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Great recessions, economic inequalities and the effectiveness of macroeconomic policies

Selma Malmberg

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L'UNIVERSITÉ DU MANS

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LA COMUE ANGERS – LE MANS

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Sciences économiques et sciences de gestion - Pays de Loire
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Par
Selma MALMBERG

Great recessions, economic inequalities and the effectiveness of macroeconomic policies

Thèse présentée et soutenue à Paris, le 26 novembre 2024

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LE MANS UNIVERSITÉ

École doctorale n°640 Sciences économiques et sciences de gestion
Laboratoire Groupe d'analyse des itinéraires et niveaux salariaux (GAINS)

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SELMA MALMBERG

Thèse de doctorat de SCIENCES ÉCONOMIQUES

Dirigée par FRANÇOIS LANGOT et PABLO WINANT

Présentée et soutenue publiquement le 26 novembre 2024 à Paris

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Introduction

A succession of major economic crises hit in the last decades, from the global financial crisis in 2008, and the COVID-19 pandemic in 2020, to the energy crisis of 2021-22, thus shaping the economies we live in. Governments reacted by injecting billions to support the economy and alleviate the rise in economic inequalities between households. However, what are the most effective (and feasible) policies to implement is still an open question. To answer them, we need to understand how macroeconomic policies –whether fiscal or monetary– work and through which channels.

Contemporary macroeconomic analyses mostly rely on dynamic stochastic general equilibrium (DSGE) models. Policies are evaluated under this framework, which assumes rational agents and generally adds new Keynesian frictions (NK) on prices or wages in the short run. Crises are then modeled as exogenous shocks affecting the economy.

One limitation of those traditional DSGE models is the representative-agent assumption. Crises do not affect uniformly the population. Rather, one’s ability to buffer the negative impacts of the crisis depends on one’s wealth, income, and employment status for instance. To account for this, heterogeneity was introduced in the seminal works of Bewley, 1983, Imrohoroglu, 1989, Aiyagari, 1994, and Huggett, 1993. As it requires keeping track of the distribution of households, it increases the size and complexifies the resolution of such models.

However, recent methodological improvements such as Reiter, 2009, Reiter, 2010, Auclert, Bardóczy, Rognlie, and Straub, 2021, and Achdou et al., 2022, made the joint analysis of macroeconomic variables and inequalities not only feasible but opened the way to a broad literature of heterogeneous-agent (HA) models.

This thesis follows this tradition, examining the joint effects of public policies on inequality and macroeconomic aggregates. This is all the more significant, that recent political events, such as the *Yellow Vests protests* in France, showed that economic inequalities were at the center of the concerns and motivations for the social unrest¹. Policies should not only stimulate growth but also assess equity. With the available quantitative tools, we can build models that replicate the heterogeneity in incomes and savings and provide clear guidance on policies that simultaneously address macroeconomic stability and inequality reduction.

¹See Algan, Beasley, et al., 2019.

Summary of the chapters

This thesis relies on four research papers, corresponding to the four chapters. The central point that links these works is the attempt to evaluate macroeconomic policies –in the French context– through the lens of heterogeneous agent models.

Chapter 1: Solving and estimating a heterogeneous-agent model with aggregate shocks

This first chapter summarises the process for solving a DSGE model with heterogeneous agents, on which all the following chapters of this thesis build. The procedure includes 1) solving the stationary state, 2) obtaining the model (linear and non-linear) impulse response functions (IRFs), and 3) estimating the parameters. It is presented step by step for a reference model, a New Keynesian Heterogeneous Agent (HANK) model with one asset and endogenous labor. Compared to the predictions of an equivalent representative-agent model, this framework allows for the simultaneous analysis of the macroeconomic and inequality consequences of aggregate shocks. In particular, the same shock has very heterogeneous implications for different types of agents.

Chapter 2: The macroeconomic and redistributive effects of shielding consumers from rising energy prices: a real-time evaluation of the French experiment

The French government implemented an energy tariff shield in 2021 to mitigate the impact of rising energy prices. This paper assesses the macroeconomic and redistributive effects of this policy using a new Keynesian business cycle model with heterogeneous agents. From a macroeconomic perspective, this policy boosts economic growth and reduces inflation but slightly increases the debt-to-GDP ratio. Regarding redistribution, the policy curtailed the increase in consumption inequality. We compare the outcomes of this policy with a redistributive policy targeted at the most vulnerable households. We also test its robustness by implementing it in different contexts, such as a labor market where wages are re-indexed to prices, with a less accommodative monetary policy, or even with a government that finances it by distortive taxes.

Chapter 3: Fiscal consolidation and debt sustainability in a HANK model

This chapter uses a Heterogeneous-Agent New-Keynesian (HANK) model to evaluate different fiscal consolidation policies. It shows that a significant reduction in the debt-to-GDP ratio can be achieved without penalizing GDP growth or worsening inequalities if the government sharply cuts social insurance-based transfers while increasing social assistance transfers. From a methodological point of view, this chapter develops a method to perform stochastic debt-sustainability analysis (SDSA) based on an estimated HANK model. The conditional forecast method is employed to identify and estimate the sequences of macroeconomic and policy shocks that underpin government forecasts provided by the Finance

Act. Using this estimated distribution of structural shocks, the benefits of fiscal consolidation policies can also be measured in terms of reducing the risk burden on public debt. The proposed policy lowers both the projected debt-to-GDP ratio in 2027 (to 105.6% against 108.3% in the Finance Act) and the risk of debt increase (there is a 25% probability of exceeding 112.6% under this policy against 116.2% under the Finance Act).

Chapter 4: Income tax fluctuations and uncertainty in France

Income tax (and income-tested benefit policies) in France have fluctuated considerably over time, mainly driven by changes in presidential leadership that I document using micro-level data. I then study the macroeconomic consequences of these fluctuations and the resulting fiscal uncertainty using a simple model of heterogeneous agents calibrated on French data. Aggregate tax uncertainty is introduced through a three-state stochastic process. This framework shows that income tax uncertainty generally generates recessive effects, as labor supply, output, and welfare decrease compared to a deterministic setup.

Chapter 1

Solving and estimating a heterogeneous agent model with aggregate shocks

Keywords: Heterogeneous agents, Computational economics.

JEL codes: C60, E21, E32, E52.

1.1 Introduction

How to solve a heterogeneous agent model? What are the most efficient methods available? This paper proposes an introduction to heterogeneous agent models by reviewing step by step the process for solving such macroeconomic models from the stationary equilibrium and impulse response functions (IRFs) following aggregate shocks to parameters' estimation. It provides the necessary state-of-the-art tools to start, explores some limitations of that literature along with potential research avenues to address these challenges.

Since the seminal works of Aiyagari, 1994, Krusell and Smith, 1998, a rich literature of models with heterogeneous agents emerged. The former is a general equilibrium model with incomplete markets where households are assumed to be *ex-ante* identical, differing only in their histories of idiosyncratic shocks they face. Krusell and Smith add aggregate risk to this benchmark heterogeneous-agent framework. It is thus possible to study the impact of aggregate shocks on the economy and extend the analysis of the standard real business cycle or New Keynesian literature that previously assumed representative agents (DSGE models).

Heterogeneous agent (HA) models rest on the idea that the heterogeneity observed between economic agents in microeconomic data has consequences at the macroeconomic level. The predictions of the models assuming representative agents may thus be modified when switching to a heterogeneous framework. The impact of macroeconomic policies may also differ. Kaplan, Moll, and Giovanni L. Violante, 2018 give an example of this. Their

two-asset Heterogeneous Agent New Keynesian (HANK) model predicts that most of a rate cut's effect stems from an increased labor demand rather than intertemporal substitution (as advocated by RANK models, for Representative Agent N-K models). Additionally, Auclert, Rognlie, and Straub, 2024 show that the policy implications between the two classes of model differ when fiscal policy is financed through deficit.

Do we need heterogeneous agent models? As Cochrane points out in his blog post “New York Times on HANK, and questions” (2023), the social welfare theorem indicates that the behavior of economic aggregates can be deduced from a representative agent's decisions. That said, the behavior of this representative agent is unlikely to be the same as the behavior of a household at the microeconomic level due to distributional considerations. However, keeping the heterogeneity (instead of aggregating it in a RA) may prove useful, especially in situations where the heterogeneous balance sheets play a bigger role. Rognlie mentions such a situation: the persistent effect of fiscal stimulus can be explained by the delayed consumption of this stimulus by the richest agents who initially saved it.

In practice, adding heterogeneity means the distribution of agents, which is potentially an infinite-dimensional object, should be tracked to retrieve the dynamic evolution of macroeconomic variables. It is one of the main challenges of this literature. Krusell and Smith, 1998 argued that a few moments of the wealth distribution (for instance, its mean) are sufficient to approximate well the macroeconomic aggregates in their HA model. When this property holds, the models featuring idiosyncratic and aggregate risks are easier to solve. However, the caveat is that such models feature properties of macro variables that are barely unchanged compared to the representative-agent framework. In particular, this seminal paper features a low degree of heterogeneity: very few households are constrained, while the vast majority have linear policy functions in asset holdings.

This approach thus does not generalize well to all HA models, especially those where the distribution plays a more significant role or when the dynamics are more complex than a basic RBC model due to price and wage rigidities for instance as in NK models *à la* Smets and Wouters, 2003, (2007). Increasing the degree of complexity increases the risk of the model being impossible to solve even numerically. Indeed, the object describing the distribution of agents then grows significantly in size. That said, many improvements allow us to solve increasingly more complex models. Therefore, HA models can offer a proper alternative to DSGE models for performing policy evaluations in a rigorous framework.

Regarding theoretical considerations and computational improvements for the resolution of HA models, two branches of the literature emerge, the continuous-time approach and the discrete-time approach. In continuous-time, Kolmogorov forward equations (also known as Fokker-Planck equations) describe the distribution's evolution. It is the approach followed for instance by Achdou et al., 2022 and Kaplan and Giovanni L. Violante, 2018. The latter proposes a quantitative method for solving such models. This paper, however, focuses on discrete-time models. Whether to solve a continuous-time or discrete-time model is largely a matter of preference, since Rendahl, 2022 showed that both approaches can achieve comparable computing times.

Reiter, 2009 proposes a very efficient method to solve discrete-time HA models. First, the model without aggregate shocks is solved in the stationary state. Then, the first-order

perturbation of the previous solution in these shocks allows getting the overall IRFs. Following this approach, one gets a solution of an HA model that is linear in aggregate shocks and nonlinear in the idiosyncratic shocks. This method is core since the following improving algorithms build on it. Most recent developments have sought ways to reduce the dimension of heterogeneity (i.e. reduce the size of the linearised system of equations) to get approximated solutions (Reiter, 2010, Winberry, 2018, Bayer and Luetticke, 2020, Ragot, 2018).

This paper displays Auclert, Bardóczy, Rognlie, and Straub, 2021’s approach to solving the dynamic response to aggregate shocks of discrete HA models in perfect foresight. They propose a quick and efficient way to compute the (sequence space) Jacobian of the linearised system around its steady state. This computation leverages the recursive properties of the value function and distribution, as well as the decomposition of the problem into several blocks. With this method and the associated Python toolbox, heterogenous-agent models can be solved without the approximate aggregation requirement and in just a few seconds. Boehl, 2023 extends the sequence space approach to allow for strongly nonlinear dynamics, in particular, when facing the zero lower bound (ZLB) or a downward nominal wage rigidity (DNWR). Note that the Auclert, Bardóczy, Rognlie, and Straub, 2021 and Boehl, 2023 methods do not explore in any way the reduction of the infinite-dimensional distribution of agents into a few statistics. However, both approaches can probably be used together to solve more sophisticated models.

Finally, I present a method for estimating HA models. When facing a state-space representation of a linearised Dynamic Stochastic General Equilibrium (DSGE) model, it is easy to get the likelihood using the Kalman filter. As a result, standard estimation techniques can be extended to HA models under state-space representations (see Winberry, 2018, for instance). Auclert, Bardóczy, Rognlie, and Straub, 2021’s method uses a sequence-space representation instead. We thus need to evaluate a closed-form expression to get the likelihood as advocated by Mankiw and Reis, 2007. Auclert and coauthors suggest reusing the Jacobians computed when solving for the dynamics of the model, for efficient and fast estimation. Once the likelihood function is computed, standard maximum likelihood or Bayesian techniques can be applied. An alternative estimation technique is also considered, the method of simulated moments.

In the same spirit as Adjemian et al., 2024’s Dynare which encompasses a set of tools to analyze DSGE models, this paper presents some necessary tools to solve and estimate a model with heterogeneous agents. Thanks to these tools, policy evaluations can be performed and new macroeconomic policies thought of using that type of model. As Sargent, 2023 states “[the] HANK revolution is not about tools but about substance”. Indeed, putting inequalities at the heart of macroeconomics, can lead to changes in the way macroeconomic policies are conducted: What redistribution do they involve? Should the objectives only be in terms of aggregate variables or should they also take inequalities into account?

The structure of this paper is as follows. Section 1.2 presents the general formulation of a heterogeneous-agent model and introduces the illustrational model that will be used throughout this work. Section 1.3 focuses on obtaining the stationary state of this model with a discussion on how to solve the microeconomic problems of the heterogeneous agents. Section 1.4 introduces a way to solve the dynamic macroeconomic problem

and compute the linear impulse response functions before estimating the model (Section 1.5). Finally, Section 1.6 discusses non-linear resolution methods, which are useful, for example, when a perfect-foresight transition between two stationary states goes through a highly non-linear regime period such as a Zero Lower Bound (ZLB).

1.2 Heterogeneous-agent model

1.2.1 Benchmark model

Before presenting the general formulation of a HA model, we introduce a benchmark model that allows macroeconomists to study the impact of monetary policy: a one-asset New Keynesian model with endogenous labor model *à la* Galí, 2015. This model is also very close to Auclert, Bardóczy, Rognlie, and Straub, 2021's one-asset HANK model. Appendix 1.8.1 fully describes the benchmark model.¹

A continuum of identical households faces idiosyncratic income shocks. Markets are incomplete as households can only save on one asset, a government bond. It results in only partial insurance against the risks they face. A household with productivity e_t and wealth holdings b_{t-1} at time t solves this recursive program:

$$\begin{aligned} v_t(e_t, b_{t-1}, \zeta_t) &= \max_{c_t, n_t} u(c_t, n_t, \zeta_t) + \beta \mathbb{E}_t v_{t+1}(e_{t+1}, b_t, \zeta_{t+1}) \\ \text{s.t. } c_t + b_t &= (1 + r_t)b_{t-1} + w_t e_t n_t - \tau_t(e_t) + d_t(e_t) \\ b_t &\geq 0 \end{aligned} \quad (1.1)$$

where we allow for an exogenous shock on preferences ζ . Both the dividends received d_t and taxes τ_t paid could be functions of the productivity state.

Introducing the joint distribution of households μ_t (across wealth b_{t-1} and idiosyncratic productivity states e_t), we can define aggregate variables:

- Asset demand: $\mathcal{A}_t \equiv \int b_t d\mu_t(e_t, b_{t-1})$
- Effective labour supply: $\mathcal{N}_t \equiv \int n_t e_t d\mu_t(e_t, b_{t-1})$
- Consumption: $\mathcal{C}_t \equiv \int c_t d\mu_t(e_t, b_{t-1})$

The production side of the model features a continuum of firms (indexed by j) in monopolistic competition. Each intermediate firm produces one good using a production function $y_{jt} = Z_t n_{jt}^{1-\alpha}$ (where n_j denotes the number of effective hours bought by firm j) and manages to charge a markup. The implied aggregate production thus is $Y_t \equiv \int_j y_{jt} dj = Z_t \int_j n_{jt}^{1-\alpha} dj = Z_t N_t^{1-\alpha}$. Besides, they face a nominal rigidity on prices following the New

¹Auclert, Rognlie, and Straub, 2024 advocate a HANK model with flexible prices and sticky wages as a canonical model instead to counter unusual countercyclical profits following a demand shock that may arise otherwise. I present the flexible wage, sticky price model, which is closest to the textbook RANK.

Keynesian tradition. Solving the firms' problem yields a New Keynesian Philipps curve, a labor demand function and an expression for the dividends paid to households (see Appendix 1.8.1).

To close the model and have a general equilibrium, we introduce a monetary policy and a fiscal authority (or a central bank and a government). The central bank sets the nominal interest rate i_t following a Taylor rule while the real interest rate r_t follows a Fisher rule.

$$\begin{aligned} i_t &= i_{ss} + \phi_\pi \pi_t + \phi_y (Y_t - Y_{ss}) + \varepsilon_t \\ 1 + r_t &= \frac{1 + i_{t-1}}{1 + \pi_t} \end{aligned}$$

Meanwhile, the government issues debt (which is the assets held by households) and defines the tax rate. The tax rate value is consistent with the tax revenues covering the interest repayment of the debt and government spending G .

$$\tau_t = r_t B + G^2$$

The economy may face several types of aggregate shocks: for instance, a productivity shock Z_t , a monetary shock ε_t , a government spending shock G_t , a public debt shock B_t , a firm's markup shock m_t or a preference shock z_t .

A system of nonlinear equations summarises this entire model:

$$\mathbf{H}_t(\mathcal{Y}, \mathcal{Z}) \equiv \begin{pmatrix} Y_t - Z_t N_t^{1-\alpha} \\ Y_t - \left[1 - \frac{m}{m-1} \frac{1}{2\kappa} (\log(1 + \pi_t))^2\right] - w_t N_t - d_t \\ r_t B + G - \tau_t \\ i_{ss} + \phi_\pi \pi_t + \phi_y (Y_t - Y_{ss}) + \varepsilon_t - i_t \\ 1 + r_t - \frac{1+i_{t-1}}{1+\pi_t} \\ \kappa \left(\frac{w_t}{(1-\alpha)Z_t N_t^{-\alpha}} - \frac{1}{m} \right) + \frac{1}{r_{t+1}} \frac{Y_{t+1}}{Y_t} \log(1 + \pi_{t+1}) - \log(1 + \pi_t) \\ \mathcal{A}_t - B_t \\ \mathcal{N}_t - N_t \end{pmatrix} = 0 \quad (1.2)$$

with $\mathcal{Y} = (Y, N, r, w, d, \pi, \tau, i)'$ gathering the time series of unknown aggregate variables and $\mathcal{Z} = (Z, \varepsilon, G, B, m, \zeta)'$ of exogenous aggregate shocks. Solving this system gives the equilibrium of the studied economy. Auclert, Bardóczy, Rognlie, and Straub, 2021 refer to this as a formulation *in sequence-space*, as opposed to the standard *state-space formulation* of a HA problem. It corresponds to the stacking of all equilibrium equations at any date t up to infinity.

1.2.2 General formulation

What differentiates a HA model from the RA framework? The presence of individual state variables in the households' Bellman equations and the aggregation over the distribution

²Note that in the benchmark calibration, $G = 0$ to match Galí's simple New-Keynesian model. I relax this in Section 1.6.

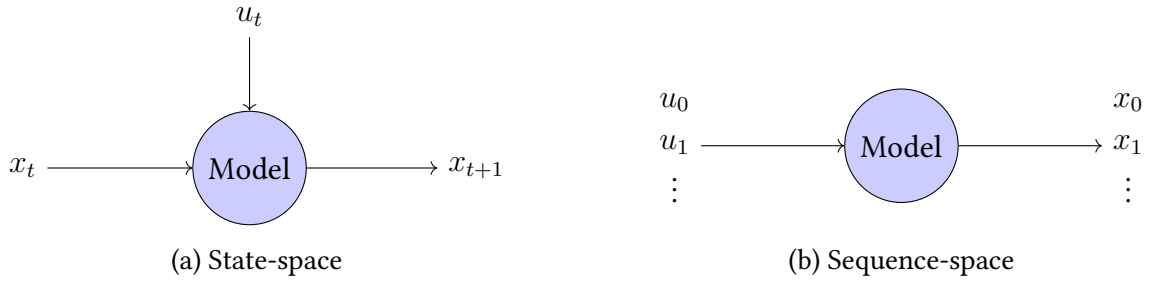


Figure 1.1: State-space and sequence-space formulations of a model.

of agents to get the macroeconomic variables.

Following the presentation of Bayer and Luetticke, 2020, let's introduce aggregate and idiosyncratic variables (both endogenous and exogenous) that characterize the model:

- Aggregate state variables (except for the distribution of agents μ_t): $S_t = (\mathcal{Z}_t, \mathcal{Y}_t)'$
- Exogenous aggregate state variables: \mathcal{Z}_t
- Endogenous aggregate state variables: \mathcal{Y}_t
- Idiosyncratic state variables: $s_t = (e_t, y_t)'$
- Exogenous idiosyncratic state variables: e_t
- Endogenous idiosyncratic state variables: y_t
- Control variables: x_t ³

The exogenous variables are assumed to follow stationary Markov chain processes. Then, a HA model is defined as:

Definition 1 (Heterogeneous-agent model). *A heterogeneous agent model is the result of the combination of:*

- A dynamic programming functional equation:

$$v_t(s_t) = \max_{x_t} u(x_t; s_t, S_t, \mu_t) + \beta \mathbb{E}_t v_{t+1}(s_{t+1})$$

s.t. Budget constraint for x_t

- Aggregation rules for selected macroeconomic variables $Y_{it} \in \mathcal{Y}_t$:

$$Y_{it} = \int y_{it} d\mu(e_t, y_t)$$

³Using those notations for the benchmark model leads to: $e_t = e_t$, $y_t = b_{t-1}$, $x_t = c_t$, $\mathcal{Z}_t = (Z_t, \varepsilon_t, G_t, B_t, m_t, \zeta_t)'$ and $\mathcal{Y}_t = (Y_t, N_t, C_t, r_t, w_t, d_t, \pi_t, \tau_t, i_t)'$.

- *The law of motion of the distribution of agents:*

$$\mu_{t+1}(e_{t+1}, y_{t+1}) = \sum_{e_t} \mu_t(e_t, y_t) \mathbb{P}(e_t \rightarrow e_{t+1}; S_t)$$

- *Prices $P_t \in S_t$ that are consistent with the production side of the economy.*
- *Market-clearing conditions.*

This heterogeneous-agent model can be summarised under Auclert, Bardóczy, Rognlie, and Straub, 2021's notation (a general formulation of equation 1.2):

$$\mathbf{H}_t(\{\mathcal{Y}_s\}, \{\mathcal{Z}_s\}) = 0. \quad (1.3)$$

This system of nonlinear equations gathers the equilibrium conditions induced by the various actors in the economy, the market-clearing conditions, the evolution of the distribution of agents, as well as, the definitions of aggregate variables.

Now that a HA model is defined, stationary equilibria result. It assumes no aggregate risk but only individual risks.

Definition 2 (Stationary equilibrium). *A stationary equilibrium is given by a value function v^* , a distribution μ^* , policy functions x^* and prices P^* such that:*

- *The policy functions maximize the Bellman equation of the households given the prices.*
- *The value function solves the Bellman equation given the policy functions.*
- *All markets clear.*
- *The distribution of agents is stationary.*

When adding aggregate shocks, the value function and the distribution are no longer time-invariant. The resulting sequential competitive equilibria are defined as follows:

Definition 3 (Sequential competitive equilibrium). *A sequential competitive equilibrium is given by sequences of value function $\{v_t\}_{t=0}^T$, distribution $\{\mu_t\}_{t=0}^T$, policy functions $\{x_t\}_{t=0}^T$, prices $\{P_t\}_{t=0}^T$ and aggregate state variables $\{S_t\}_{t=0}^T$ such that in each period t :*

- *The policy functions x_t maximise the Bellman equation of the households given the prices P_t .*
- *The value function v_t solves the Bellman equation given the policy functions x_t and the expected value of v_{t+1} .*
- *All markets clear.*
- *The distribution of agents μ_{t+1} is consistent with its previous value μ_t , the households' policy functions, and the evolution of exogenous variables.*
- *Aggregate state variables are induced by a law of motion that exploits the stochastic properties of the exogenous variables and the aggregation ones of endogenous variables.*

1.2.3 Roadmap for solving

Now that both the stationary and dynamic equilibria are defined, the model can be solved. Several approaches are available to solve such heterogeneous-agent models. Algan, Allais, et al., 2014 review some of them, whether they rely on projection or perturbation techniques. In accordance with the method first presented by Reiter, 2009 (on which Auclert, Bardóczy, Rognlie, and Straub, 2021 build their work), the model is solved in two steps:

1. Solve for the stationary state (no aggregate shocks but idiosyncratic ones).
2. Get the dynamic response as a first-order perturbation in the aggregate shocks around the stationary state.

The resulting solution is linear in aggregate shocks yet accounts for the nonlinearities in idiosyncratic shocks that matter for some households (such as borrowing constraints, for instance). The next section now deals with how to solve the stationary state.

1.3 Stationary state

1.3.1 General procedure

The process for solving the stationary state of a HA model is:

1. Fix the values of the exogenous shocks \mathcal{Z} at the stationary state values.
2. Guess the values of certain variables. For instance, make as many guesses as there are market-clearing conditions (minus one given the Walras law).
3. Solve the microeconomic problem of the households. Compute the resulting macroeconomic variables by aggregating them across the distribution of agents.
4. Check for the market-clearing conditions. If they are not satisfied, update the guesses made in step 2 and iterate.

This procedure would be the same in a representative-agent framework except for step 3 (since there is no need to solve multiple household programs and aggregate their decisions to get aggregate variables). Figure 1.2 summarises this procedure visually. In particular, solving the microeconomic problem of an individual household given prices is standard. Indeed, any global method is relevant, from value function or policy function iterations to time iteration. Provided that the problem is smooth enough to ensure differentiability and the sufficiency of the first-order conditions, time iteration can be used, exploiting the Euler equation instead of the Bellman equation. Many techniques exist that help quickly and efficiently solve such problems when using global approaches. I will briefly discuss them in the next subsection but note that all these tricks to solve the household problem

efficiently are standard and directly imputable from previous work on representative agent models (see Stokey, R. E. Lucas, and Prescott, 1989, Ljungqvist and Sargent, 2018, Judd, 1998, Stachurski, 2009, Perla, Sargent, and Stachurski, 2020).

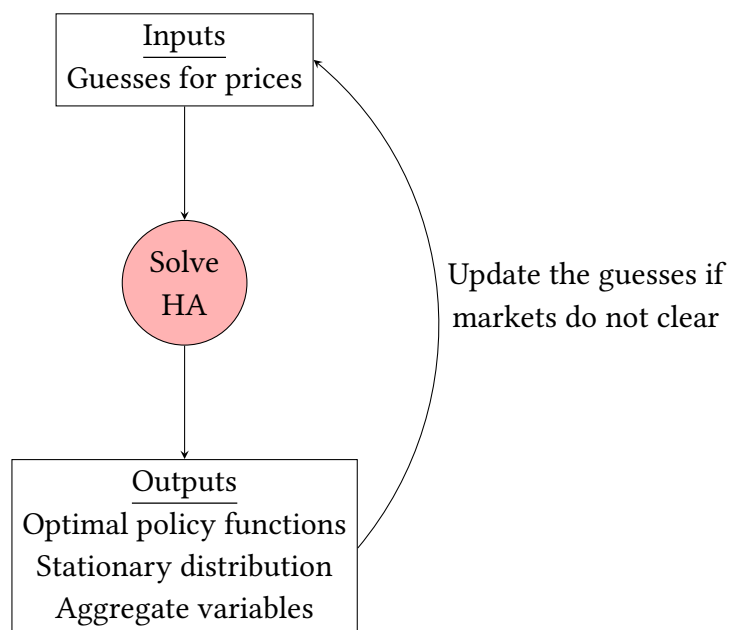


Figure 1.2: Stationary state resolution process of a heterogeneous-agent model.

1.3.2 Efficiency tools

The first thing to do is to discretize the state space. That is, instead of being continuous, the variables in s can only take a finite number of discrete values. In particular, exogenous state variables e that are generated by a continuous Markov process are discretized using the Tauchen, 1986 or Rouwenhorst, 1995 methods (if Gaussian) or the Farmer and Toda, 2017 method for non-linear non-Gaussian processes. As for the endogenous state variables y , they are assumed to take values in a finite grid. The gridpoints are generally nonuniformly spaced to have a finer mesh at locations where non-linearity occurs and the mass of agents is greater. The cross-distribution of agents across the individual state space and the values taken by the value function (or policy functions) are also discretized.

When solving the model, if an optimal value is found between two points on the grid, it is interpolated to find an approximation of its value. Various interpolation techniques (including polynomial and cubic splines interpolations) make it possible to obtain more or less precise approximations more or less quickly.

However, the truly costly step in solving the microeconomic problem of the agent is maximization, which pins down to a root-finding problem. It can nonetheless be avoided using Carroll, 2006's endogenous gridpoint technique (or its generalization by Barillas and Fernández-Villaverde, 2007). Usually, the consumption-saving program is solved for a grid of assets. On each gridpoint, corresponding to the value of the state variable, the optimal household consumption is obtained as the root of a non-linear equation corresponding to its

intertemporal choice. The endogenous gridpoint method introduces another state variable, end-of-period wealth (i.e. after consumption). Once this new state variable is introduced, consumption is no longer present on both sides of the household’s Euler equation, thus avoiding the root-finding step that is costly. For each point on the grid of end-of-period assets, the optimal consumption is obtained thanks to a simple calculus. An interpolation then allows us to identify the associated beginning-of-period wealth we are interested in. This approach obviously holds only under some conditions especially the smoothness of the value functions and policy decisions. Discrete choices (such as working or not) create kinks in value functions, and discontinuities in policy rules, that require further adaptation. Iskakov et al., 2017 explains how to use the endogenous gridpoint method when agents face such discrete and continuous choices at the same time.

1.3.3 Results

We solve the model using calibrated values for the parameters (see appendix 1.8.2 for the detail). Once the stationary state is determined, the stationary distribution of agents across the state variables’ values is known. Figure 1.3 shows that this distribution consists of a bell-shaped surface combined with a mass at the borrowing constraint.

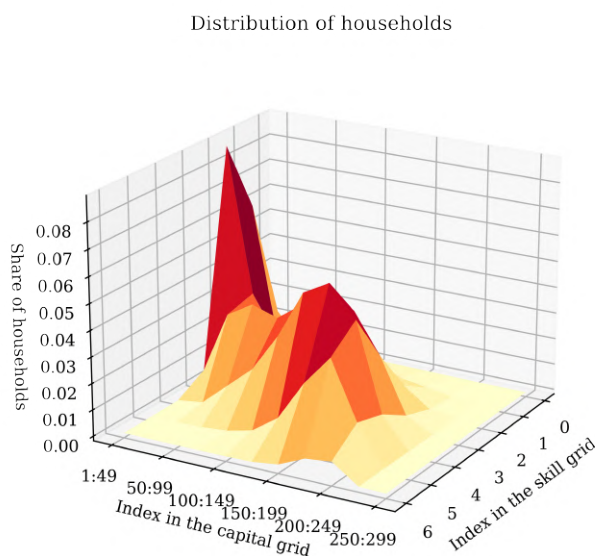


Figure 1.3: Stationary distribution of households across productivity and wealth states.

Next, a few statistics resulting from the model are compared to US wealth data (see Kuhn and Rios-Rull, 2020’s analysis of the 2019 Survey of Consumer Finances (SCF) data). This allows us to qualitatively assess the model’s ability to reproduce empirical facts.

With the calibration used, around 16.6% of the households are constrained and hold no wealth (as we assumed no borrowing is allowed). Meanwhile, the wealthiest 10% own 44.7%

Table 1.1: Wealth ratios. US data are from the SCF database and are computed by Kuhn and Rios-Rull, 2020

Wealth ratio	Model	2019 US data
$D9/D2$	185	187
$D9/D5$	6.72	9.98
$D5/D2$	27.5	18.7
Mean/Median	2.37	6.13

of all assets. In the US data, around 10% of the households have negative or zero wealth. The top-10% holds 76.4% of the wealth. This suggests that the benchmark HA model generates a few too many poor households and not enough concentration of wealth at the top of the distribution as seen in Table 1.1. This is common to most basic heterogeneous agent models. Table 1.2 reports Gini coefficients for wealth and incomes in the data and in the model. While this simple benchmark model generates realistic disposable income inequalities (as measured by the post-tax and transfer income Gini coefficient), this is not enough to generate enough wealth inequalities as observed in the data. This suggests that additional mechanisms are missing while understanding the reasons why rich households save that much is still an open question. Many possible explanations exist, that are summarised in De Nardi, 2016's talk. Among others:

- Krusell and Smith, 1998 advise using heterogeneous preferences through stochastic discount factors
- De Nardi, 2004 explores bequest motives since a large share of wealth is inherited.
- Heterogeneous asset returns may also be part of the story as studied by Gabaix, Lasry, and Moll, 2016.
- Entrepreneurs with higher returns on their investment help generate a realistic wealth distribution (see Cagetti and De Nardi, 2006).
- Guvenen, Karahan, et al., 2021 and Krebs, Krishna, and Maloney, 2019 suggest modeling more realistically the income risks faced by households.
- Education and human capital inequality may also contribute to this observation.

Table 1.2: Gini coefficients.

	Model	2019 US data	Source
Wealth	0.659	0.852	Credit Suisse (Global wealth databook 2019)
Income before taxes and transfers	0.607	0.505	OECD (market income before taxes and transfers)
Income post taxes and transfers	0.421	0.395	OECD (disposable income post taxes and transfers)
Labour income before taxes and transfers	0.247		

1.4 Linear dynamics

As advocated by Reiter, 2009 and Auclert, Bardóczy, Rognlie, and Straub, 2021, the dynamic response of the model to an aggregate shock results from the first-order perturbation of the linearised system around the stationary equilibrium. For instance, starting from equation (1.2) for a shock $Z \in \mathcal{Z}$ and a variable $Y \in \mathcal{Y}$ and differentiating it leads to:

$$\begin{aligned} \mathbf{H}(Y, Z) &= 0 \\ d\mathbf{H}(Y, Z) &= \underbrace{\frac{\partial \mathbf{H}}{\partial Y}}_{H_Y} dY + \underbrace{\frac{\partial \mathbf{H}}{\partial Z}}_{H_Z} dZ = 0 \end{aligned}$$

Thus, the response dY to a shock dZ is given by:

$$dY = -H_Y^{-1} H_Z dZ \tag{1.4}$$

where H_X denotes the Jacobian of the system of equations (1.2) with respect to the variable $X \in \{Y, Z\}$ and evaluated at the stationary state. $G^{Y,Z} \equiv -H_Y^{-1} H_Z$ denotes the general equilibrium Jacobian of the model from shock Z to output variable Y .

1.4.1 Direct acyclic graph representation

Underlying the computation of the general equilibrium Jacobian $G^{Y,Z}$ of the system are derivatives of the household distribution. As a result, solving the system may be computationally intensive. In that context, exploiting the inherent structure of the model may be efficient. Auclert and coauthors advise arranging the equations in a given order and thus representing the system as a directed acyclic graph (DAG). This may be intuitive. For instance, if the central bank sets the interest rate, households can make their saving decisions. This approach solely requires identifying equations that can be grouped together (creating a block), the input variables needed to solve each block and the output quantities thus determined. Then just order the blocks. The subtlety is to reorganize the thus obtained directed graph to make it acyclical⁴. Once the model is represented as a DAG, the Jacobians of each block are computed. By doing so, a topological sorting of the DAG enables the calculation of the Jacobian matrix for the entire system by multiplying the blocks in a specific order. This enables us to easily obtain a complicated Jacobian (that of the system), by calculating simpler intermediate Jacobians (those of each block). Auclert, Bardóczy, Rognlie, and Straub, 2021 provide a Python toolbox that automates this process⁵.

One possible representation of the benchmark HANK model as a direct acyclic graph (DAG) is shown in Figure 1.4. For the sake of concision, the exogenous shocks are not represented. They would otherwise be on arrows from the input box to all blocks (firms, central bank, fiscal, HA) and market clearing conditions. Appendix 1.8.3 details the calculation of

⁴In other words, if we go through the graph following the oriented arrows, we will never come full circle (i.e. retrace our steps).

⁵Codes are available here.

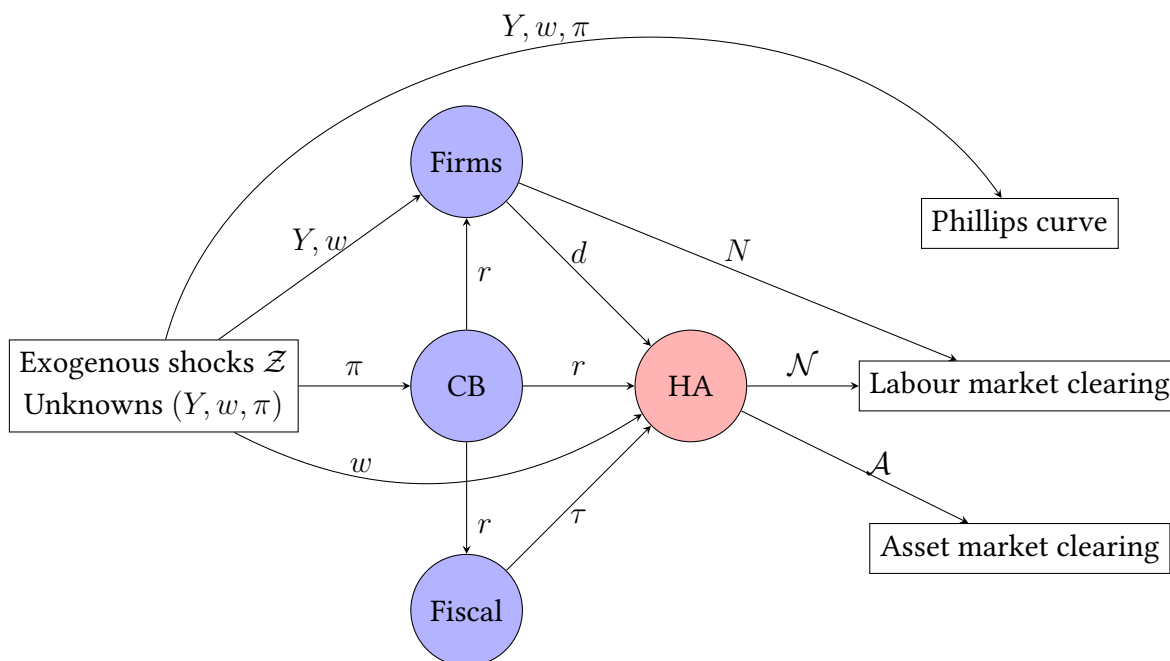


Figure 1.4: Directed acyclic graph representation of the benchmark HANK model.

the Jacobians of each block. Most blocks have easily computed Jacobians for they contain static equations. That's the blue blocks in Figure 1.4 and their Jacobians are thus sparse. The HA block is a more complicated object because due to the shock persistence, all the history matters for current decisions, while the Bellman equation makes current decisions dependent upon future values. The Jacobian of the household program can, however, be obtained leveraging on automatic differentiation as in Boehl, 2023 or by exploiting its recursive properties as in Auclert, Bardóczy, Rognlie, and Straub, 2021's *fake-news* algorithm⁶. Following an exogenous shock (preference, TFP, public spending, or monetary for instance), and assuming target values for the wage w , the output Y , and the inflation rate π , the central bank sets the interest rate r , and the firm determines the dividends d to pay to households. Given all that information, households make their decisions, which yield an aggregate labor supply and asset demand. These aggregates are compared respectively to labor demand by firms and bond emissions by the government to ensure market-clearing conditions. Note that a DAG representation of a model is not unique. Another possible (yet naive) DAG would be the one corresponding to solving the system all at once with no ingenious ordering of the equations. Such DAG is given in Figure 1.5. While both approaches yield the same result for the general equilibrium Jacobian, the naive one takes 6.60s to compute as opposed to 5.29s for the previous DAG.

⁶See Appendix 1.8.3 for the actual recursive relation and how it can be exploited to evaluate the household Jacobian.

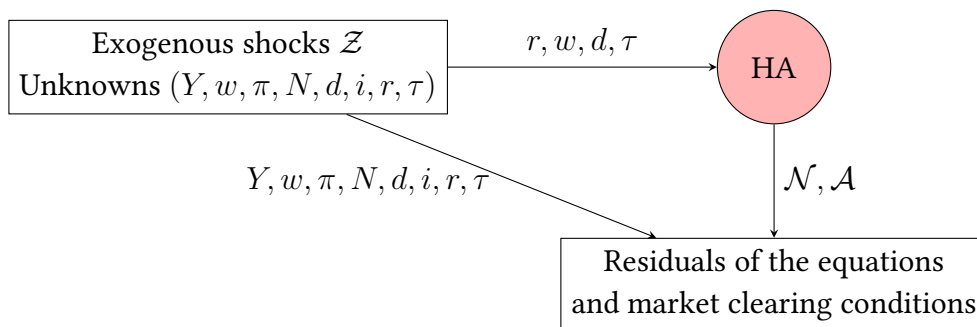


Figure 1.5: Naive directed acyclic graph representation of the benchmark HANK model.

1.4.2 Local determinacy

The Blanchard and Kahn, 1980 conditions are the standard approach to determine whether a unique solution to a linear system of difference equations with rational expectations (thus including DSGE models) exists or not. Writing the system of linearised equations defining the steady-state under matrix form, the model is locally determined if and only if, the number of eigenvalues with modulus greater than one is equal to the number of control variables in the model. This criterion, however, requires the model to be written in state space, which is not the case here.

Assessing the local determinacy and the existence of a solution of a (heterogeneous agent) model written in sequence space is still a work in progress, with Auclert, Rognlie, and Straub, 2023a proposing a criterion, the winding number, thus extending Onatski, 2006's approach. They indicate that the winding number criterion points to the same conditions as Blanchard-Kahn.

1.4.3 Impulse response functions

Once the Jacobian of the system is obtained and inverted, the impulse response functions are computed following equation (1.4). Figures 1.6, 1.7 and 1.8 display them in red. These results can be compared to the representative agent framework ones (in blue on the figures) documented by Galí, 2015.

Following a 1% rate hike (Figure 1.6), output, consumption, labor, effective labor (that is hours worked weighted by productivity) and wages respond more in a HA framework compared to the RA case. They fall more yet retrieve their steady-state values at a similar speed. Taxes and the real rate also increase more than in a representative-agent framework, while inflation diminishes further.

As for a preference shock, modeled as a 1% increase in the discount rate (Figure 1.7), the relative changes in responses are similar to the monetary shock. Taxes, inflation, and the real rate experience respond approximatively as in the representative agent framework while the other variables feature a stronger reaction when adding heterogeneous agents.

Most of the variables respond slightly more to a productivity shock (Figure 1.8) when

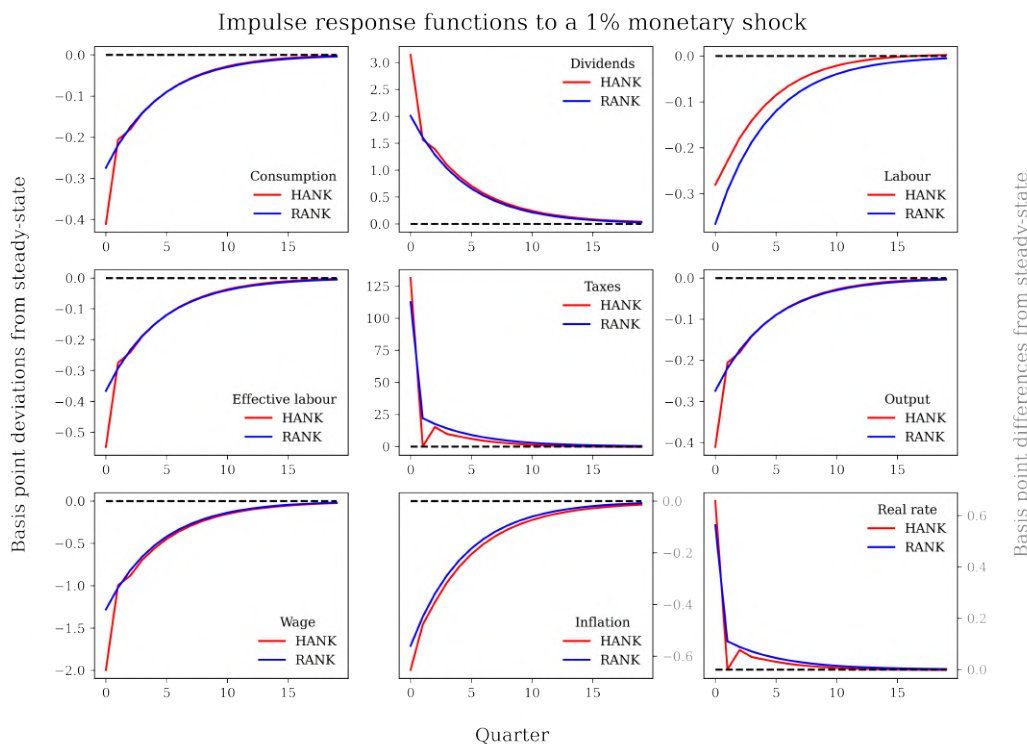


Figure 1.6: Dynamic responses to a 1% positive monetary policy shock.

featuring heterogeneous agents.

Moreover, more complex heterogeneous agent models may prove to feature more differences in aggregate responses to the representative-agent framework. Indeed, Ahn et al., 2018 shows that aggregate consumption's response to an aggregate productivity shock is different from the RA framework when considering a two-asset HA model. For the sake of illustration, we have here presented a simple model with little heterogeneity. However, the method presented can be applied to models featuring richer heterogeneity that may prove more relevant.

But more importantly, assuming HA allows for a finer analysis, not only restricted to aggregate outcomes. Figure 1.9 displays the impulse response functions for the aggregate macroeconomic variables and the households with median productivity. According to this one-asset HANK model, when the central bank hikes interest rates by 1%, aggregate labor and consumption diminish. Holdings of government debt remain unchanged since we assumed a fixed supply. Looking at the median productivity type, the story is different. He dis-saves to try to smooth consumption while working less. This graph motivates the use of HA models: behind aggregate dynamics, very different phenomena can occur.

It is also possible to analyze the differential impact of a shock on every agent type. For instance, Figure 1.10 shows the impulse responses of each idiosyncratic productivity type. Overall, wealth and consumption are increasing functions in the productivity of the household. Meanwhile, labor (without weighting by individual productivities) is a decreasing

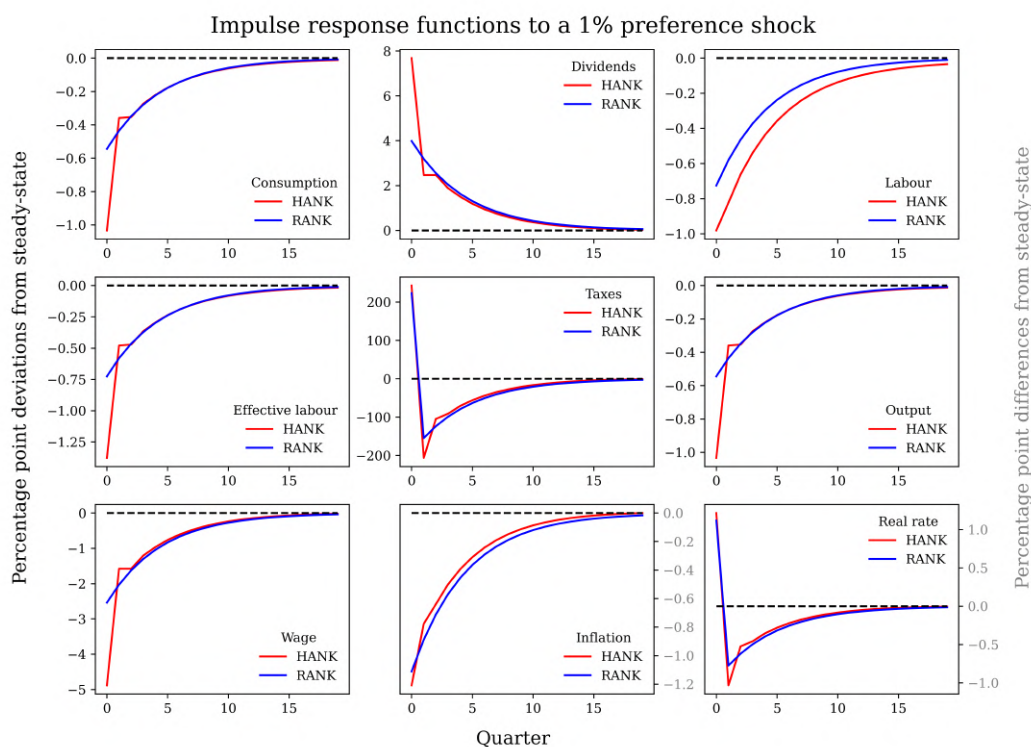


Figure 1.7: Dynamic responses to a 1% positive discount rate shock.

function. The less productive types are thus the ones working the more hours while producing the less efficient hours of labor. They are also those less consuming and holding bonds. When the central bank hikes rates, wages diminish. The less productive types have no choice but to increase their labor supply (as they hold few or no bonds). More productive households, which are also richer, will tend to work less, saving their wealth and thus benefiting from higher rates. The median type is the most affected one, from the wealth perspective. Those households decrease their labor supply (in a manner similar to more productive types), diminishing their labor income. Meanwhile, their lower asset holdings compared to richer types fail to generate enough financial income to compensate for this loss. As a result, they dissave slightly to smooth consumption. In this model and calibration, the median agent is, therefore, a pivotal agent, wishing to behave like a rich person (lowering his labor supply) but finding himself obliged to use his savings (which poorer agents cannot do).

A key observation is also that monetary policy is not neutral in terms of inequalities. Wealth dynamics differ among the various households. Moreover, these dynamics last a very long time: while the other variables converge back to the stationary state in about twenty quarters, it takes almost seventy quarters for wealth. This observation on the persistence of wealth dynamics generalizes to preference and productivity shocks as shown in Appendix 1.8.3.

Heterogeneous-agent models also allow the plot and study of the evolution of the wealth distribution over time following some aggregate shocks. This can be useful for discussing

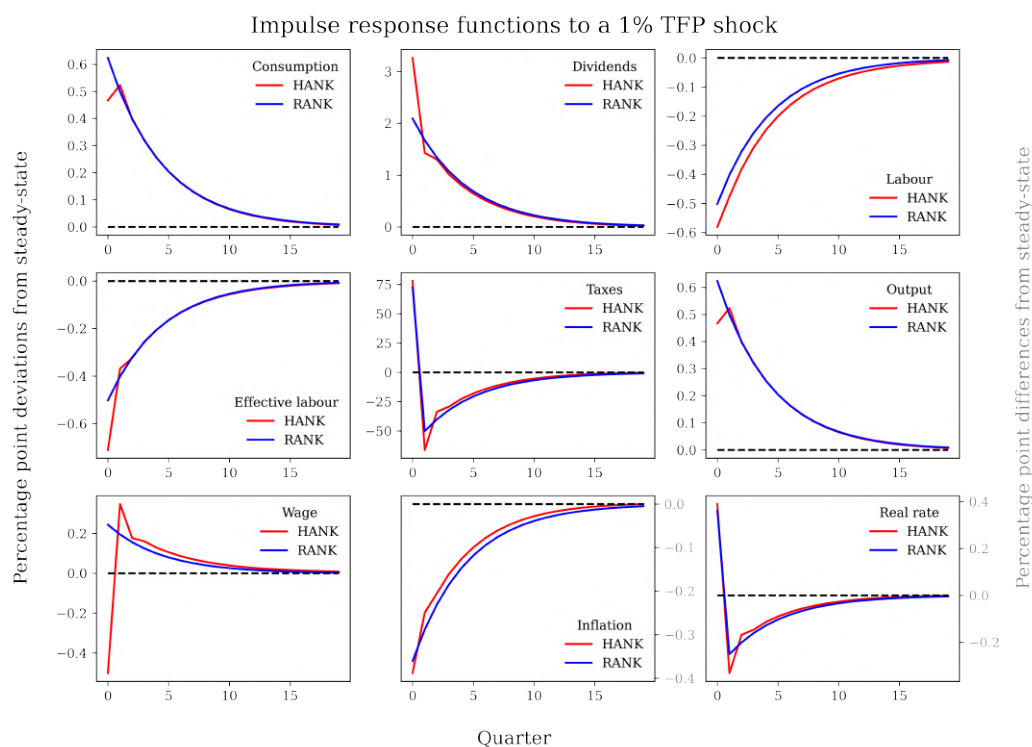


Figure 1.8: Dynamic responses to a 1% positive technological shock.

the consequences of macroeconomic policies on inequality. Table 1.3 summarises just one statistic in that regard, how the share of households stuck at the borrowing constraint react at the moment of a shock.

Table 1.3: Share of constrained agents at the impact of aggregate shocks.

	Share of constrained households	
Initial distribution	16.57%	
1pp increase of the policy rate	16.58%	+0.01pp
1% increase of TFP	17.70%	+1.13pp
1% increase of the discount factor	15.87%	−0.70pp

1.5 Estimation

So far, models solved relied on calibrated values for the parameters. While the literature provides information on the standard values of some of these parameters⁷, others may be

⁷For instance, risk aversion, intertemporal elasticity of substitution, Frisch elasticity of labor supply, capital and labor shares in production.

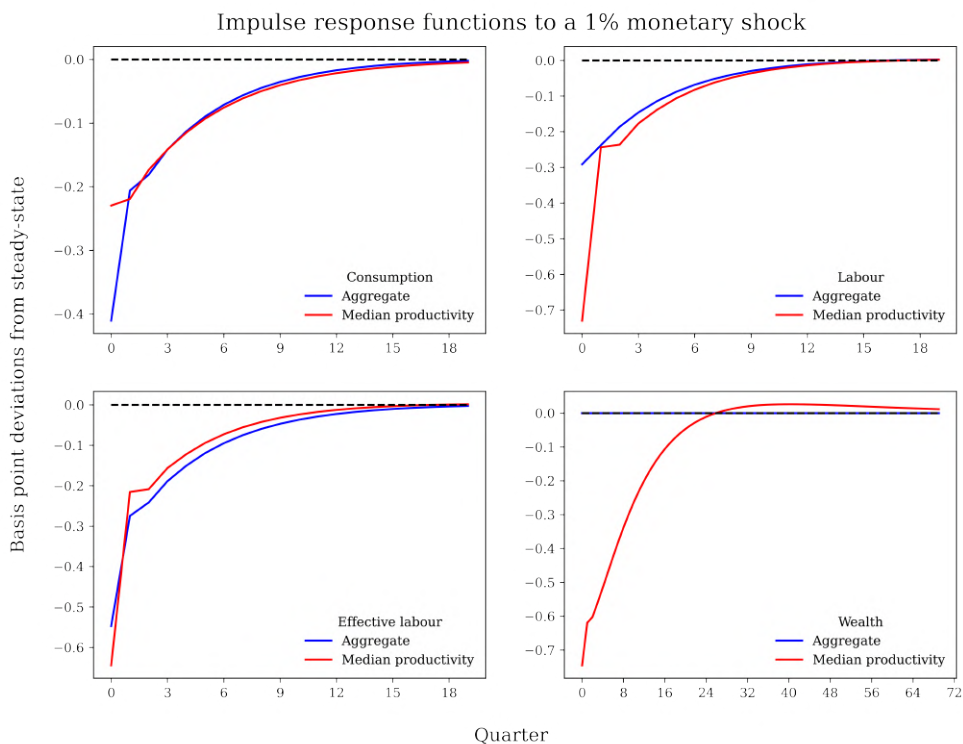


Figure 1.9: Aggregate and median-productivity type household responses to a 1% positive monetary shock.

harder to identify. Estimation can be the solution to this problem, by retrieving possible values for the parameters from available observed data. See Fernández-Villaverde and P. A. Guerrón-Quintana, 2021 for a review of the estimation methods available.

Let's denote $\vec{\Theta}$ the set of parameters we wish to estimate. It may include the parameters driving the shock process (persistence and variance, for instance) together with other structural parameters of the model.

We will estimate those parameters using three different techniques: a method of simulated moments (MSM) approach, a maximum likelihood (ML) approach, and a Bayesian approach.

1.5.1 Method of simulated moments

The method of simulated moments selects values for the parameters that minimize the distance between moments of the empirical data and these same moments obtained through the model's simulation. The procedure for estimating $\vec{\Theta}$ using \vec{Y}^{obs} as observations then is:

1. Select the moments to be considered. Here we took the standard deviations of the

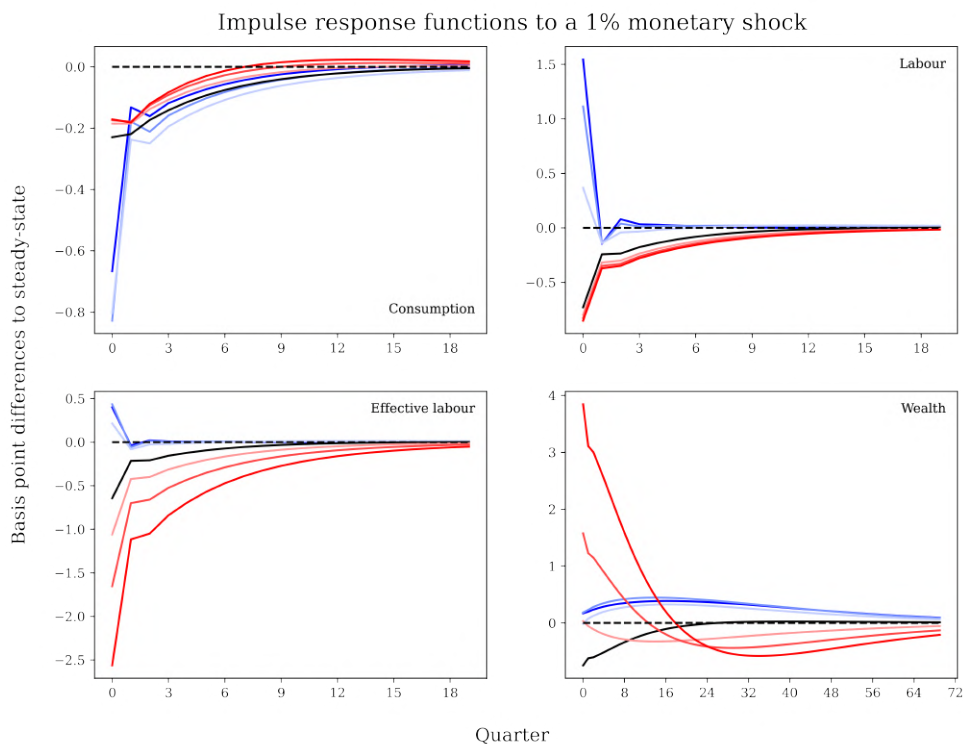


Figure 1.10: Impulse response functions to a 1% positive monetary shock by productivity types: black is the median productivity; the darker the blue, the lower the productivity compared to the median; darker red shades are associated with higher productivity than the median.

observed time series and their contemporaneous covariances. We computed them for the observations and denoted them \vec{m} .

2. Assume the values of the parameters $\vec{\Theta}$ to be estimated.
3. Simulate many time series consistent with the model's predictions given $\vec{\Theta}$.
4. Compute the estimated moments of these simulations $\hat{m}(\vec{\Theta})$ using the sample means.
5. Compute the objective function $e(\vec{m}, \hat{m}(\vec{\Theta}))^T W e(\vec{m}, \hat{m}(\vec{\Theta}))$ we want to minimise, where W is a weighting matrix and $e(\vec{m}, \hat{m}(\vec{\Theta})) = \frac{\hat{m}(\vec{\Theta}) - \vec{m}}{\vec{m}}$ are the errors.
6. The estimated parameters are those minimizing this distance measure between the simulated moments and the observed ones, i.e. minimizing the objective function.

For efficiency, the simulations re-use the sequence-space Jacobian. Estimating the parameters of the aggregate shocks only is not that costly in terms of computing time as it does not require recomputing the Jacobian of the model. The reason is that aggregate shocks intervene after solving for the stationary state and computing the Jacobian of the system

around that point. However, when estimating some structural parameters of the model, each time a different value is considered, the stationary state may need to be solved again and undoubtedly the Jacobian recomputed. Indeed, some parameters such as the Phillips curve coefficient κ or the Taylor rule ones (ϕ_y, ϕ_π) do not affect the steady state, solely the dynamical response. In that case, the Jacobian is recomputed around an unchanged steady-state. If estimating parameters that affect the steady state (such as β for instance), then it should also be solved again at each step.

I estimate the shocks' persistence and volatilities, as well as the Taylor and Phillips curve coefficients, using US data. The three shocks considered are a monetary shock, a government spending shock, and a price markup shock. It requires three time series of observed data⁸: output, inflation, and federal funds rate. I restrict to the 1Q1984-4Q2007 period, which corresponds to the Great Moderation, and retrieve the data from the DB-nomics database. The output series being nonstationary, I consider the difference in its log values.

The data moments I wish to replicate are the contemporaneous correlations (see Table 1.4) and standard deviations of the time series.

Table 1.4: Variance-covariance matrix of the observed data.

Variance-covariance matrix	Output	Inflation	Interest rate
Output	2.828×10^{-5}	-1.782×10^{-6}	2.565×10^{-6}
Inflation	*	4.809×10^{-6}	5.513×10^{-6}
Interest rate	*	*	3.649×10^{-5}

Running this estimation with 1,000 simulations on the Great Moderation period, I get:

Table 1.5: Simulated method of moments estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter		Estimate
Monetary policy shock	ρ_ε	0.360
	σ_ε	0.00806
Government spending shock	ρ_G	0.999
	σ_G	0.00999
Price markup shock	ρ_m	0.930
	σ_m	0.0200
Taylor rule	ϕ_π	1.51
	ϕ_y	0.430
Phillips curve	κ	0.00850

Table 1.5 shows that the estimated persistence of the government spending shock hits the upper bound.

⁸The number of observables is limited to three since we have three shocks in our model.

1.5.2 Maximum likelihood and Bayesian estimations

A way to overpass the failed estimation by the method of simulated moments is to move to another objective function, namely the likelihood function. The general procedure for estimating $\vec{\Theta}$ follows:

1. Choose the time series of data \vec{Y}^{obs} used for the estimation and process them (i.e. make them stationary and demean them).
2. Assume the values of the parameters $\vec{\Theta}$ to be estimated (in the case of a maximum likelihood approach) or their prior distribution $p(\vec{\Theta})$ (when performing Bayesian estimation).
3. Define measurement equations that link the observed time series to aggregate variables of the model \vec{Y} .
4. Using the previously defined relationships between $d\vec{Y}$ and $d\vec{Y}^{obs}$, compute the data's autocovariances $\text{cov}(dY_{i,t}^{obs}, dY_{j,t'}^{obs})$ and stack them into a matrix V .
5. Compute the likelihood function $\mathcal{L}(\vec{\Theta}) = -\frac{1}{2} \log \det V - \frac{1}{2} d\vec{Y}^{obs'} V^{-1} d\vec{Y}^{obs}$ using a Cholesky decomposition for V for efficiency motives.⁹
6. In a maximum likelihood estimation, the estimated parameters maximize the likelihood. The inverse of the Hessian at the optimum provides their standard errors. In a Bayesian estimation, the posterior distribution we are interested in $p(\vec{\Theta} | Y^{obs})$, is proportional to $\exp(\mathcal{L}(\vec{\Theta}))p(\vec{\Theta})$.

Calculating the autocovariance function of \vec{Y} is necessary to compute the autocovariance function of \vec{Y}^{obs} . Auclert, Bardóczy, Rognlie, and Straub, 2021 propose to exploit the Jacobians previously computed during the resolution of the dynamic response. More precisely, the calculated impulse response functions are equivalent to the MA(∞) representation of the model when the certainty equivalence property of linearization holds (see Auclert, Bardóczy, Rognlie, and Straub, 2021's Appendix C.3 for the demonstration).

Let's assume all the exogenous shocks of the model admit a MA(∞) representation as follows:

$$dZ_t = \sum_{s=0}^{\infty} \mathbf{m}_s^Z \varepsilon_{t-s}^Z$$

Then, the outputs of the HA model can be represented by a MA(∞) representation that involves the general equilibrium Jacobians of the model:

$$dY_t = \sum_{s=0}^{\infty} \sum_{\text{shock } z} [G^{Y,z} \mathbf{m}^z]_s \varepsilon_{t-s}^z$$

⁹Note that the likelihood cannot be computed using the Kalman filter which requires a linear state-space formulated problem (see Herbst and Schorfheide, 2015).

Just as before, by reusing the Jacobians, computing the objective function (here the likelihood) is quicker. The process is standard except for that. See Appendix 1.8.4 for more details on the step-by-step log-likelihood computation.

Table 1.6 (resp. 1.7) summarises the results of the maximum likelihood (resp. Bayesian) estimation when the model is estimated over the 1Q1984-4Q2007 period. For the Bayesian approach, one million draws were run using the Metropolis-Hastings algorithm, a standard method among Markov chain Monte Carlo (MCMC) approaches. This method is however not the most efficient one and one may prefer to use sequential Monte Carlo methods (i.e. particle filters, see Herbst and Schorfheide, 2015, Acharya et al., 2023). Those approaches let a great number of Monte Carlo chains evolve in parallel.

Boehl, 2022 also proposes a very promising avenue for estimating HA models using Bayesian techniques. He introduces a DIME (Differential-Independence Mixture Ensemble) MCMC sampling technique. In line with the sequential MC methods, several chains make draws. Those draws are generated from a proposal distribution that can either be local (i.e. specific to the chain considered) or global (i.e. independent from the current chain). This aspect allows the chains to move more easily away from local maxima. His approach also allows for draws outside of the priors' bounded supports. This is particularly important when some parameters to estimate take values close to those boundaries (for instance persistences close to unity) because otherwise, those draws would be systematically discarded, thus massively slashing the acceptance rate. Note that this method still uses Auclert, Bardóczy, Rognlie, and Straub, 2021's sequence-space approach to compute the Jacobians and likelihood of the model.¹⁰

This approach resolves the limits of the standard Random Walk Metropolis-Hastings (RWMH) algorithm, namely slow convergence when the distributions are ill-shaped (for instance due to parameter regions leading to indeterminacy or multimodality induced by multiple local maxima). Besides, it allows parallelizing the process, thus leveraging on the increasing computing power available. As a result, estimations are quicker and may allow the estimation of more complex parameters affecting the steady state. Boehl, 2022, for instance, estimates the intertemporal elasticity of substitution in a two-asset HANK version of the Smets and Wouters, 2007 model.

The maximum likelihood approach also fails to identify the persistence parameter for the public spending shock. Indeed, the estimate hits the upper bound of the range of values considered. The reason is that such a simple model fails to generate enough persistence. This estimation method is thus not very well adapted to such a basic model. One would prefer a Bayesian approach, that restricts the range of values considered thanks to the choice of a prior (Figure 1.11).

That said, the maximum likelihood estimates can serve as an informed starting point for the Bayesian estimation.

Note that these estimation techniques (ML, Bayesian, and MSM) rely solely on macroeconomic data. A calibration gives the parameters that govern the microeconomic behavior of the model (here, the idiosyncratic shocks).

¹⁰The codes for this improved Bayesian inference are available here.

Table 1.6: Maximum likelihood estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter		Estimate	Standard error	95% confidence interval
Monetary policy shock	ρ_ε	0.587	0.149	[0.296,0.879]
	σ_ε	0.00349	0.00144	[0.000674,0.00630]
Government spending shock	ρ_G	0.999	-	-
	σ_G	0.00841	0.00203	[0.00443,0.0124]
Price markup shock	ρ_m	0.717	0.153	[0.418,1.016]
	σ_m	0.0261	0.00851	[0.00937,0.0427]
Taylor rule	ϕ_π	2.02	1.17	[-0.283,4.32]
	ϕ_y	0.222	-	-
Phillips curve	κ	0.0426	0.0708	[-0.0962,0.181]

Table 1.7: Bayesian estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter		Prior distribution	Posterior			
			Mean	Std. dev.	5th perc.	95th perc.
Monetary policy shock	ρ_ε	$\beta(0.5, 0.2)$	0.438	0.0801	0.271	0.542
	σ_ε	$\text{inv}\Gamma(0.4, 4)$	0.00732	0.000717	0.00620	0.00867
Government spending shock	ρ_G	$\beta(0.5, 0.2)$	0.963	0.0122	0.942	0.984
	σ_G	$\text{inv}\Gamma(0.4, 4)$	0.00910	0.000745	0.00793	0.0104
Price markup shock	ρ_m	$\beta(0.5, 0.2)$	0.702	0.0645	0.600	0.803
	σ_m	$\text{inv}\Gamma(0.4, 4)$	0.0589	0.0112	0.0452	0.0850
Taylor rule	ϕ_π	$\Gamma(1.5, 0.25)$	1.75	0.276	1.31	2.27
	ϕ_y	$\Gamma(0.5, 0.25)$	0.766	0.121	0.571	1.00
Phillips curve	κ	$\Gamma(0.1, 0.1)$	0.0132	0.00392	0.00788	0.0205

Note: The parameters mentioned in the distributions are the means and standard deviations.

1.5.3 Using macro and micro data

Other economists have argued for the simultaneous use of macro and microeconomic data. Indeed, considering solely macroeconomic data only helps match a few moments at the aggregate level. Meanwhile, it forgets about the cross-sectional distributions involved in HA models. The easiest way of incorporating microeconomic data into the estimation relies on adding a few time series of cross-sectional moments. These time series are then included in the observed data set, along with standard macroeconomic data. This is the approach followed by Bayer, Born, and Luetticke, 2020 who add time series using repeated observations of the same cross-sectional moments, the shares of wealth and income held by the top 10% of households. This approach is made increasingly accessible by the availability of detailed microeconomic data at the household or firm level. These extensive databases can help construct high-frequency inequality measures (see Blanchet, Saez, and Zucman, 2022 for households, for instance).

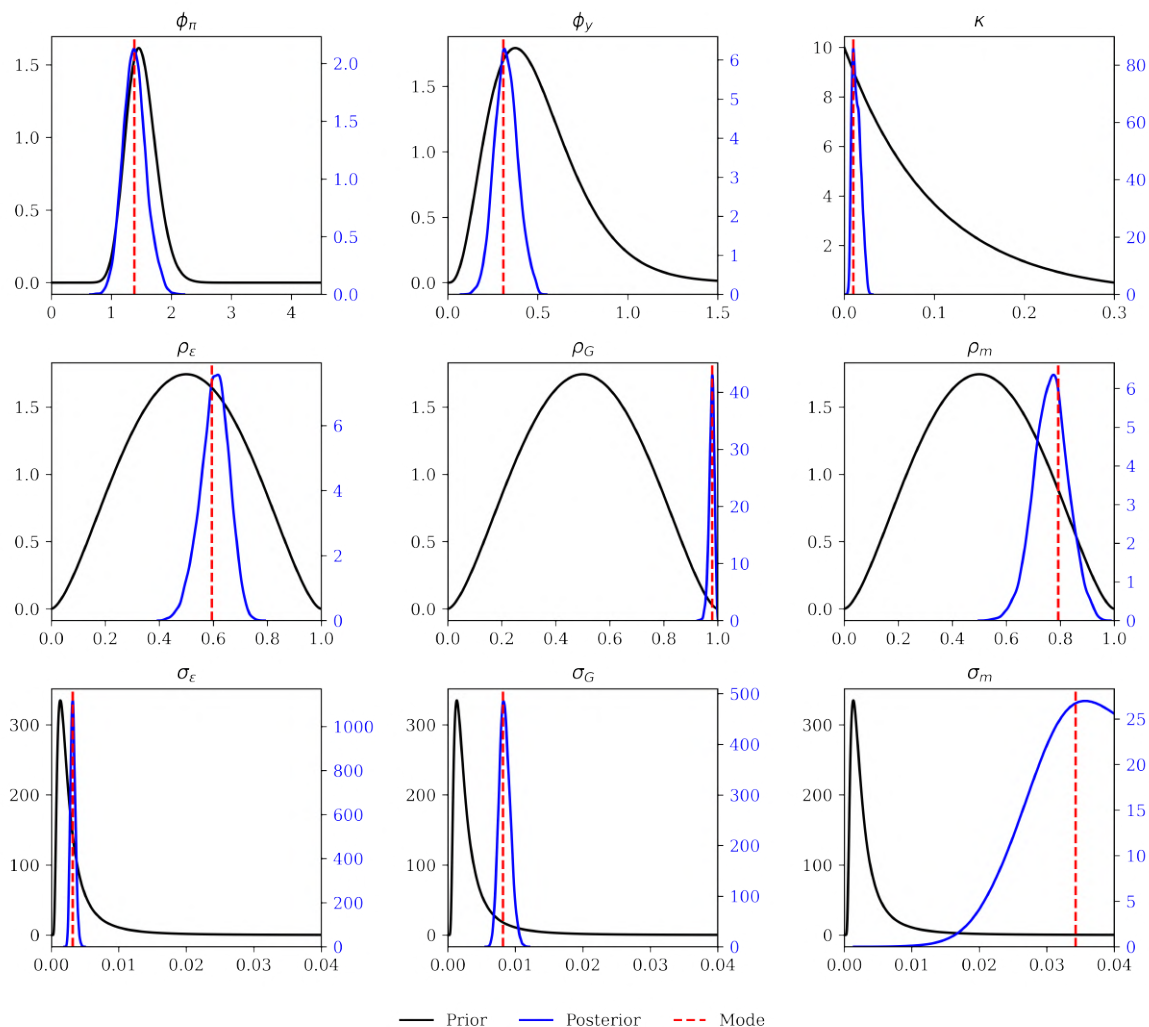


Figure 1.11: Prior and posterior density functions.

In the same spirit, Papp and Reiter, 2020 develop an estimation method that makes use of both macroeconomic aggregate data and micro-level ones, whether they are time series of cross-sectional moments or panel data. This method also works with data reported with different frequencies. In particular, when microdata are available yearly, while macro data are quarterly. In practice, it relies on the simulation of several households facing idiosyncratic shocks. This simulation is then linearised in the aggregate shocks. Liu and Plagborg-Møller, 2023 provide Matlab codes for this approach.

Note that, all these methods have been presented for models written in the state space. Applying them to the framework presented in this paper is thus not straightforward. It would require either generalizing these technical approaches to models formulated in sequence space or recovering the state-space laws of motion of the model prior to estimation.

1.6 Non linear dynamics and transitions

1.6.1 Nonlinear dynamics

What are the possible limits of the sequence space approach for solving HA models? So far, the IRFs obtained assumed a linear response in the aggregate shocks (while keeping the nonlinearities of the household's program). This may be relevant for small shocks that do not shift the economy far from its steady state. It can however exhibit limitations when experiencing big shocks or structural changes.

Solving the nonlinear dynamics of the model would require solving the whole system of equations 1.2. This can be done iteratively using a Newton method: starting from an initial guess $\mathcal{Y}^{(0)}$ (for instance the steady-state values \mathcal{Y}_{ss}), the following terms are computed as follows:

$$\mathcal{Y}^{(k+1)} = \mathcal{Y}^{(k)} - J^{-1}(\mathcal{Y}^{(k)})\mathbf{H}(\mathcal{Y}^{(k)}, \mathcal{Z}) \quad (1.5)$$

until convergence (that is the distance between $\mathcal{Y}^{(k+1)}$ and $\mathcal{Y}^{(k)}$ is arbitrarily low).

There, $J^{-1}(\mathcal{Y}^{(k)})$ denotes the inverse of the sequence-space Jacobian of the system around $\mathcal{Y}^{(k)}$. The problem is that recomputing the Jacobian in each step and inverting it is costly. Auclert, Bardóczy, Rognlie, and Straub, 2021 thus suggest using the steady-state Jacobian instead, which is computed only once and used in every subsequent step:

$$\mathcal{Y}^{(k+1)} = \mathcal{Y}^{(k)} - J^{-1}(\mathcal{Y}_{ss})\mathbf{H}(\mathcal{Y}^{(k)}, \mathcal{Z}).$$

Boehl, 2023 points out that this is called the chord method and that convergence is not guaranteed if one is far from the steady state and the function has a strong nonlinear behavior. He proposes an alternative method for constructing iteratively the $\mathbb{Y}^{(k)} = \mathcal{Y}^{(k)} - \mathcal{Y}^{(k-1)}$:

$$\mathbb{Y}^{(k+1)} = \mathbb{Y}^{(k)} - \alpha_k J^{-1}(\mathcal{Y}_{ss}) \cdot (\mathbf{H}(\mathcal{Y}^{(k)}, \mathcal{Z}) - \Lambda(\mathcal{Y}^{(k)}, \mathbb{Y}^{(k)})) \quad (1.6)$$

where $\Lambda(x, y) = J(x)y$ is simply a vector resulting from the multiplication of a Jacobian and an input vector (which is computationally easy to obtain). The α 's are positive real numbers serving as dampening factors, to ensure convergence. They are updated as follows:

$$\alpha_k = \min \left\{ \alpha_{k-1}, \frac{\gamma}{|R(J^{-1}(\mathcal{Y}_{ss})\Lambda(\mathcal{Y}^{(k)}, \mathbb{Y}^{(k)}), \mathbb{Y}^{(k)})|} \right\}.^{11}$$

$\gamma \in (0, 2)$ governs the speed of convergence. Appendix 1.8.6 provides an intuition on how to go from the standard Newton solver (equation 1.5) to the efficient version for solving HA models in equation 1.6. For a formal demonstration, refer to Boehl, 2023's paper.

1.6.2 Transition paths between two steady states

The previous methods can be used to compute the transition path between two steady states resulting from a permanent exogenous shock. In particular, starting from an initial

¹¹ R denotes the Rayleigh quotient of a matrix A and vector x : $R(A, x) = \frac{x^T A x}{x^T x}$.

steady-state featuring a government spending over GDP ratio of 35%¹², we assume the government increases this ratio by 10 percentage points. This increase is unexpected when starting in period one and occurs gradually over an approximately two-year period. Table 1.8 compares these two steady states.

Table 1.8: Comparison of the two stationary states for the transition.

Variable	Initial steady-state	Final steady-state	Difference
Output	1.0	1.07	+7.1%
Consumption	0.65	0.59	-6.1%
Labour	1.0	1.10	+9.5%
Real wage	0.625	0.611	-1.4%
Inflation	0%	0%	0%
Real rate	0.5%	0.61%	+0.11%
Nominal rate	0.5%	0.61%	+0.11%
Government spending over GDP ratio	0.35	0.45	+10%
Debt-to-GDP ratio	5.6	5.6	0%
Taxes	0.378	0.519	+14.1%
Dividends	0.375	0.401	+2.6%
Share of constrained agents	0.134	0.123	-1.1%
Wealth share of top 10%	0.408	0.394	-1.3%

The transition dynamics are obtained both using Auclert, Bardóczy, Rognlie, and Straub, 2021 and Boehl, 2023's toolboxes. Note that in both cases, the final steady-state description (not the initial one) is used for computing the Jacobians used to obtain the transition. Both approaches provide very much similar results. As government spending increases, this crowds out part of the demand from the households: consumption drops. Despite this, output increases, supported by the additional demand addressed by the government. To balance its budget, the government increases the tax rate on labor incomes. Over the transition, inflation increases (before stabilizing once again at zero when the final steady state is reached), fueled by the additional demand. This results in a temporary diminution of the real rate. Meanwhile, the nominal interest rate also adjusts downward to cool down inflation. Just as in the previous sections, we can obtain the dynamics of consumption (or any other decision variable) for the various productivity levels. All types experience a drop in their consumption levels following a permanent public spending shock. However, while the median and top productivity types manage to change their consumption paths smoothly, the lower productivity types have a more chaotic dynamic capturing that they are more likely to be financially constrained.

The transition paths given by the Auclert, Bardóczy, Rognlie, and Straub, 2021 and Boehl, 2023 methods are close (see Figure 1.12), indicating that we are not in a case where Auclert, Bardóczy, Rognlie, and Straub, 2021's approach fails. That being said, the nominal interest rate falls into negative territory during the transition period. This is not a problem if, we consider this nominal rate to be a shadow rate of the central bank (that takes into

¹²Note that this is a slight deviation from the benchmark calibration used so far that assumed no government spending.

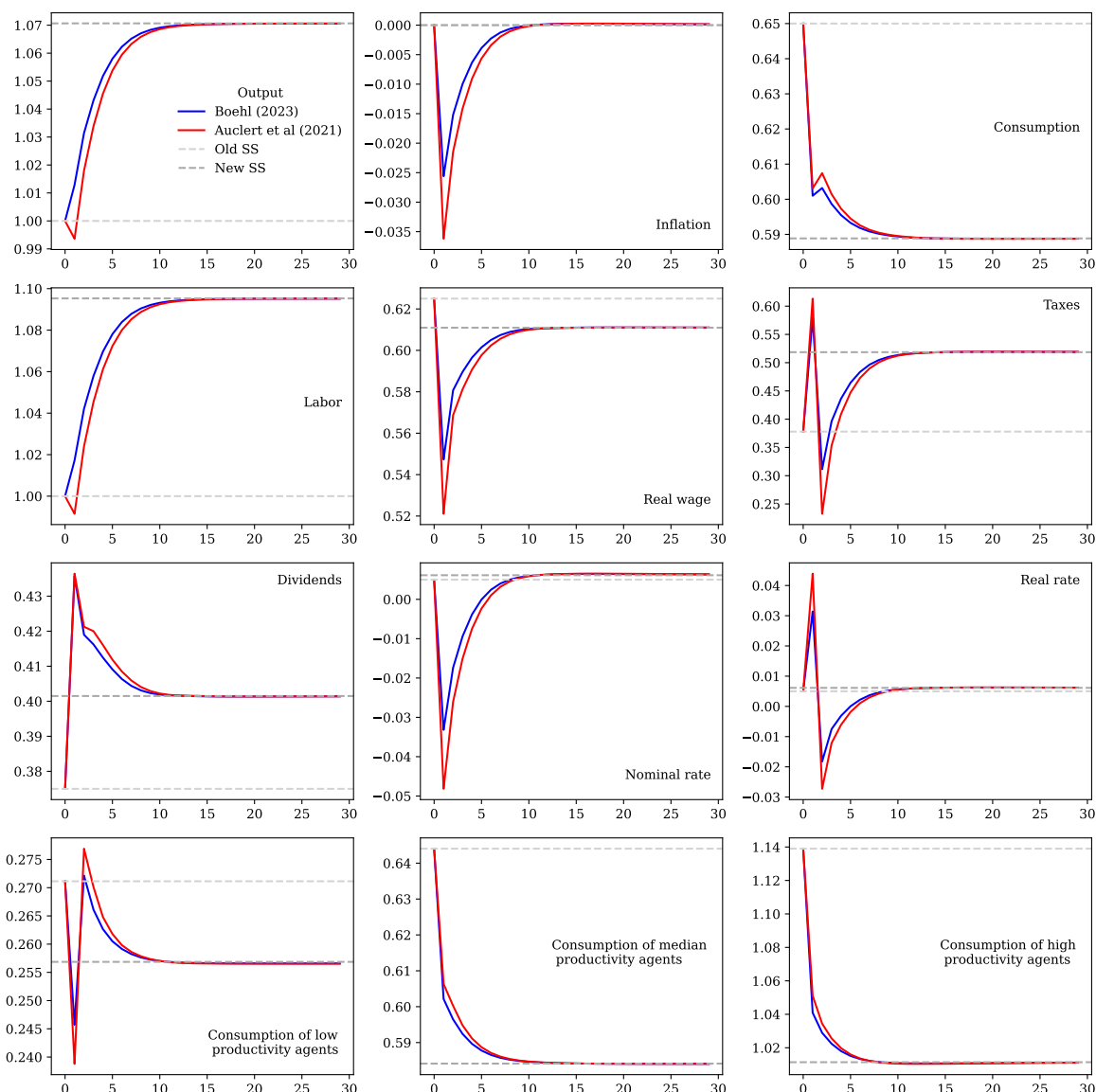


Figure 1.12: Nonlinear transition paths using Auclert, Bardóczy, Rognlie, and Straub, 2021 and Boehl, 2023's approaches.

account all possible supporting measures implemented, not just rate policy) and not a pure policy rate that cannot be negative¹³.

Boehl, 2023's approach however allows us to deal more easily with nonlinearities in the aggregate equations of the model and thus add a zero-lower bound (ZLB). By adding the following equation, to the description of our model, we can account for this ZLB:

$$i_t^{ZLB} = \max(i_t, 0).$$

Figure 1.13 displays the transition paths obtained when adding the ZLB and how they

¹³If so, when estimating the model, the federal fund rates can be replaced by J. C. Wu and Xia, 2016's shadow rate series when studying a period that features a zero-lower bound (ZLB).

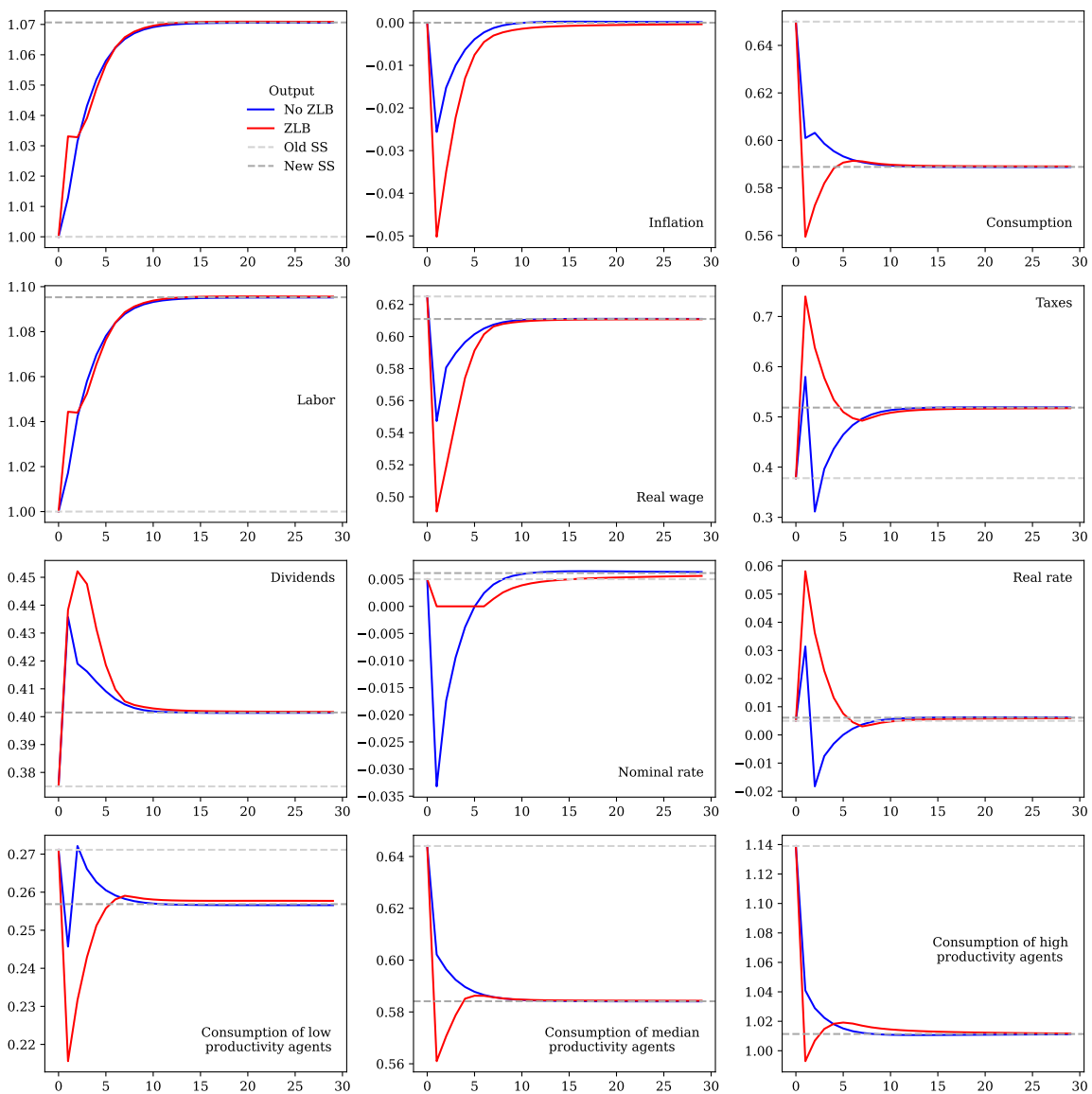


Figure 1.13: Transition paths with and without a zero-lower bound nonlinearity.

compare to the previously obtained ones. When facing the ZLB, the central bank is unable to adjust the nominal interest rate further down, losing a bit of control over inflation. The fiscal policy thus plays a bigger role and reacts more compared to the no ZLB scenario. Looking at the consumption response functions by productivity, it can be seen that the ZLB induces higher volatility. The lowest productivity types are experiencing bigger drops in their consumption compared to more productive agents. Indeed, most of those agents' revenues come from labor and the real wage experiences a massive drop in that scenario. Meanwhile, more productive types, who also have financial incomes benefit from an increased real rate which allows them to smooth consumption better.

This approach may thus be relevant to use whenever one wants to study a strong nonlinearity. In the context of a zero-lower bound, this approach can be used to study more specifically the mechanisms underlying unconventional monetary policies, such as quan-

titative easing. Indeed, with such a framework, it may be possible to study the channels specific to these policies and not just account for them as a negative nominal interest rate. This approach would be relevant to study the Great Recession period for instance. Berger, Bocola, and Dovis, 2023 show that using HA models to study that period is relevant as heterogeneity accounted for a third of output losses (compared to explaining 7% of aggregate volatility when the economy does not face a ZLB).

1.7 Conclusion

A method for solving heterogeneous agent models in discrete time and perfect foresight (i.e. agents are only surprised by shocks occurring at $t = 0$) was presented. This method is based on the method developed by Auclert, Bardóczy, Rognlie, and Straub, 2021. All the steps of the resolution and estimation are presented through a simple HANK model *à la* Galí, 2015.

The method can be applied to more complex models that will prove more interesting in studying the impacts of crises and policy implementations of the economy. Despite the models being written in sequence space, getting impulse response functions, estimating the model, making a historical shock decomposition, and so on is still possible. This may however require adapting the commonly used algorithms that usually rely on state space formulations of the models.

As with DSGE models with representative agents, the resolution of the equilibrium state and the calibration/estimation of certain model parameters can be challenging steps when there is no clear idea of where the equilibrium state lies. Errors can also occur in the creation of a directed acyclic graph of the model. Despite these possible difficulties, the methods presented allow us to easily obtain the dynamics of a heterogeneous-agent model, in the same spirit that Dynare allows us to analyze the dynamics of representative-agent models.

1.8 Appendix

1.8.1 Presentation of the benchmark HANK model

Households

Let's consider a continuum of households of mass 1. The problem of agent $i \in [0, 1]$ is given by:

$$\begin{aligned}
 v_t(e_t, b_{t-1}, \zeta_t) &= \max_{c_t, b_t, n_t} u(c_t, n_t, z_t) + \beta \mathbb{E}_t v_{t+1}(e_{t+1}, b_t, \zeta_{t+1}) \\
 \text{s.t. } p_t c_t + b_t &= (1 + r_t) b_{t-1} + w_t e_t n_t - \tau_t + d_t \\
 b_t &\geq \underline{b}
 \end{aligned}$$

where for simplicity, we drop the time and household i 's subscripts. The variables are defined as follows:

- c_{it} consumption
- n_{it} hours worked
- b_{it} holdings of one-period bonds paying r_{t+1} in interests in the next period
- e_{it} idiosyncratic productivity shock
- ζ_t preference shock
- $p_t \equiv \left(\int_0^1 p_{jt}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ aggregate price index
- p_{jt} price of good j

Solving for the consumption, savings, and labour decisions, the first-order conditions are:

$$\begin{aligned} \text{with respect to } c : u_c(c, n, \zeta) &= \lambda_t p_t \\ \text{with respect to } n : u_n(c, n, \zeta) &= -\lambda_t w_t e \\ &\Rightarrow -\frac{u_n(c, n, \zeta)}{u_c(c, n, \zeta)} = \frac{w_t}{p_t} e \\ \text{with respect to } b : \beta \mathbb{E} \frac{\partial v_{t+1}}{\partial b}(e', b, z') &= \lambda_t \\ &\Rightarrow \beta \left((1+r') \frac{u_{c'}}{u_c} \frac{p}{p'} \right) = 1 \end{aligned}$$

The utility function considered is:

$$u(c, n, \zeta) = \begin{cases} \left(\frac{c^{1-\sigma}}{1-\sigma} - \theta \frac{n^{1+\phi}}{1+\phi} \right) \zeta & \text{for } \sigma \neq 1 \\ \left(\log(c) - \theta \frac{n^{1+\phi}}{1+\phi} \right) \zeta & \text{for } \sigma = 1 \end{cases}$$

μ_t denotes the distribution of households across wealth and individual productivity states. It defines the following aggregate macroeconomic variables:

$$\begin{aligned} \text{Consumption:} \quad \mathcal{C}_t &\equiv \int c_t d\mu_t(e_t, b_{t-1}) \\ \text{Effective labour:} \quad \mathcal{N}_t &\equiv \int e n_t d\mu_t(e_t, b_{t-1}) \\ \text{Asset demand:} \quad \mathcal{A}_t &\equiv \int b_t d\mu_t(e_t, b_{t-1}) \end{aligned}$$

Firms

There is a continuum of identical firms indexed by $j \in [0, 1]$. Each firm produces one good differentiated from the other ones. The firms thus are in a monopolistic competition setting. Production requires labor only. Besides, we assume that each firm hires a representative workforce. Firm j faces the following problem:

$$J_t(p_{j,t+1}) = \max_{y_{jt}, n_{jt}, p_{jt}} \frac{p_{jt}}{p_t} y_{jt} - w_t n_{jt} - \underbrace{\frac{m}{m-1} \frac{1}{2\kappa} \log(1 + \pi_{jt})^2 Y_t}_{\psi_{jt}} + \frac{J_{t+1}(p_{jt})}{1 + r_{t+1}}$$

$$\text{s.t. } y_{jt} = Z_t n_{jt}^{1-\alpha}$$

$$y_{jt} = \left(\frac{p_{jt}}{p_t} \right)^{-\frac{m}{m-1}} Y_t$$

where $\pi_{jt} \equiv \frac{p_{jt}}{p_{j,t-1}} - 1$ denotes the inflation rate. This problem features a nominal rigidity on prices *à la* Rotemberg, which translates into a quadratic price adjustment cost ψ_{jt} . Furthermore, symmetry is assumed, i.e. the prices of all goods are the same ($p_{jt} = p_t \forall j$). Subsequently, all the j subscripts can be dropped.

