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# Strategic information provision in multidimensional environments

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INSTITUT  
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DE PARIS

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Thèse de doctorat



# Strategic information provision in multidimensional environments

Thèse de doctorat de l'Institut Polytechnique de Paris  
préparée à l'École Polytechnique

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Spécialité de doctorat : Sciences économique

Thèse présentée et soutenue à Palaiseau, le 10 Juin 2024, par

**MARGARITA KIRNEVA**

Composition du Jury :

Alessandro Riboni Professor, École Polytechnique	Président
Sidhartha Gordon Professor, University Paris-Dauphine	Rapporteur
Bruno Strulovici Professor, Northwestern University	Rapporteur
Galit Ashkenazi-Golan Assistant Professor, London School of Economics	Examinatrice
Olivier Gossner Professor, École Polytechnique	Directeur de thèse
Matías Núñez CNRS Research Fellow, École Polytechnique	Co-directeur de thèse

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# Summary

The quality of individual and collective decisions depends significantly on the amount of information available, on the ability to acquire it and on the knowledge of the incentives and preferences of other agents. This thesis aims to study how information is communicated and learned in strategic multidimensional environments, and what can be done to reduce the strategic motives and to promote truthful information provision.

The thesis can be divided in 2 parts. The first part, consisting of 2 chapters, both joint with Matías Núñez, deals with the aggregation of information regarding preference in committees in varied settings. It suggests different mechanisms for committee voting which have some desirable properties which most of the currently used mechanisms lack, and are significantly easier for the committee members than some classic mechanisms in the literature. The second part of the thesis studies the effects of the multidimensionality of the environments and of an imperfect access to information on strategic communication (Chapter 3).

Chapter 1 considers the problem of preference aggregation in a small committee which needs to choose one alternative from the set of many such as a hiring committees. We propose the first class of simultaneous voting mechanisms in which each Nash equilibrium is coalition-proof thus preventing coordination failures. The paper then studies necessary conditions for arbitrary mechanisms to implement a Pareto efficient rule ensuring that each equilibrium is coalition-proof and shows that the presence of veto rights in the mechanism is unavoidable to achieve this double implementation notion.

Chapter 2 considers the settings in which a committee needs to choose one alternative from the set of two which is of a particular relevance for the variety of political institutions such as the European Parliament or the US Congress. We design two mechanisms, one simultaneous and one sequential, that ensure that the majority preferred option wins in all equilibria. The existence of the simultaneous mechanism overcomes recent impossibility results concerning the implementation of majority rule. The sequential version of the mechanism allows to reach the (correct) outcome in significantly

fewer steps than the widely used roll call voting. The results extend to the settings in which voter can abstain or have incomplete information about the preferences of others.

Chapter 3 focuses on strategic communication in multidimensional environments. It considers a multidimensional Sender-Receiver game in which Receiver can acquire limited information after observing the Sender's signal. Depending on the parameters describing the conflict of interest between Sender and Receiver, it characterises optimal information disclosure and the information acquired by Receiver as a response. The chapter shows that in the case of partial conflict of interests (aligned on some dimensions and misaligned on others) Sender uses the multidimensionality of the environment to divert Receiver's attention away from the dimensions of misalignment of interests. The paper present applications to consumer's choice and informational lobbying.

# Resumé

La qualité des décisions individuelles et collectives dépend de manière significative de la quantité d'informations disponibles, de la capacité à les acquérir et de la connaissance des incitations et des préférences des autres agents. Cette thèse vise à étudier comment l'information est communiquée et apprise dans des environnements stratégiques multidimensionnels, et ce qui peut être fait pour réduire les motivations stratégiques et promouvoir la fourniture d'informations véridiques.

La thèse peut être divisée en deux parties. La première partie, composée de deux chapitres, tous deux coécrits avec Matías Núñez, traite de l'agrégation de l'information concernant les préférences au sein des comités dans divers contextes. Elle propose différents mécanismes de vote pour les comités, qui possèdent certaines propriétés souhaitables que la plupart des mécanismes actuellement utilisés n'ont pas, et qui sont significativement plus faciles à utiliser pour les membres du comité que certains mécanismes classiques de la littérature.

La deuxième partie de la thèse étudie les effets de la multidimensionnalité des environnements et d'un accès imparfait à l'information sur la communication stratégique (Chapitre 3).

Le chapitre 1 considère le problème de l'agrégation des préférences dans un petit comité qui doit choisir une alternative parmi un ensemble de nombreuses, comme c'est le cas des comités de recrutement. Nous proposons la première classe de mécanismes de vote simultanés dans lesquels chaque équilibre de Nash est insensible aux coalitions, empêchant ainsi les échecs de coordination. L'article étudie ensuite les conditions nécessaires pour que des mécanismes arbitraires mettent en oeuvre une règle Pareto efficace en s'assurant que chaque équilibre soit insensible aux coalitions et montre que la présence de droits de veto dans le mécanisme est inévitable pour atteindre cette double notion d'implémentation.

Le chapitre 2 s'intéresse aux situations dans lesquelles un comité doit choisir une alternative parmi deux, ce qui est particulièrement pertinent pour diverses institutions

politiques telles que le Parlement européen ou le Congrès américain. Nous concevons deux mécanismes, l'un simultané et l'autre séquentiel, qui garantissent que l'option préférée par la majorité l'emporte dans tous les équilibres. L'existence du mécanisme simultané surmonte les résultats récents d'impossibilité concernant la mise en oeuvre de la règle majoritaire. La version séquentielle du mécanisme permet d'atteindre le résultat (correct) en beaucoup moins d'étapes que le vote par appel nominal largement utilisé. Les résultats s'étendent aux situations où les électeurs peuvent s'abstenir ou ont une information incomplète sur les préférences des autres.

Le chapitre 3 se concentre sur la communication stratégique dans des environnements multidimensionnels. Il considère un jeu de l'expéditeur et du récepteur multidimensionnel dans lequel le récepteur peut acquérir des informations limitées après avoir observé le signal de l'expéditeur. En fonction des paramètres décrivant le conflit d'intérêt entre l'expéditeur et le récepteur, il caractérise la divulgation optimale de l'information et les informations acquises par le récepteur en réponse. Le chapitre montre que dans le cas d'un conflit d'intérêts partiel (aligné sur certaines dimensions et non aligné sur d'autres), l'expéditeur utilise la multidimensionnalité de l'environnement pour détourner l'attention du récepteur des dimensions où les intérêts sont non alignés. Le chapitre applique ensuite le cadre théorique au contexte d'un consommateur qui choisit une consommation optimale de plusieurs biens tout en étant incertain quant à la qualité des biens. Il démontre comment le comportement du consommateur est affecté par les motivations de diversion d'attention d'un publicitaire. Par exemple, dans le cas d'un alignement partiel des intérêts, la décision du consommateur concernant le budget total alloué est mieux informée, tandis que la décision sur la consommation relative des biens est inférieure à une situation de référence sans publicitaire.



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# General Introduction

This thesis belongs to the field of Microeconomic Theory. More precisely, it consists of two parts: the first part (Chapters 1 and 2) analyses the questions of the design of voting mechanisms while the second part (Chapter 3) deals with information design in strategic complex settings.

## Chapter 1 - Voting by Simultaneous Vetoes

This chapter, joint work with Matías Núñez, provides a new class of mechanisms for committee voting in the settings when one alternative from the set of many has to be chosen. Examples of such committees include hiring committees, ECB Governing Council etc. While generally believed to improve decisions due to aggregation of information and preferences committees offer fail to generate an efficient outcome. It has been shown theoretically (see for instance [Hurwicz and Schmeidler \(1978\)](#) and [Maskin \(1999b\)](#)) that none of the usually used voting procedures ensures the Pareto-efficiency of the outcome which would be the minimum requirement. Moreover, none of the usually used voting mechanisms is coalition-proof, thus, making it vulnerable to manipulations by groups of committee members.

**Mechanism.** In the first part of the paper we introduce a novel class of mechanisms, simultaneous veto (SV) mechanisms. They proceed as follows. Each member of the committee is endowed with some fixed amount of veto rights with the total number of veto rights being fixed at the number of alternatives to choose from minus 1. Committee members are asked to simultaneously announce the set of alternatives to veto with the size of the announced set being equal to the number of veto rights assigned. The outcome of the mechanism is a random draw from the set of non-vetoed alternatives. We show that for any eligible distribution of the endowment of veto rights such mechanism implements a corresponding Pareto-efficient Veto-by-random-priority (VRP) rule firstly introduced in [Moulin \(1983\)](#). Thus, the mechanisms achieve what we view as the minimal requirement on efficiency. Moreover, we show that each Nash equilibrium of such

mechanisms is coalition-proof. We denote this new double implementation in Nash and coalition-proof equilibria as coalitional implementation.

**Coalitional implementation.** In the second part of the paper we study coalitional implementation. Firstly, we ask which conditions a mechanism should satisfy to coalitionally implement some Pareto-efficient rule. While we do not provide the full characterization we show that the veto rights are necessary for such demanding implementation notion. Secondly, we consider which rules can be coalitionally implemented by some mechanism. We restrict our attention to the mechanisms which exhibit a No Gain from Cooperation (NGC) property which can be interpreted as a simplicity requirement, and show that any rule which can be coalitionally implemented by such mechanisms is a subset of the VRP rules.

**Contribution.** The novelty of the mechanisms proposed in the paper is the use of lotteries as possible outcomes to increase pivotality of voters and, thus, to incentivize truthful voting. Note, however, that the mechanisms are built in such way that the equilibrium outcome is always deterministic, that is the lotteries occur as outcomes only off equilibrium. This differs our approach from the virtual implementation literature (see, for instance, [Bochet and Maniquet \(2010\)](#)) which randomizes in equilibrium. We also consider a possibility of coalitional implementation of Pareto-efficient rules through deterministic mechanisms and provide an impossibility result. Thus, off-equilibrium lotteries are the key to achieve the desirable outcome.

Moreover, the implementation notion suggested (coalitional implementation) is novel and of a particular interest. It allows to avoid inefficiencies arising from voters' communication, thus, making the equilibrium outcomes more robust.

## **Chapter 2 - Legitimacy of collective decisions: a mechanism design approach**

This chapter, also a joint work with Matías Núñez, considers binary decisions - cases in which a committee of voters needs to choose one alternative from the set of two. This framework is of an extreme practical relevance as it incorporates such examples as political institutions (the EU Parliament, the US Congress), boards of directors and so on. It was pointed out in the existing literature that the decisions of such committees often end up being non-optimal in the sense that the chosen outcome does not correspond to the alternative preferred by the majority of the committee members. Several reasons are commonly considered as the potential source for such inefficiencies: abstention by the voters, conformity motives, uncertainty about the preferences of others. Even more

surprisingly, on the theoretical side there are almost no known mechanisms which can ensure (in the sense of Nash implementation) the majority preferred alternative to be elected in any equilibrium for any preference profile\*. In this paper we provide theoretical solutions to the problem. Firstly, we propose a simultaneous mechanism which Nash implements majority rule while being significantly less complicated than ones which exist in the literature so far. By doing this we overcome a recent impossibility presented in [Xiong \(2021\)](#) which shows that no voting mechanism can achieve Nash implementation of the simple majority. We show that by enlarging the class of voting mechanisms by allowing one additional element in voter's strategies, the impossibility does not hold anymore. Secondly, we consider a sequential version of the mechanism which is easier for voters while allows to reach the outcome at least almost twice quicker than the roll call vote procedure often used in the settings of political institutions.

**Simultaneous mechanism.** The first mechanism we introduce - the bloc formation (or, shortly, BF) mechanism, Nash implements majority rule through a one stage game. Denote by  $I$  the set of voters with the total size  $n = 2p + 1$ , and by  $A = \{a, b\}$  - the set of alternatives. The mechanism proceeds as follows: each voter is asked to submit a vote consisting of an alternative and to nominate a subset of other voters of a size  $p$ . That is a vote of voter  $i$  is a pair  $(v_i, c_i)$  with  $v_i \in A$  and  $c_i \subset I$  such that  $|c_i| = p$ . We say that a bloc in favor of one of the alternatives is formed if there is a majority group of voters (of size at least  $p + 1$ ) who all vote for this alternative and all nominate only the members of this group. If there is a bloc in favor of an alternative, this alternative is the final outcome. If the bloc was not formed the outcome is a lottery with the weight of an alternative increasing in the number of voters who voted for it and in the popularity of these voters (i.e. how often they were nominated by others). The Figure below gives an example of profiles in which no bloc is formed (Figure 1 (a)), and a bloc is formed (Figure 1 (b)). The circles illustrate voters,  $a, b$  - their votes and the arrows - the profile of nominations.

As in the first chapter of the thesis, the lotteries are introduced to increase the pivotality and, thus, to give the voters strict incentives to communicate their true preferences. In this mechanism each voter can increase probability of her favorite alternative to be selected by voting for it if she is nominated by any other voter or by nominating other voters who vote for this alternative. Similarly to the previous chapter, lotteries occur only off equilibrium. In any equilibrium profile a bloc in favor of the majority preferred alternative is formed.

**Sequential mechanisms.** In the second part of the paper we extend the framework

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\*The only known mechanisms which satisfy this property are integer games (see, for instance, [Maskin \(1999b\)](#)). However, their structure is too complicated to make their usage in practice feasible.

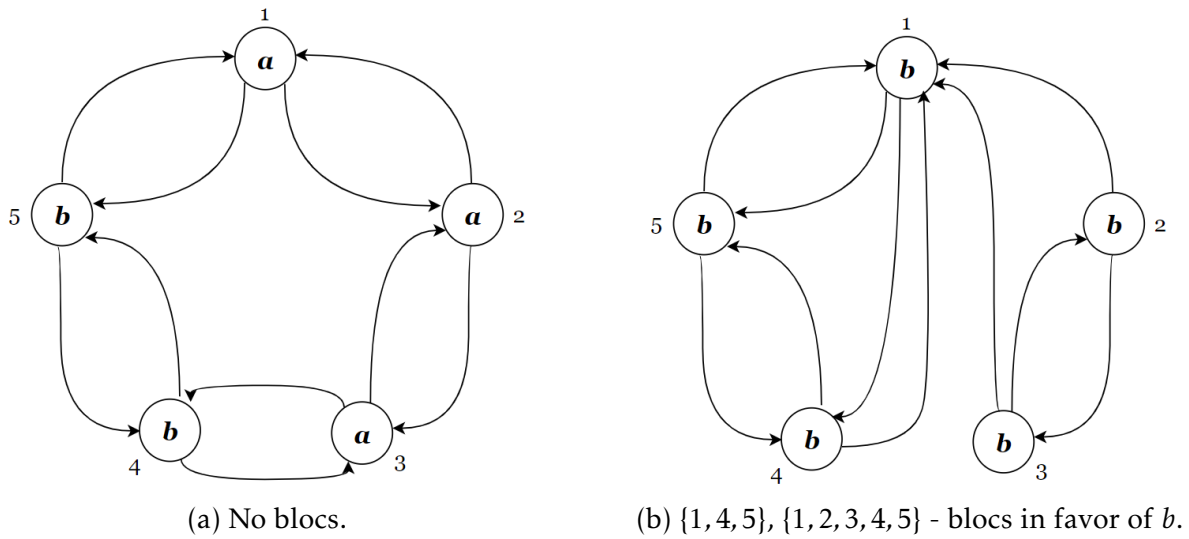


Figure 1: Voting profiles formed by the BF mechanism.

and consider a mechanism with multiple stages. There are two benefits in doing this compared to the one-stage BF mechanism. First of all, it allows for a simplification of the strategy space and, thus, makes the mechanism more transparent for voters. Secondly, the BF mechanism is quite demanding if one considers possible relaxation of the assumptions. By moving towards sequential voting procedures we allow voters to abstain and/or to have incomplete information about the preferences of others which is of importance for many real-life settings. Note that the sequential voting procedures are broadly used in some cases, for instance, a Roll-call vote in the US Senate. However, the procedure requires every committee member to vote which is time consuming. Our mechanism allows to reduce significantly the time necessary to reach the decision while keeping the majority preferred alternative the unique equilibrium outcome. The mechanism proceeds as follows: at the first stage each voter votes for one of the alternatives generating a majority winner of the first stage. At the second stage a group of voters is randomly chosen to ratify the outcome. The size of the group decreases with the support for the majority winning alternative of the first stage. The selected voters are asked one by one to ratify the first stage by submitting "Yes" or "No", and the procedure stops as soon as one of the voters submits "Yes". If none of the voters agreed the outcome is the lottery proportional to the first stage votes. We show that such mechanism subgame perfectly implements the majority rule while requiring (in the longest scenario) almost twice as little steps as the roll-call vote. Moreover, we show that the mechanism can be used to implement any supermajority rule with the corresponding adjustments.

**Contribution.** The mechanisms presented provide important theoretical insights by possessing desirable properties (theoretically) and remaining simple enough to be tested experimentally and potentially used in practice. Importantly, compared to some existing works which introduce randomization to achieve almost perfect implementation (see, for instance, [Laslier and Weibull \(2013\)](#), [Núñez and Pivato \(2019\)](#) and [Azevedo and Budish \(2019\)](#)), our mechanisms do not randomize equilibrium even though the lotteries are possible outside of the equilibria. The next step in this work is to bring the proposed mechanism closer to practice by testing them experimentally and comparing their performance to the performance of the widely used existing mechanisms in the lab.

### **Chapter 3 - Informing to Divert Attention**

This chapter considers a strategic communication between an informed expert and an uninformed decision-maker in multidimensional environments. The interaction of this type are a crucial part of everyday life. One can think of a hiring committee, lobbyist - policy-makers interactions, consumers in the presence of advertising and many others. All the above mentioned examples share two important properties: the decision problem is complex in the sense that there are several decision to make, and the decision-maker can access additional information after observing the one provided by the expert. In this case the expert's information has two potential effects on the decisions made. First of all, it directly informs the expert about the state of the world making the decisions better-informed. However, there is a second potential effect of the expert's information which is that the attention she gives to different issues affects the amount of attention the decision-maker will give to different issues in her own search for information. Thus, the expert's information directs decision-maker's search. This chapter studies the interaction between these two effects depending on the conflict of interests between the expert and the decision-maker. I find that whenever the expert wants to hide some issues while to reveal the others (partially misaligned interests) then she might optimally choose a seemingly counter-intuitive information provision strategy: she (partially) provides information about the issues she wants to hide in order to divert decision-maker's attention away from them. Moreover, I find that the information provision by the expert is possible even in the case of fully misaligned interests, i.e. when in an ideal for her scenario the decision-maker doesn't learn about any of the issues. The reason is that while the expert prefers decision-maker's uncertainty on all of the issues she still might view learning on of the issues as less costly than on the other. In this case she would prefer to partially disclose one (more costly) issue to prevent decision-maker's further learning



about it.

**Theoretical framework.** From the theoretical perspective this paper considers a two-dimensional Sender-Receiver framework with private information acquisition. There is an unknown state of the world which is a point in a 2-dimensional space ( $\theta \in \mathbb{R}^2$ ) and Receiver needs to take two actions - one for each dimension of the state of the world, thus  $a \in \mathbb{R}^2$ . Both agents share a prior belief about the state of the world which is normally distributed around prior expectation normalized to zero. I do not impose any restrictions on the variance-covariance matrix, thus, the dimensions are allowed to be correlated. Sender's and Receiver's payoffs depend on the true state of the world and the actions taken by Receiver, however, their objectives might differ. The payoffs are assumed to be quadratic in both, the state of the world and the actions of Receiver.

Prior to making the decisions, Receiver acquires information in two stages: firstly, Sender commits to provide an arbitrary amount of linear signals of an arbitrary precision. That is, she is choosing which dimension will be more under the focus and how noisy the information will be. After observing the information provided by Sender, Receiver chooses one additional potentially costly linear signal to observe.

I work in the framework with normal distributions and linear signals which makes the model tractable and allows to obtain the closed-form solution. This assumption is in line with recent literature on information and communication (see, for instance, [Liang and Mu \(2020\)](#), [Liang, Mu and Syrgkanis \(2021\)](#)). Moreover, I assume preferences quadratic in both - the state of the world and Receiver's actions. This approach, while common in the literature ([Velicheti, Bastopcu and Başar \(2023\)](#), [Sayin and Başar \(2021\)](#)), allows to be very precise about what the conflict of interests is since the ex-ante expectation of the payoffs of both agents can be written as a function of Receiver's posterior variance-covariance matrix. Thus, the interests can be characterized as aligned or misaligned depending on whether Sender wants to decrease or increase Receiver's uncertainty on the dimension in question.

**Main Results.** The main results show that Sender might choose to provide information in a counterintuitive way by partially revealing information on a dimension she wants to hide if:

- the interests are partially aligned with misalignment on the more uncertain dimension and Receiver's costs of information acquisition are sufficiently low;
- the interests are fully misaligned with stronger misalignment on initially more uncertain dimension.

The intuition for the result is the following. By construction, Receiver in the framework always chooses to acquire information about the most uncertain for her dimension. Thus, Sender affects Receiver's preference for information on different dimensions by choosing what to leave more uncertain. Hence, the diverting attention logic: if the dimension Sender wants to hide is initially the most uncertain and Receiver on her own would be able to learn a lot (due to low costs), Sender might prevent it by disclosing this dimension to make it the less uncertain one.

Moreover, Receiver might prefer to be less informed, i.e. to bear higher costs of information acquisition, in order to incentivize intuitive information provision from Sender. Indeed, she is strictly more informed ex-post whenever Sender is choosing to reveal information on a dimension of alignment rather than to divert attention. Another way for Receiver to prevent such behavior of Sender is to commit to the type or amount of information she will acquire.

**Application. Consumer Choice** In the chapter I consider two different applications for the theoretical results: consumer choice and informational lobbying.

For the consumer choice application I rely on the recent work by [Kőszegi and Matějka \(2020\)](#). They consider a consumer who is deciding optimal consumption of several goods of unknown quality while being Rationally Inattentive, which means that she bears costs of acquiring information and, thus, will never learn all possible information about all available products. They show that due to limited attention consumer focuses on either learning more about the total quality shock of the goods or the relative quality shock, and, thus, the consumption behavior exhibits either mental budgeting, or naive diversification - two behavioral phenomena discussed in the literature.

The model in [Kőszegi and Matějka \(2020\)](#) corresponds to the model of Receiver in this chapter in the absence of Sender. Thus, my framework allows to demonstrate how the behavior of such consumer might be affected by the presence of Sender with only partially aligned interests. The question is natural in the settings if we consider Sender to be an advertiser or a producer of the goods.

I consider a producer of the goods who wants to maximize the total spending of the consumer on her products while keeping the consumption as equal as possible across the goods. The assumption corresponds, for instance, to a producer facing interdependencies in the costs of producing the goods, such as dairy products or the production from crude oil. As the information designed by the producer one can think of the trials, experiences, or expert reviews. I show that the effect on the consumers' decisions depends on whether the goods are complements or substitutes. In case of the complements consumer is better off as more information is provided to her which leads to an improved

decision concerning the total budget spent on the goods. However, in the case of substitutes the effect is not straightforward. While consumer's decision about the total budget allocated is still closer to her first best, the decision about the distribution of the budget across the goods is inferior compared to the case when no information is provided by the producer. This is due to the diverting attention motives of the producer. On average however, the positive effect dominates, so overall ex-ante the consumer prefers to have access to the additional information from the producer.

**Application. Informational Lobbying.** The theoretical framework presented in this chapter shares many qualitative features with recent works on informational lobbying: multiplicity of decisions, sequential structure of the information provision, partial alignment of interests (see, for instance, [Cotton and Dellis \(2016\)](#), [Ellis and Groll \(2020\)](#) and [Cotton and Dellis \(2016\)](#)). However, most of them abstract from the information design perspective and usually focus on the question of whether informational lobbying is detrimental for the quality of the decisions. On the contrary, the theoretical framework of this chapter considers the problem of strategic information disclosure on the part of the lobbyist in the cases when the presence of the lobbyist improves the decisions of the policy maker.

The results suggest that, even though the policy-maker prefers to have the information from the lobbyist, she might prefer either to commit to more costly or less precise information acquisition process, or to contract with the lobbyist ex-ante about the type of information which will be provided.

Moreover, the theoretical results imply that if the precise costs of information acquisition (quality of information available) for the policy-maker are not known to the lobbyist, the policy-maker might have incentives to appear less informed than she is in reality to incentivize more straightforward communication from the lobbyist.

**Contribution.** First of all, the paper contributes to the theoretical literature on Bayesian persuasion. More precisely to the two recent strands: one which considers the question of multidimensional polarization without allowing Receiver to acquire additional information afterwards, and the other which allows for such information acquisition but focuses on uni-dimensional frameworks. In the former the most related works are [Tamura \(2018\)](#), [Velicheti et al. \(2023\)](#), [Sayin and Başar \(2021\)](#). The work by [Tamura \(2018\)](#) is the one which is the closest in terms of the techniques used. In the second strand the most closely related works are [Matyskova and Montes \(2023\)](#), [Bizzotto, Rüdiger and Vigier \(2020\)](#). This chapter combines the two strands as the only way to address the diverting attention motives. Indeed, to achieve this goal both are necessary: multiple dimensions to distribute the attention, and Receiver's information acquisition

to create directing search motives for Sender.

Moreover, the chapter relates to the literature on Rational Inattention since one of the costs functions for Receiver I consider is entropy costs. The closest connection is to the literature on Rationally Inattentive consumers, for instance, [Kőszegi and Matějka \(2020\)](#) and closely related to it [Hu \(2020\)](#). Compared to these works I add Sender to the framework and, thus, provide novel results on the effect of external information provision on the behavior of rationally inattentive consumers.

Finally, there is a substantial body of literature on expert advice which considers different frameworks and different applications. For instance, for the case of informational lobbying. To this literature my work contributes by introducing a new possible role for information provision which was not discussed before. Moreover, by doing so it opens a door for new empirical research.

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# Chapter 1

## Voting by Simultaneous Vetoes - joint with Matías Núñez

We propose the first class of simultaneous voting mechanisms which induce coalitional implementation, that is, double implementation in both Nash and coalition-proof equilibria. In these mechanisms, each voter vetoes a list of alternatives and the outcome is randomly selected among non-vetoed alternatives. For each specification of the veto rights, these mechanisms coalitionally implement a Veto by random priority rule introduced by [Moulin \(1981\)](#). We then discuss necessary conditions for arbitrary mechanisms to coalitionally implement a Pareto efficient rule. We show that coalitional implementation is strongly related to the presence of veto rights in the mechanism.

**JEL Codes:** D71, D72.

**Keywords:** Implementation, Voting, Vetoes, Coalition Formation, Efficiency.

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Selection committees are involved in many key decisions both in academia and in the public and the private sectors. In the former, appointments tend to be made by committees whereas in the latter boards of directors tend to manage corporate governance. Interest rates are also often determined through committees such as the LIBOR and EU-RIBOR benchmark rates. Likewise, the European Central Bank and the FED decide on monetary policies through committees.\* Behind this extended use of committees often lies the idea that taking into account the opinion of several voters can improve the quality of the final decisions. Despite the ubiquity of committees, it is commonly agreed among economists that selection committees might generate inefficiencies. There are several reasons behind these inefficiencies<sup>†</sup> such as poor information revelation, delays or status-quo bias. Moreover, classical results support this suspicion of inefficiency since no usual voting rule is strategy-proof or ensures that all equilibria are well behaved. Two main limits to the design of voting rules emerge from this literature: no usual rule guarantees that every equilibrium outcome is Pareto-efficient<sup>‡</sup> or ensures that coordination failures do not occur in equilibrium.<sup>§</sup> While the former can be thought as a theoretical problem with limited practical relevance, the latter often leads in practice to the election of minority preferred candidates (due to vote-splitting).

We contribute to this debate by designing voting mechanisms that ensure Pareto-efficiency and coordination in *any* equilibrium for *any* preference profile. These mechanisms, which we call Simultaneous Veto (SV) mechanisms, are based on vetoes: we endow voters with some (integer) amount of veto rights, we let them veto the corresponding number of alternatives and then we select the outcome randomly among non-vetoed alternatives. With a simple condition on the amount of veto rights distributed among the voters, namely that the sum of veto rights is precisely number of alternatives minus 1, each voter is always pivotal in these mechanisms and each Nash equilibrium is coalition-proof. These two properties prevent the Pareto-inefficient equilibrium outcomes and coordination failures described above. We consider the feature of SV mecha-

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\*See [Riboni and Ruge-Murcia \(2010\)](#) and [Riboni and Ruge-Murcia \(2014\)](#) for a strategic analysis of voting in monetary policy committees.

<sup>†</sup>See [Caillaud and Tirole \(2007\)](#) and [Visser and Swank \(2007\)](#) for the role of information sharing in committees and Bayesian persuasion.

<sup>‡</sup>In the extreme case of two voters, no Pareto efficient rule is Nash implementable. With three or more voters, some rules of interest are implementable (see [Hurwicz and Schmeidler \(1978\)](#) and [Maskin \(1999\)](#)) but none of them among the usual voting rules. A noteworthy exception is the kingmaker mechanism which implements Pareto-efficient alternatives, as we discuss in the main text.

<sup>§</sup>See the divided majority situation studied by [Myerson and Weber \(1993\)](#) and [Bouton and Castanheira \(2012\)](#) for a theoretical analysis and [Bouton, Castanheira and Llorente-Saguer \(2016\)](#) for experimental evidence on how voters coordinate with different voting rules.

nisms that each Nash equilibrium is coalition-proof as appealing. Indeed, it makes the set of equilibrium outcomes unaffected by the degree of communication present in the committee, and obviously, allows to avoid communication failures. We introduce the new notion of coalitional implementation, which consists of double implementation in both Nash and coalition-proof equilibria. We show that SV mechanisms coalitionally implement the Veto by random priority rules (Moulin (1981)). All the results concerning SV mechanisms do not rely on preference restrictions over the alternatives, except that we require preferences over lotteries to satisfy stochastic dominance (a fairly mild assumption).

In the second part of the paper we concentrate on coalitional implementation. We give necessary conditions that an arbitrary mechanism should satisfy in order to coalitionally implement a Pareto-efficient rule, or, in other words, for each Nash equilibrium to be Pareto-efficient and coalition-proof. We show that this form of implementation is closely associated to the veto rights through the maximality condition which states that the sum of veto rights assigned to any two coalitions which form a partition of the set of voters would be equal to the number of alternatives minus 1. This condition is close to the one which can be found in the literature on implementation in coalition-proof equilibria and on effectivity functions. Moreover, in the settings with no less alternatives than voters there should be at least one voter who has strictly positive veto rights. While a general description of all coalitionally implementable rules and implementing mechanisms seems to be a complicated task and is out of the scope of this paper, we find that if the implementing mechanism is simple enough then the Veto by random priority rules are the smallest ones which can be coalitionally implemented. As a simplicity criteria we introduce a No Gains from Cooperation (NGC) property which requires that no two coalitions can gain additional veto rights by joining their forces. This criteria is satisfied by the SV mechanisms. While the NGC property may seem desirable to make the mechanism understandable for different voters, it is impossible to construct a mechanism which satisfies NGC and allows each voter to participate in the decision in the settings with less alternatives than voters. Thus, in order to achieve coalitional implementation in this case and not to exclude any voters from the decision process, mechanisms with coalitional vetoes, i.e. which need more than 1 voter to veto a single alternative, are required. We give an example of such mechanism in the case of two alternatives by means of which we coalitionally implement the majority rule. This is, to the best of our knowledge, the first voting mechanism that implements the majority rule. Indeed, recent work by Xiong (2021) has shown that no simple mechanism can implement this rule.

In the third and final part of the paper we show that off-equilibrium lotteries are

necessary for coalitional implementation. Indeed, the mechanisms we propose in the first part, the SV mechanisms, are Deterministic-in-equilibrium (**DE**). In each equilibrium only a single alternative (a degenerate lottery) is elected, but we use lotteries as off-equilibrium threats. All the results of the second part are formulated for the **DE** implementing mechanisms. As a final result, we show that no Pareto-efficient rule (except dictatorships) is coalitionally implementable by any deterministic mechanism if there are no less alternatives than voters and by deterministic mechanisms satisfying **NGC** in general settings.

Section 1 presents a review of the literature while Section 3 introduces the setting and the definition of coalitional implementation. Section 3 shows that simultaneous veto mechanisms coalitionally implement the VRP rules. Section 4 presents the results dealing with coalitional implementation through arbitrary mechanisms, with coalitional implementation of the majority rule and finally with the impossibility results involving deterministic mechanisms.

## 1 Review of the literature

The current contribution is related to several strands of the literature, which we outline below. Since the literature is vast, we only mention the most closely related papers.

**Nash Implementation.** Hurwicz and Schmeidler (1978) and Maskin (1999) propose the idea of Nash implementation with deterministic mechanisms and provide necessary and sufficient conditions that implementable social choice correspondences (SCCs) satisfy (based on Maskin Monotonicity): that is the conditions for the sets of desirable outcomes such that there exists a deterministic mechanism for which the set of Nash equilibrium outcomes coincides with the set of the desirable outcomes. Moore and Repullo (1990) fully characterize Nash implementable SCCs. Saijo, Sjostrom and Yamato (2007) consider secure implementation, double implementation in Nash and dominant strategies, and characterize implementable SCCs through the rectangular property. Bochet (2007) and Benoît and Ok (2008) show that a rule is Maskin monotonic if and only if it is Nash implementable via a mechanism with off-equilibrium lotteries.

**Implementation in Strong and Coalition-proof equilibria.** Maskin (1978) and Suh (1997) consider double implementation under Nash and strong Nash equilibrium with deterministic mechanisms and mostly obtain negative results. Moulin (1982) considers the distribution of veto power and shows that if a neutral social choice function is partially implementable in strong equilibrium, the veto power of the various coalitions

should be maximally distributed. [Moulin and Peleg \(1982\)](#) consider the perspective of effective functions and prove that an effectivity function is representable in strong Nash equilibrium if and only if it is stable and maximal. [Keiding and Peleg \(2002\)](#) provide necessary conditions for a social choice rule to be implementable in coalition-proof equilibria in terms of effectivity functions. There is a connection between their different conditions and the ones in [Theorem 2](#), the difference being that we are concerned with double implementation in Nash and coalition-proof equilibria. Finally, a complete characterization of strongly implementable SCCs (i.e. implementable in Strong Nash) is given by [Dutta and Sen \(1991\)](#) focusing on a modification of integer games whereas [Korpela \(2013\)](#) provides a simple sufficient condition for strong implementation.

**Veto-based mechanisms.** Several existing works consider vetoes as a possible way to improve mechanisms' properties and achieve implementation. [Mueller \(1978\)](#) proposes a dynamic veto mechanism for public good provision. [Moulin \(1981\)](#) and [Moulin \(1982\)](#) consider sequential veto mechanism and introduce the proportional veto principle.\* More recently, in two-voter settings, [De Clippel, Eliaz and Knight \(2014\)](#) argues that the shortlisting mechanism, based on vetoes, improves coordination both in theory and in the lab. [Bouton, Llorente-Saguer and Malherbe \(2018\)](#) shows that majority rules with veto are superior to the usual unanimity rule. In a setting with two voters, [Laslier, Núñez and Sanver \(2021\)](#) show that any Nash implementable Pareto efficient SCC can be implemented through a veto-based mechanism. [Bogomolnaia, Holzman and Moulin \(2021\)](#) analyze bargaining settings from the perspective of vetoes.†

**Domain restrictions.** Since few attractive rules exist when preferences are unrestricted‡, an assumption of single-peaked preferences is often used. [Moulin \(1980\)](#) characterizes the generalized median rules as the class of peak-only strategy-proof rules. [Gershkov, Moldovanu and Shi \(2017\)](#) analyze voting rules that maximize the collective welfare. [Núñez and Xefteris \(2017\)](#) design approval mechanisms that coalitionally implement the median rule. The assumption of single-peakedness is meaningful both theoretically and in applications. Yet, several settings of interest are multidimensional per se, including the location of headquarters (quality of labor supply vs. taxation conditions), recruitments in an academic department (quality vs. fit of the candidate), and project

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\*See [Ianovski and Kondratev \(2020\)](#) for a computation of the size of the proportional veto core.

†See also [Macé and Treibich \(2021\)](#) who studies cooperation among heterogeneous voters and argues that veto power is needed to induce efficiency.

‡The field of social choice is plagued with impossibility results such as the Gibbard-Satterthwaite theorem. For a recent work on this area, see [Brandl, Brandt, Eberl and Geist \(2018\)](#) which proves the incompatibility of strategy-proofness and efficiency in random voting mechanisms assuming that voters' preferences over lotteries follow stochastic dominance.



investments (short-term vs. long term profitability) among others. This can crucially affect the efficiency of rules crafted for single-peaked preferences in applied settings, since it is hard to determine how preferences are ex-ante. SV mechanisms avoid this problem since their theoretical performance does not rely on preference restrictions.

## 2 Setting for Committee Design

A committee  $I = \{1, \dots, n\}$  of voters with  $n \geq 3$  needs to choose an alternative out of a finite set  $A$  of alternatives.\* Each voter  $i$  has a strict, binary and transitive preference relation  $\succ_i$  defined over the alternatives. We let  $R_i$  denote the class of possible preference relations of voter  $i$ . Let  $\succ = (\succ_1, \dots, \succ_n) \in R \equiv \prod_{i \in N} R_i$  be a preference profile. We denote by  $C^I = \{X \subseteq I\}$  the set of possible coalitions in  $I$ . For any coalition  $J \in C^I$  we denote by  $R_J \equiv \prod_{i \in J} R_i$  the set of possible joint preference profiles of the voters in the coalition  $J$ .

### 2.1 Mechanisms

We let  $\Delta$  denote the set of lotteries over  $A$  with  $\Delta = \{\beta : A \rightarrow [0, 1] \mid \sum_{a \in A} \beta(a) = 1\}$ . For each  $\beta \in \Delta$ ,  $\text{supp}(\beta)$  denotes the support of  $\beta$ , that is the alternatives that are selected with positive probability according to  $\beta$ . For each set  $X \subseteq A$ , we denote by  $\text{unif}(X)$  the uniform lottery over  $X$ , that is  $\text{unif}(X) = \{\beta \in \Delta \mid \beta(x) = \frac{1}{|X|} \text{ for any } x \in X\}$ . A mechanism is a function  $g : M \rightarrow \Delta$  that assigns to every  $m \in M$  a unique element of  $\Delta$ , where  $M = \prod_{i \in I} M_i$ , and  $M_i$  is the strategy space of voter  $i$ , which we assume to be finite. For each  $J \in C^I$  we denote by  $M_J = \prod_{i \in J} M_i$  the strategy space of coalition  $J$  with a typical element  $m_J$ ; similarly,  $m_{-J}$  denotes the message profile of coalition  $I \setminus J$ .

Let  $g(M) = \{\beta \in \Delta \mid \exists m \in M \text{ s.t. } g(m) = \beta\}$  be the range of the mechanism  $g$ , that is the set of lotteries that can be induced by the different message profiles in the mechanism  $m$ . We only consider mechanisms with a finite range. For each  $J \in C^I$ , let  $g(M_J, m_{-J}) = \{\beta \in \Delta \mid g(m_J, m_{-J}) = \beta \text{ for some } m_J \in M_J\}$  be the attainable set of a coalition  $J$  at  $m_{-J}$ , in other words, the set of lotteries that coalition  $J$  can induce when the other voters play  $m_{-J}$ .

### 2.2 Preferences over lotteries

We assume that preferences over lotteries are derived from the preferences over alternatives through preference extensions. We say that  $\succeq_i^{\text{ext}}$  is an extension of  $\succ_i$  if, for any pair  $x, y \in A$ ,  $x \succ_i y \implies x \succeq_i^{\text{ext}} y$ . For each mechanism  $g : M \rightarrow \Delta$ , we write  $R_{g(M)}$  the

\*See [Laslier et al. \(2021\)](#) for a related analysis with two voters.

class of preferences over the lotteries in  $g(M)$ . An element of  $R_{g(M)}$  is denoted by  $\succeq_i^{ext}$  and let  $\succeq^{ext}$  be a profile of preference extensions. Let  $\kappa(\succ_i) \in R_{g(M)}$  be a set of admissible preference extensions of voter  $i$  associated to  $\succ_i$ . Similarly,  $\kappa(\succ)$  is the set of admissible preference extensions associated to  $\succ$ .

We assume that preferences over lotteries satisfy stochastic dominance (**SD**). This condition is satisfied by many preference extensions since requires minimal information on the preferences of the voter. Observe that one lottery stochastically dominates another one if and only if the former yields at least as much expected utility as the latter for any von-Neumann-Morgenstern utility representation consistent with the ordinal preferences.

For any lottery  $\beta \in \Delta$  and each  $x \in A$ , we write  $\beta^+[x] = \sum_{y \in A: y \succ_i x} \beta(y)$  to refer to the probability, according to  $\beta$ , of obtaining an alternative preferred to  $x$  according to  $\succ_i$ . The preference extension  $\succeq_i^{ext}$  of  $\succ_i$  satisfies **SD** if

$$\beta \succeq_i^{ext} \gamma \iff \forall x \in A, \beta^+(x) \geq \gamma^+(x),$$

and  $\beta \succ_i^{ext} \gamma$  if some inequality is strict. The domain  $\kappa(\succ)$  satisfies **SD** if for any  $\succ \in R$ , any extension  $\succ_i^{ext} \in \kappa(\succ)$ ,  $\succ_i^{ext}$  satisfies **SD**. Since all preference extensions that we consider in the paper satisfy **SD**, we denote, in the sequel, any preference extension by  $\succeq_i^{SD}$  rather than  $\succeq_i^{ext}$ .

