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INSTITUT  
POLYTECHNIQUE  
DE PARIS

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Thèse de doctorat



# Robust Evaluation of Public Policies to Heterogeneous Effects

Thèse de doctorat de l'Institut Polytechnique de Paris  
préparée à l'École nationale de la statistique et de l'administration économique

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Thèse présentée et soutenue à Palaiseau, le 2 décembre 2024, par

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# Contents

- Introduction Générale** **6**
  
- General Introduction** **14**
  
- 1 Difference-in-Differences for Continuous Treatments and Instruments with Stayers** **23**
  - 1.1 Introduction . . . . . 23
  - 1.2 Set-up, assumptions, and building-block identification result . . . . . 27
    - 1.2.1 Set-up . . . . . 27
    - 1.2.2 Assumptions . . . . . 28
    - 1.2.3 Building-block identification result . . . . . 29
  - 1.3 Estimating the average of switchers’ slopes . . . . . 32
    - 1.3.1 Target parameter . . . . . 32
    - 1.3.2 Identification . . . . . 33
    - 1.3.3 Estimation and inference . . . . . 34
  - 1.4 Estimating a weighted average of switchers’ slopes . . . . . 35
    - 1.4.1 Target parameter . . . . . 35
    - 1.4.2 Identification . . . . . 36
    - 1.4.3 Estimation and inference . . . . . 38
  - 1.5 Instrumental-variable estimation . . . . . 40
    - 1.5.1 Notation and assumptions . . . . . 41
    - 1.5.2 Target parameter . . . . . 43
    - 1.5.3 Identification . . . . . 43
    - 1.5.4 Estimation and inference . . . . . 44
  - 1.6 Extensions . . . . . 45
    - 1.6.1 More than two time periods . . . . . 45
    - 1.6.2 Placebo tests . . . . . 46

1.6.3	Estimators robust to dynamic effects up to a pre-specified treatment lag . . . . .	47
1.7	Application . . . . .	47
1.8	Conclusion . . . . .	53
<b>2</b>	<b>Estimating heterogeneous peer effects with partial population experiments</b>	<b>55</b>
2.1	Introduction . . . . .	55
2.2	Set-up, notations and main assumptions . . . . .	59
2.2.1	Randomized experiment . . . . .	59
2.2.2	Natural Experiment . . . . .	62
2.3	Identification . . . . .	64
2.3.1	Intuition . . . . .	64
2.3.2	Randomized Experiment . . . . .	65
2.3.3	Natural Experiment . . . . .	67
2.3.4	Sufficient Conditions for Identification . . . . .	68
2.4	Estimation . . . . .	68
2.4.1	Randomized Experiment . . . . .	69
2.4.2	Natural Experiment . . . . .	70
2.5	Extensions . . . . .	72
2.5.1	Identity-specific “within” and “between” influences . . . . .	72
2.5.2	Identity orthogonal to treatment eligibility . . . . .	72
2.5.3	Including Covariates . . . . .	73
2.6	Application: Conditional cash transfers in Mexico (Progresa) . . . . .	73
2.7	Conclusion . . . . .	76
<b>3</b>	<b>Elderly Home Care Market and Spousal Informal Care Supply</b>	<b>83</b>
3.1	Introduction . . . . .	83
3.2	Institutional Setting . . . . .	87
3.2.1	French LTC Providers . . . . .	87
3.2.2	LTC Funding . . . . .	88
3.3	Model . . . . .	90
3.3.1	Households . . . . .	90
3.3.2	Firms . . . . .	92
3.4	Data . . . . .	93
3.4.1	CARE - Ménages . . . . .	93
3.5	Estimation . . . . .	101

3.5.1	A supply with constant marginal costs . . . . .	101
3.5.2	A greater reaction of male caregiving spouses to a change in the out-of-pocket formal home care price . . . . .	103
3.6	Future Developments . . . . .	104
3.7	Conclusion . . . . .	105
<b>Conclusion</b>		<b>107</b>
<b>A</b>	<b>Difference-in-Differences for Continuous Treatments and Instruments with Stayers</b>	<b>109</b>
A.1	Proofs . . . . .	109
A.1.1	Theorem 1 . . . . .	109
A.1.2	Theorem 2 . . . . .	109
A.1.3	Theorem 3 . . . . .	111
A.1.4	Theorem 4 . . . . .	114
A.1.5	Theorem 5 . . . . .	115
A.1.6	Proposition 1 . . . . .	122
A.1.7	Theorem 6 . . . . .	123
A.1.8	Theorem 7 . . . . .	125
A.1.9	Theorem 8 . . . . .	125
<b>B</b>	<b>Estimating heterogeneous peer effects with partial population experiments</b>	<b>127</b>
B.1	Proofs . . . . .	127
B.1.1	Reduced forms in the randomized experiment design . . . . .	127
B.1.2	Proposition 2 . . . . .	128
B.1.3	Proposition 2 . . . . .	130
B.1.4	Proposition 3 . . . . .	132
B.1.5	Proposition 4 . . . . .	133
B.1.6	Proposition 5 . . . . .	134
B.2	Identity-specific “within” and “between” influences . . . . .	142
B.2.1	Identification . . . . .	142
B.2.2	Estimation . . . . .	146
B.2.3	Simulations . . . . .	147
B.3	Orthogonality between Eligibility and Identity . . . . .	151
B.4	Inclusion of Covariates . . . . .	155
B.4.1	Identification . . . . .	155



B.4.2 Estimation . . . . .	157
<b>C Elderly Home Care Market and Spousal Informal Care Supply</b>	<b>159</b>
C.1 Figures . . . . .	159
C.2 Likelihood Formula . . . . .	160
C.3 Simulations . . . . .	162

# List of Figures

1.1	Gasoline tax in 1987 among 1987-to-1988 switchers and stayers . . . . .	49
1.2	Distribution of tax changes between consecutive periods . . . . .	50
2.1	Decomposition of Total Treatment Effects . . . . .	77
2.2	Population Multipliers by Population Structure . . . . .	78
2.3	Treatment effects on eligibles and non-eligibles . . . . .	79
2.4	Progresa - Common Support Assumption . . . . .	79
2.5	Results - Progresa . . . . .	80
2.5	Results - Progresa . . . . .	81
B.1	Simulations - 4 endogenous peer effects parameters - 1,000 groups . . . . .	149
B.2	Simulations - 4 endogenous peer effects parameters - 10,000 groups . . . . .	149
B.3	Simulations - 4 endogenous peer effects parameters - 1,000,000 groups . . . . .	150
B.4	Simulations - 2 endogenous peer effects parameters - 1,000 groups . . . . .	150
C.1	Linear fit between the total contract hours of home care workers employed by a SAAD and the estimated demand for home care hours for SAADs based on data from the CARE-Ménages survey . . . . .	159

# List of Tables

1.1	Comparing switchers and stayers	49
1.2	Effects of gasoline tax on quantities consumed and prices	51
1.3	Placebo effects of gasoline tax on quantities consumed and prices	52
1.4	IV estimators of the price-elasticity of gasoline consumption	53
2.1	Descriptive Statistics - Progresa	81
2.2	Estimation results - Progresa	82
3.1	<i>CARE-Ménages</i> - Summary Statistics	95
3.2	Care by level of dependency and relationship status	96
3.3	Health and feelings among caring spouses, by gender and age	98
3.4	Subsample for estimation of the demand - summary statistics	99
3.5	Subsample for the estimation of the supply - summary statistics	100
3.6	Supply - IV estimates	102
3.7	Demand - Maximum Likelihood estimates	103
C.1	Marginal Densities - Likelihood Function	161
C.2	Simulations - Estimates	163