By aggregating all intermediate goods, total labor demand and output are:

$$N_t \equiv \int_0^1 n_{jt} dj$$

$$Y_t \equiv \int_0^1 y_{jt} dj \stackrel{\text{sym}}{=} Z_t N_t^{1-\alpha}$$

The Lagrangian is:

$$\mathcal{L}_t = \frac{p_{jt}}{p_t} \left(\frac{p_{jt}}{p_t} \right)^{-\frac{m}{m-1}} Y_t - w_t n_{jt} - \frac{m}{m-1} \frac{1}{2\kappa} \log(1 + \pi_{jt})^2 Y_t + \frac{J_{t+1}(p_{jt})}{1 + r_{t+1}}$$

$$- mc_t \left(Z_t n_{jt}^{1-\alpha} - \left(\frac{p_{jt}}{p_t} \right)^{-\frac{m}{m-1}} Y_t \right)$$

The first-order condition with respect to $p_{jt} = p_t$ yields the following New-Keynesian Phillips curve (NKPC):

$$\log(1 + \pi_t) = \kappa \left(mc_t - \frac{1}{m} \right) + \frac{1}{1 + r_{t+1}} \log(1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t}$$

where:

- mc_t is a Lagrange multiplier and the marginal cost of an individual firm
- $\frac{m}{m-1}$ is the firm's markup

The marginal cost is:

$$\begin{aligned} mc_t &\equiv \frac{w_t}{MPL_t} \\ &= \frac{w_t}{(1-\alpha)Z_t N_t^{-\alpha}} \end{aligned}$$

The dividends derive from the no-profit condition:

$$d_t = Y_t - w_t N_t - \psi_t$$

Fiscal and monetary policies

A central bank sets the nominal interest rate i_t following a Taylor rule:

$$i_t = i_{ss} + \phi_\pi \pi_t + \phi_y (Y_t - Y_{ss}) + \varepsilon_t$$

where:

- i_{ss} and Y_{ss} denote the stationary state values of i_t and Y_t
- $Y_t - Y_{ss}$ is the output gap
- ε_t is an exogenous monetary policy shock

Additionally, a Fisher rule links real and nominal interest rates:

$$1 + r_t = \frac{1 + i_{t-1}}{1 + \pi_t}$$

A government supplies the one-period nominal bonds in which households invest. This quantity of sovereign bonds is B_t , exogenously set up. The government also establishes the tax rate τ_t to repay the interest on its debt.

$$\tau_t = r_t B_t$$

Market clearing

The market clearing conditions are:

$$\begin{aligned} \text{Good} &: Y_t = C_t \\ \text{Labour} &: N_t = \mathcal{N}_t \\ \text{Asset} &: B_t = \mathcal{A}_t \end{aligned}$$

1.8.2 Stationary state of the benchmark HANK model

Solving the model

1. Fix the values of the exogenous shocks Z, z, ε to 1 and the value of inflation π to 0. Set target values for the interest rate r , aggregate output Y , the supply of sovereign debt B and government spending G . From this, deduce:

$$N = \left(\frac{Y}{Z}\right)^{\frac{1}{1-\alpha}}$$

$$MPL = (1 - \alpha)ZN^{-\alpha}$$

$$w = \frac{MPL}{m}$$

$$d = Y - wN$$

$$\tau = rB + G$$

2. Make assumptions about the values of the discount factor β and the disutility of labour θ .
3. Solve the microeconomic problem of the households. Aggregate across the distribution of agents to get the supply of savings \mathcal{A} and labor \mathcal{N} .
4. If $\mathcal{A} = B$ and $\mathcal{N} = N^{14}$, the stationary state is solved. Otherwise, update β and θ and iterate.

Calibration

Table 1.9: Calibration of the HANK model. It follows Auclert, Bardóczy, Rognlie, and Straub, 2021.

Parameter		Value	Target
β	Discount factor	0.982	$r = 0.5\%$
θ	Disutility of labour	0.59	$\mathcal{N} = 1$
σ	Inverse elasticity of intertemporal substitution	2	
ϕ	Inverse Frisch elasticity	2	
\underline{b}	Borrowing constraint	0	
ρ_e	Autocorrelation of earnings	0.966	
σ_e	Cross-sectional std of log earnings	0.5	
α	Capital share	0.25	
m	Steady-state markup	1.2	
κ	Phillips curve slope	0.1	
B	Sovereign bond supply	5.6	
G	Government spending	0	
ϕ_π	Taylor rule coefficient for inflation	1.5	
ϕ_y	Taylor rule coefficient for output	0	
n_e	Number of idiosyncratic productivity states	7	
n_a	Number of points in the asset grid	500	

¹⁴If furthermore $\mathcal{C} + G = Y$, then Walras' law is verified. If not, the resolution of the stationary state suffers from a problem.

1.8.3 Dynamics of the benchmark HANK model

Jacobians of the blocks

Let's define the Jacobians of each block for the benchmark HANK model. For block b , $J_b^{o,i} \equiv \left(\frac{\partial o_t}{\partial i_s} \right)_{t,s}$ denotes the Jacobian of this block for an input i and output o . These Jacobians are matrixes where t denotes the current period and s the past and future ones. The system is assumed to go back to its stationary equilibrium after T periods, so that $t, s \in \llbracket 0, T-1 \rrbracket$.

Firm block.

- Inputs: w, π, Y, Z
- Outputs: N, d
- Equations:
$$\begin{cases} N_t = \left(\frac{Y_t}{Z_t} \right)^{\frac{1}{1-\alpha}} \\ d_t = Y_t - w_t N_t - \frac{m}{m-1} \frac{1}{2\kappa} (\log(1 + \pi_t))^2 Y_t \end{cases}$$
- Jacobians: $J_{firm}^{N,w}, J_{firm}^{N,\pi}, J_{firm}^{N,Y}, J_{firm}^{N,Z}, J_{firm}^{d,w}, J_{firm}^{d,\pi}, J_{firm}^{d,Y}, J_{firm}^{d,Z}$
Note that as N does not depend on w and π , $J_{firm}^{N,w} = J_{firm}^{N,\pi} = 0$.

Central bank.

- Inputs: π, ε
- Output: r
- Equation: $r_t = \frac{1+i_{ss}+\varepsilon_{t-1}+\phi_\pi \pi_{t-1}+\phi_y(Y_{t-1}-Y_{ss})}{1+\pi_t} - 1$
- Jacobians: $J_{CB}^{r,\pi}, J_{CB}^{r,\varepsilon}$

Fiscal block.

- Inputs: r, B
- Output: τ
- Equation: $\tau_t = r_t B_t$
- Jacobians: $J_{fiscal}^{r,\tau}, J_{fiscal}^{r,B}$

Household block.

- Inputs: w, r, d, τ, ζ
- Outputs: $\mathcal{A}, \mathcal{N}, \mathcal{C}$

- Aggregation equations:
$$\begin{cases} \mathcal{A}_t(\{r_s, w_s, d_s, \tau_s, \zeta_s\}) \\ \mathcal{N}_t(\{r_s, w_s, d_s, \tau_s, \zeta_s\}) \\ \mathcal{C}_t(\{r_s, w_s, d_s, \tau_s, \zeta_s\}) \end{cases}$$
- Jacobians: $J_{HA}^{w,A}, J_{HA}^{r,A}, J_{HA}^{d,A}, J_{HA}^{\tau,A}, J_{HA}^{\zeta,A}, J_{HA}^{w,N}, J_{HA}^{r,N}, J_{HA}^{d,N}, J_{HA}^{\tau,N}, J_{HA}^{\zeta,N}, J_{HA}^{w,C}, J_{HA}^{r,C}, J_{HA}^{d,C}, J_{HA}^{\tau,C}, J_{HA}^{\zeta,C}$

While the Jacobians of most blocks are sparse and can be analytically derived, the households' Jacobians are more complicated. Indeed, the values of aggregate savings, labor, and consumption at time t depend on the past values of input variables due to the persistence of the shocks. Via the Bellman equation, they also depend on future values. Auclert, Bardóczy, Rognlie, and Straub, 2021 efficiently calculate the Jacobians associated with the household block exploiting these recursive relationships in their *fake-news* algorithm.

Fake-news algorithm

Let's present how Auclert, Bardóczy, Rognlie, and Straub, 2021 manage to efficiently compute the households' Jacobians by exploiting the recursive nature of the problem.

The household's program is always of the following form:

$$\begin{aligned} \mathbf{Y}_t &= \sum_e \int_k y(e, k_-; V_{t+1}, \mathbf{X}_t) D_t(e, dk_-) & (\text{Aggr.}) \\ V_t(e, k_-) &= \max_k u(e, k_-, k, \mathbf{X}_t) + \beta \mathbb{E} V_{t+1}(e', k) & (\text{DP}) \\ D_{t+1}(e', K) &= \sum_e D_t(e, k_t^{*-1}(e, K)) P(e, e', \mathbf{X}_t) & (\text{LoM}) \end{aligned}$$

where \mathbf{X} are the exogenous inputs to the household's problem, \mathbf{Y} the macroeconomic aggregate variables in output and D the distribution of agents.

When discretizing, this system of equations becomes:

$$\begin{aligned} \mathbf{v}_t &= v(\mathbf{v}_{t+1}, \mathbf{X}_t) \\ \mathbf{D}_{t+1} &= \Lambda(\mathbf{v}_{t+1}, \mathbf{X}_t)' \mathbf{D}_t \\ \mathbf{Y}_t &= y(\mathbf{v}_{t+1}, \mathbf{X}_t)' \mathbf{D}_t \end{aligned}$$

where:

- \mathcal{X}_{ss} , is the steady state of any variable \mathcal{X} .
- \mathbf{v}, \mathbf{D} are the values of the value function and distribution at the gridpoints of the grids considered.
- $\Lambda(\mathbf{v}', \mathbf{X})$ is the transition matrix that discretizes the law of motion of D .

Linearising around the steady-state (and assuming that \mathbf{X} , \mathbf{Y} are one-dimensional for the sake of conciseness) leads to:

$$\begin{aligned}
d\mathbf{v}_t &= \mathbf{v}_v d\mathbf{v}_{t+1} + \mathbf{v}_x dX_t \\
d\mathbf{D}_{t+1} &= (\Lambda_v d\mathbf{v}_{t+1} + \Lambda_x dX_t)' \mathbf{D}_{ss} + \Lambda'_{ss} d\mathbf{D}_t \\
dY_t &= (\mathbf{y}_v d\mathbf{v}_{t+1} + \mathbf{y}_x dX_t)' \mathbf{D}_{ss} + \mathbf{y}'_{ss} d\mathbf{D}_t \\
&= \underbrace{d\mathbf{y}'_t \mathbf{D}_{ss}}_{\text{individual effect}} + \underbrace{\mathbf{y}'_{ss} d\mathbf{D}_t}_{\text{distributional effect}} \\
d\mathbf{v}_T &= 0 \text{ (Terminal condition)} \\
d\mathbf{D}_0 &= 0 \text{ (Initial condition)}
\end{aligned}$$

where u_x is the matrix of partial derivatives of $u \in \{\mathbf{v}, \mathbf{y}, \Lambda\}$ with respect to the variable $x \in \{x, v\}$. Obtaining the households' response to a shock (i.e. the Jacobian) requires computing dY_t .

Iterating on $d\mathbf{v}_t$:

$$\begin{aligned}
d\mathbf{v}_t &= \mathbf{v}_v d\mathbf{v}_{t+1} + \mathbf{v}_x dX_t \\
&= \mathbf{v}_v [\mathbf{v}_v d\mathbf{v}_{t+2} + \mathbf{v}_x dX_{t+1}] + \mathbf{v}_x dX_t = \mathbf{v}_v^2 d\mathbf{v}_{t+2} + \mathbf{v}_v \mathbf{v}_x dX_{t+1} + \mathbf{v}_x dX_t \\
&= \mathbf{v}_v^3 d\mathbf{v}_{t+3} + \mathbf{v}_v^2 \mathbf{v}_x dX_{t+2} + \mathbf{v}_v \mathbf{v}_x dX_{t+1} + \mathbf{v}_x dX_t \\
&\vdots \\
&= \mathbf{v}_v^{T-t} \underbrace{d\mathbf{v}_T}_0 + \mathbf{v}_v^{T-t-1} \mathbf{v}_x dX_{T-1} \dots + \mathbf{v}_v \mathbf{v}_x dX_{t+1} + \mathbf{v}_x dX_t
\end{aligned}$$

After $T - t - 2$ forward iterations, we can see that $d\mathbf{v}_t$ only depends on the current shock dX_t and future ones

$$d\mathbf{v}_t = \sum_{u=0}^{T-t-1} \mathbf{v}_v^u \mathbf{v}_x dX_{t+u}$$

The individual effect $d\mathbf{y}'_t \mathbf{D}_{ss}$ of dY_t thus depends of:

$$\begin{aligned}
d\mathbf{y}_t &= \mathbf{y}_v d\mathbf{v}_{t+1} + \mathbf{y}_x dX_t \\
&= \mathbf{y}_v \left(\sum_{u=0}^{T-t-2} \mathbf{v}_v^u \mathbf{v}_x dX_{t+1+u} \right) + \mathbf{y}_x dX_t \\
&= \sum_{u=0}^{T-t-2} (\mathbf{y}_v \mathbf{v}_v^u \mathbf{v}_x dsX_{t+1+u}) + \mathbf{y}_x dX_t \\
&= \sum_{s=0}^{T-1} \alpha_s dX_s
\end{aligned}$$

where:

$$\alpha_s = \begin{cases} 0 & \text{si } s < t \\ \mathbf{y}_x & \text{si } s = t \\ \mathbf{y}_v \mathbf{v}_v^{s-t-1} \mathbf{v}_x & \text{si } s > t \end{cases}$$

This means that only the current shock in t and future ones impact $d\mathbf{y}_t$. Future shocks are anticipated through the $\mathbf{y}_v \mathbf{v}_v^{s-t-1} \mathbf{v}_x$ term.

However, dY_t also features a distributional effect $\mathbf{y}'_{ss} d\mathbf{D}_t$ (that can be itself decomposed into a recursive $d\mathbf{D}_t$ and non-recursive part $(\Lambda_v d\mathbf{v}_{t+1} + \Lambda_x dX_t)' \mathbf{D}_{ss}$). Iterating on $d\mathbf{D}_t$ leads to:

$$\begin{aligned} d\mathbf{D}_0 &= 0 \\ d\mathbf{D}_1 &= (\Lambda_v d\mathbf{v}_1 + \Lambda_x dX_0)' \mathbf{D}_{ss} + \Lambda'_{ss} d\mathbf{D}_0 = (\Lambda_v d\mathbf{v}_1 + \Lambda_x dX_0)' \mathbf{D}_{ss} \\ d\mathbf{D}_2 &= (\Lambda_v d\mathbf{v}_2 + \Lambda_x dX_1)' \mathbf{D}_{ss} + \Lambda'_{ss} d\mathbf{D}_1 \\ &= (\Lambda_v d\mathbf{v}_2 + \Lambda_x dX_1 + \Lambda_v d\mathbf{v}_1 \Lambda_{ss} + \Lambda_x dX_0 \Lambda_{ss})' \mathbf{D}_{ss} \\ d\mathbf{D}_3 &= (\Lambda_v d\mathbf{v}_3 + \Lambda_x dX_2)' \mathbf{D}_{ss} + \Lambda'_{ss} d\mathbf{D}_2 \\ &= (\Lambda_v d\mathbf{v}_3 + \Lambda_x dX_2 + \Lambda_v d\mathbf{v}_2 \Lambda_{ss} + \Lambda_x dX_1 \Lambda_{ss} + \Lambda_v d\mathbf{v}_1 \Lambda_{ss}^2 \\ &\quad + \Lambda_x dX_0 \Lambda_{ss}^2)' \mathbf{D}_{ss} \end{aligned}$$

Finally,

$$d\mathbf{D}_t = \left(\sum_{u=0}^{t-1} \Lambda_v d\mathbf{v}_{t-u} \Lambda_{ss}^u + \Lambda_x dX_{t-u-1} \Lambda_{ss}^u \right)' \mathbf{D}_{ss}$$

We can summarise the response of dY_t using its two components:

- Individual effect $d\mathbf{y}'_t D_{ss}$ such that $\frac{d\mathbf{y}'_t D_{ss}}{dX_s} = \mathcal{Y}_{s-t}$
 where $\mathcal{Y}_u = \begin{cases} 0 & \text{if } u < 0 \\ \mathbf{y}'_x \mathbf{D}_{ss} & \text{if } u = 0 \\ (\mathbf{y}_v (\mathbf{v}_v)^{u-1} \mathbf{v}_x)' \mathbf{D}_{ss} & \text{if } u > 0 \end{cases}$
- Distributional effect $\mathbf{y}'_{ss} d\mathbf{D}_t$ such that: $\frac{d\mathbf{y}'_{ss} d\mathbf{D}_t}{dX_s} = \mathcal{P}'_0 \mathcal{D}_{s-t+1} + \mathcal{P}'_1 \mathcal{D}_{s-t+2} + \dots + \mathcal{P}'_{t-1} \mathcal{D}_s$
 with $\mathcal{P}'_u = \begin{cases} 0 & \text{if } u < 0 \\ \mathbf{y}'_{ss} (\Lambda'_{ss})^u & \text{if } u \geq 0 \end{cases}$ and $\mathcal{D}_u = \begin{cases} 0 & \text{if } u < 0 \\ \Lambda'_x \mathbf{D}_{ss} & \text{if } u = 0 \\ (\Lambda_v (\mathbf{v}_v)^{u-1} \mathbf{v}_x)' \mathbf{D}_{ss} & \text{if } u > 0 \end{cases}$

Combining the two effects gives,

$$\frac{dY_t}{dX_s} \equiv \mathcal{J}_{t,s} = (\mathcal{Y}_{s-t} + \mathcal{P}'_0 \mathcal{D}_{s-t-1} + \mathcal{P}'_1 \mathcal{D}_{s-t-2} + \dots + \mathcal{P}'_{t-1} \mathcal{D}_s)$$

The Jacobians can be built recursively since the terms $\mathcal{J}_{t,s}$ and $\mathcal{J}_{t-1,s-1}$ are identical up to an additive term:

$$\mathcal{J}_{t,s} = \begin{cases} \mathcal{F}_{t,s} & t = 0 \text{ or } s = 0 \\ \mathcal{F}_{t,s} + \mathcal{J}_{t-1,s-1} & \text{otherwise} \end{cases} \text{ where } \mathcal{F}_{t,s} = \begin{cases} \mathcal{Y}_s & t = 0 \\ \mathcal{P}'_{t-1} \mathcal{D}_s & t > 0 \end{cases}$$

Auclert, Bardóczy, Rognlie, and Straub, 2021 call \mathcal{F} a fake news matrix for it is the Jacobian of “false” shocks. At period $t = 0$, the households are informed of a sequence of shocks $\{dX_t\}_{t=0}^T$. dX_0 happens but all the future shocks do not materialize and households

are informed of that in period 1. Those shocks have however impacted the future decisions of households since they were anticipated. These responses to future shocks are then used as numerical derivatives in an approach similar to Boppart, Krusell, and Mitman, [2018](#).

The fake news algorithm saves a considerable amount of time compared with direct differentiation of the household problem. Indeed, Auclert, Bardóczy, Rognlie, and Straub, [2021](#) report that the Jacobian of the household's block in a similar one-asset HANK can be obtained in less than a second (0.646 s) with this technique compared with 176 s with the direct approach.

Impulse response functions

The combination of the Jacobians of the blocks following the directed acyclic graph of the model then provides the Jacobian of the entire system. The overall response of the model to a shock results. In practice, these computations do not require to be done by hand and are automated.

Impulse response functions per productivity types

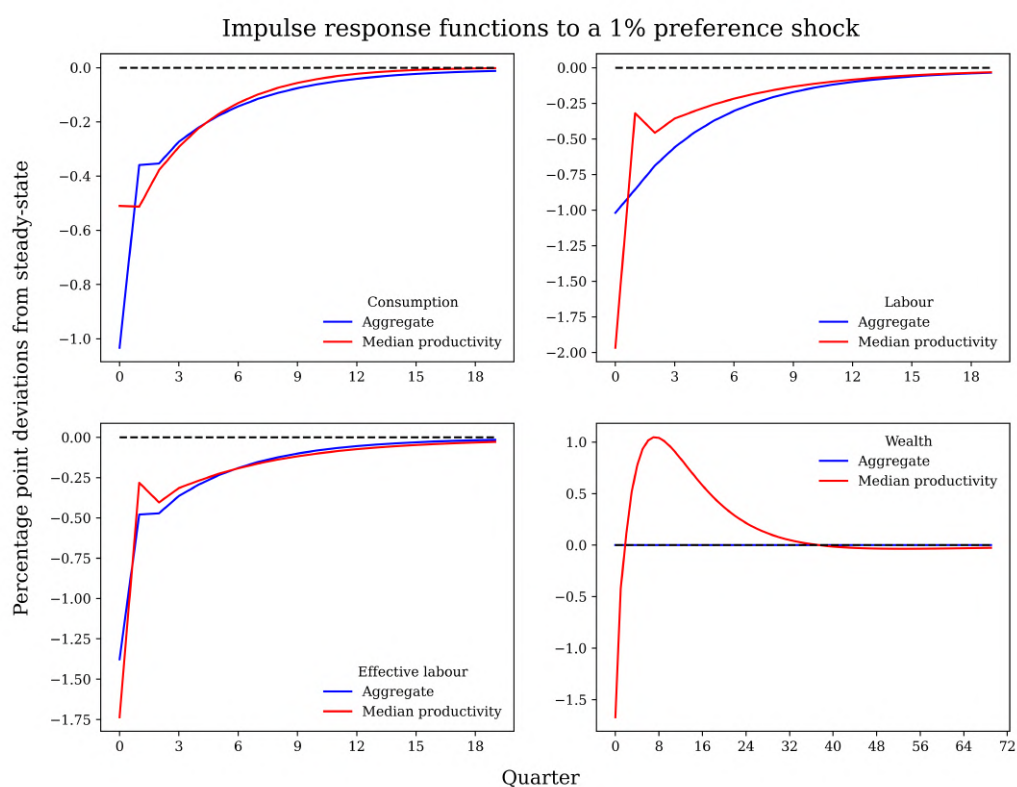


Figure 1.14: Aggregate and median-productivity type household responses to a 1% positive preference shock.

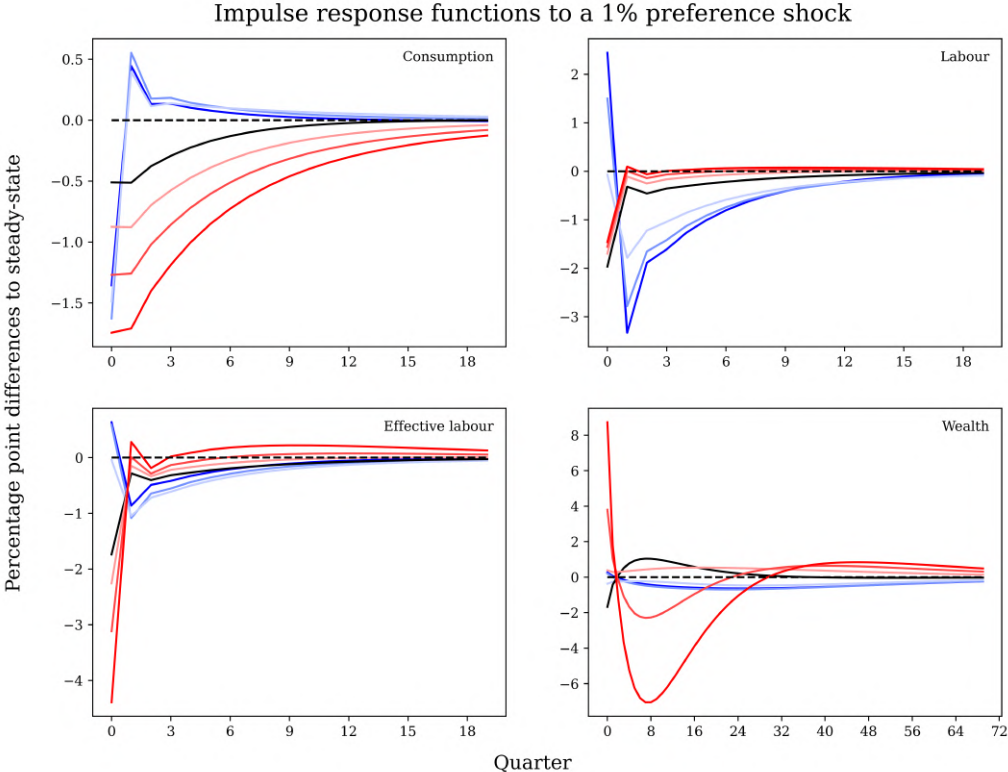


Figure 1.15: Impulse response functions to a 1% positive preference shock by productivity types: black is the median productivity; the darker the blue, the lower the productivity compared to the median; darker red shades are associated with higher productivity than the median.

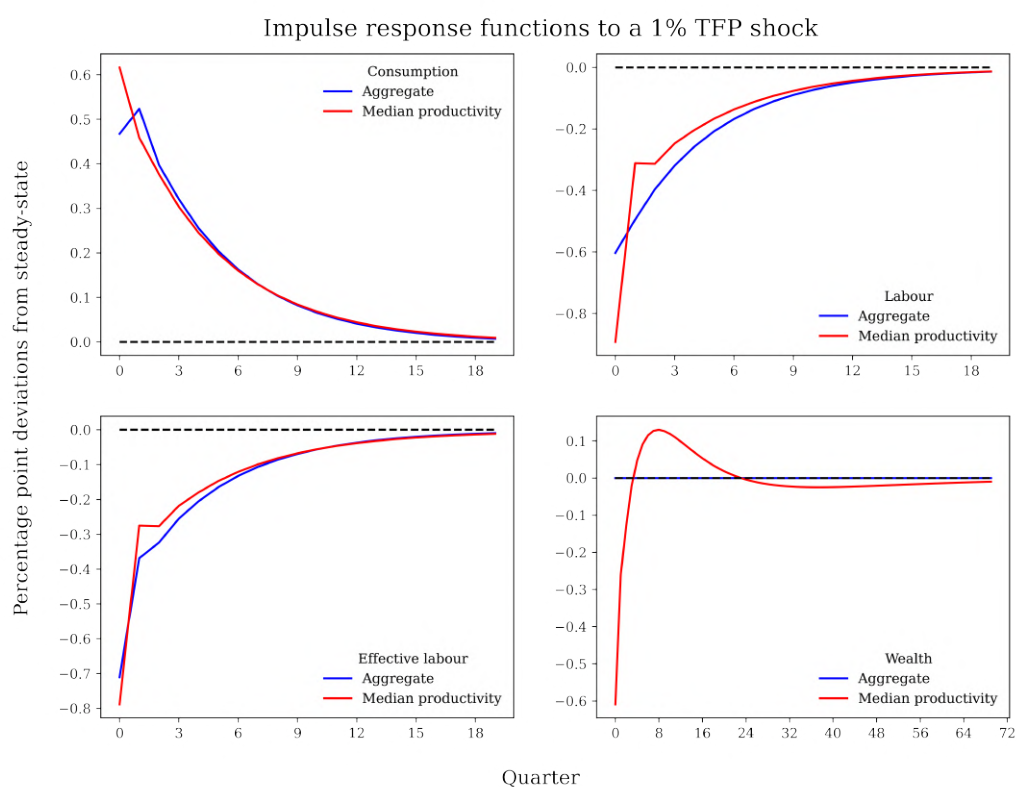


Figure 1.16: Aggregate and median-productivity type household responses to a 1% positive productivity shock.

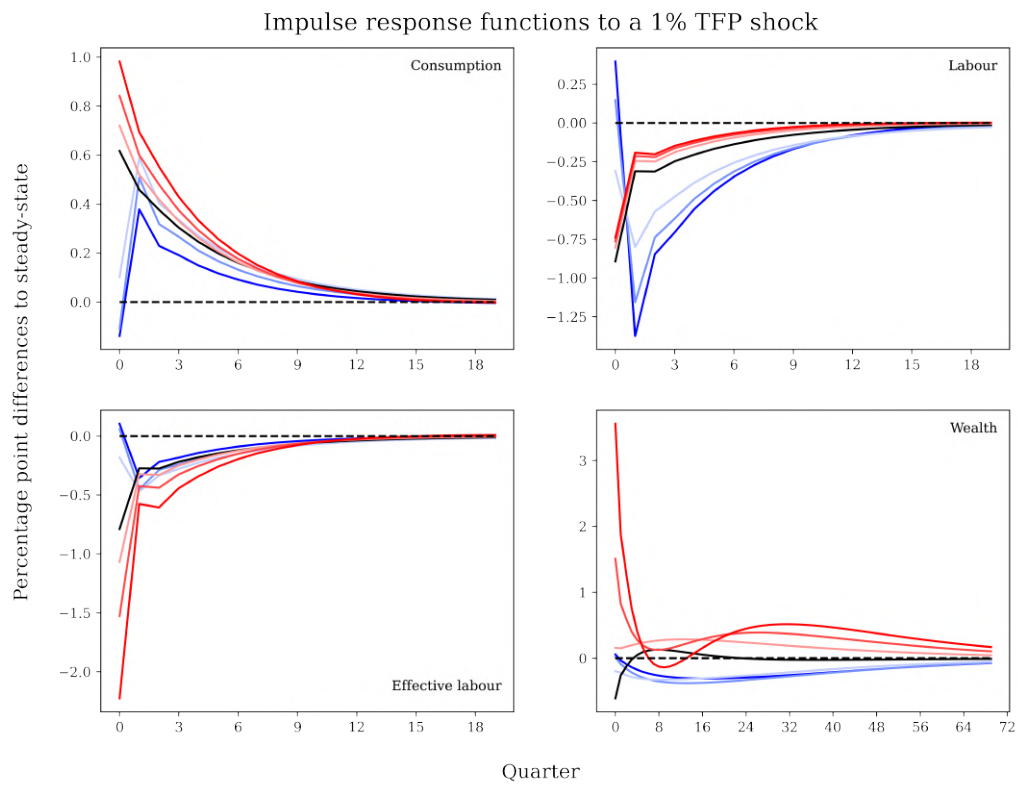


Figure 1.17: Impulse response functions to a 1% positive productivity shock by productivity types: black is the median productivity; the darker the blue, the lower the productivity compared to the median; darker red shades are associated with higher productivity than the median.

1.8.4 Estimation of the benchmark HANK model

Computing the log-likelihood

- Step 1: Write the MA representation of the shocks.

For instance, if we consider that the shocks follow AR(1) processes,

$$\begin{aligned} dZ_t &= \rho_Z dZ_{t-1} + \varepsilon_{Z,t}, & \varepsilon_Z &\sim \mathcal{N}(0, \sigma_Z^2) \text{ i.i.d.} \\ d\zeta_t &= \rho_z d\zeta_{t-1} + \varepsilon_{\zeta,t}, & \varepsilon_\zeta &\sim \mathcal{N}(0, \sigma_\zeta^2) \text{ i.i.d.} \\ d\varepsilon_t &= \rho_\varepsilon d\varepsilon_{t-1} + \varepsilon_{\varepsilon,t}, & \varepsilon_\varepsilon &\sim \mathcal{N}(0, \sigma_\varepsilon^2) \text{ i.i.d.} \end{aligned}$$

Then the moving average representations of these processes yield:

$$\begin{aligned} dZ_t &= \sum_{s=0}^t \rho_Z^s \varepsilon_{Z,t-s} = \overrightarrow{m}_Z' \cdot \overrightarrow{\varepsilon}_Z \\ d\zeta_t &= \sum_{s=0}^t \rho_\zeta^s \varepsilon_{\zeta,t-s} = \overrightarrow{m}_\zeta' \cdot \overrightarrow{\varepsilon}_\zeta \\ d\varepsilon_t &= \sum_{s=0}^t \rho_\varepsilon^s \varepsilon_{\varepsilon,t-s} = \overrightarrow{m}_\varepsilon' \cdot \overrightarrow{\varepsilon}_\varepsilon \end{aligned}$$

where $\overrightarrow{m}_X = (1, \rho_X, \dots, \rho_X^t)'$, $\overrightarrow{\varepsilon}_X = (\varepsilon_{X,1}, \dots, \varepsilon_{X,t})'$ for $X \in \{Z, z, \varepsilon\}$.

- Step 2: Write the MA representation of the model.

In a HA model, the response of an output X to a shock dZ is given by:

$$dX = G^{X,Z} dZ$$

where $G^{X,Z}$ denotes the general equilibrium Jacobian of this model.

Following the proposition by Auclert, Bardóczy, Rognlie, and Straub, 2021, the impulse response functions of the model correspond to the coefficients of its moving-average representation given the certainty equivalence. As a result,

$$dX_t = \sum_{s=0}^{T-1} \sum_{u \in \{Z, z, \varepsilon\}} m_s^{X,u} \varepsilon_{u,t-s}$$

with $\overrightarrow{m}^{X,u} = G^{X,u} \cdot \overrightarrow{m}_u$.

Note that the time horizon T must be large to ensure the system has time to converge back to the steady state. Otherwise, the variables would jump in the last periods, leading to wrong responses.

In practice, we consider three variables output Y , inflation π and the nominal interest rate i , so that:

$$\begin{aligned} dY_t &= \sum_{s=0}^{\infty} \{m_s^{Y,Z} \varepsilon_{Z,t-s} + m_s^{Y,\zeta} \varepsilon_{\zeta,t-s} + m_s^{Y,\varepsilon} \varepsilon_{\varepsilon,t-s}\} \\ d\pi_t &= \sum_{s=0}^{\infty} \{m_s^{\pi,Z} \varepsilon_{Z,t-s} + m_s^{\pi,\zeta} \varepsilon_{\zeta,t-s} + m_s^{\pi,\varepsilon} \varepsilon_{\varepsilon,t-s}\} \\ di_t &= \sum_{s=0}^{\infty} \{m_s^{i,Z} \varepsilon_{Z,t-s} + m_s^{i,\zeta} \varepsilon_{\zeta,t-s} + m_s^{i,\varepsilon} \varepsilon_{\varepsilon,t-s}\} \end{aligned}$$

Introducing, $\overrightarrow{dY}_t = (dY_t, d\pi_t, di_t)'$ and $\overrightarrow{\varepsilon}_t = (\varepsilon_{Z,t}, \varepsilon_{\zeta,t}, \varepsilon_{\varepsilon,t})'$, the three previous equations rewrite:

$$\overrightarrow{dY}_t = \sum_{s=0}^{T-1} \Phi_s \overrightarrow{\varepsilon}_s$$

with Φ_s matrixes of size 3×3 .

- Step 3: Compute the model's covariances.

Let Y_1 and Y_2 denote two variables that admit a MA representation as previously defined ($Y_1, Y_2 \in \{Y, \pi, i\}$). Then, the covariance between these two variables with a lag l is given by:

$$\text{cov}(dY_{1,t}, dY_{2,t+l}) = \sum_{u \in \{Z, \zeta, \varepsilon\}} \sigma_u^2 \sum_{s=0}^{T-1} m_s^{Y_1, u} m_{s+l}^{Y_2, u}$$

A Fast Fourier Transform (FFT) algorithm efficiently computes the covariances between every variable at all lags. The resulting covariance matrix Ω is of size $3T \times 3T$. Indeed, we stack the responses vertically $(dY_0, dY_1, \dots, dY_{T-1}, d\pi_0, \dots, d\pi_{T-1}, di_0, \dots, di_{T-1})'$ and the matrix Ω is such that:

$$\Omega = \begin{bmatrix} \Phi_0 & & & & \\ \Phi_1 & \Phi_0 & & & \\ \Phi_2 & \Phi_1 & \ddots & & \\ \vdots & & \ddots & \ddots & \\ \Phi_{T-1} & \Phi_{T-2} & \cdots & \Phi_1 & \Phi_0 \end{bmatrix}$$

- Step 4: Define the measurement equations

We also need to specify observation equations (also called measurement equations). They tell us how observed variables \overrightarrow{Y}^{obs} relate to those generated by our model \overrightarrow{Y} .

In this case, we have:

$$\begin{aligned} dY_t^{obs} &= dY_t + u_t^Y \\ d\pi_t^{obs} &= d\pi_t + u_t^C \\ di_t^{obs} &= di_t + u_t^i \end{aligned}$$

where the u_t^X are measurement errors that we assume to be null.

- Step 5: Compute the covariance matrix X

We now compute V the matrix composed of blocks $\text{cov}(dY_{1,t}^{obs}, dY_{2,s}^{obs})$. Due to the previous equations, an analytical expression links the observed data's covariances and those of the model.

For $Y_1, Y_2 \in \{Y, \pi, i\}$,

$$\begin{aligned} \text{cov}(dY_{1,t}^{obs}, dY_{2,s}^{obs}) &= \text{cov}(dY_{1,t} + u_t^{Y_1}, dY_{2,s} + u_s^{Y_2}) \\ &= \underbrace{\text{cov}(dY_{1,t}, dY_{2,s})}_{\text{computed in } \Omega} + \underbrace{\text{cov}(dY_{1,t}, u_s^{Y_2})}_0 + \underbrace{\text{cov}(u_t^{Y_1}, dY_{2,s})}_0 + \underbrace{\text{cov}(u_t^{Y_1}, u_s^{Y_2})}_{\sigma_{u_{Y_1}}^2 \mathbb{1}_{\{t=s\}} \mathbb{1}_{\{Y_1=Y_2\}}} \end{aligned}$$

Stacking the results in the following order, $(dY_0^{obs}, \dots, dY_{T-1}^{obs}, d\pi_0^{obs}, \dots, d\pi_{T-1}^{obs}, di_0^{obs}, \dots, di_{T-1}^{obs})'$, it results that,

$$V = \Omega(I_T \otimes \Sigma)\Omega'$$

where $\Sigma = \begin{pmatrix} \sigma_Z^2 & 0 & 0 \\ 0 & \sigma_\zeta^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix}$ is the covariance matrix of the shocks and V a matrix of size $3T \times 3T$.

- Step 6: Compute the log-likelihood.

Given the matrix V , the log-likelihood is:

$$\mathcal{L}(\vec{\Theta}) = -\frac{1}{2} \log \det V - \frac{1}{2} \vec{dY}^{obs'} V^{-1} \vec{dY}^{obs} \quad (1.7)$$

In practice, V is factored using a Cholesky decomposition for efficiency reasons. This requires the matrix V to be symmetric and positive-definite. A covariance matrix is always symmetric and semidefinite. For some parameter values, the positive-definite character may fail (i.e. there are some null or negative eigenvalues). Whenever this happens, the associated covariance matrix V does not make sense and we should not compute the likelihood.

Let's demonstrate equation (1.7).

Proof. For $u \in \{Z, z, \varepsilon\}$, the shocks $\varepsilon_{u,t}$ are independent and identically distributed random variables that follow a normal distribution $\mathcal{N}(0, \sigma_u^2)$.

For $X \in \{Y, \pi, i\}$, the observations dX^{obs} follow $dX^{obs} = dX$ as we assumed the measurement errors to be null.

The MA representation of dX yields that it is a function of the shocks:

$$dX^{obs} = dX = f(\{\varepsilon_{Z,t}, \varepsilon_{\zeta,t}, \varepsilon_{\varepsilon,t}\}_{t \in [0, T-1]}).$$

Thus dX_t^{obs} is a random variable that is the sum of $3T$ independent normally distributed random variables (the $\varepsilon_{u,t}$) weighted by some coefficients (those of the MA representation). The vertically staked observations thus are a multivariate normally distributed random variable:

$$(dY_0^{obs}, \dots, dY_{T-1}^{obs}, d\pi_0^{obs}, \dots, d\pi_{T-1}^{obs}, di_0^{obs}, \dots, di_{T-1}^{obs})' \sim \mathcal{N}(0_{3T}, V)$$

where V is a $3T \times 3T$ matrix (the covariance matrix constructed previously).

The density function of a multivariate normal distribution with mean vector μ and covariance matrix Σ is given by:

$$p(x) = \frac{\exp\left(-\frac{1}{2}(x - \mu)' \Sigma^{-1}(x - \mu)\right)}{\sqrt{(2\pi)^k \det(\Sigma)}}$$

where x is a vector of size k .

Applying this formula to our case ($\mu = 0_{3T}$, $\Sigma = V$, $k = 3T$), we have:

$$p(x) = \frac{\exp\left(-\frac{1}{2}x'V^{-1}x\right)}{\sqrt{(2\pi)^{3T} \det(V)}} \propto \exp\left(-\frac{1}{2}x'V^{-1}x\right) \exp\left(-\frac{1}{2} \log \det V\right)$$

The log-likelihood of our observed variables is:

$$\begin{aligned} \mathcal{L} &= \log(p(\overrightarrow{dY^{obs}})) \\ &\propto \log\left(\exp\left(-\frac{1}{2}\overrightarrow{dY^{obs}'}V^{-1}\overrightarrow{dY^{obs}}\right) \exp\left(-\frac{1}{2} \log \det V\right)\right) \\ &\propto -\frac{1}{2}\overrightarrow{dY^{obs}'}V^{-1}\overrightarrow{dY^{obs}} - \frac{1}{2} \log \det V \end{aligned}$$

□

Maximum likelihood estimation

The parameters are estimated by maximising the log-likelihood function $\mathcal{L} : \vec{\Theta} \mapsto \mathcal{L}(\vec{\Theta})$. First, we estimate only the shock parameters (Tables 1.10 and 1.11). Then we also estimate some structural parameters, namely the Taylor rule and the Phillips curve coefficients (Tables 1.12 and 1.6 in Section 1.5).

- Parameters to estimate: $\rho_Z, \rho_\zeta, \rho_\varepsilon, \sigma_Z, \sigma_\zeta, \sigma_\varepsilon, (\phi_\pi, \phi_y, \kappa)$.
- Observed variables: US output Y^{obs} , inflation rate π^{obs} and Federal fund rates i^{obs} over the 1Q1984-4Q2007 period.
- Number of periods: 95.
- Measurement equations:
$$\begin{cases} Y^{obs} = Y \\ \pi^{obs} = \pi \\ i^{obs} = i \end{cases}$$
 assuming no measurement errors.

Table 1.10: Maximum likelihood estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter		Estimate	Standard error	95% confidence interval
Monetary policy shock	ρ_ε	0.504	0.0219	[0.461,0.461]
	σ_ε	0.00329	0.000265	[0.00277,0.00380]
Preference shock	ρ_ζ	0.993	0.00968	[0.974,1.012]
	σ_ζ	0.000298	3.47×10^{-5}	[0.000230,0.000366]
Technological shock	ρ_Z	0.989	0.00151	[0.986,0.992]
	σ_Z	0.00847	0.000566	[0.00736,0.00958]

Table 1.11: Maximum likelihood estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter		Estimate	Standard error	95% confidence interval
Monetary policy shock	ρ_ε	0.545	0.0409	[0.465,0.625]
	σ_ε	0.00317	0.000275	[0.00263,0.00371]
Government spending shock	ρ_G	0.999	-	-
	σ_G	0.00937	0.000831	[0.00774,0.0110]
Price markup shock	ρ_m	0.751	0.0403	[0.672,0.830]
	σ_m	0.0231	0.00202	[0.0192,0.0271]

Table 1.12: Maximum likelihood estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter		Estimate	Standard error	95% confidence interval
Monetary policy shock	ρ_ε	0.760	0.0542	[0.654,0.866]
	σ_ε	0.00279	0.000269	[0.00226,0.00331]
Preference shock	ρ_ζ	0.649	0.0978	[0.458,0.841]
	σ_ζ	0.0153	0.00108	[0.0132,0.0174]
Technological shock	ρ_Z	0.797	0.185	[0.4333,1.16]
	σ_Z	0.0107	0.00339	[0.00404,0.0173]
Taylor rule	ϕ_π	1.5	-	-
	ϕ_y	0.240	0.0613	[0.119,0.360]
Phillips curve	κ	0.0110	0.00353	[0.00409, 0.0179]

Bayesian estimation

We then reestimate the model's parameters (with and without structural parameters) using a Bayesian approach instead. For the prior distributions, we assume beta distributions for the shock persistences, inverse gamma distributions for volatilities, and gamma distributions for structural parameters. Draws following the posterior distribution are generated using a Metropolis-Hastings algorithm. Results when estimating only the shock parameters, are reported in Tables 1.13 and 1.14. Adding structural shocks, we get Tables 1.15 and 1.7 in Section 1.5.

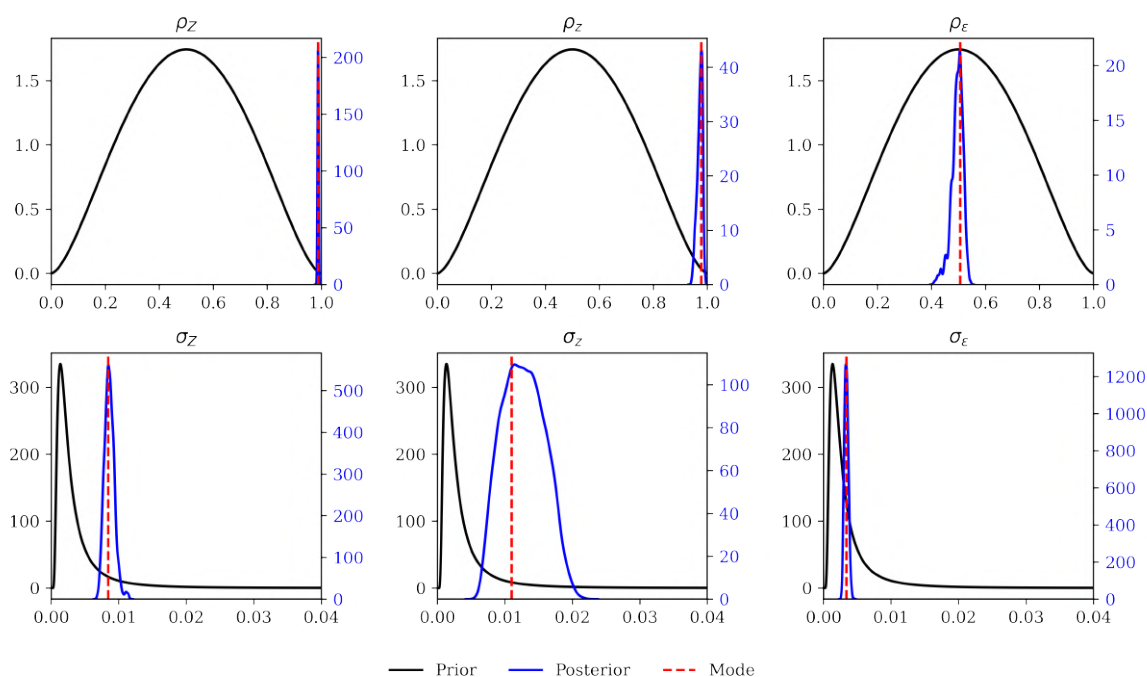


Figure 1.18: Prior and posterior density functions.

Table 1.13: Bayesian estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter	Prior distribution		Posterior			
			Mean	Std. dev.	5th perc.	95th perc.
Monetary policy shock	ρ_ϵ	$\beta(0.5, 0.2)$	0.504	0.0280	0.454	0.548
	σ_ϵ	$\text{inv}\Gamma(0.4, 4)$	0.00335	0.000291	0.00290	0.00382
Preference shock	ρ_ζ	$\beta(0.5, 0.2)$	0.974	0.0145	0.945	0.990
	σ_ζ	$\text{inv}\Gamma(0.4, 4)$	0.0131	0.00335	0.00747	0.0185
Technological shock	ρ_Z	$\beta(0.5, 0.2)$	0.986	0.00172	0.983	0.989
	σ_Z	$\text{inv}\Gamma(0.4, 4)$	0.00838	0.000673	0.00737	0.00953

Note: The parameters mentioned in the distributions are the means and standard deviations.

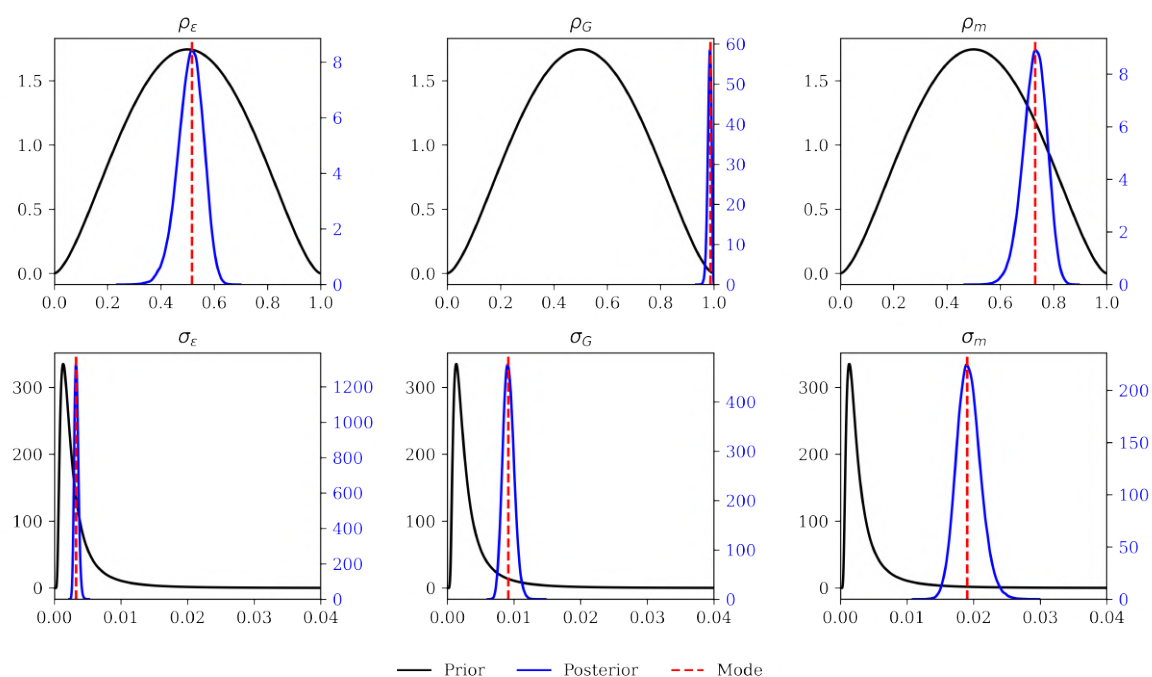


Figure 1.19: Prior and posterior density functions.

Table 1.14: Bayesian estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter		Prior distribution	Posterior			
			Mean	Std. dev.	5th perc.	95th perc.
Monetary policy shock	ρ_ϵ	$\beta(0.5, 0.2)$	0.513	0.0472	0.432	0.586
	σ_ϵ	$\text{inv}\Gamma(0.4, 4)$	0.00332	0.000303	0.00286	0.00385
Government spending shock	ρ_G	$\beta(0.5, 0.2)$	0.985	0.00743	0.972	0.997
	σ_G	$\text{inv}\Gamma(0.4, 4)$	0.00928	0.000858	0.00795	0.0108
Price markup shock	ρ_m	$\beta(0.5, 0.2)$	0.727	0.0443	0.651	0.796
	σ_m	$\text{inv}\Gamma(0.4, 4)$	0.0198	0.00171	0.0172	0.0228

Note: The parameters mentioned in the distributions are the means and standard deviations.

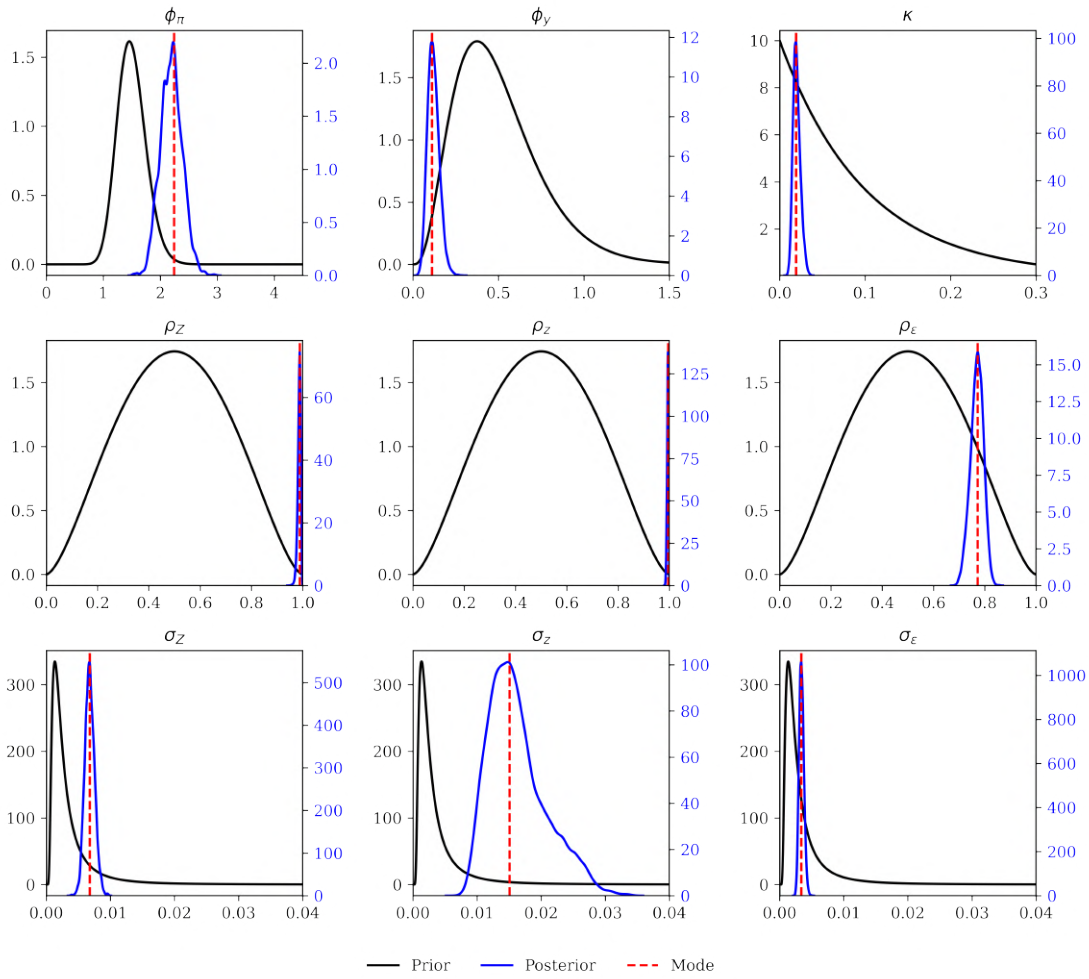


Figure 1.20: Prior and posterior density functions.

Table 1.15: Bayesian estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter	Prior distribution	Posterior				
		Mean	Std. dev.	5th perc.	95th perc.	
Monetary policy shock	ρ_ϵ	$\beta(0.5, 0.2)$	0.778	0.0431	0.720	0.855
	σ_ϵ	$\text{inv}\Gamma(0.4, 4)$	0.00332	0.000481	0.00258	0.00414
Preference shock	ρ_ζ	$\beta(0.5, 0.2)$	0.841	0.0983	0.682	0.955
	σ_ζ	$\text{inv}\Gamma(0.4, 4)$	0.0133	0.00267	0.00933	0.0176
Technological shock	ρ_Z	$\beta(0.5, 0.2)$	0.961	0.0442	0.867	0.995
	σ_Z	$\text{inv}\Gamma(0.4, 4)$	0.00997	0.00196	0.00704	0.0138
Taylor rule	ϕ_π	$\Gamma(1.5, 0.25)$	1.98	0.561	1.18	2.69
	ϕ_γ	$\Gamma(0.5, 0.25)$	0.194	0.134	0.0342	0.411
Phillips curve	κ	$\Gamma(0.1, 0.1)$	0.0125	0.00820	0.00264	0.0246

Note: The parameters mentioned in the distributions are the means and standard deviations.

Method of simulated moments estimation

Table 1.16: Simulated method of moments estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter		Estimate
Monetary policy shock	ρ_ε	0.750
	σ_ε	0.000001
Preference shock	ρ_ζ	0.548
	σ_ζ	0.00300
Technological shock	ρ_Z	0.946
	σ_Z	0.00244

Table 1.17: Simulated method of moments estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter		Estimate
Monetary policy shock	ρ_ε	0.200
	σ_ε	0.000146
Government spending shock	ρ_G	0.898
	σ_G	0.00398
Price markup shock	ρ_m	0.812
	σ_m	0.0139

Table 1.18: Simulated method of moments estimation of the HANK model over the 1Q1984-4Q2007 period.

Parameter		Estimate
Monetary policy shock	ρ_ε	0.750
	σ_ε	0.00648
Preference shock	ρ_ζ	0.763
	σ_ζ	0.0174
Technological shock	ρ_Z	0.840
	σ_Z	0.0112
Taylor rule	ϕ_π	2.31
	ϕ_y	1.00
Phillips curve	κ	0.00562

1.8.5 Benchmark RANK model

Presentation of the model

We consider a basic New Keynesian model *à la* Galí. A few modifications are proposed to allow comparison to the benchmark HANK model:

- A labour disutility coefficient θ_n is added to the utility function
- The representative agent has to pay taxes τ_t
- The government supplies a fixed quantity of debt B and public spending G in each period
- The government runs a balanced budget in each period: $\tau_t = r_t B + G$
- Price-adjustment cost *à la* Rotemberg instead of Calvo price-setting.

Calibration

Table 1.19: Calibration of the RANK model.

Parameter		Value	Target
β	Discount factor	0.99	$r = 0.5\%$
θ_n	Disutility of labour	0.63	$N = 1.0$
σ	Inverse elasticity of intertemporal substitution	2	
ϕ	Inverse Frisch elasticity	2	
\underline{b}	Borrowing constraint	0	
α	Capital share	0.25	
m	Steady-state markup	1.2	
κ	Phillips curve slope	0.1	
B	Sovereign bond supply	5.6	
G	Government spending	0	
ϕ_π	Taylor rule coefficient for inflation	1.5	
ϕ_y	Taylor rule coefficient for output	0	

1.8.6 Intuition of Boehl, 2023's nonlinear procedure

Starting from the equation of step k of a Newton method (see equation 1.5):

$$\begin{aligned}\mathcal{Y}^{(k+1)} &= \mathcal{Y}^{(k)} - J^{-1}(\mathcal{Y}^{(k)})\mathbf{H}(\mathcal{Y}^{(k)}, \mathcal{Z}) \\ \Leftrightarrow J(\mathcal{Y}^{(k)})(\mathcal{Y}^{(k+1)} - \mathcal{Y}^{(k)}) &= -\mathbf{H}(\mathcal{Y}^{(k)}, \mathcal{Z})\end{aligned}$$

Introducing $\mathbf{y} \equiv \mathcal{Y}^{(k)} - \mathcal{Y}^{(k-1)}$, then:

$$\begin{aligned}J(\mathcal{Y}^{(k)})\mathbf{y} &= -\mathbf{H}(\mathcal{Y}^{(k)}, \mathcal{Z}) \\ \Leftrightarrow [J(\mathcal{Y}^{(k)}) - \alpha_k^{-1}J(\mathcal{Y}_{ss}) + \alpha_k^{-1}J(\mathcal{Y}_{ss})]\mathbf{y} &= -\mathbf{H}(\mathcal{Y}^{(k)}, \mathcal{Z}) \\ \Leftrightarrow \alpha_k^{-1}J(\mathcal{Y}_{ss})\mathbf{y} &= -\mathbf{H}(\mathcal{Y}^{(k)}, \mathcal{Z}) - [J(\mathcal{Y}^{(k)}) + \alpha_k^{-1}J(\mathcal{Y}_{ss})]\mathbf{y} \\ \Leftrightarrow \mathbf{y} &= \mathbf{y} + \alpha_k J^{-1}(\mathcal{Y}_{ss})J(\mathcal{Y}^{(k)})\mathbf{y} - \alpha_k J^{-1}(\mathcal{Y}_{ss})\mathbf{H}(\mathcal{Y}^{(k)}, \mathcal{Z}) \\ \Leftrightarrow \mathbf{y} &= \mathbf{y} + \alpha_k J^{-1}(\mathcal{Y}_{ss}) [J(\mathcal{Y}^{(k)})\mathbf{y} - \mathbf{H}(\mathcal{Y}^{(k)}, \mathcal{Z})]\end{aligned}$$

So far, $\alpha_k > 0$ is not specified. Denoting $\Lambda(\mathcal{Y}^{(k)}, \mathbf{y}) \equiv J(\mathcal{Y}^{(k)})\mathbf{y}$, we get:

$$\mathbf{y} = \mathbf{y} - \alpha_k J^{-1}(\mathcal{Y}_{ss}) [\Lambda(\mathbf{H}(\mathcal{Y}^{(k)}, \mathcal{Z}) - \mathcal{Y}^{(k)}, \mathbf{y})]$$

This expression is very close to (1.6). It needs however to be turned properly into an iterative procedure (to replace \mathbf{y} by $\mathbb{Y}^{(k+1)}$ and $\mathbb{Y}^{(k)}$). This is the purpose of Boehl, 2023's Proposition 1.

Chapter 2

The macroeconomic and redistributive effects of shielding consumers from rising energy prices: a real-time evaluation of the French experiment

This chapter is co-authored with François Langot, Fabien Tripier and Jean-Olivier Hairault.

Keywords: HANK model, Energy crisis, Tariff shield, Policy evaluation.

JEL codes: C54, C63, E32, E65, H12, Q43.

2.1 Introduction

Relief from the COVID-19 crisis in 2021 and Russia's incursion into Ukraine in 2022 precipitated an energy shock in Europe unparalleled in the history of the Eurozone. With gas prices surging by a factor of more than five and barrel prices more than doubling since 2021, the magnitude of inflationary shocks has been huge. In 2022, inflation rates surged to 8% in Italy, 8.3% in Germany, and 12% in the Netherlands. By contrast, France demonstrated a comparatively lower inflation rate of 6.2%, distinguishing itself from its European counterparts. Since October 2021, the French government has implemented a “tariff shield” to mitigate the impact of the energy shock.¹ At the end of 2022, the French government decided to extend the tariff shield to 2023. This policy had several important implications.

¹The tariff shield comprises a freeze on gas prices at their October 2021 levels, a limit on the rise in electricity prices, and the introduction of a discount at the pump starting in April 2022. In retrospect, it seems that this policy has had a notable impact on inflation. According to the French Statistical Institute (INSEE), inflation would have been 3.1 points higher between the second quarters of 2021 and 2022. See the publication INSEE Analysis *n°75* Soaring energy prices: the “tariff shield” cuts inflation in half (in French). For an appraisal of the French policy in comparison with measures implemented in other economies, see the fiscal tracker provided by the think tank Bruegel “National fiscal policy responses to the energy crisis”.

Beyond inflation, it is expected to affect economic growth, public finances, and economic inequality considering the larger proportion of energy expenditure among the most disadvantaged households.² The objective of this study is to assess the impact of the tariff shield implemented in France by comparing it with alternative policies. To achieve this, we develop a new method for Heterogeneous-Agent New-Keynesian (HANK) models that involves identifying the shocks that rationalize the data, including forecast data, using the sequence-space method introduced by Auclert, Bardóczy, Rognlie, and Straub, 2021. This original method can be employed in real time to assist governments in making policy decisions.

Specifically, we examine the effectiveness of the tariff shield over a specific period and at the time of its announcement. What are the expected consequences of this policy on output, inflation, inequality, and public debt? How does its efficacy compare to that of alternative policies? We require a particular approach to achieve this objective in terms of policy evaluation because some policies alter the structure of the economy, rendering analysis based solely on impulse response functions (IRF) to exogenous shocks ineffective. As these policies modify the responses to all shocks, evaluating their effects requires combining the impacts of all shocks. Moreover, policy evaluation focuses on a particular period—the period that motivates the policy—that results from the history of shocks. Therefore, it is necessary to identify shocks specific to that period, because the effectiveness of an economic policy depends on the composition of the shocks it faces. Real-time policy evaluation presents additional difficulties because it is necessary to compare the anticipated effects of different policies. This implies the need to identify future shocks that define a relevant environment for comparing the policies studied.

To meet these requirements, we use the information provided by the government in its Finance Act (including spending, taxes, deficits, and debt) as well as its forecasts of GDP growth, inflation, and other relevant economic indicators.³ Thus, our method integrates historical data and forecasts into a HANK model (*i*) to estimate the sequences of the different shocks that likely hit the French economy so that the government forecasts come true, in the spirit of the conditional forecast methodology presented by Del Negro and Schorfheide, 2013^{4,5} and (*ii*) to develop counterfactual scenarios, given these sequences of shocks, providing real-time evaluations of the alternative policies needed for decision-making.

The choice of a general equilibrium model is crucial for this method. We use the HANK

²In France, the energy share in consumption is more than 10% for those with an income lower than the median and 8% for those in the top 10% of the income class.

³The forecasts contained in the Finance Act may be considered as robust as possible because they are based on the largest information set, bringing together (*i*) the qualitative expertise of numerous experts from different French ministries and the European Commission, (*ii*) the use of large non-structural econometric models, and (*iii*) statistical analyzes of surveys on French confidence (entrepreneurs and consumers).

⁴Therefore, the shocks estimated for the five-year period of forecasts (2023-2027) can be interpreted as the evolution of the economic conditions necessary to make the government's forecasts consistent within the model's framework.

⁵A challenge for conditional forecasts is that they may be exposed to the R. Lucas, 1976 critique. The Finance Act may also modify the decision rules of the government. To address this problem, we implemented a stability test of the policy rules and showed how to incorporate a possible change in policy into the evaluation method.

model because it enables us to study the impact on macroeconomic aggregate variables (such as output, inflation, or public debt) and the dynamics of inequalities across households simultaneously. At the aggregate level, the advantage of the HANK model is that it predicts the observed depressive effect of a positive shock on energy prices, unlike the representative-agent new Keynesian (RANK) model, as shown in Auclert, Monnery, et al., 2023.⁶ From a distributional perspective and since the “yellow vests” protests, the evaluation of an economic policy that changes the price of energy must consider its redistributive impacts, which is possible using a HANK model. To properly capture the greater sensitivity of the poorest to energy price shocks, our model introduces incompressible consumption of energy products. This allows the model to generate a share of energy products in the consumption basket that decreases with income, as observed in the data. Additionally, it allows for price elasticity, which increases with income, making it difficult for the poorest to avoid energy price increases. Another novelty of our model is that it allows the government to smooth policy financing by issuing new debt. Based on the calibrated parameters for the steady state, our HANK model for the French economy reproduces government forecasts (output, inflation, and public debt) conditional on government policies (expenditure and revenue) and energy prices. The solutions are obtained through a first-order approximation around the steady state (Reiter, 2009, (2010)) and the sequence-space Jacobian approach (Auclert, Bardóczy, Rognlie, and Straub, 2021). This method enables us to determine the unique sequence of unanticipated shocks that fits a given observed time series. Estimating these shocks reveals the size of the energy shock in France, allowing us to move beyond an IRF analysis and uncover the size of the budget intervention required to cope with the magnitude of this shock. Next, to evaluate an alternative policy, we maintain the paths of all the exogenous variables as given by the benchmark and only change the policy tool under consideration.

Table 2.1: Growth, inflation, indebtedness, and inequalities for various policies

Scenario	GDP growth		Inflation rate		Inequality evolution	Debt-to-GDP 2027
	2022	2023	2022	2023		
No tariff shield	1.68%	0.94%	7.3%	3.5%	2.09 → 2.28	111.3%
Tariff shield	2.85%	1.00%	6.5%	3.4%	2.09 → 2.24*	112.6%
Tariff shield & Faster wage indexation on prices	2.38%	0.77%	7.4%	4.1%	2.09 → 2.25	114.1%
Transfers	1.97%	1.13%	7.5%	4.4%	2.05 → 2.12	116.1%

* Lecture: the consumption of the top 10% earners is 2.24 times larger in 4Q2024 than those of the bottom 10% earners when the policy was implemented, whereas it was 2.09 times larger before the crisis (1Q2022).

We demonstrate that the tariff shield played a crucial role during the recent energy crisis. As illustrated in Table 2.1, the implementation of the tariff shield in 2022 and 2023 supported economic growth, averaging 1.9% per year between 2022 and 2023, while limiting inflation to 5.6% per year during the same period. Additionally, it mitigated the increase in consumption inequality in times of crisis. The fiscal cost, while significant, remains manageable in terms of public finances, as the debt-to-GDP ratio would only be 1.3 points higher compared to a scenario without a tariff shield. Comparatively, this policy outperforms al-

⁶In the representative agent model, the energy shock leads demand to shift toward the consumption of domestically produced goods, which counterfactually sustains growth.

ternative strategies such as faster wage indexation to inflation⁷ or targeted redistribution policy.⁸ Table 2.1 indicates that a more rapid wage indexation to inflation (or the implementation of a redistributive policy) would have resulted in an average growth rate of 1.57% (or 1.55%), an inflation rate of 5.74% (or 5.94%), and a larger debt-to-GDP ratio (+4.8 points and +5.8 points respectively). In other words, without a tariff shield, the French economy would have experienced lower economic growth and higher inflation in 2022 and 2023, respectively.

Literature. Our study contributes to several strands of literature. Building on the seminal contributions of Aiyagari, 1994 and Krusell and Smith, 1998, an extensive body of literature has emerged over the past few years that develops models with heterogeneous agents. These models incorporate market frictions such as price and wage rigidities, which are essential for business cycle analysis.⁹ Kaplan, Moll, and Giovanni L. Violante, 2018 and Auclert, Rognlie, Souchier, et al., 2021 demonstrated the empirical performance of these HANK models and their relevance for policy evaluations. HANK models have been widely used to analyze the business cycle and inequality dynamics of the US economy.

One of our contributions is extending these analyses to the French economy, which is distinctive because of its membership in a monetary union. We adapt the Taylor rule to reflect the fact that the European Central Bank (ECB) responds only partially to French inflation, which is only a part of European inflation. Beyond their ability to reproduce wealth inequalities, Auclert, Monnery, et al., 2023 demonstrated that HANK models can induce a recession following an energy shock, as observed through real-world data, in contrast to Representative-Agent New Keynesian (RANK) models. By applying the HANK model to French data, we also examine its ability to generate a recession during an energy crisis. Our approach is different from those of Bayer, Kriwoluzky, et al., 2023 and Auclert, Monnery, et al., 2023, who considered a two-country model within a monetary union. By contrast, our approach concentrates on a small open economy model. The benefit of focusing on a small open economy lies in the demand for a limited dataset that facilitates real-time assessments based on historical data and forecasts. However, the approach is limited in that it cannot analyze intercountry interactions. Our third contribution is that this study addresses the unequal impacts of energy price changes by examining them using an estimated model that incorporates business cycle fluctuations and short-term macroeconomic policies. Känzig, 2023¹⁰ revealed that the carbon tax is regressive, affecting the most disadvantaged house-

⁷In this scenario, the tariff shield is accompanied by a more rapid wage indexation to inflation, leading nominal wages to be indexed to consumer price inflation in less than a year.

⁸The redistribution scheme provides equal lump-sum transfers to all households, covering incompressible energy expenses, and thus benefiting the poorest disproportionately.

⁹Many new methods have been developed to use these models more easily. For continuous time, Achdou et al., 2022 popularized an approach based on solving the Kolmogorov-Fokker-Planck forward equations coupled with HJB backward equations (see, e.g., Kaplan and Giovanni L. Violante, 2018). For discrete time, Reiter, 2009, (2010), Winberry, 2018 and Bayer and Luetticke, 2020 developed methods for improving the accuracy and resolution speed of these heterogeneous agent models. The Auclert, Bardóczy, Rognlie, and Straub, 2021 approach integrates the set of tools necessary for macroeconomists to use these HANK models to make economic policy assessments, and it is possible to (i) compute the dynamic responses to aggregate shocks, (ii) check the stability of the dynamics, and (iii) estimate parameters. We use this approach here.

¹⁰Känzig, 2023 studied the impacts of the European carbon market reforms on the Euro area.

holds more severely owing to their more energy-intensive consumption and/or lower price elasticity. Pieroni, 2023 developed a HANK model similar to ours, demonstrating that (i) low-income households incur the greatest costs from an energy shock, and (ii) monetary and fiscal policies can theoretically mitigate these unevenly distributed costs.

Our study also provides new tools for stochastic debt sustainability analyses (see Bohn, 1998 and Blanchard, 2019). It allows us to analyze debt dynamics within a stochastic general equilibrium framework. Therefore, beyond evaluating the direct impact of the tariff shield on the government budget, our general equilibrium analysis incorporates the interactions between debt, inflation, GDP growth, interest rates, and so forth in a stochastic environment, providing new tools for managing new fiscal rules in the EU (see Blanchard, Leandro, and Zettelmeyer, 2021). This approach complements the semi-structural approach of Bouscasse and Hong, 2023 based on the method of McKay and Wolf, 2023, which identifies the best policy for stabilizing real debt.

Finally, we enrich the literature on policy evaluations based on conditional forecasts, which is particularly useful for developing counterfactual policy scenarios. These forecasts rely on external information to predict the evolution of certain economic variables and derive economic shocks that are consistent with these paths. Conditional forecasts primarily focus on the monetary policy interest rate in Vector Autoregression (VAR) (Waggoner and Zha, 1999 and Antolin-Diaz, Petrella, and Rubio-Ramirez, 2021) and Dynamic Stochastic General Equilibrium (DSGE) models (Leeper and Zha, 2003 and Del Negro and Schorfheide, 2013).¹¹ Our contribution involves estimating conditional forecasts based on the official government's forecasts for public finance and macroeconomic aggregates using the sequence-space Jacobian methodology (Auclert, Bardóczy, Rognlie, and Straub, 2021), and then evaluating policies using counterfactual scenarios derived from our HANK model.

The remainder of this paper is organized as follows: Section 2.2 presents the model. Section 2.3 describes the quantitative methodology used in the study. Section 2.4 outlines the model calibration and estimation. Section 2.5 analyzes the quantitative implications of the alternative policies. Section 2.6 presents the results of several robustness tests. Finally, Section 2.7 concludes the paper.

2.2 Model

The model presented in this section is similar to those in Auclert, Rognlie, and Straub, 2023b (2021), and (2023). Additional features are included to account for energy consumption and input.¹² Fiscal tools are introduced to allow the French government to fight the rise in inflation during the energy crisis, and the Taylor rule is adopted to account for the weight of the French economy in the Euro area.

¹¹See also Galí, 2011 for a critical analysis of conditional forecasts.

¹²Appendix 2.8.1 gives a graphical representation of the model structure. Our modeling of the opening of the economy is very limited: France, which does not produce any raw energy, imports raw materials, and the balance of trade is ensured on each date, assuming that these revenues are used by foreigners to finance an export request addressed to French firms. Obviously, this partial vision of the French trade balance prevents us from examining the implications of our model for real exchange rates and trade exchanges.

2.2.1 Households

In each household, the worker's productivity takes the values $e' \in \mathbf{E}$ conditional to a current value $e \in \mathbf{E}$. The transition matrix between the productivity levels is $\mathcal{P}(e, e')$. If $V_t(e, a_-)$ denotes the value function of households, $a_- \in \mathbf{A}$ its asset position at the beginning of period t and $0 < \beta < 1$ the discount rate, the household's decision rules are deduced from

$$V_t(e, a_-) = \max_{c, a \geq 0} \left\{ u(c) - v(n) + \beta \sum_{e'} \mathcal{P}(e, e') V_{t+1}(e', a) \right\}$$

$$(1 + \tau_c)c + a = (1 + r)a_- + (1 - \tau_l)w e n + \tau \bar{\tau}(e) + d \bar{d}(e) - (1 + \tau_c)(1 - s_H)p_E \underline{c}_E$$

c is the consumption basket, n is the number of hours worked (determined by unions), d is the transfer from firms (dividends), and τ is the transfer from the government. τ_c is the consumption tax rate, and τ_l is the labor income tax rate. The rule for the distribution of government transfers $\bar{\tau}(e)$ is $\bar{\tau}'(e) \leq 0$. Dividends are distributed according to rule $\bar{d}(e)$, such that $\bar{d}'(e) \geq 0$. The real wage and real interest rate are w and r , respectively, where $1 + r = \frac{1+i_-}{1+\pi}$, i_- is the pre-determined nominal interest rate, and π is the inflation rate. The subsistence energy level is \underline{c}_E and the rate at which the tariff shield reduces energy prices is s_H . Incompressible consumption \underline{c}_E is bought directly by households at the energy producer, implying that it is not in consumption basket c . Preferences¹³ are given by $u(c) = \log(c)$ and $v(n) = \varphi \frac{n^{1+\nu}}{1+\nu}$, where $\varphi > 0$ and $\nu > 0$. The consumption basket $c = \left(\int_0^1 c_i^{\frac{\varepsilon_d - 1}{\varepsilon_d}} di \right)^{\frac{\varepsilon_d}{\varepsilon_d - 1}}$ is sold at price P and is composed of imperfectly substitutable goods c_i , with ε_d the elasticity of substitution. The basket c_i is given by

$$c_i = \left(\alpha_E^{\frac{1}{\eta_E}} (c_{i,E} - \underline{c}_E)^{\frac{\eta_E - 1}{\eta_E}} + (1 - \alpha_E)^{\frac{1}{\eta_E}} (c_{i,H})^{\frac{\eta_E - 1}{\eta_E}} \right)^{\frac{\eta_E}{\eta_E - 1}} \quad (2.1)$$

where $c_{i,E}$ is energy consumption, $c_{i,H}$ is the domestically produced consumption good, η_E is the elasticity of substitution between these two goods, and α_E is the share of energy in c_i . The basket c_i satisfies the household's preferences if $c = p_H c_H + (1 - s_H)p_E (c_E - \underline{c}_E)$, i.e. $c + (1 - s_H)p_E \underline{c}_E = p_H c_H + (1 - s_H)p_E c_E$ with $p_H = P_H/P$ and $p_E = P_E/P$.¹⁴

2.2.2 Supply

Intermediate Goods Y_H are produced with energy E and labor N using technology

$$Y_H = Z \left(\alpha_f^{\frac{1}{\sigma_f}} E^{\frac{\sigma_f - 1}{\sigma_f}} + (1 - \alpha_f)^{\frac{1}{\sigma_f}} N^{\frac{\sigma_f - 1}{\sigma_f}} \right)^{\frac{\sigma_f}{\sigma_f - 1}}$$

¹³We restrict our analysis to preferences compatible with a balanced growth path as shown in King, Plosser, and Rebelo, 1988. This is consistent with the detrending method used to stationarize the data.

¹⁴The intratemporal households' choices are managed by the firms that create final goods by combining home goods and energy services satisfying the households' preferences. This allows us to introduce a Phillips curve on the consumer price index (CPI) via an adjustment cost on price adjustment paid by the retailers.

where σ_f is the elasticity of substitution between E and N and α_f is the share of energy.

Final goods Y_F are produced with intermediate goods Y_H and energy Y_E using the technology

$$Y_F = \left(\alpha_E^{\frac{1}{\eta_E}} Y_E^{\frac{\eta_E-1}{\eta_E}} + (1 - \alpha_E)^{\frac{1}{\eta_E}} Y_H^{\frac{\eta_E-1}{\eta_E}} \right)^{\frac{\eta_E}{\eta_E-1}}$$

which corresponds to the household preferences defined by equation (2.1). Perfect competition causes the prices of these two goods to equal their marginal costs: $p_j = mc_j$ for $j = H, F$ where

$$\begin{aligned} mc_H &= Z^{-\frac{1}{\sigma_f}} \left(\alpha_f p_E^{1-\sigma_f} + (1 - \alpha_f) w^{1-\sigma_f} \right)^{\frac{1}{1-\sigma_f}} \\ mc_F &= \left(\alpha_E ((1 - s_H) p_E)^{1-\eta_E} + (1 - \alpha_E) (p_H)^{1-\eta_E} \right)^{\frac{1}{1-\eta_E}} \end{aligned}$$

Retailer i produces consumption goods using final goods according to a linear production function: $Y_i = Y_{i,F}$. Households and the government have the same preferences. Thus,

their baskets, c and G , respectively, are defined by $Y = \left(\int_0^1 Y_i^{\frac{\varepsilon_d-1}{\varepsilon_d}} di \right)^{\frac{\varepsilon_d}{\varepsilon_d-1}}$ for $Y = c, G$.

Under monopolistic competition, retailers obtain markup, but support adjustment costs when they change prices. With ψ_P as the adjustment cost parameter, the price-setting rule is deduced as follows:

$$\begin{aligned} \Pi_t(P_{i,-}) &= \max_{P_i} \left\{ \frac{P_i - P_F}{P} Y_i - \frac{\psi_P}{2} \left(\frac{P_i}{P_{i,-}} - 1 \right)^2 Y + \frac{1}{1 + r_+} \Pi_{t+1}(P_i) \right\} \\ \text{s.t. } Y_i &= \left(\frac{P_i}{P} \right)^{-\varepsilon_d} Y \end{aligned}$$

This leads to the following New-Keynesian Phillips Curve (NKPC):

$$\pi_t = \kappa_P \left(mc_t - \frac{1}{\mu} \right) + \frac{1}{1 + r_{t+1}} \frac{Y_{t+1}}{Y_t} \pi_{t+1} \quad \text{given that } \pi_t = \frac{P_t}{P_{t-1}} - 1$$

where $mc_t = \frac{P_{Ft}}{P_t} = p_{Ft}$, $\kappa_P = \frac{\varepsilon_d}{\psi_P}$ and $\mu = \frac{\varepsilon_d}{\varepsilon_d-1}$.¹⁵ The dividends distributed to households by firms (their profits) are defined as $d_{Ft} = Y_t - p_{Ft} Y_{Ft} - \frac{\psi_P}{2} \left(\frac{P_t}{P_{t-1}} - 1 \right)^2 Y_t$.

2.2.3 Unions

For task k , a union sets a wage for all levels of productivity $e \in \mathbf{E}$ and asset $a \in \mathbf{A}$. Assuming that wage adjustments are costly, with ψ_W as the adjustment cost parameter, the program is

$$\begin{aligned} U_t^k(W_{k,-}) &= \max_{W_k} \left\{ \int_e \int_{a_-} [u(c(e, a_-)) - v(n(e, a_-))] d\Gamma(a_-, e) \right. \\ &\quad \left. - \frac{\psi_W}{2} \left(\frac{W_k}{W_{k,-}} - 1 \right)^2 + \beta U_{t+1}^k(W_k) \right\} \\ \text{s.t. } N_k &= \left(\frac{W_k}{W} \right)^{-\varepsilon} N \quad \text{with } W = \left(\int_0^1 W_k^{1-\varepsilon} dk \right)^{\frac{1}{1-\varepsilon}} \end{aligned}$$

¹⁵Remark that for π "small," we have $(\pi_t + 1) \pi_t \approx \pi_t \equiv \frac{P_t}{P_{t-1}} - 1$.

where N_k denotes the quantity of labor in task k , N is the aggregate employment, and ε is the degree of substitution among tasks. The equilibrium distribution of the households satisfies $\int_e \int_{a_-} d\Gamma(a_-, e) = 1$. The income after wage and consumption taxes of household i is

$$\frac{1 - \tau_l}{1 + \tau_c} e_i w n_i = \frac{1 - \tau_l}{1 + \tau_c} e_i \int_0^1 \frac{W_k}{P} n_{ik} dk = \frac{1 - \tau_l}{1 + \tau_c} e_i \int_0^1 \frac{W_k}{P} \left(\frac{W_k}{W} \right)^{-\varepsilon} N dk$$

where the second equality comes from $n_{ik} = n_{i'k} \equiv N_k$ as unions impose that every worker works the same hours. After integrating this result into the union's objective, the nominal wage set by the union leads to a new Keynesian Phillips curve for the wage inflation dynamics:

$$\pi_{Wt} = \kappa_w \left(N_t v'(N_t) - \frac{1}{\mu_w} \frac{1 - \tau_l}{1 + \tau_c} \frac{W_t}{P_t} N_t u'(C_t) \right) + \beta \pi_{Wt+1}$$

where $\mu_w \equiv \frac{\varepsilon}{\varepsilon - 1}$ and $\kappa_w \equiv \frac{\varepsilon}{\psi_W}$. Note that $\int_e \int_{a_-} u'(c) d\Gamma(e, a_-)$ is approximated by $u'(C)$ with C the aggregate consumption.

2.2.4 Government

The government collects revenue (R_t) and incurs expenditures (S_t), and the difference between revenue and expenditures is financed by issuing public debt B_t . Therefore, we have

$$\begin{aligned} P_t R_t &= P_t \tau_l w_t N_t + P_t \tau_{ct} (C_t + p_{Et} c_E) \\ P_t S_t &= P_t G_t + P_t \tau_t + P_t s_{Ht} (p_{Et} Y_{Et} + (1 + \tau_{ct}) p_{Et} c_E) \\ B_t &= (1 + i_{t-1}) B_{t-1} - P_t R_t + P_t S_t \Rightarrow b_t = (1 + r_t) b_{t-1} - R_t + S_t \end{aligned}$$

where $b = B/P$ is the real public debt and $G_t = \left(\int_0^1 g_{it}^{\frac{\varepsilon_d - 1}{\varepsilon_d}} di \right)^{\frac{\varepsilon_d}{\varepsilon_d - 1}}$ is the real government spending. To ensure the stability of the public debt dynamics, the transfers τ_t incorporate a fiscal brake $\theta \left(\frac{b_{t-1}}{b} - 1 \right)$ with $\theta > 0$, such that $\tau_t = T_t - \theta \left(\frac{b_{t-1}}{b} - 1 \right) + \vartheta_t$, where T_t represents the discretionary part of the transfers and ϑ_t is a measurement error that gives a stochastic dimension at the debt, allowing us to include it in the estimation as an observable variable. As the Finance Act does not entail any revision of tax rates to finance the tariff shield, we assume that this additional expenditure is financed through debt issuance.¹⁶

2.2.5 Monetary Policy

The monetary policy of the ECB is summarized by a Taylor rule $i_t = \rho_r i_{t-1} + (1 - \rho_r) (r_{ss} + \phi_\pi \pi_t^{EU}) + \tilde{\varepsilon}_t$, where the European inflation is defined as $\pi_t^{EU} = \mu_{FR} \pi_t + (1 - \mu_{FR}) \pi_t^{REU}$, where π_t^{REU} denotes inflation in the rest of the Euro area and μ_{FR} represents the share of the French economy. Assuming that π_t^{EU} is correlated with French inflation,

¹⁶Alternative financing schemes are presented in section 2.6.

that is, $\pi_t^{REU} = \rho_\pi \pi_t + \pi_t^{REU*}$, where π_t^{REU*} represents the specific inflation of the rest of the Euro area, and the Taylor rule becomes as follows:

$$\dot{i}_t = \rho_r \dot{i}_{t-1} + (1 - \rho_r) (r_{ss} + \phi_\pi (\mu_{FR} + (1 - \mu_{FR}) \rho_\pi) \pi_t) + \varepsilon_t$$

with $\varepsilon_t = \tilde{\varepsilon}_t + \phi_\pi (1 - \rho_r) (1 - \mu_{FR}) \pi_t^{REU*}$. Hence, ε_t is not a “pure” monetary shock but a composite shock that also contains inflation shocks that occur in the rest of the Euro area.¹⁷

2.2.6 Energy Market

Energy E_t^s is produced using raw energy \bar{E}_t through technology $E_t^s = \Upsilon \bar{E}_t^\nu$, where $0 < \nu \leq 1$ and $\Upsilon > 0$. Raw energy is purchased at an exogenous price \tilde{P}_{Et} , and the quantity of raw energy \bar{E}_t is adjusted to ensure energy market equilibrium. If $\nu < 1$, then the equilibrium energy price P_{Et} is the solution to $E_t^s = \mathcal{E}_t \equiv E_t + Y_{Et} + \underline{c}_E$ and the energy sector distributes dividends $d_{Et} = P_{Et} E_t^s - \tilde{P}_{Et} \bar{E}_t > 0$. If $\nu = 1$, then the energy price is $P_{Et} = \tilde{P}_{Et} / \Upsilon$ and $d_{Et} = 0$. Given that France does not produce any raw energy, we assume that revenues from raw energy sales $\mathcal{R}_{Et} = P_{Et} E_t^s$ are earned by a foreign representative agent who uses these to purchase goods exported by French firms $\mathcal{R}_{Et} = X_t$. Under this assumption, albeit highly simplistic, the trade balance is always at equilibrium without any financial trade with the rest of the world.¹⁸

2.2.7 Equilibrium

The market-clearing conditions used to determine the unknowns $\{r, w, p_E\}$ are:

$$\begin{aligned} \text{asset market:} \quad b = \mathcal{A} &\equiv \int_{a_-} \int_e a(a_-, e) d\Gamma(a_-, e) \\ \text{labor market:} \quad N = \mathcal{N} &\equiv \int_{a_-} \int_e en(a_-, e) d\Gamma(a_-, e) \\ \text{energy market:} \quad E^s = \mathcal{E} &\equiv E + Y_E + \underline{c}_E \end{aligned}$$

Market-clearing on the goods market can be used to check the Walras law, given $d = d_F + d_E$,¹⁹

$$Y \left(1 - \frac{\psi_P}{2} \pi^2 \right) = X + C + G$$

¹⁷The output gap is not in the Taylor rule because (i) this target is not in the ECB mandate (see Art. 127, Ch. 2, The Treaty on the Functioning of the European Union (2012)) and (ii) papers dealing with the same subject, such as Bayer, Kriwoluzky, et al., 2023 and Auclert, Monnery, et al., 2023, also choose this restriction, making it easier to compare the results.

¹⁸A version of the model where the consumption of the foreign representative agent depends only on a fraction of his current income but also on his wealth (the counterpart of international financial exchanges) indicates that our conclusions on policy ranking are not affected. Results are available in Appendix 2.8.15.

¹⁹In the case where $\nu = 1$ and thus $d_E = 0$, we have $\bar{E} = \mathcal{E} \equiv E + Y_E + \underline{c}_E$ and $Y \left(1 - \frac{\psi_P}{2} \pi^2 \right) = p_E \bar{E} + C + G$.

2.3 Quantitative Method

This section introduces an original method for “real-time” policy evaluation using HANK models. First, we provide a brief overview of the linearized model solutions. Leveraging this solution and given that any Finance Act commits the government to its expenditures and revenues based on forecasts for future years, we can then identify the shocks that would match the government forecasts with model solutions (method of conditional forecasts à la Del Negro and Schorfheide, 2013, extended to HANK models). Subsequently, using these identified shocks, we propose a method that allows us to compare the policy announced in the Finance Act with alternatives in exactly the same context. This method transcends traditional IRF analysis by enabling comparisons across scenarios in which the new policy can combine shocks and changes in the model’s multipliers. By imposing strong theoretical restrictions, this approach yields causal interpretations, albeit at the expense of focusing on a narrower set of variables than large-scale non-structural approaches.

2.3.1 Model’s Solution

The equilibrium dynamics must satisfy the following conditions.

$$\mathbf{H}_t(\mathbf{Y}, \mathbf{Z}) \equiv \begin{pmatrix} \Phi(S_{t+1}, S_t, S_{t-1}) \\ \mathcal{A}_t - b_t \\ \mathcal{N}_t - N_t \\ \mathcal{E}_t - E_t^s \end{pmatrix} = 0 \quad (2.2)$$

where \mathbf{Y} and \mathbf{Z} are the time series of all the aggregate variables and aggregate shocks, respectively. The system $\Phi(S_{t+1}, S_t, S_{t-1}) = 0$ regroups all equations describing firm, union, government, and central bank behaviors, where S_t is the vector of the aggregate variables controlled by these agents. The solution to Equation (2.2) was obtained using the approximation method developed by Auclert, Bardóczy, Rognlie, and Straub, 2021. With $[H_Y]_{t,s} \equiv \frac{\partial \mathbf{H}_t}{\partial Y_s}$ and $[H_Z]_{t,s} \equiv \frac{\partial \mathbf{H}_t}{\partial Z_s} \forall s, t$, this is given by

$$0 = \sum_{s=0}^{\infty} [H_Y]_{t,s} dY_s + \sum_{s=0}^{\infty} [H_Z]_{t,s} dZ_s \Rightarrow dY = -H_Y^{-1} H_Z dZ \equiv G dZ$$

where $dZ = Z - \bar{Z}$, \bar{Z} is the steady-state value of Z , and G is the complete Jacobian of the dynamic system. We assume that all shocks in the model have the following MA(∞) representation: $dz_t = \sum_{s=0}^{\infty} \mathbf{m}_s^z \varepsilon_{t-s}^z$ and $\forall z \in \mathcal{Z}$, where \mathcal{Z} is the set of shocks. Therefore, the solution of the HANK model can be represented by an MA(∞) that involves the Jacobians of the model:

$$dY_t = \sum_{s=0}^{\infty} \sum_{z \in \mathcal{Z}} [G^{Y,z} \mathbf{m}^z]_s \varepsilon_{t-s}^z \equiv \sum_{s=0}^{\infty} \sum_{z \in \mathcal{Z}} m_s^{Y,z} \varepsilon_{t-s}^z \quad (2.3)$$

Replacing ∞ by T “large” and using the Jacobians, one can determine the unique sequence of unanticipated shocks $\{\varepsilon_s\}_{s=0}^T$ allowing the model to fit a given sequence of $\{dY_s\}_{s=0}^T$.

2.3.2 Conditional Forecasts

In the spirit of Del Negro and Schorfheide, 2013, we assume that in period $t \in \{T+1, \dots, T+H\}$ the vector of endogenous variables $\mathcal{F} = \{Y_s, \pi_s, \frac{b_s}{Y_s}, G_s, T_s, P_{Es}\}_{s=T+1}^{T+H}$ takes the forecasted values \mathcal{F}^f reported in the Finance Act. To achieve this objective, we use Equation (2.3) to compute the vector of unanticipated shocks Ψ allowing the model solution \mathcal{F} to match the target \mathcal{F}^f , $\forall t \in \{T+1, \dots, T+H\}$. In order to satisfy the rank condition necessary for identification, the six time series in \mathcal{F}^f and the model (equation (2.3)) allow us to identify the time series of the 6 shocks $\Psi = \{\varepsilon_s^\beta, \varepsilon_s^\mu, \varepsilon_s^\vartheta, \varepsilon_s^{P_E}, \varepsilon_s^G, \varepsilon_s^T\}_{s=T}^{T+H}$ on preference (β), markup (μ), measurement error (ϑ), energy price (P_E), government expenditures (G), and transfers (T). Among the shocks Ψ , it is necessary to distinguish between the two groups of shocks.

- (i) The shocks $\{\varepsilon_s^{P_E}, \varepsilon_s^G, \varepsilon_s^T\}_{s=T+1}^{T+H}$ that affect the exogenous and observable variables $\{P_{FE}, G, T\}$. They are identified using only the forecasts for $\mathcal{F}_x^f = \{P_E, G, T\}_{s=T+1}^{T+H}$, without any filtering by the model. In order to evaluate the tariff shield, we add a supplementary shock that mimics the dynamics of the subsidies s_H provided over the $T+1, \dots, T+h$ period, with $h \leq H$. For all these exogenous variables, we assume that $dz_t = \rho^z dz_{t-1} + \varepsilon_t^z$, for $z \in \{P_E, G, T, s_H\}$.
- (ii) The shocks $\{\varepsilon_s^\beta, \varepsilon_s^\mu, \varepsilon_s^\vartheta\}_{s=T+1}^{T+H}$ that affect the unobservable variables $\{\beta, \mu, \vartheta\}$. They are identified using the model restrictions (equation (2.3)) and shocks $\{\varepsilon_s^{P_E}, \varepsilon_s^G, \varepsilon_s^T\}_{s=T+1}^{T+H}$ where dY_t contains $\{Y_s, \pi_s, \frac{b_s}{Y_s}\}$. We assume that $dz_t = \rho^z dz_{t-1} + \varepsilon_t^z$ for $z \in \{\beta, \mu, \vartheta\}$.

Given that the Finance Act may announce many policy changes, the decision rules for G and T may be unstable, biasing the policy evaluation (R. Lucas, 1976). Using the government's forecasts to identify $\{\varepsilon_s^G, \varepsilon_s^T\}_{s=T+1}^{T+H}$, we test whether these realizations are in the confidence band of the model estimated using historical data. In this case, the stability of the model parameters is not rejected. Otherwise, the unstable parameters must change and their new estimated values are based on forecasted data, implying changes in the estimation of Ψ .

We can interpret the estimated shocks Ψ as the economic context that allows for the realization of the Finance Act forecasts. To evaluate the effect of an alternative policy by considering the economy in the same context, this vector of shocks must be invariant. Therefore, economic policies are compared by controlling the economic context, as in a controlled experiment.

