# Trois applications de la théorie des jeux coopératifs

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# Three applications of cooperative game theory

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**Titre** : Trois applications de la théorie des jeux coopératifs

**Mots-clés** : Allocation, Valeur de Shapley, Application, Jeux, Coopération, Axiomatique

**Résumé** : La théorie des jeux coopératifs est un ensemble d'outils pertinents pour appréhender des situations conflictuelles dans lesquelles des agents économiques coopèrent pour ensuite récupérer une part du fruit de la coopération. La thèse s'articule autour de trois différentes applications originales ; dans le domaine de l'économie de la santé avec la mise en place de soins intégrés pour le traitement des maladies chroniques entrainant une coopération des prestataires de soins ; dans le domaine de la taxation internationale avec l'étude de la mise en place d'un taux commun harmonisé pour taxer les profits des firmes multinationales dans les pays du G7/G20 ; et dans le domaine de la finance avec l'études des signaux apportés par les donateurs dans des projets utilisant les plateformes de crowdfunding comme source de financement.

**Title** : Three applications of cooperative game theory

**Keywords** : Games, Cooperation, Application, Axiomatic, Shapley value, Allocation

**Abstract** : Cooperative game theory is a set of relevant tools to apprehend conflict situations in which economic agents cooperate in order to recover a share of the fruit of the cooperation. The thesis is articulated around three different original applications ; in the field of health economics with the implementation of integrated health care for the treatment of chronic diseases leading to a cooperation of healthcare providers ; in the field of international taxation with the study of the implementation of a common harmonized corporate tax rate to tax the profits of multinationals in the G7/G20 countries ; and in the field of finance with the study of the signals brought by contributors in projects using crowdfunding platforms as a source of financing.

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# Introduction générale

## La contextualisation économique des problèmes d'allocation des ressources

La science économique, selon la définition de Robbins (1932), consiste à « étudier le comportement humain comme une relation entre des fins et des moyens rares à usages alternatifs ». En effet, dans un monde où les ressources sont limitées et nos désirs sont illimités, l'économie nous permet de répartir ces ressources rares (revenus, productions, temps, consommations) et de les allouer aux agents économiques de la manière la plus efficace et équitable possible. Ces deux notions renvoient respectivement à des concepts d'optimalité et de justice sociale (Pareto, 1909). Pendant longtemps, les penseurs de l'économie politique classique (Adam Smith, David Ricardo, Thomas Malthus, Jean-Baptiste Say ou encore Jean de Sismondi) souhaitaient une économie reposant sur l'idée du « laissez-faire », selon laquelle une économie de marché décentralisée pourrait s'autoréguler et permettre une allocation des ressources sans redistribution ni régulation, l'Etat restant cantonné à ses fonctions régaliennes (défense, armée, justice). La justification de l'intervention de l'Etat se fera plus tard au XXème siècle avec l'apparition de l'Etat-providence, un Etat qui interviendrait pour palier les défaillances de marché. Cette justification de l'intervention des pouvoirs publics repose sur trois fonctions essentielles de l'Etat, définies par Richard Musgrave (Musgrave, 1959), que sont la fonction de redistribution, la fonction de régulation et la fonction d'allocation. Ces trois principes reposent sur une volonté de générer le plus d'efficacité et d'équité possible dans la société, le premier par une allocation optimale des ressources et le second par une régulation accrue des marchés et une redistribution plus juste des revenus. Dans une économie décentralisée, efficacité et équité sont souvent deux critères difficilement conciliables. Nous verrons dans la suite de cette introduction que cette thèse aborde également la difficulté de concilier certains principes de justice distributive dans le cadre d'une économie centralisée.

La définition de l'économie de Robbins (1932) peut s'étendre à tout type de personnes physiques ou morales engagées dans une interaction pouvant les mener à différentes finalités. Des individus, des animaux, des institutions bancaires, des pays, des entreprises et bien d'autres agents encore, doivent en effet effectuer des choix pour satisfaire au mieux leurs propres intérêts et atteindre un résultat qui soit le meilleur pour eux. La rationalité,

selon la microéconomie standard, guide et contraint ainsi chaque agent. Les agents en interaction peuvent disposer de plusieurs alternatives pour arriver à des fins différentes, comme la possibilité de pouvoir faire des alliances pour atteindre de meilleurs résultats ensemble. Cette idée de coalition pour résoudre une divergence d'intérêts portant sur une même ressource sera l'idée essentielle de ce manuscrit, pour montrer en quoi le fait de s'allier avec d'autres peut mener à des situations plus ou moins efficaces, stables, et équitables.

Ainsi, nous mobiliserons un domaine des mathématiques « indissociable de la microéconomie » (Guerrien, 2010) utilisé pour prolonger la théorie de la décision individuelle à l'interaction conflictuelle dès que 2 à $n$ agents (que l'on nommera joueurs) mettent en place des stratégies ou des accords qui auront un impact non seulement sur l'agent lui-même, mais aussi sur la satisfaction des autres agents. Il s'agit de la théorie des jeux ou « théorie de la décision interactive » pour reprendre le terme d'Aumann (2008). Cette dernière traite d'une part, des aspects non-coopératifs de l'économie , lorsque des joueurs disposant d'un pouvoir de décision sont engagés dans un processus d'interactions stratégiques, et d'autre part, des aspects coopératifs de l'économie, lorsque des joueurs sont arbitrés de manière centralisée dans un processus menant à des accords les contraignant et dont le fruit de la coopération, qu'elle soit implicite ou explicite, doit leur être alloué.

La théorie des jeux s'est structurée autour de l'ouvrage fondateur de von Neumann and Morgenstern (1944), intitulé « Theory of games and economic behavior », même si plusieurs précurseurs ont développé certains concepts avant la parution de l'ouvrage. Parmi eux, on peut notamment citer Cournot (1838) qui avait anticipé plus d'un siècle auparavant le concept d'équilibre non-coopératif entre les entreprises d'un duopole où la concurrence s'opère par les quantités. Les branches coopératives et non-coopératives sont unifiées dans l'ouvrage de von Neumann and Morgenstern (1944) mais ont ensuite connu des trajectoires séparées à travers les travaux de Nash (1950b) pour les jeux non-coopératifs et Nash (1950a) et Shapley (1953) pour les jeux coopératifs. Ces derniers ont joué un rôle décisif dans l'émergence d'applications de la théorie des jeux auxquelles la présente thèse apporte de nouvelles contributions.

# La théorie des jeux coopératifs

La théorie des jeux coopératifs, ou théorie des jeux coalitionnels, se développe en parallèle des approches stratégiques, même si des ponts existent entre les deux champs dans le cadre du « programme de Nash ». On peut se référer à Serrano (2021) pour une revue de littérature dans ce domaine. Dans cette thèse, nous ne nous focaliserons pas sur les aspects stratégiques lorsque le pouvoir de décision est donné aux agents, ni aux processus stratégiques de formation des accords et des coalitions. Aumann (1989) nous propose ainsi une justification de l'approche coopérative :

> « *La théorie coopérative commence par une formalisation des jeux (la forme de coalition) qui fait abstraction des procédures et [...] se concentre plutôt sur les possibilités d'accord [...] Il y a plusieurs raisons pour lesquelles les jeux coopératifs ont été traités séparément. L'une d'elles est que lorsqu'on intègre explicitement des procédures de négociation et d'exécution dans le modèle, les résultats d'une analyse non coopérative dépendent très fortement de la forme précise des procédures, de l'ordre dans lequel les offres et les contre-offres sont faites, etc. Cela peut être approprié dans des situations de vote où les règles précises de l'ordre parlementaire prévalent, où un bon stratège peut effectivement l'emporter. Mais les problèmes de négociation sont généralement plus flous et il est difficile de définir précisément les procédures à suivre. Plus fondamentalement, on a le sentiment que les procédures ne sont pas vraiment pertinentes, que ce sont les possibilités de former des coalitions, de promettre et de menacer qui sont décisives, plutôt que de savoir à qui revient le tour de parler... Les détails détournent l'attention de l'essentiel. Certaines choses se voient mieux de loin ; les camps romains autour de Massada sont indiscernables lorsqu'on s'y trouve, mais facilement visibles du haut de la montagne* » (Aumann, 1989).

Ainsi, l'approche coopérative peut être vue comme pertinente dès lors qu'il existe des possibilités d'accord, qu'ils soient officiels avec la signature d'un contrat contraignant les agents à coopérer, ou bien officieux, engendrant ainsi une coopération plus implicite. Le lecteur est invité à consulter Driessen (1988) et Peleg and Sudhölter (2003) pour une introduction complète aux jeux coopératifs et à leurs différentes solutions. L'idée centrale de cette approche coopérative est de formaliser une situation dans laquelle plusieurs individus s'engagent par accord (ou non), à participer à un projet commun conduisant à des gains ou des coûts. L'objectif final est de répartir la ressource engendrée ou le coût

généré parmi l'ensemble des participants. La théorie économique et la théorie des jeux coopératifs offrent de multiples manières de procéder à cette répartition, en s'appuyant sur des principes de justice distributive pertinents et variés. La thèse mettra en lumière que ces différents outils s'adaptent naturellement pour appréhender des problèmes économiques différents traitant de la coopération.

## Le modèle classique des jeux coopératifs

La thèse est centrée sur la classe des jeux coopératifs à utilité transférable qui suppose que les agents évaluent de façon identique les utilités qu'ils perçoivent. Formellement, un jeu coopératif avec utilité transférable est une paire $(N, v)$ dans laquelle $N = \{1, ..., n\}, n \in \mathbb{N}$, est l'ensemble fini des agents engagés dans le projet commun et $v$ une fonction caractéristique ou coalitionnelle qui attribue à chaque sous-ensemble d'agents $S \subseteq N$ un montant $v(S) \in \mathbb{R}$ (et par convention $v(\varnothing) = 0$) qui permet de mesurer le pouvoir d'un groupe $S$ appelé « coalition », en terme monétaire ou en terme de négociation. Ce montant $v(S)$ s'interprète comme le meilleur résultat possible engendré par les seuls membres de $S$ sans l'aide des agents de $N \backslash S$. Le montant à répartir est le résultat de la grande coalition $v(N)$ et les autres montants $v(S)$, avec $S \neq N$, peuvent être vus comme des scénarios contrefactuels qui révèlent de l'information sur ce que les membres de $S$ pourraient se répartir si seuls ces agents venaient à coopérer, ou si les membres de $S$ venaient à se dissocier de l'accord entre les $n$ agents. L'évaluation de ces différents éléments composant la fonction caractéristique est donc une première étape essentielle pour étudier le processus de coopération et la répartition de ressources correspondant.

La théorie des jeux coopératifs fournit un ensemble de méthodes et d'outils pour proposer des solutions à de tels problèmes de répartition de ressources. Pour faire émerger des règles de partage comme concepts de solutions, la méthode axiomatique permet de promouvoir certaines allocations parmi d'autres, parce qu'elles respecteraient des principes philosophiques de justice distributive, que l'on nomme axiomes. En effet, un arbitre, en tant qu'entité chargée de résoudre la divergence d'intérêts engendrée par le partage de la ressource entre les participants, sélectionne certains principes qu'il juge essentiel. Certaines propriétés sont très anciennes. A titre d'illustration, Aristote suggérait déjà un partage qui tienne compte des différences ou des similarités entre les participants. De ce fait, deux personnes complètement égales dans la génération du résultat à répartir devraient recevoir un paiement ou une facture identique. Il nomme cette propriété le traitement égalitaire des égaux ou traitement inégalitaire des inégaux. L'arbitre s'efforce donc

de faire respecter une ou plusieurs propriétés pour un résultat qui peut être différent selon la combinaison d'axiomes retenus. Cela peut donc amener à la caractérisation d'une, de plusieurs, voire d'aucune règles de partage. Pour plus de précisions sur la méthode axiomatique, le lecteur peut se référer à Thomson (2001) et Dequiedt et al. (2011).

La thèse mobilisera les deux concepts de solutions les plus étudiés dans la littérature sur la théorie des jeux coopératifs. Ces concepts seront exploités dans chacun des trois chapitres de la thèse. Le premier concept de solutions est la valeur de Shapley (Shapley, 1953). Il s'agit d'une règle de partage que l'on peut définir comme marginaliste, c'est-à-dire qu'elle propose d'allouer à chaque joueur, un montant à hauteur de ce qu'il apporte en moyenne de manière marginale à chaque coalition qu'il peut rejoindre. Formellement, en notant $s$ et $n$ le nombre d'éléments des ensembles $S$ et $N$ respectivement, la valeur de Shapley est la règle de partage notée $Sh$, qui associe à chaque jeu coopératif $(N, v)$ l'allocation

$$Sh_i(N, v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} \big(v(S \cup \{i\}) - v(S)\big), \quad \forall i \in N.$$

La fraction correspond à la pondération de la contribution en fonction de la taille de la coalition que le joueur $i$ rejoint et le terme entre parenthèses est la définition même de ce qu'apporte l'individu $i$ en rejoignant la coalition $S$.

En 70 ans, plusieurs propriétés fondamentales ont été invoquées dans les nombreuses caractérisations axiomatiques de la valeur de Shapley. La caractérisation de Myerson (1980) repose sur l'axiome de contributions équilibrées qui impose que le variation de paiement d'un premier joueur suite au départ d'un second joueur doit être identique si les deux joueurs intervertissent leurs rôles. Le principe de cet axiome est adapté dans un des chapitres de cette thèse. La caractérisation de Young (1985) consacre l'axiome de monotonie forte, qui impose que le paiement d'un joueur ne doit pas diminuer si ses contributions marginales aux coalitions ne diminuent pas. Shubik (1962) offre la caractérisation aujourd'hui considérée comme étant la plus classique de la valeur de Shapley. Il invoque quatre axiomes reflétant des principes de justice distributive différents. Désignons par $f$ une règle de partage arbitraire et par $f_i(N, v)$ le paiement qu'elle recommande pour le joueur $i$ dans le jeu $(N, v)$. Ainsi, si $f$ satisfait simultanément aux quatre axiomes suivants, alors cette règle de partage coïncide avec la valeur de Shapley :

— **Efficience**. Le résultat total engendré par la grande coalition doit être réparti

intégralement entre les différents joueurs :

$$\sum_{i \in N} f_i(N, v) = v(N).$$

— **Traitement égalitaire des égaux**. Si $i$ et $j$ sont égaux dans le jeu, c'est-à-dire si chacune de leurs contributions marginales sont égales au sens où $v(S \cup \{i\}) = v(S \cup \{j\})$ pour toute coalition $S \subseteq N \backslash \{i, j\}$, alors :

$$f_i(N, v) = f_j(N, v).$$

— **Additivité**. La somme des paiements que reçoivent les joueur dans deux jeux est la même que les paiements obtenus dans la somme de ces deux jeux. Autrement dit, si l'on considère deux jeux $(N, v)$ et $(N, w)$, et que l'on note $(N, v + w)$ le jeu obtenu par la somme vectorielle des deux fonctions caractéristiques $v$ et $w$, alors :

$$f(N, v + w) = f(N, v) + f(N, w).$$

— **Axiome du joueur nul**. Le joueur $i$ est considéré comme joueur nul dans un jeu si sa présence est neutre au sein des coalitions qu'il rejoint, ou autrement dit, si chacune de ses contributions marginales est égale à 0, c'est-à-dire si $v(S \cup \{i\}) = v(S)$ pour chaque $S \subseteq N \backslash \{i\}$. Alors, si $i$ est un joueur nul dans le jeu $(N, v)$, il reçoit un paiement nul :

$$f_i(N, v) = 0.$$

Le second concept de solution mobilisé dans cette thèse est le concept de cœur d'un jeu coopératif, longtemps attribué à Gillies (1959) avant que Zhao (2018) précise la contribution pionnière de Lloyd Shapley. Le cœur est l'ensemble des allocations qui satisfont simultanément à l'axiome d'efficience et à la condition de rationalité collective des coalitions, c'est-à-dire l'ensemble des allocations telles que l'intégralité de la ressource générée par la coopération de tous est redistribuée et que chaque coalition puisse percevoir un montant total au moins aussi élevé que ce qu'elle pourrait obtenir en faisant cavalier seul. Le cœur du jeu coopératif $(N, v)$ est formellement défini par :

$$C(N, v) = \left\{ x \in \mathbb{R}^N : \sum_{i \in S} x_i \geq v(S), S \subseteq N, \sum_{i \in N} x_i = v(N) \right\}.$$

La valeur de Shapley et le cœur sont basés sur des principes différents. D'une part,

la valeur de Shapley est souvent décrite comme une solution équitable. En effet, parmi les propriétés désirables qu'elle satisfait, plusieurs expriment la notion d'équité, soit en comparant deux joueurs distincts dans un même jeu (l'axiome de traitement égalitaire des égaux), soit en comparant un même joueur dans deux jeux distincts (l'axiome de monotonie forte), soit en comparant deux joueurs dans deux jeux distincts (l'axiome de contributions équilibrées, qui mêle un jeu et certains de ses sous-jeux). D'autre part, le cœur est constitué de l'ensemble des allocations souvent qualifiées de stables. Sur chaque allocation du cœur, aucune coalition de joueur n'a intérêt à remettre en cause sa participation au projet commun puisque le paiement total de ses membres est au moins aussi élevé que ce à quoi la coalition pourrait prétendre sans coopération avec les autres joueurs. Dans les chapitres de cette thèse, nous ferons fréquemment la distinction entre l'équité et la stabilité des allocations. La valeur de Shapley ne fait pas systématiquement partie du cœur d'un jeu coopératif. Si l'on admet que la valeur de Shapley est l'allocation équitable et que le cœur est composé des allocations stables, cela revient à dire qu'il n'existe pas systématiquement une allocation à la fois stable et équitable. La littérature sur les jeux coopératifs propose toutefois des conditions permettant de réconcilier ces deux principes. La plus connue est la condition de convexité d'un jeu coopératif qui stipule que la contribution marginale d'un joueur ne peut pas diminuer si la coalition qu'il rejoint s'élargit. Formellement, un jeu $(N, v)$ est convexe si pour tout joueur $i$ et toute paire de coalitions $S \subseteq T \subseteq N \backslash \{i\}$, on a :

$$v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T).$$

Shapley (1971) démontre que la valeur de Shapley est une allocation du cœur d'un jeu convexe. La condition de convexité émerge souvent naturellement de jeux coopératifs engendrés par des problèmes économiques.

## Jeux coopératifs enrichis d'une structure exogène et applications

Les développements évoqués dans la section précédente ont été accompagnés de nombreuses applications et variations autour du modèle de base considéré jusqu'à présent. Cette section présente ces deux aspects afin de mettre en perspective les travaux de thèse.

Premièrement, le modèle de base de jeux coopératifs a été augmenté par différents types de structures exogènes de sorte à refléter des contraintes hiérarchiques, de priorité ou de communication ou l'existence d'affinités spécifiques entre les joueurs. Aumann and

Dreze (1974) et Owen (1977) proposent le modèle de jeux avec structure de coalitions dans lequel l'ensemble des joueurs est partitionné en unions. Une interprétation possible est que la réunion des membres d'une union permet à cette union d'accroître son pouvoir de négociation dans le processus de partage de la ressource. Myerson (1977) introduit le modèle de jeux avec structure de communication dans lequel la communication bilatérale entre les paires de joueurs est modélisée par les arêtes d'un graphe non orienté dont les sommets sont les joueurs. Gilles et al. (1992) considèrent le modèle de jeux avec structure de permission représentée par un ordre partiel sur l'ensemble des joueurs qui modélise des asymétries entre ces joueurs (des besoins différents, des relations hiérarchiques, etc.). Ils interprètent une telle structure par le fait qu'un joueur a besoin de la permission de ses prédécesseurs dans la structure afin de pouvoir coopérer pleinement au sein d'une coalition. Plusieurs autres types de structures de permissions ont été envisagées et sont listées et comparées par van den Brink (2017).

Deuxièmement, les applications de la théorie des jeux coopératifs se sont multipliées depuis la contribution pionnière de Shapley and Shubik (1954) où la valeur de Shapley, ici appelée indice de Shapley-Shubik, permet d'évaluer l'importance des votants au sein du congrès des Etats-Unis. Au delà de la mesure du pouvoir de vote, ces applications incluent notamment :

— **Les problèmes de revendications**. La somme des prétentions d'un groupe d'agents sur une ressource commune est trop élevée pour pouvoir honorer toutes les revendications. Cette situation crée un problème de rationnement étudié, entre autres par O'Neill (1982), Aumann and Maschler (1985) et Curiel et al. (1987). Pour une revue plus complète de cette littérature, le lecteur peut se référer à Thomson (2019).

— **Le partage de coûts**. Un projet commun de construction génère des coûts qu'il faut imputer aux différents agents. On peut par exemple s'intéresser aux frais de nettoyage de canaux d'irrigation entre les ranchs américains (Aadland and Kolpin, 1998), aux frais d'atterrissage d'une piste d'aéroport (Littlechild and Owen, 1973), aux frais de péage d'une autoroute (Kuipers et al., 2013) ou encore aux frais de construction et de maintenance d'un réseau ferroviaire (Fragnelli et al., 1999).

— **La répartition des revenus engendrés par un programme de pass culturel**. Des institutions culturelles peuvent s'allier pour attirer des visiteurs par la création d'un pass permettant une visite moins onéreuse pour l'ensemble des services proposés. Ginsburgh and Zang (2003) se sont intéressés à l'utilisation de la

valeur de Shapley dans ce contexte, avant que d'autres règles ne soient proposées, inspirées des problèmes de revendication (Casas-Mendez et al., 2014) ou encore à la saveur plus proportionnelle (Bergantiños and Moreno-Ternero, 2015, 2016).

— **Les problèmes d'ordonnancement**. La gestion d'un calendrier ou d'une file d'attente pose la question de l'ordre dans lequel des tâches doivent être réalisées. Maniquet (2003) s'intéresse à la réorganisation collectivement optimale d'une file d'attente et mobilise la valeur de Shapley pour que les agents lésés par une telle réorganisation soient compensés par les agents qu'elle favorise.

— **Le partage des ressources en eau**. Des conflits internationaux sont fréquents lorsqu'un fleuve s'écoule à travers plusieurs pays, notamment lorsque les ressources en eau issues de ce fleuve ont un impact significatif sur les secteurs énergétique ou agricole de ces pays. Les pays situés en aval du fleuve sont en effet dépendants des agissements des pays situés en amont du fleuve. Le lecteur peut se référer à Béal et al. (2013) pour une revue de cette littérature.

— **La réparation des dommages en droit civil**. Plusieurs parties prenantes sont conjointement responsables des dommages causés à une victime qui sollicite la justice pour obtenir réparation. L'objectif est alors de déterminer la responsabilité de chacune des parties prenantes dans le préjudice de sorte à répartir le coût des dommages (Dehez and Ferey, 2013).

— **Le classement lors d'un concours**. Certaines règles de partage peuvent être mobilisées dans les problèmes de classement. La valeur de Shapley a notamment été utilisée par Ginsburgh and Zang (2012) pour classer les différents vins lors de concours internationaux.

Les structures enrichissant le modèle de jeu coopératif évoquées plus haut émergent souvent de manière explicite ou implicite dans le cadre de telles applications. A titre d'exemple, deux pays limitrophes dans le modèle de Ambec and Sprumont (2002) s'apparentent à deux joueurs connectés par une arête dans le modèle de jeux avec structure de communication de Myerson (1977). Ces applications recourent quasiment systématiquement à la valeur de Shapley comme règle de partage.

## Apport de la thèse et des différents chapitres

Cette thèse est composée de trois chapitres proposant des applications nouvelles de la théorie des jeux coopératifs à trois problèmes économiques. Ces chapitres sont indépendants sur le fond mais unifiés par la méthodologie employée. Chaque chapitre doit être considéré

comme un article de recherche à part entière qui comporte ses propres notations et définitions. Cela peut donc conduire à quelques répétitions d'un chapitre à l'autre. La première application a pour cadre de l'économie de la santé, la seconde application s'inscrit dans le domaine de la fiscalité internationale et la troisième application examine le champ de la finance. Ces applications illustrent de plusieurs façons l'éventail des outils présentés dans cette introduction.

Premièrement, les jeux coopératifs construits dans le cadre de ces applications possèdent systématiquement une structure sous-jacente qui permet de faire émerger des résultats théoriques. En ce sens, la contribution de la thèse s'inscrit pleinement dans la lignée des applications listées dans la section précédente et ne se limite pas à des calculs d'allocations dans des cas particuliers, permis par des données, contrairement à une littérature empirique foisonnante ces dernières années mais à la portée plus limitée. Le concept de chaîne de soins développé dans le premier chapitre peut être vu comme une variante naturelle des structures de permissions conjonctives chez Gilles et al. (1992). De même, le regroupement des contributeurs ayant un même timing de contribution à un projet de financement participatif au troisième chapitre peut être vu comme une variante à la fois des structures de permissions conjonctive et des structures de coalitions chez Owen (1977).

Deuxièmement, les chapitres de thèse mobilisent la méthode axiomatique à deux niveaux. Tout d'abord, le troisième chapitre offre une caractérisation axiomatique de la valeur de Shapley sur une classe de jeux particulière. Ensuite, les trois chapitres présentent plusieurs résultats dont les démonstrations reposent sur des axiomes tels que le traitement égalitaire des égaux, l'axiome du joueur nul et l'axiome d'additivité évoqués précédemment. Ces résultats exploitent aussi l'axiome de désirabilité introduit par Maschler and Peleg (1966), qui impose qu'un joueur possédant des contributions marginales au moins aussi élevées qu'un second joueur reçoive un paiement au moins aussi élevé que le second joueur.

Troisièmement, bien que la thèse propose principalement des résultat théoriques, elle ne néglige pas pour autant la dimension empirique puisque le deuxième chapitre confronte des résultats théoriques à des calculs d'allocations s'appuyant sur des données macroéconomiques réelles tirées de Tørsløv et al. (2023). Malgré la quantité de données parfois colossale nécessaire pour représenter de tels jeux (plus de 100 000 coalitions pour le cas des 17 pays du G20 sur lesquels les données sont renseignées), ces calculs pratiques

ont été rendus possibles par la structure particulière de ces jeux et des outils de programmation.

Quatrièmement, la thèse démontre que la valeur de Shapley et le cœur fournissent des allocations pertinentes dans chacun des trois contextes économiques appliqués.

Tous ces éléments soulignent que la théorie des jeux coopératifs peut être perçue comme un ensemble d'outils opérationnels que le théoricien des jeux coopératifs tient à disposition pour dialoguer avec d'autres collègues spécialisés dans différents champs de l'économie.

## Chapitre 1 : Application au système de santé

Le premier chapitre, écrit en seul auteur, est une application des jeux coopératifs au cadre de l'économie de la santé. Aujourd'hui, le système T2A (tarification à l'activité) conduit un patient atteint de problèmes de santé récurrents ou chroniques (diabète, cancers, insuffisance respiratoire, cardiaque, rénale, etc.) à subir une sorte de double peine. Tout d'abord, il doit endurer plusieurs interventions médicales, allant de la détection de la maladie jusqu'à son éventuel rétablissement. Ensuite, il doit aussi supporter le coût de chacune de ces différentes interventions. Le rapport Véran (Véran, 2017) souligne un risque d'effets inflationnistes puisque la T2A est essentiellement fondée sur le niveau d'activité des prestataires de soins. Pour modérer le nombre de ces interventions et pour en alléger le coût total pour le patient, ce rapport encourage un financement au parcours. Un tel mode de financement prend la forme d'un forfait de soins qui est l'unique paiement à la charge du patient pour l'ensemble de son parcours de soins. L'idée essentielle est que le coût du forfait soit inférieur à la somme des tarifs proposés pour l'ensemble des prestations de soins dispensées au long du parcours. Intuitivement, un tel parcours peut être représenté par une chaîne d'interventions médicales se succédant chronologiquement de la première intervention jusqu'à la guérison. Ces interventions impliquent un nombre potentiellement important de professionnels de santé, du médecin généraliste en passant par les cabinets libéraux des spécialistes, les hôpitaux, les cliniques, ou encore les instituts médico-sociaux. Malgré les recommandations du rapport Véran, le système du parcours de soin tarde à être mis en place en France.