# Introduction Générale

La science économique a connu un « tournant appliqué » (Backhouse and Cherrier (2014)) ces dernières décennies. Dans les années 1960, plus de la moitié des articles publiés dans l'*American Economic Review*, le *Journal of Political Economy* et le *Quarterly Journal of Economics* — trois des revues généralistes les plus respectées de la discipline — étaient des travaux purement théoriques. Cette part est tombée à moins de 28% au début des années 2010 (Hamermesh (2013)). L'une des causes identifiées de ce basculement a été le développement et la promotion de meilleures méthodes empiriques, permettant de mieux distinguer corrélation et causalité, dans un mouvement que Angrist and Pischke (2010) ont qualifié de « révolution de la crédibilité ». La méthode la plus fiable pour identifier des effets causaux, en termes de validité et de robustesse des estimations statistiques, repose sur les expériences avec assignation aléatoire. C'est le principe des essais cliniques en médecine : deux groupes sont générés aléatoirement à partir d'une même population, l'un bénéficiant d'un traitement et l'autre servant de groupe témoin. L'effet causal moyen du traitement peut alors être mesuré en comparant les résultats moyens entre le groupe traité et le groupe témoin. L'évaluation des politiques publiques par assignation aléatoire a démontré, par exemple, l'effet causal positif de l'assurance maladie universelle sur l'utilisation de soins et l'état de santé de diverses sous-populations (Finkelstein et al. (2012)) et a prouvé l'efficacité de nombreux programmes visant, par exemple, à améliorer l'assiduité des élèves à l'école (e.g. Miguel and Kremer (2004), Parker and Todd (2017)) ou les résultats sur le marché du travail pour les personnes au chômage (e.g. LaLonde (1986)). Toutefois, les expériences randomisées sont coûteuses, chronophages et parfois impossibles à mettre en œuvre. Il est ainsi difficile d'imaginer la mise en place d'une expérience aléatoire évaluant, par exemple, l'effet de l'immigration sur l'économie d'un pays hôte. Cependant, il existe de nombreuses situations où des groupes traités et non traités peuvent être construits à partir d'un événement externe, naturel, qui agit « comme si » la population avait été divisée aléatoirement. Étant donné que l'événement crée des conditions similaires à une assignation aléatoire, ces expériences naturelles peuvent être utilisées pour établir des relations causales. Plusieurs méthodes quasi-expérimentales d'analyse des politiques ont été développées pour concevoir des groupes témoins pertinents et crédibles. Les méthodes les plus utilisées sont les variables instrumentales (IV),

les méthodes de régression sur discontinuité (RD) et les différences-en-différences (DiD). Les modèles IV sont généralement employés lorsque l'assignation du traitement est corrélée avec d'autres facteurs confondants, ce qui empêche d'isoler les effets causaux du traitement. La variable instrumentale est un facteur tiers corrélé à l'assignation du traitement mais qui n'a aucun effet direct sur la variable d'intérêt, si ce n'est par son lien avec la variable traitement. Les méthodes RD exploitent souvent les critères d'éligibilité arbitraires d'un programme, où les unités situées au-dessus d'un seuil sont éligibles et celles en dessous ne le sont pas. Les méthodes DiD estiment l'effet causal d'un traitement en comparant les changements d'une variable d'intérêt au fil du temps entre un groupe traité et un groupe non traité, en supposant que, sans l'existence du traitement, les deux groupes auraient suivi des tendances similaires. Des estimateurs statistiques intuitifs sont associés à chacune de ces méthodes. Pour les méthodes IV, l'estimateur est celui des doubles moindres carrés (2MC) (Angrist and Imbens (1995)). Pour les méthodes RD, il s'agit des régressions linéaires locales autour du seuil de discontinuité (Thistlethwaite and Campbell (1960)). Pour les méthodes DiD, l'estimateur est celui des moindres carrés ordinaires (MCO) issu d'une régression à double effets fixes (TWFE) (e.g. Bertrand et al. (2004)).

Lorsque les effets des traitements sont homogènes, ces trois méthodes quasi-expérimentales peuvent en principe, tout comme les expériences randomisées, estimer un effet causal moyen. Cependant, cette condition est rarement satisfaite. Lorsque cette hypothèse n'est pas vraie, l'interprétation causale des estimateurs associés à ces trois méthodes quasi-expérimentales est modifiée de diverses manières. Dans les RD, les estimations peuvent être interprétées comme l'effet causal moyen pour les unités situées juste autour du seuil. Dans les modèles IV, les conséquences de l'hétérogénéité des effets du traitement sur l'interprétation causale ont été étudiées par Angrist et al. (1996). Des restrictions supplémentaires, comme l'hypothèse de monotonie, sont nécessaires pour interpréter l'estimateur des 2MC comme un effet causal. Une vaste littérature étudiant l'inférence causale en présence d'effets de traitement hétérogènes utilisant des variables instrumentales a émergé à partir de ce projet initial (voir, par exemple, Angrist et al. (2000), Heckman and Vytlacil (2005), Mogstad and Torgovitsky (2018), Heckman and Pinto (2018)). Dans le design canonique de DiD avec un traitement binaire, deux groupes et deux périodes temporelles, le coefficient associé à la variable de traitement dans une régression TWFE peut être interprété comme un effet causal moyen au sein du groupe ayant reçu le traitement lors de la deuxième période, à condition que l'hypothèse de tendance commune soit satisfaite. Cependant, dans les designs comportant plus de deux groupes et/ou plus de deux périodes temporelles, même si l'hypothèse de tendance commune est satisfaite pour toutes les périodes temporelles consécutives, De Chaisemartin and d'Haultfoeuille (2020), Imai and Kim (2021), Goodman-Bacon (2021), Sun and Abraham (2021) et Borusyak et al. (2024) ont récemment démontré que l'estimateur des moindres carrés ordinaires (MCO) fondé sur la régression TWFE n'identifie pas un effet causal pertinent lorsque les effets