The second condition, Priority Extension, deals with the richness of the domain of preference extensions. A domain  $\kappa$  satisfies a priority extension (**PREX**) in  $\Delta$  iff for all  $\succ_i \in R_i$  and for each  $x \in A$ , there is some  $\succeq_i^{SD} \in \kappa(\succ_i)$  such that for any two lotteries  $\beta, \gamma \in \Delta$

$$\beta^+[x] > 0 \text{ and } \gamma^+[x] = 0 \implies \beta \succ_i^{SD} \gamma.$$

Laslier et al. (2021) rely in this condition to characterize Nash-implementable social choice correspondences in two-player settings.

## 2.3 Nash Implementation

Given a mechanism  $g : M \rightarrow \Delta$ , the strategy profile  $m \in M$  is a Nash equilibrium of  $g$  at  $\succeq^{SD}$  if there is no  $i \in I$  with  $g(m'_i, m_{-i}) \succ_i^{SD} g(m)$  for some  $m'_i \in M_i$ . The set  $BR(\succeq_i^{SD}, m_{-i})$  contains the best responses of a voter with preferences  $\succeq_i^{SD}$  under the mechanism  $g$  when other voters play according to  $m_{-i}$ . Let  $N^g(\succeq^{SD})$  be the set of Nash equilibria of  $g$  at  $\succeq^{SD}$ . Throughout, we consider mechanisms that are deterministic or deterministic-in-equilibrium (**DE**). A mechanism  $g$  is deterministic if for any  $m \in M$ ,  $g(m) \in A$ . A

mechanism  $g$  is **DE** in the domain  $\kappa(>)$  if for any  $> \in R$ ,  $\succeq^{SD} \in \kappa(>)$  and  $m \in N^g(\succeq^{SD})$ ,  $g(m) \in A$ .

The mechanism  $g$  implements the SCC  $f$  in Nash equilibria in the domain  $\kappa(>)$  if for each  $> \in R$  and each  $\succeq^{SD} \in \kappa(>)$ ,  $N^g(\succeq^{SD}) = f(>)$ . The SCC  $f$  is Nash implementable if there exists a mechanism that implements  $f$  in Nash equilibria.

## 2.4 Coalitional Implementation

The strategy  $m'_j$  is a self-enforcing deviation for coalition  $J$  at profile  $m$  if (i)  $g(m'_j, m_{-j}) \succ_j^{SD} g(m)$  for all  $j \in J$  and (ii) there is no  $H \subseteq J$  with some  $m''_H \in M_H$  such that  $g(m''_H, m'_{J \setminus H}, m_{-J}) \succ_h^{SD} g(m'_j, m_{-j})$  for all  $h \in H$  for some preference extension  $\succeq^{SD} \in \kappa(>)$ . The strategy profile  $m \in M$  is a coalition-proof equilibrium of  $g$  at  $\succeq^{SD}$  in the domain  $\kappa(>)$  if no coalition  $J \in C^I$  has a self-enforcing deviation.\* Observe that since any unilateral deviation by a single voter is self-enforcing, any coalition-proof equilibrium is a Nash equilibrium. Let  $C^g(\succeq^{SD})$  be the set of coalition-proof equilibria of  $g$  at  $\succeq^{SD}$ . By definition  $C^g(\succeq^{SD}) \subseteq N^g(\succeq^{SD})$ .

The mechanism  $g$  coalitionally implements the SCC  $f$  in the domain  $\kappa(>)$  if for each  $> \in R$  and each  $\succeq^{SD} \in \kappa(>)$  (i)  $N^g(\succeq^{SD}) \neq \emptyset$  and  $C^g(\succeq^{SD}) \neq \emptyset$ , (ii)  $N^g(\succeq^{SD}) = C^g(\succeq^{SD})$ , and (iii)  $f(>) = \bigcup_{m \in N^g(\succeq^{SD})} g(m)$ . A SCC  $f$  is coalitionally implementable at the domain  $\kappa(>)$  if there exists a mechanism that coalitionally implements  $f$ .

## 3 Coalitional implementation via simultaneous vetoes

This section presents the simultaneous veto mechanisms and introduces the first main result of this work: simultaneous veto mechanisms coalitionally implement Veto by random priority rules. It introduces the simultaneous veto mechanisms, discusses its basic properties and, after defining the Veto by random priority rule, shows the implementation argument.

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\*Note that this definition and the classical definition for coalition-proof Nash by [Bernheim, Peleg and Whinston \(1987\)](#) are not equivalent. For self-enforcing deviation of coalition we need to consider all the coalitions of smaller size, so we can write it in the recursive form. Consider this step from the original definition: Then for a game  $\langle I, M, P^{SD} \rangle$  with  $|I| = n$ ,  $m \in M$  is self-enforcing if for all  $J \subset I$ ,  $m_j$  is a coalition-proof equilibrium in the game  $\langle J, M_j, P_j^* \rangle$ . The difference with the self-enforcing coalitional deviation of our definition is that we allow for any deviations of  $H$  while the classic definition would consider only the deviations which are self-enforcing for  $H$ . However, the proofs of all the results which use coalition-proof Nash consider either coalitions of one voter or by a group of voters with identical preferences. In these cases the two definitions of self-enforcing coalitional deviations coincide. Since the proofs proceed by contradiction, the results apply for both definitions.

### 3.1 Simultaneous veto mechanisms

Let  $v = (v_i)_{i \in N}$  be a set of non-negative integers such that:

$$\sum_{i \in N} v_i = k - 1. \quad (1.1)$$

From now on, we refer to any vector  $v$  that satisfies (1.1) as the vector of veto rights (or simply veto vector), to  $v_i$  as the veto rights of voter  $i$  and to  $v_J = \sum_{i \in J} v_i$  as the veto rights of coalition  $J$ .

In a simultaneous veto mechanism  $SV_v$ , the strategy space of voter  $i$  is given by  $M_i = \{X \subseteq A \mid |X| = v_i\}$ . If in the profile  $m \in M$ ,  $a \in m_i$  for some  $i \in I$ , then alternative  $a$  is *vetoed* in  $m$ , otherwise  $a$  is *non-vetoed*. Requirement (1.1) implies that there are less vetoes than alternatives, which ensures that for any message profile some alternative remains non-vetoed. The class of SV mechanisms includes the mechanism in which the veto rights are equally split among voters (i.e.  $v = (1, 1, \dots, 1)$  when  $k = n + 1$ ) so that all voters have the right of vetoing exactly one alternative. This class also includes dictatorships: when a voter concentrates all the veto rights (i.e.  $v = (k - 1, 0, \dots, 0)$  or a permutation of it). The rest of SV mechanisms allow some inequality on the veto rights of the different voters, even though, for any combination of voters and alternatives, there is a simultaneous veto mechanism with an almost equal distribution of vetoes:  $|v_i - v_j| \leq 1$  for any pair  $i, j$  of voters.

**Definition 1.** *The simultaneous veto mechanism  $SV_v : M \rightarrow \Delta$  with veto vector  $v = (v_1, \dots, v_n)$  associates each message profile  $m \in M$  to the lottery*

$$SV_v(m) = \text{unif}\left(A \setminus \bigcup_{i \in I} m_i\right).$$

Note that an SV mechanism selects uniformly an alternative among the non-vetoed ones.\*

Prior to formally describing voters' equilibrium strategies induced by an SV mechanism, we introduce some additional notations and definitions.

For each  $i \in I$ ,  $\succ_i \in R_i$  and  $a \in A$ , let  $L(a, \succ_i) \equiv \{b \in A \mid a \succ_i b\}$  be the lower contour set of  $a$  at  $\succ_i$  with cardinality  $l(a, \succ_i)$ . For each  $J \in C^I$ ,  $\succ_J \in R_J$  and  $a \in A$ , let  $L(a, \succ_J) \equiv \{b \in A \mid b \in$

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\*Indeed, as long as preferences over lotteries satisfy **SD** and lotteries are uniform, voters can compare the different outcomes they can induce with their vote. The assumption of uniformity can be relaxed but this would also require to impose stronger conditions than **SD** on the preference extensions to avoid bad equilibria. Hence, we prefer to use the current definition.

$L(a, \succ_i)$  for some  $i \in J$  be the lower contour set for coalition  $J$  of  $a$  for  $\succ_J$  with cardinality  $l(a, \succ_J)$ . Note that for each  $J \in C^I$  and each  $a \in A$ ,  $L(a, \succ_J) = \bigcup_{i \in J} L(a, \succ_i)$ .

A social choice correspondence (SCC) is a function  $f : R \rightrightarrows A$  that associates a set  $f(\succ)$  of alternatives to every  $\succ \in R$ . A SCC  $f$  satisfies Maskin monotonicity iff for all  $\succ, \succ' \in R$  and  $x \in A$  with  $L(x, \succ_i) \subseteq L(x, \succ'_i)$  for all  $i \in I$ , we have  $x \in f(\succ) \Rightarrow x \in f(\succ')$ . An alternative  $x$  is Pareto-efficient in the preference profile  $\succ$  if there is no  $y \in A$  such that  $y \succ_i x$  for each  $i \in N$ . A SCC  $f$  is Pareto efficient if  $f(\succ)$  only contains Pareto efficient alternatives for any  $\succ \in R$ .

For each simultaneous veto mechanism  $SV_v$ , each voter  $i$  and each  $m_{-i} \in M_{-i}$ , let  $V(m_{-i}) = \{a \in A \mid a \in m_j \text{ for some } j \neq i\}$  denote the set of vetoed alternatives by all voters other than  $i$ . It is rather intuitive that **SD** implies that, in any simultaneous veto mechanism, voter  $i$ 's best response is to veto her  $v_i$  least preferred alternatives among those in  $A \setminus V(m_{-i})$  as stated by the next result. This in turn implies that each simultaneous veto mechanism  $SV_v$  is **DE** as formalized by the next result.

**Proposition 1.** For any preference profile  $\succ \in R$  and any simultaneous veto mechanism  $SV_v$ :

1. the best response of a voter with preferences  $\succ_i$  is to veto her  $v_i$  least preferred alternatives among  $A \setminus V(m_{-i})$ , that is:

$$\forall \succeq_i^{SD} \in \kappa(\succ_i), \quad BR(\succeq_i^{SD}, m_{-i}) = \{a \in A \setminus V(m_{-i}) \mid |L(a, \succ_i) \cap (A \setminus V(m_{-i}))| < v_i\}.$$

2. the outcome is deterministic in any equilibrium so that  $SV_v(m) \in A$  for any  $m \in N^{SV_v(\succeq^{SD})}$  and any  $\succeq^{SD} \in \kappa(\succ)$ .

3. for any  $\succ \in R$  and any  $\succeq^{SD} \in \kappa(\succ)$  and any  $m \in N^{SV_v(\succeq^{SD})}$ ,  $SV_v(m)$  is Pareto-efficient.

*Proof.* 1. For any simultaneous veto mechanism, the attainable set  $g(M_i, m_{-i})$  for voter  $i$  contains any uniform lottery with support in  $A \setminus V(m_{-i})$ . Note that, for any  $m_{-i} \in M_{-i}$ ,  $|A \setminus V(m_{-i})| \geq k - \sum_{j \in I \setminus \{i\}} v_j = v_i + 1$ . Then, since  $\succeq_i^{SD}$  satisfies **SD**, the most preferred outcome in  $g(M_i, m_{-i})$  for voter  $i$  is the uniform lottery among her most preferred  $|A \setminus V(m_{-i})| - v_i$  alternatives in  $A \setminus V(m_{-i})$ . Thus, the best response of voter  $i$  is to veto her  $v_i$  least preferred alternatives in  $A \setminus V(m_{-i})$ .

2. Fix some simultaneous veto mechanism  $SV_v$  and consider any equilibrium  $m \in N^{SV_v(\succeq^{SD})}$ . Assume by contradiction that  $SV_v(m) \notin A$  so that  $\text{supp}(SV_v(m)) \geq 2$ . This, in turn, implies that there are at least two voters, denoted  $i$  and  $j$ , whose choices of vetoes intersect,  $m_i \cap m_j \neq \emptyset$ . However, as stated by 1., the best response of voter  $i$  for any  $m_{-i}$ , denoted  $BR(\succeq_i^{SD}, m_{-i})$ , is included in  $A \setminus V(m_{-i})$ . Thus, in equilibrium,  $BR(\succeq_i^{SD}, m_{-i}) \cap m_j = \emptyset$  for any voter  $j \neq i$  which contradicts  $m$  being an equilibrium as desired.

3. Consider some equilibrium profile  $m$  and let  $x \in A$  be its outcome (as shown by 1.) for any veto vector  $v$ , the  $SV_v$  mechanism is **DE**). If  $x$  is Pareto inefficient, there is some  $y$  with  $y \succ_i x$  for any voter  $i \in N$  while  $y$  is vetoed by some voter  $j$ . Then according to Proposition 1.1, voter  $j$  is not playing her best response which contradicts  $m$  being a Nash equilibrium. Thus, any equilibrium outcome of the  $SV_v$  mechanism is Pareto-efficient.  $\square$

Proposition 1 implies that, in equilibrium, a voter only vetoes alternatives that she prefers less than the outcome. Moreover, one can check that the converse also holds: each profile in which every voter  $i$  vetoes  $v_i$  unique alternatives while she prefers the outcome to each of these  $v_i$  alternatives is an equilibrium, which gives a first characterization of the set of equilibria of simultaneous veto mechanisms. More formally, for each simultaneous veto mechanism  $SV_v$  and each preference profile  $\succ$ , the set of Nash equilibria can be defined as follows:

$$N^{SV_v(\geq^{SD})} = \{m \in M \mid \forall i \in I, \forall x \in m_i \quad SV_v(m) \succ_i x \text{ and } \forall j \in I \quad m_i \cap m_j = \emptyset\}. \quad (1.2)$$

In a Nash equilibrium, no communication is assumed to arise before the strategies are chosen. If communication takes place before the vote, then it might be the case that the notion of Nash equilibrium is not well adapted. Hence, we focus on coalition-proof equilibria in which coalitions can deviate. Yet, in contrast with the strong equilibrium notion, coalitional deviations are only allowed when they are self-enforcing, that is no part of the coalition that can in turn deviate after the coalitional deviation has occurred and obtain a profitable deviation. As we now show, any Nash equilibrium of a SV mechanism is coalition-proof.

**Proposition 2.** For each veto vector  $v$  and  $\succ^{SD} \in \kappa(\succ)$ ,  $N^{SV_v(\geq^{SD})} = C^{SV_v(\geq^{SD})}$ .

The intuition for this result is related to the equilibrium strategies. Indeed, as described by Proposition 1, every voter vetoes only alternatives less preferred than the equilibrium outcome and no pair of voters vetoes common alternatives. Proposition 2 shows that this implies that coalition has a self-enforcing deviation only if one of its members has a unilateral profitable deviation. The presence of deviations is excluded by Nash equilibrium due to the veto rights.

*Proof.* In order to prove that every Nash equilibrium of  $SV_v$  is coalition-proof, take some preference profile  $\succ \in R$  and assume that the domain  $\kappa(\succ)$  satisfies **SD**. Consider some

equilibrium  $m \in N^{\text{SV}_v(\geq^{SD})}$  with  $\text{SV}_v(m) = \{a\}$ . Since (1.1) holds and any equilibrium is deterministic, exactly  $k-1$  alternatives are vetoed in the profile  $m$  so that each alternative is vetoed by exactly one voter.

First we show that self-enforcing deviations from a Nash equilibrium can not lead to non-degenerate lotteries. Assume by contradiction that some coalition  $J \in C^I$  has some self-enforcing deviation  $m'_J$  with  $|\text{supp}(\text{SV}_v(m'_J, m_{-J}))| > 1$ . In the profile  $(m'_J, m_{-J})$ , there is some voter  $j^*$  in  $J$  that vetoes the same alternative as some other voter since in the profile  $m$  each alternative is vetoed by exactly one voter. Since voter  $j^*$  has strict preferences over the alternatives, voter  $j^*$  has a profitable deviation in the profile  $(m'_J, m_{-J})$ : vetoing her  $v_{j^*}$  least preferred alternatives in  $A \setminus V(m_{-j^*})$ . This shows that coalitional deviations leading to lotteries are not self-enforcing. Hence, only deviations which lead to a deterministic outcome are self-enforcing.

Assume that the equilibrium  $m$  is not coalition-proof. This implies that some coalition  $J \in C^I$  has a self-enforcing deviation  $m'_J$ . Since  $m'_J$  is self-enforcing, as previously shown, it must be that  $\text{SV}_v(m'_J, m_{-J}) = \{b\}$  for some  $b \in A$ . Since  $m'_J$  is self-enforcing,  $b \succ_j a$  for all  $j \in J$ . Since coalition  $J$  can reach  $b$  as an outcome it must be the case that under  $m$ ,  $b$  is vetoed by some voter  $j' \in J$  which contradicts  $m$  being Nash equilibrium according to Proposition 1.2. This concludes the proof.  $\square$

To conclude the section we provide an example of Simultaneous Veto mechanisms for a committee of 3 voters. This example illustrates the way in which the mechanisms protect voters from their worst alternatives being elected.

**Example (Coordination with SV mechanisms):** Let  $I = \{1, 2, 3\}$  and  $A = \{a, b, c, d\}$  respectively denote the set of voters and alternatives. Figure 1 presents three preference profiles in order to illustrate the equilibria of the SV mechanisms. Profiles (a) and (b) both admit a Condorcet cycle, the difference being that in (a), alternative  $d$  is not part of the Condorcet Cycle whereas in (b),  $d$  is part of it and is the unique alternative not ranked last by some voter. Finally, in profile (c), voters 1 and 2 share the same preferences which are opposite to the ones of voter 3, which leads  $a$  to be the Condorcet winner. For simplicity, we assume that the veto vector coincides with  $v = (1, 1, 1)$ . In profile (a), any alternative in the Condorcet cycle ( $a$ ,  $b$  or  $c$ ) are elected in some equilibrium. In profile (b),  $d$  is the unique equilibrium outcome. Finally, in profile (c), alternative  $b$  is the unique equilibrium outcome. Observe that voters 1 and 2 coordinate their vetoes in equilibrium preventing  $c$  and  $d$  to be elected.  $\square$

In this section we have described the set of equilibria of SV mechanisms. Yet, characterization provided by (1.2) is silent on the alternatives that are selected in equilibrium.

$\succ_1$	$\succ_2$	$\succ_3$	$\succ'_1$	$\succ'_2$	$\succ'_3$	$\succ''_1$	$\succ''_2$	$\succ''_3$
$a$	$c$	$b$	$a$	$c$	$b$	$a$	$a$	$\mathbf{d}$
$b$	$a$	$c$	$b$	$a$	$c$	$b$	$b$	$c$
$c$	$b$	$a$	$\mathbf{d}$	$\mathbf{d}$	$\mathbf{d}$	$c$	$c$	$b$
$\mathbf{d}$	$\mathbf{d}$	$\mathbf{d}$	$c$	$b$	$a$	$\mathbf{d}$	$\mathbf{d}$	$a$
(a) Condorcet Cycle			(b) Compromise			(c) Coordination		

Figure 1.1: Examples of SV mechanisms

It turns out that the implemented alternatives consists of the Veto by Random Priority outcomes as discussed in the next sections.

### 3.2 Veto by random priority rule

For each profile  $\succ \in R$ , the Veto by random priority rule with veto vector  $v$ , denoted  $\text{VRP}_v$ , selects all the alternatives with a lower contour set greater than  $v_J \geq 0$  for each coalition  $J$ , that is:

$$\{a\} \in \text{VRP}_v(\succ) \iff \text{for each coalition } J \in C^I, l(a, \succ_J) \geq v_J. \quad (1.3)$$

Here we slightly abuse the notation, and consider the collection of  $(v_J)_{J \in C^I}$  - the veto rights for every possible coalition  $J$  of voters. However, as before  $v = (v_1, \dots, v_n)$  still denotes the vector of veto rights of individual voters.

When a voter has  $v_i$  veto rights, the VRP rule never selects an alternative among her worst  $v_i$  alternatives. Similarly, when a coalition has veto power  $v_J$  the outcome is never among the worst  $v_J$  alternatives of the coalition. By doing so, the VRP rule ensures a minimal level of satisfaction to each coalition of voters.

This rule is discussed in depth by [Moulin \(1983\)](#) and it is referred as the Veto core correspondence. Note that with 2 voters, this rule coincides with the Pareto-and-veto rules ([Laslier et al. \(2021\)](#) prove that, under some conditions, these rules are the only Pareto-efficient Nash implementable voting rules in settings with two voters). In a setting with three voters, the VRP rule can be illustrated as follows. For any veto vector  $v$ , an alternative  $a$  is selected if and only if it satisfies the following inequalities:

$$\underbrace{l(a, \succ_i) \geq v_i \text{ for each } i = 1, 2, 3,}_{\text{individual veto power}} \quad \underbrace{l(a, \succ_{1,2,3}) \geq v_1 + v_2 + v_3,}_{\text{Pareto efficiency}}$$

$$\underbrace{\text{and } l(a, \succ_J) \geq v_J \text{ for each } J \in \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}.}_{\text{veto coordination}}$$



The two first sets of inequalities convey a clear message: any alternative selected by the  $VRP_v$  rule is Pareto-efficient and, for each voter  $i$ , the alternative does not belong to her worst  $v_i$  alternatives. The third set of inequalities ensures that coordination among voters is effective and no veto is wasted. Indeed,  $l(a, \succ_j) \geq v_j$  implies that there are at least  $v_j$  alternatives less preferred than  $a$  for the different members of the coalition  $J$ .

The definition of the  $VRP_v$  rule, stated by (1.3), relies on the cardinal of the different lower contour-sets. While being compact such formulation is not the most intuitive. We now provide an alternative formulation of this rule which gives additional intuition for how the alternatives are selected. To state this formulation, we introduce the following notation: for each veto vector  $v$  satisfying (1.1), we denote by  $\Pi(I, v)$  the set of all possible orderings of the voters' vetoes, where each voter appears exactly  $v_i$  times. The generic element of  $\Pi(I, v)$  is denoted by  $\pi$ . We refer to  $\pi_j$  as the  $j$ 'th element in  $\pi$ . For example, if  $I = \{1, 2, 3\}$  and the veto vector equals  $v = (3, 2, 1)$  with 7 alternatives, then one possible ordering can be  $(1, 2, 1, 1, 3, 2)$  so that voter 1 appears in positions 1, 3 and 4, voter 2 in position 2 and 6 and voter 3 in the position 5 of  $\pi$ .

For each veto vector  $v$  satisfying (1.1) and each ordering  $\pi \in \Pi(I, v)$  consider the following algorithm:

$$\begin{aligned}
 B_{\pi_1} &= \text{the least preferred alternative of } \succ_{\pi_1} \text{ in } A \\
 B_{\pi_2} &= \text{the least preferred alternative of } \succ_{\pi_2} \text{ in } A \setminus B_{\pi_1} \\
 &\vdots \\
 B_{\pi_{i+1}} &= \text{the least preferred alternative of } \succ_{\pi_{i+1}} \text{ in } A \setminus (B_{\pi_1} \cup B_{\pi_2} \cup \dots \cup B_{\pi_i}) \\
 &\vdots
 \end{aligned} \tag{1.4}$$

The previous algorithm singles out an alternative for each specification of the veto rights: the set  $A \setminus (B_{\pi_1} \cup B_{\pi_2} \cup \dots \cup B_{\pi_{k-1}})$  is a singleton.

**Proposition 3.** (*Moulin (1983), Theorem 4*) For each veto vector  $v$  and each preference profile  $\succ \in R$ ,

$$VRP_v(\succ) = \bigcup_{\pi \in \Pi(I, v)} A \setminus (B_{\pi_1} \cup B_{\pi_2} \cup \dots \cup B_{\pi_{k-1}})$$

where the sets  $B_{\pi_1}, B_{\pi_2}, \dots, B_{\pi_{k-1}}$  are given by (1.4).

The former proposition claims that the set of alternatives selected by the Veto by ran-

dom priority rule,  $\text{VRP}_v$ , coincides with the set of alternatives that one can get applying the algorithm given by (1.4) to any possible ordering of the vetoes.

Note that in the case of  $\text{VRP}_v$ ,  $v_J = \sum_{j \in J} v_j$  for any coalition  $J$ . Thus, it is fully characterized by the vector  $v = (v_1, \dots, v_n)$ .

**Remark 1:** For each preference profile  $\succ \in R$  and each veto vector  $v$ , the rule  $\text{VRP}_v(\succ)$  is well-defined and Pareto-efficient (see [Moulin \(1983\)](#)).

**Remark 2:** As a by-product of Proposition 3, for each veto vector  $v$ , each  $\succ \in R$  and each  $a \in \text{VRP}_v(\succ)$ , we can find a partition  $(X_1, \dots, X_n)$  of  $A \setminus \{a\}$  such that

$$X_i \subseteq L(a, \succ_i) \text{ and } \bigcap_{i \in I} X_i = \emptyset \text{ and } |X_i| = v_i.$$

As a final result of this section, we show that no selection of a VRP rule is Maskin monotonic. This implies that one cannot implement a proper selection of it by focusing on simultaneous mechanisms.

**Proposition 4.** *For each veto vector  $v$ , if  $f \subseteq \text{VRP}_v$  and  $f$  is Maskin monotonic, then  $f = \text{VRP}_v$ .*

*Proof.* Take some  $f \subseteq \text{VRP}_v$ . If some voter has no vetoes then she plays no role in the outcome. On the contrary, if some voter has  $k - 1$  vetoes then she is a dictator. Hence in the sequel we consider w.l.o.g. a veto vector  $v$  such that  $0 < v_i < k - 1 \forall i \in I$ . For each  $a \in A$  and each vector  $v$ , define  $B_v^a = \{\succ \in R \mid l(a, \succ_i) = v_i \forall i \in I \text{ and } l(a, \succ) = k - 1\}$ . Take some  $\succ \in B_v^a$ . It follows that  $L(a, \succ_i) \cap L(a, \succ_j) = \emptyset$  for any  $i, j \in I$  and  $\{a\} \in \text{VRP}_v(\succ)$ . In order to prove that  $f(\succ) = \{a\}$ , assume that there is some  $y \neq a$  and  $y \in f(\succ)$ . In this case,  $l(y, \succ_i) \geq v_i$  for each  $i \in I$ , which implies that  $y$  Pareto dominates  $a$ , a contradiction.

Since preferences over alternatives are unrestricted, for any partition  $(X_1, \dots, X_n)$  of  $A \setminus \{a\}$  with  $|X_i| = v_i \forall i \in I$ , there is some profile  $\succ \in B_v^a$  such that  $L(a, \succ_i) = X_i \forall i \in I$ . Consider any profile  $\succ' \in R$  with  $\{a\} \in \text{VRP}_v(\succ')$ . By Remark 2, there exists a profile  $\succ \in B_v^a$  with  $L(a, \succ) \subseteq L(a, \succ')$  and  $a \in f(\succ)$ . Then by Maskin Monotonicity  $a \in f(\succ')$ , as required.  $\square$

### 3.3 Coalitional implementation via simultaneous vetoes

The next Theorem presents one of the central results of this work, that is that SV mechanisms coalitionally implement the Veto by random priority rules.

**Theorem 1.** *For each veto vector  $v$ , the simultaneous veto mechanism  $\text{SV}_v$  coalitionally implements the correspondence  $\text{VRP}_v$ , so that:*

1. for each  $\succ \in R$ ,  $\{a\} = SV_v(m)$  for some  $m \in N^{SV_v(\geq^{SD})} \iff \{a\} \in VRP_v(\succ)$  and,
2. for each  $\succ \in R$  and  $\geq^{SD} \in \kappa(\succ)$ ,  $N^{SV_v(\geq^{SD})} = C^{SV_v(\geq^{SD})}$ .

*Proof.* In order to see that any equilibrium outcome of an SV mechanism is part of a VRP rule, take some preference profile  $\succ \in R$ . Proposition 1 ensures that  $SV_v$  is **DE** so that no pair of voters vetoes common alternatives. Assume by contradiction that there is some  $a \in A$  such that  $\{a\} \notin VRP_v(\succ)$  whereas  $SV_v(m) = \{a\}$  for some  $m \in N^{SV_v(\geq^{SD})}$ . That means that there is some coalition  $J$  for which  $l(a, \succ_J) < v_J$ . Since no pair of voters vetoes common alternatives,  $l(a, \succ_J) < v_J = \sum_{j \in J} v_j$  is only possible if some voter  $j \in J$  is vetoing some alternative  $a' \in A$  such that  $a' \succ_j a$ . This means that  $j$  is not playing her best response, so that  $m$  is not a Nash equilibrium, a contradiction.

In order to understand that for any veto vector  $v$  and any  $\succ \in R$  and any alternative in  $a \in VRP_v(\succ)$ , there is an equilibrium of the  $SV_v$  mechanism that selects it, it suffices to apply Remark 2. Indeed, Remark 2 states for each veto vector  $v$ , each  $\succ \in R$  and each  $a \in VRP_v(\succ)$ , we can find a partition  $(X_1, \dots, X_n)$  of  $A \setminus \{a\}$  such that  $X_i \subseteq L(a, \succ_i)$  and  $\bigcap_{i \in I} X_i = \emptyset$  and  $|X_i| = v_i$ . Hence, consider the profile  $m$  with  $m_i = X_i$  so that  $SV_v(m) = \{a\}$ . Since  $m_i \subseteq L(a, \succ_i)$ , each voter  $i$  is playing a best response to  $m_{-i}$ , which concludes the proof.

The arguments above show that the  $SV_v$  mechanism Nash implements  $VRP_v$  rule. Therefore, applying the statement of Proposition 2, we conclude that it also coalitionally implements the  $VRP_v$  rule, completing the proof.  $\square$

## 4 Vetoes and Coalitional Implementation

In this section, we give necessary conditions for a mechanism to coalitionally implement a Pareto efficient SCC (Theorem 2). Among other things we show that coalitional implementation is closely related to the presence of veto rights. We then prove by example how to coalitionally implement the majority rule and finally show that coalitional implementation of a Pareto efficient rule is not possible with deterministic mechanisms.

### 4.1 Coalitional implementation

Up to this point, this work has shown that the  $VRP_v$  are coalitionally implementable via the SV mechanisms. This section considers the reverse questions: which are the Pareto-efficient SCCs that can be coalitionally implemented? What is the class of mechanisms that allow to achieve such implementation? As we show in the sequel, under some conditions, the idea of veto is closely related to the one of coalitional implementation.

To the best of our knowledge, if one excludes integer/modulo games and dictatorships, few mechanisms in the literature have been shown to implement a Pareto efficient rule. Among them, a salient class are the Kingmaker games (see [Maskin \(1999\)](#) and [Hurwicz and Schmeidler \(1978\)](#)) where one voter decides the identity of the voter who will, in turn, decide the outcome. As we now show, these mechanisms do not achieve coalitional implementation.

**Example (Kingmaker game):** Let  $A = \{a_1, a_2, a_3, a_4\}$  and  $N = \{1, 2, 3\}$  respectively denote the set of alternatives and the set of voters. The strategy sets of voters 1, 2 and 3 are denoted by  $M_1 = \{2, 3\}$  and  $M_2 = M_3 = A$ . Define  $g : M_1 \times M_2 \times M_3 \rightarrow A$  with  $g^{KM}(m_1, m_2, m_3) = a_{s_1}$ . In other words, the winning alternative is the alternative announced by the voter nominated by voter 1. It can be shown that the mechanism  $g^{KM}$  Nash implements the SCC  $f(\succ) = \{a \in A \mid \exists j \in \{2, 3\} \text{ s.t. } a \succ_j b \text{ for all } b \in A \setminus \{a\}\}$ . Remark that  $f$  is Pareto-efficient. Yet, this mechanism does not coalitionally implement  $f$ . Indeed, observe that the profile  $m$  with  $m_1 = 3$ ,  $m_2 = m_3 = a$  is an equilibrium for any preference profile as long as the nominated voter, voter 3, ranks  $a$  first in her preferences. Observe that voter 3 is best responding whereas neither voter 1 nor voter 2 can alter the outcome. However, if both 1 and 2 prefer  $b$  to  $a$ , they have a coalitional deviation that leads to  $a$  (i.e.  $m_1 = 2$  and  $m_2 = b$ ) which shows that this mechanism admits equilibria which are not coalition-proof.  $\square$

In order to derive conditions on coalitionally implementable rules, we impose the completeness of the preference extensions over lotteries. Observe that in the previous sections we did not require that the preferences over lotteries were complete. This is so due to the structure of SV mechanisms. Indeed, they are constructed in such way that for each voter  $i \in I$  for any profile  $m_{-i}$  there is an element of  $SV_v(M_i, m_{-i})$  which voter  $i$  strictly prefers to any other element of the attainable set under **SD**. The existence of a maximal element in the attainable set does not require that the voter can compare each pair of elements in the attainable set. Yet, since we do not know the structure of attainable sets in an arbitrary mechanism, we assume that the preference domain satisfies completeness, which definition is as follows.

**Definition 2.** For each  $i \in N$  and each  $\succ_i \in R_i$ , the set  $\kappa(\succ_i)$  of admissible preference extensions satisfies **completeness** if for each  $\succeq_i^{SD} \in \kappa(\succ_i)$  for any  $\alpha, \beta \in \Delta$ , either  $\alpha \succeq_i^{SD} \beta$  or  $\beta \succeq_i^{SD} \alpha$ . The domain  $\kappa(\succ)$  satisfies completeness if each  $\kappa(\succ_i)$  is complete for each  $i \in N$  and each  $\succ_i \in R_i$ .

For each mechanism  $g$ ,  $\text{VETO}(g, J)$  represents the sets of alternatives that coalition  $J$

can exclude from being chosen for any strategy of voters outside  $J$ . Formally, for each coalition  $J \in C^I$ ,

$$\text{veto}(g, J) = \{X \subseteq A \mid \exists m_J \in M_J \text{ s.t. } X \cap \text{supp}(g(m_J, m_{-J})) = \emptyset \forall m_{-J} \in M_{-J}\}.$$

A mechanism  $g$  is coalition-wise neutral-on-its-vetoes if and only if for every  $J \in C^I$ , if  $X \in \text{veto}(g, J)$ , then for any set  $Y$  satisfying  $|Y| = |X|$ ,  $Y \in \text{veto}(g, J)$ . In other words, if a coalition of voters has the right to exclude some subset of alternatives, it also has the right to exclude any other subset of alternatives with the same cardinality. We write  $v_g(J)$  to denote the veto rights of coalition  $J$  under the mechanism  $g$  when  $g$  is coalition-wise neutral-on-its-vetoes.

The main result of this section is as follows.

**Theorem 2.** *Assume that the domain  $\kappa(\succ)$  satisfies completeness and **PREX**. Let  $f$  be a Pareto-efficient SCC. If  $f$  is coalitionally implementable by some **DE**-mechanism  $g$ , then :*

1.  $g$  is coalition-wise neutral-on-its-vetoes,
2. for any partition  $(J, K)$  of  $I$ ,  $v_g(J) + v_g(K) = k - 1$ ,
3. if  $a \in f(\succ)$ , then  $l(a, \succ_i) \geq v_g(i)$  for each  $i \in I$ .

This theorem gives several insights regarding coalitionally implementable SCCs.

The fact that any implementing mechanism  $g$  is coalition-wise neutral-on-its-vetoes is noteworthy. It shows that any mechanism where some voter(s) has decisive power over a subset of alternatives is ruled out (such as local dictatorships). Moreover, Theorem 2 implies that for any partition  $(J, K)$  of the voters, either one of the coalitions can impose the outcome or both coalitions have positive veto power. These veto rights have implications on the rules one can expect to coalitionally implement. The property 3 shows that each voter is ensured to obtain an outcome which is not among his worst  $v_g(i)$  alternatives. This means that any coalitionally implementable rule which fails to coincide with a  $\text{VRP}_v$  rule exhibits the following property: there is some coalition  $J$  and some  $a \in f(\succ)$  such that  $l(a, \succ_J) < v_g(J)$ .

The next condition, No Gains from Cooperation (**NGC**), is a simplicity requirement for the mechanism.\* It demands that merging coalitions does not allocate supplementary veto power to the voters. This requirement is satisfied by the SV mechanisms where veto rights are additive.

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\*See , [Börger and Li \(2019\)](#) for the notion of strategic simplicity of a mechanism.

**Definition 3.** A mechanism  $g$  satisfies **No Gains from Cooperation (NGC)** if for all  $J, K \subset I$  such that  $J \cap K = \emptyset$ , for any  $X \in \text{VETO}(g, J \cup K)$  there exist  $Y \in \text{VETO}(g, J)$  and  $Z \in \text{VETO}(g, K)$  such that  $X = Y \cup Z$ .

**Proposition 5.** Under the assumptions of Theorem 2, let  $f$  be a coalitionally implementable SCC via a mechanism  $g$ . The following claims hold:

1. If  $g$  satisfies NGC, then  $f \supseteq \text{VRP}_v$  for some  $v$ .
2. If  $k \geq n$ , then at least some voter has positive veto power.

The complete proof of the result is presented in the Technical Appendix. Proposition 5 implies that the implemented rule  $f$  selects all alternatives that some VRP rule prescribes, if one requires an implementing mechanism to be simple enough, i.e. to satisfy NGC. To see why it is true observe that Theorem 2 imposes that  $v_g(J) + v_g(K) = k - 1$  for any partition of the voters. Since NGC implies that the veto powers of the voters are additive, we can prove that the mechanism  $g$  admits any of the equilibria of the SV mechanisms, which leads to the result.

Moreover, with as many alternatives as voters, we can show that at least some voter has positive veto power. This means that any coalitionally implementable SCC violates the well-known condition of No-Veto power. This condition, extensively used in implementation theory, states that whenever  $n - 1$  voters rank the same alternative on top of their preferences, "the last voter cannot prevent the alternative from being  $f$ -optimal (i.e. "he cannot veto it") as Maskin (1999) puts it. With less alternatives than voters, this need not be the case as we discuss in the next sections.

## 4.2 Coalitional implementation with two alternatives: endogenous vetoes

As shown in the previous section, whenever the number of alternatives is no less than the number of voters, there must be at least one voter with veto rights. Moreover, there are always coalitionally implementable rules if the implementing mechanism satisfies NGC. While such simplicity requirement can be reasonable for the settings with a large number of alternatives, it leads to some voters being excluded from the decision process when  $k < n$ . Indeed, it is impossible to endow each voter with at least 1 veto right when there are less alternatives than voters. However, as shown by part 2 of Theorem 2 the veto rights are required independent of the number of alternatives and the number of voters. Thus, if one wants to coalitionally implement a SCC in the environment with  $k < n$  and at the same time leave no voter aside, one is forced to consider mechanisms

which violate **NGC** and allow for coalitional gains. While the general characterization of such mechanisms is out of scope of this work, we show on the example of majority rule and 2 alternatives how coalitional implementation via mechanisms which violate **NGC** can be achieved.

The main difficulty here is to ensure that each equilibrium is coalition-proof while no voter having veto power (if a voter has veto power, he is a dictator in this setting). Our results contrast with the results of [Xiong \(2021\)](#) that shows that, in this setting, no rule (except dictatorship) is implementable through a direct mechanism or through an indirect mechanism that satisfies some regularity condition: voting for one of the alternatives is a part of each voter's strategy space. Note that our mechanism does not satisfy this regularity condition by making each voter to report not only the alternative, but also the other voters she would like to cooperate with. Thus by extending the set of allowed mechanisms we circumvent the impossibility result. Remark that our mechanism is not direct since it requires the voters to announce their preference and the voters they want to be in a coalition with. In a sense, they are related to the Kingmaker mechanisms previously described. The main difference is that in the Kingmaker mechanism, a single voter nominates other voters and the rest of voters vote for an alternative whereas, in the current mechanism, each voter both nominates other voters and votes for an alternative.

Let  $A = \{a, b\}$  be the set of alternatives and  $n = 2p + 1$  be the number of voters to ensure that for each profile  $\succ$ ,  $\text{MR}(\succ) = \{a \in A \mid |\{i \in I \mid a \succ_i b\}| > |\{i \in I \mid a \succ_i b\}|\}$  defines the unique majority preferred alternative. We let  $g^{PV}$  denote the coalitional vetoes mechanism such that the strategy  $m_i$  of each voter equals  $m_i = (n_i, c_i) \in M^i : \mathcal{I}^p \setminus \{i\} \times A$  so each voter  $i$  nominates a group of  $p$  voters excluding himself and announces an alternative. For each  $m \in M$ , let  $N(m) = \{i \in N \mid i = n_j \text{ for some } j \in N\}$  denote the set of nominated voters and  $C(m) = \{a \in A \mid a = c_i \text{ for some } i \in N(m)\}$  denote the set of announced alternatives by nominated voters. For each  $m \in M$ , the outcome of the coalitional veto mechanism  $g^{PV}$  is as follows.\*

Let  $M \in C^I$  be a majority of voters with  $|M| = p + 1$ . In the profile  $m$ , the strategies of the voters in  $M$  form a block against  $x$  if for each  $i \in M$  the strategy  $s_i$  satisfies:

1. for each  $i \in M$ ,  $n_i = M \setminus \{i\}$
2. for each  $i \in M$ ,  $c_i = x$ .

The voters in  $M$  form a block against  $x$  if they all nominate each other while announcing  $x$ . If in the profile  $m$ , there is a majority who forms a block against  $x$ , the outcome is

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\*This mechanism is discussed in greater details in Chapter 2.