2.3.3 Methodology for Real-Time Policy Evaluations

Let us index by \emptyset the economy with no policy changes, by \mathcal{S} the economy where policy changes can be summarized by additional shocks (ς), and by \mathcal{P} the one where the new policy involves parameter changes and additional shocks.²⁰ The economies \emptyset and \mathcal{S} share the same Jacobians ($G_{\mathcal{S}} = G_{\emptyset}$) and thus the same multipliers ($m_{\mathcal{S},s}^{Y,z} = m_{\emptyset,s}^{Y,z}$) because the

²⁰The analysis is conducted for changes in parameters that do not modify the steady state.

exogenous processes of shocks do not change ($\mathbf{m}_{\emptyset,s}^z = \mathbf{m}_{\mathcal{S},s}^z$) if an additional shock is introduced ($m_{\mathcal{S},s}^{Y,\varsigma} \neq 0$). Therefore, for $t \in \{T+1, \dots, T+H\}$, the evaluation of a policy change modeled as a shock is given by

$$\left. \begin{aligned} dY_{\emptyset,t} &= \sum_{s=0}^{\infty} \sum_{z \in \mathcal{Z}} m_{\emptyset,s}^{Y,z} \varepsilon_{t-s}^z \\ dY_{\mathcal{S},t} &= \sum_{s=0}^{\infty} \left[\sum_{z \in \mathcal{Z}} m_{\emptyset,s}^{Y,z} \varepsilon_{t-s}^z + m_{\mathcal{S},s}^{Y,\varsigma} \varepsilon_{t-s}^{\varsigma} \right] \end{aligned} \right\} \Rightarrow dY_{\mathcal{S},t} - dY_{\emptyset,t} = \sum_{s=0}^{\infty} m_{\mathcal{S},s}^{Y,\varsigma} \varepsilon_{t-s}^{\varsigma}$$

where only the dynamics driven by the shock ς matter, as in typical IRFs. On the contrary, the economy \mathcal{P} does not have the same multipliers as \emptyset and \mathcal{S} ($G_{\mathcal{P}} \neq G_{\emptyset}$ thus, $m_{\mathcal{P},s}^{Y,v} \neq m_{\emptyset,s}^{Y,v} \forall v = z, \varsigma$). The evaluation of policy \mathcal{P} with respect to \emptyset or \mathcal{S} is given by:

$$\begin{aligned} dY_{\mathcal{P},t} &= \sum_{s=0}^{\infty} \sum_{z \in \mathcal{Z}} \left[m_{\mathcal{P},s}^{Y,z} \varepsilon_{t-s}^z + m_{\mathcal{P},s}^{Y,\varsigma} \varepsilon_{t-s}^{\varsigma} \right] \\ dY_{\mathcal{P},t} - dY_{\mathcal{S}|\emptyset,t} &= \sum_{s=0}^{\infty} \sum_{z \in \mathcal{Z}} \left[(m_{\mathcal{P},s}^{Y,z} - m_{\emptyset,s}^{Y,z}) \varepsilon_{t-s}^z + (m_{\mathcal{P},s}^{Y,\varsigma} - m_{\mathcal{S}|\emptyset,s}^{Y,\varsigma}) \varepsilon_{t-s}^{\varsigma} \right] \quad (2.4) \\ &\text{where } m_{\emptyset,s}^{Y,\varsigma} = 0 \end{aligned}$$

Because all multipliers $m_{\mathcal{P},s}^{Y,z}$, $m_{\mathcal{P},s}^{Y,\varsigma}$ change, the evaluation of the policy \mathcal{P} depends on the sequences of all shocks by combining all IRFs. Therefore, if the multipliers are not invariant and contingent on the implemented policy, and if the sample size is limited, equation (2.4) must be used for policy evaluation, even with a linear approximation of the model dynamics.²¹ The finiteness of the sample is crucial for policy evaluation because decisions hinge on (i) a context defined by the initial conditions resulting from a particular combination of shock history and (ii) policy implications over a short horizon (generally less than five years). A finite sample requires focusing on a specific realization of shock dynamics, for which the natural candidate is Ψ because it enables the model to replicate the observed series and thereby identify relevant time dependence for policy evaluation. Conditional on beginning with the current economic state, policy evaluation must compare the benchmark scenario with a counterfactual scenario. To maintain the principle of “all things being equal,” shocks Ψ are retained when evaluating any alternative scenario, leaving no explanation for the differences other than the policy change. By focusing on particular business cycle episodes, this method assigns significant importance to the relative size of each shock, because it weighs the IRFs of each shock to determine the level of each endogenous variable. Therefore, the specific identification of shocks favors policies that interact with the largest shocks.

²¹Usually, two tools are available to the public policy evaluator: (i) IRFs provide the dynamics of equilibrium disturbed by a shock, (ii) stochastic simulations generate the ergodic distribution of endogenous variables when all shocks are considered. Obviously, for a model that remains stable over an infinite sample, analysis of the ergodic distribution via its second-order moments for a linearized model is the most exhaustive tool.

2.4 Take the Model to the Data

2.4.1 Calibration and Estimation Based on Historical Data

Income process. The log of labor income follows the $AR(1)$ process

$$\log(e_t) = \rho \log(e_{t-1}) + \eta_t$$

where $\eta_t \sim \mathcal{N}(0, \sigma)$. Following Fonseca et al., 2023, ρ is set to 0.966.²² σ is not set to its estimated value (0.014) because our modeling choices limit the ability of the model to reproduce income and wealth distributions. First, we restrict the labor income process to follow $AR(1)$, which cannot generate extreme income values, as in Guvenen, Karahan, et al., 2021. Second, our model has only one asset, while Kaplan, Moll, and Giovanni L. Violante, 2018 highlighted the importance of risky assets, in addition to riskless assets, in explaining the magnitudes of wealth inequalities. A larger value for σ can compensate for these limitations; with $\sigma = 0.5$ the income, wealth, and consumption distributions generated by the model are close to their empirical counterparts (see Appendix 2.8.3).

Parameters of the decision rules. The parameters of preference, technology, price and wage-setting rules, monetary policy, and long-run fiscal aggregates were calibrated to reproduce stylized facts about the French economy using external information (see Table 2.2).

The energy production function is calibrated by targeting the adjustment of prices in the energy market. Studies such as Meyler, 2009 and Gautier, Marx, and Vertier, 2023 demonstrated that changes in consumer energy prices are primarily driven by variations in oil prices in the short run; consumer prices for liquid fuels reflect a direct, complete, and rapid pass-through of crude oil prices. Additionally, IGU, 2015 indicated that changes in wholesale gas prices, which are highly correlated with crude oil prices during crises, are generally passed on to consumer gas prices with a short lag of three to six months. In the EU electricity market, prices are based on the costs of the fossil fuels used in production. This mechanism, known as the “economic order of precedence” principle, sets the wholesale price of electricity in the EU at the costs of the last plant called upon to ensure balance (see Chapter II of the EU directive 2019/944). As gas is a flexible resource, the electricity market is often balanced by its use. Consequently, gas prices will strongly influence EU electricity prices by 2022. All these observations lead us to calibrate $\nu = 1$ so that changes in crude oil prices largely pass through to consumer energy prices.

This calibration results in 19.6% of households being financially constrained. The Marginal Propensities to Consume (MPC) per level of income are reported in Panel (a) of Figure 2.1. As expected, agents with low income consume a larger fraction of their income. Panel (b) of Figure 2.1 shows that these agents devote a larger share of their energy expenditure, as in the data. Finally, panel (c) of Figure 2.1 shows that agents with low income have more

²²This estimation is based on the European Community Household Panel (ECHP), after (i) controlling by age and (ii) extracting a purely transitional shock treated as a measurement error, uncorrelated with the innovation of the persistent component of the labor income.

Table 2.2: Calibrated parameters

Parameter	Value	Target
Preferences		
Discount factor β	0.9922	Real interest rate $r = 0.5\%$ per quarter
Disutility of labor θ	0.6343	Aggregate labor $L = 1$
Frisch elasticity of labor supply φ	0.5	Auclert, Bardóczy, Rognlie, and Straub, 2021
Incompressible energy consumption \underline{c}	0.0370	20% of the households' energy consumption
Elasticity of substitution between tasks ε	11	Wage markup $\mu_w = 1.1$, Auclert, Bardóczy, Rognlie, and Straub, 2021
Elasticity of substitution between production inputs η_E	0.5	Negative impact on GDP of energy price shock
Share parameter (energy, intermediate good) α_E	0.025	Sharing rule: 40% of energy to households
Production		
Returns of energy production ν	1	Meyler, 2009 and Gautier, Marx, and Vertier, 2023
Elasticity of substitution between production inputs σ_f	η_E	Simplifying assumption
Share parameter (energy, labor) α_f	0.075	Sharing rule: 60% of energy to firms
Elasticity of substitution between goods c_i	6	Firm markup $\mu = 1.2$, Burstein, V. M. Carvalho, and Grassi, 2020
Aggregate targets		
Share of GDP spent on energy	3.18%	Share of energy in GDP
Public debt B	4.749	Debt-to-GDP ratio 100% with annual GDP
Public spending G	0.2374	Public spending-to-GDP ratio 20%
Transfers	0.2968	Transfers-to-GDP ratio 25%
VAT rate τ_c	20%	French VAT
Income tax rate τ_l	20%	French employee tax rate
Nominal rigidity		
Price rigidity κ	0.95	Arbitrary higher than Auclert, Rognlie, and Straub, 2023b
Wage rigidity κ_w	0.1	Auclert, Rognlie, and Straub, 2023b
Monetary policy		
Taylor rule coefficient $\phi_\pi(\mu_{FR} + (1 - \mu_{FR})\rho_\pi)$	1.2	With $\phi_\pi = 1.5$ and $\mu_{FR} = 20\%$, the $\rho_\pi = 0.75$
Persistence of monetary policy ρ_r	0.85	C. Carvalho, Nechio, and Tristao, 2021
Heterogeneity		
Persistence of productivity shocks ρ	0.966	Fonseca et al., 2023 data for France
Volatility of productivity shocks σ	0.5	To match consumption inequalities

difficulty reducing their energy consumption as prices increase. This result originated from the largest share of incompressible consumption in their energy consumption.

Parameters of the Aggregate Shocks. As in all dynamic models, the impact of each shock depends on how agents expect them to persist. The autocorrelations of these $AR(1)$ processes and standard deviations of their innovations are listed in Table 2.3.

The values for $\rho^z, \forall z \in \{\beta, \mu, \vartheta, G, T, P_E\}$, were estimated using a Bayesian method based on dataset $\{Y, \pi, \frac{b}{Y}, G, T, P_E\}$ over the 2Q1995-4Q2019 sample.²³ The autocorrelation functions of these variables are deduced from the model solution (see Equation (2.3)). These estimates show that the shocks to the residual part of transfers ϑ have a large variance, which is not surprising given the non-stationarity of the debt-to-GDP ratio during this period. Shocks to markups have a larger variance than shocks to energy prices and preferences. These estimates of the shock sizes underline that the energy shock had a large magnitude even before the period of the last crisis.

For the tariff shield, we assume that households expect the government not to remove it all at once, as provided in the Financial Act, but to take a year to remove all subsidies. Thus, even if we implement what is provided for in the Financial Act in our evaluation—that is,

²³Appendix 2.8.4 presents the data used in the study. All data are stationarized by extracting a linear trend or sample mean. Additional details on the estimation procedure are in Appendix 2.8.5.

Figure 2.1: Heterogeneity in household's behaviors (per income level)

(a) Marginal Propensity to Consume (b) Energy share in consumption (c) Price elasticity of energy demand

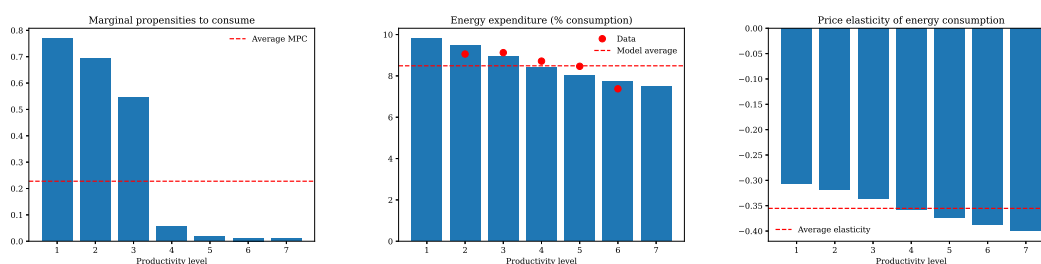


Table 2.3: Estimated parameters of the $AR(1)$ processes

Shock	Z	Persistence ρ^Z		Standard dev. σ^Z		Variance $\frac{(\sigma^Z)^2}{1-(\rho^Z)^2}$
		Mode	Mean	Mode	Mean	
Energy price	p_E	0.9563 (0.0155)	0.9548	0.1656 (0.0121)	0.1685	0.321335
Government spending	G	0.9669 (0.0156)	0.9643	0.0035 (0.0003)	0.0036	0.000184
Transfers	T	0.9079 (0.0302)	0.9062	0.0051 (0.0004)	0.0052	0.000151
Measurement error	ϑ	0.9374 (0.0048)	0.9373	0.6886 (0.0424)	0.7165	4.226374
Price markup	μ	0.7463 (0.0398)	0.7467	1.5172 (0.1052)	1.5136	5.178079
Preference	β	0.9380 (0.0127)	0.9356	0.0681 (0.0103)	0.0666	0.035583

subsidies between 1Q2022 and 4Q2023—households act in the belief that this subsidy will persist.

Finally, following Auclert, Rognlie, and Straub, 2023a, we use the Onatski, 2006 criterion for a quantitative determinacy assessment and check that the winding number is zero.

2.4.2 Shock Identification over the Forecasting Horizon

As explained in Section 2.3.3, our quantitative method requires an estimation of the relative size of all shocks identified to generate the data. After presenting the data used to estimate these shocks, we now present their estimates.

Data for Forecasted Aggregates. We estimate the sequence of shocks from 4Q2019 to 4Q2027, including both COVID-19 and energy crisis episodes, as well as the post-crisis period until 4Q2027. To do so, we use (i) observed data from 4Q2019 to 4Q2021 and (ii) forecasts from 1Q2022 to 4Q2027 published by the French government when the Prime Minister presented the Financial Act. These data²⁴ contains the government forecasts for the

- $\{G, T\}$, which are its commitments concerning its policy until 4Q2027.
- $\{P_E\}$, which is a crucial forecast on the exogenous shock that hit France at the end of 2021.
- $\{Y, \pi, \frac{b}{Y}\}$, which summarizes the objectives underpinning its policy.

The cost of the “tariff shield” is evaluated at €85 billion by the government from October 2021 to December 2023 (27 months), that is, €37 billion per year (see Sénat, 2023 and Appendix 2.8.2). However, only part of the total cost can be considered in our model. This leads us to calibrate $s_H = 0.2$ (see Appendix 2.8.2 for further details). Due to its unanticipated nature, its unknown duration as well as the unannounced modalities of its interruption (see Appendix 2.8.2), we consider it as a shock: each quarter it is a “fiscal” surprise with a persistence modeled as an $AR(1)$ process.

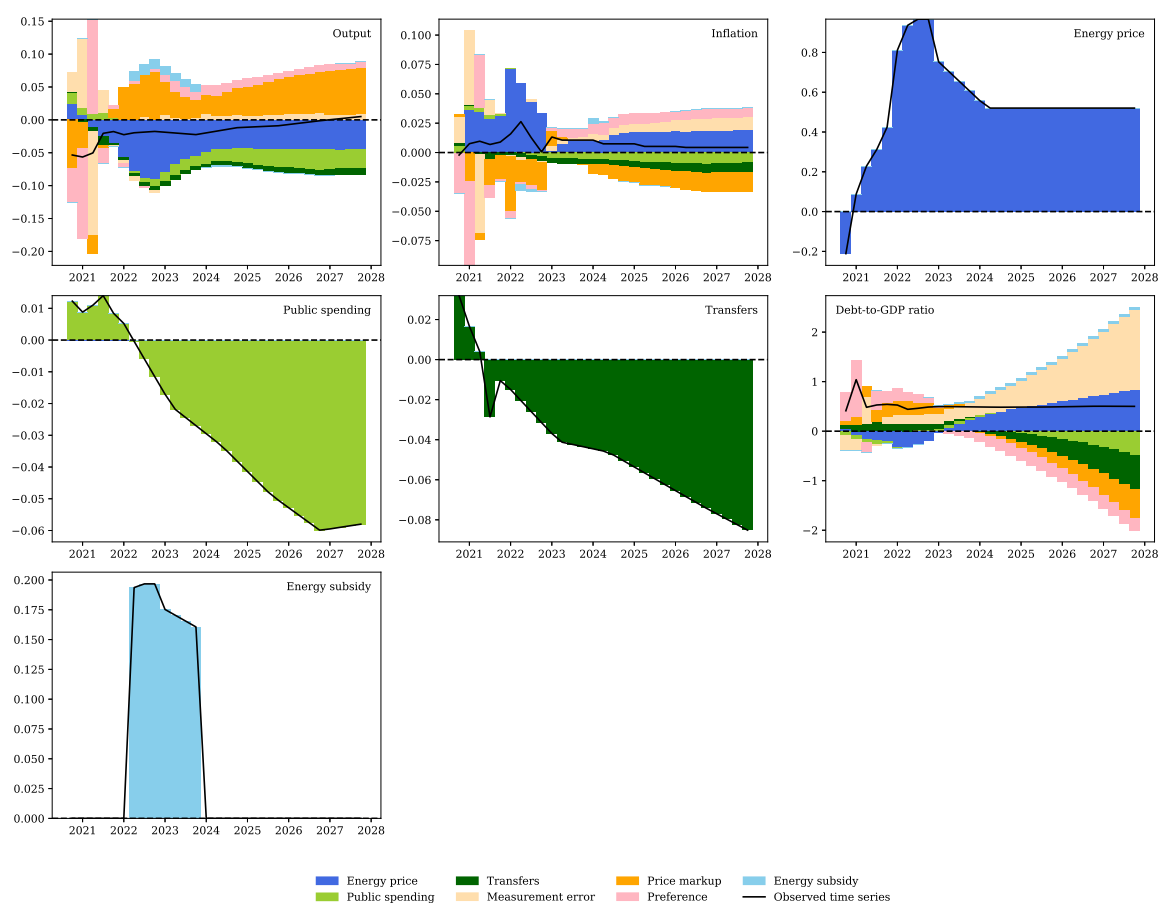
In order to estimate $\{\varepsilon_s^\beta, \varepsilon_s^\mu, \varepsilon_s^\vartheta, \varepsilon_s^G, \varepsilon_s^T, \varepsilon_s^{P_E}, \varepsilon_s^{s_H}\}_{s=4Q2019}^{4Q2027}$, we use stationarized data described in Appendix 2.8.4. We cannot reject the stability of the model parameters (see Appendix 2.8.6), suggesting that our evaluation is not biased (R. Lucas, 1976).

Estimations results. Figure 2.2 shows that the dynamics of GDP (Y) and inflation (π) are mainly driven by preference shocks (demand shocks, β) and markup shocks (supply

²⁴Data are presented in Appendix 2.8.4 (raw data). The sample spans from 4Q2019 to 4Q2027.

shocks, μ).²⁵ Shocks to the two public spending indicators (G and T) have smaller effects.²⁶ However, starting at the end of 2021, a shock to the price of energy (p_E) significantly disrupts the decomposition of the sources of macroeconomic fluctuations. Throughout 2022 and 2023, this shock accounted for as much as 60% of GDP and inflation dynamics. During these two years, only markup shocks supported the GDP (with a decrease in markups), whereas public spending and the energy price reduced it. The tariff shield also supports GDP by mitigating the recessive effects of energy shocks. The magnitude of the impact of the tariff shield is comparable to that of public spending, highlighting the significance of this policy during this particular episode of the French economic cycle.

Figure 2.2: Shock decomposition: focus after 4Q2020



Over the two years, the markup and energy price shocks explain the majority of the inflation dynamics. Only energy price shocks increase inflation during this period, whereas markup shocks reduce it. The net impact of the tariff shield on inflation seems negligible and the direct effect of moderating inflation is countered by the indirect effect of stimulating demand, which contributes to inflation.

²⁵Our estimation identifies only one supply shock, the markup shock. Hence, it can also capture the TFP shocks.

²⁶Even during the COVID-19 crisis (see Figure 2.16 in Appendix 2.8.8), the shocks $\{\beta, \mu\}$, which can be interpreted as the temporary obstacles on demand and supply induced by lockdowns, are the main sources of fluctuations. See Appendix 2.8.8 for additional details on variance decomposition.

It appears that energy shocks initially reduced the debt-to-GDP ratio until 2023. Indeed, their recessive effects on the GDP are not yet sufficiently strong, although inflation has increased since 2021. However, after 2023, the recessive effects of energy shocks became stronger while inflation declined, which explains why the debt-to-GDP ratio increased with energy shocks. In the medium term, the decline in markups partly counteracts the persistence of high energy prices, whereas the reduction in government expenditures and transfers to households contributes to public indebtedness after 2025. The impact of the tariff shield on the debt-to-GDP ratio is relatively moderate because its fiscal cost is partly offset by its positive impact on GDP.²⁷ Finally, the measurement error ϑ explains more than 50% of the debt-to-GDP ratio in the long run, providing a measure of uncertainty in the estimation and data.

Our evaluation reveals that firms are willing to reduce their markups. This has been validated by the OECD (Employment Outlook 2023), which emphasizes that France is the only country in which growth in profits was lower than growth in labor costs during this period.

2.5 Policy Analysis

This section evaluates the effectiveness of the tariff shield—a supply-side policy—for correcting market prices. Using IRFs, we show that their initial impact partially offsets the increase in energy prices, thereby mechanically reducing inflation by mitigating their energy components. Reducing the magnitude of the shock prevents its diffusion and, consequently, its recessive impact on the economy. This policy also sustains households purchasing power by lowering their energy expenditure. This enables them to avoid excessively reducing demand for domestic producers, thereby supporting economic growth. However, as shown in Section 2.3.3, an analysis based solely on IRFs is insufficient to fully account for this crisis, in which the model multipliers can change.

Alvarez et al., 2022 suggested that wages are no longer indexed to prices, implying that inflation driven by the energy shock could result in a significant reduction in purchasing power. This suggests that it could be efficient to index wages to prices more quickly to reinforce the tariff shield in the fight against the reduction of purchasing power. Therefore, we analyze the effectiveness of the tariff shield when complemented by a faster wage indexation.

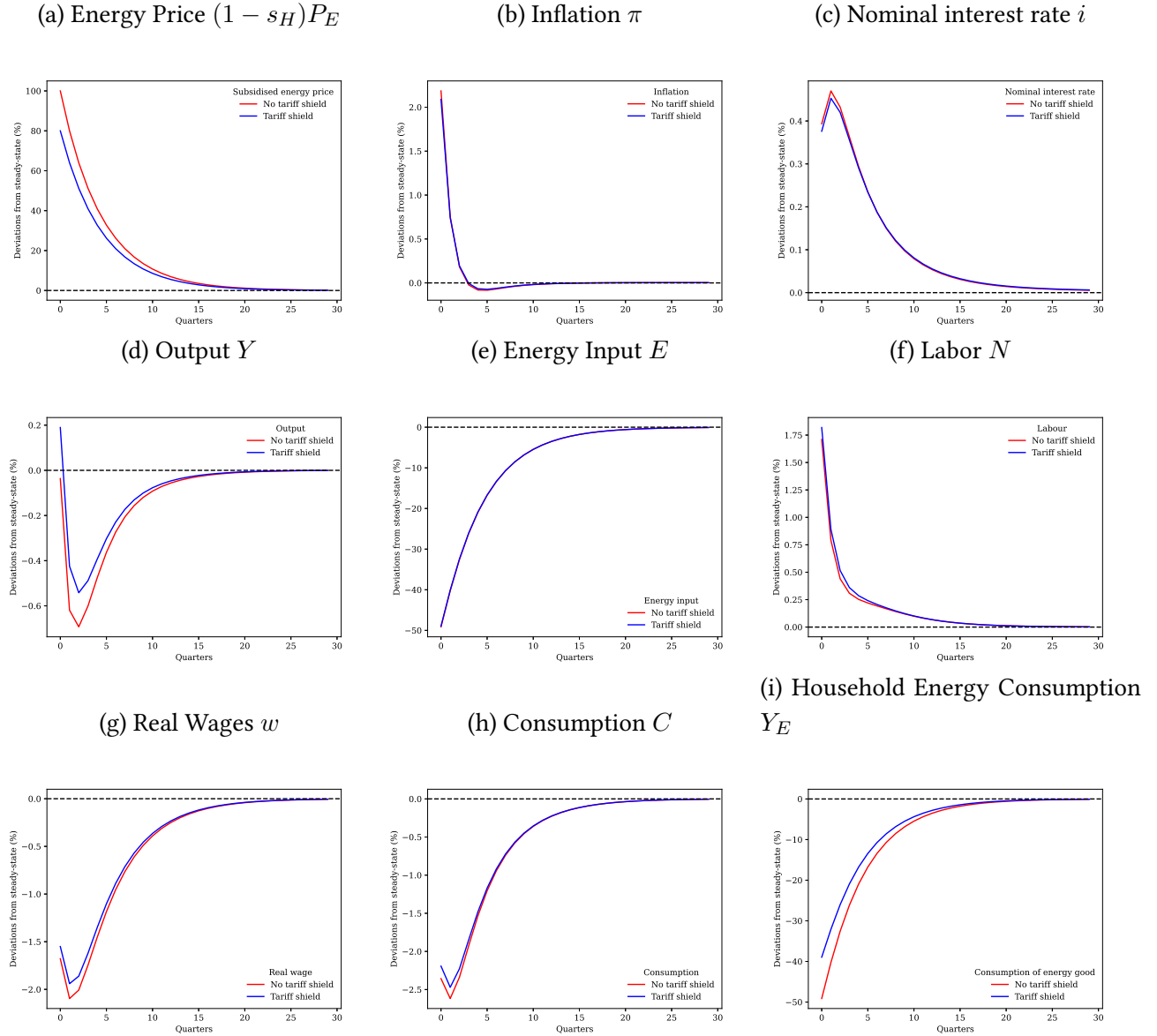
An alternative policy comprises favoring redistribution tools. This demand-oriented policy aims to fight the recession induced by an energy shock by stimulating consumption demand. Consequently, recession can also be mitigated, as the tariff shield can, but is unlikely to address inflation, contrary to what the tariff shield does.

Hereafter, we assess the macroeconomic and distributive effects of the tariff shield and compare them with two alternative scenarios.

²⁷Appendix 2.8.9 provides a measure of the implied uncertainty around these forecasts.

2.5.1 Impulse Response Functions (IRF)

Figure 2.3: Impulse Responses Functions: Energy Shock with or without Tariff Shield



Since Boppart, Krusell, and Mitman, 2018, the MIT shock analysis has provided insights into the economic mechanisms at work in the HANK models. It also evaluates a policy if perceived as a shock.²⁸ This is the case for the tariff shield; this shock's IRFs for each variable depict the policy impact. Figure 2.3 shows the IRFs for an energy shock (panel (a)) with and without a tariff shield. Therefore, the difference between the two lines in each panel indicates the impact of the tariff shield. Energy shocks lead to an increase in inflation (panel (b)), subsequently increasing the nominal interest rate (panel (c)). The tariff shield mitigates the inflation increase by 0.1 percentage point (pp) and the nominal interest rate by 0.017 pp upon impact. Both scenarios generate similar adjustment dynamics for the two variables.

²⁸See Section 2.4.2 for the details on the calibration of the tariff shield shock.

A reduction in inflation affects real wages (as nominal wages are more rigid than prices), prompting firms to increase their labor demand (panel (g)).²⁹ Concurrently, workers agreed to work more to offset the negative wealth effect of an increase in energy prices. A reduction in real wages partially offsets this increase in labor supply. By moderating inflation, the tariff shield alleviates the decline in real wages, thereby reducing the crowding-out effect on labor supply caused by real wage dynamics and explaining better performance in terms of employment and output (panels (d) & (f)). Without a tariff shield, the energy shock reduces GDP by 0.65% four quarters after the impact, whereas it is only 0.5% with a tariff shield. As the tariff shield dampens the increase in energy prices, households decrease their consumption of goods and energy to a lesser extent (panels (h) & (i)).³⁰

2.5.2 On the Effectiveness of the Tariff Shield

Table 2.4: Tariff Shield Impact

	GDP		Inflation			Debt GDP	
No tariff shield	2022	1.68%	1.31%	7.3%	5.4%	2027	111.3%
	2023	0.94%		3.5%			
Tariff shield 2022 2023	2022	2.85%	1.92%	6.5%	4.9%	2027	112.6%
	2023	1.00%		3.4%			
Tariff shield 2022 only	2022	2.85%	1.76%	6.5%	5.1%	2027	112.8%
	2023	0.70%		3.7%			

With a tariff shield in 2022 and 2023, incurring an annual cost of €37 billion and resulting in a 1.3pp increase in the debt-to-GDP ratio by 2027, Table 2.4 shows the support provided by the French government for growth during these two years.³¹ The annual growth rate for 2022-2023 would have been 1.31% without the tariff shield, compared to the forecasted 1.92% by the government. Inflation is contained as the price-wage spiral is not initiated; the annual inflation rate for 2022-2023 would have been 5.4% without the tariff shield, instead of 4.9%. Indeed, without a tariff shield, a significant increase in consumer prices triggers a strong reaction in nominal wages, fueling inflation and increasing labor costs, which explains the significantly weaker growth. If the tariff shield is not renewed by 2023, there will no longer be any growth smoothing, leading to an abrupt halt in growth by 2023. However, inflation remained because of the tariff shield operating in 2022. This partial measure does not result in budgetary savings, because it diminishes GDP growth. Thus, the tariff shield helps contain inflation while achieving higher GDP growth. Therefore, the negative shock from the rise in energy prices was cushioned, allowing for the recovery of a significant portion of the growth associated with the post-Covid crisis catch-up.

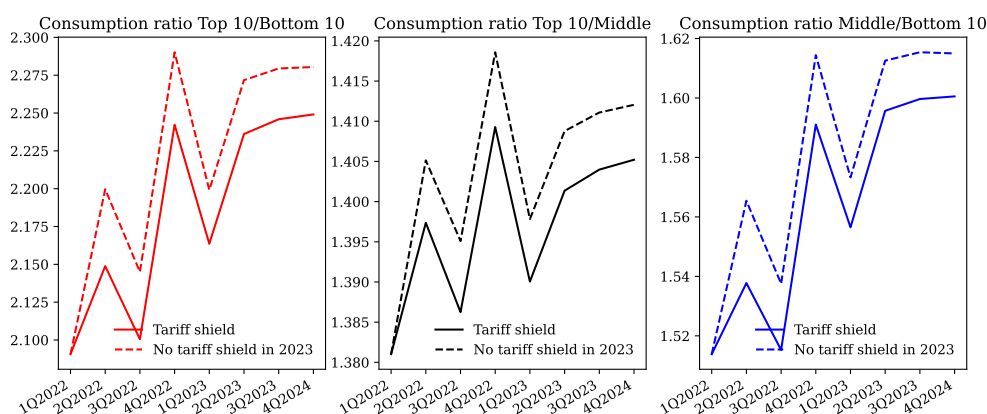
²⁹Firms that do not benefit from the tariff shield are encouraged to replace energy with workers (panels (e) & (f)).

³⁰The non-linear IRFs show that the linear approximation does not bias the results. See Appendix 2.8.7.

³¹In Appendix 2.8.10, the complete description of the quarterly path of the aggregates is presented.

Who loses the most without the tariff shield? To have a reference measure for inequalities, we use INSEE data concerning the “Household Budget”: individuals located in the Top 10% (T10) of income consume 2.21 times more than those in the Bottom 10% (B10) income (see Appendix 2.8.3).

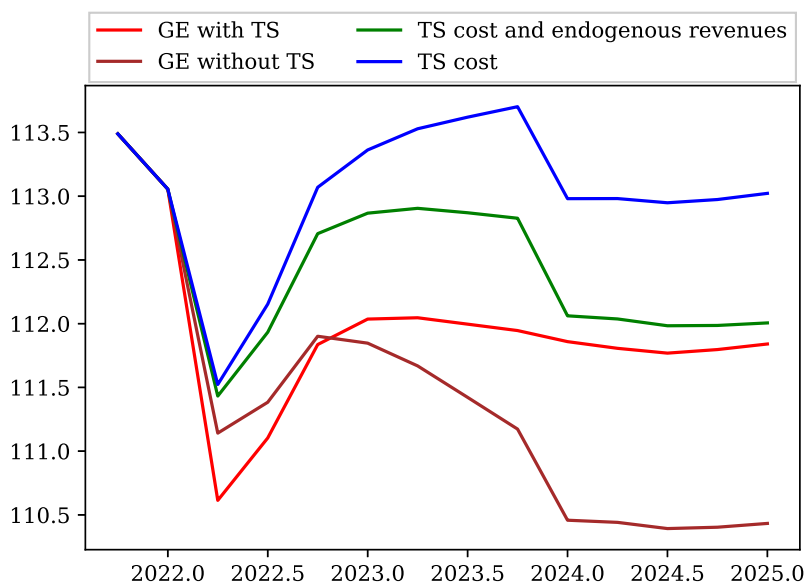
Figure 2.4: Dynamics of Inequalities with Tariff Shield



Without the tariff shield, Figure 2.4 shows that the consumption of T10, which was 2.095 times higher than that of B10 in 1Q2022, increased to 2.275 times higher than that of B10 in 4Q2022 (an 8.6% increase). Consequently, the energy crisis has exacerbated consumption inequalities. However, with the tariff shield, the consumption of the T10 would only be 2.25 times higher than that of the B10 in 4Q2022 (a 7% increase). These findings also indicate that mitigating rising inequalities primarily benefits the lower end of the distribution by sustaining the consumption of low-income households, for whom substitutions are less feasible.

General equilibrium effects on public debt dynamics. In stochastic debt sustainability analysis (see, e.g., Bohn, 1998 and Blanchard, 2019), the dynamics of public debt result from changes in government surplus, given the distributions of GDP growth, interest rates, and inflation. However, changes in government surplus also modify the distribution of GDP growth, interest rates, and inflation owing to general equilibrium (GE) effects. What is the magnitude of GE feedback on the dynamics of public debt? Figure 2.5 shows the breakdown of the evolution of the debt-to-GDP ratio. The first scenario solely reflects the tariff shield’s additional costs, with GDP, inflation, interest rates, and government revenues, following their equilibrium trajectory in an economy without a tariff shield (blue line). The second scenario (green line) depicts the trajectory of the debt-to-GDP ratio by concurrently integrating the extra costs induced by the tariff shield and the changes in the government’s revenue if it implements this policy (GDP, inflation, and interest rates following their equilibrium trajectory in an economy without a tariff shield). Finally, the third scenario (red line) represents GE, in which everything is endogenous (government expenditure and revenue, as well as GDP, inflation, and interest rates). Without the tariff shield, the debt-to-GDP ratio would have been lower, except during the year 2022 when the crisis begins because the tariff

Figure 2.5: Decomposing Public Debt-to-GDP Dynamic. GE: General Equilibrium; TS: Tariff Shield



shield absorbs the recession (see Figure 2.5). However, the GE effects significantly mitigate the impact of the tariff shield on this ratio. Indeed, relative to the simulation without the policy, the tariff shield increases the debt-to-GDP ratio by 1.2pp. Without considering the GE effects, the cost of the tariff shield would have raised the debt-to-GDP ratio by 2.5pp: thus, the GE effect reduced the cost of the tariff shield by half through the sustenance of GDP growth.

2.5.3 Would wage indexation increase the effectiveness of the tariff shield?

The tariff shield mitigates the decrease in household purchasing power caused by energy price hikes but does not alleviate those affecting goods manufactured using energy. To address these declines, faster indexation of wages to consumer prices can be considered. To assess this strategy, we calibrate the nominal wage adjustment cost parameter such that wages adjust throughout the year in response to changes in inflation.³²

Table 2.5 indicates that inflation rises significantly when faster indexation of nominal wages to prices accompanies a tariff shield.³³ This high inflation, reaching 7.4% in 2022, benefits real hourly wages, but leads to a notable reduction in employment. Despite the positive impact on real hourly wages, the negative effect on employment outweighs the impact, resulting in a decrease in household purchasing power. Consequently, this mea-

³²The parameter $\kappa_w = 0.1$ is linked to a Calvo model by $\psi_W = \frac{\theta(\varepsilon-1)}{(1-\theta)(1-\beta\theta)}$ and $\kappa_w = \frac{\varepsilon}{\psi_W}$, where θ is the probability of not switching wages. Our calibrations for $\{\mu_w, \beta, \kappa_w\}$ lead to $\psi_P = 110$ and $\theta \approx 0.74375$ implying that 30% of wages do not change over a year. When the wage indexation is modified, we set $\kappa_w = 0.5$, which leads to $\psi_P = 22$ and $\theta \approx 0.5175$ implying that only 7% of wages do not change over the year.

³³In the Appendix 2.8.11, the complete description of the quarterly path of the aggregates is presented.

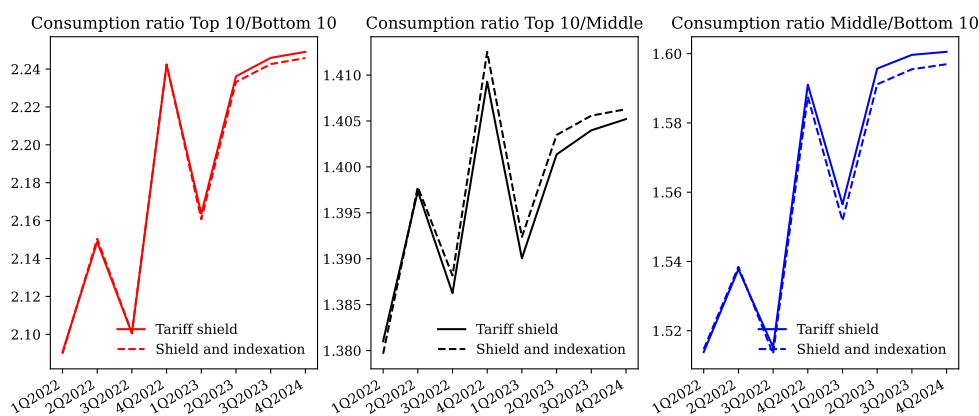
The macroeconomic and redistributive effects of shielding consumers from rising energy prices: a real-time evaluation of the French experiment

Table 2.5: Strong Wage Indexation Accompanying Tariff Shield

	GDP		Inflation		Debt/GDP	
No tariff shield	2022	1.68%	1.31%	7.3%	2027	111.3%
	2023	0.94%		3.5%		
Tariff shield 2022 2023	2022	2.85%	1.92%	6.5%	2027	112.6%
	2023	1.00%		3.4%		
Faster wage indexation & tariff shield	2022	2.38%	1.57%	7.4%	2027	114.1%
	2023	0.77%		4.1%		
Faster wage indexation & No tariff shield	2022	1.35%	1.02%	7.9%	2027	112.8%
	2023	0.70%		4.1%		

sure is less effective for growth, which declines by $(1.92 - 1.57) \times 2 = 0.7$ pp over two years compared to the reference scenario with a tariff shield in 2022 and 2023. This growth slowdown also affects government revenues, causing the debt-to-GDP ratio to increase by 1.5 points compared to the scenario with a tariff shield in 2022 and 2023.

Figure 2.6: Dynamics of Inequalities When a Strong Wage Indexation Accompanied Tariff Shield



If faster indexation of wages to prices were implemented, the pressure on labor costs would exacerbate the decline in economic activity, especially in a more inflationary context. This finding demonstrates that stronger indexation amplifies both the recessive effect of the energy shock and its inflationary impact. Moreover, Figure 2.6 shows that the redistributive effects of faster wage indexing are minimal, resulting in inequality remaining at the same level as the benchmark.

2.5.4 On the effectiveness of a redistributive demand policy

An alternative policy involves boosting the demand by redistributing transfers to households. In this approach, we suggest replacing the tariff shield with a universal transfer for all households, which represents a larger share of the budget for the most disadvantaged households. This transfer specifically targeted households to cover their essential energy consumption expenses. Consequently, we must calibrate the portion of consumption that the government considers incompressible. To implement this, we assume that the government evaluates incompressible consumption for all households as 20% of the total energy consumption of the average household. The budgetary cost of this measure is equivalent to 25% of that induced by the tariff shield. This policy is inherently redistributive because the share of incompressible energy consumption in the total energy consumption varies across income deciles, ranging from 31% for individuals in the first decile to 14% for those in the tenth decile.³⁴ Low-income households exhibit the lowest price elasticity with respect to energy, indicating that they have less flexibility in offsetting the impact of energy price shocks on consumption (see Figure 2.1). As shown in Table 2.6, this policy is less effective in

Table 2.6: Redistributive Demand Policy

	GDP		Inflation		Debt GDP		
No tariff shield	2022	1.68%	} 1.31%	7.3%	} 5.4%	2027	111.3%
	2023	0.94%		3.5%			
Tariff shield 2022 2023	2022	2.85%	} 1.92%	6.5%	} 4.9%	2027	112.6%
	2023	1.00%		3.4%			
Subsidies to incompressible consumption	2022	1.97%	} 1.55%	7.5%	} 5.9%	2027	116.1%
	2023	1.13%		4.4%			
Targeted subsidies to lowest income households	2022	2.31%	} 1.90%	7.6%	} 6.0%	2027	114.3%
	2023	1.45%		4.5%			

supporting growth: $(1.92 - 1.55) \times 2 = 0.74$ percentage points of growth are lost over two years.³⁵ Additionally, it is significantly more inflationary because it directly stimulates demand, thereby activating the price-wage spiral more strongly. This high inflation prompts the ECB to sharply increase its interest rate. Consequently, despite having a lower fiscal cost in absolute terms, the combination of weak growth and higher interest rates results in an increased debt burden, leading to a surge in the debt-to-GDP ratio compared with the tariff shield scenario.

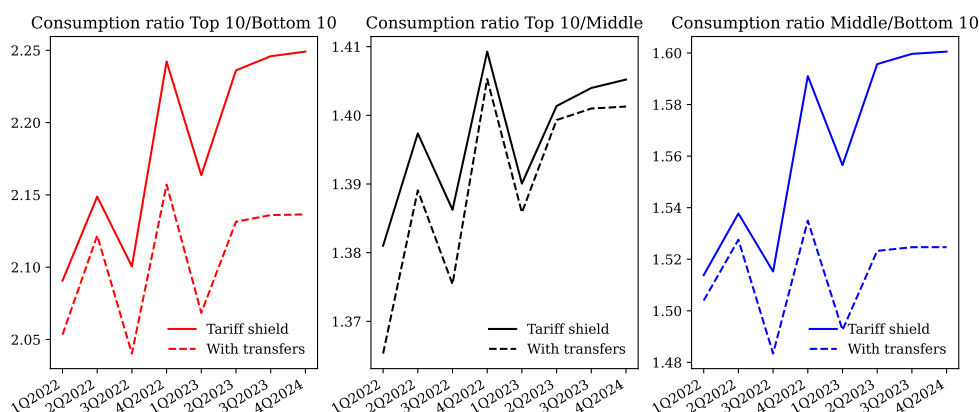
However, this redistributive demand policy has led to a greater reduction in inequality. With this transfer targeted at incompressible energy consumption, the consumption of T10, which was 2.05 times higher than that of B10 in 1Q2022, now stands at only 2.12 in 4Q2023 (a small increase of 3.4% compared to 7% with the tariff shield). The increase in the consumption ratio of the T10 group compared to that of the middle-income group is

³⁴The share of incompressible energy consumption for each decile is 31% for D1, 26% for D2, 24% for D3, 21% for D4, 20% for D5, 19% for D6, 17% for D7, 18% for D8, 16% for D9 and 14% for D10. These proportions have been computed from the INSEE's "Household Budget" survey data.

³⁵In Appendix 2.8.12, the complete description of the quarterly path of the aggregates is presented.

almost identical to that with a tariff shield. The ratio of middle-income group consumption compared to that of B10 drops from 1.60 in 4Q2023 to 1.52 (5.25% reduction). This larger reduction in inequality can be explained by the significantly higher MPC of low-income households, larger share of energy in their consumption baskets, and lower price elasticity (see Figure 2.1).

Figure 2.7: Dynamics of Inequalities: A Redistributive Demand Policy



However, this demand policy redistributes the same transfers to all households, whereas the highest-income households have lower MPC (see Figure 2.1).³⁶ Therefore, it may be preferable to target all transfers to modest households. We consider an alternative that targets transfers to households with income in the lowest 30% of the distribution. These transfers are calibrated to be two times larger than the previous ones; therefore, for an ex-ante budgetary cost, they are 40% lower. As shown in Table 2.6, these “targeted subsidies” to lowest-income households generate the same GDP growth as the tariff shield. However, this policy induces larger inflation and higher public debt than tariff shields. By strongly stimulating demand, this policy adds to the inflationary tensions induced by energy shocks. This leads to a significant increase in interest rates, resulting in a higher financial cost of public debt. Hence, although the ex-ante cost is lower than the tariff shield (only 30% of households perceive a transfer of an amount twice as large as their incompressible consumption), the effective cost of this targeted demand policy is higher because its strong inflationary effects induce a steep rise in interest rates. Therefore, although specific transfers may lead to the same GDP growth as the tariff shield, they incur significantly higher costs in terms of inflation and public debt. Therefore, tariff shields seem to be better designed to fight the recessionary effects of rising energy prices.

2.6 Robustness

In Section 2.5, we demonstrated that the tariff shield supports economic activity while mitigating inflation and reducing inequality at a cost equivalent to a 1.3 pp increase in the

³⁶Appendix 2.8.13 shows that IRFs of consumption by income levels are different.

debt-to-GDP ratio. Are the findings robust? This section presents robustness analyses of the monetary and fiscal policies that may change during the energy crisis.³⁷

2.6.1 Monetary Policy

In our benchmark, the ECB responds to European inflation by considering only French inflation, as the rest of the European inflation correlates with it. Consequently, the ECB supported the implementation of the French tariff shield by reducing interest rates. However, this could result in an overestimation of the effectiveness of the tariff shield because France is the sole country implementing a tariff shield. As other Euro area countries have not adopted such anti-inflationary policies, their inflation rates are likely to increase because of energy price inflation. Consequently, the ECB might raise its interest rate, contrary to what occurs in our benchmark. This argument also applies to evaluating transfers to low-income households. However, in this case, an adjustment of the ECB interest rate would lead to an underestimation of its effectiveness because this policy is inflationary.

To isolate the dynamics of the ECB interest rate (i_t) from policy decisions in France, we assume that i_t follows the equilibrium values determined in the model without a tariff shield. This trajectory of i_t reflects the equilibrium in which no country implements a tariff shield. For the equilibrium with policy to align with the predetermined path of i_t , we estimate the occurrence of monetary policy shocks (ε_t) to ensure consistency. Consequently, we assess the impact of the policy implemented in France by specifying that it does not affect the ECB interest rates.

Table 2.7: Robustness to Alternative Monetary Policies

	GDP		Inflation		Debt GDP		
No tariff shield	2022	1.68%	} 1.31%	7.3%	} 5.4%	2027	111.3%
	2023	0.94%		3.5%			
Monetary policy: benchmark Taylor rule							
Tariff shield	2022	2.85%	} 1.92%	6.5%	} 4.9%	2027	112.6%
2022 2023	2023	1.00%		3.4%			
Targeted subsidies to lowest income households	2022	2.31%	} 1.90%	7.6%	} 6.0%	2027	114.3%
	2023	1.45%		4.5%			
Neutral monetary policy: Taylor rule independent of French policies							
Tariff shield	2022	2.47%	} 1.78%	6.2%	} 4.5%	2027	112.9%
2022 2023	2023	1.10%		2.9%			
Targeted subsidies to lowest income households	2022	2.50%	} 2.25%	7.7%	} 6.6%	2027	108.3%
	2023	2.00%		5.5%			

The results are presented in Table 2.7. By fixing the nominal rate on a trajectory aimed

³⁷These adjustments in monetary and fiscal policy are unanticipated and take effect concurrently with the tariff shield. Therefore, these scenarios do not necessitate model re-estimation, but they do entail changes to the model multipliers.

at fighting high European inflation—without considering the disinflation achieved in France due to the tariff shield—shows an average loss of 0.14 pp in growth over the period 2022-2023, with the inflation rate being higher by 0.4 pp. This loss of growth primarily occurred in 2022, as the interest rates in this scenario were higher than those in our benchmark, prompting intertemporal substitution of consumption. These macroeconomic outcomes do not favor a reduction in public debt, which increases by 0.3 pp compared to our benchmark evaluation of the tariff shield.

This hypothetical interest rate path, independent of French policy, is favorable for inflationary policies that could be implemented in France. This is the case for demand policies, such as transfers targeted at low-income individuals. As their inflationary effects are not offset by an increase in ECB rates, the gains in growth are higher by 0.35 pp on average per year, and inflation is higher by 0.6 pp relative to our benchmark evaluation. These changes in the aggregates make it easier to achieve a low debt-to-GDP ratio, which is even lower than that without a tariff shield.

Figure 2.8: Dynamics of Inequalities With Lower Inflation Correlation and Tariff Shield

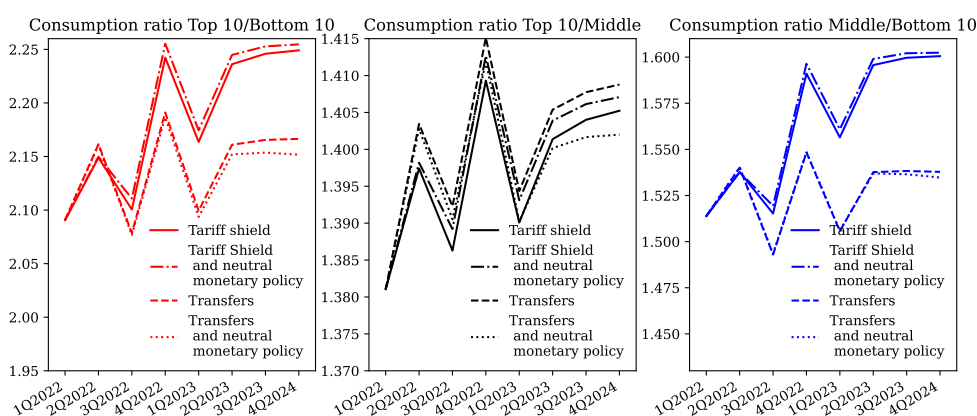


Figure 2.8 shows that a Taylor rule unaffected by the policies implemented in France provides even greater advantages for transfers in fighting inequalities. In this scenario, transfers are more expansionary (supporting low-income individuals) and lead to a more substantial reduction in the real interest rate (lowering the financial income of the wealthiest individuals).

In summary, during the tariff shield period, when the ECB followed European inflation and diverged from French inflation, the effectiveness of the tariff shield policy was likely weaker for output growth, but stronger for inflation. Conversely, the effectiveness of a transfer policy would have been affected oppositely, being stronger in terms of production, but weaker in terms of inflation. Two lessons can be drawn from these results, which extend beyond the scope of the present model but are worth highlighting. First, a coordinated policy based on tariff shields at the European level is likely to be more effective than a coordinated policy for transfers. Second, a unilateral tariff shield was less noncooperative than a transfer policy that induced inflation without internalizing the overall inflationary costs for the Euro area.

2.6.2 Fiscal Policy

In our benchmark scenario, the tariff shield is not financed by any contemporaneous tax and leads to an increase in public debt, which is consistent with French policy. To assess the robustness of the tariff shield, this section examines the case in which it is financed contemporaneously by payroll ($\Delta\tau_l$) or consumption ($\Delta\tau_c$) taxes, which are distortionary. We introduce the following budgetary adjustments: $\Delta\tau_l wN = \Xi_{sHP_E}(Y_E + (1 + \tau_c)\underline{c}_E)$ or $\Delta\tau_c C = \Xi_{sHP_E}(Y_E + (1 + \tau_c)\underline{c}_E)$, where Ξ represents the tariff shield share financed by an increase in one of the tax rates. Similar to the tariff shields, these tax increases were transitory and unanticipated. We set $\Xi = 1$ to introduce the largest distortion, which implies no debt financing.

Table 2.8: Robustness to Alternative Fiscal Policies

	GDP		Inflation		Debt GDP		
No tariff shield	2022	1.68%	1.31%	7.3%	5.4%	2027	111.3%
	2023	0.94%		3.5%			
Tariff shield	2022	2.85%	1.92%	6.5%	4.9%	2027	112.6%
	2023	1.00%		3.4%			
Tariff shield in 2022 & 2023 financed by distortive taxation							
Payroll tax ($\Delta\tau_l$)	2022	2.46%	1.64%	6.7%	5.2%	2027	111.1%
	2023	0.82%		3.7%			
Consumption tax ($\Delta\tau_c$)	2022	1.85%	1.48%	6.0%	4.4%	2027	111.0%
	2023	1.11%		2.8%			

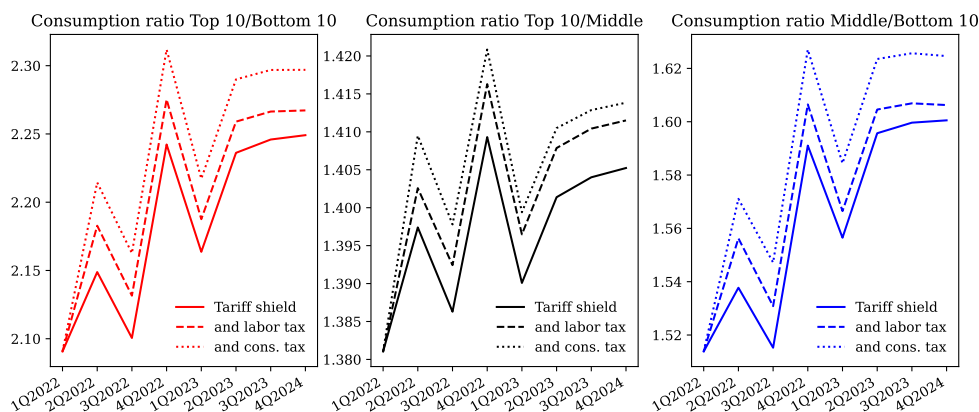
Table 2.8 shows that the introduction of this budgetary adjustment, which increased tax distortions, diminished the effectiveness of tariff shields in terms of GDP growth. However, even if the tariff shield is entirely financed by a distorting tax (either payroll or consumption tax), it remains effective. The dampening effect of tariff shields on inflation continues to have positive spillover effects across the economy.

Taxation on labor is less detrimental to growth because it has a broader base, which means that it can be increased to a lesser extent to achieve the same budgetary benefit.³⁸ The increase in payroll tax rates is inflationary because it constitutes a negative supply shock, leading to a reduction in labor supply, whereas consumption tax reduces inflation because it works as a demand shock. Table 2.8 illustrates that higher GDP growth with payroll tax adjustments than with consumption tax adjustments also results in higher inflation in the former scenario. However, the debt-to-GDP ratio is slightly lower than without the tariff shield, indicating that the subsequent stronger GDP growth provides additional dividends to a tax-financed tariff shield.

Figure 2.9 illustrates that an increase in tax rates exacerbates inequalities because they distort the labor supply, which is the primary source of income for the poorest households.

³⁸Indeed, in our simplified model, output is primarily generated through labor, and it is divided into private consumption and public consumption. Importantly, public consumption is not included in the consumption tax base.

Figure 2.9: Dynamics of Inequalities With Alternative Tax Adjustments



Consequently, even if tax adjustments diminish the efficiency of the tariff shield, they remain powerful tools for increasing GDP growth.

2.7 Conclusion

This article presents a method for conducting “real-time” policy evaluations. This approach enables the comparison of alternative policies that alter the structure of the economy during a specific period. A benchmark policy is used for identifying the shocks that shape the relevant environment for evaluating the policies under study.

Using this method, we show that the tariff shield implemented in France from 2022 to 2023 represents a favorable compromise to mitigate inflation, support growth, and dampen the increase in inequality. Its fiscal cost would amount to a 1.3 pp increase in the debt-to-GDP ratio by 2027. This policy appears to explain why France experienced lower inflation than its European counterparts. Additionally, we show that supporting this policy through wage indexation is not advisable. Finally, the tariff shield proves to be more effective than direct transfers to households (demand policy) because it provides greater support for employment by containing the rise in labor costs.

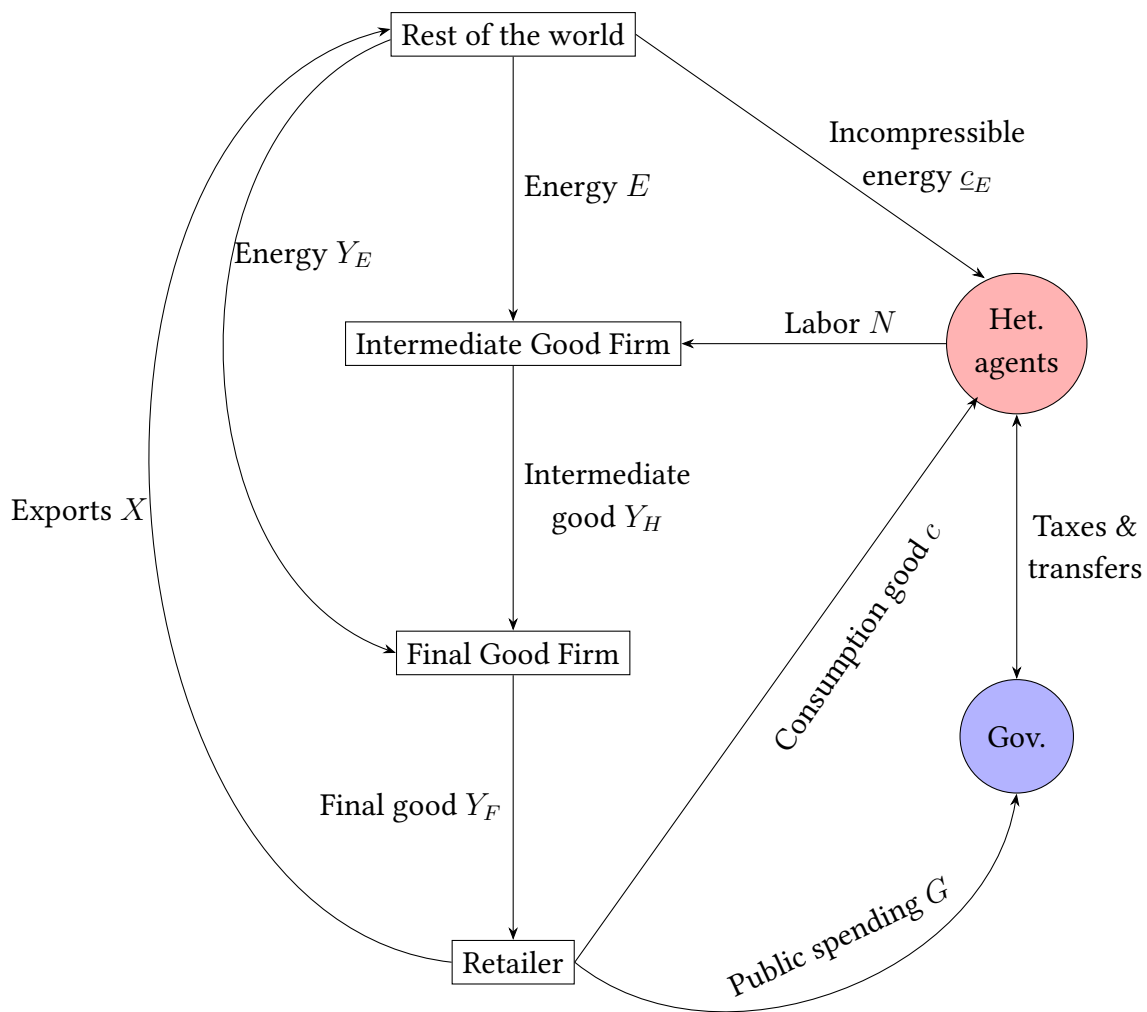
It’s worth noting that our study does not address the “free rider” problem highlighted by Auclert, Monneray, et al., 2023 and Bayer, Kriwolulzky, et al., 2023: if all countries had implemented a tariff shield, the price of energy could have risen even further, potentially canceling out the effect of the policy. However, in this crisis, which appears to have been a one-shot game, France was the first and quickest to react. This opportunistic response may provide the French economy with a temporary competitive advantage, even if the ECB follows Euro area inflation that diverges from French inflation during the crisis.

An important limitation of the tariff shield is its environmental cost. By subsidizing energy, it fails to encourage energy-saving behaviors. The rise in energy prices and its repercussions on the purchasing power of households, especially the poorest, raise ques-

tions about the acceptability of environmental policies. Our framework can be used to assess policies aimed at mitigating the adverse effects of a carbon tax on efficiency and inequalities, as shown in Langot, Malmberg, Tripier, and Jean-Olivier Hairault, 2023.

2.8 Appendix

2.8.1 Model's structure



2.8.2 The tariff shield

Model's calibration

We calibrate the total expenditures in energy as $\frac{p_E \mathcal{E}(p_E)}{Y} = 3.18\%$ where the GDP = $Y = \text{€}2,639.1$ billion in 2022 and $\mathcal{E}(p_E)$ is the quantity of consumed energy that depends on price with $\mathcal{E}'(p_E) < 0$. Hence, we have $p_E \mathcal{E}(p_E) = \text{€}83.9234 \approx \text{€}84$ billion. The cost of the “tariff shield” is evaluated at $\text{€}85$ billion by the government from October 2021 to December 2023 (27 months), i.e. $\text{€}37$ billion per year (see Sénat, 2023). The set of interventions included in

Table 2.9: Tariff shield (billions of €) – October 2021 to December 2023. Source: Sénat, 2023.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Tariffs of electricity	Tax on elect.	Nuclear elect.	Tariffs of gas	Energy vouchers	Damper on elect.	Task “Economy”	Tariffs of fuel	Univ.	Total
24.7	18	8	10	2.9	3.6	7.2	10.4	0.3	85.1

the tariff shield is given in Table 2.9. The subsidies perceived by the households (columns (1)-(5) & (8) of the Table 2.9) represent 87% of the total, i.e. $\text{€}32$ billion per year.

After the energy crisis, the energy price changes because p_E increases by dp_E , but also because the tariff shield s_H can be implemented. At the new price \hat{p}_E , the energy expenditures $\hat{p}_E \mathcal{E}(\hat{p}_E)$ can be approximated around $p_E \mathcal{E}(p_E)$ by $\hat{p}_E \mathcal{E}(\hat{p}_E) \approx p_E \mathcal{E}(p_E) \left[1 + (1 + \varepsilon) \frac{\hat{p}_E - p_E}{p_E} \right]$ where we denote $\varepsilon = \mathcal{E}'(p_E) \frac{p_E}{\mathcal{E}(p_E)}$. After the crisis, the energy price is $\hat{p}_E \equiv p_E + dp_E$ without a tariff shield, implying $\frac{\hat{p}_E - p_E}{p_E} = \frac{dp_E}{p_E}$, or $\hat{p}_E \equiv (1 - s_H)(p_E + dp_E)$ with a tariff shield, implying $\frac{\hat{p}_E - p_E}{p_E} = -s_H + (1 - s_H) \frac{dp_E}{p_E}$. Therefore, the difference between the two “after crisis” scenarios (with and without policy) identifies the impact of the tariff shield. This gap $\frac{\Delta}{Y}$ (relative to GDP) is defined by

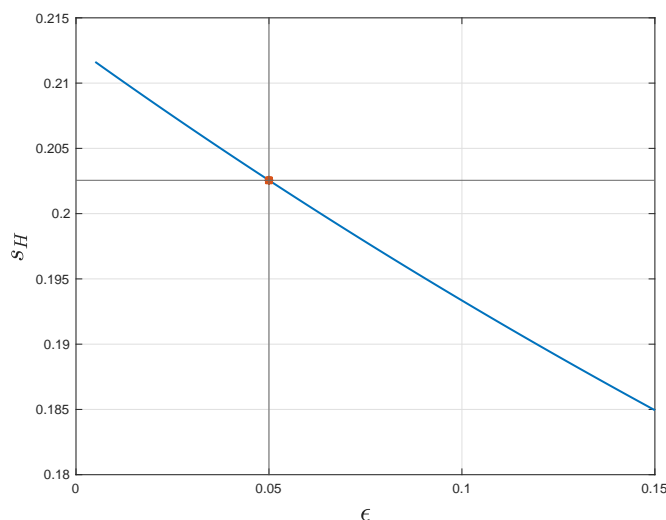
$$\frac{\Delta}{Y} \approx -s_H \frac{p_E \mathcal{E}}{Y} (1 - \varepsilon) \left(1 + \frac{dp_E}{p_E} \right) \Rightarrow s_H \approx - \frac{\frac{\Delta}{Y}}{\frac{p_E \mathcal{E}}{Y} (1 - \varepsilon) \left(1 + \frac{dp_E}{p_E} \right)}$$

With $\frac{p_E \mathcal{E}}{Y} = 0.0318 \times 40\%$, where 40% is the share of energy used by the households, $\frac{\Delta}{Y} = \frac{-37}{2,639.1} \times 87\%$, where 87% is the share of the tariff shield targeted to households (sum of the columns (1)-(5) & (8) of Table 2.9 over the total cost) and $\frac{dp_E}{p_E} = \frac{180-50}{50} = 2.6$ (an increase from $\$50$ to $\$180$ per barrel from the middle of 2020 to middle of 2022), we obtain $s_H \approx 20\%$ with $\varepsilon = 0.05$.

Figure 2.10 gives the sensitivity of s_H to the calibration for the price elasticity of energy demand. Even with the “conservative” value of 0.1 preferred by Bachmann et al., 2022, Baquee et al., 2022 and Langot, Malherbet, et al., 2022, the value of s_H is still below 0.21.³⁹

³⁹Labandeira, Labeaga, and Lopez-Otero, 2017 distinguish between short-run (within one year) and “long-

Figure 2.10: Calibration of the “tariff shield”



Implementation

We present the main features of the “tariff shield” components reported in Table 2.9. The key point is that these components were introduced on a discretionary basis, without prior planning and with numerous unanticipated changes in amounts, scope and duration.⁴⁰ This leads us to model it as a shock following a $AR(1)$ process. A forecast of its annual cost is provided for in the Finance Act, but for individuals, each quarter, it represents a “fiscal” surprise. In what follows, we detail the set of measures contained in the “tariff shield” and highlight their unanticipated nature.

Tariff shield on electricity. To mitigate the surge in electricity prices, the French government implemented a freeze on the escalation of regulated electricity sale prices. This freeze was enacted through decrees, prohibiting any increase in the regulated electricity prices that the French Energy Regulatory Commission proposes each January, following the calculation methods outlined in the Energy Code. The gap between the non-frozen regulated price levels and the decreed frozen price levels was reimbursed to suppliers by the State as part of the energy public service charges. These measures were disclosed in the Finance Act, which is annually enacted in the fall. Consequently, economic agents were only made aware of the tariff shield for 2022 towards the end of 2021, with the Finance Act not committing to extend the scheme into future years. The tariff shield for 2023 was like-

run” (otherwise) elasticity estimates. For natural gas, they find an average short-run elasticity of -0.18 and a long-run elasticity of -0.684. For heating oil, the average short- and long-run estimates are -0.017 and -0.185, respectively. For energy in general, the estimates are similar with a short-run elasticity of -0.221 and a long-run elasticity of -0.584. In our analysis, which accounts for variations in the short run, we retain the value of -0.05, closer to the “short-run” elasticity estimates of Labandeira, Labeaga, and Lopez-Otero, 2017.

⁴⁰In its report Sénat, 2023, the “commission des finances” of the French Senate states that “An initial assessment of the implementation of these measures reveals that they were conceived in a hurry and in an iterative way, even though they could have been better anticipated. The players involved are unanimous in regretting the hasty conditions that governed the design and implementation of the measures.”

wise only disclosed to economic agents at the end of 2022, upon the passing of the Finance Act. Therefore, the introduction of the electricity tariff shield resulted from two successive economic policy shocks, unanticipated by economic agents.

Tax on electricity. For both 2022 and 2023, the freeze on electricity prices was partially funded through a cut in the electricity excise duty. These decisions were enacted by decree and disclosed in the Finance Act exclusively for the current year, without a multi-year framework. Consequently, each of these decisions for 2022 and 2023 caught economic agents off guard.

Nuclear electricity. Since 2011, electricity suppliers have been granted regulated access to the energy produced by existing nuclear power plants. In 2022, the nuclear energy producer expanded the volume of regulated access to historical nuclear electricity by 20 terawatt-hours (TWh). This adjustment, applied across all electricity supply contracts, advantaged all consumers, encompassing both private individuals and businesses, extending well beyond those eligible for the shield. This decision was made spontaneously, without prior planning.

Tariff shield on gas. The gas tariff shield relies on the freezing of regulated natural gas sales prices. The gap between those regulated prices and the unfrozen regulated price, reflective of the suppliers' average costs, was covered by the government and directly compensated to gas suppliers. This arrangement enabled customers to benefit from a reduced supply price, which directly included the government's assistance under the tariff shield, even for contracts under market offers (which account for 75% of consumers). The gas tariff shield was initially announced by the Prime Minister on October 21, 2021, and subsequently enacted into law on December 30, 2021. At its announcement, the gas tariff shield was intended to last until June 30, 2022. However, diverging from the initial timeline, the "gas" tariff shield was extended for the first time on June 25, 2022, until December 31, 2022. This extension represented a deviation in the scheme's duration that economic agents could not have foreseen, which is modeled as an unanticipated shock. Another unexpected turn occurred on December 30, 2022, with a further extension from January 1, 2023, to June 30, 2023, this time capping the rise in regulated tariffs to 15% incl. tax relative to the October 2021 price. Consequently, the application period and the scope of the gas tariff shield have been altered several times at discretion. Initially limited to customers on a regulated tariff offer, it was broadened to include all customers on a market offer contract, as well as sole building owners for residential use and co-ownership syndicates of such buildings, on August 17, 2022. This expansion of the tariff shield's scope, equally unforeseen, is also incorporated as a surprise in the model.

Energy vouchers. The energy voucher is a government subsidy aimed at the lowest 20% of households. It helps them pay their energy bills (electricity, natural gas, fuels such as wood or fuel oil) or fund energy renovation projects. It is allocated based on income and the composition of the household (individuals living under the same roof) as declared to

the tax authorities. Introduced in 2018 to replace social energy prices, the energy voucher has undergone several discretionary modifications amid the energy crisis, which are subsequently modeled as unanticipated shocks. These alterations encompass both the voucher's amount – with an initial additional sum of €100 declared in 2021 and a further increase of €200 announced in 2022 – and the criteria for income eligibility. In 2022, eligibility unexpectedly expanded to include the 3rd and 4th income deciles of households, granting them €100, and also covering expenditure specifically for households using domestic fuel or wood for heating. The “fuel oil voucher” targeted the first five income deciles, providing a €200 grant for energy voucher recipients and 100 euros for others. Meanwhile, the “wood voucher” was designed for the first seven income deciles, offering an aid amounting to between €50 and €200, based on income, household size, and the type of wood fuel utilized.

Damper on electricity. The damper mechanism is intended for small and medium-sized businesses and public or semi-public sector entities that are not eligible for the electricity tariff shield. Eligible entities benefit from a reduction in the price of their electricity supply equal to an amount in €/MWh corresponding to the difference between the contract electricity price and a reference price, up to a ceiling also expressed in €/MWh. Application of the electricity damper therefore reduces the bill of eligible customers, for half the volumes consumed, by the difference between the electricity price of their contract and the strike price of €180/MWh, capped at €320/MWh. The maximum annual subsidy under this scheme is €160/MWh. This scheme, due to come into effect on December 31, 2022, has also been subject to discretionary changes in response to demands from very small businesses, with bakers at the forefront. On February 3, 2023, an “over-amortizer” was introduced to guarantee an electricity price of €230/MWh for these very small businesses.

“Economy” Task. In the French government's budget, the task “Economy” brings together several budgetary instruments designed to promote employment, growth, business competitiveness, export development, competition, and consumer protection. The funds allocated to this mission were used to set up an assistance mechanism designed to support the largest energy-consuming companies that are not eligible for the dampers. A first allocation of €1.5 billion was decided in April 2023, a second allocation of €1.5 billion in August 2022, and a third of €4 billion in November 2023. These allocations were not part of a multi-year program, but were made on a discretionary basis for each year and therefore could not be anticipated. Depending on the company's situation, the aid will be equal to a capped amount of between 30% and 70% of the company's sales over the period concerned. However, due to overly restrictive criteria and an overly complex procedure, these amounts of aid have not been fully distributed.

Tariffs on fuel. To respond to the price hike observed in spring 2022, a fuel price rebate of €0.15 (excluding tax) was implemented in March 2022. This fuel subsidy then underwent several modifications, each of which was decided independently, creating surprises for consumers who were unaware of the various changes in advance. This aid was initially scheduled to run from April to July 2022. It was then extended and even increased to €0.25

from September to November 2022, before being reduced to €0.0833 until the end of the year. All individuals were concerned by this general aid. The aid was paid directly to the operators responsible for supplying or releasing the fuels for consumption and was then passed on by them to the end consumer in the form of prices. This non-targeted fuel rebate was replaced on January 1, 2023 by a €100 allowance targeted at low-income workers who use their private vehicles to commute to work. Once again, consumers were unaware of this decision until it was implemented.

University (Univ.). The university category corresponds to the specific aid scheme set up in 2022 to assist universities in covering their energy costs.

2.8.3 Income, wealth and consumption distributions

Income inequalities. As in Bayer, Kriwoluzky, et al., 2023 and following the approach proposed by Castaneda, Diaz-Gimenez, and Rios-Rull, 1998, we calibrate the parameters governing individual labor income risk in our model to reproduce moments of the observed total income distribution. More precisely, we choose σ to minimize the difference between what the model predicts for $Q75/Q50$ ($\approx \frac{D6-D7}{D4-D5}$) and the observed values (see Table 2.10).

Table 2.10: Income inequalities (see INSEE data)

	Model		Data	
	Deciles	Deciles/Med	Deciles	Deciles/Med
Lower than D2	0.7066	0.7408	11025	0.5008
D2-D3	0.8570	0.8984	17195	0.7811
D4-D5	0.9538	1.0000	22015	1.0000
D6-D7	1.2341	1.2939	27785	1.2621
D8-D9	1.6788	1.7601	32540	1.4781
Greater than D9	2.3507	2.4645	49300	2.2394

We then obtain $\sigma \approx 0.5$, given the value of the persistence of the productivity shock estimated by Fonseca et al., 2023 on French panel data ($\rho = 0.966$).

Table 2.11: Consumption inequalities (see INSEE data)

	D1/D5	D2/D5	D3/D5	D4/D5	D6/D5	D7/D5	D8/D5	D9/D5	D10/D5	D10/D1
Data	0.6778	0.7703	0.8438	0.9517	1.1385	1.2216	1.3724	1.5078	2.0327	2.9990
Model	0.6741	0.7471	0.9140	0.9292	1.0892	1.1404	1.2310	1.3472	1.9949	2.9595

This calibration of σ also allows the consumption distribution generated by our model to be close to the one observed in the data (see Table 2.11). We are particularly attentive to consumption inequalities because they are the closest measure of household welfare.

We consider that a higher calibration of σ than the one estimated with an $AR(1)$ process on individual panel data is not unrealistic because recent work on labor income has shown that the process generating their dynamics would be more complex than an $AR(1)$. Guvenen, Karahan, et al., 2021 underline that earnings growth rates are negatively skewed and exhibit excess kurtosis. Relative to a normal distribution, their results suggest that there are more individuals with small and large earnings changes, but fewer with medium-sized earnings changes. Moreover, the large earnings changes are often negative. Ferriere, Grübener, et al., 2023 show that a productivity process modeled as a Gaussian Mixture Autoregressive ($GMAR$) process can account for these facts. This process generates more extreme income values than what is possible to obtain with an $AR(1)$ process estimated on individual panel data. However, with a larger value for σ , some of the large dispersion contained in a complex model of labor income, such as a $GMAR$, can be approximated by the simple $AR(1)$ process used in our model.

Wealth inequalities. Another challenge for heterogeneous-agent models is to replicate the observed wealth inequalities (see, e.g., De Nardi and Fella, 2017). Since the work of Kaplan, Moll, and Giovanni L. Violante, 2018, it has been known that two-asset models are better suited to generate the large wealth inequalities observed in the U.S. Table 2.12 shows that our simple one-asset model can generate a wealth distribution not far from the one observed in France, which is, of course, less unequal than in the U.S. The main issue is not

Table 2.12: Wealth inequalities (see INSEE data)

	Model		Data	
	Deciles	Deciles/Med	Deciles	Deciles/Med
Lower than D2	0.0015	0.0008	2000	0.1921
D2-D3	0.2845	0.1430	5300	0.5091
D4-D5	1.9892	1.0000	10411	1.0000
D6-D7	6.1588	3.0961	15908	1.5280
D8-D9	7.7627	3.9024	28196	2.7083
Greater than D9	10.6023	5.3299	61751	5.9313

the typical inability of the model to generate the large wealth share held by the richest individuals, but rather its incapacity to generate sufficient savings at the bottom of the distribution. This lack of saving incentive for the poorest can stem from the substantial French transfers targeted at workers beyond the median labor income, which are not conditioned to any eligibility criteria in our model, unlike the French tax-benefit system. The detailed modeling of the French tax and transfer system is highly complex, and we defer it to future research.

Consumption inequalities. To simplify the numerical resolution of the model, we compute the average consumption by labor productivity deciles. This approach is essential for generating the cyclical evolution of consumption inequalities, as it requires computing Impulse Response Functions (IRFs) for different levels of consumption. To ensure comparability of inequality measures across various policy counterfactuals, we define each consumption level relative to an exogenous state of the economy shared among all economies. A natural candidate for this is the labor productivity e at the time of the shock, which serves as a common state variable. In the model, the income process involves a distribution as shown in Table 2.13.

Table 2.13: Definition of the earning groups

Model	B1.5	B10	B33	Middle	T33	T10	T1.5
Earnings	0-1.5%	1.5-10%	10-33%	33-66%	66-90%	90-98.5%	98.5-100%

With this distribution, it is possible to calculate the consumption of individuals whose labor income is among the lowest 1.5%, or among the lowest 10%, and so on.

Table 2.14: Consumption inequalities when deciles are based on the distribution of consumption or income

	D1/Med	Q1/Med	Q2/Med	Q3/Med	D10/D5	D10/D1
Conditionally to consumption	0.7340	0.9044	1.0956	1.3498	2.1723	2.9595
Conditionally to income	0.7347	0.9034	1.0966	1.2953	1.6245	2.2112

The comparison between the two rows of Table 2.14 indicates that the differences between these two computation methods are small. It is noteworthy that the computation is contingent upon labor income being less unequal, especially at the top of the distribution, when wealth matters.

2.8.4 French Data

Observed data

Data	Web access	Providers
Population	DBnomics code	Eurostat
GDP	DBnomics code	Eurostat
CPI	DBnomics code	INSEE
Enenergy price	DBnomics code	OECD
Government consumption	DBnomics code	Eurostat
Government transfers	DBnomics code	Eurostat
Public debt	DBnomics code	Eurostat

Table 2.15: Data sources

All the raw series of Table 2.15 are quarterly and range from 2Q1995 to 4Q2021. For the population, which is an annual series, we build a quarterly series by interpolation. All these series (with the exception of prices) are divided by the population to obtain per capita variables: $\{Y, \frac{b}{Y}, G, T\}$. The Consumer Price Index (CPI) series is monthly. It is quarterlyized using a moving average, from which we derive π . Finally, the energy price (P_E) is the crude oil price.

From government forecasts to quarterly data

	2021	2022	2023	2024	2025	2026	2027
Population (15-64)	41462267	41427249	41402466	41381174	41360167	41338765	41311515
GDP growth	6,8%	2.5%	1.4%	1.6%	1.7%	1.7%	1.8%
GDP share of G	25%	23.6%	23.1%	22.7%	22.3%	22%	22.4%
GDP share of T	21.2%	19.9%	19.4%	19.2%	19.1%	19.1%	18.5%
Debt-to-GDP	112.5%	111.9%	111.7%	112.8%	113.3%	113.2%	112.5%
Energy price	\$71	\$110	\$98	\$85	\$85	\$85	\$85
CPI (inflation rate)	1.6%	4.5%	3.2%	1.9%	1.75%	1.75%	1.75%

Table 2.16: Government forecasts. Source: Financial Act

The government consumption (G) is the sum of “intermediate consumption” + “compensation of employees” + “social benefits in kind”. The transfers (T) are “Social benefits in cash”.

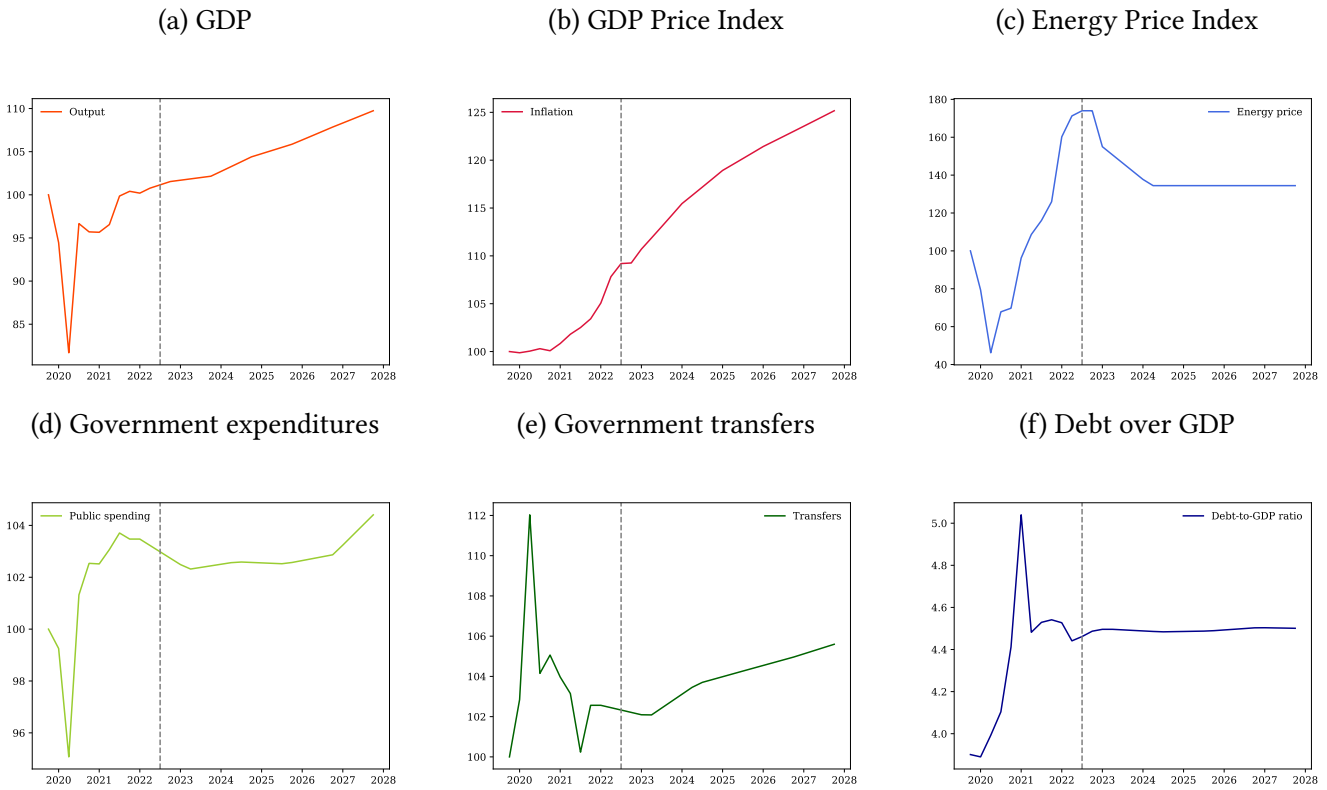
For GDP, CPI and energy price of the year τ , we compute the quarterly growth rates g_τ^z using the annual growth rates $g_{a,\tau}^z$ (forecasts of the GDP, CPI and energy price growth

rates reported in Table 2.16)⁴¹, solving

$$(1 + g_{a,\tau}^z) \times \sum_{q=1Q}^{4Q} Z_{q,\tau} = Z_{1Q,\tau+1} \times [1 + (1 + g_{\tau}^z) + (1 + g_{\tau}^z)^2 + (1 + g_{\tau}^z)^3]$$

where $Z = \text{GDP, IPC, energy price}$. We constructed quarterly data for GDP, IPC, and energy prices over the periods 1Q2022 to 4Q2027 (see panels (a), (b), and (c) of Figure 2.11). To obtain quarterly series, we interpolated the GDP shares of G (government expenditure) and T (government transfers). Subsequently, using the quarterly GDP data, we derived quarterly data for G and T over the same periods (see panels (d) and (e) of Figure 2.11). Regarding the debt-to-GDP ratio, we simply performed quarterly interpolation to construct quarterly data for $\frac{b}{y}$ over the periods 1Q2022 to 4Q2027 (see panel (f) of Figure 2.11).

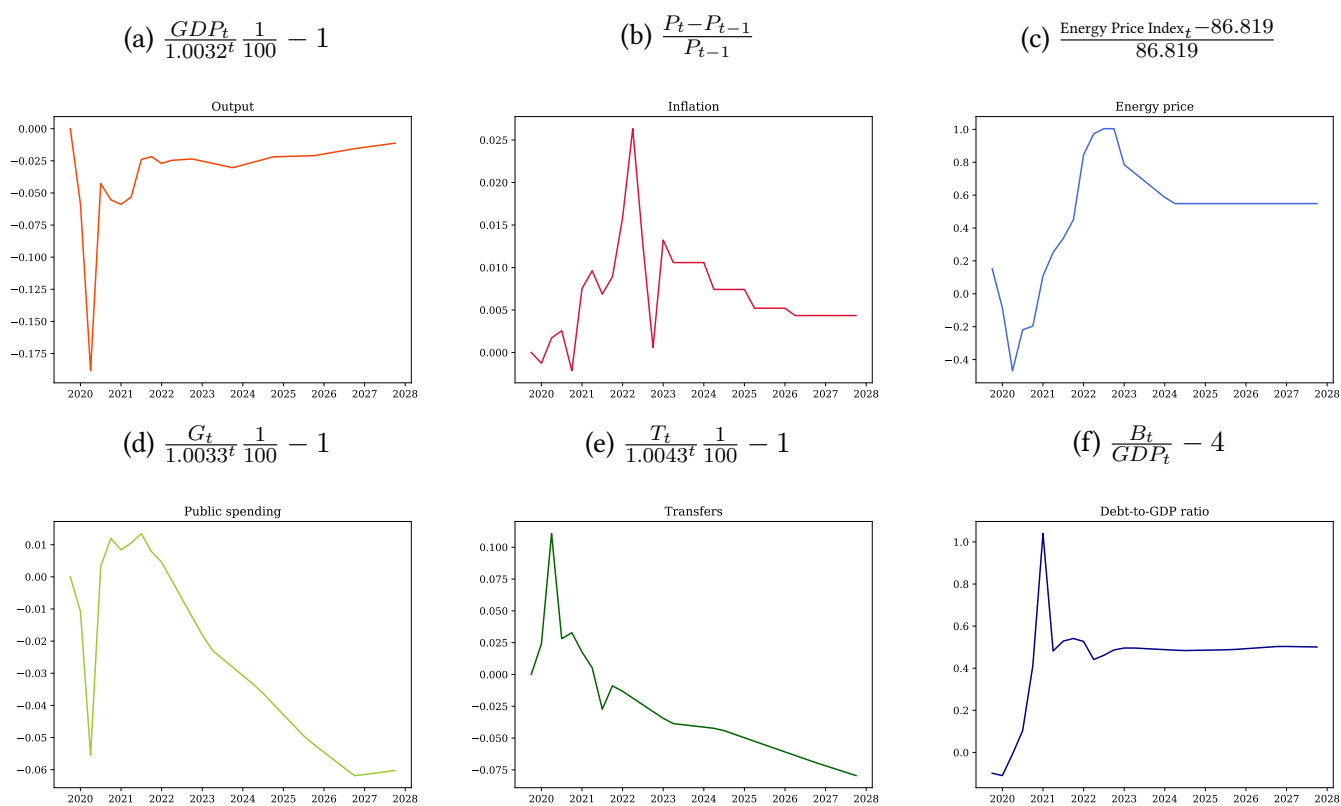
Figure 2.11: Raw Data: 4Q2019 = 100



Data are stationarized by extracting a linear trend, with the exception of the debt-to-GDP ratio, for which only the average over the sample is retrieved (see Figure 2.12).

⁴¹For the energy price, we deduce the annual growth rate from forecasts of the data in level.

Figure 2.12: Stationnarized French Data: 4Q2019 = 0



2.8.5 Estimation of the exogenous shocks processes

The persistence ρ and the standard deviation σ of the shock processes are estimated using a Bayesian procedure: based on a Metropolis-Hastings algorithm, we draw one million draws. The first half of accepted draws were burned in to correct for possible mischoice of the starting point.

The prior distributions considered are reported in Table 2.17. For energy prices (p_E), government spending (G), and transfers (T), our HANK model simply replicates the exogenous input series. Consequently, guesses for the values of these parameters can be obtained by estimating an $AR(1)$ on the time series $\{p_E, G, T\}$. These estimates are used as information to define the priors of these shocks. The remaining priors for $\{\vartheta, \mu, \beta\}$ are assumed to follow beta distributions for the persistence and inverse-gamma distributions for the standard deviation, as usual in the literature.

Table 2.17: Bayesian estimation results of the parameters of the $AR(1)$ processes

Shock		Prior	Mode	Std	95% CI
Energy price	p_E	ρ $\mathcal{N}(0.89, 0.05)$	0.956306	0.015542	[0.928103, 0.979286]
		σ $\mathcal{N}(0.13, 0.07)$	0.168543	0.012100	[0.149946, 0.189320]
Government spending	G	ρ $\mathcal{N}(0.73, 0.09)$	0.966911	0.015604	[0.934831, 0.986524]
		σ $\mathcal{N}(0.004, 0.003)$	0.003563	0.000315	[0.003200, 0.004221]
Transfers	T	ρ $\mathcal{N}(0.81, 0.07)$	0.907994	0.030241	[0.850255, 0.953381]
		σ $\mathcal{N}(0.005, 0.006)$	0.005199	0.000422	[0.004576, 0.005950]
Measurement error	ϑ	ρ $\beta(0.8, 0.05)$	0.937477	0.004867	[0.928861, 0.944730]
		σ $inv\Gamma(3.0, 1.0)$	0.688678	0.042471	[0.666099, 0.795908]
Price markup	μ	ρ $\beta(0.8, 0.05)$	0.746349	0.039882	[0.680943, 0.812585]
		σ $inv\Gamma(0.05, 1.0)$	1.517267	0.105297	[1.346353, 1.690274]
Preference	β	ρ $\beta(0.8, 0.05)$	0.938043	0.012706	[0.916814, 0.959411]
		σ $inv\Gamma(0.05, 1.0)$	0.068144	0.010359	[0.048737, 0.083068]

Because our model is not formulated in a linear state-space framework, the Kalman filter cannot be used to evaluate the log-likelihood. Instead, following the approach of Auclert et al. (2021), the log-likelihood of our model is computed using the covariance matrix linking the model's variables. This covariance matrix is based on the Jacobian of the model, which can be obtained using the sequence space method. It's worth noting that since we do not estimate structural parameters that affect the Jacobian of the system, the same Jacobian can be reused throughout the entire estimation process, resulting in computational time savings.

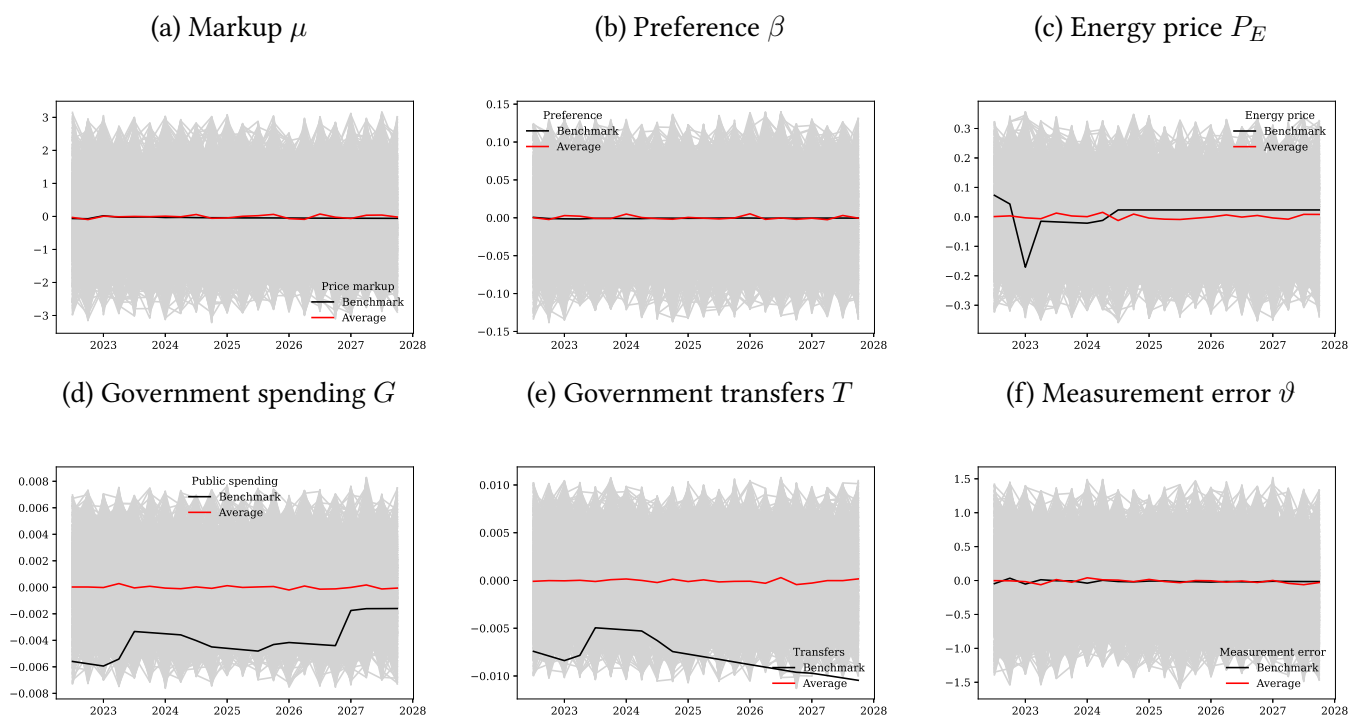
Therefore, our estimation relies on the linearized model around its steady state. Estimating based on the nonlinear model could yield different results, especially if strong nonlinearities exist in the considered regions of parameter space. However, comparing the linear and nonlinear Impulse Response Functions (IRFs) of our model (see Appendix 2.8.7), it appears that these nonlinearities are not particularly strong. This suggests that a linear approximation is reasonable and computationally efficient for estimation. Indeed, estimations based on the nonlinear version of a HANK model are still in their early stages. To the

best of our knowledge, Kase, Melosi, and Rottner, 2023 are currently the only ones doing this. Their approach relies on a neural network particle filter, treating the model's parameters as inputs to the neural networks. This method involves (i) training the neural network that maps input parameters to policy functions (as well as an additional neural network that ensures an equilibrium exists for a given parameter set), (ii) training the particle filter neural network that maps the parameters to the likelihood, and (iii) running the Bayesian estimation. Once the three neural networks are trained, the Bayesian estimation incurs almost no cost (1ms per draw in Kase, Melosi, and Rottner, 2023's example) since each draw does not require solving the model again (particularly its fully nonlinear version). However, training neural networks is computationally expensive. Therefore, we leave this for future research where the nonlinear features of the model may be more significant.

