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Essays on energy transition and macroeconomic dynamics

Daniel Esteban Bastidas Cordova

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UNIVERSITE PARIS I PANTHÉON SORBONNE

ED de Economie

Laboratoire de rattachement : CES

THÈSE

Pour l'obtention du titre de Docteur en Economie

Présenté et soutenu publiquement

le 29 Septembre 2023 par

Daniel Esteban BASTIDAS CORDOVA

Essays on energy transition and macroeconomic dynamics

Sous la direction de M. Gaël GIRAUD

Directeur de recherche CNRS

Membre du Jury

Jean-Bernard CHATELAIN	Président pressenti	Université Paris 1
Manuel Alejandro CARDENETE	Rapporteur	Universidad Loyola
Matheus GRASSELLI	Rapporteur	McMaster University
Marc LAVOIE	Membre	University of Ottawa
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Summary

The energy shift towards greener economic systems entails risks and challenges. Being able to understand the way in which those risks interact with broader macroeconomic dynamics is crucial for informed policymaking and effective climate action. This thesis engages in studying these interactions, and on modeling the macroeconomic effects resulting from the implementation of climate mitigation policies, or from energy transition scenarios. It does so by the use of both easy-to-replicate standalone analyses, and by the construction of a mid-size stock-flow consistent model endowed with a multisectoral production module. The thesis contains three chapters, the first one studies the stability properties of the Keen model when the assumed production technology changes. The second chapter focuses on Brazil's Nationally Determined Contributions (NDC) and explores the feasibility of achieving both economic growth and reduced industrial emissions. The analysis of alternative policy scenarios provides valuable information for decision-makers aiming to strike a balance between economic development and climate goals. The third chapter presents a proof-of-concept stock-flow consistent macro model for Brazil, combining a multi-sectoral input-output framework with a macroeconomic layer. The model enables the simulation of effects from different policy options on emissions, industry-level outcomes, and macro-financial aggregates.

Key words: Energy transition, Macroeconomic dynamics, Monetary economics, Climate mitigation policies, Stock-flow consistent model, Multi-sectoral input-output framework, Government debt.

Résumé

La transition vers des systèmes économiques plus verts comporte des risques et des défis. Comprendre la manière dont ces risques interagissent avec les dynamiques macroéconomiques globales est crucial pour élaborer des politiques éclairées et des mesures climatiques efficaces. Cette thèse se penche sur l'étude de ces interactions et sur la modélisation des effets macroéconomiques résultant de la mise en œuvre de politiques d'atténuation du changement climatique ou de scénarios de transition énergétique. Ce faisant, elle utilise à la fois des analyses autonomes faciles à reproduire et la construction d'un modèle de taille moyenne, cohérent avec les stocks et les flux, doté d'un module de production multisectoriel. La thèse se compose de trois chapitres : le premier étudie les propriétés de stabilité du modèle de Keen lorsque la technologie de production présumée change. Le deuxième chapitre se concentre sur les Contributions Nationales Déterminées (CND) du Brésil et explore la faisabilité de concilier à la fois la croissance économique et la réduction des émissions industrielles. L'analyse de scénarios de politiques alternatives fournit des informations précieuses aux décideurs cherchant à concilier développement économique et objectifs climatiques. Le troisième chapitre présente un modèle macroéconomique de preuve de concept, cohérent avec les stocks et les flux, pour le Brésil, combinant un cadre d'entrées-sorties multisectoriel avec une couche macroéconomique. Le modèle permet de simuler les effets de différentes options politiques sur les émissions, les résultats au niveau des industries et les agrégats macro-financiers.

Mots-clés : Transition énergétique, Dynamique macroéconomique, Économie monétaire, Politiques d'atténuation du changement climatique, Modèle stock-flux cohérent, Cadre entrées-sorties multi-sectoriel, Dette gouvernementale.

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This thesis is the result of a long journey, and certainly a challenging one at multiple levels. While a personal quest, it has benefited from the direct and indirect help of multiple people from different paths of life who have shared their time, energy, and ideas with me, making this road even more interesting, valuable and enjoyable. Moreover, I would like to acknowledge the contribution of many people outside of academia who, through their friendship, have helped make the entire experience even better for my family as well.

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Florent is the co-author in both papers replicated here under the first two chapters. I greatly appreciated our long discussions and the learning process it represented for me to collaborate on these topics with him. Needless to say, I am particularly grateful for the role he has played in this process both as a friend and a colleague.

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Introduction (in English)

Context

The Paris Agreement created a landmark signaling both the awareness of the international community about the risks of climate change, and the existence of political will to limit global warming below +2 degrees Celsius compared to pre-industrial levels. Adopted by 195 countries on December 2015, the agreement requires each country to submit –and update every five years– their national climate action plans, indicating the nationally determined contributions (NDCs) that outline their emissions reduction targets. NDC are a public commitment towards the international community, and the reference against which progress can be evaluated.

Despite being a strong step forward on the fight against Climate change, the Paris Agreement needs to be supported by ambition actions from countries regarding the implementation of mitigation policies, as well as the investment in the development of new, cleaner technologies. Ideally, under strong coordination and collaboration between countries.

The challenges that countries face to bring these ambitions to reality are not minor. Energy is at the core of every country’s economic activity, with fossil fuels being key energy products. A transition away from fossil-fuels could lead to job losses and stranded assets. The effects are amplified for countries whose economies have been heavily dependent on the extraction, processing and exporting of such energy products. Moreover, the required technologies need to be developed and deployed, with large amounts of investment required for such purposes.

Nevertheless, aside from the environmental and health benefits, and other co-benefits that would result from a transition towards green sources of energy, there can also be economic opportunities for countries to consider and embrace. The creation of new jobs in the renewable energy sector, the increased demand for exports of key products required for the new technologies to work, and even the flourishing of additional fields of activity (e.g. eco-tourism) could be considered.

While the need for action cannot be limited to a cost-benefit analysis, it may risk to be reduced to that extent at the policy-making sphere. Governments, in particular those of developing countries, may observe other social problems as more locally-pressing and urgent to attend. The large amounts of financing required for adaptation policies and investment in new technologies, often imported when considering the case of developing countries, can also be a barrier. Additionally, the economic relevance of the fossil-fuel related sectors within an economy can undermine the political will to switch away from such energy sources.

From the international cooperation and financing point of view, international institutions and national development banks with capacity to provide funding to foreign economies, can play a key role on the transition. Through technical cooperation, policy coordination and financing, these institutions can help reduce the number of barriers mentioned before.

Whether from the perspective of the local policy-maker or from a funding institution, addressing these challenges requires clarity on the type of intervention required, its magnitude, its effects, and the potential creation of winners and losers that it may have. Increased awareness of the benefits, and on the expected magnitude and distribution of losses can also help improve the policy design and accompany it with measures aimed at reducing the risks associated to the transition, and to avoid heavily concentrated losses. Moreover, having access to tools where different "what if" scenarios could be simulated may be of particular help given most policies required would be unprecedented, and historical analysis may be limited in that regard.

Having had the chance to be working as a consultant for the French Development Agency (AFD) at the time I started my research, it was soon made clear to me that the institution was concerned about these questions. Willing to provide funding for developing economies on their needs to finance the energy transition, they were aware of the potential impacts that this could have on a country. Aside from those already mentioned, large flows of financing for investment purposes could also lead to foreign imbalances affecting the evolution and stability of the local economy; create radical foreign exchange fluctuations; and increase dependencies on imported technologies, among others.

Brazil as a case study

Although my research was not being developed under the institutional frame of the AFD, I shared their interest and curiosity on these questions and engaged in a collaborative process to develop tools that could provide both quick assessments, and more detailed perspectives about these types of questions. The country of choice as a case study was Brazil.

Holder of the main part of the Amazon, Brazil is the fifth largest economy in surface and population. The country holds numerous natural assets such as agricultural lands; sources of fresh water; as well as mineral, metal, and fossil fuels resources. The economy is relatively closed, supported by a historical focus on the domestic market, with the dynamics of the economic development being mainly driven by the agriculture and mining sectors.

Brazil was an early mover in developing a National Climate Change Plan in December 2008, enacted into law in December 2009, and has shown periods of significant progress in lowering its CO₂ emissions on a voluntary basis, making it a leading player in the fight against climate change. By 2004, Brazil reached historical-high levels of CO₂ emissions, with the land use, land-use change and forestry sector (hereafter LULUCF) being the main source of them. Since 2005, the Brazilian government embarked the country on a large-scale campaign to slow deforestation. As a result, greenhouse-gas (hereafter GHG) emissions from LULUCF declined until 2010, after which a new surge was observed.

All in all, its position as an emerging economy with a leading role in the South American sub-continent; the privileged access to oil, mineral and fresh water resources; the need to

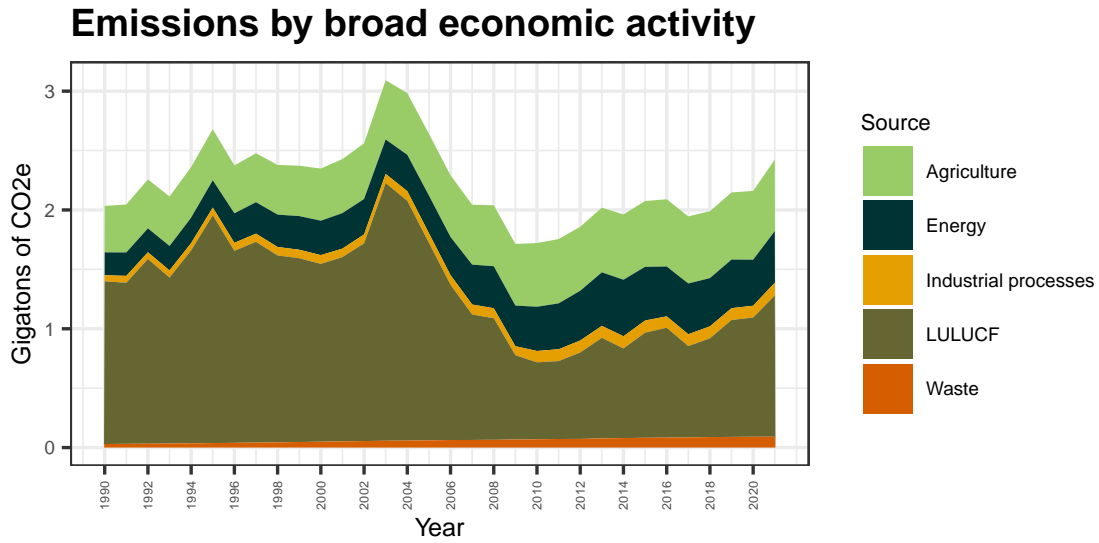


Figure 1: Total CO_2 equivalent emissions by broad economic activity.
Source: Author's calculations based on SEEG data

preserve the large surface of the Amazon region within its borders; as well as the voluntary actions taken in the past to reduce its negative contribution to climate change, make of Brazil a highly relevant and interesting country to focus as the case of study.

Motivation

This thesis was motivated by the study of two broad sets of questions:

1. How can mitigation policies affect the domestic activity of an open economy?
2. How can traditional economic policies affect industrial CO_2 emissions?

Under these broad lines, multiple questions emerged: (i) how can the timing of these policy affect the results, (ii) how can the shocks propagate within the economy, and among others, (iii) how could they affect the evolution of key macro-economic aggregates. Implicitly, these questions were driving me to engage into thought experiments where the concepts of hysteresis and path dependency would always have to be considered.

In brief, I was looking to engage into analyses that would require the use of dynamical tools, while considering the industrial inter-dependencies of an economy. Moreover, I was looking to do it without having to impose the existence of either rational agents with rational expectations, equilibria, nor market-clearing prices. Neither I wanted to model a monetary economy where the credit creation would be governed by the principle of loanable funds.

I understood the propagation could take place through multiple channels, but that the production and financial networks were key to study. I decided to focus on the former. It fits naturally into a macro analysis as the national accounts include the supply and use

tables that capture the inter-dependence at the industry levels. Exercises focusing on the latter, like [Battiston et al. \(2017\)](#) and [Battiston et al. \(2021\)](#), would often require much more granular data, and would have a different unit of analysis than the intended here.

The stock-flow-consistent framework (SFC), as suggested by my supervisor, appeared as the ideal work-horse to tackle the questions at hand. Developed over the foundations of the work of Wynne Godley and James Tobin, and further popularized by the timely publication of the work that Godley and Lavoie (see [Godley and Lavoie \(2006\)](#)), the SFC framework was already attracting the attention of a growing number of researchers trying to model more realistic monetary systems than those existing in the prevailing literature. The existence of empirical applications and whole-country SFC models (see [Cripps and Godley \(1976\)](#); [Godley and Zezza \(1992\)](#); [Zezza \(2008\)](#); [Godley \(2012\)](#); [Papadimitriou et al. \(2013\)](#), among others) have also helped increase the tool's visibility in the policy sphere.

While stock-flow consistency is not a feature unique to these types of models, the attention to tight accounting is certainly an element that defines them. In that regard, four principles govern the accounting of these models, [Nikiforos and Zezza \(2018\)](#):

- Flow consistency: all flows have a source and a destination. The income of an agent is the expense of another.
- Stock consistency: the financial assets held by an agent is the liability of another.
- Stock-flow consistency: all flows contribute to the dynamics of at least one stock. While the inverse is not necessarily required, existing stocks normally affect upcoming flows.
- Quadruple accounting: every transaction creates four accounting entries. Consider a product sale or the payment of salaries. The motive generates the first pair of transactions. The purchase of a good or the payment of salaries will be registered as outflow of funds for one agent and an income for the other. The way in which the funds are transferred leads to a second pair of entries. The paying sector will record a decrease in its assets or an increase in its liabilities, while the receiving sector will register the corresponding increase in its assets or decrease in its liabilities. This is, overall, a resulting condition from the previous three principles.

Conveniently, these correspond as well to the main accounting principles used in the System of National Accounts (SNA), the foundations of which were initially laid down by Morris Copeland [Copeland \(1949\)](#), which helped naturally integrate it as a source of information.

All in all, the work was organized around the construction of a stock-flow-consistent model for Brazil, endowed with an input-output matrix that would also enable the study of policy effects at the industry level. Several modeling choices required additional understanding of the potential consequences of one option over another. This led to multiple standalone analyses, as well as initial oversimplified versions of the model.

In that regard, most exercises applied to Brazil were built over the computable general equilibrium framework (CGE). Among other differences, most cases relied on a CES production function that allowed for capital and labor substitution in the short run. If considering instead that the substitution between capital and labor is not a result of relative factor prices but instead of technical progress, the choice of a technology closer in behavior to a Leontief production function seemed more appropriate. To better understand the potential effects of choosing one over the other, a simpler model had to be tested under both cases. Using [Grasselli and Costa Lima \(2012a\)](#) –where they study the stability properties of the [Keen \(1995\)](#) model– as reference, we endowed the same model with a CES production function and studied how these properties changed. Despite its limitations and the fact that, for instance, the Keen model partially verifies the third principle of SFC,¹ the model provided a simplified working framework allowing for analytical studies of its equilibrium properties. The results obtained in this joint work with Florent Mc Isaac and Adrien Fabre (see [Bastidas et al. \(2019\)](#)) are reported in Chapter 1.

Similarly, with Florent Mc Isaac, we worked on simplified versions of the supply and use tables to better understand the interconnections between industries within the Brazilian economy. Multiple analyses were performed on the national accounting data for that purpose (see [Mc Isaac and Bastidas \(2017\)](#)). Among those, two methods were the object of additional focus at a later stage (see [Bastidas and Mc Isaac \(2019\)](#)). The results from the latter exercise are reported in Chapter 2.

Based on these exercises, and having tested a few exploratory versions of an SFC model with Florent during an early stage of the process in which we were not yet using the financial accounts information from the national accounting system, I engaged on the building of the SFC-IO model for Brazil. The model was built from scratch, and rebuilt multiple times to ensure it was being consistent with newer releases and updates of National accounting data. Indeed, being heavily data-demanding, and trying to ensure consistency between the treatment of the historical data and the simplifying assumptions used, the model was iteratively built and re built as the historical dataset was extended or modified. Its current form corresponds to the first complete version of the running model, but it still features over-simplifying assumptions and methods in several sections. Chapter 3 presents the model, and the appendices discuss the challenges encountered and the solutions used when building it, on the expectation that these ideas could help similar exercises in growing awareness of the elements that would need to be considered and the potential limitations to encounter. Unlike the work reported in Chapters 1 and 2, the current version of the model has not been exposed to peer-review but has benefited from the feedback received in past iterations. All existing errors are, of course, my own.

¹The accumulation of debt from the producing sector has to be matched by an accumulation of assets held by the financial sector. However, the growing stock of assets is assumed not to have any feedback in the economy.

Chapter 1: Minskyan Classical Growth Cycles: Stability Analysis of a Stock-Flow Consistent Macrodynamic Model

Co-written with Florent Mc Isaac and Adrien Fabre,² Chapter 1 presents an exercise that follows [van der Ploeg \(1987\)](#)'s research program in testing both its extension of [Goodwin \(1967\)](#)'s predator-prey model and the Minsky Financial Instability Hypothesis (FIH) proposed by [Keen \(1995\)](#). While Goodwin's model features a Leontief production technology, van der Ploeg studied how the stability of the model changed if substitution between factors of production was allowed. By doing so, he showed that the possibility of substitution between capital and labor transformed the close orbit previously obtained by Goodwin, into a stable focus.

Again based on Goodwin's work, [Keen \(1995\)](#)'s model relaxed the assumption that profit is equal to investment by introducing a nonlinear investment function. His aim was to incorporate Minsky's insights concerning the role of debt finance. Inspired by van der Ploeg, we study the stability properties of the Keen's model, when endowed with a production technology allowing for capital-labor substitution. Using numerical techniques, we show that our new model keeps the desirable properties of Keen's model. However, we also demonstrate that when the economy is endowed with a class of CES production function –that includes the Cobb-Douglas and the linear technology as limit cases–, the unique stable equilibrium is an economically desirable one, and we explore the basin of attraction of such equilibrium under both the CES and the Leontief technologies.

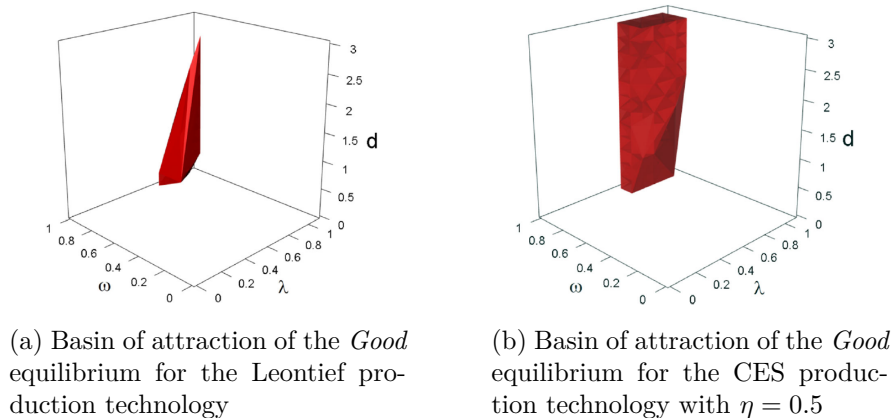


Figure 2: Basins of attraction of the good equilibrium under the Leontief and the CES ($\eta = 0.5$) cases.

Finally, we propose a modified extension that includes speculative component in the economy as in [Grasselli and Costa Lima \(2012b\)](#) and investigate its effect on the dynamics. We conclude that CES production function is a more suitable assumption for empirical purposes than the Leontief counterpart. Using numerical simulations, we show that under plausible calibration the model endowed with CES production function eventually loses the cyclical property of Goodwin's model with and without the speculative component.

²See [Bastidas et al. \(2019\)](#)

We conclude that the class of CES production functions may significantly alter the dynamical landscape of the Keen’s model. Substitution allows to mitigate the influence of wages on employment rate (and vice versa), so that both adjust more rapidly towards their equilibrium values. This result holds for speculative and nonspeculative economies.

Interestingly, numerical simulations showed that van der Ploeg’s extension plugged into Keen’s model increases the set of points that converges towards the *Good* equilibrium. Therefore, an intermediate conclusion would be that the assumed substitution between factors of production leads to the convergence of any reasonable initial condition towards the *Good* equilibrium. Provided that substitution is strong enough (*i.e.* $\eta < 0$), it even ensures that the *Bad* equilibrium is no longer stable. This conclusion is reinforced on the presence of speculative schemes. Furthermore, using numerical techniques, we showed that when substitution is allowed, the change in the volume of the basin of attraction is globally less sensitive to a change of parameters.

This paper contributed to a growing body of work that has developed theoretical models based on the Goodwin-Lotka-Volterra model. The methodology used in this paper may be seen as a starting point for further analysis. We have tested the sensibility of the model to key parameters; another extension would be to test the robustness of the results with respect to changes in the behavioral functions. These changes would presumably affect the vector field of the system more substantially.

In sum, we have seen that the basin of attraction changes substantially when substitution is allowed. Here, we simply compared various models according to the volume of the basin of attraction associated with each equilibrium.

Chapter 2: Reaching Brazil’s Nationally Determined Contributions: An Assessment of the Key Transitions in Final Demand and Employment

Co-written with Florent Mc Isaac,³ Chapter 2 results from a selection of two empirical exploratory exercises performed on Brazilian national accounting data. While aimed at informing the ongoing building process for the SFC-IO model, both methodologies constitute standalone exercises that can be used to provide quick assessments on the questions they tackle.

Brazil had achieved significant advances in climate mitigation by reducing its greenhouse gas (GHG) emissions until 2010. On top of this, the country had committed to furthering its actions through the Nationally Determined Contributions (NDC) issued during the Paris Agreement. At the same time, existing projections anticipated a significant increase in GDP in the years after the pledge. Higher economic activity is, however, often linked to larger amounts of industrial emissions.

Brazil’s reduction of overall emissions in previous years had mainly been linked to their performance on cutting land-use-related ones. As pointed by [Chen et al. \(2013\)](#)’s main conclusion, if deforestation were to be greatly reduced, the burden of cutting CO₂

³See [Bastidas and Mc Isaac \(2019\)](#)

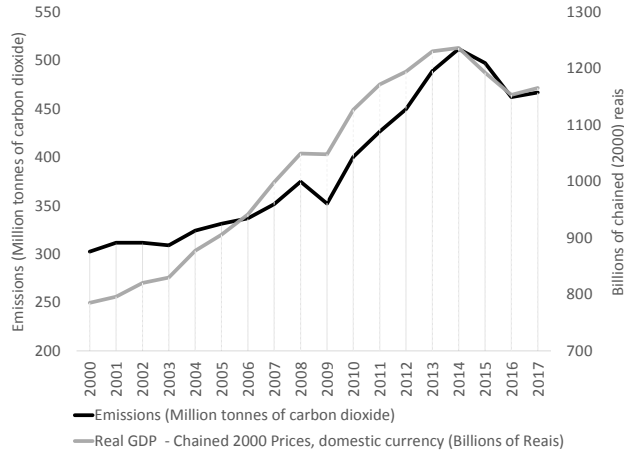


Figure 3: GDP and industrial emissions evolution in Brazil

emissions from energy use and industrial processes would be minimal. However, more recent data on land-use emissions showed that additional efforts might also be required in the energy and industrial sectors. This was, however, in apparent opposition to the expected growth paths.

The first of the exercises studied in Chapter 2 focused on analyzing the feasibility of both goals: sustained economic growth, and reduced industrial emissions. Using Brazil’s industrial structure, we evaluate the minimal changes needed in domestic final demand to meet both the NDC target and the forecasted economic growth, while not impacting overall employment.

Despite our results showing that it may be possible to meet both objectives, the new economic landscape implies a reorganization of activities where production is heavily mobilized to the service sector, to the detriment of industrial activities. The employment reduction is compensated by an increase in primary labor-intensive activities.

This, however, would imply that unless the demand adjusts its behavior to reduce consumption of industrial goods in favor of services, the effects of such a configuration would require larger imports of industrial goods, effectively outsourcing the polluting activities. This should increase the relevance of the fight against deforestation as main means to reducing emissions, while allowing the rest of the sectors to adjust in more realistic ways, while still achieving anticipated GDP growth levels.

We complemented our own analysis with an application of [Perrier and Quirion \(2016\)](#)’s methodology to further explore how to qualitatively explain the change in employment levels following demand changes. We discussed three examples from the orientations given by the NDC and from the view of orienting the economy to services. We show that shifting activities from oil refining to biofuels is likely to be a job-creating change, while going away from the extracting sector to the benefit of the utilities sector may increase unemployment.

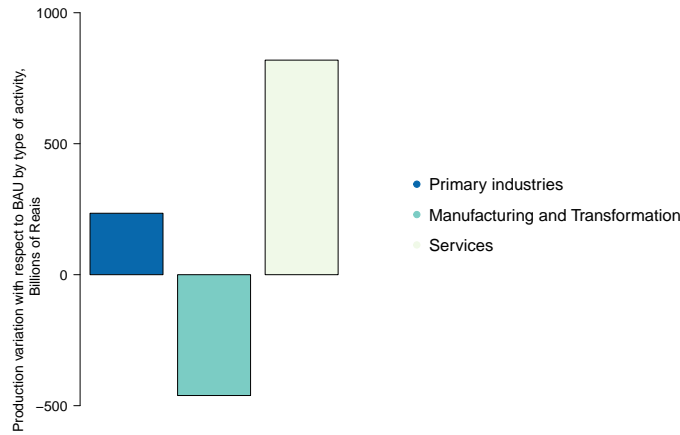


Figure 4: Difference in production, by type of activity with respect to BAU

However, we demonstrate that shifting final demand to a service oriented economy is net job-creating, except for sectors like financial intermediary, real estate, legal, accounting, among others. Moreover, we observed this transition would not be costly from the perspective of public finance as the service sectors pay more taxes than the average of the economy. Furthermore, local services can hardly be substituted by foreign services. A caveat for such a transition would be that average wages may decrease, which could generate social tensions emerging from inadequacies between job's qualification and worker's qualification.

Chapter 3: SFC-IO-BR: A proof-of-concept model for Brazil

While Brazil could achieve their NDC targets solely by fighting deforestation and reducing LULUCF emissions, it may need accompanying efforts from all other sectors. There are several challenges and opportunities to consider in that regard. First, economic dynamics in Brazil have mainly been driven by primary activities, namely mining, and agriculture. Second, more than 80% of Brazilian's electricity already comes from renewable sources (?), but the hydro-power capacity of the country could further improve. Third, the government has seen its position as a net borrower deepened in recent years, which suggests the funding needed for transition efforts may further affect the government's net borrowing position, or that the funding may not come from public accounts in the amounts required, and that additional international financing may be needed.

The exercise carried on in Chapter 3 aims at being a tool able to simulate (i) the effect that mitigation policies can have not only on emissions, but also on the dynamics of key macro-financial aggregates; (ii) the effect that traditional policies can have in the main macroeconomic aggregates, on the industrial production decisions, and as a result on emissions. As such, the model can be classified as a "what-if" exercise. It does not aim at being a forecasting tool, but instead at testing the effect that alternative policies

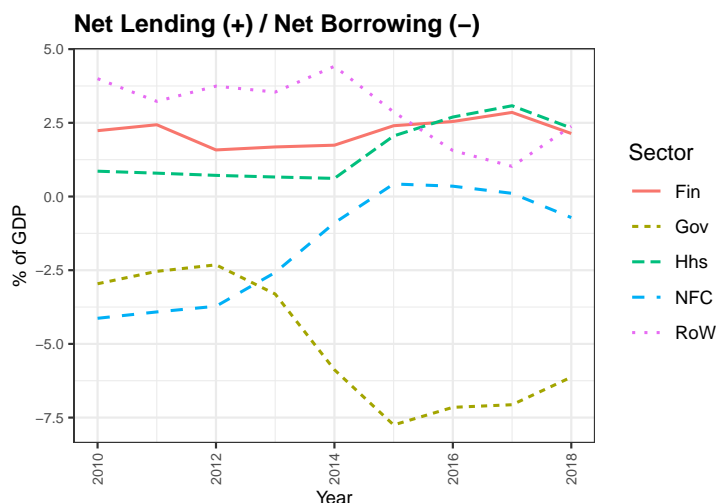


Figure 5: Evolution of the net lending (+) / net borrowing (-) positions by institutional sector

create, with respect to a business-as-usual scenario, and to explore results at the industry level of detail.

Moreover, the model does not use general equilibrium frameworks but instead simulates supply and demand behavior without imposing clearing conditions on non-financial markets, and allowing not only for mismatches between supply and demand, but also for production and investment to be funded on demand –dropping the assumption that only pre-existent savings can be lent.

The pilot is built by having two separate layers interacting with each other, and thus combining the macro-modeling approach of stock-flow consistent models (SFC) (Godley and Lavoie (2006), Nikiforos and Zezza (2018), Zezza and Zezza (2019)), with the multi sectoral perspective given by the input-output framework (Miller and Blair (2009), Mahajan et al. (2018)).

The production module is defined at the industry level of detail, containing twelve sectors of activity: agriculture, mining, mil and gas, oil derivatives, biofuels and alcohol, utilities, machinery and construction, industries (not spec. elsewhere), transportation, services (not spec. elsewhere), financial intermediation, and public administration. It describes the interactions within each of these, and between these and the final demand sectors. In doing so, it keeps track of transactions at different valuation methods (i.e. volumes, basic prices and market prices).

The results from the production module are aggregated and reported to the macroeconomic layer, which is defined at the institutional sector level of detail, featuring households, non-financial corporations, financial corporations, the government, and the rest of the World. Each of these engage with each others in transactions in the market of goods and services. While six types of financial assets are modeled, the current version of the pilot does not include a financial market, nor behavioral rules to clear supply and demand

of financial assets. These are instead allocated with the help of matrix-balancing methods.

Closely matching the structure of the national accounts, the model specifies the consumption and investment behaviors, as well as the key rules driving the dynamics of the system.

Calibrated to match projections released before COVID shocks, the model simulates three different scenarios:

- Business as usual⁴
- Carbon tax: the implementation of a gradual introduction of a carbon tax reaching levels of 70 USD/tCO₂e and 120 USD/tCO₂e by 2030 and 2035, respectively, is simulated.
- Limit government expenditure: In line with recent proposals, a cap on government expenditure is modeled.

Results can then be studied at the macro layer, or the production module layer:

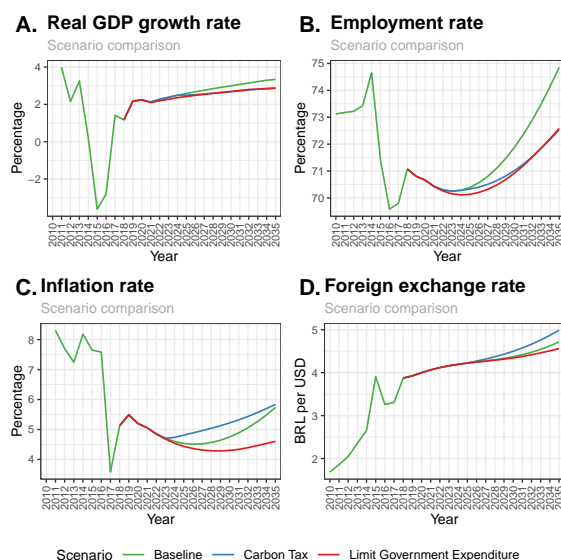


Figure 6: Evolution of the net lending (+) / net borrowing (-) positions by institutional sector

While still a pilot version, the model allows to tackle example questions in the directions desired. Its structure ensures accounting consistency within and between both layers, and allows to simulate industrial effects despite not being able to determine industry-specific balance sheets, as it would be desired in a theoretical model. The level of detail used in the production module allows for results to be tracked by industry, origin (domestic or imported), and in multiple valuation methods (real terms, basic prices and market prices), which enables the modeler to spot sources of chaotic behavior despite the size of the model.

⁴The last available observation for the national accounting data used was that of 2018. Simulations start in 2019, and no shock has been implemented to model the COVID crisis, as this has been left for further research.

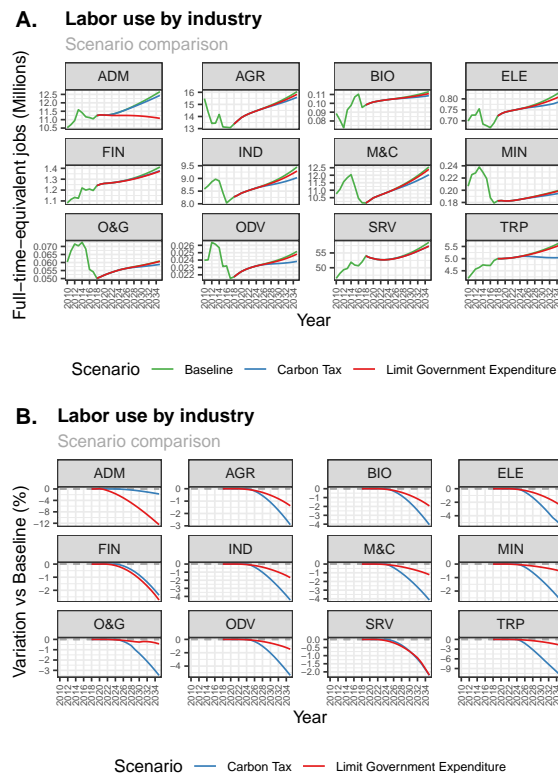


Figure 7: Evolution of the net lending (+) / net borrowing (-) positions by institutional sector

The use of key aggregate accounts for the macro layer ensures consistency with historical data can be verified at every stage of the construction process.

On the other hand, it has to be noted that a model with this level of detail is extremely data-demanding. The creation and maintenance of a dataset consistent with the assumptions used in the running version of the model are by large the most time consuming process. Simpler tools may be enough for quick assessments, but policy makers would certainly benefit from increased information on the effects of implementing one or other policy alternatives.

All in all, despite the ability to produce some results consistent with the targets set during the calibration process, there is still much room for improvement at both the technical and theoretical levels. In terms of priorities, the model should consider more iterations between years.⁵ The introduction of additional iterations will require a re-calibration of parameters. Ideally, behavioral equations could be updated to avoid extreme simplifications. Finally, with the upcoming releases of national accounting data, there will be information to evaluate the magnitude and distribution of the COVID-related shocks, and the first signs of economic reorganization as the recovery process takes place. This can certainly enrich the insights that the model can provide by ensuring the baseline considers

⁵On a future step, this could be further improved by re-calibrating the model using quarterly information if available.

these events.

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Introduction (en Français)

Contexte

L'accord de Paris a constitué un événement marquant, signalant à la fois la prise de conscience de la communauté internationale des risques du changement climatique et l'existence de la volonté politique de limiter le réchauffement climatique mondial à moins de +2 degrés Celsius par rapport aux niveaux préindustriels. Adopté par 195 pays en décembre 2015, l'accord exige que chaque pays soumette – et mette à jour tous les cinq ans – ses plans nationaux d'action climatique, indiquant les contributions déterminées au niveau national (CDN) qui définissent leurs objectifs de réduction des émissions. Les CDN sont un engagement public envers la communauté internationale et servent de référence pour évaluer les progrès réalisés.

Malgré les avancées considérables dans la lutte contre le changement climatique, l'accord de Paris doit être soutenu par des actions ambitieuses des pays en ce qui concerne la mise en œuvre de politiques d'atténuation et les investissements dans le développement de nouvelles technologies plus propres. Idéalement, cela nécessite une coordination et une collaboration solides entre les pays.

Les défis auxquels les pays sont confrontés pour concrétiser ces ambitions ne sont pas négligeables. L'énergie est au cœur de l'activité économique de chaque pays, les combustibles fossiles jouant un rôle clé. Une transition hors des combustibles fossiles pourrait entraîner des pertes d'emplois et des actifs échoués, affectant particulièrement les économies fortement dépendantes de l'extraction, du traitement et de l'exportation de ces produits énergétiques. De plus, les technologies nécessaires doivent être développées et déployées, ce qui demande d'importants investissements.

Néanmoins, en plus des avantages environnementaux et sanitaires, et d'autres avantages connexes résultant d'une transition vers des sources d'énergie vertes, il peut également y avoir des opportunités économiques à considérer et à embrasser pour les pays. Cela inclut la création de nouveaux emplois dans le secteur des énergies renouvelables, une demande accrue pour les exportations de produits essentiels requis pour les nouvelles technologies, voire le déploiement de domaines d'activité supplémentaires (par exemple, l'écotourisme).

Bien que la nécessité d'agir ne puisse être limitée à une analyse coûts-avantages, elle risque d'être réduite à ce niveau dans le domaine de l'élaboration des politiques. Les gouvernements, notamment ceux des pays en développement, peuvent considérer d'autres problèmes sociaux comme plus pressants et urgents à résoudre au niveau local. Les montants importants de financement nécessaires pour les politiques d'adaptation et les investissements dans de nouvelles technologies, souvent importées dans le cas des pays en développement, peuvent également constituer un obstacle. De plus, le poids économique des secteurs liés aux combustibles fossiles peut compromettre la volonté politique de se détourner de ces sources d'énergie.

Du point de vue de la coopération internationale et du financement, les institutions internationales et les banques nationales de développement ayant la capacité de fournir des

financements aux économies étrangères peuvent jouer un rôle clé dans la transition. Par la coopération technique, la coordination des politiques et le financement, ces institutions peuvent contribuer à réduire les obstacles mentionnés précédemment.

Que ce soit du point de vue du décideur politique local ou de l'institution de financement, relever ces défis nécessite une clarté sur le type d'intervention nécessaire, son ampleur, ses effets et l'apparition potentielle de gagnants et de perdants qu'elle pourrait engendrer. Une prise de conscience accrue des avantages, ainsi que de l'ampleur et de la répartition attendues des pertes, peut également contribuer à améliorer la conception des politiques et à les accompagner de mesures visant à réduire les risques associés à la transition, et à éviter des pertes fortement concentrées. De plus, l'accès à des outils permettant de simuler différents scénarios permettant de répondre à la question : "et que se passerait-il si?" peut être particulièrement utile étant donné que la plupart des politiques nécessaires seraient sans précédent, et que l'analyse historique est limitée à cet égard.

Ayant eu l'occasion de travailler en tant que consultant pour l'Agence Française de Développement (AFD) lorsque j'ai commencé ma recherche, il m'a été rapidement clair que l'institution se souciait de ces questions. Souhaitant fournir un financement pour les besoins des économies en développement en matière de transition énergétique, ils étaient conscients des impacts potentiels que cela pourrait avoir sur un pays. Outre ceux déjà mentionnés, d'importants flux de financement à des fins d'investissement pourraient également entraîner des déséquilibres étrangers affectant l'évolution et la stabilité de l'économie locale; créer des fluctuations radicales des taux de change; et accroître les dépendances vis-à-vis des technologies importées, entre autres.

Le choix du Brésil comme étude de cas

Bien que ma recherche n'ait pas été développée dans le cadre institutionnel de l'AFD, j'ai partagé leur intérêt et leur curiosité pour ces questions et me suis engagé dans un processus de collaboration pour développer des outils pouvant fournir à la fois des évaluations rapides et des perspectives plus détaillées sur ces types de questions. Le Brésil a été choisi comme étude de cas.

En tant que son territoire couvre la plus grande partie de l'Amazonie, le Brésil est la cinquième économie mondiale en termes de superficie et de population. Le pays détient de nombreux actifs naturels tels que des terres agricoles, des sources d'eau douce, ainsi que des ressources minérales, métalliques, et énergétiques. L'économie est relativement fermée, soutenue par une focalisation historique sur le marché intérieur, avec des dynamiques de développement économique principalement tirées par les secteurs de l'agriculture et de l'exploitation minière.

Le Brésil a été l'un des précurseurs dans l'élaboration d'un Plan National sur le Changement Climatique en décembre 2008, adopté en loi en décembre 2009, et a réalisé des périodes de progrès significatifs en réduisant volontairement ses émissions de CO₂, et devenant ainsi un acteur majeur dans la lutte contre le changement climatique. Dès 2004, le Brésil

avait en effet atteint des niveaux historiquement élevés d'émissions de CO₂, le secteur de l'utilisation des terres, du changement d'affectation des terres et de la foresterie (désormais LULUCF) étant la principale source de ces émissions. Depuis 2005, le gouvernement brésilien s'est engagé dans une vaste campagne pour ralentir la déforestation. En conséquence, les émissions de gaz à effet de serre (désormais GHG) provenant du LULUCF ont diminué jusqu'en 2010, après quoi une nouvelle augmentation a été observée.

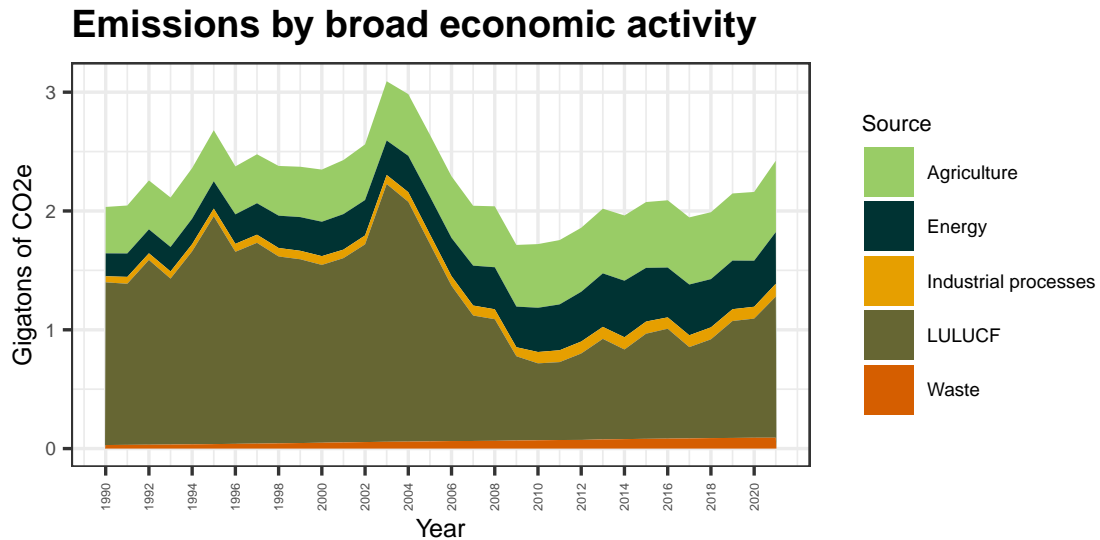


Figure 8: Émissions totales équivalentes de CO₂ par secteur économique.
Source : Calculs de l'auteur basés sur les données SEEG.

Dans l'ensemble, sa position en tant qu'économie émergente jouant un rôle de premier plan dans le sous-continent sud-américain, l'accès privilégié aux ressources pétrolières, minières et en eau douce, la nécessité de préserver la vaste superficie de la région amazonienne sur son territoire, ainsi que les actions volontaires entreprises par le passé pour réduire sa contribution négative au changement climatique, font du Brésil un excellent candidat pour tester un modèle macroéconomique quantitatif combinant une approche input-output et la simulation des émissions de gaz à effet de serre.

Motivation

Cette thèse était motivée par l'étude de deux ensembles de questions générales :

1. Comment les politiques d'atténuation peuvent-elles affecter l'activité nationale d'une économie ouverte ?
2. Comment les politiques économiques traditionnelles peuvent-elles affecter les émissions industrielles de CO₂ ?

Sous ces grandes lignes, de multiples questions ont émergé : (i) comment le calendrier de ces politiques peut-il influencer les résultats, (ii) comment les chocs peuvent-ils

se propager au sein de l'économie et entre autres, (iii) comment pourraient-ils affecter l'évolution des agrégats macro-économiques clés. Implicitement, ces questions me poussaient à m'engager dans des expériences de pensée où les concepts d'hystérésis et de dépendance au chemin devraient toujours être pris en compte.

En bref, je cherchais à m'engager dans des analyses qui nécessiteraient l'utilisation d'outils dynamiques, tout en tenant compte des interdépendances industrielles d'une économie. De plus, je souhaitais le faire sans avoir à imposer l'existence d'agents rationnels avec des attentes rationnelles, d'équilibres, ni de prix de marché équilibrés. Je ne voulais pas non plus modéliser une économie monétaire où la création de crédit serait régie par le principe des fonds prêtables.

J'ai compris que la propagation pourrait se produire par le biais de canaux multiples, mais que les réseaux de production et financiers étaient essentiels à étudier. J'ai décidé de me concentrer sur le premier. Il s'intègre naturellement dans une analyse macro-économique car les comptes nationaux incluent les tableaux des ressources et des emplois qui capturent l'interdépendance au niveau des industries. Les exercices axés sur le second, comme [Battiston et al. \(2017\)](#) et [Battiston et al. \(2021\)](#), nécessiteraient souvent des données beaucoup plus détaillées et auraient une unité d'analyse différente de celle envisagée ici.

Le cadre des modèles stock-flux cohérents (SFC), comme suggéré par mon superviseur, s'est révélé être l'outil idéal pour aborder les questions en cours. Développé sur les fondements du travail de Wynne Godley et James Tobin, et popularisé ultérieurement par la publication opportune du travail de Godley et Lavoie (voir [Godley and Lavoie \(2006\)](#)), le cadre SFC attirait déjà l'attention d'un nombre croissant de chercheurs cherchant à modéliser des systèmes monétaires plus réalistes que ceux existant dans la littérature prédominante. L'existence d'applications empiriques et de modèles SFC pour des pays entiers (voir [Cripps and Godley \(1976\)](#); [Godley and Zezza \(1992\)](#); [Zezza \(2008\)](#); [Godley \(2012\)](#); [Papadimitriou et al. \(2013\)](#), entre autres) a également contribué à accroître la visibilité de l'outil dans le domaine des politiques.

Bien que la cohérence des stocks et des flux ne soit pas une caractéristique unique de ces types de modèles, l'attention portée à la comptabilité rigoureuse est certainement un élément qui les définit. À cet égard, quatre principes régissent la comptabilité de ces modèles, [Nikiforos and Zezza \(2018\)](#):

- Cohérence des flux : tous les flux ont une source et une destination. Le revenu d'un agent est la dépense d'un autre.
- Cohérence des stocks : les actifs financiers détenus par un agent sont la responsabilité d'un autre.
- Cohérence des stocks et des flux : tous les flux contribuent à la dynamique d'au moins un stock. Alors que l'inverse n'est pas nécessairement requis, les stocks existants affectent généralement les flux à venir.

- Quadruple comptabilité : chaque transaction crée quatre entrées comptables. Prenons l'exemple d'une vente de produit ou du paiement de salaires. Cela génère la première paire de transactions. L'achat d'un bien ou le paiement de salaires sera enregistré comme une sortie de fonds pour un agent et un revenu pour l'autre. La manière dont les fonds sont transférés conduit à une deuxième paire d'entrées. Le secteur payant enregistrera une diminution de ses actifs ou une augmentation de ses passifs, tandis que le secteur recevant enregistrera la correspondante augmentation de ses actifs ou diminution de ses passifs. Cela résulte globalement des trois principes précédents.

Ceux-ci correspondent également aux principes comptables utilisés dans le Système de Comptabilité Nationale (SCN), dont les fondements ont été posés initialement par Morris Copeland [Copeland \(1949\)](#), ce qui m'a permis de l'intégrer naturellement comme source d'information.

Dans l'ensemble, le travail a été organisé autour de la construction d'un modèle stock-flux cohérent pour le Brésil, doté d'une matrice entrées-sorties permettant également l'étude des effets des politiques au niveau de l'industrie. Plusieurs choix de modélisation ont nécessité une meilleure compréhension des conséquences potentielles d'une option par rapport à une autre. Cela a conduit à de multiples analyses indépendantes, ainsi qu'à des versions initiales simplifiées du modèle.

À cet égard, la plupart des exercices appliqués au Brésil ont été construits dans le cadre de l'équilibre général calculable (CGE). Au contraire de ce qui a été fait ici, la plupart des cas reposaient sur une fonction de production CES qui permettait la substitution du capital et du travail à court terme. Si l'on considère plutôt que la substitution entre capital et travail n'est pas le résultat des prix relatifs des facteurs mais plutôt du progrès technique, le choix d'une technologie plus proche du comportement d'une fonction de production de Leontief semblait plus approprié. En outre, dans une approche *input-output*, les consommations intermédiaires sont toujours implicitement considérées comme complémentaires. Pour mieux comprendre les effets potentiels du choix de l'une ou l'autre option, un modèle plus simple devait être testé dans les deux cas. En utilisant [Grasselli and Costa Lima \(2012a\)](#) – où sont étudiées les propriétés de stabilité du modèle de [Keen \(1995\)](#) – comme référence, nous avons doté le même modèle d'une fonction de production CES et étudié comment ces propriétés ont changé. Malgré ses limitations et le fait que, par exemple, le modèle de Keen vérifie partiellement le troisième principe du SFC,⁶ le modèle a fourni un cadre de travail simplifié permettant des études analytiques de ses propriétés d'équilibre. Les résultats obtenus dans ce travail conjoint avec Florent Mc Isaac et Adrien Fabre (voir [Bastidas et al. \(2019\)](#)) sont rapportés dans le chapitre 1.

De même, avec Florent Mc Isaac, nous avons travaillé sur des versions simplifiées des tableaux des ressources et des emplois pour mieux comprendre les interconnexions entre les industries au sein de l'économie brésilienne. De multiples analyses ont été réalisées

⁶L'accumulation de la dette du secteur productif doit être compensée par une accumulation d'actifs détenus par le secteur financier. Cependant, on suppose que la croissance du stock d'actifs n'a aucun effet sur l'économie.

sur les données de comptabilité nationale à cette fin (voir [Mc Isaac and Bastidas \(2017\)](#)). Parmi celles-ci, deux méthodes ont fait l'objet d'un examen plus approfondi à un stade ultérieur (voir [Bastidas and Mc Isaac \(2019\)](#)). Les résultats de ce dernier exercice sont rapportés dans le chapitre 2.

Sur la base de ces exercices et après avoir testé quelques versions exploratoires d'un modèle SFC avec Florent au début du processus, où nous n'utilisons pas encore les informations des comptes financiers du système de comptabilité nationale, j'ai entrepris la construction du modèle SFC-IO pour le Brésil. Le modèle a été construit à partir de zéro et reconstruit à plusieurs reprises pour s'assurer qu'il était cohérent avec les nouvelles versions et mises à jour des données de la comptabilité nationale. En effet, étant très exigeant en termes de données et cherchant à assurer une cohérence entre le traitement des données historiques et les hypothèses simplificatrices utilisées, le modèle a été construit et reconstruit de manière itérative au fur et à mesure que le jeu de données historiques était étendu ou modifié. Sa forme actuelle correspond à la première version complète du modèle en cours d'exécution, mais il présente encore des hypothèses et des méthodes de simplification dans plusieurs sections. Le chapitre 3 présente le modèle, et les annexes discutent des défis rencontrés et des solutions utilisées lors de sa construction, dans l'espoir que ces idées pourraient aider des exercices similaires à prendre conscience des éléments à prendre en compte et des limitations potentielles à rencontrer. Contrairement aux travaux rapportés dans les chapitres 1 et 2, la version actuelle du modèle n'a pas été soumise à un examen par des pairs, mais elle a bénéficié des commentaires reçus lors des itérations précédentes. Toutes les erreurs existantes sont, bien sûr, de ma responsabilité.

Chapitre 1: Cycles de croissance classiques de Minsky : Analyse de stabilité d'un modèle macrodynamique stock-flux cohérent

Co-écrit avec Florent Mc Isaac et Adrien Fabre,⁷ le chapitre 1 présente un exercice qui suit le programme de recherche de [van der Ploeg \(1987\)](#) en testant à la fois son extension du modèle proie-prédateur de [Goodwin \(1967\)](#) et l'Hypothèse d'Instabilité Financière (HIF) de Minsky proposée par [Keen \(1995\)](#). Alors que le modèle de Goodwin présente une technologie de production de type Leontief, van der Ploeg a étudié comment la stabilité du modèle changeait si la substitution entre les facteurs de production était autorisée. Ce faisant, il a montré que la possibilité de substitution entre le capital et le travail transformait l'orbite étroite précédemment obtenue par Goodwin en un foyer stable.

En nous basant à nouveau sur le travail de Goodwin, le modèle de [Keen \(1995\)](#) a assoupli l'hypothèse selon laquelle le profit est égal à l'investissement en introduisant une fonction d'investissement non linéaire. Son objectif était d'incorporer les idées de Minsky concernant le rôle de la finance de la dette. Inspirés par van der Ploeg, nous étudions les propriétés de stabilité du modèle de Keen, lorsqu'il est doté d'une technologie de production permettant la substitution du capital et du travail. En utilisant des techniques

⁷Voir [Bastidas et al. \(2019\)](#)

numériques, nous montrons que notre nouveau modèle conserve les propriétés souhaitables du modèle de Keen. Cependant, nous démontrons également que lorsque l'économie est dotée d'une classe de fonctions de production CES – qui inclut les technologies Cobb-Douglas et linéaires comme cas limites –, l'équilibre stable unique est économiquement souhaitable, et nous explorons le bassin d'attraction d'un tel équilibre sous les technologies CES et Leontief.

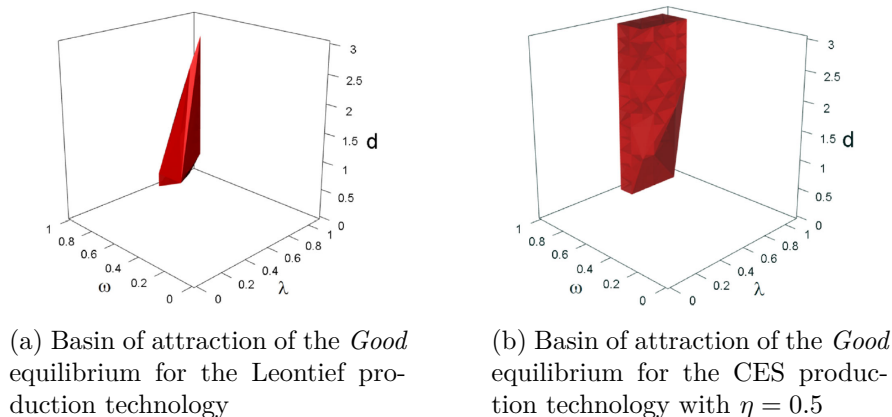


Figure 9: Basins of attraction of the good equilibrium under the Leontief and the CES ($\eta = 0.5$) cases.

Enfin, nous proposons une extension modifiée qui inclut un composant spéculatif dans l'économie, comme dans [Grasselli and Costa Lima \(2012b\)](#), et nous étudions son effet sur la dynamique. Nous concluons que la fonction de production CES est une hypothèse plus appropriée à des fins empiriques que son équivalent de Leontief. En utilisant des simulations numériques, nous montrons que, sous une calibration plausible, le modèle doté de la fonction de production CES perd finalement la propriété cyclique du modèle de Goodwin, avec ou sans le composant spéculatif.

Nous en concluons que la classe de fonctions de production CES peut modifier significativement le paysage dynamique du modèle de Keen. La substitution permet d'atténuer l'influence des salaires sur le taux d'emploi (et vice versa), de sorte que les deux s'ajustent plus rapidement vers leurs valeurs d'équilibre. Ce résultat est valable pour les économies spéculatives et non spéculatives.

Fait intéressant, les simulations numériques ont montré que l'extension de van der Ploeg intégrée au modèle de Keen élargit l'ensemble de points qui converge vers l'équilibre *Bon*. Par conséquent, une conclusion intermédiaire serait que la substitution supposée entre les facteurs de production conduit à la convergence de toute condition initiale raisonnable vers l'équilibre *Bon*. Pourvu que la substitution soit suffisamment forte (*c'est-à-dire* $\eta < 0$), cela garantit même que l'équilibre *Mauvais* n'est plus stable. Cette conclusion est renforcée en présence de mécanismes spéculatifs. De plus, en utilisant des techniques numériques, nous avons montré que lorsque la substitution est autorisée, le changement du volume du bassin d'attraction est globalement moins sensible à un changement de paramètres.

Cet article contribue à un ensemble croissant de travaux qui ont développé des mod-

èles théoriques basés sur le modèle de Goodwin-Lotka-Volterra. La méthodologie utilisée dans cet article peut être considérée comme un point de départ pour des analyses plus approfondies. Nous avons testé la sensibilité du modèle à des paramètres clés; une autre extension consisterait à tester la robustesse des résultats par rapport aux changements dans les fonctions comportementales. Ces changements affecteraient probablement le champ de vecteurs du système de manière plus substantielle.

En résumé, nous avons constaté que le bassin d’attraction change considérablement lorsque la substitution est autorisée. Ici, nous avons simplement comparé différents modèles en fonction du volume du bassin d’attraction associé à chaque équilibre.

Chapitre 2: Atteindre les Contributions Déterminées au Niveau National du Brésil : Une Évaluation des Transitions Clés de la Demande Finale et de l’Emploi

Co-écrit avec Florent Mc Isaac,⁸ le chapitre 2 résulte d’une sélection de deux exercices exploratoires empiriques réalisés à partir des données comptables nationales du Brésil. Bien que destinées à informer le processus de construction en cours du modèle SFC-IO, les deux méthodologies constituent des exercices autonomes pouvant être utilisés pour fournir des évaluations rapides sur les questions qu’elles abordent.

