness is defined as $\theta_t = \frac{v_t}{u_t}$ and a vacancy filling rate becomes

$$q_t = \mu_t \theta^{-\alpha}. \tag{A.16}$$

For the job creation side, the present value of a vacancy is defined as

$$V_t = -\kappa + \beta E_t q_t J_{t+1} + \beta E_t (1-q_t) V_{t+1}, \tag{A.17}$$

in which κ is a fixed vacancy posting cost. A free entry condition is assumed, and it implies that $V_t = 0, \forall\, t$ resulting in a traditional job creation condition, given by

$$\kappa = \beta E_t q_t J_{t+1}. \tag{A.18}$$

Finally, the aggregate output is given by

$$\begin{aligned} Y_t = \frac{n_t}{1-\phi_t^e} \int_{-\infty}^{v_t^k} (a_t - \varepsilon_t) g(\varepsilon_t) d\varepsilon_t \\ + \frac{n_t}{1-\phi_t^e} \int_{v_t^k}^{v_t^f} [(a_t - \varepsilon_t)(1 - K^*(\varepsilon_t))] g(\varepsilon_t) d\varepsilon_t \\ -n_t c_f - \frac{n_t}{1-\phi_t^e} \phi_t^e f - v_t \kappa. \end{aligned} \tag{A.19}$$

3.A.2 Wage bargaining

Wages are a result of collective bargaining, in which the labor union bargains wages for all workers jointly. If the agreement is not reached, the outside option is a strike where workers do not produce any output and the firm pays no wages. In turn, the job relationships are retained. The Nash bargaining problem is given by

$$\arg \max_{w_t} (W_t - \tilde{W}_t)^{\gamma} (F_t - \tilde{F}_t)^{1-\gamma}, \tag{A.20}$$

in which γ is the bargaining power of labor union and the value functions are

$$W_t = w_t + \beta E_t[(1-\phi_t) W_{t+1} + \phi_{t+1} U_{t+1}], \tag{A.21}$$

$$\tilde{W}_t = b_t + \beta E_t[(1-\phi_t) W_{t+1} + \phi_{t+1} U_{t+1}], \tag{A.22}$$

$$F_t = a_t - w_t - c_f + \beta E_t J_{t+1}, \tag{A.23}$$

$$\tilde{F}_t = -c_f + \beta E_t J_{t+1}, \tag{A.24}$$

and the U_t is the value of unemployment for the worker.

In case of a strike, workers are assumed to earn an outside option, which equals the unemployment benefit b_t, but the work contracts are not resigned. The solution of wage bargaining is given by

$$w_t = \gamma a_t + (1-\gamma) b_t. \tag{A.25}$$

3.A.3 Households, the government and closing the model

The households in the model are stylized and maximize utility from consumption. The household's utility function is assumed to take CRRA-form as

$$U(C_t) = \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma}\right), \tag{A.26}$$

in which σ is a risk-aversion parameter. And budget constraint is given by

$$\begin{aligned} C_t + B_{t+1} = &\underbrace{w_t \frac{n_t}{1-\rho_t^e} \int_{-\infty}^{v_t^k} g(\varepsilon_t) d\varepsilon_t}_{\text{Wages of full-time workers}} + \underbrace{w_t \frac{n_t}{1-\rho_t^e} \int_{v_t^k}^{v_t^f} (1 - K^*(\varepsilon_t)) g(\varepsilon_t) d\varepsilon_t}_{\text{Wages of STW workers}} \\ &+ \underbrace{b_t \frac{n_t}{1-\rho_t^e} \int_{v_t^k}^{v_t^f} K^*(\varepsilon_t) g(\varepsilon_t) d\varepsilon_t}_{\text{Unemployment benefits of STW workers}} + b_t u_t + R_t B_t + \Pi_t - T_t, \end{aligned} \tag{A.27}$$

in which C_t is consumption, B_t a government risk-free bond, R_t a gross return of the bond, Π_t the profits from firms which the household owns, and T_t a lump sum tax.

$$\frac{1}{R_{t+1}} = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}. \tag{A.28}$$

The government pays unemployment benefits not only to unemployed workers, but also for hours not worked to workers in STW. The government runs a balanced budget, with budget constraint given by

$$b_t \frac{n_t}{1-\rho_t^e} \int_{v_t^k}^{v_t^f} K^*(\varepsilon_t) g(\varepsilon_t) d\varepsilon_t + b_t u_t + R_t B_t = T_t + B_{t+1}. \tag{A.29}$$

All output, which is left after the frictional costs, is consumed. Hence, the aggregate budgets constraint is simply

$$Y_t = C_t. \tag{A.30}$$

3.B Estimation

3.B.1 Data

The observables used in calibration are retrieved from the Bundesbank website (https://www.bundesbank.de/en/statistics). The following lists the data series.

Unemployment rate. Unemployment registered pursuant to section 16 Social Security Code III / Germany / Social Security Code III and Social Security Code II / Rate / Calendar and seasonally adjusted
Series: BBDL1.M.DE.Y.UNE.UBA000.A0000.A01.D00.0.R00.A
Source: Seasonal adjustment based on data provided by the Federal Employment Agency.

Vacancies. Reported vacancies, total / Germany / Total / Absolute value / Calendar and seasonally adjusted
Series: BBDL1.M.DE.Y.VAC.VBA000.A0000.A00.D00.0.ABA.A
Source: Seasonal adjustment based on data provided by the Federal Employment Agency.,

Employed workers. Employed persons according to ESA 2010 / Germany / Domestic concept / Absolute value / Calendar and seasonally adjusted
Series: BBDL1.M.DE.Y.EMP.EAA000.A0000.A00.D10.0.ABA.A
Source: Seasonal adjustment based on data provided by the Federal Statistical Office.

Short-time workers. Short-time workers, basis for entitlement according to section 96 only / Germany / Social Security Code III / Absolute value / Calendar and seasonally adjusted
Series: BBDL1.M.DE.Y.LMP.LKA100.A0000.A02.D00.0.ABA.A
Source: Seasonal adjustment based on data provided by the Federal Employment Agency.

3.B.2 Detrending

Due to the so-called Hartz reforms there is a long declining trend in the German unemployment rate. A linear trend is removed from the beginning of the sample until the Covid-19, which is attached at the end of the sample. Figure 3.8 illustrates this detrending.

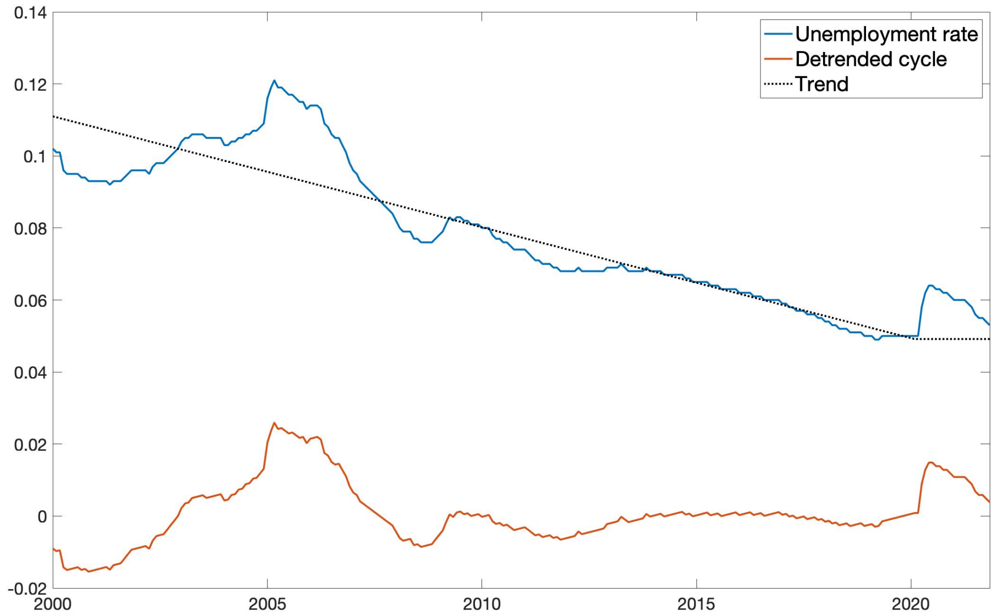


Figure 3.8: Unemployment rate, its trend and detrended cycle.

3.C Simulation

3.C.1 Shock decomposition

This section contains additional shock decomposition graphs of the two models.

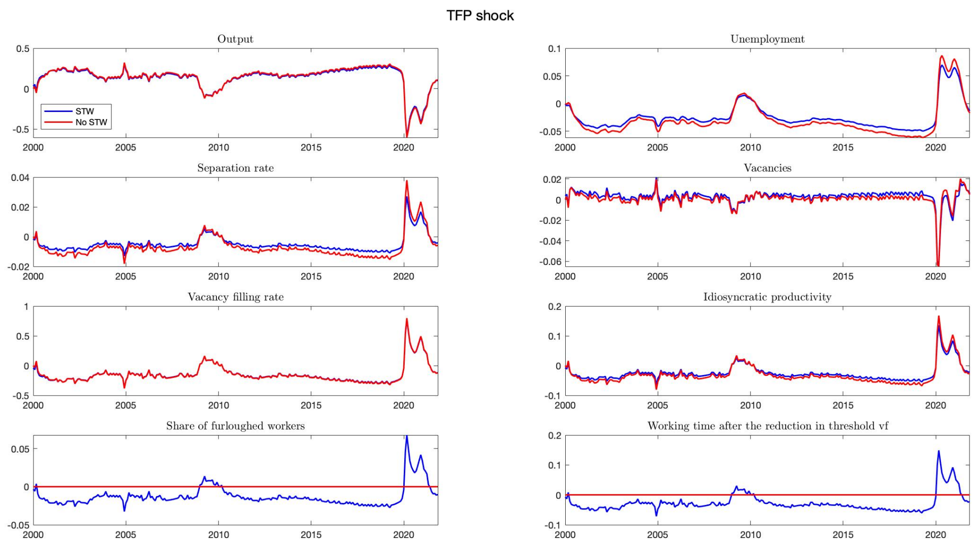


Figure 3.9: Productivity shock.

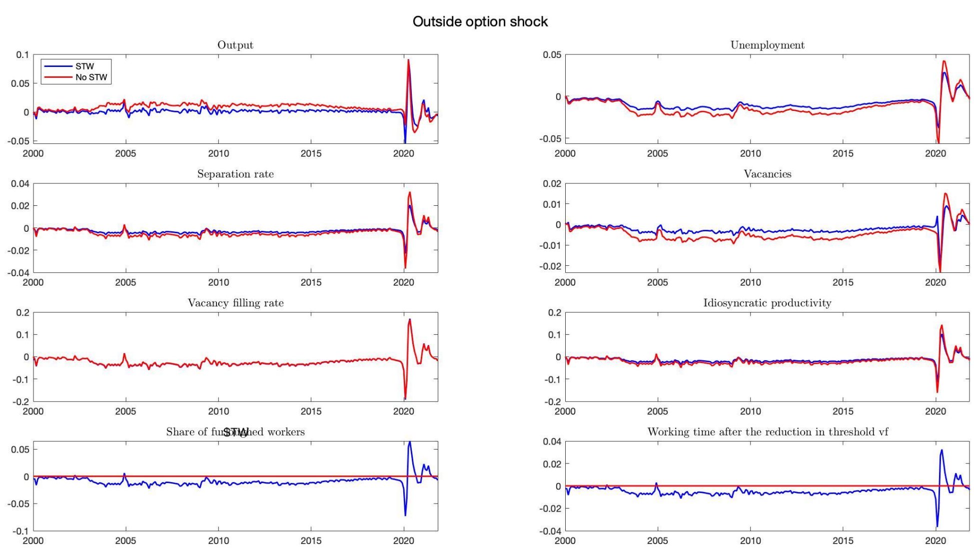


Figure 3.10: Workers' outside option shock.

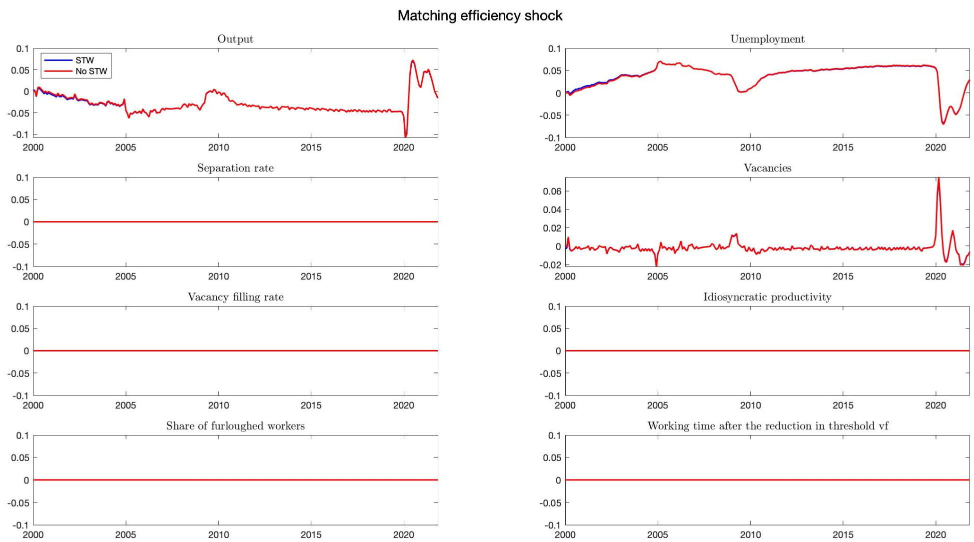


Figure 3.11: Matching efficiency shock.

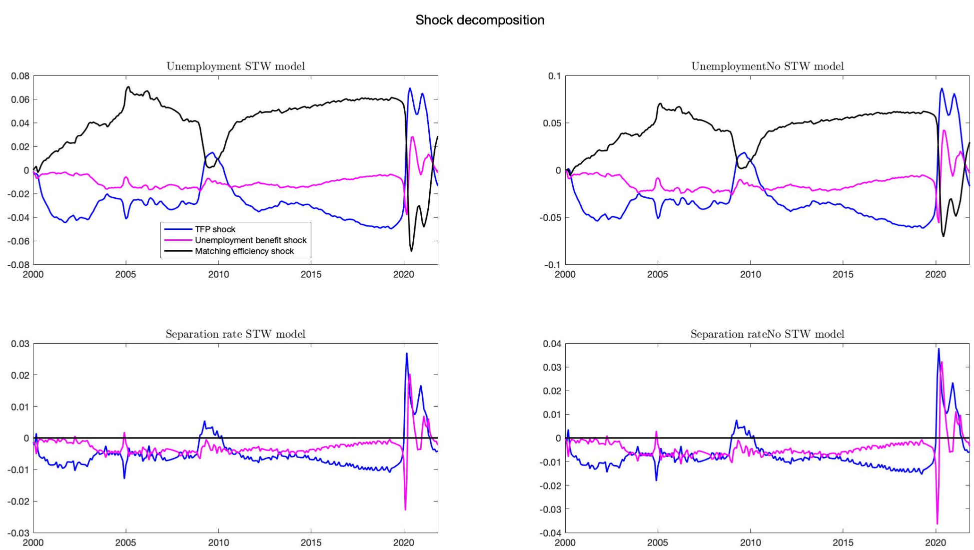


Figure 3.12: Decompositions of the unemployment and separation rates.

