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Multi-criteria decision making for sustainable engineering systems- Application for the energy renovation of buildings

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Abstract

The trend towards sustainability in the building sector is steadily gaining momentum, driven by well-established environmental regulations and the strong will of societies worldwide. While new building projects adhere to established standards, the construction of new buildings replaces only 1-3% of existing buildings annually. Renovation projects often present challenges such as the presence of uncertain input data regarding existing building conditions, the involvement of multiple stakeholders with varying interests and priorities, and the quantitative and qualitative nature of the sustainability factors. To address these limitations, this thesis intends to assist in integrating uncertainty into the decision-making process.

To achieve this, the thesis aims to develop a multi-criteria decision making (MCDM) framework based on sustainability factors that encompass economic, environmental, social, and technical aspects within the context of energy renovation for existing buildings. In contrast to existing deterministic MCDM methods, the proposed framework incorporates uncertain input data into its calculations, resulting in outcomes that are more aligned with reality.

A comprehensive state-of-the-art review of some of the well-established and commonly used MCDM methods was conducted. Furthermore, a SWOT (Strengths, Weaknesses, Opportunities and Threats) analysis was performed to justify the selection of a specific MCDM method for the research problem. The application of MCDM methods in the context of energy renovation of buildings was examined to gain an in-depth understanding of the sustainability factors under consideration. To address this issue, probability distribution and Monte Carlo simulation were employed and integrated into the MCDM methods. These tools effectively represent uncertainty, simulate human decision-making, and handle ambiguous situations.

The novel MCDM framework, probabilistic ELECTRE Tri, was developed based on the ELECTRE Tri method, probability distribution, revised Simos method, and Monte Carlo simulation. This framework was specifically developed for the classification of scenarios related to building energy renovation. To illustrate and validate the proposed framework, it was applied to a real-world case study of a social housing project located in the region of Lyon, France. A comparative analysis of the results obtained using the novel probabilistic ELECTRE Tri method and the ELECTRE Tri method highlights the significance of incorporating uncertain input data into the decision-making process.

The developed framework enhances transparency, adaptability, flexibility, and user understanding, benefiting a wide range of stakeholders including building owners, tenant managers, utility providers, and relevant regulatory and policy-making bodies. The developed MCDM framework is designed to be generic in nature and can be used by decision-makers to enhance the objectivity and consistency of the decision-making process. Additionally, it contributes to the advancement of sustainable building renovation and aligns with the broader global agenda of environmental stewardship and energy efficiency.

Keywords: multi-criteria decision making, ELECTRE Tri, uncertainty, probability distribution, criteria weighting, energy renovation, sustainability.

Résumé

La tendance à la durabilité dans le secteur du bâtiment gagne progressivement en ampleur, portée par des réglementations environnementales bien établies et la forte volonté des sociétés du monde entier. Alors que les nouveaux projets de construction respectent des normes établies, la construction de nouveaux bâtiments ne remplace que 1 à 3% des bâtiments existants chaque année. Les projets de rénovation présentent souvent des défis tels que la présence de données d'entrée incertaines concernant l'état des bâtiments existants, l'implication de multiples parties prenantes aux intérêts et priorités divergents, ainsi que la nature quantitative et qualitative des facteurs de durabilité. Pour répondre à ces limitations, cette thèse vise à intégrer l'incertitude dans le processus de prise de décision.

Pour ce faire, cette thèse se propose de développer un cadre de prise de décision multicritère (MCDM) basé sur des facteurs de durabilité englobant les aspects économiques, environnementaux, sociaux et techniques dans le contexte de la rénovation énergétique des bâtiments existants. Contrairement aux méthodes MCDM déterministes existantes, le cadre proposé intègre des données d'entrée incertaines dans ses calculs, produisant ainsi des résultats plus proches de la réalité.

Une revue approfondie de l'état de l'art des méthodes MCDM bien établies et couramment utilisées a été réalisée. De plus, une analyse SWOT (Forces, Faiblesses, Opportunités et Menaces) a été effectuée pour justifier la sélection d'une méthode MCDM spécifique adaptée à la problématique de recherche. L'application des méthodes MCDM dans le contexte de la rénovation énergétique des bâtiments a été examinée afin de mieux comprendre les facteurs de durabilité pris en compte. Pour traiter cette problématique, la distribution de probabilité et la simulation Monte Carlo ont été employées et intégrées aux méthodes MCDM. Ces outils permettent de représenter efficacement l'incertitude, de simuler la prise de décision humaine et de gérer des situations ambiguës.

Un cadre MCDM novateur, le probabilistic ELECTRE Tri, a été développé en s'appuyant sur la méthode ELECTRE Tri, la distribution de probabilité, la méthode Simos révisée et la simulation Monte Carlo. Ce cadre a été spécialement conçu pour la classification de scénarios liés à la rénovation énergétique des bâtiments. Afin d'illustrer et de valider le cadre proposé, il a été appliqué à une étude de cas réelle portant sur un projet de logement social situé dans la région de

Lyon, en France. Une analyse comparative des résultats obtenus à l'aide de la méthode probabilistic ELECTRE Tri et de la méthode ELECTRE Tri met en évidence l'importance d'intégrer des données d'entrée incertaines dans le processus de prise de décision.

Le cadre développé améliore la transparence, l'adaptabilité, la flexibilité et la compréhension par les utilisateurs, bénéficiant ainsi à un large éventail de parties prenantes, y compris les propriétaires de bâtiments, les gestionnaires de locataires, les fournisseurs d'énergie et les organismes de réglementation et de politique pertinents. Ce cadre MCDM est conçu pour être générique et peut être utilisé par les décideurs afin d'améliorer l'objectivité et la cohérence du processus de prise de décision. De plus, il contribue à l'avancement de la rénovation durable des bâtiments et s'aligne sur l'agenda global plus large de la préservation de l'environnement et de l'efficacité énergétique.

Mots-clés: prise de décision multicritère, ELECTRE Tri, incertitude, distribution de probabilité, pondération des critères, rénovation énergétique, durabilité.

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Abbreviations

(Sorted alphabetically)

AHP	Analytic hierarchy process
ANP	Analytic network process
CRITIC	Criteria importance through inter-criteria correlation
CO₂	Carbon dioxide
DM	Decision-maker
ELECTRE	Elimination and Choice Expressing Reality
FST	Fuzzy set theory
GHG	Greenhouse gas
MADM	Multi-attribute decision making
MAUT	Multi-attribute utility theory
MCDM	Multi-criteria decision making
MODM	Multi-objective decision making
PDF	Probability density function
PROMETHEE	Preference ranking organization method for enrichment evaluations
TOPSIS	Technique for order preference by using similarity to ideal solution
VIKOR	VIekriterijumsko KOmpromisno Rangiranje
WPM	Weighted product method
WSM	Weighted sum method

Notations and Symbols

$A = \{a_1, a_2, \dots, a_m\}$ Finite set of alternatives

$G = \{g_1, g_2, \dots, g_n\}$ Finite set of criteria

$W = \{w_1, w_2, \dots, w_j\}$ Weights of the criteria

$B = \{b_1, b_2, \dots, b_k\}$ Finite set of reference profiles

$C = \{C_1, C_2, \dots, C_k\}$ Finite set of categories

a_{ij} - Value of alternative i in terms of criterion j

$p_j(b_k)$ - Preference threshold indices associated with the criterion j and reference profile b_k

$q_j(b_k)$ - Indifference threshold indices associated with the criterion j and reference profile b_k

$v_j(b_k)$ - Veto threshold indices associated with the criterion j and reference profile b_k

λ Cutting level

σ Degree of credibility

b_k Reference profile

I Indifferent

P Preferred

R Incomparable

μ Mean value

σ Standard deviation

v Variance

1 Introduction

This chapter outlines the research background, motivation, objectives, questions, hypothesis, and methodology. It provides an overview of the thesis layout and current publication status, offering a comprehensive perspective on the research endeavor.

1.1 Research background

The building sector consumes around 40% of global energy demand and generates approximately 30% of greenhouse gas emissions associated with energy usage. It is anticipated to experience an annual growth rate of 3%, reaching an estimated 70% by the year 2050 under normal scenarios [1-3]. A significant proportion of residential buildings were constructed prior to the 1960s, indicating that currently, a substantial number of buildings do not comply with the basic requirements outlined in the legislative framework of the European Union [4, 5]. Given the low rate of new construction and the long lifespan of buildings, with estimates indicating that at least 75% of the current buildings in the European Union will still be standing by the year 2050, this represents a major challenge as well as an opportunity for undertaking remedial measures [6-8].

A consensus exists among experts regarding the significance of energy-efficient operation of existing buildings to achieve sustainability goals within the built environment and ensure reliable and sustainable energy [9]. The fight against climate change and the promotion of energy efficiency are closely interconnected. The European Union has established different initiatives (e.g., Green Deal, Renovation Wave, Fit for 55, and more) to significantly reduce greenhouse gas emissions and keep climate change at a manageable level [10, 11]. There is an urgent need to focus on investing in and adopting legislative guidelines for renovating existing buildings. The International Energy Agency has disseminated 25 policy recommendations specifically focusing on energy efficiency. If implemented on a global scale, these recommendations could achieve annual savings of 7.6 gigatons of carbon dioxide emissions and reduce annual energy consumption by 1,950 million tons of oil equivalent by the year 2030 [12, 13].

In France, the building sector, including residential and tertiary buildings, is presently the foremost energy-consuming sector. In the year 2018, buildings consumed approximately 63

million tons of oil equivalent, accounting for 45% of the country's total energy consumption. The expected rise in residential buildings is directly attributed to population growth and the increasing demand for household appliances. Additionally, buildings are responsible for 23% of the total greenhouse gas emissions, constituting just under one-quarter of all emissions in France. As a consequence, the building sector is the second-largest emitter of greenhouse gases, surpassed only by the transportation sector. Therefore, it contributes significantly to global warming. The building sector emits over 123 million tons of carbon dioxide annually, thereby constituting approximately 28% of the country's carbon dioxide emissions [14, 15].

Although various energy efficiency initiatives have been implemented, France energy-saving goals still fall considerably below planned objectives. Carbon dioxide emissions originating from buildings decreased by 12% between the years 1990 and 2014, whereas the targeted reduction for the year 2050 is a substantial 75%. In contrast, energy consumption increased by 17% during the same period and has only recently begun to decline since 2012, whereas the desired goal is to reduce energy consumption by approximately 50% by the year 2050 [16].

The recently proposed French Energy Transition for the Green Growth legislation has established specific goals related to the rapid progress of enhancing energy efficiency in residential buildings. It has set a target of 500,000 energy retrofits per annum, commencing with effect from 2017, with a noteworthy portion of 120,000 retrofits designated for low-income households. Unfortunately, in the year 2014, only 288,000 homes underwent energy retrofits, which amounted to a mere 1% of the global building stock. This data highlights a lack of motivation for energy efficiency initiatives [17].

The terms renovation, retrofit, and refurbishment are frequently used interchangeably. According to the formulation of European Committee for Standardization in the year 2021, these terms refer to planned substantial modifications and improvements to existing construction works. In this thesis, the general term "*renovation*" will be used to describe the enhancement of energy efficiency in existing buildings, which may involve rebuilding, refurbishing, or retrofitting. The key factors that have significant impacts on building energy renovation include policies and standards, stakeholder expectations, building characteristics, and retrofitting technologies [18-20].

Policies and standards typically establish the minimum requirements for building energy renovation and provide guidance for their implementation. These policies and standards play a fundamental role in raising public awareness, improving efficiency, and enhancing the

effectiveness of the building energy renovation [21-23]. Human expectations reflect the needs and goals of stakeholders, defining the main objectives and significant factors for building energy renovation [18]. Additionally, the effectiveness of building energy renovation also depends on building characteristics, including geographic location, climate zone, building type, age, size, envelope component, energy equipment and source, and occupancy rate [24]. Finally, retrofit technologies are energy conservation measures used to improve building energy efficiency. Selecting appropriate energy conservation is essential for enhancing efficiency and achieving energy renovation goals [25].

Properly renovating existing buildings can lead to a significant reduction in energy usage, with reductions ranging from 50% to 85% [26-30]. Moreover, renovating buildings holds the potential to lower carbon emissions, estimated to lead to an 80% decrease globally by 2050 compared to emissions in 2005 [31]. However, initiating building renovation evaluations can be challenging due to the complexity of the building and its environment, which are composed of multiple interdependent subsystems with various economic, ecological, social, technical, and aesthetic aspects to consider [32].

Energy renovation typically reduces a building's energy usage by implementing measures that improve energy efficiency. These measures may include enhancing thermal insulation, reducing thermal load, implementing solar shading, upgrading inefficient equipment, and adopting renewable energy technology. For example, a residential building located in Orlando, United States, underwent energy renovation, employing various energy efficiency measures, which resulted in an estimated annual reduction of up to 29% in energy consumption [33]. Similarly, a real-world case study in Australia demonstrated that transforming an existing dwelling into a net zero energy home significantly increased energy efficiency by 50% [34].

However, the adoption of energy-efficient practices in the building sector seems to be falling short of the goals established by public policy in various European countries, including France. Analyzing the elements that contribute to the limited acceptance of energy-efficient initiatives in the building sector, despite their cost-effectiveness and availability, has emerged as a subject of great interest. The appeal of investing in energy efficiency measures may not be as high as initially expected due to the presence of barriers that hinder their widespread adoption. This phenomenon is commonly referred to as the "*energy efficiency gap*" or "*energy efficiency paradox*". The energy efficiency gap is the discrepancy between the achievable level of cost-effective energy efficiency

and the actual level of energy efficiency observed in practice. The cause of this discrepancy is referred to as a "*barrier to energy efficiency*", which represents a mechanism that obstructs investment in technologies that are not only energy-efficient but also cost-effective for potential investors [35, 36].

Initially, special emphasis is placed on examining, describing, and classifying barriers to energy efficiency in existing buildings using a taxonomy. These barriers to the adoption of renovation in existing buildings vary across countries and can be categorized into four clusters [37, 38].

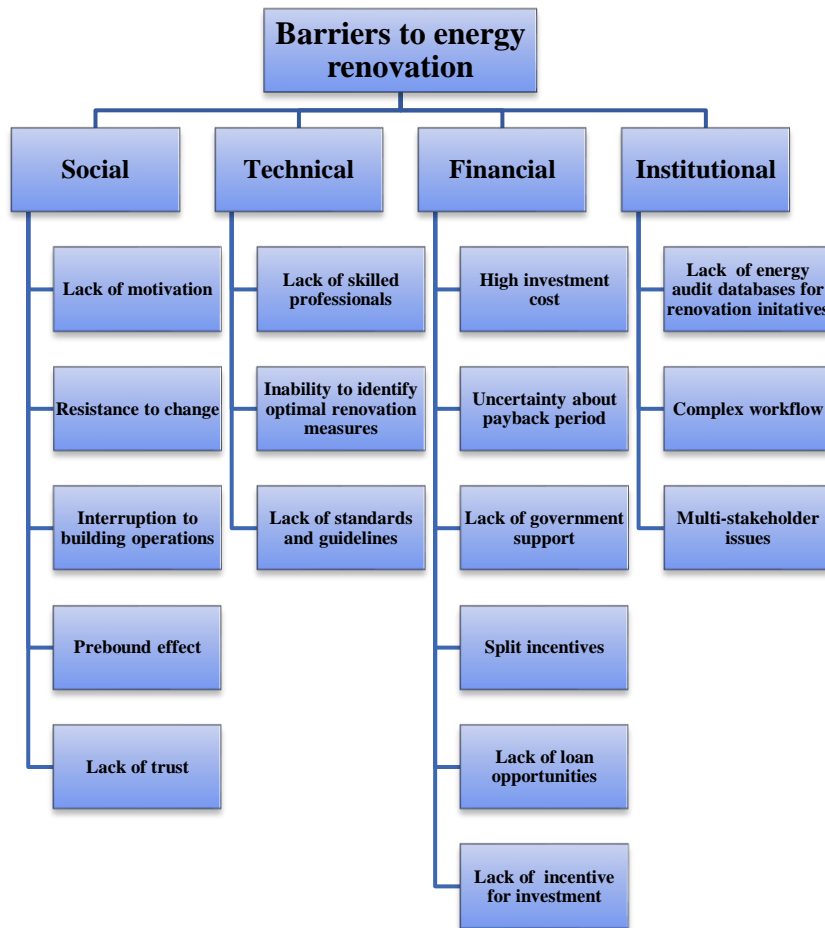


Figure 1.1. Barriers to energy renovation of buildings.

The clusters include social, technical, financial, and institutional issues as shown in Figure 1.1.

Social issues persist in residential buildings, where residents often lack motivation due to limited information and knowledge of energy efficiency measures and strategies for reducing greenhouse gas emissions. Many residents, particularly the elderly, are reluctant to adopt

improvements, perceiving energy-efficient technologies as both difficult to use and challenging to access [39, 40]. Such challenges are associated with energy renovation interventions that disrupt the normal operations of buildings, which may include temporary area shutdowns, resident relocation, or limited access to certain areas, all potentially influencing residents' decisions to proceed with renovations [41]. Furthermore, residents tend to increase their heating or fuel consumption following renovations, resulting in actual post-renovation energy usage that exceeds initial projections based on the calculated energy ratings [42, 43]. Additionally, residents often doubt the advertised benefits of renovation as outlined in cost reports and have limited trust in contractors' commitment and professionalism [39, 40, 44].

Obtaining technical support presents a significant challenge for stakeholders, primarily due to the limited availability of qualified professionals. This challenge becomes more complex due to the difficulty in determining the expertise of eligible professionals to address specific issues [23, 45]. For example, residents may seek assistance from a heating system contractor to resolve a thermal comfort problem, such as uneven home heating, despite the fact that an air sealing and insulation contractor would possess superior qualifications to efficiently resolve the issue [46]. The inability to determine the most suitable renovation measures arises due to the diverse characteristics of existing buildings in terms of building typology, construction period, and energy efficiency properties. Consequently, the unique characteristics of each building necessitate specific renovation measures [47]. Moreover, the lack of clear, enforceable standards and guidelines presents a challenge for professionals when using innovative technologies in energy renovation projects [42].

Residents have limited financial resources to allocate towards costly renovation procedures. The initial cost of building renovation is significantly higher compared to other sustainable choices, such as environmentally friendly vehicles, when measured in terms of greenhouse gas emissions reduction per ton. Due to these financial constraints, residents are often reluctant to allocate funds for renovation [48, 49]. Furthermore, the lack of assessments on the impact of price changes on energy demand can reduce the profitability of renovation investments, prolong payback periods, and necessitate investors to bear potential financial losses [42]. Governments have prioritized energy production industries over energy efficiency initiatives, as evidenced by tax policies and research and development support. This indicates a lack of governmental commitment to promoting energy efficiency efforts [50]. A challenge arises when the entity incurring the costs

of energy efficiency initiatives differs from the one enjoying the savings. For example, landlords may hesitate to invest in energy-efficient solutions because they incur the upfront costs, while residents benefit from lower energy expenses. As a result, landlords often lack the incentive to pursue energy-efficient upgrades [51]. Attractive loan opportunities are limited for residents with low to medium incomes, who are often not eligible for bank loans [50]. Additionally, the lack of commercial activity related to energy-efficient renovations also contributes to the failure to attract private sector investors. For these investors, financial returns are the main priority, rather than social and environmental benefits [52, 53].

The lack of energy audit databases for existing residences presents a significant obstacle in identifying specific residences for energy renovation initiatives [54, 55]. Although renovation professionals offer various options, none provides a comprehensive range of energy renovation services. As a result, residents must engage different professionals to carry out different renovation measures, adding complexity to the renovation process [56, 57]. The differing interests of public and private stakeholders further complicate the decision-making process for renovations. It is essential to understand the needs and expectations of these stakeholders. For example, local governments may prioritize reducing greenhouse gas emissions to comply with national climate change mitigation goals, but these priorities may not necessarily align with the interests of other stakeholders [39]. Although moderate renovations can significantly reduce energy usage, achieving substantial energy conservation through renovation demands a higher level of dedication and effort [58].

1.2 Research motivation

The growing concerns about the greenhouse gas emissions that contribute to climate change have led to ongoing efforts to minimize the environmental impact associated with the built environment. The French Government has established various targets to reduce greenhouse gas emissions, with the aim of contributing to efforts to mitigate climate change. Enhancing the energy efficiency of existing residential buildings can make a significant contribution to mitigating climate change, considering that this sector represents a substantial portion of the current building stock. Hence, it is necessary to address the obstacles and difficulties that hinder the adoption of renovation practices in the residential sector.

Building energy renovation can provide benefits from three distinct perspectives. First and foremost, it is becoming increasingly challenging to ignore the most obvious benefits of investing in energy efficiency, which are associated with the environment. The presence of more energy-efficient buildings would result in a reduction in the consumption of fossil fuels, leading to a decrease in greenhouse gas emissions. This reduction is essential for achieving the European Union's goal of decarbonizing the building stock by the year 2050 and addressing climate change caused by human activities, while simultaneously mitigating pollution [61-63].

Secondly, while the economic benefits may be less obvious, they are widespread. Economic benefits indicate that enhancing the energy efficiency of an existing building is a more cost-effective approach to increasing its asset value compared to constructing a new building, reducing energy bills, and enabling utility providers to reduce the need for new energy infrastructure. On a broader scale, improved energy efficiency in European buildings could lead to a reduced dependence on other countries for energy. Moreover, the benefits of such improvements would significantly outweigh the renovation costs [61, 64].

Finally, from a societal perspective, investments in energy efficiency present the opportunity to accomplish two objectives simultaneously. A significant number of existing buildings in Europe are in need of renovation. Energy-efficient renovations can address this issue by reducing energy consumption, while simultaneously enhancing the quality of living environments, as energy-efficient residences are commonly associated with higher levels of warmth and lower susceptibility to mold growth compared to energy-inefficient counterparts. Moreover, energy-efficient buildings improve air quality. The presence of less harmful surroundings not only improves overall well-being but also stimulates economic progress [65, 66].

On a global scale, the implementation of energy efficiency and effective energy renovation programs undoubtedly contribute to the accomplishment of the 13th Sustainable Development Goal (SDG) established by the United Nations. These programs act as a means to address the challenge of climate change and its consequences. However, the impact of a comprehensive energy renovation plan extends well beyond this objective. Through its multidimensional approach, which includes the reduction of energy demand, mitigation of adverse environmental and health effects, enhancement of living conditions, affordability of energy services, promotion of innovation, assurance of economic prosperity, creation of employment opportunities, and advancement of decent work, energy efficiency can bring various benefits and make a significant contribution to

several other sustainable development goals. It challenges humanity to separate economic growth from issues related to climate change, poverty, and inequality [67, 68].

Among the various assessment tools and methods that exist, MCDM stands out as a highly preferred approach for facilitating sustainable decision-making processes. MCDM is widely recognized as a comprehensive and user-friendly methodology that holds high esteem in both academic research and real-world projects. The research carried out in this thesis on energy renovation of buildings has specifically employed the MCDM approach due to its ability to adapt and facilitate a trade-off among various conflicting and opposing criteria based on economic, environmental, social, and technical factors, while also promoting effective communication among stakeholders.

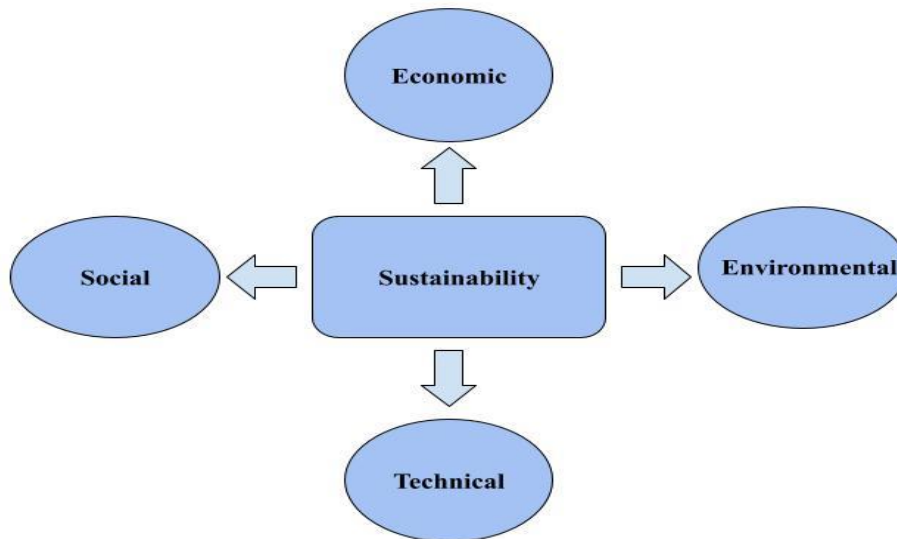


Figure 1.2. The four key pillars of sustainability for energy renovation of buildings [69].

The renovation strategies should account for the diverse interests of stakeholders and adhere to the four sustainability principles: economic viability, environmental responsibility, social acceptability, and technical feasibility as shown in Figure 1.2.

The MCDM framework proposed in this thesis has the potential to assist in exploring the trade-offs and finding the right balance among stakeholder expectations when implementing energy renovation projects and gaining acceptance. Furthermore, it would be beneficial for relevant regulatory and policy-making bodies to adopt an evidence-based framework to promote the formulation of renovation policies and incentive programs for the building sector.

1.3 Research aim and objectives

The aim of this research is to develop a multi-criteria decision making (MCDM) framework based on sustainability factors in terms of economic, environmental, social, and technical aspects to classify energy alternatives/scenarios in the context of energy renovation for existing buildings, ensuring a transparent evaluation during different operational stages by incorporating the input data uncertainty.

The specific objectives of the research are presented in Table 1.1:

- To review the current state-of-the-art related to MCDM to understand the fundamental theories and methodologies used to develop a MCDM framework and analyze their suitability and application limits.
- To assess sustainability factors in terms of economic, environmental, social, and technical aspects, taking into account both quantitative and qualitative criteria, and to develop a MCDM framework that incorporates quantitative and qualitative perspectives simultaneously.
- To apply the MCDM framework to a real-world case study, in which potential scenarios are proposed, sustainability criteria are defined, an algorithm is developed, and results are obtained and discussed.

1.4 Research questions

Four main research questions related to the classification of energy alternatives/scenarios within the context of energy renovation for existing buildings were identified and are presented in Table 1.1:

- What are the research gaps and issues in the existing MCDM methods used in energy renovation for existing buildings and how it can be improved?
- Which MCDM methods and tools are valuable and applicable to address the research gaps and issues associated with energy renovation for existing buildings?
- How can MCDM methods and tools be employed to propose a MCDM framework for the classification of energy alternatives/scenarios within the context of energy renovation for existing buildings?
- How efficient and valuable are the MCDM methods and tools in the classification of energy alternatives/scenarios within the context of energy renovation for existing buildings?

1.5 Research hypothesis

The general hypothesis is that employing a MCDM approach that integrates probability distribution will more effectively address input data uncertainty by incorporating probabilistic values into the decision-making process, as opposed to the crisp (precise) values used in most of the existing MCDM methods, leading to improved decision-making.

Table 1.1. Traceability matrix of research objectives, questions and chapters addressed.

Research Objectives		Research Questions		Chapters Addressed
RO1	To review the current state-of-the-art related to MCDM to understand the fundamental theories and methodologies used to develop a MCDM framework and analyze their suitability and application limits.	RQ1	What are the research gaps and issues in the existing MCDM methods used in energy renovation for existing buildings and how it can be improved?	Chapter 2 Chapter 3
		RQ2	Which MCDM methods and tools are valuable and applicable to address the research gaps and issues associated with energy renovation for existing buildings?	
RO2	To assess sustainability factors in terms of economic, social, environmental, and technical aspects, taking into account both quantitative and qualitative criteria, and to develop a MCDM framework that incorporates quantitative and qualitative perspectives simultaneously.	RQ3	How can MCDM methods and tools be employed to propose a novel MCDM framework for the classification of energy alternatives/scenarios within the context of energy renovation for existing buildings?	Chapter 3 Chapter 4
RO3	To apply the MCDM framework to a real-world case study, in which potential scenarios are proposed, sustainability criteria are defined, an algorithm is developed, and results are obtained and discussed.	RQ4	How efficient and valuable are the MCDM methods and tools in the classification of energy alternatives/ scenarios within the context of energy renovation for existing buildings?	Chapter 4

Legend: RO=Research Objective; RQ=Research Question

1.6 Research methodology overview

A brief overview of the methodological approaches employed in this study to address the research questions is presented herein. However, this section does not go into detail concerning the specific MCDM methods and tools used and the rationale justifications for their adoption in this study, as these are thoroughly described in the subsequent Chapters.

The research methodology comprises multiple stages, with the completion of each stage marking the commencement of the next stage. Hence, the initial stage included a comprehensive overview of the research to be conducted, comprehension of the fundamental concepts and terms involved, the research motivation, the formulation of the research objectives and questions, as well as the proposed hypothesis.

In the subsequent stage, a critical review of the state-of-the-art on MCDM methods, especially within the context of energy renovation for buildings, was undertaken. This included the methodologies of the most commonly used MCDM methods, a SWOT (Strengths, Weaknesses, Opportunities, and Threats) analysis of these methods, approaches for criteria weighting, and the application of MCDM methods that integrate both quantitative and qualitative criteria in real-world case studies. Based on this analysis, the rationale for selecting the ELECTRE Tri method for the research and the revised Simos method for criteria weighting is presented. To address the issue of input data uncertainty, probabilistic values were employed instead of crisp (precise) values. Consequently, this thesis proposes an improved probabilistic ELECTRE Tri MCDM method, which is based on ELECTRE Tri, probability distribution, revised Simos method, and Monte Carlo simulation.

A case study is a commonly used approach for validating and evaluating the feasibility of a proposed model, particularly when the research aims to understand a contemporary phenomenon within its real-world context [70]. In this study, the proposed model was implemented in a social housing project located in the region of Lyon, France. The findings of the case study were thoroughly discussed and validated, followed by the formulation of conclusions and prospects for future research endeavors.

1.7 Thesis layout

The **descriptive part** is presented in Chapters 1 and 2. These two chapters describe the introduction and state-of-the-art of this research, respectively.

Chapter 1 presents the research background, motivation, research aim and objectives, research questions, and research hypotheses. In the last section, the thesis structure is outlined, focusing on the content and role of subsequent chapters.

Chapter 2 presents a comprehensive state-of-the-art review of commonly used MCDM methods. It addresses key topics such as an overview of various MCDM methods, a SWOT analysis of these MCDM methods, the application of MCDM methods to real-world case studies in the context of energy renovation for existing buildings. Additionally, methods for determining criteria weights during the decision-making process are explored.

The **operational part** presents a novel MCDM model developed and applied to a real-world case study of a social housing project located in the region of Lyon, France. This part includes Chapters 3 to 5.

Chapter 3 presents the proposed methodological framework for the development of a novel MCDM model, the probabilistic ELECTRE Tri. This model integrates the ELECTRE Tri method, probability distributions, the revised Simos method, and Monte Carlo simulation.

Chapter 4 presents the application of the proposed probabilistic ELECTRE Tri in the classification of energy scenarios/alternatives into predefined categories within the context of energy renovation for buildings, using a real-world case study. The results obtained through the probabilistic ELECTRE Tri are compared with those obtained using the ELECTRE Tri method.

The primary aim is to highlight the application and effectiveness of the proposed model in achieving the research objectives. The chapter explains the method for data gathering, which includes identifying alternatives/scenarios, selecting sustainability criteria, determining criteria weights, establishing threshold parameters, discussing outcomes, and validating the findings.

Finally, **Chapter 5** presents a summary of the conclusions and main contributions of the research, along with recommendations for future research endeavors.

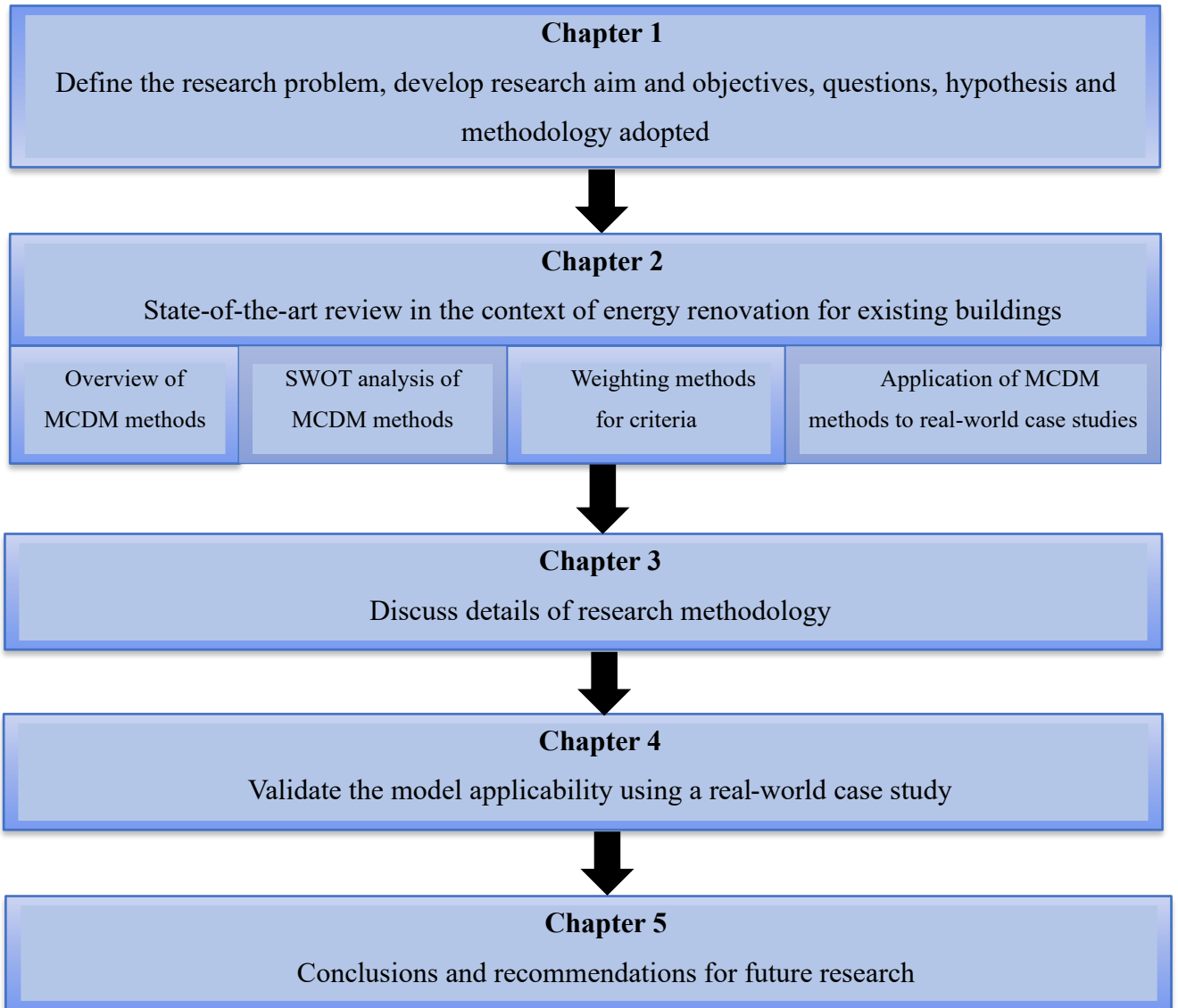


Figure 1.3. Thesis structure.

1.8 List of publications

The research presented in this PhD thesis has resulted in the following publications in peer-reviewed journals:

- Baseer, M., Ghiaus, C., Viala, R., Gauthier, N., Daniel, S. pELECTRE-Tri: Probabilistic ELECTRE-Tri Method—Application for the Energy Renovation of Buildings. *Energies* 2023, *16*, 5296. <https://doi.org/10.3390/en16145296>.
- Baseer, M., Ghiaus, C., & Kachalla, I. A. An Overview of Barriers and Potential Solutions for Energy Renovation of Existing Buildings. *Journal of Electrical Systems*, 2024, *20* (11s), pp.53-58.

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2 State-of-the-Art

This chapter presents the current state-of-the-art review of the most commonly used MCDM methods and their application in the context of energy renovation for existing buildings.

It focuses on the following key topics:

- Introduction to MCDM
- Overview of MCDM methods
- SWOT analysis of MCDM methods
- Weighting methods for criteria
- The application of MCDM to real-world case studies
- Rationale for the selection of the ELECTRE Tri method for the research and the revised Simos method for the criteria weights

2.1 Introduction to MCDM

Decision-makers in the energy sector often encounter complex challenges related to sustainability factors across economic, environmental, social, and technical dimensions [1]. Addressing these challenges involves evaluating trade-offs between conflicting and opposing criteria, which highlights the insufficiency of traditional single criterion models. The results provided by these methods can be measured in terms of a single attribute, such as money, or any other quantifiable criterion. The rationale behind employing single criterion models in decision analysis is to optimize the benefits of a specific criterion while minimizing associated costs. However, real-world decisions involve diverse attributes, aspects, and dimensions.

Therefore, tools are required that provide flexibility in managing a broad spectrum of factors to facilitate effective decision-making [2-4]. To fulfill this requirement, a tool is needed that integrates both quantitative and qualitative analysis in a scientific manner, rather than solely relying on intuition and experience. Consequently, integrating both quantitative and qualitative analysis in the energy sector is imperative. This approach enables making well-informed decisions and maximizing the efficient use of available resources [5, 6].

MCDM is a branch of operational research that aids decision-making in complex scenarios involving conflicting and opposing criteria. It facilitates the incorporation of quantitative and

qualitative analysis in a scientific manner [7-9]. The optimal solution is formulated by taking into account the available information and constraints specific to the decision-making context, rather than depending on the "true" result that certain engineering methods might yield. The effectiveness of an MCDM application is measured by the enhanced understanding of the decision problem and the acceptance of the final recommendation [10, 11].

MCDM based decisions are justifiable and transparent, supported by detailed documentation for traceability, leading to their widespread use in supporting sustainability assessment in the energy sector [12-14]. The optimal solution involves a trade-off among diverse energy and non-energy considerations including economic, environmental, social, technical, and regulatory aspects. Although the social aspect in sustainable buildings is often neglected, some studies have highlighted building features that enhance social sustainability [15, 16].

It is essential to emphasize that a building cannot excel in all sustainability factors simultaneously due to their inherent contradictions. Therefore, achieving sustainable buildings necessitates carefully balancing the impacts on these factors throughout the building life cycle. Sustainability assessments play a vital role in collecting and disseminating information to support decision-making [17-19].

2.1.1 Multiple objective and multiple attribute decision making

MCDM methods can be divided in two branches as presented in Table 2.1:

Table 2.1. Features of multiple objective and multiple attribute decision making [20].

Features	MODM	MADM
Criteria	Objectives	Attributes
Objectives	Explicit	Implicit
Attributes	Implicit	Explicit
Constraints	Explicit	Implicit
Alternatives	Implicit	Explicit
Number of alternatives	Infinite (large)	Finite (small)
Decision-makers control	Significant	Limited
Decision modeling paradigm	Process-oriented	Outcome-oriented
Relevance	Design/Search	Evaluation

- **Multiple objective decision making (MODM)**

In MODM, objectives refer to the desired goals that need to be maximized or minimized. An MODM problem involves optimizing multiple, often conflicting objectives simultaneously.

It typically involves an infinite number of alternatives, such as portfolio optimization or engineering design problems. Alternatives in MODM are not given directly at the beginning of the problem. Instead, a mathematical framework is given to define the set of alternatives, and to represent trade-offs between multiple criteria using objective and constraint functions. Typically, the mathematical framework is expressed through continuous functions.

The objective of MODM is to identify the optimal solution within the feasible set defined by design constraints. This class of problems is also called a continuous multi-criteria decision problem.

Several optimization models, such as goal programming, compromise programming, the constraint method, and fuzzy multi-objective programming, are employed to address MODM problems.

- **Multiple attribute decision making (MADM)**

In MADM, attributes refer to the measurable characteristics used to evaluate and compare different alternatives. MADM problems involve selecting the best alternative from a finite set of alternatives while considering multiple, usually conflicting and opposing attributes.

The decision attributes in MADM are discrete, and the objective is to classify, rank, or select the most preferred alternative by considering the trade-offs between the different attributes. The number of alternatives is finite and explicitly known from the beginning of the problem. This class of problems is also called a discrete multi-criteria decision problem.

In this thesis, the model developed addresses a type of problem whose characteristics are compatible with the concept of MADM (all alternatives and criteria are well-defined at the beginning of the decision-making process). The focus will solely be on MADM methods, making any further discussion on MODM unnecessary.

2.1.2 General stages of MCDM

The decision-making process can be outlined through the following steps as shown in Figure 2.1 [21-24]:

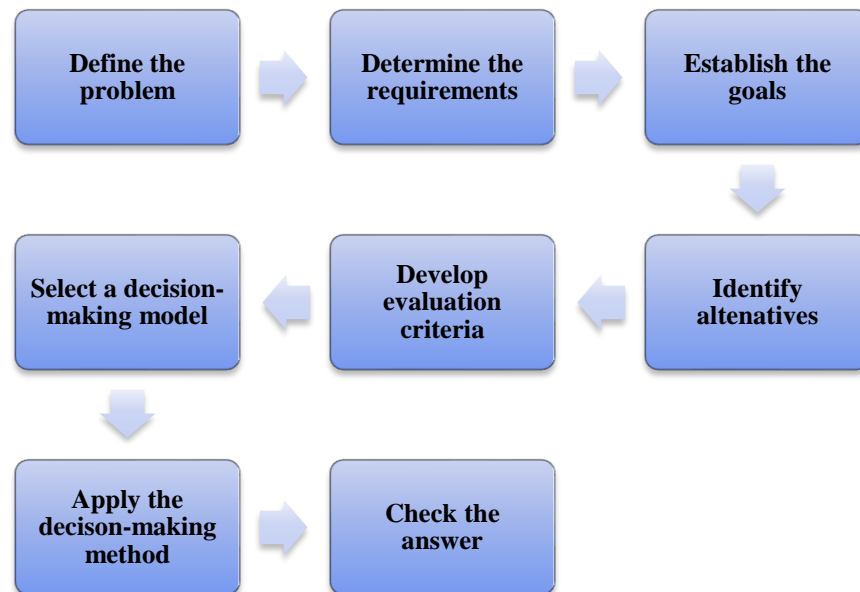


Figure 2.1. General stages of decision-making process [24].

Step 1. Define the problem

The initial step is to clearly articulate the issue by outlining both the current and desired conditions. While a concise statement is ideal, complex decision problems often necessitate a more detailed and comprehensive problem statement. It must be clear, concise, and mutually accepted by all stakeholders, even if achieving such consensus requires an iterative process.

At a minimum, the problem statement should identify the root causes, limiting assumptions, system and organizational boundaries, and any stakeholder concerns.

This step establishes the foundation for a well-defined decision-making process, ensuring that all stakeholders have a shared and thorough understanding of the problem.

Step 2. Determine the requirements

Requirements outline the prerequisites that must be fulfilled by any viable solution, defining the anticipated characteristics. They may cover both quantitative and qualitative aspects of the problem, providing clear standards for acceptability. In mathematical terms, requirements specify

the potential solutions to the decision problem, ensuring that solutions operate within defined boundaries.

In contrast, criteria are the measurable attributes used to evaluate and compare alternative solutions. These criteria may include both quantitative and qualitative variables, depending on the nature of the issue and stakeholders' preferences. While quantitative criteria involve measurable quantities, such as costs or performance metrics, qualitative criteria may encompass attributes such as usability, aesthetics, or social impact. Regardless of their nature, criteria act as the dimensions along which solutions are assessed to determine their effectiveness in meeting the established requirements.