Le Brésil avait réalisé des avancées significatives en matière de réduction des émissions de gaz à effet de serre (GES) jusqu’en 2010. De plus, le pays s’était engagé à renforcer ses actions par le biais des Contributions Déterminées au Niveau National (CDN) émises lors de l’Accord de Paris. En même temps, les projections existantes prévoyaient une augmentation significative du PIB dans les années suivant l’engagement. Cependant, une activité économique accrue est souvent liée à une augmentation des émissions industrielles.

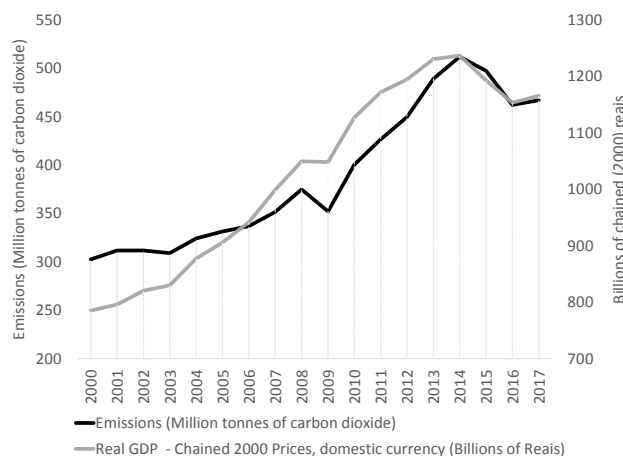


Figure 10: GDP and industrial emissions evolution in Brazil

⁸Voir Bastidas and Mc Isaac (2019)

La première des études réalisées dans le chapitre 2 s’est concentrée sur l’analyse de la faisabilité des deux objectifs : une croissance économique soutenue et une réduction des émissions industrielles. En utilisant la structure industrielle du Brésil, nous évaluons les changements minimaux nécessaires dans la demande finale domestique pour atteindre à la fois l’objectif de la CDN et la croissance économique prévue, sans affecter l’emploi global.

Malgré nos résultats montrant qu’il pourrait être possible de réaliser les deux objectifs, le nouveau paysage économique implique une réorganisation des activités où la production est fortement mobilisée vers le secteur des services, au détriment des activités industrielles. La réduction de l’emploi est compensée par une augmentation des activités primaires intensives en main-d’œuvre.

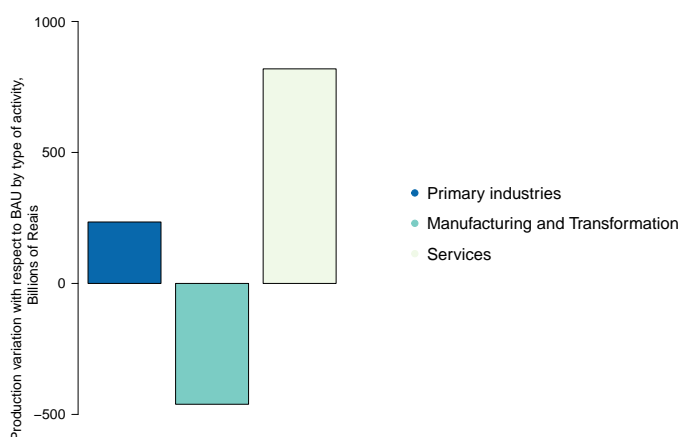


Figure 11: Difference in production, by type of activity with respect to BAU

Cependant, cela impliquerait que, sauf si la demande ajuste son comportement pour réduire la consommation de biens industriels au profit des services, les effets d’une telle configuration nécessiteraient des importations plus importantes de biens industriels, externalisant ainsi effectivement les activités polluantes. Cela devrait accroître l’importance de la lutte contre la déforestation en tant que principal moyen de réduire les émissions, tout en permettant aux autres secteurs de s’ajuster de manière plus réaliste tout en atteignant les niveaux de croissance anticipés du PIB.

Nous avons complété notre propre analyse par une application de la méthodologie de [Perrier and Quirion \(2016\)](#) pour explorer davantage comment expliquer qualitativement le changement des niveaux d’emploi suite aux changements de la demande. Nous avons discuté de trois exemples basés sur les orientations données par la CDN et sur l’orientation de l’économie vers les services. Nous montrons que le passage des activités de la raffinerie de pétrole aux biocarburants est susceptible de créer des emplois, tandis que le passage du secteur extractif au profit du secteur des services publics pourrait augmenter le chômage.

Cependant, nous démontrons que l’orientation de la demande finale vers une économie axée sur les services crée nettement des emplois, sauf pour des secteurs tels que les in-

termédiaires financiers, l'immobilier, le juridique, la comptabilité, entre autres. De plus, nous avons observé que cette transition ne serait pas coûteuse du point de vue des finances publiques, car les secteurs des services paient plus d'impôts que la moyenne de l'économie. De plus, les services locaux peuvent difficilement être remplacés par des services étrangers. Une mise en garde pour une telle transition serait que les salaires moyens pourraient diminuer, ce qui pourrait entraîner des tensions sociales découlant d'inadéquations entre les qualifications des emplois et celles des travailleurs.

Chapitre 3 : SFC-IO-BR : Un modèle de preuve de concept pour le Brésil

Bien que le Brésil puisse atteindre ses objectifs de CDN en luttant uniquement contre la déforestation et en réduisant les émissions de LULUCF, il pourrait avoir besoin d'efforts supplémentaires de tous les autres secteurs. Plusieurs défis et opportunités doivent être pris en compte à cet égard. Premièrement, la dynamique économique au Brésil a principalement été soutenue par des activités primaires, à savoir l'exploitation minière et l'agriculture. Deuxièmement, plus de 80% de l'électricité brésilienne provient déjà de sources renouvelables (?), mais la capacité hydroélectrique du pays pourrait être améliorée. Troisièmement, le gouvernement a vu sa position de débiteur net s'approfondir ces dernières années, ce qui suggère que le financement nécessaire aux efforts de transition pourrait affecter davantage la position de débiteur net du gouvernement, ou que le financement ne viendra pas des comptes publics dans les montants requis, et qu'un financement international supplémentaire pourrait être nécessaire.

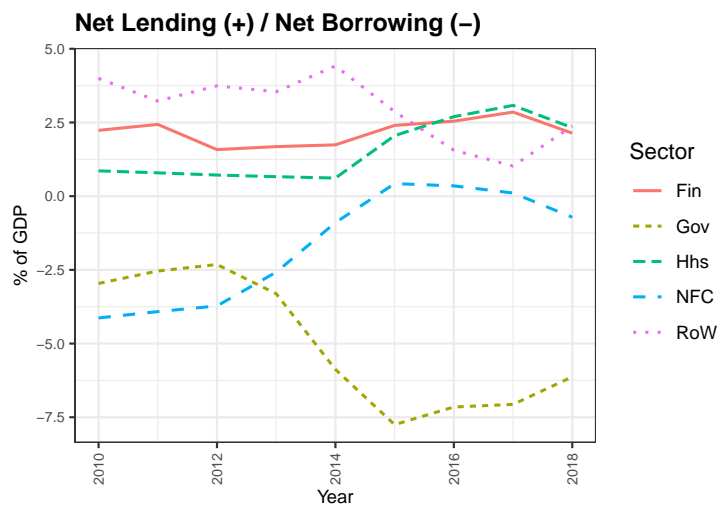


Figure 12: Evolution of the net lending (+) / net borrowing (-) positions by institutional sector

L'exercice réalisé dans le Chapitre 3 vise à être un outil capable de simuler (i) l'effet que les politiques d'atténuation peuvent avoir non seulement sur les émissions, mais aussi sur la dynamique des principales agrégations macro-financières; (ii) l'effet que les politiques traditionnelles peuvent avoir sur les agrégats macroéconomiques principaux, sur

les décisions de production industrielle, et par conséquent sur les émissions. En tant que tel, le modèle peut être classé comme un exercice "et si". Il ne vise pas à être un outil de prévision, mais plutôt à tester l'effet que les politiques alternatives créent par rapport à un scénario de maintien des activités habituelles, et à explorer les résultats au niveau détaillé des secteurs industriels.

De plus, le modèle n'utilise pas de cadres d'équilibre général, mais simule plutôt le comportement de l'offre et de la demande sans imposer de conditions de compensation sur les marchés non financiers, et en permettant non seulement les déséquilibres entre l'offre et la demande, mais aussi le financement de la production et de l'investissement selon la demande - abandonnant ainsi l'hypothèse selon laquelle seules les épargnes préexistantes peuvent être prêtées.

Le modèle pilote est construit en ayant deux couches séparées qui interagissent entre elles, combinant ainsi l'approche macro-modélisation des modèles stock-flux cohérents (SFC) (Godley and Lavoie (2006), Nikiforos and Zezza (2018), Zezza and Zezza (2019)) avec la perspective multi-sectorielle donnée par le cadre d'entrées-sorties (Miller and Blair (2009), Mahajan et al. (2018)).

Le module de production est défini au niveau détaillé de l'industrie, comprenant douze secteurs d'activité : agriculture, extraction minière, pétrole et gaz, dérivés du pétrole, bio-carburants et alcool, services publics, machines et construction, industries (non spécifiées ailleurs), transport, services (non spécifiés ailleurs), intermédiation financière et administration publique. Il décrit les interactions au sein de chacun de ces secteurs et entre eux et les secteurs de demande finale. Ce faisant, il suit les transactions selon différentes méthodes de valorisation (c'est-à-dire les volumes, les prix de base et les prix du marché).

Les résultats du module de production sont agrégés et rapportés à la couche macro-économique, qui est définie au niveau détaillé du secteur institutionnel, comprenant les ménages, les sociétés non financières, les sociétés financières, le gouvernement et le reste du monde. Chacun d'eux s'engage dans des transactions sur le marché des biens et services. Bien que six types d'actifs financiers soient modélisés, la version actuelle du pilote n'inclut pas de marché financier, ni de règles comportementales pour compenser l'offre et la demande d'actifs financiers. Ces actifs sont plutôt alloués à l'aide de méthodes d'équilibrage de matrice.

Étroitement lié à la structure des comptes nationaux, le modèle spécifie les comportements de consommation et d'investissement, ainsi que les règles clés qui régissent la dynamique du système.

Calibré pour correspondre aux projections publiées avant les chocs de la COVID, le modèle simule trois scénarios différents :

- Maintien des activités habituelles⁹

⁹La dernière observation disponible des données comptables nationales utilisées était celle de 2018. Les simulations commencent en 2019 et aucun choc n'a été mis en œuvre pour modéliser la crise de la COVID, car cela est réservé à des recherches ultérieures.

- Taxe carbone : la mise en œuvre progressive d'une taxe carbone atteignant 70 USD/tCO₂e et 120 USD/tCO₂e d'ici 2030 et 2035, respectivement, est simulée.
- Limite des dépenses gouvernementales : Conformément aux propositions récentes, un plafond des dépenses gouvernementales est modélisé.

Les résultats peuvent ensuite être étudiés au niveau macroéconomique ou au niveau du module de production :

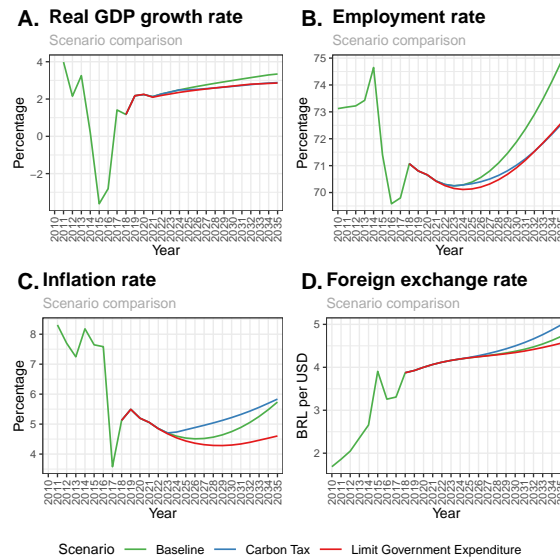


Figure 13: Evolution of the net lending (+) / net borrowing (-) positions by institutional sector

Bien que toujours à l'état de version pilote, le modèle permet d'aborder des questions d'exemple dans les directions souhaitées. Sa structure assure une cohérence comptable au sein des deux couches et entre elles, et permet de simuler les effets industriels malgré l'incapacité de déterminer des bilans spécifiques à chaque industrie, comme on le souhaiterait dans un modèle théorique. Le niveau de détail utilisé dans le module de production permet de suivre les résultats par industrie, origine (domestique ou importée) et selon plusieurs méthodes de valorisation (en termes réels, prix de base et prix du marché), ce qui permet au modélisateur de repérer les sources de comportement chaotique malgré la taille du modèle. L'utilisation de comptes agrégés clés pour la couche macro assure que la cohérence avec les données historiques peut être vérifiée à chaque étape du processus de construction.

D'un autre côté, il convient de noter qu'un modèle avec ce niveau de détail exige énormément de données. La création et la maintenance d'un ensemble de données cohérent avec les hypothèses utilisées dans la version en cours du modèle sont largement les processus les plus chronophages. Des outils plus simples peuvent suffire pour des évaluations rapides, mais les décideurs politiques bénéficieraient certainement d'une information accrue sur les effets de la mise en œuvre de telles ou telles alternatives politiques.

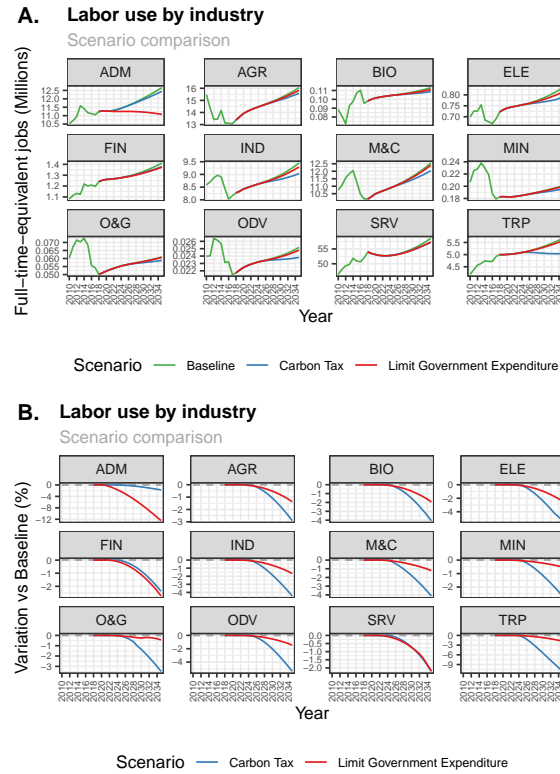


Figure 14: Evolution of the net lending (+) / net borrowing (-) positions by institutional sector

Dans l'ensemble, malgré la capacité à produire des résultats cohérents avec les objectifs fixés lors du processus de calibration, il reste encore beaucoup de place pour l'amélioration tant au niveau technique que théorique. En termes de priorités, le modèle devrait envisager davantage d'itérations entre les années.¹⁰ L'introduction d'itérations supplémentaires nécessitera une nouvelle calibration des paramètres. Idéalement, les équations comportementales pourraient être mises à jour pour éviter les simplifications extrêmes. Enfin, avec les prochaines publications des données comptables nationales, il y aura des informations pour évaluer l'ampleur et la répartition des chocs liés à la COVID, ainsi que les premiers signes de réorganisation économique à mesure que le processus de reprise se met en place. Cela peut certainement enrichir les perspectives que le modèle peut fournir en veillant à ce que la ligne de base tienne compte de ces événements.

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¹⁰Dans une étape future, cela pourrait être amélioré en recalibrant le modèle en utilisant des informations trimestrielles si disponibles.

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Chapter 1

Minskyan Classical Growth Cycles: Stability Analysis of a Stock-Flow Consistent Macrodynamic Model

1.1 Introduction

It has been half a century since [Goodwin \(1967\)](#) developed a model of endogenous real growth cycles. Based on a simple and well known dynamic—the nonlinear Lotka-Volterra prey-predator model—Goodwin’s model appeals in its simplicity and can be easily applied by researchers in a variety of fields (physics, biology, etc.). Goodwin’s growth cycle is a simple dynamic model of distributional shares of output¹ and of (un)employment. In his model, he shows how accumulation takes the form of a cycle. The solution of the model describes a family of closed cycles in the state variables (workers’ share and employment).²

In the 1980s, Van der Ploeg merged what he called “the neo-Keynesian” and the “neo-classical views.” He extended the Goodwin-Lotka-Volterra model by incorporating constant elasticity of substitution, or CES production technology, thus capital and labor no longer being complementary factors of production. By doing so, he relaxed the original assumption of a constant capital-to-output ratio made by Goodwin. The advantage of this relaxation is that both the Leontief technology underlying Goodwin’s model and the more general technology underlying Solow’s model, [Solow \(1956\)](#), are incorporated as special cases. Furthermore, van der Ploeg showed that the choice of incorporating CES technology destroys the conservative nature of the Goodwin system, i.e. transforms a closed orbit in a stable focus. Therefore, perpetual cycles are replaced by damped cycles or monotonic convergence to the balanced growth equilibrium. The primary economic rationale of van der Ploeg’s extension would be that improved profit will stimulate investment and thus increase output. This gain in output would in turn stimulate jobs and push wages up by wage negotiation. Through internal financing, firms would substitute labor by capital by firing and installing new capital. For example, in Goodwin’s model, a higher wage share results in lower profits. The latter will negatively impact output growth and employment due to lower investment. This lower investment will eventually lead to a new boom in the “class struggle” cycle, which in turn will lead to a decrease in wages and increase in profits. In van der Ploeg’s model, when substitution between factors of production is considered, a given period of high employment rate—the apex of the cycle—will induce a substitution of labor by capital rather than an increase in wages. Such substitution will allow for the dampening of employment and wage variations.

In van der Ploeg’s framework, the equality between investment and profits always holds. In this paper, we propose to relax the equality by incorporating a Minskyan framework including debt. Minsky’s Financial Instability Hypothesis (hereafter: FIH) links the expansion of credit with the rise of asset prices and the inherent fragility of the financial system. Although Minsky made his points clear, aided by convincing diagrams and incisive exploration of data, he refrained from presenting his ideas as a mathematical model. This task was taken up by [Keen \(1995\)](#), where a system of differential equations was proposed

¹In the sense of the GDP at factor cost, where the income approach of the GDP is summarized as the distribution of wages and profits.

²For a complete overview of Goodwin’s modern dynamics and its economic interpretation, we refer the reader to the paper of [Grasselli and Costa Lima \(2012\)](#).

as a simplified model incorporating the basic features of Minsky’s hypothesis. By adding a new dimension to Goodwin’s model, Keen’s dynamic model changes a conservative system into a dissipative one in which the dynamics display sensitive dependence on initial conditions. Specifically, this new model has a stable equilibrium defined in terms of the employment rate, the profit rate, and the debt-to-output ratio. Additionally, the system will converge to this equilibrium if the initial conditions are sufficiently close to what we will call the *Good* equilibrium. However, for other initial conditions, the model bifurcates (see [Pomeau and Manneville \(1980\)](#)) and undergoes an unstable cyclical breakdown towards what we will call the *Bad* equilibrium.

This paper shows that when the economy is endowed with CES production function, the properties of Keen’s model are preserved. Except that, under the assumption of a class of production function being bounded by the Cobb-Douglas and the linear, the equilibrium that led to the collapse (i.e., the *Bad* equilibrium) is no longer locally stable. We also show that the basin of attraction towards the *Good* equilibrium is substantially larger when substitution is allowed. Finally, we indicate that the model with CES production function would be a more suitable candidate for estimation purposes than the Leontief counterpart. All these conclusions hold whenever speculation is added to the economic landscape.

This paper is organized as follows: Section 2 outlines the model that departs from [Keen \(1995\)](#) by incorporating CES production technology. Section 3 introduces the equilibria and the study of their local stability. Section 4 presents different properties deduced from the study of the basins of attraction. Section 5 extends the model by allowing for speculation. Finally, Section 6 offers concluding remarks and suggestions for further works.

1.2 The model

[Keen \(1995\)](#) assumed Leontief production technology in which the inputs of production—capital and labor—are not substitutable. The production function is defined as follows³

$$Y^k := \min \left(\frac{K^k}{\nu}, aL^k \right),$$

where Y^k is the real output, K^k the stock of productive capital, L^k the labor force and $a > 0$ the labor productivity. In this framework, the capital-to-output parameter, $\nu > 0$, is constant and technology is assumed to be used at its maximal capacity. [van der Ploeg \(1985\)](#) relaxed the assumption that capital and labor cannot be substituted with each other by endowing the economy with CES production function

$$Y = C \left[bK^{-\eta} + (1 - b)(e^{a_t}L)^{-\eta} \right]^{-\frac{1}{\eta}}, \quad (1.1)$$

where $C > 0$ is the factor productivity (or efficiency parameter) and $b \in (0; 1)$ reflects the capital intensity of the production. The short-run elasticity of substitution between

³The superscript k stands for Keen.

capital and labor is given by $\sigma_\eta := \frac{1}{1+\eta}$. The labor productivity is driven by a constant rate of growth a_l . It is worth recalling that the CES production function allows for three limit cases: (i) when $\eta \rightarrow +\infty$, one retrieves the Leontief production function; (ii) $\eta \rightarrow 0$ leads to the Cobb-Douglas production; (iii) if $\eta \rightarrow -1$ one recovers the linear production function.^{4,5}

Real wages are set according to a short-run Phillips curve and we assume that the production sector behaves like a large oligopoly which adjusts the quantity produced so as to maximize its profit.⁶ The first-order condition that characterizes the profit maximizing behavior implies that real wages equal their marginal return

$$\frac{\partial Y}{\partial L} = w.$$

Thus, van der Ploeg broke Goodwin's assumption of a constant capital-to-output ratio, making it time-varying and defined by

$$\nu_\eta(t) := \frac{K(t)}{Y(t)} = \frac{1}{C} \left(\frac{1 - \omega(t)}{b} \right)^{-\frac{1}{\eta}},$$

where $\omega := wL/Y$ is the wage-to-output ratio. We endow ν_η with a subscript η since the dynamics will strongly depend upon η 's value. Whenever $\eta \rightarrow +\infty$, the capital-to-GDP ratio is constant as in the Leontief case. The full model derivation of the Minskyan classical growth cycle model is available in Appendix 1.A. It boils down to a three-dimensional system

$$\begin{cases} \dot{\omega} &= \omega \left[\left(\frac{\eta}{1+\eta} \right) [\Phi(\lambda) - a_l] \right] \\ \dot{\lambda} &= \lambda \left[\kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta - a_l - \beta - \frac{1}{\eta(1-\omega)} \dot{\omega} \right] \\ \dot{d} &= d \left[r - \kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \delta + \frac{\dot{\omega}}{(1-\omega)\eta} \right] + \kappa(\pi) - (1 - \omega) \end{cases} \quad (1.2)$$

where N is the active population, assumed to grow at a constant rate β , and $\lambda := L/N$ is the employment rate, which determines the level of wages through the short-term Phillips curve : $\frac{\dot{w}}{w} = \Phi(\lambda)$. The second aggregate behavior is given by the function $\kappa(\cdot)$, that controls the investment-to-output ratio and depends upon the profit share $\pi := 1 - \omega - rd$. In the latter, d stands for the debt-to-output ratio and $r > 0$ for the constant short term interest rate. Finally, the parameter $\delta > 0$ refers to the depreciation rate of capital.

⁴We confine ourselves to $\eta \in]0, +\infty[$ (that is an elasticity of substitution that lies in the set $]0, 1[$). The reason is twofold: (i) such short term elasticity would imply an above unity substitution between capital and labor in a very short term that is very unlikely (see Klump et al. (2012)); and (ii) such values values would break the predator-prey logic of the clockwise behavior suggested by Goodwin and shown by the data (Solow (1990); Harvie (2000); Mohun and Veneziani (2006); or Mc Isaac (2017)).

⁵It is worth mentioning that the consumption price is normalized to 1 throughout the paper.

⁶This minimal rationality argument is analogous to the assumption in Goodwin's model that the allocation of capital and labor is always at the diagonal of the (K^k, L^k) -plan, so that we have not only $Y^k = \min \left(\frac{K^k}{\nu}, aL^k \right)$ but also $Y^k = \frac{K^k}{\nu} = aL^k$.

In system (1.2), whenever $\eta \rightarrow +\infty$ and $C = 1/\bar{\nu}$, Keen (1995) model is retrieved. Thus, we recover Leontief production function and, as previously stated, a constant capital-output ratio.

The growth rate of the economy is given by⁷

$$g := \frac{\dot{Y}}{Y} = \kappa(\pi)C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta - \frac{\dot{\omega}}{(1-\omega)\eta}.$$

Here, as in both the van der Ploeg and Keen models, the behavior of households is fully accommodating in the sense that, given investment $I := \kappa(\pi)Y$, consumption is determined by the macro balance

$$C := Y - I = (1 - \kappa(\pi))Y$$

precluding a more general specification of household saving propensity.⁸ Table 1.1 makes the stock-flow consistency of the model explicit. For this paper, we adopt the convention that

$$D := L - M^f,$$

where the net borrowing D is the difference between firm loans, L , and firms deposits, M^f . Furthermore, the model (1.2) is retrieved from Table 1.1 by assuming $r = r^M = r^L$.

	Households	Firms		Banks	Sum
Balance Sheet					
Capital stock		K			K
Deposits	M^h	M^f		$-M$	
Loans		$-L$		L	
Sum (net worth)	X^h	X^f		X^b	X
Transactions					
		current	capital		
Consumption	$-C$	C			
Investment		I	$-I$		
Accounting memo [GDP]		[Y]			
Wages	W	$-W$			
Interests on deposits	$r^M M^h$	$r^M M^f$		$-r^M M$	
Interests on loans		$-r^L L$		$r^L L$	
Financial Balances	S^h	Π	$-I$	S^b	
Flow of funds					
Gross Fixed Capital Formation		I			I
Change in Deposits	\dot{M}^h	\dot{M}^f		$-\dot{M}$	
Change in loans		$-\dot{L}$		\dot{L}	
Column sum	S^h	Π		S^b	I
Change in net worth	$\dot{X}^h = S^h$	$\dot{X}^f = \Pi + -\delta K$		$\dot{X}^b = \Pi^b$	\dot{X}

Table 1.1: Balance sheet, transactions, and flow of funds in the economy

⁷See Appendix 1.A for the computation.

⁸Studying the consequences of dropping Say's law will be the task of a forthcoming paper.

With this set-up, we can readily verify that the accounting identity that requires investment to be equal to savings always holds.⁹

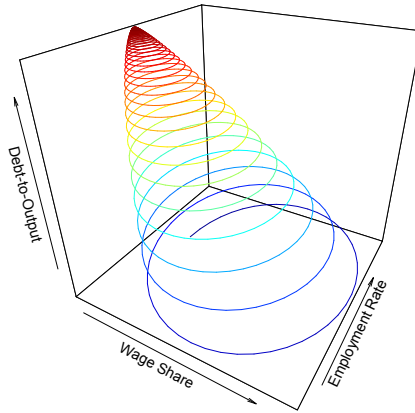


Figure 1.1: Keen’s model with Leontief production function and initial condition in the neighborhood of the *Good* equilibrium.

For the numerical simulations that follow, most of the parameters are borrowed from Keen (2013) and are explained in Appendix 1.B. Figure 1.1 shows the (ω, λ, d) -space, or the phase space, of Keen’s model converging towards the *Good* equilibrium. Qualitatively, we observe that Goodwin’s cyclical behavior is dampened when the trajectory approaches its equilibrium value. Remember that, in the CES setting, Figure 1.1 refers to the limit case

$$\lim_{\eta \rightarrow +\infty} \nu_\eta(t) = \nu,$$

of a constant capital-to-output ratio.

Turning to the time-varying behavior of the capital-to-output ratio, Figures 1.2 and 1.3 show the counterpart model where the elasticity of substitution for capital and labor are

⁹We note that according to Nguyen-Huu and Pottier (2016), the channel of debt financing is not fully determined by the model. Indeed, it does not distinguish between loanable funds and endogenous money creation since both rationales induce the same set of equations.

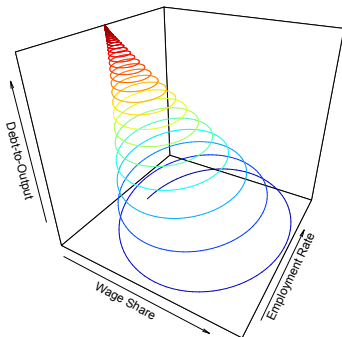


Figure 1.2: CES model with $\eta = 500$ and initial conditions in the neighborhood of the *Good* equilibrium.

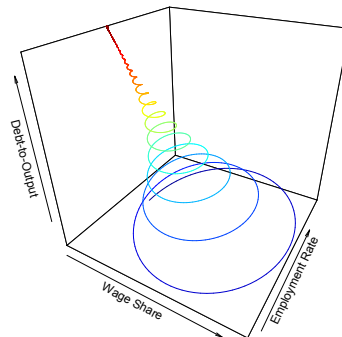


Figure 1.3: CES model with $\eta = 100$ and initial conditions in the neighborhood of the *Good* equilibrium.

$\sigma_{500} = \frac{1}{1+500} \approx 0.2\%$ and $\sigma_{100} = \frac{1}{1+100} \approx 1\%$, respectively. By allowing for substitution between capital and labor (while maintaining other parameters and beginning again near the *Good* equilibrium), we observe that the cycles are more muted when η is lower. This characteristic echoes the stable focus behavior demonstrated by van der Ploeg as opposed to the closed orbit showed by Goodwin's dynamics. A primary rationale to explain the

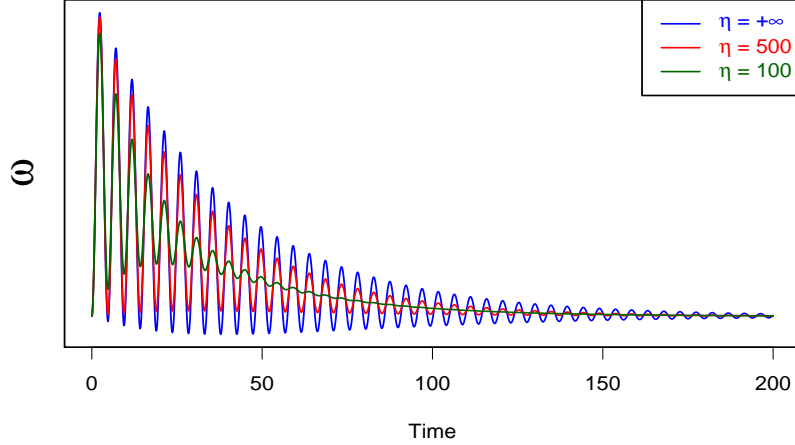


Figure 1.4: Evolution of ω in the previous simulations. In blue: $\eta = +\infty$. In red: $\eta = 500$. In green: $\eta = 100$.

difference might be that a gain in profit boosts investment and output. Thus, employment will increase and so will total wages. Indeed, when $\eta > 0$, the wage share, ω , increases according to the wage negotiation curve

$$\frac{\dot{\omega}}{\omega} = \frac{\eta}{1+\eta} (\Phi(\lambda) - a_l).$$

This rise in ω will increase the capital-to-output ratio ν_η since

$$\frac{\dot{\nu}_\eta}{\nu_\eta} = \frac{\dot{\omega}}{(1-\omega)\eta}.$$

It follows that firms will tend to substitute labor by capital by firing and installing new capital, because the growth of the capital-output ratio enters negatively in the evolution of the employment rate (see Appendix 1.A).

Figure 1.4 shows the time series for the wage share ω as obtained by the three different settings used for the production function. These simulations clarify the difference in the evolution of the wage share and its cyclical properties (especially the amplitude of the cycles). The blue curve represents the Leontief technology. Interestingly, it demonstrates a more volatile behavior than its counterparts: (i) the CES with $\eta = 500$ (in red); and (ii) the CES with $\eta = 100$ (in green). We qualitatively observe that the amplitude of cycles decreases as η increases, while their periodicity remains similar. In other words, the qualitative change induced by the allowance of the capital-labor substitution affects

only the amplitude of cycles, remaining unchanged the long-run trend.¹⁰ Finally, Figure 1.5 illustrates the behavior of the system in a case with low substitution ($\eta = 100$), with the usual parameters (see Appendix 1.B).

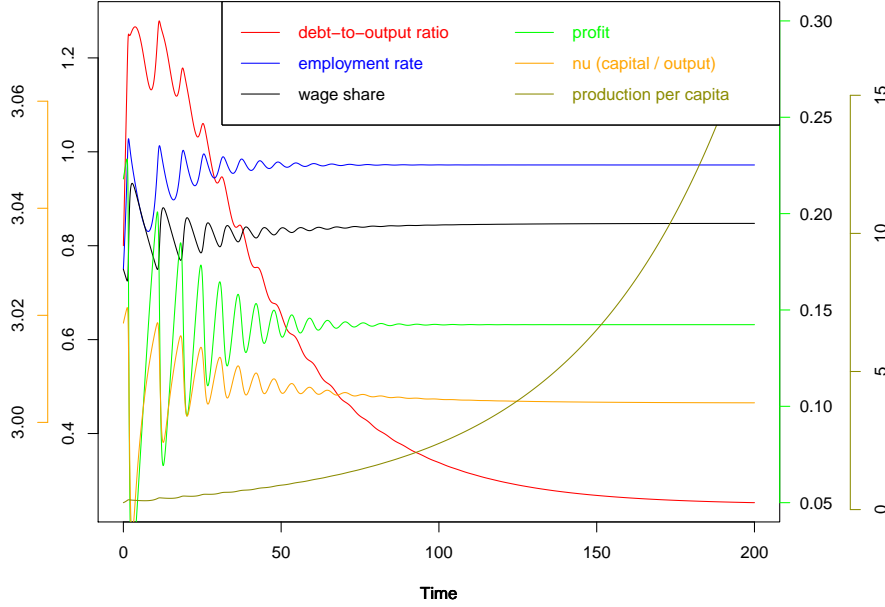


Figure 1.5: Illustration of the system dynamics with benchmark parameters and $\eta = 100$.

1.3 Equilibria

All equilibria can be found by an exhaustive examination of three cases. First, $\omega_1 \neq 0$, meaning that the wage share is not zero. This point will be labeled as the *Good* equilibrium. Second, $\omega_2 = 0$, $\lambda_2 \neq 0$, which provides an economically meaningless equilibrium. Finally, $\omega_3 = 0$, $\lambda_3 = 0$, which provides two equilibria: an *Trivial* and a *Bad* equilibrium.

1.3.1 The *Good* equilibrium

Prior to the derivation of this equilibrium, it is worth mentioning that the point $(\omega_1, \lambda_1, d_1)$ refers to the *Good* equilibrium. Using the equilibrium condition of ω_1 , we find that $\lambda_1 = \Phi^{-1}(a_l)$. Next, using the equilibrium condition of λ_1 , $\kappa(\cdot)$ can be written in terms of ω_1 so that

$$\kappa(\pi_1) = \frac{a_l + \beta + \delta}{C} \left(\frac{1 - \omega_1}{b} \right)^{-\frac{1}{\eta}} =: \zeta_1(\omega_1). \quad (1.3)$$

¹⁰In addition, it is worth mentioning that the equilibrium points differ slightly depending on the value of η .

Plugging equation (1.3) into the equilibrium condition of d_1 yields d_1 as a function of ω_1 :

$$d_1(\omega_1) = \frac{1 - \omega_1 - \zeta_1(\omega_1)}{r - a_l - \beta}. \quad (1.4)$$

Plugging equation (1.4) into the $\kappa(\cdot)$ of equation (1.3) leads to a nonlinear equation that ω_1 should satisfy. Depending on the specification chosen for κ ,¹¹ we solve this equation numerically so that we get ω_1 . Next, we find d_1 using equation (1.4). As with the Goodwin and Solow models, at the equilibrium point, the real growth rate of the economy is given by

$$g = \frac{\dot{Y}}{Y} = \kappa(\pi_1)C \left(\frac{1 - \omega_1}{b} \right)^{1/\eta} - \delta = a_l + \beta.$$

One can note that, under reasonable specifications, this equilibrium exists. The right hand side of equation (1.3) is a function of ω_1 that equals 0 at $\omega_1 = 1$ and $\zeta_1(0) = \frac{a_l + \beta + \delta}{C} b^{1/\eta}$ at 0; while the left hand side equals κ_0 for $\omega_1 = 1$ and $\kappa \left(1 - \frac{r}{r - a_l - \beta} (1 - \zeta_1(0)) \right)$ for $\omega_1 = 0$. Since both sides are continuous function of ω , it suffices that $\kappa \left(1 - \frac{r}{r - a_l - \beta} (1 - \zeta_1(0)) \right) < \zeta_1(0)$ to ensure the existence of the *Good* equilibrium. As $\eta > 0$, $1 - \frac{r}{r - a_l - \beta} (1 - \zeta_1(0))$ is often negative (it is negative as long as $\frac{a_l + \beta + \delta}{C} > \frac{1}{1 - \zeta_1(0)}$), so that $\kappa_0 < \zeta_1(0)$ ensures the existence of the equilibrium (because κ is increasing).¹² As $\zeta_1(0) \in \left[0; \frac{a_l + \beta + \delta}{C} \right]$ in this case, assuming e.g. $\kappa(0) = 0$ is a sufficient condition for the existence of the *Good* equilibrium.

In addition, one can show that, under reasonable specifications, ω_1 is positive. Indeed, as equation (1.3) rewrites $\omega_1 = 1 - b \left(\frac{C\zeta_1}{a_l + \beta + \delta} \right)^{-\eta}$, one just needs that $\left(\frac{C\zeta_1}{a_l + \beta + \delta} \right)^\eta > b$ to insure a non negative ω_1 . As $\eta > 0$, this is equivalent to ζ_1 being above $\frac{a_l + \beta + \delta}{C} b^{1/\eta}$.¹³ Hence, as long as the image of κ is entirely on the right side of this value, ω_1 is positive.

1.3.2 The *Slavery* equilibrium

This second equilibrium is economically meaningless. It would suggest that wages are null while employment is still positive, and would be interpreted as characterizing *Slavery*. Its derivation can be sketched in the same manner as before. The following function $\zeta_2(\cdot)$ can be derived from system (1.2) so that

$$\kappa(1 - rd_2)Cb^{-\frac{1}{\eta}} - \delta = a_l + \beta + \frac{\Phi(\lambda_2) - a_l}{1 + \eta} =: \zeta_2(\lambda_2). \quad (1.5)$$

Equation (1.5) together with system 1.2 gives

$$d_2(\lambda_2) = \frac{1 - \frac{b^{1/\eta}}{C} (\zeta_2(\lambda_2) + \delta)}{r - \zeta_2(\lambda_2)}. \quad (1.6)$$

¹¹An example with an affine function is provided in Appendix 1.D. It shows that, with the simplest case, a closed-form expression for the equilibrium could not be written.

¹²For $\eta < 0$, $\zeta_1(0)$ takes high values (often above 1), so that the previous inequality holds.

¹³This equivalence would be the opposite for a negative value of η .

Finally, plugging equation (1.6) into equation (1.5) gives a value satisfied by λ_2 . Hence, d_2 can be deduced from equation (1.6). Note that whenever $\eta \rightarrow +\infty$ we retrieve [Grasselli and Costa Lima \(2012\)](#) results. We show in Appendix 1.E that for our benchmark specification, this equilibrium exists.

1.3.3 The *Trivial* and the *Bad* equilibria

This condition gives us two kinds of equilibria. On the one hand, *trivial* equilibria can be found by solving system 1.2, so that

$$(\bar{\omega}_3, \bar{\lambda}_3, \bar{d}_3) = (0, 0, \bar{d}_3),$$

is an equilibrium point for (1.2), and with d_3 being any solution of

$$d \left[r - \kappa(1 - rd)C \left(\frac{1}{b} \right)^{1/\eta} + \delta \right] + \kappa(1 - rd) - 1 = 0 \quad (1.7)$$

Note that this condition for finding the equilibrium is similar to [Grasselli and Costa Lima \(2012\)](#) if we identify $C(1/b)^{-1/\eta} = \nu$. Therefore, we expect this equilibrium to be unstable in the same way that $(\bar{\omega}_3, \bar{\lambda}_3)$ is a saddle point for the Goodwin model.

On the other hand, another equilibrium with infinite debt can be found using the change of variable: $\frac{1}{d} =: u$. We get that

$$\begin{cases} \dot{\omega} &= \omega \left[\left(\frac{\eta}{1+\eta} \right) [\Phi(\lambda) - a_l] \right], \\ \dot{\lambda} &= \lambda \left[\kappa(\pi(u))C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta - a_l - \beta - \frac{1}{(1-\omega)} \left(\frac{1}{1+\eta} \right) [\Phi(\lambda) - a_l] \right], \\ \dot{u} &= -u \left[r - \kappa(\pi(u))C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \delta + \frac{\omega}{(1-\omega)} \left(\frac{1}{1+\eta} \right) [\Phi(\lambda) - a_l] \right] - u^2 (\kappa(\pi(u)) - (1 - \omega)). \end{cases} \quad (1.8)$$

with $\pi(u) := 1 - \omega - r/u$. Hence, the point $(\bar{\omega}_3, \bar{\lambda}_3, \bar{u}_3) = (0, 0, 0)$, or $(\bar{\omega}_3, \bar{\lambda}_3, \bar{d}_3) = (0, 0, +\infty)$ is an equilibrium. It can be interpreted as representing the collapse of the economy induced by over-indebtedness, where employment and wage converge towards 0 and debt increases constantly towards infinity. This equilibrium is labeled as the *Bad* equilibrium.

1.3.4 Local stability study

This subsection seeks to present the Jacobian matrices of the two models previously presented (both with and without the change of variable). This step will help us to analyze the local stability of the equilibria displayed above. Note that behavioral functions $\Phi(\cdot)$ and $\kappa(\cdot)$ are similar to those of [Keen \(2013\)](#). For the sake of clarity, in what follows, we

use $\kappa(\pi) := \kappa(1 - \omega - rd)$. The Jacobian associated with system (1.2) is

$$J(\omega, \lambda, d) = \begin{bmatrix} \left(\frac{\eta}{1+\eta}\right) [\Phi(\lambda) - a_l] & \omega \left(\frac{\eta}{1+\eta}\right) \Phi'(\lambda) & 0 \\ \epsilon_1 & \epsilon_2 & \lambda \left[-r\kappa'(\pi)C \left(\frac{1-\omega}{b}\right)^{1/\eta}\right] \\ \epsilon_3 & \frac{d}{1+\eta} \left(\frac{\omega}{1-\omega}\right) \Phi'(\lambda) & \epsilon_4 \end{bmatrix},$$

with

$$\begin{aligned} \epsilon_1 &= \lambda \left[-\kappa'(\pi)C \left(\frac{1-\omega}{b}\right)^{1/\eta} - \frac{1}{\eta} \kappa(\pi) \frac{C}{b^{1/\eta}} (1-\omega)^{1/\eta-1} - \frac{1}{(1-\omega)^2} \left(\frac{1}{1+\eta}\right) [\Phi(\lambda) - a_l] \right], \\ \epsilon_2 &= \kappa(\pi)C \left(\frac{1-\omega}{b}\right)^{1/\eta} - \delta - a_l - \beta - \frac{1}{(1-\omega)} \left(\frac{1}{1+\eta}\right) [\Phi(\lambda) - a_l] - \lambda \frac{1}{(1-\omega)} \frac{1}{1+\eta} \Phi'(\lambda) \\ \epsilon_3 &= d \left[\kappa'(\pi)C \left(\frac{1-\omega}{b}\right)^{1/\eta} + \frac{1}{\eta} \kappa(\pi) \frac{C}{b^{1/\eta}} (1-\omega)^{1/\eta-1} + \frac{1}{1+\eta} (\Phi(\lambda) - a_l) \frac{1}{(1-\omega)^2} \right], \\ &\quad -\kappa'(\pi) + 1, \\ \epsilon_4 &= r - \kappa(\pi)C \left(\frac{1-\omega}{b}\right)^{1/\eta} + \delta + \frac{\omega}{(1-\omega)} \left(\frac{1}{1+\eta}\right) [\Phi(\lambda) - a_l] + rd\kappa'(\pi)C \left(\frac{1-\omega}{b}\right)^{1/\eta} \\ &\quad - r\kappa'(\pi). \end{aligned}$$

At the equilibrium point $(\bar{\omega}_0, \bar{\lambda}_0, \bar{d}_0) = (0, 0, \bar{d}_0)$, the Jacobian moves down to a lower triangular matrix

$$J(0, 0, d_0) = \begin{bmatrix} \left(\frac{\eta}{1+\eta}\right) [\Phi(0) - a_l] & 0 & 0 \\ 0 & \epsilon'_2 & 0 \\ \epsilon'_3 & 0 & \epsilon'_4 \end{bmatrix}$$

with

$$\begin{aligned} \epsilon'_2 &= \kappa(1 - rd_0)C \left(\frac{1}{b}\right)^{1/\eta} - \delta - a_l - \beta - \left(\frac{1}{1+\eta}\right) [\Phi(0) - a_l], \\ \epsilon'_3 &= d \left[\kappa'(1 - rd_0)C \left(\frac{1}{b}\right)^{1/\eta} + \frac{1}{\eta} \kappa(1 - rd_0) \frac{C}{b^{1/\eta}} + \frac{1}{1+\eta} (\Phi(0) - a_l) \right] - \kappa'(1 - rd_0) + 1, \\ \epsilon'_4 &= \left[r - \kappa(1 - rd_0)C \left(\frac{1}{b}\right)^{1/\eta} + \delta \right] + rd_0 \kappa'(1 - rd_0)C \left(\frac{1}{b}\right)^{1/\eta} - r\kappa'(1 - rd_0). \end{aligned}$$

The Jacobian's real eigenvalues are given by diagonal entries and it is not easy to determine the sign of d_0 . Nevertheless, we note that, for a sufficiently large value of $\pi_0 := 1 - rd_0$, ϵ'_2 is positive, whereas a sufficiently small value of π_0 (i.e., a large enough value of d_0) makes ϵ'_4 non-negative. We conclude that this equilibrium point is likely to be unstable.

Although [Grasselli and Costa Lima \(2012\)](#) could analytically retrieve all equilibria,

our model is too intricate to do so. Thus, we will later compute the eigenvalues of the Jacobian matrices corresponding to each equilibrium under reasonable calibration.

In order to analyze the local stability of the *Bad* equilibrium point $(0, 0, +\infty)$, we denote $\kappa(\pi(u)) := \kappa(1 - \omega - r/u)$ and the corresponding Jacobian matrix of system (1.8) is

$$J(\omega, \lambda, u) = \begin{bmatrix} \left(\frac{\eta}{1+\eta}\right) [\Phi(\lambda) - a_l] & \omega \left(\frac{\eta}{1+\eta}\right) \Phi'(\lambda) & 0 \\ \epsilon_1 & \epsilon_2 & \lambda \left[\kappa'(\pi(u)) C \left(\frac{1-\omega}{b}\right)^{1/\eta} \frac{r}{u^2} \right] \\ \epsilon_3'' & \frac{-u}{1+\eta} \left(\frac{\omega}{1-\omega}\right) \Phi'(\lambda) & \epsilon_4'' \end{bmatrix},$$

with

$$\begin{aligned} \epsilon_3'' &= -u \left[\kappa'(\pi(u)) C \left(\frac{1-\omega}{b}\right)^{1/\eta} + \kappa(\pi(u)) C \frac{(1-\omega)^{1/\eta-1}}{b^{1/\eta}} \frac{1}{\eta} + \right. \\ &\quad \left. \left(\frac{1}{1+\eta} (\Phi(\lambda) - a_l)\right) \frac{1}{(1-\omega)^2} \right] - u^2 [-\kappa'(\pi(u)) + 1], \\ \epsilon_4'' &= - \left[r - \kappa(\pi(u)) C \left(\frac{1-\omega}{b}\right)^{1/\eta} + \delta + \frac{\omega}{(1-\omega)} \left(\frac{1}{1+\eta}\right) [\Phi(\lambda) - a_l] \right] \\ &\quad + \frac{r}{u} \kappa'(\pi(u)) C \left(\frac{1-\omega}{b}\right)^{1/\eta} - 2u [\kappa(\pi(u)) - (1-\omega)] - \kappa'(\pi(u)) r. \end{aligned}$$

Despite the previous comment on the equilibrium points, since $\bar{\omega} = 0$ and $\bar{\lambda} = 0$, the Jacobian matrix is diagonal at this point. Thus, its eigenvalues are summarized by the diagonal terms

$$\begin{aligned} \left(\frac{\eta}{1+\eta}\right) [\Phi(0) - a_l], \quad \kappa_0 C \left(\frac{1}{b}\right)^{1/\eta} - (\delta + a_l + \beta) - \left(\frac{1}{1+\eta} (\Phi(0) - a_l)\right), \\ -(r - \kappa_0 C \left(\frac{1}{b}\right)^{1/\eta} + \delta). \end{aligned}$$

The sign of the eigenvalues will depend on the parameter η , which is assumed to belong to the set $]0; +\infty[$. Indeed, when assuming $\Phi(0) < 0$, where wages decrease below some positive employment rate threshold, the first eigenvalue has the opposite sign of η . Furthermore, the remaining eigenvalues are negative if and only if

$$\kappa_0 = \inf_{\pi \in \mathbb{R}} \{\kappa(\pi)\} < \frac{b^{1/\eta}}{C} \min \left(r + \delta, \quad a_l + \delta + \beta + \frac{1}{1+\eta} (\Phi(0) - a_l) \right).$$

Finally, given that $\Phi(0) < 0$, the *Bad* equilibrium is stable if and only if $\eta > 0$ and the previous conditions are fulfilled.¹⁴

It is worth mentioning that depending on η and due to the condition of $\kappa_0 = \inf_{\pi \in \mathbb{R}} \kappa \in \mathbb{R}$, boundary condition of phenomenological function $\Phi(\cdot)$ will have a significant effect on the

¹⁴Thus, if the elasticity of substitution is too high, i.e. below that of Cobb-Douglas (as in the linear case e.g.), the *Bad* equilibrium is unstable.

stability of the *Bad* equilibrium. Indeed, by taking reasonable values for r and δ , if $\Phi(0) < -(1 + \eta)(a_l + \delta + \beta) + a_l$ then $\kappa_0 < 0$ if one wants to guarantee the local stability of the *Bad* equilibrium.

To numerically study the properties of other equilibria, we use a baseline calibration that closely follows Keen (2013).¹⁵ Our model differs from Keen (2013)’s in that we assume that the productive sector is endowed with CES technology. We remind the reader that the capital-to-output ratio is not constant and equals

$$\nu_\eta(t) = \frac{1}{C} \left(\frac{1 - \omega(t)}{b} \right)^{-\frac{1}{\eta}}.$$

In building the capital stock dataset, database makers assume an initial capital-to-output stock of about 3 (see Mc Isaac (2017)). In order to retrieve similar results for the Leontief case, we will assume that $\nu_\eta(t) \rightarrow 3$ whenever $\eta \rightarrow +\infty$, which implies that $C = 1/3$. On the other hand, the ratio $(1 - \omega(t))/b$ should oscillate around 1 since b represents the share of capital in the production function in equation (1.1). In order to find a reasonable value, we will assume that $b = 1 - \omega^*$, where ω^* is the *Good* equilibrium value of the Leontief model counterpart. Finally, the parameter η will be tested for different values of $\sigma = 1/(1 + \eta)$. First, we will test $\sigma = 0$, i.e. the Leontief case, where $\eta \rightarrow +\infty$. Second, the case where $\sigma \approx 1^+$ —close to the Cobb-Douglas case, i.e. $\eta \rightarrow 0$ —will be investigated. Third, the intermediary case between the last two, where $\sigma = 1\%$ (or $\eta = 100$) will be displayed.

Table 1.2: The summary table of stability for all equilibria.

Value of η	<i>Good</i>	<i>Trivial</i>	<i>Slavery</i>	<i>Bad</i>
$+\infty$	o	x	x	o
100	o	x	x	o
0	o	x	x	o

Table 1.2 provides a summary of the tables available in Appendix 1.C. An “o” means that the equilibrium is stable whereas an “x” stands for local instability. As expected, the *Good* is always stable while the *trivial* and the *slavery* are not. Figure 1.6 provides an example of a trajectory that converges towards a *Good* equilibrium. The simulation begins at a point slightly behind the convergent spiral, at the bottom of the figure. At the beginning of the simulation, the trajectory displays ample cycles with a debt-to-output ratio that increases over the time. In other words, it shows large fluctuations of ω , the wage share, and λ , the employment rate. However, cycles will be less pronounced over time and after reaching a given level of debt-to-output, the productive sector begins deleveraging until it reaches its equilibrium point, at which of the state variable displays a finite value.

Figure 1.7 gives an example where the economy is no longer attracted by the *Good*

¹⁵See Appendix 1.B for the details.

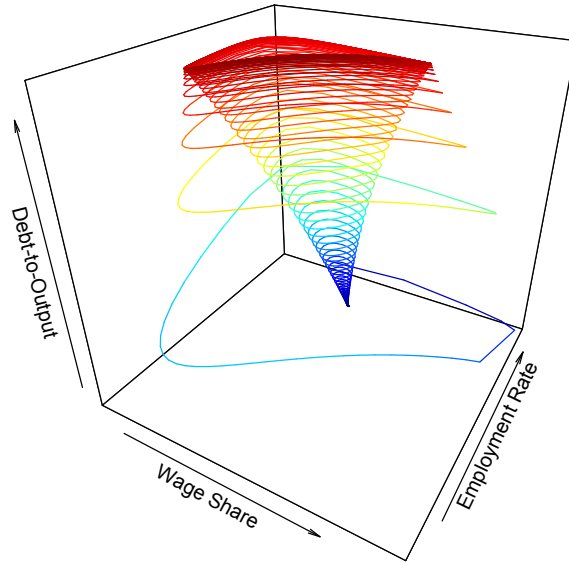


Figure 1.6: Simulation of a trajectory converging towards the *Good* equilibrium with the calibration of Appendix 1.B and $\eta = 500$. The initial conditions for λ and d are set to their equilibrium values while ω is initiated at 0.8055877 (at 0.06 from its equilibrium value). The initial condition is located at the bottom of the figure.

equilibrium. The only difference between Figures 1.6 and 1.7 is the initial condition of ω , which results in slightly higher profits. In the short to medium run, we observe a trajectory that is qualitatively similar to Figure 1.6. However, when the system arrives in a region where the economy should deleverage, as in this example, the debt burden is too high and the productive sector can no longer reduce its debt. At this point, the nonfinancial corporate's debt grows indefinitely. Moreover, starting from this debt burden moment, the wage share is getting closer to unity at the apex of each cycle. As shown by system (1.2), the decrease in employment and the increase of the debt-to-output ratio (that is the convergence towards the *Bad* attractor) will speed-up as $\omega \rightarrow 1^-$.¹⁶

¹⁶It is worth mentioning that the variation of λ is unbounded as $\omega \rightarrow 1^-$. Therefore, it is very likely that the Lyapunov function shows unbounded variation making the variational domain be in $\mathcal{D}_{(\omega, \lambda, d)} = [0, 1] \times [0, 1] \times \mathbb{R}$. We leave the proof for further research. For more insights see [Grasselli and Costa Lima \(2012\)](#) or [Nguyen-Huu and Costa-Lima \(2014\)](#)

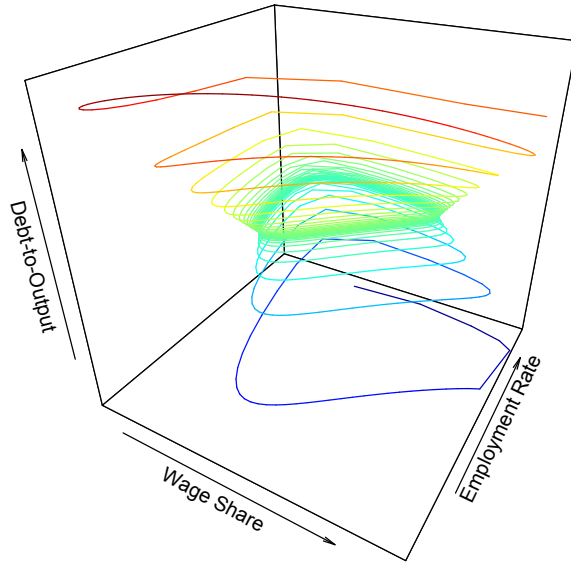


Figure 1.7: Simulation of a trajectory towards the *bad* equilibrium with the calibration of Appendix 1.B and $\eta = 500$. The initial conditions corresponds to the darkest blue point and is the same as in Figure 1.6 except for $\omega = 0.8055876$

1.4 Numerical study of the basins of attraction

This section aims at quantitatively and qualitatively analyzing the specificity of each model presented above according to the respective basins of attraction. We consider three cases: (i) $\eta \rightarrow +\infty$; (ii) $\eta = 100$; and (iii) $\eta = 0.5$;¹⁷ Furthermore, we perform a sensitivity analysis of the following key parameters: a_l , the rate of labor productivity growth; β , the rate of population growth; δ , the depreciation rate of capital; and r the short term interest rate.