La première contribution de ce chapitre est une modélisation de problème de répartition du forfait de soin entre les différent prestataires de soins. A notre connaissance, il s'agit

de la première modélisation du parcours de soin dans la littérature. Pour cela, chaque prestation médicale apparaissant le long du parcours de soin est caractérisée par un coût pour le professionnel de santé et un prix (plus élevé que le coût précédent) que le patient devrait payer pour cette prestation si le parcours de soin n'était pas mis en place. Nous supposons que le forfait est suffisamment élevé pour couvrir l'ensemble des coûts des prestations médicales du parcours, mais demeure inférieur à la somme des prix des prestations pour le patient. Par conséquent, il semble naturel qu'une allocation du forfait de soin dans un tel cadre attribue une part du forfait de soin à chaque professionnel de santé au moins aussi élevée que la somme des coûts de ses prestations. Une fois ce principe adopté, il reste simplement à déterminer l'allocation du reliquat du forfait de soins. Ce premier chapitre se concentre sur cette dernière question. Pour y répondre, il est essentiel d'évaluer l'importance de chaque professionnel de santé le long du parcours de soin. Cette évaluation dépend essentiellement de trois paramètres : le nombre d'interventions du professionnel, le prix unitaire de ces interventions et la position de ces interventions au sein de la chaîne de soins. Ces éléments peuvent être utilisés pour estimer ce que chaque professionnel pourrait légitimement revendiquer. Nous construisons à cet effet quatre jeux coopératifs distincts qui tiennent compte ou non de deux aspects : les positions des prestations de soins dans le parcours et le poids financier de chaque professionnel, mesuré par la somme des prix de ses interventions. Certains de ces jeux empruntent des éléments à la littérature sur les problèmes de revendications et à la littérature sur la gestion des cours d'eau internationaux (où la chaîne de soins peut s'apparenter au sens naturel d'écoulement de l'eau d'un fleuve). Le chapitre s'écarte néanmoins substantiellement de ces littératures dans d'autres dimensions. Par exemple, la possibilité qu'un professionnel de soins puisse intervenir à de multiples reprises le long du parcours reste inédite à notre connaissance.

Nous montrons que trois ce ces quatre jeux sont convexes, ce qui implique que la valeur de Shapley est une allocation du cœur comme nous l'avons évoqué précédemment. L'allocation ainsi obtenue est à la fois équitable et stable dans le sens où, lorsque le forfait est partagé, aucune coalition de professionnels ne reçoit au total moins que ce qu'elle pourrait légitimement revendiquer en l'absence des autres professionnels de soins. Nous étudions ensuite trois règles de partage illustrant trois principes très différents :

— La valeur de Shapley (Shapley, 1953). En fonction du type de jeu coopératif retenu, nous obtenons des propriétés supplémentaires de la valeur de Shapley. A titre d'exemple, imaginons que la chronologie des interventions médicales est pris en compte dans la construction du jeu coopératif. Par ailleurs, supposons que

les premières interventions sont particulièrement importantes (par exemple par ce qu'un bon diagnostic initial peut ensuite conditionner la longueur du parcours de soin). Dans un tel cadre, la valeur de Shapley du jeu ainsi obtenue est telle que les paiements perçus par les professionnels de santé sont ordonnés par la position de la première intervention les impliquant. Un résultat similaire est démontré lorsque le pouvoir de négociation des coalitions d'un jeu coopératif est calculé à partir de la marge totale de chaque professionnel, i.e. la somme des marges (différence entre le prix et le coût) de ses interventions. Ici aussi, la valeur de Shapley fournit des paiements tels que les professionnels de santé sont rangés par ordre croissant de marge totale.

— La règle de priorité introduite par Moulin (2000), qui propose de répartir le reliquat du forfait en remboursant totalement les interventions dans l'ordre chronologique jusqu'à épuisement du forfait. Ainsi, les premiers intervenants sont systématiquement mieux traités que les prestataires tardifs dans le parcours. Nous prouvons que l'allocation recommandée par la règle de priorité fait également partie du cœur des trois mêmes jeux coopératifs que la valeur de Shapley.

— Une règle de partage proportionnelle, qui répartit le reliquat du forfait en remboursant chaque acteur en proportion de sa marge totale. Ainsi, les professionnels dont la marge totale est importante seront les mieux traités. Nous montrons que cette règle fournit une allocation du cœur dans deux des quatre jeux coopératifs (et dans lesquels les deux autres règles de partage engendrent également des allocations stables).

Il ressort de ces résultats que la manière de concevoir le pouvoir de négociation des professionnels de santé n'est pas neutre quant aux propriétés des allocations que l'on peut proposer. Le cœur de deux des quatre jeux coopératifs étudiés contient les trois allocations précédentes, ce qui laisse de la flexibilité concernant le choix d'une allocation du reliquat du forfait de soin. Pour conclure, ce chapitre fournit des premiers enseignements sur un éventuel passage au système de parcours.

## Chapitre 2 : Application à la taxation des firmes multinationales

Le deuxième chapitre, écrit en collaboration avec Alexandre Chirat (U. Paris Nanterre), est une application principalement empirique de la théorie des jeux coopératifs à un problème de fiscalité internationale. Nous nous intéressons à la mise en place d'un taux commun d'imposition pour les firmes multinationales au niveau du G7 et du G20. En 2021, Joe Biden a souhaité réformer le taux d'imposition minimal sur les profits des firmes

multinationales. Cela implique de passer d'un système actuel où chaque pays taxe les profits des firmes multinationales à hauteur de son taux domestique à un système où chaque pays adopterait un seul et même taux d'imposition. Ce dernier système permettrait en outre de pouvoir lutter contre l'évasion fiscale dans les paradis fiscaux. Plusieurs taux harmonisés sont envisageables. Deux ont retenu notre attention : celui de 15% résultant d'un accord signé entre 136 pays en octobre 2021 et celui 25% proposé par les économistes Saez and Zucman (2019). Un enjeu de ce chapitre est de tester la crédibilité en terme de stabilité et d'équité des différents taux sur la coopération des pays du G7/G20.

Ce travail est, à notre connaissance, la première application des jeux coopératifs aux problèmes de taxation internationale. Nous construisons trois jeux qui s'appuient potentiellement sur les paramètres suivants : pour chaque pays, les profits des firmes multinationales reportés dans le pays, le taux d'imposition domestique du pays et les profits du pays transférés vers les paradis fiscaux, le taux commun d'imposition et le taux moyen d'imposition dans les paradis fiscaux. Chaque jeu traduit la capacité des coalitions de pays à récolter des revenus issus de la taxation. Le premier jeu qualifié de « contraint » oblige les pays à utiliser le taux commun dès lors qu'au moins deux pays coordonnent leurs efforts au sein d'une coalition. Le second jeu qualifié de « non-contraint » permet aux coalitions contenant au moins deux pays d'arbitrer entre l'adoption du taux commun et l'utilisation des taux domestiques de ses membres ; l'option finalement retenue étant celle qui permet de collecter les revenus les plus importants. Le troisième jeu, qualifié de jeu de récupération des « pertes », est tel que la récupération des pertes associées aux paradis fiscaux est possible si et seulement si l'accord est signé par l'ensemble des pays participants, i.e. uniquement pour la grande coalition du jeu.

Deux types de résultats sont obtenus. Premièrement, en utilisant les données de l'année 2015 fournies par Tørsløv et al. (2023), nous obtenons des résultats concluants sur la structure du jeu (premier résultat ci-après) et l'application des concepts de solution (deuxième et troisième résultats) :

— Au niveau du G7, la grande coalition génère un surplus (comparativement à la somme des revenus perçus séparément par chacun des pays membres via son taux domestique) dès que le taux commun est supérieur ou égal à 20%. De plus, lorsque le taux commun est de 25% (proposition de Saez and Zucman, 2019), plus le nombre de pays est grand au sein d'une coalition, plus ce surplus est important, notamment car les Etats-Unis déclarent 46.7% des profits des firmes multinationales à l'échelle

du G7 et appliquent un taux domestique de 21%.

— Au niveau du G7, la crédibilité de la proposition de taux commun à 25% de Saez and Zucman (2019) est testée en répartissant les revenus au moyen de la valeur de Shapley. Il en ressort que l'allocation obtenue n'est pas dans le cœur du jeu « non-contraint » mais fait partie du cœur de la somme de jeu « non-contraint » et du jeu de récupération des pertes. Ce résultat met en lumière que la coopération de tous les pays contre l'évasion fiscale est décisive pour s'assurer de la crédibilité d'une application du taux de 25%.

— Au niveau du G20, l'extension du résultat précédent n'est pas possible pour un taux commun de 25%. Plus précisément, l'allocation prescrite par la valeur de Shapley ne fait pas partie du cœur du jeu « non-contraint » et du cœur de la somme du jeu « non-contraint » et du jeu de récupération des « pertes ». Ce résultat s'explique de la manière suivante. Le total des pertes récupérées des paradis fiscaux augmente lorsque le G7 est élargi au G20 mais insuffisamment. Ainsi, puisque la valeur de Shapley du jeu de récupération des pertes est une allocation purement égalitaire, la part moyenne par pays diminue, conduisant à une effet de dilution.

Deuxièmement, nous présentons des résultats théoriques généraux sur les propriétés des différents jeux. Le jeu « non-contraint » d'une part et la somme du jeu « non-contraint » et du jeu de récupération des « pertes » d'autre part sont monotones, ce qui signifie que l'élargissement des coalitions ne fait jamais diminuer les revenus que ces coalitions sont susceptibles d'amasser. Il s'avère que le jeu « non contraint » n'est pas systématiquement suradditif, ce qui signifie que la réunion de deux coalitions de pays disjointes n'engendre pas nécessairement des revenus fiscaux plus importants que la somme des revenus des deux coalitions de départ. Enfin, le cœur de la somme du jeu « non-contraint » et du jeu de récupération des « pertes » est non vide à condition que le taux commun retenu soit au moins aussi élevé que chacun des taux domestiques (par exemple, cette condition imposerait un taux au moins égal à 35% si l'on tenait compte des taux domestiques reportés dans Tørsløv et al., 2023). Dans ce cas, l'accord sur la mise en place d'un tel taux commun peut paraître crédible en terme de stabilité, même si la valeur de Shapley n'appartient pas systématiquement au cœur.

## Chapitre 3 : Application au financement participatif

Le troisième chapitre, écrit en collaboration avec Sylvain Béal (U. Franche-Comté), Marc Deschamps (U. Franche-Comté) et Catherine Refait-Alexandre (U. Franche-Comté), est une application dans le domaine du financement participatif, également appelé *crowd-*

*funding*. Un projet de financement participatif est le plus souvent ouvert aux dons/contributions sur une période de temps limitée, et les dons donnent lieu à une récompense dont la valeur est positivement corrélée au niveau du don. Certains résultats empiriques mettent en évidence que les contributeurs réalisent l'essentiel des dons sur les premiers jours du projet, avant une chute spectaculaire du volume quotidien des dons dans les jours suivants. Il semble également que plus les contributions précoces sont importantes, plus la baisse subséquente du volume des contributions est atténuée. Cela laisse à penser que les premiers contributeurs (qualifiés de précoces par la suite) émettent des signaux positifs quant à la qualité du projet et pourraient être récompensés à ce titre. Autrement dit, si deux contributeurs réalisent la même contribution mais à deux moment différents, alors le plus précoce des deux pourrait recevoir une récompense plus élevée. Le chapitre est centrée sur cette propriété que nous qualifions d'effet signal.

Le chapitre propose une modélisation d'un projet de crowdfunding qui s'écarte des pratiques courantes dans deux dimensions. Premièrement, un principe de récompense consacrant l'effet signal n'existe pas aujourd'hui, à notre connaissance, dans les plateformes de financement participatif : la valeur d'une récompense dépend actuellement uniquement de la valeur de la contribution, pas de sa chronologie. Nous remettons en cause ce principe dans notre travail. Deuxièmement, nous introduisons un système de génération collective de récompense. Plus précisément, en pratique, le niveau de récompense d'un contributeur dépend fortement du niveau de sa propre contribution mais peu ou pas du niveau de contribution des autres contributeurs. Dans ce chapitre, au contraire, nous proposons un nouveau modèle dans lequel la récompense de chaque contributeur peut dépendre de toutes les contributions au projet grâce à une procédure en deux étapes : (1) une **fonction de récompense** associe à chaque contribution totale accumulée une récompense totale qui doit être répartie entre les contributeurs et ensuite (2) une **règle de récompense** associe à chaque montant total de récompense la part revenant à chaque contributeur. Nous pensons que le principe de génération d'une récompense collective est pertinent pour prendre en compte les synergies potentielles que des contributions distinctes peuvent apporter à la valeur d'un projet. Il semble plus juste que le bénéfice retiré de ces synergies revienne, au moins partiellement, aux contributeurs et ne soit pas intégralement capté par l'entrepreneur. Une fois encore, une telle modélisation semble inédite dans la littérature.

Sur cette classe de problèmes de crowdfunding, nous invoquons deux axiomes qui nous semblent pertinents :

— **Axiome d'efficience**. La totalité de la récompense collectivement engendrée par l'ensemble des contributions doit être intégralement reversée aux contributeurs.
— **Axiome de récompenses équitables**. L'impact du retrait d'un contributeur $i$ sur la récompense attribuée à un second contributeur $j$ est identique à l'impact qu'aurait le retrait du contributeur $j$ sur la récompense attribuée au joueur $i$.

Ce second axiome ne fait sens que lorsque l'on précise ce qu'il advient d'un projet de crowdfunding dans le scénario hypothétique où un ou plusieurs contributeurs seraient absents. Comme point de référence, nous adoptons le point de vue plutôt extrême selon lequel un contributeur restant contribue de la même manière que dans la situation où tous les contributeurs sont présents si et seulement si aucun des contributeurs ayant une chronologie de contribution plus précoce ne fait partie des absents. Dans le cas contraire, sa contribution est nulle. Autrement dit, nous supposons que les contributeurs sont considérablement influencés par les contributions passées qu'ils observent au moment d'effectuer leur propre contribution. Nous nuançons cette hypothèse à la fin du chapitre. Le premier résultat est qu'il existe une unique règle de récompense satisfaisant aux axiomes d'efficience et de récompenses équitables. Nous montrons par ailleurs que cette règle de récompense coïncide avec la valeur de Shapley d'un jeu coopératif particulier. Notons que l'axiome de l'axiome de récompenses équitables n'est pas qu'une transposition immédiate aux problèmes de crowdfunding de l'axiome de contributions équilibrées de Myerson (1977).

Les autres résultats du chapitre décrivent des propriétés du jeu coopératif précédent et de sa valeur de Shapley pour certaines spécifications générales de la fonction de récompense.

— Lorsque la fonction de récompense est strictement croissante, la règle de récompense que nous caractérisons véhicule l'effet signal : si deux contributeurs effectue la même contribution au projet, alors celui qui contribue en premier reçoit une récompense plus importante. Pour cette raison, cette règle de récompense est appelée la règle de signal. De plus, si deux contributeurs sont caractérisés par une chronologie de contribution identique, alors celui qui réalise la contribution la plus importante reçoit une récompense plus importante.
— Si la fonction de récompense est convexe, alors le jeu coopératif associé est également convexe. A l'instar des autres chapitres, la convexité implique la non-vacuité du cœur et l'appartenance de la règle de signal à celui-ci. Par le corollaire 2 de Sprumont (1990), on en déduit que si la coalition des contributeurs s'élargit, alors la récompense obtenue par chaque contributeur présent avant l'élargissement augmente également. La règle de signal ne lèse donc personne en cas d'arrivée de

nouveaux contributeurs.

— Si la fonction de récompense est une fonction à seuil, qui associe une récompense totale nulle en deçà d'un certain seuil de contributions et une récompense totale strictement positive mais constante une fois ce seuil atteint, alors les contributeurs peuvent être répartis en trois groupes en fonction des récompenses attribuées par la règle de signal. Le premier est l'ensemble des contributeurs les plus précoces dont la somme des contributions ne permet pas d'atteindre le seuil. Le second groupe est un ensemble de contributeurs intermédiaires ayant un même timing et dont les contributions, cumulées avec celles des contributeurs du premier groupe, permettent de, mais ne sont pas nécessaires pour, atteindre le seuil. Le troisième groupe contient les contributeurs restants, les plus tardifs. Ce partitionnement met en évidence les résultats suivants. Si deux contributeurs appartiennent au premier groupe, alors ils reçoivent la même récompense. La récompense des contributeurs du second groupe est croissante avec le montant de leur contribution. Les récompenses obtenues sont ordonnées par les groupes : les membres du premier groupe sont mieux récompensés que les membres du second groupe, et les membres du second groupe sont eux même mieux récompensés que les membres du troisième groupe (dont la récompense s'avère nulle).

# Chapitre 1

# A cooperative game approach to integrated healthcare

This article focuses on the sharing of a bundled payment for integrated healthcare. We model this problem by means of cooperative game theory. Various approaches are considered, each of which gives rise to a particular cooperative game, and make it possible to take the chronology of medical events into account. The Shapley value, a priority rule and a proportional allocation rule are used to (partially) refund the healthcare professionals on the basis of the fee paid by the patient. We establish some properties of these allocation rules. We also show that the core of some of these aformentioned games is non-empty and can contain these allocation rules.

## 1.1 Introduction

Nowadays, we observe a worrying increase of patients who have chronic diseases (Hackbarth et al., 2008) in different health systems from several countries. This highlights a real problem because the countries must react to treat these patients the most efficiently, within health systems which are very fragmentated (Brekke et al., 2022). Generally, we identify three types of health professionals, from ambulatory medicine (physicians and specialists) to clinics/hospitals and social centers (retirement homes and rest houses for instance) to treat the patient with different degrees of coordination and different market structures including the possibility to have insurers such as in France or Switzerland for instance or not such as in the United States. The patients who have chronic disease or disease which require different chronological interventions will meet these different health professionals to recover in a process of healthcare which is defined from the identification

of the chronic disease to the recovering (or death if the disease is uncurable). There are a lot of such chronic diseases from heart, respiratory or renal failure to diabete, but also Covid-19 in certain cases in recent periods. These chronic diseases reduce the life quality and can be very expensive for patients (van Dijk et al., 2014) especially within fee-for-services health systems where the healthcare professionals could have a moral hazard to overuse in order to serve their own interests. Indeed, the healthcare professionals do not always have the best incentives to treat the patients, and this can lead to a multiplication of consultations, even useless consultations. This problem is all the more relevant as the prospects of OECD (2017) concerning the ageing population are worrying with 9% of the population above 65 years old in 1960 to approximately 25% in two thirds of the OECD countries. Since 2020, Covid-19 has decreased the trend but it is still a challenge for countries and health systems to fight against a trend of a growing number of patients.

Therefore, in order to increase the quality and coordination of healthcare services, the implementation of a bundled payment should be a credible solution for the future (Porter and Lee, 2013). In such a bundled payment, the patient only pays a fee which has to be shared among the participating healthcare professionals. Indeed, the integrated healthcare has a lot of advantages, like a better coordination between the healthcare professionals and between the latters and the patients. It can also reduce the problem of fragmentation between services by cooperation (HCAAM, 2015). The implementation of a single bundled payment which covers all healthcare provided should increase the global quality of care (Brekke et al., 2022) and should give a better experience for patients (Stokes et al., 2018). Bundled payments in integrated healthcare are surveyed by Rocks et al. (2020) who show that "integrated care is likely to reduce cost and improve outcome" through a meta-analysis of 34 studies. The effects of this topical issue are analyzed by some experimentations in a lot of countries since a decade. We refer to HCAAM (2015) for experiments in France, Busse and Stahl (2014) for Germany and England, Struijs and Baan (2011), de Bakker et al. (2012) and Busse and Stahl (2014) for the Netherlands that show an improvement of the coordination of the care. In this article, we focus exclusively on the problem of compensating the healthcare professionals via the fee, but we do not discuss the strategic aspects underlying the behavior of these actors. The two approaches are therefore complementary in the sense that the compensation mechanisms highlighted in our work will influence the behavior of the healthcare professionals.

In this article, we study this problem by means of cooperative game theory, which is a set of tools relevant to analyse situations where payoffs or costs are generated by a group of agents who have mutual and conflicting interests. Cooperative game theory

has been extensively used in applications in recent years. Here we focus on the final sharing of the bundled payment among the health professionals who participate in patient recovering. Our modelling of bundled payment problem is inspired by the literature on bankruptcy games (O'Neill, 1982; Aumann and Maschler, 1985; Young, 1987). A classical bankruptcy game is constructed from an estate which must be shared among several claimants with different claims, the total of which exceeds the available estate. In the context of a bundled payment in healthcare, the claimants are the healthcare professionals, their claims correspond to the total benefit of their respective consultations and the estate is the fee paid by patient. We will create four different bankruptcy games depending on which criteria we want to highlight more in the integrated healthcare context : the benefit on medical events and the timeline for recovery. They will also depend on what an healthcare professional claims, i.e her total benefit or what remains of the fee if all other healthcare professionals are already paid. For instance, we show that taking the chain into account gives a very important weight to the earliest healthcare professionals.

This article contains three contributions. Firstly, the games that we propose have new structure and depart from the literature on bankruptcy games in two directions. We allow for the possibility of an healthcare professional to act several times during the process of recovering. Furthermore, we take into account the chain of medical events in the sense that the evaluation of the bargaining power of coalitions can depend on the positions of its members in this chain. Secondly, we study the main properties of our four cooperative games. In particular we prove that three out of four are convex. Convex games possess numerous interesting properties. For example, the core is non-empty and the Shapley value (Shapley, 1953) yields a core allocation. The core allocations are considered as stable allocations in the sense that, in our context, the part of the fee obtained by each coalition of healthcare professionnals is not less than what this coalition can expect without the help of the other healthcare professionals. Thirdly, for the above reason, we rely on the Shapley value in order to design an allocation to our bundled payment problem. We describe several properties of the resulting allocation depending on which bankruptcy game the Shapley value is applied to. As an illustration, if the Shapley value is applied to a specific bankruptcy game, it tends to reward more the healthcare professionals who act at the beginning of the process. For the sake of comparison, we also study the priority rule (Moulin, 2000) and a proportional rule (See Moulin (1987) for an application of the proportional principle to a general class of problems and Zou et al. (2021) for an axiomatization in the context of cooperative games), two other allocation rules based on very different principles. For each of these three allocation rules we determine whether

the resulting payoffs belong to the core of our four cooperative games.

Combining sequentiality with the possibility to act several times in the process is a feature that does not appear in any other article to the best of our knowledge. Nonetheless, our article can related to other approaches. For instance, the literature on claims problems takes into account sequentiality principles at two distinct levels : within of the structure of the considered sharing problem and in the design of an allocation rule. Ansink and Weikard (2012), Mianabadi et al. (2014) and Estévez-Fernández et al. (2021) study models in which the players sequentially put claims on the estate. These works focus on a context of water sharing along a river where agents are linearly ordered. To the contrary, Moulin (2000) and Sanchez-Soriano (2021) consider classical bankruptcy problems (without sequential aspects) but examine allocation rules in which the players are treated sequentially. Liability problems with sequential structures are also studied in Dehez and Ferey (2013) and Gudmundsson et al. (2020) in the context of sharing damages and losses from the conflict between victims and injurers.

The rest of the article is organized as follows. Section 1.2 presents the model. Section 1.3 provides definitions of the tools and the approaches that we use and studies the properties of the games. Section 1.4 is the study of the aforementioned allocation rules. Section 1.5 concludes.

## 1.2 The model

A **bundled payement problem** is described by a quintuplet $(N, \{\hat{p}_i, m_i\}_{i\in N}, C, \hat{F})$. The finite set $N = \{1, \dots, n\}$ contains the **healthcare professionals** that a chronic patient needs either for a general appointment or a surgical intervention. Each healthcare professional $i \in N$ charges the patient a price $\hat{p}_i$ for any visit whereas the cost to the healthcare professional of such a visit is $m_i$, with $0 < m_i < \hat{p}_i$. From chronic disease identification to total recovery of the patient (or death if the chronic disease is incurable), the sequence of medical events can be represented by a **chain** $C$ which specifies the order in which the healthcare professionals provide services to the patient. A given healthcare professional can be involved in several **positions** of chain $C$. Formally, the chain is a finite $k-$dimensional vector :

$$C = (c_1, \dots, c_k),$$

where, for each position $q \in \{1, \dots, k\}$, $c_q \in N$ stands for the unique healthcare professional involved in position $q$. Moreover, let $q(i)$ denote the first position of $C$ involving an healthcare professional $i \in N$. Finally, in the context of an integrated healthcare, the

patient eventually only pays a fee $\hat{F}$. We assume that :

$$\sum_{q\in\{1,\dots,k\}} m_{c_q} \leq \hat{F} < \sum_{q\in\{1,\dots,k\}} \hat{p}_{c_q}. \tag{1.1}$$

This means that $\hat{F}$ is enough to cover the cost of all medical services but is strictly less than the sum of the prices for medical events. We consider that this property is unavoidable, so that an allocation $x = (x_1, ..., x_n)$ of $\hat{F}$ must satisfy :

$$x_i \geq m_i |C^{-1}(i)|, \quad \forall i \in N, \tag{1.2}$$

where $C^{-1}(i)$ is the set of positions in chain $C$ involving $i$. Hence, $|C^{-1}(i)|$ is the number of visits to the healthcare professional $i$. As a consequence, without any loss of generality, the bundled payment problem can be completely described without reference to the costs by the quadruplet $B = (N, \{p_i\}_{i\in N}, C, F)$ where

$$F = \hat{F} - \sum_{q\in\{1,\dots,k\}} m_{c_q},$$

and

$$p_i = \hat{p}_i - m_i.$$

From (1.1) and the definitions of $F$ and $p_i$, we get that

$$0 < F < \sum_{q\in\{1,\dots,k\}} p_{c_q}. \tag{1.3}$$

In this new problem $B$, we only allocate $F$, i.e. what remains of the original fee once each healthcare professional $i$ has been refund its total cost, on the basis of the margin/benefit $p_i$ that this healthcare professional can still claim. Therefore, from (1.2), an allocation for problem $B$ is just a positive vector adding up to $F$.

## 1.3 A cooperative game approach

### 1.3.1 Definitions

A **cooperative game** on a player set $N = \{1, ..., n\}$ is a characteristic function $v$ which assigns a worth $v(S)$ to each coalition $S \subseteq N$, and such that $v(\varnothing) = 0$. The worth of a coalition is the total payoff that its members can secure by themselves. In other words, it represents the best outcome that each subset of the participants ('players') can

achieve being unaided. The coalition $N$ of all players is called the grand coalition and is considered as actually formed, the others coming from counterfactual scenarios.

An **allocation** for $v$ is a payoff vector $x = (x_1, ..., x_n)$ which assigns a payoff $x_i \in \mathbb{R}$ to each player $i \in N$ in order to reflect her participation to game $v$. It is efficient if $\sum_{i \in N} x_i = v(N)$. The **core** of game $v$ is the set $O(v)$ of all efficient allocations $x$ such that no coalition of players $S$ gets a total payoff $\sum_{i \in S} x_i$ smaller than its worth $v(S)$, that is to say not smaller than what it can secure for its members. Formally :

$$O(v) = \left\{ x \in \mathbb{R}^N : \sum_{i \in S} x_i \geq v(S), S \subseteq N, \sum_{i \in N} x_i = v(N) \right\}.$$

The core can be empty. If $x$ is a core allocation, then it is in the interest of no coalition of players to split from the grand coalition. Hence the grand coalition can be considered as stable if its members are paid according to such a core allocation. Stability is sometimes incompatible with fairness considerations.