To ensure clarity and avoid ambiguity, requirements should be expressed in exact quantitative terms whenever possible. This enables a definitive determination of whether a solution meets the requirements or not. By documenting both the requirements and the criteria, potential disputes can be avoided, and stakeholders can have a clear understanding of the basis for decision-making.

Thus, while requirements define the standards for acceptable solutions, criteria provide the means to assess how well each solution meets the established requirements, whether they involve quantitative or qualitative variables.

Step 3. Establish the goals

Goals are comprehensive statements expressing purpose and preferred programmatic principles. Goals exceed basic requirements and include preferences and aspirations related to the problem. In contrast to requirements, which impose constraints, goals act as objectives to strive toward. From a mathematical perspective, goals are similar to objectives, while requirements are more similar to constraints.

Understanding both requirements and goals is essential for defining alternatives effectively. Requirements establish the boundaries within which solutions must operate to meet basic standards, while goals provide the overarching aims and aspirations guiding the decision-making process.

Criteria, in turn, are measurable attributes used to evaluate and compare alternatives against both goals and requirements. Criteria can be classified into two main types: objective and subjective. Objective criteria are quantifiable and typically include factors such as cost, quality, reliability, and performance metrics. Subjective criteria, on the other hand, are more qualitative and may involve factors such as user satisfaction, environmental impact, or social responsibility.

It is essential to determine relevant criteria and evaluate their respective significance in the decision-making process.

Step 4. Identify alternatives

Alternatives are various options available for accomplishing a specified goal. These alternatives represent different courses of action that can be selected to transform the current condition into the desired condition. Whether an alternative already exists or exists solely in the mind, each alternative must fulfill the necessary requirements [25].

In cases where there are a finite number of possible alternatives, each one can be evaluated individually to determine if it satisfies the requirements. Any infeasible alternatives are eliminated from further consideration, resulting in a clear list of viable options.

In scenarios with an infinite number of potential alternatives, the set of alternatives is defined by the solutions that meet the constraints outlined in the mathematical representation of the requirements.

Step 5. Develop evaluation criteria

Evaluation criteria are the specific attributes used to assess and compare the alternatives under consideration. These criteria provide the basis for evaluating the performance of each alternative in achieving the goals and fulfilling the requirements of the decision problem. It is essential to establish a distinct set of criteria that is carefully balanced to reflect the future performance of the project. The evaluation, if possible, should allow for a comprehensive order or ranking to facilitate comparisons.

Criteria identification should fulfill the following requirements [26-28]:

- Be relevant to the objective of the problem, reflecting the important aspects and issues to be addressed.
- Be non-redundant and non-overlapping, avoiding the reflection of the same concept to avoid double counting.
- Be clear and explicit to ensure simple understanding and interpretation.
- Discriminate among the solutions by prioritizing them based on their importance, with some holding more weight than others.
- Be independent of each other, so that the variation of one criterion does not directly influence another criterion.
- Be flexible enough to accommodate potential variations in decision-makers preferences.

- Be limited in number to ensure the dimensions of the problem remain manageable.

Step 6. Select a decision-making method

The selection of a suitable decision-making method is influenced by various factors:

- The suitability of various MCDM methods is often influenced by the complexity of the decision problem. When the problems are relatively simple with few alternatives and factors, simple methods might be adequate. However, for more complex problems with various alternatives and criteria, more advanced methods may be more appropriate.
- The data availability and quality play significant roles in selecting the appropriate MCDM method. Some methods require precise data for criteria weights and alternative evaluations. If the data is readily available and reliable, these methods can be effectively applied. However, in situations where data is uncertain, it may be necessary to employ MCDM approaches that can handle imprecision, vagueness, or qualitative assessments from stakeholders.
- Stakeholder preferences and decision problem context also influence the selection of the MCDM method. The decision context, including the level of uncertainty, risk tolerance, and stakeholder involvement, may favor certain methods over others.
- Methods that provide transparent and interpretable results facilitate understanding and consensus-building among stakeholders, whereas complex methods with opaque algorithms or subjective judgments may be perceived as less credible.
- The ability to conduct sensitivity analysis and evaluate the robustness of the decision outcomes is another factor to consider in method selection. Some MCDM methods allow for sensitivity analysis to examine the impact of changes in criteria weights or alternative evaluations on the final decision rankings. Robust methods that are less sensitive to variations in input parameters may be preferred in situations where uncertainty or variability is high.

Step 7. Apply the decision-making method

To facilitate well-informed decisions, it is essential to assess the alternatives according to the given criteria. The assessment can be objective, based on widely accepted measurement scales, such as money or weight. On the other hand, subjective assessment presents more challenges and can be expressed in three distinct ways [29]:

- An ordinal ranking represented by numbers (e.g., 1, 2, 3, ...) or symbols (e.g., + + +).
- A nominal scale mirroring the attributes of choices represented by color types.
- A binary scale comprising solely two responses, represented by a "Yes" or "No".

Once the evaluations are completed, the method can be used to select, rank or classify the alternatives.

Step 8. Check the answers

The alternatives identified through the decision-making methodology need to be validated to ensure they comply with the requirements and goals of the decision problem. It is essential to ensure the accurate application of the decision-making method.

When handling complex issues, additional goals or requirements may need to be incorporated into the decision-making method.

Figure 2.2 outlines the step-by-step process, demonstrating how the process can be divided into three main sections (the three circles on the left):

A. Problem structuring: Data is gathered and managed according to the specific features of the problem to facilitate its definition. Steps 1 to 3 are included in this section.

B. Model building: This involves selecting the decision-making model, incorporating data from previous stages. Steps 4 to 6 are included in this section.

C. Using the model to inform and challenge the thinking: Once results are obtained, it is essential to evaluate and decide on the solution based on selection/ranking/classification. Steps 7 to 8 are included in this section.

Identifying the problem marks the beginning of the process, while outlining an action plan signifies its end. The downward arrows represent the process flow, indicating the sequence of steps/phases. In contrast, dotted arrows are visible in the opposite direction, denoting the continual adjustments that could (and should) be made throughout the process to enhance and consequently achieve superior outcomes.

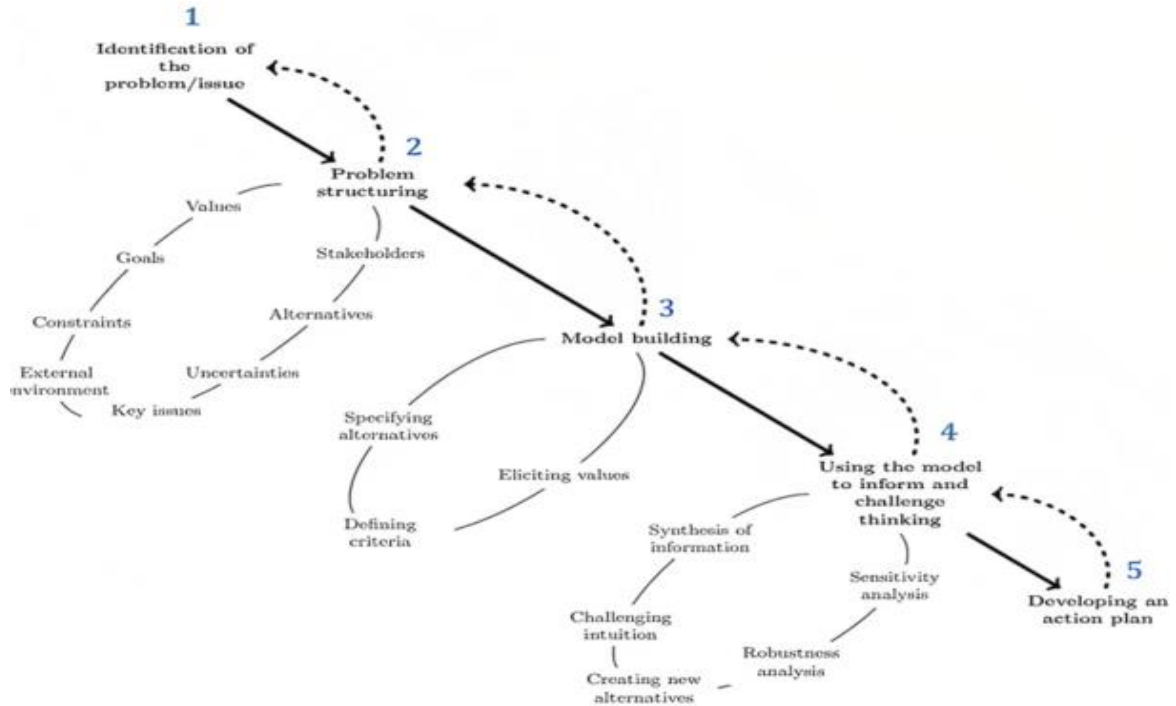


Figure 2.2. Step by step procedure of decision-making process [30].

2.2 Overview of MCDM methods

There is a broad spectrum of MCDM methods, each with its own theoretical framework and perspective. Several methods have also generated multiple variants to address new issues or overcome specific limitations. Moreover, hybrid approaches integrate two or more methods to leverage their strengths and address complex decision-making problems more effectively.

MCDM has been developed to address four types of problems: choice, ranking, sorting, and description problems as shown in Figure 2.3. The first three types of problems involve evaluating alternatives in order to reach a specific outcome. On the other hand, description problems are encountered in situations where the alternatives and their unique features are described to characterize the problem [31-33].

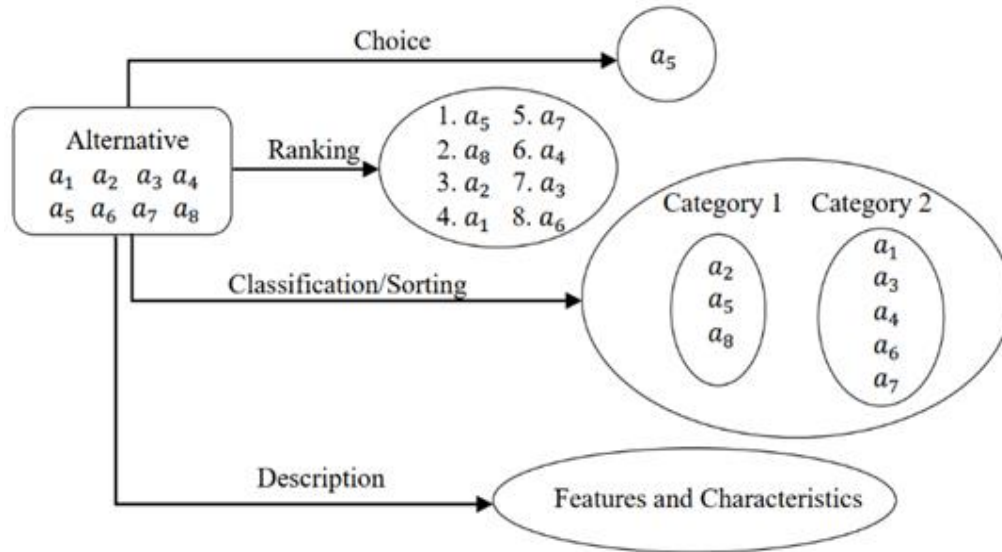


Figure 2.3. Types of MCDM problems [34].

- In a **choice (or selection)** problem, a choice has to be made between different alternatives evaluated on multiple attributes. The result of the MCDM helps in determining the optimal alternative from all existing alternatives. For example, developing an energy plan for a country, selecting an energy contract for a residential house.
- In a **ranking (or prioritization)** problem, the alternatives are ordered in a sequence based on preference, from the most preferred to the least preferred. There may be cases where two or more alternatives are considered equally preferable. These rankings can be either partial or full, depending on whether ties between rankings are allowed or not. For example, country rankings based on energy sustainability, ranking different energy-efficient building designs for a house.
- In a **sorting (or classification)** problem, alternatives are categorized into predefined groups, which are ranked from the least preferred to the most preferred. The result of such methods is the allocation of alternatives to these specific categories. These categories are distinct from each other and defined by boundaries. For example, the classification of energy alternatives into three categories: high, medium, and low.

- The **description** problem involves elaborating on the alternatives, establishing a set of criteria, and assessing the performance of all or certain alternatives while considering additional relevant information.

Since the 1970's, MCDM research has experienced significant development, resulting in its widespread application in various fields such as environmental management [35], energy sector [36], management science [37, 38], computer science and artificial intelligence [39], health care [40], and transportation [41].

2.2.1 School based classification of MCDM methods

Two major schools of thought have emerged in MCDM, the American school of thought and the European school of thought, each characterized by distinct approaches to handling decision problems: the aggregation-based approach and the outranking-based approach, respectively.

In the American school of thought, spearheaded by Keeney and Raiffa, the primary focus is on aggregation-based methods. These methods emphasize the quantification and aggregation of criteria using utility functions or value functions to determine the overall desirability of alternatives. Aggregation-based methods combine individual criteria evaluations into a comprehensive assessment of alternatives.

In the European school of thought, pioneered by Bernard Roy, the primary focus is on outranking-based methods. These methods evaluate the relative superiority of alternatives without aggregating criteria into a single score. Outranking methods involve the pairwise comparison of alternatives to identify those that outrank others based on predefined criteria [42-44].

2.2.1.1 American school of thought

The methods involved in the American school of thought assign numerical values to alternatives by assessing each criterion and then synthesizing them into a global score. These methods are based on compensatory preferences, allowing low performance on one criterion to be offset by high performance on another. Consequently, this approach simplifies the evaluation and comparison of all available alternatives.

Typically, these approaches incorporate the preferences of decision-makers through criteria weights or value/utility functions. The set of methodologies is closely associated with a value-focused approach that aims to maximize an overall value/utility function.

Some of the commonly used MCDM methods under the American school of thought are explained below.

2.2.1.1.1 Multi-Attribute Utility Theory (MAUT)

The primary hypothesis of multi-attribute utility theory is that alternatives are assessed by evaluating their performance on multiple criteria and then integrating these evaluations into an overall utility or value function [45, 46].

In the preliminary phases of the process, the precise nature of this function might not be evident, necessitating its definition. The utility function of a measurement reflects preferences concerning different attribute levels while considering uncertainties through probabilities and expectations.

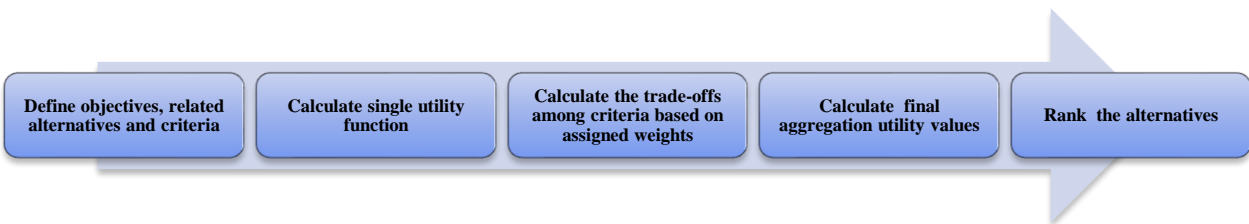


Figure 2.4. Steps of the MAUT method.

The MAUT method involves the following steps as shown in Figure 2.4:

Step 1: The relevant criteria are identified and defined, which are essential for evaluating alternatives. The selection of criteria is necessary and often involves input from experts and stakeholders to ensure a comprehensive analysis of the decision problem.

Step 2: Preferences for different levels of each attribute are expressed through utility functions, which assign numerical values, often on a scale, to represent the satisfaction associated with each level of an attribute. These utility functions capture subjective preferences and trade-offs between different attribute levels.

Typically, these utility functions exhibit an increasing trend since the common objective is often maximization (e.g., higher energy output corresponds to a higher utility score; increased energy efficiency results in a higher utility score). However, there are cases where decreasing utility functions are employed, especially when the objective is minimization (e.g., as in the energy consumption or cost of a product).

Step 3: The process involves assessing and understanding the trade-offs that must be considered when evaluating alternatives based on multiple criteria.

By quantifying preferences through utility functions, it becomes possible to explicitly evaluate and analyze these trade-offs, assess the impact of improving one attribute on the overall utility, and consider the implications of prioritizing certain attributes over others.

Step 4: The utility scores across all criteria are aggregated to determine the overall utility of each alternative. This aggregation process involves combining individual utility scores using mathematical techniques.

Aggregation ensures a comprehensive evaluation of alternatives that considers multiple dimensions of performance simultaneously.

Step 5: The overall utility scores enable decision-makers to rank or prioritize alternatives and select the most preferred one that maximizes the overall utility. This ranking provides valuable insights into the relative performance of each alternative across multiple criteria, enabling the selection of the most desirable alternatives.

In this method, the issue of incomparability is non-existent, as utility scores are inherently comparable and transitive:

- If alternative A is favored over alternative B, and alternative B is favored over alternative C, it follows that alternative A is favored over alternative C.
- If alternative A and alternative B have equal scores, A and B are considered indifferent to each other.

Sensitivity analysis allows decision-makers to evaluate the robustness of the decision outcomes by varying input parameters such as criteria weights or utility functions. This helps identify critical factors that significantly influence the final recommendations. By examining how the results change with variations in inputs, it becomes possible to better understand the uncertainty associated with decisions and make more informed and robust decisions.

2.2.1.1.2 Analytic Hierarchy Process (AHP)

The analytic hierarchy process is a widely recognized MCDM method formulated by Saaty [47]. In AHP, comparative judgment involves pairwise comparisons between all elements at a particular level of the hierarchical structure [48].

Pairwise comparisons are often preferred over evaluating multiple alternatives simultaneously.

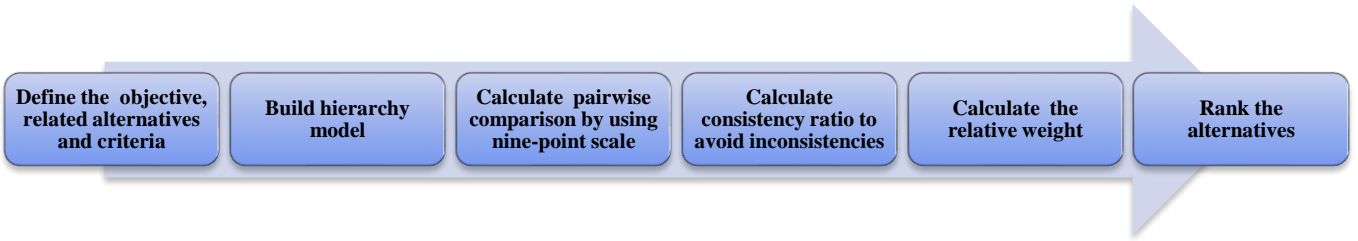


Figure 2.5. Steps of the AHP method.

The process of addressing a problem using the AHP method can be divided into five distinct steps as shown in Figure 2.5:

Step 1: This step involves identifying the objective, selecting the most relevant alternatives, and defining the evaluation criteria to achieve the objective effectively.

Step 2: A model is developed to analyze the essential elements of a problem in a hierarchical manner such as decision alternatives, attributes, and the goal.

The hierarchical structure used in the AHP methodology has a three-level structure: an overall goal, criteria/attributes, and alternatives/options as shown in Figure 2.6.

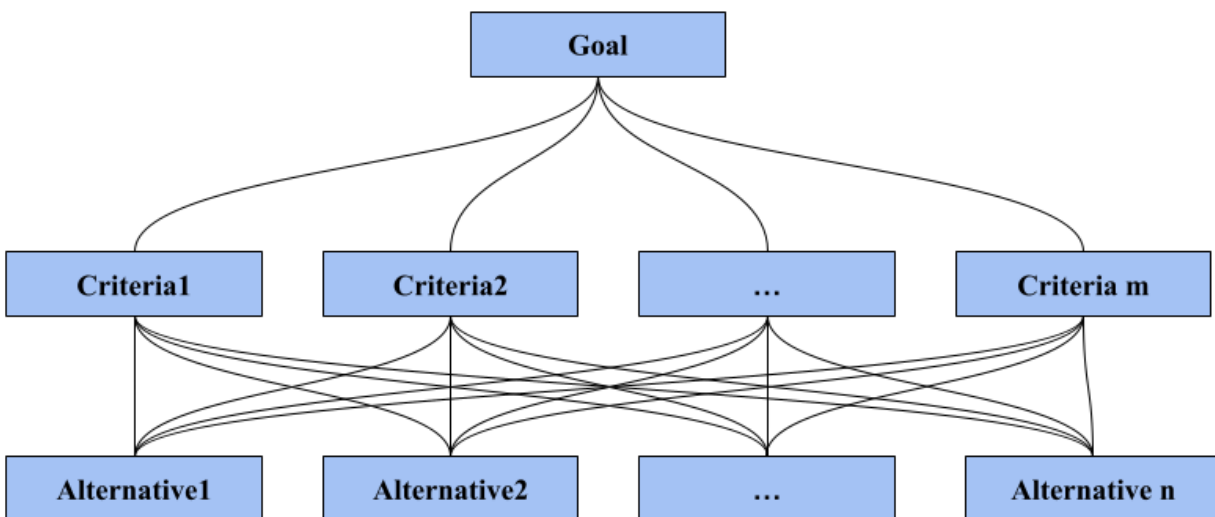


Figure 2.6. AHP basic hierarchical structure [49].

Step 3: The pairwise comparison employs a mathematical framework developed to conduct a pair comparison of each criterion relative to another criterion. Comparisons between criteria are conducted in pairs with respect to the project goal.

A pairwise comparison matrix (A) is established, in which each entry a_{ij} in the matrix A is computed by comparing the row element a_i with the column element a_j [50].

Pairwise comparisons are usually evaluated on the fundamental 1 to 9 scale as presented in Table 2.2. The nine-point scale is employed to transform the decision-makers' judgments into numerical quantities that represent the values of a_{ij} .

Table 2.2. AHP pairwise comparison scale.

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective.
3	Moderate importance	Experience and judgment slightly favor one activity over another.
5	Strong importance	Experience and judgment strongly favor one activity over another.
7	Very strong importance	An activity is strongly favored over another; its dominance is clearly demonstrated in practice.
9	Extreme importance	The evidence affirming one activity over another is of the highest possible order.
2,4,6,8	Intermediate values between the two adjacent judgments	When compromise is needed.

Once the pairwise comparison matrix (also called the judgmental matrix) has been formed, the local priorities of the alternatives with respect to each criterion are obtained.

The local priority weights are typically derived by employing the eigenvector method, which involves determining the principal eigenvector of the judgmental matrix.

Step 4: This step involves assessing the consistency of the judgments provided in Step 3. Even though the aim is to be consistent, a certain level of inconsistency may still arise due to the subjective nature of the judgments.

To minimize inconsistencies, a consistency ratio (CR) is computed to measure the degree of discrepancies in judgment. The consistency level for each matrix is determined.

The formula for calculating the consistency ratio is

$$CR = CI / RI \quad (2.1)$$

where:

$CI = (\lambda_{max} - n) / (n - 1)$ - consistency index;

λ_{max} - principal eigen value of the pairwise comparison matrix;

n - number of alternatives being compared;

RI - random index, a predefined value obtained from the dimensions of the pairwise comparison matrix, as specified by Saaty.

If the consistency ratio is 0.10 or below, the results are considered valid, otherwise the results are inconsistent and need to be recalculated until a satisfactory level of consistency is achieved

Step 5: Finally, the alternatives' overall performance scores are calculated by multiplying each criterion's relative normalized weight (w_i) by the corresponding normalized performance score for each alternative and subsequently summing these products across all criteria for each alternative.

2.2.1.1.3 Analytic Network Process (ANP)

The analytic network process is an extension of the analytic hierarchy process, developed by Thomas L. Saaty, enabling the modeling of complex decision problems with interdependencies and feedback loops among alternatives and criteria [51, 52].

ANP demonstrates how each element in the system is interconnected, showing their influence and control over one another. In contrast to a simple hierarchical configuration, ANP effectively illustrates the interconnections among the elements as shown in Figure 2.7.

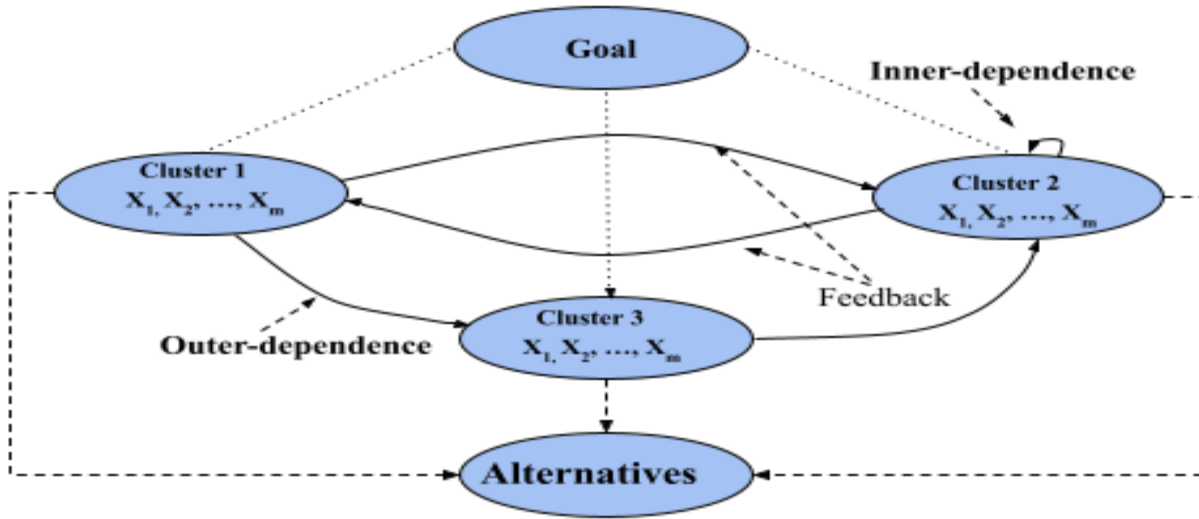


Figure 2.7. Typical ANP framework and process [53].

ANP involves two distinct layers: the upper layer acts as a control mechanism, while the lower layer acts as a network framework. Typically, the control layer includes objectives, while the network layer consists of clusters and their elements. The control layer governs the clusters and indicators within the network layer.

The clusters and internal indicators mutually influence each other, establishing the network structure. It is important to highlight that a fundamental ANP framework necessitates a "goal," two or more "clusters," and the "elements" of these clusters [54].

ANP requires the construction of more complex and elaborate structures for problem analysis, often demanding a substantial investment of time and effort.

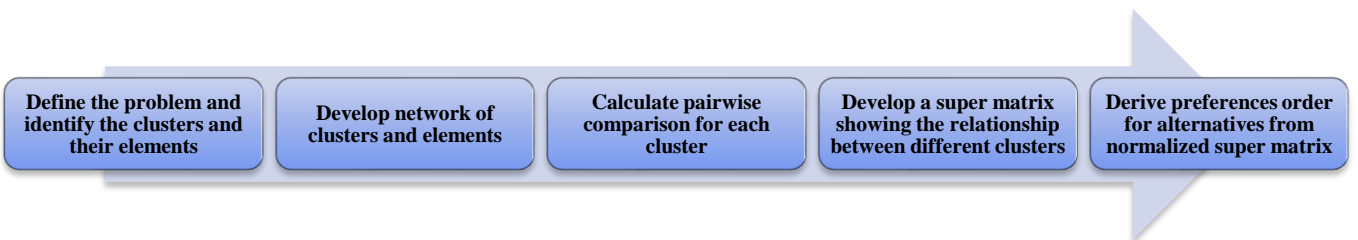


Figure 2.8. Steps of the ANP method.

The ANP method involves the following steps as shown in Figure 2.8:

Step 1: Initially, the model is structured into a network showing the various elements and their interconnections, establishing the foundation for further analysis.

Step 2: Each cluster within the network is subjected to pairwise comparison to determine its relative importance. Additionally, pairwise comparisons are conducted among criteria within each cluster. This process facilitates a detailed comprehension of the significance of each cluster and criterion.

Step 3: A super matrix is developed based on the network structure. This super matrix reflects the relationships between different clusters within the system. It acts as a comprehensive representation of the interdependencies and influences among the various elements.

Step 4: Finally, the preference order for alternatives is derived from the normalized super matrix. This synthesis process integrates the insights gained from the previous steps to establish a coherent and prioritized ranking of alternatives based on their suitability and alignment with the identified elements and clusters.

2.2.1.1.4 Weighted Sum Method (WSM)

The weighted sum method is the most commonly used technique in practice, especially for single dimensional problems [19, 55].

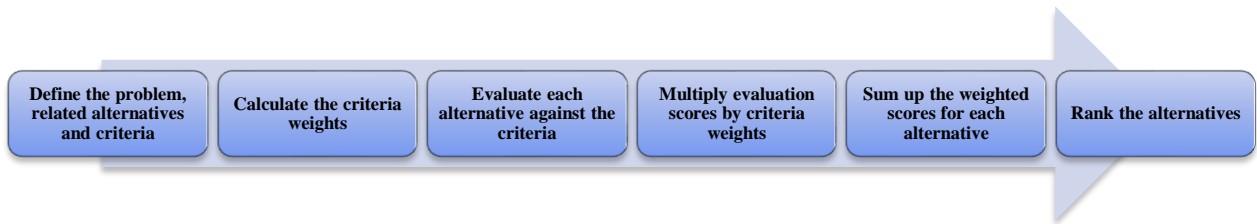


Figure 2.9. Steps of the WSM.

In single-dimension cases, where all performance scores are measured in the same unit, the WSM can be applied effectively to rank alternatives by calculating a global score through the weighted arithmetic mean as shown in Figure 2.9.

$$A_{WSM-score} = \sum_{j=1}^n a_{ij}w_j \quad (2.2)$$

where:

$A_{WSM-score}$ - weighted sum score of each alternative;

a_{ij} - value of i^{th} alternative in terms of the j^{th} criterion;

w_j - weight of the j^{th} criterion;

n - number of decision criteria.

Challenges arise when employing this method for decision-making problems involving multiple dimensions. The assumption of additive utility is violated when integrating various dimensions and consequently different units.

The WSM examines all the criteria values of an alternative and employs basic arithmetic operations of addition and multiplication. Furthermore, it is essential that the criterion values and their respective weights are both quantifiable and commensurable. This method depends on the fundamental principle that criteria are independent of each other, implying that the impact of one criterion on the overall score is not influenced by other criteria values.

In situations where there are interactions between different criteria, employing the simple additive weighted method might result in a biased result [56, 57].

2.2.1.1.5 Weighted Product Method (WPM)

The weighted product method employs multiplication measures instead of addition to rank the alternatives as shown in Figure 2.10.

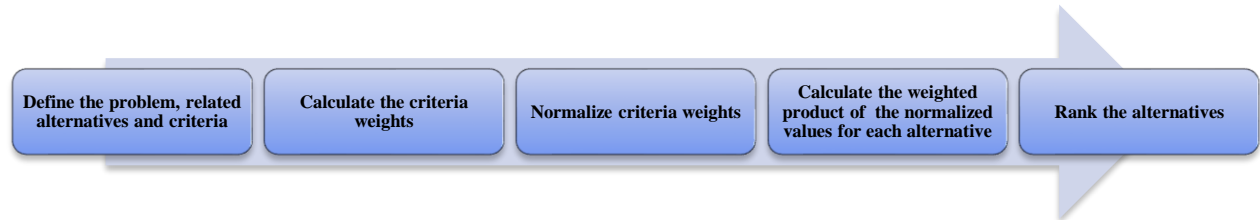


Figure 2.10. Steps of the WPM.

The WPM was proposed to address the multi-dimensional challenges associated with the WSM and can be applied to both single and multi-dimensional problems [58].

$$S(A_k/A_i) = \prod_{j=1}^n (A_{kj}/A_{ij})^{w_j} \quad (2.3)$$

where:

$S(A_k/A_i)$ - pairwise weighted product of alternatives criterion comparison ratios;

A_{kj} - value of the k^{th} alternative in terms of the j^{th} criterion;

A_{ij} - value of the i^{th} alternative in terms of the j^{th} criterion;

w_j - weight of the j^{th} criterion;

n - number of criteria.

If $S(A_k/A_i)$ exceeds one, it implies that alternative A_k is preferable to alternative A_i .

The benefit of this method lies in its dimensionless nature, enabling the use of relative values instead of measured ones.

WPM has a limitation in its restriction to ranking alternatives pair by pair. To address this issue, a variation of this method involves solely considering the use of products, without incorporating the ratios [59]. In this variation, the total performance of alternative A_k , when all the criteria are considered is

$$P(A_k) = \prod_{j=1}^n (a_{kj})^{w_j} \quad (2.4)$$

where $P(A_k)$ measures the total performance or score of alternative A_k when all the criteria are considered, compared to the normal version $S(A_k/A_i)$ where the relative total performance between two pairs is measured.

2.2.1.1.6 Techniques for Order Preference by Using Similarity to Ideal Solution (TOPSIS)

The techniques for order preference by using similarity to ideal solution method was originally developed by Hwang and Yoon [60]. In TOPSIS, each of the n alternatives evaluated under m criteria is considered a point in an m -dimensional space. It depends on the fundamental assumption that each criterion can be assessed under monotonically increasing or decreasing utility functions.

As a result, when provided with a performance data set, it is relatively simple to define a positive ideal solution (PIS) and a negative ideal solution (NIS). The ideal alternative should be located closest to the PIS and farthest from the NIS, based on geometrical distance measures.

To measure each alternative's relative proximity to the PIS and NIS, a Euclidean distance approach is used.

Consider a MCDM problem having m alternatives (A_1, A_2, \dots, A_m) , and n criteria (C_1, C_2, \dots, C_n) , the decision matrix is

$$D_{n,m} = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{bmatrix}$$

where the elements of the matrix x_{ij} represent the performance of alternative i in terms of the criterion j .

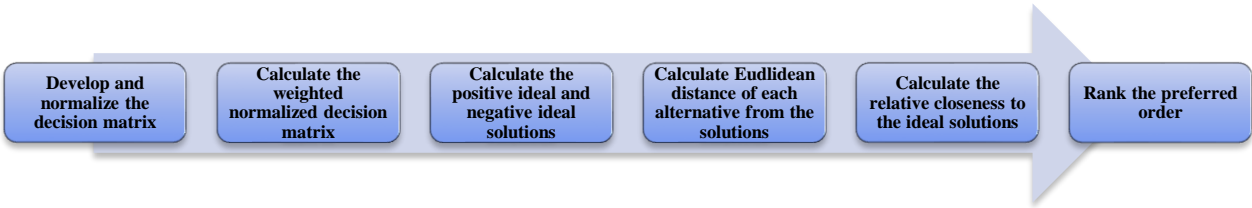


Figure 2.11. Steps of the TOPSIS method.

The TOPSIS method consists of the following steps as shown in Figure 2.11:

Step 1: Calculate the normalized decision matrix.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^n x_{ij}^2}} \quad (2.5)$$

where $i = 1, 2, 3, \dots, m$

$j = 1, 2, 3, \dots, n$

The normalization process is carried out to facilitate comparison by converting various attributes into a standardized unit.

Step 2: Calculate the weighted normalized decision matrix by multiplying the normalized decision matrix with the assigned weights for each criterion.

$$v_{ij} = w_j * r_{ij} \quad (2.6)$$

Step 3: Calculate the positive ideal solution (A^+) and negative ideal solution (A^-).

Let C^+ be the subset of positive criteria (e.g., benefits, gains), and C^- be the subset of the negative criteria (e.g., cost, losses), then

$$\begin{aligned}
A^+ &= \{v_1^+, v_2^+, \dots, v_m^+\} \\
&= \{(max v_{ij}, |C_j \in C^+)\}, \{(min v_{ij}, |C_j \in C^-\}\}
\end{aligned} \tag{2.7}$$

$$\begin{aligned}
A^- &= \{v_1^-, v_2^-, \dots, v_m^-\} \\
&= \{(min v_{ij}, |C_j \in C^+)\}, \{(max v_{ij}, |C_j \in C^-\}\}
\end{aligned} \tag{2.8}$$

These two alternatives could be considered hypothetical since they do not form part of the set of alternatives under consideration. One represents the Positive Ideal Solution (PIS) that is desired to achieve, while the other exemplifies the Negative Ideal Solution (NIS) that is intended to avoid.

Step 4: Calculate Euclidean distance of each alternative from PIS and NIS.

$$d_i^+ = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^+)^2} \tag{2.9}$$

and

$$d_i^- = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^-)^2} \tag{2.10}$$

for $i = 1, 2, 3, \dots, n$

Step 5: Calculate the relative closeness to the ideal solution.

$$c_i = \frac{d_i^-}{d_i^+ + d_i^-} \tag{2.11}$$

The numerical value of c_i varies between 0 and 1, with a value of 1 representing the maximum nearness of the relevant decision point to the PIS, and a value of 0 representing the maximum nearness of the relevant decision point to the NIS.

Step 6: Rank alternatives based on their closeness values and obtain the final preference rank order.

2.2.1.1.7 Vlekriterijumsko KOpromisno Rangiranje (VIKOR)

The Vlekriterijumsko KOpromisno Rangiranje method provides a compromise ranking list, compromised solution, and weight stability intervals to ensure preference stability.

This method is developed to manage conflicting criteria, facilitate in ranking, and assist in selecting among various alternatives. It introduces a multi-criteria ranking index that assesses proximity to the optimal solution, particularly beneficial in scenarios where there is uncertainty or difficulty in expressing preferences during the initial phases of the problem [61].

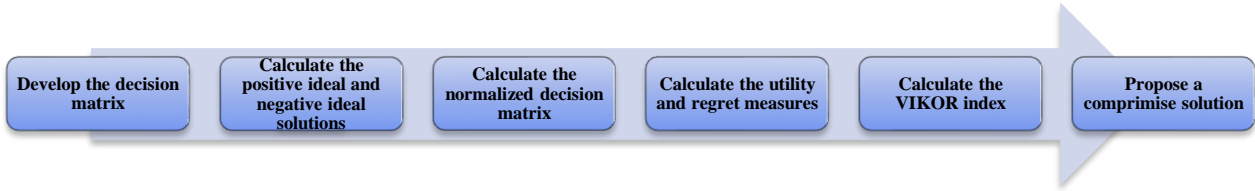


Figure 2.12. Steps of the VIKOR method.

The VIKOR method consists of the following steps as shown in Figure 2.12:

Step 1: Calculate the best x_j^* and the worst x_j^- values of all criterion functions.

$$x_j^* = \max_j x_{ij} \quad (2.12)$$

$$x_j^- = \min_j x_{ij} \quad (2.13)$$

where $j = 1, 2, 3, \dots, n$

Once these values are obtained, normalization is calculated.

$$r_{ij} = \frac{x_j^* - x_{ij}}{x_j^* - x_j^-} \quad (2.14)$$

Step 2: Calculate the values of S_j and R_j where $i = 1, 2, 3, \dots, m$.

$$S_j = \sum_{i=1}^n w_j \frac{x_j^* - x_{ij}}{x_j^* - x_j^-} \quad (2.15)$$

$$R_j = \max \left[w_j \left(\frac{x_j^* - x_{ij}}{x_j^* - x_j^-} \right) \right] \quad (2.16)$$

where:

S_j - utility measure for the alternative x_j ;

R_j - regret measure for the alternative x_j ;

w_j - weight of the j^{th} criterion.

Step 3: Calculate the value of S^* and R^* .

$$S^* = \min (S_j), S^- = \max (S_j) \quad (2.17)$$

$$R^* = \min (R_j), R^- = \max (R_j) \quad (2.18)$$

where $j = 1, 2, 3, \dots, m$

Step 4: Calculate the value of Q_j and rank the alternatives by the values of Q_j .

$$Q_j = v \left(\frac{S_j - S^*}{S^- - S^*} \right) + (1 - v) \left(\frac{R_j - R^*}{R^- - R^*} \right) \quad (2.19)$$

where:

v - weight for the strategy of maximum group utility;

$(1 - v)$ - weight of individual regret. The usual setting for v is "0.5" [62].

Step 5: Sort the values S , R and Q in descending order and rank the alternatives accordingly, resulting in three ranking lists.

Step 6: The alternative a_1 is ranked as the best solution by the measure Q_j if the following two conditions are satisfied:

Condition 1. "Acceptable advantage":

$$Q(a_2) - Q(a_1) \geq \frac{1}{m-1} \quad (2.20)$$

where:

a_2 - second-best option according to Q_j ;

m - number of alternatives.

Condition 2. "Acceptable stability in decision making":

Alternative a_1 should also be ranked the best according to value R and/or S . The solution is considered stable if the value of v for the case:

- "Ruling by majority" is larger than 0.5;
- "Voting by consensus" is equal to 0.5;
- "Considering veto" is smaller than 0.5.

A set of compromise solutions is suggested if any of these conditions are not met.

- If condition 1 is satisfied and condition 2 is not satisfied, then both a_1 and a_2 are proposed as comprised solutions.
- If condition 1 is not satisfied, then a set of alternatives a_1, a_2, \dots, a_m is proposed as the comprised solutions and represented as

$$Q(a_m) - Q(a_1) < \frac{1}{m-1} \quad (2.21)$$

The VIKOR method was subsequently expanded by Obricovic and Tzeng in order to assess the stability of the weights [62]. This expansion includes a technique for determining the stability interval of the weights and a process for reaching a compromise in situations where decision-makers have differing opinions.

2.2.1.2 European school of thought

In the European school of thought, the solution aligns more closely with human reasoning compared to the American school of thought. The methods involved in the European school of thought do not necessitate assigning specific numerical values or utility functions to the alternatives. The fundamental premise is that decision-makers assess the outranking relation that "*alternative A is at least as good as alternative B*" for each pair of alternatives.

These methods use an outranking relational model that captures the inherent imprecision and uncertainty in human judgments, which is often difficult to represent using numerical values or utility functions. This outranking relationship involves pairwise comparisons of alternatives to assess the extent to which one alternative is superior to another based on predefined criteria, resulting in a complete or partial ranking of the alternatives.

Some of the commonly used MCDM methods under the European school of thought are explained below.

2.2.1.2.1 Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE)

The preference ranking organization method for enrichment evaluations, initially introduced by Brans for ranking and selecting a finite set of options, was later extended by Brans and Vincke [63]. This method evaluates a limited number of predetermined alternatives against multiple criteria, assigning a specific weight to each criterion to reflect its significance.

An appropriate preference function is selected to express the level of preference for different evaluations, measuring the relative preference for one alternative over another based on their evaluations of a particular criterion. This preference function translates the difference into a preference degree, ranging from 0 (no preference) to 1 (strict preference) [64].

Since its introduction, the PROMETHEE family has evolved into six distinct variants, each developed to address specific MCDM problems.

PROMETHEE I

PROMETHEE I was initially developed for the purpose of partial ranking of alternatives by considering the preference, indifference and incomparability relationships among them. Partial ranking involves ties or incomparability, as opposed to a complete preorder or total order [65].

PROMETHEE II

PROMETHEE II was developed to provide a comprehensive ranking of the alternatives through the calculation of the net flow, which summarizes the overall preference of one alternative compared to all others. The net flow indicates the disparity between the outgoing and incoming flows.

- Outgoing flow indicates the degree to which an alternative a_i is preferred over the other alternatives.
- Incoming flow indicates the degree to which the other alternatives are preferred over alternative a_i .

A higher net flow indicates a more favorable alternative, providing a clear and effective means of identifying the optimal choice.