1.4.1 Methodology for the basins of attraction

We adopt a simple approach to evaluate the basins of attraction. We simulate our system taking initial conditions at each points of a grid in the (ω, λ, d) space (typically, this grid consists of 325 points in the cuboid $[0.5; 0.95] \times [0.6; 1] \times [0; 3]$). At the time horizon of $t = 200$, we compute the Euclidian distance of the set of simulated variables to their equilibrium values. We consider that a simulation converges towards the equilibrium whenever that distance falls below a chosen precision (0.5 in practice). Although our programs are flexible and allow us to evaluate many basins of attraction, depending on the choices of the equilibrium, the grid, the precision, the time horizon and the time-steps (for the Runge-Kutta fourth-order method used in the simulations), we stick to

¹⁷The case $\eta = 0.5$ is identified as being the closest to the Cobb-Douglas. However, as shown in Appendix 1.F, when we derive the model with Cobb-Douglas production technology, we found that the wage share is no longer time-varying and equals, at all times, the output elasticity $1 - b$. Therefore, the original Goodwin prey-predator (between the employment rate and the wage share) logic does not hold anymore, as previously eluded.

the specification described above and evaluate only the basin of attraction of the *Good* equilibrium, the only one which is always stable. We finally plot the basins and compute their volume using Delaunay triangulation.

1.4.2 Main results

Figure 1.8 plots the basin of attraction of the *Good* equilibrium in the Leontief case. In other words, every economy that is initialized in that set will numerically converge towards the *Good* equilibrium. When the debt is low, the wage share must be high in order to keep the profit share at reasonable levels. In other words, if the wage share is not high enough, profit will be high and the investment share would skyrocket as a consequence. This would drive the economy to the Minskyan paradigm. In contrast, when the debt is high, the wage share should remain low. A consequence of low wage share would be higher profit, which provides a suitable situation for financial instability through the investment function since most of the value added would go to the debt services and no longer to workers and investment. A high wage share coupled with a high debt will prevent firms from deleveraging and, therefore from reducing their debt.

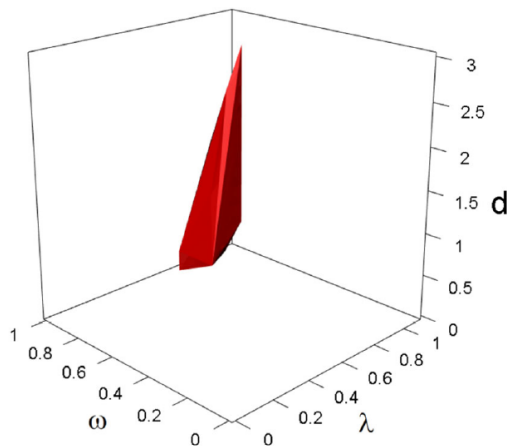


Figure 1.8: Basin of attraction of the *Good* equilibrium for the Leontief production technology. The *Good* equilibrium point is $(\omega, \lambda, d) = (0.865, 0.972, 1.50)$.

Figure 1.9 shows the example of the basin of attraction of an economy endowed with CES production function where $\eta = 0.5$ (that is close to the Cobb-Douglas production function). In the numerical analysis of the set of points under consideration, nearly all of the explored state space is covered. However, the area where wage share, debt ratio, and employment levels are high does not lead to a convergence towards the *good* equilibrium. The latter case represents low profits and the incapacity of the corporate sector to deleverage, preventing the economy from converging towards the *Good* equilibrium.

Figure 1.10 displays the basin of attraction of the Bad equilibrium for the system of Eq. (1.8) with $\eta = 0.5$ (that is close to Cobb-Douglas technology). Recall that, in this system $u = 1/d$. The basin of attraction of the Bad equilibrium lies in an area where $u \leq 0.1$ (i.e. $d \geq 10$). The numerical analysis shows that, at such high levels of debt, the

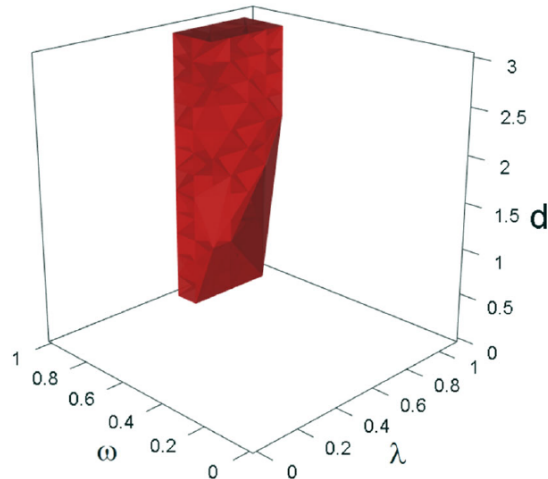


Figure 1.9: Basin of attraction of the *Good* equilibrium for the CES production technology with $\eta = 0.5$. The *Good* equilibrium point is $(\omega, \lambda, d) = (0.865, 0.972, 1.50)$.

system will not converge to the *Bad* equilibrium only at low level of wage share together with low level of employment—that is when income distribution is favoring banks (through debt repayment) and firm’s self-funded investment.

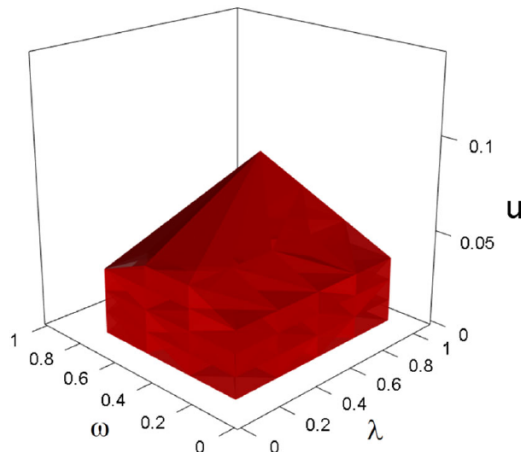


Figure 1.10: Basin of attraction of the *Bad* equilibrium for the CES production technology with $\eta = 0.5$. The *Bad* equilibrium point is $(\omega, \lambda, u) = (0, 0, 0)$.

1.4.3 Sensitivity analysis

Table 1.3 shows the percentage of initial condition points that converge towards the *Good* equilibrium. For instance, 70 represents the percentage of points within the considered area that converges under the benchmark calibration. Given a higher elasticity of substitution $\sigma = 1/(1 + \eta)$, the basin of attraction is usually larger. This is well illustrated by Figure 1.8, the basin of attraction of the Leontief case. Figure 1.9 plots the basin of attraction of the CES function with $\sigma = 2/3$. Note that CES production technology qualitatively shows a significantly larger basin of attraction than the Leontief one, even with very low

Table 1.3: Parameter sensitivity of basins of attraction: percentage of points from the simulation grid falling into the basin of attraction of the *Good* equilibrium. Values are reported in percentages.

parameters	benchmark: (a_l, r, δ, β) $= (2, 4, 1, 1)$	a_l		r		δ		β	
		$\bar{1}$	\dagger_3	$\bar{1}$	\dagger_5	$\bar{0.5}$	\dagger_2	$\bar{0}$	\dagger_3
$\eta = 0.5$	52	52	52	41	54	52	52	52	52
$\eta = 100$	51	28	54	40	50	50	45	33	53
$\eta = 10^{15}$	9	3	12	0	13	8	6	2	24

substitution. In general, the lower values of η are also associated with larger basins of attraction. Indeed, the basin of attraction of $\eta = 0.5$ shows uniformly a higher basin of attraction. Moreover, for the case $\eta = 0.5$, the only parameter that changes the value reported in Table 1.3 is the real interest rate r . For a higher value of r , the basin of attraction generally increases in volume, while for a lower value, it significantly decreases. The higher sensitivity of debt in the profits induced by the higher interest rate prevents, in almost all cases, the economy from falling into a state of the economy that is not desired. Interestingly, an increase in the depreciation rate, δ , increases the size of the basin of attraction on the studied space for the first two cases of η , while decreasing the size of the basin of attraction for $\eta = 10^{15}$. Finally, this exercise provides a first approach to sensitivity analysis. The study of the size change for the basin of attraction under multiple simultaneous parameter variations is left for further research.

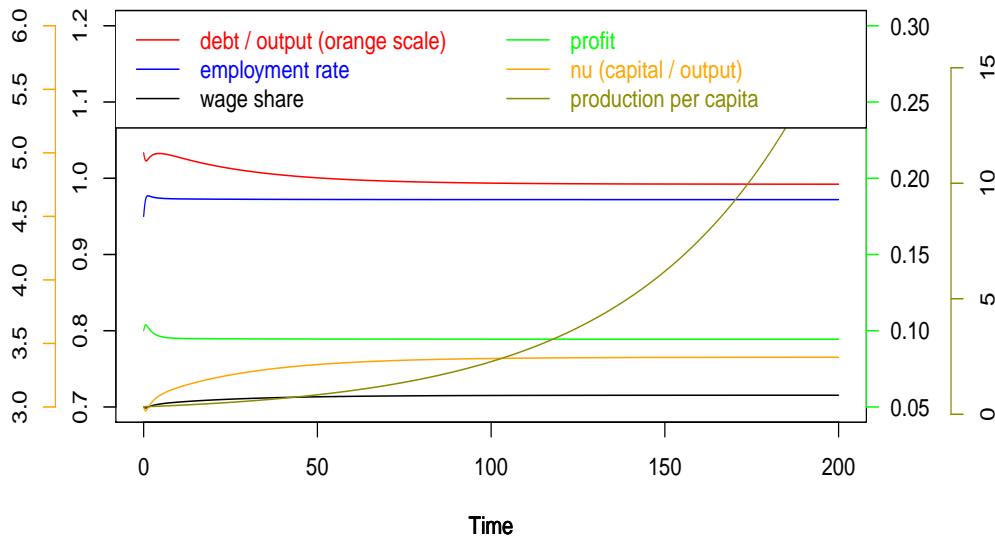


Figure 1.11: Illustration of the system dynamics with benchmark parameters except $b = 0.4$ and $\eta = 0.5$.

In system (1.2), an increase of the labor productivity growth rate, the depreciation rate of capital, or population growth negatively impact the employment rate dynamics. Thus, in the case with almost no substitution ($\eta = 10^{15}$), an increase in one of these parameters moderates booms in the business cycles. This is similar to what substitution would induce (a decrease of the employment rate due to a lower relative cost of labor). This explains why in the Leontief case, Table 1.3 exhibits larger basins of attraction as a_l and β increase. As soon as we allow for a limit degree of substitution, from almost 0% to $1/(100 + 1) \approx 1\%$, the surface of the basin of attraction substantially increases and highlights the transformation recorded by difference of the Leontief to the CES case. This change is somewhat similar to van der Ploeg’s results for the model’s trajectory behaviors. However, these parameters only have a modest effect on the size of the basins of attraction when substitution is higher. Indeed, Figure 1.11 plots the phase portrait of the state variable with the baseline calibration (see Appendix 1.B) and a coefficient $\eta = 0.5$. In this simulation, the calibrated elasticity of substitution between capital and labor is $\frac{1}{1+0.5} \approx 66.66\%$, which is consistent with most of the empirical findings surveyed by Klump et al. (2012).¹⁸ However, this simulation no longer shows the primary rationale of the Goodwin’s endogenous economic fluctuations. In other words, under what is ostensibly the most realistic calibration, our finding is that the model is no longer able to replicate the business cycles after the introduction of a reasonable substitution between factors of production. More research is needed to understand whether this is an inherent shortcoming feature of Goodwin-Keen inspired models or if the a cyclical behavior (see Solow (1990), or Harvie (2000)) could be recovered obtained with some additional refinement.

1.5 Speculation

This section aims at analyzing the destabilizing property of speculation. Indeed, as emphasized by Grasselli and Costa Lima (2012) for the Keen’s model, the model presented so far fails even more at capturing Minsky’s well known conclusion that “stability—or tranquility—, in a world with a cyclical past and capitalist financial institutions is destabilizing.” To see whether we can retrieve the destabilizing impact found by Grasselli and Costa Lima (2012), we endow the system with the same speculation scheme specification, that is to rewrite the debt evolution so that

$$\dot{D} = \kappa(\pi)Y - \pi Y + S,$$

with, S , the speculative component whose dynamics is

$$\frac{\dot{S}}{S} = \Psi(g_K(\omega, d)),$$

¹⁸Klump et al. (2012) surveyed a number of studies intended for developed countries in various timeframes (ca. 1800-2000). Almost 75% of the estimated elasticities showed a value between 0.5 and 1.

where $g_K(\omega, d) = \frac{\dot{K}}{K}$ is the growth of the productive capital.¹⁹ It is worth mentioning that, as noticed by [Grasselli and Nguyen-Huu \(2015\)](#), the additional flow of credit, S , corresponds to a purchase of existing financial assets. The additional source of net borrowing of firms from banks is used in turn to buy financial assets held by the Firms sector itself. S does not appear either in the consolidated balance sheet for the Firms sector of Table 1.1, nor in the transactions and flow of funds matrices as the purchase is made through the intra-firm market. Defining the ratio of speculation to economic output by, $s := S/Y$, it is easy to see that the additional speculative term leads to the following four-dimensional system

$$\begin{cases} \dot{\omega} &= \omega \left[\left(\frac{\eta}{1+\eta} \right) [\Phi(\lambda) - a_l] \right], \\ \dot{\lambda} &= \lambda \left[\kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta - a_l - \beta - \frac{1}{\eta(1-\omega)} \frac{\dot{\omega}}{\omega} \right], \\ \dot{d} &= d \left[r - \kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \delta + \frac{\dot{\omega}}{(1-\omega)\eta} \right] + \kappa(\pi) - (1-\omega) + s, \\ \dot{s} &= s \left[\Psi \left(\kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta \right) - \kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \delta \right]. \end{cases} \quad (1.9)$$

1.5.1 Equilibria

We follow [Grasselli and Costa Lima \(2012\)](#)'s guideline in deriving the equilibria of the model augmented by the speculative component. Therefore, we restrain ourselves into the meaningful ones, corresponding to the locally stable equilibria previously examined, as a four-dimensional system shows three options based on equilibria that are locally stable in Subsection 1.3.4.

First, it is easy to show that the equilibrium $(\omega_1, \lambda_1, d_1, 0)$ exists and is a endomorphic embedding of the *Good* equilibrium found in subsection 1.3.1. It is worth mentioning that this equilibrium point corresponds to an economy without speculation, as one may expect.

Second, possible high level debt is captured through the change of variable $u = 1/d$, with which the system becomes

$$\begin{cases} \dot{\omega} &= \omega \left[\left(\frac{\eta}{1+\eta} \right) [\Phi(\lambda) - a_l] \right], \\ \dot{\lambda} &= \lambda \left[\kappa(\pi(u)) C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta - a_l - \beta - \frac{1}{(1-\omega)} \left(\frac{1}{1+\eta} \right) [\Phi(\lambda) - a_l] \right], \\ \dot{u} &= -u \left[r - \kappa(\pi(u)) C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \delta + \frac{\omega}{(1-\omega)} \left(\frac{1}{1+\eta} \right) [\Phi(\lambda) - a_l] - u^2 [\kappa(\pi(u)) - (1-\omega) + s] \right], \\ \dot{s} &= s \left[\Psi \left(\kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta \right) - \kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \delta \right]. \end{cases} \quad (1.10)$$

we can observe that $(\bar{\omega}_2, \bar{\lambda}_2, \bar{u}_2, \bar{s}_2) = (0, 0, 0, 0)$ is an equilibrium of System (1.10). The latter is equivalent to the previous *Bad* equilibrium as it corresponds to collapsing wages and employment, and exploding debt, and the absence of speculation.

¹⁹Note that in [Grasselli and Costa Lima \(2012\)](#), the growth rate of output equals that of capital, as ν is constant. We chose to align the growth in speculation with the growth rate of capital, as its objects is precisely existing assets. However, aligning the speculation with output would have been equally compatible with [Grasselli and Costa Lima \(2012\)](#).

Third, we study the former system's counterpart to study for high level of speculation. That implies the change of variable $x = 1/s$, so that

$$\begin{cases} \dot{\omega} &= \omega \left[\left(\frac{\eta}{1+\eta} \right) [\Phi(\lambda) - a_l] \right], \\ \dot{\lambda} &= \lambda \left[\kappa(\pi(u))C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta - a_l - \beta - \frac{1}{(1-\omega)} \left(\frac{1}{1+\eta} \right) [\Phi(\lambda) - a_l] \right], \\ \dot{u} &= -u \left[r - \kappa(\pi(u))C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \delta + \frac{\omega}{(1-\omega)} \left(\frac{1}{1+\eta} \right) [\Phi(\lambda) - a_l] \right] - u^2 (\kappa(\pi(u)) - (1-\omega) + 1/x), \\ \dot{x} &= x \left[-\Psi \left(\kappa(\pi(u))C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta \right) + \kappa(\pi(u))C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta \right]. \end{cases} \quad (1.11)$$

It is not clear that $(0, 0, 0, 0)$ could be an equilibrium of System (1.11) because of u^2/x , we therefore introduce the ratio $v = \frac{s}{d} = \frac{u}{x}$, leading to the system

$$\begin{cases} \dot{\omega} &= \omega \left[\left(\frac{\eta}{1+\eta} \right) [\Phi(\lambda) - a_l] \right] \\ \dot{\lambda} &= \lambda \left[\kappa(\pi(v, x))C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta - a_l - \beta - \frac{1}{(1-\omega)} \left(\frac{1}{1+\eta} \right) [\Phi(\lambda) - a_l] \right] \\ \dot{v} &= -v \left[r - \Psi \left(\kappa(\pi(v, x))C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta \right) + \frac{\omega}{(1-\omega)} \left(\frac{1}{1+\eta} \right) [\Phi(\lambda) - a_l] \right] - v^2 (x\kappa(\pi(v, x)) - x(1-\omega) + 1) \\ \dot{x} &= x \left[-\Psi \left(\kappa(\pi(v, x))C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta \right) + \kappa(\pi(v, x))C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta \right], \end{cases}$$

with $\pi(v, x) = 1 - \omega - r/vx$. We observe that, under the same condition of [Grasselli and Costa Lima \(2012\)](#), both $(0, 0, 0, 0)$ and $(0, 0, \Psi \left(\kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} - \delta \right) - r, 0)$ are equilibria of System (1.12). In terms of the original system, the equilibrium $(0, 0, 0, 0)$ of system (1.12), is the point

$$(\bar{\omega}_3, \bar{\lambda}_3, \bar{d}_3, \bar{s}_3) = (0, 0, +\infty, \pm\infty)$$

where wages and employment are collapsing while debt and speculation become infinitely large with debt that is growing faster than the speculation since in the case $v \rightarrow 0^+$ or at the same pace for $v \rightarrow \Psi \left(\kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} - \delta \right) - r$.

1.5.2 Modified local stability

We first derive the Jacobian of System (1.9):

$$J(\omega, \lambda, d, p) = \begin{pmatrix} 0 \\ J(\omega, \lambda, d) \\ 1 \\ \zeta_1 & 0 & \zeta_2 & \zeta_3 \end{pmatrix},$$

with

$$\begin{aligned}\zeta_1 &= -p \left(\Psi' \left(\kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta \right) - 1 \right) \times \\ &\quad \left[\kappa'(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \frac{1}{\eta} \kappa(\pi) C \left(\frac{1}{b} \right)^{1/\eta} (1-\omega)^{1/\eta-1} \right], \\ \zeta_2 &= -p \left(\Psi' \left(\kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta \right) - 1 \right) \kappa'(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta}, \\ \zeta_3 &= \Psi \left(\kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta \right) - \kappa(\pi) C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \delta.\end{aligned}$$

Let suppose that $p_3(y)$ is the characteristic polynomial of the Jacobian $J(\omega_1, \lambda_1, d_1)$ at the *Good* equilibrium point. Therefore, the characteristic polynomial of the $J(\omega, \lambda, d, p)$ at the point $(\omega_1, \lambda_1, d_1, 0)$ is

$$p_4(y) = p_3(y)(\zeta_3 - y).$$

We previously numerically showed that the three-dimensional model under consideration has roots of p_3 that are negative. Assuming that the previous *Good* equilibrium's growth rate of the economy holds, a necessary and sufficient condition for $(\omega_1, \lambda_1, d_1, 0)$ to be asymptotically locally stable is equivalent to

$$\Psi(a_l + \beta) < a_l + \beta.$$

This founding is similar to Grasselli and Costa-Lima: the speculation can destabilize the *Good* equilibrium as its growth is greater than the growth of the economy.

Furthermore, when evaluating the Jacobian matrix for the modified System (1.10) at the equilibrium point $(0, 0, 0, 0)$, one finds

$$J(0, 0, 0, 0) = \begin{pmatrix} \iota_1 & 0 & 0 & 0 \\ 0 & \iota_2 & 0 & 0 \\ 0 & 0 & \iota_3 & 0 \\ 0 & 0 & 0 & \iota_4 \end{pmatrix},$$

with

$$\begin{aligned}\iota_1 &= \frac{\eta}{1+\eta} (\phi(0) - a_l), \\ \iota_2 &= \kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} - (a_l + \beta + \delta) - \frac{1}{\eta+1} (\phi(0) - a_l), \\ \iota_3 &= \kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} - (r + \delta), \\ \iota_4 &= \Psi \left(\kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} - \delta \right) - \kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} + \delta.\end{aligned}$$

Denoting the growth rate of the economy at infinite debt by

$$g_0 = \kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} - \delta,$$

we note that a condition for this equilibrium $(\bar{\omega}_2, \bar{\lambda}_2, \bar{u}_2, \bar{p}_2)$ to be stable is

$$\Psi(g_0) < g_0.$$

In other words, the speculation can have a destabilizing property on the *Bad* equilibrium whenever the speculative component decreases faster than the growth rate of the economy. Indeed, this mechanism would allow the debt-to-output ratio to stay at finite levels.

Finally, the Jacobian matrix of System (1.12) at the point $(0, 0, 0, 0)$ reduces to

$$J(0, 0, 0, 0) = \begin{pmatrix} \iota_1 & 0 & 0 & 0 \\ 0 & \iota_2 & 0 & 0 \\ 0 & 0 & \Psi(g_0) - r & 0 \\ 0 & 0 & 0 & -\Psi(g_0) + g_0 \end{pmatrix},$$

whereas at $(0, 0, \Psi \left(\kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} - \delta \right) - r, 0)$, with the additional assumption,

$$\lim_{x \rightarrow +\infty} x \kappa'(-x) = 0,$$

we have

$$J(0, 0, \Psi \left(\kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} - \delta \right) - r, 0) = \begin{pmatrix} \iota_1 & 0 & 0 & 0 \\ 0 & \iota_2 & 0 & 0 \\ 0 & 0 & -\Psi(g_0) + r & -(\Psi(g_0) - r)^2 (\kappa_0 - 1) \\ \gamma_1 & 0 & 0 & -\Psi(g_0) + g_0 \end{pmatrix},$$

with

$$\gamma_1 = \left(\Psi \left(\kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} - \delta \right) - r \right) \left[\Psi' \left(\kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} - \delta \right) \left[-\frac{1}{\eta} \left[\kappa_0 C \left(\frac{1}{b} \right)^{1/\eta} \right] - \frac{1}{1 + \eta} (\phi(0) - a_l) \right] \right].$$

Therefore, provided that $g_0 < \Psi(g_0)$, and given that we assume $\iota_1 < 0$ and $\iota_2 < 0$, the equilibrium $(\bar{\omega}_3, \bar{\lambda}_3, \bar{d}_3, \bar{p}_3) = (0, 0, +\infty, \pm\infty)$ is stable. As in Grasselli and Costa-Lima's paper, this equilibrium remains stable either $\Psi(g_0) < r$ or $\Psi(g_0) > r$, the only difference would be that the ratio, p/d , will be attracted by 0 instead of $\Psi(g_0) - r$. Either way, $d \rightarrow +\infty$ and $p \rightarrow +\infty$ are guaranteed since v remains positive.

1.5.3 Numerical study of the basins of attraction

This section reproduces the methodology in Section 1.4 for System (1.9).

The basins of attraction

Using the same calibration as in Section 1.4, we compare the results in terms of basins of attraction. For compatibility with the previous exercise, we select four initial points of the speculation-to-output ratio, s , and discuss the geometrical deformation.

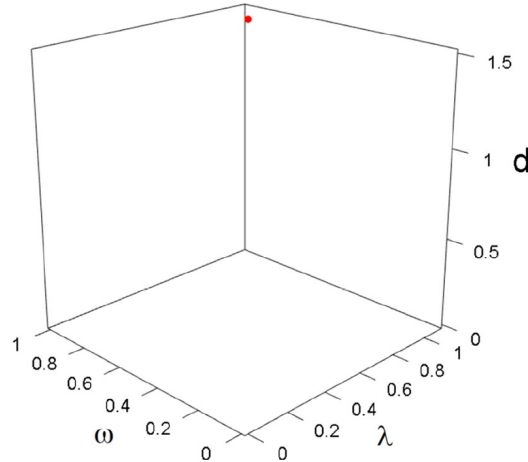


Figure 1.12: Projection of the basin of attraction of the *Good* equilibrium for the Leontief production technology on the hyperplane $(\omega, \lambda, d, s = 0.1)$. Projections on other values for s give similar results: no visible convex set of points in the *good* attractor given the distance between points of the grid. The *Good* equilibrium point is $(\omega, \lambda, d, s) = (0.865, 0.972, 1.50, 0)$.

1.5.4 Sensitivity analysis

The sensitivity analysis follows the same methodology as in the previous section. We used a quadri-dimensional grid of 320 for $(\omega, \lambda, d, s) \in [0.65; 0.95] \times [0.7; 1] \times [0; 3] \times [0.01; 0.1]$.

Table 1.4: Parameter sensitivity of basins of attraction: percentage of points from the simulation grid falling into the basin of attraction of the *Good* equilibrium. Values are reported in percentages.

parameters	benchmark: $(a_l, r, \delta, \beta) = (2, 4, 1, 1)$	a_l		r		δ		β	
		$\bar{1}$	\dagger_3	$\bar{1}$	\dagger_5	$\bar{0.5}$	\dagger_2	$\bar{0}$	\dagger_3
$\eta = 0.5$	42	41	26	25	43	41	43	42	6
$\eta = 100$	11	8	0	10	10	11	7	8	0
$\eta = 10^{15}$	2	2	0	0	0	3	1	1	0

Table 1.4 is the speculative augmented model counterpart of Table 1.3. Similarly, the higher the η is, the bigger the basin of attraction is. However, each entry of Table 1.4 is strictly lower than its counterpart. This shows the destabilizing effect of speculation in

the economy. Interestingly, the numerical simulations show different reaction of a change of parameter. Indeed, almost all the changes lead to an opposite response compare to Table 1.3, to the exception of an increase of the interest rate, r , in the Leontief case. A higher a_l , β , or δ decreases the volume of the basin of attraction. For the case of δ , the result is not surprising. Indeed, an increase of δ will have a direct impact on the economic growth driven by capital accumulation. Therefore, a shrinking δ in a speculative economy implies less economic outcomes for potentially high debt repayments. For the case of a_l , one can see from the wage share equation of System 1.9 that an economy with a high a_l (comparatively to an economy with a low one) will steer wages down and, thus, boost profits. This is likely to lead the economy towards a highly speculative one and therefore potentially unstable. For the case of β , the channel would be slight different from the one of a_l as the direct impact of the increase in the population growth rate is through the employment rate, λ . As a result, the lowered employment rate likely decrease the wage share and lead to the same narrative than for the previous case.

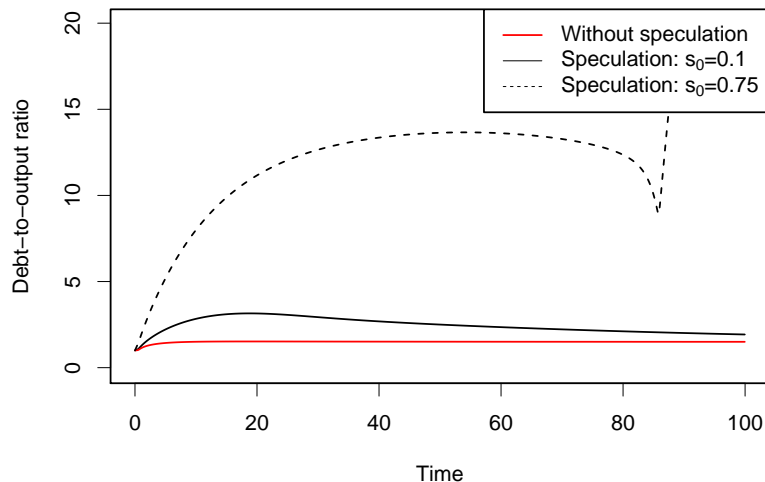


Figure 1.13: Trajectories of debt for different values of initial speculation and for $(\omega_0, \lambda_0, d_0) = (0.85, 0.85, 1)$.

For the sake of illustration, Figure 1.13 compares the trajectories of the model with and without speculation. As the initial condition of the speculation goes up, the debt-to-output rate shows higher levels that potentially lead to unsustainable trajectories.

1.6 Conclusions and Suggestions for Further Work

This paper presented multiple insights on how the class of CES production functions may significantly alter the dynamical landscape of Keen’s model. The moderating effect induced by the flexibility of the use of factors of production on the dynamics of the system is a primary rationale of this paper’s model. Substitution allows to mitigate the influence of wages on employment rate (and vice versa), so that both adjust more rapidly towards their equilibrium values. This result holds for speculative and nonspeculative economies.

Interestingly, numerical simulations showed that van der Ploeg's extension plugged into Keen's model increases the set of points that converges towards the *Good* equilibrium. In other words, given the same behavioral conditions, the volume of the basin of attraction of the economically desirable equilibrium is greater for economies where some substitution can take place. Therefore, an intermediate conclusion would be that substitution between factors of production leads to the convergence of any reasonable initial condition towards the *Good* equilibrium. Provided that substitution is strong enough (*i.e.* $\eta < 0$), it even ensures that the *Bad* equilibrium is no longer stable. This conclusion is reinforced in the presence of speculative schemes.

Furthermore, using numerical techniques, we showed that when substitution is allowed, the change in the volume of the basin of attraction is globally less sensitive to a change of parameters. Consequently, a model with CES function would be more suitable as a model for estimation. Indeed, when using statistical methods, the exact inference of a parameter is strongly unlikely. Thus, an economy endowed with CES production shows more robust properties to cope with numerical errors of the type inherent to the statistical analysis or flaws in the data. Additionally, realistic calibration of the model showed monotonic convergence rather than dampened cycles. Since the emergence of business cycles is an empirical fact (at least for some advanced economies), this inability to replicate business cycles under some calibration either suggests that the Keen model relies on complementary factors or that the phenomenological functions are not well calibrated.

This paper contributed to a growing body of work that has developed theoretical models based on the Goodwin-Lotka-Volterra model. The methodology used in this paper may be seen as a starting point for further analysis. We have tested the sensitivity of the model to key parameters; another extension would be to test the robustness of the results with respect to changes in the behavioral functions. These changes would presumably affect the vector field of the system more substantially. Another natural extension might include testing the properties of CES production function with three inputs (for instance by adding energy as in [Acurio Vasconez \(2015\)](#)), two of which can be substituted with each other, while the third remains complementary. This would shed light on possibilities for the recovery of endogenous cycles in the model.

A different sort of application would involve dropping Say's law and modeling the behavior of the demand side. This would require either the introduction of the utilization rate of capital or an allowance for inventories. Such an application has been made by [Grasselli and Nguyen-Huu \(2016\)](#), who showed that the economy does not converge towards an equilibrium and exhibit limit cycles. This extension may be a way to reconcile the CES technology with the endogenous cycle theory within the Goodwin framework. Furthermore, another way to capture cycles in case of high sustainability between factors would be to introduce money with a Taylor rule. This extension may be able to cope with monetary cycles that in turn could influence the real economy through that pattern.

In sum, we have seen that the basin of attraction changes substantially when substitution is allowed. Here, we simply compared various models according to the volume of

the basin of attraction associated with each equilibrium. An alternative viewpoint would consist of measuring the “strength” with which an equilibrium is attracting the economy. This could be achieved, e.g., by relying on the Freidlin–Wentzell theory (see [Freidlin and Wentzell \(1998\)](#)). Converting the model into stochastic differential equations by adding a Brownian motion would allow for the study of the probability that a given sample path will remain far from an equilibrium. In other words, we would be able to compute the probability that the system shifts from the basin of attraction of the *Good* equilibrium to the *Bad* equilibrium. An expected result would be that the probability of straying out of the *Good* equilibrium is lower when substitution is higher. The stochastic model would also—through Malliavin calculus—allow for a sensitivity test and would help to develop a sturdy understanding of the sensitivity of the model to a change of parameter. This research may strengthen the conclusion of the paper concerning the capability of CES technology to cope with a change of parameter.

Two final extensions are worth mentioning. First, one might study the structural stability of the dynamical system of each model (CES versus Leontief) in order to test whether the qualitative behavior of the trajectories is affected by C^1 -small perturbations. Second, one might analyze possible Hopf bifurcation after introducing lags in the Phillips curve (or in the capital accumulation equation). This change transforms the system from ordinary differential equations to delay differential equations.

Appendices for Chapter 1

1.A Getting the reduced form of the system

We assume that the productive sector is endowed with CES technology so that

$$Y = C \left[bK^{-\eta} + (1-b)(e^{a_t}L)^{-\eta} \right]^{-\frac{1}{\eta}}. \quad (1.13)$$

Additionally, we make the assumption that wages are set at marginal rate of productivity, so that:

$$\frac{\partial Y}{\partial L} = w.$$

For simplicity, we define $L^e := e^{a_t}L$, so that the following relationship holds

$$\frac{\partial Y}{\partial L^e} = \frac{\partial Y}{\partial L} e^{-a_t}. \quad (1.14)$$

By using equations (1.13) and (1.14)

$$\frac{\partial Y}{\partial L^e} = \frac{(1-b)}{C^\eta} \left(\frac{Y}{L^e} \right)^{1+\eta}.$$

By taking the derivative of (1.13) and using (1.14)

$$\begin{aligned} \left(\frac{\omega}{1-b} \right)^{\frac{1}{\eta}} C &= \frac{Y}{L^e} \\ \Leftrightarrow \left(\frac{\omega}{1-b} \right)^{\frac{1}{\eta}} C e^{a_t} &= \frac{Y}{L} \end{aligned} \quad (1.15)$$

with ω , the share of total real wages ($W := wL$) in the production:

$$\omega := \frac{wL}{Y}.$$

Let $a := Y/L$ be the labor productivity, one has $\omega = w/a$. The growth rate of the wage share is given by

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{w}}{w} - \frac{\dot{a}}{a}.$$

Using equation (1.15), one gets the following growth rate for labor productivity

$$\frac{\dot{a}}{a} = \frac{1}{\eta} \frac{\dot{\omega}}{\omega} + a_t.$$

Suppose that the growth rate of wages is given by a short-term Phillips Curve

$$\frac{\dot{w}}{w} = \Phi(\lambda).$$

The dynamic of the wage share is given by

$$\frac{\dot{\omega}}{\omega} = \left(\frac{\eta}{1+\eta} \right) [\Phi(\lambda) - a_l].$$

The population grows according to

$$\frac{\dot{N}}{N} = \beta \geq 0.$$

The employment rate is defined by $\lambda := \frac{L}{N}$, while the capital-output ratio is given by $\nu := \frac{K}{Y}$. Hence, the employment rate dynamic

$$\begin{aligned} \frac{\dot{\lambda}}{\lambda} &= \frac{\dot{L}}{L} - \frac{\dot{N}}{N} \\ &= \frac{\dot{K}}{K} - \frac{\dot{a}}{a} - \frac{\dot{\nu}}{\nu} - \frac{\dot{N}}{N}. \end{aligned}$$

The profit share in the production is given by

$$\pi := 1 - \omega - rd,$$

where r is the short-term interest rate set by the central bank, and paid by producers, while d is the ratio of real debt-to-production (i.e. $\frac{D}{Y}$). The capital accumulation is given by

$$\begin{aligned} \dot{K} &= \kappa(\pi)Y - \delta K, \\ \frac{\dot{K}}{K} &= \frac{\kappa(\pi)}{\nu} - \delta \end{aligned}$$

where δ is the depreciation rate of capital, $\kappa(\pi)$ is a function of the profit share, and ν is the time-varying capital-to-output ratio. From the expression of $\frac{\partial Y}{\partial K}$ and knowing that Y is homogeneous of degree one, we obtain

$$\nu = \left(\frac{1-\omega}{b} \right)^{-\frac{1}{\eta}} \frac{1}{C}.$$

Its growth rate is given by

$$\frac{\dot{\nu}}{\nu} = \frac{\dot{\omega}}{(1-\omega)\eta}.$$

Therefore, the growth rate of employment is

$$\frac{\dot{\lambda}}{\lambda} = \kappa(\pi)C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta - a_l - \beta - \frac{1}{\eta(1-\omega)} \frac{\dot{\omega}}{\omega}.$$

The debt dynamic is the difference between investment and the profits made by the corporate sector, in other words

$$\dot{D} = \kappa(\pi)Y - \pi Y.$$

The growth rate of production is given by

$$g := \frac{\dot{Y}}{Y} = \kappa(\pi)C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta - \frac{\dot{\omega}}{(1-\omega)\eta}.$$

Thus, the ratio of real debt on production is

$$\frac{\dot{d}}{d} = \frac{\dot{D}}{D} - \frac{\dot{Y}}{Y} = \frac{\kappa(\pi) - \pi}{d} - \kappa(\pi)C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \delta + \frac{\dot{\omega}}{(1-\omega)\eta}.$$

Hence, its dynamic is

$$\dot{d} = d \left\{ r - \kappa(\pi)C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \delta + \frac{\dot{\omega}}{(1-\omega)\eta} \right\} + \kappa(\pi) - (1-\omega).$$

To wrap up, and for the sake of clarity, the tree-dimensional system is

$$\begin{cases} \dot{\omega} &= \omega \left[\left(\frac{\eta}{1+\eta} \right) [\Phi(\lambda) - a_l] \right] \\ \dot{\lambda} &= \lambda \left[\kappa(\pi)C \left(\frac{1-\omega}{b} \right)^{1/\eta} - \delta - a_l - \beta - \frac{1}{\eta(1-\omega)} \frac{\dot{\omega}}{\omega} \right] \\ \dot{d} &= d \left[r - \kappa(\pi)C \left(\frac{1-\omega}{b} \right)^{1/\eta} + \delta + \frac{\dot{\omega}}{(1-\omega)\eta} \right] + \kappa(\pi) - (1-\omega) \end{cases}$$

1.B Parameter values

The calibration is almost entirely borrowed from [Keen \(2013\)](#) to the exception of the speculation function that is borrowed from [Grasselli and Costa Lima \(2012\)](#). The time frequency between t and $t + 1$ is considered to be one year. The same generalized expo-

Var./ Param.	Description	Value or inputs
a_l	Rate of the labor productivity growth	2%
β	Rate of population growth	1%
δ	Depreciation rate of capital	1%
$\Phi(\cdot)$	Nonlinear Phillips curve's params.	$\Phi(\lambda, -0.01, 2.35 * 10^{-23}, 50)$
$\kappa(\cdot)$	Investment function's params.	$\kappa(\pi, 0.05, 0.05, 1.75, 0)$
r	Interest rate paid by producers	4%
η	Control the elasticity of substitution	$\approx 0; 1; +\infty$
C	The factor productivity	1/3
b	K share in the production function	$1 - 0.865 = 0.135$
$\Psi(x)$	The speculation function	$\Psi(x) = -0.25 + 0.25e^{-0.36+12x}$

Table 1.5: Calibration for the numerical estimations.

nential function is used for both the relationship between investment as a share of output, and the short term Phillips Curve

$$\Phi(\lambda, \phi_0, \phi_1, \phi_2) = \Phi(\lambda) = \phi_0 + \phi_1 * e^{\phi_2 * \lambda}.$$

Figure 1.15 displays the behaviors of the phenomenological functions using the calibration given by Table 1.5.

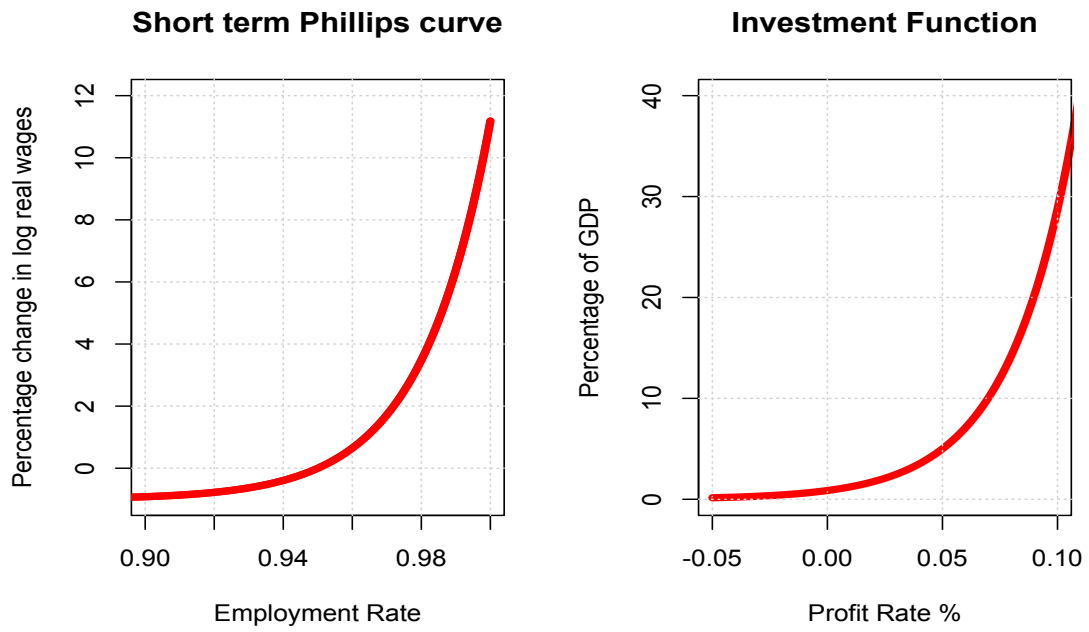


Figure 1.14: Phenomenological functions behaviors according to the parameters in Table 1.5.

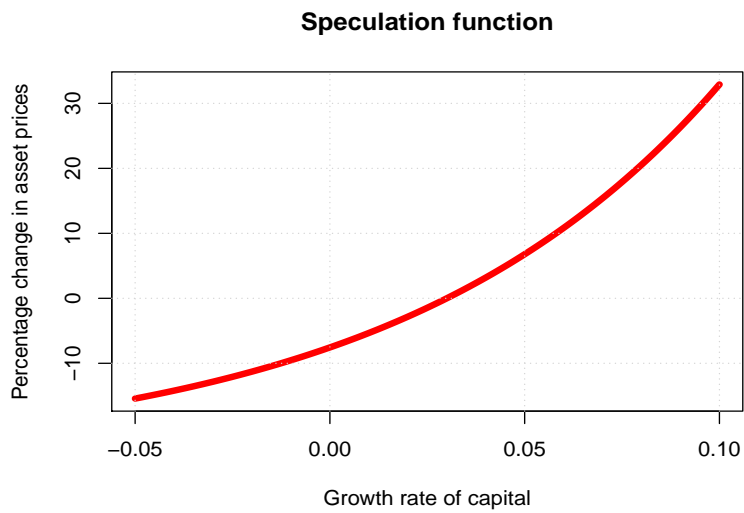


Figure 1.15: Phenomenological functions behaviors according to the parameters in Table 1.5.

1.C Numerical Results for the Stability of Equilibrium

Value of η	Eigen. 1.	Eigen. 2.	Eigen. 3.
10^{15}	-0.022+1.33i	-0.022-1.33i	-0.030
100	-0.075+1.33i	-0.075-1.33i	-0.030
0.1	-9.69	-0.162	-0.037
-1/2	-0.0498+0.023i	-0.0498-0.023i	-21.575

Table 1.6: The numerical eigenvalues of all the models at their *Good* equilibrium point $(\omega_1, \lambda_1, d_1)$.

Table 1.6 displays the numerical eigenvalues of the Jacobian at the *Good* equilibrium point, where $(\omega_1, \lambda_1, d_1)$ are all finite. Remember, to be locally stable, the Jacobian matrix at this equilibrium point has to be negative definite. This would mean that the eigenvalues are all non-positive. In this exercise, they all show local stability at the *Good* equilibrium value.²⁰ For the sake of completeness, Tables 1.7 and 1.8 show respectively the eigenvalues

Value of η	Eigen. 1.	Eigen. 2.	Eigen. 3.
10^{15}	6.8e+10	-6.88+10	-0.03
100	0.05	-0.04	-0.03
0.1	0.074	-0.012	-0.003
-1/2	0.03	0.02	1.69e-08

Table 1.7: The numerical eigenvalues of all the models at their *Trivial* equilibrium point $(\bar{\omega}_3, \bar{\lambda}_3, \bar{d}_3)$.

Value of η	Eigen. 1.	Eigen. 2.	Eigen. 3.
10^{15}	0.05	-0.04	-0.03
100	0.05	-0.04	-0.03
0.1	1.27	-0.52	-0.002
-1/2	-0.88	0.021	-0.0016

Table 1.8: The numerical eigenvalues of all the models at the *slavery* equilibrium point $(\bar{\omega}_2, \bar{\lambda}_2, \bar{d}_2)$.

of the *Trivial* and the *Slavery* equilibria. We observe that, as expected, non of these trials display a local stability for that equilibrium point. The eigenvalues of the *Bad* equilibrium are shown in Table 1.9 and confirm that, whenever $\eta \in]-1; 0[$, the only stable equilibrium that may be asymptotically globally stable is the *good*.

²⁰In the simulation, the value 0 would lead to numerical errors, therefore we choose the value 0.1 as the lowest value that does not show numerical error. Similarly, $+\infty$ has been approached by 10^{15} to show the behavior of the the model near the Leontief case.

Value of η	Eigen. 1.	Eigen. 2.	Eigen. 3.
10^{15}	-0.03	-0.04	-0.05
100	-0.03	-0.04	-0.05
0.1	-0.003	-0.01	-0.05
$-1/2$	0.03	0.02	-0.05

Table 1.9: The numerical eigenvalues of all the models at their *Bad* equilibrium point $(\omega_3, \lambda_3, d_3)$.

1.D Equilibria for an Affine Investment Function

For the sake of completeness, what follows derives theoretically the equilibrium points under the assumption of an affine investment function.

On the one hand, one can easily find the case $\omega_1 \neq 0$, one has

$$\bar{\lambda}_1 = \Phi^{-1}(a_l).$$

For what follows, we will consider that $\kappa(\cdot)$ is an affine function, i.e $\kappa(x) = K_0 + K_1x$. By using the third equation of the system (1.2), one has

$$\begin{aligned} -\bar{d}_1(\beta + a_l) + \kappa(\pi) - \pi &= 0 \\ \Leftrightarrow -K_0 - K_1\pi + \pi + \bar{d}_1(\beta + a_l) &= 0 \\ \Leftrightarrow (1 - K_1)\pi - K_0 + \bar{d}_1(\beta + a_l) &= 0 \end{aligned}$$

where $\pi := 1 - \bar{\omega}_1 - r\bar{d}_1$ and K_0 and K_1 are constants. Thus,

$$\begin{aligned} (1 - \bar{\omega}_1)(1 - K_1) + \bar{d}_1(\beta + a_l - r(1 - K_1)) - K_0 &= 0 \\ \frac{K_0 + (1 - \bar{\omega}_1)(K_1 - 1)}{\beta + a_l + r(K_1 - 1)} &= \bar{d}_1. \end{aligned}$$

By adding the last equality of the second equation of the system (1.2), one has that $\bar{\omega}_1$ is such that satisfies with

$$\begin{aligned} (\delta + a_l + \beta) b^{1/\eta} [\beta + a_l + r(K_1 - 1)] \\ = [K_0(\beta + a_l - r) + K_1(1 - \bar{\omega}_1)(\beta + a_l)] C(1 - \bar{\omega}_1)^{1/\eta} \end{aligned}$$

Consider another non-trivial equilibrium taking the form of

$$(\bar{\omega}_2 = 0, \bar{\lambda}_2 \neq 0, \bar{d}_2 \neq 0)$$

After plugging the first equation of system (1.2) into the second and third equations, the

evaluation of the mentioned equilibrium into the second equation of (1.2), yields

$$\kappa(1-rd)C\left(\frac{1}{b}\right)^{1/\eta} - \delta - a_l - \beta - \left(\frac{1}{1+\eta}\right) [\Phi(\lambda) - a_l] = 0$$

Likewise, from the third equation in (1.2) it follows that:

$$d\left[r - \kappa(1-rd)C\left(\frac{1}{b}\right)^{1/\eta} + \delta\right] + \kappa(1-rd) - 1 = 0$$

Solving the of these equations for $\kappa(1-rd)C\left(\frac{1}{b}\right)^{1/\eta}$ and replacing it in the second equation, produces

$$d\left[r - a_l - \beta - \left(\frac{1}{1+\eta}\right) [\Phi(\lambda) - a_l]\right] + \kappa(1-rd) - 1 = 0$$

In the same line as before, consider the case where $\kappa(x) = K_0 + K_1x$, for which the expression above would become

$$\begin{aligned} & d\left[r - a_l - \beta - \left(\frac{1}{1+\eta}\right) [\Phi(\lambda) - a_l] - rK_1\right] + K_0 + K_1 - 1 = 0 \\ \iff \bar{d}_2 &= \frac{1 - K_0 - K_1}{r(1 - K_1) - a_l - \beta - \left(\frac{1}{1+\eta}\right) [\Phi(\lambda) - a_l]} \end{aligned}$$

It can be seen by plugging this into the second equation of system (1.2), that the equilibrium employment will be any $\bar{\lambda}_2$ satisfying:

$$\frac{rK_0 + (K_0 + K_1)\xi}{r(1 - K_1) + \xi} \frac{C}{b^{1/\eta}} - \delta + \xi = 0$$

with ξ being itself an expression in terms of $\bar{\lambda}_2$

$$\xi := -a_l - \beta - \left(\frac{1}{1+\eta}\right) [\Phi(\bar{\lambda}_2) - a_l].$$

1.E Existence of the *Slavery* equilibrium

Let us take the last two equations of the main system when $\omega \rightarrow 0$.

$$\dot{\lambda} \simeq \lambda \left[\kappa(1-rd)Cb^{-\frac{1}{\eta}} - \frac{\Phi(\lambda) - a_l}{1+\eta} - (\delta + a_l + \beta) \right] \quad (1.16)$$

$$\dot{d} \simeq d \left(r - \kappa(1-rd)Cb^{-\frac{1}{\eta}} + \delta + \omega \frac{\Phi(\lambda) - a_l}{1+\eta} \right) + \kappa(1-rd) - 1 \quad (1.17)$$

At the equilibrium, whenever $\lambda > 0$, one finds, by injecting 1.16 into 1.17:

$$0 \simeq d \left(r - \kappa(1 - rd) C b^{-\frac{1}{\eta}} + \delta + \omega \left(\kappa(1 - rd) C b^{-\frac{1}{\eta}} - \delta - a_l - \beta \right) \right) + \kappa(1 - rd) - 1$$

As κ is bounded, one can neglect the term in ω . Defining $s = C b^{-\frac{1}{\eta}}$, $e = r + \delta > 0$ and $z(d) = -\kappa(1 - rd) \in [-1; 0]$ one thus obtains

$$d(sz(d) + e) = 1 + z(d) \in [0; 1]$$

If $s < e$, the left hand side is a continuous and non-negative function of d which passes through the origin and is equivalent to $d(e - s\kappa_0)$ at $+\infty$. Hence, the equation has a solution and $s < e$ is a sufficient condition for the existence of a *Slavery* equilibrium.

For $\eta < 0$, s converges decreasingly towards 0 when η tends to 0, so there exists an interval $] -\eta_{min}; 0[$ within which the existence of the equilibrium is insured.

Note that for $\eta > 0$, as $b < 1$, s decreases with η and converges towards C , so that if there is a substitutability η_0 such that $s_{\eta_0} < e = r + \delta$, the existence of the *Slavery* equilibrium is insured below this substitutability (for $\eta > \eta_0$). That being said, for the benchmark specification, this inequality does not hold. One can then derive a less restrictive sufficient condition: $e - s\kappa_0 > 0$. Finally, if $\kappa_0 < b^{\frac{1}{\eta}(r+\delta)}/C$, a *Slavery* equilibrium exists (with an equilibrium value for the debt potentially very high). For our benchmark specification with $\eta = 1$ (resp. $\eta = \infty$, resp. $\eta = -0.5$), it suffices that $\kappa_0 < 0.02$ (resp. $\kappa_0 < 0.15$, resp. $\kappa_0 < 8.2$) for the equilibrium to exist.

1.F Getting the reduced form of the system with Cobb-Douglas production function

We assume that the productive sector is endowed with a constant return to scale Cobb-Douglas technology so that

$$Y = C \left[K^b (e^{a_l t} L)^{(1-b)} \right]. \quad (1.18)$$

Additionally, we make, as previously, the assumption that wages are set at marginal rate of productivity, so that:

$$\frac{\partial Y}{\partial L} = w.$$

For simplicity, we define $L^e := e^{a_l t} L$, so that the following relationship holds

$$\frac{\partial Y}{\partial L^e} = \frac{\partial Y}{\partial L} e^{-a_l t}. \quad (1.19)$$

By using equations (1.18) and (1.19)

$$\frac{\partial Y}{\partial L^e} = (1 - b) \left(\frac{Y}{L^e} \right).$$

By taking the derivative of (1.13) and using (1.14)

$$\omega = (1 - b) \tag{1.20}$$

with ω , the share of total real wages ($W := wL$) in the production:

$$\omega := \frac{wL}{Y}.$$

Let $a := Y/L$ be the labor productivity, one has $\omega = w/a$. The growth rate of the wage share is given by

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{w}}{w} - \frac{\dot{a}}{a}.$$

Using equation (1.20), one gets the following growth rate for labor productivity

$$\frac{\dot{a}}{a} = \frac{\dot{w}}{w}.$$

Suppose that the growth rate of wages is given by a short-term Phillips Curve

$$\frac{\dot{w}}{w} = \Phi(\lambda).$$

The dynamic of the wage share is given by

$$\boxed{\frac{\dot{\omega}}{\omega} = 0}.$$

The population grows according to

$$\frac{\dot{N}}{N} = \beta \geq 0.$$

The employment rate is defined by $\lambda := \frac{L}{N}$, while the capital-output ratio is given by $\nu := \frac{K}{Y}$. Hence, the employment rate dynamic

$$\begin{aligned} \frac{\dot{\lambda}}{\lambda} &= \frac{\dot{L}}{L} - \frac{\dot{N}}{N} \\ &= \frac{K}{K} - \frac{\dot{a}}{a} - \frac{\dot{\nu}}{\nu} - \frac{\dot{N}}{N}. \end{aligned}$$

The profit share in the production is given by

$$\begin{aligned} \pi &:= 1 - \omega - rd, \\ &= b - rd \end{aligned}$$

where r is the short-term interest rate set by the central bank, and paid by producers, while d is the ratio of real debt-to-production (i.e. $\frac{D}{Y}$). The capital accumulation is given by

$$\begin{aligned}\dot{K} &= \kappa(\pi)Y - \delta K, \\ \frac{\dot{K}}{K} &= \frac{\kappa(\pi)}{\nu} - \delta\end{aligned}$$

where δ is the depreciation rate of capital, $\kappa(\pi)$ is a function of the profit share, and ν is the time-varying capital-to-output ratio. By dividing equation 1.18, by the output, Y , we are able to obtain

$$\nu = \left(\frac{1}{C}\right)^{1/b} \left(\frac{a}{e^{a_l t}}\right)^{\frac{1-b}{b}}.$$

Its growth rate is given by

$$\frac{\dot{\nu}}{\nu} = \frac{1-b}{b} (\Phi(\lambda) - a_l).$$

Therefore, the growth rate of employment is

$$\boxed{\frac{\dot{\lambda}}{\lambda} = \frac{\kappa(\pi)}{\nu} - \delta - \Phi(\lambda) - \beta - \frac{1-b}{b} (\Phi(\lambda) - a_l)}.$$

The debt dynamic is the difference between investment and the profits made by the corporate sector, in other words

$$\dot{D} = \kappa(\pi)Y - (\pi)Y.$$

The growth rate of production is given by

$$g := \frac{\dot{Y}}{Y} = \frac{\kappa(\pi)}{\nu} - \delta - \frac{1-b}{b} (\Phi(\lambda) - a_l).$$

Thus, the ratio of real debt on production is

$$\frac{\dot{d}}{d} = \frac{\dot{D}}{D} - \frac{\dot{Y}}{Y} = \frac{\kappa(\pi) - \pi}{d} - \frac{\kappa(\pi)}{\nu} + \delta + \frac{1-b}{b} (\Phi(\lambda) - a_l).$$

Hence, its dynamic is

$$\boxed{\dot{d} = d \left\{ r - \frac{\kappa(\pi)}{\nu} + \delta + \frac{1-b}{b} (\Phi(\lambda) - a_l) \right\} + \kappa(\pi) - b}.$$

To wrap up, and for the sake of clarity, the three-dimensional system is

$$\begin{cases} \dot{\nu} &= \nu \left(\frac{1-b}{b} [\Phi(\lambda) - a_l] \right) \\ \dot{\lambda} &= \lambda \left[\frac{\kappa(\pi)}{\nu} - \delta - \Phi(\lambda) - \beta - \frac{1-b}{b} (\Phi(\lambda) - a_l) \right] \\ \dot{d} &= d \left\{ r - \frac{\kappa(\pi)}{\nu} + \delta + \frac{1-b}{b} (\Phi(\lambda) - a_l) \right\} + \kappa(\pi) - b \end{cases}$$

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Chapter 2

Reaching Brazil's Nationally Determined Contributions: An Assessment of the Key Transitions in Final Demand and Employment

2.1 Introduction

Given the increasing awareness about climate change and the growing concern about the potential negative consequences of a temperature increase beyond $+1.5^{\circ}\text{C}$ above pre-industrial levels [Masson-Delmotte et al. \(2014\)](#), countries around the globe adopted a historic international climate agreement in Paris in December 2015. In anticipation, countries published their *Intended Nationally Determined Contributions* (hereafter INDCs)–converted into *Nationally Determined Contributions* (NDC)–that outlined the goals in term of climate actions they intended to take post-2020. However, most of the NDCs remain intentionally vague regarding the short term actions that would be taken up by governments towards achieving any political milestones. For example, Brazil does not commit to achieving any objective before 2025,¹ [Fed. Rep. of Brazil \(2016\)](#). This allows time for local governments to adjust their policies to be in line with a $+2^{\circ}\text{C}$ decarbonization pathway. In this paper, we will investigate policy options that would allow Brazil to meet its NDC targets with a focus on their impacts in terms of employment.