de traitement sont hétérogènes entre les groupes et/ou les périodes temporelles. Cet estimateur identifie une moyenne pondérée des différents effets de traitement, avec des poids qui, bien que sommant à un, peuvent être négatifs. Par conséquent, il est possible d'obtenir un estimateur négatif alors même que les effets de traitement pour chaque groupe et pour toutes les périodes, sont positifs. De nouveaux estimateurs robustes aux effets de traitement hétérogènes ont été mis au point pour des designs de différences-en-différences avec une adoption d'un traitement binaire potentiellement échelonnée dans le temps à travers les groupes (Callaway and Sant'Anna (2021), De Chaisemartin and d'Haultfoeuille (2020), Sun and Abraham (2021), Borusyak et al. (2024)). Toutefois, le cadre des différences-en-différences avec plusieurs groupes et/ou périodes temporelles a également été utilisé pour évaluer les effets de variables de traitement continues, comme des tarifs douaniers (par exemple, Fajgelbaum et al. (2020)) ou des taxes sur des biens de consommation (par exemple, Li et al. (2014)). Pourtant, l'estimateur MCO issu d'une régression TWFE se comporte également mal en termes d'inférence causale dans ce cadre, lorsque les effets de traitement sont hétérogènes entre les groupes, comme l'ont récemment démontré Callaway et al. (2024). Le premier chapitre de cette thèse, rédigé en collaboration avec Clément de Chaisemartin, Xavier D'Haultfoeuille, Doulo Sow et Gonzalo Vasquez-Bare, développe deux nouveaux estimateurs de paramètres causaux pertinents dans un cadre de différences-en-différences avec une variable de traitement continue. Le scénario classique envisagé est celui d'un économètre disposant d'un panel de nombreux groupes observés sur deux périodes. Tous les groupes peuvent être déjà traités lors de la première période, mais à des doses différentes. Entre les deux périodes, certains groupes, appelés *switchers*, subissent une modification de la dose de traitement qu'ils reçoivent. En revanche, d'autres groupes, appelés *stayers*, conservent la même dose de traitement au cours des deux périodes. Le premier paramètre considéré, appelé AS, correspond à la pente moyenne de la fonction des "résultats potentiels" pour les *switchers* pendant la seconde période, entre leur dose de traitement de la première période et celle de la seconde période. Le second paramètre, appelé WAS, est une version pondérée de cette moyenne, où chaque *switcher* reçoit un poids proportionnel à la valeur absolue du changement de sa dose de traitement entre les deux périodes. L'identification et l'estimation de ces deux paramètres reposent sur une nouvelle hypothèse de tendance commune : en l'absence de modification de leur dose de traitement entre les deux périodes, les groupes *switchers* auraient, en moyenne, connu la même évolution de leur variable de résultat que les groupes *stayers* ayant reçu la même dose initiale de traitement. Cette hypothèse de tendance commune présente l'avantage d'être testable par une méthode "placebo", en comparant les évolutions des résultats des *switchers* et des *stayers* avant les modifications de la dose de traitement des *switchers*. D'un point de vue économique, les paramètres AS et WAS n'ont pas vocation à servir les mêmes objectifs. Sous des restrictions de forme sur la fonction des résultats potentiels, le paramètre AS peut être utilisé pour inférer l'effet d'autres modifications des doses

de traitement que celles observées entre les périodes 1 et 2. Le WAS peut être utilisé pour effectuer une analyse coût-bénéfice des modifications des doses de traitement effectivement observées. Les résultats concernant l'identification et l'estimation de ces paramètres sont ensuite étendus à d'autres configurations. En particulier, ils sont adaptés à un cadre avec variables instrumentales. Ce type de design est couramment utilisé dans les études visant à estimer l'élasticité-prix de la demande pour un bien. Par exemple, les taxes variant entre différentes régions peuvent être utilisées comme instrument pour les prix de vente moyens du bien dans les régions. Nous montrons que le rapport entre l'effet WAS de l'instrument sur la variable de résultat et l'effet WAS de l'instrument sur la dose de traitement correspond à une moyenne pondérée des pentes des résultats des *switchers* par rapport au traitement, les *switchers* dont la dose de traitement réagit le plus fortement aux variations de l'instrument recevant un poids plus élevé. Ces résultats sont appliqués pour estimer l'élasticité-prix de la consommation d'essence, en utilisant un panel de données au niveau des États pour la période 1966-2008 compilé par Li et al. (2014).

Dans la grande majorité des études évaluant des politiques, qu'elles reposent sur un design expérimental ou une expérience naturelle, l'interprétation causale des quantités estimées repose sur une condition essentielle : la *Stable Unit Treatment Value Assumption* (SUTVA). Cette hypothèse stipule que le statut de traitement d'une unité n'affecte pas les résultats des autres unités de la population, qu'elles reçoivent ou non le traitement. Lorsque cette condition est violée, le groupe de contrôle — composé d'unités ne recevant pas le traitement — ne peut plus servir de base de comparaison valide, car ses membres ont pu modifier leur comportement en réponse à l'existence du traitement. Cependant, les effets de débordement (*spillover effects*), qui violent l'hypothèse SUTVA, sont courants en sciences sociales, du fait de l'existence d'effets de pairs et d'interactions sociales. Tenir compte de ces effets de pairs est donc essentiel pour évaluer l'impact causal de politiques ou de programmes. Prenons, par exemple, une intervention dans un lycée visant à réduire le taux d'abandon scolaire. L'intervention a un impact direct sur les élèves qui y participent, et certains décident de rester dans le système éducatif alors qu'ils auraient abandonné autrement. Mais les élèves sont également influencés par les décisions de leurs pairs : voir un ami rester dans le système scolaire peut inciter un autre élève à faire de même. Ainsi, l'impact initial de l'intervention est amplifié par les effets de pairs. Dans d'autres contextes, cependant, les interactions sociales peuvent atténuer les effets d'une politique. Cette dynamique est connue sous le nom d'effet du multiplicateur social (*social multiplier effect*). Un décideur politique a intérêt à connaître le sens et l'ampleur de ces effets de pairs afin de concevoir au mieux les programmes et maximiser leurs effets recherchés. Pour autant, les effets de pairs sont complexes. Premièrement, les effets de pairs peuvent prendre différentes formes. Le comportement d'un individu peut répondre soit aux caractéristiques exogènes du groupe auquel il appartient, soit au comportement du groupe. Manski

(1993) qualifie le premier type d'effets de pairs exogènes, et le second d'effets de pairs endogènes. Deuxièmement, les effets de pairs peuvent découler de divers mécanismes sociaux, avec des sens et des magnitudes différents. L'exemple précédent reflète un phénomène de conformisme, où les élèves imitent le comportement de leurs pairs. Mais d'autres mécanismes existent. Dans certains cas, un groupe d'individus (type A) influence un autre groupe (type B) sans réciprocité : les individus de type A servent alors de modèles. Dans d'autres cas, le groupe peut se polariser : les individus de type A ont des comportements conformistes entre eux mais leurs comportements divergent de ceux adoptés par les individus de type B, et vice versa. Les outils économétriques standards pour évaluer les effets de pairs présentent des limites. Bien que les modèles basés sur les réseaux puissent théoriquement tenir compte de la complexité des interactions sociales (voir Bramoullé et al. (2020) pour une revue de cette littérature), ils nécessitent des données détaillées sur l'ensemble du réseau, souvent indisponibles. Par conséquent, la plupart des études s'appuient sur le modèle linéaire en moyennes (*Linear-In-Means*, LIM) introduit par Manski (1993). Dans le modèle de Manski, le comportement d'un individu dépend linéairement du comportement moyen du groupe, des caractéristiques moyennes du groupe et de facteurs inobservés. Cependant, ce modèle impose l'hypothèse restrictive selon laquelle les individus répondent de manière homogène aux changements de comportement de leurs pairs, ce qui le rend mal adapté pour saisir la diversité des mécanismes sociaux. Le deuxième chapitre de cette thèse, fruit d'un travail conjoint avec Pauline Rossi et Zheng Wang, étend le modèle LIM en introduisant une hétérogénéité dans la composante endogène. Notre modèle permet aux individus d'être influencés différemment par le comportement de leurs pairs, selon qu'ils partagent ou non la même identité. Cela distingue notre approche de celle de Masten (2018), premier travail à notre connaissance qui considère également un modèle LIM avec effets endogènes hétérogènes. Dans le modèle de Masten (2018), les individus sont influencés à des degrés divers par leurs pairs, mais l'influence des pairs est homogène quelle que soit leur identité. Dans la version de base de notre modèle, l'identité est définie par l'éligibilité à un programme mis en œuvre au niveau du groupe. Nous développons une méthodologie fondée sur une expérimentation en population partielle (*partial population experiment*) pour identifier les différentes composantes de l'effet causal moyen total du programme. Concrètement, nous supposons qu'un échantillon de plusieurs groupes est observé. Au sein de chaque groupe, des individus sont éligibles au programme. Cette part d'individus éligibles est connue et varie entre les groupes. Lorsqu'un groupe est sélectionné pour faire partie du programme, tous les individus éligibles participent au programme tandis qu'aucun des individus non-éligibles ne le fait. Notre stratégie compare ensuite les groupes traités et non traités ayant la même proportion d'individus éligibles. Cette approche est compatible avec des expériences randomisées ainsi qu'avec des designs de différences-en-différences, à condition qu'une hypothèse de tendance commune soit respectée. Cette stratégie d'identification est par ailleurs adaptée à des contextes où



