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Strategies in problem-solving: a model to cluster problem-solving behaviours

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SPÉCIALITÉ : Informatique

Par **Axel PALAUDE**

Stratégies pour la résolution de problèmes : Un modèle de
classification de comportements de résolution

Sous la direction de : **Thierry VIEVILLE**

Soutenance le 6 décembre 2024

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Stratégies pour la résolution de problèmes : Un modèle de classification de comportements de résolution

Résumé :

La régulation de l'apprentissage est l'ensemble des compétences permettant de faciliter l'apprentissage. Cela inclut des compétences motivationnelles, émotionnelles, ainsi que des compétences de métacognition, comme le contrôle des objectifs et des stratégies d'apprentissage. Les situations d'apprentissage sont présentes au quotidien, et peuvent prendre la forme de problèmes ouverts, problèmes dont on ne connaît pas de solution ou bien de méthode pour y parvenir. La résolution de problème ouvert nécessite l'apprentissage des éléments inconnus, ce qui sollicite des compétences de régulation. Comprendre les stratégies employées par des apprenants pour résoudre des problèmes ouverts permet donc de mieux comprendre les processus de régulation et donc le processus d'apprentissage. Les processus de régulation peuvent être influencés et entraînés, et cette compréhension des processus pourrait mener, en plus, à des méthodes pédagogiques toujours plus adaptées.

Cette thèse présente un modèle représentant le processus d'apprentissage d'un apprenant confronté à un problème ouvert, ainsi qu'une méthode permettant, à partir d'observations externes d'actions faites par des apprenants, de grouper des séquences de résolution de problème ouvert en fonction des stratégies employées. Les méthodes d'analyse ont recours à des algorithmes de regroupement utilisant l'alignement temporel comme mesure de la proximité entre des séquences de résolution de problèmes. Cet alignement temporel s'applique sur des données symboliques comparables grâce à une notion de métrique préalablement définie. Dans le cadre de petits jeux de données, une méthode de metaclustering est présentée à des fins de robustesse. Deux études de cas utilisent la méthode définie ici pour montrer les potentiels usages d'une telle méthode mais également en discuter ses limitations. La première, CreaCube, est l'étude d'une tâche créative de résolution de problème, dans laquelle nous essayons de cerner les processus de la créativité. La deuxième, Outer Wilds, essaye d'étendre l'analyse à des problèmes longs.

Mots-clés : Résolution créative de problème, Stratégies, Régulation, Alignement temporel

Strategies in problem-solving : a model to cluster problem-solving behaviours

Abstract:

Learning regulation is the set of skills that facilitate learning. This includes motivational and emotional skills, as well as meta-cognition skills, such as monitoring goals and strategies.

Learning situations are present in every-day life, and mostly take the form of open problems, problems for which there is an unknown solution or method to achieve them. Open problem solving requires learning unknown elements, which requires regulation skills.

Understanding the strategies used by learners to solve open-ended problems allows us to better understand the regulation processes and therefore the learning process. Regulatory processes can be influenced and trained, and this understanding of the processes could lead, to better-suited teaching methods.

This thesis presents a model representing the learning process of a learner confronted with an open problem, as well as a method allowing, based on external observations of actions carried out by learners, to group problem-solving sequences depending on the strategies used.

The analysis methods use clustering algorithms, which using dynamic time warping as a measure of similarity between problem-solving sequences. This dynamic time warping measure is applied to symbolic data, which can be compared through the use of proper metrics. In the context of small datasets, a meta-clustering method is presented for robustness purposes.

Two case studies use the method defined here to show the potential uses of such a method but also to discuss its limitations. The first, CreaCube, is the study of a creative problem-solving task, in which we try to understand the processes of creativity. Second, Outer Wilds tries to extend the analysis to long problems.

Keywords: Creative problem solving, Strategies, Regulation, Dynamic Time Warping

RÉSUMÉ ÉTENDU

CONTEXTE Les compétences transversales (créativité, pensée informatique, résolution de problèmes) sont des enjeux majeurs dans le domaine de l'éducation aujourd'hui. Nous postulons que l'enseignement de ces compétences transversales pourraient bénéficier d'une compréhension toujours plus fines des comportements des apprenants dans les activités sollicitant les compétences transversales. Nous postulons également que, plus généralement, cette compréhension de l'application des compétences transversales peut également bénéficier directement aux apprenants pour les aider à comprendre leurs mécanismes de régulation. Pour cela, nous travaillons dans une action exploratoire tridisciplinaire qui mêle sciences de l'apprentissage, neurosciences cognitives et informatique, pour essayer de modéliser et comprendre des tâches de résolution de problème, qui peuvent également solliciter d'autres compétences transversales.

Le travail présenté ici se concentre donc sur la modélisation de la résolution de problème en général, et à la modélisation des problèmes mal défini en particulier. Les problèmes mal définis correspondent aux problèmes pour lesquels il existe chez le sujet une incertitude concernant la ou les méthodes permettant de résoudre le problème et/ou à quoi ressemble la ou les solutions du problème.

En particulier, nous nous intéressons aux différents comportements que les personnes appliquent pour pouvoir résoudre ces problèmes. D'une manière générale, nous nous attendons à des alternances entre exploration (expérimentation de nouvelles alternatives, génération de nouveaux stimuli) et exploitation (utilisation des connaissances, qu'elles soient sémantiques ou procédurales). Ces comportements sont à mettre en lien avec la double notion de pensée convergente et divergente, qui ont été décrites comme étant au coeur de la cognition créative (Alexandre, 2020a) : la première génère de nouvelles idées, tandis que la seconde sélectionne une des idées comme solution du problème.

REPRÉSENTATION DES PROBLÈMES MAL DÉFINIS Pour représenter ces problèmes mal définis, nous partons d'une définition classique des problèmes bien définis comme présenté par Newell et Simon (1979):

- Un espace de problème, qui contient tous les états possibles d'un problème, et d'un ensemble d'opérateurs employables par le sujet pour passer d'un état à un autre.
- Un problème est constitué d'un ensemble d'états initiaux (le début du problème), d'états finaux (les différentes solutions), et d'un ensemble de contraintes limitant l'emploi des opérateurs.

Nous étendons ce modèle aux problèmes mal définis en considérant les états d'un espace de problème comme le couple contenant à la fois l'état du problème, mais également l'état "mental" du sujet, à savoir l'ensemble de connaissances actuel dont il ou elle dispose sur le problème pour pouvoir le résoudre. Comme il est difficile d'accéder à l'état mental d'un sujet, nous proposons également une représentation d'un problème en cours de résolution par un sujet comme l'espace des états physiques de l'activité sur lequel on ajoute un **brouillard de guerre** : pour chaque état possible de l'espace (connu ou non du sujet), on applique un "score d'incertitude", compris entre 1 (quand l'état n'est pas découvert) et 0 (quand l'état est connu), comme illustré Figure 1. Ce score n'est pas binaire, et peut permettre de prendre en compte à la fois des évolutions dans l'activité qui ne passent pas par des changements physiques: le sujet peut en apprendre plus sur les états possibles sans agir matériellement sur l'activité, diminuant alors l'incertitude, ou bien oublier l'existence de certains états ou les manières de les atteindre, augmentant alors l'incertitude.

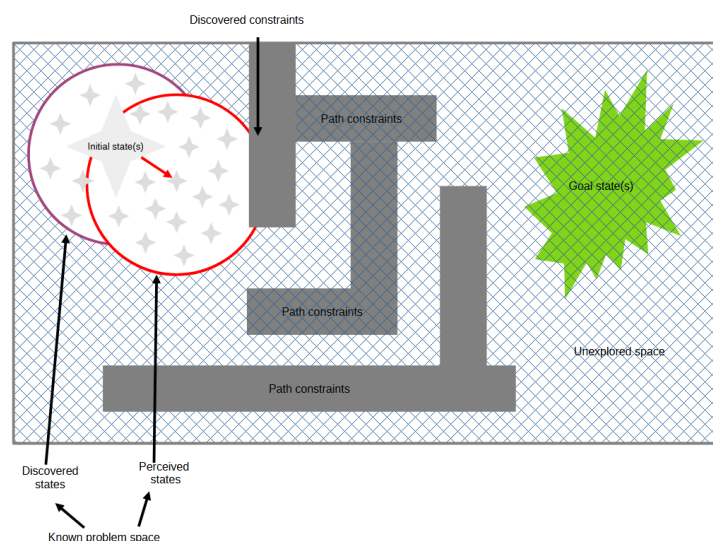


Figure 1: Représentation de l'espace de problème connu et inconnu d'un sujet à un moment de la résolution

MÉTHODE DE CLASSIFICATION DES COMPORTEMENTS DANS DES PROBLÈMES MAL DÉFINIS En se basant sur notre représentation des problèmes ouverts, nous proposons une méthode pour représenter les différentes résolutions de problèmes effectuées par différents sujets et permettre de les classer afin d'essayer d'obtenir de potentiels comportements-types de résolution de problème.

Pour cela, nous nous proposons de représenter la résolution de problème d'un sujet par une séquence d'évènements. Ces évènements sont représentés sous la forme de symboles, ce qui donne lieu à une représentation d'une résolution de problème d'un sujet par une séquence de symboles. Cette séquence de symboles nous permet de retracer l'avancée du sujet dans l'espace de problème défini : chaque sujet est donc associé à une trajectoire dans l'espace de problème. Notre objectif étant de comparer ces trajectoires, nous devons d'abord définir une distance entre les différents symboles et groupes de symboles. Cette distance nous permet d'avoir une mesure de "l'éloignement" des différents ensembles de symboles. Cette distance nous sert de base pour obtenir une mesure de la dissimilarité entre deux trajectoires, c'est-à-dire à quel point deux trajectoires sont éloignées.

A partir de cette mesure, nous sommes alors capables de calculer l'éloignement entre les différentes trajectoires d'un jeu de données de multiples résolutions d'un même problème. Nous pouvons alors appliquer des méthodes de partitionnement comme le **k-medoids clustering** pour pouvoir générer des groupes de trajectoires, sensées représenter différents comportements-types.

Cependant, comme les expériences des sciences de l'apprentissage s'effectuent généralement à notre connaissance sur des corpus réduits, nous appliquons également du **metaclustering**. Au lieu d'utiliser comme mesure pour réaliser les partitionnements la mesure de dissimilarité entre les trajectoires, nous calculons une autre mesure de similarité. Cette autre mesure est calculée en appliquant le **k-medoids clustering** sur toutes les configurations initiales possibles : plus deux trajectoires sont placées souvent dans le même groupe, plus elles seront proches.

Cet algorithme permet d'effacer une partie des biais concernant le choix des conditions initiales pour un corpus réduit, mais ne peut s'appliquer que sur des corpus réduits et sur un nombre limité de groupes différents.

Nous avons alors décidé d'appliquer notre méthode sur deux expériences différentes.

CREACUBE : UNE TÂCHE DE RÉOLUTION CRÉATIVE DE PROBLÈME Romero, David et al. (2019) propose une tâche de résolution de problème appelée CreaCube, centrée autour de la résolution créative de problèmes et la pensée informatique. Dans cette tâche, le sujet se voit présenter deux points de repère et quatre cubes robotiques modulaires qu'il peut assembler, pour construire une structure répondant à la consigne donnée, à savoir "construire un véhicule autonome", comme illustré Figure 2. Ce problème est un problème mal défini, dont on peut représenter l'évolution par un ensemble de symboles nommés **observables**. Par exemple, chaque possibilité d'assemblage des 4 cubes correspond à une observable spécifique. Les observables ne sont pas limités aux informations matérielles, mais elles contiennent également des suppositions sur l'état mental du sujet. En particulier, l'expérience se concentre sur la découverte de ce que Gibson (1979) nomme "affordances", c'est-à-dire la découverte de potentialités d'action, comme la découverte de roues sur un cube qui suggère de placer la face des roues sur le sol.



Figure 2: Etat initial de CreaCube

En découpant les trajectoires à différents moments et en essayant de classifier ces trajectoires, nous essayons de déterminer si certains moments impliquent plus que d'autres des changements dans les trajectoires des sujets. En particulier, nous considérons les affordances comme des points d'inflexion, à partir desquels il y a un changement de comportement du sujet et par extension une modification de la trajectoire. Cela tend à renforcer la découverte d'affordances comme des **insights** (des moments "eurêka" qui modifient la conception que se fait le sujet du problème).

OUTERWILDS : UNE LONGUE TÂCHE DE RÉOLUTION DE PROBLÈME Nous nous proposons d'étudier le jeu vidéo Outer Wilds sous le prisme de la résolution de problème. Dans le jeu, le sujet est mis face à un grand espace de jeu sans objectif bien défini et doit découvrir comment accéder à la fin du jeu en rassemblant des informations d'intrigues différentes, qui peuvent être découvertes et suivies dans n'importe quel ordre et de manière fragmentée. En considérant les découvertes de nouvelles informations (représentées de manière visible dans le jeu) comme les symboles, les sujets suivent donc différentes trajectoires de résolution, tout en se dirigeant finalement vers la même fin.

En découpant les trajectoires à différents moments ou à la découverte de certaines informations-clés, nous remarquons que, contrairement à CreaCube, il semble difficile d'associer certains points de découpe à des points d'inflexion à partir desquels la trajectoire change. Ou plutôt, la majorité des points de découpe permettent, après application de la méthode de regroupement, de classer dans des groupes différents les trajectoires pré- et post-découpe. Cela tend à renforcer une observation qui est que la résolution du jeu tend à se faire sur une compréhension progressive et diffuse du problème, ce qui correspond à de l'**intuition**. A l'inverse de l'insight, il n'y a pas de moment de changement soudain de conceptualisation de problème (par exemple par la découverte d'affordances).

CONCLUSION Bien qu'elle en soit à un stade préliminaire, une telle formalisation des problèmes mal définis ainsi que notre méthode d'analyse pourraient ouvrir la voie à une meilleure compréhension de la complexité des processus de résolution de problèmes :

- En permettant une spécification de problèmes mal définis et des observables d'intérêt à collecter pendant les tâches liées à ces problèmes;
- En proposant une représentation dynamique de la compréhension d'un problème mal défini de la part du sujet;
- En proposant une méthode permettant de réaliser des études préliminaires sur la présence d'évènements semblables à des insights, ou à la présence d'un apprentissage du problème par une intuition diffuse.

Cela pourrait permettre, à terme, à une conception d'activités pédagogiques toujours plus adaptées à l'étude de l'enseignement des compétences transversales, tout en tenant compte des différences individuelles que peuvent avoir les apprenants dans leur apprentissage.

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I am very thankful for the advices I received from my advisors, Thierry Viéville and Frédéric Alexandre, and I want to extend this gratitude to Margarida Romeo who was also of great help during these three years. Their expertise in the numerous topics of this thesis were a never-ending source of inspiration, and the discussions we had were of great interest for my work, and for me personally. In particular, I can't be thankful enough to Thierry, whose support for research but also for my career was truly invaluable. I also want to acknowledge the work of students that I (co-)supervised within school projects : Waris Radji, Corentin Léger, Lucas Bardisbanian, Paul Bernard, Benjamin Hate, Morgane Laval, Lucie Bannay and Baptiste Bernard. Their contributions were inspiring for the development of my work, and helped me develop my work as well as some mentoring skills.

I also want to greatly thank Chloé Mercier for everything. As a PhD student on a very similar topic on the same exploratory action while also having started her PhD one year before me, she was a wonderful and inspiring colleague. Our discussions on our common topic were often insightful, and her advices as a more experienced PhD student were invaluable for PhD-related work, problems and anxieties. I can't thank her enough for her presence.

I also wish to greatly thank Nathan Trouvain for his patience with me and my unstoppable will to create educational games, for the mediation activity we discussed together and for the passionate conversations we had together about pretty much anything.

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I also want to say thank you to Thomas Vitry, whose presence as a temporary room-mate and colleague was of great value. He is a good friend and I can't thank him enough for his help while developing the Outer Wilds experiment. By extension, I want to thank everyone who was involved in this crazy project that is the Outer Wilds experiment.

On a more personal note, I want to thank my family for their support when it was needed. This thesis is also dedicated to Elite, our family dog who passed away at the age of 16 during the writing of this thesis.

I also want to thank all my friends for their presence during these three years. Whether it was online or in real life, our role-playing games were my main way of resting after work, whether it was Wandar, La Vallée or else : Rémi, Théo, Nicolas, Julie, Arthur, Quentin, Priscillia, Youen, Béatrice, Luca, Eléonor, Hugo, Paul, Anthony, Eric, Nathan, Emilien, Antoine...

Finally, I want to express my deepest feelings for my partner Cécile, whose patience, encouragement, understanding and courage helped me go through these three (eight ?) years of my life.

DISCLAIMER : HOW TO READ THIS THESIS ?

As you may have noticed, the format of this thesis is unusual. The font and spacing are larger. Therefore this thesis may look bigger than others. Don't be fooled by the format!

Why are we using it, then ?

I always have been interested in accessibility of science. This accessibility can come in multiple forms : popularization events, scientific outreach, etc.

For this thesis in particular, I wanted to create something that was accessible to a large audience. Obviously, it is not for everyone, as it is the result of a 3-years work on the topic.

One way of making this thesis accessible is the readability. Reading scientific articles is often difficult, with specific vocabulary and complicated sentences. I decided to write this thesis in respect to the recommendations of the **British dyslexia association**¹. The recommendations include:

- A font size of 12-14 points, larger than usual;
- Larger inter-letter and character spacing;
- The use of bold for emphasis instead of italics;
- Left align text without justification and no multiple columns;
- Avoidance of green, red and pink for references (for color blind people).

A difficult recommendation to follow is about the writing style. They encourage to be concise and avoid long and dense paragraphs. This part is difficult to follow in scientific papers with a lot of in-sentences citations with specific terminology.

To face this problem, I decided to create a boxed-header in each section. This boxed-header will sum up the content of the section, with the specific writing style recommended by the British dyslexia association:

- Avoidance of long paragraphs;
- Concision;

¹See **here** for the complete recommendation guide.

DISCLAIMER : HOW TO READ THIS THESIS ?

- Clear language using every day words;
- Bullet points for clarity;
- No double negatives;
- No abbreviations.

The content of each boxed-header will be less precise than the content of the section, but I hope they will be enough to follow the process behind my work. If you are used to the literature, these boxed-headers may not appear very precise.

In the end, I decided to choose Helvet as a font. It is a compromise between readability and style : Readability because sans serif fonts are more readable; Style because fonts designed for dyslexia are often unpleasant for non-dyslexic people, like comic sans and open dyslexic.

Thank you Quentin for your help regarding this choice.

HOW SHOULD YOU READ THIS THESIS ? If you are reading this thesis you are obviously free to read it the way you want, but here are some suggestions depending on your profile :

- **You are used to scientific papers** I would suggest skipping the boxed-headers if you're looking for a complete reading. Boxed-headers can act as little incomplete summaries of sections to help you find the notions you are looking for while browsing the thesis. If you are dyslexic and used to scientific papers, I hope the format suits you.
- **You are not used to scientific papers** Maybe you are here out of curiosity, or maybe you want to understand what are the results of the experiment you took part in. I suggest you read mostly boxed-headers, and deep dive into a section if you are curious to get details about a specific one.

INTRODUCTION

Education, teaching and learning are three different things that overlap with each other.

Teaching is the process of transmitting skills or knowledge.

Learning is the process of acquiring new skills, knowledge or understanding.

Education is the transmission of knowledge, skills and norms. Education can be formal or informal. Formal education is greatly related to teaching, structured in institutional frameworks like schools. Informal education refers to what is acquired outside of these frameworks, for example during daily-life experience in a family.

Here, we are interested in learning. We want to know how learning works, because we think that if people know how their learning process works, they will be able to learn more efficiently.

This process of organizing our way of learning is called *regulation*.

Education is the transmission of knowledge, skills, and character traits and comes in many forms. Formal education happens in a complex institutional framework, like public schools. Non-formal education is also structured but takes place outside the formal schooling system, while informal education is unstructured learning through daily experiences. Formal and non-formal education are divided into levels that include early childhood education, primary education, secondary education, and tertiary education. Other classifications focus on the teaching method, like teacher-centered and student-centered education, and on the subject, like science education, language education, and physical education. The term "education" can also refer to the mental states and qualities of educated people and the academic field studying educational phenomena.

Teaching is the transmission of knowledge, know-how or interpersonal skills by a teacher to one (or more) learner. It is performed in educational settings, as part of education.

Learning is the process of acquiring new understanding, knowledge, behaviours, skills, values, attitudes, and preferences. Humans, animals, and some machines have the ability to learn; there is also evidence for some kind of learning in certain plants. Some learning is immediate, induced by a single event (e.g. being burned by a hot stove), but much skill and knowledge accumulate from repeated experiences. The changes induced by learning often last a lifetime, and it is hard to distinguish

learned material that seems to be "lost" from that which cannot be retrieved.

The learning process is a never-ending one. It occurs at any stage of life, and it is not limited to learning through formal education. In particular, we postulate that learning can occur at any moment in everyday life. Each problem that a person needs to solve is an occasion to learn. Problems can be faced because of necessity (e.g. because a person needs to eat, they need to learn how to gather food, or cook) or be self-imposed (e.g. because a person wants to know more about a specific topic), with different degrees in the middle of both ends. The learning process is also mainly individual. Even if learning occurs often in situations in which other people are involved, we see it as a self-improvement process.

However, we also need to take into account that, even if the learning process is mainly individual, the means to learning are not evenly distributed. For instance, formal education in schools is not exactly the same for everyone. Formal education systems change over time, are not identical from culture to culture, country to country, or even from school to school in one country. As such, other processes are at work in the educational process of one's learning.

For instance, the process of social reproduction describes the reproduction of social structures and systems, based on preconditions in demographics, education and inheritance of material property or legal titles, leading to the maintenance and continuation of existing social relations. It is a variation of Karl Marx's economic reproduction, and has been studied, e.g., by Pierre Bourdieu and Jean-Claude Passeron. Such preconditions consist of **capitals** that are transmitted from one generation to another. Bourdieu distinguishes four types of capital :

- **Economic**: The income and wealth of a person
- **Cultural**: Outlooks, beliefs, knowledge and skills passed between generations
- **Human**: Education and job training a person receives
- **Social**: The social network to which one belongs

Those capitals influence each other, e.g. wealthy people are more likely to offer better education opportunities (with personal teachers for instance). Included in these capitals, we can find knowledge, skills and

education in cultural and human capitals. As such, a part of our learning opportunities are inherited.

Our main goal in this thesis is to foster learning. By focusing on the individual level, we are missing central points for the individual's development. However, we also think that understanding learning at the individual level, in addition to a better understanding of ourselves as humans, is a good opportunity to fight some of the inequalities implied by social positions. We think that a good understanding of our learning process naturally leads to a better **regulation** of it, and thus leads to a more efficient learning process. Regulation processes are detailed in this thesis, but consists partly of metacognitive processes (organisation of learning, monitoring the advancement of the learning process) for which their understanding leads naturally to a better use of them, e.g. knowing how our brain learns new information can help us organize our learning in a way that favours such information learning.

The experiments done in this thesis are playful activities. Play has been approached by several theorists as a form of learning. Children experiment with the world, learn the rules, and learn to interact through play. Lev Vygotsky agrees that play is pivotal for children's development, since they make meaning of their environment through playing educational games. There, our activities are also a way to illustrate what informal learning is, as learning occurs in playful situations that are apart from formal learning in, e.g., schools.

In this thesis, we will start by defining what regulation of learning is. We will consider the individual aspects and put them in relation to collaborative aspects of regulation for learning. We will finally introduce what are problem-solving activities for us and for learning.

With these concepts in mind, as our goal is to understand learning, we will try to understand how do we learn in problem-solving activities. We will then present a model of problem-solving representation, as well as a way to represent a subject's actions as a trajectory in the defined model. This will allow us to create a measure of similarity between different subjects' trajectories, that we will use to create multiple classifications of subjects' trajectories. Our main assumption is that these classifications may correspond to different "learning strategies" that subjects adopt, thus leaning us to a better understanding of problem-solving strategies implemented by learners.

INTRODUCTION

Finally, we will put this model to the test with two different experiments. The first one, Creacube, has been extensively studied by, e.g., Romero et al. (2021), and we will try to implement the model based on previous observations on the activity. We can also use the model as a way to make preliminary observations about what can be inferred about learning strategies from activities. The second experiment, Outer Wilds, is an illustration of this.

Thanks to these experiments, we will discuss the pros and cons of the model we developed as a preliminary model that can be applied to understand problem-solving activities.

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1 REGULATION OF LEARNING

1.1 REGULATION

1.1.1 SELF-REGULATION

Learning is the acquisition of new knowledge, skills and know-hows. Regulation is the process of facilitating learning. It allows the learner to have the skills and confidence to take control of their own learning. This process is mainly individual, and the personal regulation process is called **self-regulated learning (SRL)**. Self-regulated learning refers to metacognitive skills that allow the learner to reach the learning goal. Successful self-regulated learning consists of the piloting of :

- cognition, to set goals and plans relative to the learning situation.
- behaviour, to monitor and control behaviour in line with goals.
- emotion, to associate the learning process to positive emotions.
- motivation, to stay motivated to achieve the learning process.

B. Zimmerman (2008) identifies three phases of SRL models :

- a pre-task preparation phase : The learner analyses the task, plans et sets goals.
- an execution phase : The learner performs the action and monitor their progress and performance.
- a post-task reflection phase : The learner evaluates their performance and adapts accordingly.

Self-regulated learning processes are skills that can be trained and improved. Our work focuses on self-regulated learning skills applied to problem-solving tasks. In particular, I am interested in cognition and behaviour:

What kind of strategy a learner can adopt when facing a problem, and what do these strategies imply for learning ?

Answering this question will help us better understand learners.

Learning often occurs in social situations : we rarely learn alone. The environment plays an important role in learning, in particular when learning situations are built upon group situations such as the classroom. However, we can't ignore the personal skills of individuals that facilitate the learning process. Regulation of learning "refers to the ways that students take strategic control of their behaviour, cognition, motivation and emotions toward achieving an optimal learning objective or completing a learning task" (S. Zhang et al., 2021). Regulation occurs at different levels (individual, group) and thus can be represented as a multi-layered process, in which each layer influences one another.

At the most restrained level, **self-regulated learning (SRL)** consists of the mastering of a student's learning processes (B. J. Zimmerman, 2015). During SRL, students pilot their behaviour, cognition, emotions and motivation to achieve a task or a learning objective (Hadwin et al., 2011). It is an adaptive process rooted in the recognition of successes and failures (Sanchez, 2017). It generally implies goal-driven, achievement and optimisation-focused behaviours (Piotrkowicz et al., 2021), that is the narrowing of the gaps between current condition and stated goals by elaborating and adjusting strategies (Sun et al., 2022). Puustinen and Pulkkinen (2001) conducted a review on SRL models and concluded that three phases are identifiable on said models : a preparatory phase which includes task analysis, planning, and goal setting; a performance phase in which the actual task is done with progress and performance monitoring; and an appraisal phase, in which the student reflects and adapts for future performances, for instance to consolidate learning gains (Schraw et al., 2006). For instance, B. Zimmerman (2008) suggests three phases during SRL: the pre-task preparation (including orientation and planning), the execution phase (monitoring), and the post-task reflection phase (evaluation).

Multiple other models of SRL exist concomitantly to this general decomposition, as discussed in e.g. (Panadero, 2017). For instance, Boekaerts (2011) proposed a Dual processing model in which strategies either protect the learner's ego (triggering negative cognitions) or reinforce the interest (triggering positive cognitions) depending on the learner's goals and needs. Hadwin et al. (2011) decompose SRL in four phases : task definition, goal setting and planning, enacting study tactics and strategies, and metacognitively adapting studying. Pintrich (2000)'s proposes

a similar four-phase decomposition : (1) Forethought, planning and activation; (2) Monitoring; (3) Control; and (4) Reaction and reflection. Additionally, the Metacognitive and affective mode of Self-Regulated Learning model (MASRL) of Efklides (2011) separates the macrolevel of the person (and general skills of SRL associated to it), and a microlevel (with task-specific interactions between the learner and the subject). Finally, another relevant model is the socially-shared regulation of learning (SSRL), as developed by e.g. Hadwin et al. (2011). This model shows that despite the advantages of collaboration and computer-supported collaboration for learning (Dillenbourg and Schneider, 1995b), collaboration poses cognitive, motivational, social, and environmental challenges (Järvelä et al., 2013; Koivuniemi et al., 2017), and necessitates another level of regulation. The SSRL model (Hadwin et al., 2011) proposed the existence of three modes of regulation in collaborative settings: self-regulation (SRL), co-regulation (CoRL), and shared regulation (SSRL). In this model, SRL refers to the individual learner's regulatory actions (cognitive, metacognitive, motivational, emotional, and behavioral) that involve adapting to the interaction with the other group members. We will develop the other levels in section 1.1.2.

As the piloting of behaviour, motivation and cognition are crucial for the execution of the SRL phases, emotional piloting is also playing a role in SRL as higher SRL is related to more positive emotions (Lajoie, 2021). Boekaerts (2011), for instance, takes emotions into account to explain behaviours like ego-preservation (protection of self-esteem).

High levels of SRL also allow a better use of learning tools. For instance, in suitable conditions, digital environments are known to improve the performance of knowledge workers (Järvelä et al., 2007) by giving access to a lot of self-regulation tools (for planning, monitoring, etc.), but may also be less-suited to people with low levels of SRL. MOOCs (Massive Open Online Course) and other platforms of online learning require good self-regulation skills of individuals to be efficient (Fan et al., 2021).

Understanding self-regulation processes is then a way, on the one hand, to create more accessible tools for learning (i.e. that do not require high levels of SRL) and, on the other hand, to be able to develop methods to help people work on their SRL skills. This goal requires to consider other aspects of regulation. Indeed, as previously said, learning is a process

that is rarely achieved alone, and we need to consider effects of collaboration on learning. We will discuss these effects in section 1.1.2.

We want to focus on a specific part of self-regulated learning skills, namely SRL skills related to problem-solving tasks. We are convinced that those skills are solicited often, as problems appear frequently in everyday life, especially ill-defined problems (see section 1.3.1). Thus, studying them will allow us to reach a better understanding of our everyday-life learning processes. We think that a person conscious of their learning processes will be able to improve their SRL skills.

In this thesis, our goal is to reach a better understanding of problem-solving strategies done by human learners by the behavioural analysis of problem-solving activities.

1.1.2 COLLABORATIVE LEARNING

Different levels of regulation exist. Self-regulation is personal, but regulation exists also at the interpersonal level and at the group level.

- Regulation through interpersonal interactions is called **co-regulation**. In co-regulation, people engage in each other's metacognitive processes. It contributes to deepen the learning process by sharing. People can elaborate on others' contributions. They can explain, question and provide feedback to each other. They can also share regulating processes like task planning and execution.
- Regulation within a group is called **socially-shared regulation**. The group regulates everyone by fixing goals, conducts, etc. It allows members to share the cognitive load of the task. Members can share knowledge and understanding. They can also help to maintain a positive socioemotional atmosphere. Socially-shared regulation becomes more difficult as the size of the group increases. For instance, the bigger a classroom, the more difficult the socially-shared regulation can maintain the motivation of its members.

These three levels are interdependent (S. Zhang et al., 2021). For instance, a better engagement in the social space implies better individual achievement. My focus on self-regulated learning implies that we need to concentrate on individual activities.

Collaboration in the classroom is widely used by teachers as a means to enhance learning (Freinet, 1993) as well as a means for inclusion (Mulholland and O'Connor, 2016). More than just taking into account the group as a constraint (given that there are more students than teachers), the presence of a group and the creation of group activities can lead to collaborative learning.

COLLABORATIVE LEARNING Collaborative learning can be defined as "any instructional method in which students work together in small groups toward a common goal" (Prince, 2004). We can view it as the set of all group-based methods for learning. For Dillenbourg and Fischer (2010), collaborative learning is "the co-construction of shared understanding". The members of the group each make the individual effort to try to understand what the others mean, and this effort leads to cognitive activities

such as dialogue which in turn allow each individual to make cognitive changes in relation to their own understanding. These efforts provide access to shared understanding. As such, student interactions can increase group performance and individual learning outcomes (Diziol et al., 2010), and at the engagement level, group work "facilitates learning through social interactions and increases the students personal engagement in the learning process" (Marfisi-Schottman and George, 2014). For the case of students, reciprocal peer tutoring increases learning in classroom settings, because students are more inclined to reflect, elaborate and feel more accountable for their knowledge (Roscoe and Chi, 2007). For instance, in the field of programming education, as well as educational robotics, Zhong et al. (2017) showed that pair learning improves students' confidence, productivity, achievement and the quality of cooperation. As such, collaborative learning has multiple pedagogical benefits (Law et al., 2021). It helps to foster deep learning and build knowledge. It is considered as one of the 21st-century skills, and helps to develop other 21st-century skills such as creativity, critical thinking or problem-solving. In particular, for problem-solving, collaborative problem-solving refers to problem solving activities that involve interaction between individuals in a group. In educational settings, collaborative problem-solving is "a process in which two or more collaborative parties interact with each other to share and negotiate ideas and a priori experiences, jointly regulate and coordinate behaviours and learning activities, and apply social strategies to sustain the interpersonal exchanges to solve a shared problem" . Thus, there are a variety of approaches to learning, as described by Stahl (2005). They vary from individually-centered in which individual learning is assisted by collaboration, like the zone of proximal development (Vygotsky and Cole, 1978), to socially-centered in which all learning is social or collaborative as it resides in culture and history, like distributed cognition (J. Zhang and Norman, 1994). Grounded in the theoretical contributions of these theories, socially-shared regulation tries to model the regulation of learning by a group for collaborative learning.

As groups require the investment of individuals, different levels of regulation must be considered, from the self-regulation of an individual (SRL) to the regulation of an entire group. In addition to self-regulation, regulation processes that consider collaborative processes exist. S. Zhang et al. (2021) describes two different supplementary levels for collaborative

learning, as they unfold different collaborative processes : co-regulation and socially-shared regulation.

CO-REGULATION Co-regulation learning is a transitional process through interpersonal interactions (S. Zhang et al., 2021). Students influence one another's regulatory process. It implies to engage in two dialogical spaces : cognitive and relational (Saqr and López-Pernas, 2021). Cognitive space encompasses both cognitive and metacognitive activities like discussing concepts, while relational space encompasses expressions of ideas, negotiations, communications... Both spaces play a role in the achievement of a shared understanding about concepts used in the cognitive space. Interactions allow deep-level metacognitive activities (depth) and the building and elaboration on other members' metacognitive contributions (sharedness) while discussing, explaining, questioning each other (Piotrkowicz et al., 2021). Co-regulation effects are present at the participant's contributions (how a student responds, connects and engages with other's contribution) and at the level of reasoning and argumentation strategies (Saqr and López-Pernas, 2021).

SOCIALLY-SHARED REGULATION Socially-shared regulation, that we will also call group regulation, consists of the regulation of everyone by the group, by fixing goals, conducts etc. (Zheng et al., 2019). It integrates cognitive, motivational and emotional components. There is a need for group regulation during collaboration: members need to be able to work on shared goals, communicate to share knowledge and understanding and monitor their progress and process. Group members react to peer's previous metacognitive contributions in order to achieve a shared understanding of the task but also in order to ensure a shared planning of the execution of the task or to ensure coordination between members (Malmberg et al., 2021). Regulation intervenes also to maintain a positive socioemotional atmosphere to facilitate collaboration. Socially-shared regulation becomes more difficult as the size of the group increases. In a classroom for instance, collaborative learning becomes challenging. In class-wide collaboration practices, "students rarely achieve a deepening of knowledge" (Chen et al., 2021).

These different types of regulation (SRL included) are interdependent, starting from the point that there is a construction of shared understand-

ing in co-regulation and socially-shared regulation that require the use of self regulation skills for interaction (Järvelä et al., 2007). Self regulation can make use of social-regulation skills for an effective collaborative learning and is correlated to positive collaborative learning performance (S. Zhang et al., 2021), and in the other way, engagement in the social space is positively related to individual achievement (Slof et al., 2021). Given this interdependence, we need to consider the different mechanisms of collaboration that may be impactful on an individual's learning, that is the subject of the next section.

COLLABORATION MECHANISMS We will now describe different processes of collaboration among the literature that are helpful to consider collaborative mechanisms outside of only individuals' contributions. The theory of common ground of Clark and Brennan (1991) explains that collaboration, communication and collective actions are built on common ground and its accumulation. While critiqued by for instance Koschmann and LeBaron (2003), this theory explains the concept of "grounding", which is the process of updating the common ground in order to reach an (unattainable) mutual belief. At least, group members must develop a shared mental model to specify how they will communicate and coordinate their actions to share the group knowledge (Järvelä and Rosé, 2022). This coordination can take place during different processes. The first one is (self-)explanation, in which a more knowledgeable other does an explanation to help the group. This cognitive activity is beneficial for the whole group (Dillenbourg and Schneider, 1995a), because learning can occur when there is reinterpretation of another one's appropriated knowledge (appropriation), and because verbal interactions have intrinsic learning effects (internalization). Explanatory and argumentative interactions that play a role in the co-construction of scientific knowledge are called epistemic interactions (Sanchez, 2017). These epistemic interactions can take place via peer feedback (Tan and Chen, 2022), or more generally peer contributions like peer-assessment, peer-feedback and peer-rating (Kollar and Fischer, 2010). There are different characteristics of feedback:

- Unidirectional or bi-directional: The process of giving feedback benefits students as observation and comparison can lead to improvement (Chang et al., 2012);

1 Regulation of learning

- Number of individuals, from dyads to a classroom : Class-wide interventions could provide a wider learning scope, but the management task is more difficult;
- Specific versus nonspecific feedback : The specificity of the feedback concerns the contextual relevance, specific feedback being more effective because they give a more appropriate amount of information;
- Affective or cognitive : Affective feedback (positive or negative feelings shared) is generally less specific than cognitive feedback (summarizing, specifying, etc).

