



**HAL**  
open science

# Risk management of Stable Value-type pension guarantees and study of liquidity problems in derivatives products

Behzad Alimoradian

► **To cite this version:**

Behzad Alimoradian. Risk management of Stable Value-type pension guarantees and study of liquidity problems in derivatives products. Business administration. Université Claude Bernard - Lyon I, 2023. English. NNT : 2023LYO10174 . tel-04650952

**HAL Id: tel-04650952**

**<https://theses.hal.science/tel-04650952>**

Submitted on 17 Jul 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

**THESE de DOCTORAT DE  
L'UNIVERSITE CLAUDE BERNARD LYON 1**

**Ecole Doctorale N° ED 486**

**Discipline : Sciences Économiques et Gestion**

Soutenue publiquement le 27/09/2023, par :

**Behzad Alimoradian**

---

**Gestions de risque des garanties de  
pension de type Stable Value et  
Etude sur des problèmes de liquidité  
dans les produits dérivés**

---

Devant le jury composé de :

Planchet, Frédéric	Professeur à l'Université Lyon 1	Président
Barrieu, Pauline	Professeure à London School of Economics	Rapporteure
Hillairet, Caroline	Professeure à l'ENSAE Paris	Rapporteure
Bauer, Daniel	Professeur à l'Université du Wisconsin-Madison	Examineur
Loisel, Stéphane	Professeur à l'Université Lyon 1	Directeur de thèse
Salhi, Yahia	Maitre de Conférences à l'Université Lyon 1	Co-directeur de thèse
Jakubiak, Jeffrey	Gérant Associé à Valerian Capital Group	Invité

# Contents

<b>Résumé</b>	<b>1</b>
<b>Summary</b>	<b>4</b>
<b>Acknowledgements</b>	<b>6</b>
<b>I Introduction</b>	<b>9</b>
<b>General Introduction</b>	<b>10</b>
<b>I.1 Synthetic GIC Asset Liability Modeling</b>	<b>13</b>
I.1.1 What is a Stable Value Fund? . . . . .	14
I.1.1.1 Types of Stable Values Offering . . . . .	15
I.1.1.2 Stable Value Funds' Historical Development . . . . .	16
I.1.1.3 Stable Value Return Stabilization Mechanism . . . . .	18
I.1.1.3.1 Exploring Research on Guaranteed Funds . . . . .	18
I.1.1.4 Mathematical modeling framework . . . . .	19
I.1.1.4.1 Book Value Wrap Accounting . . . . .	20
I.1.1.5 Academic Literature on Stable Value . . . . .	22
I.1.1.6 Regulatory Guidance on Quantitative Asset Liability Man- agement of the Synthetic GIC . . . . .	23
I.1.1.6.1 U.S. GAAP . . . . .	24
I.1.1.6.2 Stochastic Exclusion Test . . . . .	26
I.1.1.6.3 Reserves . . . . .	26
I.1.1.6.4 Capital Charges . . . . .	27
I.1.1.7 Market Practice . . . . .	29
I.1.2 Fixed Income Asset Modeling . . . . .	31
I.1.2.1 Fixed Income Securities . . . . .	32
I.1.2.2 Three Important Characteristics of Fixed Income Securities .	33
I.1.2.2.1 Yield . . . . .	33
I.1.2.2.2 Duration . . . . .	34
I.1.2.2.3 Spread . . . . .	35
I.1.2.3 Fixed Income Funds: A Portfolio of Fixed Income Securities	36
I.1.2.4 Our Modelling Approach to Fixed Income Funds . . . . .	37
I.1.3 Liability Participant Cash Flows Modeling . . . . .	37

I.1.3.1	Review of Lapse Hypothesis and Their Relevance to Stable Values . . . . .	37
I.1.3.1.1	The Influence of Employers on Investment Behavior . . . . .	38
I.1.3.1.2	401(k) Investor’s Inattentiveness . . . . .	38
I.1.3.1.3	Flight-to-Safety . . . . .	40
I.1.3.1.4	Moneyness Hypothesis . . . . .	41
I.1.3.1.5	Rate Deficit Hypothesis . . . . .	41
I.1.3.2	Ecosystem, Fiscal, and Legal Considerations Impacting Stable Value Participants’ Behavior . . . . .	43
I.1.3.3	Data for Research . . . . .	47
I.1.3.4	Conclusion and the Liability Model . . . . .	47
I.1.4	Asset Liability Modeling . . . . .	48
I.1.4.1	Literature Review of Asset-Liability Models for Guaranteed Products . . . . .	48
I.1.4.2	Synthetic GIC Asset-Liability Management . . . . .	49
I.1.4.3	Conclusions . . . . .	51
<b>I.2</b>	<b>Derivatives Under Market Impact</b>	<b>54</b>
I.2.1	Market Impact Theories . . . . .	55
I.2.1.1	Three Types of Market Impact . . . . .	55
I.2.2	Market Impact and Information . . . . .	58
I.2.3	Derivatives under Market Impact . . . . .	61
I.2.3.1	Full Hedge Approach . . . . .	61
I.2.3.2	Market Manipulation Regulation in Derivative Strategies . . . . .	63
I.2.3.3	Partial Hedging Approach . . . . .	65
<b>I.3</b>	<b>Summary of Major Contributions</b>	<b>67</b>
I.3.1	A Quantitative Framework for Asset Liability Risk Management of Synthetic GIC . . . . .	67
I.3.2	Derivatives under Market Impact . . . . .	69
<b>II</b>	<b>Synthetic GIC Asset Liability Modeling</b>	<b>70</b>
<b>II.1</b>	<b>A Model for Fixed Income Funds</b>	<b>71</b>
II.1.1	Introduction . . . . .	71
II.1.2	Fixed Income Funds Modeling Framework . . . . .	72
II.1.3	Empirical results . . . . .	75
II.1.4	Model Limitations, Conclusions, and Prospects for Further Refinement . . . . .	80
<b>II.2</b>	<b>Understanding Key Drivers of Participant Cash Flows for Individually Managed Stable Value Funds</b>	<b>82</b>
II.2.1	Introduction . . . . .	82



II.2.2	Literature Review on Lapse Behavior, Contextualizing It into the Stable Value Ecosystem, and Hypothesis Development . . . . .	84
II.2.2.1	Individually Managed Stable Value Regulation and Ecosystem	84
II.2.2.1.1	The 401(k) Withdrawal Treatment . . . . .	85
II.2.2.1.2	The 401(k) Inner Transfers and Investment Options	86
II.2.2.2	Literature Review and Hypothesis Development . . . . .	87
II.2.3	Methodology, and Data Collection and Cleansing . . . . .	91
II.2.4	Observations . . . . .	93
II.2.4.1	Trends . . . . .	93
II.2.4.2	Plan Sponsor’s Ecosystem . . . . .	97
II.2.4.3	Rate Deficit Arbitrage . . . . .	99
II.2.4.4	Herd Behavior . . . . .	101
II.2.4.5	Flight-to-Safety Behavior . . . . .	102
II.2.4.6	Moneyness Hypothesis . . . . .	105
II.2.5	Discussion and Conclusions . . . . .	107
II.2.A	Appendix II.2.A . . . . .	110
II.2.B	Appendix II.2.B . . . . .	111
II.2.C	Appendix II.2.C . . . . .	112
<b>II.3</b>	<b>Risk Assessment for Synthetic GICs: A Quantitative Framework for Asset-Liability Management</b>	<b>115</b>
II.3.1	Introduction . . . . .	116
II.3.2	Stable Value Guaranteed Insurance Contract . . . . .	118
II.3.2.1	Book Value Wrap Accounting . . . . .	119
II.3.2.2	Insurance Guarantee . . . . .	121
II.3.3	Asset Model . . . . .	122
II.3.4	Liability Cash Flow Model . . . . .	123
II.3.4.1	Regime Changing Trend Component . . . . .	124
II.3.4.2	Herd Component . . . . .	126
II.3.4.3	Rate Deficit Component . . . . .	126
II.3.4.4	Flight-to-safety Component . . . . .	127
II.3.5	Wrap Risk Results . . . . .	127
II.3.6	Risk Scenarios to be Considered by an Insurer . . . . .	130
II.3.6.1	Inflation . . . . .	130
II.3.6.2	Yield Spike . . . . .	133
II.3.7	Path Dependence of Synthetic GIC Risk and the Suitability of the Liability Model . . . . .	135
II.3.8	Conclusions . . . . .	137
II.3.A	Appendices . . . . .	139
<b>II.4</b>	<b>Stable Value Wrap: A Risk Perspective for a Post Pandemic Market</b>	<b>142</b>
II.4.1	Managers Chasing Yields Amid Post-Pandemic Inflation Expectations	142
II.4.2	Behavioral Bias in Assessing the Risk of Stable Value Guarantees . .	143

II.4.2.1	Tendency to Underestimate Remote Risks . . . . .	143
II.4.2.2	Tendency to View the World Linearly . . . . .	143
II.4.3	Quantifying the Wrap Tail Risk . . . . .	144
II.4.4	Conclusions . . . . .	145
<b>III</b>	<b>Derivatives under Market Impact</b>	<b>146</b>
<b>III.1</b>	<b>Derivatives under Market Impact: Disentangling Cost and Information</b>	<b>147</b>
III.1.1	Introduction . . . . .	147
III.1.2	Market Ecosystem Description . . . . .	150
III.1.3	The Market Impact Model . . . . .	151
III.1.4	Option Hedges for the Insider Trader . . . . .	153
III.1.5	Large Trading Program . . . . .	158
III.1.5.1	Trading Portfolio of Cash and Stock . . . . .	158
III.1.6	Physical Delivery Options and Hedging Strategies for Large Traders	161
III.1.6.1	Manipulating the Payoff Constraints and Problem Reduction	162
III.1.7	Numerical Results . . . . .	164
III.1.7.1	Transaction Cost Parametrization . . . . .	164
III.1.7.2	Solving the Partial Differential Equation . . . . .	166
III.1.7.3	Numerical Results . . . . .	167
III.1.8	Conclusion . . . . .	171
III.1.A	Appendices . . . . .	173
<b>IV</b>	<b>Conclusions and Perspectives</b>	<b>175</b>

# Résumé

Cette thèse examine des défis rencontrés dans les secteurs de la finance et de l'assurance, notamment dans des contextes où les approches de modélisation standard s'avèrent inadéquates. Les motivations de cette recherche proviennent entre autres de problématiques rencontrées par les praticiens et de divergences observées par l'auteur entre la modélisation théorique et la réalité du terrain.

Le premier cas porte sur une garantie d'assurance complexe : le contrat d'investissement garanti synthétique (GIC synthétique). Ce produit, significativement présent dans le secteur des retraites aux États-Unis, est examiné dans cette thèse.

Le deuxième cas concerne la couverture des produits dérivés sur un marché financier lorsque l'actif sous-jacent est illiquide, ce qui entraîne une insuffisance de la théorie d'arbitrage. La thèse examine de manière exploratoire certains défis qui en découlent. Les années d'expérience de l'auteur en tant qu'analyste quantitatif dans le secteur financier ont mené à une réflexion sur un problème moins exploré et lié au rôle de l'information dans les marchés et à des problèmes de liquidité.

## **Gestion des risques des contrats de garanties de pensions de type stable value**

Les fonds de pension offrent une stabilité de performance d'investissement et un ratio rendement-risque avantageux en raison de leur diversification. Cependant, cette stabilité peut être compromise par les risques de marché, une gestion d'investissement insatisfaisante ou d'autres facteurs. Dans de telles situations, les sociétés d'assurance peuvent jouer un rôle important en tant que garants pour préserver la sécurité financière.

Cette recherche porte sur certaines garanties de fonds de pension nommées « stable value », régulées par l'Internal Revenue Service du Département du Trésor des États-Unis, couvrant plus de 800 milliards de dollars d'actifs sur le marché des retraites des États-Unis. Parmi ces garanties d'assurance, on retrouve le contrat synthétique d'investissement garanti, le « GIC synthétique », couvrant plus de 400 milliards de dollars d'actifs sur le marché de la retraite des États-Unis.

Malgré la complexité de ce produit et sa part significative sur le marché américain, il existe un manque de recherche universitaire dans ce domaine, ce qui a motivé l'entreprise de ce projet de recherche et de cette thèse. Cela est d'autant plus important que la probabilité

d'activation de la garantie, et donc de paiement par l'assureur des garanties du GIC synthétique est faible, bien que la valeur nominale du paiement potentiel puisse être très élevée. Pour ces raisons, il existe une incertitude - partagée par les praticiens - quant à la mesure du risque.

L'objectif de cette thèse est de développer une théorie économique et mathématique pour évaluer le risque de ces garanties d'investissement et déterminer les scénarios les plus défavorables qu'il convient d'étudier. L'un des objectifs est de créer un modèle de gestion actif/passif hybride qui intègre à la fois les facteurs de risque actuariels et financiers. Pour atteindre cet objectif, plusieurs étapes doivent être franchies. Nous expliquons brièvement ci-dessous chaque étape et la contribution de cette thèse.

La première étape est une étude des fonds sous-jacents de la garantie GIC synthétique. C'est le sujet abordé dans le chapitre II.1. Ce chapitre introduit un modèle mathématique pour modéliser les fonds obligataires. Contrairement à la plupart de la littérature qui modélise un fonds obligataire à l'aide de la formule de la somme des paiements de coupons, dans ce manuscrit, la dynamique est modélisée en utilisant la formule de la duration. La pertinence statistique du modèle présenté est examinée et confrontée à d'autres modèles alternatifs ; il est constaté que le modèle possède une capacité explicative similaire à celle de la somme des paiements des coupons lors de la comparaison des rendements réalisés par le fonds obligataire.

Le SGIC est un produit d'actuariat, et le risque associé dépend du comportement des retraits effectués par les bénéficiaires. Il est donc primordial de comprendre le comportement des flux de trésorerie des participants. C'est le sujet du chapitre II.2. Dans ce chapitre, une base de données de flux de trésorerie, représentant environ 80% du marché du GIC synthétique, a été analysée. En exploitant ces données uniques et en étudiant l'écosystème 401(k), plusieurs hypothèses comportementales sur les rachats ont été examinées. Il a été démontré que les flux de trésorerie présentent des tendances non monotones à moyen et long terme. Il a été aussi observé que des facteurs internes à l'écosystème de l'employeur, tels que la croissance de l'emploi, les options par défaut du 401(k) et l'introduction de nouvelles options d'investissement, ont un impact significatif sur ces tendances. De plus, il a été constaté que le comportement de "fuite vers la sécurité" joue un rôle prédominant lors des crises du marché financier. Des retraits massifs dus à des problèmes de réputation ont été aussi observés dans les données historiques avec une faible probabilité d'occurrence. Ainsi, certaines hypothèses comportementales abordées dans la littérature, telles que l'hypothèse de la "valeur intrinsèque", semblent moins pertinentes dans ce contexte.

Finalement, avec une compréhension des risques de rachat et des risques financiers, un modèle de gestion actif/passif est établi. C'est le sujet du chapitre II.3. À travers ce modèle, deux scénarios de perte principaux sont identifiés : le scénario *inflationniste* et le scénario *pic de rendement puis retour à la moyenne*. Ces scénarios ont été testés contre des événements historiques aux États-Unis, tels que la Grande Dépression, la période inflationniste des années 1980, la crise financière de 2008 et la pandémie de 2020.

Dans le chapitre II.3, les mesures de risque obtenues par simulation du modèle sont également comparées avec l'approche du scénario unique formulée par l'Association Nationale des Régulateurs d'Assurance américains NAIC (2015). Cette comparaison révèle que, selon la formule réglementaire américaine, les provisions et les capitaux nécessaires sont souvent évalués à zéro. Cette incohérence met en lumière les limites potentielles des mesures de risque actuarielles basées sur la réglementation.

Finalement, le chapitre II.4 met en lumière les implications pratiques du modèle quantitatif présenté dans les chapitres précédents. Il comprend une lettre distribuée parmi les acteurs de l'industrie, servant d'exemple concret sur la façon dont la méthodologie quantitative présentée dans ce manuscrit peut être employée pour examiner les problématiques d'actualité. Plus précisément, il se concentre sur la réaction des assureurs face aux demandes des gestionnaires de fonds dans le contexte d'un marché post-pandémique.

## Couverture des options sur les marchés illiquides

Suite à la discussion sur les garanties GIC synthétiques, la thèse s'oriente vers le deuxième thème de recherche, portant sur la couverture des options sur les marchés illiquides. Ce volet de la recherche a été stimulé par une situation réelle rencontrée dans une banque d'investissement. Un trader a consulté l'auteur pour obtenir des conseils sur l'évaluation et la gestion des risques d'un produit dérivé de taille significative sur un marché peu liquide.

Dans le chapitre III.1, nous explorons les implications de l'activité d'un grand trader ayant un impact significatif sur un marché. Nous adoptons le point de vue d'un autre trader, informé de la stratégie de trading du grand trader. À travers cette perspective, nous établissons l'existence de mesures dites *information-neutre*, dans lesquelles l'actif actualisé est considéré comme un processus de martingale pour ce trader informé. Une stratégie de couverture pour les dérivés est alors élaborée à l'attention du grand trader, prenant en compte à la fois les coûts de transaction et les activités de couverture propres au trader, tout en visant à atténuer le risque de manipulation du marché. Ce volet de la thèse vise à explorer comment un grand trader peut couvrir efficacement un produit dérivé, sans provoquer involontairement une manipulation du marché.

# Summary

In this thesis, themes drawn from the author's experience in the finance and insurance industries are explored, motivated by the observation of gaps in both theoretical and practical models in the literature relating to certain aspects of these sectors.

## The Synthetic GIC

The primary theme explored by the author concerns a specific pension fund guarantee structure known as "stable value." Regulated by the U.S. Department of Treasury's Internal Revenue Service, stable values cover \$800 billion in assets within the U.S. retirement market, for which approximately \$400 billion account for Synthetic Guaranteed Investment Contracts commonly referred to as *Synthetic GICs*.

Because of very low payout probability and potentially large notional value of stable value guarantees in case of payment, there is a lot of uncertainty among practitioners about risk measurement best practices. Therefore, this thesis aims to develop a quantitative framework to assess the risk associated with these investment guarantees. One of the objectives is to create an Asset Liability Management (ALM) model that incorporates both actuarial and financial risk factors. Achieving this goal requires several steps, each of which has led to new findings and insights.

The Synthetic GIC, as a financial product, inherently depends on the financial market. Therefore, in order to truly understand the Synthetic GIC, one must comprehend its underlying fixed-income funds. As such, in Chapter [II.1](#), a novel mathematical model is introduced for these fixed-income funds. This new approach challenges conventional literature on the sum of coupon payments, which predominantly emphasizes secondary risks.

The Synthetic GIC is fundamentally an actuarial product, and the withdrawal behaviors of participants significantly influence the level of risk associated with it. In Chapter [II.2](#), key behavioral assumptions, such as flight-to-safety behaviors during global market crises and the impact of employment growth and investment options on cash flow, are investigated using unique aggregate cash flow data representing around 80% of the market.

Thanks to the foundational understanding established in Chapters [II.1](#) and [II.2](#), we construct an Asset-Liability Management (ALM) model for Synthetic GICs in Chapter [II.3](#). We conduct a scenario analysis based on the ALM model, identifying two primary loss

scenarios: the inflationary scenario and the yield spike scenario. Furthermore, we examine regulatory aspects, comparing the tail risk model with other regulatory frameworks, such as the single scenario approach by the National Association of Insurance Commissioners. Our findings suggest potential limitations inherent in these regulatory measures.

Chapter II.4 highlights the practical implications of the quantitative risk framework in place in this thesis. It corresponds to a letter distributed among industry participants, exemplifying how our quantitative methodology can be leveraged to confront and analyze prevailing issues. Specifically, it focuses on insurers' response to managers' demands in the context of a post-pandemic market.

## **Derivatives under market impact**

Another area addressed in this thesis is derivatives pricing theory under market impact. Chapter III.1 examines the implications of a large trader's activity on the martingale pricing theory. Motivated by a practical issue faced by the author at an investment bank, this research examines the hedging strategy for a large trader. This involves balancing transaction costs while being aware of the market impact of their own activities. Taking such an approach, we consider a certain type of insider trader and show that there exists a set of *information-neutral probability measures*, under which the discounted asset is a martingale process for the insider trader. We propose a derivative hedging policy for the large trader while mitigating market manipulation.

# Acknowledgements

Beginning my PhD journey four years ago as a more mature student, I have now reached 40; within this journey of life there were people who believed in me, offered help along the way, and added meaning to my life. This thesis is a reflection of the bonds I have formed and the trust and openness bestowed upon me by those people. I feel a profound duty to express my gratitude, not merely as a formal requirement of a Ph.D. thesis, but if I were to omit this, I would have missed grasping and remembering the true significance of their presence. I hope I can play the same role for the people around me, too.

I would like to first express my deepest gratitude to my grandfather, Hossein. Our time together was brief, as our paths on this earth didn't overlap for long. I vividly remember visiting him in the hospital when I was ten years old, the day I saw a reflection of his joy in his brightening eyes upon seeing me. This him in me continues to live. A friend once told me that a person dies twice: first when he dies, and when the last person who remembers him dies. As probably the last person who remembers him, I hope these words extend his memory as a counter to his death so that it is scripted of how a genuine human being he was.

I am deeply grateful to Mr. Ali Asghar Eskandar Bayati, who guided me towards Sharif University after my university entrance exam. At Sharif, I forged a strong connection with him. He is one of the most vibrant, intelligent, and caring people I met there. I am so thankful for his interest in my activities, and his care left a lasting impression. I recall his thoroughness when he read an entire newspaper featuring an article about a Philosophy reading group I organized at the university –. It was through his introduction to Ecole Polytechnique alumni that I found myself visiting this institution during a summer trip to France; otherwise, I might have never moved to a French university.

My profound thanks go to my PhD supervisor, Stephane Loisel. From our initial meeting in London, his trust and openness have been invaluable, making me feel an integral part of the academic community from the very beginning. I thank him for his great empathy, which was crucial during the challenging phases of my PhD journey. His distinctive way of handling complex situations, reminiscent of a mathematician's methodical style, reassuringly stating, "There is a problem, we will find a solution.", was a management lesson for me.

I am deeply grateful to Jeff Jakubiak, who has been much more than a manager to me, a mentor through all these years. After our first meeting in George Town at the Indian



restaurant by the old HSBC building, filled with my questions about the stable value business, I felt a special connection. I appreciate the lessons learned from him in cultivating authentic and caring human interactions; his approach, emphasizing genuine connection over business, inspires loyalty and respect, willing to work out of true allegiance to his compassionate and authentic nature, and observing this through the years was a great learning lesson for me. I appreciate his support for this research project.

My gratitude extends to my second supervisor, Yahia Salhi, for his invaluable brainstorming sessions. His analytical perspective on complex issues has significantly enhanced my research methodology. Additionally, his contribution to creating a welcoming and friendly work environment is deeply appreciated.

I am thankful to Professors Pauline Barrieu and Caroline Hillairet for their insightful feedback, which significantly enhanced the quality of my manuscript, and to Prof. Frédéric Planchet for his engaging discussions and pertinent questions during my defense. I also extend my gratitude to Prof. Bauer for his thought-provoking questions and his acceptance to participate in the jury despite the time zone challenges.

I am grateful to my co-authors Anne Eyraud-Loisel and Karim Barigou. Your insights transformed an industry insight into great research work. Our collaboration was marked by complementary skills, creating a vibrant and enjoyable atmosphere in every meeting and discussion, which made our interactions a joyful and productive experience.

I also thank Scott Sokol for his support, the trust he made in me, and the effort he made for the research to be visible within the stable value community. I am thankful to my colleague Peter Torsson for the great effort he makes to keep and maintain the data and for his great positive attitude. I am thankful to Joe Brickner for all the interactions and help in getting me acquainted with the stable value business.

I am also grateful to Alvin Ngugi for the engaging and insightful discussions we have shared. Our working relationship has been both enjoyable and fruitful, marked by a strong synergy that has significantly contributed to our collaborative efforts. Elio Farina for helping me improve my latex skills.

I also would like to thank my Ph.D. student peers, Sarah Bensalem and Pierre Montesinos, for their support in helping me acclimate to university life and for fostering great friendships. I am thankful for the camaraderie of Natalia, Tachfine, and Carine, whose vibrancy and joy greatly enriched my PhD journey. Tragically, we lost Carine during this time, a profound loss that deeply affected us all. Her spirit remains a cherished part of my academic and personal experience. I would like to thank Etienne for his friendship and his generosity in sharing with me his source code and knowledge. I am thankful to Julian Vedani for fostering a warm and welcoming environment; being part of his trekking training adventures added a unique and enjoyable dimension to my journey during Covid. To Nicolas Leboine, Alexis Bienvenue, Denys Pommeret, Pierre-Olivier Goffard, Xavier

Milhaud, and Djana Mokhtari for the friendly environment and help over the last four years.

I also would like to thank my friends and colleagues Dominique Bang for his great support and mentorship during my investment banking years and since then, and Artur Sepp, who helped me along the way to present my research at international conferences.

My thanks also extend to Matheus Tanha and Amir Esmaili, who have patiently listened to my endless musings over the years. To Robert, for the profound philosophical discussions during our tennis matches amid the pandemic; to Chris, for the memorable beach days and our adventurous times sneaking into the Westin's jacuzzi. To Nicholas Pairaudeau (Nick) and Victoria for fostering such a wonderful community and friendship over the past four years, and Nick for organizing such great Crypto events. I am grateful for Shabnam's enduring support and patience over the last year; her unwavering presence and heartwarming assistance have been invaluable to me, providing strength throughout my journey.

I extend my thanks to my sister Neguine for always enduring my brotherly jokes with grace and to my niece Chloe for having made her first Ph.D. defense and for her patience during the lengthy defense session. My cousin Caroline's vibrant spirit and infectious joy have been a constant source of happiness. To my parents, who have given me unconditional love: as I write this, I find myself at a loss for words. Our relationship is so profound that a few lines cannot fully express my feelings and gratitude.

I would also like to acknowledge my heritage and those who are not here with us. I would like to express my gratitude to my grandma Derakhshandeh, whom I greatly missed during the days my sister and I spent at her house in Tehran during our childhood.

I am grateful for the rich cultural heritage that has ingrained in me a poetic understanding of life and helped me appreciate meaningful connections and friendships in life. As the Persian poet Saadi expressed:

I was not a being yet, when I first fell in love.

## **Part I**

# **Introduction**

# General Introduction

Over my 18 years of experience as a quantitative risk analyst in the insurance and investment banking industry, I have been exposed to practical challenges, some of which have not been addressed with sufficient scientific rigor in the industry. Sometimes, temporary fixes are implemented as practical solutions, but over time, they become a legacy and are passed down through generations of professionals. Conversely, while some of these problems might have been touched upon in academia, they occasionally seem misinterpreted, leading to a disconnect between the approach and the actual challenges faced in the field. My research started with an intellectual curiosity to address this gap. As such, two themes are addressed in this thesis. The main theme relates to a type of Guaranteed Investment Contract (GIC) in the U.S. market known as stable value. This thesis results from a partnership between Université Claude Bernard Lyon 1 and my current employer, Valerian Capital, which provides consultancy services to insurance companies offering stable value insurance guarantees to the pension industry. The second theme concerns derivatives pricing under market impact and is inspired by my previous experience as a quantitative analyst in a bank in London.

The thesis is structured into four parts. Part **II** delves into the primary theme: asset-liability modeling of the Guaranteed Investment Contracts mentioned earlier. The secondary theme, derivatives under market impact, is explored in Part **III**. Part **IV** concludes the thesis with a discussion on prospective future research. As the thesis is paper-based<sup>1</sup>, it is common practice in our research environment to present the context, the motivation, the pre-existing literature and market practice, as well as the main contributions of the thesis in an introduction. Part **I** plays this role.

Following the same flow as the corpus of the thesis, the introduction, Part **I**, is itself split into various chapters to introduce each theme separately. Chapter **I.1** is an introduction to the primary theme of Guaranteed Investment Contracts asset-liability quantitative risk framework, while Chapter **I.2** introduces the secondary theme of derivatives under market impact. Lastly, Chapter **I.3**

---

<sup>1</sup>A paper-based thesis is a collection of inter-related articles on one or several specialized topics authored or co-authored by the Ph.D. candidate.

provides a summary of the major contributions made in this thesis once main concepts have been introduced<sup>2</sup>.

The primary theme of this thesis, covered in Part II, revolves around the asset-liability modeling of the so-called Synthetic Guaranteed Investment Contract (Synthetic GIC)<sup>3</sup>. This section's principal objective is to develop a quantitative risk framework for this type of contract, a goal we accomplish in Chapter II.3 of Part II. The path to this goal necessitated the exploration and comprehension of several intriguing aspects, including a new modeling approach for fixed-income assets (Chapter II.1) and the modeling of participants' cash flow liabilities (Chapter II.2). Lastly, to illustrate the practical utility of our quantitative risk modeling approach, Chapter II.4 presents a letter disseminated to several industry stakeholders. It serves to demonstrate how our quantitative approach can be used to assess and address real-world issues of a post-pandemic market.

The introduction in Chapter I.1 follows the same structure as Part II. Section I.1.1 serves as an introduction to Stable Values, whereas Section I.1.2 leads into Chapter II.1 by discussing the modeling of assets, particularly fixed-income funds. Section I.1.3 gives an overview of participant cash flow liabilities and leads into Chapter II.3. Finally, Section I.1.4 introduces quantitative risk analysis and modeling for the Synthetic GIC, acting as a preamble to Chapter II.3. These introductory sections are designed to provide comprehensive context, an overview of past literature, and a summary of our contributions. However, detailed methodology discussions are reserved for their respective chapters.

The secondary theme of this thesis, inspired by the author's experience as a quantitative analyst, addresses the influence of market liquidity on derivatives hedging. This exploration emerged from practical issues surrounding pricing and hedging large derivative positions and further opened investigations into the large trader's market manipulative activities. This supplemental theme is encapsulated in Part III, with a more detailed examination in Chapter III.1. The introductory notes in Chapter I.2 provide a detailed discussion of the context of the problem and the past literature. Section I.2.1 is an introduction to the market impact theories. Section I.2.2 discusses the relationship between market impact and information, while Section I.2.3 sheds light on derivatives pricing under market impact.

Lastly, as this paper-based thesis deals with two subjects treated independently from each other in different articles, there might be instances of overlapping

---

<sup>2</sup>This common practice aims to ease the work of the Ph.D. committee members. Other readers may skip this chapter.

<sup>3</sup>One of the three types of Guaranteed Investment Contracts described in Section I.1.1.1

mathematical notation; a specific notation used for a variable in Part II might be replicated for a different variable in Part III. We acknowledge this potential for confusion and apologize in advance. Given the exhaustive list of concepts introduced in this thesis, we could have employed non-standard Greek letters to ensure unique notation for each concept. Still, that approach might have introduced its own set of complexities. However, readers should be aware that the notations in Chapter I.1 align with Part II and that the notations in Chapter I.2 are consistent with Part III.

# I.1 Synthetic GIC Asset Liability Modeling

The U.S. retirement landscape offers participants' defined contribution investment schemes. These include individual retirement accounts [IRS \(a\)](#), which are managed individually, and employer-associated plans managed via an employer plan-sponsor. Additionally, there are profit-sharing plans [IRS \(b\)](#), where a fraction of an employer's profit is contributed to employees' retirement schemes [US Department of Labor \(2023\)](#). Of the employer-associated schemes, 401(k)s are the most prominent in terms of assets. These are private employer-regulated plans, while 403(b) plans are designed for public sector employees and are regulated under the [Erisa \(2020\)](#) Act. Both allow employees to designate a portion of their salary for retirement investment.

Within employer-related schemes, a variety of investment options are often presented to participants. While the stable value funds aim to preserve the principal and provide consistent returns, many plans also offer stock or equity mutual funds that invest primarily in stocks and might range from broad indices like the S&P 500 to sector-specific or international stock funds. Bond or fixed income funds are another option, primarily investing in corporate or government bonds to provide steady interest income with typically lower volatility than stock funds. Additionally, some participants might opt for target date or lifecycle funds, which automatically adjust their asset allocation as the investor approaches her targeted retirement date, balancing risk and reward as necessary.<sup>1</sup>

Notably, due to their popularity, stable value options also exist in non-retirement schemes like 529(a), which cater to educational schemes. In this thesis, we primarily concentrate on the stable value fund within 401(k) schemes. This has the largest industry size, followed by 529(a) and 403(b).<sup>2</sup> As of the first quarter of 2022, stable value funds held USD \$901 billion in assets, representing 8% of defined contribution plan assets, according to the Investment Company Institute's Retirement Plan Assets [Stable Value \(2022\)](#).

---

<sup>1</sup>See Section [I.1.3.2](#) for a typical example of investment options.

<sup>2</sup>Nevertheless, some results from this study can be generalized to the 403(b) and 529(a) schemes.

Stable value funds provide a secure investment option to the market, guaranteeing a short-term return and capital. One variant of the stable value offerings is known as Synthetic GIC, where an insurer guarantees the investment returns. This section aims to establish a quantitative modeling framework to evaluate the risk associated with these types of guarantees.

### I.1.1 What is a Stable Value Fund?

Stable value funds are financial products designed to provide investors with a low-risk investment option. They have been popular in the US since the 1980s, offering stable returns and principal preservation to plan participants. Many defined contribution retirement plans offer these investment options and are generally characterized by very low risk with liquidity and principal preservation, like money market funds.

In effect, stable value funds are fixed income investments in which investors receive a return close to the return earned on a medium-term investment grade bond fund without the corresponding risks. They are invested in a high-quality, short-duration, diversified pool of fixed-income assets. They are protected from interest rate volatility by book value wraps, which guarantee withdrawals and transfers to plan participants at book value. For more details about stable value funds, the interested reader is referred to the comprehensive handbook on stable value investments by [Fabozzi \(1998\)](#) and [Stable Value Investment Association \(2020\)](#).

One key characteristic of stable values is that they are synthetic and do not exist independently. Instead, they are created by synthesizing other investment funds, where a fund is wrapped into a guaranteed scheme and sold as a new product. This process of bundling, also known as synthesizing, allows investors to benefit from the combined strengths of funds' performance while minimizing risk (see [Figure I.1.1](#)).

FIGURE I.1.1



Figure [I.1.2](#) shows a pictorial representation of the stable value ecosystem. In search of low-risk and principal-preserving investments, participants make their contributions or distributions to a retirement account managed by a



bankruptcy-remote trust. These are done through an employer or a pool of employers. An investment entity then invests the assets. These key components comprise the stable value ecosystem, as shown in Figure I.1.2. The Figure also shows cash flows by company plan sponsor/employer. These are generally standard contributions, such as bonus payments made by the employer, but also one-off events, such as company plan-sponsor corporate actions.

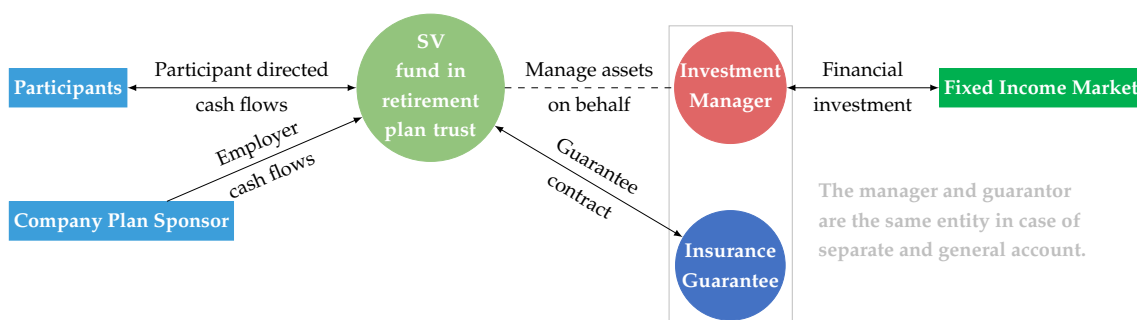


FIGURE I.1.2: Stable value ecosystem.

Figure I.1.2 should be interpreted from left to right: assets originating from participant-directed cash flows, in conjunction with employer contributions, are securely held in a bankruptcy-remote retirement trust. These assets are then entrusted to an investment manager. A significant proportion of these investments are typically directed towards fixed income securities. Additionally, the retirement trust establishes a contract with an insurance company to cover any discrepancies between assets and liabilities. As indicated by the rectangle in the diagram, in certain forms of stable value offerings (specifically, separate and general account stable values), the insurance company and the asset manager might be unified as a single global entity.

Figure I.1.2 also shows cash by company plan sponsor/ employer. These are generally contributions, like bonus payments by the employer, but also one-off events of restructuring due to merger acquisition, company bankruptcy, layoffs, or other company plan-sponsor corporate actions.<sup>3</sup>

### I.1.1.1 Types of Stable Values Offering

Stable value funds are sold through different types of stable value contracts offered by banks and/or insurance companies:

- **Synthetic GICs:** In the case of Synthetic GICs, the retirement plan trusts contracts with one or more guarantors, usually insurance companies, to

<sup>3</sup>These cash flows are generally larger in size and can result in a sudden increase or decrease in the SV fund size. The insurance guarantee may have strict contractual terms for these cash flows and may not cover them at *book value payment* under the *wrap agreement* or may cover them at up to a limit.

match its assets and liabilities. Synthetic stable value funds, particularly Synthetic GICs, are often referred to as *wraps* because they wrap other investment products into a single, low-risk investment option.

- **Separate Account Guaranteed Investment Contracts:** Under a separate account, a segregated portfolio of assets is held in an insurance company's separate account to support the company's obligation to pay principal and interest to plan participants at a rate that, as with GICs and Synthetic GICs, can be fixed or reset periodically. If the fund's obligations or liabilities exceed the value of the separate account's assets, the insurance company's general account backstops the excess liabilities.
- **Insurance Company General Account Portfolio Rate Products ("Insurance Company General Accounts"):** In the case of general account products, the investment and guarantor are the same entity, typically an insurance company. Under these arrangements, the general account of an insurance company supports the insurance company's obligations to pay principal and interest to plan participants.

Additionally, stable value funds can represent one employer or a pool of employers. In the case where the plan sponsor is a single employer, the fund is known as *individually managed*, whereas *pooled* funds allow small and medium-sized plan sponsors to bring their stable value funds together in a single fund formed under federal or state banking laws. According to the Stable Value Investment Association's quarterly survey, \$162 billion of the stable value wrap market is represented by stable value *pooled* funds, and \$270 billion are *individually managed* (SVIA (2020b))<sup>4</sup>.

While some conclusions in this thesis can be generalized to stable value funds, the primary focus is on individually managed Synthetic GICs. In Chapter II.2, we will examine the historical liability cash flow data exclusively for these types of offerings. Consequently, our models are specifically tailored to explain and manage risk for individually managed Synthetic GIC issuances. However, given their similar nature, many findings can be extended to other stable value fund issuances. Throughout the thesis, we will identify areas where our findings can be applied more broadly.

### I.1.1.2 Stable Value Funds' Historical Development

Stable value funds and stable value fund wrap products have evolved significantly since their inception in the 1970s and 1980s. Initially, the funds emerged as a popular investment option in response to the high inflation and interest rate environment of the time and the desire for a low-risk alternative to

---

<sup>4</sup>See (Stable Value Investment Association, 2020) for more details.

money market funds. Bankers Trust and JP Morgan introduced early versions of the stable value concept, with the latter mirroring the former in product comparability.

The initial stable value fund wrap product was designed to contractually cover a single security sold to the plan, with the fund's wrap provider required to buy the individual security and pay book value. This purchase/sale of assets was referred to as "non-participating" since the stable value fund received the book value and did not participate in any market value gain or loss on the sale of the security. However, as managers negotiated for lower fees given the low probability of a benefit-responsive event, contractual terms changed such that the employees or the fund shared the risk of benefit-responsive payments. The portfolio would sell securities in the market and amortize any gain/loss into the future crediting rate within the wrap. This new contractual provision, termed "participating," ensured the plan/employees participate in any gains and losses due to benefits responsive provisions. This is now the market convention for all wrap contracts.

In the early 1990s, the Stable Value Investment Association (SVIA) was established. It became a non-profit organization to educate retirement plan sponsors and the public about how stable value funds can contribute towards a financially secure retirement. The SVIA caters to a diverse audience, including consultants, insurers, and other stakeholders in the stable value market. It publishes a biannual newsletter called "*The Stable Time*" that covers industry concerns and association activities, as well as quarterly and annual surveys and articles about the stable value market, see [SVIA \(2019b\)](#) and [SVIA \(2019a\)](#). The SVIA also organizes annual seminars that bring together investment managers, insurers, and stable value consultants to discuss various topics related to the stable value marketplace.

The same decade also saw a growth in competition and a decrease in wrap fees driven by several factors, including increased competition, changes in product mechanics, and provider experience with the product. Additional challenges and risks, such as liquidity and interest rate risk, emerged in the same period. Regulatory requirements have evolved since then, with the Department of Labor issuing guidance on how stable value funds should be structured and managed in ERISA plans.

In terms of the reserves calculation for an insurance company guaranteeing these funds, the National Association of Insurance Commissioners, appointed as the regulatory support body by chief regulators from the 50 states, published a guideline, A-695 [NAIC \(2015\)](#), for estimating Synthetic GIC reserves. Their methodology is principle-based but uses deterministic stress scenarios. Commonly, Federal regulations co-exist with State regulations, e.g., the State

of New York and Nebraska. [New York State \(1999\)](#). In 2007, [Roseman, B. and Gaskel, G. \(2007\)](#) published a revised paper on the Capital charge calculation. Their suggested methodology is based on the S&P Capital Model, which is a combination of stress and projected scenarios. Their methodology provides a different C-3 capital charge for a typical Synthetic GIC product.

The 2008 financial crisis is another important time point in the stable value fund chronology that had a significant impact, with some funds experiencing significant withdrawals and other challenges. Despite this, no wrap providers exited the business due to losses but rather due to economies of scale or changes in a strategic direction related to institutional product focus or mergers.

Due to the financial crisis, the Dodd-Frank Wall Street Reform and Consumer Protection Act 2010 set a regulatory framework for increasing transparency and reducing systemic risk in the financial system. One implication is that market participants are subject to margin and collateral requirements as well as record-keeping and reporting duties [PWC \(2011\)](#). In August 2011, the Commodity Futures Trading Commission (CFTC) and Securities and Exchange Commission (SEC) also announced a joint study to ascertain whether stable value contracts should fall within the definition of a swap and, if so, whether they should be exempt from enhanced regulations. The results of this study are still outstanding as of the date of this publication.

### **I.1.1.3 Stable Value Return Stabilization Mechanism**

Regardless of the variety of market offerings, all stable value funds have common characteristics that transform a volatile (riskier) fund into a more stable (less risky) one. At their core, stable value funds can be described as the 'wrapped' versions of existing, more volatile fixed-income funds, presented under a book value accounting framework. In this section, we will delve deeper into the book value mechanism. We will also review the literature on investment fund return stabilization and then explain how this is done in practice for the case of stable values.

#### **I.1.1.3.1 Exploring Research on Guaranteed Funds**

Investment funds with return stabilization features have long been a financial and insurance industry pillar. Established products such as traditional annuities, defined benefit pensions, with-profit funds, and unit-linked schemes all encompass elements of return stabilization, as outlined in [Gale et al. \(2016\)](#).

Moreover, guaranteed investments have maintained prominence in many forms, especially in pension-related products across different countries. The significance of these investments extend to regulatory guidance on maximum

rate-of-return guarantee, as detailed in the European Union's Third Directive on Life Assurances [European Commission \(2002\)](#).

Academic literature reflects this focus on guaranteed investment products as well. [Miltersen and Persson \(2003\)](#), for example, investigated the cash buffer and crediting rate concepts in fixed-term investment contracts void of redemption through a stochastic control method. [Kling et al. \(2007\)](#) delved into how various surplus distribution mechanisms could impact a life insurer's sustainability, particularly when products contain interest rate guarantees. [Døskeland and Nordahl \(2008\)](#) focused on the optimal crediting rates and reserve surpluses for fixed-term contracts. In contrast, [Goecke \(2013\)](#) proposed a mathematical structure for a smoothing mechanism in the design of a collective reserve fund. More recently, [van Bilsen and Linders \(2019\)](#) developed a mathematical framework to integrate a shock-absorbing mechanism, thereby distributing asset performance into annuity payouts.

In the context of stable value, return stabilization is achieved through a mechanism known as book value accounting. Before explaining why such a framework ensures return stability and less sensitivity to market fluctuations, and discussing practicalities of book value wrap accounting and the other concepts used in this part, let us precise a typical mathematical framework that we shall use in this part, even if some definitions and results would remain valid with different bricks of models (this is the case for the dynamics of the risk-free interest rate and of the spread). Practitioners might skip this mathematical subsection if they prefer to discover the model with a step-by-step approach.

#### I.1.1.4 Mathematical modeling framework

Let  $(\Omega, \mathcal{F}, \mathbb{P})$  be a complete probability space, equipped with a filtration  $\mathbb{F} = \{\mathcal{F}_t\}_{t \geq 0}$  representing the evolution of the available information over time. All stochastic processes in this part will be  $\mathbb{F}$ -adapted.

- $\{M_t\}_{t \geq 0}$  represents the evolution of the market value of the fund;
- $\{B_t\}_{t \geq 0}$  represents the evolution of the book value of the fund;
- $\{\gamma_t\}_{t \geq 0}$  corresponds to the evolution of the rate, also known as the crediting rate, that represents the growth of the book value of the fund;
- $\{C_t\}_{t \geq 0}$  represents the (cumulated) cash flow activities in/from the fund.
- $\{y_t\}_{t \geq 0}$  represents the evolution of the underlying yield over time.

**Definition 1.** *The dynamics of the stochastic processes of interest are described by the following equations: for  $t \geq 0$ ,*

$$y_t = r_t + s_t, \tag{I.1.1}$$

$$\begin{aligned} dr_t &= \kappa_r(r_t - r_\infty)dt + v_r\sqrt{r_t}dW_t^r, \\ ds_t &= \kappa_s(s_t - s_\infty)dt + v_s\sqrt{s_t}dW_t^s, \end{aligned} \quad (\text{I.1.2})$$

$$\begin{aligned} \langle W_t^s, W_t^r \rangle &= \rho t, \\ dB_t &= B_t\gamma_t dt + dC_t, \end{aligned} \quad (\text{I.1.3})$$

$$dM_t = M_t(y_t + \delta)dt - M_t\theta_t dy_t - pB_t dt + dC_t, \quad (\text{I.1.4})$$

$$dC_t = B_t \left( \underbrace{\beta_t}_{\text{structural trend including herd behavior}} + \underbrace{g(r_t - \gamma_t)}_{\text{rate deficit}} + \underbrace{f(s_t)}_{\text{flight-to-safety}} \right) dt, \quad (\text{I.1.5})$$

$$\gamma_t = \left( \frac{1}{\theta_t} \ln \frac{M_t}{B_t} + y_t - p \right)_+, \quad (\text{I.1.6})$$

where  $r_\infty$  and  $s_\infty$  are the asymptotic value of the rate  $r_t$  and spread  $s_t$  for a large  $t$ ,  $v_r$  and  $v_s$  are the standard deviation of rate and spread, respectively and  $\kappa_r$  and  $\kappa_s$  are the corresponding rate of mean reversion, and  $(W_t^s)_{t \geq 0}$  and  $(W_t^r)_{t \geq 0}$  are two (correlated) Brownian motions, and where  $p \in \mathbb{R}^+$  is the insurance premium while the yield of the fund  $\{y_t\}_{t \geq 0}$  and the duration  $\{\theta_t\}_{t \geq 0}$  (expressed in years) are time-dependent processes to be defined in Equation (II.1.10). Note that  $C_0 = 0$ ,  $B_0 > 0$ ,  $M_0 > 0$ ,  $r_0 > 0$  and  $s_0 > 0$ . The process  $\{\beta_t\}_{t \geq 0}$  is a regime switching process whose switching probabilities may depend on the evolution of the ratio  $M_t/B_t$  over time.

The definition describes how the market value of the fund can grow or shrink based on the fund's cash flows ( $C_t$ ), taking into account the insurance premium ( $p$ ) as a cash outflow, as well as how the market value of the fund can grow or shrink based on the fund's cash flows ( $C_t$ ), taking into account the insurance premium ( $p$ ) as a cash outflow. We will explain and discuss the main components of our modeling framework. Please note that the most specific ones will be only discussed in detailed in Part II for the sake of concision.

#### I.1.1.4.1 Book Value Wrap Accounting

In the stable value fund context, return stabilization is achieved by a *book value wrap accounting* mechanism.

The book value satisfies:

$$dB_t = B_t\gamma_t dt + dC_t. \quad (\text{I.1.7})$$

Note that the *book value* may grow/shrink by the liability cash flow activities  $dC_t$  over the period  $[t, t + dt)$ .

To ensure capital preservation, the crediting rate  $\gamma_t$  is set to always be non-negative. For general and most separate accounts stable value funds,  $\gamma_t$  is defined at the insurance company's discretion. However, for Synthetic GICs and some separate account products,  $\gamma_t$  is defined through a formula. While there are variations in the crediting formulas used within the market, The most common formula in its continuous form is expressed in Equation I.1.8. In some contracts, the crediting rate formula may differ from Equation I.1.8, but in their continuous form, such formulas converge or have a residual difference to the equation.

As  $\{M_t\}_{t \geq 0}$  represents the market value of the fund, the crediting rate  $\{\gamma_t\}_{t \geq 0}$  satisfies

$$\gamma_t = \left( \frac{1}{\theta_t} \ln \frac{M_t}{B_t} + y_t - p \right)_+, \quad (\text{I.1.8})$$

where  $p \in \mathbb{R}^+$  is the insurance premium,  $\{y_t\}_{t \geq 0}$  is a time-dependent process representing the yield of the fund while  $\{\theta_t\}_{t \geq 0}$  would be the duration of the assets for most contracts. We will precisely define the yield and duration in Chapter II.1.

**Remark 1.** *In financial terms, Equation I.1.8 aims to make the book value converge to the market value during the stable value fund's duration  $\theta_t$  (expressed in years). In other words, Given that the growth speed of the market value of assets is  $y_t$ , see Chapter II.1 minus the insurance costs  $p$ , the right side of Equation I.1.8 would be the approximation for the crediting rate desired for the book value to converge to the market value in  $\theta_t$  years. In other words, we assume that there are no cash inflows and outflows, i.e. if  $dC_u = 0$  during the period  $[t, t + \theta_t]$ , hence  $B_t$  is only a function of the crediting rate  $\gamma_t$ . Let us also assume that there is no market risk, i.e. the market value grows at a pace of  $y_t - p$ , then even if there is asset-liability mismatch at  $t$ , i.e.  $B_t \neq M_t$ , we desire the two to be the same in  $\theta$  years. Obviously, for this approximation to hold, the right side of Equation I.1.8 has to be positive by asking  $\gamma_t > 0$ .*

Mathematically speaking one can verify that under Equation I.1.8, when the market value of the assets is below the value of liabilities (bad days), then the average return of the book value (the crediting rate) is below the yield minus insurance premium (the average return of the market value). Reciprocally, when assets are exceeded over liabilities, the crediting rate (book value returns) will be higher than the asset returns (yield minus the premium). Simply put, Equation I.1.8 means that to further mitigate short-term market volatility, any realized or unrealized market value decline in assets is amortized against crediting rates  $\gamma_t$ . This means that in bad markets, participants effectively bear substantially all market risk through a reduction in their crediting rate:



$$\begin{aligned} \text{if } M_t < B_t, \text{ then } \gamma_t &> y_t - p; \\ \text{if } B_t > M_t, \text{ then } \gamma_t &< y_t - p. \end{aligned} \tag{I.1.9}$$

Figure I.1.3 visualizes the phenomena described above. Here, "good days" refer to periods when the market value is rising, while "bad days" indicate times when the market value is decreasing. As observed from the graph, when the market value of the assets (represented by the green line) increases sharply, the book value (depicted by the red line) also rises at a more measured pace. Conversely, even when the market value of the assets dips (green line descending - bad periods), the book value continues its ascent, consistently outperforming the market during these bad periods.

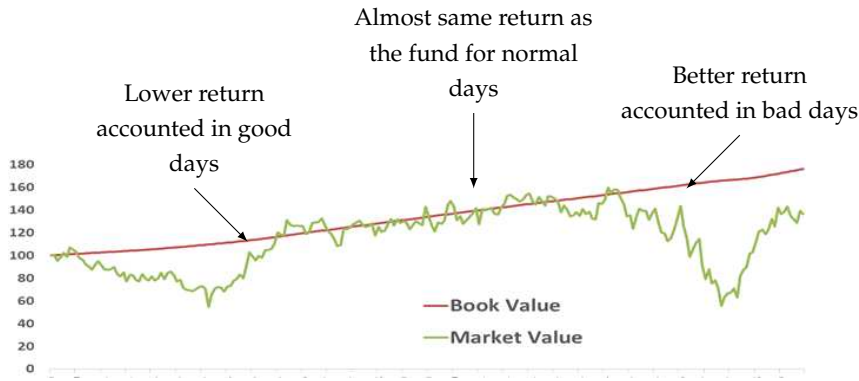


FIGURE I.1.3: Book Value Accounting Mechanism. Note that in this graph, we have assumed that the cash flows process  $C_t = 0$  for all  $t \geq 0$ . This graph is simulated for illustration purposes.

**Remark 2.** Note that at first glance, Figure I.1.3 appears to suggest that the book value  $B_t$  is deterministic. However, this is not the case. When there is no cash inflow or outflow, i.e.  $C_t = 0$  for all  $t \geq 0$ , the book value becomes a very smooth, finite variation process that gives the illusion of being deterministic. This smoothness is due to the crediting rate  $\gamma_t$  formula and is one main reason for the attractiveness of stable value funds by the investors.

### I.1.1.5 Academic Literature on Stable Value

There is limited research that focuses on the evolution of stable value funds as 401(k) investment options selected by individual pensioners, particularly the transition from traditional GICs in the early 1990s as discussed by Fabozzi (1998) to Synthetic GICs as discussed by Tobe (2004). Tobe (2004) offers a useful consultant's perspective on the historical development of stable value funds.



In terms of risk management, [Kwun et al. \(2009\)](#) and [Kwun et al. \(2010\)](#) published their research in which they examined the risks associated with stable value and Synthetic GICs, as well as the guaranteed risk from the insurer's perspective. They employed a Heath-Jarrow-Morton interest rate model and a dynamic lapse formula contingent on the crediting rate deficit.