$y$ . Otherwise, for each message profile  $m$ , we write  $p(i, m) = |\{j \in J \mid n_j = i\}|$  to denote the score of voter  $i$  at profile  $m$ , i.e. the number of voters who nominate  $i$ . The outcome of the coalitional veto mechanism  $g^{PV}$  coincides with the lottery  $\beta(\cdot, m)$  which equals:

$$\beta(x, m) = \frac{\overbrace{\sum_{i \in I} p(i, m) \times \mathbb{1}\{c_i = y\}}^{\text{score of } y \text{ by nominated voters}}}{\underbrace{\sum_{i \in I} p(i, m)}_{\text{total number of nominations}}}, \text{ for any } x \in A.$$

Remark that  $\beta(x, m)$  coincides with the frequency with which  $y$  is announced among nominated voters taking into account the scores of these voters.

This means that the probability that  $x$  is selected equals the frequency with which  $x$  is *not* announced by the nominated voters, or, in other words, the frequency with which  $y$  is announced by the nominated voters. When a voter votes for an alternative, this decreases the probability with which this alternative is selected to be the outcome. This is similar to the idea of SV mechanisms in which a veto for an alternative decreases the probability with which this alternative is selected.

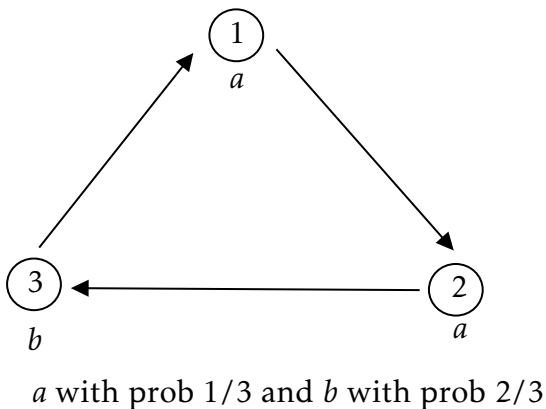


Figure 1.a: No block

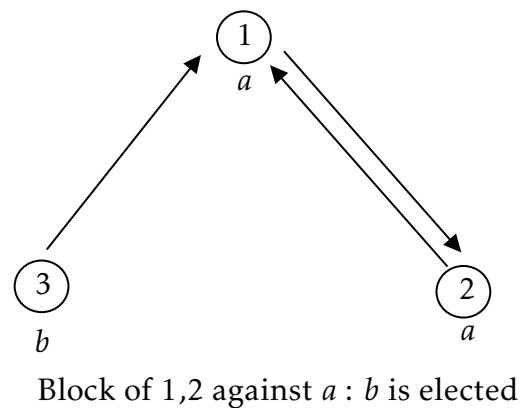


Figure 1.b: Block of voters

Figure 1: A mechanism with coalitional vetoes

Figure 1 represents two possible message profiles in the mechanism with three voters (i.e. when  $p = 1$ ). In the figure, the arrows represent the nominations whereas the letters



denote the alternatives announced by the voter. In Figure 1.a, the profile  $m$  is depicted with  $m_1 = (2, a)$ ,  $m_2 = (3, a)$  and  $m_3 = (1, b)$ . This leads to a lottery where  $a$  is selected with probability  $1/3$  and  $b$  with probability  $2/3$  since  $a$  is announced by both 1 and 2 (both are nominated) and  $b$  is announced by 3 (nominated by 2). Figure 1.b presents the profile  $m'$  with a block of voters with  $m'_1 = (2, a)$ ,  $m'_2 = (1, a)$  and  $m'_3 = (1, b)$ . In this case, 1 and 2 form a block against  $a$  and alternative  $b$  is selected.

At any profile in which no block is formed, any nominated voter strictly prefers to announce her least preferred alternative since this will strictly decrease the probability that this alternative wins. At any profile in which there is a block a member of the block prefers to stay in it only if the block is formed against her least preferred alternative. These two properties lead to each equilibrium selecting the alternative preferred by majority.

Observe that in the mechanism  $g^{PV}$ , every voter and every coalition with less than  $p+1$  voters has zero veto power whereas any coalition with at least  $p+1$  voters has a veto power of 1.

**Theorem 3.** *Let  $A = \{a, b\}$ . Assume that the domain  $\kappa(>)$  satisfies SD. The coalitional vetoes mechanism  $g^{PV}$  coalitionally implements the majority rule  $MR(>)$ .*

*Proof.* W.l.o.g. assume that any voter  $i$  in  $\{1, \dots, p+1\}$  is such that  $a >_i b$  so that  $a$  is the majority preferred alternative. Observe that the profile  $m$  where every voter in  $\{1, \dots, p+1\}$  nominates a subset of  $\{1, \dots, p+1\}$  and every such voter announces  $b$  is an equilibrium that selects  $a$  since it creates a block against  $b$ . No block of voters against  $a$  can be an equilibrium since there at most  $p$  voters who prefer  $b$  to  $a$ .

To see that  $a$  is the unique equilibrium outcome, consider the two following cases.

First, if all voters prefer  $a$  to  $b$ , the claim is immediate, since whenever a voter is nominated she announces  $b$ , so that only  $a$  can be elected at equilibrium.

Assume now that this is not the case, so that there is some voter, say  $h$ , with  $b >_h a$ .

We first claim that, in equilibrium, any voter  $i$  with  $a >_i b$  announces  $b$ . This is immediate if the voter  $i$  is nominated since this decreases the probability that  $b$  is elected. Assume, by contradiction, that  $i$  is not nominated in some equilibrium  $m$  while announcing  $a$ . The voter  $h$ , with  $b >_h a$ , nominates  $p$  voters by definition. Since there are at most  $p$  voters (including voter  $h$ ) who prefer  $b$  to  $a$ , this means that  $h$  nominates some voter, say  $j$ , with  $a >_j b$ . Yet, in equilibrium,  $j$  announces  $b$ . This means that voter  $h$  prefers to deviate and nominate  $i$  (who chooses  $a$ ) since this will strictly increase the probability that  $b$  is elected, showing that  $m$  is not an equilibrium.

Finally, it suffices to see that any voter who prefers  $a$  to  $b$  only nominates voters who prefer  $a$  to  $b$ . Indeed, as previously argued, any voter who prefers  $a$  to  $b$  announces  $b$  in any equilibrium. Moreover, any voter who prefers  $b$  to  $a$  announces  $a$  if nominated. Thus, any voter who prefers  $a$  to  $b$  only nominates voters who prefer  $a$  to  $b$ . Therefore, there is always a block against  $b$ , which shows that the unique equilibrium outcome is  $a$ , the majority preferred alternative.  $\square$

### 4.3 Coalitional implementation via deterministic mechanisms

This section shows that coalitional implementation via deterministic mechanisms is severely restricted. We first show with an example that veto-based mechanisms with deterministic tie-breaking rules tend to generate Pareto dominated equilibria. We then provide some sufficient conditions that ensure that the only coalitionally implementable rule with deterministic mechanisms is dictatorship.

**Example (Pareto inefficiency of deterministic mechanisms):** Let  $I = \{1, 2, 3\}$  and  $A = \{a, b, c, d\}$  respectively denote the set of voters and the set of alternatives. Assume that the preferences of the voters are the ones presented in Figure 1.2.

$\succ_1$	$\succ_2$	$\succ_3$	Simultaneous veto
<b>a</b>	$d$	$d$	· $d$ is vetoed in any equilibrium (worst for 1)
$b$	<b>a</b>	<b>a</b>	· $c$ is vetoed in any equilibrium (worst for 2 and 3)
$c$	$b$	$b$	· $b$ is vetoed by either 2 or 3 in any equilibrium
$d$	$c$	$c$	· $a$ unique winner in equilibrium
(a) Preference Profile	(b) Pareto efficiency of SV mechanisms		

Figure 1.2: An example of an SV mechanism

*Veto-rank mechanism.* We consider the mechanism in which each voter selects one alternative to veto and then ranks the remaining alternatives. The alternative ranked first receives a score of 2, the alternative ranked second receives a score of 1 and the alternative ranked third gets a score of 0. The veto-rank mechanism then selects the alternative with the highest sum of the voters scores from the nonvetoed alternatives, with ties being broken alphabetically.\* Assume that voter 1 vetoes  $d$  while voters 2 and 3 both veto  $a$ . Observe that any profile with these vetoes that leads to the election of  $b$  is a Nash equilibrium. Indeed, in this case  $b$  is the best alternative for each voter from among the non-vetoed alternatives. Therefore, no player has a profitable unilateral

\*This mechanism is applied in arbitration settings, see [De Clippel et al. \(2014\)](#) for an analysis.

deviation so that  $b$  is selected in equilibrium. However,  $b$  is Pareto-dominated by  $a$ . In this case voters' aggressive behavior, vetoing other voter's top alternative, leads them to an inefficient equilibrium.

*SV mechanism.* Letting  $v = (1, 1, 1)$  denote the veto vector, it suffices to see that  $a$  is the unique outcome of the SV mechanism in this preference profile since  $a$  is the unique Pareto efficient alternative not ranked last by some voter (see Figure 2(b)).  $\square$

We now prove that no deterministic mechanism which satisfies **NGC** can coalitionally implement a Pareto-efficient rule. Moreover, in the settings with no less alternatives than voters, we show that no deterministic mechanism independently of its simplicity can coalitionally implement a Pareto-efficient SCC. This implies that lotteries off-equilibrium are key to obtain coordination among voters. It should be emphasized that other results in this direction exist in the literature. Indeed, [Maskin \(1978\)](#) considers implementation in Strong equilibria and [Suh \(1997\)](#) is concerned with double implementation in Nash and strong Nash equilibria. This is contrast with [Moulin \(1983\)](#) proves that any VRP rule is implementable in strong equilibria.

**Theorem 4.** *Let  $f$  be a Pareto-efficient SCC coalitionally implementable by some deterministic mechanism  $g$ . If either  $k \geq n$  or  $g$  satisfies **NGC**, then  $f$  is dictatorial.*

The main logic of this impossibility result is rather simple. The notion of coalitional implementation imposes that either a voter is a dictator (veto power of  $k - 1$ ) or at least two voters have positive veto power. In the latter case, we prove that there exists Pareto dominated equilibria as for instance the example dealing with the veto-rank mechanism. This theorem proves that off-equilibrium lotteries are useful to avoid Pareto inefficiencies. The complete proof is presented in the Technical Appendix.

## 5 Conclusion

This paper contributes to the design of voting rules by proposing simple simultaneous mechanisms that achieve coalitional implementation while ensuring that every equilibrium outcome is Pareto-efficient.\* As we show, this notion of implementation is closely

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\*A natural application of our mechanisms is to use them as screening devices: namely, to use these mechanisms as a first step in a selection process. Indeed, it seems that removing alternatives that are deemed as undesirable by the different voters is a necessary first step for making a reasonable collective choice (see [Manzini and Mariotti \(2007\)](#) for a similar idea in decision theory and [Horan and Sprumont \(2021\)](#) for a two-stage collective choice procedure).

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related to the introduction of veto rights in the implementing mechanism. With more alternatives than voters, at least some voter has veto rights in any mechanism that achieves coalitional implementation. With less alternatives than voters, it can be the case that only coalitions have veto rights; for instance, this is the case in the coalitional vetoes mechanism that implements the majority rule and in which coalitions are formed endogenously. Further work on the endogenous formation of coalition in voting environments, probably in a dynamic setting, seems to be a promising venue of research.

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# 1 Appendix

It presents the proof of Theorem 2 that provides necessary conditions on the veto structure of a coalitionally implementable rule. It also shows the arguments behind the impossibility result of Proposition 5 and Theorem 4.

## 1.1 Proof of Theorem 2

The proof is divided in several lemmas. Lemma 1 establishes that the implementing mechanism endows the different coalitions with some veto power. Lemma 2 shows that a coalitionally implementable SCC must be coalition-wise neutral-on-its vetoes. Lemma 3 establishes the partition property.

**Lemma 1.** *Assume that the preference domain  $\kappa(\succ)$  satisfies completeness and **PREX**. If the mechanism  $g$  coalitionally implements the SCC  $f$ , then, for any partition  $(X, Y)$  of  $A$  and any partition  $(J, H)$  of  $I$  either  $Y \in \text{VETO}(g, J)$  or  $X \in \text{VETO}(g, H)$ , but not both.*

*Proof.* Let  $(X, Y)$  be a partition of  $A$  and  $(J, H)$  be a partition of  $I$ . Consider the following preference profile  $\succ$  and its extension  $\succeq^{SD}$  (which exists due to **PREX**):

1.  $\forall j, j' \in J, \succ_j = \succ_{j'}$  and  $\forall h, h' \in H, \succ_h = \succ_{h'}$ ;
2.  $\forall j \in J, \forall h \in H, \forall x \in X$  and  $\forall y \in Y, x \succ_j y$  and  $y \succ_h x$ ;
3.  $\forall j, j' \in J, \succeq_j^{SD} = \succeq_{j'}^{SD}$  and  $\forall h, h' \in H, \succeq_h^{SD} = \succeq_{h'}^{SD}$ ;
4.  $\forall j \in J, \forall \beta, \gamma \in g(M)$  with  $\sum_{x \in X} \beta(x) > 0$  and  $\sum_{x \in X} \gamma(x) = 0, \beta \succ_j^{SD} \gamma$ ;
5.  $\forall h \in H, \forall \beta, \gamma \in g(M)$  with  $\sum_{y \in Y} \beta(y) > 0$  and  $\sum_{y \in Y} \gamma(y) = 0, \beta \succ_h^{SD} \gamma$ .

Let  $g$  be a mechanism that coalitionally implements  $f$ . It follows that there is some  $m \in N^g(\succeq^{SD}) = C^g(\succeq^{SD})$  with  $g(m) = \{a\}$  with some  $a \in A$  with either  $a \in X$  or  $a \in Y$ . Assume by contradiction that  $Y \notin \text{VETO}(g, J)$  and  $X \notin \text{VETO}(g, H)$ .

- $a \in X$ . Since  $Y \notin \text{VETO}(g, J)$ , there is some  $m'_H \in M_H$  such that  $\text{supp}(g(m_J, m'_H)) \cap Y \neq \emptyset$ . If  $|H| = 1$ , there is some  $h$  with  $H = \{h\}$ . Then, by construction of  $\succ$ , the strategy  $m'_H$  is a profitable deviation for voter  $h$ , which contradicts  $m$  being an equilibrium. If  $|H| > 1$ , a similar logic applies. Since the preferences of all members of coalition  $H$  coincide, there is a self-enforcing deviation for  $H$  at  $m$ .

To see why observe first that for each voter  $h \in H$  there exists  $\tilde{\beta} \in g(M_H, m_J)$  such that  $\tilde{\beta} \succeq_h^{SD} \gamma$  for all  $\gamma \in g(M_H, m_J)$  (since  $\succeq^{SD}$  is complete and  $g(M)$  is finite by



assumption). Since there is  $m'_H \in M_H$  such that  $\text{supp}(g(m_J, m'_H)) \cap Y \neq \emptyset$ , it follows  $\text{supp}(\tilde{\beta}) \cap Y \neq \emptyset$  due to 4. Denoting by  $m'_H$  the strategy such that  $g(m_J, m'_H) = \tilde{\beta}$ , observe that  $m'_H$  is self-enforcing for coalition  $H$  since the preference extension of all members of coalition  $H$  coincide and hence no coalition  $H' \subset H$  can achieve an outcome preferred to  $\tilde{\beta}$ . This contradicts the fact that  $m$  is a coalition-proof equilibrium.

- $a \in Y$ . By the same logic, a similar contradiction arises.

Thus either  $Y \in \text{veto}(g, J)$  or  $X \in \text{veto}(g, H)$ . Since the mechanism  $g$  is well-defined, it is impossible that both conditions hold simultaneously, concluding the proof.  $\square$

**Lemma 2.** *Assume that the preference domain  $\kappa(>)$  satisfies completeness and **PREX**. If the mechanism  $g$  coalitionally implements a Pareto-efficient SCC  $f$ , then  $g$  is coalition-wise neutral-on-its-vetoes.*

*Proof.* Consider some partition  $(J, H)$  of  $I$  and let  $X \in \text{veto}(g, J)$ ,  $x \in X$  and  $x' \in A \setminus X$ . Thus, there exists  $m_J \in M_J$  that vetoes  $X$ . Define the set  $X' = X \setminus \{x\} \cup \{x'\}$  and observe that  $|X| = |X'|$ . Write  $Y = A \setminus (X \cup \{x'\})$ , so that  $A$  can be partitioned as follows:

$$A = (X \setminus \{x\}) \cup \{x\} \cup \{x'\} \cup Y = X' \cup \{x\} \cup Y.$$

Suppose, by contradiction, that  $X' \notin \text{veto}(g, J)$ . Lemma 1 implies that  $Y \cup \{x\} \in \text{veto}(g, H)$ . Therefore, there exists  $m_H \in M_H$  that vetoes  $Y \cup \{x\}$ . Since  $x'$  is neither vetoed by  $m_J$  nor by  $m_H$ ,  $g(m_J, m_H) = \{x'\}$ . Consider a unanimous preference profile  $> = (>_1, \dots, >_n)$  such that  $x >_i x' >_i y$  for any  $y \neq x, x'$  and any  $i = 1, \dots, n$ . Note that  $x'$  is Pareto dominated by  $x$  but at  $(m_J, m_H)$ , both coalitions veto  $x$ . No unilateral deviation is hence possible since no deviation can add  $x$  to the support of the lottery outcome. Hence, **SD** implies that  $(m_J, m_H)$  is a Nash equilibrium that selects a Pareto dominated alternative, a contradiction.

The proof of the lemma can be derived by applying the same logic to a finite sequence of sets  $X = X_1, \dots, X_s = X'$  with  $|X_i \cap X_{i+1}| = |X| - 1$  for each  $i \in \{1, \dots, s-1\}$ .  $\square$

Lemmas 1 and 2 jointly prove that, for each Pareto efficient and implementable SCC, the implementing mechanism  $g$  is coalition-wise neutral-on-its-vetoes.

For each coalition  $J$ , we denote by  $U(a, >_J) = \{x \in A \mid x >_j a \forall j \in J\}$  the upper contour set of  $a$  at  $>_J$ . Thus, the upper contour set of  $a$  for coalition  $J$  includes all the alternatives which each member of  $J$  prefers to  $a$ , or  $U(a, >_J) = A \setminus (\{a\} \cup L(a, >_J))$ .

The next lemma shows that for any partition in two of the set of voters, the sum of their respective veto powers adds up to  $k - 1$ .

**Lemma 3.** *Assume the preference domain  $\kappa(>)$  satisfies completeness and **PREX**. If the mechanism  $g$  coalitionally implements a Pareto-efficient SCC  $f$ , then for any partition  $(J, H)$  of  $I$ ,  $v_g(J) + v_g(H) = k - 1$ .*

*Proof.* Consider a partition  $(J, H)$  of  $I$ . Consider the following preference profile  $>$ :

1.  $\forall j, j' \in J, >_j = >_{j'}$  and  $\forall h, h' \in H, >_h = >_{h'}$
2.  $\forall a \in A, U(a, >_J) = L(a, >_H)$ , so the preference of  $J$  and  $H$  are opposed
3.  $\forall j, j' \in J, \succeq_j^{SD} = \succeq_{j'}^{SD}$  and  $\forall h, h' \in H, \succeq_h^{SD} = \succeq_{h'}^{SD}$

Observe that for each  $a \in A, U(a, >_J) \cap U(a, >_H) = \emptyset$  with  $|U(a, >_J) \cup U(a, >_H)| = k - 1$ . This implies that  $|U(a, >_J)| + |U(a, >_H)| = k - 1$ .

Consider some equilibrium  $m$  of the mechanism  $g$  with  $g(m) = \{a\}$  for some  $a \in A$ . Assume that for each  $i \in I, \succeq_i^{SD}$  satisfies **PREX** for  $a$ .

Since the preferences of all members in each coalition coincide, then coalition  $H$  vetoes  $U(a, >_J)$  by playing the equilibrium strategy  $m_H$ . To see why, assume by contradiction that  $m_H$  does not veto  $U(a, >_J)$ . Then, there is some  $m'_J$  such that  $\text{supp}(g(m'_J, m_{-J})) \cap U(a, >_J) \neq \emptyset$ . Yet, since  $\forall j, j' \in J, \succeq_j^{SD} = \succeq_{j'}^{SD}$  by the same logic as in the proof of Lemma 1 there exists a self-enforcing deviation for coalition  $J$ , proving that  $m$  is not coalition-proof. Thus,  $v_g(H) \geq |U(a, >_J)|$ . Similarly, we can prove that the equilibrium strategy  $m_J$  of coalition  $J$  vetoes  $U(a, >_H)$  and, therefore,  $v_g(J) \geq |U(a, >_H)|$ . As previously argued,  $|U(a, >_J)| + |U(a, >_H)| = k - 1$ , so that  $v_g(J) + v_g(H) \geq k - 1$ . In order for  $g$  to be well-defined,  $v_g(J) + v_g(H) \leq k - 1$  (at least some alternative should be left non-vetoed). Thus,  $v_g(J) + v_g(H) = k - 1$ .  $\square$

*Proof of Theorem 2.*

Lemmas 1-3 jointly show that  $g$  is coalition-wise neutral-on-its-vetoes, and that for each partition  $(J, K)$  of the voters their respective veto power  $v_g(J)$  and  $v_g(K)$  adds up to  $k - 1$ .

As a final claim, we show that for any preference profile  $>$ , an equilibrium outcome is not among the  $v_g(i)$  worse alternatives for any player  $i$ . To prove the claim, let  $m \in N^g(\succeq^{SD})$  be an equilibrium and  $x$  its outcome. Assume that  $x$  is among the  $v_1$  worst alternatives of player 1. As previously argued, player 1 has a veto power of  $v_g(1)$ . Let  $m'_i$  denote the strategy of player  $i$  that vetoes his worst  $v_i$  alternatives. For any strategy  $m_{-i} \in M_{-i}$ , the support of  $g(m'_i, m_{-i})$  is included in player  $i$ 's top  $k - v_g(i)$  alternatives.

However, due to **PREX**, player  $i$  prefers any such lottery to  $x$ , which proves that player  $i$  has a profitable deviation and thus  $m$  is not an equilibrium, which concludes the proof of the theorem.

## 1.2 Proof of Proposition 5

The proof of the proposition is split in two independent parts.

**If  $g$  satisfies NGC, then  $f \supseteq \text{VRP}_v$  for some  $v$ .**

In order to prove the first part of the proposition, the next lemma proves that if  $g$  is the implementing mechanism in Theorem 2 and satisfies **NGC** then  $v_g(J)$  is additive, so that for each pair of coalitions  $J_1, J_2$  with  $J_1 \cap J_2 = \emptyset$  and  $J = J_1 \cup J_2$ ,  $v_g(J_1 \cup J_2) = v_g(J_1) + v_g(J_2)$ . Finally, Lemma 5 proves that any coalitionally implementable  $f$  selects all the alternatives prescribed by some  $\text{VRP}_v$  rule.

**Lemma 4.** *Assume that the preference domain  $\kappa(>)$  satisfies completeness and **PREX**. If the mechanism  $g$  coalitionally implements a Pareto-efficient SCC  $f$  and satisfies **NGC**, then  $v_g(\cdot)$  is additive.*

*Proof.* Consider any two disjoint coalitions  $J_1$  and  $J_2$  such that  $J_1 \cup J_2 = J$ . Consider some  $X \subset A$  such that  $X \in \text{veto}(g, J)$  and  $|X| = v_g(J)$ . According to **NGC**, there are  $Y, Z \subset A$  such that  $Y \in \text{veto}(g, J_1)$ ,  $Z \in \text{veto}(g, J_2)$  and  $Y \cup Z = X$ . Then there are 2 possibilities:

1. either  $v_g(J_1) = |Y|$  and  $v_g(J_2) = |Z|$
2. or  $v_g(J_1) > |Y|$  (or  $v_g(J_2) > |Z|$ )

If  $v_g(J_1) > |Y|$  or  $v_g(J_2) > |Z|$  then  $v_g(J_1) + v_g(J_2) > v_g(J)$ , since the mechanism  $g$  is neutral-on-its-vetoes by Lemma 2. Thus, it means that the joint coalition has less veto rights than its parts separately. However, this cannot be the case. To see this, take some profile  $m$  such that  $m_{J_1}$  and  $m_{J_2}$  are vetoing two disjoint sets of sizes  $v_g(J_1)$  and  $v_g(J_2)$  respectively. Such profile exists since  $g$  is coalition-wise neutral-on-its-vetoes and  $v_g(J_1) + v_g(J_2) \leq k - 1$  since  $g$  is well-defined. Since the set of alternatives vetoed by  $J_1 \cup J_2$  should at least include the alternatives vetoed by  $J_1$  and  $J_2$ ,  $v_g(J) \geq v_g(J_1) + v_g(J_2)$ . Then the only possibility is that  $v_g(J_1) = |Y|$  and  $v_g(J_2) = |Z|$ . Thus  $v_g(J) = v_g(J_1 \cup J_2) = v_g(J_1) + v_g(J_2)$ , as required.  $\square$

**Lemma 5.** *Assume that domain  $\kappa(>)$  satisfies **PREX**. If a Pareto-efficient SCC  $f$  is coalitionally implementable by some **DE** mechanism satisfying **NGC** then  $f \supseteq \text{VRP}_v$  for some  $v$ .*

*Proof.* Consider some mechanism  $g$  satisfying **NGC** and coalitionally implementing  $f$ .

By Theorem 2, the mechanism  $g$  is coalition-wise neutral-on-its-vetoes, additive and for each partition  $J, K$  of  $I$ ,  $v_g(J) + v_g(K) = k - 1$ . Thus, there exists some veto vector  $v = (v_1, \dots, v_n)$  such that  $v_g(i) = v_i$  and  $v_g(J) = \sum_{j \in J} v_j$ .

Consider some preference profile  $\succ$ . Take some alternative  $a$  such that  $\text{SV}_v(m) = \{a\}$  for some  $m \in N^{\text{SV}_v(\geq^{SD})}$ .

Now consider a strategy profile  $\tilde{m}$  under the mechanism  $g$  such that each voter  $i$  vetoes under  $\tilde{m}$  the alternatives in  $m_i$ . In other words, we construct a profile such that each voter vetoes the same alternatives as in an equilibrium profile for the mechanism  $\text{SV}_v$ . Such a profile exists since  $g$  is coalition-wise neutral-on-its-vetoes and  $v_g(i) = v_i$  for each voter  $i \in I$ . Then  $g(\tilde{m}) = \{a\}$  since all other alternatives are vetoed.

If there is some voter  $i$  with a profitable deviation, say  $\tilde{m}'_i$  then  $\text{supp}(g(\tilde{m}'_i, m_{-i})) \cap U(a, \succ_i) \neq \emptyset$ . However, by construction of  $\tilde{m}$  all alternatives in  $U(a, \succ_i)$  are vetoed by voters in  $I \setminus \{i\}$ . Thus,  $\tilde{m}$  is a Nash equilibrium of the mechanism  $g$  at  $\geq^{SD}$ .

But then we have for every  $\succ \in R$ :

$$f(\succ) = \bigcup_{\tilde{m} \in N^g(\geq^{SD})} g(\tilde{m}) \supseteq \bigcup_{m \in N^{\text{SV}_v(\geq^{SD})}} \text{SV}_v(m) = \text{VRP}_v(\succ)$$

The equalities follow respectively from the fact that  $g$  coalitionally implements  $f$  and  $\text{SV}_v$  coalitionally implements  $\text{VRP}_v$ .  $\square$

**If  $k \geq n$ , then at least some voter has positive veto power.**

The second part of Proposition 5 shows that when  $k \geq n$ , at least some voter has positive veto rights. We start with the simple case of 3 voters and 3 alternatives and show that at least 1 voter should have at least one veto right. Then by increasing the number of voters and alternatives we arrive to the general result.

1.  $n = k = 3$ .

Consider the preference profile in Table 1 that describes a Condorcet cycle:

Assume that  $\geq^{SD}$  satisfies **PREX** and let  $g$  be a mechanism that coalitionally implements a Pareto-efficient SCC. Assume that  $a$  is an equilibrium outcome. Thus, there exists some  $m \in N^g(\geq^{SD})$  with  $g(m) = \{a\}$ .

Notice that since  $a$  is the worst alternative for voter 2 it must be that  $\{b, c\} \in \text{VETO}(g, 1 \cup 3)$ . Additionally, due to **PREX** it must be that  $c$  is vetoed by  $m_{1 \cup 2}$ , so  $\{c\} \in \text{VETO}(g, 1 \cup 2)$ .

$\succ_1$	$\succ_2$	$\succ_3$
$a$	$b$	$c$
$b$	$c$	$a$
$c$	$a$	$b$

Table 1

There are 2 possibilities:

1.  $c$  is vetoed by voter 1 under  $m$ . Thus  $\{c\} \in \text{veto}(g, 1)$ .
2.  $c$  is not vetoed by voter 1 under  $m$ , but it is vetoed by coalitions  $1 \cup 2$  and  $1 \cup 3$ . Then consider the preference profile in Table 2, in which 2 and 3 share identical preferences.

$\succ'_1$	$\succ'_2$	$\succ'_3$
$a$	$c$	$c$
$b$	$a$	$a$
$c$	$b$	$b$

Table 2

.Assume that  $\succeq_2^*$  and  $\succeq_3^*$  are identical and satisfy **PREX**. We claim that  $m$  is a Nash equilibrium in the profile  $\succ'$ . Voter 1 has no profitable deviation since  $a$  is her most preferred alternative. Voter 2 gets her best alternative among non-vetoed ones by voters 1 and 3, since  $c$  is vetoed by coalition  $1 \cup 3$ . The same applies to voter 3. However,  $m$  is not coalition-proof. Indeed, by the same logic as in the proof of Lemma 1, the coalition  $2 \cup 3$  has a self-enforcing deviation  $(m'_2, m'_3)$  such that  $\{c\} \in \text{supp}(g(m_1, m'_2, m'_3))$ . Thus, it is not possible that in the profile  $\succ$  under equilibrium  $m$   $c$  is vetoed by coalitions  $1 \cup 2$  and  $1 \cup 3$  but not by voter 1 individually.

Then,  $\{c\} \in \text{veto}(g, 1)$ . Since, according to Theorem 2,  $g$  is coalition-wise neutral-on-its-vetoes, it follows that  $v_g(1) \geq 1$ .

Notice that if  $b$  or  $c$  are equilibrium outcomes in Table 1, a symmetric logic applies and leads to  $v_g(2) \geq 1$  and  $v_g(3) \geq 1$  correspondingly, concluding the proof for the case  $n = k = 3$ .

In the next step we consider 3 voters and 4 alternatives and show that the sum of individual veto rights should be at least 2.

2.  $n = 3$  and  $k = 4$

Consider the preference profile  $\succ$  of Table 3, assume that  $\succeq^{SD}$  satisfies **PREX** and let  $g$  be a mechanism that coalitionally implements a Pareto-efficient SCC.

Since the SCC is Pareto-efficient, only  $a$ ,  $b$  or  $c$  can be equilibrium outcomes.

$\succ_1$	$\succ_2$	$\succ_3$
$a$	$b$	$c$
$b$	$c$	$a$
$c$	$a$	$b$
$d$	$d$	$d$

Table 3

W.l.o.g. assume that  $a$  is an equilibrium outcome. Then, by the same logic as in the first case, we obtain that  $v_g(1) \geq 1$ .

Now consider the preference profile  $\succ'$  depicted by Table 4 and assume that  $\succeq'^{SD}$  satisfies **PREX**.

$\succ'_1$	$\succ'_2$	$\succ'_3$
$a$	$d$	$d$
$b$	$b$	$c$
$c$	$c$	$a$
$d$	$a$	$b$

Table 4

Note that in this profile  $d$  is not an equilibrium outcome since  $v_g(1) \geq 1$ . Then there are 2 possibilities:

1. There is some  $m \in N^g(\succeq'^{SD})$  such that  $g(m) = \{a\}$ . Then  $\{d, c\}$  should be vetoed by coalition  $1 \cup 2$ ,  $\{d, b, c\}$  should be vetoed by coalition  $1 \cup 3$ . If we assume that  $d$  is not vetoed by voter 1 individually, but only by 2 coalitions then we arrive to the same contradiction as in the case  $n = k = 3$ . Similarly,  $c$  must be vetoed by voter 1 individually. Thus  $v_g(1) \geq 2$ .

2.  $a$  is not an equilibrium outcome. Then, w.l.o.g assume that there is some  $m' \in N^g(\succeq'^{SD})$  such that  $g(m') = \{b\}$ . Then  $a$  must be vetoed by coalition  $2 \cup 3$ ,  $d$  must be vetoed by coalition  $(1, 3)$  and  $d, c, a$  must be vetoed by coalition  $1 \cup 2$ . If  $a$  is not vetoed individually by voter 2 then we can use the same logic as before to show that there exists a profile in which  $m'$  is a Nash equilibrium, but not coalition-proof. Thus,  $a$  must be vetoed by voter 2 individually. Thus,  $v_g(2) \geq 1$ . Combining this with  $v_g(1) \geq 1$  we get that  $\sum_{i \in I} v_g(i) \geq 2$ .

Now we can generalize that for arbitrary settings with  $k \geq n$ .

3.  $k \geq n$ .

The first preference profile to consider is included in Table 5. In this profile, denoted

$\succ$ , the first  $n$  alternatives form a Condorcet cycle whereas all the remaining alternatives are Pareto-dominated.

$\succ_1$	$\succ_2$	$\succ_3$	...	$\succ_n$
$a_1$	$a_2$	$a_3$	...	$a_n$
$a_2$	$a_3$	$a_4$	...	$a_1$
$a_3$	$a_4$	$a_5$	...	$a_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_n$	$a_1$	$a_2$	...	$a_{n-1}$
$a_{n+1}$	$a_{n+1}$	$a_{n+1}$	...	$a_{n+1}$
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
$a_k$	$a_k$	$a_k$	...	$a_k$

Table 5

We assume that  $\succeq^{SD}$  satisfies **PREX**. W.l.o.g, assume that  $a_1$  is an equilibrium outcome of  $g$  at  $\succeq^{SD}$ . Then alternative  $a_n$  must be vetoed by all coalitions of size  $n-1$  which include voter 1. Then, as in the case  $n=k=3$ , there are 2 possibilities:

1.  $a_n$  is vetoed by voter 1. Then,  $v_g(a_n) \geq 1$ .
2.  $a_n$  is vetoed by all coalitions of size  $n-1$  which include voter 1, but not by voter 1 individually. In this case consider the preference profile  $\succ'$  in Table 6.

$\succ'_1$	$\succ'_2$	$\succ'_3$	...	$\succ'_n$
$a_1$	$a_n$	$a_n$	...	$a_n$
$a_n$	$a_1$	$a_1$	...	$a_1$
$a_2$	$a_2$	$a_2$	...	$a_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_k$	$a_k$	$a_k$	...	$a_k$

Table 6

Assume that  $\succeq'^{SD}$  satisfies **PREX**. Assume also that for voters  $2,3,\dots,n$ , their preference extension over lotteries is identical.

In this case,  $m$  is a Nash equilibrium. However, since  $\succeq'^{SD}$  satisfies **PREX**, the preferences of voters  $2,3,\dots,n$  are identical and  $a_n$  is not vetoed by voter 1, coalition  $2 \cup 3 \cup \dots \cup n$  has a profitable self-enforcing deviation. Thus,  $m$  is not coalition-proof.

Then we can conclude that  $v_g(1) \geq 1$ .

In the next step we consider the preference profile  $\succ''$  included in Table 7. Assume that  $\succeq''^{SD}$  satisfies **PREX**.

$\succ''_1$	$\succ''_2$	$\succ''_3$	...	$\succ''_n$
$a_1$	$a_{n+1}$	$a_{n+1}$	...	$a_{n+1}$
$a_2$	$a_2$	$a_3$	...	$a_n$
$a_3$	$a_3$	$a_4$	...	$a_1$
$a_4$	$a_4$	$a_5$	...	$a_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_n$	$a_n$	$a_1$	...	$a_{n-2}$
$a_{n+2}$	$a_1$	$a_2$	...	$a_{n-1}$
$a_{n+3}$	$a_{n+2}$	$a_{n+2}$	...	$a_{n+2}$
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
$a_{n+1}$	$a_k$	$a_k$	...	$a_k$

Table 7

In this profile, the alternatives  $a_1, \dots, a_n$  form a Condorcet cycle and the alternatives  $a_{n+2}, \dots, a_k$  are Pareto-dominated. However, alternative  $a_{n+1}$  is the worst one for voter 1 while it is the best one for all other voters.

Note that since  $v_g(1) \geq 1$ ,  $a_{n+1}$  is not an equilibrium outcome. Assume that there is some profile  $m'' \in N^g(\succ''^{SD})$  such that  $g(m'') = \{a_1\}$ . Then  $a_n$  and  $a_{n+1}$  must be vetoed by each coalition of size  $n-1$  which includes voter 1 since these alternatives are preferred to  $a_1$  by all voters except voter 1. Thus, applying the same logic as before we can conclude that  $v_g(1) \geq 2$ .

If there is no such  $m''$  consider w.l.o.g. some  $m''' \in N^g(\succ''^{SD})$  with  $g(m''') = \{a_2\}$ . Then  $a_{n+1}$  must be vetoed by all coalitions of size  $n-1$  which include voter 1 and  $a_1$  must be vetoed by all coalitions of size  $n-1$  which include voter 2. Applying the same logic as before that leads to  $v_g(1) \geq 1$  and  $v_g(2) \geq 1$ .

According to the results of this step we move to the next profile  $\succ'''$ . If at this stage  $v_g(1) \geq 2$  then  $\succ'''$  is as indicated in Table 8:

Since  $v_g(1) \geq 2$ , the alternatives  $a_{n+1}$  and  $a_{n+2}$  are not equilibrium outcomes. Thus only the alternatives that form the Condorcet cycle can be equilibrium outcomes. Then, by the same logic as before one voter has additional veto rights.

If at the previous step,  $v_g(1) \geq 1$  and  $v_g(2) \geq 1$  then consider the profile  $\succ''''$  in Table 9.

Then again only the alternatives in the Condorcet cycle can be equilibrium outcomes, and so we get an additional veto for some voter.

After repeating this procedure we finally arrive to the situation in which the sum of vetoes of individual voters is  $k-n$  and we can not construct a profile with Pareto-



$\succ_1'''$	$\succ_2'''$	$\succ_3'''$	$\dots$	$\succ_n'''$
$a_1$	$a_{n+1}$	$a_{n+1}$	$\dots$	$a_{n+1}$
$a_2$	$a_{n+2}$	$a_{n+2}$	$\dots$	$a_{n+2}$
$a_3$	$a_2$	$a_3$	$\dots$	$a_n$
$a_4$	$a_3$	$a_4$	$\dots$	$a_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_n$	$a_{n-1}$	$a_n$	$\dots$	$a_{n-3}$
$a_{n+3}$	$a_n$	$a_1$	$\dots$	$a_{n-2}$
$a_{n+4}$	$a_1$	$a_2$	$\dots$	$a_{n-1}$
$a_{n+5}$	$a_{n+3}$	$a_{n+3}$	$\dots$	$a_{n+3}$
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
$a_{n+1}$	$a_{k-1}$	$a_{k-1}$	$\dots$	$a_{k-1}$
$a_{n+2}$	$a_k$	$a_k$	$\dots$	$a_k$

Table 8

dominated alternatives. Indeed in the last stage, the alternatives  $a_1, \dots, a_n$  form a Condorcet cycle, for each voter  $i$  there are  $v_g(i)$  alternatives less preferred than any alternative in the Condorcet cycle, and each voter  $i$  prefers the remaining alternatives to each alternative in the Condorcet cycle. By the usual logic, if one alternative in the Condorcet cycle is elected then one voter has additional veto which leads to  $(k - n + 1)$ 'st veto assigned, as desired. It shows that whenever  $k \geq n$  there is at least one voter who has veto rights.

### 1.3 Proof of Theorem 4

Let  $f$  be a Pareto-efficient SCC coalitionally implementable by some deterministic mechanism  $g$ .

If  $f$  satisfies **NGC**, we now show that  $f$  is dictatorial. Note that Lemmas 1, 2 3 and 4 apply also for deterministic mechanisms. Indeed, by focusing on deterministic mechanisms,  $g(M) \subseteq A$  which implies the voters' preferences over  $g(M)$  directly satisfy completeness. Moreover, the role played by **SD** and **PREX** in these lemmas is replaced by the strict preferences of the voters over the alternatives since the mechanism is deterministic.

Then there are two possibilities:

- one voter, say voter  $i$ , is a dictator, so  $v_g(i) = k - 1$ ;
- or there are at least 2 voters with some veto rights.

$\succ_1'''$	$\succ_2'''$	$\succ_3'''$	$\dots$	$\succ_n'''$
$a_{n+2}$	$a_{n+1}$	$a_{n+1}$	$\dots$	$a_{n+1}$
$a_1$	$a_2$	$a_{n+2}$	$\dots$	$a_{n+2}$
$a_2$	$a_3$	$a_3$	$\dots$	$a_n$
$a_3$	$a_4$	$a_4$	$\dots$	$a_1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_n$	$a_1$	$a_2$	$\dots$	$a_{n-1}$
$a_{n+2}$	$a_{n+1}$	$a_1$	$\dots$	$a_{n+1}$
$a_{n+3}$	$a_{n+3}$	$a_{n+2}$	$\dots$	$a_{n+2}$
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
$a_k$	$a_k$	$a_{k-1}$	$\dots$	$a_{k-1}$
$a_{n+1}$	$a_{n+2}$	$a_k$	$\dots$	$a_k$

Table 9

Indeed, if voter  $i$  has strictly less than  $k - 1$  vetoes there is some coalition which does not include voter  $i$  which has at least 1 veto. Then by Lemma 4, at least one voter in this coalition has veto rights.