It is now generally accepted that shaping a low-carbon economy at a large scale, as urged by [Masson-Delmotte et al. \(2014\)](#), requires decoupling between economic activity and fossil fuel combustion, see [Kaya \(1997\)](#), by transforming the energy sector based on fossil fuel into renewable energies, among other possibilities. However, as noted by [Chen et al. \(2013\)](#), unlike most developed countries, Brazil’s CO_2 emissions in 2005 was predominantly the result of land-use change, and especially stemmed from converting forests to crop and pasture lands.

If deforestation was greatly reduced, the burden of cutting CO_2 emissions from energy use and industrial processes would be minimal. [Chen et al. \(2013\)](#)

Prior to the Paris Agreement and under the scope of the Law 12.187 passed by the Brazilian government in December 2009, [Chen et al. \(2013\)](#) argued in 2013 that if recent trends continued, Brazil’s energy sector CO_2 emissions would triple by 2040. However, if the deforestation policy were to be implemented as planned, through 2040, there would be no need to cut CO_2 emissions from energy use and industrial processes until 2035. On the other hand, recent data (August 2017 and July 2018) show that (illegal) deforestation is increasing [INPE \(2018\)](#)–although the levels are below those recorded in the early 2000s–, casting doubts on Brazil’s potential to meet international agreement goals without cutting CO_2 emissions from energy use and industrial processes.

Several studies have explored the impact of implementing climate policies–mainly taxes–on energy use and industrial processes, CO_2 emissions, GDP and employment by using macroeconomic Computable General Equilibrium (CGE) or partial equilibrium models (see e.g. [Lefèvre et al. \(2018\)](#); [La Rovere et al. \(2014\)](#) or [MACHADO-FILHO \(2009\)](#); [De Gouvello et al. \(2010\)](#)). However, as employment is not the primary focus of these

¹Brazil intends to commit to reducing its greenhouse gas emissions by 37% (resp. 43%) below 2005 levels in 2025 (resp. 2030).

papers, only rough explanations are provided to explain the difference in employment due to energy shift scenarios. We aim at complementing the existing literature by providing a data-driven analysis that better accounts for the factors explaining the differences in employment between sectors.

To understand how differences emerge regarding the level of employment within each sector, we utilize a growing body of literature in employment decomposition that belongs to the tradition of Input-Output framework—see, for instance [Perrier and Quirion \(2016\)](#) and [Perrier and Quirion \(2018\)](#). We present a qualitative assessment of the employment shifts between sectors that would be required to meet the Paris Agreement as applied to Brazil. This descriptive approach allows for qualitative distinctions of inter-branch substitutions caused by the energy transition. In this paper, energetic transition is understood to shift final demand from one CO₂ emitting sector to a low carbon content sector—being either energy-to-energy or industry-to-industry.

The paper is structured as follows: Section 2.2 outlines the characteristics of the current Brazilian energy sector, its contribution in Brazil’s GHG emissions, and its challenges. Section 2.3 suggests an optimization program that minimizes the sectoral demand changes in order to meet the Paris Agreement objective while ensuring expected economic growth. Section 2.4 presents a qualitative assessment of employment shifts between sectors that are required to meet the Paris Agreement as applied to Brazil. Our main conclusions are outlined in the final section.

2.2 Overview of Brazil’s energy sector

Brazil’s endowment of energy resources is sufficient to meet the country’s need and to comply with international environmental agreements. According to the Sustainable Energy for All database [World Bank \(2018\)](#), in 2016, 100% of the Brazilian population had access to electricity. Most importantly, around 80% of that total electricity supply was generated by renewable sources [BP \(2018b\)](#). In contrast, two-thirds of Brazil’s primary energy consumption was based on fossil energy.

Figure 2.1 shows that the primary energy consumption in Brazil increased by 28% in the decade 2007-2017, going from 229.6 to 294.4 million of tonnes of oil equivalent (hereafter mtoe). However, the relative share of consumption by energy source remained fairly constant over that period, with fossil fuel representing approximately 60%. It is worth noting that the increase of 6 points in renewable resources was insufficient to offset the reduction of 8 points in hydropower.

Similar dynamics can be seen in the electricity generation, as shown in Figure 2.2. Indeed, in the same decade, 2007-2017, the reduction in the share of hydropower (84% to 63%) has been partially compensated by the increased use of other renewable sources (4% to 17%), along with an expansion in the share of natural gas (3.5% to 11%).

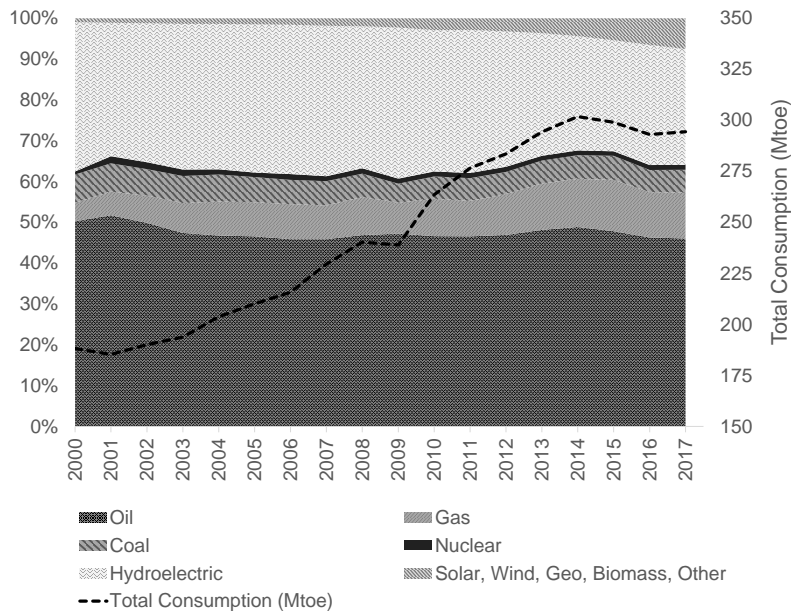


Figure 2.1: Brazil's primary energy demand 2000-2017. Total levels and composition by source. Source: BP (2018b)

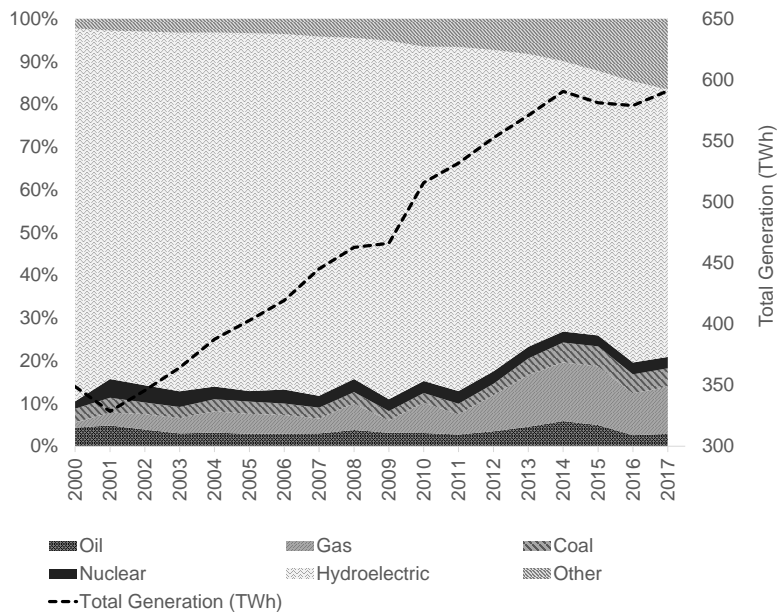


Figure 2.2: Brazil's electricity generation 2000-2017. Total levels and composition by source. Source: BP (2018b)

Brazil's early determination to move forward with alternatives to fossil fuels was a natural choice due to the country's large hydropower potential, their agricultural capacities, as well as the rising concerns over energy security. However, the large "pre-salt" oil and gas discoveries in the Santos basin beginning in 2006 have raised Brazil's potential and

ambitions in the oil sector once again [World Bank \(2016\)](#). The record high production of oil in 2017 (2.7 Mb/d), as well as Brazil's largest growth in gas production in its history (+ 3.2 bcm) during the same year [BP \(2018a\)](#), confirm this.

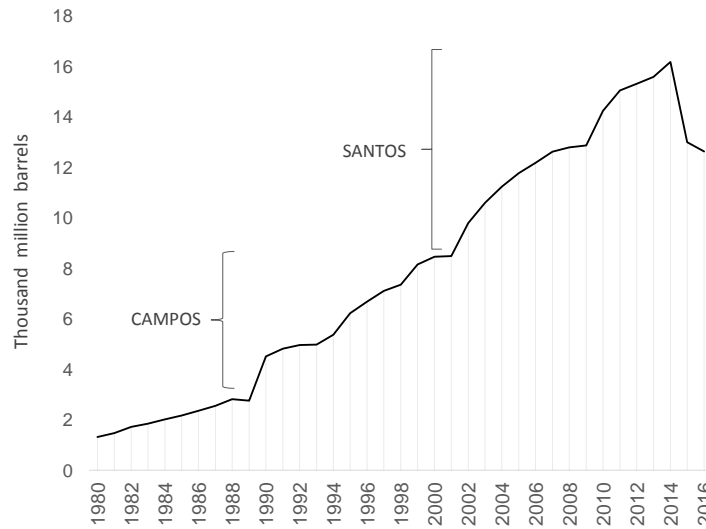


Figure 2.3: Evolution of Brazil's proven oil reserves. Source: [BP \(2018b\)](#)

The growing use of carbon intensive energy sources in Brazil decreases the country's chance to meet their CO₂ emissions targets in either 2025 or 2030. Indeed, as can be seen in Figure 2.4, during the last decade the overall CO₂ emissions have only decreased in times of contracting economic activity. This trend suggests that Brazil's economic growth is still coupled with burning fossil fuel.² No sign of decoupling has been shown by the data. This delay in the necessary decoupling is yet another potential detriment for the achievement of the Paris Agreement.

²When analyzing GDP growth and CO₂ emissions from 2000 on, we conclude that there is a positive link between the two in which a 1% increase in real GDP is correlated to a 1.4% increase in CO₂ emissions.

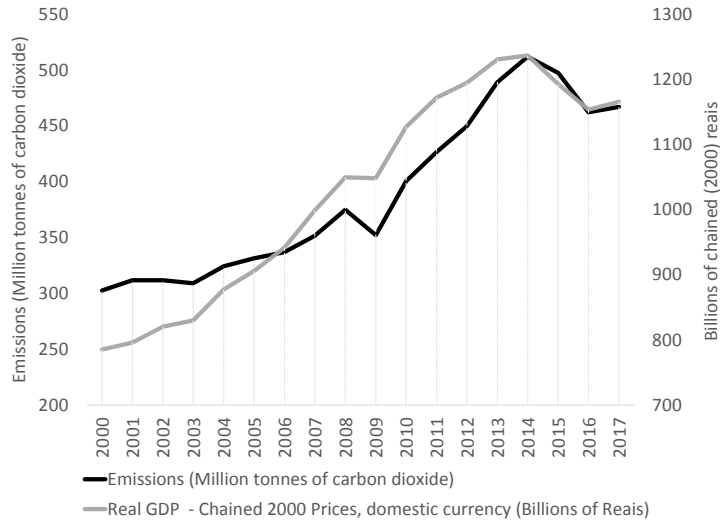


Figure 2.4: Brazil’s CO₂ emissions and real GDP 2000-2017. Sources: BP (2018b) and World Bank.

2.3 Reconciling NDC with projected economic growth through final demand

In its NDC, Brazil intends to commit to reducing its GHG emissions by 37% (resp. 43%) below 2005 levels in 2025 (resp. 2030). According to BP (2018b), CO₂ emissions by fossil fuels in 2017 were 41% greater than those in 2005. Although improvements has been made in 2014, where emissions were nearly 55% higher than in 2005, there are further efforts to be done if the NDC commitments are going to be fulfilled.

The objective of this section is to understand the minimal changes in the final demand by sector that are required in order to meet the 2025 NDC³ CO₂ emissions target. We use the information contained in Brazil’s 2014 supply use tables (SUT), and its corresponding transformation to input-output tables (IOT). We combine them with the emissions by country and industry reported in the Eora global supply chain database.⁴

We proceed with the assumption that the productive inter-relations remain those of 2014, the latest available SUT at the desired level of details. Moreover, we assume that for each constant monetary unit (*reais*) of production, the structure of intermediate consumption of energy by source is also represented by that of 2014. As such, the reduction of GHG emissions could be achieved by redirecting the final demand towards those sectors that rely less on carbon-intensive energy sources and whose inputs are less carbon-intensive.

³Note that GHG emissions from land-use are not taken into account in this paper, as they are known to result from illegal activities.

⁴See Lenzen et al. (2012, 2013) for further details.

Furthermore, we assume a constant improvement in energy efficiency for all processes and energy sources, which translates into a decreasing rate of emissions for each unit of energy used through technological progress, as detailed shortly.

In addition to achieving the emissions target, we assume that the minimal required changes in the final demand should satisfy a real GDP in 2025 that is consistent with the projected economic growth of Brazil, and should provide a higher employment rate in 2025 than in 2014. Thus, the minimal changes in demand will result from the solution of the following program:

$$\begin{aligned}
& \text{Min } \left\| (y_{2025} - y_{2014})y_{2014} \right\| \\
& \text{Subject to:} \\
& \sum_{j=1}^{68} Y_{2014,j} \prod_{t=2015}^{2025} (1 + \alpha_t) = \sum_{j=1}^{68} Y_{2025,j} \\
& \text{em} \cdot g_{2025} \times (1 - a_{em})^{2025-2014} \leq \text{EM}_{\text{NDC}} \\
& \frac{l_q \cdot g_{2025} \times \frac{1}{(1+a_l)^{2025-2014}}}{N_{2014} \times (1 + a_N)^{2025-2014}} \geq \frac{J_{2014}}{N_{2014}} \\
& y_{2025,j} := \frac{Y_{2025,j}}{\sum_j Y_{2025,j}} > 0, \quad \forall j = 1, \dots, 68 \\
& g_{2025} = (\mathbf{I} - A_d)^{-1} \times Y_{2025}
\end{aligned}$$

where $Y_{t,j}$ is the final demand of sector $j \in \{1, \dots, 68\}$ at time t ; y is the vector of shares of final demand by industry whose entries sum to 1; J stands for the total amount of full-time-equivalent jobs in the economy; g is the vector of domestic production by industry; A_d is the matrix of domestic technical coefficients; em is the CO_2 emission intensity; EM_{NDC} is the NDC's 2025 emission target after subtracting what would correspond to land-use emissions;⁵ a_{em} is the assumed annual rate of decrease of emissions across all industries; a_l is the assumed annual growth rate of average labor productivity across all industries; N stands for the population, with a_N being its assumed annual growth rate; l_q is the vector containing the amount of full-time-equivalent jobs required per monetary unit produced; α_t is the projected annual growth rate of the economy; and \cdot stands for the element-wise vector division operation.

In this optimization program, the 2014 final demand is selected as the optimal share that policymakers should target. We argue that these 2014 shares in final demand are the consequences of past socioeconomic forces that have led the economy to this particular distribution. Therefore, a deviation from this target would require a complete reorientation of the industrial process, and thus steer the economy away from its historical socioeconomic equilibrium. As climate policy is adding new constraints to the system, the objective function of policymakers would be to minimize the cost of redirecting the economy and to create new opportunities such as increasing employment rate.

⁵We assume land-use emissions remain constant with respect to 2014 levels.

We proceed in line with traditional static input-output analysis by assuming fixed prices and a fixed technical coefficients matrix. Furthermore, we assume a steady adoption and dissemination of cleaner technologies reducing industrial CO₂ intensity per unit of output at an average annual rate of 1.52% ($a_{em} = 0.0152$) as in Nordhaus (2017). In addition, we use observed annual growth rates until 2018, after which we assume an average growth rate of real GDP of 2%, in line with the projections⁶ for Brazil until 2025. The labor productivity growth, a_l , is assumed to be the compounded average growth rate (CAGR) for labor productivity,⁷ for the decade 2009-2017. Similarly, a_N is assumed to correspond to the CAGR of the Brazilian population, for the period 2009-2017.⁸ The minimization problem is solved numerically.⁹

Figure 2.5 shows the relative changes in the shares of final demand in 2025 with respect to 2014 for each of the 68 sectors. Under the redistribution of the demand structure, emissions satisfies the Paris Agreement target for 2025. In contrast, if the final demand structure remained unchanged, emissions would be approximately 6.4% higher than the ones targeted in the NDC. We observe a large drop ($< -75\%$) in final demand applied to the domestic industries such as coal extraction, oil refining, mechanical machinery, and utilities (including gas and water distribution).¹⁰ Most of these sectors are manufacturing industries and are key sectors to the Brazilian economy (see Marconi et al. (2016)). Results indicate that the required changes in the economy are substantial and would have a significant impact on both future capital investments and jobs. Moreover, these large decreases in final demand concern domestic industries. These decreases in CO₂ intensive consumption goods may be partially compensated by imports, which would translate into outsourcing of production-based CO₂ emissions Davis and Caldeira (2010). Such mechanisms are left for further research.

Figure 2.6 shows the variation in the domestic production (in Billions of *reais* in 2014) that result from changes in final demand. The variation of the domestic production takes into account both direct and indirect—through the intermediate consumption—effects on industrial output of final demand changes. For example, the 2025 production of the coal extraction sector would have to remain roughly equivalent to that of 2014. Therefore, according to this result, the investments needed throughout the period 2014-2025 in this economic sector should not exceed the consumption of fixed capital and the potential decrease in the efficiency of the mines. In Figure 2.5, we observe that primary industries such as agriculture and livestock; services such as accommodation and food services, public and private health, as well as public and private education are booming, and thus would require

⁶Forecasts from 2019 to 2025 refer to those from the Western Hemisphere Regional Economic Outlook, from International Monetary Fund (2018)

⁷We considered the labor productivity per person employed (2011 PPP Dollars) at constant prices, from Conference Board (2019)

⁸See Instituto Brasileiro de Geografia e Estatística - IBGE (2018) for further details.

⁹The algorithm has $68 - 1 = 67$ degrees of freedom, which allow us to consider one given sector as a residual. We apply the algorithm for every possible case, and thus explore a wide variety of initial conditions.

¹⁰A 100% drop in final demand for a given sector does not imply that its production vanishes; goods produced by this sector would still be consumed as intermediate goods.

additional investments and adequate labor forces. On the other hand, the production of extracting activities, most manufacturing activities, and some of the services will see a decline in production. It is worth noting that our results indicate that all of the energy sectors (extraction, oil refinery, utilities, and biofuels) will be negatively impacted by the required changes in final demand, putting the production of 2025 below that of 2014.¹¹ However, a sector like utilities, which contains activities such as distribution of gas, may experience an above average decrease in carbon intensity of the period as the country moves away from fossil energies. Finally, the results show that the sectors within which economic activities would decrease the most include oil and gas extraction, oil refining, mechanical machines, vehicles, and real estate.

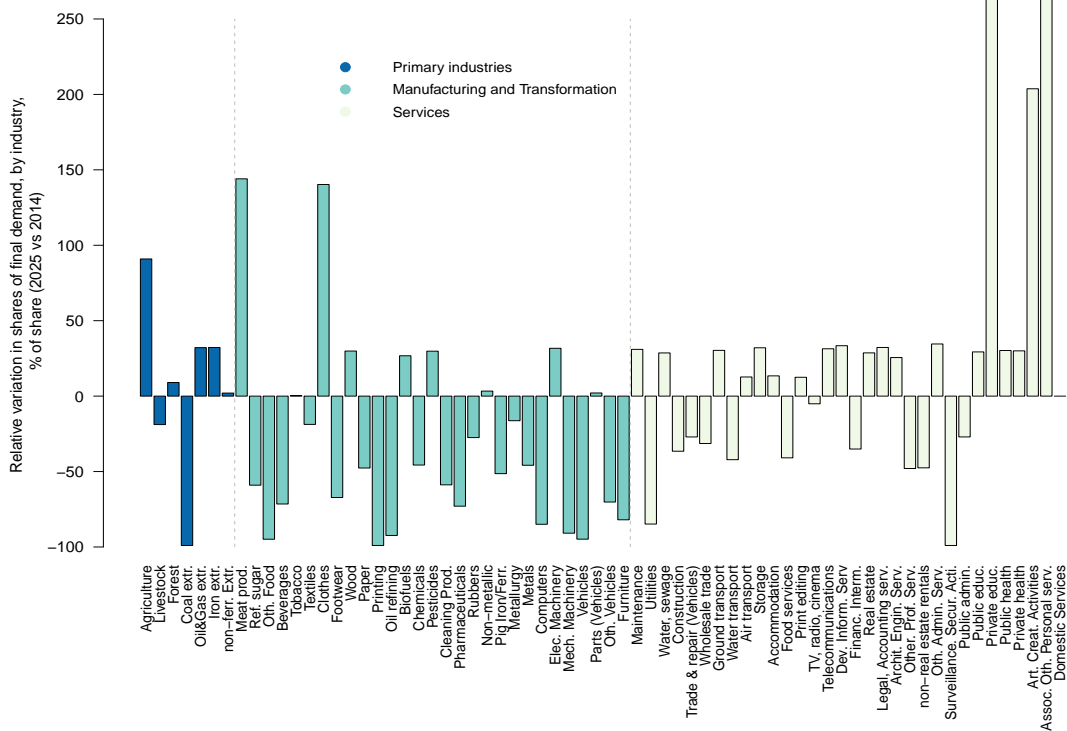


Figure 2.5: Relative variation in final demand structure (2025 vs 2014. % of share)

¹¹Of course, this result depends on the fact that we assumed the same reduction in emission intensity for every sector.

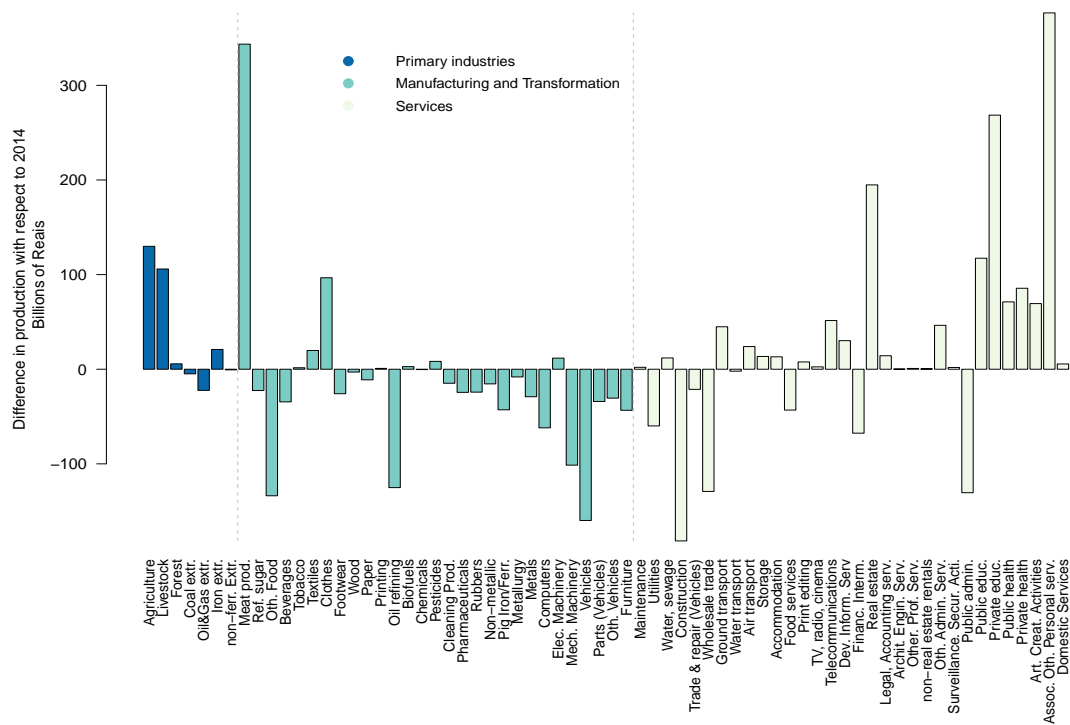


Figure 2.6: Difference in production with respect to 2014. Millions of Reais.

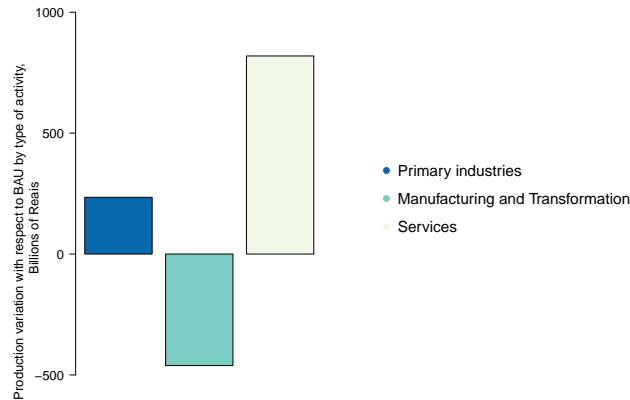


Figure 2.7: Difference in production, by type of activity, with respect to BAU

Figure 2.7 displays an aggregate picture of the results that are shown in Figure 2.6 by reorganizing the sectors in three sets (primary, manufacturing and transformation, and services). It shows the difference in production resulting from the optimization program in millions of *reais* of 2014 and the production of a Business-as-Usual (BaU) scenario. The BaU scenario represents a homothetic increase in activity for each sector with respect to the GDP growth. In other words, if the 2025 final demand shares were to remain consistent with those of 2014, every set would show a zero value since every sector is attaining its

potential growth. Figure 2.7 shows that the manufacturing and transformation is the only sector that would suffer (drastically) from a change in final demand. The transfer of activity would mostly run from this set into that of services, and into primary industries to a lesser extent.

These results operate in conjunction with those presented in [Marconi et al. \(2016\)](#), which argues that agriculture and extracting industries have weak backward linkages and shorter supply chains. Therefore, their ability to boost the rest of the economy is weak, which makes these sectors a secondary option when moving away from manufacturing activities. Moreover, [Marconi et al. \(2016\)](#) identifies that the oil-related and the mineral extraction industries among the key sectors of the Brazilian economy, which means that they have the ability to boost both upstream and downstream industries.

Such results highlight one of the greatest challenges for Brazil in its path towards a low carbon economy. Indeed, by moving away from carbon intensive activities and towards a service-oriented economy, Brazil would be renouncing its key industries in favor of those with lower interconnections and hence with lower potential to boost economic activity across sectors. However, if this shift is oriented towards services, it would generate more jobs since these sectors have a higher employment content. In the next section, we push our analysis of the employment effect a step further by providing a qualitative assessment of the composition of the employment to understand the mechanisms at stake when it comes to analyzing the shift between activities.

2.4 Employment decomposition following a low-carbon shift

Following [Perrier and Quirion \(2016\)](#), we take the definition of employment content as the amount of direct and indirect jobs created for each monetary unit of final demand addressed to an industrial sector. The results presented in this section are normalized. Therefore, they are independent of the optimal industrial structure of the Brazilian economy obtained in section 2.3. We use an average employment content as reference point.¹² The spread between each industry's employment content and the Brazilian average content is explained by the differences of the following factors:

- import rate of final demand ($\Delta \overline{\mathbf{mf}}$),
- import rate of intermediate consumption ($\Delta \mathbf{MI}$),
- taxes and subsidies ($\Delta \overline{\mathbf{t}}$)
- wage share in value added ($\Delta \overline{\omega}$), and
- the inverse of wages ($\Delta \overline{\mathbf{s}}$).

¹²See Appendix 2.A for further technical details.

The allocation of taxes and imports by product and industry is key in this methodology. Hence, to avoid any bias due to conversion methods, we employ the most recent official input-output tables for Brazil (2010), which include all required matrices.

We first consider the standard deviation of each factor derived in Appendix 2.A.2. The higher the value is, the greater the variation it explains.

Factors	Stand. dev
$\Delta\bar{s}$	17.69
$\Delta\bar{\omega}$	7.42
$\Delta\mathbf{MI}$	1.67
$\Delta\bar{\mathbf{mf}}$	1.27
$\Delta\bar{\mathbf{t}}$	1.02

Table 2.1: Standard deviation of each factor ranked in descending order, 2010.

Table 2.1 shows the result of [Ang \(2005\)](#)'s LMDI decomposition applied to 66 sectors of the Brazilian economy in 2010.¹³ We note that the most significant factor that explains discrepancies in employment between sectors is (the inverse of) wages ($\Delta\bar{s}$), followed by the wage share in value added ($\Delta\bar{\omega}$). The other factors have smaller explanatory power and are the import rate of products for the intermediate consumption ($\Delta\mathbf{MI}$) and the import rate of final products ($\Delta\bar{\mathbf{mf}}$). Interestingly, net taxes ($\Delta\bar{\mathbf{t}}$) play a rather minor role.¹⁴

The detailed results for the application of this methodology to the Brazilian input-output tables (2010) are shown in Figure 2.8.¹⁵ A sector that shows a net bar above zero has an employment content that is above the average. The results suggest that the employment content of most manufacturing of industrialized goods is below the national average. Exceptions include clothing, the food and wood industries, and the production of biofuels. In addition to the fact that manufacturing sectors have below average employment content, we observe from Figure 2.8 that these sectors have a roughly similar decomposition that shows above average wages and cost of capital, higher taxes, and that is more reliant on imported intermediate consumption. Moreover, most of the sectors that would be negatively impacted by changes in final demand show a below average employment content. As a result, the changes in demand that we find in the previous section would be net job creating. Indeed, with respect to a scenario in which the demand shares remain unchanged, the optimizing structure of the final demand would create 16.5% more full-time-equivalent jobs, which would represent an increase of 8 points in the employment

¹³Domestic services are not included because the data does not show any transactions with the other industries in the economy. Furthermore, IOT for 2010 merge whole sale and retail sectors into a single one. We do not perform a sensitivity analysis with the other years as the assumptions made on these data are not suitable for the decomposition under consideration.

¹⁴Interestingly, this ranking is consistent with the findings of [Perrier and Quirion \(2016\)](#) who have applied this methodology to France.

¹⁵The numerical values are reported in Appendix 2.B.

rate of 2025. It is worth mentioning that most of the sectors for which final demand should increase (agriculture, livestock and food services, among others) have well below average wages. For the remainder of the paper, we will further break down Figure 2.8 by highlighting some sectors in order to provide examples of how policymakers can incentive certain job substitutions.

The contrasting results for the oil refining industry and that of biofuels provides a first scenario of substitution to consider.

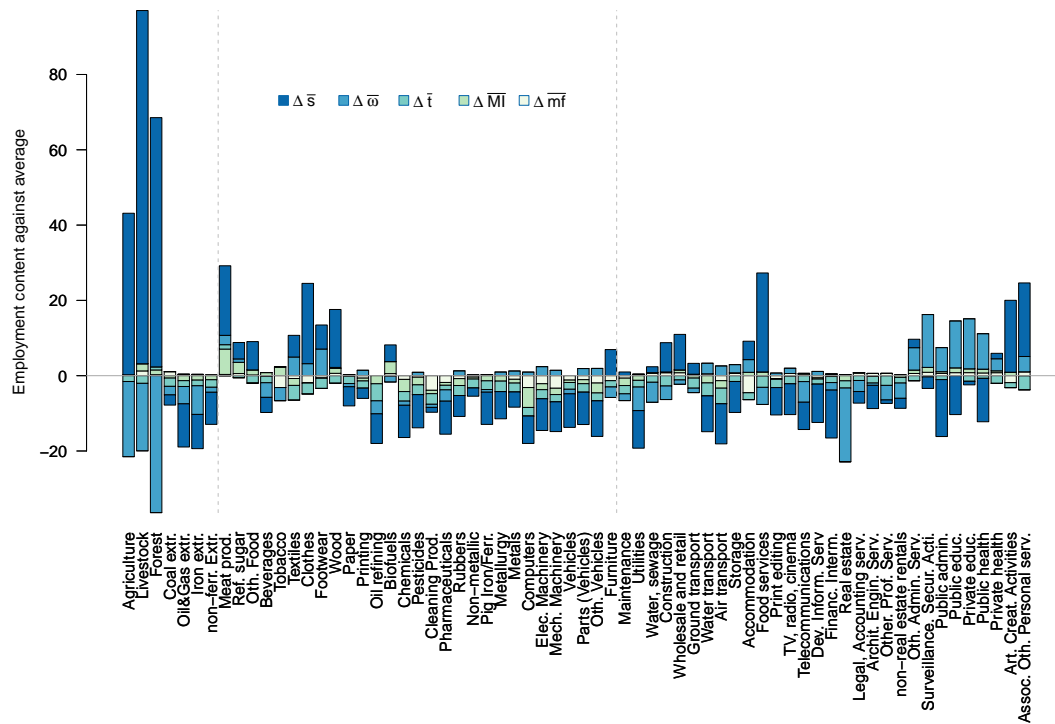


Figure 2.8: Employment content difference with respect to the mean - decomposition

Example 1: increase the share of sustainable biofuels In its NDC, Brazil is committed to increasing the share of sustainable biofuels in the national energy mix to approximately 18% by 2030. This might be achieved by expanding biofuel consumption, increasing ethanol supply -which implies increasing the share of advanced second generation biofuels-, and increasing the share of biodiesel in the diesel mix. In terms of our sectoral representation, this would mean that the demand of the sector manufacturing biofuel will increase to the detriment of the oil refining and coke ovens.

Before starting the analysis, a caveat arises; an increase in the demand of one *real* in the sector biofuel would not cancel out one *real* of demand for the sector oil refining. The idea is to provide a first order approximation of what would be the employment content in the short run when substituting, say, a tributary incentive to foster the demand switch of one *real* from one sector to the other.

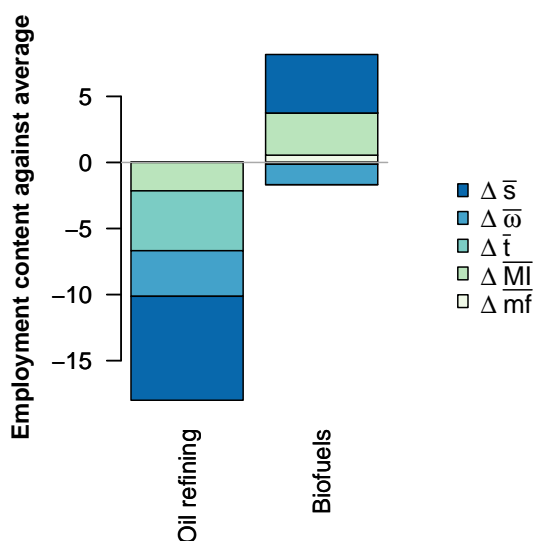


Figure 2.9: Employment content decomposition for oil refining and manufacture of biofuels

Figure 2.9 shows the employment content of the two sectors under consideration. We note that the sector of biofuel has a positive employment content against a negative one for oil refining sector. We explain the difference by more attractive wages for the oil refining sector. Second, oil refining pays more taxes than the average level while biofuel is at the average. Third, we see that the import rate is negatively impacting the oil refining industry, while the biofuel is above average since it relies comparatively more on national products. We conclude that a policy that targets biofuel rather than refined oil is more likely to create jobs, to the extent of 20 jobs per million of *reais*. However, it would translate into reduced wages, less tax revenue collected by the government, a boost for the domestic activities, and possibly lead the economy to a sub-optimal path as the final demand for biofuels would likely decrease. Again, as pointed out by [Perrier and Quirion \(2016\)](#), it is worth mentioning that this methodology does not take into account intersectoral frictions or the inertia in the good and the labor market when final demand shifts from one sector to another.

Example 2: moving towards renewables Brazil hopes to achieve 45% of renewables in the energy mix by 2030. Their strategy includes expanding the use of renewable energy sources other than hydropower in the total energy mix to between 28% and 33% by 2030. Another strategy is the expand the use of non-fossil fuel energy sources domestically, increasing the share of renewables (other than hydropower) in the power supply to at least 23% by 2030, including by raising the share of wind, biomass, and solar. In the sectoral decomposition of the Brazilian economy, it is impossible to distinguish what is nuclear, wind, or solar in the electricity sector.¹⁶ However, comparing the utilities sector to

¹⁶Despite the results in the last section of decrease in demand for the utilities, we assume that if the energy mix of Brazil reaches 45% of renewable, the sector of utilities would see a faster decrease in its CO₂ intensity that previously assumed, and thus a higher demand for this sector may still be in line with

the extraction of oil and the extraction of coal, and non-metallic minerals provide insight on the impact of such policy within the labor market.

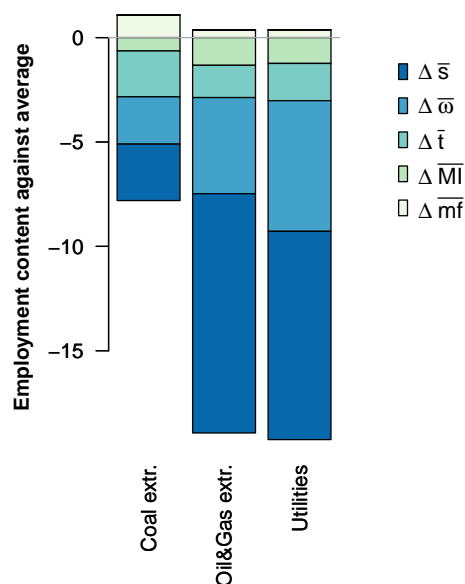


Figure 2.10: Employment content decomposition for coal extraction, oil extraction, and utilities

Figure 2.10 displays the employment content of the three sectors under consideration. Interestingly, the sector of utilities and the one of oil extraction look alike. Despite the fact that all three sectors are below average, shifting one million *reais* from the sector of coal extraction to the utilities causes the destruction of almost 12 jobs as wages are higher than average and the cost of capital is significantly higher in the utilities sector. To conclude, shifting jobs from oil extraction to utilities seems straightforward, while shifting from coal extraction to utilities would require more human and physical capitals.

Example 3: shifting demand to a services-oriented economy In Section 2.3, we showed that a global shift from the carbon intensive sectors towards services would assist in advancing both economic growth and GHG reduction. In this last example, the motivation behind the choice of these sectors is that construction is the sector in which production increases the most. Financial intermediate represents the type of service that would be impacted most negatively. Metallurgy is the representative sector among the manufacturing are transformation sectors as it mimics most of the employment decomposition of its category. Also, both food services and private education are representative in the services in terms of employment decomposition. The results should to be compared with those for oil refining, coal extraction, and oil extraction in Figures 2.9 and 2.10.

reaching the NDC.

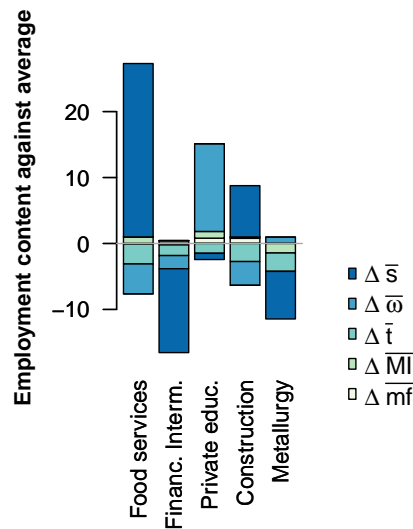


Figure 2.11: Employment content decomposition of Food industry, Financial intermediary, Private education, Construction, and Metallurgy

Figure 2.11 shows the employment decomposition of the five sectors under consideration. Among the five sectors, three sectors are systematically above average in terms of employment content, while the financial intermediary and the metallurgy appear to be below average. This can be explained either by wages that are below average, like in the Food and Construction sectors, or by lower capital intensity, as is the case for Private education. Interestingly, as with the carbon intensive sector, all service sectors under consideration pay more taxes than average. Therefore, transitioning from oil extraction to a service sector should not be costly from the perspective of public finance. Moreover, we note that services rely less than average on imported intermediate consumption, which is beneficial to the country's balance of trade. Finally, the financial intermediary, among all the service sectors under consideration, is the one that has nearly the same patterns as both oil refining and oil extraction, aside from the fact that finance intermediary requires less imports.

2.5 Concluding remarks

By combining input-output data, GHG data, and employment data, we provide a qualitative explanation of changing activities in terms of employment as this shift contributes to both the Paris Agreement and economic growth objectives. To our knowledge, this paper is the first one applied to Brazil that is able to study the factors explaining the differences in employment of shifting activities.

We began our analysis by exploring whether achieving NDC targets in 2025 with a forecasted yearly economic growth of 2% are still possible if final demand shifts from coal extraction, oil refining and carbon intensive manufacturing industries (such as the food-manufacturing and the vehicles industries) to services sectors (such as the provision of

health and education, or the tourism-related industries). In other words, if the economy is becoming increasingly service-oriented, both objectives can be met. Moreover, we show that such a shift in demand could be net job-creating.

However, it should be noted that the new economic landscape that would result from such configuration, would imply a reduction in the production of certain goods domestically. In particular industrial ones. Unless the demand adjusts its behavior correspondingly, the reduced production would only lead to larger imports, and to an effective outsourcing of polluting activities. The results remain of interest, as they show the drastic magnitude of industrial reorganization required to meet the NDC objectives by reducing industrial emissions alone. The fight against deforestation could allow for less dramatic changes, while still reducing emissions and achieving projected GDP growth levels.

We have complemented our own analysis with [Perrier and Quirion \(2016\)](#)'s employment decomposition analysis to further explore how to qualitatively explain the change in employment levels following demand changes. We discussed three examples from the orientations given by the NDC and from the view of orienting the economy to services. We show that shifting activities from oil refining (1991) to biofuels (1992) is likely to be a job-creating change, while going away from the extracting sector to the benefit of the utilities sector may increase unemployment. However, we demonstrate that shifting final demand to a service oriented economy is net job-creating, except for sectors like financial intermediary, real estate, legal, accounting, among others. Moreover, we observed this transition would not be costly from the perspective of public finance as the service sectors pay more taxes than the average of the economy. Furthermore, local services can hardly be substituted by foreign services. A caveat for such a transition would be that average wages may decrease, which could generate social tensions emerging from inadequacies between job's qualification and worker's qualification.

Appendices for Chapter 2

2.A Data and methodologies

2.A.1 Data and notations

The Brazilian statistical agency, *Instituto Brasileiro de Geografia e Estatística* (hereafter IBGE), computes and publishes the Brazilian input-output tables (hereafter IOT) at a frequency of five years. To date, the latest release was published in 2016 using data from 2010. In the absence of newer and highly detailed official information, the IBGE publishes supply and use tables (hereafter SUT) every year with a lag of 2 years. However, SUT neither include the use tables at basic prices nor the valuation matrices to transform from basic into purchasers prices. Instead, use tables are presented in purchaser's prices, with valuation vectors only. In this paper, the employment content analysis applied to Brazil is implemented with the most exhaustive tables in order to benefit from the existence of official valuation matrices required on the computation of symmetric input-output tables. This includes the possibility to correctly distinguish how imports are distributed to industries and institutional sectors, which can influence the results of the domestic technical coefficients.

Throughout the paper, SUT are transformed into industry-by-industry input-output tables (IIOT) using standard conversion methods. In the cases in which basic price information is not available, we follow the methodology detailed in [Guilhoto et al. \(2005\)](#) to allocate taxes and margins proportionally to the flows of goods depicted in the use table and the final demand matrix. The allocation of taxes and trade and transportation margins to each industry j is computed at the *prorata* according to the shares by product i as given by:

$$\alpha_{ij} := \frac{Z_{ij}}{\sum_j Z_{ij}}$$

where $\sum_j Z_{ij}$ represents the total demand of good i in market prices. Similarly, under the assumption that imports follow the same distribution as the domestic production, we allocate the vector of imports to intermediate consumption and final demand according to the structure of the flows with respect to the total demand. Taxes on imports are assumed to follow the same shares as that of imports of goods.

The rectangular use table and final demand matrix at basic prices are obtained by subtracting the values in the valuation matrices (i.e., matrices showing the distribution of taxes and trade and transportation margins) from the original information at purchasers price. The principle of proportionality by which the valuation matrices are constructed ensures that the total supply and total demand will be balanced.

For the year 2010, the official input-output table published by the IBGE is composed of 67 industries and 127 products. The industry and production corresponding to *domestic services* are withdrawn from the analysis,¹⁷ resulting in 66 industries and 126 products.

¹⁷*Domestic services* is not an input for any other sector and does not require intermediate consumption to produce. It uses only labor for its production and all value added is transferred to households as compensation to employees.

Although the final demand is decomposed into six institutional sectors, we only considered one aggregate sector in this analysis, resulting in a final demand vector that includes stock variation. In brief, Table 2.2 summarizes the data (and notations) used in the computation of symmetric input-output tables and technical coefficients.¹⁸

Var.	Variable names
\mathbf{V}^T	Supply table at basic prices (prod \times ind)
\mathbf{U}	Use table at basic prices (prod \times ind)
\mathbf{U}_d	Use table of domestic goods (prod \times ind)
\mathbf{U}_m	Use table of imported goods (prod \times ind)
\mathbf{y}	Final demand vector (by prod)
\mathbf{y}_d	Final demand vector of domestic goods (by prod)
\mathbf{y}_m	Final demand vector of imported goods (by prod)
$\mathbf{T}_{Ud}^{\text{net}}$	Net tax on dom. interm. goods (prod \times ind)
$\mathbf{T}_{Um}^{\text{net}}$	Net tax on imp. interm. goods (prod \times ind)
$\mathbf{T}_{Yd}^{\text{net}}$	Tax on dom. final goods (by prod)
$\mathbf{T}_{Ym}^{\text{net}}$	Taxes on imp. final goods (by prod)
\mathbf{W}	Value added, GDP revenues, and emp.
$\mathbf{q} - \mathbf{m}$	Total domestic production (by prod)
\mathbf{g}	Total domestic production (by ind)

Table 2.2: Matrices notations of the official IOT (2010)

The IBGE provides as well matrix of domestic technical coefficients, for symmetric input-output table (industry-by-industry). The procedure detailed in [Perrier and Quirion \(2016\)](#) (see Appendix 2.A.2) requires, in addition, the square matrices denoting imported and aggregate technical coefficients. For this purpose, we follow [Beutel \(2008\)](#) to compute an industry-by-industry input-output table under their so-called *model D*. This process matches domestic matrices of technical coefficients with the matrices published by the IBGE, and yields additional useful matrices as detailed below.

Following standard input-output conventions, and by using the *Model D* from [Beutel \(2008\)](#), we transform the product-by-industry supply use tables into industry-by-industry input-output tables.

We decompose the throughput by industry, \mathbf{g} into:

$$\mathbf{g} = \mathbf{A}_d \mathbf{g} + \mathbf{f}_d, \quad (2.1)$$

where \mathbf{A}_d stands for the domestic matrix of technical coefficients, and \mathbf{f}_d for the vector of final demand by industry. This can be re-written as

$$\mathbf{g} = (\mathbf{I}_n - \mathbf{A}_d)^{-1} \mathbf{f}_d,$$

¹⁸To go further on the input-output methodology, see [Beutel \(2008\)](#)

where, $\mathbf{L}_d := (\mathbf{I}_n - \mathbf{A}_d)^{-1}$, denotes the inverse of the Leontief matrix for domestic production. Similarly, we define $\mathbf{L} := (\mathbf{I}_n - \mathbf{A})^{-1}$ to represent the Leontief matrix for the global economy, with \mathbf{A} standing for the matrix of technical coefficients including both, domestic and imported inputs.

The information from the IBGE is complemented with that of the EORA global supply chain database, which provides input-output tables augmented for emissions by industry and country. Although it uses 55 industries –in contrast to the 68 reported in the tables published by the IBGE – the matching between both matrices is straightforward. EORA may cluster similar sectors and provide a single value of CO₂ emissions for the entire cluster. We proceed by distributing this among the clustered industries prorata of the production levels, under the assumption that those similar processes had also similar emission patterns. The obtained vector of industrial emissions EM is then transformed into a vector of emission intensities, em, such that $em_j := EM_j/g_j$ for any industry $j = 1, \dots, 68$.

2.A.2 Establishing the employment impact

Perrier and Quirion (2016) put forward a methodology to decompose the sectorial employment content into five factors: (i) import rate of final product; (ii) import rate of intermediate consumption; (iii) net taxes rate; (iv) labor intensity; and (v) wage share in the value added.

The intermediary objective of the methodology is to define the employment content of a given economy, as the absolute number of direct and indirect full-time equivalent jobs (**fte**) that are necessary to produce 1 million *reais* of final demand towards a given sector j at a given year.

Let, e , be the column vector denoting employment intensity (the direct employment per monetary unit of production). For each industry j , the element e_j is given by

$$e_j := \frac{\mathbf{fte}_j}{\mathbf{g}_j}.$$

The domestic total employment factor associated to industry j (the direct and indirect employment per monetary unit of domestic final demand addressed to sector j), is then the row vector, $\bar{\mathbf{e}}^T$, defined by

$$\bar{\mathbf{e}}^T := e^T \mathbf{L}_d,$$

where \mathbf{L}_d stands for the domestic leontief inverse matrix, and T for the matrix transpose operator. In other words, each element j in $\bar{\mathbf{e}}$ will be given by

$$\bar{e}_j = \sum_i e_i \mathbf{L}_{dij}.$$

Multiplying the employment factor, e_j , by the final domestic demand addressed to industry j , \mathbf{f}_{dj} , we obtain the total number of direct and indirect jobs created by that

particular demand vector.

Let the final demand addressed to domestic firm j be a share $\overline{\mathbf{mf}}_j$ of the total final demand for domestic and imported goods of type j , \mathbf{f}_j , so that

$$\mathbf{f}_{d_j} = \overline{\mathbf{mf}}_j \mathbf{f}_j,$$

and let the final demand vector, \mathbf{f} , be normalized to a vector of ones. Then, the employment content, \mathbf{ce}_j , defined as the total number of direct and indirect jobs created by industry j for each unit of final demand for goods of type j , is given by

$$\mathbf{ce}_j := \overline{\mathbf{mf}}_j \sum_i e_i \mathbf{L}_{d_{ij}}.$$

Let \mathbf{va} be a row vector of gross value added by industry. We define \mathbf{va}^d as the row vector representing value added augmented by taxes so that \mathbf{va}_j^d corresponds to its national accounts interpretation. Each element j of \mathbf{va}^d is obtained by adding the gross value added of a sector to the total net taxes on domestic and imported inputs of firm j , and the net taxes that final demand paid for both, domestic and imported goods of type j

$$\mathbf{va}^d := \mathbf{va} + \mathbf{1}_n^T (\mathbf{T}_{Ud}^{\text{net}} + \mathbf{T}_{Um}^{\text{net}}) + \mathbf{t}_{Yd}^{\text{net}} + \mathbf{t}_{Ym}^{\text{net}},$$

where, $\mathbf{1}_n$, is a column vector of ones, and $\mathbf{t}_{Yd}^{\text{net}} + \mathbf{t}_{Ym}^{\text{net}}$ yields a column vector whose entry j aggregates the net indirect taxes paid on good j by all institutional sectors that compose the final demand.

Let \mathbf{va}^{at} represent the gross value added after considering the other net indirect taxes on production, $t^{O.net}$. In other words, \mathbf{va}_j^{at} is what will be allocated to the compensation of employees and the gross operating surplus. Thus, it represents the GDP at basic prices,

$$\mathbf{va}^{at} := \mathbf{va} - t^{O.net}.$$

Finally, let \mathbf{comp} be the row vector of compensation to employees. The employment intensity for each firm j can now be expressed as follows

$$\begin{aligned} e_j &:= \frac{\mathbf{fte}_j}{\mathbf{g}_j} = \frac{\mathbf{L}_{ji}}{\mathbf{L}_{ji}} \frac{\mathbf{va}_j^d}{\mathbf{g}_j} \frac{\mathbf{va}_j^{at}}{\mathbf{va}_j^d} \frac{\mathbf{comp}_j}{\mathbf{va}_j^{at}} \frac{\mathbf{fte}_j}{\mathbf{comp}_j} \\ &= \frac{\mathbf{L}_{ji}}{\mathbf{L}_{ji}} \frac{\mathbf{va}_j^d}{\mathbf{g}_j} \bar{\mathbf{t}}_j \bar{\omega}_j \bar{\mathbf{s}}_j \end{aligned}$$

where $\bar{\mathbf{t}}_j$, $\bar{\omega}_j$, and $\bar{\mathbf{s}}_j$ be defined and interpreted as

- $\bar{\mathbf{t}}_j := \mathbf{va}_j^{at} / \mathbf{va}_j^d$, the weight of taxes in the gross value added of industry j ;
- $\bar{\omega}_j := \mathbf{comp}_j / \mathbf{va}_j^{at}$, the relative importance of compensation to labor with respect to that of capital (i.e., the wage share) for sector j ; and,
- $\bar{\mathbf{s}}_j := \mathbf{fte}_j / \mathbf{comp}_j$, the inverse of the mean wage paid in sector j .

Let \mathbf{MI} be the matrix defined as the element-wise ratio of the inverse of the domestic Leontief, and the global Leontief inverse, so that for each pair (i, j)

$$\mathbf{MI}_{ij} := \frac{\mathbf{L}_{dij}}{\mathbf{L}_{ij}}.$$

Finally, by defining, $\mathbf{V}_{ij} := \mathbf{L}_{ij} (\mathbf{va}_i^d / \mathbf{g}_i)$, the employment content for industry j becomes

$$\mathbf{ce}_j := \overline{\mathbf{mf}}_j \sum_i \mathbf{V}_{ij} \mathbf{MI}_{ij} \bar{\mathbf{t}}_i \bar{\omega}_i \bar{\mathbf{s}}_i.$$

Set as a reference decomposition, [Perrier and Quirion \(2016\)](#) introduce the mean employment content for industry j , $\mathbf{ce}_{m,j}$

$$\mathbf{ce}_{m,j} = \overline{\mathbf{mf}}_m \sum_i \mathbf{V}_{m,ij} \mathbf{MI}_{m,ij} \bar{\mathbf{t}}_m \bar{\omega}_m \bar{\mathbf{s}}_m,$$

where,

$$\begin{aligned} \overline{\mathbf{mf}}_m &:= \frac{\sum_j \mathbf{f}_{dj}}{\sum_j \mathbf{f}_j}, \\ \bar{\mathbf{t}}_m &:= \frac{\sum_j \mathbf{va}_j^{\text{at}}}{\sum_j \mathbf{va}_j^d}, \\ \bar{\omega}_m &:= \frac{\sum_j \mathbf{comp}_j}{\sum_j \mathbf{va}_j^{\text{at}}}, \\ \bar{\mathbf{s}}_m &:= \frac{\sum_j \mathbf{fte}_j}{\sum_j \mathbf{comp}_j}, \\ \mathbf{V}_{m,ij} &:= \mathbf{L}_{m,ij} \frac{\sum_j \mathbf{va}_j^d}{\sum_j \mathbf{g}_j}, \end{aligned}$$

with $\mathbf{L}_{m,ij} := \sum_{i,j} \mathbf{L}_{ij} / (\#ofsectors) \forall i, j$, and with the matrix \mathbf{MI}_m built so that $\forall i$, $\mathbf{MI}_{m,ii} = (a^d / (1 - n a^d)) / (a / (1 - n a))$, and $\mathbf{MI}_{m,ij} = ((1 - (n - 1)a^d) / (1 - n a^d)) / ((1 - (n - 1)a) / (1 - n a))$, elsewhere. Here, n stands for the number of industries, while a^d and a for the average technical coefficients of the domestic and the total technical coefficient matrices, respectively.