l'identité pertinente pour les effets de pairs hétérogènes est orthogonale à l'éligibilité au programme. En reprenant l'exemple précédent, si l'intervention visant à réduire le risque d'abandon scolaire cible des élèves issus de milieux socio-économiques défavorisés, notre stratégie de base estime un effet causal total qui tient compte des effets de pairs hétérogènes entre élèves défavorisés et élèves davantage favorisés. Cependant, si le genre des individus constitue l'identité la plus pertinente pour les effets de pairs hétérogènes, notre approche alternative permet d'aborder cette complexité. Ces deux dernières caractéristiques distinguent notre méthode de celle de Arduini et al. (2020), qui, à notre connaissance, est la seule autre étude à considérer un modèle LIM avec effets de pairs endogènes hétérogènes basés sur l'identité des pairs. À partir de ces stratégies d'identification, nous développons des estimateurs par la méthode des moments généralisés (*Generalized Method of Moments*, GMM) et démontrons la convergence et la normalité asymptotique de ces estimateurs. Nous appliquons notre méthodologie à l'étude de Progresa, un programme de transferts monétaires conditionnels mis en place au Mexique visant à réduire l'abandon scolaire chez les élèves issus d'un milieu financièrement défavorisé. Notre analyse estime l'effet direct du transfert monétaire sur les élèves issus d'une famille pauvre, l'effet indirect sur les élèves issus de familles non pauvres, ainsi que l'amplification de ces effets par les interactions sociales.

De nombreuses politiques publiques ne peuvent pas être évaluées à l'aide de méthodes expérimentales ou quasi-expérimentales, faute de groupe de contrôle approprié, ou du moins observable. Ce défi est amplifié lorsqu'il existe des interactions complexes entre les agents impliqués dans l'expérience, comme illustré dans le chapitre précédent, où il devient difficile de trouver un groupe de contrôle adéquat. Dans de tels cas, la solution consiste à introduire plus de structure dans le modèle, en s'appuyant sur la théorie économique. Bien que l'interprétation causale des résultats issus de modèles structurels repose généralement sur des hypothèses fortes, ajouter de la structure présente des avantages. Les paramètres estimés peuvent être utilisés pour évaluer les effets *ex ante* de politiques publiques alternatives. Le dernier chapitre de cette thèse développe un tel modèle structurel pour évaluer les effets des politiques publiques modifiant les conditions d'accès à l'aide à domicile non-médicalisée pour les personnes âgées dépendantes, sur différentes sous-populations de ménages. Traditionnellement, les membres de la famille nucléaire sont les premiers à fournir le soutien nécessaire lorsqu'une personne âgée n'est plus en mesure de réaliser de manière autonome les tâches essentielles du quotidien (voir, par exemple, Capuano (2018)). Cependant, le vieillissement de la population, la baisse des taux de fécondité, l'augmentation de la participation des femmes au marché du travail et la complexification des structures familiales réduisent la disponibilité des soins informels sur lesquels une personne âgée dépendante peut compter. Cette situation exerce une pression accrue sur les aidants familiaux restants, en particulier les conjoints, s'ils sont présents, car ils tendent à être les plus disponibles. En raison de ces évolutions sociales

et démographiques, un marché formel de prise en charge de la dépendance a émergé dans les pays développés. Dans ce marché, des organisations fournissent des services d'aide et d'accompagnement à domicile. Les services rendus par les employés de ces organisations sont totalement substituables à l'aide informelle fournie par les proches de la personne âgée dépendante. Par conséquent, les personnes âgées et leurs familles prennent en compte cette aide professionnelle, également dite formelle, lorsqu'ils planifient la prise en charge de la dépendance. L'intervention publique joue un rôle clé pour garantir l'accès à l'aide à domicile formelle pour l'essentiel des ménages. Cependant, avec l'augmentation continue de la population âgée, le débat sur la conception de politiques plus soutenables financièrement prend de l'ampleur. Pour ce faire, il est essentiel de considérer tous les effets générés par une hausse des financements publics sur ce type d'aide. Plusieurs études, basées sur des expériences naturelles, ont montré qu'un meilleur accès au marché formel de l'aide à domicile non-médicalisée augmente l'offre de travail des filles des personnes âgées dépendantes (voir, par exemple, Løken et al. (2017), Shen (2021), Chen and Lin (2022)) tout en améliorant la santé des personnes âgées (voir, par exemple, Massner and Wikström (2023), Frimmel et al. (2020)). Cependant, peu d'études ont examiné l'impact des changements de ces conditions d'accès sur l'aide informelle fournie par le conjoint d'une personne âgée dépendante, bien que ce dernier soit souvent le principal aidant lorsque la personne âgée est en couple. De plus, plusieurs travaux ont démontré que le rôle d'aidant a un effet négatif sur la santé du conjoint (voir, par exemple, Wagner and Brandt (2018), Barbosa et al. (2020)). Étant donné que les conjoints sont eux-mêmes souvent âgés, fournir de l'aide informelle de manière intensive peut conduire à une dépendance plus rapide ou à une utilisation accrue de services de santé, qui sont coûteux pour le senior mais aussi pour la puissance publique. Comprendre comment l'aide informelle apportée par le conjoint réagit à des variations du coût de l'aide formelle à la charge des ménages, selon le type de ménage considéré, est crucial pour concevoir des politiques publiques alternatives efficaces. C'est précisément l'objet du troisième chapitre de cette thèse. La principale contribution de ce chapitre est le développement, dans un cadre statique, d'un modèle d'offre et de demande pour le marché de l'aide à domicile non-médicalisée. À notre connaissance, il s'agit du premier projet à prendre en compte l'offre. Du côté de la demande, les ménages composés d'une personne âgée dépendante et de son conjoint décident ensemble de la consommation d'aide formelle et de la provision d'aide informelle fournie par le conjoint. Le bien-être de la personne âgée dépendante dépend de l'ensemble de l'aide reçue, les aides formelle et informelle étant considérée comme entièrement substituables. Les ménages sont hétérogènes dans leurs caractéristiques observables et leurs préférences. Du côté de l'offre, le modèle tient compte d'une certaine différenciation horizontale entre les fournisseurs d'aide à domicile professionnelle non-médicalisée. Les ménages peuvent choisir entre payer une prestation auprès d'un service d'aide et d'accompagnement à domicile (SAAD) ou d'employer directement une aide à domicile

professionnelle. Un marché de concurrence parfaite est supposé au sein de chaque type de fournisseur d'aide professionnelle. L'identification et l'estimation de ce modèle d'offre et de demande exploitent le fait que le marché de l'aide à domicile non-médicalisée est administré localement, ce qui génère une variation géographique exogène des prix à la charge des ménages pour une heure d'aide à domicile professionnelle. Les paramètres structurels du modèle de demande sont estimés par maximum de vraisemblance, en utilisant les données de l'enquête *CARE-Ménages*. Cette enquête représentative de la population française de plus de 60 ans fournit des informations détaillées sur les aides humaines et financières reçues par les personnes âgées pour les activités quotidiennes, qu'il s'agisse d'aide fournie par la famille ou auprès de prestataires professionnels. Les résultats préliminaires suggèrent que, toutes choses égales par ailleurs, les conjoints fournissant de l'aide informelle de sexe masculin et les conjoints âgés sont plus sensibles à une variation du prix de l'aide à domicile non-médicalisée professionnelle. Les paramètres structurels du modèle d'offre sont estimés par la méthode des doubles moindres carrés, en utilisant des données sur les marchés locaux, y compris l'enquête *CARE-Ménages*, des données démographiques publiques et la base administrative française appariant les employeurs et les employés. Les résultats préliminaires indiquent que les services de soins personnels et sociaux à domicile fonctionnent à coûts marginaux constants.