2.8.6 Testing the stability of the processes driving exogenous variables

In Figure 2.13, we present the estimated values of each innovation used to align the time series of government forecasts (black line). Subsequently, we examine whether these estimated realizations fall within the confidence interval of the innovations for the processes driving these exogenous variables. The red line depicts the mean of the distribution of these innovations, and the gray cloud represents the 95% confidence band.

Figure 2.13: Impulse Responses Functions: Only tariff shield



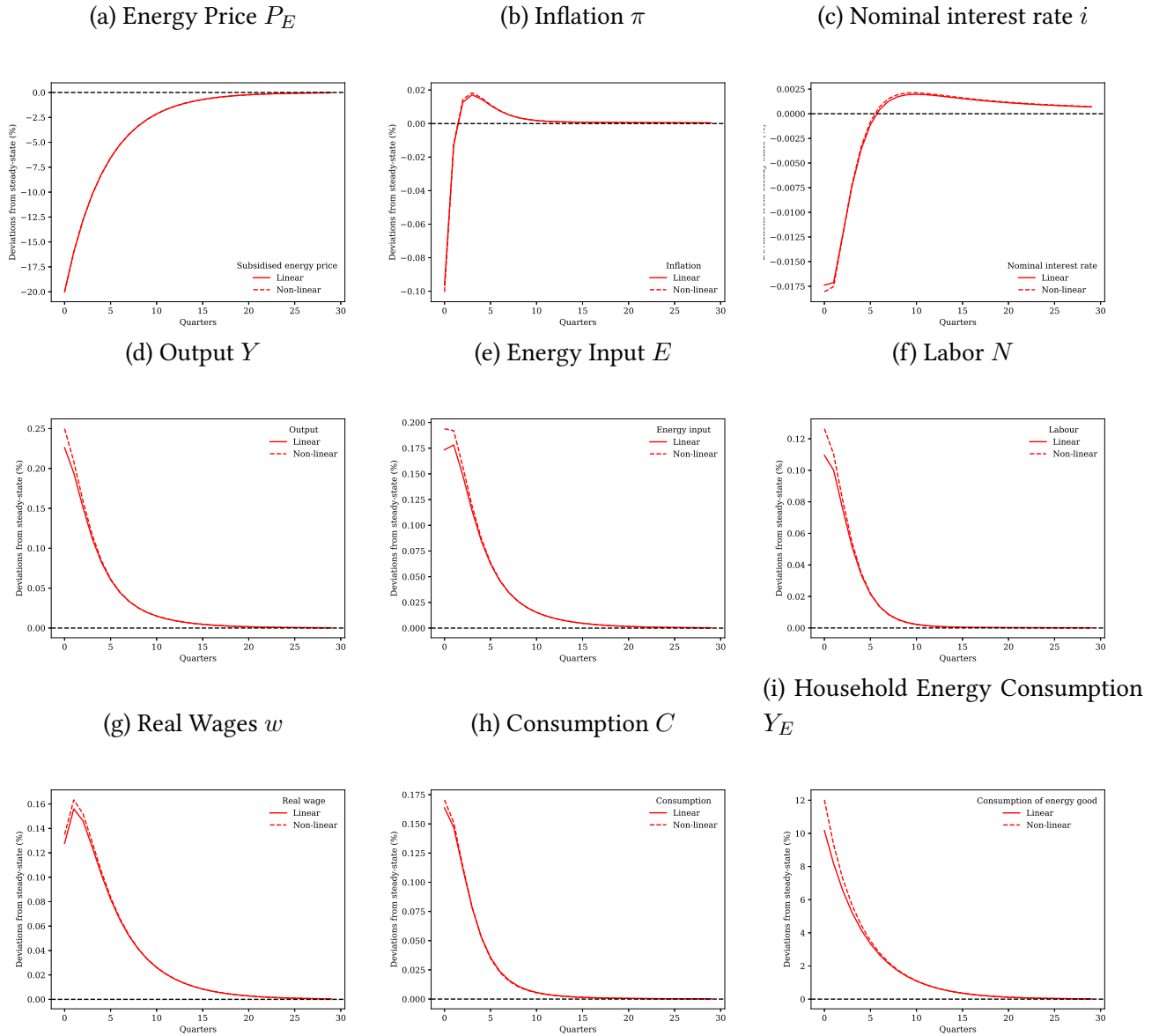
All the estimated innovations used to align the government forecasts fall within their respective confidence bands. This suggests that the policy rules of the government, summarized by ρ^G , ρ^T , σ^G , σ^T , are stable. In other words, the new information on government policy contained in the Finance Act supports the view that the government's policy rule will not change. Thus, our estimation is not biased by the R. Lucas, 1976 critique.

Finally, these results indirectly indicate that the stationarity of the time series used for model estimation cannot be rejected. Indeed, they suggest that stationary processes for the model's shocks enable the generation of the observed series, given that they were made stationary.

2.8.7 Non-linear IRFs

As the size of the tariff shield shock is not so large, the gaps between a linear and a non-linear approximation of the model dynamic remain small (see Figure 2.14).

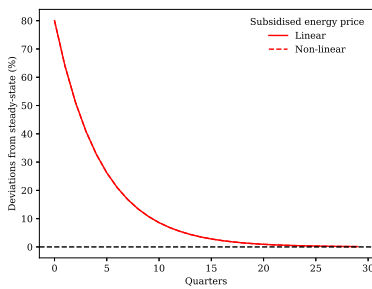
Figure 2.14: Impulse Responses Functions: Only tariff shield



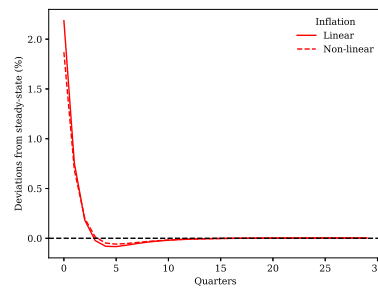
Even with a larger shock, namely the energy shock accompanied by the tariff shield, the gaps between the linear and the non-linear approximations of the model dynamic are still small, except for the energy used by firms or consumed by households (see Figure 2.15). Nevertheless, as the share of energy in the GDP is relatively small, these gaps in energy dynamics seem to be not of first order.

Figure 2.15: Impulse Responses Functions: Energy price shock and tariff shield

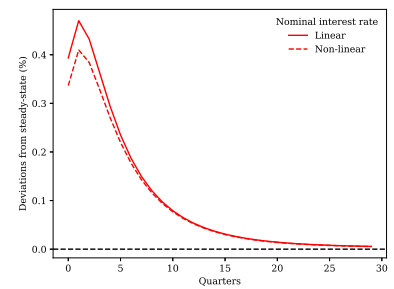
(a) Energy Price P_E



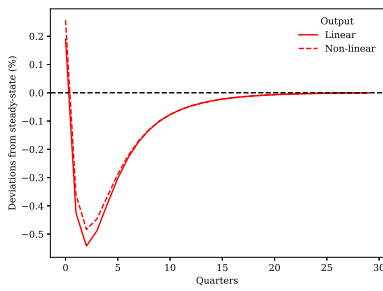
(b) Inflation π



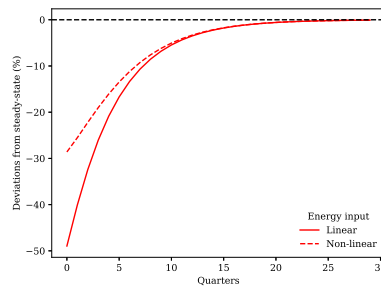
(c) Nominal interest rate i



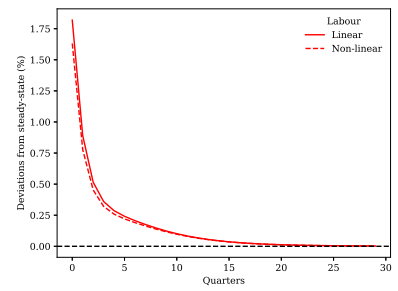
(d) Output Y



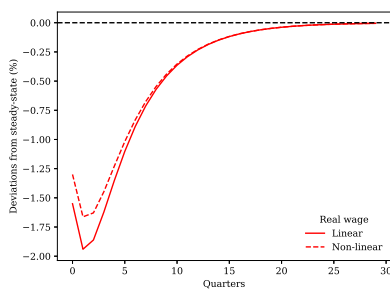
(e) Energy Input E



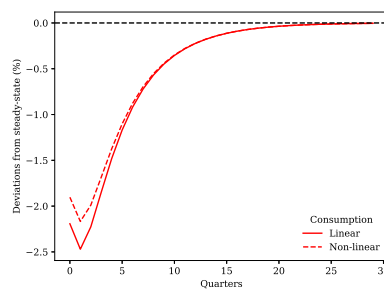
(f) Labor N



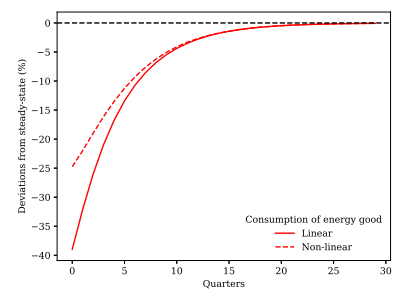
(g) Real Wages w



(h) Consumption C



(i) Household Energy Consumption Y_E

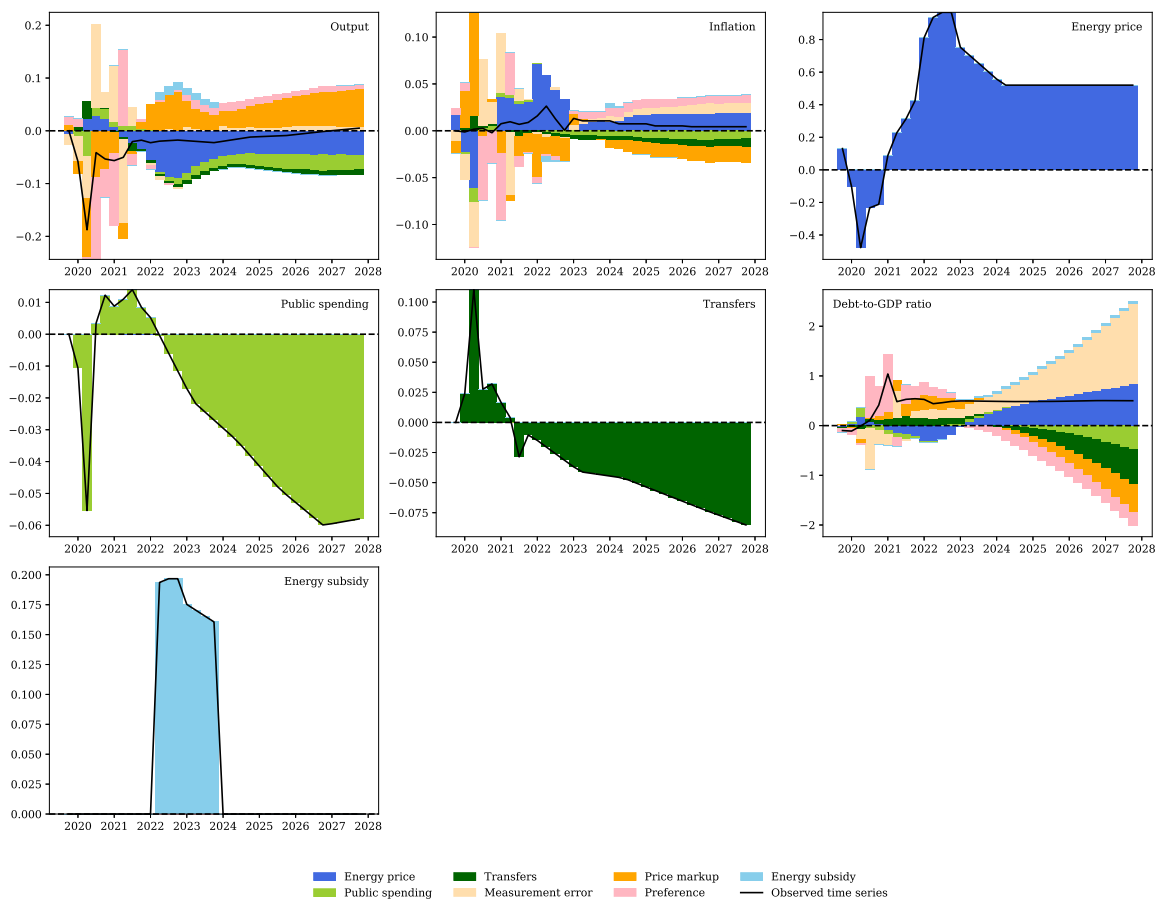


2.8.8 Shock decomposition

Table 2.18: Variance decomposition. For each variable $\{y, \pi, \frac{b}{y}\}$ and each period, the table provides the share deviation from the steady state explained by each shock $\{\beta, \mu, \vartheta, G, T, P_E, s_H\}$

	2022				2023				2024	2025	2026	2027	
	1Q	2Q	3Q	4Q	1Q	2Q	3Q	4Q	2Q	2Q	2Q	2Q	
y	β	0.83	0.11	0.04	0.12	1.24	3.87	4.08	4.19	7.36	2.75	1.24	1.09
	μ	42.09	32.85	34.17	38.80	26.51	22.62	19.92	17.25	27.00	45.77	52.91	59.30
	ϑ	0.41	0.45	0.16	0.08	0.61	0.12	0.11	0.17	0.74	0.58	1.24	0.94
	G	0.01	0.15	0.46	0.84	2.01	3.69	4.84	6.38	8.92	11.74	12.72	10.81
	T	0.27	0.23	0.23	0.25	0.44	0.65	0.66	0.70	0.72	0.92	1.01	1.09
	p_E	56.39	64.11	63.05	58.31	67.44	67.00	68.01	68.67	55.16	38.22	30.88	26.77
	s_H	0.00	2.11	1.90	1.61	1.75	2.06	2.37	2.65	0.10	0.00	0.00	0.00
	π	β	0.54	0.11	0.57	0.03	3.87	24.78	23.49	23.51	25.90	10.96	7.48
μ		29.29	9.10	20.20	32.82	69.36	4.95	0.05	0.03	21.94	28.35	27.71	28.24
ϑ		0.02	0.18	0.25	0.04	8.71	4.66	3.43	3.72	7.06	5.35	10.55	11.91
G		0.00	0.04	0.26	0.75	9.46	16.80	13.93	13.76	9.58	8.25	9.93	8.54
T		0.07	0.18	0.39	0.73	7.37	11.59	8.91	8.17	4.96	4.10	5.42	6.81
p_E		70.08	89.66	78.17	65.61	1.12	37.17	50.18	50.81	30.13	42.99	38.92	37.28
s_H		0.00	0.73	0.14	0.02	0.10	0.05	0.01	0.00	0.44	0.00	0.00	0.00
$\frac{b}{y}$		β	21.06	11.69	13.70	8.01	1.76	2.20	6.75	9.52	13.89	9.52	4.54
	μ	28.78	28.25	28.34	27.86	24.45	8.11	1.46	0.02	0.52	2.78	5.31	6.76
	ϑ	9.48	13.10	14.29	29.20	54.64	71.63	64.67	54.34	43.25	48.70	54.03	58.20
	G	0.35	0.01	0.14	0.85	3.38	3.80	2.10	0.90	0.02	0.62	2.34	4.32
	T	7.84	8.27	9.09	10.04	14.54	7.31	1.77	0.07	1.08	5.34	8.70	10.35
	p_E	32.48	38.56	34.41	24.05	1.07	6.52	22.66	34.44	40.35	32.67	24.92	18.05
	s_H	0.00	0.11	0.03	0.00	0.16	0.41	0.60	0.70	0.87	0.37	0.17	0.08

Figure 2.16: Shock decomposition since 4Q2019

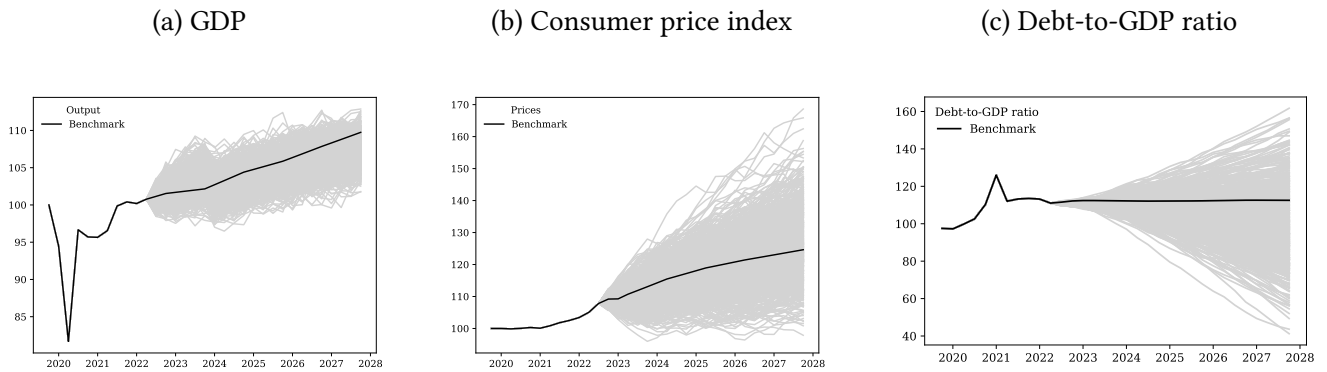


2.8.9 Forecasting

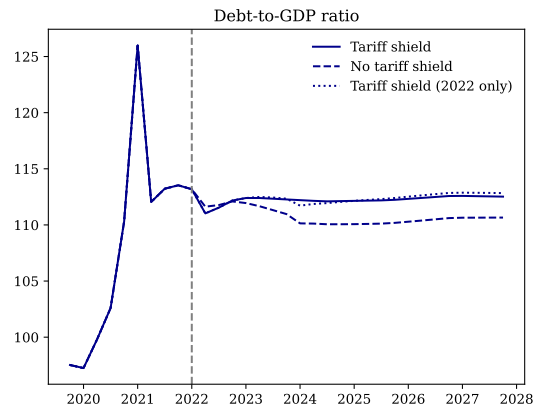
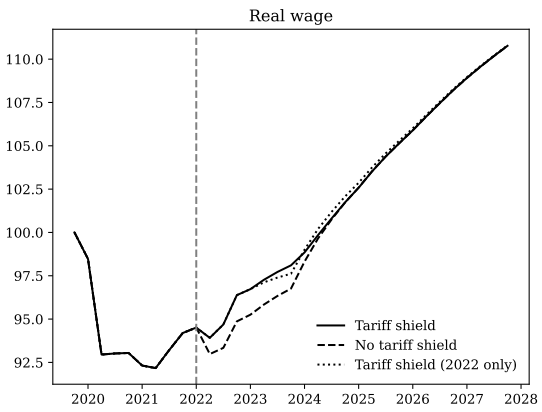
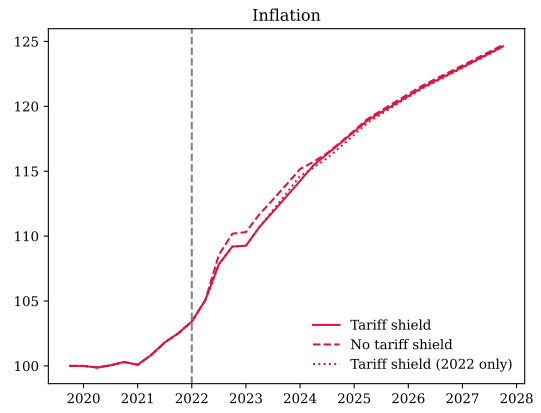
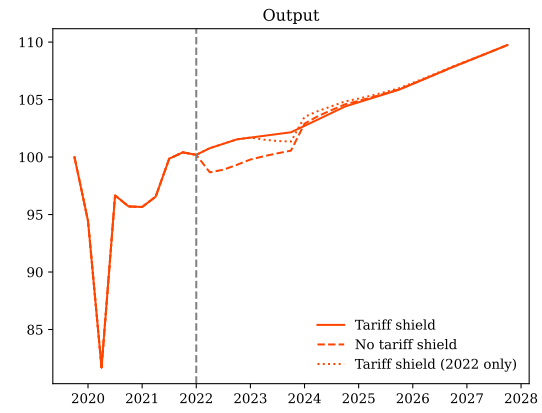
The shocks obtained in the variance decomposition are used as inputs to the model to construct the economy’s response for all macroeconomic variables. Since all shocks are assumed to have normally distributed innovations ($\varepsilon^Z \rightsquigarrow N(0, \sigma_Z^2)$ for $Z = \beta, \mu, \vartheta, G, T, P_E$), we use the standard deviation of these estimated shocks over the sample period 1Q2022-4Q2027 to compute the confidence intervals of the model’s forecasts, under the assumption that the subsidy on energy consumption has no uncertainty.

Firstly, given that the standard deviation of government and transfer innovations (ε^G and ε^T) are small, the large areas of the confidence bands reported in Figure 2.17 underscore that the innovations of the shocks on $\beta, \mu, \vartheta, P_E$ have a large variance, leading to uncertainty in forecasts.

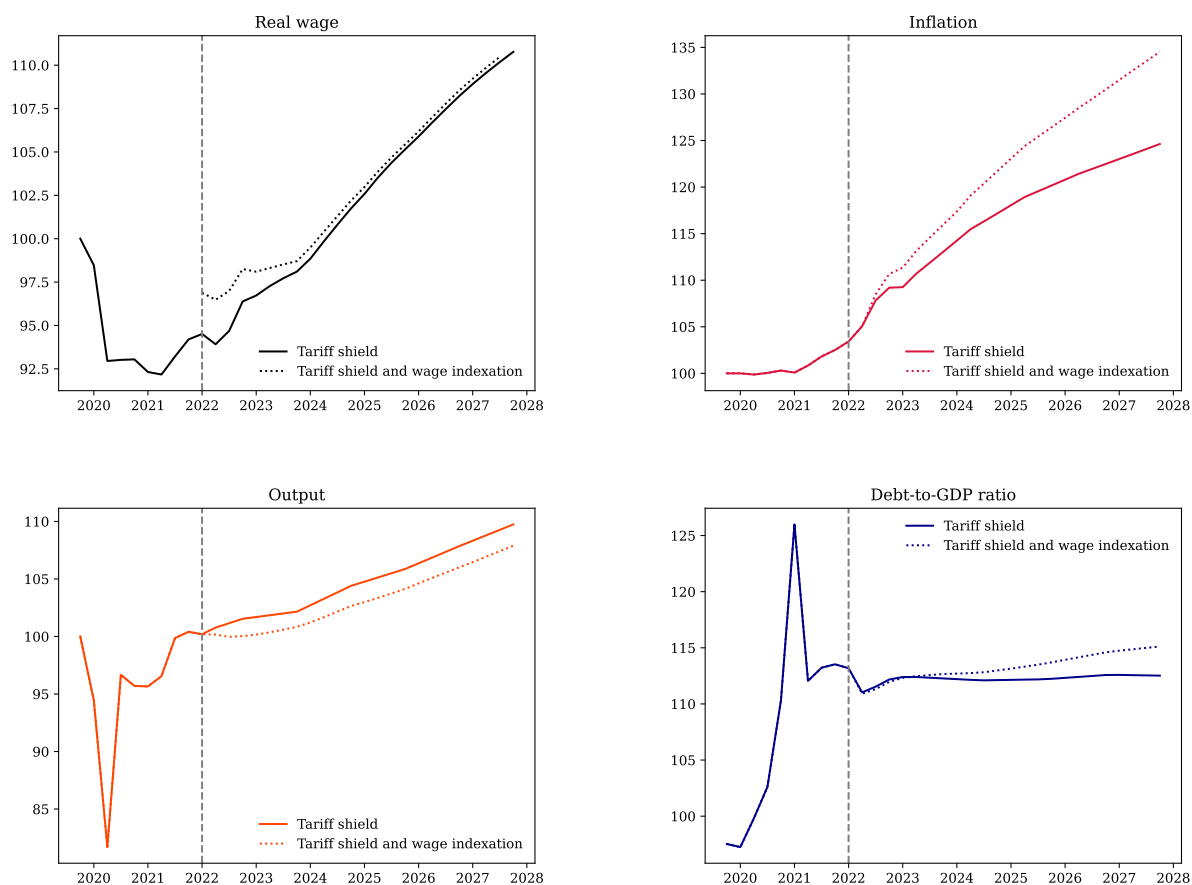
Figure 2.17: Uncertainty on Model’s Forecasts



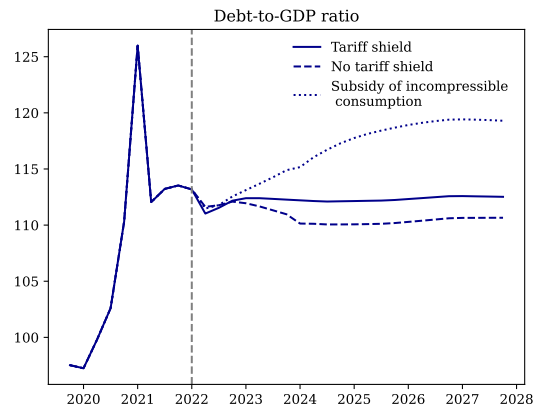
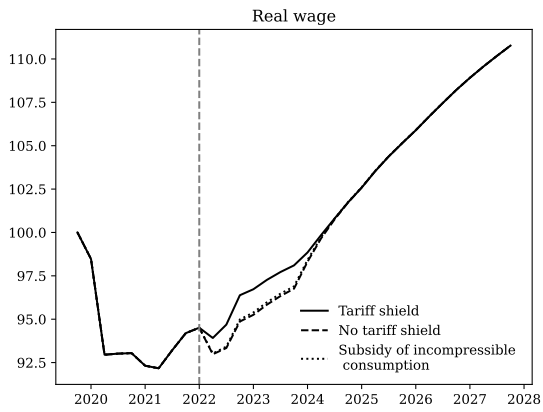
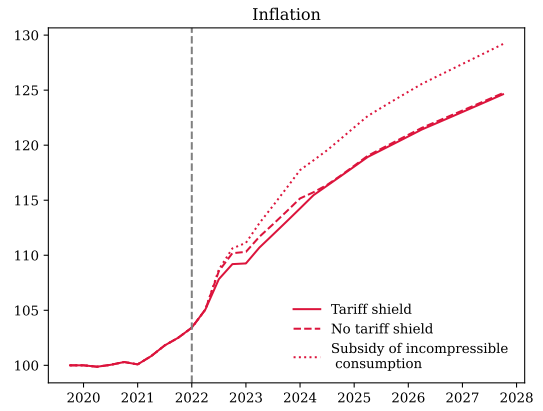
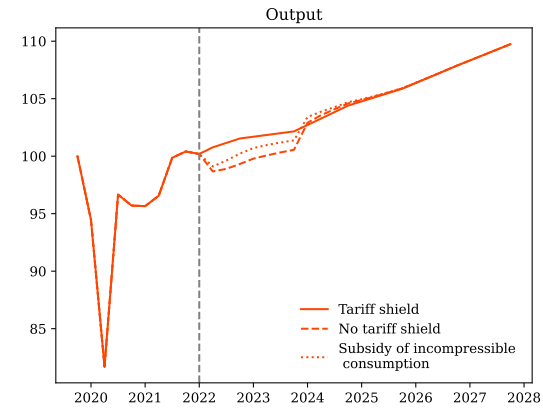
2.8.10 Tariff shield: Aggregates since 4Q2019



2.8.11 Re-activating the price-wage spiral: Aggregates since 4Q2019



2.8.12 Subsidizing incompressible energy consumption: Aggregates since 4Q2019



2.8.13 IRFs by productivity levels

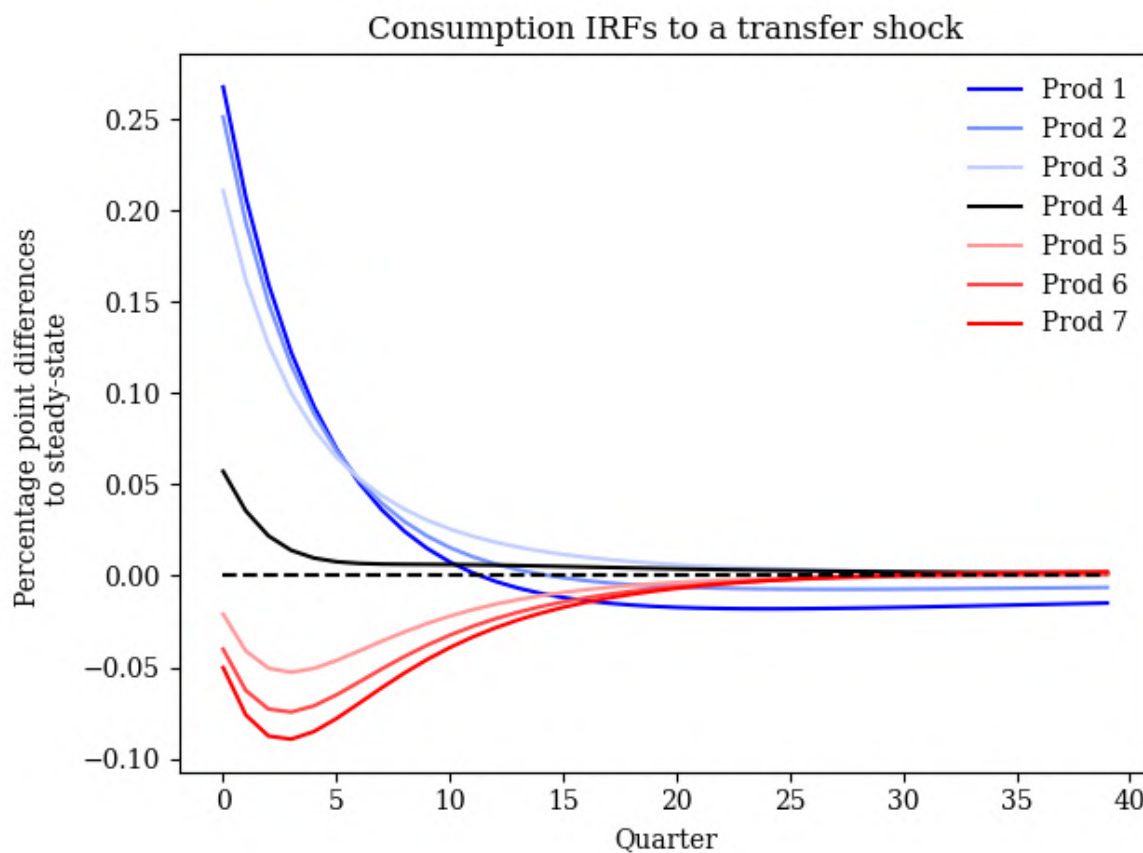
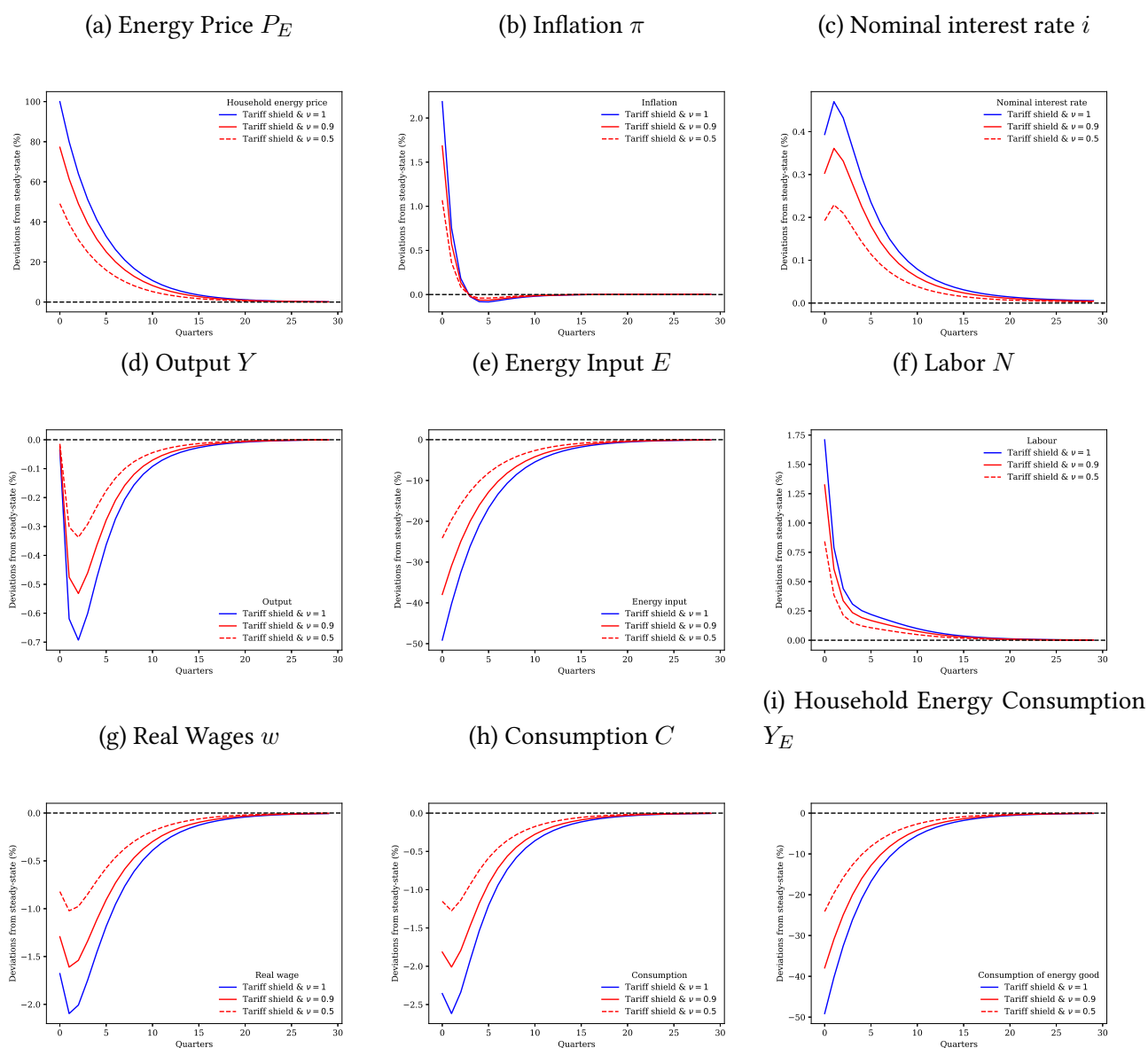


Figure 2.18: Consumption impulse response function of each type of household.

2.8.14 Sensitivity to the Energy Production Function

This appendix presents the IRFs of our model for different values of the parameter ν of the energy production function. In the main text, this parameter is calibrated at 1. However, we demonstrate that for values that remain admissible given the estimates of the quick pass-through of raw energy prices on energy prices ($\nu = 0.9$), the IRFs remain very close to those of our benchmark model. They only begin to diverge significantly for "unrealistic" values such as $\nu = 0.5$.

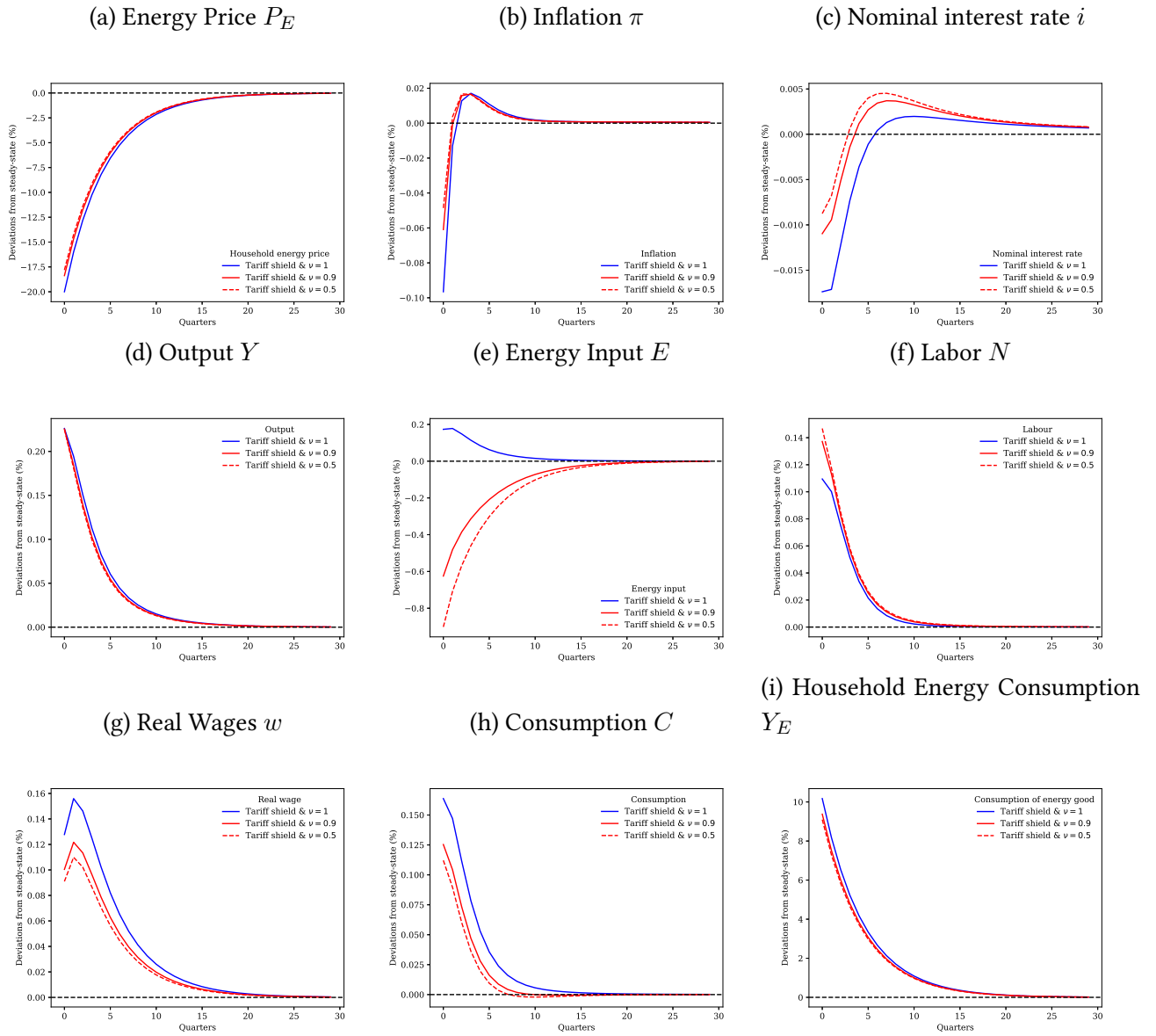
Figure 2.19: IRFs to Energy Shock. Solid blue lines for $\nu = 1$, solid red lines for $\nu = 0.9$, and dotted red lines for $\nu = 0.5$.



Following an energy price shock, Figure 2.19 demonstrates that these alternative calibrations of the energy market generate Impulse Response Functions (IRFs) that are not significantly different from those of our benchmark model. In particular, the introduction

The macroeconomic and redistributive effects of shielding consumers from rising energy prices: a real-time evaluation of the French experiment

Figure 2.20: IRFs to Tariff Shield. Solid blue lines for $\nu = 1$, solid red lines for $\nu = 0.9$, and dotted red lines for $\nu = 0.5$.



of an endogenous energy price ('elastic supply'), which distributes the impact of the shock between a price effect and a quantity effect, does not have a strong repercussion on the other aggregates. This emphasizes that it is the value of energy (price \times quantity) that ultimately matters in decision-making processes.

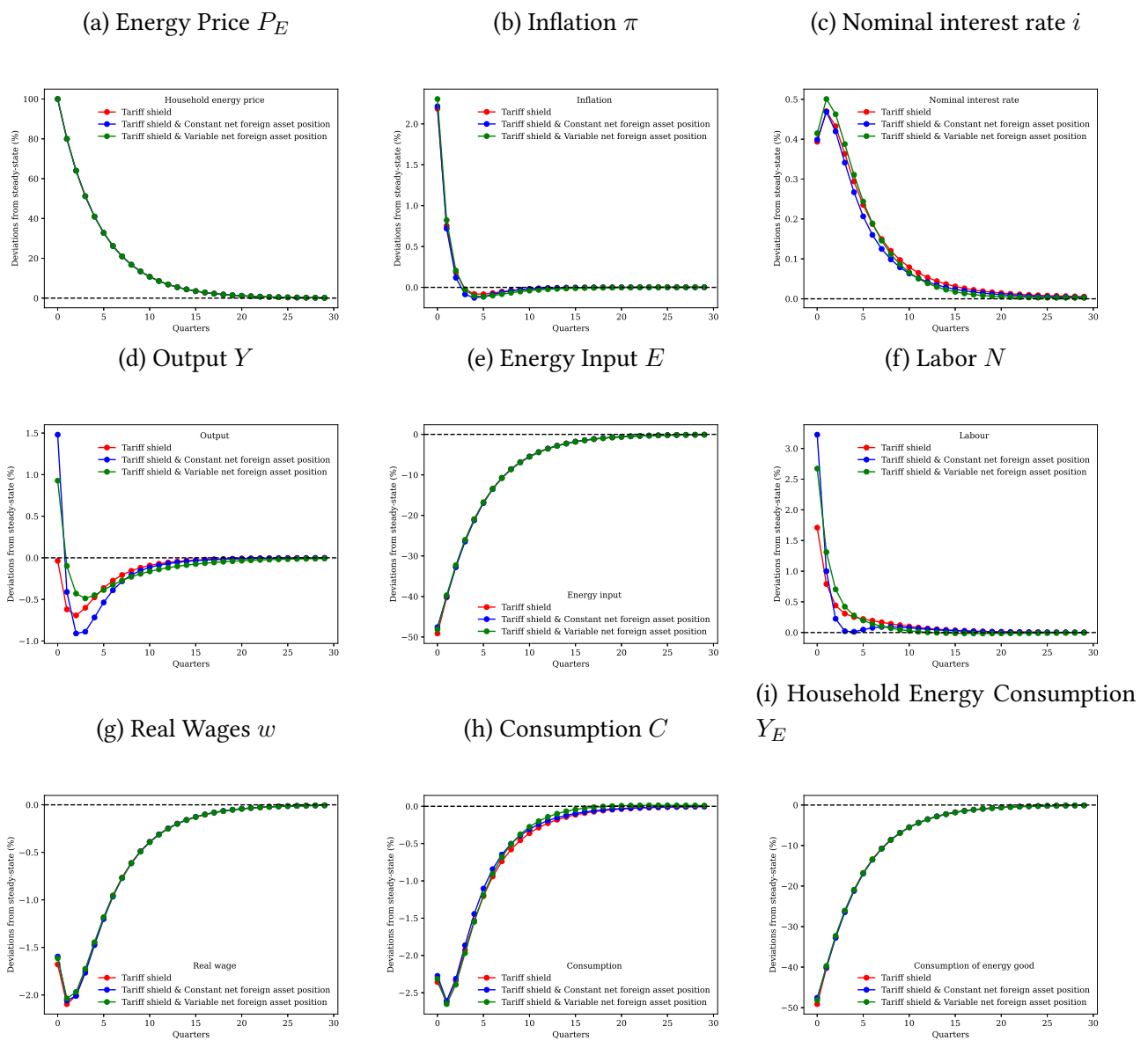
However, what is crucial for our analysis is the comparison of these different models as part of the evaluation of the tariff shield. Figure 2.20 demonstrates that these alternative calibrations of the energy production function generate similar Impulse Response Functions (IRFs) to those of our benchmark model. An endogenous energy price only alters the price-quantity adjustments in the energy market, with all other variables showing the same IRFs as in our baseline model. Therefore, our basic model appears to be more parsimonious without biasing the results of the tariff shield evaluation.

2.8.15 A model with an ad-hoc demand for French exports

In our simplistic modeling of trade, France’s exports to raw energy-producing countries exactly match their income from French purchases of their raw materials. This may appear as a strong restriction. To address this issue, we propose to break this perfect correlation by introducing an international financial asset.

In our benchmark model, energy revenues $p_{Et}\bar{E}_t$ are used each period to purchase French goods, denoted as X_t (exports for France). This results in variations in demand addressed to French firms that exactly mirror variations in the value of imported energy. One way to

Figure 2.21: IRFs to Energy Shock



de-correlate these demand variations from the ones of energy sale revenues is to introduce foreign assets and an endogenous variation of demand for export addressed to French firms.

In this case, the budgetary constraint of foreign agents is given by

$$b_{x,t+1} = (1 + r_t)b_{x,t} - p_{Et}\bar{E}_t + X_t$$

where $b_{x,t}$ represents their net foreign asset position which is a real credit vis-à-vis French economy, $p_{Et}\bar{E}_t$ their revenues from the raw energy sales and X_t their demand addressed to the French firms. For de-correlating X_t from $p_{Et}\bar{E}_t$, several modeling strategies are possible but two of them can summarize the different possible outcomes.

- **Constant net foreign asset position.** In this first, let's assume that foreign households chose the fraction $1 - s_{xt}$ of their revenues $p_{Et}\bar{E}_t$ devoted to demand addressed to French firms such that their real credit vis-à-vis French households stays constant, i.e. $b_{x,t+1} = b_{x,t} \equiv \bar{b}_x, \forall t$. This leads to $X_t = (1 - s_{xt})p_{Et}\bar{E}_t$ with $s_{xt} = \frac{r_t \bar{b}_x}{p_{Et}\bar{E}_t}$. The value of \bar{b}_x is deduced from the steady state restriction $\bar{b}_x = \frac{\bar{s}_x \bar{p}_E \bar{E}}{\bar{r}}$, given a calibration for \bar{s}_x and the steady state values $\{\bar{p}_E, \bar{E}, \bar{r}\}$. Therefore s_{xt} splits the energy income between goods and financial markets. Our benchmark model corresponds to $s_{xt} = 1, \forall t$. For the numerical example, we choose $\bar{s}_x = 0.9$.
- **Variable net foreign asset position.** In this second strategy, let's assume that the expenditures of foreign households are given by $X_t = -\nu_b b_{xt} + \nu_m p_{Et}\bar{E}_t$. This expression can be viewed as a "reduced form" of the decision rule of a foreign household. At the steady state, we have $\bar{b}_x = \frac{1 - \nu_m}{\bar{r} - \nu_b} \bar{p}_E \bar{E}$ given $\{\bar{p}_E, \bar{E}, \bar{r}\}$. When energy revenues transitory rise, then the demand for exports addressed to French firms increases, but less than revenues and with lags. For the numerical example, we choose $\nu_b = 0.15$ and $\nu_m = 0.5$.

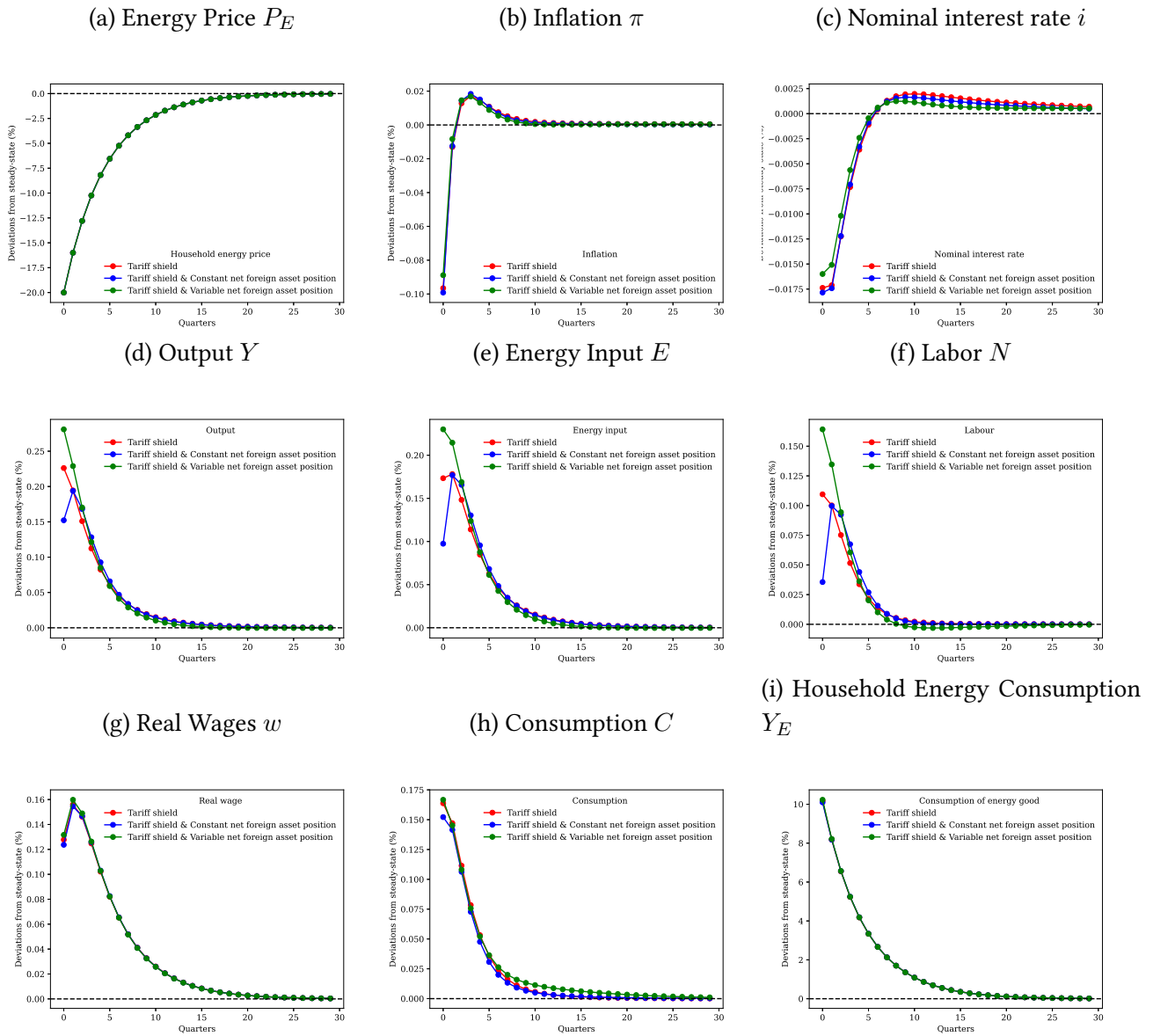
For both models, the asset market equilibrium is $b_t + b_x = \mathcal{A}_t$, whereas on the goods market, the equilibrium is $Y_t \left(1 - \frac{\psi_P}{2} \pi_t^2\right) = \mathcal{C}_t + G_t + X_t$.

Outcomes. After an energy price shock, Figure 2.21 illustrates that these alternative models of French exports generate Impulse Response Functions (IRFs) that are not significantly different from our benchmark model. The introduction of a short-term imbalance in the trade balance helps mitigate the recessionary effect of the increase in energy prices in the first quarter, regardless of the modeling of export demand.

This is because expenditures (exports for French firms) are less sensitive than income (French import of energy), resulting in foreign agents becoming wealthier due to the energy price shock. As a result, this increase in foreign savings reduces the real interest rate, which is perceived as a negative wealth effect by French households. Consequently, they are encouraged to work more, leading to an adjustment that is beneficial for GDP. This adjustment dominates the crowding-out effect induced by a propensity to consume of foreign households that is lower than one, partly because the crowding-out effect on demand is a steady-state argument.

However, what is crucial for our analysis is the comparison of these different models as part of the evaluation of the tariff shield. Figure 2.22 demonstrates that these alternative models of French exports generate similar Impulse Response Functions (IRFs) to those of our

Figure 2.22: IRFs to Tariff Shield



benchmark model. The introduction of a trade balance imbalance only marginally influences the response of GDP during the first 3 quarters. Therefore, our basic model appears to be the most parsimonious without biasing the results of the tariff shield evaluation.

Chapter 3

Fiscal consolidation and debt sustainability in a HANK model

This chapter is co-authored with François Langot, Jocelyn Maillard, Fabien Tripier and Jean-Olivier Hairault.

Keywords: HANK model, Policy evaluation, Fiscal policy, Debt sustainability.

JEL codes: C54, C63, E32, E62, H63, H68.

3.1 Introduction

The succession of large crises (subprime, European sovereign debt, COVID-19, Russian invasion of Ukraine) has led governments to strongly sustain economic activity by raising public debt. Now, the time has come to implement fiscal consolidation programs aimed at reducing public debt. However, experience has shown that many of these programs have failed to achieve their objectives for two primary reasons. Firstly, the recessionary impact of these programs may be so substantial that there is no reduction in the debt-to-GDP ratio (Blanchard and Leigh, 2013; Blanchard and Leigh, 2014, IMF, 2023). Secondly, they tend to exacerbate inequality (Ball et al., 2013, Brinca et al., 2021)), potentially leading to social unrest and electoral outcomes that undermine the continuity of these programs (Brender and Drazen, 2008).¹ These two reasons underscore the need to evaluate fiscal consolidation programs based on a model that considers household heterogeneity. Accurately assessing the macroeconomic impact of these programs is crucial, as it depends significantly on fiscal/budgetary multipliers, which are often inadequately measured in representative agent models (Kaplan, Moll, and Giovanni L. Violante, 2018, Auclert, Bardóczy, and Rognlie, 2023). Moreover, anticipating the distributional effects of these programs is essential, as they impact households differently based on their income and wealth levels.

Therefore, we develop a Heterogeneous-Agent New-Keynesian (HANK) model to assess public debt sustainability and fiscal consolidation programs. This model incorporates

¹See Alesina, Ciminelli, et al., 2021 for a discussion on this issue.

energy in consumption and production as well as heterogeneous levels of education. The first advantage of a HANK model is that it can predict the observed depressive effect of a positive energy-price shock on output (see e.g. Blanchard and Galí, 2007), contrary to a Representative-Agent New-Keynesian (RANK) model, as shown in Auclert, Monnery, et al., 2023.² The second is that it allows us to capture the greater sensitivity of the poorest to many shocks, particularly of energy prices, thanks to an incompressible consumption of energy products.³ We extend the HANK model à la Auclert, Rognlie, and Straub, 2023b to account for heterogeneous responses of labor supply by introducing three types of labor. Therefore, with nominal-wage rigidities resulting from a nominal wage set by a union monopoly specific to each educational group, labor markets react differently depending on income levels. This allows the model to generate differentiated wage dynamics, hours worked by employees, and the number of jobs in each segment as in the data.

Another challenge in evaluating fiscal consolidation programs is defining the economic context within which the program is implemented. Fiscal consolidation not only directly impacts the government's public finances but also affects agent decision rules due to potential structural adjustments. To tackle this challenge, we employ the conditional-forecast method, previously used in VAR models (Waggoner and Zha, 1999, Antolin-Diaz, Petrella, and Rubio-Ramirez, 2021), as well as in New-Keynesian models featuring representative agents (Del Negro and Schorfheide, 2013). Langot, Malmberg, Tripier, and Jean-Olivier Hairault, 2023 extend this approach to HANK models using the sequence-space method introduced by Auclert, Bardóczy, Rognlie, and Straub, 2021.⁴ This method enables us to estimate the sequences of shocks that allow the model to match government forecasts, given a structural model that defines the economy.⁵ By retaining these estimated sequences of shocks in all experiments, we can then compare the implications of different fiscal consolidation programs by immersing them in a strictly identical context. By doing so, we provide a new tool for stochastic debt-sustainability analysis (SDSA) that allows us to compare different fiscal consolidation programs given the same particular initial context as well as the same characteristics of the business cycle that is expected to occur.⁶ The estimation of the distribution of future shocks using conditional forecasts allows us to determine distributions for the future debt trajectory in accordance with Blanchard, Leandro, and Zettelmeyer,

²In a RANK model, the energy shock leads households to substitute energy goods by domestically-produced goods, which counterfactually sustains GDP growth in the home country. For energy shocks in HANK models, see also Bayer, Kriwoluzky, et al., 2023 and Langot, Malmberg, Tripier, and Jean-Olivier Hairault, 2023.

³The model generates a consumption share of the energy decreasing with household incomes as in the data, but also a price elasticity increasing with incomes, making it difficult for the poorest to avoid energy-price increases.

⁴Solutions are obtained thanks to the first-order approximation method developed by Reiter, 2009, (2010).

⁵We take advantage that when a government announces its spending and revenue for the years to come in a Finance Act, it also provides forecasts about economic aggregates (GDP, inflation, interest rates, public debt,...) in order to attest to the budgetary sustainability of its policies. These forecasts are based on a large information and use a mixture of non-structural models and also statistical information and informal knowledge of forecasters. These forecasts dominate those based on a structural model because they use a larger information set and fewer restrictions.

⁶The dynamics of public debt depend on the fiscal situation (specifically, the level of public deficit) and the macroeconomic environment described by the real interest rate (the difference between debt-service rate and inflation) and the economic-growth rate (see Bohn, 1998 and Blanchard, 2019).

2021's recommendations for SDSA.⁷ These assessments are of primary importance for public policy because the assessment of the sustainability of a country's debt partly determines the interest rate at which it borrows, and therefore the budgetary burden of repaying its debt.

Using calibrated parameters for the steady state and estimated parameters for the shocks' processes over the 2Q2003-4Q2019 period, our HANK model for the French economy identifies the sequences of each structural shock allowing it to reproduce government forecasts (output, inflation, interest rates, hours worked, wages and public debt) conditionally on government policies (expenditures and revenues) and energy prices for the future period up to 4Q2027. The challenge for conditional forecasts, when applied to economic policies, is that they may be exposed to the R. Lucas, 1976 critique. To tackle this problem, we implement a stability test of policy rules and show how to incorporate a possible shift in policy into the evaluation method. Our estimations reveal that only government decision rules are unstable between the pre and after-Finance Act periods. It suggests that after twenty years of rising public debt in France, a new fiscal policy will be implemented by the government to prevent a future debt increase. Based on this method, our assessment indicates that strong firm-markup reductions (large and persistent negative markup shocks) coupled with large increases in labor supplies (strong and persistent negative shocks on the disutility of working) are necessary for government forecasts to materialize. Thus, this very particular scenario suggests that there is a large uncertainty surrounding the 2024 Finance Act and also underlines that the government lets the stabilization of its debt depend on favorable supply shocks, rather than on adjustments in its spending and revenues. Even in this favorable context, the debt-to-GDP ratio is reduced by only 2.3pp (from 110.6 to 108.3%).

Without penalizing economic growth or exacerbating inequality, could France reduce its indebtedness more sharply than forecasted in the Finance Act? Using the structural shocks estimated through conditional forecasts, we can compare the future trajectories of the French economy according to several fiscal consolidation rules that lead to a reduction of expenditures by €20 billion/year until 2027 in the same economic context.⁸ We show that a strategy based on deeper cuts in public consumption expenditure would not be effective because of the recession it would cause. This economic slowdown would disproportionately affect the most disadvantaged households, heavily reliant on earned income, compared to the more affluent households, who also receive financial income. As such, it would slightly increase consumption inequalities. A strategy based on homogeneous deeper cuts in public transfers has small detrimental effects on economic growth, as the increase of low and middle-skill labor supply almost compensates for the decrease in transfers. It strongly exacerbates inequalities as low-skill workers own little assets and need those transfers to be able to consume. However, we show that if the reduction in public transfers only concerns

⁷A key difference between the tool of SDSA proposed in this article and the one developed in institutions (such as the European Commission, see Bouabdallah et al., 2017) is that the distributions of shocks we are using are based on the estimation of a general-equilibrium model and not on econometric models without economic structure.

⁸A reduction of €20 billion per year represents a decrease in expenditures (public spending and transfers) from 49.6% of GDP to 48.9%. The head of the Court of Auditors, Pierre Moscovici, calls for a cut of between €50 and €60 billion in total over the next three French government budgets.

transfers based on social-security contributions (i.e. Bismarckian type), and if assistance transfers (i.e. Beveridgian type) are increased with respect to the 2024 Finance Act, then it is possible to achieve a larger reduction in the debt-to-GDP ratio (5.0pp) with a stronger GDP and lower consumption inequalities than in the 2024 Finance Act. Indeed, reducing Bismarckian-type insurance transfers strongly stimulates labor supply across all household categories and consequently boosts economic growth. Besides, increasing Beveridge-type assistance transfers, supports consumption among the most disadvantaged households, resulting in lower consumption inequalities.⁹

Beyond this expected debt trajectory, our approach also makes it possible to extend the SDSA to HANK models: the uncertainty over GDP growth, inflation, and interest rates determines the distribution of debt trajectories, all verifying the general equilibrium and therefore taking into account all interactions between the government and market actors. Our approach provides an assessment of the risks weighing on public debt as defined by Blanchard, Leandro, and Zettelmeyer, 2021 for who a SDSA “*would generate a distribution of paths of the debt ratio (sometimes called a ‘fan chart’), based on forecasts for the drivers of the debt dynamics, which are themselves stochastic. (...) The forecasts would also take into account the policy intentions of the authorities, as well as the interactions between growth and fiscal policies. The result would be a distribution for the debt ratio n years out (...) conditional on expected policies (but allowing for uncertainty about these policies would affect the economy).*” The Government Finance Act provides projections that are not accompanied by any measure of risk surrounding this projection. Our method therefore allows us first to determine the distribution of the debt-to-GDP ratio given the projections in the Finance Act and our estimated HANK model. The median value of the distribution of the debt-to-GDP ratio in 2027 is 104.7%, below the forecast value of 108.3%, and there is 25% probability of exceeding the value of 116.2% at this horizon. Then, we can assess the benefits of alternative policies in terms of reduction in the risk of debt increase. Under the policy described above, based on a reallocation of transfers, the risk of debt increase falls by 3.6pp: there is a 25% probability of exceeding 112.6% under this policy against 116.2% following the Finance Act.

Literature. Our paper contributes to several strands of the literature. We contribute to the literature on the quantitative analysis of HANK models. Following Kaplan, Moll, and Giovanni L. Violante, 2018 and Auclert, Rognlie, Souchier, et al., 2021 that have demonstrated the empirical performance of these HANK models and their relevance for macroeconomic policy evaluations¹⁰, a set of papers illustrate how to apply these models to macroeconomic policy analysis. Auclert, Rognlie, and Straub, 2023b provide a complete analysis of fiscal and monetary multipliers in HANK models and discuss the advantages of these models compared to usual RANK models. Auclert, Monneray, et al., 2023, Pieroni, 2023, Bayer, Kriwoluzky, et al., 2023 and Langot, Malmberg, Tripier, and Jean-Olivier Hairault, 2023 use

⁹Using model-based simulations, we also provide a decomposition of debt-to-GDP ratio dynamics between partial and general-equilibrium effects.

¹⁰These works follow Aiyagari, 1994 and Krusell and Smith, 1998, and develop models with heterogeneous agents which also included market frictions, as price and wage rigidities, relevant for business cycle analysis.

HANK models to analyze the impact of the recent energy crisis.¹¹ Our paper extends these analyses by estimating a HANK model to explain the business cycle and inequality dynamics of the French economy which is a member of a monetary union. We also contribute to the literature on policy evaluations based on conditional forecasts by identifying the structural shocks of a HANK model using the conditional-forecast method and data provided by a government’s Finance Act forecasts, taking advantage of the solution method provided in Auclert, Bardóczy, Rognlie, and Straub, 2021’s sequence-space Jacobian.¹² Finally, we contribute to the literature on fiscal consolidation programs, which has been extensively studied empirically (e.g. Ball et al., 2013; Blanchard and Leigh, 2013; Alesina, Favero, and Giavazzi, 2015; IMF, 2023), in New-Keynesian models with representative agents (e.g. Erceg and Lindé, 2013) and by Brinca et al., 2021 in a neoclassical macro-model with heterogeneous agents and incomplete insurance markets. As Brinca et al., 2021, we analyze the incidence of fiscal consolidation on output and inequality, but in an estimated model that takes into account nominal rigidity and monetary policy.

The remainder of the article is organized as follows. Section 3.2 presents the model. Section 3.3 describes the quantitative methodology and the estimation results. Section 3.4 presents the method of conditional forecasts and quantitative results are provided in Section 3.5. Section 3.5.1 is devoted to the stability test of fiscal policies. Section 3.5.2 discusses the debt sustainability implied by the Finance Act and Section 3.5.3 the induced path of inequalities. Section 3.5.4 assesses alternative fiscal consolidation programs. Section 3.5.5 quantifies the level of risk associated with the debt-to-GDP ratio in the Finance Act and shows how it can be reduced through alternative fiscal consolidation programs. Finally, Section 3.6 concludes.

3.2 Model

This model extends the one presented in Chapter 2.2 by adding permanent skill heterogeneity on top of idiosyncratic income risk. Households can now be either low, medium, or highly skilled. Three unions are introduced, that set the wage of each skill category. The model thus features three labor markets, which can exhibit differentiated dynamics.

Another addition is a more detailed fiscal policy in the model. Instead of a flat tax on labor income, households now face a progressive income tax *à la* Heathcote, Storesletten, and Giovanni L. Violante, 2017, on top of a flat tax on labor income that represents social security contributions. Dividends received by the firms are also taxed –according to another

¹¹In Auclert, Monneray, et al., 2023, there is no incompressible consumption and no public debt dynamics, whereas Pieroni, 2023 introduces incompressible consumption, but not public debt dynamics. These two papers present theoretical analyses of hypothetical policies. The analysis of Bayer, Kriwoluzky, et al., 2023 does not provide an estimation of the model based on time series of aggregates.

¹²Many methods have been developed to easily use HANK models. Achdou et al., 2022 have proposed an approach based on forward-backward equations (See e.g. Kaplan and Giovanni L. Violante, 2018). In discrete time, Reiter, 2009, (2010), Winberry, 2018 and Bayer and Luettkicke, 2020 have developed methods to improve the accuracy and resolution speed. The Auclert, Bardóczy, Rognlie, and Straub, 2021 approach integrates a set of tools allowing us (i) to compute the dynamic responses to aggregate shocks, (ii) to check the stability of the dynamics and (iii) to estimate parameters and shock realizations.

flat rate– which is a proxy for corporate tax.

Finally, the transfers that households receive from the government are now broken down between social-security contributions (or Bismarckian transfers) and assistance (or Beveridgian) transfers. This corresponds to pensions and unemployment benefits, which are proportional to income, and the rest of transfers (poverty, family...), which decrease with income.

A set of parameters of the model are thus calibrated to reproduce some stylized facts on the three labor markets, as well as the more detailed fiscal system. Those parameters, their values, and the associated moment they try to match are reported in Table 3.2.

3.2.1 Households

There are three types of household skills $s \in \{l, m, h\}$: low (l), medium (m), and high, (h). Within each skill group, households experience idiosyncratic productivity shocks that will take values $e' \in E$ conditionally to a current value $e \in E$. The transition matrix between productivity levels is $\mathcal{P}(e, e')$.

Household decision rules are independent of skill-type s . Hence, we drop the s subscript. These decisions are deduced from the following maximization problem

$$V_t(e, a_-) = \max_{c, a \geq 0} \left\{ u(c) - v(n) + \beta_t \sum_{e'} \mathcal{P}(e, e') V_{t+1}(e', a) \right\}$$

$$s.t. \quad a_t = a_{t-1} + y_t(a, e) - (1 + \tau_c)[c_t + (1 - s_{H,t})p_{E,t}c_E]$$

where the subsistence level for the energy consumption is c_E , s_H stands for a tariff shield, and p_E is the relative price of energy. The discount factor β_t changes over time to account for demand shocks and is assumed to follow a stationary $AR(1)$ process. The net income $y(a, e)$ and the taxable labor income net of social contributions $z(e)$ are defined as follows

$$y_t(a, e) = r_t a_{t-1} + (1 - \tau_f) d_t \bar{d}(e) + (1 - \mu) z_t(e)^{1-\lambda} + \tau_t \bar{\tau}(e)$$

$$z_t(e) = (1 - \tau_l) w_t e_t n_t + T_t \bar{T}(e)$$

where $1 + r_t = \frac{1+i_{t-1}}{1+\pi_t}$, $\pi_t = \frac{P_t}{P_{t-1}} - 1$ and dividends d are distributed non-uniformly across households according to the function $\bar{d}(e) > 0$. The tax rate on firms' profits is τ_f . The social contributions are proportional to labor incomes, with a contribution rate τ_l . A first transfer, T , is indexed to labor income and taxable (pensions and unemployment benefits), the indexation being defined by $\bar{T}(e) > 0$. It is thus the Bismarckian part of transfers. The second transfers, τ , is larger for poorer households, the indexation being given by $\bar{\tau}(e) < 0$. It corresponds to the Beveridgian part of transfers. The changes in T and τ follow stationary $AR(1)$ processes, and the government can also modify the distributions of these two transfers. Total progressive taxes are $\mathcal{T}_{I,t}(e) = z_t(e) - (1 - \mu)(z_t(e))^{1-\lambda}$, where λ determines the degree of progressivity. A tax scheme is commonly labeled progressive (regressive) if the ratio of marginal to average tax rates ($\frac{\mathcal{T}'_I(z)}{\mathcal{T}_I(z)/z} = \frac{1-(1-\mu)(1-\lambda)z^{-\lambda}}{1-(1-\mu)z^{-\lambda}}$) is larger (smaller) than one for every level of income z . Note that the progressive taxes become transfers if $\mathcal{T}_I(z) < 0$ i.e. when $z < (1 - \mu)^{\frac{1}{\lambda}}$.

We assume that $u(c) = \log(c)$, $v(n) = \varphi_{s,t} \frac{n^{1+\nu}}{1+\nu}$. The labor disutility parameters $\varphi_{s,t}$, which are skill specific, change over time in order to account for labor supply shocks and are assumed to follow a stationary $AR(1)$ process. Households buy at price P a basket c composed by differentiated but imperfectly substitutable goods c_i given by $c_t = \left(\int c_{i,t}^{\frac{\varepsilon_d-1}{\varepsilon_d}} di \right)^{\frac{\varepsilon_d}{\varepsilon_d-1}}$, where ε_d measure the elasticity of substitution between goods i . Each of these differentiated goods c_i is a basket given by

$$\begin{aligned} c_{i,t} &= \left(\alpha_E^{\frac{1}{\eta_E}} (c_{iE,t} - \underline{c}_E)^{\frac{\eta_E-1}{\eta_E}} + (1 - \alpha_E)^{\frac{1}{\eta_E}} (c_{iH,t})^{\frac{\eta_E-1}{\eta_E}} \right)^{\frac{\eta_E}{\eta_E-1}} \\ &= \left(\alpha_E^{\frac{1}{\eta_E}} (\tilde{c}_{iE,t})^{\frac{\eta_E-1}{\eta_E}} + (1 - \alpha_E)^{\frac{1}{\eta_E}} (c_{iH,t})^{\frac{\eta_E-1}{\eta_E}} \right)^{\frac{\eta_E}{\eta_E-1}} \end{aligned}$$

where c_{iE} is the energy consumption good, which should exceed the subsistence level \underline{c}_E , and c_{iH} the domestically produced consumption good. The basket c_i satisfies the household's preferences if $c = p_H c_H + (1 - s_H) p_E (c_E - \underline{c}_E)$, i.e. $c + (1 - s_H) p_E \underline{c}_E = p_H c_H + (1 - s_H) p_E c_E$ with $p_H = P_H/P$ and $p_E = P_E/P$.¹³

3.2.2 Unions

For each type $s \in \{l, m, h\}$, a union sets a unique wage by task k whatever the levels of productivity $e \in E$ and wealth $a \in A$. The union's program for the skill group s is

$$\begin{aligned} U_{k,t}^s(W_{k,t-1}^s) &= \max_{W_{k,t}^s} \left\{ \int_e \int_a [u(c_{i,t}^s(e_i, a)) - v(n_{i,t}^s(e_i, a))] d\Gamma^s \right. \\ &\quad \left. - \frac{\psi_W^s}{2} \left(\frac{W_{k,t}^s}{W_{k,t-1}^s} - 1 \right)^2 + \beta U_{k,t+1}^s(W_{k,t}^s) \right\} \\ \text{s.t. } N_{k,t}^s &= \left(\frac{W_{k,t}^s}{W_t^s} \right)^{-\varepsilon^s} N_t^s \end{aligned}$$

where $W_t^s = \left(\int_k (W_{k,t}^s)^{1-\varepsilon^s} dk \right)^{\frac{1}{1-\varepsilon^s}}$ and ψ_W^s is the wage adjustment cost parameter. Note that Γ^s is the distribution of workers with skill s , which sums to 1. As a result, the total equilibrium distribution satisfies $\sum_s \omega^s \int_e \int_a d\Gamma^s = 1$ with ω^s , where ω^s is the fraction of s -type in the population. The unions' decisions for the nominal wages lead to the NKPCs:

$$\pi_{W,t}^s = \kappa_W^s \left(N_t^s v'(N_t^s) - \frac{1}{\mu_w^s} t d_t^s \frac{W_t^s}{P_t} N_t^s \tilde{u}'_t(c^s, \mathcal{T}^s) \right) + \beta \pi_{W,t+1}^s$$

with tax distortion $t d_t^s = \frac{(1-\mu)(1-\tau_l)}{1+\tau_c}$, union markup $\mu_w^s = \frac{\varepsilon_s}{\varepsilon_s-1}$ and wage rigidity parameters $\kappa_w^s \equiv \frac{\varepsilon^s}{\psi_W^s}$ that are specific to each s . Remark that the average value of a net wage increase in terms of consumption units, $\tilde{u}'_t(c^s, \bar{\mathcal{T}}^s) \equiv \int_e \int_a u'(c^s) e (1 - \bar{\mathcal{T}}^s) d\Gamma^s(e, a)$ with $\bar{\mathcal{T}}^s$ the average tax rate, is deduced from the households behaviors and computed at the equilibrium.¹⁴

¹³The intratemporal households' choices are managed by the firms that create final goods by combining home goods and energy services satisfying the households' preferences. This allows us to introduce a Phillips curve on the consumer price index (CPI) via an adjustment cost on price adjustment paid by the retailers.

¹⁴See Appendix 3.7.1 for details on the derivation of the Phillips curve.

3.2.3 Goods Supply

Basic-good producers produce Y_N using only labor and minimize their production costs

$$\begin{aligned} & \min_{n_{i,t}^l, n_{i,t}^m, n_{i,t}^h} \{W_t^l N_t^l + W_t^m N_t^m + W_t^h N_t^h\} \\ \text{s.t.} \quad & \begin{cases} Y_{N,t} \leq \left(\alpha_l^{\frac{1}{\varepsilon_N}} (A_t^l N_t^l)^{\frac{\varepsilon_N-1}{\varepsilon_N}} + \alpha_m^{\frac{1}{\varepsilon_N}} (A_t^m N_t^m)^{\frac{\varepsilon_N-1}{\varepsilon_N}} + \alpha_h^{\frac{1}{\varepsilon_N}} (A_t^h N_t^h)^{\frac{\varepsilon_N-1}{\varepsilon_N}} \right)^{\frac{\varepsilon_N}{\varepsilon_N-1}} \\ N_t^s = \sum_i \omega^s \pi_i^s e_{i,t}^s n_{i,t}^s \quad \forall s \in \{l, m, h\} \end{cases} \end{aligned}$$

where $\sum_i \pi_i^s e_i^s = \varpi^s$ is the average productivity of each population and A^s a s -type productivity shock assumed to follow a stationary AR(1) process $\forall s \in \{l, m, h\}$. The optimal labor demands are

$$\begin{aligned} N_t^s &= \frac{\alpha_s}{A_t^s} \left(\frac{W_t^s / (A_t^s \varpi^s)}{MC_{N,t}} \right)^{-\varepsilon_N} Y_{N,t} \\ \text{with } MC_{N,t} &= \left(\sum_s \alpha_s \left(\frac{W_t^s}{A_t^s \varpi^s} \right)^{1-\varepsilon_N} \right)^{\frac{1}{1-\varepsilon_N}} \quad \forall s \in \{l, m, h\} \end{aligned}$$

As all s -type employees work the same number of hours (Unions), then $n_{i,t}^s = n_{i',t}^s \equiv n_t^s$, $\forall i, i'$. After normalizing $n_t^s = 1$ (α_s are found to match this restriction),¹⁵ $N_t^s = \sum_i \omega^s \pi_i^s e_{i,t}^s$, knowing $\sum_s \omega_s = 1$, where π_i^s is the mass of workers of skill s and productivity e_i^s . Assuming perfect competition leads to:

$$\begin{aligned} \Pi_{N,t} = (W_t - MC_{N,t})Y_{N,t} = 0 &\Rightarrow W_t = MC_{N,t} \\ \Leftrightarrow w_t = mc_{N,t}, \text{ with } \begin{cases} w_t &= \frac{W_t}{P_t} \\ mc_{N,t} &= \frac{MC_{N,t}}{P_t} \end{cases} \end{aligned}$$

Intermediate-good producers produce Y_H with energy E and basic goods Y_N while minimizing their production costs

$$\min_{E_t, Y_{N,t}} \{W_t Y_{N,t} + P_{FE,t} E_t\} \quad \text{s.t.} \quad Y_{H,t} \leq Z_t \left(\alpha_f^{\frac{1}{\sigma_f}} E_t^{\frac{\sigma_f-1}{\sigma_f}} + (1 - \alpha_f)^{\frac{1}{\sigma_f}} Y_{N,t}^{\frac{\sigma_f-1}{\sigma_f}} \right)^{\frac{\sigma_f}{\sigma_f-1}}$$

The optimal demands of production factors are:

$$Y_{N,t} = (1 - \alpha_f) \left(\frac{W_t}{MC_{H,t}} \right)^{-\sigma_f} Y_{H,t}, \quad E_t = \alpha_f \left(\frac{P_{FE,t}}{MC_{H,t}} \right)^{-\sigma_f} Y_{H,t}$$

with a marginal cost defined as follows

$$MC_{H,t} = Z_t^{-\frac{1}{\sigma_f}} \left(\alpha_f (P_{FE,t})^{1-\sigma_f} + (1 - \alpha_f) W_t^{1-\sigma_f} \right)^{\frac{1}{1-\sigma_f}}$$

¹⁵See Appendix 3.7.2 for more details

Assuming perfect competition leads to:

$$\begin{aligned} \Pi_{H,t} = (P_{H,t} - MC_{H,t})Y_{H,t} = 0 &\Rightarrow P_{H,t} = MC_{H,t} \\ \Leftrightarrow p_{H,t} = mc_{H,t}, &\text{ with } \begin{cases} p_{H,t} = \frac{P_{H,t}}{P_t} \\ mc_{H,t} = \frac{MC_{H,t}}{P_t} \end{cases} \end{aligned}$$

Final-good producers combine goods in order to satisfy the households' preferences. They minimize their production costs

$$\begin{aligned} \min_{Y_{H,t}, Y_{FE,t}} &\{P_{H,t}Y_{H,t} + (1 - s_{H,t})P_{FE,t}Y_{FE,t}\} \\ \text{s.t. } Y_{F,t} &\leq \left(\alpha_E^{\frac{1}{\eta_E}} (Y_{FE,t})^{\frac{\eta_E-1}{\eta_E}} + (1 - \alpha_E)^{\frac{1}{\eta_E}} (Y_{H,t})^{\frac{\eta_E-1}{\eta_E}} \right)^{\frac{\eta_E}{\eta_E-1}} \end{aligned}$$

The optimal decisions satisfy

$$Y_{FE,t} = \alpha_E \left(\frac{(1 - s_{H,t})P_{FE,t}}{MC_{F,t}} \right)^{-\eta_E} Y_{F,t}, \quad Y_{H,t} = (1 - \alpha_E) \left(\frac{P_{H,t}}{MC_{F,t}} \right)^{-\eta_E} Y_{F,t}$$

with a marginal cost defined as follows

$$MC_{F,t} = \left(\alpha_E ((1 - s_{H,t})P_{FE,t})^{1-\eta_E} + (1 - \alpha_E) (P_{H,t})^{1-\eta_E} \right)^{\frac{1}{1-\eta_E}}$$

Assuming perfect competition leads to:

$$\begin{aligned} \Pi_{F,t} = (P_{F,t} - MC_{F,t})Y_{F,t} = 0 &\Rightarrow P_{F,t} = MC_{F,t} \\ \Leftrightarrow p_{F,t} = mc_{F,t}, &\text{ with } \begin{cases} p_{F,t} = \frac{P_{F,t}}{P_t} \\ mc_{F,t} = \frac{MC_{F,t}}{P_t} \end{cases} \end{aligned}$$

Retailer i produces according to a linear production function, $Y_{i,t} = Y_{i,t,F}$, an imperfectly substitutable good. The households and the government have the same preferences

and thus their baskets, respectively c and G , are defined by $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\varepsilon_{d,t}-1}{\varepsilon_{d,t}}} di \right)^{\frac{\varepsilon_{d,t}}{\varepsilon_{d,t}-1}}$,

for $Y = c, G$. We assume the elasticity of substitution across goods changes over time, i.e. $\varepsilon_{d,t}$ depends on t . These variations in $\varepsilon_{d,t}$ lead to price-markup shock (a disturbance to the desired markup of retailers' prices over their marginal costs) because the markup is given by $\mu_t = \frac{\varepsilon_{d,t}}{\varepsilon_{d,t}-1}$. We assume that μ_t follows a stationary $AR(1)$ process. The i -retailer's sets its price (Monopolistic competition) to maximize its profits

$$\begin{aligned} \Pi(P_{i,t-1}) = \max_{P_{i,t}} &\left\{ \frac{P_{i,t} - P_{F,t}}{P_t} y_{i,t} - \frac{\psi_P}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 Y_t + \frac{1}{1 + r_{t+1}} \Pi(P_{i,t}) \right\} \\ \text{s.t. } y_{i,t} &= \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon_d} Y_t \end{aligned}$$

This leads to the following NKPC:

$$\pi_t = \kappa_P \left(mc_t - \frac{1}{\mu_t} \right) + \frac{1}{1 + r_{t+1}} \frac{Y_{t+1}}{Y_t} \pi_{t+1}$$

with $mc_t = \frac{P_{F,t}}{P_t}$ and $\kappa_P = \frac{\varepsilon_d}{\psi_P}$. The firm's profit (its dividends) is defined by

$$D_t = P_t Y_t - P_{F,t} Y_{F,t} - \frac{\psi_P}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 P_t Y_t,$$

knowing that with a linear production, we have $Y_t = Y_{F,t}$.

3.2.4 Central Bank

The central bank, here the ECB, follows a monetary rule:

$$i_t^* = \rho_r i_{t-1}^* + (1 - \rho_r) (i_{ss}^* + \phi_\pi \pi_t^{EU}) + \tilde{\varepsilon}_t$$

with the European inflation defined as

$$\pi_t^{EU} = \mu_{FR} \pi_t + (1 - \mu_{FR}) \pi_t^{REU} \quad \text{where} \quad \pi_t^{REU} = \rho_\pi \pi_t + \pi_t^{REU*}$$

where π_t^{REU} is the inflation in the rest of the Euro area, μ_{FR} the share of the French economy in the Euro area, and π_t^{REU*} the uncorrelated component of EU inflation with French inflation (an *iid* process by assumption). Therefore, the "effective" Taylor rule for the French economy is

$$\begin{aligned} i_t^* &= \rho_r i_{t-1}^* + (1 - \rho_r) (i_{ss}^* + \phi_\pi (\mu_{FR} + (1 - \mu_{FR}) \rho_\pi) \pi_t) + \varepsilon_t \\ \text{with } \pi_t &= \frac{P_t}{P_{t-1}} - 1 \quad \text{and} \quad \varepsilon_t = \tilde{\varepsilon}_t + \phi_\pi (1 - \rho_r) (1 - \mu_{FR}) \pi_t^{REU*} \sim AR(1) \end{aligned}$$

3.2.5 Interest rate and risk premium

The interest rate decided by the central bank i_t^* may differ from the effective interest rate on the French government debt. Let us define the effective nominal interest rate

$$i_t = i_t^* + \vartheta_t$$

where ϑ_t is an exogenous wedge that can be either positive (due to risk premium) or negative (due to the maturity composition of government debt). We assume that ϑ_t follows a stationary *AR*(1) process. The Fisher rule leads to $1 + r_t = \frac{1+i_{t-1}}{1+\pi_t} = \frac{1+i_{t-1}^*+\vartheta_{t-1}}{1+\pi_t}$.

3.2.6 Government

Government revenues and expenditures are denoted respectively R_t and D_t . Public debt (B_t) finances the differences:

$$\begin{aligned} R_t &= \sum_s \int_a \int_e \mathcal{T}_I^s(e) d\Gamma^s(a, e) + \tau_l \sum_s \int_a \int_e w_t^s n_t^s e d\Gamma^s(a, e) + \tau_c (c_t + p_{FE,t} \underline{c}_{FE}) \\ D_t &= \left(\int g_{i,t}^{\frac{\varepsilon_d-1}{\varepsilon_d}} di \right)^{\frac{\varepsilon_d}{\varepsilon_d-1}} + \Xi_t + s_{H,t} p_{FE,t} (Y_{FE,t} + (1 + \tau_c) \underline{c}_{FE}) \\ b_t &= (1 + r_t) b_{t-1} - R_t + D_t \end{aligned}$$

where $b = B/P$ is the real public debt and Ξ_t the real transfers. To ensure the stability of public debt dynamics stability, the lump-sum transfer incorporates a fiscal brake

$$\Xi_t = \Upsilon_t - \theta \left(\frac{b_{t-1}}{b} - 1 \right) + e_{\tau,t} \quad \text{with } \Upsilon_t = \sum_s \int_a \int_e [\tau \bar{\tau}(e) + T \bar{T}(e)] d\Gamma^s(a, e)$$

such that Ξ_t is reduced when debt is larger than its steady-state level. Υ_t is the observed transfers paid by the government to households (Beveridgian $\sum_s \int_a \int_e \tau \bar{\tau}(e) d\Gamma^s(a, e)$ and Bismarckian $\sum_s \int_a \int_e T \bar{T}(e) d\Gamma^s(a, e)$ components of transfers) and e_{τ} is a measurement error. We assume that e_{τ} follow stationary $AR(1)$ processes.

3.2.7 Energy Market

The energy E_t^s is produced using raw energy \bar{E}_t through the technology $E_t^s = \Upsilon \bar{E}_t^\nu$, where $0 < \nu \leq 1$ and $\Upsilon > 0$. The raw energy is purchased at an exogenous price \tilde{P}_{Et} , and the quantity of raw energy \bar{E}_t adjusts to ensure energy market equilibrium. Assuming that $\nu = 1$, then the energy price is $P_{Et} = \tilde{P}_{Et}/\Upsilon$, and the energy sector distributes dividends $d_{Et} = 0$. Given that France does not produce any raw energy, we assume that the revenues from raw energy sales $\mathcal{R}_{Et} = P_{Et} E_t^s$ are earned by a foreign representative agent who uses them to purchase the goods exported by French firms $\mathcal{R}_{Et} = X_t$. With this assumption, albeit highly simplistic, the trade balance is always at equilibrium without any financial trade with the rest of the world.¹⁶ We assume that the relative price of energy P_{Et} follows a stationary $AR(1)$ process.

3.2.8 Equilibrium

Market-clearing conditions used to determine the unknowns $\{N, w, p_{FE}\}$ are

$$\begin{aligned} \text{asset market:} \quad & b_t = \mathcal{A}_t \equiv \sum_s \int_a \int_e a_t^s(a, e) d\Gamma^s(a, e) \\ \text{labor market:} \quad & N_t = \mathcal{N}_t \equiv \sum_s \int_a \int_e n_t^s(a, e) d\Gamma^s(a, e) \\ \text{energy market:} \quad & \bar{E}_t = \mathcal{E}_t \equiv Y_{Et} + \underline{c}_E + E_t \end{aligned}$$

and the market clearing condition on the goods market can be used to check the Walras law:

$$Y_t \left(1 - \frac{\psi_P}{2} \pi^2 \right) = X_t + \mathcal{C}_t + G_t$$

¹⁶Meyler, 2009 and Gautier, Marx, and Vertier, 2023 demonstrate that changes in consumer energy prices are primarily driven by variations in oil prices in the short run: consumer prices for liquid fuels reflect a direct, complete, and rapid pass-through of crude oil prices, thus leading us to calibrate $\nu = 1$ so that changes in crude oil prices largely pass through to consumer energy prices.

3.3 Model Estimation

3.3.1 Approximation of the equilibrium dynamic

The equilibrium defined in section 3.2.8 can be summarized by the following system:

$$\mathbf{H}_t(\mathbf{Y}, \mathbf{Z}) \equiv \begin{pmatrix} \Phi(S_{t+1}, S_t, S_{t-1}) \\ \mathcal{A}_t - b_t \\ \mathcal{N}_t - N_t \\ \mathcal{E}_t - \bar{E}_t \end{pmatrix} = 0 \quad (3.1)$$

where $\Phi(S_{t+1}, S_t, S_{t-1}) = 0$ regroups all the equations describing firm, union, government, and central-bank behaviors, with S_t the vector of aggregate variables controlled by these agents, \mathbf{Y} gathering the time series of unknown aggregate variables and \mathbf{Z} of exogenous aggregate shocks.¹⁷

In step 1, we calibrate parameters that determine the steady state of the economy using (3.1) for $x_{t+1} = x_t = x, \forall x \in \{S, \mathcal{A}, b, \mathcal{N}, N\}$. In step 2, we estimate the parameters that govern the dynamics of the aggregate shocks,

$$\mathbf{Z} \in \mathcal{Z} \equiv \{\beta, \mu, P_E, \varepsilon, \vartheta, \{\varphi_s\}_{s=l,m,h}, \{A_s\}_{s=l,m,h}, G, T, e_\tau\},$$

using Bayesian techniques and a linear approximation of (3.1) given by

$$0 = \sum_{s=0}^{\infty} [H_Y]_{t,s} dY_s + \sum_{s=0}^{\infty} [H_Z]_{t,s} dZ_s \quad \text{where } [H_Y]_{t,s} \equiv \frac{\partial H_t}{\partial Y_s} \quad \text{and } [H_Z]_{t,s} \equiv \frac{\partial H_t}{\partial Z_s}$$

$$\Rightarrow d\mathbf{Y} = \mathbf{G}d\mathbf{Z} \quad \text{with } \mathbf{G} = -H_Y^{-1}H_Z, \quad d\mathbf{Y} = \mathbf{Y} - \bar{\mathbf{Y}}, \quad \text{and } d\mathbf{Z} = \mathbf{Z} - \bar{\mathbf{Z}}$$

where $\bar{\mathbf{Y}}$ and $\bar{\mathbf{Z}}$ are the steady state values of \mathbf{Y} and \mathbf{Z} . \mathbf{G} is the complete Jacobian of the dynamic system. \mathbf{G} depends on calibrated parameters determined in step 1. If all the exogenous shocks of the model have the following MA(∞) representation, $dZ_t = \sum_{s=0}^{\infty} \mathbf{m}_s^Z \varepsilon_{t-s}^Z$, then the solution of the HA model can be represented by a MA(∞) that involves \mathbf{G} and \mathbf{m} :

$$dY_t = \sum_{s=0}^{\infty} \sum_{Z \in \mathcal{Z}} [\mathbf{G}^{Y,Z} \mathbf{m}^Z]_s \varepsilon_{t-s}^Z = \sum_{s=0}^{\infty} \sum_{Z \in \mathcal{Z}} m_s^{Y,Z} \varepsilon_{t-s}^Z \quad (3.2)$$

where \mathcal{Z} is the set of shocks. Replacing ∞ by a “large” finite integer and using the Jacobians, one can estimate the parameters $(\rho^Z)^s = [\mathbf{m}^Z]_s$ if the Z_t follow $AR(1)$ processes, using a Bayesian method and a data set. This representation of the model’s solution allows us to decompose the variances of endogenous variables as well as to analyze the historical decomposition of time series.

Let the model solution be $\mathcal{Y}_t = \mathcal{M}(\mathcal{E}_t | \Theta, \Phi)$. The growth-path equilibrium gives a first set of restrictions that allows us to calibrate Φ via $\bar{\mathcal{Y}} = \mathcal{M}_{ss}(0 | \Phi) \rightarrow \Phi = \mathcal{M}_{ss}^{-1}(\bar{\mathcal{Y}})$, where \mathcal{M}_{ss} denote the set of model’s restrictions at the steady state. Deviations around the trend provide a second set of restrictions that are used to estimate Θ through $\mathcal{Y}_t = \mathcal{M}(\mathcal{E}_t | \Theta, \Phi)$ and the data $\{\mathcal{Y}_t\}_{t=t_0}^T$.

¹⁷The dynamic paths of this economy are solved using the method developed by Auclert, Bardóczy, Rognlie, and Straub, 2021.

3.3.2 Calibrated parameters Φ

The first subset $\Phi_1 \in \Phi$ is calibrated using external information. Results are reported in Table 3.1.