Exchanges can also lead to conflicts that cannot be ignored, but can lead to decreasing confirmation biases (alternative proposal) and a shared cognitive load (Dillenbourg and Schneider, 1995b). Epistemic interactions are also transformed by an evolving social exchange. The group has a corporate identity residing in their past and their composition. Crook (2022) calls the effect group awareness. The group can reduce group members' efforts to coordinate their actions (Zhong and Xia, 2022). However, it has a tendency to converge in process, more than in knowledge (Dillenbourg and Fischer, 2010). It can also be detrimental to the entire group : S. Zhang et al. (2021) found that low-performing groups (for on-line collaborative learning activities) tend to have more "routine-level" regulatory behaviours, focusing more on task planning and process monitoring (superficial aspects) rather than content monitoring. Collaborative mechanisms are not the only mechanisms that occur when adding the social environment of a group. General dynamics on groups are known to highly influence individual behaviours. We also need to take that into account.

Kauffeld and Lehmann-Willenbrock (2012) propose the input-process-output model to represent group dynamics in general :

- Input : incentives, size of the group, personalities, constraints (time for instance);
- Process : activities mediation relationship between input and output such as planning monitoring behaviour, managing conflict and commitment
- Output : productivity, satisfaction, effectiveness

We will first focus on inputs and outputs to analyse the general characteristics of a group. First, there are general characteristics of a group, representing elements that exist only within the existence of the group for the duration of the task, and that can influence group members' behaviours and ultimately outputs. Those parameters are :

- The size : For Dillenbourg and Schneider (1995a), large groups may be detrimental to some members that tend to be "asleep" or excluded from interactions. Large groups tend to work differently than small groups, with the creation of sub-groups, that is for instance a common organization in classrooms.
- The heterogeneity of group's members : Hoffman (1965) describes heterogeneity of members as their intrinsic heterogeneity : Age, gender or religion are important factors of heterogeneity; and can trigger conflicts and imply a difficult social grounding, which is the mechanism by which an individual attempts to maintain the belief that their partner has understood what they meant. There is also heterogeneity in regard to the task that can be beneficial to the group's performance : Differences in prior knowledge (Chiu, 2000), general level of group-working skills (systematic sharing of information, critique of ideas and not people etc.) or interdependence of knowledge or material between members (Johnson et al., 1994) are also influencing factors. Detrimental effects may also happen with this heterogeneity. For instance, expert students may become a leader rather than a peer, interrupting teammates' cognitive process (Sanchez and Mandran, 2017).
- Past experience within and outside of the group : Hoffman (1965) indicates past experience within the group may encourage members perceived as "better" to speak more and encourage more of their ideas. Past experience may also influence the level of collaboration within members given their prior relations from individual work with sharing to total collaboration (Chiu, 2000, Bales and Strodtbeck, 1951).
- Pre-existing structures : The group's usual functioning is influenced by its structure: Hierarchical structures, horizontal structures, distributed roles structures etc. For instance, Qu and Liu (2021) showed that the presence of a leader moderates the relationship between

idea generation and implementation. This influence is perceptible even when the structure is artificial. In education, assigning roles tends to support and stimulate students in undertaking specific activities that they would otherwise not engage in (De Wever and Strijbos, 2021).

- Environmental influences : The environment of the group influences its functioning, whether because of cultural norms, physical environment, financial or time constraints (Bales and Strodtbeck, 1951). For instance Sanchez and Mandran (2017) points out that time-pressure factors may lead to anxiety and pressure, inhibiting learning, and that a competitive environment affects the self-conception of students.
- Group mood : A mood is a diffuse feeling state without clear antecedent or cause (Barsade and Gibson, 2007), in relation to individuals' affects and emotions. It is a collective phenomenon as there is mood convergence between group members through communicative reactions and mutual emotional contagion (Lehmann-Willenbrock et al., 2011). It is partly responsible for circles of complaints and actions (Kauffeld and Lehmann-Willenbrock, 2012) : complaining cycles are linked to a passive group mood and pronounced negative effects such as dysfunctional communication, whether interest-in-change cycles are correlated with an active group mood and the contagion of positive affect leads to improved cooperation, decreased conflict, and enhanced creativity among other. It is not a simple dichotomy, as sometimes a negative mood can improve team performance with an increase of conflicts and a more careful assessment of task problems (Jordan et al., 2006).

At the individual level, different characteristics are able to influence the group functioning. Characteristics relative to the heterogeneity of the group : age, gender, religion (Hoffman, 1965), prior knowledge, group-working skills (Chiu, 2000) etc. Whether an individual is extrovert, talkative or shy will influence the individual's participation in the group. In particular, certain personalities are more inclined to adapt to group effects such as searching for uniformity or agreeing to the majority (Hoffman, 1965). Finally, an individual's affect (composed of feeling states and traits) and emotions (feeling states accompanied with physiological reactions) are

responsible for part of the group mood as there are emotional reactions between members of the group (Barsade and Gibson, 2007).

Given all characteristics of collaborative learning, we can consider multiple ideas on what can be done to foster learning. Although the work of this thesis focuses solely on individual learning, it was very important to put it in context both to show the limits of our study but also to open up its perspectives. To this end, Chapter 5 will discuss possible future work that can be done or existing work that we can be inspired by given the way collaborative learning works in relation to self-regulation.

1.2 CREATIVITY : A CASE OF SELF-REGULATION

1.2.1 CREATIVITY AT AN EDUCATIONAL LEVEL

Creativity is both a skill to develop and a skill to solicit. Kupers et al. (2019) defines it as an iterative process. The person interacts with the task and the social environment. These interactions allow to create iteratively a creative output. This is based on the 4 P's of creativity from Rhodes (1961) :

- Person (the person doing the task)
- Process (the creative process of the person)
- Press (the environment in which creativity occurs)
- Product (the creative output)

Creativity can be considered at multiple levels. We can study it at the group level, for instance in classrooms. We can study it at the individual level. We can also consider a macro-level : creativity is then a static trait of the personality of individuals. However, we are more interested in the micro-level : the way the creative process works at the level of one task.

In education, creativity is both a skill to develop and a skill to solicit. On the one hand, a creative approach to disciplinary learning at both the teacher and student levels is of great benefit, for instance, to optimize student engagement, or to widen the ways to better learn (Leroy et al., 2021). On the other hand, creativity on its own is one of the important 21st century skills, and is thus a competence in itself (Engeness, 2020). In Kupers et al. (2019), creativity is defined, considering child students, as an iterative process of interrelations between the child, the task, and

the social environment, with "constraints" and "emergences". It is based on the 4 P's of creativity from Rhodes (1961) as reviewed in, e.g., Gruszka and Tang (2016) : person, product, process, press organized in a hierarchical way, and considers that they are interrelated to create "constraints" in the top-down direction and "emergences" in the bottom-up direction.

Another clarification in the framework proposed by Kupers et al. (2019) concerns the different levels of creativity : micro-level within a task, in a dynamic view on an evolving and measurable "creative act" (behaviours, strategies), macro-level considering "creative personalities" in a static way) and how it can be evaluated (e.g., by assessment or personality questionnaires), whether the study is static (measurement at a given time) or dynamic (measurement of the evolution of creativity over time), and whether possibly proposed intervention is unidirectional (simple evaluation of its effect on creativity) or within a causality process (examining the chain of causes and effects over time). Here, our geometric formulation of CPS is rather micro, dynamic and in a causality process, therefore complementary to what is generally studied, according to Kupers et al. (2019) for children or Thurlings et al. (2015) at the level of teacher innovative behaviours and pedagogical innovation. A step further, Mansfield et al. (1978) highlights the fact that studies of the effectiveness of creativity training methods generally suffer from methodological weaknesses, e.g. the Hawthorne effect (the fact that individuals modify aspects of their behaviour in response to their awareness of being observed). Considering creativity training, they observe that only considering divergent thinking as a creativity process yields biased results, whereas it is a complete process influenced by cognitive, motivational, personality and situational factors.

While cognitive neuroscience advances the understanding on how our brains function in creative tasks (Alexandre, 2020, Daikoku et al., 2021), educational and psychology researchers developing creative studies develop a diversity of approaches in which we can find studies at the cognitive level (Hao et al., 2016, Lubart and Sternberg, 1995, Radel et al., 2015), the behavioural level (Nemiro et al., 2017), the small group level (**cassoneGroupProcessesCreative2020**, Kelly and Karau, 1999, Paulus, 2000, Sarmiento and Stahl, 2008) and the organizational level (Selkrig and Keamy, 2017). Learning scientists more often focus on situated

learning tasks in ecological contexts of education to analyse in some cases the creative process, as in Savic (2016), considering the emergence of the creative process at the individual, small group, or classroom level (Mathisen et al., 2004, Stinkeste et al., 2021) and observing differences with respect to culture, personality and recognized values (Basantia, 2017, Lubart, 1998). These approaches also lead to applications in pedagogy as reported in Ni et al. (2014) where the theoretical approach allowed to successfully help students in improving divergent thinking. These different levels of analysis of creativity create a diversity not only of conceptual but also methodological models in the study of this phenomenon in education. In order to advance in the consideration of a multilevel approach of creativity, we consider a pluralistic epistemological approach (Turkle and Papert, 1992) in the study of creativity in domain specific tasks, thus restrained to an activity oriented approach (Albero and Guérin, 2014, Romero et al., 2021) and focus on ill-defined problem-solving tasks engaging one learner. Despite this specificity, we acknowledge that studying human creativity requires to consider also a socio-cultural perspective to analyse how knowledge is being shaped by prior experiences, cultural context and the activity system in which the subjects are developing their creative processes.

1.2.2 CREATIVITY AT A NEURAL LEVEL

Different kind of memory intervene in the creative process :

- **Working memory:** A system temporarily storing chunks of information. It can receive stimuli, and even process information, controlled by the **prefrontal cortex**.
- **Semantic memory :** A long-term memory that stores concepts, built from experience. Concepts are hierarchical. Relations between concepts can be used for creative thinking.
- **Episodic Memory :** A system storing episodes, based on the **hippocampus**. Episodes are neural activity patterns corresponding to experiences personal situations. The hippocampus can recall episodes to make predictions about future outcomes.
- **Procedural Memory :** A system storing know-hows, skills. Automated know-how are called procedures. Procedural Memory can associate objects with memory of what to do with it.

When in need of these memories, different regions of the brain can be solicited. Some regions are dedicated to monitor stimuli, internal information and external information. But networks of regions are activated differently between a deliberate search for ideas, and the spontaneous emergence of an idea.

Whereas children's creativity is mainly conceptualized today as a static and stable trait, it is probably more realistic and fruitful to study it at the micro-level, as a dynamic process with moment-to-moment interactions between the child and his or her environment (Kupers et al., 2019). In addition, authors claim that interventions could be more beneficial if they were related to chains of causes and effects in the underlying processes. Elucidating the neural bases of creativity is an interesting way to propose such a description at the micro and dynamic level. In addition, such a study can also bring a lot to related questions like understanding the influences between creativity and motivation or considering the role of behavioural disorders (e.g. hyperactivity or autism spectrum disorders) in children's creativity.

MEMORY

The role of memory in creativity has been extensively studied (Gerver et al., 2023), more precisely the interactions between memories that are of several types (Alexandre, 2020). A distinction is traditionally made between short-term (i.e., working) memory and long-term explicit (or declarative) and implicit memories, with regard to how the information is stored in the brain. Different types of memory intervene in the creative process.

WORKING MEMORY Working memory is primarily a system allowing the temporarily storage of chunks of information. It is also a processing system (Cowan, 2008). Stimuli received are encoded in a "sensory buffer" or sensory memory (Cowan, 2010) for a limited period of time. Stimuli can be either external or internal received information that activated sensory areas. Sensory areas include exteroception (the perception of the situation by, e.g. the visual or auditory cortical areas), proprioception (the perception of one's own body and movement) and interoception (the sensation of pain, pleasure, and emotions in the insular cortex). From immediate perception, regularities and rules are extracted from past experience, and different kinds of learning shape cerebral circuits' ability to be exploited in creative problem-solving tasks. After being filtered through attentional mechanisms, some of them can be temporarily retained in working memory for future use and manipulation. This is mainly controlled by the prefrontal cortex (PFC). For example, when solving a problem, working memory can be used to store traces useful to the processes involved in carrying out the task, as well as updating them as the task progresses. Because these processes often require prior information, working memory also enables retrieval from longer-term memories.

SEMANTIC MEMORY Semantic memory is a declarative long-term memory in which the organization of knowledge extends beyond the simple association of frequently linked stimuli: a hierarchy of concepts is built from experience and can be found in a semantic network, with both the idea of hierarchy (one specific class belonging to a more general class, like a cat being an animal, the relation being "is-a-kind-of") and the idea of associated relations and properties (visual or auditory characteristics but also more abstract as a name or a link to the owner). Building on this,

the sensory cortex is associated with the elaboration of semantic memory, It is also worth noting that, on the surface of the sensory cortex, the features and properties related to the identification of the corresponding "objects" are mapped onto the ventral regions (called the *what* pathway), whereas those related to their localization or use are mapped onto the dorsal regions (the *how* pathway). The associative nature of semantic memory is believed to be of primary importance in creative thinking (Dietrich, 2004; Jung et al., 2013; Gerver et al., 2023) as a function of both educational experience (Denervaud et al., 2021) and lifespan (Cosgrove et al., 2021).

EPISODIC MEMORY Episodic memory involves a cerebral region called the hippocampus (or more generally the hippocampal formation) and corresponds to memorizing episodes, i.e. multimodal neural activity patterns related to personally experienced situations within their spatio-temporal contexts and associated responses and outcomes. Episodes including a strong emotional dimension (for example associated with an error, novelty, or other kinds of reinforcement) correspond to stronger traces, and episodes in which nothing special happened have the tendency to be forgotten quickly. To store such information, the hippocampus receives as an input a compressed summary of the activities of most regions of the cortex at the moment of the episode and has a powerful mechanism associated with its recurrent architecture that allows the binding of the references to these multimodal elements to form a unique trace: frequently repeated segments of temporal inputs are concatenated into single conceptual units (Gobet and Sala, 2019). Episodic memory retrieval is a dynamic process that draws upon the sequential ability to reconstruct past experiences from corresponding cues, by a phenomenon called replay (Rabinovich et al., 2023). Later, when a similar episode (or a part of it, referred to as a cue) is experienced, the hippocampus is able to recall the full initial episode and to reactivate the corresponding cortical regions, yielding the same various sensations associated with that episode. Furthermore, when similar episodes are stored, an internal mechanism within the hippocampal formation allows the brain to detect that these episodes bind common features. In a process called consolidation (Eichenbaum, 2017), if this binding is not yet represented in the semantic memory, these episodes will be sent back by replay to the cor-

tex during specific resting moments (particularly during sleep), to train the cortex actively and help in the formation of new concepts binding these features, improving the quality of the information representation in the semantic network. More, the hippocampus is not only able to recall some full initial episode or some episode with common features but to also replay a partial episode or composite episodes made of several pieces. This allows to do prediction about the future outcome by simulation of unrealized episodes (by an imaginative process). This capability provides another way to decide which behaviour to select in a certain situation, apart from the dominant behaviour, by remembering similar cases in which dominant behaviour was applied or not. The capability to decide which behaviour to select is essential in creative behaviour. When subjects have a creative intention and aim to engage in creative behaviour, they should develop a metacognitive judgment of candidate ideas based on their creative properties (mainly novelty and usefulness) and select those that can lead to an efficient and creative process or outcome. This yields a regulatory process to behave in a creative way instead of reusing existing habitual behaviours, especially when these become inappropriate.

PROCEDURAL MEMORY Procedural memory is an example of implicit memory: it stores know-how and integrated procedures that have been automatized, such as motor skills. These procedures are gradually learned through a transfer from declarative and working memories. In problem-solving tasks, the brain figures out the problem situations by associating its sensory representations with a physical or mental behaviour encoded in the motor and premotor cortex (respectively for elementary and integrated action plans) and in the orbito-frontal cortex (for giving an emotional value to a situation and making a decision accordingly). More precisely, this goal-directed organization can be broken down into what–why–where–how loops (Alexandre, 2021) in which the value of an action is evaluated in terms of general preference (what) and motivation (why) to decide both a general goal and a next-step sub-goal, allowing the generation of rules for action (where and how). The elaborated multiscale distributed representation of sensory information is associated through training with a variety of behaviours, corresponding to responses that can be given in the physical and mental worlds. What is

specifically considered here is the capacity to anticipate the resulting situation when the response is triggered. This is the case for procedural learning, mainly associating the motor and premotor cortex with the dorsal (how) cortical regions. Throughout this learning, the consequences of actions in the real world can be anticipated. This gives rise to the concept of affordances (Gibson, 1977), whereby the perception of a situation can pre-activate possible actions and anticipate their outcomes (see section 3.1). Each object is always associated with some "what to do with" properties. This is also the case for respondent conditioning, associating the orbito-frontal cortex with the what cortical regions. Here, the decision to give a certain emotional value to a situation allows us to anticipate the corresponding reinforcement (reward or punishment). In both cases, such learning involves loops associating the cortex with the basal ganglia. After a certain time, this process results in what is called the dominant behaviour, the behaviour generally triggered in the corresponding situation. After extensive training, it can even become a habitual behaviour in the sense that the response is automatically triggered when the situation is perceived, with no anticipation of the forthcoming outcome. This response is called stimulus-based behaviour, as traditionally opposed to goal-directed behaviour. Such automatic behaviours, which are very frequent in a stable and predictable world, are exactly the opposite to what is sought in a creative intention oriented towards novelty and appropriateness.

A REPRESENTATION OF THE BRAIN NETWORKS IN THE CREATIVE PROCESS

Observing brain activity with imaging devices (e.g., fMRI and MEG) during elementary cognitive tasks has led to the definition of large-scale brain networks, associating widespread brain regions. Among them, three are particularly active in some steps of the creative process (Beaty et al., 2016), illustrated in Figure 1.1. The **default mode network** (DMN) corresponds to brain regions that are active during spontaneous thought and mind wandering. It includes regions surrounding the hippocampus (precisely the precuneus cortex), together with the ventro-medial prefrontal cortex and the lateral parietal cortex; a primary function of this network is episodic memory retrieval. The **central executive network** (CEN) includes the dorso-lateral prefrontal cortex and is activated for the control of sustained attention and planning, thus playing a key role

in working memory. The **salience network** (SN) mainly consists of the insular cortex and the anterior cingulate cortex (ACC). It monitors the salience of stimuli, integrating a variety of internal and external information. It is hypothesized to play the role of mediator activating a switch between the DMN and CEN (Uddin, 2015). Beaty et al. (2016) reported a major association between the DMN and the CEN during creativity, in which the DMN proposes candidate ideas (divergent phase) and the CEN stands for the evaluation of their appropriateness (convergent phase).

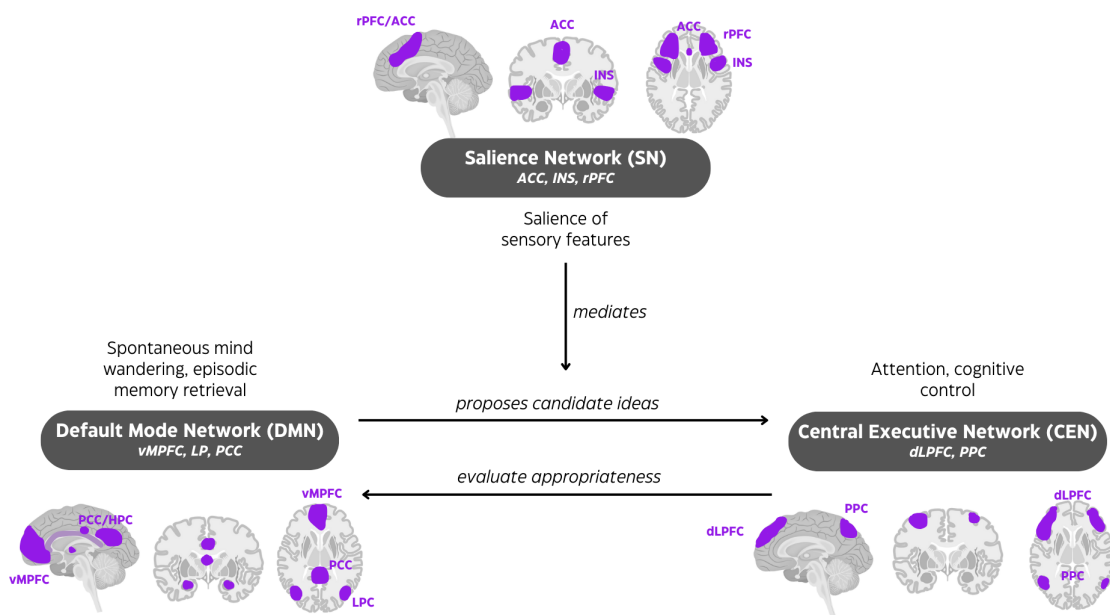


Figure 1.1: Large-scale brain networks involved in CPS, according to Beaty et al. (2016) and supported by Dietrich (2004), Jung et al. (2013) and Kounios and Beeman (2014).

The DMN, involved in mind wandering and idea generation, is mainly composed of the ventro-medial prefrontal cortex (vMPFC), the lateral parietal cortex (LPC) and the precuneus cortex (PCC). Thanks to the connection of the PCC to the hippocampal regions (precisely the caudal part of the parahippocampus), it enables episodic memory retrieval and replay. The CEN, involved in cognitive control, is mainly composed of the dorso-lateral prefrontal cortex (dLPFC) and the posterior parietal cortex (PPC). There is a slight asymmetry between the right and left hemisphere, the dLPFC and PPC of the dominant hemisphere (left here) being more active in convergent mode. Finally, the SN, involved in attention and control, is mainly composed of the rostral prefrontal cortex (rPFC), the anterior cingulate cortex (ACC) and the insular cortex (INS - primarily the anterior insula AI).

SPONTANEOUS AND DELIBERATE MODES We can make a distinction between the spontaneous emergence of an idea and a deliberate search for new ideas, including questioning existing beliefs or knowledge about the problem to solve.

In a spontaneous mode, new ideas are generated from the default network and episodic memory (Kounios and Beeman, 2014), subsequently evaluated by the CEN. In a more deliberate mode, the SN and CEN can take into account the characteristics of the task and orient the DMN toward a more systematic exploration of possible ideas. Dietrich (2004) related the first mode to intuition and discussed the correlation of age with this dual process of imposing constraints to be appropriate and relaxing them to be creative, with a more spontaneous approach observed in childhood. Kounios and Beeman (2014) also reported a neural inhibition of visual inputs by the CEN to reduce distracting inputs, facilitate the retrieval of weak solutions, and evoke actors influencing insight (see below), like mood (positive affects and reduced anxiety favouring a long-term view and broadened semantic processing; see also Diamond and Ling (2016) on these topics).

These modes are different but they do not lead to a single dichotomy in processes. For van Ede et al. (2020), the distinction proposed between goal-directed and stimulus based behaviours is highly related to sources of attentional selection, that is, to focus voluntarily on things that are relevant to our goals rather than involuntarily capturing salient events in the external world. These two sources jointly influence the selection of internal memory representations. Thus, we cannot limit stimulus-based mechanisms to a simple bottom-up process because attentional mechanisms of the brain are, from the beginning, modulated by top-down processing: in the posterior (temporal/parietal) cortex, this top-down process is modulatory and maps the injection of prior information onto the involved perceptual stimulus processing (Friston, 2003). Following this track, in line with W. Zhang et al. (2020)'s metacontrol approach, both persistent (i.e., characterized by a strong top-down bias and competition between goals) and flexible (i.e., characterized by some weak top-down bias and weak competition goals) behaviours are goal-directed behaviours. Furthermore, goal-directed behaviour is hierarchical as soon as the problem-solving task becomes complex, (Egge et al., 2022). According to W. Zhang et al. (2020), creative cognition in divergent and con-

vergent thinking is modulated by metacontrol states. Divergent thinking and insight solutions are enhanced by goal flexibility whereas convergent thinking seems to benefit from metacontrol biases toward goal persistence. Flexibility towards the goal is characterized by a weak top-down bias and a weak mutual competition between alternative decisions. W. Zhang et al. (2020) report that flexibility is promoted by a weak activation of the dorsolateral prefrontal cortex (dLPFC) together with the temporal/parietal cortex (T/PC) while the inferior frontal gyrus (IFG) is strongly activated in the dominant hemisphere (represented on the left here) in comparison with the non-dominant hemisphere. The opposite pattern is observed for goal persistence.

To this end, the prefrontal cortex is a major cerebral region to behave beyond stimulus driven dominant behaviour and to promote more flexible behaviour driven by internal analysis. This is achieved through two internal processes (O'Reilly, Hazy, et al., 2014): the medial part monitors errors, suggesting that the dominant behaviour might not be adopted in certain circumstances, and aims to predict when these errors might occur to inhibit the dominant behaviour accordingly; while the lateral part learns new contextual rules, which are better adapted to new specific cases, based on interactions with episodic memory. In both processes, this is implemented with the specific mechanism of working memory, in which prefrontal neurons display sustained activity to evaluate histories of activity, maintain constant inhibition of the dominant behaviour, and bias the activity of the sensory cortex to promote other more adapted rules instead. In particular, the biasing might correspond to increasing the saliency of some (classically supposed) minor features to orient the behaviour toward responses dealing with them. Behaviour switching is much more efficient than learning and unlearning a new behaviour and is crucial in CPS. The prefrontal cortex implements biasing activity to orient other cortical regions toward non-dominant behaviour, rather than implementing a behavioural rule per se. The consequence is genericity, and, in the framework of reasoning by analogy, for example, it should be rather easy to adapt the contextual rule to another similar biasing of activity in another cortical region. The task set model reviewed below proposes a model for such behaviour switching.

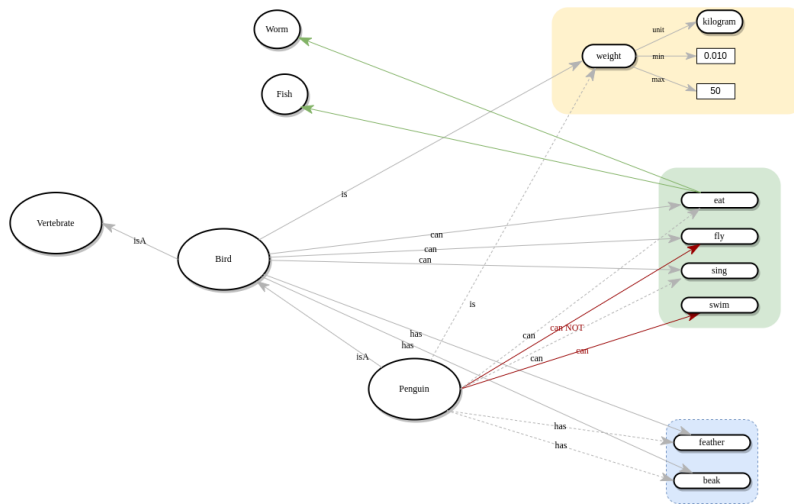
IDEA GENERATION AND EVALUATION Jung et al. (2013) insisted on the important role of episodic memory in the replay of retrospective and prospective memories and of interoception in the insular cortex to set the selection of replays toward more original ideas. Amnesic patients suffering from episodic memory loss due to hippocampal damage have been reported to perform poorly on tests to assess creativity (Jung-Beeman et al., 2004). More, the hippocampus is not only able to recall a full initial episode or an episode with common features but episodes also able to replay a partial episode or composite episodes made up of several pieces, in other words to predict unrealized episodes through simulation (Stachenfeld et al., 2017). The prefrontal cortex plays a fundamental biasing role towards hippocampal replay, these two structures being deeply interconnected (Eichenbaum, 2017). In one direction, the hippocampus can provide arbitrary cue binding to the PFC, allowing to refine the contexts in which the dominant behaviour should be inhibited and replaced by specific rules. In the other direction, the prefrontal cortex can control the retrieval of memories in the hippocampus in certain contexts by suppressing the recall of inappropriate memories. Altogether, this forms the basis of prospective memory (Buckner, 2010), a phenomenon whereby the prefrontal cortex can control the hippocampus step by step to make it produce a virtual (i.e., not really experienced) trajectory within the mapping of previous episodes. This generative capability of the hippocampus thus allows for imagination and generation of candidate ideas, for which metacognitive judgement will help selecting those that can lead to an efficient and creative process or outcome. Altogether, as pointed out by Schlichting and Preston (2015), it is interesting to observe that imagination is achieved through the recombination of prior memories and is consequently not so novel. Such recombinations are performed under the control of both spontaneous and deliberate regulatory processes, allowing to behave in a creative way instead of reusing existing habitual behaviours, especially when these become inappropriate. At this stage, a rather subtle difference is to be made between regulation and evaluation: during the divergent thinking phase, the role of the deliberate system is not to evaluate the result of the spontaneous generation process but rather to regulate the way in which the generation process is performed (Tubb and Dixon, 2014). This involves judgment along both "hot" (affective) and "cold" (analytic) dimensions (Dietrich, 2004), emo-

tion allowing the comparison of different signals using emotional value as a common currency (Levy and Glimcher, 2012). Tubb and Dixon (2014) maps this distinction onto an explicit-implicit dimension: implicit judgement is fast and affective, resulting in what is sometimes called a "gut feeling", whereas explicit judgement is more demanding but provides additional information such as the value function gradient.

Regarding the control of idea appropriateness, the same distinction between the ventral (what, semantic representation) and the dorsal (how, perception for action) regions of the sensory cortex can be made for the PFC (Bunge, 2004). In the PFC, the former is in charge of biasing the activity of the ventral part of the sensory cortex, setting the focus on "objects" and dimensions that will be of specific interest for the contextual rule, and keeping the representation of the desired goal active. The latter takes care of the organization of the generated behaviour, including over time, so that the global behaviour obeys the constraints of the ongoing task and displays appropriate temporal characteristics. More specifically, Dietrich (2004) reported that the dorsal lateral PFC is involved in its syntactic aspect, whereas the dorsal medial PFC is for the control and inhibition of common ideas (Mayseless et al., 2015) (also useful in the regulation of divergent thinking phases, as reviewed earlier). In addition, the posterior to anterior axis of the lateral PFC seems to correspond to a concrete to abstract axis. At the how level, it has been observed (Badre, 2008) that more concrete and proximal in time rules are represented on the posterior side, whereas more anterior regions display more abstract and temporally extended rules, leading to a hierarchical view of the representation of rules, in which more abstract rules can control more concrete ones. At the what level, this concrete to abstract axis corresponds to the level of abstraction of the representation (O'Reilly, Bhattacharyya, et al., 2014).

1.2.3 CREATIVITY AS A SET OF COMPUTATIONAL PROCESSES

We want to implement divergent thinking and convergent thinking phases. For that, we create a data structure of knowledge. This data structure consists of **symbols**. For instance, a **bird** is a symbol in this data structure. Symbols are associated to sensori-motor features. Features are properties that can be numerical or not. For a bird, a numerical features can be its weight. Another feature can be the capability to fly. We can associate symbols with each other to create meaning. Associations are done hierarchically :



For instance, bird and vertebrate are symbols. We can associate them with a **is a** property : A bird is a vertebrate.

For divergent thinking, the creative process extends the data structure to create creative results. Some methods are :

- **Reasoning by analogy** : "Robin is to Batman what Sancho is to Don Qixote". We can use the relation between Batman and Robin to imagine the relation between Sancho and Don Qixote.
- **Interpolation** : By taking features from an elephant and a penguin, we can imagine what is an eleguin. By varying which feature comes from which animal, we can imagine different versions of eleguin.

SYMBOLIC REPRESENTATION

The symbolic representation of creativity can have multiple representation at the computational level. We present here an example computation of an approach based on the litterature.

In order to implement the idea generation, i.e., divergent thinking process and evaluation, i.e. convergent thinking process, we propose to represent the information related to these concepts using a minimal and unified data structure, which is quite common in computational approaches of creativity. This representation allows us to define the notion of a concept region as well as a distance between two concepts, thus turning any set of concepts into a structured concept metric space. Practically, such a metric is useful for evaluation, as it provides a basis for deciding how similar two concepts are.

The unified data structure corresponds to a symbolic representation of human knowledge. In order to properly use it, we need to define what is a symbol. A first approach, at the syntactic level, is to consider a symbol as an "atom of knowledge", which is no more than the label (or identifier) of an object in the wide sense. It has a "meaning" in the sense of Harnad (1990), as reviewed and discussed in Taddeo and Floridi (2005), when it is semantically grounded, in which grounding is the process of embedding symbolic computations onto real-valued features (Badreddine et al., 2021), thus providing a semantic interpretation or model of the symbolic system (in the sense of a model of a set of logical assertions), which involves the capacity to pick referents of concepts and also a notion of consciousness. This includes affordances, i.e. not only features but also the capability of interaction with it, to attain an objective, and receive some outcome. This means that it is no longer an abstract set of assessments (potentially without any concrete implementation) but something that corresponds to a real object.

As discussed in, e.g., Raczaszek-Leonardi and Deacon (2018), in relation to the emergence of symbolic thinking (De Villiers, 2007), the key problem is "ungrounding", i.e., how to represent symbols from sensorimotor features and interaction with the environment. This aspect of the emergence of symbols, i.e., the fact that a symbolic representation emerges from a biological or any physical system in interaction with its environment, is enlightened by the semiotic approach as reviewed in De Villiers (2007), first considering a wider notion of "sign" and introducing a hierarchical meaning of an "icon" built only from sensorimotor features, i.e. at the level of the likeness with the object, e.g., with features such as color or smell, structures at an "index" level built by concrete relationships between given objects, i.e., at the level of a relation with the

object, e.g., a weathercock indexing the wind direction and strength, this giving rise to a "symbol" in the semiotic sense, which corresponds to abstract general relationships between concrete concepts or sensorimotor features, but with a qualitative break-up with respect to concrete object features, e.g., a road sign, as schematized in Figure 1.2.

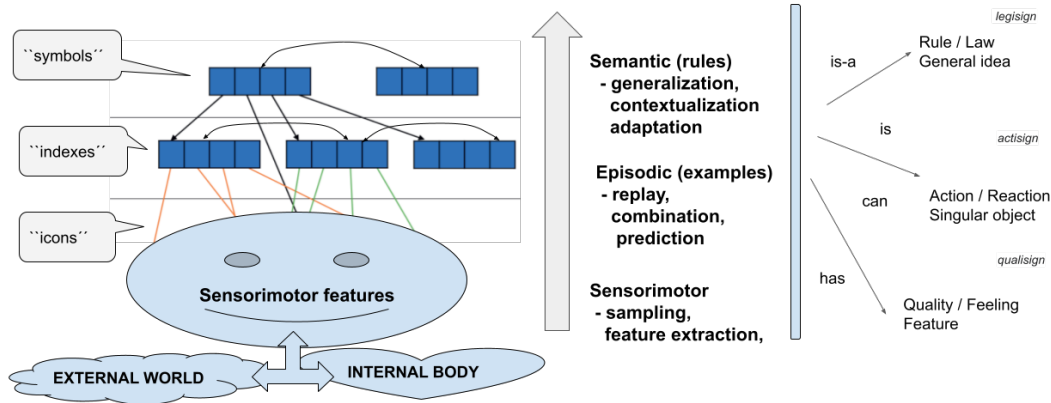


Figure 1.2: The semiology hierarchy of sign and symbols

At the computing level, we aim at manipulating the symbolic representation of knowledge of the form shown in Figure 2.2, as introduced in Mercier and Vieville (2023). In our context, we represent concepts as a hierarchical data structure. Concepts are anchored in an input/output, i.e., stimulus/response, framework, which might consist of sensorimotor feature spaces (colored regions) corresponding, for example, to different sensor modalities. Inherited features (e.g., the penguin "is-a" bird and thus inherits the features of a bird) are shown with dotted lines, while red lines represent overwritten values (e.g., a penguin can also swim but cannot fly). Green arrows point toward concepts that are themselves attributes of other concept features, accounting for inter-concept relationships. Values are completed by meta-information that is not explicitly manipulated by the agent but is used for process specification or interpretation (e.g., the weight unit and bounds).

At the modeling level, we follow Gärdenfors (2004), with the simple idea that an individual resource can be defined by "feature dimensions", i.e., attributes with some typed value. For instance, a bird could be the following. The used syntax is a weak form of the JSON syntax. For instance, using a weak form of the JSON syntax, we can represent a bird as the following representation :

```
bird: {
```

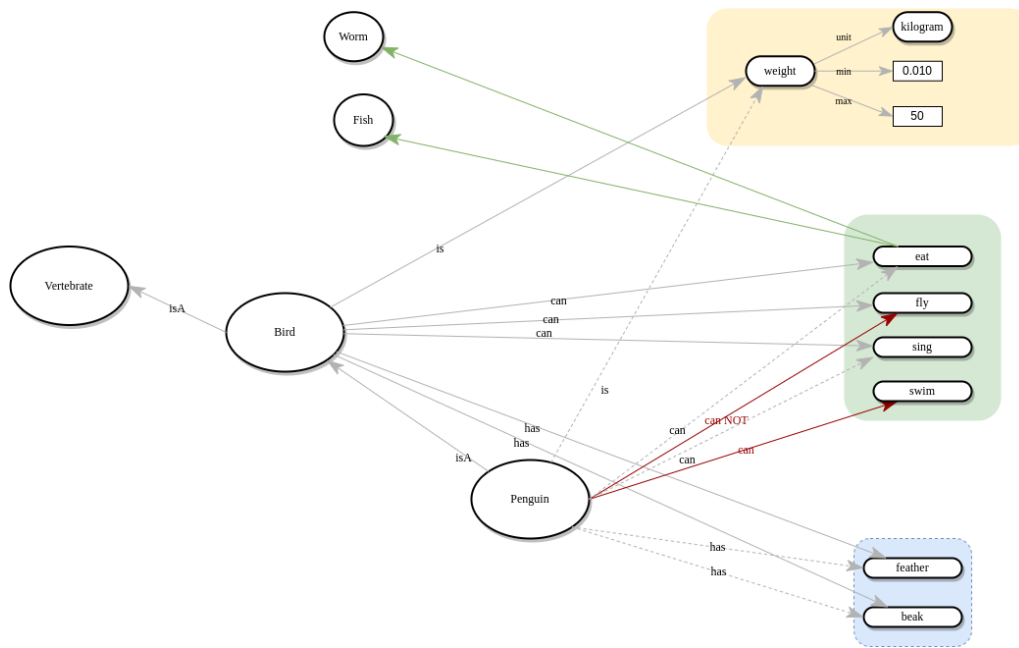


Figure 1.3: Hierarchical data structure representing concepts, see text. From Mercier and Vieville, 2023.

```

is_a: vertebrate
can: { sing fly eat: { worm fish } }
has: { feather beak }
is: { weight: { min: 0.010 max: 50 unit: kilogram } }
},

```

with some exceptions like penguins:

```

penguin: {
  is_a: bird
  can: { fly: false walk }
}.