# General Introduction

Economics has known, in the last decades, an "applied turn" (Backhouse and Cherrier (2014)). In the 1960s, more than half of the articles published in the *American Economic Review*, the *Journal of Political Economy* and the *Quarterly Journal of Economics* - three of the discipline's most respected generalist journals - were purely theoretical works. This share fell to less than 28% in the early 2010s (Hamermesh (2013)). One of the identified causes for this shift was the development and the advocacy of better empirical research designs, that allow to better distinguish between correlation and causation, in a movement that Angrist and Pischke (2010) coined the "credibility revolution". The best design for uncovering causal effects, in terms of reliability and validity of statistical estimates, are experiments with random assignment. It is the principle of clinical trials in medicine. Two groups are randomly selected from the same population. One benefits from a treatment while the other serves as a control. The average causal effect of the treatment can be measured by comparing the average outcome within the group that benefited from the treatment with the average outcome within the control group. Random assignment policy evaluation has demonstrated, for instance, the positive causal effect of health insurance on healthcare use and health outcomes (Finkelstein et al. (2012)) and has proven the efficacy of numerous programs that aimed at, for example, at improving school attendance (e.g. Miguel and Kremer (2004), Parker and Todd (2017)) or labor market outcomes for unemployed people (e.g. LaLonde (1986)). But randomised experiments are time-consuming, expensive, and may not always be practical. It's difficult to imagine the setting-up of a randomised experiment to evaluate, for instance, the effect of immigrants on the economy of the host country. Nevertheless, there exist many situations in which a treatment group and a control group can be formed based on an "as-if random" naturally occurring external event. Since the event create conditions that are similar to random assignment, these natural experiments can be used to make causal claims. Several quasi-experimental policy-analysis designs have been developed to conceive relevant and credible control groups. The most often used designs in studies exploiting a natural experiment are instrumental variables (IV), regression discontinuity methods (RD) and difference-in-differences (DiD). IV designs are typically used when treatment assignment is correlated with some other confounding factors, which prevents from isolating the causal effects of the treatment.

The instrumental variable is a third factor that is correlated with treatment assignment but that has no direct effect on the outcome, except through its connection with treatment assignment. RD designs exploit, most often, the arbitrary eligibility criteria of a program, whereby units above a certain threshold are eligible to the program while those below are not. The average outcome of units that are just above the cutoff is compared with the average outcome of units that are just below. Since those near the threshold are similar in most ways except for taking part in the program, any sudden change in outcomes at the cutoff is attributed to the effect of the program itself. DiD designs estimate the causal effect of a treatment by comparing the changes in outcomes over time between a group that receives the treatment and a group that does not. The main identification assumption is that, without the treatment, both groups would have followed similar trends. By looking at the difference in outcomes before and after the intervention for both groups, the design isolates the effect of the treatment from other factors that may have influenced the outcomes. Intuitive statistical estimators have been associated with each of these designs. For IV designs, it is the double least squares (2SLS) estimator (Angrist and Imbens (1995)). For RD designs, this estimator is based on local linear regressions around the discontinuity threshold (Thistlethwaite and Campbell (1960)). For DiD designs, it is the ordinary least squares (OLS) estimator from a two-way fixed effect (TWFE) linear regression (e.g. Bertrand et al. (2004)).

If treatment effects are homogeneous, these three quasi-experimental methods can in principle, just like randomised experiments, estimate an average causal effect of the treatment. But this condition is, most of the time, very unlikely to be satisfied. When this assumption fails, the causal interpretation of the estimators associated with these three quasi-experimental designs is modified in different ways. The RD design estimates can be interpreted as the average causal effect of the treatment for units that are located at the cutoff, as long as units that are just above and just below this threshold are still similar. In the context of instrumental variables, the consequences of the presence of heterogeneous treatment effects on the causal interpretation of the double least squares estimator were highlighted by Angrist et al. (1996). The addition of new restrictions on the design of the experiment, such as the monotonicity condition, is necessary to be able to interpret the 2SLS estimator as a sensible causal effect. A vast literature studying causal inference in the presence of heterogeneous treatment effects using instrumental variables has stemmed from this initial project (e.g. Angrist et al. (2000), Heckman and Vytlačil (2005), Mogstad and Torgovitsky (2018), Heckman and Pinto (2018)). In the baseline difference-in-difference design with a binary treatment with two groups and two time periods, the coefficient associated to the treatment variable in the TWFE regression can be interpreted as an average causal effect within the group that received the treatment for the second period, as long as the common trend condition holds. But in DiD designs with more than two groups and/or more than two time periods, even if the common trend condition holds for each time period, De Chaisemartin and d'Haultfoeuille (2020), Imai and Kim

(2021), Goodman-Bacon (2021), Sun and Abraham (2021) and Borusyak et al. (2024) have recently demonstrated that the OLS estimator based on the TWFE regression does not identify a valuable causal effect if treatment effects are heterogeneous across groups and/or across time periods. The estimator identifies a weighted average of the different treatment effects with weights that sum to one but that can be negative. It is therefore possible to obtain a negative estimator while the treatment effects of every group, across all time period, are positive. New estimators that are robust to heterogeneous treatment effects have been introduced for these DiD designs with potentially staggered adoption (Callaway and Sant'Anna (2021), De Chaisemartin and d'Haultfoeuille (2020), Sun and Abraham (2021), Borusyak et al. (2024)). However, the difference-in-difference framework with multiple groups and/or time periods has also been used to evaluate the effects of continuous treatments such as custom tariffs (e.g Fajgelbaum et al. (2020)) or taxes on consumption goods (e.g. Li et al. (2014)). Yet, the OLS estimator from a TWFE regression also behaves poorly in terms of causal inference within this framework when treatment effects are heterogeneous across groups, as shown recently by Callaway et al. (2024). The first chapter of this thesis, that is joint work with Clément de Chaisemartin, Xavier D'Haultfoeuille, Doulo Sow and Gonzalo Vasquez-Bare, introduces two new estimators of valuable causal parameters within a difference-in-differences design with a continuous treatment variable. In its basic version, the observational study relies on a panel of numerous groups observed over two periods. The groups may already all be treated in the first period, albeit at different doses. Between the two periods, some groups, called switchers, experience a change in the dose of treatment they receive. In contrast, some groups, called stayers, keep the same treatment dose over both periods. The first considered parameter, called AS, corresponds to the average slope of the "potential outcome" function in the second period for switchers, between their treatment dose in the first period and that in the second period. The second parameter, called WAS, is a weighted version of the previous average, where each switcher receives a weight proportional to the absolute value of the change in their treatment dose between the two periods. The identification and estimation of these two parameters rely on a novel common trend assumption: in the absence of a change in their treatment dose between the two periods, the switcher groups would have experienced, on average, the same evolution in their outcome variable as the stayer groups with the same initial treatment dose. This parallel trends assumption has the advantage of being placebo testable, by comparing switchers' and stayers' outcome evolutions before switchers' treatment changes. Economically, the AS and WAS parameters can serve different purposes. Under shape restrictions on the potential outcome function, the AS parameter can be used to infer the effect of other treatment changes than those that took place from period one to two. The WAS can be used to conduct a cost-benefit analysis of the treatment changes that effectively took place. The results on the identification and estimation of these parameters are then extended to other designs. In particular, they are adapted to a design with instrumental variables. This