The PROMETHEE II method exhibits a characteristic referred to as "*compensation*", where superior performance on one criterion by an alternative may offset its inferior performance on another criterion. This phenomenon arises due to PROMETHEE II utilization of a weighted sum approach to combine the criteria, facilitating trade-offs among them. The rationale behind this compensation behavior is the way PROMETHEE II assigns' weights to each criterion, which determines the relative significance of each criterion in the final ranking. Consequently, a high weight assigned to a particular criterion has the potential to offset low performance on another criterion with a lower weight.

To address the issue of criteria compensation, the PROMETHEE for Sustainability Assessment (PROSA) method was developed as an extension of PROMETHEE II. PROSA eliminates the compensatory nature between criteria by employing a non-compensatory aggregation method. This ensures equal consideration of all criteria in the final ranking, promoting a more equitable and sustainable decision-making process [66, 67].

PROMETHEE III

PROMETHEE III was developed to enhance the perception of indifference by determining alternative rankings using overlapping intervals derived from interval flows. PROMETHEE I and II methods typically depend on a single threshold value to represent indifference among alternatives. However, these methods may have limitations since they might not fully capture the perception of indifference, which is more accurately represented by a range or interval instead of a single value.

In PROMETHEE III, alternatives are ranked based on preference and indifference intervals, ensuring a more accurate evaluation. The interval is calculated using the mean value of the net flows, with the upper and lower bounds of indifference established by adding and subtracting the standard deviation of the net flows multiplied by a coefficient γ . The choice of the coefficient γ is influenced by the application to ensure that it does not hinder the differentiation among alternatives.

PROMETHEE III presents an uncertain estimate of the net flows, whereas PROMETHEE II can be considered a specific case of PROMETHEE III with $\gamma = 0$ [65, 68, 69].

PROMETHEE IV

PROMETHEE IV was developed to fully or partially prioritize alternatives in situations featuring a continuous range of feasible options, as opposed to discrete ones. The primary

enhancement of PROMETHEE IV, compared to earlier variants, is its ability to thoroughly prioritize a continuous set of alternatives.

While PROMETHEE I, II, and III were effective with discrete sets of alternatives, real-world scenarios often involve continuous sets of alternatives. The earlier versions of PROMETHEE method were inadequate for scenarios with continuous alternatives due to their inability to fully or partially prioritize them. PROMETHEE IV addresses this limitation by expanding the approach to handle continuous decision spaces.

It enables ranking alternatives within a continuous range, resulting in a more comprehensive evaluation and ranking of viable options. This characteristic makes PROMETHEE IV particularly valuable for optimization and resource allocation problems involving continuous set of alternatives [68].

The original developers of the initial four PROMETHEE techniques introduced a visual interactive tool known as GAIA. This method builds upon the concepts of the previous four techniques while emphasizing graphical visualization.

PROMETHEE V

PROMETHEE V was developed with the objective of optimizing the overall outranking of alternatives to find the best subset of alternatives from a discrete set while considering constraints, such as segmentation constraints, to maximize the net outranking flow.

The set of alternatives is not simply listed, but rather categorized into clusters or segments. Moreover, constraints exist that need to be satisfied within each cluster or among different clusters. The initial versions of PROMETHEE method did not have the capability to handle such segmentation constraints, treating the set of alternatives as an unstructured list without considering clustering constraints. PROMETHEE V methodology addresses problems involving segmented alternatives and additional constraints.

It enables decision-makers to:

- Define clusters or segments within the alternative set.
- Specify constraints that must be satisfied within each cluster (e.g., maximum number of alternatives available within a cluster).
- Specify constraints that must be satisfied between different clusters (e.g., minimum number of alternatives available within a group of clusters).

PROMETHEE V employs an optimization approach to identify the optimal subset of alternatives that satisfies all the specified constraints while optimizing the PROMETHEE outranking flows (which measure the preference of one alternative over others based on multiple criteria). In the absence of PROMETHEE V, segmentation constraints would either have to be ignored (resulting in suboptimal solutions) or addressed using alternative, complex optimization techniques.

By integrating segmentation constraints directly into the PROMETHEE framework, PROMETHEE V provides a more comprehensive and practical solution for complex multi-criteria decision making problems involving clustered alternatives and additional requirements [70].

PROMETHEE VI

PROMETHEE VI introduces a methodology to represent the "*human brain*" in the decision-making process by incorporating fuzzy preferences and handling incomplete information. The key limitation of earlier versions of PROMETHEE method was their inability to adequately address the uncertainty and subjectivity inherent in human decision-making.

Specifically, PROMETHEE VI employs a preference function that models the decision-maker's cognitive mechanisms, allowing for partial indifference, preference thresholds, and expression of hesitation between alternatives. This is essential as decision-making in real-world scenarios often involves ambiguity and incomplete information, areas that earlier versions of PROMETHEE method did not fully address.

By integrating these aspects, PROMETHEE VI enhances its ability to capture the nuances of human judgment and preference structures. Consequently, it emerges as a more robust and realistic MCDM tool, particularly for complex decisions involving multiple stakeholders and conflicting criteria. The ability to simulate the "*human brain*" in the analysis is a significant advantage of PROMETHEE VI over its predecessors [71].

Table 2.3. PROMETHEE types and the nature of problems addressed.

PROMETHEE type	Nature of the problem addressed
PROMETHEE I	Partial ranking
PROMETHEE II	Complete ranking
PROMETHEE III	Ranking based on intervals
PROMETHEE IV	Ranking based on continuous solution
PROMETHEE V	Problems involving segmentation constraints
PROMETHEE VI	Model uncertainty specific to human judgement

2.2.1.2.2 Elimination and Choice Translating the Reality (ELECTRE)

The Elimination and Choice Translating the Reality is a family of MCDM methods originally developed by Bernard Roy and his colleagues at the SEMA consultancy firm in the 1960s [72]. Initially focused on selecting the optimal alternative within a given set, the ELECTRE method has subsequently evolved to address three main types of decision problems: choosing, ranking, and sorting [73].

The primary distinction between PROMETHEE and ELECTRE can be attributed to their methodologies. PROMETHEE employs precise preference functions for evaluating differences among alternatives, while ELECTRE depends on the use of thresholds and pairwise comparisons to establish an outranking relationship. ELECTRE is particularly suitable for scenarios that necessitate veto thresholds and instances where specific criteria cannot be strongly opposed, even if others support the outranking.

The ELECTRE method consists of several variations that follow the same core principles but differ in their procedures and intended purposes. Each method within the ELECTRE family addresses a specific purpose based on the nature of the problem under consideration.

ELECTRE I

The ELECTRE I method was developed specifically to address the choice problem. The main purpose behind the adoption of ELECTRE I is predominantly theoretical and educational in nature. Its theoretical significance lies in its status as one of the initial MCDM methods developed, establishing the foundation for the outranking approach. It has significantly influenced the development of various other MCDM methods, particularly those based on outranking concepts.

ELECTRE I is frequently used in educational environments to introduce students to the principles of outranking methods. Its relatively simple structure and intuitive approach make it a valuable pedagogical tool for teaching MCDM principles.

Despite its merits, the complexities of practical applications, characterized by a wide range of quantitative and qualitative criteria, limits the significance of this method. This leads to the establishment of conflicting and highly varied criteria incorporating numerical and ordinal scales. Additionally, a certain degree of vagueness, ambiguity, or uncertainty consistently accompanies the data obtained from real-world case studies [74].

ELECTRE II

ELECTRE II was developed to address the ranking problem. The primary distinction between ELECTRE I and ELECTRE II is the introduction of two distinct outranking relations in ELECTRE II: strong outranking and weak outranking. These relations provide a more nuanced approach to establishing preferences between alternatives [75, 76].

These relations are established by comparing concordance indices and discordance indices against two sets of threshold values:

(α^*, β^*) and (α^-, β^-)

- If the criterion $C(a_i, a_j)$ is greater than or equal to α^* and the criterion $D(a_i, a_j)$ is less than or equal to β^* , and $C(a_i, a_j)$ is greater than or equal to $C(a_j, a_i)$, then alternative a_i is considered a strong outranking alternative a_j .

If $C(a_i, a_j) \geq \alpha^*$ and $D(a_i, a_j) \leq \beta^*$

where: $0.5 < \alpha^* < 1$, $0 < \beta^* < 1$, and

$C(a_i, a_j) \geq C(a_j, a_i)$

- If the criterion $C(a_i, a_j)$ is greater than or equal to α^- and the criterion $D(a_i, a_j)$ is less than or equal to β^- , and $C(a_i, a_j)$ is greater than or equal to $C(a_j, a_i)$, then alternative a_i is considered a weak outranking alternative a_j .

If $C(a_i, a_j) \geq \alpha^-$ and $D(a_i, a_j) \leq \beta^-$

where: $0.5 \leq \alpha^- < 1$, $0 < \beta^- < 1$, and

$C(a_i, a_j) \geq C(a_j, a_i)$

ELECTRE IS

The ELECTRE IS was developed by incorporating both an indifference and a veto threshold into ELECTRE I, respectively.

The indifference threshold defines a range within which differences between alternatives are considered insignificant, while the veto threshold specifies a level of difference beyond which an alternative is considered unacceptable, regardless of its performance on other criteria. These thresholds help address challenges associated with heterogeneous scales.

Irrespective of the scale type, this approach consistently aids in identifying the optimal action or set of actions [72].

ELECTRE III

ELECTRE III was developed to address the ranking problem by formulating a fuzzy outranking relation. While ELECTRE III follows the fundamental principles of ELECTRE II, it establishes indifference and preference thresholds for each criterion, either as a constant or as a function of the corresponding criterion value. By employing these thresholds, it becomes feasible to represent the outranking relations in a fuzzy manner.

In contrast to ELECTRE IS, where the discordance index is binary, ELECTRE III incorporates fuzziness in both concordance and discordance indices. It introduces an outranking credibility degree, resulting from the combination of discordance indices and the global concordance indices. This calculation is based on criteria where discordance index values surpass the global concordance index values, instead of defining a single global discordance index.

ELECTRE III categorizes alternatives using credibility degrees and a fuzzy outranking relation, applying both ascending and descending distillation processes [77].

ELECTRE IV

ELECTRE IV, an extension of ELECTRE III, was developed to address scenarios where defining the weights of criteria is challenging or intentionally omitted. The process of establishing outranking relations in ELECTRE III and ELECTRE IV differs in terms of their preference domains.

However, the most significant distinction between the two methods lies in their respective distillation processes for ranking alternatives. The distillation process in ELECTRE III is based on credibility degrees computed from the concordance and discordance indices, whereas the

distillation process in ELECTRE IV takes into account the number of criteria within each preference domain [78].

ELECTRE III-H

ELECTRE III-H was developed to assist in establishing specific preference models for each level of the hierarchical structure of criteria. This flexibility allows for the customization of preference modeling based on the unique characteristics and priorities at each hierarchical level.

By organizing the problem structure according to expertise and breaking it down into more manageable sub-problems, each node of the criteria hierarchy can be addressed individually.

Results (rankings or classifications) are obtained not only at the overall comprehensive level but also at each intermediate level of the hierarchy, enhancing the decision-making process [79].

ELECTRE Tri

The most recent additions to the ELECTRE family have been developed to address sorting problems. This process involves assigning alternatives to predefined categories based on their performance across various criteria.

- ELECTRE Tri involves sorting alternatives into predefined categories by comparing their performance to reference profiles.
- ELECTRE Tri-C assesses alternatives by comparing their performance to characteristic profiles for each category.
- ELECTRE Tri-nC evaluates alternatives by comparing their performance to multiple characteristic reference profiles for each category.

These variations provide flexibility for handling different types of sorting problems, making ELECTRE Tri suitable for a wide range of decision-making scenarios [80, 81].

ELECTRE I and ELECTRE II handle criteria considered as "*true*" without involving thresholds, while ELECTRE III, ELECTRE IS, ELECTRE IV, ELECTRE Tri, ELECTRE Tri-C, and ELECTRE Tri-nC handle criteria considered as "*pseudo*" and involve the incorporation of preference and indifference thresholds [82-84].

The use of pseudo criteria assists in addressing issues such as imprecise data, incomplete information, or conflicting preferences. By introducing these thresholds, the ELECTRE methods provide a more comprehensive framework for evaluating alternatives under uncertainty or imperfect information. However, a limitation of ELECTRE methods is their inability to address uncertainty associated with input data values.

Table 2.4. ELECTRE types and the nature of problems addressed.

ELECTRE type	Nature of the problem addressed	Aggregation nature
ELECTRE I ELECTRE IS	Choice	Non-Compensatory
ELECTRE II ELECTRE III ELECTRE IV	Ranking	
ELECTRE Tri ELECTRE Tri-C ELECTRE Tri-nC	Sorting	

2.2.1.3 Hybrid MCDM methods

Using a sole MCDM method is often insufficient in addressing decision challenges faced in the real world. By integrating two or more MCDM methodologies and leveraging the benefits of each, a more reliable outcome can be achieved. It is essential to identify the similarities between the methodologies and highlight the benefits of each approach when integrating MCDM methods [26]. A variety of hybrid MCDM approaches, integrating multiple MCDM methodologies, have been employed to address decision-making problems in various fields [85, 86].

The hybrid MCDM methods can be classified into three types:

- MCDM method integrated with Fuzzy Set Theory (FST);
- Integration of two or more MCDM methods;
- MCDM method integrated with methodologies from other domains.

One common enhancement involves the integration of MCDM approaches with Fuzzy Set Theory (FST). When faced with uncertainty, it becomes challenging to employ deterministic MCDM methods to assess the relative importance of criteria and alternatives performance. This challenge arises because evaluations can only be expressed using linguistic terms rather than deterministic values. Further advancement in deterministic MCDM methods is necessary to address this challenge. FST was introduced to the realm of MCDM by Bellman and Zadeh [87] and Zimmermann [88] and has been applied across various domains [89]. Ever since FST was introduced into MCDM (FMCDM), various methods have been formulated within this framework to address a range of decision problems, producing encouraging results. Examples of such FMCDM methods include fuzzy AHP [90], fuzzy TOPSIS [91], fuzzy ELECTRE [92], fuzzy

PROMETHEE [93], an integration of fuzzy ANP, fuzzy DEMATEL, and fuzzy VIKOR [94], and an integration of fuzzy MOORA and fuzzy AHP [95].

Researchers have frequently proposed and implemented the integration of an MCDM approach with other MCDM method(s), either with or without the inclusion of FST. The main aim of such integration is to develop a more robust methodology by mitigating the limitations of one method through the incorporation of the strengths of another method into an integrated proposed approach [26]. A common characteristic of this integration is the utilization of one method for assigning criteria weights, while another is employed for alternatives ranking [94, 96, 97].

Similarly, the integration of MCDM methods with other techniques or methods has been proposed to address limitations that cannot be resolved by combining MCDM methods alone or with FST. This integration aims to overcome specific limitations by incorporating the strengths of complementary methods into an integrated framework [98, 99].

2.3 SWOT analysis of the MCDM methods

To identify the most suitable MCDM method relevant to the topic of this thesis, a SWOT analysis of some of the most commonly used MCDM methods was conducted and is presented in Table 2.5.

Table 2.5. The SWOT analysis of commonly used MCDM methods.

Method	Strengths	Weaknesses	Opportunities	Threats
MAUT	Enables detailed and comprehensive evaluation of alternatives by considering multiple attributes and their utility values, making it suitable for complex decision-making problems. Supports rational decision-making by explicitly modeling preferences and trade-offs among different attributes, leading to more informed and justifiable decisions.	Defining utility functions for each attribute can be subjective, potentially introducing bias, especially if stakeholder representation is unequal or if the process lacks transparency. Requires extensive data collection and analysis, which can be challenging in cases where data is scarce or unreliable.	Potential for integration with other decision-making or data analytics techniques to enhance its capabilities and broaden its applications. Increased use of participatory approaches can improve stakeholder engagement, making decisions more inclusive and representative of diverse interests.	Competition with other decision-making approaches may limit its use in specific contexts, potentially restricting broader adoption. Uncertainty and variability in data and stakeholder preferences can present significant challenges to the reliability and robustness of decision outcomes.
AHP	Decomposes complex decision problems into a hierarchical structure of criteria and alternatives,	Rank reversal issues may occur, potentially impacting the reliability of the rankings. Pairwise comparisons can introduce subjectivity and	Increasing adoption due to extensive testing and proven reliability in a wide range of applications.	The method's subjective nature may be seen as a limitation by decision-makers who prefer more objective approaches.

	<p>simplifying both understanding and analysis. Accommodates both qualitative and quantitative factors, allowing for a comprehensive evaluation of alternatives.</p> <p>Incorporates a mechanism to assess the consistency of judgments in the pairwise comparison process, which enhances the robustness of decision outcomes.</p>	<p>bias, as they rely heavily on the decision-maker's judgments.</p> <p>The hierarchical structure may not suit all decision problems, particularly those involving interdependencies among criteria or alternatives.</p>	<p>Effective in participatory decision-making processes, as it allows stakeholders to incorporate their judgments and preferences directly.</p>	<p>As the number of criteria and alternatives grows, the number of required pairwise comparisons increases rapidly, making the process computationally intensive and time-consuming for large-scale problems.</p>
ANP	<p>Handles complex decision problems involving interdependencies, feedback loops, and hierarchical or non-hierarchical relationships among elements, allowing for a comprehensive decision model.</p> <p>Accounts for interdependencies among criteria and alternatives, providing a more realistic and flexible framework for decision-making.</p>	<p>Complexity arises due to the need to account for interdependencies, making it more challenging to implement and understand.</p> <p>Consideration of interdependencies and feedback loops can increase computational intensity, especially in large-scale problems.</p> <p>Requires extensive data and information to accurately model interdependencies, which may be difficult to obtain in certain decision contexts.</p>	<p>Increasing recognition of network-based approaches in decision-making is enhancing its relevance and broadening its applications.</p> <p>Supports participatory decision-making by incorporating stakeholder judgments and preferences, making decisions more representative.</p> <p>Integration with other decision models can provide a robust and comprehensive analysis, enhancing its versatility.</p>	<p>Potential resistance due to perceived complexity and the need for specialized knowledge, which can limit its broader adoption.</p> <p>High data requirements and complex computational needs may restrict its use to contexts with sufficient resources and expertise.</p>
WSM	<p>Transparent and easy to explain to stakeholders, as it uses a straightforward additive process, where the weighted scores for each criterion are aggregated to determine the final score for each alternative. Ideal for decision problems involving single-dimensional criteria.</p>	<p>The additive nature may not be appropriate for all decision problems, especially those where criteria are interdependent or where interactions between criteria must be considered.</p> <p>The results are highly sensitive to assigned weights, meaning that if weights are inaccurately determined, the final decisions may be biased.</p>	<p>Increasing the use of participatory approaches can enhance stakeholder engagement, making decisions more inclusive and representative of diverse interests.</p>	<p>Potential for biased results if criteria are assigned equal or arbitrary weights without proper justification.</p> <p>Limited applicability for complex decision problems involving interdependent criteria, which may require more sophisticated methods.</p>

WPM	Employs a multiplicative approach for evaluating criteria, which can provide a nuanced view of alternatives. Capable of handling both independent and dependent criteria, making it flexible for a variety of decision-making problems.	The process of assigning weights to criteria can be subjective, potentially introducing bias, especially if all stakeholders are not equally represented or if the weighting process lacks transparency. Normalization is required to ensure comparability, adding additional steps that may introduce errors in the decision-making process.	Potential for integration with other methods to develop a more robust decision-making framework, allowing it to handle complex problems effectively. Increasing the use of participatory approaches can improve stakeholder engagement, making decisions more inclusive and representative of diverse interests.	Risk of overemphasis on certain criteria, which may skew results and reduce objectivity in the final decision. Potential for bias from subjective weighting, which can impact the reliability and acceptance of outcomes, especially if not transparently managed.
TOPSIS	Provides a clear quantitative analysis of alternatives, enabling objective decision-making based on numerical data. Simple to understand and implement due to its approach of ranking alternatives based on their closeness to an ideal solution, making it accessible to a wide range of users.	Lacks guidelines for determining criteria weights, assuming that decision-makers can weigh criteria accurately, which may lead to inconsistencies. Neglects uncertainty in decision data, limiting its applicability in scenarios characterized by uncertain information. The final decision can be sensitive to the normalization method used, potentially impacting the outcome and reliability of results.	Potential for integration with other methods to develop more robust decision-making framework. Ongoing research focused on enhancing sensitivity analysis and addressing uncertainty, which could make it more adaptable to dynamic environments.	Resistance to adoption may arise due to its perceived simplicity, despite the need for specialized knowledge in accurately determining weights and selecting normalization methods. Potential limitations in handling data variability and complex weighting requirements could restrict its broader acceptance in situations requiring sophisticated analysis.
VIKOR	Focuses on identifying compromise solutions, which is particularly useful in group decision-making problems where stakeholders have conflicting preferences. Highly effective in balancing conflicting criteria, making it suitable for decision problems involving trade-offs between various criteria.	Interdependency issues may occur, which can complicate the analysis when criteria are not independent. Challenges in handling conflicting situations and sparse data can limit its effectiveness in complex scenarios. Inappropriate parameter selection can significantly affect the final decision, requiring careful calibration.	Increasing the use of participatory approaches can enhance stakeholder engagement, making decisions more inclusive and reflective of diverse interests. Ongoing research is focused on reducing sensitivity to input changes, which could make the method more robust in handling variations in data.	Limited adoption may result from its perceived complexity and the need for specialized knowledge in accurately determining weights and parameters. The presence of uncertainty and variability in data, along with diverse stakeholder preferences, presents significant challenges to the reliability of outcomes. Difficulty in clearly communicating the process and results to non-experts can limit its broader application in decision-making processes.
PROMETHEE	User-friendly interface with clear visualization of the decision-making process, making it accessible and understandable for users. Capable of handling various types of	The selection of preference functions can be subjective, potentially influencing outcomes and introducing bias into the decision-making process.	Broad potential for application in preference-based decision-making problems, particularly those requiring nuanced comparison between alternatives. Ongoing research is focused on improving scalability and	Complex preference models and the need for specialized knowledge can make it challenging to communicate effectively with non-experts, potentially hindering broader adoption.

	quantitative and qualitative criteria and preferences, providing a robust framework suitable for complex decision problems.	Rank reversal issues may occur, affecting the consistency of rankings under certain conditions. Normalization is required to ensure comparability, adding additional steps that may introduce sources of error.	reducing subjectivity, which could enhance its usability in larger and more complex problems.	The reliance on user-specified parameters may limit its applicability in situations where standardized settings are preferred.
ELECTRE	Effectively handles both quantitative and qualitative criteria, making it versatile for a wide range of decision-making problems. Robust in dealing with imprecise and uncertain data, ensuring reliable outcomes. Applicable in cases where incomparable alternatives exist, allowing for flexibility in complex scenarios.	The use of threshold values can significantly influence the final solution. Not ideally suited for problems with a large number of alternatives, as computational complexity increases rapidly, making it less efficient in such cases.	Potential for wider application in decision problems involving imprecise information, enhancing decision-making in uncertain environments. Ongoing research is focused on improving robustness to parameter changes, which may increase its flexibility. Integration with other MCDM methods or advanced techniques can expand its capabilities and help address current limitations.	Selecting the optimal parameters for various situations can be challenging, requiring domain-specific expertise. The presence of uncertainty and variability in data, along with differing stakeholder preferences, can challenge the reliability and robustness of the outcomes. Explaining the process and results in clear, simplified terms for non-experts can be difficult, potentially limiting its accessibility and acceptance in broader contexts.

2.4 Weighting methods for evaluation of criteria

The criteria weighting is a key element in the decision-making process. The main benefit of using a weighting method is that it determines the priorities of each criterion in order to clearly show their significance. By assigning appropriate weights to the criteria, the weighting methodology helps identify the optimal choice among alternatives by considering their overall performance across all criteria.

The simplest approach is to consider all criteria as equally weighted without assigning any specific weights. The equal weight method has been used in many decision problems [100, 101].

Assigning appropriate weights to criteria is an essential step that requires careful consideration in decision-making problems, as establishing the values of these weights is one of the main challenges in the process [102, 103].

The approaches for establishing the weights of the criteria can be broadly classified into three distinct groups: subjective, objective, and integrated weighting methods as presented in Table 2.6 [104-106].

Table 2.6. Weighting methods for evaluation of criteria.

Subjective weighting methods	Objective weighting methods	Integrated weighting methods
Trade-off method	Shannon's entropy method	Optimal weighting based on sum of squares
Direct rating method	Standard deviation method	Optimal weighting based on relational coefficient of graduation
Point allocation method	Criteria importance through inter-criteria correlation	Multiplication synthesis
Swing method	Statistical variance procedure	Additive synthesis
Nominal group technique		
Ratio method		
Simos method		
Pairwise comparison method		
Simple multi-attribute ranking technique		

2.4.1 Subjective weighting methods for evaluation of criteria

In the determination of criteria weights using subjective methods, decision-makers play a pivotal role. The aggregation of perspectives from decision-makers is a necessary aspect of multi-criteria analysis during the decision-making process. Specifically, identifying criteria weights based on stakeholders' expressed preferences involves significant considerations. These weights substantially influence the analysis outcome, raising concerns that defining the criteria weights through subjective judgments could potentially compromise the reliability of the evaluation results.

Factors such as time constraints, inadequate information, stakeholder awareness, linguistic ambiguity, and the specific method employed for preference collection all influence the resultant findings.

Given these highlighted issues, there is no universally accepted procedure. Defining the relevance of evaluation criteria based on stakeholders' preferences is a complex activity that is susceptible to cognitive biases. The information provided by decision-makers and stakeholders is often imprecise, inconsistent, and influenced by personal biases.

The factors that influence the outcome of the subjective weighting methods include:

- The procedure used for eliciting preferences;
- The vagueness of human language;
- Lack of information/awareness among stakeholders regarding the decision-making problem;
- Personal biases;
- Potential lack of rationality in stakeholders' viewpoints;
- The procedure used for aggregating preferences from individuals.

To ensure an effective and reliable decision-making process that minimizes undesirable influences, it is advisable to employ strategies that incorporate objectivity in defining the importance of the evaluation criteria [107-109].

Trade-off method

The trade-off method is established on pairwise comparison of criteria. Two artificial alternatives are constructed for each pair of criteria, differing only in the performance level of those criteria. The decision-makers are obliged to select one of the criteria initially and then their readiness to compromise on one criterion in favor of the other is examined. The determination of trade-off weights among criteria is influenced by the behavior of the decision-makers.

The method is criticized for its complexity and inconsistency in gathered preferences. Furthermore, the computational workload increases as the number of criteria grows, resulting in an increased number of pairwise comparisons [110].

Direct rating method

The direct rating method involves the decision-makers initially ranking all the criteria based on their level of significance.

In contrast to fixed-point scoring methods, the rating method does not impose limitations on the decision-maker's responses. This means that one can modify the importance of a specific criterion without having to adjust the weight of another criterion [111].

Point allocation method

The point allocation method presents a simple solution for assigning criteria weights based on their relative priority. Decision-makers assign a specific number of points to each criterion, with a higher number of points indicating greater importance. In this case, the decision-makers must allocate a total of 100 points among the criteria ensuring that the sum equals 100.

Even though this approach is simple to standardize, it should be noted that the weights obtained through point allocation may lack precision. Additionally, implementing the method becomes increasingly challenging when dealing with six or more criteria [112].

Swing method

The swing method is based on the analysis of two artificial options: Option A, which exhibits the lowest level of performance across all criteria, and Option B, which demonstrates the highest level of performance across all criteria. The relative weights assigned to the criteria are derived through an iterative process, during which the extent to which the swing from the performance

level of option A to that of option B is determined. The criteria importance is determined by the chronological sequence of these choices.

The swing method is straightforward and less susceptible to inconsistencies in preferences. Furthermore, its computational effort is less influenced by the number of criteria [113].

Nominal group technique

The nominal group technique is a systematic brainstorming method employed to generate a substantial quantity of ideas related to a specific matter, while ensuring equal participation of all stakeholders. In addition to its capacity to generate a plethora of ideas, this technique also facilitates the prioritization of these ideas. In conclusion, the ideas that gain majority support are selected [114].

Ratio method

The ratio method involves prioritizing the relevant criteria based on their relative importance. In this approach, the criterion of least importance is allocated a value of 10, while the other criteria are allocated multiples of 10. Subsequently, the derived weights are then normalized to ensure that the total sum equals one [115].

Simos method

The Simos method involves associating each criterion with a "*playing card*". Initially, decision-makers prioritize these criteria, represented by colored playing cards, based on their level of significance. When multiple criteria are considered equally significant, they are assigned the same rank. Subsequently, decision-makers are provided with a set of white cards to position between two adjacent colored cards. These white cards indicate the disparity in importance among the criteria denoted by the colored cards. If no white cards are placed between colored cards, the criteria's significance is measured using the default unit of the interval [116].

Pairwise comparison method

The pairwise comparison method is employed to compare multiple populations in pairs to identify significant differences between them. It can also be described as a method where decision-makers assess each criterion against one another in order to establish preference levels.

To facilitate the assessment of the significance of favoring one criterion over another, an ordinal scale ranging from 1 to 9 is employed. The analytic hierarchy process is one of the frequently used techniques that involves pairwise comparisons [117].

Simple multi-attribute ranking technique

In the simple multi-attribute ranking technique, decision-makers are required to prioritize the criteria through ordinal ranking based on their relative significance. The criterion with the least significance is assigned 10 points, whereas the one with the most significance is assigned 100 points. Subsequently, the remaining criteria are allocated points based on their perceived significance. To determine the weight of the criteria, the sum of points is normalized to one.

This method is developed for both allocating criteria weights and evaluating the alternatives' performance [118].

2.4.2 Objective weighting methods for evaluation of criteria

Objective weighting methods are based on data collected from each criterion, using a mathematical function to determine the weights without any influence from decision-makers [119-121].

Shannon's entropy method

The entropy method is established on the concept of entropy, which was defined by Shannon as a measurement of disorder. In the field of information theory, entropy is used as a metric to quantify uncertainty in a discrete probability distribution. As the level of uncertainty within a set of data increases, so does the value of entropy.

When presented with a decision matrix that contains information about different options, the entropy method can be employed to obtain the criteria ranking, or in other words, the weighting values. It can be stated that the information value will be higher if the distribution of decision criteria requires less probable information, and conversely, the information value will be lower if the distribution is highly probable [122].

Standard deviation method

The standard deviation method is employed to assess the significance of different criteria when making decisions. Initially, calculating the standard deviation for each criterion is considered essential. This involves measuring the extent to which the values of each criterion deviate from the average. If the values of a criterion are closely clustered together, its standard deviation will be minimal. Conversely, if the values are more dispersed, the standard deviation will be greater. Subsequently, the criteria weights are determined. The standard deviation of each criterion is divided by the sum of all the standard deviations. This yields a proportion for each criterion. An

increase in standard deviation results in a corresponding increase in the weight assigned to the specific criterion.

The standard deviation method highlights criteria that show greater variability in values, as these criteria exert a more substantial influence on the decision-making process [123].

Criteria importance through inter-criteria correlation

The criteria importance through inter-criteria correlation method depends on the standard deviation and employs correlation analysis to assess the significance of each criterion [124]. Initially a decision matrix is developed and normalized. Subsequently, the correlation coefficient is calculated for each pair of criteria, indicating the strength of their relationship. When two criteria exhibit a strong correlation, it implies that they hold significant influence in the decision-making process. Finally, the allocation of criteria weights is based on their correlation coefficients, with higher correlations resulting in greater weight [125].

Statistical variance procedure

The statistical variance procedure is used as a means to establish the criteria weights in the decision-making process. Initially, the statistical variance for each criterion is calculated. It measures the extent to which the values of each criterion deviate from their average value. A higher variance indicates a greater dispersion of values, whereas a lower variance implies that they are closer together. Subsequently the criterion weight is calculated by dividing the statistical variance of each criterion by the sum of all the variances. This results in a proportion being assigned to each criterion. The magnitude of the variance directly influences the magnitude of the weight assigned to that specific criterion.

The statistical variance procedure assigns greater weight to criteria with values that exhibit greater dispersion from their average, as criteria with higher variance tend to have a more substantial influence on the decision-making process [126].

2.4.3 Integrated weighting methods for evaluation of criteria

The integrated weighting methods constitute a comprehensive approach used to establish the criteria weights in decision-making methods. In contrast to purely subjective or objective approaches, the integrated method amalgamates subjective judgments with objective information to establish criteria weights. This implies that it considers the perspectives of decision-makers and the available quantitative data to reach a trade-off.

By integrating subjective and objective information, this method mitigates the limitations that may arise from solely relying on one approach. For example, purely subjective methods can be susceptible to cognitive biases, while purely objective methods may ignore significant qualitative considerations. The integrated approach aims to find a balance between these methods.

Researchers have proposed diverse approaches for integrating subjective and objective information in determining criteria weights.

Overall, the integrated weighting methods provides a comprehensive and well-balanced approach to determining criteria weights by incorporating both subjective judgments and objective data, thereby enhancing the decision-making process [127-129].

2.5 Application of MCDM methods for energy renovation of buildings

Table 2.7 presents a brief overview of the recent research papers examining the application of MCDM methods. It highlights the criteria used in these real-world case studies within the context of energy renovation for buildings.

Table 2.7. Application of MCDM methods for energy renovation of buildings.

Author/ Ref	MCDM	Application area	Criteria	Country
Yilan et al.,2020 [130]	MAUT	Ranking electricity generation technologies based on performance score.	Economic, technical, environmental, socio-economical.	Turkey
Medal et al.,2021 [131]	AHP	Prioritizing factors for decision-making in energy efficiency retrofits for buildings.	Payback period, life cycle cost, funding mechanism, carbon dioxide emissions, educational programs, demand pressure.	USA
Kamarauzza man et al.,2018 [132]	AHP	Sustainability assessment schemes for building refurbishment.	BREEAM, LEED, CASBEE, BEAM PLUS, HQE, GBLS, Green Mark, Green Star, GBI, MyCrest.	Malaysia
AbdelAzim et al.,2017 [133]	AHP	A classification method for assessing the energy performance of pre-existing structures based on specific criteria.	Utilization of renewable energy sources, optimization of building envelope efficiency strategies, deployment of energy-efficient equipment, adoption of environmentally friendly refrigerants, adherence to operation and maintenance, installation of metering and sub-metering systems within buildings, and assessment of the transportation impact associated with buildings.	Egypt
Golic et al.,2023 [134]	AHP	Facilitating and enhancing the efficiency of the design process for energy-efficient renovation of buildings.	Energy consumption per year, carbon dioxide emissions per year, payback period, investment costs of envelope energy renovation.	Serbia
Xu and Chan, 2013 [135]	ANP	Developing a modeling strategy for optimizing building energy retrofits within the context of energy performance contracts.	No particular criteria were employed; this constituted an initial examination of strategies for organizing the issue.	China
Mirzee et al.,2019 [136]	SMART	Decision-makers' preferences for subjective criteria in designing sustainable and resilient buildings.	Social, economic, environmental.	USA

Romani et al.,2022 [137]	Comparison of WSM, min-max and Pareto concept	Applying multi-criteria analysis in sustainable building renovation, integrating various criteria across the building's life cycle.	Social, economic, environmental.	France
Salvadó et al.,2022 [138]	Decision support system algorithm	A decision-support system considering uncertain and conflicting criteria within the refurbishment process.	Profitability, carbon dioxide emissions reduction, comfort, human health, ecosystem quality, resources consumption.	France
Jahangiri et al.,2023 [139]	Comparison of multi-criteria analysis: EDAS, ARAS, WASPAS, TOPSIS	Proposing a framework for adopting building-integrated photovoltaics with a focus on optimizing the solar cells arrangement on the building facade while accounting for current weather conditions.	Financial and technical.	Iran
Wang,2015 [140]	TOPSIS	Benchmarking building energy efficiency.	Energy consumption intensity, efficiency in cooling degree day, efficiency in heating degree day, efficiency in overall degree day, efficiency focused on occupants, overall efficiency focused on room, bathroom-oriented efficiency.	USA
Jalaei et al.,2015 [141]	TOPSIS+BIM	Developing a methodology to assist the design team in selecting the most appropriate sustainable building components.	Economic, environmental, social well-being.	Canada
Sarmas et al.,2022 [142]	TOPSIS	Supporting and identifying the financial procedure for energy efficiency investment in building projects.	Energy reduction per cost, age of building, energy consumption per unit of heating area.	Latvia
Perara et al.,2018 [143]	TOPSIS	Identifying the most preferred building retrofits and incentive programs for households.	Economic and environmental.	Canada
Silva et al.,2016 [144]	TOPSIS+AHP	Enhancing the thermal and energy efficiency of a residential building.	Thermal comfort and energy usage.	Brazil
Caruso et al.,2023 [145]	TOPSIS+AHP	Identifying the most suitable renovation strategies for buildings.	Installation cost, total cost post-retrofit life cycle, carbon emissions post-retrofit life cycle, probability of annual failure, aesthetic influence, necessity for specialized labor, design expertise, essential modification at the foundation level.	Italy
Seyis and Ergen, 2017 [146]	TOPSIS+MAUT	Facilitating the selection of applicable green residential building certification credits.	Time wasted and costs incurred in both the design and construction phases.	Turkey
Starynina and Ustinovichius, 2020 [147]	Integrated MADM framework— COPRAS- TOPSIS-SAW	A knowledge-based decision support system for sustainable public building renovation, focused on optimizing energy efficiency.	Thermal transmittance, forecasted lifetime of the measure, possible construction works despite the seasonality, price of the modernization, simple payback period, annual savings, thickness of the structure.	Lithuania
Seddik et al.,2016 [148]	PROMETHEE	Ranking different renovation solutions for masonry buildings.	Energy consumption, investment cost, potential for compromising historical architectural aesthetics, and threat to the structural integrity.	Algeria
D.Dirutigliano et al.,2018 [149]	PROMETHEE	Ranking various options for retrofitting buildings at both the building and district levels.	Economic, social, environmental.	Italy
Taillandier et al., 2016 [150]	ELECTRE III	Quantifying the performance of residential housing renovation alternatives.	Economic, social, environmental.	France
Rey, 2004 [151]	ELECTRE III	Assessing strategies for retrofitting office buildings using a ranking system.	Annual energy consumption for heating, yearly carbon dioxide emissions, and cost for renovations.	Switzerland
H.Bernardo et al., 2017 [152]	ELECTRE Tri	Categorizing energy performance for school buildings.	Energy consumption, greenhouse gas emissions, operation cost, maintenance cost, indoor air quality, thermal comfort.	Portugal
Napoli et al.,2020 [153]	ELECTRE Tri-nC	Classifying energy retrofitting measures for public buildings into specific categories.	Energy efficiency, financial feasibility.	Italy

Pinzon et al.,2022 [154]	TOPSIS +Pairwise comparison	Ranking and selecting the optimal renovation option for a building.	Environmental, economic, social.	Spain
Ongpeng et al.,2022 [155]	AHP+VIKOR	Evaluating and determining a balanced building renovation scenario through stakeholder engagement.	Environmental, economic, technical.	Philippines

It is evident that MCDM methods are extensively employed to address the challenges associated with the energy renovation in existing buildings. These methods focus on the selection, ranking, and classification of various energy alternatives/scenarios, constituting the core application areas.

Criteria selection is an essential phase in the decision-making process. To develop a comprehensive set of criteria for a specific decision problem, the initial step involves reviewing previous case studies to identify the criteria used. This review evaluates the appropriateness, reliability, practicability, and limitations of the criteria, thereby ensuring a robust decision-making framework.

Energy related decision-making problems necessitate the consideration of various main criteria. The most commonly used criteria categories include economic, environmental, social, and technical aspects. Each of these categories typically comprises several sub-criteria or indicators [156-158].

While the list of authors who have identified relevant criteria for building energy renovations is by no means exhaustive, it highlights the major inconsistencies and lack of consensus on a comprehensive set of evaluation factors. This lack of consensus suggests that the criteria selection for building energy renovation decisions is a complex and context-dependent process, requiring researchers and practitioners to carefully evaluate the specific decision-making context, stakeholder preferences, and data availability.

2.6 Selection and justification of MCDM method and weighting method

In recent decades, various new MCDM methods have been introduced, accompanied by improvements in the maturity of the existing methods, thereby enhancing their applicability to real-world problems. These advancements have focused on addressing specific challenges encountered in practical scenarios that necessitate enhanced decision-making approaches [159]. Despite the development of various methods, the effectiveness of these approaches in the decision-making process is contingent upon the characterization of the decision problems. It is essential to

understand that there is no "*one-size-fits-all*" solution, and no individual method can address all potential decision-making situations effectively. Consequently, decision-makers often face challenges in justifying their choice of a specific method for their decision problem [160].

The adoption of a multi-criteria approach is commonly based on subjective considerations, such as decision-makers' experience with a particular method or the availability of software that facilitates its use [46, 161, 162]. Similar concerns arise when selecting MCDM software. Typically, decision-makers select decision support software with which they are already acquainted [163]. Thus, the selection of a MCDM method for a problem is not determined exclusively by the method itself, but rather by adjusting the decision problem to align with the chosen MCDM method. Therefore, determining the most appropriate method to address a specific problem type presents a challenging question [164-166].

Selecting the appropriate MCDM method for a particular decision situation carries significant importance as different methodologies may produce varied outcomes for the same issue. These methods differ in several aspects, such as the complexity level of the problems they can address, the methods used to assign criteria weights, thresholds representation, the incorporation of data uncertainty, and the techniques used to aggregate data [167, 168].

MCDM methods share several fundamental principles, including the identification of a set of potential alternatives to address the decision problem, the establishment of at least two distinct criteria to assess each alternative, the assignment of relative weights to reflect the significance of each criterion, and the designation of a decision-maker responsible for reviewing the analysis and making the final decision, considering all relevant factors [169].

During the decision-making process, a systematic and comprehensive approach is necessary, involving problem definition and selection of an appropriate MCDM method, executed rigorously to obtain an accurate recommendation [170].

To identify the most appropriate MCDM method for this thesis, a state-of-the-art review of the most commonly used MCDM methods, including a SWOT analysis, was undertaken. This review greatly facilitated the comparison and analysis of the various MCDM methods. Based on the comprehensive assessment, the ELECTRE Tri method was determined to be the most suitable approach for developing and implementing a new MCDM framework for the following reasons:

- The evaluation process involves a diverse set of measurement scales, including quantitative and qualitative aspects. This flexibility allows the direct use of the original data in the decision-making process, eliminating the need for transforming it into artificial numerical scales.
- The existence of significant heterogeneity among criteria, such as duration, noise, distance, etc., can be attributed to the inherent nature of evaluations, eliminating the need for normalization techniques. The heterogeneity of scales presents a significant challenge for many decision support systems, as they usually necessitate a uniform measurement scale for all criteria. This feature is of great significance in real-world applications.
- Alternatives are classified based on their performance instead of being ranked from most preferred to least preferred, which facilitates a more subjective ranking of alternatives.
- The ability to employ reference profiles and categorization provides valuable perspectives on the overall performance of alternatives, enabling an absolute classification for each alternative, rather than solely relying on relative performance. This feature is particularly beneficial when identifying an alternative that exhibits exceptional overall performance.
- The following holds true for at least one criterion: small variations in evaluations are insignificant for preferences, however, when multiple small variations accumulate, they can become significant.
- The method incorporates the concept of incomparability between a pair of alternatives, which occurs when one alternative outperforms the other on certain criteria but underperforms on other, making it impossible to establish a relationship between them.
- No direct evaluations are made between alternatives. As a result, if a new alternative needs to be included in the classification procedure, there is no need to reclassify the existing alternatives as the new alternative can be compared to the reference profiles.
- The method incorporates thresholds and a user-defined cutting level, considering uncertainties in computations and performance assessments while prohibiting compensatory trade-offs. Decision-makers may find it inappropriate to compensate for a loss on one criterion by a gain on another. In these cases, non-compensatory aggregation is essential.