While [Kwun et al. \(2009\)](#) addresses product risk from the insurer's vantage point, [Xiong and Idzorek \(2012\)](#) investigates the risk from the individual investor's perspective by categorizing it into credit and illiquidity risk. Regarding the investor's liquidity risk, the authors contend that restrictions or penalties on withdrawals of some guaranteed products constrain investor access to funds and impede their ability to rebalance their overall portfolio to achieve the target asset allocation.

David Babbel and Miguel Herce have contributed significantly to the research on stable value funds, with their recent work [Babbel and Herce \(2018\)](#) building upon previous findings about stable value returns. Investigating performance from 1973 to 2017, they discuss stable value performance against alternative low-risk investment products. Their analysis explores factors contributing to past performance and evaluates the likelihood of future continuation, culminating in a discussion on including stable value funds in target date funds.

Despite the existing research on stable value funds, there is a limited amount of literature specifically addressing the quantitative modeling aspects of these funds and no recent research paper. Considering the product's significance, especially its prominence in the U.S. market, one would expect more comprehensive and recent research. The reasons for this shortfall are unclear. However, this highlights the importance and relevance of this thesis in bridging the gap in the literature.

### **I.1.1.6 Regulatory Guidance on Quantitative Asset Liability Management of the Synthetic GIC**

Given the limited literature on stable value, there is not a body of research for us to position our research against. Yet, our focus has been redirected toward positioning this thesis within the regulatory guidelines and prevailing market practices.

This section draws heavily from my firsthand experience in the industry over the past seven years. Hence, the subsequent observations should be understood as interpretations influenced by my journey rather than exhaustive or definitive assessments of regulatory guidance. I am neither an accountant nor a qualified U.S. actuary. As such, the insights shared should be approached from a Quant's viewpoint, especially when discussing U.S. regulations and market practices related to Synthetic GICs.

For Synthetic GICs, market practice and regulatory guidance are closely linked. This becomes clear when considering that, under typical market conditions, regulatory guidelines often result in no required reserves and zero Capital for these products. Consequently, the industry's approach to quantitative risk management is somewhat relaxed, given the zero monetary obligations. The underlying reasons for the zero-value Capital and reserves will be discussed in more detail later.

#### I.1.1.6.1 U.S. GAAP

While the primary focus of this thesis is not the accounting principles surrounding Synthetic GIC reporting—I am not an accountant, nor is that my area of expertise—there is merit in addressing certain aspects, given the interdisciplinary nature of this research.

One significant finding discussed in this subsection pertains to the valuation of the insurer's guarantee. As we will delve into, there exists an accounting discrepancy that makes the Synthetic GIC an attractive offering. The accounting rules governing this product make it a favorable proposition for both the insurers and the retirement plan sponsor trust.

The Financial Accounting Standards Board (FASB) guidelines for reporting SVF Wrap contracts at book or contract value can be found in the most recent update of the Accounting Standards Codification. This update builds on earlier guidance from the Statement of Position (FASB, 2005), see also AICPA (2013), from the 401(k) plan retirement trust perspective<sup>5</sup>:

"The wrapper contract would be valued at the difference between the fair value of the trust assets and the contract value attributable by the wrapper to such assets."

Therefore, in mathematical terms, at any valuation date  $t$ , the wrapper contract value from the 401(k) retirement plan trust is:

$$(M_t - B_t)_+ \tag{I.1.10}$$

However, under Statement 133 FASB (1998) classifies Synthetic GICs as a derivative instrument. This classification stems from the similarities between Synthetic GICs and put options. In essence, in exchange for a set fee, the insurer agrees to compensate if the proceeds from the total asset sale do not cover the promised benefit payments to employees, provided all managerial and contractual terms are met. Under U.S. GAAP, Synthetic GICs are treated

<sup>5</sup>It is important to note that the language mentioned is derived from the (FASB, 2005), which proposes changes to the regulatory document. However, as of the time of this writing, the final version of these changes post-proposal has not been located, and it is unclear whether the proposal has been officially adopted.

as derivatives and are therefore subject to fair value accounting, in line with Accounting Standards Codification Topic 815, also known as Derivatives and Hedging (ASC 815) [FASB \(2022\)](#). Sections 815-10-05-8 to 815-10-05-16 and 815-10-55-63 of ASC 815 offer insights into Synthetic GICs.

One notable statement in these sections clarifies:

"From the perspective of the issuer of the contract, synthetic guaranteed investment contracts are derivative instruments as defined in this Subtopic."

For Synthetic GICs, as they are derivatives, the principle of fair value accounting is applied. This requires insurers to develop a method to determine this fair value. When derivatives are frequently traded, the common practice is to use the replacement cost based on market prices. However, as Synthetic GICs are not widely traded, I believe the industry often leans towards a model-based approach.

Mathematically, from our read of the GAAP accounting principle and of the market practice, at any date  $t$ , the fair value of the wrap contract can be represented by the present value at time  $t$  of the future payoff

$$G_T = (B_{\hat{T}} - M_{\hat{T}})_+, \quad (\text{I.1.11})$$

to be paid by the insurer at the stochastic time

$$\hat{T} = T_{max} \wedge \inf \{u \geq t, \quad M_u \leq 0\}. \quad (\text{I.1.12})$$

where  $T_{max}$  is a time horizon. In theory, there is no fixed maturity for this contract; however, given the fact that parties have the opportunity to terminate the contract early - under certain legal clauses defined by each contract - it is common practice to introduce this time horizon  $T_{max}$ . The analysis of optimal strategies of the insurer with respect to this early termination option is left for further research.

The divergence in accounting principles between the insurer and the 401(k) plan trust offers insights into the attractiveness of this product. Looking at the broader market perspective, the combined value of  $(M_t - B_t)_+$  and the present value  $(B_{\hat{T}} - M_{\hat{T}})_+$  does not cancel out. In simpler terms, under GAAP, the economy is not operating on a zero-sum basis.

The term  $\hat{T}$  in the wrap valuation for the insurer effectively reduces its value, making it more favorable. This is particularly advantageous for insurers since, in situations where the market value is lower than the book value, they are not obligated to report significant deficits. Concurrently, the accounting method is

beneficial for the plan sponsor trust as it consistently portrays assets liability mismatch at time  $t$ .

This mutual accounting benefit creates a win-win situation for both parties involved in the transaction. It is plausible to argue that this favorable accounting framework significantly contributes to the product's existence, as it presents accounting advantages for both sides.

#### **I.1.1.6.2 Stochastic Exclusion Test**

Once more, it is important to highlight that the primary objective of this thesis is not an exhaustive review of RBC principles for Synthetic GIC. Yet, considering the interdisciplinary nature of this topic and the findings of my research, it appears that the prevailing market practices for the quantitative method in ALM of the Synthetic GIC have been influenced by certain regulatory standards. Therefore, this thesis finds it necessary to discuss specific terminologies and concepts. A noteworthy terminology is the "Stochastic Exclusion Test," [NAIC \(2018\)](#) which essentially determines whether a particular block of business requires a stochastic simulation.

If a block of policies satisfies the stochastic exclusion test criteria, the insurer can utilize the deterministic calculations for reserve for that specific block of policies. In this situation, there is no need for stochastic simulation.

In most market conditions, synthetic GICs typically meet the requirements of the stochastic exclusion test. This is predominantly because the scenarios utilized for these tests are different than the peculiar scenarios we identified in Chapter [II.3](#). Again, by 'market conditions,' we are referring to situations where interest rates lie between 0% and 10% and where the market-to-book value ratio exceeds 90%. The implications of this for synthetic GICs are significant.

#### **I.1.1.6.3 Reserves**

The regulation around reserves calculation in the U.S. insurance sector goes beyond the discussion of this research as it is very exhaustive to mention. However, we only note that the National Association of Insurance Commissioners, designated as the regulatory support by the chief insurance regulators from all 50 states, enacted guideline A-695 [NAIC \(2015\)](#) for the estimation of Reserves for the Synthetic GIC. Although their methodology is fundamentally principle-based, it employs deterministic scenarios. Beyond national regulations, certain states have devised their regulatory frameworks, notably including New York and Nebraska. For instance, New York State Regulation 128, [New York State \(1999\)](#), also adopts a deterministic scenarios methodology for Reserves computation.

For reserve reporting, most insurers predominantly adopt either the NAIC model regulation or the NY Regulation 128. Both methodologies adhere to the following rudimentary formula: The reserves are defined as the positive part of the difference between the present value of projected guaranteed liabilities and the market value of assets.

In the execution of the formula, as mentioned earlier, insurers generally project liability cash flows, presupposing a minimum guaranteed rate, typically set at 0% for *individually managed* Synthetic GICs. Since liabilities almost invariably possess a 0% crediting rate floor, the present value of liabilities predominantly falls below the market value of assets. Therefore, in non-crisis market environments, the reported Reserve for individually managed Synthetic GICs is almost always 0, and the Reserve Margin is positive. The Reserves Margin is defined as the positive part of the difference between the market value of assets and the present value of projected guaranteed liabilities.

It is imperative to highlight that the NAIC model regulation methodology for reserves associated with Synthetic GICs linked to stable value pooled funds is different. Although this topic extends beyond this thesis, such a methodology necessitates the projection of both liability and assets predicated on a postulated market yield, with reserves determined by any anticipated future deficit.

#### I.1.1.6.4 Capital Charges

The National Association of Insurance Commissioners (NAIC) introduced the Risk-Based Capital (RBC) model as a regulatory measure to ensure insurance companies in the U.S. sustain adequate capital levels relative to their distinct risk profiles. By addressing the financial stability of insurance entities, the RBC model serves as a cornerstone in the regulatory landscape. The primary RBC formula (before Operational Risk) for insurance companies is given by:

$$\begin{aligned} \text{RBC} = & C0 + C4a \\ & + \sqrt{(C1o + C3a)^2 + (C1cs + C3c)^2 + C2^2 + C3b^2 + C4b^2} \end{aligned} \quad (\text{I.1.13})$$

Within this equation,  $C0$  represents the base capital requirement. Asset risks are captured by  $C1o$ , pertaining to all other assets, and  $C1cs$ , tied to unaffiliated common stock.  $C2$  delineates the insurance risk. Market risks are categorized by  $C3a$  for interest rate risk,  $C3b$  for health credit risk, and  $C3c$  for a broader market risk. The formula further incorporates business risks, with  $C4a$  addressing the premium and liability components and  $C4b$  combining business risk with the health administrative expense risk. The design of the formula tells of the NAIC's approach to risk. Consolidating different risks under a

square root accounts for the interplay between them, suggesting that they may compound or possibly counterbalance each other.

In regards to C-3 Capital charges, The NAIC regulation [NAIC \(2021\)](#) encompasses different phases for the C-3 risk (Capital calculation for the interest rate risk): Phase I, II, and III, see [American Academy of Actuaries Life Reserves and Capital Practice Note Work Group \(2008\)](#). The Phases are components of the NAIC Risk-Based Capital (RBC) model, designed to measure the risks of interest rate and market fluctuations. However, key distinctions separate the phases. Phase I, introduced in 1995, adopts a deterministic approach for interest rate risk calculation, excluding considerations of future interest rate uncertainties. On the other hand, Phase II, crafted in 2006, employs a stochastic approach, using a Monte Carlo simulation to consider the uncertainty of future rates.

The primary differentiation between Phase I and II is in their treatment of interest rate risk. As we previously discussed, a majority of the Synthetic GIC block of business clears the Stochastic Exclusion Test, and therefore, Phase II is not required.

On another note, The NAIC risk-based capital rules treat Synthetic GICs akin to guaranteed separate accounts, as deduced from [NAIC \(2021\)](#):

"Synthetic GICs are contracts with provisions similar to separate accounts with guarantees, except that the insurance company does not own the assets. For businesses of this type, the C1 and C-3 risk-based capital is determined to be the same as if the insurance company owned the assets and provided the same guarantees as in a guaranteed separate account."

Further elaboration on guaranteed separate accounts interprets C-1 and C-3 components, with potential offsets by the reserve margin. [NAIC \(2021\)](#) statement:

"Non-Indexed Separate Accounts: .... For contracts valued using the fair value of assets and the fair value (at current interest rates) of liabilities, risk-based Capital is calculated as the excess of the regular C-1 and C-3 standards over the applicable reserve margins...."

It is worth noting the ambiguous nature of this language. The Appendix for RBC Capital in [Fabozzi \(1998\)](#) does not explicitly mention this offsetting effect. However, one interpretation posits that any computed C-1 and C-3 Capital can be offset against any reserve margins. Given the deterministic assumptions, an individually managed Synthetic GIC generally possesses some reserve margin, effectively neutralizing the C-3 capital impact. Therefore, due to this potential

offset of both C-1 and C-3 risks by the reserve margin, risk-based Capital typically culminates in zero allocation.

We have conducted a thorough analysis using Valerian's proprietary tool designed for the computation of Reserves and RBC under the NAIC deterministic scenario based on our interpretation of the regulatory guidelines. Our findings suggest that under a wide variety of market conditions, encompassing most normal and even several severe scenarios; the regulations would necessitate neither reserves nor RBC Capital. Specifically, by 'market conditions,' we are referring to situations where interest rates lie between 0% and 10% and where the market-to-book value ratio exceeds 90%. To provide a qualitative context to this range, these market conditions reflect what has been observed over the past two decades, with a few exceptions during the 2008 financial crisis. Hence, for nearly all these periods, our interpretation of the regulations would require no Reserves and Capital.

Given that there are zero statutory RBC (Risk-Based Capital) estimates, insurers frequently maintain discretionary Capital or resort to an economic capital measure for both pricing and managerial reporting. For some insurers, this practice boils down to adhering to the guidelines of rating agencies, such as those set forth by Standard & Poor's (S&P). In 2007, S&P issued an updated document concerning the computation of capital charges [Roseman, B. and Gaskel, G. \(2007\)](#). This proposed methodology stems from the S&P Capital Model. To our understanding, this model amalgamates both stress testing and forecasted scenarios. The result is a capital charge requirement for a typical SGIC transaction. We will explore the shortcomings of this approach in more depth in Chapter [II.3](#). Still, it is pivotal to clarify a prevalent misconception: the S&P's capital charges estimates were conceived at a time when the market had not seen a significant asset-liability mismatch since the Market-to-book ratio for the industry was close to 100%.

#### **I.1.1.7 Market Practice**

Owing to the relaxed regulation on the Synthetic GIC by the NAIC, a multitude of cases resulted in zero Reserves and Capital. While regulatory requirements don't enforce intricate models, it is noteworthy, based on my experiences, that many insurers opt for advanced models for management reporting. The prevalent tools and the positioning of this research in the context of the market are elaborated below:

##### **Asset Scenarios**

Existing tools for asset projection are the Academy Scenario generator from [American Academy of Actuaries \(2020\)](#) and the C-3 Phase II scenarios, with a subset of insurers also employing enterprise-specific scenarios.



The AAA scenarios distinctly emphasize interest rate risk over the corporate credit bond index risk. This has led to a scenario where many insurers do not include intricate credit spread stochastic scenarios, particularly for complex assets such as securitized products. As we delve deeper into this thesis, especially in Chapter II.3, the limitations of relying solely on interest rate scenarios become evident. The 2008 financial crisis, for instance, highlighted credit risks that were possibly overlooked by some scenarios. This aspect becomes even clearer in Chapter II.4, where we examine the risks emanating from the financial crisis for a typical Synthetic GIC.

### Withdrawal Models

There is significant diversity in how insurers approach liability/lapse models. While some maintain constant withdrawal rates, a significant number of insurers merge a deterministic withdrawal model with a dynamic one linked to the crediting rate deficit to an interest rate. The most commonly observed formula (in its generic form) expresses the withdrawal rate at any time  $t$  as

$$B_t \times (a + b \times g(\gamma_t - r_t)), \quad (\text{I.1.14})$$

where  $g : \mathbb{R} \rightarrow \mathbb{R}$  is the rate deficit function, further discussed and analyzed in Chapter II.3. Note that  $a, b \in \mathbb{R}^+$  are other factors depending on the *stable value plan characteristics* and that  $r_t$  corresponds to a benchmark interest rate at time  $t$ . For the set of stable value plans, the insurer may choose different values for coefficients  $a$  and  $b$ .

It is crucial to emphasize that these observations stem from my personal market experience. Moreover, the intricacies in the withdrawal models derive influence from various factors:

- Plan characteristics: More than half of the industry utilizes experience data in their withdrawal modeling, anchoring their projections in historical experiences, as such, to determine the value of factors  $a$  and  $b$  for a specific plan in Equation I.1.1.7.

Most insurers factor in demographic data, specific plan details, and pooled fund considerations. While demographics stand out as the primary determinant, considerations related to pooled funds and specific plan details are also significant.

In this thesis, we explore the impact of plan sponsor characteristics. In Chapter II.2, we detail how the plan sponsor ecosystem interacts with withdrawal patterns, highlighting the various factors within the ecosystem that shape these trends. Furthermore, we underscore the importance



of incorporating a stochastic component, recognizing that the plan sponsor ecosystem evolves. It is imprudent to assume past characteristics will persist unchanged in future projections.

- **Other Market Dynamics:** Some insurers also consider a factor dependent on market-to-book value ratios and equity market returns in their withdrawal rate projections [I.1.1.7](#).

The nuanced relationships between equity markets and participant cash flows will be further delved into in subsequent sections of this thesis. In Chapter [II.2](#), we conclude that this relationship is attributed to the *flight-to-safety behavior* during the Equity market crisis, an indirect influence on the stable value cash flows, which will be examined in more depth.

Regarding the market-to-book ratio, our findings in Chapter [II.2](#) suggest that even though no clear statistical relationship between the ratio and cash flows has been observed. However, we also discuss that it is imperative not to dismiss the potential existence of such a relationship, as it can become particularly pertinent considering the reputational risks associated with it, which could precipitate mass lapses.

### Stress Testing

While there is variation in stress scenarios, we have observed that most of these scenarios are aligned with ERM practices common in the industry or the stress scenarios as part of the stochastic exclusion test in [American Academy of Actuaries \(2020\)](#). These scenarios, however, are enhanced with either a deterministic withdrawal rate or the model described in Equation [I.1.1.7](#). This is despite that, as detailed in Chapter [II.3](#), the timing of cash flows is pivotal; to manifest a loss, this timing becomes essential. Without the right timing, the scenario may not reveal the tail risk.

The market scenarios for the stress test employed involve withdrawal during inflationary times. Yet, as we will detail in section [II.3](#), the Synthetic GIC risk is dependent. Withdrawals must align with the withdrawal rates. Also, mean reverting rate scenarios can cause losses for the insurers, which are left mostly undetected in common rate scenarios by the [American Academy of Actuaries \(2020\)](#).

## I.1.2 Fixed Income Asset Modeling

Stable values are the outcome of synthesizing fixed income funds. Therefore, to comprehend stable value funds more thoroughly, it is beneficial to familiarize oneself with the domain of fixed income investments. Primarily, fixed income funds are aggregations of fixed income securities. As such, a concise

introduction to fixed income securities will be given before delving into the specifics of fixed income funds.

### I.1.2.1 Fixed Income Securities

Fixed income securities are debt instruments that provide returns in the form of regular or fixed interest payments and principal repayments. When an investor purchases a fixed income security, they essentially lend money to the security issuer for a set period, during which they will receive interest payments at a predetermined rate. At the end of the maturity period, the investor is repaid the principal amount they initially invested [Fabozzi and Mann \(2012\)](#).

Fixed-income securities are attractive to investors seeking a reliable source of income. They are less risky than equities because they provide a fixed return. However, they may be vulnerable to changes in interest rates, inflation, and credit risk, which can affect their market value and yield ([Brealey and Myers, 2016](#)).

Different types of securities are considered fixed income, such as:

- **Corporate bonds:** Corporate bonds are a type of debt security issued by corporations to raise capital from investors.
- **Securitized assets:** Securitized assets refer to assets that are pooled with other financial assets to create a new security that can then be divided up, repackaged, and sold to investors. Examples of these are:
  - **Mortgage-backed securities:** These structured finance products provide investors securitization on a pool of mortgages. The type of mortgages can vary depending on the type of mortgage securities (i.e., commercial *CMBS* or residential *RMBS*), whether the cash flows are passed to the investor as pass-through or have some restructuring of the securities (pass-through mortgages or collateralized mortgage obligations *CMO*), and whether U.S. government agencies provide them. In the U.S., several government agencies provide such securities, including Fannie Mae, Freddie Mac, and Ginnie Mae [Mark Henricks \(2022\)](#).
  - **Asset-backed securities:** Asset-backed securities refer to a securitized pool of asset-backed loans, such as credit cards or automobile loans.
- **Municipal bonds:** Municipal bonds are another form of debt security issued by or on behalf of a local authority to raise capital from investors to finance expenditures.

- Treasury bonds: Treasury bonds are a type of debt security issued by the U.S. Treasury to support government spending.
- Sovereign bonds: Sovereign bonds are a type of debt security similar to treasury bonds but are instead issued by sovereign government entities other than the U.S. Treasury to support government spending.
- Agency bonds: Agency bonds are another type of debt security issued by U.S. government agencies other than the U.S. Treasury.

### I.1.2.2 Three Important Characteristics of Fixed Income Securities

This section examines three vital statistics of fixed-income funds. These statistics are common across all types of fixed income securities, enabling an overview of the risk-return profile of the security.

#### I.1.2.2.1 Yield

Let  $\{c_{t_i}\}_{t_1 \leq t_i \leq t_n}$  representing a time series of deterministic coupon payments for fixed income security, where  $t_i$  is the time of the  $i$ -th coupon. Let  $D_t(\cdot)$  be the valuation (at some time  $t < t_1$ ) of a security depending on the valuation rate  $r$ , defined by the equation below:

$$D_t(r) = \sum_{i=1}^n e^{-r(t_i-t)} c_{t_i}, \quad (\text{I.1.15})$$

Let  $M_t$  represent the market value of a security at time  $t$ . The yield  $y_t$  of a fixed-income security is the rate of return that an investor would receive if they bought the security at its current market price and held it until its last payment. For a fixed coupon bond, at each time  $t$ , the yield of the security is defined by the solution of the following equation:

$$M_t = D_t(y_t). \quad (\text{I.1.16})$$

We emphasize that the valuation function  $D_t$  introduced in Equation (I.1.16) corresponds to the Laplace transform applied to the yield function, whose exact definition is provided in Chapter II.1 by means of Equation (II.1.2).

Note that the cash flows  $c_{t_i}$  are fixed and terminate at  $t_n$ . This is what constitutes the definition of the yield to maturity. Yield to Maturity (YTM) measures the total return anticipated on a bond if it is held until it matures. YTM considers the bond's current market price, par value, coupon rate, and time remaining until maturity. It assumes that all coupon payments are made on time and reinvested at the same rate as the current yield. YTM assumes that the bond will not be called before its maturity date. However, another definition of the

yield is known as the Yield to Worst (YTW). The YTW calculates the lowest yield an investor can expect to earn on a bond, given a set of possible scenarios. These scenarios may include the possibility of the bond being called or the possibility of default. YTW assumes that the bond issuer will exercise any call options it has most unfavorably for the investor, leading to the lowest possible yield that the investor can receive. In other words, YTW considers the worst-case scenario of a bond's expected returns. Since YTM assumes that the bond will be held to maturity and all coupon payments will be made, while YTW considers the possibility of the bond being called early or defaulting, YTW is typically a more conservative estimate of a bond's expected returns.

In this thesis, given that the focus is on a portfolio of securities within a fixed-income fund and not on individual securities, the definition of the yield presented later is a variation of the formula I.1.16. The reader can jump to Chapter II.1 for more details.

#### I.1.2.2.2 Duration

An important characteristic of fixed-income securities is their price sensitivity to changes in interest rates, referred to as *duration*. There are two common definitions of duration: the *Macauley duration* and the *effective duration*. Fix  $t < t_1$ .

The Macauley duration is defined as:

$$\text{Macauley Duration} = \frac{1}{D_t(y_t)} \sum_{i=1}^n t_i e^{y_t(t_i-t)} c_{t_i}, \quad (\text{I.1.17})$$

where  $y_t$  is the fixed income securities' yield, note that Equation I.1.17 implies that the cash flows are deterministic; otherwise, the Macauley duration would be stochastic and not simply an estimate. Therefore, Macauley duration is insufficient when the upcoming cash flows are stochastic, i.e., floating rate notes. The concept of the *effective duration* emanates from this limitation, mathematically defined as:

$$\text{Effective Duration} = \frac{1}{D_t} \left. \frac{\partial D_t(r)}{\partial r} \right|_{r=y_t}. \quad (\text{I.1.18})$$

In most cases, these two durations (Macauley and effective) are the same. However, they often differ regarding securities with non-deterministic cash flows. This leads to a disparity between the Macauley duration and the effective duration. In this thesis, in the absence of other precision, duration corresponds to effective duration.

### I.1.2.2.3 Spread

The spread of fixed income security is the difference between the yield of the security and the yield of a benchmark security with the same maturity. The benchmark security is usually a Treasury security of the same maturity, which is considered to be a risk-free investment. The spread is an important measure of credit risk, as it reflects the additional compensation that investors demand for bearing the credit risk of the security issuer. The spread can vary depending on the creditworthiness of the issuer, the supply and demand dynamics of the market, and other factors that affect the perceived riskiness of the security. Generally, the higher the spread, the greater the perceived credit risk of the security.

Let  $t \geq 0$  be a fixed time, and let  $s_t$  be the spread at time  $t$ . Let  $r_t$  be the benchmark interest rate at time  $t$ , and let  $y_t$  be the security yield at time  $t$ . Then, by definition, the spread at time  $t$  is equal to

$$s_t := y_t - r_t. \quad (\text{I.1.19})$$

As it may be inferred from Equation I.1.19, the definition of the spread depends on how the yield  $y_t$  and the rates  $r_t$  are defined. This is why different definitions of the spread exist, with some common definitions being:

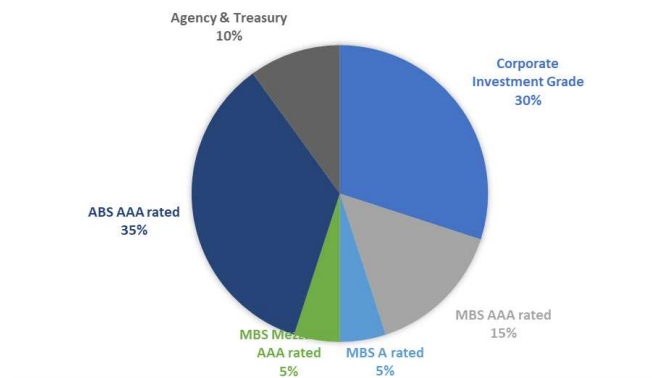
- **Yield spread:** This refers to the difference between a fixed-income security yield and a benchmark security yield with the same maturity and credit rating.
- **Option-adjusted spread (OAS):** This is the yield spread adjusted to consider the embedded option(s) in a bond, such as a call or put option. This is a spread added to the benchmark yield curve such that the fair price of a fixed-income security with an embedded option and the market price are equal.
- **Credit spread:** This is the difference between a fixed-income security yield and a risk-free security yield, such as a U.S. Treasury security, with the same maturity.
- **Z-spread:** This is the constant spread added to each point on the spot yield curve to make the bond's price equal to its current market price. The Z-spread is also referred to as the static spread.

Investors use these different definitions of spreads to evaluate the relative value and risk of fixed income securities. Nevertheless, for the sake of this thesis, the definition of spread is the yield's spread to a risk-free rate  $r_t$  as defined in Equation I.1.19.

### I.1.2.3 Fixed Income Funds: A Portfolio of Fixed Income Securities

A fixed-income fund essentially aggregates a variety of fixed-income securities. As an illustration, Figure I.1.4 depicts the percentage allocations of different financial instruments within a hypothetical fixed income fund. This chart provides a visual representation of the various investment categories discussed earlier. For instance, it reveals that 30% of the fund's market value is invested in corporate bonds. Additionally, 35% is invested in ABS securities, with an AAA rating from credit agencies. For a deeper understanding of credit ratings, one can refer to [Credit rating \(2023\)](#). The fund's composition plays a pivotal role in shaping its overall risk-return profile.

FIGURE I.1.4: Pie chart illustrating sector allocation proportions within a hypothetical fixed income fund. Note: This representation is for illustrative purposes only.



In practice, all the statistics of fixed income security are also descriptive of fixed income funds. In mathematical terms, let us assume we have a fund composed of  $n$  securities, each invested with  $w_i$  proportion of the market value of the fixed income fund, where  $i$  is the  $i$ -th fixed income security index.

An example of a weighting methodology is the notional weighted duration approach. With this backdrop, if the statistic is the duration of the fund, we have:

$$\text{Fund duration} = \sum_{i=1}^n w_i \times \text{Duration of security } i,$$

where  $w_i$  is the proportion of portfolio market value in  $i$ -th security.

If the statistic is the yield of the fund, the weight would be expressed differently as:

$$\text{Fund yield} = \sum_{i=1}^n w_i \times \text{Yield of security } i.$$

This thesis uses a very specific definition of the fund's yield, duration, and spread. This definition aligns with, but may deviate slightly from, the average

weighting method presented here. The reader is referred to Chapter II.1 for the exact definition.

#### I.1.2.4 Our Modelling Approach to Fixed Income Funds

To summarize, we model  $\{\theta_t\}_{t \geq 0}$  as a stochastic process representing the duration of the fixed income fund at any time  $t \geq 0$  while  $\{\tilde{M}_t\}_{t \geq 0}$  and  $\{y_t\}_{t \geq 0}$  are stochastic processes representing respectively the market value and the yield of the funds, as per exact definition provided in Chapter II.1. Note that we use the notation  $M_t$  instead of  $\tilde{M}_t$  in Chapter II.1. In Chapter II.1, a mathematical framework is given for the following formulation of a fixed-income asset under certain regularity conditions discussed in the same section. This formulation (without participant cash flows compared to other parts of Part II), which provides a simplified depiction of fluid dynamics, suggests that the fund's volatility has a linear relationship with the volatility of its yield, moderated by its duration. Moreover, it proposes that the fund's average return can be approximated by its yield adjusted by a  $\{\delta_t\}_{t \geq 0}$ , a stochastic process representing an adjustment factor:

$$\frac{d\tilde{M}_t}{\tilde{M}_t} = (y_t + \delta_t)dt - \theta_t dy_t. \quad (\text{I.1.20})$$

### I.1.3 Liability Participant Cash Flows Modeling

To establish a comprehensive asset-liability quantitative framework, it is necessary to model both the assets and liabilities. While the previous section introduced the literature and approach to fixed-income asset modeling, this section focuses on the liability model. This section explores the existing lapse hypotheses in the literature and their pertinence to stable value. Subsequently, the conclusions are presented, and a liability lapse model is developed based on the participants' behavioral stylized facts. This section, in essence, is a revision of II.2, but with a more detailed examination of the past literature on lapse modeling.

#### I.1.3.1 Review of Lapse Hypothesis and Their Relevance to Stable Values

Literature extensively discusses investor withdrawal behavior, including elements like the ecosystem and influence of plan sponsors [Madrian and Shea \(2001\)](#); [Mitchell et al. \(2006\)](#), rate deficit hypothesis [Barsotti et al. \(2016\)](#), herd behavior and mass lapse hypothesis [Loisel and Milhaud \(2011\)](#); [Barsotti et al. \(2016\)](#), moneyness hypothesis [Bacinello et al. \(2011\)](#); [Cheng et al. \(2019\)](#), flight-to-safety [Baur and Lucey \(2010\)](#); [Dorn and Huberman \(2005\)](#). These hypotheses

were initially formed to explain the lapse behavior in financial products like annuities and savings accounts. This thesis tests the applicability of these hypotheses in the realm of stable value funds, using historical data, case studies, and cash flow trend analysis during different periods, including the COVID-19 pandemic.

#### **I.1.3.1.1 The Influence of Employers on Investment Behavior**

Understanding employers' influence on their employees' investment behavior is crucial in assessing stable value behavior. Research indicates that the employer's role in guiding investment options can significantly impact the savings and investment habits of the participants.

In a comprehensive study, [Madrian and Shea \(2001\)](#) investigated the influence of employers on stable value behavior. Their findings emphasize two key points: Firstly, participation in 401(k) schemes is significantly higher under automatic enrollment. Secondly, they found that the default contribution rate and investment allocation set by companies under automatic enrollment heavily influence the saving behavior of 401(k) participants.

In the case of Synthetic GICs, it will be seen in Chapter [II.2](#) that the employer ecosystems and their influence result in medium to long-term trends in participant cash flows, which will be the backbone of a stochastic trend model, modeled by the variable  $\beta$  representing the trend of cash flows. Full details of the  $\beta$  model can be found in Chapter [II.3](#).

#### **I.1.3.1.2 401(k) Investor's Inattentiveness**

Inattentiveness is a prevalent aspect of human behavior, particularly influencing financial and resource allocation decision-making. This phenomenon has been studied in behavioral economics and financial literature as "rational inattention" [Maćkowiak et al. \(2021\)](#); [Sims \(2003\)](#). The distinction between available and internalized information implies that inattentiveness may be a rational response, as individuals prioritize their attention across various topics and activities. Information economics suggests that agents maximize utility subject to the cost of acquiring information, responding to costs and benefits of attention [Gabaix \(2019\)](#); [Stigler \(1961\)](#).

In various financial contexts, such as mortgage refinancing, inattention and inertia have been observed, with individual characteristics influencing the level of attention [Andersen et al. \(2015\)](#); [Deng and Quigley \(2012\)](#); [Agarwal et al. \(2016\)](#). For instance, [Deng and Quigley \(2012\)](#) found that more than half of the mortgages in their sample missed at least one profitable exercise of the call option. In contrast, [Agarwal et al. \(2016\)](#) discovered that approximately 59% of borrowers refinanced sub-optimally.



Factors like sluggish reactions to macro news, procrastination, and models involving sticky action and information can also contribute to inattentiveness [Gabaix \(2019\)](#). [Gabaix \(2019\)](#) cites many papers that provide evidence of reaction to macro news with a lag in several contexts.

Studies focusing on defined contribution retirement plans, such as 401(k) plans, have consistently found evidence of inattentiveness. Low trading activity and limited understanding of plan benefits are common among plan participants [Mitchell et al. \(2006\)](#); [Agnew et al. \(2003\)](#); [Choi et al. \(2002\)](#); [Investment Company Institute \(2008\)](#); [Young \(2018\)](#); [Plansponsor Staff \(2016\)](#). For example, [Mitchell et al. \(2006\)](#) found that 80% of participants engaged in no trading activity in their 401(k) plans, while [Young \(2018\)](#) reported that only 8% of Vanguard participants traded within their accounts in 2017. [Eberhardt et al. \(2021\)](#) mentioned that people spend less time planning for retirement than they devote to the buying processes for a television or a tablet. [Agnew et al. \(2003\)](#) concluded that even within the most active age cohort (55-64 years of age), trading activity averages only 0.6 trades per year. [Choi et al. \(2002\)](#) stated that employees are likely to do whatever requires the least current effort, often following the path of least resistance. [Investment Company Institute \(2008\)](#) observed that, even with the market turmoil beginning in the later half of 2008, individuals made very few changes to their 401(k) contribution elections and balances, with fewer than 14% making any changes to balances and fewer than 10% making changes to their contributions. A survey from the International Foundation of Employee Benefit Plans [Plansponsor Staff \(2016\)](#) suggests that nearly 50% of participants fail to comprehend the information on their plan's benefits, and most ignore the information provided to them. Further, only 19% of employers believe that their employees understand their benefits. This demonstrates the pervasiveness of inattentiveness in financial decision-making, even in crucial areas like retirement planning.

The research presented suggests that inattentiveness is a prevalent characteristic among participants of 401(k) plans, specifically affecting stable value cash flows. In the scope of this thesis, it is important to note that, due to the unavailability of individual participant cash flow data, inattentiveness cannot be empirically evaluated. However, it is indirectly observed by noting that some of the lapse hypotheses, which will be further elaborated upon in this subsection, are less prevalent in the context of stable values. These include the moneyness hypothesis and the rate deficit hypothesis, both of which would be more pronounced if participants were more attentive to their investment options.

### I.1.3.1.3 Flight-to-Safety

In the realm of finance and investment, a hypothesis known as "flight-to-safety" postulates that during periods of market instability, investors have a propensity to reallocate their assets from high-risk categories to safer ones, such as gold or government bonds [Baur and Lucey \(2010\)](#); [Dorn and Huberman \(2005\)](#). This behavior is motivated by the instinct to safeguard capital and avert further losses.

The author's examination of historical data provides suggestive evidence for flight-to-safety behavior in stable value funds. Although extended periods of crises have not been encountered over the past 20 years, short-term impacts indicate that investors view stable value funds as a safe harbor during turbulent times, as evidenced by positive inflows during such periods. When markets stabilize, these funds often see an outflow as participants transfer their balances back to equities and other investment options.

The aggregate industry cash flows<sup>6</sup> illustrate these trends (see [Figure I.1.5](#)). The figure shows that aggregated industry cash flows increased by 10% during the 18 months of the 2008 financial crisis, by 3% in 9 months during the 2015-2016 stock market sell-off, and by 10% in 10 months during the 2020 COVID-19 pandemic. After these crises, participants transferred funds back into equities, as shown by the index dropping 5% in 9 months after the financial crisis, 5% in 9 months after the 2015-2016 stock market sell-off, and 6% in 12 months after the COVID-19 pandemic.

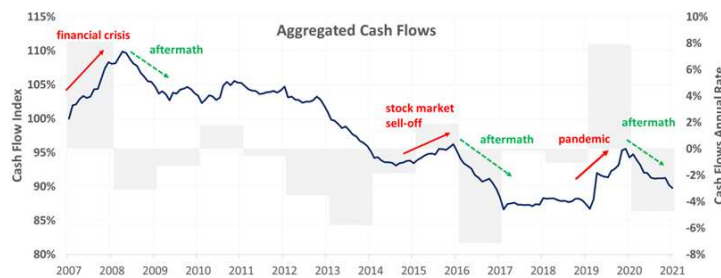


FIGURE I.1.5: Aggregated cash flows since 2007. Data source: proprietary data from Valerian Capital Group LLC.

<sup>6</sup>The aggregated cash flow index is formulated through the book value weighted average of cash flows across each fund. Suppose  $0 \leq k \leq n$  denotes the indices of funds present in our dataset, with  $n$  being the total number of funds. Let  $\{t_k; 0 \leq k \leq m\}$  symbolize the monthly time discretization. Given that  $C_{t_{i+1}}^k - C_{t_i}^k$  represents the participant cash flows for fund  $k$  during month  $i$ , and  $B_{t_i}^k$  stands for the book value of fund  $k$  at the start of the  $i$ -th month, the aggregated cash flows rate for month  $t_i$  can be expressed as

$$1 + \frac{\sum_{k=0}^m C_{t_{i+1}}^k - C_{t_i}^k}{\sum_{k=0}^m B_{t_i}^k}.$$

The aggregate index depicted in [Figure I.1.5](#) is then crafted by the cumulative product of the aggregated cash flow rate. For more details about the dataset, please refer to the Appendix section of [Chapter II.2](#)

#### I.1.3.1.4 Moneyness Hypothesis

The literature on asset-liability modeling often focuses on identifying the optimal lapse strategy for all, or a subset of, rational investors. This is typically based on the concept known as the 'moneyness hypothesis.' For instance, [Bacinello et al. \(2011\)](#) formulated a stochastic optimization problem relating to policyholders' lapses, defined by a specific utility function. Similarly, [Cheng et al. \(2019\)](#) presented an optimization problem in which a segment of investors termed "pros" make withdrawal decisions based on the degree to which the payoff is 'in-the-money.' This is interpreted as the comparative value of the guarantees against the paid premium.

In this thesis, the moneyness of stable value products is assessed through the market-to-book value (MBV) ratio, which can also be referred to as the asset-liability ratio. The aim is to investigate the potential relationship between the MBV and participants' cash flows. However, as revealed in [Chapter II.2](#), the historical data does not suggest any significant correlation between the market-to-book value and participants' cash flows.

To conclude, while the 'moneyness hypothesis' is certainly applicable in scenarios involving active trading (such as trading American stock options), it may not be the prevailing factor when dealing with financial products designed to guarantee deep tail risk. This is due to the reason that, if participants were always acting based on moneyness, they would likely have withdrawn all their stable value investment already since, as noted in [Chapter I.1](#), the product is deep out-of-the-money. In the context of stable value funds, the product offers insurance protection rather than a trading opportunity. As such, the nature of its insurance structure, which ensures protection, could render the moneyness of the product less influential, thereby limiting the applicability of this hypothesis.

#### I.1.3.1.5 Rate Deficit Hypothesis

A significant portion of existing literature considers the risk of participant withdrawals due to the presence of alternative investment options with superior returns.

In the European life insurance market, for instance, the French regulatory body, *Autorité de Contrôle Prudentiel (ACP (2013))*, recommends using a piecewise affine, S-shaped curve for dynamic surrender rates of French savings contracts. These contracts include a mix of unit-linked and Euro-linked savings contracts. The regulator suggests that the maximum annual surrender rate should be between 20% and 40%. An excerpt from [ACP \(see pages 5–6 of 2013\)](#) (presented in French) shows that the highest yearly surrender rate resulting

from competing products should range between 20% and 40% for the French life insurance industry.

$$\text{Surrender Rate} = \begin{cases} \text{RC}_{\max} & \text{si } \gamma_t - r_t < \alpha \\ \text{RC}_{\max} \frac{(\gamma_t - r_t - \beta)}{\alpha - \beta} & \text{si } \alpha < \gamma_t - r_t < \beta \\ 0 & \text{si } \beta < \gamma_t - r_t < \eta \\ \text{RC}_{\min} \frac{(\gamma_t - r_t - \eta)}{\delta - \eta} & \text{si } \eta < \gamma_t - r_t < \delta \\ \text{RC}_{\min} & \text{si } \gamma_t - r_t > \delta \end{cases}$$

	$\alpha$	$\beta$	$\eta$	$\delta$	$\text{RC}_{\min}$	$\text{RC}_{\max}$
Max Ceiling	-4%	0%	1%	4%	-4%	40%
Min Ceiling	-6%	-2%	1%	2%	-6%	20%

FIGURE I.1.6: Surrender rate from French regulator for French savings contracts as a function between the crediting rate  $\gamma_t$  and the competitive actuarial rate  $r_t$ . Extract from [ACP \(2013\)](#).

In the realm of stable value funds, the typical design of a 401(k) plan substantially restricts potential arbitrage due to the limited number of funds offered to participants, along with the implementation of the so-called equity wash requirements. Under this plan provision, participants are not allowed to directly transfer their assets from the stable value fund to a competing capital preservation fixed income product that might offer a higher return in a rising interest rate environment.

Most equity wash provisions necessitate that funds be invested for a minimum of 90 days before a transfer to any competing fund can take place. This effectively prevents participants from swiftly moving their assets from the stable value fund to another competing investment fund by mandating a preliminary transfer to a non-competing investment fund for a specific duration.

Generally, competing investment funds are those where the likelihood of principal loss is minimal, such as a money market fund or a conservative short-term bond fund. This protective measure is put in place to discourage participants from conducting frequent transfers in pursuit of better rates in alternative capital preservation fixed income markets.

In Chapter II.2 of this thesis, the investigation of historical data failed to detect any tangible evidence of participants partaking in rate arbitrage behaviors. Furthermore, it was observed that communication from plan sponsors generally revolves around past performance, which can result in delayed participant responses to rate deficits among investment options. Consequently, this diminishes the immediate relevance of this hypothesis. Nevertheless, the hypothesis

might be justified if there is a sustained rate deficit over a lengthy period, particularly for higher deficits.

With these assumptions in mind, a  $g(\cdot)$  function was developed in Chapter I.1, represented visually in Figure I.1.7. This function assumes that the rate deficit must reach a critical level to trigger participant reaction. For instance, in Figure I.1.7, the critical level is approximately  $\pm 4\%$ , with a maximum withdrawal rate of 20%. It should be noted that these figures are merely inputs and can not be empirically verified.

The figure also overlays historical data points available for this thesis on the parametric model displayed in the top-left corner. This was done to highlight the consistency of the function  $g(\cdot)$  with the historical observations.

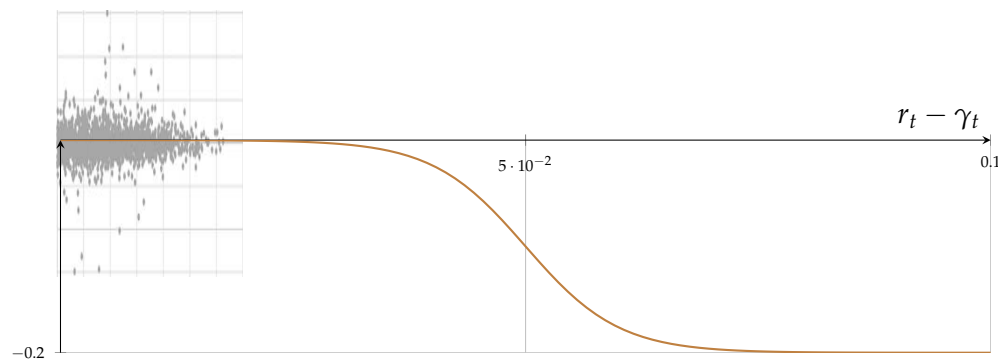


FIGURE I.1.7: Cash flow rate as a function of the crediting rate deficit. The empirical data points have been superimposed to the theoretical function. Empirical data source: proprietary data from Valerian Capital Group LLC.

### I.1.3.2 Ecosystem, Fiscal, and Legal Considerations Impacting Stable Value Participants' Behavior

An analysis of the governing laws and regulations is essential to fully comprehend the dynamics of withdrawals from stable value funds in 401(k) retirement plans. As such, we begin our discussion with a brief introduction to the regulatory framework governing withdrawals from 401(k) plans and transfers away from stable value plans.

This subsection revisits some of the topics discussed in Chapter II.2. If the reader is already familiar with that content, they may choose to skip this subsection or refer directly to the mentioned chapter. In Chapter II.2, and in this subsection we explore how our understanding of this regulatory context guides us in the formulation of hypotheses about participant cash flow behaviors. These hypotheses will subsequently undergo empirical testing.

Withdrawals from stable value funds can be classified into two categories: outer-plan activities and inner-plan activities. Outer-plan activities pertain to

withdrawing funds from a 401(k) plan. Inner-plan activities, on the other hand, concern the reallocation of funds within a plan.

#### **Out of 401(k) Plan Activities**

401(k) plans, popular retirement investment vehicles regulated under the Employee Retirement Income Security Act, offer tax advantages to individuals contributing towards them. However, premature withdrawals before retirement age typically invoke penalties, generally 10% pre-tax as per the status at the time of this research. Certain exceptions to these tax penalty rules exist, for which a shortlist can be found in Table I.1.1 with further details available in [IRS \(2023a\)](#). Withdrawals can be categorized into early withdrawals, hardship withdrawals, and loans.

TABLE I.1.1: 401(k) withdrawal penalties exceptions

<b>Early Withdrawal</b>	
Age	After participant/IRA owner reaches age 59½
Death	After the death of the participant/IRA owner
Disability	Total and permanent disability of the participant
Domestic Relations	To an alternate payee under a Qualified Domestic Relations Order
	Medical health and insurance premiums paid while unemployed, amount of non-reimbursed medical expenses up to a limit
Rollover	In-plan Roth rollovers or eligible distributions contributed to another retirement plan or IRA
<b>Hardship</b>	
Medical	Medical care expenses for the employee, the employee's spouse, dependents, or beneficiary
Housing	Costs directly related to the purchase of an employee's principal residence (excluding mortgage payments) Payments necessary to prevent the eviction of the employee from the employee's principal residence or foreclosure on the mortgage on that residence Certain expenses to repair damage to the employee's principal residence
Education	Tuition, related educational fees, and room and board expenses for the next 12 months of post-secondary education for the employee or the employee's spouse, children, dependents, or beneficiary  Death and funeral expenses for the employee, the employee's spouse, children, dependents, or beneficiary
<b>Loans</b>	
Loans	The maximum amount a participant may borrow from the plan is 50% of the account balance or \$50,000, whichever is less

### Inner Plan Activities

The Employee Retirement Income Security Act (ERISA) is a federal statute stipulating minimum requirements for employee benefit plans, including retirement schemes. Per ERISA, those overseeing retirement plans must relay

specific information about the plan’s investment alternatives and associated fees to participants. It is worth noting that ERISA sets the minimum standard, and plan administrators and consultants may provide more comprehensive details to participants. Table I.1.2 outlines the investment options and corresponding statistics that need to be shared under [U.S. Department of Labor \(2020\)](#).

Name/ Type of Option	Average Annual Total Return as of 12/31/XX				Benchmark			
	1yr.	5yr.	10yr.	Since Inception	1yr.	5yr.	10yr.	Since Inception
<b>Equity Funds</b>								
A Index Fund/ S&P 500	26.5%	0.34%	-1.03%	9.25%	26.46%	0.42%	-0.95%	9.30%
					S&P 500			
B Fund/ Large Cap	27.6%	0.99%	N/A	2.26%	27.80%	1.02%	N/A	2.77%
					US Prime Market 750 Index			
C Fund/ Int'l Stock	36.73%	5.26%	2.29%	9.37%	40.40%	5.40%	2.40%	12.09%
					MSCI EAFE			
D Fund/ Mid Cap	40.22%	2.28%	6.13%	3.29%	46.29%	2.40%	-0.52%	4.16%
					Russell Midcap			
<b>Bond Funds</b>								
E Fund/ Bond Index	6.45%	4.43%	6.08%	7.08%	5.93%	4.97%	6.33%	7.01%
					Barclays Cap. Aggr. Bd.			
<b>Other</b>								
F Fund/ GICs	0.72%	3.36%	3.11%	5.56%	1.8%	3.1%	3.3%	5.75%
					3-month US T-Bill Index			
G Fund/ Stable Value	4.36%	4.64%	5.07%	3.75%	1.8%	3.1%	3.3%	4.99%
					3-month US T-Bill Index			
Generations 2020/ Lifecycle Fund	27.94%	N/A	N/A	2.45%	26.46%	N/A	N/A	3.09%
					S&P 500			
					23.95%	N/A	N/A	3.74%
<hr/>								
			Return	Term	Other			
H 200X/ GIC			4%	2 Yr.	The rate of return does not change during the stated term.			
I LIBOR Plus/ Fixed-Type Investment Account			LIBOR +2%	Quarterly	The rate of return on 12/31/xx was 2.45%.			
J Financial Services Co./ Fixed Account Investment			3.75%	6 Mos.	The rate of return on 12/31/xx was 3.75%.			

TABLE I.1.2: Communication requirements with stable value as an investment option, extract from [U.S. Department of Labor \(2020\)](#).



We explore this ecosystem further in Chapter II.2. In this chapter, we will see how this investigation, combined with a review of the existing lapse literature, will assist us in generating lapse hypotheses to be tested empirically.

### I.1.3.3 Data for Research

This research studies monthly cash flows, which are anonymously sourced, and a proprietary data set of monthly aggregate cash flows for all participants. The structure of the collected data includes key factors such as the *end of month date*, the *aggregated book value* in dollars, *net participants' cash flows*, the *crediting rate* (which denotes the annualized rate of return of the fund as a percentage), and the *market value* (representing the end-of-month market value of the fund in dollars).

The research utilized 41,742 data points spanning over US \$222 billion in book value across 288 funds. Although each plan's start and end dates differ, the earliest available data dates back to 1997. Detailed summary statistics and histograms of the data are presented in Table I.1.3 :

TABLE I.1.3: Summary of cash flow data

Historical period	Book value balances (U.S. \$)	Number of plans	Number of data points
Jan 17 – Dec 21	\$ 132 billion	172	27,421
Jan 14 – Dec 21	\$ 110 billion	137	24,416
Jan 08 – Dec 21	\$ 78 billion	38	15,710
Nov 97 – Dec 21	\$ 222 billion	297	41,742

In Chapter II.2 of this thesis, statistical analyses on the dataset are performed with a cross-examination of the same with pre-existing literature. If a hypothesis has an observed pattern within the data, the hypothesis is statistically validated. Conversely, if no current literature mirrors the identified pattern, a new behavioral hypothesis is constructed, and its logical coherence within the framework of the 401(k) ecosystem is examined.

### I.1.3.4 Conclusion and the Liability Model

Based on the analysis and conclusions from the participant experience study in Chapter II.2, it was concluded that any lapse model built for projecting adverse withdrawal scenarios should consider several important behavioral components. To model this, recall that  $\{C_t\}_{t \geq 0}$  is the stochastic process representing

the participants' cash flow rates as per Equation I.1.21:

$$dC_t = B_t \left( \underbrace{\beta_t}_{\text{structural trend including herd behavior}} + \underbrace{g(r_t - \gamma_t)}_{\text{rate deficit}} + \underbrace{f(s_t)}_{\text{flight-to-safety}} \right) dt. \quad (\text{I.1.21})$$

More justification for this model can be found in Chapter I.1. As it can be interpreted from Equation I.3.2, the model is composed of key components:

- A structural, non-monotonic regime-changing trend in cash flows related to the nature of plan sponsors' ecosystem, which indirectly influences participants' behavior. In the model, this is captured by the term  $\beta_t$ .
- A rate deficit risk factor that is triggered by deficits significantly higher than 1% and sustained for a period longer than one year. This is represented in the model by the term  $g(s_t)$ .
- A herd behavior component, where the plausibility of this behavior could potentially be influenced by reputational damage due to a low market-to-book value. This component is also included in the model's  $\beta_t$  term.
- The cash flow risk-mitigating effect of flight-to-safety behavior during a crisis. In the model, this is captured by the term  $f(s_t)$ , where  $s_t$  is the spread of the fixed income fund.

## I.1.4 Asset Liability Modeling

The prior two sections have established an asset and a liability model. In this section, the goal is to merge these two models and discuss the insurer's contingent claims and potential risk scenarios. Existing literature on asset-liability modeling within the context of guaranteed structures is examined, followed by an elaboration on how our problem fits within the scope of stable value guarantees. This section serves as a concise preview of the more comprehensive discussion in Chapter I.1.