Table 3.1: **Parameters Φ_1 based on external information**

	Values	Targets
Preferences		
Discount factor β	0.9888	Real interest rate $r = 0.74\%$ per quarter
Frisch elasticity of labor supply φ	1	Chetty et al., 2012
Elasticity of intertemporal substitution σ	1	Log-utility for consumption
Incompressible energy consumption \underline{c}	0.041	20% of households' energy consumption
Wage markup μ_w	1.1	Auclert, Bardóczy, Rognlie, and Straub, 2021
Low-skill labor disutility ϕ_l	0.3634	Low-skill wage
Middle-skill labor disutility ϕ_m	0.3278	Middle-skill wage
High-skill labor disutility ϕ_h	0.1482	High-skill wage
Elasticity of substitution between production inputs η_E	0.5	Negative impact on GDP of energy-price shock
Share parameter (energy, intermediate good) α_E	0.043	Sharing rule: a half of energy to households
Production		
Elasticity of substitution between production inputs σ_f	η_E	Simplifying assumption
Share parameter (energy, labor) α_f	0.056	Sharing rule: a half of energy to firms
Firm markup μ	1.2	Auclert, Bardóczy, Rognlie, and Straub, 2021
Productivity parameters A_s	1	Normalization
Energy price	0.105	Share of energy in GDP of 3.18%
Government		
Public debt B	5.418	Debt-to-GDP ratio 100% with annual GDP
Public spending G	0.236	Public spending-to-GDP ratio = 17.5%
Transfers	0.192+0.242	Transfers-to-GDP ratio (Bev. + Bism.) = 32.1%
Nominal rigidity		
Price rigidity κ	0.95	Arbitrary higher than Auclert, Rognlie, and Straub, 2023b
Wage rigidity κ_w	0.1	Auclert, Rognlie, and Straub, 2023b
Monetary policy		
Taylor rule coefficient $\phi_\pi(\mu_{FR} + (1 - \mu_{FR})\rho_\pi)$	1.2	With $\phi_\pi = 1.5$, $\mu_{FR} = 20\%$, and $\rho_\pi = 0.75$
Persistence of monetary policy ρ_r	0.85	C. Carvalho, Nechio, and Tristao, 2021

The second subset $\Phi_2 \in \Phi$ takes values based on some steady-state restrictions of the model and are deduced in order to solve

$$\min_{\Phi_2} [\Psi_s(\Phi_2) - \Psi_d] W [\Psi_s(\Phi_2) - \Psi_d]' \quad \text{with } W = Id$$

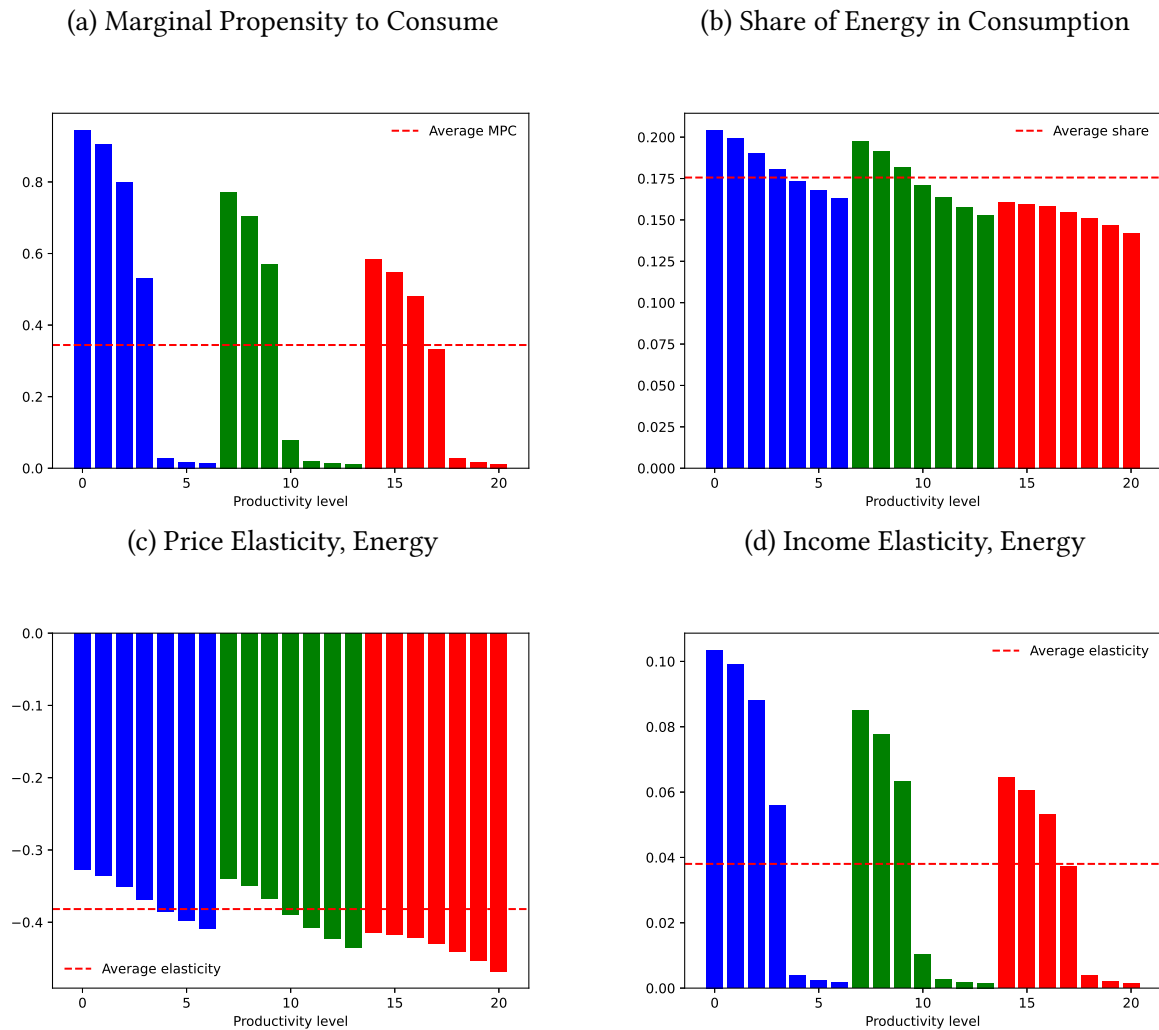
where $\Psi_z, \forall z = s, d$, is the set of simulated/targeted moments. The results are reported in Table 3.2.

The calibrations implied by the model steady state are described in Appendix 3.7.2. The Marginal Propensities to Consume (MPC) per level of income are reported in panel (a) of Figure 3.1. As expected, the agents with low incomes consume a larger fraction if their income increases. Panel (b) of Figure 3.1 shows that the agents devote a larger share of their expenditures to energy, as in the data. Panel (c) of Figure 3.1 shows that the agents with low incomes have more difficulty reducing their energy consumption when the price increases. This result comes from the largest share of incompressible consumption in their energy consumption. Finally, panel (d) of Figure 3.1 shows that the energy MPCs decline with income. Finally, this calibration results in 31% of households being financially constrained.

Table 3.2: Parameters Φ_2 based on steady-state restrictions

Productivity & dividends		Value	Moment Ψ_z	Data	Model
Productivity-persistence low-skill		$\rho_l = 0.97$	Gross income D10/D1	11.67	11.64
Productivity-persistence middle-skill		$\rho_m = 0.965$	Gross income D5/D1	2.94	2.73
Productivity-persistence high-skill		$\rho_h = 0.94$	Average productivity persistence	0.966	0.966
Productivity-variance low-skill		$\sigma_l = 0.36$	Net consumption D10/D1	3.07	3.12
Productivity-variance middle-skill		$\sigma_m = 0.64$	Net Consumption D5/D1	1.49	1.49
Productivity-variance high-skill		$\sigma_h = 1.4$	Net income D10/D1	4.16	3.72
Dividends rule $\bar{d}(e) = e^{a_{div}}$		$a_{div} = 1.775$	Dividends D10/D1	66.25	65.34
Government		Value	Moment Ψ_z	Data	Model
Beveridgian transfer rule $\bar{r}(e) = e^{a_{beve}}$		$a_{beve} = -0.47$	Beveridgian Transfer D10/D1	0.36	0.36
Bismarckian transfer rule $\bar{T}(e) = e^{a_{bism}}$		$a_{bism} = 0.815$	Bismarckian Transfer D10/D1	5.43	5.43
Level of the income tax $(1 - \tau_z)z^{1-\lambda}$		$\lambda = 0.089$	Net income D5/D1	1.57	1.42
Progressivity of the income tax $(1 - \tau_z)z^{1-\lambda}$		$\tau_z = 0.35$	Income-tax revenues/GDP	0.115	0.115
Level of VAT		$\tau_c = 0.213$	VAT revenues/GDP	0.17	0.17
Level of social security contribution		$\tau_l = 0.242$	Social-security contribution revenues/GDP	0.195	0.195
Level of the corporate tax		$\tau_f = 0.35$	Corporate-tax revenues/GDP	0.045	0.045

Figure 3.1: Heterogeneous households' behaviors



3.3.3 Estimated parameters Θ

As in all dynamic models, the impact of each shock depends on how the agents expect them to persist. The vector of parameters

$$\Theta = \{\rho^Z, \sigma^Z | \text{for } Z \in \mathcal{Z} \equiv \{\beta, \mu, P_E, \varepsilon, \vartheta, \{\varphi_s\}_{s=l,m,h}, \{A_s\}_{s=l,m,h}, G, T, e_\tau\}\}$$

is estimated using a Bayesian method

$$\hat{\Theta} = \operatorname{argmax} \mathcal{L}(\Theta | \{\mathcal{Y}_t\}_{t=t_0}^T, \Phi) \quad \text{where } \mathcal{Y}_t = \mathcal{M}(\mathcal{E}_t | \Theta, \Phi)$$

given that \mathcal{M} is deduced from equation (3.2). In order to have a “just-identified” system, the number of time series used in the estimation is equal to the number of shocks introduced in the model. Therefore, to identify the 14 shocks, we use the data set

$$\mathcal{Y} = \left\{ Y_{t,t}, \pi_t, P_{E,t}, i_t^*, i_t, \{N_{s,t}\}_{s=l,m,h}, \{\pi_{s,t}^w\}_{s=l,m,h}, G_t, T_t, \frac{b_t}{Y_t} \right\}_{t=t_0}^T$$

over the sample $t_0 = 2Q2003$ to $T = 4Q2019$.^{18,19} Table 3.3 gives this mapping between data and shocks. The energy-consumption subsidies (the tariff shield) are present between 1Q2022 and 4Q2023.

Table 3.3: Identification: Data (\mathcal{Y}) – Shocks (\mathcal{E})

\mathcal{Y}	GDP		Int. rates		Energy	Government		Debt-to-
	Y_t	π_t	BCE	debt	price	cons.	trans.	GDP
\mathcal{E}	β	μ	ε	ϑ	P_E	G	T	e_τ
\mathcal{Y}	Hours			Wages				
	l	m	h	l	m	h		
\mathcal{E}	φ_l	φ_m	φ_h	A_l	A_m	A_h		

The autocorrelations of these $AR(1)$ processes and the standard deviations of their innovations are reported in Table 3.4 (see details in Appendix 2.8.5).

Based on these estimations of the shock processes, the variance decomposition²⁰ indicates that the shocks to household demand are the main source of output fluctuations (44%). Next, shocks to productivity and firm-markup shocks each account for around 16%

¹⁸Appendix 3.7.2 presents the data used in the paper. All data are stationarized by extracting a linear trend, except the debt-to-GDP ratio where only its average over the sample is extracted.

¹⁹Additional details on the estimation procedure can be found in appendix 2.8.5. See also Langot, Malmberg, Tripier, and Jean-Olivier Hairault, 2023

²⁰The variance decomposition, as well as the historical decomposition of aggregates with respect to shocks, are presented in Appendix 3.7.4.

Table 3.4: Estimated parameters of the $AR(1)$ processes (Standard errors in parenthesis)

Shock	Z	Persistence ρ^Z	Standard dev. σ^Z	Variance
		Mean	Mean	$\frac{(\sigma^Z)^2}{1-(\rho^Z)^2} \times 100$
Preference	β	0.778482 (0.025231)	0.009143 (0.001438)	0.02121870
Price markup	μ	0.827839 (0.025517)	0.023420 (0.002331)	0.17430147
Energy price	P_E	0.793492 (0.022682)	0.116112 (0.009058)	3.64013832
Monetary policy	ε	0.575467 (0.040994)	0.006381 (0.000498)	0.00608774
Spread	ϑ	0.832970 (0.034203)	0.001172 (0.000126)	0.00044864
Disutility l	φ_l	0.771044 (0.048294)	0.020303 (0.002861)	0.10165741
Disutility m	φ_m	0.772314 (0.046330)	0.018561 (0.002492)	0.08537402
Disutility h	φ_h	0.692028 (0.058132)	0.032577 (0.003967)	0.20365890
Productivity l	A^l	0.819040 (0.029585)	0.024635 (0.002089)	0.18436577
Productivity m	A^m	0.795198 (0.029001)	0.018561 (0.001954)	0.09370358
Productivity h	A^h	0.844523 (0.032298)	0.025028 (0.002205)	0.21842485
Government consumption	G	0.714310 (0.056981)	0.004220 (0.000379)	0.00363613
Transfers	T	0.797681 (0.044078)	0.005315 (0.000465)	0.00776707
Measurement error	e_τ	0.773476 (0.047124)	0.015915 (0.004046)	0.06304835

of these variations. Fluctuations in oil prices account for 6% of production variations.²¹ More than half of inflation fluctuations are explained by oil-price fluctuations (56%), and 15% by markup fluctuations. The high contribution of energy to inflation fluctuations is due to price rigidity, which means that domestic prices only react with delay to shocks, particularly those that are very persistent, whereas energy prices are determined by very volatile shocks. Indeed, the estimated variance of oil-price shocks is ten times higher than that of markup shocks and a hundred times higher than that of preference shocks (respectively 3.64, 0.17, and 0.021, see Table 3.4).²² Debt dynamics are determined by these main determinants of output and inflation fluctuations. Its variations are explained by shocks to energy prices (29%), consumer demand (26%), and markup (21%), followed by shocks to labor supply (5%), productivity (4%), and interest rates (2%). Finally, the three main macroeconomic

²¹Other shocks play a minor role: 5% for labor-supply shocks and interest-rate shocks and 1.6% for public spending shocks.

²²Productivity shocks explain 9% of inflation variations, followed by demand shocks for less than 8% and labor supply for 5%.

shocks (oil price, demand, and markup) also explain most of the fluctuations in labor markets. They account for 74% of variations in unskilled jobs, 75% in intermediate jobs, and 66% in skilled jobs.

3.4 Methodology

Our objective is to evaluate various strategies that could enable the government to reduce its debt in the future, assuming “all things being equal”. Since public policy extends beyond mere shocks, these strategies may entail adjustments to the model’s multipliers. In particular, if the government announces that it will consolidate its debt according to a specific rule, then the behaviors of economic agents are modified. The model adapts to this new rule, leading to changes in both the risk and the level of the debt, even if the overall environment remains unchanged. In this paper, we will consider that the government implements its fiscal consolidation program through the following rule that can be applied to each of its policy instruments:

$$\Upsilon_t = \rho \Upsilon_{t-1} - \tau^\Upsilon (b_{t-1} - b) + \varepsilon_t^\Upsilon \quad (3.3)$$

with $\Upsilon \in \{G, T, bism, beve\}$. This means that instead of following a simple AR(1) process, the level of each policy instrument also depends on the difference between the level of debt at the beginning of the period and its steady state. If τ^Υ equals zero then we are back to the usual AR(1) process. But, if τ^Υ is positive, then a debt higher than at its steady state will automatically lead to a decrease in this policy instrument’s level.

With these changes in the fiscal rule, relying on analysis based on impulse response functions (IRFs) becomes impractical. Instead, we propose utilizing the method outlined by Langot, Malmberg, Tripier, and Jean-Olivier Hairault, 2023, which involves a two-stage process. First, we identify sequences of future shocks that enable the model to replicate specific forecasts (an extension of the conditional forecast method à la Del Negro and Schorfheide, 2013 to HANK models). In the second stage, these shocks establish an invariant economic context in which each economy will operate. Any disparities observed in the simulated series during this latter stage can then be attributed solely to differences in the policies implemented. Moreover, the originality of our quantitative method is to propose a real-time evaluation, i.e. a method that can be used at the time when governments make their economic-policy decisions.

3.4.1 Conditional Forecasts and the R. Lucas, 1976 Critique

Using Equation (3.2), we first estimate the model \mathcal{M} using the dataset \mathcal{Y}_T :

$$\mathcal{Y}_T = \mathcal{M}(\mathcal{E}_T | \hat{\Theta}) \quad \text{with} \quad \hat{\Theta} = \arg \max \mathcal{L}(\Theta | \mathcal{Y}_T)$$

where \mathcal{E}_T is the innovations of the shocks. A conditional forecast uses information on future realizations of \mathcal{Y}_{T+H} , in order to reveal the sequences of \mathcal{E}_{T+H} satisfying $\mathcal{Y}_{T+h} = \mathcal{M}(\mathcal{E}_{T+h} | \hat{\Theta})$, $\forall h \in \{1, \dots, H\}$. Given that the Finance Act can propose new policies, the government’s decision rules can change compared to the ones observed in the past and

estimated over the sample $t \in [0, T]$. Therefore, to take seriously this potential problem, first underlined by R. Lucas, 1976, we must distinguish two cases.

1. **Stability of the government's decision rules.** First, the government announces its policy by committing itself on $\{y_{T+h}^g\}_{h=1}^H \in \mathcal{Y}_{T+H}$. Assume that its decision rule is such that y^g is an exogenous variable following an $AR(1)$ process: $y_{T+h}^g = \rho_g y_{T+h-1}^g + \sigma_g \varepsilon_{T+h}^g$. Therefore, the sequence of $\{\varepsilon_{T+h}^g\}_{h=1}^H \in \mathcal{E}_{T+H}$ satisfying its announcement is given by $\widehat{\varepsilon}_{T+h}^g = \frac{1}{\widehat{\sigma}_g} (y_{T+h}^g - \widehat{\rho}_g y_{T+h-1}^g)$. This forecast of the innovations is admissible if and only if $\widehat{\varepsilon}_{T+h}^g \in CI$ of $\mathcal{N}(0, 1)$. If this is the case, then the stability of the parameters $\{\rho_g, \sigma_g\}$ is not rejected by the new announcements of the government. Therefore, the sequence \mathcal{E}_{T+H} can be deduced from the model inversion:

$$\mathcal{E}_{T+H} = \mathcal{M}^{-1}(\mathcal{Y}_{T+H} | \widehat{\Theta})$$

If $\mathcal{E}_{T+h} \in CI$ of $\mathcal{N}(0, 1)$, then the stability of household and firm behaviors is also ensured. If this is the case, then the R. Lucas, 1976 critique is not quantitatively relevant and the sequence \mathcal{E}_{T+H} is not biased.

2. **Instability of the government's decision rules.** If the government's decision rules are unstable ($\mathcal{E}_{T+h} \notin CI$ of $\mathcal{N}(0, 1)$), then they can be rewritten as follows:

$$y_t^g = \begin{cases} \widehat{\rho}_g y_{t-1}^g + \widehat{\sigma}_g \varepsilon_t^g & \text{if } t \leq T \quad \text{Old policy rule} \\ \widetilde{\rho}_g y_{t-1}^g + \widetilde{\sigma}_g \varepsilon_t^g & \text{if } t > T \quad \text{New policy rule} \end{cases}$$

Therefore, when the government announces $\{y_{T+h}\}_{h=1}^H$, the sequence of $\{\varepsilon_{T+h}\}_{h=1}^H$ must be identified using $\varepsilon_{T+h} = \frac{1}{\widetilde{\sigma}_g} (y_{T+h} - \widetilde{\rho}_g y_{T+h-1})$. The parameters $\widetilde{\Theta}_g = \{\widetilde{\rho}_g, \widetilde{\sigma}_g\}$ can be re-estimated using the government commitments $\{y_{T+h}\}_{h=1}^H$ and we will have necessarily $\varepsilon_{T+h} \in CI$ of $\mathcal{N}(0, 1)$. Therefore, the identification of \mathcal{E}_{T+H} is now made through

$$\mathcal{E}_{T+h} = \mathcal{M}^{-1}(\mathcal{Y}_{T+h} | \{\widehat{\Theta}_{-g}, \widetilde{\Theta}_g\})$$

In this case, the R. Lucas, 1976 critique is quantitatively relevant and the identification process of the shock innovations must be "corrected" to be unbiased.

The estimated shocks \mathcal{E}_{T+H} can be interpreted as the economic context allowing the realization of the Finance Act forecasts \mathcal{Y}_{T+H} . To evaluate the impact of an alternative policy by always considering the economy in an identical context, this vector of shocks must be invariant. The comparison of economic policies is therefore done by controlling the economic context as in a controlled experiment.

3.4.2 Changes in Model Multipliers and Policy Evaluation

Let us index by \emptyset the economy with no policy changes and by \mathcal{P} the one where the new policy involves parameter changes and additional shocks.²³ Using Equation (3.2), the eval-

²³The analysis is conducted for changes in parameters that do not modify the steady state.

uation of the policy \mathcal{P} with respect to \emptyset is given by

$$\begin{aligned}
 dY_{\emptyset,t} &= \sum_{s=0}^{\infty} \sum_{Z \in \mathcal{Z}} m_{\emptyset,s}^{Y,Z} \varepsilon_{t-s}^Z \quad \text{and} \quad dY_{\mathcal{P},t} = \sum_{s=0}^{\infty} \sum_{Z \in \mathcal{Z}} \left[m_{\mathcal{P},s}^{Y,Z} \varepsilon_{t-s}^Z + m_{\mathcal{P},s}^{Y,\zeta} \varepsilon_{t-s}^{\zeta} \right] \\
 dY_{\mathcal{P},t} - dY_{\emptyset,t} &= \sum_{s=0}^{\infty} \sum_{Z \in \mathcal{Z}} \left[(m_{\mathcal{P},s}^{Y,Z} - m_{\emptyset,s}^{Y,Z}) \varepsilon_{t-s}^Z + m_{\mathcal{P},s}^{Y,\zeta} \varepsilon_{t-s}^{\zeta} \right] \tag{3.4}
 \end{aligned}$$

As all multipliers $m_{\mathcal{P},s}^{Y,Z}$ have changed, evaluating policy \mathcal{P} depends on sequences of all shocks by combining all IRFs.²⁴ Therefore, if the multipliers are not invariant due to policy changes and if the sample size is limited, Equation (3.4) must be used for policy evaluation, even with a linear approximation of the model dynamics. The finiteness of the sample requires focusing on a specific realization of the shock dynamics, for which the natural candidate is \mathcal{E}_{T+H} since it enables the model to match \mathcal{Y}_{T+H} and thereby identifies relevant time dependence for policy evaluation. Conditional on beginning from the current economic state, policy evaluation must compare the benchmark scenario to a counterfactual one. To maintain the principle of “all things being equal”, shocks \mathcal{E}_{T+H} are retained when evaluating any alternative scenario, leaving no explanation for the differences other than the policy change.²⁵

3.4.3 Stochastic Debt-Sustainability Analysis

Our HANK model allows us to generate the distribution of the debt-to-GDP ratio, given the estimated distribution of shocks. Using conditional-forecast techniques, it is also possible to measure the risk that the debt-to-GDP ratio would be larger than the one announced in the Finance Act. Our HANK model enables us to generate the distribution of the debt-to-GDP ratio based on the estimated distribution of structural shocks. Using conditional-forecast techniques, it is also possible to assess the risk that the debt-to-GDP ratio could exceed the one announced in the Finance Act for 4Q2027. Specifically, our methodology allows us to account for the contribution of each shock to the uncertainty surrounding debt-to-GDP forecasts. These analyses contribute to extending our setup of the stochastic debt-sustainability analysis (SDSA) as defined by Blanchard, Leandro, and Zettelmeyer, 2021.

How does the debt risk change when a fiscal consolidation policy is implemented?

If the government neither announces a new fiscal rule nor a new policy in advance, but decides to adjust its spending levels in a discretionary manner at the beginning of each period, then, debt risk remains the same as outlined in the Finance Act. However, if the government announces that it will consolidate its debt according to the rule (3.3), then the behavior of economic agents is modified. This is what will occur in the fiscal consolidation cases we will study in the next section.

²⁴When multipliers do not change, $m_{\emptyset,s}^{Y,z} = m_{\mathcal{P},s}^{Y,z}$ and thus $dY_{\mathcal{P},t} - dY_{\emptyset,t} = \sum_{s=0}^{\infty} m_{\mathcal{P},s}^{Y,\zeta} \varepsilon_{t-s}^{\zeta}$ is an IRF.

²⁵This method gives significant importance to the relative size of each shock, as it weighs the IRFs of each to determine the level of each endogenous variable. Therefore, a specific identification of shocks favors policies that interact with the largest shocks.

An alternative measure of risk on public debt. We denote $b_{t+n}^\alpha(x)$ the debt-at-risk measure, which satisfies

$$F(b_{t+n}^\alpha(x)|\mathcal{I}_t) = \alpha,$$

where $F(b_{t+n}^\alpha(x)|\mathcal{I}_t)$ is the conditional distribution of the debt-to-GDP ratio estimated by the HANK model, given the information set \mathcal{I}_t , which includes the model parameters $\{\Phi, \Theta\}$, and the policy $x, \forall x \in \mathcal{X}$, announced as of time t for $t + n$ periods. According to our model, the value of $b_{t+n}^\alpha(x)$ is the debt-to-GDP ratio that the economy will exceed with a probability $1 - \alpha\%$ in n periods given the policy x .²⁶ The use of a structural model allows us to evaluate counterfactual scenarios for reducing the risk of public debt. We define

$$\Delta_{t+n}^\alpha(x, y) = b_{t+n}^\alpha(x) - b_{t+n}^\alpha(y) \quad (3.5)$$

as debt-at-risk changes, measured in percentage points, induced by a shift from policy x to y . Applied to the fiscal consolidation programs we are studying, this measure makes it possible to assess the risk reduction they bring in relation to the policy announced by the government.

3.5 Quantitative Results

3.5.1 Assessing Changes in Government Policy Rules

Following the method described in Section 3.4.1, we compare the simulations based on the models where $\{\rho^Z, \sigma^Z\}, \forall Z$, are or are not re-estimated.

Table 3.5: Estimated parameters over 4Q2019 to 4Q2027

Z	G	T
$\tilde{\rho}^Z$	0.95953	0.90391
$\tilde{\sigma}^Z$	00867	0.02299

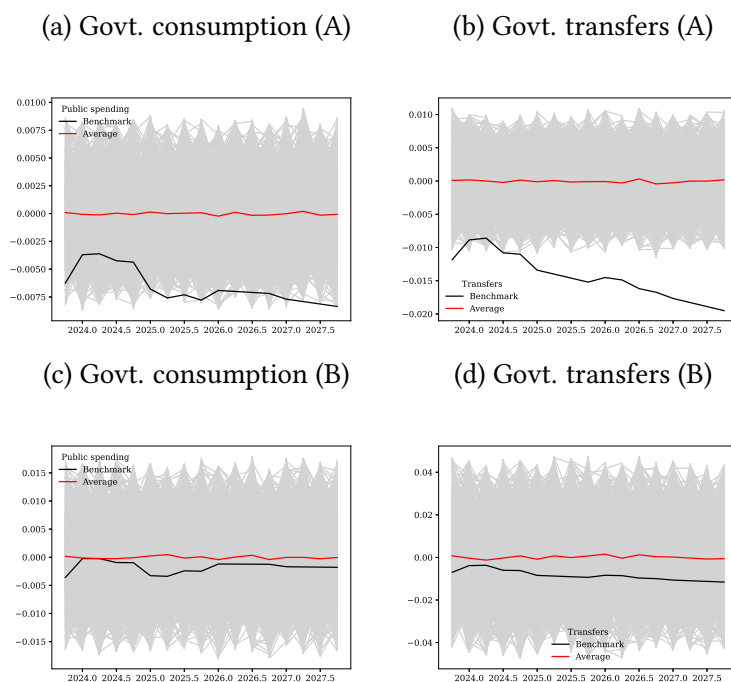
Indeed, except for government-consumption and government-transfer shocks (see the innovations of household and firm shocks displayed in Figure 3.23 in Appendix 3.7.5), it is not possible to reject the assumption of invariance of the laws of these innovations. Therefore, we need to reestimate only the shock parameters for the government consumption G and transfer T shocks. We estimate those on the data $\{G_t, T_t\}_{t=4Q2019}^{4Q2027}$ corresponding to the government's commitments as presented in the Finance Act. In the following evaluations of the French Finance Act, based on conditional forecasts, we use the new estimated values for $\{\tilde{\rho}^Z, \tilde{\sigma}^Z\}$, for $Z = G, T$, reported in Table 3.5. Those values are used from then since

²⁶The underlying definition of risk is similar to the one developed by Adrian, Boyarchenko, and Giannone, 2019 for growth at risk, the key difference being that our predictive distributions are based on the estimation of an estimated structural model rather than on quantile regressions.

they allow the agents' expectations to be modeled satisfactorily.²⁷

Figure 3.2 shows the confidence intervals for the innovations of the government decision rules (grey areas), the mean of these shocks (red line), and the sequences of the innovations (black lines) identified by the models to allow it to match the government forecasts. For the other shocks of the model, although in the confidence interval, the estimated

Figure 3.2: Innovations of government decision rules. (A) before and (B) after re-estimation.



shocks reveal that the Finance Act is highly optimistic regarding markups and the high-skill worker labor-market paths whereas the underlying sequence of productivity shocks declines continuously.²⁸

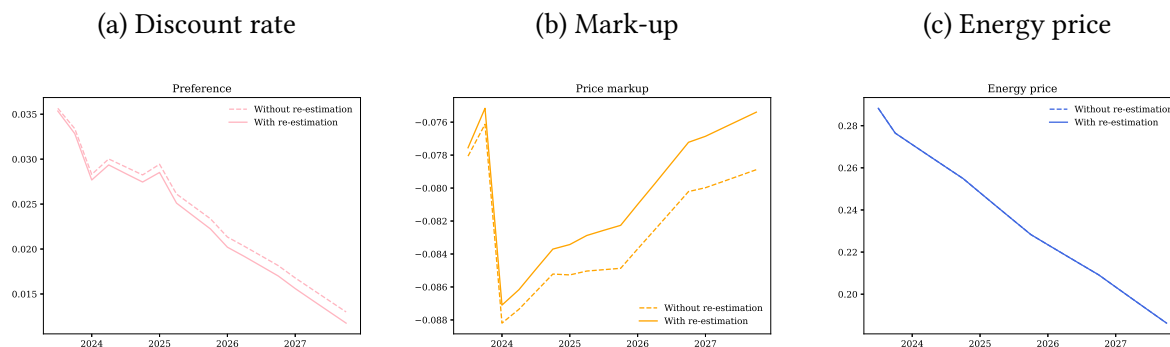
3.5.2 Shocks Underlying the Government's Forecasts

Figures 3.3 to 3.6 represent the deviations from the steady-state values of all time series in \mathcal{E}_{T+H} .

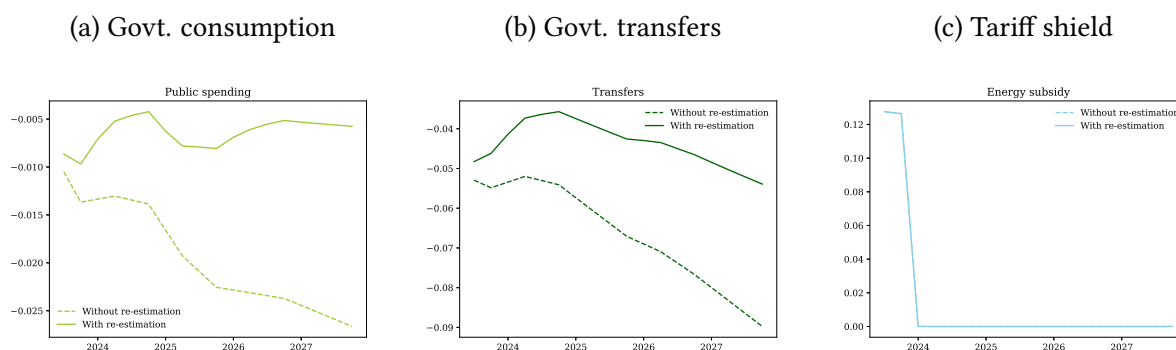
Panels (a), (b), and (c) of Figure 3.3 show that the discount factor (β_t) and the energy price ($P_{E,t}$) take values above their steady-state, whereas the price markup (μ_t) takes values below its steady-state. The estimate for β_t underlines that the households' demand is depressed over the forecast periods by an exceptionally high preference for saving. Concerning the markup, our estimates suggest that it is almost divided by 2 over the forecast

²⁷Figures 3.32 to 3.35 in the Appendix 3.7.8 show the forecasts of endogenous variables and their confidence intervals under the assumption that all innovations of the exogenous variables are drawn in their estimated distributions.

²⁸See panels (d), (i), (m), (p), (s) and (v) in Figure 3.23 in Appendix 3.7.5.

Figure 3.3: Estimated shocks $\{\beta, \mu, P_E\}$. (A) before (dotted line) and (B) after re-estimation (line)

period (from 20% to 11% or 12%). Finally, our estimate of oil price corresponds to a variation going from €50.3 to €46 (in prices of 1995) from 3Q2023 to 4Q2027, given that the average value of oil price between 1Q2003 to 4Q2019 is equal to €38.9.

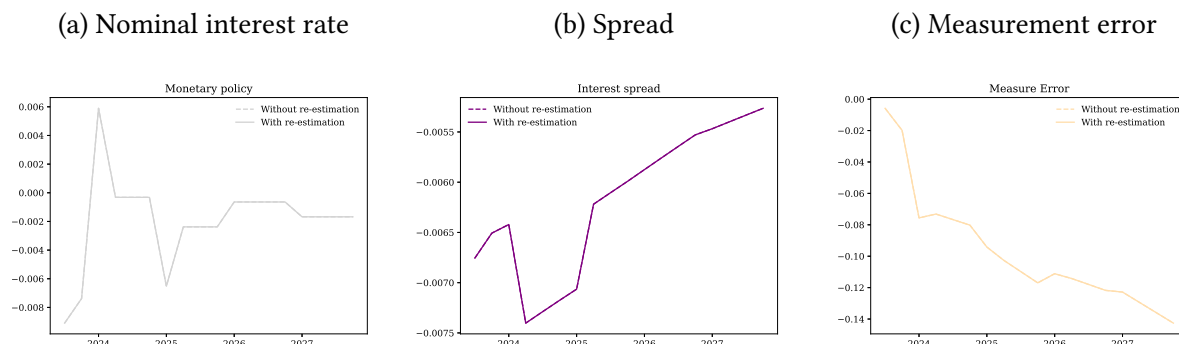
Figure 3.4: Estimated shocks $\{G, T, s_H\}$. (A) before (dotted line) and (B) after re-estimation (line)

Panels (a) and (b) of Figure 3.4 show that the government consumption (G_t) and transfers (T_t) take values below their steady state over the entire forecast period. From these data, we deduce that G_t will take values consistently around -0.005 over the forecast period, meaning a reduction of around 0.5% of the government consumption with respect to its 4Q2019 value, whereas T_t is 4% to 5% below their 4Q2019 value. The values of energy subsidies (the tariff shield) correspond to an energy-price cut approximately equal to 12% until 4Q2023 (see panel (c) of Figure 3.4).

If the monetary policy shock oscillates around zero, suggesting very little variation in it (see panel (a) of Figure 3.4), the risk premium paid by the government when it repays its debt is on the other hand continuously below its long-term value, even if it rises towards it at the end of the forecast period (see panel (b) of Figure 3.4).²⁹ Over the forecast period, the spread will be lower by 0.6 percentage points per quarter on average (2.4 percentage points per year) compared to its long-term value which is 0.24 % per quarter (1% per year). The real interest rate paid by the government to repay its debt (this rate will be on average 0.6%

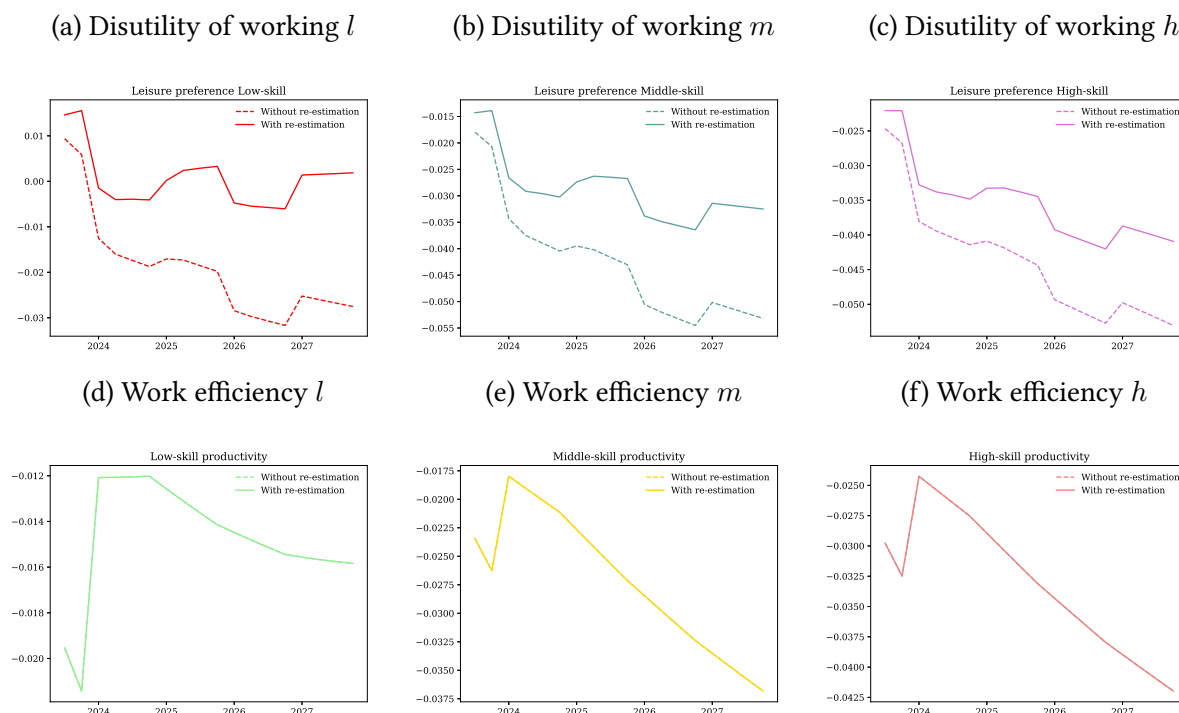
²⁹The interest rate is quarterly and corresponds to an annual interest of 3%, with an annual spread equal to 1%, thus implying a BCE annual interest rate equal to 2%.

Figure 3.5: Estimated shocks $\{\varepsilon, \vartheta, e\}$. (A) before (dotted line) and (B) after re-estimation (line)



per year) will therefore be lower than the real ECB-interest rate over the entire forecast period (this rate will be on average 2%).

Figure 3.6: Estimated shocks $\{A^s, \varphi^s\}_{s=l,m,h}$. (A) before (dotted line) and (B) after re-estimation (line)



Finally, panels (d), (e) and (f) of Figure 3.6 show that the labor productivity of all employees declines (when removing the historical trend) over the forecast period: between 1 and 2% for workers receiving low wages, and between 2 and 4% for those receiving middle and high salaries. If the decline in productivity is shared by all types of workers, the same is not true for the labor-disutility evolution (see Panels (a), (b), and (c) of Figure 3.6). It is stable for low-wage workers while it decreases up to 4% for those with middle and high wages.

When these shocks are put in the model, it appears that the budgetary decisions only explain around 20% of the variance of the GDP forecast. Therefore, the Finance Act forecasts

rely heavily on shocks not controlled by the government (see Appendix 3.7.6 for more details on variance decomposition over the forecasted period). The contribution of the markup shocks explains the strong forecasted GDP growth, whereas the shocks on the disutility of working are necessary to sustain the rise in total hours worked.³⁰

3.5.3 Induced Inequality Dynamics

In addition to reproducing the expected evolution of macroeconomic aggregates, our model also makes it possible to estimate the evolution of inequalities compatible with this equilibrium trajectory. Changes in consumption inequalities are reported in Figure 3.8. Panel (a) gives the quarterly consumption of workers who only have access to low-skill jobs. Among these low-skill jobs, a worker may have remuneration among the lowest 10% (called Bottom 10%), equal to the median (Median), or even among the highest 10% (Top 10%). The figure then reports the consumption levels associated with these three wage levels for the distribution of low-skill jobs. Panels (b) and (c) of Figure 3.8 report the same information but for workers with access to wages in middle and high-skilled jobs. Finally, panel (d) of Figure 3.7 makes the connection between the extremes, the top 10% of high wages compared to the bottom 10% of low wages. It also reports the consumption ratios between the bottom 10% of low earners and the median of the middle class, and between the Top 10% and the median of the middle class.

Increases in the real interest rate and high wages are factors in increasing inequalities, while strong GDP growth further favors the consumption of the poorest through employment increases. Given that government announces that it will reduce transfers³¹ the model gives the induced rise in inequalities: in 2023, a well-off worker consumes 4.36 times more than a poor worker, while he would consume 4.82 times more in 2027. The drop in transfers penalizes households with the lowest incomes who are the main beneficiaries and who have no savings to compensate for these losses of income. In this scenario, strong growth, which nevertheless favors the consumption of the most disadvantaged, is not sufficient to contain the increase in inequalities.

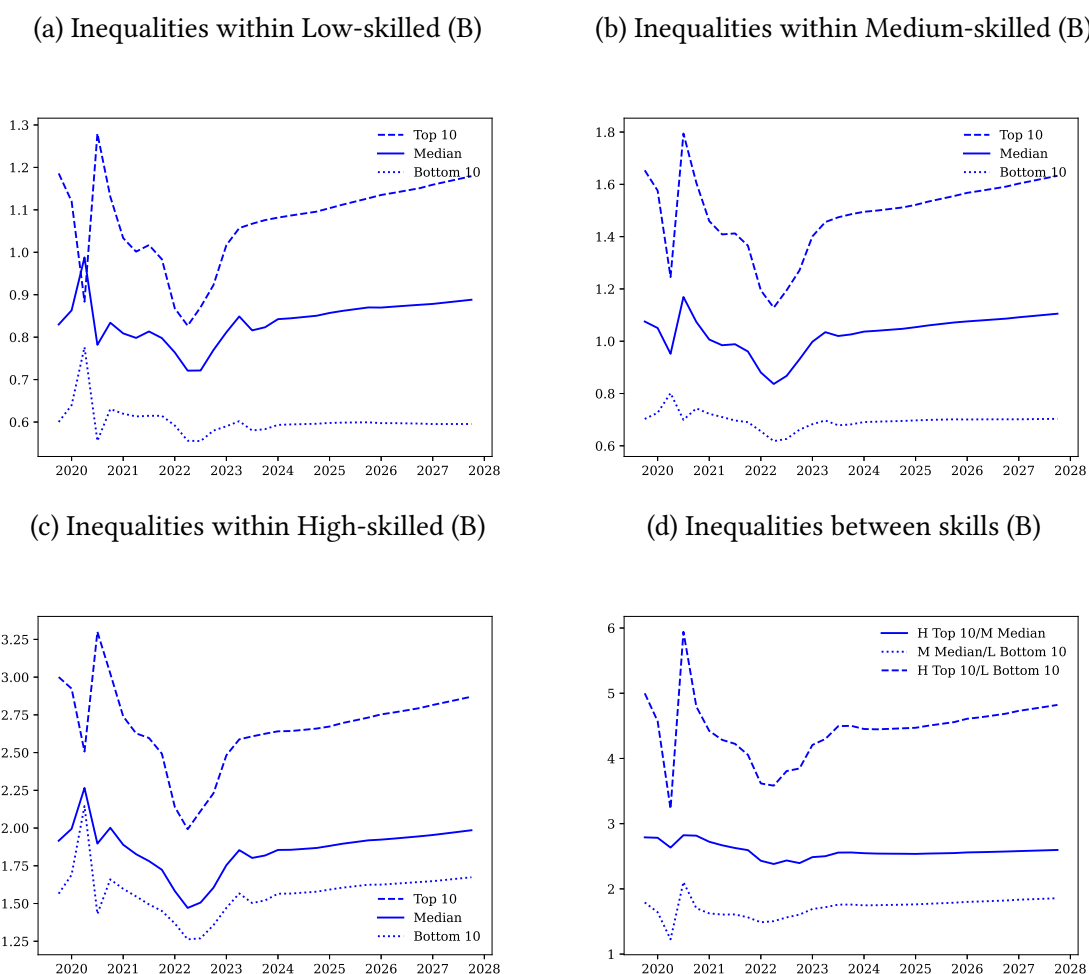
3.5.4 Assessing Fiscal Consolidation Programs

In its Finance Act, the French government plans for a small decrease in the debt-to-GDP ratio by 2027 to 108.3% thanks to a strong increase in GDP growth. But, this growth relies largely on particular realizations of structural shocks. It would therefore be more efficient to decrease debt through more substantial fiscal consolidation programs that would allow France to achieve a larger reduction in its public debt. Moreover, fiscal space is required to face new challenges such as climate change (Blanchard, 2023). However, the difficulties of decreasing debt have been underlined for years. Empirical studies have notably found that

³⁰Appendix 2.8.9 proposes a robustness analysis of the Finance Act based on uncertainty around the shock realizations.

³¹Without any precision on the implementation of this reduction, we assume that the Bismarckian and the Beveridgian components of transfers are reduced homogeneously.

Figure 3.7: Consumption ratios: comparison across two different workers' types. (B) after re-estimation.



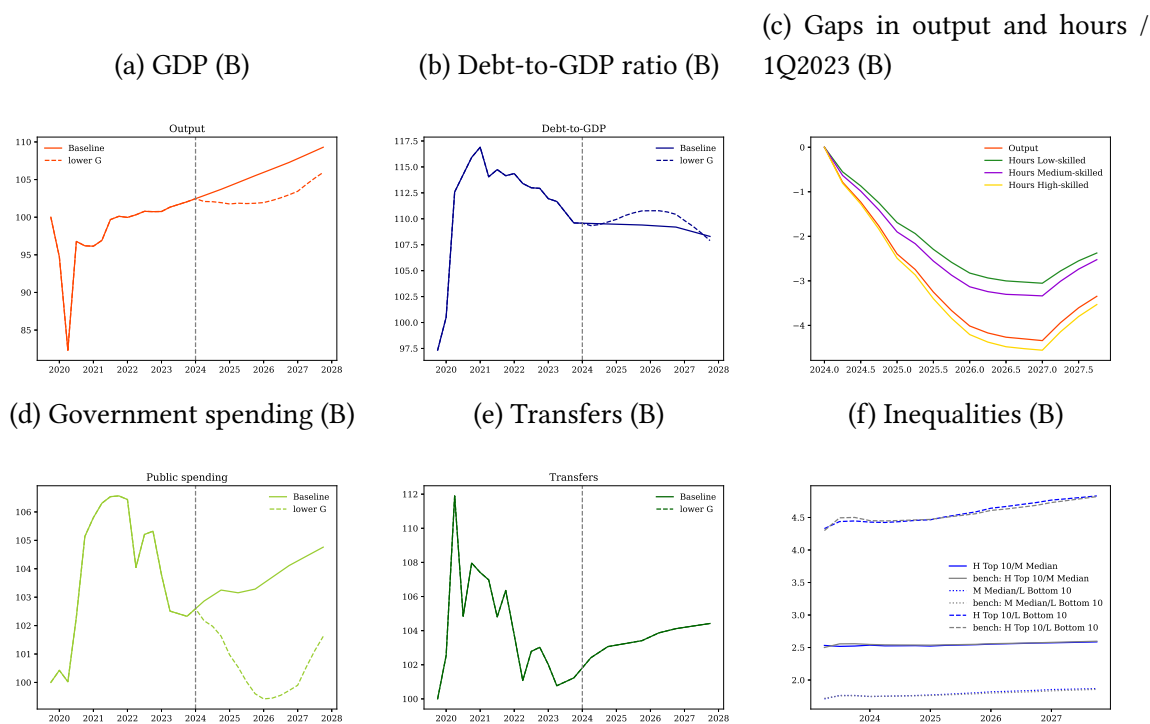
policies focusing on decreasing debt have often hindered economic growth (Blanchard and Leigh, 2013, Alesina, Favero, and Giavazzi, 2015 and IMF, 2023) while Ball et al., 2013 and Brinca et al., 2021 show how fiscal consolidation programs are usually correlated with an increase in inequalities, calling into question the social and political acceptability of these policies. Therefore, the implementation of the 2024 Finance Act could be blocked, even if it reduces only modestly the debt-to-GDP ratio. The challenge is thus to improve the Finance Act by targeting three more ambitious objectives: causing a large decrease in public debt, sustaining stronger GDP growth, and fighting more intensively against inequalities.

How to reduce debt without damaging growth or worsening inequality?

Thanks to our decomposition of public spending between public consumption, Bismarckian (proportional to wage), and Beveridgian transfers (inversely proportional to wage) we can evaluate the consequences of different fiscal consolidation policies. Furthermore, with our method we take into account the general equilibrium effects of those programs and the

heterogeneous-agent structure of the model gives us a precise vision of inequality dynamics induced by those policies. This way our model allows us to go further into the analysis of those policies than previous studies. Using the structural shocks estimated for the forecast period, we look at the future trajectories of the French economy according to several fiscal and budgetary policies in the same economic context defined by these structural shocks and compare them to the forecasts of the Finance Act.

Figure 3.8: Counterfactual: €20 billion reduction in government spending. (B) after re-estimation.



Simple policies. We first consider two different programs that aim at reducing government expenditures by €20 billion.

1. Scenario 1: A reduction in public consumption expenditure G .
2. Scenario 2: A reduction in public transfers Ξ keeping unchanged the breakdown between Bismarckian (T) and Beveridgian (τ) components of transfers.³²

Figure 3.8 shows the consequences of a fiscal consolidation based solely on a reduction in public consumption. It fails at strongly reducing French public debt while sustaining economic growth or decreasing inequalities. Debt-to-GDP first increases and finally only gets 0.4pp lower than in the Finance Act at the end of 2027. The fall in public consumption causes a decrease in aggregate demand. As such, output growth is strongly damaged with

³²This means that in Scenario 1 τ^G will be chosen to reach a yearly average reduction of 20 billion euros (similarly with τ^T in the second scenario).

an average annual growth of 1.01% (versus 1.64% in the Finance Act). This also leads to a reduction in employment for all workers, regardless of their qualifications (annual growth of 0.22% versus 0.71% in the Finance Act). As the most disadvantaged households are the most dependent on labor income, this would also lead to a small increase in inequality. As such, a fiscal consolidation program based on decreasing public consumption does not seem to be effective.

Figure 3.9: Counterfactual: €20 billion reduction in Transfers. (B) after re-estimation.

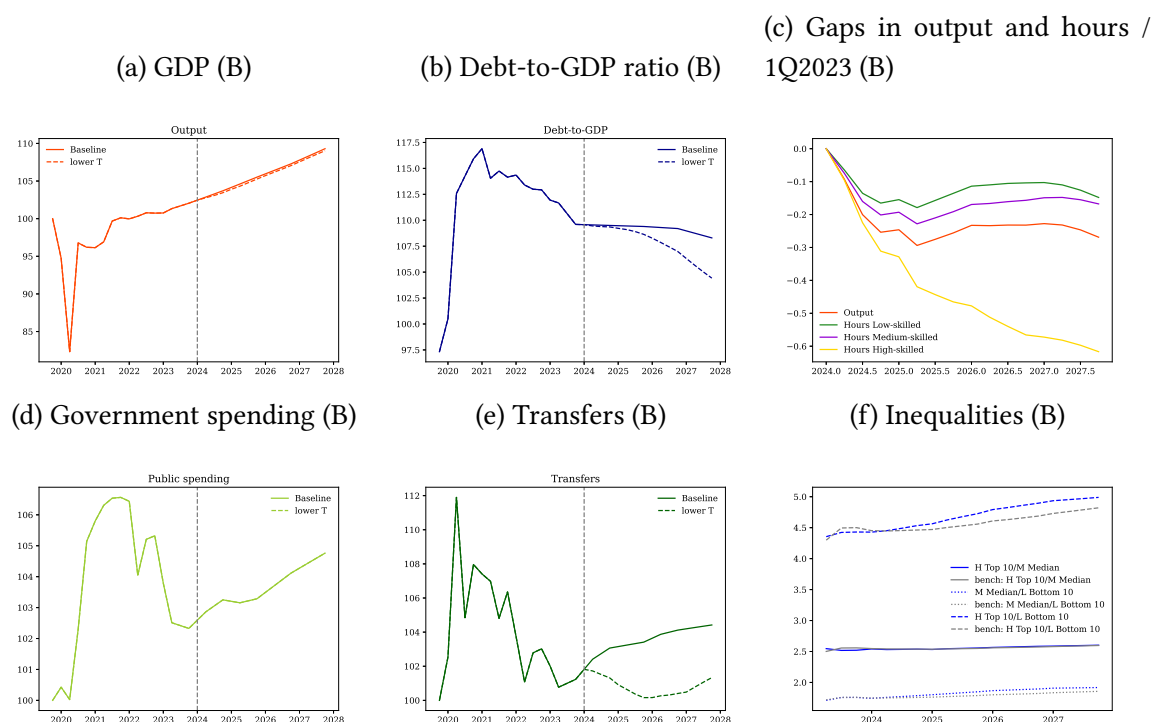


Figure 3.9 shows the consequences of the second program. Unlike the previous program, cuts in public transfers significantly reduce the debt-to-GDP ratio (it falls to 104.4% of GDP in 4Q2027). It is not recessionary. However, economic growth will be slightly lower than projected in the Finance Act (the average annual growth will be 1.59%, versus 1.64% in the Finance Act). Low and middle-skill workers increase their supply of labor, compensating partially for the decrease in income coming from the fall in transfers. The main issue is that this program leads to a sharp rise in inequalities. The extreme consumption-inequality ratio rises to 4.99 in 2027 (versus 4.82 in the Finance Act). Low-skill households are strongly dependent on Beveridgian transfers that represent the majority of their income. Therefore, their fall has a very strong impact on those disadvantaged households. Despite increasing their supply of labor, it is not enough to compensate for the suffered income loss and their consumption falls more sharply than for other households. Thus, despite the strong decrease in public debt, the increase in inequality this program entails would be a considerable obstacle to its social acceptability.

Policy mix. We have seen that those simple policies are not enough to meet our three conditions. As such, we will now try to find whether there is a combination of Bismarckian

and Beveridgian-transfer changes that manage to decrease public debt by at least 5 pp while maintaining output growth and without worsening inequalities.

Figure 3.10: Grid of different combinations of Beveridgian and Bismarckian transfers

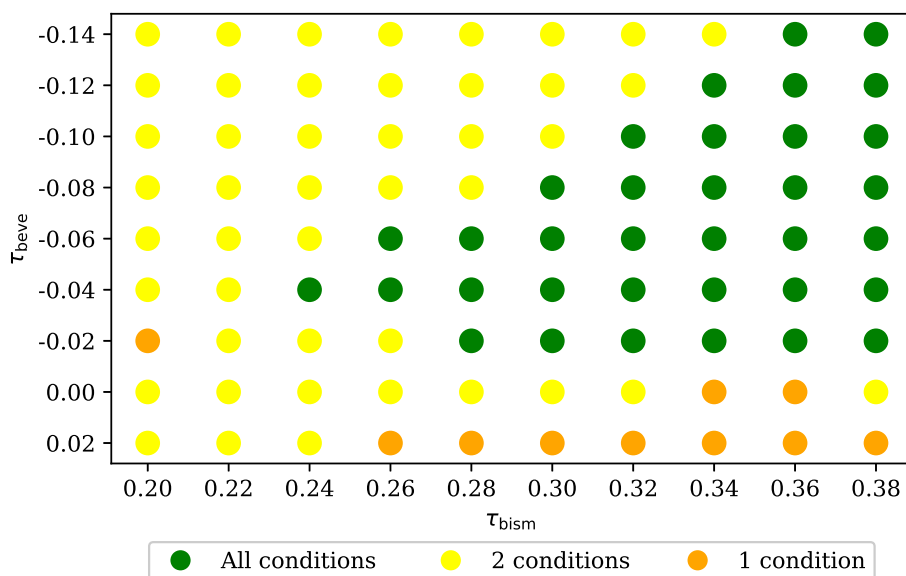
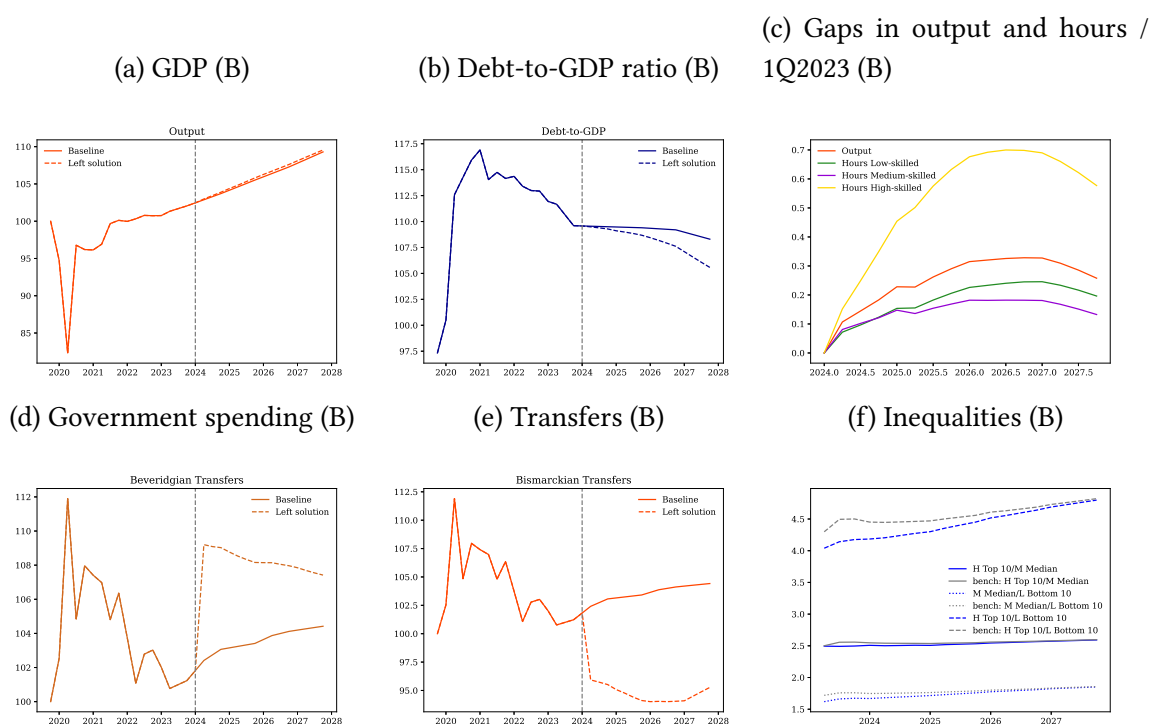


Figure 3.10 shows the results of different combinations of τ^{beve} and τ^{bism} and their consequences for our criteria. As we can see, there are several combinations (indicated in green) that allow us to obtain the required fiscal consolidation conditions. What is necessary to respect our three conditions is an increase in the level of Beveridgian transfers (that decrease with income) combined with a decrease of Bismarckian transfers (that increase with income). Indeed, the decrease in Bismarckian transfers leads to a decrease in the debt level but it must be compensated by a strong enough increase in Beveridgian transfers to avoid a worsening of consumption inequality. We will now show the results for the most left point in the figure that allows obtaining the three conditions (that is for $\tau^{beve} = -0.04$ and $\tau^{bism} = 0.24$) as this is the way to reach our threefold objective while requiring the smallest reforms.

Thus, Figure 3.11 shows the consequences of the third program, based on a stronger reduction in Bismarckian transfers than the second program (€35 billion per year on average), but partially compensated by an increase in Beveridgian transfers (of €15 billion per year). With this program, a larger reduction in the debt-to-GDP ratio than in the benchmark occurs (105.6% of GDP in 2027Q4 versus 108.3% in the benchmark), which is a decrease of exactly 5pp compared to 2023. Economic growth is higher than in the Finance Act (1.69% per year versus 1.64%). Indeed, as transfers decrease, households lose part of their income and decide to work more to compensate for this loss. This change in labor supply occurs for all types of households as Bismarckian transfers fall. However, the incentive is all the stronger the higher the income, because Bismarckian transfers are taxed by a progressive tax: lowering them further reduces the progressiveness of the tax on high incomes who can then compensate for these losses of income by a greater increase in their labor supply. Finally, the increase in Beveridgian transfers –favoring the more disadvantaged households –compensates for the strong decrease in Bismarckian transfers, supporting the consump-

Figure 3.11: Counterfactual: €35 billion reduction in Bismarckian transfers accompanied by €15 billion increase in Beveridgian transfers. (B) after re-estimation.



tion of low-income workers. As such, inequalities follow a similar path as in the Finance Act (4.80 versus 4.82 in 2027 for the extreme consumption-inequality measure). Thus, this program reconciles the three objectives: a sharp reduction in the debt-to-GDP ratio, no change in inequality compared to the benchmark situation, and sustained economic growth. As such, it seems possible to reduce debt without worsening inequalities or damaging economic growth. What matters is the type of transfers to cut. The government must decrease transfers proportional to income as pensions or unemployment insurance.

General versus partial-equilibrium effects

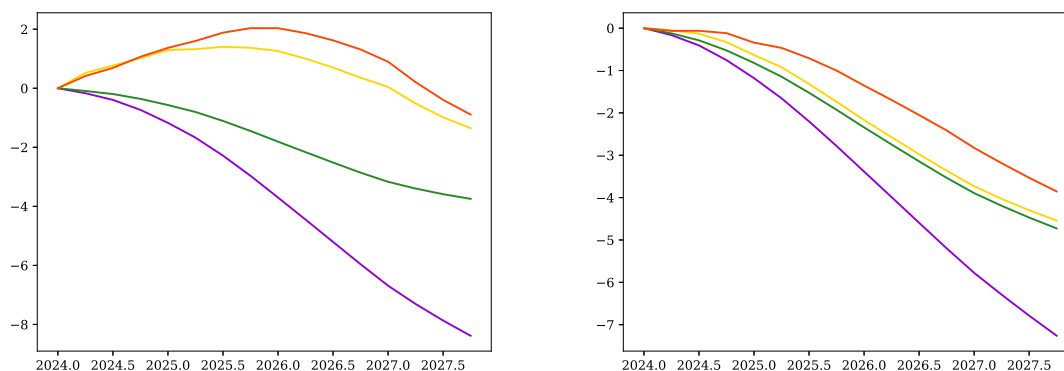
The advantage of our method is to give us the general equilibrium effect of each of the government policies we studied. We can decompose this general-equilibrium effect between different partial-equilibrium effects. Figure 3.12 shows these effects for the debt-to-GDP dynamics for the three policy scenarios. The results are expressed as percentage-point differences with respect to the forecasts of the Finance Act. First, the impact of the decrease in government expenditures on the debt-to-GDP ratio is given by assuming that all the other variables keep their paths deduced from the benchmark scenario (purple lines): this can be viewed as the direct effect of the policy. Second, we add to this decrease in government expenditures the modification in government revenue corresponding with the analyzed policy scenario (green line). Indeed, the decrease in government expenditures can induce a change in public revenue. Third, we compute the debt-to-GDP ratio by also taking into account the changes in GDP dynamics as well as those in public spending and revenue. In

this case, the real interest rate is the only variable that remains as in the benchmark scenario (yellow line). Finally, the general-equilibrium effect takes into account all endogenous changes (orange line).

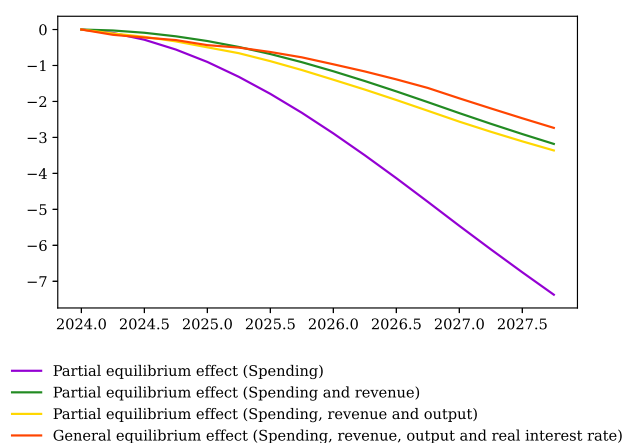
Figure 3.12: Partial versus general equilibrium effects of public policies on debt-to-GDP ratio (absolute difference with respect to the Finance Act forecasts). (B) after re-estimation.

(a) Decrease of public consumption – scenario 1

(b) Decrease of transfers – scenario 2



(c) Decrease of Bismarckian transfers and increase in Beveridgian transfers – scenario 3



Whatever the policy scenario considered, the direct effect of the policy (the sole impact of the decrease in public expenditures) is the largest. It implies a reduction in the debt-to-GDP ratio with respect to the benchmark scenario of more than 7 percentage points for all policies. For the policy decreasing Bismarckian transfers and increasing Beveridgian transfers (scenario 3), this 7 pp difference means that this partial-equilibrium effect is about four times bigger than the general-equilibrium effect (see panel (c) of Figure 3.12). When taking into account the implied change in government revenue, the impact on debt diminishes, the reduction of the debt-to-GDP ratio thus being limited between -3 pp and -5 pp. In scenarios 1 and 2, the revenue falls because the activity is reducing. For scenario 3, activity is maintained, but the reduction in Bismarckian transfers decreases the income taxes perceived by the government whereas the increase in Beveridgian transfers increases the cost of this policy. When changes in GDP dynamics are taken into account, the decrease of the

debt-to-GDP ratio is lessened for scenario 1 as the decrease in government consumption is strongly contractionary. For the policy that reduces transfers homogeneously (scenario 2), the impact on the debt-to-GDP ratio is slightly lessened as this policy slightly reduces GDP. The opposite occurs for scenario 3 as GDP growth is larger than in the benchmark, meaning that taking into account GDP adjustment reduces the debt-to-GDP ratio (from -3 pp to -3.3 pp difference with respect to the baseline). Finally, taking into account interest rate adjustments, the last component to consider to obtain general equilibrium, shows that all these policies to reduce public spending lead to an increase in real interest rates in the short term. Indeed, these policies reduce inflation. But this reduction is integrated with a delay by the ECB, which leads the real interest rate to increase in the short term. As the reduction of inflation is of similar magnitude for the three scenarios, this last effect is of similar size.

3.5.5 Stochastic Debt-Sustainability Analysis

In this section, we present forecasts of the debt-to-GDP ratio conditional on the economic policy pursued.³³ We make forecasts by drawing 1000 realizations from the estimated distributions of all shocks. We then plot the forecast paths of the debt-to-GDP ratio and calculate the values of $b_{t+n}^\alpha(x, y)$ for $\alpha = \{16.7, 25, 50, 75, 83.3\}$ in percentage.

Table 3.6: Debt-at-risk $b_{t+n}^\alpha(x)$ conditional to the policy x with (a) the benchmark policy (Finance Act), (b) Scenario 1, (c) Scenario 2, and (d) Scenario 3.

Debt-to-GDP values in 4Q2027

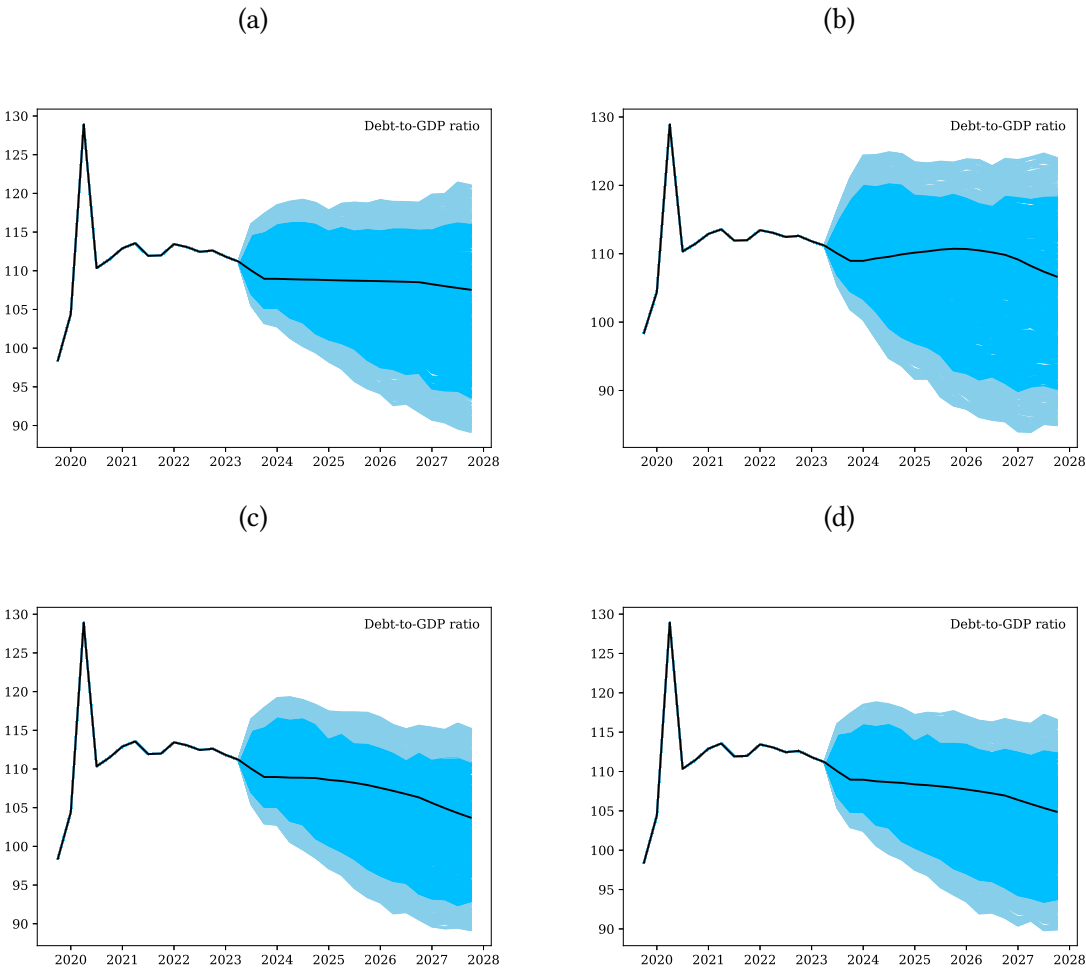
Policy x	Policy forecast	Distribution of forecasts for percentiles α				
		16.7%	25%	50%	75%	83.3%
(a) Benchmark (Finance Act)	108.3%	89.2%	93.7%	104.7%	116.2%	121.3%
(b) Scenario 1	107.9%	84.9%	90.2%	104.5%	118.5%	124.2%
(c) Scenario 2	104.4%	89.2%	92.9%	101.3%	111.0%	115.4%
(d) Scenario 3	105.6%	89.9%	93.8%	102.7%	112.6%	116.6%

Panel (a) of Figure 3.13 shows the distribution of debt-to-GDP ratio (in shapes of blue) as well as the trajectory planned by the government (in the black line) when policy $\mathcal{P}_t(a)$ is implemented (Finance Act). The path planned by the government is not in the middle of the distribution: the median of the forecast is 104.7% while the government forecasted 108.3%. This means that the government is rather pessimistic in the sense that its debt-to-GDP forecast tends to under-estimate this ratio. According to our model, there is a 25% chance that debt will exceed the value of 116.2% (debt at risk at the 25% threshold is $b_{t+n}^{0.75}(\mathcal{P}_t) = 116.2\%$). There is also a 25% chance that government debt will fall below 93.7% of GDP by 2027.

We now compare the policy $\mathcal{P}_t(0)$ associated with the Finance Act with alternative policies $\mathcal{P}_t(x)$, for $x \in \{1, 2, 3\}$. Results reported in Figure 3.13 and Table 3.6 make it possible

³³See Section 3.7.8 for the unconditional forecasts of all macroeconomic variables considered in the estimation.

Figure 3.13: Distribution of debt-to-GDP forecasts conditional to the policy (in blue) and debt-to-GDP planned in the policy scenario (in black). Panel (a): Benchmark (Finance Act). Panel (b): Scenario 1. Panel (c): Scenario 2. Panel (d): Scenario 3. Shaded color corresponds to the distribution percentiles [16.7; 83.3], [25; 75] in percentage.



to assess how fiscal consolidation programs reduce the risk burden on debt. For scenario 1, committing to a more intensive debt-reduction path through public consumption adjustment only increases the risk to the debt. Indeed, the decrease in government consumption is strongly recessionary, leading to a strong increase in the debt-to-GDP ratio in the short term. This increases the size of the variations along the path. Therefore, the interquartile risk increases from 22.5pp in the baseline to 28.3pp. There is a 25% probability that debt exceeds 118.5% of GDP at the end of 2027, meaning that scenario 1 increases the debt-to-risk value (at the 25% level of risk) by $\Delta_{t+20}^{0.75}(0, 1) = 2.3\text{pp}$. On the contrary, scenarios 2 and 3 decrease the risk of debt. Decreasing transfers have no recessionary impact, leading to a decrease of the interquartile risk that reaches 18.1 and 18.8pp for scenarios 2 and 3 respectively. They also logically reduce the debt-at-risk value. Scenario 2 reduces it by $\Delta_{t+20}^{0.75}(0, 2) = -5.2$ percentage points while scenario 3 reduces it by $\Delta_{t+20}^{0.75}(0, 3) = -3.6$ percentage points.

3.6 Conclusion

We develop a methodology that rigorously assesses the debt sustainability implied by Finance Acts and evaluates the implications of alternative fiscal consolidation programs. Using the conditional forecasts of a HANK model, we identify all future shocks consistent with the Finance Act proposed by the government. Then, we measure the contribution of fiscal decisions in the projections and compare it to the contributions of tailwinds and headwinds, thus revealing the optimism or pessimism of the forecast. The uncertainties about these macroeconomic projections may cast doubt on France's ability to reduce its public debt. We therefore compare alternative fiscal consolidation programs to the one proposed in the government Finance Act. The results show that fiscal consolidation via a reduction in public consumption has the disadvantage of generating significant recessionary effects, unlike fiscal consolidation via a reduction in public transfers. The choice of which transfers are subject to cuts is of crucial importance in terms of consumption inequalities between households. We show that it is possible to reduce debt and support growth without increasing inequality if the reduction concerns insurance transfers (of the Bismarckian type) are partially compensated by increasing assistance transfers (of the Beveridgian type). Our analysis also shows the importance of studying debt sustainability in general equilibrium: fiscal policies should not be decided only looking at partial equilibrium effects. In all studied cases, the impact on debt in partial equilibriums exceeds largely their impacts measured in general equilibrium.

3.7 Appendix

3.7.1 Union and Phillips Curve

A union sets a unique nominal wage W_k^s by task k , $\forall s \in \{l, m, h\}$. The union's program is:

$$U_{k,t}^s(W_{k,t-1}^s) = \max_{W_{k,t}^s} \left\{ \int_{e,a_-} u(c^s(e, a_-)) - v(n_k^s(e, a_-)) d\Gamma^s(e, a_-) \right. \\ \left. - \frac{\Psi^s W}{2} \left(\frac{W_{k,t}^s}{W_{k,t-1}^s} - 1 \right)^2 + \beta U_{k,t+1}^s(W_{k,t}^s) \right\}$$

$$\text{s.t. } N_{k,t}^s = \left(\frac{W_{k,t}^s}{W_t^s} \right)^{-\varepsilon_s} N_t^s$$

$$\Leftrightarrow U_{k,t}^s(W_{k,t-1}^s) = \max_{W_{k,t}^s} \left\{ \int_{e,a_-} u(c^s(e, a_-)) - v(n_k^s(e, a_-)) d\Gamma^s(e, a_-) \right. \\ \left. - \frac{\Psi^s W}{2} \left(\frac{W_{k,t}^s}{W_{k,t-1}^s} - 1 \right)^2 \left(\frac{W_{k,t}^s}{W_t^s} \right)^{-\varepsilon_s} + \beta U_{k,t+1}^s(W_{k,t}^s) \right\}$$

where $W_t^s = \left(\int_k (W_{k,t}^s)^{1-\varepsilon_s} dk \right)^{\frac{1}{1-\varepsilon_s}}$ and $N_t^s = \left(\int_k (N_{k,t}^s)^{\frac{\varepsilon_s-1}{\varepsilon_s}} dk \right)^{\frac{\varepsilon_s}{\varepsilon_s-1}}$. Furthermore, the union k imposes that every worker works the same amount of hours, so that: $n_k^s = N_k^s$. An agent of type s :

- Provides $n_{k,t}^s$ hours of work for the task k and a total number of hours $n^s = \int n_{k,t}^s dk$
- Gets income $z_t^s = \int_k (1 - \lambda_{inc}) ((1 - \tau_{ssc}) W_{k,t}^s e_t n_{k,t}^s + A)^{1-\tau_{inc}} dk$
- Has before tax income $\tilde{z}_t^s = \int_k ((1 - \tau_{ssc}) W_{k,t}^s e_t n_{k,t}^s dk + A$
- Pays the progressive tax $\tilde{z}_t^s - z_t^s$
- where $A = (1 - \tau_f)d(e) + \bar{T}(e)$

$$z_t^s = \int_k (1 - \lambda_{inc}) ((1 - \tau_{ssc}) W_{k,t}^s e_t n_{k,t}^s + A)^{1-\tau_{inc}} dk \\ = \int_k (1 - \lambda_{inc}) \left((1 - \tau_{ssc}) W_{k,t}^s e_t \left(\frac{W_{k,t}^s}{W_t^s} \right)^{-\varepsilon_s} N_t^s + A \right)^{1-\tau_{inc}} dk$$

FOC w.r.t W_k^s of the labor union's problem:

$$0 = \underbrace{\frac{\partial}{\partial W_k^s} \left(\int_{e,a_-} u(c^s(e, a_-)) - v(n_k^s(e, a_-)) d\Gamma^s(e, a_-) \right)}_{\text{part 1}} \\ - \Psi^s W \frac{1}{W_{k,t-1}^s} \left(\frac{W_{k,t}^s}{W_{k,t-1}^s} - 1 \right) + \beta \underbrace{\frac{\partial U_{k,t+1}^s(W_k^s)}{\partial W_k^s}}_{\text{part 2}}$$

Part 1.

$$\begin{aligned}
 \frac{\partial}{\partial W_k^s} \left(\int_{e,a_-} u(c^s(e, a_-)) - v(n_k^s(e, a_-)) d\Gamma(e, a_-) \right) &= \int_{e,a_-} \left(\frac{\partial u}{\partial c^s} \frac{\partial c^s}{\partial W_k^s} - \frac{\partial v}{\partial n_k^s} \frac{\partial n_k^s}{\partial W_k^s} \right) d\Gamma^s(e, a_-) \\
 &= \int_{e,a_-} \left(u'(c^s) \frac{\partial c^s}{\partial W_k^s} - v'(n_k^s) \frac{\partial n_k^s}{\partial W_k^s} \right) d\Gamma^s(e, a_-) \\
 &= \int_{e,a_-} \left(u'(c^s) \frac{\partial c^s}{\partial W_k^s} + v'(n_k^s) \varepsilon_s \frac{n_k^s}{W_k^s} \right) d\Gamma^s(e, a_-)
 \end{aligned}$$

Indeed,

$$\frac{\partial n_k^s}{\partial W_k^s} = \frac{\partial}{\partial W_k^s} \left(\left(\frac{W_{k,t}^s}{W_t^s} \right)^{-\varepsilon_s} N_t^s \right) = -\varepsilon_s \frac{N_t^s}{W_k^s} \left(\frac{W_{k,t}^s}{W_t^s} \right)^{-\varepsilon_s} = -\varepsilon_s \frac{N_k^s}{W_k^s}$$

and we have

$$\begin{aligned}
 \frac{\partial c^s}{\partial W_k^s} &= \frac{1}{(1 + \tau_c) P_t} \frac{\partial z^s}{\partial W_k^s} \\
 &= \frac{1}{1 + \tau_c} \frac{\partial}{\partial W_k^s} \int_k (1 - \lambda_{inc}) \left((1 - \tau_{ssc}) W_{k,t}^s e_t n_{k,t}^s + A \right)^{1 - \tau_{inc}} dk \\
 &= \frac{1}{(1 + \tau_c) P_t} \int_k (1 - \lambda_{inc}) (1 - \tau_{inc}) (1 - \tau_{ssc}) \left(e_t n_{k,t}^s + W_{k,t}^s e_t \frac{\partial n_k^s}{\partial W_k^s} \right) \left((1 - \tau_{ssc}) W_{k,t}^s e_t n_{k,t}^s + A \right)^{-\tau_{inc}} dk \\
 &= \frac{1}{(1 + \tau_c) P_t} \int_k (1 - \lambda_{inc}) (1 - \tau_{inc}) (1 - \tau_{ssc}) \left(e_t n_{k,t}^s - \varepsilon_s W_{k,t}^s e_t \frac{N_k^s}{W_k^s} \right) \left((1 - \tau_{ssc}) W_{k,t}^s e_t n_{k,t}^s + A \right)^{-\tau_{inc}} dk \\
 &= \frac{1}{(1 + \tau_c) P_t} \int_k (1 - \lambda_{inc}) (1 - \tau_{inc}) (1 - \tau_{ssc}) (1 - \varepsilon_s) e_t n_{k,t}^s \left((1 - \tau_{ssc}) W_{k,t}^s e_t n_{k,t}^s + A \right)^{-\tau_{inc}} dk
 \end{aligned}$$

Part 2. Envelope theorem: $\frac{\partial U_{k,t+1}^s}{\partial W_k^s}(W_k^s) = \Psi_W^s \left(\frac{W_{k,t+1}^s}{W_{k,t}^s} - 1 \right) \frac{W_{k,t+1}^s}{(W_{k,t}^s)^2} = \Psi_W^s \frac{1}{W_{k,t}^s} \left(\frac{W_{k,t+1}^s}{W_{k,t}^s} - 1 \right) \frac{W_{k,t+1}^s}{W_{k,t}^s}$

At the symmetric equilibrium, we have $W_k^s = W_{k'}^s = W^s$ and $n_k^s = n_{k'}^s = N^s \forall k, k'$

$$\frac{\partial c^s}{\partial W_k^s} = \frac{1}{(1 + \tau_c) P_t} (1 - \tau_{inc}) (1 - \tau_{ssc}) (1 - \varepsilon_s) e_t N_t^s \underbrace{\left((1 - \tau_{ssc}) W_t^s e_t N_t^s + A \right)^{-\tau_{inc}}}_{1 - \text{average tax rate} = 1 - \bar{\tau}_t^s}$$

Using $\pi_{W,t}^s = \frac{W_t^s}{W_{t-1}^s} - 1$, the FOC rewrites:

$$\begin{aligned}
 0 &= \frac{1}{(1 + \tau_c) P_t} (1 - \tau_{inc}) (1 - \tau_{ssc}) (1 - \varepsilon_s) N_t^s \int_{e,a_-} u'(c^s) e_t (1 - \bar{\tau}_t^s) d\Gamma^s(e, a_-) \\
 &\quad + v'(N^s) \frac{N^s}{W^s} \varepsilon_s - \Psi_W^s \frac{1}{W_{t-1}^s} \pi_{W,t}^s + \beta \Psi_W^s \pi_{W,t+1}^s (1 + \pi_{W,t+1}^s) \frac{1}{W_t^s}
 \end{aligned}$$

Notation. Assume that we can compute $\tilde{u}'_t(c^s, \bar{\mathcal{T}}^s) \equiv \int_{e, a_-} u'(c^s) e_t (1 - \bar{\mathcal{T}}_t^s) d\Gamma^s(e, a_-)$, therefore we deduce:

$$\begin{aligned} 0 &= \frac{1}{(1 + \tau_c) P_t} (1 - \tau_{inc})(1 - \tau_{ssc})(1 - \varepsilon_s) N_t^s \tilde{u}'_t(c^s, \bar{\mathcal{T}}^s) + v'(N_t^s) \frac{N_t^s}{W_t^s} \varepsilon_s \\ &\quad - \Psi_W^s \frac{1}{W_{t-1}^s} \pi_{W,t}^s + \beta \Psi_W^s \frac{1}{W_t^s} \pi_{W,t+1}^s (1 + \pi_{W,t+1}^s) \\ \Leftrightarrow 0 &= (1 - \varepsilon_s) \frac{(1 - \tau_{inc})(1 - \tau_{ssc})}{(1 + \tau_c) P_t} W_t^s N_t^s \tilde{u}'_t(c^s, \bar{\mathcal{T}}^s) + v'(N_t^s) N_t^s \varepsilon_s \\ &\quad - \Psi_W^s (1 + \pi_{W,t}^s) \pi_{W,t}^s + \beta \Psi_W^s \pi_{W,t+1}^s (1 + \pi_{W,t+1}^s) \end{aligned}$$

The resulting New-Keynesian Phillips curve for sector s is:

$$\begin{aligned} \pi_{W,t}^s &= \frac{\varepsilon_s}{\Psi_W^s} \left[\frac{1 - \varepsilon_s}{\varepsilon_s} \frac{(1 - \tau_{inc})(1 - \tau_{ssc})}{(1 + \tau_c) P_t} W_t^s N_t^s \tilde{u}'_t(c^s, \bar{\mathcal{T}}^s) + v'(N_t^s) N_t^s \right] + \beta \pi_{W,t+1}^s \\ &= \kappa_W^s \left(N_t^s v'(N_t^s) - \frac{1}{\mu_w^s} t d_t^s \frac{W_t^s}{P_t} N_t^s \tilde{u}'_t(c^s, \bar{\mathcal{T}}^s) \right) + \beta \pi_{W,t+1}^s \end{aligned}$$

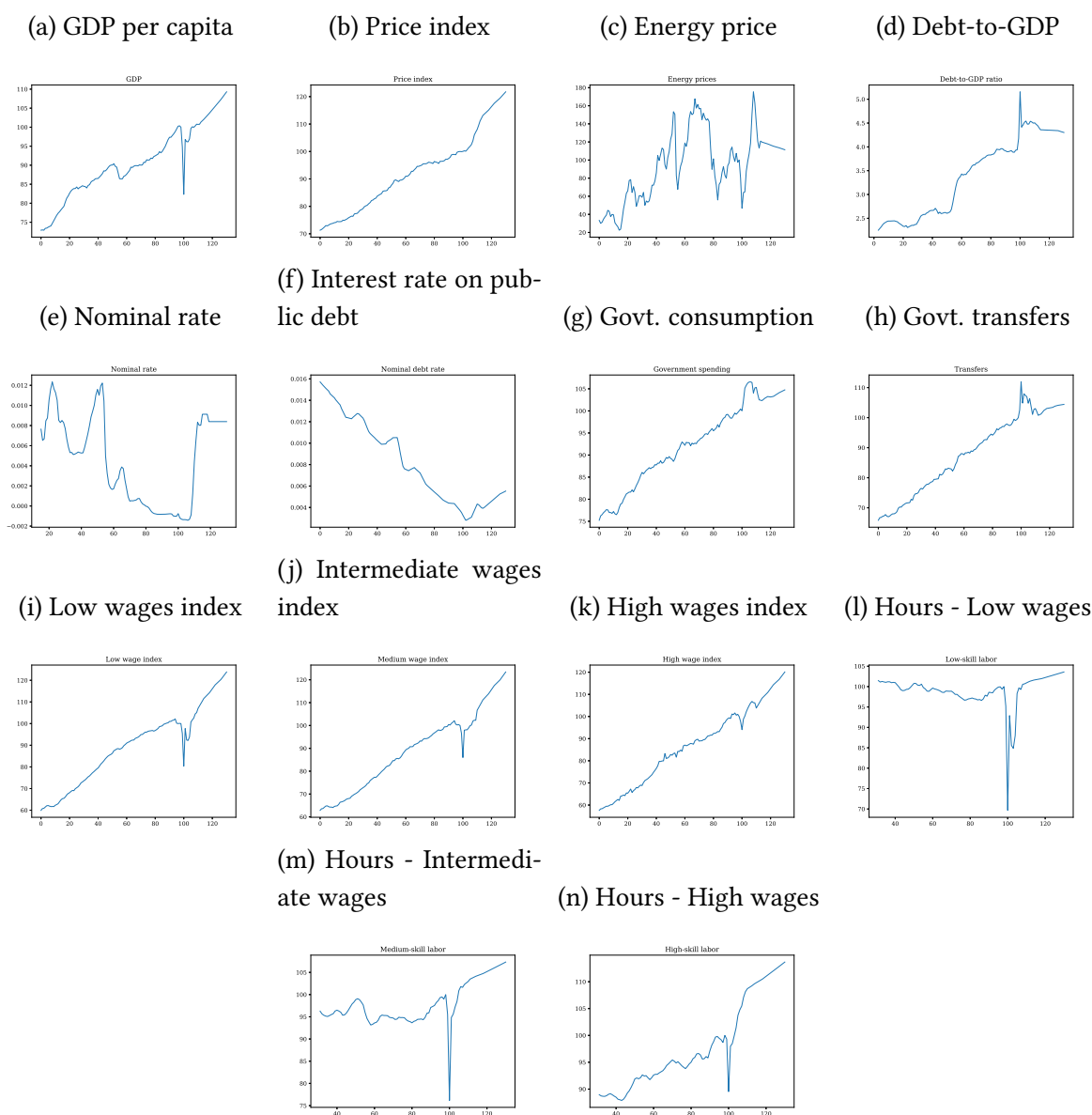
with tax distortion $t d_t^s = \frac{(1 - \tau_{inc})(1 - \tau_{ssc})}{1 + \tau_c}$, $\mu_w^s = \frac{\varepsilon_s}{\varepsilon_s - 1}$, $\kappa_W = \frac{\varepsilon_s}{\Psi_W^s}$, $\pi_W (1 + \pi_W) \approx \pi_W$ and the average level of e is 1 leading to $W_t^s e_t N_t^s = W_t^s N_t^s$.

3.7.2 Data, Steady State and Calibrations

Raw data

All the raw series of Table 3.7 are quarterly and range from 2Q1995 to 4Q2021, except for Euribor which starts in 1999, and the employment rate, available only from 2Q2003. For the population and interest charges, which are annual series, we build quarterly series by interpolation. Some series are divided by the population to obtain per-capita variables: $\{Y, \frac{b}{Y}, G, T\}$. The consumer-price index series is monthly. It is quarterlyized using a moving average, from which we derive π . The energy price (P_{FE}) is the crude oil price in Euro. Finally, the data of hours and wages by worker types are constructed using data described in Appendix 3.7.2. All times series are plotted in Figure 3.14.

Figure 3.14: Raw data (100 in 4Q2019)



Data	Web access	Providers
Population	DBnomics code	Eurostat
GDP	DBnomics code	Eurostat
CPI	DBnomics code	INSEE
Energy price	DBnomics code	INSEE
Government consumption	DBnomics code	Eurostat
Government transfers	DBnomics code	Eurostat
Public debt	DBnomics code	Eurostat
Employment rate	DBnomics code	INSEE
Employment in Agriculture	DBnomics code	Eurostat
Employment in Wholesale and Retail Trade	DBnomics code	Eurostat
Employment in Construction	DBnomics code	Eurostat
Employment in Real estate	DBnomics code	Eurostat
Employment in Science and Administration	DBnomics code	Eurostat
Employment in Industry	DBnomics code	Eurostat
Employment in Finance	DBnomics code	Eurostat
Employment in Information and Communication	DBnomics code	Eurostat
Total Compensation in Agriculture	DBnomics code	Eurostat
Total Compensation in Wholesale and Retail Trade	DBnomics code	Eurostat
Total Compensation in Construction	DBnomics code	Eurostat
Total Compensation in Real estate	DBnomics code	Eurostat
Total Compensation in Science and Administration	DBnomics code	Eurostat
Total Compensation in Industry	DBnomics code	Eurostat
Total Compensation in Finance	DBnomics code	Eurostat
Total Compensation in Information and Communication	DBnomics code	Eurostat
Total hours in Agriculture	DBnomics code	Eurostat
Total hours in Wholesale and Retail Trade	DBnomics code	Eurostat
Total hours in Construction	DBnomics code	Eurostat
Total hours in Real estate	DBnomics code	Eurostat
Total hours in Science and Administration	DBnomics code	Eurostat
Total hours in Industry	DBnomics code	Eurostat
Total hours in Finance	DBnomics code	Eurostat
Total hours in Information and Communication	DBnomics code	Eurostat
Euribor	DBnomics code	BdF
Interest charges	DBnomics code	AMECO

Table 3.7: Data sources

For the estimation, the time series of per-capita GDP, price index, Govt. consumption, Govt. transfers, and wages are stationarized around a linear trend. The other times series are demeaned by their average over the sample.

Energy Market

Data for consumption gives us the distribution of consumption by type of goods. We consider that the categories "Housing, water, gas, electricity and other fuels" ("Logement, eau, gaz, électricité et autres combustibles") and "fuels and lubricants, antifreeze" ("Carburants et lubrifiants, antigel") belong to the energy part of consumption. Making the assumption that the demand for energy is equally shared between households and firms, we obtain that the shares of firm and household energy demand over GDP are each 1.59%.

Assuming further that $\sigma_f = \eta_E$, $Z = 1$ and $s_H = s_F = 0$. Then, using $mc = 1/\mu$, we can set $mc_F = mc$ and $p_F = mc_F$. Hence, we deduce

$$Y_{FE} = \alpha_E \left(\frac{p_{FE}}{p_F} \right)^{-\eta_E} Y \quad \Leftrightarrow \quad \frac{p_{FE} Y_{FE}}{Y} = \alpha_E \left(\frac{1}{p_F} \right)^{-\eta_E} p_{FE}^{1-\eta_E}$$

$$E = \alpha_f (1 - \alpha_E) \left(\frac{p_{FE}}{p_F} \right)^{-\eta_E} Y \quad \Leftrightarrow \quad \frac{p_{FE} E}{Y} = \alpha_f (1 - \alpha_E) \left(\frac{1}{p_F} \right)^{-\eta_E} p_{FE}^{1-\eta_E}$$

which gives us:

$$\alpha_f = \frac{\alpha_E \frac{p_{FE} E}{Y}}{1 - \alpha_E \frac{p_{FE} Y_{FE}}{Y}}$$

$$p_{FE} = \left(\frac{(\frac{p_{FE} Y_{FE}}{Y}) + \frac{p_{FE} E}{Y}}{\alpha_E + \alpha_f (1 - \alpha_E)} mc^{-\eta_E} \right)^{\frac{1}{1-\eta_E}}$$

Labor Market.