```

This general approach of semantic knowledge representation using a hierarchical taxonomy ("is_a") with capability features ("can"), including those related to other resources (McClelland and Rogers, 2003), extrinsic features ("has"), and intrinsic features ("is").

Some features are properties, and others are relations. A property can be qualitative, e.g., the "is-covered-by" property takes a value in an enumeration (e.g., "sing", "fly"), or quantitative (e.g., the "weight"). The features can be hierarchical, either because the value is an enumeration (e.g., "can") or because the value has some features (e.g., "weight"). Such a data structure defines a "concept" in the sense of Gärdenfors, 2004 (e.g., "a bird"), which is both a convex region of the state space (e.g., the region of all birds) and a prototype: Each feature has a default value, and this also defines a prototype (e.g., a typical, i.e., prototypical, bird). It corresponds to the third cognitive memory architecture, as

proposed by Eichenbaum, 2017. At the programming level, it is going to be implemented as a "type". At the geometric level, data value corresponds to points and concept to regions, but with tricky property: Any data structure is the prototype of a region. When defining such data structure, there are several design choices and the following general recommendation that might be useful:

- **Atomic value:** It is always better to decompose the information as much as possible in atomic irreducible elements (e.g., `<tt>family_name: Smith first_names: [John Adam]</tt>` instead of `<tt>name: 'Smith, John Adam'</tt>`) for algorithmic processing.
- **Maximal tree structure:** It is always better to organize features in sub-structures than to present flattened information (e.g., create a sub-structure for the name, birth date, etc.) in order to maximize modularity.

At the implementation level, it is always preferable to choose explicit and standard names for features, i.e., look at already established vocabulary, otherwise avoid acronym or abbreviation, i.e., choose the most common word for the feature to name.

Using Vector Symbolic Architecture implemented at the neural spiking assembly level thanks to the Neural Engineering Framework (Eliasmith, 2013), such a cognitive symbolic data structure can be implemented as biologically plausible memory, allowing to manipulate it conjointly at both a symbolic and numeric level (Mercier and Vieville, 2023).

Based on such representation, we can now define creativity as a set of divergent and convergent thinking processes allowing reasoning.

DIVERGENT THINKING PROCESSES

Creative problem solving requires divergent thinking as analyzed in Romero et al. (2024) and formalized in Alexandre et al. (2024). Divergent thinking requires not only interpolating but to extrapolate new resources from existing ones. Let us illustrate these opportunities by considering three non-exclusive examples, as illustrated in Figure 1.4:

- **Projective divergent extrapolation:** Given a present state represented as a data structure, we can extrapolate another state, only

constrained by some requirement (e.g., we want to invent a "penguinemu" which is a penguin morphing towards an emu) which the only constraints that it has some of the emu's features.

- **Sequential extrapolation:** Let us consider two resources and the sequence defining a path from the former to the latter, for instance, the yesterday state and the today state, re-applying the editing sequence on the today state allows us to extrapolate what could be the tomorrow state, under the assumption that the evolution will be the same. We could also add some randomness to explore alternatives or introduce some constraints, as in the previous example, to conform to any requirement.
- **Reasoning by analogy:** Following the definition of analogy reasoning proposed by Han et al. (2018), e.g. reasoning of the form "Robin is to Batman what Sancho is to Don Quixote" we can use the editing sequence from Robin to Batman in the source context, to re-apply it to Sancho in the target context in order to generate by analogy qualities that could apply to Don Quixote. The mapping from Robin to Sancho, i.e., from the source to the target domain, forms a commutative diagram with the source and target relations. This mechanism is iterative in Han et al. (2018) and performs only at a symbolic (ontologic, see Mercier (2022)) level.

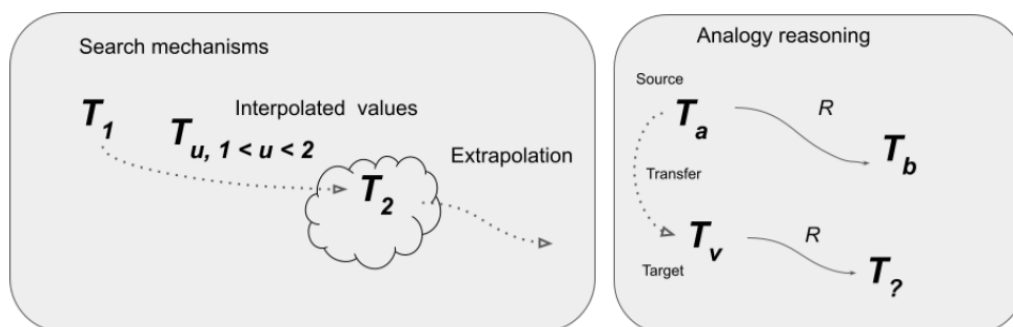


Figure 1.4: A schematic representation of two kinds of generative processes. Left: Search, by extrapolation in the mirror of an interpolation between two structures. Right: Reasoning by analogy, as formalized in Han et al. (2018).

STATE EVALUATION AND BAYESIAN REASONING In a geometric model, both the current state approximative estimation and the targeted goal correspond to regions of the state space. Each of them is estimated with regard to both bottom-up, external stimuli and top-down, internal informa-

tion (that is, prior knowledge as well as emotional and cognitive considerations). Each state may be assigned some valuations called rewards, which can be either extrinsic (external and typically tangible, such as food or money, but also social, such as fame or praise) or intrinsic (i.e. fulfilling an intrinsic need such as curiosity, e.g. the learning progress). Furthermore, the estimated value is both imprecise, i.e., partially defined, and approximate, i.e., bound to some uncertainty (in the probabilistic case) or precision (in the deterministic case). Given these two ingredients, in a probabilistic framework, the notions of a priori versus a posteriori probability, e.g. in Bayesian approaches, implement such ideas with a numeric data representation. Such a computing mechanism (Viéville, 2021) corresponds to a geometric projection of the incoming stimulus onto the region specified by prior information. In other words, given a stimulus and prior information, the estimated value is the state value as close as possible to the stimulus and compatible with prior information. We can also consider the notion of partial knowledge and its related degree of belief, i.e., the estimation of the available knowledge by a subject, which seems to be closer to cognitive representations. We need to define this notion of belief considering necessity and possibility, "distinguishing what is plausible from what is less plausible, what is the normal course of things from what is not, what is surprising from what is expected" (Dencœux et al., 2020). At the data representation level, we simply add an attribute with a belief value, while the chosen theory provides rules to combine different belief values. This simply means that the estimated value is now weighted by a level of belief combining the belief in a priori information and the level of confidence in the stimulus. These elements allow us to offer a complete operational description of how a partially defined and partially observable state value is estimated, and how to evaluate to what extent the targeted goal has been attained or not.

REGULATION We now have to discuss not only how we evaluate, but also how we manage the subgoals and goals. Choosing and keeping track of goals is a key point in regulation processes. Goal setting takes into account both external and internal stimuli, which are considered with respect to extrinsic motivations (e.g. physiological needs) and intrinsic motivations. In problem-solving tasks, there may be concurrent goals, e.g.

directed either towards performance (solving the task efficiently) or towards a better mastery of the problem, like developing task-related self-improvement, which corresponds to some intrinsic motivation (Poortvliet et al., 2007).

Following the proposal of Mercier, Roux, et al. (2021) we propose to evaluate the cost towards a given goal from the previously defined distance between a current state and the estimated subgoal state. Regarding performance related goal, we can make the strong assumption that the shorter this distance, the lower the potential cost, i.e. that editing distance operations can be related to actions generating a trajectory in the state space. We also consider that the reward provided by a subgoal is related to the estimated distance to the desired goal, the shorter this distance, the higher the reward. Combining cost and reward provides a rule to choose the goal: the one with the maximal reward at minimal cost. Such a common currency is directly related to emotion (Alexandre, 2021, Dietrich, 2004)

1.3 PROBLEM-SOLVING ACTIVITIES AND STRATEGIES

1.3.1 PROBLEM-SOLVING ACTIVITIES

Problem solving is the process by which individuals attempt to overcome difficulties, achieve plans that move them from a starting situation to a desired goal, or reach conclusions. It requires the use of higher mental functions, such as reasoning and creative thinking.

We mainly separate problems in two categories:

- **Well-defined problems** are problems with clear starting positions, clear goals and clear allowable actions. They can be usually solved with precise methods and algorithms. For instance, jigsaw puzzles are well-defined : The goal is to start from the stack of pieces and, by fitting pieces into each other, reach the final picture.
- **Ill-defined problems** are lacking one or more properties of well-defined problems. The goal may be unclear, and it may exist multiple solutions. The potential actions possible may be unknown to the subject beforehand. Drawing a beautiful picture is ill-defined : the goal is fuzzy, subjective and the method to reach the goal is not specified. Ill-defined problems often require creativity.

Most problems of everyday life are ill-defined problems. Ill-defined problems generally require the discovery or understanding of solutions, or actions to take. This implies a process of learning : learning the rules, identify a viable solution, etc.

We think that understanding how we solve ill-defined problems will help us foster our learning process. Ill-defined problems will be our focus from now on.

Problem solving is the process by which individuals attempt to overcome difficulties, achieve plans that move them from a starting situation to a desired goal, or reach conclusions. For Popper (1971), all life is problem-solving: Humans engages in processes of conjecture and refutation to learn. Problem-solving is nowadays central in most of the competency frameworks of the OECD countries. It requires the use of higher mental functions, such as reasoning and creative thinking.

For Newell (1979), problem solving is the "fundamental organizational unit of all human goal oriented symbolic activity". This has led to the definition of well-defined problems, that have strong assumptions about

the learner's capabilities to solve the problem. In particular, they can be solved through a path that is known or deduced from the initial problem-space. In either case, problems can be solved by working forward or backwards to isolate the path to take to solve the problem. Thus, such problems can be different, depending on if the problem has a solution model that is known or not from the subject. Uncertainty in subject's knowledge is a way to separate problems into different categories.

We mainly separate problems in two categories:

- **Well-defined problems** are problems with clear starting positions, clear goals and clear allowable actions. They can be usually solved with precise methods and algorithms. For instance, jigsaw puzzles are well-defined : The goal is to start from the stack of pieces and, by fitting pieces into each other, reach the final picture. A more detailed definition of well-defined problems will be done in section 2.1.
- **Ill-defined problems** are lacking one or more properties of well-defined problems. The goal may be unclear, and it may exist multiple solutions. The potential actions possible may be unknown to the subject beforehand. Drawing a beautiful picture is ill-defined : the goal is fuzzy, subjective and the method to reach the goal is not specified. Ill-defined problems often involve creativity.

Most problems of everyday life are ill-defined problems. Ill-defined problems generally require the discovery or understanding of solutions, or actions to take. Considering that uncertainty in path is a parameter of ill-defined problems, what is a well-defined problem for someone can be an ill-defined problem for another one, falling back to the separation between people knowing a solution model or not.

The solving process of an ill-defined problem (except for a completely random strategy) often implies a process of learning, whether it is learning the rules, or learning what can be a viable solution to the problem.

Ill-defined problems are at the core of learning. We postulate that a better understanding of how we solve ill-defined problems will help us foster our learning process. Thus, ill-defined problems will be our focus from now on.

1.3.2 STRATEGIES IN PROBLEM-SOLVING

Strategies are a set of actions oriented to achieve a goal. For instance, a strategy can be :

- Completely random : the subject takes a random action, hoping they get a relevant feedback.
- Trial-and-error : the subject tries to reach a solution by a specific sequence of actions (trial), and get potentially relevant feedback from failure (error).

However, there is no definitive classification of the different strategies someone can apply to solve a problem, and strategies above are just potential examples. For instance, some strategies can be relative to specific domain. In mathematics, a proof by contradiction is a strategy in which the subject tries to prove a statement true by showing that the negative is contradictory.

Yet, we can consider two general categories of strategies :

- **Exploration** strategies are used to discover attributes of the problem. It includes the search for features, limitations, functions and attributes. It also includes abstract conceptualizations of the problem. For instance, taking random actions to get feedback is an exploration strategy. We associate these strategies to divergent thinking phases of the creative process, more in a stimulus-driven way.
- **Exploitation** strategies are used to evaluate prior knowledge and present information, like trial-and-error strategies. We associate these strategies to convergent thinking phases of the creative process, more in a goal-driven way.

A problem-solving strategy is "a plan of action used to find a solution" (Amit and Portnov- Neeman, 2017).

In well-defined problem-solving tasks, prior knowledge can support the identification of the problem and the use of existing relevant knowledge to solve the problem (Brand-Gruwel et al., 2005). On the contrary, in ill-defined problems, prior knowledge and early ideas of the solution may need to be inhibited in order to think outside-the-box. Thus, the learner needs to be aware of this need (Leroy et al., 2021).

As ill-defined problems are our focus, we consider situations in which prior knowledge does not allow to engage in a conservative behaviour, i.e. to only exploit prior knowledge and skills, but requires to generate novel ideas and implement them to succeed in solving an unknown problem situation (Romero, Lille, et al., 2018).

A first model, described by Polya (1945), can fit the needs of an ill-defined problem solving approach :

- Understanding the problem (estimating the actual situation and choosing a goal or subgoal)
- Devising the plan
- Carrying out the plan
- Looking back to compare predicted result and observed result of the plan.

This echoes the different processes of creativity and more generally self-regulation. A strategy, in this case, corresponds to the set of actions enacted by the subject to transform their environment. For instance, trying a random action is a strategy.

Strategies in problem solving has usually been considered as domain-specific. For instance, in writing studies, the study of strategies have permitted to identify different types of strategies at the sentence generation and revision (Flower and Hayes, 1981) which are specific to the writing tasks. As they are learning strategies, they can be specific to the field of learning. Biggs (2012) tried to propose three main approaches to learning :

- Deep approach in which critical evaluation and syntheses if information are performed. It is driven by intrinsic motivation.
- Surface approach in which superficial cognitive strategies are performed. It is driven by extrinsic motivation.
- Strategic approach, which alternates between deep and surface approaches.

This model is mainly focused on motivation. We postulate that, in ill-defined problem solving, strategies are not only related to motivation but also constrained by the ill-definition of the problem, in order to form a broader learning strategy.

In neuroscience, Womelsdorf et al. (2021) separates learning strategies in two :

- Slow learning, using prediction errors between planning and judging the outcome to slowly update value expectations to improve choices;
- Fast learning, using working memory of recent rewards to guide choices.

Gubenko et al. (2021) differentiates **generation** and **exploitation** strategies. Generative strategies are defined as the integration of prior knowledge with present information, while exploration strategies are more attribute finding in the environment. We can consider generative strategies as more goal driven while exploration strategies are more stimulus driven focusing the attention into the features the environment can provide. These strategies, as shown Figure 1.5, are complementary.

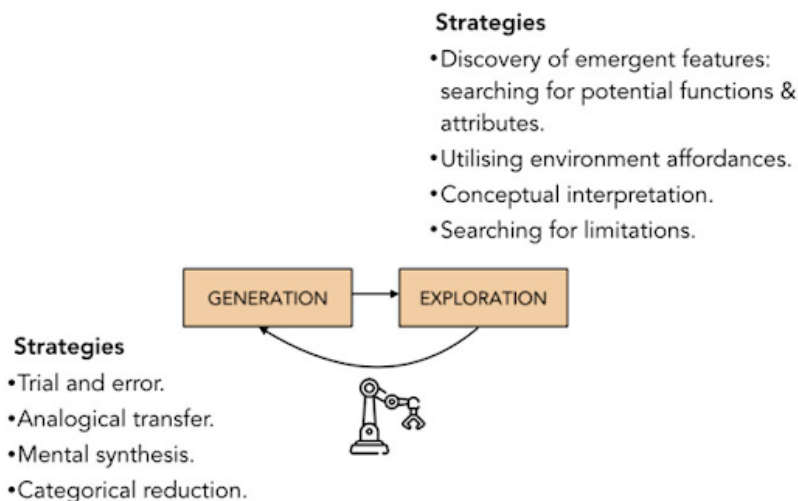


Figure 1.5: Gubenko et al. (2021)'s strategies

Thus, Gubenko et al. (2021) defines four generation strategies :

- Trial-and-error
- Analogical transfer
- Mental Synthesis
- Categorical reduction

And four exploitation strategies building upon obtained knowledge :

- Discovery of emergent features

- Conceptual interpretation
- Using environment's affordances
- Searching for limitations

It is important to note that strategies can be applied at multiple levels, and their use is not set in stone : learners can freely adapt their strategies, this is one of the main use of self-regulated skills.

There are a lot of different approaches to the categorization of problem-solving strategies, leading to multiple classification of learners' behaviours. For instance, Chiew and Wang (2004) proposed seven "cognitive approaches to problem solving" (p. 2) : facts (knowing the solution by facts), hill climbing (advancing with small steps), working backward (starting from the known solution and then working step-by-step back to the initial problem), algorithm application, exhaustive search, heuristic approaches (which can be divided into three subcategories, namely rule of thumb, mean-ends analysis, and brainstorming) and analogy approach.

Learning problem-solving skills is about learning perseverance and changing the attitudes towards problems. For instance, Wismath et al. (2014) reported that "almost all the students began the course reporting limited confidence in their abilities to tackle problems, and expressed a belief that the key attributes involved in successful problem solving were intelligence and creativity". While these are important, it is in fact often persistence which leads to the most success: knowing what to do when you are stuck, and being willing to keep going rather than giving up (Polya, 1945). Gaining a deep understanding that most problems are solvable with sustained effort we suggest marks a fundamental and irreversible threshold in students' development of problem solving skills. As such, Wismath et al. (2014) reported an increase in reported levels of acquired problem-solving skill and confidence in those skills among student participants.

POSITIONING In this chapter, we have produced a state of the art of the regulation of learning, from self-regulation to collaborative learning, and considered the particular case of creativity. Here, we are particularly interested in regulation during the problem-solving process. As mentioned in the introduction, we think that the learning process occurs at any stage of life, and is not limited to formal education. It can occurs at any point,

at any moment in everyday life and, as stated in section 1.3.1, they are often ill-defined problems.

In the literature, well-defined problems are extensively studied. Models of representation of such problems exist, and they can be used to study learning strategies. Unfortunately, there are at the best of our knowledge no general models of representation of ill-defined problems. It does not mean that models do not exist. For instance, in chapter 3, we will refer to existing work on an activity called Creacube and existing models used to represent and study it. There is no such thing as an "activity-agnostic" model of representation of ill-defined problems that exist for well-defined problems (developed in section 2.1).

We think that developing such model of representation could be beneficial to the domain. This model should take into account the richness of ill-defined problems (which is the subject's knowledge and lack of knowledge to solve the problem). With such model, comparing different ill-defined problems in order to find similarities and specificities of each activities will lead to a better understanding of general mechanisms of learning.

In this thesis, we propose to tackle the creation of such model. Chapter 2 will develop our method to reach this goal. The idea is to start from the well-known model of representation of well-defined problems representing each possible state of a problem as a state that can be attained from near other states. We will then extend it to ill-defined problems by considering the subject's knowledge and skills as part of the state of the problem. This representation is impossible to grasp fully because we can't access the subject's complete set of knowledge and skills, but it can be partially reconstructed after the activity by looking at the problem-solving process.

If this created model is valid, we will be able to use it to discover potential similarities and/or differences between different ill-defined problems. For this, we will use two different activities, namely Creacube in chapter 3 and Outer Wilds in chapter 4.

2 A MODEL OF PROBLEM-SOLVING REPRESENTATION

2.1 WELL-DEFINED PROBLEM-SPACES

Newell (1981b) proposed a formalization of well-defined problems. This formalization allows to represent problems by a **problem space** and a **problem**.

A problem space consists of :

- A set including all possible states of the problem.
- A set of operators. An operator is an action a person can take to change the state of the problem.

A problem consists of :

- A set of all possible initial states. A person solving the problem starts in one of these state.
- A set of all possible goal states. A person solving the problem wins by reaching one goal state.
- A set of path constraints. Path constraints are all the rules that limit the possible actions a person can take.

A person solving a well-defined problem starts at an initial state and moves state-by-state until they reach a goal state. They need to respect all rules given by the path constraints. See Figure 2.1 for an example.

In Human Problem Solving, Newell and Simon (1972) proposed an information processing approach to problem solving along with a formalization of well-defined problems. Newell (1981b) illustrated it with the well-known problem of the Tower of Hanoi: Considering three towers, with disks on tower 1, move all disks from tower 1 to tower 3. Disks can be moved only one at a time, must be placed on a tower, but not on smaller disks.

This approach, based on the computer metaphor, consists of applying a search algorithm to a mental representation of what they call the problem space. This problem space, once specified, allows the use of heuristics, i.e. decision methods based on a search of the best possible alternative at each branching based on available information. Such heuristic may not be always the best because of, for instance, the size of the problem can make impossible an exhaustive computation.

For Newell (1981b), the problem space consists of:

- A set of symbolic structures called the states of the space. They allow to represent at least each possible situation that may happen during the problem-solving process.
- A set of operators over the space. Operators are functions that are (maybe partially) defined over the state space and produce states as outputs. Sequences of operators define paths that can be interpreted into sequences of states.

With this definition, we can then define a problem as :

- A set of initial set, subset of the states of the space.
- A set of goal states, subset of the states of the space.
- A set of path constraints, reducing the set of operators over the state space.

Solving a problem consists of finding a path in the problem space, starting at any initial state and ending at any goal state while satisfying path constraints (Newell, 1981a). This information processing approach allows to test problem solving algorithms using computer simulations. This has been done with the General Problem Solver (Newell and Simon, 1972 and later on other computational cognitive architectures like SOAR (J. Laird et al., 1986; J. E. Laird et al., 2017) or ACT-R (Anderson et al., 1997)

The key principle, which the General Problem Solver relies on, is the physical symbol system hypothesis (Newell and Simon, 1972), that is, the problem space is represented as units of information called symbols that are manipulated and combined into structures (expressions) using logical rules. This is the basis for what we now call symbolic artificial intelligence.

Let's consider the Hanoi Tower representation. Figure 2.1 shows a representation of the Tower of Hanoi problem.

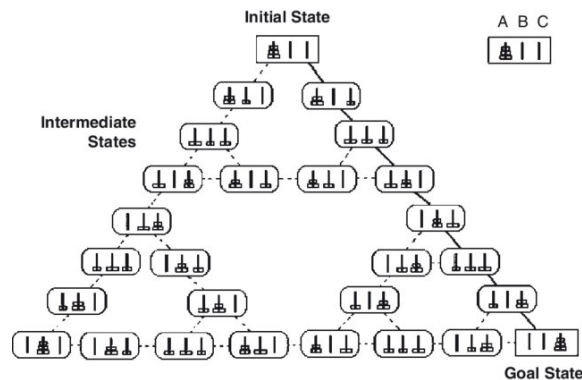


Figure 2.1: An exhaustive search of possible states of the Tower of Hanoi problem. Credits to Vinod Goel for the image.

Following the previous definition, we can determine what applies to the Tower of Hanoi problem :

- States : Configurations of the disks on the three pegs.
- Operators :
 - Move a disk by placing it from one peg on another
 - Recognize a configuration as an instance of a pattern
- Initial state : The configuration shown at the top in Figure 2.1, that is the state in which all disks are on the first peg.
- Goal state : The state in which all disks are on the third peg.
- Path constraints : No disk can be placed on a smaller disk.

Despite being a significant advance in the computational study of problem solving, Newell and Simon (1972)'s approach remains limited to well-defined problems such as the Tower of Hanoi.

In the next section, we propose to extend this formalization to ill-defined problems. This will also allow to represent divergent and convergent processes as discussed in section 1.2.3.

2.2 THE THREE-PROBLEM-SPACE FRAMEWORK AND THE FOG OF WAR ANALOGY

The framework presented in this section consists on an iterative process of a construction of a computable problem space for ill-defined problem

solving. As such, the exact problem space (section 2.2.1) serves as a limit in order to approximate the well-defined problem space from Newell and Simon (1972), that is indeed never reached. The subject-related problem space (section 2.2.2) reduces it to plausible situations, while the observer problem-space (section 2.2.3) reduces it to a computable problem space for an external observer, thanks to what we call a fog of war analogy.

2.2.1 EXACT PROBLEM-SPACE

We create an extension of well-defined problem spaces to ill-defined problems : the **exact problem-space**.

In this model, we can represent everything at any given time:

- The physical representation of all matter in the universe.
- A complete understanding of the mental representation of every intelligent being.

In this model, a state is just the description of everything at a given time. Constraints prevents impossible states to happen.

This model is absurd to use: it is an abstraction to approximate the well-defined problem-space. We are not able to represent the entire state of the material universe. We do not have access to complete mental representations of intelligent beings.

This model is useful because an ill-defined problem becomes a well-defined problem, because abstract representations that make ill-defined problems are entirely known. With this representation, we can find a path from an initial state to a solution.

However, we can use it as a complete model to reduce. We want to reduce it to a model that we can compute.

We want to extend the well-defined problem representation of Newell to ill-defined problems. Such representation should consider every possibility of ill-defined problems, which will lead to computational problems.

For now, let's try to construct a problem representation based on Newell. But first, let's consider two problems that will serve to illustrate the construction.

A PROBLEM OF ORIENTATION We consider the following open problem: A person arrives at the train station for the first time, and must meet an-

other person at a monument of the city they arrived at, not knowing what does the monument look like. Every other element of the problem is considered unknown for now.

We will extend it along the creation of what could be the representation of an ill-defined problem. It is an ill-defined problem as the goal and the path to reach the goal are unknown to the subject beforehand.

WHAT IS A STATE? WHAT IS AN OPERATOR ? In order to define what is a state in an ill-defined problem, we need to understand what is the purpose of a state. For well-defined problems like the Tower of Hanoi problem, a state is an abstract representation of what could be the situation of all **relevant** elements of the problem : on which towers are the disks. No matter if the disks are made of wood or of steel, the abstract configuration stays the same. No matter either if the disks are put on the towers upside-down, because the orientation of the disk is not relevant to the problem. A state of the problem ignores any change in the real situation that does not change the abstract representation defined. This abstract representation is possible thanks to the well-defined property of the problem. As all possible states and operators are defined beforehand, the subject may know that any other action will have no effect on the problem-solving process.

From this, we postulate that a state of a problem should include all relevant features of the environment for the problem. The difficulty for ill-defined problem is that, as there exists uncertainty and unknowns, it is difficult for a subject to precisely determine which feature is relevant or not. In fact, one main relevant feature of an ill-defined problem is the degree of knowledge of the obfuscated states or operators. Thus, everything that can lead to disambiguation or learning of previously unknown states or operators should be considered as a relevant element of the problem.

Unfortunately, everything present in the environment can be, for the subject, a potentially relevant element of the problem (with different degree of uncertainty concerning the relevance of the problem). As such, for an ill-defined problem, every change in the environment may have an effect on the problem-solving process, and thus two slightly different environments should be considered as two different states.

Moreover, the knowledge of the subject, i.e. what they know (or don't know), is relevant to solve ill-defined problems. Knowing the code of a locked door can allow the subject to pass through it. If the code was to be learned during the problem, then the different states leading to the subject knowing the code and using it are useful.

As such, any possible change in the material environment of the problem or in the abstract representation of the subject is susceptible to be a relevant change in the problem-solving process.

Let's consider our orientation problem. The action of asking a stranger the way is a valid operator that can update the knowledge of the subject in order for them to learn what does a goal state look like. The state goes from "unknown monument" to "known monument". Still using the stranger example, maybe the stranger will offer to guide the person to the monument. The problem will be solved this way, and the goal state may be still unknown (the subject does not have to know what does the monument look like). However, we can consider that this change of the environment is a state change, because it reveals the path to the monument by having someone leading the way.

Let's then define a state for an ill-defined problem space.

- An **exact state** is an exact description of the entire physical world and all abstract representations of intelligent beings present during a problem at a given time.

With this definition of an exact state, we can define simply an operator by the shift from one state to another.

- An operator is a couple of exact states (a, b) , a being the exact starting state, b being the exact arrival state

From these definitions, we can deduce that an ill-defined problem consists of :

- A set of all possible initial exact states that is a subset of all exact states of the problem.
- A set of all possible goal exact states that is a subset of all exact states of the problem.
- A set of path constraints, that contains all the operators that can't happen.

These definitions are far too general. However, we consider them as the necessary definitions to precisely define ill-defined problems. To properly represent a problem in real-life, we should be able to represent not only the entire universe but also exact characterizations of the internal states of every sentient being in the universe. More, we would need an infinite amount of states to store each of these representations. This is absurd in terms of computation (and even in terms of imagining).

However, to represent the absurdity of the situation, let's consider some extreme situations of our orientation problem. Consider this part as funny little thoughts experiments :

- **Why not a priori exclude any physical element ?** Let's consider that the subject slips on a banana peel or is hit by a car. They are rescued by a paramedic that will bring them to an hospital near the monument. Looking through the window, they can discover the path they have to take to go to the monument.
- **Why not a priori exclude any macro description item of the entire universe ?** Let's consider that the subject only knows that the monument shines in the dark. A sudden eclipse illuminates it, rendering it visible to the subject, now knowing where the monument is.
- **Why include the physical state of the subject ?** Let's consider that the person is hungry and decide to go to a restaurant. There, they find a tourism flyer of the city, and are able to go to the correct monument.
- **Okay, but what about things on the other side of earth ?** Let's consider that a catastrophe happens on the other side of earth. The phone rings, it's a notification of the incident. The subject takes their phone, and remembers at this moment that they can use the map app on their phone.
- **Why include the internal representation of the subject ?** In every previous example, we can also have the situation in which the subject is completely unaware of their surrounding, thus missing every occasion to discover unknown states or the goal.
- **Why include the internal representation of other sentient beings ?** Let's consider the friend the subject needs to find. Maybe

they will grow tired of waiting, and go in search of the missing subject. Thus, the real goal states change.

- **Why operators do not correspond to actions relative to the subject anymore ?** Let's consider the previous situation. The friend, tired of waiting, come directly to see the subject. The problem is solved without any actions taken. Also, it is difficult to determine what is an action.
- **What is an operator anyway ?** Let's consider the operator "asking someone". We can consider that, while the actions is at the beginning of its execution, the subject, having a little bit of social anxiety, suddenly remembers, thanks to stress, that they can use their app on their phone. The action is not performed, but the situation changed.

We will call this definition of the problem and the problem space the **exact problem and problem space**.

PURPOSE OF THE EXACT PROBLEM AND PROBLEM SPACE As stated previously, this model is a useless abstraction as it : it is for instance impossible to compute. We do not have the capacity to store the entire material universe, neither can we access precisely a subject's internal state. We need to reduce the size of the model to be more accessible.

However, we think that this general model is still relevant for our understanding of ill-defined problems representation. It is a limit case that shows that we cannot exclude elements a priori in ill-defined problems. The iterative method of construction applied here led us to the consideration of the subject's mental state, absent from the well-defined problem model.

This general representation allows us to re-evaluate the problem and problem-space of the Hanoi tower problem.

First, let's consider problems that we will call problems of deception : problems in which the goal state or the operators are, in addition to be partially unknown, misled by wrong or purposefully incomplete instructions. For instance, let's consider that the goal state of the Tower of Hanoi problem is given by a picture that the subject needs to reproduce. The real goal is not to solve the Tower of Hanoi problem but also to reproduce the image. For instance, this time the disks may have a top

and a bottom, and all disks must be put on their bottom face to reproduce the image. The subject may think that solving the tower of Hanoi problem is enough and face incomprehension when it isn't, leading them to re-evaluate their knowledge about the problem, operators and state. This problem is technically not the Tower of Hanoi problem, because the relevant states are different. However, their representation for the learner may be the same. Including the subject's internal state to the state seems relevant in this case, because a shift needs to occur for the subject to solve the problem.

This doesn't explain why the tower of Hanoi can be an ill-defined problem. For this, let's consider the problem the other way around. The problem is a simple Tower of Hanoi problem, well-defined. However, the subject, used to problems of deception, may consider it as a deception problem, and stop advancing the problem to start and search for other ways to solve the problem. For the subject, the problem transformed into an ill-defined problem.

CONCLUSION There is a significant difference between an ill-defined and a well-defined problem: the latter considers the subject. A well-defined problem has a definition that excludes the subject, whereas an ill-defined problem can't afford to exclude it, because there is a need to characterize the degree of knowledge of the hidden states or operators. An ill-defined problem with no uncertainty for a subject could even be considered as a well-defined problem. This observation implies that, within a state of an ill-defined problem, we can consider two distinct parts : the physical state that is, and therefore has no degree of uncertainty, and the subject's internal state, that includes knowledge about the problem and the problem space with different degree of uncertainty. The consideration of other sentient being's internal states is out of the scope of this thesis.

We can try to characterize both to be able to represent the state of the problem given the internal state of the subject. This is the purpose of the next section.

2.2.2 SUBJECT PROBLEM-SPACE

We reduce the exact problem-space to a model that considers the subject, creating what we call a **subject problem-space**. First, we represent the material environment, by :

- **Limiting the environment** : We limit the environment of the task to a realistic environment of features we can observe. For data collection, we need to eliminate data in which unobserved features play a role in the problem solving process.
- **Creating a symbolic structure** : We can reduce multiple features into one or more symbols. We describe a bicycle by the bicycle symbol instead of all physical properties of the matter it is composed of.
- **Ignoring irrelevant features** : For instance, if we can't control the height of a subject, but in most cases, we can suppose that their height is not relevant in the problem-solving process.

Reducing the environment requires a level of assumption about what is relevant to the subject and what is not.

Based on a symbolic data structure, we can create the representation of the learner.

In a subject problem-space, a state is composed of a pair of states : the state of the material environment, and the mental state of the learner. The problem is that we can't access the mental state of the learner. We need to reduce it to a representable level.

Thanks to our elaboration on the abstract notion of exact problem state, we have made explicit two key aspects regarding the representation of an ill-defined problem:

- Model the material environment of the learner
- Model the internal representation of the learner.

MODELING THE MATERIAL ENVIRONMENT In the last section, the "perfect" problem state was iteratively constructed to consider the entire universe as a material space because of the need to consider the learner's evolving mental representation. However, in order to reach a modelling that is computable, we need to reduce this "perfect" problem in order to be

able to study specific ill-defined problems. We will do it in three steps, ordered by the number of assumptions required.

- **Limiting the environment:** The first step is to limit the physical state to the material environment that is realistically in the scope of the chosen experiment. For instance, considering a problem that is presented in a room without any way to access the outside of the room, we can ignore the material environment outside of the room, because any change in the outside material state cannot be perceived by the learner, and thus cannot influence their internal representation.

For instance, the temperature of the room or its brightness because of the sun are still parameters that are considered at this point. However, other outside elements can be unrepresented in the state of the problem. Classically, any experiment in which the outside environment forces a change in the representation (e.g. a little earthquake shaking the room) should be removed from the dataset.

This doesn't mean that the outside of the room should not be considered. If the subject knows that someone is waiting outside of the room, their behaviour may differ from a case in which no one is waiting. However, this kind of feature has to be considered in the internal representation of the learner that we will describe in the next part.

- **Reducing the state to interpretable symbolic data:** In the last section, we didn't elaborate on the way to represent the "perfect" states. If we consider an atomist vision, the potential representation of the material environment could be given by the precise position of each atom of the material environment. A reasonable reduction is the interpretation of the material space into a state of symbols. For instance, a piece of cake with or without a little piece of crumb can still be considered as the same piece of cake. We can turn every element of the material space into symbols that can have a numerical value. For instance, a cube can have a color (a symbol) or a size (a number). This echoes the computational representation of the creative process presented in section 1.2.3.
- **Making assumptions about relevant features:** Lastly, it is possible to make assumptions about what is important and what isn't in the

problem solving process. For instance, using a previous example, we can assume that temperature doesn't affect the problem solving process.

The first step is part of the definition of the experiment. However the two next steps necessitate a level of assumption about what is relevant for the subject and what is not. It is necessary to point it out as this can be a source of error or imprecision in experimental results.

REPRESENTING THE LEARNER'S INTERNAL REPRESENTATION At the computing level, we aim at manipulating the symbolic representation of knowledge of the form shown in Figure 2.2, as introduced in Mercier and Vieville (2023). In our context, we represent concepts as a hierarchical data structure. Concepts are anchored in an input/output, i.e., stimulus/response, framework, which might consist of sensorimotor feature spaces (colored regions) corresponding, for example, to different sensor modalities. Inherited features (e.g., the penguin "is-a" bird and thus inherits the features of a bird) are shown with dotted lines, while red lines represent overwritten values (e.g., a penguin can also swim but cannot fly). Green arrows point toward concepts that are themselves attributes of other concept features, accounting for inter-concept relationships. Values are completed by meta-information that is not explicitly manipulated by the agent but is used for process specification or interpretation (e.g., the weight unit and bounds).