### I.1.4.1 Literature Review of Asset-Liability Models for Guaranteed Products

Guaranteed investment products, such as Defined Benefit (DB) schemes, have been a focus of substantial research. Particularly in the late 1990s and early 2000s, numerous studies explored guaranteed benefit pension schemes. This period also saw the rise of option pricing models and arbitrage pricing theory, with many papers leveraging these tools to evaluate the put option value of the guaranteed schemes.

Prominent early research in this area includes [Marcus \(1987\)](#)'s work, which implemented partial differential equations for option pricing to estimate the value of the Pension Benefit Guaranty Corporation. Subsequently, [Persson and Aase \(1997\)](#) offered a valuation of the price of a fixed-term guaranteed contract using an Ornstein-Uhlenbeck process, and [Miltersen and Persson \(1999\)](#) utilized a Heath-Jarrow Morton process. As with-profit guaranteed investments gained traction in the UK in the late 1990s, [Ballotta et al., \(2006\)](#) designed a Merton-type process for valuation.

The shift from DB to Defined Contribution (DC) plans in the early 2000s prompted a shift in research focus. [Lachance et al. \(2003\)](#) explored the cost implications of transitioning from a DB to a DC plan using option pricing methods. Simultaneously, empirical studies such as [Munnell et al. \(2009\)](#) delved into the pricing of fixed-income guarantees in the financial market, providing both retrospective and prospective estimates of guarantee costs.

The Guaranteed Minimum Withdrawal Benefit (GMWB) is a product that has been thoroughly investigated in this realm due to its resemblance to stable values and its susceptibility to both lapse and financial risks. [Milevsky and Salisbury \(2006\)](#) analyzed GMWBs employing a Gaussian-type withdrawal strategy and used both static and dynamic perspectives. [Bauer et al. \(2008\)](#) put forward a more comprehensive model, while [Bacinello et al. \(2011\)](#) introduced a novel method for resolving the GMWB variable annuities pricing and risk, leveraging the Longstaff-Schwartz ([Longstaff and Schwartz, 2001](#)) method.

When it comes to the asset-liability management of the Synthetic GIC guarantees, the method proposed in this thesis aligns with the line of literature discussed earlier. The key differences, however, lie in the modeling of the asset and liability, as well as the definition of the insurance payoff. The following subsection offers a more detailed explanation of this insurance payoff definition.

#### **I.1.4.2 Synthetic GIC Asset-Liability Management**

In the context of asset-liability management, on one side of the stable value fund, a retirement plan trust manages assets using an investment entity. On the other side, the trust is liable for the book value of the assets to the retirement plan participants. In order to ensure asset-liability parity, the trust enters into a guarantee contract with an insurance company and pays insurance premiums. By the asset-liability matching principle, from the retirement plan trust perspective, the book value should always be at par with the assets.

Table [I.1.4](#) below shows a simplified T-account from the 401(k) trust's perspective. On the left side, the trust manages assets using an investment entity. On the right side, the trust is liable for the book value of the assets to 401(k) plan

participants. In order to ensure asset-liability parity, the trust enters into a guarantee contract with an insurance firm and pays insurance premiums.  $B_t$  and  $M_t$  represent the aggregated *book value* and *market value* for the sum of all participants (investors) contracts, respectively. By the asset-liability matching principle, from the 401(k) trust perspective, the book value plus the sum of insurance premiums should always be at par with the assets. Further, the sum of upcoming insurance premiums  $P_t$  grows at the rate of an insurance premium  $p$ .

TABLE I.1.4: T-Account Table From The Stable Value 401(k) Trust Perspective.

Assets	Liabilities
Fund's asset ( $M_t$ )	Book value ( $B_t$ )
Insurance guarantee	sum of upcoming insurance premiums

The insurance guarantee does not, as a matter of fact, legally cover all scenarios that may cause an asset-liability mismatch. The contract terms have some protections for the wrapper (insurer), some of which are discussed below:

- Tax, regulatory, and legislative changes: Any material changes to the regulatory, tax, or statutory legal framework for retirement plans are excluded under most book value wrap contracts as these changes may adversely affect the structural barriers that inhibit outflows.
- In practice, losses arising from credit defaults of assets are excluded from the book value calculation. Therefore, the wrap provider is not exposed to credit default risk. However, changes in market value that occur due to credit deterioration are covered as long as assets remain within eligible investment guidelines.

**Definition 2.** We define time  $\bar{T}$ , the stopping time of the last resort, when the market value reaches zero ( $M_{\bar{T}} = 0$ ) - the last remaining participant withdraws. We define the insurance payoff, denoted as  $G_{\bar{T}}$ , by the asset-liability mismatch at the time of the last resort:

$$G_{\bar{T}} = (B_{\bar{T}} - M_{\bar{T}})_+ = B_{\bar{T}} \quad (\text{I.1.22})$$

$$\bar{T} = T_{max} \wedge \inf \{t > 0, \quad M_t = 0\}.$$

where  $T_{max}$  is a time horizon. In theory, there is no fixed maturity for this contract; however, given the fact that parties have the opportunity to terminate

the contract early - under certain legal clauses defined by each contract - it is common practice to introduce this time horizon  $T_{max}$ . The analysis of optimal strategies of the insurer with respect to this early termination option is left for further research.

As can be deciphered from Equation I.1.22, the timing of the insurance payment,  $\bar{T}$ , is stochastic. This is the key differentiating characteristic of stable value funds guarantee - that the insurance company acts as the *last resort*, meaning that as long as the last individual (or the last pool of individuals) has not withdrawn their assets from the fund, the current participants invested in the fund, and not the insurer, will pay for any past fund shortage.<sup>7</sup> The insurer will make a claim payment only when the assets of the stable value fund have been exhausted due to participant-directed behavior. The *last resort* concept is the core advantage of stable value funds from other guaranteed investment structures and makes the insurance guarantee cheaper and more affordable.

### I.1.4.3 Conclusions

One of the aims of this thesis is to provide guidance to insurers in navigating potential risk scenarios associated with these guaranteed contracts. To achieve this, a Monte Carlo framework was utilized to project assets and liabilities, subsequently analyzing instances where losses were observed. This exploration led us to the identification of two recurring patterns: the *yield spike mean reverting scenario* and the *inflationary scenario*. This section delves into these scenarios, employing historical back tests as a means to evaluate their plausibility.

Figure I.1.8 portrays the average yield and withdrawal rates for the 0.1% percentile loss simulated through a Monte Carlo framework. From this Figure, it can be inferred that the riskiest scenarios occur when yields are rising alongside strong concurrent withdrawal rates. These scenarios are named as the *inflationary scenarios*.

---

<sup>7</sup>There also exists an additional provision for a stable value fund, known as an unwinding period or extended termination. The wrap provider can force the contract into extended termination. This will allow the crediting rate to be modified such that over a set period, such as the duration of the portfolio, the market value and book value will converge.

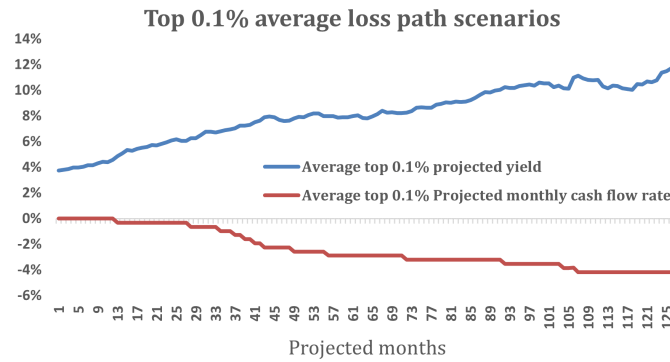


FIGURE I.1.8: The *inflation scenario*: Top 0.1% average loss path scenarios. Data source: Monte Carlo simulation.

Figure I.1.9 presents another pattern discerned within the 1% highest loss scenarios, where interest rates rise in conjunction with withdrawals, followed by a sustained phase of lower yields, termed as the *yield spike mean reverting scenarios*.

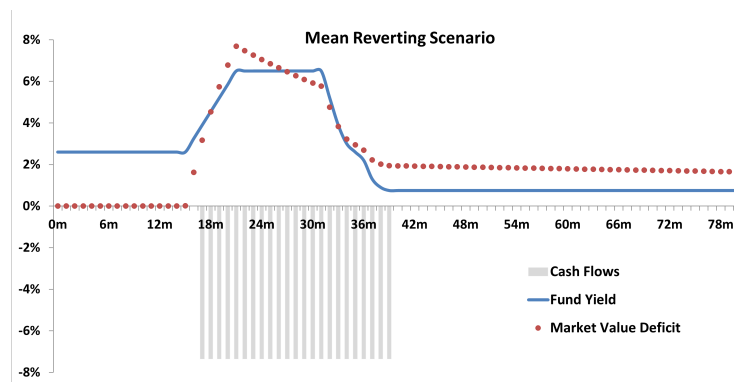


FIGURE I.1.9: The *yield spike mean reverting scenarios* observed within the 1% of loss scenarios. Data source: single scenario simulation.

A *mean reverting scenario* unfolds when yields experience a sudden hike, only to later fall back to more customary levels. Despite the recovery of market conditions hinting at risk reduction, this scenario turns risky if it coincides with significant withdrawals during periods of high yield. The two scenarios mentioned above are comprehensively discussed in Chapter I.1.

Another notable observation from the study is the inadequacy of the National Association of Insurance Commissioners’ formula-based approach to assessing the tail risk of this product. The U.S. National Association of Insurance Commissioners (NAIC) uses a Risk-Based Capital (RBC) approach, presenting a formulaic method for capital charges of Synthetic GICs, as detailed in (NAIC, 2012). The model regulations for *Synthetic GICs* and *separate accounts* are outlined in (NAIC, 2015) and (NAIC, 2017), respectively. However, the

highly path-dependent nature of risk scenarios for the Synthetic GIC necessitates a more complex model that generates flexible values for risk factors. For instance, the NAIC formulaic approach results in zero capital for a typical contract despite the estimated actual tail risk of the product being non-zero.

As an illustration, the NAIC capital charges for a U.S. insurer are examined using the formulaic approach under the assumptions of reserve estimates regulation NAIC (2015). For a typical Synthetic GIC contract as defined in the Appendix of Chapter II.3, the NAIC *formulaic* risk-based capital assessment is zero:

NAIC RBC Capital	99% CTE of PV of loss-premium	Loss frequency	Avg. loss size
0%	0.35%	0.28%	1.38%

TABLE I.1.5: Monte Carlo based estimation of the Synthetic GIC risk for a typical Synthetic GIC contract as defined in the Appendix of Chapter II.3.

This analysis indicates a discrepancy between the formulaic approach and the actual risks associated with the stable value fund, emphasizing the need for a more flexible, dynamic risk model such as the one derived from this thesis.

## I.2 Derivatives Under Market Impact

This research project was initiated by my intellectual curiosity to respond to the question of a trader in an investment bank. The trader was repurchasing a sizable derivatives portfolio from an institutional entity. He was concerned that the current option prices and the volatility surface did not account for the market impact he would create when hedging this substantial position, as such information was confidential. Consequently, he could not rely on the option prices to calibrate its volatility. He approached me with the central question: "What price should I offer?"

My initial exploration of the literature led to full hedge models, such as the one presented by [Liu and Yong \(2005\)](#). However, due to certain cyclical phenomena discussed in this thesis, the pricing model suggested by [Liu and Yong \(2005\)](#) leads to scenarios where traders find it unpractical to fully hedge their positions during significant market fluctuations, as the repercussions would be amplified.

As an alternative, I considered the partial hedge methods, such as those outlined in [Guéant and Pu \(2017\)](#), which employ stochastic optimization techniques. However, as detailed in this thesis, such models can inadvertently lead to market manipulation.

Given these challenges, it became evident that a more practical approach was required. This quest for an alternative practical solution forms the core of this research project.

To clarify, the question is how a large trader can manage a derivative position without inadvertently causing market manipulation. My expertise and research interest lie in risk management.

In this Chapter, we offer an overview of the literature related to market impact. We then explore the option pricing under illiquid markets and subsequently outline our model. We will present a framework in which the market is studied from the perspective of one insider trader who is knowledgeable about the



activities of another large trader. This section serves as an introduction to Chapter III.1, where we detail our methodology.

**Definition 3.** We consider a finite, deterministic time horizon  $T \in \mathbb{R}^+$ . Let  $(\Omega, \mathcal{F}, \mathbb{P})$  be a complete probability space. Let  $\{W_t\}_{0 \leq t \leq T}$  be a standard Brownian motion and  $\{N_t\}_{0 \leq t \leq T}$  be a homogeneous Poisson process with intensity  $\phi > 0$ . We assume that the processes  $\{W_t\}_{0 \leq t \leq T}$  and  $\{N_t\}_{0 \leq t \leq T}$  are independent. The probability measure  $\mathbb{P}$  is traditionally called a real-world probability measure. Let  $\mathbb{F} = \{\mathcal{F}_t\}_{t \in [0, T]}$  be the filtration jointly generated by the processes  $\{W_t\}_{0 \leq t \leq T}$  and  $\{N_t\}_{0 \leq t \leq T}$ .

## I.2.1 Market Impact Theories

Over the past two decades, the field of optimal liquidation has experienced significant growth, sparked by the foundational work of [Almgren and Chriss \(2001\)](#). Their framework centered around decomposing market impact into permanent and temporary components, which has since been widely adopted and adapted by researchers and practitioners alike.

Therefore, the literature predominantly refers to two types of market impact: the temporary impact, which is also known as slippage or transaction cost, and the permanent impact.<sup>1</sup> Recently, a new strand of literature has emerged focusing on transient market impact, which models the resilience of the order book. The following sections introduce each of these types of market impact.

### I.2.1.1 Three Types of Market Impact

#### Temporary

Temporary impact refers to the immediate effect on the price due to a trade, also known as instantaneous market impact or execution cost. When a large order is executed, it may temporarily cause the price to rise (for a buy) or fall (for a sell) due to liquidity constraints. However, unlike the permanent impact, the temporary impact is short-lived, and the price is expected to revert back to its original level shortly after the trade. This impact is mainly associated with the cost paid by the trader to take liquidity from the market and is assumed not to have a long-term effect on prices. Both the [Almgren and Chriss \(2001\)](#) framework and [Alfonsi et al. \(2012\)](#) studies incorporate temporary market impact.

A key ingredient for our analysis concerns the annualized rate, which we consider to be externally computed by the large trader: we do not have a

<sup>1</sup>Transaction cost and temporary impact terminology have been used sometimes in the literature interchangeably, see ([Alfonsi et al., 2012](#)). As an example, [Huberman and Stanzl \(2004\)](#) uses the concept of temporary market impact as the slippage cost. Some other literature, e.g., [Gatheral \(2010\)](#), [Bouchaud et al. \(2003\)](#), also consider a temporal price impact decaying over time.

prescribed dynamic for  $\chi_t$  since we assume it is decided by the large trader at time  $t$ . For example, if the large trader is hedging an option, it can be directly computed by solving the optimization problem of the same kind as the one described in (I.2.12).

**Definition 4.** The  $\mathbb{F}$ -adapted process  $\{\chi_t\}_{0 \leq t \leq T}$  representing the evolution of the annualized rate of shares executed by the large trader over time, the  $\mathbb{F}$ -adapted process  $\{V_t\}_{0 \leq t \leq T}$  representing the evolution of the number of shares held by the large trader over time and the  $\mathbb{F}$ -adapted underlying spot price process  $\{S_t\}_{0 \leq t \leq T}$  are jointly defined as

$$\begin{cases} dV_t = \chi_t dt, \\ \chi_t = \tilde{\chi}(t, S_t, V_t), \\ \frac{dS_t}{S_{t-}} = \mu_t dt + \sigma dW_t + \Lambda(\chi_t) dN_t, \end{cases} \quad (\text{I.2.1})$$

where  $\tilde{\chi} : \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R} \rightarrow \mathbb{R}$  is a piecewise continuous and bounded function describing the speed of execution of the large trader and  $\Lambda : \mathbb{R} \rightarrow \mathbb{R}$  is a non-decreasing, square integrable function describing the permanent market impact of the large trader, with  $\Lambda(0) = 0$ . We require positive initial conditions  $S_0 > 0$   $V_0 \geq 0$ . We also assume that  $t \rightarrow \mu_t$  is a deterministic function taking values in  $\mathbb{R}$  (representing the drift of the underlying asset), that the volatility  $\sigma \in \mathbb{R}^+$  is deterministic and constant.

At this point, we also introduce a stochastic process  $\{C_t\}_{0 \leq t \leq T}$  representing the transactional cost borne by the large trader. In our formulation, the dynamic of  $C_t$  reads the following:

$$\begin{cases} C_0 = 0 \\ dC_t = \eta \left( \frac{\chi_t}{\chi_t + A} \right)^\alpha \chi_t dt, \end{cases} \quad (\text{I.2.2})$$

where  $A \in \mathbb{R}$  is the (annualized) average volume,  $\eta \in \mathbb{R}^+$  is the impact factor, and  $\alpha \in [0, 1]$  is the exponential factor.

This formulation of the temporary cost of trading is consistent with previous studies such as [Grinold and Kahn \(2000\)](#) and [Zarinelli et al. \(2015\)](#) where  $\alpha$  is often set to  $1/2$ , corresponding to a square root formula.

### Transient

This concept is more recent in the literature and is related to the resilience of the order book. Transient impact is the intermediate effect of a trade on the asset price, which occurs between the temporary and permanent impacts. It captures the price movement as the order book gradually recovers from the

initial liquidity shock caused by the trade. The transient impact represents the dynamics of the order book as it adapts to the new supply and demand conditions and may partially revert over time. This concept has gained attention in recent years as researchers began to explore models of order book resilience, with several papers contributing to the understanding of transient market impact.

[Gatheral et al. \(2012\)](#) introduced a transient market impact model that incorporated price reversion over time, deriving closed-form expressions for optimal liquidation strategies. [Klöck et al. \(2014\)](#) studied a market impact model including a public exchange and a dark pool, analyzing the transient impact of order flow and the conditions for the absence of price manipulation. In contrast, [Cont et al. \(2013\)](#) empirically investigated the transient price impact of different order book events, providing valuable real-world evidence to the discussion of market impact concepts.

In this thesis, a transient market impact is not considered.

### Permanent

This refers to the long-lasting effect of a trade on the asset price. When a large buy (sell) order is executed, it can cause the asset price to permanently increase (decrease) due to the change in supply and demand dynamics. The permanent impact represents a lasting change in the price level, which does not revert over time. In the [Almgren and Chriss \(2001\)](#) framework, the permanent impact is a key component of their model.

**Remark 3.** *For the sake of simplicity, we have assumed that the permanent impact intensity  $\phi$  is not influenced by the execution rate  $\chi_t$ . The relationship between  $\phi$  and  $\chi_t$  is complex and has not been extensively studied in the empirical literature. Most academic literature so far investigated the relationship between  $\chi_t$  and the average price impact, i.e., they studied the  $\Lambda$  function, but not the variation of the price impact, i.e., any relationship between  $(N_t)_{0 \leq t \leq T}$ , and  $(\chi_t)_{0 \leq t \leq T}$ .*

At an infinitesimal level,  $dN_t$  represents the number of events that occur in the infinitesimal time interval  $[t, t + dt]$ . It follows a Poisson distribution with mean  $\phi dt$ . Therefore, the Equation [I.2.1](#) could be interpreted as follows: the large trader requests shares at the rate of  $\chi_t$  at any given time  $t$ . Due to the size of  $\chi_t$ , there may be a lack of liquidity for this order. It is assumed that within a small time interval  $[t, t + dt)$ , there is either enough market depth or not. If there is enough market depth, there will be no market impact. However, if there is a lack of depth, there will be a market impact of size  $\Lambda(\chi_t)$ . The probability of being in a market with no depth during the interval  $[t, t + dt)$  is  $\phi dt$ , where  $\phi$  is a positive constant.

The model presented in Equation I.2.1 is based on the assumption that the liquidity depth of the market is stochastic and changes over time. Equation I.2.1 postulates that the permanent price differs for each scenario path depending on when the shares were executed and the depth of the market on that day. The idea of stochastic liquidity has been recently explored in the literature, as shown in various papers including Klöck (2012), Almgren (2012), Ma et al. (2020), and Fruth et al. (2019) on modeling a stochastic transient impact, and in Barger and Lorig (2019) on modeling a stochastic permanent impact. Note that the market impact could be positive or negative depending on whether the shares are bought or sold. In the case of no trading activity by the large trader, Equation (I.2.1) reduces to the standard Black-Scholes dynamics.

Note that while the results of this study are extendable to the case of non-constant volatility, i.e., local volatility, for the sake of simplicity and for the sake of focusing discussion on the market impact component, the volatility  $\sigma$  is taken to be constant.

## I.2.2 Market Impact and Information

An area of research that has received less attention is the relationship between the permanent impact and information. The permanent impact can be interpreted as an informational component, representative of market activity that could result in a liquidity imbalance. The entities that possess this information gain a unique probabilistic perspective on the market, as this information carries implications for future market activities. This perspective is supported by the work of Eyraud-Loisel (2005) and Eyraud-Loisel (2013), who contribute to the understanding of filtration enlargement due to private information. It is also echoed in the studies by Cetin et al. (2006) and Cetin et al. (2010), who interpret the permanent market impact in the context of the asset supply curve.

However, an essential point is that the information itself is not measurable. Instead, what is measurable is the generated market impact of the information. As information is inherently non-measurable, practitioners have adopted the concept of the *impact factor* when discussing the implications of information (news). A tangible case example of this point can be seen in Figure I.2.1, captured from a market news website called City Falcon, where the effect of news is quantified through the use of the impact factor.

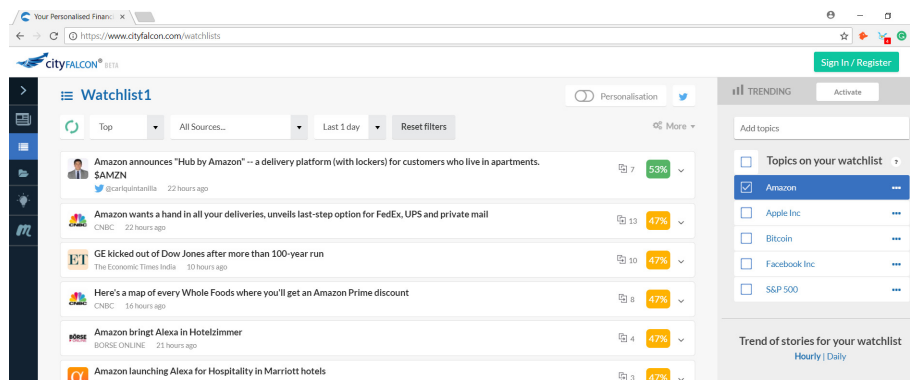


FIGURE I.2.1: Information on this website is shown with a respective estimated impact factor on the stock.

As a result, any theory of information inevitably results in a corresponding theory of market impact. In this research, the aim is to translate the intricate qualitative and interpretive relationship between information and market impact into a structured mathematical framework.

To achieve this, a market ecosystem featuring an informer needs to be defined resulting in an ecosystem that contains three types of traders:

- The *large trader*, who not only impacts the market but is also cognizant of the impact of their actions on market dynamics.
- The *insider trader*, who shares the knowledge base of the large trader but is too small to generate any market impact.
- The *average trader*, who neither sways the market nor is privy to the activities of the large trader. Note that this trader is not a research subject of this thesis.

Note that here, we aim to explore how a large trader can effectively manage a derivative position in practice without unintentionally causing market manipulation. While this research area is expansive, certain topics, such as the impact of filtration enlargement and its effects on uninformed traders, will not be our primary focus. My expertise centers on risk management for derivatives. While the interplay between informed and uninformed traders, influenced by filtration enlargement, holds significance in areas like algorithmic and statistical trading, these areas are not ones in which I have direct professional experience.

Before going into the insight of the following theorem, we extend the general model stated in (I.2.1), by introducing the following quantities: a deterministic rate  $r_t$ , the dividend yield  $q$  and the cost of carry  $b \in \mathbb{R}$  that is the adjustment rate between the futures market and the current value of the stock.

We are now in the position to state one of the main results of this part. We refer to Chapter III.1 for more details and a comprehensive proof of the Theorem.

**Theorem I.2.2.1.** *We assume the following regularity condition*

$$\Lambda(\chi_t)(\mu_t + (q - b) - r_t) < \sigma^2$$

holds  $d\mathbb{P} \times dt$ -a.s.,  $0 \leq t \leq T$ . Then, the following holds:

1. There exists a set of probability measures  $\mathcal{I}$  where for any measure  $I \in \mathcal{I}$  the returns of the discounted asset

$$S_t = \mathbb{E}^I \left[ e^{-\int_t^T r_s ds} e^{(q-b)(T-t)} S_T \middle| \mathcal{F}_t \right] \quad (\text{I.2.3})$$

are martingales under the measure  $I$ .

2. An option  $\mathcal{O}$  bought/sold by the insider trader

$$\mathcal{O}_t = \mathbb{E}^I \left[ e^{-\int_t^T r_s ds} \mathcal{O}_T \middle| \mathcal{F}_t \right]$$

is also a martingale under probability  $I \in \mathcal{I}$ .

3. We call the set  $\mathcal{I}$ , the set of information-neutral probabilities. Then, among all the information-neutral measures, there exists a probability measure  $\mathcal{I}^*$  that minimizes the quadratic variation of the difference between the hedging portfolio and option value. We call  $\mathcal{I}^*$  the minimal variation information-neutral probability measure.
4. The discounted return of the asset is a martingale under measure  $\mathcal{I}^*$ .

Moreover, the dynamic of the spot under  $\mathcal{I}^*$  is described by

$$\frac{dS_t}{S_{t-}} = (r_t - (q - b) - \Lambda(\chi_t)\omega_t) dt + \sigma dW_t^{\mathcal{I}^*} + \Lambda(\chi_t) dM_t, \quad (\text{I.2.4})$$

where  $\{M_t\}_{0 \leq t \leq T}$  is a doubly stochastic Poisson process whose stochastic intensity process  $\{\omega_t\}_{0 \leq t \leq T}$  verifies that for each  $0 \leq t \leq T$ ,

$$\omega_t = \phi \left( \frac{\sigma^2 - \Lambda(\chi_t)(\mu_t + (q - b) - r_t)}{\sigma^2 + \phi \Lambda(\chi_t)^2} \right).$$

5. Furthermore, if the insider trader has entered into a transaction modeled by a payoff  $\mathcal{O}(S_T)$ , we call  $\mathcal{O}(t, S, V)$ , the price of this option at time  $t$  and spot level  $S$ . Then, for all  $0 \leq t \leq T$ , the option price  $\mathcal{O}$  can be computed via the following

partial differential equation:

$$\begin{aligned} r_t \mathcal{O} = & \frac{\partial \mathcal{O}}{\partial t} + (r_t - (q - b) - \Lambda(\chi_t)\omega_t) S \frac{\partial \mathcal{O}}{\partial S} + \frac{\partial \mathcal{O}}{\partial V} \chi_t dt \\ & + \frac{1}{2} \frac{\partial^2 \mathcal{O}}{\partial S^2} \sigma^2 S^2 + (\mathcal{O}(t, S + S\Lambda(\chi_t), V) - \mathcal{O}(t, S, V)) \omega_t. \end{aligned} \quad (\text{I.2.5})$$

**Remark 4.** We refer to this measure as “information-neutral” rather than “risk-neutral”. This distinction arises because, for the large trader, it is not truly risk-neutral in nature. Given that the large trader is also an insider, they have complete control over the process  $\chi_t$ . As such,  $\Lambda(\chi_t)d\pi_t$  isn’t purely an exogenous source of risk. Instead, it is a risk over which the large trader exerts influence. Thus, the term “risk-neutral” does not fully capture the essence of the concept in this context.

## I.2.3 Derivatives under Market Impact

In general, there are two schools of thought with regard to hedging policies for derivatives: one suggesting a full hedge policy and the other partial hedging. The full hedge approach generally results in option prices that are too expensive in illiquid markets simply because the cost of rebalancing the *delta* of the option can be considerable when getting closer to the strike and expiry date.

The alternative method is partial hedging. In the case of a partial hedge, the principal approach has been optimizing a gain function for the large trader, e.g., [Guéant and Pu \(2017\)](#). However, in optimizing a utility function, the solutions may manipulate the payoff and the underlying stock in favor of the large trader. Therefore, it is important to formulate the problem such that market abuse does not become inherent in the model. In that regard, this thesis is one of the first to introduce and emphasize the issue of market manipulation in derivatives hedging.

### I.2.3.1 Full Hedge Approach

In this section, the full hedge model is explored as exemplified by [Liu and Yong \(2005\)](#). This methodology integrates the permanent impact into an extension of the Black-Scholes model, resulting in a parabolic partial differential equation. Further notable works on the permanent impact in derivatives include [Loeper et al. \(2018\)](#), [Bordag and Frey \(2008\)](#), and [Abergel and Loeper \(2017\)](#).

**Case 1.** [Liu and Yong \(2005\)](#) model

[Liu and Yong \(2005\)](#) proposed stock dynamics represented by Equation I.2.6, which assumes a linear market impact model:

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma dW_t + \kappa(t, S_t) dV_t, \quad (\text{I.2.6})$$



where  $\kappa$  denotes the market impact factor. Unlike our framework, the model by [Liu and Yong \(2005\)](#) is a deterministic liquidity model and assumes that it is designed as such to fully hedge, i.e., option payoff to be almost surely replicated using a portfolio comprised of the underlying asset and the risk-free rate.

Let us assume the option price under the [Liu and Yong \(2005\)](#) model to be  $v(t, S_t)$ , with the terminal payoff at  $T$  being  $v(T, S_T)$ . [Liu and Yong \(2005\)](#) prove that under a full hedge approach, the option price can be determined by a partial differential equation [I.2.7](#). Under some regularity detailed in [Liu and Yong \(2005\)](#) which ensures the stability criterion  $\left| \kappa(t, S_t) S_t \frac{\partial^2 v}{\partial S_t^2} \right| < 1$ , they prove that the option price follows the following partial differential equation:

$$\frac{\partial v}{\partial t} + rS_t \frac{\partial v}{\partial S_t} + \frac{1}{2} \frac{\partial^2 v}{\partial S_t^2} \sigma^2 S_t^2 \left( \frac{1}{1 - \kappa(t, S_t) S_t \frac{\partial^2 v}{\partial S_t^2}} \right)^2 - rv = 0. \quad (\text{I.2.7})$$

Also, the number of shares by the large trader to be executed to fully hedge its position would be:

$$\frac{\partial v}{\partial S_t}(t, S_t) \quad (\text{I.2.8})$$

The expression  $\frac{1}{1 - \kappa(t, S_t) S_t \frac{\partial^2 v}{\partial S_t^2}}$  in Equation [I.2.7](#) might evoke in a reader the concept of a geometric series summation. Under the full hedge approach framework, a trader is expected to purchase or sell  $d \frac{\partial v}{\partial S}$  shares at any delta rebalancing dates, which by Ito's lemma it can be expanded by:

$$d \frac{\partial v}{\partial S} = \frac{\partial^2 v}{\partial t \partial S} dt + \frac{\partial^2 v}{\partial S^2} dS_t + \frac{1}{2} \frac{\partial^3 v}{\partial S^3} (dS_t)^2 + o(t) \quad (\text{I.2.9})$$

This leads us to a cyclical phenomenon due to delta hedging, illustrated in [Figure I.2.2](#). A spot movement of  $\delta$  compels the trader to purchase additional shares, thereby inducing an additional spot movement, and so on.

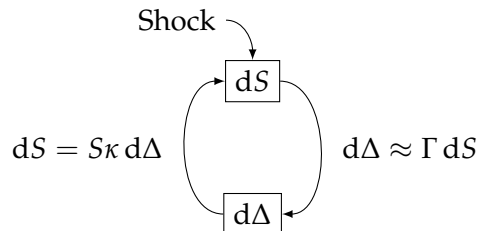


FIGURE I.2.2: The cyclic phenomenon generated by the market impact of delta hedging a derivative



Let us assume a market shock of  $d\delta_t^0 = S_t\sigma dW_t$  in the infinitesimal period of  $dt$ , the cyclic phenomenon results in the total shock  $\sum_{i=0}^{i=\infty} \delta_t^i$ , where  $\delta_t^n = \kappa(t, S_t)S_t \frac{\partial^2 v}{\partial S^2} \delta_{n-1}$  for all  $n \in \mathbb{N}$ . Upon convergence, assured by the stability condition given by  $\left| \kappa(t, S_t)S_t \frac{\partial^2 v}{\partial S^2} \right| < 1$ , the series sums up to  $\sigma \frac{1}{1 - \kappa(t, S_t)S_t \frac{\partial^2 v}{\partial S^2}} dW_t$ .

$$\sum_{i=0}^{\infty} \delta_t^i = \sigma \frac{1}{1 - \kappa(t, S_t)S_t \frac{\partial^2 v}{\partial S^2}} dW_t.$$

This sum of the series equation enables the reformulation of the stock dynamics. Subsequently, by employing the Feynman-Kac formula, the non-linear partial differential equation is derived as depicted in Equation I.2.7.

Note that a similar interpretation of [Liu and Yong \(2005\)](#) model is also presented in [Glover et al. \(2010\)](#), and [Said \(2020\)](#).

Upon implementation of the [Liu and Yong \(2005\)](#) model's partial differential equation, we observed that in a market with significant liquidity in the underlying assets, the full hedge approach proves to be challenging; the main catalyst for this is the cyclic phenomenon mentioned earlier. This phenomenon brings about two main challenges: Firstly, it creates scenarios in which the impact of its hedging can be so profound that, at times, the trader might choose not to hedge. Secondly, this cyclic effect results in [Liu and Yong \(2005\)](#) giving too expensive prices - for some options - so that they become untenable due to a lack of market demand for such exorbitantly priced options.<sup>2</sup>

Considering these challenges, a more viable solution is the partial hedging approach, which will be the primary focus for the remainder of this thesis. Before delving into that, however, it is essential to highlight and discuss the regulatory challenges a large trader might encounter when adopting a partial hedging strategy.

### I.2.3.2 Market Manipulation Regulation in Derivative Strategies

The following notes explain two regulatory concepts pertinent to market manipulation while hedging derivatives. We will explain further in Chapter [III.1](#) how these regulatory standards will guide us in defining a derivative hedging program.

#### Abusive Squeeze

An abusive squeeze occurs when a trader sequentially buys (or sells) an asset, only to sell (or buy) the same asset within a specified time frame. This action is intended to generate profits from the resulting market impact caused by such trading activity.

<sup>2</sup>This statement is verified from my implementation of the model at the investment bank.

According to regulatory standards, this behavior is flagged as *market abuse*. As delineated in section 1.6.4 of the Market Abuse directive of the FCA handbook ([Financial Conduct Authority, 2005](#)):

*...an abusive squeeze - a scenario where an individual ... exerts a significant influence over the supply, demand, or delivery mechanisms of a qualifying investment, related investment, or the foundational product of a derivative contract...*

### Payoff Manipulation

In effect, if a large trader influences the option's payoff through their hedging strategy's market impact, it would be categorized as market abuse, as per Article 12.1.a.ii of the EU market manipulation regulation ([European Parliament and of the Council, 2014](#)) and as illustrated in the FCA handbook on Market Abuse ([Financial Conduct Authority, 2005](#)) (MAR 1.6.15):

*... engaging in a transaction, initiating an order to trade, or any other conduct which: ii) establishes, or has the potential to establish, the price of one or multiple financial instruments ... at a distorted or artificial level ...*

*... manipulative techniques can span both spot and derivatives markets. Activity in financial instruments, including commodity derivatives, may manipulate related spot commodity contracts. Conversely, spot commodity contracts might be used to influence corresponding financial instruments. The prohibition against market manipulation should encompass these interactions ...*

Furthermore, the directive elaborates:

*... suppose a trader possesses a short position, which would be profitable if a specific financial instrument—currently a component of an index—drops out of that index. If this trader places a substantial sell order for this instrument just before the trading session closes, their intention might be to artificially alter the price of the instrument. This could force the instrument out of the index, thereby realizing a profit ...*

**Example 1.** Consider a large influential trader who offloads a very short-term out-of-the-money Knock-out call option. The option's delta approximates 0%. Now, if the stock experiences a sudden upswing, the delta would shift closer to 100%. Presented with a choice, the trader could delta hedge the position by procuring shares at an elevated price or engage in short selling to pull the market downwards, returning the delta to a 0% state. Alternatively, the trader might opt to buy a vast quantity of stocks, knowing that their significant market impact could activate the Knock-out strike price. In essence, the trader possesses the capability—either intentionally or inadvertently—to tamper with the payoff.

### I.2.3.3 Partial Hedging Approach

**Definition 5.** We define the hedging strategy account the couplet  $\{H_t, V_t\}_{0 \leq t \leq T}$ , remind that  $\{V_t\}_{0 \leq t \leq T}$  is representing the number of shares holding account, and  $\{H_t\}_{0 \leq t \leq T}$  is an  $\mathbb{F}$ -adapted process representing the cash account, with  $V_0$  as a constant and  $H_0 = 0$ . The dynamics of these processes are described as follows:

$$\begin{aligned} dV_t &= \chi_t dt, \\ dH_t &= H_t r_t dt + V_t S_t (q - b) dt - S_t dV_t - d\langle S, V \rangle_t - dC_t, \end{aligned} \quad (\text{I.2.10})$$

being  $\{C_t\}_{0 \leq t \leq T}$  the transaction costs process paid by the large trader.

We assume a physical delivery European option with an expiry date of  $T$  and a settlement date of  $T + \tau$ .  $Q(S_T)$  denotes the terminal number of shares to be either delivered or received, and  $\mathcal{H}(S_T)$  is the terminal cash transaction.

The final profit and loss (P&L) at maturity is given by:

$$P\&L = \underbrace{\mathcal{H}(S_T) + S_T Q(S_T)}_{\text{Payoff}} + \underbrace{H_T + S_T V_T}_{\text{Accounting value of hedges}} - \underbrace{S_T \bar{C}_T(Q(S_T) + V_T, \tau)}_{\text{Terminal execution cost of missing shares}}, \quad (\text{I.2.11})$$

where  $\bar{C}_T(Q(S_T) + V_T, \tau)$  is the execution cost of buy/selling  $Q(S_T) + V_T$  amount of shares during the period of expiry  $T$  and settlement  $T + \theta$  of the option. We give a precise definition and parametrization of the function  $\bar{C}_T$  in Chapter III.1.

In essence, the trader's primary objective is to optimize the P&L described in Equation (I.2.11). However, this ambition is bounded by regulatory restrictions. To effectively address these constraints associated with market abuse, certain modifications to the stochastic optimization problem are necessary. First, to counteract the risk of payoff manipulation, the payoff component of Equation (I.2.11) is excluded from the optimization framework. Additionally, to prevent potential arbitrage from trading activities, such as buying and selling shares for hedging, the stochastic control problem is formulated under an information-neutral probability measure. As will be elaborated in Chapter III.1, the expected P&L of the hedging activities under this measure is zero.

Among all the information-neutral measures, this thesis specifically focuses on the minimum variance information-neutral measure. Therefore, the expected utility of the optimization problem is assessed under this particular probability measure. This leads us to the cost optimization problem as per Equation

(I.2.12):

$$J(t, V_t, S_t) = \inf_{\{V_s\}_{s \geq t}} \mathbb{E}^{I^*} \left[ e^{-\int_t^T r_s ds} S_T \bar{C}_T (Q(S_T) + V_T, \tau) + \int_t^T e^{-\int_t^s r_u du} S_s dC_s \middle| \mathcal{F}_t \right]. \quad (\text{I.2.12})$$

The cost in Equation (I.2.12) consists of two parts: the transaction cost of executing missing shares between the expiry date  $T$  and the settlement date and the transaction cost of executing shares before the expiry date. These two terms act against each other, and the trader needs to strike a balance between buying shares too early and getting closer to a full delta hedge with fewer missing shares to recover in the future but paying extra costs because of executing the shares too quickly, or leaving its position partially unhedged. If lucky, the market will move in the trader's direction, and the partial hedge will be closer to the full hedge. Otherwise, the trader will have some time between the expiry date  $T$  and the settlement date  $T + \tau$  to execute the missing shares. Note that at the expiry date, the number of required shares is fixed and is no longer uncertain.

In Part III, a stochastic differential equation for the cost function  $J$  is derived. The same Part provides numerical examples comparing the prices and hedges of our model against the benchmark given by the Black & Scholes model.

## I.3 Summary of Major Contributions

This research contributes to two areas of study: *Synthetic GIC Asset-Liability Modeling* and *Derivatives under Market Impact*. This section briefly outlines these contributions.

### I.3.1 A Quantitative Framework for Asset Liability Risk Management of Synthetic GIC

This thesis seeks to introduce a simpler, Black & Scholes-like model for fixed-income bond funds, aiming to effectively capture their primary risk factors. This approach contrasts with the complex foundational models typically used in fixed income literature. Instead, this model focuses on primary risk factors, being the yield of the fund, providing a holistic approach to capture fund dynamics. This asset model is developed in Chapter II.1 is as follows:

$$\frac{dM_t}{M_t} = (y_t + \delta_t)dt - \theta_t dy_t. \quad (\text{I.3.1})$$

Our findings indicate that our model provides explanatory power equivalent to that of the traditional sum of reinvested coupon payments model. The simplicity and robustness of our model provide a compelling case for its use in academia and practice for risk management of fixed-income funds.

One other major contribution of this research is the detailed examination of participant cash flows for stable value funds. A unique dataset was leveraged, which represents approximately 80% of the large employer plan market. The analysis, presented in Chapter II.2, reveals that:

- Cash flows exhibit medium to long-term non-monotonic trends. Various elements within the plan sponsor's ecosystem, such as employment growth, default 401(k) plan options, and the introduction of new investment options, indirectly yet significantly influence these trends. These observations contribute to the existing body of literature on participant behavior in retirement plans.

- The research also uncovers the dominant role of flight-to-safety behavior during global market crises, enriching and offering a new case study for the existing literature on the flight-to-safety phenomenon.
- While the risk of mass lapses due to reputational issues is observed, the findings indicate that the probability of such occurrences is low.
- Other behavioral hypotheses, such as the moneyness hypothesis, are deemed less relevant in the context of stable value funds.

Therefore, in this thesis, we enhance existing literature on the lapse hypothesis, contextualizing it within the unique nuances of the stable value ecosystem, identifying the most impactful factors, and introducing new hypotheses as needed. Furthermore, as an outcome of the research, a multi-component model is presented to project scenarios and assess the lapse risk of stable value fund liabilities. This model, discussed in Chapter II.3, is critical for risk assessment, bridging the gap between empirical behavioral findings and risk management:

$$dC_t = B_t \left( \underbrace{\beta_t}_{\text{structural trend including herd behavior}} + \underbrace{g(r_t - \gamma_t)}_{\text{rate deficit}} + \underbrace{f(s_t)}_{\text{flight-to-safety}} \right) dt. \quad (\text{I.3.2})$$

Using the asset and liability model above, the study provides a quantitative framework for asset liability management of the Synthetic GIC. The complex path-dependent nature of the risk for this product is demonstrated in Chapter II.3. Two critical risk scenarios for the insurer are identified – the inflationary scenario and the yield spike scenario. The model’s suitability is assessed, and the analysis shows that it is flexible enough to detect and generate the riskiest scenarios from the insurer’s perspective.

In addition, the study offers detailed discussions on solvency and capital margins. In Chapter II.3, the sensitivity of wraps’ tail risk to parameters like asset duration and credit quality is highlighted, a finding that is in stark contrast with current U.S. regulatory approaches for which the capital reserves estimate based on their methodology is almost always zero. This divergence prompts a call for more nuanced models that could mimic the most critical scenarios inferred from the path-dependent nature of the risk of this product.

Hence, this thesis addresses a significant research gap by introducing a quantitative modeling framework and scenario analysis for risk management of stable value fund guarantee products, an essential part of the U.S. financial market due to their substantial size.

### I.3.2 Derivatives under Market Impact

This thesis addresses the practical issue of model inadequacy for large derivative transactions, considering the perspective of an insider trader who is privy to a large trader's activities.

In Chapter III.1, the existence of *information-neutral probability measures* is established. Building on this framework, a derivatives hedging policy for the large trader is developed. The research also provides a detailed discussion of market manipulation in derivatives and one methodology on the design of the quantitative hedging framework in order to mitigate any payoff manipulation.

This research thus contributes to understanding how a large trader can effectively manage a derivative position under an illiquid market and addresses the often overlooked issue of the risk of market manipulation in derivative hedging strategies.

## **Part II**

# **Synthetic GIC Asset Liability Modeling**



## II.1 A Model for Fixed Income Funds

*This Chapter, while essential to this thesis, does not follow an article format. Instead, it serves as the foundational asset model used for the quantitative analysis in subsequent sections. The content of this Chapter, co-authored with Alvin Ngugi, contributes to the modeling of fixed income funds. Consequently, post-thesis, we plan to restructure this Chapter into a standalone article for future presentation and publication.*

### II.1.1 Introduction

As significant components of the financial landscape, fixed income funds or mutual bond funds make up approximately 20% of the floating assets. Despite their substantial presence, existing literature offers no straightforward approach to model these funds' risk. Current models typically represent a bond fund as an aggregate of cash flows, such as the summation of coupon payment structures by [Ringer and Tehranchi \(2006\)](#), a blend of zero-coupon bonds by [Andersson and Lagerås \(2013\)](#), or a configuration of reinvested coupon payments as demonstrated by [Kwun et al. \(2010\)](#) and [Alfonsi et al. \(2019\)](#).

In addition, these models often utilize arbitrage-free interest rate models for simulating the cash flow. These types of models, such as the Heath-Jarrow-Morton and Ornstein-Uhlenbeck processes, developed by practitioners and academics since the early 1990s, primarily catered to investment banks' needs for pricing and hedging trading instruments like swaptions and caplets and introduced unnecessary complexity to understanding fixed-income fund risk. By overly emphasizing secondary risks and downplaying primary risks, these models may be unsuitable for fixed-income funds.

Contrastingly, the literature for equity funds is much simpler, primarily because the groundwork was laid with an uncomplicated model - the Black & Scholes. The complexities of this model were incrementally added over time. However, the fixed-income literature did not start with a simple model, resulting in a more complex foundation. Consequently, this paper aims to

reintroduce simplicity by presenting the 'Black & Scholes equivalent for fixed-income bond funds. This simple model aims to capture the primary risk factors associated with these securities effectively.

Our model is based on two key metrics pertinent to any fixed income fund - *yield* and *duration*. It aims to elucidate bond fund dynamics using these interpretive statistics, offering a simpler yet robust alternative to traditional methods of sum of coupons.

We contrast our model with the conventional method of summing coupons for post-model validation. Despite its simplicity, we demonstrate that our model provides an explanatory power that matches, if not surpasses, the conventional method.

The rest of the paper unfolds as follows: Section II.1.2 presents proof and justification for our fixed-income fund model. Section II.1.3 applies the model to historical data on fixed-income funds using anonymized aggregate indices. Section II.1.4 concludes the paper, underscoring key findings and potential avenues for future research.

## II.1.2 Fixed Income Funds Modeling Framework

**Definition 6.** Let  $(\Omega, \mathcal{F}_t, \mathbb{P})$  be a probability space, and  $\{M_t\}_{0 \leq t}$  be a continuous stochastic process representing the market value of the fixed income fund. Let  $\mathbb{P}$  be the historical probability measure,  $\mathbb{F} = (\mathcal{F}_t)_{t \in [0, T]}$  be the filtration generated by fixed income funds' market values  $\{M_t\}_{t \geq 0}$ . Let's also assume that  $\{M_t\}_{t \geq 0}$  is an Ito process, even though we will not need to expand its dynamics any further.

The  $\mathcal{F}$ -adapted stochastic process  $\{M_t\}_{t \in [0, T]}$  describes the market value of the fund at time  $t$ . In real applications, this value is set by the market and is known to correspond to the weighted sum of the prices of each security in the fund.

**Definition 7.** For each time  $t \geq 0$ , let  $c_t(u) : \mathbb{R}^+ \rightarrow \mathbb{R}$  be a piecewise continuous exponential order function. That is, there exist constants  $c$ ,  $t_0$ , and  $a$  such that:

$$|c_t(u)| \leq ce^{au} \quad (\text{II.1.1})$$

for all  $u > t_0$ .

For each time  $t \geq 0$ ,  $c_t(u)$  represents the coupon promised at time  $u$ . Note that we assume that  $c_t(u)$  is a piecewise continuous function, constructed and rebalanced by the manager at any time  $t$ , therefore  $c_t(u)$  is known at  $t$ . In the case of a large fixed-income fund, the assumption of piecewise continuity is reasonable since a well-managed fixed-income fund consists of a large variety

of bonds with different coupon payment dates, creating the typical continuous piecewise shape.

**Definition 8.** For any given time  $t \geq 0$ , the present value of the coupon structure  $D_t : \mathbb{R}^+ \rightarrow \mathbb{R}$  is defined by the Laplace transform of the coupon function:

$$D_t(r) = \mathcal{L}\{c_t\}(r) = \int_t^\infty c_t(u-t)e^{-(u-t)r} du. \quad (\text{II.1.2})$$

Here,  $r \in \mathbb{R}^+$  represents a fixed reference interest rate.

**Definition 9.** The yield  $y_t$  of the fund corresponds to an  $\mathcal{F}_t$ -adapted process, which is the reference rate that equates the Laplace transform of the coupon function to the market value of the fund, namely, hence

$$D_t(y_t) = M_t, \quad (\text{II.1.3})$$

for any time  $t \geq 0$ . Moreover, since the coupon function piecewise continuous exponential we know from Schiff (1999), that  $D$  is invertible. Hence, also the following equality holds

$$y_t = D_t^{-1}(M_t) \quad (\text{II.1.4})$$

**Proposition II.1.2.1.** For a given  $M_t$  and given function  $c_t(\cdot)$  conforming with the regularity conditions explained above,  $y_t$  exists, and it is unique.

*Proof.* For any given time  $t \geq 0$ , one can prove that under condition II.1.1, the Laplace transform is unique, and the reverse Laplace function exists therefore, there is a homeomorphism between  $\{M_t(\omega) \leftrightarrow y_t(\omega), \omega \in \Omega\}$ . Proof of unicity, existence, and homeomorphism is provided in Schiff (1999).  $\square$

By applying the Ito formula to the function  $D_t(y_t)$ , we derive the following equation :

$$dD_t(y_t) = \frac{\partial D_t(y_t)}{\partial t} dt + \frac{\partial D_t(y_t)}{\partial r} dy_t + \frac{\partial^2 D_t(y_t)}{\partial r^2} d\langle y \rangle_t \quad (\text{II.1.5})$$

Note that at any time  $t$ , we do have the following dynamics:

$$dM_t = dD_t(y_t) + c_t(t)dt \quad (\text{II.1.6})$$

In financial terms, the Equation II.1.6 means that the coupon received at  $t$ , i.e.  $c_t(t)dt$  will be kept in the fund account as cash - and reinvested in some new securities -.

On the other hand, one can easily verify that for any  $r : c_t(t)dt + \frac{\partial D_t(r)}{\partial t} = D_t(t)r$ . Therefore, the following equation holds:

$$dM_t = D_t(y_t) \cdot y_t dt + \left. \frac{\partial D_t(r)}{\partial r} \right|_{r=y_t} dy_t + \left. \frac{\partial^2 D_t(r)}{\partial r^2} \right|_{r=y_t} d\langle y \rangle_t \quad (\text{II.1.7})$$

**Definition 10.** For any given time  $t \geq 0$ , we define the duration function  $\theta_t : \mathbb{R}^+ \rightarrow \mathbb{R}$  and the convexity function  $\eta_t : \mathbb{R}^+ \rightarrow \mathbb{R}$  functions for a given reference rate  $r$ , as the first and second order derivatives, respectively, of the Laplace transform, namely:

$$\begin{aligned} \theta_t(r) &= -\frac{1}{D_t(r)} \frac{\partial D_t(r)}{\partial r}, \\ \eta_t(r) &= \frac{1}{D_t(r)} \frac{\partial^2 D_t(r)}{\partial r^2} \end{aligned} \quad (\text{II.1.8})$$

We take the perspective of an investment manager overseeing a portfolio of fixed-income securities. The manager may continuously re-evaluate the portfolio at any  $t \geq 0$ . During each restructuring time, the manager determines a desired cash flow structure, denoted by  $c_t(u)$ , where  $u$  specifies the time of the upcoming coupon designated by the manager at the specific time  $t$ .

The manager can adjust this coupon structure by selling existing securities, purchasing new ones, or rebalancing upcoming coupons. Such restructuring is driven by two primary metrics in mind: duration and convexity. Duration is the first-order sensitivity of the portfolio's market value with respect to a change in yield, while convexity is the second-order sensitivity.

The objective of the investment manager is to maintain the duration  $\theta_t$  and the convexity  $\eta_t$  at desired levels  $\bar{\theta}_t$  and  $\bar{\eta}_t$  respectively, for all times  $t$  in  $[0, T]$ . We assume that  $\bar{\theta}_t$  and  $\bar{\eta}_t$  are  $\mathcal{F}_t$ -adapted process, known and decided by the portfolio manager at time  $t$  based on the filtration  $(\mathcal{F}_t)$ . These constraints lead to the following system:

$$\begin{aligned} \theta_t(y_t) &= \bar{\theta}_t, \\ \eta_t(y_t) &= \bar{\eta}_t. \end{aligned} \quad (\text{II.1.9})$$

Hence, the manager is asked to solve (II.1.10) to compute for every time  $t$  the yield that allows having the duration and the convexity at the desired targets. The manager has control of the accounting balance of  $M_t$ ; it is asked to compute the manager has to find an appropriate coupon structure  $c_t^*(u)$ , such that duration and convexity can be set as the target values.

**Proposition II.1.2.2.** At any time  $t \geq 0$ , for a given fixed income market value  $\{M_t\}$ , and for a set  $\bar{\theta}(t), \bar{\eta}(t)$ , the system (II.1.10) combined with Eq. (II.1.3) has at least

one coupon structure solution. Thus from Eq. (II.1.4), we get the following:

$$\begin{aligned}\theta_t(y_t^*) &= \bar{\theta}_t, \\ \eta_t(y_t^*) &= \bar{\eta}_t.\end{aligned}\tag{II.1.10}$$

being  $y_t^*(\cdot)$  the yields corresponding to  $c_t^*(\cdot)$  that lets the manager achieve the prescribed targets.