Assume there is no dictator. Let  $J \in C^I$  be the set of voters with strictly positive veto rights. Lemma 2 implies that  $g$  is coalition-wise neutral-on-its-vetoes so that any coalition  $K \in C^I$  can veto any set of alternatives with cardinality at most  $v_g(K)$ . Consider the following strategy profile  $m$ :

1.  $a_1 \notin g(M_{-j}, m_j)$  for all  $j \in J$  for some  $a_1 \in A$ ;
2.  $g(m) = x$  for some  $x \in A$ .

Point 1 is possible since set  $J$  contains at least 2 voters. Thus, if they both veto  $a_1$  in the profile, this alternative is not in the attainable set of any of these two voters. Point 2. is possible since  $g$  is deterministic, thus the outcome is a singleton for any profile  $m$ .

Take now the following preference profile: for each  $i \in I$ ,  $a_1 \succ_i x \succ_i y$  for any  $y \neq a_1, x$ . Then for every voter  $j$ , for any  $m'_j \in M_j$ ,  $a_1 \notin g(m'_j, m_{-j})$ . That means that after any deviation of any voter  $j$ , alternative  $a_1$  is still vetoed by other voters. Then, the best outcome any voter can achieve by deviation is  $x$ . Since  $g(m) = x$ ,  $m$  is a Nash equilibrium that implements  $x$ , an alternative Pareto-dominated by  $a_1$ . This contradicts  $f$  being Pareto-efficient.

If  $k \geq n$ , the proof of Proposition 5 directly applies and shows that at least some voter has strictly positive veto rights. Hence, by a similar logic to the one exposed in this proof when  $g$  satisfies NGC, we can prove that, again,  $f$  is a dictatorship.

## Chapter 2

# Legitimacy of collective decisions: a mechanism design approach - joint with Matías Núñez

We design two mechanisms that ensure that the majority preferred option wins in all equilibria. The first one is a simultaneous game where agents choose other agents to cooperate with on top of the vote for an alternative, thus overcoming recent impossibility results concerning the implementation of majority rule. The second one adds sequential ratification to the standard majority voting procedure allowing to reach the (correct) outcome in significantly fewer steps than the widely used roll call voting. Both mechanisms use off-equilibrium lotteries to incentivize truthful voting. We discuss different extensions, including the possibility for agents to abstain.

**JEL Codes:** D71, D72.

**Keywords:** Majority, Voting, Implementation, Lottery, Random Sample.

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## 1 Introduction

In legislative, referenda, and committee settings, majority voting is commonly used and is based on simple and intuitive axioms. This method plays a crucial role in analyzing democratic institutions; moreover it is particularly simple to use since voting sincerely is a weakly dominating strategy. However, there is the question of the legitimacy of the decision. For instance, if few voters take part in the election, the outcome might not reflect the preferences of the electorate and can be considered illegitimate, as argued by [Qvortrup \(2005\)](#) in the context of referenda.\* Practitioners often introduce participation quorums to ensure that a sufficient number of voters participate, raising the likelihood that the collective decision is legitimate. The U.S. Senate, the U.K House of Commons, the U.N. General assembly and many other organizations use these quorums. However, the literature suggests that quorums may modify the incentives that voters face, leading to potential negative effects as the existence of quorum-busting, where the minority abstains to prevent the quorum to be reached (see [Herrera and Mattozzi \(2010\)](#) in an equilibrium model and by [Aguiar-Conraria, Magalhães and Vanberg \(2016\)](#) in an experimental setting).

Since the effectiveness of participation quorums to raise legitimacy is dubious, we consider the legitimacy issue through another perspective: the lens of implementation theory. The objective is to design voting mechanisms where all equilibria select some desirable alternative: an implementation problem *À la* [Maskin \(1999\)](#). The situation involves two options, labeled as  $a$  and  $b$ , which a group of voters must choose between. Each voter strictly prefers one option over the other. The goal of the designer is to select the option specified by a social choice rule (such as majority rule), without knowing the preference profile. However, voters are not forced to vote truthfully, so the designer aims to have a mechanism that ensures that the desired option wins in all equilibria of the game. [Xiong \(2021\)](#) demonstrates the existence of a two-alternative implementation problem. This problem states that, with two alternatives, a social choice rule (among which majority rule is the most salient one) is implementable via a simultaneous voting mechanism<sup>†</sup> if and only if it is dictatorial. The category of voting mechanisms is large and covers most of the currently used ones. Our contribution is the design of two classes of mechanisms that circumvents this implementation problem.

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\*Moreover, surveys find that voters often express regret for not participating in the election (see [Blais, Feitosa and Sevi \(2017\)](#) for a recent contribution).

<sup>†</sup>A voting mechanism allows each agent to vote for each option and is monotonic in the usual sense: if  $x$  wins at some profile and gets additional support from some agent,  $x$  remains the winner.

Our first contribution is the design of the Bloc formation mechanism, the first simultaneous one, different than an integer game, that Nash implements majority rule. The mechanism is not a pure voting mechanism because it requires each agent to vote for one of the two options and to nominate  $p$  agents. The outcome depends on whether a majority group of voters vote for the same alternative, say  $x$ , while nominating only voters in the group. In such case, a bloc in favor of  $x$  is formed, and  $x$  wins. If no bloc is formed, the outcome is an endogenous lottery that incentivizes truth-telling. This mechanism's definition actually allows us to see a voting profile as a directed network, making the outcome of the vote depending on the network structure generated by the voting profile. The implementation result extends to a setting with an even number of voters.

Our second contribution deals with sequential voting mechanisms. Remark that sequential majority voting (or roll call voting) subgame perfect implements majority rule when votes are mandatory. This is no longer the case under a participation quorum due to quorum busting. Moreover, this system can be lengthy as the number of steps required goes from  $p + 1$  (the first voters all agree) to  $2p + 1$  ( $p + 1$  prefer one option and  $p$  the other). Our contribution is the design of the Majority with Random confirmations, or RC mechanism that reduces the numbers of steps. Each voter votes for one of the two options (the voting stage). Half plus one of the voters are (randomly) selected, and each one, consecutively, declares whether or not he approves of the winner of the vote (the confirmation stage). If one of them approves, the procedure terminates and that option is elected. If all disapprove, a lottery is cast between the two options, where the probability of each option is proportional to the votes it received in the voting stage. The main advantage of the RC mechanism with respect to the sequential majority one is that the number of steps required is at most  $p + 1$  and might be smaller (since it terminates with the first approval declaration from the  $p + 1$  selected people). We extend the mechanism and the implementation result in two directions: (i) one where agents can abstain in any of the stages making the strategic problem richer and (ii) a setting with incomplete information where agents do not know the preferences of the rest of agents. Result (i) is important since it shows that there is no need of introducing participation quorums to ensure that the outcome is correct. Result (ii) shows that the implementation via RC mechanism does not depend on complete information. As a final result, we extend the logic of the RC mechanism beyond majority rule, as any supermajority rule can be implemented via a simple modification of the RC mechanism.

This work is organized as follows. After laying out the model in Section 3, Section 4 considers the Bloc formation mechanism. Section 5 analyzes the Majority voting with

Random confirmations and its different extensions and Section 6 concludes. The appendix contains the proof of the implementation via the BF mechanism as well as the analysis of three extensions of the RC mechanism (abstention, shares revelation and even number of voters).

## 2 Review of the literature

This paper belongs to a new strand of the literature in implementation where the aim is to find attractive implementing mechanisms that could be tested experimentally rather than analyzing whether a social goal is implementable as in the classic strand following Maskin (1999). Among the papers in this literature, we could cite the recent contribution by Chen, Holden, Kunimoto, Sun and Wilkening (2023) showing that any social choice rule is implementable via two-stage mechanisms involving transfers and lotteries and the one by Echenique and Núñez (2022) which proves that a two-stage mechanism involving prices suffices to implement efficient outcomes.

Our focus is on mechanisms with off-equilibrium lotteries. These mechanisms are known to be more permissive than deterministic mechanisms. See Benoît and Ok (2008), Bochet (2007) and Laslier, Núñez and Sanver (2021) for recent contributions.

Börger and Smith (2014) develop a similar idea to show that one can achieve Pareto improvements over random dictatorship through simultaneous mechanisms. In the related framework of the Condorcet jury theorem, Laslier and Weibull (2013) proposes the introduction of a "slightly randomized majority rule" to ensure that the unique equilibrium is informative. Our mechanism shares a similar spirit to this idea but without randomization in equilibrium. Likewise, Núñez and Pivato (2019) and Azevedo and Budish (2019) describe similar ideas for large populations of agents. See Moore and Repullo (1988) and Vartiainen (2007) for an analysis of rules which are subgame perfect implementable as well as Aghion, Fudenberg, Holden, Kunimoto and Tercieux (2012) for the robustness of subgame perfect implementation to information perturbations.

## 3 Model

We consider a finite set  $I = \{1, \dots, n\}$  of agents, with generic element  $i$ , who need to choose one option out of the set  $A = \{a, b\}$  with generic element  $x$ . We assume that  $n$  is odd ( $n = 2p+1$ ) except in Section 4.4 and in Appendix 1.4. Each agent has strict and complete preferences over  $A$  where  $aR_i b$  denotes that  $a$  is strictly preferred to  $b$ . A vector  $R =$

$(R_1, \dots, R_n) \in \mathcal{R}^n$  denoted the preference profile where  $\mathcal{R}$  is the set of strict preference relations over  $A$ . A social choice function (SCF) is a mapping  $f : \mathcal{R}^n \rightarrow A$  that selects a single option for each profile  $R$ . The majority rule, denoted  $Maj$ , is the SCF that selects the majority preferred option : for each preference profile  $R$ ,

$$Maj(R) = \begin{cases} a & |\{i \in I : aR_i b\}| \geq p + 1 \text{ and } \\ b & \text{otherwise.} \end{cases}$$

We let  $\Delta$  denote the set of lotteries over  $A$  with  $\Delta = \{\beta : A \rightarrow [0, 1] : \sum_A \beta(a) = 1\}$ . A simultaneous *mechanism* is a function  $g : M \rightarrow \Delta$  that assigns to every  $m \in M$  a unique element of  $\Delta$ , where  $M = \prod_{i \in I} M_i$ , and  $M_i$  is the strategy space of agent  $i$ .

We assume that preferences over lotteries satisfy stochastic dominance (**SD**). In our setting, **SD** requires that an agent (weakly) prefers lottery  $\beta$  over lottery  $\eta$  if and only if  $\beta$  assigns (weakly) higher probability to her preferred option  $x$  :

$$\beta \tilde{R}_i^{\text{SD}} \eta \iff \beta(x) \geq \eta(x) \text{ and } \beta R_i^{\text{SD}} \eta \iff \beta(x) > \eta(x),$$

where  $\beta \tilde{R}_i^{\text{SD}} \eta$  means that agent  $i$  weakly prefers  $\beta$  to  $\eta$  and  $\beta R_i^{\text{SD}} \eta$  implies that she strictly prefers the former to the latter. This definition implies that a lottery  $\beta$  stochastically dominates lottery  $\eta$  when  $\beta$  yields at least as much expected utility as  $\eta$  for any von-Neumann Morgenstern utility representation consistent with the ordinal preferences.

### 3.1 Implementation notions

A simultaneous mechanism specifies a game-form: this means that, when the mechanism is coupled with preferences over options for each of the agents, it defines a normal-form game. A Nash equilibrium of the mechanism  $g$  is a profile  $m \in M$  such that  $g(m) \tilde{R}_i^{\text{SD}} g(m'_i, m_{-i})$  for each  $i \in I$  and any  $m'_i \in M_i$ . For a mechanism  $g$ , let  $\text{NE}^g(R)$  denote the set of Nash equilibria at preference profile  $R$ . A mechanism Nash implements a social choice function  $f$  if for any  $R$ , the outcome of any member of  $\text{NE}^g(R)$  is an element of  $f(R)$  and any element of  $f(R)$  is the outcome of some member of  $\text{NE}^g(R)$ .

A sequential mechanism is an extensive game form  $\Gamma = (\mathcal{H}, M, \mathcal{Z}, g)$  where  $\mathcal{H}$  is the set of all histories,  $M = M_1 \times \dots \times M_n$  is the message space with  $M_i = \prod_{h \in \mathcal{H}} M_i(h)$  for all  $i$  where  $M_i(h)$  is the set of available messages for  $i$  at history  $h$ ;  $\mathcal{Z}$  describes the history that immediately follows history  $h$  given that  $m$  has been played; and  $g$  is the outcome that maps the set of terminal histories. The notation  $g(m; h)$  denotes the outcome that obtains when agents use strategy profile  $m$  starting from history  $h$ .

There is an initial history  $\emptyset \in \mathcal{H}$  and  $h_t = (\emptyset, m^1, m^2, \dots, m^{t-1})$  is the history at the end of period  $t$ , where for each  $k$ ,  $m^k \in M(h_k)$ . If for  $t' \geq t + 1$ ,  $h_{t'} = (h_t, m^t, \dots, m^{t'-1})$ , then  $h_{t'}$  follows history  $h_t$ . Since  $\Gamma$  contains finitely many stages, there is a set of terminal histories  $H_T \subset \mathcal{H}$  such that  $H_T = \{h \in \mathcal{H} : \text{there is no } h' \text{ following } h\}$ . A subgame-perfect equilibrium for the game  $\Gamma(R)$  is an element  $m \in M$  such that, for each agent  $i$ ,  $g(m; h) \tilde{R}_i^{\text{SD}} g(m'_i, m_{-i}; h)$  for all  $m'_i \in M_i$  and all  $h \in \mathcal{H} \setminus H_T$ . The set  $SPNE^g(R)$  denotes the set of subgame-perfect equilibria of the game  $\Gamma(R)$ . We say that a mechanism implements the SCR  $f(R)$  in subgame-perfect equilibria, if for each  $R$ , the outcome of any member of  $SPE(\Gamma(R))$  is an element of  $f(R)$  and any element of  $f(R)$  is the outcome of some member of  $SPE(\Gamma(R))$ .

A similar idea applies to the incomplete information setting where the equilibrium notion upon which we rely is Perfect Bayesian equilibrium denoted PBE in the sequel (a formal definition is skipped \*).

### 3.2 A discussion on the Majority mechanism

The following two mechanisms are relevant in both theory and practice, as discussed in the introduction. The majority mechanism, denoted  $\theta_M : A^n \rightarrow A$ , requests each agent to vote for one of the two options and selects  $Maj(m)$  as the winner. The majority mechanism with quorum  $Q$ , denoted  $\theta_Q : (A \cup \text{abs})^n \rightarrow A$ , requires that each voter either announces their vote for an option in  $A$  or abstains. For each profile  $m$ , the outcome  $\theta_Q(m)$  can be expressed as:

1. if  $n_{abs} \leq Q$ ,  $a$  wins if  $n_a \geq \frac{n - n_{abs}}{2}$ ,  $b$  wins otherwise,
2. if  $n_{abs} \geq Q$ ,  $b$  is the winner.

where  $n_a$ ,  $n_b$ , and  $n_{abs}$  respectively represent the number of votes for  $a$ ,  $b$ , and abstentions

As we now discuss, both mechanisms do not implement the majority-preferred option.

Remark first that the mechanism  $\theta_M$  fails to Nash implement the majority rule. Since the mechanism is strategy-proof, it has an equilibrium in sincere strategies where  $Maj(R)$  is the winner. Although this equilibrium is focal, the mechanism has many other equilibria for each preference profile  $R$ , many of which do not select  $Maj(R)$ . For instance, the strategy profile  $m$  with  $m_i = b$  for all  $i \in I$  is an equilibrium for any  $R$  since no player can prevent the victory of  $b$ . While theoretically possible, there is a widely

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\*For the formal definition see [Fudenberg and Tirole \(1991\)](#)



held belief that such equilibria rarely arise in practice: indeed, with the mechanism  $\theta_M$ , some agents in the majority need to vote for their worst-preferred option to allow for the defeat of the majority option.

A similar logic to the one with the Majority mechanism shows that the mechanism  $\theta_Q$  fails to implement  $Maj(R)$ . However, this failure to implement the majority rule is more credible than with  $\theta_M$ . Indeed, the majority winner of the recorded votes may fail to select the majority preferred option of the electorate since every voter decides whether to participate. This means that there are equilibria where every participating agent votes sincerely (i.e. for their preferred alternative) while  $Maj(R)$  loses.

Regarding implementation via sequential mechanisms, remark that the dynamic counterpart of  $\theta_M$  does implement majority rule in subgame-perfect equilibria. Indeed, voters correctly anticipate the moves of the successors which ends up in the correct option being selected. However, the dynamic  $\theta_Q$  (much more used in practice than  $\theta_M$ ) does not follow the same logic since adopting the strategy of not showing-up the minority of voters can prevent the victory of the majority-preferred option.

## 4 A simultaneous mechanism

In this section, we introduce the Bloc formation mechanism (BF mechanism), the first mechanism, beyond integer games, that Nash implements the majority rule\*. We also comment about its interpretation as a network formation game and show its strategic behavior with an even number of agents.

### 4.1 Simultaneous blocs

In the BF mechanism, the message  $m_i$  of agent  $i$  consists of (1.) a vote for an option  $v_i$  and (2.) a nomination of  $p$  agents excluding herself (denoted  $c_i$ ). Formally, the mechanism is denoted  $\chi_{BF} : M \rightarrow \Delta$  with, for all  $i \in I$ ,  $M_i := A \times 2_p^{-i}$  where  $2_p^{-i}$  denotes all the sets of  $p$  agents different from  $i$ .

The central notion of this mechanism is the idea of a bloc of agents. For each option  $x$ , a bloc in favor of option  $x$  is a majority group of agents, denoted  $B$ , such that each agent votes for  $x$  while nominating only agents in  $B$ . This can be formally defined as follows.

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\*Recall that in integer games voters submit the full profile of preference and an integer. Integers submitted are used to determine the outcome if the submitted profiles differ.

**Definition 1.** For each  $x \in A$ , any set  $B$  of agents with  $|B| \geq p + 1$  forms a **bloc** in favor of option  $x$  in the profile  $m$  if:

1.  $v_i = x \forall i \in B$  (only votes for  $x$ ) and,
2.  $c_i \subset B \forall i \in B$  (only nominations in  $B$ ).

The outcome of the BF mechanism depends on whether the message profile has a bloc. Denote by  $B^m$  the set of blocs formed in profile  $m$ . By definition, all blocs in a profile (if any) favor the same option since each bloc contains a majority of agents. Therefore, for any profile  $m$  in which there is a bloc in favor of option  $x$ ,  $\chi_{BF}(m) = x$ .

If the profile  $m$  does not contain a bloc, the outcome is the lottery  $\eta(m)$  over  $A$  with, for each  $x \in A$ :

$$\eta^x(m) = \sum_{i \in I} \eta_i(m) \mathbb{1}\{v_i = x\} \quad \text{with} \quad \eta_i(m) = \frac{|\{j \in I \setminus \{i\} : i \in c_j\}|}{np}.$$

To see the logic behind this formula, we let  $\eta_i(m)$  be the weight of agent  $i$ , that is the share of nominations of  $i$  in the total nominations  $np$ . By construction,  $\sum_{i \in I} \eta_i(m) = 1$  for any  $m \in M$ . When all the other agents nominate  $i$ , agent  $i$  has the maximal possible weight of  $\eta_i(m) = \frac{n-1}{np}$  whereas  $\eta_i(m) = 0$  when none of the other agents nominate  $i$ .

We thus interpret  $\eta^x(m)$  as the sum of the weights of the agents who vote for  $x$  so that, by construction,  $\eta^a(m) + \eta^b(m) = 1$ . Notice that the weight  $\eta^x(m)$  is strictly increasing in the number of nominations for agents voting for  $x$  and, thus, in the number of agents voting for  $x$  among nominated agents.

The previous rules of the mechanism can be summarized as follows. For each message profile  $m$ , the outcome of the mechanism  $\chi_{BF}$  coincides with:

$$\chi_{BF}(m) = \begin{cases} a & \text{if } m \text{ admits a bloc in favor of } a, \\ b & \text{if } m \text{ admits a bloc in favor of } b, \\ \eta(m) & \text{otherwise.} \end{cases}$$

To conclude the description of the BF mechanism notice that it is strategy-proof, that is for any agent  $i$  with  $aR_i b$  (resp.  $bR_i a$ ), any nomination  $c_i \in 2_p^{-i}$  and any message  $m_{-i}$ , agent  $i$  weakly prefers to vote for  $a$  (resp.  $b$ ) since:

$$\chi_{BF}(a, c_i, m_{-i}) \tilde{R}_i^{SD} \chi_{BF}(b, c_i, m_{-i}).$$

In what follows, we show that in any equilibrium of the BF mechanism, most agents strictly prefer to vote truthfully, ensuring that the majority-preferred option wins. This majority of voters that strictly prefer to vote honestly represents the main advantage of the BF mechanism for the usual majority voting one where, in some equilibria, all voters may be indifferent between their two votes.

## 4.2 Voting profile as a directed graph

It is useful to consider blocs in terms of the graph theory. Notice that for any message profile  $m = (v, c)$  the nomination profile  $c$  creates a directed graph in which the vertices are the agents and the edges their nominations. Formally, for each message profile  $m = (v, c)$  denote by  $G_m = (I, C)$  the directed graph formed by  $c$  where the set  $I$  of agents coincides with the set of vertices, and  $C$  is the adjacency matrix such that  $C_{ij} = 1$  if  $j \in c_i$  and  $C_{ij} = 0$  otherwise.

We can formulate an option definition of bloc using the adjacency matrix.

**Definition 2.** For each  $x \in A$ , a set  $B$  of agents with  $|B| \geq p + 1$  forms a **bloc** in favor of option  $x$  in profile  $m$  if:

1.  $v_i = x \forall i \in B$  and,
2. the restriction of  $C$  to the set  $B$ , denoted  $C_B$ , is such that  $\sum_{h \in B} C_{ih} = p \forall i \in B$ .

It follows from Definition 2 that if a set  $B$  of agents forms a bloc in favor of  $x$ , then there is no path from any agent  $i \in B$  to any agent  $j \in I \setminus B$  in the associated graph  $G_m$ . Remark that Definition 1 is equivalent to Definition 2. Firstly, both definitions require at least  $p + 1$  agents to vote for the same option. To show that  $\sum_{h \in B} C_{ih} = p \forall i \in B$  is equivalent to agents in  $B$  voting only for other agents in  $B$ , observe that the row  $i$  of the adjacency matrix  $C$  gives the nominations of agent  $i$ . Thus,  $\sum_{h \in I} C_{ih} = p$  follows by definition. Since, according to Definition 2,  $\sum_{h \in B} C_{ih} = p \forall i \in B$ ,  $C_{ih} = 0$  for all  $h \in I \setminus B$ , that is no agent  $i \in B$  is nominating an agent outside  $B$  which proves the equivalence.

We need some additional definitions to formulate the main results regarding blocs.

We say that  $G_J$  is a subgraph of a graph  $G$  induced by the set  $J \subseteq I$  of vertices if it includes all vertices in  $J$  and its adjacency matrix  $C_J$  is the restriction of  $C$  to  $J$  (i.e. includes only rows and columns corresponding to vertices in  $J$ ).

**Definition 3.** A subgraph  $G_J$  for some  $J \subseteq I$  of a graph  $G$  is strongly connected if there exists a path in each direction between any pair  $i, j$  of vertices with  $i, j \in J$ .

**Definition 4.** A bloc  $B \subseteq I$  in favor of  $x$  is effective iff it is strongly connected.

Each vertex in Figure 2.1 represents an agent, the letters within represent their votes, and the arrows indicate nominations. In Figure 2.1a, there are no blocs in the profile. The only potential bloc involves agents  $\{1, 2, 3\}$ , as they all vote for  $a$  while the others vote for option  $b$ . Agent 1 and agent 3 nominate agent 5 and agent 4, respectively, which violates the conditions required to form a bloc.

In Figure 2.1b, the profile admits two blocs:  $\{1, 4, 5\}$  and  $\{1, 2, 3, 4, 5\}$ . Indeed, in both of these subsets all agents vote for  $b$  and nominate only agents within the bloc. However, only the bloc  $\{1, 4, 5\}$  is effective because it is strongly connected. Notice that there is no path from agent 4 to agent 2, which prevents  $\{1, 2, 3, 4, 5\}$  from being an effective bloc.

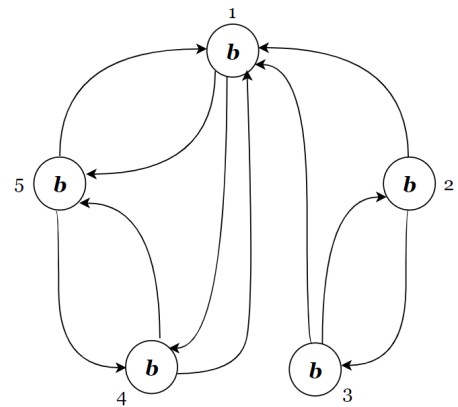
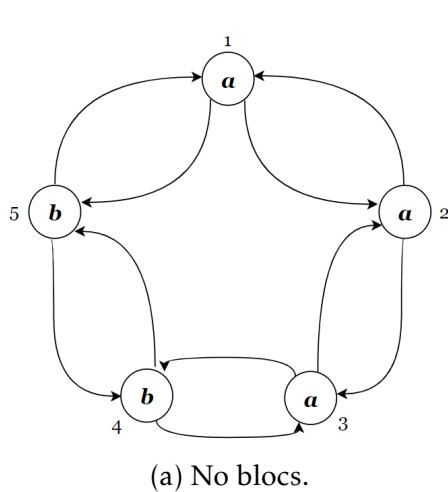


Figure 2.1: Voting profiles formed by the BF mechanism.

The next proposition shows the existence and uniqueness of effective blocs.

**Lemma 1.** Any profile  $m$  admitting a bloc also admits an effective bloc  $B^*$ . Moreover, the effective bloc  $B^*$  is unique and satisfies  $B^* = \bigcap_{B \in B^m} B$ .

*Proof. Existence.* Let  $m$  be some profile with  $B^m \neq \emptyset$  and consider w.l.o.g. that all blocs are in favor of  $a$ . Assume, for the sake of contradiction, that there is no effective bloc in  $m$ . This means that any bloc  $B \in B^m$ , is not effective and therefore not strongly connected. It follows that there are 2 vertices, namely  $i$  and  $j$ , with no path from  $i$  to  $j$ , from  $j$  to  $i$ , or in both directions. W.l.o.g. assume that there is no path from  $i$  to  $j$ . It follows that we can find a partition  $(B', B \setminus B')$  of  $B$  such that: (1)  $i \in B'$ , (2) there is no path from any agent in  $B'$  to agent  $j$  and (3) there is a path from any agent in  $B \setminus B'$  to  $j$ . The existence of this partition implies that agents in  $B'$  only nominate agents in  $B'$  so that  $c_h \subset B' \forall h \in B'$ .

Moreover, since each agent nominates  $p$  agents we have that  $|B'| \geq p + 1$ . We have thus proved that  $B'$  is a bloc.

It follows that if  $B$  is a bloc which is not strongly connected, it contains another bloc of smaller size. Thus, since the minimal size of a bloc is  $p + 1$ , for each bloc  $B$  which fails to be strongly connected, there is a bloc contained in  $B$  which is strongly connected.

**Uniqueness.** Assume by contradiction that for some profile  $m$  there are two non-identical effective blocs  $B^*$  and  $B'^*$ . Since each bloc consists of at least  $p + 1$  agents,  $B^* \cap B'^* \neq \emptyset$ . Thus, there is some agent  $i$  such that  $i \in B^* \cap B'^*$ . By definition of a bloc, there is no path from  $i$  to any  $j \in B^* \setminus (B^* \cap B'^*)$  since  $i \in B'^*$ . Likewise, there is no path from  $i$  to any  $h \in B'^* \setminus (B^* \cap B'^*)$ . By assumption, blocs  $B^*$  and  $B'^*$  are effective and, thus, strongly connected. It follows that there is a path between any two vertices of an effective bloc, reaching the desired contradiction.

$B^* = \bigcap_{B \in \mathcal{B}^m} B$ . We have shown that each bloc which is not effective includes an effective bloc. We have also shown that the effective bloc is unique. The claim follows directly from the two observations.  $\square$

Lemma 1 shows that in any profile  $m$  with blocs, the intersection of the blocs is non-empty and is a bloc itself. Moreover, this intersection is strongly connected meaning that there is no agent which can be removed from it in such way that the profile still admits a bloc. This property has an important implication on the strategic behavior, as summarized by the next result: for any profile  $m$  admitting a bloc, any agent in the effective bloc has a strategy  $m'_i$  that allows her to break all blocs in  $m$  (i.e. no bloc in  $(m'_i, m_{-i})$ ).

**Lemma 2.** For any profile  $m$  admitting a bloc, any agent  $i$  in the effective bloc  $B^*$  has a strategy  $m'_i$  such that  $B^{(m'_i, m_{-i})} = \emptyset$ .

*Proof.* Take some  $m$  with  $B^m \neq \emptyset$ . W.l.o.g. assume that all blocs in  $B^m$  are in favor of  $x$  and consider some agent  $i$  in the effective bloc  $B^*$ .

Observe first that there is a path from any  $j \in B$  to  $i$ . Indeed,  $|c_j| = p + 1$  by definition, and therefore  $c_j \cap B^* \neq \emptyset$  so that there is a path from  $j$  to some agent  $h \in c_j \cap B^*$ . Moreover, since  $B^*$  is strongly connected, there is a path from  $h$  to  $i$  since both belong to  $B^*$ : the existence of a path from  $j$  to  $i$  follows.

Consider a deviation  $m'_i = (y, c_i)$ , that is agent  $i$  votes for  $y$  instead of  $x$  while keeping her nominations unchanged. After such deviation,  $i$  cannot be a part of a bloc in favor of  $x$  since she votes for  $y$ .

By definition of a bloc, for any bloc  $B$  there is no path from the members of the bloc to the agents in  $I \setminus B$ . However, as stated before, since  $i \in B^*$ , there is a path to  $i$  from any member of any bloc in  $B^m$ . Thus, there is no bloc in favor of  $x$  in profile  $(m'_i, m_{-i})$ .

Notice also that since  $B^m \neq \emptyset$  and since  $c_i$  includes only agents voting for  $x$ , there can be no bloc in favor of  $y$  in  $(m'_i, m_{-i})$ . Thus,  $B^{(m'_i, m_{-i})} = \emptyset$ , ending the proof.  $\square$

### 4.3 Nash implementation

The main result of this section is as follows.

**Proposition 1.** *The Bloc formation mechanism Nash implements the majority rule.*

The formal proof of Proposition 1 can be found in the Appendix, but we provide some informal explanation in the following paragraphs. The existence of an equilibrium selecting the majority preferred option is simple. If there are at least  $p + 1$  agents who prefer option  $a$  and these agents vote for  $a$  and nominate each other, this creates a bloc in favor of  $a$ . This profile is an equilibrium since no agent within the bloc wants to deviate (as they obtain their most preferred outcome) and no agent outside the bloc can alter the outcome (by definition).

To discard the existence of an equilibrium with an outcome being a lottery with full support, observe that the weight with which each option wins is strictly increasing (1) in the number of votes it obtains from agents with positive weight and (2) in the number of nominations that agents voting for this option get. Therefore, it is optimal for agents to vote truthfully and nominate as many agents voting for their preferred option as possible, leading to the formation of a bloc in favor of option  $a$ .

Finally, we can argue that no bloc can be formed in favor of  $b$ . Assume, by contradiction, that such a bloc exists. As shown in Section 4.2, an effective bloc exists and includes some majority agent. Then, according to Lemma 2, a majority agent who is a member of the effective bloc can break all the blocs in favor of option  $b$  in the profile leading to a lottery being the outcome. Such a deviation is profitable for a majority agent, contradicting the existence of an equilibrium in which a bloc in favor of option  $b$  is formed. The following example illustrates the logic of the mechanism on this precise point.

*Example 2:* Consider a profile  $R$  with agents 1, 2, 3 preferring  $a$  and agents 4, 5 preferring  $b$  so that  $Maj(R) = a$ . Remark that no equilibrium profile admits a bloc in favor of  $b$ . Indeed, let  $m = (c, v)$  be the profile where each agent votes  $b$  (i.e.  $v_i = b \forall i$ ) and nominations are as follows:  $c_1 = 4, 5$ ,  $c_2 = 1, 3$ ,  $c_3 = 1, 2$ ,  $c_4 = 1, 5$  and  $c_5 = 1, 4$ . The profile  $m$  admits two blocs:  $\{1, 4, 5\}$  and  $\{1, 2, 3, 4, 5\}$ . The bloc  $\{1, 4, 5\}$  is the effective one since

one cannot find a smaller group that nominate each other while voting  $b$ . If any agent  $i \in \{1, 4, 5\}$  deviates to  $m'_i = (a, c_i)$ , the profile  $(m'_i, m_{-i})$  admits no bloc and the outcome is a lottery between  $a$  and  $b$ . Since agent 1 prefers  $a$  to  $b$ , she has a profitable deviation and thus the profile  $m$  is not an equilibrium.

#### 4.4 Bloc formation with an even number of voters

In this section we show that the implementation result for BF mechanism extends to the case when the number of agents is even, i.e.  $n = 2p$ .

In order to incorporate the possibility of even number of agents we need to extend the notion of the majority rule. In this section  $Maj$  is a social choice correspondence (SCC) such that for each preference profile  $R$ :

$$Maj(R) = \begin{cases} a & |\{i \in I : aR_i b\}| \geq p + 1, \\ b & |\{i \in I : bR_i a\}| \geq p + 1, \\ \{a, b\} & \text{otherwise.} \end{cases}$$

We extend the notion of Nash implementation and say that a mechanism  $g$  Nash implements  $Maj(R)$  if for all  $R$  with  $|Maj(R)| = 1$ , the outcome of any  $NE^g(R)$  is  $Maj(R)$ ; and if  $Maj(R) = \{a, b\}$ , the outcome of any  $NE^g(R)$  is any lottery over the two options.

The definition of BF mechanism remains the same as introduced in Section 4.1., notably, the minimal size of a bloc is still  $p + 1$ .

**Lemma 3.** *With an even number of agents, the BF mechanism Nash implements the majority rule. For all  $R \in \mathcal{R}$  such that  $Maj(R) = A$ , the unique equilibrium outcome is a lottery with equal weights.*

*Proof.* Remark first that for any profile  $R$  where  $Maj(R)$  is uniquely defined, the logic of the proof of Proposition 1 applies verbatim. The only case that remains is the one when  $Maj(R) = A$ . We show that the unique equilibrium outcome in such case is a lottery that selects each of the options with equal probability.

**Step 1: no bloc can be formed in equilibrium.** Consider some equilibrium  $m$  and assume  $B^m \neq \emptyset$ . W.l.o.g. assume that a bloc is formed in favor of  $a$ . Then according to Lemma 1 the effective bloc  $B^*$  exists. Since  $|B^*| \geq p + 1$  there is some  $i \in B^*$  such that  $bR_i a$ . Then according to Lemma 2, agent  $i$  has a deviation  $m'_i = (b, c_i)$  which breaks all the blocs in  $B^m$  and thus switches the outcome to a lottery. Thus, the deviation is profitable for agent  $i$ , showing that  $m$  is not an equilibrium.

**Step 2: no lottery which assigns higher probability to one of the options can be an equilibrium outcome.** By contradiction assume that a lottery  $\eta(m)$  is an outcome for some equilibrium profile  $m$  with  $\eta^a(m) > 1/2$ . That means  $\sum_{i \in I} \eta_i(m) \mathbb{1}\{v_i = a\} > \sum_{i \in I} \eta_i(m) \mathbb{1}\{v_i = b\}$ , that is the weights of  $a$ -agents exceed the weights of  $b$ -agents. Then one of the following statements is true:

- There is some agent  $i$  with  $bR_i a$  such that  $\eta_i(m) > 0$  and  $v_i = a$ . In this case agent  $i$  has a profitable deviation  $m'_i = (b, c_i)$ ;
- There is some agent  $i$  with  $bR_i a$  such that  $\exists j \in c_i$  with  $v_j = a$  and  $\exists h \notin c_i$  with  $v_h = b$ . That is there is an agent who prefers  $b$  to  $a$  but votes for  $a$ -agent when a  $b$ -agent is available. In this case agent  $i$  has a profitable deviation  $m'_i = (b, c_i \setminus \{j\} \cup \{h\})$ .
- None of the above holds, but there is some agent  $i$  with  $bR_i a$  with  $\eta_i(m) = 0$  and  $v_i = a$ . Consider some agent  $j$  with  $aR_j b$ . Since there is no agent who prefers  $b$  to  $a$ , votes for  $a$  and has positive weight it must be that there is  $h \in c_j$  with  $v_h = b$  (since  $|c_j| = p$ ). Then agent  $j$  has a profitable deviation  $m'_j = (a, c_j \setminus \{h\} \cup \{i\})$  contradicting that  $m$  is an equilibrium.  $\square$

## 5 A sequential mechanism

This section presents the Majority voting mechanism with Random confirmations (RC mechanism). This mechanism combines the features of the standard simultaneous and sequential majority procedures; this combination reduces the length of all equilibria compared to the sequential majority voting (with the shortest equilibrium including just 2 stages) while keeping the equilibrium outcome unique (in contrast to the one-shot majority voting).

The rules of the RC mechanism are as follows.

*Voting stage:* Each agent  $i$  votes for an option  $v_i \in A$ .

The profile of votes  $v$  is publicly announced. We can also relax this step of the mechanism by revealing only the shares of votes for each option which can be more suitable for practical use. We discuss this relaxation in Appendix C.

*Outcome of the Voting stage:* The option with most votes in  $v$  is denoted the winner of the Voting stage.

*Confirmation stage:* A subset of agents of size  $p + 1$  is randomly chosen and ordered through a uniform draw (recall that  $n = 2p + 1$ ). We denote the order by  $\pi = (\pi_1, \dots, \pi_{p+1})$ . At each stage  $t \in \{1, \dots, p + 1\}$ , agent  $\pi_t$  announces  $Y$  or  $N$ .

*Outcome of the Confirmation stage::*



Whenever an agent  $\pi_t$  announces  $Y$ , the game ends, the outcome being the winner of the Voting stage. If no agent announces  $Y$ , this means that all agents in  $\{\pi_1, \dots, \pi_{p+1}\}$  announce  $N$ . In this case, the outcome is the lottery  $\beta(v)$  that assigns to each option its share of Voting stage votes, so that

$$\beta_a(v) = \frac{|\{i \in I : v_i = a\}|}{n} \text{ and } \beta_b(v) = 1 - \beta_a(v).$$

The purpose of this lottery is to give incentives to agents to vote for their most preferred option. Notice that the Confirmation stage announcements  $Y$  and  $N$  can be viewed as agreement and disagreement with the Voting stage outcome respectively. That is, if some agent in  $\{\pi_1, \dots, \pi_{p+1}\}$  agrees with the outcome being the winner of the Voting stage, this option is the outcome. On the other hand, if no one agrees, the outcome is the previously mentioned lottery.

We now establish the implementation under complete and incomplete information and discuss the extension to abstention.

## 5.1 Complete information

To provide a better understanding of the mechanism, we first present an example that demonstrates its logic before proceeding with a formal argument that encompasses the whole argument.

*Example 1:* Consider a preference profile  $R$  with agents 1, 2, 3 preferring  $a$  to  $b$  and agents 4, 5 preferring  $b$  to  $a$  so that  $\text{Maj}(R) = a$ . If all agents vote  $b$  in the Voting stage (i.e.  $v_i = b$  for all  $i \in I$ ), the outcome is  $b$  independently of the Confirmation stage votes. Notice that in case the outcome was determined by simultaneous majority, such profile  $v$  would be an equilibrium selecting a minority-preferred option. We now demonstrate that under RC mechanism there is always a voter who has a profitable deviation given such Voting stage profile. Consider agent 1's deviation from  $v_1 = b$  to  $v'_1 = a$  so that  $(v'_1, v_{-1}) = (a, b, b, b, b)$ . After this deviation, the outcome depends on the votes of agents in  $\{\pi_1, \pi_2, \pi_3\}$  in the Confirmation stage. If either agent 4 or agent 5 is in this set,  $b$  is the outcome since both agents prefer  $b$  to  $a$  and, thus, their best response is to announce  $Y$ . Otherwise, the set  $\{\pi_1, \pi_2, \pi_3\}$  equals  $\{1, 2, 3\}$  up to a permutation. Notice that such order  $\{\pi_1, \pi_2, \pi_3\}$  occurs with strictly positive probability. The unique best response of any of these agents is to announce  $N$  if  $Y$  was not announced before. Thus, in the unique SPE of the Confirmation stage,  $N$  is announced by all three agents and the outcome of the mechanism is a lottery that selects  $a$  with probability  $1/5$ . Therefore, by deviating

from  $v_1$  to  $v'_1$ , agent 1 induces a lottery that assigns  $a$  a positive probability; by **SD**, agent 1 prefers to deviate showing that any strategy profile in which  $v_i = b$  for every agent  $i$  cannot be an equilibrium.

Table 2.1 illustrates this example. The left part represents the Voting stage profiles: unanimous in the first case, and after the deviation of agent 1 afterwards. The right part illustrates the SPE outcome of the Confirmation stage given the Voting stage profiles and the set  $\{\pi_1, \pi_2, \pi_3\}$ .