The Shapley value of a game (Shapley, 1953) is traditionally seen as a fair allocation rule. It is efficient, additive (the Shapley value in the sum of 2 games is the sum of the Shapley values in these 2 games), assigns a null payoff to any player whose marginal contributions to coalitions are null, and assigns equal payoffs to players characterized by identical marginal contributions to coalitions. More specifically, the Shapley value is uniquely characterized by these four properties or axioms, and assigns to a player $i$ in a game $v$ a payoff $Sh_i(v)$ which is a weighted average of all her marginal contributions. If we denote the cardinal of coalition $S$ by $s = |S|$, then the Shapley value of game $v$ is given by :

$$Sh_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} \big( v(S \cup \{i\}) - v(S) \big), \quad \forall i \in N,$$

where $v(S \cup \{i\}) - v(S)$ is the marginal contribution of $i$ to the coalition $S$.

A game is convex if the marginal contributions are non-decreasing with the size of the coalition, that is :

$$v(S) - v(S \setminus \{i\}) \leq v(T) - v(T \setminus \{i\}) \quad \forall i \in S \subseteq T \subseteq N. \tag{1.4}$$

Convexity is a sufficient conditon for the nonemptiness of the core. Shapley (1971) even shows that the Shapley value of a convex game lies on its core.

A **bankruptcy problem** $(N, E, (\alpha_i)_{i \in N})$ is a situation in which an estate $E$ must be

shared among players in $N$ who have claims $\alpha_i \geq 0$, $i \in N$, satisfying :

$$E > \sum_{i \in N} \alpha_i.$$

To each bankruptcy problem, following O'Neill (1982), Aumann and Maschler (1985) and Curiel et al. (1987), it is possible to associate a cooperative game.
If a coalition hopes to recover as much of its members' claims as the estate allows, then we get the following game :

$$v(S) = \min \left\{ E; \sum_{i \in S} \alpha_i \right\}, \quad \forall S \subseteq N, \quad S \neq \varnothing,$$

and $v(\varnothing) = 0$. In this case, "the worth of a coalition is the sum of its members' claims, as long as this sum does not exceed the total available amount. If each member of a coalition has a right to his claim, then the coalition would seem to have a right to the sum of the claims, as long as that is feasible" (Aumann, 2010).
To the contrary, if a coalition only expects to secure what remains of the estate once the other claimants have obtained their claims, then we get a second game :

$$v(S) = \max \left\{ 0; E - \sum_{i \in N \setminus S} \alpha_i \right\}, \quad \forall S \subseteq N.$$

In the first game, the coalitions are optimistic about their opportunities whereas in the second game they are rather pessimistic. We refer to Thomson (2019) for a survey on bankruptcy problems.

### 1.3.2 Integrated healthcare games

In this section, to each $B = (N, \{p_i\}_{i \in N}, C, F)$, we will associate several cooperative games. We proceed in three steps : we begin with extra definitions regarding the chain $C$, then we introduce 4 types of cooperative games, finally we study their convexity.

#### The chain

How to define what can be the best result for a coalition $S \subseteq N$ when its members cooperate without the other players $N \setminus S$ ? In other words, where should the chain stop if only the healthcare professionals in $S$ act ? The maximal chain for $S$ denoted by $C(S)$ is precisely the set of all positions from the beginning of the chain to the first position involving an healthcare professional outside of $S$. Note that a position involving a heal-

thcare professional $i \in S$ does not belong to $C(S)$ if this position is located after the first position involving an healthcare professional outside of $S$.

There is the possibility of all healthcare professionals to act more than once in the chain. Therefore, we need a correspondance $N \longrightarrow \{1, ..., k\}$ that associates to each $i \in N$ one or more positions in the chain $C$. This is done by the inverse function $C^{-1}(i)$ defined as :

$$C^{-1}(i) = \left\{ q \in \{1, ..., k\} : c_q = i \right\}, \quad \forall i \in N.$$

Hence, the total margin of medical events involving $i$ is :

$$\sum_{q \in C^{-1}(i)} p_{c_q} = |C^{-1}(i)| p_i.$$

This total margin can be interpreted as the legitimate claim of healthcare professional $i$ or its bargaining power when sharing $F$ (once she has been refund of its costs as underlined in Section 1.2) which refers naturally to the bankruptcy approach.

For each $S \subseteq N$, let

$$C^{-1}(S) = \bigcup_{i \in S} C^{-1}(i),$$

so that, the maximal complete chain for $S$ is :

$$C(S) = \max_{q \in \{1,...,k\} : \{1,...,q\} \subseteq C^{-1}(S)} (c_1, ..., c_q). \tag{1.5}$$

The last position in the maximal complete chain for $S$ is denoted as $q_S$ and the remainder of the chain is called the complementary chain $C \backslash C(S)$, which can contain whoever, even some healthcare professionals in $S$. Thus :

$$C(S) = (c_1, ...., c_{q_S}) \quad \text{and} \quad C \backslash C(S) = (c_{q_{S+1}}, ..., c_k).$$

These concepts of maximal complete chain and complementary chain will be useful to describe two of the four games in the next section.

**The different games**

In order to apprehend the problem $B$, we need to evaluate the bargaining power of all coalitions. Four possibilities can be obtained by answering the two following natural questions :

— Shall we account for the position of the healthcare professionals within the process of recovering ?

— Should a coalition look at its opportunities with an optimistic or pessimistic view ?

We can answer positively to the first question because it makes sense to think that the quality of a patient's care is determined by the first interventions she receives. In particular, the diagnosis is established at the beginning of the process and conditions the subsequent treatment. For example, if the physician refers the patient to a cardiologist for chest pains while the heart is not the source of the problem, it can lead to useless consultations and wrong guidance such as laboratory blood tests, radiology or even emergency service consultation which may lead to nothing. For these reasons, it may be interesting to provide incentives for first healthcare professionals in the process to be particularly efficient. We materialize this idea by relying, for each coalition of healthcare professionals on the maximal chain associated with this coalition. To the contrary, if one does not want to take into acccount the positions of healthcare professionals in the recovering process then, the bargaining power of coalitions can be determined by the full margin of their members and not just by the margin induced by the maximal chain of the coalition.

The second question is classical in the literature on bankruptcy problems (O'Neill, 1982; Aumann and Maschler, 1985). The vision of a coalition is optimistic if the coalition expects to get back the portion of its margin covered by the fee without taking into account the claims of the other healthcare professionals. The vision of a coalition is pessimistic if it expects to obtain only what is left of the fee after each other healthcare professional is refunded the amount of her margin, if possible. The four approaches are summarized in the following table :

| | Optimistic vision | Pessimistic vision |
|---|---|---|
| Chain | $w_B^C(S) = \min\left\{F; \sum_{c_q \in C(S)} p_{c_q}\right\}$ | $v_B^C(S) = \max\left\{0; F - \sum_{c_q \in C\setminus C(S)} p_{c_q}\right\}$ |
| Not Chain | $u_B(S) = \min\left\{F; \sum_{i\in S} p_i \lvert C^{-1}(i)\rvert\right\}$ | $z_B(S) = \max\left\{0; F - \sum_{i\in N\setminus S} p_i \lvert C^{-1}(i)\rvert\right\}$ |

Games $u_B$ and $z_B$ can be considered as classical bankruptcy games in which the estate is the fee $F$ and in which the claims are the margin of healthcare professionals. Games $w_B^C$ and $v_B^C$ are not bankruptcy games because the total claim of a coalition is not equal to the sum of the individual claims of its members.

**Example 1** Consider a patient who has a lung cancer. The problem $B = (N, (p_i)_{i\in N}, C, F)$ to treat this patient involves three healthcare professionals, i.e. $N = \{1, 2, 3\}$. They are

respectively the practician, the specialist and the hospital, and we set $p_1$ = \$25, $p_2$ = \$50 and $p_3$ = \$80. Let $F$ = \$345. The patient needs the following chronological treatment $C$ :

$$1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 3 \longrightarrow 3 \longrightarrow 3 \longrightarrow 1.$$

The maximal complete chain, its complementary chain, and the number of elements of the chain for each non-empty coalition are presented in the following table :

| $S$ | $C(S)$ | $C \backslash C(S)$ | $q_S$ |
|---|---|---|---|
| $\{1\}$ | 1 | $2 \to 1 \to 3 \to 3 \to 3 \to 1$ | 1 |
| $\{2\}$ | $\varnothing$ | $1 \to 2 \to 1 \to 3 \to 3 \to 3 \to 1$ | 0 |
| $\{3\}$ | $\varnothing$ | $1 \to 2 \to 1 \to 3 \to 3 \to 3 \to 1$ | 0 |
| $\{1,2\}$ | $1 \to 2 \to 1$ | $3 \to 3 \to 3 \to 1$ | 3 |
| $\{1,3\}$ | 1 | $2 \to 1 \to 3 \to 3 \to 3 \to 1$ | 1 |
| $\{2,3\}$ | $\varnothing$ | $1 \to 2 \to 1 \to 3 \to 3 \to 3 \to 1$ | 0 |
| $\{1,2,3\}$ | $1 \to 2 \to 1 \to 3 \to 3 \to 3 \to 1$ | $\varnothing$ | 7 |

Consider the coalition $\{1,2\}$. The maximal complete chain for $\{1,2\}$ is composed of the first three positions (without discontinuity) on the chain because $1 \in S$ and $2 \in S$. The fourth position is assigned to the healthcare professional $3 \notin S$. Therefore, the cardinal of $C(S)$ is the number of elements in $C(S)$ or, equivalently, the position of the last element of $C(S) : q_S = 3$. The complementary chain is $C \backslash C(S) = 3 \to 3 \to 3 \to 1$, which contains all remaining positions.
The four games that we propose in the context of this example are described below.

| $S$ | $\{1\}$ | $\{2\}$ | $\{3\}$ | $\{1,2\}$ | $\{1,3\}$ | $\{2,3\}$ | $\{1,2,3\}$ |
|---|---|---|---|---|---|---|---|
| $v_B^C(S)$ | 5 | 0 | 0 | 80 | 5 | 0 | 345 |
| $w_B^C(S)$ | 25 | 0 | 0 | 100 | 25 | 0 | 345 |
| $u_B(S)$ | 75 | 50 | 240 | 125 | 315 | 290 | 345 |
| $z_B(S)$ | 55 | 30 | 220 | 105 | 295 | 270 | 345 |

Remember that the worth of each coalition reflects the subjective evaluation of their barganinig power in the games. This translates the power relations between the different coalitions. Observe that $u_B(\{2\}) + u_B(\{1,3\}) > u_B(\{1,2,3\})$ which implies that the core of $u_B$ is empty. The core of three other games is non-empty as a consequence of the corollary 1 in the next section. □

**On the convexity of integrated healthcare games**

It is known from Curiel et al. (1987) that game $z_B$ is convex and that $z_B$ and $u_B$ are connected by the duality relation (Aumann and Maschler, 1985). The dual of a game $v$ is

the game $v^D$ such that for each $S \subseteq N, v^D(S) = v(N) - v(N \backslash S)$. Since the dual of a convex game is a concave game (Bilbao, 2000), $u_B$ is not a convex game. In this section, we show that both $v_B^C$ and $w_B^C$ are convex which implies that they are not the dual of each other.

**Proposition 1** *For any integrated healthcare problem $B$, $(i)$ the game $v_B^C$ is convex and $(ii)$ the game $w_B^C$ is convex.*

The proof of this result is in the Appendix. Convexity property in integrated healthcare games can be interpredted as follows : the extra bargaining power brought by any healthcare professional to a coalition she joins is increasing with the size of this coalition. That shows the importance of being as many healthcare professionals as possible within a coalition for complentarity reasons and justify the cooperation in the integrated healthcare problem. Proposition 1 and Shapley (1971) implies other interesting properties and results, especially the non-emptiness of three of the four considered games.

**Corollary 1** *The cores of the games $v_B^C$ and $w_B^C$ (and also $z_B$) are non-empty and contain the Shapley value.*

This means that if we accept that $v_B^C$, $w_B^C$ and $z_B$ correctly model the bargaining power of coalitions of healthcare professionals during the process, there are allocations which can be stable (in the sense of the core). The part of the fee obtained by each coalition of healthcare professionnals is not less than what this coalition can expect without the help of the other healthcare professionals according to its bargaining power. Since the Shapley values of these three games are in their cores, it is interesting to reward the healthcare professionals by means of this allocation rule. In the next section, we investigate some properties of the resulting allocations, but we also study two other allocation rules based on alternative (fairness) principles.

## 1.4 The allocation rules

### 1.4.1 The Shapley value

The Shapley value satisfies numerous desirable axioms. The desirability axiom (Maschler and Peleg, 1966) is one of them. It states that if a first healthcare professional has marginal contributions to coalitions at least as large as the marginal contributions of a second healthcare professional, then she should obtain a payoff at least as large as the payoff of the second healthcare professional. Let $f$ be an arbitrary allocation rule on any

class of games $G$.

**Desirability** : For each $v \in G$, for each pair of distinct players $i, j \in N$, such that for each $S \subseteq N \backslash \{i, j\}$, $v(S \cup \{i\}) \geq v(S \cup \{j\})$, then $f_i(v) \geq f_j(v)$.

We make use of this property to prove the following result.

**Proposition 2** *The payoffs provided by the Shapley value of game* $(N, v_B^C)$ *are ordered by the position of the first medical event involving each healthcare professional :*

$$q(i) < q(j) \Longrightarrow Sh_i(v_B^C) \geq Sh_j(v_B^C)$$

**Proof.** Consider a distinct pair of healthcare professionals $i, j \in N$ such that the first position involving the healthcare professional $i$ is previous to the first position involving the healthcare professional $j$ in the chain, i.e.

$$q(i) < q(j). \tag{1.6}$$

Let $S \subseteq N \backslash \{i, j\}$. From (1.6) we get

$$C(S \cup \{j\}) \subseteq C(S \cup \{i\}),$$

which implies

$$C \backslash C(S \cup \{j\}) \supseteq C \backslash C(S \cup \{i\}).$$

Hence

$$F - \sum_{c_q \in C \backslash C(S \cup \{i\})} p_{c_q} \geq F - \sum_{c_q \in C \backslash C(S \cup \{j\})} p_{c_q},$$

which is equivalent to

$$v_B^C(S \cup \{i\}) \geq v_B^C(S \cup \{j\}).$$

Therefore, the healthcare professional $i$ is at least as desirable as $j$ so that $Sh_i(v_B^C) \geq Sh_j(v_B^C)$ since $Sh$ satisfies the desirability axiom. ■

Thus, game $v_B^C$ shows that the chronology of medical events is important if the Shapley value is used, because healthcare professionals involved at the beginning of the chain earn not less than subsequent professionals. This allocation can be used to provide incentives for the first healthcare professionals in the process to be particularly efficient.

The next result deals with the following specific situation. Consider the healthcare professional whose first intervention is the latest in the recovery process. Suppose that the total margin that follows this intervention in the chain is at least as large as the fee. Such a case is more likely to occur when the treatment is long. We show that the Shapley value of game $v_B^C$ provides the same payoffs to all healthcare professionals.

**Proposition 3** *Let $q^* = \max_{j \in N} q(j)$. Assume that $\sum_{q \geq q^*} p_{c_q} > F$, then the Shapley value of $v_B^C$ provides equal payoffs to all healthcare professionals.*

**Proof.** Consider the case where $q^*$ is the first position involving the healthcare professional who acts for the first time the latest in the chain :

$$q^* = \max_{j \in N} q(j).$$

Denote by $i$ the healthcare professional who acts in this position $q^*$. Assume that $\sum_{q \geq q^*} p_{c_q} > F$. The complementary chain of $C(N \backslash \{i\})$ is equal to the set of all positions after the first intervention of $i$ :

$$C \backslash C(N \backslash \{i\}) = \{c_q : q \geq q^*\}.$$

Pick $S \neq N$ and consider the following two cases :

• Let $S \not\ni i$, then $\left\{c_q : q \geq q^*\right\} \subseteq C \backslash C(S)$. Thus, the sum of the margins charged from the position $q^*$ to the end of the chain is greater than the fee $F$. Formally :

$$\sum_{c_q \in C \backslash C(S)} p_{c_q} \geq \sum_{q \geq q^*} p_{c_q} > F. \tag{1.7}$$

• Let $S \ni i$. The maximal chain of $S$ is included in the maximal chain of $N \backslash \{i\}$. Formally :

$$C(S) \subseteq C(N \backslash \{i\}) \Rightarrow C \backslash C(N \backslash \{i\}) \subseteq C \backslash C(S). \tag{1.8}$$

From (1.7) and (1.8), this implies that whatever the positions $q$ occuring after the position $q^*$ :

$$\left\{c_q : q \geq q^*\right\} \subseteq C \backslash C(S),$$

which means that :

$$\sum_{c_q \in C \backslash C(S)} p_{c_q} > F.$$

By definition on $v_B^C$, this implies that $v_B^C$ is a symmetric game for each $S \subseteq N$ :

$$v_B^C(S) = \begin{cases} 0 & \text{if } s \neq n, \\ F & \text{if } s = n. \end{cases}$$

All players are equal in a symmetric game. Since $Sh$ satisfies the well-known axiom of equal treatment of equals, we get :

$$Sh_j(v_B^C) = \frac{F}{n}.$$

for each $j \in N$. ■

Note that Proposition 2 is compatible with Proposition 3 in the sense that the situation of Proposition 2 is an extreme case in which all inequalities are weak.

The games $u_B$ and $z_B$ take into account the benefit of the healthcare professionals. The greater your margins and number of medical events, the greater your benefit and your payoff using the Shapley value. That rewards better the most expensive healthcare professionals and the ones who have the greatest number of medical events.

**Proposition 4** *The payoffs provided by the Shapley value of games $u_B$ and $z_B$ are ordered by the amount of margin involving each healthcare professionals. For each $j \in N\backslash\{i\}$ :*

$$p_i|C^{-1}(i)| \geq p_j|C^{-1}(j)| \Longrightarrow \begin{cases} Sh_i(u_B) \geq Sh_j(u_B), \\ Sh_i(z_B) \geq Sh_j(z_B). \end{cases}$$

**Proof.** Consider $i, j \in N$ such that $p_i|C^{-1}(i)| \geq p_j|C^{-1}(j)|$. Let $S \subseteq N\backslash\{i, j\}$. Then :

$$\sum_{k \in S \cup \{i\}} p_k|C^{-1}(k)| \geq \sum_{k \in S \cup \{j\}} p_k|C^{-1}(k)|,$$

or equivalently,

$$\sum_{k \in S} p_k|C^{-1}(k)| + p_i|C^{-1}(i)| \geq \sum_{k \in S} p_k|C^{-1}(k)| + p_j|C^{-1}(j)|.$$

This shows that $u_B(S \cup \{i\}) \geq u_B(S \cup \{j\})$. The healthcare professional $i$ is at least as desirable than the healthcare professional $j$. By the axiom of desirability :

$$Sh_i(u_B) \geq Sh_j(u_B).$$

Since $z_B = (u_B)^D$, $Sh(z_B) = Sh(u_B)$ because $Sh$ is self-dual which means that we obtain the same payoffs for players using the Shapley value in a game and in its dual. Hence, the result also holds for $z_B$. ■

We recall that the Shapley value is in the core of $v_B^C$, $w_B^C$ and $z_B$ according to convexity property, but the Shapley value may not be in the core of $u_B$ since the latter can be empty as shown is Example 1.

### 1.4.2 The priority rule

The priority rule (Moulin, 2000) is the allocation rule $x^P$ which rewards the healthcare professionals in the order of their interventions until the fee $F$ is depleted. Thus, all medical events are refunded from the beginning of the process as long as $F$ can be used up. Denote by $\hat{q}$ the penultimate medical event which is refunded, so that $\hat{q}_{+1}$ will be the last (possibly partially) refunded medical event. We have :

$$\hat{q} = \text{argmax}\left\{q \in \{1, ..., k\} : \sum_{r=1}^{q} p_{c_r} < F\right\}.$$

The set of all healthcare professionals who act before the depletion of $F$ is :

$$\hat{S} = \left\{i \in N : q(i) \leq \hat{q}_{+1}\right\}.$$

For an healthcare professional $i \in N$, $x_i^P(B)$ refunds all medical events involving $i$ before the depletion of $F$ and possibly a part of a medical event if there is a residue of $F$ in the remaining medical event involving $i$ :

$$x_i^P(B) = \begin{cases} \sum\limits_{q \leq \hat{q} : c_q = i} p_{c_q} & \text{if } c_{\hat{q}_{+1}} \neq i, \\ \sum\limits_{q \leq \hat{q} : c_q = i} p_{c_q} + F - \sum\limits_{q=1}^{\hat{q}} p_{c_q} & \text{if } c_{\hat{q}_{+1}} = i. \end{cases}$$

In this section, we show that the priority rule is a core allocation in all games except $u_B$ exactly as the Shapley value.

**Proposition 5** *The payoffs provided by the priority rule $x^P$ in problem $B$ are in the core of the game $w_B^C$.*

**Proof.** Let $S \subseteq N$, we show that $\sum\limits_{i \in S} x_i^P(B) \geq w_B^C(S)$. We consider three cases depending on the link between $S$ and $\hat{S}$.

— Suppose that $S \cap \hat{S} = \varnothing$. Then $C(S) = \varnothing$, so that :

$$\sum_{i \in S} x_i^P(B) = w_B^C(S) = 0.$$

— Suppose that $\hat{S} \subseteq S$. Then $C(\hat{S}) \subseteq C(S)$, which implies, by definition of $\hat{S}$, that :

$$F \leq \sum_{q \leq q_{\hat{S}}} p_{c_q} \leq \sum_{q \leq q_S} p_{c_q}.$$

Therefore, it holds that :

$$w_B^C(S) = F = \sum_{i \in \hat{S}} x_i^P(B) = \sum_{i \in S} x_i^P(B).$$

— Suppose that $S \cap \hat{S} \neq \{\varnothing; \hat{S}\}$. Then :

$$\sum_{i \in S} x_i^P(B) = \sum_{i \in S \cap \hat{S}} x_i^P(B) \geq \sum_{q \leq q_{S \cap \hat{S}}} p_{c_q} = w_B^C(S \cap \hat{S}) = w_B^C(S).$$

This concludes the proof. ■

**Proposition 6** *The payoffs provided by the priority rule $x^P$ in problem $B$ are in the core of the game $v_B^C$.*

**Proof.** Let $S \subseteq N$, we show that $\sum_{i \in S} x_i^P(B) \geq v_B^C(S)$. We consider three cases :

— Suppose that $S \cap \hat{S} = \varnothing$. Then $C(S) = \varnothing$, so that :

$$\sum_{i \in S} x_i^P(B) = v_B^C(S) = 0.$$

— Suppose that $\hat{S} \subseteq S$. Then $C(\hat{S}) \subseteq C(S)$, which implies :

$$\sum_{i \in S} x_i^P(B) = \sum_{i \in \hat{S}} x_i^P(B) = F \geq v_B^C(S).$$

— Suppose that $S \cap \hat{S} \neq \{\varnothing; \hat{S}\}$. We have :

$$\sum_{i \in S} x_i^P(B) = \sum_{i \in \hat{S} \cap S} x_i^P(B) = \sum_{i \in \hat{S}} x_i^P(B) - \sum_{i \in \hat{S} \setminus S} x_i^P(B).$$

The difference above can be rewritten as :

$$F - \sum_{i \in \hat{S} \setminus S} x_i^P(B). \tag{1.9}$$

From (1.9) :

$$F - \sum_{i \in \hat{S} \setminus S} x_i^P(B) \geq F - \sum_{c_q \in C \setminus C(\hat{S} \cap S)} p_{c_q} \geq F - \sum_{c_q \in C \setminus C(S)} p_{c_q}.$$

Hence

$$\sum_{i \in S} x_i^P(B) \geq v_B^C(S).$$

This concludes the proof. ■

**Proposition 7** *The payoffs provided by the priority rule $x^P$ in any problem $B$ are in the core of the game $z_B$.*

**Proof.** Fix any bundled payment problem $B$. We already know that $x^P(B)$ is an efficient allocation. Now, let $S \subseteq N$ be any coalition. Since $\sum_{i \in S} x_i^P(B) \geq 0$, proving that $\sum_{i \in S} x_i^P(B) \geq z_B(S)$ amounts to show that $\sum_{i \in S} x_i^P(B) \geq F - \sum_{i \in N \setminus S} p_i |C^{-1}(i)|$. We have

$$\sum_{i \in S} x_i^P(B) = F - \sum_{i \in N \setminus S} x_i^P(B) \geq F - \sum_{i \in N \setminus S} p_i |C^{-1}(i)|,$$

where the equality comes from the fact that $x^P(B)$ is an efficient allocation and the inequality from the fact that the overall margin of the professionals in $S$ is not less that what the priority rule allocates to these players. Conclude that $x^P(B) \in O(z_B)$. ■

From example 1, in which the core of game $u_B$ is empty, we get that the priority rule is not always a core allocation of game $u_B$.

### 1.4.3 The proportional allocation rule

The proportional allocation rule $y^P$ is the allocation rule which refunds the healthcare professionals in proportion to their margins. This allocation rule refunds more healthcare professionals who have the highest benefit in the process of recovering. Formally :

$$y_i^P(S) = \frac{\sum_{i \in S} p_i |C^{-1}(i)|}{\sum_{j \in N} p_j |C^{-1}(j)|} \times F.$$

In this section, we prove that the proportional allocation rule belongs to the cores of $v_B^C$ and $z_B$ but not always to the core of the two other games.

**Proposition 8** *The payoffs provided by the proportional allocation rule $y^P$ in any problem $B$ are in the core of the game $v_B^C$.*

**Proof.** Fix any bundled payment problem $B$. We already know that $y^P(B)$ is an efficient allocation. Now, let $S \subseteq N$ be any coalition. Since $\sum_{i\in S} y_i^P(B) \geq 0$, proving that that $\sum_{i\in S} y_i^P(B) \geq v_B^C(S)$ amounts to show that $\sum_{i\in S} y_i^P(B) \geq F - \sum_{c_q\in C\setminus C(S)} p_{c_q}$. By definition of the proportional allocation rule and the fact that the overall benefit of the professionals in $S$ is not less that part of this benefit they obtain in $C(S)$, we can write

$$\sum_{i\in S} y_i^P(B) = \sum_{i\in S} \frac{p_i|C^{-1}(i)|}{\sum_{j\in N} p_j|C^{-1}(j)|}F = \frac{\sum_{i\in S} p_i|C^{-1}(i)|}{\sum_{c_q\in C} p_{c_q}}F \geq \frac{\sum_{c_q\in C(S)} p_{c_q}}{\sum_{c_q\in C} p_{c_q}}F.$$

The last expression can be rewritten as

$$\frac{\sum_{c_q\in C} p_{c_q} - \sum_{c_q\in C\setminus C(S)} p_{c_q}}{\sum_{c_q\in C} p_{c_q}}F = \left(1 - \frac{\sum_{c_q\in C\setminus C(S)} p_{c_q}}{\sum_{c_q\in C} p_{c_q}}\right)F \geq \left(1 - \frac{\sum_{c_q\in C\setminus C(S)} p_{c_q}}{F}\right)F,$$

where the inequality comes from assumption (1.3). Simplifying, we get

$$\sum_{i\in S} y_i^P(B) \geq F - \sum_{c_q\in C\setminus C(S)} p_{c_q},$$

as desired, proving that $y^P(B) \in O(v_B^C)$. ∎

**Proposition 9** *The payoffs provided by the proportional allocation rule $y^P$ in any problem $B$ are in the core of the game $z_B$.*

**Proof.** The proof is similar to that of Proposition 8. Fix any bundled payment problem $B$ and let $S \subseteq N$ be any coalition. From the same arguments as those at beginning of the proof of Proposition 8, it is enough to show that $\sum_{i\in S} y_i^P(B) \geq F - \sum_{j\in N\setminus S} p_j|C^{-1}(j)|$. We have

$$\sum_{i\in S} y_i^P(B) = \sum_{i\in S} \frac{p_i|C^{-1}(i)|}{\sum_{j\in N} p_j|C^{-1}(j)|}F = \frac{\sum_{j\in N} p_j|C^{-1}(j)| - \sum_{j\in N\setminus S} p_j|C^{-1}(j)|}{\sum_{j\in N} p_j|C^{-1}(j)|}F,$$

or equivalently

$$\sum_{i\in S} y_i^P(B) = \left(1 - \frac{\sum_{j\in N\setminus S} p_j|C^{-1}(j)|}{\sum_{j\in N} p_j|C^{-1}(j)|}\right)F.$$

Using assumption (1.3), we obtain that

$$\left(1 - \frac{\sum_{j\in N\setminus S} p_j|C^{-1}(j)|}{\sum_{j\in N} p_j|C^{-1}(j)|}\right)F \geq \left(1 - \frac{\sum_{j\in N\setminus S} p_j|C^{-1}(j)|}{F}\right)F = F - \sum_{j\in N\setminus S} p_j|C^{-1}(j)|,$$

so that

$$\sum_{i\in S} y_i^P(B) \geq F - \sum_{j\in N\setminus S} p_j|C^{-1}(j)|,$$

which proves that $y^P(B) \in O(z_B)$. ■

Two examples are enough to show that the proportional allocation rule can lie outside of a core of the two other games. The first one is Example 1 in which the core of game $u_B$ is empty. The second one is provided below.