The spread with respect to the mean employment content for industry j is then defined as

$$\begin{aligned} \mathbf{ce}_j - \mathbf{ce}_{m,j} &:= \overline{\mathbf{mf}}_j \sum_i \mathbf{V}_{ij} \mathbf{MI}_{ij} \bar{\mathbf{t}}_i \bar{\omega}_i \bar{\mathbf{s}}_i \\ &\quad - \overline{\mathbf{mf}}_m \sum_i \mathbf{V}_{m,ij} \mathbf{MI}_{m,ij} \bar{\mathbf{t}}_m \bar{\omega}_m \bar{\mathbf{s}}_m \end{aligned}$$

As in [Perrier and Quirion \(2016\)](#), we follow the logarithmic mean Divisia index (LMDI) approach of [Ang \(2005\)](#) to decompose the difference in the sum of terms, where each

of them represents the contribution of a coefficient to the global change in employment content for a given industry i

$$\mathbf{ce}_j - \mathbf{ce}_{m,j} := \Delta \overline{\mathbf{mf}}_j + \Delta \mathbf{MI}_j + \Delta \bar{\mathbf{t}}_j + \Delta \bar{\omega}_j + \Delta \bar{\mathbf{s}}_j$$

where,

$$\begin{aligned} \Delta \overline{\mathbf{mf}}_j &= \sum_i \gamma_j \ln \left(\frac{\overline{\mathbf{mf}}_j}{\overline{\mathbf{mf}}_m} \right), \\ \Delta \mathbf{MI}_j &= \sum_i \gamma_j \ln \left(k_{ij} \frac{\mathbf{MI}_{ij}}{\mathbf{MI}_m} \right), \\ \Delta \bar{\mathbf{t}}_j &= \sum_i \gamma_j \ln \left(k_{ij} \frac{\bar{\mathbf{t}}_i}{\bar{\mathbf{t}}_m} \right), \\ \Delta \bar{\omega}_j &= \sum_i \gamma_j \ln \left(k_{ij} \frac{\bar{\omega}_i}{\bar{\omega}_m} \right), \\ \Delta \bar{\mathbf{s}}_j &= \sum_i \gamma_j \ln \left(k_{ij} \frac{\bar{\mathbf{s}}_i}{\bar{\mathbf{s}}_m} \right), \\ k_{ij} &= \left(\frac{\mathbf{V}_{ij}}{\mathbf{V}_{m,ij}} \right)^{1/4}, \\ \gamma_i &= \frac{\mathbf{ce}_i - \mathbf{ce}_{m,i}}{\ln(\mathbf{ce}_i) - \ln(\mathbf{ce}_{m,i})}. \end{aligned}$$

2.B Employment decomposition

It is worth mentioning here that the average employment content is 24.671.

Industry	Empl. cont.	$\Delta\bar{mf}$	$\Delta\bar{MI}$	$\Delta\bar{t}$	$\Delta\bar{w}$	$\Delta\bar{s}$
Agriculture	46.30	0.15	0.08	-1.55	-19.97	42.92
Livestock	101.69	1.27	1.91	-2.04	-17.93	93.81
Forest	56.84	0.26	1.26	0.82	-36.37	66.20
Coal extr.	17.94	1.08	-0.64	-2.20	-2.26	-2.71
Oil&Gas extr.	6.10	0.37	-1.33	-1.55	-4.61	-11.46
Iron extr.	5.63	0.34	-1.18	-1.50	-7.61	-9.09
non-ferr. Extr.	11.89	0.15	-1.09	-1.99	-1.33	-8.52
Meat prod.	53.86	0.33	6.74	1.16	2.48	18.48
Ref. sugar	32.94	0.57	2.98	-0.54	0.90	4.36
Oth. Food	31.73	0.20	1.34	-1.97	-0.02	7.52
Beverages	15.73	-0.17	0.83	-1.70	-3.88	-4.01
Tobacco	20.37	-3.15	2.33	0.02	-3.47	-0.03
Textiles	28.90	-0.75	-1.84	-3.90	4.96	5.75
Clothes	44.33	-1.95	-0.02	-2.90	3.19	21.33
Footwear	34.75	-0.63	-0.05	-2.70	7.08	6.38
Wood	40.25	0.63	1.32	-2.00	0.25	15.39
Paper	16.93	0.28	-0.16	-1.93	-0.84	-5.09
Printing	20.08	-0.65	-0.75	-1.92	1.44	-2.72
Oil refining	6.68	0.01	-2.15	-4.53	-3.45	-7.87
Biofuels	31.16	0.55	3.19	-0.14	-1.55	4.43
Chemicals	8.25	-0.99	-3.23	-2.54	-1.10	-8.56
Pesticides	11.73	-0.41	-2.02	-2.60	0.90	-8.81
Cleaning Prod.	14.96	-3.88	-0.89	-2.82	-0.83	-1.29
Pharmaceuticals	9.13	-1.80	-0.75	-1.21	-3.01	-8.77
Rubbers	15.13	-0.79	-1.82	-2.66	1.29	-5.56
Non-metallic	19.47	-0.38	-0.52	-2.37	0.25	-2.18
Pig Iron/Ferr.	12.05	0.31	-1.37	-2.30	-0.71	-8.55
Metallurgy	14.20	-0.01	-1.44	-2.79	0.98	-7.21
Metals	17.57	-0.96	-1.00	-2.37	1.25	-4.03
Computers	7.64	-3.15	-5.29	-2.28	0.96	-7.27
Elec. Machinery	12.52	-2.14	-1.58	-2.42	2.41	-8.42
Mech. Machinery	11.26	-3.34	-1.67	-1.94	1.43	-7.89
Vehicles	10.94	-1.17	-0.64	-1.80	-1.21	-8.91
Parts (Vehicles)	13.60	-1.06	-1.07	-2.26	1.90	-8.58
Oth. Vehicles	10.47	-1.95	-2.63	-2.10	1.93	-9.45
Furniture	25.82	-1.33	0.03	-1.66	-2.79	6.91

Industry	Empl. cont.	$\Delta\mathbf{mf}$	$\Delta\mathbf{MI}$	$\Delta\mathbf{t}$	$\Delta\bar{w}$	$\Delta\bar{s}$
Maintenance	19.04	-0.66	-1.96	-2.20	-1.80	0.98
Utilities	5.79	0.37	-1.24	-1.78	-6.26	-9.98
Water, sewage	19.98	0.67	0.13	-1.75	-5.32	1.57
Construction	27.10	0.78	0.21	-2.74	-3.60	7.78
Wholesale and retail	33.35	0.79	0.75	-1.13	-1.16	9.43
Ground transport	23.45	0.40	-0.64	-2.68	-1.18	2.88
Water transport	13.11	0.50	-1.92	-3.45	2.81	-9.50
Air transport	9.16	-1.33	-1.99	-4.14	2.60	-10.65
Storage	17.85	0.62	0.15	-1.54	2.17	-8.22
Accommodation	27.47	-4.54	0.90	-1.82	3.37	4.89
Food services	44.31	-0.10	1.00	-3.01	-4.54	26.28
Print editing	14.95	-0.77	-0.20	-2.20	0.71	-7.27
TV, radio, cinema	16.32	0.58	-0.14	-2.02	1.40	-8.17
Telecommunications	10.98	0.41	0.20	-1.58	-5.48	-7.25
Dev. Inform. Serv	13.38	-0.57	-0.34	-1.34	1.13	-10.17
Financ. Interm.	8.56	0.43	-0.22	-1.61	-1.99	-12.72
Real estate	2.05	0.28	-1.37	-1.91	-19.57	-0.05
Legal, Accounting serv.	18.23	0.65	0.20	-1.27	-2.94	-3.08
Archit. Engin. Serv.	16.54	0.60	-0.20	-1.75	-0.61	-6.17
Other. Prof. Serv.	17.91	0.61	-0.05	-2.53	-3.86	-0.92
non-real estate rentals	16.18	0.18	-0.45	-1.51	-4.06	-2.66
Oth. Admin. Serv.	33.00	0.58	0.92	-1.34	5.94	2.24
Surveillance. Secur. Acti.	37.53	0.90	1.30	-0.34	14.04	-3.05
Public admin.	15.97	0.60	0.47	-1.04	6.39	-15.12
Public educ.	28.88	0.80	1.28	-0.02	12.46	-10.31
Private educ.	37.35	0.80	1.02	-1.50	13.29	-0.94
Public health	23.59	0.73	1.02	-0.72	9.41	-11.52
Private health	28.57	0.79	0.50	-2.04	3.19	1.45
Art. Creat. Activities	41.52	-1.85	0.86	-1.32	0.07	19.09
Assoc. Oth. Personal serv.	45.55	1.03	-0.07	-3.69	4.06	19.54

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Chapter 3

SFC-IO-BR: A proof-of-concept model for Brazil

3.1 Introduction

In 2022 Brazil updated its Nationally Determined Contribution (NDC) to the Paris Agreement, committing to reduce its greenhouse gas emissions (GHG) by 43% below 2005 levels by 2030.¹ Despite initial reductions, Brazilian emissions in the last decade have shown a continuous increase, raising questions about the country’s ability to achieve its NDC targets.

The national efforts to reduce deforestation at the beginning of the century were translated into a decrease in total emissions led by the contraction of those from Land Use, Land Use Change and Forestry (LULUCF) between 2003 and 2010 (See Figure 3.1). LULUCF emissions, however, have shown a steady increase ever since, without a compensating reduction from the rest of the sectors of activity.

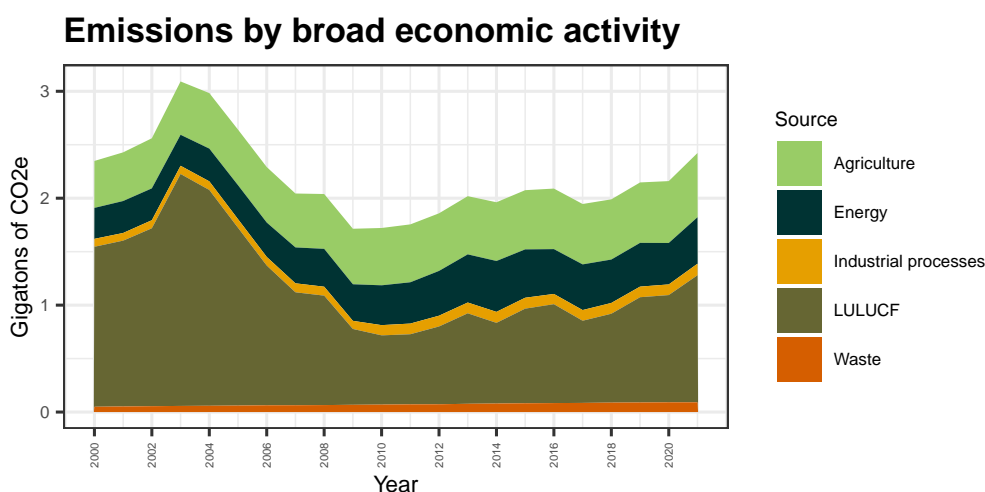


Figure 3.1: Total CO2 equivalent emissions by broad economic activity. Source: Author’s calculations based on SEEG data

By 2021, emissions by broad economic activity came mainly from LULUCF (49%), Agriculture (25%), Energy (18%), Industrial processes (4%), and Waste (4%). When compared to the NDC reference year, in 2021 total emissions were 92% of the ones observed in 2005, while these were at a historic low of 65% in 2010. While Brazil could achieve its NDC targets solely by fighting deforestation and reducing LULUCF emissions, it may need accompanying efforts from all other sectors. There are several challenges and opportunities to consider in that regard. First, economic dynamics in Brazil have mainly been driven by primary activities, namely mining, and agriculture (see Figure 3.2). A shock to these will have economy-wide effects, especially given the reliance of the country on these sector’s exports. Second, more than 80% of Brazil’s electricity already comes from renewable sources (BP (2018)), but the hydro-power capacity of the country could further improve, reducing its reliance on other fuels. Third, the government has seen its position

¹According to their submission to the United Nations Framework Convention on Climate Change (UNFCCC), Brazil has also set a goal of reaching net zero emissions by 2050.

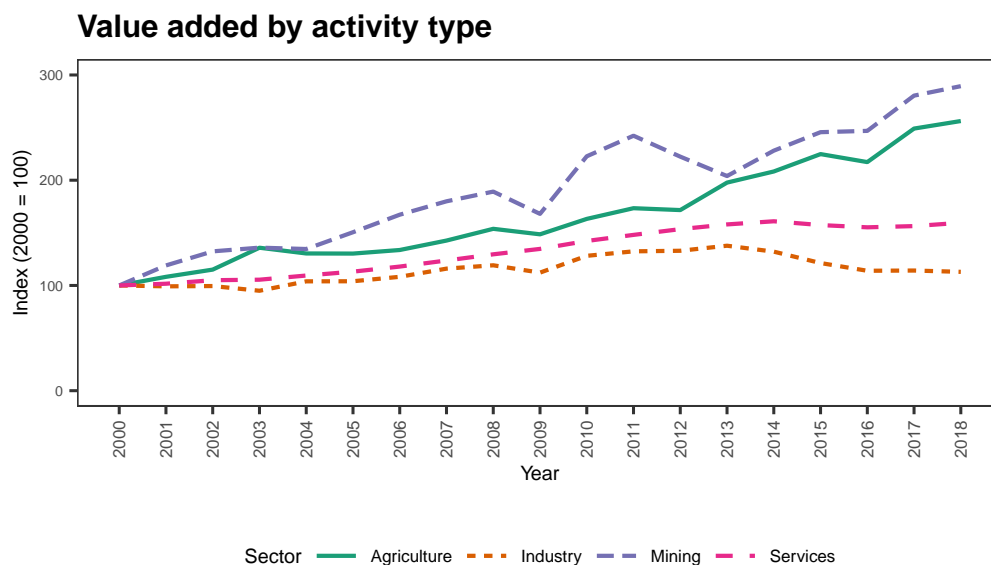


Figure 3.2: Value added by activity type (Index 2000 = 100).
Source: Author’s calculations based on IBGE data.

as a net borrower deepened in recent years (see Figure 3.3), which suggests the funding needed for transition efforts may further affect the government’s net borrowing position, or that the funding may not come from public accounts in the amounts required, and that additional international financing may be needed. Despite these challenges, Brazil has the potential to be a leader in the fight against climate change. To realize this potential, it is crucial to foster widespread awareness of the need for action while simultaneously providing information and tools to enhance understanding of the complex effects that mitigation policies can have on the economy. These effects vary across economic activities and can be spread through production and financial networks, requiring an in-depth study of multiple transmission channels. By gaining a better understanding of these dynamics, policymakers can better design mitigation policies, as well as compensation mechanisms that effectively address the needs of vulnerable and affected sectors. Emphasizing the necessary efforts and potential benefits can also contribute to increased political will for the timely implementation of mitigation measures.

Along those lines, the main objectives driving the construction of the model can be seen from a two-way perspective. On the one hand, this pilot version aims at exploring how macroeconomic policies or exogenous shocks can affect the domestic economy and, given its industrial structure, the evolution of CO_2 emissions. On the other hand, it looks to assess the effects that climate mitigation policies, such as the introduction of a carbon tax, could have at the industry and at the macro-financial level.

Several research programs have contributed to similar efforts through the construction of integrated assessment models (IAM) calibrated to Brazilian data. IMACLIM (Wills and Lefèvre (2012); Wills (2013); Lefèvre (2016)), BeGreen(Magalhaes (2013)), EFES(MCTIC (2017)), B-MARIA (Domingues et al. (2010)), and EPPA-BR (Silva and Gurgel (2012);

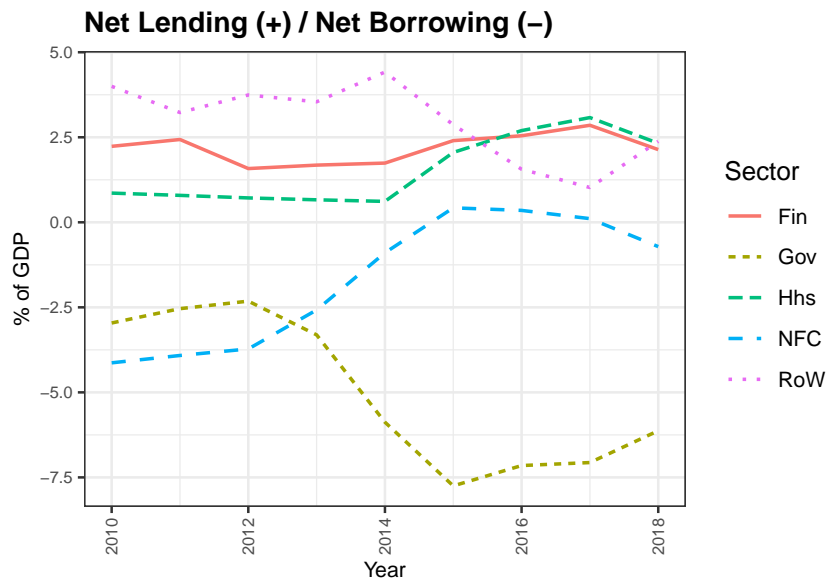


Figure 3.3: Evolution of net lending (+) or net borrowing (-) positions by sector.
Source: Author’s calculations based on IBGE data.

Prinn and Reilly (2017)), are the most prominent examples. These models exploit the information available in both input-output tables and energy balances. As a result, they all feature multisectoral models which include a set of energy products described in more detail than in standard input-output tables. Unlike the current exercise, they are all built under the computable general equilibrium framework (CGE), so their consumption and production decisions are determined by optimal choices given relative prices, and bounded according to certain substitution possibilities.² While featuring higher levels of detail with regards to the energy supply and demand, the macroeconomic and especially the macro-financial layers are less developed, with the focus often being set on estimating the effects of mitigation policies on GDP and employment (Cattan and McIsaac (2021)).

While similar in terms of objectives, the pilot model presented here differs in both focus and methodology. The IAM models mentioned before have been constructed with greater detail on the energy side of the model. The macroeconomic layers, however, have remained less developed. The current exercise features the opposite balance, with the energy sector developed to exploit national accounting data without increasing the detail to match the one from energy balances. On the macro side, however, the interactions are not limited to key aggregates, but also to the modeling of the main macro-financial accounts and their evolution.

Regarding the methods, the model does not rely on a CGE framework and does not impose equilibrium conditions nor the existence of price clearing mechanisms. It instead simulates supply and demand behavior without imposing clearing conditions on non-financial markets and allowing not only for mismatches between supply and demand, but also for

²Rafael Cattan and Florent McIsaac provide a detailed comparison of these models and a critical assessment of their approach (Cattan and McIsaac (2021)).

production and investment to be funded on-demand –dropping the assumption that only pre-existent savings can be lent. Moreover, this exercise proposes a two-layered tool, combining the macro-modeling approach of stock-flow consistent models (SFC) (Godley and Lavoie (2006), Nikiforos and Zezza (2018), Zezza and Zezza (2019)), with the multi-sectoral perspective given by the input-output framework (Miller and Blair (2009), Mahajan et al. (2018)).

As such, the model is part of a growing set of literature that relies on the SFC framework to analyze questions related to ecological macroeconomics, while being one of the few that has done so by coupling the analysis with the input-output framework for a specific country. Regarding the former, pure SFC models have already tackled similar questions either from a theoretical or global perspective (Naqvi (2015)), (Monasterolo and Raberto (2018)), and Bovari et al. (2018)), or from a country-specific point of view (Jackson and Victor (2020)). On the other hand, SFC models extended with the input-output framework have either been built with a focus on physical input-output tables (IOT) (Dafermos et al. (2017)), or monetary ones (? , Valdecantos (2021), and Jackson (2019)).

In this regard, the closest model in objectives and approach is that of Valdecantos (Valdecantos (2021)), who builds –to the best of my knowledge– one of the sole empirical SFC-IO models to date for a whole country, calibrated to Argentina. Unlike his work, which exploits in more detail the macro-financial relationships by studying the evolution of the relevant variables for a longer time window and with higher frequency, the Brazilian pilot has focused on the few available annual data points. In contrast, the pilot has been built based on a richer dataset describing the production network. Indeed, the time series of input-output data used for calibration was constructed to keep coherence with the key assumptions of the model, and respect the consistency with national accounting data. The discussion of the challenges encountered and the solutions proposed along that process are presented in the Appendices, with the expectation that they could provide new researchers engaging in similar exercises with enough information so that they can be aware of the existence of key modeling choices that have to be made from early stages of the process.

The rest of the chapter is structured as follows. Section 3.1.1 describes the level of detail used in each layer, section 3.2 presents the model’s equation and accounting conventions, section 3.3 provides results for the baseline simulations, present the policies implemented, and comments on the results obtained; and section 3.4 concludes, discusses the current model’s limitations, and presents potential lines of improvement and future research. Finally, Appendix 3.A provides additional figures, and Appendix 3.B discusses the sources of information and the process used to build the dataset on which the model is calibrated.

3.1.1 Level of detail

The model features two layers, with the economic activity presented at different levels of detail in each. The macro-financial layer focuses on all monetary transactions between agents, as well as the evolution of their balance sheets. At this level, the economy is subdivided into five institutional sectors:

1. Households (H)
2. Non-Financial Corporations (C)
3. Financial Corporations (F)
4. Government (G)
5. Rest of the World (W)

At a second layer, the production module expands these sectors in order to model the interactions between industries. The module takes the evolution of the key aggregate indicators as an input, computes the relevant variables at a higher level of detail, and feeds aggregate information back into the macro layer. In the module, the institutional sector of Non-Financial Corporations is dis-aggregated into industrial activities,³ which along with the provision of services by the Financial Corporations and the Public Administration, account for a total of twelve industrial clusters. For simplicity, we will refer to these interchangeably as industries, industrial clusters, or clusters, even when including those in charge of the provision of public administration services.

1. Agriculture (AGR):
2. Mineral extraction (MIN)
3. Oil and gas extraction (O&G)
4. Oil derivatives (ODV)
5. Bio-fuels and alcohol (BIO)
6. Utilities, including generation and distribution of electricity (ELE)
7. Machinery and construction (M&C)
8. Industries not specified elsewhere (IND)
9. Transportation (TRP)
10. Services not specified elsewhere (SRV)

³The correspondence between the industries reported in the National Accounts and the ones used in the model is provided in Appendix 3.B.3

11. Financial intermediation (FIN)
12. Public administration (ADM)

While remaining close to the some sector aggregations commonly used in national accounts, this configuration sets a greater focus on the industrial activities related to carbon-intensive sectors and to the energy generation and distribution.⁴

The input-output approach is used here to provide details regarding the transactions on the markets of goods, services, and factors of production. The aggregate layer takes these inputs and completes them with information about the rent generation, the primary and secondary income allocation, the rent use, and the transactions regarding fixed capital. Once considering all these, the financing capacity –or financing requirements– per institutional sector can be retrieved and studied.

The financing capacity is looked from a portfolio perspective that allows us to build a set of financial accounts. These accounts present the stock and the evolution of the asset and liability positions for each institutional sector and by each type of financial instrument. As for the latter, six types of financial instruments are considered in the model:

1. Reserves (R): gold and Special Drawing Rights
2. Cash (H)
3. Deposits (D)
4. Bonds (B): bonds, loans, and other obligations subject to interest payments.
5. Shares (S): equities, and participation in investment or pension funds.
6. Other financial assets (Z)

3.2 The macroeconomic model

3.2.1 The production module: an input-output production economy.

The production module explores the transactions of goods, services, and factors of production at the industrial level. Depending on the case, these interactions are modeled using different valuation methods: real terms,⁵ basic prices, or market prices.⁶

⁴The clustering strategy can be adjusted so that the aggregation of industries responds to a different focus of interest. For instance, if aiming at analyzing recovery policy alternatives as a response to the COVID pandemic, one may want to explicitly model the public and private provision of health services separately, as well as the production of chemical and pharmaceutical products, among others. Along these lines, while the pandemic is still evolving, the extent and magnitude of its heterogeneous effects across the economy are far from being fully identified. Not being the purpose of the current version of the model, this analysis and the study of potential recovery plans, as well as the use of other potential industrial aggregations are left for further research.

⁵Computed using the basic price of the base year, 2010, equal to unity.

⁶Within the module, the subscripts R , BP or MP , will signal when a variable is expressed in real terms, basic prices or market prices, respectively. For the aggregate layer, such distinctions will not be needed and a unique valuation will be used for each variable.

This section details the different blocks of the module. First, the production and pricing decision process is presented. Then, the demand by the different types of agents is described. In the third step, total supply and demand, as well as mismatches between both sides are discussed. Then, the additional dynamics occurring at the production module level are presented. Subsequently, the Value Added per industry, as well as its components, is computed. Finally, the aggregation of key variables to be reported to the macro layer is shown.

The production and pricing decisions

Domestic production

Each industry i produces a single commodity using capital, labor, and inputs from the other industries. The maximum *throughput* or gross production in real terms, is denoted $Q_{R,i,t}^{max}$, and depends linearly on the existing stock of capital for that cluster at the beginning of period t , $K_{R,i,t}$:⁷

$$Q_{R,i,t}^{max} = \frac{K_{R,i,t}}{\nu_{i,t}} \quad (3.1)$$

where $\nu_{i,t} > 0$ is the capital-to-output ratio for industry i during period t .

The *desired* production in sector i , $\tilde{Q}_{R,i,t}$, is assumed to follow expected total demand in real terms, and defines the *desired* rate of capacity utilization as:

$$\tilde{u}_{i,t} = \frac{TD_{R,i,t}^{EXP}}{Q_{R,i,t}^{max}} \quad (3.2)$$

The *actual* rate of capacity utilization is bounded above, and is assumed to be subject to frictions preventing its full adjustment towards the desired one as follows:

$$u_{i,t} = \min \{1, \lambda^u u_{i,t-1} + (1 - \lambda^u) \tilde{u}_{i,t}\} \quad (3.3)$$

Finally, producing one unit of i requires not only some existing installed capital, but also $a_{ji,t} \geq 0$ units of commodity j , as well as $a_{li,t} \geq 0$ units of labor.⁸ These are all used according to a Leontief technology:

$$Q_{R,i,t} = \min \left\{ \frac{u_{i,t} K_{R,i,t}}{\nu_{i,t}}, \frac{L_{i,t}}{a_{li,t}}, \frac{IC_{R,ij,t}^{max}}{a_{ji,t}} \right\} \quad (3.4)$$

where $IC_{R,ij,t}^{max}$ is the maximum quantity of good j available to sector i during period t , and $L_{i,t}$ is the amount of labor employed (full-time equivalent jobs) in sector i for that same period, and where the amount of good j required for a unit of i to be produced during time

⁷This considers the potential production by an industry operating at full capacity.

⁸Commodities used as inputs can be of domestic or foreign origin, while labor is assumed to be domestic only.

t , $a_{ji,t}$, is an element of the matrix of deflated technical coefficients, $A_t \in \mathcal{M}_{N \times N}(\mathbb{R}^+)$:⁹

$$A_t = \begin{bmatrix} a_{11,t} & \dots & a_{1N,t} \\ \vdots & \ddots & \vdots \\ a_{N1,t} & \dots & a_{nN,t} \end{bmatrix},$$

Pricing

Industries are assumed to set their basic or pre-tax prices by applying their *desired* markup over their expected unitary cost.¹⁰¹¹

$$\overrightarrow{P_{BP,t}} = \left(1 + \overrightarrow{\mu_t^D}\right) \odot \overrightarrow{UC_t^E}$$

where the \odot operator denotes the Hadamard product or element-wise multiplication.

The vector of expected cost per unit, $\overrightarrow{UC_t^E}$, is on itself dependent on expected domestic and foreign market prices, on the cost of labor per unit of production, and on the last known structure of the matrix of deflated technical coefficients:

$$\overrightarrow{UC_t^E} = \left(A_{t-1}^d\right)^T \times \overrightarrow{P_{MP,t}^{d,E}} + E_t \cdot \left(A_{t-1}^f\right)^T \times \overrightarrow{P_{MP,t}^{f,E}} + \overrightarrow{w_t} \odot \overrightarrow{a_{l,t}} \quad (3.5)$$

where E_t represents the foreign exchange rate in quantities of domestic currency per unit of foreign one, A_{t-1}^d and A_{t-1}^f stand for the domestic and imported components of the technical coefficients' matrix from the previous period, so that $A_{t-1} = A_{t-1}^d + A_{t-1}^f$, $\overrightarrow{w_t}$ denotes the wage rate vector in time t per labor unit, and $\overrightarrow{a_{l,t}}$ the vector of labor required per unit produced. These variables are all known at the beginning of period t , unlike the market prices for domestic and foreign goods, for which the industries only have an expected value $\overrightarrow{P_{MP,t}^{d,E}}$ and $\overrightarrow{P_{MP,t}^{f,E}}$, respectively.

Let agents assume the expected inflation equals the last observed one. The domestic and foreign expected market prices can be then built as:

$$\overrightarrow{P_{MP,t}^{d,E}} = \left(\left(1 + \pi_{t-1}^d\right) \cdot \overrightarrow{P_{BP,t-1}^d}\right) \odot \left(1 + \overrightarrow{\tau_t}\right) \quad (3.6)$$

$$\overrightarrow{P_{MP,t}^{f,E}} = \left(\left(1 + \pi_{t-1}^f\right) \cdot \overrightarrow{P_{BP,t-1}^f}\right) \odot \left(1 + \overrightarrow{\tau_t}\right) \odot \left(1 + \overrightarrow{c_{f,t}}\right) \quad (3.7)$$

with $\overrightarrow{\tau_t}$ and $\overrightarrow{c_{f,t}}$ being the vectors of indirect tax rates and customs fees, respectively.

Once industries have computed the expected market prices, they can reconstruct their

⁹Despite the time indexing, for simplicity the matrix will be assumed to be constant, and the study of the evolution of its components is left for further research. This assumption is fairly consistent with the observed evolution of deflated technical coefficients, as it is depicted in Figure 3.43 from Appendix 3.B.1.

¹⁰Since prices are determined simultaneously, industries cannot directly estimate the *actual* cost of their intermediate consumption. They, hence, work with expected values based on the known information to define the prices they will post for the current period. Their *actual* markup will be also obtained ex post.

¹¹Notice that this simplified behavior abstracts from the fact that some industrial sectors, especially primary-extractive ones, may instead be international-price-takers. Such variations are left for further research.

expected unitary cost, and with it define the *actual* basic prices they will post for the current period. Given that tax rates and customs fees are known, agents will also be able to compute the *actual* market prices for the period:

$$\overrightarrow{P_{MP,t}^d} = \overrightarrow{P_{BP,t}^d} \odot (1 + \overrightarrow{\tau}_t) \quad (3.8)$$

$$\overrightarrow{P_{MP,t}^f} = \overrightarrow{P_{BP,t}^f} \odot (1 + \overrightarrow{\tau}_t) \odot (1 + \overrightarrow{c}_{f,t}) \quad (3.9)$$

With basic prices being defined, and with indirect tax rates and customs fees known, industries will now be able to compute their *actual* unit cost and the markup rate that verifies:

$$\overrightarrow{UC}_t = (A_t^d)^T \times \overrightarrow{P_{MP,t}^d} + E_t \cdot (A_t^f)^T \times \overrightarrow{P_{MP,t}^f} + \overrightarrow{w}_t \odot \overrightarrow{a}_{l,t} \quad (3.10)$$

$$\overrightarrow{P_{BP,t}} = (1 + \overrightarrow{\mu}_t) \odot \overrightarrow{UC}_t \quad (3.11)$$

The demand side

Households' consumption

Each period, households are assumed to define the total budget they will allocate for the consumption of each type of good, in market prices. They do so by indexing their prior budget with the expected nominal growth of the economy, $g_t^{Y_{NOM}, E}$, adjusted by the last known relative change of the wage share, ω_t .

$$\overrightarrow{C_{MP,t}} = \overrightarrow{C_{MP,t-1}} \cdot (1 + g_t^{Y_{NOM}, E} + \lambda^\omega \cdot g_{t-1}^\omega) \quad (3.12)$$

where λ^ω represents the sensitivity of the consumption budget's growth rate, g_{t-1}^ω , with respect to the lagged relative variation of the wage share, $\frac{\omega_t - 1}{\omega_{t-2}} - 1$. For simplicity, it is assumed that λ^ω is constant in time and across goods.

For each type of product i , the share $\theta_{i,t}^d$ of the budget in market prices will be spent on domestic goods. The complement will be spent on imported goods purchased from domestic industries acting as intermediaries. The budget devoted to domestic consumption in market prices, $\overrightarrow{C_{MP,t}^d}$ verifies:

$$\overrightarrow{C_{MP,t}} = \overrightarrow{C_{MP,t}^d} + \overrightarrow{C_{MP,t}^f} \quad (3.13)$$

$$\overrightarrow{C_{MP,t}^d} = \text{diag}(\overrightarrow{\theta}_t^d) \times \overrightarrow{C_{MP,t}} \quad (3.14)$$

where $\text{diag}(\overrightarrow{x})$ is the operator transforming vector \overrightarrow{x} into a diagonal matrix.

Consumption can then be retrieved in real terms, by origin:

$$\begin{aligned}\overrightarrow{C_{R,t}^d} &= \left(\text{diag} \left(\overrightarrow{P_{MP,t}^d} \right) \right)^{-1} \times \overrightarrow{C_{MP,t}^d} \\ &= \left(\text{diag} \left(\overrightarrow{P_{MP,t}^d} \right) \right)^{-1} \times \text{diag} \left(\overrightarrow{\theta_t^d} \right) \times \overrightarrow{C_{MP,t}^d}\end{aligned}\quad (3.15)$$

$$\begin{aligned}\overrightarrow{C_{R,t}^f} &= \left(\text{diag} \left(E_t \cdot \overrightarrow{P_{MP,t}^f} \right) \right)^{-1} \times \overrightarrow{C_{MP,t}^f} \\ &= \left(\text{diag} \left(E_t \cdot \overrightarrow{P_{MP,t}^f} \right) \right)^{-1} \times \left(\mathbf{I}_N - \text{diag} \left(\overrightarrow{\theta_t^d} \right) \right) \times \overrightarrow{C_{MP,t}^f}\end{aligned}\quad (3.16)$$

with \mathbf{I}_N representing an identity matrix of order N . It follows that the total consumption in real terms per type of good will be given by:

$$\begin{aligned}\overrightarrow{C_{R,t}} &= \overrightarrow{C_{R,t}^d} + \overrightarrow{C_{R,t}^f} \\ &= \Gamma_t \times \overrightarrow{C_{MP,t}}\end{aligned}\quad (3.17)$$

where $\Gamma_t \in \mathcal{M}_{N \times N}(\mathbb{R}^+)$ is given by:¹²

$$\Gamma_t = \left(\text{diag} \left(\overrightarrow{P_{MP,t}^d} \right) \right)^{-1} \times \text{diag} \left(\overrightarrow{\theta_t^d} \right) + \left(\text{diag} \left(E_t \cdot \overrightarrow{P_{MP,t}^f} \right) \right)^{-1} \times \left(\mathbf{I}_N - \text{diag} \left(\overrightarrow{\theta_t^d} \right) \right)\quad (3.18)$$

Gross fixed capital formation by industry and product

The model allows different products to be used as capital goods by different industries. Investment by cluster and type of product used is recorded in the capital absorption matrix. For any given period, flows in real terms are assumed to be given by:

$$I_{R,t}^K = I_{R,t-1}^K \times \text{diag} \left(1 + g_t^{Y_{R,E}} + \delta^K + \kappa^\nu \cdot g_t^\nu + \kappa^u \cdot g_t^u + \kappa^\omega \cdot g_t^\omega \right)\quad (3.19)$$

with $I_{R,t}^K \in \mathcal{M}_{N \times N}(\mathbb{R}^+)$ denoting the capital absorption matrix in real terms for period t , and δ^K the average depreciation rate. The terms g_t^ν , g_t^u , and g_t^ω stand for the growth rate of the capital-to-output ratio, the utilization rate, and the wage share, respectively. The accompanying κ coefficients represent the sensitivity of the investment growth in real terms with respect to these elements.

Once the total investment in real terms is determined by the type of good and industry, the total values in market prices can be found. Subsequently, the classification by origin

¹²The vector of domestic shares of consumption, $\overrightarrow{\theta_t^d}$, is defined to be applied over variables in market prices. The shares in real terms will certainly differ. Hence, the distribution of consumption by origin can only be done once the total consumption per good in market prices is obtained. For some types of agents and consumption purposes, the model assumes this is the starting point, as the consumption choices are determined in market prices, divided by origin, and only then converted into real terms as a last step. For other purposes –like intermediate consumption–, the process will be applied backward. The matrix Γ_t becomes handy when implementing these processes.

can take place, first, in market prices, and then in real terms:

$$I_{MP,t}^K = \Gamma_t^{-1} \times I_{R,t}^K \quad (3.20)$$

$$I_{MP,t}^{d,K} = \text{diag} \left(\overrightarrow{\theta}_t^d \right) \times I_{MP,t}^K \quad (3.21)$$

$$I_{MP,t}^{f,K} = \left(\mathbf{I}_N - \text{diag} \left(\overrightarrow{\theta}_t^d \right) \right) \times I_{MP,t}^K \quad (3.22)$$

$$I_{R,t}^{d,K} = \left(\text{diag} \left(\overrightarrow{P}_{MP,t}^d \right) \right)^{-1} \times I_{MP,t}^{d,K} \quad (3.23)$$

$$I_{R,t}^{f,K} = \left(\text{diag} \left(E_t \cdot \overrightarrow{P}_{MP,t}^f \right) \right)^{-1} \times I_{MP,t}^{f,K} \quad (3.24)$$

Government's consumption

The government's consumption of final goods and services in market prices is assumed to evolve according to the expected nominal GDP growth rate.

$$\overrightarrow{G}_{MP,t} = \left(1 + g_t^{Y_{NOM},E} \right) \times \overrightarrow{G}_{MP,t-1} \quad (3.25)$$

The distribution by origin follows the same procedure as the one presented for households' consumption.

$$\overrightarrow{G}_{MP,t} = \overrightarrow{G}_{MP,t}^d + \overrightarrow{G}_{MP,t}^f \quad (3.26)$$

$$\overrightarrow{G}_{MP,t}^d = \text{diag} \left(\overrightarrow{\theta}_t^d \right) \times \overrightarrow{G}_{MP,t} \quad (3.27)$$

Information in real terms by origin can now be obtained.

$$\begin{aligned} \overrightarrow{G}_{R,t}^d &= \left(\text{diag} \left(\overrightarrow{P}_{MP,t}^d \right) \right)^{-1} \times \overrightarrow{G}_{MP,t}^d \\ &= \left(\text{diag} \left(\overrightarrow{P}_{MP,t}^d \right) \right)^{-1} \times \text{diag} \left(\overrightarrow{\theta}_t^d \right) \times \overrightarrow{G}_{MP,t} \end{aligned} \quad (3.28)$$

$$\begin{aligned} \overrightarrow{G}_{R,t}^f &= \left(\text{diag} \left(E_t \cdot \overrightarrow{P}_{MP,t}^f \right) \right)^{-1} \times \overrightarrow{G}_{MP,t}^f \\ &= \left(\text{diag} \left(E_t \cdot \overrightarrow{P}_{MP,t}^f \right) \right)^{-1} \times \left(\mathbf{I}_N - \text{diag} \left(\overrightarrow{\theta}_t^d \right) \right) \times \overrightarrow{G}_{MP,t} \end{aligned} \quad (3.29)$$

Aggregating the components, the total government consumption in real terms is obtained.

$$\overrightarrow{G}_{R,t} = \overrightarrow{G}_{R,t}^d + \overrightarrow{G}_{R,t}^f \quad (3.30)$$

$$= \Gamma_t \times \overrightarrow{G}_{MP,t} \quad (3.31)$$

Intermediate consumption

Industries purchase commodities or services to be used as inputs in their production processes. Once the actual level of desired production is determined, the matrix of in-

intermediate consumption in real terms required to fulfill that throughput can be obtained as:

$$IC_{R,t} = A_t \times \text{diag}(\overrightarrow{Q_{R,t}}), \quad (3.32)$$

$$= (A_t^d + A_t^f) \times \text{diag}(\overrightarrow{Q_{R,t}}) \quad (3.33)$$

$$= IC_{R,t}^d + IC_{R,t}^f \quad (3.34)$$

where each element $IC_{R,ji,t}$ of the matrix states the total volume of inputs used from good of type j in the entire production of industry i during period t .

Using Γ_t and θ_t^d , we can obtain the equivalent intermediate consumption matrix in market prices, and split it by origin:

$$IC_{MP,t} = \Gamma_t^{-1} \times IC_{R,t} \quad (3.35)$$

$$IC_{MP,t}^d = \text{diag}(\overrightarrow{\theta_t^d}) \times IC_{MP,t} \quad (3.36)$$

$$IC_{MP,t}^f = \left(\mathbf{I}_N - \text{diag}(\overrightarrow{\theta_t^d}) \right) \times IC_{MP,t} \quad (3.37)$$

$$IC_{R,t}^d = \left(\text{diag}(\overrightarrow{P_{MP,t}^d}) \right)^{-1} \times IC_{MP,t}^d \quad (3.38)$$

$$IC_{R,t}^f = \left(\text{diag}(\overrightarrow{E_t \cdot P_{MP,t}^f}) \right)^{-1} \times IC_{MP,t}^f \quad (3.39)$$

The results obtained from equations (3.38) and (3.39) verify (3.34). Similarly, the results from equations (3.35), (3.36) and (3.37) will be consistent with:

$$IC_{MP,t} = \text{diag}(\overrightarrow{P_{MP,t}^d}) \times IC_{R,t}^d + E_t \cdot \text{diag}(\overrightarrow{P_{MP,t}^f}) \times IC_{R,t}^f \quad (3.40)$$

Exports

Exports of goods and services in real terms are assumed to evolve according to the foreign real GDP growth, the variation of the relative prices between foreign and domestic goods, and the lagged change in the Net Financing Capacity of the Rest of the World, $Agg_t^{B9,W}$, such that:

$$\overrightarrow{X_{R,t}} = \overrightarrow{X_{R,t-1}} \odot \left(1 + g_{R,t}^{Y^f} + \lambda^{RelPr} \cdot g_t^{RelPr} + \lambda^{B9,W} \cdot \left(\frac{Agg_{t-1}^{B9,W}}{Agg_{t-2}^{B9,W}} - 1 \right) \right) \quad (3.41)$$

where each i element from the vector denoting the growth rate of the relative prices is

given by $g_{i,t}^{RelPr} = \frac{E_t \cdot P_{i,t}^f}{E_{t-1} \cdot P_{i,t-1}^f} \cdot \frac{P_{i,t-1}}{P_{i,t}} - 1$. Finally, in basic prices,¹³ exports would be:

$$\overrightarrow{X_{BP,t}} = \overrightarrow{P_{BP,t}^d} \odot \overrightarrow{X_{R,t}} \quad (3.42)$$

Imports

The industry producing good of type i will also act as the intermediary agent importing foreign good i . Imports are assumed to take place on demand. Their valuation in real terms, basic prices and market prices is given respectively by:

$$\overrightarrow{M_{R,t}} = \overrightarrow{C_{R,t}^f} + \left(I_{R,t}^{f,K} + IC_{R,t}^f \right) \times \overrightarrow{1_N} + \overrightarrow{G_{R,t}^f} \quad (3.43)$$

$$\begin{aligned} \overrightarrow{M_{BP,t}} &= E_t \cdot \overrightarrow{P_{BP,t}^f} \odot \overrightarrow{M_{R,t}} \\ &= \overrightarrow{C_{BP,t}^f} + \left(I_{BP,t}^{f,K} + IC_{BP,t}^f \right) \times \overrightarrow{1_N} + \overrightarrow{G_{BP,t}^f} \end{aligned} \quad (3.44)$$

$$\begin{aligned} \overrightarrow{M_{MP,t}} &= E_t \cdot \overrightarrow{P_{MP,t}^f} \odot \overrightarrow{M_{R,t}} \\ &= \overrightarrow{C_{MP,t}^f} + \left(I_{MP,t}^{f,K} + IC_{MP,t}^f \right) \times \overrightarrow{1_N} + \overrightarrow{G_{MP,t}^f} \end{aligned} \quad (3.45)$$

where $\overrightarrow{1_N}$ is a column unit vector, of dimension $N = \text{number of industries}$.¹⁴

Total supply, demand and inventories

Total supply results from the addition of total local production and imports. In real terms, basic prices and market prices, this yields:

$$\overrightarrow{TS_{R,t}} = \overrightarrow{Q_{R,t}} + \overrightarrow{M_{R,t}} \quad (3.46)$$

$$\overrightarrow{TS_{BP,t}} = \overrightarrow{Q_{BP,t}} + \overrightarrow{M_{BP,t}} \quad (3.47)$$

$$\overrightarrow{TS_{MP,t}} = \overrightarrow{Q_{MP,t}} + \overrightarrow{M_{MP,t}} \quad (3.48)$$

Total demand addresses to domestic industries groups the flows of intermediate consumption and the final demand flows from both domestic and foreign agents. In real terms and

¹³In general, since there are no trade and transportation margins, the only difference between basic and market prices will be the indirect tax rates and customs fees. For all other flows, the valuation in basic prices is not explicitly presented but can be readily obtained. Regarding exports and imports, the valuation in basic prices is relevant for the computation of the expenditure and receipts of the Rest of the World.

¹⁴Note that since both $(I_{BP,t}^{f,K}$ and $IC_{BP,t}^f$ are matrices, post-multiplying them by $\overrightarrow{1_N}$ yields their row sums, and thus the total imported demand of each good for either investment or intermediate consumption purposes.

basic prices, it can be expressed as:¹⁵

$$\overrightarrow{TD}_{R,t}^d = \overrightarrow{C}_{R,t}^d + \left(I_{R,t}^{d,K} + IC_{R,t}^d \right) \times \overrightarrow{1}_N + \overrightarrow{G}_{R,t}^d + \overrightarrow{X}_{R,t} \quad (3.49)$$

$$\overrightarrow{TD}_{BP,t}^d = \overrightarrow{C}_{BP,t}^d + \left(I_{BP,t}^{d,K} + IC_{BP,t}^d \right) \times \overrightarrow{1}_N + \overrightarrow{G}_{BP,t}^d + \overrightarrow{X}_{BP,t} \quad (3.50)$$

Since imports take place on demand, the domestic component of the total supply will be just domestic production, which does not necessarily match the demand addressed to domestic industries. The resulting discrepancies will result in stock variations:¹⁶

$$\overrightarrow{SV}_{R,t} = \overrightarrow{Q}_{R,t} - \overrightarrow{TD}_{R,t}^d \quad (3.51)$$

$$\overrightarrow{SV}_{BP,t} = \overrightarrow{Q}_{BP,t} - \overrightarrow{TD}_{BP,t}^d \quad (3.52)$$

Value added and its components

The vector of value added for the domestic industries is defined by:

$$\overrightarrow{VA}_t = \overrightarrow{Q}_{BP,t} - \overrightarrow{1}_N \times IC_{MP,t} \quad (3.53)$$

Compensation of employees in industry i is given by the amount of labor used, L_i , and the wage rate on that sector w_i :¹⁷

$$\begin{aligned} \overrightarrow{CE}_t &= \overrightarrow{L}_t \odot \overrightarrow{w}_t \\ &= \overrightarrow{L}_t \odot \overrightarrow{w}_{t-1} \cdot (1 + g_t^w) \end{aligned} \quad (3.54)$$

Net other taxes are assumed to be the result of a constant vector of rates expressing the net other taxes, charged on the current level of production:

$$\overrightarrow{NTP}_t = \overrightarrow{\tau}^{NTP} \odot \overrightarrow{Q}_{BP,t} \quad (3.55)$$

The gross operating surplus by industry is obtained as a residual:

$$\overrightarrow{GOS}_t = \overrightarrow{VA}_t - \left(\overrightarrow{CE}_t + \overrightarrow{NTP}_t \right) \quad (3.56)$$

Additional dynamics within the module

Capital accumulation

At any given period, the installed capacity each industry has, by type of good in real

¹⁵It should be noted that exports are not charged with taxes, so their valuation in market prices is the same as in basic prices. Moreover, total demand is not computed in market prices, as not all demand may be fulfilled, and hence not all desired purchases will end up in a transaction subject to taxation.

¹⁶Inventories are valued at replacement cost, which is assumed to be in basic prices and hence excluding taxes. Thus, no market price valuation for them is computed.

¹⁷It is assumed that all wages have the same growth rate, which is determined at the macro layer.

terms, is given by the capital stock matrix, $KS_{R,t}$:

$$KS_{R,t} = (1 - \delta^K) \cdot KS_{R,t-1} + I_{R,t-1}^K \quad (3.57)$$

For simplicity, the production function is built using instead a capital index vector, with a single entry per industry, defined as:

$$\vec{K}_t = (KS_{R,t})^T \times \vec{1}_N \quad (3.58)$$

Technical progress

Both capital and labor are assumed to be subject to technical progress. The evolution of the capital-to-output ratio and the labor requirements per unit produced are assumed to be exogenous and equal for all industries, so that:

$$\vec{\nu}_t = (1 + g_t^\nu) \cdot \vec{\nu}_{t-1} \quad (3.59)$$

$$\vec{a}_{\ell,t} = (1 + g_t^{a_\ell}) \cdot \vec{a}_{\ell,t-1} \quad (3.60)$$

Markup dynamics

Industries set a target ratio for inventories as a share of production. They adjust their *desired* markup for the current period depending on how the observed ratio from the previous one differed from the target:

$$\vec{\mu}_t^D = \vec{\mu}_{t-1}^D \odot \left(1 + \lambda^\mu \cdot \left(\overrightarrow{SVQ}_{t-1} - \overrightarrow{SVQ} \right) \right) \quad (3.61)$$

where \overrightarrow{SVQ}_t is the lagged ratio of stock variation relative to production, so that for each industry i , we have $SVQ_{i,t} = \frac{SV_{R,i,t-1}}{Q_{R,t,t-1}}$, and \overrightarrow{SVQ} is the target ratio, which needs not be the same across industries.

Industrial CO_2 emissions and emissions' dynamics

Productive activities are assumed to produce CO_2 emissions based on a linear relationship between the output level and industry-specific emission factors. The emission vector by industry can be expressed as:

$$\vec{EM}_t = \vec{Q}_{R,T} \odot \vec{em}_t^{CO_2} \quad (3.62)$$

where the vector of emission factors, $\vec{em}_t^{CO_2}$, is assumed to be subject to emission improvements, and hence decreases annually at a steady pace:

$$\vec{em}_t^{CO_2} = \frac{1}{1 + g^{em}} \cdot \vec{em}_{t-1}^{CO_2} \quad (3.63)$$

Aggregation of key variables

Once all relevant variables are computed at the industry level with the required valuations, a subset of these is aggregated to feed the macro layer of the model. In most cases, variables on the macro layer represent detailed who-to-whom monetary transactions between institutional sectors and will be treated as multi-dimensional arrays.¹⁸

In that regard, the underlying assumption that households are not direct producers of goods or services allows the implementation of a direct matching between all industries from the production module and three institutional sectors from the macro layer.

The institutional sector of Financial Corporations (F) is matched to the industry of Financial intermediation (FIN). Similarly, the Government (G) institutional sector is assumed to match the industry in charge of the provision of services linked to the Public administration (ADM). All other industries are grouped under the Non-Financial Corporations (C) institutional sector.

Consumption of final goods and services

At the macro layer, consumption represents -in a single matrix for any given year- the set of all market transactions regarding the consumption of the households, government, and the rest of the world that was satisfied with both local production or imports. It follows, that the consumption matrix will also represent all funds that the respective producing sectors received in exchange for the final products they have either manufactured and sold or imported for their clients.

Consider the example below, where the entry $C_t^{C,H}$ represents the consumption by households of all goods of the types produced or imported by the Non-Financial Corporations (C), and corresponds to the following aggregation of the consumption vector from the production module:

$$C_t^{C,H} = \sum_{i \in \mathcal{A}^C} C_{MP, i, t}$$

where $\mathcal{A}^C = \{\text{AGR, MIN, O\&G, ODV, BIO, ELE, M\&C, IND, TRP, SRV}\}$.

It should be noted that the market price valuation is used. This is the case for both domestically produced and imported goods, as they are purchased by local agents subject to domestic indirect taxes. This won't be the case for consumption of the rest of the World, as exports entering the consumption matrix are exempt from local indirect taxes and recorded in basic prices only.

This accounting convention suggests that producing -and importing- sectors are acting as collectors of indirect taxes and customs fees, which will need to be subsequently transferred to the government.

¹⁸To avoid confusion, notation is defined to have multiple indices on each variable whenever the who-to-whom structure is used. For monetary flows and balance sheet accounts, the notation has been set to use consistent rules. The order of indices indicates (1) the provider of the good or service, or the issuer of financial liability, and (2) the purchaser of the goods or services, or holder of the financial asset.

Gross fixed capital formation

The capital absorption matrix obtained in the production module, in market prices, is reported to the macro layer once the aggregation of industries into institutional sectors is done according to the mapping previously described.

The resulting array called $GFCE_t$ is such that the entry $GFCE_t^{C,F}$, for example, expresses the market value of the investment done by the Financial corporations (F), using goods produced or imported by the Non-financial corporations (C).

As expected, institutional sectors can both be providers or purchasers of goods serving as capital, as will be explicitly shown in Table 3.3.

The assumption of non-producing households needs to be considered here as well. This simplification does not match reality. Certain households indeed act as producers and they require capital goods for such purposes. They invest, and hence this outflow of funds needs to be accounted for to avoid misestimations of their net financing capacity at the end of a given period. To correct the effect of the simplifying assumption, adjustment accounts are introduced on the macro layer and are calibrated to historical information. These adjustments are shown in Table 3.3, and verify:

$$ADJ_t^{GFCE,H} = ADJ_t^{GFCE,C} + ADJ_t^{GFCE,F} + ADJ_t^{GFCE,G}$$

Intermediate consumption

As for the capital absorption matrix, the intermediate consumption matrix computed on the production module is converted by aggregating industries into institutional sectors given the defined mapping. Again, transactions here group the purchases done in market prices of both domestic and imported goods, produced or distributed by local sectors. The entry $IC_t^{C,F}$, for instance, corresponds to the total value in market prices of the domestic and imported goods of the types produced by the Non-financial corporations, and purchased by the Financial corporations (F) during the period t .

Imports and stock variations

Up to here, the demand for imported goods for investment, intermediate and final consumption purposes has been addressed to the domestic industries acting as intermediaries. The latter, in turn, will incur in the purchase of these goods from the Rest of the World, which is done in basic prices. In brief, the total imports M_t will be carried on by the three producing and importing sectors:

$$M_t = M_t^C + M_t^F + M_t^G$$

Stock variations recorded on the production module are aggregated by institutional sector and produce the SV_t array. For any given period, t , the slice SV_t of the array verifies: $\text{diag}(SV_t) = \overrightarrow{SV_{BP,t}}$. These apply only to domestic production, as it has been assumed that imports take place on demand. Here, the values obtained from the production module

need to be adjusted to match the national accounts. The Adj_t^{SV} in Table 3.3 account for this and are calibrated according to historical observations.

Compensation of employees and employment

The production module makes no distinction between the localization of the workers and hence assumes all salaries are paid to domestic households. In practice, these salaries go partially to foreign agents, and local households also receive salaries from foreign sources. For simplicity, and given that historically salaries are a net receipt for the Brazilian economy, it has been assumed that all salaries registered on the production module are given to domestic households only.

The aggregation, hence, is directly done using the mapping between industries and institutional sectors. It is complemented by assuming that the local households are net receivers of salaries from the Rest of the World. These, are taken to grow according to foreign GDP, and enter the accounting expressed in domestic currency.

In brief, salaries received by households will correspond to those aggregated from the production module, plus the net receipts from the Rest of the World, $W_t^{H,W}$:

$$W_t^{H,\Sigma} = W_t^{H,C} + W_t^{H,F} + W_t^{H,G} + W_t^{H,W} \quad (3.64)$$

On its side, total employment, LAB_t , is taken to be the addition of the amount of full-time-equivalent jobs created in each industry. The employment rate, η_t , can then also be defined:

$$LAB_t = \vec{L}_t \cdot \vec{1}_N \quad (3.65)$$

$$\eta_t = \frac{LAB_t}{N_t} \quad (3.66)$$

where N_t stands for the working-age population, whose growth, g_t^N , is assumed to be constant during the simulated period.