design is commonly encountered in studies that aim to estimate the price elasticity of demand for a good. For instance, taxes that vary across regions can be used as an instrument for prices. We demonstrate that the reduced-form WAS effect of the instrument on the outcome, divided by the first-stage WAS effect is equal to a weighted average of switchers' outcome-slope with respect to the treatment, with greater weight given to switchers who experience a larger first-stage effect. These findings are applied to estimate the price elasticity of gasoline consumption, using the state-level panel dataset from 1966 to 2008 compiled by Li et al. (2014).

In the vast majority of policy analysis studies, whether they are based on an experimental design or on a natural experiment, the interpretation of the estimates as valuable causal quantities relies on an essential condition: the Stable Unit Treatment Value Assumption (SUTVA). SUTVA asserts that the treatment status of one unit does not affect the outcomes of other units in the population, whether or not they receive the treatment. When this condition is violated, the control group—composed of units that do not receive the treatment—can no longer serve as a valid comparison because its members may alter their behavior in response to the treatment's existence. However, spillover effects, which violate SUTVA, are common in the social sciences, particularly due to peer effects and social interactions. Considering these effects is crucial when assessing the causal impact of policies or programs. For example, consider an intervention in high school aimed at reducing dropout rates. The intervention directly impacts the students who participate, and some decide to stay in school when they might have dropped out otherwise. But students are also influenced by their peers' decisions. Seeing a friend stay in school may motivate another student to do the same. In this way, the initial impact of the intervention is amplified by peer effects. In other contexts, however, social interactions can dampen the effects of a policy. This dynamic is known as the social multiplier effect. A policymaker may want to understand the magnitude and direction of these peer effects in order to design interventions that minimize dropout risk. Yet, peer effects are complex and multifaceted. Firstly, peer effects can take different forms. An individual's behavior may respond either to the exogenously determined characteristics of the group they belong to or to the group's behavior. Manski (1993) describes the first type as exogenous peer effects and the second as endogenous peer effects. Secondly, peer effects can arise from various social mechanisms, with differing signs and magnitudes. The earlier example reflects a conformist pattern, where students imitate their peers' behavior. But other mechanisms exist. In some cases, one group of individuals (type A) influences another (type B) without reciprocation. Here, type A individuals serve as role models. In yet other cases, the group may polarize: type A individuals conform to each other but diverge from type B, and vice versa. Standard econometric tools for evaluating peer effects face limitations. While network-based models can, in theory, account for the complexity of social interactions (see Bramoullé et al. (2020) for a review), they require detailed data on the entire network, which is often unavailable. Consequently, most studies rely on the linear-in-means

(LIM) model introduced by Manski (1993). In Manski's model, an individual's behavior depends linearly on the group's average behavior, the group's average characteristics, and unobserved factors. However, this model imposes the restrictive assumption that individuals respond homogeneously to changes in their peers' behavior, making it ill-suited for capturing the diversity of social mechanisms. The second chapter of this thesis, joint work with Pauline Rossi and Zheng Wang, extends the LIM model by introducing heterogeneity in the endogenous component. Our model allows individuals to be influenced differently by their peers' behavior, depending on whether or not they share the same identity. This distinguishes our approach from that of Masten (2018), the first work to our knowledge that also considers a LIM model with heterogeneous endogenous effects. In Masten (2018)'s model, individuals are more or less influenced by their peers to varying degrees, but this influence remains homogeneous across peers. In the baseline version, identity is defined by eligibility for a program implemented at the group level. We develop a partial population experiment design to identify the different components of the total average causal effect of the program. Specifically, we observe several groups, each with a known share of eligible individuals, which varies across groups. When a group is selected to benefit from the program, all eligible individuals participate, while none of the ineligible ones do. Our strategy then compares treated and untreated groups with the same share of eligible individuals. This approach works with both randomized experiments and DiD designs, provided a common trend condition holds. Moreover, this design can be adapted to settings where the relevant identity for peer effects is orthogonal to program eligibility. Returning to the earlier example, if the intervention targets students from disadvantaged socio-economic backgrounds, our baseline strategy estimates the total causal effect while accounting for heterogeneous peer effects between poor and non-poor students. However, if gender is the more relevant identity, our extended approach addresses this complexity. These two last features of our method set it apart from that of Arduini et al. (2020), which, to our knowledge, is the only other work to consider a LIM model with heterogeneous endogenous peer effects based on peer identity. From these identification strategies, we develop Generalized Method of Moments (GMM) estimators and establish their consistency and asymptotic normality. We apply our methodology to the study of Progresa, a conditional cash transfer program in Mexico aimed at reducing school dropout among poor students. Our analysis estimates the direct effect of the cash transfer on poor students, the indirect effect on non-poor students, and the extent to which these effects are amplified by social interactions.

Many public policies cannot be evaluated using quasi-experimental methods because no suitable control group exists, or at least none is observed. This challenge is amplified when there are complex interactions between agents involved in the experience, as illustrated in the previous chapter, where finding an appropriate control group becomes difficult. In such cases, the solution is to introduce more structure into the model, that is based on economic theory. While the causal interpretation of results



from structural models typically relies on strong assumptions, adding structure has its advantages. The estimated parameters can be used to assess the *ex-ante* effects of alternative public policies. The final chapter of this thesis develops such a structural model to evaluate the effects of public policies that modify access conditions to personal and social home care for dependent elderly individuals on different subpopulations. Traditionally, nuclear family members are the first to provide the necessary support when a senior can no longer perform essential daily tasks independently (e.g. Capuano (2018)). However, population aging, declining fertility rates, increasing female labor market participation, and the complication of family structures are reducing the availability of informal care that a dependent senior can rely on. This situation puts additional pressure on the remaining family caregivers, particularly on the spouses, if present, as they tend to be the most available. Due to these social and demographic changes, a formal market for long-term care (LTC) has emerged in developed countries. Within this market, organizations provide personal and social home (PSH) care services. The services delivered by the home care workers in these organizations are fully substitutable for the informal help provided by the senior's relatives. As a result, seniors and their families take this formal alternative into account when organizing care. Public intervention plays a critical role in ensuring access to formal home LTC for households. However, as the elderly population continues to grow, there is increasing debate on how to design more financially sustainable policies. To achieve this, it is crucial to consider all the effects that additional public spending on this type of care generates. Several studies, based on natural experiments, have shown that better access to the formal PSH care market increases labor supply among the daughters of dependent seniors (e.g., Løken et al. (2017), Shen (2021), Chen and Lin (2022)) while also improving the health of the senior (e.g., Massner and Wikström (2023), Frimmel et al. (2020)). However, few studies have examined the impact of changes in access conditions on the informal care provided by the spouse of a dependent senior, despite the fact that the spouse is often the primary caregiver when the senior is in a couple. Moreover, several studies have demonstrated that being a caregiver has a negative effect on the spouse's health (e.g., Wagner and Brandt (2018), Barbosa et al. (2020)). Given that spouses are typically elderly themselves, providing informal care too intensively can lead to a faster decline into dependency or increased reliance on costly health care services. Understanding how spousal caregiving responds to changes in the out-of-pocket costs of formal care in different types of households is therefore crucial for designing effective public policies. This is precisely the focus of the third chapter of this dissertation. The main contribution is the development in a static framework of a supply and demand model of the PSH care market. To our knowledge, it is the first project that accounts for the supply. On the demand side, households consisting of a dependent senior and their spouse jointly decide on their consumption of formal care and the amount of informal care provided by the spouse. The senior's well-being is influenced by both types of care, which are treated as substitutes. Households are heterogeneous in