**Example 2** Three healthcare professionals are involved in the process of recovering of a patient who needs the following chain :

$$1 \longrightarrow 3 \longrightarrow 2 \longrightarrow 2 \longrightarrow 1$$

The margins are $p_1$ = \$10, $p_2$ = \$5 and $p_3$ = \$20, and the fee $F$ = \$30. The game $w_B^C$ is given by the following table :

| $S$ | $\{1\}$ | $\{2\}$ | $\{3\}$ | $\{1,2\}$ | $\{1,3\}$ | $\{2,3\}$ | $\{1,2,3\}$ |
|---|---|---|---|---|---|---|---|
| $w_B^C(S)$ | 10 | 0 | 0 | 10 | 30 | 0 | 30 |

Concerning coalition $\{1,3\}$, the proportional allocation rule yields $y_1^P(w_B^C) + y_3^P(w_B^C) = 24 < 30 = w_B^C(\{1,3\})$. This shows that the proportional rule does not belong to the core of $w_B^C$. □

We conclude this section with a calculation of the allocation rules in the context of Example 1.

| | $Sh(v_B^C)$ | $Sh(w_B^C)$ | $Sh(u_B)$ | $Sh(z_B)$ | $x^P$ | $y^P$ |
|---|---|---|---|---|---|---|
| Practician (player 1) | 130.83 | 144.17 | 68.33 | 68.33 | 55 | 70.9 |
| Specialist (player 2) | 125.83 | 119.17 | 43.33 | 43.33 | 50 | 47.26 |
| Hospital (player 3) | 88.33 | 81.67 | 233.33 | 233.33 | 240 | 226.84 |

This table clearly underlines that the choice to take into account the chain has a significant influence on the structure of the constructed game, and in turn, on the payoffs prescribed by the Shapley value. In Example 1, player 1 is involved in two of the three first positions in the chain whereas player 3 has the highest total margin. In the two games based on the chain, player 1 gets the most important shares to the detriment of player 3. In the two other games, the situation is reversed.

## 1.5 Conclusion

Cooperative game theory allows to apprehend the problem of integrated healthcare by assuming the cooperation of the different healthcare professionals involved in the process. The exogenous chain generated by the treatment of the disease and the four different games offer the possibility to the actor in charge of the final sharing to choose between different criteria in order to apprehend a same problem, either the financial criteria for which each healthcare professional would like to be refunded with the highest amount, or the timeline process for which the earliest heathcare professionals in the process are the most refunded. There are a lot of different allocation rules and among them, we applied three allocation rules with distinct principles, an allocation rule with marginality principles (the Shapley value), an allocation rule with priority principles (the Priority rule) and a proportional allocation rule. The table below summarizes all our results on the relationship between the allocation rules and each of our four games :

| | $O(v_B^C)$ | $O(w_B^C)$ | $O(u_B)$ | $O(z_B)$ |
|---|---|---|---|---|
| $Sh$ | + | + | − | + |
| $x^P$ | + | + | − | + |
| $y^P$ | + | − | − | + |

The symbol " + " means that the allocation rule always belongs to the core of the considered game, the symbol " − " has the converse meaning. The principles underlying the three allocation rules are very different. Nonetheless, the payoffs recommanded by these allocation rules may all belong to the core as in games $v_B^C$ and $z_B$. Hence, the actor in charge of the final sharing has two degrees of flexibility : he can choose in the core any of the three allocation rules and he can choose to incorporate or not the chain of medical events into the modelling of bargaining power of the coalitions. Moreover, this actor is able to respect simultaneously stability (in the sense of the core) and some fairness criterion (in the sense of the principles behind the three allocation rules). The literature on integrated healthcare shows that patients should benefit because of the improvement of the quality of care (Rocks et al., 2020).

Future works can be envisaged by relying on alternative approaches to the problem. Our results based on the chain of medical events sometimes give excessive importance to the first positions of the chain. Mixing the two approaches, with or without the chain, is perhaps a possibility to design less extreme allocations. Therefore, we could imagine some less extreme allocations which take into account the chain of the process and the benefit at the same time. Another road could be that $F$ varies from one coalition to

another. Moreover, instead of relying on bankruptcy games, it could make sense to use the literature on bargaining with claims (see Chun and Thomson, 1992; Bossert, 1993; Herrero et al., 1999). At last, in order to model a process that is not totally sequential, one possibility is perhaps to allow several players to act at a given position in the chain. This would imply that these players do not act in a specified order. These alternatives approaches are left for future work as they go beyond the scope of this article.

# Appendix

The proof of proposition 1 relies on a lemma. First we need some definitions. Let $A$ be the difference between $F$ and the total cost of chain $C$ :

$$A = F - \sum_{c_q \in C} p_{c_q} < 0.$$

Define a function $\psi : 2^N \longmapsto \mathbb{R}$ which assigns to each $S \subseteq N$, the real number

$$\psi(S) = \sum_{c_q \in C(S)} p_{c_q}.$$

Then, for any $S \subseteq N$, $v_B^C(S)$ and $w_B^C(S)$ can be rewritten as :

$$v_B^C(S) = \max\left\{0; A + \psi(S)\right\}, \quad \text{and} \quad w_B^C(S) = \min\left\{F; \psi(S)\right\}.$$

**Lemma 1** *For any integrated healthcare problem $B$, $\psi$ is a convex game.*

**Proof.** Recall that for each $R \subseteq N$, $q_R$ is the last position in $C(R)$, and that $q(i)$ stands for the first position held by $i$ in the chain $C$.
We want to prove (1.4). So choose $S, T \subseteq N$ and $i \in N$ such that $S \subseteq T \subseteq N \backslash \{i\}$. We consider two cases :

• Suppose $q(i) > q_S$. Then $q_T$ can be smaller or greater than $q_i$ in the following possible chains :

$$q_1 \longrightarrow q_2 \longrightarrow .... \longrightarrow q_S \longrightarrow .... \longrightarrow q(i) \longrightarrow .... \longrightarrow q_T \longrightarrow ..... \longrightarrow q_N,$$

$$q_1 \longrightarrow q_2 \longrightarrow .... \longrightarrow q_S \longrightarrow .... \longrightarrow q_T \longrightarrow .... \longrightarrow q(i) \longrightarrow ..... \longrightarrow q_N.$$

In each case, the marginal contribution of an healthcare professional $i$ to coalition $S$ is null. Since $\psi$ is obviously monotonic, we obtain :

$$\psi(S) - \psi(S\backslash\{i\}) = 0 \leq \psi(T) - \psi(T\backslash\{i\}),$$

• Suppose $q(i) \leq q_S$. Then the chain looks like :

$$q_1 \longrightarrow q_2 \longrightarrow .... \longrightarrow q(i) \longrightarrow .... \longrightarrow q_S \longrightarrow .... \longrightarrow q_T \longrightarrow ..... \longrightarrow q_N,$$

where the case $q_S = q_T$ is possible. The marginal contribution of $i$ to $S$ is the sum of all margins charged between $q(i)$ and $q_S$ along the chain and the marginal contribution of $i$ to $T$ where $S \subseteq T$ is the sum of all margins charged between $q(i)$ and $q(T)$ :

$$\psi(S) - \psi(S\backslash\{i\}) = \sum_{q=1}^{q_S} p_{c_q} - \sum_{q=1}^{q_{(i)}-1} p_{c_q} = \sum_{q=q(i)}^{q_S} p_{c_q},$$

$$\psi(T) - \psi(T\backslash\{i\}) = \sum_{q=1}^{q_T} p_{c_q} - \sum_{q=1}^{q_{(i)}-1} p_{c_q} = \sum_{q=q(i)}^{q_T} p_{c_q}.$$

Furthermore, it holds that $q_S \leq q_T$ because $S \subseteq T$. This implies that :

$$\sum_{q=q(i)}^{q_S} p_{c_q} \leq \sum_{q=q(i)}^{q_T} p_{c_q} \Leftrightarrow \psi(S) - \psi(S\backslash\{i\}) \leq \psi(T) - \psi(T\backslash\{i\}),$$

as desired. ■

**Proof. (Proposition 1) Part** $(i)$ : To demonstrate that the game $v_B^C$ is convex, we adopt the proof technique in Curiel et al. (1987). For each $S \subseteq N$, we show that :

$$v_B^C(S \cup \{i\}) - v_B^C(S) \leq v_B^C(T \cup \{i\}) - v_B^C(T)$$

$$\Leftrightarrow v_B^C(S \cup \{i\}) + v_B^C(T) \leq v_B^C(T \cup \{i\}) + v_B^C(S),$$

which is equivalent to

$$\max\left\{0; F - \sum_{c_q \in C\backslash C(S\cup\{i\})} p_{c_q}\right\} + \max\left\{0; F - \sum_{c_q \in C\backslash C(T)} p_{c_q}\right\}$$
$$\leq \max\left\{0; F - \sum_{c_q \in C\backslash C(T\cup\{i\})} p_{c_q}\right\} + \max\left\{0; F - \sum_{c_q \in C\backslash C(S)} p_{c_q}\right\}.$$

Rearranging :

$$\max\left\{0; F - \sum_{c_q \in C\setminus C(T)} p_{c_q}; F - \sum_{c_q \in C\setminus C(S\cup\{i\})} p_{c_q}; 2F - \sum_{c_q \in C\setminus C(S\cup\{i\})} p_{c_q} - \sum_{c_q \in C\setminus C(T)} p_{c_q}\right\}$$
$$\leq \max\left\{0; F - \sum_{c_q \in C\setminus C(S)} p_{c_q}; F - \sum_{c_q \in C\setminus C(T\cup\{i\})} p_{c_q}; 2F - \sum_{c_q \in C\setminus C(T\cup\{i\})} p_{c_q} - \sum_{c_q \in C\setminus C(S)} p_{c_q}\right\}.$$

Using $A$ and $\psi$, we can rewrite the above inequation as :

$$\max\left\{0; A + \psi(T); A + \psi(S \cup \{i\}); 2A + \psi(S \cup \{i\}) + \psi(T)\right\}$$
$$\leq \max\left\{0; A + \psi(S); A + \psi(T \cup \{i\}); 2A + \psi(T \cup \{i\}) + \psi(S)\right\}.$$

Observe that :

— $A + \psi(T) \leq A + \psi(T \cup \{i\})$ by monotonicity of $\psi$,
— $A + \psi(S \cup \{i\}) \leq A + \psi(T \cup \{i\})$ by monotonicity of $\psi$,
— $2A + \psi(S \cup \{i\}) + \psi(T) \leq 2A + \psi(T \cup \{i\}) + \psi(S)$ by Lemma 1.

Since all terms are non-negative, the proof is complete.

**Part** $(ii)$ : We proceed with two cases to prove that :

$$w_B^C(S) - w_B^C(S\setminus\{i\}) \leq w_B^C(T) - w_B^C(T\setminus\{i\}), \quad i \in S \subseteq T.$$

• Suppose $q(i) > q_S$. Then $C(S \cup \{i\}) = C(S)$ which implies that :

$$w_B^C(S) - w_B^C(S\setminus\{i\}) = 0.$$

Furthermore, by monotonicity of $w_B^C$, we have :

$$w_B^C(T) \geq w_B^C(T\setminus\{i\}) \iff w_B^C(T) - w_B^C(T\setminus\{i\}) \geq 0,$$

as desired.

• Suppose $q(i) \leq q_S$. In this case, we have :

$$C(S\setminus\{i\}) = C(T\setminus\{i\}) = \left\{c_1, ...., c_{q_{(i)-1}}\right\}$$

This implies :

$$\psi(S\backslash\{i\}) = \psi(T\backslash\{i\}).$$

Hence

$$\min\left\{F; \psi(S\backslash\{i\})\right\} = \min\left\{F; \psi(T\backslash\{i\})\right\},$$

or equivalently $w_B^C(S\backslash\{i\}) = w_B^C(T\backslash\{i\})$. Similarly as before, by monotonicity of $w_B^C$, we get :

$$w_B^C(T) \geq w_B^C(S).$$

Thus :

$$w_B^C(T) - w_B^C(T\backslash\{i\}) \geq w_B^C(S) - w_B^C(S\backslash\{i\}).$$

Conclude that $w_B^C$ is convex. ■

# Chapitre 2

# Assessing the credibility and fairness of international corporate tax rate harmonization via cooperative game theory

This article uses the main tools of cooperative game theory, the core of a game and the Shapley value, to tackle the challenge posed by corporate tax harmonization in order to fight tax competition and profit shifting. These tools are applied to provide a counterfactual evaluation and to assess the credibility of Saez and Zucman (2019) proposal to establish a minimum rate at 25% at the G7/G20 level. Based on the empirical data of Tørsløv et al. (2020a), our main results are the following. First, at the G7 level, the more countries involved in the agreement, the more efficient it would be. Second, stability of cooperation at the G7 level can be achieved without giving up fairness consideration in the distribution of the surplus. We then extend our application to the G20 and show that these results do not hold anymore. Third, from our original methdological approach, we confirm that not only the target rate matters in the perspective of international tax cooperation, but also the numbers of participants and their current effective rates.

## 2.1 Introduction

On February 19, 2021, G7 leaders argue that they will "strive to reach a consensus-based solution on international taxation by mid-2021 within the framework of the OECD". This statement is in line with OECD/G20 project on "Base Erosion and Profit Shifting". Launched in 2013, the project results in reforms proposals based on two pillars (OECD,

2020a,b). The first aims at changing the taxation rules to tackle the challenge posed by the digitalization of the economy. The second aims at establishing a minimum tax rate to multinationals. This second pillar should "reduce the differences in effective tax rates across jurisdictions", which are "one of the main drivers of profit shifting", in order to "reduce multinationals' incentives to shift profit to low-tax jurisdictions" (OECD 2020a, p.17). Few months ago, President Biden puts forward his Made in America Tax Plan whose similar aim is to "stop profit shifting, and ensure other nations won't gain a competitive edge by becoming tax haven". Biden's administration proposed to raise corporate tax rate up to 28% in the United States and to increase the minimum tax on US corporations to 21% [1]. It also "encourages" other countries "to adopt strong minimum taxes on corporations". On June 5, 2021, G7 finance ministers announced to have reach an agreement on a minimum statutory corporate tax rate of 15%. This lead to an agreement on a Global Minimum Tax of 15% between 136 countries in october 2021. These various rates often seem to come as a rabbit out of the hat, particularly because "the appropriate level of minimum tax rate" was and remains "extremely contentious" (Hebous and Keen, 2022). Yet this paper shows that the level of rate targeted crucially matters ! Indeed, not all rate levels can ensure credible international tax cooperation.

The 2008 financial crisis had already strengthened the will of G20 members to fight profit shifting. A mean advocated in April 2009 was to put an end to "the era of bank secrecy". Johannesen and Zucman (2014) however documented that the strategy based on exchange of bank information proved rather unsuccessful. The measures taken have led to "a relocation of bank deposits between tax havens but have not triggered significant repatriation of funds" (2014, p.89). This failure explains why Saez and Zucman (2019) put forward another strategy, in line with OECD and G20 second pillar, for reforming international taxation. They advocate tax cooperation around a minimum statutory corporate tax rate at the G20 level. With two aims in mind, they propose a rate of 25%. First, such cooperation should prevent tax competition and counter the well-documented downward trend in statutory corporate tax rate (Devereux and Loretz, 2013; Keen and Konrad, 2013). Second, this minimum corporate tax rate would serve as a benchmark to recover tax deficit due to profit shifting into tax havens as soon as members of the G20 accept to police their multinationals [2]. For year 2015, Tørsløv et al. (2020b) evaluate the

1. The Biden's administration claims wanting to "create a more level playing field between domestic companies and multinationals". See "Fact Sheet : The American Jobs plan", Statements and releases, March 31, 2021, White House.

2. Devereux and Loretz (2013, p.746) distinguish three forms of competition : (*i*) competition for flows of capital, which depend on effective marginal tax rate ; (*ii*) competition for flows of firms, which depends on average tax rate ; and (*iii*) competition for flows of profits, which depends on statutory tax rates. The

effective corporate tax rate in non-OECD tax havens at 7% [3]. By comparison, the average effective corporate tax rate in OECD countries is 19%. At first glance, their proposal seems attractive both to fight against tax competition and to recover tax revenue losses due to profit shifting. First, contrary to the strategy of information exchange, this one does not require the cooperation of tax havens. Second, the mechanism advocated does not imply double taxation.

> "To understand how this could work, let's consider a concrete example. Imagine that, by shifting intangibles and manipulating intragroup transactions, the Italian automaker Fiat had managed to make $1 billion in profits in Ireland, taxed at 5%, and $1 billion in Jersey, one of the Channel Islands, taxed at 0%. There's a problem here : Fiat pays much less tax than it should ; much less, in particular, than domestic Italian businesses. We call this a tax deficit. The good news is that nothing prevents Italy from curbing this deficit itself, by collecting the taxes that tax havens choose not to levy. Concretely, Rome could tax Fiat's Irish income at 20%. It could tax its Jersey bounty at 25%. More generally, it could easily impose remedial taxes such that Fiat's effective tax rate, in each of the countries where it operates, equals 25%" (Saez and Zucman 2019, 115-116).

While stressing the administrative viability of their proposal, Saez and Zucman believe that "it is probably too optimistic to expect that all G20 countries will agree to police their own multinationals, join the club of tax collectors of last resort, and apply sanctions against tax havens" (2019, p.125). This judgment might result from the fact that neither the level of the minimum statutory tax rate they advocate nor the credibility of the agreement is discussed in their book. Assessing that credibility is the aim of this article. If Saez and Zucman advocate that the 25% statutory tax rate acts as a *minimum*, a way to assess the credibility of such an agreement is to evaluate whether G7 countries as a first step, and then G20, have incentive to harmonize their corporate tax rate, so that the 25% corporate statutory tax rate acts as a *common* rate. If it does, these countries would a fortiori have individual and collective interests to consider a 25% corporate statutory tax rate as a minimum one. After reaching an agreement on a common rate, each country

reform advocated by Saez and Zucman (2019) addresses this third form of corporate tax competition, which can be defined as horizontal competition between countries for corporate profits. For simplicity, we do not discuss the issue of vertical competition between jurisdictions of the same country.

3. There are increasing attempts to estimate profit shifting in tax havens and tax losses (Janský and Palanský, 2019). There is obviously no methodological consensus. Strong criticisms have even been raised against the methods used in these attempts (Blouin and Robinson, 2020). The data provided by Tørsløv et al. (2020a, 2020b) prove to be the most operational ones yet.

of the coalition would indeed be free to rise its own corporate statutory tax rate above 25%, while this rate would continue to serve as a benchmark to recover tax deficit in tax havens.

This paper contributes to the growing litterature on the global minimum tax rate (Janeba and Schjelderup, 2022; Hebous and Keen, 2022; Hindriks and Nishimura, 2022; Johannesen, 2022; Konrad and Thum, 2021). Its methodological contribution lies in the use of cooperative game theory as a tool to assess the credibility of such a tax harmonization. Cooperative game theory enables designing rules for allocating payoffs among coalition members. Applied to Saez and Zucman's proposal, cooperative game theory allows to evaluate the tax revenues resulting from the cooperation of all countries of the G7/G20 to test if smaller coalitions of countries have an interest to depart from the grand coalition, whose *raison d'être* is to establish a common corporate tax rate of 25% acting as a minimum to fight tax competition and as a benchmark to recover tax deficit, and, consequently, reduce the incentive of multinationals, in the futur to shifted profits. We focus on the two main classical tools of cooperative game theory, namely the core of a game (Shapley, 1955) and the Shapley value (Shapley, 1953). These tools can be used to evaluate the stability of cooperation and the fairness [4] of surplus and recovered tax deficit distribution, respectively. By surplus, we refer to the additional tax revenues that are generated by cooperation.

The aim of our work is not to assess the credibility of Saez and Zucman proposal by theoretically determining whether the equilibrium outcome of strategic interactions can lead o the cooperation on a common statutory corporate tax rate at 25%. We rather consider cooperation to be effective and examine the allocation of profits resulting from cooperation. Therefore, the reader should not expect to find in this article the study of determinants of cooperation among countries. Our modelling strategy aims at reasoning *as if* the bidding agreement were the equivalent of a social contract *à la Rousseau* between countries forming a coalition (Moulin, 2002) [5]. In this specific case, the cooperative games that we propose have been built as if G7/G20 members act as a social planner applying Saez and Zucman's reform. Reasoning as if confers a counterfactual value to our modelization, in which we hypothetically envisage tax rate harmonization in situations where some but not all G7/G20 members agree to cooperate [6]. The whole point of our

4. The fairness, in Shapley's vision, can be seen as fair treatment of all participants depending on their own marginal contribution. The player "is paid out his fair share of the value from the coalition for having joined the coalition" (Chakravarty et al., 2015)

5. Our study takes a "multilateralism" rather than a "gradualism" approach (Konrad and Thum 2021).

6. The goal of counterfactualizing, as DeMartino (2020) recently argues, is indeed "to identify alter-

framework is to consider that the target rate is an exogenous data, proposed by expert or produced by the political process, whose relevance should be studied. This is, indeed, the way the rates proposed by Saez and Zucman and the G7 members were defined. Taking the rules of the reform as given is more realistic than acting as if they were the result of individual optimization process. The 2015 data recently provided by Tørsløv, Wier et Zucman (2020a, 2020b) represent the factual, namely the individual amount of tax revenues effectively recovered by G7 countries [7]. Our cooperative games provide the counterfactual since they lead to reason as if the agreement on a common statutory corporate tax rate was implemented.

Our main results are the following. First, at the G7 level, **the more countries involved in the agreement, the more efficient it would be**. For every coalition with two or more countries of the G7 containing the United States, the agreement on a common statutory corporate tax rate of 25% generates a surplus of tax revenues. This result also applies for all coalitions with five or more countries, no matter which countries of the G7 are considered. Second, **stability of cooperation can be achieved without giving up fairness consideration in the distribution of the surplus**. Considering the Shapley value as an allocation rule, we show that the cooperation of G7 countries around a common statutory corporate tax rate of 25% belongs to the core of our game. Because of this second result, Saez and Zucman's proposal seems more credible than expected at first glance at the G7 level. This achievement is the result of a combination of a relatively high rate of 25% generating a surplus and the recovering of tax deficit. The third result confirms, from another methodological viewpoint, the thesis that **not only the target rate matters in the perspective of establishing international cooperation, but**

native worlds, where things could go very differently from what we might expect". Then, "counterfactual scenarios" could serve as "instruments for managing better in a world that we can't ultimately know, and that we can influence but never control" (2020, p.9).

7. Raw data used in the article are presented in Appendix 2.1. Data have been precisely built to "be used to quantify the tax revenues that individual countries could gain under different tax reform scenarios" (Tørsløv and al. 2020a, 4).

**also the numbers of participants and their current effective rates** [8]. At the G20 [9] level, the agreement on a common statutory corporate tax rate of 25% still generate a surplus of tax revenues. But the principle according to which the more countries involved in the agreement, the more efficient it would be, does not hold anymore. The stability of cooperation under such a common corporate tax rate cannot be reconciled with fairness consideration (in the sense of Shapley value). The Shapley value is not a core allocation because increasing the number of coalition members generates a dilution effect in the distribution of recovered tax deficits.

To the best of our knowledge, this article represents the first attempt to apply cooperative game theory, on an empirical basis, to international tax cooperation issues. Despites recognizing that international tax cooperation is "a natural response" to limit the effect of tax competition (Gresik, 2001) and that this "natural appeal" will generally be more efficient than decentralized decision-making (Keen and Konrad, 2013, p.288, p.317), the literature dealing with international corporate taxation in a context of globalization has focused on competition rather than cooperation [10] . Even when focusing on cooperation on the choice of a common tax rate, these works are embedded in the non cooperative framework (Konrad and Schjelderup, 1999; Konrad, 2009; Hindriks and Nishimura, 2022). The issue of sharing the revenues generated by such a cooperation on a common rate has not been adressed so far. In this paper, we fill in this gap, and in order to do so, we use cooperative game theory. Wilson (1999) argues that "at the international level, there do not exist strong institutions for coordinating the activities of sovereign nations" and

8. The literature on tax harmonization emphazises on the benefits to cooperate and is a good solution but only if there are no big differences between countries, especially in terms of rates (Bucovetsky, 1991) but also in terms of degree of harmonization. Indeed, total harmonization (all countries) and partial harmonization (coalition of countries) lead to different results. Conconi et al. (2008) studied these different level of cooperation within a European Union application and partial harmonization is more "sustainable" and "desirable" than total harmonization which is almost never beneficial, unless there are very strong and credible commitments from countries (Conconi et al., 2006). Partial harmonization with subgroup of countries is also advocated by Konrad and Schjelderup (1999) with "Pareto-improving" situation in which effective tax rates strategically complement each other. In general, tax harmonization fails if there are too many asymmetries between countries in terms of rates, size, population (Haufler and Lülfesmann, 2015; Kanbur and Keen, 1993) but also in terms of temporal inconsistency (Kehoe, 1989) which can create different incentives and dynamics reactions, for countries with different domestic tax policies and different national organization (centralized or federal organization).

9. If we refer to the G20, we compute our games for 16 countries only. Data for Argentina, Indonesia and Saudi Arabia are not available. Moreover, we do not consider the European Union as a single entity.

10. The main result of the theoretical literature on tax competition, under the assumption of (perfect) mobility of capital, is that small open countries tend to lower their statutory tax rate to enlarge their tax base (Zodrow and Mieszkowski, 1986; Wilson, 1986). These models provide a good approximation of the actual behavior of countries that are called tax havens. Tax havens, which are generally little countries (Dharmapala and Hines Jr, 2009), cannot attract activities requiring physical location, whereas they easily attract profits.

Kehoe (1989) also advocated the implementation of an institution that guarantees strong commitments and cooperation. Devereux and Loretz add that "tax cooperation so as to reduce corporate tax competition has proved to be difficult to implement on international basis" so that the large countries should seek to "improve attractiveness of the country as a corporate location" in order to be able to maintain relatively high tax rate and revenues (2013, p.765). Keen and Konrad explain that tax coordination "requires the absence of asymmetric information and it typically requires full commitment", so that agreements should be written "prior to any possible unilateral action by which a single player can tilt the cooperative outcome in their own favor" [11] . They conclude on this basis that tax cooperation requires "more than one can expect" in an "international context with sovereign countries" (2013, p.287). But it is precisely because cooperation seems unlikely that we propose to account for it in a counterfactual and static way rather than basing it on a model of strategic interactions. And in any case, the fact that tax coordination requires full commitment from sovereign countries is not *per se* an obstacle to its study. Indeed, cooperative game theory is a relevant starting point to help resolving the main problem identified concerning the economic analysis of international tax coordination, which is "to explain what makes commitment feasible and credible at the stage when countries commit on their timing" (Keen and Konrad 2013, p.280).