Indirect taxes

Producing sectors pay taxes and receive subsidies from the government. Moreover, in the market of goods and services, they sell their products at market prices, which includes net indirect taxes on products. This means they are the primary collectors of these taxes. Each period they register the net indirect taxes on products that they have to transfer to the government. These can be retrieved by checking the sales to local counterparts of domestic and foreign goods to each sector in market prices minus the sales and imports in basic prices. The total net indirect taxes that will be transferred to the government corresponds to the sum of the taxes paid by each collecting sector.¹⁹

¹⁹Note below that the symbol Σ in the superscript should hereafter be understood as the sum of all counterparts.

$$NITP_t^{G,\Sigma} = NITP_t^{G,C} + NITP_t^{G,F} + NITP_t^{G,G} \quad (3.67)$$

Similarly, producing sectors have to pay for the Net other taxes on production, $NOTP_t$. These result from aggregating the vector \overrightarrow{NTP}_t according to the industry-sector correspondence. The resulting array satisfies:

$$NOTP_t^{G,\Sigma} = NOTP_t^{G,C} + NOTP_t^{G,F} + NOTP_t^{G,G} \quad (3.68)$$

The addition of both sets of taxes is reported in the Net Indirect Taxes array, with verifies:

$$NIT_t^{G,\Sigma} = NIT_t^{G,\Sigma} + NOTP_t^{G,\Sigma} \quad (3.69)$$

Gross domestic product, GDP deflator, and inflation

The expenditure approach to compute the Gross Domestic Product allows us to obtain it in both real and nominal terms:

$$GDP_t = \left(\overrightarrow{C}_{MP,t} + I_{MP,t}^K \times \overrightarrow{1}_N + \overrightarrow{G}_{MP,t} + \overrightarrow{X}_{BP,t} - \overrightarrow{M}_{BP,t} \right) \cdot \overrightarrow{1}_N \quad (3.70)$$

$$GDP_{R,t} = \left(\overrightarrow{C}_{R,t} + I_{R,t}^K \times \overrightarrow{1}_N + \overrightarrow{G}_{R,t} + \overrightarrow{X}_{R,t} - \overrightarrow{M}_{R,t} \right) \cdot \overrightarrow{1}_N \quad (3.71)$$

The year-to-year growth of both variables will be denoted, g_t^{YNOM} and g_t^{YR} , respectively. Finally, the GDP deflator can also be obtained, and its relative change is taken to define inflation:²⁰

$$GDP_t^{deflator} = \frac{GDP_t}{GDP_{R,t}} \quad (3.72)$$

$$\pi_t = \frac{GDP_t^{deflator}}{GDP_{t-1}^{deflator}} - 1 \quad (3.73)$$

Gross operating surplus and mixed income

The Gross Operating Surplus obtained within the module has to be adjusted to compensate for the assumption that households don't produce.²¹ The surplus of the production that industries did on behalf of households is transferred back to the latter by means of the of account ADJ_t^{GOS} , as shown in Table 3.3.²²

$$ADJ_t^{GOS,H} = ADJ_t^{GOS,F} + ADJ_t^{GOS,C} \quad (3.74)$$

²⁰This abstracts from the fact that GDP is normally service-intensive while the basket of goods consumed by families taken as reference for the IPC index calculation is more goods-intensive.

²¹Assuming otherwise would require the definition of differentiated capital stocks, tax rates, customs fees, technical coefficients, and emission factors, among others, for corporations and producing households. There is not enough publicly available information to estimate these easily, and any related exercises are left for further research.

²²It should be noted that, unlike most variables on the macro layer, the adjustment accounts and the main macro aggregates are presented as vectors only.

The macro-layer version of the Gross Operating Surplus -which includes mixed income, and net indirect taxes as an income for the Government sector- can then be retrieved as:²³

$$\overrightarrow{GOS_t^{Macro}} = \overrightarrow{GOS_t} + \overrightarrow{ADJ_t^{GOS}} \quad (3.75)$$

3.2.2 The macro layer

At the macro layer, and for any period t , the starting point of the analysis is the stock of financial assets and liabilities by the end of $t - 1$ in current value.²⁴ This is computed for each institutional sector, by type of instrument and counterpart -a balance sheet regarding only financial assets and liabilities at the end of the previous accounting period, re-valuated to current prices. At the beginning of t , a summary report of the net stock of financial assets in current value by institutional sector presents a structure as the one depicted in Table 3.1.^{25, 26}

Fin. Asset	Households (H)	Non Fin. Corp. (C)	Fin. Corp. (F)	Gov. (G)	RoW (W)	Sum
Reserves (R)			$R_{t-1,t}$		$-R_{t-1,t}$	0
Cash (H)	$H_{t-1,t}^{\Sigma,H}$	$H_{t-1,t}^{\Sigma,C}$	$H_{t-1,t}^{W,F} - H_{t-1,t}^{F,\Sigma}$		$-H_{t-1,t}^{W,\Sigma}$	0
Deposits (D)	$D_{t-1,t}^{\Sigma,H}$	$D_{t-1,t}^{\Sigma,C}$	$D_{t-1,t}^{\Sigma,F} - D_{t-1,t}^{F,\Sigma}$	$D_{t-1,t}^{\Sigma,G} - D_{t-1,t}^{G,\Sigma}$	$D_{t-1,t}^{\Sigma,W} - D_{t-1,t}^{W,\Sigma}$	0
Bonds (B)	$B_{t-1,t}^{\Sigma,H} - B_{t-1,t}^{H,\Sigma}$	$B_{t-1,t}^{\Sigma,C} - B_{t-1,t}^{C,\Sigma}$	$B_{t-1,t}^{\Sigma,F} - B_{t-1,t}^{F,\Sigma}$	$B_{t-1,t}^{\Sigma,G} - B_{t-1,t}^{G,\Sigma}$	$B_{t-1,t}^{\Sigma,W} - B_{t-1,t}^{W,\Sigma}$	0
Shares (S)	$S_{t-1,t}^{\Sigma,H}$	$S_{t-1,t}^{\Sigma,C} - S_{t-1,t}^{C,\Sigma}$	$S_{t-1,t}^{\Sigma,F} - S_{t-1,t}^{F,\Sigma}$	$S_{t-1,t}^{\Sigma,G}$	$S_{t-1,t}^{\Sigma,W} - S_{t-1,t}^{W,\Sigma}$	0
Oth. F.A. (Z)	$Z_{t-1,t}^{\Sigma,H} - Z_{t-1,t}^{H,\Sigma}$	$Z_{t-1,t}^{\Sigma,C} - Z_{t-1,t}^{C,\Sigma}$	$Z_{t-1,t}^{\Sigma,F} - Z_{t-1,t}^{F,\Sigma}$	$Z_{t-1,t}^{\Sigma,G} - Z_{t-1,t}^{G,\Sigma}$	$Z_{t-1,t}^{\Sigma,W} - Z_{t-1,t}^{W,\Sigma}$	0
Net Fin. Pos.	$NFP_{t-1,t}^H$	$NFP_{t-1,t}^C$	$NFP_{t-1,t}^F$	$NFP_{t-1,t}^G$	$NFP_{t-1,t}^W$	0

Table 3.1: Net stock of financial assets, by type of instrument and holding sector, at the end of period $t - 1$

As mentioned before, the interaction between the layers is such that the production module is first run, computing all required variables at the industry-level, and aggregating them at the sector-level, as described in section 3.2.1.

The production module provides, among others, the required information regarding the transactions on the markets of goods and services, as well as the primary distribution

²³Notice that, to avoid confusion, the production module-related vectors use sector indices as subscripts, while macro-layer ones use them as superscripts. Moreover, while the national accounts do not explicitly compute this for the Rest of the World, the values are recorded here to simplify upcoming calculations.

²⁴Interest rates and exchange rates to be applied during period t are defined at the end of period $t - 1$, which allows this revaluation of the balance sheet to be obtained.

²⁵The interaction between sectors occurs at different levels and the notation can be easily confusing as a result. The balance sheet belongs to the aggregate layer of the model, and its notation is particular to it. When dealing with stocks and variations of stocks of financial assets, the superscripts accompanying the type of instrument will refer to the issuer and the holder of the asset, in that order. The symbol Σ in the superscript should hereafter be understood as the sum of all counterparts. Finally, assets and liabilities have to be revalued to account for price changes. The indices used in Table 3.1 suggest that these correspond to stocks at the end of the period $t - 1$ valued using asset prices from t . This means the balance sheet presented already accounts for the revaluation of assets and liabilities. As an example, the notation $B_{t-1,t}^{\Sigma,H}$ corresponds to the total stock of bonds -from all potential issuers- held by the household sector, H , at the end of period $t - 1$, valued with prices from period t .

²⁶It should be noted that the Brazilian national accounts include the Central Bank's balance sheet and operations as part of the Financial Corporations, which is why this sector appears as an issuer of cash.

of income -with a focus on salaries and indirect taxes. This is complemented by the modeled behavior of the accounts detailing the primary income allocation, and the secondary distribution of income, done at the macro layer. Altogether, these allow to compute the net financing capacity or requirements.

Primary income allocation

All institutional sectors have interest-paying liabilities in the form of loans or bonds, all of which will be hereafter referred to as bonds for simplicity. The bonds issued by an institutional sector will have a counterpart-specific interest rate assigned to it. For instance, the outstanding current value of the stock of bonds issued by households that are held by the financial corporations at time t , $B_t^{H,F}$, will pay an interest rate of $r_{t+1}^{H,F}$ during $t + 1$. This rate needs not be the same as the one that households pay to the Non-Financial Corporations for holding the bonds that the former have issued.

Aside from bonds, Financial Corporations and the Rest of the World are also issuers of deposits, on which interests are assumed to be paid. In this case, the interest rate each issuer pays does not change by counterpart. All in all, the interest received and paid by a sector j will be given by equations (3.76) and (3.77), respectively:

$$INT_t^{\Sigma, j} = \sum_{h \neq W} r_t^{h, j} B_{t-1}^{h, j} + rd_t^{F, j} D_{t-1}^{F, j} + E_t(r_t^{W, j} B_{t-1}^{W, j} + rd_t^{W, j} D_{t-1}^{W, j}) \quad (3.76)$$

$$INT_t^{j, \Sigma} = \begin{cases} \sum_h r_t^{j, h} B_{t-1}^{j, h}, & \text{if } j \in \{H, C, G\} \\ \sum_h (r_t^{j, h} B_{t-1}^{j, h} + rd_t^{j, h} D_{t-1}^{j, h}), & \text{if } j = F \\ E_t \sum_h (r_t^{j, h} B_{t-1}^{j, h} + rd_t^{j, h} D_{t-1}^{j, h}), & \text{if } j = W \end{cases} \quad (3.77)$$

where r_t stands for the interest rate on Bonds, rd_t for the rate paid on deposits, and E_t for the exchange rate expressed in domestic currency per units of foreign one. As noted before, the outflow corresponding to the payment of interest on deposits only applies to sectors F and W .

In addition to interest payments, the primary income allocation also considers dividends. While all sectors can receive dividends on their stocks held, only the stock issuers (F , C and W) will register a non-null outflow of funds for this concept. As a result, the dividends received and paid by sector j , respectively, will be given by:

$$DIV_t^{\Sigma, j} = \sum_{h \in \{C, F\}} \varphi_t^{h, j} S_{t-1}^{h, j} + E_t \varphi_t^{W, j} S_{t-1}^{W, j} \quad (3.78)$$

$$DIV_t^{j, \Sigma} = \begin{cases} \sum_h \varphi_t^{j, h} S_{t-1}^{j, h}, & \text{if } j \in \{C, F\} \\ E_t \sum_h \varphi_t^{j, h} S_{t-1}^{j, h}, & \text{if } j = W \\ 0, & \text{otherwise} \end{cases} \quad (3.79)$$

Where $\varphi_t^{j, h}$ represents the rate paid per share. This is a issuer-holder specific rate

applied to the stock of sector j held by h at the end of the previous period, $S_{t-1}^{j,h}$.

Finally, Households and Non-financial corporations will transfer funds to the government on each period under the concept of rent from natural resources, RNR_t . These transfers are assumed to evolve at the same pace as nominal GDP:

$$RNR_t^{j,G} = RNR_{t-1}^{j,G} \cdot (1 + g_t^{YNOM}), \quad \text{where } j \in \{H, C\} \quad (3.80)$$

Aggregate: Gross national income (B.5)

Following national accounting identities, the vector of Gross National Income for time t is defined in the model as:²⁷

$$\begin{aligned} \overrightarrow{GNI}_t = \overrightarrow{GOS}_t^{Macro} + (INT_t + DIV_t + RNR_t + W_t + NIT)^T \times \overrightarrow{1}_S \\ - (INT_t + DIV_t + RNR_t) \times \overrightarrow{1}_S \end{aligned} \quad (3.81)$$

where $\overrightarrow{1}_S$ is a column vector of ones, of dimension $S = \text{number of institutional sectors}$.

Secondary distribution of income

Aside from indirect taxes on products and production, agents also pay direct taxes on income, which along with social contributions, social benefits, and with other current transfers account for the secondary distribution of income.

Taxes on income are assumed to result from a sector-specific tax rate charged on Gross National Income of domestic private agents. This array has non-zero entries only on the column corresponding to the Government sector, whose elements verify:

$$YTAX_t^{G,j} = \tau^{INC,j} \cdot GNI_t^j, \quad \text{for each } j \in \{H, C, F\} \quad (3.82)$$

Social Contributions, on its side, are assumed to follow a linear relationship with the total amount of salaries received by households. The *SOC* slice for time t is hence determined by:

$$SOC_t = (1 + g_t^W) \cdot SOC_{t-1} \quad (3.83)$$

where $g_t^W = \frac{W_t^{H,\Sigma}}{W_{t-1}^{H,\Sigma}} - 1$, represents the relative variation of the total salaries received by households with respect to $t - 1$.

Social Benefits, SOB_t , and Other Current Transfers, OCT_t , are in turn assumed to follow a linear relationship with nominal GDP, and evolve at the same pace:

$$SOB_t = (1 + g_t^{YNOM}) \cdot SOB_{t-1} \quad (3.84)$$

$$OCT_t = (1 + g_t^{YNOM}) \cdot OCT_{t-1} \quad (3.85)$$

²⁷The variables not explicitly noted as vectors here, correspond to arrays. The slice corresponding to any given period t , is a who-to-whom matrix.

Aggregate: Gross disposable income (B.6)

Gross Disposable Income, GDI_t , is obtained by adding the net receipts from Taxes on Income, Social Benefits and Social Contributions, to the Gross National Income:

$$\begin{aligned} \overrightarrow{GDI}_t &= \overrightarrow{GNI}_t + (YTAX_t + SOB_t + SOC_t + OCT_t)^T \times \overrightarrow{1}_S \\ &\quad - (YTAX_t + SOB_t + SOC_t + OCT_t) \times \overrightarrow{1}_S \end{aligned} \quad (3.86)$$

Adjustments due to intellectual property rights

For simplicity, adjustments due to intellectual property rights are also assumed to be linearly dependent on nominal GDP, evolving at the same pace:

$$IPR_t = \left(1 + g_t^{YNOM}\right) \cdot IPR_{t-1} \quad (3.87)$$

Aggregate: Gross savings (B.8)

Gross Savings are obtained after subtracting the public and private consumption expenditures on final goods from the Gross Disposable Income for both domestic and foreign agents, and adding the net receipts of funds from intellectual property rights adjustments. For the Rest of the World, this aggregate is obtained after also adding the imports paid in basic prices.²⁸ Given the array structure of all the detailed flows, this vector can be written as:

$$\overrightarrow{GS}_t = \overrightarrow{GDI}_t + \left((M_t)^T - C_t + (IPR_t)^T - IPR_t \right) \times \overrightarrow{1}_S \quad (3.88)$$

Net transfers of capital and net transfers of non-produced, non-financial assets

Net transfers of capital are also assumed to depend linearly on nominal GDP, growing at the same pace:

$$NKT_t = \left(1 + g_t^{YNOM}\right) \cdot NKT_{t-1} \quad (3.89)$$

Aggregate: Net lending or net borrowing (B.9)

Finally, the net financing capacity or requirements, $\overrightarrow{Agg_t^{B9}}$, can be computed. A positive value would imply the sector has a net lending position at the end of the current period, while a negative value means it is a net borrower. The vector is obtained by subtracting the net investment expenditures -including investment devoted to GFCF, but also the one resulting from increasing inventories- from the Gross Savings, and then adding

²⁸Recall here, that the slice C_t includes consumption of domestic and foreign agents, and hence groups the data from the vectors $\overrightarrow{C_{MP,t}}$, $\overrightarrow{G_{MP,t}}$ and $\overrightarrow{X_{BP,t}}$, so there is no need to further include exports in the calculation.

the net transfers of capital:

$$\begin{aligned} \overrightarrow{Agg_t^{B\dot{g}}} &= \overrightarrow{GS_t} - (GFCF_t + SV_t) \times \overrightarrow{1_S} \\ &\quad - \left(\overrightarrow{ADJ_t^{GFCF}} + \overrightarrow{ADJ_t^{SV}} \right) \\ &\quad + \left((NKT_t)^T - NKT_t \right) \times \overrightarrow{1_S} \end{aligned} \quad (3.90)$$

Finally, table 3.3 shows the monetary flows by transaction types between institutional sectors. The vertical accounting of it must verify the Net Financing Capacity or Requirements as obtained here.

These resulting positions will then need to be matched by net changes in assets and liabilities during the period, as will be presented in section 3.2.2.²⁹

Interest rates

All interest rates are assumed to be pre-determined. They are defined at the end of a given period and are used to compute the interest payments of the upcoming one. This implies that the values reported in Table 3.1 for this period are indeed known when the period starts. The new rates and foreign exchange rate will be used to compute such table for $t + 1$.

Let r_{t+1}^{REF} be the policy rate defined by the Central Bank for the next period, according to an inflation targeting rule:

$$r_{t+1}^{REF} = \max \{ r^* + \pi_t + \phi \cdot (\pi_t - \pi^*) , 0 \} \quad (3.91)$$

where r^* stands for the real interest rate parameter, π^* is the target inflation rate, and $\phi > 0$ measures the degree of reactivity of the monetary policy.

While deposit rates are assumed to be issuer-specific, the interest rates on bonds are issuer-and-holder-specific. In both cases, the domestic rates are assumed to evolve to keep a constant spread with respect to the policy rate. Foreign interest rates both on deposits and on bonds are assumed to be constant in foreign currency. Hence:

$$rd_{t+1}^{h,j} = rd_t^{h,j} + \left(r_{t+1}^{REF} - r_t^{REF} \right), \quad \forall j, \text{ and, } \forall h \in F, G \quad (3.92)$$

$$r_{t+1}^{h,j} = r_t^{h,j} + \left(r_{t+1}^{REF} - r_t^{REF} \right), \quad \forall j, \text{ and, } \forall h \neq W \quad (3.93)$$

$$rd_{t+1}^{W,j} = rd_t^{W,j}, \quad \forall j \quad (3.94)$$

$$r_{t+1}^{W,j} = r_t^{W,j}, \quad \forall j, \text{ and, } \forall h \neq W \quad (3.95)$$

²⁹This simplification implies indeed that transactions in the market of financial assets will have no effect on the current's period Macroeconomic activity, but only on the next. This, as these transactions are assumed to take place ex-post, once to clear flow imbalances resulting from prior balance sheet positions and transactions on the market of goods and services.

Foreign exchange rate

At the end of each period, given the observed flows and net financing positions, the exchange rate for the upcoming period will be determined. The exchange rate, expressing the amount of domestic currency per unit of foreign one, is assumed to evolve according to the inflation differentials, and to the net lending or borrowing position of the Rest of the World to the domestic agents as a share of nominal GDP.

$$E_{t+1} = E_t \cdot \left(1 + \lambda^{\pi,E} \cdot (\pi_t - \pi_t^f) + \lambda^{B9,E} \cdot \frac{Agg_t^{B9,W}}{GDP_t} \right) \quad (3.96)$$

Revaluation of financial assets

Reserves, cash and deposits

The domestic holdings of monetary gold and SDR are referred to as (international) reserves. For simplicity, gold prices are assumed to remain constant and hence the revaluation of reserves, \hat{R}_t , is taken to be only explained by fluctuations in the foreign exchange rate:

$$\hat{R}_t = R_{t-1} \cdot \frac{E_t}{E_{t-1}} \quad (3.97)$$

Domestic cash and domestic deposits are assumed to keep their valuation at par in domestic currency. These are hence subject to no revaluation. Deposits abroad, and holdings of foreign cash are, on the contrary, subject to revaluation when expressed in domestic currency.

$$\hat{H}_t^{W,j} = H_{t-1}^{W,j} \cdot \frac{E_t}{E_{t-1}}, \quad \forall j \quad (3.98)$$

$$\hat{D}_t^{W,j} = D_{t-1}^{W,j} \cdot \frac{E_t}{E_{t-1}}, \quad \forall j \quad (3.99)$$

Bonds

Bonds are issued with an issuer-holder-specific interest rate. The revaluation of each of such combinations is assumed to depend on the relative change of the interest rate with respect to the previous period. An increase (decrease) on the rate that will be paid by the issuer, h , of a bond to its holder, j with respect to the previous period, will make the existing stock of bonds issued by h and held by j decrease (increase) in value so that the yield produced by the bonds in circulation matches the one that will be provided with the

bonds issued during t .³⁰

$$\hat{B}_t^{h,j} = B_{t-1}^{h,j} \cdot \left(\frac{r_{t-1}^{h,j}}{r_t^{h,j}} - 1 \right), \quad \forall h \neq W, \text{ and } \forall j \quad (3.100)$$

$$\hat{B}_t^{W,j} = B_{t-1}^{W,j} \cdot \left(\frac{r_{t-1}^{W,j}}{r_t^{W,j}} - 1 \right) \cdot \frac{E_t}{E_{t-1}}, \quad \forall j \quad (3.101)$$

Shares and other financial assets

For simplicity, domestic shares and other financial assets are assumed not to change in value, nor to be subject of defaults or write-offs. The shares and other financial assets issued by the Rest of the World, however, are subject to revaluation when expressing the outstanding value in domestic currency.

$$\hat{S}_t^{W,j} = \hat{S}_{t-1}^{W,j} \cdot \frac{E_t}{E_{t-1}}, \quad \forall j \quad (3.102)$$

$$\hat{Z}_t^{W,j} = \hat{Z}_{t-1}^{W,j} \cdot \frac{E_t}{E_{t-1}}, \quad \forall j \quad (3.103)$$

Portfolio allocations

The current section describes the numerical approximation that has been implemented to produce balanced who-to-whom matrices for all asset classes and institutional sectors.³¹

The model assumes that net lenders will devote a constant fraction γ^{NL} of their new variations to the purchase of new financial assets. The changes in liabilities will correspond so that the net lending during the period is respected. The extreme case of $\gamma^{NL} = 1$, assumes that net lenders do not borrow, which is however not consistent with reality. A value of $\gamma^{NL} > 1$ is more sensible and would imply that net lenders will also borrow.

Likewise, net borrowers are assumed to devote a constant fraction γ^{NB} of their new variations to the issuance of new financial liabilities. The changes in assets will correspond so that the net borrowing during the period is respected. As before, the extreme case of $\gamma^{NB} = 1$, assumes that net borrowers cannot purchase new assets during the period. As alternatives, $\gamma^{NB} < 1$ represents the case where net borrowers sell assets they hold not to incur in excessive borrowing, while $\gamma^{NB} > 1$ would imply that net borrowers are still able to purchase financial assets, but would require additional borrowing to ensure the net targets are respected.

Both γ^{NL} and γ^{NB} are assumed not to correspond to the extreme cases. Given they are modeled as constant, they are set so that the global stocks of assets and liabilities are not depleted and that the net borrowing of a period matches the net lending.

³⁰The interest rate fluctuations are assumed to be the only explanation for the revaluation of domestic bonds, abstracting from other elements such as defaults and write-offs. Moreover, it should be noted that holders of a class of bond from an issuer, won't see their value change if only other interest rates change. This simplification has the underlying assumption that not all subsets of an institutional sector have access to all asset classes and issuer counterparts, or that they disregard the developments of these other assets.

³¹This is indeed a simplification that verifies all accounting rules and is consistent with the structure of the model, but abstracts from sector-specific portfolio or funding behaviors.

The total purchases of new assets and the total issuance of new liabilities by institutional sector can be then defined as:

$$\overrightarrow{\Delta A_t^{TOT}} = \gamma^{NL} \cdot \overrightarrow{Agg_t^{B9+}} - (1 - \gamma^{NB}) \cdot \overrightarrow{Agg_t^{B9-}} \quad (3.104)$$

$$\overrightarrow{\Delta L_t^{TOT}} = (1 - \gamma^{NL}) \cdot \overrightarrow{Agg_t^{B9+}} - \gamma^{NB} \cdot \overrightarrow{Agg_t^{B9-}} \quad (3.105)$$

where the superscripts + and - denote we only consider, respectively, the non-negative or non-positive values on the vector.

With the total changes in assets and liabilities being defined, this has now to be translated into a who-to-whom structure by asset class. The RAS algorithm described in Appendix 3.B.2 is implemented here for such a purpose.

Once the aggregate final stock and variations matrix are obtained, they are allocated by asset class according to the weights of each class on the aggregate stocks from last period.

Finally, the balance sheet at the end of the period, valued in current prices can be seen in Table 3.2.

Fin. Asset	Households (H)	Non-Fin. Corp. (C)	Fin. Corp. (F)	Gov (G)	RoW (W)	Sum
Reserves (R)			R_t		$-R_t$	0
Cash (H)	$H_t^{\Sigma,H}$	$H_t^{\Sigma,C}$	$H_t^{W,F} - H_t^{F,\Sigma}$		$-H_t^{W,\Sigma}$	0
Deposits (D)	$D_t^{\Sigma,H}$	$D_t^{\Sigma,C}$	$D_t^{\Sigma,F} - D_t^{F,\Sigma}$	$D_t^{\Sigma,G} - D_t^{G,\Sigma}$	$D_t^{\Sigma,W} - D_t^{W,\Sigma}$	0
Bonds (B)	$B_t^{\Sigma,H} - B_t^{H,\Sigma}$	$B_t^{\Sigma,C} - B_t^{C,\Sigma}$	$B_t^{\Sigma,F} - B_t^{F,\Sigma}$	$B_t^{\Sigma,G} - B_t^{G,\Sigma}$	$B_t^{\Sigma,W} - B_t^{W,\Sigma}$	0
Shares (S)	$S_t^{\Sigma,H}$	$S_t^{\Sigma,C} - S_t^{C,\Sigma}$	$S_t^{\Sigma,F} - S_t^{F,\Sigma}$	$S_t^{\Sigma,G}$	$S_t^{\Sigma,W} - S_t^{W,\Sigma}$	0
Other F.A. (Z)	$Z_t^{\Sigma,H} - Z_t^{H,\Sigma}$	$Z_t^{\Sigma,C} - Z_t^{C,\Sigma}$	$Z_t^{\Sigma,F} - Z_t^{F,\Sigma}$	$Z_t^{\Sigma,G} - Z_t^{G,\Sigma}$	$Z_t^{\Sigma,W} - Z_t^{W,\Sigma}$	0

Table 3.2: Net stock of financial assets, by type of instrument and holding sector, at the end of period t

Transaction	Households	NFC Current	NFC Capital	Fin. Current	Fin. Capital	Gov. Current	Gov. Capital	RoW	Sum
Final Consumption	$-C_t^{\Sigma,H}$	$C_t^{C,\Sigma}$		$C_t^{F,\Sigma}$		$C_t^{G,\Sigma}$	$-C_t^{\Sigma,G}$	$-C_t^{\Sigma,W}$	0
Investment (GFCF)		$GFCF_t^{C,\Sigma}$	$-GFCF_t^{\Sigma,C}$	$GFCF_t^{F,\Sigma}$	$-GFCF_t^{\Sigma,F}$	$GFCF_t^{G,\Sigma}$	$-GFCF_t^{\Sigma,G}$		0
Adjustment of GFCF	$-ADJ_t^{GFCF,H}$		$ADJ_t^{GFCF,C}$	$ADJ_t^{GFCF,F}$	$ADJ_t^{GFCF,F}$	$ADJ_t^{GFCF,G}$	$ADJ_t^{GFCF,G}$		0
Imports		$-M_t^C$		$-M_t^F$		$-M_t^G$		M_t	0
Stock Variation		SV_t^C	$-SV_t^C$	SV_t^F	$-SV_t^F$	SV_t^G	$-SV_t^G$		0
Adjustment of SV		$-ADJ_t^{SV,C}$		$ADJ_t^{SV,F}$		$ADJ_t^{SV,G}$			0
Interm. Consumption		$IC_t^{C,\Sigma} - IC_t^{\Sigma,C}$		$IC_t^{F,\Sigma} - IC_t^{\Sigma,F}$		$IC_t^{G,\Sigma} - IC_t^{\Sigma,G}$			0
Acc. memo 1 [GDP]		$[AM1]_t^C$		$[AM1]_t^F$		$[AM1]_t^G$			[GDP]
Acc. memo 2 [-GDP]		$-[AM2]_t^C$		$-[AM2]_t^F$		$-[AM2]_t^G$		$[AM2]_t^W$	$[-GDP]$
Wages	$W_t^{H,\Sigma}$	$-W_t^{H,C}$		$-W_t^{H,F}$		$-W_t^{H,G}$		$-W_t^{H,W}$	0
Net Ind. Taxes		$-NIT_t^{G,C}$		$-NIT_t^{G,F}$		$-NIT_t^{G,G} + NIT_t^{G,\Sigma}$			0
Adjustment of GOS	$ADJ_t^{GOS,H}$	$-ADJ_t^{GOS,C}$		$-ADJ_t^{GOS,F}$					0
Interests	$INT_t^{\Sigma,H} - INT_t^{H,\Sigma}$	$INT_t^{\Sigma,C} - INT_t^{C,\Sigma}$		$INT_t^{\Sigma,F} - INT_t^{F,\Sigma}$		$INT_t^{\Sigma,G} - INT_t^{G,\Sigma}$		$INT_t^{\Sigma,W} - INT_t^{W,\Sigma}$	0
Dividends	$DIV_t^{\Sigma,H}$	$DIV_t^{\Sigma,C} - DIV_t^{C,\Sigma}$		$DIV_t^{\Sigma,F} - DIV_t^{F,\Sigma}$		$DIV_t^{\Sigma,G}$		$DIV_t^{\Sigma,W} - DIV_t^{W,\Sigma}$	0
Rent from Nat. Res.	$-RNR_t^{H,G}$	$-RNR_t^{C,G}$				$RNR_t^{\Sigma,G}$			0
Taxes on income	$-YNTAX_t^{G,H}$	$-YNTAX_t^{G,C}$		$-YNTAX_t^{G,F}$		$YNTAX_t^{G,\Sigma}$			0
Social Contributions	$-SOC_t^{\Sigma,H}$			$SOC_T^{F,H}$		$SOC_t^{G,H}$			0
Social Benefits	$SOB_t^{\Sigma,H}$			$SOB_t^{F,H}$		$-SOB_t^{G,H}$			0
Other Current Transf.	$OCT_t^{H,\Sigma} - OCT_t^{\Sigma,H}$	$OCT_t^{C,\Sigma} - OCT_t^{\Sigma,C}$		$OCT_t^{F,\Sigma} - OCT_t^{\Sigma,F}$		$OCT_t^{G,\Sigma} - OCT_t^{\Sigma,G}$		$OCT_t^{W,\Sigma} - OCT_t^{\Sigma,W}$	0
Int. Prop. Rights	$IPR_t^{H,\Sigma}$			$-IPR_t^{H,F}$		$-IPR_t^{H,G}$			0
Oth. Net K. Transf.	$OCT_t^{H,\Sigma} - OCT_t^{\Sigma,H}$	$OCT_t^{C,\Sigma} - OCT_t^{\Sigma,C}$		$OCT_t^{F,\Sigma} - OCT_t^{\Sigma,F}$		$OCT_t^{G,\Sigma} - OCT_t^{\Sigma,G}$			0
Net financing capacity (B.9)	$Agg_t^{B9,H}$	$Agg_t^{B9,C}$		$Agg_t^{B9,F}$		$Agg_t^{B9,G}$		$Agg_t^{B9,W}$	0

Table 3.3: Monetary flows in domestic currency by transaction type and institutional sector.

3.3 Simulations

3.3.1 Baseline projections

The model parameters have been set according to historical information, and fitted to obtain GDP growth rates between 2.5 and 3% until 2030, consistent with what official and international forecasts published in 2019 suggested.³² The parameter values used for the simulations presented here are detailed in Table 3.4.

Description	Parameter	Value
Stickiness of utilization rate	λ^u	0.8
Elast. of consumption wrt wage share growth	λ^ω	0.15
Depreciation rate of Capital	δ^K	0.05
Elast. of investment wrt the K/Y ratio	κ^ν	-0.05
Elast. of investment wrt utilization rate	κ^u	1.05
Elast. of investment wrt wage share	κ^ω	-0.75
Elast. of exports wrt relative prices	λ^{RelPr}	0.01
Elast. of exports wrt lagged net borrowing	$\lambda^{B9,W}$	0.001
Tech progress (K)	g_t^ν	0.02 $\forall t$
Tech progress (Labor saving)	$g_t^{a_\ell}$	0.02 $\forall t$
Target ratio of stock variation over output	$S\bar{V}Q$	0.01 \forall non-service
Elast. markup wrt SV/Q gap vs target	λ^μ	-0.01
Emission efficiency gains	g^{em}	0.01
Population growth	g_t^N	0.004
Real interest rate	r^*	0
Degree of reactivity of Monetary Policy	ϕ	1.15
Target inflation rate	π^*	0.045
Elast. Forex wrt Net Lending / GDP	$\lambda^{B9,E}$	-0.5
Sens. Forex wrt Inflation differential	$\lambda^{\pi,E}$	1.25
New fin assets as a share of Net lending	γ^{NL}	1.05
New fin liabilities as a share of Net Borrowing	γ^{NB}	0.95

Table 3.4: Parameter values

The resulting baseline simulations present a growing economy with increasing employment, inflation rates remaining below the two digits, and a currency depreciating against the dollar at a steady but slower pace than in previous years.³³ Regarding the employment rate, the model suggests a level above that observed in historical years could be achieved towards the end of the simulation period as the result of real GDP growing faster than the population and labor-saving technical progress.

As for the historical period, industrial CO_2 emissions follow closely the evolution of the real GDP. This is tamed by an annual efficiency improvement that assumes industrial

³²As mentioned before, no shocks to simulate the COVID crisis or its recovery process have yet been implemented.

³³An slower depreciation rate was aimed at for stability purposes of the simulations given the current simplified behavioral assumptions, and as a way to compensate for the fact that the foreign exchange rate is defined here only against the US dollar, and it does not capture the depreciation that other regional currencies from key trade partners may have against the Brazilian real.

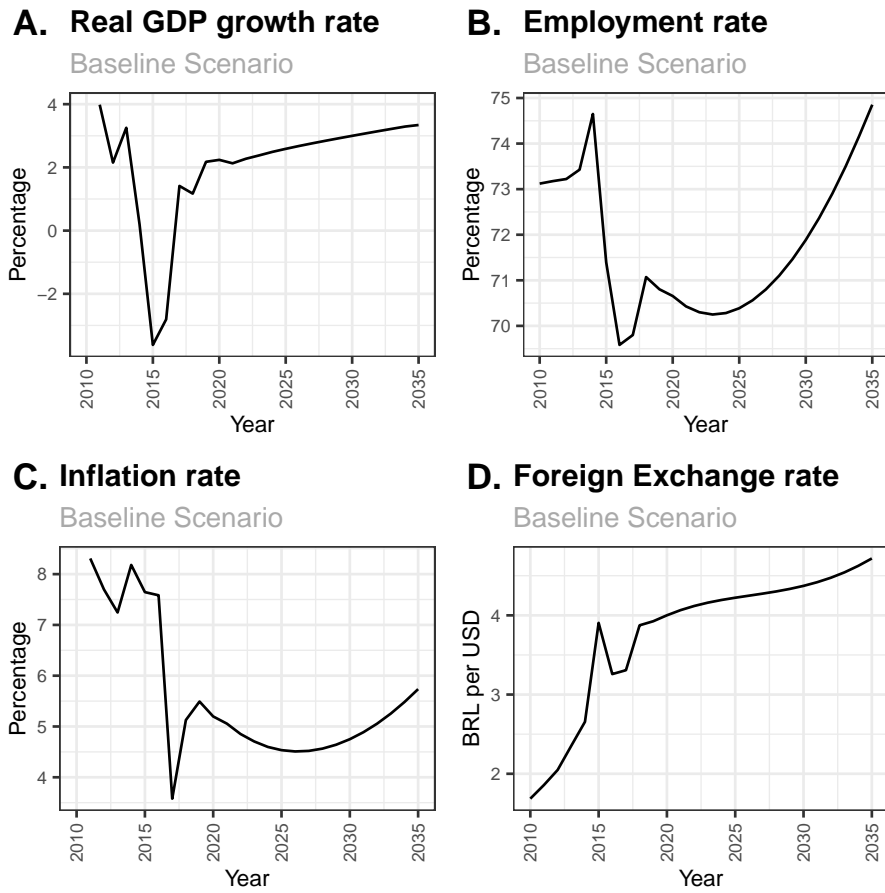


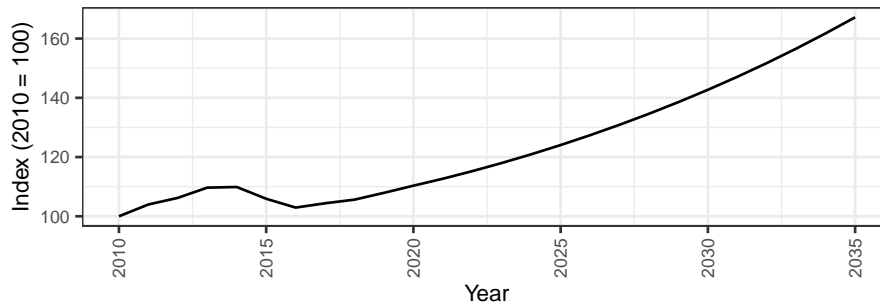
Figure 3.4: Real GDP growth, employment, inflation, and foreign exchange rates.

processes decrease their pollution per unit of output in time. As a result, when comparing 2035 vs 2010, the baseline suggests GDP would have increased by nearly 65%, and emissions by 35%.

Under the baseline, production by industry is set to grow at a relatively steady pace across all industries (Figure 3.6), with all of them increasing the amount of full-time-equivalent jobs required for their activities (Figure 3.7) whenever the production growth exceeds the improvement pace of labor-saving technical progress, which is assumed to be the same across industries.

A. Real GDP

Baseline Scenario



B. Industrial CO2 emissions

Baseline Scenario

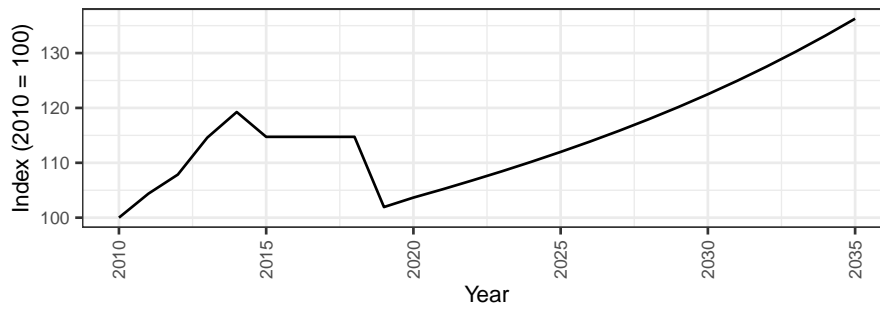


Figure 3.5: Evolution of real GDP and industrial emissions

Production by industry

Baseline Scenario

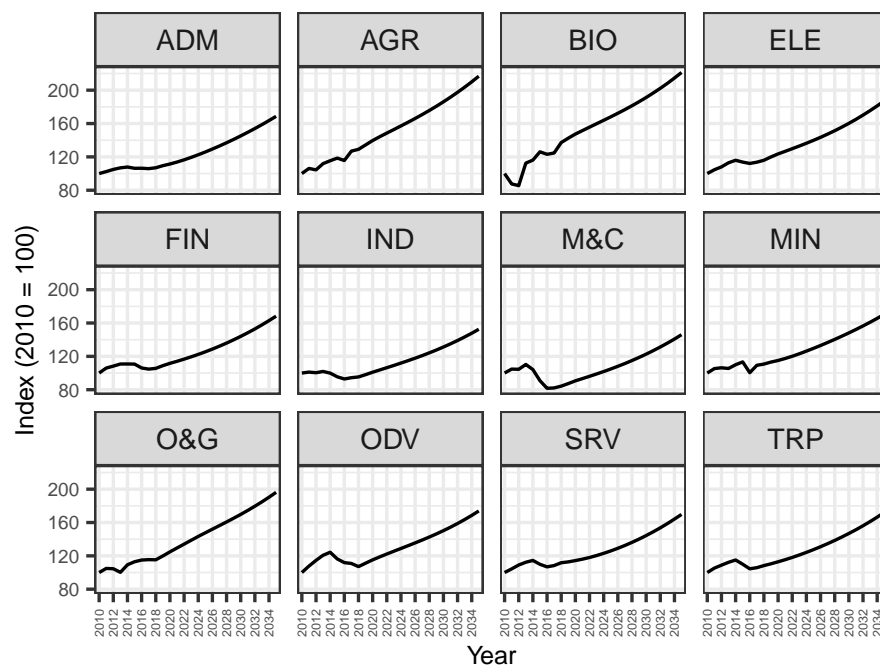


Figure 3.6: Production index, by industry

Labor use by industry

Baseline Scenario

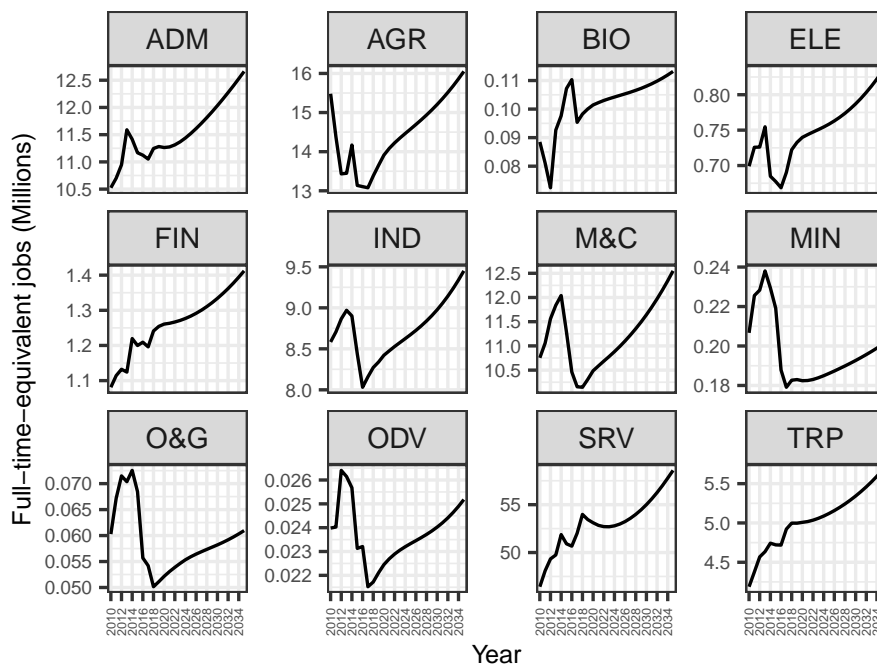


Figure 3.7: Labor use by industry

On the demand side, the model produces stable participation of the different components of GDP by the expenditure approach (Figure 3.8). It should be noted that, given the current simplified configuration, it also shows a trajectory where industries over-invest, and the current balancing mechanism only tames but does not reverse the process. At

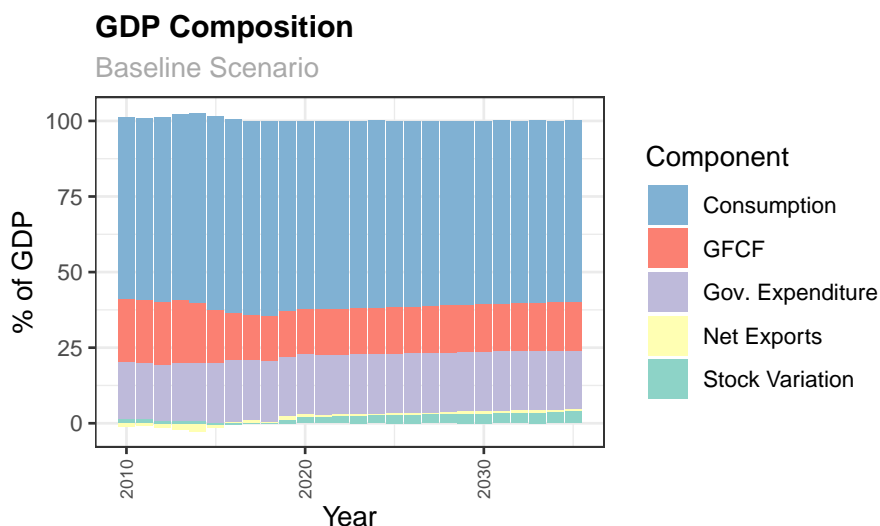


Figure 3.8: GDP composition - expenditure approach

the institutional sector level, the gross disposable income, net savings, and net financing capacity (net lending or net borrowing), as detailed in equations (3.86), (3.88) and (3.90), respectively, are also simulated. Figure 3.9 shows the evolution of these accounts for each institutional sector. As it can be seen, the model suggests the government will remain a net borrower, with its financing requirements remaining close to 7.5% of GDP. The magnitude of the net borrowing from the Government, allows the other sectors to hold the status of net lenders along the simulations. These results are mirrored in the net variation of assets and liabilities, which then contributes to the existing stocks. As mentioned before, this is currently implemented by means of numerical methods to balance the total supply and demand of financial assets by considering prior portfolio positions, as well as the total new issuance and existing stocks by type of instrument. Being the net lenders, the domestic private sectors and the rest of the World will show an increase in the net stock of assets. The evolution of the stock of assets and liabilities by type of instrument is shown in Figures 3.10 and 3.12, respectively. These profiles are then expanded by counterpart in Figures 3.11 and 3.13.

Non-financial corporations show negative net stocks of all instruments except deposits. The sector increases its holding of bonds initially by purchasing those issued by the government sector and the financial corporations while selling most of the latter in the second stage. Their financing comes from issuing bonds –mainly held by the financial corporations and the rest of the World–, and by issuing shares held mainly by all non-government sectors.

The financial corporations increase their portfolio of obligations from all other sectors

Net financial flows

Baseline Scenario

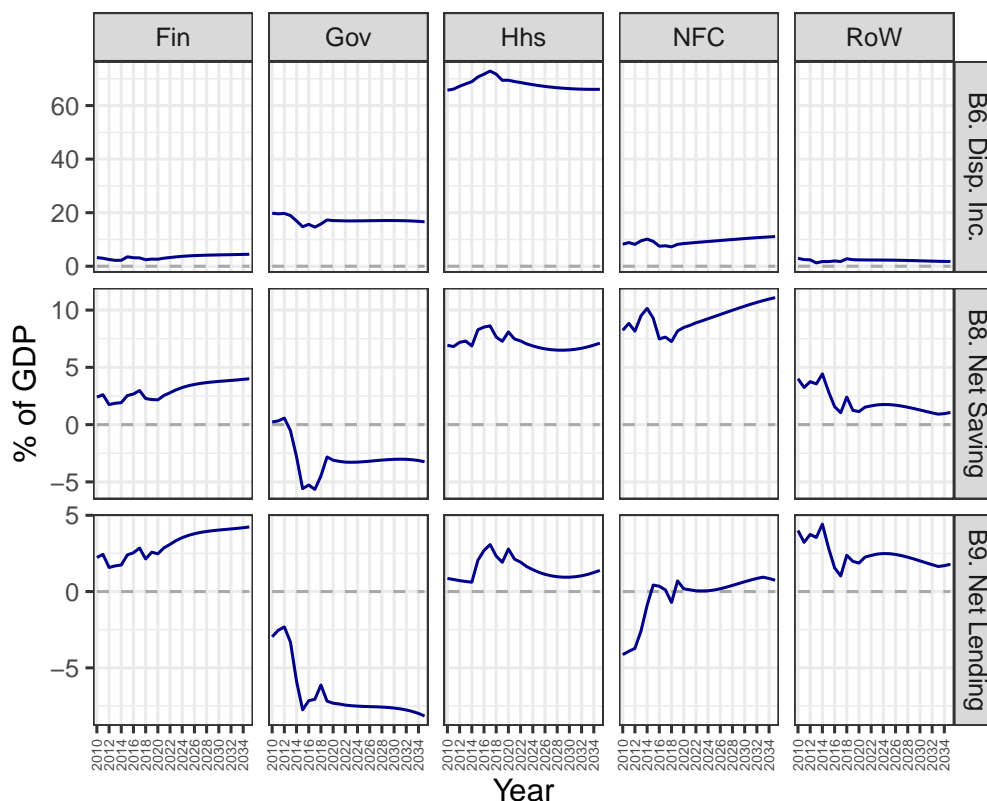


Figure 3.9: Key (net) financial flows

(Bonds), among which the government is their main counterpart. Despite their net lender position, they issue bonds and shares. The former are mainly held by the rest of the World and by non-financial corporations, while the latter are mainly in the hands of households and the rest of the World.

The government exhibits a sustained status of net borrower. Despite this, it initially increases its holdings of bonds, deposits, and shares from other sectors, which it finances by increasing its liability position in bonds, mainly held by the financial sector and the rest of the World.

Households increase their liabilities by issuing additional debt obligations –mainly to the local financial sector–, which enables them to further increase their asset position. They do so by purchasing government bonds and shares from financial corporations.

Finally, the rest of the World decreases its liability position in bonds, while increasing its issuance of shares –mainly to the financial corporations and households–. This enables a further expansion of its portfolio holdings, which is translated into additional stocks of government bonds and a moderate increase in shares from both domestic corporate sectors.

Evolution of Stocks of Assets

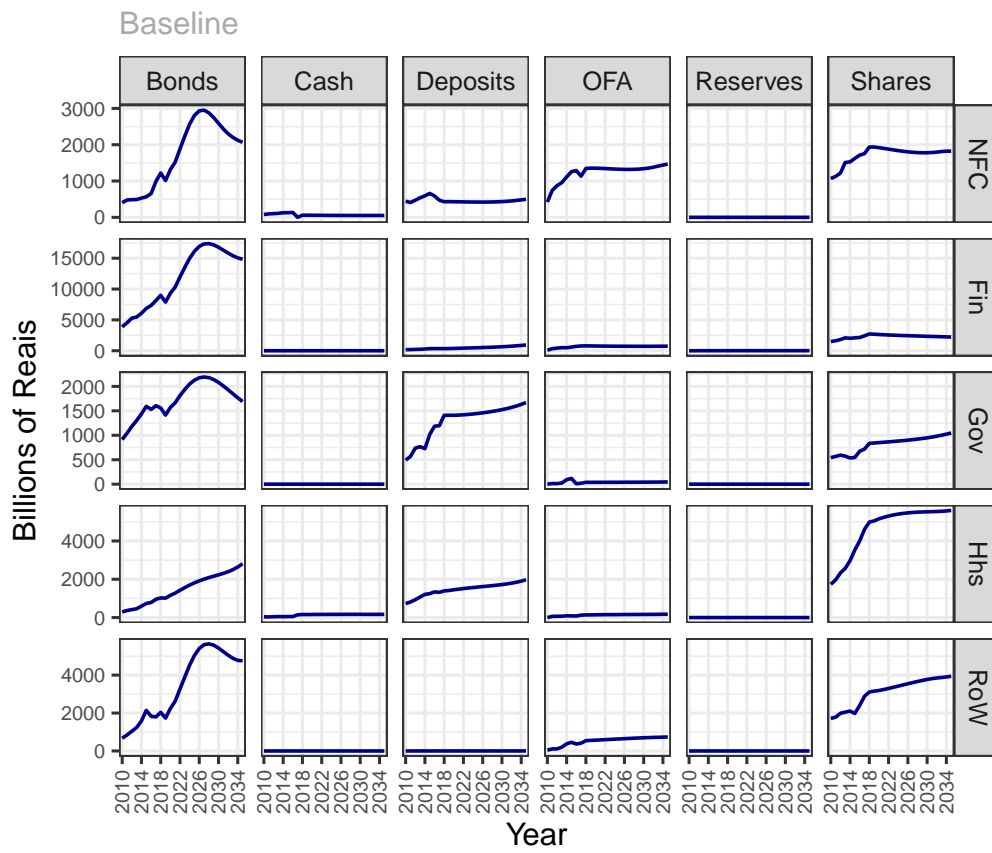


Figure 3.10: Evolution of the total stock of financial assets

Stock of Assets by issuing counterpart

Baseline

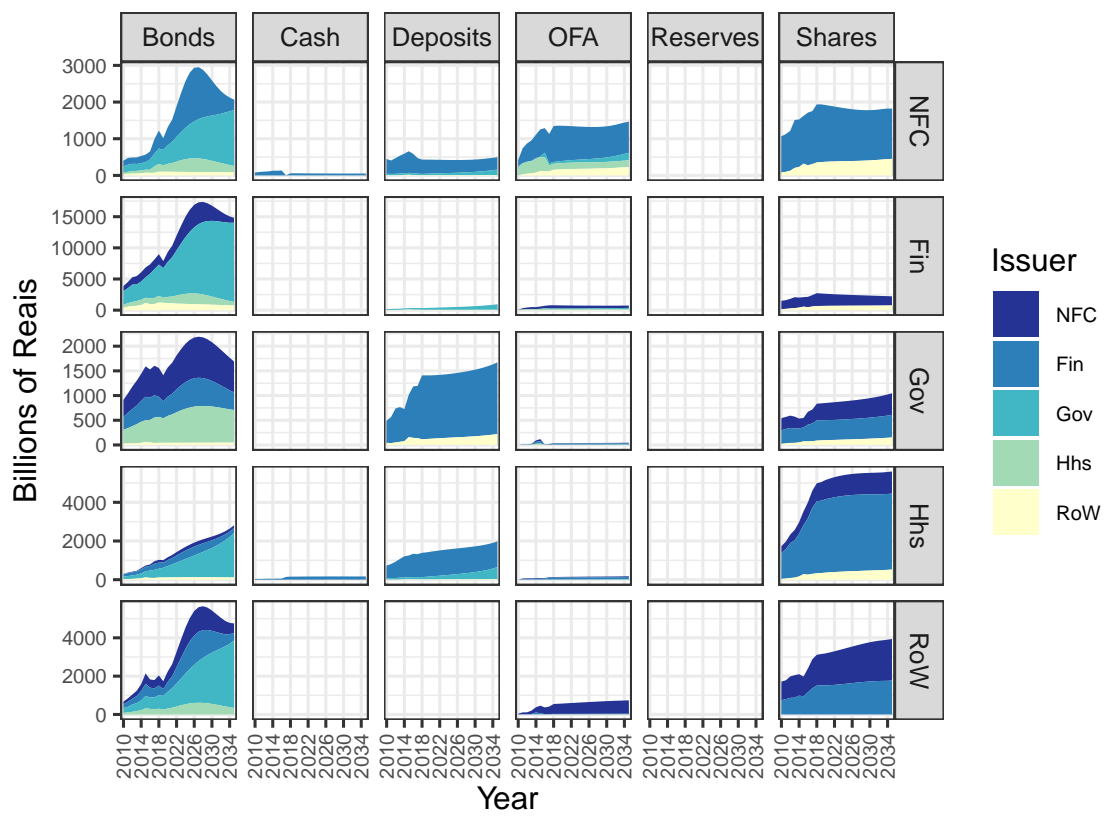


Figure 3.11: Total stock of financial assets by issuing counterpart

Evolution of Stocks of Liabilities

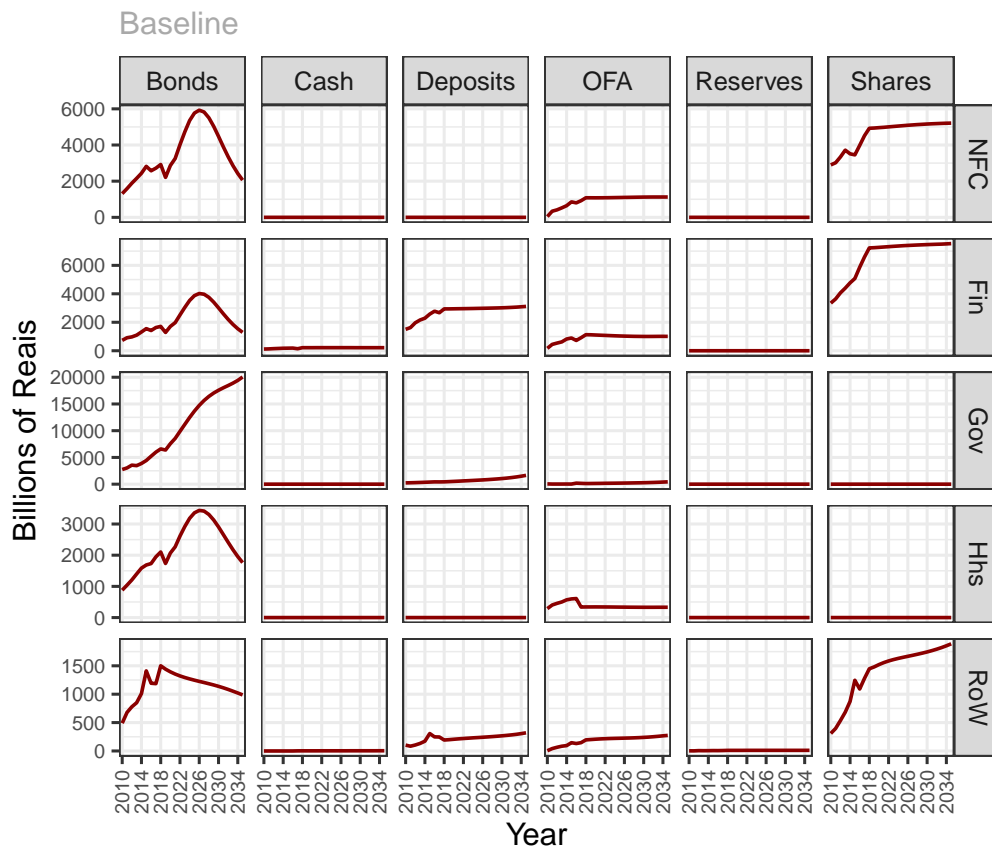


Figure 3.12: Evolution of the total stock of financial liabilities

Stock of Liabilities by holding counterpart

Baseline

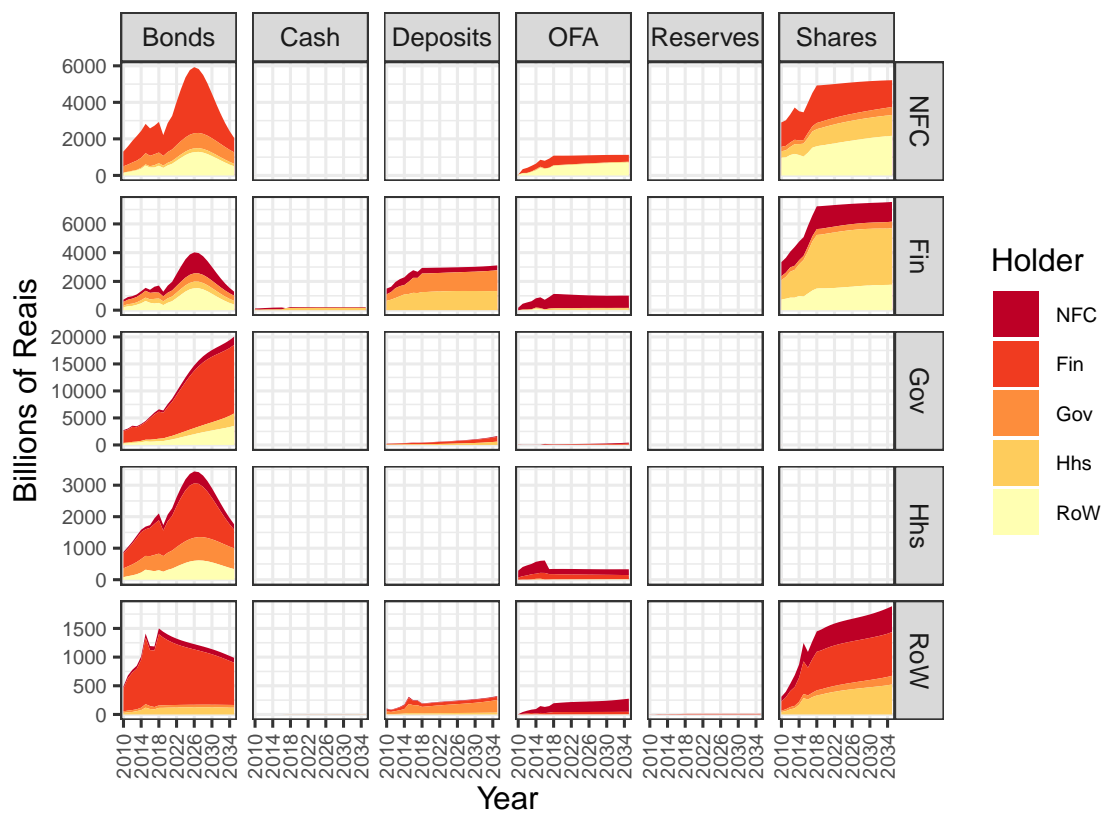


Figure 3.13: Total stock of financial liabilities by issuing counterpart

3.3.2 Policy scenarios

Description of scenarios

Two policy interventions are simulated to test the model’s capabilities, namely, the introduction of a carbon tax, and the implementation of a rule that limits the growth of government expenditure in final goods and services.