Chapter 4

On the efficiency of labor markets with short-time work policies

Abstract

This paper evaluates the social optimality of labor markets with search frictions and a job retention policy, named short-time work (STW), which has been applied in developed economies during the Covid-19 crisis. In a general equilibrium model, costs related to the STW cannot be internalized in wages, creating a systematic inefficiency through which job creation is too low. Government transfers to redistribute output to correct the inefficiency are proposed in the model. Furthermore, a calibration exercise matching the German economy over the period 2000-2021 suggests that transfers required for the social optimality are 1.9% of output. In addition, the unemployment rate is 1.8 percentage points lower in the presence of optimal transfers.

Keywords: Search and matching, Short-time work policies, Constraint efficiency, Endogenous separations.

JEL: E24, J64.

4.1 Introduction

Short-time work (STW) is a policy program by which employers can temporarily reduce the working time of full-time workers. During the Covid-19 recession 2020-2021, governments in developed economies supported an unprecedented participation in the STW schemes, in two dimensions. First, the number of workers in STW was larger than ever before. For instance in Germany, 19% of employed workers were in STW during Spring 2020, compared with 4% during the Great Recession (Organisation for Economic Co-operation and Development (OECD), 2020) and an average of 0.78% over the past 20 years.[1] Second, the reduction of working hours also reached a record level. More specifically, the working time of German workers in STW was reduced on average by almost 50% during Covid-19, while it was by less than 30% in the Great Recession (Herzog-Stein et al., 2021).

Governments are subsidizing the participation in STW by providing workers a compensation of hours not worked, which usually corresponds to unemployment benefits. In addition, firms receive subsidies, such as the reimbursements of social security contributions, in order to incentivize higher participation in STW programs.[2] The goal of these policies is to retain jobs, in order to preserve the value of the job relationships and to stabilize unemployment.

The economic literature has indeed found that STW participates in preventing job destruction and limiting unemployment increase during recessions (Boeri and Bruecker (2011), Aiyar and Dao (2021), Cahuc et al. (2021), Kopp and Siegenthaler (2021)), but so far relatively little is known about the optimal level of these policies. This paper contributes to this gap by asking i) what is the socially efficient level of STW, and ii) under which conditions a competitive economy can reach this optimum. In order to answer these questions, I build on the decentralized general equilibrium search and matching labor market model by Balleer et al. (2016) and compare it with a constraint-efficient social-planner solution. I also introduce two government subsidies and evaluate their capacity to restore the decentralized efficiency.

The intuition is as follows. Jobs are subject to an idiosyncratic productivity shock. With low values of this shock, workers may generate profit losses. In this case, a firm has two alternatives. First, it can participate in STW and decrease the working hours of low productive workers. Second, it can endogenously separate from these workers. If the firm chooses separation, it loses the value of a filled job, which arises from both costly vacancy creation and a fixed separation cost. If the net value of a job occupied by worker in STW is positive, then the firm participates in STW and keeps the worker employed. In addition, the extent by which the working time is reduced in STW is also costly. Without this cost, the firm would always reduce the working hours to zero in order to avoid any loss without losing the value of a filled job. Consequently, separation would never occur, which is counterfactual. Finally, the costs related to STW and separation have an impact on job creation. When a firm considers opening a vacancy, it has an expectation about the idiosyncratic productivity and, as a result, the STW and separation costs. Therefore, the firm opens less vacancies in the presence of these costs.

This paper considers a collective wage bargaining, in which the outside option is a

[1]Source: Bundesbank (https://www.bundesbank.de/en/statistics) and author's calculations.

[2]See section 3 for more details of the institutional framework.

strike. During a strike, job relationships are preserved, but STW and separation costs still hold.[3] Hence, in general the firm cannot internilize these costs in the collective bargaining. Therefore they create a systematic inefficiency in a competitive economy. In turn, the inefficiency has two consequences. First, competitive firms are creating less than the socially optimal amount of new jobs. Second, a competitive level of STW is lower than the socially optimal level. Both of these are a result of wages which are systematically too high. If the STW and separation costs would be internalized, the workers would obtain lower wages.

The social optimality can be restored under three conditions. The first is the so-called Hosios rule (Hosios, 1990), which is related to vacancy creation. More specifically, each new vacancy has a negative externality on the other firms and a positive externality on the unemployed workers. The Hosios rule states that these search externalities are internalized in wages if and only if the elasticity of matching function equals the bargaining power of workers. In addition, I show that two other conditions define the size of government transfers to firms. These transfers correct the socially inefficient output sharing of the competitive wage bargaining outcome. In other words, the transfers cover a fraction of STW and separation costs. More precisely, a STW transfer must equal the bargaining power of workers times the cost of working hour reduction, and a separation transfer must equal the bargaining power times the separation cost.

The different effects of the two transfers are investigated in a numerical exercise. At the steady-state, the STW transfer increases the hour reduction in STW and job creation simultaneously. On the contrary, the separation transfer only increases job creation without a change in the working hours in STW. The reason is that, the firm chooses the working time based on the job-specific productivity and wage. Furthermore, the firm is aware of this optimal working time when it decides about endogenous separation. Consequently, the working time choice affects separation, but not vice versa, and hence the working hours in STW are independent of the separation transfer.

Finally, the model is calibrated to match the German economy over the period of 2000-2021. In this calibration exercise, the transfers required for social optimality are found to be equal to 1.9% of output, which is substantial, considering that these transfers are the only subsidies in the model. As a comparison, in 2019, before the Covid-19 period, the labor market policy expenditures in Germany were 1.3% of GDP.[4] Furthermore, with the optimal transfers, the average unemployment rate would have been 6.0% instead of 7.8%

[3]In an individual bargaining, workers' reservation wage would include the costs, since the workers avoid them in their outside option of unemployment.

[4]Source: European Commission, Directorate-General for Employment Social Affairs and Inclusion report, named Labour Market Policy - Expenditure and Participants, Data 2019, ISSN: 2467-4443.

without the transfers, suggesting a substantial improvement. In addition, the increased employment suggests that there are also fiscal benefits to finance the transfers.

The rest of the paper is organized as follows. Section 4.2 discusses the related literature. Section 4.3 describes the institutional framework of STW policies in Europe. Section 4.4 presents the model. Section 4.5 includes the theoretical, and Section 4.6 the quantitative analysis. Section 4.7 concludes.

4.2 Related literature

This paper investigates the social efficiency of labor markets with search and matching frictions. A fundamental rule of constraint efficiency in this class of models is derived by Hosios (1990) and further discussed in labor market context in Pissarides (2000). Hosios (1990) and Pissarides (2000) argue that competitive wage bargaining may not internalize externalities that vacancy postings have on the other searching firms and the unemployed workers in the same labor markets, and show that these congestion externalities are internalized in wages if and only if the elasticity of matching function equals the bargaining power of workers. In turn, if wages are determined by a wage-posting process instead of bargaining, and workers can direct their search towards specific wages, Moen (1997) and Acemoglu and Shimer (1999) show that the decentralized economy is always socially efficient in these so-called competitive search models.

In this paper, the model encompasses a Nash bargaining of wages and a random search. Consequently, the Hosios rule is necessary but not sufficient condition for optimality, due to the STW and separation costs. In this regard, this paper belongs to a large group of literature in which the optimality under the Hosios condition breaks down when additional features are included in the model. Some of the recent papers in this category are Flórez (2019) in which the inefficiency arises from addition of an informal sector, Cai (2020) with externalities from the on-the-job search, Albrecht et al. (2020) where the inefficiency is a consequence of workers sending multiple applications, Wilemme (2021) with externalities from a directed search, Laureys (2021) in which the inefficiency results from human capital depreciation, and Griffy and Masters (2022) with the inefficiency from participation externality.

To the best of my knowledge, the social optimality of labor markets with STW has received very little attention in the literature so far. Two early contributions are FitzRoy and Hart (1985) and Burdett and Wright (1989). FitzRoy and Hart (1985) show that the implementation of payroll taxes, that finance an unemployment insurance can explain the choice between layoffs and working-time reductions, and argue that even though STW has an effect, it is quantitatively insufficient to explain the difference between the US and Europe. Burdett and Wright (1989) build a labor contract model, where wages and unemployment benefits are taxed with separate rates, and compare the unemployment insurance with a policy where also hours not worked are compensated. They show that the contract choice of agents is dictated by the ratio of the two tax rates, and that the policy choice can distort the economy to lower than full employment, or lower than full working time.

This paper also contributes to the theoretical literature of STW, including Boeri and Bruecker (2011), Niedermayer and Tilly (2016), Balleer et al. (2016), Osuna and Pérez (2021) and Albertini et al. (2022). Boeri and Bruecker (2011) discuss the theory of STW in a stylized model where employment and hour supply choices are substitutes and conclude that the STW design should encourage hour adjustments during recessions, but may suffer from moral hazard in expansions. The importance of incentives for STW participation is also present in my paper. Niedermayer and Tilly (2016) build a partial equilibrium life-cycle model, where STW is modeled with flexible working hours and temporary unemployment. In turn, the model here is in general equilibrium, and STW has specific policy and cost functions. Osuna and Pérez (2021) have a model of STW with the Spanish characteristics, such as temporary work contracts and temporary layoffs, both of which are not considered here. Albertini et al. (2022) build a dynamic partial equilibrium model, which is calibrated to match the French economy especially during the Covid-19 pandemic. The model encompasses, for instance, heterogeneous household wealth, human capital and rare disaster shocks, which are not part of the model in this paper. Furthermore, the analysis here is static, while the different STW policy implementations are compared in a dynamic setting in Albertini et al. (2022).

The model in this paper is based on Balleer et al. (2016). They use a dynamic stochastic general equilibrium model, to show that STW mitigates unemployment increase as a response to negative productivity shocks. Furthermore, they demonstrate that a discretionary change in STW policy does not have impact on unemployment, since the firms cannot anticipate the change. My paper builds on the decentralized model of Balleer et al. (2016), by deriving a social planner solution, and by adding government transfers in order to restore the decentralized efficiency.

So far, the research of STW has been mainly empirical. Abraham and Houseman (1994) use a linear model to estimate the employment and hours adjustment speed in the German, Belgian and French economies to a change in output, and compare it with the US. The Great Recession has inspired numerous articles. Hijzen and Venn (2011), Boeri and Bruecker (2011) do a cross-country analysis of OECD and developed economies. Balleer et al. (2016), Niedermayer and Tilly (2016) and Cooper et al. (2017) use data from Germany, Cahuc et al. (2021) from France and Kopp and Siegenthaler (2021) from

Switzerland. Later, the Covid-19 recession caused an unprecedented take-up of STW policies and inspired a large quantity of research. These include Osuna and Pérez (2021) who analyze Spain, Aiyar and Dao (2021), Dengler and Gehrke (2021), Herzog-Stein et al. (2021) and Teichgräber et al. (2022) who research Germany, Benghalem et al. (2023) who investigate France and Hijzen and Salvatori (2022) who study Switzerland.

The empirical literature supports the contribution of STW in preventing job losses and stabilizing unemployment over business cycles, but also points out externalities including working hour distortions (Cooper et al., 2017), windfall effects, i.e. the heterogeneous effects of STW in different types of firms (Cahuc et al., 2021), deadweight costs, i.e. the excess cost of STW when the number of saved jobs is smaller than the number of workers participating in the program, (Boeri and Bruecker, 2011), and displacement effects, i.e. that STW maintains jobs which are not viable at all without subsidies (Osuna and Pérez, 2021). Many of these inefficiencies are related to the optimal level of STW, which this paper analyzes from a theoretical perspective.

4.3 Institutional framework

This section discusses the STW and job retention programs in Europe, in order to present the important institutional features that the model in this paper is build to capture. STW policies have two important aspects. First, STW includes subsidies for firms and workers, in order to incentivize the participation in the program. Consequently, the level of STW is affected by the generosity of the subsidies. Second, governments are setting criteria for the participation. Furthermore, the control of these criteria requires administrative effort from firms. In conclusion, the level of STW is a result of, on the one hand, the gains from participation in the program, and on the other hand, the costs and the limiting criteria of the participation.

The number of countries implementing STW jumped during the Covid-19 pandemic, but the policy itself has a long history, for instance in Germany and Switzerland where it has existed since the 1920s (Müller and Schulten, 2020). However, the coverage, eligibility and generosity of established STW programs have been adjusted during the large economic crises such as the Great Recession or the Covid-19 pandemic (Mosley, 2020).

The main part of STW policies is a compensation paid to workers for hours not worked, since the workers are also earning only a fraction of their normal full wage. In general, this compensation is equivalent with an unemployment insurance, but can be more or less generous, having varied from a 100% replacement rate in Denmark and Netherlands to a 50% in Poland during Covid-19 (Müller and Schulten, 2020). Most commonly, the government covers all the STW allowances from the unemployment insurance systems, social security funds, or from the government budget (Mosley, 2020). However, some countries, e.g. Sweden and Italy, also require some fraction of the allowance from the employer. In addition, the payment method varies between countries. In most countries, the STW compensation is paid as a wage allowance to the employer, who then pays it to the worker along with the part of the wage. However, for instance in Finland and Spain, the STW allowance is paid directly to the employee (Müller and Schulten, 2020).

The second type of STW-related transfers are subsidies to firms. The two main subsidies are a compensation of social security payments and a support for training of the workers in STW. These two can also be combined. For instance, in Spain the government subsidizes 50% of social security contributions of a worker in STW, which is increased to 80% if the worker participates in training (Osuna and Pérez, 2021).[5] These subsidies decrease the cost of STW to firms, while the training maintains human capital during the hours not worked.

The criteria for STW vary considerably between different countries. Generally, there are criteria for major events that are covered by the STW program, most notably, a temporary economic downturn, a bad weather in construction industry, restructuring or force majeure. In addition, there are restrictions for both firms and workers. For the firms, these relate most commonly to the reduction in working time and the number of employees, but also to the firm size or business sectors. For the workers, for instance fixed-term contracts or recently hired workers may be excluded. (Mosley, 2020)

The details of the administrative processes in different countries vary even more than the STW criteria. However, the main part is an application usually to some dedicated government office. The application process may require negotiations between the firm and the national or local authorities, trade-unions or employee representatives (Mosley, 2020). In some cases, such as Germany, the workers' approval is required before their wages can be reduced in STW (Balleer et al., 2016).