Voting stage					Confirmation stage					Outcome
1	2	3	4	5	1	2	3	4	5	
<b>Unanimous vote for <math>b</math>, <math>\{\pi_1, \pi_2, \pi_3\} = \{1, 2, 3\}</math></b>										
$b$	$b$	$b$	$b$	$b$	$N$	$N$	$N$	-	-	$b$
<b>Deviation to <math>v'_1 = a</math>, <math>\{\pi_1, \pi_2, \pi_3\} = \{1, 2, 5\}</math></b>										
$a$	$b$	$b$	$b$	$b$	$N$	$N$	-	-	$Y$	$b$
<b>Deviation to <math>v'_1 = a</math>, <math>\{\pi_1, \pi_2, \pi_3\} = \{1, 2, 3\}</math></b>										
$a$	$b$	$b$	$b$	$b$	$N$	$N$	$N$	-	-	$1/5a + 4/5b$

Table 2.1: Majority voting with Random confirmations

A similar logic to the one described in the example shows that at least  $p + 1$  agents who prefer the majority option are sincere in the Voting stage of *any* equilibrium which leads to the implementation result, stated formally as follows. While the proof here is written with an odd number of agents, it can be extended to situations with an even number of them by properly modifying the mechanism (as detailed in appendix 1.4).

**Proposition 2.** *The RC mechanism subgame perfect implements the majority rule.*

*Proof.* We start solving the game backwards from the Confirmation stage. If the profile  $v$  is unanimous, the Confirmation stage does not affect the outcome. However, if  $v$  is not unanimous, we denote by  $x$  the winning option of the Voting stage and by  $y$  the remaining option.

Next we consider agent  $\pi_{p+1}$  and assume that no agent from  $\{\pi_1, \dots, \pi_p\}$  announced  $Y$ . If  $xR_{\pi_{p+1}}y$ , the unique best response for agent  $\pi_{p+1}$  is  $Y$ ; otherwise, it is  $N$ .

Moving on to agent  $\pi_p$  and assuming no  $Y$  was announced before, the best response for  $\pi_p$  is:

- $Y$  if  $xR_{\pi_p}y$  and  $yR_{\pi_{p+1}}x$ ;
- $N$  if  $yR_{\pi_p}x$  and  $yR_{\pi_{p+1}}x$ ;
- $\{Y, N\}$  otherwise.

This logic can be extended to earlier agents in  $\pi$  in the following way. For any agent  $\pi_i$  with  $i \leq p + 1$  the best response in the Confirmation stage is:

- $Y$  if  $xR_{\pi_i}y$  and  $yR_{\pi_j}x$  for all  $i < j \leq p + 1$ ;
- $N$  if  $yR_{\pi_j}x$  for all  $i \leq j \leq p + 1$ ;
- $\{Y, N\}$  otherwise.

Then the SPE outcome of the Confirmation stage is the following one.

**Lemma 4.** *For any non-unanimous profile  $v$  of the Voting stage with  $x$  being the winner, the SPE outcome of the Confirmation stage is:*

- $x$  if  $xR_iy$  for some  $i \in \{\pi_1, \dots, \pi_{p+1}\}$ ;
- the lottery  $\beta(v)$  otherwise.

Consider the Voting stage assuming that  $Maj(R) = a$  with  $b$  being the minority preferred option. Remark that if  $a$  wins the Voting stage, the equilibrium outcome is  $a$ . This follows from Lemma 4 and the fact that for any order  $\pi$ , the subset  $\{\pi_1, \dots, \pi_{p+1}\}$  includes some agent who prefers  $a$  to  $b$ .

We claim that in any equilibrium the score of  $a$  in the profile  $v$  is greater than or equal to  $p + 1$ . Suppose, by contradiction, that this is not the case, and there are fewer than  $p + 1$  votes in favor of  $a$  in  $v$ . Then there are only two possible cases in which  $b$  wins with positive probability.

**Case 1:**  $v$  is unanimous in favor of  $b$ . This logic of this case is analogous to the one in Example 1. Consider some agent  $i$  with  $aR_i b$ . If she deviates to  $v'_i = a$ , then if  $\{\pi_1, \dots, \pi_{p+1}\}$  contains only majority agents, the outcome is a lottery according to Lemma 4 and it assigns positive probability to  $a$ . Thus, such deviation is profitable for agent  $i$ . It follows that the unanimous profile  $v$  in favor of  $b$  is not an equilibrium.

**Case 2:** There are some votes for  $a$  in profile  $v$ , but less than  $p + 1$ . This implies that there is some agent  $i$  with  $v_i = b$  while  $aR_i b$ . For this agent, deviating to  $v'_i = a$  is profitable. Indeed, if after such deviation,  $a$  is the winner of the Voting stage, then  $a$  is the outcome. If this is not the case,  $b$  is the winner of  $(v'_i, v_{-i})$ . By Lemma 4, if  $\{\pi_1, \dots, \pi_{p+1}\}$  includes only majority agents, which occurs with strictly positive probability, the outcome is the lottery  $\beta(v'_i, v_{-i})$ . Deviation by  $i$  to  $v'_i = a$  increases the probability of  $a$  being selected in the lottery:

$$\beta_a(v) = \frac{|\{i \in I : v_i = a\}|}{n} < \frac{|\{i \in I : v_i = a\}| + 1}{n} = \beta_a(v'_i, v_{-i}).$$

Therefore, due to **SD**, the agent  $i$  finds profitable to deviate to  $v'_i$ .

Thus, we have eliminated all profiles  $v$  in which less than  $p + 1$  agents vote  $a$  as potential equilibria and this completes the proof.  $\square$

## 5.2 Incomplete information

We now prove that the RC mechanism implements the majority rule when we relax the assumption of complete information. It is assumed that each agent knows her own preference over the options and has some beliefs over the preferences of other agents. For simplicity of the argument we assume that each agent believes that the preferences of other agents are *i.i.d.* and assigns probability  $q_a$  (resp.  $1 - q_a$ ) to each agent preferring  $a$  to  $b$  (resp.  $b$  to  $a$ ). Later we show that the *i.i.d.* assumption can be relaxed without affecting the result. A strategy for an agent  $i$  is a mapping  $\sigma_i = (\sigma_i^1, \sigma_i^2)$  where  $\sigma_i^1 : \mathcal{R} \rightarrow A$  and  $\sigma_i^2 : \mathcal{R} \times H_i \rightarrow \{Y, N\}$  stand for the strategies in each of the stages with  $h_1 = v$  -the Voting stage profile, and  $h_t \in \mathcal{R} \times \{Y, N\}^{t-1}$  for all  $t \in \{2, \dots, p + 1\}$ . We denote the conditional beliefs of agent  $i$  about preferences of agent  $j$  by  $\mu_i(R_j | h_i)$  with  $\mu_i(aR_j b | \emptyset) = q_a$ . A first-stage vote of agent  $i$ ,  $v_i^1$ , is revealing given strategy  $\sigma_i^1$  if  $\mu_j(aR_i b | v_i^1) = 1$  if  $aR_i b$  and  $\mu_j(aR_i b | v_i^1) = 0$  otherwise for any agent  $j \neq i$ , that is if the preference of  $i$  is uniquely determined given her first-stage vote.

**Proposition 3.** *Under incomplete information, the RC mechanism implements the majority rule in PBE.*

*Proof.* Fix any strategy profile  $\sigma^1$ , vote profile  $v$  and order  $\pi$  of agents. Denote by  $x$  the winner of the Voting stage given  $v$  and by  $y$  the remaining option. Recall that in the Confirmation stage, an agent votes if and only if all previous votes are  $N$ , since otherwise (whenever a player announces  $Y$ ) the game ends.

The best response for the last mover, denoted agent  $\pi_{p+1}$  is to vote  $Y$  if  $xR_{\pi_{p+1}}y$  and  $N$  otherwise.

The best response of any agent  $\pi_t$  with  $t < p + 1$  in the confirmation stage is as follows:

- $Y$  if  $xR_{\pi_t}y$  and  $\mu_{\pi_t}(yR_{\pi_{t+1}}x, \dots, yR_{\pi_{p+1}}x) | h_{\pi_t}) > 0$ , that is, agent  $\pi_t$  assigns positive probability to the event that all agents in  $\{\pi_{t+1}, \dots, \pi_{p+1}\}$  prefer  $y$  to  $x$  (all successors have opposing preference);
- $N$  if  $yR_{\pi_t}x$  and  $\mu_{\pi_t}(yR_{\pi_{t+1}}x, \dots, yR_{\pi_{p+1}}x) | h_{\pi_t}) > 0$ , that is, agent  $\pi_t$  assigns positive probability to the event that all successors have the same preference;
- $\{Y, N\}$  otherwise.

Note that based on our assumption on prior beliefs, if for some agent  $\pi_t$ ,  $\mu_{\pi_t}(yR_{\pi_{t+1}}x, \dots, yR_{\pi_{p+1}}x) | h_{\pi_t}) = 0$  then, the same applies to the rest of the other agents.

More precisely, this occurs only if some agent in  $\{\pi_{t+1}, \dots, \pi_{p+1}\}$  prefers  $x$  to  $y$  and her Voting stage strategy was revealing. It follows that, in case of non-revealing strategies for agents in  $\{\pi_{t+1}, \dots, \pi_{p+1}\}$ , agent  $\pi_t$  strictly prefers to be truthful, i.e. to announce  $Y$  if the winner of  $v$  is her preferred option and  $N$  otherwise. Thus, we can summarize the outcome of the Confirmation stage as follows.

**Lemma 5.** *For any strategy profile  $\sigma^1$  and Voting stage profile  $v$ , the PBE outcome of the Confirmation stage is:*

- $x$  if  $v_i = x$  for all  $i \in I$ ,
- $x$  if some agent in  $\{\pi_1, \dots, \pi_{p+1}\}$  prefers the winner at  $v$ ,
- the lottery  $\beta(v)$  otherwise.

Consider the Voting stage of the mechanism, an agent  $i \in I$  and an arbitrary profile  $\sigma_{-i}^1$ . We now show that  $i$  strictly prefers to vote for her most preferred option.

Assume w.l.o.g. that  $aR_i b$ . Notice that there is no  $v_{-i}$  for which  $i$  strictly prefers to vote  $b$ . However, voting  $b$  may be a best response if  $i$  is indifferent between voting  $a$  or voting  $b$  for all possible realizations of preferences  $R_{-i}$  and votes  $v_{-i}$  of other agents given strategies  $\sigma_{-i}^1$ .

Consider some realization  $R_{-i}$  and  $v_{-i}$  where  $Maj(R)$  is not the unique option getting the most votes in  $v_{-i}$ . In this case,  $i$  strictly prefers to vote  $a$  independently of whether  $Maj(R) = a$  or  $Maj(R) = b$ . Indeed, when  $i$ 's vote is the  $p + 1^{th}$  in favor of  $a$ , by voting  $a$  rather than  $b$ , she induces the outcome to be  $a$  rather than a lottery (if  $Maj(R) = a$ ) or a lottery which selects  $a$  with higher probability under  $v_i = a$  than  $v_i = b$  (if  $Maj(R) = b$ ).

If for some strategy profile  $\sigma_{-i}^1$ , the option  $Maj(R)$  gets the most votes in  $v_{-i}$  for all realizations  $R_{-i}$  and  $v_{-i}$  that occur with positive probability, voting  $b$  is a best response for  $i$ . We now show that no such strategy profile exists.

Consider some profile  $R_{-i}$  such that  $p - 1$  agents prefer  $a$  to  $b$  and the remaining  $p + 1$  agents prefer  $b$  to  $a$  so that  $Maj(R) = b$ . The profile  $R_{-i}$  occurs with positive probability by assumption. Moreover, given  $\sigma_{-i}^1$ , any realization of  $v_{-i}$  is such that the majority of agents in  $I \setminus i$  vote for  $b$  (since  $Maj(R) = b$ ). Now consider a different profile  $R'_{-i}$  such that  $R_h = R'_h$  for all  $h \in I \setminus \{i, j\}$  with  $bR_j a$  and  $aR'_j b$ . That is, the profile  $R'$  is such that  $Maj(R) = a$  (since  $aR_i b$ ) and the only difference with  $R_{-i}$  is the preference of agent  $j$ . By assumption, in the profile  $R'_{-i}$  for any realization  $v_{-i}$  the majority of agents in  $I \setminus \{i\}$  vote  $a$ . Notice, however, that each agent can condition her strategy only on her preference since this is the information available to agents in the Voting stage. Thus, for all agents in  $I \setminus \{i, j\}$  the probability to vote for  $a$  or  $b$  remains the same when moving from  $R_{-i}$  to  $R'_{-i}$ . Thus, the only change in votes occurs for agent  $j$ . Assume that either in  $R_{-i}$  with  $bR_j a$  or

in  $R'_{-i}$  with  $aR'_j b$  agent  $j$  randomizes, i.e. votes for  $a$  and  $b$  with positive probability. In this case, there is some profile  $v_{-i}$  which occurs with positive probability under  $R_{-i}$  and  $R'_{-i}$ . However, this contradicts the assumption that for any realization of  $v_{-i}$  the majority preferred option obtains the majority of the votes. Thus, agent  $j$  votes  $a$  when  $aR_j b$  and  $b$  when  $bR_j a$ : she votes sincerely. Notice that agent  $j$  was random so that the same logic applies to any agent in  $I \setminus \{i\}$ . Thus,  $i$  is indifferent between voting  $a$  or  $b$  only if all other agents are truthful.

Consider now a profile  $R_{-i}$  such that exactly  $p$  agents prefer  $a$  to  $b$  and  $p$  remaining agents prefer  $b$  to  $a$ . Since they are truthful there are  $p$  votes for  $a$  and  $p$  votes for  $b$ . In this case if  $v_i = a$  the outcome is  $a$  as prescribed by Lemma 5 whereas if  $v_i = b$ , the outcome is a lottery which assigns positive probability to  $b$ . Thus,  $i$  strictly prefers to be truthful. This completes the proof.  $\square$

Notice that our initial assumption on prior beliefs being *i.i.d.* was unnecessarily demanding. If the prior beliefs satisfy the following weaker conditions, the result remains valid:

- Each agent assigns a positive probability to the event  $T_a$  (the event where  $p$  agents other than  $i$  prefer  $a$  to  $b$  and  $p$  agents prefer  $b$  to  $a$ );
- For any subset of agents  $I' \subset I$  and any agent  $j \in I \setminus I'$ , for any  $x, y \in A$ ,  $x \neq y$ ,  $\mu_j(\forall i \in I', xR_i y \mid yR_j x) > 0$ .

The first assumption implies that if the rest of agents vote truthfully, an agent believes she is pivotal with positive probability since there might be exactly  $p$  voters of each type. The second assumption ensures that at least some agent will not be indifferent between voting  $Y$  and  $N$  if her preferred alternative is the winner of the Voting stage independently of the strategy profile  $\sigma_{-i}$ .

### 5.3 Abstention

We now discuss an extension of the RC mechanism where we allow the agents to abstain. To distinguish from the original mechanism we call it RC mechanism with abstention. The possibility of abstention makes the strategic problem richer. Indeed, the abstention of many majority agents can induce the victory of the minority and make agents indifferent between abstaining or voting for any of the options.\* In order to deal with

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\*If one considers the simultaneous majority mechanism with abstention, there is a plethora of equilibria where the turnout of minority agents is larger than the one of majority agents and the minority preferred option wins.

abstention, we extend the definition of the mechanism as follows.

*Voting stage:* Each agent  $i$  votes for an option or abstains, that is  $v_i \in A \cup \{abs\}$ . The profile of votes  $v$  is publicly announced. We call the option which gets most votes in  $v$  the winner of the Voting stage. If no agent participates, i.e.  $v_i = abs$  for all  $i \in I$ , or if the number of votes for  $a$  is equal to the number of votes for  $b$ , the outcome is a lottery which assigns probability  $1/2$  to each of the options.

*Confirmation stage:*

A subset of agents of size  $p + 1$  is randomly chosen and ordered through a uniform draw (from all the agents independent of whether they participated or abstained in the Voting stage). We denote the order by  $\pi = (\pi_1, \dots, \pi_n)$ . At each stage  $t \in \{1, \dots, p + 1\}$ , agent  $\pi_t$  announces  $Y$  or  $N$  as long as  $Y$  was not announced before.

*Outcome:*

If the two options are tied in the Voting stage (including the case in which all agents abstain) the outcome is the lottery which assigns probability  $1/2$  to each option. If there is a single winner in the Voting stage and some agent  $\pi_t$  announces  $Y$ , the game ends, the outcome being the winner of the Voting stage. Finally, if all agents in  $\{\pi_1, \dots, \pi_{p+1}\}$  announce  $N$  or abstain in the Confirmation stage the outcome is a lottery  $\beta(v)$ , which assigns to each option its share of first-stage votes, so that:

$$\beta_a(v) = \frac{|\{i \in I : v_i = a\}|}{|\{i \in I : v_i = a\}| + |\{i \in I : v_i = b\}|} \quad \text{and} \quad \beta_b(v) = 1 - \beta_a(v).$$

**Proposition 4.** *The RC mechanism with abstention subgame perfect implements the majority rule in the presence of abstention.*

The proof of the result can be found in the Appendix. As a final comment on this mechanism, observe the existence of the following equilibrium. Consider a strategy profile in which only 2 agents vote in the Voting stage and both vote for the majority preferred option, and all agents vote  $Y$  in the Confirmation stage whenever asked to vote. This is a subgame perfect equilibrium for any preference profile where the outcome is reached after only three votes and two steps. Indeed, no deviation is possible in the Voting stage since the winner is not altered by adding or subtracting one vote. In the Confirmation stage, there is always a majority agent among the ones taking part so that she strictly prefers to announce  $Y$ . Thus, only two initial votes suffice to guarantee that the majority option is elected.

## 5.4 Supermajorities and RC mechanisms

We now consider a final extension of the baseline model where both options are not treated symmetrically. The set of options consists of a status quo  $sq$  and an alternative policy  $x$ , so that  $A = \{sq, x\}$ . Consider the supermajority voting rule  $Maj_k$  with:

$$Maj_k(R) = \begin{cases} x & |\{i \in I : xR_i sq\}| \geq p + k \text{ and } , \\ sq & \text{otherwise.} \end{cases}$$

where  $1 \leq k \leq p + 1$ . That is, for the alternative policy  $x$  to be selected it needs to be preferred to status quo by at least  $p + k$  agents. Notice that in case  $k = 1$ , the rule  $Maj_k$  is the simple majority rule considered in the rest of the paper, whereas when  $k = p + 1$ ,  $Maj_k$  is the unanimity rule.

For each supermajority rule  $Maj_k$ , we provide an extension of the RC mechanism, denoted  $RC_k$ , that implements it in subgame-perfect equilibria. Its formal definition follows:

*Voting stage:* Each agent  $i$  votes simultaneously for one of the options,  $v_i \in A$ . The profile of votes  $v$  and the winner (based on the supermajority rule  $Maj_k$ ) are publicly announced.

*Confirmation stage:* A subset of agents of size  $\bar{t}$  is chosen and ordered through a uniform draw with  $\bar{t} = p + k$  if  $sq$  is the winner of the first stage and  $\bar{t} = p + 2 - k$  otherwise. An order  $\pi$  of agents is randomly chosen through a uniform draw. At each stage  $t \in \{1, \dots, \bar{t}\}$  agent  $\pi_t$  announces  $Y$  or  $N$ .

As in the baseline model, the mechanism ends at stage  $t \leq \bar{t}$  if agent  $\pi_t$  announces  $Y$ , the winner of the first stage being the outcome. If all agents in  $\{\pi_1, \dots, \pi_{\bar{t}}\}$  announce  $N$  the outcome is the lottery with weights  $\beta_a(v)$  and  $\beta_b(v)$  given by the share of Voting stage votes.

The main difference with the benchmark  $RC$  mechanism is that the number of agents selected for the Confirmation stage varies as a function of the winner of the Voting stage. For instance, in the case of the unanimity rule ( $k = p + 1$ ), the  $RC_{p+1}$  mechanism only requires one agent in the Confirmation stage if the status quo wins (to be certain that at least some agent prefers  $x$  to  $sq$ ) whereas it requires that all agents take part in the Confirmation stage when  $x$  wins initially (to be sure that all agents indeed have  $x$  as their preferred option). As we now show, this simple modification of the  $RC$  mechanism suffices to implement any supermajority rule.

**Proposition 5.** *For any  $1 \leq k \leq p$ , the  $RC_k$  mechanism subgame perfect implements the su-*



*permajority voting rule*  $Maj_k$ .

*Proof.* Notice that the logic of the Confirmation stage holds for any supermajority rule, thus, the result of Lemma 4 applies. That is, in any profile  $v$  with winner  $x$ ,  $x$  is the SPE outcome of the Confirmation stage if at least one agent in  $\{\pi_1, \dots, \pi_{\bar{t}}\}$  prefers  $x$  to  $y$  with  $y \neq x$ , and a lottery otherwise.

Consider then the Voting stage. Assume first that some profile with  $Maj_k(R) = sq$  (that is, less than  $p + k$  agents prefer  $x$  to  $sq$ ) admits an equilibrium which selects  $x$  with positive probability. Given the equilibrium outcome of the Confirmation stage discussed above it follows that in such equilibrium  $x$  is the winner of the Voting stage. Thus, there is some agent  $i$  such that  $sqR_i x$  and  $v_i = x$ . Then, this agent has a profitable deviation since the set  $\{\pi_1, \dots, \pi_{\bar{t}}\}$  includes only agents who prefer  $sq$  to  $x$  with positive probability. Indeed, this holds since there at least  $2p + 1 - (p + k - 1) = p + 2 - k$  such agents and  $\bar{t} = p + 2 - k \leq p + k$  by definition. In this case, the outcome is a lottery that assigns higher probability to  $sq$  under  $(v'_i, v_{-i})$  than under the initial profile  $v$ .

Notice that a symmetric logic applies if we consider a preference profile in which at least  $p + k$  agents prefer  $x$  to  $sq$  and the existence of an equilibrium which selects  $sq$  with positive probability, which completes the proof.  $\square$

## 6 Conclusion

The main contribution of this work is the design of simple mechanisms that implement majority rule. The Bloc formation mechanism follows a different idea: agents need to avoid coordination problems by nominating each other and forming blocs. The RC mechanism requests voters to confirm their vote after expressing a preference between two options. Additionally, our paper demonstrates that lotteries can be used to design mechanisms with fewer steps compared to existing methods, reinforcing the argument stated by [Abreu and Sen \(1991\)](#) that lotteries can lead to more permissive implementation.

We acknowledge that future empirical research is needed to determine the welfare gains of the proposed mechanisms compared to traditional voting procedures. A natural extension of our work on the theoretical front would be to investigate the implementation of efficient rules subject to incentive compatibility, such as the weighted majority rules as characterized by [Azrieli and Kim \(2014\)](#). On the experimental front, we plan to explore the role of these mechanisms in participation games (see [Kirneva, Núñez and Xefteris \(2023\)](#)), as well as in other settings such as public good provision ones.

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# 1 Appendix

## 1.1 Bloc formation mechanism

Proof of Proposition 1.

W.l.o.g. assume that any agent  $i$  in  $\{1, \dots, p+1\}$  is such that  $aR_i b$  so that  $a$  is the majority-preferred option and  $b$  the minority-preferred one. Any agent  $i$  with  $aR_i b$  is a majority agent. We need to prove that (A.) there is an equilibrium implementing  $a$  and that (B.) any equilibrium selects  $a$ .

A. Existence of an equilibrium selecting  $a$ .

Consider the set  $J = \{1, \dots, p+1\}$  that consists only of majority agents. Take the strategy profile  $m$  where for each  $i \in J$ ,  $v_i = a$  and  $c_i \subset J \setminus \{i\}$  so that coalition  $J$  forms a bloc in favor of  $a$ . It follows that  $\chi_{BF}(m) = a$ . To see why  $m$  is an equilibrium, remark that each agent in  $J$  prefers  $a$  to  $b$  (and  $a$  to any lottery with both  $a$  and  $b$  in its support by **SD**) and hence does not want to deviate. Each agent outside  $J$  cannot affect the outcome since the bloc formed by  $J$  is formed independently of the deviation of any agent outside  $J$ . This shows the existence of an equilibrium selecting  $a$ .

B. Any equilibrium implements  $a$ .

For the sake of clarity, we divide this part of the proof in two sections. In section B.1, we show that there is no bloc in favor of  $b$  in equilibrium. In section B.2, we show that any strategy profile that leads to a full-support lottery cannot be an equilibrium, concluding the proof.

B.1. No bloc in favor of  $b$  in equilibrium.

Take any profile  $m$  with a bloc  $B$  in favor of  $b$ ; hence  $\chi_{BF}(m) = b$ . The definition of a bloc means that at least  $p+1$  agents vote for  $b$  and nominate only agents in  $B$ . Consider the effective bloc  $B^*$  which exists and is unique according to Lemma 1. Since  $a$  is the majority option, there is some agent  $i \in B^*$  with  $v_i = b$  in the profile  $m$  and  $aR_i b$ .

Assume that  $m$  is an equilibrium. Suppose that agent  $i$  deviates from  $m_i = (b, c_i)$  to  $m'_i = (a, c_i)$ . This means that  $B^*$  is not anymore an effective bloc in favor of  $b$  in the profile  $(m'_i, m_{-i})$ . Moreover, since  $B^* = \bigcap_{B \in \mathcal{B}^m} B$ , there is no other remaining bloc in the profile  $(m'_i, m_{-i})$  as shown by Lemma 1; thus the outcome  $\chi_{BF}(m'_i, m_{-i})$  is a lottery with support  $a$  and  $b$  with  $a$  being selected with positive probability since  $\eta_i(m) > 0$  and thus  $\eta_i(m'_i, m_{-i}) > 0$  ( $i$  was nominated by some other agent in  $m$ , being part of  $B^*$ ). Thus, by **SD**,  $m'_i$  is a profitable deviation for  $i$  since it increases the probability of  $a$  being selected,

proving that  $m$  is not an equilibrium.

B.2. There is no equilibrium which selects  $b$  with positive probability.

Assume that there is some equilibrium  $m$  where the outcome is a full-support lottery.

Notice that the following two statements hold for any equilibrium profile  $m$  with the outcome being a lottery:

(1) any agent  $i$  who is nominated ( $\eta_i(m) > 0$ ) is sincere.

(2) any agent nominates the largest number of agents who announce her preferred option. In other words, if  $aR_i b$  then  $|\{j : v_j = a \text{ and } j \in c_i\}| = \min\{p, |\{h \in I : v_h = a\}|\}$ .

Indeed, (1) holds since with  $\eta_i(m) > 0$  the vote of agent  $i$  affects the final outcome, thus, voting sincerely increases the probability of  $i$ 's favorite option being selected. Statement (2) holds since the weight  $\eta^x(\cdot)$  is increasing in the sum of the weights of  $x$ -agents and each agent's weight strictly increases on the number of votes that she receives.

Given that (1) and (2) hold since  $m$  is an equilibrium and that  $B^m = \emptyset$ , there is some majority agent which votes  $b$  and is not nominated. Indeed, assume this is not the case and such agent does not exist. According to (1) all nominated agents vote sincerely. It follows from (2) then that all majority agents nominate only other majority agents who are also sincere. This means that a bloc in favor of  $a$  exists contradicting  $B^m = \emptyset$ . Consider then some minority agent  $j$ , i.e.  $bR_j a$ . Since (1) holds,  $c_j$  does not include any majority agent who votes  $b$ , that is  $|\{h \in c_j : v_h = b\}| < p$ . Then since  $\eta^b(m)$  is increasing in the number of nominations of  $b$ -agents, agent  $j$  has a profitable deviation: to nominate agent  $i$  in  $c_j$  rather than some  $a$ -agent. Formally,  $m'_j = (b, c'_j)$  with  $c'_j = (c_j \setminus \{h\}) \cup \{i\}$  for some  $h$  with  $v_h = a$ . This contradicts  $m$  being an equilibrium, and concludes the proof.

## 1.2 RC mechanism with abstention

Proof of Proposition 4

In the Confirmation stage, all agents in  $\{\pi_1, \dots, \pi_{p+1}\}$  are indifferent between announcing  $N$  or abstaining. Indeed, by construction the mechanism treats equally these announcements and, in the Confirmation stage, the best response does not depend on the previous announcements.

Assume first that the Voting stage admits a unique winner. In this case, the Confirmation stage outcome coincides with the one presented in Lemma 4.

Assume now that both  $a$  and  $b$  are tied in the profile  $v$ . Thus, the outcome is a lottery which assigns probability of  $1/2$  to each of the options. Therefore, all agents involved in

the Confirmation stage are indifferent between all 3 possible announcements. Then, the counterpart of Lemma 4 can be formulated as follows.

**Lemma 6.** *For any non-unanimous profile  $v$  of the Voting stage, the SPE outcome of the Confirmation stage is:*

- $x$  if  $x$  is the unique winner in  $v$  and  $xR_i y$  for some  $i \in \{\pi_1, \dots, \pi_{p+1}\}$ ,
- a lottery  $\beta(v)$  if  $x$  is the unique winner in  $v$  and  $yR_i x$  for any  $i \in \{\pi_1, \dots, \pi_{p+1}\}$ ,
- a lottery which assigns equal probabilities to both options if  $v$  does not admit a unique winner.

Consider now the Voting stage of the mechanism. We show that there is no equilibrium which selects  $b$  (the minority preferred option) with positive probability. By contradiction, assume that such equilibrium exists.

**Case 1:** The outcome is deterministic and selects  $b$  with probability 1 for all orders  $\pi$ . In this case, given Lemma 6, one of the following statements holds:

- all participating majority agents vote  $b$ . If any of these agents deviates to  $v'_i = a$ , this is a profitable deviation since there is positive probability that only majority agents are selected at the Confirmation stage and, by Lemma 6, the outcome in this case is a lottery;
- no majority agent participates. Then for any majority agent  $i$  with  $v_i = abs$ , deviating to  $v'_i = a$  is profitable since it leads to a lottery as an outcome with positive probability.

**Case 2:** The outcome is  $b$  with positive probability. Notice, that if  $a$  is the winner of the Voting stage,  $a$  is the outcome for all possible orders  $\pi$  since some majority agent is among the first  $p+1$  agents at the Confirmation stage. Thus, if  $b$  is selected with positive probability, she is the winner of  $v$ , or that  $v_j = \emptyset$  for all  $j \in I$ . If  $b$  is the winner of  $v$  then there is some majority agent who either abstains or votes for  $b$  in the Voting stage.

- Assume  $v_j = abs$  for all  $j \in I$ . In this case any agent has incentives to enter and vote for her favorite option since this option will be the outcome with only one agent present at the Voting stage.

- Assume  $b$  is the winner and there is some majority agent  $i \in I$  with  $v_i = b$ . Then the deviation to  $v'_i = a$  is profitable. Indeed, if after this deviation  $a$  is the winner of  $v$ ,  $a$  is the outcome of the mechanism. Otherwise, the outcome is a lottery based on the Voting stage profile for any  $\pi$ . The deviation from  $v_i$  to  $v'_i = a$  increases the probability of  $a$  in such lottery.

- Assume  $b$  is the winner and there is some majority agent  $i \in I$  with  $v_i = abs$ . Then the deviation to  $v'_i = a$  is profitable. Indeed, if after such deviation  $a$  is the winner of the Voting stage,  $a$  is the equilibrium outcome. If this is not the case,  $b$  is the winner of  $(v' - i, v_{-i})$ . Thus, if some minority agent is in  $\{\pi_1, \dots, \pi_{p+1}\}$ ,  $b$  is the outcome by Lemma 6.

However, if only majority agents are in  $\{\pi_1, \dots, \pi_{p+1}\}$ , the outcome is the lottery  $\beta(v'_i, v_{-i})$ . Deviation by  $v$  to  $v'_i = a$  increases the probability of  $a$  in such lottery: assuming that the number of votes for  $a$  in  $v$  is  $n_a$  and the total number of the Voting stage participants is  $n$  the probabilities are the following:

$$\beta_a(v) = \frac{n_a}{n} < \frac{n_a + 1}{n + 1} = \beta_a(v'_i, v_{-i}).$$

This concludes the proof.

### 1.3 RC with shares revelation

In this section we prove that the implementation results presented in Section 4 are robust to a simplification of RC mechanism where only the shares of votes for alternatives are revealed at the end of the voting stage.

Note first that this relaxation of the mechanism does not affect the logic of the complete information stage, that is the proof of Proposition 1 remains correct. Indeed, since we work with complete information, the agents cannot extract any additional information from knowing the entire profile  $v$  compared to knowing only the shares of votes for different options.

With incomplete information, however, the agents can extract more information about the other agents' preferences from the full profile  $v$  rather than from the shares. In other words, the validity of Lemma 5 is not guaranteed to hold. In what follows we show that this is indeed the case. For simplicity we continue to denote by  $x$  the majority winner of the Voting stage and by  $y$  the remaining option.

If in the RC mechanism only vote shares are revealed, for any strategy profile  $\sigma^1$  and Voting stage profile  $v$ , the PBE outcome of the Confirmation stage is:

- $x$  if  $v_i = x$  for all  $i \in I$ ,
- $x$  if  $xR_i y$  for some  $i \in \{\pi_1, \dots, \pi_{p+1}\}$ ,
- the lottery  $\beta(v)$  otherwise.

Assume some non-unanimous profile  $v$  (so the agents know that the shares of both options are positive) and consider, firstly, agent  $\pi_{p+1}$ . It is the last agent to cast a vote, thus, she has the information necessary to determine the outcome. Thus, her strategy remains unchanged, she votes  $Y$  is  $xR_{\pi_{p+1}} y$  and  $N$  otherwise.

Consider now the agent  $\pi_p$ . There are two possible cases:

**Case 1:** Agent  $\pi_p$  knows the preference of  $\pi_{p+1}$  based on the strategy profile  $\sigma^1$ , on  $\{|i \in I : v_i = a|\}$  and on the fact that no predecessor voted  $Y$  in the Confirmation stage. \* In this case, she strictly prefers to vote  $Y$  if  $xR_{\pi_p}y$  and the preference of  $\pi_{p+1}$  are opposing, and to vote  $N$  if  $yR_{\pi_p}x$  and the preference of  $\pi_{p+1}$  are the same. Otherwise, agent  $\pi_p$  is indifferent between voting  $Y$  and  $N$ .

**Case 2:** Agent  $\pi_p$  does not know the preference of  $\pi_{p+1}$  prior to her vote, that is she assigns positive probability to  $x$  being both the preferred and the least preferred option of agent  $\pi_{p+1}$ . In this case  $\pi_p$  strictly prefers to vote  $Y$  if  $xR_{\pi_p}y$  and  $N$  if  $yR_{\pi_p}x$ , that is to vote according to her true preference to maximize the probability of her favorite option being elected.

Considering an arbitrary agent  $i \in \{\pi_1, \dots, \pi_{p+1}\}$  she is indifferent between voting  $Y$  and  $N$  if

- $xR_iy$  and she assigns 0 probability to an event in which all agents to vote after her in the Confirmation stage have opposing preference;
- $yR_ix$  and she assigns 0 probability to an event in which all agents to vote after her in the Confirmation stage have the same preference.

Indeed, if none of the 2 cases holds, agent  $i$  either prefers  $x$  to  $y$  and believes that she is the last person to vote with such preference with positive probability (so no subsequent agent will vote  $Y$ ), or she prefers  $y$  to  $x$  and believes that none of the subsequent agents will vote  $Y$  with positive probability (due to identical to  $i$ 's preference).

Otherwise, agent  $i$  strictly prefers to vote according to her preference, that is to vote  $Y$  if  $xR_iy$  and to vote  $N$  if  $yR_ix$ .

Note that since in equilibrium agents know the strategies of others, they assign probability 1 to an event only if the event takes place effectively. This completes the proof of the Lemma.

## 1.4 RC with an even number of agents

The RC mechanism for the case of an even number of voters is extended by adding the following step: in case the Voting stage has a unique winner, then proceed to Confirmation stage as before; if both options get the same number of votes in the Voting stage, the

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\*Note that there exist combinations of strategy profiles and Voting stage vote shares such that the knowledge of  $\pi_{p+1}$  is possible: for instance, if all agents vote truthfully in the Voting stage, and all agent  $\pi_p$  was the only vote to vote for the preferred alternative.



outcome is a lottery which assigns equal weights to both options (no Confirmation stage needed).

With an even number of agents, the RC mechanism subgame perfect implements the majority rule with any lottery being an equilibrium outcome for all  $R \in \mathcal{R}$  such that  $Maj(R) = A$ .

*Proof.* First of all, note that the logic of Lemma 4 holds independently of the number of agents whenever the winner of the Voting stage is well-defined (and the outcome is an equal weight lottery otherwise).

In the Voting stage, the argument is identical to the case of an even number of voters for each preference profile  $R$  for which  $Maj(R)$  is a singleton. For any  $R$  such that  $Maj(R) = A$ , our extended implementation notion allows any lottery between the two options to be an equilibrium outcome. Thus, to show the existence of equilibrium is sufficient. One such possible equilibrium is the one in which each agent votes in the Voting stage according to her preference and the outcome is a lottery assigning equal weights to both options. To see that such equilibrium is not unique consider the Voting stage profile in which all agents vote  $a$ . Since among  $p + 1$  agents participating in the Confirmation stage there exists some agent  $i$  such that  $aR_i b$ , the outcome is  $b$ . Note that no agent who prefers  $b$  to  $a$  has a profitable deviation, since  $a$  is still the winner of the Voting stage after any such deviation.  $\square$

# Chapter 3

## Informing to divert attention

I study a multidimensional Sender-Receiver game in which Receiver can acquire limited information after observing the Sender's signal. Depending on the parameters describing the conflict of interest between Sender and Receiver, I characterise optimal information disclosure and the information acquired by Receiver as a response. I show that in the case of partial conflict of interests (aligned on some dimensions and misaligned on others) Sender uses the multidimensionality of the environment to divert Receiver's attention away from the dimensions of misalignment of interests. Moreover, there is negative value of capacity to acquire information in the sense that Receiver would be better off if she could commit not to extract private information or to have access to information of lower quality. I present applications to consumer's choice and informational lobbying.

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## 1 Introduction

Economic agents, whether individuals, firms or politicians, must make decisions concerning issues on which they do not possess full knowledge. In these circumstances the agents need to rely on the expertise of the more informed parties. While having access to more complete and better information these informed experts might, however, be self-interested and, thus, provide information strategically to influence the resulting decisions. Apart from affecting decisions directly by making decision-makers more informed, experts' information transmission also has an indirect effect by changing decision-makers' preference for information and, thus, by altering their own search for information.

In this paper, I study this new role for information provision - directing the decision-maker's search for information when this search is limited by some exogenous constraints. I show that, whenever the decision-maker cannot obtain information on the issues separately, the expert's best strategy might be counterintuitive: to provide some information on the issue which she wants to hide in order to divert the decision-maker's attention toward another issue. Moreover, in cases when the expert would not want the decision maker to learn anything on either of the issues (misaligned interests), it still might be optimal for her to provide some amount of information on one of the issues to divert the decision-maker's attention towards a more favorable issue.

From the perspective of the decision-maker she always benefits from the expert's information compared to the case when she makes decisions on her own. However, she might prefer to face stronger limitations on her information acquisition process (higher costs) to benefit from more information disclosure by the expert. In other words, for some types of conflict of interest, the decision-maker is facing a negative value of capacity to acquire information.

From a technical perspective, I study a multi-dimensional Sender-Receiver framework with quadratic preferences and Receiver's access to additional information after she observes Sender's signals. In the spirit of Bayesian Persuasion literature, Sender commits to a collection of linear signals before the state of the world is realized. In her turn, Receiver, upon observing the realization of Sender's signal(-s), can obtain one additional linear signal with the weights and precision of her choice. Receiver's signal is assumed to be costly, with the cost function being represented either by entropy costs as in Rational Inattention literature, or by a convex increasing precision-dependent function which is standard for many applications.

In such a framework, Receiver's choice of private information depends on the amount

of attention (i.e. relative weights) given to different dimensions by Sender and the quality (i.e. precision) of Sender's information. In the case of quadratic preferences and the multiplicity of Receiver's actions, Receiver always chooses to obtain information on the most uncertain dimension. Thus, by strategically changing the relative uncertainty of dimensions, Sender affects Receiver's preference for information and, hence, her learning process.

The results show that the motives for information transmission depend on the conflict of interest. Due to the assumption of quadratic preferences, the objectives of Sender and Receiver can be described in terms of the posterior uncertainty of Receiver. Hence, the conflict of interests on each dimension can be pinned down by whether Sender benefits from Receiver's learning of this dimension or not.

In the presence of a partial conflict of interests (the case in which Sender wants to reveal one dimension but to hide another one), she faces two competing strategies. The first one is intuitive: Sender reveals the dimension on which interests are aligned and Receiver (partially) learns the other dimension. The second strategy is less intuitive: Sender partially provides information on the dimension where interests are misaligned to change Receiver's preferences for information and to make her obtain information on the dimension of alignment. Whenever Receiver has sufficiently low costs of information acquisition and is thus able to obtain a precise signal, the second strategy is preferred by Sender. Hence, Sender provides information with the goal of diverting Receiver's attention away from the dimension of misalignment of interests.

With fully misaligned interests, I show that contrary to the standard intuition, information transmission is possible and is also driven by attention diverting motives of Sender. Indeed, with fully misaligned interests the trade-off is either to reveal nothing and Receiver obtains information on the more uncertain dimension or to partially reveal this more uncertain dimension to switch Receiver's focus away from it. Depending on the relative conflict of interests on the dimensions the second type of solution might be chosen by Sender.