At the modeling level, we follow Gärdenfors (2004), with the simple idea that an individual resource can be defined by "feature dimensions", i.e., attributes with some typed value. For instance, a bird could be the following. The used syntax is a weak form of the JSON syntax.

```
bird: {
  is_a: vertebrate
  can: { sing fly eat: { worm fish } }
  has: { feather beak }
  is: { weight: { min: 0.010 max: 50 unit: kilogram } }
},
```

with some exceptions like penguins:

```
penguin: {
  is_a: bird
  can: { fly: false walk }
}.
```

2.2 The three-problem-space framework and the Fog of war analogy

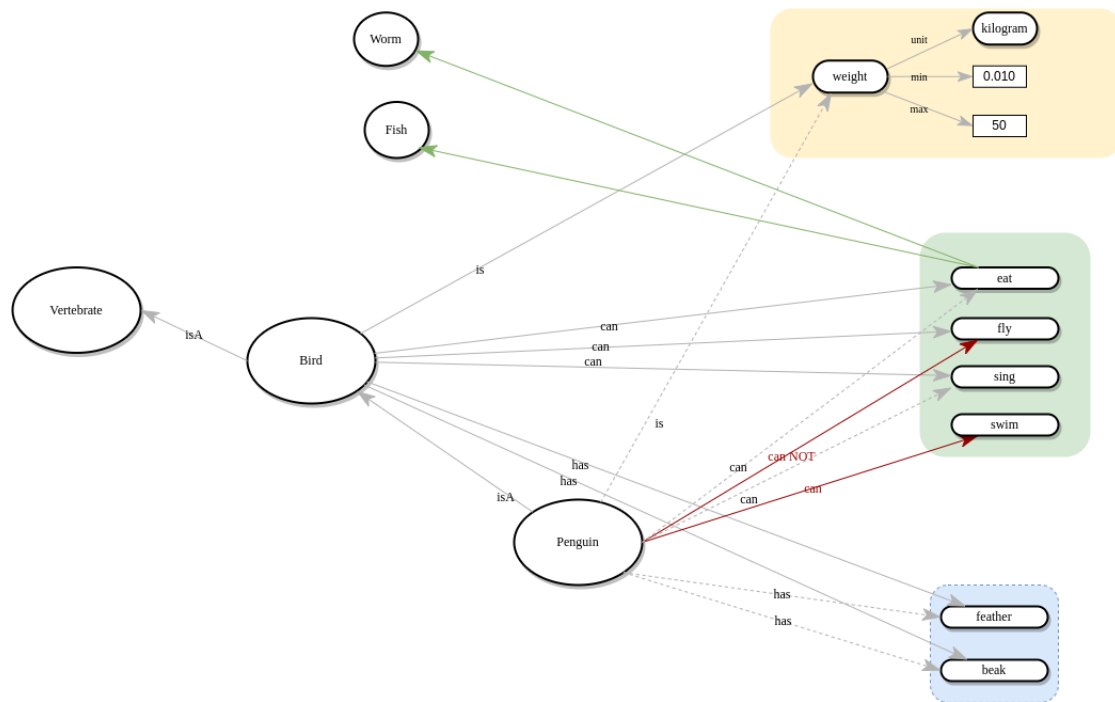


Figure 2.2: Hierarchical data structure representing concepts, from Mercier and Vieville (2023)

Here, we choose the general approach of semantic knowledge representation using a hierarchical taxonomy (is-a) with capability features (can), including those related to other resources, extrinsic features (has), and intrinsic features (is) (McClelland and Rogers, 2003). This illustrative example is sufficient to allow us to detail the main characteristics of our representation. Some features are properties, and others are relations. A property can be qualitative, e.g., the is-covered-by property takes a value in an enumeration (e.g., "sing, fly"), or quantitative (e.g., the weight). The features can be hierarchical, either because the value is an enumeration (e.g., can) or because the value has some features (e.g., weight).

Such a data structure defines a "concept" in the sense of Gärdenfors (2004) (e.g., "a bird"), which is both a convex region of the state space (e.g., the region of all birds) and a prototype: Each feature has a default value, and this also defines a prototype (e.g., a typical, i.e., prototypical, bird). It corresponds to the third cognitive memory architecture, as proposed by Eichenbaum (2017). At the programming level, it is going to be implemented as a "type". At the geometric level, data value corresponds to points and concept to regions, but with tricky property: Any data structure is the prototype of a region.

When defining such data structure, there are obviously several design choices and the following general recommendation might be useful:

- **Atomic values** : It is always better to decompose the information as much as possible in atomic irreducible elements (e.g., `<tt>family_name: Smith first_names: [John Adam]</tt>` instead of `<tt>name: 'Smith, John Adam'</tt>`) for algorithmic processing.
- **Maximal tree structure**: It is always better to organize features in sub-structures than to present flattened information (e.g., create a sub-structure for the name, birth date, etc.) in order to maximize modularity.
- We already mentioned the importance of providing as much as a possible default value, and this is a design requirement at several levels, see for instance Appendix B for a discussion at the numeric data representation level.
- We also point out, at the very concrete implementation level, that it is always preferable to choose explicit and standard names for features, i.e., look at already established vocabulary, otherwise avoid acronym or abbreviation, i.e., choose the most common word for the feature to name.

Using Vector Symbolic Architecture implemented at the neural spiking assembly level thanks to the Neural Engineering Framework Eliasmith, 2013, such a cognitive symbolic data structure can be implemented as biologically plausible memory, allowing to manipulate it conjointly at both a symbolic and numeric level Mercier and Vieville, 2023.

A (SIMPLIFIED) PROBLEM OF ORIENTATION A learner is put into a labyrinth and needs to find the exit. If they were to look to the plan pinned on the map on their left, they'll have access to the plan of the labyrinth to leave more easily.

We can represent the material environment in a graphical 2-D space if we consider that no other action than moving will have a change on the material representation. We exclude from the experiment those who try to break the walls (limiting the environment), we do not consider the outside of the labyrinth (limiting the environment), we assume that all walls are identical and immovable to be considered as only one object (reducing the state to interpretable symbolic data) and we assume that the wind from the exit of the labyrinth will not help the learner to find the solution (making assumptions about relevant features).

A representation of this material environment can be reduced to Figure 2.3

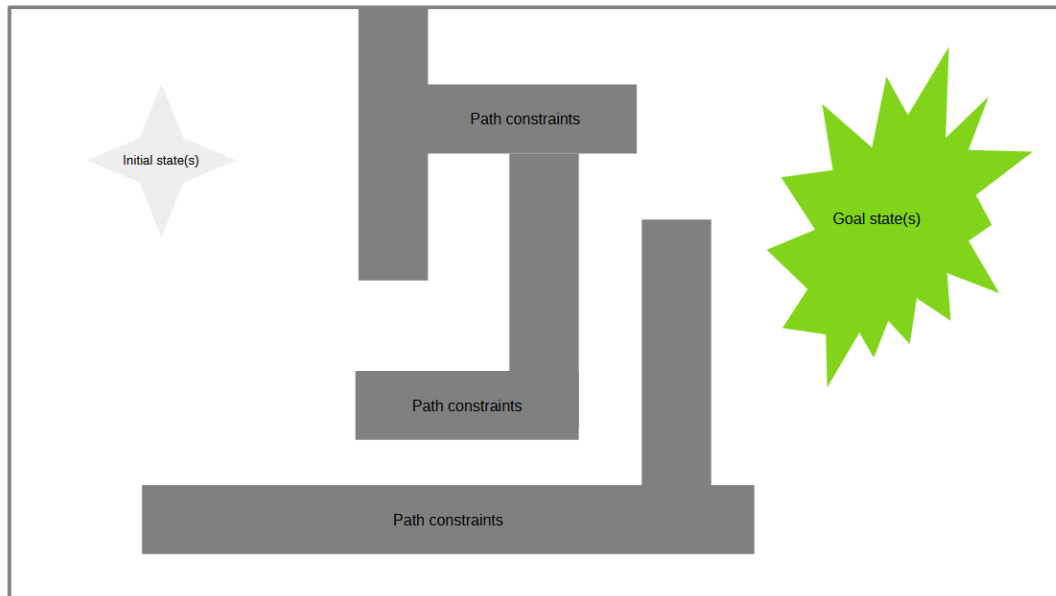


Figure 2.3: The representation of the material environment of our simplified problem of orientation

Note that a state is technically the entire material environment plus the actual position of the learner in the labyrinth, that we make coincide graphically to the map of the labyrinth plus the position of the learner. As for the internal representation of the subject, we can consider the previously defined representation.

HOW DO WE USE THESE CONCOMITANT PROBLEM-SPACES? As the task advances, both the material and the internal representation progress. They may not change at the same time (no material change can happen while the subject thinks for instance). Initial and goal states set can be defined as set of state couples (from one and the other).

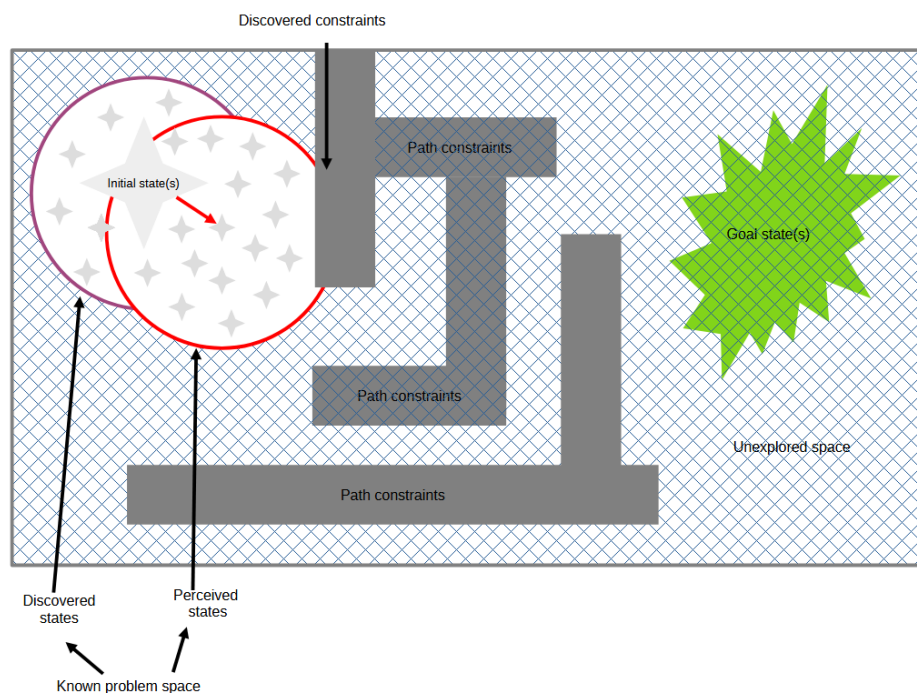
We think that the main goal of learning science is the understanding of the internal representation of the learner, while we have mostly access to the material representation (only mostly because there are methods to get a glimpse at the internal representations, e.g. post-task interviews). This duality of approaches between the material representation and the representation of the learner is extremely important and makes it possi-

ble to introduce the notion of computational learning science (Romero et al., 2020) that we try to formalize here.

The next section propose a way to adapt these spaces to be used by an external observer.

2.2.3 OBSERVER PROBLEM-SPACE

We can keep the material representation of the subject problem-space : we choose to represent a state by a set of **observables**. An observable is a feature of the environment or the player that can be computed by an external observer. We can't access the mental representation of the learner. We will reduce it to their knowledge of the material problem-space at any given moment. We apply a **fog of war** on the states of the material environment. The fog of war disappear when unknown spaces are explored or learned. The fog of war may reappear on zone that the subject forgets. We can represent it this way :



The fog of war is not just "present or not" : it allows to consider uncertainty from subjects.

We think that, based on the trajectory we can observe on a material state, we could represent the fog of war of a subject a posteriori.

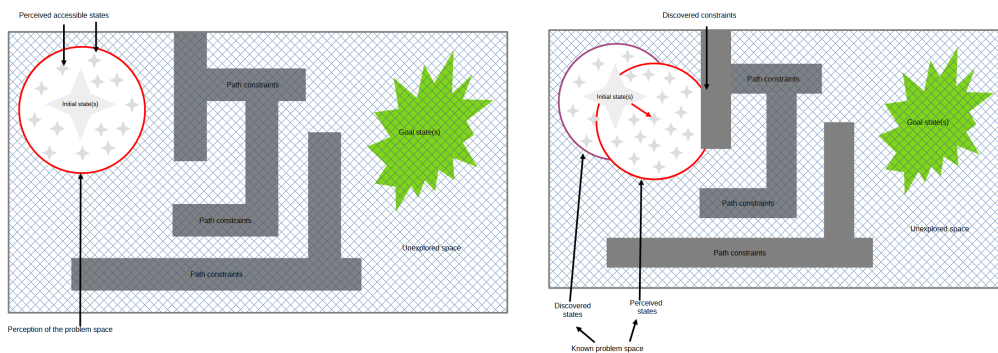
In the previous section, we proposed to represent problem solving with two concomitant problem and problem space for one problem : the environment and the learner. However, it is difficult to represent the learner's

state of their internal representation. As we have mostly access to the environment space, we propose a reduction of the internal representation with a **fog of war analogy**.

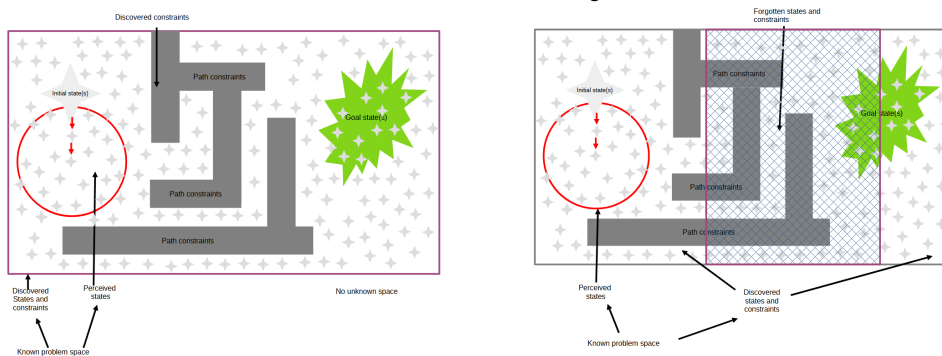
THE FOG OF WAR ANALOGY The fog of war is a term referring to the uncertainty in situational awareness. It was experienced by participants in military operations, but it is also commonly used in mechanics of video games.

We will define the fog of war as a continuous process of "the actual state of known information, with a degree of uncertainty". In the problem space, the fog of war exists on the states that are partially or totally hidden, and disappears as new information about the states or operators are discovered.

Let's take the example of the simplified problem of orientation defined in the previous section. Figure 2.4 shows different possible situations of the learner and the associated fog of war.



(a) Initial perceived material states and fog (b) Fog of war update after a material state change



(c) Discovery of all the material environment in one state (d) Forgetting of previous knowledge, fog of war reappears

Figure 2.4: A possible evolution of the observer problem-space with the fog of war analogy

- Figure 2.4a is the beginning : the learner can only perceive their surroundings. Therefore, the rest of the material state is unknown. The learner cannot represent it, and thus the fog of war covers everything that is not in their surroundings.
- Figure 2.4b happens when the learner takes the action of movement (one of the operators). By moving, the surroundings change. Therefore, the fog of war is gradually revealed by the learner, as they update their knowledge of the environment, in particular the path constraints (the walls). The initial state may not be perceivable anymore, but the learner knows what is present there, and thus the fog of war is still revealed in the initial state.
- Figure 2.4c shows a situation in which the learner goes at a point where they can find a map of the entire labyrinth. By looking at it (an other operator), they learn the entire map, and thus the entire space is revealed.
- Figure 2.4d is a possible continuation of the discovery. The learner moves towards the goal, but they forget a part of the map. By forgetting, the fog of war covers once again the parts that are no longer known by the learner.

The analogy proposed in this example can be extended to any material problem space. This way, the internal representation of the learner is applied to the material state space through the fog of war. Note that two different states of the learner's internal representation may lead to the same fog of war : it is another reduction of the space.

We can apply this analogy more precisely by adding a "degree of certainty" behind the fog of war. The fog of war can be partially present on a state if, e.g. two different possibilities are considered by the learner. If one of them is true and the other is false, the fog of war is partially removed.

We insist on the fact that the fog of war corresponds to the real meaning of the learner's internal representation. If the learner makes a bad assumption about the situation, then the fog of war is not revealed even if the learner thinks that there is something there. For instance, let's take once again our simplified problem of orientation. Instead of having access to the map of the labyrinth, the learner has access to a wrong map

of the labyrinth. The learner may think they know the map, but the fog of war is not revealed in the area that are wrong in the fake map.

OBSERVABLES FOR EXPERIMENTS As an external observer, we need a way to access the problem space of the activity. This requires an access to the material space, which may be easy, and the internal representation, which is more difficult.

To represent the material environment, there is a need to define **observables** relative to it. These observables do generally correspond to the features that were chosen in the last section : all the relevant features that exist in the material environment. All features are observables, and the state of each observable correspond to the value (symbolic or numeric) of the associated feature. To be considered as an observable, it must be potentially subject to change. A proper set of observables is then a set of observables that suffice to determine in which material state the learner is at any moment during the activity.

We also need a way to characterize the learner's internal representation. When choosing observables for this, this is more difficult. Observables are features that must be perceivable by an external observer. They are more subject to experimenters' interpretation. Defining the learner's observables is a challenge, and deciding what is important to observe and what is not is the main questioning that must be answered when designing an experiment.

For instance, let's come back to the simplified problem of orientation. There is no need to observe the position of the walls during the activity, as their position will not change. However, the position of the subject in the labyrinth is relevant to observe (and their position only is enough to determine in which material state the learner actually is). The set of observables is, in this case, composed only of the position of the learner. The observables relative to the learner are more difficult to infer. Our proposition, for this experiment, is to consider if the learner looked at the map or not. For an interpretation, a learner that looked up the map would be considered as someone who knows the entire labyrinth, while a learner that did not look it up would be considered as someone who knows only the places in the labyrinth that they visited.

COMPARING FOG OF WARS With a perfect definition of observables relative to the learner, we would be able to understand the learner's internal representation. However, it is not possible right now to do that : it would require the understanding of human learning, which is exactly what we aim for in this kind of research. Our definition of the observables relative to the learner are then assumptions about human learning. This is where the fog of war analogy can help, by comparing two different kind of fog of wars : the "real" fog of war and the "supposed" fog of war.

The real fog of war is the one that is deduced by the internal representation of the learner. As we cannot access it, our observables are able to generate another kind of fog of war, based on the hypothesis of the research. Let's consider the orientation problem : In Figure 2.4, we proposed in the fifth step that the learner may have forgotten the map that they discovered. As our only observable is if the learner looked up the map or not, we are unable to characterize what parts of the labyrinth they did forget. Two person with the same observable, one with forgetting and the other without, will then not behave the same way. Thus, characterizing, for instance, the moments when the learner stops (to think, or try to remember) may be a relevant observable to add.

DISCUSSION : HOW CAN THIS MODEL BE USED ? In the last sections, we created, step-by-step, a model of representation of ill-defined problems and problem spaces, starting from the largest considerations up to a reasonable observable space that we could use in experiments. Which kind of questions can this model help to answer ? We propose here some use-cases of this model :

- **Testing relevant observables** When defining relevant observables, observations of multiple experiments can allow to determine whether an observable has an influence or not on the behaviour. It can be a material observable or not. For instance, if an observable is considered but the value of this feature has no influence on the task performance, this can be an indicator that the observable is not relevant.
- **Associating observables with behaviours** Doing a classification of outcomes or behaviours can lead to have interesting results if specific observables are present in a category and not in the other. This can lead to a better understanding of the reasons why we adopt behaviours. This use-case will be developed in section 3

- **Characterizing oversight** Observables can help to identify when there was an oversight of the learner. This may help to characterize oversight in general. This will be developed in section 4.

With this model in mind, we will now develop the methods and algorithm we will use to generate experiments on ill-defined problem solving.

2.3 OPERATIONALIZING ABSTRACT PROBLEM-SPACES

2.3.1 DEFINING DISTANCES IN SYMBOLIC DATA STRUCTURES

We need to define a **distance** between different states of a problem space.

A distance is a function d that satisfies three properties :

- **separation** : The distance between two different states is always strictly positive, and if the distance between two states is null, then those two states are identical.
- **symmetry** : The distance between state x and y is the same as the distance between state y and state x , i.e., moving from x to y is the same as moving from y to x .
- **triangle inequality** : Moving from x to z and then z to y , instead of moving directly from x to y , can not shorten the path distance.

For instance, the shortest time to go from one city to another by car is a distance. The shortest time to go from one city to another by walking is also a distance. They are different, but are defined on the same space. This is coherent with our common conception of this concept.

This can be applied to abstract symbolic problem spaces. An usual distance for this is the editing distance. The editing distance is the number of different editions necessary to transform one state into another. For instance, with one letter change and one letter addition, we can transform **power** into **flower**. The edit distance between power and flower is then 2 : $d(\text{flower}, \text{power}) = 2$

The pivotal idea of a geometric definition of problem-spaces is to consider an abstract state space where each point is a symbolic data structure, representing contextual information about the physical space, and even about the agent's internal state. Performing an action corresponds to deciding, as a step in the problem-solving process, to modify some characteristics of the state space both at the external level (e.g., moving

an object) and at the internal level (i.e., modifying the internal representation).

Based on the framework defined in the previous section, this section will consider the observer problem-space of section 2.2.3. Before applying the fog of war analogy to represent player knowledge, we need first to consider the data structure of the material environment of the problem-space. This environment is a symbolic data structure : a state in this structure is composed of multiple symbolic information, linked (or not) to numerical values.

As a symbolic data structure, there is no direct way to operationalize this data structure. The operationalization requires to be able to compare multiple states of this data structure. This can be done by the definition of a **distance** between different states.

A distance is a function d that satisfies three properties :

- **symmetry** : The distance between state x and y is the same as the distance between state y and state x :

$$d(x, y) = d(y, x)$$

- **separation** : The distance between two different states is always strictly positive. If the distance between two states is null, then those two states are identical.

$$d(x, y) = 0 \iff x = y$$

- **triangle inequality** : It is impossible to reduce the distance between x and y by stopping by any other state z . Adding a step cannot shorten the path :

$$d(x, y) \leq d(x, z) + d(z, y)$$

Defining a distance is a key process to represent path constraints of the environment. When finding a path from one state to another, path constraints are factors of distance increase. Having a properly defined distance allows the existence of a metrizable symbolic data structure, giving access to a notion of proximity : a state easily attainable from another translates into a small distance between these two states, whereas states distant from each other (higher distance) are separated by path constraints and are not easily attainable from each other.

There is no such thing as an unique distance. To take the example of maps, shortest time to go from one city to another by car is a distance,

while the shortest time to go from one city to another by walking is also a distance. They are different, but are defined on the same space.

Distances are usually used on numerical spaces like the euclidian distance. For symbolic data structures, we can define distances that are relevant for particular problems. However, there is a well-known distance that we can apply on most (if not all) problem spaces : the **edit distance**. An edit distance between two symbolic states x and y is the minimal number of operations to transform x into y . Possible operations are :

- **Addition** : Adding a symbol to the state
- **Suppression** : Removing a symbol from the state
- **Edition** : Changing a symbol into another. This is not necessary to include, as it could be represented by a suppression of the old symbol and the addition of the new symbol.

For instance, if we consider a word as a symbolic state, with symbols being letters, we can transform **power** into **flower** by doing:

- A transformation of **p** into **l**, creating **lower**;
- An addition of **f**, creating **flower**.

Thus, the edit distance is : **$d(\text{power}, \text{flower}) = 2$** .

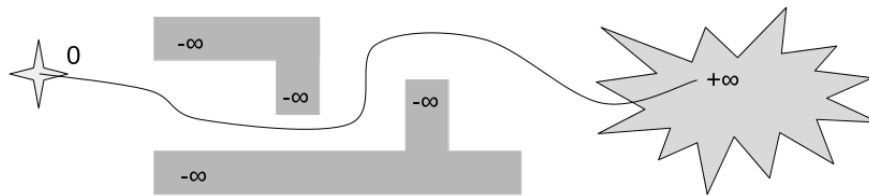
For symbolic structures, it may still be easier sometimes to convert a symbolic space into one or more numerical values. In this case, the notion of distance can obviously still be defined. However, it is important to know whether the multiple numerical values of a state are **dependent** (The modification of a value affects others) or **independent** (Each value may evolve separately without affecting each other), or a mix. We will discuss this case in section 2.4.1.

With the notion of distance can come the notion of trajectory, as a way to represent problem-space exploration (and so problem-solving process).

2.3.2 PROBLEM-SPACE EXPLORATION ON METRIC SPACES

A trajectory is a way to represent the advancement of a subject over time on a problem. It is a sequence of states from a problem space. If subjects act differently during the problem-solving process, their trajectories will differ. This is done on a metric space, that can be numerical or, as defined with the editing distance, symbolic.

If we put reward on the goal states, and negative rewards to constraints, we can mimic problem-solving trajectories. We can create algorithms that solve the problem : the algorithm tries to move from state to state, in order to reach the maximal possible reward. It will naturally avoid negative rewards and move towards the goal :



Starting with Newell (1981b), a general problem solver can be stated in the form of a trajectory generator, including at a biologically plausible algorithmic level (Viéville, 2002), including hierarchic in terms of scale and precision as developed by Pizlo (2006). Such representation allows to represent a conceptual space in the sense of Gärdenfors (2004).

Such complex problem is solved in several steps by generating sub-goals on the trajectory, as discussed in details in Alexandre et al. (2024). The open-ended goal is not well defined but simply constrained by certain requirements, making the final state not unique but corresponding to a region. Such a goal and obstacles are not known from the beginning while discovered during exploration, while the initial state is only partially known (and observable) to the learner, and there is no explicit operation to move from one state to another, but potential actions are discovered by the learner when interacting with the environment.

The key point is that such a formalism yields to a universal algorithm to solve a problem formalized that way, considering a reward null at the initial state, unbounded and negative of obstacles and unbounded and positive when a goal is reached, i.e., scalar-field, written:

$$V(x) \stackrel{\text{def}}{=} \sum_i V_i(x) - \sum_j V_j(x), V_i(x) \stackrel{\text{def}}{=} \frac{\omega(x_i)}{\delta(x, x_i)}, \omega(\cdot) > 0$$

where x_i and x_j are points respectively sampled by projection on the goal regions and on the obstacles or constraints regions by the algorithm. Here, $\delta(\cdot)$ is an editing distance as developed previously. Goals can be weighted by $\omega(\cdot)$, i.e., given with a certain reward value that corresponds to their importance. Similarly, obstacles can be weighted, i.e., given a certain repulsion value that corresponds to the importance to avoid them. We always can choose $\omega(\cdot) = 1$ as the default value, if there is no benefit to introducing a priori information at this stage.

In order to reach a goal from a given position, taking constraint into account, a more generic method is to consider a **reward potential**, i.e, very high on the goal position or the goal region, say $+\infty$, and very low when on obstacle or where a given constraint is not satisfied, say $-\infty$. In this context, the notion of obstacles to avoid is equivalent here to path constraints to apply: The former is a geometric interpretation of the latter. Then finding a "global plan", i.e. a trajectory, from any initial position towards a goal location, reduces to locally maximizing the reward potential, thus moving towards the goal while avoiding the obstacles.. Usually we consider the minimization of such potential, thus the opposite potential (i.e., $-V(x)$ instead of $V(x)$) but, here, we consider the potential as a reward thus increasing when approaching the goal.

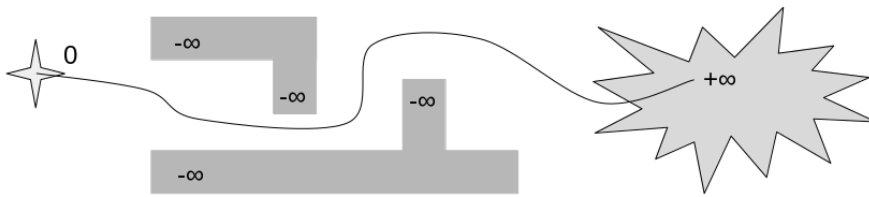


Figure 2.5: A valid trajectory from an initial state to some final state satisfying path constraints.

In a numerical space, given a set of points $\{x_1 \cdots x_i \cdots x_I\}$ on the goal regions and points $\{x_1 \cdots x_j \cdots x_J\}$ on the obstacles, we can define the reward potential as

$$V(x) \stackrel{\text{def}}{=} \sum_{x_i \in \{Goals\}} \frac{1}{\|x - x_i\|^d} - \gamma \sum_{x_j \in \{Obstacles\}} \frac{1}{\|x - x_j\|^d}$$

with $d > 0, \gamma > 0$

with obviously $\lim_{x \rightarrow x_i} V(x) = +\infty$ and $\lim_{x \rightarrow x_j} V(x) = -\infty$, i.e., is infinitely high on points belonging to the goal and infinitely low on points belonging to the obstacles.

Given such a reward potential, maximizing this potential drives toward the goal, and avoiding obstacles.

Given this variational formulation, solving the problem reduces to the global maximization of this scalar field from the symbolic data structures to the reward values. Such a potential corresponds up to a constant to the reward represented in Fig. 2.5, while it attains a maximum on a goal state, maximization avoiding obstacles. It also means that a gesture or a plan is within this framework encoded by the goal and constraints, as it is the case in the hippocampus for navigation or in arm gesture as reviewed by Höge (2020), showing that such representation is biologically coherent.

We talk about reward here because the potential is directly linked to reinforcement learning. However, our formalism is not like reinforcement learning based on the optimization of an average gain but on obtaining a maximum deterministic gain which is estimated better and better as the generation of the trajectory. It is more to be linked to concepts of optimal control more effectively at such a level of abstraction (Viéville, 2002).

We thus have a general mechanism of trajectory generation in a symboling space. A demonstration of such mechanism is accessible at:
<https://line.gitlabpages.inria.fr/aide-group/symboling/Trajectory.html>

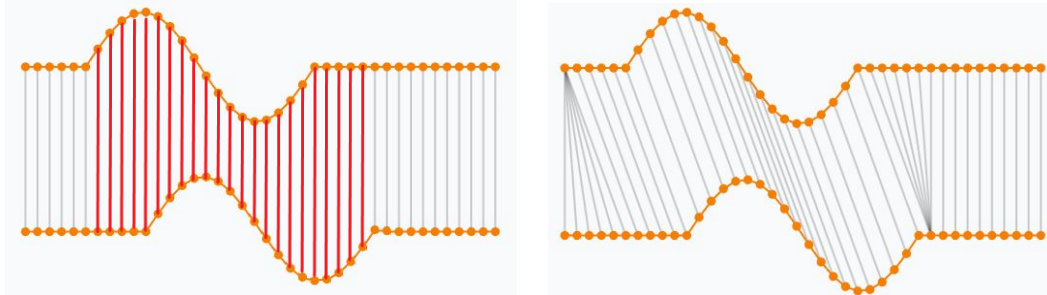
2.4 DYNAMIC TIME WARPING AND CLUSTERING AS A METHOD OF BEHAVIOUR CLASSIFICATION

2.4.1 DYNAMIC TIME WARPING : EXTENDING DISTANCE OF STATE TO SIMILARITY OF SEQUENCE

We want to compare trajectories instead of points. For this, we create a measure of **dissimilarity**. If the value is low, trajectories are similar. If the value is high, they are very different.

A basic way to do it is to compare each trajectory, point by point. This is called the Euclidian match. However, this can lead to similar trajectories, with minor time differences, with a very high dissimilarity. We want to avoid that.

The measure of dissimilarity we use is called **Dynamic Time Warping**. It can associate one point with multiple points from the other sequence. In comparison, the left is the Euclidian match and the right is Dynamic Time Warping :



Dynamic Time Warping always tries to match different points with the following rules :

- The first state of each sequence are matched together.
- The last state of each sequence are matched together
- Each state must be matched with one or more states from the other sequence
- A state from sequence 1 cannot be matched with a state i from sequence 2 if a previous state of sequence 1 was already matched to a state j after state i in sequence 2.

In the previous section, the definition of a distance in a symbolic state space allows us to generate **trajectories**, representing a path towards the goal state(s). As such, a trajectory is a sequences of states from the corresponding data structure. We consider in this section that we

defined an adapted metric of distance between states from the symbolic representation of the observer problem state.

One of our goals in this research is to be able to classify behavioural data on multiple subject's trajectories. A subject's trajectory is the result of their actions over the course of an activity, and can represent a good abstraction of that subject's behaviour(s) performed during this activity. There are a lot of multiple classification algorithms. Clustering techniques allow to create groups of data points based on measures of distance between those points. Classifying trajectories, however, requires to consider trajectories as data points. Thus, the notion of distance must be extended from points to sequences of points. To be more precise, we need to build upon a distance between points in order to create a distance for trajectories.

As we are mainly working with trajectories performed by humans, we need to take into account multiple elements considering unavoidable differences between subjects. In particular, subjects may perform the same actions, but with a difference in execution speed. To account these differences, we are choosing to use **Dynamic Time Warping**, as presented in "Dynamic Time Warping" (2007).

Dynamic Time Warping (DTW) is an algorithm used for measuring similarity between two temporal sequences which may vary in speed ("Dynamic Time Warping", 2007). This is important to compare behaviours, as we want to ignore differences due to execution speed of subjects' actions. The DTW method consists on finding an optimal match between two given time sequences by associating points from a time sequence to one or more points from the other sequences, with some restrictions :

- The first state of each sequence are matched together.
- The last state of each sequence are matched together
- Each state must be matched with one or more states from the other sequence
- A state i from sequence 1 cannot be matched with state j from sequence 2 if a state k before state i in sequence 1 was already matched to a state k after state j in sequence 2.

With these restrictions, it is possible to create a lot of different matches between two sequences. We are interested in the optimal match. The

optimal match between two given time sequences is then the mapping of associated points that satisfies all of the previous restrictions and has the minimal cost of satisfying mappings, the minimal cost being the sum of absolute differences between values of each paired points.

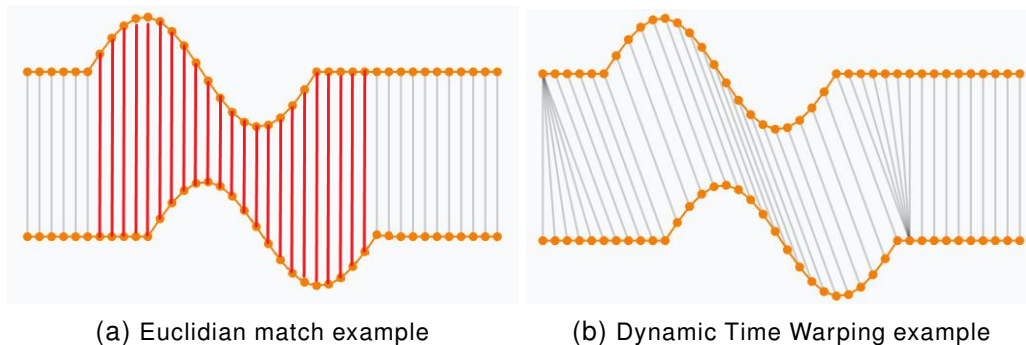


Figure 2.6: Different matching between two sequences shifted in time. Credits to Tavenard (2021)

Let us compare, Figure 3.5, the difference between Euclidian match and Dynamic Time Warping for 2-dimensional sequences. By matching only points of the same timestamp value, Figure 2.6a associates multiple points (associations in red) that are different in value, leading to a bigger total distance than with Figure 2.6b.

The main benefit of using a dynamic matching algorithm like DTW is that it is able to take into account little variations between two similar sequences. Another benefit is that this method allows to compare trajectories of different total time.

We will now detail the algorithm for the dynamic time warping algorithm. Let us consider two sequences $x = x_1, \dots, x_n$ and $y = y_1, \dots, y_m$ of respective lengths n and m . We also consider a distance d between two points of the state space from which sequences x and y happened.

DTWDistance (x : array [1...n], y : array [1...m]):

```

DTW = matrix [0...n, 0...m]
for i in range 0 to n:
  for j in range 0 to m:
    DTW[i, j] = infinity

DTW[0, 0] = 0
for i in range 1 to n:
  for j in range 1 to m:
    cost = d(x[i], y[j])
    DTW[i, j] = cost + min(DTW[i-1, j], DTW[i, j-1], DTW[i-1, j-1])

return DTW[n, m]
```


In this algorithm, $DTW[n, m]$ is the total distance between x and y with the best alignment. More precisely, $DTW[i, j]$ is the distance between subsequences $x' = x_1, \dots, x_i$ and $y' = y_1, \dots, y_j$. This algorithm dynamically computes the best possible match, but doesn't get the optimal set of matches. This can be done by knowing which option was chosen for the minimum for the update value, respectively insertion, deletion and match.

This algorithm is entirely based on the proper notion of distance, in particular for symbolic data structures. As previously noted, we will sometimes want to convert symbolic states into one or more numeric values. Thus, a trajectory may be composed of multiple numerical trajectories. Note that the same argument can be done for symbolic states, but the numerical case corresponds to applications that will be explained in chapters 3 and 4.

Let us consider a trajectory as a set of multiple time sequences. It is important to adapt Dynamic Time Warping to multiple time sequences, see Stübinger and Walter (2022) or Olivares-Atarcos et al. (2019) :

- **If sequences are independant**, which means that no modification on one sequence has any effect on another, then we can sum Dynamic Time Warping on each individual pair of sequences from the pair of trajectories to get the general Dynamic Time Warping result on two trajectories.
- **If sequences are dependent**, which means that a modification on one sequence can effect another of the same trajectory, then we need to apply Dynamic Time Warping the complete trajectory : The distance used for it is then a distance that can compare pairs, triplets, etc. instead of individual values.