The next section of this paper describes the model. For the modeling perspective, the institutional setting is summarized in two key points. First, the subsidies to firms encourage them to increase the level of STW above what they would choose when only considering the labor cost reductions. Second, the administrative process requires effort, making STW costly for the firms in a non-pecuniary sense. These costs along the criteria by the government limits the level of STW. Both of the two elements will be part of the model. On the one hand, the costs of STW, in both dimension, the number of workers and the number of working hours, are modeled as cost functions. On the other hand, there are transfers reducing these costs, which are also analyzed in order to correct the

[5] Workers can also receive higher STW allowances if they participate in training, e.g. in France 100% of hourly wage instead of 70% (Mosley, 2020).

social inefficiencies in the model.

4.4 Model

The model is based on Balleer et al. (2016). The economy features labor markets with search and matching frictions and endogenous job destruction and creation processes à la Mortensen and Pissarides (1994). The labor market model is supplemented with a short-time work policy (STW). The government collects a lump-sum tax from households to finance transfers to firms, which are studied to correct the social inefficiencies from STW.

The economy encompasses households, firms and the government. Households provide labor, consume and save to government risk free bonds. Household members are either employed or unemployed. Firms post vacancies to hire unemployed workers to produce a homogeneous consumption good using labor as the productive input. Job productivity is subject to an idiosyncratic shock. For the low values of this shock firms can decide to destroy these jobs as in Mortensen and Pissarides (1994), or reduce the working hours by participating in STW. Full details of the model are in the appendix 4.A.

4.4.1 The firm

Each job within a firm has an idiosyncratic productivity, which is drawn each period from a time-invariant distribution. In addition, all the jobs share a common productivity component. The total productivity of a job is a combination of the two, as (Details in Appendix 4.A.1)

$$y_{i,t} = A_t - \varepsilon_{i,t}, \tag{1}$$

where $y_{i,t}$ is output of job i, A_t is total productivity, and $\varepsilon_{i,t}$, an idiosyncratic productivity. Based on the total productivity of the job, i.e. $A_t - \varepsilon_{i,t}$ the workers are divided into three groups. The first group with the lowest productive workers are endogenously separated and become unemployed. The second group are the workers that are sent to STW, and the third group are the workers who work full-time.

Figure 4.1 illustrates the idiosyncratic productivity draw and the division of workers into three groups. The endogenously separated workers become unemployed. Hence, the total employment stock, is a sum of employees in full-time work and in STW, given by

$$n_t = n_t^{FT} + n_t^{STW}, \tag{2}$$

where n_t is the total employment, n_t^{FT} the stock of workers working full time, and n_t^{STW}

the stock of workers in STW.

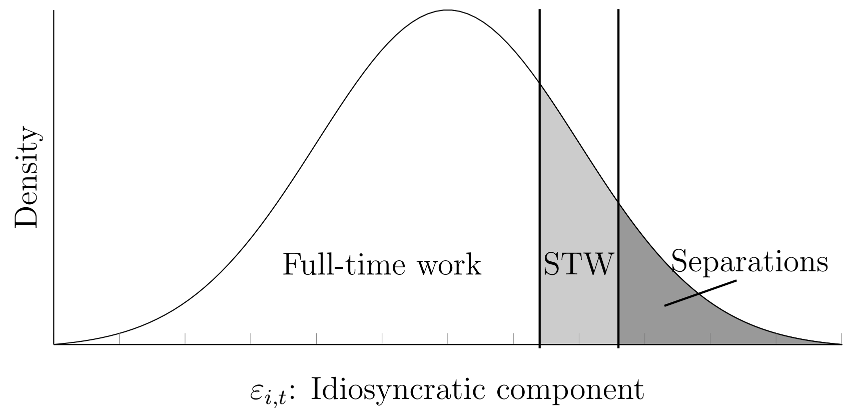


Figure 4.1: The job productivity is combined by a common and idiosyncratic components as $A_t - \varepsilon_{i,t}$. The latter is drawn from a time-invariant distribution. This draw divides workers into three groups: separation, STW , and full-time work.

As a difference to the standard search and matching model, the employment evolution is presented separately for the two worker groups. The stock of full-time workers evolves as

$$n_t^{FT} = (1-\rho^x)(1-\rho_t^e)(1-\chi_t)\left[n_{t-1}^{FT} + n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1}\right], \tag{3}$$

where ρ^x is a rate of exogenous separation, ρ_t^e a rate of endogenous separation, χ_t a rate of workers in STW, $q(\theta_t)$ a vacancy filling rate and v_t the number of vacancies. As in the canonical employment evolution equations, the current period employment consists of the previous period employed workers, here $n_{t-1}^{FT} + n_{t-1}^{STW}$, and of new matches, $q(\theta_{t-1})v_{t-1}$. The former means that the full-time workers can remain in the full-time stock, but also the previous period workers in STW can become full-time workers due to a new productivity draw.

Symmetrically, the stock of workers in STW, evolves as

$$n_t^{STW} = (1-\rho^x)(1-\rho_t^e)\chi_t\left[n_{t-1}^{FT} + n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1}\right]. \tag{4}$$

Again, the period $t-1$ full-time worker can be sent to STW in period t. The new matches from period $t-1$ are subject to the same transition probabilities as the existing matches and these workers can also be sent to STW or separate endogenously immediately in their first employment period.

Figure 4.2 depicts the probability tree of separation, STW and full-time work rates. The exogenous separation can be considered to occur first, after which the idiosyncratic productivity is drawn. The firm chooses endogenous separation and STW. The workers who are not endogenously separated nor sent to STW, work full time. Note that, this definition gives a total separation rate as in Den Haan et al. (2000), i.e.

$$\rho_t = \rho^X + (1 - \rho^X)\rho_t^e. \tag{5}$$

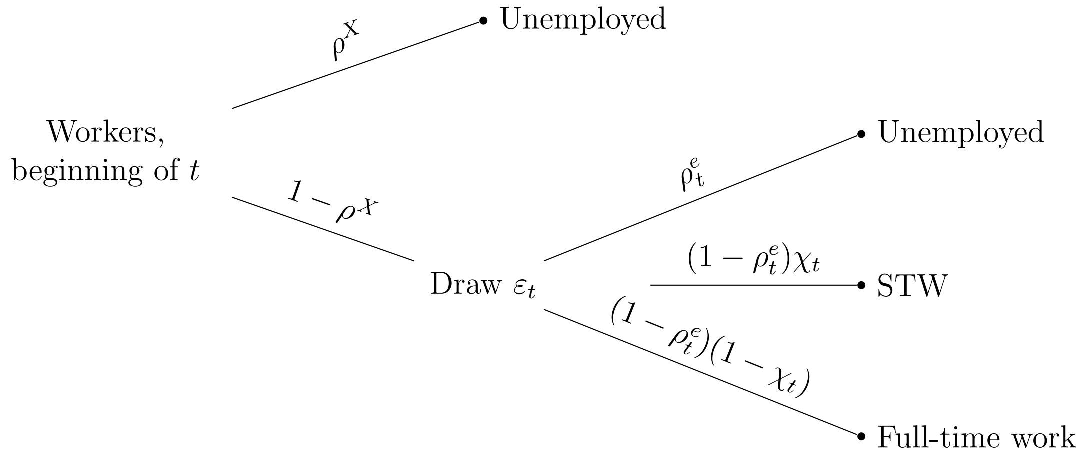


Figure 4.2: The sequence of separation and STW, where ρ^X and ρ_t^e are the exogenous and endogenous separation rates respectively, and χ_t is the STW rate. Exogenous separation occurs first, followed by the idiosyncratic draw, which results in the firm's decision about endogenous separation and STW.

The firm's choice of sending workers to STW is considered in two dimensions, each one with a cost. The first is the number of workers sent to STW, captured by χ_t above. A cost related to χ_t is given by

$$X(\chi_t) = \frac{c_\chi}{2}\chi_t^2, \tag{6}$$

where c_χ is a scale parameter. A quadratic cost function is chosen to allow an interior solution, as compared to a linear cost which would result in the firm sending either all or none of the workers to STW. The cost function $X(\chi_t)$ can be interpreted as a policy criterion which the government imposes on the STW. For instance, a lower scale parameter c_χ would imply more lax policy and increase the number of workers in STW.

The second dimension is the number of working hours reduced from the workers in STW. A cost in the number of hours is given by

$$C(K_t^*) = \frac{c_k - \tau^K}{2}(K_t^*)^2, \tag{7}$$

where K_t^* is an optimal hour reduction, c_k is a fixed cost, and τ^K is a STW transfer from the government. A full working time is normalized to one, and the hour reduction K_t^* is a fraction of the full time continuous between 0 and 1. Likewise with the cost function (6), the quadratic form here enables the optimal choices other than only 0 or 1.

The firm produces homogeneous consumption good using labor as the input of production. The profits of the firm are given by

$$\begin{aligned}\Pi_t = {} & \underbrace{n_t^{FT} A_t^{FT} + n_t^{STW} A_t^{STW}(1-K_t^*)}_{\text{Total output}} - \underbrace{w_t^{FT} n_t^{FT} - w_t^{STW} n_t^{STW}(1-K_t^*)}_{\text{Total wages}} \\ & \underbrace{-C(K_t^*)n_t^{STW} - X(\chi_t) - \frac{\rho_t^e}{1-\rho_t^e} n_t(f-\tau^f) - \kappa v_t,}_{\text{Costs and transfers}}\end{aligned} \tag{8}$$

where A_t^{FT} and A_t^{STW} are aggregate outputs and w_t^{FT} and w_t^{STW} are aggregate wages of full-time and STW workers respectively, f is a separation cost and κ a vacancy cost. The separation cost is paid for the endogenously separated workers. It is important to note that the existence of STW does not require the separation cost f. This cost is added to allow calibrations to match relevant levels of STW. Since vacancy posting is costly, all preserving matches have a positive value and consequently some workers generating profit losses would still have a positive match value, and would be sent to STW even in the absence of separation cost.

Furthermore, I consider an additional policy tool, namely a separation transfer τ^f. This transfer, alike the STW transfer in the cost function (7), is considered here in order to compensate social inefficiencies due to these costs. Wages are bargained collectively, and they contain the productivity dependent component.

The firm chooses 6 control variables, namely the number of vacancies v_t, the number of workers in full-time work n_t^{FT}, the number of workers in STW n_t^{STW}, the hour reduction in STW K_t, the share of workers in STW χ_t, and the rate of endogenous separation ρ_t^e so as to maximize the expected discounted real profits as

$$\max_{v_t, n_t^{FT}, n_t^{STW}, K_t, \chi_t, \rho_t^e} E_t \sum_{t=0}^{\infty} \beta^t \Big(\Pi_t\Big), \tag{9}$$

subject to the employment evolution equations (2), (3), (4) and the STW cost functions (6) and (7).

The first-order conditions for vacancies, v_t, full-time workers, n_t^{FT}, and STW workers, n_t^{STW}, are combined to form a job creation condition (JCC) as

$$
\begin{aligned}
\frac{\kappa}{q(\theta_t)} = \beta E_t \Bigg\{ (1-\rho^x) \Bigg[& \underbrace{(1-\rho^e_{t+1})(1-\chi_{t+1})(A^{FT}_{t+1} - w^{FT}_{t+1})}_{\text{Profits of full-time work}} \\
+ & \underbrace{(1-\rho^e_{t+1})\chi_{t+1}[(1-K^*_{t+1})(A^{STW}_{t+1} - w^{STW}_{t+1}) - C(K^*_{t+1})]}_{\text{Profits of STW}} \Bigg] \\
& - \underbrace{(1-\rho^x)\rho^e_{t+1}(f-\tau^f)}_{\text{Separation costs}} + \underbrace{(1-\rho_{t+1})\frac{\kappa}{q(\theta_{t+1})}}_{\text{Continuation value}} \Bigg\}.
\end{aligned} \tag{10}
$$

JCC equates the cost of hiring a new worker with the expected gains from a new job. The gains consist of the expected full-time work and STW profits, and the continuation value, net of costs. The costs are related to hour reductions in STW, separation and vacancy posting. Note that, if there is no STW, i.e. $\chi_{t+1} = 0$, the model corresponds to a canonical search and matching model. This can be seen from the job creation condition (10) where the second line disappears.

The first-order condition for hour cut K_t results in an optimal hour reduction condition given by

$$
K^*_t = -\frac{A^{STW}_t - w^{STW}_t}{c_k - \tau^K}. \tag{11}
$$

The worker who generates contemporaneous deficits, i.e. $A^{STW}_t < w^{STW}_t$, can be sent to STW. The optimal choice of K^*_t is negatively related to productivity and positively to wage. For instance, more productive workers work more hours in STW, and higher wages lead to higher hour reductions. The cost c_k has negative impact on K^*_t, i.e. the higher the cost the smaller the working time cut. The transfer τ^K is studied later as a part of the optimal policy.

The first-order condition for the share of workers in STW χ_t is formulated as the following condition

$$
\chi_t = (\lambda^{STW}_t - \lambda^{FT}_t)\frac{n_t}{c_\chi} \tag{12}
$$

where λ^{FT}_t and λ^{STW}_t are Lagrange multipliers of constraints related to the full-time and the STW workers' evolution equations (3) and (4). It presents the marginal gain for the firm from having a worker in STW instead of in full-time work. If sending a marginal worker to STW increases profits of the firm more than having her working full time, i.e. $\lambda^{STW}_t > \lambda^{FT}_t$, the share χ_t increases and vice versa. The cost c_χ for choosing the STW share is firm not worker specific and hence the total impact of the marginal gain is scaled by the size of employment n_t. One way to consider this is that the cost is an effort lost when a firm negotiates about the χ_t with the government. The negotiation process is the same for different sized firms, hence the larger firms gain more from STW than the smaller firms and choose a larger share χ_t.

A first order condition for the share of endogenous separation is formulated as the following condition

$$\rho_t^e = \frac{f - \tau^f}{\lambda_t^{FT}(1-\chi_t) + \lambda_t^{STW}\chi_t} + 1. \tag{13}$$

The firm takes into account the possibility to send low productive workers to STW, and reduce the deficits which they generate. Hence, the cost of separation, i.e. $f - \tau^f$, is relative to a change in the marginal values of full-time and STW workers. This equation is not interpreted or developed further here, but compared with the social planner condition in the analysis.

Wages are bargained collectively, which corresponds to many European countries for instance Germany (Balleer et al., 2016). A labor union bargains wages for all workers. An outside option is a strike. During the strike, workers remain matched with the firm, but no production occurs. The workers do not earn wages but the home production b. The firm also holds the match value during the strike. The union is assumed to bargain a merit based wage such that the more productive workers earn higher wages. The Nash bargaining problem is (Details in the appendix 4.A.5)

$$\arg\max_{w_t} (W_t - \tilde{W}_t)^{\gamma}(F_t - \tilde{F}_t)^{1-\gamma}, \tag{14}$$

where γ is the bargaining power of workers, W_t the worker's value when working, $\tilde{W}_t$ the workers value on strike, F_t the firms value of production and $\tilde{F}_t$ the firms value of strike.