To this aim, we provide preliminaries on cooperative game theory and introduce the games modelling Saez and Zucman's scenario of reform (Section 2.2). Next we compute our games for G7 countries and presents the main results (Section 2.3). We show that even if some countries of the G7 have current statutory and effective corporate tax rate superior to 25%, this latter could act as a common rate acceptable for all G7 members because of surplus generated (Section 2.3.1). The credibility and stability of cooperation depend on the various allocation rules of the surplus of tax revenue and tax deficit recovered that could be implemented. We consider the most studied one, because of its properties in terms of efficiency and fairness, namely the Shapley value (Section 2.3.2). We then extend this application to the G20 (Section 2.4). We finally echo currents events by providing quantitive evaluations based on Biden's administration plan to raise corporate tax rate to 28% and set a minimum rate on US multinationals at 21% on the one hand and G7 announcement on a minimum rate of 15% on the other. We compute our games under these values to provide comparison with previous results (Section 2.5). In conclusion, we sum up the main results and discuss some limitations of our work that should be overcome in future works.

11. Hence the importance of the notion of fairness.

## 2.2 A cooperative game theoretic approach

In this section, we first present the framework of cooperative game theory. We then illustrate how tax cooperation can be formalized with cooperative game theory.

### 2.2.1 Preliminaries on cooperative games

A cooperative game is a pair $(N, v)$ where $N = \{1, ..., n\}$ is a finite set of $n$ players and $v$ is a characteristic function which assigns a worth $v(S)$ to each coalition $S \subseteq N$, and such that $v(\varnothing) = 0$. The worth of a coalition is the payoff that members of the coalition $S$ can secure by themselves. In other words, it represents "the best outcome that each subset of the participants ('players') can achieve being unaided" (Shapley and Shubik, 1967). With $n$ players, a cooperative game is composed of $2^n$ potential coalitions (including the empty coalition). The coalition $N$ of all players is called the grand coalition and is considered as actually formed, the others coming from counterfactual scenarios.

An *allocation* for a game $(N, v)$ is a payoff vector $x = (x_1, ..., x_n)$ which assigns a payoff $x_i \in \mathbb{R}$ to each player $i \in N$ in order to reflect her participation to game $(N, v)$. It is (Pareto) efficient if $\sum_{i \in N} x_i = v(N)$. The *core* of game $(N, v)$ is the set $C(N, v)$ of all efficient allocations $x$ such that no coalition of players $S$ gets a total payoff $\sum_{i \in S} x_i$ smaller than its worth $v(S)$, that is to say not smaller than what it can secure for its members. Formally :

$$C(N, v) = \left\{ x \in \mathbb{R}^N : \sum_{i \in S} x_i \geq v(S), S \subseteq N, \sum_{i \in N} x_i = v(N) \right\}.$$

The core can be empty. If $x$ is a core allocation, then it is in the interest of no coalition of players to split from the grand coalition. Hence the grand coalition can be considered as stable if its members are paid according to such a core allocation. Stability is sometimes incompatible with fairness considerations.

The Shapley value of a game is traditionally seen as a fair allocation rule. It is efficient, additive (the Shapley value in the sum of 2 games is the sum of the Shapley values in these 2 games), assigns a null payoff to any player whose marginal contributions to coalitions are null, and assigns an equal payoff to players characterized by identical marginal contributions to coalitions. More specifically, the Shapley value is uniquely characterized by these four properties or axioms, and assigns to a player $i$ in a game $(N, v)$ a payoff $Sh_i(N, v)$ which is a weighted average of all her marginal contributions. If we denote the

cardinal of coalition $S$ by $s = |S|$, then the Shapley value is formally :

$$Sh_i(N,v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} \big(v(S \cup \{i\}) - v(S)\big), \quad \forall i \in N.$$

### 2.2.2 The games induced by tax harmonization

Currently, every country forming the G7 discretionary determines its statutory corporate tax rate. Mainly because of profit shifting, there is a divergence between the tax revenues effectively collected and the application of statutory rates to the reported corporate profits. For this reason, Tørsløv, Wier and Zucman (2020) calculated an effective corporate tax rate for each country that differs in various extents from statutory rate (Appendix 2.1). With tax cooperation as we modelled it, every country abandons either its discretionary power to set its own statutory rate (common rate) or to set it without supranational constraint (minimum rate). If tax cooperation is implemented, one stake concerns the terms of the agreement related to the sharing of the surplus generated by cooperation. Another concerns the tax deficit that could be recovered. Such a problem of tax cooperation could be described by the sextuplet $(N, (\pi_i, t_i, \pi_i^L)_{i \in N}, t, \bar{t})$ where :

— $N = \{1, ..., n\}$ represents the countries of the G7 or G20.
— $\pi_i$ represents the corporate reported profits in country $i$, for each $i \in N$.
— $t_i$ represents the effective corporate tax rate in country $i$, for each $i \in N$.
— $\pi_i^L$ represents the profit shifts to tax havens from country $i$, for each $i \in N$.
— $t$ represents a common corporate statutory tax rate — in our case 25%.
— $\bar{t}$ represents the average statutory corporate tax rate in tax havens — in our case, 8.77%.

It should be clear that $t_i$ can be more or less than $t$ for a given country $i$ and that $\bar{t}$ is less than $t$. We propose two different games to model the first part of the economic problem posed by Saez and Zucman's proposal. The first one is a constrained game whereas the second opens the possibility of cooperation to recover tax deficit but without applying the common rate if unfavorable for some countries. The first game $(N, v)$ assigns to each one-country coalition its current individual tax revenue, the corporate reported profits in one country multiplied by its effective tax rate (the factual). For each coalition $S$ with two or more countries, it assigns the sum of the corporate reported profits in each country of the coalition multiplied by the common corporate statutory taxe rate (counterfactuals).

Formally, the worth $v(S)$ is :

$$v(S) = \begin{cases} \pi_i t_i & \text{if } s = 1, \\ \left( \sum_{i \in S} \pi_i \right) t & \text{if } s > 1. \end{cases}$$

This worth $v(S)$ is the tax revenue that $S$ could achieve if the profits reported in its members are taxed at the common rate $t$, except for singletons ($s = 1$) for which the revenue is calculated from the current domestic effective rate. In other words, we assume that as soon as two countries agreed to form a coalition, they have to adopt the common rate $t$. Since we consider the targeted rate as exogenously determined, for the reasons mentionned above, we do not consider the possibility that each coalition has its own targeted rate. As an alternative to game $(N, v)$, we introduce a game $(N, v^*)$. It consists in considering the maximum payoffs (tax revenues) that each coalition could pretend to with or without implementing the common rate $t$. Singletons are treated as in $(N, v)$. For coalitions $S$ with two or more countries, they apply the common rate $t$ only if it leads to a total revenue at least as large as what its members would obtain from their respective current domestic rate. Formally, the worth $v^*(S)$ is :

$$v^*(S) = \begin{cases} \pi_i t_i & \text{if } s = 1, \\ \max \left\{ \left( \sum_{i \in S} \pi_i \right) t; \sum_{i \in S} \pi_i t_i \right\} & \text{if } s > 1. \end{cases}$$

The game $(N, v^*)$ is an interesting complement to $(N, v)$ for two reasons. First, it better takes the rationality of coalitions of country into account. By introducing a maximization, game $(N, v^*)$ models a situation where the total revenue of coalition of countries cannot decrease if a new country joins the coalition. In other words, $(N, v^*)$ is monotonic game (see Appendix 2.4 in which extra properties are provided). Thus, the meaning of cooperation depends on the value attributed to $t$. If it is relatively low, countries cooperate without setting a common rate but only to define a benchmark rate to recover tax deficit. If it is high enough, games $(N, v)$ and $(N, v^*)$ coincide. Countries cooperate by setting a common rate that also serve as a benchmark rate to recover tax deficit. Second, the difference between $v^*(S)$ and $v(S)$ immediately provides the tax revenue losses generated if $S$ accepts the common statutory corporate tax rate $t$ (Appendix 2.2).

As a reminder, one of the two main aims of the reform advocated by Saez and Zucman is to provide a benchmark to recover profit losses due to profit shifting in tax havens. This recovering of tax deficit is modeled by a third game $(N, w)$. For singletons, the worth $w(S)$

| G7 | USA | JAP | GER | FRA | UK | ITA | CAN |
|---|---|---|---|---|---|---|---|
| $t_i$(effective corporate tax rate in %) | 0.21 | 0.26 | 0.11 | 0.27 | 0.17 | 0.18 | 0.35 |
| $\pi_i$(corporate reported profits in billion \$) | 1889 | 634 | 553 | 188 | 425 | 212 | 143 |
| $\pi_i^L$ (profit shift to tax havens in billion \$) | 142 | 28 | 55 | 32 | 62 | 23 | 17 |

FIGURE 2.1 – The data for G7 (2015)

is equal to zero (the factual). Without a coalition of countries signing a bidding agreement to fight against profit shifting into tax haven, it is unrealistic that a country manage to police its multinationals [12]. We apply the same reasoning for all coalitions except the grand one. Indeed, any country could have incentive to free ride. If for instance only six of the seven G7 countries agree on policing their multinationals using the common rate $t$ as a benchmark, the free-riding country would potentially attract flow of firms, capitals and profits. That is why unanimity is required for a credible recovering of tax deficit, which translates into a game such that only the worth of the grand coalition is different from zero. The latter amount is equal to the sum of profit losses of each G7 countries multiplied by the difference between the common statutory corporate tax rate $t$ and the average tax rate in fiscal havens $\bar{t}$. Formally, the worth $w(S)$ is :

$$w(S) = \begin{cases} 0 & \text{if } s < n, \\ \sum\limits_{i \in N} \pi_i^L (t - \bar{t}) & \text{otherwise.} \end{cases}$$

## 2.3 Application to G7

### 2.3.1 Computation

As a first step, we calculate the worth $v(S)$, $v^*(S)$, and $w(S)$ for the G7 case, that is to say the tax revenues of the 127 potential coalitions [13] . The results are presented in Appendix 2.2. To do that, we only needed the empirical data of Figure 2.1 below, extracted from Tørsløv and al. (2020b), and to set $t = 0.25$.

These raw data are worth a comment. The corporate profits of firms in the United States represents 46.7% of corporate profits reported by corporations of the G7 and its current effective corporate tax rate is 21%. That explains a first important result. For every coalitions with two or more countries containing the United States, the agreement

12. If the French government only decide to police its multinationals while other governments of the G7 do not, French population would be prone to criticize its government for penalizing its multinationals.

13. The worth of the empty coalition is zero.

on a common statutory corporate tax rate of 25% would generate a surplus, that is to say more tax revenues compared to the sum of individual current tax revenues of countries participating in the coalition. This counterfactual assessment leads to conclude that the government of the United States could assume a leadership in the perspective of a real agreement.

On the contrary, Japan, France and Canada have current effective corporate tax rates above the common statutory rate of 25%. That explains the second important result. Coalitions experiencing a loss under the bidding agreement, loss quantified by column $v^*(S) - v(S)$ in Appendix 2.2, always contains some of these countries. For instance, if Canada, France, and Japan cooperate (the coalition nº 50 in Appendix 2.2), the loss of tax revenues would amount to $24.4 billion of dollars. We can however note that among the 120 coalitions with two or more countries, only 7 generate a loss of tax revenues. We also observe that no coalition of five or more countries, no matter which countries are considered, experiences a loss under the bidding agreement on a common rate at 25%. This is the first main result.

**Result 1** *The more G7 countries involved in this agreement, the more efficient it would be.*

Another result is that the grand coalition of all G7 countries would generate, under the bidding agreement, an amount of tax revenues equal to $1011 billion. Compared to what they currently generate separately (the sum of the worth of singletons), the agreement generate a surplus that amounts to $177.42 billion. The existence of such a surplus is a necessary condition, though not a sufficient one, to conceptualize a stable allocation of payoffs between members of the grand coalition. Another information is noteworthy. We calculated that the common statutory corporate tax rate should be at least 20% to ensure that the grand coalition generates a surplus compared to the sum of current individual tax revenues. This means that the profits generating by the participation of the United States, whose current effective rate is 21%, are not necessary to generate a surplus. In light of this result, the rate of 15% advocated by G7 ministers of finance, yet only as a benchmark to recover tax deficit, is not a credible option as a common rate. If 136 countries agreed on a global minimum rate of 15% in october 2021, it is only because being low, it did not prevent tax competition nor eliminate any incentive of multinationals to shift profit abroad. Such a low bound might even generate a race to that bottom (Kiss, 2012).

We also compute the game $(N, w)$ in which we assume that cooperation between all G7 members is required to recover tax deficit and prevents free riding. The amount of tax deficit recovered if the members of the grand coalition police their multinationals is

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| | $v(\{i\}) = v^*(\{i\})$ | $Sh(N, v)$ | $Sh(N, v^*)$ | $Sh(N, w)$ | $Sh(N, v^* + w)$ |
| USA | 369.69 | 463.88 | 463.09 | 8.31 | 471.39 |
| JAP | 164.84 | 163.78 | 164.67 | 8.31 | 172.98 |
| GER | 60.83 | 129.57 | 128.78 | 8.31 | 137.08 |
| FRA | 50.76 | 51.85 | 52.61 | 8.31 | 60.92 |
| UK | 72.25 | 104.81 | 104.01 | 8.31 | 112.32 |
| ITA | 38.16 | 54.75 | 54.27 | 8.31 | 62.57 |
| CAN | 50.05 | 42.36 | 43.57 | 8.31 | 51.88 |
| Total | 833.58 | 1011 | 1011 | 58.15 | 1069.15 |

FIGURE 2.2 – The Shapley value for G7 (in billion $)

$58.15 billion. Almost 40% comes from US multinationals. This second surplus is also taken into account in our work on the Shapley value as an allocation rule to share the whole surplus of tax cooperation.

### 2.3.2 The Shapley value

We computed the Shapley value of game $(N, v)$ for $t = 25\%$ (Figure 2.2, Column 2). The resulting allocation does not belong to the core of the game since it assigns less to Japan and Canada than their worth as singletons. The same applies to game $(N, v^*)$ (Figure 2.2, Column 3). On the contrary, the United states, because of hosting a large share of corporate profits reported in G20 countries, and Germany, because of a current effective rate (11%) far below the common statutory corporate rate (25%), are the main beneficiaries if this allocation rule is used. From the case of Germany, we observe that, in its application to international fiscal cooperation, the Shapley value benefits more countries which are currently the more involved in tax competition. In other words, it provides these countries substantial incentives in favor of cooperation rather than competition. But if the Shapley value has to be used as an allocation rule in tax cooperation, it should be applied to both the surplus and the recovered tax deficit.

The Shapley value, as an allocation rule, does not ensure the stability of the cooperation between the G7 countries for both games $(N, v)$ and $(N, v^*)$. Between $Sh(N, v)$ and $Sh(N, v^*)$, the difference is tiny, but not marginal. The Shapley value of $(N, v^*)$ attributes a higher allocation for countries having current effective corporate tax rate higher than 25%. The explanation lies in considering the maximum payoffs that each coalition could pretend with or without implementing the common rate in game $(N, v^*)$. In the rest of the

article, we consider only the Shapley value of this latter, assuming that it provides a better benchmark. For each country $i$, the difference between $Sh_i(N, v^*) - v^*(\{i\})$ evaluates the gain or loss between the payoffs assigned by the Shapley value and the current effective tax revenues of country $i$. The payoff assigns to Canada and Japan should respectively be \$6.48 and \$0.17 billion higher to just equal their current domestic tax revenues.

Since an agreement between G7 countries would foster the recovering of their tax deficit, we computed the Shapley value of the game $(N, w)$. With an average tax rate of 8.77% in tax havens and a common statutory corporate tax rate of 25%, we previously showed that G7 countries could recovered \$58.15 billion in tax deficit. Since we assume that such a recovering is credible only if all G7 countries would be willing to cooperate, the Shapley value of the game $(N, w)$ is equivalent to an equal division of the tax deficit recovered. It assigns \$8,31 billion to each G7 country (Figure 2.2, Column 4). From the additivity of the Shapley value, we then deduced $Sh_i(N, v^* + w)$ (Figure 2.2, Column 5). If we compare the amount assigned by this Shapley value to the individual worth of country in game $(N, v^* + w)$, which is equal to $v^*(\{i\})$ since $w(\{i\})$ is equal to zero, every country receives more than its current tax revenues. Furthermore, we compute that the Shapley values $(N, v^* + w)$ belongs to the core of games $(N, v^* + w)$ for $t = 0.25$. This is the second main result.

**Result 2** *Stability of cooperation between G7 countries can be achieved without giving up fairness consideration in the distribution of the surplus.*

The Shapley value of game $(N, w)$, because of being a symmetric game, is equivalent to an equal division rule. This later is also a common allocation rule on cooperative game theory (van den Brink, 2007). Contrary to the Shapley value, it does not consider the marginal contributions of countries to put the emphasis on solidarity between members of a coalition. One might wonder why the United States, who would contribute to recover \$23 of the \$58 billon of tax deficit, would accept such an allocation rule (Figure 2.3). Some arguments could be put forward. First and foremost, there is a theoretical argument, namely that the fight against profit shifting in tax havens requires the collaboration of all members of the grand coalition. Each country implementing the common rate as a benchmark to recover losses due to profit shifting would benefit each members of the coalition. An equal division of tax deficit is moreover prone to foster both cooperation and commitment. Second, the potential feeling in the US to be aggrieved in the short run could be compensated by understanding that, as soon as the playing field has been leveled in terms of statutory corporate tax rate, it might benefit in the longer run its greater ability to attract flows of capital and firms. Third, beyond a principle of cooperation, the

| | 1 | 2 |
|---|---|---|
| | $Sh_i(N, w)$ | Profit recovered by country $i$ |
| USA | 8.31 | 23.05 |
| JAP | 8.31 | 4.52 |
| GER | 8.31 | 8.91 |
| FRA | 8.31 | 5.21 |
| UK | 8.31 | 9.98 |
| ITA | 8.31 | 3.65 |
| CAN | 8.31 | 2.80 |
| Total | 58.15 | 58.15 |

FIGURE 2.3 – Allocation of profit recovered (in billion $)

equal division appear, in a normative perspective, relevant in international context since it echoes the principle that the voice of one sovereign country equal another.

## 2.4 Application to G20

In line with Saez et Zucman's proposal, we extend our application at the G20 level. Since data are not available for Argentina, Indonesia and Saudi Arabia (Appendix 2.1) and since we do not consider the European Union as a single entity to avoid duplication with its members, we take 16 countries into account. We computed our games with $t$ equal to 25%, which is pratically possible through a computer program since there are $2^{16} - 1$ non empty coalition. Results are presented in Figure 2.4.

For a common statutory corporate tax rate of 25%, the grand coalition of the G16 countries would generate, under the bidding agreement, an amount of tax revenues equal to $2023.58 billion. Compared to what they currently generate, the sum of the worth of singletons, the agreement would generate a first surplus of $474.08 billion. However, tax deficit recovered, contrary to what happened at the G7 level, does not ensure stability of the cooperation if the Shapley value is used as an allocation rule. $Sh_i(N, v^* + w)$ is less than $(v^* + w)(\{i\})$ for two countries, namely Australia and Canada. The increase from 7 to 16 countries yields a dilution effect in the distribution of recovered tax deficits.

We also computed our games for $t$ equal to 21% and 28%, namely the values mentioned in Biden's tax plan (Appendix 2.3). For $t$ equal 28%, the payoff attributed by the Shapley value to Canada is still inferior to its worth as a singleton. Ensuring a stable cooperation

| Countries | $v^*(\{i\})$ | $Sh(N, v^*)$ | $Sh(N, w)$ | $Sh(N, v^* + w)$ | $Sh(N, v^* + w) - (v^* + w)(\{i\})$ |
|---|---|---|---|---|---|
| Australia | 53.7 | 47.64 | 4.9 | 52.54 | -1.16 |
| Canada | 50.05 | 39.16 | 4.9 | 44.06 | -5.99 |
| France | 50.76 | 49.44 | 4.9 | 54.34 | 3.58 |
| Germany | 60.83 | 134.93 | 4.9 | 139.83 | 79 |
| Italy | 38.16 | 53.88 | 4.9 | 58.78 | 20.62 |
| Japan | 164.84 | 161.17 | 4.9 | 166.07 | 1.23 |
| Korea | 44.64 | 62.7 | 4.9 | 67.6 | 22.96 |
| Mexico | 39 | 80.28 | 4.9 | 85.18 | 46.18 |
| Turkey | 12.78 | 52.39 | 4.9 | 57.29 | 44.51 |
| UK | 72.25 | 105.83 | 4.9 | 110.73 | 38.48 |
| USA | 396.69 | 469.05 | 4.9 | 473.95 | 77.26 |
| Brazil | 54.8 | 69.5 | 4.9 | 74.4 | 19.6 |
| China | 413.8 | 512.2 | 4.9 | 517.1 | 103.3 |
| India | 37.6 | 92.08 | 4.9 | 96.98 | 59.38 |
| Russia | 40.6 | 72.22 | 4.9 | 77.12 | 36.52 |
| South Africa | 19 | 21.11 | 4.9 | 26.01 | 7.01 |
| Total | 1549.5 | 2023.58 | 78.4 | 2101.98 | |

FIGURE 2.4 – The Shapley value for G20 (in billion $)

at the G16 level around a common statutory corporate tax rate with the Shapley value as an allocation rule does not ensure cooperation for all statutory corporate tax rate under 28%. But running our program, we did not find any value of $t$, even higher than 28%, such that the corresponding Shapley value is a core allocation at the G16 level. The main explanation lies in the fact that some G20 countries, in particular Germany, Turkey and India, have current effective corporate tax rate so low, respectively 11%, 6% and 10%, that their marginal contributions in case of an agreement on a common rate would mechanically be relatively high, as illustrated by the differences between column $v^*(\{i\})$ and $Sh_i(N, v^*)$ in Figure 2.4. This lead to the third main result, which is consistent with the literature based on non-cooperative game theory.

**Result 3** *Not only the target rate matters in the perspective of establishing international cooperation, but also the numbers of participants and their current effective rates.*

Even if the Shapley value is not a core allocation at the G16 level, the core is non-empty if the common tax rate is high enough. This is a consequence of a general result provided in Appendix 2.4.

## 2.5 Tax rates matters !

The results at the G20 level invites to reconsiderate the case of the G7 for other value attributed to $t$. We computed our games for a common statutory corporate tax rate $t = 0.28$ and $0.21$ (Figure 2.5). In the scenario with $t = 0.28$, the surplus generated by the grand coalition of G7 members compared to the sum of individual effective tax revenues would amount to \$298.74 billion. The tax deficit that could be recovered in the case of 28% acting as a benchmark would amount to \$69 billion instead of \$58 billion at 25%. As an allocation rule that is judged both efficient and fair, we goes on computing the Shapley value for our game $(N, v^* + w)$ for $t$ equals 28%. Then we quantify the global tax revenues generated by a potential agreement, by making the difference, for each G7 country, between the payoff assigned by the Shapley value and its worth as a singleton. An agreement is credible, in the sense that the Shapley value of $(N, v^* + w)$ belong to its core.

Since President Biden's plan is however calibrated on a minimum tax on US multinationals equal to 21%, such a rate seems politically more credible in a near future. That is why we compute our games for a common statutory corporate tax rate $t = 0.21\%$. In this scenario, the surplus generated by the grand coalition would amount to \$15.66 billion. The tax deficit that could be recovered in the case of 21% acting as a benchmark would amount to \$43 billion. But applying the Shapley value as an allocation rule, Japan and Canada would respectively loose \$9 and \$3 billion of tax revenues compared to their current situation. As a consequence, the Shapley value does not belong to the core of the game $(N, v^* + w)$ for $t = 21\%$ — whereas it does for $t = 25\%$ and 28%. These differences reminds us of a simple fact. The targeted rate in the perspective of reaching a credible international cooperation to fight profit shifting cannot be chosen randomly. The rate critically matters !

On June 5, 2021, G7 finance ministers announced to have reached an agreement on a minimum statutory corporate tax rate of 15%. Such a low rate would not ensure the stability of the coalition if considered as a common statutory corporate tax rate. The preceding insights thus lead to conclude that its aims could only be to recover tax deficits, but on an individual basis, but in no way to fight tax competition. To go on providing quantitive evaluation, Figure 2.6 summarizes the amount of tax deficit that would be recovered by each G7 countries for the different rates mentioned above as well as the value in case of an egalitarian sharing. With a minimum corporate tax rate on 15%, G7 countries would recover \$22.31 billions of dollars in 2015.

| | $Sh_i(N, v^* + w) - (v^* + w)(\{i\}))$ | | |
|---|---|---|---|
| $t$ | 21% | 25% | 28% |
| USA | 8.13 | 74.70 | 127.07 |
| JAP | -9.06 | 8.14 | 27.44 |
| GER | 33.05 | 76.25 | 95.22 |
| FRA | 1.83 | 10.16 | 18.71 |
| UK | 17.17 | 40.07 | 55.83 |
| ITA | 11.47 | 24.41 | 34.54 |
| CAN | -3.11 | 1.83 | 8.80 |
| Total | 59.48 | 235.57 | 367.62 |

FIGURE 2.5 – Indiviudal tax revenues surplus (in billions of $ )

| Countries | 15% | 21% | 25% | 28% |
|---|---|---|---|---|
| USA | 8.85 | 17.37 | 23.05 | 27.32 |
| JAP | 1.73 | 3.41 | 4.52 | 5.36 |
| GER | 3.42 | 6.71 | 8.91 | 10.56 |
| FRA | 2 | 3.92 | 5.20 | 6.17 |
| UK | 3.83 | 7.52 | 9.98 | 11.82 |
| ITA | 1.41 | 2.78 | 3.68 | 4.37 |
| CAN | 1.07 | 2.11 | 2.80 | 3.31 |
| Total | 22.31 | 43.82 | 58.14 | 68.91 |
| Equalitarian sharing | 3.19 | 6.26 | 8.31 | 9.84 |

FIGURE 2.6 – Tax deficit recovered by each country (in billions of $)

## 2.6 Conclusion

At the G7 level, the main results of this exploratory study are the following ones. First, for every coalitions with two or more countries containing the United States, the agreement on a common statutory corporate tax rate of 25% yields a surplus of tax revenues. Second, no coalition of five or more countries, no matter which countries are considered, experiences a loss under such a bidding agreement. Third, if the Shapley value is considered as an allocation rule, the cooperation of G7 countries around a common statutory corporate tax rate of 25% is stable as soon as tax deficits recovered are incorporated. These results have been computed only for the year 2015 (Tørsløv and al. 2020). Thus, the empirical next step is to replicate our approach for other years and for other datasets as soon as more such precise data would be available. If Saez and Zucman's proposal is credible at the G7 level, it is not at the G20 level. In this case, the stability of cooperation under a common corporate tax rate is no possible under the Shapley value. The two main explanations are the extreme divergencies between effective corporate tax rates and the dilution effect in the distribution of recovered tax deficits. A theoretical next step of our research will be to reflect on the Owen value (Owen, 1977) as an alternative allocation rule. It is a generalization of the Shapley value that takes into account a pre-existing organization of countries into mutually disjoint sub-coalitions. These sub-coalitions would aim to reflect proximities between some countries of the G20, such as the proximities of their current effective corporate tax rate as well as their commercial or geopolitical affinities. Furthermore, other questions in the case of a bidding agreement and full committment are interesting to investigate. An example is the question of how to use the surplus generated by the grand coalition. It could be used for instance to finance a supranational agency in charge of establishing accounting standards and recovering tax deficit. Indeed, tax cooperation would require the establishment of common norms to calculate multinationals profits and effective tax rate Vicente (2021) and ressources to enforce the recovering of tax deficit (Hindriks et Nishimura, 2022).

To conclude, we would like to comment on some points regarding our methodology. A challenge addressed to our modelization is that the implementation of tax cooperation on an agreement on a statutory corporate rate tax would have dynamic effects that are not apparent in our results. In fact, our applications provide a static evaluation of the consequences of an agreement. But since this particular agreement aimed at leveling the playing field and providing a benchmark in order to recover profit losses due to profit shifting, there are good reasons to believe that profit shifting generated by this

agreement will be in favor of G7 countries and at the expenses of tax havens. Indeed, if the reform is implemented, knowing that its aim is to provide a benchmark for policing multinationals, it will eliminate the discrepancy in corporate tax rate that generates profit shifting. Studying such dynamic effects could be done by providing microeconomic foundations to the framework of our cooperative games, i.e introducing a non cooperative game theory model of tax competition, in which the payoff function of each country of the coalition would rely on the allocation rule adopted to share profits under cooperation.