Scenario: Carbon tax

Evaluate the progressive introduction of a carbon levy on domestic industrial CO_2 emissions. The carbon price is set to be introduced in 2023, growing linearly to reach levels of 70 USD/tCO₂e and 120 USD/tCO₂e by 2030 and 2035, respectively. Furthermore, the scenario assumes that 90% of the revenues created by the tax are recycled back to the economy through three channels: Transfers to households (50%), reduction of corporate income taxes (20%), and reduction of personal income taxes (20%).

Scenario: Limit government expenditure

Considers the deteriorating position of the government as a net lender, and simulates the effect of limiting the government expenditure on final goods and services to grow at a maximum pace of 1.5% above the last observed inflation.³⁴

The next section presents the main results from these simulations. Additional figures comparing the results from the three scenarios can be seen in Appendix 3.A.1. Other scenario-specific results are also included in Appendix 3.A.2.

Simulation results

As it can be seen in Figure 3.14, both policies have a negative effect on the GDP. Under the rule to limit the government expenditure’s growth rate, the GDP by 2035 would be 2% lower than that of the Baseline, with a minimal reduction of emissions during the simulated period (Figure 3.15).

The carbon tax scenario also produces a negative effect on GDP, which is partially tamed by the revenue recycling mechanisms used. Under this scenario, GDP would be 1.6% lower than the Baseline in 2035. This is, however, compensated by the reduction in emissions, which reach levels of 7.2% below the baseline for 2035. The cap on expenditures produces a minimal reduction in emissions.

³⁴This is taken in line with proposals discussed in early 2023 to set a rule that would allow the government expenditure to increase at a pace between 0.6% and 2.5% above inflation. See for instance: <https://www.gov.br/planalto/en/latest-news/brazil2019s-new-fiscal-rules-include-the-poorest-in-the-government-budget-and-foresee-a-surplus-for-2025>

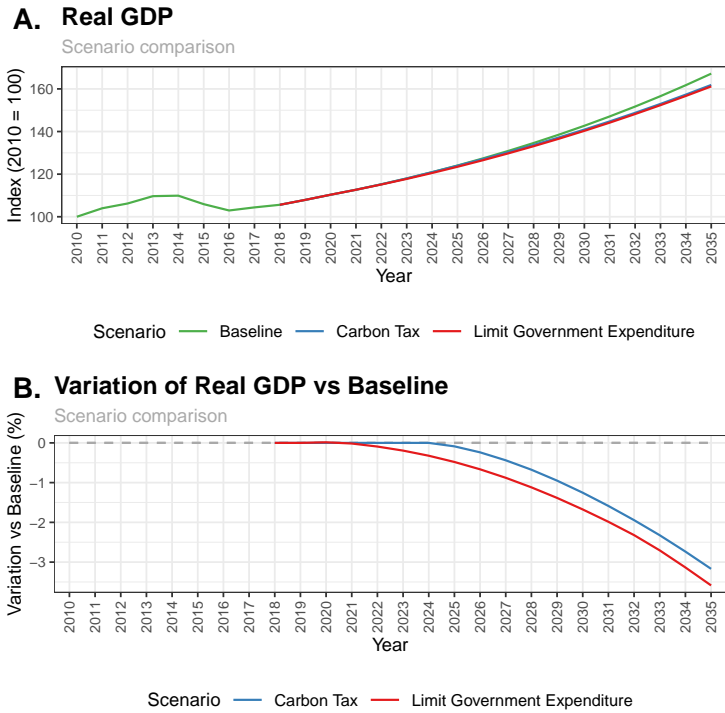


Figure 3.14: Real GDP evolution and variation vs Baseline

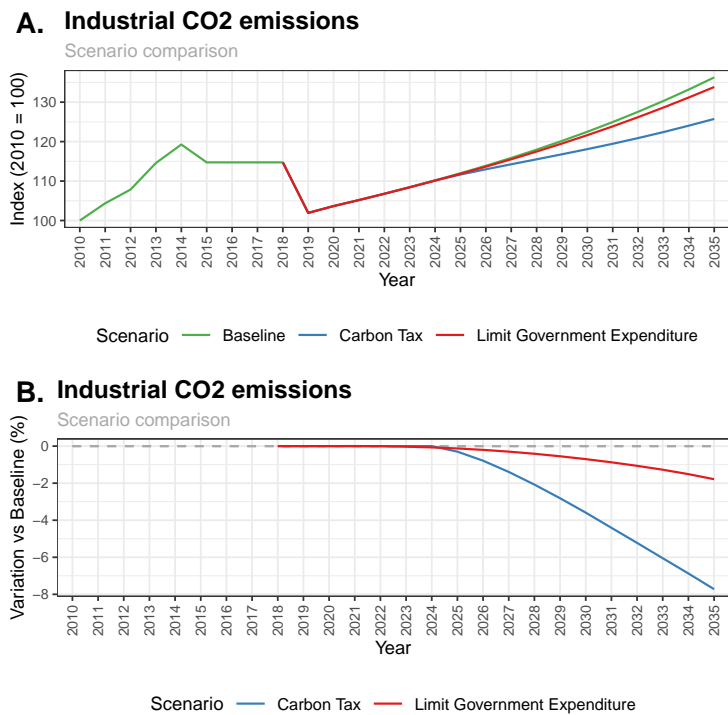


Figure 3.15: Evolution of CO2 emissions, and variation vs Baseline

The GDP trajectory is explained by a small decrease in the GDP growth rate, which still remains positive and around 2.5% in both scenarios, (Figure 3.16, A). By 2035, the reduced activity resulting from the higher domestic prices, or the cap on demand from the government, is reflected by the employment rate decreasing 1.5 percentage points with respect to the baseline (Figure 3.16, B).

Inflation increases under the carbon tax scenario given the effect the levy has on domestic prices, but decreases on the scenario under limited government expenditure given the relative shrink in demand for goods and services. In all cases, inflation remains above 4% under the current configuration (Figure 3.16, C). The increase of more than 1 percentage point on inflation under the carbon tax scenario, also leads to a slight depreciation of the currency (Figure 3.16, D).

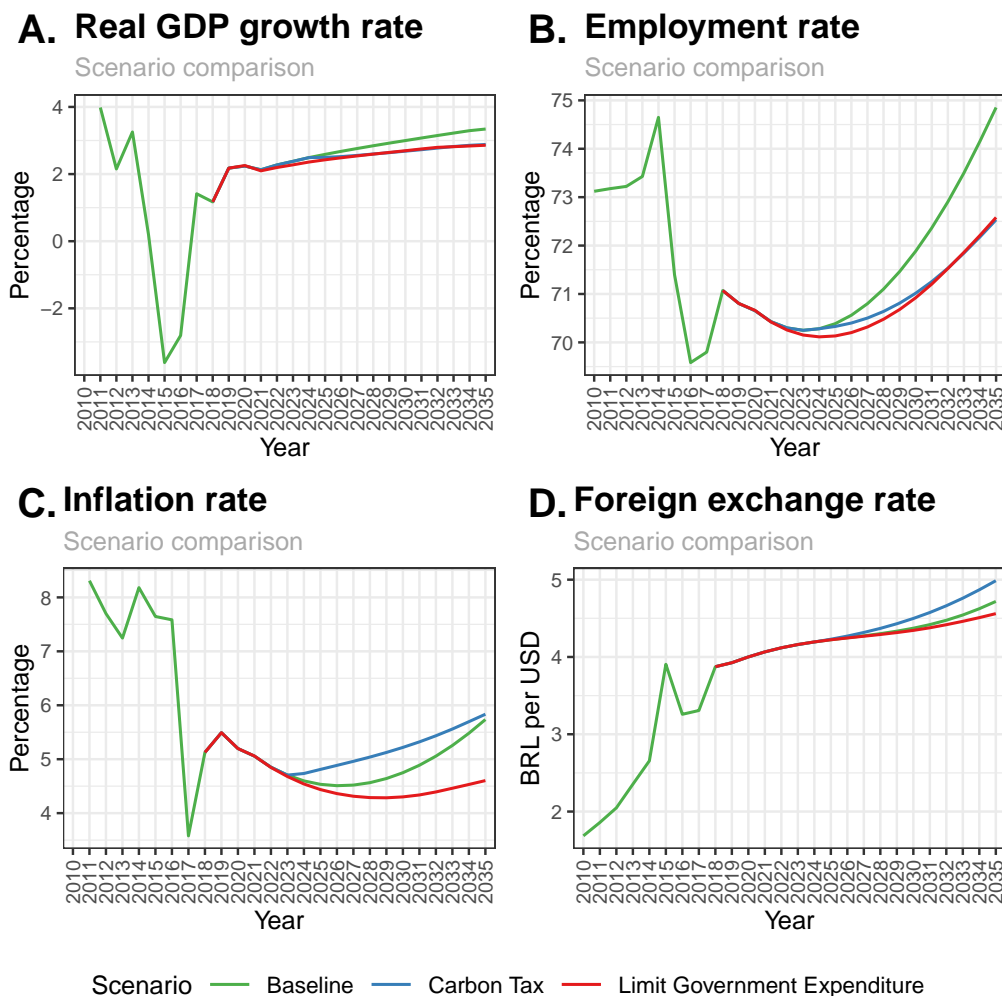


Figure 3.16: Real GDP growth, employment, inflation and exchange rates, by scenario.

As for the net financing capacity or requirements (see Figure 3.17), the revenue recycling mechanisms simulated under the carbon tax scenario appear to correct the trajectory for households, which remain net lenders, with annual net financing capacities of about 2% of GDP. Domestic corporations, despite being taxed on industrial emissions, remain

as a whole partially unaffected given the compensation received through reductions in corporate income tax. The rest of the World, on the other hand, experiences a reduced total demand, which paired with the slow substitution of domestic goods by foreign ones, results in a slight decrease in its position as net towards the end of the simulated period. The government remains fairly unaffected given the tax is not implemented with fiscal objectives, and only 10% of the revenues raised remain under its accounts.

The scenario setting a cap on government expenditures, on the other hand, improves the government’s position, reducing its borrowing needs. This, however, is done at the expense of the domestic households and non-financial corporations, both of which appear as net borrowers towards 2035.

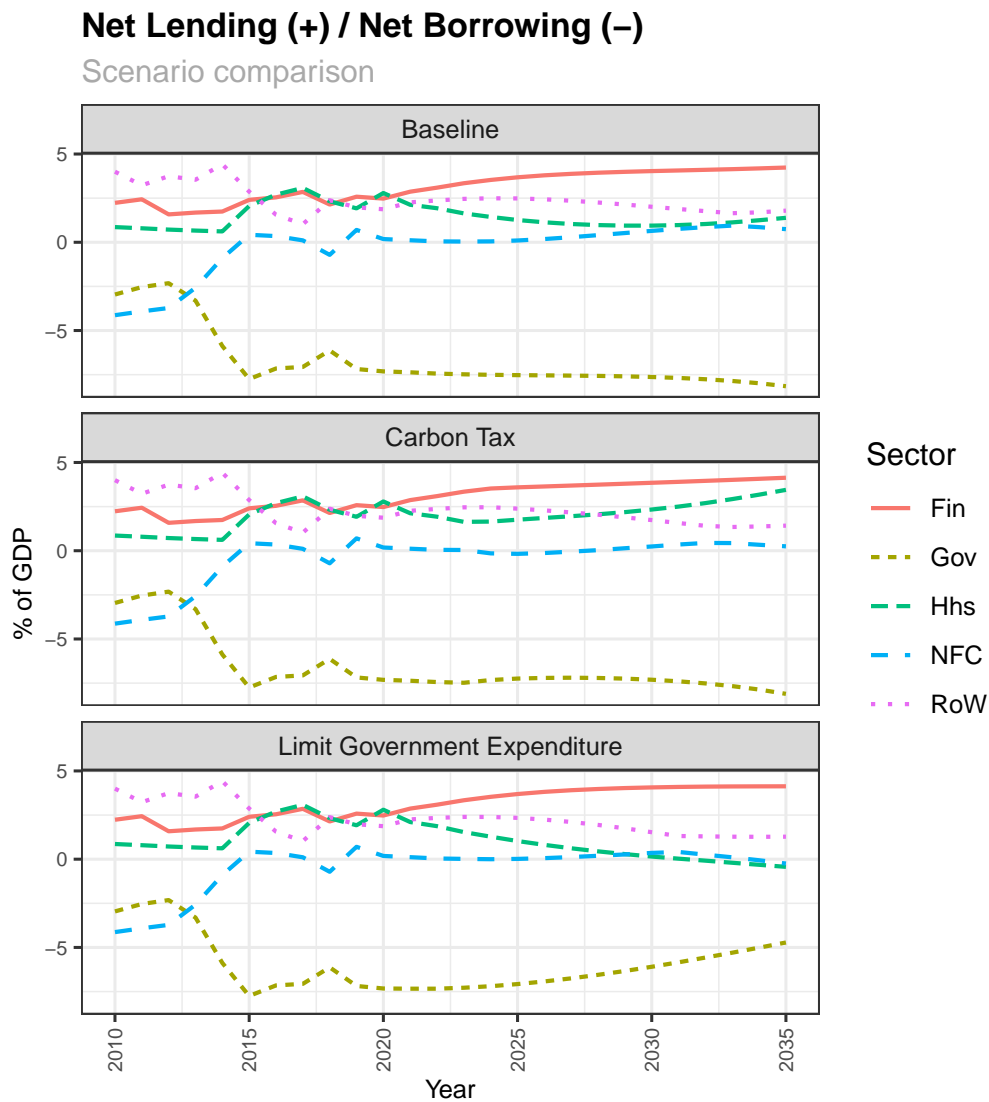


Figure 3.17: Net financing capacity or requirements by scenario and sector.

3.4 Discussion, limitations, and future lines of improvement

While still a pilot version, the model allows to tackle example questions in the directions desired. First, the model projects the evolution of macro-financial variables as a result of climate mitigation policies (carbon tax); and second, the model can evaluate not only macroeconomic performance but also the evolution of industrial emissions following a fiscal policy not directly aimed at controlling them. Its structure ensures accounting consistency within and between both layers and allows the simulation of industrial effects despite not being able to determine industry-specific balance sheets, as it would be desired in a theoretical model.

The level of detail used in the production module allows for results to be tracked by industry, origin (domestic or imported), and multiple valuation methods (real terms, basic prices, and market prices), which enables the modeler to spot sources of chaotic behavior despite the size of the model.

The use of key aggregate accounts for the macro layer ensures consistency with historical data can be verified at every stage of the construction process. Again, given the size and complexity of the model, this is particularly useful during the stages where the behavioral equations and the historical datasets are built in an inter-dependent and iterative way.

Furthermore, this is done without imposing optimizing behaviors, rational expectations, or market-clearing prices. As such, the flexibility of modeling choices is greater than with other frameworks relying on similar data requirements and tackling the same questions.

On the other hand, it has to be noted that a model with this level of detail is extremely data-demanding. The creation and maintenance of a dataset consistent with the assumptions used in the running version of the model are by large the most time-consuming process. Simpler tools may be enough for quick assessments, but policymakers would certainly benefit from increased information on the effects of implementing one or other policy alternatives.

The data is indeed a constraint. Having used annual data only, and starting from the year for which the new conventions and base year for the national accounts in Brazil was introduced, there were too few yearly observations to perform econometric analysis. The calibration was hence done by fitting the model to existing macro forecasts. The lack of detailed information on the effects of COVID at the time of building the dataset was another constraint. Preliminary data has already been published, and upcoming releases could help switch the last observed year to 2021 instead of 2018.

Regarding the simulations, given the layered structure of the model, and for traceability purposes on its debugging stages, the implementation has been done through a set of recursive equations not solved simultaneously. This means the model will not look for a fix-point solution, and that the system of equations will need variables to be sequentially determined.

Moreover, the time step for the simulations has been set to be one year. While helped speed up the calibration process of the pilot version, it certainly has several drawbacks that need to be acknowledged. Among the most important ones, the multiplier effects is most likely underestimated given (i) the lack of iterations within each year, and (ii) the lack of a fixed point solution that could compensate for the former; overshooting or undershooting behavioral responses to external shocks can have drastic balancing replies otherwise unseen when simulating multiple iterations within each year. This is for instance observed on the exports of services (SRV), as well as the imports of utilities (ELE) and transportation (TRP) under the carbon tax scenario. More generally, the elements described here contribute to creating chaotic behavior whenever simulations for the other scenarios are produced for a longer time horizon (e.g. 2050).

It should be noted as well that the model has been run under multiple simplifying assumptions, turning off more realistic behavioral rules and feedback mechanisms to speed up calibration and to produce a running pilot version. An example of this is the numerical process used to deal with the allocation of supply and demand of financial assets, which has produced simulations in which all accounting principles are respected, but no portfolio preferences or constraints were explicitly set.

All in all, despite the ability to produce some results consistent with the targets set during the calibration process, there is still much room for improvement at both the technical and theoretical levels. In terms of priorities, the model should consider more iterations between years.³⁵ The introduction of additional iterations will require a re-calibration of parameters. Ideally, behavioral equations could be updated to avoid extreme simplifications. Finally, with the upcoming releases of national accounting data, there will be information to evaluate the magnitude and distribution of the COVID-related shocks, and the first signs of economic reorganization as the recovery process takes place. This can certainly enrich the insights that the model can provide by ensuring the baseline considers these events.

³⁵On a future step, this could be further improved by re-calibrating the model using quarterly information if available.

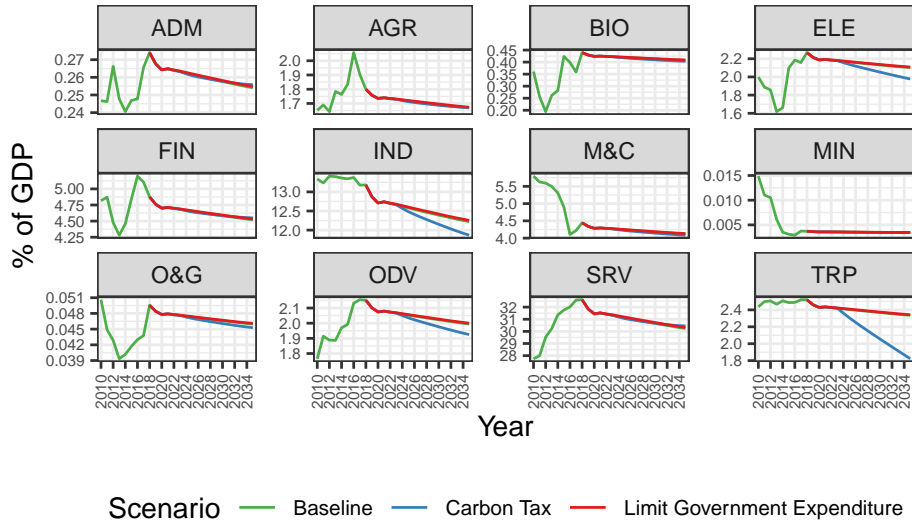
Appendices for Chapter 3

3.A Additional figures

3.A.1 Comparing scenarios

A. Household consumption by industry

Scenario comparison



B. Household consumption by industry

Variation vs Baseline

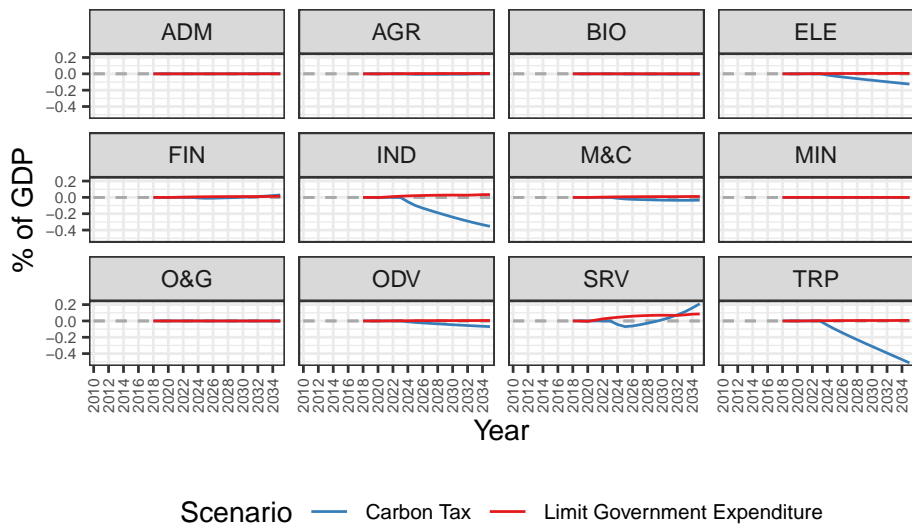
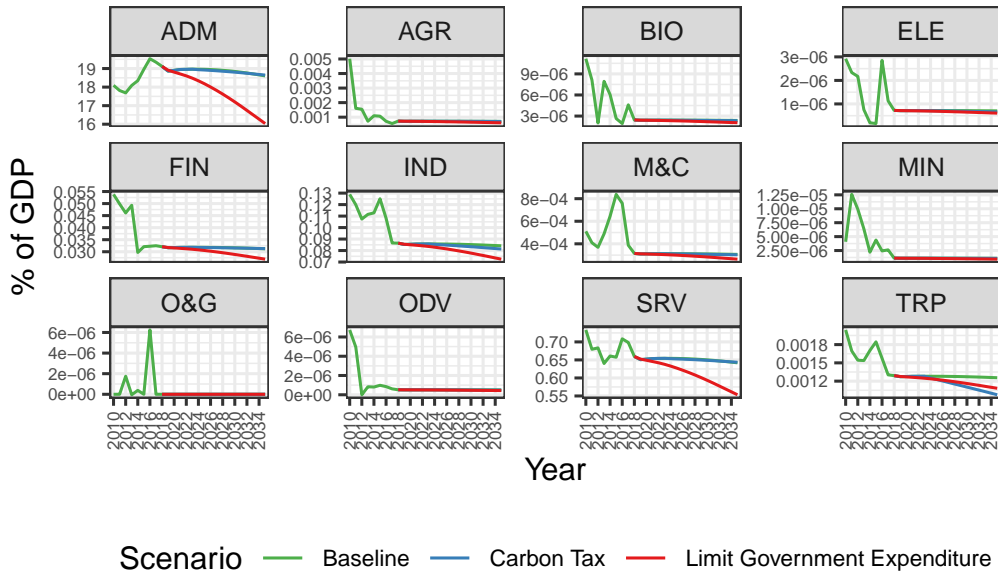


Figure 3.18: Consumption by type of good. Level and variations with respect to Baseline.

A. Government expenditure by industry

Scenario comparison



B. Government expenditure by industry

Variation vs Baseline

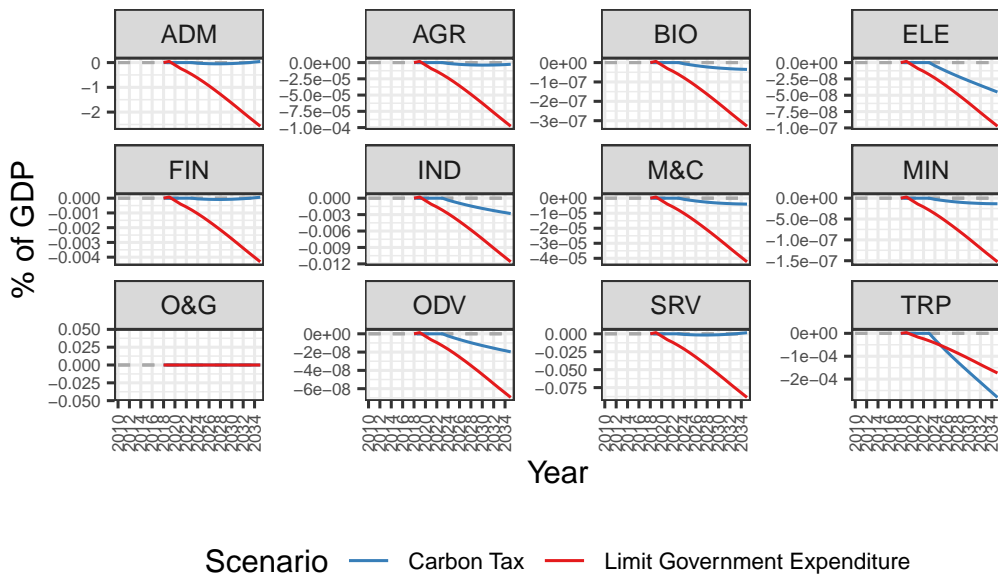
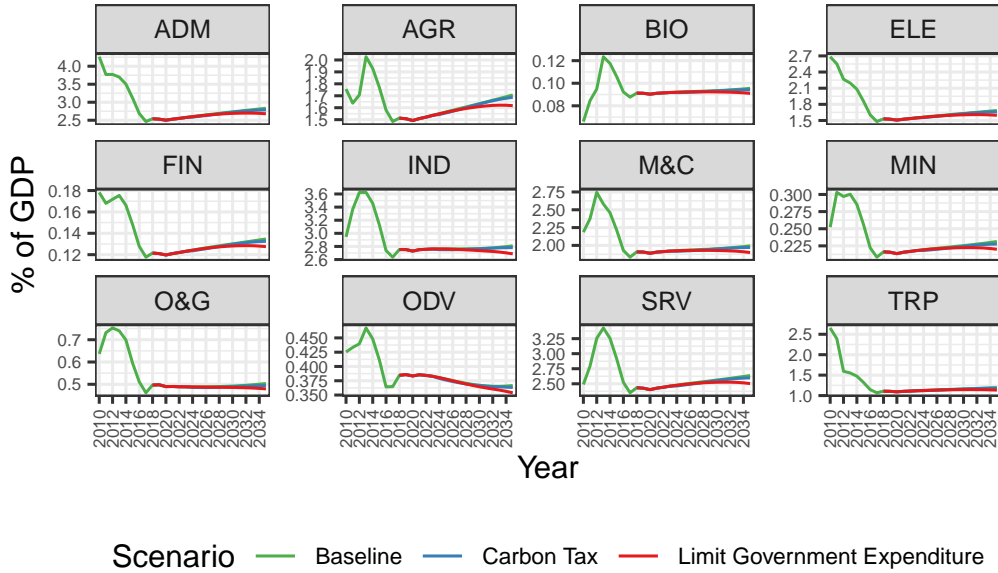


Figure 3.19: Government expenditure by type of good. Level and variations with respect to Baseline.

A. GFCF by investing industry

Scenario comparison



B. GFCF by investing industry

Variation vs Baseline

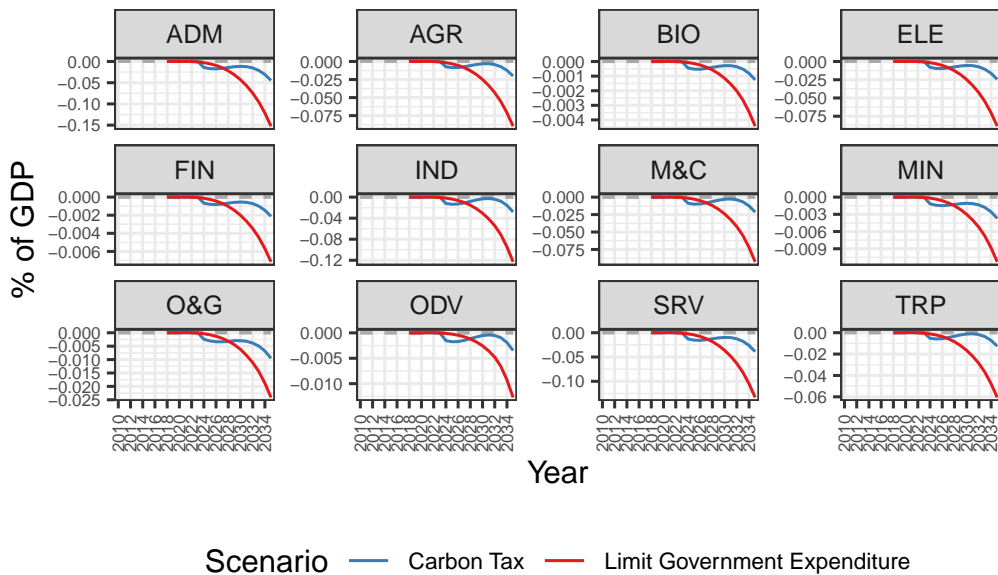
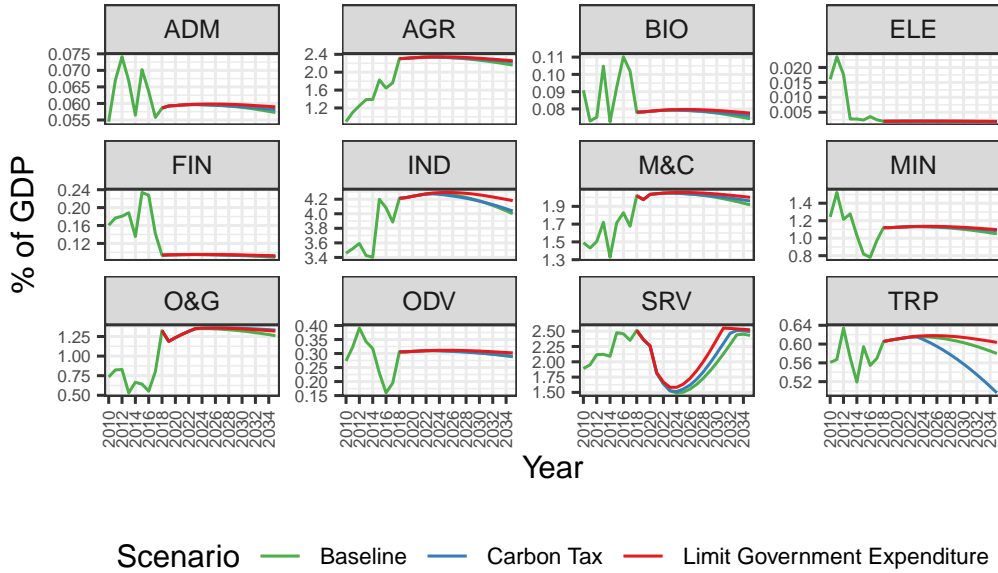


Figure 3.20: Gross fixed capital formation by investing industry. Level and variations with respect to Baseline.

A. Exports by industry

Scenario comparison



B. Exports by industry

Variation vs Baseline

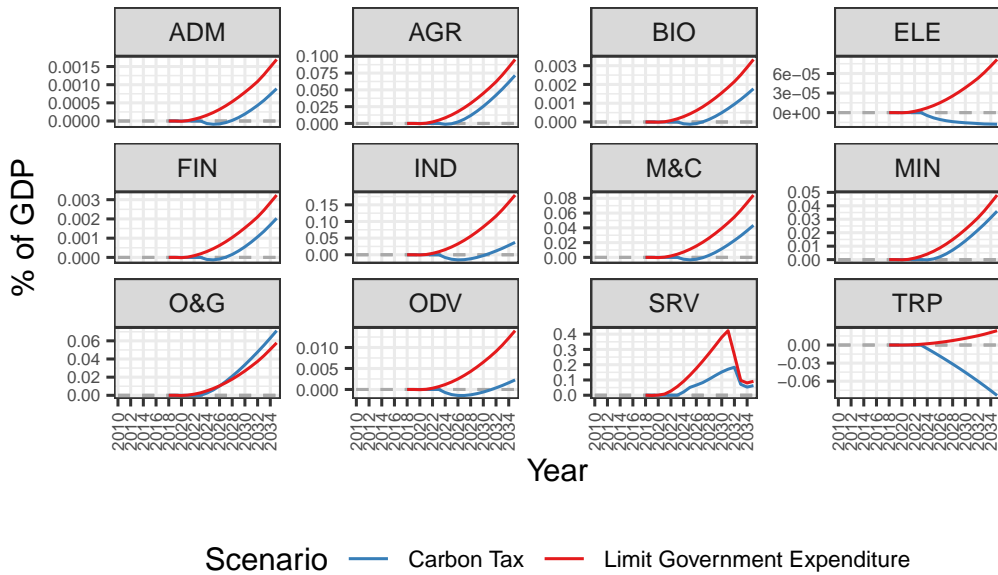
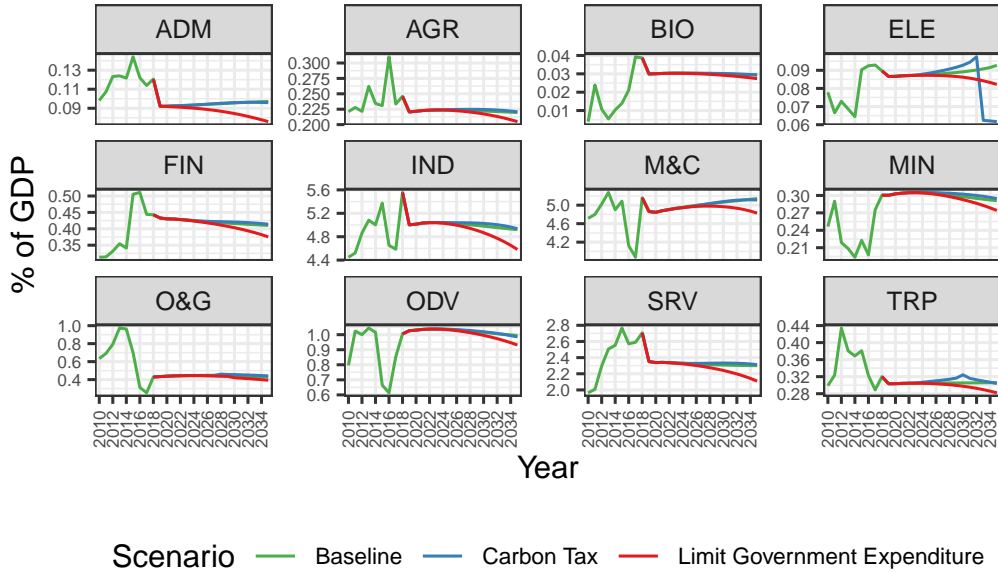


Figure 3.21: Exports by type of good. Level and variations with respect to Baseline.

A. Imports by type of good

Scenario comparison



B. Imports by type of good

Variation vs Baseline

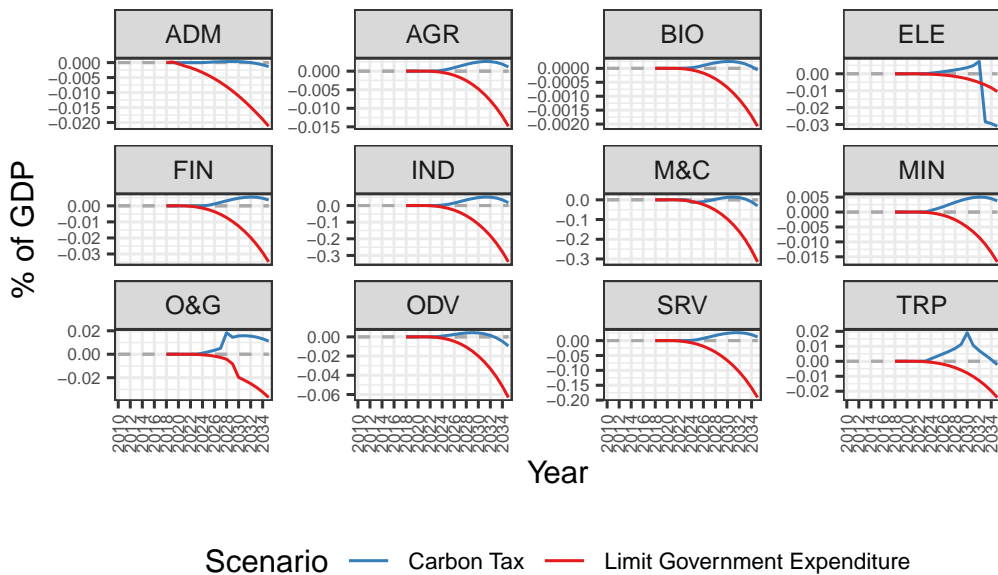


Figure 3.22: Imports by type of good. Level and variations with respect to Baseline.

3.A.2 Additional policy-specific figures

Scenario: Carbon Tax

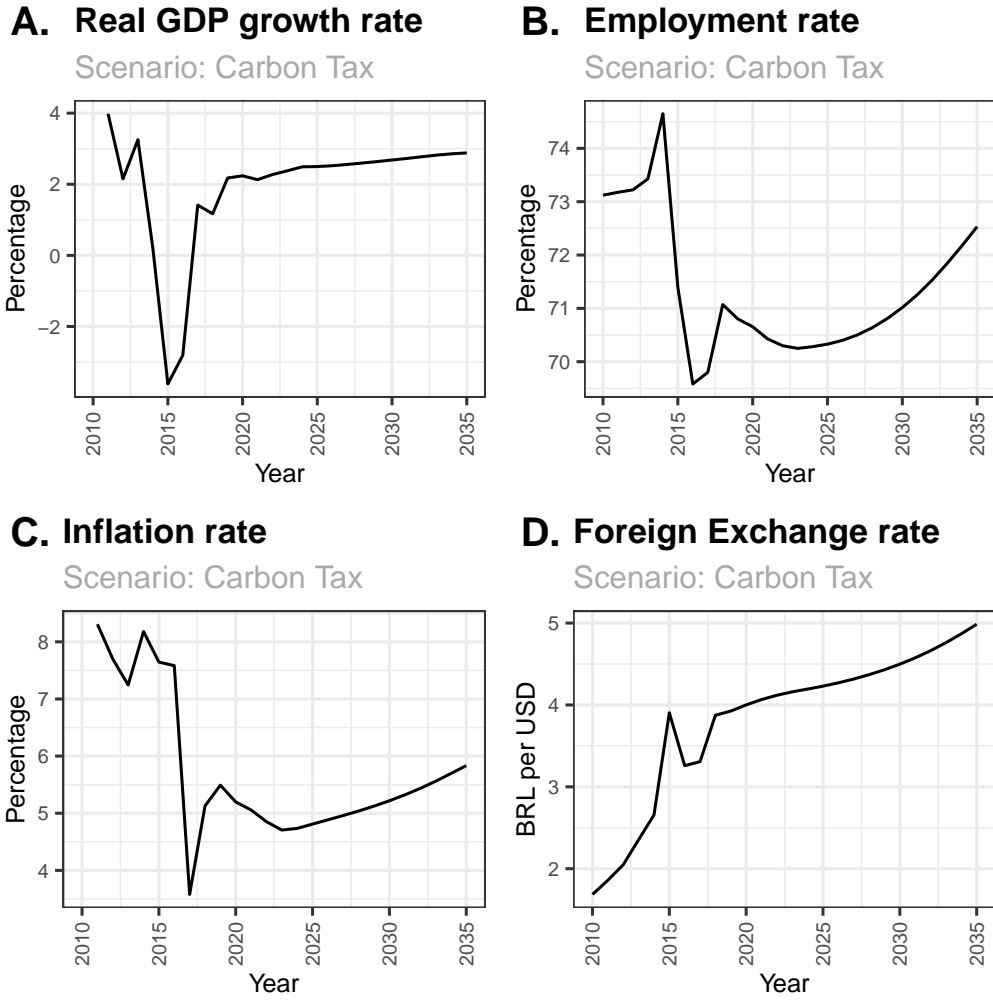


Figure 3.23: Real GDP growth, employment, inflation, and foreign exchange rates.

Labor use by industry

Scenario: Carbon Tax

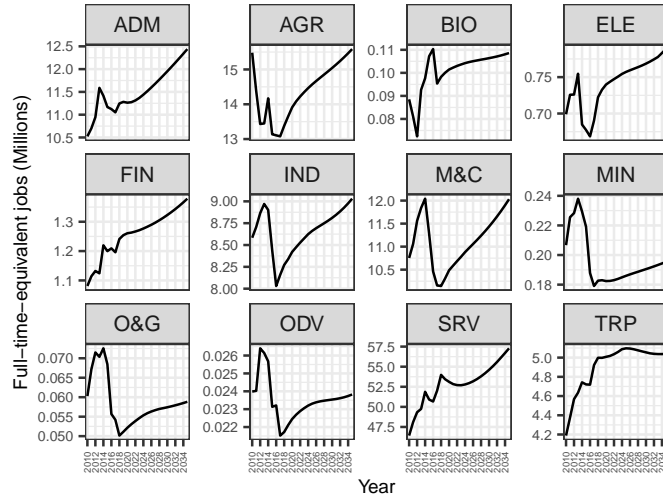


Figure 3.24: Labor use by industry

GDP Composition

Scenario: Carbon Tax

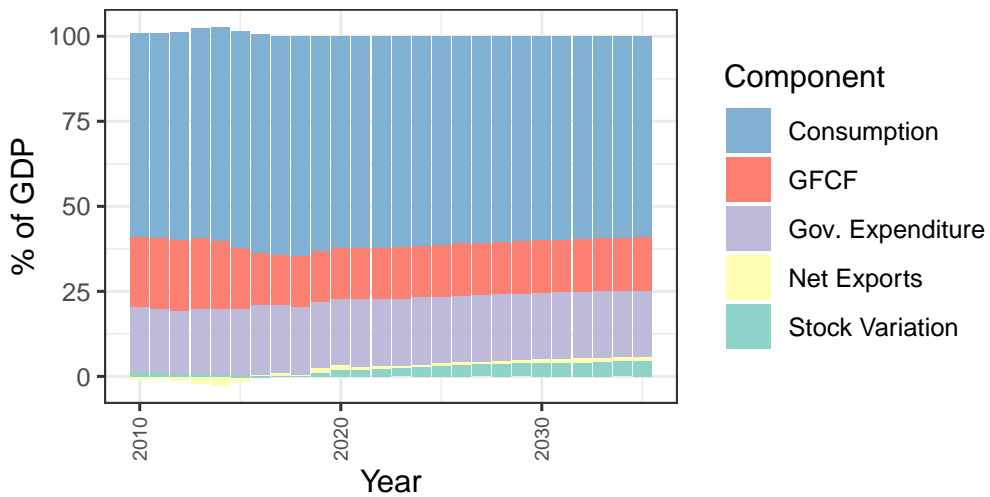


Figure 3.25: GDP by expenditure composition

Net financial flows

Scenario: Carbon Tax

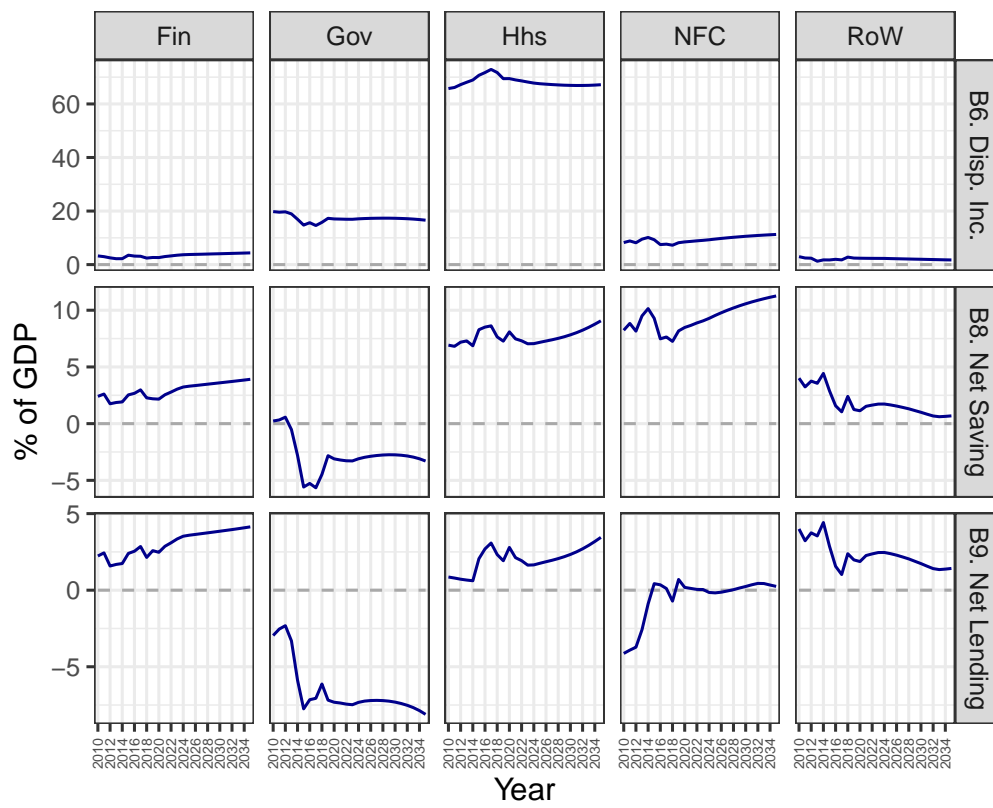


Figure 3.26: Key (net) financial flows

Evolution of Stocks of Assets

Carbon Tax

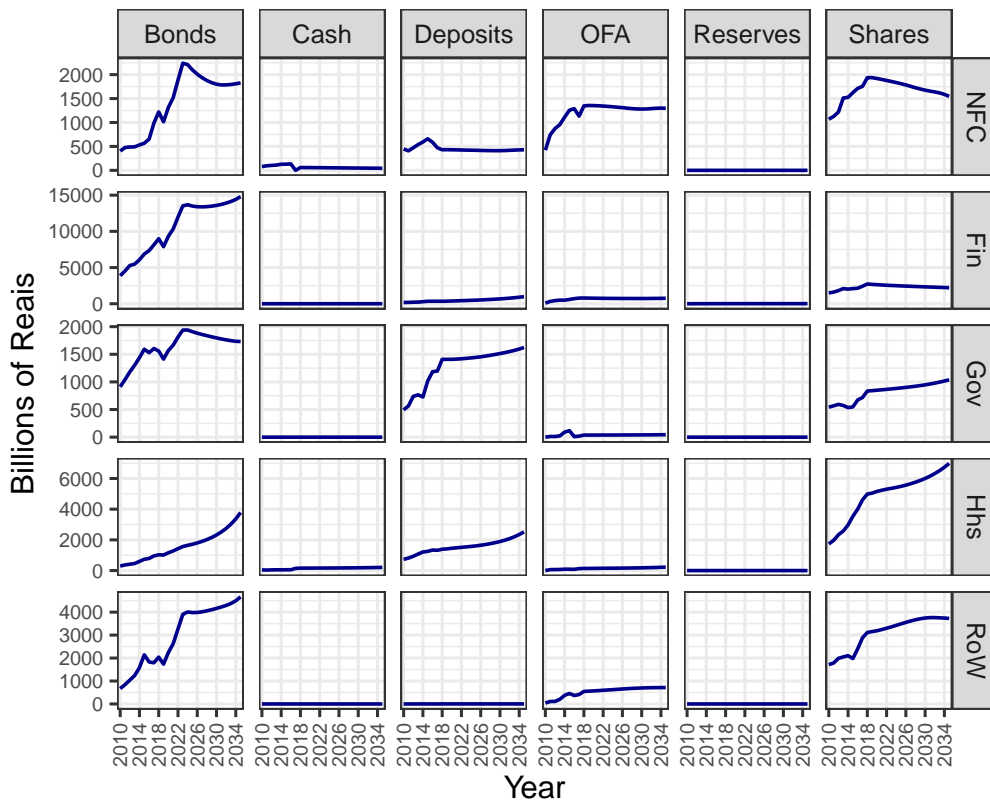


Figure 3.27: Evolution of the total stock of financial assets

Stock of Assets by issuing counterpart

Carbon Tax

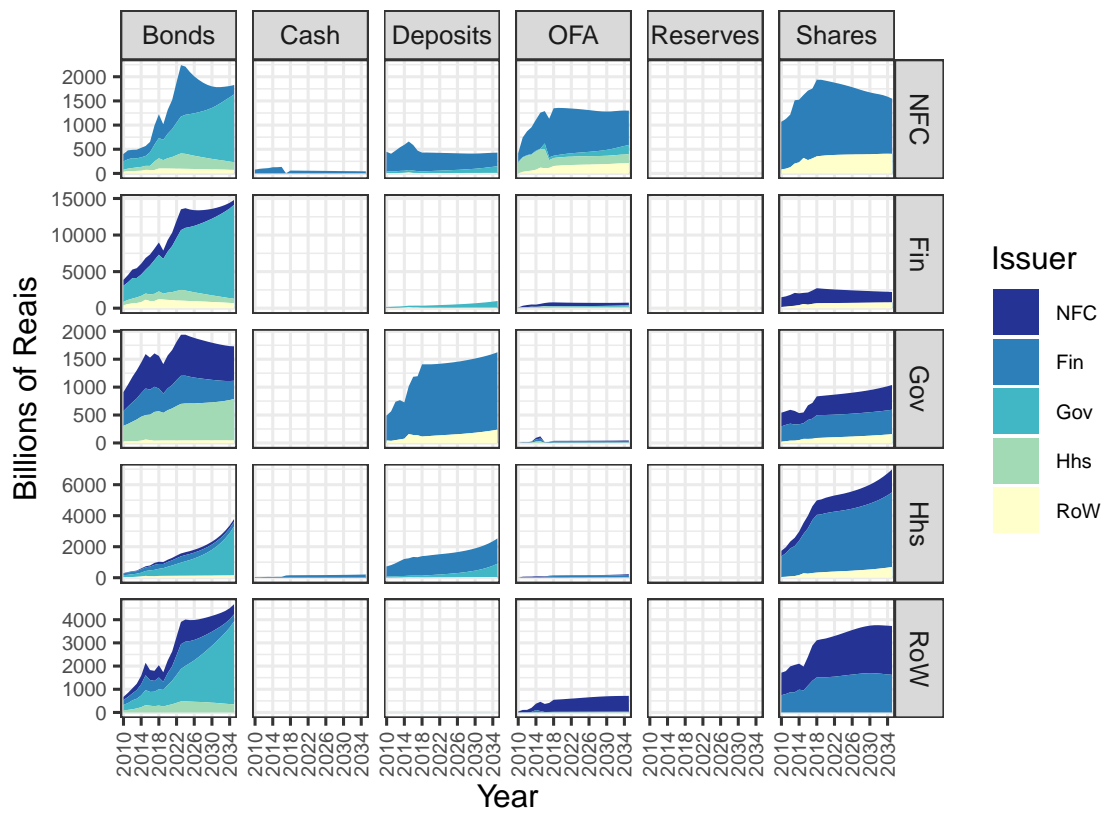


Figure 3.28: Total stock of financial assets by issuing counterpart

Evolution of Stocks of Liabilities

Carbon Tax

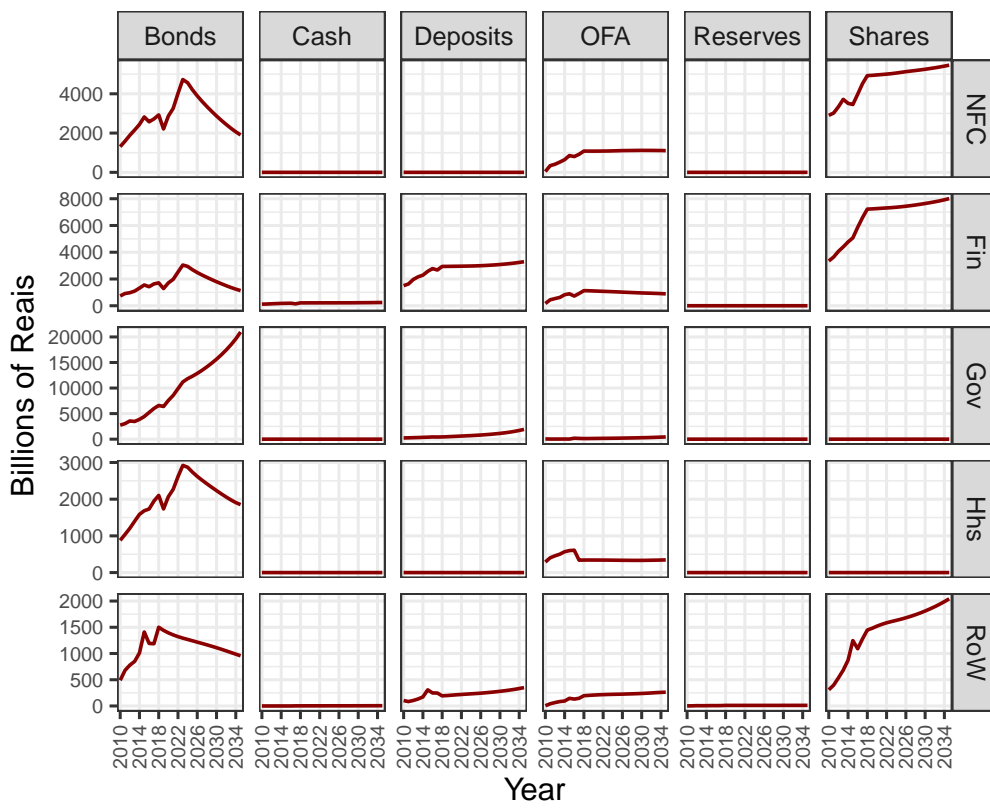


Figure 3.29: Evolution of the total stock of financial liabilities

Stock of Liabilities by holding counterpart

Carbon Tax

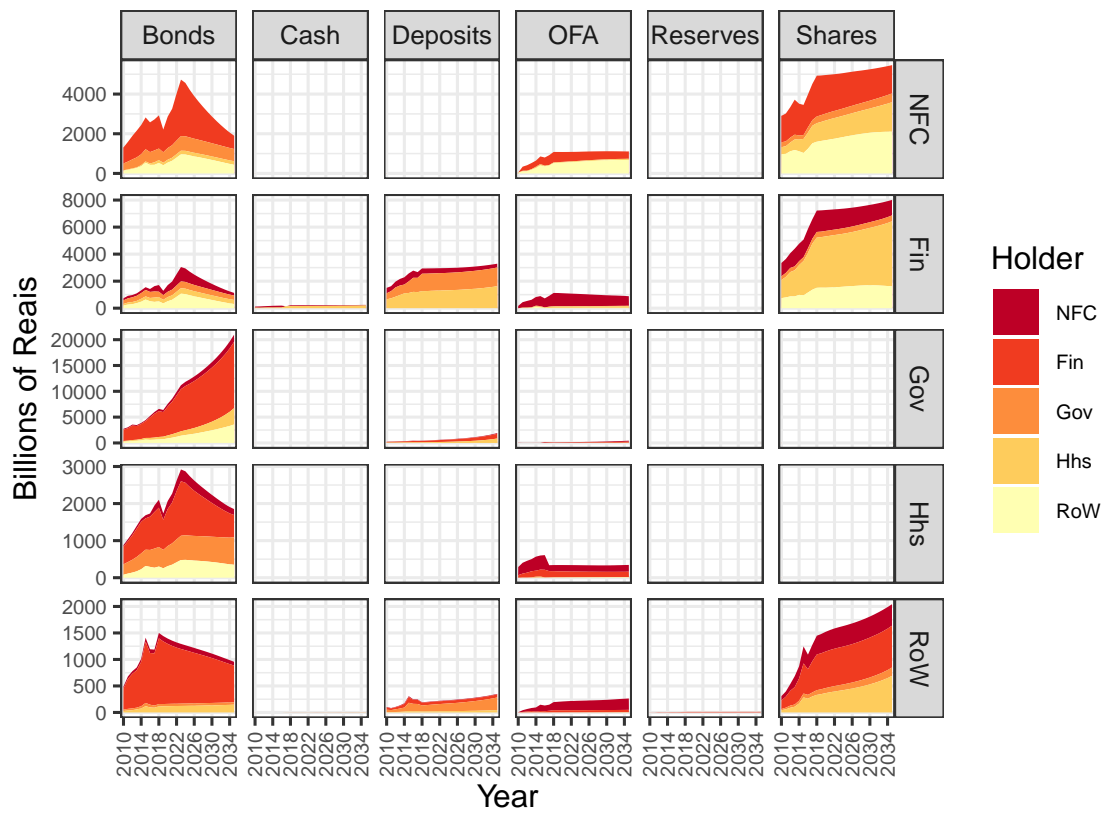
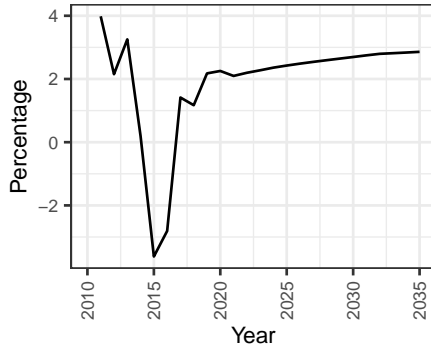


Figure 3.30: Total stock of financial liabilities by issuing counterpart

Scenario: Limit Government Expenditures

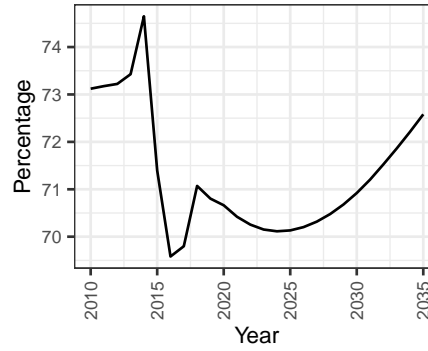
A. Real GDP growth rate

Scenario: Limit Gov. Exp.