*Proof.*  $c_t(\cdot)$  is the inverse Laplace transform of any continuous function  $F : \mathbb{R}^+ \rightarrow \mathbb{R}$ ,  $F \in \mathcal{L}^\infty(\mathbb{R}^+)$  satisfying the following conditions:  $F(x) = M_t$ ,  $F'(x) = M_t \bar{\theta}_t$ ,  $F''(x) = M_t \bar{\eta}_t$ .  $F$  is a function, that its value at  $x$  is  $M_t$ , its first derivative at  $x$  is  $M_t \bar{\theta}_t$ , and its second derivative at  $x$  is  $M_t \bar{\eta}_t$ . There are a multitude of  $\mathcal{L}^\infty(\mathbb{R}^+)$  continuous functions  $F$ , and a multitude of variables  $x$  that satisfy this system of equations. The inverse Laplace of  $F$  is the functions  $c_t(\cdot)$ , and the solution  $x$  is the variable  $y_t$ . Hence, there are a multitude of functions  $c_t(\cdot)$  and variable  $y_t$  that satisfy this fixed point problem. But note that as explained in proposition II.1.2.1, once the coupon structure function  $c_t(\cdot)$  is chosen by the manager, there would be only one unique solution  $y_t^*$ .

□

By combining Equations II.1.10, and II.1.7, we get:

$$dM_t = M_t y_t dt - M_t \bar{\theta}_t dy_t + \frac{1}{2} M_t \bar{\eta}_t d\langle y \rangle_t \tag{II.1.11}$$

Finally, if we assume that  $M_t$  and consequently  $y_t$  are Ito processes, we obtain Equation II.1.12. This formulation, which provides a simplified depiction of the fund dynamics, suggests that the fund's volatility has a linear relationship with the volatility of its yield, moderated by its duration. Moreover, it proposes that the fund's average return can be approximated by its yield adjusted by a  $\delta_t$  factor ( the convexity adjustment factor).

$$\frac{dM_t}{M_t} = (y_t + \delta_t) dt - \bar{\theta}_t dy_t. \tag{II.1.12}$$

### II.1.3 Empirical results

The data utilized for this study is sourced from anonymized indices, providing comprehensive coverage of fixed income products with a global scope. These indices serve as comprehensive benchmarks, capturing the investment-grade, US dollar-denominated, fixed-rate taxable bond market. Due to this wide-ranging tracking, the indices represent the fixed income market, adhering to a monthly drop and select methodology to maintain the fund's rating level.

The sample period for our study spans from February 2000 to March 2021. Our overall sample consists of the US aggregate index and two rated related indices. Closing prices of the business day, the final traded prices for the respective indices, were utilized. In the event of missing prices, the most recent non-missing values were employed, which arose for only eight data points.

In the subsequent sections, we examine the actual indices' comparability with the evolution generated by our model and present the evolution when employing a sum-of-coupons model. The figures below demonstrate the results of applying our baseline model to the Aggregate Bond Index and its related indices. The impairment rate was determined by fitting the data to our model and finding the impairment rate that maximizes the model fit, as measured by the coefficient of determination ( $R^2$ ).

Figure II.1.1 compares the historical data of the US Aggregate Index and the outputs produced by both our model and the par coupon model. Visually, the model appears to represent our data satisfactorily, as the time series produced by the model closely aligns with the actual fund time series.

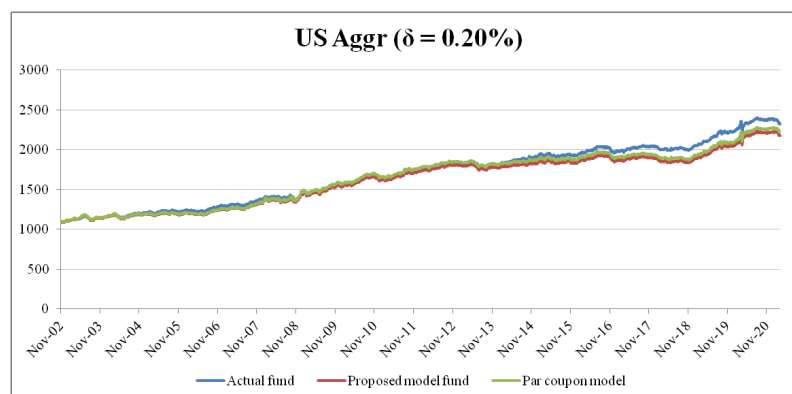


FIGURE II.1.1: Comparison of fitted model with historical data: US Aggr. Historical data from Barclays (nd) fixed income indices.

Figure II.1.2 and II.1.3 present the historical data of two rated indices, the US Aggregate AA and Aggregate Baa, alongside the outputs of our model and the par coupon model. Once again, for the selected  $\delta$ , the model visually appears to be suitable.

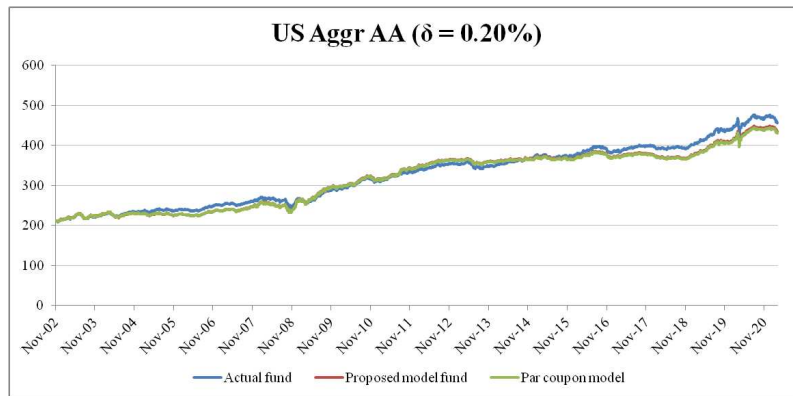


FIGURE II.1.2: Comparison of fitted model with historical data: US Aggr AA. Historical data from Barclays (nd) fixed income indices.

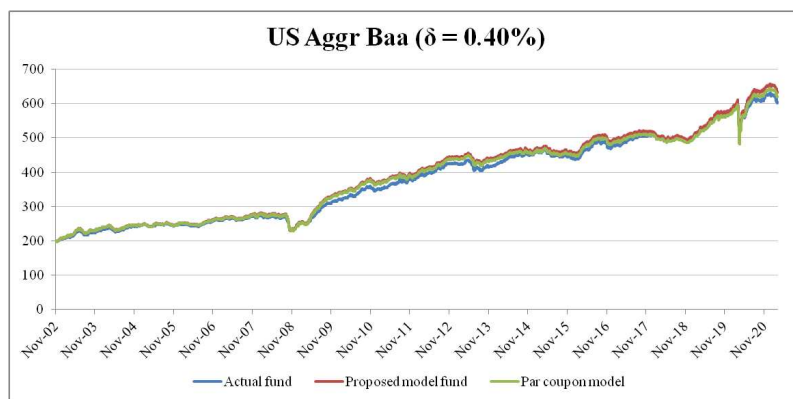


FIGURE II.1.3: Comparison of fitted model with historical data: US Aggr Baa. Historical data from Barclays (nd) fixed income indices.

To confirm the perceived consistency between the actual data and the model’s predictions, we conduct several statistical tests based on monthly and semi-annual returns of the fund. This approach aligns with our understanding that investors generally have a long-term investment horizon and are more interested in long-term movements rather than daily fluctuations. Tables II.1.1 and II.1.2 below summarize key statistical evaluation metrics, testing the model’s fit to the actual fund. We also provide fit statistics for the par coupon model relative to the fund.

Metric	US Aggr ( $\delta = 0.20\%$ )	US Aggr AA ( $\delta = 0.20\%$ )	US Aggr Baa ( $\delta = 0.40\%$ )
$R^2$	91.24%	88.75%	93.77%
Kendall correlation	87.08%	82.20%	85.95%

TABLE II.1.1: Evaluation metrics for the proposed model: Monthly data

Metric	US Aggr ( $\delta = 0.20\%$ )	US Aggr AA ( $\delta = 0.20\%$ )	US Aggr Baa ( $\delta = 0.40\%$ )
$R^2$	88.91%	77.77%	92.83%
Kendall correlation	86.08%	78.07%	86.41%

TABLE II.1.2: Evaluation metrics for the proposed model: Semi-annual data

Metric	US Aggr ( $\delta = 0.20\%$ )	US Aggr AA ( $\delta = 0.20\%$ )	US Aggr Baa ( $\delta = 0.40\%$ )
$R^2$	91.32%	88.78%	93.94%
Kendall correlation	86.97%	82.22%	85.92%

TABLE II.1.3: Evaluation metrics for the par coupon model: Monthly data

Metric	US Aggr ( $\delta = 0.20\%$ )	US Aggr AA ( $\delta = 0.20\%$ )	US Aggr Baa ( $\delta = 0.40\%$ )
$R^2$	87.53%	77.81%	92.86%
Kendall correlation	85.60%	78.06%	86.24%

TABLE II.1.4: Evaluation metrics for the par coupon model: Semi-annual data

These statistical evaluations reinforce the suitability of the proposed model for simulating the evolution of rated fixed income funds, with both  $R^2$  and Kendall correlation figures indicating high degrees of fit. The metrics substantiate the high accuracy of the selected model.

It is crucial to emphasize that one of the primary structures used in academic literature to replicate a fixed income fund is the par coupon model, even though a multitude of variations in coupon structures do exist. We aim to demonstrate that our market value model is as competent as some of these well-established benchmarks. Considering the vast array of alternatives to the par coupon—including parsed, ascending, or descending coupons with different growth rates—it becomes impractical to assess our model against each one, given the exhaustive nature of the list. As such, we have opted to compare our model with the most commonly used variation, which is the par coupon.

The residual plots for the US Aggregate and US Aggregate Baa indices, shown below, illustrate residuals concentrated around zero. This indicates that the model's predictions are, on average, accurate, with only a few outliers observed during the 2008/09 financial crisis and the onset of the COVID-19 pandemic.

The residual plot exhibits increased clustered volatility during the financial crisis and the beginning of the pandemic period. A comprehensive exploration of this volatility's underlying factors necessitates further qualitative and quantitative research. However, a non-exhaustive list of the potential drivers are:



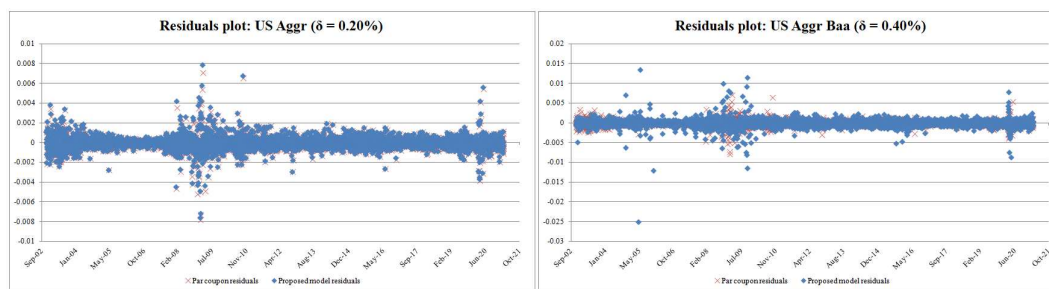


FIGURE II.1.4: Residuals plot. Source: Backtested simulation against Barclays (nd) fixed income indices data.

1. **Trading Frequency and Density:** Other factor tied to the funds' structure, i.e., target rating, duration, etc., requires managers to engage in more frequent buy and sell actions during crisis periods. The dense clustering around the pandemic and financial crisis periods could be indicative of this elevated trading activity. In other words, the time in Figure II.1.4 should not be viewed linearly; the time is not linear. There is just more going on during a crisis than during normal days. While this hypothesis might explain the clustered residual density during these intervals of crisis, it does not clarify their augmented size.
2. **Operational Volatility:** With heightened market volatility and frequent trading during crises, managerial decision-making becomes critical to the daily performance of the fund. Also, the risk of operational mistakes or misguided buy/sell decisions introduces another source of volatility. Validating this hypothesis would necessitate a comparative analysis of multiple funds managed by different managers across these volatile periods.
3. **Convexity Shifts:** Other risk factors to consider include changes in asset convexity during these periods, especially for securitized products that may be subject to different prepayment assumptions due to market turmoil. Even though Figure II.1.4 represents a US corporate aggregate fund, which is not subject to prepayment, its convexity still could be influenced by sharp yield movements.
4. **Default and Credit Risks:** Another risk heightened during these periods is the risk of default and credit migration. This could result in assets being impaired or assessed at losses. However, this hypothesis would suggest primarily negative residuals during these crises, which is not consistently observed in Figure II.1.4.
5. **Aggressive Liquidation and Acquisitions:** In volatile market conditions, such as during the 2008 financial crisis or early pandemic days, asset values are heavily influenced by aggressive liquidation and acquisitions,

causing price instability. This hypothesis can be seen as plausible since the residuals' oscillates between positive and negative values during the period of crisis are in Figure II.1.4. Full validation of such a hypothesis would require examining each security's volatility within the fund and trading volumes to discern any aggressive liquidation or acquisition trends.

In conclusion, while the drivers behind the residual plot require more in-depth analysis, the fact that our model readily captures these nuances underlines its effectiveness. We have managed to simplify the modeling of a fixed income fund's primary risks, eliminating the need for complex interest rate structure models and calibrations to swaption and floor/cap markets. Instead, both professionals and academics can perceive the risk factors during periods of crisis without getting entangled in modeling intricacies,

In conclusion, our findings demonstrate that the proposed model exhibits high accuracy when representing the evolution of rated fixed income funds, offering both visual and statistical confirmation. The par coupon model also displays a satisfactory fit, adding further robustness to our analysis.

## II.1.4 Model Limitations, Conclusions, and Prospects for Further Refinement

Throughout this paper, we have presented and explored a novel model to enhance the management and forecasting capabilities of fixed-income fund portfolios. The model was validated through comparison with the sum of coupons method, revealing comparable, if not superior, predictive capability, indicating its potential applicability in the sector.

The model was tested on a historical data set derived from the anonymized indices. The results were largely positive, with the model demonstrating high consistency with actual data and error metrics on par with the sum of coupons method. This confirms the model's potential and indicates a promising direction for future research.

Nevertheless, a notable limitation of our model is its inability to accurately represent fluctuations during periods of financial crisis, as evidenced in Figure II.1.4. It is important to recall that our model shares this limitation with the benchmarked industry-standard sum-of-coupons method. A potential solution to this problem could be the introduction of a more complex self-exciting dynamic, such as a Hawkes or GARCH process, for the  $\delta_t$  parameter instead of merely a deterministic value. This area warrants further research and subsequent enhancement of our model.

Additional limitations and potential extensions of this model pertain to considering investment strategies where managers place multiple duration constraints on the portfolio instead of a singular one, as defined in Equation II.1.10. This strategy, called 'barbell', allows a fund manager to assign extra weight to cash flows around specific durations to capitalize on the yield curve.

In conclusion, the model proposed in this paper contributes an innovative modeling approach fixed income funds. It is a potent tool that academics and practitioners can use to project and simulate risk scenarios for fixed income funds.

## II.2 Understanding Key Drivers of Participant Cash Flows for Individually Managed Stable Value Funds

### II.2.1 Introduction

Stable value funds are a common low-risk investment option in many defined contribution U.S. retirement plans. They provide liquidity and principal preservation while offering slightly higher returns than money market funds (SVIA, 2020a). As a result, these funds have become increasingly popular over the years, with assets under management reaching USD 888 billion as of the third quarter of 2020 (SVIA, 2020b). However, despite their importance in the U.S. market, some critical attributes of stable value funds remain unstudied to date.

Previous studies, such as Babbel and Herce (2007) and Babbel and Herce (2018), have provided statistical analyses of stable value fund performance relative to alternative investment options such as money market instruments. Tobe (2004) offers a clear explanation of stable value funds intended for retirement consultants. Xiong and Idzorek (2012) studied stable value funds from the investor's perspective and concluded that the funds might bear credit and liquidity risk for the participants. Kwun et al. (2009) and Kwun et al. (2010) examined the guarantee risk and proposed an asset-liability model for insurance guarantees under a dynamic lapse formula. However, limited research exists on the behavioral properties of stable value cash flows.

The exploration into the primary drivers of stable value funds cash flows is not merely an academic exercise but a practical necessity. A nuanced understanding of these drivers, particularly in terms of participant cash flows, is crucial for asset liability management within insurance companies. The misapplication or oversimplification of lapse models used in the industry, borrowed from similar but unrelated financial products such as annuities, can lead to a poor understanding of the risk scenarios involved in the liability model. The end

result is a risk management strategy that is ill-equipped to handle the specific dynamics of stable value liabilities.

This paper aims to bridge this gap by providing a contextualized analysis of the lapse hypothesis, specifically tailored to the nuances of stable value funds within 401(k) plans. To achieve this, we will employ three main tools: (1) a review of the literature on lapse models, (2) an in-depth analysis of the stable value and 401(k) context, and (3) an exploratory analysis of a unique dataset containing historical monthly cash flows for stable value funds, with data spanning from 1997 to 2021.

We aim to complement the existing literature on lapse models by relevance of some previous lapse hypotheses in the context of stable value funds, identifying the most dominant ones, and introducing new, more contextually relevant, hypotheses where applicable. However, given the absence of specific literature on lapses for stable value funds, we expanded our research to include lapse hypotheses associated with other financial products, such as savings accounts and annuities. Some of these hypotheses are the influence of plan sponsors [Madrian and Shea \(2001\)](#); [Mitchell et al. \(2006\)](#), rate deficit hypothesis [Barsotti et al. \(2016\)](#), herd behavior and mass lapse [Loisel and Milhaud \(2011\)](#); [Barsotti et al. \(2016\)](#), moneyness hypothesis [Bacinello et al. \(2011\)](#); [Cheng et al. \(2019\)](#), and flight to safety [Baur and Lucey \(2010\)](#); [Dorn and Huberman \(2005\)](#). We will test the relevance of these hypotheses using historical data, case studies, and analyses of cash flow trends during different periods, including the COVID-19 pandemic.

The necessity for a contextually tailored approach to lapse modeling becomes even more important in light of events such as the bank collapses of 2023. These events highlighted the unique participant demographics within certain banks. As highlighted by [Vo and Le \(2023\)](#), these demographics had a significant concentration “...in a small group of depositors, many of whom work in the venture capital industry. As a result, they are likely to know each other, increasing the risk of a bank run...”. Indeed, single-employer stable value plans bear similarities with these banks, notably in their participant demographics: they are typically composed of individuals who are either current or former colleagues. This web of connections among participants can potentially shape their withdrawal tendencies.<sup>1</sup> Consequently, context-specific investigation of participant cash flows in stable value funds is a necessity.

The outline of the paper is as follows: Section [II.2.2](#) enumerates lapse hypotheses from the academic literature that have been used to model withdrawals in the context of products other than stable value funds. The same section

---

<sup>1</sup>In fact, these connections contribute to non-monotonic trends in participant cash flows, a phenomenon we will delve into in Section [II.2.4.1](#).

provides an overview of the regulatory and fiscal environment for stable value funds within a 401(k) investment plan, helping readers better understand the context. Section II.2.3 describes the data and research methodology used for our empirical assessment. In Section II.2.4.1, we identify a dominant behavioral factor observed in the data: the trend component. Section II.2.4 presents our testing and verification of various lapse hypotheses, informed by the literature on lapses for other products (Section II.2.2), the context of stable value funds and 401(k) schemes (Section II.2.4.1), and our exploratory study of the stable value cash flow data (Section II.2.4). Finally, Section II.2.5 summarizes the main takeaways from this study in terms of the applicability of lapse hypotheses to stable value products and offers guidance on modeling lapses for generating risk scenarios.

It should be noted that while some conclusions in this paper can be generalized to stable value funds as a whole, the primary focus is on individually managed synthetic GICs, also known as wraps. These are individually managed 401(k) plans where the plan sponsor is a single employer. For clarity, a 401(k) is a tax-advantaged, defined-contribution retirement account offered by many employers to their employees. Workers can make contributions to their 401(k) accounts through automatic payroll withholding and their employers can match some or all of those contributions.

## **II.2.2 Literature Review on Lapse Behavior, Contextualizing It into the Stable Value Ecosystem, and Hypothesis Development**

In this section, we delve into the stable value ecosystem and its intersection with established lapse hypotheses from existing literature, predominantly designed for financial instruments such as annuities and savings accounts. Some of the literature has been extensively reviewed in works like that of [Eling and Kochanski \(2013\)](#). We initiated our research by examining the stable value ecosystem and reviewing pertinent lapse studies. Drawing from our read of the regulatory framework and previous research, we then formulated our hypotheses. These hypotheses set the stage for further empirical analysis in subsequent sections of our study.

### **II.2.2.1 Individually Managed Stable Value Regulation and Ecosystem**

Understanding the regulatory framework, as highlighted by [Cumming and Dai \(2009\)](#), is fundamental due to its influence on cash flow behaviors. They observed distinct behavioral trends within hedge fund capital flows across

various regulatory environments, emphasizing the impact regulatory landscapes can have. This underscores the necessity of deep-diving into laws and regulations when assessing lapse risk, particularly for comprehending withdrawals from stable value funds within 401(k) retirement plans.

With that in mind, the regulation surrounding stable value fund withdrawals can be classified into outer-plan and inner-plan activities. Outer-plan activities involve withdrawing funds from a 401(k) plan, such as rollovers or distributions. In contrast, inner-plan activities involve reallocating funds within a plan, such as rebalancing or switching between stable value funds and other options. Sections II.2.2.1.1 and II.2.2.1.2 discuss the treatment rules for outer- and inner-plan activities.

### II.2.2.1.1 The 401(k) Withdrawal Treatment

The 401(k) plan is a popular retirement investment option regulated by the Employee Retirement Income Security Act (ERISA). These plans provide tax benefits to individuals who make contributions with early withdrawals before retirement, usually resulting in penalties. As of the date of this study, the penalties for early withdrawals are generally 10% pre-tax. While detailed rules on 401(k) withdrawals can be found in IRS (2023b), it is essential to note that there are exceptions to the tax penalty rules. Table II.2.1 provides a summary of these exceptions (see more details in IRS (2023a)). The table classifies three categories of withdrawals: early withdrawals, hardship withdrawals, and loans.<sup>2</sup>

TABLE II.2.1: The 401(k) withdrawal penalty exceptions.

<b>Early Withdrawal</b>	
Age	After participant/IRA owner reaches age 59½
Death	After the death of the participant/IRA owner
Disability	Total and permanent disability of the participant
Domestic relations	To an alternate payee under a Qualified Domestic Relations Order
Medical health	Insurance premiums paid while unemployed, amount of non-reimbursed medical expenses up to a limit
Rollover	In-plan Roth rollovers or eligible distributions contributed to another retirement plan or IRA
<b>Hardship</b>	
Medical	Medical care expenses for the employee, the employee’s spouse, dependents, or beneficiary

<sup>2</sup>In regulatory terminology, the term “hardship” (IRS, 2023c) refers to situations where a participant faces financial difficulties. In cases of immediate and substantial financial need, participants can withdraw a portion of their assets without incurring penalties.

TABLE II.2.1: *Cont.*

<b>Hardship</b>	
Housing	Costs directly related to the purchase of an employee’s principal residence (excluding mortgage payments) Payments necessary to prevent the eviction of the employee from the employee’s principal residence or foreclosure on the mortgage on that residence Certain expenses to repair damage to the employee’s principal residence
Education	Tuition, related educational fees, and room and board expenses for the next 12 months of post-secondary education for the employee or the employee’s spouse, children, dependents, or beneficiary
Death	Funeral expenses for the employee, the employee’s spouse, children, dependents, or beneficiary
<b>Loans</b>	
Loans	The maximum amount a participant may borrow from the plan is 50% of the account balance or USD 50,000, whichever is less

It is worth noting that certain withdrawal activities delineated in Table II.2.1 may not be viable contributors to a mass withdrawal phenomenon. For example, a group qualifying domestic order, which typically arises in scenarios such as divorce settlements, is improbable to transpire in mass among a plan’s participants. Upon analysis of the potential causes presented in Table II.2.1, it becomes evident that rollovers and retirement age withdrawals are the most likely out-of-plan activities resulting in a mass lapse.

#### II.2.2.1.2 The 401(k) Inner Transfers and Investment Options

The Employee Retirement Income Security Act (ERISA) is a federal law that sets minimum standards for employee benefit plans, including retirement plans, to ensure proper management and funding. Under ERISA, retirement plan administrators must communicate certain information to participants about the plan’s investment options and fees. The tables in Appendix II.2.C present extracts of these communication requirements and guidelines.<sup>3</sup> Table II.2.15 in Appendix II.2.C presents the investment options available within a 401(k) plan, along with additional guidance on the investment-related information that should be disclosed. One notable aspect of Table II.2.15 is the limited and summarized nature of the information concerning investment choices.

It is important to highlight that, in the majority of 401(k) plans offering stable value funds, no other capital preservation funds compete with stable value.

<sup>3</sup>ERISA’s requirements form the lowest bar regarding the level of detail and quality of communication expected, and plan administrators may provide more detailed information to participants.



This means that participants seeking capital preservation typically only have the option of stable value funds within their plan. For the few plans that do feature competing capital preservation funds, an equity wash provision is implemented to deter participants from engaging in interest rate arbitrage. This provision requires participants to transfer monies out of a stable value fund to a non-capital preservation investment option for a period of at least 90 days before it could then be transferred to another capital preservation option. The rule aims to discourage participants from rate deficit withdrawals, which could negatively impact the stable value fund's performance. Intriguingly, these provisions form the basis of what [Xiong and Idzorek \(2012\)](#) refers to as the "illiquidity risk" of stable value funds, suggesting that participants face certain frictions when attempting to reallocate these funds.

### II.2.2.2 Literature Review and Hypothesis Development

In various studies, such as [Barsotti et al. \(2016\)](#), withdrawals from financial products are typically modeled using an interest rate differential between the product under scrutiny and competing interest rates. This concept is widely referred to as the rate deficit hypothesis. [Dar and Dodds \(1989\)](#) pioneered empirical research by investigating British household saving patterns via life insurance. They found that endowment policies responded to alternative investment return rates. [Kuo et al. \(2003\)](#) corroborated this view, identifying a dominant long-term relationship influenced by the rate deficit hypothesis in their study of ordinary life insurance policies from 1951 to 1998—a period marked by diverse interest rate fluctuations. Similar phenomena were observed by [Sierra Jimenez \(2012\)](#) in their study on equity fund flows, which showed a delayed response to changes in certain components of the interest rates. Likewise, [Kim \(2005\)](#) found evidence of policyholder surrender behaviors in the Korean market being responsive to the interest rates of alternative products. Other empirical papers verifying this hypothesis are [Outreville \(1990\)](#); [Tsai et al. \(2002\)](#); [Floryszczak et al. \(2016\)](#); [Barucci et al. \(2020\)](#); [Alfonsi et al. \(2019\)](#).

This observation suggests that, in the majority of cases, the option for immediate transfer between a stable value fund and a less risky fixed-income asset class is unavailable. This insight supports the idea of considering potential lags when analyzing the impact of the rate deficit hypothesis on cash flows. Therefore this serves as the foundation for Hypothesis 1.

Also, it is worth noting that, based on our read of the 401(k) ecosystem, the tables in Appendix II.2.C demonstrate that participants receive more information about the past performance of investment options and less about their future returns. The focus on past performance stems from its role as the sole comparable metric between investment options, such as in the case of equity

funds, where the concept of yield is not applicable. Consequently, if participants were to make allocations based on the rate deficit hypothesis, they might rely on past performance data rather than current and future yield information. This also suggests that participants should anticipate a delay in responding to rate deficits. This observation indicates that, when conducting a statistical study of participant cash flows and their relation to a rate deficit hypothesis, it is necessary to consider delayed indicators for the rate deficit, incorporating potential lags to account for this phenomenon. As such, we set forth to test the rate deficit hypothesis:

**Hypothesis 1.** *Due to equity wash restrictions across almost all plans and the lack of competing capital preservation investment options in many plans, participants do not respond to short-term and minor rate deficits. However, they could potentially react to a prolonged rate deficit, albeit with a delay.*

The flight-to-safety hypothesis is another well-established concept in financial literature. This hypothesis postulates that, during periods of market turbulence, investors are inclined to shift their investments from high-risk assets to lower-risk ones, such as government bonds or precious metals. As discussed in [Baur and Lucey \(2010\)](#); [Dorn and Huberman \(2005\)](#), this hypothesis is driven by the desire to preserve capital and avoid further losses. In the context of retirement planning, an empirical analysis by [Butrica and Smith \(2016\)](#) revealed that, during the 2009 global financial crisis, the likelihood of 401(k) participants investing in stocks fell from 63% in 2006, prior to the crisis, to 52%. This trend suggests a move away from riskier assets during periods of financial instability. Given that stable value is considered a safe investment choice, the “flight-to-safety” hypothesis seems applicable. Another paper studying this phenomenon during the COVID-19 pandemic is [Ji et al. \(2020\)](#).

Despite equity wash provisions deterring participants from moving to competing investment options, there are no barriers to rebalancing between stable values and riskier assets. As such, we set forth to test the flight-to-safety hypothesis:

**Hypothesis 2.** *The flight-to-safety phenomenon was observed within the stable value investment in a 401(k), specifically during the 2008 global financial crisis and the COVID-19 pandemic.*

The *moneyiness hypothesis* is another established concept in the literature, primarily used in asset-liability modeling to determine the optimal lapse strategy for rational investors. This hypothesis generally suggests that investors tend to react based on the “moneyiness” of their investment, typically interpreted as the option value of the guarantees compared to the premium paid. For instance, [Bacinello et al. \(2011\)](#) defined a stochastic optimization problem using a utility function to determine policyholders’ lapse behavior. Similarly, [Cheng](#)

et al. (2019) devised an optimization problem to capture the withdrawal behavior of a subset of their investors, termed “pros,” contingent on the moneyness of their payoff. Knoller et al. (2016), through their empirical analysis of variable annuities, noted that holders of larger policies are more sensitive to the moneyness of their embedded derivatives. In the context of stable value funds, the market-to-book value (MBV) ratio, also referred to as the asset–liability ratio, could be used as an indicator of the “moneyness” of the funds.

Based on our understanding of the 401(k) ecosystem, we have found that, while the minimum communication requirement is met, the MBV ratio is generally not disclosed to the participants, as evident from the tables in Appendix II.2.C. Consequently, it is plausible that the participants’ actions may not be influenced by the MBV ratio, given the lack of easily accessible information. We verify a relationship between the MBV and participants’ cash flows:

**Hypothesis 3.** *Historically, there exists no significant relationship between participants’ cash flow behavior and the MBVs.*

Previous research in the literature also emphasized the significance of employers in the allocation and withdrawal behavior of plan participants. For instance, Madrian and Shea (2001) analyzed individual 401(k) account data from June 1997 to June 1999 and concluded that the employer’s role is critical in investment allocation decisions. Similarly, Mitchell et al. (2006) analyzed historical data on individual 401(k) accounts from 2003 to 2004 and found that the plan sponsor influenced approximately 10% of activities.<sup>4</sup>

Eberhardt et al. (2021) further underscore the importance of the plan sponsor’s communications on participants’ behavior, indicating that various aspects of the ecosystem significantly influence participant actions. This point aligns with findings from Kalantonis et al. (2021), who examine the role of sentiment in corporate investment decisions. This idea could be extended to stable value sentiments and the potential influence of plan sponsor communications on them. Moreover, the importance of the plan sponsor role is echoed in studies like Tang et al. (2010) and particularly in Mitchell and Utkus (2022), where the examination of target-date funds as investment options offer further insights.

Upon examining Table II.2.1, we find that the most plausible causes for withdrawals, namely rollovers and retirement age withdrawals, are intricately linked to the plan sponsor’s employment ecosystem, such as employee count trends. This observation suggests that a detailed analysis of the plan sponsor’s employment dynamics could provide valuable insights. For example, a plan sponsor experiencing an increase in employment might be associated with a reduced risk of fund withdrawal compared to one experiencing a downturn in

---

<sup>4</sup>In this paper, we will use the terms plan sponsor, employer, and company interchangeably, even though they may sometimes refer to different legal entities.

workforce size. Based on this understanding, we verify the following hypothesis:

**Hypothesis 4.** *The plan sponsor's ecosystem impacts the participants' behavior. Big changes in the pattern of participants' cash flows coincide with a change in the plan sponsor's ecosystem.*

The literature presents a phenomenon referred to as the herd behavior and mass lapse hypothesis, which could be particularly relevant in the context of stable value investments. This hypothesis, as discussed by [Loisel and Milhaud \(2011\)](#), proposes that peer influences can lead to correlated withdrawal behavior among policyholders, resulting in mass lapses. Additionally, [Barsotti et al. \(2016\)](#) expands on this hypothesis, explaining how both self-excited endogenous and exogenous factors can trigger a contagion effect, leading to mass withdrawal behavior.

Further evidence of herd behavior is presented in studies such as [Chiang and Zheng \(2010\)](#), which observed and compared herd behavior cross-country for equity markets. Other similar phenomena are also discussed in [Shin \(2009\)](#), who studied bank runs for saving accounts, and [Hirshleifer and Hong Teoh \(2003\)](#), who provided more insight into the dynamics of information cascades.

In the specific context of 401(k) plans, herd behavior could be potentially amplified, given that all participants are colleagues or former colleagues. This setup can strengthen the information cascade, as explained by [Hirshleifer and Hong Teoh \(2003\)](#).

Other factors worth considering include the potential for significant fluctuations in mass lapse rates due to hardships or other factors causing individuals to move in and out of the 401(k) plan. These fluctuations could be influenced by elements such as reputational concerns about the investment manager, the option to switch to a self-directed brokerage, or mass withdrawals due to a network effect among employees of the same company. As such, we set forth the following hypothesis:

**Hypothesis 5.** *Herding behavior is plausible for stable value participants' cash flows.*

Lastly, factors such as the plan sponsor ecosystem and macroeconomic variables may lead to observable trends within cash flow data and induce serial correlation. Studies such as [Phillips Jr et al. \(1985\)](#) have observed these phenomena in behavioral lapse data. For instance, a multitude of empirical papers investigating lapse through a cointegration approach indirectly implies the existence of trends and autocorrelation within participant behavior [De Giovanni \(2010\)](#); [Kuo et al. \(2003\)](#); [Barucci et al. \(2020\)](#). This pattern emerges due to the significant time-dependent serial correlation observed in many economic

descriptive variables, which in turn indirectly influence the serial correlation in lapse rates. As such, we set forth the following hypothesis:

**Hypothesis 6.** *Participant cash flows demonstrate non-monotonic trends over varying durations.*

### II.2.3 Methodology, and Data Collection and Cleansing

In this study, our methodology incorporates several stages, including data collection, data cleansing, and statistical analysis.

**Data Collection:** We have collected anonymous and proprietary, aggregated data, which represents the monthly cash flows for all participants within each of 288 stable value funds, spanning from 1997 onwards. This dataset comprises 41,742 data points across USD 222 billion worth of book value, for which summary statistics and histograms can be found in Table II.2.13 of Appendix II.2.A. These represent the sum of monthly aggregated cash flows for all participants within the plan and do not contain any personal information. The schema of the data collected includes end of month date, aggregated book value in dollar amount, net participants' cash flows, crediting rate (the annualized rate of return of the fund as a percentage), and market value (the market value of the fund at the end of the month in dollar amount).

With respect to data availability, the dataset for this research is anonymous and proprietary, sourced from multiple large wrappers and pension funds. This dataset is not publicly accessible due to confidentiality. However, comparable data might be obtainable from other financial institutions. Despite our specific dataset not being available for replication, the methodologies and conclusions of this paper are applicable to similar datasets within the field.

**Data Cleaning and Preparation:** These data are inherently reliable as they are derived from actual transactions within stable value funds. However, to further enhance the credibility of our dataset, we cross-verified these figures with additional resources, including the record of communication between the investment managers and insurers, and conducted a thorough quality check to rectify any inconsistencies.

For example, the original data included activities that were company plan sponsor-initiated, such as plan disbursements, mergers, and spin-offs. These activities were carefully separated from participant activities to isolate participant cash flow data. This required careful review and adjustment of 213 data points. As a result, we have obtained an accurate record of participant-only cash flow time series. This step was critical for enhancing the accuracy and relevance of our data to our study.

**Statistical Analysis:** Our empirical analysis strategy consists of two steps. Initially, we analyze a subset of our data representing worst-case scenarios (plans with the highest withdrawal rates or plans with the highest change in the pattern of withdrawal) to understand the behavioral component behind the most significant withdrawals. We believe that studying these dynamics can provide valuable insights into the overall patterns of cash flow behavior. Subsequently, we extend our analysis to encompass the entire dataset, aiming to verify whether the identified patterns remain consistent across different scenarios.

In our methodology, we have taken multiple steps to ensure the robustness of our results and mitigate any potential biases. One such bias could involve the tendency to identify patterns where none exist, akin to perceiving trends in Brownian motion, a concept highlighted by [Mahdavi-Damghani \(2012\)](#). To address this, we implemented two strategies: firstly, we used non-parametric tests to examine whether our observations reflected true patterns rather than random fluctuations; secondly, we conducted in-depth analyses for specific plans that exhibited strong trends or changes in trends, seeking to understand whether the observed patterns were grounded in underlying factors and not merely statistical artifacts.

Linear regression models were our primary tool in investigating our hypotheses. However, we recognize that alternative methods such as quantile regression [Yang et al. \(2018\)](#) could also be helpful, especially when dealing with variables like MBVs and rate deficit, and their relationship with cash flows. But, upon qualitative examination of the data, we did not find compelling evidence of strong dependencies, even non-linear ones, that would necessitate such an approach. For instance, in the case of low MBVs and their relationship with cash flows, we did not discern a clear correlation.

Note that more sophisticated models, such as quantile regression or cointegration models, are not presented in this paper. Given the qualitative evidence (or lack thereof) and the large list of hypotheses we are testing, we opted not to use these tools in this paper in the interest of conciseness. In conclusion, the methodology employed in this paper—while not exhaustive—provides a sound basis for the analysis. We did not add sophistication to our models when there was no qualitative or data observation justifying the need for it. Future work could certainly explore more sophisticated models to build upon our findings, especially as more data becomes available post the interest rate rise of 2022.



## II.2.4 Observations

With the objective of comprehending the key drivers behind participants' behavior in investment plans, we performed empirical analysis on the hypotheses developed in Section II.2.2. The below notes summarize our findings.

### II.2.4.1 Trends

In our preliminary exploratory empirical data analysis, and drawing from our practical experience with cash flows, we identify non-monotonic trends within the historical data. Some examples of these non-monotonic trends are shown in Appendix II.2.B. To validate these trends, we first examine the independence of the monthly cash flow data since establishing non-independence is a necessary condition for the presence of such trends.

Therefore, we aim to assess time dependency in participant cash flows of stable value funds for the plan sponsors using statistical tests, specifically the Ljung–Box test and the Durbin–Watson test. The dataset consists of participant cash flows for various plan sponsors, and the tests are applied to each sponsor's data to test for serial correlation. For the Ljung–Box test a lag of 1 was chosen to capture potential monthly patterns and a lag of 12 was chosen to represent a full calendar year.

The results of the tests indicate that serial correlations are consistently present across some plan sponsors. Specifically, the Ljung–Box test with a lag of 12 found that 133 out of 318 plan sponsors (42%) have a  $p$ -value less than 0.05, indicating the rejection of the null hypothesis and suggesting significant autocorrelation in those plan sponsor cash flows. The Durbin–Watson test found that 184 out of 318 plan sponsors (58%) have a  $p$ -value less than 0.05, indicating the rejection of the null hypothesis and suggesting the presence of significant autocorrelation or serial correlation in those plan sponsor cash flows. The Ljung–Box test with a lag of 1 found that 156 out of 318 plan sponsors (49%) have a  $p$ -value less than 0.05, indicating the rejection of the null hypothesis and suggesting a significant serial correlation in those plan sponsor cash flows. The remaining plan sponsors do not show evidence of significant serial correlation. Table II.2.2 shows the results of the Ljung–Box and Durbin–Watson tests for participant cash flows in stable value funds.

TABLE II.2.2: Results of the Ljung–Box and Durbin–Watson tests for participant cash flows in stable value funds.

Test	Number of Plans Not Rejected	Number of Plans Rejected
Durbin–Watson	134	184
Ljung–Box (Lag 12)	185	133
Ljung–Box (Lag 1)	162	156

Table II.2.2 suggests that time dependency in participant cash flows may be a common phenomenon among many stable value funds. To better understand this time dependency, we implement a test for non-monotonic trends, as our observation hints at the existence of such a type of trend for many of the funds. For this test, we use only funds with more than five years of historical data. We then deploy a bootstrap version of the WAVK test, as described in Lyubchich et al. (2013) and Wang et al. (2008). The test results are summarized in Table II.2.3, with the null hypothesis being that there is no trend among the data. Out of 280 plans, 126 plans (45%) exhibit a  $p$ -value of less than 0.05, rejecting the null hypothesis, providing evidence of non-monotonic trends. The remaining 154 plans (55%) have higher  $p$ -values than 0.05 and the test concludes that they do not show evidence of non-monotonic trends.

TABLE II.2.3: Results of the WAVK test for non-monotonic trends.

Test	Number of Plans Not Rejected	Number of Plans Rejected
WAVK	154	126

The results suggest that a significant proportion of the plans exhibit non-monotonic trends. We thus deploy the changepoint algorithm explained in Chen and Gupta (2011) to infer these trends from the monthly cash flows. This algorithm is based on hypothesis testing and the bisection method to capture multiple changepoints. The hypothesis tests the statistics of the variance difference between the sub-sectioned time series and the entire series. Interested readers may refer to Hawkins (1977) for additional details. We then manually verify each of these trends to ensure that the algorithm does not over-fit and that there is, at least visually, an observable trend present. After applying the changepoint algorithm, we infer 565 trends for all plans, with their basic statistics described in Table II.2.4. Trend behavior is observed in almost all of the plans we analyze, and we present some of these plans in Appendix II.2.B. The figures in the Appendix clearly show that the discussed trend is dominant across these plans.

TABLE II.2.4: Statistics for participant trends.

	Minimum	Maximum	Average	Standard Deviation	Skewness	Kurtosis
Trends' size *	-0.32	0.52	-0.0058	0.0251	1.17	13.2
Trends' duration	6 months	24.5 years	7.5 years	5.3 years	0.53	-0.39

\* The measures for the trend sizes are annualized percentages.

Figures II.2.1 and II.2.2 show the relationship and empirical copula of the duration of the trend against the trends' sizes. A key takeaway from these two Figures II.2.1 and II.2.2 is the hump-shaped relationship between regime trends and their duration. In other words, the empirical data demonstrate that



very large trends of inflows or outflows do not persist for extended periods, while mid-size flow trends can persist for longer periods, with some extending for more than 25 years.

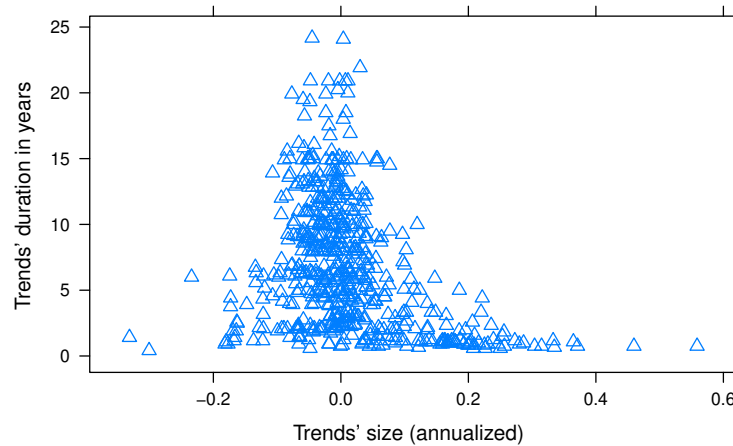


FIGURE II.2.1: Empirical data for trends and their respective duration.<sup>5</sup>

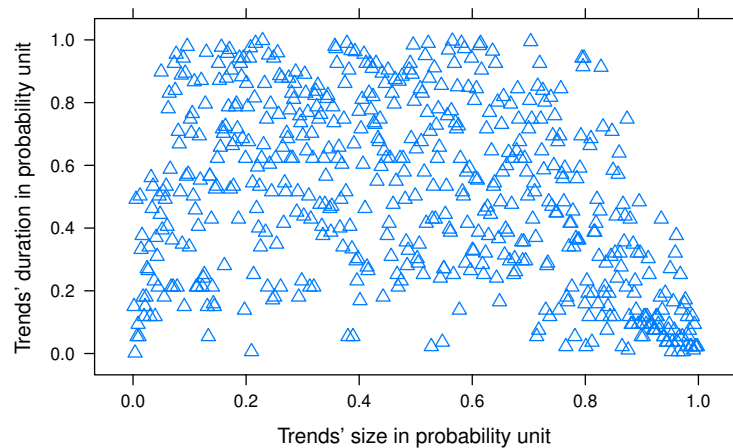


FIGURE II.2.2: Empirical copulas for trends and their respective duration.<sup>6</sup>

We then perform an analysis of the homogeneity of trend data amongst plans. The homogeneity of trend data is a critical factor in making any generic statement about the statistics of trends as a whole. To ensure that the trends are homogeneous among different plans, we analyze the dataset by dividing it into arbitrary subgroups and subjecting them to statistical tests. Our goal is to determine whether the trends were heterogeneous among plans or within groups of plans. To test for homogeneity of variance, we use Levene's test, Bartlett's test, and the *F*-test of equality of variance between two normally

distributed sets.<sup>7</sup> To avoid bias in subgroup selection, we performed the tests on multiple arbitrary groups. Table II.2.5 shows the recorded lowest  $p$ -value we observe in terms of acceptance of the null hypothesis. The test statistics had reasonably high  $p$ -values and, as a result, we cannot reject the hypothesis of homogeneity of trends among plans.

TABLE II.2.5: Test of homogeneity for participants' cash flow trends.

Test	Test Stat.	$p$ -Value
$F$ -test	1.3243	0.06
Bartlett's test	3.824	0.06
Levene test	1.7823	0.1825
Chi-square test	3.7830	0.2394

We also seek to understand the relationship between trends before and after a changepoint. To explore this, we generated empirical copulas for trend sizes before and after the changepoint, as depicted in Figure II.2.3. It is important to note that, due to the changepoint algorithm employed, no data points are present around the diagonal in the figure. The statistics are summarized in Table II.2.6. Our analysis led us to conclude no strong relationship exists between trends before and after a changepoint.

TABLE II.2.6: Statistics for correlation of trends before and after a changepoint.

	Spearman Corr	Kendall Tau
Current trend size and next trend size	-0.083	-0.029
Current duration and next trend size	0.048	0.034

Table II.2.6 and Figure II.2.3 show no strong correlation statistics for trends before and after a changepoint. We attribute this lack of relationship to the following economic intuition: the trends are indicative of both idiosyncratic factors affecting the plan sponsor and systemic marketplace conditions, and any changes are often structural and unpredictable. These changes depend on factors such as the financial health of the plan sponsor and management decisions affecting employment, which go beyond quantifiable economic variables; predicting the direction of a trend change caused by an unquantifiable and unpredictable corporate disruption or material change poses significant challenges. This may explain why we do not observe a strong correlation between the size and duration of trends before and after a changepoint. The next section introduces some of the corporate and economic factors contributing to changes in the trends.

<sup>7</sup>However, we should note that  $F$ -test statistics assume a normal distribution for both sets, which is not valid for cash flow trends due to their higher tail kurtosis compared to a normal distribution (as indicated in Table II.2.4).

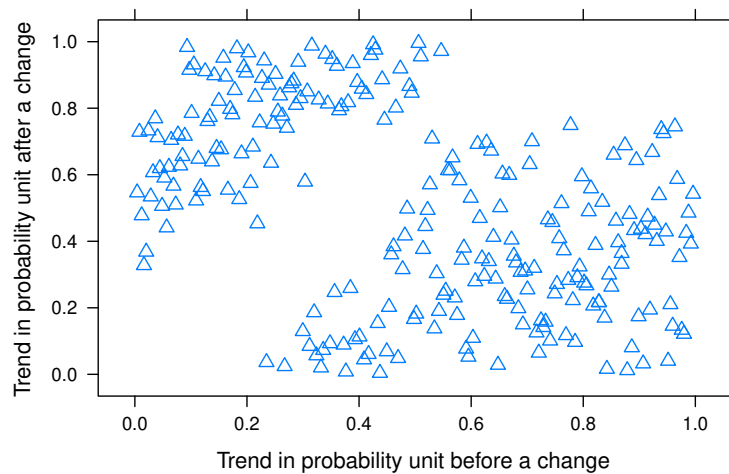


FIGURE II.2.3: Empirical copulas for trends before and after a changepoint.<sup>8</sup>

### II.2.4.2 Plan Sponsor’s Ecosystem

Here, we evaluate the relevance of the plan sponsor’s ecosystem hypothesis by focusing on instances of significant changes in trends. Our goal is to determine whether the plan sponsor plays a role in these changes.

To test the hypothesis, we analyze the historical communication for all data points located in the bottom right and top left corners of Figure II.2.3 at the 0.1 centile level (i.e., the two squares from 0.1 to 0.9 centiles and 0.9 to 0.1 centiles). These 30 data points represent the largest change in trends. Our objective is to ascertain whether these changes coincide with a change in the plan sponsors’ ecosystem.

Upon reviewing the corporate communications at the time of the trend change, we find that these changes are associated with macroeconomic, systematic, or corporate-related events. We categorize these events into six common groups as shown in Table II.2.7. Out of these groups, four are related to the plan sponsor ecosystem: “Bankruptcy” (3 out of 30 plans), “Introduction of new investment options” (2 plans), “Post spinoff/merger participant transfer” (6 plans), and “Strong employment growth or reduction” (7 plans). It is worth noting that among these seven plans with strong employment growth or reduction, all of the plans experiencing strong employment growth exhibit a positive trend, while all of the plans with a reduction in employment size demonstrate negative trends. In total, 18 out of the 30 plans with the largest change in trends experience a change in the plan sponsor’s ecosystem. This evidence is sufficient to reject the null hypothesis.

However, we acknowledge that a more robust test would involve comparing the frequency of plan sponsor-related events during the period of the trend

change with the frequency of such events for plans/periods without changes in trends. A more persuasive argument would be to verify whether plan sponsor-related events occur less frequently for the other 297-minus-30 plans with weaker changes in trend or during periods without trend changes, compared to the top 30 plans experiencing the largest trend changes. Nonetheless, given the time-consuming nature of examining corporate communications, we are unable to conduct this stronger test.

TABLE II.2.7: Plans that experienced the largest change in trend with explanations found in corporate communications.

Event Occurring during the Period of a Drastic Change in Trend	Number of Plans	Trend Sign
Bankruptcy	3	Negative
Employment growth or reduction	7	Respectively + and –
Flight to safety during crisis	11	Positive
Introduction of new investment options with being the default options	2	Negative
Post spinoff/merger participant voluntary transfer to/from new investment scheme	6	Spinoffs: +, Mergers + or –
Reputational issues leading mass withdrawal	1	Negative

Figure II.2.4 presents a case study of a plan that experienced outflows from their stable value option when the plan sponsor introduced a target date fund to the investment options offered in their 401(k) plan. Our review of the plan sponsor’s communications with participants revealed that they actively promoted the new target date fund as a default option for those who had not chosen a specific investment option. This promotion likely contributed to the increased cash flows into the target date fund.

In summary, our empirical study demonstrates the impact of the employer’s ecosystem on participant cash flows. Factors such as available investment options and changes to these can result in varying behavioral trends. These observations, and our case study, align with the findings of Mitchell and Utkus (2022), who also noted the influence of alternative options, such as target date funds, on 401(k) cash flows. Other studies on the impact of the plan sponsor ecosystem and default options, like Tang et al. (2010), further corroborate our findings.

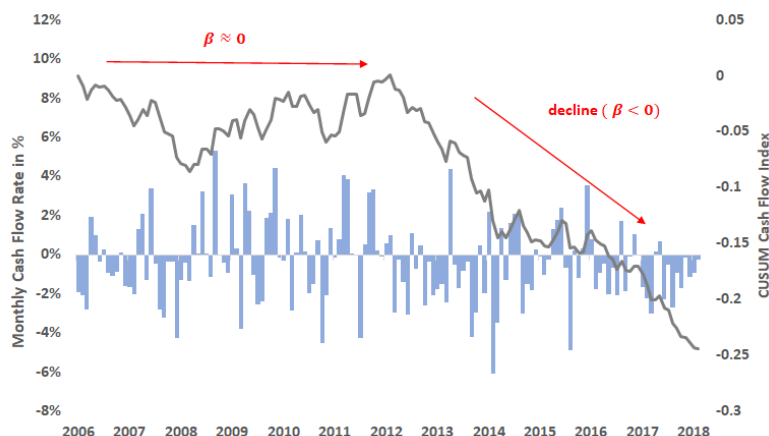


FIGURE II.2.4: Change in trends in withdrawal from a stable value fund due to the introduction of target-date funds within the plan. <sup>9</sup>

### II.2.4.3 Rate Deficit Arbitrage

We define the rate deficit as the spread between crediting rates and UST rates (i.e., rate deficit), which serves as an indicator to assess the competitiveness of stable value returns compared to competing funds, such as money market and short-term bond funds. We will use this indicator to test the hypothesis that participants transfer balances from stable value funds to competing funds with higher returns.

To verify this hypothesis, we regress the monthly cash flows for all plans against their respective rate deficit. For this analysis, we use the 1-year UST rate from [Federal Reserve Bank of St. Louis \(2020\)](#). Table II.2.8 summarizes the linear regression statistics for rate deficits and stable value cash flows. The table also assesses the regression results for a 1-year lag. As discussed earlier in Section II.2.2, because the communication of the plan sponsor is based on past performances, there are reasons to consider participants reacting to the rate deficit with a delay. The table shows that the coefficients of determination are low, suggesting that the regressions do not demonstrate a statistically significant linear relationship between the rate differential and stable value cash flows. Figure II.2.5 compares the monthly cash flow data with the 1-year rate deficit, with the range rate deficit observed in the historical data ranging between 1.04% (stable value fund having lower return) and -5% (stable value fund having higher returns).