# Appendix

**Appendix 2.1 : Data 2015 for the G20 countries (in billion $)**

| Countries | Effective corporate taxe rate ($t_i$) | Reported corporate profit ($\pi_i$) | Total profit shift to tax haven ($\pi_i^L$) |
|---|---|---|---|
| Australia | 30% | 179 | 12.00 |
| Canada | 35% | 143 | 17.23 |
| France | 27% | 188 | 32.08 |
| Germany | 11% | 553 | 54.90 |
| Italy | 18% | 212 | 22.70 |
| Japan | 26% | 634 | 27.85 |
| Korea | 18% | 248 | 4.44 |
| Mexico | 12% | 325 | 12.12 |
| Turkey | 6% | 213 | 4.61 |
| UK | 17% | 425 | 61.50 |
| USA | 21% | 1889 | 142.04 |
| Brazil | 20% | 274 | 13.24 |
| China | 20% | 2069 | 54.64 |
| India | 10% | 376 | 8.75 |
| Russia | 14% | 290 | 11.35 |
| South Africa | 25% | 76 | 3.83 |
| Argentina | No data | No data | No data |
| Saudi Arabia | No data | No data | No data |
| Indonesia | No data | No data | No data |
| Source : Tørsløv et al. (2020b) | Appendix A.6 | Appendix A.6 | Appendix C.4 |

**Appendix 2.2 : Results of $(N, v)$, $(N, v^*)$ for G7 and $t = 25\%$**

| Number | Coalition $S$ | $v(S)$ | $v^*(S)$ | $v^*(S) - v(S)$ |
|---|---|---|---|---|
| 1 | {USA} | 396.69 | 396.69 | 0 |
| 2 | {JAP} | 164.84 | 164.84 | 0 |
| 3 | {GER} | 60.83 | 60.83 | 0 |
| 4 | {FRA} | 50.76 | 50.76 | 0 |
| 5 | {UK} | 72.25 | 72.25 | 0 |
| 6 | {ITA} | 38.16 | 38.16 | 0 |
| 7 | {CAN} | 50.05 | 50.05 | 0 |
| 8 | {USA,JAP} | 630.75 | 630.75 | 0 |
| 9 | {USA,GER} | 610.5 | 610.5 | 0 |
| 10 | {USA,FRA} | 519.25 | 519.25 | 0 |
| 11 | {USA,UK} | 578.5 | 578.5 | 0 |
| 12 | {USA,ITA} | 525.25 | 525.25 | 0 |
| 13 | {USA,CAN} | 508 | 508 | 0 |
| 14 | {JAP,GER} | 296.75 | 296.75 | 0 |
| 15 | {JAP,FRA} | 205.5 | 215.6 | 10.1 |
| 16 | {JAP,UK} | 264.75 | 264.75 | 0 |
| 17 | {JAP,ITA} | 211.5 | 211.5 | 0 |
| 18 | {JAP,CAN} | 194.25 | 214.89 | 20.64 |
| 19 | {GER,FRA} | 185.25 | 185.25 | 0 |
| 20 | {GER,UK} | 244.5 | 244.5 | 0 |
| 21 | {GER,ITA} | 191.25 | 191.25 | 0 |
| 22 | {GER,CAN} | 174 | 174 | 0 |
| 23 | {FRA,UK} | 153.25 | 153.25 | 0 |
| 24 | {FRA,ITA} | 100 | 100 | 0 |
| 25 | {FRA,CAN} | 82.75 | 100.81 | 18.06 |
| 26 | {UK,ITA} | 159.25 | 159.25 | 0 |
| 27 | {UK,CAN} | 142 | 142 | 0 |
| 28 | {ITA,CAN} | 88.75 | 88.75 | 0 |
| 29 | {USA,JAP,GER} | 769 | 769 | 0 |
| 30 | {USA,JAP,FRA} | 677.75 | 677.75 | 0 |

| Number | Coalition $S$ | $v(S)$ | $v^*(S)$ | $v^*(S) - v(S)$ |
|---|---|---|---|---|
| 31 | {USA,JAP,UK} | 737 | 737 | 0 |
| 32 | {USA,JAP,ITA} | 683.75 | 683.75 | 0 |
| 33 | {USA,JAP,CAN} | 666.5 | 666.5 | 0 |
| 34 | {USA,GER,FRA} | 657.5 | 657.5 | 0 |
| 35 | {USA,GER,UK} | 716.75 | 716.75 | 0 |
| 36 | {USA,GER,ITA} | 663.5 | 663.5 | 0 |
| 37 | {USA,GER,CAN} | 646.25 | 646.25 | 0 |
| 38 | {USA,FRA,UK} | 625.5 | 625.5 | 0 |
| 39 | {USA,FRA,ITA} | 572.25 | 572.25 | 0 |
| 40 | {USA,FRA,CAN} | 555 | 555 | 0 |
| 41 | {USA,UK,ITA} | 631.5 | 631.5 | 0 |
| 42 | {USA,UK,CAN} | 614.25 | 614.25 | 0 |
| 43 | {USA,ITA,CAN} | 561 | 561 | 0 |
| 44 | {JAP,GER,FRA} | 343.75 | 343.75 | 0 |
| 45 | {JAP,GER,UK} | 403 | 403 | 0 |
| 46 | {JAP,GER,ITA} | 349.75 | 349.75 | 0 |
| 47 | {JAP,GER,CAN} | 332.5 | 332.5 | 0 |
| 48 | {JAP,FRA,UK} | 311.75 | 311.75 | 0 |
| 49 | {JAP,FRA,ITA} | 258.5 | 258.5 | 0 |
| 50 | {JAP,FRA,CAN} | 241.25 | 265.65 | 24.4 |
| 51 | {JAP,UK,ITA} | 317.75 | 317.75 | 0 |
| 52 | {JAP,UK,CAN} | 300.5 | 300.5 | 0 |
| 53 | {JAP,ITA,CAN} | 247.25 | 253.05 | 5.8 |
| 54 | {GER,FRA,UK} | 291.5 | 291.5 | 0 |
| 55 | {GER,FRA,ITA} | 238.25 | 238.25 | 0 |
| 56 | {GER,FRA,CAN} | 221 | 221 | 0 |
| 57 | {GER,UK,ITA} | 297.5 | 297.5 | 0 |
| 58 | {GER,UK,CAN} | 280.25 | 280.25 | 0 |
| 59 | {GER,ITA,CAN} | 227 | 227 | 0 |
| 60 | {FRA,UK,ITA} | 206.25 | 206.25 | 0 |
| 61 | {FRA,UK,CAN} | 189 | 189 | 0 |
| 62 | {FRA,ITA,CAN} | 135.75 | 138.97 | 3.22 |
| 63 | {UK,ITA,CAN} | 195 | 195 | 0 |

| Number | Coalition $S$ | $v(S)$ | $v^*(S)$ | $v^*(S) - v(S)$ |
|---|---|---|---|---|
| 64 | {USA,JAP,GER,FRA} | 816 | 816 | 0 |
| 65 | {USA,JAP,GER,UK} | 875.25 | 875.25 | 0 |
| 66 | {USA,JAP,GER,ITA} | 822 | 822 | 0 |
| 67 | {USA,JAP,GER,CAN} | 804.75 | 804.75 | 0 |
| 68 | {USA,JAP,FRA,UK} | 784 | 784 | 0 |
| 69 | {USA,JAP,FRA,ITA} | 730.75 | 730.75 | 0 |
| 70 | {USA,JAP,FRA,CAN} | 713.5 | 713.5 | 0 |
| 71 | {USA,JAP,UK,ITA} | 790 | 790 | 0 |
| 72 | {USA,JAP,UK,CAN} | 772.75 | 772.75 | 0 |
| 73 | {USA,JAP,ITA,CAN} | 719.5 | 719.5 | 0 |
| 74 | {USA,GER,FRA,UK} | 763.75 | 763.75 | 0 |
| 75 | {USA,GER,FRA,ITA} | 710.5 | 710.5 | 0 |
| 76 | {USA,GER,FRA,CAN} | 693.25 | 693.25 | 0 |
| 77 | {USA,GER,UK,ITA} | 769.75 | 769.75 | 0 |
| 78 | {USA,GER,UK,CAN} | 752.5 | 752.5 | 0 |
| 79 | {USA,GER,ITA,CAN} | 699.25 | 699.25 | 0 |
| 80 | {USA,FRA,UK,ITA} | 678.5 | 678.5 | 0 |
| 81 | {USA,FRA,UK,CAN} | 661.25 | 661.25 | 0 |
| 82 | {USA,FRA,ITA,CAN} | 608 | 608 | 0 |
| 83 | {USA,UK,ITA,CAN} | 667.25 | 667.25 | 0 |
| 84 | {JAP,GER,FRA,UK} | 450 | 450 | 0 |
| 85 | {JAP,GER,FRA,ITA} | 396.75 | 396.75 | 0 |
| 86 | {JAP,GER,FRA,CAN} | 379.5 | 379.5 | 0 |
| 87 | {JAP,GER,UK,ITA} | 456 | 456 | 0 |
| 88 | {JAP,GER,UK,CAN} | 438.75 | 438.75 | 0 |
| 89 | {JAP,GER,ITA,CAN} | 385.5 | 385.5 | 0 |
| 90 | {JAP,FRA,UK,ITA} | 364.75 | 364.75 | 0 |
| 91 | {JAP,FRA,UK,CAN} | 347.5 | 347.5 | 0 |
| 92 | {JAP,FRA,ITA,CAN} | 294.25 | 303.81 | 9.56 |
| 93 | {JAP,UK,ITA,CAN} | 353.5 | 353.5 | 0 |
| 94 | {GER,FRA,UK,ITA} | 344.5 | 344.5 | 0 |
| 95 | {GER,FRA,UK,CAN} | 327.25 | 327.25 | 0 |
| 96 | {GER,FRA,ITA,CAN} | 274 | 274 | 0 |

| Number | Coalition $S$ | $v(S)$ | $v^*(S)$ | $v^*(S) - v(S)$ |
|---|---|---|---|---|
| 97 | {GER,UK,ITA,CAN} | 333.25 | 333.25 | 0 |
| 98 | {FRA,UK,ITA,CAN} | 242 | 242 | 0 |
| 99 | {USA,JAP,GER,FRA,UK} | 922,25 | 922.25 | 0 |
| 100 | {USA,JAP,GER,FRA,ITA} | 869 | 869 | 0 |
| 101 | {USA,JAP,GER,FRA,CAN} | 851.75 | 851.75 | 0 |
| 102 | {USA,JAP,GER,UK,ITA} | 928.25 | 928.25 | 0 |
| 103 | {USA,JAP,GER,UK,CAN} | 911 | 911 | 0 |
| 104 | {USA,JAP,GER,ITA,CAN} | 857.75 | 857.75 | 0 |
| 105 | {USA,JAP,FRA,UK,ITA} | 837 | 837 | 0 |
| 106 | {USA,JAP,FRA,UK,CAN} | 819.75 | 819.75 | 0 |
| 107 | {USA,JAP,FRA,ITA,CAN} | 766.5 | 766.5 | 0 |
| 108 | {USA,JAP,UK,ITA,CAN} | 825.75 | 825.75 | 0 |
| 109 | {USA,GER,FRA,UK,ITA} | 816.75 | 816.75 | 0 |
| 110 | {USA,GER,FRA,UK,CAN} | 799.5 | 799.5 | 0 |
| 111 | {USA,GER,FRA,ITA,CAN} | 746.25 | 746.25 | 0 |
| 112 | {USA,GER,UK,ITA,CAN} | 805.5 | 805.5 | 0 |
| 113 | {USA,FRA,UK,ITA,CAN} | 714.25 | 714.25 | 0 |
| 114 | {JAP,GER,FRA,UK,ITA} | 503 | 503 | 0 |
| 115 | {JAP,GER,FRA,UK,CAN} | 485.75 | 485.75 | 0 |
| 116 | {JAP,GER,FRA,ITA,CAN} | 432.5 | 432.5 | 0 |
| 117 | {JAP,GER,UK,ITA,CAN} | 491.75 | 491.75 | 0 |
| 118 | {JAP,FRA,UK,ITA,CAN} | 400.5 | 400.5 | 0 |
| 119 | {GER,FRA,UK,ITA,CAN} | 380.25 | 380.25 | 0 |
| 120 | {USA,JAP,GER,FRA,UK,ITA} | 975.25 | 975.25 | 0 |
| 121 | {USA,JAP,GER,FRA,UK,CAN} | 958 | 958 | 0 |
| 122 | {USA,JAP,GER,FRA,ITA,CAN} | 904.75 | 904.75 | 0 |
| 123 | {USA,JAP,GER,UK,ITA,CAN} | 964 | 964 | 0 |
| 124 | {USA,JAP,FRA,UK,ITA,CAN} | 872.75 | 872.75 | 0 |
| 125 | {USA,GER,FRA,UK,ITA,CAN} | 852.5 | 852.5 | 0 |
| 126 | {JAP,GER,FRA,UK,ITA,CAN} | 538.75 | 538.75 | 0 |
| 127 | {USA,JAP,GER,FRA,UK,ITA,CAN} | 1011 | 1011 | 0 |

## Appendix 2.3 : Results of $v^*(\{i\})$, $Sh(N, v^* + w)$ for G20

***Table A2.3.1 :*** $t$ ***= 21%***

| $t = 0.21$ | $v^*(\{i\})$ | $Sh(N, v^*)$ | $Sh(N, w)$ | $Sh(N, v^* + w)$ | $Sh(N, v^* + w) - (v^* + w)(\{i\})$ |
|---|---|---|---|---|---|
| Australia | 53.7 | 41.18 | 3.69 | 44.87 | -8.83 |
| Canada | 50.05 | 34.31 | 3.69 | 38 | -12.05 |
| France | 50.76 | 42.25 | 3.69 | 45.94 | -4.82 |
| Germany | 60.83 | 111.14 | 3.69 | 114.83 | 54 |
| Italy | 38.16 | 44.54 | 3.69 | 48.23 | 10.07 |
| Japan | 164.84 | 139.67 | 3.69 | 143.36 | -21.48 |
| Korea | 44.64 | 51.95 | 3.69 | 55.64 | 11 |
| Mexico | 39 | 65.42 | 3.69 | 69.11 | 30.11 |
| Turkey | 12.78 | 41.63 | 3.69 | 45.32 | 32.54 |
| UK | 72.25 | 87.82 | 3.69 | 91.51 | 19.26 |
| USA | 396.69 | 397.66 | 3.69 | 401.35 | 4.66 |
| Brazil | 54.8 | 58.09 | 3.69 | 61.78 | 6.98 |
| China | 413.8 | 432.61 | 3.69 | 436.3 | 22.5 |
| India | 37.6 | 75.02 | 3.69 | 78.71 | 41.11 |
| Russia | 40.6 | 59.07 | 3.69 | 62.76 | 22.16 |
| South Africa | 19 | 17.39 | 3.69 | 21.08 | 2.08 |
| Total | 1549.5 | 1699.75 | 59.04 | 1758.79 | |

***Table A2.3.2 :*** $t = 28\%$

| $t = 0.28$ | $v^*(\{i\})$ | $Sh(N, v^*)$ | $Sh(N, w)$ | $Sh(N, v^* + w)$ | $Sh(N, v^* + w) - (v^* + w)(\{i\})$ |
|---|---|---|---|---|---|
| Australia | 53.7 | 53.42 | 5.81 | 59.23 | 5.53 |
| Canada | 50.05 | 43.83 | 5.81 | 49.64 | -0,41 |
| France | 50.76 | 55.54 | 5.81 | 45.94 | 10.59 |
| Germany | 60.83 | 151.54 | 5.81 | 114.83 | 96.52 |
| Italy | 38.16 | 60.91 | 5.81 | 48.23 | 28.56 |
| Japan | 164.84 | 179.64 | 5.81 | 143.36 | 20.61 |
| Korea | 44.64 | 70.75 | 5.81 | 55.64 | 31.92 |
| Mexico | 39 | 90.5 | 5.81 | 69.11 | 57.31 |
| Turkey | 12.78 | 59.48 | 5.81 | 45.32 | 52.51 |
| UK | 72.25 | 118.85 | 5.81 | 91.51 | 52.41 |
| USA | 396.69 | 523.07 | 5.81 | 401.35 | 132.19 |
| Brazil | 54.8 | 78.22 | 5.81 | 61.78 | 29.23 |
| China | 413.8 | 571.25 | 5.81 | 436.3 | 163.26 |
| India | 37.6 | 103.73 | 5.81 | 78.71 | 71.94 |
| Russia | 40.6 | 81.47 | 5.81 | 62.76 | 46.68 |
| South Africa | 19 | 24.15 | 5.81 | 21.08 | 10.96 |
| Total | 1549.5 | 2266.35 | 92.96 | 2359.31 | |

## Appendix 2.4 : Properties of game $(N, v^*)$ and $(N, v^* + w)$

**Proposition 10** *The game $(N, v^*)$ is a monotonic game, i.e., for each $S \subseteq T \subseteq N$, it holds that $v^*(S) \leq v^*(T)$.*

**Proof.** It is enough to prove that $v^*(S) \leq v^*(S \cup \{i\})$ for each $i \in N$ and each $S \subseteq N \backslash \{i\}$. We consider the case where $s > 1$. By definition,

$$v^*(S \cup \{i\}) = \max\left\{\left(\sum_{j \in S \cup \{i\}} \pi_j\right)t; \sum_{j \in S \cup \{i\}} \pi_j t_j\right\} = \max\left\{\left(\sum_{j \in S} \pi_j\right)t + \pi_i t; \sum_{j \in S} \pi_j t_j + \pi_i t_i\right\}.$$

Since $\pi_i t_i > 0$ and $\pi_i t > 0$, we can write that :

$$\max\left\{\left(\sum_{j \in S} \pi_j\right)t + \pi_i t; \sum_{j \in S} \pi_j t_j + \pi_i t_i\right\} \geq \max\left\{\left(\sum_{j \in S} \pi_j\right)t; \sum_{j \in S} \pi_j t_j\right\},$$

which implies that $v^*(S \cup \{i\}) \geq v^*(S)$, as desired. The case where $s = 1$ is similiar and is omitted. ■

**Proposition 11** *The game $(N, v^*)$ may not be super-additive., i.e., $v^*(S) + v^*(T) \leq v^*(S \cup T)$ may not hold for all $S, T \subseteq N$ such that $S \cap T = \varnothing$*

**Proof.** We consider the following numerical example, which we construct from the tax harmonization problem $(N, (\pi_i, t_i, \pi_i^L)_{i \in N}, t, \bar{t})$ such that $\{1, 2, 3\} \subsetneq N$, $t = 0.3$ and :

| $i$ | 1 | 2 | 3 |
|---|---|---|---|
| $\pi_i$ | 60 | 40 | 150 |
| $t_i$ | 0.25 | 0.2 | 0.4 |

We straightforwardly get that $v^*(\{3\}) = 60$, $v^*(\{1, 2\}) = 30$ and $v^*(\{1, 2, 3\}) = 83$. Hence,

$$v^*(\{1, 2\}) + v^*(\{3\}) > v^*(\{1, 2, 3\}),$$

which shows that the game $(N, v^*)$ is not superradditive. ■

Propositions 10 and 11 highlight other properties of a game $(N, v^* + w)$. Since $(N, w)$ is (trivially) monotonic, $(N, v^* + w)$ is monotonic too (as the sum of two monotonic games). Since $w(S) = 0$, $\forall S \subsetneq N$, $(N, v^* + w)$ may not be superadditive as well.

**Proposition 12** *If $t \geq \max_{i \in N} t_i$, then the core of $(N, v^* + w)$ is nonempty.*

**Proof.** As a start, if $t \geq \max_{i \in N} t_i$, remark that $(v^* + w)$ can be rewritten as follows : for each $S \subseteq N$,

$$(v^* + w)(S) = \begin{cases} \pi_i t_i & \text{if } s = 1, \\ \left( \sum_{i \in S} \pi_i \right) t & \text{if } 2 \leq s \leq n - 1, \\ \left( \sum_{i \in N} \pi_i \right) t + \sum_{i \in N} \pi_i^L \left( t - \bar{t} \right) & \text{if } s = n, \end{cases}$$

and keep in mind that $t > \bar{t}$. Next, note that $(N, v^* + w) = (N, r + u)$ where, for each $S \subseteq N$,

$$r(S) = \left( \sum_{i \in S} \pi_i \right) t,$$

and

$$u(S) = \begin{cases} \pi_i\big(t_i - t\big) & \text{if } s = 1, \\ 0 & \text{if } 2 \le s \le n-1, \\ \sum\limits_{i \in N} \pi_i^L\big(t - \bar{t}\big) & \text{if } s = n. \end{cases}$$

It is clear that $(N, r)$ is an additive game so that its core contains a single allocation $(\pi_i t)_{i \in N}$. Regarding $(N, u)$, observe that $u(\{i\}) \le 0$ for each $i \in N$ since $t \ge t_i$ by assumption, that $u(S) = 0$ if $2 \le s \le n-1$ and that $u(N) > 0$ from the fact that $t > \bar{t}$. Hence, any allocation $x \in \mathbb{R}^N$ such that

$$x_i \ge 0 \text{ and } \sum_{i \in N} x_i = u(N) \tag{2.1}$$

yields that $\sum_{i \in S} x_i \ge 0 \ge u(S)$ for each $S \subsetneq N$. We proved that $x$ is a core allocation in $(N, u)$. Furthermore, it is well-known that the core is covariant under strategic equivalence (see Peleg and Sudhölter (2003) for instance), which implies that the core of $(N, r+u)$ is composed of the allocations of the form $z + (\pi_i t)_{i \in N}$, $z \in C(N, r)$. Therefore, we obtain that the core of $(N, r+u)$ contains the allocations $y$ such that that, for each $i \in N$, $y_i = x_i + \pi_i t$, where $x$ is described by (2.1). From $v^* + w = r + u$, conclude that the core of $(N, v^* + w)$ is nonempty, although the Shapley value may not be a core allocation. ■

# Chapitre 3

# Early contributors and fair rewards in crowdfunding

We address the issue of fairly rewarding contributors to a crowdfunded project. We develop a theoretical non-strategic model, and introduce a new reward rule : a contributor's reward depends both on her financial contributions and the timing of her contribution. Following the axiomatic method used in models of resource sharing, we characterize this new reward rule by a pair of natural axioms. The resulting rewards coincide with the Shapley value of a suitable cooperative game built from the crowdfunding project. This allocation rule conveys a signaling effect : if two contributors make the same financial contribution, the earlier obtains a greater reward.

## 3.1 Introduction

Crowdfunding is now an essential tool for financing small- and medium-sized businesses. Crowdfunding aims to attract a large number of contributors or funders (who can be called the "crowd"). To achieve this, most crowdfunding methods offer contributors compensation that can take the form of a reward or a share in the company. According to Miglo (2022), the theoretical literature has developed around five issues :

— To what extent does crowdfunding allow a company to better understand market demand, especially through pre-orders ? (see for instance Chemla and Tinn, 2020) ;

— To what extent does crowdfunding allow a company whose project quality is not known with certainty to attest its quality to contributors ? (see for instance Chen et al., 2018) ;

— To what extent does crowdfunding allow the creation of network effects, in particular through increased information exchange between contributors or between

contributors and the company ? (see for instance Belleflamme et al., 2014) ;
— To what extent does crowdfunding mitigate moral hazard problems ? (see Schwienbacher, 2018; Belavina et al., 2020, for the choice to make an effort by the entrepreneur and the issue of funds diversion) ;
— What roles do behavioral biases play in crowdfunding ? (see for instance Miglo, 2021) ;

In this article, we address the issue of fairly rewarding the participating contributors by designing a suitable reward rule. In particular, we focus on two aspects of crowdfunding that deviate from current practices.

The first one is that if two contributors make identical financial contributions to the project but at different time, the earlier of the two contributors can obtain a greater reward. There are multiple reasons which justify such a difference in treatment. The signal sent by these two contributions is different. According to the notion of information cascade, two identical signals do not have the same effect if they are not sent at the same time. Rewarding the earliest contributor allows us to emphasize that the impact of their contribution (a distinct externality in the sense of Hu et al., 2015) on the success of the crowdfunding campaign is not the same. An early contribution is more valuable to an entrepreneur who seeks to estimate the uncertain demand for his asset (on this issue, see Strausz, 2017; Ellman and Hurkens, 2019, among others). Furthermore, an early contribution allows the contributor to promote the project to her network and thus encourage new contributions. Cason et al. (2021) also insist on the critical role of the first contributions from the point of view of potential contributors who are waiting to evaluate the dynamics of the growth of the amount fundraised.

We collected data from over 1 694 reward-based projects from three platforms (Miimosa, Tudigo, Winefunding) between 03/02/2013 to 10/16/2021. These platforms were chosen because of the availability of data on the timing of contributions. In total, 1 279 projects have a fundraising time less than 50 days, 415 have a fundraising time more than 50 days, and the average collected amount per project is 5 017.3 euros. Figures 3.1 and 3.2 report the daily amount collected and the daily number of contributors (on the vertical axis), respectively, as a function of the fundraising time in days (on the horizontal axis). The empirical findings support the results in the literature that the first days are critical : the daily number of contributors and the daily amount collected decreased over time. Based on this evidence, we design a reward rule which encapsulates the aforementioned

signaling effects.

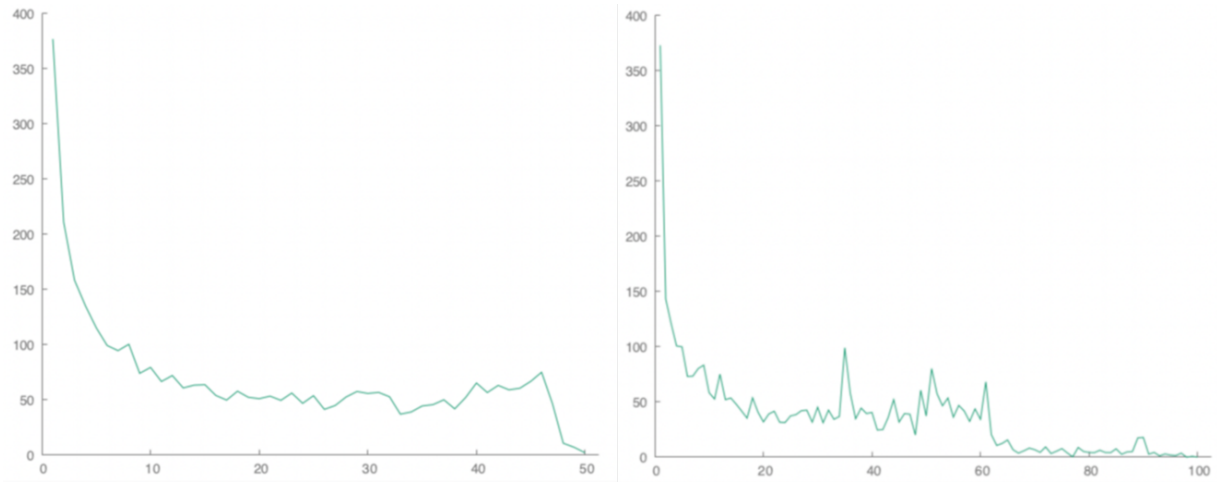


FIGURE 3.1 – Daily amount collected in euros – left (< 50 days), right (> 50 days)

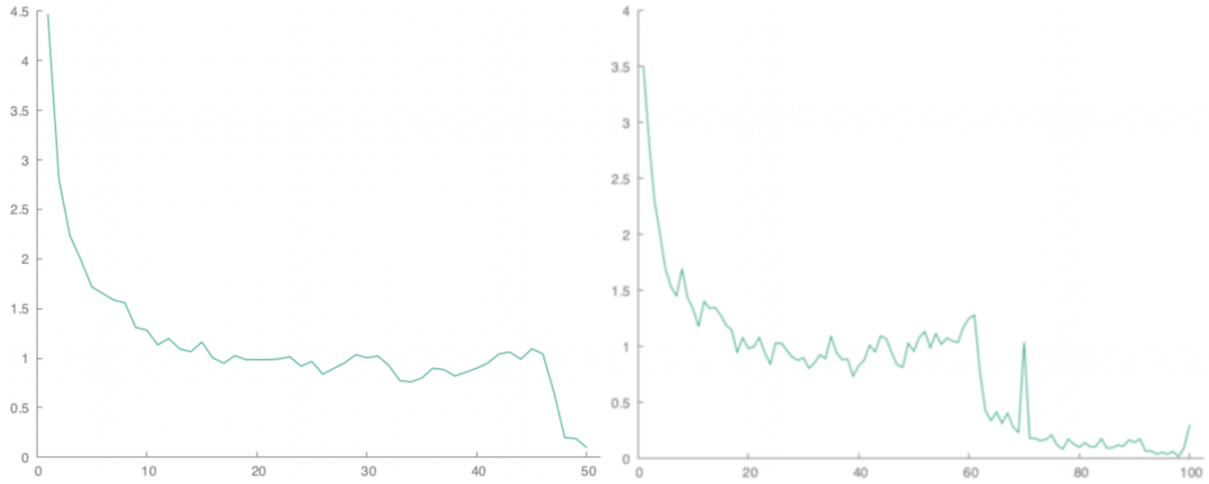


FIGURE 3.2 – Daily contributors – left (< 50 days), right (> 50 days)

The second aspect on which we focus is a principle of collective reward generation. In practice, the reward level of a given contributor depends heavily on the level of her own contributions but only slightly on the contribution level of the other contributors. Here, to the contrary, we propose a new model in which the reward of each contributor can depend on all contributions to the project through a two-step procedure : (1) the total accumulated contribution yields the total reward that must be shared among the participating contributors and then (2) the previous amount of reward is distributed among the contributors according to our reward rule. We think that the principle of collective

reward generation is relevant for taking into account the potential synergies that distinct contributions can bring to the value of a project. At the very least, it seems fairer that these synergies should benefit contributors and not be captured solely by the entrepreneur.