I extend the baseline framework in several directions. Firstly, I consider the case in which Receiver needs to make a unique decision based on the two dimensions of the state of the world. This set up is particularly important as it is highly relevant for multiple real life applications: optimal funding based on different features of a project, optimal grade, design of optimal rankings etc. I show that if Receiver can observe only one of the two dimensions, but not a mixture of them, then the main intuitions hold, that is Sender still diverts Receiver's attention under some conflicts of interest. Secondly, I extend the main framework of the paper to allow Receiver to observe multiple signals

while facing a budget constraint. In this case, Sender does not divert attention anymore but chooses a more aggressive strategy - to reveal no information even when there is a partial alignment of interests in order to complicate Receiver's learning.

An important application of the results is consumer's choice in the presence of taste shocks and information acquisition constraints. [Kőszegi and Matějka \(2020\)](#) build a theory of mental budgeting and naive diversification and show that consumers either keep the budget unchanged and vary its share spent on different goods (mental budgeting) if the goods are substitutes, or vary the budget keeping its division between the goods (naive diversification) if the goods are complements. The results are generated by the assumption that consumers cannot learn the taste shocks for all the goods due to information costs and, thus, decide to focus either more on relative tastes for the goods or the total taste. The theoretical framework of this paper naturally extends the one by [Kőszegi and Matějka \(2020\)](#) by adding a Sender, i.e. an advertiser or a producer. In Section 5.1 I consider an advertiser who wants to maximize the total spendings of consumers while minimizing the difference in spendings on different products. I show that, while the naive diversification logic for complements stands, consumers do not use mental budgeting in the presence of the advertiser due to the information policy of the latter. Moreover, I show that the advertiser might find a diverting attention strategy optimal, i.e. to emphasize the difference in tastes between the two goods to make consumers question the total taste. Also, even though in expectation consumer is always better off in the presence of Sender even in the presence of a conflict of interests, she is ex-post worse off compared to the no Sender benchmark if her prior beliefs are sufficiently correct.

Another important example to which the theoretical results of the paper apply is the case of informational lobbying. Most of the time, as in my framework policy-makers face multiple decisions on different issues. Moreover, as empirical evidence suggests (see, for instance, [Bertrand, Bombardini and Trebbi \(2014\)](#)) lobbyists tend to tailor the type of information they provide to the preference and expertise area of the policy-maker they are facing. The classic lobbying literature is mostly concerned with the question of whether informational lobbying is detrimental to the decision-making. In contrast, this paper looks at the question of optimal information provision and optimal policy-maker's access to information. While in my setting the policy-maker is always better informed in the presence of a lobbyist, she might receive less information than possible if the lobbyist is convinced the policy-maker is well informed and the interests are only partially aligned. Moreover, the theoretical results in the paper suggest that the lobbyist might take into account the subsequent information search of the policy-maker. In this scenario, she would either underprovide information on an issue with shared preference

or strategically provide information on unfavorable issue. In this case, a policy-maker can benefit from ex-ante committing to the type of information she is going to obtain or from artificially decreasing the quality/quantity of information available. These theoretical intuitions provide the basis for future empirical research on the frequency of such behavior in real world settings.

The rest of the paper proceeds as follows. Section 1.1 discusses the related literature, Section 2 provides a simple example illustrating the main results, Section 3 provides the general model and extends the results, Section 4 presents various extensions and numerical illustrations, Section 5 applies the results to the consumer's choice problem a la [Kőszegi and Matějka \(2020\)](#) and Section 6 concludes.

## 1.1 Literature Review

This work contributes to several strands of the literature.

The commitment assumption imposed on Sender relates to the literature on Bayesian persuasion starting from [Kamenica and Gentzkow \(2011\)](#) and [Rayo and Segal \(2010\)](#). In particular, there are two recent blocks of research: one focuses on multidimensional persuasion under different assumptions (without allowing Receiver access to additional information of her choice), the other considers the uni-dimensional persuasion problem with ex-post information acquisition by Receiver.

In the first of the two blocks, [Tamura \(2018\)](#) extends the classic Bayesian persuasion settings to the multidimensional case. Among other results it shows the optimality of the linear signals under Gaussian prior beliefs. [Velicheti, Bastopcu and Başar \(2023\)](#) extends the framework by introducing multiple senders with possibly different objectives under Gaussian beliefs and quadratic payoffs. [Sayin and Başar \(2021\)](#) provide analysis of persuasion with state-dependent quadratic payoffs for general distributions. [Farokhi, Teixeira and Langbort \(2016\)](#) and [Sayin and Başar \(2018\)](#) are other important contributions to the literature. [Jain \(2018\)](#) considers a two-dimensional Sender-Receiver framework in which commitment (Bayesian persuasion) is possible on one dimension while on the other dimension communication is in the form of cheap talk. [Khantadze, Kremer and Skrzypacz \(2021\)](#) study persuasion of multiple Receivers in a binary multidimensional framework with one action per dimension.

In the literature on persuasion with private information acquisition, [Bizzotto, Rüdiger and Vigier \(2020\)](#) and [Matyskova and Montes \(2023\)](#) show that negative value of information may arise in a uni-dimensional setting in which Receiver has access to an additional signal afterwards. [Bizzotto et al. \(2020\)](#) study a binary framework with fixed

precision of Receiver's signal. [Matyskova and Montes \(2023\)](#) fully solve the model with Shannon entropy costs of private information for Receiver and show that Receiver's equilibrium payoff is not necessarily monotonic in the level of informativeness (costs parameter).

This paper combines the two strands described above as the only way to study the diverting attention motives for Sender. Indeed, in a uni-dimensional framework there is no other dimension to divert Receiver's attention to, while in multidimensional frameworks without Receiver's own search for information, Sender does not need to take into account the effect of her information on Receiver's information strategy.

One of the information cost functions I allow for Receiver is entropy costs which relates this paper to the literature on rational inattention which starts from [Sims \(2003\)](#). From the recent contributions, [Kőszegi and Matějka \(2020\)](#) consider a consumer's multiproduct consumption problem in the presence of taste and price shocks. Information acquisition about the shocks is costly, thus, the consumer strategically decides which of them to observe and to which extent. In the 2 goods example with the taste shocks, [Kőszegi and Matějka \(2020\)](#) shows that (under some restrictions on the available signals), if the goods are substitutes, consumers do not gather information on the total taste and thus keep the total spending fixed. However, in the case of complements, the consumer diversifies and varies the total spending while keeping the consumption of the 2 goods equal. In Section 5 I discuss this example in more detail and show how the introduction of Sender (for example, an advertiser, or a producer) with potentially different objectives from the consumer changes the consumption decision of the latter.

[Hu \(2020\)](#) considers a multidimensional Sender-Receiver framework with Rationally inattentive Receiver. While the solution to the generalized Receiver's information acquisition problem is similar to this paper, the role of Sender in [Hu \(2020\)](#) is different: Sender can either change the relative importance assigned to the dimensions by Receiver, or prevent Receiver from acquiring information on one of the dimensions. However, compared to the framework of the current paper, Sender does not decide on the optimal amount of information provision, hence, they exclude diverting attention motives.

From a technical point of view, there is an extensive literature which relies on Gaussian beliefs and a linear signals structure. See for example, [Liang and Mu \(2020\)](#), [Liang, Mu and Syrgkanis \(2021\)](#).

This paper is closely related to the literature on informational lobbying. [Cotton and Dellis \(2016\)](#) study a binary two-dimensional framework, but they assume that the information on both sides is either perfect or absent. Thus, the work abstracts from the diverting attention motives which are the main focus of this work. [Ellis and Groll \(2020\)](#)

consider a uni-dimensional setting in which a budget-constrained lobbyist can either provide information or subsidies (or both) after which a budget-constrained policy-maker can search for information herself. The policy-maker may benefit from being more budget-constrained in their setting. [Cotton and Li \(2018\)](#) consider a framework in which a politician may obtain information about the policy issues before lobbyists make the decision on monetary funding. They show that the policy-maker may prefer to commit to information of lower quality to induce a competition between lobbyist leading to higher monetary transfers. Other important contributions with a similar approach to informational lobbying are [Dellis and Oak \(2019\)](#), [de Bettignies and Zabojnik \(2019\)](#) and [Hirsch, Kang, Montagnes and You \(2019\)](#). This work contributes to the field by being the first to focus attention on the lobbyist's optimal information provision rather than whether the lobbying itself is detrimental.

Finally, the theoretical framework of the paper allows interpretation of the results in relation to many other applications. The literature has used closely related frameworks to study different issues. For political economy and media competition see, for instance, [Duggan and Martinelli \(2011\)](#), [Perego and Yuksel \(2022\)](#) and [Yuksel \(2022\)](#). In these works policies are multidimensional and the learning technology for citizens or information provided by media are linear signals. The important role of multidimensionality was also pointed out in the question of bonus remuneration ([Bénabou and Tirole \(2016\)](#) and [Fehr and Schmidt \(2004\)](#)) and in a career concerns framework ([Dewatripont, Jewitt and Tirole \(1999\)](#)).

## 2 The Model

There are 2 agents - Sender and Receiver. There is a 2-dimensional state of the world  $\theta = (\theta_1, \theta_2) \in \mathbb{R}^2$ . Receiver needs to take 2 actions, one per each dimension of the state of the world:  $a = (a_1, a_2) \in \mathbb{R}^2$ .

**Payoffs.** I assume quadratic payoffs for Sender and Receiver with:

$$u_i(a, \theta) = -\|Q_i^\theta \theta + Q_i^a a\|^2 \quad (3.1)$$

for some arbitrary  $Q_i^\theta$  and  $Q_i^a$  of size  $2 \times 2$  for  $i \in \{R, S\}$ .

**Information.**

Sender and Receiver have a common prior over the state of the world (prior expecta-



tions are normalized to 0):

$$\theta \stackrel{E}{\sim} \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} v_1 & \rho\sqrt{v_1v_2} \\ \rho\sqrt{v_1v_2} & v_2 \end{pmatrix}\right) = \mathcal{N}(\mu, \Sigma)$$

with  $\mu = (0, 0)^T$ .

Throughout the paper it is assumed that  $v_1 > v_2$ . The special case of  $v_1 = v_2$  will be discussed later when the intuition for the main results will be provided.

Prior to making a decision Receiver obtains information from 2 sources sequentially.

Firstly, Sender commits ex-ante (before learning the state) to send a *set* of linear signals of the form:

$$S_S = \alpha_S \cdot \theta + \varepsilon_S$$

with  $\varepsilon_S \sim \mathcal{N}(0, \sigma_S^2)$ . That is, Sender chooses  $\alpha_S$  - a matrix of  $n \times 2$  for an arbitrary  $n \in \mathbb{Z}_+$  and a noise  $\sigma_S^2$  - a vector of length  $n$ . I denote the interim beliefs of Receiver, after the realization of  $S_S$  is observed by  $(\check{\mu}, \check{\Sigma})$ .

After observing the realization of Sender's signals, Receiver obtains *one* additional linear signal of her choice:

$$S_R = \alpha_R \cdot \theta + \varepsilon_R$$

with  $\varepsilon_R \sim \mathcal{N}(0, \sigma_R^2)$ . That is, Receiver chooses  $\alpha_R$  - a vector of dimension 2 and a noise  $\sigma_R^2$ . I denote the posterior beliefs of Receiver by  $(\tilde{\mu}, \tilde{\Sigma})$ .

I impose directly the assumption of linear signals. Extensive literature studying persuasion in multidimensional settings shows the optimality of linear signals for Sender (in the absence of the additional information acquisition on Receiver's side) in case of Gaussian beliefs and quadratic preference. See, [Tamura \(2018\)](#), [Sayin and Bařar \(2021\)](#), [Akyol, Langbort and Bařar \(2016\)](#) for references. However, it is important to note that the linear signals are not necessarily optimal for Sender whenever Receiver has additional private information.

Beyond technical convenience linear signals provide also a meaningful economic interpretation, namely the amount of focus given to each of the dimension. For instance, if Receiver chooses  $\alpha_R$  such that  $\alpha_{R_1} \gg \alpha_{R_2}$ , the signal she observes is much more informative about dimension 1 of the state than dimension 2. So, for instance, [Heidhues, Johnen and Kőszegi \(2021\)](#) demonstrate that consumers, when searching for information on different products, tend to either focus on one product learning its characteristic

in details, or browse through information on all the products without learning anything deeply. This pattern corresponds well to the predictions obtained by imposing linear signals. Moreover, in the spirit of [Kőszegi and Matějka \(2020\)](#) whenever the state of the world reflects a consumer's taste over two different products, it might natural to assume that the consumer might observe her taste for one or the other, or the relative taste, but not to observe both separately.

In the baseline framework I assume that Sender can provide an arbitrary number of signals while Receiver can choose only one signal later. This assumption is suitable for multiple applications. As was discussed above in the case of a consumer limitations to her search can be natural and go in line with the empirical evidence. However, the advertiser is able to provide multiple information in different form, thus, multiple signals are possible. As another example, one can think of a regulator conducting their own check of a new pharmaceutical product, in which they can run one type of experiment for an arbitrary sample size, but have no resources to design separate experiments. Another example can be job hiring process - the candidate is often free in submitting any supporting information for her portfolio, while the hiring side is often restricted in the number of interviews/tests it can conduct.

**Costs.** Receiver is facing costs of information acquisition. I consider the 2 following cost specification:

- (entropy costs)  $c(\check{\Sigma}, \tilde{\Sigma}) = -\frac{\lambda}{2} \log \left( \frac{|\tilde{\Sigma}|}{|\check{\Sigma}|} \right)$ .
- (precision-dependent costs)  $c(1/\sigma_R^2) = \lambda f(1/\sigma_R^2)$  with  $f : \mathbb{R}_+ \rightarrow \mathbb{R}$  with  $f'(\cdot) > 0$ ,  $f''(\cdot) > 0$  and some  $\lambda > 0$ .

Thus, the total payoff of Receiver is given by:

$$\tilde{u}_R(a, \theta, S_R) = u_i(a, \theta) - c(\cdot).$$

**Timing.** To summarize, the timing of the model is the following:

- $t = 1$ : Sender commits to  $(\alpha_S, \sigma_S^2)$  observed by Receiver;
- $t = 2$ : The state is realized;
- $t = 3$ :  $S_S$  are realized and observed by Receiver, who updates her beliefs and selects  $(\alpha_R, \sigma_R^2)$ ;
- $t = 4$ :  $S_R$  is realized, Receiver chooses an action  $a \in \mathbb{R}^2$ .

### 3 Illustrative Example

In this section I present and solve a simplified version of the framework which captures the main intuition for the general results.

#### 3.1 Setting

I assume  $\alpha_i^T \in \{(1,0), (0,1)\}$ , that is, both Sender and Receiver are restricted to choose a unique signal which reveals (partially) one of the dimensions\*

I also assume the following payoff functions:

$$u_R(a, \theta) = -(a_1 - \theta_1)^2 - (a_2 - \theta_2)^2. \quad (3.2)$$

$$u_S(a, \theta) = \sum_{i \in \{1,2\}} \left( -\beta_i (a_i - \theta_i)^2 - (1 - \beta_i) (a_i - a_i^*)^2 \right).^\dagger \quad (3.3)$$

To fix ideas, consider that Receiver is a policy maker who has to make decisions on two policy issues and wants them to be appropriate for the state of the world ( $\theta$ ). Sender, an informational lobbyist, only partially shares the interests of Receiver and would prefer the action to be “distorted” towards  $a_i^*$ . Parameters  $\beta_i$  capture the extent to which lobbyist’s incentives are aligned with the ones of the policy maker on dimension  $i$ .

I assume Receiver faces entropy costs of information acquisition. Hence, if she chooses to observe a signal on dimension  $i$  the costs are:

$$c(\check{v}_i, \tilde{v}_i) = -\frac{\lambda}{2} \log \left( \frac{\tilde{v}_i}{\check{v}_i} \right).$$

Finally, assume that dimensions of the state of the world are not correlated, hence,  $\rho = 0$ ; w.l.o.g. assume  $v_1 > v_2$ .

#### 3.2 Optimal information provision

The strategic interaction is analyzed by backward induction. Given Receiver’s payoff (3.2), the optimal action conditional on the information obtained is:

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\*The notation  $x^T$  refers to a transpose of vector  $x$  throughout the paper.  $X^T$  refers to the transpose of the matrix  $X$ .

†Alternative Sender’s payoff leading to exact same analysis is  $u_S(a, \theta) = -\sum_i (a_i - \beta_i \theta_i)^2$ .

$$a_R = \tilde{\mu}$$

Taking this into account, ex-ante expected payoffs of Receiver (3.2) and Sender (3.3) can be written as:

$$\mathbb{E}u_R(a, \theta) = -\tilde{v}_1 - \tilde{v}_2$$

and

$$\mathbb{E}u_S(a, \theta) = \text{const} - (2\beta_1 - 1)\tilde{v}_1 - (2\beta_2 - 1)\tilde{v}_2 \quad (3.4)$$

correspondingly. Note, that such representation of expected payoffs makes the conflict of interest apparent: if the coefficient in front of  $\tilde{v}_i$  ( $-2\beta_i - 1$ ) is positive then Sender benefits from Receiver's uncertainty and, thus, is not interested in disclosing any information on this dimensions. Conversely, if the coefficient is negative, Sender would ideally induce full learning. This follows directly from (3.4) and has an intuitive interpretation: the interests are *aligned* iff Sender puts relatively higher weight to the decision matching the true state of the world ( $\beta_i > 1/2$ ), i.e. to the Receiver's objective, and are *misaligned* otherwise ( $\beta_i < 1/2$ ). I distinguish the three following possibilities:

- Interests are *fully aligned* if  $\beta_1 > 1/2$  and  $\beta_2 > 1/2$ ;
- are *fully misaligned* if  $\beta_1 < 1/2$  and  $\beta_2 < 1/2$ ;
- are *partially aligned* if  $(\beta_1 - 1/2)(\beta_2 - 1/2) < 0$ .

One additional definition is needed before stating the optimal information provision in this framework.

**Definition 1.** *Sender diverts Receiver's attention with her signal if:*

- *Sender provides information on a dimension where interests are misaligned;*
- *Receiver would obtain information on this dimension in the absence of Sender's signal and obtains information on the other dimension after observing Sender's signal.*

I now formulate the optimal Sender's strategy, and prove the result in the next section.

**Proposition 1.** *Sender diverts Receiver's attention by partially disclosing dimension 1 iff*

1. *Incentives of Sender and Receiver are partially aligned with  $\beta_1 < 1/2, \beta_2 > 1/2$  and*

$$\frac{\lambda}{2} < \frac{-(2\beta_1 - 1)}{2(\beta_2 - \beta_1)} v_2 \equiv \frac{\lambda^*}{2},$$

or

2. *Incentives of Sender and Receiver are fully misaligned ( $\beta_1 < 1/2$  and  $\beta_2 < 1/2$ ) and*

$$(v_2 - \lambda/2)(\beta_1 - \beta_2) < 0.$$

Proposition 1 pins down the set of cases in which Sender prefers to use seemingly counter intuitive strategy of diverting attention. This is driven by the effect which Sender's choice of signal has on the Receiver's focus on different dimensions when Receiver's cost of information acquisition are low. Indeed, in the cases underlined in the Proposition 1 Sender reveals the dimension she wants to hide. The intuition for this is the following: whenever Receiver is able to obtain sufficiently precise signal on her own, it is too costly for Sender to allow Receiver to learn the dimension of misalignment. Thus, using the fact that Receiver can obtain only one signal, she adjusts Receiver's uncertainty to force Receiver to learn the dimension where interests are aligned.

The argument is similar in the case of full misaligned of interests when the misalignment is higher on initially more uncertain dimension (case 2 in Proposition 1). In this case, information provision is possible for diverting attention reasons even though ideally Sender would prefer to reveal no information.

Such counter-intuitive strategic behavior of Sender has implications for Receiver's welfare as described by the next result. To illustrate that I fix some arbitrary  $V_1 < 0, V_2 > 0, v_1$  and  $v_2$  and alter the costs parameter  $\lambda$ . I slightly abuse the notation and write  $\mathbb{E}[u_R(\lambda)]$  for the expected payoff of Receiver given the optimal strategies described above and costs  $\lambda$ .

**Proposition 2.** *R's utility is non-monotonic in her costs of information acquisition: there exists an interval  $(\underline{\lambda}, \lambda^*)$  such that  $\mathbb{E}[u_R(\lambda)] < \mathbb{E}[u_R(\bar{\lambda} + \delta)]$  for all  $\lambda \in (\underline{\lambda}, \lambda^*)$  and some  $\delta \rightarrow 0$ .*

*Proof.* The result follows intuitively from Proposition 1. Indeed, the payoff from the diverting attention solution at  $\lambda^*$  is:

$$u_R(a, \theta) - c(\cdot) = -v_2 - \lambda^*/2 + \frac{\lambda^*}{2} \log \left( \frac{\lambda^*}{2v_2} \right)$$

The payoff from the intuitive solution at  $\lambda^*$  is:

$$u_R(a, \theta) - c(\cdot) = -\lambda^*/2 + \frac{\lambda^*}{2} \log\left(\frac{\lambda^*}{2v_1}\right)$$

Note that at  $\lambda^*$  Sender is indifferent between the two solutions.

The payoff from the intuitive solution at the threshold  $\lambda^*$  exceeds the payoff from the diverting attention solution at this threshold if:

$$-v_2 < \lambda^*/2 \log\left(\frac{v_2}{v_1}\right) \quad (3.5)$$

If (3.5) holds, then the diverting attention solution generates lower utility for Receiver also in the region of  $\lambda^*$  which completes the proof.  $\square$

Note that in the interval  $(\underline{\lambda}, \lambda^*)$  described in Proposition 2 Receiver's expected utility is decreasing in  $\lambda$ . Moreover it is decreasing for any  $\lambda > \lambda^*$ . However, at  $\lambda^*$  - the point at which Sender changes her strategy from diverting attention to intuitively revealing dimension of alignment, the expected utility is discontinuous and jumps upwards. Figure 3.1 illustrates this point.

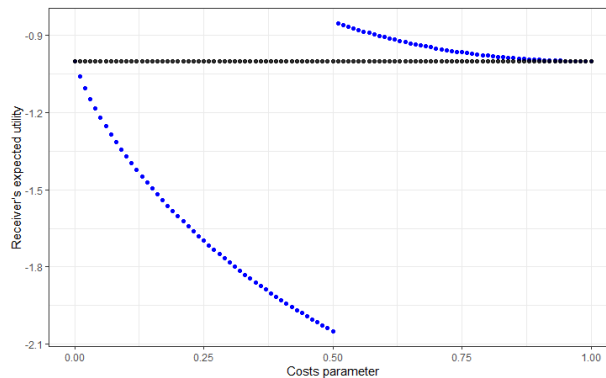


Figure 3.1: Receiver's expected utility (in blue) vs. no Sender benchmark (in black) as a function of the costs parameter  $\lambda$ . Parameters:  $\beta_1 = 0.25$ ,  $\beta_2 = 0.75$ ,  $v_1 = 1.5$ ,  $v_2 = 1$ .

### 3.3 Solution

This section presents the full solution including Sender's and Receiver's strategies in the cases discussed by Proposition 1. To begin with, note that since Sender, by assumption, can provide a (possibly noisy) signal on one dimension only there are only two types of interim uncertainties she can induce on the Receiver's side:

- $(\check{v}_1, v_2)$  with  $\check{v}_1 \leq v_1$  if  $S_S = \theta_1 + \varepsilon_S$
- $(v_1, \check{v}_2)$  with  $\check{v}_2 \leq v_2$  if  $S_S = \theta_2 + \varepsilon_S$

Receiver's utility function is given by the sum of quadratic losses and she bears entropy costs of information collection. Hence, given any pair of interim uncertainties  $(\check{v}_1, \check{v}_2)$ , she learns the more uncertain dimension with a signal which makes the posterior variance equal to  $\lambda/2$ . That is the posterior beliefs are:

- $(\lambda/2, \check{v}_2)$  if  $\check{v}_1 > \check{v}_2$  and  $\check{v}_1 > \lambda/2$
- $(\check{v}_1, \lambda/2)$  if  $\check{v}_1 < \check{v}_2$  and  $\check{v}_2 > \lambda/2$
- $(\check{v}_1, \check{v}_2)$  if  $\check{v}_1 < \lambda/2$  and  $\check{v}_2 < \lambda/2$

The set of attainable posterior beliefs is presented in Figure 3.2.

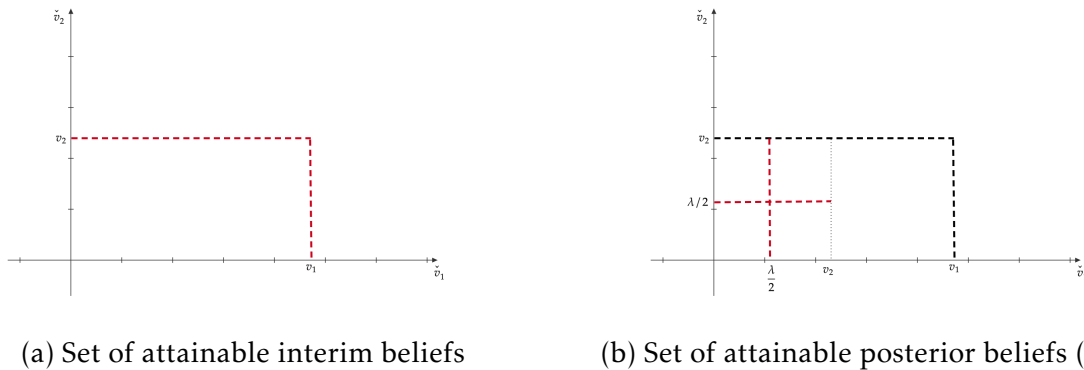


Figure 3.2: Attainable beliefs when Sender and Receiver have access to a single signal each

Notice that the attainable set of posterior beliefs is non-convex. For each possible alignment/misalignment of interests one of the four extreme points of the state will be the optimal solution for Sender. The information flow to reach each of these posteriors is the following:

- *Solution A*: Sender fully reveals dimension 1 ( $S_S = \theta_1$ ) and Receiver learns dimension 2 until  $\lambda/2$ ;
- *Solution B*: Sender doesn't reveal any information, Receiver learns dimension 1 until  $\lambda/2$ ;

- *Solution C*: Sender reveals dimension 1 until  $v_2$ , Receiver learns dimension 2 until  $\lambda/2$ ;
- *Solution D*: Sender reveals dimension 2 ( $S_S = \theta_2$ ) and Receiver learns dimension 1 until  $\lambda/2$ .

Notice that in solution C the only goal of information provision for Sender is to divert Receiver's attention from dimension 1 by making it more certain.

There are two main take-aways from Proposition 1. Firstly, with partial alignment of interests Sender might prefer to disclose the dimension on which interests are misaligned to divert Receiver's attention instead of fully disclosing the dimension of alignment of interests. The intuition is the following: in case Sender discloses dimension on which interests are aligned, Receiver will choose a signal informative about the other dimension. In case the costs of information acquisition for Receiver are low, her chosen signal is very precise, and, thus, costly for Sender. Thus, in the case of well informed Receiver, Sender prefers to settle for only partial revelation of the dimension of alignment of interests to prevent information acquisition on the other dimension by Receiver.

Secondly, in the case of fully misaligned interests (Sender prefers Receiver not to learn any of the dimensions), the information transmission is still possible counter to the first intuition. Sender prefers to reveal some information if, in the absence of it, Receiver learns the dimension on which interests are more misaligned (lower  $|\beta_i|$ ). In this case, information is revealed to divert Receiver's attention from the dimension of higher misalignment of interests.

An important case is the one with  $\lambda = 0$ , that is the information is costless for Receiver and the only restriction is that the signal must contain information about one of the two dimensions exclusively. Then whenever  $\beta_1 < 1/2$  and  $\beta_2 > 1/2$  (interests are misaligned on the more uncertain dimension and aligned on the other one) the unique solution for Sender for all parameters is the one based on the attention diversion.

Another important takeaway from the simple framework is the Receiver's choice of information to acquire: namely, she chooses to observe the dimension which is least known to her at the moment. This feature is in contrast to what is often obtained in the search literature (see, for instance, [Gossner, Steiner and Stewart \(2021\)](#)) where once Receiver's attention was focused on one item she is relatively more likely to observe it in the future. The reason for this difference lies in the combination of the utility function of Receiver - she wants to learn the state and treats equally both dimensions, and the cost function\*.

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\*There is a big range of cost functions which allow such dynamics (given the symmetry of  $u_R$ ). Any



## 4 General results

Now I go back to the original formulation of the problem. Section 4.1 reformulates the problem as a linear programming, Section 4.2 provides the solution to Receiver's information acquisition problem. Section 4.3 gives the main general results.

### 4.1 LP reformulation of the problem

From (3.1) it follows that the optimal action of Receiver is given by:

$$a_R = -(Q_R^{a^T} Q_R^a)^{-1} Q_R^{a^T} Q_R^\theta \tilde{\mu}.$$

Then as is established in the literature (see, for instance, [Velicheti et al. \(2023\)](#) or Lemma 1 in [Tamura \(2018\)](#)) there exist symmetric  $2 \times 2$  matrices  $V_R$  and  $V_S$  such that

$$\mathbb{E}[u_i(a^*, \theta)] = \mathbb{E}[\tilde{\mu}^T V_i \tilde{\mu}] + \text{const}, \quad (3.6)$$

for  $i \in \{R, S\}$ .

From (3.1) it is possible to link  $V_i$ s to the original parameters of the model in the following way:

$$V_R = Q_R^{\theta^T} Q_R^\theta \quad (3.7)$$

and

$$V_S = \Lambda^T \Lambda - \Lambda^T Q_S^\theta - Q_S^{\theta^T} \Lambda$$

with  $\Lambda = Q_S^a (Q_R^{a^T} Q_R^a)^{-1} Q_R^{a^T} Q_R^\theta$  (see, for instance, [Tamura \(2018\)](#) or [Velicheti et al. \(2023\)](#) for the derivation).

In the example of Section 3 the corresponding matrices  $V_S$  and  $V_R$  are:

$$V_S = \begin{pmatrix} 2\beta_1 - 1 & 0 \\ 0 & 2\beta_2 - 1 \end{pmatrix} \quad V_R = \begin{pmatrix} 1 & 0 \\ 0 & 1. \end{pmatrix}$$

### 4.2 Receiver's optimal information acquisition

For the main results I focus on Receiver with the payoff such that  $V_R = I$  which allows us to have the closed-form solution for both Receiver's and Sender's problems. Then (3.7)

---

non-decreasing precision-dependent cost function leads to the learning of the most uncertain dimension.

implies that  $Q_R^\theta$  must be of the form:

$$Q_R^\theta = \begin{pmatrix} q_1 & q_2 \\ -q_2 & q_1 \end{pmatrix}$$

with  $q_1^2 + q_2^2 = 1$ . Note that the assumption on  $V_R$  does not impose any restriction on  $Q_R^a$ .

In Section 5.1 the relaxation of this restriction on  $V_R$  will be discussed.

Such  $V_R$  implies that uncertainty on different dimensions is equally costly for Receiver and she wants to learn the state of the world as precisely as possible. Thus, she wants to reduce the sum of the posterior uncertainties the most as stated in the following Lemma.

From now on I denote by  $v(\alpha) = \alpha^T \Sigma \alpha$  the variance on dimension  $\alpha \cdot \theta$ .

**Lemma 1.** *Assume  $V_R = I$ . Receiver optimally obtains a signal  $S_R = \alpha_R \cdot \theta + \varepsilon_R$  such that  $v(\alpha_R)$  is maximized given  $\|\alpha_R\| = 1$ .*

*Proof.* **Step 1:** Given (3.6) the ex-ante expected payoff of Receiver can be written as:

$$\mathbb{E}u_R(a, \theta) = -\tilde{v}_1 - \tilde{v}_2 + \text{const.} \quad (3.8)$$

Assume first that there are no costs of information acquisition for Receiver on top of the restriction of a unique signal being available. Consider arbitrary interim beliefs  $\check{\Sigma}$ . There exists a rotation matrix  $U$  such that  $\check{\Sigma}^U = U^T \check{\Sigma} U$  is diagonal\*. That is, there exists another basis in which the dimensions are not correlated. Then the payoff in (3.8) can be rewritten in the following way:

$$\mathbb{E}u_R(a, \theta) = -[U^T \check{\Sigma}^U U]_{11} - [U^T \check{\Sigma}^U U]_{22} + \text{const} = -\tilde{v}_1^U - \tilde{v}_2^U + \text{const.}$$

Then in this new basis  $U$  the problem is trivial - the optimal learning strategy of Receiver is to learn a dimension  $i^U = \arg \max_i v_i^U$ .

Note that the dimension  $i^U = \arg \max_i v_i^U$  corresponds to the eigenvector of  $\check{\Sigma}$  with the highest eigenvalue. Thus, the corresponding signal indeed discloses this dimension, i.e.  $S_R = \max_{\alpha: \|\alpha\|=1} \text{Var}(\alpha \cdot \theta)$ .

**Step 2:** Now I take the costs into consideration. Consider arbitrary precision dependent cost function  $\lambda c(1/\sigma_R^2)$  with  $c(\cdot)$  being an increasing function. Notice that for any fixed costs  $\bar{c}$  (i.e. for a fixed precision of a signal) the uncertainty is reduced the most if

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\*The rotation matrix has a form  $U = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$  where  $a$  is the co-sinus of the rotation angle and  $b$  is the sinus of the rotation angle.

the signal is on the dimension  $\alpha_R = \arg \max_{\alpha: \|\alpha\|=1} \text{Var}(\alpha \cdot \theta)$ . Thus, the statement of the lemma for precision-dependent costs.

Next consider the entropy costs  $c(\check{\Sigma}, \tilde{\Sigma})$ . Fix some costs  $c$  and denote  $C = e^c$ . Then if Receiver observes a signal  $S_R$  with  $\alpha_R$  such that it generates costs  $c$  it must satisfy:

$$\text{Var}(S_R) = \frac{v(\alpha_R)}{1 - C}.$$

This follows from the fact that given entropy costs  $C = \sigma^2/(v(\alpha_R) + \sigma^2)$  where  $\sigma^2$  is the noise in  $S_R$  and that  $\text{Var}(\alpha_R) = v(\alpha_R) + \sigma^2$ .

Then, for given costs the signal which maximizes the uncertainty reduction is the solution of:

$$\max_{\alpha_R} (1 - C) \frac{\text{Cov}^T(S_R, \theta) \text{Cov}(S_R, \theta)}{v(\alpha_R)}$$

The solution is independent of  $C$  and coincides with  $\arg \max_{\alpha_R} v(\alpha_R)$ . Thus, for each targeted costs, the signal which minimizes the ex-post uncertainty is the one which uncovers the most uncertain dimension. This completes the proof.  $\square$

I do not impose any restrictions on  $V_S$ , thus, any quadratic preference are allowed for Sender. Note, however, that there exists a rotation matrix  $U'$  such that  $V_S^U = U'^T V_S U'$  is diagonal. That is, there exists a new basis such that Sender cares exclusively about the posterior uncertainties of Receiver about the associated axes, but not the correlations. Also note that since  $V_R = I$  by assumption,  $V_R^U = U'^T V_R U' = I$ , thus in the new basis Receiver also wants to minimize the sum of residual uncertainties, i.e. to learn both dimensions as precisely as possible.

Thus, Receiver's solution satisfies Lemma 1, that is she observes the most uncertain dimension. The optimal noise in the signal can be obtained as the solution to:

$$\max_{\sigma^2} -\frac{v\sigma^2}{v + \sigma^2} - c(\cdot)$$

where  $v = \max_{\alpha} v(\alpha)$ .

I denote the resulting optimal posterior beliefs given the interim beliefs  $\check{\Sigma}$  by  $\tilde{\Sigma}(\check{\Sigma})$ . Notice that with entropy costs posterior beliefs on the dimension  $\arg \max_{\alpha} v(\alpha)$  are equal to  $\min\{v(\alpha), \lambda/2\}$  while for the convex increasing precision-dependent costs it is an increasing continuous function of  $v(\alpha)$ .

To simplify notations I am going to assume directly  $V_S$  to be diagonal which is, as described above w.l.o.g. Sender's problem can be written then in the following way:

$$\begin{aligned} \max_{\Sigma'} -e^T V_S \circ \tilde{\Sigma}(\Sigma') e \\ \text{s.t.} \\ \Sigma - \Sigma' \geq 0 \end{aligned} \quad (3.9)$$

Note that according to (3.9) Sender is choosing the interim beliefs while it is standard in the literature to consider optimization over posteriors. While the problem is easily rewritten as an optimization over posteriors the formulation with interim beliefs is more convenient given the solution method to obtain the main results.

For better understanding of the results another formulation is also useful:

$$\begin{aligned} \max_{U_S, \Sigma'} -e^T V_S \circ \tilde{\Sigma}(U_S \Sigma' U_S^T) e \\ \text{s.t.} \\ \Sigma - U_S \Sigma' U_S^T \geq 0 \\ \text{and} \\ \Sigma' \text{ is diagonal with } \Sigma'_{11} > \Sigma'_{22}. \end{aligned} \quad (3.10)$$

Formulation (3.10) states that Sender can choose a dimension of maximal uncertainty for Receiver (that is the rotation  $U_S$  of beliefs) and the interim beliefs in this basis.

Denote by  $\tilde{v}(v)$  the solution to the following unidimensional problem of Receiver:

$$\tilde{v}(v) = \max_{v'} -v' - c(\cdot)$$

That is, if Receiver decides to obtain a signal on a dimension with uncertainty  $v$ , the optimal posterior belief is  $\tilde{v}(v)$ . Note that due to the assumptions on the costs of information acquisition  $\tilde{v}(v)$  is non-decreasing.

In the benchmark framework of Section 3, notion of the conflict of interest was direct in the sense that a dimension  $i$  with  $V_i > 0$  was considered a dimension of alignment and a dimension with  $V_i < 0$  - a dimension of misalignment. For the general results, however, the extended notion of alignment is needed.

**Definition 2.** *The interests on a dimension  $x \cdot \theta$  with  $\|x\| = 1$  are aligned if  $V_1 x_1^2 + V_2 x_2^2 > 0$  and misaligned otherwise.*

The definition partitions the space of the dimensions in 2 parts. The intuitive mean-

ing of it is as follows: if Receiver would learn some arbitrary dimension  $x \cdot \theta$  it would reduce her uncertainty on both original dimensions at the same time. Thus, there are 2 potential effects for Sender: the positive effect of learning on the original dimension of alignment and the negative effect of learning on the original dimension of misalignment. Depending on which effect dominates a dimension  $x \cdot \theta$  is either a dimension of alignment or misalignment.

To see that such Definition comes naturally, consider the problem of Sender as in (3.10). In the maximizing pair  $(U_S, \check{\Sigma}^{U_S})$ ,  $\check{\Sigma}^{U_S}$  is diagonal, thus, the problem of the expected payoff of the Sender can be written as:

$$\mathbb{E}u_S(a, \theta) = -(V_1 a^2 + V_2 b^2) \tilde{v}_1^{U_S} - (V_2 a^2 + V_1 b^2) \tilde{v}_2^{U_S}$$

where  $a, b$  are the entrances of the rotation matrix  $U_S$  and  $\tilde{\Sigma}(\check{\Sigma}^{U_S}) = \begin{pmatrix} \tilde{v}_1^{U_S} & 0 \\ 0 & \tilde{v}_2^{U_S} \end{pmatrix}$ .

Hence,  $V_1^{U_S} \equiv (V_1 a^2 + V_2 b^2)$  and  $V_2^{U_S} \equiv (V_2 a^2 + V_1 b^2)$  are the coefficients with which dimensions  $1^{U_S}$  and  $2^{U_S}$  enter the decision problem. Thus, as in the case of the reduced problem presented in Section 3, when  $V_1^{U_S} > 0$  the interests on the dimension  $1^{U_S}$  are aligned in the sense that in Sender's ideal scenario the posterior uncertainty of Receiver on this dimension is 0. On the contrary, when  $V_1^{U_S} < 0$  the interests on the dimension  $1^{U_S}$  are misaligned in the sense that ideally Sender prefers Receiver's posterior uncertainty on this dimension to be as high as possible. The same holds for the dimension  $2^{U_S}$ .

### 4.3 Main results

The first theorem addresses the entropy costs: it provides the complete characterization of the optimal solution. The second theorem includes the statement for convex precision-dependent costs. While it does not provide the complete characterization it demonstrates the existence of the region with the diverting attention solution.

In this section I always use  $U$  for the rotation which diagonalizes prior beliefs and  $U_S$  for the rotation of interim (and thus posterior) beliefs - the choice variable of Sender according to the formulation (3.10). Also for any rotation  $U'$  I write  $1^{U'}$  for the dimension 1 in the basis associated with  $U'$  and  $2^{U'}$  for the orthogonal dimension. For any rotation  $U'_S$  I assume the interim beliefs chosen are such that  $\check{v}_1^{U'_S} > \check{v}_2^{U'_S}$ . Otherwise, one can switch the rotation to the orthogonal one. In other words, by choosing  $U_S$  Sender chooses the dimension of maximal uncertainty in interim beliefs.

The following notation is necessary for the results:

$$v^* = \frac{1}{2} \left( v_1 + v_2 - \sqrt{(v_1 - v_2)^2 + 4v_1 v_2 \rho^2} \right). \quad (3.11)$$

Intuitively,  $v^*$  is the uncertainty on the dimension of minimal uncertainty (dimension  $2^U$ ).