In conclusion, our analysis did not reveal a strong linear relationship between participant cash flows and the rate deficit, even when a one-year lag was applied. However, it is essential to note that, during the historical period covered by our cash flow data (1997 to 2021), there were no prolonged periods of substantial rate deficits, with the maximum observed rate deficit being only 1.04%. As a result, it can be inferred that participants did not exhibit significant

TABLE II.2.8: Results of cash flows against 1-year UST rate deficit, with a one-year lag.

Lag	Correlation	R <sup>2</sup>	Coefficient of Regression	p-Value	Number of Data Points
0y	1.8%	0.03%	2.9%	0.04%	36,259
1y	2.7%	0.06%	3.5%	0.06%	36,259

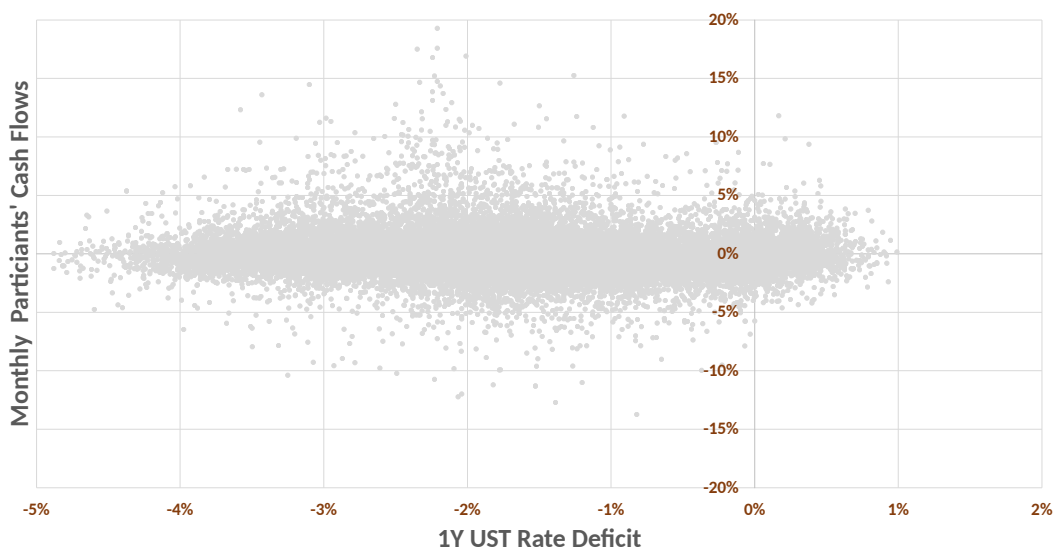


FIGURE II.2.5: Historical data of the relationship between cash flow rate as a function of the crediting rate deficit to the 1-year UST rates.<sup>10</sup>

sensitivity to the rate deficit of stable value funds for deficits below 1%.

However, our analysis does not preclude the possibility of a dynamic relationship emerging with larger rate deficits than 1%. Therefore, our findings do not contradict those of [Kuo et al. \(2003\)](#), nor do they refute the rate deficit hypothesis as posited by [Sierra Jimenez \(2012\)](#). It is possible that participant responses might be lagged, although this phenomenon has not been observed within our data period spanning from 1997 to 2021.

The post-2022 period, marked by a shift into a new interest rate regime, provides an intriguing context for further exploration of this hypothesis. Notably, traditional linear regression models may fall short of accurately capturing the dynamics of this altered financial environment. Therefore, it might be advantageous to deploy quantile analysis and regression methods, as suggested by [Chen et al. \(2022\)](#); [Yang et al. \(2018\)](#), or a generalized dynamic factor model similar to [Yang \(2022\)](#), on more recent data from this period.

### II.2.4.4 Herd Behavior

We define a herding event as a situation in which a substantial number of investors rapidly withdraw funds from a specific investment vehicle within a short period. We consider strong withdrawal rates, defined as any annualized rate above 50% over a 6-month period, as a necessary condition to identify such events. This threshold is based on the concept of herding, which involves the potential for mass withdrawals to rapidly deplete a fund if not addressed. The bottom left of Figure II.2.2 illustrates the location of this threshold in our analysis. By focusing on this specific definition of strong withdrawal rates, we can accurately identify herding events and assess their potential impact on the investment vehicle.

During our analysis of historical data at the plan sponsor level, we discovered one instance where panic-driven behavior in stable value plans led to a significant withdrawal of over 8% of the fund in a single month due to a reputational issue (see Figure II.2.6).

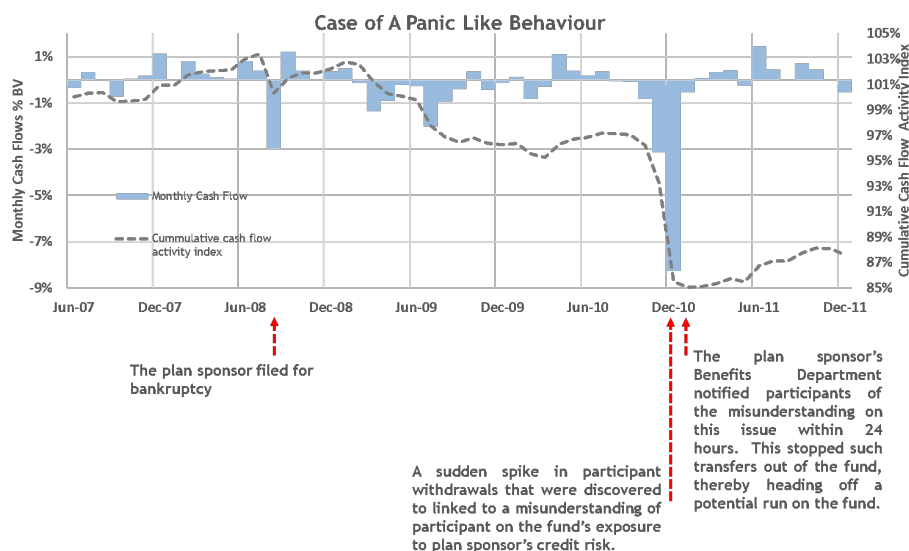


FIGURE II.2.6: Case of panic-like withdrawal behavior by plan participants.<sup>11</sup>

The plan sponsor successfully mitigated the issue by effectively communicating with plan participants within 24 hours. The communication reassured participants, addressed their concerns, and ultimately prevented further panic-driven withdrawals. It is worth noting that this behavior raised concerns about the potential amplification of such patterns if not mitigated. Therefore, we considered the possibility of herding behavior, similar to a bank run, leading to the fund's full or partial depletion.

This case study highlights the importance of efficient plan sponsor communication in mitigating the risks associated with herding behavior, reinforcing the



findings of [Eberhardt et al. \(2021\)](#). The significance of the network effect among participants and its potential to trigger herding behavior, as highlighted by [Loisel and Milhaud \(2011\)](#), is also underscored in this case. Moreover, this network effect could potentially be intensified among 401(k) plan participants, as they are all current or former employees of the same company—a scenario akin to bank runs as examined by [Vo and Le \(2023\)](#).

#### II.2.4.5 Flight-to-Safety Behavior

In this subsection, we investigate the flight-to-safety hypothesis, which postulates that, during periods of crisis, investors tend to shift towards more secure investment alternatives to safeguard their assets. Our analysis concentrates on two major crises: the 2008 financial crisis and the COVID-19 pandemic.

We define the financial crisis period as occurring between September 2007 and December 2009, and the COVID-19 pandemic as lasting from January 2020 to June 2021. These definitions are based on our judgment of the market consensus, as opposed to utilizing statistical techniques and financial market indicators, as demonstrated in [El-Shagi et al. \(2013\)](#). We begin our analysis by partitioning the data into three distinct subgroups: the global financial crisis, the COVID-19 pandemic, and other non-crisis periods. We then classify each trend data within these subgroups. A trend data point is regarded as part of a global financial crisis subgroup if its period (start of the trend and ending of the trend) overlaps with our definition of the global financial crisis; the same rule is applied to the COVID-19 pandemic subgroup. However, if a changepoint occurred for a plan during the period of the global financial crisis, then, between the trend data point before or after the changepoint, only the point that had the shortest duration will be part of the global financial crisis subgroup; the same rule is applied to the COVID-19 pandemic subgroup.

Table [II.2.9](#) outlines the data for each subgroup, illustrating the number of trend data points in each category. Some data points may pertain to multiple subgroups, given that the duration of certain trends may coincide with more than one subgroup period. The table enumerates the aggregate number of trend data points for the global financial crisis, the COVID-19 pandemic, and other periods, as well as the number of data points shared between the various subgroups.

Following the data categorization, we aim to test the hypothesis that the distribution of trend data during the global financial crisis and the COVID-19 pandemic is higher than in other normal periods. To do so, we administer a test of stochastic dominance of order 1 among the three subgroups. In this subsection, we present the results of the stochastic dominance tests performed on cash flow samples from three different periods: non-crisis periods, the



TABLE II.2.9: Subgroups of segregated data.

Subgroup	Number of (Trend) Data Points
Group 1: global financial crisis	158
Group 2: COVID-19 pandemic	207
Group 3: non-crisis periods	489
Common in Group 1 and 3	0
Common in Group 2 and 3	178
Common in Group 1, 2, and 3	0

COVID-19 pandemic, and the global financial crisis (GFC). Specifically, we examined cash flow samples from these three distinct periods using the one-sided Kolmogorov–Smirnov (K-S) test, the Mann–Whitney U test, and the KSPA test [Hassani and Silva \(2015\)](#). While the KSPA test, which is performed on the absolute value of the cash flow trends, does not directly validate the flight-to-safety hypothesis, it can provide valuable insight into whether the size of the trends, regardless of their direction, differs during crisis and non-crisis periods. The results of these tests are presented in [Table II.2.10](#).

TABLE II.2.10: Stochastic dominance test results.

Test	Comparison	$H_0$	Test Statistic	$p$ -Value	Decision
K-S (Greater)	Non-crisis vs. COVID-19	$F_x \geq F_y$	$D^+ = 0.138$	$5.79 \times 10^{-3}$	Reject
K-S (Greater)	Non-crisis vs. GFC	$F_x \geq F_z$	$D^+ = 0.229$	$1.11 \times 10^{-5}$	Reject
K-S (Greater)	COVID-19 vs. GFC	$F_y \geq F_z$	$D^+ = 0.109$	$9.83 \times 10^{-1}$	Not Reject
Mann–Whitney U	Non-crisis vs. COVID-19	$P(x > y) = 0.5$	$W = 42905$	$2.34 \times 10^{-3}$	Reject
Mann–Whitney U	Non-crisis vs. GFC	$P(x > z) = 0.5$	$W = 29249$	$7.44 \times 10^{-6}$	Reject
Mann–Whitney U	COVID-19 vs. GFC	$P(y > z) = 0.5$	$W = 14916$	$1.46 \times 10^{-1}$	Not Reject
KSPA	Non-crisis vs. COVID-19	$F_{ x } \geq F_{ y }$	$D^+ = 0.00409$	$9.96 \times 10^{-1}$	Not Reject
KSPA	Non-crisis vs. GFC	$F_{ x } \geq F_{ z }$	$D^+ = 0.00971$	$9.83 \times 10^{-1}$	Not Reject
KSPA	COVID-19 vs. GFC	$F_{ y } \geq F_{ z }$	$D^+ = 0.10414$	$4.31 \times 10^{-2}$	Reject

Based on a significance level of 0.05, the null hypothesis ( $H_0$ ) is either accepted or rejected. The results of the one-sided Kolmogorov–Smirnov test indicate that the cash flow distributions during non-crisis periods significantly dominate those during the COVID-19 pandemic and the global financial crisis periods. However, there is no significant difference between the cash flow distributions during the COVID-19 pandemic and the global financial crisis.

The Mann–Whitney U test results reveal a significant difference between the cash flow distributions during non-crisis periods and those during the COVID-19 pandemic and the global financial crisis periods. However, no significant difference is found between the cash flow distributions during the COVID-19 pandemic and the global financial crisis.

The KSPA test indicates that the magnitude of cash flow trends during non-crisis periods does not significantly deviate from those during the COVID-19 pandemic or the global financial crisis. It is important to clarify, however, that

this test does not directly refute or confirm the flight-to-safety hypothesis. This is because the KSPA test is conducted on the absolute values of the cash flow trends, without considering their directional signs.

We have also plotted the empirical cumulative distribution function (ECDF) to visually verify the stochastic dominance among the three samples. The plot is shown in Figure II.2.7.

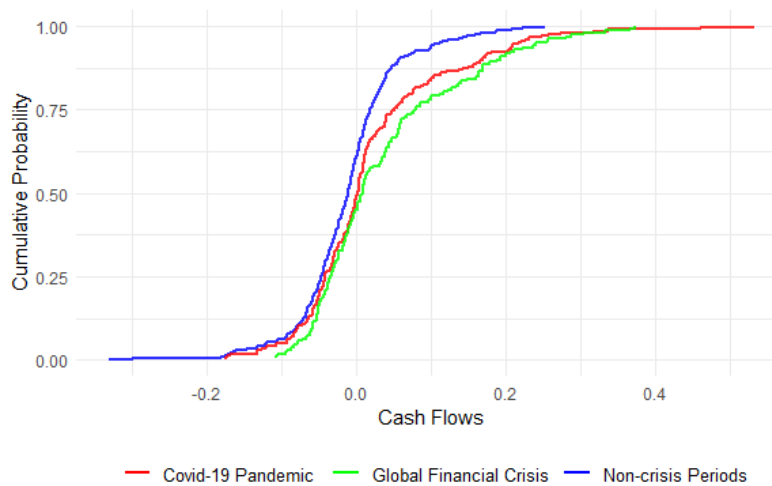


FIGURE II.2.7: ECDF of cash flows during non-crisis, COVID-19 pandemic, and global financial crisis periods.<sup>12</sup>

The ECDF plot in Figure II.2.7 provides a visual representation of the cash flow distributions during the non-crisis, COVID-19 pandemic, and global financial crisis periods. The plot confirms the results of the stochastic dominance tests. The cash flow distribution during non-crisis periods (blue line) dominates those during the COVID-19 pandemic (red line) and the global financial crisis (green line), as the blue line is consistently above the red and green lines. The cash flow distributions during the COVID-19 pandemic and the global financial crisis do not show a clear pattern of dominance, which is consistent with the test results.

In essence, our findings suggest that cash flow distributions during non-crisis periods exhibit stochastic dominance over those during crisis periods like the COVID-19 pandemic or the global financial crisis. This lack of significant difference between cash flow distributions during the COVID-19 pandemic and the global financial crisis lends credibility to the “flight-to-safety” hypothesis during these times of uncertainty. Our analysis provides another instance of flight-to-safety behavior besides traditional safe havens like gold and other precious metals, as discussed in [Baur and Lucey \(2010\)](#). Furthermore, our findings align with the research on the flight-to-safety effect during the COVID-19 pandemic as investigated by [Ji et al. \(2020\)](#).

Lastly, to ascertain the robustness of our findings, we conducted sensitivity analyses. The outcomes reveal that minor modifications to the definitions of the crisis periods do not significantly impact the results, further validating our analysis.

### II.2.4.6 Moneyness Hypothesis

As discussed in Section II.2.2, many works of literature on asset–liability management relate lapsation to the moneyness of the financial product. In this paper, we utilize the market-to-book value (MBV) ratio, which also serves as an asset–liability ratio, as an indicator of the moneyness of a stable value. A lower ratio implies that the assets are more “in the money” in comparison to the liabilities.

To test this, we perform regression testing on the monthly cash flow data. We find no significant relationship between cash flows and MBV ratios, as demonstrated by the low  $R^2$  values in Table II.2.11.

TABLE II.2.11: Results of market-to-book value ratio regression against fund-level cashflows.

Correlation	Coefficient of Determination	$p$ -Value	Number of Observations
$-2.03 \times 10^{-2}$	$4.12 \times 10^{-4}$	$1.46 \times 10^{-4}$	35,049

We also observe a stronger relationship between cash flows and MBVs during the global financial crisis, as shown in Table II.2.12. However, the  $R^2$  values remained insignificant, suggesting that, even for these periods, the relationship was not substantial. Note that the relationship between aggregated cash flows and MBV ratios was more pronounced during the global financial crisis due to the “flight-to-safety” effect of stable value investments. Therefore, this relationship is spurious, with the common cause being the “flight-to-safety” effect.

TABLE II.2.12: MBV ratios regressed against cash flows during the global financial crisis and the COVID-19 pandemic.

Period	Correlation	Coefficient of Determination	$p$ -Value	Number of Observations
Global Financial Crisis	$-1.78 \times 10^{-1}$	$3.18 \times 10^{-2}$	0.00	4893
COVID-19 Pandemic	$-3.40 \times 10^{-2}$	$1.2 \times 10^{-3}$	$2.97 \times 10^{-2}$	4108

To further support this lack of statistical significance, we study the funds with extremely low MBV ratios, which are potential outliers. We find no statistically significant relationship between MBVs and cash flows in these cases. Figure II.2.8 displays the MBV ratios for 18 plans with extremely low MBV ratios for

at least six months (the “Extreme MBV Plans”) and compares them with their quarterly cumulative participant cash flows.

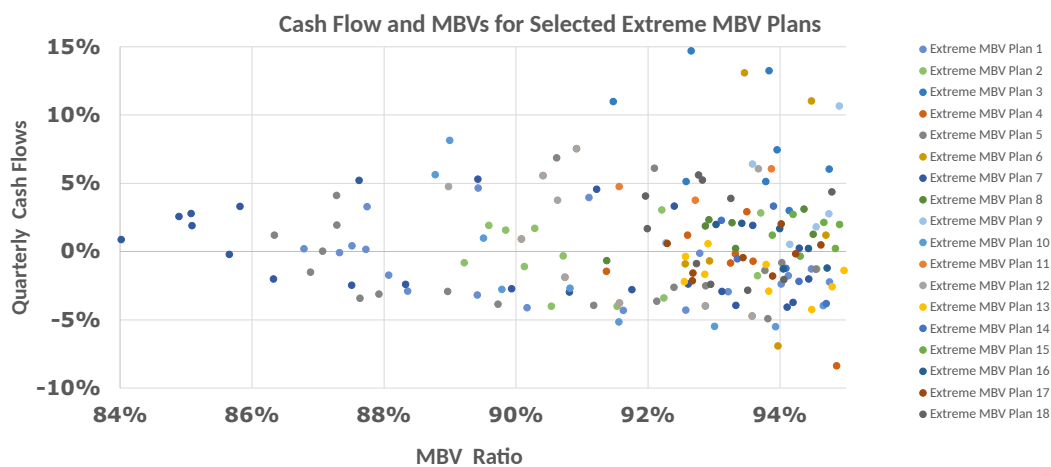


FIGURE II.2.8: Quarterly cash flows for the lowest MBV ratio observed.<sup>13</sup>

In conclusion, we did not find a strong relationship between participant cash flows and market-to-book values. While the  $p$ -values and coefficients of regression were minimal, they were slightly more pronounced during the global financial crisis period. However, we attribute this to the flight-to-safety hypothesis rather than participants being directly sensitive to the market-to-book values.

Our analysis does not align with the findings of Knoller et al. (2016), which discusses the propensity of large policyholders to react to embedded options in variable annuities. We lack individual policyholder data to verify this. However, given that the product we are considering is an insurance product, we tend to believe that the “moneyness” hypothesis may not be as applicable in this context.

Given that the level of market-to-book values (MBVs) observed in our study are all above 85%, a potential alternative approach could have been to employ quantile regression, as described by Yang (2022), or an autoregressive copula model such as Yang and Hamori (2021). However, based on our observations from the graphical analysis above, we argue that even using quantile or non-linear regression methods would not have resulted in significant outcomes in our case. In addition, one unexplored area of improvement in our analysis is to complement with conditional correlation similar to what was carried out in Yang et al. (2018); such an approach could have been used in analyzing the relationship between the MBV and cash flows during different periods like COVID-19 and non-crisis.

## II.2.5 Discussion and Conclusions

Having conducted a study on participant cash flows in stable value funds using aggregated fund-level data that represents 80% of the individual stable value plans in the market, we found that non-monotonic trends are the dominant factors in stable value cash flow data. We then investigated several hypotheses from the literature that could explain these cash flow trends, including the flight-to-safety effect, rate deficit, herd behavior, mass lapse, and moneyiness hypotheses.

On the flight-to-safety effect, our findings indicated that, during crises, participants tend to transfer funds to stable value options as they perceive them as relatively safer.

For the rate deficit hypothesis, we found no evidence of rate arbitrage behaviors in our historical dataset. The absence of direct competitive alternatives within numerous 401(k) plans, coupled with the presence of the equity wash rule, acts as a deterrent for participants to arbitrage their portfolio based on rate deficits. Additionally, plan sponsors typically emphasize past performances in their communication to participants, resulting in delays in responding to rate deficits between investment options, thereby diminishing the significance of this hypothesis. It is also worth noting that we observed limited rate deficit instances from 2000 to 2021 due to stable interest rates. However, our analysis does not provide a conclusion on the impact of high inflationary rates on stable value cash flows, nor the impact of the U.S. interest rates spike post-2022.

Regarding herd behavior and mass lapse, our data shows one case of mass lapse due to a reputational issue. Based on our understanding of the stable value ecosystem, we believe that negative news or rumors about a particular stable value fund or its underlying investments could lead to a loss of confidence among investors, prompting them to withdraw their investments en masse. Therefore, the risk of a reputational mass lapse is plausible but with a low probability of occurrence.

Using the market-to-book value ratio (asset-liability ratio) as an indicator of the moneyiness of the stable value products, we verified the moneyiness hypothesis. We observe no significant relationship between the market-to-book values and participants' monthly cash flows. We justify the result obtained by the hypothesis that the moneyiness hypothesis is plausible when active trading, such as trading American stock options, is involved. However, the protection offered by stable value funds is an insurance structure rather than a trading one, limiting the relevance of this hypothesis. It is worth noting that a low market-to-book value could potentially be perceived as a weakness and hypothetically lead to reputational issues, increasing the likelihood of herd behavior (even though the

case example in Section II.2.4.4 had a market-to-book value close to 100% and the reputational issue was unrelated to its market-to-book).

We conclude that factors such as plan sponsor communication and management decisions, financial health, industry sector, employment policies, growth or layoffs, plan demographics, and default options can impact participant cash flow trends. Therefore, it is crucial to consider these factors when assessing the risk of mass lapses. These factors can influence medium-to-long-term cash flow trends. For instance, our case study in Section II.2.4 reveals participants transferring a portion of their stable value funds to invest in target-date funds when the plan sponsor integrates this option as a default. This finding is consistent with Mitchell and Utkus (2022), who observed a significant proportion of participants transferring funds into target-date funds within the first year of their adoption in 401(k) plans.

Based on our analysis and the conclusions drawn, we suggest that any lapse model built for projecting adverse withdrawal scenarios should consider at least the following risk factors:

- The trend in cash flows is related to the nature of the plan sponsors' ecosystem, which indirectly influences participants' behavior.
- A herd behavior component, where the plausibility of this behavior could potentially be influenced by reputational damage<sup>14</sup>.
- The cash flow risk-mitigating effect of flight-to-safety behavior during a crisis.

Therefore, in terms of future work, an interesting extension of our study could be to integrate our findings within an asset-liability management (ALM) model. This could be particularly useful for evaluating the guarantee risk associated with insurance products. Such a framework could leverage our empirical findings to provide more nuanced risk assessments and strategic insights for both plan sponsors and insurers.

While the statistical evidence did not support a rate deficit effect, there was no historical data available to examine extremely large rate deficits for a long period of time. Given this, a lapse model should also likely consider a rate deficit risk factor that is triggered by deficits significantly higher than 1% and sustained for a period longer than one year.

Also, with inflation on the rise since 2022, it could be insightful to apply a quantile analysis and regression approach to our analysis. This could help us better understand the sensitivity of participant cash flows to rates under the rate deficit hypothesis, following methodologies laid out in studies like

---

<sup>14</sup>A low market-to-book value could potentially increase the chances of a reputational issue.

Yang et al. (2018) and Chen et al. (2022). However, as of the time of this paper, proprietary data for this period remains unavailable and unprocessed. Given the potential for a time lag in participant responses to rate deficits, this future research opportunity might need more time to come to fruition.

The “flight-to-safety” phenomenon identified in our analysis is a novel insight within the context of stable value funds. A deeper examination of this behavior could offer valuable insights, contributing to our comprehension of participant responses in volatile markets. Specifically, the use of transfer data might shed more light on how participants react under varying market conditions, especially given the direct relevance of the “flight-to-safety” effect to internal transfers within a 401(k) plan between riskier and less risky assets. Regrettably, in this current study, we had access only to net cash flow data (the sum of internal and external transfers within a 401(k) plan), precluding a full examination of transfer data that might more clearly illuminate the “flight-to-safety” effect. Future research that incorporates such data could provide a more nuanced understanding of these patterns.

## II.2.A Basic Data Statistics

The summary statistics of the participant cash flow data are shown in Table II.2.14. A histogram of the data for the underlying monthly cash flows is shown in Figure II.2.9.

TABLE II.2.13: Summary of cash flow data.

Historical Period	Book Value Balances (USD)	Number of Plans	Number of Data Points
Jan 17–Dec 21	132 billion	172	27,421
Jan 14–Dec 21	110 billion	137	24,416
Jan 08–Dec 21	78 billion	38	15,710
Nov 97–Dec 21	222 billion	297	41,742

TABLE II.2.14: Statistics of participant-related monthly cash flows.

Minimum	Maximum	Average	Standard Deviation	Skewness	Kurtosis
-14.6%	19.9%	-0.06%	1.55%	1.02	11.51

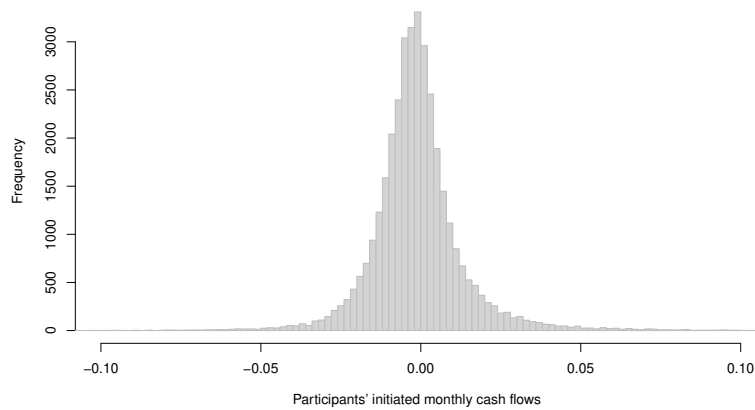
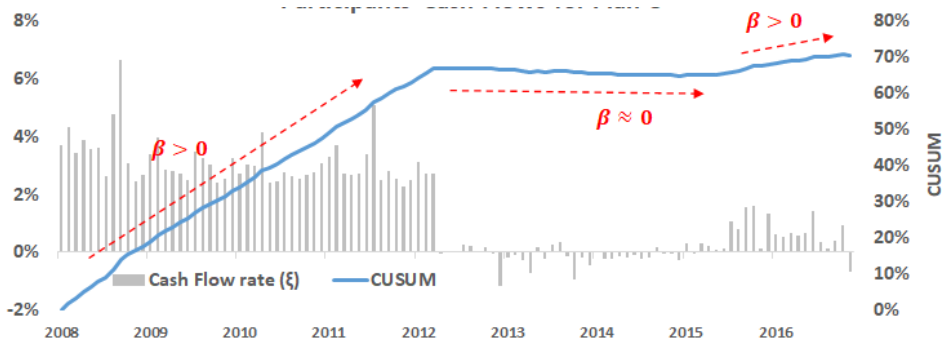
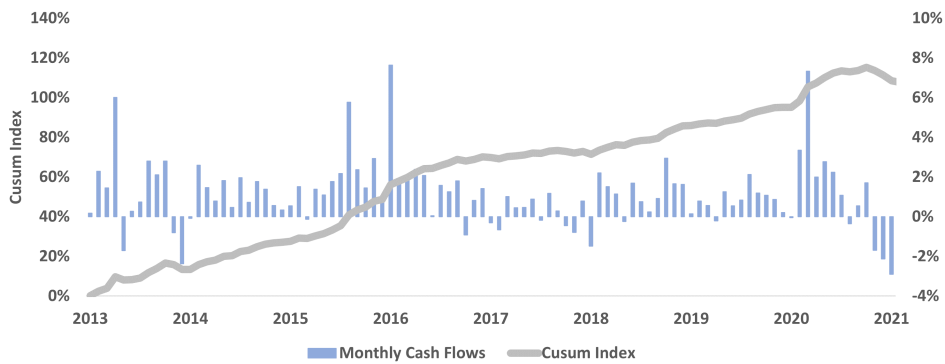
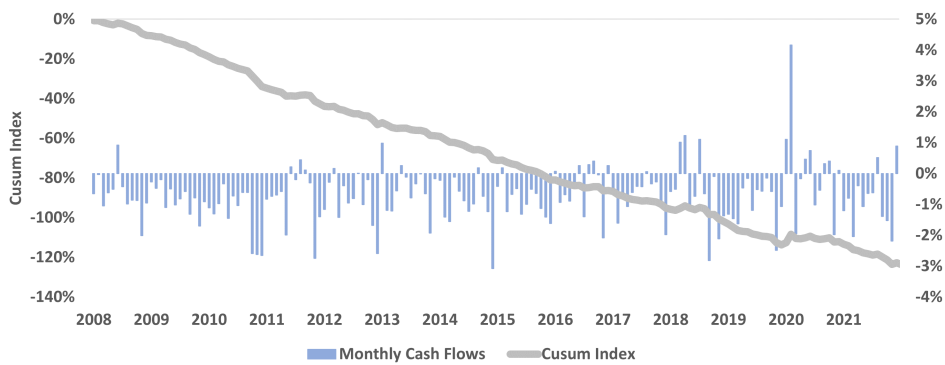
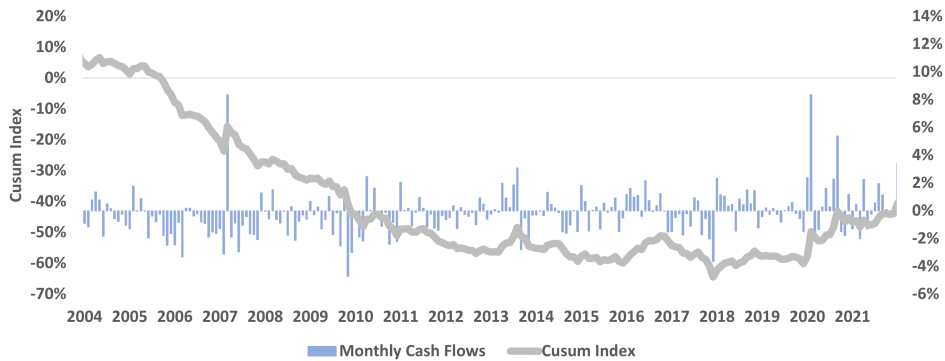


FIGURE II.2.9: A histogram of monthly cash flows representing data for individual plans. Each observation is expressed as a percentage of the month's initial book value. Source: Valerian proprietary data.



## II.2.B Example Plans' Cash Flows with Their Respective Trends



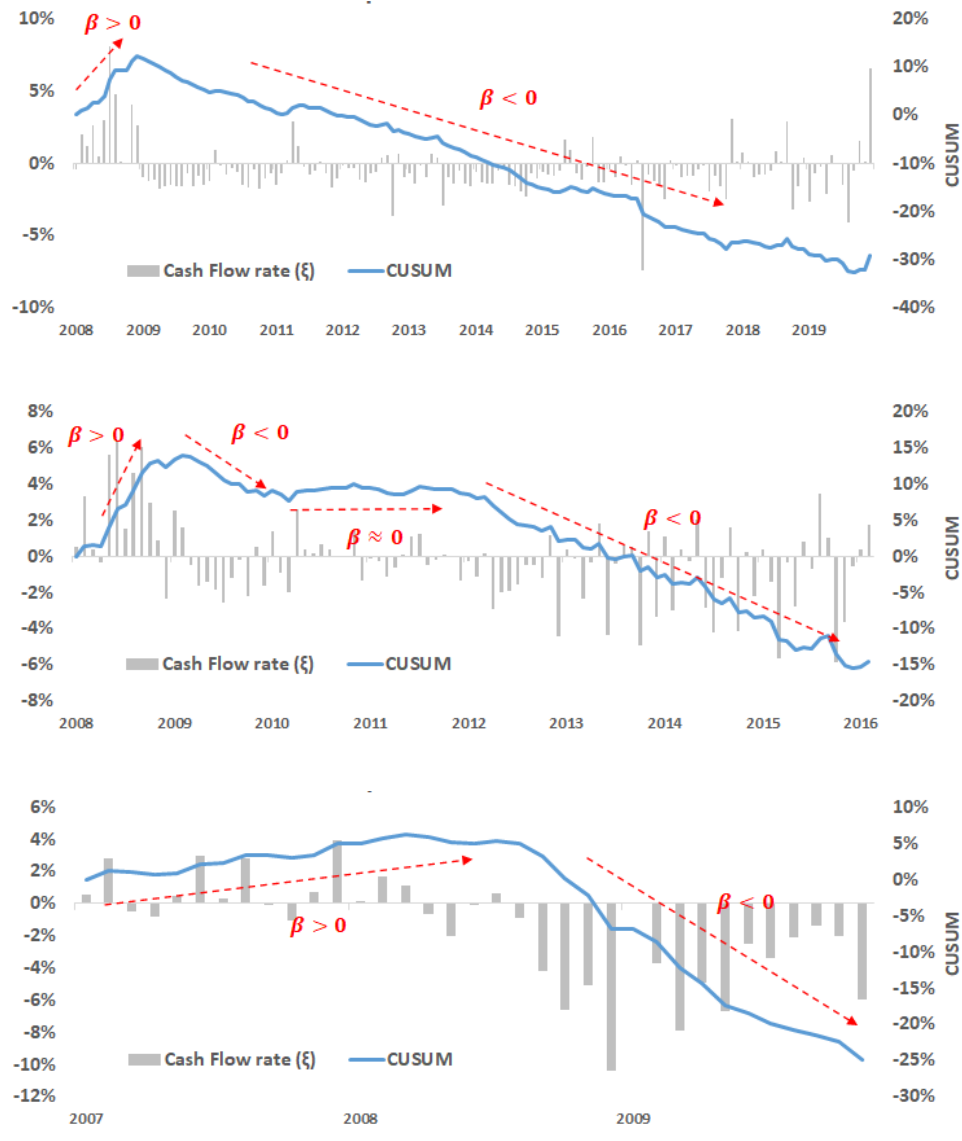


FIGURE II.2.10: Source: Valerian proprietary data.

### II.2.C Plan sponsor Communication

The below shows an extract of communication requirements from [U.S. Department of Labor \(2020\)](#) perspective.

Model Comparative Chart  
**ABC Corporation 401k Retirement Plan**  
 Investment Options – January 1, 20XX

This document includes important information to help you compare the investment options under your retirement plan. If you want additional information about your investment options, you can go to the specific Internet Web site address shown below or you can contact [insert name of plan administrator or designee] at [insert telephone number and address]. A free paper copy of the

information available on the Web site[s] can be obtained by contacting [insert name of plan administrator or designee] at [insert telephone number].

## Document Summary

This document has 3 parts. Part I consists of performance information for plan investment options. This part shows you how well the investments have performed in the past. Part II shows you the fees and expenses you will pay if you invest in an option. Part III contains information about the annuity options under your retirement plan.

## Part I. Performance Information

Table II.2.15 focuses on the performance of investment options that do not have a fixed or stated rate of return. Table II.2.15 shows how these options have performed over time and allows you to compare them with an appropriate benchmark for the same time periods. Past performance does not guarantee how the investment option will perform in the future. Your investment in these options could lose money. Information about an option’s principal risks is available on the Web site[s].

TABLE II.2.15: Variable Return Investments

Name/ Type of Option	Average Annual Total Return as of 12/31/XX				Benchmark			
	1yr.	5yr.	10yr.	Since Inception	1yr.	5yr.	10yr.	Since Inception
<b>Equity Funds</b>								
A Index Fund/ S&P 500www. website address	26.5%	0.34%	-1.03%	9.25%	26.46%	0.42%	-0.95%	9.30%
							S&P 500	
B Fund/ Large Capwww. website address	27.6%	0.99%	N/A	2.26%	27.80%	1.02%	N/A	2.77%
							US Prime Market 750 Index	
C Fund/ Int'l Stockwww. website address	36.73%	5.26%	2.29%	9.37%	40.40%	5.40%	2.40%	12.09%
							MSCI EAFE	
D Fund/ Mid Capwww. website address	40.22%	2.28%	6.13%	3.29%	46.29%	2.40%	-0.52%	4.16%
							Russell Midcap	
<b>Bond Funds</b>								
E Fund/ Bond Indexwww. website address	6.45%	4.43%	6.08%	7.08%	5.93%	4.97%	6.33%	7.01%
							Barclays Cap. Aggr. Bd.	
<b>Other</b>								
F Fund/ GICswww. website address	0.72%	3.36%	3.11%	5.56%	1.8%	3.1%	3.3%	5.75%
							3-month US T-Bill Index	
G Fund/ Stable Valuwww. website address	4.36%	4.64%	5.07%	3.75%	1.8%	3.1%	3.3%	4.99%
							3-month US T-Bill Index	
Generations 2020/ Lifecycle Fundwww. website address	27.94%	N/A	N/A	2.45%	26.46%	N/A	N/A	3.09%
							S&P 500	
					23.95%	N/A	N/A	3.74%
							Generations 2020 Composite Index*	

\* Generations 2020 composite index is a combination of a total market index and a US aggregate bond index proportional to the equity/bond allocation in the Generations 2020 Fund.

Table II.2.16 focuses on the performance of investment options that have a fixed or stated rate of return. Table II.2.16 shows the annual rate of return of each such option, the term or length of time that you will earn this rate of return, and other information relevant to performance.

TABLE II.2.16: Fixed Return Investments

<b>Name/ Type of Option</b>	<b>Return</b>	<b>Term</b>	<b>Other</b>
H 200X/ GIC www. website address	4%	2 Yr.	The rate of return does not change during the stated term.
I LIBOR Plus/ Fixed-Type Investment Account www. website address	LIBOR +2%	Quarterly	The rate of return on 12/31/xx was 2.45%. This rate is fixed quarterly, but will never fall below a guaranteed minimum rate of 2%. Current rate of return information is available on the option's Web site or at 1-800-yyy-zzzz.
J Financial Services Co./ Fixed Account Investment www. website address	3.75%	6 Mos.	The rate of return on 12/31/xx was 3.75%. This rate of return is fixed for six months. Current rate of return information is available on the option's Web site or at 1-800-yyy-zzzz.

## II.3 Risk Assessment for Synthetic GICs: A Quantitative Framework for Asset-Liability Management

*This chapter revisits the article co-authored with Jeffrey Jakubiak, Stephane Loisel, and Yahia Salhi.*

### **Abstract**

This study addresses a research gap in quantitative modeling framework and scenario analysis for the risk management of stable value fund wraps, a crucial segment of the U.S. financial market with over USD \$400 billion in assets. In this paper, we present an asset-liability model that encompasses an innovative approach to modeling the assets of fixed-income funds coupled with a liability model backed by empirical analysis on a unique data set covering 80% of the stand-alone plan sponsor market, contrasting with models based solely on regular deterministic cash flows and interest rate differences. Our model identifies and analyzes two critical risk scenarios from the insurer's perspective: inflationary and yield spike. Our approach demonstrates that the tail risk of wraps, used as an economic capital measure, is sensitive to characteristic parameters of the fund, such as the duration and credit quality of assets. This finding contrasts starkly with U.S. regulatory approaches, such as the NAIC's formulaic single scenario approach requirements, which frequently produce a zero capital result. These findings reveal limitations in current actuarial risk and profitability metrics for U.S. insurers and argue that a more sophisticated risk model reproducing the two critical scenarios is necessary.

**Keywords:** Stable Value, Synthetic GIC, Asset Liability Management, Wraps, Book Value Accounting, Crediting Rate, 401(k) Investment Plans, Benefit Responsive

### II.3.1 Introduction

Stable value funds are an essential investment choice within 401(k) plans, functioning as financial vehicles that ensure capital preservation and augment the range of investment options for investors. The funds' notoriety is highlighted by the third quarter of 2020, where stable value funds managed over USD \$888 billion of assets [SVIA \(2020a\)](#). A study by [MetLife \(2022\)](#) observes that over 82% of defined contribution plans provide stable value funds to their participants. The practical utility of stable value funds is also highlighted by [Babbel and Herce \(2018\)](#), who found that stable value funds outperform money market instruments and can improve the return of a retirement portfolio through better diversification.<sup>1</sup> Despite their significant role and size in the U.S. market, academic studies exploring the asset-liability management of stable value funds from an insurer's perspective are scarce, with the only literature being [Kwun et al. \(2009\)](#) and [Kwun et al. \(2010\)](#).

Stable value market trends underline the relevance of our study. Over time, investment managers of stable value funds have increasingly allocated assets towards lower credit quality and higher duration fixed-income assets. These gradual shifts, which began post the 2008 financial crisis, have recently allowed some stable value funds to invest in high-yield bonds and lower-graded securities. Consequently, the shift towards riskier stable value fund assets warrants a robust quantitative method to evaluate the related wrap insurance risk. Given these market developments, a quantitative risk framework for asset liability management of stable value funds is more critical than ever.

The recent technical defaults of several banks in 2023, attributed to inadequate Asset-Liability Management (ALM) risk management as outlined by [Dinh \(2023\)](#), underscores the continued need for a robust ALM framework. Such financial setbacks might have been mitigated or even prevented by employing a quantitative approach to comprehend the ALM mismatch risk, using tools such as stress and reverse stress tests. Furthermore, it is worth noting that the banks that defaulted were characterized by unique target participant demographics (deposit holders), as documented by [Vo and Le \(2023\)](#). They report a concentration "...in a small group of depositors, many of whom work in the venture capital industry. As a result, they are likely to know each other, increasing the risk of a bank run..." This instance underscores the importance of understanding the behavioral aspects of participant cash flows and incorporating an appropriate model for asset-liability management and scenario

---

<sup>1</sup>Although some conclusions from this paper may be applicable to stable value funds in general, our primary focus is on individually managed synthetic guaranteed investment contracts (GICs), also known as *wraps*. According to [SVIA \(2020a\)](#), about USD \$400 billion is insured via wrap contracts.

generation.<sup>2</sup> Therefore, one of our primary objectives in this paper is to propose a liability model that reflects the behavior of cash flows, thus addressing these significant concerns in the field.

The studied liability model is inspired by [Alimoradian et al. \(2023\)](#), where the authors empirically examine a dataset of 44000 monthly cash flows for 297 stable value plans. Their study concludes that a suitable model for projecting participants' cash flow scenarios should incorporate a rate deficit hypothesis, reputational mass lapse, flight-to-safety behavior, and a regime-switching trend model representing the plan sponsor's influence and ecosystem. The authors also noticed that hypotheses like the *moneyness hypothesis* are less relevant in the context of stable values. These components constitute the building blocks of our model in this paper.

To further justify our liability cash flow model, we also assess its suitability in Section II.3.7. This process involves identifying and discussing the loss paths from the insurance perspective.<sup>3</sup> We will see in the same section that the existence of a stochastic trend component in our liability model enables the generation of adverse loss scenarios. However, models without an independent stochastic component, like the model proposed by [Kwun et al. \(2009\)](#) where withdrawal is directly linked to the crediting rate deficit, fall short in generating high withdrawal rates at the right timing, leading to missing some of the riskiest scenarios. Additionally, the U.S. [NAIC \(2015\)](#) regulatory approach to reserve calculation does not adequately consider the complex path-dependent nature of this product's insurance risk. Therefore, the riskiest scenarios may not be included in the projected cash flow scenarios of the [NAIC \(2015\)](#) either.

In addition to our liability cash flow model, we also developed a simple yet robust asset model that encapsulates the primary risk factors of fixed-income funds. Since the early 1990s, interest rate models have been developed to cater to the needs of investment banks. Although initially designed to price and hedge trading instruments, these models are also used for projecting fixed income funds' cash flows. [Kwun et al. \(2010\)](#) used different interest rate models to estimate the present value of these coupons, defining a structure of reinvested coupon payments. However, such models may not adequately capture primary risk factors, focusing instead on secondary risks like term-structure basis spread risk in interest rates. Our paper proposes a more generic approach to risk when dealing with diversified portfolios by modeling the

---

<sup>2</sup>These plans are typically provided to a single employer's employees and are individually managed within the broader 401(k) plan.

<sup>3</sup>Regulators, such as those cited in [EIOPA \(2015\)](#), stress the significance of this analysis and advise incorporating reverse stress tests in their Own Risk and Solvency Assessment (ORSA) process.

yield, the primary risk factor of fixed-income funds.<sup>4</sup>

The seminal work of Ermanno Pitacco on variable annuities [Bacinello et al. \(2011\)](#) has had a profound influence on our research, serving as a guiding beacon and a source of inspiration for this paper. Pitacco's pioneering approach, which strived to provide a unified framework for modeling insurance-linked products, has resonated with us, inspiring our approach to analyzing and studying similar products. Particularly striking was his deep understanding and detection of risk scenarios for variable annuities [Bacinello et al. \(2010\)](#). His novel approach left an indelible impression on us, prompting us to apply similar methodologies in our work. Pitacco's unique blend of theoretical and practical research has had a far-reaching impact, making significant contributions to the field of insurance and beyond. He was able to ground his theoretical insights in practical realities, a trait that we believe can be seen in our research. His work has influenced daily practices in the insurance industry, especially in the structuring of various finance products, and can also be discerned in the area of stable values. The existence of our current research is a testament to the continuity of Pitacco's way of thinking. His spirit of innovation and curiosity lives on in our work, guiding our efforts as we seek to unravel the complexities of stable value funds.

The rest of the paper is structured as follows: Section [II.3.2](#) introduces the stable value ecosystem, elucidates its structural mechanism, and explains the efficiency of such an investment structure. Section [II.3.3](#) and Section [II.3.4](#) present the asset model and the participants' lapse liability model, respectively. Section [II.3.5](#) overlays the numerical simulation results while Section [II.3.6](#) discusses the risk scenarios for the insurer. Section [II.3.7](#) in particular discusses the path-dependent nature of the risk of this product and the suitability of our cash flow model. The paper concludes with a discussion of the economic capital requirements and best practices in stable value fund guarantee risk management. It provides recommendations for practitioners, rating agencies, and regulators on estimating stable value fund wrap risk in Section [II.3.8](#).

## II.3.2 Stable Value Guaranteed Insurance Contract

Stable value funds are investment options offered by many defined contribution retirement plans, generally characterized as very low-risk investment options with liquidity and principal preservation similar to money market

---

<sup>4</sup>The asset model we propose here, which focuses on yield as the risk factor for fixed-income funds, aligns with industry practices for participant disclosure and stable value fund evaluation, where yield is a significant factor in the fund's description.



funds but with slightly higher returns.<sup>5,6,7</sup>

Figure II.3.1 is a pictorial representation of the stable value ecosystem. Participants, in search of low-risk and principal-preserving investments, make their contributions or distributions to a retirement account managed by a bankruptcy-remote trust. These contributions or distributions are made through an employer or a pool of employers, and an investment entity then invests those assets appropriately.<sup>8</sup> Under the terms of the insurance contract, the assets are guaranteed at book value responsive payment. The details of this book value mechanism will be discussed in the following subsection.

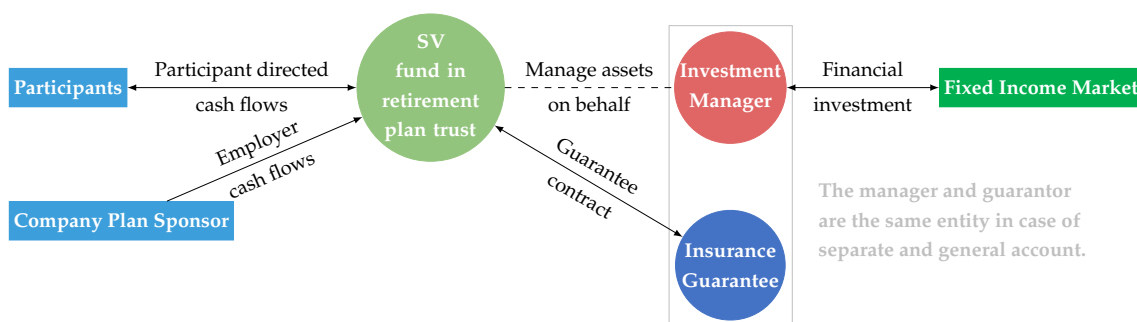


FIGURE II.3.1: Stable value ecosystem.

### II.3.2.1 Book Value Wrap Accounting

This subsection presents the mathematical formulation for the book value wrap accounting concept in the context of stable value funds. This specific accounting practice allows the fund participants to treat their investments at

<sup>5</sup>For more details about stable value funds, we refer the reader to the [Stable Value Investment Association \(2020\)](#) and [Fabozzi \(1998\)](#) comprehensive handbook on stable value investments.

<sup>6</sup>According to the Stable Value Investment Association’s quarterly survey, USD \$162 billion of the stable value wrap market is represented by stable value pooled funds, and USD \$270 billion are *individually managed* (SVIA, 2020b). See ([Stable Value Investment Association, 2020](#)) for more details.

<sup>7</sup>Generally speaking, there are three major types of stable value products: *General Account* where the assets of the funds are managed by and invested in the insurance company’s general account, *Separate Account* where the insurance company is also the manager of the fund, but the assets of the funds will be segregated from the insurance company’s general account, and *Synthetic Guaranteed Investment Contracts (Synthetic GICs)* where the insurance company is an independent entity from the manager of the funds (see [SVIA \(2020a\)](#) for more details). Stable value funds can also either represent one employer or a pool of employers. In the case where the plan sponsor is one employer, the fund is known as *individually managed*. In contrast, pooled funds allow small and medium-sized plan sponsors to bring their stable value funds together in a single fund formed under federal or state banking laws.

<sup>8</sup>Figure II.3.1 also shows cash flows by company plan sponsor/employer. These cash flows are usually standard contributions, like bonus payments made by the employer. Still, they can also include once-off cash flows due to restructuring after a merger or acquisition, company bankruptcy, layoffs, or other company plan-sponsor corporate actions. The insurance guarantee may have strict contractual terms for these cash flows and may not cover them at *book value payment* under the *wrap agreement* or may cover them up to a certain, predetermined limit. In the case of general and separate account products, the investment and the guarantor roles are performed by the same entity, typically an insurance company. However, with Synthetic Guaranteed Investment Contracts (GICs), the retirement plan trust engages one or more guarantors, typically insurance companies, to ensure a balance between its assets and liabilities.

their original cost plus any credited interest, as opposed to their current market value. This key feature, which allows transactions at book value, provides the principal protection characteristic inherent to stable value funds.

$B_t$  and  $M_t$  represent the aggregated book value and market value for the sum of all participant contracts, respectively.

**Definition 11.** We define the following time-dependent processes for each  $t \geq 0$ :

- $\{M_t\}_{t \geq 0}$  represents the market value of the fund;
- $\{B_t\}_{t \geq 0}$  denotes the book value of the fund;
- $\{\gamma_t\}_{t \geq 0}$  corresponds to the rate, also known as the crediting rate, that represents the growth of the book value of the fund;
- $\{C_t\}_{t \geq 0}$  represents the cash flow activities in/from the fund.

The book value of the fund evolves according to the following dynamic

$$dB_t = B_t \gamma_t dt + dC_t, \quad (\text{II.3.1})$$

while we assume crediting  $\{\gamma_t\}_{t \geq 0}$  to be described by the following equation:

$$\gamma_t = \left( \frac{1}{\theta_t} \ln \frac{M_t}{B_t} + y_t - p \right)_+, \quad (\text{II.3.2})$$

where  $p \in \mathbb{R}^+$  is the insurance premium while the yield of the fund  $\{y_t\}_{t \geq 0}$  and the duration  $\{\theta_t\}_{t \geq 0}$  (expressed in years) are time-dependent processes previously defined in Equation (II.1.10).

**Remark 5.** For general accounts and most separate stable value funds,  $\gamma_t$  can be defined at the discretion of the insurance company. However, for Synthetic GICs and some separate account products,  $\gamma_t$  can be defined by means of an exact formula. While there are several versions for the crediting formulas used within the market, one of the most used formulas corresponds to Eq. (II.3.2).

Other contracts may use a different crediting rate expression. Still, when expressed in a continuous formulation, they always converge to Eq. (II.3.2) (or equivalent have a residual difference w.r.t. this formula).

Note that Eq. (II.3.2) is aiming at making the book value converge to the market value in  $\theta_t$  years. Hence, for a given yield  $y_t$  (resembling the growth speed of the market value  $M_t$ ) and insurance costs  $p$ , the right side of Equation II.3.2 approximates the crediting rate to achieve the convergence of the book value to the market value in  $\theta_t$  years.

**Example 2.** If we assume no cash inflows/outflows, i.e., if  $dC_u = 0$  during the period  $[t, t + \theta_t]$ ,  $B_t$  depends only on the crediting rate  $\gamma_t$ . Moreover, if we neglect market risk,

*i.e., the market value grows at a pace of  $y_t - p$ , then even if there is an asset-liability mismatch at  $t$ , i.e.,  $B_t \neq M_t$ , we desire the two to be the same over  $\theta_t$  years.<sup>9</sup>*

However, in reality, both cash inflows and outflows occur, and the yield of the fund changes over time. Furthermore, if  $\gamma_t \leq 0$ , convergence is no longer guaranteed. This is the motivation behind the fact the fund needs to recourse to an insurance company to cover any losses due to an asset-liability mismatch, which will be further explained in the following Section.

### II.3.2.2 Insurance Guarantee

In the context of stable value funds, the retirement plan trust manages assets using an investment entity while being liable for the book value of the assets to the retirement plan participants. To ensure asset-liability parity, the trust enters into a guarantee contract with an insurance company and pays insurance premiums.

Table II.3.1 illustrates a simplified T-account from the 401(k) trust’s perspective. On the left side, the trust manages assets using an investment entity. On the right side, the trust is liable for the book value of the assets to 401(k) plan participants. To ensure asset-liability parity, the trust enters into a guarantee contract with an insurance firm and pays insurance premiums. According to the asset-liability matching principle, the book value plus the sum of insurance premiums should always be at par with the assets.

TABLE II.3.1: Simplified T-Account from the 401(k) trust’s perspective.