Figure 3.15: Raw labor-market statistics

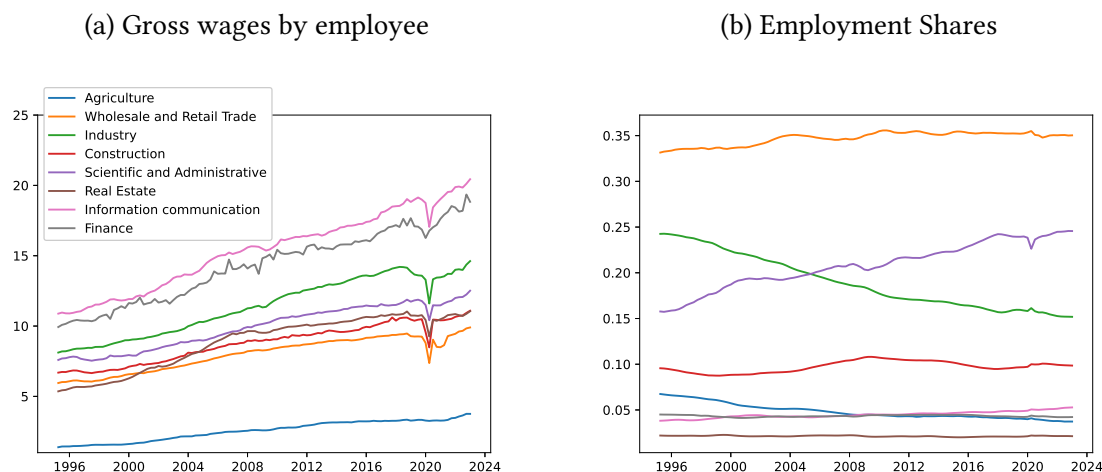
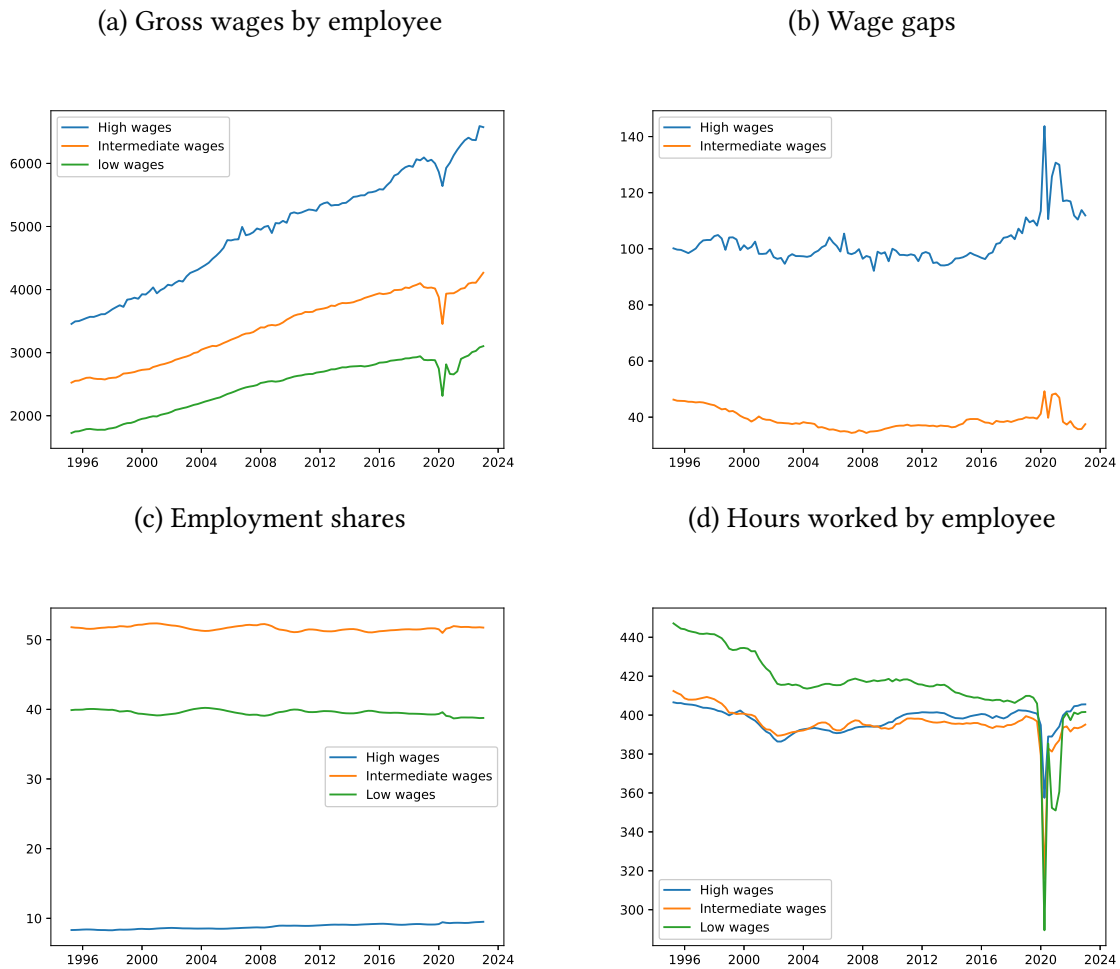


Figure 3.15 shows the employment shares and gross wage by sector of the French economy. For our model, we divide the labor market into three submarkets, $s = l, m, h$:

- the submarket of high wages ($s = h$): Information Communication + Finance
- the submarket of intermediate wages ($s = m$): Industry + Construction + Scientific and Administrative + Real Estate
- the submarket of low wages ($s = l$): Wholesale and Retail Trade + Agriculture

Figure 3.16: Labor-market statistics



We obtain the distributions shown in Figure 3.16. Between 1Q2003 and 4Q2019, the employment rate were $\tilde{N}_L = 25.5\%$, $\tilde{N}_M = 32.5\%$ and $\tilde{N}_H = 5\%$. Therefore, the aggregate employment rate is $\sum_s \tilde{N}_s = \tilde{N} = 63\%$. This allows to deduce the relative employment sizes s : $\tilde{N}_s/\tilde{N} \in \{40.5; 51.5; 8\}\%$.

To determine the employment rate by s , we need to use the participation rates and the employment rates for each s . We approximate these rates by assuming that low-wage workers have a diploma lower than the “baccalauréat”, those with an intermediate wage have

Sector s	L	M	H	Mean
Shares (\tilde{N}_s/\tilde{N})	45.2%	48.6%	6.2%	-
Wages	1	1.4	2	1.3
Employment rates	56%	66.25%	81%	63%
Hours worked	1	0.98	1.02	0.99
\hat{N}_s (employment rates \times Share)	25.3	31.6	5.1	-

Table 3.8: Labor-market statistics

a diploma between the “baccalauréat” and two years of higher education, and high-wage workers have a bachelor or more. Data for the participation rates, and data for unemployment lead to

- Participation rates (1985-2016): $P_l = 71\%$, $P_m = 76.25\%$ and $P_h = 87.0\%$
- Unemployment rates (2016): $U_l = 15\%$, $U_m = 10\%$ and $U_h = 6\%$.

This leads to employment rates equal to $N_l = 56\%$, $N_m = 66.25\%$ and $N_h = 81\%$, consistent with a total employment rate equal to 63% with the shares N_s/N .

Beyond the employment rate (N), we also use the number of hours worked by workers (h) to build the aggregate hours $Nh = N \times h$.

- Information provided by the data
 - the values of $\{\omega^s, n^s\} \forall s$. Given that efficient labor is $N^s = \sum_i \omega^s \pi_i^s e_i^s n_i^s$, where ω^s is the size of each population, $\sum_i \pi_i^s e_i^s = \varpi^s$ is the average productivity of each population, with π_i^s the mass of type- s workers with productivity e_i . $n^s(i) = n^s(i') \equiv n^s, \forall i, i'$ is the homogenous aggregate hours worked $\forall s$ (restriction implied by unions) by each population, we deduce that $N^s = \varpi^s \omega^s n^s = \varpi^s \hat{n}^s$.
 - the relative wages $\{1, l^m, l^h\}$
- Unknown parameters determined by steady-state restrictions: $\{\alpha_s, \varpi_s\}_{s=l,m,h}$

The production function is

$$\begin{aligned}
 Y_N &= \left(\alpha_l^{\frac{1}{\varepsilon_N}} (\varpi^l \omega^l n^l)^{\frac{\varepsilon_N-1}{\varepsilon_N}} + \alpha_m^{\frac{1}{\varepsilon_N}} (\varpi^m \omega^m n^m)^{\frac{\varepsilon_N-1}{\varepsilon_N}} + \alpha_h^{\frac{1}{\varepsilon_N}} (\varpi^h \omega^h n^h)^{\frac{\varepsilon_N-1}{\varepsilon_N}} \right)^{\frac{\varepsilon_N}{\varepsilon_N-1}} \\
 &= \left(\alpha_l^{\frac{1}{\varepsilon_N}} (\varpi^l \hat{n}^l)^{\frac{\varepsilon_N-1}{\varepsilon_N}} + \alpha_m^{\frac{1}{\varepsilon_N}} (\varpi^m \hat{n}^m)^{\frac{\varepsilon_N-1}{\varepsilon_N}} + \alpha_h^{\frac{1}{\varepsilon_N}} (\varpi^h \hat{n}^h)^{\frac{\varepsilon_N-1}{\varepsilon_N}} \right)^{\frac{\varepsilon_N}{\varepsilon_N-1}}
 \end{aligned}$$

where it is assumed that there are no TFP shocks at the steady state: $A^s = 1 \forall s$. The entrepreneur controls the aggregate hours by skill \hat{n}^s . The firm's FOC are, $\forall s \in \{l, m, h\}$:

$$\begin{aligned} \frac{\partial Y_N}{\partial \hat{n}^s} &= \alpha_s^{\frac{1}{\varepsilon_N}} \varpi^s (\varpi^s \hat{n}^s)^{\frac{\varepsilon_N-1}{\varepsilon_N}-1} \left(\alpha_l^{\frac{1}{\varepsilon_N}} (\varpi^l \hat{n}^l)^{\frac{\varepsilon_N-1}{\varepsilon_N}} + \alpha_m^{\frac{1}{\varepsilon_N}} (\varpi^m \hat{n}^m)^{\frac{\varepsilon_N-1}{\varepsilon_N}} + \alpha_h^{\frac{1}{\varepsilon_N}} (\varpi^h \hat{n}^h)^{\frac{\varepsilon_N-1}{\varepsilon_N}} \right)^{\frac{\varepsilon_N}{\varepsilon_N-1}-1} \\ &= \frac{W^s}{W} \\ \Rightarrow \hat{n}^s &= \alpha_s (\varpi^s)^{\varepsilon_N-1} \left(\frac{W^s}{W} \right)^{-\varepsilon_N} Y_N \end{aligned}$$

where W is the total marginal cost:

$$\begin{aligned} W = MC_N &= \left(\sum_s \alpha_s \left(\frac{W^s}{\varpi^s} \right)^{1-\varepsilon_N} \right)^{\frac{1}{1-\varepsilon_N}} \\ &= W_l \left(\alpha_l \left(\frac{1}{\varpi^l} \right)^{1-\varepsilon_N} + \alpha_m \left(\frac{l^m}{\varpi^m} \right)^{1-\varepsilon_N} + \alpha_h \left(\frac{l^h}{\varpi^h} \right)^{1-\varepsilon_N} \right)^{\frac{1}{1-\varepsilon_N}} \\ &\equiv W_l \Gamma(\alpha_l, \alpha_m, \alpha_h, \varpi^l, \varpi^m, \varpi^h) \end{aligned}$$

This leads to

$$\begin{aligned} \hat{n}^s &= \alpha_s (\varpi^s)^{\varepsilon_N-1} \left(\frac{W^s}{W_l \Gamma(\alpha_l, \alpha_m, \alpha_h, \varpi^l, \varpi^m, \varpi^h)} \right)^{-\varepsilon_N} Y_N \\ &= \alpha_s (\varpi^s)^{\varepsilon_N-1} \Gamma(\alpha_l, \alpha_m, \alpha_h, \varpi^l, \varpi^m, \varpi^h)^{\varepsilon_N} \left(\frac{W^s}{W_l} \right)^{-\varepsilon_N} Y_N \end{aligned}$$

Therefore, we deduce

$$\hat{n}^l = \alpha_l (\varpi^l)^{\varepsilon_N-1} \Gamma(\alpha_l, \alpha_m, \alpha_h, \varpi^l, \varpi^m, \varpi^h)^{-\varepsilon_N} Y_N \quad (3.6)$$

$$\hat{n}^m = \alpha_m (\varpi^m)^{\varepsilon_N-1} (l^m)^{-\varepsilon_N} \Gamma(\alpha_l, \alpha_m, \alpha_h, \varpi^l, \varpi^m, \varpi^h)^{-\varepsilon_N} Y_N \quad (3.7)$$

$$\hat{n}^h = \alpha_h (\varpi^h)^{\varepsilon_N-1} (l^h)^{-\varepsilon_N} \Gamma(\alpha_l, \alpha_m, \alpha_h, \varpi^l, \varpi^m, \varpi^h)^{-\varepsilon_N} Y_N \quad (3.8)$$

where $\{\varpi^l, \varpi^m, \varpi^h, \alpha_l, \alpha_m, \alpha_h\}$ are the 6 unknowns. Remark that the homogeneity of the production function implies that

$$\sum_s W_s n^s = W Y_N \Rightarrow Y_N = \frac{1}{\Gamma} (n^l + l^m n^m + l^h n^h)$$

this equation being a linear combination of equations (3.6), (3.7) and (3.8). **Thus, we have 6 unknowns for only 3 independent equations (3.6), (3.7) and (3.8).**

Additional restrictions

1. $\sum_s \alpha_s = 1 \Rightarrow \alpha_h = 1 - \alpha_l - \alpha_m$
2. $\varpi_m = 1$ because we only observe 2 relative wages, $\{W_m/W_l, W_h/W_l\}$

3. $\sum_s \omega^s \varpi^s = 1 \Rightarrow \varpi^l = \frac{1 - \omega^m \varpi^m - \omega^h \varpi^h}{\omega^l}$. With 2. this leads to $\varpi^h = \frac{1 - \omega^l \varpi^l - \omega^m}{\omega^h}$
 \Rightarrow **The remaining 3 unknowns are** $\{\varpi^l, \alpha_l, \alpha_m\}$.

Using the definition of the function Γ , we deduce

$$\Gamma(\alpha_l, \alpha_m, \alpha_h, \varpi^l, \varpi^m, \varpi^h) = \left(\alpha_l \left(\frac{1}{\varpi^l} \right)^{1-\varepsilon_N} + \alpha_m \left(\frac{l^m}{\varpi^m} \right)^{1-\varepsilon_N} + \alpha_h \left(\frac{l^h}{\varpi^h} \right)^{1-\varepsilon_N} \right)^{\frac{1}{1-\varepsilon_N}}$$

$$\Leftrightarrow \tilde{\Gamma}(\alpha_l, \alpha_m, \varpi^l) = \left(\alpha_l \left(\frac{1}{\varpi^l} \right)^{1-\varepsilon_N} + \alpha_m (l^m)^{1-\varepsilon_N} + (1 - \alpha_l - \alpha_m) \left(\frac{\omega^h l^h}{1 - \omega^l \varpi^l - \omega^m} \right)^{1-\varepsilon_N} \right)^{\frac{1}{1-\varepsilon_N}}$$

The solution can be obtained using:

$$\hat{n}^l = \alpha_l (\varpi^l)^{\varepsilon_N - 1} \tilde{\Gamma}(\alpha_l, \alpha_m, \varpi^l)^{(\varepsilon_N - 1)} (\hat{n}^l + \hat{l}^m \hat{n}^m + \hat{l}^h \hat{n}^h) \quad (3.9)$$

$$\hat{n}^m = \alpha_m (\hat{l}^m)^{-\varepsilon_N} \tilde{\Gamma}(\alpha_l, \alpha_m, \varpi^l)^{(\varepsilon_N - 1)} (\hat{n}^l + \hat{l}^m \hat{n}^m + \hat{l}^h \hat{n}^h) \quad (3.10)$$

$$\hat{n}^h = \alpha_h (\varpi^h)^{\varepsilon_N - 1} (\hat{l}^h)^{-\varepsilon_N} \tilde{\Gamma}(\alpha_l, \alpha_m, \varpi^l)^{(\varepsilon_N - 1)} (\hat{n}^l + \hat{l}^m \hat{n}^m + \hat{l}^h \hat{n}^h) \quad (3.11)$$

where the variables with a “hat” are the observable data. Solutions must satisfy $\varpi^l > 0$, $0 < \alpha_l < 1$, $0 < \alpha_m < 1$ with $\alpha^l + \alpha^m < 1$. Moreover, the solution would be such that $\varpi^l < \varpi^m < \varpi^h$. Results (parameters and wage distributions) are reported in Figure 3.17.

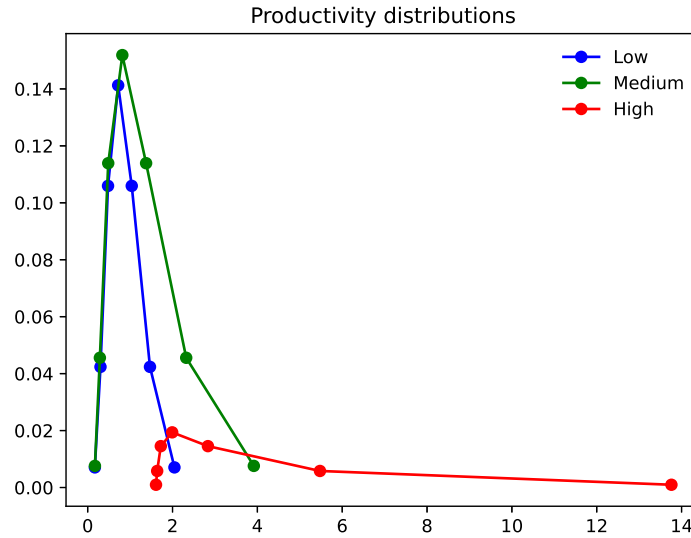


Figure 3.17: Productivity Distribution by Workers' Type s

Given the solutions for $\{\varpi^l, \varpi^m, \varpi^h\}$, where $\sum_s \omega^s \varpi^s = 1$, we define $\varpi^s = \Delta^s + \sum_i e_i^s \pi_i^s$, with $\sum_i e_i^s \pi_i^s = 1, \forall s$. Using the parameters $\{\rho^s, \sigma^s\}$, the grids of the idiosyncratic productivity shocks $[e_1^s, \dots, e_N^s]$ and the Markov matrix $[\tilde{\pi}_{ii'}^s]$ are constructed, and satisfy $\sum_i e_i^s \pi_i^s = 1$ where π_i^s is the eigenvector associated to the eigenvalue 1 of the matrix $[\tilde{\pi}_{ii'}^s]$.

Finally, equilibria on labor markets allow us to deduce the values of the disutility of working $\varphi_s, \forall s = l, m, h$ (see the Table 3.9)).

s	l	m	h
	CES parameters of Y_N		
α_s	0.3133	0.5230	0.1636
ϖ_l	0.7808	1	2.5978
	Income Risks		
ρ_s	0.97	0.965	0.94
σ_s	0.36	0.64	1.4
	Disutility of working		
φ_s	0.3634	0.3278	0.1482

Table 3.9: Specific Parameters by Workers' Types s

Calibration of taxation and transfers

We use several moments of the decile distribution to calibrate the transfer and income tax parameters in particular. Those moments are computed from the data in Accardo et al., 2021 for 2018. We consider that Bismarckian transfers encompass pensions and unemployment insurance transfers ("retraite" et "chômage et revenus de remplacement"). Beveridgian transfers include the remaining monetary transfers as well as in-kind health, social action, and housing transfers ("santé" et "action sociale et logement"). This way, we obtain the share of transfers over GDP (17.9% for Bismarckian and 14.2% for Beveridgian) and their distribution by decile. For gross income, we consider primary income ("revenu primaire élargi") to which we remove social contributions ("cotisations sociales") and add Bismarckian transfers. For net income, we use disposable income ("revenu disponible") plus health and social action and housing transfers in kind. Data for consumption gives us the distribution of consumption by decile. Finally, we obtain data for the distribution of dividends from André, Germain, and Sicsic, 2023, using the dividend and mixed revenue category ("dividendes et revenus mixtes").

From government forecasts to quarterly data

The transfers (T) are the sum of "Social benefits in cash" + "Social benefits in kind". The energy price is first divided by the EUR/USD exchange rate and by the inflation level. For GDP, IPC, energy price and wage of the year τ , we compute the quarterly growth rates g_τ^z using the annual growth rates $g_{a,\tau}^z$ (forecasts of the GDP, IPC, energy price and wage growth rates reported in the Table 3.10)³⁴, solving

$$(1 + g_{a,\tau}^z) \times \sum_{q=1Q}^{4Q} Z_{q,\tau} = Z_{1Q,\tau+1} \times [1 + (1 + g_\tau^z) + (1 + g_\tau^z)^2 + (1 + g_\tau^z)^3]$$

where Z = GDP, IPC, energy price, and wage. We built the quarterly data for GDP, IPC, and energy price over the periods 1Q2022 to 4Q2027 (see panels (a), (b) & (c) of Figure 3.18).

³⁴For the energy price, we deduce the annual growth rate from forecasts of the data in level.

	2022	2023	2024	2025	2026	2027
Population (15-64)	41427249	41402466	41381174	41360167	41338765	41311515
GDP growth	2.6%	1.0%	1.6%	1.7%	1.7%	1.8%
GDP share of G	23.7%	23.2%	22.8%	22.5%	22.3%	22.1%
GDP share of T	25.7%	24.9%	25.0%	24.8%	24.6%	24.3%
Debt-to-GDP	111.6%	109.6%	109.5%	109.4%	109.2%	108.3%
Energy price	\$101	\$83	\$83	\$83	\$83	\$83
IPC (inflation rate)	5.2%	4.9%	2.6%	2.0%	1.8%	1.8%
Employment	2.4%	0.8%	0.6%	0.8%	0.8%	0.8%
Wage	5.0%	4.8%	3.0%	2.8%	2.4	2.5%
Short-term interest rate	0.3%	3.1%	3.7%	3.4%	3.4%	3.4%
GDP share of interest charges	1.9%	1.7%	1.9%	2.1%	2.3%	2.4%
Exchange rate EUR/USD	1.05	1.07	1.07	1.07	1.07	1.07

Table 3.10: Government forecasts. Source: Finance Act

We get quarterly series by interpolating the GDP share of G (government consumption) and the GDP share of T (government transfers). Then, using the quarterly data of GDP, we built quarterly data for G and T over the periods 1Q2022 to 4Q2027 (see panels (a) & (b) of the Figure 3.19). Concerning the debt-to-GDP ratio, the GDP share of interest charges and employment, we simply perform quarterly interpolation to construct quarterly data over the periods 1Q2022 to 4Q2027 (see panel (c) of the Figure 3.19), considering that the Finance Act gives the end-of-year value. For the short-term interest rate, we use the annual value for each quarter of the corresponding year.

Projected data for the labor markets. On the labor market, the government forecasts give aggregate projections for the employment rate (N) and the real wage (w). In order to construct hours worked (employment rate multiplied by number of hours worked per employee, $H = N \times h$) and real wages by salary level over the period 1Q2023 to 4Q2027, we use the government forecasts for N and w and the projections of the trends of N_s , h_s and w_s observed over the period 2Q2003 to 4Q2022. To do this, we assume that:

- (i) the distribution of jobs remains stable, ie. $N_{s,t} = \omega_s N_t$, for $t \in [1Q2023; 4Q2027]$. Hence, using the values of ω_s , we build $N_{s,t}$, consistent with N_t provided by the government forecasts.
- (ii) the ratio between the salary of a job of type l and the aggregate salary continues to evolve as it did between 2Q2003 to 4Q2022, $w_{l,t} = \mu_t w_t$ with μ_t observed between 2Q2003 and 4Q2022 then projected linearly between 1Q2023 to 4Q2027. This provides the data for $w_{l,t}$, given the government forecasts for w_t .
- (iii) the wage gaps between m , h and l remain stable, i.e. $w_{s,t} = \gamma_s w_{l,t}$, for $s = m, h$.
- (iv) the number of hours worked by type of employee $s = l, m, h$ evolves in the future (3Q2023 to 4Q2027) continuing the linear trend observed since 2Q2003.

Using this set of restrictions, we built total hours worked H_s and real wage w_s for each labor market $s = l, m, h$ for the sample 1Q2023 to 4Q2027. Data for the labor market are in the Figure 3.20.

Figure 3.18: Data and Govt. Forecasts – Goods Market, Financial Market

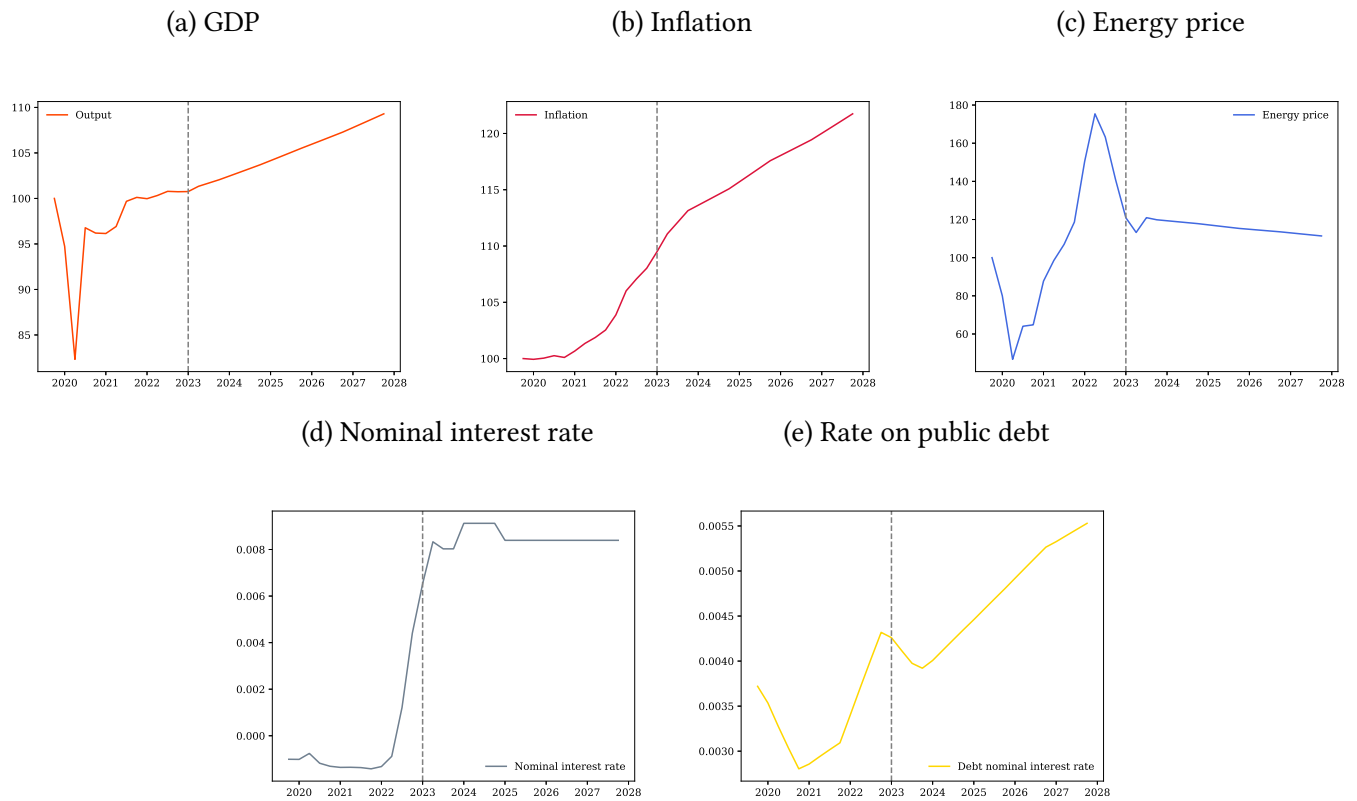
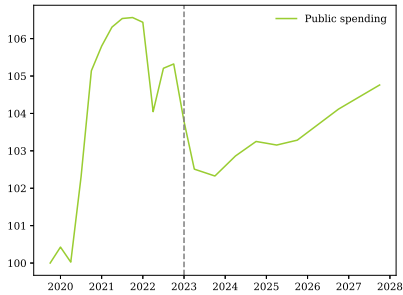
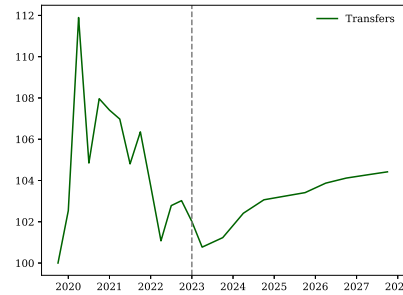


Figure 3.19: Data and Govt. Forecasts – Government Accounts

(a) Govt. consumption



(b) Govt. transfers



(c) Debt-to-GDP

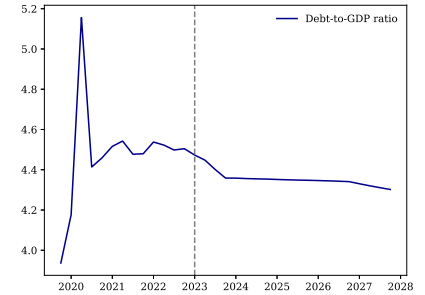
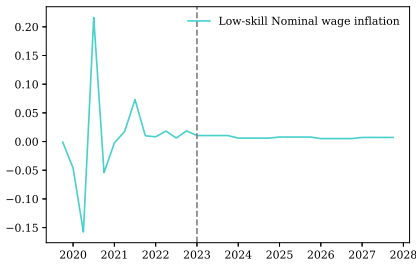
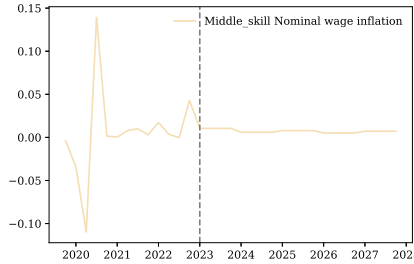


Figure 3.20: Data and Govt. Forecasts – Labor Markets

(a) Wage inflation l



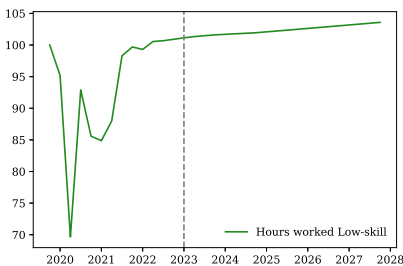
(b) Wage inflation m



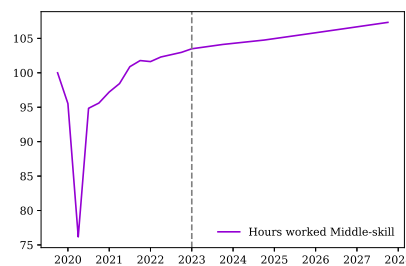
(c) Wage inflation h



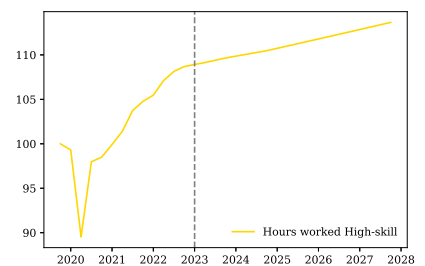
(d) Hours worked l



(e) Hours worked m



(f) Hours worked h



3.7.3 Estimation of the Exogenous Shocks Processes

The persistence ρ and the standard deviation σ of the shock processes are estimated using a Bayesian procedure: based on a Metropolis-Hastings algorithm, we draw one million draws. The first half of accepted draws were burned in to correct for possible mischoice of the starting point.

The prior distributions considered are reported in Table 3.11. For energy prices (p_{FE}), government spending (G), and transfers (T), our HANK model simply replicates the exogenous input series. Consequently, guesses for the values of these parameters can be obtained by estimating an $AR(1)$ on the time series $\{p_{FE}, G, T\}$. These estimates are used as information to define the priors of these shocks. The remaining priors for

$$\{\beta, \mu, \varepsilon, \vartheta, \{\varphi_s\}_{s=l,m,h}, \{A_s\}_{s=l,m,h}, e_\tau\}$$

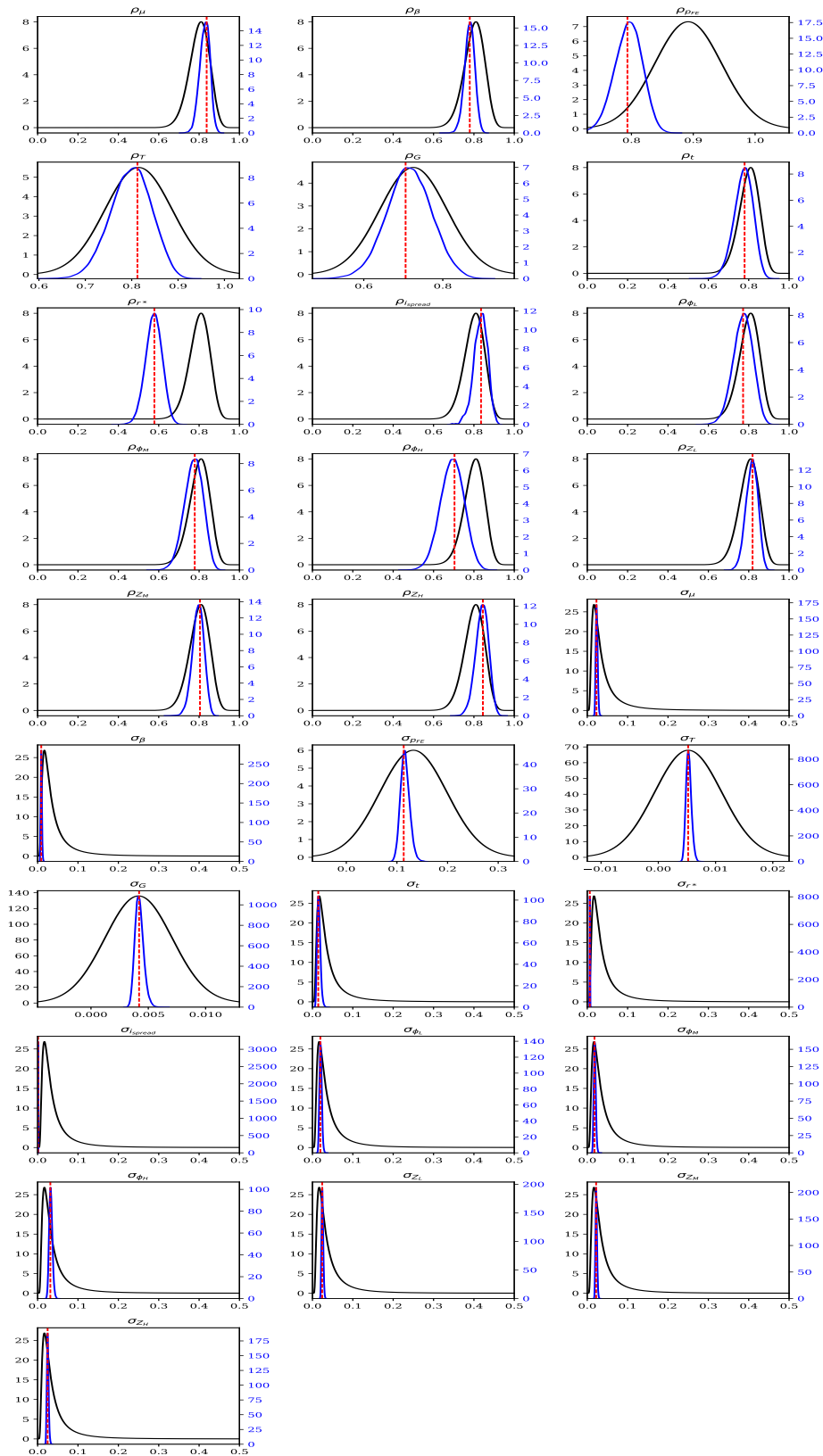
are assumed to follow beta distributions for the persistence and inverse-gamma distributions for the standard deviation, as usual in the literature. The results of the estimation are in Table 3.11 and in Figure 3.21.

Table 3.11: Bayesian estimation results of the parameters of the $AR(1)$ processes

Shock		Prior	Mode	Mean	Std	95% CI
Preference	β	ρ $\beta(0.8, 0.05)$	0.779433	0.778482	0.025231	[0.735657, 0.818507]
		σ $inv\Gamma(0.05, 1.0)$	0.008922	0.009143	0.001438	[0.006991, 0.011675]
Price markup	μ	ρ $\beta(0.8, 0.05)$	0.797303	0.789200	0.027489	[0.784003, 0.866901]
		σ $inv\Gamma(0.05, 1.0)$	0.022775	0.023420	0.002331	[0.019887, 0.027508]
Energy price	p_{FE}	ρ $\mathcal{N}(0.89, 0.054)$	0.793492	0.794674	0.022682	[0.755927, 0.830306]
		σ $\mathcal{N}(0.13, 0.067)$	0.113373	0.116112	0.009058	[0.102126, 0.131723]
Monetary policy	ε	ρ $\beta(0.8, 0.05)$	0.578370	0.575467	0.040994	[0.506296, 0.641365]
		σ $inv\Gamma(0.05, 1.0)$	0.006279	0.006381	0.000498	[0.005615, 0.007241]
Spread	ϑ	ρ $\beta(0.8, 0.05)$	0.835287	0.832970	0.034203	[0.773230, 0.884632]
		σ $inv\Gamma(0.05, 1.0)$	0.001158	0.001172	0.000126	[0.000981, 0.001391]
Disutility l	φ_l	ρ $\beta(0.8, 0.05)$	0.772068	0.771044	0.048294	[0.688632, 0.847175]
		σ $inv\Gamma(0.05, 1.0)$	0.019574	0.020303	0.002861	[0.015940, 0.025250]
Disutility m	φ_m	ρ $\beta(0.8, 0.05)$	0.778410	0.772314	0.046330	[0.692780, 0.844720]
		σ $inv\Gamma(0.05, 1.0)$	0.017942	0.018561	0.002492	[0.014766, 0.022911]
Disutility h	φ_h	ρ $\beta(0.8, 0.05)$	0.703815	0.692028	0.058132	[0.594230, 0.785392]
		σ $inv\Gamma(0.05, 1.0)$	0.031485	0.032577	0.003967	[0.026451, 0.039417]
Productivity l	A^l	ρ $\beta(0.8, 0.05)$	0.819040	0.815107	0.029585	[0.764506, 0.861222]
		σ $inv\Gamma(0.05, 1.0)$	0.024080	0.024635	0.002089	[0.021438, 0.028305]
Productivity m	A^m	ρ $\beta(0.8, 0.05)$	0.804328	0.795198	0.029001	[0.745639, 0.840641]
		σ $inv\Gamma(0.05, 1.0)$	0.021964	0.022278	0.001954	[0.019318, 0.025698]
Productivity h	A^h	ρ $\beta(0.8, 0.05)$	0.844523	0.839992	0.032298	[0.784530, 0.890326]
		σ $inv\Gamma(0.05, 1.0)$	0.024729	0.025028	0.002205	[0.021721, 0.028933]
Government spending	G	ρ $\mathcal{N}(0.726, 0.085)$	0.705917	0.714310	0.056981	[0.619684, 0.807090]
		σ $\mathcal{N}(0.0041, 0.0029)$	0.004198	0.004220	0.000379	[0.003650, 0.004887]
Transfers	T	ρ $\mathcal{N}(0.815, 0.073)$	0.812377	0.797681	0.044078	[0.722558, 0.867606]
		σ $\mathcal{N}(0.0052, 0.0059)$	0.005212	0.005315	0.000465	[0.004602, 0.006132]
Measurement error	e_τ	ρ $\beta(0.8, 0.05)$	0.779472	0.773476	0.047124	[0.691685, 0.846046]
		σ $inv\Gamma(0.05, 1.0)$	0.014396	0.015915	0.004046	[0.010032, 0.023161]

Because our model is not formulated in a linear state-space way, the Kalman filter cannot be used to evaluate the log-likelihood. Instead, and consistently with Auclert, Bardóczy, Rognlie, and Straub, 2021, the log-likelihood of our model is computed using the covariance matrix linking the model's variables. This covariance matrix relies on the Jacobian of the model which can be obtained using the sequence-space method. Note that because we do not estimate structural parameters that affect the Jacobian of the system, the same Jacobian can be reused throughout the entire process of estimation, which saves some computing time.

Figure 3.21: Prior and Posterior Distributions of the Estimated Parameters



3.7.4 Shock Decomposition of Historical Data

This appendix presents the variance decomposition of the estimated model for the period of estimation 1Q2003-4Q2019. The variance decomposition of the main macroeconomic variables allows us to evaluate which are the dominant shocks for explaining the business cycle over the entire period. It gives the contribution of each shock in explaining the deviations of each endogenous variable from its long-term value (see Table 3.12).³⁵

Table 3.12: Variance decomposition. Share of the deviation from the steady state explained by each shock: mean value for the sample 1Q2003-4Q2019.

	output	inflation	debt	int. rate	debt rate	empl L	empl M	empl H	wage L	wage M	wage H
β	43.8%	7.6%	25.8%	14.4%	14.2%	28.3%	27.5%	23.9%	13.8%	14.9%	10.5%
μ	16.0%	15.2%	20.6%	11.5%	11.3%	10.5%	9.4%	9.5%	6.4%	7.2%	6.6%
P_E	5.9%	56.1%	29.3%	50.4%	50.3%	35.4%	37.9%	32.8%	41.5%	40.5%	35.5%
ε	4.0%	1.4%	4.2%	2.2%	1.9%	3.0%	2.9%	1.9%	3.7%	3.7%	2.1%
ϑ	0.8%	0.2%	1.3%	0.3%	2.4%	0.6%	0.6%	0.3%	0.6%	0.5%	0.3%
φ_l	1.0%	1.0%	0.8%	1.2%	1.1%	6.8%	0.0%	0.0%	13.5%	0.5%	0.4%
φ_m	3.8%	3.3%	1.7%	2.8%	2.4%	0.3%	7.6%	0.1%	1.3%	15.4%	0.9%
φ_h	0.7%	0.6%	0.4%	0.7%	0.6%	0.0%	0.0%	13.3%	0.3%	0.3%	25.8%
A^l	4.4%	2.2%	2.0%	3.2%	3.1%	9.2%	0.2%	0.2%	4.3%	1.9%	1.7%
A^m	10.1%	5.9%	1.6%	4.3%	4.1%	0.4%	8.1%	0.4%	2.5%	4.7%	2.2%
A^h	1.7%	0.7%	0.2%	1.0%	0.9%	0.0%	0.0%	14.2%	0.5%	0.4%	3.5%
G	1.6%	0.1%	0.1%	0.0%	0.0%	1.1%	1.1%	0.8%	0.2%	0.2%	0.1%
T	0.1%	0.2%	0.2%	0.3%	0.2%	0.1%	0.1%	0.1%	0.6%	0.5%	0.2%
e_τ	6.2%	5.7%	11.9%	7.7%	7.3%	4.3%	4.6%	2.6%	10.9%	9.4%	10.2%

Based on the sequences of shocks identified by the estimation procedure, our model enables us to identify the macroeconomic channels through which the financial crisis impacted the French economy. Figure 3.22 reports results for all variables used in the estimation. The first channel is household demand. In 2008, we observed a reversal of shocks to consumer demand. Whereas demand was sustaining the economy before the crisis, during the crisis it caused a decline in GDP. Therefore, our model attributes a significant part of the drop in demand to an increase in households' patience, leading them to postpone their consumption, which can be explained by the very high level of uncertainty surrounding the resolution of the banking crisis.

The slowdown of economic activity at the world level is also the result of the oil shock for the period between 3Q2008 and 1Q2009. However, the impact of the energy price shock must be decomposed. Indeed, the substitution effect leads the output to increase in response to a positive energy price shock because consumers substitute energy for domestically produced goods.³⁶ This effect can be over-compensated by the income effect that leads output to decline in response to a rise in energy prices. The income effect is driven by both the

³⁵Fluctuations represent the differences between the current values of economic variables and their long-term values.

³⁶In a RA DSGE model, as consumer spending remains constant (result implied by the combination of the Backus-Smith condition with the real UIP condition), the entire output response is driven by expenditure switching.

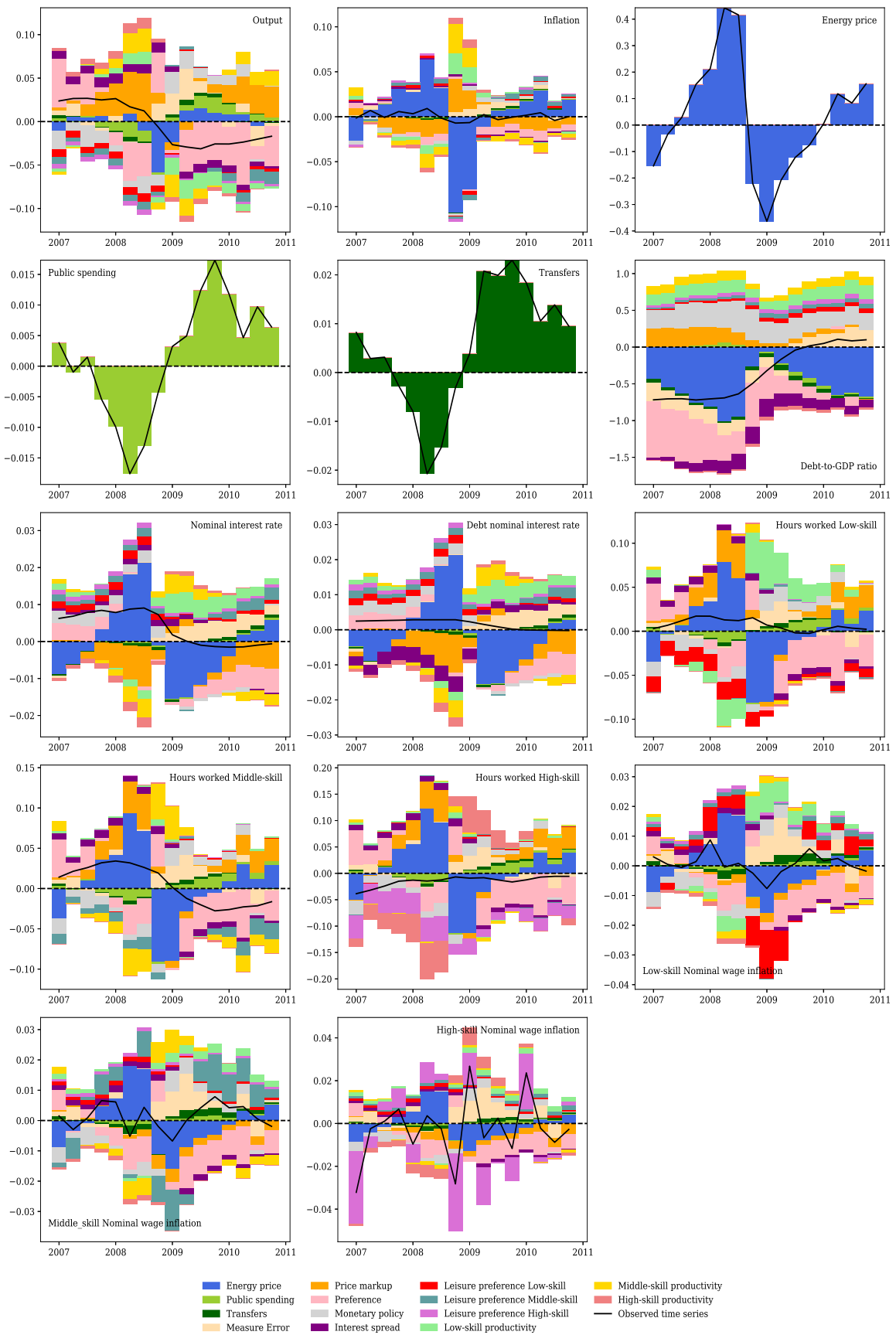


Figure 3.22: Historical decomposition.

decline of the purchasing power and a potential reduction in activity and thus in income distributed to the households (the Keynesian multiplier). If agents can't perfectly smooth their consumption thanks to financial markets as in a RA DSGE model, then the negative income effect induced by the increase in energy price can generate a recession, magnified by the Keynesian multiplier.³⁷ Obviously, the size of the income effect depends on the impact of energy price on the consumer price index: if the nominal rigidities are strong and if the persistence of the shock is small enough, this effect can be small and then dominated by the substitution effect. The highly negative oil price shock in 4Q2008 reduces production as the substitution effect dominates, then from 2Q2009 onward, as prices adjust, the income effect prevails, stimulating economic activity.³⁸

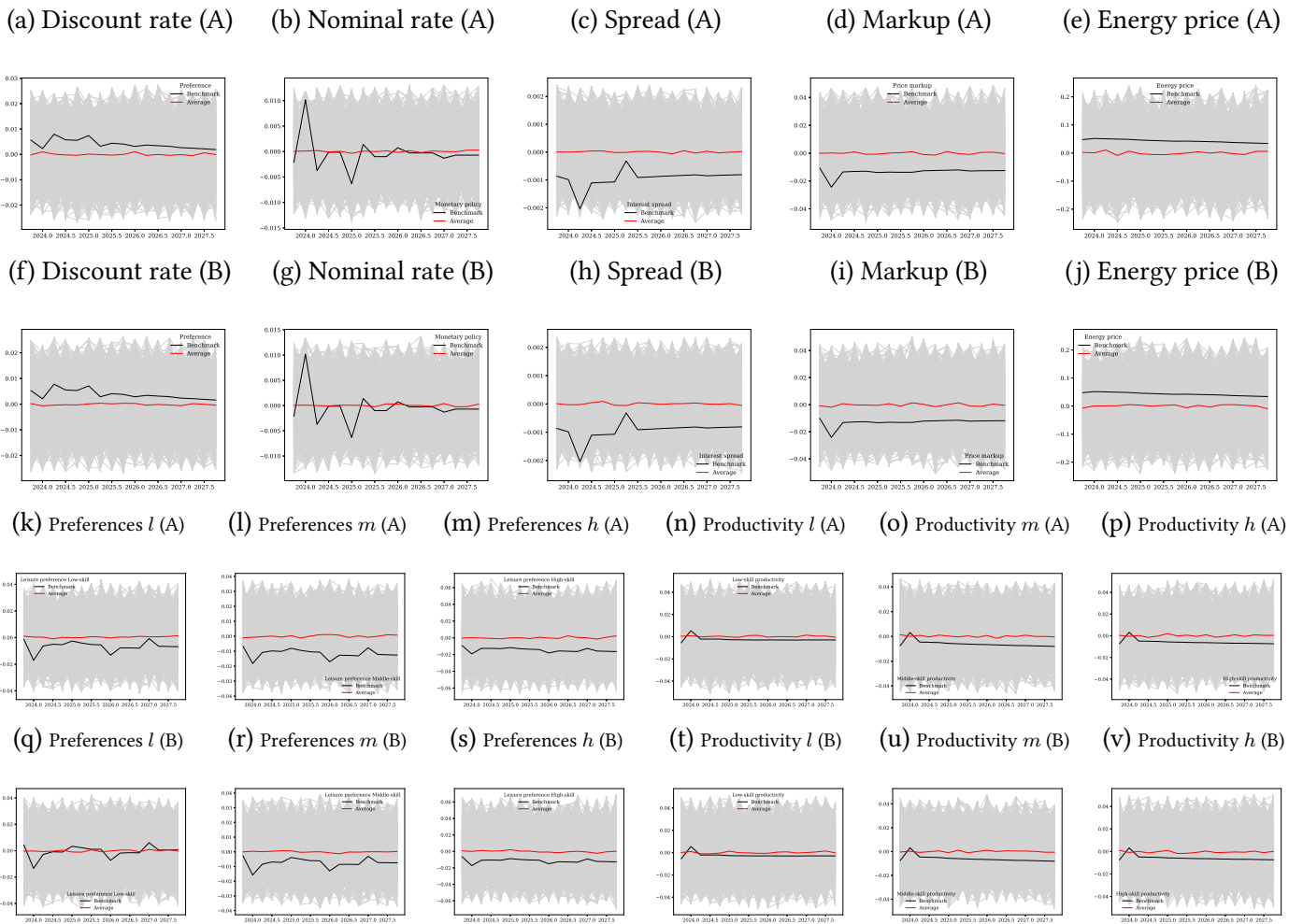
The onset of a recession is also the joint product of a fall in productivity (for all workers) and a reversal in firm markup. The recession was then mitigated by monetary and fiscal policy shocks, which contributed positively to output growth as of 1Q2009.

³⁷The advantage of HANK models to predict the observed depressive effect of positive shock on energy price, contrary to what predict a Representative-Agent New-Keynesian (RANK) model, is demonstrated in Auclert, Monnery, et al., 2023.

³⁸Structural VARs show the recessionary effects of a rise in oil prices in the medium term. In the short term, the response is most often not significantly different from zero and may even be positive in some cases (e.g. Blanchard and Galí, 2007 and Kilian, 2008).

3.7.5 Innovations Before and After Estimation

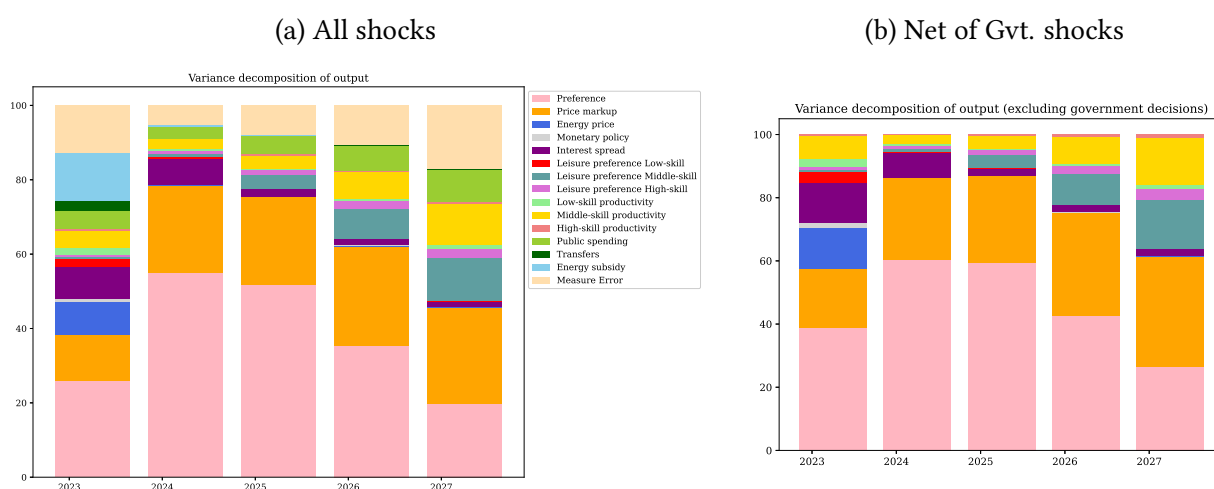
Figure 3.23: Innovations of households and firms shocks. (A) before and (B) after re-estimation.



3.7.6 Shock Decomposition of Forecasts

Figure 3.24 (panel (a)) indicates that budgetary decisions only explain around 20% of GDP forecast during the period. Therefore, the Finance Act forecasts rely heavily on shocks not controlled by the government. The panel (a) of Figure 3.24 gives the exhaustive decomposition of the variance of GDP according to the 15 sources of fluctuation integrated into the model. In 2027, the sum of the contributions of public consumption and transfer shocks, to which we also add the contribution of measurement error, amounts to $8.4\% + 0.4\% + 17.0\% = 25.8\%$.³⁹

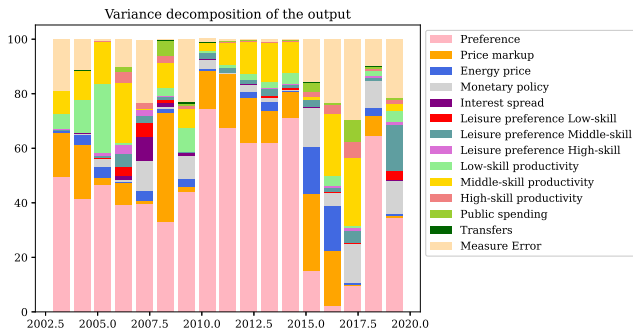
Figure 3.24: Variance decomposition: Output. After re-estimation (B)



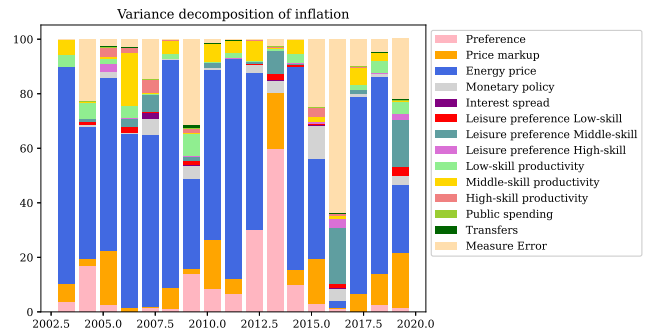
To extract the shocks that are not in the government budget, we then report in panel (b) of Figure 3.24 the remaining effects (which are then re-normalized to 100%). If we exclude budgetary shocks, shocks to demand, markup, energy price, labor supply, and productivity explain the majority of variations in GDP. Monetary policy shocks and risk premiums on public debt have, for their part, only a modest contribution.

Figure 3.26 gives the contribution of each shock in the quarterly forecast of each variable. Four variables are purely exogenous: public consumption, public transfers, energy subsidy (tariff shield), and energy prices. It appears that energy-price shocks have a large contribution to inflation dynamics (price and wages). Demand and markup shocks explain a large part of output dynamics. Finally, the shocks on the disutility of working and productivity have significant contributions to total hours worked.

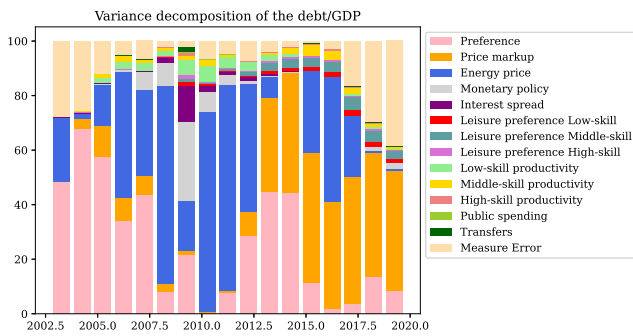
³⁹The complete statistics of the variance decomposition are in the Tables 3.13 and 3.14.



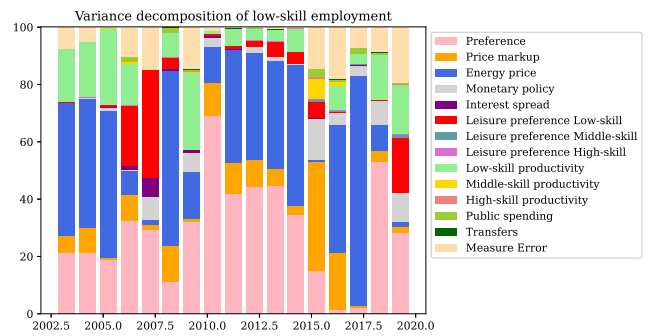
(a) Output



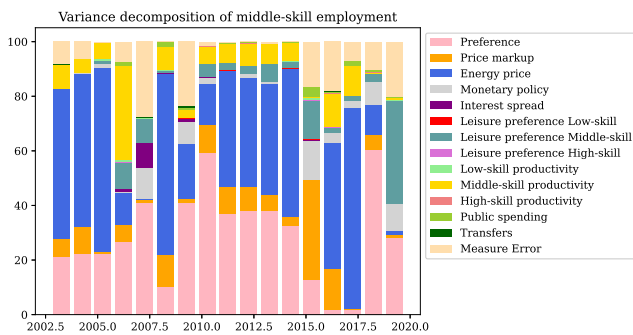
(b) Inflation



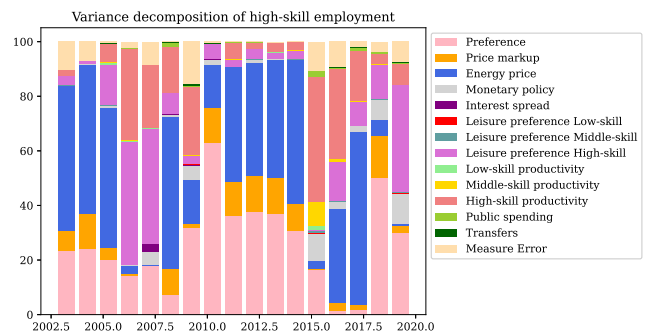
(c) Debt-to-GDP



(d) Hours worked l



(e) Hours worked m



(f) Hours worked h

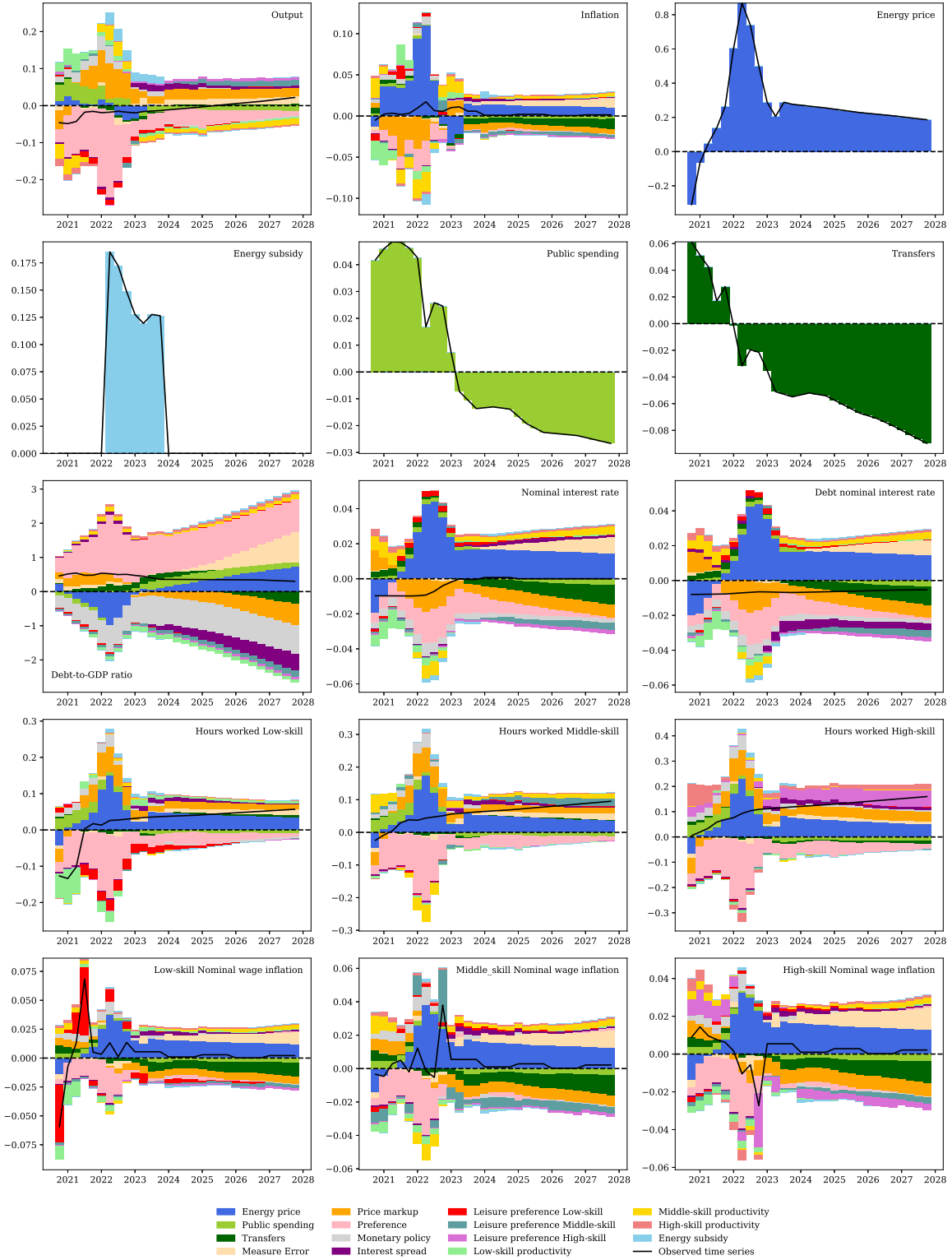
Table 3.13: Variance decomposition. After re-estimation (B). Share of the deviation from the steady state explained by each shock

	2023				2024	2025	2026	2027	
	1Q	2Q	3Q	4Q	2Q	2Q	2Q	2Q	
<i>y</i>	β	32.8%	26.0%	49.4%	50.5%	55.0%	51.6%	35.3%	19.7%
	μ	28.4%	12.5%	10.6%	12.1%	23.6%	23.9%	26.8%	25.9%
	P_{FE}	21.4%	8.7%	0.4%	0.1%	0.0%	0.0%	0.0%	0.1%
	ε	0.5%	0.9%	3.9%	3.4%	0.1%	0.0%	0.2%	0.1%
	ϑ	2.3%	8.6%	9.9%	7.2%	7.0%	2.1%	1.8%	1.5%
	φ_l	2.3%	2.3%	1.3%	1.3%	0.5%	0.1%	0.0%	0.0%
	φ_m	1.3%	0.4%	0.0%	0.0%	0.8%	3.8%	8.0%	11.5%
	φ_h	0.2%	0.6%	0.5%	0.5%	0.7%	1.1%	2.0%	2.7%
	A^l	0.1%	1.8%	1.3%	1.3%	0.5%	0.5%	0.7%	0.9%
	A^m	0.0%	4.7%	3.9%	4.7%	2.6%	3.5%	7.1%	11.0%
	A^h	0.0%	0.4%	0.3%	0.3%	0.2%	0.3%	0.5%	0.8%
	G	0.1%	5.0%	7.4%	7.4%	3.1%	4.8%	6.6%	8.4%
	T	0.9%	2.7%	0.5%	0.1%	0.2%	0.2%	0.4%	0.4%
	s_H	9.4%	12.7%	10.4%	10.7%	0.5%	0.1%	0.0%	0.0%
e_τ	0.3%	12.9%	0.1%	0.5%	5.2%	8.0%	10.5%	17.0%	
π	β	0.7%	0.0%	0.8%	1.0%	1.1%	1.3%	0.1%	0.1%
	μ	16.5%	20.7%	10.6%	9.2%	18.1%	11.6%	9.3%	9.2%
	P_{FE}	58.7%	48.6%	63.4%	59.9%	53.3%	48.7%	40.4%	30.4%
	ε	0.1%	0.1%	1.6%	1.6%	0.1%	0.0%	0.1%	0.1%
	ϑ	0.1%	0.7%	3.1%	2.8%	2.1%	0.7%	0.3%	0.2%
	φ_l	0.0%	0.1%	0.2%	0.3%	0.0%	0.0%	0.0%	0.0%
	φ_m	0.1%	1.7%	0.6%	0.8%	4.0%	2.7%	4.5%	2.9%
	φ_h	0.1%	0.3%	0.2%	0.2%	0.6%	0.6%	0.9%	0.7%
	A^l	3.6%	3.9%	0.1%	0.2%	0.1%	0.0%	0.1%	0.1%
	A^m	19.1%	18.2%	2.4%	3.4%	0.4%	2.4%	3.1%	3.5%
	A^h	0.5%	0.8%	0.1%	0.3%	0.1%	0.3%	0.3%	0.3%
	G	0.0%	0.1%	3.0%	3.8%	1.5%	2.6%	2.9%	3.1%
	T	0.6%	2.4%	13.6%	15.8%	10.3%	16.3%	22.1%	29.1%
	s_H	0.1%	0.1%	0.1%	0.1%	2.4%	0.0%	0.0%	0.0%
e_τ	0.0%	2.4%	0.2%	0.6%	5.9%	12.8%	15.8%	20.4%	
$\frac{b}{y}$	β	69.0%	64.0%	60.5%	57.8%	56.4%	49.2%	38.7%	28.6%
	μ	0.1%	0.5%	0.8%	1.2%	2.1%	4.9%	7.8%	9.7%
	P_{FE}	0.4%	0.3%	0.1%	0.5%	1.9%	5.9%	10.3%	13.3%
	ε	27.1%	30.8%	31.8%	33.5%	32.0%	29.8%	25.7%	20.6%
	ϑ	0.0%	0.0%	0.3%	0.6%	1.9%	4.0%	5.7%	6.6%
	φ_l	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	φ_m	0.2%	0.2%	0.5%	0.5%	0.4%	0.5%	0.6%	0.8%
	φ_h	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.1%	0.2%
	A^l	0.8%	1.0%	0.9%	0.8%	0.7%	0.5%	0.4%	0.2%
	A^m	0.1%	0.0%	0.0%	0.1%	0.1%	0.2%	0.3%	0.4%
	A^h	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%
	G	0.5%	1.2%	2.8%	3.2%	3.2%	2.7%	1.7%	0.8%
	T	1.5%	1.7%	2.0%	1.6%	0.8%	0.0%	0.5%	2.3%
	s_H	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
e_τ	0.0%	0.1%	0.0%	0.0%	0.1%	1.9%	7.9%	16.3%	

Table 3.14: Variance decomposition. After re-estimation (B). Share of the deviation from the steady state explained by each shock

	2023				2024	2025	2026	2027	
	1Q	2Q	3Q	4Q	2Q	2Q	2Q	2Q	
N_l	β	31.3%	20.0%	23.5%	24.4%	26.1%	26.2%	16.4%	8.6%
	μ	23.2%	8.0%	4.3%	5.0%	9.8%	10.3%	10.6%	9.8%
	P_{FE}	14.4%	19.9%	54.2%	53.3%	52.2%	53.6%	60.2%	63.2%
	ε	0.2%	0.2%	1.1%	0.9%	0.4%	0.2%	1.0%	1.0%
	ϑ	2.0%	6.1%	4.7%	3.5%	3.4%	1.0%	0.7%	0.4%
	φ_l	17.8%	15.9%	5.7%	5.9%	2.9%	0.7%	0.1%	0.0%
	φ_m	0.2%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%
	φ_h	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	A^l	7.6%	14.7%	2.0%	2.5%	0.7%	0.8%	1.2%	1.4%
	A^m	0.8%	0.0%	0.1%	0.1%	0.0%	0.0%	0.0%	0.1%
	A^h	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	G	0.1%	3.2%	3.5%	3.5%	1.4%	2.3%	2.9%	3.8%
	T	0.7%	1.6%	0.1%	0.0%	0.4%	0.7%	1.3%	1.7%
	s_H	1.5%	1.3%	0.6%	0.6%	0.3%	0.1%	0.0%	0.0%
e_τ	0.3%	8.8%	0.0%	0.2%	2.5%	4.2%	5.6%	10.0%	
N_m	β	31.2%	18.7%	22.6%	24.2%	26.5%	25.5%	16.0%	8.9%
	μ	26.5%	7.9%	3.9%	4.8%	9.7%	9.7%	9.6%	8.7%
	P_{FE}	19.7%	25.8%	60.0%	58.0%	52.9%	47.6%	47.4%	43.9%
	ε	0.5%	0.7%	1.9%	1.8%	0.0%	0.0%	0.0%	0.0%
	ϑ	2.6%	7.6%	4.9%	3.6%	3.4%	1.0%	0.8%	0.7%
	φ_l	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	φ_m	3.2%	0.6%	0.0%	0.0%	1.5%	6.2%	12.3%	17.5%
	φ_h	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	A^l	0.0%	0.2%	0.2%	0.2%	0.1%	0.0%	0.0%	0.0%
	A^m	12.9%	18.2%	1.7%	2.6%	1.2%	2.6%	4.6%	6.8%
	A^h	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	G	0.1%	4.6%	3.7%	3.8%	1.5%	2.6%	3.2%	4.0%
	T	1.0%	2.4%	0.3%	0.1%	0.1%	0.1%	0.2%	0.2%
	s_H	2.0%	1.7%	0.7%	0.7%	0.3%	0.0%	0.0%	0.0%
e_τ	0.3%	11.5%	0.0%	0.2%	2.8%	4.6%	5.8%	9.2%	
N_h	β	35.7%	22.8%	23.0%	23.7%	24.1%	21.6%	13.3%	7.5%
	μ	25.6%	10.0%	5.6%	6.7%	11.9%	13.4%	14.7%	14.9%
	P_{FE}	20.9%	24.4%	49.4%	47.4%	42.7%	36.0%	32.5%	27.7%
	ε	2.0%	2.4%	3.2%	3.2%	0.2%	0.7%	0.3%	0.4%
	ϑ	1.6%	4.5%	3.4%	2.7%	2.7%	1.1%	1.0%	1.0%
	φ_l	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	φ_m	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%
	φ_h	8.6%	16.0%	10.1%	10.6%	14.5%	21.9%	31.9%	40.5%
	A^l	0.1%	0.2%	0.2%	0.1%	0.1%	0.0%	0.0%	0.0%
	A^m	0.5%	0.0%	0.1%	0.0%	0.0%	0.0%	0.1%	0.1%
	A^h	2.4%	6.8%	1.0%	1.6%	0.8%	1.6%	2.7%	3.6%
	G	0.1%	2.7%	2.7%	2.8%	1.3%	1.9%	2.1%	2.4%
	T	0.8%	2.1%	0.6%	0.5%	0.1%	0.3%	0.5%	0.9%
	s_H	1.4%	1.1%	0.5%	0.5%	0.2%	0.0%	0.0%	0.0%
e_τ	0.3%	7.0%	0.0%	0.2%	1.4%	1.5%	1.0%	1.0%	

Figure 3.26: Shock Decomposition. After re-estimation (B)



3.7.7 Evaluating the Uncertainty Around a Finance Act

For the Finance Act to be realized, the previous sections have shown that the markup, the spread, and the disutility of labor must be sustainably below their stationary states, which favors activity. On the other hand, activity would be slowed down by a preference for the present and productivity below their stationary states. In order to evaluate the sensitivity of the government forecasts to the particular shock realizations that could sustain its policy, we construct counterfactual scenarios in which the sequence of one of these estimated shock is multiplied by 75%. We can then express in terms of GDP losses or gains (or debt increase or reduction) the macroeconomic impact of government overconfidence (or underconfidence) in future economic context. A summary of the results is shown in Table 3.15 while details are presented in the following subsections of the Appendix.

	Benchmark	Markup	Spread	Disutility	Preference	Productivity
Output	1.64%	1.53%	1.62%	1.53%	1.73%	1.76%
Debt-to-GDP	108.3%	111.0%	111.1%	109.0%	105.7%	107.8%
inflation rate	2.42%	3.05%	2.29%	2.79%	2.81%	2.08%
Total employment	0.71%	0.62%	0.69%	0.63%	0.78%	0.65%
Low-skill employment	0.52%	0.45%	0.51%	0.52%	0.58	0.49%
Middle-skill employment	0.83%	0.75%	0.81%	0.72%	0.90%	0.76%
High-skill employment	0.90%	0.74%	0.85%	0.62%	0.99%	0.79%

Table 3.15: Uncertainty assessment: impact of smaller shocks

Note: Results are average annual growth over the period 1Q2023-4Q2027 for output and employment. Inflation is given annual percentage while the debt-to-GDP ratio corresponds to the value in 4Q2027.

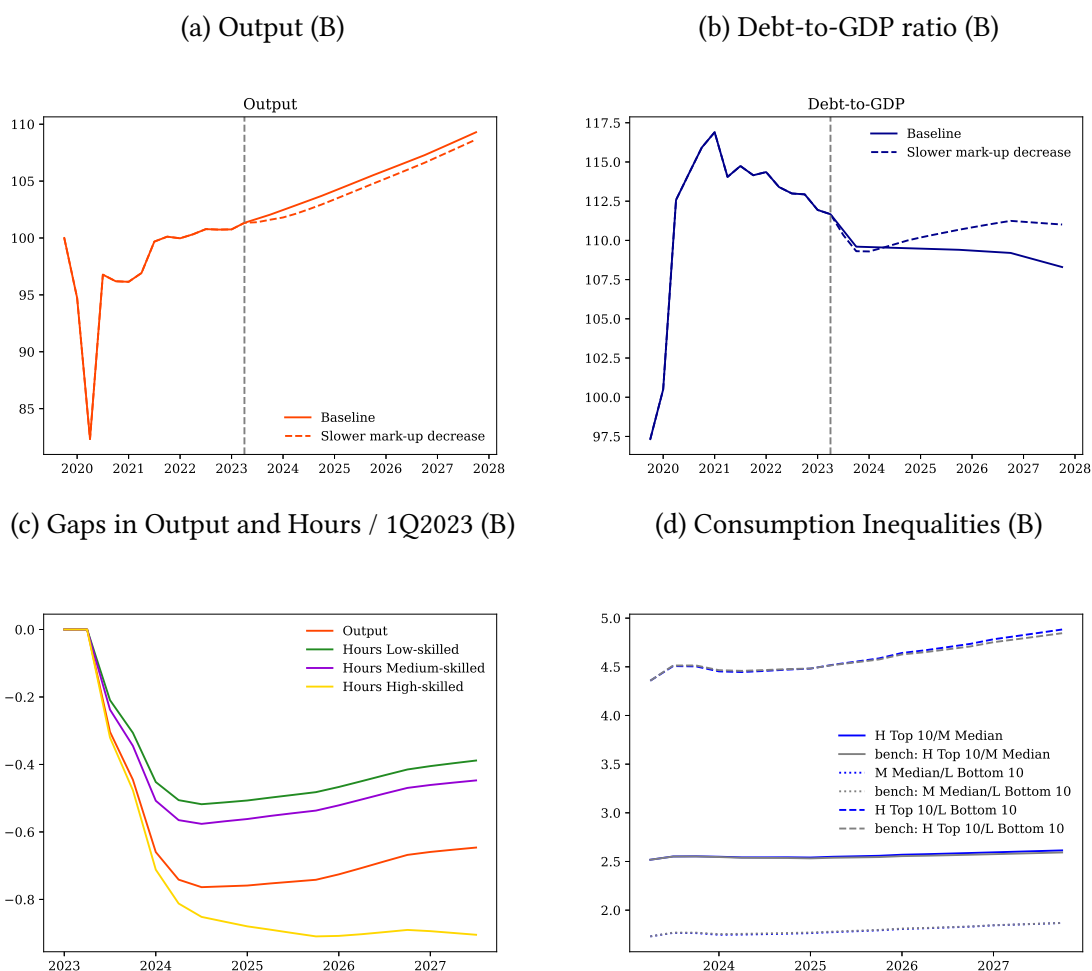
The Finance Act forecasts a rather optimistic annual increase of GDP of 1.64% between 2003 to 2027. As we can observe, the output growth can vary between +0.12pp to -0.11pp depending on the scenario. A decrease in price markup or labor disutility shocks (i.e. an increase in markup and labor disutility) would cause the highest fall while the diminution of the productivity shock (i.e. an increase in productivity) would lead to an important increase in GDP. The government plans for the debt-to-GDP ratio to decrease to 108.3% in 4Q2027. However, a decrease of labor disutility or interest spread shocks by 25% would completely overturn any planned decrease, with the debt-to-GDP ratio remaining at 111%. On the contrary, a diminution of the preference shock, meaning households increasing consumption, would allow the government to decrease debt more easily, to 105.7% of GDP. Then, annual inflation is forecasted to be at 2.42% over the considered period. However, an increase in the markup would make it go above 3%. A better-than-expected productivity would diminish labor costs and as such have the opposite effect, bringing inflation down to 2.08%. Finally, the government forecasts an increase in employment of 0.71% per year. Preferences more favorable to consumption would help employment growth, causing it to increase to 0.78%. On the contrary, a higher interest spread or higher disutility of labor would logically bring employment down.

Thus, the size of shocks is of great importance for reaching the government forecasts. Even rather small changes can lead to large consequences in terms of output, debt, inflation, or employment.

Smaller Decreases in Firms' Markups

The first “favorable wind” concerns the fall in firms’ markups. These markup reductions contribute to 26% of the 2027 GDP forecast (see figure 3.24). These contractions in markups

Figure 3.27: Counterfactual: Smaller Decreases in Firms’ Markups. (B) after re-estimation.



are essential to the overall coherence of the Finance Act which indicates a rapid exit, from 2024, from the current period of high inflation and a return to moderate inflation occurring in a context of strong employment growth. To reconcile the dynamism of the labor market with the slowdown in inflation, our estimation indicates that the firms’ markups should fall by 9 percentage points. Contractions in the markups of French firms, favorable to growth, were observed during the inflationary episode that began in 2021 (see Barnard and Ollivaud, 2023). But French companies were the only ones within the OECD to have reacted in this way, which makes the hypothesis of reproduction of this behavior fragile, especially with this magnitude. These contractions in markups may be partly induced by the announced reductions in taxes on business added value: their decrease from 1.50% to 0.75% reduces the markup by almost 1pp, and their disappearance would reduce it by 1.5pp, leaving the remaining 7.5pp burning to the firms.

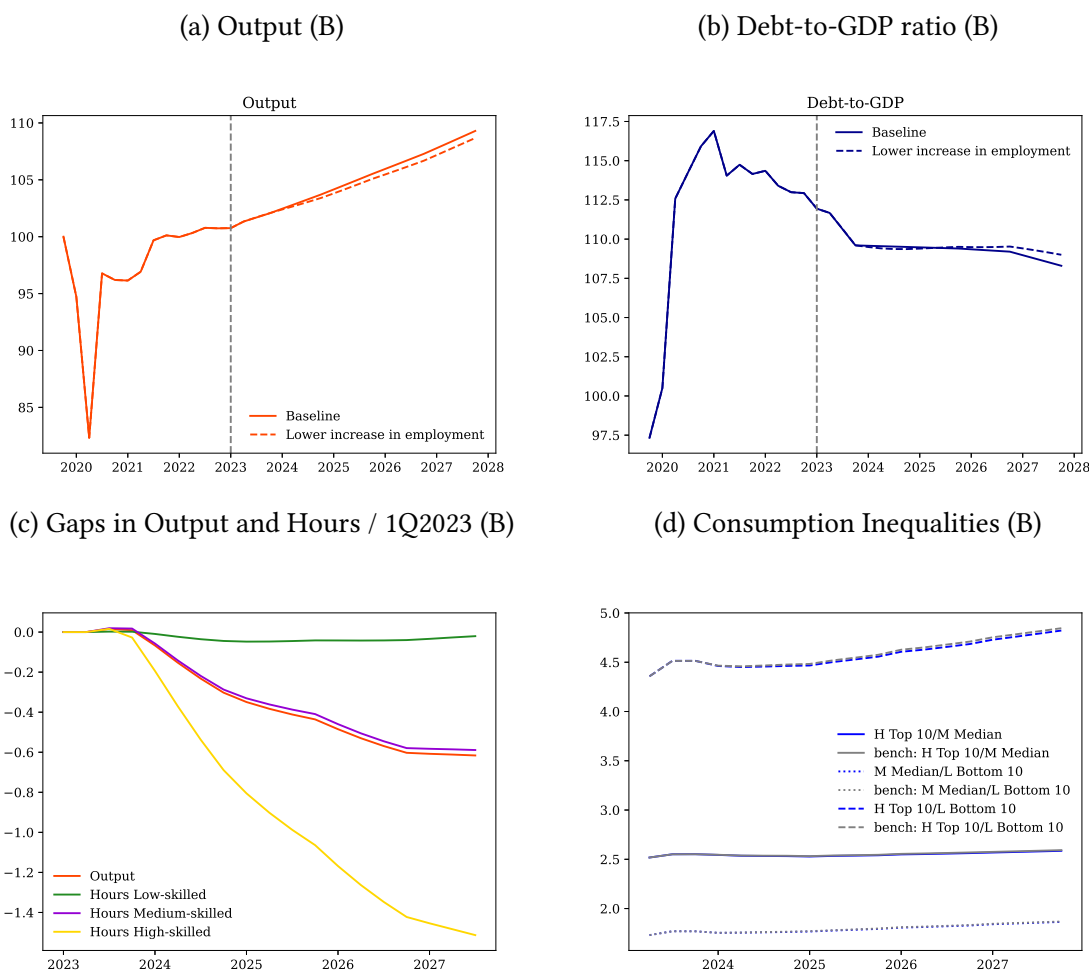
Finance Act forecasts predict a rapid exit, from 2024, from the current period of high

inflation thanks in part to a reduction in corporate margin rates. If we reduce these expected declines in firms' markups by 25%, then, greater inflationary pressures (the inflation rate would be on average 3.05% per year compared to 2.42% in the Finance Act) would lead to an increase in interest rates and a fall in real wages, reducing growth (1.53% per year compared to 1.64% in the Finance Act). The debt would then amount to 111.0% of GDP in 2027. The employment growth rate would be at 0.62% per year, compared to 0.64% in the Finance Act.

Less Favorable Labor-Market Adjustments

The second “favorable wind” concerns the evolution of the labor market. Indeed, the Finance Act projections require strong employment growth which cannot occur, according to our model, without changes in labor-supply behavior. Our estimates indicate that these

Figure 3.28: Counterfactual: Smaller Decline in the Disutility of Working. (B) after re-estimation.



changes in labor supply contribute to 14% of the 2027 GDP forecast. They also indicate that the adjustments are different from one labor market to another: the drop in transfers, provided for in the Finance Act, encourages low-wage employees to compensate for these

reductions in income by increasing their hours worked. As the hours forecasted for this type of employee will increase as much as what this mechanism generates, the model then identifies that their disutility of work remains stable. On the contrary, the expected number of hours worked by middle and high-wage employees increases, while they are only little affected by the drop in transfers: an exogenous increase in the labor supply is then necessary (the disutility of work falls) to generate the expected increase in their hours worked. Thus, beyond the repercussions of reductions in transfers on hours worked, it does not seem unrealistic to envisage that pension and unemployment insurance reforms will result in variations in the employed population and hours worked which are different depending on remuneration. Indeed, social measures accompany these structural reforms via modulations in favor of the more modest workers (for example, the “early retirement for long career” procedure of the latest pension reform). Exogenous and differentiated variations in labor supply can therefore be interpreted as the impact of these modulations.

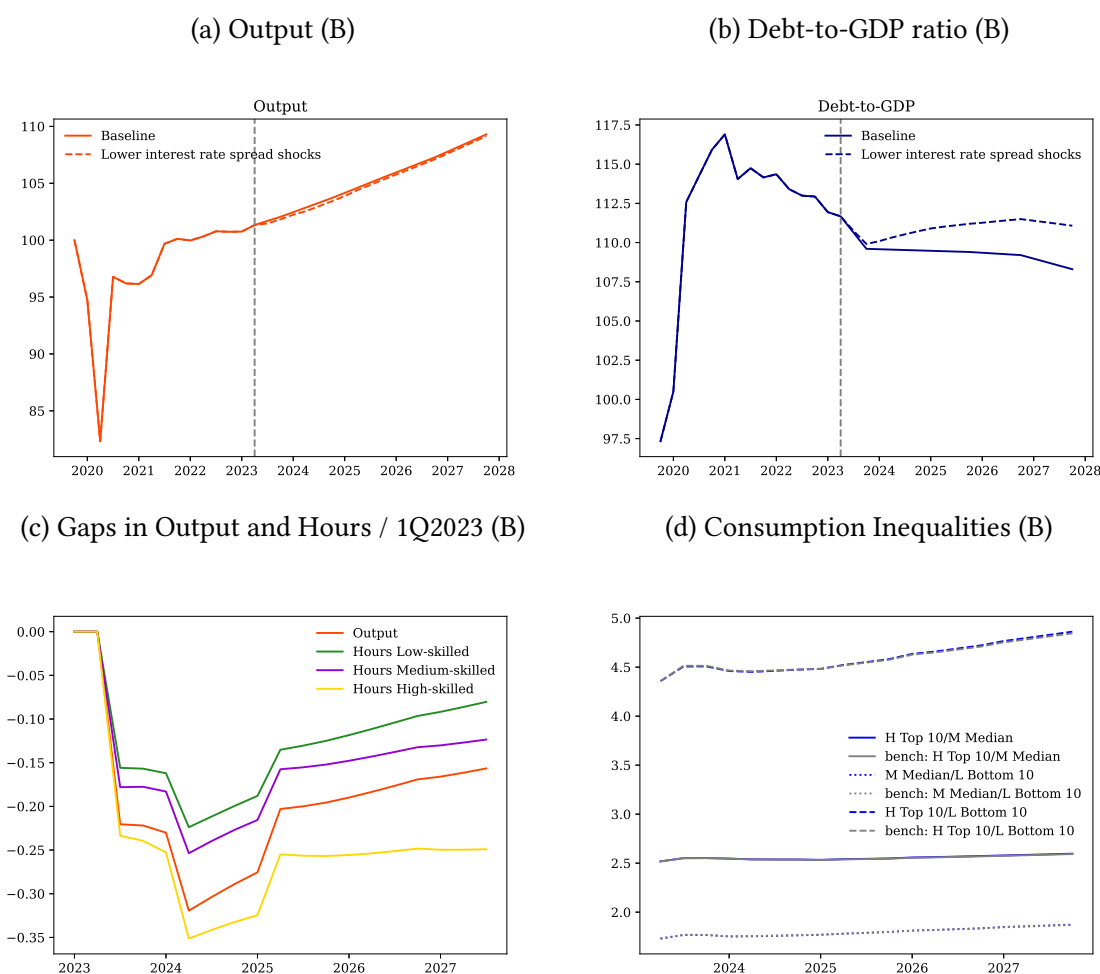
Thus, our model suggests that middle and high-wage workers are willing to work more for unchanged wage levels. Our estimate reveals that the success of these policies is necessary for the Finance Act forecasts to be realized. If the magnitude of these changes in the labor supply of middle and high earners were reduced by 25%, growth would be lower (1.53% per year on average, a loss of 0.11pp compared to the Finance Act) and the increase in employment of high earners would be slowed down, going from 0.9% per year to 0.62%. When the labor supply is less stimulated, inflation is higher (2.79% per year on average between 2023 and 2027 compared to 2.42% in the Finance Act), which partly cushions the increase in the debt ratio which would represent 109.0% of GDP at the end of 2027 (an increase of 0.7 points compared to the Finance Act).

A Less Favorable Interest Rate Spread Dynamics

The third “favorable wind” concerns the evolution of the interest rate. Finance Act projections require low interest rates, with a slow transmission of the ECB-rate increases, that occurred in 2022 and 2023, to the apparent debt rate. Given the fairly long maturity of the French debt (8.5 years in August 2023), the government is not directly exposed to the rise in short-term rates. However, in the event of tension in the sovereign debt market, the gap between the ECB and debt rates could narrow more quickly and weaken the projections.

By being less optimistic than the government about the evolution of the interest rate on the public debt, the annual growth rate would be lower by only 0.02pp over the period 2023-2027 (the economy would be mainly impacted in 2024) but the debt-to-GDP ratio would increase by 2.8pp, rising to 111.1% at the end of 2027.

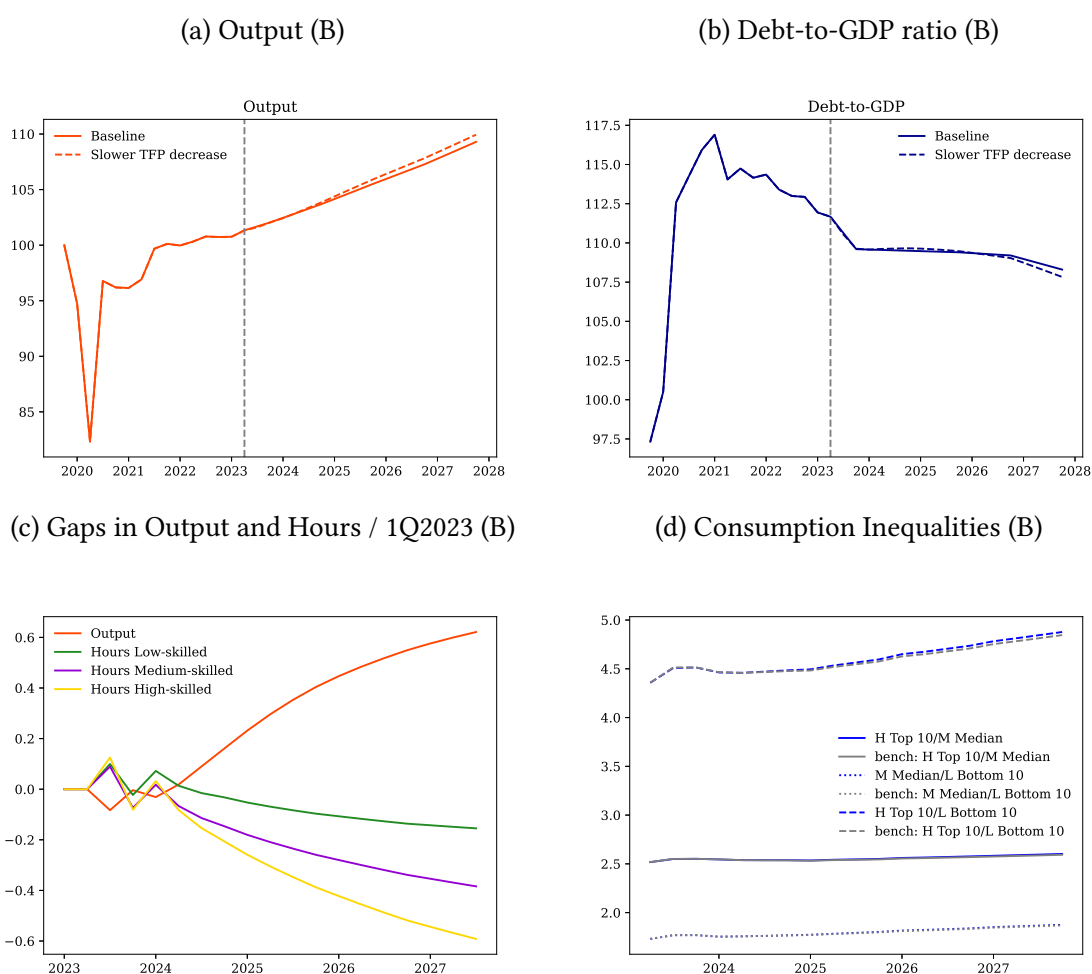
Figure 3.29: Counterfactual: Higher Interest Rate on Public Debt. (B) after re-estimation.



More Moderate Productivity Declines

Our assessment of the Finance Act also reveals that “unfavorable winds” must be taken into account to see Finance Act forecasts occurring. The first is a slowdown in labor productivity which would contribute to 13% of the realization of GDP in 2027. This strong slowdown in productivity in France is also underlined in the OECD forecasts. Our model also indicates that this drop in productivity growth is observed for all wage levels.

Figure 3.30: Counterfactual: Smaller Declines in TFP. (B) after re-estimation.

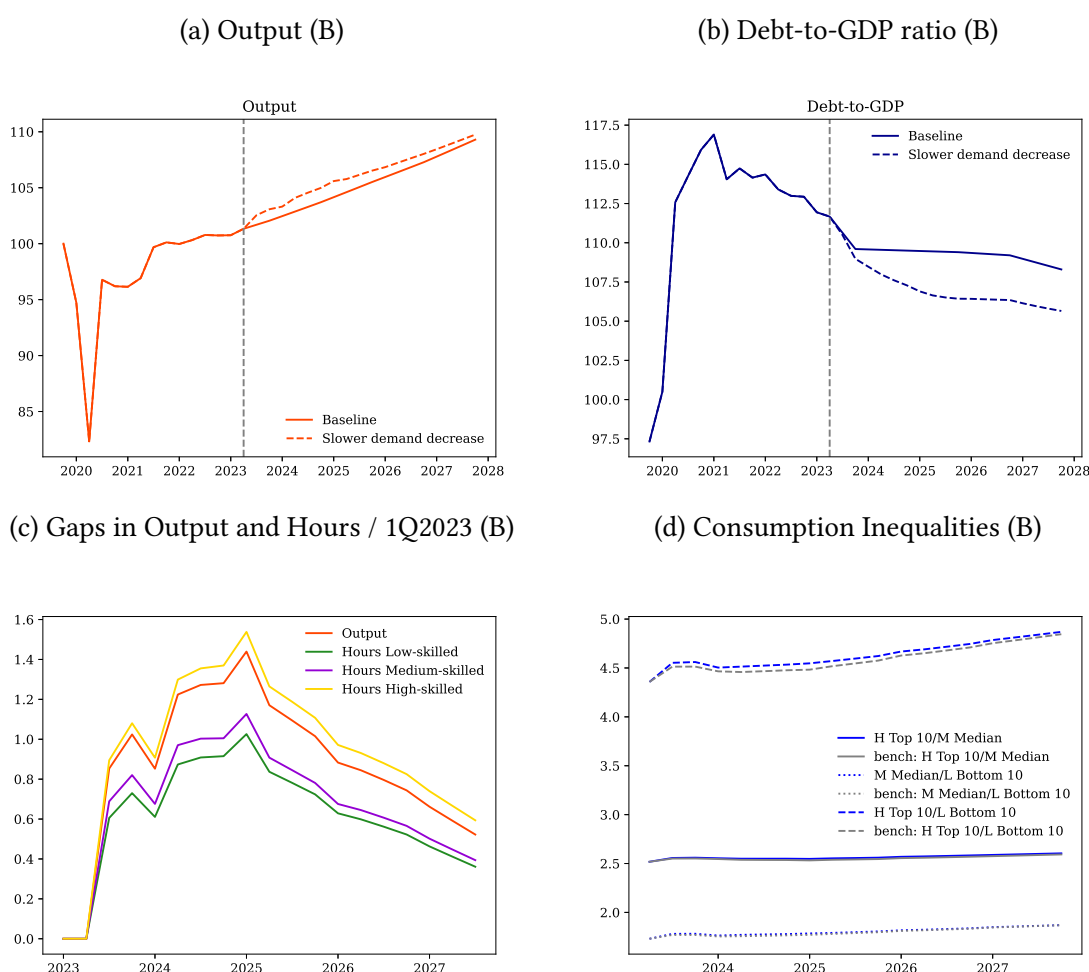


Finance Act forecasts assume very strong employment growth. As a result, labor-productivity growth would decline: the new-filled jobs, for each salary level, would be less productive than current jobs. If these productivity losses were lower, then growth could be 1.76% per year (compared to 1.64%), in a context of very low inflation (2.08% per year compared to 2.42%). The debt would then slightly decrease to 107.8% of GDP in 2027. But these better results regarding GDP, inflation, and debt would come at the cost of a lower job creation. With a more favorable productivity trajectory, fewer jobs would be needed to produce: the employment growth rate would only be 0.65% per year.