An essential aspect of the decision-making process involving multiple criteria is the assignment of weights to each criterion within the MCDM model [171]. Since the process of

assigning weights is inherently subjective, it is necessary to select a suitable weighting method for this thesis. Therefore, a comprehensive overview of several well-known weighting methods was conducted. As a result, the revised Simos method, a systematic method developed to obtain consensus among stakeholders, was selected for the following reasons:

- A reliable, precise, and user-friendly method for measuring weights, frequently employed in various research studies.
- Easily adjustable and integrable with various MCDM methods.
- The process can be conducted either online or in person, offering flexibility for stakeholders who are unable to convene in one location simultaneously. Preferences can still be obtained using an electronic version of the Simos method, such as a worksheets or presentations. This flexibility ensures a smooth progression of the process, minimizing disruptions caused by planning or logistical issues.

Traditional MCDM approaches generally assume that the evaluation data for decision-making is both readily accessible and accurate. However, in reality, information is often uncertain, imprecise, and subjective, which limits the applicability of MCDM methods [172-175]. Decision-makers encounter challenges when expressing their opinions using crisp (precise) values due to various factors including human cognitive limitations, measurement errors, incomplete or missing information, and data approximations. These factors introduce inherent ambiguity into the decision-making process. For instance, economic uncertainties may arise from fluctuating raw materials prices, while environmental uncertainties may originate from imprecise quantification of environmental impacts. Despite these issues, uncertainty is often neglected in the decision-making process to avoid complexities, resulting in solutions that deviate significantly from reality. Decision-makers may encounter situations where the available data is incomplete, lacking either comprehensive assessments of the alternatives under consideration or detailed information about the significance of the evaluation criteria, thereby impeding a detailed analysis. Due to the complex nature, imprecision, and inherent uncertainties in the process, it is essential to incorporate uncertainty into the decision-making process to address these challenges effectively, thereby gaining acceptance from the stakeholders involved [176].

Various theoretical frameworks have been proposed to estimate assessment ranges using the existing knowledge and the available data of the decision problem under consideration [177].

The ELECTRE Tri method addresses the concept of fuzziness by assigning alternatives into specific categories. Fuzzy logic is a mathematical approach that handles the vagueness inherent in linguistic terms and subjective judgments, which often lack sharp boundaries. In MCDM, vague values arise when criteria do not have precise values but instead lie within ranges with ambiguous boundaries. Fuzzy sets are a tool within fuzzy logic that help model these vague criteria. Unlike traditional sets, where an element either fully belongs or does not belong at all, a fuzzy set allows partial membership. Each element is assigned a membership degree that ranges from 0 to 1, where 0 means no membership and 1 means full membership. This approach enables a more detailed assessment of alternatives, as it expresses the degree of satisfaction for each criterion with more flexibility. A fuzzy set allows decision-makers to define the level of performance as partially meeting certain criteria, rather than meeting them entirely or not at all.

In the context of energy renovation, a fuzzy set could assess the environmental performance of different renovations alternatives. For example, carbon emission reductions could be evaluated using fuzzy sets with membership functions defining "*low*," "*medium*," and "*high*" emission reductions. Each renovation alternative would receive a membership value within these categories, enabling a more comprehensive evaluation. Nevertheless, it is essential to note that although fuzzy values allow for nuanced assessments, these values are crisp rather than imprecise. This means that, after processing through the fuzzy sets, the final decision-making results are definitive rather than imprecise, providing clear categorizations of alternatives based on the fuzzy assessments used during the analysis.

An imprecise value is characterized by a lack of precision or specificity, meaning it provides only a limited level of detail or exactness. Imprecision often arises from factors such as measurement errors, data approximations, and incomplete or missing information, all of which can obscure the true value and introduce uncertainty. In contrast to fuzzy values, which inherently involve assigning membership degrees within a set, imprecise values focus on indicating the level of uncertainty associated with the value itself without the need for such membership degrees. To better understand and quantify this precision, probability density functions are used. These functions describe the likelihood of different values occurring within a specified range, effectively capturing the spread or variability of possible outcomes. This allows for modeling the probability of a value being close to certain points within the range, providing deeper insights into the degree of certainty or uncertainty with the value.

In the context of energy renovation of buildings, probabilistic distributions can be used to model the economic uncertainties related to renovation alternatives. For example, initial investment costs, energy savings, and payback period are all aspects that could vary due to fluctuating market conditions, material costs, or energy price changes. To handle this variability, probabilistic distributions such as the normal distribution can be employed. This enables consideration of the possible economic outcomes along with their probabilities within a specified range.

These probabilistic approaches provide a more comprehensive and realistic assessment of the various alternatives, resulting in well-informed decisions that consider the complex and dynamic nature of the issue.

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3 Methodology

This chapter presents a comprehensive description of the methodology proposed and validated in this thesis. By applying these methodological principles, the research problem can be effectively addressed. As a result, this chapter outlines the underlying assumptions of the research, based on the insights obtained from the state-of-the-art review conducted in the preceding chapter. The review also facilitated the identification of the research gap. The methodology employed to address this research gap is explained, alongside the philosophical stance adopted. The chapter includes an overview of following methods: ELECTRE Tri method, the revised Simos method, the probability density function, and the Monte Carlo simulation. Subsequently, the novel probabilistic extension of ELECTRE Tri method is explained.

3.1 Research framework

The framework for the proposed probabilistic ELECTRE Tri is shown in Figure 3.1, providing a structured overview of its key components. To fully comprehend this framework, it is essential to examine the MCDM methods and tools employed in its development. These methods and tools are described in the subsequent sections, highlighting their roles and contributions to the framework.

3.2 Description of the ELECTRE Tri method

The ELECTRE Tri method employs an outranking approach to evaluate alternatives based on pre-defined reference profiles, resulting in the sorting and categorization of alternatives according to their ability to fulfill specific conditions. In contrast to pairwise comparisons, ELECTRE Tri assigns alternatives to distinct categories by assessing their similarity to the reference profiles. This evaluation necessitates carefully defined parameters to ensure a robust categorization procedure [1, 2].

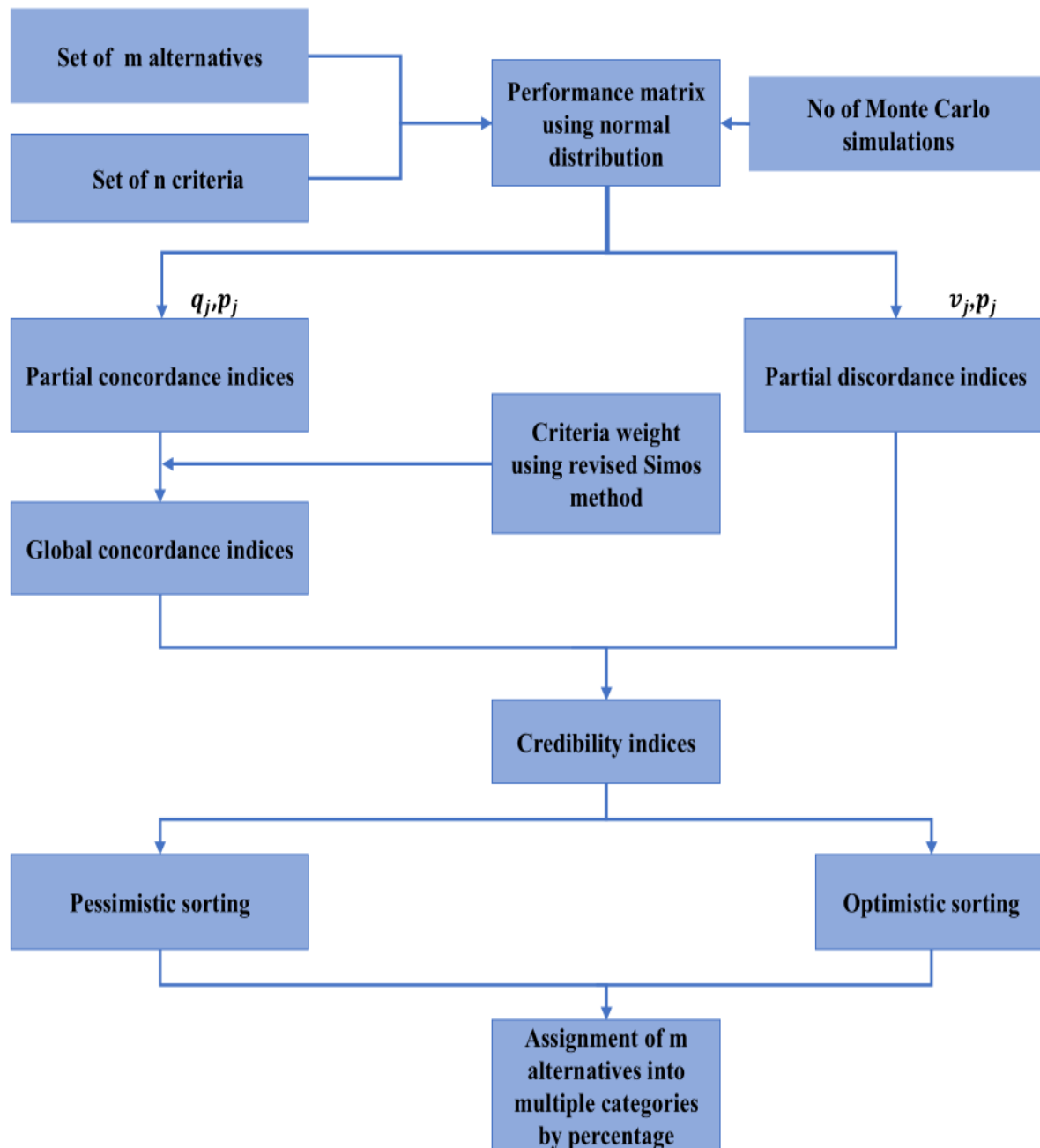


Figure 3.1. The framework for the proposed probabilistic ELECTRE Tri method.

To apply the ELECTRE Tri method, it is essential to define distinct and mutually exclusive ordered categories. These categories are represented by reference profiles $B = \{b^0, b^1, \dots, b^k\}$ to establish a set consisting of k categories. Each category C^h ($h=1, \dots, k$) is defined by means of two reference profiles: an upper reference profile b^k and a lower reference profile b^{k-1} . The lower reference profile of a higher category also functions as the upper reference profile for the immediately preceding lower category. These categories should be arranged in a specified order (e.g., from the least preferred to the most preferred, or from least significant to the most significant, etc.) represented by $C = \{C^1, C^2, \dots, C^k\}$. By convention, C^1 denotes the least preferred category, and C^k the most preferred category as shown in Figure 3.2 [3].

To effectively implement the ELECTRE Tri methodology, it is essential to accomplish two main phases [4].

The initial phase is to establish an outranking relation between alternatives and reference profiles, which is accomplished through the computation of concordance and discordance indices. These indices are calculated using the parameters associated with a pseudo-criterion: specifically, the indifference threshold q_j , preference threshold p_j , and veto threshold v_j .

The indifference threshold (q_j) is the maximum degree of difference between the evaluations of two alternatives on a particular criterion g_j , below which they are considered indifferent for that criterion. If the performance difference is less than or equal to the indifference threshold q_j , the alternatives are considered equivalent on criterion g_j . It represents the maximum performance difference that is considered negligible. For energy savings, if two projects result in an energy efficiency improvement difference of less than 2% (e.g., one project achieves 22% improvement and another achieves 24%), this difference is so minor that it doesn't influence the decision-maker's choice. Thus, 2% would be the indifference threshold for this criterion.

The preference threshold (p_j) is the minimum degree of superiority required for an alternative to be preferred over another on a particular criterion g_j . It represents the minimum performance difference required to prefer one alternative over another on criterion g_j . If an improvement of 5% in energy savings (e.g., 25% improvement compared to 30%) is sufficient for the decision-maker to have a clear preference for one project over another, then 5% acts as the preference threshold. Here, a project with 30% improvement is decisively better than one with 25%.

The veto threshold (v_j) is the maximum allowable performance difference on a particular criterion g_j beyond which an alternative cannot be considered as outranking another, irrespective of its performance on other criteria. If the performance difference between two alternatives exceeds the veto threshold, any potential outranking relationship is nullified, indicating a substantial performance gap that prevents one alternative from being preferred over the other on criterion g_j , regardless of its performance on other criteria. It represents the maximum performance disparity deemed unacceptable for an outranking relationship. If any renovation project achieves less than 15% in energy savings, it is deemed unacceptable and excluded from consideration. This 15% acts as the veto threshold; any project with energy savings below 15% is immediately ruled out, even if it performs well on other criteria.

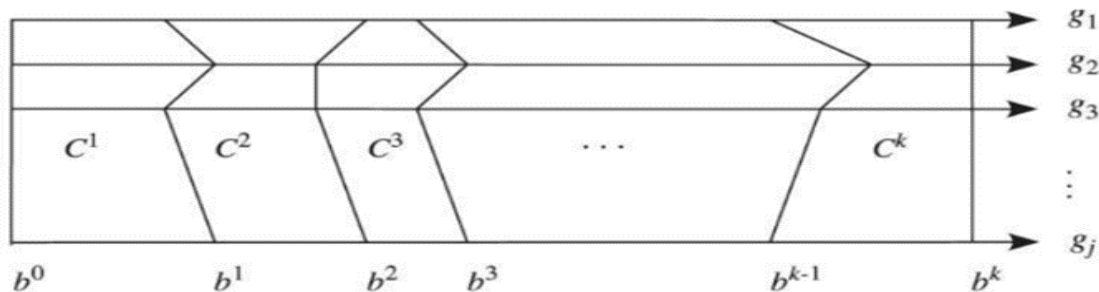
The veto v_j , preference p_j and indifference q_j thresholds follow this rule, where $v_j \geq p_j \geq q_j$.

The subsequent phase involves assigning alternatives to their respective categories based on the outranking relation established in the initial phase, which is determined through the computation of the pessimistic procedure and optimistic procedure.

The pessimistic procedure compares each alternative sequentially to the reference profiles of the categories, starting from the highest category. The alternative is assigned to the highest category such that it is at least as good as the reference profile of that category.

The optimistic procedure compares each alternative sequentially to the reference profiles of the categories, starting from the lowest category. The alternative is assigned to the lowest category such that it is at least as good as the reference profile of that category.

When the assignments from the pessimistic and optimistic procedures differ, it indicates an ambiguity in the classification of the alternative.



Legend:

$C^1, C^2, C^3, \dots, C^k$ = Number of categories, $b^0, b^1, b^2, \dots, b^k$ = Reference profiles for the categories, $g_1, g_2, g_3, \dots, g_j$ = Number of criteria

Figure 3.2. Ordered categories defined by reference profile for each criterion [5].

Once the essential characteristics of the ELECTRE Tri method are defined, its application necessitates specific inputs and parameters [6]:

Alternatives

Alternatives refer to the scenarios, options, actions, strategies or plans to be evaluated in the decision-making process. Alternatives are represented as

$$A = \{a_1, a_2, \dots, a_m\}$$

where A is the finite set of alternatives and m is the total number of alternatives in A that needs to be sorted.

Criteria

Criteria refer to the attributes/factors used to evaluate the alternatives in a decision problem. The criteria may be quantitative, qualitative, or binary in nature. Criteria are represented as

$$C = \{g_1, g_2, \dots, g_n\}$$

where C is the finite set of criteria and n is the total number of criteria.

Weights

Weights are used to establish the significance of each criterion. An importance vector is assigned to each criterion, representing its weight. Weights are represented as

$$W = \{w_1, w_2, \dots, w_j\}$$

where w_j is the weight of the j_{th} criterion, which expresses the relative significance of criterion g_j compared to the other criteria. By convention, weights are non-negative values, and their sum equals 1.

Performance matrix

The performance matrix is typically presented in a structured format, such as a table or matrix. The rows represent the alternatives, while the columns correspond to the criteria, offering a comprehensive overview of each alternative's performance based on each criterion.

For criteria with a preference for increasing performance, a higher evaluation of the alternative on that criterion indicates better performance. To ensure consistency in calculations, criteria with a decreasing preference are transformed by multiplying their values by "-1", effectively adjusting them with the increasing preference direction of the other criteria [7].

$g_j(a_i)$ represents the performance of alternative a_i on criterion g_j .

Cutting level

The cutting level is the minimum credibility index value required to validate the assertion alternative a_i is superior to the reference profile b_k . The default cutting level value, unless specified otherwise, is commonly set at 0.75. A higher cutting level value (e.g., 0.75) makes it more difficult for a_i to outrank b_k , as it necessitates that the criteria favoring the outranking relationship constitute at least 75% of the total criteria weight.

This value can be adjusted to accommodate the specific case study or decision-making problem under consideration. The determination of a suitable cutting level is influenced by various factors such as the objectives to be achieved, the desired level of precision in ranking alternatives, and the inherent limitations associated with the problem at hand.

ELECTRE Tri involves the following steps:

Step 1: Calculate partial concordance indices

The partial concordance index is a measure of the degree of agreement between an alternative a_i and a reference profile b_k based on a specific criterion. It expresses the degree of evidence in favor of the hypothesis that alternative a_i is at least as good as reference profile b_k on a fuzzy scale ranging from 0 (alternative a_i is significantly inferior to reference profile b_k in terms of that criterion) to 1 (alternative a_i is at least as good as reference profile b_k in terms of that criterion).

It is necessary to calculate two partial concordance indices, $c_j(a_i, b_k)$ and $c_j(b_k, a_i)$, for each alternative a_i and reference profile b_k in order to establish the outranking relationship.

The partial concordance index $c_j(a_i, b_k)$ represents the degree to which alternative a_i is at least as good as the reference profile b_k , while $c_j(b_k, a_i)$ represents the degree to which the reference profile b_k is at least as good as alternative a_i .

The partial concordance function $c_j(a_i, b_k)$ is defined by three segments as shown in Figure 3.3:

- A constant value of 1 when $[g_j(b_k) - g_j(a_i)] \leq q_j(b_k)$, indicating a high degree of concordance.
- A constant value of 0 when $[g_j(b_k) - g_j(a_i)] \geq p_j(b_k)$, indicating no concordance.
- A decreasing linear segment between $g_j(b_k)$ and $p_j(b_k)$, indicating a gradual decline in concordance.

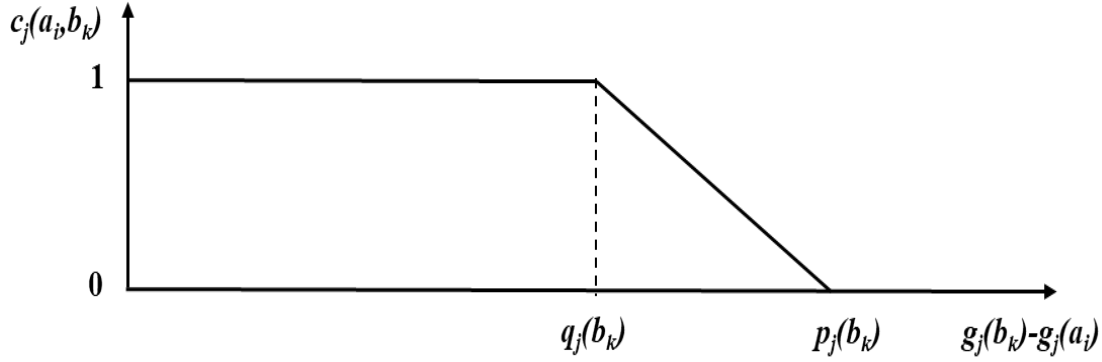


Figure 3.3. Illustration of partial concordance index.

Calculating both partial concordance indices is necessary because the outranking relationship between a_i and b_k is not always symmetric. These two partial concordance indices are not merely opposites of each other due to the non-reciprocal nature of the outranking relationship. The values of these indices can differ, reflecting the asymmetric nature of the outranking relation.

The calculation of both partial concordance indices enables the establishment of the overall outranking relation between a_i and b_k , which forms the basis for assigning alternatives to categories. Using only one partial concordance index would be insufficient to capture the complete outranking relationship between the alternative a_i and the reference profile b_k [8].

The partial concordance indices $c_j(a_i, b_k)$ and $c_j(b_k, a_i)$ are

$$c_j(a_i, b_k) = \begin{cases} 0 & \text{if } [g_j(b_k) - g_j(a_i)] \geq p_j(b_k) \\ 1 & \text{if } [g_j(b_k) - g_j(a_i)] \leq q_j(b_k) \\ \frac{g_j(a_i) - g_j(b_k) + p_j(b_k)}{p_j(b_k) - q_j(b_k)} & \text{otherwise} \end{cases} \quad (3.1)$$

and

$$c_j(b_k, a_i) = \begin{cases} 0 & \text{if } [g_j(a_i) - g_j(b_k)] \geq p_j(b_k) \\ 1 & \text{if } [g_j(a_i) - g_j(b_k)] \leq q_j(b_k) \\ \frac{g_j(b_k) - g_j(a_i) + p_j(b_k)}{p_j(b_k) - q_j(b_k)} & \text{otherwise} \end{cases} \quad (3.2)$$

where:

$g_j(b_k)$ - value of the reference profile b_k for the criterion j ;

$g_j(a_i)$ - alternative a_i evaluation on the criterion j ;

$p_j(b_k)$ - preference threshold for the criterion j and reference profile b_k ;

$q_j(b_k)$ - indifference threshold for the criterion j and reference profile b_k .

Step 2: Calculate global concordance indices

The global concordance index provides an overall assessment of how one alternative compares to another across all criteria. The global concordance indices are computed by aggregating the partial concordance indices for all criteria, while considering the significance of the weights assigned to each criterion.

It is necessary to calculate two global concordance indices, $c(a_i, b_k)$ and $c(b_k, a_i)$, to ensure consistency and symmetry in the assessment of the alternative's performance across criteria.

The global concordance indices $c(a_i, b_k)$ and $c(b_k, a_i)$ are

$$c(a_i, b_k) = \frac{\sum_{j=1}^n w_j c_j(a_i, b_k)}{\sum_{j=1}^n w_j} \quad (3.3)$$

and

$$c(b_k, a_i) = \frac{\sum_{j=1}^n w_j c_j(b_k, a_i)}{\sum_{j=1}^n w_j} \quad (3.4)$$

where:

n - number of criteria;

w_j - weight of criterion j .

In ELECTRE Tri, the weights represent relative importance coefficients or votes in favor of the criteria. The significance of a criterion increases with a higher weight.

Step 3: Calculate the partial discordance indices

The partial discordance index is a measure of the degree of disagreement between an alternative a_i and a reference profile b_k based on a specific criterion. It expresses the degree of evidence against the hypothesis that alternative a_i is at least as good as reference profile b_k based on a specific criterion on a fuzzy scale ranging from 0 (alternative a_i is at least as good as reference profile b_k in terms of that criterion) to 1 (alternative a_i is significantly inferior to reference profile b_k in terms of that criterion).

It is necessary to calculate two discordance indices, $d_j(a_i, b_k)$ and $d_j(b_k, a_i)$ for each alternative a_i and reference profile b_k in order to establish the outranking relationship, similar to the partial concordance indices.

Ensuring the calculation of both global concordance and discordance indices is necessary to comprehensively establish the outranking relation between alternatives and reference profiles, forming the basis for categorizing alternatives. Depending solely on one type of index would not capture the complete relationship [8].

The partial discordance $d_j(a_i, b_k)$ is defined by three segments as shown in Figure 3.4:

- A constant value of 1 when $[g_j(b_k) - g_j(a_i)] > v_j(b_k)$, indicating a high degree of discordance.
- A constant value of 0 when $[g_j(b_k) - g_j(a_i)] \leq p_j(b_k)$, indicating no discordance.
- A decreasing linear segment between $p_j(b_k)$ and $v_j(b_k)$, indicating a gradual decline in discordance.

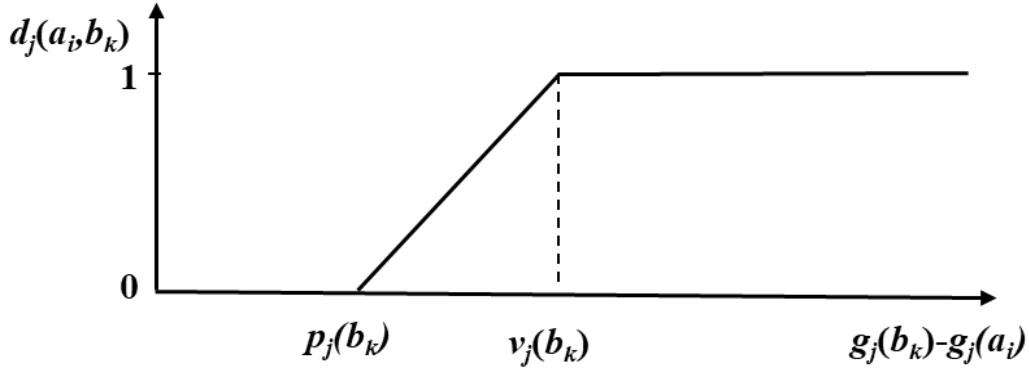


Figure 3.4. Illustration of partial discordance index.

The discordance index $d_j(a_i, b_k)$ and $d_j(b_k, a_i)$ are

$$d_j(a_i, b_k) = \begin{cases} 0 & \text{if } [g_j(b_k) - g_j(a_i)] \leq p_j(b_k) \\ 1 & \text{if } [g_j(b_k) - g_j(a_i)] > v_j(b_k) \\ \frac{g_j(b_k) - g_j(a_i) - p_j(b_k)}{v_j(b_k) - p_j(b_k)} & \text{otherwise} \end{cases} \quad (3.5)$$

and

$$d_j(b_k, a_i) = \begin{cases} 0 & \text{if } [g_j(a_i) - g_j(b_k)] \leq p_j(b_k) \\ 1 & \text{if } [g_j(a_i) - g_j(b_k)] > v_j(b_k) \\ \frac{g_j(a_i) - g_j(b_k) - p_j(b_k)}{v_j(b_k) - p_j(b_k)} & \text{otherwise} \end{cases} \quad (3.6)$$

where $v_j(b_k)$ is the veto threshold for the criterion j and reference profile b_k .

Step 4: Calculate the credibility index of the outranking relation

Calculating the degree of credibility is an essential step in determining the outranking relationships between alternatives a_i and reference profiles b_k . This calculation measures the degree of closeness between each alternative a_i and the corresponding reference profile b_k . The resulting credibility values are then compared to a cutting level threshold. When the credibility

value surpasses this threshold, it indicates a preference for the alternative a_i over the reference profile b_k , establishing an outranking relationship.

The degree of credibility is determined based on the global concordance and the discordance indices in both directions $\sigma(b_k, a_i)$ and $\sigma(a_i, b_k)$, to fully establish the outranking relationships between the alternatives and reference profiles [9].

The credibility index $\sigma(a_i, b_k)$ and $\sigma(b_k, a_i)$ are

$$\sigma(a_i, b_k) = C(a_i, b_k) \prod_{j \in J} \frac{1 - d_j(a_i, b_k)}{1 - C(a_i, b_k)} \quad (3.7)$$

where:

j - all the criteria for which $d_j(a_i, b_k) > C(a_i, b_k)$;

$d_j(a_i, b_k)$ - partial discordance between alternatives a_i and reference profiles b_k for criterion j ;

$C(a_i, b_k)$ - global concordance between alternative a_i and reference profile b_k .

and

$$\sigma(b_k, a_i) = C(b_k, a_i) \prod_{j \in J} \frac{1 - d_j(b_k, a_i)}{1 - C(b_k, a_i)} \quad (3.8)$$

where:

j - all the criteria for which $d_j(b_k, a_i) > C(b_k, a_i)$;

$d_j(b_k, a_i)$ - partial discordance between reference profiles b_k and alternatives a_i for criterion j ;

$C(b_k, a_i)$ - global concordance between reference profile b_k and alternative a_i .

In cases where the veto threshold is considered inappropriate, the credibility index aligns with the global concordance index.

Once all the credibility indices have been calculated, they are assessed against the cutting level threshold, which typically falls within the range of [0.5-1.0] and must surpass [1- highest weight/total weight] [10, 11].

This correlation between the credibility indices and cutting level facilitates the process of determining the outranking relation.

Step 5: Exploiting the outranking relation

The relationship between the performance of each alternative a_i and the reference profile b_k can be categorized into four types, depending on the values of $\sigma(a_i, b_k)$, $\sigma(b_k, a_i)$ and λ , as presented in Table 3.1.

Table 3.1. Relationship between the alternative and reference profile.

	$\sigma(a_i, b_k) \geq \lambda$	$\sigma(a_i, b_k) < \lambda$
$\sigma(b_k, a_i) \geq \lambda$	$a_i I b_k$	$a_i < b_k$
$\sigma(b_k, a_i) < \lambda$	$a_i > b_k$	$a_i R b_k$

- If $\sigma(a_i, b_k) \geq \lambda$ and $\sigma(b_k, a_i) \geq \lambda$,

Then $a_i I b_k$: a_i is indifferent to b_k .

If both $a_i S b_k$ and $b_k S a_i$ are true, then a_i and b_k are considered indifferent ($a_i I b_k$). This indicates an inability to express a strict preference between a_i and b_k . Both $a_i S b_k$ and $b_k S a_i$ being true (i.e., a_i outranks b_k and b_k outranks a_i) indicates an inability to express a strict preference between a_i and b_k .

- If $\sigma(a_i, b_k) \geq \lambda$ and $\sigma(b_k, a_i) < \lambda$,

Then $a_i > b_k$: a_i is preferred to b_k .

The preferred ($>$) outranking relation S is a weak preference relation, where $a_i S b_k$ means " a_i is at least as good as b_k ". If $a_i S b_k$ is true and $b_k S a_i$ is false, then a_i is strictly preferred to b_k ($a_i > b_k$).

- If $\sigma(a_i, b_k) < \lambda$ and $\sigma(b_k, a_i) \geq \lambda$,

Then $a_i < b_k$: a_i is not preferred to b_k .

If $a_i S b_k$ is true but $b_k S a_i$ is false, then a_i is preferred to b_k ($a_i > b_k$). Conversely, if $b_k S a_i$ is true but $a_i S b_k$ is false, then b_k is preferred to a_i ($b_k > a_i$).

- If $\sigma(a_i, b_k) < \lambda$ and $\sigma(b_k, a_i) < \lambda$,

Then $a_i R b_k$: a_i is incomparable to b_k .

If neither $a_i S b_k$ nor $b_k S a_i$ is true, then a_i and b_k are considered incomparable ($a_i R b_k$). This indicates a lack of sufficient information to determine the preference relation between a_i and b_k .

Indifference and incomparability are both symmetric relations, represented as follows:

$$a_i I b_k \Leftrightarrow b_k I a_i \text{ and } a_i R b_k \Leftrightarrow b_k R a_i$$

The presence of incomparability is one of the main characteristics that distinguishes MCDM methods, such as ELECTRE, from classical decision theory, where all alternatives must be comparable [13]. The ELECTRE Tri method incorporates fuzziness through the use of weak preference relations, indifference thresholds, and the ability to declare alternatives as incomparable. This method better captures the inherent fuzziness of criteria in multi-criteria decision problems.

The outranking relation does not directly assign an alternative to a specific and predefined category. Instead, it is essential to use either the optimistic or pessimistic exploitation procedures. These assignment procedures assess the relative comparison of an alternative a_i with the reference profiles b_k , thereby facilitating the determination of the appropriate category for the assignment of the alternative. The results of these two procedures may vary, providing a range of possible category assignments.

- The optimistic procedure assigns an alternative a_i to the highest category C_h for which a_i outranks the lower reference profile b_{k-1} . The procedure progresses from the lowest to the highest category, stopping at the first category where the condition is satisfied. This approach tends to assign alternatives to higher categories when there is ambiguity in the outranking relations.
- The pessimistic procedure assigns an alternative a_i to the lowest category C_l such that the reference profile b_k is preferred to a_i . The procedure progresses from the highest to the lowest category, stopping at the first category where the condition is satisfied. This approach tends to assign alternatives to lower categories when there is ambiguity in the outranking relations.

A summary table can be constructed to consolidate the results by assigning alternatives to categories based on the two procedures.

When the two assignment procedures are compared, discrepancies may arise when an alternative is found incomparable to one or more reference profiles. In such cases, the pessimistic

assignment procedure places the alternative in a lower category than the optimistic assignment procedure [14].

The pessimistic procedure categorizes alternatives in a more conservative manner, assigning alternatives to a lower category than determined by the reference profile. Conversely, the optimistic procedure adopts a less conservative approach, resulting in alternatives being assigned to higher categories [2].

Regardless of the assignment procedure used, it is essential to meet the following seven requirements [15, 16]:

- No alternative can exhibit indifference to more than one reference profile.
- Each alternative must be assigned exclusively to a single category (uniqueness/unicity).
- The assignment of a particular alternative to its designated category is independent of the assignment of any other alternative (independence).
- The procedure for assigning alternatives to categories must be consistent with the design of the reference profiles themselves (conformity).
- If two alternatives share the same outranking relationship with a particular reference profile, the alternatives must be assigned to the same category (homogeneity).
- If alternative a' outranks alternative a , then alternative a' must be assigned to a category that is at least as good as, or better than, the category of a (monotonicity).
- Grouping two neighboring categories must not affect the assignment of alternatives to categories that are not involved in the grouping (stability).

Illustrative example

A 2x2 matrix is employed to demonstrate the functionality of ELECTRE Tri method. This method analyzes two alternatives, each evaluated on two criteria, with the objective of assigning them to predefined categories using specific weights and thresholds.

Given data:

- **Alternatives:** a_1 and a_2
- **Criteria:** c_1 and c_2

The performance matrix based on alternatives and criteria is as follows:

	c_1	c_2
a_1	12	18
a_2	14	16

- **Weights:**
 $w_1=0.7$ for c_1
 $w_2=0.3$ for c_2
- **Thresholds for both criteria:**
Indifference threshold (q_j): 2
Preference threshold (p_j): 4
Veto threshold (v_j): 8

- **Categories:**

Alternatives are classified into the following categories based on their performance:

Poor

Mediocre

Excellent

- **Reference profiles:**

	c_1	c_2
b_1	10	15
b_2	15	20

Steps in the ELECTRE Tri method:

Step 1: Calculate partial concordance indices

Alternative a_i is at least as good as reference profile b_k ; concordance agrees with this statement.

Partial concordance indices $c_j(a_i, b_k)$ and $c_j(b_k, a_i)$ are calculated by

$$c_j(a_i, b_k) = \begin{cases} 0 & \text{if } [g_j(b_k) - g_j(a_i)] \geq p_j(b_k) \\ 1 & \text{if } [g_j(b_k) - g_j(a_i)] \leq q_j(b_k) \\ \frac{g_j(a_i) - g_j(b_k) + p_j(b_k)}{p_j(b_k) - q_j(b_k)} & \text{otherwise} \end{cases}$$

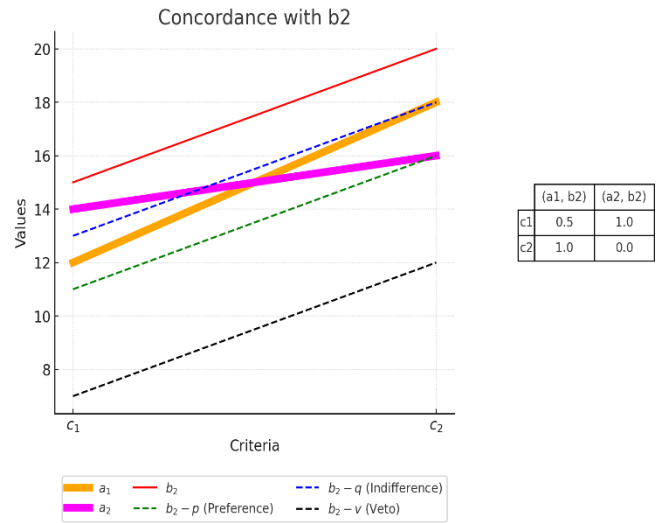
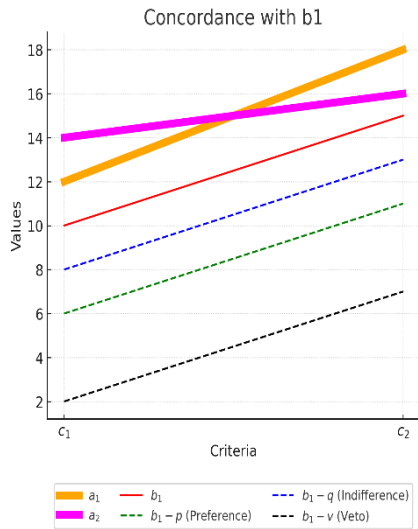
$$c_j(b_k, a_i) = \begin{cases} 0 & \text{if } [g_j(a_i) - g_j(b_k)] \geq p_j(b_k) \\ 1 & \text{if } [g_j(a_i) - g_j(b_k)] \leq q_j(b_k) \\ \frac{g_j(b_k) - g_j(a_i) + p_j(b_k)}{p_j(b_k) - q_j(b_k)} & \text{otherwise} \end{cases}$$

	(a_1, b_1)	(a_2, b_1)	(b_1, a_1)	(b_1, a_2)
c_1	1	1	1	0
c_2	1	1	0.5	1

	(a_1, b_2)	(a_2, b_2)	(b_2, a_1)	(b_2, a_2)
c_1	0.5	1	1	1
c_2	1	0	1	1

$$\frac{g_2(b_1) - g_2(a_1) + p_2}{p_2 - q_2}$$

$$15 - 18 + 4/4 - 2 = 0.5$$



Step 2: Calculate global concordance indices

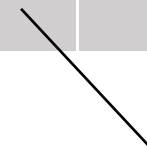
Alternative a_i is at least as good as reference profile b_k across all criteria by considering the weights assigned to each criterion. Global concordance indices $c(a_i, b_k)$ and $c(b_k, a_i)$ are calculated by

$$c(a_i, b_k) = \frac{\sum_{j=1}^n w_j c_j(a_i, b_k)}{\sum_{j=1}^n w_j}$$

$$c(b_k, a_i) = \frac{\sum_{j=1}^n w_j c_j(b_k, a_i)}{\sum_{j=1}^n w_j}$$

	(a_1, b_1)	(a_2, b_1)	(b_1, a_1)	(b_1, a_2)
C	1	1	0.85	0.30

	(a_1, b_2)	(a_2, b_2)	(b_2, a_1)	(b_2, a_2)
C	0.65	0.70	1	1



$$c(a_1, b_2) = \frac{(w_1 * c_1(a_1, b_2) + w_2 * c_2(a_1, b_2))}{w}$$

$$= \frac{(0.7 * 0.5 + 0.3 * 1)}{1} = 0.65$$

Step 3: Calculate partial discordance indices

Alternative a_i is at least as good as reference profile b_k ; discordance disagrees with this statement. Partial discordance indices $d_j(a_i, b_k)$ and $d_j(b_k, a_i)$ are calculated by

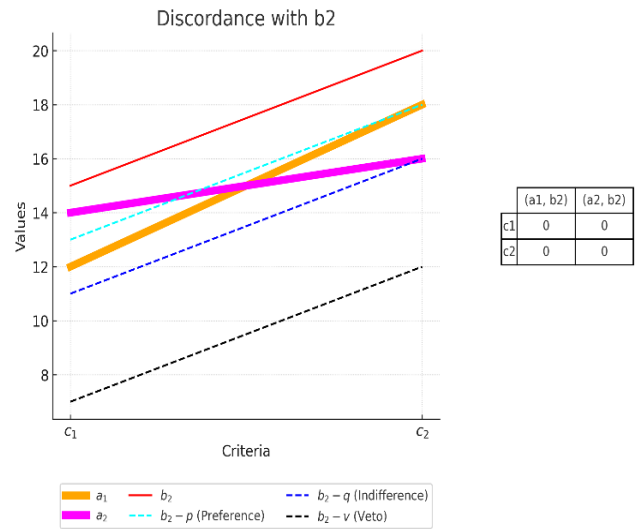
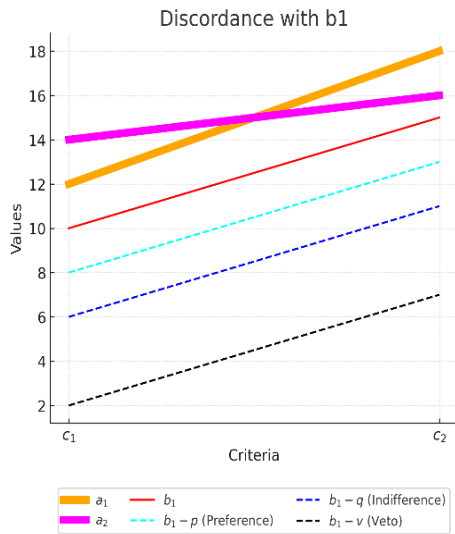
$$d_j(a_i, b_k) = \begin{cases} 0 & \text{if } [g_j(b_k) - g_j(a_i)] \leq p_j(b_k) \\ 1 & \text{if } [g_j(b_k) - g_j(a_i)] > v_j(b_k) \\ \frac{g_j(b_k) - g_j(a_i) - p_j(b_k)}{v_j(b_k) - p_j(b_k)} & \text{otherwise} \end{cases}$$

$$d_j(b_k, a_i) = \begin{cases} 0 & \text{if } [g_j(a_i) - g_j(b_k)] \leq p_j(b_k) \\ 1 & \text{if } [g_j(a_i) - g_j(b_k)] > v_j(b_k) \\ \frac{g_j(a_i) - g_j(b_k) - p_j(b_k)}{v_j(b_k) - p_j(b_k)} & \text{otherwise} \end{cases}$$

	(a_1, b_1)	(a_2, b_1)	(b_1, a_1)	(b_1, a_2)
d_1	0	0	0	0
d_2	0	0	0	0

	(a_1, b_2)	(a_2, b_2)	(b_2, a_1)	(b_2, a_2)
d_1	0	0	0	0
d_2	0	0	0	0

$(g_1(b_1) - g_1(a_1))$
 $(10-12) = -2 < 4$



Step 4: Calculate credibility index

The credibility index combines both concordance and discordance indices, providing a final assessment of whether an alternative a_i genuinely outranks a reference profile b_k . Credibility index $\sigma_j(a_i, b_k)$ and $\sigma_j(b_k, a_i)$ are calculated by

$$\sigma(a_i, b_k) = C(a_i, b_k) \prod_{j \in J} \frac{1 - d_j(a_i, b_k)}{1 - C(a_i, b_k)}$$

$$\sigma(b_k, a_i) = C(b_k, a_i) \prod_{j \in J} \frac{1 - d_j(b_k, a_i)}{1 - C(b_k, a_i)}$$

	(a_1, b_1)	(a_2, b_1)	(b_1, a_1)	(b_1, a_2)		(a_1, b_2)	(a_2, b_2)	(b_2, a_1)	(b_2, a_2)
σ	1	1	0.85	0.3	σ	0.65	0.70	1	1

Step 5: Exploit the outranking relation

Assuming a cutting level of $\lambda = 0.75$

If $\sigma(a_i, b_k) \geq \lambda$ and $\sigma(b_k, a_i) \geq \lambda$

a_i is indifferent to b_k ($a_i I b_k$)

If $\sigma(a_i, b_k) \geq \lambda$ and $\sigma(b_k, a_i) < \lambda$

a_i is preferred to b_k ($a_i > b_k$)

If $\sigma(a_i, b_k) < \lambda$ and $\sigma(b_k, a_i) \geq \lambda$

a_i is not preferred to b_k ($a_i < b_k$)

If $\sigma(a_i, b_k) < \lambda$ and $\sigma(b_k, a_i) < \lambda$

a_i is incomparable to b_k ($a_i R b_k$)

	$\sigma(a_i, b_k) \geq \lambda$	$\sigma(a_i, b_k) < \lambda$
$\sigma(b_k, a_i) \geq \lambda$	$a_i I b_k$	$a_i < b_k$
$\sigma(b_k, a_i) < \lambda$	$a_i > b_k$	$a_i R b_k$

	b_1	b_2
a_1	I	$<$
a_2	$>$	I

Step 6: Classification results

ELECTRE Tri can classify each alternative using both optimistic and pessimistic approaches once the credibility indices are calculated.