The solution results in a wage rule that is dependent on the idiosyncratic productivity. The job-specific wage is given by

$$w_t = \gamma(A_t - \varepsilon_t) + (1-\gamma)b. \tag{15}$$

The wage rule is aggregate for the firm's problem over the idiosyncratic component, to result the wages for the full-time and STW workers.

The above definition of wages fully internalizes the idiosyncratic productivity of jobs. This definition of wages is used to simplify the comparison of decentralized and social planner solutions. I acknowledge that this wage may, in some cases, result in an incentive incompatibility for the workers in STW, even though the worker's quitting decision is not modeled in detail. However, this wage definition allows to pin down the social inefficiency from STW, even when the wages are adjusting to low productivities. As a comparison, Appendix 4.A.7 considers the additional inefficiencies that arise from the idiosyncratic productivity when the same wage is paid to all workers.

4.4.2 The social planner

The socially optimal solution in this section follows a standard approach to constraint efficiency in markets with search and matching frictions, e.g. Pissarides (2000). The social welfare consists of household's consumption. An aggregate consumption is a sum of output and home production net of real costs. The households are assumed risk-neutral, and hence, the social optimality depends only on the level of expected consumption. As a comparison, Jung and Kuester (2015) consider optimality when workers are risk-averse. They present the social welfare as a sum of worker's utility when employed and unemployed. This utilitarian definition of welfare with risk-neutral workers would result in the same conditions of optimality as presented here.

The social planner is subject to the same labor market frictions as the competitive firm, including the costs related to STW. The social planner optimizes welfare as

$$\begin{aligned} U_t^{SP} = \max_{v_t, n_t^{FT}, n_t^{STW}, K_t, \chi_t, \rho_t^e} E_t \sum_{t=0}^{\infty} \beta^t \Big[& \underbrace{n_t^{FT} A_t^{FT} + n_t^{STW} A_t^{STW} (1 - K_t)}_{\text{Output}} \\ & + \underbrace{b(1 - n_t + K_t n_t^{STW})}_{\text{Home production}} \\ & \underbrace{- C(K_t) n_t^{STW} - X(\chi_t) - \frac{\rho_t^e}{1 - \rho_t^e} n_t f - \kappa v_t}_{\text{Costs}} \Big], \end{aligned} \tag{16}$$

subject to the same conditions as the firm (2), (3), (4), (6) and (7). The social planner problem does not include wages. Wages are only redistributing output between agents, and by assumption, the distributional considerations are excluded from the social welfare function. For the same reason, home production is considered as an outside option for workers instead of unemployment benefits. The unemployment benefits would also only redistribute output without an effect on welfare.

The first-order conditions for vacancies, v_t, full-time workers, n_t^{FT}, and STW workers, n_t^{STW} are combined to form a job creation condition (JCC) as

$$\begin{aligned} \frac{\kappa}{q(\theta_t)(1 - \xi_t)} = \beta E_t \Big\{ & (1 - \rho^x) \Big[(1 - \rho_{t+1}^e)(1 - \chi_{t+1})(A_{t+1}^{FT} - b) \\ & + (1 - \rho_{t+1}^e) \chi_{t+1} [(1 - K_{t+1}^*)(A_{t+1}^{STW} - b) - C(K_{t+1}^*)] \Big] \\ & - (1 - \rho^x) \rho_{t+1}^e f + (1 - \rho_{t+1}) \frac{\kappa}{q(\theta_{t+1})(1 - \xi_{t+1})} \Big\}, \end{aligned} \tag{17}$$

where ξ_t is defined as $-\xi_t \equiv \frac{\theta_t q'(\theta_t)}{q(\theta_t)}$ and $q'(\theta_t)$ is the derivative of $q(\theta_t)$ w.r.t. θ_t. More intuitively put, ξ_t is the elasticity of unemployment in the matching function. The social JCC is compared with the competitive JCC in the optimality analysis later.

The first-order condition for hour reduction K_t results in an optimal condition given by

$$K_t^* = -\frac{A_t^{STW} - b}{c_k}. \tag{18}$$

The home production b is directly present in the social hour reduction condition (18), as compared with the competitive condition (11). It has a positive impact on K_t^*. The reason is that the social optimality is a balance between output and home production. By the definition of social welfare function (16), the share of hours K_t^* that an STW worker is not working, she uses for home production. If, ceteris paribus, b increases, the household's utility from one hour of home production exceeds the same hour in output production. As a result, the socially optimal hour reduction increases.

The first-order condition for the share of workers in STW, χ_t is formulated as

$$\chi_t = (\phi_t^{STW} - \phi_t^{FT})\frac{n_t}{c_\chi}, \tag{19}$$

where ϕ_t^{FT} and ϕ_t^{STW} are the Lagrange multipliers of the constraints of employment evolutions (3) and (4). Likewise the JCC, this condition (19) is compared with the competitive solution in the optimality analysis. The same applies to the endogenous separation condition which is formulated as

$$\rho_t^e = \frac{f}{\phi_t^{FT}(1-\chi_t) + \phi_t^{STW}\chi_t} + 1. \tag{20}$$

4.5 Analysis

This section compares the competitive and social planner solutions analytically. The first part of the section shows that there are inefficiencies, and detects their size and sign. The second part proposes sufficient conditions to correct the inefficiencies in decentralized economy. The optimality conditions include the transfers presented in the model, and this section also investigates the size of these transfers.

4.5.1 Size and sign of inefficiencies

In order the compare job creation conditions, the wage equations for full-time and STW workers are inserted into the decentralized JCC equation (10). Then the decentralized JCC (10) is subtracted from the social planner's condition (17) yielding (details in appendix 4.A.6)

$$E_t\beta\frac{(1-\rho_{t+1})\kappa}{q(\theta_{t+1})}\underbrace{\left(\frac{1}{1-\xi_{t+1}}-\frac{1}{1-\gamma}\right)-\left(\frac{1}{1-\xi_t}-\frac{1}{1-\gamma}\right)}_{\text{The congestion externality, (+/-)}}\frac{\kappa}{q(\theta_t)}$$

$$-\underbrace{\left(f-\frac{f-\tau^f}{1-\gamma}\right)\beta E_t[(1-\rho^x)\rho^e_{t+1}]}_{\text{Inefficiency from separation cost } f,(-)} \tag{21}$$

$$-\underbrace{\left(c_k-\frac{c_k-\tau^K}{1-\gamma}\right)\frac{1}{2}E_t\beta(K^*_{t+1})^2}_{\text{Inefficiency from STW cost } c_k,(-)}=0.$$

In this comparison of dynamic equations, the time-varying variables, such as the labor-market tightness θ are assumed to be identical between the two solutions. Equation (21) shows the three sources of inefficiency in the decentralized economy: the congestion externality, separation costs and STW costs. In general, the inefficiencies are negative and result in too little job creation, except if the congestion externality has considerably high positive effect.

The congestion externality is a familiar result from models with search and matching frictions. When a firm posts a vacancy, the labor-market tightness increases. The other firms which are searching for workers become worse off, since their vacancy filling becomes less likely. On the other hand, the unemployed workers who are searching for a job become better off, since they are more likely to match with an open vacancy. The wage bargaining between firms and workers may not internalize the impact that the vacancy posting has on labor markets, thus wages can be either too high or too low compared with the socially optimal outcome. As a result competitive firms end up creating too many or too few new jobs. Further on, unemployment is not on the socially optimal level.

The other inefficiencies from the separation cost and the STW cost have unarguably negative effect. When a firm is hiring a new worker, the outcome of the match is unknown, i.e. the idiosyncratic productivity has not realized yet. There is a chance that the idiosyncratic productivity is low, and the firm has to either send the worker to STW or, in the worst case, separate with the worker. Both of these actions are costly to the firm. Since, the competitive firm has an expectation about the idiosyncratic draw, it will create less jobs the higher these costs.

The STW and separation costs are not internalized in the wage bargaining. These costs materialize only if the firm chooses to send workers to STW or to separate. Hence, the wage bargaining could only include the expectation of the costs. In the collective bargaining process, the outside option is a strike. When a strike occurs, the expected STW and separation costs remain, since they depend on the idiosyncratic draw which is independent from striking. As a result, the firm cannot bargain lower wages with these costs, because in the strike the costs would persist but no production occurs. [6] In the individual competitive bargaining the firm could deduct the costs from a wage, but even in this case, the worker would lose the cost when employed but avoid it when unemployed. Hence, the cost would increase the reservation wage as part of the worker's outside option in the bargaining process.

The size of inefficiencies depend on the bargaining power of workers and is expressed by the multipliers inside of the brackets on each line of equation (21). The size of congestion externality is dictated by the difference of elasticity terms ξ_t and ξ_{t+1} and the bargaining power of workers γ. In search and matching models, the matching function is often defined to have a constant elasticity, for instance using a Cobb-Douglas form, which makes a comparison with a constant bargaining power straightforward.

The separation cost and the STW cost create a negative externality with an equal relative size. While the social planner only considers costs f and c_k, the competitive firm multiplies these costs with an inverse of the firm's bargaining power, i.e. $1/1-\gamma$, which by definition of the bargaining power is greater than 1. These costs cannot be internalized in wages and the firm bears all of them. When the workers have positive bargaining power, the firm only obtains fraction of the production surplus and has less resources to allocate to job creation. Higher the bargaining power, the more the expected STW and separation costs decrease the number of vacancies. Another difference between the socially optimal and competitive solutions is the transfers τ^f and τ^K, which are considered in the next section to correct the inefficiencies.

4.5.2 Optimal policy

The previous section detected social inefficiencies by comparing the social planner solution with the decentralized solution. As a result, the sources and sizes of externalities are determined in the model. Consequently, it is possible to solve conditions under which the decentralized economy can restore the social optimality. These conditions for social optimality are formulated in the following proposition.

[6] More formally, the costs appear in both of the firm's value functions F_t and $\tilde{F}_t$ and cancel each other when the two are deducted, $F_t - \tilde{F}_t$ in the Nash problem.

Proposition 1 *The decentralized level of job creation is socially optimal if the following three sufficient conditions hold simultaneously: $\xi_t = \gamma$, $\forall t$, i.e. the elasticity of matching function with respect to unemployment must equal the bargaining power of workers (Hosios condition), $\tau^f = \gamma f$, i.e. the separation transfer must equal worker's bargaining power times the separation cost and $\tau^K = \gamma c_k$, i.e. the STW transfer must equal worker's bargaining power times the unit cost of hour reduction.*

PROOF, appendix 4.A.6

Proposition 1 contains three sufficient conditions for the optimality. The first condition for the optimality is the famous Hosios rule (see Hosios (1990) and Pissarides (2000)). It states that if and only if the bargaining power of workers equals the elasticity of matching function with respect to unemployment, the bargaining results in wages that are optimal by internalizing the impact of vacancy postings on the searching firms and workers on the same labor market.

The second condition is related to the separation cost f. The condition dictates that the optimal separation transfer is the bargaining power times the separation cost, i.e. $\tau^f = \gamma f$. Because the firm needs to bear all this cost, it will create less jobs when the cost exists. The government can increase job creation by subsidizing firms. The optimal transfer cancels out the bargaining outcome by redistributing a share of output, which is relative to the bargaining power of workers, to the firm. The optimal transfer enables the firm to create equal amount of new jobs than it would create when obtaining all the production surplus. This corresponds to the choice of the social planner, since the planner is optimizing output without redistribution considerations.

The third condition is related to the cost of STW. Analogically to the separation cost and transfer, the STW transfer must equal the bargaining power of workers times the unit cost of hour reduction, i.e. $\tau^K = \gamma c_k$. The competitive firm compares the cost of reducing working hours with the reduction of deficits, which the low productive worker generates, and decides the working time accordingly. Higher bargaining power means that the firm can reduce more wage costs when cutting the working time in STW, but it also means that the profitable full-time workers obtain a larger share of output, leaving the firm with less recourses to use for the fixed STW costs. The social planner only compares the utility gained from output with the utility from home production without distortions from surplus sharing. As a result, the social planner would allocate more to job creation and higher employment by reducing more working hours in STW than the decentralized firms which consider the surplus sharing.

The above presented three conditions, the Hosios rule, the separation cost rule and the STW cost rule are sufficient but not necessary conditions for the constraint efficiency. Other combinations of conditions may also lead to the social optimality. From the sufficient conditions, it is rather straightforward to see that if no workers are in STW, the Hosios rule and the optimal separation transfer are sufficient to ensure the social optimality. As mentioned in the previous section, if there is no STW, the model corresponds to a canonical search and matching model with the separation cost. If the separation cost does not exist, the Hosios condition alone is sufficient for the optimality.

Moreover, at least in a theoretical sense, the third option to reach the social optimality is based on the different signs of the externalities. The violation of Hosios condition can result in a positive or a negative externality, meaning too many or too few new vacancies are opened. On the contrary, the separation and STW costs are creating solely negative externalities. Hence, in theory it is possible that a positive externality from the violation of Hosios condition is outweighed by the negative impact of separation and STW costs. This case is rather theoretical, and acknowledged here, but a quantification of the magnitudes of the different conditions are not studied further.

In addition, it is worth to emphasize that under the sufficient conditions, there are no other inefficiencies from STW and separation choices by the firm. More specifically, the choices which the firm makes, i.e. the vacancy posting, the share of workers in STW and the rate of separation, are conducted by comparing marginal costs of these choices. The marginal values consist of the profit functions for the competitive firm, which are equal for all the labor market choices, hence, also the inefficiencies in all choices are equivalent. As a result, the sufficient conditions ensure the efficiency of vacancy creation, STW participation and separation decisions of competitive firms.

4.6 Quantitative analysis

This section is analyzing the empirical relevance of the model, with a calibration exercise matching quarterly values of German economy between 2000-2021. The German economy is chosen for three main reasons. First, labor market data including STW statistics are readily available. Second, the German STW program is well established, most researched and has inspired other European policymakers when they have implemented their own job retention policies. Third, the model in this paper builds on Balleer et al. (2016) which is primarily constructed to model the German labor markets allowing a cross-checking of the calibration.

4.6.1 Calibration exercise

The essential factor of social optimality are the costs of STW and separation. As these costs cannot be directly observed from the data, the first part of the calibration exercise is to detect the values of cost parameters in the model. In order to accomplish this task, the steady-state of the model is calibrated to match the quarterly values of German economy between 2000-2021.

The strategy is to choose the steady-state targets and parameter values, which have clear counterparts in the data or established values in the literature. Based on these calibrated values, the model equations are used to detect the values of the cost parameters. Finally, these costs dictate the optimal transfers.