More Moderate Demand Decreases

The second “unfavorable wind” is a low level of household demand, which contributes to 20% of the achievement of GDP in 2027. According to the French Institute of Statistics (INSEE), the savings rate of French households was 16% on average between 1950 and 2022, reaching a record level of 27% in 2Q2021. It fell until 2Q2022 when it was 16.6% but has since risen to 18.8% in 1Q2023. In accordance with these data, our model identifies a preference shock that shifts household decisions in favor of savings which remains sustainably above its average in the next two years.

Figure 3.31: Counterfactual: Smaller Rise in Discount Factor (smaller contraction in Demand). (B) after re-estimation.



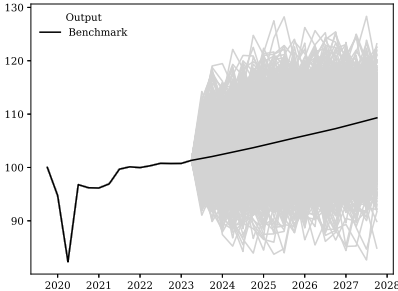
Our results indicate that households would, between 2024 and 2025, have a strong propensity to save while moderating their consumption, despite the growth in their income. With a diminution of these variations in demand, there would be an increase in consumption in 2024-2025 and therefore also in economic activity and state resources. Thus, if we moderate this contraction in household demand, the average GDP growth rate between 2023 and 2027 would be 1.73% per year, with inflation at 2.81% (compared to 2.42%) and

a debt-to-GDP ratio of 105.7% in 2027. Employment would also grow stronger (0.78% per year vs 0.71% in the Finance Act).

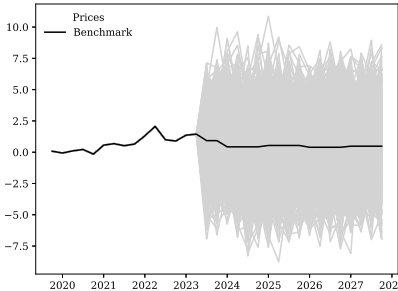
3.7.8 Forecasts

Figure 3.32: Predicted endogenous variables. (A) before and (B) after re-estimation.

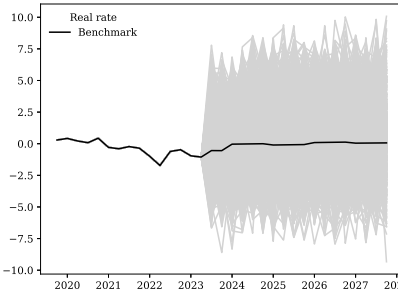
(a) Output (A)



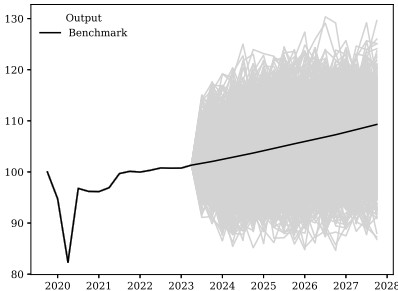
(b) Inflation (A)



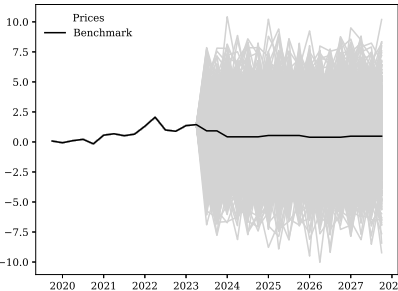
(c) Real interest rate (A)



(d) Output (B)



(e) Inflation (B)



(f) Real interest rate (B)

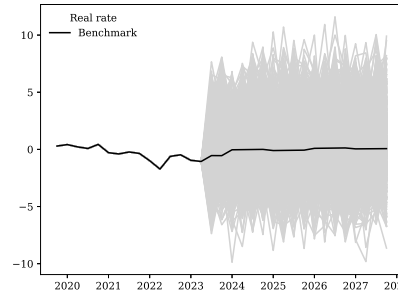
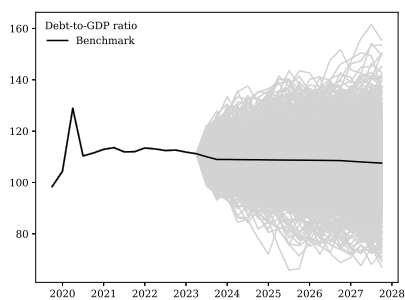
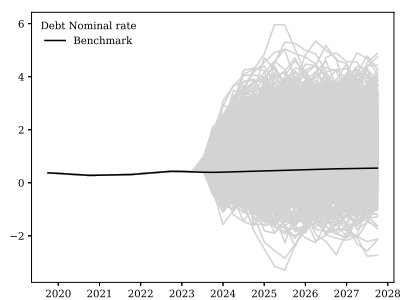


Figure 3.33: Predicted endogenous variables. (A) before and (B) after re-estimation.

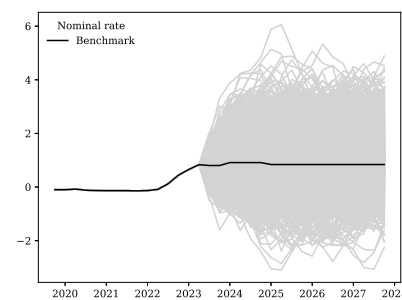
(a) Debt-to-GDP ratio (A)



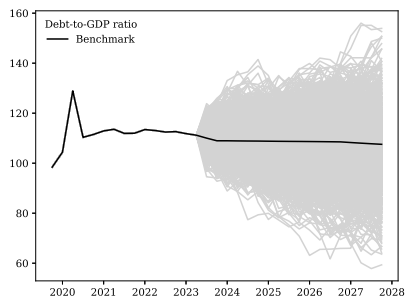
(b) Nominal rate on debt (A)



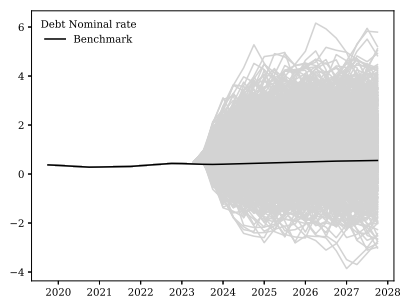
(c) Nominal rate (A)



(d) Debt-to-GDP ratio (B)



(e) Nominal rate on debt (B)



(f) Nominal rate (B)

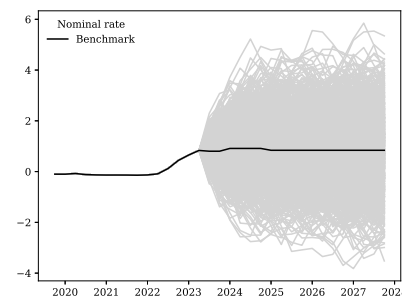
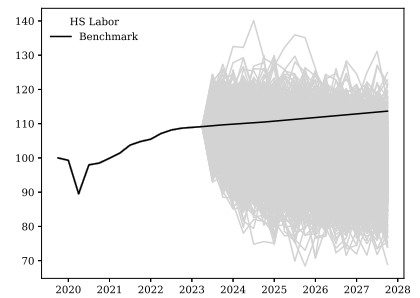
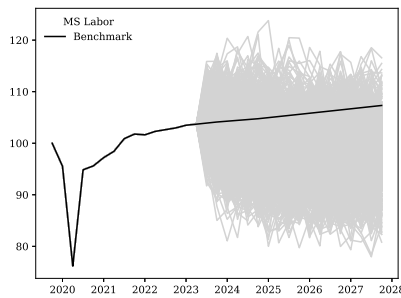
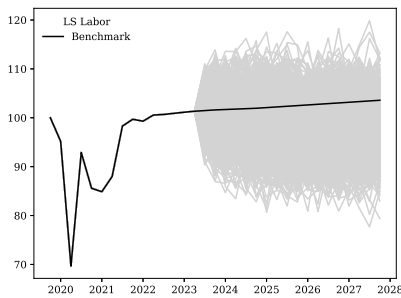


Figure 3.34: Predicted endogenous variables. (A) before and (B) after re-estimation.

(a) Hours worked - low wage workers (A)
 (b) Hours worked - medium wage workers (A)
 (c) Hours worked - high wage workers (A)



(d) Hours worked - low wage workers (B)
 (e) Hours worked - medium wage workers (B)
 (f) Hours worked - high wage workers (B)

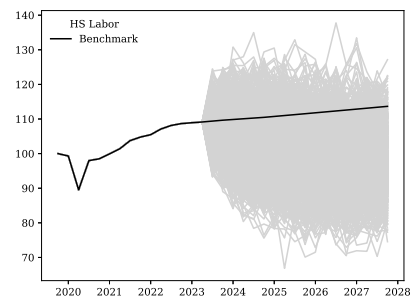
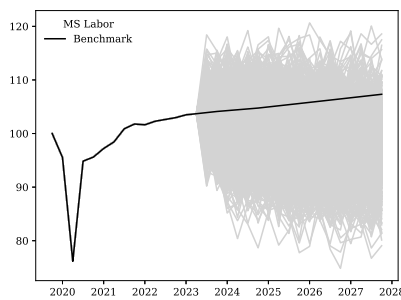
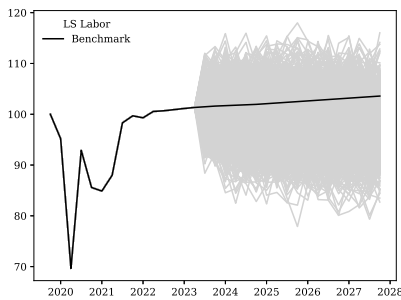
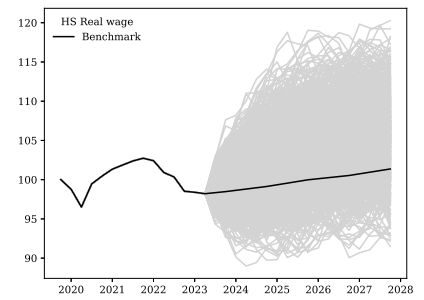
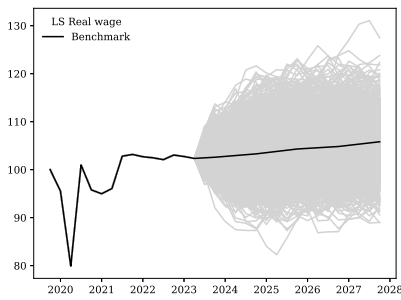


Figure 3.35: Predicted endogenous variables. (A) before and (B) after re-estimation.

(b) Real wage - medium wage workers

(a) Real wage - low wage workers (A) (A)

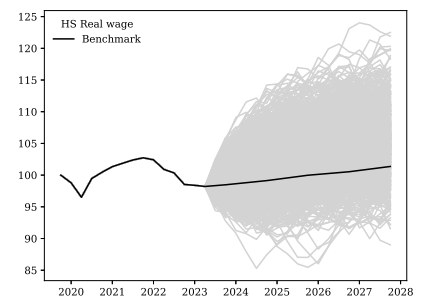
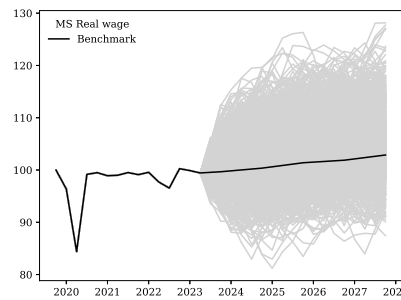
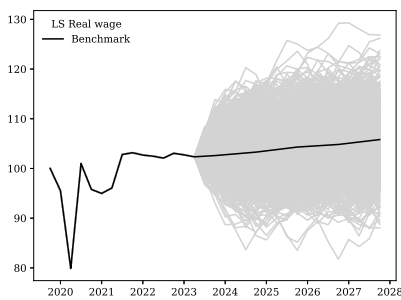
(c) Real wage - high wage workers (A)



(e) Real wage - medium wage workers

(d) Real wage - low wage workers (B) (B)

(f) Real wage - high wage workers (B)



Chapter 4

Income tax fluctuations and uncertainty in France

Keywords: Heterogeneous agents, Taxation, Fiscal policy, Uncertainty.

JEL codes: E21, E62, H20, H31.

4.1 Introduction

Progressive income taxes –and income-tested benefits– are a central tool in the tax toolkit available to governments to raise revenues and redistribute across households. In France, those taxes account for a significant proportion of total tax revenue, around 55%.¹ However, their design requires careful consideration of the efficiency-redistribution trade-off. Indeed, while more progressivity redistributes income across households, thus lowering post-tax and transfer income inequalities, it may disincentivize labor supply, potentially reducing overall output. On the other hand, since it reallocates demand towards households with higher marginal propensities to consume (MPCs), this additional demand could boost output growth.

Following Ramsey, 1927's work, a wide literature has focused on uncovering the optimal tax parameters a government should adopt for a given and specific tax function. Mirrlees, 1971 expanded this approach to identifying optimal nonparametric tax functions². Despite these results, governments may be guided by political choices when it comes to arbitrating between efficiency and redistribution and deciding on the terms of a tax.

Income tax policies in France are no exception and have fluctuated considerably over time, influenced by changes in presidential leadership (Figure 4.1).³ I have estimated this

¹In 2022, social security contributions accounted for 33.9% of the French total tax revenues, while (progressive and flat) income taxes represented 21.6% according to the INSEE.

²See Stantcheva, 2020 for a recent review of dynamic taxation studies.

³The volatility of income tax progressivity has already been documented in the US context by Borella et al., 2022, where they estimate over time the progressivity and level parameters of a progressive taxation

tax progressivity parameter by fitting a standard tax functional form on French statutory income tax schedules over the period 1950-2023 and collected by the *Institut des Politiques Publiques (IPP)*. This figure reveals that every French president has modified the income tax. Large-scale tax reforms, as in 1987 and 2006, when the number of income brackets and the marginal tax rates were lowered, are relatively rare. Yet, other tools such as the non-indexation or partial indexation of threshold incomes to inflation (bracket creep) also generate some volatility. See Appendix 4.6.1 for details on how Figure 4.1 is derived.

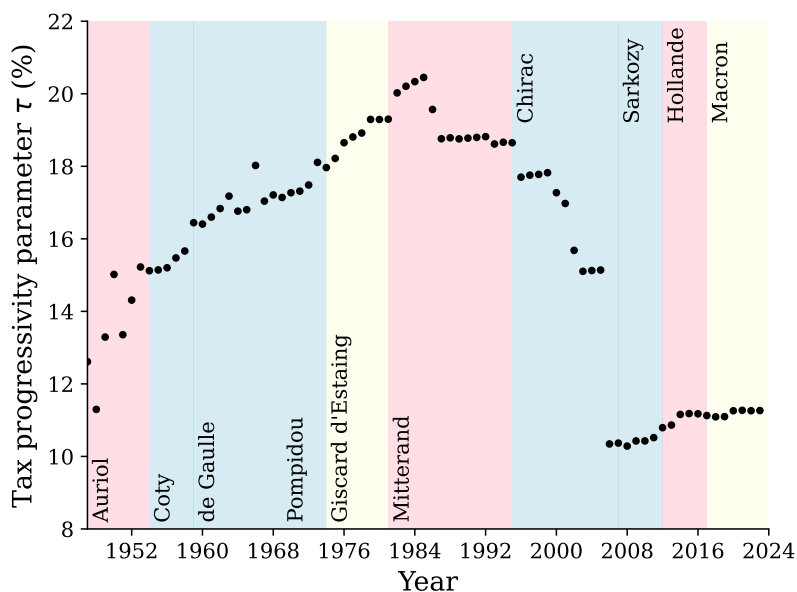


Figure 4.1: Evolution of the income tax progressivity parameter. *Source: Author's calculations based on IPP data.*

Figure 4.1 looks only at the progressive income tax (*impôt sur le revenu (IR)*) which accounts only for a small share of taxes and benefits households face that are levied on their earned income. First, I will document, using micro-level OECD data, the changes in all taxes and benefits affecting households over the 2001-2024 period. Then, I will estimate, standard taxation parameters to bring these observations to the model. It shows that the tax system's progressivity has fluctuated consistently with the motivational fact. What are the macroeconomic implications of these income tax changes? In particular, these frequent fluctuations in income tax policy induce a general feeling of uncertainty among households, as the ever-changing landscape of tax reforms adds an unpredictable element to consider when making their consumption-labor-saving decisions. Borella et al., 2022 acknowledge that uncertainty regarding future taxes should have an impact but do not study it as they assume perfect foresight. This paper attempts to address this issue by relaxing the perfect foresight hypothesis. How does the resulting tax uncertainty affect the economy? To

function à la Feldstein, 1969, Bénabou, 2000, Heathcote, Storesletten, and Giovanni L. Violante, 2017. Qiu and Russo, 2024 provides a cross-country comparison of estimated similar tax parameters (including for France). They cover a 1980-2019 timespan albeit at a five or three-year frequency. The comparison of their results to this paper is discussed in Appendix 4.6.4.

answer these questions, I propose a simple model of heterogeneous agents, calibrated on French data, to assess the effects of fiscal fluctuations and uncertainty.

In practice, I model fiscal uncertainty in a tractable way where the government opts for either a low, average, or high progressivity income tax regime by controlling the transition matrix between these states. In my setup, this tax progressivity will affect the economy through labor supply distortions, which have been proven to be significant by Zidar, 2019 in the US context and Lehmann, Marical, and Rioux, 2013 in the French context. Both studies suggest that tax changes impact both the extensive and intensive margins of labor supply. Those effects are not concentrated only on top earners but rather can lead to big responses among median-income households. Heathcote, 2005 shows in a heterogeneous agent model with incomplete markets that income tax changes affect the economy not only through labor distortions but also through savings decisions. Indeed, incomplete insurance together with borrowing constraints prevents some households from smoothing consumption.

I also consider a static framework, to have an intuition of the impact of tax uncertainty and derive analytical expressions of the channels at play. It demonstrates that uncertainty hampers labor supply, output, and overall economic welfare compared to a deterministic steady state due to the concave nature of these macroeconomic variables with respect to the tax progressivity parameter. Besides, I show that the uncertainty cost depends on the idiosyncratic income processes. As such, the microeconomic risk from the household perspective and the macroeconomic risk interact, highlighting a possible amplification of fiscal uncertainty.

The fact that macro-fiscal and micro-labor risk interact relates to Schaab, 2020 which shows that uncertainty's impact on the economy is all the more significant when the micro and macro uncertainty interact. In his framework, however, this interaction relies on job separation and finding rates (micro risk for the households) changing along the business cycle (macro risk). Households react to recessions not so much because the real wage is affected but primarily because the risk of losing one's job increases. In my framework, the macro fluctuations of the tax progressivity parameter endogenously induce a change in disposable incomes (micro risk).

Literature. This work is part of the literature on the effects of household taxation, specifically progressive income taxation: Heathcote, Storesletten, and Giovanni L. Violante, 2017, Ferriere and Navarro, 2023, Pizzo, 2023. McKay and Reis, 2016 quantifies the size of the automatic stabilization ensured by the US fiscal policy. While they assess the business cycle implications of progressive income taxation, the level of its progressivity is held constant in their exercise. Zoi, 2023 integrates a heterogeneous agent with a time-varying progressive income tax and income risk volatility into a business cycle model. Studying the impulse response function to an unexpected TFP shock, she finds that the government's optimal policy is to adjust tax progressivity along the business cycle to wave off some of the negative consequences of the shock. Compared to her framework, I add tax shocks as a source of risk.

This paper also relates to the uncertainty literature and its impact on the business cycle. Bloom, 2009 proves empirically that uncertainty has high fluctuations. Policymakers

(from central banks to governments) often cite uncertainty as a concern for the economic outlook, while Backus, Ferriere, and Zin, 2015 shows it interacts with the business cycle. However, the literature on the business cycle consequences of uncertainty usually found small or ambiguous results. Basu and Bundick, 2017 shows that demand uncertainty shocks amplified the Great Recession, explaining 0.6% of output decline (1.5% when accounting for the zero lower bound (ZLB)). Fernández-Villaverde, P. Guerrón-Quintana, et al., 2015 and Born and Pfeifer, 2014 both consider the consequences of fiscal policy uncertainty. The first argues that fiscal uncertainty shocks significantly impact macroeconomic variables while the latter concludes with minor consequences. Both approaches rely on a representative-agent framework which fails to capture that fiscal multipliers depend on which part of the distribution the tax burden lies as demonstrated by Ferriere and Navarro, 2023. Ábrahám, Brendler, and Cárceles-Poveda, 2024 studied capital tax duration uncertainty in a heterogeneous firm setting and found a significant impact through the tax arbitrage opportunities it generates.

The closest to this paper is Ferriere, Grübener, et al., 2023. They derive an optimal income tax-and-transfer schedule (albeit without the uncertainty discussion) in a heterogeneous-agent model and estimate it on US data. I build on this framework to evaluate the uncertainty cost of income tax risk analytically. Chang and C. Wu, 2023 study the optimal fiscal policy a government should adopt when the households' preference and income process parameters are uncertain. Contrary to Chang and C. Wu, 2023, I assume the uncertainty lies in the government's intentions regarding tax progressivity, while the government is assumed to have a perfect understanding of the mechanisms at play. Examining the interplay of those sources of uncertainty is an interesting research avenue albeit outside the scope of this paper.

The rest of the paper is organized as follows. Section 4.2 presents the empirical evidence of income tax fluctuations in France. Section 4.3 introduces a static model to understand the consequences of fiscal uncertainty before extending it to the full model (Section 4.4).

4.2 Empirical evidence

Progressive tax and transfer systems are commonly modeled via a functional form introduced by Feldstein, 1969 and popularised by Bénabou, 2000 and Heathcote, Storesletten, and Giovanni L. Violante, 2017, hereafter denoted HSV function. This choice is convenient because it provides a good approximation of observed tax data while being parsimonious, relying solely on two parameters. The first, $\tau \in [-1, 1]$ governs the tax system's progressivity, while the second, λ , defines the overall taxation level.

Pre-income tax income y and post-income tax income \tilde{y} are related through

$$\tilde{y} = (1 - \lambda)y^{1-\tau}.$$

I aim to estimate the parameters (τ, λ) that best approximate the French tax and transfer system and document the changes in those parameters over the years. While Figure 4.1 only

accounted for the progressive income tax (*Impôt sur le revenu* or IR), I will now consider all taxes and benefits indexed on labor income.

4.2.1 Tax and transfer data

The OECD's tax-benefit (TaxBEN) simulator reports the amounts of income-tested taxes and transfers faced by French households for each year between 2001 and 2024. It includes the employee social security contributions, income taxes as well as in-work, social assistance, and housing benefits⁴. Since this simulator is cross-country, I will be able to compare the French income tax profiles to the ones in the US, for instance. The profiles of tax and transfer amounts along the labor income distribution are modified throughout time, following the various government reforms. Using a k -means clustering method, I classify years with similar tax profiles. Each cluster represents a common spirit of the institutional framework. In France, all taxes and benefits can be grouped into two clusters (except for the income tax which exhibits three clusters). Figure 4.2 displays only four years representative of each cluster's extremums.

The progressive part of income tax profiles changed as governments changed the marginal rates, the number of tax brackets, and their thresholds. But more importantly, at the end of the 1990s, two additional income taxes were introduced on top of the historic progressive income tax, the CSG (for *Contribution sociale généralisée*) and CRDS (*Contribution pour le remboursement de la dette sociale*). They are low flat rates applied on a broad taxable base that includes all labor income. This explains why even the bottom of the distribution faces a non-zero yet positive income tax (Figure 4.2a).

Regarding social contributions (Figure 4.2b), the biggest change happened in 2019 when Macron's government reduced them at the bottom of the distribution and made them less regressive. The reform compensated for the implied decrease in tax revenues with an increase in the CSG (*basculé cotisation-CSG*). Note that, 2018 is an in-between year since the reform was implemented as of 1 October 2018.

Housing benefits (Figure 4.2d) have remained very similar over the 2001-2024 period regarding who benefits (those earning less than 45% of the average income), the amounts perceived, and the phasing-out rate. Partial indexation of those benefits to inflation may explain why the amounts decreased over the years.⁵

Social assistance benefits (Figure 4.2e) lately reverted to their early 2000 nature: concentration on those earning less than 20% of the average income, quick phasing-out, and less generous amounts (compared to 2009 and 2015). More generous in-work benefits compensate for this decrease in social assistance benefits, with households earning up to 80% of the average income receiving such transfers (Figure 4.2e). The amounts have also significantly increased. See Appendix 4.6.2 for a brief description of the main reforms that occurred.

⁴For now, I abstract from family and unemployment benefits that require keeping track of the household composition (number of adults and children as well as their ages) and employment history (previous wage, time in employment...) respectively.

⁵A 5 € reduction in the monthly allowance for each beneficiary was also implemented from the end of 2017.

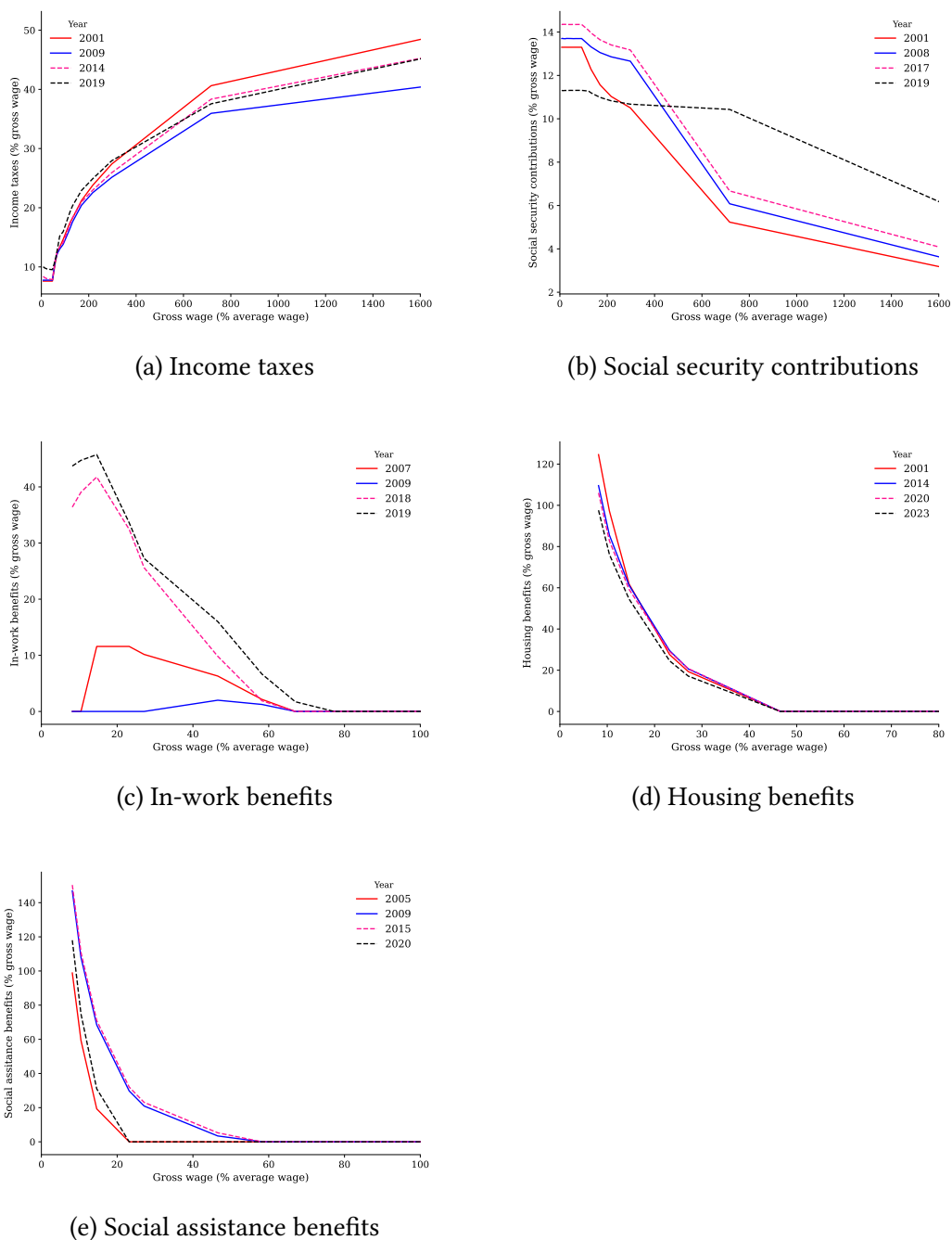


Figure 4.2: Taxes and transfers (as % of gross income) along the income distribution in France.

Figure 4.24 (in Appendix 4.6.5) shows equivalent profiles for the US. Usually, the taxes and benefits have changed less often than in the French case. Most years display very similar profiles except when the fiscal authority reacts to crises. For instance, the income tax featured negative taxes at the bottom of the distribution when hit by the Great Financial Crisis. Similarly, in-work benefits increased substantially in 2009 and then 2021 as the COVID-19 pandemic hit. These changes are usually temporary and a response to an ex-

ogenous economic shock, while French changes are more long-lasting and seem driven by changes in political leadership. Note that the US does not feature housing benefits.

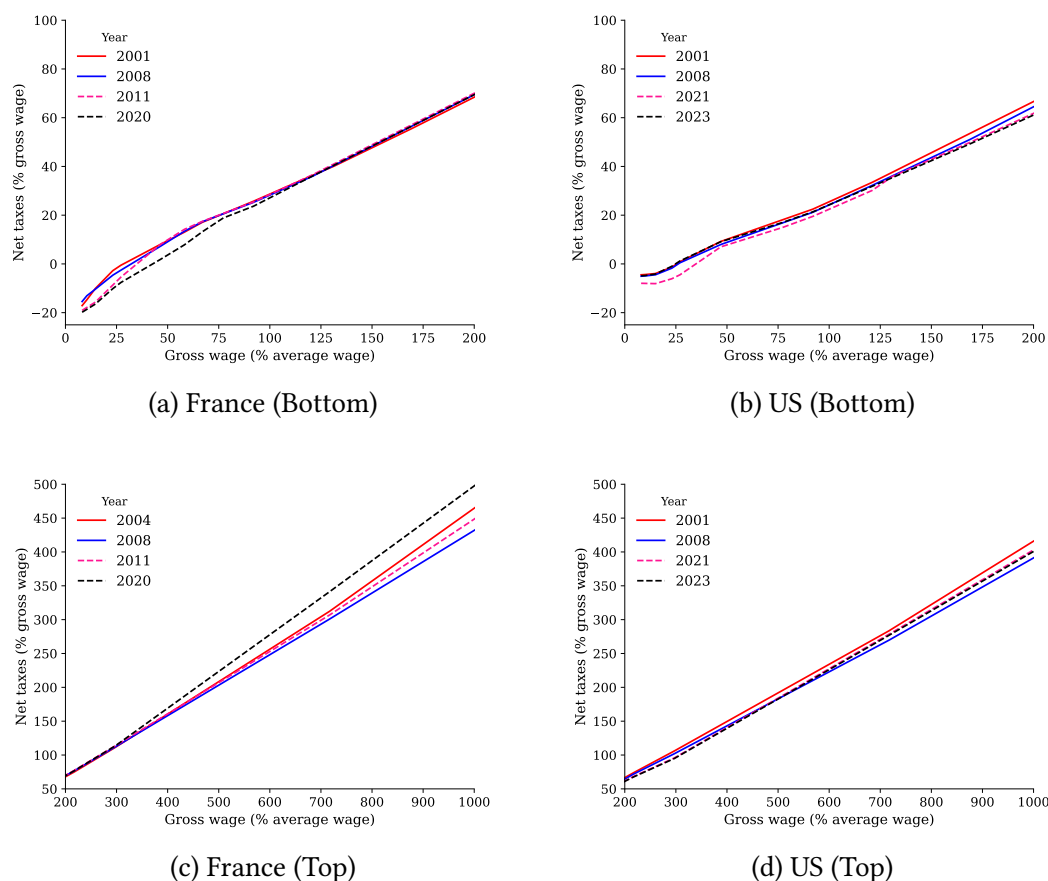


Figure 4.3: Total taxes and transfers (as % of average wage) along the income distribution.

4.2.2 Net tax function

Aggregating all these taxes and transfers, one can compute the net taxes a French (or American) household would pay given his labor income (Figure 4.3). It confirms that French taxes changed more than in the US, both at the bottom and top of the income distribution.

The estimates for each year between 2001 and 2024 of the parameters (λ, τ) are reported in Figure 4.4.⁶ It shows that the tax function parameters fluctuate through time, consistently with the motivational fact regarding the tax progressivity τ . Heathcote, Storesletten, and G. Violante, 2020 show that underlying income inequality trends may call for progressivity changes. In particular, increased residual inequalities should lead to higher progressivity. However, in the French case, these changes in tax parameters do not seem to be a consequence of policymakers reacting to changing pre-income tax inequalities for they seem to

⁶See Appendix 4.6.4 for a detailed presentation of the estimation procedure.

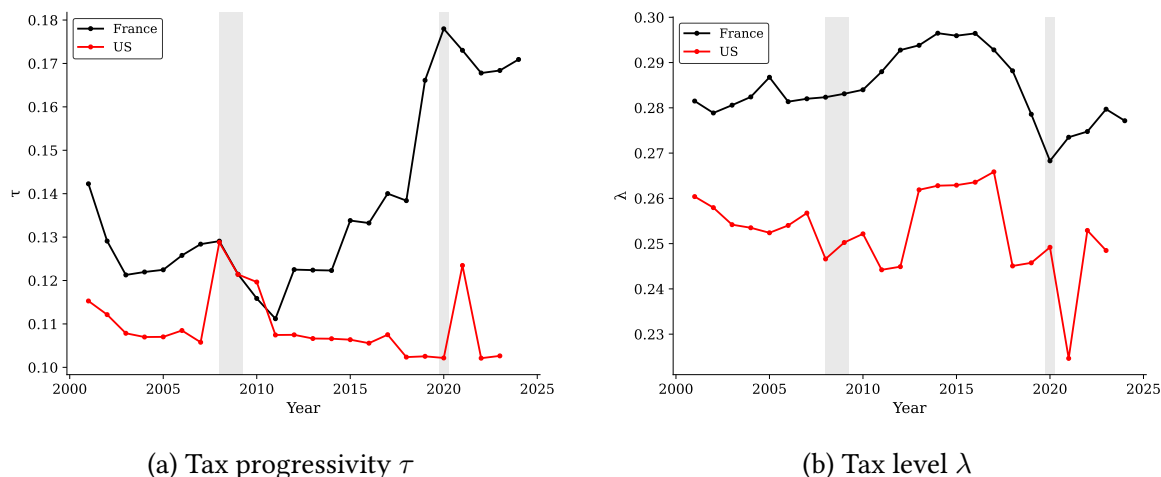


Figure 4.4: Estimation of the HSV tax function parameters on French and US data. *Source: Author's calculations on OECD TaxBEN data.*

be quite stable in France (contrary to the US) over the period (Figure 4.5)⁷. While some of the tax parameter changes occurred during the Great Financial Crisis, they tend to be long-lasting in France, suggesting they are not just temporary adjustments to the business cycle, unlike in the US.



Figure 4.5: Pre-tax income inequality indicators. *Source: World Inequality Database (WID) and GRID (Global Repository of Income Dynamics) data.*

France's fluctuating tax progressivity τ can create uncertainty for households as to whether they will face high or low tax progressivity tomorrow. The following sections will look at the impact of this fiscal uncertainty on progressivity, first in a static framework (Section 4.3) and then in a quantitative heterogeneous-agent model (Section 4.4).

⁷As Guvenen, Pistaferri, and Giovanni L Violante, 2022a indicate, the Gini coefficients are computed on gross labor earnings (reported in the GRID database). Meanwhile, the WID indicators are computed on pre-tax income that includes labor and capital earnings, as well as social benefits net of the contributions associated. It thus partly accounts for redistribution but not fully as other taxes and transfers are absent.

In practice, the uncertainty on fiscal progressivity will be captured thanks to a three-state Markov process,⁸ for the low, average, and high progressivity states observed on panel (a) of Figure 4.4. Such process can reproduce the fluctuations observed as confirmed in Figure 4.6.

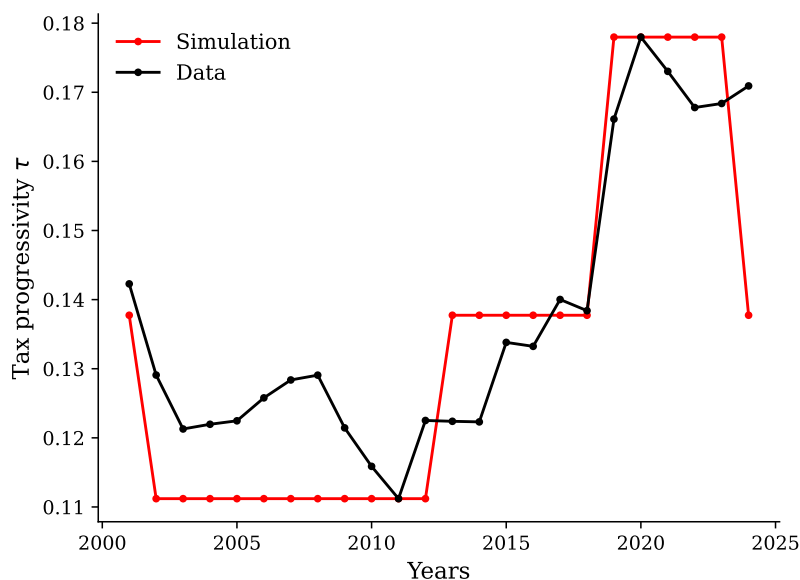


Figure 4.6: Approximation of tax progressivity changes by a Markov process.

4.3 Static model

How does income tax uncertainty impact households' decisions and aggregate macroeconomic variables? To answer this, a simple static and analytical model can be considered.

4.3.1 Optimal income tax progressivity without uncertainty

Benchmark. Without uncertainty, the considered analytical model is just as in Ferriere, Grübener, et al., 2023. A continuum of hand-to-mouth households i face a static problem:

$$\begin{aligned} \max_{c_i, n_i} \quad & \ln(c_i) - \theta \frac{n_i^{1+\nu}}{1+\nu} \\ \text{s.t.} \quad & c_i = (1 - \lambda)(e_i n_i)^{1-\tau} \end{aligned}$$

where the idiosyncratic labor productivities e_i are log-normally distributed, $e_i \sim \ln - \mathcal{N}\left(-\frac{\sigma_e^2}{2}, \sigma_e^2\right)$ ⁹.

⁸The calibration of this Markov process is presented in Table 4.4.

⁹As a result, $\mathbb{E}(e_i) = 1$ and $\text{Var}(e_i) = \exp(\sigma_e^2) - 1$.

The government raises taxes $\mathcal{T}(\tau) = \int (e_i n_i - (1 - \lambda)(e_i n_i)^{1-\tau}) di$ to finance government spending G ,

$$G = \mathcal{T}(\tau).^{10}$$

The firm produces according to $Y = L$. The labor market clearing yields

$$L = \mathcal{N}^e \equiv \int e_i n_i di.$$

The good market clearing condition writes

$$Y = G + \mathcal{C}$$

where $\mathcal{C} = \int c_i di$. Solving this model we get the following expressions for labor supply, output, income tax level, and welfare:

$$\begin{aligned} n_0(\tau) &= \left(\frac{1 - \tau}{\theta} \right)^{\frac{1}{1+\nu}} \\ Y_0(\tau) &= n_0(\tau) \\ \lambda_0(\tau) &= 1 - \frac{Y_0(\tau) - G}{Y_0(\tau)^{1-\tau}} \exp\left(\frac{\sigma_e^2}{2} \tau(1 - \tau) \right) \\ W_0(\tau) &= \ln(n_0(\tau) - G) - \frac{1 - \tau}{1 + \nu} - \frac{(1 - \tau)^2 \sigma_e^2}{2} \end{aligned}$$

where the subscript zero indicates that the expressions were obtained without additional exogenous income (just labor income).

With an exogenous income. What would happen if some agents receive an exogenous income y_i ? For simplicity, we assume a fraction m (representative of the entire distribution for σ_e) of agents rely only on labor income, while a mass $1 - m$ receives an exogenous income y on top of that. This extension of the previous framework allows us to see how the results would be modified when accounting for a proxy of financial incomes, while not fully incorporating a saving strategy. Because of this formulation, individual labor decisions are no longer identical for every household. Those having $y = 0$ will behave as previously, supplying n_0 hours of work while those benefiting from an additional income will lower their labor supply due to the wealth effect (see Appendix 4.6.6 for the detailed derivations).

In that case, integrating $\int e_i n_i di$ gives the output

$$Y(y, m, \tau) = Y_0(\tau) - (1 - m) \frac{y}{1 + \nu} \frac{Y_0(\tau)}{Y_0(\tau) - G} \exp(-\tau(1 - \tau)\sigma_e^2)$$

This expression reveals that output is depressed compared to Y_0 due to the wealth effect of households receiving an additional income. Naturally, this depressing effect is all the more muted that the share m of households relying solely on their labor is high. Figure 4.7 confirms this.

¹⁰Note that the value of the income tax level parameter λ is unique once the government has chosen a tax progressivity τ if it wants to raise G .

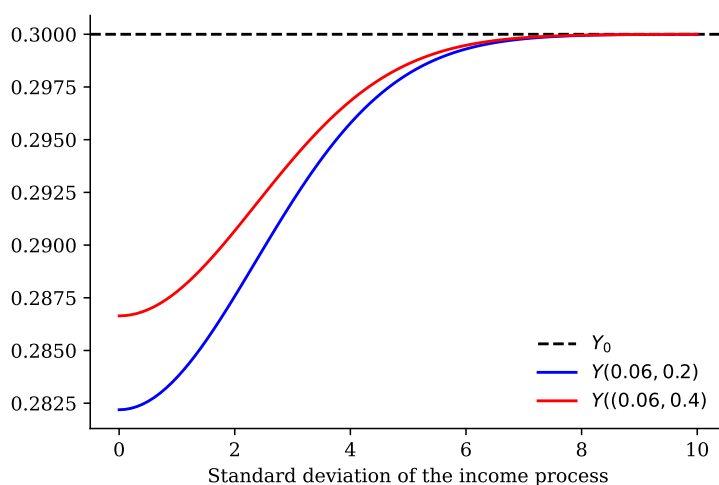


Figure 4.7: Output when only a share $m = 0.2$ (blue) or $m = 0.4$ (red) of households receives labor income only.

This opens the door to increased output if tax progressivity uncertainty depresses financial incomes and thus attenuates the wealth effect. Richer households would then supply more hours of work to smooth consumption.

Non-log utility. Choosing a log utility is convenient for obtaining tractable results. How are the results modified when deviating from $\sigma = 1$ in the utility function?

With $\sigma \neq 1$, labor supply is no longer invariant across the idiosyncratic productivity distribution. The level parameter of income taxation λ affects the labor supply decisions as well as the progressivity parameter τ . Finally, both households' preference parameters (σ , ν) matter:

$$n_i(\tau, \lambda, \sigma, \nu) = \left(\frac{(1 - \tau)(1 - \lambda)^{1 - \sigma} e_i^{(1 - \tau)(1 - \sigma)}}{\theta} \right)^{\frac{1}{\nu + \tau + \sigma(1 - \tau)}}.$$

Integrating over the distribution, we can get the aggregate labor supply that equates to output

$$Y(\tau, \lambda, \sigma, \nu) = \left(\frac{(1 - \tau)(1 - \lambda)^{1 - \sigma}}{\theta} \right)^{\frac{1}{\nu + \tau + \sigma(1 - \tau)}} \exp \left(\frac{\sigma_e^2 (1 - \tau)(1 + \nu)(1 - \sigma)}{2 (\nu + \tau + \sigma(1 - \tau))^2} \right).$$

The fact that the government must have a balanced budget makes it possible to determine λ and, by integrating the utilities across the distribution of households, we can obtain the welfare expression. See Appendix 4.6.6 for the details. Figure 4.8 shows how output Y , the level of income taxation λ , and welfare W are modified when departing from $\sigma = 1$ (black dots). In particular, as risk aversion σ increases, households are willing to work less, leading to smaller output. Welfare is lower compared to the log case.

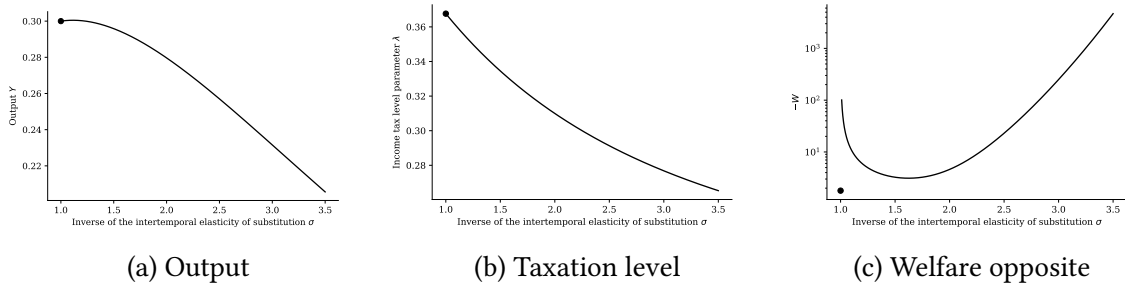


Figure 4.8: Output Y , taxation level λ , and welfare opposite $-W$ as functions of the inverse of the intertemporal elasticity of substitution σ . The black dots report the log case ($\sigma=1$) values.

4.3.2 Optimal income tax progressivity with uncertainty

I now enrich Ferriere, Grübener, et al., 2023’s simple framework (in the log-utility and no exogenous income case) to account for fiscal uncertainty in the form of income tax progressivity uncertainty. The government may switch between two fiscal regimes a low τ^- and a high progressivity τ^+ one (with $\tau^- < \tau^+$). The low progressivity state occurs with a probability p (the high progressivity one with a probability $1 - p$). Since labor supply is a decreasing function of income tax progressivity τ , we have that

$$\left(\frac{1 - \tau^+}{\theta}\right)^{\frac{1}{1+\nu}} \equiv n^+ < n^- \equiv \left(\frac{1 - \tau^-}{\theta}\right)^{\frac{1}{1+\nu}}$$

$$Y^+ < Y^-$$

Whether welfare is higher in the low or high progressivity state depends on the specific values considered as welfare is an increasing and then decreasing function of tax progressivity τ . As a result, $W^- \gtrless W^+$ is ambiguous.

However, for every variable $X \in \{Y, n, C, W, \tau, \lambda\}$, its expected value given the fiscal uncertainty is given by

$$\mathbb{E}(X) = pX^- + (1 - p)X^+.$$

The impact of uncertainty on variable X , denoted by u^X is given by the difference between this expected value $\mathbb{E}(X)$ and the value of variable X evaluated at the expected tax progressivity $X(\bar{\tau})$ where $\bar{\tau} \equiv \mathbb{E}(\tau)$. This difference will be non-null as long as X is a nonlinear function in τ ¹¹

$$u^X \equiv \mathbb{E}(X) - X(\mathbb{E}(\tau)).$$

Because labor supply and welfare are concave functions of tax progressivity (see Appendix 4.6.6), uncertainty is detrimental to labor supply, output, and aggregate welfare in the economy. Figure 4.9 shows this for welfare.

¹¹In fact, u^X is the Jensen gap and will be positive (resp. negative) if X is a convex (resp. concave) function of τ .

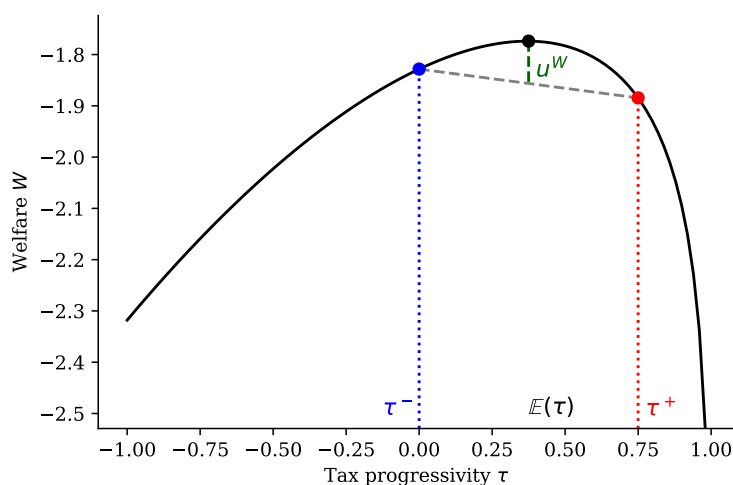


Figure 4.9: Impact of fiscal uncertainty on welfare.

How large are these uncertainty gaps? What channels drive the size of these uncertainty effects? I derive, as much as possible, analytical expressions to get the size of these uncertainty gaps and their decompositions in several channels.

Following Ferriere, Grübener, et al., 2023, the welfare can be decomposed into a size S , a labor disutility L^D and a redistribution R channel

$$W(\tau) = \underbrace{\ln(Y(\tau) - G)}_{\equiv S(\tau)} - \underbrace{\frac{1 - \tau}{1 + \nu}}_{\equiv L^D(\tau)} - \underbrace{\frac{(1 - \tau)^2 \sigma_e^2}{2}}_{\equiv R(\tau)}.$$

As a result, the cost of uncertainty on welfare u^W is the sum of the Jensen gaps of each channel

$$u^W = u^S + u^{L^D} + u^R.$$

Table 4.1 shows what channels drive the cost of tax uncertainty on welfare. The labor disutility term being linear in τ , it bears zero cost.

	Impact on welfare uncertainty cost		
	Size	Labor disutility	Redistribution
Income volatility $\sigma_e \uparrow$	No impact	No impact	\uparrow
Government spending $G \uparrow$	\uparrow	No impact	No impact
Tax progressivity volatility \uparrow	\uparrow	No impact	\uparrow
Labor flexibility \uparrow ($\nu \downarrow$)	\uparrow	No impact	No impact
Labor disutility $\theta \uparrow$	\uparrow	No impact	No impact

Table 4.1: Drivers of the uncertainty cost on welfare.

Impact of tax volatility. As expected, the cost of tax uncertainty increases with the size of the support $\tau^+ - \tau^-$. This goes through both the size and redistribution channels. The size channel is quantitatively bigger than the redistribution channel.

Impact of pre-tax income volatility. The cost of tax uncertainty increases with the volatility of the pre-tax income σ_e . In practice, the cost increases linearly in the variance σ_e^2 .

Impact of changes in government spending. Changing government spending G affects the welfare cost of uncertainty only through the size channel. As public spending increases, so do the taxes to be levied, making distortions in the labor market all the more costly.

Impact of preference parameters. The size channel also reveals that the uncertainty gap depends on the Frisch elasticity of labor supply and labor disutility

$$u^S = \ln \left(\frac{\left(\left(\frac{1-\tau^-}{\theta} \right)^{\frac{1}{1+\nu}} - G \right)^p \left(\left(\frac{1-\tau^+}{\theta} \right)^{\frac{1}{1+\nu}} - G \right)^{1-p}}{\left(\left(\frac{1-p\tau^- - (1-p)\tau^+}{\theta} \right)^{\frac{1}{1+\nu}} - G \right)} \right)$$

As might be expected, greater labor supply flexibility (reflected in a lower value for the ν parameter) leads to a greater adjustment in labor supply response, making the cost of uncertainty greater (Figure 4.10a).

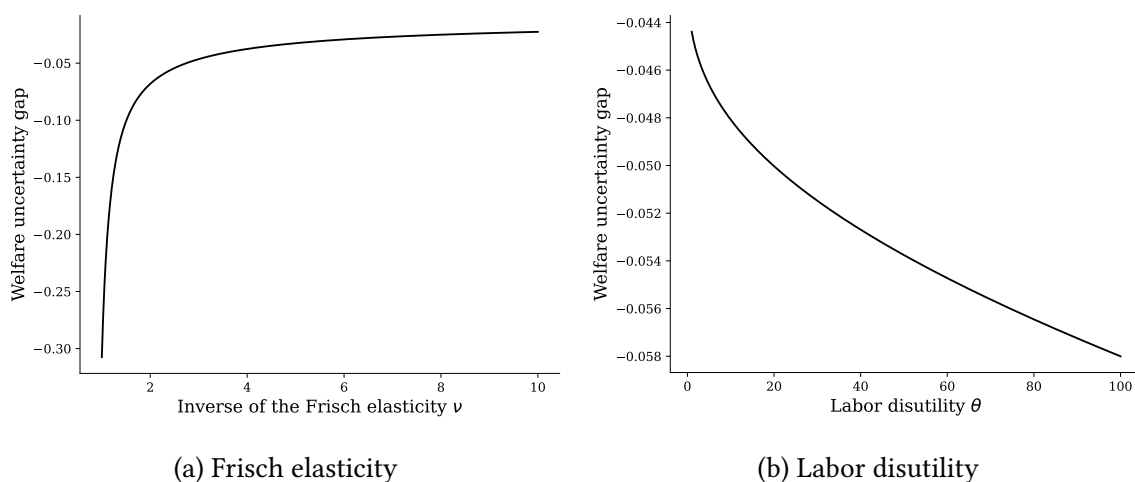


Figure 4.10: Impact of the ν and θ parameters on the welfare uncertainty gap (for $p = 0.25$).

As the labor disutility θ decreases, additional hours of labor impact less negatively the households' utility, lowering the cost of fiscal uncertainty (Figure 4.10b).

4.4 Quantitative model

4.4.1 Model

I now consider income tax uncertainty in a fully-fledged heterogeneous-agent model *à la* Aiyagari, 1994. Appendix 4.6.7 provides a graphical description of the model.

Households. A continuum of *ex-ante* identical households of mass 1 faces idiosyncratic income risk $e \in \mathcal{E}$. The transition matrix from e to e' is Q_e . The tax and transfer system is modeled by a progressive fiscal scheme defined by (τ, λ) where both the level λ and progressivity τ parameters are time-varying. The households can only save on one asset, the government bond.

$$\begin{aligned} v_t(e, a_-) &= \max_{c, a, n} u(c, n) + \beta \mathbb{E} v_{t+1}(e', a) \\ \text{s.t.} \quad &c + a = (1 + r) a_- + (1 - \lambda)(w e n)^{1-\tau} + \vartheta(e) \\ &a \geq 0 \end{aligned}$$

where the utility function is $u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \theta \frac{n^{1+\nu}}{1+\nu}$ and ϑ is a lump-sum transfer whose amount is invertly proportional to e .

Aggregating over the distribution Γ_t , the following aggregates are defined:

$$\begin{aligned} \mathcal{C}_t &= \int_{e, a_-} c_t(e, a_-) d\Gamma_t(e, a_-) \quad (\text{Consumption}) \\ \mathcal{A}_t &= \int_{e, a_-} a_t(e, a_-) d\Gamma_t(e, a_-) \quad (\text{Wealth}) \\ \mathcal{N}_t &= \int_{e, a_-} n_t(e, a_-) d\Gamma_t(e, a_-) \quad (\text{Labor}) \\ \mathcal{N}_t^e &= \int_{e, a_-} e_t n_t(e, a_-) d\Gamma_t(e, a_-) \quad (\text{Effective labor}) \\ \mathcal{T}_t &= \int_{e, a_-} T(w_t e_t n_t(e, a_-), \lambda_t, \tau_t) d\Gamma_t(e, a_-) \quad (\text{Income tax revenues}) \end{aligned}$$

where $T(x, \lambda, \tau) = x - (1 - \lambda)x^{1-\tau}$ gives the amount of (net) taxes a household with labor income x has to pay given the tax system (λ, τ) .

Firm. The firm produces using labor only: $Y_t = Z_t L_t$. As a result, $w_t = Z_t$.

Fiscal policies. The government raises taxes (and distributes transfers) through a progressive income tax \mathcal{T} defined by (λ, τ) to reimburse the exogenous debt B and finance government spending G and lump-sum transfers ϑ :

$$B_t + \mathcal{T}_t = (1 + r_t) B_{t-1} + G_t + \vartheta_t$$

Assuming a constant debt and transfers, this rewrites:

$$\mathcal{T}_t = r_t B + G_t + \vartheta.$$

Note that across the various experiments, the fiscal burden on households \mathcal{T} (and ϑ) is held constant. That is when tax progressivity τ changes, the tax level λ adjusts so $\mathcal{T}_t = \mathcal{T}_{ss}$ in every period. This allows a study of the impact of progressivity shocks without interference from a change in tax levels. This comes at a cost of government spending being time-varying. Indeed, since the real interest rate changes, one quantity has to adjust for the government budget constraint to clear.¹²

Market clearing. The asset, labor, and good market-clearing conditions lead to:

$$\begin{aligned} B_t &= \mathcal{A}_t \\ L_t &= \mathcal{N}_t^e \\ Y_t &= \mathcal{C}_t + G_t \end{aligned}$$

4.4.2 Calibration

Income process. Using AR(1) processes for idiosyncratic labor productivities makes it impossible to match key moments of the income risk distribution as was shown by Guvenen, Karahan, et al., 2021 in the US context. Indeed, income risk as measured by labor income growth is negatively skewed and has more kurtosis than an underlying normal distribution would imply. Following Ferriere, Grübener, et al., 2023, I assume that idiosyncratic productivities follow a Gaussian Mixture Autoregressive (GMAR) process

$$\begin{aligned} \log(e_t) &= \rho_e \log(e_{t-1}) + \eta_t \\ \eta_t &\sim \begin{cases} \mathcal{N}(\mu_1, \sigma_1^2) & \text{with probability } p_1 \\ \mathcal{N}(\mu_2, \sigma_2^2) & \text{with probability } 1 - p_1 \end{cases} \end{aligned}$$

where the value of μ_2 is deduced from the $\mathbb{E}\eta = 0$ restriction. This process is discretized using Farmer and Toda, 2017's method. Besides and similarly to Hubmer, Krusell, and Smith, 2020, I modify the top values of e so that the distribution features a Pareto tail. The Pareto tail coefficient as well as parameters $(\rho_e, p_1, \mu_1, \sigma_1, \sigma_2)$'s values are obtained thanks to Guvenen, Pistaferri, and Giovanni L Violante, 2022b's GRID database (for *Global Repository of Income Dynamics*). This database compiles observations from the French DADS (*Déclaration annuelle des données sociales unifiée*) and reports yearly estimates of some key statistics of the earning growth distribution. According to this database, the income distribution average 5% Pareto tail index over the 1991-2015 period is $\kappa = 2.8$. The values of the five income process parameters $(\rho_e, p_1, \mu_1, \sigma_1, \sigma_2)$ are chosen to reproduce five moments: four moments of the labor income growth distribution and one of the labor income distribution. Those moments and their 1991-2015 averages are standard deviation (0.47), the difference between the ninetieth and tenth percentiles (1.64), skewness (-1.11), and kurtosis (14.08). The remaining target is the labor income share of the top 10% earners (26%).

¹²Alternatively, one may want to fix government spending G and let λ adjust, making \mathcal{T} time-varying: $\mathcal{T}_t = r_t B + G + \vartheta$. This alternative, however, leads to indeterminacy as briefly discussed in Appendix 4.6.7.

National accounts aggregates. Public debt and public spending are calibrated to correspond to a ratio of 100% and 19% of GDP respectively. The tax progressivity parameter τ takes values among those estimated in Section 4.2. Tax levels λ adjust so that the government's budget balance. As shown in Appendix 4.6.7, net taxes on average amounted to 23.9% of GDP over the period, the lump-sum transfer ϑ ensures the order of magnitude is met, while distributing further transfers at the bottom of the distribution. Indeed, the HSV function alone does not redistribute enough at the bottom compared to the observations (Figure 4.11).¹³

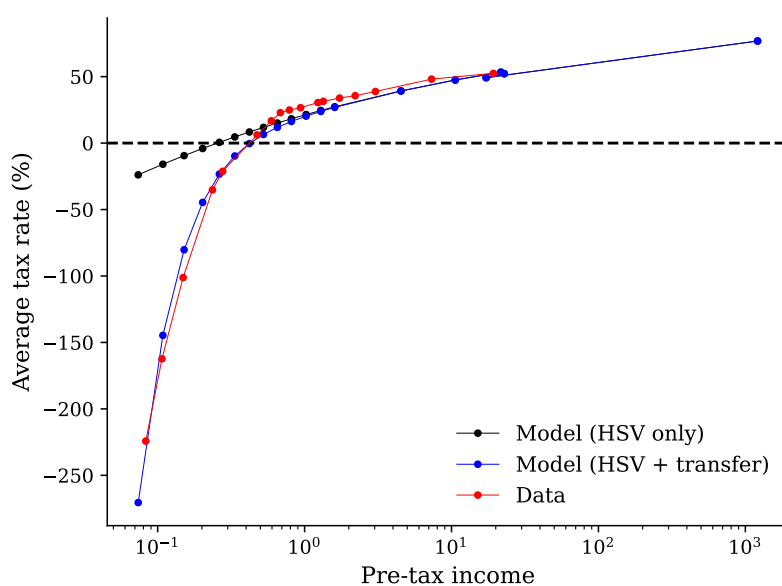


Figure 4.11: Average tax rates along the income distribution. *Source: OECD TaxBEN.*

Household preferences. Risk aversion is set to 2 and the Frisch elasticity of labor supply to 0.5. I calibrate first a reference steady-state with $\tau \approx 17.30\%$ corresponding to the 2021 estimation. The discount factor β is calibrated to match a 3% annual real interest rate target while ensuring the asset market clears. The labor disutility θ is chosen to match a normalization of effective labor supply of unity. Table 4.2 reports all the parameter values considered. The other steady-states would take the same values for (β, θ) while r and Y will adjust.

4.4.3 Deterministic steady states

Performance. Table 4.3 shows the model's performance at matching the income and wealth distributions. The GMAR process choice allows us to generate reasonable labor income inequalities. Note that only the D10 gross labor income value is a targeted moment. In terms of wealth distribution, the model generates too much concentration of wealth at

¹³Further details on this can be found in Appendix 4.6.7.

Household		Income process			
Discount factor	β	0.916	Persistence	ρ_e	0.895
Risk aversion	σ	2.000	Probability of the first normal shock	p_1	0.745
Inverse of the Frisch elasticity	ν	2.000	Mean of the first normal shock	μ_1	0.100
Labor disutility	θ	1.832	Standard dev. of the first normal shock	σ_1	0.100
			Standard dev. of the second normal shock	σ_2	0.325
			Pareto tail parameter (for top 5%)	κ	2.766
Government			Number of productivity levels	n_e	19
Debt-to-GDP ratio	$\frac{B}{Y}$	1.000			
Government spending-to-GDP ratio	$\frac{G}{Y}$	0.190			
Lump-sum transfer-to-GDP ratio	$\frac{y}{Y}$	0.019			

Table 4.2: Parameter values.

the top of the distribution to the detriment of the middle of the distribution compared with the data. Income inequality alone does not seem to be able to explain and generate the wealth inequalities observed.

Gross labor income	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
Data	1.5%	3.0%	4.6%	6.4%	8.0%	9.5%	11.0%	13.1%	15.8%	27.0%
Model	3.6%	5.4%	6.3%	6.9%	8.2%	8.9%	10.5%	11.6%	13.5%	26.7%

Net worth	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
Data	-0.2%	0.2%	0.6%	1.6%	3.6%	5.7%	8.5%	12.1%	18.0%	49.9%
Model	0.0%	0.0%	0.0%	0.1%	0.8%	1.8%	3.6%	6.4%	12.5%	74.9%

Table 4.3: Gross labor income and net worth shares by deciles generated in the model and observed in French data. *Source: INSEE, comptes nationaux distribués 2019, enquête Histoire de vie et Patrimoine 2021.*

Impact of tax progressivity. Comparing several deterministic steady states where only the tax progressivity τ differs leads to Figure 4.12.

As τ increases, contributors to the tax system tend to reduce their labor participation as the taxes they have to pay increase. Those benefitting from the redistribution, as they receive more transfers, will also tend to reduce their labor participation. As a result, the aggregate labor in the economy (which is also output) tends to decline. Consumption tends to fall slightly less as low-income/high-marginal propensity to consume households receive more transfers under a higher progressivity regime and thus consume more. With hours worked falling more than consumption (supported by redistribution), when moving from τ^- to τ^+ , the total effect is slightly positive in terms of welfare. From the saving attitudes, we see that highly productive households (who own most of the total assets) wish to save less in the high-progressivity state to smooth consumption. Since the supply of assets, the government debt B is fixed, the real interest rate adjusts upwards.

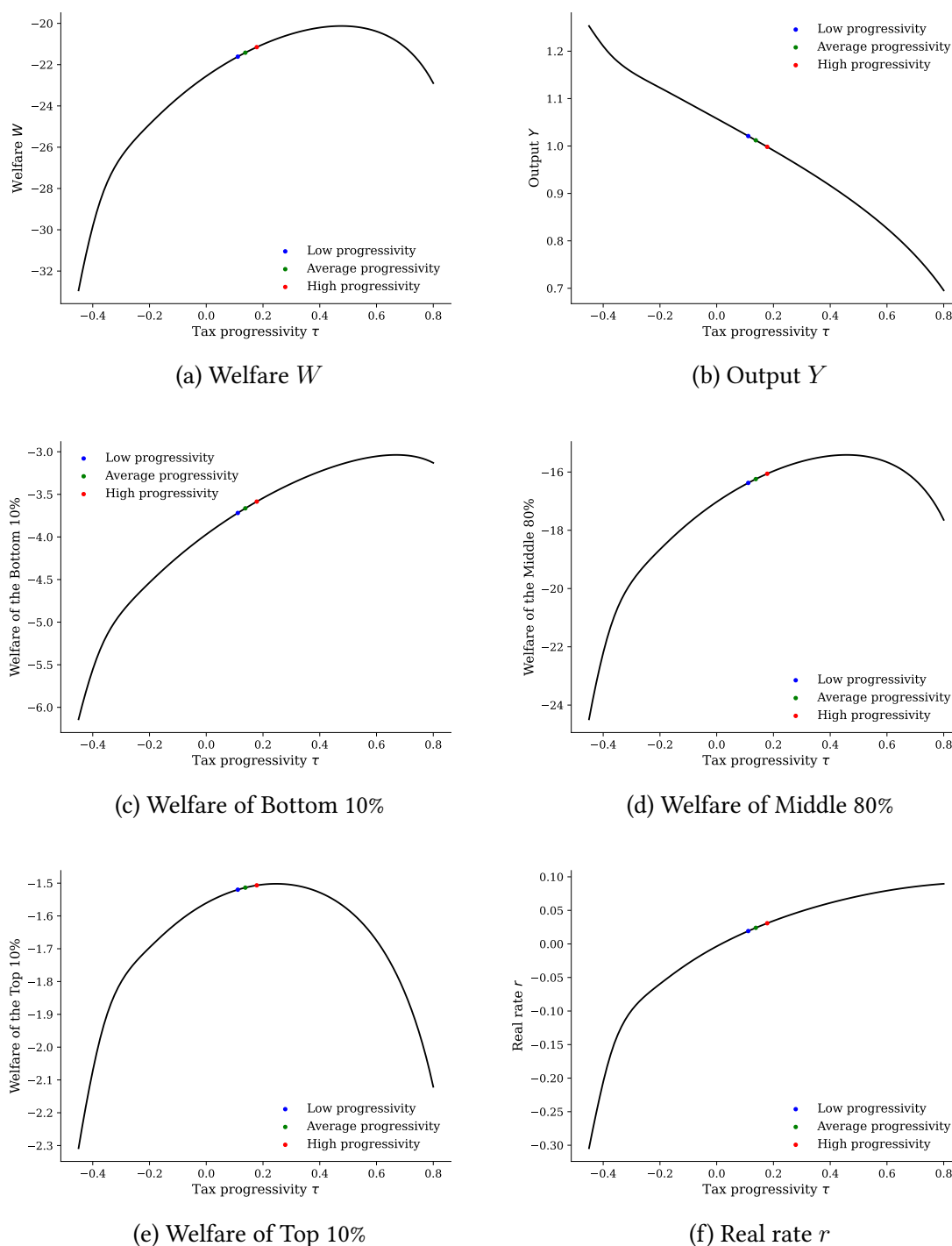


Figure 4.12: Impact on deterministic steady-state values of tax progressivity τ .

4.4.4 Tax uncertainty

What if the government does not commit to one tax progressivity anymore? I consider three fiscal regimes, a low τ^- , an average $\bar{\tau}$ and a high τ^+ progressivity (where $\tau^- \leq \bar{\tau} \leq \tau^+$). The government can either commit to a fiscal regime and thus keep the taxation schedule

over time or switch from one regime to another. In that case, the transition matrix Q_τ is arbitrarily fixed by the government:

$$Q_\tau = \begin{pmatrix} p_- & 1 - p_- & 0 \\ p & \bar{p} & 1 - \bar{p} - p \\ 0 & 1 - p_+ & p_+ \end{pmatrix}$$

The values for p_- , p_+ , p and \bar{p} (Table 4.4) are chosen to reproduce some key moments observed in Figure 4.4a.

Moment for τ	Target	Parameter	Estimated value	Associated moment
Mean	0.1377	Probability to stay in τ_-	p_- 0.8345	0.1333
Standard deviation	0.0205	Probability to stay in τ_+	p_+ 0.6896	0.0208
Autocorrelation	0.9160	Probability to stay in $\bar{\tau}$	\bar{p} 0.8214	0.7598
Skewness	0.8096	Probability of $\bar{\tau} \rightarrow \tau_-$	p 0.1087	0.8096

Table 4.4: Parameters of the transition matrix Q_τ and associated moments.

τ is thus assumed to follow a three-state Markov process. The values taken are $\tau^- = 11.1\%$, $\bar{\tau} = 13.8\%$ and $\tau^+ = 17.8\%$, which correspond to the historical low, mean and high values observed in the 2001-2024 period.

The government adjusts in every period the associated value for the level parameter λ to run a balanced budget under all fiscal states of the world. That is, the government is assumed to have a perfect knowledge of the implications of any tax reform, including changes in labor supply and household distribution.

From a microeconomic perspective, households make their consumption, saving and labor decisions conditionally on their productivity e and the tax regime (τ, λ) as well as their expectations upon these variables. In that context, the household's program rewrites:

$$\begin{aligned} v_t(e, a_-; \tau, \lambda) &= \max_{c, a, n} u(c, n) + \beta \sum_{e'} \sum_{(\tau', \lambda')} \mathbb{P}(e', \tau' | e, \tau) v_{t+1}(e', a; \tau', \lambda') \\ \text{s.t.} \quad &c + a = (1 + r)a_- + (1 - \lambda)(wen)^{1-\tau} \\ &a \geq 0 \end{aligned}$$

where the probabilities $\mathbb{P}(e', \tau' | e, \tau)$ are given by the transition matrix $\Pi \equiv Q_\tau \otimes Q_e$.

4.4.5 Dynamics with aggregate uncertainty

Solving the model with aggregate risk requires more complex tools than a first-order perturbation approach. Second-order approximations of heterogeneous-agent models are still preliminary with notable proposals by Reiter, 2023, Bhandari et al., 2023 and Lee, 2024. Another way to deal with aggregate risk in such models is to use a global solution method. Due to household heterogeneity, the state of the economy includes the distribution Γ_t which is an infinite dimensional object. In their seminal paper, Krusell and Smith, 1998 circumvent

this difficulty by considering the law of motion of a few moments instead of keeping track of the whole distribution.¹⁴ I adapt this approach to solve the model with uncertain tax progressivity. In particular, Krusell and Smith, 1998 considers the aggregate capital's law of motion for households to form their expectations. Since this model has bonds only and no physical capital, the law of motion has to apply to the price of the asset, the real interest rate r since it has no analytical expression.

1. Guess the laws of motion of aggregate variables (r, λ) .
2. Solve for the households' policy functions $\{c, n, a\}$ given those laws of motion.
3. Given those policy functions and the transition matrix Q_e for idiosyncratic shocks, update the distribution of agents and solve the aggregate variables $\{r_t, \lambda_t\}_t$ for the equilibrium conditions to hold.
4. Update the laws of motion for (r, λ) . If those laws of motion have converged, stop, otherwise iterate from step 1 onwards.

In practice, I assume that a state-dependent VAR relationship links the moments (r, λ) . The states being given by the current tax progressivity τ

$$\begin{aligned} r_{t+1} &= \beta_{00} + \beta_{01}r_t + \beta_{02}\lambda_t + \varepsilon_{0t} \\ \lambda_{t+1} &= \beta_{10} + \beta_{11}r_t + \beta_{12}\lambda_t + \varepsilon_{1t} \end{aligned}$$

For the simulation part, I adopt the non-stochastic approach proposed by Young, 2010, which updates a histogram representation of the household distribution. This approach is deemed more suitable than a Monte Carlo simulation of many households. Specifically, given the income process I have chosen, the high-productivity state is a rare occurrence (an extreme tail event). As such, simulating a large number of households would be necessary to achieve reasonable precision in the aggregate variables, which would significantly hamper computational efficiency.

Stochastic steady state. I simulate my model over a long period with progressivity shocks drawn according to the Q_τ transition matrix being the only source of aggregate risk. The averages of the macroeconomic variables over that period correspond to the stochastic steady-state values, which account for the expected risk on tax progressivity. These values are compared to a deterministic steady state with constant tax progressivity, whose level is the sample average. Table 4.5 compares those two steady states, thus uncovering fiscal uncertainty's impact. Note that these are preliminary results obtained with less volatility in tax progressivity τ compared to the observations (and $\vartheta = 0$).

¹⁴Other recent developments regarding global solution methods for solving heterogenous-agent models include: Young, 2010's projection technique that allows a non-stochastic simulation of households, Pröhl, 2019's use of generalized chaos polynomials for projection, Schaab, 2020's use of adaptative sparse grids for finer approximation around mass points of the distribution and Fernández-Villaverde, Hurtado, and Nuño, 2023's neural network approach.

Table 4.5: Steady-state impact of fiscal uncertainty ($p_- = p_+ = 0.99$, $\bar{p} = p = 0$, $\vartheta = 0$).

	Stochastic SS	Deterministic SS	Deviation
Tax progressivity τ	19.4%	19.4%	0.00%
Tax level λ	20.3%	20.1%	0.88%
Real rate r	2.897%	2.902%	-0.15%
Consumption \mathcal{C}	0.8014	0.8011	0.03%
Output Y	1.0024	1.0021	0.03%
Labor \mathcal{N}^e	1.0024	1.0021	0.03%
Hours \mathcal{N}	0.8310	0.8305	0.05%
Government spending G	0.20102	0.20098	0.02%
Government spending ratio $\frac{G}{Y}$	20.04%	20.05%	-0.06%

Fiscal uncertainty does impact macroeconomic variables. Households here adjust for this additional risk by increasing their savings in the stochastic steady-state compared to the deterministic one. Since the asset supply is fixed, the real rate slightly falls, depressing financial incomes. To compensate, households supply more labor and can consume more. That said, as consumption increases less than hours, fiscal uncertainty is likely to hamper welfare. These results are just an illustration of a specific calibration of fiscal uncertainty. Future work will look at the cost of fiscal uncertainty for a more realistic calibration. This may require changing the moments used to forecast future expectations.

Note that while the perfect foresight response of the economy to the same path of tax progressivity τ can be informative, it generates too extreme responses of the economic agents. As a tax change is to happen, households anticipate it and adjust massively their labor supply and savings, driving a high volatility in the interest rate. See Figure 4.37 in Appendix 4.6.9.

4.5 Conclusion

This paper shows how fiscal uncertainty on tax progressivity may affect macroeconomic aggregate variables through the labor distortions and changes in saving behaviors it may generate. Fiscal uncertainty usually has recessionary effects, depressing labor supply, output, and consumption compared to a deterministic tax environment. The size of this recession depends on the volatility of the idiosyncratic productivity process, which interacts with the probabilities of staying in a given fiscal regime, as shown in the static framework.

Fiscal uncertainty may slightly boost output compared to a deterministic framework if the additional precautionary saving it creates pushes the real rate down and depresses financial incomes. In that case, the high-productive households may supply more labor to smooth their consumption. However, welfare is likely to be smaller under uncertainty as consumption increases less than hours worked.

How much these results carry over to the fully-calibrated quantitative model is part

of the next steps. Changes in the income process' volatility or the amount of liquidity in the economy (governed by the size of the public debt) will likely influence the size of the uncertainty costs. Indeed, both will impact the share of constrained agents having non-linear consumption decisions. In turn, it will impact the concavity of the aggregate functions with respect to tax progressivity τ and thus the size of the cost of tax progressivity uncertainty.

4.6 Appendix

4.6.1 Motivational facts

The *Institut des Politiques Publiques (IPP)* built a database gathering French income tax schedule evolutions since its creation in 1915. In particular, they report the income thresholds and marginal tax rates, defining the statutory income tax scales that are in place for each fiscal year. These income thresholds are expressed in nominal terms. I express all amounts in 2021 euros after adjusting for yearly inflation (using the INSEE's consumer price index) over the period. I also converted French francs (for years before 2000) amounts to euros. Figure 4.13 shows that the households' average tax rates have changed over time.

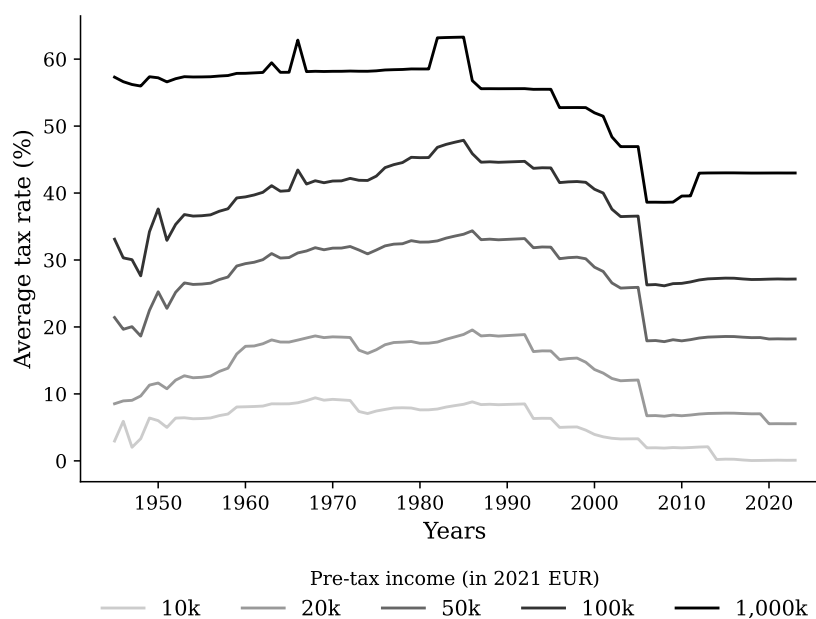


Figure 4.13: Evolution of the average tax rates for various pretax incomes.

Starting from a uniform grid of pre-tax incomes y_i , I compute for each year the associated post-tax income $\tilde{y}_i = (1 - \lambda)y_i^{1-\tau}$ using these statutory income tax scales. Regressing pre-tax incomes on post-tax incomes (in logs)

$$\log(\tilde{y}) = \log(1 - \lambda) + (1 - \tau) \log(y)$$

I estimate the (λ, τ) parameters for each year using ordinary least squares. The resulting values for tax progressivity τ are those reported in Figure 4.14. Note that only the tax progressivity is easily interpretable. The tax levels λ are informative for their variations, the levels in themselves can't be interpreted, because it would require normalizing the data to the average income for each year.

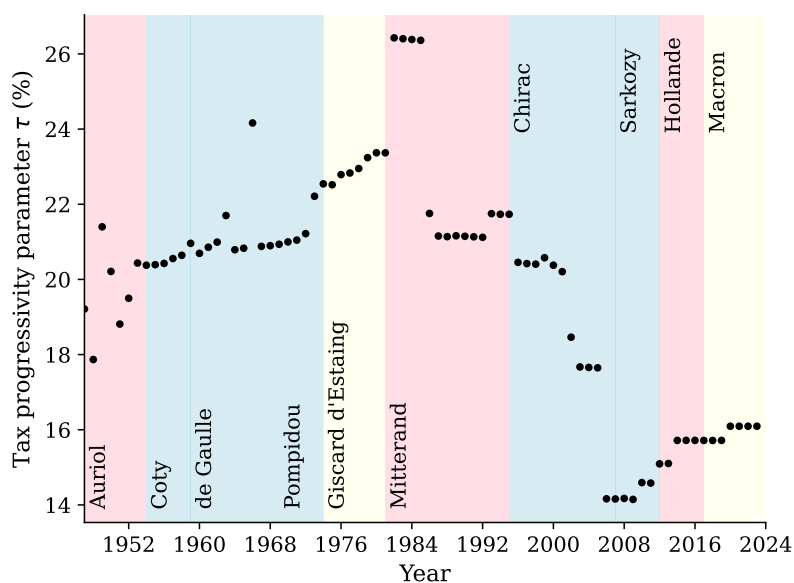


Figure 4.14: Evolution of the income tax progressivity parameter. *Source: Author's calculations based on IPP data.*

Using a standard OLS without weights on the observations may lead to the estimates capturing the variations in taxation at the top of the distribution only. The resulting τ fluctuations would be driven by those of the top marginal rates only. Instead, tax progressivity changes also happened for the rest of the distribution. Putting weights on how likely a pre-tax income is to be found in the distribution would correct for this.

In practice, I do not have detailed information on how the income distribution has distorted over such a long period (from the 1950s to today). I assumed the distribution remained stable and took the weights associated with the 2016 distribution. This leads to Figure 4.1.

4.6.2 A brief history of French income taxation and income-tested benefits

This section presents briefly the main taxes and income-tested benefits covered in this paper and how they evolved over the studied period, that is 2001-2024. Table 4.15 gives the overall structure of the taxes and benefits.

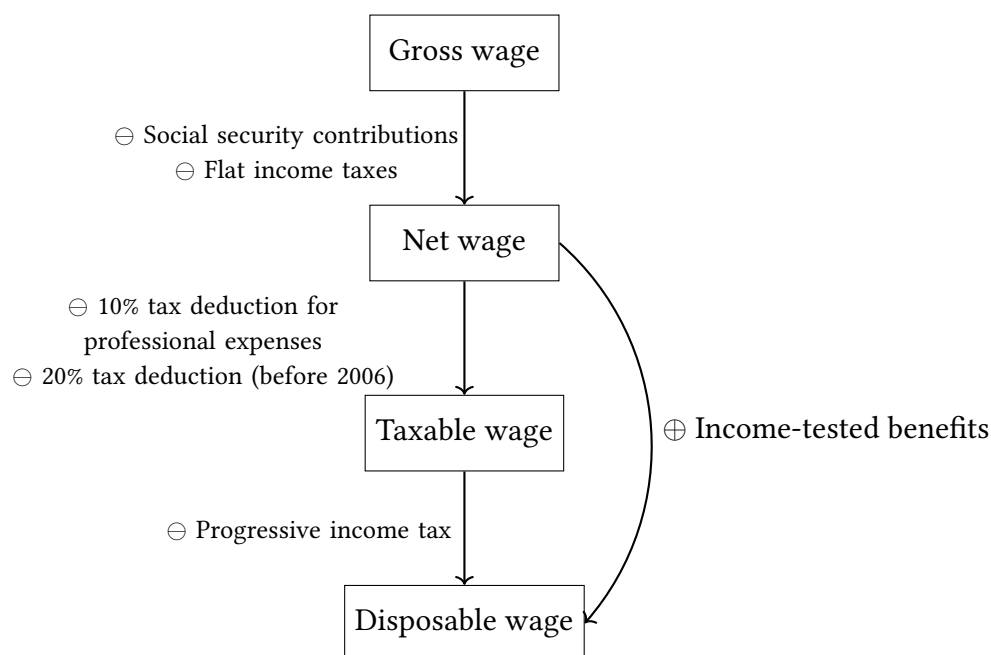


Figure 4.15: Structure of the French taxes and benefits indexed on labor income.

Income taxes

On 15 July 1914, the French Senate voted to introduce a new fiscal tool, a progressive income tax. This tax was implemented for the first time on the revenues of 1916. Since then, various reforms have modified the scope and rules of its application. This section describes its main evolutions in recent history (i.e. since the 2000s). See Piketty, 2016 for a more thorough description of the French income tax system before 1998.

General idea. French incomes are taxed using a progressive tax scale featuring increasing marginal rates. The value of thresholds and associated marginal rates are set to change from year to year as reforms are introduced. If no major reforms are implemented, the thresholds are usually increased in line with the expected inflation rate of the year. Table 4.6 shows an example of such a tax scale for the year 2022.

Incomes are declared to the tax administration and taxed at the household level. Based on the declared income, a gross amount of income taxes to pay is computed. If this gross tax amount is below a certain threshold (that is defined each year), then the total amount of taxes to pay is reduced through a discount mechanism (*décote* in French). This discount has

Table 4.6: Progressive scale for 2022 income. *Source: DGFIP.*

Income bracket	Tax rate on the income bracket
Up to 10,777€	0%
From 10,778€ to 27,478€	11%
From 27,479€ to 78,570€	30%
From 78,571€ to 168,994€	41%
Over 168,994€	45%

evolved throughout time but has existed for a long time, having been indirectly introduced as early as the 1950 income tax system. Figure 4.16 illustrates that tax reduction for 2022.

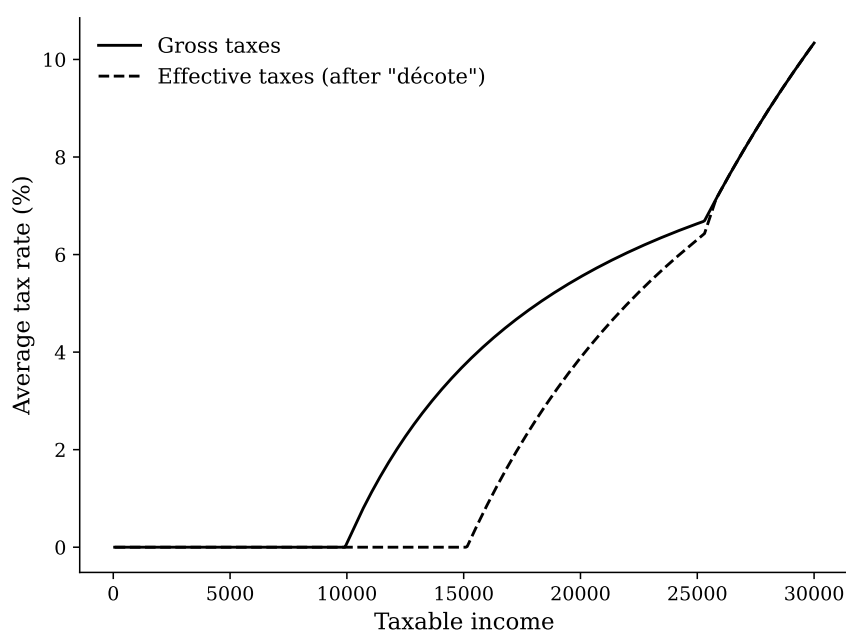


Figure 4.16: Average tax rate before and after the *décote* tax reduction in 2022.

History of income tax reforms. 1991. The universal social contribution (*Contribution sociale généralisée* in French, CSG for short) was introduced in 1991 to generate additional tax revenues to fund the French social protection system. It is a flat rate on all incomes (not only labor earnings, but also capital incomes, pensions, and unemployment benefits, albeit at a reduced rate for the last two). It initially had a low rate of 1.1% but has kept increasing, reaching 9.2% since 2018.

It is not considered a social security contribution since it does not give entitlement to benefits, but an income tax (by the OECD for instance). The importance of this tax in terms of revenues for the government has kept increasing, more than doubling the income tax revenues compared to the progressive income tax alone (Figure 4.17).

1996. Since 1996, all revenues (labor, capital, pension, unemployment benefit...) are also subject to the contribution to the reimbursement of social debt (*Contribution pour le*



Figure 4.17: Income tax revenues as a share of French GDP.

remboursement de la dette sociale in French or CRDS). It is a flat rate of 0.5% to reduce the debt of the social security system. Like the CSG, it is considered an income tax by the OECD.

2006. The de Villepin government implemented a consequent fiscal reform in 2006, documented in Didier, 2014. First, they reduced the progressivity by suppressing some tax brackets and lowering the top marginal rates. They also lowered the overall marginal rates to compensate for the suppression of the 20% tax deduction on wages that was automatic before.

2012. An exceptional contribution on high incomes was added to the progressive income tax scale under François Hollande's presidency (defined in the *LOI n° 2011-1977 du 28 décembre 2011 de finances pour 2012*). Table 4.7 reports the amounts at play. According to the law, this contribution is set to disappear when the public deficit becomes null.

Table 4.7: Exceptional contribution on high incomes.

Income bracket	Tax rate for a single person	Tax rate for a couple
Up to 250,000€	0%	0%
From 250,000€ to 500,000€	3%	0%
From 500,000€ to 1,000,000€	4%	3%
Over 1,000,000€	4%	4%

Social security contributions

General idea. Social Security contributions are of two kinds, depending on whether they are paid by the employee or the employer. In practice, they are broken down into several contributions of different rates and taxable bases. Some are levied on wages below a certain threshold only, the *plafond annuel de la Sécurité sociale (PASS)*. In this case, the contributions are regressive.

Employer social security contributions. The OECD's TaxBEN simulator only accounts for the employee part of social security contributions. The employer contributions also fluctuated over time, especially in the 2001-2005 and 2013-2020 periods, as the tax exemptions for low-wage earners evolved as shown by Bozio and Wasmer, 2024. Figure 4.18 documents those evolutions. These fluctuations are not yet accounted for in my estimates of (τ, λ) . However, this paper focuses on the labor supply response to tax changes, while Lehmann, Marical, and Rioux, 2013 shows in the French context that the bargaining between workers and firms seems to operate on the wage net of employer contributions.

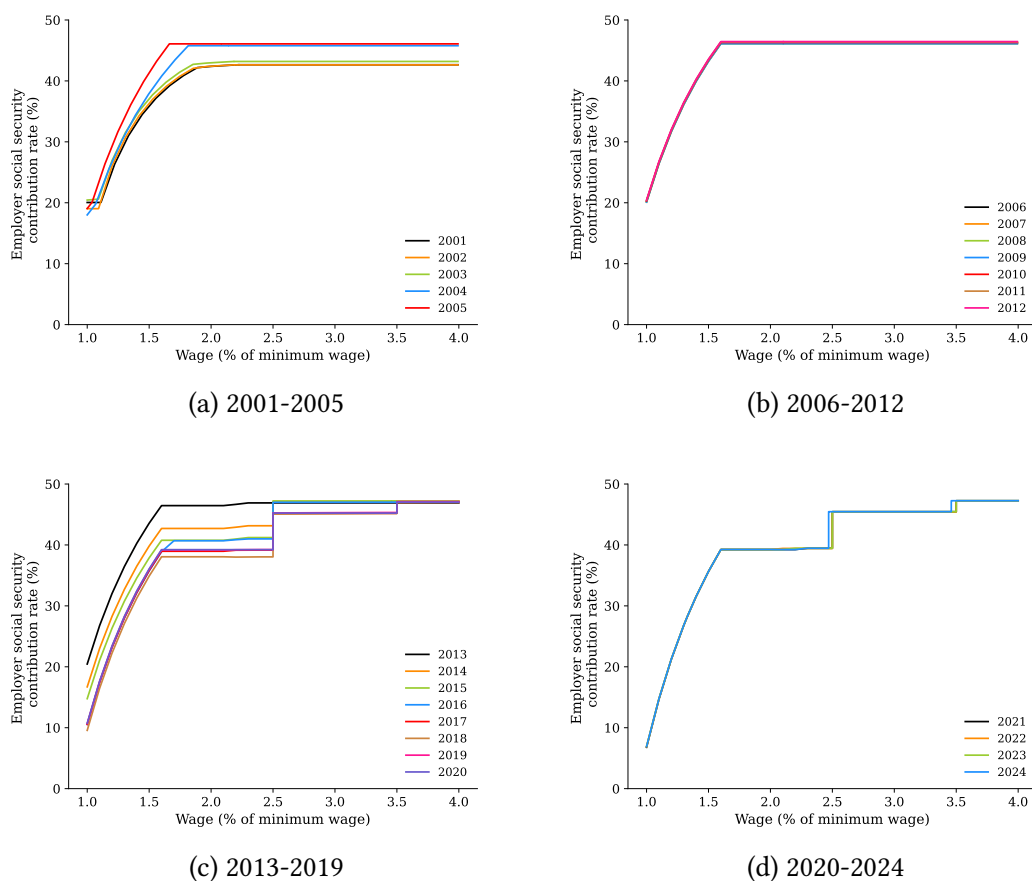


Figure 4.18: Evolution of the employer social security contributions along the income distribution. *Source: Dares, Bozio and Wasmer, 2024.*

Social assistance benefits

General idea. Social assistance benefits in France rely on non-taxable transfers to households to ensure they receive a minimum disposable income. The names and amounts of these benefits have evolved through time.

History of social benefit reforms. 2009. The *Revenu de solidarité active (RSA)* was introduced, in replacement of the previous *Revenu minimum d'insertion (RMI)*, which was invented in 1988. In 2007 and 2008, a transition benefit existed.

2015. Since 1 April 2015, the benefits have increased yearly by the average inflation rate over the past twelve months. Before, it was increased by the expected inflation rate for the year.

2022. Following the energy crisis and the resulting high inflation, the RSA was increased discretionarily by 4% as of 1 July 2022 (on top of the automatic indexation on inflation). This was an attempt to protect the purchasing power of the poorest households. Such discretionary increases also occurred every year between 2013 and 2017.

In-work benefits

General idea. In-work benefits are transfers to low-income households, that they perceive in addition to their wage. They are non-taxable and the amounts are adjusted annually for inflation.

History of in-work benefit reforms. 2016. The *Prime d'activité (PA)* is introduced in replacement of the previous *RSA activité* and *Prime pour l'emploi (PPE)*. These two schemes were not considered in-work benefits because the *RSA activité* was rather a social assistance benefit, while the PPE, which was an earned income tax credit was aggregated in the income taxes.

2022. On top of the automatic indexation on inflation, the amounts perceived were increased by 4% as of August 2022 to preserve purchasing power in a high-inflationary context.

Housing benefits

General idea. Housing benefits (*Allocations logement* in French) are benefits to low-income households to help them pay rent. Some low-income first-time homebuyers may also receive some to help them with mortgage payments. It is not taxed. The amounts are slightly reduced if the household also receives in-work or social assistance benefits. There is an automatic indexation based on expected inflation.

2017. As of 1 October 2017, every beneficiary saw the amount of their housing benefit decrease by 5€.

2022. On top of the automatic inflation indexation, housing benefits were discretionarily increased by 4% in August 2022.

4.6.3 HSV tax function

Let's recall the properties of the tax function introduced in Feldstein, 1969, Bénabou, 2000 and Heathcote, Storesletten, and Giovanni L. Violante, 2017 (denoted HSV for short) which is commonly used to model progressive tax and transfer fiscal systems.

General formulation

Let's define y as the pre-tax income. The tax function is

$$T(y) = y - (1 - \lambda)y^{1-\tau}$$

where λ governs the level of taxation and τ the progressivity. In particular, λ is the tax rate applied to a household of income 1.