- **Optimistic classification**

From low to high.

$$a_1: a_1 I b_1 \quad a_1 < b_2 = a_1 \in (b_1, b_2)$$

$$a_2: a_2 > b_1 \quad a_2 I b_2 = a_2 > b_2$$

	<i>Poor</i> < b_1	<i>Mediocre</i> (b_1, b_2)	<i>Excellent</i> > b_2
a_1		X	
a_2			X

Optimistic classification a_1 : *Mediocre*

a_2 : *Excellent*

- **Pessimistic classification**

From high to low.

$$a_1: a_1 < b_2 \quad a_1 I b_1 = a_1 < b_1$$

$$a_2: a_2 I b_2 \quad a_2 > b_1 = a_2 \in (b_1, b_2)$$

	<i>Poor</i> < b_1	<i>Mediocre</i> (b_1, b_2)	<i>Excellent</i> > b_2
a_1	X		
a_2		X	

Pessimistic classification a_1 : *Poor*

a_2 : *Mediocre*

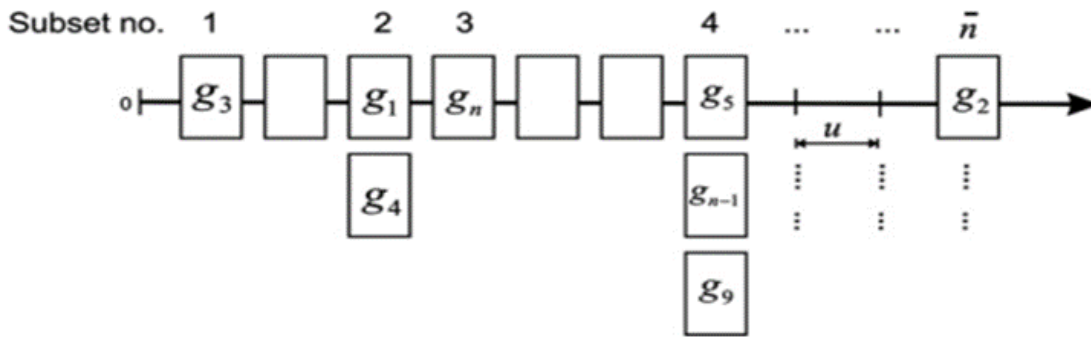
The ELECTRE Tri method assigns each alternative to the most appropriate category based on concordance and discordance values, employing both optimistic and pessimistic approaches. Overall, the ELECTRE Tri method provides a structured and transparent approach to multi-criteria decision making, supporting comprehensive evaluations that consider both opportunities and caution in the classification of alternatives.

3.3 Description of the revised Simos method

The revised Simos method has become a widely used technique for determining criteria weights, particularly in the context of MCDM applications. This method is considered an indirect elicitation method, as decision-makers do not explicitly state their own weightings. Instead, they are guided through a process of expressing their preferences, resulting in a more intuitive and understandable outcome [17-19].

The procedure is outlined in the following steps as shown in Figure 3.5:

- The decision-makers are presented with a set of cards (equivalent to the number of criteria being considered) that contain the names and brief descriptions of each criterion. These cards should not contain any information that could potentially influence decision-makers' preferences.
- The decision-makers proceed to rank the cards based on their relative importance, starting from the least important and ending with the most important.
- If decision-makers perceive specific criteria as equally important, these criteria are considered as "*ex aequo*" and grouped together at the same level of importance.
- Additional white cards are provided to the decision-makers, which represent larger differences in importance between subsequent item cards within the ordered set.
- The decision-makers have the ability to insert one or more white cards between item cards that are separated by white cards to express a greater importance gap. The presence of two successive item cards implies an importance gap of one unit, denoted as u . Similarly, two item-cards separated by a single white card implies an importance gap of $2u$, and so on.
- Finally, the decision-makers indicate the absolute importance gap between the most important and the least important item cards. This gap is represented by a numerical value, which provides valuable insight into the extent to which the most important card surpasses the least important ones. This ratio functions as a parameter for the calculation process and is denoted as z .



Legend:

$g_1, g_2, g_3, \dots, g_n$ = Number of criteria, u = Importance gap, n = Criteria rank

Figure 3.5. Schematic representation of Simos method [20].

The revised Simos method is used to calculate the weights of criteria once the necessary data is obtained. The algorithm assigns a numerical value to each criterion's weight. Specifically, it computes two weights for each criterion g_i , for $i = 1, \dots, n$.

The procedure for determining the criteria weights consists of the following steps:

Step 1: The non-normalized weight $k(r)$ is calculated.

For each *ex aequo* subset, based on their respective rank. These weights are denoted by $k(1), \dots, k(r), \dots, k(n^*)$.

$$\left\{ \begin{array}{l} e_r = e'_r + 1 \quad \forall r = 1, \dots, n^* - 1 \\ e = \sum_{r=1}^{n^*-1} e_r \\ u = \frac{z - 1}{e} \end{array} \right. \quad (3.9)$$

where:

e'_r - number of white cards between rank r and the immediately following rank $r + 1$;

e - total number of white cards, defined as the sum of the gaps between each consecutive rank;

n^* - total number of ranks. By convention, $k(1) = 1$;

z - significance of the last criterion compared to the first one in the ranking;

u - quantifies the spacing between the ranks of the criteria in the ranking relative to the total number of gaps between all ranks.

For example, if z (the rank of the last criterion) closely approximates 1, indicating it is ranked similarly to the first criterion, u assumes a small value. Conversely, when z is significantly higher, indicating a substantial gap between the ranks, u assumes a larger value.

For u , it is recommended to retain up to six decimal places.

The non-normalized weights $k(r)$ are calculated using

$$k(r) = 1 + u(e_0 + \dots + e_{r-1}) \quad (3.10)$$

Weights should be rounded to two decimal places. It is imperative that the *ex aequo* criteria maintain the same weight $k(r)$.

Step 2: The normalized weight k_i is calculated.

Consider g_i as a criterion with rank r and k'_i its non-normalized weight, expressed as $k'_i = k(r)$.

$$\begin{cases} K' = \sum_{i=1}^n k'_i \\ k_i^* = \frac{100}{K'} k'_i \end{cases} \quad (3.11)$$

In some cases, rounding techniques result in a total weight less than 100, which causes undesirable distortions that are considered unacceptable within the MCDM framework. In order to address this weight distortion, the following algorithm is proposed to mitigate its effect.

The normalized weight k_i from k_i^* is rounded by one of the three possible options, represented by w .

$$\begin{cases} w = 0, & \text{No figures after the decimal point} \\ w = 1, & \text{One figure after the decimal point} \\ w = 2, & \text{Two figures after the decimal point} \end{cases}$$

Using a limited number of decimal points allows adjustment to k_i^* to ensure that the sum of the final (normalized weights) k_i equals 100.

$$\sum_{i=1}^n k_i = 100 \quad (3.12)$$

Where k_i is the normalized weight of the criterion g_i , for $i = 1, \dots, n$.

3.4 Description of the probability density function (PDF)

The probability density function is a mathematical representation that effectively characterizes the probability distribution of a continuous random variable. Its versatility allows it to represent a diverse range of phenomena, including but not limited to the Poisson, uniform, normal and exponential distributions. These functions facilitate the representation and assessment of uncertainties inherent in input data while considering the associated parameter values [21].

Although any probability distribution may be employed, this discussion primarily focuses on the use of the normal distribution due to its ability to generalize the methodology effectively.

The selection of the normal distribution is supported by two primary factors:

- It is widely used to model real-world values across various domains, including natural sciences, social sciences, and economics, particularly when the actual data distribution is uncertain.
- When dealing with large sample sizes, the normal distribution often acts as a suitable approximation for the data distribution, irrespective of the underlying distribution. According to the Central Limit Theorem, the sampling distribution of the mean tends towards a normal distribution as sample size increases, provided it has a finite mean and variance [22]. This empirical finding has been supported by renowned statisticians, such as Pierre-Simon Laplace, who observed a tendency towards normality in data as sample size increases, through extensive experimentation across various fields.

The normal distribution, symbolically expressed as $N(\mu, \sigma^2)$, is characterized by two parameters: the mean value μ , which represents the average value of the dataset, and the standard deviation σ , which measures the dispersion of observations (i.e., the dataset) from the mean as shown in Figure 3.6. The variance is represented by $v = \sigma^2$. As a result, calculations are performed using μ and v as input parameters [23].

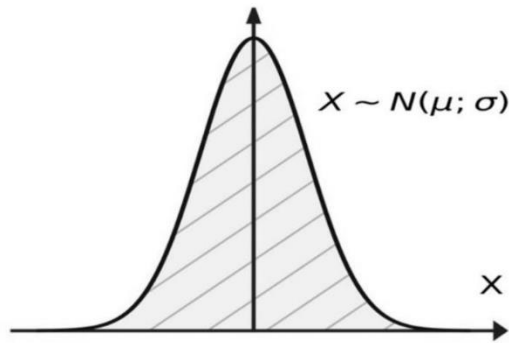


Figure 3.6. Illustration of a normal distribution.

3.5 Description of the Monte Carlo simulation

Simulation methods are employed in real-life scenarios to enhance the understanding of complex systems and support informed decision-making. Simulation involves replicating genuine scenarios, processes, or activities, whether existing or under development, to construct computer models [24, 25].

Monte Carlo simulation, which incorporates random variables obtained from probability distributions, is particularly valuable. It is a statistical technique that estimates operations in complex systems by randomly sampling from probability distributions that represent real-world scenarios. In this method, a dataset is generated, where a single data point is sampled from each probability distribution. Based on the available information, an appropriate probability distribution is selected to align with expectations, and all relevant parameters are defined.

Through iterative repetition, Monte Carlo simulation produces results that differ for each run of the model, effectively representing the inherent distributions of the input data. This variability enables the method to capture and reflect uncertainties in the data, providing more robust and realistic insights. As a result, Monte Carlo simulation facilitates decision-making by accounting for the uncertainties present in the data [26, 27].

A detailed analysis of how the proposed methodology can be implemented is provided in a step-by-step manner.

Step 1: Modeling uncertainty of input values through probability density functions

The fundamental assumption is that the uncertainty associated with input values is characterized through probability density functions instead of crisp (precise) values. Consequently, the assessment of each alternative based on each criterion incorporates the inherent uncertainty in the input data by treating it as a probability distribution. The performance of alternative a with respect to criterion j is represented as

$$S_a(C_j) = F_j(x) \quad (3.13)$$

where:

$S_a(C_j)$ - alternative a performance with respect to criterion j ;

F_j - probability density function for criterion j ;

x - variable that influences the performance of alternative a on criterion j .

Initially, a random value is selected from each probability distribution, forming a new performance matrix using these selected values. This process simulates various scenarios, enabling a more comprehensive evaluation of the alternatives under different conditions.

Step 2: Applying ELECTRE Tri to obtain optimistic and pessimistic classification

Once the dataset is obtained, ELECTRE Tri is applied to determine optimistic and pessimistic classification for each alternative. These classifications assign the alternatives to distinct categories for both optimistic and pessimistic procedures. Table 3.2 and Table 3.3 illustrate this process with 3 alternatives and 5 categories.

Table 3.2. Illustration of optimistic classification obtained in distinct categories.

Alternatives	C1	C2	C3	C4	C5
a1		X			
a2				X	
a3	X				

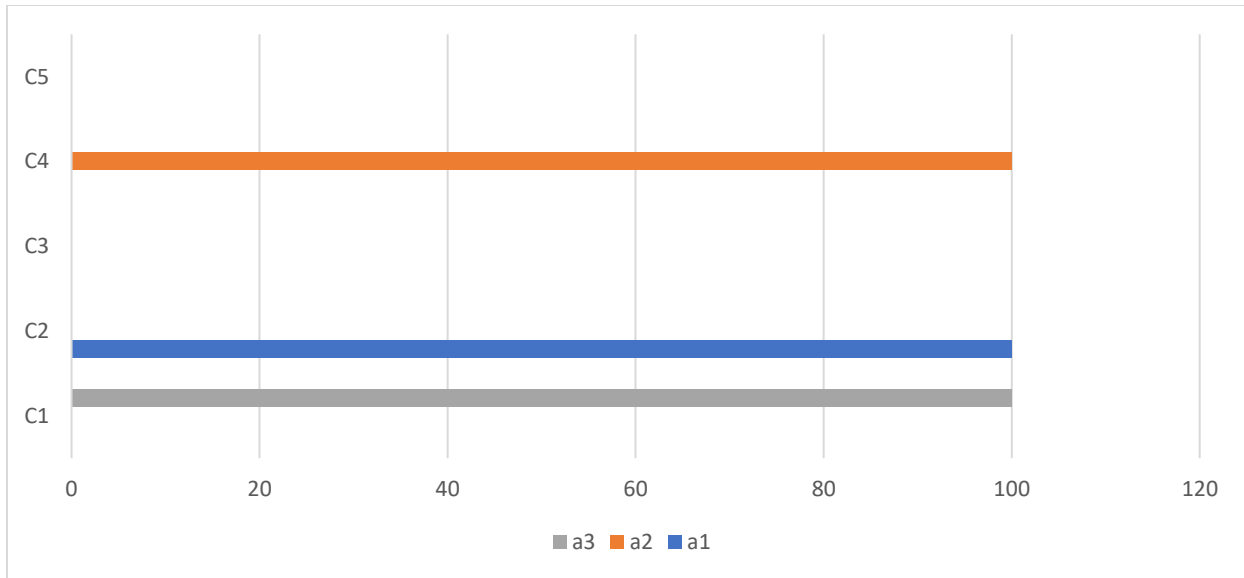


Figure 3.7. Illustration of optimistic classification obtained in distinct categories.

Table 3.3. Illustration of pessimistic classification obtained in distinct categories.

Alternatives	C1	C2	C3	C4	C5
a1	X				
a2			X		
a3	X				

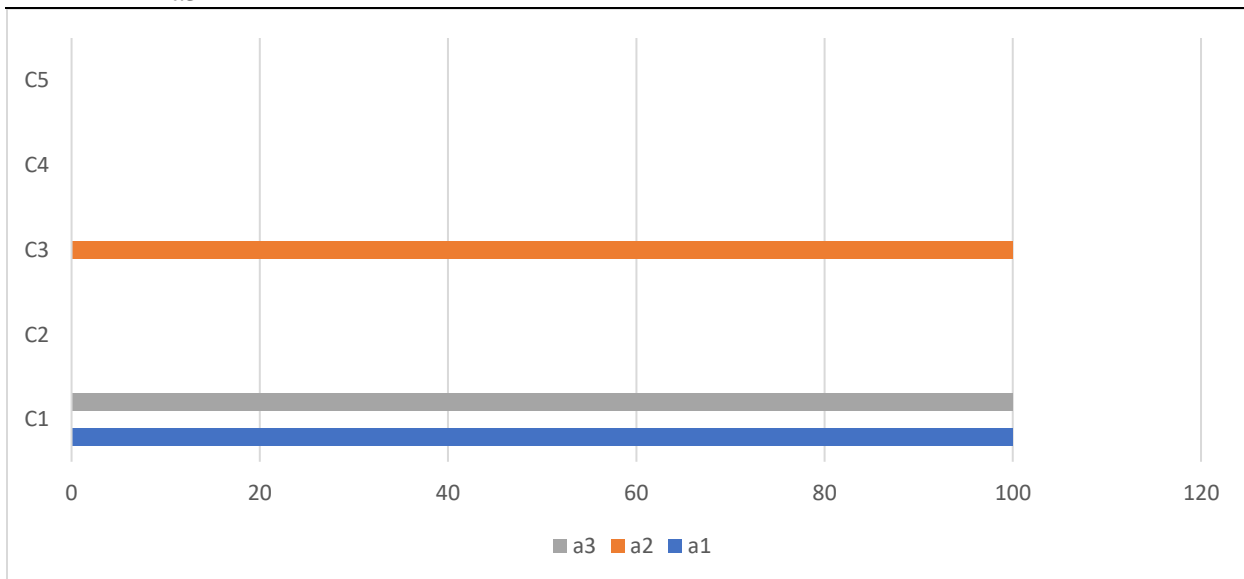


Figure 3.8. Illustration of pessimistic classification obtained in distinct categories.

Step 3: Employing iterative Monte Carlo simulation for robust classification analysis

The final step involves iterating the initial two steps multiple times using Monte Carlo simulation. In each iteration, a unique random value is selected from the probability distribution for each input parameter to produce a significant number of optimistic and pessimistic classification. The frequency of classifying each alternative into each category is determined by recalling the classification of each alternative across all iterations. This approach effectively captures the uncertainty in the input data.

Each alternative's classification is then expressed as percentage values, resulting in tables that illustrate the likelihood of each alternative being assigned to a specific category. Table 3.4 and Table 3.5 present these results, providing a clear depiction of the probabilities for each alternative's classification.

Table 3.4. Illustration of optimistic classification across multiple categories.

Alternatives	C1	C2	C3	C4	C5
a1	10.2	25.9	63.9	0	0
a2	0	0	100	0	0
a3	0	2.5	45.7	39.3	12.5

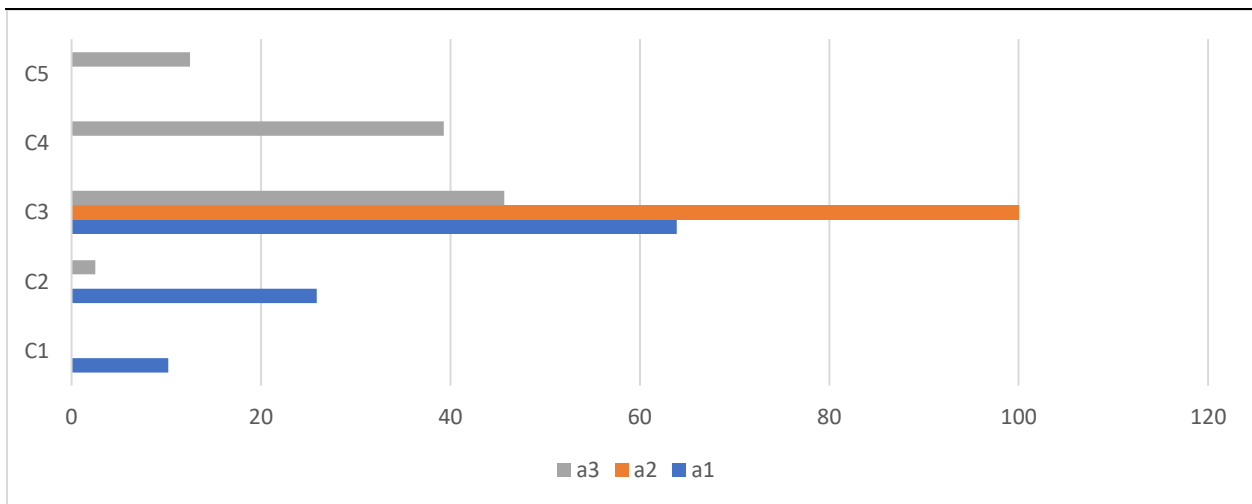


Figure 3.9. Illustration of optimistic classification across multiple categories.

Table 3.5. Illustration of pessimistic classification across multiple categories.

Alternatives	C1	C2	C3	C4	C5
a1	10.2	25.9	63.9	0	0
a2	0	100	0	0	0
a3	2.5	45.7	39.3	12.5	0

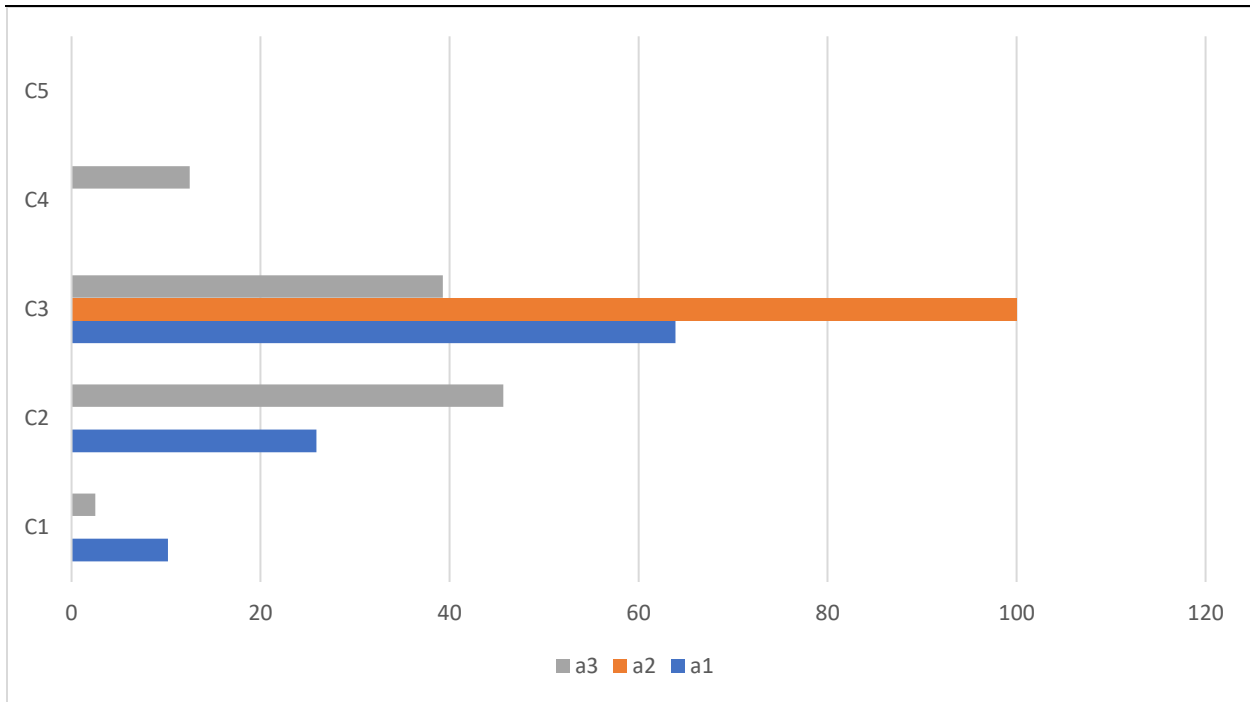


Figure 3.10. Illustration of pessimistic classification across multiple categories.

The outlined methodology offers the significant benefit of yielding more comprehensive results. Furthermore, these results not only classify the alternatives but also reveal their sensitivity to variations in the input data.

An automated algorithmic approach was developed to implement the probabilistic ELECTRE Tri method, resulting in enhanced processing capabilities and optimized computations. This algorithm systematically delineates upper and lower reference profiles, ensuring a uniform evaluation process and facilitating the classification of alternatives based on their performance relative to criteria.

The strength of this approach lies in its flexibility, allowing the reference profiles to be specified in accordance with the particular criteria and objectives of the decision-making process. This adaptability enables tailored adjustments that correspond to the unique context of each scenario,

ensuring that the categorization reflects the nuances and priorities inherent in the evaluation criteria.

The performance of each alternative, as measured against the established reference profiles, acts as a benchmark for determining the ranking categories. Through rigorous analysis and comparison, the algorithm simplifies complex data into actionable insights, thereby facilitating informed decision-making processes.

Overall, the integration of this computational algorithm enhances the efficiency and accuracy of the probabilistic ELECTRE Tri method. It provides a robust framework for evaluating and categorizing options in various decision-making contexts.

The selection of the Python programming language for implementing this method was influenced by several factors:

- Its renowned readability ensures that the code is comprehensible and easy to follow, facilitating smoother collaboration and maintenance.
- Its user-friendly interface reduces the learning curve, enabling decision-makers and users with diverse levels of technical expertise to interact seamlessly with the system.
- Its open-source nature and widespread adoption provide free accessibility and access to a dynamic community for support and development.
- Its flexibility, particularly in terms of cross-platform compatibility, reinforces its suitability for diverse computing environments.

These features make Python an optimal choice for implementing the probabilistic ELECTRE Tri method. Appendix D: *Python code implementation for the probabilistic ELECTRE Tri methodology*.

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4 Application to the case study

This chapter presents a detailed case study demonstrating the application of the probabilistic ELECTRE Tri method within a social housing building project located in the region of Lyon, France. It commences with a brief overview of the project context and case study, followed by a description of the data collection process. Subsequently, a performance matrix is developed to incorporate all collected data, and procedures are applied to establish the relevant parameters. The proposed methodology, the probabilistic ELECTRE Tri method, as explained in Chapter 3, is then implemented using the aforementioned data. Finally, the results are thoroughly analyzed and discussed.

4.1 Project context

Immobilière Rhône-Alpes (IRA), a subsidiary of 3F (Action Lodgment group) owns 20,000 housing units, while 3F itself owns 250,000 housing units. IRA initiated the Innovation in Energy Renovation (InER) project, which was awarded funding from the ALINOV innovation initiative by 3F to address energy inefficiency and the significant environmental impact within the social housing sector.

The InER project is an ambitious effort to promote sustainable renovation practices for electrically heated residential buildings, aligning with broader national energy transition goals. Through this project, IRA aims to enhance energy efficiency, reduce greenhouse gas emissions, maintain tenant affordability and comfort, and develop replicable solutions for similar housing units within the 3F group.

The InER project is driven by several objectives to ensure long-term economic, environmental, social, and technical benefits including:

- Enhancing energy efficiency labels for social housing units;
- Reducing energy consumption and optimizing greenhouse gas emissions;
- Incorporating renewable energy production and ecological energy storage;
- Optimizing cost control and system management;
- Enhancing tenant comfort and providing options for monitoring energy usage;
- Ensuring scalability and adaptability for similar housing renovations.

4.1.1 Project consortium

The InER project consortium consists of key organizations, each contributing specialized skills and knowledge as shown in Figure 4.1:

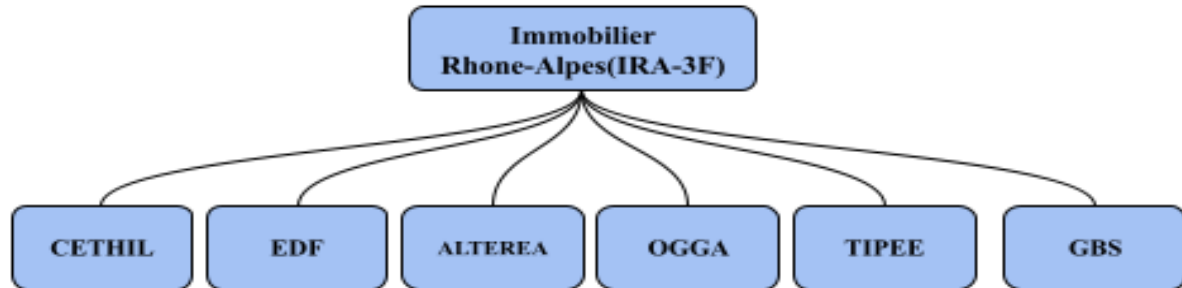


Figure 4.1. InER project consortium.

- **Immobilière Rhone-Alpes (IRA-3F):**
The project coordinator, responsible for overall management and feasibility assessments.
- **CETHIL (Centre for Energy and Thermal Sciences of Lyon):**
A research laboratory within INSA Lyon, providing expertise in multi-criteria decision making and experimental performance evaluations.
- **EDF (Electricity France):**
A leading electric utility company, contributing innovative solutions and overseeing implementation.
- **ALTEREA (Engineering technical firm):**
An engineering technical firm, specializing in building energy performance, leading technical coordination and cost estimation for scenarios.
- **OGGA (Energy management solutions firm):**
An energy management solutions firm, developing intelligent energy management systems and supporting the technical infrastructure for experimental evaluation.
- **TIPEE (Engineering technical firm):**
An engineering technical firm, focusing on sustainable building solutions, conducting thermal simulations and techno-economic evaluations.
- **GBS (Sociology firm):**
A sociology firm, providing insights into the social and organizational aspects of energy transition, ensuring tenant needs and expectations are incorporated.

Together, this consortium integrates economic, environmental, social, and technical perspectives, ensuring that the project's solutions are both comprehensive and feasible for the unique needs of social housing.

4.1.2 Project phases

The InER project is organized into different phases, focusing on establishing the groundwork for the project, from defining the research scope to selecting the optimal renovation solutions.

- **Phase 1: Project launch**

Defines the research scope and develops an analysis framework for evaluating potential renovation solutions.

- **Phase 2: Building characterization**

Involves conducting detailed audits of the selected buildings, assessing aspects such as energy usage, environmental impact, tenant comfort, and costs. This phase also evaluates the technical state of existing installations.

- **Phase 3: Solution identification**

Identifies potential renovation solutions based on the analysis framework, comparing these solutions against predefined criteria.

- **Phase 4: Prequalification**

Presents the results of the building characterization and potential renovation solutions to narrow down a shortlist for further study, selecting the most viable options for the next phase.

- **Phase 5: Qualification**

Involves conducting an in-depth analysis of each renovation solution, focusing on:

- Feasibility in terms of regulatory, technical, and implementation factors;
- Detailed cost analysis, covering investment, operational, and maintenance costs;
- Calculations of energy efficiency improvements and greenhouse gas reductions;
- Lifecycle analyses to understand long-term sustainability and impact.

- **Phase 6: Decision-making**

Concludes the initial studies phase with a final selection of solutions, chosen based on a comprehensive evaluation of feasibility, cost, and impact.

Through these structured phases, the InER project establishes a solid foundation for advancing sustainable social housing renovations.

The primary focus of this thesis is the application of the probabilistic ELECTRE Tri method, a robust decision-making framework developed to evaluate and classify proposed solutions within the context of a case study. This method rigorously assesses each solution's alignment with the project's goals, employing a data-driven approach to prioritize impactful and sustainable renovation strategies. The integration of this methodology ensures a systematic and effective pathway for achieving the project's objectives.

4.2 Case study

The social housing project consists of three neighboring buildings, designated B1, B2, and B3, located in the region of Lyon, France. These buildings were constructed in the year 1973 and comprise a total of 67 apartments that range in size from studio apartments to larger apartments with up to 5 rooms, covering an occupied area of 4,815 square meters as shown in Figure 4.2, Figure 4.3, and Figure 4.4.



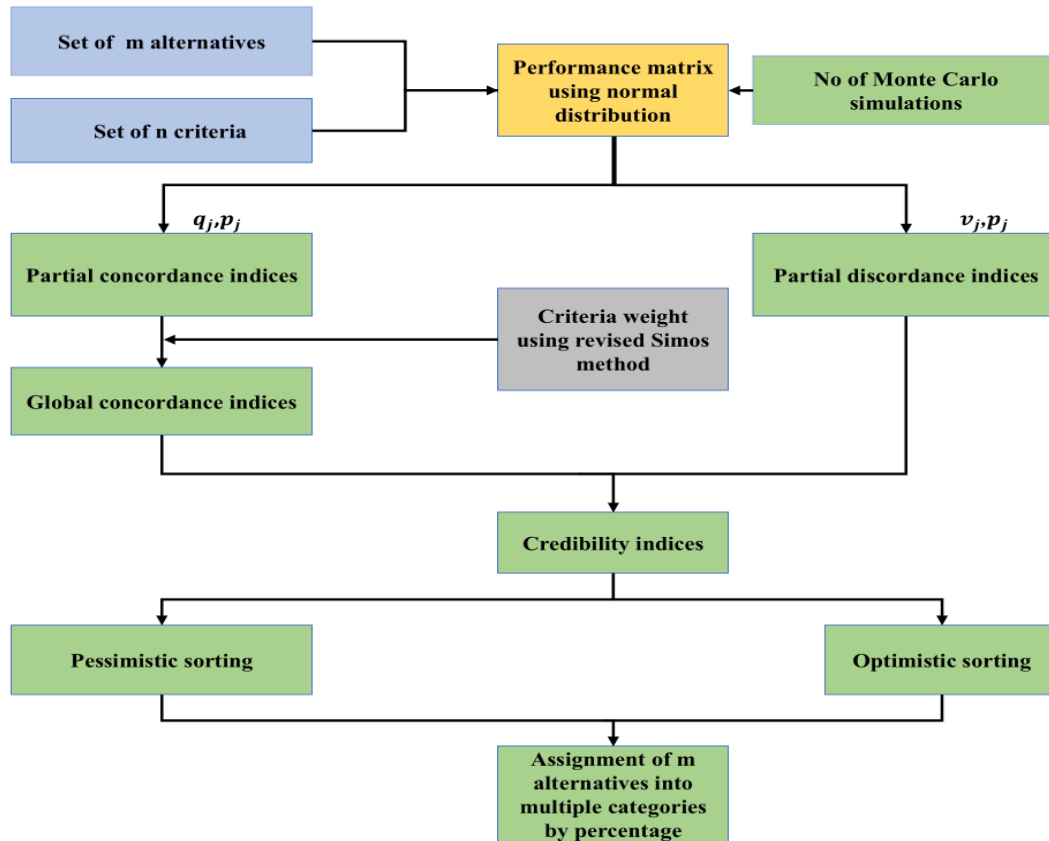
Figure 4.2. Social housing project overview used to demonstrate the methodology [1].



Figure 4.3. Social housing project east side [1].



Figure 4.4. Social housing project top view [1].



Legend:





	Input	Output	The responsible consortium firm
	Set of m alternatives, set of n criteria	Performance matrix	ALTEREA, OGGA, TIPEE, GBS
	Mean, variance	Normal distribution	CETHIL
	Revised Simos method	Weights of criteria	ALTEREA, OGGA, TIPEE, GBS CETHIL
	PELECTRE Tri Performance matrix, normal distribution, thresholds, weights of criteria, number of Monte Carlo simulations	Assignment of m alternatives into multiple categories by percentage	CETHIL

Figure 4.5. Application of the methodology from Chapter 3 to the case study.

A thermal assessment study was conducted by the engineering thermal firm to estimate the primary energy consumption and greenhouse gas emissions of the buildings. This estimation was conducted using a methodological framework approved by the relevant regulatory authority as presented in Table 4.1. The buildings were classified based on their energy consumption and greenhouse gas emissions as presented in Table 4.2. The firm identified potential improvement areas to enhance the energy efficiency of the buildings.

Table 4.1. Summary of regulatory approved thermal calculations [1].

Building	Net floor area (m ²)	Primary energy consumption (kWh/(m ² .year))	Primary energy consumption distribution based on use (kWh/(m ² .year))			
			Heat	DHW	Light	Vents
B1	1756	323	240	67	6	10
B2	1760	326	243	67	6	10
B3	1299	365	280	69	6	10

Table 4.2. Summary of primary energy consumption and greenhouse gas emissions [1].

Building	Primary energy consumption kWh/(m ² .year)		Greenhouse gas emissions kgeqco ₂ /(m ² .year)	
B1	323	231 to 330 E	18	11 to 20 C
B2	326	231 to 330 E	18	11 to 20 C
B3	365	331 to 365 E	20	11 to 20 C

The thermal performance assessment revealed deficiencies in the building envelopes of all three residential buildings. To determine the overall heat transfer coefficient H , a comparison was made with a reference coefficient H_{max} acting as a baseline for evaluating the overall heat transfer coefficient of the building envelope as presented in Table 4.3. This comparison clearly demonstrated insufficient insulation in the building envelopes.

Table 4.3. Summary of overall heat transfer coefficient [1].

Building	H_{max} (W/(m ² .year))	H (W/(m ² .year))	Difference
B1	1.00	1.25	-25%
B2	0.92	1.19	-29%
B3	0.86	1.15	-34%

The preliminary evaluation highlighted that the thermal losses in the buildings were similar as shown in Figure 4.6. The two primary sources of losses were identified as air infiltration and windows, accounting for approximately 28% and 25% of the total losses, respectively. These findings were attributed to the presence of outdated single-glazed wooden windows within the building envelopes and single-flow ventilation systems with self-regulating exhaust units in the bathroom spaces.

Thermal bridges and external walls also contributed significantly to the losses, accounting for approximately 21% and 19%, respectively. The thermal bridges, particularly those occurring at the junction of walls and floors, present a major concern due to significant heat losses resulting from the operation of an electric underfloor heating system.

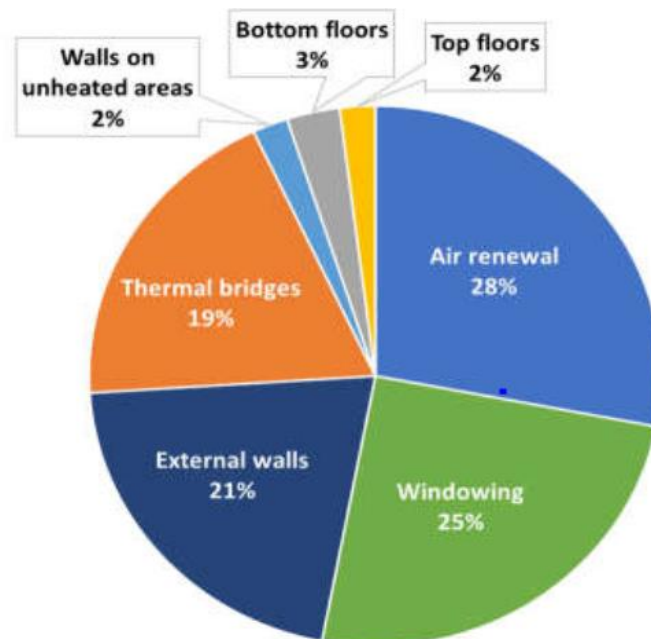


Figure 4.6. Heat loss distribution of the buildings [1].

The primary step in any renovation project focuses on implementing passive strategies that aim to reduce heat transfer. Examples of these measures include:

- Upgrading thermal insulation materials to decrease heat transfer through the building envelope.
- Installing an effective ventilation system to optimize air circulation and indoor air quality.
- Eliminating thermal bridges by applying external insulation layers to prevent heat loss through structural elements that penetrate the insulation.

These approaches aim to reduce energy consumption while maintaining comfortable indoor conditions and promoting sustainable building practices.

4.3 Identification of potential renovation scenarios

Various renovation scenarios were examined, each involving elementary renovation actions with specific objectives. These actions were thoughtfully selected in consultation with manufacturers and experts to align with the project's goals. The elementary renovation actions included modifications to specific building components or systems aimed at enhancing building performance.

The elementary renovation actions were consolidated to establish global renovation scenarios, ensuring coherence and compatibility. The objective was to develop global renovation scenarios that effectively address the project objectives and could be evaluated according to the relevant criteria.

To develop potential global renovation scenarios, the elementary renovation actions were classified into seven distinct families as presented in Table 4.4. The purpose of these elementary renovation actions was to improve thermal comfort and occupant satisfaction while simultaneously reducing energy usage and carbon emissions. These seven distinct families involve specific measures for enhancing energy performance as defined in the European Union's Directive 2010/31/EU on the Energy Performance of Buildings and the European Union's Directive 2012/27/EU on Energy Efficiency [2].

Table 4.4. The basic elementary actions [1].

Class	Options
Electric floor heating	Connected Disconnected
Auxiliary heating	Existing electric radiant heater Smart electric radiant heater Smart electric storage heater Low temperature hot water radiators Automated towel warmer
Domestic hot water production	Existing individual electric hot water tank Smart individual electric hot water tank Smart individual thermodynamic hot water tank Individual condensing gas boiler Collective thermodynamic Individual solar centralized hot water tank
Controlled mechanical ventilation system	Existing controlled mechanical ventilation Controlled mechanical ventilation system with single-flow humidity-sensitive exhaust units
On-site energy production	None Roof mounted photovoltaic solar panels Roof mounted thermal solar panels Roof mounted hybrid solar panels
Exterior joinery replacement	None All exterior joinery replaced with double glazing All exterior joinery replaced with triple glazing Only balcony windows replaced with double glazing
Exterior wall insulation	Existing wall insulation Installing exterior thermal insulation

The development of the seven global renovation scenarios involved integrating various elementary renovation actions, each tailored to meet the project objectives:

- S1: Maintenance of existing individual electric heating and domestic hot water systems.
- S2: Enhancement of individual electric heating and domestic hot water systems (version 1).
- S3: Enhancement of individual electric heating and domestic hot water systems (version 2).
- S4: Individual electric heating and thermodynamic domestic hot water systems.
- S5: Individual gas heating and domestic hot water systems.
- S6: Individual electric heating and collective thermodynamic domestic hot water systems.
- S7: Individual electric heating and collective solar domestic hot water systems.

Each of the seven global renovation scenario Families was further subdivided into four distinct variants, denoted as $S_{i,1}$ to $S_{i,4}$, where $i = 1, 2, \dots, 7$. These variants were subjected to comprehensive assessments to measure their respective performance impacts. Adjustments were made to the on-site renewable energy production and joinery in each variant, while the space heating and domestic hot water systems remained unchanged. For instance, certain renovation scenarios involved the implementation of roof-mounted photovoltaic solar panels or the replacement of balcony windows, whereas others involved the substitution of exterior windows and the installation of roof-mounted hybrid solar panels.

Overall, a total of 28 potential scenarios were considered, with each main Family (7 in total) having 4 variants, resulting in a comprehensive exploration of various global renovation scenarios aimed at identifying optimal performance outcomes while maintaining project objectives and sustainability goals as shown in Figure 4.7.

While formulating the seven global renovation scenario Families, the focus was on ensuring the consistency and coherence of the combinations through individual elementary renovation actions. The focus was specifically on scenario 'S1.1', which represents the building in its original state, without any modifications to the building envelope or technical systems. This particular scenario acts as a baseline for assessing other scenarios and includes the installation of heated towel rails in bathrooms to enhance thermal comfort in a simple and cost-effective manner. Furthermore, these scenarios uniformly incorporate exterior thermal insulation as a primary measure to reduce the building's energy consumption.