For instance, if we consider two points $x_i = (x_{i,a}, x_{i,b}, x_{i,c})$ and $y_i = (y_{j,a}, y_{j,b}, y_{j,c})$ from two sequences (on which we apply DTW), for independent DTW, we'll use separately $d_a(x_{i,a}, y_{j,a})$, $d_b(x_{i,b}, y_{j,b})$, $d_c(x_{i,c}, y_{j,c})$, while dependent DTW would require $d(x_i, y_i)$.

To sum up, Dynamic Time Warping is a way to compute similarity between temporal sequences. The similarity score can be then used as a measure of proximity between these trajectories : the higher the similarity, the more we can consider that two trajectories should be classified

in a same group in classification approaches. Amongst different classification methods, we opted for k-medoids clustering.

2.4.2 K-MEDOID CLUSTERING

K-medoids clustering is a clustering algorithm that creates k groups of points in a dataset based on how close they are to each other. It is an incremental algorithm consisting of multiple steps :

- Step 0 : Decide of a number of groups (the value of k) and choose k arbitrary points in the dataset. They are the **medoids**, one for each k groups.
- Step 1 : For each point of the dataset, compute the distance between the point and each medoid. Put the point in the group corresponding to the closest medoid: This is the medoid with the minimal distance to the point.
- Step 2 : For each k group, compute the distance of each point with each other point in the group. Then, for each point, compute the sum of every distance between the point and the other points. The point with the minimum sum is the new medoid of the group.
- Step 3 : Repeat step 1 and 2. Stop if medoids do not change anymore or after a predetermined number of repetitions.

With this algorithm, each group updates its most central point after each cycle. By updating its central points, a point that was previously in another group may be closest to a new central point and then change its group. When medoids do not change anymore, groups can't change either.

This algorithm requires the use of points and an appropriate way to compare them. For the clustering of behaviours, we use trajectories as points (each point represent one subject solving the problem). We use dynamic time warping as a measure of similarity to determine which trajectory medoid is the most similar.

AVERAGE SEQUENCE VERSUS MEDOID SEQUENCE In classification methods, it is often interesting to look at what could be a representative data point for each group. For trajectories, it corresponds to a "prototypical" trajectory, that is the sequence that represents the group. For behavioural

analysis, one goal can be to associate different "prototypical" trajectory from different groups with different behaviours.

The choice of the "prototypical" trajectory can be discussed. We can mainly consider two possibilities : an average sequence or a medoid.

An average sequence would be an hypothetical trajectory (not necessarily present in the dataset), based on the "mean" of data points from one cluster. This is easy to grasp for numerical sequences : for instance, when comparing two time sequences of the same length, we can consider the average sequence as the sequence resulting of the mean of each pair of points at the same time.

When considering Dynamic Time Warping, Petitjean et al. (2011) proposed an algorithm called Dynamic Time Warping Barycenter Averaging (DBA) in order to iteratively compute an average sequence with DTW, on which it is possible to apply k-means clustering as in Ferreira et al. (2012). The method is the following :

The algorithmic method then consists of the following steps : (Initialisation) : Create an initial average sequence A. This sequence can be random, preprocessed or even a sequence from the corpus For each sequence S from the corpus, compute the DTW algorithm between S and A. For each point a from A, create a set $s(a)$ containing all points associated with a from all sequences from the DTW algorithm. For each point a from the average sequence A, compute the barycenter of all points from the $s(a)$ set, coordinate by coordinate. The resulting barycenter becomes the new point a, thus modifying all points from sequence A. Repeat 2 and 3 until the average sequence A is stable (does not change during step 3) or until a determined number of iterations. (End) A is the average sequence.

The method is discussed in details in Palaude and Viéville (2023). The main flaw of this method is its application to symbolic data : the barycenter of multiple symbolic value doesn't have a clear meaning. One explored solution was a temporary translation into numerical values. However, this leads to explainability problems : An average sequence contains data-points that may not belong to a valid state (not existing at all, or belonging to forbidden states by path constraints).

As such, a medoid approach seems to be better-suited. A **medoid** is a representative point of a data set, while being a point from the data set itself. It is the datapoint whose sum of dissimilarities to all the data

points in its cluster is minimal. In other words, for $X = x_1, \dots, x_n$ a data set and d a dissimilarity measure (including DTW), the medoid is defined as

$$x_{medoid} = \underset{y \in X}{\operatorname{argmin}} \sum_{i=1}^n d(y, x_i)$$

As it is a point from a dataset, there is not the same problem of explainability. For dataset of individuals performing problem-solving tasks, a medoid is the trajectory resulting of one individual.

K-MEDOIDS CLUSTERING **K-medoids clustering** is a process of partitioning dataset into k groups in order to solve k -medoid problems (Kaufman and Rousseeuw, 1990). Multiple heuristic solutions exist to create such partitions. We are going to present one of them, based on the creation of a **distance matrix**, as presented, e.g., in Park and Jun (2009):

- Step 0 : Create distance matrix M that keeps all the similarity values between each pair of points : For x_i and x_j two points in the dataset, $M_{i,j} = d(x_i, x_j)$
- Step 1 : Decide of a number of groups (the value of k) and choose k arbitrary points in the dataset. They are the initial **medoids**, one for each k groups.
- Step 2 : For each point of the dataset, using the distance matrix M , put it in the group corresponding to the closest medoid: This is the medoid with the minimal value to the point.
- Step 3 : For each point, compute the sum of every distance between the point and the other points. The point with the minimum sum becomes the new medoid of the group.
- Step 4 : Repeat step 2 and 3. Stop if medoids do not change anymore or after a predetermined number of repetitions.

With this algorithm, each group updates its most central point after each cycle. By updating its central points, a point that was previously in another group may be closest to a new central point and then change its group. When medoids do not change anymore, groups can't change either. Note that this algorithm, as an "alternating" heuristic, tends to produce sub-optimal results, mainly because the search space is smaller

(medoids will slowly move towards their position during each step), it is greatly influenced by which points are chosen as initial points.

Other methods, such as Partitioning Around Medoids (PAM) by Kaufman and Rousseeuw (1990), are usually more optimal, but the runtime complexity is too important for bigger dataset. This is technically not a problem for our datasets in general. For instance, datasets from chapters 3 and 4 are composed of maximum 200 points. However, we are not necessarily interested in optimal matching. It is mainly due to the fact that our datasets are, in fact, usually small. An optimal matching may not be in our best interest. Our goal is not to determine precisely which sequence belongs to which group, but instead to try to interpret such groups for educational research. As such, we are more interested in groups in themselves, but rather in their relations to each other.

This is the same reason why an exhaustive search is possible (and will be done in the next section) but not to obtain the optimal clusters.

2.4.3 META-CLUSTERING

Clustering algorithm are better suited to large datasets. For smaller datasets, we use **Meta-clustering** for more robust results. Robust, in this case, means that we are less likely to get clusters that correspond to unwanted particular cases.

- We create a **meta-matrix**. This is a matrix of n lines and columns, n being the number of points in the dataset. Each cell of the matrix is initialized at 0.
- We apply a **k-medoid algorithm** with a predetermined set of initial medoids.
- We update the meta-matrix. Given two points i and j from the dataset, if i and j are in the same cluster we increase $M(i, j)$ by one. Otherwise, we do nothing.
- We repeat Step 2 and 3 until we applied the k-medoid algorithm on all possible initial medoids.

In the final matrix, if two point i and j were clustered together often, $M(i, j)$ will be a high number. If they were not, $M(i, j)$ will be a low number. If an execution of the k-medoid algorithm gives an unwanted particular case, the increase it implies is negligible in the final matrix. We can then apply a clustering algorithm on the dataset with a new measure of similarity : The higher the score of two points in the meta-matrix, the closest they are to each other.

One of the main problems of our clustering method is the different results obtained when having access to different initial points. The **meta-clustering** approach, as inspired by Zeng et al. (2002), or Caruana et al. (2006)), takes into account the fact that different clustering methods will give different results. The idea is to consider multiple unique clustering solutions as an ensemble from which we can create a **similarity metamatrix**. This matrix contains, for each pair of observations, the probability of being in the same cluster.

We propose to define our own metamatrix based on the application of k-medoids clustering on all possible initial points (with a fixed value for k).

For this, we adopt the following method :

- Step 0 : We create an empty **metamatrix**. This is a matrix of n lines and columns, n being the number of points in the dataset. Each cell of the matrix is initialized at 0.
- Step 1 : We apply a **k-medoid algorithm** with a set of different initial medoids.
- Step 2 : We update the meta-matrix. Given two points i and j from the dataset, if i and j are in the same cluster we increase $M(i, j)$ by one. Otherwise, we do nothing.
- We repeat Step 2 and 3 until we applied the k-medoid algorithm with a new and different set of initial medoids, until all possible initial medoids are treated.

In the final matrix, considering two point i and j were clustered together often, $M_{i,j}$ corresponds to the number of times i and j were clustered together among the different initial medoids. If they were not, $M_{i,j}$ will be a low number, up to 0 if i and j were systematically clustered in different groups.

We can convert M into a real distance metamatrix by associating the number of each cell to the "proximity" of each point, with the simple operation :

$$M'_{i,j} = 1 - \frac{M_{i,j}}{M_{i,i}}$$

This operation transforms the metamatrix into a similarity metamatrix. We are not calling it a distance, as at least the separation is not verified. On one hand, if an execution of the k-medoid algorithm gives an unwanted particular case, the increase it implies is negligible in the final similarity metamatrix. This metamatrix is also a percentage metamatrix : Based on all iterations, given that we choose the initial medoids randomly, $M_{i,j}$ gives the probability that points i and j are not in the same cluster by the end of the execution.

As we want to interpret such similarity matrix, we can apply hierarchical clustering on it to create a hierarchy of clusters. However, this requires to transform the similarity metamatrix into a proper distance metamatrix. This can be done by converting the matrix into a square-form distance matrix.

For more details on the implementation on both the square-form transformation and the hierarchical clustering implementation, see the associated **scipy** modules : `spatial.distance` and `cluster.hierarchical`

As an illustrative example, to apply hierarchical clustering, we can use a bottom-up approach :

- Each data point starts in its own cluster
- Until each data point is in one cluster, compute the distance between each clusters, for instance by considering the minimal distance between points from each cluster. The two nearest clusters are merged with each other.

Example of hierarchical clustering results will be discussed in chapters 3 and 4.

To sum up the different algorithms we presented, our proposed method for analyzing datasets of subject's trajectories on symbolic state space is the following :

- Using **dynamic time warping**, we measure the similarity between the different trajectories. Dynamic time warping allows to measure similarity of sequences that may vary in speed. This allows to ignore execution speed differences of subject during problem-solving tasks.
- Using a **k-medoid clustering** algorithm for a set value of k , we create an arbitrary number of groups of similar sequences, based on the similarity given by the dynamic time warping algorithm.
- We apply **metaclustering** in order to reinforce the robustness of our result. It creates clusters of trajectory based on a similarity given by how often two trajectories were clustered together in during multiple executions of the previous step.

We designed our method as an observational method : our goal is to get different representations of datasets in order to make observations about the data or the model. For instance, we can apply the method on sub-trajectories. Sub-trajectories are parts of a trajectory. For instance, if we cut a trajectory halfway through the task, we have two sub-trajectory for each subject : first half and second half. In chapter 3, we cut trajectories at different points and try to create clusters on these sub-trajectories, in

2 A model of problem-solving representation

order to determine if the cutting point is a moment that imply a shift in the strategy of most subjects or not.

In order to show potential pros and cons of the method, we will use it on two different problem-solving activities, in chapters 3 and 4. The code of our work is available at <https://gitlab.inria.fr/line/aide-group/creadata>

3 CREACUBE : AN ILL-DEFINED PROBLEM-SOLVING TASK

3.1 PRESENTATION OF THE ACTIVITY

Creacube is an ill-defined robotic game-based activity : Four cubes are presented on a table with a black point and a red point on it. The instructions are repeatable at will : **Build a vehicle made up of four pieces that moves by itself from the red point to the black point.**

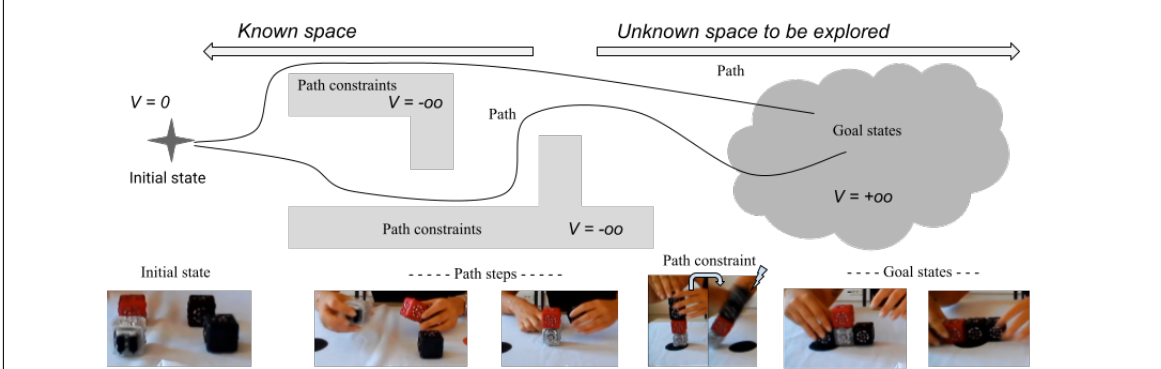
This is a creative problem-solving task. The subject assembles the cubes in order to create a vehicle. Each cube has a different property, unknown to the subject at first. They are also magnetic and can be assembled.

Their roles and names are :

- **Wheels:** the white cube has wheels that can be powered up to move.
- **Battery:** the blue cube has a switch that power the other connected cubes.
- **Sensors:** the black cube has sensors that look like eyes that send a value to the wheels to make them move.
- **Inverter:** The red cube changes the value sent by sensors depending on its position in the four-cube configuration.

Only the first three are necessary to create a vehicle but the fourth must be added to succeed. Four visible features can be discovered in this activity. They are called **affordances** and they suggest how cubes can be used : magnets on all cubes, switch on battery, wheels on wheels and eyes on sensors.

A problem space representation of this task may be as follows :



CreaCube is an ill-defined robotic game-based activity (Leroy et al., 2021; Romero, DeBlois, and Abadjyan, 2018). Building with modular bricks is based on visual constructive play objects (Ness and Farenga, 2007). They are used to make learners engage into tangible programming brick games (Kalmpourtzis and Romero, 2020; McNerney, 2004) with modular educational robots (Leroy et al., 2021; Romero, 2019). Robotic elements will influence the actions of the participant (Jamone et al., 2016) towards the objects, leading to observation, assembly, etc. In the context of an ill-defined problem solving task, these legal operators are not specified beforehand.

In the Creacube task, the participant is invited to manipulate four cubes of different colors. Each cube has different properties such as wheels or sensors, but these technological properties are unknown to the subject when starting the activity. The goal of this CPS task is to build a vehicle composed of four pieces that can move autonomously from one point to another, without any further initial information about the problem.

Figure 3.1 shows the initial configuration of the activity : Four different cubelets (red, white, blue and black) are presented on a table (with a black point and a red point on it) to a player. The instructions, repeatable at will, consist in one sentence : "build a vehicle made up of four pieces that moves by itself from the red point to the black point".



Figure 3.1: The initial configuration of the Creacube task

In this task, operators are suggested by the physical features of the artifacts. For example, the blue cube has a switch on one of its faces, appealing to be activated. Gibson (1977) defined such suggestions given by the environment as **affordances**. Norman (2013) adapted the con-

cept of affordances to human-computer interaction to refer to "action possibilities" that are perceivable by the subject. In Norman's definition, affordances depend not only on the physical capabilities of the subject, but also on their goals, beliefs, and past experiences. Through their interactions with the environment, the learner relates the perceived physical properties of the artifacts to prior knowledge (e.g., "a switch can be flipped to activate a device") and generates new hypotheses. Affordances are a way to make sense of stimuli, and a key point in the creative process is to be able to think of different appropriate ways to use the material to solve the problem, using mechanisms such as analogy to transfer knowledge from a task to another (Alexandre, 2020; Guilford, 1967). Considering the models of creativity developed in section 1.2, we can associate this to the preparation phase, whereby one recognizes a problem from both the external stimuli and the internal goals and prior knowledge, thus combining both topdown and bottom-up mechanisms. As for the Creacube task, each cube has a set of affordances. One of them is common to every cube : they have magnetic faces. Magnets suggest the way to connect the cubes with each other, by putting one magnetic face on another. In addition to this affordance, we can focus on each cube individually :

- The white cube has wheels and can receive electrical power to move. The main affordance here is the wheel, suggesting the way the vehicle can move. An example of action to take when discovering this affordance is to put the white cube on the table, wheels on the table.
- The blue cube has a switch that can power other connected cubes. The main affordance here is the switch, suggesting the activation of the battery.
- The black cube has sensors that look like eyes. Two different but similar affordances were identified for this cube : sensors and eyes. They serve a similar effect but depending on the subject, the "eyes" were not identified as such but directly as "sensors", leading to positioning of the sensors at different positions than, for instance, the front suggested by the "eyes". However, the "eyes" affordance is here a slightly more precise affordance than sensors, and discovering it is often equivalent to sensors (see section 3.2).

- The red cube doesn't have any affordance other than the magnets. It is the only cube with all 6 faces with magnets. That said, it has an effect on the structure.

In the following sections, we will refer to these for cubes by respectively **Wheels**, **Battery**, **Sensors** and **Inverter**. For a structure to properly function, the Battery switch needs to be activated to power the wheels. The Sensors, depending on its distance to an obstacle, sends a signal to the Wheels, making the wheels move with a corresponding speed. The behaviour of this can be altered by the presence of the Inverter.

As for the problem space of this activity, we have here an example (illustrated Figure 3.2) of a problem structure with one initial state and multiple goal states (i.e. structures of four that respect the conditions) with different path constraints corresponding to different types of problems (imbalance, unactivated battery etc.).

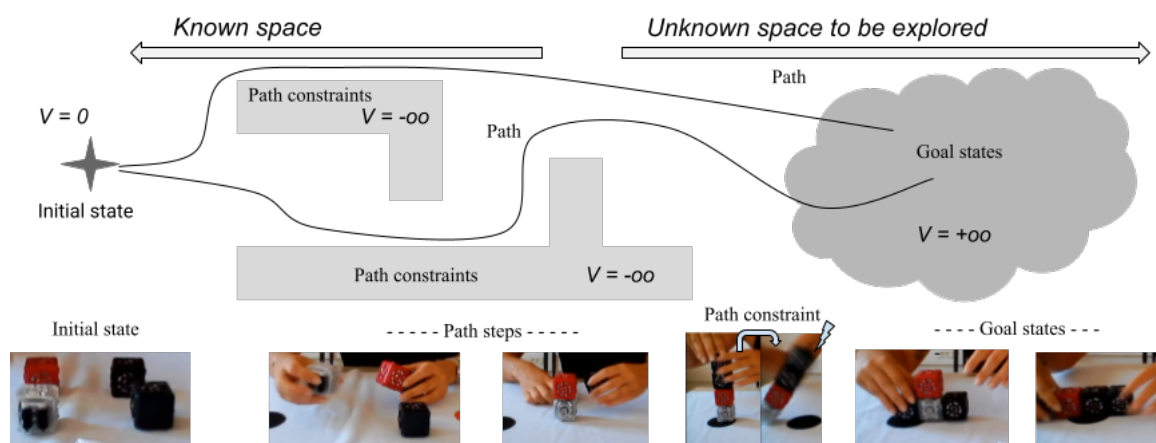


Figure 3.2: An abstract representation of the problem space of Creacube

We can use the CreaCube activity in order to study some transversal competencies, also known as 21st-century skills, including problem solving, collaboration, creativity and computational thinking (Romero et al., 2017)

We aim to use the method presented in chapter 2 on the Creacube activity. We aim to use it as an observation experiment to show the capabilities and limits of the metaclustering method on the activity.

3.2 DATA COLLECTION

To assess the creative process, we adapt criteria of Guilford (1967) into a triplet:

- **Fluidity**: The number of final structures done by the subject;
- **Flexibility**: The number of conceptually different ideas, here the number of different shapes ;
- **Innovation**: The number of uncommon ideas amongst participants. Uncommon shapes are done by less than 5 percent of subjects.

Following Mercier (2022), we have a set of observables for Creacube:

CREACUBE		2. Activity							
AS00		F000	F002	F010	F011	F012	F013	F014	F020
AS01		F021	F022	F024	F030	F034	F040	F042	F044
AS02		F050	F051	F060	F061	F064	FXX		
AS03									
F000-SBIW-T	F000-SIWB-T	F000-SWBI-T	F000-BSIW-T	[...] (every permutation is encoded for each shape)				Example: F020-IBWS-T	
F000-BIWS-T	F000-BWSI-T	F000-IWBS-T	F000-IBWS-T					P01. Imbalance	
F000-ISWB-T	F000-WBSI-T	F000-WSBI-T	F000-WIBS-T					P02. Rotation	
U00. Play instructions	U01. Stop playing instructions	U02. Question instructions	U03. No cubes in hand (no manipulation)	U04. Hands up with 1 cube	U05. Hands up with 2 cubes	U06. Hands up with 3 cubes	U07. Hands up with 4 cubes	FL01 Rotate the cube without changing its position	P03. Wrong direction
E01. Ecstasy/Joy/Serenity	E02. Admiration/Trust/Acceptance	E03. Terror/Fear/Apprehension	E04. Amazement/Surprise/Distracted	E05. Grief/Sadness/Pensiveness	E06. Loathing/Disgust/Boredom	E07. Rage/Anger/Annoyance	E08. Vigilance/Anticipation/Interest	FL02 Reposition the cube into the same configuration	P04. Reverse (outward)
A01. Wheels	A02. Magnets	A03. Switch	A04. "Eyes"	A05. Sensors	S01	S02	T01. No test	T02. Drop Out / Abandon	P05. Reverse (to the person)
									P06. Colour association
									P07. Connection
									P08. Does not move (wheels)
									P09. Does not move (switch)
									P10. Does not move (sensor)
									P11. Does not move (inv)

At a given moment, only some observables are activated. We are interested in two observables in particular :

- **F** observables are all **final structures**. Only one can be activated at a time. We can associate an observable with a color code corresponding to the ordering of the cubes in the structure.
- **A** observables are all **affordances**. They are activated from the moment they are discovered for the first time.

A sequence of a subject will be a sequence of these triplets with increasing values.

Different tests have been developed to assess creativity, like the Alternative Uses Test (AUT) (Guilford, 1967), the Remote Associates Test (Mednick, 1968) or the Torrance test of Creative Thinking (Torrance, 1974, Torrance, 1990). They were created to assess different aspects of creativity.

The Torrance Test of Creative Thinking (TTCT) is one of the most used creativity assessment tools (Cramond et al., 2005, Besançon et al., 2011) without being an universally praised model, with Almeida et al., 2008, for instance, pointing out the format, content and demand on TTCT specific tasks to explain some factors of the test, while Kim, 2006 critiquing the originality measure.

The Torrance Test of Creative Thinking consists on sets of verbal and figural tasks, with stimuli or not. For instance, a task in the TTCT can be finding unusual uses of objects, creating a story, a drawing, etc. In its first versions, the TTCT was based on the four criteria of the Alternative Uses Test of Guilford, 1967 :

- Fluency (or fluidity) : The number of ideas the subject comes up with;
- Flexibility : The number of conceptual domains the answers relate to;
- Originality (or novelty) : A measure of how uncommon the response of a subject is, compared to, say, other subjects. The less common it is, the more original it is;
- Elaboration : A measure of how detailed the answers of a subject are.

Later versions of the TCTT did not have flexibility (because of a high correlation between fluency and flexibility), but instead a measure of resistance to premature closure (a measure of tolerance to ambiguity), abstractness of titles (the level of abstraction of titles for figural tasks e.g. tasks in which pictures are drawn and named), and a checklist of thirteen creative strengths.

In the CreaCube task, DT dimensions have been assessed as follows (Kohler and Romero, 2023):

- **Fluidity**: the number of configurations made, regardless of differences between configurations;

3 Creacube : an ill-defined problem-solving task

- **Flexibility**: the number of different configurations created by a subject.
- **Originality** (often called **Innovation**): the number of configurations appearing in less than 5 percent of the total number of configurations done by all subjects. This threshold is subjective, but this value may be re-evaluated later, see section 3.3.

We work mainly on the model of Guilford (1967) as the TTCT model includes also measures not related to divergent thinking measure. The measure of elaboration is not considered here as there is no need for the subject to detail an answer and so to measure the level of detail of the answer. One solution may be to consider that evaluation happens when a configuration is tested.

The data collection of CreaCube experiments consists of sets of observables, and a set of observables represents the state of the activity at a given time. In this context, an observable represents a specific element of the scene : the actual configuration of cubes or the player's behaviour. Observables require a decision on the way direct analysis of the situation or indirect one (through video analysis) or other data such learning analytics provided by the modular cubes can inform these observables. The set of observables used for this study is based on the work of Mercier (2022), as seen in Figure 3.3.

CREACUBE 2. Activity									
AS00	F000	F002	F010	F011	F012	F013	F014	F020	
AS01									
AS02	F021	F022	F024	F030	F034	F040	F042	F044	
AS03	F050	F051	F060	F061	F064	FXX			
F000-SBIW-T	F000-SIWB-T	F000-SWBI-T	F000-BSIW-T	[...] every permutation is encoded for each shape		Example: F020-BWS-T		P01. Imbalance	
F000-BIWS-T	F000-BWSI-T	F000-IWBS-T	F000-IBWS-T					P02. Rotation	
F000-ISWB-T	F000-WBSI-T	F000-WSBI-T	F000-WIBS-T					P03. Wrong direction	
U00. Play instructions	U01. Stop playing instructions	U02. Question instructions	U03. No cubes in hand (no manipulation)	U04. Hands up with 1 cube	U05. Hands up with 2 cubes	U06. Hands up with 3 cubes	U07. Hands up with 4 cubes	FL01 Rotate the cube without changing its position	P04. Reverse (outward)
E01. Ecstasy/Joy/Serenity	E02. Admiration/Trust/Acceptance	E03. Terror/Fear/Apprehension	E04. Amazement/Surprise/Distracted	E05. Grief/Sadness/Pensiveness	E06. Loathing/Disgust/Boredom	E07. Rage/Anger/Annoyance	E08. Vigilance/Anticipation/Interest	FL02 Reposition the cube into the same configuration	P05. Reverse (to the person)
A01. Wheels	A02. Magnets	A03. Switch	A04. "Eyes"	A05. Sensors	S01	S02	T01. No test	T02. Drop Out / Abandon	P06. Colour association
								T03. Succeed	P07. Connection
									P08. Does not move (wheels)
									P09. Does not move (switch)
									P10. Does not move (sensor)
									P11. Does not move (inv)

Figure 3.3: The set of observables of the CreaCube experiment, see Mercier (2022)

- **Final Configurations - F F** observables represent each a configuration. For each structure shape, there are a lot of different configurations, depending on the position of each cube and where specific cube faces are situated (mostly wheels and eyes). For instance, we will consider the structure **F000**, where all cubes are aligned. There are 24 different configurations, some of them are detailed in Figure 3.3. For each configuration, the wheels may touch the ground or not, and sensors can see obstacles (if directed to the bottom where the table is) or not (if placed in front of the line for instance). For each configuration, goal structures can exist but are not systematic (depending on the orientation. Four letters **S** (sensors), **W** (wheels), **B** (Battery) and **I** (inverter) are used to represent the order, and the letter **T** or **F** (True or False) indicates if the structure can meet the requirements to solve the problem.
- **Partial Structures - AS AS** observables indicate how many cubes are connected with each other while not forming a configuration: none (**AS00**), two (**AS01**), three (**AS02**) and four but with two unconnected pairs (**AS03**).
- **Switch - S S** observables indicates if the switch is on or off.
- **Affordances - AF AF** observables indicates whether each affordance detailed in section 3.1 is discovered or not.
- **Emotions - E E** observables are based on a specific model of emotion representation using Plutchik's wheel of emotions The model has a classification of 8 main types of emotions, and observables only take the main types into account.
- **User - U U** observables are related to actions taken by the subject, both for understanding of instructions (playing the instructions) and manipulation of cubes (number of cubes in hands).
- **Testing, problems and rearranging - T, P and FL T** observables indicates whether the subject is testing a configuration, succeeding or giving up. While testing, the subject can face problems. **P** observables correspond to main problems identified by observers that can occur during an activity : imbalance of the structure, the structure moving in the wrong direction, etc. **FL** observables correspond to

changes of a structure without changing the shape in order to try to solve a problem.

In Palaude et al. (2022), we discussed different ways to refine these observables to get a better representation of the evolution of the material environment by the sequence of observables, leading to observables less related to subjective interpretation of the situation (for FL observables for instance). Some suggestions are the consideration of autonomous vehicles of three pieces, shapes of partial structures, and the consideration of which cubes are in hand at any moment.

However, the data collection process was done beforehand, thanks to the work of the ANR CreaMaker. Based on a subset of the CreaCube corpus, selecting only children (7-12 years old) who performed the task individually (93 subjects), we have for each experiment a data set including a time sequence starting when the subject hears the instructions for the first time and stopping when the subject succeeds or gives up. In each sequence, key events that can be observed have been annotated, resulting in 83 different types of observables annotated across all experiments. Those time sequences are thus the symbolic trajectories. Using the DT dimensions given by Kohler and Romero (2023), another representation of the subject's sequence can be specified, as shown in Figure 3.4.

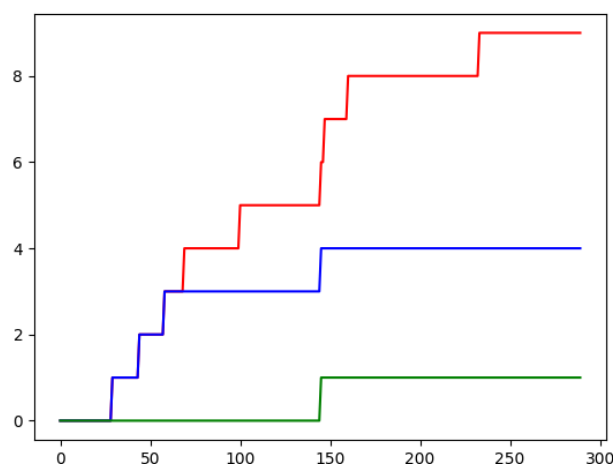


Figure 3.4: A time sequence (in seconds) from the Creacube dataset (fluidity is red, flexibility is blue and innovation is green)

Our dataset also includes, for each children, a second experiment done right after the first one. Children had to perform the same task with the same instructions. The main difference is that they did solve it once.

However, as other dimension than learning are considered here (mostly motivational), this second experiment is out of the scope of this thesis, but could be a welcome addition in future work.

Based on the method presented in section 2.4, we will now visualize the dataset in order to make observations about the subject's trajectories during the task.

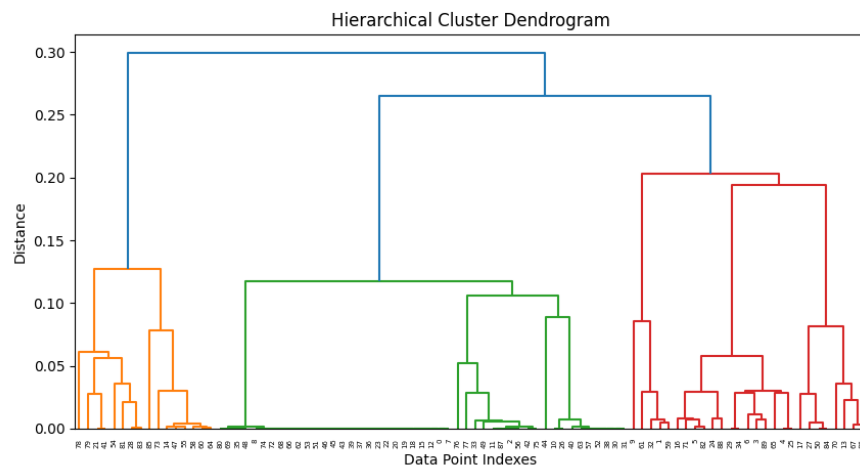
3.3 DATA ANALYSIS

3.3.1 METACLUSTERING CREACUBE SEQUENCES

We apply Dynamic Time Warping on trajectories from the Creacube dataset. We use sequences composed of the three Divergent-Thinking components : innovation, fluidity, and flexibility.

We put every value into a matrix S : $S_{i,j}$ is the similarity between sequence i and j . We apply our metaclustering method : we compute possible results of every possible execution of k-medoid clustering for a given value of k . Then we create a proximity matrix M' : $M'_{i,j}$ will be low (near 0) if sequences i and j are clustered together often. $M'_{i,j}$ will be high (near 1) if sequences i and j are clustered often in different groups.

Based on it, we can see the hierarchy of proximity between sequences :



For instance, green trajectories are similar between each other, but are very different from yellow and red trajectories.

Given the DT dimensions used to represent a subject's sequence, we do have three numerical sequences that are increasing over time, depending on if a configuration of four cubes counts towards fluidity, flexibility and innovation. These three dimensions are not independent, but

this representation allows us to apply dependent Dynamic Time Warping to the different sequences. This will be our measure of distance for the metaclustering we are going to apply. Figure 3.5 shows an example of DTW between two sequences of Creacube.

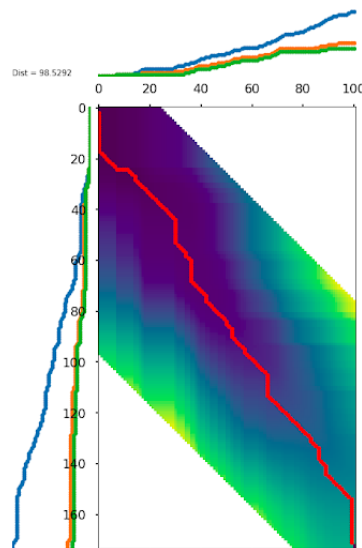


Figure 3.5: DTW applied to two different Creacube trajectories. Each point of the first sequence is matched as to a point of the second sequence. Fluidity is blue, flexibility is green and innovation is orange

Now we have a distance, we can apply our metaclustering method on k-medoids executions. We will apply the method on $k = 3$.

The first step is to create a matrix M (the metamatrix for $k = 3$) of size 90 by 90, the size being the number of different sequences. $M_{1,2}$ corresponds to the number of times sequences 1 and 2 were clustered together. The metamatrix is empty at the start of the metaclustering method. Note that the matrix is symmetrical.

The second step is to compute every possible outcome of a k-medoid algorithm. Assuming there is no loop in a single k-medoid execution, we have a maximum of $\binom{90}{3}$ different potential outcomes (we skipped combinations if two points were identical, resulting in the loss of one cluster during the execution). Let us consider a result in which s_1 and s_2 are in the same cluster, but in a different cluster than s_3 .

The third step is to update the metamatrix to add the results of each specific execution. $M_{1,2}$ is increased by 1, but neither $M_{1,3}$ or $M_{2,3}$ are increased, because s_3 was not in the same cluster. We do it for every other outcome, even if they are identical to a previous one (as long as the initial points are different).

For our Creacube corpus, the resulting metamatrix is shown Figure 3.6.

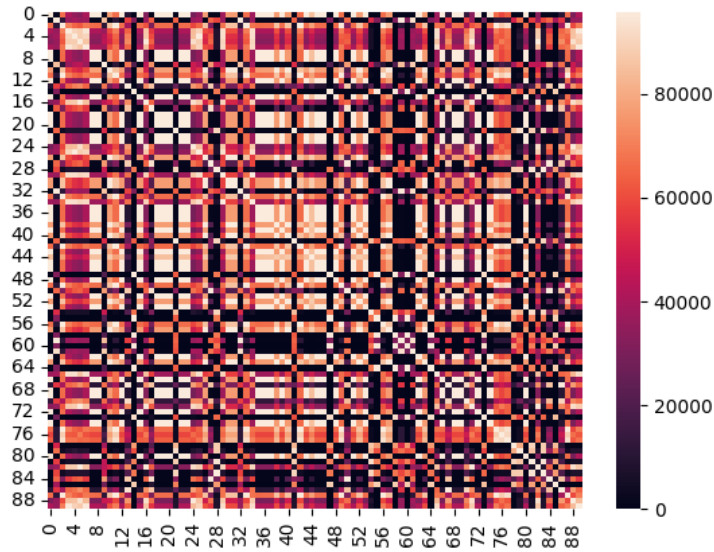


Figure 3.6: The metamatrix of all executions of k-medoid clustering for $k = 3$ of the Creacube dataset. The brighter the color, the more often the points were clustered together.

For instance, it is normal to have a really bright diagonal : a point is necessarily clustered with itself. We need to convert this metamatrix into a distance matrix, because the number of times two points are clustered together is not a distance.

The associated distance is easy to compute: we can divide everything by the total number of executions, and then the distance is one minus this result. The resulting matrix M' (of size 90 by 90) is then

$$M'_{i,j} = 1 - \frac{M_{i,j}}{M_{1,1}}$$

1

Each value is between 0 (the point to itself or identical sequences) and 1 (two points that were never clustered together).

We can then apply hierarchical clustering on this M' matrix. The results specifically for $k = 3$ are shown Figure 3.7.

This can also be done for other values of k , namely 2, 4 and 5. We need to choose which metamatrix is the more relevant for our clustering purpose.

¹As a point is necessarily clustered with itself, any other point of the diagonal is the same.