Assets	Liabilities
Fund’s asset ( $M_t$ )	Book value ( $B_t$ )
Insurance guarantee	Sum of upcoming insurance premiums

A key characteristic of a stable value fund guarantee is that the insurance company acts as the last resort, meaning that as long as the last individual (or the last pool of individuals) has not withdrawn their assets from the fund, the current participants invested in the stable value fund - and not the insurer - will pay for any past fund shortage. In other words, the insurer will make a claim payment only when the assets of the stable value fund have been fully exhausted. The last resort concept is the core advantage of stable value funds over other guaranteed investment structures, making the insurance guarantee relatively more affordable since, because of the last resorting, the embedded put option becomes very remote and out of the money.

<sup>9</sup>In other words, if the right side of Equation II.3.2 is positive ( $\gamma_t > 0$ ), and if the yield is constant over time, and there are no cash inflows and outflows, ( $dC_t = 0$ ), then the market value and book value will converge in  $\theta_t$  years.

**Definition 12.** The market value deficit  $G_t$ , also known as asset-liability mismatch, is defined as the difference between the book value and market value, i.e.,  $G_t = B_t - M_t$  representing the intrinsic value of an insurance payoff.

Another key tool is given by the resort time.

**Definition 13.** Given a time horizon  $T_{max}$ , the last resort time  $\hat{T}$  is defined as the time when the market value reaches below zero  $M_{\hat{T}} \leq 0$ , thus

$$\hat{T} = T_{max} \wedge \inf_{t \geq 0} \{t : M_t \leq 0\} .$$

Moreover,  $G_{\hat{T}}$  as the insurance loss paid at time  $\hat{T}$ , the asset-liability mismatch at the last resort when the last participant withdraws, hence the following

$$G_{\hat{T}} = (B_{\hat{T}} - M_{\hat{T}})_+ = B_{\hat{T}} , . \quad (II.3.3)$$

holds.

We emphasize that the stochastic resort time  $\hat{T}$  corresponds to a random variable.

In theory, there is no fixed maturity for this contract; however, given the fact that parties have the opportunity to terminate the contract early - under certain legal clauses defined by each contract - it is common practice to introduce a time horizon  $T_{max}$ . The analysis of optimal strategies of the insurer with respect to this early termination option is left for further research.

### II.3.3 Asset Model

This section of the paper focuses on modeling the dynamics of the market value of stable value funds, an essential aspect of the asset-liability management framework. Diverging from the methodology of modeling fixed income funds based on the sum of coupons, the model presented in this paper adopts a more generic approach, emphasizing yield as the primary risk factor. This approach is inspired by the way practitioners assess the dynamics and returns of these types of funds.

The following equation describes how the market value of the fund can grow or shrink based on the fund's cash flows ( $C_t$ ), taking into account the insurance premium ( $p$ ) as a cash outflow.

$$dM_t = M_t(y_t + \delta)dt - M_t\theta_t dy_t - pB_t dt + dC_t. \quad (II.3.4)$$

In the model described by Eq. II.3.4, the yield  $y_t$  is crucial as it represents the return on investment. The adjustment rate  $\delta$  accounts for the increase/decrease in the value of the securities within the fund due to factors such as the convexity of the fund, impaired securities, operational volatility, etc. Therefore, in our model, the net expected growth rate of the fund is captured by  $y_t + \delta$ .

By concentrating on the yield as the primary risk factor and incorporating the effective duration, this model provides a more suitable approach for understanding and managing the risks associated with fixed-income funds.

To complete our model, we have to define the dynamics of the yield  $\{y_t\}_{t \geq 0}$ . A natural approach to modeling the yield of a fixed income fund involves decomposing it into the sum of a risk-free rate  $r_t$  and a spread  $s_t$  given by :

$$y_t = r_t + s_t. \quad (\text{II.3.5})$$

The spread is commonly known as an option-adjusted spread, or OAS (see [Risk Management Task Force \(2004\)](#)). The instantaneous interest rates and OAS evolves according to the following Cox-Ingersoll-Ross (CIR) Model

$$\begin{aligned} dr_t &= \kappa_r(r_t - r_\infty)dt + \nu_r\sqrt{r_t}dW_t^r \\ ds_t &= \kappa_s(s_t - s_\infty)dt + \nu_s\sqrt{s_t}dW_t^s \\ \langle W_t^s, W_t^r \rangle &= \rho t. \end{aligned} \quad (\text{II.3.6})$$

where  $r_\infty$  and  $s_\infty$  are the asymptotic value of the rate  $r_t$  and spread  $s_t$  for a large  $t$ ,  $\nu_r$  and  $\nu_s$  are the standard deviation of rate and spread, respectively and  $\kappa_r$  and  $\kappa_s$  are the corresponding rate of mean reversion while the Brownian motions  $W_t^s$  and  $W_t^r$  represents the source of randomness for the correlated dynamics.

The term  $\rho$  represents the stochastic correlation between the Brownian motions of interest rates and the credit spread of the fund. The relationship between interest rates and credit spreads is highly regime-dependent and non-stationary, with a generally negative correlation observed (see [Duffee \(1998\)](#) for more details). A higher correlation between OAS and interest rates increases the tail risk for the insurance guarantee, as it generates higher volatility for the yield and market value of the stable value fund.

### II.3.4 Liability Cash Flow Model

In this section, we present a model for the liability cash flow dynamics of the stable value fund, focusing on the cash flows generated by the participants'

contributions and withdrawals. Recall that  $C_t$  is the stochastic process representing the accumulated cash flows by the participants of the stable value fund. A positive  $C_t$  indicates the growth of the stable value fund, while a negative  $C_t$  signifies withdrawals from the fund. We define the dynamics of  $\{C_t\}_{t \geq 0}$  as per Equation II.3.7:

$$dC_t = B_t \left( \underbrace{\beta_t}_{\text{structural trend including herd behavior}} + \underbrace{g(r_t - \gamma_t)}_{\text{rate deficit}} + \underbrace{f(s_t)}_{\text{flight-to-safety}} \right) dt. \quad (\text{II.3.7})$$

As it can be deciphered from Equation II.3.7, the model is composed of key components whose relevance to liability cash flows will be elaborated upon in the rest of this section.

- A structural, non-monotonic regime-changing trend in cash flows related to the nature of plan sponsors' ecosystem, which indirectly influences participants' behavior. In our model, this is captured by the term  $\beta_t$ .
- A rate deficit risk factor: This is represented in our model by the term  $g(r_t - \gamma_t)$ . The rate deficit hypothesis, as illustrated in Alimoradian et al. (2023); Barsotti et al. (2016), suggests that withdrawals from financial products can be modeled using interest rate differentials between the product in question and other competing interest rates.
- A herd behavior component: This component is also included in our model's  $\beta_t$  term.
- The cash flow risk-mitigating effect of flight-to-safety behavior during a crisis: In our model, this is captured by the term  $f(s_t)$ .

### II.3.4.1 Regime Changing Trend Component

The regime-changing trend component is important for capturing the dynamics of different market conditions and plan-sponsor ecosystem and their impact on the participant's cash flows. Alimoradian et al. (2023) conducted empirical data analysis and identified non-monotonic trends within the historical participant cash flow data. Alimoradian et al. (2023) observe that these trends are stochastic, varying in both size and duration, with notable change points. Importantly, they find no strong correlation between trends before and after a change point. This lack of correlation is attributed to an economic intuition that these trends are indicative of the prevailing market environment and the

overall health of the plan-sponsor ecosystem<sup>10</sup>. Alimoradian et al. (2023) observations align with the findings of Madrian and Shea (2001) and Mitchell et al. (2006), who analyzed individual 401(k) account data and concluded that the plan-sponsor's role is critical in investment allocation decisions. They found that approximately 10% of liability cash flow activities by participants of the fund were found to be influenced by the plan sponsor. Notably, these changes in trends are often structural and unpredictable, contingent on factors like management decisions that extend beyond quantifiable economic variables. For instance, a company with a history of stable employment may continue this trend in the subsequent years. However, if there is a regime change, the trend could swing in any direction.

We define the set  $\{S_i, \tau_i\}$  as the  $i$ -th state and its time of switching to this state. With this notation, Equation II.3.8 relates the trends  $\beta$  with the states. In addition, the duration of each state ( $\tau_{i+1} - \tau_i$ ) is stochastic, and we assume that it follows an exponential distribution with the intensity parameter  $\frac{1}{d_i}$ . Note that  $d_i$  does have a simple economic interpretation; it is the average duration in years of being in the regime  $S_i$ . Therefore, we have:

$$\begin{aligned} \forall \tau_i \leq t \leq \tau_{i+1}, \quad \beta_t &= b_i \\ P(\tau_{i+1} - \tau_i \geq x) &= e^{-\frac{x}{d_i}} \\ P(S_i = s) &= p_s. \end{aligned} \tag{II.3.8}$$

As discussed earlier, in Equation II.3.9, we assume that the trend of the next regime,  $\beta_{i+1}$ , is independent of the trend of the current regime,  $\beta_i$ .

$$(\beta_{i+1}, \tau_{i+1}) \perp\!\!\!\perp (\beta_i, \tau_i), \tag{II.3.9}$$

In other words, we assume the parameters of the next regime do not depend on those of the current regime.

To round out our model, we discretize the state process into four states: growth, stability, and decline, and we include an additional herd state. This is depicted in Table II.3.2. The growth state signifies a positive cash flow trend for the fund, whereas the decline state signifies a trend of withdrawals. Note that in Table II.3.2, the probability of occurrence of the herd state is a function of the market-to-book value ratio, while the probabilities of occurrence of other states are considered constant.

<sup>10</sup>Factors such as plan sponsor communication and management decisions, financial health, industry sector, employment policies, growth or layoffs, plan demographics, and default options can all impact participant cash flow trends.

State	Respective trend	Probability of occurrence	Average duration
Stable state	$b_0$	$1 - p_1 - p_2 - p_3(\frac{M_t}{B_t})$	$d_0$ years
Decline state	$b_1$	$p_1$	$d_1$ years
Growth state	$b_2$	$p_2$	$d_2$ years
Herd state	$b_3$	$p_3(\frac{M_t}{B_t})$	$d_3$ years

TABLE II.3.2: Participant trends probability of occurrence and their average duration

Table II.3.2 presents a regime-switching model with three main states and an additional state to account for herd behavior. A three-state model offers the benefit of alternating between high, stable, and low withdrawal rates at random intervals. It is worth noting that the goal here is not to prescribe the exact number of states needed for cash flow projections. Instead, we aim to provide a generic model for liability cash flows that aligns with behavioral studies, as seen in Alimoradian et al. (2023). Therefore, an insurance industry practitioner could choose to apply to fewer or more states based on the specific experience of the plan.

### II.3.4.2 Herd Component

The herd state is an important component in our model because it accounts for the potential impact of mass withdrawals on the insurer’s liability. This behavioral component was observed in stable values by Alimoradian et al. (2023). Further, according to studies by Loisel and Milhaud (2011) and Barsotti et al. (2016), peer-to-peer influence and contagion effects can lead to mass correlation in withdrawals among policyholders’ cash flows, resulting in mass lapses. Alimoradian et al. (2023) also note that a low market-to-book value could potentially be perceived as a weakness and hypothetically lead to reputational issues, increasing the likelihood of herd behavior. Therefore, in Table II.3.10, the herd behavior is shown in the last trend component  $p(\frac{M_t}{B_t})$  as a function of market-to-book value.

### II.3.4.3 Rate Deficit Component

Equation II.3.7 also has a rate deficit component shown by the function  $g(\cdot)$  as the difference between the yield of the fund and the crediting rate of the stable value fund. In Alimoradian et al. (2023), the authors analyze historical data for the years 2000 to 2021 to observe any relationship between the participant’s lapses and the interest rates. However, the authors did not observe any relationship between the rate deficits and the cash flow rates. Still, they pointed out that the historical period of 2000 to 2021 was limited as the market was not exposed to an inflationary period where interest rates rose significantly.



#### II.3.4.4 Flight-to-safety Component

The flight-to-safety hypothesis, as discussed in [Baur and Lucey \(2010\)](#) and [Dorn and Huberman \(2005\)](#), posits that during times of market stress, investors tend to shift their investments from riskier assets to safer assets. The risk-mitigating effect of flight-to-safety was empirically observed in stable values by the study of [Alimoradian et al. \(2023\)](#). This factor should thus be considered when modeling stable value fund liability cash flows to account for potential shifts in participant behavior during market crises.

In this work, we model the 'flight-to-safety' effect using the function  $f(s_t)$ , where  $s_t$  represents the credit spread of the fund. One might question why we have chosen  $s_t$  as an indicator of market distress and the flight to safety rather than other indicators, such as equity performance. We have two primary reasons for this choice.

First, stable value funds are diversified investment portfolios encompassing a large portion of U.S. fixed income markets. Therefore, these funds are highly correlated with and representative of the U.S. fixed income market. A common indicator of the U.S. financial market entering a crisis is a widening global credit spread, as discussed in [Salhi and Théron \(2014\)](#). Given its diversified nature, a stable value fund would also experience this widening of spreads during a crisis.

Secondly, the credit spread is an internal risk factor to the system, impacting both assets and liabilities. Using an internal risk factor is more coherent with the model structure and ensures that the flight-to-safety component is organically integrated into the system. If we chose an external factor, it would be equivalent to adding an independent stochastic component to the model. For instance, if equity performance was used as the argument to the function  $f(\cdot)$ , it would only be material to the risk measurement if there is a significant correlation between interest rates and equities. Assuming no strong correlation exists, incorporating equities would merely add an independent stochastic factor that would be more appropriately introduced directly into the stochastic trend component as an extra trend state representing process.

### II.3.5 Wrap Risk Results

This section outlines the main results from the Monte Carlo simulation of the model we described earlier. Before diving into the specifics, we will introduce an added assumption to the model. In our model framework, the effective duration, represented by  $\theta_t$ , indicates how sensitive an asset is to changes in its yield. For the discussions in this paper, we will assume that the effective

duration remains constant. This is a reasonable assumption because stable-value funds often have fixed durations due to the investment constraints set by their respective insurance companies. So, for the rest of this paper, we assume that  $\forall t \geq 0 \quad \theta_t = \theta$ .

Table II.3.3 shows the loss frequency under different parameter assumptions for the probability of transitioning into a decline ( $p_1$ ) and different definitions of a decline ( $b_1$ ). Other parameter inputs are as described in Appendix II.3.A. Table II.3.3 also shows the convexity of the wrap risk resulting when the loss frequency is low, an indication of the remoteness of the risk.

	$p_1 = 1/200$	$p_1 = 1/100$	$p_1 = 1/50$	$p_1 = 1/20$	$p_1 = 1/10$
$b_1 = 1/4$	0.98%	1.03%	1.23%	1.63%	2.46%
$b_1 = 1/3$	0.98%	1.11%	1.36%	2.19%	3.64%

TABLE II.3.3: Loss frequency for different probabilities of entering a declining state in a year

	$p_1 = 1/200$	$p_1 = 1/100$	$p_1 = 1/50$	$p_1 = 1/20$	$p_1 = 1/10$
$b_1 = 1/4$	0.12%	0.20%	0.57%	0.91%	1.48%
$b_1 = 1/3$	0.32%	0.50%	1.24%	1.95%	3.05%

TABLE II.3.4: 99% CTE of loss comparison for the different probabilities of entering a declining state in a year

Another aspect we consider is the credit quality of the assets within the fund. In terms of asset defaults, most stable value wraps do not cover, or only partially cover, the default risk of the assets. Regardless of the contractual terms, in practice, investment managers can sell the assets before any default occurs. However, the fund will still likely experience a market value loss due to credit rating migration. Therefore, the main risk factor to consider is credit migration, and in order to study this further, we assume that the credit ratings downgrade occurs at a transition probability as discussed in [Caouette et al. \(1998\)](#). We, therefore, calibrated the parameter  $\delta$  in Equation II.3.4 based on the tables from [Caouette et al. \(1998\)](#). For example, for AA-rated investments, we obtain  $\delta = 0.07\%$ . Our risk estimates in Table II.3.5 show that stable value funds with lower credit quality have significantly more tail risk than stable value funds with higher credit quality assets, which insurance companies may need to consider when evaluating risk.

Avg. OAS	Assets avg. S&P grading	Migration haircut $\delta$	99% CTE of PV of loss - premium	Avg. loss size
1.1%	AA	-0.07%	0.17%	1.38%
2.5%	A	-0.20%	0.48%	2.25%
3.2%	BBB	-0.50%	1.14%	2.98%

TABLE II.3.5: Guarantee risk of funds with different investment structures for a wrap contract specified in Section II.3.A

Apart from a stable value fund's average rating, one other factor that has a strong impact on the tail risk of the guarantee is the duration. Table II.3.6 shows that funds with higher duration have significantly greater tail risk. As an example, the average loss size increases from 0.03% to 10.28% from a 3-year to a 9-year stable value fund duration.

Duration in years	Avg OAS	99% CTE of PV of loss - premium	Loss frequency	Avg. loss size
3	0.8%	0.03%	0.06%	0.54%
4	1.1%	0.17%	0.28%	1.38%
5	1.4%	0.60%	1.30%	1.43%
6	1.9%	2.18%	2.09%	2.57%
9	2.4%	10.28%	4.23%	6.21%

TABLE II.3.6: Guarantee risk for a diversified AA-rated fund with different durations

Lastly, we also reviewed the statutory capital requirements for the wrap contract and compared it with the tail risk of the model presented here. The U.S. National Association of Insurance Commissioners (NAIC) Risk Based Capital (RBC) introduces a formulaic approach for capital charges of Synthetic GICs detailed in (NAIC, 2012). Model regulation for *Synthetic GICs* and *separate accounts* are described in (NAIC, 2015) and (NAIC, 2017) respectively. Since the emergence of the principles based approach, especially *Solvency I* and *II* by the European regulators, NAIC has also adopted a *principles based approach*, which applies stochastic modeling under *C3 Phases* (NAIC, 2008). This approach corroborates with the International Association of Insurance Supervisors (IAIS) initiative to develop an International Capital Standard (ICS) and a comparable measure of capital across different jurisdictions.<sup>11</sup> Table II.3.7 illustrates the NAIC capital charges for a U.S. insurer under the formulaic approach and the

<sup>11</sup>Currently, large global insurance groups are subject to different capital standards that make it difficult to compare their solvency positions across jurisdictions. The main objectives of the ICS are to establish a common language for supervisors to discuss the solvency of IAIGs and to enhance global convergence among the group capital standards that are in place.

assumption of reserve estimates regulation NAIC (2015). Here, we first considered a typical stable value guarantee contract specified in Appendix II.3.A. For completeness, we also assessed the 99% conditional tail expectation (CTE), the loss frequency, and the average loss size. The NAIC formulaic risk-based capital is 0.

NAIC A-695 Reserve	99% CTE of PV of loss-premium	Loss frequency	Avg. loss size
0%	0.35%	0.28%	1.38%

TABLE II.3.7: Monte Carlo based estimation of the stable value fund wrap risk

### II.3.6 Risk Scenarios to be Considered by an Insurer

The main objective of this paper is to introduce the readers to the risk scenarios from the insurer’s perspective. In order to do this, we projected assets and liabilities within a Monte Carlo framework and studied the scenarios in which losses are observed. This exercise helped us to identify two common patterns: the *yield spike mean reverting scenario* and the *inflationary scenario*. In this section, we discuss these scenarios and provide historical backtests to assess their validity.

#### II.3.6.1 Inflation

Figure II.3.2 shows the average yield and withdrawal rates for the loss threshold of 0.1% centile. This is the highest threshold of losses, and, as can be seen, the most extreme scenarios occur when the yields are rising with strong withdrawal rates occurring simultaneously. We call these scenarios the *inflationary scenarios*.

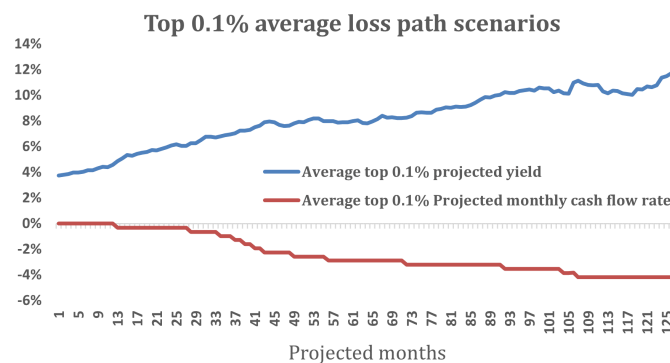


FIGURE II.3.2: Top 0.1% average loss path scenarios. Data source: Monte Carlo simulation.

*Inflationary* yield rises have already happened historically, specifically during the inflationary period of the late 1970s/early 1980s and during the Great Depression Period of the 1930s. Although stable value funds did not exist at the time, evaluating a hypothetical wrap risk during these periods may provide insight into the magnitude of the risk. Therefore, without any explicit assumptions, we back-test a wrap contract against these periods.

**1970s Inflationary Period:** In the late 1970s and into the early 1980s, there was an inflationary period that experienced sustained double-digit interest rates for approximately three years. From the perspective of stable value fund assets, this scenario would have exhibited a longer period of adverse market conditions than the 2008 financial crisis. However, higher yields also allow the crediting rate reset mechanism to potentially converge to differences between market values and book values. Figure II.3.4 below shows a hypothetical loss scenario over this period. Although stable value funds did not exist at the time, we used historical data from the Federal Reserve to construct this hypothetical scenario. It shows an insurance loss scenario with cash withdrawal rates of 40% annually (~ 4.16% monthly) for a sustained period of 7 years. In the figure, the fund yield is the historical yield level obtained by summing the 3y UST rates and the credit spreads. Interest rates were based on ([Board of Governors of the Federal Reserve System, 2020](#)), and credit spreads were estimated using US Moody’s equivalent Baa corporate bonds available from ([Federal Reserve Bank of St. Louis, 2020](#)).

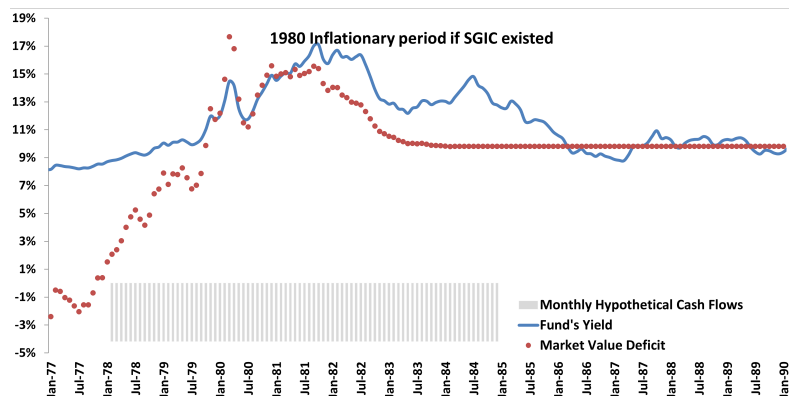


FIGURE II.3.3: Hypothetical Synthetic GIC contract with losses during the Inflationary Period if stable value funds had existed at the time. Data source: Simulated backtest against historical data by ([Federal Reserve Bank of St. Louis, 2020](#)).

**Great Depression Period:** Interest rates also reached double digits during the *Great Depression*, starting in the late 1920s. With the Great Depression lasting approximately ten years, the adverse conditions would have lasted much longer than the 2008 financial crisis. Figure II.3.4 shows a hypothetical loss scenario over this period. The graph shows the losses incurred for a 60%

annualized withdrawal rate for a sustained period of 3 years. Once again, high yields combined with worsening withdrawal rates produce an environment that can result in sustained losses.

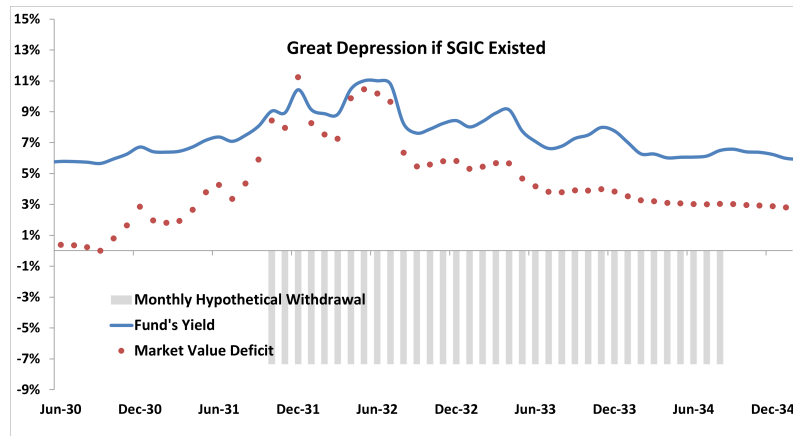


FIGURE II.3.4: Great Depression hypothetical scenario. The same data sources as the inflationary period were used for this analysis. Data source: Simulated backtest against historical data by (Federal Reserve Bank of St. Louis, 2020).

**COVID-19 Pandemic:** While the long-term effect of the COVID-19 pandemic is not yet known, the U.S. economy has experienced high inflation and rising interest rates during much of the aftermath of the pandemic with stable value funds consequently experiencing asset-liability mismatches that could result in a loss when coupled with a high withdrawal rates. Figure II.3.5 shows the consequence of the aftermath of the pandemic period for a very high withdrawal scenario on a hypothetical fund. The long-term effect of the pandemic is not yet known, but for the example wrap contract, there is an 11% market value to book value deficit, so the wrap may incur losses if it experiences a high withdrawal rate in the future.

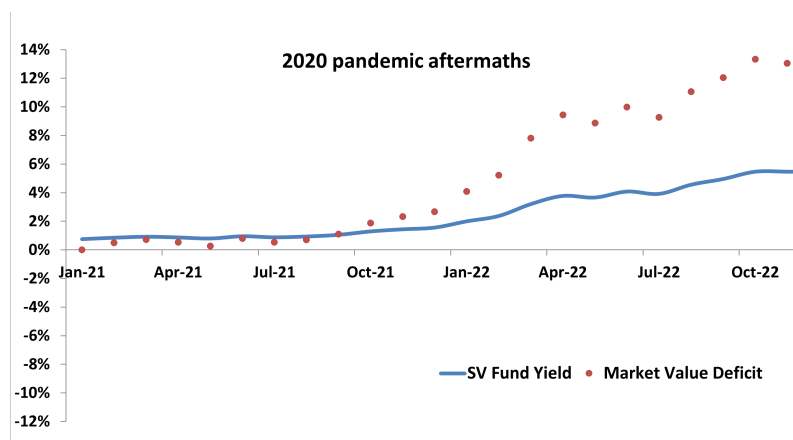


FIGURE II.3.5: Post COVID-19 pandemic inflationary scenario assuming no liability cash flows (positive or negative). Data source: Valerian Capital Group LLC proprietary data for the SV Fund Yield.

### II.3.6.2 Yield Spike

Figure II.3.6 shows another pattern observed within the 1% loss scenarios. The pattern shows a period in which yields rise and withdrawals co-occur, followed by sustained lower yields, which we refer to as *yield spike mean reverting scenarios*.

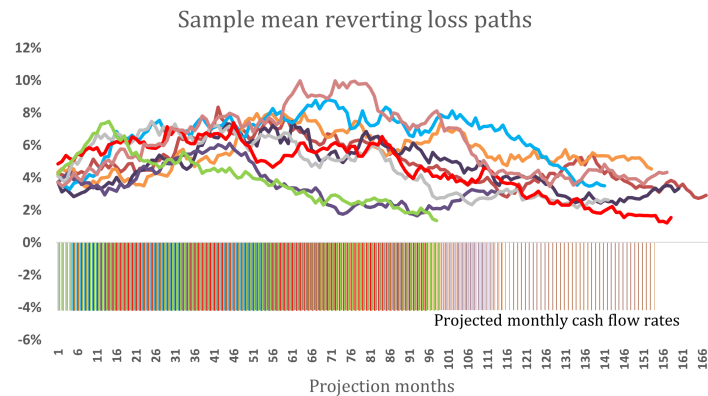


FIGURE II.3.6: The *yield spike mean reverting scenarios* observed within the 1% of loss scenarios. Data source: simulated single scenario.

The *mean reverting scenario* is when yields rise sharply and then subsequently revert back to more normalized levels. While the normalization of market conditions would likely suggest the mitigation of risk, this scenario is risky if it coincides with very large withdrawals during high yield periods. Suppose large withdrawals do not occur during high yield periods. In that case, mean reversion will have a lesser impact since there will be capital gains that offset any capital losses that occurred during the original increase in yields. As such, the path to a loss is a “run” on the stable value fund with a large proportion of the book value balances withdrawn over a relatively short period of time. The interrelationship in this scenario is path dependent; higher yields followed by mean reversion and withdrawals combine to create a low-yielding environment without a likelihood of market-to-book value convergence.

**Financial Crisis:** The reader may again realize that mean-reverting scenarios have also happened historically, specifically during the 2008 financial crisis. During the financial crisis, there were adverse conditions that reduced the market value of stable value fund assets. However, most of the funds did not experience net cash outflows, so there were no actual wrap contract losses experienced during the financial crisis.<sup>12</sup> Furthermore, as shown below in Figure II.3.7, the market value of stable value fund assets rose significantly by mid-2009 as shown by the negative market value deficit, i.e., by mid-2009, the market value had returned to a level equal to or greater than the book value.

<sup>12</sup>Source: Aggregated cash flow index from Valerian Capital Group



Figure II.3.7 shows a loss scenario under a potential cash flow withdrawal rate. The graph shows a loss scenario generated by a 60% (~ 7.35% monthly) annualized withdrawal rate occurring for a sustained period of 18 months.

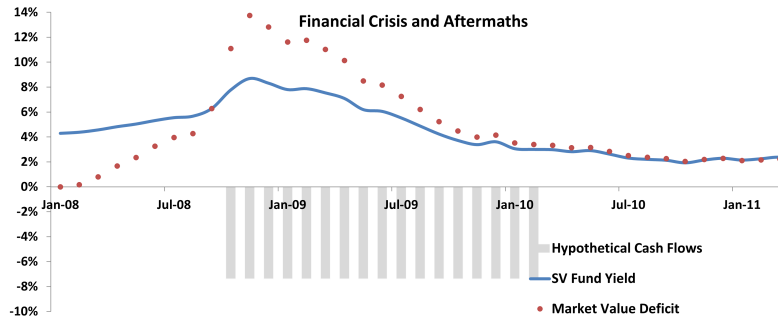


FIGURE II.3.7: Withdrawal scenario that would have generated losses as a result of the financial crisis. Data source: Simulated backtest on Valerian Caoital Group LLC proprietary data.

The *inflation* and *mean reverting yield spike* are the two riskiest scenarios observed within our projected paths. In both of these scenarios, a common characteristic is that an increase in yields aligns with significant cash flow withdrawals. When yields rise, the market value of assets, as well as the gap between assets and liabilities, expands. If this coincides with substantial withdrawals, the gap becomes irrecoverable, leading to losses for the insurer. Another way to describe the economics behind the above scenarios is through the time of recovery of an asset-liability mismatch. In the discussion in Section II.3.2.1, we showed that if there is no market risk, and no cash flow risk, and the crediting rate formula has not reached its plateau of 0%, then book and market value will converge over  $\theta$  years. This last condition in our previous statement is crucial. If the market-to-book ratio is very low, one can see that crediting rate formula II.3.2 will result in a 0% crediting rate and therefore, the recovery time will be much longer. In addition, when we have strong withdrawals coinciding with a low market-to-book value, each withdrawal will worsen the market-to-book ratio hence in mathematical terms we have:  $\frac{M_t}{B_t} < 1 \Rightarrow \frac{M_t - \Delta}{B_t - \Delta} < \frac{M_t}{B_t}$ . Therefore, the riskiest scenarios from the insurance perspective are withdrawals occurring during low market-to-book ratio, because they will make the convergence of market-to-book to par longer, if not impossible. Such a scenario of low market-to-book ratio as such can occur only if the yields rises. This is the basis of the *inflation* and *mean reverting yield spike*.



### II.3.7 Path Dependence of Synthetic GIC Risk and the Suitability of the Liability Model

This section investigates the path-dependent nature of Synthetic GIC risk and our model’s aptitude for generating scenarios that replicate the riskiest paths.

In section II.3.6, we identified inflation and mean-reverting yield spikes as the primary risk scenarios. When these scenarios coincide with large withdrawals, they lead to insurer losses. However, the timing of withdrawals is crucial—they must follow the yield increase, but not too distantly.

Figure II.3.8 presents three variations of the mean-reverting scenario. The left graph shows a scenario with early withdrawals, and the right graph depicts late withdrawals. Neither of these scenarios results in losses. Conversely, the central scenario represents a substantial loss payment, demonstrating the significant impact of withdrawal timing on insurance loss.

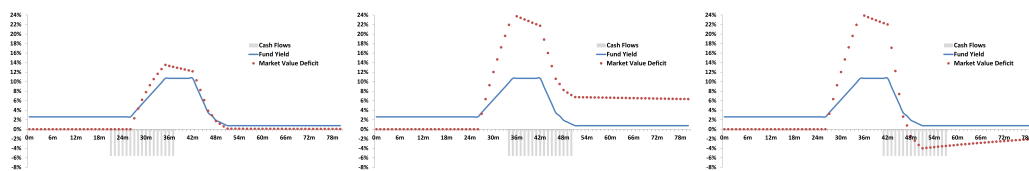


FIGURE II.3.8: Three variations of the mean reverting scenario. Data source: simulated backtest. Data source: simulated single scenario.

Indeed, not only is the timing of withdrawals crucial but so too are the periods without withdrawals. Figure II.3.9 compares the same mean reverting scenario with a deterministic model with constant withdrawal rates and one where the same withdrawal rates only occur during a yield spike. A severe loss of about 11% of the original notional results is evident when the withdrawal rate occurs within 24 months (left graph). Conversely, if a deterministic trend model were used, the loss would be just below 2% (right graph).

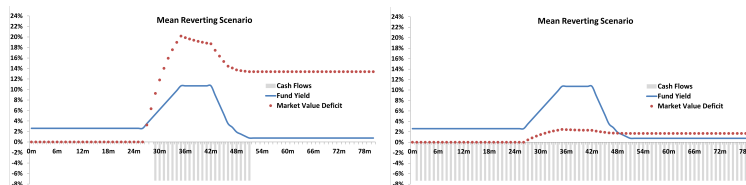


FIGURE II.3.9: A mean-reverting scenario example. If a deterministic model were used instead, the loss size would be less than 2% of the initial book value. Conversely, combining cash flow withdrawals with the stable value fund’s significant market value decrease results in an approximately 11% loss of the initial book value. Data source: simulated backtest.

Figure II.3.10 further emphasizes the impact of synchronizing the yield spike period with the withdrawal period by comparing the insurance loss size for various withdrawal starting dates. The same yield spike scenario forms the basis for this comparison, with annual withdrawal rates of 20%, 40%, and 60% over 48 months. The x-axis marks the withdrawal starting period relative to the yield spike initiation date. Negative values on the x-axis denote withdrawals starting before the yield increase, and positive values indicate withdrawals beginning after the yield increase. The most substantial insurance loss arises when the withdrawal kick-off aligns with the yield spike onset.

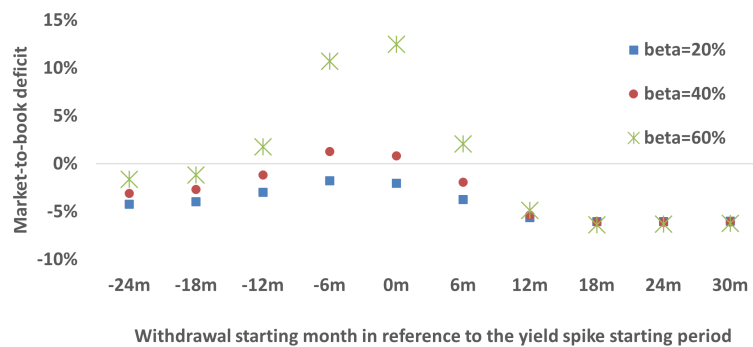


FIGURE II.3.10: Insurance loss size (Market-to-Book value deficit) as a function of the withdrawal starting date relative to the yield spike initiation date. Data source: simulated backtest.

The stochastic state trend component of our cash flow model provides flexibility in high withdrawal rate durations and starting periods, as well as non-withdrawal periods. Therefore, our model can generate the riskiest paths due to this stochastic trend component. However, if our model only incorporated a rate-deficit component, akin to the model in Kwun et al. (2010), the mean-reverting scenario depicted in Figure II.3.11 would generate no losses as withdrawals would not be correctly timed.

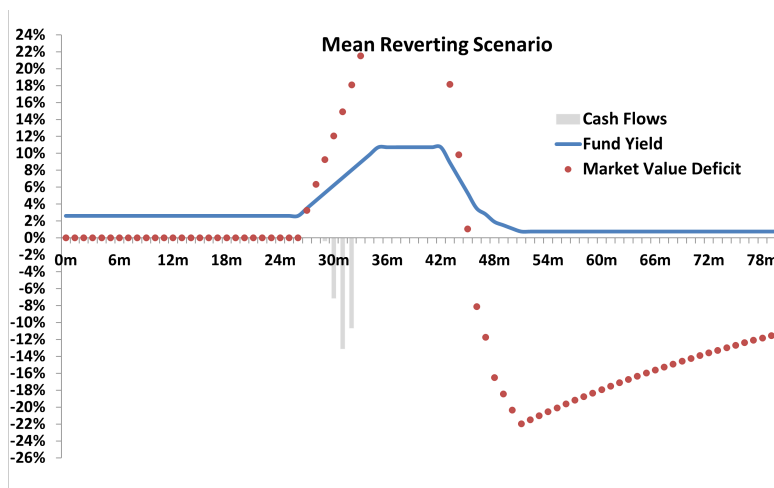


FIGURE II.3.11: Mean reverting scenario with a linear rate-deficit withdrawal model  $g(\cdot)$ . Data source: simulated backtest. Data source: simulated backtest.

We should note that while the examples given primarily focused on the *mean-reverting yield spike scenario*, we also tested the *inflationary scenario*. Our findings confirm that for both scenarios, the highest insurance loss occurs when periods of high withdrawals coincide with the yield spike or inflationary period.

In conclusion, our cash flow model, with its stochastic multi-state *beta* component, proves an adaptable tool for the valuation and risk management of insurance liabilities arising from stable value wrap contracts. The path-dependent nature of Synthetic GIC risk necessitates a model capable of generating a wide variety of loss scenarios with variable withdrawal timings and durations. Our model's flexibility, combined with its grounding in empirical participant behavior, allows it to generate nuanced risk scenarios critical for effective risk management of stable value funds.

## II.3.8 Conclusions

The paper describes a methodology for insurance companies to use within their ERM framework to better understand risk scenarios, assess tail risk, and determine capital requirements for stable value guarantees. Hence, our model can be used to estimate the capital requirements for the insurance company to maintain a certain low probability of insufficient capital for loss payments. In this research, we proposed and tested an asset and liability model for stable value fund wraps.

On the asset side, we suggested an innovative approach to modeling fixed income funds that offers a compelling alternative to traditional arbitrage-free interest rate models, which we argue are ill-suited to handle large portfolios of

fixed-income funds. Our model's core strength lies in its ability to capture the primary risk factor for fixed-income funds, the yield, despite its simplicity.

On the liability side, we introduced a multi-component model encapsulating a rate deficit, regime switching trends, herd behavior, and a flight-to-safety component. We argue this is the most suitable model for detecting risk scenarios from the insurer's perspective. Furthermore, our model is consistent with empirical observations that detect long-to-medium-term trends within historical data.

Through our research, we identified two critical scenarios: the inflationary scenario and the yield spike scenario. The importance of these scenarios lies in their ability to provide comparative notes on different risk sectors, aiding companies in assessing their aggregate risk exposures.

Economic capital requirements were a focus of our research. Our findings showed that the tail CTE, as a measure of economic capital, is starkly different from the NAIC's formulaic single scenario approach, which almost always results in zero capital. We noted the significance of understanding the differing statutory capital requirements under various regulations. While we did not explicitly calculate the Solvency II capital for a typical fund, our analysis suggests that it would likely be near zero for a 100% market-to-book fund due to less frequent loss occurrences. In contrast, the Swiss solvency test's 99% expected shortfall might demonstrate a non-zero capital charge.

The research highlights the vital influence of duration on tail risk. Our findings show an almost threefold increase in estimated loss frequency for a six-year fund guarantee compared to a four-year fund, with the loss size being nearly twice as large. We also discussed asset ratings' impact on tail risk and provided tail risk metrics for guarantees based on the different average credit quality of stable value funds' assets.

In conclusion, our research underscores the necessity for a model that can generate variable withdrawal scenarios in response to fluctuations in yield. We demonstrated that our multi-state *beta* component model outperforms deterministic trend models and rate-deficit function models in capturing the complex interplay between yield spikes, inflation, and participant withdrawal behavior. Our research contributes to a comprehensive understanding of Synthetic GIC risk management, with our model's flexibility accommodating additional factors specific to the insurance contract. This model serves as a vital step towards better industry practices in managing Synthetic GIC risks and a robust foundation for further research.

## II.3.A Appendices

### Benchmark Stable Value Guarantee Contract

Table II.3.8 shows the most important flavors of a typical stable value guaranteed contract.

Policy flavor	Value	Description
Initial market to book value	100%	
Fund duration	4 years	
Assets rating	AA	The average rating of the assets that make up the composition of the fund
Contract term	30 years	Generally, contracts are into perpetuity, but given that the Monte Carlo simulation cannot handle an infinity term, our analysis shows that - within the assumptions of our model - increasing the terms of the contract any further has an insignificant impact on the tail risk
Extended termination period	10 years	Also known as the wind-down period, this clause allows the insurer to impose stronger limits on the insured assets of the portfolio. We assume that the insurer will trigger such a clause at the 30 <sup>th</sup> year anniversary of the contract

TABLE II.3.8: Features of a typical guaranteed contract.

### Other Parameters Assumptions

Considering the non-stationarity of the correlation between interest rates and OAS spreads over varying historical periods and the overall tendency to be negative, see Duffee (1998), for the rest of this paper, we assume  $\rho = 0$  as a conservative and prudent approach. Also, using a log-likelihood method for the calibration of the interest rates and credit spreads on 3-month increments with a historical calibration window of 20 years for the credit spreads and 50 years for the interest rates, we obtain the following model parameters as of Table II.3.9.

	$\kappa$	$\sigma$	$\theta$
Interest rates	0.08	0.054	0.035
AA rated OAS spreads	0.16	0.050	0.011

TABLE II.3.9: CIR estimates, used in this paper, from the log-likelihood method.

Figure II.3.12 shows function  $g(\cdot)$  as an S-shape structure. The graph assumes that the rate deficit shall reach a critical level for participants to react. For example, the critical level in Figure II.3.12 is around  $\pm 4\%$ , with a maximum withdrawal rate of 20%. These numbers are just inputs and are not empirically verified. Figure II.3.12 also overlays the graph of historical data points from Alimoradian et al. (2023) with the parametric model presented by the authors in its top-left corner. This is to emphasize that the function  $g(\cdot)$  is consistent with the historical observations noted by Alimoradian et al. (2023).

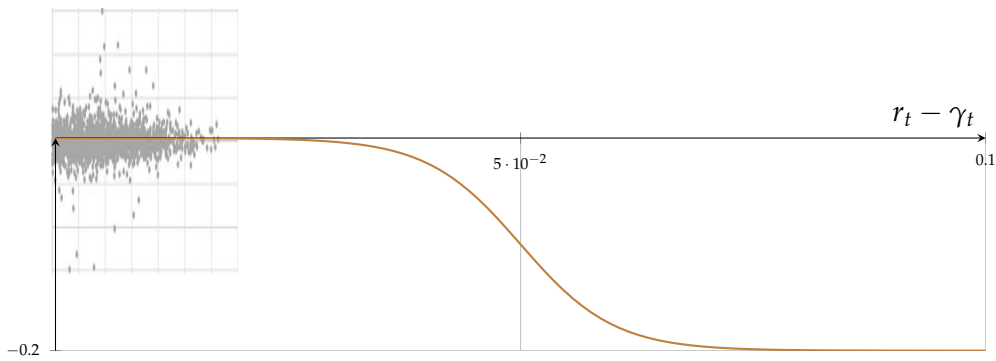


FIGURE II.3.12: Cash flow rate as a function of the crediting rate deficit. The empirical data points have been superposed to the theoretical function. Historical data source: Valerian proprietary data.

The parametrization assumption for the rate deficit component ( $g(\gamma_t - r_t)$ ) is as below :

$$g(x) = 0.1 \times \tanh(-100x + 5) + 0.1 \times \tanh(-100x - 5) \tag{II.3.10}$$

Therefore, the flight-to-safety component is represented by the following function:

$$f(s) = 0.2 \times \mathbb{1}_{s \geq 0.03}. \tag{II.3.11}$$

State	Respective trend	Probability of occurrence	Average duration
Stable state	0	$0.93 - 0.0003 \times \mathbb{1}_{\frac{M_t}{B_t} < 1.0}$	9 years
Decline state	-1/6	0.05	5 years
Growth state	+1/10	0.05	5 years
Herd state	-90%	$0.0003 \times \mathbb{1}_{\frac{M_t}{B_t} < 1.0}$	0.25 years

TABLE II.3.10: Parameters input assumption for the trend model

## II.4 Stable Value Wrap: A Risk Perspective for a Post Pandemic Market

*This chapter has been extracted from an industry letter co-authored with Jeffrey Jakubiak and Stephane Loisel and shared among various stakeholders within the Synthetic GIC market.*

### Abstract

Due to competitive pressures in the stable value marketplace, investment managers of stable value funds have gradually been increasing exposure to lower credit quality and higher duration fixed income assets. While these changes have been incremental since the financial crisis, recent trends now allow some stable value funds to invest in high-yield bonds and structured finance securities that historically have had higher price volatility than stock markets. Many insurance companies write stable value wraps, i.e., Synthetic GICs. They could be exposed to potential reserve strain or losses in historically adverse conditions due to these changes in investment strategy.

### II.4.1 Managers Chasing Yields Amid Post-Pandemic Inflation Expectations

Since the COVID-19 pandemic, the U.S. economy has experienced a new paradigm with high inflation prospects and a low-interest rate environment, potentially a consequence of quantitative easing and capital injection into the financial market. Consequently, the U.S. fixed-income market has experienced challenges with a return on investment behind inflation. Figure II.4.1 shows the yield of the U.S. aggregate fixed income index adjusted by the CPI urban inflation. The index is the lowest ever since the 1990s.

This new paradigm poses a challenge for investment managers who aim to achieve higher investment returns to outpace inflation. In response, they may adopt more aggressive strategies by investing in lower-credit securities. As a



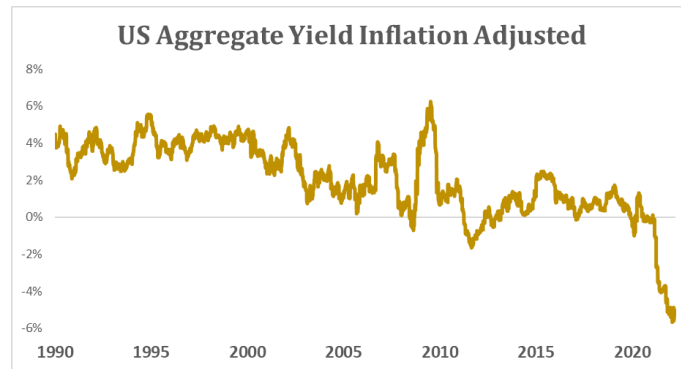


FIGURE II.4.1: Inflation-adjusted US Aggregate fixed income index, the level of investment yield has been negative since the pandemic. Data source: Barclays fixed income indices.

result, there is increased pressure on stable value wrappers to insure riskier funds. It is important to note that stable value guarantees are sensitive to the credit quality and duration of the underlying assets. If wrappers make concessions in ensuring these riskier funds, there is a potential for significant insurance losses in the event of an adverse scenario.

## II.4.2 Behavioral Bias in Assessing the Risk of Stable Value Guarantees

### II.4.2.1 Tendency to Underestimate Remote Risks

Behavioral studies [Slovic et al. \(1978\)](#) show that when an event's risk is very remote, the cognitive bias could act as if the risk is nonexistent. Our informal industry survey concluded that some stable value practitioners share the same cognitive bias; they consider the chance zero. Another significant problem with such judgment is that they lose the ability to distinguish between low-risk and high-risk funds, as they all are perceived with zero risk. This is especially problematic since investment managers have recently requested higher-risk securities.

The presence of this behavioral bias underscores the significance of employing quantitative methods for risk assessment. Without such methods, subjectivity can dominate the process, leading some practitioners to perceive the risk as zero while others may overestimate it as overly conservative.

### II.4.2.2 Tendency to View the World Linearly

Figure [II.4.2](#) shows two 12-inch pizzas v.s. one 16-inch pizza; the proportional dimension illustrated in Figure [II.4.2](#) is accurate, and the reader may ask which of the two options is the best to choose from. The cognitive bias is that the

pizzas are seen linearly from their horizontal axis, so two 12-inch pizza looks bigger. However, using the  $\pi r^2$  the formula for the 16-inch pizza is bigger.



FIGURE II.4.2: The larger pizza has a lower sphere than 2× the smaller pizza, but the cognitive bias to see the world linearly makes most people choose 2× the smaller pizza.

Another manifestation of this cognitive bias is the frequent assumption that introducing 10% lower quality assets into a fund will merely double the wrap risk profile compared to the inclusion of 5% lower quality assets. However, this perception is biased, as wrap risk does not exhibit a linear but a non-linear, convex relationship to the credit quality of the assets wrapped.

### II.4.3 Quantifying the Wrap Tail Risk

Here, we compared two funds: high-credit quality asset funds, representative of what is commonly wrapped by insurers, and one lower credit quality asset, representing investment managers seeking higher yields. Figure II.4.3 shows the summary asset allocation for the two; the lower credit quality fund has a higher concentration in securities that experienced a higher historical drawdown, especially during the 2008 financial crisis.

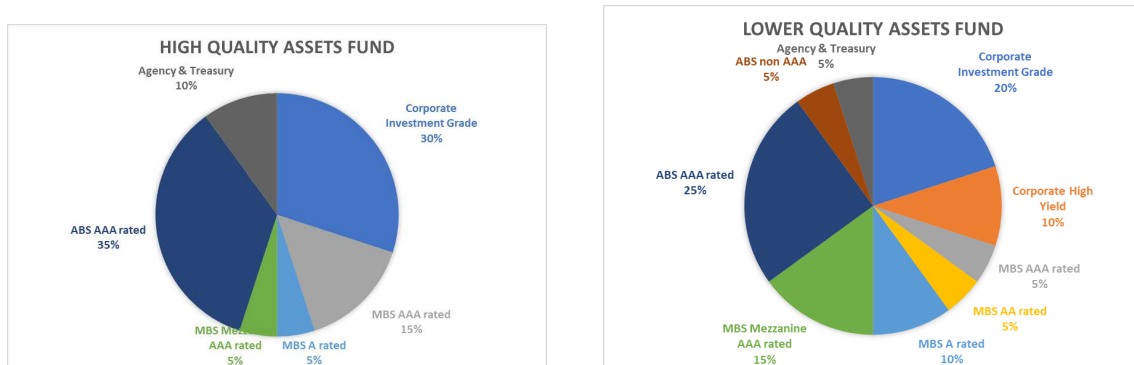


FIGURE II.4.3: Asset Allocation for high-quality vs. lower-quality assets. Data source: hypothetical example for illustration purposes only.

Figure II.4.4 shows a backtest of the *market-to-book* value ratio of the two funds. It shows that the higher quality fund had experienced a 3-month drawdown of 12% during the financial crisis, while the lower quality fund lost 25% of its value. Considering this potential drawdown, the net present value of the insurer’s losses is estimated, and the impact is shown in Table II.4.1. As can

be deciphered from this Table, both the loss frequency and severity increase significantly for the lower credit quality fund.

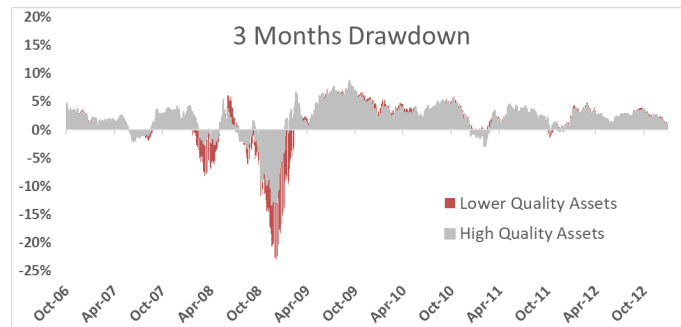


FIGURE II.4.4: 3 months drawdown. Data source: historical backtest against Valerian Capital Group proprietary data.

TABLE II.4.1: Summary of risk results after the 3-month drawdown similar to the financial crisis for a 100% Market-to-Book transaction calculated based on the asset-liability model.

	High-quality assets	Low-quality assets
Market to Book after the 3-month drawdown	88%	75%
Loss frequency	5.4%	17.8%
Loss severity in avg	2.5%	5.9%

### II.4.4 Conclusions

While the risk of insurance loss on stable value wraps remains low, its relationship with market factors like duration and credit quality of assets is highly non-linear and convex. Historical backtesting suggests that riskier stable value funds could face a 25% drop in market values in a crisis similar to the 2008 financial downturn. This letter, cognizant of our inherent human bias to downplay deep tail risks, probes deeper into potential hazards associated with introducing riskier assets into the \$400 billion stable value wraps market.

## **Part III**

# **Derivatives under Market Impact**

# III.1 Derivatives under Market Impact: Disentangling Cost and Information

*This chapter revisits the article co-authored with Karim Barigou and Anne Eyraud-Loisel.*

## Abstract

In this paper, we examine the implications of a large trader's activity and its market impact on the option pricing theory. This framework builds upon considering the perspective of an insider trader, who is aware of the large trader's trading policy but does not bear any transaction costs. Through this framework, we establish the existence of *information-neutral probability measures*, under which the discounted asset is a martingale process for the insider trader. We then develop a derivatives hedging policy for the large trader that takes into consideration both the transaction costs and the permanent impact of its own hedging activities while mitigating market manipulation. The paper concludes with numerical results that showcase the optimal delta-hedging strategy for an out-of-the-money call option and compare it to the Black-Scholes model.