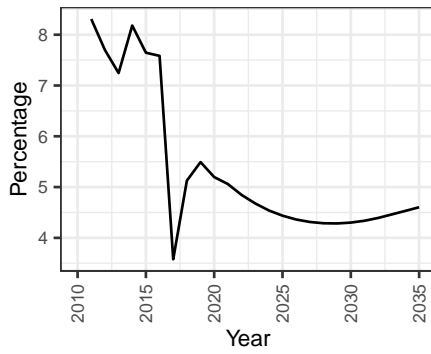
B. Employment rate

Scenario: Limit Gov. Exp.



C. Inflation rate

Scenario: Limit Gov. Exp.



D. Foreign Exchange rate

Scenario: Limit Gov. Exp.

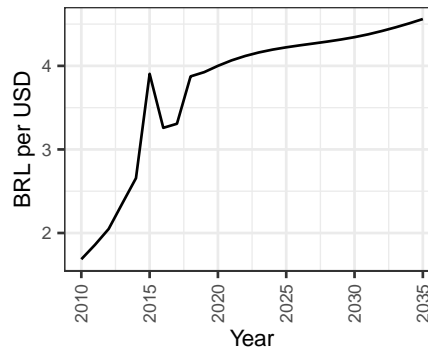
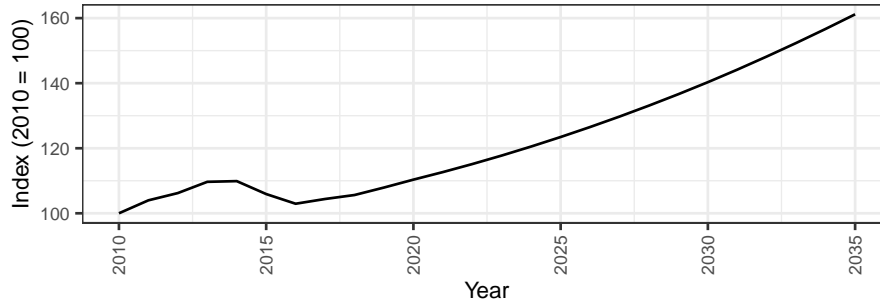


Figure 3.31: Real GDP growth, employment, inflation, and foreign exchange rates.

A. Real GDP

Scenario: Limit Gov. Expenditure



B. Industrial CO2 emissions

Scenario: Limit Gov. Expenditure

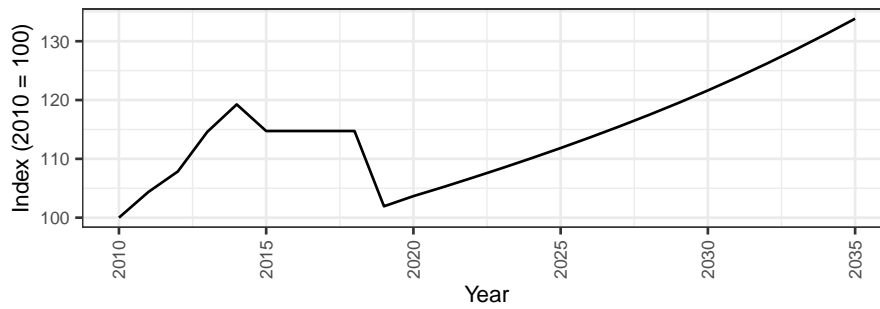


Figure 3.32: Evolution of real GDP and industrial emissions

Labor use by industry

Scenario: Limit Gov. Expenditure

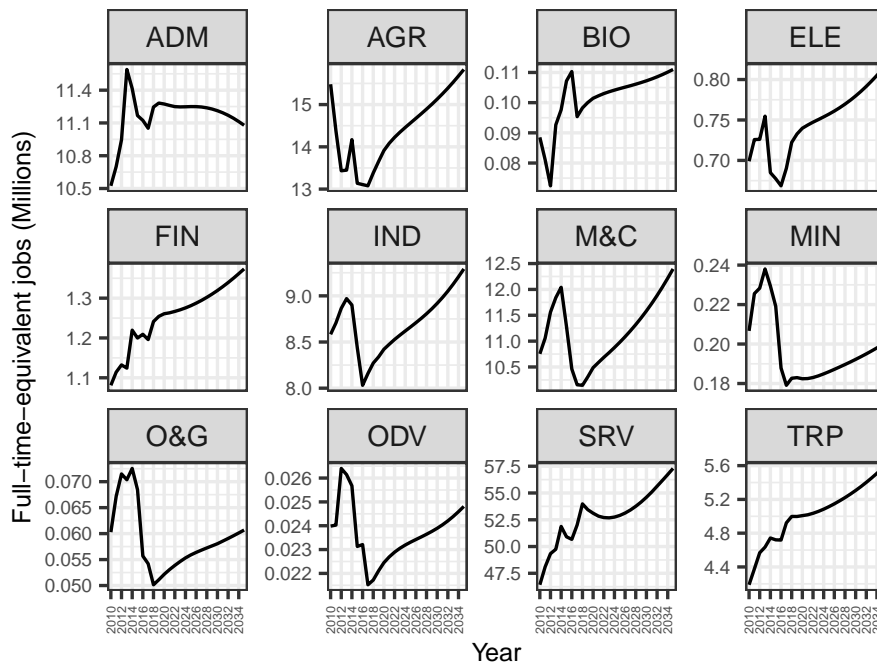


Figure 3.33: Labor use by industry

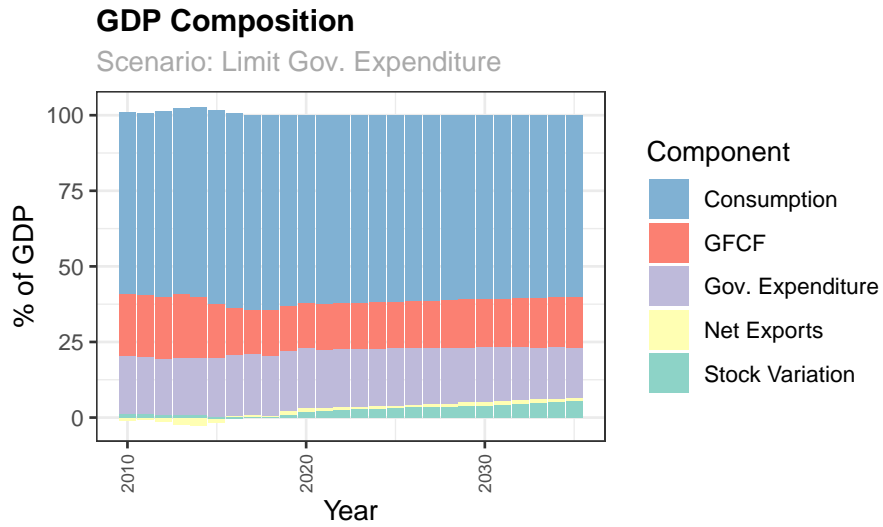


Figure 3.34: GDP by expenditure composition

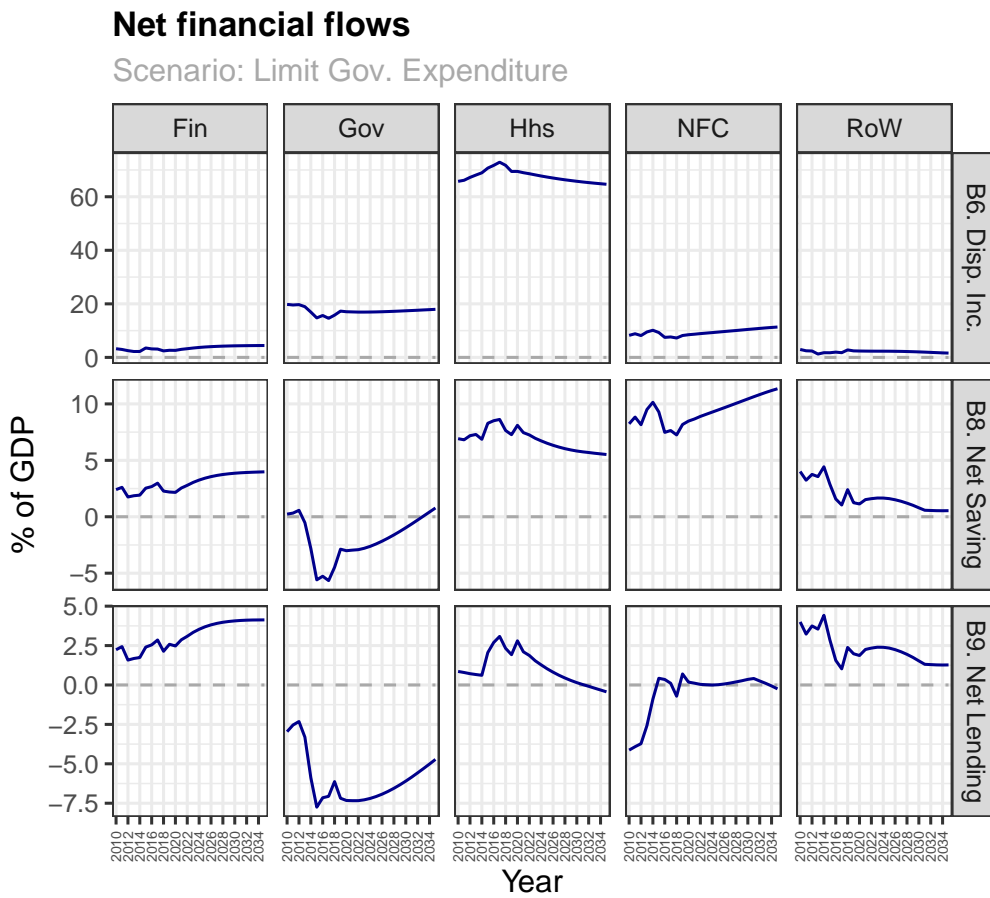


Figure 3.35: Key (net) financial flows

Evolution of Stocks of Assets

Limit Government Expenditure

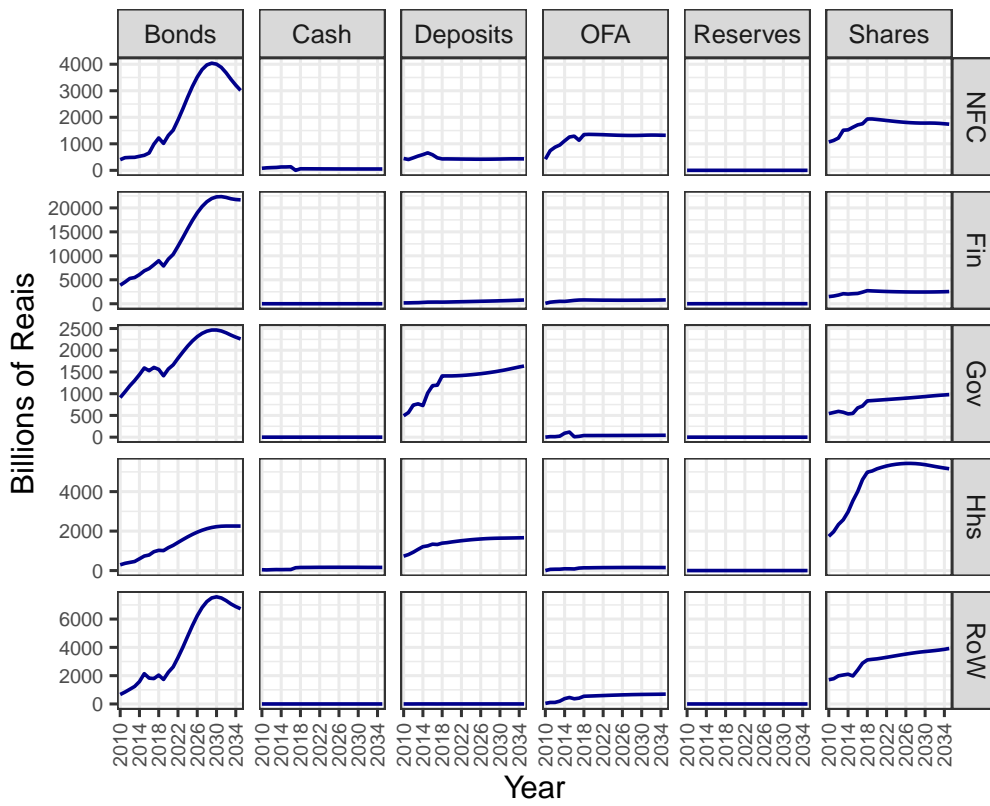


Figure 3.36: Evolution of the total stock of financial assets

Stock of Assets by issuing counterpart

Limit Government Expenditure

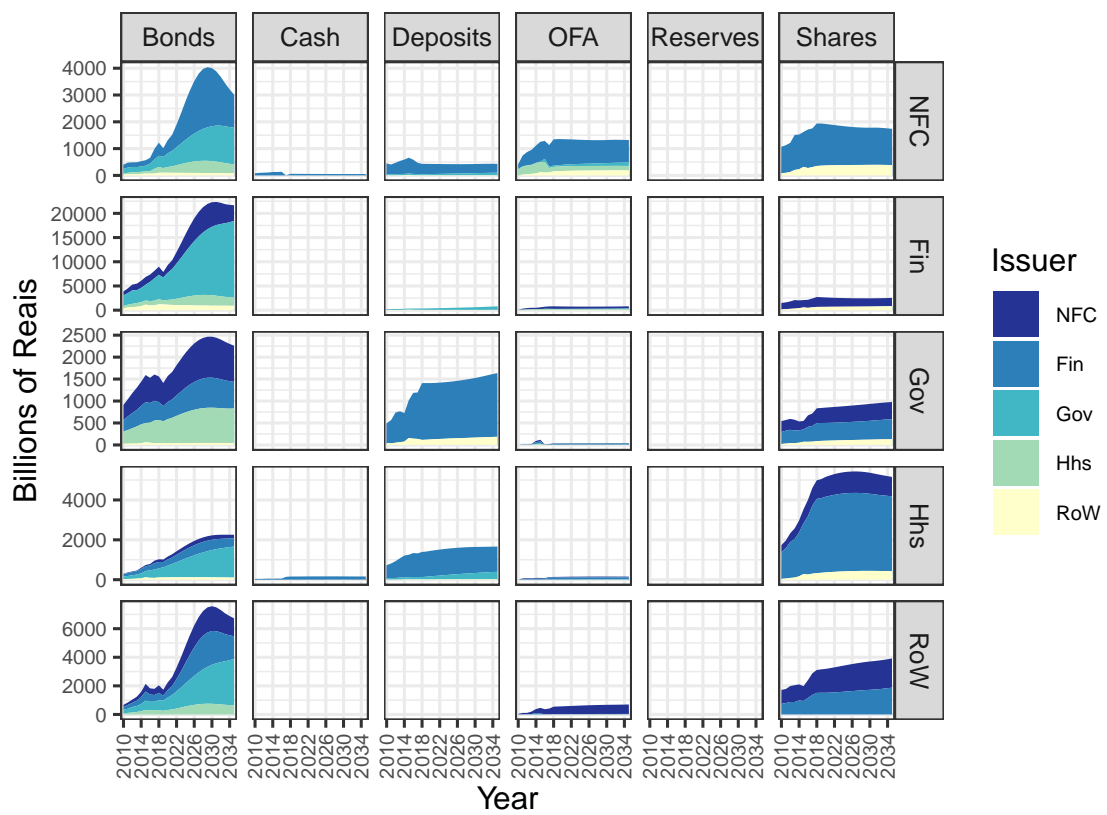


Figure 3.37: Total stock of financial assets by issuing counterpart

Evolution of Stocks of Liabilities

Limit Government Expenditure

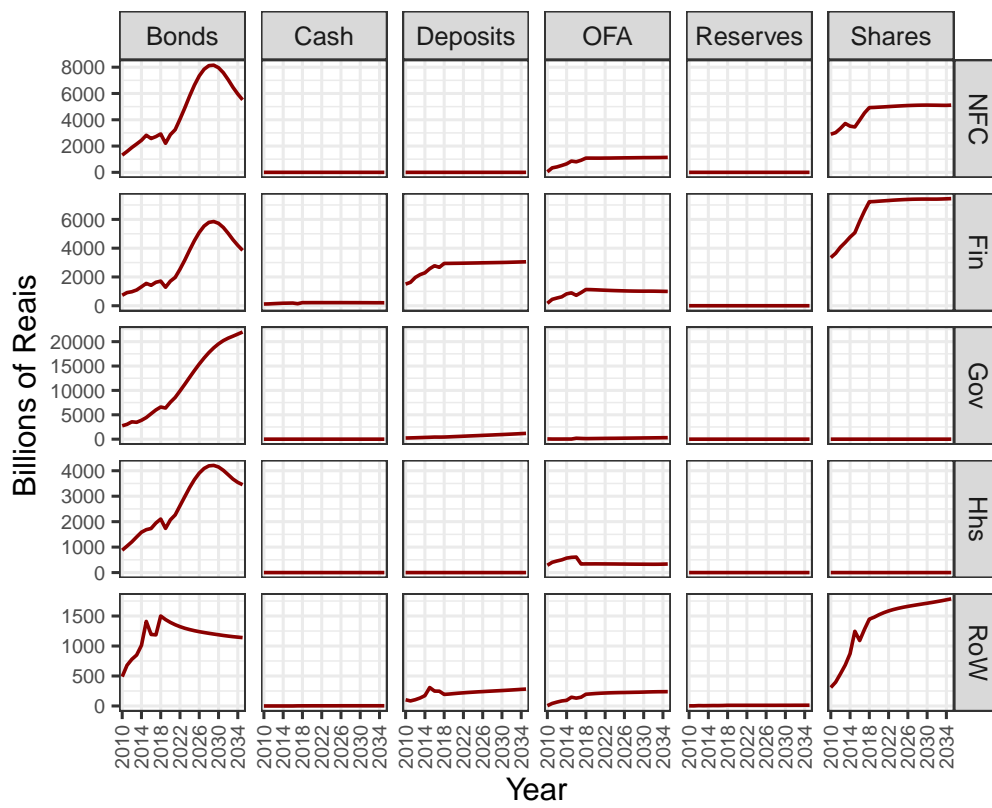


Figure 3.38: Evolution of the total stock of financial liabilities

Stock of Liabilities by holding counterpart

Limit Government Expenditure

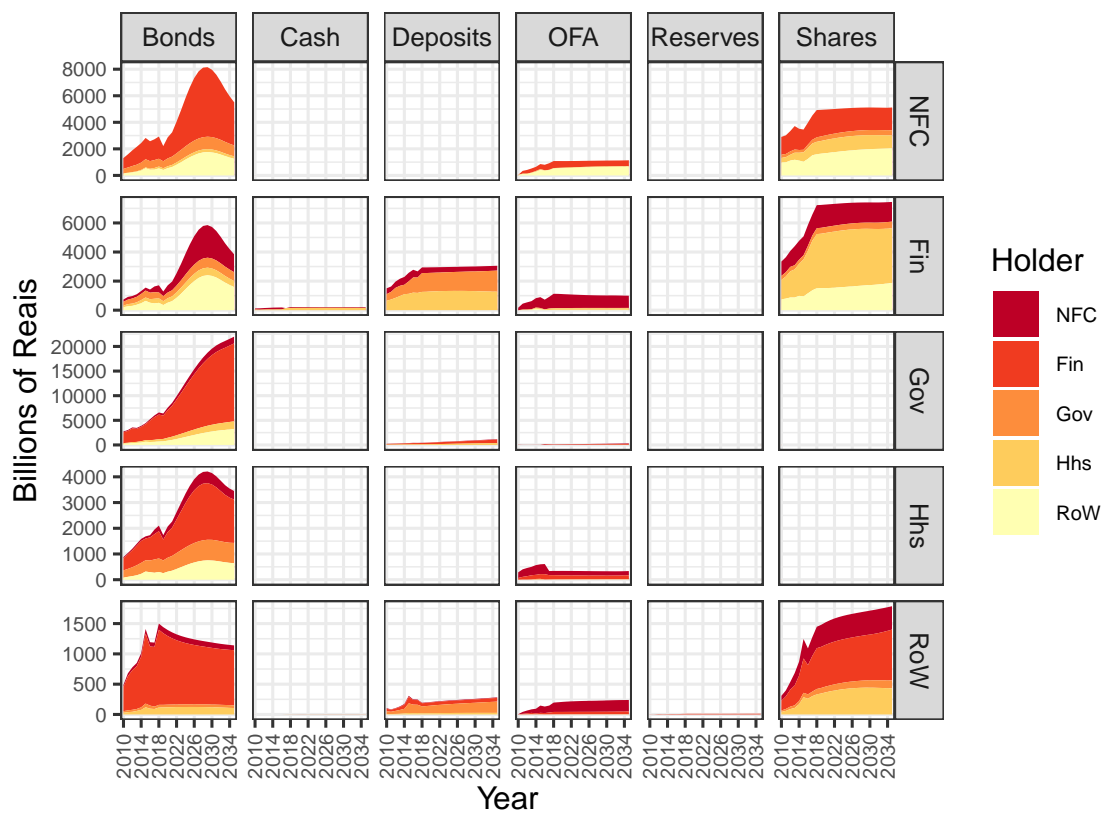


Figure 3.39: Total stock of financial liabilities by issuing counterpart

3.B Historical data, sources and transformations

The Brazilian national accounts, as reported by the *Instituto Brasileiro de Geografia e Estatística* (IBGE), are the prime source of information for our purposes. Among the sets of data published by the IBGE, the Supply-and-Use Tables (SUT) and the Integrated Economic Accounts (IEA) are the most useful for building the structure and dynamics of the model. These are expanded and complemented with the help of other domestic and international sources to achieve the coverage and level of detail desired.

The information from the IEA framework provides a detailed description of multiple accounts regarding production, generation, distribution, and redistribution of income, as well as the balance sheets by institutional sector, for any given period, [Lequiller and Blades \(2014\)](#). The production accounts within the IEA framework, however, do not have the level of detail desired, as these are presented at the level of institutional sectors, and do not exhibit the production network at the industry level. These are hence re-computed based on the information from the SUT framework under the assumption that households provide the labor required for production but do not engage directly in production, delegating this activity to industries. As such, the cost of all inputs and the proceeds from all products sold are faced by industries, who later transfer the corresponding part to households to match the production and the gross operating surplus they hold under the IEA information.

The production information comes from the SUT, which are transformed into input-output-tables (IOT) and expanded to account for industrial CO_2 emissions, and to provide the level of detail required for all relevant variables, and then aggregated into institutional sectors to be reconciled with the macro information in the IEA. The rest of the macro transactions come from the IEA, but are also expanded into who-to-whom tables for each account. This, as the IEA presents how an operation is a source and/or a use of funds for a given sector, but does not explicitly indicate its counterpart. While this is trivial in some cases, it is not a simple task for some other accounts where all sectors interact. A process of matrix balancing is applied to all subaccounts to create these who-to-whom matrices, which are then aggregated into larger key accounts providing information on counterparts for each transaction, which also allows for a detailed set of balance sheets to be obtained, where counterpart information is provided.

This exercise requires the use of multiple data sources, which often produce discrepancies that have to be reconciled. Moreover, the simplifying assumptions put in place require transformations and adjustments to be applied. These are done in such a way that the main macroeconomic aggregates are not altered, and that the within-model consistency is respected. The establishment of a hierarchy of data sources contributes towards that objective.

3.B.1 Expanding the macroeconomic information: the production network

Supply-use-tables

The SUT provide an integrated framework to present and check the consistency of national accounting data. They rely on the principle that the total supply of a good or service matches its total uses. The SUT allow to retrieve value added and aggregate demand in a detailed and consistent way. They also provide rich information on the production network at the industry and/or commodity levels and are the fundamental source of data to build the tables required for input-output analysis. A standard set of SUT could be represented in the integrated form as follows:³⁶

		Products				Industries				Final Use					Total
		c_1	c_2	...	c_M	i_1	i_2	...	i_N	C	I	G	X	SV	
Domestic Products	c_1	/				U_d				Y_d					x
	c_2														
	...														
	c_M														
Imported Products	c_1	/				U_m				Y_m					m
	c_2														
	...														
	c_M														
Industries	i_1	V				/				/					g
	i_2														
	...														
	i_N														
Value Added	Ce	/				W				/					w
	Nt														
	GOS														
Total		x^T				g^T				y					

Figure 3.40: Integrated supply-and-use framework

where sub-matrices should be read as:

- U_d and U_m : Use matrices for intermediates by origin of the products. Intermediate consumption of domestic and foreign goods by domestic industries for their production activities (product-by-industry).
- x : Column vector of total use of domestic products (product output).
- Y_d and Y_m : Final use matrices by origin of the products. Final consumption of domestic and foreign goods by category or purpose, including the accumulation of inventories (product-by-category).
- m : Column vector of total use of imported products (product imports).
- V : Make matrix. Describes the products that each industry supplies (industry-by-product). It is the transpose of the supply matrix.

³⁶The notation used here is the standard one employed to present the SUT framework. It is, however, only used for didactic purposes in this section and should not be confused with the variables introduced so far as part of the model accounting. Whenever the elements of this framework will be needed as part of the model, they will be given a model-consistent notation.

- g : Column vector of industrial production (industry output).
- W : Gross value added matrix. Among its components, it normally includes the compensation to employees, the net indirect taxes on production, and the gross operating surplus (component-by-industry).
- w : Column vector of gross value added (national total for each component).
- g^T : Row vector of industry output (industry output).
- y : Row vector of final use by purpose or category (total by category).

To ensure consistency, all of these matrices should be expressed under the same valuation, either in market or in basic prices. For the model, the first aim is to reconstruct the integrated SUT framework in basic prices. The IBGE provides most matrices needed to do this for the Brazilian case.

On the supply side, the IBGE reports the V^T matrix –the production of each good by industry– and the vector of imported products, m , both in basic prices. Regarding the use side, the intermediate consumption and final demand matrices are provided, U and Y respectively, without distinction by origin of the products. While these are both given in market prices, the data released includes the vectors of tax, transportation, and trade margins that can be used to estimate the information in basic prices. Finally, the value-added matrix, W , and the summary vector w are both provided, including the number of full-time-equivalent jobs required during the period by each industry as one of their components.

This information is published on an annual basis, with four alternative levels of disaggregation: 12, 20, 51 or 68 industries. For all of these, there is information from 2010 up to 2018. However, the retropolation that includes data from 2000 to 2010 only existed for the 12 or 51-industry level at the time of building the dataset. The latter has been chosen to increase the time coverage and the level of detail. Finally, these tables are published with current and last year’s prices, which allows for a reconstruction of price indices at the product or industry level.

The Input-Output tables

The input-output tables provide a detailed image of the productive structure of an economy. They are built from the SUT, and they allow to analyze the effects of supply or demand-sided shocks through the identification of technical coefficients. In the case of Brazil, the IBGE publishes a set of IOT each five years, with a delay of two. The last release was done in 2017, with information from 2015. Given the low frequency for its publication, the annual IOT will be constructed from the released SUT, using the official IOT as a reference only.

To better understand the difference between SUT and IOT, one should focus on the production matrix. The make matrix, V , from the SUT framework describes which prod-

ucts are manufactured by which industries. It is often a rectangular matrix, as the System of National Accounts (SNA) may register a higher number of commodities than industries. Moreover, it allows for a single product to be manufactured by firms from different industries, or for a single industry to manufacture multiple types of products. It is, hence, not necessarily a square nor a diagonal matrix.

The IOT framework, instead, assumes a diagonal production matrix where a single sector is responsible for the production of a unique type of output. The transformation of the production matrix can be done to express these input-output links in terms of commodity-by-commodity (CIOT), or industry-by-industry (IIOT). The choice will depend on the research question and on the desired focus. One should consider that depending on the choice made, a different subset of matrices from the SUT framework will remain unaltered, while the rest will be modified. This will be a key factor to take into account when matching these results with other sources of information. A comparison between Figure 3.40 and Figure 3.41 can help illustrate this:

		Products				Final Use					Total
		c_1	c_2	...	c_M	C	I	G	X	SV	
Domestic Products	c_1	S_d	Y_d					x			
	c_2										
	...										
	c_M										
Imported Products	c_1	S_m	Y_m					m			
	c_2										
	...										
	c_M										
Value Added	Ce	E	/ / / / / / / / / /					w			
	Nt										
	Gos										
	Total		x^T	y							

		Industries				Final Use					Total
		i_1	i_2	...	i_N	C	I	G	X	SV	
Domestic Industries	i_1	B_d	F_d					g			
	i_2										
	...										
	i_N										
Imports from Industries	i_1	B_m	F_m					m			
	i_2										
	...										
	i_N										
Value Added	Ce	W	/ / / / / / / / / /					w			
	Nt										
	Gos										
	Total		g^T	y							

(a) CIOT
(b) IIOT

Figure 3.41: Commodity-by-commodity (CIOT) and industry-by-industry (IIOT) input-output tables

Matrices S and B are the IOT equivalent of the use matrix, U , from the SUT framework. They express the total expenditure on each input purchased by a manufacturing sector for its productive process. Like the production matrix, the use matrix is always transformed when moving from the SUT to the IOT framework.

The final demand matrix, Y , is expressed in terms of products by category, so it remains unaltered under the CIOT framework. This is not the case for the IIOT, where this matrix will have to be adjusted to express the final sales in terms of the industry from where these products came. In contrast, the value-added matrix remains unaltered under the IIOT, but it is transformed under the CIOT to ensure accounting consistency.

For our purposes, the main criterion to select the focus is the ability to match the resulting data with the dis-aggregation used for the Integrated Economic Accounts, which presents the information by institutional sector. A correspondence between institutional

sectors and industries is much more clear than one between institutional sectors and products. The IIOT is, thus, a better fit for the model.

Retrieving a set of IIOT in basic prices

The data in the SUT is transformed into basic prices prior to building the input-output tables. The goal is to retrieve not only the values in basic and market prices but also in deflated terms with all relevant pieces of information to move from one valuation method to the other.

As mentioned before, the IBGE already reports the information on the supply side of the SUT –the make matrix and the vector of imports– in basic prices. The challenge is to transform the information on the use side to basic prices. In general terms, the total supply in basic prices, TS^{BP} , should equal the total demand (including accumulation of inventories) in basic prices, TD^{BP} . The latter can be retrieved as:

$$TD^{BP} = TD^{MP} - TD^{td.mg} - TD^{tp.mg} - TD^{nit} - TD^{cf}$$

with TD^{BP} and TD^{MP} representing the valuation in basic and market prices, respectively, and where $TD^{td.mg}$, $TD^{tp.mg}$, TD^{nit} , and TD^{cf} stand for the trade margins, the transportation margins, the net indirect taxes on products, and the customs fees on imports, respectively. All of these are provided by the IBGE with each release of the supply-and-use tables. They are, however, reported as vectors only, and the allocation of each of these margins and taxes to each category of the intermediate and final demand is required.

The allocation of trade and transportation margins is done according to the process detailed in [Guilhoto and Sesso Filho \(2005\)](#). A separate method is applied to allocate the taxes and customs fees. The goal in that case is to obtain a single vector for each of these two margins. This follows the simplifying assumption that, for each product or service, there is a unique indirect tax rate –and a unique customs fee– that is paid irrespective of the domestic agent purchasing it.³⁷

The case of imported goods needs to be approached first. Domestic taxes on imported goods are paid on top of the value of the good once customs fees have been charged to it. Hence, to allocate taxes one should first allocate customs fees, and to do that, one should have already distributed the vector of imports across domestic consumers.

Imports are assumed to be absorbed by all domestic agents *pro rata* of the existing structure of total demand –excluding exports and stock variation.³⁸ This allows to split the final demand matrix, Y , by domestic or imported origin, Y_d and Y_m . The same applies to the intermediate consumption matrix, U , which can now be dissected into U_d and U_m .

³⁷The focus is restricted to domestic agents as exports are assumed to be exempt from domestic taxes.

³⁸It has been assumed that no re-exports take place, and when they do, they are first transformed locally to account instead for a domestic good being exported. Moreover, the accumulation of inventories is subtracted from total demand for this analysis, as it is assumed that inventories are only made of domestic goods.

The average customs fee rates are computed straightforwardly as the ratio of customs fees to total imports, by product. Once Y_m and U_m have been obtained, the customs fees can also be allocated by multiplying each element of those matrices by its corresponding rate. Similarly, the indirect tax rates are then computed as the ratio of the net indirect taxes to the total demand –excluding exports and stock variations, and augmented by customs fees– by product. Once the tax rates are retrieved, the matrices of taxes charged to intermediate consumption and final demand can be built.

As mentioned before, the IBGE produces the SUT in current and last year's prices. The procedure to transform from market to basic prices is applied to both sets of files. With all information expressed in basic prices, one can proceed to cluster the industries in the newly obtained matrices to retrieve the information aggregated according to the industrial clusters defined by the user.³⁹

Once the clustering is done, the SUT –in current and last year's prices– are transformed into two sets of IIOT for each year. For this, let T be a transformation matrix representing the participation of each industry in the total output of a given type of product, and defined as:

$$T := V \times (\text{diag}(x))^{-1}$$

Each major component of the IIOT detailed in Figure 3.41b can then be defined as:

$$\begin{aligned} B_d &= T \times U_d \\ B_m &= T \times U_M \\ W &= W \\ F_d &= T \times Y_d \\ F_m &= T \times Y_m \end{aligned}$$

This process corresponds to the "Model D" defined in the United Nations' Handbook of supply-and-use-tables [Mahajan et al. \(2018\)](#). The "Model D" is one of the two options to transform the SUT into a set of IIOT [Mahajan et al. \(2018\)](#). It provides industry-by-industry IOT based on the fixed product sales structure assumption. This implies that each product is assumed to have its own specific sales structure, irrespective of the industry where it is produced. Unlike the alternative method to obtain a set of IIOT, "Model D" produces no negative values within the transformed matrices.⁴⁰

The results resemble now the structure presented in Figure 3.41b. These are expressed in basic prices, with all required components to reconstruct the relevant elements of the demand matrices in market prices. For simplicity, in what follows the industrial output retrieved in the industry-by-industry input-output tables, will be referred to as products, or commodities. This should not be confused with the actual division of products or

³⁹The Appendix 3.B.3 presents the mapping between the officially reported industries and the aggregate ones used in the model.

⁴⁰Readers interested in a detailed description of the process and its alternatives, as well as in the presentation of applied examples, would find Chapter 12 of [Mahajan et al. \(2018\)](#) particularly useful.

commodities reported in the original supply-and-use tables.

Deflating values and obtaining price indices

With all information in basic and market prices, the focus now goes on deflating the retrieved values to find a measure of volumes produced and consumed. For this purpose, let $X_{t,t}$ be an element of the IIOT valued at current basic prices, and $X_{t,t-1}$ be the same element valued at the previous year's basic prices, such that:

$$\begin{aligned} X_{t,t} &= \bar{X}_t P_t \\ X_{t,t-1} &= \bar{X}_t P_{t-1} \end{aligned}$$

where \bar{X} represents the deflated value –hereafter referred to as the real value or volume– of the variable X , and P its basic price at the corresponding time. Evidently, the ratio between these two will indicate the gross price increase between both periods, as:

$$\frac{X_{t,t}}{X_{t,t-1}} = \frac{P_t}{P_{t-1}}$$

Let P_0 be the level of basic prices at the reference year, and set $P_0 = 1$. Given that P_0 has been fixed, and that the left-hand side of the equation above is known, one can retrieve P_1 by using the results from this ratio. The recursive application of this process allows us to reconstruct a price index for all years. It should be noted, however, that the process will be applied to all products. This means that all prices, and hence all relative prices, will be equal to one for the reference year.⁴¹

Moreover, it should be considered that the price indices obtained with such an exercise need not be the same for intermediate or final consumption, investment, exports, or others. This, as the basket of disaggregated goods devoted to each segment of demand may not be the same, and neither does the vector of prices associated with it. In fact, if this process is applied to different elements of the IIOT, the results would yield a different evolution of prices for the same product depending on who purchases it. In other words, the direct application of this procedure to all elements would generate a matrix of prices instead of a single vector with each element being the price of a product for each time period. A simplifying assumption is adopted to avoid modeling a different price level for each combination of product and consumer. The evolution of domestic prices for all products is assumed to be given by the application of the process detailed above to the production vector only. The evolution of imported prices –expressed in domestic currency– is computed by applying the same procedure to the imports vector. These are then transformed into foreign currency units by using the average exchange rate for each year. All other elements are deflated using the obtained price indices. This allows us to

⁴¹The reference year is set to be 2010, as it is the baseline year used by the IBGE for building and reinterpolating their National Accounting data. It is also the first year for which we have complete data consistently reported under the most recent format of the IBGE's Integrated Economic Accounts.

retrieve a consistent set of IIOT in volumes and in basic prices.

Figure 3.42 shows the evolution of both domestic and imported basic price indices obtained for the twelve industrial categories under the IIOT framework for the period 2010-2018.

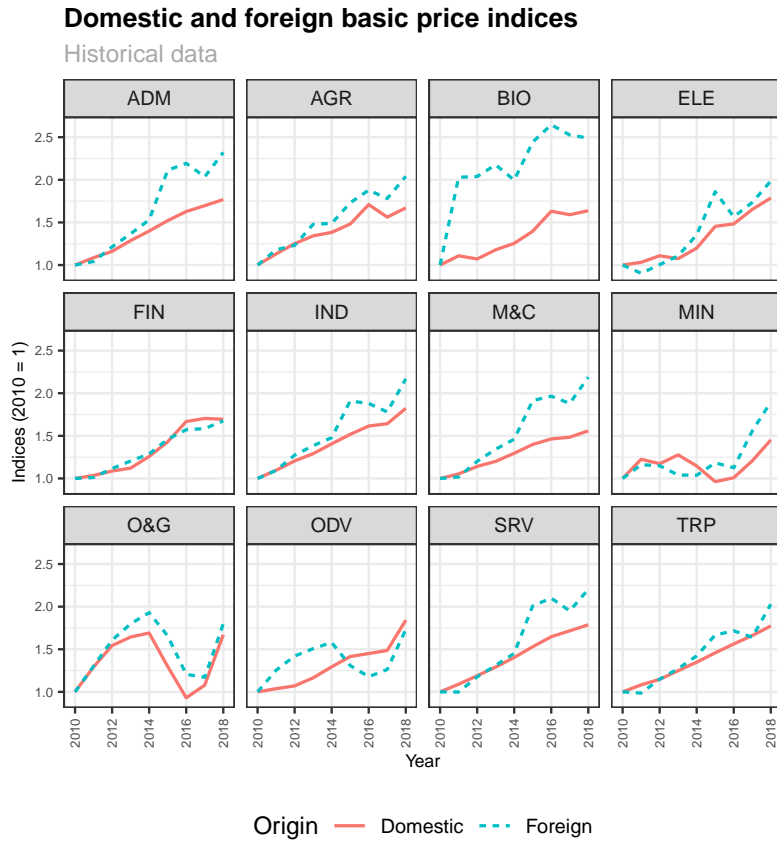


Figure 3.42: Basic price indices, by industry and origin.

One particular benefit of deflating the values in the input-output tables is that one can retrieve a matrix of technical coefficients in what we will refer to as volumes. In general, for the IIOT framework, this matrix is obtained by dividing the inputs used by each industry per its of output:

$$A = B \times (\text{diag}(g))^{-1}$$

This can be applied both to the information on basic prices or in volumes. The latter is much more convenient, as it cleans the results from the effects of the evolution of relative prices. The evolution of both deflated and undeflated technical coefficients can be seen in Figure 3.43. As expected, deflated technical coefficients are more stable. This is particularly the case for the inputs used by oil-related sectors (*O&G* and *ODV*), whose undeflated technical coefficients present larger fluctuations at the beginning of the decade given the volatility of relative prices of oil and oil derivatives versus other products.

Deflated and un-deflated technical coefficients

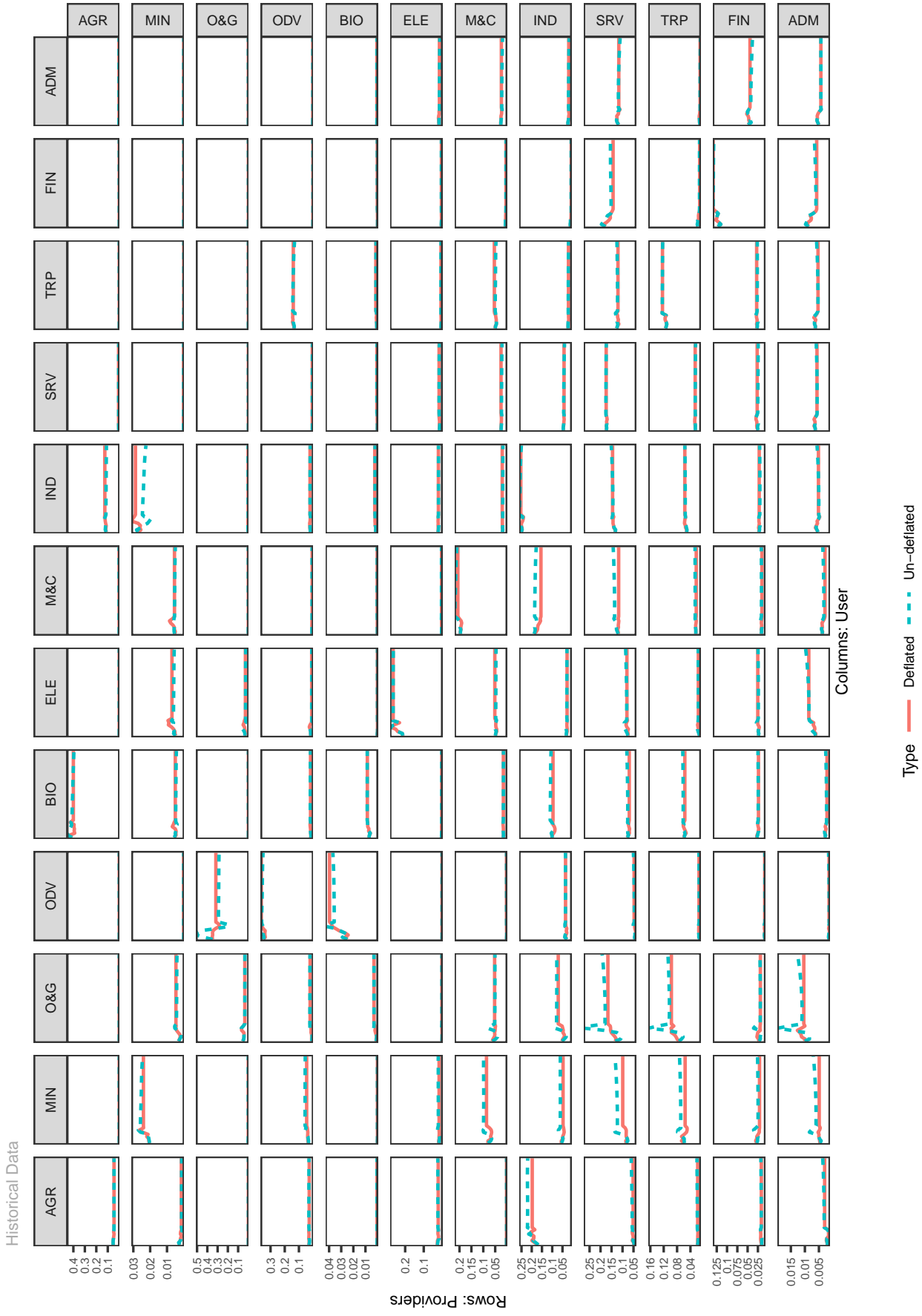


Figure 3.43: Evolution of the deflated and un-deflated technical coefficients, from 2010 to 2018.

Expanding the input-output-tables

At this stage, input-output-tables are obtained, and the main flows can be evaluated in real terms, basic prices, and market prices. The current dataset is then complemented with information from other sources to achieve the variable coverage and level of detail desired. In particular, industrial CO_2 emissions are still to be included, and the gross fixed capital formation can be disaggregated, allowing to compute an index of outstanding capital stock.

The industrial CO_2 emissions are taken from the Brazilian country tables of the EORA dataset, see [Lenzen et al. \(2012\)](#) and [Kanemoto et al. \(2011\)](#). For simplicity, the focus is set on producing activities and only industrial emissions are considered. Given the close consistency of the industrial sectors in the EORA dataset and the official SUT, the matching is straightforwardly done. The ratio of emissions to real production is called the emission factor, and it can be seen as an additional technical coefficient detailing the amount of pollution "needed" for the output of an industry to be produced.

Regarding investment, the supply use tables present it as a vector with as many entries as products defined in the national accounts. This means that GFCF describes which goods are being used as capital, but it does not specify who is carrying on the investment. Ideally, For each year, this information should be instead presented as capital absorption matrices, by good and industrial activity. Using the official SUT, industrial surveys, and customs data, [Miguez et al. \(2014\)](#) and [Miguez and Freitas \(2021\)](#) build such matrices for Brazil.

The results are available until 2013, and the last versions of the tables are consistent with the updated 2010 national accounting conventions. However, given official national accounts are revised after the year of publication, discrepancies exist. These are corrected by expanding or contracting each row of the capital absorption matrices so that row sums match the vector of GFCF. Finally, for reference purposes, the resulting matrices are aggregated to compute the average participation of each industry in the usage of each good as capital. These ratios are then used to convert the GFCF vectors for the years after 2013 into capital absorption matrices.

To build the indices of installed capacity, the capital stock by country from the Penn World Table database, [Feenstra et al. \(2015\)](#), is used along with the time series of the capital absorption matrices, and an average depreciation rate which is assumed to be the same across goods and sectors. Given the initial conditions, the flow and the depreciation rate are assumed as known, the matrix of capital stock by good and sector can be obtained for any given year. For practical purposes, the production function requires an index of capital for each industry instead of a vector of the different types of capital goods the industry uses. The index is assumed to be the addition of the capital stock in real terms across all the goods that each sector is using.

3.B.2 Building who-to-whom matrices

As mentioned before, the IEA framework shows how an account may represent a source and/or a use of funds for a given sector, but it does not explicitly mention the counterparts for each flow (or stock of financial assets). Ideally, for each operation type, we should have an $N \times N$ matrix, where N would be the number of institutional sectors, stating on the rows which sectors are using the funds and in the columns which sectors are receiving them. The Integrated economics accounts framework only provides the total row and column sums for this matrix, but the elements within the matrix are to be obtained. This is often done by applying matrix-balancing methods. The most widely used of this is the RAS algorithm, and it is mostly required on transactions related to financial assets, or revenues from financial assets (interest payments and dividends).

The RAS is a matrix-balancing algorithm that takes as inputs the row sums and the column sums of the desired resulting matrix, and an initial guess for that matrix. It then iteratively proceeds to scale rows, and columns to match targeted totals until it converges to a matrix that respects the desired constraints, [Miller and Blair \(2009\)](#). In this case, the row sums correspond to the total uses or liabilities -transactions, stocks, variations or revaluations depending on the array in question-, and the column sums correspond to the total receipts or assets. The initial guess is taken to be the current known structure. In the case of Brazil, the only official who-to-whom table for stocks of financial assets dates from 2009 and was built under the previous national accounting framework. It does not include all required accounts, and it is subject to different conventions, but it is still a useful guide for several assets.

For all other transactions not related to the balance sheet of each sector, the initial guess is a trivial allocation of row values according to column weights. The resulting matrix verifies row and column sums, but most often will include values in the $[RoW, RoW]$ element of the matrix, which should be 0. This entry is normally forced to be 0, and the new matrix is used as the initial guess for the RAS algorithm.

While convergence may be obtained, the resulting matrices from applying the RAS algorithm when the inputs mix positive and negative entries can make no intuitive sense, and would not be of use for a model like this. Here, this is indeed an issue as the variations of assets and liabilities may not be positive for all sectors.

Recall that the model builds an array for each asset, which is a time-series of who-to-whom matrices for that asset class. An slice of the array for any given time, t , would be the who-to-whom structure for that period.

Let the who-to-whom matrices for any asset class $\mathcal{X} \in \{R, H, D, B, S, Z\}$ for the stock at the end of period t , the variation due to sales and purchases during t , and the revaluation at the beginning of the period given new prices be denoted respectively by: \mathcal{X}_t , $\Delta\mathcal{X}_t$, and $\hat{\mathcal{X}}_t$.

It follows that, at the end of any period, the outstanding who-to-whom structure for asset \mathcal{X} will depend on its previous structure, the revaluation that took place at the

beginning of the period, and the transactions during the period:

$$\mathcal{X}_t = \mathcal{X}_{t-1} + \hat{\mathcal{X}}_t + \Delta\mathcal{X}_t \quad (3.106)$$

Here, it should be noted that both $\hat{\mathcal{X}}_t$ and $\Delta\mathcal{X}_t$ can have negative entries. We have already obtained the former based on the variations of the interests and exchange rates, but the latter remains to be identified. The application of the RAS algorithm to identify it would, however, produce non-sensible results.

To deal with this, one should recall that, while the revaluation of an asset may be negative, the total evaluated value of it must not. A negative value would imply a short position, which is unlikely to occur at the aggregate level of the institutional sectors.

Taking a look at equation (3.106), this implies that the result of $(\mathcal{X}_{t-1} + \hat{\mathcal{X}}_t)$ will not have negative entries. Moreover, since short sales are not realistic for an institutional sector as a whole, we know that any negative entry on $\Delta\mathcal{X}_t$ is bounded below. Agents cannot sell more than what they currently have in current values. Or said otherwise, that \mathcal{X}_t will only have non-negative entries $\forall t$.

With this in mind, the RAS algorithm can be used to obtain \mathcal{X}_t , and only afterward compute $\Delta\mathcal{X}_t$ as a residual, given the other components of \mathcal{X}_t are known.

The inputs for the algorithm would be such that the column and row targets would be given by the total assets (liabilities) held (issued) in current value by each institutional sector, plus the variation that will take place during this period, and that was defined in equations (3.104) and (3.105).

$$\text{ColTarget} = \left(\Sigma_{\mathcal{X}} \left(\mathcal{X}_{t-1} + \hat{\mathcal{X}}_t \right) \right)^T \times \vec{1}_S + \overrightarrow{\Delta A_t^{TOT}} \quad (3.107)$$

$$\text{RowTarget} = \left(\Sigma_{\mathcal{X}} \left(\mathcal{X}_{t-1} + \hat{\mathcal{X}}_t \right) \right) \times \vec{1}_S + \overrightarrow{\Delta L_t^{TOT}} \quad (3.108)$$

Finally, it should be noted that the model abstracts from the financial intermediation indirectly measured (FISIM) logic, see [UN \(2008\)](#) and [Lequiller and Blades \(2014\)](#). As a result, the model would build average interest rates paid for each combination of user-holder of interest-paying obligations (bonds). To do this, the who-to-whom matrices of the interest payments, the change in bonds, and the stock of bonds at the end of a given period, were determined under stricter constraints that would ensure consistency of accounting identities and on revenue dynamics.

3.B.3 Industry mapping

N.	Industry from the Brazilian National Accounts	Model
1	Agricultura silvicultura exploração florestal	AGR
2	Pecuária e pesca	AGR
3	Petróleo e gás natural	O&G
4	Minério de ferro	MIN
5	Outros da indústria extrativa	MIN
6	Alimentos e Bebidas	IND
7	Produtos do fumo	IND
8	Têxteis	IND
9	Artigos do vestuário e acessórios	IND
10	Artefatos de couro e calçados	IND
11	Produtos de madeira - exclusive móveis	IND
12	Celulose e produtos de papel	IND
13	Jornais revistas discos	IND
14	Refino de petróleo e coque	ODV
15	Álcool	BIO
16	Produtos químicos	IND
17	Fabricação de resina e elastômeros	IND
18	Produtos farmacêuticos	IND
19	Defensivos agrícolas	IND
20	Perfumaria higiene e limpeza	IND
21	Tintas vernizes esmaltes e lacas	IND
22	Produtos e preparados químicos diversos	IND
23	Artigos de borracha e plástico	IND
24	Cimento e outros produtos de minerais não-metálicos	IND
25	Fabricação de aço e derivados	IND
26	Metalurgia de metais não-ferrosos	IND
27	Produtos de metal - exclusive máquinas e equipamentos	IND
28	Máquinas e equipamentos inclusive manutenção e reparos	M&C
29	Eletrodomésticos e material eletrônico	M&C
30	Máquinas para escritório aparelhos e e material eletrônico	M&C
31	Automóveis camionetas caminhões e ônibus	M&C
32	Peças e acessórios para veículos automotores	M&C
33	Outros equipamentos de transporte	M&C
34	Móveis e produtos das indústrias diversas	M&C
35	Produção e distribuição de eletricidade gás água esgoto e limpeza urbana	ELE
36	Construção civil	M&C
37	Comércio	SRV
38	Transporte armazenagem e correio	TRP
39	Serviços de informação	SRV
40	Intermediação financeira seguros e previdência complementar e serviços relacionados	FIN
41	Atividades imobiliárias e aluguéis	SRV
42	Serviços de manutenção e reparação	SRV
43	Serviços de alojamento e alimentação	SRV
44	Serviços prestados às empresas	SRV
45	Educação mercantil	SRV
46	Saúde mercantil	SRV
47	Serviços prestados às famílias e associativas	SRV
48	Serviços domésticos	SRV
49	Educação pública	ADM
50	Saúde pública	ADM
51	Administração pública e seguridade social	ADM

Table 3.5: List of industries from the Brazilian national accounts, and their correspondence to model's industries.

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