observed characteristics and in preferences. On the supply side, the model accounts for some horizontal differentiation of formal personal and social care providers. Households can choose between paying for personal and social care services from a firm or hiring a self-employed home care assistant. A market of perfect competition within a type of provider is assumed. The identification and estimation of this supply and demand model exploit the fact that the PSH care market is administered at a local level, which leads to exogenous geographic variation in out-of-pocket prices for an hour of formal PSH care. The structural parameters of the demand model are estimated using maximum likelihood, drawing on data from the *CARE-Ménages* survey. It is a representative survey of the French population over 60 which provides detailed information on the human and financial assistance seniors receive for daily activities, whether through family or formal services. Preliminary results suggest that, all other things being equal, male and older caregiving spouses are more responsive to a variation in the price of formal PSH care. The structural parameters of the supply model are estimated using two-stage least squares (2SLS), drawing from data on local markets, including data from the *CARE-Ménages* survey, publicly available demographic data, and the French matched employer-employee administrative database. Preliminary results indicate that personal and social home care services operate under constant marginal costs.



# Chapter 1

## Difference-in-Differences for Continuous Treatments and Instruments with Stayers

with Clément de Chaisemartin, Xavier D’Haultfœuille, Doulo Sow  
and Gonzalo-Vasquez-Bare

We propose difference-in-differences estimators in designs where the treatment is continuously distributed at every period, as is often the case when one studies the effects of taxes, tariffs, or prices. We assume that between consecutive periods, the treatment of some units, the switchers, changes, while the treatment of other units remains constant. We show that under a placebo-testable parallel-trends assumption, averages of the slopes of switchers’ potential outcomes can be nonparametrically estimated. We generalize our estimators to the instrumental-variable case. We use our estimators to estimate the price-elasticity of gasoline consumption.

### 1.1 Introduction

A popular method to estimate the effect of a treatment on an outcome is to estimate a two-way fixed effects (TWFE) regression that controls for unit and time fixed effects:

$$Y_{i,t} = \alpha_i + \gamma_t + \beta_{TWFE} D_{i,t} + u_{i,t},$$

where  $D_{i,t}$  is the treatment of unit  $i$  at time  $t$ . de Chaisemartin and D’Haultfoeuille (2023a) find that 26 of the 100 most cited papers published by the American Economic Review from 2015 to 2019 have estimated at least one TWFE regression. De Chaisemartin and d’Haultfoeuille (2020), Goodman-Bacon (2021), and Borusyak et al. (2024) have shown that under a parallel trends assumption, TWFE regressions are not robust to heterogeneous effects: they may estimate a weighted sum of treatment effects across periods and units, with some negative weights. Owing to the negative weights,  $\beta_{TWFE}$  could be, say, negative, even if the treatment effect is positive for every unit  $\times$  period. Importantly, the result in De Chaisemartin and d’Haultfoeuille (2020) applies to binary, discrete, and continuous treatments.

Several alternative heterogeneity-robust difference-in-difference (DID) estimators have been proposed (see Table 2 of de Chaisemartin and D’Haultfoeuille (2023b)). Some apply to binary and staggered treatments (see Sun and Abraham (2021), Callaway and Sant’Anna (2021), Borusyak et al. (2024)). Some apply to designs where all units start with a treatment equal to 0, and then get treated with heterogeneous, potentially continuously distributed treatment intensities (see de Chaisemartin and D’Haultfoeuille (2023a), Callaway and Sant’Anna (2021), de Chaisemartin and D’Haultfoeuille (2024)). However, treatments continuously distributed at every period, including the first one in the data, are ubiquitous in applied work. For instance, taxes (see Li et al. (2014)) or tariffs (see Fajgelbaum et al. (2020)) are often continuously distributed at all periods. No difference-in-difference estimator robust to heterogeneous effects is available for such designs. Proposing such estimators is the purpose of this paper.

We assume that we have a panel data set, whose units could be geographical locations such as counties. We start by considering the case where the panel has two time periods. From period one to two, the treatment of some units, hereafter referred to as the switchers, changes. On the other hand, the treatment of other units, hereafter referred to as the stayers, does not change. We consider two target parameters. The first one is the average slope of switchers’ period-two potential outcome function, from their period-one to their period-two treatment, hereafter referred to as the Average of Slopes (AS). Our second target is a weighted average of switchers’ slopes, where switchers receive a weight proportional to the absolute value of their treatment change, hereafter referred to as the Weighted Average of Slopes (WAS). We propose a novel parallel trends assumption on the outcome evolution of switchers and stayers with the same period-one treatment, in the counterfactual where switchers’ treatment would not have changed. Because it conditions on units’ period-one treatment, this parallel-trends assumption does not impose any restriction on effect heterogeneity. This parallel trends assumption is also placebo testable, by comparing switchers’ and stayers’ outcome evolutions before switchers’ treatment changes. We view the possibility of placebo-testing it as an important advantage of our assumption, as placebo tests are

an essential step in establishing the credibility of an identifying assumption in observational studies (Imbens et al. (2001), Imbens and Xu (2024)). We show that under our placebo-testable parallel-trends assumption, the AS and the WAS are identified. This contrasts with other target parameters, like the dose-response function or the average marginal effect, which can only be identified under non-placebo-testable assumptions.

Economically, the AS and the WAS can serve different purposes, so neither parameter dominates the other. Under shape restrictions on the potential outcome function, the AS can be used to infer the effect of other treatment changes than those that took place from period one to two. Instead, the WAS can be used to conduct a cost-benefit analysis of the treatment changes that effectively took place. On the other hand, when it comes to estimation, the WAS unambiguously dominates the AS. First, we show that it can be estimated at the standard parametric rate even if switchers can experience an arbitrarily small change of their treatment between consecutive periods. Second, we show that under some conditions, the asymptotic variance of the WAS estimator is strictly lower than that of the AS estimator. Third, unlike the AS, the WAS is amenable to doubly-robust estimation, which comes with a number of advantages.