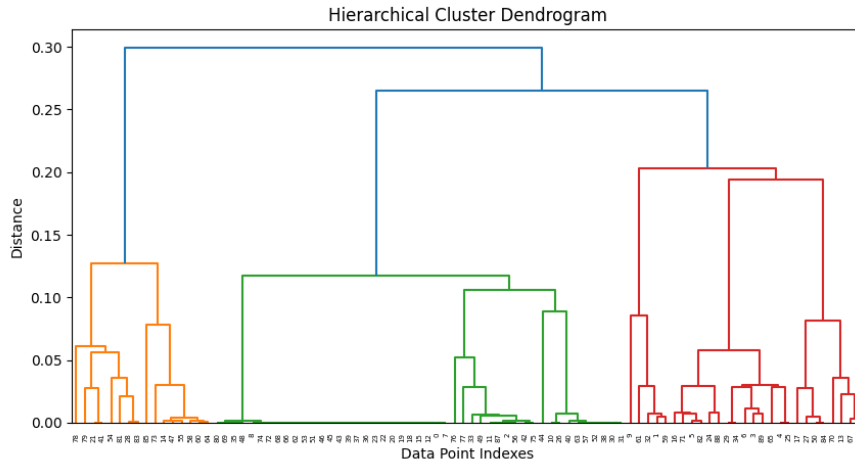


Figure 3.7: Hierarchical clustering of k-medoids clustering of the Creacube dataset for $k = 3$

For this, we decide to compute the silhouette coefficient of each result. The silhouette coefficient is a measure of how similar an object is to its own cluster (cohesion) compared to other clusters (separation). The silhouette goes from -1 to $+1$, and a high value indicates that the object is well matched to its own cluster and poorly matched to neighboring clusters. If most objects have a high value, then the clustering configuration is appropriate.

Let's consider a data point i in the cluster C_i , the cohesion is :

$$co(i) = \frac{1}{|C_i|-1} \sum_{j \in C_i, i \neq j} M'_{i,j}$$

Similarly, the separation is :

$$\min_{i \neq j} \frac{1}{|C_j|} \sum_{j \in C_i} M'_{i,j}$$

Finally, the silhouette S of sequence i is :

$$S_i = \frac{co(i) - s(i)}{\max\{co(i), se(i)\}}, \text{ if } |C_i| > 1$$

The silhouette coefficient, as adapted from Kaufman and Rousseeuw (1990) is then the mean silhouette of each point of the dataset.

For this metamatrix, the silhouette coefficient is 0.59. We want to choose a value of k that is small while having a good silhouette coefficient. For this, we compute the silhouette coefficient for each metamatrix we can compute, from 2 to 5. The results are shown in Figure 3.8.

Now that we need to choose which k to choose, we are using a simple elbow method (taking the value at the elbow of the curve). For the rest of this section, we will use the metamatrix for $k = 4$. The associated dendrogram is shown in Figure 3.9).

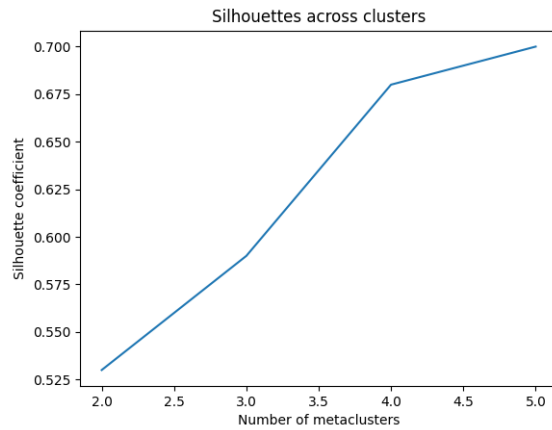


Figure 3.8: Silhouette coefficient of metaclustering on k-medoids executions on the Creacube dataset for multiple values of k

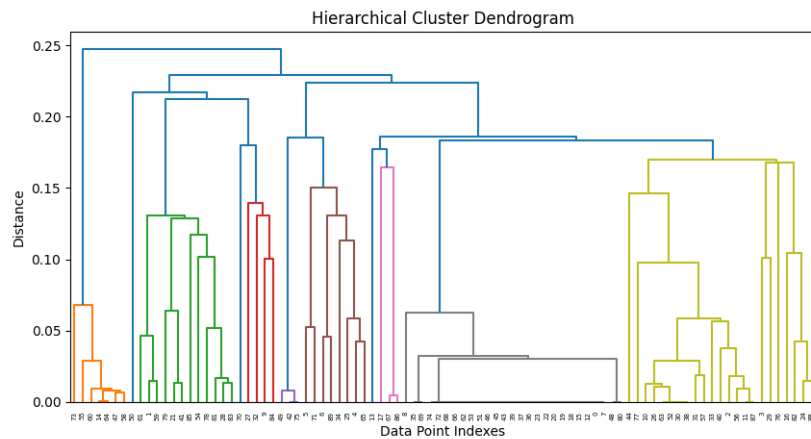
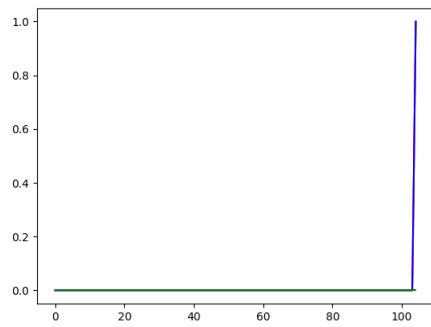


Figure 3.9: Hierarchical clustering of k-medoids clustering of the Creacube dataset for $k = 4$. Silhouette of 0.68

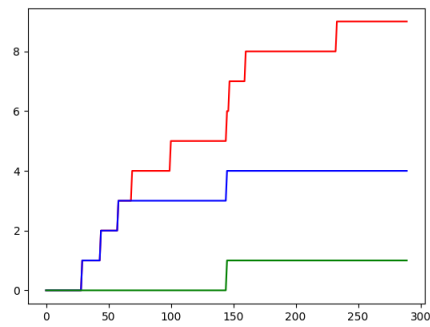
ANALYSING OUR CLUSTERS The first thing that we can notice with this method is the apparition of identical sequences. They are at a distance of 0, and there is an entire subgroup of sequences that are identical. These sequences correspond to sequences in which the subjects finish the task with just one try : the first configuration of four is sufficient to solve the problem. Subjects did not take the same time to do so, but thanks to Dynamic Time warping, they are properly associated with each other.

Our goal is to determine if the clusters we created here can correspond to well-known strategies in the literature. For this, we decide to take a look at the different medoids, and for each medoid, the point within their group that is at the farthest distance from it. Figure 3.10 shows each of these points.

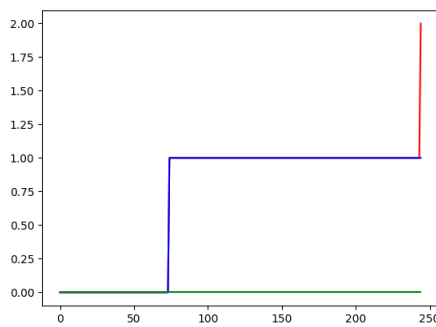
3 Creacube : an ill-defined problem-solving task



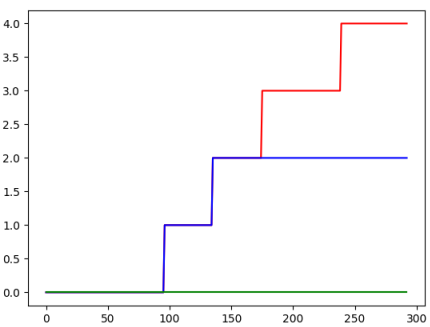
(a) Medoid of group 1



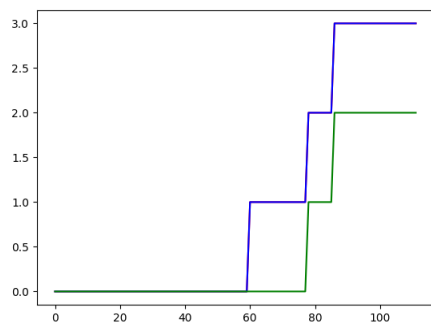
(b) Farthest point away within group 1



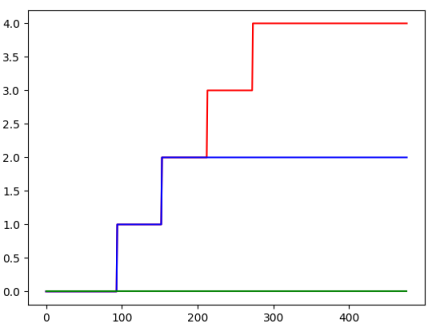
(c) Medoid of group 2



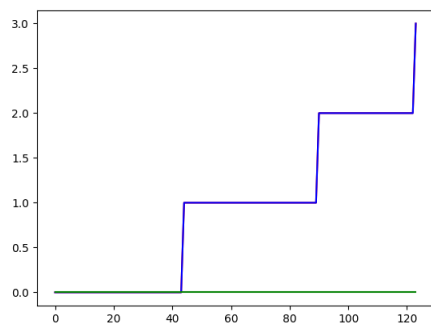
(d) Farthest point away within group 2



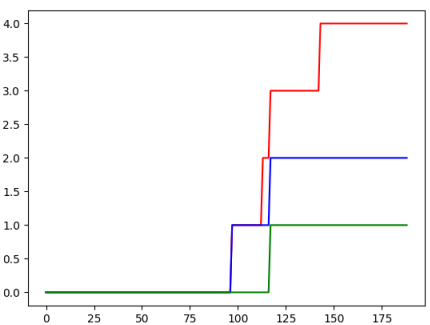
(e) Medoid of group 3



(f) Farthest point away within group 3



(g) Medoid of group 4



(h) Farthest point away within group 4

Figure 3.10: Medoids of each group and their farthest away points within the group. Fluidity is red, flexibility is blue and innovation is green

Unfortunately, this method is not very conclusive in terms of explainability. Medoids look more similar to each other than to points from their own cluster.

This can be explained by the nature of the Dynamic Time Warping algorithm. As DTW tries to match points from sequences to points from the other sequence, poor sequences (with little to no changes over time) tend to be less distant to rich sequences (i.e. with multiple increases of DT components over time). This leads to clusters centered around poor sequences. It is then difficult to associate different problem-solving profiles to each clusters as we wanted to do. The only thing we can do is to compare the different poor sequences with each other, and, except for the first medoid that corresponds to immediate success (which is clearly not the case for the farthest point away in its own cluster).

The method was not efficient for obtaining behavioural clusters. However, we will now try to verify on subsequences of Creacube if the model is relevant for identifying insights.

3.3.2 SEARCHING FOR INFLEXION POINTS WITHIN THE CREACUBE TASK

Another idea is to cut sequences in two : pre-cut and post-cut sequences. We can cut them at a certain time (in half or at the quarter for instance). We can also cut it when specific events are triggered.

For Creacube, the idea is to cut sequences at the discovery moment of an affordance, for instance the wheels.

When we apply metaclustering on it, we can compute a **derivation** score. The derivation score is based on the number of pre-cut *pre* and post-cut *post* sequences on each cluster C_i

$$d(C_i) = \left| 0.5 - \frac{pre(i)}{pre(i) + post(i)} \right|$$

If the derivation is 0.5, metaclustering properly separates pre-cut and post-cut sequences. The lower the derivation is, the worst the separation between pre-cut and post-cut sequences is.

For cuts at the discovery moment of wheels, switch and magnets, we can observe that the derivation score is better than when we cut sequences at the mean time of discovery. It means that these affordances are important for trajectories' shape.

An insight is a phenomenon corresponding to a sensation that we often call a "Aha effect" or a eureka moment. Insight occurs when "what" could be the solution emerges. This process is generally defined as immediate, as it is the result of a restructuring of thoughts.

A question that we want to answer about the Creacube task is the presence of such insights :

Are there any insight moments within Creacube sequences ?

In particular, within the Creacube task, we postulate that insight may occur after the discovery of an affordance, allowing to shift the perception of the activity. For instance, finding the wheels can change the mental process from "how do I make a vehicle" to "How do I make these wheels move". Such insights can occur immediately, later, or not happen at all. The question we will try to answer now is the following :

Can we associate the discovery of affordances to insight moments within the Creacube task ?

In order to answer this question, we need to determine what could be an insight for our sequences. This requires a strong assumption : we assume that an insight implies a change in problem-solving behaviour. The restructuring of thoughts allows the subject to adapt their strategy to better fit updated knowledge. This way, whenever an insight happens, a shift in strategy may become visible.

Do affordances lead to shifts in Creacube sequences' trajectories that are significant enough to be associated to insight moments ?

Our hypothesis is that all necessary affordances (that are required to find to create a valid structure) can be associated to insight moments. The goal of this second experiment is to determine if, by using our meta-clustering method, we can reach a conclusion that direct us towards the validation or invalidation of this hypothesis.

CUTTING SEQUENCES FOR METACLUSTERING We are still using sequences containing the three DT components trajectories used in previous section : fluidity, flexibility and innovation. We will define the notion of inflexion point as a point in a sequence from which the sequence that happens after is different from the sequence that happened before. An inflexion

point can not necessarily associated to an insight. For instance, if we consider the point happening after the first second of the task of each sequence and cut each sequence at this moment, pre-cut sequences will all be one-second long (and for Creacube, nearly all of them would be empty) and will be very different from post-cut sequences.

We will consider the discovery of each affordance as a potential inflexion point. For each of them, we create a new dataset composed of pre-cut sequences and post-cut sequences with the cutting point being the discovery of such affordance. For instance, for the affordance AF03 (switch), the dataset contains twice the number of sequences from the Creacube dataset : half are sequences starting at the beginning of the task and finishing at the discovery of affordance AF03, and the other half are sequences starting at the discovery of affordance AF03 and finishing at the end of the task. For post-cut sequences, we need to subtract each DT-component by their number at the start of the post-cut sequence, because we want to compare the shape of the evolution and not the value

For example, let's consider the following sequence :

```
fluidity : {0, 0, 0, 1, 1 (discovery of switch), 1, 2, 2, 3, 3, 4, 4, 5}
flexibility : {0, 0, 0, 1, 1 (discovery of switch), 1, 1, 1, 2, 2, 2, 2, 3}
innovation : {0, 0, 0, 0, 0 (discovery of switch), 0, 0, 0, 1, 1, 1, 1, 1}
```

would be cut in two separate sequences, with the post-cut sequence being reinitialized to 0.

pre-cut with cut at AF03

```
fluidity : {0, 0, 0, 1, 1}
flexibility : {0, 0, 0, 1, 1}
innovation : {0, 0, 0, 0, 0}
```

post-cut with cut at AF03

```
fluidity : { 0, 1, 1, 2, 2, 3, 3, 4}
flexibility : {0, 0, 0, 1, 1, 1, 1, 2}
innovation : {0, 0, 0, 1, 1, 1, 1, 1}
```

Doing this for each affordance gives each time a different new dataset. We decided to create 5 specific cuts : AF01 (magnets), AF02 (wheels), AF03 (switch), discovery of the subject's first affordance, and discovery of the subject's last affordance. It is not possible to do it for the two other affordances (eyes and sensors), are they are not found often enough to still have a dataset to work on.

Doing so also revealed sequences from which affordances were not annotated. Some affordances must be found in order to solve the problem

(the switch needs to be activated for instance to allow the cubes to be powered), so sequences lacking those were removed. This reduced our dataset from 90 sequences to 66 sequences.

We want to see if these cuts are better inflexion points than random cuts done on sequences. We will then also perform cuts at specific time in order to compare cuts. We will compare with multiple datasets for which cuts were done not on the happening of specific events, but on a proportion of the total time of the sequence, ranging from 10 % to 90% (before and after that, some sequences are too short to be properly cut in pre- and post-cut sequences).

PERFORMING METACLUSTERING ON K-MEDOIDS CLUSTERING TO COMPUTE THE DERIVATION Our next step is to apply our clustering method on newly created datasets. The idea behind the method is that the more pre-cut and post-cut sequences are different, the greater the DTW distance between those sequences. Thus, when creating clusters from our datasets, if pre-cut and post-cut sequences are classified in different clusters, this is an indication that they were different enough to be categorized differently. Let us take the example of the AF03 (switch) cut dataset. We can apply metaclustering on k-medoids execution for different values of k . For each of them, similarly to the previous section, we compute the silhouette coefficient for each of them. The results are shown Figure 3.11.

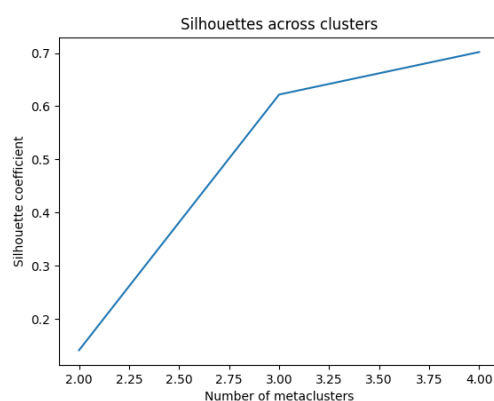


Figure 3.11: Silhouettes coefficient of k-medoids metaclustering on the AF03-cut dataset for multiple k values

Using the same elbow technique as we did in the previous section, we decided to go with $k = 3$. The resulting hierarchical clustering is shown Figure 3.7. The 66 first sequences are pre-cut sequences, while the rest are post-cut sequences.

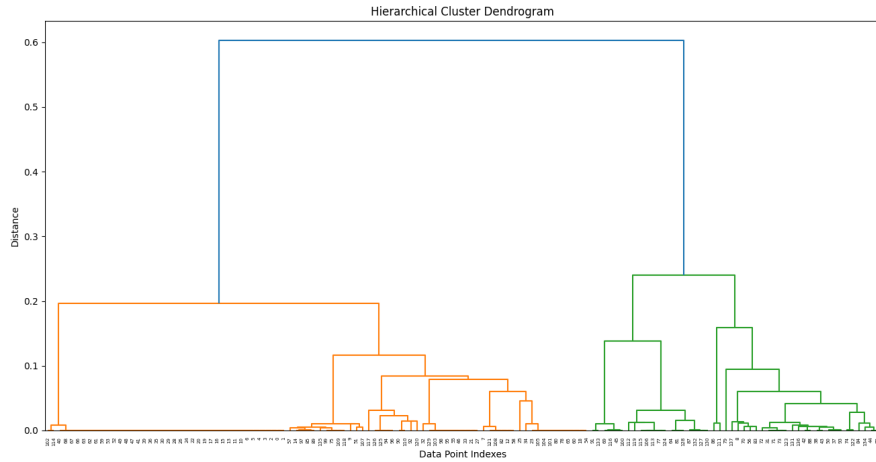


Figure 3.12: Hierarchical clustering of k-medoids metaclustering on the AF03-cut dataset for $k = 3$

Our goal now is to verify, for each cluster, the proportion of each pre-cut and post-cut sequence. For this, we will compute an indicator that we are calling the **derivation**.

DERIVATION COMPUTATION Derivation is a measure on how clusters are properly separating pre-cut sequences to post-cut sequences. The goal is to verify if pre-cut sequence and post-cut sequence are in different clusters. This is a way to take into account that subjects may have different behaviours, and a trajectory at the start of the task for a subject can correspond to the trajectory at the end of the task for another.

Let $pre(i)$, $post(i)$ be respectively the number of pre-cut sequences and post-cut sequences in cluster C_i . The derivation score of cluster C_i is :

$$d(C_i) = \left| 0.5 - \frac{pre(i)}{pre(i) + post(i)} \right|$$

To get the general derivation score, we compute the weighted mean of clusters' derivation scores, weighted by the number of sequences.

Derivation scores range from 0 (both parts of complete sequences are systematically in same clusters) to 0.5 (no pre-cut sequence is in the same cluster as its associated post-cut sequence).

Figure 3.13 shows the different derivation scores obtained with cuts being a proportion of the total time sequence, that we will compare to the resulting derivations of AF01, AF02 and AF03 cuts, that have a derivation of, respectively, 0.33, 0.37, 0.34.

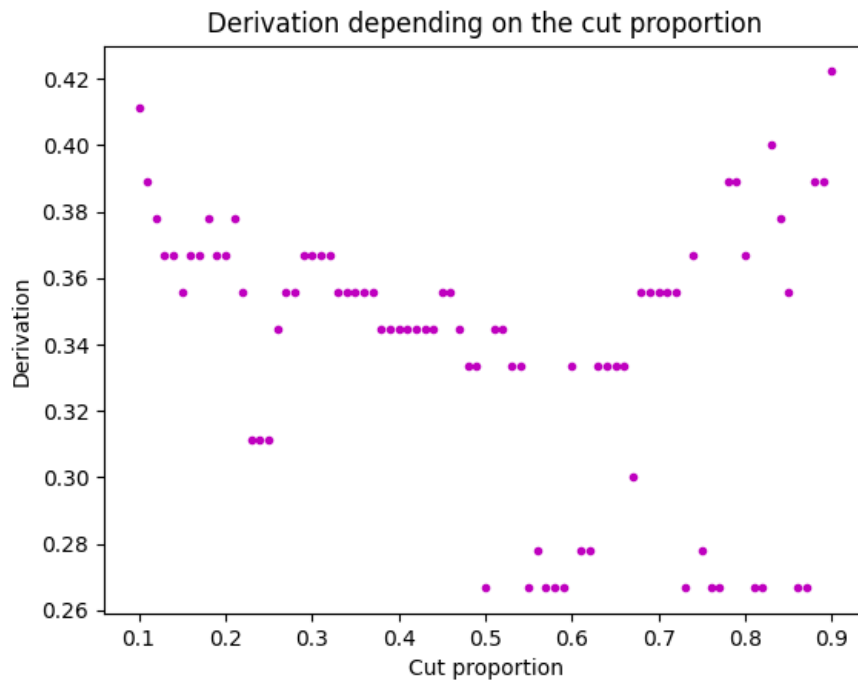


Figure 3.13: Derivation scores of different proportional cuts of the Creacube dataset

The first thing to note is that the better derivation scores are at the extreme cut proportions. This is coherent, considering that even if DTW can ignore small time differences, fluidity, flexibility and innovation increase a lot more in the longer sequences. The more equivalent in size pre-cut and post-cut sequences are, the less this effect is present.

The mean derivation of all time cuts is 0.34 with a standard derivation of 0.04.

- The mean time of discovery of AF01 (magnets) is at 32.2 % of a sequence. The derivation of AF01-cut is 0.33, and 0.37 for the 0.32-cut.
- The mean time of discovery of AF02 (wheels) is at 27.7 % of a sequence. The derivation of AF02-cut is 0.37, and 0.36 for the 0.32-cut.
- The mean time of discovery of AF03 (switch) is at 41.7 % of a sequence. The derivation of AF03-cut is 0.34, and 0.34 for the 0.32-cut.

These cuts do not have a significantly better derivation than the mean derivation of all time cuts, or than the cut at their mean time of discovery. However, most good derivation scores (except the extreme points)

happen during the first half of the task. This is coherent with the fact that most important discoveries must be made at the beginning of the task anyway : It is impossible to create a structure without sticking the magnets together (thus discovering the affordance) and the wheels are more apparent than other affordances and usually put on the table before the beginning of the first configuration. While not indicating that specifically one affordance is an inflexion point that changes a trajectory's shape, this shows that a shift occurs within the first part of the activity, and this shift happens after that most affordances are discovered.

We think that these affordances may be associated to insights : they allow the subject to create their first configurations, allowing a shift in behaviours from random or observational approaches to potential trial-and-error methods for instance. This is purely illustrative, as we are not able to determine what kind of strategies the subject apply over the course of the activity at this point of our study.

We will put these results in perspective with results from the next experiment of chapter 4, and use them to discuss the potential use of the method as a way to find insights in chapter 5.

3.3.3 DISCUSSION : FIRST OBSERVATIONS ABOUT THE METHOD

We can make multiple observations of our results :

- Different clusters of trajectories do not necessarily correspond to different problem-solving behaviours.
- Our method tend to create clusters with medoids that are poor trajectories. Poor trajectories have less changes of states. They have usually low dissimilarity scores with any other trajectories. This explains our first observation.
- Clustering cut sequences tend to show that discovering specific affordances change the trajectories' shape. This confirms that discovering the switch, the wheels and the magnets have a big influence on the problem-solving behaviour : they trigger insights. Insights are immediate reconfiguration of the problem's representation of the learner.

In this chapter, we tried to use a metaclustering method in order to visualize and analyse the Creacube dataset, discovering uses and limits of the method that we may extend to a more general case :

- As a partitioning method, medoids tend to be as "easy to associate" to other sequences by the Dynamic Time Warping algorithm. This implies that medoids will be usually poor sequences, that are sequences with minimal symbolic state shifts. At the other end of the spectrum, furthest neighbors of medoids will be usually rich sequences, that are sequences with multiple symbolic state-shifts. This may be corrected by choosing a more appropriate measure than the DT-component for this specific use. For instance, having access to multiple trajectories with similar ends can help, as we will have for the Outerwilds experiment in next section 4. This doesn't mean that correcting this method will allow, to get clusters of similar trajectories that are interpretable and explainable, but rather this change can help to lean towards this goal.
- From observation of partitioning pre-cut and post-cut sequences cut at different times, we associated the precision decrease of our clustering method to a natural decrease of precision around the center of a task (not necessarily the middle). We can imagine that the derivation computed on multiple time-cuts will show a sort of parable, going down from a maximal derivation score of 0.5 (meaning a perfect separation) to the central moment of the task before going up to a maximal derivation score at the end, with perfect derivation scores decreasing only some time after the beginning and increasing back to maximal score at some time before the end. We will discuss this hypothesis in chapter 5.
- From observation of partitioning pre-cut and post-cut sequences, we think that the three mandatory affordances that are AF01 (magnets), AF02 (wheels) and AF03 (switch) and their discovery (or the moment where all three are discovered) are key moments of the activity. As pre-affordances and post-affordances trajectories are more easily separated by our method with affordances rather than their associated mean time of discovery, we postulate that this is an indicator that these discoveries are more than inflexion points by being insights, allowing a change in the mental state of the learner.

4 OUTER WILDS : LONG TASKS AND ADAPTIVE BEHAVIOURS

4.1 OUTERWILDS, DATA COLLECTION AND POTENTIAL BIASES

Outer Wilds is a video game about exploring a solar system. It is also an ill-defined problem : there is no other obstacle to finish the game than the subject's knowledge of the method to solve it.

We want to observe long sequences of ill-defined problem-solving. It has been possible to collect a rather small dataset of 14 subjects on which we annotated their playthroughs. We also conducted multiple semi-directed interview to try to get their strategies.

The experiment bias is present here. Self-Regulation contains motivation. Players often reported a motivational boost from their participation :some would have given up if they weren't part of it.

Outer Wilds is a video-game developed by Mobius digital and published by Annapurna Interactive. It is a puzzle-game in which the player is an astronaut that is going into space, in order to unravel the secrets of an ancient, long-gone species called the nomais that lived in their solar system. **Before continuing this presentation section (and the analysis we are going to do on the game), we highly recommend the reader to try and play the game themselves. The only kind of progress in the game is the acquisition of new knowledge via the exploration of this solar system. This PhD thesis will not go into all the details of the game, but it will certainly alter the experience for the reader if they were to play it after.** The game progression is interesting in two points :

- Every time the player dies in the game, or every 22 minutes of play without death, the game resets to its original state (the exception being the ship log, as developed below). In the game, this is presented via a time loop.

- It is possible to reach the end from the very start of the game, but it is very unlikely, as the process is unknown from the player at the beginning. The only way to progress in the game is through the acquisition of new knowledge. This knowledge is discovered over time. Once every major piece of information is discovered, the player can infer the process to finish the game.

Three important pieces of information needs to be gathered in order to reach the conclusion :

- The player must learn the location (and the way to go to such location) of the winning state. In the game, this corresponds to the discovery of The Vessel, the spaceship of the previous ruling species of the solar system.
- The player must power the vessel with a source of energy that it is possible to find in the center of a planet called Ash Twin.
- The player must learn a code to enter in the vessel once the energy source from Ash Twin powers it. This can be obtained in a location called the Probe Tracking Module.

From these three pieces of information, the player must infer that they need to take the power source from Ash Twin in order to bring it to the Vessel in order to enter the code to finish (and – in our case – win) the game. Each of these pieces of information is the culmination of three different intrigues, that are separated in the game by different color codes, respectively red, orange and green. A fourth and optional intrigue exists, associated to the purple color, that reach its conclusion by the encounter with a nomai called Solanum. Finally, other pieces of informations unrelated to a specific intrigue is colorless. A typical playthrough of this videogame is around 20 hours of play. As such, it is rarely achieved in only one session of play, and players are invited to finish it through multiple sessions. There is no particular order in which any information can be obtained. All planets are accessible from the start of the game and, apart from the uncertainty on how to reach some places, the player can go wherever they want From this description of the game concept, we can associate it with an open problem, in which, at the start, neither the actions that are possible, nor the way to reach the end are known to the player. However, it differs from Creacube in one major point, which is the

time required to finish the task. We want to study this task as we want to determine if, thanks to our model, we are able to find a potential influence of the pauses in-between sessions on the behaviour of a player.

DATA COLLECTION We collected and annotated the playthroughs of 14 different players. From this little corpus, 2 of them (17%) are female and all of them (100%) are french. 7 of them (50%) are between 18 and 25 years old, and 7 of them (50%) are between 26 and 30 years-old. Finally, 83% of them have a high-level of education (license's degree) while 17% are working in the artistic industry. All of these informations are based on a questionnaire answered by subjects at the start of the experiment. The goals of this research was completely explained to the players beforehand, and they knew that their playthrough was being recorded. They were also asked to participate to semi-directed questionnaires before and after each session. The details of the questionnaires are accessible in Appendix C.

BIASES We need to point that our corpus is highly imbalanced in terms of parity, level of education and even age. These imbalances, when studying problem-solving behaviors, may lead to the loss or underestimation of the prevalence of some problem-solving behaviors. Our goal with this experiment is to do a preliminary study of the task as, unlike Creacube, Outer Wilds is not studied a lot. Our dataset of annotated playthroughs will be accessible online for whoever wanting to iterate on it. For now, this preliminary work's purpose is to determine properties and generate observations on the task thanks to our method. The potential extensions of this work will be discussed in section 4.4. From the semi-directed questionnaires, we can also point the well-known observation bias : players reported that they were behaving differently from what they would have done if they were not part of a scientific project.

« Je pense que je me suis comporté différemment de ce que j'aurais fait d'habitude parce que je savais que j'étais observé. »

(I think I was behaving differently from usual because I knew I was recorded.)

In particular, it seems that the main perceived difference was a difference in motivation.

« Si je n'avais pas fait partie d'une expérience, j'aurais abandonné il y a un moment. Je serais allé voir une solution pour continuer ! »

(If i wasn't part of an experiment, i would have given up a long time ago. I would have checked a solution to advance!)

As motivational processes are part of self-regulation skills, this is an important point to consider. For now, we make the assumption that this motivational change was not responsible for significant changes in the session-to-session planning and execution of the task.

4.2 COLLECTING AFFORDANCES AND CHOOSING RELEVANT MEASURE

Important annotations we did was about the update of what is called a ship log in the game. The ship log contains pieces of informations discovered by the learner. It is a sort of external memory for them. Every time a player discovers something new, the ship log is updated. For trajectories, we considered multiple sequences based on multiple observables :

- **Color sequences** : ship log updates can be of 5 colors : green, red, orange, purple and colorless. They correspond to 4 intrigues, the purple one is optional. Colorless updates do not relate to a particular intrigue.
- **Location sequences** : ship log updates can come from multiple locations. We reduced them to 8 different class of locations. They mainly correspond to each different planet the player can explore. Details can be accessed in Appendix D.

We study trajectories based on color sequences, location sequences, or both.

AFFORDANCES AND PLAYTHROUGHS' ANNOTATIONS In *Outer Wilds*, the player can finish the game from the start, but the method to do so is unknown. The progression of the game consists only of the gathering of new information. The game has already an in-game method to measure such gathering : the ship log. It starts with a limited amount of information, and is updated with each piece of information that the game considers as a relevant piece of information, the moment the game considers

that the information is obtained. The player may already know an information without having collected it. It is also possible that the player may not consider a piece of information even if the game considers that they should. Our main assumption for the actual knowledge of the player is strong : we will consider from now on that the knowledge of a player at any given time corresponds to the state of the ship log at this very same time.

Figure 4.1 shows a completed ship log with the number of different informations written as sentences accessible on each element of the graph¹.

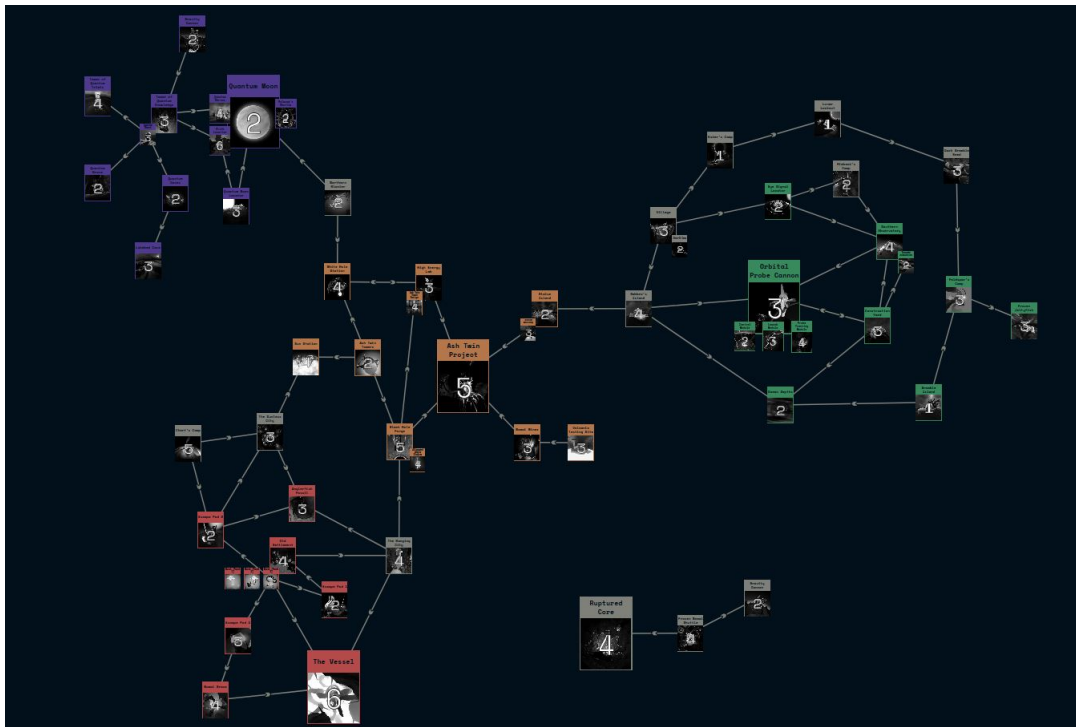


Figure 4.1: The Outer Wilds shiplog completed. Each color corresponds to a different intrigue. The biggest rectangle on each color correspond to the main location of the intrigue, not necessary the final one.

The update of a shiplog allows to determine when a player gains new knowledge. Usually, whenever the ship log updates itself, a sound and a textual notifications happen. The exception is whenever the astronaut doesn't have their spatial suit on, which can happen at any time but mostly happen in the first hour of the game, before the player has access to it. We decided to annotate every time an update happens. Our first affordances are what we call **update** affordances. To each update, we associate the general location in which the information was discovered (from a total of 8 different general locations), which color corresponds to the update (from a total of 5, including colorless) and what part of the

¹For more details, the ship log is accessible at <https://outerwilds.ventures>

ship log was updated precisely. In case of multiple squares updated at the same time, all updates are annotated. Even without notifications, it is possible to annotate updates by witnessing the trigger of the update. For more details on how annotations were performed, see Appendix D.

In addition to the acquisition of new knowledge, the player can perform actions. We decided to annotate some of them (see D for more details). Each annotations includes the general location in which the action was performed :

- **talk** : Whenever the player talks to a character, the name of the character is noted. While the player stays facing the character and starts talking to them again after finishing one dialogue, a new **talk** is not annotated.
- **enter** : This is annotated every time the player enters a different planet. Entering a planet means to land on it or to enter its atmosphere, depending on the planet.
- **teleport** : This is similar to enter, except teleportation can happen only on specific locations of planets and is then more precise than a simple **enter**.
- **die** variants : Every time the player is sent back to the beginning of the time loop. Most of the time, it is by dying in different ways, but it corresponds to every way to start a new loop, for instance leaving the game back to the menu and restart it is a **die** variant.
- **ship_log** and **end_ship_log** : Every time a player looks at the ship log is annotated. This way, we are capable of
- Other, more minor actions (like sleeping at a campfire or roasting a marshmallow) were annotated in case future works need them.

For each annotation, the timestamp of the session and of the entire playthrough was annotated.

RESEARCH QUESTIONS AND HYPOTHESIS Our first goal, as for Creacube, is to see if it is possible, with the metaclustering approach, to determine different classes of behaviours. However, we think that, given how small our corpus is, it is very unlikely.

Each playthrough of Outer Wilds can be decomposed into multiple gaming sessions. In total, if we consider sessions instead of entire playthroughs, we have a dataset of 132 sequences. With this decomposition, our goal is to determine if it is possible to classify different session-to-session behaviors. In particular, if such classification can be done, we will try to identify if there is a special moment, an inflexion point that could be associated to insight, that could corresponds to a change in player behaviour.

Is it possible to classify playthroughs or sessions in terms of player behaviour, and if so, is there any sort of insight moment inside such sequences ?

At this sake, we need first to determine a proper measure of the advancement of the player within the game, in order to apply dynamic time warping on sequences or subsequences and allowing us to use a metaclustering technique on k-medoids executions, as described in section 2.