**Theorem 1.** *Assume Receiver faces entropy costs of information acquisition. Then Sender always induces diagonal  $\check{\Sigma}$ . Moreover, Sender diverts Receiver's attention away by providing partial information on one of the dimensions of misalignment if:*

- *The conflict of interests is partial with the misalignment on the more uncertain dimension ( $V_1 < 0, V_2 > 0$ ) and*

$$\frac{\lambda}{2} \leq -\frac{-V_1}{-V_1 + V_2} v^* \quad (3.12)$$

- *The interests are fully misaligned ( $V_1 < 0, V_2 < 0$ ) and  $|V_1| > |V_2|$ .*

*If these conditions are not satisfied, Sender fully reveals a dimension of alignment of interests in case of the partial conflict, and provides no information in the case of fully misaligned interests.*

The proof of the result is given in the Appendix. The main part of the proof demonstrates that Sender always prefers to get rid of the correlation between the original dimensions by setting  $\check{\Sigma}$  to be diagonal. After this step the problem becomes identical to the one discussed in Section 3.1.

The diverting attention solution is obtained in the following way: to reveal the dimension of maximal uncertainty  $1^U$  until the uncertainty level  $v^*$ . Since by assumption  $V_1 < 0, V_2 > 0$  and  $v_1 > v_2$ , the dimension of maximal uncertainty is a dimension of misalignment. Hence, the solution is indeed diverting attention according to Definition 1. In contrast when conditions of Theorem 1 are not satisfied Sender chooses an intuitive solution: to fully reveal dimension 2.

The next result is an analog of Theorem 1 for the case of Receiver facing a convex precision-dependent costs of information acquisition.

**Theorem 2.** *Assume Receiver faces convex precision-dependent costs of information acquisition. Then Sender diverts Receiver's attention by providing information on one of the dimensions of misalignment for any partial conflicts of interests when Receiver is sufficiently well informed, that is  $\lambda$  is low enough. Moreover, when the relative conflict of interest on the dimension of misalignment is high and the dimensions are strongly correlated Sender might prefer to reveal no information.*

The formal proof of the result is left for the Appendix.

Note that Theorem 2 uses the generalized notion of alignment of interests. That is, even for non-diverting attention solution the information might be provided on both original dimensions. However, what matters is if the combined information is the one Sender overall wants to reveal or hide (hence, the generalized notion). On top of that there are qualitatively new possible solutions Sender might find optimal compared to the case of entropy costs of information acquisition.

Next Lemma describes in detail the structure of possible equilibrium strategies of Sender.

**Lemma 2.** *There are three strategies for Sender which can occur in equilibrium when Receiver is facing convex costs of information acquisition:*

- (intuitive) To reveal fully one of the dimensions of alignment of interests  $\alpha \cdot \theta$  such that\*:

$$\max_{\alpha} (\alpha^{\perp} \cdot V) \tilde{v}(v_1^{U_S})$$

- (diverting attention) To reveal the dimension of maximal uncertainty until  $v^*$ ;
- To choose  $U_S$  such that  $V_1^{U_S} < 0$ ,  $V_2^{U_S} < 0$  forcing Receiver to learn a dimension of misalignment. It is done by providing partial information either on a dimension of alignment, or on a dimension of misalignment or by providing no information if  $U_S = U$ .

There are several take-away(s) from Lemma 2. First, notice that the third type of solution (not intuitive or diverting attention) includes strategies such that Sender provides information on a dimension of misalignment and Receiver learns herself some other dimension of misalignment. However, since Receiver would learn on her own the dimension of highest uncertainty, these solutions do not satisfy Definition 1.

The other fact to notice is that the third type of the solution includes no information provision. This might be optimal for Sender to reveal no information to Receiver even when interests are partially aligned. This can occur if the conflict of interests on dimension 1 is relatively high ( $|V_1| \gg V_2$ ) and/or correlation between the dimensions is too strong.

An important implication of Theorem 2 is that whenever Receiver is facing low costs of information acquisition (low  $\lambda$ ) the diverting attention solution dominates any other solution.

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\* $\alpha^{\perp}$  stands for a vector orthogonal to  $\alpha$ .

The Figures below illustrate the intuition above. Figure 3.3 illustrates the case in which the conflict of interests on dimension 1 is relatively strong and the dimensions are highly correlated. The horizontal axis represents rotation  $U_S$  given by the co-sinus of the rotation angle ( $b$ ). The vertical axis presents Sender's expected payoff given by (3.10). In all of the figures the black dotted vertical line represents the dimension of maximal uncertainty. That is  $V_1^U < 0$  and  $V_2^U < 0$  in this example. The red solid line gives the threshold such that, for all  $U_S$  with  $b$  higher than this threshold (to the right from the red line),  $V_2^{U_S} > 0$ . The values for the  $U_S$  with  $b$  above the red line threshold are the intuitive solution from Lemma 2, the solutions on the left are of the third type from Lemma 2. The cost function used is:

$$c(1/\sigma^2) = \left(\frac{1}{\sigma^2}\right)^{10},$$

which is indeed convex. It follows then that for high costs of information acquisition for Receiver, Sender chooses the third type of solution from Lemma 2, that is the solution which is neither intuitive nor diverting attention. The intuition is the following: Sender wants to reveal some information since the interests are partially aligned, however, the correlation is too high, thus, revealing the dimension of alignment is too costly. For the intermediate level of costs she chooses to reveal nothing. This is intuitive: on the one hand the dimensions are highly correlated, thus, Sender doesn't want to reveal too much. On the other hand the costs are still high enough to prevent Sender from diverting Receiver's attention. Finally, Figure 3.3c shows that whenever Receiver is sufficiently well informed diverting attention solution generates the highest payoff for Sender.

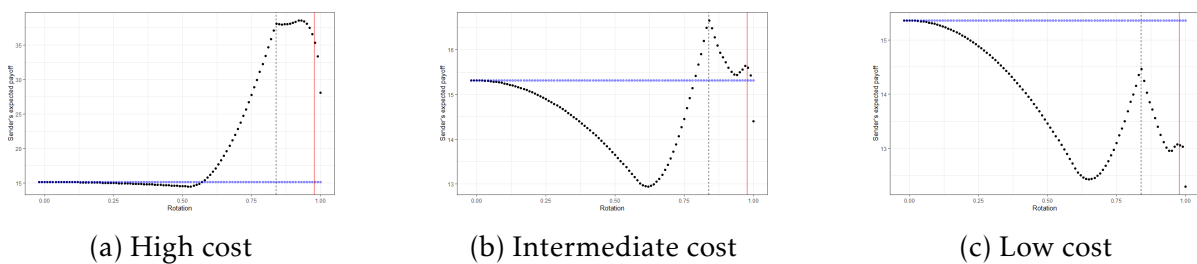


Figure 3.3: Sender's payoff from different candidate solutions for different costs levels.  $|V_1|/V_2 = 20$ ,  $\rho = 0.8$

Figure 3.4 provides the same illustration for the case when the conflict of interests is relatively weak and the dimensions are less correlated with each other. In this case, as discussed above, there are only two possible equilibrium solutions: an intuitive one



when costs of information acquisition are high as illustrated in Figure 3.4a, and a diverting attention one when the costs are low as illustrated in Figure 3.4b.

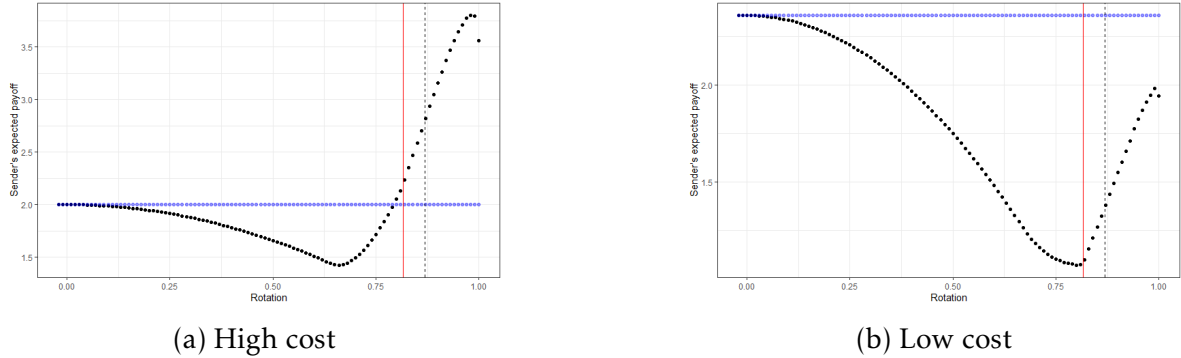


Figure 3.4: Sender's payoff from different candidate solutions for different costs levels.  $|V_1|/V_2 = 2$ ,  $\rho = 0.6$

The discussion of the case with the strong conflict of interests  $|V_1| > V_2$  in Theorem 2 provides the intuition for the case of fully misaligned interests ( $V_1 < 0$ ,  $V_2 < 0$ ). The next result provides the set of possible equilibrium strategies for Sender in this case.

**Corollary 1.** *When the interests are fully misaligned ( $V_1 < 0$ ,  $V_2 < 0$ ) there are 3 types of solution which maximize Sender's payoff depending on the parameters:*

- (no information) To provide no information;
- (diverting attention) To partially reveal the dimension of maximal uncertainty until  $v^*$ ;
- (diverting attention) To partially reveal some dimension of misalignment (different from the one of maximal uncertainty).

*If one of the last 2 solutions is chosen in equilibrium, Sender diverts Receiver's attention.*

This result follows directly from the case of  $|V_1| > V_2$  in the proof of Theorem 2. Note that with fully misaligned interests all dimensions are dimensions of misalignment, thus, any information provision in equilibrium has diverting attention motives as its goal.

The final step in the section is to show that in the case of a partial conflict of interests Receiver might prefer to face higher costs of information acquisition. In this case Sender has more incentives to disclose information in a non-strategic way, that is to choose an intuitive solution.

**Proposition 3.** *Assume Receiver is facing entropy costs of information acquisition. There exists an interval  $(\underline{\lambda}, \bar{\lambda})$  such that for every  $\lambda \in (\underline{\lambda}, \bar{\lambda})$ , Receiver's payoff net of costs is higher at some  $\bar{\lambda} + \delta$  with  $\delta \rightarrow 0$ .*

*Proof.* The result is a direct consequence of Proposition 2. Indeed, note that if the result holds for the comparison of the diverting attention solution and the intuitive solution with  $U_S = I$  then it holds for the optimal intuitive solution (since the costs of information for Receiver increase while the gains remain the same).  $\square$

The similar result holds in the case of convex precision-dependent costs of information acquisition for Receiver.

**Proposition 4.** *Assume a partial conflict of interests with  $V_1 < 0$ ,  $V_2 > 0$  and that Receiver is facing a convex precision-dependent costs of information acquisition. Then for some values of parameters there exists an interval  $(\underline{\lambda}, \bar{\lambda})$  such that for every  $\lambda \in (\underline{\lambda}, \bar{\lambda})$ , Receiver's payoff net of costs is higher at  $\bar{\lambda}$ .*

*Proof.* The result is a consequence of the two following observations.

Firstly, as was already established earlier for sufficiently low costs of information acquisition Sender always chooses a diverting attention solution.

Secondly, in the region around  $\rho = 0$  Receiver is facing a negative value of information for some parameters of the model. Indeed, assume  $\rho = 0$ . Sender chooses the “diverting attention” solution if

$$-V_1 v_2 - V_2 \tilde{v}(v_2) \geq -V_1 \tilde{v}(v_1).$$

Since the LHS is decreasing in  $\lambda$  and the RHS is increasing in  $\lambda$  there is a threshold  $\bar{\lambda}$  such that Sender chooses the diverting attention solution for all  $\lambda < \bar{\lambda}$ . Thus, Receiver is facing a negative value of information if:

$$-v_2 - \tilde{v}(v_2) - \bar{\lambda}c(1/\sigma^2) < -\tilde{v}(v_1) - \bar{\lambda}c(1/\sigma'^2),$$

where  $\sigma^2$  is such that  $\tilde{v}(v_2) = v_2 \sigma^2 / (v_2 + \sigma^2)$  and  $\sigma'^2$  is such that  $\tilde{v}(v_1) = v_1 \sigma'^2 / (v_1 + \sigma'^2)$ . This holds for some costs functions and prior beliefs.

The same logic holds with the increase of the  $|\rho|$ .  $\square$

The last remark of the section deal with the linear precision-dependent costs of information acquisition, that is  $c(1/\sigma^2) = \lambda/\sigma^2$ . Note that this cost function corresponds to the LLR costs introduced in Pomatto, Strack and Tamuz (2018). Facing such costs of information acquisition Receiver behaves in the same way as if facing entropy costs: she

learns the most uncertain dimensions until  $\sqrt{\lambda}$ . Thus, results of Theorem 1 and Proposition 3 apply.

## 5 Applications

In this section I discuss some important though not exhaustive applications of the theoretical results presented above to the real life situations. Section 5.1 studies the choice of optimal consumption bundle by a rationally inattentive consumer adding an information provider to the framework presented in recent study by [Kőszegi and Matějka \(2020\)](#). Section 5.2 discusses the implications of the theoretical results to informational lobbying framework.

### 5.1 Consumer's choice

In this section I apply my results to study the effect of the presence of advertiser or producer's information on consumer choice using the model presented in [Kőszegi and Matějka \(2020\)](#) (I focus on the example of Section II). First, I present the consumer's side model and then add an advertiser.

In the model of [Kőszegi and Matějka \(2020\)](#) a consumer needs to choose the consumption level of 2 goods facing taste shocks. Her utility of consumption of the goods is quadratic in tastes and consumption levels and takes the following form:

$$u_C(a, \theta) = (\bar{\theta} + \theta_1)a_1 + (\bar{\theta} + \theta_2)a_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \gamma a_1 a_2 - (a_1 + a_2)$$

where

- $\bar{\theta} > 1$  is the average taste for 2 goods;
- $\theta_1, \theta_2$  are independent random taste shocks (state of the world) distributed according to  $\mathcal{N}(0, v_{\theta_i})$ ;
- $\gamma \in (-1, 1)$  is a substitutability parameter with the goods being substitutes when  $\gamma > 0$  and complements when  $\gamma < 0$  (neither for  $\gamma = 0$ );
- prices of both goods are normalized to 1.

The consumer can observe one of the tastes  $\theta_1, \theta_2$ , the relative taste  $\theta_- = \theta_1 - \theta_2$  or the total taste  $\theta_+ = \theta_1 + \theta_2$  but not several of these at the same time.

The consumer's problem can then be written in terms of relative and total tastes  $\theta_-$  and  $\theta_+$  and solved for  $a_- = a_1 - a_2$ ,  $a_+ = a_1 + a_2$ . The optimal consumption of the consumer is given by:

$$a_- = \frac{\tilde{\mu}_-}{1 - \gamma} \quad \text{and} \quad a_+ = \frac{2(\bar{\theta} - 1) + \tilde{\mu}_+}{1 + \gamma}$$

where  $\tilde{\mu}_-$  and  $\tilde{\mu}_+$  are posterior expectations of the relative taste  $\theta_-$  and the total taste  $\theta_+$  correspondingly. Then the consumer's expected utility rewrites as follows:

$$\mathbb{E}u_C(a, \theta) = -\frac{1}{1 - \gamma} \tilde{v}_- - \frac{1}{1 + \gamma} \tilde{v}_+$$

where  $\tilde{v}_-$  and  $\tilde{v}_+$  are posterior uncertainties about the relative and the total taste.

In such framework [Kőszegi and Matějka \(2020\)](#) shows that if the tastes are uncorrelated (i.e.  $\Sigma$  diagonal):

- and the goods are substitutes ( $\gamma > 0$ ), consumer observes  $\theta_-$  and, thus  $a_+$  is fixed with  $a_+ = \frac{2(\bar{\theta}-1)}{1+\gamma}$ , however the relative consumption  $a_-$  varies depending on the signal. The phenomena is known as "mental budgeting";
- and the goods are complements ( $\gamma < 0$ ), consumer observes  $\theta_+$  and, thus  $a_-$  is fixed with  $a_- = 0$ , however the total consumption  $a_+$  varies depending on the signal. The phenomena is known as "naive diversification".

The natural second step is to ask if and how the consumer's behavior changes in the presence of an additional information provided by an advertiser or a producer prior to consumer's own information search. One relevant example is a presentation of new products, for instance, in hi-tech industry, where the information provided by the producer comes before consumer has access to any other search.

Prior to solving this question, I modify slightly the example by introducing correlations between the taste shocks (which is a natural assumption for products of the same class, or produced by the same manufacturer). The presence of  $\rho \neq 0$  leads to  $v_- \neq v_+$ , thus, the consumer is unequally uncertain initially about the relative taste and the total taste with  $v_- < v_+$  if  $\rho > 0$  and  $v_- > v_+$  otherwise. On top of that, assume that the consumer faces the entropy costs of information acquisition with scaling parameter  $\lambda$  which goes in line with the main assumption of [Kőszegi and Matějka \(2020\)](#). In this case the consumer chooses to observe the relative taste  $\theta_-$  if:

$$\frac{1}{1 - \gamma} v_- - \frac{1}{1 + \gamma} v_+ > \frac{\lambda}{2} \log \left[ \frac{1 + \gamma v_-}{1 - \gamma v_+} \right]$$

and she learns the dimension until  $\lambda/2$ .

I now add Sender which can be an advertiser or a producer of the goods. Assume that Sender wants to increase the total spendings of the consumer while having them heterogeneous enough across the goods. For instance, a producer of the goods enjoys high total gain but would prefer consumers to buy both rather than to leave one good on the shops shelves. Thus she would prefer to sell an equal amount of both goods. Such payoff of Sender can be written as:

$$u_S(a, \theta) = a_+^2 - a_-^2$$

It translates into the expected payoff:

$$\mathbb{E}u_S(a, \theta) = \frac{1}{(1-\gamma)^2} \tilde{v}_- - \frac{1}{(1+\gamma)^2} \tilde{v}_+$$

Thus, there is a partial conflict of interests between Sender and the consumer. Given that  $\theta_-$  and  $\theta_+$  are uncorrelated by assumption there are 2 possible solutions for Sender:

- To communicate perfectly the total taste for the goods  $\theta_+$  while remaining silent about the relative taste  $\theta_-$ . For instance, in practice this can be done by communicating the properties and the benefits of the consumption of the goods from the category while remaining silent on the relative advantages of the two products.
- To reveal part of the information on the relative taste, to force the consumer to force on the total taste. In practice, in case of the products-substitutes the sender can provide some information on the relative benefits of one product over the other on some features (but not all of them) without focusing on the benefits of the products from this category.

Figures 3.5 and 3.7 present the consumption behavior of the consumer as a function of the substitutability of the goods in the presence of Sender. In both the black dots correspond to the case when Sender is present, blue dots to the benchmark case with no Sender. Finally, red dots show the optimal consumption level if full information would be available. Figure 3.5 shows the consumption patterns of an optimistic consumer compared to the realized state. One can see that the pattern for the relative consumption remains the same compared to the baseline scenario without Sender: she observes no information on the relative taste when the goods are substitutes and receives such information otherwise. However the behavior of the consumer changes with respect to the total consumption when goods are compliments. While for substitutes the pattern

resembles the one of the no Sender benchmark, for the compliments the mental budgeting never occurs: consumer is always observing some information about the total taste due to diverting attention strategy of Sender. Also consumer is making better choices on average in the presence of Sender in terms of total consumption, but worse choices (further from ideal points) in terms of relative consumption. This goes in line with the diverting attention intuition - since Sender is interested in consumer's learning of total taste but wants to hide the relative taste the switch in the behavior occurs towards more attention to total taste.

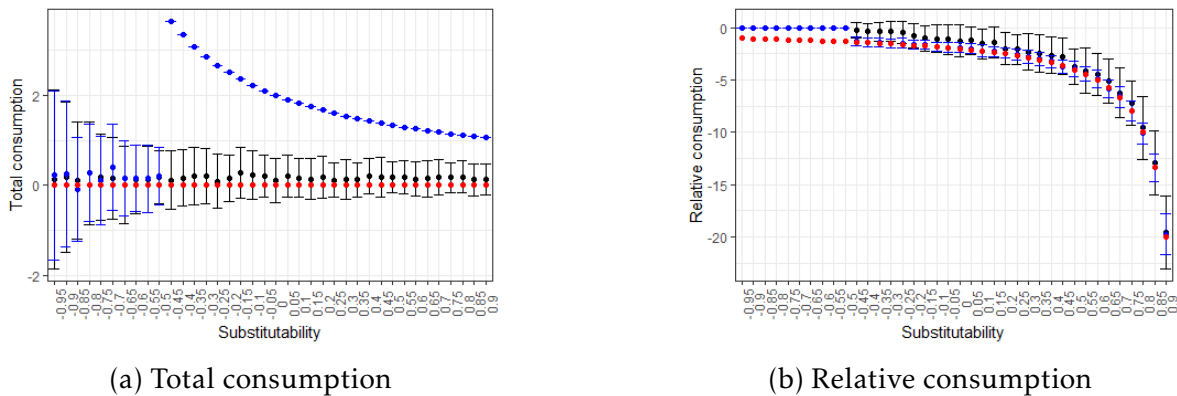


Figure 3.5: Total and relative consumption of the consumer optimistic towards relative taste of good 1 and total taste

Figure 3.6 demonstrates the weights (attention) assigned to  $\theta_-$  by the consumer and Sender. Figure 3.6a shows the benchmark case of no Sender: consumer learns  $\theta_+$  when the goods are complements and  $\theta_-$  when the goods are substitutes (which corresponds to her benchmark consumption behavior). Figure 3.6b shows the attention of the consumer in the presence of Sender. She never chooses to obtain information on  $\theta_-$  and focuses 100% of her attention on  $\theta_+$ . This happens due to Sender's information provision strategy (Figure 3.6c): when the goods are substitutes and the consumer would observe the relative taste on her own, Sender chooses to partially reveal  $\theta_-$  to make the consumer switch her attention to the total taste  $\theta_+$ . Thus Sender chooses to divert the consumer's attention.

The same holds for a pessimistic consumer. This is the case due to the fact that Sender commits to her strategy before learning the state, and thus the uncertainties are the optimization variables for both, Sender and the consumer.

One can see that the presence of Sender, which is natural in the optimal consumption problem, can strongly affect the consumer's behavior and significantly decrease the

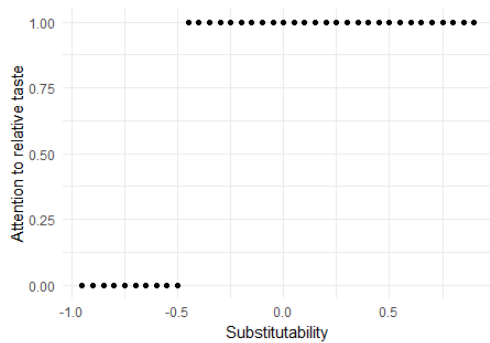
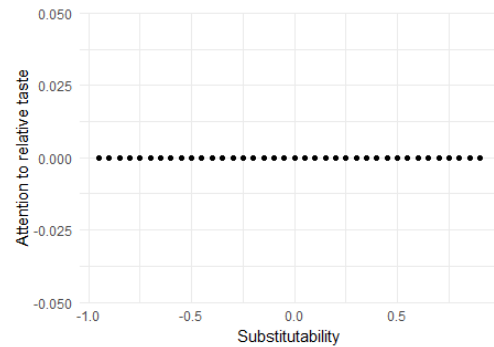
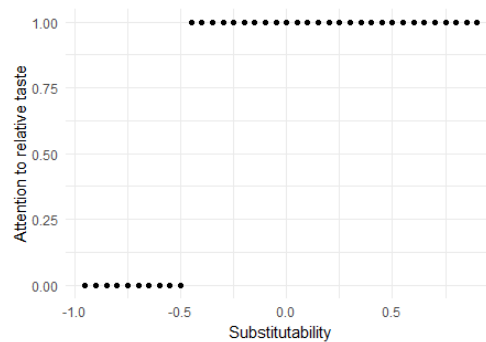
(a) Receiver's attention to  $\theta_-$  in the absence of Sender(b) Receiver's attention to  $\theta_-$  in the presence of Sender(c) Sender's attention to  $\theta_-$ 

Figure 3.6: Total and relative consumption of the consumer pessimistic towards relative taste of good 1 and total taste

presence of mental budgeting. Moreover, Sender's information does not necessarily reveal the information about the goods which is beneficial for Sender: whenever the goods are substitutes she provides information to change the consumer's focus rather than to reveal information in an intuitive way.

Next I illustrate implications for consumer's welfare caused by the presence of Sender in terms of realized payoffs. First I fix some value of  $\theta_+$  and vary the relative taste  $\theta_-$ .

As seen in Figure 3.6 there are 3 qualitatively different cases: when the goods are substitutes ( $\gamma > 0$ ), goods are complements with  $\gamma$  sufficiently close to 0 and complements with  $\gamma \ll 0$ . I compare consumer realized payoffs for these 3 cases which are described in Table 3.1. Intuitive and Diverting attention solutions differ by the cost parameter for consumer ( $\lambda = 3$  versus  $\lambda = 1$ ). The benchmark columns represent the case of no Sender (thus, the initial [Kőszegi and Matějka \(2020\)](#) framework): 'W' corresponds to the weight assigned to the relative taste  $\theta_-$  and 'N' to the amount of noise ( $\sigma_R^2$ ) chosen by the consumer. The persuasion columns represent the framework with Sender. 'W S' stands for

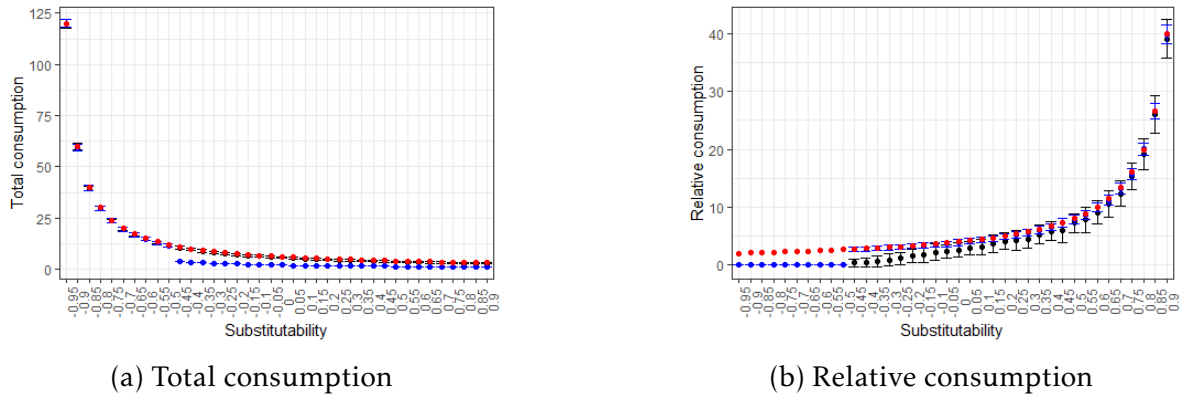


Figure 3.7: Total and relative consumption of the consumer pessimistic towards relative taste of good 1 and total taste

the weight assigned to the relative taste  $\theta_-$  by Sender and 'N S' for the noise  $\sigma_S^2$  chosen by Sender. 'W' and 'N' stand for the similar choice variables for the consumer (chosen upon observing Sender's information). Note that it is assumed directly here that Sender sends a unique signal, the previous discussion in the section shows that it is indeed optimal.

Type of goods	$\gamma$	Type of solution	Benchmark		Persuasion			
			W	N	W S	N S	W	N
Substitutes	0.5	Intuitive	1	0.83	0	0	1	0.83
		Diverting	1	0.26	1	1.14	0	1
Complements	-0.4	Intuitive	1	2.85	0	0	1	2.85
		Diverting	1	0.77	1	55.9	0	0.33
	-0.8	Intuitive	0	0.33	0	0	1	4.08
		Diverting	0	0.1	0	7.41	0	0.1

Table 3.1: Cases studied for fixed  $\theta_+$  with the corresponding attention and noise in the main framework and no Sender benchmark

As was shown before for the case when goods are substitutes in the absence of Sender the consumer observes the relative taste  $\theta_-$ , intuitively the signal is more precise when  $\lambda$  is lower. In the case of high costs for the consumer Sender fully reveals the total taste (intuitive solution), but in the case of low costs she provides just enough information on the relative taste to discourage consumer's further search in this direction.

Figures 3.8-3.10 represent the realized outcomes for Benchmark and Persuasion cases for the parameters described above. In all this illustrations  $\theta_+$  is fixed and  $\theta_-$  varies from -10 to 10 (horizontal axis in the graphs). In each graph black lines represent the distribution (mean+standard deviation) in the Persuasion case and blue lines - in the



Benchmark no Sender case.

Figure 3.8 (first column) illustrates the case when goods are substitutes. In Figures 3.8a and 3.8b consumer has high costs of information acquisition, thus the solution chosen by Sender is intuitive (the top line of Table 3.1). In the case of Figure 3.8a, however, consumer has incorrect prior beliefs about the total taste  $\theta_+$  (that is  $\mu_+ \neq \theta_+$ ), while in the Figure 3.8b her beliefs are correct. With incorrect beliefs she benefits from the presence of Sender. Indeed, Sender reveals fully the total taste, thus, correcting mistake in the priors. In the absence of Sender, however, the consumer never learns the total taste and, thus, always sets the wrong total consumption. On the other hand, when the prior beliefs are correct the consumer is indifferent to the presence of Sender since her total consumption is always correct.

In Figures 3.8c and Figures 3.8d consumer has high costs of information acquisition, thus Sender diverts consumer's attention away from the relative taste  $\theta_-$  (the second line of Table 3.1). Again Figure 3.8c shows the case when prior beliefs about the total taste are incorrect and Figure 3.8d case when the prior beliefs are correct. The consumer is relatively better off in the presence of Sender when beliefs are incorrect. However, in case of correct beliefs, or only a small mistake, consumer is better off without Sender. Indeed, as Table 3.1 shows, in this case consumer learns much less on relative taste with the addition of Sender (the noise in the signal is 1.14 compared to 0.26 in the absence of Sender). If beliefs are correct this difference is not compensated by learning more about the total taste.

Figures 3.9 and 3.10 provide the similar analysis for the case of complements. In Figure 3.9 even though the goods are complements, the consumer still chooses to observe the relative taste in the absence of Sender. Thus, the patterns and interpretations are similar to the case of substitutes in Figure 3.8.

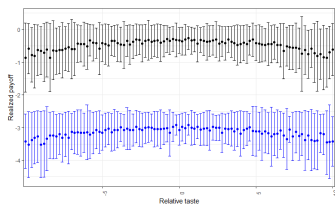
In the case presented in Figure 3.10 the consumer learns herself the total taste, thus, the diverting attention solution as defined by Definition 1 does not exist. In case of intuitive solution Sender reveals the total taste entirely. Hence, the consume benefits from the presence of Sender in case of incorrect prior beliefs (Figure 3.10a). On the other hand, whenever the consumer has too low costs of information acquisition Sender prefers to reveal total taste just enough to keep consumer's focus on the same question. The consumer neither benefits nor loses from the presence of Sender (Figures 3.10c-3.10d).

Figures 3.11 and 3.12 fix the value of relative taste  $\theta_-$  and vary the total taste  $\theta_+$ . Figure 3.11 looks at the case of goods being substitutes and Figure 3.12 - at the case when goods are complements with high  $|\gamma|$ . If the consumer learns the relative taste in

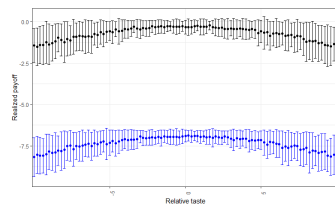
the absence of the Sender (substitutes as in Figure 3.11 of weak complements) consumer benefits a lot from the presence of Sender unless her beliefs on total taste are correct (as was shown in Figures 3.8d and 3.9d).

However, when the goods are strong complements and consumer would learn total taste in the absence of Sender (Figure 3.12) she benefits less from the presence of Sender, especially she is indifferent facing low costs of information acquisition.

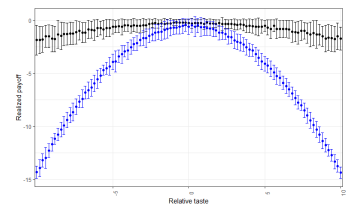
Overall, the consumer benefits from the presence of Sender when her prior beliefs are far from the realized state, but can be worse off if her initial beliefs on one of the dimensions (total taste or relative taste) are confirmed while happen to be wrong on the other. In this case she loses more if she is well-informed in the sense of lower costs of information acquisition. Note, however, that, as theoretical results show, in expectation the consumer is always weakly better off in the presence of Sender, that is in expectation the positive effects outweigh the negative ones.



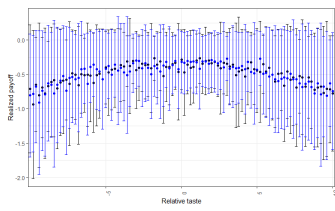
(a) High costs, incorrect prior belief



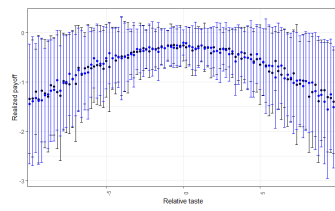
(a) High costs, incorrect prior belief



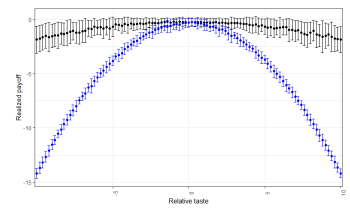
(a) High costs, incorrect prior belief



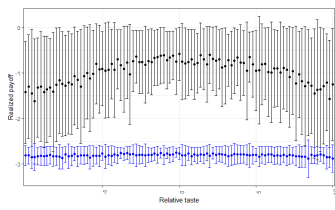
(b) High costs, correct prior belief



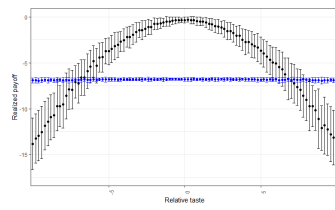
(b) High costs, correct prior belief



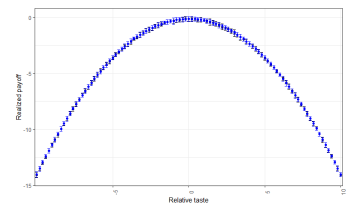
(b) High costs, correct prior belief



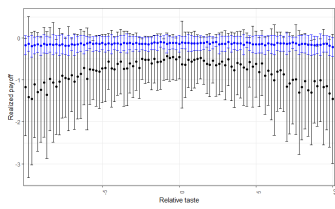
(c) Low costs, incorrect prior belief



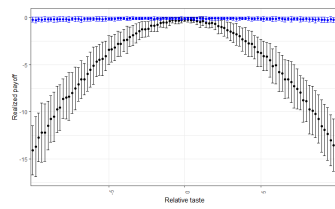
(c) Low costs, incorrect prior belief



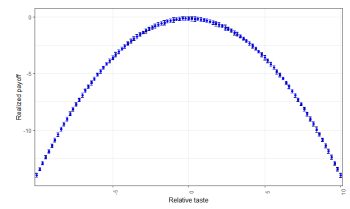
(c) Low costs, incorrect prior belief



(d) Low costs, correct prior belief



(d) Low costs, correct prior belief

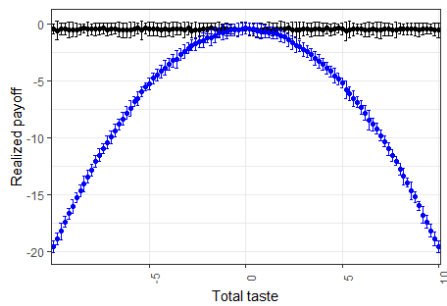


(d) Low costs, correct prior belief

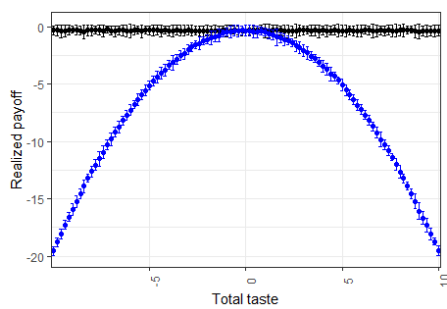
Figure 3.8: Consumer's realized payoffs when goods are substitutes

Figure 3.9: Consumer's realized payoffs when goods are complements, low  $|\gamma|$

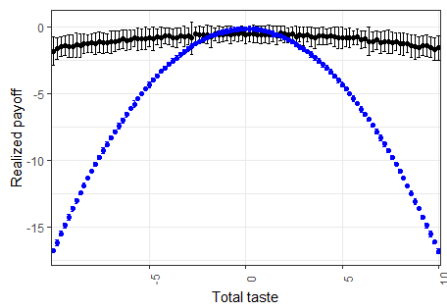
Figure 3.10: Consumer's realized payoffs when goods are complements, high  $|\gamma|$



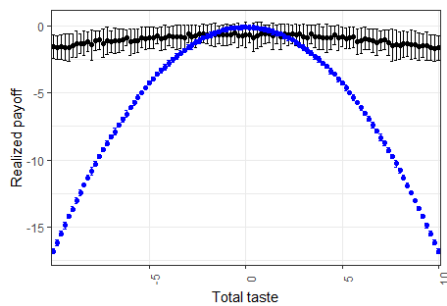
(a) High costs, incorrect prior belief



(b) High costs, correct prior belief

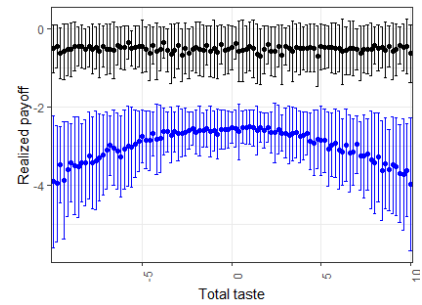


(c) Low costs, incorrect prior belief

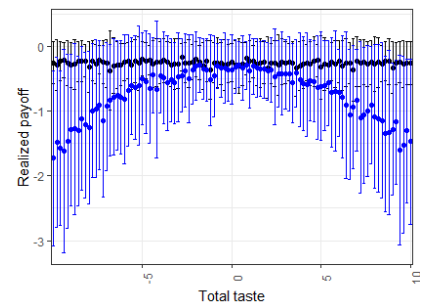


(d) Low costs, correct prior belief

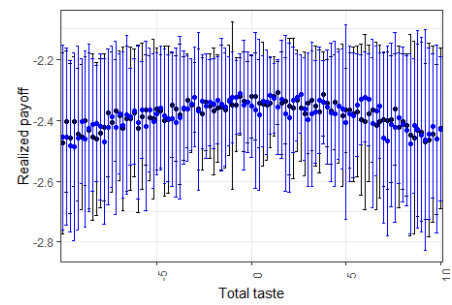
Figure 3.11: Consumer's realized payoffs when goods are substitutes



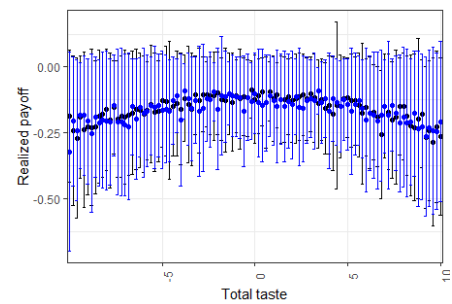
(a) High costs, incorrect prior belief



(b) High costs, correct prior belief



(c) Low costs, incorrect prior belief



(d) Low costs, correct prior belief

Figure 3.12: Consumer's realized payoffs when goods are complements

## 5.2 Implications for lobbying

As the results of the previous sections show in the case a policy-maker needs to decide several issues on some of which her interests are conflicting with the ones of a lobbyist, the latter might decide to provide some information she would not disclose ideally just to divert the policy-maker attention. Note, however, that contrary to the majority of existing literature on informational lobbying the results do not question if the presence of a lobbyist is harmful for the policy-maker. The policy-maker in my settings is always at least weakly better-off in the presence of a lobbyist even in the presence of a conflict of interests.

However, the payoff of the policy-maker depends on her costs of information acquisition in 2 ways:

- Directly, by affecting how much information the policy-maker can acquire;
- Indirectly, by affecting the choice of information of the lobbyist.

While through the first channel the policy-maker always benefits from having lower costs of information acquisition, as theoretical results demonstrate the second effect is not that straightforward.

Policy-maker has several possible solutions to overcome a negative secondary effect. One is to collect all the information she wants prior to the interaction with the lobbyist if it is possible. In this case there is no diverting attention motives for the lobbyist so she either reveals fully the issues where interests are aligned or reveals nothing if there is no such issues. Note that in this case policy-maker needs to either be able to commit to no ex-post information acquisition (by timing of the decision, for example) or to have full access to potential information without interaction with the lobbyist (so lobbyist's information doesn't change the policy-maker's abilities to process/acquire information).

However, it might be that information acquisition is not possible for the policy-maker before the interaction with the lobbyist. In this case she can commit to the direction of her private learning by specifying the type of research she will conduct/information source she will use (for instance, by a contract or a public agenda).

On the contrary, creating precise agenda for the communication with the lobbyist (that is fixing the issues/dimensions on which information will be provided) might not be effective, but kill lobbyist's incentives to provide high quality (precise) information.