N°	Basic renovation actions	Global renovation scenarios																											
		S1.1	S1.2	S1.3	S1.4	S2.1	S2.2	S2.3	S2.4	S3.1	S3.2	S3.3	S3.4	S4.1	S4.2	S4.3	S4.4	S5.1	S5.2	S5.3	S5.4	S6.1	S6.2	S6.3	S6.4	S7.1	S7.2	S7.3	S7.4
a ₁	Electric floor heating																												
a _{1.1}	Connected	•	•	•	•	•	•	•	•																				
a _{1.2}	Disconnected									•	•	•	•	•	•	•	•	•	•	•	•								
a ₂	Auxiliary heating																												
a _{2.1}	Existing electric radiant heater	•	•	•	•	•	•	•	•																				
a _{2.2}	Smart electric radiant heater					•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a _{2.3}	Smart electric storage heater									•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a _{2.4}	Low temperature hot water radiators																	•	•	•	•								
a _{2.5}	Automated towel warmer					•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a ₃	Domestic hot water production																												
a _{3.1}	Existing individual electric hot water tank	•	•	•	•	•	•	•	•																				
a _{3.2}	Smart individual electric hot water tank					•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a _{3.3}	Smart individual thermodynamic hot water tank																												
a _{3.4}	Individual condensing gas boiler																	•	•	•	•								
a _{3.5}	Collective thermodynamic																					•	•	•	•				
a _{3.6}	Individual solar centralized hot water tank																									•	•	•	•
a ₄	Controlled mechanical ventilation system																												
a _{4.1}	Existing controlled mechanical ventilation	•				•	•	•	•																				
a _{4.2}	CMV with single flow humidity sensitive exhaust units	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a ₅	On-site energy production																												
a _{5.1}	None	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a _{5.2}	Roof mounted photovoltaic solar panels					•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a _{5.3}	Roof mounted thermal solar panels																									•			
a _{5.4}	Roof mounted hybrid solar panels																									•	•	•	
a ₆	Exterior joinery replacement																												
a _{6.1}	None	•				•	•	•	•																				
a _{6.2}	All exterior joinery replaced with double glazing	•	•			•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
a _{6.3}	All exterior joinery replaced with triple glazing			•																						•			
a _{6.4}	Balcony windows replaced with double glazing				•																						•		
a ₇	Exterior wall insulation																												
a _{7.1}	Existing wall insulation	•				•	•	•	•																				
a _{7.2}	Exterior thermal insulation		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	

Figure 4.7. Identification of global renovation scenarios from the basic elementary actions [1].

4.4 Identification of sustainability criteria

A detailed set of evaluation criteria was identified to assess the performance of various energy scenarios and achieve the project objectives. The process of identifying these criteria for evaluating energy renovation scenarios involved a comprehensive review of the state-of-the-art, consultation with industry experts, and feedback from academic researchers. This process was instrumental in finalizing the criteria list, eliminating irrelevant ones, and resulting in a practical yet meaningful number of criteria suitable for sustainability assessment.

These decision criteria are classified into four primary categories:

- Economic
- Environmental
- Social
- Technical

Each criterion was assessed through quantitative, qualitative, or binary methods, based on the specific assessment unit. Various sets of criteria can be employed to evaluate the economic, environmental, social, and technical aspects of sustainability. The selection of criteria is influenced by the preferences and priorities of stakeholders. The classification of the criteria into categories is particularly beneficial in situations involving a large number of criteria. This classification further facilitates a comprehensive examination of the issue by highlighting the complementary nature of the scenarios across different sustainability factors.

In this research, a total of sixteen evaluation criteria were selected to assess the energy renovation scenarios across the four sustainability factors. Table 4.5 provides a detailed list of these criteria, including their data type, units of measurement, and preference direction (maximization or minimization). However, it is essential to acknowledge that this list is not universally applicable, and users have the flexibility to modify or adapt it based on specific location, project nature, and stakeholder preferences. Moreover, Appendix B: *Means of criteria evaluation* presents a detailed description of the means used to evaluate the criteria in the case study.

Table 4.5. Identification of sustainability criteria.

Category	Criteria	Data type	Unit	Preference
Economic	Investment cost	Quantitative	€ excl. taxes	Minimize
	Reinvestment cost over a period of 30 years	Quantitative	€ excl. taxes	Minimize
	Possibility of financial aid	Quantitative	€ excl. taxes	Maximize
	Operation cost	Quantitative	€ excl. taxes	Minimize
	Energy cost effectiveness of the solution	Quantitative	€ excl/kWh/ (m ² . year)	Minimize
Technical	Ease to integrate into existing building	Qualitative	5-point scale	Maximize
	Implementation in occupied space	Binary	Yes/No	Maximize
	Maintenance/Serviceability	Qualitative	5-point scale	Maximize
	Ease of monitoring/Energy management	Qualitative	5-point scale	Maximize
Social	Impact of the cost on the tenant	Quantitative	€ excl. taxes	Minimize
	Level of thermal comfort	Qualitative	5-point scale	Maximize
	Level of acoustic comfort	Qualitative	5-point scale	Maximize
	Aesthetics and space requirements	Qualitative	5-point scale	Maximize
Environmental	Energy consumption reduction	Quantitative	kWh/m ² . year	Maximize
	Carbon dioxide emissions avoided	Quantitative	Ton of CO ₂ /year	Maximize
	Production place	Quantitative	Km	Minimize

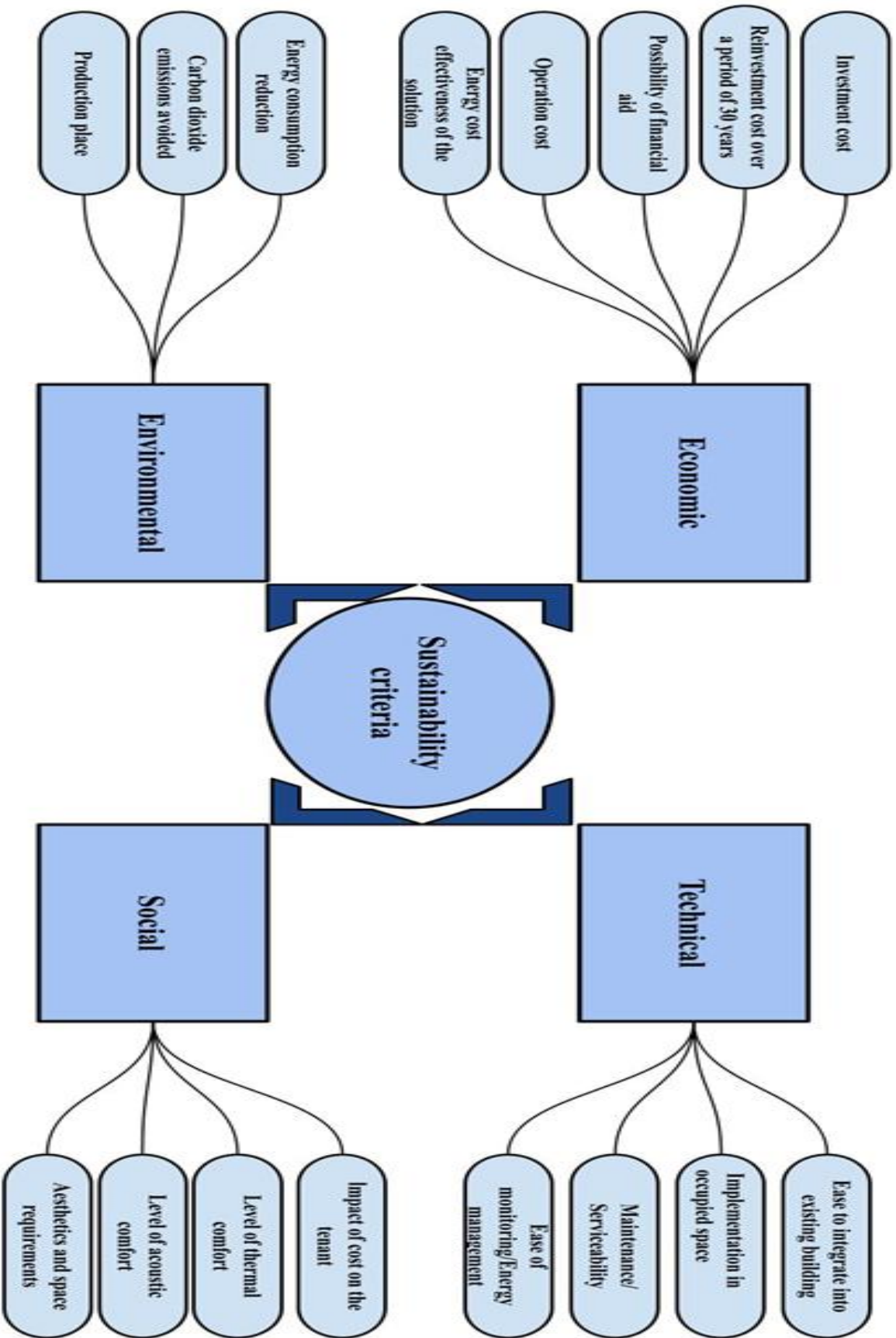


Figure 4.8. Identification of sustainability criteria.

Table 4.6. Description of sustainability criteria.

Category	Criteria	Description
Economic	Investment cost	The initial capital cost required for implementing the proposed energy renovation solution is evaluated against the available project budget to assess its economic viability.
	Reinvestment cost over a period of 30 years	The projected cost of replacing equipment and components approaching the end of their useful life is distributed over a period of 30 years.
	Possibility of financial aid	The potential financial assistance and economic incentives, such as grants, subsidies, or tax credits from the government, are examined to determine the availability of specific incentives for implementing the proposed energy renovation solution.
	Operation cost	The recurring costs associated with the operation, maintenance, repairs, replacements, and consumables required for the proposed energy renovation solution are evaluated.
	Energy cost effectiveness of the solution	The correlation between the costs associated with the proposed energy renovation solution and the benefits obtained through reduced overall energy consumption is analyzed.
Technical	Ease to integrate into existing building	The proposed energy renovation solution's capability to integrate effectively into an existing building is assessed. This includes evaluating whether there is adequate space for system installation, such as sufficient space on the roof or façade, and considering architectural modifications such as creating openings or establishing new hydraulic and electrical networks.
	Implementation in occupied space	The feasibility of carrying out the installation and implementing the proposed energy renovation solution in occupied spaces is examined. It may involve temporary area shutdowns, occupant relocation, or limited access to certain areas.
	Maintenance/Serviceability	Maintenance requirements are assessed based on factors such as maintenance frequency, system accessibility, and the availability of spare parts or components for replacement or repair.
	Ease of monitoring/Energy management	The simplicity of control mechanisms (e.g., valves, power regulators, etc.) and the ability to monitor and meter energy produced and distributed to individual tenants are evaluated.

Social	Impact of cost on the tenant	The financial impact of the proposed energy renovation solution on tenants is assessed, considering additional costs incurred and potential savings.
	Level of thermal comfort	The operation of the proposed energy renovation solution is evaluated for its ability to provide sufficient thermal comfort to tenants.
	Level of acoustic comfort	The operation of the proposed energy renovation solution is evaluated for potential noise pollution and its effects on tenants.
	Aesthetics and space requirements	The proposed energy renovation solution is evaluated for its effective and aesthetic integration into the residential units.
Environmental	Energy consumption reduction	Energy savings are quantified by comparing post-renovation energy consumption with the pre-renovation state.
	Carbon dioxide emissions avoided	The amount of carbon dioxide emissions avoided is calculated based on reduced energy consumption and the type of energy source used.
	Production place	The average geographical distance between the production sites of the proposed energy renovation solution components and the implementation site is evaluated for its environmental impact.

4.5 Weights of sustainability criteria

It is necessary to determine the significance of each criterion during the decision-making process due to differences in their impact on the overall outcome. Assigning weights to each selected criterion is essential for achieving this objective. Moreover, the significance of criteria may differ among decision-makers.

The revised Simos method, a widely used weighting method in the domain of outranking methods, was employed to compute these weights. To enhance the efficiency of determining criteria weights, criteria are classified into various categories. This method facilitates the initial allocation of weights among the categories and subsequently to each criterion within its respective category. Additionally, grouping criteria into categories helps manage a large number of criteria.

The calculation of weights for criteria involves three distinct steps:

1. The initial step involves ranking the four categories of criteria and assigning absolute weights to each category using the revised Simos method as presented in Table 4.7.
2. The subsequent step involves ranking the criteria independently within each category and assigning relative weights to each criterion within its corresponding category.
3. Finally, the absolute weight of each criterion is determined by multiplying its relative weight by the absolute weight of its respective category as presented in Table 4.8, Table 4.9, Table 4.10, and Table 4.11.

When using this method, the significance of each criterion is taken into account in the decision-making process. Appendix A: *Weighting of evaluation criteria using the revised Simos method* presents the calculation of criteria weights in more detail.

Table 4.7. Weights of sustainability criteria.

Criteria	Category	Absolute weight
g1	Economic	45.00
g2	Technical	25.00
g3	Social	15.00
g4	Environmental	15.00

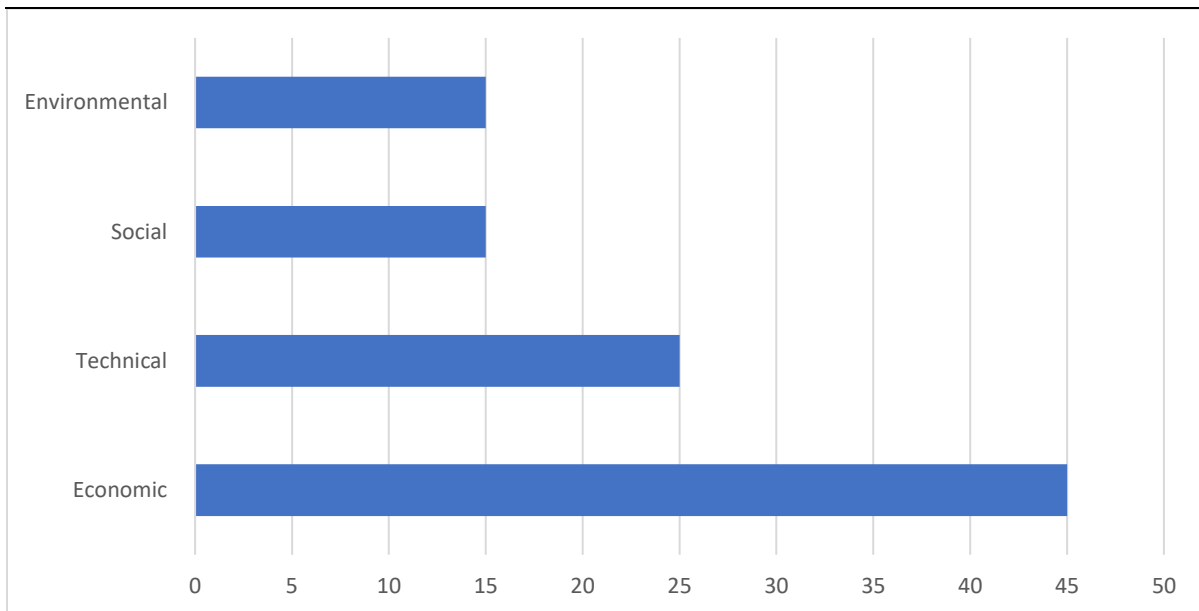


Figure 4.9. Weights of sustainability criteria.

Table 4.8. Weights of economic criteria.

Criteria	Description	Relative weight	Absolute weight
g1.1	Investment cost	27.27	12.27
g1.2	Reinvestment cost over a period of 30 years	23.64	10.64
g1.3	Possibility of financial aid	9.09	4.09
g1.4	Operation cost	27.27	12.27
g1.5	Energy cost effectiveness of the solution	12.73	5.73

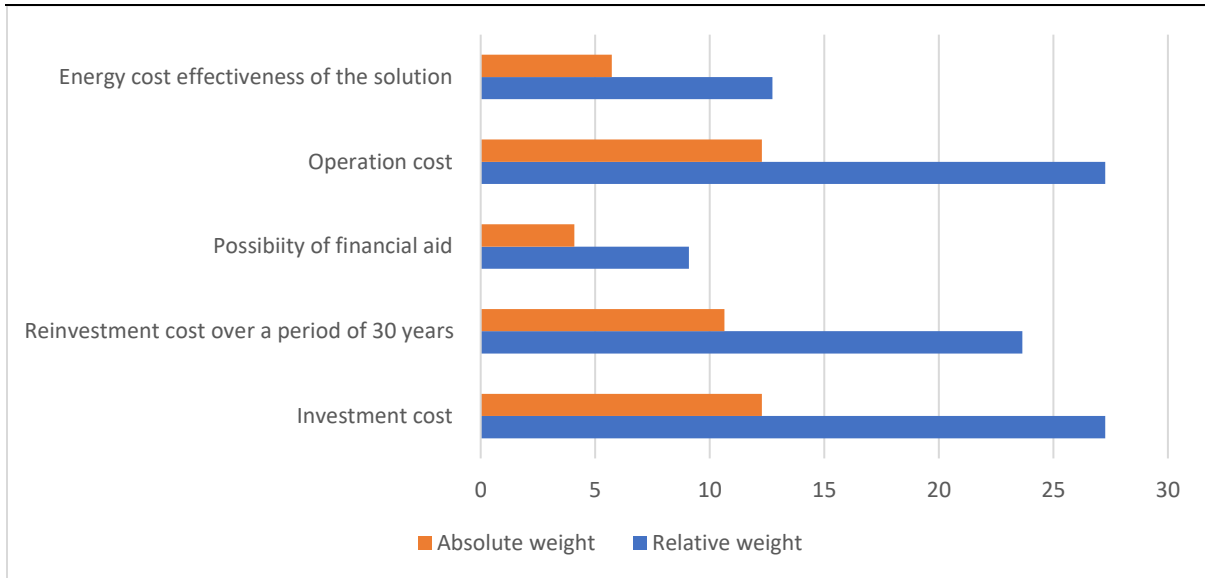


Figure 4.10. Weights of economic criteria.

Table 4.9. Weights of technical criteria.

Criteria	Description	Relative weight	Absolute weight
g2.1	Ease to integrate into existing building	28.57	7.14
g2.2	Implementation in occupied space	42.85	10.71
g2.3	Maintenance/Serviceability	14.29	3.57
g2.4	Ease of monitoring/Energy management	14.29	3.57

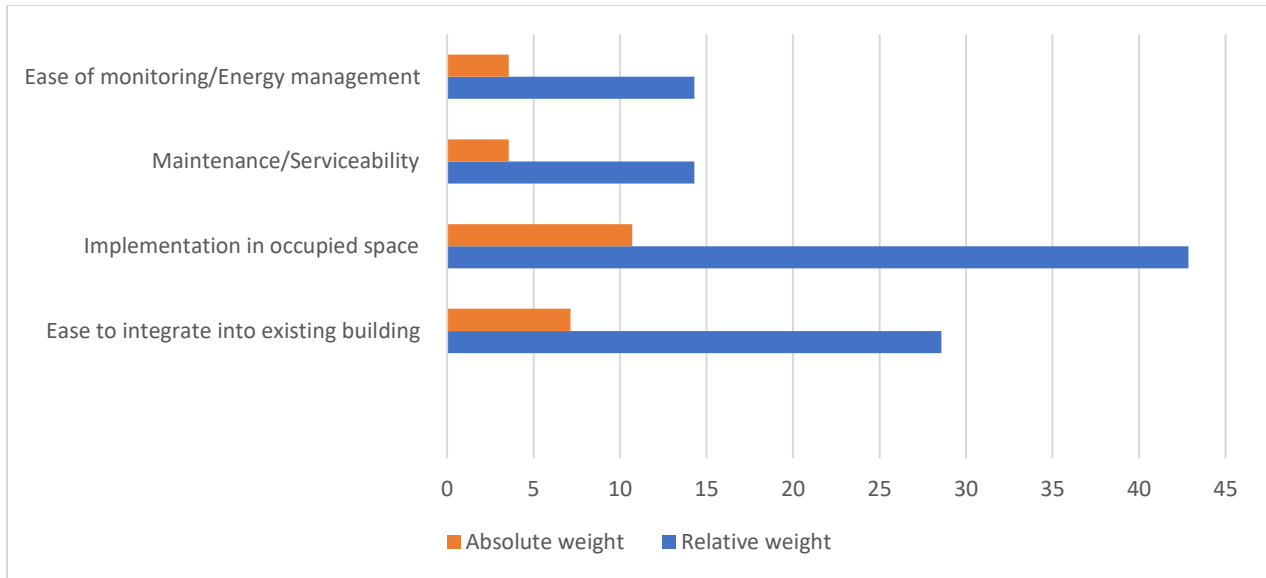


Figure 4.11. Weights of technical criteria.

Table 4.10. Weights of social criteria.

Criteria	Description	Relative weight	Absolute weight
g3.1	Impact of the cost on the tenant	40.90	6.14
g3.2	Level of thermal comfort	31.82	4.77
g3.3	Level of acoustic comfort	13.64	2.05
g3.4	Aesthetics and space requirements	13.64	2.05

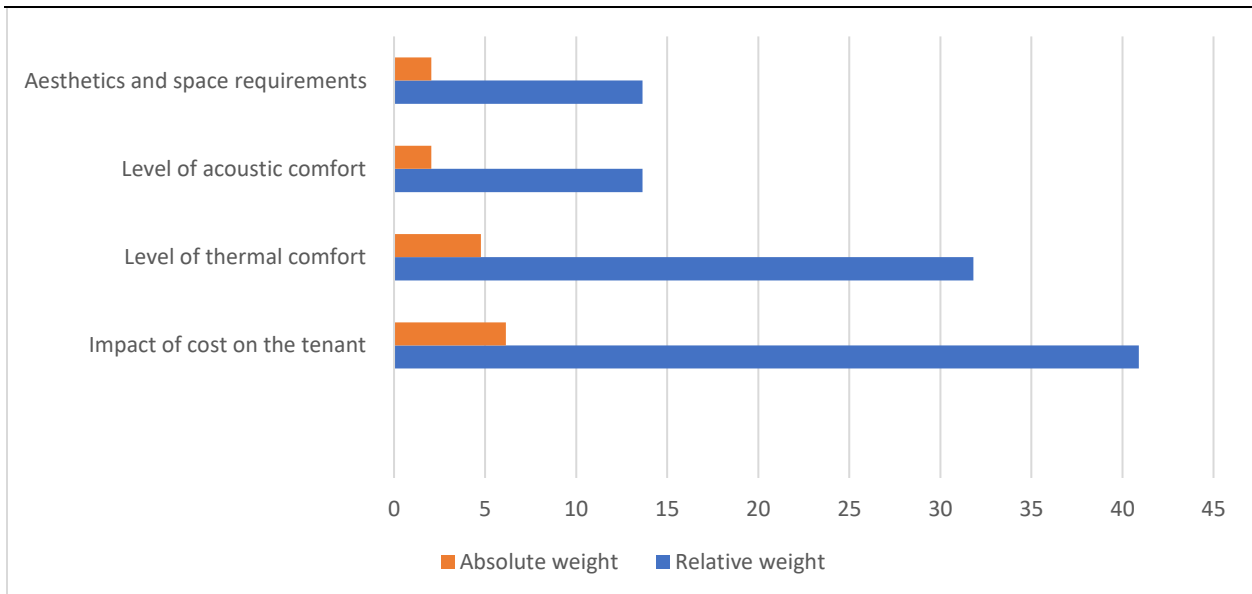


Figure 4.12. Weights of social criteria.

Table 4.11. Weights of environmental criteria.

Criteria	Description	Relative weight	Absolute weight
g4.1	Energy consumption reduction	50.00	7.50
g4.2	Carbon dioxide emissions avoided	33.33	5.00
g4.3	Production place	16.67	2.50

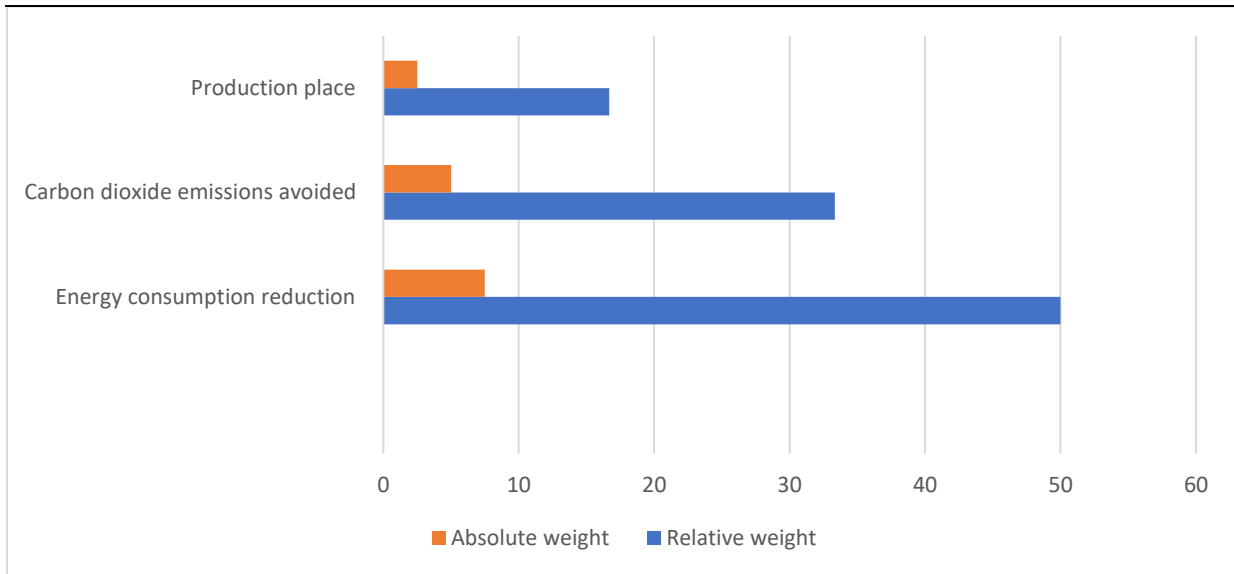


Figure 4.13. Weights of environmental criteria.

4.6 Performance matrix

The performance matrix evaluates each global renovation scenario according to various criteria. This matrix was developed through a collaborative evaluation process involving vendors, installation professionals, engineering technical firms, a sociology consultancy firm, and rental property management specialists. It includes the evaluation of each global renovation scenario as shown in Figure 4.7, according to the criteria defined in Table 4.5.

To evaluate the economic feasibility of each scenario, data were collected from vendors and service providers regarding procurement, implementation, and maintenance costs, which were used to calculate the total cost for each scenario. Appendix C: *Estimation of cost of different materials* presents some of the cost data used in assessing the economic criteria. Moreover, reinvestment costs over a thirty-year period were estimated based on the average lifespan of major components within the scenarios, which aligns with the standard building maintenance intervals.

Experts in the fields of engineering and sociology firms were engaged for consultation, specifically focusing on building energy transition, in order to evaluate the technical and social

effectiveness of the scenarios. The qualitative criteria were assessed for each elementary action within the respective system category, such as domestic hot water production, using a rating scale of 0 to 5. This methodology was determined using a linear distribution scale for evaluating each scenario. Consequently, a performance rating using a 5-point scale was conducted to determine the effectiveness of each scenario with respect to the criteria g2.1, g2.3, g2.4, g3.2, g3.3, and g3.4.

A comparative assessment of the energy performance of the various scenarios was conducted in collaboration with engineering consultants. Simulation tools were employed to model the building's behavior, including its various components and systems, to analyze the primary and final energy consumption of each scenario while incorporating variations in input parameters.

To address the issue of uncertainty in input data, probability density functions were used, with a specific emphasis on the normal distribution, which is subsequently defined for various scenarios associated with each criterion. The normal distribution is characterized by mean value μ and the variance $v = \sigma^2$. The mean value is obtained from the performance matrix, while the variance is influenced by the specific criterion under consideration.

In order to ensure model consistency and reduce complexity in the decision-making approach, a uniform variance value of 10% of the mean was assigned to all the criteria, ensuring uniform treatment of uncertainty across all scenarios. Variance is represented by "*Var*" within the performance matrix presented in Table 4.12 and Table 4.13. For future assessments, statistical models that consider data evolution over time may be employed to reassess the variance value.

Table 4.12. Global renovation scenarios performance matrix-(Part 1).

Scenarios	Economic					Technical			
	g1.1	g1.2	g1.3	g1.4	g1.5	g2.1	g2.2	g2.3	g2.4
	€ excl.	€ excl.	€ excl.	€ excl.	€/kwh/m ² .year	0...5	Yes/No	0...5	0...5
S1.1	0.00	1,757,134	0.00	82,701	0.00	5.00	Yes	4.38	3.57
S1.2	1,008,654	551,661	952,088	41,346	5933	3.33	Yes	5.00	3.57
S1.3	1,260,874	711,361	991,697	36,119	6591	3.00	Yes	5.00	3.57
S1.4	905,165	551,661	917,604	42,268	5457	3.67	Yes	5.00	3.57
S2.1	1,239,757	551,661	955,885	40,971	6771	2.33	Yes	4.38	4.29
S2.2	1,397,247	625,566	996,168	28,230	6736	1.67	Yes	3.75	3.57
S2.3	1,636,981	785,266	1,035,489	23,290	7185	1.33	Yes	3.75	3.57
S2.4	1,282,394	390,966	995,521	28,762	6255	2.00	Yes	3.75	3.57
S3.1	1,344,232	610,011	957,187	41,692	7140	2.00	Yes	4.38	5.00
S3.2	1,522,330	683,916	1,031,776	28,197	7087	1.33	Yes	3.75	4.29
S3.3	1,751,844	843,616	1,070,386	23,760	7526	1.00	Yes	3.75	4.29
S3.4	1,385,515	449,316	1,030,933	28,758	6545	1.67	Yes	3.75	4.29
S4.1	1,531,907	775,596	1,070,213	32,638	6586	1.67	Yes	3.75	5.00
S4.2	1,635,756	849,501	1,110,816	19,664	6342	1.00	Yes	3.13	4.29
S4.3	1,899,185	1,009,201	1,149,305	15,369	6895	0.67	Yes	3.13	4.29
S4.4	1,517,360	614,901	1,109,597	20,507	5980	1.33	Yes	3.13	4.29
S5.1	1,427,140	787,239	1,124,178	26,341	5284	1.33	No	2.50	2.86
S5.2	1,555,978	861,144	1,131,706	12,639	5229	0.67	No	1.88	2.14
S5.3	1,796,800	1,020,844	1,204,438	10,007	5867	0.33	No	1.88	2.14
S5.4	1,492,598	626,544	1,131,267	12,444	5027	1.00	No	1.88	2.14
S6.1	1,372,026	811,447	1,030,582	35,809	6507	1.67	Yes	3.13	2.86
S6.2	1,526,016	885,352	1,071,164	22,827	6466	1.00	Yes	2.50	2.14
S6.3	1,717,070	1,045,052	1,109,732	18,439	6767	0.67	Yes	2.50	2.14
S6.4	1,377,529	650,752	1,070,150	23,678	5934	1.33	Yes	2.50	2.14
S7.1	1,404,394	847,315	995,961	37,880	6790	1.00	No	0.63	0.71
S7.2	1,533,952	921,220	1,070,209	24,669	6603	0.33	No	0.00	0.00
S7.3	1,771,679	1,080,920	1,109,007	20,125	7057	0.00	No	0.00	0.00
S7.4	1,432,939	686,620	1,035,825	25,267	6251	0.67	No	0.00	0.00
Var	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

g1.1=Investment cost, g1.2=Reinvestment cost over a period of 30 years, g1.3=Possibility of financial aid g1.4=Operation cost, g1.5=Energy cost effectiveness of the solution
g2.1=Ease to integrate in occupied space, g2.2=Implementation in occupied space, g2.3=Maintenance/serviceability g2.4=Ease of monitoring/Energy management

Table 4.13. Global renovation scenarios performance matrix-(Part 2).

Scenarios	Social				Environmental		
	g3.1	g3.2	g3.3	g3.4	g4.1	g4.2	g4.3
	<i>€ excl.</i>	<i>0...5</i>	<i>0...5</i>	<i>0...5</i>	<i>kWh/m².year</i>	<i>Ton of CO₂/year</i>	<i>Km</i>
S1.1	368	0.00	5.00	4.00	0.00	0.00	0.00
S1.2	182	2.78	5.00	4.00	170	65	418
S1.3	159	3.33	5.00	4.00	191	74	690
S1.4	186	2.22	5.00	4.00	166	64	418
S2.1	168	4.44	4.00	4.00	183	71	2368
S2.2	141	4.44	4.00	4.00	207	80	2448
S2.3	118	5.00	4.00	4.00	228	88	2720
S2.4	144	3.89	4.00	4.00	205	79	2448
S3.1	162	2.22	3.00	2.00	188	73	2368
S3.2	133	2.22	3.00	2.00	215	83	2448
S3.3	113	2.78	3.00	2.00	233	90	2720
S3.4	136	1.67	3.00	2.00	212	82	2448
S4.1	114	2.22	1.00	0.00	233	90	2314
S4.2	86	2.22	1.00	0.00	258	99	2394
S4.3	66	2.78	1.00	0.00	275	106	2666
S4.4	91	1.67	1.00	0.00	254	98	2394
S5.1	164	0.56	0.00	1.00	270	5	2425
S5.2	0	0.56	0.00	1.00	298	33	2505
S5.3	113	1.11	0.00	1.00	306	50	2777
S5.4	137	0.00	0.00	1.00	297	32	2505
S6.1	138	3.89	5.00	5.00	211	81	2618
S6.2	110	3.89	5.00	5.00	236	91	2698
S6.3	90	4.44	5.00	5.00	254	98	2970
S6.4	114	3.33	5.00	5.00	232	89	2698
S7.1	142	4.44	3.00	3.00	207	80	2548
S7.2	114	4.44	3.00	3.00	232	89	2628
S7.3	93	5.00	3.00	3.00	251	97	2900
S7.4	117	3.89	3.00	3.00	229	88	2628
Var	0.1	0.1	0.1	0.1	0.1	0.1	0.1

g3.1=Impact of the cost on the tenant, g3.2=Level of thermal comfort, g3.3=Level of acoustic comfort, g3.4=Aesthetics and space requirements
g4.1=Energy consumption reduction, g4.2=Carbon dioxide emissions reduction, g4.3=Production place

4.7 Identification of thresholds

In order to evaluate each scenario's performance across different criteria, it is necessary to establish tolerance thresholds that determine whether a scenario is indifferent, preferred, not preferred, or incomparable. These thresholds consider the imperfect nature of data and subjective elements that may influence the definition of each criterion. Therefore, establishing tolerance thresholds for each specific criterion is essential.

The probabilistic ELECTRE Tri method facilitates the establishment of thresholds based on individual preferences. However, defining these thresholds becomes challenging in the presence of multiple criteria and limited expertise in certain criteria.

In the case study, tolerance thresholds were defined as a proportion of the mean performance across various scenarios for each criterion. This approach was adopted due to pragmatic factors, including the large number of criteria and the experimental nature of the approach. Furthermore, defining the thresholds in this manner facilitates adjustments by modifying the parameter γ , enabling an analysis of its impact on the classification of scenarios.

The indifference threshold is

$$q_j = \gamma_q \cdot \mu_j \quad (4.1)$$

where $\gamma_q = 5\%$ for the indifference threshold.

μ_j represent the mean performance of all the scenarios on a given criterion defined by

$$\mu_j = \mu(g_j) = \frac{\sum_{i=1}^m g_j(S_i)}{m} \quad (4.2)$$

The preference threshold is

$$p_j = \gamma_p \cdot \mu_j \quad (4.3)$$

where $\gamma_p = 15\%$ for the preference threshold.

The veto threshold is

$$v_j = \gamma_v \cdot \mu_j \quad (4.4)$$

where $\gamma_v = 30\%$ for the veto threshold.

4.8 Identification of boundary reference profiles and categories

The identification of six boundary reference profiles $B = \{b^0, b^1, b^2, b^3, b^4, b^5\}$, was essential in establishing five distinct categories $C = \{C1, C2, C3, C4, C5\}$.

These categories are represented as follows:

- $C1 = \text{Very Low}$
- $C2 = \text{Low}$
- $C3 = \text{Medium}$
- $C4 = \text{High}$
- $C5 = \text{Very High}$

The boundary reference profiles range from b^0 , representing the lowest boundary reference profile, to b^5 , representing the highest boundary reference profile. These profiles establish thresholds between consecutive categories, where the upper boundary of a less-preferred category becomes the lower boundary of the subsequent, more-preferred category.

By clearly defining these transitions, each boundary reference profile ensures consistent and systematic classification across various scenarios, enhancing the robustness and reliability of the categorization process.

In the ELECTRE Tri method, each criterion is classified based on its ideal performance type:

- **Direct-type criterion:** For this criterion, higher values indicate better performance; thus, maximization is preferred.
- **Indirect-type criterion:** For this criterion, lower values indicate better performance; thus, minimization is preferred.

The boundary reference profiles are mathematically derived based on the performance levels observed across scenarios, specifically focusing on the most and least preferred levels for each criterion g_j :

The lowest boundary reference profile $g_j(b^0)$ is

$$g_j(b^0) = \begin{cases} \min(g_j(S_i)) - p_j, & \text{if } g_j \text{ is an indirect criterion} \\ -\max(g_j(S_i)) - p_j, & \text{if } g_j \text{ is a direct criterion} \end{cases} \quad (4.5)$$

Here, the preference threshold p_j is applied to establish a distinct separation between categories, ensuring clear and strict classification boundaries. This threshold ensures that each category has a clearly defined boundary, enhancing the clarity and precision of the classification.

The highest boundary reference profile $g_j(b^5)$ is

$$g_j(b^5) = \begin{cases} -\min(g_j(S_i)) + p_j, & \text{if } g_j \text{ is an indirect criterion} \\ \max(g_j(S_i)) + p_j, & \text{if } g_j \text{ is a direct criterion} \end{cases} \quad (4.6)$$

This equation sets the uppermost boundary, ensuring that performances falling into this profile represent the highest level of preference within the evaluation scale.

The intermediate reference profiles b^1 , b^2 , b^3 , and b^4 delineate the transitions between consecutive categories (e.g., b^1 defines the boundary between $C1$ and $C2$, b^2 between $C2$ and $C3$, and so on). These profiles are defined statistically using

$$g_j(b^k) = \begin{cases} -(\mu_j + \delta_j^k \sqrt{V_j}), & \text{if } g_j \text{ is an indirect criterion} \\ \mu_j + \delta_j^k \sqrt{V_j}, & \text{if } g_j \text{ is a direct criterion} \end{cases} \quad (4.7)$$

where:

k - intermediate reference profile $k=1,2,3,4$;

μ_j - mean performance value across scenarios for criterion g_j ;

δ_j^k - statistical coefficient applied for profile b^k ;

V_j - variance, reflecting the spread of performance values across scenarios.

In this study, a statistical method was employed to establish consistent boundary definitions across all criteria. This statistical approach ensures that all criteria, whether direct or indirect, adhere to a systematic and uniform method for boundary definition. Furthermore, this approach reduces the time and resource demands traditionally associated with developing boundary profiles, which often require intensive collaboration among decision-makers.

By enabling reproducible and robust boundary definitions, this statistical approach supports effective classification across complex, data-intensive decision-support scenarios. The selection of five categories provides a balance between simplicity and precision, providing distinct classification levels that facilitates the efficient evaluation of scenarios.

Table 4.14. Identification of reference profiles and thresholds.

Criteria	Type	b^0	b^1	b^2	b^3	b^4	b^5	q_j	p_j	v_j
g1.1	Indirect	-2,112,010	-1,667,495	-1,528,955	-1,418,833	-1,330,025	-212,825	70,942	212,825	468,215
g1.2	Indirect	-1,874,915	-928,689	-837,382	-680,856	-602,593	-273,185	39,260	117,781	259,118
g1.3	Direct	-152,458	963,728	1,031,134	1,071,156	1,111,178	1,356,896	50,819	152,458	335,408
g1.4	Indirect	-87,010	-38,545	-28,729	-24,522	-18,913	-5698	1436	4309	9480
g1.5	Indirect	-8,452	-6853	-6717	-6376	-5491	926	309	926	2037
g2.1	Direct	-0.23	0.71	1.20	1.54	2.09	5.00	0.08	0.23	0.51
g2.2	Binary	0.00	0.00	0.00	1.00	1.00	1.00	0.00	1.00	2.00
g2.3	Direct	-0.45	1.93	2.60	3.26	4.00	5.00	0.15	0.45	0.98
g2.4	Direct	-0.45	1.98	2.35	3.30	4.28	5.00	0.15	0.45	0.99
g3.1	Indirect	-389	-160	-137	-115	-99	-46	7.00	21	45
g3.2	Direct	-0.43	1.78	2.23	3.74	4.19	5.00	0.14	0.43	0.94
g3.3	Direct	-0.45	1.00	2.10	3.36	4.08	5.00	0.15	0.45	0.99
g3.4	Direct	-0.41	0.59	1.87	3.14	4.41	5.00	0.14	0.41	0.90
g4.1	Direct	-33	194	223	234	257	340	11	33	74
g4.2	Indirect	-11	60	74	82	90	117	4.00	11	24
g4.3	Indirect	-3308	-2737	-2657	-2496	-2376	338	113	338	744

4.9 Degree of credibility index

The degree of creditability index is determined by measuring how closely each scenario aligns with its corresponding reference profile and comparing it against a predetermined cutting level. Through an iterative process, a cutting level of 0.75 was identified as suitable for this case study. It should be noted that selecting a higher cutting level could result in scenarios becoming incomparable. This occurs because a higher cutting level increases the rigor of the credibility assessment, potentially causing more scenarios to fail to meet the required threshold for establishing a clear outranking relationship.

4.10 Sorting procedure

In the final stage, the probabilistic ELECTRE Tri method employs two specific sorting procedures to assign scenarios to categories: The "*optimistic procedure*" and the "*pessimistic procedure*". These procedures take different approaches to address the issue of incomparability between scenarios and categories.

In the optimistic procedure, if a scenario is incomparable to a category, the scenario is placed in the subsequent higher performance category. This procedure favors a more optimistic interpretation of the scenario's performance by assigning it to a higher category.

In contrast, in the pessimistic procedure, if a scenario is incomparable to a category, the scenario is placed in the subsequent lower performance category. This procedure adopts a more conservative interpretation of the scenario's performance by assigning it to a lower category.

Using both the optimistic and pessimistic procedures offers a balanced view of each scenario's potential performance. The optimistic procedure provides an upper boundary of performance (the best possible placement), while the pessimistic procedure gives a lower boundary (the most cautious placement). Together, these perspectives allow a comprehensive understanding of the range of possible outcomes for each scenario, facilitating more informed judgments about their relative strengths and weaknesses.

The results obtained using the probabilistic ELECTRE Tri method and ELECTRE Tri method are presented in Table 4.15, Table 4.16, Table 4.17, and Table 4.18.

Table 4.15. Optimistic classification obtained using the probabilistic ELECTRE Tri method for all scenarios.

Scenarios	C1	C2	C3	C4	C5	Total
S1.1		100				100
S1.2		28.4	71	0.6		100
S1.3		0.7	65.8	33.3	0.2	100
S1.4		41.9	58	0.1		100
S2.1		14.5	82.6	2.9		100
S2.2		0.2	3.1	53.9	42.8	100
S2.3		2.2	7.3	65.7	24.8	100
S2.4		0.1	3	57.8	39.1	100
S3.1		14.4	82.3	3.3		100
S3.2		1.3	4.3	94.4		100
S3.3		7.4	34.9	57.7		100
S3.4			6.4	93.6		100
S4.1			100			100
S4.2		0.4	99.6			100
S4.3		33	67			100
S4.4			100			100
S5.1		100				100
S5.2		100				100
S5.3		92.9	7.1			100
S5.4		100				100
S6.1		0.1	40.7	58.1	1.1	100
S6.2		0.3	11.6	87.9	0.2	100
S6.3		21.7	75.9	2.4		100
S6.4		0.3	0.7	94	5	100
S7.1		100				100
S7.2		100				100
S7.3		100				100
S7.4		100				100

Table 4.16. Pessimistic classification obtained using the probabilistic ELECTRE Tri method for all scenarios.