**Keywords:** Option Pricing, Market Impact, Illiquid Markets, Transaction Costs, Stochastic Optimal Control.

## III.1.1 Introduction

The traditional option pricing framework, as outlined in the seminal paper by [Black and Scholes \(1973\)](#), assumes a frictionless and liquid financial market. However, this framework may not hold when dealing with large notional derivatives on illiquid assets that are subject to market impact. When a significant hedge is executed, it can consume the market's liquidity, which in turn can have an impact on the underlying asset's spot price. Therefore, a new

mathematical framework is needed to effectively hedge derivatives in these illiquid markets.

Market impact is a key topic in the literature, especially in the context of the optimal liquidation problem, also known as Delta-1 strategies. The literature typically describes two types of market impact: temporary impact and permanent impact, see [Almgren and Chriss \(1997\)](#).<sup>1</sup> The temporary impact represents the slippage or transaction costs and refers to the short-term deviation from the price due to the costs incurred in executing the trade. The permanent impact, however, refers to the lasting effect on the price of the underlying asset resulting from trade.

The prominent framework of [Almgren and Chriss \(2001\)](#) considers a deterministic permanent market impact function to cumulative share amount, and later [Gatheral \(2010\)](#) proves that one sufficient condition for no-arbitrage is the linearity of the permanent market impact model. However, later works such as [Guéant \(2013\)](#) suggested other plausible forms of permanent impact, which are non-linear. Our permanent impact model, in this paper, is based on the assumption that the permanent market impact is stochastic and varies over time. Stochastic liquidity has garnered interest in recent literature, with numerous papers exploring modeling stochastic transient impact ([Klöß, 2012](#); [Almgren, 2012](#); [Ma et al., 2020](#)) and stochastic permanent impact ([Barger and Lorig, 2019](#)). Our model assumes a stochastic permanent impact, exhibiting a positive but non-isomorphic relationship with accumulated shares. As a result, the permanent price may differ across scenario paths based on share execution time and market depth on a given day, even if the same number of cumulated shares are purchased at time  $t$ .

In this paper, we also present a novel interpretation of the permanent impact as the knowledge about the trading activities of one or multiple traders leading to a prospective liquidity imbalance. Any entity with access to this information will have a different perception of the spot price and will apply a correction to it. This correction, we argue, is what constitutes the permanent impact. Once the trade is executed, this information becomes public, and the permanent impact becomes a reality, contributing to the explanation of asset price variation in the market. To frame this approach within a quantitative model, we introduce a hypothetical "insider trader" who is aware of the permanent impact but does not bear any transaction costs.

In this paper, our primary aim is to explore how a large trader, aware of its market activities, can hedge significant derivatives positions with consideration of market manipulation. While our objective differs, we acknowledge foundational works on the advantages of insider traders. We have been influenced by

---

<sup>1</sup>More recently, the transient impact has also been introduced as a decaying function over time.

concepts from research such as [Hillairet \(2004\)](#), which examined information levels and martingale pricing theories. The work by [Gorud and Pontier \(1998\)](#) delved into how insider information can be integrated into self-financing portfolios. [Amendinger et al. \(1998\)](#) investigated how such information modifies the utility function of a trading portfolio to benefit the insider. Further, both [Eyraud-Loisel \(2005, 2013\)](#) provided insights into filtration enlargement due to private information. Lastly, the studies conducted by [Cetin et al. \(2006\)](#) and [Cetin et al. \(2010\)](#) analyzed the continuous market impact by focusing on the asset's supply curve. While we borrow concepts from and leverage upon these studies, our research focuses on a similar question in the domain but with different practicality. As such, our aim in this paper is not to assess the impact of the large trader activity on the uninformed, and therefore, we do not develop a theory for the rest of the market.

In this paper, we present a novel modeling approach for permanent market impact utilizing a jump model. The selection of this modeling approach is theoretically robust, ensuring an arbitrage-free roundtrip trading framework. However, it is important to note that the main contribution of this study is not the specific market impact model employed, as similar insights can be derived using alternative models. The primary contribution of our research lies in the methodology employed to separate information from other factors impacting price, as well as the conceptualization of the insider trader. This conceptualization leads to valuable mathematical insights that enhance our understanding of the role of information in option pricing theory.

In the field of derivatives, hedging policies have received significant attention, with two main trends emerging. On the one hand, the full hedge approach, such as the seminal paper of [Liu and Yong \(2005\)](#), seeks to fully offset the risk of a derivative instrument. Their approach has been further developed and expanded upon in recent papers by [Loeper et al. \(2018\)](#), [Said \(2019\)](#), [Bordag and Frey \(2008\)](#), and [Abergel and Loeper \(2017\)](#). On the other hand, partial hedging, as in [Jonsson and Sircar \(2002\)](#), aims to balance the cost of hedging against the benefit of retaining some exposure to the underlying asset. Full hedge strategies can result in high option prices in illiquid markets due to the cost of adjusting the delta as the option approaches expiration. This is particularly relevant in markets with limited liquidity. This paper focuses on partial hedging, a common approach of which is optimizing a gain function for the large trader, as seen in [Guéant and Pu \(2017\)](#). However, this approach also presents the risk of market manipulation, as the large trader's utility function may drive them to act in their self-interest. To avoid market manipulation, we conclude that the stochastic optimization problem shall be reduced to a transaction cost optimization problem. On that note, this paper highlights the model risk of market abuse in derivatives. It offers a discussion on how to

address it, making it one of the first to introduce the issue in the context of partial hedging in derivatives.

The ecosystem presented in this paper is designed for a scenario with a single large trader, which is a common situation in illiquid markets<sup>2</sup>. In fact, this research project originated from a real-life practical experience where a trader consulted one of the authors of this paper due to concerns about the inability of existing models to hedge and price a new large derivative transaction unknown to the market. The trader observed that the current state-of-the-art derivatives models are usually calibrated based on the implied volatility surface. However, when the market lacks knowledge about new transactions, the implied volatility surface can become biased and require correction. Furthermore, some of the results obtained in this paper can be extended to the case of multiple large traders as long as one considers the aggregated effect of the large traders and conceptualizes them as one group. This is particularly useful since most large traders, as discussed in (Cont and Schaanning, 2017), adopt the same directional strategies during crisis periods.

This paper is organized into seven sections. Section III.1.2 provides an overview of the market ecosystem, including the presence of one large and one insider trader. Section III.1.3 focuses on the stock dynamics and how they are affected by market impact. In Section III.1.4, the pricing equation for the insider trader is derived, and the concept of information-neutral probabilities, which determine whether the asset is a martingale, is explained. In Section III.1.5 we study the trading portfolio dynamics of the large trader. We also define the concept of a round trip and propose conditions for which a round trip is arbitrage-free within the stochastic market model in this paper. Section III.1.6 deals with the derivation of the optimal hedging strategy for the large trader and the important issue of market manipulation in the context of the stochastic control problem. Section III.1.7 presents a numerical solution to the problem through a finite difference method and compares the strategies of both the large and insider traders. Overall, the paper provides a comprehensive analysis of the market ecosystem, stock dynamics, and optimal hedging strategies for both large and insider traders, with a special emphasis on the issue of market manipulation. All proofs can be found in the Appendix.

### III.1.2 Market Ecosystem Description

We examine a market that is composed of many small active traders, but limited liquidity. We assume that a large trader enters this market with a trading strategy  $\{\chi_t\}_{t \geq 0}$ , where  $\chi_t$  is the (annualized) speed of the shares

---

<sup>2</sup>The reader is referred to the introduction section of Almgren and Li (2016) for a lucid description that such case is common.



execution program by the large trader at time  $t$ . To conceal its order size, the large trader keeps its trading strategy hidden from the market. Therefore, the market will not know about  $\chi_t$  until it is executed.<sup>3</sup> However, there is one insider trader who is privy to the large trader's trading strategy. This market thus has three types of traders:

- The *large trader* who has market impact and is cognizant of the effect of its own actions on the market.
- The *insider trader* who possesses the same knowledge as the large trader but lacks any market impact.
- The *average trader* who is too small to influence the market and is unaware of the large trader's activities. This group of traders is not the focus of this paper.

It is worth noting that, while here we considered only one large trader, some of the results obtained in this paper can be generalized in the case of multiple large traders as long as the insider trader has knowledge of the aggregated effect of the large traders' activities. It is also important to note that while the findings of this paper can be extended to any trading strategy, later in this paper, we assume that the large trader's objective is to hedge derivative exposure.

### III.1.3 The Market Impact Model

**Definition 14.** We consider a finite, deterministic time horizon  $T \in \mathbb{R}^+$ . Let  $(\Omega, \mathcal{F}, \mathbb{P})$  be a complete probability space. Let  $\{W_t\}_{0 \leq t \leq T}$  be a standard Brownian motion and  $\{N_t\}_{0 \leq t \leq T}$  be a homogeneous Poisson process with intensity  $\phi > 0$ . We assume that the processes  $\{W_t\}_{0 \leq t \leq T}$  and  $\{N_t\}_{0 \leq t \leq T}$  are independent. The probability measure  $\mathbb{P}$  is traditionally called a real-world probability measure. Let  $\mathbb{F} = \{\mathcal{F}_t\}_{t \in [0, T]}$  be the filtration jointly generated by the processes  $\{W_t\}_{0 \leq t \leq T}$  and  $\{N_t\}_{0 \leq t \leq T}$ .

While the assumption of a homogeneous Poisson process with a constant intensity  $\phi$  simplifies the model for tractability, it is a restrictive assumption. In more complex market environments, one could consider a Cox process with an intensity process that is  $\mathbb{F}$ -adapted and thus dependent on market factors. However, for the sake of simplicity and tractability, this thesis considers the simpler case of a homogeneous Poisson process.

**Definition 15.** The  $\mathbb{F}$ -adapted underlying spot price process  $\{S_t\}_{0 \leq t \leq T}$ , the  $\mathbb{F}$ -adapted process  $\{\chi_t\}_{0 \leq t \leq T}$  representing the evolution of the annualized rate of shares

<sup>3</sup>While this goes beyond the scope of this paper, there are many algorithmic methods to hide the order size; the most simple one is a price taker. Another one may be iceberg type limit orders. See [Christensen and Woodmansey \(2013\)](#) for more details.

executed by the large trader over time, and the  $\mathbb{F}$ -adapted process  $\{V_t\}_{0 \leq t \leq T}$  representing the evolution of the number of shares held by the large trader over time are jointly defined as

$$\begin{cases} S_0 > 0 \text{ and } V_0 \geq 0, \\ \frac{dS_t}{S_{t^-}} = \mu_t dt + \sigma dW_t + \Lambda(\chi_t) dN_t, \\ dV_t = \chi_t dt, \\ \chi_t = \tilde{\chi}(t, S_t, V_t), \end{cases} \quad (\text{III.1.1})$$

where  $\tilde{\chi} : \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}$  is a piecewise continuous and bounded function representing the speed of execution of the large trader, and  $\Lambda : \mathbb{R} \rightarrow \mathbb{R}$  is a non-decreasing, square-integrable function describing the permanent market impact of the large trader, with  $\Lambda(0) = 0$ . We also assume that  $t \rightarrow \mu_t$  is a deterministic function taking values in  $\mathbb{R}$  (representing the drift of the underlying asset) and that the volatility  $\sigma \in \mathbb{R}^+$  is deterministic and constant.

In Equation III.1.1, the size of the jump is denoted by  $\Lambda(\chi_t)$ , as opposed to  $\Lambda(\chi_{t^-})$ . The notation  $t^-$  is typically used to ensure the validity of the closed-form exponential representation of the process  $\{S_t\}_{0 \leq t \leq T}$ , which is a product of the impact of all jumps and the exponential Gaussian process, as detailed in Doléans-Dade (1970). In simpler terms, for the exponential formulation to hold, jumps need to be applied first to the spot price early at  $t^-$ , followed by other sources of variation. However, in our model, an exponential expansion is not employed. From a financial perspective, this implies that while the jump increments are applied to the price at  $S^{t^-}$  before other variations, its actual magnitude is determined at time  $t$ , not at  $t^-$ .

The large trader executes shares at the speed of  $\chi_t$  shares at any given time  $t$ . We assume that within a small time interval  $[t, t + dt)$ , there is either enough market depth or not. If there is enough market depth, there will be no permanent market impact. However, if there is a lack of depth, there will be a permanent market impact of size  $\Lambda(\chi_t)$ . At each of the Poisson jumps, the impact on the asset price is permanent and a function of the instantaneous trading rate. One could, of course, argue that price impact may be present continuously and not only at random times. Likewise, permanent price impact may depend on the cumulative amount of shares traded, while immediate impact may depend on the trading rate, in contrast to the assumption in our paper. The goal of this paper is to have a first look at models where market depth materializes only at random instants, following the impression of practitioners whose practical challenges motivated this initial work. Combining our (unconventional) form of market impact with more classical ones and analyzing similar problems in a generalized model would be interesting but is

left for further research.<sup>4</sup>

Note that the market impact could be positive or negative depending on whether the shares are bought or sold. In the case of no trading activity by the large trader, Equation (III.1.1) reduces to the standard Black-Scholes dynamics. Note that Equation (III.1.1) postulates that the permanent price differs for each scenario path depending on when the shares were executed and the depth of the market on that day.<sup>5</sup>

Note that while the results of this paper are extendable to the case of non-constant volatility, i.e., local volatility, for the sake of simplicity and for the sake of focusing our discussion on the market impact component, we consider the volatility  $\sigma$  to be constant within this paper.

### III.1.4 Option Hedges for the Insider Trader

The insider trader has access to the information about the trading policy  $\{V_t, \chi_t\}_{0 \leq t \leq T}$  of the large trader over time. Let  $\{r_t\}_{0 \leq t \leq T}$  be a deterministic interest rate process. We also assume that the holder of the stock benefits from a dividend yield of  $q \geq 0$ , as well as the presence of a securities lending market and a future market for the underlying stock. The holder of the stock bears a cost of carry  $b$ , which is the adjustment rate between the future market and the current value of the stock. These markets, although subject to liquidity issues, enable the holder to enter into short selling positions.<sup>6</sup>

**Theorem III.1.4.1.** *We assume the following regularity condition:*

$$\Lambda(\chi_t)(\mu_t + (q - b) - r_t) < \sigma^2 \quad (\text{III.1.2})$$

holds  $d\mathbb{P} \times dt$ -a.s.,  $0 \leq t \leq T$ . Then, the following holds:

1. *There exists a set of probability measures  $\mathcal{I}$  where for any measure  $I \in \mathcal{I}$  the returns of the discounted asset*

$$S_t = \mathbb{E}^I \left[ e^{-\int_t^T r_s ds} e^{(q-b)(T-t)} S_T \middle| \mathcal{F}_t \right] \quad (\text{III.1.3})$$

<sup>4</sup>It is worth mentioning that for the sake of simplicity, we assume that the impact rate of the order ( $\phi$ ) is independent of its size ( $\chi_t$ ). The relationship between  $\phi$  and  $\chi_t$  has received less attention in the empirical literature because most academic literature so far investigates the average price impact rather than its variations. We make the point that while our model can be generalized by integrating this relationship, our intention is not to define a very comprehensive model but to study the role of information in an exploratory model inspired by the market practice and discussions with traders.

<sup>5</sup>The stochastic permanent impact has been a recent trend in the literature; see, for example, [Barger and Lorig \(2019\)](#).

<sup>6</sup>It is important to note that our model does not take into account the variability of the borrowing spread in illiquid markets. This spread can exhibit significant fluctuations, especially during market events such as a short squeeze.

are martingales under the measure  $I$ .

2. An option  $\mathcal{O}$  bought/sold by the insider trader

$$\mathcal{O}_t = \mathbb{E}^I \left[ e^{-\int_t^T r_s ds} \mathcal{O}_T \mid \mathcal{F}_t \right]$$

is also a martingale under probability  $I \in \mathcal{I}$ .

3. We call the set  $\mathcal{I}$ , the set of information-neutral probabilities. Then, among all the information-neutral measures, there exists a probability measure  $\mathcal{I}^*$  that minimizes the quadratic variation of the difference between the hedging portfolio and option value. We call  $\mathcal{I}^*$  the minimal variation information-neutral probability measure.
4. The discounted return of the asset is a martingale under measure  $\mathcal{I}^*$ .

Moreover, the dynamic of the spot under  $\mathcal{I}^*$  is described by

$$\frac{dS_t}{S_{t-}} = (r_t - (q - b) - \Lambda(\chi_t)\omega_t) dt + \sigma dW_t^{\mathcal{I}^*} + \Lambda(\chi_t) dM_t, \quad (\text{III.1.4})$$

where  $\{M_t\}_{0 \leq t \leq T}$  is a doubly stochastic Poisson process whose stochastic intensity process  $\{\omega_t\}_{0 \leq t \leq T}$  verifies that for each  $0 \leq t \leq T$ ,

$$\omega_t = \phi \left( \frac{\sigma^2 - \Lambda(\chi_t) (\mu_t + (q - b) - r_t)}{\sigma^2 + \phi \Lambda(\chi_t)^2} \right).$$

5. Furthermore, if the insider trader has entered into a transaction modeled by a payoff  $\mathcal{O}(S_T)$ , we call  $\mathcal{O}(t, S, V)$ , the price of this option at time  $t$  and spot level  $S$ . Then, for all  $0 \leq t \leq T$ , the option price  $\mathcal{O}$  can be computed via the following partial differential equation:

$$\begin{aligned} r_t \mathcal{O} = & \frac{\partial \mathcal{O}}{\partial t} + (r_t - (q - b) - \Lambda(\chi_t)\omega_t) S \frac{\partial \mathcal{O}}{\partial S} + \frac{\partial \mathcal{O}}{\partial V} \chi_t dt \\ & + \frac{1}{2} \frac{\partial^2 \mathcal{O}}{\partial S^2} \sigma^2 S^2 + (\mathcal{O}(t, S + S\Lambda(\chi_t), V_t) - \mathcal{O}(t, S, V)) \omega_t. \end{aligned} \quad (\text{III.1.5})$$

*Proof.* By Itô formula applied to a process with jumps, we can approximate the option price dynamics as:

$$\begin{aligned} d\mathcal{O}_t = & \frac{\partial \mathcal{O}}{\partial t} dt + \frac{\partial \mathcal{O}}{\partial S} (\mu_t dt + \sigma dW_t) S_{t-} + \frac{1}{2} \frac{\partial^2 \mathcal{O}}{\partial S^2} \sigma^2 S_{t-}^2 dt \\ & + \frac{\partial \mathcal{O}}{\partial V} \chi_t dt + [\mathcal{O}(t, S_{t-} + S_{t-}\Lambda(\chi_t), V_t) - \mathcal{O}(t, S_{t-}, V_t)] dN_t, \end{aligned}$$

where  $\mathcal{O}(t, S_{t-} + S_{t-}\Lambda(\chi_t), V_t) - \mathcal{O}(t, S_{t-}, V_t)$  is the jump in the price of the option due to large traders' activity  $\chi_t$ , and  $S_t(\mu_t dt + \sigma dW_t)$  is the continuous part of the spot movement.

On the other hand, the insider trader's option hedging portfolio at time  $t$  is given by:

$$P_t = \delta_t S_t + \overbrace{(P_t - \delta_t S_t)}^{\text{Cash Account}},$$

where the insider trader has  $\delta_t$  in shares and  $P_t - \delta_t S_t$  in cash amount. Due to the fact that the notional is small, the insider trader - contrary to the large trader - can hedge its position without additional transaction costs. The dynamics of the portfolio  $P_t$  are as follows:

$$dP_t = \delta_t (dS_t + S_t^- q dt - S_t^- b dt - S_t^- r_t dt) + r_t P_t dt.$$

In our incomplete market, the option cannot be perfectly hedged by a self-financing portfolio. Therefore, we search for a hedging strategy that satisfies the following properties:

- $\delta_t = \arg \inf_{\delta} d \langle (P - \mathcal{O}) \rangle_t$ : The second order option price movement is reduced and optimally minimized.

The stochastic part of  $\mathcal{O}_t - P_t$  can be formulated as:

$$\begin{aligned} d(\mathcal{O}_t - P_t) &= \dots \times dt + \left( \frac{\partial \mathcal{O}}{\partial S} - \delta_t \right) S_t^- \sigma dW_t \\ &+ [\mathcal{O}(t, S_t^- + S_t^- \Lambda(\chi_t), V_t) - \mathcal{O}(t, S_t^-, V_t) - \delta_t S_t^- \Lambda(\chi_t)] dN_t. \end{aligned}$$

Therefore, quadratic variation of  $\mathcal{O}_t - P_t$  reads as follows:

$$\begin{aligned} d \langle (P_t - \mathcal{O}_t) \rangle &= \left( \frac{\partial \mathcal{O}}{\partial S} - \delta_t \right)^2 S_t^{-2} \sigma^2 dt \\ &+ \left[ \delta_t - \left[ \frac{\mathcal{O}(t, S_t^- + S_t^- \Lambda(\chi_t)) - \mathcal{O}(t, S_t^-)}{S_t^- \Lambda(\chi_t)} \right] \right]^2 \phi S_t^{-2} \Lambda(\chi_t)^2 dt. \end{aligned}$$

If the market is complete, it is possible to find  $\delta_t$  such that the quadratic variation  $d \langle (P - \mathcal{O}) \rangle_t$  is always zero. This would have been the case for the Black-Scholes model without any market impact. However, it is impossible due to the jump component of the market impact. One can only find a strategy that minimizes the quadratic variation, but it will not neutralize it. The insider trader minimizing the quadratic variation will obtain the following formula:

$$\delta_t S_t^- = \frac{\sigma^2}{\sigma^2 + \phi \Lambda(\chi_t)^2} \frac{\partial \mathcal{O}}{\partial S} S_t^- + \frac{\phi \Lambda(\chi_t)}{\sigma^2 + \phi \Lambda(\chi_t)^2} (\mathcal{O}(t, S_t^- + S_t^- \Lambda(\chi_t), V_t) - \mathcal{O}(t, S_t^-, V_t)).$$

- $\mathbb{E}^{\mathbb{P}} [dP_t | \mathcal{F}_t] = \mathbb{E}^{\mathbb{P}} [d\mathcal{O}_t | \mathcal{F}_t]$ : The average ( first order) option price movement is fully hedged by the portfolio.

We can derive the following partial differential equation:

$$\frac{\mathbb{E}^{\mathbb{P}} [dP_t | \mathcal{F}_t]}{dt} = r_t P_t + (\mu_t + q - b - r_t + \Lambda(\chi_t)\phi) \delta_t S_{t-}.$$

On the other hand, we have

$$\begin{aligned} \frac{\mathbb{E}^{\mathbb{P}} [d\mathcal{O}_t | \mathcal{F}_t]}{dt} &= \frac{\partial \mathcal{O}}{\partial t} + \frac{\partial \mathcal{O}}{\partial S} S_{t-} \mu_t + \frac{1}{2} \frac{\partial^2 \mathcal{O}}{\partial S^2} \sigma^2 S_{t-}^2 + \frac{\partial \mathcal{O}}{\partial V} \chi_t dt \\ &+ [\mathcal{O}(t, S_{t-} + S_{t-} \Lambda(\chi_t), V_t) - \mathcal{O}(t, S_{t-}, V_t)] \phi. \end{aligned}$$

- $P_t = \mathcal{O}_t$ : As this is not a self-financing portfolio, at each time step, there will be cash injection/depletion so that the hedging portfolio value is equal to the theoretical value of the option.

We conclude that:

$$\begin{aligned} r_t P_t &= \frac{\partial \mathcal{O}}{\partial t} + \left( \mu_t - \frac{\sigma^2 (\mu_t + q - b - r_t + \Lambda(\chi_t)\phi)}{\sigma^2 + \phi \Lambda(\chi_t)^2} \right) S_{t-} \frac{\partial \mathcal{O}}{\partial S} + \frac{1}{2} \frac{\partial^2 \mathcal{O}}{\partial S^2} \sigma^2 S_{t-}^2 + \frac{\partial \mathcal{O}}{\partial V} \chi_t dt \\ &+ \phi \left( 1 - \frac{\Lambda(\chi_t)}{\sigma^2 + \phi \Lambda(\chi_t)^2} (\mu_t + q - b - r_t + \Lambda(\chi_t)\phi) \right) (\mathcal{O}(t, S_{t-} + S_{t-} \Lambda(\chi_t), V_t) - \mathcal{O}(t, S_{t-}, V_t)). \end{aligned} \quad (\text{III.1.6})$$

We simplify Equation III.1.6 by defining  $\omega_t$  as

$$\omega_t = \phi \frac{\sigma^2 - \Lambda(\chi_t) (\mu_t + q - b - r_t)}{\sigma^2 + \phi \Lambda(\chi_t)^2}. \quad (\text{III.1.7})$$

We also assume the regularity condition of III.1.2 holds; this condition is necessary as otherwise  $\omega_t$  will not be positive almost surely, and the following extension Feynman-Kac could not be applied as the intensity of the Poisson process will not be positive. One can easily verify that based on the definition of  $\omega_t$  and based on the assumption that we have made on the value of the hedging portfolio being cash injected/depleted to satisfy  $P_t = \mathcal{O}_t$ , we obtain:

$$\begin{aligned} r_t \mathcal{O} &= \frac{\partial \mathcal{O}}{\partial t} + (r_t - q + b - \Lambda(\chi_t)\omega_t) S_{t-} \frac{\partial \mathcal{O}}{\partial S} + \frac{\partial \mathcal{O}}{\partial V} \chi_t dt + \frac{1}{2} \frac{\partial^2 \mathcal{O}}{\partial S^2} \sigma^2 S_{t-}^2 \\ &+ \omega_t (\mathcal{O}(t, S_{t-} + S_{t-} \Lambda(\chi_t), V_t) - \mathcal{O}(t, S_{t-}, V_t)). \end{aligned} \quad (\text{III.1.8})$$

Having the partial differential equation above, the most common technique would be to use the Feynman-Kac formula. However, the stochastic process derived is not an exponential Levy, so one has to be careful in using the Feynman-Kac technique for non-exponential-Levy processes. Using the Feynman-Kac formula, the equivalent stochastic equation for the spot price  $S_t$  under a probability measure that we call minimal-variance information neutral

probability and note it as  $\mathcal{I}^*$  :

$$\frac{dS_t}{S_{t-}} = (r_t + b - q - \chi_t \Lambda(\chi_t) \omega_t) dt + \sigma dW_t + \Lambda(\chi_t) dM_t, \quad (\text{III.1.9})$$

where  $\{M_t\}_{0 \leq t \leq T}$  is a doubly stochastic Poisson process whose stochastic intensity process  $\{\omega_t\}_{0 \leq t \leq T}$ . Also, the following equations hold under the  $\mathcal{I}^*$  probability:

$$\begin{aligned} S_t &= \mathbb{E}^{\mathcal{I}^*} \left[ e^{-\int_t^T r_s ds} e^{(q-b)(T-t)} S_T \middle| \mathcal{F}_t \right] \\ \mathcal{O}(t, S_t, V_t) &= \mathbb{E}^{\mathcal{I}^*} \left[ e^{-\int_t^T r_s ds} \mathcal{O}(T, S_T, V_T) \middle| \mathcal{F}_t \right]. \end{aligned} \quad (\text{III.1.10})$$

To verify that our usage of the Feynman-Kac technique was just, we prove that having the stochastic process of Equation III.1.9 and the function  $\mathcal{O}$  of Equation III.1.10 would result in the partial differential equation in III.1.8.

On one hand, based on Equation III.1.10, we have:

$$\mathbb{E}^{\mathcal{I}^*} [d\mathcal{O}_t | \mathcal{F}_t] = r_t \mathcal{O}_t dt \quad (\text{III.1.11})$$

On the other hand, applying Ito on function  $\mathcal{O}$ :

$$\begin{aligned} \mathbb{E}^{\mathcal{I}^*} [d\mathcal{O}_t | \mathcal{F}_t] &= \frac{\partial \mathcal{O}}{\partial t} dt + \frac{\partial \mathcal{O}}{\partial S} \mathbb{E}^{\mathcal{I}^*} [dS_t^C] + \frac{\partial^2 \mathcal{O}}{\partial S^2} \mathbb{E}^{\mathcal{I}^*} [d \langle S^C \rangle_t] + \frac{\partial \mathcal{O}}{\partial V} \chi_t dt \\ &\quad + S_t E[dM_t] (\mathcal{O}(t, S_{t-} + S_{t-} \Lambda(\chi_t), V_t) - \mathcal{O}(t, S_{t-}, V_t)), \end{aligned} \quad (\text{III.1.12})$$

where  $S^C$  is the continuous part of the diffusion of  $S$  in the dynamics III.1.9. Now, expanding the dynamics of III.1.9 in III.1.12, and III.1.11, we will obtain again the partial differential equation III.1.8. Therefore, our usage of the Feynman-Kac technique was valid.

Note that we have proved the existence of at least one element in the set of information-neutral probability measures, which is the  $\mathcal{I}^*$ . The existence of  $\mathcal{I}^*$  is sufficient proof for the existence of the set of information neutral measures  $\mathcal{I}$  and the fact that it is not empty.

□

It is worth noting that due to market incompleteness, the condition for the uniqueness of the martingale measure is not satisfied. As further elaborated in Tankov and Voltchkova (2009), when jumps coexist with a Brownian motion, the market becomes incomplete. This means that one cannot perfectly hedge all sources of variability using a conventional hedging portfolio comprising the

underlying asset and a risk-free instrument. This also indicates the existence of numerous martingale measures.

We will discuss in Section III.1.7.2 how the three-dimensional partial differential equation from III.1.5 can be numerically converted into multiple two-dimensional PDEs during implementation. In other words, the term  $\frac{\partial \mathcal{O}}{\partial V} \chi_t dt$  will be removed from the equation III.1.5. Instead, the option price solution at each point in the grid is adjusted based on the outcomes of the other two-dimensional PDEs executed concurrently. While a comprehensive explanation of this technique is outside the scope of this paper, it is a widely recognized method, particularly in the pricing of Asian options. For further details, readers are referred to Rogers and Shi (1995).

The proof of Theorem III.1.4.1 utilizes the methods of quadratic hedging and local risk-minimization akin to the approach of Föllmer et al. (1990). The proof makes use of the Girsanov Theorem, which is applied to both the jump process and the Brownian process. This leads to a change in the drift from  $\mu_t$  to  $r_t - (q - b)$  when applied to the Brownian process and a change in the jump intensity when applied to the Poisson process. It is important to note that when either  $\chi_t = 0$  for all  $t \in [0, T]$  or  $\phi = 0$ , Equation (III.1.5) reverts to the standard Black & Scholes partial differential equation.

### III.1.5 Large Trading Program

Before delving into the derivative hedging policy for a large trader, it is crucial to gain a comprehensive understanding of the trading program within our model in the context of non-derivative Delta-1 strategies. Specifically, we need to enhance our comprehension of the trading profit and loss (P&L) and determine if the model is arbitrage-free, as well as identify the conditions under which this is the case. To achieve this, we will first introduce the transaction costs which are borne by the larger trader, contrary to the insider trader. Subsequently, we will present a formulation for the trading portfolio P&L.

#### III.1.5.1 Trading Portfolio of Cash and Stock

**Definition 16.** We define the hedging strategy account the couplet  $\{H_t, V_t\}_{0 \leq t \leq T}$ , remind that  $\{V_t\}_{0 \leq t \leq T}$  is representing the number of shares holding account, and  $\{H_t\}_{0 \leq t \leq T}$  is an  $\mathbb{F}$ -adapted process representing the cash account, with  $V_0$  as a constant and  $H_0 = 0$ . The dynamics of these processes are described as follows:

$$\begin{aligned} dV_t &= \chi_t dt, \\ dH_t &= H_t r_t dt + V_t S_t (q - b) dt - S_t dV_t - d\langle S, V \rangle_t - dC_t, \end{aligned} \tag{III.1.13}$$



where the process  $\{C_t\}_{0 \leq t \leq T}$  represents the transaction costs process by the large trader and is defined by  $C_0 = 0$  and  $dC_t = g(\chi_t)dt$ , where  $g$  is a sufficiently regular function (we will provide further details on the possible parametric modeling of  $g$  in Section III.1.7.1).

Note that the Equation III.1.13 assumes that the large trader executes  $dV_t$  assets for the cash value of  $S_t dV_t + d\langle S, V \rangle_t + dC_t$ .

**Remark 6.** Note that  $\{V_t\}_{0 \leq t \leq T}$  is a finite variation process, therefore  $d\langle V, S \rangle_t$  is naturally equal to 0. However, we have not explicitly removed this term from Equation III.1.13 because we will see that this component will be canceled out naturally when calculating the accounting value of the trading activity at the terminal date  $H_T + V_T S_T$  and also because (due to the execution time needed to complete the transaction) it is more reasonable to assume that total cost depends on the price at the end of the infinitesimal execution period, not at the beginning; thus  $S_t dV_t + d\langle S, V \rangle_t$ , the realized price at the end of the execution period, is used.

At any time  $0 \leq s \leq T$ ,  $H_s + V_s S_s$  is the accounting value of the trading activity, as the sum of the shares accounts  $V_s S_s$  and the cash account  $H_s$ . Note that this is just the accounting value, not the after-liquidation value. If the large trader had to transform the whole portfolio into cash, it would not achieve the amount of  $H_s + V_s S_s$  due to the cost of liquidation (the permanent and temporary impact).

**Definition 17.** We define the  $P\&L_{trading}$  as the product-sum of  $V_u$ , which is the number of shares held at time  $u$ , and the discounted asset return over the interval  $du$ .

$$P\&L_{trading} = \int_t^T V_u d \left( e^{-\int_t^u (r_s - (q-b)) ds} S_u \right) \quad (\text{III.1.14})$$

Essentially, this can be seen as the profit-and-loss resulting from delta rebalancing trading activities. Under any information-neutral measure  $I \in \mathcal{I}$ , the expectation is given by:

$$\mathbb{E}^I [P\&L_{trading} \mid \mathcal{F}_t] = 0 \quad (\text{III.1.15})$$

This is because the discounted asset returns function as a martingale under such a probability measure and its corresponding filtration. However, this relationship does not hold under the historical measure  $\mathbb{P}$ , implying that:

$$\mathbb{E}^{\mathbb{P}} [P\&L_{trading} \mid \mathcal{F}_t] \neq 0 \quad (\text{III.1.16})$$

If the  $P\&L_{trading}$  is positive in expectation in the historical measure, it suggests

that there are dynamic arbitrage opportunities in the market that can be exploited with the right knowledge. This kind of practice is flagged as market abuse under section 1.6.4 of the Market Abuse directive of the FCA handbook [Financial Conduct Authority \(2005\)](#). Specifically, this is termed an "abusive squeeze," which manifests when an individual or entity exerts notable control over the supply, demand, or delivery processes of a qualifying or related investment.

**Proposition III.1.5.1.** *The accounting value of the trading activity at the terminal date  $T$  follows the following equation:*

$$e^{\int_t^T -r_s ds} (H_T + S_T V_T) - (H_t + V_t S_t) = P\&L_{trading} - \int_t^T e^{-\int_t^u r_s ds} S_u dC_u \quad (III.1.17)$$

*Proof.* Let  $b_u = e^{\int_t^u -r_s ds}$  denote the discount factor or the zero coupon bond price,  $\bar{H}_u$  the discounted cash  $\bar{H}_u = b_u H_u$ , and  $\bar{S}_u = b_u S_u$  the discounted asset. Therefore:

$$\begin{aligned} d\bar{H}_u &= -r_u \bar{H}_u du + b_u dH_u = V_u \bar{S}_u (q - b) du - \bar{S}_u dV_u - b_u d\langle S, V \rangle_u - b_u S_u dC_u \\ d\bar{S}_u &= -r_u \bar{S}_u du + b_u dS_u. \end{aligned} \quad (III.1.18)$$

On the other hand:

$$d(V_u \bar{S}_u) = d\langle V, \bar{S} \rangle_u + V_u d\bar{S}_u + \bar{S}_u dV_u = b_u d\langle V, S \rangle_u + V_u d\bar{S}_u + \bar{S}_u dV_u. \quad (III.1.19)$$

Therefore, we have:

$$d(V_u \bar{S}_u + \bar{H}_u) = V_u d\bar{S}_u + V_u \bar{S}_u (q - b) du - b_u dC_u (\chi_u). \quad (III.1.20)$$

Let's now call  $\hat{S}_t = e^{\int_t^u (q-b) ds} S_u$ , then one can write:

$$d(V_u \bar{S}_u + \bar{H}_u) = V_u d\hat{S}_u - b_u dC_u. \quad (III.1.21)$$

Now, taking the integration we obtain:

$$e^{-\int_t^T r_s ds} (S_T V_T + H_T) = H_t + S_t V_t + \int_t^T V_u d \left( e^{-\int_t^u (r_s - (q-b)) ds} S_u \right) - \int_t^T e^{-\int_t^u r_s ds} dC_u.$$

□

**Remark 7.** *The left side of the Equation III.1.17 represents the discounted value of the shares and cash holding portfolio by the large trader at time  $T$  minus the same value at time  $t$ ; it is the accounting value of profit and loss from the hedging activities portfolio between  $t$  and  $T$ . This value is equal to the P&L of trading activities, as defined earlier,*

minus the transaction costs, which financially make sense.

### III.1.6 Physical Delivery Options and Hedging Strategies for Large Traders

Let us assume that the large trader is hedging an option with a terminal payoff of cash and physical delivery of shares. Physical delivery is the most prevalent transaction type for stock options, wherein the buyer receives the underlying asset upon expiration. This differs from cash settlement, where the payoff is settled in cash. For further details on option types and settlement methods, refer to [Cboe Inc. \(2022\)](#). From now on, we slightly change the notation: all stochastic processes are defined up to a deterministic time horizon  $\tilde{T} > 0$ , and we consider two deterministic, positive amounts of time  $T$  and  $\tau$  such that  $T + \tau \leq \tilde{T}$ .

**Definition 18.** We assume a physical delivery European option with an expiry date of  $T$  and a settlement date  $T + \tau$ .  $Q(S_T)$  denotes the terminal number of shares to be either delivered or received, and  $\mathcal{H}(S_T)$  is the terminal cash transaction.

It is important to recall that  $V_T$  represents the quantity of shares held by the large trader at time  $T$ . As such, the collective sum of  $Q(S_T)$  and  $V_T$  reflects the remaining shares in the large trader's account at the terminal date. For effective hedging,  $Q(S_T)$  and  $V_T$  should have opposite signs. Specifically, if  $Q(S_T)$  is positive, then  $V_T$  should be negative, and conversely.

Any residual shares, represented as  $Q(S_T) + V_T$ , need to be executed between the expiry and settlement dates. This execution cost is denoted as  $\bar{C}(Q(S_T) + V_T, \tau)$ . Notably, at time  $T$ , the cost associated with executing these shares between  $T$  and  $T + \tau$  is considered known. Hence,  $\bar{C}(Q(S_T) + V_T, \tau)$  is  $\mathcal{F}_T$ -adapted.

To hedge their position, the large trader maintains a portfolio consisting of  $H_t$  cash and  $V_t$  shares at time  $t$ , using the same notation as in the previous section. Given the high hedging costs, the large trader may opt to leave some positions unhedged, anticipating favorable market movements to offset any potential issues arising from the unhedged portion.

The final profit and loss at maturity is given by:

$$P\&L = \overbrace{\mathcal{H}(S_T) + S_T Q(S_T)}^{\text{Payoff}} + \overbrace{H_T + S_T V_T}^{\text{Accounting value of hedges}} - \overbrace{S_T \bar{C}(Q(S_T) + V_T, \tau)}^{\text{Terminal execution cost of missing shares}}. \quad (\text{III.1.22})$$

Equation (III.1.22) has three main components: the option payoff, the accounting value of the hedging portfolio, and the terminal cost of executing any

missing shares between the option expiry time  $T$  and the settlement date  $T + \tau$  for physical delivery options.

By combining Equations (III.1.17) and (III.1.22), we can obtain the following formulation for the terminal  $P\&L$ :

$$P\&L = e^{\int_t^T -r_s ds} (\mathcal{H}(S_T) + S_T Q(S_T)) + P\&L_{trading} + \left( \int_t^T e^{-\int_t^u r_s ds} S_u dC_u + e^{\int_t^T -r_s ds} S_T \bar{C}(Q(S_T) + V_T, \tau) \right) + H_t + S_t V_t. \quad (\text{III.1.23})$$

On the right side of this equation, the first component represents the option payoff, and the second term is the profit or loss from trading activities due to changes in the value of the underlying asset during the option's lifetime. The third term represents the total temporary transaction cost of the delta hedging program, while the last term represents the accounting value of the hedging portfolio at expiry.

### III.1.6.1 Manipulating the Payoff Constraints and Problem Reduction

While the large trader's goal is to optimize the  $P\&L$  of Equation (III.1.23), they must exercise caution to avoid manipulating the option's payoff, as doing so would constitute market abuse, according to article 12.1.a.ii of the EU market manipulation regulation (European Parliament and of the Council, 2014) and the FCA handbook example of Market abuse (Financial Conduct Authority, 2005) (MAR 1.6.15).

Therefore, to effectively address the constraints related to market abuse, we need to refine the stochastic optimization problem. First, to address the payoff manipulation, we remove the payoff component of Equation (III.1.23) from the optimization problem. Next, to prevent abusive squeeze arbitrage, we will estimate the expected utility under information-neutral measures. As discussed earlier, such measures will see the expectation of the  $P\&L$  of trading as zero, which is a desired property. Amongst all information-neutral measures, we have specifically studied the minimum variance information-neutral measure. Therefore, we will optimize the expected utility of our optimization problem under this probability measure. Consequently, we simplify the problem formulation to Equation (III.1.24), with function  $J$  acting as its solution.

$$J(t, V_t, S_t) = \inf_{\{V_s\}_{s \geq t}} \mathbb{E}^{I^*} \left[ e^{-\int_t^T r_s ds} S_T \bar{C}(Q(S_T) + V_T, \tau) + \int_t^T e^{-\int_t^s r_u du} S_s dC_s \middle| \mathcal{F}_t \right]. \quad (\text{III.1.24})$$

Equation (III.1.24) consists of two parts: the transaction cost of executing missing shares between the expiry date  $T$  and the settlement date and the

transaction cost of executing shares before the expiry date. These two terms act against each other, and the trader needs to strike a balance between buying shares too early and getting closer to a full delta hedge with fewer missing shares to recover in the future but paying extra costs because of executing the shares too quickly, or leaving its position partially unhedged. If lucky, the market will move in the trader's direction, and the partial hedge will be closer to the full hedge. Otherwise, the trader will have some time between the expiry date  $T$  and the settlement date  $T + \tau$  to execute the missing shares. Note that at the expiry date, the number of required shares is fixed and is no longer uncertain.

Theorem III.1.6.1 establishes the relationship between the cost function  $J$  and the partial differential equation (PDE) that governs it. The theorem states that  $J$  can be deduced from the PDE expressed in Equation (III.1.25), subject to the terminal condition in Equation (III.1.26).

**Theorem III.1.6.1.** *Let  $\mathcal{J}^*$  be the solution to the Equation (III.1.24), it is a function that depends on the optimal number of shares to be executed during a period, denoted by  $\chi_t^*$ . The solution to the PDE, given by  $\mathcal{J}_\chi(t, V, S)$ , provides the optimal strategy for the large trader to hedge its option position.*

The cost function  $J$  can be deduced from the following partial differential equation:

$$\frac{\partial \mathcal{J}^*}{\partial t} + rS \frac{\partial \mathcal{J}^*}{\partial S} + \frac{1}{2} \frac{\partial^2 \mathcal{J}^*}{\partial S^2} \sigma^2 S^2 + \frac{\mathcal{J}^* - J}{dt} \Big|_{(t,V,S)} = rJ(t, V, S), \quad (\text{III.1.25})$$

with the terminal condition

$$J(T, V, S) = S_T C(Q(S_T) - V_T, \tau), \quad (\text{III.1.26})$$

where  $\mathcal{J}^*$  is defined as follows:

$$\begin{aligned} \mathcal{J}_\chi(t, V, S) &= J(t, V + \chi dt, S + S\Lambda(\chi_t)) \omega_t dt + J(t, V + \chi dt, S)(1 - \omega_t dt) + SdC_t \\ \mathcal{J}^*(t, V, S) &= \inf_{\chi} \{ \mathcal{J}_\chi(t, V, S) \}, \end{aligned} \quad (\text{III.1.27})$$

and  $\chi_t^*$  is the optimal number of shares to be executed during the period  $dt$  and obtained from

$$\chi_t^* = \arg \inf_{\chi} \{ \mathcal{J}_\chi(t, V, S) \}. \quad (\text{III.1.28})$$

*Proof.* Knowing that  $dV_t = \chi_t dt$ , the equation III.1.24 can be reformulated as:

$$J(t, V_t, S_t) + rJ(t, V_t, S_t) dt = \inf_{\chi} E^{I^*} \left[ J(t + dt, V_t + \chi dt, S_t + dS_t) \chi dt + SdC_t(\chi) \right]. \quad (\text{III.1.29})$$

Let us define  $\mathcal{J}_\chi$  as the solution to the following equation:

$$\mathcal{J}_\chi(t, V_t, S_t) = J(t, V_t + \chi_t dt, S(1 + \Lambda(\chi_t)dN_t)) + dC_t.$$

Then, applying the Itô formula with jumps leads to the following equation where  $dS^C$  represents the non-jump part of the asset return:

$$\mathcal{J}_\chi(t + dt, V_t, S_t + dS_t^C) = \mathcal{J}_\chi + \frac{\partial \mathcal{J}_\chi}{\partial t} dt + \frac{\partial \mathcal{J}_\chi}{\partial S} dS^C + \frac{1}{2} \frac{\partial^2 \mathcal{J}_\chi}{\partial S^2} (dS^C)^2 \Big|_{(t, V_t, S_t)}. \quad (\text{III.1.30})$$

And let us also define the optimal  $\chi^*$  such that:

$$\chi_t^* = \operatorname{arg\,inf}_\chi \mathbb{E}[J(t, V_t + \chi dt, S_t(1 + \Lambda(\chi_t)dN_t)) + dC_t].$$

In such a case, Equation (III.1.29) becomes:

$$rJ(t, V_t, S_t)dt = \mathcal{J}_{\chi^*} - J + \frac{\partial \mathcal{J}_{\chi^*}}{\partial t} dt + \frac{\partial \mathcal{J}_{\chi^*}}{\partial S} E[dS^C] + \frac{1}{2} \frac{\partial^2 \mathcal{J}_{\chi^*}}{\partial S^2} E[(dS^C)^2] \Big|_{(t, V_t, S_t)}. \quad (\text{III.1.31})$$

Hence, expanding the expectation of (III.1.31), we obtain the partial differential equations as formulated in (III.1.25) and (III.1.26).  $\square$

It is important to note that  $\omega_t$  is a function that also depends on  $\chi$ , which in turn represents the optimal speed of execution during the period  $dt$  and is obtained through Equation (III.1.28). Additionally, when there is no market impact,  $\Lambda$  is equal to 0, which results in Equation (III.1.25) reducing to the standard Black-Scholes equation.

## III.1.7 Numerical Results

### III.1.7.1 Transaction Cost Parametrization

**Definition 19.** Let  $C_t$  be a stochastic process with the dynamics defined by the Equation III.1.32. The process  $\{C\}_{0 \leq t \leq T}$  represents the transactional cost borne by the large trader :

$$\begin{aligned} C_0 &= 0 \\ dC_t &= \eta \left( \frac{\chi_t}{\chi_t + A} \right)^\alpha \chi_t dt, \end{aligned} \quad (\text{III.1.32})$$

where  $A \in \mathbb{R}$  is the (annualized) average volume,  $\eta \in \mathbb{R}^+$  is the impact factor, and  $\alpha \in [0, 1]$  is the exponential factor.

This formulation is consistent with previous studies such as [Grinold and Kahn \(2000\)](#) and [Zarinelli et al. \(2015\)](#) where  $\alpha$  is often set to  $1/2$ , corresponding to a square root formula.

For longer periods between the expiry and settlement date, an optimal purchase/liquidation algorithm can be applied, as explored in the literature (e.g. [Almgren and Chriss \(1997\)](#)). However, in this study, we use a constant rate program for simplicity. In this case, the expected value of function  $\bar{C}(x, \tau)$  - the terminal cost of executing  $x$  at time  $\tau$  between the expiry and settlement - can be expressed as Equation (III.1.33).

**Proposition III.1.7.1.** *Assume the trader needs to execute  $X$  shares within a period of  $\tau$  and chooses to do so at a constant execution speed of  $\frac{X}{\tau}$ <sup>7</sup>. Let us also assume that there are no dividends and lending costs during the period of  $T$  to  $T + \tau$ . Under these conditions, the expected value of the cost under an information-neutral measure  $I \in \mathcal{I}$  is described by the following equation:*

$$\mathbb{E}^{\mathcal{I}} [\bar{C}(x, \tau) | \mathcal{F}_T] = \eta \times \frac{\left(\frac{x}{\tau}\right)^{\alpha}}{\left(\frac{x}{\tau} + A\right)^{\alpha}} x. \quad (\text{III.1.33})$$

Overall, our cost model allows us to capture the trade-off between the size of the trade ( $x$ ) and the time until the settlement date ( $\tau$ ). Specifically, as  $x$  increases, so does the cost of execution, but at a decreasing rate governed by the exponent  $\alpha$ . In contrast, as  $\tau$  increases, the cost of execution decreases due to the larger time frame for trading.

*Proof.* Having the Equation III.1.32 for the short-term cost, and with the strong assumptions that all shares requested will be executed, the long-term cost of execution of  $X$  shares in  $\tau$  time is

$$\bar{C}(X, \tau) = \int_0^{\tau} S_t dC_t.$$

Note that in our case,  $\frac{X}{\tau}$  is the constant execution rate. In case we have Equation (III.1.32) for the short-term cost, the long-term cost of execution is as

---

<sup>7</sup>While optimal purchase/liquidation algorithms for longer purchase periods are well-researched topics, as seen in works like [Almgren and Chriss \(1997\)](#), our focus here is on the constant rate program. There are compelling reasons for selecting a constant purchase program over a more intricate one. Notably, most derivatives desks do not prioritize optimizing their execution cost, placing greater emphasis on volatility instead. Thus, even if optimal algorithms are available, they might not be employed in derivatives trading. Our goal here is not to suggest how the purchasing should be executed but to understand the behavior in the most prevalent scenario. This reasoning drives our choice for a constant rate program.

follows:

$$\int_T^{T+\tau} S_t d\mathcal{C}_t = \eta \times \frac{\left(\frac{X}{\tau}\right)^{\alpha+1}}{\left(\frac{X}{\tau} + A\right)^\alpha} \int_T^{T+\tau} e^{-rt} \left(1 + \Lambda\left(\frac{X}{\tau}\right)\right)^t S_t dt,$$

$$E^I \left[ \int_T^{T+\tau} S_t d\mathcal{C}_t \right] = \eta \frac{\left(\frac{X}{\tau}\right)^{\alpha+1}}{\left(\frac{X}{\tau} + A\right)^\alpha} S_T \tau.$$

□

### III.1.7.2 Solving the Partial Differential Equation

To numerically solve the two-dimensional partial differential Equation (III.1.25), we can use the dynamic programming approach described in Algorithm 1. First, we initialize the cost function  $J(T, S, V)$  to the terminal condition  $\bar{C}(Q(S) - V, \tau)$  at the expiration time  $T$ . Then, we discretize time into  $\Delta t$  intervals and iterate backward in time from  $T$  to 0, updating  $\mathcal{J}$  at each time step using Equations (III.1.25), (III.1.27) and (III.1.28). We repeat this process for  $n$  iterations to ensure convergence.

---

**Algorithm 1** Methodology for solving the large trader's cost function

---

- 1:  $J(T, S, V) \leftarrow \bar{C}(Q(S) + V, \tau)$ ;
  - 2: Discretize the time to  $\Delta t$  intervals;
  - 3: **for**  $t = T; t \leftarrow t + \Delta t; t = 0$  **do**
  - 4:     Using  $J(t + \Delta t, \dots)$  apply Equation (III.1.28) and solve  $\chi_t^*$ ;
  - 5:     Using  $\chi_t^*$ , apply Equation (III.1.27) and calculate  $\mathcal{J}^*(t + \Delta t, \dots)$ ;
  - 6:     Using  $\mathcal{J}^*(t + \Delta t, \dots)$ , solve Equation (III.1.25) and obtain  $J(t, \dots, V)$ ;
  - 7:     **for**  $iter = 0; iter \leftarrow iter + 1; iter < n = \text{numiter}$  **do**
  - 8:         Using  $J(t, \dots)$  apply Equation (III.1.28) and solve  $\chi_t^*$ ;
  - 9:         Using  $\chi_t^*$ , apply Equation (III.1.27) and recalculate  $\mathcal{J}^*(t + \Delta t, \dots)$ ;
  - 10:         Using  $\mathcal{J}^*(t + \Delta t, \dots)$ , solve Equation (III.1.25) and obtain  $J(t, \dots, V)$ ;
  - 11:     **end for**
  - 12:     Given  $\chi_t^*$ , solve the insider's trader option price  $\mathcal{O}$  using a finite difference scheme of the partial differential in Equation (III.1.5);
  - 13: **end for**
  - 14: Obtain  $J(0, S_0, 0)$  and  $\mathcal{O}(0, S_0, V_0)$
- 

Once we have obtained the optimal hedging strategy  $\chi^*$ , we can use it to solve for the insider's trader option price  $\mathcal{O}$  using a finite difference scheme for the partial differential in Equation (III.1.5). Finally, we obtain the values of  $\mathcal{J}$  and  $\mathcal{O}$  at time 0, denoted by  $J(0, S_0, 0)$  and  $\mathcal{O}(0, S_0, V_0)$ , respectively. More details on the finite difference scheme used to solve Equation (III.1.25) are provided in Appendix III.1.A.

Note that in order to solve  $\mathcal{O}$  in Equation (III.1.5), we use a partial differential equation akin to the Asian option techniques; therefore, the algorithm runs several 2-dimensional finite difference scheme in parallel. Each scheme represents a discretization of  $V$ . The algorithm runs PDEs for  $(\mathcal{O})(t, S, \cdot)$  and



then, at a second stage, alters between the finite difference schemes in order to take into account the impact of the change in  $V$ . A full explanation of this well-known method is beyond this paper, but the reader is referred to [Rogers and Shi \(1995\)](#).