Table 4.1 summarizes the steady-state targets and calibrated parameters. The unemployment rate u is set to 7.8% and the STW rate χ to 0.78%, which are the averages from the German data between 2000 and 2021. The STW rate is slightly higher than in Balleer et al. (2016), who target a value 0.69%, due to the high rates during Covid-19. The aggregate reduction in working hours K^* is one third from the full working time, which is a long term average in Germany according to Balleer et al. (2016) and Herzog-Stein et al. (2021).

Parameter	Description	Value	Source
β	Discount factor	0.99	Annual risk-free rate 4%
u	Unemployment rate	0.078	Average 2000-2021
m	Matching efficiency	0.5	To match $q = 0.70$ and $v = 0.03$
μ	Matching elasticity	0.5	$q = 0.70$, $v = 0.03$ and the Hosios condition
χ	Rate of STW	0.0078	Average 2000-2021
K^*	Working time reduction	0.33	Balleer et al. (2016)
γ	Workers' bargaining power	0.5	The Hosios condition
ρ^X	Exogenous separations	0.02	Balleer et al. (2016) and $v = 0.03$
ρ^e	Endogenous separations	0.011	Balleer et al. (2016)
A	TFP	1	Standard in the literature
b	Home production	0.75	Losses in STW profits in FT
σ	STD of idiosyncratic distribution	1.65	Losses in STW profits in FT

Table 4.1: Calibration

The matching function parameters, the elasticity $\mu = 0.5$ and the efficiency $m = 0.5$ are calibrated to target a vacancy filling rate close to 0.7 and a vacancy rate to 0.03, which both follow Balleer et al. (2016). In addition, the bargaining power of workers is set equal to the matching elasticity to make the Hosios condition to hold. The bargaining power value of 0.5 is symmetric between the agents, and hence uninformative, which also makes it a common value in the literature.

The total separation rate is approximated to 3% in Balleer et al. (2016), with a division of 1/3 in the endogenous and 2/3 in the exogenous rate. I choose a slightly higher endogenous rate of 0.011 in order to increase the vacancy filling rate closer to the target. The total productivity $A = 1$ is standard in the literature. The home production is set to $b = 0.75$. Finally, the idiosyncratic productivity is chosen as normally distributed with a mean 0 and a standard deviation of $\sigma = 1.65$. Both home production and idiosyncratic productivity are calibrated to result in the STW workers to generate losses while the full-time workers generate profits simultaneously. In addition, home production is a flow value of unemployment to workers capturing the utility from all outside of work activities. That is why the value of b is considered higher than an unemployment insurance alone would imply.

Parameter	Description	Value	Source
n	Employment	0.922	$1-u$, labor force normalized to 1
ρ	Total separation rate	0.0308	$\rho = \rho^X + (1-\rho^X)\rho^e$
v	Vacancies	0.044	Steady-state matches and matching function
θ	Tightness	0.56	v/u
$q(\theta)$	Vacancy filling rate	0.67	Matching function calibration
c_k	Working-time reduction cost	10.55	The cost function $C(K)$
c_χ	STW-share scalar parameter	-367.32	The cost function $X(\chi)$
f	Separation cost	0.84	Job destruction condition
κ	Vacancy cost	1.45	Matching function

Table 4.2: Values implied by the calibration.

In addition to these calibrated parameters, I implement the wage rule by Balleer et al. (2016) to the calibrated model (see details in Appendix 4.A.7). This wage is chosen for two reasons. First, it is better in line with the German economy. Second, it makes workers in STW more expensive for firms incentivizing more endogenous separation. The latter effect is necessary later when the optimal transfers are imposed in the model. Otherwise, the flexible wages for STW workers together with the optimal transfers would imply zero, or even negative endogenous separation rates with plausible calibration values, making a comparison of different cases impossible.[7]

Table 4.2 presents the values which are implied by the calibration in Table 4.1 and the model equations. The vacancy rate v is higher than the target, but on the contrary the vacancy filling rate q is slightly lower. These are still in an acceptable level, given that the rest of the calibration is feasible with these values. Essential parameters for the analysis are the costs c_k, c_χ and f. These values are arbitrary, and their sizes cannot be

[7]The other option could be to model the endogenous quitting decision of workers. In which case, low wages would increase total separation through increase in quits. I leave this extension for future research.

interpreted as such, but they are used to indicate the optimal transfers. However, the scalar c_χ of the cost function of the STW rate may seem unusual as it is negative. The intuition is that it captures a government's STW policy criterion, i.e. the magnitude of losses which the worker needs to generate before participation in STW is accepted.

The results from the calibration exercise show that transfers required for the social optimality are 1.9% of output. As a comparison, all labor market expenditures in Germany during 2019 were 1.3% of GDP.[8] Since there are no other labor market subsidies in the model, the relative size of STW and separation transfers can be considered acceptable. Even though, in reality the share of STW related subsidies from the total labor market expenditures is relatively small, for instance the average was 2.94% between 2007-2017 in Germany (Mosley, 2020). In addition, the model fails to capture this feature as the relative size of STW transfer is approximately 3 times larger than the separation transfer.

4.6.2 Calibration of the optimal policy

The next part of the calibration exercise uses the costs which were solved above. These costs are now taken as given, and related optimal transfers are applied in the model. The new steady states are solved given the transfers, both jointly and separately. Table 4.3 summarizes the steady-state results.

Variable	Long name	No policy	Optimal policy	Only τ^K	Only τ^f
u	Unemployment	0.078	0.060	0.050	0.100
v	Vacancies	0.044	0.033	0.026	0.062
θ	Tightness	0.564	0.547	0.516	0.620
q	Job filling rate	0.666	0.676	0.716	0.635
ρ	Separations	0.031	0.023	0.020	0.042
χ	STW rate	0.0078	0.0068	0.0080	0.0070
K^*	Hour cut	0.33	0.77	0.85	0.29
Y	Output	0.971	0.970	0.965	0.982
C	Consumption	0.966	0.968	0.967	0.966

Table 4.3: Comparison of steady-states with optimal transfers.

In the presence of optimal transfers, the steady-state unemployment rate decreases significantly from 7.8% to 6%. At the same time, the level of STW is higher as the steady-sate hour reduction increases from one third to approximately three fourth of the full working time. As a consequence of increase in STW, the separation rate drops from 3.1% to 2.3%. More surprisingly, the share of workers in STW, i.e. χ is lower with the

[8]Source: European Commission, Directorate-General for Employment Social Affairs and Inclusion report, named Labour Market Policy - Expenditure and Participants, Data 2019, ISSN: 2467-4443.

optimal policy. The reason is that both, the higher hour cut, and the transfers subsidizing costs, result in a higher profitability of the firm. Consequently, the firm is able to have more STW workers back to full time work. Simultaneously, more low productive workers can be employed in STW instead of separation. However, the former group of workers is larger than the latter one, leading to a lower STW rate.

Another, perhaps unexpected result is a decrease in the vacancy rate, which falls from 4.4% with the baseline calibration to 3.3% with the optimal transfers. However, the steady-state labor market tightness and consequently the job filling rate remain relatively close to their original values. The former decreases from 0.56 to 0.55 and the latter increases from 0.67 to 0.68. These two variables capture the general state of labor markets, which remains close between the two steady states. Consequently, the considerably lower unemployment rate is accompanied by a lower vacancy rate, to yield approximately equal labor market tightness in the presence of the optimal policy.

Finally, the results about output and consumption confirm that the optimal policy indeed is welfare enhancing. Even though, output slightly decreases when the optimal transfers are applied, consumption is still higher than without the policy due to home production. Thus, in general, the optimal policy divides labor input more efficiently between output production and home production than in the decentralized equilibrium.

Table 4.3 also has the steady states when the two transfers are applied separately. The two steady states differ significantly, which confirms the previous analysis, that the transfers affect the economy through different channels. Unsurprisingly, the STW transfer τ^K, increases the hour reduction, since it decreases the related cost. Consequently, the lower productive workers can be kept employed in STW, and the separation rate decreases. More interestingly, the unemployment rate falls even more than with the optimal policy, as the firms are also able to keep more workers employed in full time work, due to higher firm-level profitability.

The steady state with only separation transfer τ^f has also many expected results. Because the separation costs are lower with the transfer, the separation rate increases. Even though, the higher separation rate is accompanied by a considerably higher vacancy rate, the latter is not sufficient to prevent unemployment from increasing from the baseline calibration, due to higher labor market tightness and a lower vacancy filling rate. As expected, the separation transfer alone does not cause much changes in the level of STW. Especially, the hour reduction decreases only by 4 percentage points.

As a conclusion, the two transfers give incentives for firms to optimize their production in different ways. The STW transfer subsidizes labor hoarding, i.e. the firms optimize output by keeping a large proportion of workers employed, by also cutting a large fraction of working hours from those participating in STW. In turn, the separation transfer makes the firms cut a large proportion of low productive workers, in order to create fewer high productive jobs instead. Both transfers have tradeoffs. The STW transfer results in low unemployment but low productivity as well. On the contrary, the separation transfer induces high unemployment accompanied with high productivity. In general, the calibration exercise suggests that labor market policies should be designed as a whole, since individual subsidies may have desired effects on some dimensions, but unexpected tradeoffs in the others.

4.7 Conclusion

This paper studies, in a theoretical context, the social optimality of labor markets with search frictions and short-time work policy. Workers with low productivity may generate contemporaneous profit losses, in which case firms can participate to STW and reduce working hours. However, participation in STW is costly to the firms because the administrative process requires effort and governments are regulating the STW programs. Due to these costs, the firms are creating too few new jobs, and the level of STW is too low from a social point of view. This paper proposes transfers in the model to redistribute a fraction of output to firms in order to correct the inefficiencies.

European policies, which for instance subsidize social security contributions and offer training for workers, are aiming to increase the level of STW especially during economic downturns. The transfers in this paper can be considered to capture a part of these existing policy measures in many European countries. The transfers are able to restore the social efficiency of decentralized economy, which suggests that the European governments are aware of the potential shortfalls in the level of STW without additional incentives.

The STW programs aim to prevent excess job losses and unemployment fluctuations in particular during recessions, and are shown to contribute in this task. This paper considers one potential trade-off of the policy, lower job creation. Indeed, when the firms are optimizing their output production keeping current employees in STW, the investments in new jobs can be neglected. Another potential trade-off is information asymmetry and moral hazard. The asymmetric information is a consequence of firms' ability to observe the profitability of their employees, which the government is unable to detect. Consequently, the choice of the efficient STW criterion and subsidies is complicated, and may end up in an overly generous policy. Hence, the firms may be tempted to send profitable workers to STW in order to collect additional profits on the expense of the government. The model of this paper contains elements such as job specific productivity, government transfers and STW criteria, that allow the model with feasible extensions to address also these drawbacks of STW. I leave this analysis for future research.

Appendix

4.A Model derivation

This section contains the details of the derivation of the model, the decentralized and competitive solutions and their comparisons. In addition, an alternative wage rule and its efficiency is discussed in subsection 4.A.7.

4.A.1 Aggregation and notation

The productivity of each job consists of a common component which is the same for all jobs, and an idiosyncratic component which is job specific. The output of a job i with idiosyncratic productivity $\varepsilon_{i,t}$ is given by

$$y_{i,t} = A_t - \varepsilon_{i,t}, \tag{A.1}$$

where A_t is common productivity component, and $\varepsilon_{i,t}$ is drawn from a time-invariant distribution with PDF $g(\varepsilon)$.

The detailed conditions for endogenous separations and STW are derived later, both of which induces a threshold value of idiosyncratic productivity. These thresholds are namely STW threshold v_t^k and separation threshold v_t^f. Workers with idiosyncratic productivity $\varepsilon_t < v_t^k$ are normal full-time workers. For the clarity of expression, an aggregate output of full-time workers is named A_t^{FT}. It is a conditional mean of the idiosyncratic component, conditioned on the STW threshold, given by

$$A_t^{FT} = \int_{-\infty}^{v_t^k} A_t - \varepsilon_t \quad g(\varepsilon) d\varepsilon. \tag{A.2}$$

Respectively, the aggregate output of STW workers is named A_t^{STW}, and given by

$$A_t^{STW} = \int_{v_t^k}^{v_t^f} A_t - \varepsilon_t \quad g(\varepsilon) d\varepsilon. \tag{A.3}$$

In other words, workers with idiosyncratic productivity $v_t^k < \varepsilon_t < v_t^f$ are in STW.

Using the thresholds above the share of endogenously separated workers becomes

$$\rho_t^e = \int_{v_t^f}^{\infty} g(\varepsilon) d\varepsilon, \tag{A.4}$$

in which ρ_t^e denotes the endogenous separation rate. Respectively, the rate of workers in

STW, named χ_t is given by

$$\chi_t = \int_{v_t^k}^{v_t^f} g(\varepsilon) d\varepsilon. \tag{A.5}$$

4.A.2 Households and the government

The household is risk neutral and makes consumption savings decisions to maximize utility given by

$$\max_{C_t, B_{t+1}} = \sum_{t=0}^{\infty} E_t \beta^t U(C_t) \tag{A.6}$$

subject to a budget constraint given by

$$\begin{aligned} C_t + B_{t+1} &= n_t^{FT} w_t FT + n_t^{STW} w_t^{STW} (1 - K_t^*) \\ &+ (1 - n_t) b + n_t^{STW} K_t^* b + (1 + R_t) B_t + \Pi_t - T_t, \end{aligned} \tag{A.7}$$

in which C_t is consumption, B_t a one period risk-free government bond, T_t a lump-sum tax and Π_t profits from the firm.