DEFINING RELEVANT MEASURES Given these different annotations, we need to define a way to represent each playthrough by one or more trajectories the same way that was done in the previous experiment (see section 3). The first thing to note is that we are not able to define trajectories based on the same idea of the Guilford AUT from previous section, in particular when it comes to innovation. We observed that most people did find nearly all of the knowledge accessible in the different locations, with the exception of one specific location (the volcanic testing site was found by less than half of the player). This impossibility comes from the fact that Outer Wilds is an open problem-solving activity that was not specifically designed to allow creative outcomes. Creative processes are solicited over the course of a playthrough, and creative solutions can be found for specific subtasks, some puzzles inside the game having multiple solutions. We focus on open problem analysis, so it is not a problem for our research questions, but it has to be noted.

talk, enter, teleport and **die** variants will not be our focus. Based on the results of the previous chapter 3, we observed that actions for which the number varies greatly from one playthrough to another could lead to the same problem of clusters with sequences with little to no actions taken instead of a separation in, say, different behaviours implying different trajectories. The actions may not occur the same amount between play-

ers depending on the skill of each player, that is why we decide to not consider them.

From the annotations of the player's progression, we can focus on both characteristics of a ship log update :

- Given that each update has a color associated to it, we can define 5 different sequences associated with 5 different colors, tracking progression on each intrigue : red, green, orange, purple and colorless. Every time an update happens in the ship log, we can increase the "progress value" of the corresponding color. We could add different weights to different updates, because some informations are less important than other. However, it would be difficult to do so, because, even if some informations are clearly less important, it is not usually obvious to the player which of two updates was more important. To sum up, every time an update happens, we can increase by one the "intrigue-color" sequence associated with it.
- Following the same logic, we can create different sequences based on the location where the update was found, for a total of 8 different increasing sequences.
- We can also consider both at the same time, abstracting a playthrough by the trajectory of 13 different increasing sequences.

From these decompositions, we can expect two extreme behaviours and multiple variants in-between those behaviours : a planet-by-planet or an intrigue-by-intrigue method, and other, unrelated to both, methods.

It is not relevant to consider each ship log entry as a separate sequence, as it would lead to 60 sequences that would be relevant once or twice only.

4.3 RESULTS : WHEN INSIGHT DIFFER FROM INTUITION

4.3.1 K-MEDOIDS METACLUSTERING FOR PLAYTHROUGHS AND SUBSEQUENCES

We observed that :

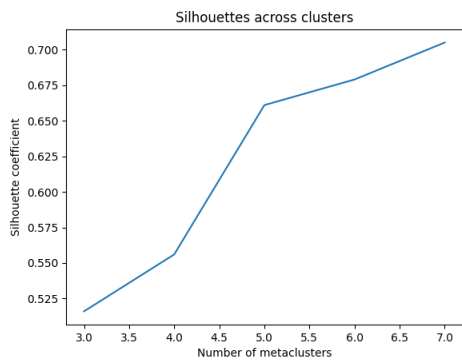
- We are not able to create categories of player behaviour based on our trajectories. However, thanks to the semi-directed interviews, we observed mostly a planet-by-planet method or a more freeform method. Freeform methods correspond to players following clues as they come, not following a particular order.
- We separated each one's entire playthrough into all different gaming sessions done by that player. We were not able to create a proper classification. The problem is that Dynamic Time Warping cannot erase too big differences of playtime. A player finds more information in a 5-hour session rather than in a 1-hour session, in general. We can compare entire playthrough because the total number of new information is roughly the same between subjects. This is not the case for gaming sessions

OBSERVATIONS ON ENTIRE PLAYTHROUGHS Given how small our dataset is, it is difficult to extract something else than observations about the different clustering methods applied here.

Figure 4.2 shows silhouette's evolution over the number of clusters for the different measures we defined previously : color, location and color-location trajectories. We will use the elbow method as a way of choosing a potentially more relevant number of clusters. As such, for each method, we decided to go with 5 clusters for color trajectories, and 4 for location and color-location trajectories. The hierarchical clustering based on the metaclustering of k-medoids executions is shown in Figure 4.3.

As stated previously, this does not allow us to reach a general conclusion about general strategies. However, from our semi-directed interviews, it seems that, while planet-by-planet method seems to exist, intrigue-by-intrigue methods are less apparent. We have several explanations for these :

4 Outer Wilds : Long tasks and adaptive behaviours



(a) Silhouettes for color trajectories



(b) Silhouettes for location trajectories



(c) Silhouettes for color-location trajectories

Figure 4.2: Silhouettes of k-medoids metaclustering for three different representations of players' trajectories

- Planet-by-planet methods are easily formulated by players. In pre and post-session interviews, several players stated that they were following such method :

« Aujourd'hui, je vais essayer de finir Léviathe. »

(Today, I'll try to finish Giant's Deep.)

- Intrigue-by-intrigue are less formulated by players. As they are not sure that they correspond to different, mostly unrelated intrigues, the method they formulate is to follow clues as they come. This implies that, while they could be following an intrigue for some time, they may be put on the road of another one without noticing a change (or without caring), and jump from intrigue to intrigue during the play.

« Je vais me laisser porter par le jeu. // Je me suis laissé porté par ce que je lisais. »

(I'm going to let myself be carried away by the game. // I let myself be carried away by what I read.)

4.3 Results : When insight differ from intuition

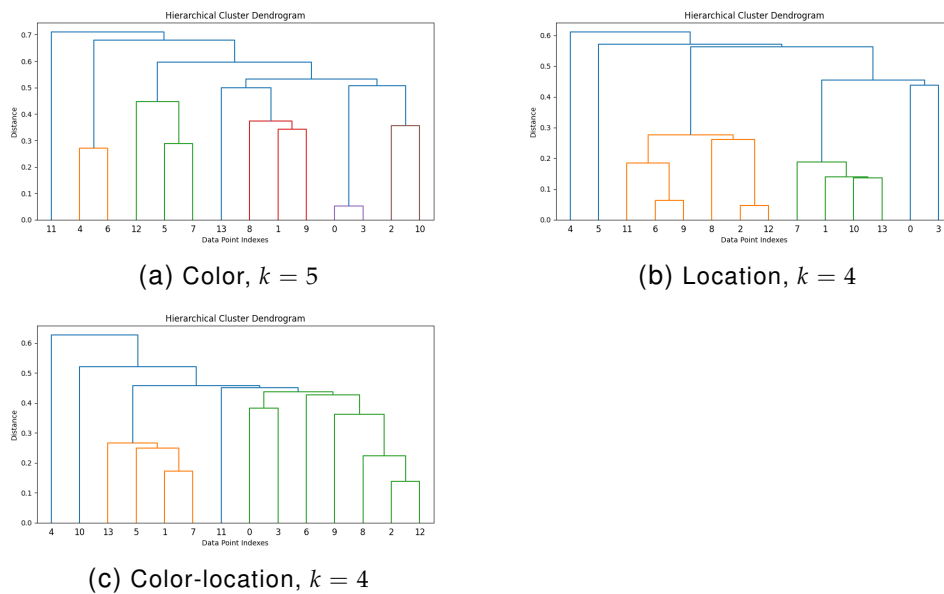


Figure 4.3: Hierarchical clustering of metaclustering on k-medoids executions for values of k determined with the elbow method

- The planet-by-planet method is easier to apply. Most locations in each general location can be accessed without needing knowledge coming from another planet. As such, it is easier to spend multiple sessions on the same planet to try to "finish" it. Even if not everything is found on a planet, moving from a planet to another brings the knowledge discovered in the previous one, allowing complete more easily the planets that would require knowledge from other ones.
- The intrigue-by-intrigue strategy is also difficult to follow with our system of annotation. Even if a player was formalizing such "wandering along the game" method, it is easy for them to stumble upon places unrelated to the intrigue that they complete, and even if they were voluntarily ignoring other intrigues, they are not usually capable of predicting if the new place they reach is related to their current intrigue or not.

In addition to this observation, we can observe that neither intrigue-by-intrigue and planet-by-planet methods were fully followed for the entirety of a playthrough. This is normal, as knowledge to solve puzzles inside one planet or for one intrigue can be obtained on other planets, or while following other intrigues. This process can be cut by insight or intuition processes, but there are multiple occurrences of such obstacles, and it greatly reduces the probability that a player finishes it following only one

of the aforementioned method from start to finish. This is similar to well-known exploration-exploitation strategies. The methods we were hypothesizing at the beginning of this section can refer to exploration strategies, alongside random exploration : there is a first phase of exploration in which a player gathers as much knowledge as possible, by following different methods (trying to investigate each planet fully, going at different places every time they reset, trying to follow the path they think the game gives them etc.) and an exploitation phase in which the player tries to complete their understanding of all of the intrigues for the completion of the game, or the completion of the ship log (a complete ship log not being required to finish the game). We will keep this observation in mind when tackling sequence cutting in section 4.3.2

OBSERVATIONS ON GAMING SESSIONS We cut our dataset of playthroughs into 154 gaming sessions of varying lengths (the players were asked to play between 40 minutes and 6 hours every time). Each session has every sequence (color or location) reset to 0. This means that we have trajectories for each individual gaming sequence. Our goal is to see if, by getting different clusters, we are able to determine if there are redundancies in, say, the beginning of each player in terms of obtained knowledge. This may lead us to understand potential session-to-session behaviours. Instead, the results showed a limit to the model that is worth discussing : the problem of high differences in terms of sequence length.

To illustrate this, we are going to use our hierarchical metaclustering method on k-medoids clustering only for color trajectories. The silhouette coefficients are accessible on Figure 4.4.

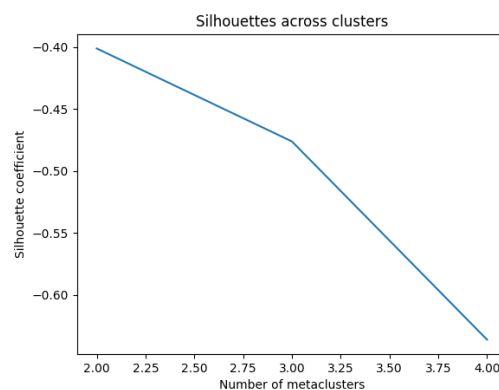


Figure 4.4: Silhouette coefficients of metaclustering on k-medoids executions for different values of k

Given how bad the silhouette coefficients are, we decided fix k to 2. Figure 4.5 shows the results we obtained by applying hierarchical clustering on metaclustering on 2-medoids executions.

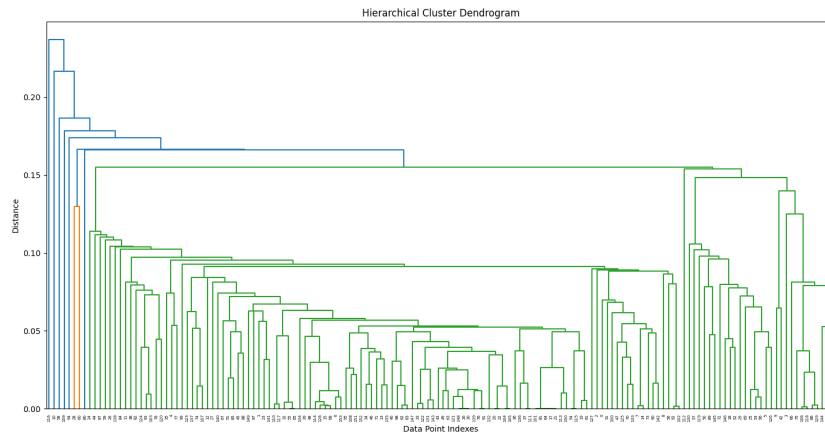


Figure 4.5: Hierarchical clustering on metaclustering for $k = 2$ on color trajectories

From this result, we should consider subsequence 116 as an entire set alone, while the rest is in the other cluster. This can be explained by cluster length. The subsequences that are clustered alone from the rest (namely subsequence 116, 10, 58 and 109) were longer than other subsequences. A typical subsequence and gaming session was around one or two hours of game time, while those sessions are all exceeding the five hours mark. This has one major implication : in a 5-hours session, a player tends to discover more knowledge than in a 1-hour one, even when erasing gameplay speed. To recall our method, Dynamic Time Warping, the algorithm we use to compare trajectories, more precisely dependent Dynamic Time Warping, allows to erase small differences in terms of speed. For instance, for creacube, two players doing nearly the same thing with a difference of 5 seconds are nearly identical in terms of DTW distance. The same is true here : if a player find in a 2-hours session what another found in a 1-and-a-half session, the DTW distance between them is small. The problem is that DTW erases only differences in execution speed, but not in terms of quantity of information obtained over time. When a 5-hours session has multiple discoveries that could not have been discovered in a 1-hour session, the difference between both sessions cannot be erased. The only comparable trajectories, in this case, would potentially be between the 5-hours session of a player and the fusion of multiple 1-hour sessions into one for another. Another

possibility would be to arbitrarily cut longer sessions into smaller ones. However, in any case, this would not correspond anymore to "session-to-session" trajectories.

To conclude, for our dataset specifically, we observed that it does not seem possible to extract session-to-session behaviours with our meta-clustering method. We will propose multiple improvements of what we've done in the discussion section 4.4.

4.3.2 TRYING TO FIND INFLEXION POINTS IN PLAYTHROUGHS

We apply the same cutting method that we used for Creacube in section 3.3.2. Our main cutting ideas were the following :

- **Cutting at the end of an intrigue** : Does the player behave differently after ending an intrigue ? Our observation tend to say no. This is not surprising. Players do not know usually that they reached the end of the intrigue. Thus, they do not change their strategies.
- **Cutting based on the time passed in the ship log** : Usually, players do not need the ship log in the first part of the game. Plenty of unexplored locations are accessible. Players can easily progress without it. In the second part of the game, finding new locations becomes difficult. This could mean a switch from an exploration phase to an exploitation phase. Our hypothesis is that players are reading the ship log more often and longer in the second part of the game. This is not true for every player.

Based on this observation, we considered that, after a given proportion of the total time spent into the ship log, we switch from the first to the second part.

We cut sequences at different given proportions because we don't know when players go from one phase to another.

However, we show that we do not find any cut that is better than its mean-time of happening. We think that this is because *Outer Wilds* is a game of intuition. **Intuition** is different than insight. Intuition is a slow process. Changes in the mental representation of the learner is progressive. This would be why there is no clear moment of switch in trajectories' shape : evolution is slow and not perceivable.

EXPLORATION AND EXPLOITATION In this section, similarly to what we did for Creacube (see section 3.3.2), we want to cut entire playthroughs into two different sequences, in order to see if the cutting point is a good indicator of a shift in strategies. Based on what we observed in the previous section, our first way of cutting sequences will be based on an understanding of exploration and exploitation. For this, we are going to use the ship log, based on what some players where explaining at the end of the playthrough.

Au début de regardais pas du tout le journal de bord, mais quand je suis arrivé à la fin et que ça devenait difficile d'avancer, j'ai commencé à le regarder pour savoir quoi faire ensuite.

At the start I was not reading the ship log at all, but when I was reaching the end and it was becoming difficult to progress, I started to read it to know what to do next.

Given this, our hypothesis is that the time reading the ship log is longer over time, because it becomes difficult to find where to explore next. This corresponds to an exploration-exploitation strategy, in which the player starts to read the ship log once the first exploratory part is done and the exploitation part begins.

Figure 4.6 shows an example of the cumulative percentage of ship log accesses over sessions. We can observe a change around session 6 or 7, from which the player spends more time inside the ship log than before.

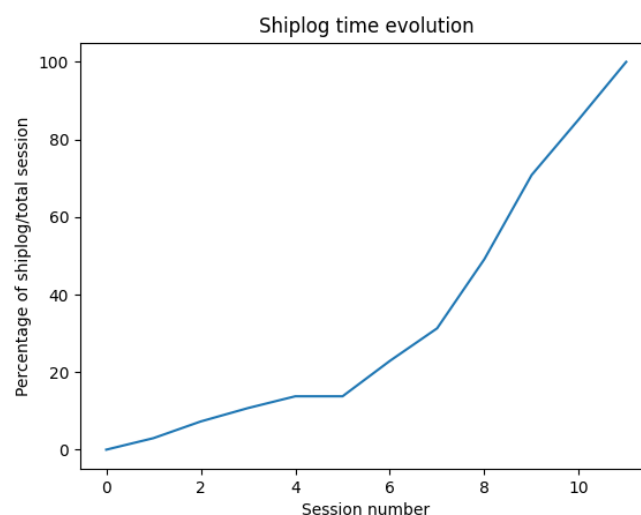


Figure 4.6: Cumulative percentage of ship log accesses over sessions

To transform this observation into a cut, we define a threshold separating what we imagine is the exploratory part and the exploitation part. This threshold may differ from one to another, so we will apply our method to multiple values of threshold, namely 20, 30, 40 and 50 %.

END-OF-INTRIGUES CUT Another idea is to cut sequences whenever a player reaches the conclusion of an intrigue (solanum, ash twin, orbital probe cannon or the vessel). Unfortunately, we face two caveats with this method :

- First, most intrigues are finished near the end of the game. From the sequences we have in our dataset, most players reach the end of the intrigues nearly at the same time, when they discovered most of all the informations there is to know in the game. This would imply the same problem as the one we faced in previous section 4.3.1 : Pre-cut sessions would be significantly longer than post-cut sessions, in terms of time and in terms of number of updates. Our method would separate pre-cut and post-cut sequences into two different groups.
- As external observers, we know that these moments are the end of an intrigue, but this may not be clear to the player, potentially leading to nothing.

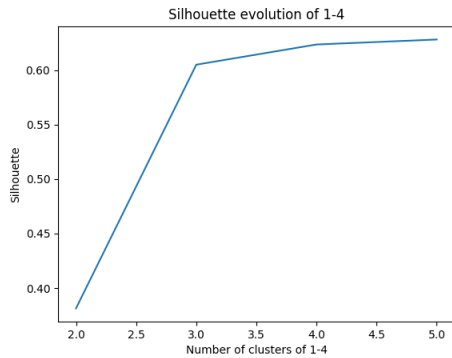
In the end, we decided to try once by cutting at the meeting with Solanum (end of the purple intrigue), in addition to other cutting methods, to be able to put other results in perspective.

STANDARD CUTS As we did in Creacube, we will create multiple cuts depending on nothing else but the time spent on the activity. For instance, sequences will be cut in half in the middle of the sequence (in terms of time played). This will allow to get standard results to compare other cuts. The different cuts will do are the following: $\frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{1}{2}, \frac{4}{7}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}$.

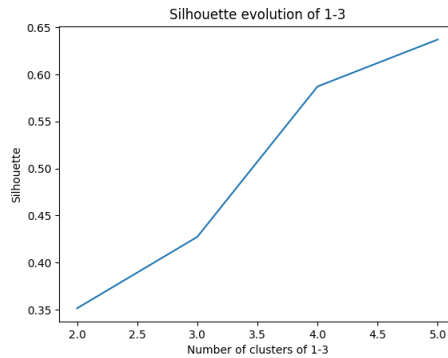
CHOOSING THE CORRECT NUMBER OF CLUSTERS For each cut defined previously, we create the evolution of the silhouette coefficient of the application of metaclustering on dataset consisting of pre-cut and post-cut sequences. As a reminder, we consider that a post-cut trajectory has

all of their sequences reset to 0, in order to compare only what happens within the trajectory to other trajectories.

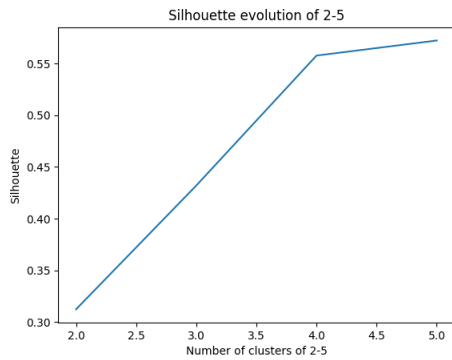
Figure 4.8 shows the set of silhouette coefficient for standard cuts.



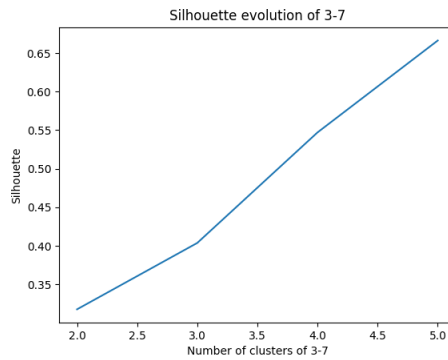
(a) Color-location, cut = $\frac{1}{4}$, choice: 3



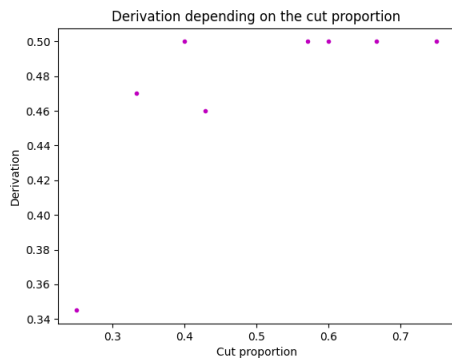
(b) Color-location, cut = $\frac{1}{3}$, choice: 4



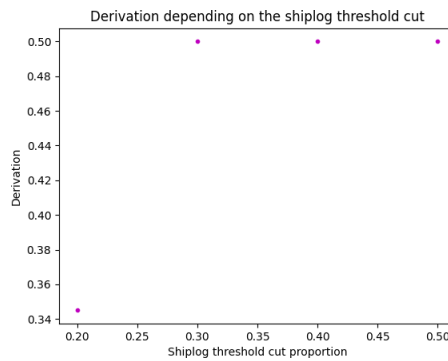
(c) Color-location, cut = $\frac{2}{5}$, choice: 4



(d) Color-location, cut = $\frac{3}{7}$, choice: 4



(a) Derivation score depending on the time cut proportion



(b) Derivation score depending on the shiplog completion threshold

Figure 4.8: Derivation scores of hierarchical metaclustering of k-medoids executions for each cut

When facing unusual value, as we want to separate roughly two categories of sequences, we opted for 2 as a value for k . We then did the same for the solanum and thresholds cuts, as shown in Figure 4.10 and 4.11

From these silhouettes, we obtain results when applying metaclustering for the chosen values of k available Figures 4.12, 4.13 and 4.14. Using the derivation method explained in section 3.3.2, we can compute the derivation for each result.

As a reminder, the derivation is an indicator of how properly separated are pre-cut and post-cut sequences : a cluster with either pre-cut sequences or post-cut sequences has a derivation of 0.5, while a cluster with an equal mix of both has a derivation of 0. The derivation scores are represented in Figure 4.8 and has an interesting shape.

In comparison to Creacube results, most of the standard cuts have a derivation of 0.5, which means that pre-cut and post-cut are successfully classified in different clusters. This is true even for cuts in half, so we can dismiss the problem of sequences of different sizes. This has two different implications :

- First, it is not possible to really compare specific cuts like threshold cuts or intrigue cuts to standard cuts as we did before, as most standard cuts allow to properly separate pre-cut and post-cut sequences. It doesn't mean that such cut cannot exist, but rather than our method cannot help to notice such cut.
- On the other hand, this also means that, based on what we see of a sequence, we may be able to determine whether it is at the beginning or at the end of a playthrough. This is interesting, as it is more related to the notion of intuition rather than insight : there could be no particular inflexion point, but rather, a diffuse evolution that leads the player from start to finish. This is coherent with the progressive understanding of the game's big picture from players: a progressive understanding on what is going on in the game, even with specific pieces locked behind puzzles locked, puzzles, that, on the contrary, necessitate a ah-ah moment to be solved.

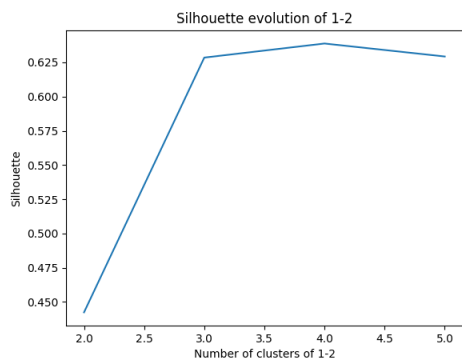
In addition, the quarter cut and the 20% threshold cut have a smaller derivation score in comparison to the rest, which means that some pre-cuts were classified inside post-cuts groups and/or vice-versa. This does not lead us to a strong conclusion, but rather an other clue of a potential exploration-exploitation strategy : the entire solar system has undiscovered knowledge at the start, the first part of a playthrough (in terms of ship log consultation or just time passed in the game) are richer in terms

of number of updates, leading to similar increasing numbers between pre-cut and post-cuts. The exploration strategy does have a decreasing value over time, as the number of unexplored areas decrease, and a random exploration cannot lead to new knowledge anymore.

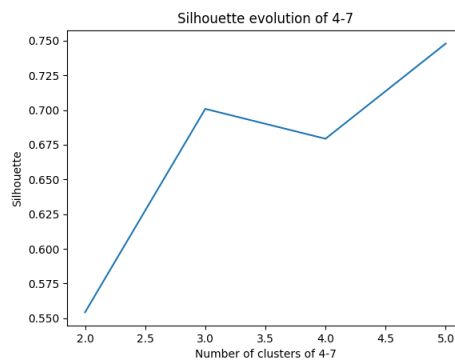
Again, this is just an observation of what could be interpreted from the results given how the game and our annotations work. Our method doesn't allow to make strong claims about what can be observed, but rather give more clues for reaching understanding of tasks and task-related behaviours.

We will now sum up our observations and discuss what could be done to improve and elaborate on our work.

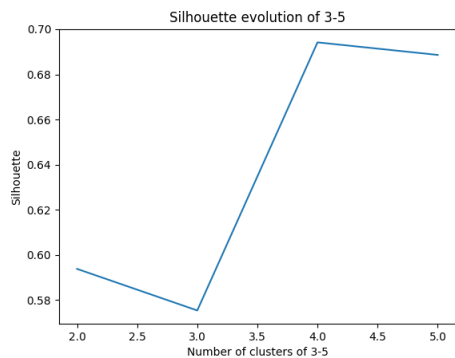
4 Outer Wilds : Long tasks and adaptive behaviours



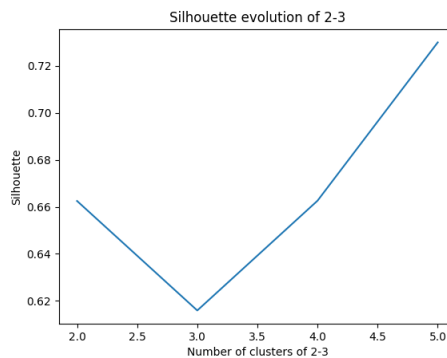
(a) Color-location, cut = $\frac{1}{2}$, choice: 3



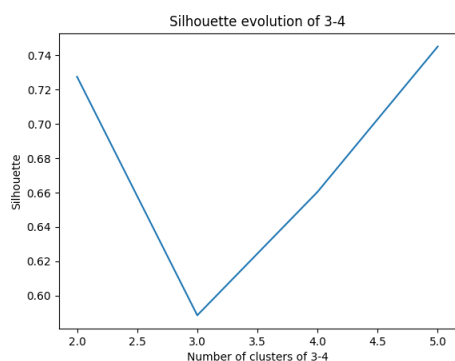
(b) Color-location, cut = $\frac{4}{7}$, choice: 3



(c) Color-location, cut = $\frac{3}{5}$, choice: 4



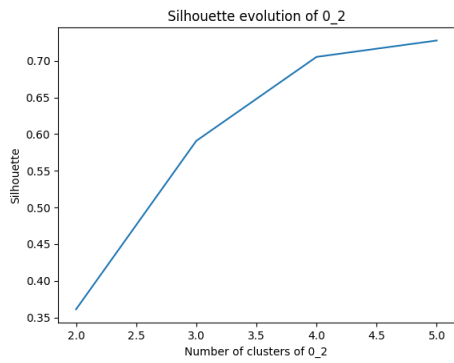
(d) Color-location, cut = $\frac{2}{3}$, choice: 2



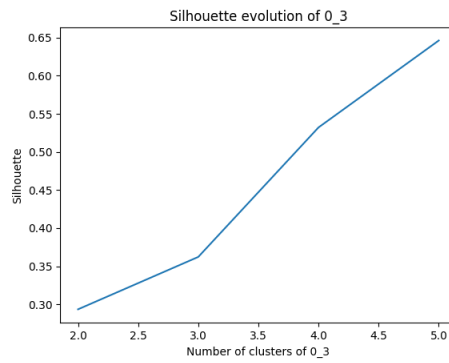
(e) Color-location, cut = $\frac{3}{4}$, choice: 2

Figure 4.9: Silhouette coefficient for multiple values of k for k -medoid metaclustering (standard cuts)

4.3 Results : When insight differ from intuition



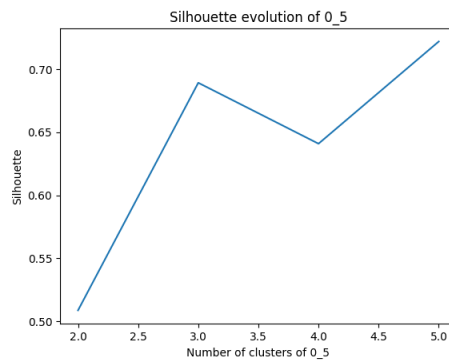
(a) Color-location, cut = 20%, choice: 4



(b) Color-location, cut = 30%, choice: 4



(c) Color-location, cut = 40%, choice: 4



(d) Color-location, cut = 50%, choice: 3

Figure 4.10: Silhouette coefficient for multiple values of k for k-medoid metaclustering (threshold cuts)

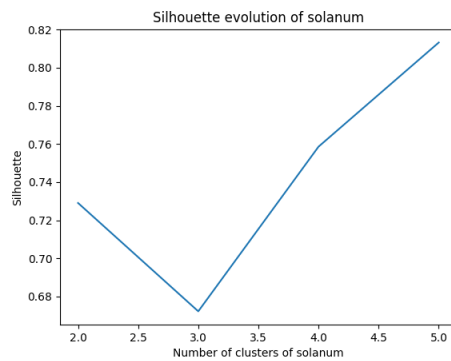
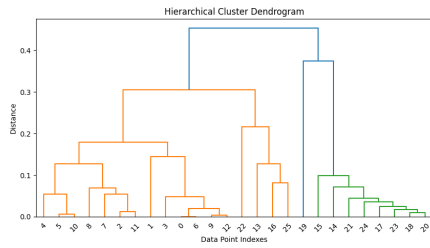
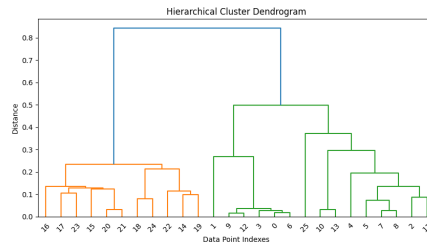


Figure 4.11: Silhouette coefficient for multiple values of k for k-medoid metaclustering (solanum cut, color-location choice: 2)

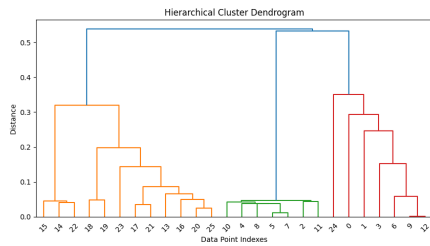
4 Outer Wilds : Long tasks and adaptive behaviours



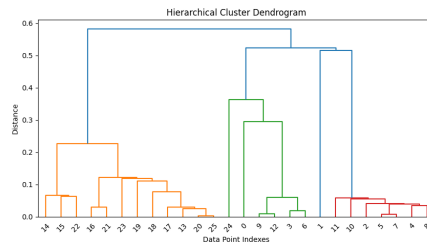
(a) Color-location, cut = $\frac{1}{4}$, k : 3, derivation : 0.35



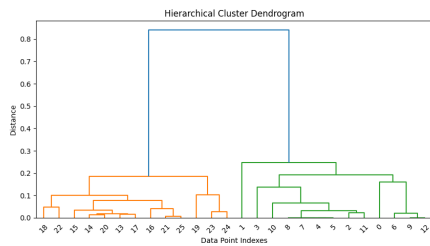
(b) Color-location, cut = $\frac{1}{3}$, k : 4, derivation : 0.46



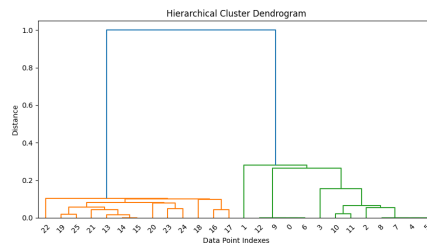
(c) Color-location, cut = $\frac{2}{5}$, k : 4, derivation : 0.5



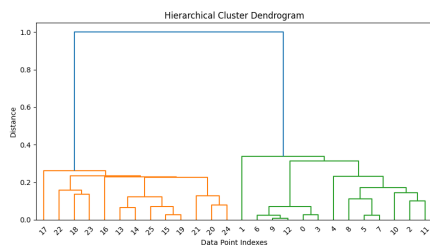
(d) Color-location, cut = $\frac{3}{7}$, k : 4, derivation : 0.46



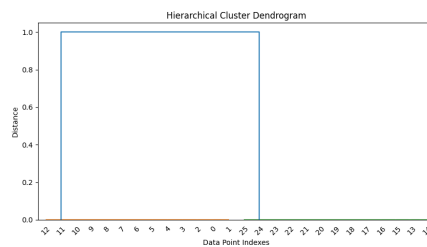
(e) Color-location, cut = $\frac{1}{2}$, k : 3, derivation : 0.5



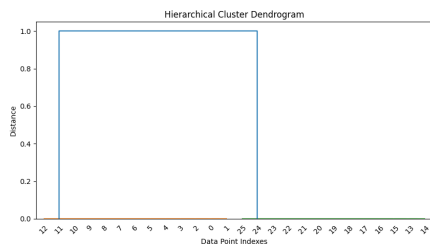
(f) Color-location, cut = $\frac{4}{7}$, k : 3, derivation : 0.5



(g) Color-location, cut = $\frac{3}{5}$, k : 4, derivation : 0.5



(h) Color-location, cut = $\frac{2}{3}$, k : 2, derivation : 0.5



(i) Color-location, cut = $\frac{3}{4}$, k : 2, derivation : 0.5

Figure 4.12: Hierarchical metaclustering of k-medoids executions (standard cuts)

4.3 Results : When insight differ from intuition

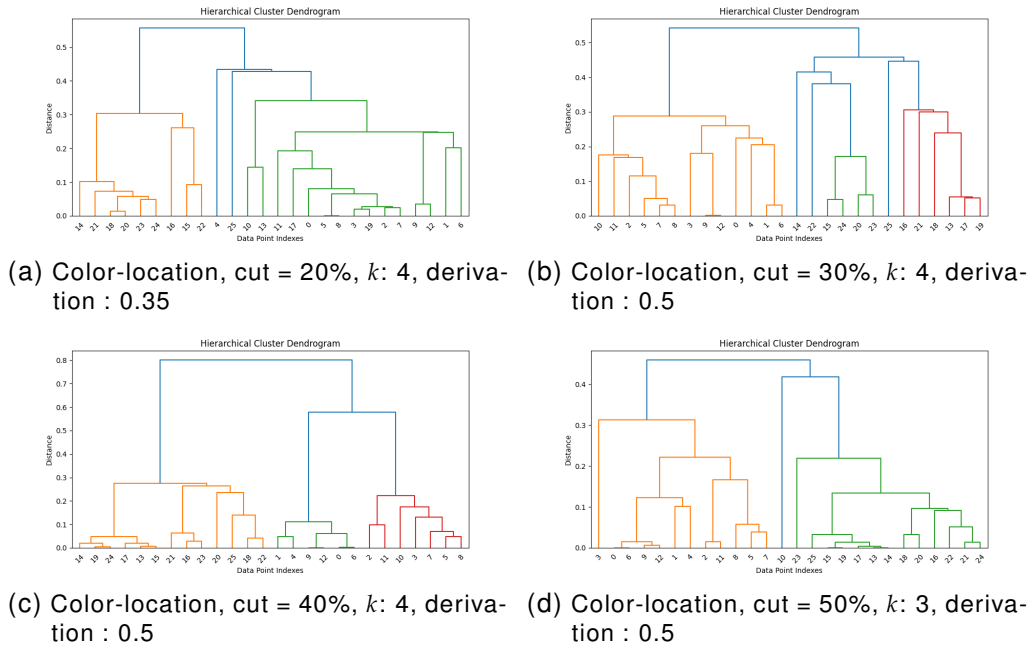


Figure 4.13: Hierarchical metaclustering of k-medoids executions (threshold cuts)

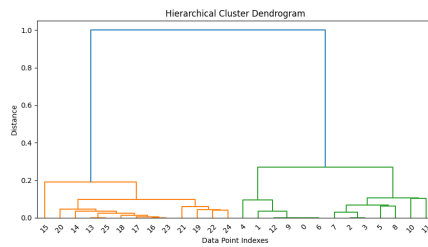


Figure 4.14: Hierarchical metaclustering of k-medoids executions (solanum cut, color-location, k : 2, derivation : 0.5)

4.4 DISCUSSION : OBSERVATIONS AND IMPROVEMENTS

- On such a small dataset, it is difficult to make preliminary observations and classify entire playthroughs of players.
- It is not relevant to use metaclustering on gaming sessions as the differences in terms of length of play is too significant between 1-hour and 5-hours sessions, the difference in execution speed being irrelevant when a player can do much less in the first one than in the second one.
- It doesn't seem that there is an insight moment at the scale of the entire game, but rather a diffuse progression corresponding to a slow understanding of the game's big picture.
- The amount of discovered knowledge in the first two or three sessions can match the amount of the entire following sessions, orienting us to an exploration-exploitation method that could be inherent to the game.

We can imagine multiple improvements to build upon this experiment. For instance, the dataset could be enriched with new points. As another idea, future experiments could limit the playtime of each gaming session.

To conclude on the analysis on our small *Outer Wilds* corpus, we have in total 4 observations that we want to discuss :

- On such a small dataset, it is difficult to make preliminary observations and classify entire playthroughs of players.
- It is not relevant to use metaclustering on gaming sessions as the differences in terms of length of play is too significant between 1-hour and 5-hours sessions, the difference in execution speed being irrelevant when a player can do much less in the first one than in the second one.
- It doesn't seem that there is an insight moment at the scale of the entire game, but rather a diffuse progression corresponding to a slow understanding of the game's big picture.
- The amount of discovered knowledge in the first two or three sessions can match the amount of the entire following sessions, orient-

ing us to an exploration-exploitation method that could be inherent to the game.