To summarize, our model rewards the contributors through three aspects : (1) the individual performance of the contributor, measured by the level of her contribution, (2) the collective performance of the contributors, measured by the final value of the project, and (3) the signaling effect, measured by the (early) timing of her contribution. To the best of our knowledge, the crowdfunding platforms only rely on the first effect.

Our results are based on an original non-strategic model of crowdfunding that incorporates the essential components of the problem : contributions, their timing, and a non-decreasing reward function that maps total contributions onto total reward. An instance of our model should be seen as a snapshot of a funded crowdfunding project. Therefore, the reader should not expect to find in this article the reasons for the success of a campaign or the intensity with which contributors participate. Rather, the model is calibrated to determine what fair rewards should be for contributors. In order to achieve this objective, we rely on the theory of cooperative games, which has been extensively used in the past years to apprehend various applications in economics and finance (see Graham et al., 1990; Ambec and Sprumont, 2002; Maniquet, 2003; Baloga et al., 2017, among others), but not yet in the context of crowdfunding, to the best of our knowledge.

We rely on the two main tools of cooperative game theory, namely the Shapley value and the core. Our findings can be summarized as follows.

Firstly, we impose two desirable axioms for an arbitrary reward rule. The axiom of *Full distribution* states that the total available reward is fully distributed among the participating contributors. The axiom of *Fair rewarding* requires that the withdrawal of a first contributor must have the same impact on the reward of a second contributor as if we measured the impact of the withdrawal of the second contributor on the reward of the first contributor. It should be clear that this last axiom does not imply that two contributors are always equally important to the project. The withdrawal of a large contributor will result in a sharp decline in the value of the project and the total amount of reward to be distributed to the remaining contributors. The impact on a small contributor, however, will be moderate. Conversely, the withdrawal of a small contributor will result in a small

decrease in the total amount of reward to be distributed to the remaining contributors. Hence, although a large contributor will absorb a significant portion of this change, the overall effect on that contributor will be moderate as well. We show that there is a unique reward rule satisfying the combination of the two axioms, and it turns out that this reward rule coincides with the Shapley value of a specific cooperative game built from the studied crowdfunding problem. In order to define this cooperative game, we rely on an assumption about the contributions that a contributor could make if some contributors withdraw – the so-called willingness to contribute. In this last situation, the contributor's contribution remains the same only if she still observes all signals relevant to her, i.e. if all contributors preceding her indeed make their contributions.

Secondly, as we have already underlined, the characterized reward rule conveys the signaling effect : an early contributor obtains a greater reward than a late contributor if they make the same financial contribution. In the final part of the article, we point out that this result remains valid for a wide range of assumptions about the contributors' willingness to contribute.

Thirdly, we obtain additional results on our reward rule when the reward function exhibits extra properties. If the reward function is convex, we show that the associated cooperative game is convex/supermodular, which means in the context of crowdfunding that the marginal contribution of each contributor in the game increases with the size of the group of contributors she joins. In this class of games, our reward rule is a core allocation. Therefore, if this cooperative game is used to describe the value of the project in terms of the participation of each conceivable group of contributors, then it means that none of these groups could provide its members with a higher reward than the one that our reward rule assigns them. In this sense, our reward rule can be considered as stable. If the reward function is a threshold function, i.e. the project is worth nothing until a certain threshold of contribution is reached, then the contributors can be partitioned into three groups : the earliest contributors (group 1), the intermediate contributors (group 2) and the latest contributors (group 3). Our reward rule is such that rewards are decreasing with time : the members of group 1 get more than the members of group 2 who get more than the members of group 3. Furthermore, the rewards within each group are equal except within group 2 in which they increase with financial contributions. Finally, if the reward function is additive and if there are no contributors contributing at the same time, then we show that the cooperative game associated with a crowdfunding problem is the same

as the cooperative game arising from the so-called pure sequential liability situations in Dehez and Ferey (2013).

The rest of the article is organized as follows. Section 3.2 and 3.3 introduce cooperative games and crowdfunding problems, respectively. Section 3.4 motivates our assumption regarding the contributors' willingness to contribute and presents the axiomatic characterization of our reward rule. Section 3.5 provides the other aforementioned results for specific reward functions. Alternative assumptions about the willingness to contribute are discussed in section 3.6. We conclude in section 3.7 by discussing the operational aspects of our model. All proofs are relegated to the appendix.

## 3.2 Preliminaries on cooperative games

Where $A$ is a finite set, we use the lower case $a$ to denote its cardinality $|A|$. A **cooperative game** (with transferable utility) is a pair $(N, v)$ such that $N = \{1, \dots, n\}$ is a finite set of players and $v$ is a characteristic function which assigns to each coalition of players $S \in\subseteq N$ a worth $v(S)$ and such that $v(\varnothing) = 0$. For a game $(N, v)$ and a nonempty coalition $S \subseteq N$, the **subgame** of $(N, v)$ induced by $S$ is the game $(S, v_{|S})$ such that, for each $T \subseteq S$, $v_{|S}(T) = v(S)$.

A player $i \in N$ is a **null player** in a game $(N, v)$ if, for each $S \subseteq N\backslash\{i\}$, it holds that $v(S \cup \{i\}) = v(S)$. A player $i \in N$ is a **necessary player** in a game $(N, v)$ if, for each $S \subseteq N\backslash\{i\}$, it holds that $v(S) = 0$. A player $i \in N$ is **at least as desirable** as a player $j \in N$ in a game $(N, v)$ if, for each $S \subseteq N\backslash\{i, j\}$, it holds that $v(S \cup \{i\}) \geq v(S \cup \{j\})$. Two players $i, j \in N$ are **equal players** in a game $(N, v)$ if each is at least as desirable as the other. Two necessary players are equal but the converse is not always true.

A game $(N, v)$ is **monotonic** if, for each pair of coalitions $S \subseteq T \subseteq N$, it holds that $v(S) \leq v(T)$. A game $(N, v)$ is **convex** if, for each pair of coalitions $S, T \subseteq N$, it holds that $v(S \cup T) + v(S \cap T) \geq v(S) + v(T)$.

An **allocation rule** is a function $f$ which assigns to each game $(N, v)$ an allocation $f(N, v) \in \mathbb{R}^N$ specifying the payoff $f_i(N, v)$ obtained by each player $i \in N$ for her participation in game $(N, v)$. The **Shapley value** (Shapley, 1953) is the allocation rule $Sh$ which assigns to each game $(N, v)$ and to each player $i \in N$ the payoff

$$Sh_i(N, v) = \sum_{S \subseteq N\backslash\{i\}} \frac{s!(n - s - 1)!}{n!} \big(v(S \cup \{i\} - v(S)\big). \tag{3.1}$$

It is well known that the Shapley value satisfies the following axioms, stated for an arbitrary allocation rule $f$.

**Null player axiom.** For each game $(N, v)$ and each null player $i \in N$ in $(N, v)$, $f_i(N, v) = 0$.

**Desirability.** (Maschler and Peleg, 1966) For each game $(N, v)$ and each pair of players $i, j \in N$ such that $i$ is at least as desirable as $j$ in $(N, v)$, $f_i(N, v) \geq f_j(N, v)$.

**Equal treatment of equal players.** For each game $(N, v)$ and each pair of equal players $i, j \in N$ in $(N, v)$, $f_i(N, v) = f_j(N, v)$.

The **core** of a game $(N, v)$ is the (possibly empty) set of allocations $C(N, v)$ distributing the worth of the grand coalition in such a way that each coalition gets at least as much as its worth, i.e.

$$C(N, v) = \left\{ x \in \mathbb{R}^N : \sum_{i \in S} x_i \geq v(S) \text{ for each } S \subseteq N \text{ and } \sum_{i \in N} x_i = v(N) \right\}.$$

Core allocations are often considered as stable whereas the Shapley value is often considered as a fair allocation rule.

## 3.3 Crowdfunding environments

In this section we introduce crowdfunding environments and discuss some special cases that will be taken up in the following sections. A **crowdfunding environment** models a completed crowdfunding project and is given by a four-tuple $C = (N, B, R, (c_i)_{i \in N})$ such that

- $N = \{1, \ldots, n\}$ is a finite set of **contributors**. Typical contributors are denoted by $i$ and $j$;
- $B = (B_1, \ldots, B_k)$ is an ordered partition of $N$, $1 \leq k \leq n$, i.e. $\bigcup_{q \in \{1,\ldots,k\}} B_q = N$ and for each $q, q' \in \{1, \ldots, k\}$, $B_q \cap B_{q'} = \varnothing$, which models the **contribution timing**. If $i \in B_q$ and $j \in B_{q'}$, $q < q'$, this means that $i$ is an earlier contributor than $j$. Each set $B_q$ can be considered as a time window containing contributors who cannot be distinguished with respect to the timing of their contribution. The case in which $B_q = \varnothing$ is allowed. Furthermore, let $q(i)$ be the index of the sole element of $B$

containing contributor $i$ ;

— $R : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is a **reward function** which assigns to each total of contributions $c \in \mathbb{R}_+$ a total reward $R(c) \in \mathbb{R}_+$ that must be used to reward contributors. We simply assume that $R(0) = 0$ and that $R$ is non-decreasing. Function $R$ can be considered as a proxy for the value of the project or not ;

— for each contributor $i \in N$, $c_i \in \mathbb{R}_+$ is her **contribution**.

The general form of the reward function $R$ includes the following special cases. A **threshold** reward function $R$ is such that there are two real numbers $c^*$ and $r^*$ such that, for each $c \in \mathbb{R}$

$$R(c) = \begin{cases} r^* & \text{if } c \geq c^* \\ 0 & \text{otherwise.} \end{cases} \tag{3.2}$$

A **convex** reward function is such that $R$ is a convex function. The **additive** reward function $R$ is such that for each $c \in \mathbb{R}$, $R(c) = c$. The additive function is special instance of convex functions. Figure 3.3 illustrates these three types of reward functions.

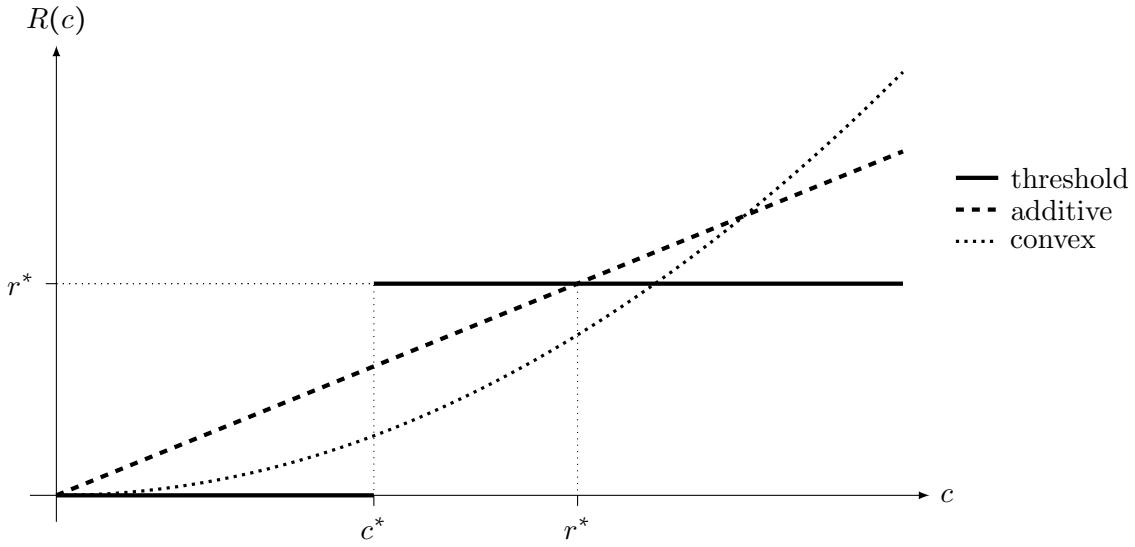


FIGURE 3.3 – An illustration of three types of reward functions

For each crowdfunding environment $C = (N, B, R, (c_i)_{i \in N})$, the objective is to determine, for each contributor $i \in N$, a fair reward for her participation in $C$. This reward aims to assess the importance of the contributor's role in the success of the campaign, which may depend on the value of her contribution, but also on the timing of this contribution. In this article, we would like to highlight the fact that early contributors can be particularly important because of the signal their contribution sends to future contributors

about the value of the project.

## 3.4 The willingness to contribute and an axiomatic study

In order to deal with the aforementioned problem, we adopt counterfactual reasoning, which considers what would have happened in the absence of some contributors. Starting from a crowdfunding environment $C = (N, B, R, (c_i)_{i \in N})$ and any nonempty subgroup of contributors $S \subseteq N$, this means that we have to define the crowdfunding situation arising from the crowdfunding environment $C$ if the other contributors in $N \backslash S$ leave or are absent. There are several ways to proceed, depending on the factors that are considered necessary for a contributor to be willing to contribute. Put differently, the crowdfunding situation on $S$ is shaped by the impact that the leaving contributors may have on the remaining contributors, and in particular on the (late) contributors that no longer receive the signal of the (early) missing contributions. Should we assume that these contributors will maintain their contributions or, on the contrary, that they will renounce them ?

We answer this question by retaining the following principle, which captures the fact that contributors are influenced by the situation they observe when deciding whether to contribute : to contribute when $S$ is the set of contributors, a contributor needs all contributors prior to her in $C$ to be made also in the crowdfunding situation on $S$. This principle can be considered as a pessimistic/prudent view in that a contributor of $S$ gives up contributing as soon as she does not observe the same contributions prior to hers. While this principle is adapted to bring out significant effects, we also discuss in section 3.6 alternative principles to model the contributors' willingness to contribute.

To formalize these concepts, we need the following definitions. For each $S \subseteq N$ and each $i \in S$, let $P_i^S$ denote the set of contributors in $S$ who contribute before $i$, that is

$$P_i^S = \{j \in S : q(j) < q(i)\}.$$

Furthermore, define $c(P_i^S)$ as the total contribution of these contributors in $P_i^S$, that is

$$c(P_i^S) = \sum_{j \in P_i^S} c_j.$$

In the crowdfunding environment $C = (N, B, R, (c_i)_{i\in N})$, note that $P_i^N$ corresponds to the set of contributors whose contributions contributor $i$ observes when making her own contribution, and so $c(P_i^S)$ indicates the total funding of the project when $i$ contributes.

From the crowdfunding environment $C = (N, B, R, (c_i)_{i\in N})$ and any coalition of contributors $S$, we define the crowdfunding situation $C^S = (S, B^S, R, (c_i^S)_{i\in S})$ induced by $S$ in which

— $B^S = (B_1^S, \ldots, B_k^S)$ with $B_q^S = B_q \cap S$ for each $q \in \{1, \ldots, k\}$. In words, $B^S$ is just the restriction of $B$ to $S$;

— for each $i \in S$, $c_i^S = c_i$ if $P_i^N \subseteq S$ and $c_i^S = 0$ otherwise.

The second item means that contributor $i$ makes her original contribution $c_i$ in $C^S$ only if all contributors in $P_i^N$ are in $S$ as well. We denote by $\mathcal{C}$ the set of all crowdfunding situations arising from the original crowdfunding environment $C = (N, B, R, (c_i)_{i\in N})$. Hence $\mathcal{C}$ also contains $C$ since $C = C^N$. Next, we introduce two axioms for a reward rule $f$ on $\mathcal{C}$. The first one imposes that the sum of the contributors' rewards is equal to the total reward generated by the sum of all their contributions.

**Full Distribution.** For each $C^S = (S, B^S, R, (c_i^S)_{i\in S}) \in \mathcal{C}$,

$$\sum_{i\in S} f_i(C^S) = R\left(\sum_{i\in S} c_i^S\right). \tag{3.3}$$

The second axiom aims at translating fairness considerations into a property for a reward rule. More specifically, we impose that the withdrawal of a first contributor must have the same impact on the reward of a second contributor as if we measured the impact of the withdrawal of the second contributor on the reward of the first contributor.

**Fair Rewarding.** For each $C^S = (N, B^S, R, (c_i^S)_{i\in S}) \in \mathcal{C}$ and each $i, j \in S$,

$$f_i(C^S) - f_i(C^{S\setminus\{j\}}) = f_j(C^S) - f_j(C^{S\setminus\{i\}}). \tag{3.4}$$

An alternative interpretation of the axiom in terms of threats is possible. Let us imagine that a contributor considers threatening to give up his contribution in order to obtain a larger share of the total rewards. Each other contributor contemplates the consequences that this threat would have on their own share of the rewards. The axiom simply states that each other contributor can neutralize such a threat by making the same threat of withdrawal to the first contributor, who would suffer the same impact as that created by

his threat.

It should be clear that the axiom of Fair Rewarding does not imply that two contributors are always equally important to the success of a campaign. The withdrawal of a large contributor will result in a sharp decline in both the value of the project and the total amount of reward to be distributed to the remaining contributors. The impact on a small contributor who can legitimately claim a small share of the total rewards, however, will be moderate. Conversely, the withdrawal of a small contributor will result in a small decrease in the total amount of reward to be distributed to the remaining contributors. Hence, although a large contributor will absorb a significant portion of this change, the overall effect on that contributor will be moderate as well. The axiom of Fair Rewarding requires precisely that the two effects be identical. It is part of a long tradition of axioms in the literature on cooperative games which require similar balanced effects (see Myerson, 1977, 1980; Herings et al., 2008; Kamijo and Kongo, 2010; Béal et al., 2016; Yokote and Kongo, 2017; Yokote et al., 2018).

It turns out that there is a unique reward rule satisfying these two axioms, and that this reward rule coincides with the Shapley value of a specific cooperative game. This result is inspired by the characterization of the Shapley value provided by Myerson (1980).

**Proposition 13** *There is a unique reward rule $f^*$ on $\mathcal{C}$ that satisfies Full Distribution and Fair Rewarding. For each $C^S = (S, B^S, R, (c_i^S)_{i \in S}) \in \mathcal{C}$, it is given by $f^*(C^S) = Sh(S, v_{C^S})$ where $(S, v_{C^S})$ is the cooperative game defined, for each coalition of contributors $T \subseteq S$, as*

$$v_{C^S}(T) = R\left( \sum_{i \in T : P_i^S \subseteq T} c_i^S \right). \tag{3.5}$$

In order to understand the cooperative game given by (3.5), assume that the set of participating contributors is $S \subseteq N$ and that $T \subseteq S$ is the coalition under consideration. Then, $v_{C^S}(T)$ indicates the total rewards that should go to the members of $T$ in this counterfactual situation, which corresponds to the total rewards calculated when the only contributions are those of $T$ members whose predecessors in the original crowdfunding campaign (with $N$ as the set of contributors) are also present in $T$.

It is easy to see that the axioms invoked in Proposition 13 are logically independent. The **null** reward rule which assigns unconditionally a null reward to each contributor

satisfies Fair Rewarding but not Full Distribution. The **equal split** reward rule $ES$ such that for each $C^S = (S, B^S, R, (c_i^S)_{i\in S}) \in \mathcal{C}$ and each $i \in S$,

$$ES_i(C^S) = \frac{1}{s} R\left(\sum_{i\in S} c_i^S\right)$$

satisfies Full Distribution but not Fair Rewarding.

**Remark 1** *From definition 3.5 of function $v_{C^S}$, it is obvious that $(S, v_{C^S})$ is a monotonic game for each nonempty $S \subseteq N$. Reformulated in the context of crowdfunding, this property has the intuitive meaning that the ability of a coalition to claim a significant share of the total available reward cannot decrease if the coalition expands.* □

## 3.5 Properties and special reward functions

In this section, we point out several interesting properties of our allocation rule $f^*$ that are valid either for all crowdfunding environments and/or for special crowdfunding environments.

### 3.5.1 The signaling effect

The **signaling effect** reflects the phenomenon that early contributors are more important than late contributors, all other things being equal. We measure this effect by comparing the result attributed by a reward rule to two contributors with the same contribution level but at different times. Formally, a reward rule $f$ conveys the signaling effect if for each crowdfunding environment $C = (N, B, R, (c_i)_{i\in N})$ and each pair of contributors $i, j \in N$ such that $c_i = c_j$ but $q(i) < q(j)$, it holds that $f_i(C) > f_j(C)$.

**Proposition 14** *If $R$ is (strictly) increasing, then the reward rule $f^*$ conveys the signaling effect.*

In the same vein, it is easy to determine that the rewards of two contributors with the same contribution timing are ranked by our reward rule according to their contributions : if $q(i) = q(j)$ and $c_i > c_j$, then $f_i^*(C) > f_j^*(C)$.

### 3.5.2 Convex reward functions

Whenever the reward function is convex, the associated cooperative game possesses extra properties.

**Proposition 15** *For each crowdfunding environment* $C = (N, B, R, (c_i)_{i\in N})$ *such that the reward function* $R$ *is convex, the associated cooperative game* $(N, v_C)$ *is convex.*

Proposition 15 implies that the Shapley value of game $(N, v_C)$ is a core allocation : if a group of contributors $N\backslash S$ eventually decide not to contribute to the project, then the group of remaining contributors $S$ cannot end up in a better situation. More specifically, the members of $S$ get a total reward according to the Shapley value which is not less than the total reward that they could obtain on their own, that is

$$\sum_{i\in S} Sh_i(N, v_C) \geq v_C(S) = v_{C^S}(S).$$

Another consequence of Proposition 15 is that another relevant allocation belongs to the core. In fact, from Shapley (1971), it is core stable to reward the contributors according to the extra value they add to the project at the time of their contribution (the contributors belonging to a given time unit $B_q$ can be sorted in any way).

Proposition 15 obviously holds for any crowdfunding situation $C^S = (S, B^S, R, (c_i^S)_{i\in S})$, $S \subseteq N$. As a consequence, in this context we get from Sprumont (1990, Corollary 2) that the Shapley value is a population monotonic allocation scheme. In our framework, this means that for each $i \in N$ and each pair of coalitions $S \subseteq T$ such that $i \in S$, $f_i^*(C^S) \leq f_i^*(C^T)$, that is, the reward obtained by any contributor cannot increase after some other contributors leave or, equivalently, that the reward received by a contributor is weakly increasing in the population of participating contributors.

### 3.5.3 The threshold function

We now examine the specific shape of the allocation $f^*(C)$ for crowdfunding environment $C = (N, B, R, (c_i)_{i\in N})$ in which $R$ is a threshold reward function. In order to state this result, let $q^*$ be the minimal $q \in \{1, \ldots, k\}$ such that

$$\sum_{i\in\bigcup_{q'\leq q} B_{q'}} c_i \geq c^*,$$

that is, $B_{q^*}$ is the earliest time unit during which the total of contributions reaches the threshold $c^*$. We partition the contributors into three groups :

$$G_1 = \bigcup_{q<q^*} B_q \cup \left\{ i \in B_{q^*} : \sum_{j\in\bigcup_{q\leq q^*} B_q} c_j - c_i < c^* \right\}.$$

$$G_2 = \left\{ i \in B_{q^*} : \sum_{j \in \bigcup_{q \leq q^*} B_q} c_j - c_i \geq c^* \right\}.$$

$$G_3 = \bigcup_{q > q^*} B_q.$$

Group $G_1$ contains the contributors that are needed to reach the threshold $c^*$. Group $G_2$ contains some contributors in $B_{q^*}$ without whom the earliest contributors up to $B_{q^*}$ can still reach the threshold. Group $G_3$ contains the remaining (later) contributors. Figure 3.4 provides a schematic representation of these groups in which the nodes within group $B_{q^*}$ represent the contributors belonging to this group, who are assumed to be positioned from left to right in ascending order of individual contributions.

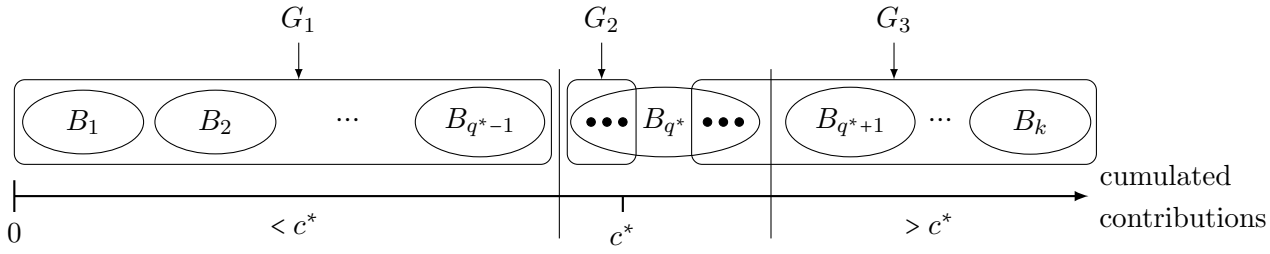


FIGURE 3.4 – Illustration of groups $G_1$, $G_2$ and $G_3$

The next result shows that contributors in $G_1$ get the same reward, which is greater than the reward allocated to contributors in $G_2$, that the rewards of contributors in $G_2$ are non-decreasing with respect to their contributions, and that contributors in $G_3$ are not rewarded at all.

**Proposition 16** *For each crowdfunding environment $C = (N, B, R, (c_i)_{i \in N})$ such that the reward function $R$ is the threshold function given by (3.2), the allocation $f^*(C)$ satisfies the following properties :*

*(i) if $i, j \in G_1$, then $f_i^*(C) = f_j^*(C)$ ;*

*(ii) if $i, j \in G_2$ and $c_i \geq c_j$, then $f_i^*(C) \geq f_j^*(C)$ ;*

*(iii) if $i \in G_1$, $j \in G_2$ and $l \in G_3$, then $f_i^*(C) > f_j^*(C) > f_l^*(C) = 0$.*

### 3.5.4 Additive reward functions

In this section we establish links between our model and two other models in the literature : games arising from liability situations (Dehez and Ferey, 2013) and from aircraft landing fee problems (Littlechild and Owen, 1973).