### III.1.7.3 Numerical Results

In this section, we present numerical results assuming market parameters described in [Table III.1.1](#) and a large trader who is the seller of a call option with parameters given in [Table III.1.2](#). Our assumptions about market impact are in line with the empirical study by [Jarrow et al. \(2018\)](#). The parameters chosen for the call option are also consistent with what one might expect from over-the-counter (OTC) derivatives.

$\mu$	$\sigma$	$\eta$	$\phi$	$\lambda$	$S_0$	$A$	$r$	$\alpha$	$q$	$b$
0.02	0.25	0.1	20.0	2.0	100	5e+6	0.01	2.0	0.0	0.0

TABLE III.1.1: Market parameters

$T$	$\tau$	Call strike	Notional
120 days	15 days	110	1e+6

TABLE III.1.2: Large Trader's option

[Figure III.1.1](#) depicts the terminal cost function  $J$  for the large trader's call option. The graph illustrates different scenarios depending on the final stock price and the number of shares  $V_T$  to be delivered at expiration. When the option is in the money, i.e., the stock price is above the strike price, the cost function is zero since the large trader delivers the shares at no additional cost. Conversely, when the option is out of the money, i.e., the stock price is below the strike price, the large trader needs to purchase the notional amount of shares before the settlement date, leading to higher costs. As shown in the graph, the cost function varies across different corners but gradually flattens as we move backward in time, as observed in [Figure III.1.2](#). These results are consistent with our cost model in [Equation \(III.1.26\)](#), which assumes a linear relationship between the cost and the spot price of the stock.

[Figures III.1.3](#) and [III.1.4](#) show the optimal amount of shares to be executed ( $\chi^*$ ) at time 0 and just before expiry, respectively. Our findings suggest that the optimal  $\chi^*$  is much lower than the Black-Scholes Delta due to the high cost of a full hedge strategy. As shown in [Figure III.1.5](#), the difference in Delta between the two strategies is significant. The large trader executes only a small percentage of  $\chi^*$  at time 0, whereas the Black-Scholes Delta is 100% for the same spot levels. However, as the expiry date approaches, the large trader's

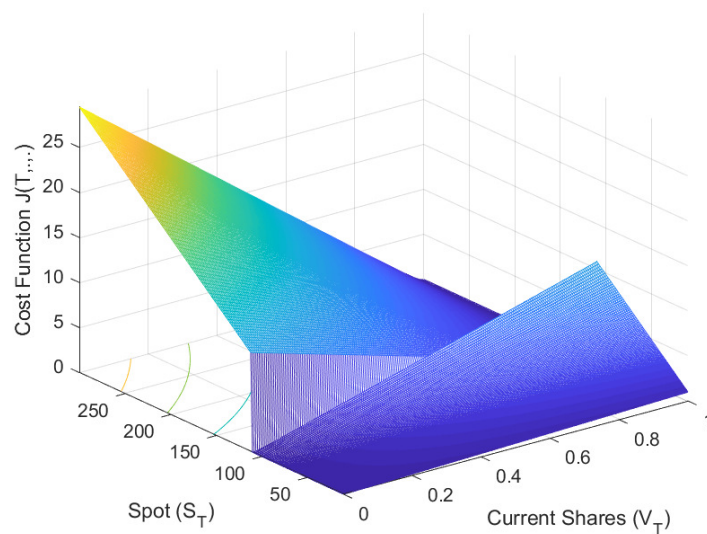


FIGURE III.1.1: The terminal cost function  $J$  for a hedged option.  $V_T$  shall be read as a multiple of the notional.

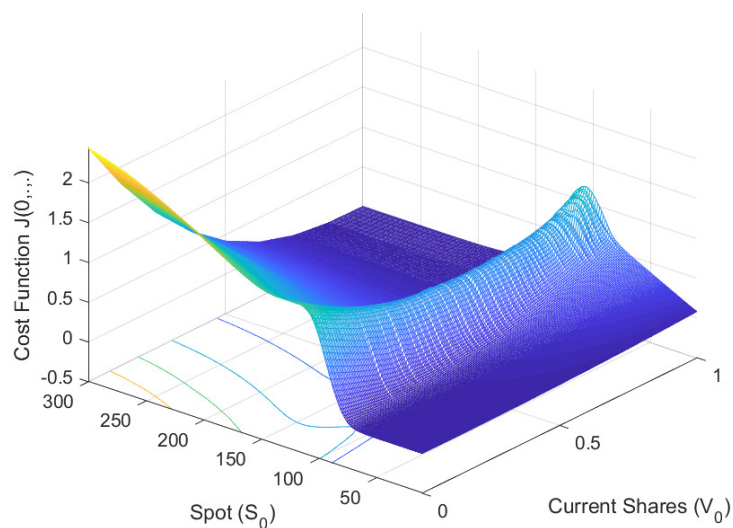


FIGURE III.1.2: The cost function  $J$  at time 0 for a hedged call option. Data source: Matlab simulation.

optimal strategy gets closer to the Black-Scholes Delta, as illustrated in Figure III.1.6. The number of shares to be purchased by the large trader is still smaller than the Black-Scholes Delta, but it reaches 50% of the B-S Delta for high spot prices. The limited time frame near expiry means that the large trader has to overcome a significant stock shortage, making the execution of the remaining shares challenging.

Examining the impact of the large trader's hedging strategy on the insider trader's option price, as defined in Table III.1.3, provides valuable insights. Figure III.1.7 compares the insider trader's option price with the Black-Scholes

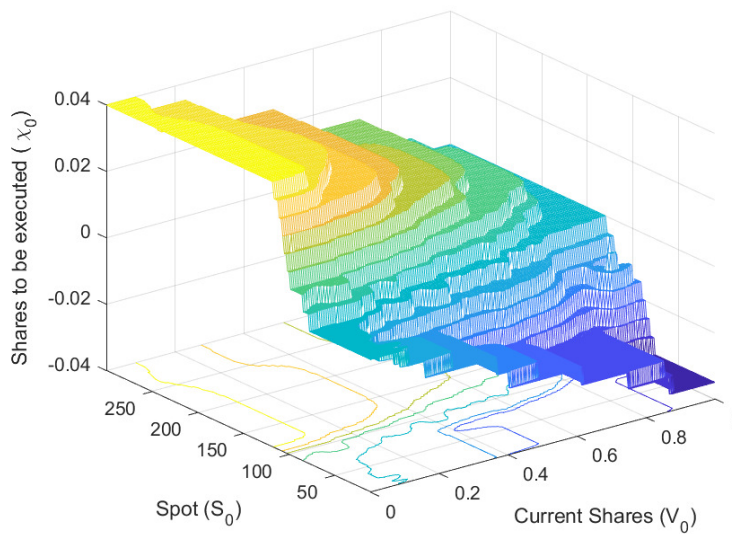


FIGURE III.1.3: The optimal shares strategy function at time 0 for a call option. Data source: Matlab simulation.

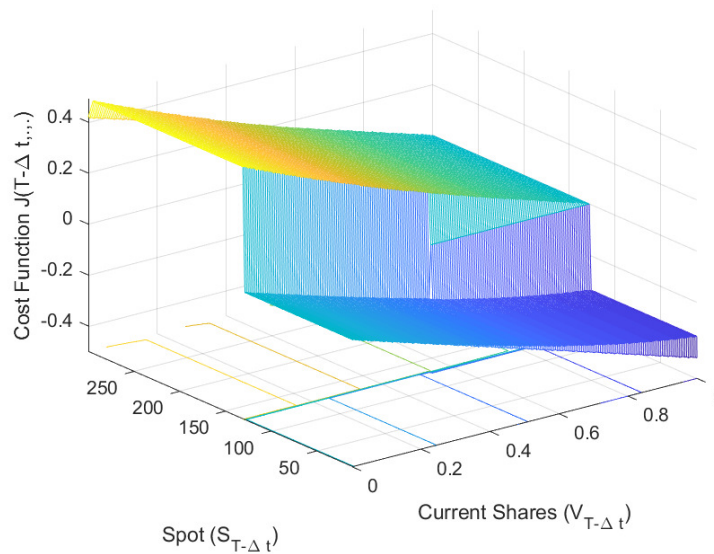


FIGURE III.1.4: The optimal shares strategy function at time  $T - dt$  for a call option. Data source: Matlab simulation.

model for the same call option. The graph highlights the option's increased convexity around the money and the higher prices than the Black-Scholes model when the option is in the money, surpassing even the intrinsic value. This phenomenon is due to the directional market impact of the large trader. If the spot price is in the money, the large trader will purchase shares, driving the spot price deeper into the money. The insider trader's option price, therefore, reflects the market impact of the large trader, resulting in a higher option price than predicted by the standard Black-Scholes model.

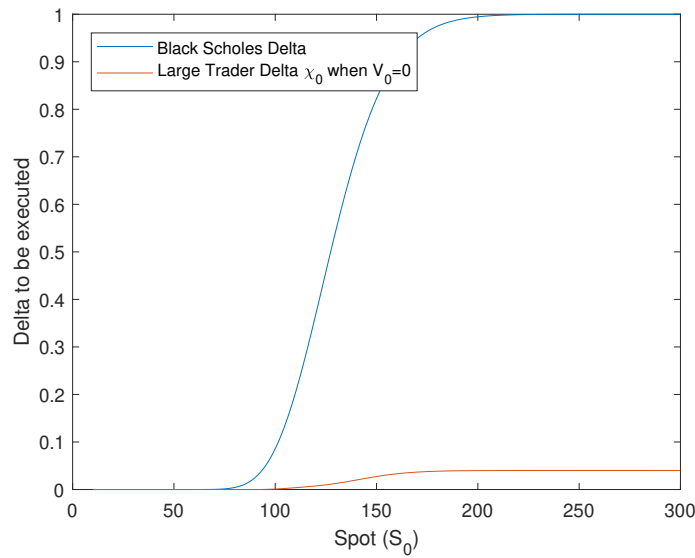


FIGURE III.1.5: Comparison between BS delta and optimal shares for the large trader at inception, assuming  $V_0 = 0$ .

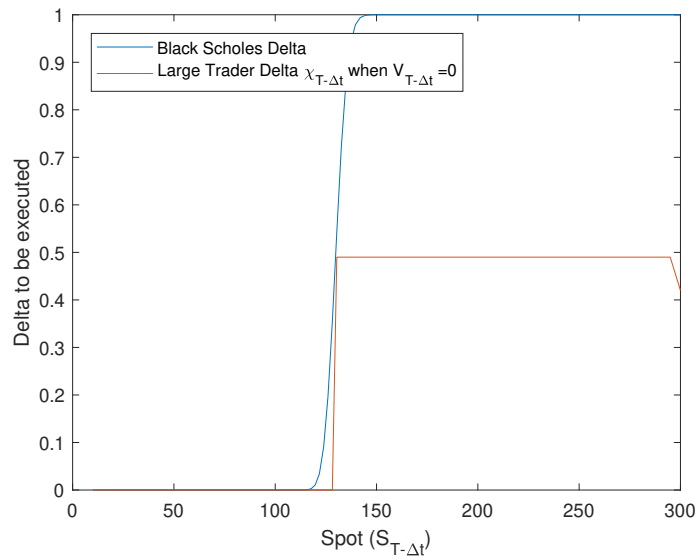


FIGURE III.1.6: Comparison between BS delta and optimal shares for the large trader at expiry, assuming  $V_T = 0$ . Data source: Matlab simulation.

$T$	Call strike
120 days	110

TABLE III.1.3: Small trader's option.

We would also like to highlight the fact that the insider option price is sensitive to parameters  $\phi, \eta, \lambda$  as shown in Table III.1.4. Economically speaking, an increase in the market impact parameters  $\phi$  and  $\lambda$  results in a higher call option price for the insider trader (for the option defined in Table III.1.2). Recall that

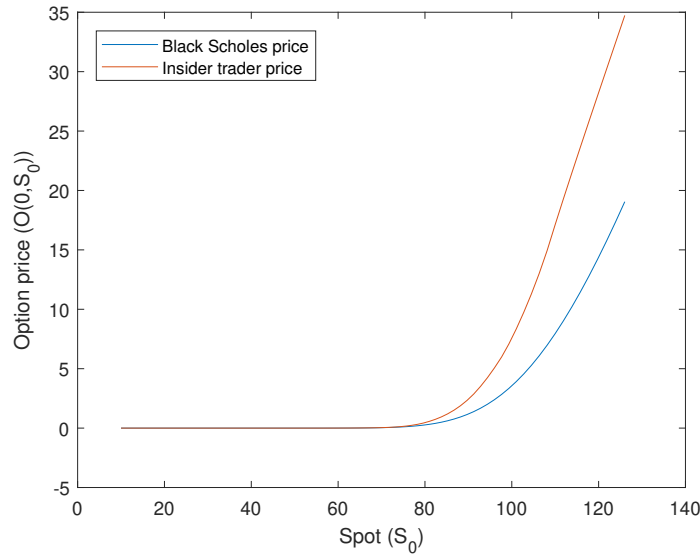


FIGURE III.1.7: Comparison between Black and Scholes price and insider trader price for a call option. Data source: Matlab simulation.

$\eta$  is the coefficient of the transaction cost (temporary impact); the higher this coefficient is, the less conservative the large trader will become and the closer the option price will be to the one with no market impact (Black & Scholes price).  $\lambda$  is the market impact factor, and  $\phi$  is the frequency of the market impact function. Higher values of these two parameters result in a higher market impact function and, accordingly, more divergence from the Black & Scholes price.

$\phi$	$\lambda$	$\eta$	Option price
10.0	0.5	0.2	4.55
10.0	1.0	0.5	4.53
20.0	1.0	0.2	5.58
20.0	0.5	0	5.46
20.0	0.0	0.2	3.59
0.0	0.0	0	3.59

TABLE III.1.4: Call option price for insider trader at  $t = 0$ .

### III.1.8 Conclusion

In conclusion, this paper introduced a novel model framework for martingale pricing theory that takes into account the impact of large traders on the market. Our analysis revealed that the negligent hedging activities of large traders could potentially cause market manipulation, highlighting the need for robust mathematical models that do not inadvertently promote such strategies. To address this issue, we proposed a hedging policy for the large trader

that considers both the costs and risks of hedging activities, while avoiding manipulative practices.

Our proposed pricing model for an insider trader, who possesses knowledge of the large trader's trading policy, is a significant contribution of this paper. We derived the pricing equation for such a trader and noted that there are information-neutral measures in which the assets' return is a martingale for the insider.

The paper concludes with numerical results that showcase the optimal delta-hedging strategy for an out-of-the-money call option. Our analysis indicates that the model proposes a more relaxed delta hedging strategy compared to the Black and Scholes model. Additionally, the price of the call option for the insider trader is observed to be higher under market impact than the Black and Scholes model. Thus, this paper provides a practical and effective model for pricing derivatives under market impact that can be easily implemented. The findings of this research can be utilized by financial market participants to better understand and navigate the dynamics of market impact and pricing of derivatives.

### III.1.A Appendices

#### Numerical Implementation of the Large Trader PDE

In terms of log returns, and assuming that  $\chi^*$  is already obtained, Equation (III.1.25) can be written as

$$\begin{aligned} x &= \log(S), \\ Y(t, x, V) &= J(t, V, S), \\ Y^*(t, x, V) &= \mathcal{J}^*(t, V, S), \\ \bar{\Lambda}(\cdot) &= \ln(1 + \Lambda(\cdot)), \\ rJ &= \frac{\partial Y^*}{\partial t} + \left( r - (q - b) - \frac{1}{2}\sigma^2 \right) \frac{\partial Y^*}{\partial x} + \frac{1}{2} \frac{\partial^2 Y^*}{\partial x^2} \sigma^2 \\ &\quad + (Y^*(t, V, x + \bar{\Lambda}(\chi^*)) - Y(t, V, x)) \frac{1}{dt}. \end{aligned}$$

We define the boundary levels  $x_{min}, x_{max}$  for the log spot and  $d_{min}, d_{max}$  for the shares. We define the discretization  $\{(t_i, x_l, d_k)\}$  with

$$\begin{aligned} (0, 0, 0) &\leq (i, l, k) \leq (I, L, K), \\ (0, x_{min}, d_{min}) &\leq (0, x_k, d_l) \leq (T, x_{max}, d_{max}). \end{aligned}$$

Here, we assume a uniform grid, with intervals of triplet  $(\Delta t, \Delta x, \Delta d)$ . At the maturity level, we define the following equation:

$$Y_{I,l,k} = C(d_k - Q(e^{x_l}), \tau)$$

At  $t_{i+1}$ , we first calculate the optimal value in discretized terms. Assuming that we have the optimal  $\chi^*$  we define  $i^* = \frac{\chi^*}{\Delta d}$  and  $L_{i,l,k}^*$  as below equation for the gridpoint  $(t_i, x_l, d_k)$ .

$$L_{i,l,k}^* = (Y_{i+1,l,k+i^*}^* - Y_{i+1,l,k}) (1 - \omega^* \Delta t) + \left( Y_{i+1,l+\lfloor \frac{\Delta l^* \Delta d}{\Delta x} \rfloor, k+i^*}^* - Y_{i+1,l,k} \right) \omega^* \Delta t + e^{x_l} dC(i \Delta d). \quad (\text{III.1.34})$$

We apply a fully implicit scheme, and the discretization will be as of below:

$$\begin{aligned} \left( 1 + r \Delta t + \frac{\Delta t}{(\Delta x)^2} \sigma^2 \right) Y_{i,l,k} &+ \left( (r - (q - b) - \frac{1}{2}\sigma^2) \frac{\Delta t}{2\Delta x} - \frac{1}{2} \frac{\Delta t}{(\Delta x)^2} \sigma^2 \right) Y_{i,l-1,k} \\ &+ \left( -(r - (q - b) - \frac{1}{2}\sigma^2) \frac{\Delta t}{2\Delta x} - \frac{1}{2} \frac{\Delta t}{(\Delta x)^2} \sigma^2 \right) Y_{i,l+1,k} = Y_{i+1,l,k}^* + L_{i,l,k}^*. \end{aligned}$$

With Neumann boundary condition, we have:

$$\begin{aligned} Y_{i,l,k} = Y_{i,0,k} &= C(N_T - Q(e^{x_{min}}), \tau) \approx -C(d_k - Q(e^{x_{min}}), \tau + T - t_i) \\ Y_{i,L-1,k} = Y_{i,L,k} &\approx C(d_k - Q(e^{x_{max}}), \tau + T - t_i). \end{aligned}$$



## **Part IV**

# **Conclusions and Perspectives**

Stable value guarantees correspond to a crucial portion of the financial industry, given the vast amount of assets they manage. However, there is a surprising lack of exhaustive academic research in this area, which motivated me to undertake this research project and this thesis. As a quant, I noticed a shortage of quantitative studies on the risk management of stable value contracts and felt compelled to address this issue.

My goal was to contribute to the community of insurers, actuaries, fund managers, consultants, and other professionals dealing with stable values. With over \$800 billion in assets managed by this market, it is imperative to have a better understanding of the quantitative aspects of stable values in order to make informed decisions and manage risks more effectively.

In addition, I aspire for this research project to serve an educational purpose, extending beyond a purely academic study. It is my intention for it to serve as a valuable resource for individuals entering this market, fostering a deeper comprehension of the quantitative aspects of stable values. By doing so, I aim to empower both current and future professionals to effectively manage assets, make well-informed decisions, and ultimately deliver improved outcomes for the pension holders we serve.

In this part, rather than rehashing the contributions, which have already been presented in the overall abstract, the general introduction, the recap of major contributions, and the conclusions of each chapter, we pivot to offer several research prospects that build on the work we have presented.

## **Flight-to-Safety**

As outlined in the introduction Chapter I.1, "flight-to-safety" is a well-documented behavioral phenomenon in financial markets. During periods of market turbulence, some investors shift their resources to safer, secure investment options. In Chapter II.3, we found this pattern present in the stable value market as well.

One question to consider is whether investors adjust their strategies based on experiences of previous market shocks. Although available historical data is limited, it would be intriguing to observe investors' reactions if two market crises occur in succession. For instance, if the initial "flight-to-safety" resulted in the selling of the riskiest assets at their lowest value, which later recovered, it would be interesting to see if the subsequent crisis would witness a reduced "flight-to-safety" response. This scenario occurred in the 2020 pandemic, followed by a financial crisis in early 2022. Investigating whether investors learned from their previous "flight-to-safety" would offer valuable insights.

Given the significant role this subject plays in the global financial economy, it certainly warrants further exploration. However, additional data-cleansing work is required for this project to be fully realized. For this thesis, we use net cash flow data, which is a sum of outer-plan and inner-plan activities, both of which have been extensively discussed in Chapter II.2. To delve deeper into the flight-to-safety behaviors in stable value funds, a thorough examination of inner-plan transfer cash flow data is necessary. It should be noted that the inner-plan transfer data requires additional cleaning to fully enable the analysis of cash flow data. This data cleansing step is crucial for gaining a comprehensive understanding of the flight-to-safety dynamics within stable value funds.

## Multiple Wrap Contract Diversification

Underwriting multiple Synthetic GIC contracts that are managed by different investment managers leads to risk diversification for the insurer. The primary factors contributing to this diversification include:

1. *Diversification benefit of underwriting contracts with several managers:* Concentrating risk with a few managers can be problematic while diversifying the business portfolio among several managers can provide significant risk benefits. This is because different managers, especially those in charge of all 401(k) options on behalf of the plan sponsor, can create a diverse ecosystem with a range of available options. This diversity reduces the influence of the ecosystem that one single manager can create.

Moreover, the risk is further diversified as reputational or creditworthiness issues affecting one manager are likely to influence only the subset of wraps managed by them. Such issues can prompt participants to withdraw their balances from the affected funds, potentially leading to losses for the Synthetic GIC issuer. By spreading the risk across multiple managers, the overall impact of such a scenario on the insurer can be minimized.

2. *Diversification benefit of multiple plan sponsor:* Diversification benefits are particularly noticeable on the liability cash flow side when wrapping multiple contracts for various plan sponsors. Each plan sponsor presents a unique demographic profile, and this variation reduces the potential impact of any one plan sponsor's idiosyncratic risk on the insurer's overall portfolio. This form of diversification can, therefore help to stabilize the liability cash flows.

## Evaluating the Optimality of Stable Value Fund Structures

Our earlier discussion in Chapter I.1 highlighted the concept of 'last resort,' which effectively minimizes the cost of insurance guarantees. Under this approach, an insurer is only required to make a payment when the final investor withdraws their funds. This is a deviation from the initial iterations of Guaranteed Investment Contracts (GICs), where insurers had to regularly cover asset-liability mismatches. This evolution from the early GICs to the current structure signifies a significant advancement in structured finance.

Beyond this structural evolution, the optimality of the crediting rate formula also invites analysis. Pivotal questions emerge: Is the current formula optimal? How can we effectively measure its optimality given various stakeholder perspectives? From whose viewpoint is the structure more optimal - the pension holder or the insurer?

The standard stochastic control problem, as primarily exemplified in fixed-term guaranteed contracts, is well-documented in works such as [Miltersen and Persson \(2003\)](#) and [Kling et al. \(2007\)](#). However, the stochastic control problem becomes significantly more complex when the guarantee's term or 'last resort' date is stochastic in nature. This added complexity necessitates a more nuanced approach to fully appreciate the mathematical intricacies involved.

In our context, new considerations surface: For which type of policyholder should the stochastic control problem optimize? A so-called 'arbitrage' policyholder, who cashes out as soon as the moneyness hypothesis is invalidated? A 'loyal' policyholder who only withdraws funds as a last resort? Or a policyholder who aligns somewhere between these extremes?

These fundamental questions must be addressed before employing stochastic optimization techniques. The responses will direct our research toward proposing new guarantee structures for the industry or verifying the optimality of existing contracts in the financial markets.

## Universal Insider Knowledge and Contingent Liquidity

The current study has primarily focused on examining the market perspective of an insider trader who has access to the knowledge of a large trader's activities. In this context, we have concluded that a large trader's prospective activities indeed have a significant effect on the financial market, which can be viewed as convexity or skew correction to the volatility surface having the knowledge of the insider trader.

An intriguing direction for future research would be to flip this question and ponder the market's perception when all participants are informed of one or several large traders' activities. That is to say, in a scenario where the market is entirely comprised of insider traders, how would the skew and convexity of the market be affected?

This shift in focus would allow for a more in-depth exploration into the extent to which the skew and convexity can signal a potential liquidity imbalance. A question that emerges from this line of thought is whether vanilla prices can reveal future liquidity imbalances. If all market participants are informed, and thus, the market is a true reflection of this collective information, would the vanilla prices hold indicators for prospective liquidity imbalances? Moreover, could a part of the volatility smile be explained by contingent market impact?

## Permanent Impact Model Assumptions

The analysis in Part III of this thesis serves as an exploratory study on the impact of information and market dynamics on derivatives pricing theory. This study is informed by the author's personal experiences and observations of industry challenges. However, it is important to note that the model developed in this study relies on certain simplifications and assumptions that may be considered somewhat rigid and may not fully align with the complexities of real-world scenarios.

The model in Part III employs a Poisson process for the stochastic liquidity component. This process, known for its discontinuity, has the limitation of being discrete with potential zero jumps at infinitesimal levels. The thesis interprets periods with zero jumps as times when the market depth is sufficient to absorb large transactions without creating a permanent impact. However, for significantly large transactions, in reality, it is unlikely that such periods without permanent market impact exist. Alternative processes, like an exponential process, which do not share this limitation, may offer a more realistic representation. Investigating and contrasting the outcomes of these alternative processes with the Poisson process presents an intriguing opportunity for further mathematical research and analysis.

Another assumption in our model is the absence of a correlation between the stochastic liquidity process  $\{N_t\}_{0 \leq t \leq T}$  and other external sources of spot volatility  $\{W_t\}_{0 \leq t \leq T}$ . In practice, days with higher volatility are often associated with greater challenges in executing stock trades without significantly impacting the market. A more realistic model might assume some degree of correlation between  $\{W_t\}_{0 \leq t \leq T}$  and  $\{N_t\}_{0 \leq t \leq T}$ .

# Bibliography

- Abergel, F. and Loeper, G. (2017). Option pricing and hedging with liquidity costs and market impact. In *Econophysics and Sociophysics: Recent Progress and Future Directions*, pages 19–40. Springer.
- ACP (2013). Orientations Nationales Complémentaires aux Spécifications Techniques pour l'exercice 2013 de préparation à Solvabilité II. Technical report, Banque de France.
- Agarwal, S., Rosen, R. J., and Yao, V. (2016). Why do borrowers make mortgage refinancing mistakes? *Management Science*, 62(12):3494–3509.
- Agnew, J., Balduzzi, P., and Sunden, A. (2003). Portfolio choice and trading in a large 401 (k) plan. *American Economic Review*, 93(1):193–215.
- AICPA (2013). Stable Value Funds and Investment. [https://perkinsaccounting.com/wp-content/uploads/2020/12/stable\\_value\\_funds\\_investment\\_contracts-Feb-2012.pdf](https://perkinsaccounting.com/wp-content/uploads/2020/12/stable_value_funds_investment_contracts-Feb-2012.pdf).
- Alfonsi, A., Cherchali, A., and Infante, J. (2019). A full and synthetic model for Asset-Liability Management in life insurance, and analysis of the SCR with the standard formula. *Available at SSRN 3431151*.
- Alfonsi, A., Schied, A., and Slynko, A. (2012). Order book resilience, price manipulation, and the positive portfolio problem. *SIAM Journal on Financial Mathematics*, 3(1):511–533.
- Alimoradian, B., Jakubiak, J. A., Loisel, S., and Salhi, Y. (2023). Understanding Key Drivers of Participant Cash Flows in Stable Value Wraps. *Risks*, xx:xx.
- Almgren, R. (2012). Optimal trading with stochastic liquidity and volatility. *SIAM Journal on Financial Mathematics*, 3(1):163–181.
- Almgren, R. and Chriss, N. (2001). Optimal execution of portfolio transactions. *Journal of Risk*, 3:5–40.
- Almgren, R. and Chriss, N. A. (1997). Optimal liquidation. *Available at SSRN 53501*.
- Almgren, R. and Li, T. M. (2016). Option hedging with smooth market impact. *Market microstructure and liquidity*, 2(01):1650002.

- Amendinger, J., Imkeller, P., and Schweizer, M. (1998). Additional logarithmic utility of an insider. *Stochastic processes and their applications*, 75(2):263–286.
- American Academy of Actuaries (2020). Economic Scenario Generators. <https://www.actuary.org/content/economic-scenario-generators>.
- American Academy of Actuaries Life Reserves and Capital Practice Note Work Group (2008). C3 Phase III practice note. [https://www.casact.org/library/studynotes/NAIC\\_White\\_Paper-Financial\\_Solvency\\_Framework.pdf](https://www.casact.org/library/studynotes/NAIC_White_Paper-Financial_Solvency_Framework.pdf).
- Andersen, S., Campbell, J. Y., Nielsen, K. M., and Ramadorai, T. (2015). Inattention and inertia in household finance: Evidence from the Danish mortgage market. *Available at SSRN 2627635*.
- Andersson, P. and Lagerås, A. N. (2013). Optimal bond portfolios with fixed time to maturity. *Insurance: Mathematics and Economics*, 53(2):429–438.
- Babbel, D. and Herce, M. (2018). An Update on Stable Value Funds Performance through 2017. *Journal of Financial Service Professionals*, 72(6).
- Babbel, D. F. and Herce, M. (2007). A closer look at stable value funds performance. Working Paper 07-21, Wharton Financial Institutions Center Working Paper.
- Bacinello, A. R., Millosovich, P., Olivieri, A., and Pitacco, E. (2011). Variable annuities: A unifying valuation approach. *Insurance: Mathematics and Economics*, 49(3):285–297.
- Bacinello, A. R., Olivieri, A., Pitacco, E., and Millosovich, P. (2010). Variable annuities: risk identification and risk assessment. *CAREFIN Research Paper*, 14.
- Ballotta, L., Haberman, S., and Wang, N. (2006). Guarantees in With-Profit and Unitized With-Profit Life Insurance Contracts: Fair Valuation Problem in Presence of the Default Option. *Journal of Risk and Insurance*, 73(1):97–121.
- Barclays (nd). Barclays Fixed income indices. <https://indices.cib.barclays/IM/12/en/indices/welcome.app>.
- Barger, W. and Lorig, M. (2019). Optimal liquidation under stochastic price impact. *International Journal of Theoretical and Applied Finance*, 22(2):1850059.
- Barsotti, F., Milhaud, X., and Salhi, Y. (2016). Lapse risk in life insurance: Correlation and contagion effects among policyholders' behaviors. *Insurance: Mathematics and Economics*, 71:317–331.
- Barucci, E., Colozza, T., Marazzina, D., and Rroji, E. (2020). The determinants of lapse rates in the Italian life insurance market. *European Actuarial Journal*, 10:149–178.

- Bauer, D., Kling, A., and Russ, J. (2008). A universal pricing framework for guaranteed minimum benefits in variable annuities. *ASTIN Bulletin: The Journal of the IAA*, 38(2):621–651.
- Baur, D. G. and Lucey, B. M. (2010). Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *Financial review*, 45(2):217–229.
- Black, F. and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3):637–654.
- Board of Governors of the Federal Reserve System (2020). Selected Interest Rates. <https://www.federalreserve.gov/releases/h15/data.htm>.
- Bordag, L. A. and Frey, R. (2008). Pricing options in illiquid markets: symmetry reductions and exact solutions. *Nonlinear Models in Mathematical Finance: New Research Trends in Option Pricing*, pages 103–129.
- Bouchaud, J.-P., Gefen, Y., Potters, M., and Wyart, M. (2003). Fluctuations and response in financial markets: the subtle nature of random price changes. *Quantitative finance*, 4(2):176.
- Brealey, R. A. and Myers, S. C. (2016). *Principles of Corporate Finance*. The Irwin/McGraw-Hill series in finance, insurance, and real estate. McGraw-Hill Education, 7 edition.
- Butrica, B. A. and Smith, K. E. (2016). 401 (k) participant behavior in a volatile economy. *Journal of Pension Economics & Finance*, 15(1):1–29.
- Caouette, J. B., Altman, E. I., Narayanan, P., et al. (1998). *Managing Credit Risk: The Next Great Financial Challenge*. John Wiley & Sons.
- Cboe Inc. (2022). Equity Options Product Specifications. [https://www.cboe.com/exchange\\_traded\\_stock/equity\\_options\\_spec](https://www.cboe.com/exchange_traded_stock/equity_options_spec), Accessed: 2022-01-30.
- Cetin, U., Jarrow, R., Protter, P., and Warachka, M. (2006). Pricing options in an extended Black Scholes economy with illiquidity: Theory and empirical evidence. *The Review of Financial Studies*, 19(2):493–529.
- Cetin, U., Jarrow, R. A., and Protter, P. (2010). Liquidity risk and arbitrage pricing theory. In *Handbook of Quantitative Finance and Risk Management*, pages 1007–1024. Springer.
- Chen, J. and Gupta, A. K. (2011). *Parametric statistical change point analysis: with applications to genetics, medicine, and finance*. Springer Science & Business Media.
- Chen, Q., Zhu, H., Yu, D., and Hau, L. (2022). How does investor attention matter for crude oil prices and returns? evidence from time-frequency



- quantile causality analysis. *The North American Journal of Economics and Finance*, 59:101581.
- Cheng, C., Hilpert, C., Miri Lavasani, A., and Schaefer, M. (2019). Surrender Contagion in Life Insurance: Modeling and Valuation. *Available at SSRN*.
- Chiang, T. C. and Zheng, D. (2010). An empirical analysis of herd behavior in global stock markets. *Journal of Banking & Finance*, 34(8):1911–1921.
- Choi, J. J., Laibson, D., Madrian, B. C., and Metrick, A. (2002). Defined contribution pensions: Plan rules, participant choices, and the path of least resistance. *Tax Policy and the Economy*, 16:67–113.
- Christensen, H. L. and Woodmansey, R. (2013). Prediction of Hidden Liquidity in the Limit Order Book of GLOBEX Futures. *The Journal of Trading*, 8(3):68–95.
- Cont, R., Kukanov, A., and Stoikov, S. (2013). The Price Impact of Order Book Events. *Journal of Financial Econometrics*, 12(1):47–88.
- Cont, R. and Schaanning, E. (2017). Fire sales, indirect contagion and systemic stress testing. *Indirect Contagion and Systemic Stress Testing (June 13, 2017)*.
- Credit rating (2023). Credit rating — Wikipedia, the free encyclopedia. [Online; accessed 9-September-2023].
- Cumming, D. and Dai, N. (2009). Capital flows and hedge fund regulation. *Journal of Empirical Legal Studies*, 6(4):848–873.
- Dar, A. and Dodds, C. (1989). Interest rates, the emergency fund hypothesis and saving through endowment policies: Some empirical evidence for the uk. *Journal of Risk and Insurance*, pages 415–433.
- De Giovanni, D. (2010). Lapse rate modeling: a rational expectation approach. *Scandinavian Actuarial Journal*, 2010(1):56–67.
- Deng, Y. and Quigley, J. M. (2012). Woodhead behavior and the pricing of residential mortgages. *NUS Institute of Real Estate Studies Working Paper Series IRES2012-025*.
- Dinh, H. T. (2023). Lessons from the silicon valley bank crisis. Technical report, Policy Center for the New South.
- Doléans-Dade, C. (1970). Quelques applications de la formule de changement de variables pour les semimartingales. *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, 16:181–194.
- Dorn, D. and Huberman, G. (2005). Talk and action: What individual investors say and what they do. *Review of Finance*, 9(4):437–481.

- Døskeland, T. M. and Nordahl, H. A. (2008). Optimal pension insurance design. *Journal of Banking & Finance*, 32(3):382–392.
- Duffee, G. R. (1998). The relation between treasury yields and corporate bond yield spreads. *The Journal of Finance*, 53(6):2225–2241.
- Eberhardt, W., Brügggen, E., Post, T., and Hoet, C. (2021). Engagement behavior and financial well-being: The effect of message framing in online pension communication. *International Journal of Research in Marketing*, 38(02):448–471.
- EIOPA (2015). Guidelines on own risk and solvency assessment. [https://www.eiopa.europa.eu/system/files/2022-10/eiopa\\_guidelines\\_on\\_orsa\\_en.pdf](https://www.eiopa.europa.eu/system/files/2022-10/eiopa_guidelines_on_orsa_en.pdf).
- El-Shagi, M., Knedlik, T., and von Schweinitz, G. (2013). Predicting financial crises: The (statistical) significance of the signals approach. *Journal of International Money and Finance*, 35:76–103.
- Eling, M. and Kochanski, M. (2013). Research on lapse in life insurance: what has been done and what needs to be done? *The Journal of Risk Finance*, 14(4):392–413.
- Erisa (2020). Employee Retirement Income Security Act (ERISA). <https://www.dol.gov/general/topic/retirement/erisa>.
- European Commission (2002). Directive 2002/83/EC concerning life assurance.
- European Parliament and of the Council (2014). Regulation (EU) No 596/2014 on market abuse (market abuse regulation).
- Eyraud-Loisel, A. (2005). Backward stochastic differential equations with enlarged filtration: Option hedging of an insider trader in a financial market with jumps. *Stochastic processes and their Applications*, 115(11):1745–1763.
- Eyraud-Loisel, A. (2013). Quadratic hedging in an incomplete market derived by an influential informed investor. *Stochastics An International Journal of Probability and Stochastic Processes*, 85(3):412–430.
- Fabozzi, F. J. (1998). *The Handbook of Stable Value Investments*, volume 38. John Wiley & Sons.
- Fabozzi, F. J. and Mann, S. V. (2012). *The handbook of fixed income securities*. McGraw-Hill Education.
- FASB (1998). Statement of Financial Accounting Standards No. 133.
- FASB (2005). FSP AAG INV-1 & SOP 94-4-1.
- FASB (2022). Derivatives and Hedging (Topic 815).

- Federal Reserve Bank of St. Louis (2020). Moody's Seasoned Baa Corporate Bond Yield. <https://fred.stlouisfed.org/series/BAA>.
- Financial Conduct Authority (2005). FCA Handbook : MAR 1.6.15 Market abuse .
- Floryszczak, A., Le Courtois, O., and Majri, M. (2016). Inside the solvency 2 black box: net asset values and solvency capital requirements with a least-squares monte-carlo approach. *Insurance: Mathematics and Economics*, 71:15–26.
- Föllmer, H., Schweizer, M., et al. (1990). *Hedging of contingent claims under incomplete information*. Rheinische Friedrich-Wilhelms-Universität Bonn.
- Fruth, A., Schöneborn, T., and Urusov, M. (2019). Optimal trade execution in order books with stochastic liquidity. *Mathematical Finance*, 29(2):507–541.
- Gabaix, X. (2019). Behavioral inattention. In Bernheim, B. D., DellaVigna, S., and Laibson, D., editors, *Handbook of Behavioral Economics – Foundations and Applications 2*, Handbook of Behavioral Economics, chapter 4, pages 261–343. Elsevier.
- Gale, W. G., John, D. C., and Kim, B. (2016). You Get What You Pay For: Guaranteed Returns in Retirement Saving Accounts. *Brookings Economic Studies*, 401.
- Gatheral, J. (2010). No-dynamic-arbitrage and market impact. *Quantitative finance*, 10(7):749–759.
- Gatheral, J., Schied, A., and Slynko, A. (2012). Transient Linear Price Impact and Fredholm Integral Equations. *Mathematical Finance*, 22(3):445–474.
- Glover, K. J., Duck, P. W., and Newton, D. P. (2010). On nonlinear models of markets with finite liquidity: some cautionary notes. *SIAM Journal on Applied Mathematics*, 70(8):3252–3271.
- Goecke, O. (2013). Pension saving schemes with return smoothing mechanism. *Insurance: Mathematics and Economics*, 53(3):678–689.
- Grinold, R. C. and Kahn, R. N. (2000). *Active portfolio management*. McGraw Hill New York.
- Grorud, A. and Pontier, M. (1998). Insider trading in a continuous time market model. *International Journal of Theoretical and Applied Finance*, 1(03):331–347.
- Guéant, O. (2013). Permanent market impact can be nonlinear. *arXiv preprint arXiv:1305.0413*.

- Guéant, O. and Pu, J. (2017). Option pricing and hedging with execution costs and market impact. *Mathematical Finance*, 27(3):803–831.
- Hassani, H. and Silva, E. S. (2015). A kolmogorov-smirnov based test for comparing the predictive accuracy of two sets of forecasts. *Econometrics*, 3(3):590–609.
- Hawkins, D. M. (1977). Testing a sequence of observations for a shift in location. *Journal of the American Statistical Association*, 72(357):180–186.
- Hillairet, C. (2004). *Equilibres sur un marché financier avec asymétrie d'information et discontinuité des prix*. PhD thesis, Université Paul Sabatier-Toulouse III.
- Hirshleifer, D. and Hong Teoh, S. (2003). Herd behaviour and cascading in capital markets: A review and synthesis. *European Financial Management*, 9(1):25–66.
- Huberman, G. and Stanzl, W. (2004). Price manipulation and quasi-arbitrage. *Econometrica*, 72(4):1247–1275.
- Investment Company Institute (2008). Retirement Saving in Wake of Financial Market Volatility.
- IRS. Individual Retirement Arrangements. <https://www.irs.gov/retirement-plans/individual-retirement-arrangements-iras>.
- IRS. Profit Sharing Plan. <https://www.irs.gov/retirement-plans/choosing-a-retirement-plan-profit-sharing-plan>.
- IRS (2023a). Retirement Plan Distributions: Exceptions to 10% Additional Tax. <https://www.irs.gov/pub/irs-pdf/p5036.pdf>.
- IRS (2023b). 401k Resource Guide Plan Participants General Distribution Rules. <https://www.irs.gov/retirement-plans/plan-participant-employee/401k-resource-guide-plan-participants-general-distribution-rules>.
- IRS (2023c). Retirement Topics - Hardship Distributions. <https://www.irs.gov/retirement-plans/plan-participant-employee/retirement-topics-hardship-distributions..>
- Jarrow, R., Fung, S., and Tsai, S.-C. (2018). An empirical investigation of large trader market manipulation in derivatives markets. *Review of Derivatives Research*, 21(3):331–374.
- Ji, Q., Zhang, D., and Zhao, Y. (2020). Searching for safe-haven assets during the covid-19 pandemic. *International Review of Financial Analysis*, 71:101526.

- Jonsson, M. and Sircar, K. R. (2002). Partial hedging in a stochastic volatility environment. *Mathematical Finance*, 12(4):375–409.
- Kalantonis, P., Kallandranis, C., and Sotiropoulos, M. (2021). Leverage and firm performance: new evidence on the role of economic sentiment using accounting information. *Journal of Capital Markets Studies*.
- Kim, C. (2005). Modeling surrender and lapse rates with economic variables. *North American Actuarial Journal*, 9(4):56–70.
- Kling, A., Richter, A., and Ruß, J. (2007). The impact of surplus distribution on the risk exposure of with profit life insurance policies including interest rate guarantees. *Journal of Risk and Insurance*, 74(3):571–589.
- Klöck, F. (2012). Regularity of Market Impact Models with Stochastic Price.
- Klöck, F., Schied, A., and Sun, Y. (2014). Price manipulation in a market impact model with dark pool.
- Knoller, C., Kraut, G., and Schoenmaekers, P. (2016). On the propensity to surrender a variable annuity contract: An empirical analysis of dynamic policyholder behavior. *Journal of Risk and Insurance*, 83(4):979–1006.
- Kuo, W., Tsai, C., and Chen, W.-K. (2003). An empirical study on the lapse rate: The cointegration approach. *Journal of Risk and Insurance*, 70(3):489–508.
- Kwun, D., Mohebbi, C., Braunstein, M., and Line, A. (2010). A risk analysis of 401k stable value funds. *International Journal of Applied Decision Sciences*, 3(2):151–167.
- Kwun, D., Mohebbi, C., Line, A., and Tsai, Y.-T. (2009). A risk analysis of stable value protection for bank-owned life insurance. *International Journal of Applied Decision Sciences*, 2(4):406–421.
- Lachance, M.-E., Mitchell, O. S., and Smetters, K. (2003). Guaranteeing defined contribution pensions: the option to buy back a defined benefit promise. *Journal of Risk and Insurance*, 70(1):1–16.
- Liu, H. and Yong, J. (2005). Option pricing with an illiquid underlying asset market. *Journal of Economic Dynamics and Control*, 29(12):2125–2156.
- Loeper, G. et al. (2018). Option pricing with linear market impact and nonlinear black–scholes equations. *Annals of Applied Probability*, 28(5):2664–2726.
- Loisel, S. and Milhaud, X. (2011). From deterministic to stochastic surrender risk models: Impact of correlation crises on economic capital. *European Journal of Operational Research*, 214(2):348–357.

- Longstaff, F. A. and Schwartz, E. S. (2001). Valuing American options by simulation: a simple least-squares approach. *The Review of Financial Studies*, 14(1):113–147.
- Lyubchich, V., Gel, Y. R., and El-Shaarawi, A. (2013). On detecting non-monotonic trends in environmental time series: A fusion of local regression and bootstrap. *Environmetrics*, 24(4):209–226.
- Ma, G., Siu, C. C., Zhu, S.-P., and Elliott, R. J. (2020). Optimal portfolio execution problem with stochastic price impact. *Automatica*, 112:108739.
- Maćkowiak, B., Matějka, F., and Wiederholt, M. (2021). Rational inattention: A review. *ECB Working Paper*.
- Madrian, B. C. and Shea, D. F. (2001). The power of suggestion: Inertia in 401(k) participation and savings behavior. *The Quarterly journal of economics*, 116(4):1149–1187.
- Mahdavi-Damghani, B. (2012). Utope-ia. *Wilmott*, 2012(60):28–37.
- Marcus, A. J. (1987). Corporate Pension Policy and the Value of PBGC Insurance. In Wise, D. A., Shoven, J. B., and Bodie, Z., editors, *Issues in Pension Economics*, chapter 3, pages 49–80. University of Chicago Press.
- Mark Henricks (2022). Ginnie Mae vs. Fannie Mae: Key Differences. <https://smartasset.com/mortgage/ginnie-mae-vs-fannie-mae>.
- MetLife (2022). 2022 Stable Value Study. <https://www.metlife.com/retirement-and-income-solutions/insights/stable-value-study/>.
- Milevsky, M. A. and Salisbury, T. S. (2006). Financial valuation of guaranteed minimum withdrawal benefits. *Insurance: Mathematics and Economics*, 38(1):21–38.
- Miltersen, K. R. and Persson, S.-A. (1999). Pricing rate of return guarantees in a Heath–Jarrow–Morton framework. *Insurance: Mathematics and Economics*, 25(3):307–325.
- Miltersen, K. R. and Persson, S.-A. (2003). Guaranteed investment contracts: Distributed and undistributed excess return. *Scandinavian Actuarial Journal*, 2003(4):257–279.
- Mitchell, O. S., Mottola, G. R., Utkus, S. P., and Yamaguchi, T. (2006). The Inattentive Participant: Portfolio Trading Behavior in 401(K) Plans. Working Paper WP 2006-115, Michigan Retirement Research Center.
- Mitchell, O. S. and Utkus, S. P. (2022). Target-date funds and portfolio choice in 401(k) plans. *Journal of Pension Economics & Finance*, 21(4):519–536.

- Munnell, A. H., Golub-Sass, A., Kopcke, R. W., and Webb, A. (2009). What does it cost to guarantee returns? Report 9-4, Center for Retirement Research at Boston College.
- NAIC (2008). NAIC White Paper, the U.S. National state-based system of insurance financial regulation and the solvency modernization initiative. [https://www.actuary.org/sites/default/files/files/C3\\_Phase\\_III.4.pdf/C3\\_Phase\\_III.4.pdf](https://www.actuary.org/sites/default/files/files/C3_Phase_III.4.pdf/C3_Phase_III.4.pdf).
- NAIC (2012). 2012 NAIC Life Risk-Based Capital Report.
- NAIC (2015). MDL-695 Synthetic Guaranteed Investment Contracts Model Regulation.
- NAIC (2017). MDL-200 Separate Accounts Funding Guaranteed minimum benefits under group contracts model regulation.
- NAIC (2018). Valuation Manual. [https://www.naic.org/documents/prod\\_serv\\_2018\\_valuation\\_manual.pdf](https://www.naic.org/documents/prod_serv_2018_valuation_manual.pdf).
- NAIC (2021). 2021 NAIC Life and Fraternal Risk-Based Capital Report.
- New York State (1999). New York State Regulation 128: Separate Account Agreements.
- Outreville, J. F. (1990). Whole-life insurance lapse rates and the emergency fund hypothesis. *Insurance: Mathematics and Economics*, 9(4):249–255.
- Persson, S.-A. and Aase, K. K. (1997). Valuation of the minimum guaranteed return embedded in life insurance products. *Journal of Risk and Insurance*, pages 599–617.
- Phillips Jr, J. M., Schweig, B. B., and Scott, J. P. (1985). Explaining whole life insurance lapse rates. *The Journal of Insurance Issues and Practices*, pages 32–40.
- Plansponsor Staff (2016). Most Participants Do Not Read Benefits Communications. <https://www.plansponsor.com/most-participants-do-not-read-benefits-communications/>.
- PWC (2011). Unfinished Business: Dodd Frank entering year two. <http://www.pwc.com/us/en/financial-services/regulatory-services/publications/assets/closer-look-dodd-frank-year-two.pdf>.
- Ringer, N. and Tehranchi, M. (2006). Optimal portfolio choice in the bond market. *Finance and Stochastics*, 10:553–573.
- Risk Management Task Force (2004). Risk Metric Definitions: Option-Adjusted Spread (OAS).

- Rogers, L. C. G. and Shi, Z. (1995). The value of an asian option. *Journal of Applied Probability*, 32(4):1077–1088.
- Roseman, B. and Gaskel, G. (2007). Criteria Revised For Determining Risk-Based Capital Charges For Synthetic GICs. Technical report, Standard & Poors.
- Said, E. (2019). How Option Hedging Shapes Market Impact. *arXiv preprint arXiv:1910.05056*.
- Said, E. (2020). *Market Impact in Systematic Trading and Option Pricing*. PhD thesis, Université Paris-Saclay.
- Salhi, Y. and Thérond, P. (2014). Alarm system for credit losses impairment. Available at SSRN 2378374.
- Schiff, J. L. (1999). *The Laplace transform: theory and applications*. Springer Science & Business Media.
- Shin, H. S. (2009). Reflections on northern rock: The bank run that heralded the global financial crisis. *Journal of economic perspectives*, 23(1):101–119.
- Sierra Jimenez, J. A. (2012). Consumer interest rates and retail mutual fund flows. Working paper 2012-39, Bank of Canada.
- Sims, C. A. (2003). Implications of rational inattention. *Journal of Monetary Economics*, 50(03):665–690.
- Slovic, P., Fischhoff, B., and Lichtenstein, S. (1978). Accident probabilities and seat belt usage: A psychological perspective. *Accident Analysis & Prevention*, 10(4):281–285.
- Stable Value (2022). The Facts About Stable Value Funds. <https://www.stablevalue.org/the-facts-about-stable-value-funds/>. Accessed: August 2022.
- Stable Value Investment Association (2020). Stable Value FAQ. [https://www.stablevalue.org/media/misc/Stable\\_Value\\_FAQ](https://www.stablevalue.org/media/misc/Stable_Value_FAQ).
- Stigler, G. (1961). The economics of information. *Journal of Political Economy*, 69(03):213–225.
- SVIA (2019a). Investment Contract Basics. Stable value investments synthetic, SVIA.
- SVIA (2019b). SVIA Investment & Policy Survey for 2018. Annual investment & policy report, SVIA.
- SVIA (2020a). Stable Value at a Glance. <https://www.stablevalue.org/knowledge/stable-value-at-a-glance>.



- SVIA (2020b). Stable Value Quarterly Characteristic Survey.
- Tang, N., Mitchell, O. S., Mottola, G. R., and Utkus, S. P. (2010). The efficiency of sponsor and participant portfolio choices in 401 (k) plans. *Journal of Public Economics*, 94(11-12):1073–1085.
- Tankov, P. and Voltchkova, E. (2009). Jump-diffusion models: a practitioner's guide. *Banque et Marchés*, 99(1):24.
- Tobe, C. B. (2004). The Consultants Guide to Stable Value. *Journal of Investment Consulting*, 7(1).
- Tsai, C., Kuo, W., and Chen, W.-K. (2002). Early surrender and the distribution of policy reserves. *Insurance: Mathematics and Economics*, 31(3):429–445.
- U.S. Department of Labor (2020). Model Comparative Chart - ABC Corporation 401k Retirement Plan. <https://www.dol.gov/sites/dolgov/files/ebsa/about-ebsa/our-activities/resource-center/publications/providing-information-in-participant-directed-plans-model-chart.pdf>.
- US Department of Labor (2023). Types of Retirement Plans. <https://www.dol.gov/general/topic/retirement/typesofplans>.
- van Bilsen, S. and Linders, D. (2019). Affordable and adequate annuities with stable payouts: Fantasy or reality? *Insurance: Mathematics and Economics*, 86:19–42.
- Vo, L. V. and Le, H. T. (2023). From hero to zero-the case of silicon valley bank. Available at SSRN 4394553.
- Wang, L., Akritas, M. G., and Van Keilegom, I. (2008). An anova-type non-parametric diagnostic test for heteroscedastic regression models. *Journal of Nonparametric Statistics*, 20(5):365–382.
- Xiong, J. X. and Idzorek, T. M. (2012). Estimating credit risk and illiquidity risk in guaranteed investment Products. *Journal of Financial Planning*, 25(7):38–47.
- Yang, L. (2022). Idiosyncratic information spillover and connectedness network between the electricity and carbon markets in europe. *Journal of Commodity Markets*, 25:100185.
- Yang, L. and Hamori, S. (2021). The role of the carbon market in relation to the cryptocurrency market: Only diversification or more? *International Review of Financial Analysis*, 77:101864.
- Yang, L., Tian, S., Yang, W., Xu, M., and Hamori, S. (2018). Dependence structures between chinese stock markets and the international financial market: Evidence from a wavelet-based quantile regression approach. *The North American Journal of Economics and Finance*, 45:116–137.

- Young, J. (2018). How America Saves 2018: Vanguard 2017 Defined Contribution Plan Data. Report, Vanguard Center for Investor Research.
- Zarinelli, E., Treccani, M., Farmer, J. D., and Lillo, F. (2015). Beyond the square root: Evidence for logarithmic dependence of market impact on size and participation rate. *Market Microstructure and Liquidity*, 1(02):1550004.