We can build on these observations to create better-suited experiments to work on the game and to judge the relevance of our model.

4.4.1 ENTIRE PLAYTHROUGHS AND LIMITED DATASETS

Our dataset was smaller than the Creacube dataset, and it didn't allow us to find explainable clusters in terms of the ways players progress through the game. A possible extension would be to expand the dataset to have access to more different playthroughs. In order to build upon our dataset, a good starting point would be to try to get different profiles of players. This could lead to a more diverse set of player behaviour. For instance, amongst high-education players of our corpus working on computer-science related topics, some of them were able to formalize easily the type of method they wanted to use and associate it with well-known methods and algorithms :

« J'ai fait une sorte de parcours en largeur. »
(I did sort of a breadth-first search.)

This cannot help us but to imagine that players less aware of the execution of such algorithms could behave differently.

Another thing to consider is the way the data was collected : Subjects were free to progress at their own pace. This has led to difficult to uneven session time, with wide ranges of time between each sessions, from half a day to entire months. We wanted to let players advance at their own pace to try to not interfere too much with motivational self-regulatory processes, however, given that this motivation self-regulatory process is already modified by the participation of the experiment, adding other constraints may not change the outcomes too much. Limiting gaming sessions to 1 to 2-hours sessions could be a proper starting point to try to substitute difference in terms of quantity of different explorations done for difference in terms of execution speed.

4.4.2 WORKING ON MULTIPLE SUBSECTIONS

The actual problem with this dataset's decomposition into gaming session is the difference in time between different sessions. We tackled

in the previous part the upgrade that can be done by having access to a better dataset, but we can also imagine different ways of cutting different sequences. We do not think that cutting sequences at each loop would be relevant, as a significant proportion of them would be empty or nearly empty, and a difference not in execution speed but in execution skill could appear more. Another possible future work could be to cut sessions in subparts of, say, one hour or one hour rolling. This could allow to suppress length differences. However, the possible conclusions are not the same, as it could be used to conclude on session-to-session behaviours, but instead on hour-to-hour behaviours. However, they may be related. A dataset of sessions of approximately the same length would allow a better exploration of our research question.

4.4.3 FINDING INTUITION ON THE MODEL AND IN THE GAME

We observed that, by cutting at different times of an entire playthrough, our metaclustering technique allowed us to properly classify pre-cut sequences and post-cut sequences into different clusters. This is partially explained by the difference in length of sequences when they are cut in extremes (for instance, isolating the last quarter of each sequence, a part in which not a lot of new knowledge is discovered, allows an easy classification) but this is not the only criterion (as seen with cuts near the middle of each playthrough). We think that having properly classified sequences between pre-cut and post-cut sequences can be explained by two other factors :

- In a best-case scenario, this can be explained by intuition, as pre-cut and post-cut sequences are each time different enough to be classified differently, without identifying an only turning point. This corresponds to intuition.
- In a worst-case scenario, this difference is partially or totally explained by the way we annotated knowledge, creating patterns of knowledge gathering that override general patterns. An example can be found in the orbital cannon that is accessible near Giant's deep in the game. It is a closed location in which most updates that can happen are green, and in addition to that, there is not only one or two new updates that can happen. If this exploration was to happen systematically at the beginning of most playthroughs, this could lead

to a classification that overrides differences in process and separate sequences on if the area was explored within the sequence or not. In this fake scenario in which the orbital probe cannon is explored at the start of each playthrough, that would imply that the metaclustering method would separate pre-cut and post-cut with disregard to the cut itself, because pre-cut sequences would systematically have a common point.

We can try to find the correct scenario by adding specific questions to pre-sessions and post-sessions semi-directed interviews, that would try to find the presence - or absence - of such intuition.

Finally, in relation to exploration-exploitation strategies, we could also modify consequently the semi-directed interviews to isolate such methods, and if so, the moment there is a shift from the first to the other.

Other hypothesis may also be tested. For instance, as for creativity, there are maybe multiple occurrences of exploration-exploitation strategies, e.g. for players exploring with a planet-by-planet method.

5 DISCUSSION AND FINAL CONCLUSION

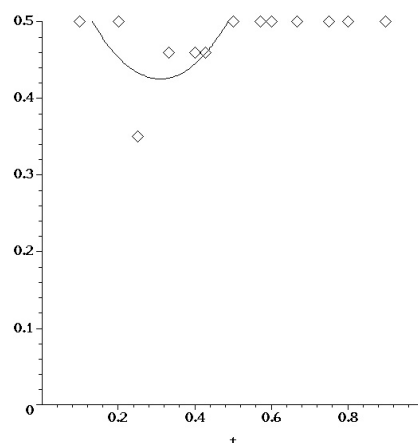
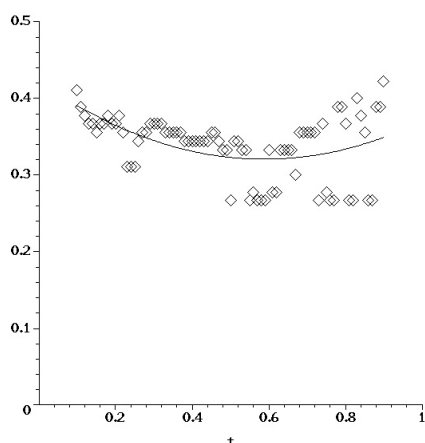
5.1 DISCUSSION

Symbolic approaches to problem solving can be put in relation with numerical approaches of problem-solving. They are usually easier to interpret. Numerical approaches are usually more powerful. Coupling them can lead to better understanding of complex problem-solving.

Our framework of multiple problem space could be used, for instance, in computer-supported learning science.

We can see two main uses of our metaclustering method based on trajectories :

- We can use it on in-development activities. The model can lead to preliminary observations. For instance, the model can help identify insight or intuition processes with the derivation score. Both cases were observed here :



Smaller parabolas could be associated to intuition. Parabolas with a high amplitude can be used to compare potential insight moments with their respective mean time of discovery.

- Trajectories can be more than just one subject at a time. We can imagine trajectories composed of multiple simultaneous subjects. This can be a way to represent collaborative learning, with a focus on changes of behaviours because of subjects' interactions.

The main goal of this thesis is to build links between computational representation and learning sciences as a way to better understand transversal skills like creativity. As we discussed in section 1.2 for creativity, there are strong links between learning science, neuroscience and computational approaches. We postulate that this tridisciplinarity can extend more generally to self-regulation mechanisms, and this thesis focused on ill-defined problem solving as a specific example of necessary regulatory skills. Studies from a computational point of view allow us to consider what are the limits of natural intelligence : for instance, the field of generative machine learning developed powerful models, including creative abilities, despite having no access to sensorimotor interactions (Harnad, 2024).

Numerical and symbolic approaches can be complementary for computational approaches. Numerical approaches can be more efficient, while symbolic representations can help with interpretability and explainability. Coupling both approaches can help for complex problem solving, and to model complex brain functions. This coupling can take different forms. In their review, d'Avila Garcez and Lamb (2023) presented different levels of couplings :

- Type 1 consists of a standard numeric computation with symbolic input/output like word2vec (Rong, 2016): Symbolic inputs are embedded in a numerical space.
- Type 2 and 3 consists of hybrid systems with symbolic and numerical interactions, like AlphaGo.
- Type 4 and 5 are compilations of symbolic knowledge into Symbolic architectures like Vector Symbolic Architectures (Eliasmith, 2013).

Our approach also consisted, by the translation of symbolic states into numerical values (DT components for Creacube and shiplog updates for Outer Wilds). A complementary approach and discussion of these topics can be found in **mercierphd2024<empty citation>**.

THE THREE-SPACE PROBLEM FRAMEWORK We proposed a framework of symbolic representation that is generic for representing problem solving. This iterative approach can help the construction of observer framework. In particular, there is a need for a separation between the material environment and the learner's internal state, because they do not require the

same level of assumption. As such, the fog of war analogy allows to consider the learner's internal state completely in a different way than the material environment. Bayesian values associated to a fixed symbolic state space is a way to represent a learner's internal state with minimal parameters. This is useful when building a general understanding on the evolution of the problem-space exploration. However, this simplicity has a major caveat : this doesn't really allow to create a symbolic representation of a learner; In such representation, there is no way of representing the knowledge of a car : this information is diluted in the bayesian values' computation, along other effects like oversight. As such, this representation doesn't construct representation of what is known but rather what can be known. Let's suppose that the notion of car is necessary, temporary oversight of this notion doesn't mean that the learner doesn't know what a car is. This implies that this model is mainly a descriptive model. Future work could build upon the model and try to infer hidden formula of Bayesian values over time, but this is way beyond the scope of this thesis.

Another advantage of spatializing symbolic spaces is the possibility to use pathfinding solutions to solve ill-defined problems. We already presented a general trajectory generation method, but this can also be a good representation for the use of pathfinding algorithms like A-star or Dijkstra for solving ill-defined problems, in addition to other machine learning methods. As proposed in Mercier, Alexandre, and Viéville (2021), we can consider a problem-solving task at a geometric level, considering being located somewhere in a symbolic state-space, with the goal of having an autonomous behaviour in interaction with an environment. We can then consider a reinforcement learning mechanism, in one of its simplest instance: Q-Learning. Considering the Q-learning mechanism is of interest to us, targeting cognitive modeling, because it is a basic ingredient of effective high-level cognitive functions such as the PROBE model and its extensions (Donoso, 2013, A. Collins and Koechlin, 2012, Domenech and Koechlin, 2015).

This framework could maybe be used efficiently in computer-supported learning sciences. Digital environments are known to improve the performance of knowledge workers (Järvelä et al., 2007), for instance by giving access to a lot of self-regulation tools (for planning, monitoring etc.). These tools help to structure and regulate collaborative learning,

however Dillenbourg and Fischer (2010) indicate that computers do not constitute effective educational tools per se. This field could benefit from another way to analyse multiple digital learning analytics as observables in our model.

METACLUSTERING AND PARTITIONING TRAJECTORIES The method developed in this thesis can be used as a visualization tool for preliminary research, as the required amount of data is small. The data collection process is very important. For instance, unless specifically targeted, sequences should contain similar amount of information. There is a risk of a trade-off between trajectory length and trajectory shape : if changes in trajectory is unbounded, sequences of different lengths could be partitioned in different clusters only because of the amount of changes in the trajectory. Such partitioning may not be relevant for behavioural study. As we saw in chapter 4, some annotations could happen differently not because of behaviour but because of player skill.

This method seems to be a good starting point to determine the presence of insight or intuition within the studied task. Let us consider both derivation scores of CreaCube and Outer Wilds. Derivation scores were used to make assumptions about the presence (or absence) of insight points, with inflexion points. The method was to create clusters on datasets created by cutting sequences in two parts, pre-cut and post-cut, the cutting point being either a proportion of total time or other indicators. If resulting clusters separate properly pre-cut and post-cut sequences, that means that there is a notable difference in trajectories between pre-cut and post-cut sequences. In this case, the derivation score can go up to 0.5 (pre-cut and post-cut sequences are perfectly clustered).

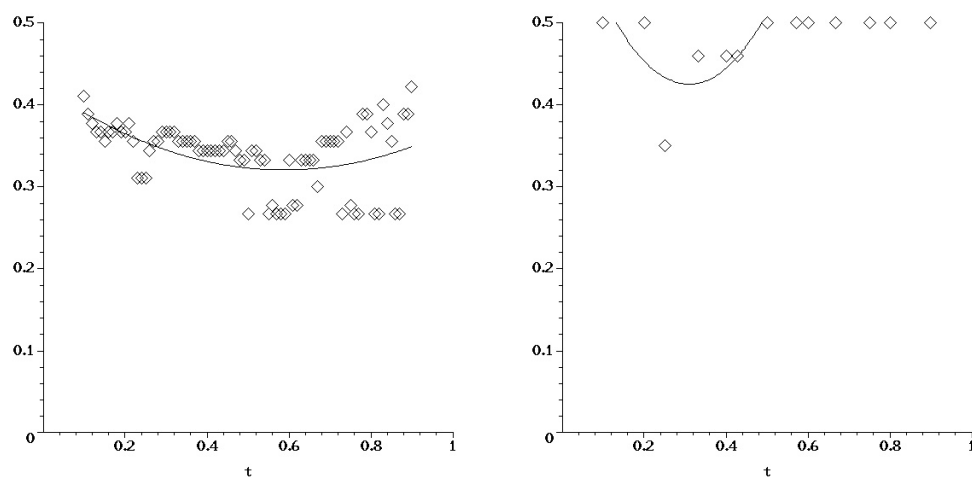
The first thing to note is that this method of metaclustering will create a parabola of derivation scores for clustered time-cut sequences :

- Time cuts near the beginning of the activity will approach or attain 0.5, as pre-cut and post-cut sequences do have incomparable lengths : the clustering method tends to create separate clusters for sequences with a big difference in length.
- The same argument applies to time cuts near the end of the activity.
- If we consider a basic case of sequences with the same trajectory evolution over time, cutting sequences in the middle can lead to

5 Discussion and final conclusion

make pre-cut and post-cut indiscernible, leading to a score of 0 or near 0.

When plotting the CreaCube corpus and the Outer Wilds corpus with derivation scores of multiple time-cuts, we can fit a parabola as shown in Figures 5.1a and 5.1b.



(a) Derivation scores and associated parabola for the Creacube corpus

(b) Derivation scores and associated parabola for the Outer Wilds corpus

Figure 5.1: Derivation scores evolution when cutting at different proportions of the total time of each sequences

From this observation, we can try to compute such parabola as a "standard deviation score evolution" and compare other derivation scores from time-unrelated cuts to the derivation score predicted on the parabola at the mean cutting time of time-unrelated time cuts. Better derivation scores are clues of possible insights, that are key moments in the sequence that change the internal representation of the learner and thus their trajectory in the problem space.

Also, the height and width of the parabola can also be an indicator of if the game does contains insights or not. In particular, parabolas with small width and height are more likely to represent activities in which there is no clear moment of behavioural change and more of a continuous process. In the case of ill-defined problem solving, a continuous process is more likely to correspond to the process of intuition, that is the slow understanding of the problem space. In the fog of war analogy, an insight would be an immediate diminution of the fog of war in a big area at a key moment, while insight corresponds more to a slow reveal of the fog of war by a continuous moving from state to state.

Using this method for this use was not its main focus when we started to develop it. However, these results completely fall into the use-case that we wanted to get by developing such model : with this model, we are capable of finding a difference in process between two ill-defined problems. This could be particularly useful for extending our research to collaborative learning : finding moments of change within a sequence could be used to find moment of co-construction and sharing of knowledge between multiple learners, each having their own trajectories (or influencing each other in one big trajectory). As our social organization today revolves a lot around classroom-wide teaching methods, being able to extend methods to collaborative learning is a step towards a better understanding on how we can better learn and teach.

5.2 PERSPECTIVE FOR COMPUTER SCIENCE AND LEARNING SCIENCE

We think that our method follows important principles for experimental design :

- Reducing the study environment
- Noting and fixing unnecessary elements of the study environment. This is done to avoid unexpected differences across experiments.

Our model is inspired partly from wargames (and by extension videogames) and AI models inspired by these domains could be considered in the future.

Our method could be extended to experiments with groups instead of just different individual subjects. For instance, we could try to use it to analyse learners' progress within a classroom while identifying key points of their understanding.

For Creacube, multiple work were done to continue this research :

- Xlim (in Poitiers) tried to automatize the annotation process of the Creacube dataset.
- Le Meudec et al. (2024) created a VR implementation of the activity. This is a new way to collect data, and could be used to train artificial agents on the problem.

For Outer Wilds, the dataset can be used for future work on it.

PERSPECTIVES OF THE MODEL OF ILL-DEFINED PROBLEM REPRESENTATION

We iteratively created a model of representation for ill-defined problems, following different ideas that are, in our opinion, in adequacy with other experiment design principles, and that we want to emphasize here:

- It is necessary to reduce the environment of the experiment to a reasonable observable state.
- Non-necessary elements of the environment for the experiment must be taken into account for study (in case they become relevant for subjects), but should be identical for all subjects to avoid unexpected behavioural differences across experiments.

We decided to call our analogy **fog of war** to represent the learner's uncertainty. This is to be put in relation with (non-necessary research) domains that consider fog of war as a concept. The fog of war is a notion corresponding to the uncertainty to military operations, and was appropriated to wargames, videogames and roleplaying (tabletop) games, once again to represent uncertainty. The state is in this case physical (corresponding to the map the player is playing in) and reveals itself during the exploration, leading to a difference of uncertainty of 3 states : the perceivable environment is certain, the unexplored environment is completely uncertain (i.e. the player has no idea of what is in a part of the map before exploring it) and the rest is in-between (i.e. the player can access the map to see where they were, but they cannot determine if, for instance, a monster to fight appeared in a previously visited zone). This relation takes us back to computer science, as we can consider in these domains which algorithms are used to simulate an AI needing to explore a space with fog of war. Such algorithms could be considered to simulate or to associate with learning patterns in the future.

PERSPECTIVES ON THE ANALYSIS METHOD In our opinion, the method developed here has the potential to be used for preliminary analysis on other experiment. The method can be adapted to other activity as a way to determine what could be the potential inflexion points of an activity. We already discussed the benefit of determining if an activity is more solved by insight or intuition. As such, this could also be a way to better design activities in general, depending on what effect is aimed for.

More generally, we can imagine the use of the method to determine if there are pivotal moments in the trajectory of a group (e.g. a classroom of students), i.e. understanding, for instance, which part of a teaching sequence is pivotal to the learning process of students. This can be an interesting addition to add to the toolbox of learning science researchers, that can be done in addition to other relevant methods of analysis. From my personal experience with computer science teachers and the learning science field, this could be used as an introductory analysis tool for teachers (mostly STEM) that are more reluctant to participate in learning science studies (because of a certain rejection of learning science methods, rejection that still exist today).

FUTURE WORK The studies of Creacube and Outer Wilds could be more detailed in the future. For instance, we could compare the use of the model on Creacube with another way to represent trajectories instead of the DT components of creativity. The same can be done for Outer Wilds, in addition to more data. We believe that this model could be extended to form a more complete preliminary tool for ill-defined problem solving analysis. The present work was bound to the exploratory research action AIDE (Artificial Intelligence for Education) and is indeed still at an exploratory stage, with numerous perspectives and only preliminary results. As for the date of this thesis, multiple works were done to continue to explore this field :

- Thanks to Xlim in Poitiers, we are looking at algorithms to automatize the annotation of the CreaCube dataset¹. This may lead to a more precise dataset, without loss of data, and even a bigger dataset.
- Thanks to Le Meudec et al. (2024)'s work, a VR implementation of Creacube is now available, allowing another way to collect data with multiple new (and maybe different) potential observables, and also to potentially train or test artificial agents on the solving of such creative problem-solving task, potentially in a more embodied way.

FINAL WORDS This present work was part of the exploratory research action AIDE (Artificial Intelligence for Education) and is indeed at an exploratory stage, with numerous perspectives and only preliminary results.

¹A first prototype can be accessed here as the result of Théo Carmes' internship

As an exploratory phase, this thesis could be considered one of the divergent thinking phase of the tridisciplinary approach developed here. We believe that this approach constitutes a promising step towards a better collaboration for a better understanding of complex problem-solving processes.

A DUAL NOTIONS IN INTERACTION DURING CPS

The Creative Problem-Solving literature produced a variety of models describing the CPS process. However, despite their differences in terminology, they are rather similar in how they decompose the process in several stages, as shown in Figure A.1.

Models	Analysis phase				Generation phase			Evaluation phase	Communication / implementation phase			
Helmholtz (1826)	Saturation				Incubation	Illumination		X	X			
Dewey (1910)	A felt difficulty	Definition and location of difficulty			Develop some possible solutions			Implications of solutions through reasoning	Experience collaboration of conjectural solution			
Wallas (1926)	Preparation				Incubation	Illumination		Verification	X			
Kris (1952)	X				Inspiration			Elaboration	Communication			
Polya (1957)	Understanding the problem	Devising a plan			Carrying out the plan			Looking Back	X			
Guilford (1957)	X				Divergence			Convergence	X			
Buhl (1960)	Recognition	Definition	Preparation	Analysis	Synthesis			Evaluation	Presentation			
Osborn (1963)	Fact-finding				Idea-finding			Solution-finding	X			
Parnes (1967)	Problem, challenge, opportunity	Fact-finding	Problem-finding		Idea-finding			Solution-finding	Acceptance-finding	Action		
Jones (1970)	Divergent			Transformation		Convergent			X			
	Search for data	Understand the problem		Pattern finding	Flashes of insight		Judgement					
Stein (1974)	X				Hypothesis formulation			Hypothesis testing	Communication of results			
Parnes (1981)	Mess finding	Problem-finding		Idea-finding			Solution-finding	Acceptance-finding				
Amabile (1983)	Problem or task presentation		Preparation		Response generation			Response validation	Outcome			
Barron and Harrington (1981)	X				Conception	Gestation	Parturition	X			Bring up the baby	
Isaksen et al. (1994)	Constructing opportunities	Exploring data	Framing problem		Generating ideas			Developing solutions	Building acceptance	Appraising tasks	Designing process	
Couger et al. (1993)	Opportunity, delineation, problem definition		Compiling information		Generating ideas			Evaluating, prioritising ideas	Developing an implementation plan			
Shneiderman (2000)	Collect				Relate			Create			Donate (communicate)	
Basadur et al. (2000)	Problem finding	Fact finding	Problem defn.		Idea finding			Evaluate and select	Plan	Acceptance	Action	
Kryssanov et al. (2001)	Functional requirements		Structural requirements		Diverge – converge at each stage			Functional solutions	Analogies, metaphors	Reinterpretation		X

Figure A.1: Creative process models, as compared by Howard et al. (2008)

Beyond the previous well-accepted dichotomy between divergent and convergent phases, authors make the distinction between other dual modes in interaction, sometimes called implicit versus explicit, or spontaneous versus deliberate, to be put in relation with stimulus-based versus goal-directed creative behaviour, likely influenced by the dual processing model of fast-thinking versus slow-thinking view of Kahneman (2011). The general idea is to differentiate explicit CPS that can be reported and consciously organized, experimented and analysed from an implicit CPS with possibly the same level of efficiency but carried out automatically

from the expression of the problem with no access to its constituents and its motivations. Cognitive science researchers agree on such distinctions, but when getting to details, their descriptions differ, leading to an established effect on the scientific community, as discussed in Zander et al. (2016). On the one hand, as summarized by Tubb and Dixon (2014) and detailed in Table A.2, there is the idea that these complementary dual-processes can all be related one to the other. Such a view is not far from the taoist concepts of yin and yang, as discussed in Deborah Frisch commentary in Stanovich and West (2000). We assume that this is an interesting integrated view, but that it should be considered as an inspired outline, while (i) both systems can act conjointly and interact, (ii) some qualities may also be related to the other system, for instance we may consciously (thus explicitly) partially control our implicit system, as discussed by Kühberger in Stanovich and West (2000).

System 1 (implicit)	System 2 (explicit)
Spontaneous mode	Deliberate mode
Associative reasoning	Rule-based reasoning
Parallel processing	Sequential processing
Holistic dialectical reasoning (understanding a system as a whole, large-scale patterns, re-enactions of such patterns)	Analytic reasoning (thinking about system parts and how they work together)
Automatic processing	Controlled processing
Machine learning numeric processing implementation	Machine learning symbolic processing implementation
Short-term reactions	Long-term planning
Relatively undemanding	Cognitively demanding
Large associative memory	Limited working memory capacity
Acquisition through biology and experience	Acquisition through cultural and formal tuition
Slow learning (quantity to extract regularities and adapt)	Faster learning (a few examples can be enough)
Fast retrieval thanks to acquisition of automatisms	Slow retrieval, requires more cognitive load
Phylogenetically evolved first	Evolved recently

Figure A.2: The S1/S2 system, reproduced from Romero et al. (2024), with contributions from Stanovich and West (2000), d'Avila Garcez and Lamb (2020) and Tubb and Dixon (2014)

On the other hand, as summarized in Tables A.3 and A.4, other dual aspects are to be taken into account which can not be projected on the S1/S2 dual axis, thus showing that cognitive processes involved in creative complex ill-defined problem solving are multi-dimensional.

Divergent / convergent thinking	These are two temporally separated phases in creativity, in fact forming a triad with the preparation phase, as analysed by Amabile (1996).
Fast thinking (S1) / slow thinking (S2)	As shown in Table A.2 and discussed by Tubb and Dixon (2014) and Augello et al. (2015). They considered that the divergent process decomposes into exploratory versus reflective mechanisms, respectively, with regard to the S1 versus the S2 system, and the convergent process decomposes into tacit versus analytic mechanisms, respectively.
Emotional / cognitive	According to Dietrich (2004), the emotional versus cognitive knowledge domain is crossed with the spontaneous versus deliberate (thus S1 versus S2) domain to produce four basic types of creativity at the cognitive neuroscience level.
Semantic / syntactic	In the study by Alexandre (2020), the standard S1/S2 distinction was discussed at the neuro-cognitive level, with different perspectives making the difference between semantic and syntactic aspects. Furthermore, semantic value was related to emotional valuation.
Exploration / exploitation	This polysemic notion has a precise definition in reinforcement learning and has been related to divergent versus convergent processes by several authors, as reviewed by Quillien (2019); to the best of our understanding, it has a different meaning. The divergent process in implicit (S1) mode was also named exploration by Tubb and Dixon (2014) and subsequent authors, and this seems to be yet another concept.
Model-free / model-based	In problem solving involving reinforcement learning, as discussed by, for example, A. G. E. Collins and Cockburn (2020), learning and decision making may involve a predictive mechanism of the environment either explicitly or implicitly (with or without a model). When considering a creative process, these aspects interact with other dimensions mentioned here, as discussed by the authors.

Figure A.3: Other dual notions encountered in CPS, reproduced from Alexandre et al. (2024) (1)

Stimulus-driven / goal-directed	Alexandre (2021) considered a systemic approach, distinguishing two different neuro-cognitive modalities, relating to the attention to external versus internal goals (van Ede et al., 2020). This is also related to top-down versus bottom-up processes, which are a relevant but polysemic notion.
Flexibility / persistence	W. Zhang et al. (2020), following Nijstad et al. (2010) and Hommel and Wiers (2017), drew a link between divergent thinking and cognitive flexibility, promoting "loose thinking" and creative thought as a process introducing more positive mood states, as opposed to convergent thinking, which is associated with cognitive persistence, for instance narrowing the focus of attention and increasing the top-down control. As discussed by A. Collins and Koechlin (2012), flexibility is related to a form of exploration (as detailed above) when referring to the choice of task sets and related actions when attempting to complete a task, while the persistence of using a given task set is related to exploitation, the link with the corresponding reinforcement learning mechanisms being made by the authors at the implementation level.
Default mode network (DMN) / central executive network (CEN)	As reviewed by Alexandre (2020) and Dietrich and Haider (2017), the DMN network is involved in the spontaneous bottom-up processing mode of creativity, while the CEN network is used in creativity that emanates from the explicit system, a deliberate top-down mode.

Figure A.4: Other dual notions encountered in CPS, reproduced from Alexandre et al. (2024) (2)

B NUMERIC SCALAR DATA

Numerical quantity related to a sensory input or an algorithm numerical value, corresponds to a bounded value (between minimal and maximal bounds), up to a given precision threshold (above which two values may differs and below which two values are indistinguishable, being equal or not), an approximate neighbourhood sampling size or "step" (below which two distinct values are in the same local area), a default value (used in initialization, or to avoid undefined value), expressed in a given unit (e.g., second, meter, etc), if any, as schematized in Figure B.1.

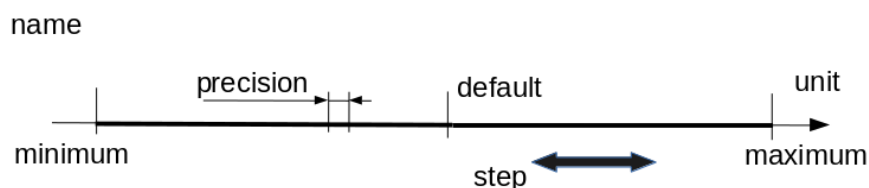


Figure B.1: Specification of a numerical value. Metadata includes a name, a unit, a default value, bounds (minimum and maximum), a precision under which two values are not distinguishable, and a step value corresponding to a neighbourhood size used to cover the value range.

Such specification is important to properly manipulate quantitative information. In particular, the values can be normalized (e.g., mapped onto the $[-1, 1]$ interval) and mapped onto a finite set of relevant values. One consequence is that algorithm precision thresholds can be deduced (often using first-order approximations), spurious values can be detected, numerical conditioning of algorithms is enhanced, and so on (see Viéville et al., 2001 for a discussion). At the computational specification level, these parameters define a sub-type of usual numerical types, yielding a better definition of the related code.

This concerns numerical sensory data and internal quantitative data (e.g., derived data, calculation output, etc.). A step further, symbolically coded data can always be sampled (e.g., a vector font drawn on a canvas and then sampled as pixels) at a given precision.

It is obvious that any quantitative measure is bounded (e.g. physical velocity magnitude stands between 0, for a still object, and the light speed) and is given up to a given precision (e.g., a localization in an image is given up to one-pixel size, a school ruler up to 1-millimeter graduation). The key point is that it is useful to make explicit this obvious fact (e.g., that any measurement device has a given precision and a measurement range) at the specification level instead of using it implicitly when required.

The notion of the positive sampling step, in order to define a local neighbourhood size, is used to weight distance calculation, and to properly sample the data space: The underlying idea is that the state space is locally convex so that in a given neighborhood local search of an optimum yields to the optimal local value. This idea has been implemented in the stepsolver variational solver, i.e., optimizer and controller.

This specification induces a pseudometric:

$$d(x, x') = \frac{|x - x'|}{step}, |x - x'| > precision \Rightarrow x \neq x',$$

in words, the distance is weighted by the *step*, i.e., the neighborhood approximate size, while if two differ by a quantity below the precision threshold, they are indistinguishable (thus either equal or not), so that we can decide if two values are different but not decide about their equality.

C SEMI-DIRECTED QUESTIONNAIRE METHODOLOGY

This appendix contains every base question of each interview performed with subjects of the Outer Wilds experiment of chapter 4. Questions are in french with an immediate translation in english. The goal of these interviews was to incite people to elaborate on their answers.

C.1 PRE-EXPERIMENT INTERVIEW

- **As-tu déjà entendu parler du jeu ? (If yes) Qu'est-ce que tu sais de celui-ci ?**
- Did you heard of this game ? (If yes) What do you know about it ?
- **Comment est-ce que tu décrirais ton profil de joueur ?**
- How would you describe the type of gamer you are ?
- **Comment est-ce que tu comptes aborder le jeu ?**
- How do you want to play this game ?

C.2 PRE-SESSION INTERVIEW

- **(Except first session) As-tu réfléchi au jeu depuis la dernière session ? A quoi as-tu pensé ?**
- (Except first session) Did you think about the game since last session ? What did you think about ?
- **Quel est ton plan pour cette session de jeu ?**
- What is your plan for this session ?
- **(Except first session) Est-ce que tu te souviens du plan donné à la fin de la dernière session ? Est-ce que tu dirais que ton plan actuel correspond à cet ancien plan ?**

- (Except first session) Do you remember the plan that you gave me at the end of last session ? Do you think that your actual plan corresponds to this previous plan ?

C.3 POST-SESSION INTERVIEW

- **Est-ce que tu peux me décrire ce qu'il s'est passé pendant ta session ?**
- Can you describe what happened during your session ?
- **De quelle manière as-tu joué au jeu pendant cette session ? (Est-ce que tu as adopté une stratégie particulière ?)**
- How did you play during this session ? (Did you adopt a particular strategy ?)
- **Comment (ré-)expliquerais-tu l'intrigue actuellement ?**
- How would you explain the story of the game so far ?
- **Est-ce que tu as suivi le plan que tu avais indiqué au début de la session ? (If no) Quelle a été ta nouvelle stratégie ? Qu'est-ce qui a déclenché ce changement ?**
- Did you follow your plan of the pre-session interview ? (If no) What was your new strategy ? What did cause the change ?
- **(Except last session) Quel est ton plan pour la prochaine session ?**
- (Except last session) What is your plan for next session ?

C.4 POST-EXPERIMENT INTERVIEW

- **Est-ce que tu peux m'expliquer ce que tu as compris des intrigues du jeu ?**
- Can you tell me what did you understand of the stories of the game ?
- **En repensant aux différentes sessions que tu as faite, est-ce que tu dirais avoir suivi une stratégie générale particulière ?**

- Looking back at your sessions, do you think that you followed a general strategy ?
- **Y a-t-il des éléments qui, selon toi, ont influencé la manière dont tu as joué au jeu ?**
- Is there anything that, according to you, influenced the way you played the game ?

D PRECISIONS ABOUT OUTER WILDS ANNOTATIONS

updates were notified by a sound and a visual cue. Each time a notification happens, it is noted with its location, color, and specific ship log update:

- For locations in which multiple updates can happen at the same time (mainly by discussing with other travelers on the different planets), updates are done in order of appearance (e.g. in dialogue), starting at the notification point, second by second. This is not exactly the moment of discovery, but the difference is in seconds. This is taken into account by Dynamic Time Warping little time differences erasure.
- Even when the update is done on the ship without a suit, there is a notification sound and visual cue on the ship board (for instance for entering giant's deep **ocean depths**).
- In case player discovers elements without the suit on (which allows notifications), multiple playthroughs and ship log consultations allow to determine what was the update moment a posteriori. This is particularly true for the beginning of the game : before getting to the ship log and the suit, the player has the opportunities to get updates without the suit. Some are mandatory (mainly updates about Timber Hearth's **village**) but other are optional, depending on some dialogue options (consultation of the piece of nomai writing in the museum for **eye signal locator**, specific dialogue with Hornfels for **esker's camp** and discussion post-statue activation with Hal for **gabbro's island**).

Table D.1 contains the list of **die** variants annotated. Table D.2 contains the list of other keys annotated. Table D.3 contains the description of locations considered for each location sequence of the Outer Wilds trajectory processing. Finally , Table D.4 contains a description of when each **enter** observable was noted.

Die variant	How to get it
Supernova	End of the loop by Sun's explosion
Retrieve	End of the loop outside of Sun's supernova's reach
Oxygen	In space or water without a suit or with no oxygen left in the suit
Fire	On campfires, Hollow's Lantern and on the Sun (via interloper, sun station or when going into the sun)
Fall	Fall from high places, but also crashes with different objects (e.g. island crashing on the player on Giant's Deep)
Crushed	Crushed by the elevator of Timber Hearth or sand on Hourglass Twins
Fish	Swallowed by an angler fish
Meditate	Go to next loop with the hidden "meditate" option
Menu	Restart the loop by quitting and reloading the save
Thorns	Damage from thorns
Electrocution	Touching Jellyfishes or Feldspar's ship
Ghost Matter	Damage from Ghost Matter
Explosion	Explosion of the ship
Time-travel	Going back in time in the Ash Twin Project

Figure D.1: Annotated die variants and how to trigger them

Annotation	How to get it
talk	Talking to an hearthian or Solanum (name added in comment of the action)
launch codes	Getting launch codes from Hornfels (mandatory)
pick DG	Picking the distortion generator (first step of end sequence)
put DG	Putting the distortion generator on the Vessel (second step of end sequence)
coordinates added	Adding the coordinates found in Giant's Depp (third step of end sequence)
win	Teleport to the Eye of the Universe (we are not interested in what happens after)
giving up	Go to next loop with the unlockable "meditate" option
ship log	Entering the ship log
end ship log	Exiting the ship log
stop session	Stopping a session
marshmallow	Roasting a marshmallow at a campfire
sleep	Sleep at a camp-fire to accelerate the passing of time
new signal	Unlocking a signal by using the signalscope near objects or people emitting signals (signal name added in comment of the action)
projection	Using a projection pool (location is the location of the player when activating it)
enter	Entering a planet or big object (see Table D.4)
teleport	Entering a planet using teleportation (location is the arrival location)
vessel	Annotated when entering the seed in which the Vessel resides.
solanum	Annotated when solanum is on screen for the first time
coordinates	Annotated when coordinates are displayed on screen for the first time
ash twin	Annotated when entering the Ash Twin project for the first time

Figure D.2: Other annotation keys and their significations

Location classes	Contains
Giant's Deep	Giant's Deep, Orbital Probe Cannon
Ash Twin	Ash Twin, Sun station
Ember Twin	Ember Twin
Timber Hearth	Timber Hearth, Attlerock
Quantum Moon	Quantum Moon
Brittle Hollow	Brittle Hollow, Hollow's Lantern, White Hole, White Hole Station
Dark Bramble	Dark Bramble
Other	Interloper, Sun, Space, Deep Space Satellite, Outer Space Nomai Satellite ¹

Figure D.3: Classification of locations for location trajectories of Outer Wilds trajectory analysis

Locations to enter	When to note it (whichever comes first)
Giant's Deep	Going through the clouds
Orbital Probe Cannon	Parking the ship / enter one of the modules
Ash Twin	Going through the visible atmosphere / parking the ship on the sand
Sun Station	Going inside it
Ember Twin	Ship touches the ground / Entering the middle hole
Timber Hearth	Ship touches the ground
Attlerock	Ship touches the planet
Quantum Moon	Going through the clouds
Brittle Hollow	Ship touches the ground / Moving below the crust
Hollow's Lantern	Ship touches the ground
White Hole	Teleportation only
Brittle Hollow	Brittle Hollow, Hollow's Lantern, White Hole, White Hole Station
Dark Bramble	Ship touches the main seed / Entering the seed
Interloper	Ship touches the ground
Other	Not noted

Figure D.4: Moments of **enter** annotations

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