Resulting in a consumption Euler equations as

$$\frac{1}{R_{t+1}} = \beta. \tag{A.8}$$

The government runs a balanced budget as

$$\tau^f + \tau^K = T_t + B_t - R_t B_{t-1}. \tag{A.9}$$

4.A.3 Firm's problem

A firm maximizes intertemporal profits by

$$\begin{aligned} \max_{v_t, n_t^{FT}, n_t^{STW}, K_t, \chi_t, \rho_t^e} E_t \sum_{t=0}^{\infty} \beta^t \Bigg(& n_t^{FT} A_t^{FT} + n_t^{STW} A_t^{STW} (1 - K_t^*) \\ & - w_t n_t^{FT} - w_t n_t^{STW} (1 - K_t^*) \\ & - C(K_t^*) n_t^{STW} - X(\chi_t) - \frac{\rho_t^e}{1 - \rho_t^e} n_t (f - \tau^n) - \kappa v_t \Bigg), \end{aligned} \tag{A.10}$$

such that

$$
\begin{aligned}
n_t^{FT} &= (1-\rho^x)\left[(1-\rho_t^e)(1-\chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW})\right], \\
n_t^{STW} &= (1-\rho^x)\left[(1-\rho_t^e)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT})\right], \\
n_t &= n_t^{FT} + n_t^{STW} \\
X(\chi_t) &= \frac{c_\chi}{2}\chi_t^2 \\
C(K_t^*) &= \frac{c_k - \tau^K}{2}(K_t^*)^2,
\end{aligned}
\tag{A.11}
$$

FOCs w.r.t v_t, n_t^{FT}, n_t^{STW}, χ_t and ρ_t^e with Lagrange multipliers λ^{FT} and λ^{STW} for employment evolution constraints:

$$
\begin{aligned}
&-\kappa + \beta E_t \lambda_{t+1}^{FT}(1-\rho^x)(1-\rho_{t+1}^e)(1-\chi_{t+1})q(\theta_t) \\
&+\beta E_t \lambda_{t+1}^{STW}(1-\rho^x)(1-\rho_{t+1}^e)\chi_{t+1}q(\theta_t) = 0
\end{aligned}
\tag{A.12}
$$

$$
\begin{aligned}
&A_t^{FT} - w_t - \frac{\rho_t^e}{1-\rho_t^e}(f-\tau^n) - \lambda_t^{FT} \\
&+\beta E_t \lambda_{t+1}^{FT}(1-\rho^x)(1-\rho_{t+1}^e)(1-\chi_{t+1}) + \beta E_t \lambda_{t+1}^{STW}(1-\rho^x)(1-\rho_{t+1}^e)\chi_{t+1} = 0
\end{aligned}
\tag{A.13}
$$

$$
\begin{aligned}
&A_t^{STW}(1-K_t^*) - w_t(1-K_t^*) - C(K_t^*) - \frac{\rho_t^e}{1-\rho_t^e}(f-\tau^n) - \lambda_t^2 \\
&+\beta E_t \lambda_{t+1}^{FT}(1-\rho^x)(1-\rho_{t+1}^e)(1-\chi_{t+1}) + \beta E_t \lambda_{t+1}^{STW}(1-\rho^x)(1-\rho_{t+1}^e)\chi_{t+1} = 0
\end{aligned}
\tag{A.14}
$$

$$
\begin{aligned}
&-\lambda_t^{FT}(1-\rho^x)(1-\rho_t^e)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) + \\
&\lambda_t^{STW}(1-\rho^x)(1-\rho_t^e)(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) - c_\chi \chi_t = 0
\end{aligned}
\tag{A.15}
$$

$$
\begin{aligned}
&-\frac{(f-\tau^f)n_t}{(1-\rho_t^e)^2} - \lambda_t^{FT}(1-\rho^x)(1-\chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
&-\lambda_t^{STW}(1-\rho^x)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) = 0
\end{aligned}
\tag{A.16}
$$

Intermediate results:

$$
\frac{\kappa}{q(\theta_t)} = \beta E_t \lambda_{t+1}^{FT}(1-\rho^x)(1-\rho_{t+1}^e)(1-\chi_{t+1}) + \beta \lambda_{t+1}^{STW}(1-\rho^x)(1-\rho_{t+1}^e)\chi_{t+1},
$$

$$\Rightarrow$$

$$
\lambda_t^{FT} = A_t^{FT} - w_t - \frac{\rho_t^e}{1-\rho_t^e}(f-\tau^f) + \frac{\kappa}{q(\theta_t)}
\tag{A.17}
$$

and

$$
\lambda_t^{STW} = A_t^{STW}(1-K_t^*) - w_t(1-K_t^*) - C(K_t^*) - \frac{\rho_t^e}{1-\rho_t^e}(f-\tau^f) + \frac{\kappa}{q(\theta_t)}
$$

Resulting in job creation condition as

$$\frac{\kappa}{q(\theta_t)} = \beta E_t\Bigg(A^{FT}_{t+1} - w_{t+1} - \frac{\rho^e_{t+1}}{1-\rho^e_{t+1}}(f-\tau^f) + \frac{\kappa}{q(\theta_{t+1})}\Bigg)$$
$$(1-\rho^x)(1-\rho^e_{t+1})(1-\chi_{t+1})$$
$$+\beta E_t\Bigg(A^{STW}_{t+1}(1-K^*_{t+1}) - w_{t+1}(1-K^*_{t+1}) - C(K^*_{t+1}) -$$
$$\frac{\rho^e_{t+1}}{1-\rho^e_{t+1}}(f-\tau^f) + \frac{\kappa}{q(\theta_{t+1})}\Bigg)$$
$$(1-\rho^x)(1-\rho^e_{t+1})\chi_{t+1} \tag{A.18}$$
$$\Leftrightarrow$$
$$\frac{\kappa}{q(\theta_t)} = \beta E_t\Bigg[(1-\rho_{t+1})\Bigg\{(1-\chi_{t+1})(A^{FT}_{t+1} - w_{t+1})$$
$$+\chi_{t+1}[(1-K^*_{t+1})(A^{STW}_{t+1} - w_{t+1}) - C(K^*_{t+1})]\Bigg\}$$
$$-(1-\rho^x)\rho^e_{t+1}(f-\tau^n) + (1-\rho_{t+1})\frac{\kappa}{q(\theta_{t+1})}\Bigg]$$

Deriving χ_t

$$-\lambda^{FT}_t(1-\rho^x)(1-\rho^e_t)(n^{FT}_{t-1} + q(\theta_{t-1})v_{t-1} + n^{STW}_{t-1})+$$
$$\lambda^{STW}_t(1-\rho^x)(1-\rho^e_t)(n^{STW}_{t-1} + q(\theta_{t-1})v_{t-1} + n^{FT}_{t-1}) - c_\chi\chi_t = 0$$
$$\Leftrightarrow$$
$$c_\chi\chi_t = (\lambda^{STW}_t - \lambda^{FT}_t)(1-\rho^x)(1-\rho^e_t)(n^{FT}_{t-1} + q(\theta_{t-1})v_{t-1} + n^{STW}_{t-1})$$
$$\Leftrightarrow \tag{A.19}$$
$$c_\chi\chi_t = (\lambda^{STW}_t - \lambda^{FT}_t)(1-\rho_t)(n^{FT}_{t-1} + q(\theta_{t-1})v_{t-1} + n^{STW}_{t-1})$$
$$\Leftrightarrow$$
$$\chi_t = (\lambda^{STW}_t - \lambda^{FT}_t)\frac{n_t}{c_\chi}$$

Deriving ρ_t^e

$$-\frac{(f-\tau^f)n_t}{(1-\rho_t^e)^2} - \lambda_t^{FT}(1-\rho^x)(1-\chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW})$$
$$-\lambda_t^{STW}(1-\rho^x)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) = 0$$
$$\Leftrightarrow$$
$$\frac{(f-\tau^f)n_t}{(1-\rho_t^e)^2} = -\lambda_t^{FT}(1-\rho^x)(1-\chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW})$$
$$-\lambda_t^{STW}(1-\rho^x)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT})$$
$$\Leftrightarrow$$
$$\frac{(f-\tau^f)n_t}{1-\rho_t^e} = -\lambda_t^{FT}(1-\rho^x)(1-\rho_t^e)(1-\chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW})$$
$$-\lambda_t^{STW}(1-\rho^x)(1-\rho_t^e)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT})$$
$$\Leftrightarrow$$
$$\frac{(f-\tau^f)n_t}{1-\rho_t^e} = -[\lambda_t^{FT}(1-\chi_t) + \lambda_t^{STW}\chi_t](1-\rho^x)(1-\rho_t^e)(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT})$$
$$\Leftrightarrow$$
$$\frac{(f-\tau^f)n_t}{1-\rho_t^e} = -[\lambda_t^{FT}(1-\chi_t) + \lambda_t^{STW}\chi_t]n_t$$
$$\Leftrightarrow$$
$$f-\tau^f = -[\lambda_t^{FT}(1-\chi_t) + \lambda_t^{STW}\chi_t](1-\rho_t^e)$$
$$\Leftrightarrow$$
$$\frac{f-\tau^f}{\lambda_t^{FT}(1-\chi_t) + \lambda_t^{STW}\chi_t} + 1 = \rho_t^e \tag{A.20}$$

Applying the definitions of λ_t^{FT} and λ_t^{STW}, the condition can be developed further as

$$\frac{(f-\tau^f)}{1-\rho_t^e} = -[\lambda_t^{FT}(1-\chi_t) + \lambda_t^{STW}\chi_t]$$

$$\Leftrightarrow$$

$$\frac{(f-\tau^f)}{1-\rho_t^e} = -\Bigg\{\left[A_t^{FT} - w_t - \frac{\rho_t^e}{1-\rho_t^e}(f-\tau^f) + \frac{\kappa}{q(\theta_t)}\right](1-\chi_t) +$$
$$\left[A_t^{STW}(1-K_t^*) - w_t(1-K_t^*) - C(K_t^*) - \frac{\rho_t^e}{1-\rho_t^e}(f-\tau^f) + \frac{\kappa}{q(\theta_t)}\right]\chi_t\Bigg\}$$

$$\Leftrightarrow$$

$$\frac{(f-\tau^f)}{1-\rho_t^e} = -\Bigg\{[A_t^{FT} - w_t](1-\chi_t) +$$
$$[(A_t^{STW} - w_t)(1-K_t^*) - C(K_t^*)]\chi_t - \frac{\rho_t^e}{1-\rho_t^e}(f-\tau^f) + \frac{\kappa}{q(\theta_t)}\Bigg\} \tag{A.21}$$

$$\Leftrightarrow$$

$$\frac{(1-\rho_t^e)(f-\tau^f)}{1-\rho_t^e} = -\Bigg\{[A_t^{FT} - w_t](1-\chi_t) +$$
$$[(A_t^{STW} - w_t)(1-K_t^*) - C(K_t^*)]\chi_t + \frac{\kappa}{q(\theta_t)}\Bigg\}$$

$$\Leftrightarrow$$

$$f-\tau^f = -\Bigg\{[A_t^{FT} - w_t](1-\chi_t) +$$
$$[(A_t^{STW} - w_t)(1-K_t^*) - C(K_t^*)]\chi_t + \frac{\kappa}{q(\theta_t)}\Bigg\}$$

Result of the equation (A.21) gives the value of threshold productivity. At the threshold the value of the job is equal with the separation cost. Furthermore, there are three separate cases. The first is that there is no STW, i.e. $\chi_t = 0$. Hence, all the separations happen to full-time workers. The idiosyncratic productivity threshold is given by

$$f-\tau^f = -\left[A_t^{FT} - w_t + \frac{\kappa}{q(\theta_t)}\right]$$

$$\Rightarrow$$

$$f-\tau^f = -\left[A_t - v_t^f - w_t + \frac{\kappa}{q(\theta_t)}\right] \tag{A.22}$$

$$\Leftrightarrow$$

$$v_t^f = f - \tau^f + A_t - w_t + \frac{\kappa}{q(\theta_t)},$$

which corresponds the job destruction condition from the canonical search and matching model with endogenous separations.

The second case is that all workers are doing STW, i.e. $\chi_t = 1$. Then, the idiosyncratic productivity threshold is given by

$$f - \tau^f = -\left[(A_t^{STW} - w_t)(1 - K_t^*) - C(K_t^*) + \frac{\kappa}{q(\theta_t)}\right]$$

$$\Rightarrow$$

$$f - \tau^f = -\left[(A_t - v_t^f - w_t)(1 - K_t^*) - C(K_t^*) + \frac{\kappa}{q(\theta_t)}\right] \tag{A.23}$$

$$\Leftrightarrow$$

$$v_t^f = A_t - w_t + \frac{1}{1 - K_t^*}\left[f - \tau^f - C(K_t^*) + \frac{\kappa}{q(\theta_t)}\right],$$

which equals the threshold equation derived in Balleer et al. (2016). The equation (A.23) also gives the separation threshold when making the additional assumption that endogenously separated workers are always those who are in STW.

The third case is that a fraction of workers are in short-time work and the rest in work full time, i.e. $1 > \chi_t > 0$. The idiosyncratic productivity threshold is then given by

$$\begin{gathered} f - \tau^f = -\Big\{[A_t - v_t^f - w_t](1 - \chi_t) + \\ [(A_t - v_t^f - w_t)(1 - K_t^*) - C(K_t^*)]\chi_t + \frac{\kappa}{q(\theta_t)}\Big\} \end{gathered} \tag{A.24}$$

4.A.4 Social planner's problem

$$
\begin{aligned}
U_t^{SP} = \max_{v_t, n_t^{FT}, n_t^{STW}, K_t, v_t^k, v_t^f} E_t \sum_{t=0}^{\infty} \beta^t \Bigg(& n_t^{FT} A_t^{FT} + n_t^{STW} A_t^{STW} (1 - K_t^*) \\
& + b(1 - n_t) + bK_t n_t^{STW} \\
& - X(\chi_t) - C(K_t) n_t^{STW} - \frac{\rho_t^e}{1 - \rho_t^e} n_t f - \kappa v_t \Bigg),
\end{aligned} \tag{A.25}
$$

such that

$$
\begin{aligned}
n_{t+1}^{FT} &= (1 - \rho^x) \left[(1 - \rho_{t+1}^e)(1 - \chi_{t+1})(n_t^{FT} + q(\theta_t) v_t + n_t^{STW}) \right], \\
n_{t+1}^{STW} &= (1 - \rho^x) \left[(1 - \rho_{t+1}^e) \chi_{t+1} (n_t^{STW} + q(\theta_t) v_t + n_t^{FT}) \right], \\
n_{t+1} &= n_{t+1}^{FT} + n_{t+1}^{STW} \\
X(\chi_t) &= \frac{c_\chi}{2} \chi_t^2 \\
C(\chi_t) &= \frac{1}{2} \chi_t^2,
\end{aligned} \tag{A.26}
$$

The derivation of job creation condition requires first order condition with respect to vacancies v_t and employment n_t^{FT} and n_t^{STW}. Denoting ϕ_t^{FT} and ϕ_t^{STW} as Lagrange multipliers of the employment evolution constraints, the FOCs of v_t, n_t^{FT} and n_t^{STW} are respectively

$$
\begin{aligned}
& -\kappa + \beta \phi_{t+1}^{FT} [(1 - \rho^x)(1 - \rho_t^e)(1 - \chi_t)(q(\theta_t) + \theta_t q'(\theta_t))] \\
& + \beta \phi_{t+1}^{STW} [(1 - \rho^x)(1 - \rho_t^e) \chi_t (q(\theta_t) + \theta_t q'(\theta_t))] = 0
\end{aligned} \tag{A.27}
$$

$$
\begin{aligned}
& A_t^{FT} - b - \frac{\rho_t^e}{1 - \rho_t^e} f - \phi_t^{FT} \\
& + \beta \phi_{t+1}^{FT} (1 - \rho^x)(1 - \rho_t^e)(1 - \chi_t) + \beta \phi_{t+1}^{STW} (1 - \rho^x)(1 - \rho_t^e) \chi_t = 0
\end{aligned} \tag{A.28}
$$

$$
\begin{aligned}
& A_t^{STW} (1 - K_t^*) - C(K_t^*) - b + K_t^* b - \frac{\rho_t^e}{1 - \rho_t^e} f - \phi_t^{STW} \\
& + \beta \phi_{t+1}^{FT} (1 - \rho^x)(1 - \rho_t^e)(1 - \chi_t) + \beta \phi_{t+1}^{STW} (1 - \rho^x)(1 - \rho_t^e) \chi_t = 0
\end{aligned} \tag{A.29}
$$