Scenarios	C1	C2	C3	C4	C5	Total
S1.1		100				100
S1.2		28.4	71	0.6		100
S1.3		0.7	65.8	33.3	0.2	100
S1.4		41.9	58	0.1		100
S2.1		14.5	82.6	2.9		100
S2.2		0.2	3.1	53.9	42.8	100
S2.3		2.2	7.3	65.7	24.8	100
S2.4		0.1	3	57.8	39.1	100
S3.1		14.4	82.3	3.3		100
S3.2		1.3	4.3	94.4		100
S3.3		7.4	34.9	57.7		100
S3.4			6.4	93.6		100
S4.1			100			100
S4.2		0.4	99.6			100
S4.3		33	67			100
S4.4			100			100
S5.1		100				100
S5.2		100				100
S5.3		92.9	7.1			100
S5.4		100				100
S6.1		0.1	40.7	58.1	1.1	100
S6.2		0.3	11.6	87.9	0.2	100
S6.3		21.7	75.9	2.4		100
S6.4		0.3	0.7	94	5	100
S7.1		100				100
S7.2		100				100
S7.3		100				100
S7.4		100				100

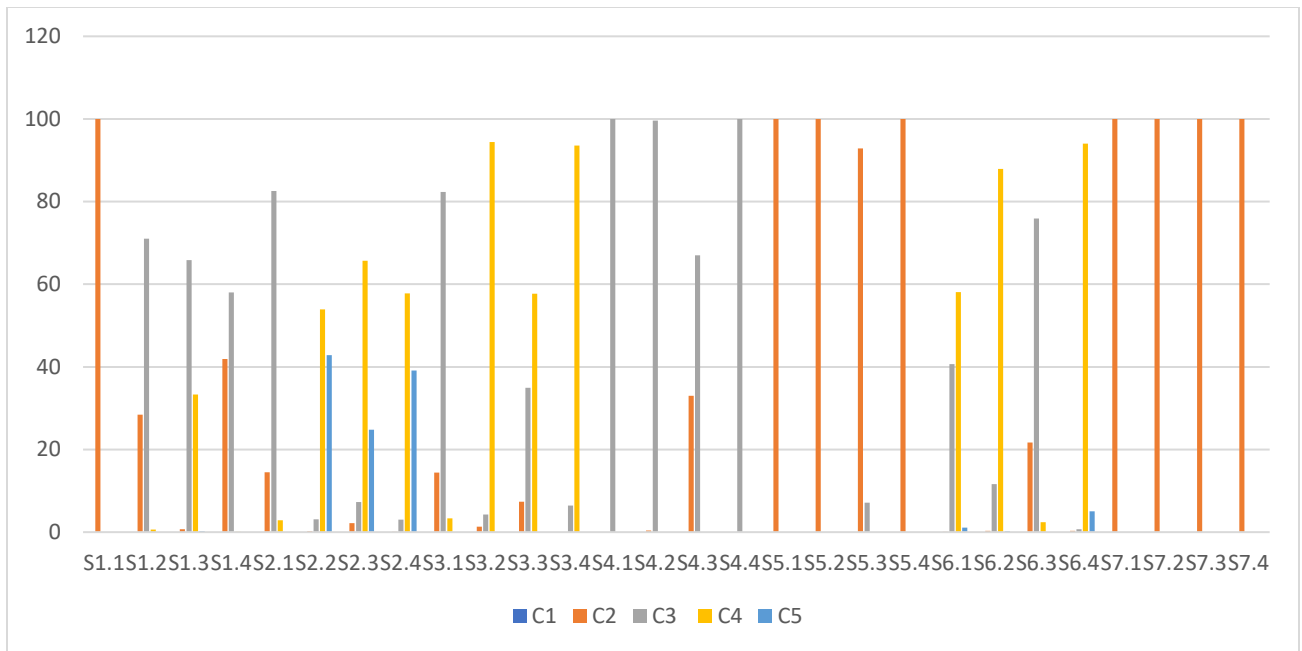


Figure 4.14. Optimistic classification obtained using the probabilistic ELECTRE Tri method for all scenarios.

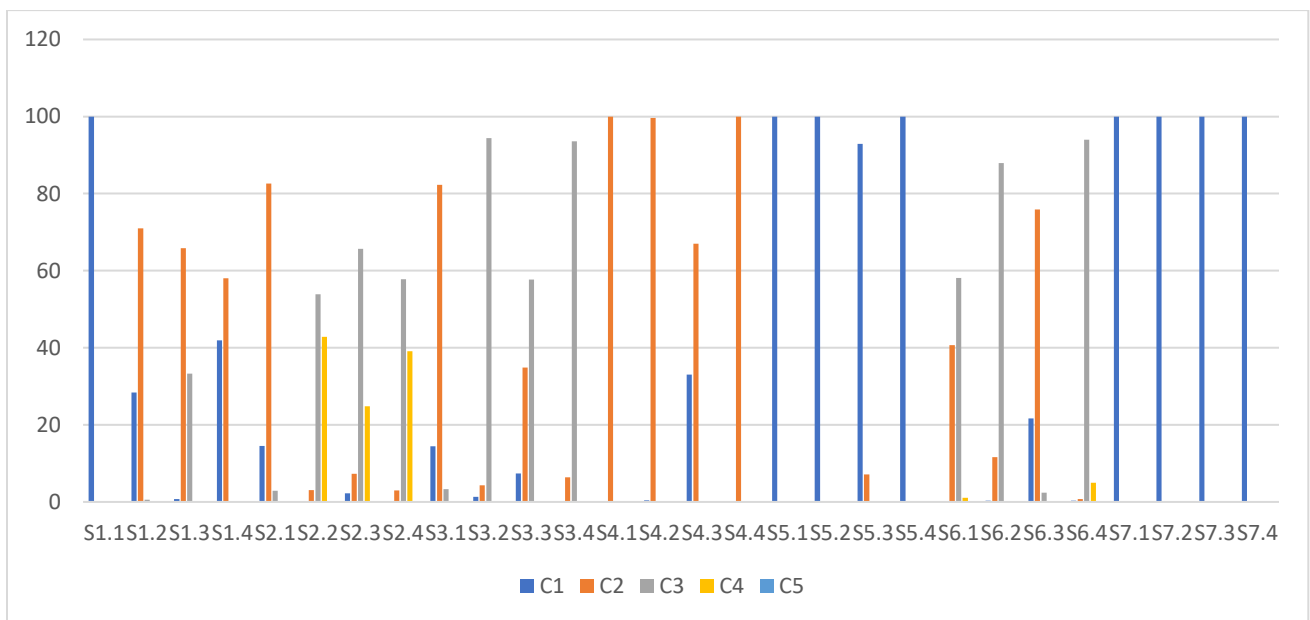


Figure 4.15. Pessimistic classification obtained using the probabilistic ELECTRE Tri method for all scenarios.

Table 4.17. Optimistic classification obtained using the ELECTRE Tri method for all scenarios.

Scenarios	C1	C2	C3	C4	C5
S1.1		X			
S1.2			X		
S1.3			X		
S1.4			X		
S2.1			X		
S2.2					X
S2.3				X	
S2.4				X	
S3.1			X		
S3.2				X	
S3.3				X	
S3.4				X	
S4.1			X		
S4.2			X		
S4.3			X		
S4.4			X		
S5.1		X			
S5.2		X			
S5.3		X			
S5.4		X			
S6.1				X	
S6.2				X	
S6.3			X		
S6.4				X	
S7.1		X			
S7.2		X			
S7.3		X			
S7.4		X			

Table 4.18. Pessimistic classification obtained using the ELECTRE Tri method for all scenarios.

Scenarios	C1	C2	C3	C4	C5
S1.1	X				
S1.2		X			
S1.3		X			
S1.4		X			
S2.1		X			
S2.2					X
S2.3			X		
S2.4			X		
S3.1		X			
S3.2			X		
S3.3			X		
S3.4			X		
S4.1		X			
S4.2		X			
S4.3		X			
S4.4		X			
S5.1	X				
S5.2	X				
S5.3	X				
S5.4	X				
S6.1			X		
S6.2			X		
S6.3		X			
S6.4			X		
S7.1	X				
S7.2	X				
S7.3	X				
S7.4	X				

4.11 Results and discussion

This section presents the results obtained using the probabilistic ELECTRE Tri method. The findings are displayed as a percentage-based classification for different scenarios within each category. Subsequently, the results obtained from the probabilistic ELECTRE Tri method are compared and contrasted with those obtained using the ELECTRE Tri method. This comparative analysis aims to provide additional perspectives and evaluate the suitability of the methods. Moreover, the analysis included an evaluation of the implications of considering data variability to determine whether its inclusion had a uniform effect across the scenarios.

The resulting table illustrates the percentage distribution of each scenario across the specified categories, providing a novel perspective on presenting the findings. The outcomes obtained through the use of ELECTRE Tri and the probabilistic ELECTRE Tri procedure are presented in Table 4.19 and Table 4.20, respectively. This analysis focused on four scenarios of the optimistic procedure in the case study: 'S2.1', 'S2.2', 'S2.3', and 'S2.4'. However, it is essential to note that this analytical framework holds relevance for all other scenarios, as well as the pessimistic procedure.

Table 4.19. Optimistic classification obtained using the ELECTRE Tri for Family 2 scenarios.

Scenarios	C1	C2	C3	C4	C5
S2.1			X		
S2.2					X
S2.3				X	
S2.4				X	

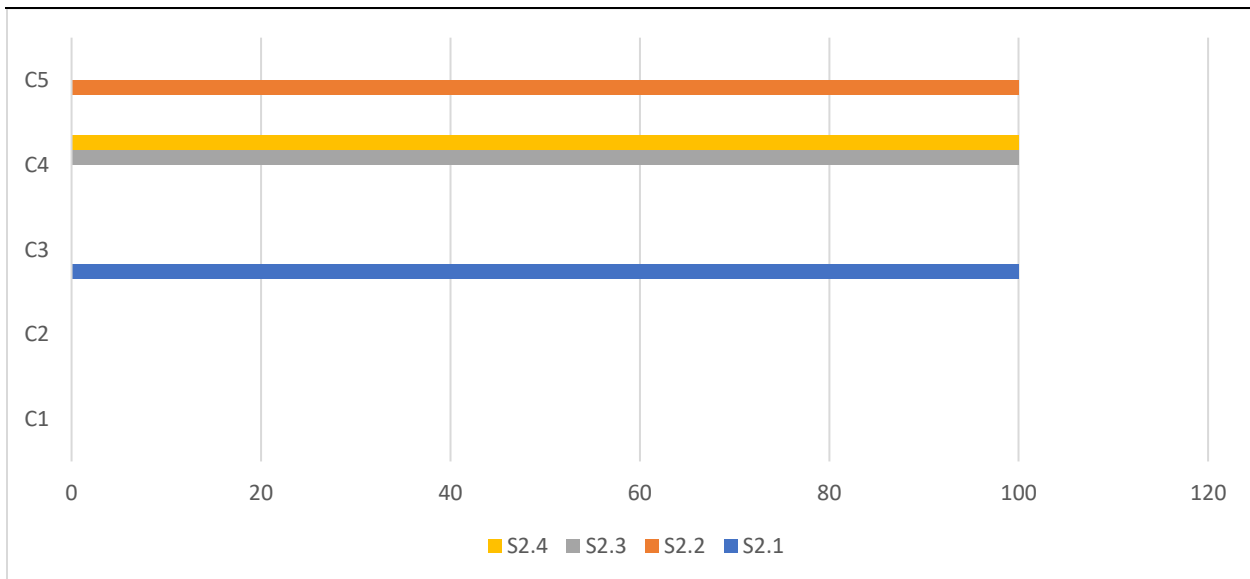


Figure 4.16. Optimistic classification obtained using the ELECTRE Tri for Family 2 scenarios.

Table 4.20. Optimistic classification obtained using the probabilistic ELECTRE Tri for Family 2 scenarios.

Scenarios	C1	C2	C3	C4	C5	Total
S2.1	0	14.5	82.6	2.9	0	100
S2.2	0	0.2	3.1	53.9	42.8	100
S2.3	0	2.2	7.3	65.7	24.8	100
S2.4	0	0.1	3	57.8	39.1	100

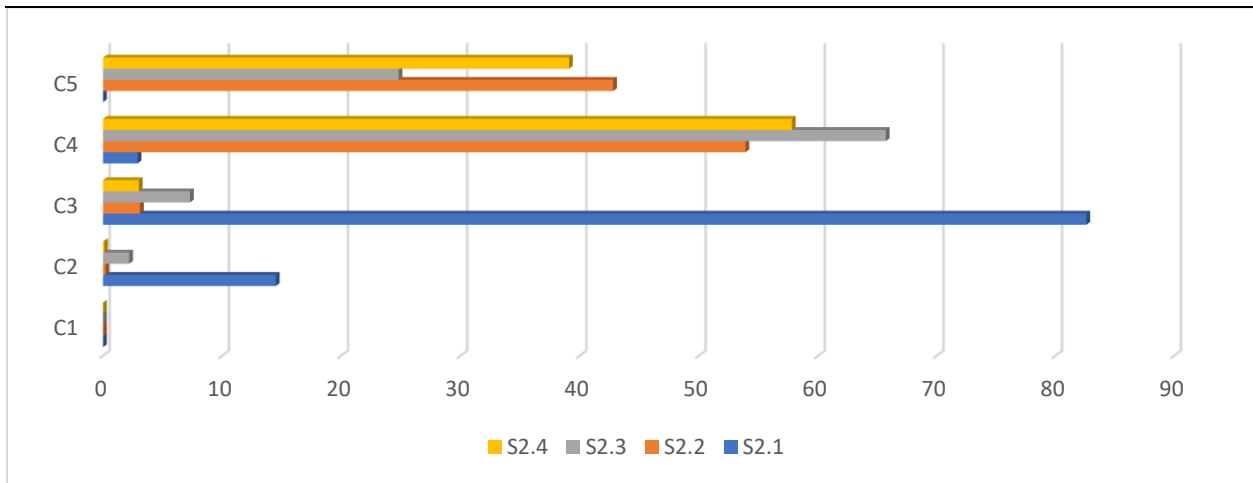


Figure 4.17. Optimistic classification obtained using the probabilistic ELECTRE Tri for Family 2 scenarios.

Table 4.20 presents valuable insights into the scenarios through the illustration of probabilistic outcomes obtained using the probabilistic ELECTRE Tri method.

When analyzing scenario 'S2.1', it is evident that it should be assigned to category 3 according to the ELECTRE Tri method. However, the probabilistic ELECTRE Tri method reveals that, when considering data fluctuation, 'S2.1' is distributed as 14.5% in category 2, 82.6% in category 3, and 2.9% in category 4.

The probabilistic classification provides valuable insights into the potential for classifying scenarios amid variability and uncertainty. In cases where two scenarios appear to have the same classifications, the percentage distribution across other categories aids in differentiation. For instance, scenarios 'S2.2' and 'S2.4' are initially classified into a single category using the ELECTRE Tri method. However, the probabilistic ELECTRE Tri method classifies scenarios 'S2.2' and 'S2.4' into multiple categories (2, 3, 4, and 5) due to the inclusion of input data uncertainty. Scenario 'S2.2' is more frequently classified in category 5 with a rate of 42.8%, compared to scenario 'S2.4', which is 39.1% in category 5. These findings suggest that scenario 'S2.2' outperforms scenario 'S2.4'.

While both scenarios are initially classified into different categories using the ELECTRE Tri method, the probabilistic ELECTRE Tri method highlights similarities in their performance. It provides a more nuanced comparison, revealing that scenario 'S2.2' tends to perform better than scenario 'S2.4' across multiple categories, particularly in category 5. This differentiation enables a clearer understanding of performance differences and facilitates a more comprehensive analysis of the scenarios.

This probabilistic allocation of scenarios across categories enables a more in-depth analysis of elementary actions. These elementary actions are carefully developed to assess their impact on the classification of the scenarios. This information helps determine whether an elementary action improves the overall performance of a scenario. By examining the performance of these elementary actions within each scenario, valuable insights can be gained into the complex interactions among different components and their effects on overall scenario performance.

The results obtained from four distinct scenarios, namely 'S2.1', 'S2.2', 'S3.1', and 'S3.2' are summarized in Table 4.21. Family 2 scenarios include an electric radiant panel and an existing electric floor heating system, whereas Family 3 scenarios include electric storage heating, excluding electric floor heating. Despite these differences, both sets of scenarios are essentially similar, except for the presence of roof solar panels, which distinguishes the index 2 scenarios ('S2.2' and 'S3.2') from the index 1 scenarios ('S2.1' and 'S3.1'). Scenarios with index 1 do not possess an independent energy system.

The analysis reveals that scenarios incorporating solar panels tend to be categorized in higher performance categories. The inclusion of solar panels introduces an element of renewable energy production, thereby enhancing the overall sustainability and efficiency of the scenarios. This information aids in assessing the effectiveness of incorporating renewable energy sources, such as solar panels, into scenario formation and evaluating their impact on overall performance. By understanding the contribution of these elementary actions, more informed decisions can be made about the optimal configuration of the scenarios.

Table 4.21. Optimistic classification obtained for four different scenarios.

Scenarios	C1	C2	C3	C4	C5	Total
S2.1	0	14.5	82.6	2.9	0	100
S2.2	0	0.2	3.1	53.9	42.8	100
S3.1	0	14.4	82.3	3.3	0	100
S3.2	0	1.3	4.3	94.4	0	100

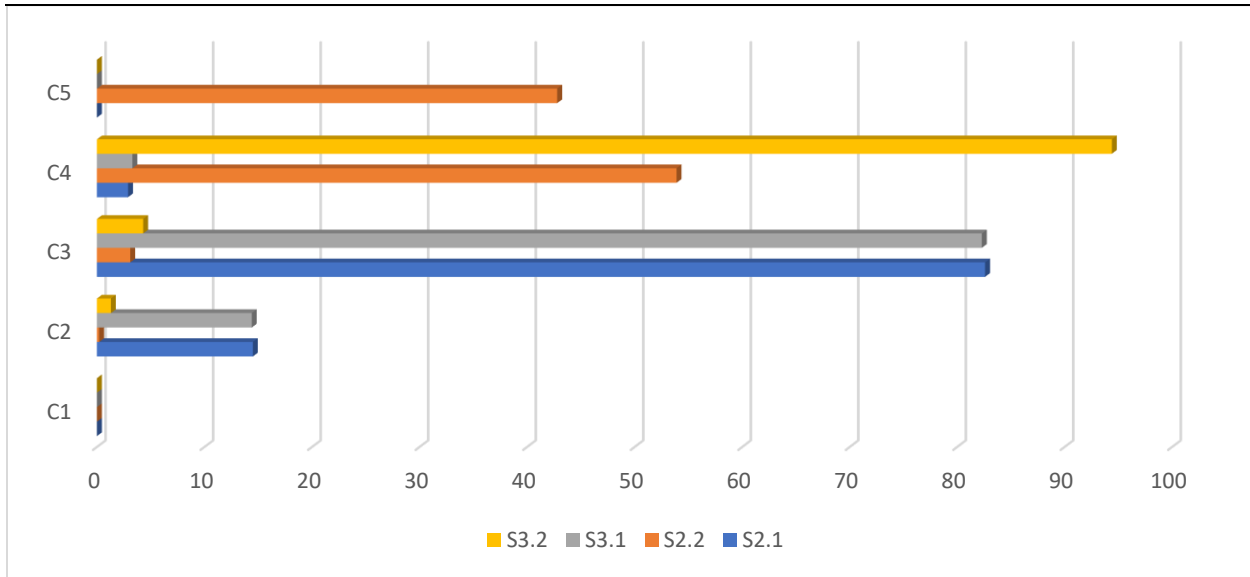


Figure 4.18. Optimistic classification obtained for four different scenarios.

Scenarios exhibit inconsistent and varied responses to the performance matrix distributions when analyzed using the probabilistic ELECTRE Tri method. While some scenarios are categorized into multiple categories, others fall within a single category.

Table 4.22 and Table 4.23 present the optimistic classification results obtained from the implementation of the probabilistic ELECTRE Tri for scenarios in Family 2 ('S2.1', 'S2.2', 'S2.3', and 'S2.4') and Family 7 ('S7.1', 'S7.2', 'S7.3', and 'S7.4'). Both families of scenarios display comparable levels of variability, characterized by fluctuations in their respective values. Family 2 scenarios are classified into four distinct categories, whereas Family 7 scenarios consistently fall within a single category based on these fluctuations. This discrepancy in classification is influenced by several factors.

Family 2 scenarios tend to exhibit more diverse responses, resulting in their classification into multiple categories, whereas Family 7 scenarios display more consistent performance, resulting in classification within a single category. Similarly, the performance of scenarios relative to the established thresholds plays an essential role. Even minor variations in performance can cause a

scenario to cross a threshold, resulting in a different classification. Family 2 scenarios experience greater fluctuations that cross multiple thresholds, resulting in their classification into multiple categories. In contrast, Family 7 scenarios consistently align within or close to a single threshold range, thereby remaining in a single category.

Moreover, the weight assigned to each criterion is highly significant, as variations in the performance of a highly weighted criterion can greatly influence the overall results. The weights assigned to each criterion influence the overall evaluation. If certain criteria carry higher weights and the scenarios within Family 7 consistently perform well or poorly on those criteria, this could result in consistent classification within a single category. Conversely, if Family 2 scenarios exhibit more variability in their performance across criteria with different weights, they are more likely to be classified into multiple categories.

Table 4.22. Optimistic classification obtained for Family 2 scenarios.

Scenarios	C1	C2	C3	C4	C5	Total
S2.1	0	14.5	82.6	2.9	0	100
S2.2	0	0.2	3.1	53.9	42.8	100
S2.3	0	2.2	7.3	65.7	24.8	100
S2.4	0	0.1	3	57.8	39.1	100

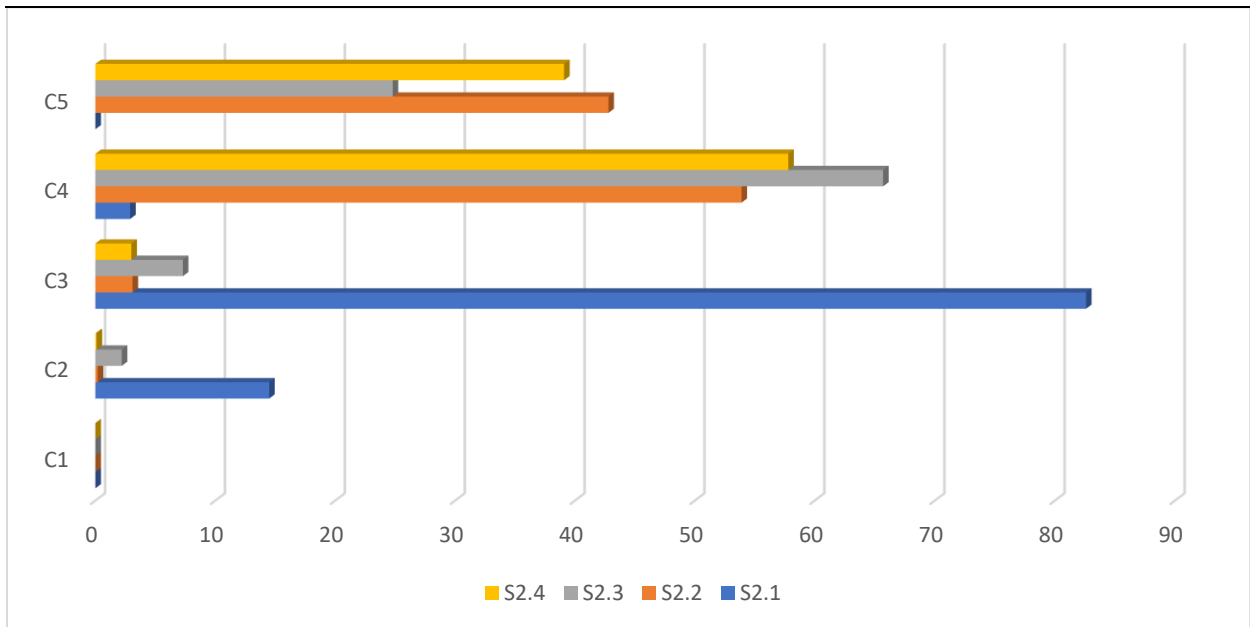


Figure 4.19. Optimistic classification obtained for Family 2 scenarios.

Table 4.23. Optimistic classification obtained for Family 7 scenarios.

Scenarios	C1	C2	C3	C4	C5	Total
S7.1	0	100	0	0	0	100
S7.2	0	100	0	0	0	100
S7.3	0	100	0	0	0	100
S7.4	0	100	0	0	0	100

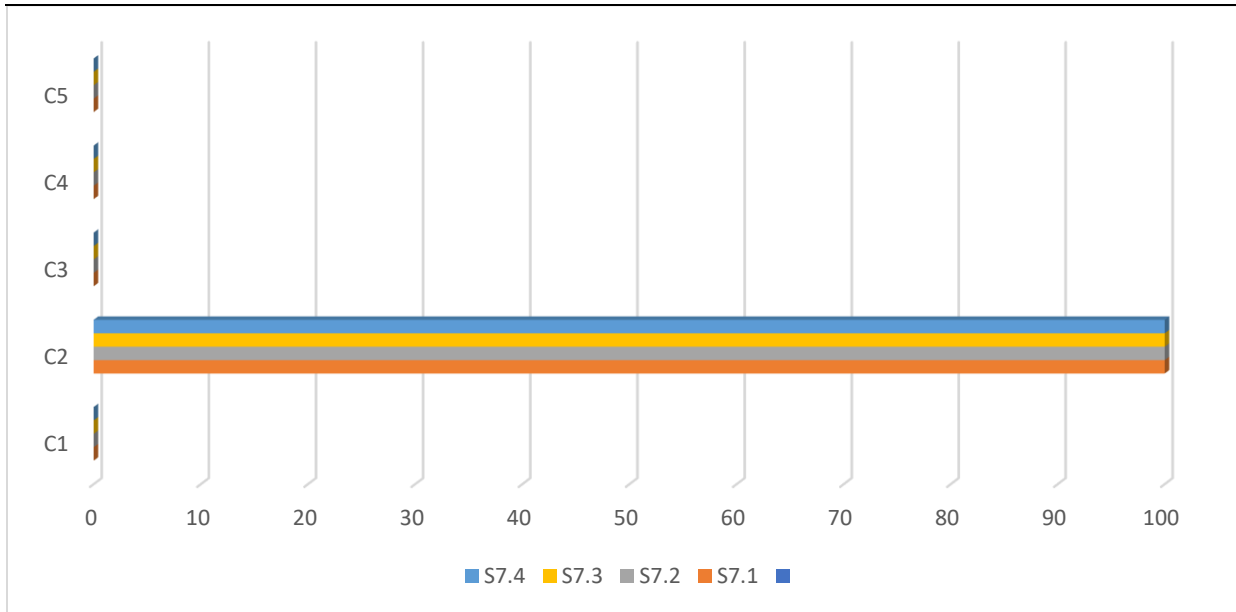


Figure 4.20. Optimistic classification obtained for Family 7 scenarios.

4.12 Validation

Validating a new methodology is a critical aspect of the model development process, as it enhances confidence in the model and increases its value. The verification of consistency in the acquired results is essential for the validation process. This necessitates that outcomes with minimal variation accurately correspond to those obtained using the crisp (precise) values.

Initially, the algorithm was executed using the ELECTRE Tri method, excluding the Monte Carlo simulation. The algorithm was then re-executed, this time incorporating the probabilistic ELECTRE Tri method, using an extremely low variance of 0.000001 and 10,000 samples.

As a result, each scenario was classified in the identical manner as when employing the ELECTRE Tri method. This consistency in classification can be attributed to the diminished variance introduced by the probabilistic ELECTRE Tri procedure with a large number of Monte Carlo simulations.

Chapter references

1. Daniel, S. and C. Ghiaus, *Multi-Criteria Decision Analysis for Energy Retrofit of Residential Buildings: Methodology and Feedback from Real Application*. *Energies*, 2023. 16(2): p. 902.
2. Olasolo-Alonso, P., L.M. López-Ochoa, J. Las-Heras-Casas, and L.M. López-González, *Energy performance of buildings directive implementation in southern European countries: a review*. *Energy Buildings*, 2023. 281: p. 112751.

5 Conclusions and recommendations for future research

This chapter presents the findings obtained from the research conducted, highlighting the potential contributions and recommendations for future work of this research.

It focuses on the following key topics:

- How the aim and objectives of this study have been achieved.
- How the research conducted has addressed the research questions.
- Finally, recommendations for future research directions.

5.1 Achievement of the research aim and objectives

This section elaborates on how the thesis has effectively addressed the aim and objectives of this research outlined in the Introduction Chapter.

The aim of this research was to develop a comprehensive MCDM framework based on sustainability factors in terms of economic, environmental, social, and technical aspects for the classification of energy alternatives/scenarios within the context of energy renovation of existing buildings. The framework ensures a transparent evaluation during different operational stages while incorporating input data uncertainty.

The proposed framework was developed through a comprehensive review of various MCDM and weighting methods. A SWOT (Strengths, Weaknesses, Opportunities and Threats) analysis was performed to gain insights into the benefits, limitations, potential applications, and challenges associated with these methods. Subsequently, a set of quantitative and qualitative criteria was defined through an in-depth review of the current state-of-the-art, consultations with industry experts, and feedback from academic researchers.

The novel probabilistic ELECTRE Tri framework integrates the ELECTRE Tri method, probability distribution, the revised Simos method, and the Monte Carlo simulation to address the classification problem and input data uncertainty in the existing research study.

The framework addresses a critical challenge in decision-making: uncertainty in input data, which can result from various sources, such as measurement errors, incomplete or missing data, sampling variability, and inherent randomness in the data-generation process. Traditional decision-

making methods often struggle with uncertain input data, resulting in outcomes that may significantly differ from reality. In ELECTRE Tri, criteria fuzziness is addressed using indifference, preference, and veto thresholds to model the inherent imprecision, vagueness, or ambiguity in evaluating alternatives. A fuzzy region exists between the indifference and preference thresholds, where determining the superiority of one alternative over another is uncertain. This region captures the difficulty in making clear distinctions between alternatives when differences are minimal. In contrast, the veto threshold establishes a clear-cut situation, where one alternative is considered unsuitable compared to another.

By incorporating these thresholds, ELECTRE Tri effectively handles the inherent fuzziness in criteria definitions and improves the evaluation of alternatives under uncertain conditions.

However, the novel framework proposed in this research employs advanced techniques such as probability distribution, the revised Simos method, and Monte Carlo simulation. By integrating these methods with the ELECTRE Tri methodology, the framework offers a robust solution for effectively managing input data uncertainty. Certain input variables in decision models are inherently uncertain or probabilistic in nature. Instead of using crisp (precise) values, these inputs are better represented as probability distributions to capture variability and uncertainty.

Probabilistic sensitivity analysis methods, such as Monte Carlo simulations, provide deep insights into model uncertainty and potential outcome ranges. By incorporating probabilistic inputs and simulating a large number of scenarios, these techniques yield a distribution of possible results and associated probabilities.

This approach enables a more realistic representation of the decision problem and accounts for potential risks, uncertainties, and trade-offs. The framework produces more accurate and reliable outcomes, even in the presence of uncertain input data. As a result, the decision-making process is significantly enhanced, leading to more well-informed outcomes.

Furthermore, the proposed framework ensures transparency throughout the decision-making process. The methodology is well-documented, enabling users to understand the process and how input data uncertainty is addressed. This clarity enhances decision-makers confidence, instilling a sense of trust when applying the framework to real-world problems. As a result, decisions are based on sound principles and comprehensive analysis, leading to improved outcomes.

5.2 Addressing the research questions

This section presents a detailed explanation of how the thesis effectively addressed the research questions in order to fulfill the aim and objectives of this study, as outlined in the Introduction chapter.

The **first research question** focused on “**What are the research gaps and issues in the existing MCDM methods used in energy renovation for existing buildings and how it can be improved?**”

In order to address this research question, a comprehensive review of the current state-of-the-art of the most commonly used MCDM methods was carried out, followed by a SWOT analysis of these methods, and their application to real-world case studies involving both quantitative and qualitative criteria. Additionally, the challenges and issues associated with the classification of energy alternatives/scenarios in the context of energy renovation for existing buildings were identified.

The findings from the review highlighted that the classification problem within this study constitutes a decision-making problem characterized by input data uncertainty within this specific research domain, as defined in Chapter Two.

Moreover, these aforementioned efforts align with the research aim, which seeks to explore the complexities involved in the context of energy renovation for existing buildings, address research gaps and issues, and establish the necessary methodology for this study.

The **second research question** focused on “**Which MCDM methods and tools are valuable and applicable to address the research gaps and issues associated with energy renovation for existing buildings?**”

In order to address this research question, an analysis of the current state-of-the-art was conducted in Chapter Two. This analysis revealed that the primary research methodology involves the adoption of the MCDM approach in the classification of energy alternatives/scenarios in the context of energy renovation for existing buildings. It was concluded that this constitutes a complex and challenging MCDM problem characterized by uncertain input data.

Chapters Two and Three focus on exploring and analyzing MCDM methodologies and tools that can enhance the classification problem. A comprehensive review and analysis of existing

MCDM methods and tools were carried out to gain a thorough understanding of their limitations and benefits in handling input data uncertainty, criteria weights, and classification of alternatives/scenarios. The adopted MCDM methods and tools include the ELECTRE Tri method for addressing the classification problem, probability distributions to model the uncertain variables in the performance matrix, the revised Simos method to determine criteria weights, and Monte Carlo simulations to generate random samples of uncertain variables. These samples were subsequently used to simulate model responses, providing insights into how input data uncertainties affect the model outputs of the proposed novel MCDM framework.

These aforementioned tasks and methodologies also contribute to the fulfillment of the following research objective:

- To explore and analyze the existing MCDM methods and tools reviewed in the state-of-the-art, assessing their effectiveness in addressing classification problems and their potential to incorporate input data uncertainty.

The **third research question** focused on “**How can MCDM methods and tools be employed to propose a novel MCDM framework for the classification of energy alternatives/scenarios within the context of energy renovation of buildings?**”

In order to address this research question, the MCDM framework presented in this research integrates various MCDM methods and tools, including the ELECTRE Tri method, which is used for the classification of the alternatives/scenarios under consideration. ELECTRE Tri, an outranking method in MCDM, effectively addresses the complexities inherent in decision-making problems. Probability distributions, particularly the normal distribution, are used to model the probabilistic nature of the input data, accounting for inherent uncertainty in the input variables. The revised Simos method is used to calculate the weights of the criteria, incorporating the decision maker's preferences and judgments in assigning weights to different criteria. Monte Carlo simulation is employed to generate multiple iterations by randomly sampling the input variables based on their probability distributions, enabling a robust assessment of the model outputs.

To the best of the author's knowledge, no prior research has addressed this classification problem using an integrated model that incorporates the ELECTRE Tri, probability distribution, the revised Simos method, and Monte Carlo simulation, while taking into account the input data uncertainty.

The primary contribution of this proposed method lies in its incorporation of probabilistic data, distinguishing it from most of the existing decision-making methods, which predominantly depend on crisp (precise) data. By incorporating the uncertainty in the input data, the proposed framework provides a more comprehensive and realistic evaluation of the decision problem, resulting in enhanced and robust decision-making outcomes.

The **fourth research question** focused on “**How efficient and valuable are the MCDM methods and tools in the classification of energy scenarios/alternatives within the context of energy renovation for existing buildings?**”

In order to address this research question, the proposed novel MCDM method was employed using data from a social housing project located in the region of Lyon, France, to evaluate the empirical validity of the method.

Chapter Four defines and discusses various scenarios/alternatives, focusing on sustainability factors based on economic, environmental, social, and technical aspects related to energy renovation of the existing buildings. The main aim of the study was the classification of the scenarios/alternatives into distinct categories by taking into account the input data uncertainty.

The ELECTRE Tri method was employed for the classification problem. The normal distribution was used as the underlying probability distribution model for the uncertain variables. The criteria weights were determined using the revised Simos method. Monte Carlo simulation was employed to generate random samples of uncertain variables, simulating model responses to analyze how input data uncertainty influenced model outputs.

The outcomes of the case study facilitated the formulation of multiple recommendations based on the final findings. Furthermore, a validation process was conducted to assess the reliability of the final outcome obtained from implementing the novel probabilistic ELECTRE Tri method.

In conclusion, the application of the proposed novel probabilistic ELECTRE Tri method in the classification of energy scenarios/alternatives is indeed highly valuable.

5.3 Recommendations for future research

This section presents various potential improvements resulting from either the limitations of the current research or areas beyond the scope of the present study.

Recommendations for addressing these limitations in future studies are outlined as follows:

The proposed framework is not strictly limited to the classification of energy scenarios /alternatives within the context of energy renovation of existing buildings. Its primary purpose is to address the problem of input data uncertainty. Certain inputs are characterized by uncertainty (probabilistic), such as prices, while others exhibit imprecision (fuzzy), such as the acceptance of proposed solution into existing buildings. In the existing research, fuzzy inputs were inadequately addressed; therefore, incorporating fuzzy logic or similar techniques to effectively manage these imprecise inputs and ensure the analysis accounts for their inherent ambiguity.

It is valuable to explore the potential and viability of employing the proposed framework across diverse fields. This adaptability highlights the multidisciplinary nature of the framework.

If the actual variance for all criteria in the performance matrix is calculated, the results could be compared with those obtained from the proposed framework to assess its effectiveness.

Although the normal distribution is selected for the proposed framework, exploring other distribution types may provide further insights.

Only one case study from France has been examined for analysis. In order to evaluate the impact of sustainability factors in terms of economic, environmental, social, and technical aspects across different regions, it is necessary to expand the geographical scope beyond France and include additional real-world case studies from various diverse regions. This will result in a broader comprehension of the research subject, facilitating the identification of optimal methodologies and highlighting potential areas for enhancement in various regions, thereby consolidating the robustness of the framework. Hence, it is recommended that the generalizability across different contexts be explored.

The list of sustainability factors considered in this thesis is not exhaustive. Sustainability is a dynamic concept, with new factors being continually incorporated into the list. The inclusion of new factors would enhance the proposed framework's robustness, enabling its applicability in different geographic regions.

A significant limitation exists in decision analysis under uncertain conditions, primarily due to challenges in implementing established approaches for characterizing uncertainty. The main challenge is the lack of comprehensive documentation for these methods. If these methods are well documented, it would greatly enhance reproducibility, providing other professionals and researchers' valuable guidance and feedback for refining the framework.

In real-world decision problems, especially those involving uncertainties, simplicity within the MCDM framework is essential to ensure transparency. If the framework is overly complex, policy-makers may perceive it as a "*black box*" approach and hesitate to use its outcomes. Therefore, maintaining a simple framework promotes closer collaboration among key stakeholders, resulting in a more productive, dynamic, and efficient approach.

Additional MCDM methods and tools could be incorporated to develop novel MCDM frameworks for future investigations. Subsequently, the outcomes obtained from these newly developed frameworks can be contrasted with the findings obtained from the framework proposed in this research.

There is an opportunity to refine and adapt the framework based on user feedback, aiming to make it more user-friendly, efficient and industry-relevant.

To bridge the gap between research and industry, forthcoming endeavors may concentrate on enhancing closer collaboration between academia and industry stakeholders. This goal can be accomplished by establishing platforms for knowledge sharing, forming industry-academia partnerships, and implementing joint research initiatives. Through the engagement of industry-experts and academic-researchers in collaborative discourse, research outcomes can be effectively

translated into practical applications, facilitating the adoption of sustainable and efficient practices within the built environment.

It is imperative to conduct research aimed at delving into the perspectives of various key stakeholders, such as proprietors and regulatory bodies. This undertaking has the potential to yield a more profound comprehension of the factors that influence the decision-making processes within the context of energy renovation of buildings. The primary aim is to streamline these decisions in alignment with sustainability principles in the building sector, thus making a significant contribution to the field.

The research, while being completed at this stage, has unveiled novel avenues for future research initiatives and has provided valuable insights into potential areas for achieving the overarching objective of advancing and enhancing sustainable practices within the built environment.

Appendices

To enhance the readability of the main text, comprehensive tables and figures containing detailed method inputs and results are presented in this section. The main text includes condensed versions of these tables and figures, providing concise excerpts or summaries.

Appendix A: Weighting of evaluation criteria using the revised Simos method.

Appendix B: Means of criteria evaluation.

Appendix C: Estimation of cost of different materials.

Appendix D: Python code implementation for the probabilistic ELECTRE Tri methodology.

Appendix A: Weighting of evaluation criteria using the revised Simos method.

This appendix presents the process for weighting the criteria associated with the four families. The objective is twofold: first, to prioritize the criteria within each family, and second, to compare the criteria across families. Each criterion's weight is calculated based on the established hierarchy, using the revised Simos method.

Initially, each criterion is assigned to its respective family. The criteria are then arranged in order from most to least important, based on individual perspectives. If multiple criteria are deemed equally important, they are given the same rank. If a criterion is considered significantly more important than the one following it, leaving blank lines between them is acceptable. Finally, a Z ratio is required for each table to determine the importance ratio between the most and least important criteria.

The next step involves ranking the four families using the same approach. Families are ordered by importance, from most to least important based on individual perspectives. Flexibility is provided to assign equal importance or differences among them. Finally, relative weights are assigned to each criterion within its respective family. To calculate the absolute weight of each criterion, its relative weight is multiplied by the absolute weight of its corresponding family.

Weighting of evaluation criteria using the revised Simos method			
1.Economic criteria-Ranking			
Rank	g1.1 Investment cost	g1.2 Reinvestment cost over a period of 30 years	g1.3 Possibility of financial aid
	g1.4 Operation cost	g1.5 Energy cost effectiveness of the solution	
Most important	1	Investment cost	Operation cost
	2	Reinvestment cost over a period of 30 years	
	3		
	4		
	5	Energy cost effectiveness of the solution	
	6	Possibility of financial aid	
	7		
	8		
	9		
	10		
	11		
	12		
	13		
	14		
Least important	15		
Ratio Z		3	
2.Technical criteria-Ranking			
Rank	g2.1 Ease to integrate into existing buildings	g2.2 Implementation in occupied site	g2.3 Maintenance/ Serviceability
	g2.4 Ease of monitoring / Energy management		
Most important	1	Implementation in occupied site	
	2	Ease to integrate into existing buildings	
	3	Maintenance/ Serviceability	Ease of monitoring / Energy management
	4		
	5		
	6		
	7		
	8		
	9		
	10		
	11		
	12		
	13		
	14		
Least important	15		
Ratio Z		4	

Figure. Ranking the importance of economic and technical criteria.

3. Social criteria-Ranking			
Rank	g3.1 Impact of cost on the tenant		g3.2 Level of thermal comfort
	g3.4 Aesthetics and space requirements		g3.3 Level of acoustic comfort
Most important	1	Impact of cost on the tenant	
	2	Level of thermal comfort	
	3		
	4	Aesthetics and space requirements	Level of acoustic comfort
	5		
	6		
	7		
	8		
	9		
	10		
	11		
	12		
	13		
	14		
Least important	15		
Ratio Z		4	
4. Environmental criteria-Ranking			
Rank	g4.1 Energy consumption reduction		g4.2 Carbon dioxide emissions avoided
	g4.3 Production place		
Most important	1	Energy consumption reduction	
	2	Carbon dioxide emissions avoided	
	3	Production place	
	4		
	5		
	6		
	7		
	8		
	9		
	10		
	11		
	12		
	13		
	14		
Least important	15		
Ratio Z		3	

Figure. Ranking the importance of social and environmental criteria.

5. Families of criteria-Ranking

		Economic criteria	Technical criteria	Social criteria
		Environmental criteria		
Rank Most important	1	Economic criteria		
	2			
	3	Technical criteria		
	4	Social criteria	Environmental criteria	
	5			
	6			
	7			
	8			
	9			
	10			
	11			
	Least important	12		
Ratio Z		4		

Figure. Ranking the importance of four families of criteria.