Then the post-tax income \tilde{y} is

$$\tilde{y} \equiv y - T(y) = (1 - \lambda)y^{1-\tau}.$$

The marginal tax rate is

$$T'(y) = 1 - (1 - \lambda)(1 - \tau)y^{-\tau}.$$

The average tax rate is

$$\frac{T(y)}{y} = 1 - (1 - \lambda)y^{-\tau}.$$

Rescaling the income distribution

Let's consider a rescaled income distribution $Y = \alpha y$ with $\alpha \neq 0$. The marginal and average tax rates are:

$$\begin{aligned} \frac{T(y)}{y} &= 1 - (1 - \lambda)y^{-\tau} \\ \frac{\bar{T}(Y)}{Y} &= 1 - (1 - \bar{\lambda})Y^{-\bar{\tau}} \\ T'(y) &= 1 - (1 - \lambda)(1 - \tau)y^{-\tau} \\ \bar{T}'(Y) &= 1 - (1 - \bar{\lambda})(1 - \bar{\tau})Y^{-\bar{\tau}} \end{aligned}$$

In both cases, the marginal and average tax rates should be equal leading to:

$$\begin{aligned} \bar{\tau} &= \tau \\ \bar{\lambda} &= 1 - \alpha^\tau(1 - \lambda) \end{aligned}$$

Convexity/Concavity

$$T'(y) = 1 - (1 - \lambda)(1 - \tau)y^{-\tau}$$

$$T''(y) = (1 - \lambda)(1 - \tau)\tau y^{-(\tau+1)}$$

$$\begin{aligned} T''(y) \geq 0 &\Leftrightarrow (1 - \lambda)(1 - \tau)\tau \geq 0 \\ &\Leftrightarrow \tau \geq 0 \text{ (if } \lambda \leq 1) \end{aligned}$$

This means the HSV tax function is concave (resp. convex) if and only if $\tau < 0$ (resp. $\tau > 0$). That is, by construction, all progressive income tax systems are convex: as income increases, the taxes paid increase faster.

Coefficient of residual income progression.

The progressivity of a tax system is usually measured using the coefficient of residual income progression (CRIP) first introduced by Musgrave and Thin, 1948. Jakobsson, 1976 showed it is the most appropriate measure.

The CRIP is the elasticity of post-tax income \tilde{y} to pre-tax income y and can be computed using the average tax rate (ATR) and marginal tax rate (MTR)

$$CRIP(y) = \frac{1 - MTR(y)}{1 - ATR(y)}.$$

A tax system is progressive at income level y , if the CRIP is lower than one. The smaller the value, the more progressive the system is.

In the case of a HSV tax function

$$\begin{aligned} CRIP(y) &= \frac{1 - T'(y)}{1 - T(y)/y} \\ &= \frac{(1 - \lambda)(1 - \tau)y^{-\tau}}{(1 - \lambda)y^{-\tau}} \\ &= 1 - \tau, \end{aligned}$$

the CRIP is constant at all income levels, equal to $1 - \tau$. The system is thus progressive as long as $1 - \tau < 1 \Leftrightarrow \tau > 0$.

4.6.4 Estimation on French data

Income distribution

As discussed in Appendix 4.6.1, weights must be applied when estimating the tax function parameters to ensure the function offers better approximations where the mass of households is. It thus requires some information about the income distribution.

Guvenen, Pistaferri, and Giovanni L Violante, 2022b's GRID database reports the log incomes that define various percentiles of the French income distribution for years between 1991 and 2016 (Figure 4.19).

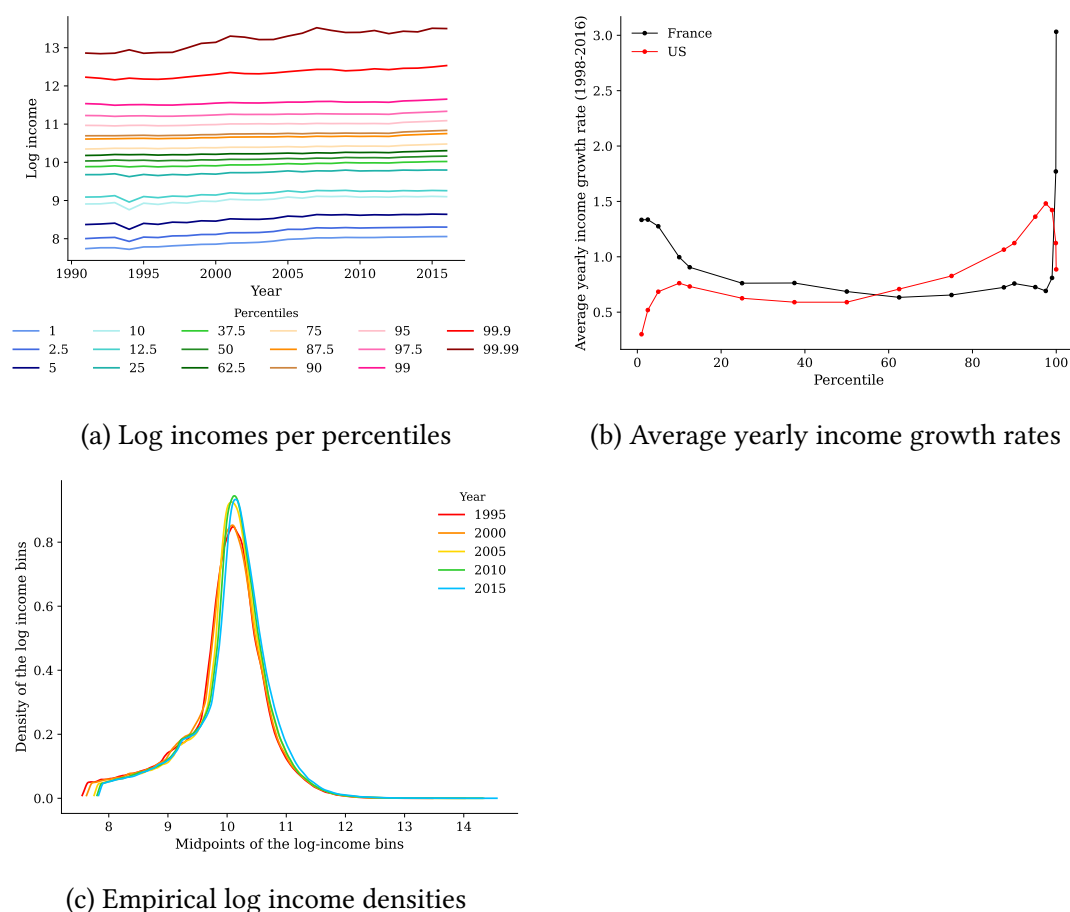


Figure 4.19: Source: GRID database.

My analysis so far relies on the income thresholds from 2016 at various percentiles (P1, P2.5, P5, P10, etc.). I assign weights to each percentile based on the distance between them. To estimate income thresholds for other years, I adjust the 2016 distribution by applying the average wage growth rate, thus translating the entire distribution without further modifications. This implies that I assume the distribution itself does not deform.

The OECD's simulator requires pre-tax and transfer incomes to be expressed as percentages of the average wage of that year. I thus expressed the 2016 income thresholds

obtained in GRID as a percentage of the average wage. Table 4.8 reports those amounts.

Percentile	2016 income (EUR)	% of average wage
1	3,155	8.3%
2.5	4,040	10.7%
5	5,646	14.9%
10	8,957	23.6%
12.5	10,480	27.7%
25	17,990	47.5%
37.5	22,494	59.4%
50	25,896	68.4%
62.5	29,858	78.8%
75	35,551	93.9%
87.5	46,709	123.3%
90	50,810	134.1%
95	65,510	173.0%
97.5	83,708	221.0%
99	114,997	303.6%
99.9	277,121	731.7%
99.99	730,136	1,927.7%

Table 4.8: 2016 French income distribution. *Source: GRID, OECD TaxBEN.*

Income-tested taxes and benefits

The OECD's TaxBEN simulator gives the taxes and benefits a household is entitled to depending on its characteristics (age, marital situation, wage, number of children...). To abstract from the family composition, I looked only at the fiscal situation faced by a single taxpayer aged 40 in full-time employment. I also consider that the housing costs (which will determine the entitlement in terms of housing benefits for low-wage households) are worth 30% of the average wage. This value stems from the institutional framework: renters usually need to earn three times the rent in wages, while mortgage monthly repayments should not exceed 35% of the disposable income. But, the bottom of the distribution also dedicates a higher share of their income to housing as confirmed by the INSEE data (Figure 4.20). They define the net housing expenditures as including rent, mortgage interest, and capital repayments, as well as condominium fees, rental fees, energy and water expenses, taxes (housing, property, garbage collection), and home insurance, while housing subsidies are deducted. These expenses are compared with disposable income that includes labor and capital incomes, pensions, unemployment benefits net of taxes and social contributions.

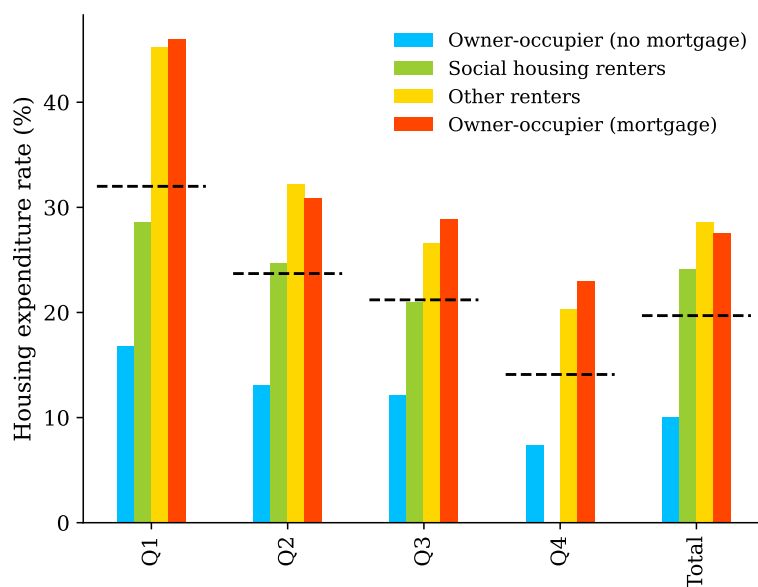


Figure 4.20: Household net housing expenditure rate by occupancy status and disposable income in 2017. *Source: INSEE, Enquête Budget de famille 2017, Enquête Revenus fiscaux et sociaux 2017.*

Clustering

I use a clustering method to identify for each type of tax or benefit, the years that share similar profiles.

Figure 4.21 shows the various clusters per type of tax or benefit. For social assistance, the first cluster regroups all years except for the 2009-2015 period, which featured a broader range of eligible households (all those earning less than 60% of the average wage). Both housing benefit clusters are very similar and hard to disentangle except for a slight fall in amounts at play since 2015. In-work benefits were reformed in 2015, the clusters identify this break. The amounts were considerably increased (to compensate for lower social assistance benefits). Income taxes can be separated into three clusters: 2006-2011 with lower marginal rates at the top of the distribution compared to the 2001-2005 and 2012-2017 periods. Finally, since 2019, another cluster emerged with an increased income tax rate especially at the bottom of the distribution (due to the rise in flat income tax rates). Finally, social security contributions were reformed in 2019 with lower but less regressive rates.

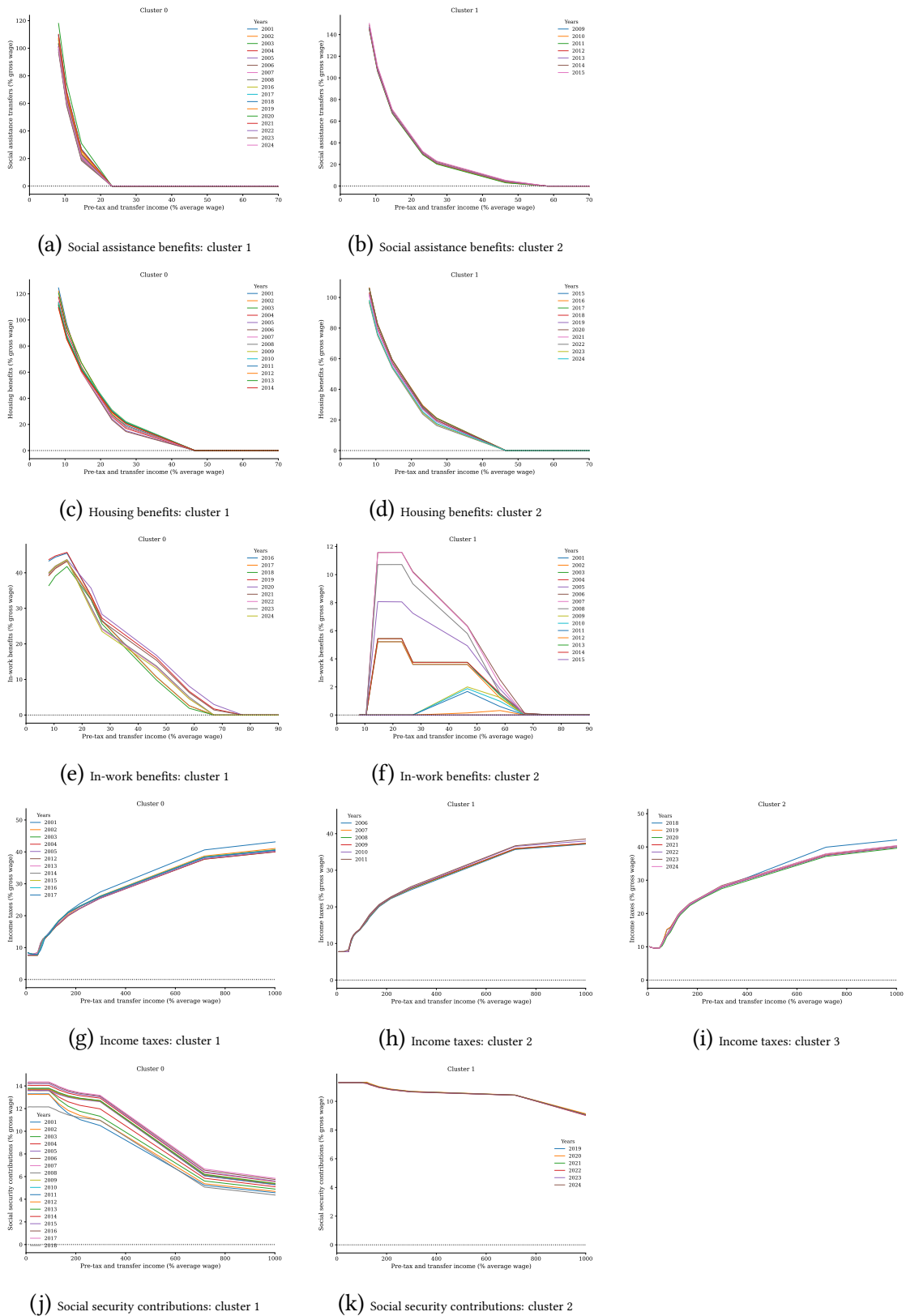


Figure 4.21: Clusters of the French taxes and benefits.

Estimation procedure

I estimate the HSV function's parameters $\{\lambda, \tau\}$ for each year. The moments I observe m_{obs} are the net taxes paid by the different households. $m(\Theta)$ gives the net taxes these households would pay according to the tax function and set of parameters considered. The estimated parameters minimize the normalized mean absolute error between the predictions and the observations

$$\hat{\Theta} = \arg \min_{\Theta} \sum_i w_i \left| \frac{m_i(\Theta) - m_{obs,i}}{m_{obs,i}} \right|$$

where w_i is the weight applied on observation i . For now, the weights $w_i = 1$ for all i . I normalize the errors to ensure that the optimization is balanced across the different moments. Otherwise, bigger observations, that is the top of the distribution, would dominate the impact on the objective function.

In practice, I solve this optimization problem using a global method, the differential evolution (DE) algorithm. As this method is stochastic and does not guarantee an optimal solution is found, I run this method a hundred times with different seeds. For each year, I select as an estimate the one with the lowest error according to my metric.

Comparison to the literature

Qiu and Russo, 2024 also estimated the HSV parameters for France (for single households only). Figure 4.22 compares their results to the one obtained in Section 4.2. However, they do not provide λ estimates for singles only.

The estimates may differ due to the reasons presented in Table 4.9. Besides the differences in the nature of data used (micro-simulation versus survey data), the notion of pre-tax income considered differs notably. Qiu and Russo, 2024's measure already includes some part of redistribution, whether by the government or private institutions (charities for instance).

Additionally, the estimates are at a wave level so may aggregate observations from close yet different years.

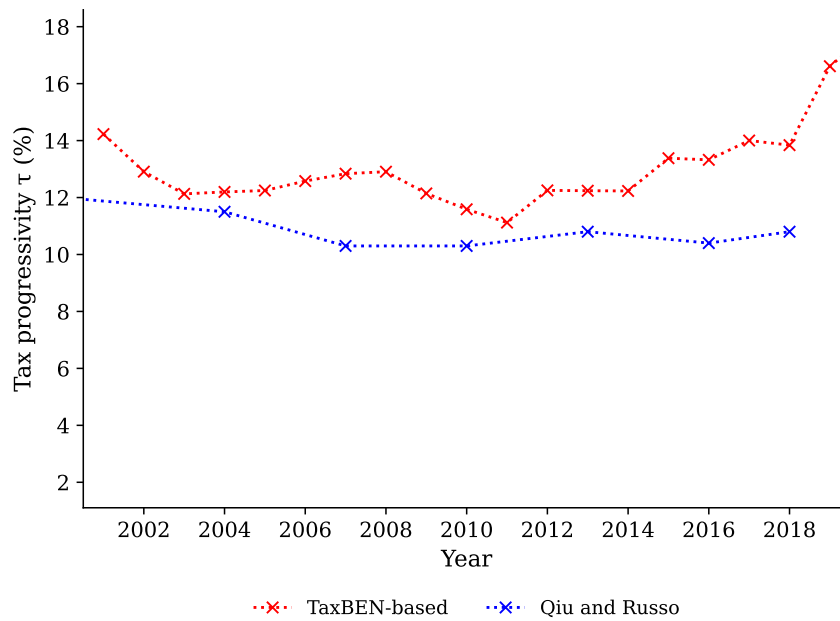


Figure 4.22: Comparison of tax progressivity estimates with the literature (Qiu and Russo, 2024).

	This paper	Qiu and Russo, 2024
Data source	OECD TaxBEN	Luxembourg Income Study (LIS)
Data type	“Hypothetical family”	Survey
Pre-tax income	Labor income	Labor income + Capital income + Pensions + Public social benefits + Private transfers
Post-tax income	Pre-tax income – Income tax – Social security contributions	Pre-tax income – Income tax – Social security contributions
Estimation frequency	Yearly	Wave (may aggregate several years)

Table 4.9: Differences in the data and estimation procedures.

4.6.5 Comparison to US data

I apply the same method used for French data and described in Appendix 4.6.4. Besides, the reference American households I consider are in the same income distribution positions as the French households I considered above (see Table 4.10). The resulting clusters and estimates differ only due to the difference in institutional contexts and reforms across the two countries.

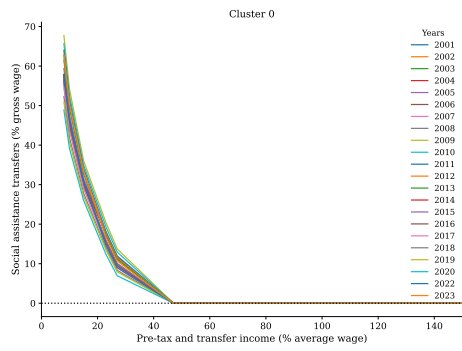
Percentile	2016 income (USD)	% of average wage
1	4,311	8.3%
2.5	5,558	10.7%
5	7,740	14.9%
10	12,259	23.6%
12.5	14,389	27.7%
25	24,674	47.5%
37.5	30,856	59.4%
50	35,531	68.4%
62.5	40,933	78.8%
75	48,777	93.9%
87.5	64,049	123.3%
90	69,659	134.1%
95	89,866	173.0%
97.5	114,800	221.0%
99	157,707	303.6%
99.9	380,086	731.7%
99.99	1,001,355	1,927.7%

Table 4.10: 2016 French income distribution. *Source: GRID, OECD TaxBEN.*

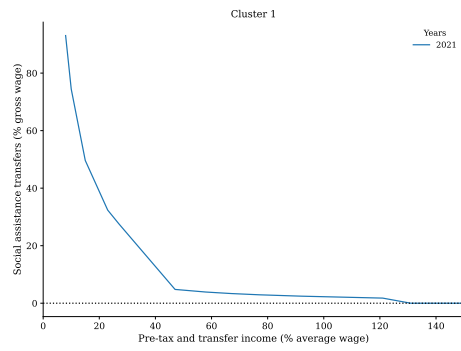
Clustering

I use a clustering method to identify the years that share similar profiles for each type of tax or benefit. Figure 4.23 displays those clusters in the US case. Most of the years are very similar unless a crisis hits and the fiscal authority adapts temporarily to counter the negative effects. For instance, social assistance transfers were increased and available for a broader base of households following the COVID-19 pandemic (those earning less than 130% of the average wage versus 50% otherwise). Similarly, in-work benefits were doubled during the Great Financial Crisis and the COVID-19 crisis.

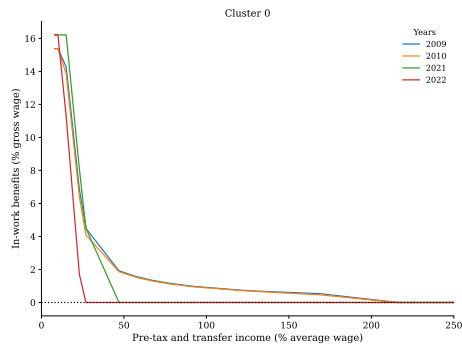
Those clusters can be summarized by just a few years' profiles as in Figure 4.24.



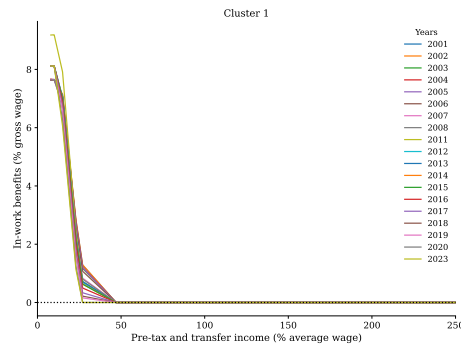
(a) Social assistance benefits: cluster 1



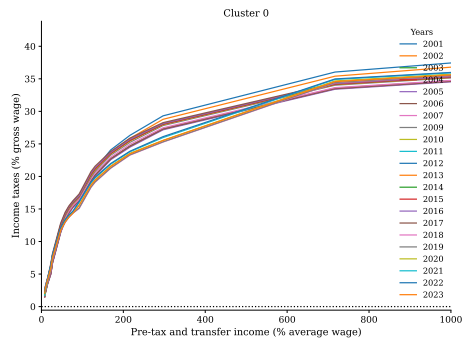
(b) Social assistance benefits: cluster 2



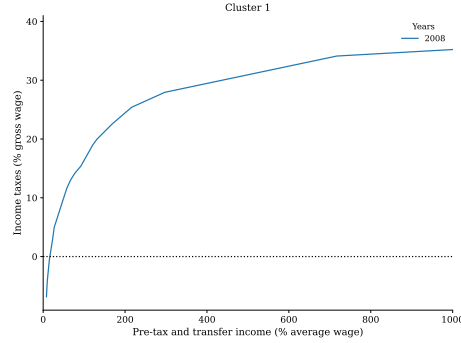
(c) In-work benefits: cluster 1



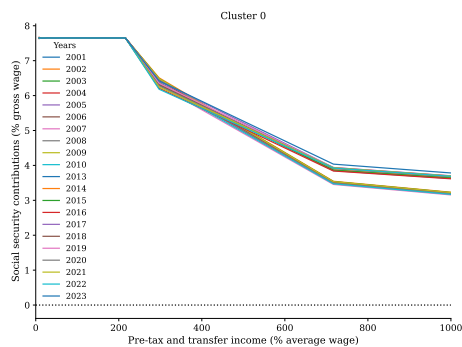
(d) In-work benefits: cluster 2



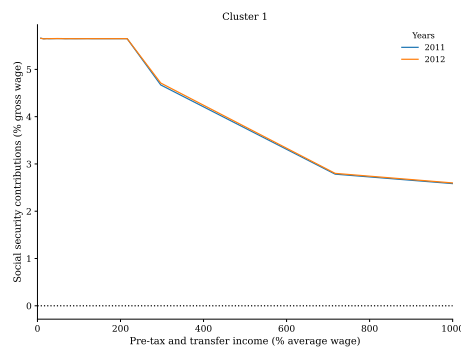
(e) Income taxes: cluster 1



(f) Income taxes: cluster 2

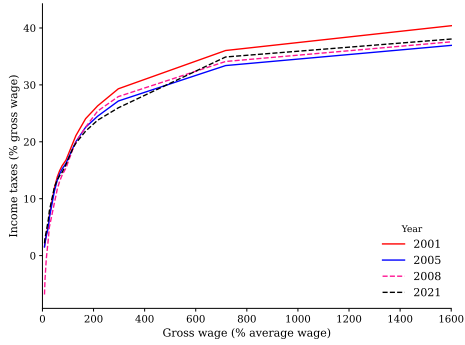


(g) Social security contributions: cluster 1

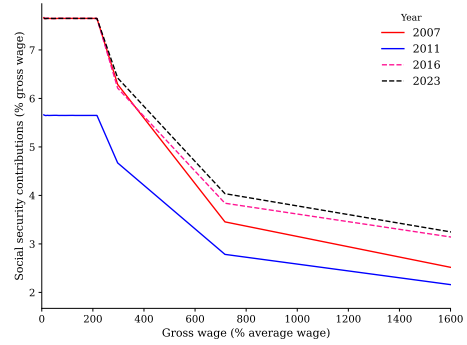


(h) Social security contributions: cluster 2

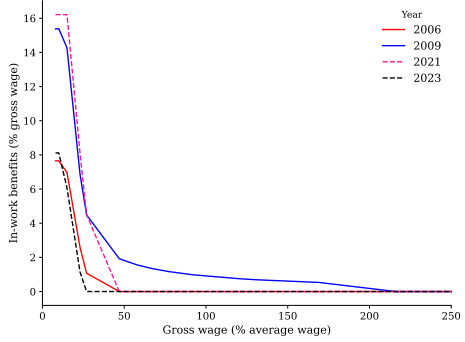
Figure 4.23: Clusters of the US taxes and benefits.



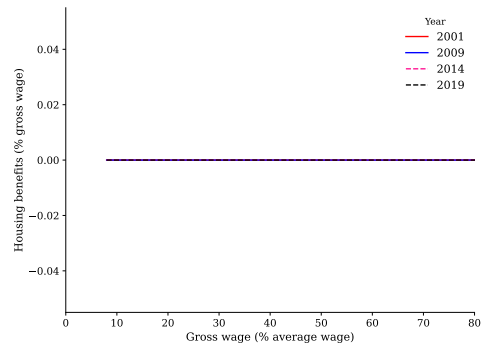
(a) Income taxes



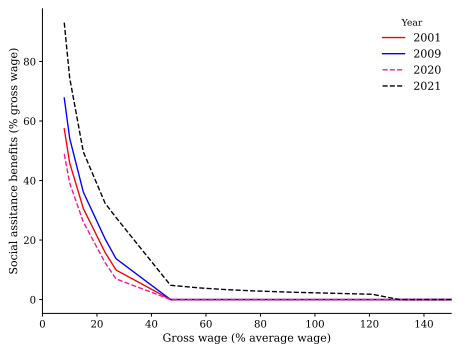
(b) Social security contributions



(c) In-work benefits



(d) Housing benefits



(e) Social assistance benefits

Figure 4.24: Taxes and transfers (as % of gross income) along the income distribution in the US.

Comparison to the literature

How do the estimates relate to previous estimates of the literature? Figure 4.25 shows this. Note that, I report estimates for single households only. For Qiu and Russo, 2024, the tax level λ parameter is not reported.

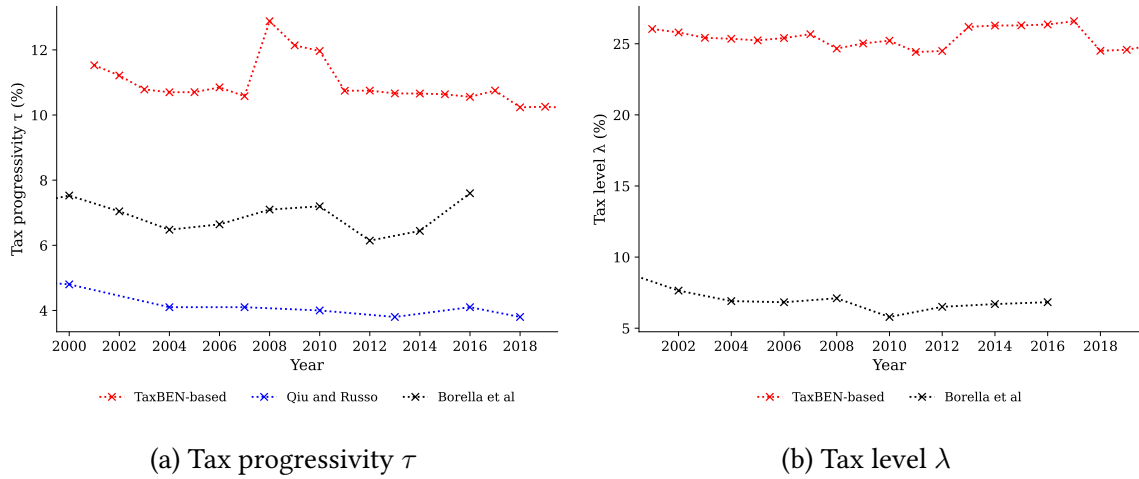


Figure 4.25: Comparison of tax progressivity estimates with the literature (Qiu and Russo, 2024).

4.6.6 Details on the static model

Optimal income tax progressivity without uncertainty

Without exogenous income

Log utility case ($\sigma = 1$). The first-order conditions of the household problem are:

- w.r.t. c : $\frac{1}{c} = \mu$
- w.r.t. n :

$$\begin{aligned} \theta n^\nu &= \mu(1-\tau)(1-\lambda)e(en)^{-\tau} \\ \Leftrightarrow n &= \left(\frac{(1-\tau)(1-\lambda)e^{1-\tau}}{\theta c} \right)^{\frac{1}{\nu+\tau}} \\ \Leftrightarrow n &= \left(\frac{(1-\tau)(1-\lambda)e^{1-\tau}}{\theta(1-\lambda)(en)^{1-\tau}} \right)^{\frac{1}{\nu+\tau}} \\ \Leftrightarrow n_0(\tau) &= \left(\frac{1-\tau}{\theta} \right)^{\frac{1}{1+\nu}} \end{aligned}$$

That is $n_i = n$ for all households as labor supply is independent on idiosyncratic productivity.

$$\begin{aligned} Y_0(\tau) &= \int e_i \left(\frac{1-\tau}{\theta} \right)^{\frac{1}{1+\nu}} di \\ &= \left(\frac{1-\tau}{\theta} \right)^{\frac{1}{1+\nu}} \underbrace{\int e_i di}_{=1} \\ &= n_0(\tau) \\ G &= \int (e_i n - (1-\lambda)(e_i n)^{1-\tau}) di \\ &= n_0(\tau) - (1-\lambda)n_0(\tau)^{1-\tau} \underbrace{\int e_i^{1-\tau} di}_{=\exp\left(-\frac{\sigma_e^2}{2}\tau(1-\tau)\right)} \\ &= Y_0(\tau) - (1-\lambda)Y_0(\tau)^{1-\tau} \exp\left(-\frac{\sigma_e^2}{2}\tau(1-\tau)\right) \\ \Leftrightarrow \lambda_0(\tau) &= 1 - \frac{Y_0(\tau) - G}{Y_0(\tau)^{1-\tau}} \exp\left(\frac{\sigma_e^2}{2}\tau(1-\tau)\right) \end{aligned}$$

The welfare writes:

$$\begin{aligned}
W_0(\tau) &= \int \left(\ln(c_i) - \theta \frac{n_0(\tau)^{1+\nu}}{1+\nu} \right) di \\
&= \int \ln((1-\lambda)(e_i n_0(\tau))^{1-\tau}) di - \theta \frac{n_0(\tau)^{1+\nu}}{1+\nu} \\
&= \ln(1-\lambda_0(\tau)) + (1-\tau) \ln(n_0(\tau)) + (1-\tau) \underbrace{\int \ln(e_i) di}_{=-\frac{\sigma_e^2}{2}} - \theta \frac{n_0(\tau)^{1+\nu}}{1+\nu} \\
&= \ln(n_0(\tau) - G) - (1-\tau) \ln(n_0(\tau)) + \frac{\tau(1-\tau)\sigma_e^2}{2} + (1-\tau) \ln(n_0(\tau)) - \frac{(1-\tau)\sigma_e^2}{2} - \frac{1-\tau}{1+\nu} \\
&= \ln(n_0(\tau) - G) - \frac{1-\tau}{1+\nu} - \frac{(1-\tau)^2\sigma_e^2}{2}
\end{aligned}$$

These are exactly the results derived in Ferriere, Grübener, et al., 2023 except for a few notational changes.

Non-log utility case ($\sigma \neq 1$). In case of a utility function $u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \theta \frac{n^{1+\nu}}{1+\nu}$ with $\sigma \neq 1$, the households first-order conditions become:

$$\begin{aligned}
\theta n_i^\nu &= c_i^{-\sigma} (1-\tau)(1-\lambda) e_i (e_i n_i)^{-\tau} \\
\Leftrightarrow n_i &= \left(\frac{(1-\tau)(1-\lambda) e_i^{1-\tau}}{\theta (1-\lambda)^\sigma (e_i n_i)^{\sigma(1-\tau)}} \right)^{\frac{1}{\nu+\tau}} \\
\Leftrightarrow n_i^{1+\frac{\sigma(1-\tau)}{\nu+\tau}} &= \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma} e_i^{(1-\tau)(1-\sigma)}}{\theta} \right)^{\frac{1}{\nu+\tau}} \\
\Leftrightarrow n_i &= \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma} e_i^{(1-\tau)(1-\sigma)}}{\theta} \right)^{\frac{1}{\nu+\tau+\sigma(1-\tau)}}
\end{aligned}$$

Labor supply is now dependent upon the household's idiosyncratic productivity e and λ , the income taxation level parameter.

Output becomes

$$\begin{aligned}
Y &= \int e_i n_i di \\
&= \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma}}{\theta} \right)^{\frac{1}{\nu+\tau+\sigma(1-\tau)}} \int e_i e_i^{\frac{(1-\tau)(1-\sigma)}{\nu+\tau+\sigma(1-\tau)}} di \\
&= \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma}}{\theta} \right)^{\frac{1}{\nu+\tau+\sigma(1-\tau)}} \int e_i^{\frac{1+\nu}{\nu+\tau+\sigma(1-\tau)}} di \\
Y(\lambda) &= \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma}}{\theta} \right)^{\frac{1}{\nu+\tau+\sigma(1-\tau)}} \exp \left(\frac{\sigma_e^2 (1-\tau)(1+\nu)(1-\sigma)}{2 (\nu+\tau+\sigma(1-\tau))^2} \right)
\end{aligned}$$

When $\sigma = 1$, we fall back on the log case and output equals the number of hours worked that is the same for all agents. Otherwise, output depends on the productivity distribution in the economy.

$$\begin{aligned}
 \int (e_i n_i)^{1-\tau} di &= \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma}}{\theta} \right)^{\frac{1-\tau}{\nu+\tau+\sigma(1-\tau)}} \int e_i^{1-\tau} e_i^{\frac{(1-\tau)(1-\sigma)(1-\tau)}{\nu+\tau+\sigma(1-\tau)}} di \\
 &= \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma}}{\theta} \right)^{\frac{1-\tau}{\nu+\tau+\sigma(1-\tau)}} \int e_i^{\frac{(1-\tau)(1+\nu)}{\nu+\tau+\sigma(1-\tau)}} di \\
 &= \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma}}{\theta} \right)^{\frac{1-\tau}{\nu+\tau+\sigma(1-\tau)}} \\
 &\quad \times \exp \left(-\frac{\sigma_e^2 (1-\tau)(1+\nu) [\nu+\tau - (1-\tau)(1+\nu-\sigma)]}{2(\nu+\tau+\sigma(1-\tau))^2} \right) \\
 &= AY(\lambda)^{1-\tau}
 \end{aligned}$$

where $A = \exp \left(\frac{\sigma_e^2 (1+\nu)(1-\tau)[(1+\nu)(1-\tau)+(1-\sigma)(1-\tau)-\nu-\tau-\sigma(1-\tau)]}{2(\nu+\tau+\sigma(1-\tau))^2} \right)$ is independent of λ .

The condition for the government to run a balanced budget writes

$$\begin{aligned}
 G &= Y(\lambda) - (1-\lambda) \int (e_i n_i)^{1-\tau} di \\
 \Leftrightarrow G &= Y(\lambda) - (1-\lambda)AY(\lambda)^{1-\tau} \\
 \Leftrightarrow (1-\lambda)AY(\lambda)^{1-\tau} - Y(\lambda) + G &= 0
 \end{aligned}$$

Solving this last equation for λ is not straightforward unless G is expressed as a fixed fraction of output $G = gY(\lambda)$. Then,

$$\begin{aligned}
 (1-\lambda)AY(\lambda)^{-\tau} - 1 + g &= 0 \\
 \Leftrightarrow (1-\lambda)Y(\lambda)^{-\tau} &= \frac{1-g}{A} \\
 \Leftrightarrow (1-\lambda)^{1-\frac{(1-\sigma)\tau}{\nu+\tau+\sigma(1-\tau)}} \left(\frac{1-\tau}{\theta} \right)^{\frac{-\tau}{\nu+\tau+\sigma(1-\tau)}} \exp \left(-\frac{\sigma_e^2 \tau(1-\tau)(1+\nu)(1-\sigma)}{2(\nu+\tau+\sigma(1-\tau))^2} \right) &= \frac{1-g}{A} \\
 \Leftrightarrow 1-\lambda &= \left[\frac{1-g}{A} \left(\frac{1-\tau}{\theta} \right)^{\frac{\tau}{\nu+\tau+\sigma(1-\tau)}} \exp \left(\frac{\sigma_e^2 \tau(1-\tau)(1+\nu)(1-\sigma)}{2(\nu+\tau+\sigma(1-\tau))^2} \right) \right]^{\frac{\nu+\tau+\sigma(1-\tau)}{\nu+\sigma}} \\
 \Leftrightarrow \lambda &= 1 - \left[\frac{1-g}{A} \left(\frac{1-\tau}{\theta} \right)^{\frac{\tau}{\nu+\tau+\sigma(1-\tau)}} \exp \left(\frac{\sigma_e^2 \tau(1-\tau)(1+\nu)(1-\sigma)}{2(\nu+\tau+\sigma(1-\tau))^2} \right) \right]^{\frac{\nu+\tau+\sigma(1-\tau)}{\nu+\sigma}} \\
 \Leftrightarrow \lambda &= 1 - \left(\frac{1-\tau}{\theta} \right)^{\frac{\tau}{\nu+\sigma}} \left(\frac{1-g}{A} \right)^{\frac{\nu+\tau+\sigma(1-\tau)}{\nu+\sigma}} \exp \left(\frac{\sigma_e^2 \tau(1-\tau)(1+\nu)(1-\sigma)}{2(\nu+\sigma)(\nu+\tau+\sigma(1-\tau))} \right) \\
 \Leftrightarrow \lambda &= 1 - \left(\frac{1-\tau}{\theta} \right)^{\frac{\tau}{\nu+\sigma}} (1-g)^{\frac{\nu+\tau+\sigma(1-\tau)}{\nu+\sigma}} \\
 &\quad \times \exp \left(\frac{\sigma_e^2 (1-\tau)(1+\nu)(\nu+\tau+\sigma(1-\tau)) - (1-\sigma) - (1+\nu)(1-\tau)}{2(\nu+\sigma)(\nu+\tau+\sigma(1-\tau))} \right)
 \end{aligned}$$

Once again, if we set $\sigma = 1$, we fall back on the expression under a log utility function.

Welfare in that context is given by

$$\begin{aligned}
W &= \int \frac{c_i^{1-\sigma}}{1-\sigma} - \theta \frac{n_i^{1+\nu}}{1+\nu} di \\
&= \underbrace{\int \frac{(1-\lambda)^{1-\sigma} (e_i n_i)^{(1-\sigma)(1-\tau)}}{1-\sigma} di}_{w_c} + \underbrace{\int -\theta \frac{n_i^{1+\nu}}{1+\nu} di}_{w_n} \\
w_c &= \frac{(1-\lambda)^{1-\sigma}}{1-\sigma} \int (e_i n_i)^{(1-\sigma)(1-\tau)} di \\
&= \frac{(1-\lambda)^{1-\sigma}}{1-\sigma} \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma}}{\theta} \right)^{\frac{(1-\sigma)(1-\tau)}{\nu+\tau+\sigma(1-\tau)}} \int e_i^{(1-\sigma)(1-\tau)(1+(1-\sigma)(1-\tau))} di \\
&= \frac{(1-\lambda)^{1-\sigma}}{1-\sigma} \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma}}{\theta} \right)^{\frac{(1-\sigma)(1-\tau)}{\nu+\tau+\sigma(1-\tau)}} \\
&\quad \times \exp \left(-\frac{\sigma_e^2}{2} (1-\sigma)(1-\tau)(1+(1-\sigma)(1-\tau))(1-(1-\sigma)(1-\tau)(1+(1-\sigma)(1-\tau))) \right) \\
w_n &= -\frac{\theta}{1+\nu} \int n_i^{1+\nu} di \\
&= -\frac{\theta}{1+\nu} \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma}}{\theta} \right)^{\frac{1+\nu}{\nu+\tau+\sigma(1-\tau)}} \int e_i^{(1-\tau)(1-\sigma)(1+\nu)} di \\
&= -\frac{\theta}{1+\nu} \left(\frac{(1-\tau)(1-\lambda)^{1-\sigma}}{\theta} \right)^{\frac{1+\nu}{\nu+\tau+\sigma(1-\tau)}} \\
&\quad \times \exp \left(-\frac{\sigma_e^2}{2} (1-\tau)(1-\sigma)(1+\nu)(1-(1-\tau)(1-\sigma)(1+\nu)) \right)
\end{aligned}$$

With exogenous income We now consider that household i receives an exogenous non-labor income y_i . The program rewrites

$$\begin{aligned}
\max_{c_i, n_i} \quad & \ln(c_i) - \theta \frac{n_i^{1+\nu}}{1+\nu} \\
\text{s.t.} \quad & c_i = (1-\lambda)(e_i n_i)^{1-\tau} + y_i
\end{aligned}$$

The first-order condition with respect to labor supply combined with the household's budget constraint leads to:

$$\begin{aligned}
c_i \theta n_i^\nu &= (1-\tau)(1-\lambda) e_i^{1-\tau} n_i^{-\tau} \\
\Leftrightarrow ((1-\lambda)(e_i n_i)^{1-\tau} + y_i) \theta n_i^\nu &= (1-\tau)(1-\lambda) e_i^{1-\tau} n_i^{-\tau} \\
\Leftrightarrow (1-\lambda) e_i^{1-\tau} n_i^{1-\tau+\nu} \theta + y_i \theta n_i^\nu &= (1-\tau)(1-\lambda) e_i^{1-\tau} n_i^{-\tau} \\
\Leftrightarrow \theta n_i^{1+\nu} + \frac{\theta y_i}{1-\lambda} \frac{n_i^{\tau+\nu}}{e_i^{1-\tau}} - (1-\tau) &= 0
\end{aligned}$$

This result is similar to Ferriere, Grübener, et al., 2023's case with non-null transfer T . The difference between T and y is however that while T is a transfer funded by the

government that thus needs to raise additional taxes accordingly to fund it, y should be thought of as an exogenous non-labor income. Besides, while T is unique for all households, y_i depends on the type i . It could for instance be a proxy for financial incomes households perceive although there is no saving in that particular setup. This last equation can rewrite $F(y_i, \lambda, n_i(y_i, \lambda)) = 0$.

From the work in the previous subsection, we already know one solution to this equation when $y = 0$. n_0 and λ_0 denote the obtained solutions for labor supply and income tax level when there is no additional exogenous income. We apply the implicit function theorem to equation $F(y_i, \lambda, n_i(y_i, \lambda)) = 0$ around the point $(y, \lambda, n_i) = (0, \lambda_0, n_0)$, which will be useful to obtain an approximated solution. Differentiating this equation with respect to y_i

$$\begin{aligned} & \theta(1 + \nu) \frac{\partial n_i}{\partial y_i} n_i^\nu + \frac{\theta}{1 - \lambda} n_i^{\tau + \nu} e_i^{\tau - 1} + \frac{\theta y_i}{1 - \lambda} (\tau + \nu) \frac{\partial n_i}{\partial y_i} n_i^{\tau + \nu - 1} e_i^{\tau - 1} = 0 \\ \Leftrightarrow & \frac{\theta}{1 - \lambda} n_i^{\tau + \nu} e_i^{\tau - 1} + \left[\theta(1 + \nu) n_i^\nu + \frac{\theta y_i}{1 - \lambda} (\tau + \nu) n_i^{\tau + \nu - 1} e_i^{\tau - 1} \right] \frac{\partial n_i}{\partial y_i} = 0 \\ \Leftrightarrow & \frac{\partial n_i}{\partial y_i}(y_i, \lambda, n_i) = - \frac{n_i^\tau e_i^{\tau - 1}}{(1 - \lambda)(1 + \nu) + y_i(\tau + \nu) n_i^{\tau - 1} e_i^{\tau - 1}} \end{aligned}$$

and evaluating in $(0, \lambda_0, n_0)$

$$\frac{\partial n_i}{\partial y_i}(0, \lambda_0, n_0) = - \frac{-n_0^\tau e_i^{\tau - 1}}{(1 - \lambda_0)(1 + \nu)} = - \frac{1}{1 + \nu} \frac{n_0}{n_0 - G} \frac{\exp(-\tau(1 - \tau) \frac{\sigma_e^2}{2})}{e_i^{1 - \tau}}$$

The labor supply expression as a function of y_i can be approximated by

$$\begin{aligned} n_i(y_i) & \approx n_0 + y_i \frac{\partial n_i}{\partial y_i}(0, \lambda_0, n_0) \\ & \approx n_0 - \frac{y_i}{1 + \nu} \frac{n_0}{n_0 - G} \frac{\exp(-\tau(1 - \tau) \frac{\sigma_e^2}{2})}{e_i^{1 - \tau}} \leq n_0 \end{aligned}$$

This expression shows that households receiving an additional non-labor income will work less than if they received nothing, due to the wealth effect.

In that new context, output is given by

$$\begin{aligned} Y(y, m, \tau) & = \int e_i n_i(y_i) di \\ & \approx \int e_i n_0 - e_i \frac{y_i}{1 + \nu} \frac{n_0}{n_0 - G} \frac{\exp(-\tau(1 - \tau) \frac{\sigma_e^2}{2})}{e_i^{1 - \tau}} di \\ & \approx n_0 - \frac{1}{1 + \nu} \frac{n_0}{n_0 - G} \exp(-\tau(1 - \tau) \frac{\sigma_e^2}{2}) \int y_i e_i^\tau di \end{aligned}$$

From the $\int y_i e_i^\tau di$ term, it appears that Y will depend on the cross-distribution of exogenous incomes y_i and idiosyncratic productivities e_i . For the sake of simplicity, we will assume now that a mass m of households representative of the entire idiosyncratic income distribution receives nothing, while the rest receives an amount y independent of their productivities, then

$$\begin{aligned} Y(y, m, \tau) &= mn_0 + (1 - m) \left[n_0 - \frac{y}{1 + \nu} \frac{n_0}{n_0 - G} \exp(-\tau(1 - \tau)\sigma_e^2) \right] \\ &= n_0 - (1 - m) \frac{y}{1 + \nu} \frac{n_0}{n_0 - G} \exp(-\tau(1 - \tau)\sigma_e^2) \end{aligned}$$

If all households receive an exogenous income ($m = 0$), we return to the case of Ferriere, Grübener, et al., 2023 with a uniform transfer. Otherwise, output is less depressed than in their case because only a $1 - m$ fraction of the population benefits from the wealth effect and thus further reduces their labor supply.

The government still has to raise G through the income tax. Using a first-order Taylor expansion of $n_i^{1-\tau}$ and the balanced-budget condition, we retrieve the value for the income tax level parameter λ .

$$\begin{aligned} G &= \int (e_i n_i(y_i) - (1 - \lambda)(e_i n_i(y_i))^{1-\tau}) di \\ &= Y(y, m, \tau) - (1 - \lambda) \int (e_i n_i(y_i))^{1-\tau} di \\ &= Y(y, m, \tau) - (1 - \lambda) \left[\underbrace{mn_0^{1-\tau} \exp(-\tau(1 - \tau)\frac{\sigma_e^2}{2})}_{\equiv \eta} + (1 - m) \int (e_i n_i(y))^{1-\tau} di \right] \\ n_i(y)^{1-\tau} &\approx n_0^{1-\tau} - (1 - \tau) \frac{y}{1 + \nu} \frac{n_0^{1-\tau}}{n_0 - G} \frac{\eta}{e_i^{1-\tau}} \\ G &\approx Y(y, m, \tau) - (1 - \lambda) \left[mn_0^{1-\tau} \eta + (1 - m) \int e_i^{1-\tau} n_0^{1-\tau} - (1 - \tau) \frac{y}{1 + \nu} \frac{n_0^{1-\tau}}{n_0 - G} \eta di \right] \\ &\approx Y(y, m, \tau) - (1 - \lambda) \left[mn_0^{1-\tau} \eta + (1 - m) n_0^{1-\tau} \eta \left(1 - (1 - \tau) \frac{y}{1 + \nu} \frac{1}{n_0 - G} \right) \right] \\ &\approx Y(y, m, \tau) - (1 - \lambda) n_0^{1-\tau} \eta \left(1 - (1 - m)(1 - \tau) \frac{y}{1 + \nu} \frac{1}{n_0 - G} \right) \\ \lambda(y, m, \tau) &\approx 1 - \frac{Y(y, m, \tau) - G}{n_0^{1-\tau} \exp(-\tau(1 - \tau)\frac{\sigma_e^2}{2}) \left[1 - (1 - m)(1 - \tau) \frac{y}{1 + \nu} \frac{1}{n_0 - G} \right]} \end{aligned}$$

Finally, welfare writes

$$\begin{aligned} W(y, m, \tau) &= \int \ln(c_i) - \theta \frac{n_i(y_i)^{1+\nu}}{1 + \nu} di \\ &= mW_0 + (1 - m) \left[\int \ln((1 - \lambda(y, m, \tau))(e_i n_i(y))^{1-\tau} + y) - \theta \frac{n_i(y)^{1+\nu}}{1 + \nu} di \right] \end{aligned}$$

Optimal income tax progressivity with uncertainty

Without exogenous income Starting from the expressions of labor supply and welfare derived in the no uncertainty case (and with $\sigma = 1, y = 0$),

$$\begin{aligned} n'_0(\tau) &= -\frac{1}{\theta(1+\nu)} \left(\frac{1-\tau}{\theta} \right)^{-\frac{\nu}{1+\nu}} < 0 \\ n''_0(\tau) &= -\frac{\nu}{\theta^2(1+\nu)^2} \left(\frac{1-\tau}{\theta} \right) < 0 \end{aligned}$$

As a result, labor supply n_0 is a decreasing function of tax progressivity τ . So is output since $Y_0 = n_0(\tau)$. Besides, labor supply and output are concave functions in τ . As a result, fiscal uncertainty will always have a detrimental impact on output (and hours).

$$\begin{aligned} W'_0(\tau) &= \frac{n'_0(\tau)}{n_0(\tau) - G} + \frac{1}{1+\nu} + (1-\tau)\sigma_e^2 \\ &= \frac{n'_0(\tau)}{\mathcal{C}_0(\tau)} + \frac{1}{1+\nu} + (1-\tau)\sigma_e^2 \\ W''_0(\tau) &= \frac{n''_0(\tau)\mathcal{C}_0(\tau) - (n'_0(\tau))^2}{(\mathcal{C}_0(\tau))^2} - \sigma_e^2 < 0 \end{aligned}$$

$W''_0(\tau) < 0$ tells us that the welfare is a concave function of tax progressivity τ . It also yields that W'_0 is a decreasing function of τ and thus that welfare is an increasing function of τ until a certain point where it starts to decrease. Just as for output, welfare is lower when facing uncertainty.

Uncertainty gaps. Welfare can be divided into size S , labor disutility L^D , and redistribution R channels. Likewise, the cost of uncertainty u^W can be decomposed

$$\begin{aligned} u^W &\equiv \mathbb{E}[W(\tau)] - W(\bar{\tau}) \\ &= u^S + u^{L^D} + u^R \end{aligned}$$

The labor disutility term is linear in τ so the cost of progressivity uncertainty is null: $u^{L^D} = 0$.

Through the redistribution channel, the cost of uncertainty is,

$$\begin{aligned} u^R &= -\frac{\sigma_e^2}{2} \mathbb{E}[(1-\tau)^2] + \frac{\sigma_e^2}{2} (1-\bar{\tau})^2 \\ &= -\frac{\sigma_e^2}{2} (\mathbb{E}[\tau^2] - \mathbb{E}[\tau]^2) \\ &= -\frac{\sigma_e^2}{2} \text{Var}(\tau) \end{aligned}$$

As a result,

- $u^R \leq 0 \Rightarrow$ uncertainty is always detrimental to welfare due to the redistribution channel
- $|u^R|$ increase in $\sigma_e \Rightarrow$ the cost of uncertainty increases when the volatility of pre-tax income increases
- $|u^R|$ increase in $Var(\tau) \Rightarrow$ bigger uncertainty on τ increases the cost of uncertainty

The fact that the cost of uncertainty on welfare increases with pre-tax income volatility σ_e relates to changes in the curvature κ of the welfare function.

$$\begin{aligned}\kappa &\equiv \left| \frac{W_0''}{(1 + W_0'^2)^{3/2}} \right| \\ &= |(W_0''(1 + O(W_0'^2)))| \\ &\approx |W_0''|\end{aligned}$$

Citanna and Tvede, 2024 study the optimality of tax uncertainty. They show that if the households' decisions (consumption for instance) are less concave than the welfare function, tax uncertainty may be Pareto improving. The tax uncertainty will generate uncertainty on disposable income and boost labor supply, as a response, which in turn will increase consumption which will more than compensate for the increased disutility of labor. Moving from a representative-agent framework ($\sigma_e = 0$) to a representative-agent one ($\sigma_e > 0$), the households' consumption decisions become more concave since it increases the share of constrained agents that have non-linear consumption decisions. This would rather push into the direction of tax uncertainty being more costly, or not Pareto improving.

Finally, for the size channel, the welfare cost of tax progressivity uncertainty is

$$\begin{aligned}u^S &= \mathbb{E}[\ln(n(\tau) - G)] - \ln(n(\bar{\tau}) - G) \\ &= p \ln(n(\tau_-) - G) + (1 - p) \ln(n(\tau_+) - G) - \ln(n(p\tau_- + (1 - p)\tau_+) - G) \\ &= p \ln\left(\left(\frac{1 - \tau_-}{\theta}\right)^{\frac{1}{1+\nu}} - G\right) + (1 - p) \ln\left(\left(\frac{1 - \tau_+}{\theta}\right)^{\frac{1}{1+\nu}} - G\right) - \ln\left(\left(\frac{1 - p\tau_- - (1 - p)\tau_+}{\theta}\right)^{\frac{1}{1+\nu}} - G\right) \\ &= \ln\left(\frac{\left(\left(\frac{1 - \tau_-}{\theta}\right)^{\frac{1}{1+\nu}} - G\right)^p \left(\left(\frac{1 - \tau_+}{\theta}\right)^{\frac{1}{1+\nu}} - G\right)^{1-p}}{\left(\left(\frac{1 - p\tau_- - (1 - p)\tau_+}{\theta}\right)^{\frac{1}{1+\nu}} - G\right)}\right)\end{aligned}$$

As a result,

- Uncertainty does not depend on the income volatility σ_e through the size channel
- If $\frac{\partial u^S}{\partial G} = \frac{1}{n(\bar{\tau}) - G} - \frac{p}{n(\tau_-) - G} - \frac{1-p}{n(\tau_+) - G} < 0$, increased government spending G increases the uncertainty cost. More public spending requires more taxes to finance them,

leading to further distorting of the labor supply. Figure 4.26 confirms this happens in the considered calibration.¹⁵

- Figure 4.26 also shows that the standard deviation of τ impacts negatively the uncertainty cost. This effect seems quantitatively bigger through the size channel than the redistribution one.

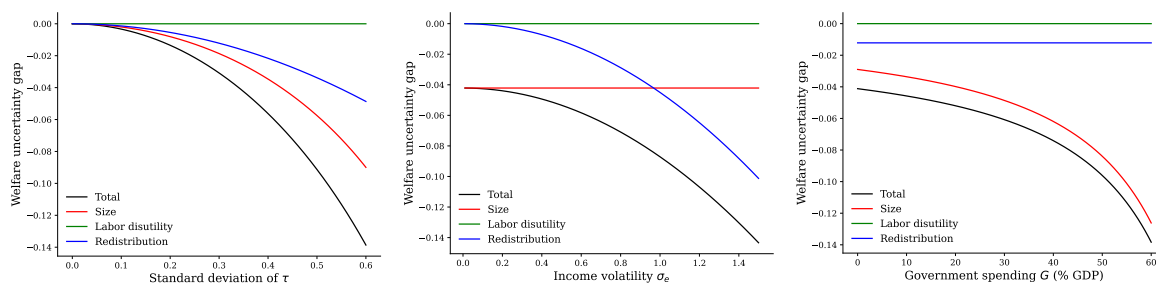


Figure 4.26: Welfare uncertainty gaps and contributions of the various channels.

¹⁵I considered the same calibration as Ferriere, Grübener, et al., 2023, $\sigma_e = 0.52$, $\sigma = 1$, $\nu^{-1} = 0.4$, $\theta = 55.4$ (to normalize $L = 0.3$), $G/Y = 23\%$. For tax uncertainty, I chose $\tau^- = 0\%$, $\tau^+ = 60\%$, $p = 0.5$.

4.6.7 Description of the model

Calibration

Income process. Figure 4.27 displays the density of productivities resulting from the chosen GMAR process.

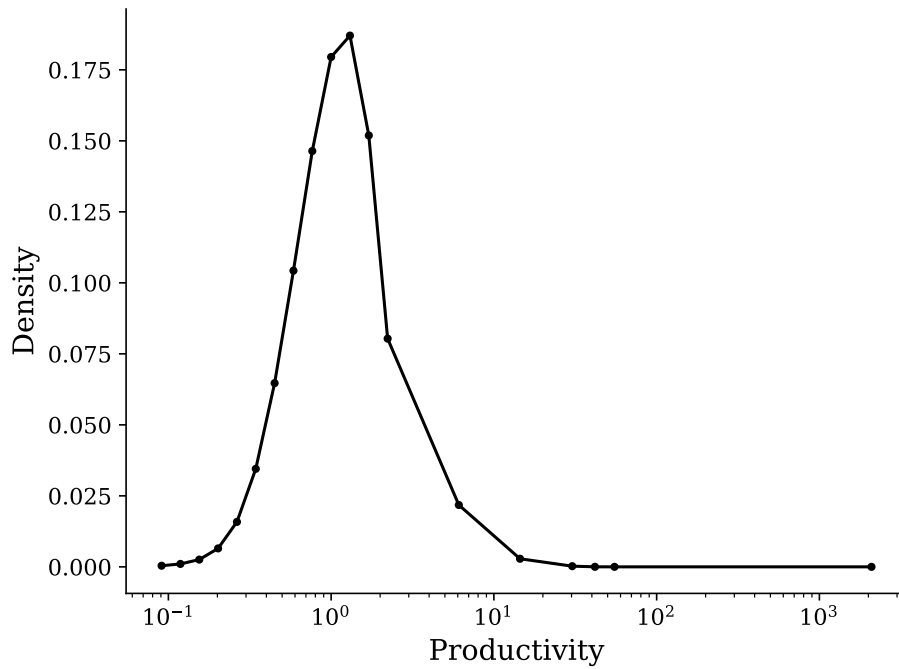


Figure 4.27: Distribution of productivities.

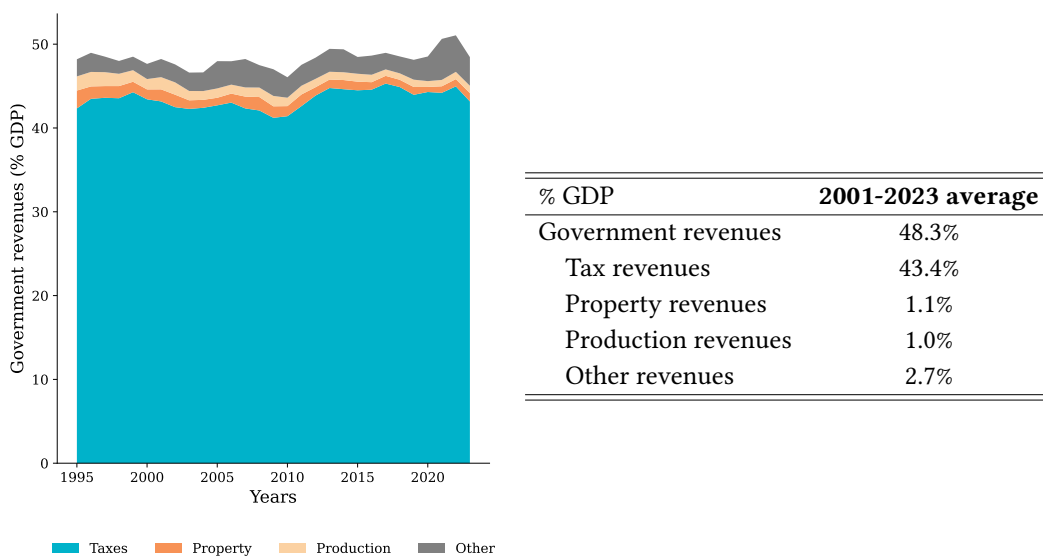


Figure 4.28: Sources of government revenues and average values. *Source: INSEE.*

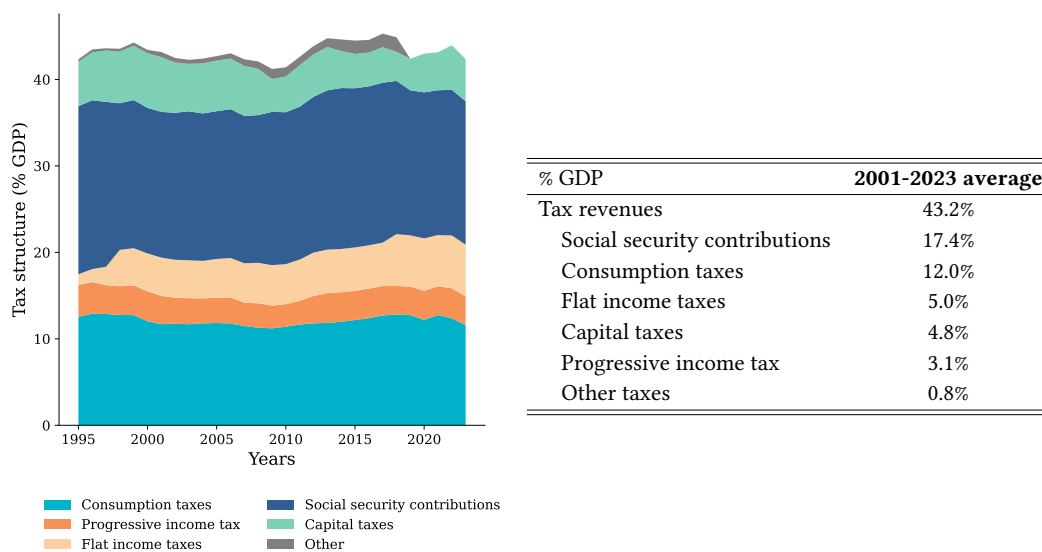


Figure 4.29: Structure of tax revenues and average values. *Source: INSEE.*

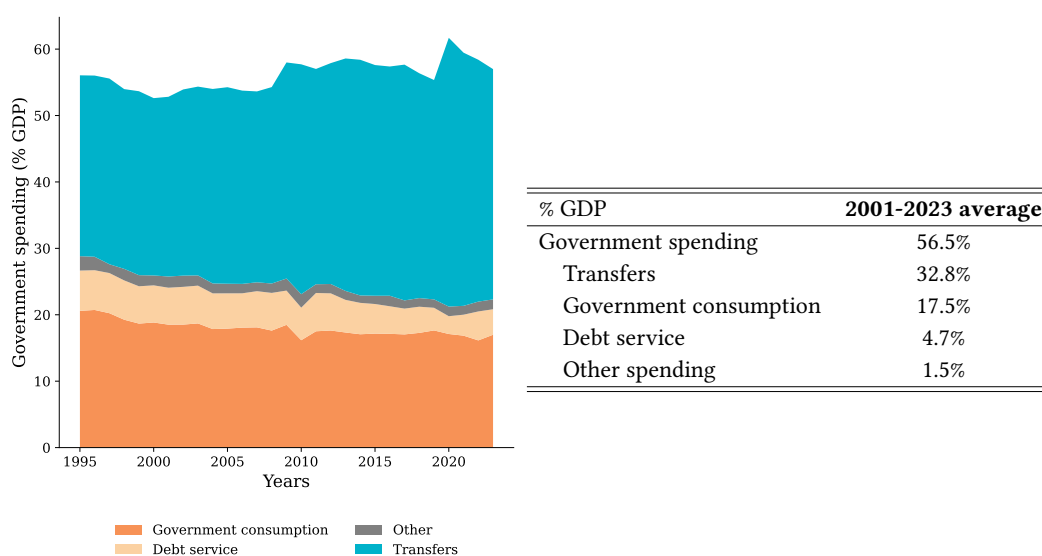


Figure 4.30: Types of government spending and average values. *Source: INSEE.*

National accounts aggregates. Following Figure 4.29, net income tax revenues represent social security contributions (17.4%), flat income taxes (5.0%), progressive income taxes (3.1%), and other taxes (0.8%), net of housing benefits ($-0.026 \times 32.8 \approx -0.9\%$) and other benefits ($-0.023 \times 32.8 \approx -0.8\%$). As a result, the ratio $T/Y \approx 23.9\%$.

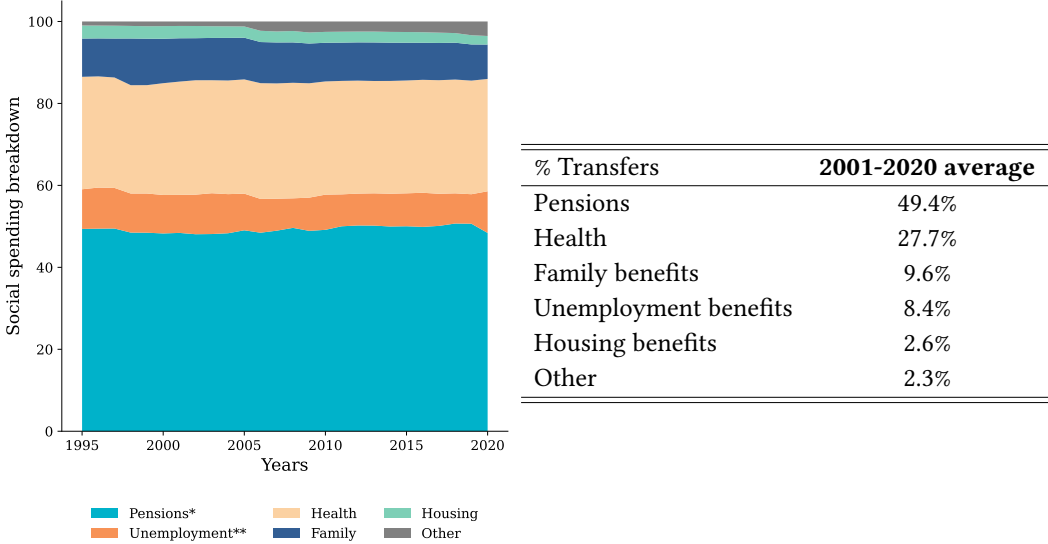


Figure 4.31: Types of government transfers and average values. *Source: OECD.*

Income and wealth distributions The lump-sum transfer ϑ allows us to lower the effective average tax rate at the bottom of the distribution, as shown in Figure 4.11. The HSV function alone would not be able to be as realistic. Indeed, it does not transfer enough to the bottom 6.1% approximatively of the distribution. The amounts perceived via lump-sum transfer ϑ , the fiscal cost for the government and the resulting average tax rates are reported in Table 4.11.

Prod. index	Masses	Pre-tax income	Post-tax income	ATR	Lump-sum transfer	Cumulative cost (% GDP)	Effective ATR
e	π_e	wen	$(1-\lambda)(wen)^{1-\tau}$	$1 - (1-\lambda)(wen)^{-\tau}$	$\vartheta(e)$	$\vartheta(e)\pi_e$	$ATR - \frac{\vartheta(e)}{wen}$
0	0.0394	0.0737	0.0911	-23.5%	0.1763	0.0070	-262.5%
1	0.1031	0.1085	0.1253	-15.5%	0.1350	0.0209	-140.0%
2	0.2561	0.1513	0.1650	-9.0%	0.1034	0.0474	-77.4%
3	0.6497	0.2025	0.2099	-3.7%	0.0792	0.0988	-42.8%
4	1.5836	0.2630	0.2606	0.9%	0.0607	0.1950	-22.2%
5	3.4524	0.3349	0.3183	5.0%	0.0465	0.3555	-8.9%
6	6.4691	0.4210	0.3845	8.7%	0.0356	0.5859	0.2%
7	10.430	0.5250	0.4616	12.1%	0.0273	0.8705	6.9%
8	14.642	0.6526	0.5527	15.3%	0.0209	1.1765	12.1%
9	17.952	0.8132	0.6631	18.5%	0.0160	1.4639	16.5%
10	18.706	1.0237	0.8022	21.6%	0.0123	1.6934	20.4%
11	15.190	1.2891	0.9707	24.7%	0.0094	1.8361	24.0%
12	8.0355	1.6033	1.1623	27.5%	0.0072	1.8939	27.1%
13	2.1787	4.5283	2.7417	39.5%	0.0026	1.8997	39.4%
14	0.2854	10.609	5.5390	47.8%	0.0011	1.9000	47.8%
15	0.0242	21.677	9.9794	54.0%	0.0005	1.9000	54.0%
16	0.0021	22.861	10.251	55.2%	0.0004	1.9000	55.2%
17	0.0008	17.215	7.9576	53.8%	0.0003	1.9000	53.8%
18	0.0007	1211.6	278.25	77.0%	0.0000	1.9000	77.0%

Table 4.11: Lump-sum transfer, fiscal cost, and average tax rates.

Note that when adding this lump-sum transfer ϑ , the CRIP is modified since the average tax rate is modified. The marginal tax rate is unchanged since the lump-sum transfer does not affect the taxable income.

$$CRIP(y) = \frac{(1-\lambda)(1-\tau)y^{-\tau}}{(1-\lambda)y^{-\tau} + \frac{\vartheta}{y}} = \frac{1-\tau}{1 + \frac{\vartheta}{(1-\lambda)y^{1-\tau}}}$$

Each decile's income and wealth shares are computed using the raw data reported in Tables 4.12 and 4.13.

Gross labor income	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	Total
Levels	4810	9520	13,470	17,530	21,730	25,700	29,970	35,430	41,960	69,160	269,280
Shares	1.8%	3.5%	5.0%	6.5%	8.1%	9.5%	11.1%	13.2%	15.6%	25.7%	

Table 4.12: Gross labor incomes per deciles and gross labor income shares. *Source: INSEE, comptes nationaux distribués 2018, André, Germain, and Sicsic, 2023.*

Net wealth	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	Total
Levels	-4,700	6,400	17,300	43,900	97,400	157,100	231,500	330,500	493,700	1,366,300	2,739,400
Shares	-0.2%	0.2%	0.6%	1.6%	3.6%	5.7%	8.4%	12.1%	18.0%	49.9%	

Table 4.13: Net worth per deciles and net worth shares. *Source: INSEE, enquête Histoire de vie et Patrimoine 2021.*

Model equations and structure

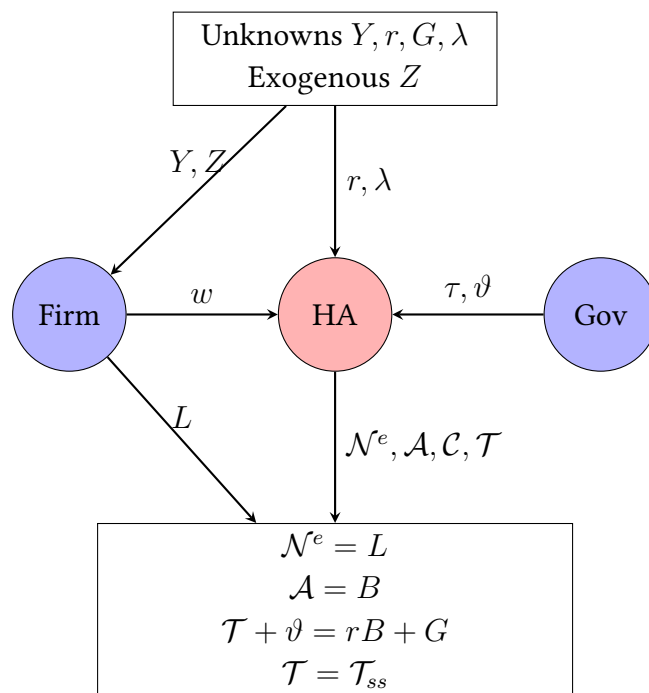


Figure 4.32: Directed acyclic graph of the model.

The full set of equations of the model is

$$\mathbf{H} \left(\left\{ \vec{Y}_s \right\}_{s=0}^{+\infty}, \left\{ \vec{Z}_s \right\}_{s=0}^{+\infty} \right) \equiv \begin{pmatrix} \vdots \\ Y_t - Z_t L_t \\ w_t - Z_t \\ \mathcal{T}_t + \vartheta - r_t B - G_t \\ \mathcal{A}_t - B \\ \mathcal{N}_t^e - L_t \\ \mathcal{T}_t - \mathcal{T}_{ss} \\ \vdots \end{pmatrix} = 0 \quad (4.1)$$

where \vec{Y} is the set of variables of the model and \vec{Z} the set of shocks.

Determinacy

Auclert, Rognlie, and Straub, 2023a builds a winding number criterion for models written in sequence-space. It conjectures that the Jacobian \mathbf{J} of the system is generically invertible if and only if the winding number of the determinant of its symbol $\mathbf{j}(z)$ is zero, where

$$\begin{aligned} \mathbf{j}(z) &= \sum_{k=-\infty}^{+\infty} \mathbf{j}_k z^k, \quad z \in \mathbb{C} \\ \mathbf{j}_k &= \lim_{s \rightarrow +\infty} \mathbf{J}_{s+k,s} \end{aligned}$$

To compute the winding number, one has to plot in the complex plane the curve described by $\det \mathbf{j}(z)$ when z covers the unit circle, $z = e^{2i\pi\theta}$ for $\theta \in [0, 1]$. Graphically, the absolute value of the winding number counts the number of times the curve wraps around zero. Whether the winding number is negative or positive depends on the way it rotates: clockwise (resp. counter-clockwise) circlings are counted negatively (resp. positively).

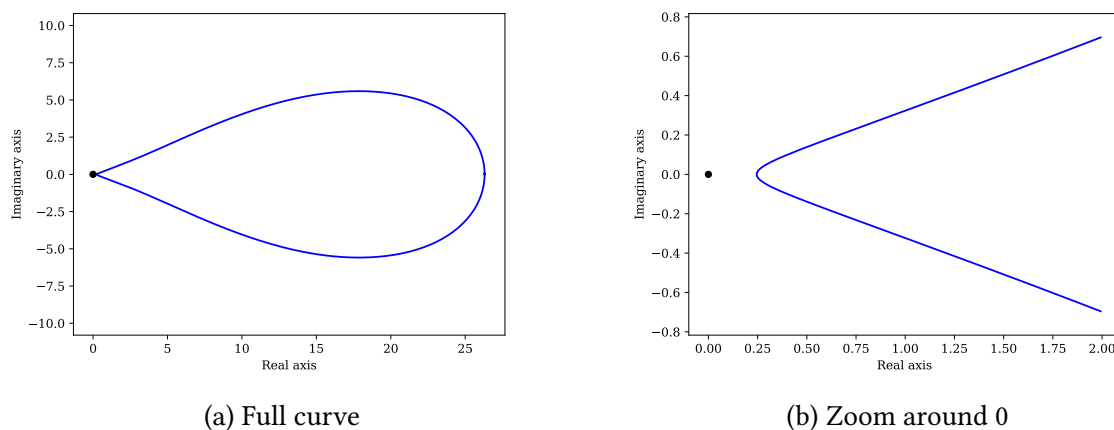


Figure 4.33: Curve described by $\det \mathbf{j}(z)$ in the complex plane when evaluated along the unit circle for the model of Section 4.4.

The model considered here has a zero winding number as shown in Figure 4.33. The Jacobian thus exists and is determinate, ensuring invertibility. Solutions to that model exist and are locally determined.

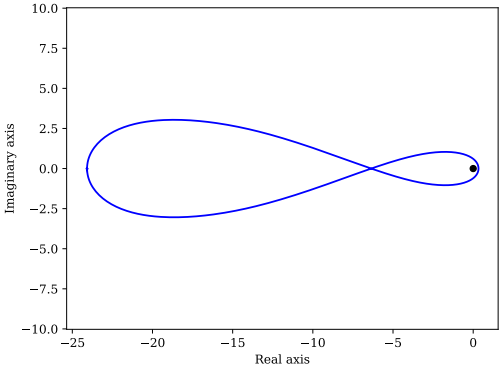
Alternative model: fixed government spending

The alternative model to equation (4.1) is given by

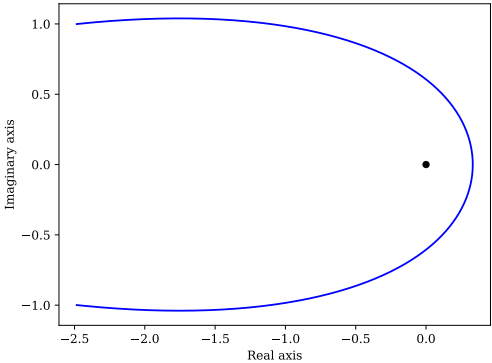
$$\mathbf{H} \left(\left\{ \vec{Y}_s \right\}_{s=0}^{+\infty}, \left\{ \vec{Z}_s \right\}_{s=0}^{+\infty} \right) \equiv \begin{pmatrix} \vdots \\ Y_t - Z_t L_t \\ w_t - Z_t \\ \mathcal{T}_t + \vartheta - r_t B - G_t \\ \mathcal{A}_t - B \\ \mathcal{N}_t^e - L_t \\ G_t - G_{ss} \\ \vdots \end{pmatrix} = 0 \quad (4.2)$$

This specification, however, yields indeterminacy as suggested by the negative winding number.

This issue does not seem to be caused by the heterogeneity since the equivalent models with a representative agent yield the same results. When the taxes are held constant, the Blanchard and Kahn, 1980 conditions are met and break down when fixing public spending instead.



(a) Full curve



(b) Zoom around 0

Figure 4.34: Curve described by $\det \mathbf{j}(z)$ in the complex plane when evaluated along the unit circle for the alternative model (G fixed).

4.6.8 Dynamic experiment

To get a first idea of the dynamic impact of fiscal uncertainty on macroeconomic variables, we consider an MIT shock on a variant of the model. Solving for the dynamic response of the model is done using Auclert, Bardóczy, Rognlie, and Straub, 2021's sequence-space Jacobian method. The determinacy and existence of solutions to this model are verified thanks to their winding number criterion (Auclert, Rognlie, and Straub, 2023a).

I consider for now that the income tax progressivity parameter is idiosyncratic such that household i faces the following program

$$\begin{aligned}
 v_t(e_{it}, \tau_{it}, a_{i,t-1}) &= \max_{c_{it}, a_{it}, n_{it}} u(c_{it}, n_{it}) + \beta \mathbb{E}_t v_{t+1}(e_{i,t+1}, \tau_{i,t+1}, a_{it}) \\
 \text{s.t.} \quad c_{it} + a_{it} &= (1 + r_t)a_{i,t-1} + (1 - \lambda_t)(w_t e_{it} n_{it})^{1-\tau_{it}} + \vartheta(e_i) \\
 a_{it} &\geq 0
 \end{aligned}$$

.

By choosing a grid of values for τ as well as a transition matrix across those states, I can control the fiscal uncertainty at the household level.

For now, it is considered that the idiosyncratic tax progressivity parameter τ_i is a function of the idiosyncratic progressivity e_i and in particular that it is increasing linearly from $\bar{\tau} - s$ to $\bar{\tau} + s$ where $\bar{\tau} = 11.1\%$ is the average tax progressivity and s governs the standard deviation around this value.

Starting from a steady state where everybody faces the same constant tax progressivity ($s = 0$), an unexpected shock occurs at $t = 0$. s rises to 3.6% before reverting to 0% according to an AR(1). Meanwhile, the government raises a constant amount of taxes in each period t so that λ is adapted. That is, the government changes the progressivity of the tax system without affecting the aggregate level of it.

Figure 4.35 reports the impulse response functions following this shock. When s increases, high-productivity households face higher taxation, lowering their labor supply. Low-productivity households receive fewer transfers, negatively affecting their consumption. Both phenomena contribute to generating a recession. As the real rate increases, government spending has to decline for the government to run a balanced budget while keeping tax revenues constant. Since effective labor (which is also output) falls more than consumption, this is welfare-improving.

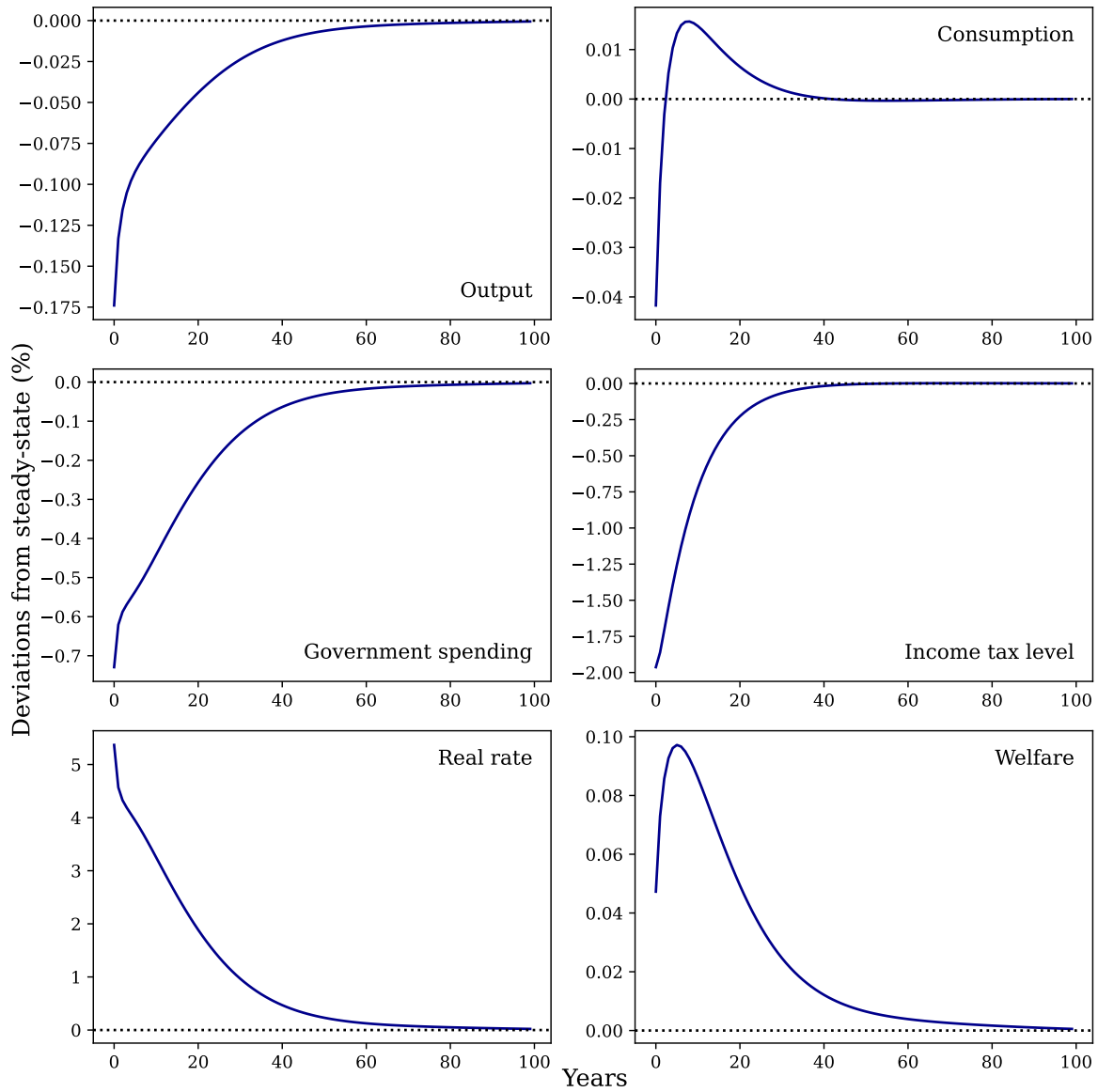


Figure 4.35: Impulse response functions to an uncertainty shock (% deviations from the final steady-state values).

4.6.9 Stochastic steady-state simulation

Tax progressivity path

I simulate a path of tax progressivity shocks τ over 1,100 periods. The shocks are driven according to Q_τ with $p_- = p_+ = 0.96$, $\bar{p} = 0.99$, and $p = 0.005$. This means less volatility than observed in the data to ensure easier algorithm convergence.

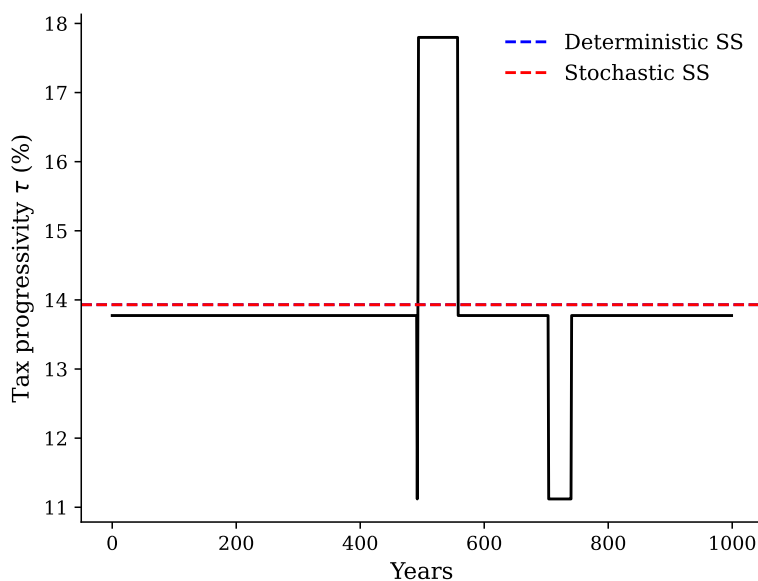
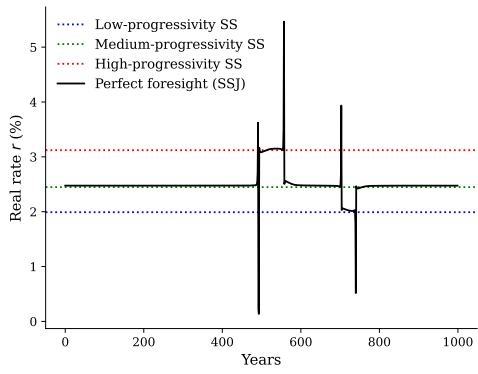


Figure 4.36: Path for the tax progressivity τ .

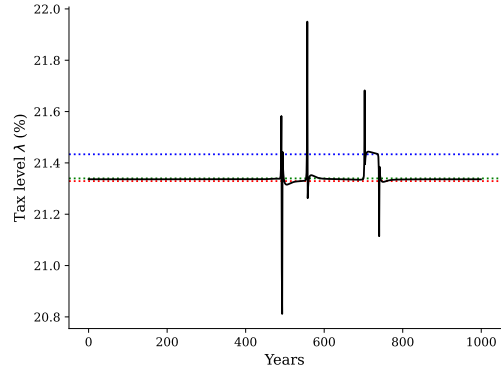
Perfect foresight response

By using the Krusell and Smith, 1998 algorithm, we relax the perfect foresight assumption. However, perfect foresight can inform us of the mechanisms at play.

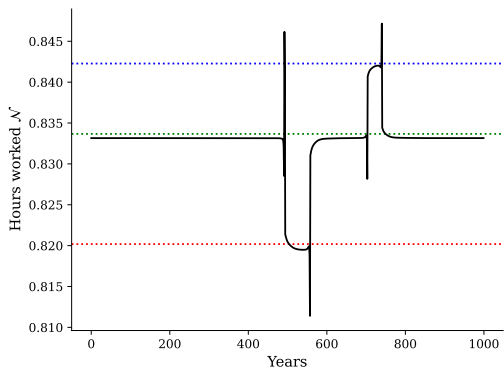
I thus solve the model's perfect foresight response to the path of shocks of Figure 4.36. For this, I use the sequence-space Jacobian method introduced by Auclert, Bardóczy, Rognlie, and Straub, 2021 and in particular, their non-linear solver. Figure 4.37 displays the responses. It shows that before a tax change, households anticipate it and react strongly.



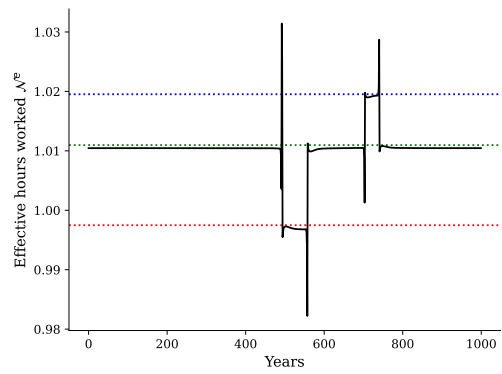
(a) Real rate



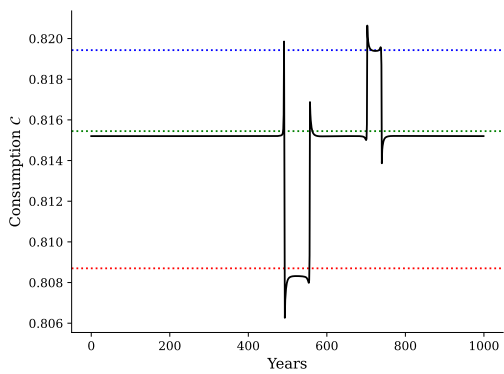
(b) Tax level



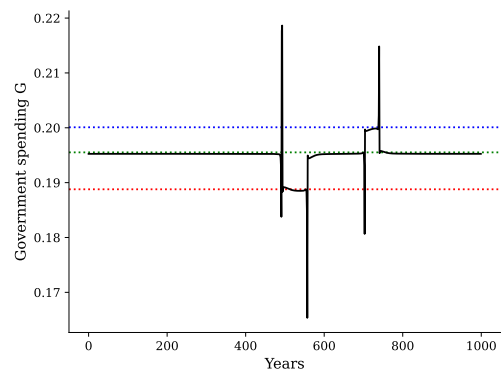
(c) Hours



(d) Effective hours



(e) Consumption



(f) Government spending

Figure 4.37: Perfect-foresight responses of the model.

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Titre : Grandes récessions, inégalités économiques et efficacité des politiques macroéconomiques

Mots-clés: Macroéconomie; Agents hétérogènes; Evaluation des politiques publiques; Inégalité sociale; Cycles économiques.

Résumé : Cette thèse explore l'interaction entre les chocs macroéconomiques, les inégalités économiques et l'efficacité de diverses politiques publiques dans le contexte français. Les crises affectent les ménages différemment selon la distribution des revenus et des richesses. En outre, les politiques publiques peuvent s'avérer plus efficaces lorsqu'elles ciblent leurs actions sur les ménages les plus affectés ou les plus réactifs. Par exemple, la politique fiscale peut stimuler efficacement la demande en concentrant les transferts vers les ménages ayant une forte propension marginale à consommer, plutôt que de les distribuer à l'ensemble des ménages. Pour ce faire, on utilise des modèles d'équilibre général dynamique et stochastique intégrant des agents hétérogènes et des rigidités néo-keynésiennes.

Le premier chapitre présente comment résoudre et estimer ce type de modèles, méthodologie sur laquelle repose les chapitres suivants. Le chapitre 2 évalue la réponse du gouvernement français à la récente crise énergétique, l'implémentation d'un bouclier tarifaire. Cette

politique est comparée à des scénarios alternatifs : des transferts ciblés et l'indexation des salaires sur les prix. Le bouclier tarifaire a soutenu la croissance économique, réduit l'inflation et freiné l'augmentation des inégalités de consommation au prix d'une légère augmentation du ratio dette/PIB. Le chapitre 3 évalue différentes politiques d'assainissement budgétaire dans le contexte français. Ces politiques peuvent échouer parce que leurs effets récessifs font baisser encore davantage le ratio dette/PIB ou parce que l'augmentation des inégalités qui en résulte entraîne des troubles sociaux et des résultats électoraux qui compromettent la continuité de ces programmes. Toutefois, ce chapitre montre qu'il est possible de réduire le ratio dette/PIB sans pénaliser la croissance ni aggraver les inégalités en réduisant les transferts d'assurance sociale tout en augmentant les aides sociales. Le dernier chapitre examine les conséquences macroéconomiques des fluctuations de l'impôt sur le revenu et de l'incertitude fiscale qui en résulte.

Title: Great recessions, economic inequalities and the effectiveness of macroeconomic policies

Keywords: Macroeconomics; Heterogeneous agents; Public policy evaluation; Inequalities; Business cycles.

Abstract: This PhD thesis explores the interplay between macroeconomic shocks, economic inequalities, and the effectiveness of various policy interventions in the French context. Crises affect households differently across income and wealth distribution. Besides, public policies can become more efficient by targeting their actions on the most affected and most responsive households. For example, fiscal policy can support demand at a reduced cost by transferring money to households with a high marginal propensity to consume instead of all households. For this purpose, Dynamic Stochastic General Equilibrium (DSGE) models with heterogeneous agents and new Keynesian features are used.

The first chapter outlines the methodology for solving and estimating such models, on which the following chapters build. Chapter 2 evaluates the French government's response to the recent energy crisis, implementing a tariff shield. This policy is compared to alternative

scenarios, including targeted transfers, and wage indexation on prices. The tariff shield supported economic growth, reduced inflation, and curtailed the increase in consumption inequalities at the cost of a slight rise in the debt-to-GDP ratio. Chapter 3 assesses different fiscal consolidation policies in the French context. Such policies may fail because their recessive effects further depress the debt-to-GDP ratio or because the resulting rising inequalities, lead to social unrest and electoral outcomes that undermine the continuity of these programs. However, this chapter shows that reducing the debt-to-GDP ratio can be achieved without penalizing GDP growth or worsening inequalities by reducing social insurance-based transfers while increasing social assistance transfers. The last chapter examines the macroeconomic consequences of income tax fluctuations and the resulting tax uncertainty.