Intermediate results:

$$\frac{\kappa}{q(\theta_t)(1-\xi_t)} = \beta E_t \phi_{t+1}^{FT}(1-\rho^x)(1-\rho_{t+1}^e)(1-\chi_{t+1}) + \beta \phi_{t+1}^{STW}(1-\rho^x)(1-\rho_{t+1}^e)\chi_{t+1},$$

$$\Rightarrow$$

$$\phi_t^{FT} = A_t^{FT} - b - \frac{\rho_t^e}{1-\rho_t^e} f + \frac{\kappa}{q(\theta_t)(1-\xi_t)}$$

$$\phi_t^{STW} = A_t^{STW}(1-K_t^*) - C(K_t^*) + (-1+K_t^*)b - \frac{\rho_t^e}{1-\rho_t^e} f + \frac{\kappa}{q(\theta_t)(1-\xi_t)} \tag{A.30}$$

Combining the results to job creation condition yields

$$\begin{aligned}\frac{\kappa}{q(\theta_t)(1-\xi_t)} = \beta E_t \Bigg[&(1-\rho_{t+1} \Bigg\{ (1-\chi_{t+1})(A_{t+1}^{FT} - b) \\ &+ \chi_{t+1}[(1-K_{t+1}^*)(A_{t+1}^{STW} - b) - C(K_t^*)] \Bigg\} \\ &-(1-\rho^x)\rho_{t+1}^e f + (1-\rho_{t+1}) \frac{\kappa}{q(\theta_{t+1})(1-\xi_{t+1})} \Bigg].\end{aligned} \tag{A.31}$$

Deriving the share of STW workers, requires FOC w.r.t χ_t.
Deriving χ_t

$$\begin{gathered} -\phi_t^{FT}(1-\rho^x)(1-\rho_t^e)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) + \\ \phi_t^{STW}(1-\rho^x)(1-\rho_t^e)(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) - c_\chi \chi_t = 0 \\ \Leftrightarrow \\ c_\chi \chi_t = (\phi_t^{STW} - \phi_t^{FT})(1-\rho^x)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\ \Leftrightarrow \\ \chi_t = (\phi_t^{STW} - \phi_t^{FT})\frac{n_t}{c_\chi} \end{gathered} \tag{A.32}$$

Deriving ρ_t^e

$$
\begin{aligned}
&-\frac{fn_t}{(1-\rho_t^e)^2} - \phi_t^{FT}(1-\rho^x)(1-\chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
&\quad -\phi_t^{STW}(1-\rho^x)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) = 0 \\
&\Leftrightarrow \\
&\frac{fn_t}{(1-\rho_t^e)^2} = -\phi_t^{FT}(1-\rho^x)(1-\chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
&\quad -\phi_t^{STW}(1-\rho^x)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) \\
&\Leftrightarrow \\
&\frac{fn_t}{1-\rho_t^e} = -\phi_t^{FT}(1-\rho^x)(1-\rho_t^e)(1-\chi_t)(n_{t-1}^{FT} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{STW}) \\
&\quad -\phi_t^{STW}(1-\rho^x)(1-\rho_t^e)\chi_t(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) \\
&\Leftrightarrow \\
&\frac{fn_t}{1-\rho_t^e} = -[\phi_t^{FT}(1-\chi_t) + \phi_t^{STW}\chi_t](1-\rho^x)(1-\rho_t^e)(n_{t-1}^{STW} + q(\theta_{t-1})v_{t-1} + n_{t-1}^{FT}) \\
&\Leftrightarrow \\
&\frac{fn_t}{1-\rho_t^e} = -[\phi_t^{FT}(1-\chi_t) + \phi_t^{STW}\chi_t]n_t \\
&\Leftrightarrow \\
&f = -[\phi_t^{FT}(1-\chi_t) + \phi_t^{STW}\chi_t](1-\rho_t^e) \\
&\Leftrightarrow \\
&\frac{f}{\phi_t^{FT}(1-\chi_t) + \phi_t^{STW}\chi_t} + 1 = \rho_t^e
\end{aligned}
\tag{A.33}
$$

4.A.5 Wage bargaining

A labor union bargains wages for all workers. An outside option is a strike when no production is done. The union bargains a merit based wage rule such that the more productive workers ear higher wages. The Nash bargaining problem is

$$\arg\max_{w_t} (W_t - \tilde{W}_t)^\gamma (F_t - \tilde{F}_t)^{1-\gamma}, \tag{A.34}$$

where

$$W_t = w_t + \beta E_t[(1-\rho_t)W_{t+1} + \rho_{t+1}U_{t+1}], \tag{A.35}$$

$$\tilde{W}_t = b_t + \beta E_t[(1-\rho_t)W_{t+1} + \rho_{t+1}U_{t+1}], \tag{A.36}$$

$$F_t = (A_t - \varepsilon_t) - w_t + \beta E_t \frac{\kappa}{q(\theta_{t+1})}, \tag{A.37}$$

$$\tilde{F}_t = \beta E_t \frac{\kappa}{q(\theta_{t+1})}, \tag{A.38}$$

Solution of A.34 is

$$(W_t - \tilde{W}_t) = \frac{\gamma}{1-\gamma}(F_t - \tilde{F}_t), \tag{A.39}$$

and inserting value functions results in

$$\begin{aligned} (w_t - b) &= \frac{\gamma}{1-\gamma}(A_t - \varepsilon - w_t) \\ \Leftrightarrow w_t &= \gamma(A_t - \varepsilon) + (1-\gamma)b. \end{aligned} \tag{A.40}$$

Then aggregate wages for full time and short time workers become

$$\begin{aligned} w_t^{FT} &= \int_{-\infty}^{v_t^k} \gamma(A_t - \varepsilon)g(\varepsilon)d\varepsilon + (1-\gamma)b \\ \Leftrightarrow w_t^{FT} &= \gamma A_t^{FT} + (1-\gamma)b, \end{aligned} \tag{A.41}$$

and similarly

$$w_t^{STW} = \gamma A_t^{STW} + (1-\gamma)b. \tag{A.42}$$

4.A.6 Proof of Proposition 1

Now, in order to compare the differences in the two JCC equations, I subtract the decentralized solution from the social planner solution with wage bargaining outcome included into the decentralized condition.

Next, the wage is replaced by its definition, for instance $w_t^{FT} = \gamma A_t^{FT} + (1-\gamma)b$. Using the wage rule and after some reorganizing, the competitive solution becomes

$$\begin{aligned} 0 = \beta E_t \Bigg(&(1-\rho^x)\Big[(1-\rho_{t+1}^e)(1-\chi_{t+1})(A_{t+1}^{FT} - b) \\ &+ (1-\rho_{t+1}^e)\chi_{t+1}(A_{t+1}^{STW}(1-K_{t+1}^*) - (1-K_{t+1}^*)b - \frac{1}{(1-\gamma)}C(K^*)\Big] \\ &- \frac{1}{(1-\gamma)}(1-\rho^x)\rho_{t+1}^e(f-\tau^n) + (1-\rho_{t+1})\frac{\kappa}{q(\theta_{t+1})(1-\gamma)}\Bigg) \\ &- \frac{\kappa}{q(\theta_t)(1-\gamma)}. \end{aligned} \tag{A.43}$$

The result of the subtraction becomes

$$E_t\beta(1-\rho^x)(1-\rho_{t+1})\frac{\kappa}{q(\theta_{t+1})}\left(\frac{1}{1-\xi_{t+1}}-\frac{1}{1-\gamma}\right)-\frac{\kappa}{q(\theta_t)}\left(\frac{1}{1-\xi_t}-\frac{1}{1-\gamma}\right)$$

$$-\left(f-\frac{f-\tau^f}{1-\gamma}\right)\beta E_t[(1-\rho^x)\rho^e_{t+1}] \tag{A.44}$$

$$-\left(c_k-\frac{c_k-\tau^K}{1-\gamma}\right)\frac{1}{2}E_t\beta(K^*_{t+1})^2=0$$

Where all the individual conditions of optimality are: $\xi_t=\gamma$, $\tau^f=\gamma f$ and $\tau^K=\gamma c_k$.

Comparing STW conditions

Equality in hour reductions (the decentralized vs. the social planner solution)

$$\begin{gathered}
-\frac{(1-\gamma)(A_t^{STW}-b)}{c_k-\tau^K}=-\frac{A_t^{STW}-b}{c_k}\\
\Leftrightarrow\\
-\frac{(1-\gamma)}{c_k-\tau^K}=-\frac{1}{c_k}\\
\Leftrightarrow\\
-(1-\gamma)c_k=-c_k+\tau^K\\
\Leftrightarrow\\
-c_k+\gamma c_k=-c_k+\tau^K\\
\Leftrightarrow\\
\tau^K=\gamma c_k.
\end{gathered} \tag{A.45}$$

The hour reduction conditions coincides with the other conditions related to the transfers from section above.

No STW case

Applying the definitions of λ_t^{FT} and λ_t^{STW}, the decentralized condition (A.19) can be developed further as

$$\chi_t = (\lambda_t^{STW} - \lambda_t^{FT})\frac{n_t}{c_\chi}$$
$$\Leftrightarrow \tag{A.46}$$
$$\chi_t = \left[(A_t^{STW} - w_t)(1 - K_t^*) - C(K_t^*) - (A_t^{FT} - w_t)\right]\frac{n_t}{c_\chi}.$$

The profits of full-time workers, i.e. $(A_t^{FT} - w_t)$ are always positive or zero. By assumption STW workers are generating deficits. Consequently, the value in brackets, comparing the output of STW and full-time workers is negative. In turn, the share of workers in STW, i.e. χ_t cannot be negative, hence the cost $c_\chi < 0$, or the definition of the share is assumed as

$$\chi_t = \max((\lambda_t^{STW} - \lambda_t^{FT})\frac{n_t}{c_\chi}, 0), \tag{A.47}$$

and if c_χ is set to positive value there is no STW.

Separation and STW conditions

The STW share of workers from decentralized and social planner solutions are compared as $\chi_t^{SP} = \chi_t^{decentralized}$ resulting in

$$(-\phi_t^{FT} + \lambda_t^{FT}) + (\phi_t^{STW} - \lambda_t^{STW}) = 0, \tag{A.48}$$

where the equality holds for certainty when the shadow values from the two solutions are equal. The comparison of marginal values results in

$$-\phi_t^{FT} + \lambda_t^{FT} = 0$$
$$\Leftrightarrow$$
$$A_t^{FT} - b - \frac{\rho_t^e}{1-\rho_t^e}f + \frac{\kappa}{q(\theta_t)(1-\xi_t)} = \tag{A.49}$$
$$(1-\gamma)(A_t^{FT} - b) - \frac{\rho_t^e}{1-\rho_t^e}(f - \tau^f) + \frac{\kappa}{q(\theta_t)},$$

which results in the subset of the same optimality conditions as the comparison of job creation conditions, i.e. $\xi_t = \gamma$, $\tau^f = \gamma f$.

The second shadow values compare as

$$
\begin{aligned}
-\phi_t^{STW(SP)} + \lambda_t^{STW} &= 0 \\
&\Leftrightarrow \\
(1-K_t^*)(A_t^{STW}-b) - C(K_t^*) - \frac{\rho_t^e}{1-\rho_t^e} f + \frac{\kappa}{q(\theta_t)(1-\xi_t)} &= \\
(1-\gamma)(1-K_t^*)(A_t^{STW}-b) - C(K_t^*) - \frac{\rho_t^e}{1-\rho_t^e}(f-\tau^f) + \frac{\kappa}{q(\theta_t)}&,
\end{aligned}
\tag{A.50}
$$

which holds when all the conditions from job creation comparison are met, i.e. $\xi_t = \gamma$, $\tau^f = \gamma f$, $\tau^K = \gamma c_k$.

Equations (A.49) and (A.50) show that the Lagrange multipliers from the two problems are equal, i.e. $\phi_t^{FT} = \lambda_t^{FT}$ and $\phi_t^{STW} = \lambda_t^{STW}$, when the conditions of Proposition 1 hold. Furthermore, this implies that the same conditions are sufficient to yield the optimality of endogenous separation choice ρ_t^e.

4.A.7 Alternative wage rule and its efficiency

The wage defined above internalizes job specific productivity by taking into account the idiosyncratic component. As a consequence, the idiosyncratic productivity does not give rise to social inefficiencies with this wage definition. This section discusses an alternative wage rule, in which the wage is not internalizing the job specific productivity.