1. Economic criteria - Weighting																											
Rank	Criteria			e _r '			e _r	Ratio z	No of criteria	k _i '	Criteria	k _i '	k _i *	k _i ''	d _i	d _{i bar}	M	d _i = {M}	L	→ L	J _{i bar} ≠ {M}	L _{bar}	→ L _{bar}	F+	F-	k _i	
1	g1.3	0	0	0	0	0	1	3	1	1.000000	1	g1.1	3.000000	27.272727	27.27	0.000267	0.000100	1	0.000267	0.000000	0		0.000000	0	0	1	27.27
2	g1.5	0	0	1	2	2	3		1	1.400000	2	g1.2	2.600000	23.636364	23.63	0.000154	0.000269	0			0	0.000269		0	1	0	23.64
3	0	0	0	1	1	0	0		-	-	3	g1.3	1.000000	9.090909	9.09	0.001000	0.000100	1	0.001000		0			0	0	1	9.09
4	0	0	0	0	0	0	0		-	-	4	g1.4	3.000000	27.272727	27.27	0.000267	0.000100	1	0.000267		0			0	0	1	27.27
5	g1.2	0	0	0	0	0	1		1	2.600000	5	g1.5	1.400000	12.727273	12.72	0.000214	0.000571	0			0	0.000571		0	1	0	12.73
6	g1.1	g1.4	0	0	0	0	0		2	3.000000		n	K'	w	K''	v		m		(v+m-n)			(n-v-m)		 F+ 	 F- 	
											5	11.000000	2	99.98	2		3		0			0		2	3	100.00	
	n						e	u																			
	5						5	0.4																			
2. Technical criteria - Weighting																											
Rank	Criteria			e _r '			e _r	Ratio Z	No of criteria	k _i '	Criteria	k _i '	k _i *	k _i ''	d _i	d _{i bar}	M	d _i = {M}	L	→ L	J _{i bar} ≠ {M}	L _{bar}	→ L _{bar}	F+	F-	k _i	
1	g2.3	g2.4	0	0	0	0	1	3	2	1.000000	1	g2.1	2.000000	28.571429	28.57	0.000300	0.000050	1	0.0003	0.000000	0		0.000167	0	0	1	28.57
2	g2.1	0	0	0	0	0	1		1	2.000000	2	g2.2	3.000000	42.857143	42.85	0.000067	0.000167	0			0	0.000167		1	0	1	42.85
3	g2.2	0	0	0	0	0	0		1	3.000000	3	g2.3	1.000000	14.285714	14.28	0.000300	0.000400	0			0	0.000400		0	1	0	14.29
4	g2.4	0	0	0	0	0	0		1	3.000000	4	g2.4	1.000000	14.285714	14.28	0.000300	0.000400	0			0	0.000400		0	1	0	14.29
	n						e	u			n	K'	w	K''	v		m		(v+m-n)			(n-v-m)		 F+ 	 F- 		
	4						2	1			4	7.000000	2	99.98	2		1		-1			1		2	2	100.00	
3. Social criteria - Weighting																											
Rank	Criteria			e _r '			e _r	Ratio Z	No of criteria	k _i '	Criteria	k _i '	k _i *	k _i ''	d _i	d _{i bar}	M	d _i = {M}	L	→ L	J _{i bar} ≠ {M}	L _{bar}	→ L _{bar}	F+	F-	k _i	
1	g3.4	g3.3	0	1	1	1	2	3	2	1.000000	1	g3.1	3.000000	40.909093	40.90	0.000022	0.000222	0		0.000000	0	0.000222	0.000222	1	0	1	40.90
2	0	0	0	0	0	0	0		-	-	2	g3.2	2.333333	31.818179	31.81	0.000057	0.000257	0			0	0.000257		0	1	0	31.82
3	g3.2	0	0	0	0	0	1		1	2.333333	3	g3.3	1.000000	13.636364	13.63	0.000267	0.000467	0			0	0.000467		0	1	0	13.64
4	g3.1	0	0	0	0	0	0		1	3.000000	4	g3.4	1.000000	13.636364	13.63	0.000267	0.000467	0			0	0.000467		0	1	0	13.64
	n						e	u			n	K'	w	K''	v		m		(v+m-n)			(n-v-m)		 F+ 	 F- 		
	4						3	0.6667			4	7.333333	2	99.97	3		0		-1			1		3	1	100.00	

Figure. Weight calculations based on the rankings of importance for economic, technical, and social criteria.

4.Environmental criteria - Weighting																											
Rank	Criteria			e _r '			e _r	Ratio Z	No of criteria	k _i '	Criteria	k _i '	k _i *	k _i ''	d _i	d _{i bar}	M	d _i = {M}	L	→ L	d _{i bar} ≠ {M}	L _{bar}	→ L _{bar}	F+	F-	k _i	
1	g4.3	0	0	0	0	0	1		1	1.000000	1	g4.1	3.000000	50.000000	50.00	0.000200	0.000000	1	0.0002	0.000000	0		0.000000	1	0	1	50.00
2	g4.2	0	0	0	0	0	1	3	1	2.000000	2	g4.2	2.000000	33.333333	33.33	0.000200	0.000100	1	0.0002		0			0	0	1	33.33
3	g4.1	0	0	0	0	0	0		1	3.000000	3	g4.3	1.000000	16.666667	16.66	0.000200	0.000400	0			0	0.000400		0	1	0	16.67
	n			e			u		-		n	K'	w	K''	v		m	(v+m-n)			(n-v-m)			 F+ 	 F- 		
	3						2	1			3	6.000000	2	99.99	1		2	0			0			1	2	100.00	
5. Families of criteria - Weighting																											
Rank	Criteria			e _r '			e _r	Ratio Z	No of criteria	k _i '	Criteria	k _i '	k _i *	k _i ''	d _i	d _{i bar}	M	d _i = {M}	L	→ L	d _{i bar} ≠ {M}	L _{bar}	→ L _{bar}	F+	F-	k _i	
1	F3	F4	0	0	0	0	1		2	1.000000	1	F1	3.000000	44.999998	44.99	0.000000	0.000222	0		0.000000	0	0.000222	0.000000	0	1	0	45.00
2	F2	0	0	1	1	1	2	3	1	1.666667	2	F2	1.666667	25.000004	25.00	0.000400	0.000000	1	0.0004		0			0	0	1	25.00
3	0	0	0	0	0	0	0		-	-	3	F3	1.000000	14.999999	14.99	0.000000	0.000667	0			0	0.000667		0	1	0	15.00
4	F1	0	0	0	0	0	0		1	3.000000	4	F4	1.000000	14.999999	14.99	0.000000	0.000667	0			0	0.000667		0	1	0	15.00
	n			e			u		-		n	K'	w	K''	v		m	(v+m-n)			(n-v-m)			 F+ 	 F- 		
	4						3	0.6667			4	6.666667	2	99.97	3		1	0			0			3	1	100.00	

Figure. Weight calculations based on the rankings of importance for environmental criteria and families of criteria.

Weighting method-Result			
N°	Families criteria weight-Result		
F1	45	Economic	
F2	25	Technical	
F3	15	Social	
F4	15	Environmental	
	100.00		
N° 1. Economic criteria weight-Result			
g1.1	27.27	Investment cost	12.27
g1.2	23.64	Reinvestment cost over a period of 30 years	10.64
g1.3	9.09	Possibility of financial aid	4.09
g1.4	27.27	Operating cost	12.27
g1.5	12.73	Energy cost effectiveness of the solution	5.73
	100.00		
N° 2. Technical criteria weight-Result			
g2.1	28.57	Ease to integrate into existing building	7.14
g2.2	42.85	Implementation in occupied space	10.71
g2.3	14.29	Maintenance/Serviceability	3.57
g2.4	14.29	Ease of monitoring/Energy management	3.57
	100.00		
N° 3. Social criteria weight-Result			
g3.1	40.9	Impact of cost on the tenant	6.14
g3.2	31.82	Level of thermal comfort	4.77
g3.3	13.64	Level of acoustic comfort	2.05
g3.4	13.64	Aesthetics and space requirements	2.05
	100.00		
N° 4.Environmental criteria weight-Result			
g4.1	50	Energy consumption reduction	7.50
g4.2	33.33	Carbon dioxide emissions avoided	5.00
g4.3	16.67	Production place	2.50
	100.00		100.00

Figure. Summary of the weighting results.

Appendix B: Means of criteria evaluation.

This appendix presents a structured framework for assessing various aspects across economic, technical, social, and environmental dimensions. Each criterion within these categories is evaluated using a method specifically formulated to capture its unique characteristics and relevance to the project. By assigning each criterion to its relevant category, this framework ensures a holistic and balanced evaluation, integrating economic, technical, social, and environmental considerations to support informed and comprehensive decision-making.

Means of criteria evaluation

1. Economic		
N°	Criteria	Means of evaluation
g1.1	Investment cost	Calculated directly based on studies conducted by the Bureau d'Études Techniques (BET), providing a cost evaluation.
g1.2	Reinvestment cost over a period of 30 years	Calculated by considering the system's expected lifetime and initial investment cost, allowing a long-term financial assessment.
g1.3	Possibility of financial aid	Information regarding available financial aid is gathered from online resources, such as government and energy provider websites, to assess the feasibility of subsidies or support for the project. → www.photovoltaique.info/fr/tarifs-dachat-et-autoconsommation/autres-aides-financieres/appels-a-projets-et-aides-regionales/#contexte → https://www.edf.fr/collectivites/le-mag/le-mag-collectivites/strategie-energetique-territoriale/autoconsommation-les-aides-disponibles-pour-les-collectivites
g1.4	Operating cost	Operating costs are challenging to estimate precisely, consultations with maintenance teams and P3 maintenance contracts are used to develop an informed approximation.
g1.5	Energy cost effectiveness of the solution	Calculated from BET estimates, this metric assesses the solution's effectiveness in reducing energy costs over time.
2. Technical		
N°	Criteria	Means of evaluation
g2.1	Ease to integrate into existing buildings	A qualitative comparison of different solutions based on component compatibility, rated subjectively by weighing the pros and cons.
g2.2	Implementation in occupied space	Initially evaluated based on the proposed solution, in consultation with specialized companies. This criterion is binary (feasible or not feasible).
g2.3	Maintenance /Serviceability	Qualitatively assessed using information from BET and partner companies. Ratings are assigned based on subjective comparisons, considering ease and complexity of maintenance.
g2.4	Ease of monitoring / Energy management	A qualitative criterion that compares different solutions based on their energy management features, rated by considering the ease of monitoring and control.

Figure. Evaluation of economic and technical criteria.

3. Social		
N°	Criteria	Means of evaluation
g3.1	Impact of the cost on the tenant	An estimate partially provided by the BET, calculated as a percentage reduction in the tenant's energy bills by comparing pre- and post-renovation scenarios.
g3.2	Level of thermal comfort	Assessed by examining heat diffusion methods, associated comfort levels, and humidity, using PMV (Predicted Mean Vote) and PPD (Predicted Percentage of Dissatisfied) indices. Evaluation is conducted through a rating system.
g3.3	Level of acoustic comfort	Based on sound levels of installed equipment, as provided in technical data sheets, and evaluated through a rating system to reflect the perceived acoustic impact.
g3.4	Aesthetics and space requirements	A subjective criterion based on tenant feedback, evaluated through sociological studies to assess tenant preferences by presenting possible scenarios.
4. Environmental		
N°	Criteria	Means of evaluation
g4.1	Energy consumption reduction	Calculated based on energy savings data from BET studies and FDES reports.
g4.2	Carbon dioxide emissions avoided	Calculated by multiplying the amount of energy recovered (kWh) by the carbon dioxide content of the avoided energy. This method takes into account the French energy mix and carbon dioxide emissions per kWh.
g4.3	Production place	Evaluated using FDES reports, the INIES database, and ACERMI certification sheets to ensure sustainable sourcing and production.

Figure. Evaluation of social and environmental criteria.

Appendix C: Estimation of cost of different materials.

This appendix presents a detailed analysis of expenses related to heating systems, water heaters, solar installations, windows, and insulation. Each component's cost estimate includes both the unit price and installation fees, calculated according to the building's specific needs. For heating systems, various models and power ratings of radiant panels and individual domestic hot water tanks are assessed. Solar thermal and photovoltaic systems include additional costs related to supports, inverters, and integration with the energy network. Window replacements and insulation materials are priced based on their size and material type, with adjustments made according to the required surface area. This comprehensive approach ensures an accurate and well-structured budget that aligns with the project's goal for energy efficiency and long-term sustainability.

	Model	P [W]	Price [€ incl. VAT]	Price [€ ex VAT]	Price [€ excl. VAT / W]	Quantity		
Radiant panels Atlantic TATOU PI Connected								
https://www.atlantic.fr/Chauffer-le-logement/Chauffage-electrique/Panneau-rayonnant/Tatou-Pilotage-Intelligent-Connecte		500	283 €	257 €	166 €	0.51	0	0
		750	283 €	257 €	166 €	0.34	147	62181
		1000	300 €	273 €	176 €	0.27	0	0
		1250	328 €	298 €	192 €	0.24	0	0
		1500	349 €	317 €	204 €	0.21	0	0
		2000	418 €	380 €	245 €	0.19	75	46875
								Total 109056
Neomitis Lamelys Ecosens 2 towel dryer								
https://www.neomitis.com/Lamelys-EcoSens%C2%B2		500	813 €	739 €	250 €	1.48	No of devices	67
		750	906 €	824 €	250 €	1.10	Global cost	71958
		1000	1,027 €	934 €	250 €	0.93		
Atlantic Lunis towel dryer								
https://www.atlantic-thermor-prescription.fr/nos-produits/seche-serviettes-electrique-lunis-atlantic-thermor		500	341 €	310 €	130 €	0.62	No of devices	67
		750	396 €	360 €	157 €	0.48	Global cost	34639
Individual DHW tank Atlantic Vizengo								
https://www.atlantic.fr/Chauffer-l-eau/Chauffe-eau/Chauffe-eau-electrique/Vizengo		150 L	6.453	460 €	418 €	500 €	918	Number of devices 150L 37
		200 L	12.675	551 €	501 €	500 €	1001	Number of devices 200L 30
								Global cost 63996
Atlantic Zeneo individual DHW tank								
https://www.atlantic.fr/Chauffer-l-eau/Chauffe-eau/Chauffe-eau-electrique/Zeneo		150 L	6.453	870 €	791 €	396 €	1187	Number of devices 150L 37
		200 L	12.675	980 €	891 €	446 €	1337	Number of devices 200L 30
								Global cost 84029
Individual DHW tank Atlantic Lineo								
https://www.atlantic.fr/Chauffer-l-eau/Chauffe-eau/Chauffe-eau-electrique/Lineo		65 L	6.678	569 €	517 €	265 €	782	Number of devices 150L 37
		80 L	6.91	609 €	554 €	280 €	834	Number of devices 200L 30
								Global cost 53954
Viessmann Vitocell 100-W individual solar DHW calorifier								
https://www.viessmann.fr/fr/chauffage-maison-individuelle/chauffe-eau/bivalent-solaire/vitocell-100-u-w.html		200 L	6.678	2,200 €	2,000 €	500 €	2500	Number of devices 200L 37
		300 L	6.91	2,750 €	2,500 €	600 €	3100	Number of devices 300L 30
								Global cost 185500
Storage heater Atlantic Galapagos PI Connected								
https://www.atlantic.fr/Chauffer-le-logement/Chauffage-electrique/Radiateur-connecte/Radiateurs-Classiques/Galapagos-Pilotage-Intelligent-Connecte		Horizontal	750	543 €	494 €	372 €	0.66	0
		Horizontal	1000	583 €	530 €	399 €	0.53	0
		Horizontal	1250	670 €	609 €	459 €	0.49	0
		Horizontal	1500	726 €	660 €	497 €	0.44	0
		Horizontal	2000	880 €	800 €	603 €	0.40	75
		Vertical	1000	761 €	692 €	521 €	0.69	0
		Vertical	1500	869 €	790 €	595 €	0.53	0
		Vertical	1800	1,047 €	952 €	717 €	0.53	0
		Low	700	605 €	550 €	414 €	0.79	0
		Low	1000	671 €	610 €	459 €	0.61	0
		Low	1400	838 €	762 €	574 €	0.54	0
								Total 105225

Solar thermal collectors-Buildings B1,B2 & B3					
	U	Qty	VAT	Unit price	Amount excluding tax
Solar thermal collectors	u	171		750 €	128,250 €
Supports & integration	set	1		3,600 €	3,600 €
Primary network					19,650 €
Network including heat insulation, balancing purge,	u	1		3,800 €	3,800 €
Pump	u	1		1,700 €	1,700 €
Security groups - expansion	u	1		750 €	750 €
1000l solar tank + flooded exchanger	u	3		2,800 €	8,400 €
Miscellaneous (Local layout in the attic)	set	1		5,000 €	5,000 €
Secondary network					15,300 €
Network including heat insulation, balancing p	set	1		3,500 €	3,500 €
DANFOSS MTCV valves	set	3		350 €	1,050 €
Network trim finishes	set	1		3,500 €	3,500 €
Water treatment (electromagnetic))	set	1		6,500 €	6,500 €
CE electrical resistance control	set	3		250 €	750 €
Regulation		1		3,200 €	3,200 €
Various					3,350 €
Studies - administrative file	set	1		850 €	850 €
DOE - plans	set	1		850 €	850 €
Commissioning tests and adjustments	set	1		850 €	850 €
Consuel	set	1		800 €	800 €
Maintenance contract	set	1			800 €
				Tot.	174,150 €
				TVA	34,830 €
Photovoltaic solar collectors-Buildings B1,B2 & B3					
	U	Qty	VAT	Unit price	Amount excluding tax
Photovoltaic sensors	u	171		245 €	41,895 €
Supports & integration	set	1		7,500 €	7,500 €
Inverters					10,060 €
SOLAREEDGE SE 12,5K	u	1		3,800 €	3,800 €
Optimizers	u	56		60 €	3,360 €
Extended Warranty	set	1		2,900 €	2,900 €
Wiring and pathways					9,400 €
DC wiring	set	1		900 €	900 €
AC wiring	set	1		4,700 €	4,700 €
pathways	set	1		1,100 €	1,100 €
DC protection	set	1		900 €	900 €
AC protection	set	1		900 €	900 €
Grounding	set	1		900 €	900 €
Monitoring		1		1,100 €	1,100 €
Various					3,350 €
Studies - administrative file	set	1		850 €	850 €
DOE - plans	set	1		850 €	850 €
Commissioning tests and adjustments	set	1		850 €	850 €
Consuel	set	1		800 €	800 €
Maintenance contract	set	1			600 €
				Tot.	73,905 €
				VAT	14,781 €

Appendix D: Python code implementation for the probabilistic ELECTRE Tri methodology.

This appendix presents a comprehensive explanation and documentation of the Python code implementation for the probabilistic ELECTRE Tri methodology. The code is developed to facilitate the application of this multi-criteria decision-making method, effectively addressing input data uncertainty by incorporating probabilistic data.

The implementation includes the following key components:

Input data processing: Functions to handle input data, including criteria values, weights, thresholds, and probability distributions for uncertain parameters.

Credibility index calculation: Algorithms to compute concordance and discordance indices, which are fundamental to the ELECTRE Tri methodology.

Monte Carlo simulation: Integration of probabilistic data using normal distribution through Monte Carlo simulation to address uncertainty in the input data.

Classification of alternatives: Procedures for assigning alternatives to categories based on both optimistic and pessimistic approaches.

In [1]:

```
import csv
import pandas as pd
import numpy as np
from numpy import random, vstack, empty
import math
```

Import data from csv file as a Pandas DataFrame

The input for the analysis is a `csv` file containing 16 rows and 39 columns.

The 16 rows correspond to the 16 criteria. The row indices are therefore the names of the criteria: `g1.1, g1.2, g1.3, g1.4, g1.5, g2.1, g2.2, g2.3, g2.4, g3.1, g3.2, g3.3, g3.4, g4.1, g4.2, g4.3`.

The columns contain the following information:

- The **mean performance value** of each scenario for each criterion (columns 0 to 27)
Names of the columns:
`'S1.1','S1.2','S1.3','S1.4','S2.1','S2.2','S2.3','S2.4','S3.1','S3.2','S3.3','S3.4',`
`'S4.1','S4.2','S4.3','S4.4','S5.1','S5.2','S5.3','S5.4','S6.1','S6.2','S6.3','S6.4',`
`'S7.1','S7.2','S7.3','S7.4'`
- The **weight** of each criterion (column 28)
Column name: `Weights`
- The **variance** of each criterion (column 29)
Column name: `Var`
- The **6 reference profiles**: `b0, b1, b2, b3, b4` and `b5` (columns 30 to 35)
Columns name: `b0, b1, b2, b3, b4` and `b5`
- The **3 thresholds**: `q` (the indifference threshold), `p` (the preference threshold), `v` (the veto threshold) (columns 36 to 38)
Columns name: `q, p` and `v`

The file is imported as a DataFrame `d`.

Two others parameters are also defined:

- `λ`: The **cut-off threshold**
- `repeat`: The **number of repetitions** for the Electre Tri method

In [2]:

```
d = pd.read_csv('Input_data_veto.csv')
λ = 0.75
repeat = 10000
```


Monte Carlo function

Twenty-eight scenarios were analyzed based on sixteen criteria. Computing all possible combinations of performance values is not feasible. To obtain more robust results, using the Monte Carlo method is proposed to generate data sets from distributions, which are then applied to the Electre Tri method.

Monte-Carlo simulation is commonly used in complex systems to estimate outcomes by using random sampling and statistical modeling.

1. Select a value from probability distribution functions.
2. Perform the calculation multiple times: (Electre Tri).
3. Obtain a set of results to be analyzed.

The first step involves providing probability distribution functions as inputs, where all values are represented as normal distributions. Two parameters are required to describe these distributions:

- ♦ The mean value: `m`
- ♦ The variance: `variance`

These values are provided in the `d` DataFrame.

The following function performs the steps below:

1. Using the variance and mean value, it creates the normal distribution for each performance.
2. Selects a random value from each distribution.
3. Returns a DataFrame (also called `d`) containing the randomly selected values.

*The returned Data Frame will also contain all the parameters initially present in the input `d` DataFrame.

In [3]:

```
def MCarlo(d):
    variance = d['VAR'].values
    m = d.iloc[:, 0:28].values #for each scenario : columns 0 to 27
    v = np.abs(m * variance[:, np.newaxis]) # calculation for the variance v
    perf = np.random.normal(m, v # pick a random value from the normal distribution with v
    d.iloc[:, 0:28] = perf
    return d
```

Partial concordance indices

This function takes as input the `d` DataFrame containing all the performances as well as all the other parameters and inputs required by the method. However, only the performance values, reference profiles, and thresholds will be used. The objective is to calculate, for each criterion j , the concordance between each pair of alternatives a_i and reference profiles b_k in both directions:

- The concordance $C_j(a_i, b_k)$
- The concordance $C_j(b_k, a_i)$

If the value of the concordance is greater than one, it is replaced by 1, and if it is less than zero, it is replaced by 0.

Finally, the function returns two DataFrames:

- `dconca`: The concordance between the performances and the reference profiles $C_j(a_i, b_k)$
- `dconcb`: The concordance between the reference profiles and the performances $C_j(b_k, a_i)$

In [4]:

```
def conce(d):
    new_df = pd.DataFrame() #DataFrame that will contain Cj(ai,bk)
    new_df2 = pd.DataFrame() #DataFrame that will contain Cj(bk,ai)
    for sc in d.iloc[:, 0:28]: # for each scenario : columns 0 to 27
        for pr in d.iloc[:, 30:36]: # for each reference profile : columns 30 to 35
            alpha = (d[sc]-d[pr]+d[d.columns[37]])/(d[d.columns[37]]-d[d.columns[36]]) #
            beta = (d[pr]-d[sc]+d[d.columns[37]])/(d[d.columns[37]]-d[d.columns[36]]) #
            new_df = pd.concat([new_df, alpha], axis=1, ignore_index=True)
            new_df2 = pd.concat([new_df2, beta], axis=1, ignore_index=True)
    # replace the negative values by zero and the one higher than one by one in both Dat
    new_df[new_df<0]=0
    new_df[new_df>1]=1
    new_df2[new_df2<0]=0
    new_df2[new_df2>1]=1
    return new_df, new_df2
```

Partial discordance indices

This function takes as input the `d` DataFrame containing all the performances as well as all the others parameters and inputs required by the method. However, only the performance values, reference profiles, and thresholds will be used. The objective is to calculate, for each criterion j , the discordance between each pair of alternatives a_i and reference profiles b_k in both directions:

- The discordance $D_j(a_i, b_k)$
- The discordance $D_j(b_k, a_i)$

If the difference is less than the indifference threshold α_j , the partial concordance will be equal to 1, as this corresponds to accepting the assertion "a_i is at least as good as b_k". On the other hand, if this difference is greater than the preference threshold β_j , the partial concordance will be equal to 0, meaning that the assertion "a_i is at least as good as b_k" is not concordant. If the value of discordance is greater than one it is replaced by 1, and if it is smaller than zero, it is replaced by 0.

Finally, the function returns two DataFrames:

- `ddiscoa`: The discordance between the performances and the reference profiles $D_j(a_i, b_k)$
- `ddiscob`: The discordance between the reference profiles and the performances $D_j(b_k, a_i)$

In [5]:

```
def disco(d):
    new_df = pd.DataFrame() # DataFrame that will contain Dj(ai,bk)
    new_df2 = pd.DataFrame() # DataFrame that will contain Dj(bk,ai)
    for sc in d.iloc[:, 0:28]: # for each scenario : columns 0 to 27
        for pr in d.iloc[:, 30:36]: # for each reference profile : columns 30 to 35
            alpha = (d[pr]-d[sc]-d[d.columns[37]])/(d[d.columns[38]]-d[d.columns[37]]) #
            beta = (d[sc]-d[pr]-d[d.columns[37]])/(d[d.columns[38]]-d[d.columns[37]]) #
            new_df = pd.concat([new_df, alpha], axis=1, ignore_index=True)
            new_df2 = pd.concat([new_df2, beta], axis=1, ignore_index=True)
        # replace the negative values by zero and the one higher than one by one in both Da
    new_df[new_df<0]=0
    new_df[new_df>1]=1
    new_df2[new_df2<0]=0
    new_df2[new_df2>1]=1
    return new_df, new_df2
```

Global concordance indices

The aim of this step is to calculate the global concordance of each scenario for each criterion. In other words, it indicates the extent to which the performances of a_i and b_k across all criteria are concordant with the assertion "a_i outranks b_k". The function requires the following inputs:

- The weights for each criterion, located in the `d` DataFrame in the column 28, named `weights`.
- The concordance matrix, divided into 2 DataFrames previously: `dconca` and `dconcb`.

The objective is to calculate the following global concordance for each scenario:

This function needs to be used twice:

- Once with `dconca` as input, returning `dgconca` as output: $C(a_i, b_k)$
- Once with `dconcb` as input, returning `dgconcb` as output: $C(b_k, a_i)$

In [6]:

```
def global_conc(d,dconcl):
new_df = pd.DataFrame(index=['b0', 'b1', 'b2', 'b3', 'b4', 'b5'], columns=['S1.1','S i = 0
for j in range(0, len(dconcl.columns),6): # for each scenario : one line out of 6 #
C(ai,bk) for the scenario for each reference profile
a = sum(dconcl[j]*d[d.columns[28]])/sum(d[d.columns[28]])
b = sum(dconcl[j+1]*d[d.columns[28]])/sum(d[d.columns[28]])

c = sum(dconcl[j+2]*d[d.columns[28]])/sum(d[d.columns[28]])
dr = sum(dconcl[j+3]*d[d.columns[28]])/sum(d[d.columns[28]])
e = sum(dconcl[j+4]*d[d.columns[28]])/sum(d[d.columns[28]])
f = sum(dconcl[j+5]*d[d.columns[28]])/sum(d[d.columns[28]])
th = [a,b,c,dr,e,f]
new_df[new_df.columns[i]]= th # add the global concordance as a new column
i = i+1

return new_df
```

Degree of credibility

The degree of credibility is calculated using:

- The global concordance of each scenario with each reference profile $C(a_i, b_k)$ named `dgconca`.
- The discordance matrix, divided in two DataFrames `ddiscoa` containing the discordance $D_j(a_i, b_k)$ and `ddiscob` containing the discordance $D_j(b_k, a_i)$.

Since the degree of credibility must be calculated both by comparing scenario performance with reference profiles and by comparing reference profiles with scenario performance, the following function should be executed twice:

- Once for the comparison of scenario performance with reference profiles (a_i, b_k)
 - Input: `dgconca` for the global concordance $C(a_i, b_k)$ and `ddiscoa` : the discordance: $D_j(a_i, b_k)$
 - Output: `dcreda`, the credibility $\delta(a_i, b_k)$
- Once for the comparison of reference profiles with scenario performances (b_k, a_i)
 - Input: `dgconcb` for the global concordance $C(b_k, a_i)$ and `ddiscoa`: the discordance: $D_j(b_k, a_i)$
 - Output: `dcredb`, the credibility $\delta(b_k, a_i)$

```
In [7]: def credibility(dgconc, ddisc):
#initialization
dcred = pd.DataFrame(index=['b0', 'b1', 'b2', 'b3', 'b4', 'b5'], columns=['S1.1', 'S1
for j in range(0, len(ddisc.columns), 6):
sc = int(j/6)
degree = [0, 0, 0, 0, 0, 0]
for pr in range(len(dcred.index)):
#verification if all  $D_j < C$ 
verif = sum(ddisc[j+pr][c] > dgconc[dgconc.columns[sc]][pr] for c in ddisc.i
#case 1
if verific == 0:
degree[pr] = dgconc[dgconc.columns[sc]][pr]
#case 2
else:
degree[pr] = (((1-ddisc[j+pr][ddisc[j+pr] > dgconc[dgconc.columns[sc]][p
dcred[dcred.columns[sc]] = degree
return dcred
```

Outranking

The objective of this step is to establish preference relationships between performances and reference profiles. These relationships are determined based on the degree of credibility calculated and the cut-off threshold λ . The value of this cutting threshold can vary.

There are four types of relationships that can be established between each a_i and each b_k .

- ♦ $a_i \ I \ b_k$: a_i is Indifferent to b_k
- ♦ $a_i \ > \ b_k$: a_i is preferred to b_k
- ♦ $a_i \ < \ b_k$: a_i is not preferred to b_k
- ♦ $a_i \ \mathbf{R} \ b_k$: a_i incomparable to b_k

The function will return a Dataframe, `dranking`, containing all these relationships between performances and reference profile.

In [8]:

```
def out_ranking_relations(creda, credb, λ):
    #initialization
    new_df = pd.DataFrame(index=['b0', 'b1', 'b2', 'b3', 'b4', 'b5'], columns=['S1.1', 'S
    classement_a = creda.apply(lambda x: x-λ)
    classement_b = credb.apply(lambda x: x-λ)
    #1 if outperformance (S), 0 if not
    classement_a[classement_a > 0] = 1
    classement_a[classement_a < 0] = 0
    classement_b[classement_b > 0] = 1
    classement_b[classement_b < 0] = 0

    mask = (classement_a == classement_b) & (classement_a == 1)
    new_df = new_df.mask(mask, "I")
    mask = (classement_a == classement_b) & (classement_a == 0)
    new_df = new_df.mask(mask, "R")
    mask = (classement_b != 0) & (classement_a == 0)
    new_df = new_df.mask(mask, "<")
    mask = (classement_a != 0) & (classement_b == 0)
    new_df = new_df.mask(mask, ">")
    return new_df
```

Sorting

The relationships established allow to achieve the final goal of the method, i.e. assigning each alternative a category. Two sorting procedures are performed: optimistic and pessimistic sorting. The primary difference between the two is that the pessimistic sorting "*pushes the alternative downward*" starting from the best category, while the optimistic sorting "*pushes the alternative up*" starting from the worst category.

A median ranking can be obtained as the average of these two rankings.

Pessimistic sorting

The following function performs pessimistic sorting using the outranking relationships established. The objective is to place each scenario in one of the five predefined categories. This type of sorting "*pushes the alternative down*".

The sorting works as follows:

The six reference profiles b_0, b_1, b_2, b_3, b_4 and b_5 define 5 categories:

C_1, C_2, C_3, C_4 and C_5 , where C_5 is the best and C_1 is the worst.

For each scenario, the categories are considered from best to worst (from C_5 to C_1). For each reference profiles encountered, the credibility $\delta(a_i, b_k)$ is compared to the cutting threshold λ :

- $\delta(a_i, b_k) > \lambda$: The scenario is ranked in the category with the same number as b_k .
- $\delta(a_i, b_k) < \lambda$: Continue to the next reference profile b_k .

In [9]:

```
def pessimistic_sort(drinking, pessi):  
    for sc in drinking:  
        step = pessi[sc]  
        for pr in reversed(range(len(drinking.index))):  
            if drinking[sc][pr] == '>' or drinking[sc][pr] == 'I':  
                step[step.index[pr]] = step[step.index[pr]] + 1 #classified break  
        pessi[sc] = step  
    return pessi
```

Optimistic sorting

The following function performs optimistic sorting using the outranking relationships established.

The sorting works as follows:

The six reference profiles define five categories, with C5 as the best and C1 as the worst.

The difference in this sorting is that, for each scenario, the categories are considered from worst to best (from C1 to C5). For each reference profiles encountered, the outranking relationships is analyzed:

- If $a_i < b_k$: The scenario is assigned to the category corresponding to b_k .
- If $a_i > b_k$, $a_i \sim b_k$ or $a_i \perp b_k$: Proceed to the next reference profile.

In [10]:

```
def optimistic_sort(drinking, opti):  
    for sc in drinking:  
        step = opti[sc]  
        for pr in (range(len(drinking.index))):  
            if drinking[sc][pr] == '<' or drinking[sc][pr] == 'R':  
                step[step.index[pr]] = step[step.index[pr]] + 1 #classified  
                break  
        opti[sc] = step  
    return opti
```

Electre Tri application function

This final method executes all previous steps to perform the Electre Tri method.

Input:

d: The input Dataframe containing performances, weights, variance, reference profiles, and thresholds.

rep: The number of times the Electre Tri method will be run, defined at the beginning of the code.

Two DataFrames are established:

Pessi_sort: Stores the pessimistic classification obtained in each iteration of the method.

Opti_sort: Stores the optimistic classification obtained in each iteration of the method.

Both DataFrames are structured similarly:

Rows: Represent the 5 categories, indexed as 'C1', 'C2', 'C3', 'C4', 'C5'. Columns: Represent the 28 scenarios, with column names: 'S1.1', 'S1.2', 'S1.3', 'S1.4', 'S2.1', 'S2.2', 'S2.3', 'S2.4', 'S3.1', 'S3.2', 'S3.3', 'S3.4', 'S4.1', 'S4.2', 'S4.3', 'S4.4', 'S5.1', 'S5.2', 'S5.3', 'S5.4', 'S6.1', 'S6.2', 'S6.3', 'S6.4', 'S7.1', 'S7.2', 'S7.3', 'S7.4'.

Initially, both DataFrames are filled with zeros.

The following functions are executed sequentially, repeated `reptimes` as defined at the beginning of the code:

(Each function is defined with detailed explanations of input and output data)

- ♦ `MCarlo`: Monte Carlo function

Input: DataFrame `d`

Output: Modified dataframe `d`: The mean values are replaced by performances values

- ♦ `conce`: Partial Concordance function

Input: DataFrame `d`

Output: Two concordance matrices `dconca`, and `dconcb`

- ♦ `disco`: Discordance function

Input: DataFrame `d`

Output: Two discordance DataFrames `ddisca`, and `ddiscb`

- ♦ `global_con`: Global Concordance function

This function is called twice: Input: DataFrame `d` and concordance DataFrame `dconca`

Output: Global concordance DataFrame `dgconca`

- Second call Input: DataFrame `d` and the concordance DataFrame `dconcb`

Output: Global concordance DataFrame `dgconcb`

- ♦ `credibility`: Credibility Degree function

This function is also called twice: Input: Global concordance and discordance DataFrames `dgconca` and

`ddisca`

Output: Credibility DataFrame `dcreda`

- Second call Input: Global concordance and discordance DataFrames `dgconcb` and `ddiscb`

Output: Credibility DataFrame `dcredb`

- ♦ `out_ranking_relations`: Out ranking function

Input: Credibility DataFrames `dcreda`` and `dcredb``

Output: Outranking DataFrame `d-ranking``

- ♦ `optimistic_sorting`: Optimistic sorting function

Input: Outranking DataFrame `dranking`` and the optimistic sorting DataFrame from the previous iteration `opti_sort``

Output: Modified optimistic sorting DataFrame, updated with the latest optimistic sorting results

`opti_sort`

- ♦ `pessimistic_sorting`: Pessimistic sorting function

Input: Outranking DataFrame `d-ranking` and the pessimistic sorting DataFrame `pessi_sort`

Output: Modified pessimistic sorting DataFrame, updated with the latest pessimistic sorting results

`pessi_sort`

In [11]:

```
Def      electre_tri      (d,rep):
#initialization
pessi   = np.zeros((5,28))
opti    = np.zeros((5,28))
pessi_sort = pd.DataFrame(pessi, index=['C1', 'C2', 'C3', 'C4', 'C5'], columns=['S1.
opti_sort = pd.DataFrame(opti, index=['C1', 'C2', 'C3', 'C4', 'C5'], columns=['S1.1'
#repetitions
for i in range(rep) :
    d = MCarlo(d)
    dconca, dconcb = conce(d)
    ddisca, ddiscb = disco(d)
    dgconca = global_conc(d,dconca)
    dgconcb = global_conc(d,dconcb)
    dcreda = credibility(dgconca, ddisca)
    dcredb = credibility(dgconcb, ddiscb)
    drankng = over_ranking_relations(dcreda, dcredb, λ)
    opti_sort = optimistic_sort(drankng,opti_sort)
    pessi_sort = pessimistic_sort(drankng,pessi_sort)
pessi_sort = pessi_sort.apply(lambda x: (x/rep)*100) #%
opti_sort = opti_sort.apply(lambda x: x/rep*100) #%
return opti_sort, pessi_sort, drankng
```

The `electre_tri` function is executed, returning two DataFrames: `opti_sort` and `pessi_sort`

Subsequently, two csv files are generated, containing the distribution of scenarios across categories as percentages:

- ♦ `pessimistic_sorting.csv` represents the result for the pessimistic sorting.
- ♦ `optimistic_sorting.csv` represents the result for the optimistic sorting.

In [12]:

```
opti_sort, pessi_sort, drankng = electre_tri (d, repeat)

#pessi_sort.to_csv('pessimistic_sorting.csv')
#opti_sort.to_csv('optimistic_sorting.csv')

print(opti_sort)
print(pessi_sort)
```

	S1.1	S1.2	S1.3	S1.4	S2.1	S2.2	S2.3	S2.4	S3.1	S3.2	...
C1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...
C2	100.0	28.4	0.7	41.9	14.5	0.2	2.2	0.1	14.4	1.3	...
C3	0.0	71.0	65.8	58.0	82.6	3.1	7.3	3.0	82.3	4.3	...
C4	0.0	0.6	33.3	0.1	2.9	53.9	65.7	57.8	3.3	94.4	...
C5	0.0	0.0	0.2	0.0	0.0	42.8	24.8	39.1	0.0	0.0	...

	S5.4	S6.1	S6.2	S6.3	S6.4	S7.1	S7.2	S7.3	S7.4
C1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
C2	100.0	0.1	0.3	21.7	0.3	100.0	100.0	100.0	100.0
C3	0.0	40.7	11.6	75.9	0.7	0.0	0.0	0.0	0.0
C4	0.0	58.1	87.9	2.4	94.0	0.0	0.0	0.0	0.0
C5	0.0	1.1	0.2	0.0	5.0	0.0	0.0	0.0	0.0

[5 rows x 28 columns]

	S1.1	S1.2	S1.3	S1.4	S2.1	S2.2	S2.3	S2.4	S3.1	S3.2	...
C1	100.0	28.4	0.7	41.9	14.5	0.2	2.2	0.1	14.4	1.3	...
C2	0.0	71.0	65.8	58.0	82.6	3.1	7.3	3.0	82.3	4.3	...
C3	0.0	0.6	33.3	0.1	2.9	53.9	65.7	57.8	3.3	94.4	...
C4	0.0	0.0	0.2	0.0	0.0	42.8	24.8	39.1	0.0	0.0	...
C5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...

	S5.4	S6.1	S6.2	S6.3	S6.4	S7.1	S7.2	S7.3	S7.4
C1	100.0	0.1	0.3	21.7	0.3	100.0	100.0	100.0	100.0
C2	0.0	40.7	11.6	75.9	0.7	0.0	0.0	0.0	0.0
C3	0.0	58.1	87.9	2.4	94.0	0.0	0.0	0.0	0.0
C4	0.0	1.1	0.2	0.0	5.0	0.0	0.0	0.0	0.0
C5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

[5 rows x 28 columns]

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THESE DE L'INSA LYON, MEMBRE DE L'UNIVERSITE DE LYON

NOM: BASEER	DATE de SOUTENANCE: 16/10/2024
Prénoms: Majid	
TITRE: Multi-criteria decision making for sustainable engineering systems-Application for the energy renovation of buildings	
NATURE : Doctorat	Numéro d'ordre : 2024ISAL0084
Ecole doctorale: Mécanique, énergétique, génie civil, acoustique (MEGA)	
Spécialité: Génie Mécanique	
RESUME:	
<p>The thesis aims to develop a multi-criteria decision making (MCDM) framework based on sustainability factors that encompass economic, environmental, social, and technical aspects within the context of energy renovation for existing buildings. In contrast to existing deterministic MCDM methods, this framework will consider uncertain data in its calculations, resulting in outcomes more aligned with reality.</p> <p>The trend towards sustainability in the building sector is steadily gaining momentum, driven by well-established environmental regulations and the strong will of societies worldwide. Although new building projects adhere to established standards, the construction of new buildings replaces only 1-3% of existing buildings annually. Renovation projects often present challenges such as the presence of uncertain data regarding existing building conditions, involvement of multiple stakeholders with different interests and priorities, and the quantitative and qualitative nature of the sustainability factors. To address these limitations, this thesis intends to assist in integrating uncertainty into the decision-making process.</p> <p>A state-of-the-art review was conducted of some of the commonly used MCDM methods. Furthermore, a SWOT (Strengths, Weaknesses, Opportunities and Threats) analysis was performed to justify the selection of a specific MCDM method for the research problem. The application of MCDM methods in the context of energy renovation of buildings was examined in order to gain in depth understanding of the sustainability factors under consideration. To address this issue, probability distribution and Monte Carlo simulation are employed and integrated with MCDM methods. These tools have the capability to represent uncertainty, simulate human decision-making, and handle ambiguous situations.</p> <p>The novel MCDM framework, probabilistic ELECTRE Tri, was specifically developed for the classification of scenarios for building energy renovation. To illustrate and validate the proposed framework, it was applied to a real-world case study of a social housing project located in the region of Lyon, France. The results obtained through the novel probabilistic ELECTRE Tri method are compared with those obtained using ELECTRE Tri, highlighting the significance of incorporating uncertainty in the decision-making process. The developed MCDM framework is designed to be generic in nature and can be used by decision-makers to enhance objectivity and consistency of the decision-making process. Additionally, it contributes to the advancement of sustainable building renovation and aligns with the broader global agenda of environmental stewardship and energy efficiency.</p>	
MOTS-CLÉS: multi-criteria decision making, ELECTRE Tri, uncertainty, probability distribution, criteria weighting, energy renovation, sustainability	
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