A **liability situation** $L$ on a player set $N$ (the set of tortfeasors) is given by an ordered partition $B^L = (B_1^L, \ldots, B_k^L)$ of $N$, as for a crowdfunding situation, which reflects the sequence of wrongful acts to a victim, and a vector of damages $d = (d_1, \ldots, d_k)$ where $d_q$, $q \in \{1, \ldots, k\}$ is the extra damage caused by the group of tortfeasors $B_q$ to the victim. Let $D_q$ be the cumulative damage induced by the first $q$ groups of tortfeasors in the sequence, i.e.

$$D_q = \sum_{h=1}^{q} d_h.$$

Then, the corresponding liability game $(N, v_L)$ is such that, for each $S \subseteq N$,

$$v_L(S) = D_q,$$

where $q$ is the largest integer such that $\cup_{h=1}^{q} B_h^L \subseteq S$.

An **aircraft landing fee problem** $A$ on a player set $N$ (the set of aircrafts which are to land on a runway) is given by an ordered partition $B^A = (B_1^A, \ldots, B_k^A)$ of $N$, which groups aircraft by type, from the smallest to the largest. The cost of building the runway depends upon the largest plane for which the runway is designed. The cost vector $a = (a_1, \ldots, a_k)$ specifies, for each $q \in \{1, \ldots, k\}$, the cost $a_q$ necessary to make the runway suitable for landings by planes of type $q$. Then, the corresponding airport game $(N, v_A)$ is such that, for each $S \subseteq N$,

$$v_A(S) = \max_{q \in \{1,\ldots,q\}: S \cap B_q^A \neq \emptyset} a_q.$$

In both situations, the pure sequential case is the one in which $|B_q^L| = 1$ and $|B_q^A| = 1$ for each $q \in \{1, \ldots, k\}$, respectively. Similarly, a crowdfunding environment $C = (N, B, R, (c_i)_{i \in N})$ is called purely sequential if $|B_q| = 1$ for each $B_q \in B$.

**Proposition 17** *The cooperative games arising from purely sequential crowdfunding environments in which the reward function is additive are purely sequential liability games.*

Furthermore, Dehez and Ferey (2013) show that liability games and airport games are dual to each other, where the dual of a game $(N, v)$ is the game $(N, v^*)$ such that, for each $S \subseteq N$, $v^*(S) = v(N) - v(N \backslash S)$, and it is well known that the Shapley value prescribes the same allocation in a game and in its dual. Hence, in the specific case described in Proposition 17 and for purely sequential aircraft landing fee problems, our reward rule $f^*$ prescribes the same allocation as the Shapley value of the corresponding liability and

airport games.

Proposition 17 does not extend to the non-sequential case, as illustrated by the next example.

**Example 3** Consider the crowdfunding environment $C = (N, B, R, (c_i)_{i \in N})$ such that $N = \{1, 2, 3\}$, $B_1 = \{1\}$ and $B_2 = \{2, 3\}$. Assume further that $c_i > 0$ for each $i \in N$ and that $R$ is a (strictly) increasing function. In the corresponding game $(N, v_C)$, it holds that $v_C(\{1\}) = R(c_1)$, $v_C(\{1, 2\}) = R(c_1 + c_2)$, $v_C(\{1, 3\}) = R(c_1 + c_3)$, $v_C(\{1, 2, 3\}) = R(c_1+c_2+c_3)$ and $v_C(S) = 0$ for each other coalition $S$. Next, assume that there is a liability situation $L = (N, B^L, d)$ from which the corresponding liability game $(N, v_D)$ coincides with $(N, v_C)$. From $v_C(\{1\}) = R(c_1) > 0$, it must be that $R(c_1) = d_1$ and $B_1^L = \{1\}$. From $v_C(\{1, 2\}) = R(c_1 + c_2) > R(c_1) = v_C(\{1\})$, it must be that $d_2 = R(c_1 + c_2) - R(c_1)$ and $B_2 = \{2\}$. But from $v_C(\{1, 3\}) = R(c_1 + c_3) > R(c_1) = v_C(\{1\})$, it must be that $d_3 = R(c_1 + c_3) - R(c_1)$ and $B_2 = \{3\} \neq \{2\}$, a contradiction which shows that there is no liability situation $L$ such that $(N, v_L) = (N, v_C)$. □

## 3.6 Towards more flexible models

In this section, we discuss alternative principles which can be used to model the willingness to contribute and which generalize the principle considered in section 3.4. The first two principles discussed here have in common that they personalize the conditions that must be met for a contributor to actually decide to contribute to the project in the counterfactual scenario in which some other contributors are absent. The first is possibly based on the identity of contributors who preceded a given contributor. The second depends only on an observed amount of contribution. So, for the rest of this section, we fix some crowdfunding environment $C = (N, B, R, (c_i)_{i \in N})$ and consider any crowdfunding (counterfactual) situation $C^S = (S, B^S, R, (c_i^S)_{i \in S})$, $S \subseteq N$.

In order to describe the first alternative principle, for each $i \in S$, denote by $E_i^S \subseteq P_i^S$ the set of contributors preceding $i$ whose contribution must be effective for $i$ to make its own contribution when the set of contributors is $S$. This approach is flexible : the principle considered in section 3.4 can be obtained by setting $E_i^S = P_i^S$ for each $i \in S$, but $E_i^S$ and $E_j^S$ can be different for two contributors $i$ and $j$ belonging to the same time window. In particular, this principle allows us to highlight the identity of contributors in $E_i^S$ in order to reflect, for example, the leading role of an influencer.

In order to describe the second alternative principle, for each $i \in S$, denote by $e_i^S$ the accumulated contribution that the project must display in order for $i$ to make its own contribution when the set of contributors is $S$. This principle implies that the identity of the contributors preceding $i$ does not matter ; only the level of contributions is relevant from the perspective of $i$. Once again, it generalizes the principle considered in section 3.4 which is the special case obtained by setting $e_i^S = \sum_{j \in P_i^S} c_j^S$.

The two approaches allow for the benchmark case in which each contributor makes exactly the same contribution in the original crowdfunding environment and when the set of contributors is restricted to $S$ by setting $E_i^S = \varnothing$ and $e_i^S = 0$ for each $i \in S$, respectively. Finally, note that under these two general principles it is necessary to adapt the cooperative game defined in equation 3.5.

These two principles take up the idea developed in section 3.4 that if a contributor does not observe certain signals, then she does not contribute at all. It is also possible to relax this assumption so that a contributor contributes partially if it partially observes the original signals. A natural way to take this principle into account is to assume that when the set of contributors is $S$, a contributor contributes in proportion to her original contribution, where the proportion is measured by the ratio between the contributions of her predecessors in $S$ and the contributions of its original predecessors in $N$.

Finally, we remark that in all these principles, the contributions when the set of contributors is $S$ are at most equal to (and often strictly less than) the original contributions for the participating contributors. An interesting consequence is that the signaling effect highlighted in Proposition 14 is robust to the adoption of any of these alternative principles.

## 3.7 Conclusion

To summarize, our model suggests an innovative way to reward contributors on a crowdfunding platform, based on the principles of cooperative games. In addition to the traditional reward, which depends on individual contribution, we propose two other types of contributions. The first one depends on the final value of the project, depending on the overall amount raised. The second one depends on the timing of the contribution :

the earlier the contribution, the higher the reward. Thus, our model highlights aspects of distributive justice and incentives for early contributions. Beyond these aspects, an obvious question is whether our reward procedure could be implemented in a practical way by the crowdfunding platforms. We believe so for the following reasons.

Firstly, it is common for platforms to distribute rewards only after a certain funding threshold has been reached, or even at the end of the fundraising campaign. Currently, at the time the project is known to be funded, a contributor receives her reward based solely on the individual amount contributed. Nonetheless, a platform has often enough information so that the reward of contributors also depends on their collective performance on the project's value.

Secondly, even if the reward system we propose takes into account both the signaling effect and the collective performance, these two aspects can be dissociated. In other words, it is possible to adapt our model in order to take into account the collective performance of the contributors but not the signaling effect and vice versa. In the same vein, the variants of the model discussed in section 3.6 offer flexibility to platforms to meet their specific needs or to play on the strength with which the signaling effect and the collective performances are taken into account.

The implementation of our model requires that the platforms evaluate their own collective reward function. In practice, we think it is likely that the platforms retain stair functions of which the threshold function studied in subsection 3.5.3 is a special case. The platforms will also have to revise the online display of their reward system to specify that (1) each amount of contribution entitles, in the event of financing of the project, at least to a certain reward (the individualized part) which can be improved if the total of contributions is sufficient (the collective part) and (2) that early contributions will qualify for additional rewards (the part materializing the signaling effect).

All in all, we consider that the principles underlying our model can be seen either as complements to current systems or as possible new standards.

# Appendix

**Proof.** (**Proposition 13**) We split the proof into two parts. Firstly, we begin by proving that the reward rule $f^*$ which assigns to each crowdfunding situation $C^S = (S, B^S, R, (c_i^S)_{i \in S})$ the Shapley value of the game $(S, v_{C^S})$ satisfies the axioms of Full

Distribution and Fair Rewarding. Regarding Full Distribution, we have

$$\sum_{i\in S} f_i^*(C^S) = \sum_{i\in S} Sh_i(S, v_{C^S}) = v_{C^S}(S) = R\Bigg(\sum_{i\in S: P_i^S\subseteq S} c_i^S\Bigg) = R\Bigg(\sum_{i\in S} c_i^S\Bigg),$$

as desired, where the second equality comes from the fact that the Shapley value is always an efficient allocation and the last equality follows from the fact that $P_i^S \subseteq S\backslash\{i\}$ for each $i \in S$.

Regarding the axiom of Fair Rewarding, we start with the following observation. For the original crowdfunding environment $C = (N, B, R, (c_i)_{i\in N})$, the cooperative game defined by (3.5) is the game $(N, v_C)$ such that, for each $S \subseteq N$,

$$v_C(S) = R\Bigg(\sum_{i\in S: P_i^N\subseteq S} c_i\Bigg).$$

By definition of the subgame $(S, v_{C|S})$ of $(N, v_C)$ induced by $S$, we have, for each $T \subseteq S$, $v_{C|S}(T) = v_C(T)$. Now, the cooperative game $(S, v_{C^S})$ defined by (3.5) for the crowdfunding situation $C^S = (S, B^S, R, (c_i^S)_{i\in S}) \in \mathcal{C}$ is such that, for each $T \subseteq S$,

$$v_{C^S}(T) = R\Bigg(\sum_{i\in T: P_i^S\subseteq T} c_i^S\Bigg).$$

By definition of $c_i^S$, the previous expression can be rewritten as

$$v_{C^S}(T) = R\Bigg(\sum_{i\in T: P_i^N\subseteq T, P_i^S\subseteq T} c_i\Bigg) = R\Bigg(\sum_{i\in T: P_i^N\subseteq T} c_i\Bigg) = v_C(T),$$

where the second equality comes from the fact that $P_i^S \subseteq P_i^N$. We have shown that $v_{C^S} = v_{C|S}$ for each $S \subseteq N$. Next, this equality can be used to write that, for each $S \subseteq N$ and each $i, j \in S$,

$$f_i^*(C^S) - f_i^*(C^{S\backslash\{j\}}) = Sh_i(S, v_{C^S}) - Sh_i(S\backslash\{j\}, v_{C^{S\backslash\{j\}}}) = Sh_i(S, v_{C|S}) - Sh_i(S\backslash\{j\}, v_{C|S\backslash\{j\}}).$$

From Myerson (1980), it is known that the Shapley value satisfies the axiom of balanced contributions, which imposes, for each pair $\{i, j\} \subseteq S$, that $Sh_i(S, v_{C|S}) - Sh_i(S\backslash\{j\}, v_{C|S\backslash\{j\}}) =$

$Sh_j(S, v_{C|S}) - Sh_j(S\backslash\{i\}, v_{C|S\backslash\{i\}})$. As a consequence, we can write that

$$\begin{aligned} & f_i^*(C^S) - f_i^*(C^{S\backslash\{j\}}) \\ = \; & Sh_i(S, v_{C|S}) - Sh_i(S\backslash\{j\}, v_{C|S\backslash\{j\}}) \\ = \; & Sh_j(S, v_{C|S}) - Sh_j(S\backslash\{i\}, v_{C|S\backslash\{i\}}) \\ = \; & f_j^*(C^S) - f_j^*(C^{S\backslash\{i\}}), \end{aligned}$$

from which we conclude that $f^*$ satisfies the axiom of Fair Rewarding.

Secondly, we prove that if an arbitrary reward rule $f$ on $\mathcal{C}$ satisfies the two axioms of Full Distribution and Fair Rewarding, then it must be that $f = f^*$. We proceed by induction on the number of contributors.

Initial step. Consider any $i \in N$ and the crowdfunding situation $C^{\{i\}} = (\{i\}, B^{\{i\}}, R, c_i^{\{i\}})$. By Full Distribution, it is clear that $f_i(C^{\{i\}}) = R(c_i^{\{i\}}) = f_i^*(C^{\{i\}})$.

Induction hypothesis. Assume that $f(C^S) = f^*(C^S)$ for all crowdfunding situations $C^S = (S, B^S, R, (c_i^S)_{i\in S}) \in \mathcal{C}$ such that $s \leq m$, $1 \leq m < n$.

Induction step. Consider any crowdfunding situations $C^S = (S, B^S, R, (c_i^S)_{i\in S}) \in \mathcal{C}$ such that $s = m + 1$. For any pair of players $i, j \in S$, we have that

$$\begin{aligned} & f_i^*(C^S) - f_j^*(C^S) \\ = \; & f_i^*(C^{S\backslash\{j\}}) - f_j^*(C^{S\backslash\{i\}}) \\ = \; & f_i(C^{S\backslash\{j\}}) - f_j(C^{S\backslash\{i\}}) \\ = \; & f_i(C^S) - f_j(C^S). \end{aligned}$$

where the first and third equalities come from the fact that both $f^*$ and $f$ satisfy Fair Rewarding and the second equality follows from our induction hypothesis. Hence, we have $f_i^*(C^S) - f_i(C^S) = f_j^*(C^S) - f_j(C^S)$ for each $i, j \in S$. Summing on $j \in S$ and using the fact that both $f$ and $f^*$ satisfy Full Distribution, we get

$$\begin{aligned} & \textstyle\sum_{j\in S}\big(f_i^*(C^S) - f_i(C^S)\big) = \sum_{j\in S}\big(f_j^*(C^S) - f_j(C^S)\big) \\ \iff \; & s\big(f_i^*(C^S) - f_i(C^S)\big) = R\Big(\textstyle\sum_{i\in S} c_i^S\Big) - R\Big(\textstyle\sum_{i\in S} c_i^S\Big) \\ \iff \; & f_i^*(C^S) - f_i(C^S) = 0, \end{aligned}$$

for each $i \in S$, which proves that $f(C^S) = f^*(C^S)$ and completes the proof. ∎

**Proof.** (**Proposition 14**) Consider any crowdfunding environment $C = (N, B, R, (c_i)_{i\in N})$ and any pair of contributors $i, j \in N$ such that $c_i = c_j$ but $q(i) < q(j)$. Assume that $R$

is an increasing function. We have to show that $f_i^*(C) > f_j^*(C)$. By Proposition 13, this inequality is equivalent to $Sh_i(N, v_C) > Sh_j(N, v_C)$. By definition (3.1) of the Shapley value, it is enough to show that $i$ is at least as desirable as $j$ in $(N, v_C)$, with a strict inequality $v_C(S \cup \{i\}) > v_C(S \cup \{j\})$ for at least one coalition $S \subseteq N \backslash \{i, j\}$. As a start, note that $q(i) < q(j)$ implies that $i \in P_j^N$,

$$P_i^N \subsetneq P_j^N, \tag{3.6}$$

and

$$\left[P_l^N \subseteq S \cup \{j\}\right] \Longrightarrow \left[P_l^N \subseteq S \cup \{i\}\right] \quad \forall S \subseteq N \backslash \{i, j\}, \forall l \in S. \tag{3.7}$$

Using (3.6) and (3.7), we can consider two cases.

Firstly, if $P_j^N \subseteq S \cup \{j\}$, then $P_i^N \subseteq S \cup \{i\}$ and so

$$v_C(S \cup \{j\}) = R\left(c_j + \sum_{l \in S : P_l^N \subseteq S \cup \{j\}} c_l\right) \leq R\left(c_i + \sum_{l \in S : P_l^N \subseteq S \cup \{i\}} c_l\right) = v_C(S \cup \{i\}).$$

Secondly, if $P_j^N \nsubseteq S \cup \{j\}$, then it holds that

$$v_C(S \cup \{j\}) = R\left(\sum_{l \in S : P_l^N \subseteq S \cup \{j\}} c_l\right) \leq R\left(\sum_{l \in S : P_l^N \subseteq S \cup \{i\}} c_l\right) \leq v_C(S \cup \{i\}).$$

We conclude that $v_C(S \cup \{i\}) \geq v_C(S \cup \{j\})$ holds for any $S \subseteq N \backslash \{i, j\}$. Finally, since $i \in P_j^N$, it follows that

$$v_C(P_i^N \cup \{i\}) = R\left(c_i + \sum_{l \in P_i^N} c_l\right) > R\left(\sum_{l \in P_i^N} c_l\right) = v_C(P_i^N \cup \{j\}),$$

which is the desired strict inequality. ■

**Proof.** (**Proposition 15**) Consider any crowdfunding environment $C = (N, B, R, (c_i)_{i \in N})$ and assume that $R$ is a convex function. From Shapley (1971), we have to show that for each $i \in N$ and each $S \subseteq T \subseteq N \backslash \{i\}$,

$$v_C(S \cup \{i\}) - v_C(S) \leq v_C(T \cup \{i\}) - v_C(T). \tag{3.8}$$

Recall that for each $S \subseteq N$, from (3.5), we have

$$v_C(S) = R\Bigg(\sum_{j \in S : P_i^N \subseteq S} c_j\Bigg).$$

In order to save on notation, let $c_S$ stand for $\sum_{j \in S : P_i^N \subseteq S} c_j$. It is easy to figure out that for two coalitions $S \subseteq T$, $\{j \in S : P_i^N \subseteq S\} \subseteq \{j \in T : P_i^N \subseteq T\}$, so that we get the following inequalities :

$$c_S \leq c_{S \cup \{i\}} \leq c_{T \cup \{i\}}, \tag{3.9}$$

and

$$c_S \leq c_T \leq c_{T \cup \{i\}}. \tag{3.10}$$

Both cases $c_{S \cup \{i\}} \leq c_T$ and $c_T \leq c_{S \cup \{i\}}$ are possible but it does not matter for the rest of the proof. Furthermore, remark that

$$\Big(\big\{j \in S \cup \{i\} : P_j^N \subseteq S \cup \{j\}\big\} \backslash \big\{j \in S : P_j^N \subseteq S\big\}\Big) \subseteq \Big(\big\{j \in T \cup \{i\} : P_j^N \subseteq T \cup \{j\}\big\} \backslash \big\{j \in T : P_j^N \subseteq T\big\}\Big),$$

which implies that

$$c_{S \cup \{i\}} - c_S \leq c_{T \cup \{i\}} - c_T. \tag{3.11}$$

From (3.9), (3.10) and the convexity of $R$, we get

$$\frac{R\big(c_{S \cup \{j\}}\big) - R\big(c_S\big)}{c_{S \cup \{i\}} - c_S} \leq \frac{R\big(c_{T \cup \{j\}}\big) - R\big(c_T\big)}{c_{T \cup \{i\}} - c_T} \leq \frac{R\big(c_{T \cup \{j\}}\big) - R\big(c_T\big)}{c_{S \cup \{i\}} - c_S},$$

where the last inequality comes from (3.11). Therefore, it follows that

$$R\big(c_{S \cup \{j\}}\big) - R\big(c_S\big) \leq R\big(c_{T \cup \{j\}}\big) - R\big(c_T\big),$$

which is equivalent to (3.8) as desired. ∎

**Proof.** (**Proposition 16**) Part *(i)*. We start by showing that any player in group $G_1$ is a necessary player in the associated game $(N, v_C)$. So pick any $i \in G_1$ and any $S \subseteq N \backslash \{i\}$. Then,

$$\sum_{j \in S : P_j^N \subseteq S} c_j \leq \sum_{j \in S \cap G_1} c_j < c^*,$$

where the strict inequality comes from the definition of $G_1$ and the fact that $i \in G_1 \backslash S$. Hence, since $R$ is the threshold function, we obtain that $v_C(S) = 0$, proving that $i$ is

a necessary player in $(N, v_C)$. Because $i$ was chosen arbitrarily in $G_1$, conclude that all players in $G_1$ are necessary in $(N, v_C)$, which in turn implies that these players are all equal in $(N, v_C)$. By the axiom of equal treatment of equals, we get that $Sh_i(N, v_C) = Sh_j(N, v_C)$ for each $i, j \in G_1$, which is equivalent to $f_i^*(C) = f_j^*(C)$ for each $i, j \in G_1$ as desired.

Part *(ii)*. We prove that if $i, j \in G_2$ and $c_i \geq c_j$, then $i$ is at least as desirable as $j$ in $(N, v_C)$. So, pick any $S \subseteq N \backslash \{i, j\}$. Because $i$ and $j$ are in $G_2$, they belong to the same time unit $B_{q^*}$. This implies that $P_i^N = P_j^N$ and so that

$$\sum_{l \in S \cup \{i\} : P_l^N \subseteq S \cup \{i\}} c_l \geq \sum_{l \in S \cup \{j\} : P_l^N \subseteq S \cup \{j\}} c_l.$$

The latter inequality is equivalent to $v_C(S \cup \{i\}) \geq v_C(S \cup \{j\})$, proving that $i$ is at least as desirable as $j$ in $(N, v_C)$. From the axiom of desirability, we get that $Sh_i(N, v_C) \geq Sh_j(N, v_C)$ for each $i, j \in G_2$ such that $c(i) \geq c(j)$, or equivalently that $f_i^*(C) \geq f_j^*(C)$ for each $i, j \in G_2$ such that $c(i) \geq c(j)$.

Part *(iii)*. We proceed in two steps. Firstly, we show that any contributor in $G_3$ is a null player in game $(N, v_C)$. So let $i \in G_3$ and choose any $S \subseteq N \backslash \{i\}$. Two cases can be distinguished. If $\sum_{j \in S : P_j^N \subseteq S} c_j \geq c^*$, then obviously $v_C(S) = r^* = v_C(S \cup \{i\})$. To the contrary, if $\sum_{j \in S : P_j^N \subseteq S} c_j < c^*$, then it must be that $(G_1 \cup G_2) \nsubseteq S$, which implies that $P_i^N \nsubseteq S \cup \{i\}$ since $(G_1 \cup G_2) \subseteq P_i^N$. Similarly, if $j \in S$ is such that $P_j^N \nsubseteq S$, then $P_j^N \nsubseteq S \cup \{i\}$ as well. Therefore, $\sum_{j \in S : P_j^N \subseteq S} c_j = \sum_{j \in S \cup \{i\} : P_j^N \subseteq S \cup \{i\}} c_j$, leading once again to the equality $v_C(S) = v_C(S \cup \{i\})$. Conclude that $i$ is a null player in $(N, v_C)$.

Secondly, we show that if $i \in G_1$, $j \in G_2$ and $l \in G_3$, then $i$ is at least as desirable as $j$ and $j$ is at least as desirable as $l$ in $(N, v_C)$. Consider any $S \subseteq N \backslash \{i, j\}$. From part *(i)*, we have that $v_C(S \cup \{j\}) = 0$ since contributor $i$ is necessary in $(N, v_C)$. By monotonicity of $(N, v_C)$ (see remark 1) and $v_C(\varnothing) = 0$, we obtain that

$$v_C(S \cup \{j\}) = 0 \leq v_C(S \cup \{i\}), \tag{3.12}$$

proving that $i$ is at least as desirable as $j$ in $(N, v_C)$. Next, pick any $S \subseteq N \backslash \{j, l\}$. We have already proved that $l$ is a null player in $(N, v_C)$ at the beginning of part *(iii)*. Together

with the monotonicity of $(N, v_C)$, this implies that

$$v(S \cup \{l\}) = v(S) \leq v(S \cup \{j\}), \tag{3.13}$$

proving that $j$ is at least as desirable as $l$ in $(N, v_C)$. It is easy to show that the previous two inequalities can be strict by choosing $S = P_i^N$ in (3.12) and $S = P_j^N$ in (3.13). It follows that $Sh_i(N, v_C) > Sh_j(N, v_C) > Sh_l(N, v_C) = 0$ or, equivalently, $f_i^*(C) > f_j^*(C) > f_l^*(C)$. ■

**Proof.** (**Proposition 17**) Suppose that the reward function of a crowdfunding environment is additive and assume further that each time window contains one and only one contributor. For simplicity, assume that the contribution timing is consistent with the natural order on natural numbers, i.e. $B = (B_1, \ldots, B_n)$ with $B_i = \{i\}$ for each $i \in N$. In such a case, let us show that the corresponding cooperative game $(N, v_C)$ coincides with the liability game obtained from the pure sequential liability situation in which $d_i = c_i$ and $B_i^L = \{i\}$ for each $i \in N$. In fact, letting $i_S$ be the greatest player in $S$ for which $P_i^N \subseteq S$, for each $S \subseteq N$, we have

$$v_C(S) = R\left(\sum_{i \in S : Pi^N \subseteq N} c_i\right) = R\left(\sum_{i=1}^{i_S} c_i\right) = \sum_{i=1}^{i_S} c_i,$$

where the second equality follows from the fact that $B = B^L$ contains $n$ singletons and the third equality comes from the additivity of $R$. This completes the proof. ■

# Conclusion générale

La thèse permet de mettre en évidence la richesse des différents outils ou concepts de solution que nous propose la théorie des jeux coopératifs en application à des contextualisations économiques originales ; à l'économie de la santé, aux problèmes de taxation internationale et au financement participatif. De nombreux résultats théoriques, empiriques et sur la méthode axiomatique sont présentés et émergent des modèles utilisées ou des données mobilisées. A travers les trois différents chapitres, nous développons donc un point de vue pour appréhender certains problèmes et conseiller sur la méthode de partage à employer. Le choix repose sur des critères d'équité et/ou de stabilité qui offrent des possibilités intéressantes à l'arbitre pour choisir à bon escient la règle de partage.

Mais il existe encore beaucoup de champs d'applications qui nous permettraient de mobilisr les outils et les concepts de solution, notamment un projet actuel concernant l'économie digitale. En effet, pendant ma dernière année de doctorat, j'ai pu m'intéresser à une quatrième application originale, en collaboration avec Florian Navarro (U. Angers) dans un projet en cours d'écriture permettant d'utiliser les outils de la théorie des jeux coopératifs dans le domaine de l'économie numérique. Ainsi, nous nous intéressons à la répartition des revenus des ayants-droits sur les plateformes de streaming musical (Spotify, Deezer, Apple Music, etc.) dont l'étude a été commandée par le rapport du Centre National de la musique. En effet, les artistes présents sur les catalogues offerts par ces plateformes peuvent être rémunérés de plusieurs façons. Une répartition « à la part de marché » a longtemps été le mode de partage pour rémunérer les artistes. Cependant, on s'aperçoit assez vite des limites d'une telle répartition puisqu'elle néglige les écoutes individualisées, ce qui engendre une sorte d'inéquité puisque chaque artiste gagne plus en fonction de sa part de marché, au détriment des jeunes artistes ou des artistes peu connus. Prenons dès lors un exemple, si un consommateur dépense chaque mois 20 euros pour son abonnement à l'une des plateformes utilisant ce mode de répartition, et que dans le même temps, un artiste possède 50% des parts de marché, alors ce dernier pourra prétendre recevoir 50% et donc 10 euros de l'abonnement de l'utilisateur, et ce, que l'artiste soit écouté ou non ! Ainsi, l'idée serait donc de créer de nouvelles règles de partage plus équitable et prenant plus en considération les écoutes et non les parts de marché. Dès lors, nous pouvons mobiliser les jeux coopératifs et la valeur de Shapley pour proposer une alternative. Une analyse également empirique mobilisant des données de l'ARCOM ou des différentes plateformes est également envisageable pour tester notre modèle et notre règle de partage.

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