In Balleer et al. (2016) the wage is constant for all workers. The wage is defined at the unconditional mean of the idiosyncratic productivity, and further, by assuming that this mean is zero. As a result, the wage depends on the aggregate productivity component, and the outside option for workers. This changes the value function of the firm to

$$F_t^{alt} = A_t - w_t + \beta E_t \frac{\kappa}{q(\theta_{t+1})}, \tag{A.51}$$

and the wage becomes

$$w_t = \gamma A_t + (1-\gamma)b. \tag{A.52}$$

Using this wage rule, the profits of a worker working full time π_t^{FT} are given by

$$\begin{aligned}
\pi_t^{FT} &= A_t^{FT} - w \\
&\Leftrightarrow \\
\pi_t &= A_t^{FT} - (\gamma A_t + (1-\gamma)b) \\
&\Leftrightarrow \\
\pi_t^{FT} &= \int_{-\infty}^{v_t^k} (A_t - \varepsilon_t) g(\varepsilon) d\varepsilon - (\gamma A_t + (1-\gamma)b) \\
&\Leftrightarrow \\
\pi_t^{FT} &= (1-\gamma)(A_t + b) - \int_{-\infty}^{v_t^k} \varepsilon_t g(\varepsilon) d\varepsilon.
\end{aligned} \tag{A.53}$$

The same wage rule for the profits of a STW worker yields

$$\pi_t^{STW} = \left[(1-\gamma)(A_t + b) - \int_{v_t^k}^{v_t^f} \varepsilon_t g(\varepsilon) d\varepsilon\right](1 - K_t^*). \tag{A.54}$$

In order to investigate the social efficiency of this alternative wage rule, the same notation as in equations above is applied to the social planner solution. Specifically, the social planner solution of output and home production for full-time and STW workers are respectively y_t^{FT} and y_t^{STW}, given by

$$y_t^{FT} = A_t + b - \int_{-\infty}^{v_t^k} \varepsilon_t g(\varepsilon) d\varepsilon. \tag{A.55}$$

and

$$y_t^{STW} = \left[A_t + b - \int_{v_t^k}^{v_t^f} \varepsilon_t g(\varepsilon) d\varepsilon\right](1 - K_t^*). \tag{A.56}$$

As is done in section 4.A.6, I first apply the equations (A.53) and (A.54) in the decentralized job creation condition (A.18), in order to get a job creation condition with the functional form of wage, as

$$\begin{aligned}
\frac{\kappa}{q(\theta_t)(1-\gamma)} = \beta E_t \Bigg((1-\rho_t) \Bigg[& (1-\chi_{t+1})(A_{t+1} - b - \frac{1}{(1-\gamma)} \int_{-\infty}^{v_t^k} \varepsilon_{t+1} g(\varepsilon) d\varepsilon) \\
& + \chi_{t+1}(1 - K_{t+1}^*)(A_{t+1} - b - \frac{1}{(1-\gamma)} \int_{-\infty}^{v_t^k} \varepsilon_{t+1} g(\varepsilon) d\varepsilon) - \frac{1}{(1-\gamma)} C(K^*) \Bigg] \\
& - \frac{1}{(1-\gamma)} (1-\rho^x) \rho_{t+1}^e (f - \tau^n) + (1-\rho_{t+1}) \frac{\kappa}{q(\theta_{t+1})(1-\gamma)} \Bigg).
\end{aligned} \tag{A.57}$$

Second, I compare this decentralized job creation condition with the social planner condition, as in section 4.A.6, by subtracting the decentralized condition in (A.57) from the social planner condition (A.31). The subtraction yields

$$
\begin{gathered}
E_t\beta(1-\rho_t)\underbrace{(1-\chi_{t+1})\left(1-\frac{1}{1-\gamma}\right)\int_{-\infty}^{v_t^k}\varepsilon_{t+1}g(\varepsilon)d\varepsilon}_{\text{Full time workers' productivity (+/-)}}+ \\
E_t\beta(1-\rho_t)\underbrace{\chi_{t+1}\left(1-\frac{1}{1-\gamma}\right)\int_{v_t^k}^{v_t^f}\varepsilon_{t+1}g(\varepsilon)d\varepsilon}_{\text{STW workers' productivity (+/-)}} \\
+\Lambda=0,
\end{gathered}
\tag{A.58}
$$

in which Λ is the left hand side of the equation (A.44), i.e. it contains the inefficiencies from congestion externality, STW costs and separation cost. These two additional inefficiencies arise from the wage bargaining not internalizing the job specific productivity. When workers have bargaining power, the multiplier $\frac{1}{1-\gamma}>1$, and the idiosyncratic component ε_{t+1} gives rise to externalities on job creation. These externalities can be positive or negative depending on the assumptions about the distribution of idiosyncratic productivity $g(\varepsilon)$.

For simplicity, let us make the following assumptions about the idiosyncratic distribution: i) the unconditional mean is zero, and ii) the distribution is symmetric with respect to the mean. In other words, the positive and negative values of ε_t have equal probability. We can consider normal distribution as an example, which is depicted in Figure 4.1 of the main text. As in the figure, the distribution of productivities for full-time workers is the original distribution from which the endogenous separations and STW cut off the right tail. Hence, the mean ε_t for the full-time workers, i.e. conditional mean, is negative for all the positive shares of endogenous separations or STW. On the contrary, for the STW workers the mean is positive.[9] More precisely, the integral over the idiosyncratic component ε_t in equation (A.53) is negative and in equation (A.54) positive.

The inefficiency is the following. When the wage is constant to all workers according to (A.52), it is too low for those working full-time, and too high for those in STW, as compared with their productivities. Hence, there is a positive externality on job creation from the full-time workers' productivity, i.e. firms are creating too many new jobs from the social welfare perspective. On the contrary, there is a negative externality on job creation from the STW workers, i.e. the firms are creating too few new jobs. Which of

[9] Only in the extreme case in which the share of full-time workers is smaller than the endogenous separation rate, the mean for STW workers could become negative.

these externalities is larger depends on the calibration of the model, most notable the shares of the workers in STW and full-time work.

4.B Numerical illustration

The following presents a numerical illustration of the decentralized and socially optimal economies. The impact of the transfers proposed above are studied separately, to demonstrate some of the difference between them. Notice that this numerical exercise is not based on an empirically relevant calibration, but artificial configuration which is chosen to present some key features of the model.

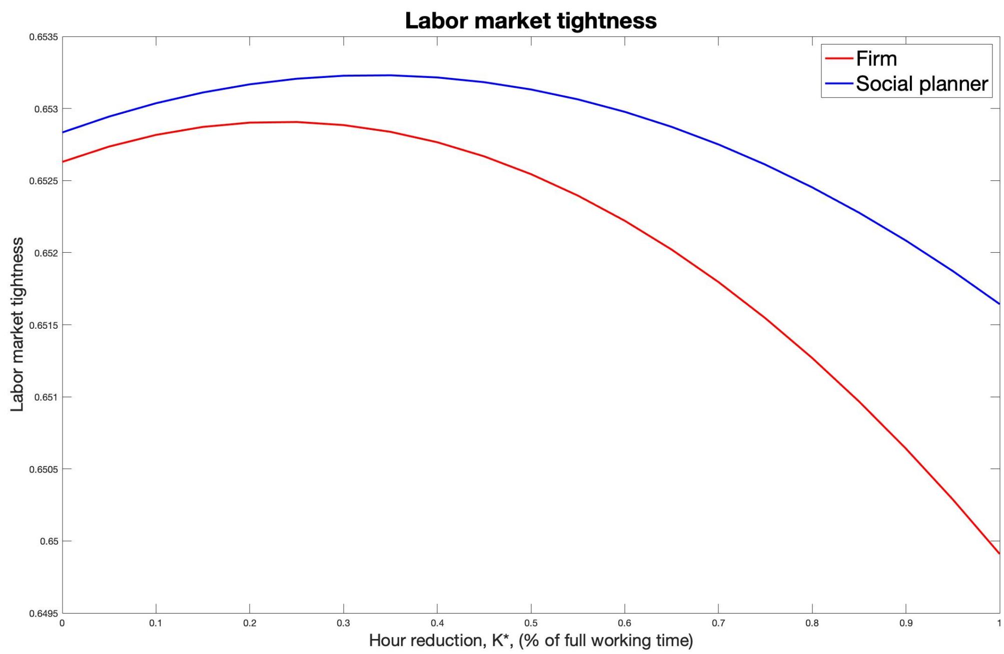
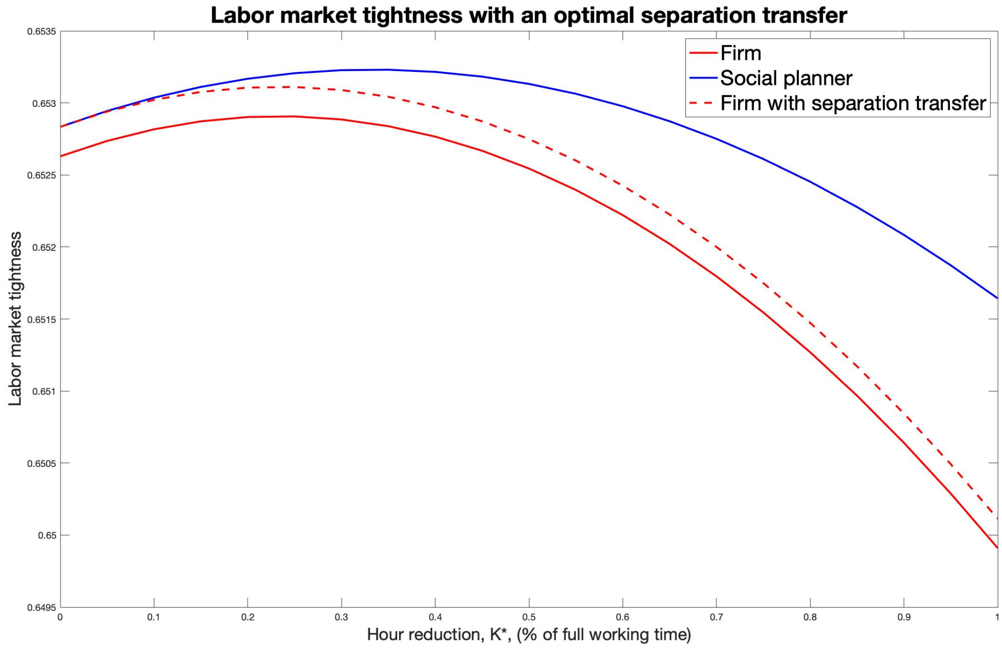


Figure 4.3: Labor market tightness with the different levels of hour reductions. The blue line is the social planner vacancy creation curve from equation (17) and the red line the competitive economy, i.e. equation (10). The Hosios condition is assumed to hold, but there are no transfers.

Figure 4.3 presents the relationship of labor market tightness and working-hour reductions in steady-state. A systematic inefficiency is shown as the curve of the competitive firm is below the social planner's curve with any level of working time. The curves are increasing at first, because the convex cost of reducing working time is small and at the same time the hour reductions are linear.



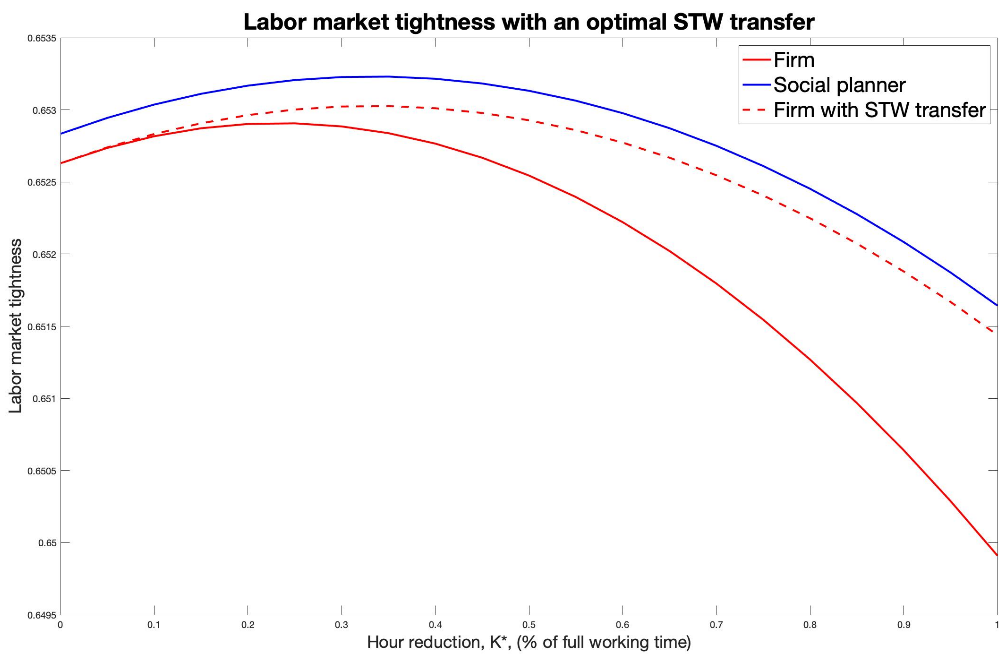


Figure 4.4: Labor market tightness and hour reductions with an optimal separation transfer above and an STW transfer below. The Hosios condition is assumed to hold. The blue line is the social planner vacancy creation curve from equation (17), the red line the competitive economy and the red dashed line the competitive economy with the transfer, i.e. equation (10) with $\tau^f = \gamma f$ first and $\tau^K = \gamma c_K$ in second.

The gains from adjusting the working time are larger than the costs from it. When the hour reduction increases, the cost increases according to a quadratic function and becomes larger than the benefit from hour reductions. This turns the curves downward sloping and in total results in a convex shape.

Figure 4.4 presents the comparison of optimal transfer derived in previous section. The first graph shows the impact of separation transfer. This transfer moves the competitive curve directly upward, making the decentralized economy closer to the socially optimal with any level of working time in STW. In addition, if there is no STW, i.e. the hour reduction $K^* = 0$, the decentralized economy is optimal. When the hour reductions are increasing, the inefficiency of the competitive economy, i.e. the distance of the curves, increases as well.

The lower part of Figure 4.4 shows the impact of STW transfer. The optimal STW transfer incentivizes the firm to reduce more working hours. Furthermore, there are more resources available for creating vacancies. As a result, the curve of the firm with STW transfer moves up and right, where more vacancies are created and the level of STW is increased. However, when the hour reduction K^* is small, the impact of the transfer on job creation is negligible since the cost is also small.

The cases in Figure 4.4 illustrate the difference in channels through which the transfers affect labor markets. The separation transfer reduces the cost of separations, which increases the total profits of the firm. A fraction of these profits is used to increase vacancy postings. The cost of hour reduction by the firm does not depend on the separation cost, hence the optimal choice of the working time in STW is unaffected. The STW transfer subsidizes the hour reductions and incentivizes firms to increase the level of STW. There are two consequences. The first is analogical with the separation transfer, in which the cost reduction increases profits. The second is the decreased working time of low productive workers, which decreases losses generated by these workers, allowing larger surplus to be used in vacancy creation.

4.C Calibration

4.C.1 Data

The observables used in calibration are retrieved from the Bundesbank website (https://www.bundesbank.de/en/statistics). The following lists the data series. The period considered is 2000-2021, and the steady-state targets in the calibration exercise are calculated as simple averages over the sample period.

Unemployment rate. Unemployment registered pursuant to section 16 Social Security Code III / Germany / Social Security Code III and Social Security Code II / Rate

/ Calendar and seasonally adjusted
Series: BBDL1.M.DE.Y.UNE.UBA000.A0000.A01.D00.0.R00.A
Source: Seasonal adjustment based on data provided by the Federal Employment Agency.

Vacancies. Reported vacancies, total / Germany / Total / Absolute value / Calendar and seasonally adjusted
Series: BBDL1.M.DE.Y.VAC.VBA000.A0000.A00.D00.0.ABA.A
Source: Seasonal adjustment based on data provided by the Federal Employment Agency.,

Employed workers. Employed persons according to ESA 2010 / Germany / Domestic concept / Absolute value / Calendar and seasonally adjusted
Series: BBDL1.M.DE.Y.EMP.EAA000.A0000.A00.D10.0.ABA.A
Source: Seasonal adjustment based on data provided by the Federal Statistical Office.

Short-time workers. Short-time workers, basis for entitlement according to section 96 only / Germany / Social Security Code III / Absolute value / Calendar and seasonally adjusted
Series: BBDL1.M.DE.Y.LMP.LKA100.A0000.A02.D00.0.ABA.A
Source: Seasonal adjustment based on data provided by the Federal Employment Agency.,

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