## 6 Extensions

### 6.1 Single action

Until now the paper focused on the case when Receiver needs to take multiple decisions. While reasonable for some applications, in others it is more suitable to assume that Receiver needs to take one action based on the information about different dimensions of the state of the world. The examples include, for instance, an optimal investment in a project with multiple (unknown) features, an optimal bonus for an employer, an optimal consumption of a product and so on. To model these problems assume that the payoff of Sender is given in the following way:

$$u_R(a, \theta) = -(a - \gamma_R \cdot \theta)^2 \quad (3.13)$$

with  $\|\gamma_R\| = 1$ . That is Receiver believes that the optimal action is matching the state with the vector of weights  $\gamma$ . Assume that the objective of Sender is similar but she has a different view on the weights which should be assigned to different dimensions of the state of the world:

$$u_S(a, \theta) = -(a - \gamma_S \cdot \theta)^2$$

with  $\|\gamma_S\| = 1$ . In this section I assume that  $\Sigma$  is diagonal, i.e. the dimensions are uncorrelated.

Note that if Receiver has access to any signal which can be a linear combination of the dimensions, she always focuses on observing  $\gamma_R \cdot \theta$ , thus, no diverting of attention is possible. However, this logic does not hold anymore if the available signals are on one of the two dimensions but no mixtures are allowed.

Given the diagonal prior beliefs and the payoff function specified above, the pair of ex-ante expected payoffs can be written as follows:

$$\begin{aligned} \mathbb{E}u_R &= -\gamma_{R_1}^2 \tilde{v}_1 - \gamma_{R_2}^2 \tilde{v}_2 \\ \mathbb{E}u_S &= -(2\gamma_{S_1}\gamma_{R_1} - \gamma_{R_1}^2)\tilde{v}_1 - (2\gamma_{S_2}\gamma_{R_2} - \gamma_{R_2}^2)\tilde{v}_2. \end{aligned} \quad (3.14)$$

I keep the assumption of  $v_1 > v_2$  w.l.o.g. Then the interests of Sender and Receiver are partially aligned with the misalignment on the more uncertain dimension if Sender assigns sufficiently more weight to the dimension 2 in determining the correct action.

**Proposition 5.** *Under partial misalignment of interests and unique action for Receiver, Sender diverts Receiver's attention if:*

- *In the absence of Sender's information Receiver learns dimension 1, if the following condition is satisfied:*

$$\gamma_{R_1}^2 (v_1 - \frac{\lambda}{2}) > \gamma_{R_2}^2 (v_2 - \frac{\lambda}{2}), \quad (3.15)$$

- *diverting attention generates higher payoff for Sender compared to fully revealing dimension of alignment of interests.*

*Sender provides no information if:*

- *(3.15) does not hold, and*
- *no information provision generates higher payoff for Sender compared to fully revealing dimension of alignment of interests.*

*In these cases Receiver faces the negative value of information.*

The formal proof and the similar statement for the convex costs is left for the Appendix.

Thus diverting attention motives are present even in the case when Receiver has only one decision to make if the signal space is restricted to include only non-mixing signals. Similar result holds in the case of fully misaligned interests (presented in the Appendix).

An important case is when Sender just wants to maximize the action of Receiver, while Receiver holds the same preference as in (3.13). This is the case of fully misaligned interests with respect to posterior uncertainties. For instance, if Sender's payoff is given by  $a^2$ , the expected payoff is:

$$\mathbb{E}u_S = \gamma_{R_1}^2 \tilde{v}_1 + \gamma_{R_2}^2 \tilde{v}_2.$$

That is, the incentives of Sender and Receiver are opposing (0-sum game). In this case Sender might prefer to reveal some information if either Receiver learns dimension 1 in the absence of Sender and Receiver assigns sufficiently high relative weight to dimension 1 ( $\gamma_{R_1}^2/\gamma_{R_2}^2$ ), or if Receiver learns dimension 2 in the absence of Sender and Receiver assigns sufficiently high relative weight to dimension 2 ( $\gamma_{R_2}^2/\gamma_{R_1}^2$ ). Thus, information provision is possible even in the case of opposing interests. Moreover, given that the payoffs are as in a 0-sum game, Receiver would prefer no information provision from Sender.

The last example can be a good description of an interaction in a job hiring process, where the candidate provides portfolio first, and then the firm decides which abilities to

test further. In this case, the firm might prefer to announce ex-ante which type of test it wants to provide, to discourage strategic information provision by the candidate.

## 6.2 Budget constraint on Receiver's private information acquisition

So far the paper assumed that Receiver has access to a unique signal while Sender can commit to send any number of linear signals. In this section this assumption is replaced by a budget constraint: Receiver can observe any number of costly linear signals, but the total costs of obtaining this information cannot exceed a certain exogenous threshold. For clarity we assume in this section that dimensions are not correlated, that is  $\rho = 0$ .

Consider entropy costs of information acquisition. The budget constraint takes then the following form: for any collection of signals  $S_R = (S_{R_1}, S_{R_2}, \dots, S_{R_k})$

$$-\sum_{i=1}^k \log \frac{v'_k}{v_k} \leq C \quad (3.16)$$

where  $C$  is some constant and  $v'_k$  and  $v_k$  are the posterior and prior uncertainties for signal  $k$  correspondingly.

Receiver problem then writes:

$$\begin{aligned} \max_{S_R} & -\tilde{v}_1 - \tilde{v}_2 \\ \text{s.t.} & (3.16). \end{aligned}$$

Consider some interim beliefs of Receiver  $(\check{v}_1, \check{v}_2)$  and denote  $i = \arg \max_{i \in \{1,2\}} \check{v}_i$  and by  $j$  the remaining dimension. The optimal strategy for Receiver given interim beliefs  $(\check{v}_1, \check{v}_2)$  is then:

- if  $\log \frac{v_j}{v_i} \leq C$  Receiver chooses posterior beliefs such that  $\tilde{v}_1 = \tilde{v}_2 = v$  with

$$\log \frac{\tilde{v}_1}{\check{v}_1} + \log \frac{\tilde{v}_2}{\check{v}_2} = C; \quad (3.17)$$

- if  $\log \frac{v_j}{v_i} > C$  Receiver observes dimension  $i$  (more uncertain) with

$$\log \frac{\tilde{v}_i}{\check{v}_i} = C$$

setting posterior beliefs to  $(\tilde{v}_i, v_j)$ .



Thus Receiver equalizes the uncertainty on the dimensions if possible (making it as small as costs allow), and reduces the uncertainty on the most uncertain dimension otherwise.

Assume a partial conflict of interests, that is  $V_1 < 0$ ,  $V_2 > 0$ . In this case there are 2 types of solution available for Sender:

- To reveal no information;
- To reveal fully dimension 2 so that Receiver learns dimension 1.

The first solution generates the expected payoff for Sender of  $v(-V_1 - V_2)$  if  $\log(v_2/v_1) \leq C$  with  $v$  given by (3.17) and of  $-V_1 \tilde{v}_1(v_1) - V_2 v_2$  otherwise. The second solution generates the payoff of  $-V_1 \tilde{v}_1(v_1)$ . If  $-V_1(v - \tilde{v}(v_1)) < V_2 v$  Sender prefers to send no information to let Receiver obtain all the information on her own. This result generalizes for any convex precision-dependent cost function.

**Proposition 6.** *If the interests of Sender and Receiver are partially aligned with misalignment on the more uncertain dimension and*

$$-V_1(v - \tilde{v}(v_1)) > V_2 v$$

*with  $v$  given by (3.17), Sender provides no information to Receiver.*

It follows that with a budget-constrained Receiver Sender cannot benefit from diverting attention. Instead, she prefers to provide no information even if the interests are partially aligned. The intuition for this result lies in the fact that Receiver always tries to smooth the uncertainty. Thus she is not facing a trade-off of what to observe, so diverting attention becomes impossible.

## 7 Conclusion

Information provision from an informed to an uninformed party is a part of almost all economic interactions. There are multiple possible reasons to reveal information: improving the quality of the decisions made, reputation concerns, etc. In this paper I uncover a new role for information provision: to divert the attention of the receiving side from unfavorable issues.

For these purposes, I consider a multidimensional Sender-Receiver framework with commitment in which Sender provides information to Receiver, and the latter may extract some additional information afterwards. In such setting information from Sender

has two effects: the standard *persuasion* effect and the effect of *directing the subsequent search* for information.

I show that different reasons for information provision dominate depending on the conflict of interests and Receiver's cost of information acquisition. Whenever interests are partially aligned or fully misaligned Sender might prefer to reveal some information on the dimension where interests are misaligned (stronger misaligned) to divert Receiver's attention away from this dimension and force Receiver to search information on the other dimension. One of the main reasons for such counter intuitive strategy lies in the Receiver's learning dynamics: given any beliefs, it is optimal for her to obtain information on the dimension of maximal uncertainty. Given that, in case the interests are strongly misaligned on the initially more uncertain dimension, Sender prefers to give some information about it so that Receiver would seek information on the other dimension.

Moreover, if Receiver bears costs of information acquisition, in the case described above she faces negative value of information: whenever the costs are sufficiently low Receiver prefers to have the costs increased.

The theoretical results obtained in the paper shed new light on many economic situations. One set of questions to which the results are of a particular interest is of a consumer choice of optimal bundle. While the literature demonstrates how the consumption patterns are affected due to limited attention of consumer in the presence of taste shocks (see [Kőszegi and Matějka \(2020\)](#)), I show the effect which the presence of strategic Sender of information (advertiser or producer) has on these patterns. Among others I show that "mental budgeting" is much less likely to happen if Sender wants to incentivize as high spendings as possible while the "naive diversification" persists.

The framework is natural for the studies of the environments which involve expert advice. In particular, it is applicable to the case of informational lobbying as it often includes multidimensionality and the sequential information acquisition structure (see, for instance, [Cotton and Dellis \(2016\)](#) and [Ellis and Groll \(2020\)](#)) For this case my paper provides a new perspective relative to the literature: while most of the existing papers are concerned with whether informational lobbying is detrimental for the quality of the decision making, I study the optimal lobbyist's and policy-maker's behavior in the settings where the lobbying has on average a positive effect. The applicability of the results is not restricted to this particular environment. One can think of consultants for government bodies, financial advice, hiring processes, etc. This is particularly the case taking into account the extension of the framework which allows Receiver to take a single action based on the combination of the features (dimensions).

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By uncovering new motives for information provision, this paper opens the door to a wide range of follow-up questions. On the technical side, an important issue is determining the conditions under which Receiver is willing to learn a more uncertain dimension which is the building bloc for the optimality of attention diversion for Sender. Another important exercise is to extend the framework beyond the continuous world and normal distributions. From the applied perspective, the use of the framework for consumer choice problem allows to study the simultaneous decision of firms on the pricing and information policy. I leave these questions for future research.

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# 1 Appendix

## 1.1 Proof of Theorem 1

Consider first Receiver who is facing entropy costs of information acquisition. As was shown Sender's problem can be decomposed in finding a rotation matrix  $U_S$  and beliefs  $\check{\Sigma}$  such that  $U_S \check{\Sigma} U_S^T$  is diagonal. For each given rotation  $U_S$  the set of feasible interim beliefs (that is such that  $\Sigma - \check{\Sigma} \geq 0$ ) is given by the following inequality:

$$\check{v}_2^{U_S} \leq \frac{\check{v}_1^{U_S}(v_1 b^2 - 2ab\rho\sqrt{v_1 v_2} + v_2 a^2) - v_1 v_2(1 - \rho^2)}{\check{v}_1^{U_S} - v_1 a^2 - 2ab\rho\sqrt{v_1 v_2} - v_2 b^2} = \phi^{U_S}(\check{v}_1^{U_S}) \quad (18)$$

where  $a, b$  are the components of  $U_S$ :

$$U_S = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

with  $a^2 + b^2 = 1$ . Note, that (18) takes into account the fact that there is no correlation between the dimensions of the basis  $U_S^T U_S$ . Note also that for each rotation the boundary on the RHS of (18) includes the point  $(v_1^{U_S} = v^*, v_2^{U_S} = v^*)$  with  $v^*$  given by (3.11).

With respect to the basis  $U_S^T U_S$  the expected payoff of Sender can be formulated as:

$$\mathbb{E}u_S = \text{const} - \tilde{v}_1^{U_S} (V_1 a^2 + V_2 b^2) - \tilde{v}_2^{U_S} (V_1 b^2 + V_2 a^2) = \text{const} - \tilde{v}_1^{U_S} V_1^{U_S} - \tilde{v}_2^{U_S} V_2^{U_S} \quad (19)$$

with  $V_1^{U_S} \equiv V_1 a^2 + V_2 b^2$ ,  $V_2^{U_S} \equiv V_1 b^2 + V_2 a^2$ . Then there are 4 possible cases.

**Case 1:**  $V_1^{U_S} > 0$ ,  $V_2^{U_S} > 0$ . The solution is then full revelation of  $1^{U_S}$  and  $2^{U_S}$  leading to deterministic 0 payoff.

**Case 2:**  $V_1^{U_S} < 0$ ,  $V_2^{U_S} > 0$ . Thus, Sender wants to induce the highest possible uncertainty in dimension  $1^U$  and the lowest possible in dimension  $2^U$ .

There are 2 types of potential solution (in terms of  $\tilde{v}_1^{U_S}$  and  $\tilde{v}_2^{U_S}$ ) in such case:

- To fully reveal dimension  $2^{U_S}$  without revealing any information on dimension  $1^{U_S}$  so that Receiver learns dimension  $1^{U_S}$  herself. This leads to posterior beliefs  $(\tilde{v}_1^{U_S}, \tilde{v}_2^{U_S}) = (\lambda/2, 0)$  if  $v_1^{U_S} \geq \lambda/2$  and  $(\tilde{v}_1^{U_S}, \tilde{v}_2^{U_S}) = (v_1^{U_S}, 0)$  if  $v_1^{U_S} < \lambda/2$ . Then this type of solution generates the highest payoff for  $U_S = I$  which satisfies the condition of  $V_1^{U_S} < 0$ ,  $V_2^{U_S} > 0$ . Indeed  $V_1^{U_S}$  is minimized at  $U_S = I$  while  $V_2^{U_S}$  is maximized at this point.

- To reveal some information on dimension  $1^{U_S}$  to make it less uncertain and force Receiver to learn dimension  $2^{U_S}$ . Then the posterior beliefs are  $(\tilde{v}_1^{U_S}, \tilde{v}_2^{U_S}) = (v^*, \lambda/2)$ . Thus, vector  $(\tilde{v}_1^{U_S}, \tilde{v}_2^{U_S})$  is independent of  $U_S$ , so the rotation  $U_S$  enters Sender's utility only through the coefficients  $V_1^{U_S}$  and  $V_2^{U_S}$ . Then the maximum is attained for  $U_S = I$ .

**Case 3:**  $V_1^{U_S} > 0, V_2^{U_S} < 0$ . Note that this case is fully symmetric to Case 2 and the same solutions apply with  $U_S$  being 90 degrees rotation matrix which corresponds to the renaming of the axis.

**Case 4:**  $V_1^{U_S} < 0, V_2^{U_S} < 0$ .

There are 2 candidate solutions (in terms of posterior uncertainties):  $(\lambda/2, v_2^{U_S})$  and  $(v_2^{U_S}, \lambda/2)$ . It follows from Cases 2 and 3 that the utility of Sender from these 2 solutions is maximized when  $U_S = I$  which contradicts  $V_1^{U_S} < 0, V_2^{U_S} < 0$ . Thus, there is no solution in this region.

In result, one can conclude that for entropy costs of information acquisition it is always possible for Sender to remove correlation from Receiver's belief, that is,  $\check{\rho} = 0$ .

Then the two pairs of equilibrium beliefs which can be achieved in equilibrium are  $(\tilde{v}_1^{U_S}, \tilde{v}_2^{U_S}) = (\lambda/2, 0)$  and  $(\tilde{v}_1^{U_S}, \tilde{v}_2^{U_S}) = (v^*, \lambda/2)$  if  $\lambda/2 < v^*$  and only one pair  $(\tilde{v}_1^{U_S}, \tilde{v}_2^{U_S}) = (\lambda/2, 0)$  otherwise. Then it is easy to see that Sender prefers the diverting attention solution if condition 3.12 is violated.

## 1.2 Proof of Theorem 2, Lemma 2

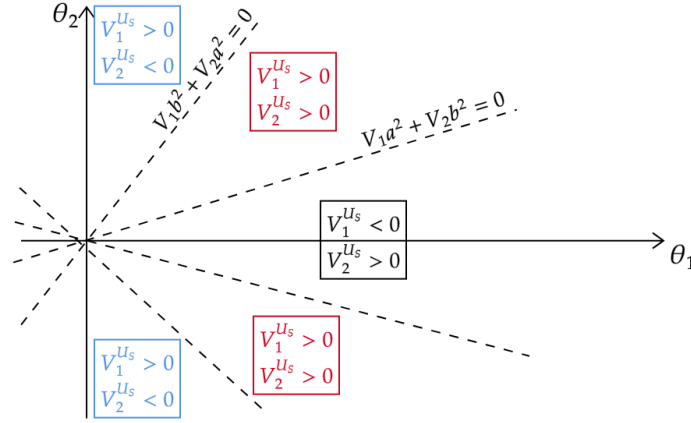
Consider now a convex precision-dependent costs. First of all notice that  $V_1 < 0, V_2 > 0$  and the definition of the rotation such that dimension  $1^{U'}$  is the one with the highest uncertainty,  $V_1^{U'} < 0$ .

Following the same logic as in the proof of Theorem 1 for each possible rotation  $U_S$  chosen by Sender equation (19) holds. There are then 2 different cases:  $|V_1| < V_2$ , that is, relative conflict of interests on dimension 1 is smaller than the agreement on dimension 2; and  $|V_1| > V_2$ , that is, relative conflict of interests on dimension 1 is bigger than the agreement on dimension 2.

**Case 1:**  $|V_1| < V_2$ . That means that for any  $U_S$  either  $V_1^{U_S} < 0$  and  $V_2^{U_S} > 0$ , or  $V_1^{U_S} > 0$  and  $V_2^{U_S} > 0$ , or  $V_1^{U_S} > 0$  and  $V_2^{U_S} < 0$ . Figure 13 illustrates this statement.

First step is to show that in equilibrium  $\text{sgn } b = \text{sgn } \rho$  - that is Sender chooses rotation  $U_S$  in the direction of maximal uncertainty and not away from it. In other words, Sender does not change the sign of the correlation between the dimensions for Receiver. Notice



Figure 13: Coefficient sign as a function of rotation  $U_S$ 

first, that optimal  $U_S$  cannot be such that  $V_1^{U_S} > 0$  and  $V_2^{U_S} > 0$ . Indeed, the maximal payoff Sender can achieve in this case is 0 while strictly positive payoffs are attainable for other choices of  $U_S$ .

Assume now that Sender optimally sets some  $U_S$  such that  $V_1^{U_S} < 0$  and  $V_2^{U_S} > 0$ . In this case Sender chooses one of the 2 solutions (in terms of interim beliefs induced): either  $\check{v}_1^{U_S} = (\phi^{U_S})^{-1}(0)$ ,  $v_2^{U_S} = 0$  or  $v_1^{U_S} = v_2^{U_S} = v^*$ . Note that  $v^*$  does not depend on the choice of the  $U_S$ , thus, the highest payoff for solution the second solution is achieved for  $U_S = I$  (it follows from (19)). Thus, if Sender optimally chooses  $U_S \neq I$  such that  $V_1^{U_S} < 0$  and  $V_2^{U_S} > 0$  she implements the posterior beliefs  $v_1^{U_S} = (\phi^{U_S})^{-1}(0)$ ,  $v_2^{U_S} = 0$ .

Assume now that  $\text{sgn } b \neq \text{sgn } \rho$  for the optimal  $U_S$  and the optimal interim beliefs are of the type  $\check{v}_1^{U_S} = (\phi^{U_S})^{-1}(0)$ ,  $v_2^{U_S} = 0$ . Thus, there exists a symmetric  $U'_S$  with an entry  $b' = -b$  such that  $V_1^{U_S} = V_1^{U'_S}$  and  $V_2^{U_S} = V_2^{U'_S}$ . Then it follows from (18) that  $(\phi^{U_S})^{-1}(0) < (\phi^{U'_S})^{-1}(0)$  leading also to a higher posterior belief  $\tilde{v}((\phi^{U'_S})^{-1}(0)) > \tilde{v}((\phi^{U_S})^{-1}(0))$ . Thus  $U_S$  does not maximize Sender's payoff contradicting the assumption that it is the optimal choice of the rotation.

For the remaining case, assume that Sender optimally sets some  $U_S$  such that  $V_1^{U_S} > 0$  and  $V_2^{U_S} < 0$ . By definition, rotation  $U_S$  is such that dimension  $1^{U_S}$  is the dimension of the higher uncertainty (relative to  $2^{U_S}$ ). Then there are 2 possible solutions for Sender for each fixed  $U_S$ : to induce interim beliefs  $v_1^{U_S} = v_2^{U_S} = v^*$  or  $v_1^{U_S} = 0$ ,  $v_2^{U_S} = v^*$ . Note that both solutions do not depend on the sign of the  $b$ , that is, on the sign of induced correlation. Moreover, both solutions generate the highest payoff whenever  $b^2 = 1$  (that

is  $U_S$  is 90 degree rotation).

Thus, restricting attention to the cases when  $\text{sgn } b = \text{sgn } \rho$  is without loss of generality.

In the next step consider all possible candidate equilibrium solutions for Sender discussed above:

- **Solution 1:**  $U_S$  rotates by 90 degrees setting the interim uncertainties in the new basis to  $(0, v^*)$ . Generated payoff:  $-V_1 \frac{v^* \sigma^2}{v^* + \sigma^2}$ ;
- **Solution 2:**  $U_S$  rotates by 90 degrees setting the interim uncertainties in the new basis to  $(v^*, v^*)$ . Generated payoff:  $-V_2 \frac{v^* \sigma^2}{v^* + \sigma^2} - V_1 v^*$ ;
- **Solution 3:** Setting  $U_S = I$  with the interim uncertainties  $(v^*, v^*)$ . Generated payoff:  $-V_1 v^* - V_2 \frac{v^* \sigma^2}{v^* + \sigma^2}$ ;
- **Solution 4:** Setting  $U_S$  to  $\arg \max_{U'} -V_1^{U'} (\phi^{U'})^{-1}(0) \sigma^2 / ((\phi^{U'})^{-1}(0))$  setting the interim beliefs to  $((\phi^{U'})^{-1}(0), 0)$ .

Note that Solutions 2 and 3 are identical. Also Solution 1 is dominated by Solution 4 for Sender: Putting  $U_S = I$  in Solution 4 generates a higher payoff for Sender than Solution 1. Thus, the optimal solution is given by  $U_S$  such that  $V_1^{U_S} < 0, V_1^{U_S} > 0$ .

The next step is to show that optimal  $U_S$  is strictly in between  $U$  (the dimension of maximal uncertainty) and  $I$ . By contradiction, assume that the optimal rotation is not in this region and  $U_S$  rotates beliefs of Receiver away from  $U = I$ . By previous argument, it is still in the region such that  $V_1^{U_S} < 0, V_2^{U_S} > 0$  (note that it means that  $V_1^U < 0, V_2^U > 0$ ). Consider now a rotation  $U'_S$  which is symmetric to  $U_S$  with respect to  $U$ . Such rotation leads to the same attainable set of pairs of  $(\tilde{v}_1^{U'_S}, \tilde{v}_2^{U'_S})$  as for  $U_S$ . Thus, the optimal interim beliefs given  $U_S$  are the same - that is to induce  $((\phi^{U'_S})^{-1}(0), 0)$ . Note, however, that  $|V_1^{U'_S}| > |V_1^{U_S}|$  leading to a higher payoff for Sender.

Solution 3 is obtained in the following way: to reveal dimension 1<sup>U</sup> until  $v_1^U = v^*$ . Thus, if  $V_1^U < 0$  Sender finds it optimal to reveal the dimension where interests are misaligned according to Definition 2.

For Solution 4, however, Sender fully reveals dimension 2<sup>U<sub>S</sub></sup> with  $U_S = \arg \max_{U'} -V_1^{U'} \tilde{v}((\phi^{U'})^{-1}(0))$ . As was proven before, payoff-maximizing  $U_S$  is such that  $V_1^{U_S} < 0, V_2^{U_S} > 0$ , thus, information is provided on the dimension where interests are aligned.

**Case 2:** Assume now  $|V_1| > V_2$ , that is, the disagreement on the dimension 1 is stronger than the agreement on the dimension 2. That is there are 3 zones for the  $U_S$

(illustrated on Figure 14): such that  $V_1^{U_S} < 0, V_2^{U_S} > 0$ ; such that  $V_1^{U_S} < 0, V_2^{U_S} < 0$  and that  $V_1^{U_S} > 0, V_2^{U_S} < 0$ . Again it is never strictly beneficial for Sender to change the sign of the correlation between the dimensions. For rotations  $U_S$  such that  $V_1^{U_S} V_2^{U_S} < 0$  the same argument as in Case 1 applies. I now show that it also applies for the remaining case of  $V_1^{U_S} < 0, V_2^{U_S} < 0$ .

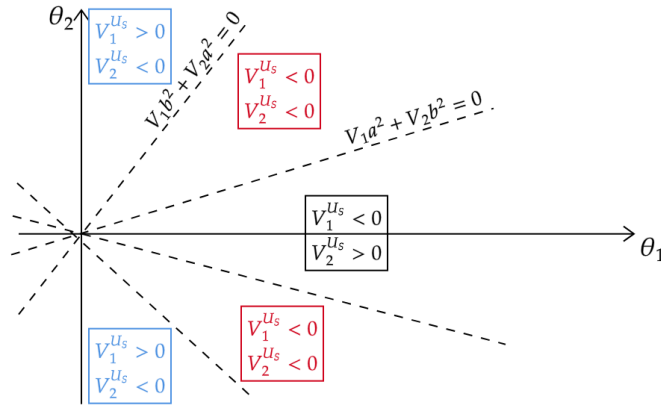


Figure 14: Coefficient sign as a function of rotation  $U_S, |V_1| > V_2$

Consider some  $U_S$  such that  $V_1^{U_S} < 0, V_2^{U_S} < 0$  and  $\text{sgn } b = \text{sgn } \rho$  and another  $U'_S$  which is symmetric to  $U_S$ , that is  $a' = a, b' = -b$ . That leads to  $V_1^{U'_S} = V_1^{U_S}, V_2^{U'_S} = V_2^{U_S}$ . Note that by assumption  $\check{v}_1^{U_S} > \check{v}_2^{U_S}$  for any  $U_S$  (thus, it is also true for  $U'_S$ ). Also note that  $v_1^{U_S} > v_1^{U'_S}$  (and  $v_2^{U_S} < v_2^{U'_S}$ ). Thus for each  $\check{v}_2^{U'_S} = \check{v}_2^{U_S}$  Sender can induce as an interim belief,  $(\phi^{U'_S})^{-1}(\check{v}_2^{U'_S}) \leq (\phi^{U_S})^{-1}(\check{v}_2^{U_S})$ . Together with the convexity of the costs function it means that the set of attainable posterior beliefs at  $U'_S$  is a subset of the corresponding set at  $U_S$ . Thus, Sender cannot obtain strictly higher payoff by setting  $U'_S$  with  $\text{sgn } b' \neq \text{sgn } \rho$  allowing to focus on the solutions in which Sender induces interim correlation of the same sign as in the prior beliefs.

For the same reason as in Case 1, among the  $U_S$  such that  $V_1^{U_S} V_2^{U_S} < 0$  there are only 2 candidate solutions: either Sender diverts attention by revealing dimension 1<sup>U</sup> until  $v^*$ , or intuitively reveals dimension 2.

Hence, the remaining case with a potential solution(s) is to set  $U_S$  such that  $V_1^{U_S} < 0$  and  $V_2^{U_S} < 0$ . Figures 15a-15b show the set of attainable beliefs taking into account the fact that  $\check{v}_1^{U_S} > \check{v}_2^{U_S}$ . Figure 15a presents the case when  $U_S \neq I$ , and Figure 15b - the case when  $U_S = I$ , that is the beliefs are not rotated.

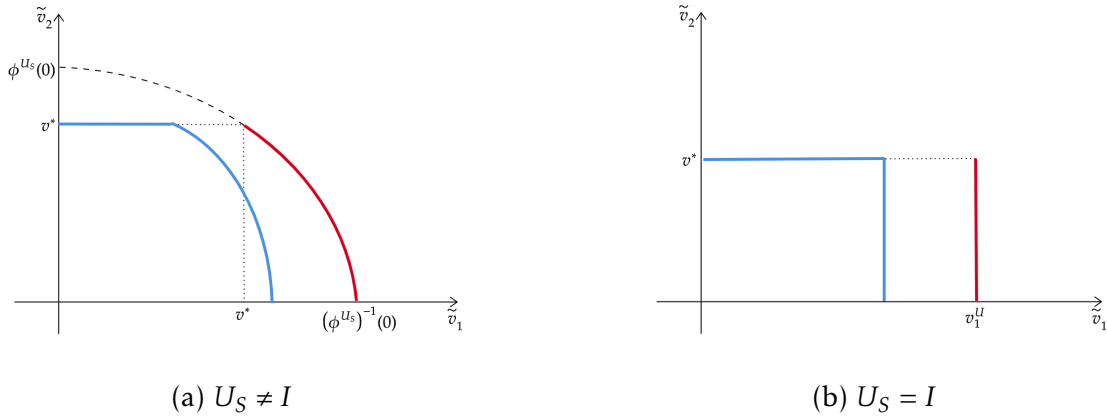


Figure 15: Attainable posteriors for  $U_S$  such that  $V_1^{U_S} < 0, V_2^{U_S} < 0$

Since in this region  $V^{U_S} < 0$  and  $V_2^{U_S} < 0$ , the best pair of posteriors for Sender for each fixed  $U_S$  belongs to the frontier (in blue on Figure 15). Note first that for any  $U_S$ , any solution  $(\tilde{v}_1^{U_S}, \phi(\tilde{v}_1^{U_S}))$  such that  $\tilde{v}(\tilde{v}_1^{U_S}) < v^*$  is dominated by the diverting attention solution. Indeed, Sender's payoff from inducing interim beliefs  $(\tilde{v}_1^{U_S}, \phi(\tilde{v}_1^{U_S}))$  generates the payoff of:

$$\begin{aligned}
 -V_1^{U_S} \tilde{v}(\tilde{v}_1^{U_S}) - V_2^{U_S} \phi(\tilde{v}_1^{U_S}) &< -V_1^{U_S} v^* - V_2^{U_S} v^* = (V_1 - V_1^{U_S} - V_2^{U_S})v^* - V_1 v^* = \\
 &= -V_1 v^* - V_2 v^* < -V_1 v^* - V_2 \tilde{v}(v^*)
 \end{aligned}$$

where the last term is Sender's payoff from the diverting attention solution. The first equality uses the fact that  $V_1^{U_S} + V_2^{U_S} = V_1 + V_2$  for any  $U_S$ .

Moreover, Solution 3 exists only if  $U$  is such that  $V_1^U < 0$  and  $V_2^U < 0$  that is correlation between the dimensions is sufficiently high or  $|V_1| \gg V_2$ . Indeed, consider some  $U_S$  such that  $V_1^{U_S} < 0$  and  $V_2^{U_S} < 0$  while  $V_1^U < 0$  and  $V_2^U > 0$ , and assume that some pair of posteriors  $(\tilde{v}_1^{U_S}, \tilde{v}_2^{U_S})$  maximizes Sender's payoff for given  $U_S$ . Now consider some other  $U'_S$  such that  $V_1^{U'_S} < 0$ ,  $V_2^{U'_S} < 0$  and  $V_1^{U'_S} < V_1^{U_S}$ . Note that the pair of posteriors  $(\tilde{v}_1^{U_S}, \tilde{v}_2^{U_S})$  is attainable under  $U'_S$  since the set of attainable posterior beliefs grows. Indeed,  $(\phi^{U'_S})^{-1}(0) > (\phi^{U_S})^{-1}(0)$ . Note moreover that  $V_1^{U_S} + V_2^{U_S} = V_1^{U'_S} + V_2^{U'_S} = V_1 + V_2$ . Thus,  $V_1^{U_S} - V_1^{U'_S} = V_2^{U'_S} - V_2^{U_S} = \Delta > 0$ . Then Sender's payoff from choosing the posterior

uncertainties  $(\tilde{v}_1^{U_S}, \tilde{v}_2^{U_S})$  under  $U'_S$  is:

$$\begin{aligned} -V_1^{U'_S} \tilde{v}_1^{U'_S} - V_2^{U'_S} \tilde{v}_2^{U'_S} &= -(V_1^{U_S} - \Delta) \tilde{v}_1^{U_S} - (V_2^{U'_S} + \Delta) \tilde{v}_2^{U_S} = \\ &= -V_1^{U_S} \tilde{v}_1^{U_S} - V_2^{U_S} \tilde{v}_2^{U_S} + \Delta(\tilde{v}_1^{U_S} - \tilde{v}_2^{U_S}) \end{aligned} \quad (20)$$

Note that according to the previous argument,  $U_S$  can be optimal only if  $\tilde{v}_1^{U_S} > v^*$ . Thus,  $\tilde{v}_1^{U_S} - \tilde{v}_2^{U_S} > 0$ . Then it follows from (20) that  $U'_S$  generates strictly higher payoff for Sender.

Moreover, the optimal  $U_S$  can only be in between  $U$  and  $I$ . For the  $U_S$  such that  $V_1^{U_S} < 0$  and  $V_2^{U_S} < 0$  and  $V_1^{U_S} > V_1^U$  any solution is dominated by the non revealing one. Indeed, non-revealing solution generates payoff of:

$$-V_1^U \tilde{v}(v_1^U) - V_2^U v^*$$

Now consider any other solution  $U_S$  with  $V_1^{U_S} > V_1^U$  and optimal posteriors  $(v_1^{U_S}, v_2^{U_S})$ . The payoff is:

$$\begin{aligned} -(V_1^U + \Delta) \tilde{v}(v_1^{U_S}) - (V_2^U - \Delta) v_2^{U_S} &\leq -V_1^U \tilde{v}(v_1^{U_S}) - V_2^U v^* + \Delta(v_2^{U_S} - \tilde{v}(v_1^{U_S})) < \\ &= -V_1^U \tilde{v}(v_1^{U_S}) - V_2^U v^* < -V_1^U \tilde{v}(v_1^U) - V_2^U v^* \end{aligned}$$

The second to last inequality comes from the fact that otherwise solution  $U_S$  is dominated by the diverting attention one if  $v_2^{U_S} - \tilde{v}(v_1^{U_S}) > 0$ .

Then there are 3 possible solutions:

- **Solution 1:** Setting  $U_S = I$  with the interim uncertainties  $(v^*, v^*)$ . Generated payoff:  $-V_1 v^* - V_2 \frac{v^* \sigma^2}{v^* + \sigma^2}$ ;
- **Solution 2:** Setting  $U_S$  to  $\arg \max_{U'} -V_1^{U'} (\phi^{U'})^{-1}(0) \sigma^2 / ((\phi^{U'})^{-1}(0))$  setting the interim beliefs to  $((\phi^{U'})^{-1}(0), 0)$  (optimal  $U_S$  such that  $V_1^{U_S} < 0, V_2^{U_S} > 0$ );
- **Solution 3:** Setting  $U_S$  to  $\arg \max_{U', v} -V_1^{U'} (\phi^{U'})^{-1}(v) \sigma^2 / ((\phi^{U'})^{-1}(v) - V_2 v)$  with interim beliefs  $(v, \phi^{U_S})^{-1}(v)$  (optimal  $U_S$  such that  $V_1^{U_S} < 0, V_2^{U_S} < 0$ ).

As was discussed for Case 1, Solution 1 is the diverting attention Solution while Solution 2 is the one in which Sender fully reveals a dimension on which interests are aligned.

Finally, note that if  $\tilde{v}(v_1^U) < v^*$  then for any  $v' < \tilde{v}(v_1^U)$ ,  $\tilde{v}' < v^*$  where  $v_1^U$  is the uncertainty on the dimension of maximal uncertainty. This happens if costs are sufficiently low. In this case Solution 1 dominates any other solution.

Moreover, for sufficiently low of information acquisition there Solution 2 is also dominated by the diverting attention Solution 1. Indeed the payoff from Solution 1 (assuming some optimal  $U'_S$ ) is  $-V_1^{U'_S} \tilde{v}(v_1^{U'_S})$  and it converges to 0 as  $\lambda \rightarrow 0$  (the cost parameter).

The payoff from the diverting attention solution is given by  $-V_1 v^* - V_2 \tilde{v}(v^*)$ , thus, it is bounded away from 0 by  $-V_1 v^*$  which completes the proof.

### 1.3 Proof of Proposition 5

Notice first that since Sender can induce any pair of interim beliefs such that  $\check{v}_1 \leq v_1$  and  $\check{v}_2 < v_2$  she cannot benefit from inducing correlations for Receiver. Thus, it is possible to focus on diagonal interim beliefs.

Condition (3.15) follows directly from the problem (3.14) of Receiver. Note that  $v_2$  such that (3.15) is an equality is an increasing linear function of  $v_1$ .

Then, Figure 16a represents the set of attainable posteriors if condition (3.15) is satisfied and Figure 16b - when it is not satisfied. Blue dotted ligne shows the constraint itself. In Figure 16a  $\bar{v}_1 = (v_2 - \lambda/2) \gamma_{R_2}^2 / \gamma_{R_1}^2 + \lambda/2$ . In Figure 16b  $\bar{v}_2 = (v_1 - \lambda/2) \gamma_{R_1}^2 / \gamma_{R_2}^2 + \lambda/2$

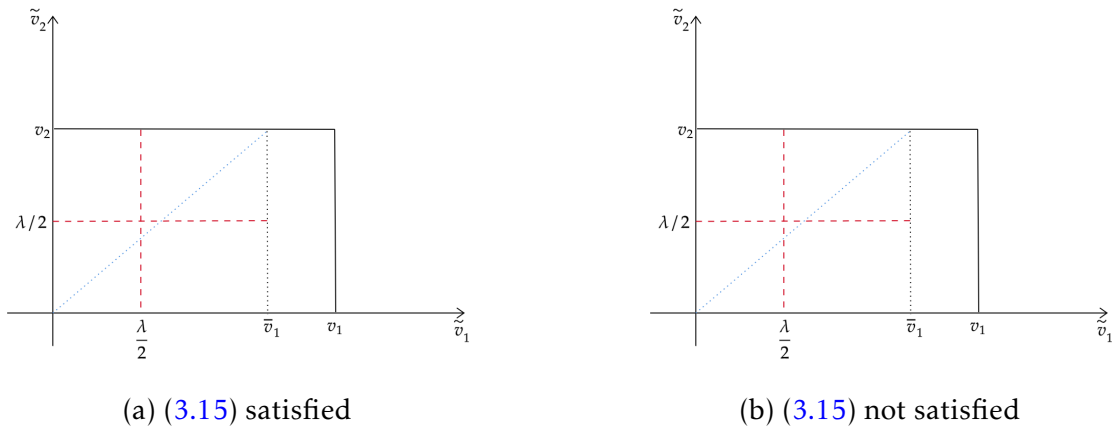


Figure 16: Attainable posterior beliefs, single action for Receiver

The solution then is similar to the one presented in Section 3.2. Note, however, that in the case of condition (3.15) being not satisfied, that is if in the absence of Sender's information Receiver learns dimension 2, there are 2 possible equilibrium solutions: intuitive disclosure of the dimension of alignment and no disclosure at all. That is, no diverting attention solution is present.

For convex precision-dependent costs, condition (3.15) rewrites as:

$$\gamma_{R_1}^2(v_1 - \tilde{v}(v_1)) > \gamma_{R_2}^2(v_2 - \tilde{v}(v_2))$$

Note that due to the convexity of the costs, boundary  $v_2$  is still an increasing function of  $v_1$ . Moreover, it is never optimal for Receiver to learn a dimension fully. Then the same argument holds.

**Titre :** Fourniture é d'informations dans des environnements multidimensionnels

**Mots clés :** theorie de jeux, conceptions d'information, choix social, économie politique

**Résumé :** La qualité des décisions individuelles et collectives dépend de manière significative de la quantité d'informations disponibles, de la capacité à les acquérir et de la connaissance des incitations et des préférences des autres agents. Cette thèse vise à étudier comment l'information est communiquée et apprise dans des environnements stratégiques multidimensionnels, et ce qui peut être fait pour réduire les motivations stratégiques et promouvoir la fourniture d'informations véridiques.

La thèse peut être divisée en deux parties.

La première partie, composée de deux chapitres, tous deux coécrits avec Matías Núñez, traite de l'agrégation de l'information concernant les préférences au sein des comités dans divers contextes. Elle propose différents mécanismes de vote pour les comités, qui possèdent certaines propriétés souhaitables que la plupart des mécanismes actuellement utilisés n'ont pas, et qui sont significativement plus faciles à utiliser pour les membres du comité que certains mécanismes classiques de la littérature.

**Title :** Strategic information provision in multidimensional environments

**Keywords :** game theory, information design, social choice, political economy

**Abstract :** The quality of individual and collective decisions depends significantly on the amount of information available, on the ability to acquire it and on the knowledge of the incentives and preferences of other agents. This thesis aims to study how information is communicated and learned in strategic multidimensional environments, and what can be done to reduce the strategic motives and to promote truthful information provision.

The thesis can be divided in 2 parts. The first part, consisting of 2 chapters, both joint with Matías

Núñez, deals with the aggregation of information regarding preference in committees in varied settings. It suggests different mechanisms for committee voting which have some desirable properties which most of the currently used mechanisms lack, and are significantly easier for the committee members than some classic mechanisms in the literature. The second part of the thesis studies the effects of the multidimensionality of the environments and of an imperfect access to information on strategic communication (Chapter 3).