Then, we consider the instrumental-variable (IV) case. For instance, one may be interested in estimating the price-elasticity of a good's consumption. If prices respond to demand shocks, the counterfactual consumption trends of units experiencing and not experiencing a price change may not be parallel. On the other hand, the counterfactual consumption trends of units experiencing and not experiencing a tax change may be parallel. Then, taxes may be used as an instrument for prices. In such cases, we show that the reduced-form WAS effect of the instrument on the outcome divided by the first-stage WAS effect is equal to a weighted average of switchers' outcome-slope with respect to the treatment, where switchers with a larger first-stage effect receive more weight. Hereafter, we refer to this effect as the IV-WAS effect. The ratio of the reduced-form and first-stage AS effects is also equal to a weighted average of slopes, with arguably less natural weights, so in the IV case the WAS seems both economically and statistically preferable to the AS. Importantly, we show that the reduced-form parallel-trends assumption implicitly restricts treatment-effect heterogeneity. Such restrictions can be alleviated by controlling for groups baseline treatment in the IV specification, but the resulting estimator still restricts effects' heterogeneity across units.

We consider other extensions. First, we extend our results to applications with more than two time periods. Importantly, with several time periods our estimators rely on a parallel-trends assumption over consecutive periods, rather than over the entire duration of the panel. Second, we propose a placebo estimator of the parallel-trends assumption underlying our estimators.

Finally, we use the yearly, 1966 to 2008 US state-level panel dataset of Li et al. (2014) to estimate

the effect of gasoline taxes on gasoline consumption and prices. Using the WAS estimators, we find a significantly negative effect of taxes on gasoline consumption, and a significantly positive effect on prices. The AS estimators are close to, and not significantly different from, the WAS estimators, but they are also markedly less precise: their standard errors are almost three times larger than that of the WAS estimators. This shows that our theoretical result on the precision ranking of the WAS and AS estimators, which we derive under strong assumptions, still holds in a real-life application where those assumptions probably do not hold. The precision losses attached to using the AS have consequences. The AS estimator of the effect of taxes on prices is not statistically significant, so with that estimator one cannot use taxes as an instrument to estimate the price-elasticity of consumption, because the instrument does not have a first-stage. This contrasts with the WAS, whose first-stage t-stat is around 7. We compute an IV-WAS estimator of the price elasticity of gasoline consumption, and find a fairly small elasticity of -0.67, in line with previous literature (for instance, Hausman and Newey (1995) find a long-run elasticity of -0.81). Our placebo estimators are small, insignificant, and fairly precisely estimated, thus suggesting that our parallel trends assumption is plausible.

Our estimators are computed by the `did_multiplyt_stat` Stata package, available from the SSC repository. Our package allows estimators with control variables and weights, see the help file and the package's companion paper for further details.

## Related Literature

Our paper builds upon several previous papers in the panel data literature. Chamberlain (1982) seems to be the first paper to have proposed an estimator of the AS parameter. Under the assumption of no counterfactual time trend, the estimator therein is a before-after estimator. Then, our paper is closely related to the work of Graham and Powell (2012), who also propose DID estimators of the AS (see their Equation (21)) when the treatment is continuously distributed at every time period. Their estimators rely on a linear effect assumption and assume that units experience the same evolution of their treatment effect over time, a parallel-trends-on-treatment-effects assumption. By contrast, our estimator of the AS does not place any restriction on treatment effects. But our main contribution to this literature is to introduce the WAS, and to contrast the pros and cons of the AS and WAS estimators. Our results are also related to Hoderlein and White (2012), who consider the average marginal effect of a continuous treatment with panel data. However, their target parameters and identifying assumptions are different. For instance, they rule out systematic changes of the outcome over time.

With respect to the aforementioned heterogeneity-robust DID literature, we make two contributions. First, in the non-IV case we propose estimators that can be used even if units' treatment varies at baseline.

Thus we usefully complement previous literature, that has mostly focused on the case where all units have a baseline treatment equal to zero (see Sun and Abraham (2021), Callaway and Sant’Anna (2021), Borusyak et al. (2024), Callaway and Sant’Anna (2021), de Chaisemartin and D’Haultfoeuille (2024)). One exception predating this paper is De Chaisemartin and d’Haultfoeuille (2020), who allow for a non-binary discrete treatment at baseline in their Web Appendix, and propose estimators comparing switchers and stayers with the same baseline treatment. However, that paper does not allow for continuously distributed treatments, and comparing switchers and stayers with the same baseline treatment is no longer feasible with a continuously distributed treatment.<sup>1</sup> Second, in the IV case, previous IV-DID literature has only considered classical designs with two periods and binary instrument and treatment (de Chaisemartin (2010), Hudson et al. (2017)), as well as fuzzy DID designs, a special case of IV-DIDs (de Chaisemartin and D’Haultfoeuille (2018)). Instead, this paper proposes broadly applicable IV-DID estimators, and also highlights that IV-DID estimators impose restrictions on treatment effect-heterogeneity, which can be mitigated by controlling for the baseline treatment.

Importantly, our estimators require that there be some stayers, whose treatment does not change between consecutive periods. This assumption is unlikely to be met when the treatment is say, precipitations: for instance, US counties never experience the exact same precipitations over two consecutive years. In de Chaisemartin et al. (2023), we discuss the (non-trivial) extension of the results in this paper to applications without stayers.

**Organization of the paper.** In Section 1.2, we present the set-up, introduce notation and discuss our main assumptions. In Section 1.3, we introduce the AS and discuss its identification and estimation. Section 1.4 then turns to the WAS. Section 1.5 extends our previous results to an instrumental variable set-up. We consider other extensions in Section 1.6. Finally, our application is developed in Section 1.7. The proofs are collected in the appendix.

## 1.2 Set-up, assumptions, and building-block identification result

### 1.2.1 Set-up

A representative unit is drawn from an infinite super population, and observed at two time periods. This unit could be an individual or a firm, but it could also be a geographical unit, like a county or a region.<sup>2</sup> All expectations below are taken with respect to the distribution of variables in the super population. We

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<sup>1</sup>Another exception, posterior to this paper, is de Chaisemartin and D’Haultfoeuille (2023a), who extend the estimators in this paper to models with dynamic effects in Section 1.10 of their Web Appendix.

<sup>2</sup>In that case, one may want to weight the estimation by counties’ or regions’ populations. Extending the estimators we propose to allow for such weighting is a mechanical extension.



are interested in the effect of a continuous and scalar treatment variable on that unit's outcome. Let  $D_1$  (resp.  $D_2$ ) denote the unit's treatment at period 1 (resp. 2), and let  $\mathcal{D}_1$  (resp.  $\mathcal{D}_2$ ) be the set of values  $D_1$  (resp.  $D_2$ ) can take, i.e. its support. Let  $S = 1\{D_2 \neq D_1\}$  be an indicator equal to 1 if the unit's treatment changes from period one to two, i.e. if they are a switcher.

For any  $d \in \mathcal{D}_1 \cup \mathcal{D}_2$ , let  $Y_1(d)$  and  $Y_2(d)$  respectively denote the unit's potential outcomes at periods 1 and 2 with treatment  $d$ , and let  $Y_1$  and  $Y_2$  denote their observed outcomes at periods 1 and 2. Our potential outcome notation assumes that  $Y_1$  does not depend on units' period-two treatment, thus ruling out anticipation effects, a commonly-made assumption in the DID literature. Our notation also assumes that  $Y_2$  does not depend on units' period-one treatment, thus ruling out dynamic effects. When the treatment is continuously distributed at period one, allowing for dynamic effects opens up the so-called initial-conditions problem. As units receive heterogeneous doses at period one, they may have experienced treatment changes before period one. With dynamic effects such changes may still affect their outcome over the study period, but they cannot be accounted for because they are not observed. Ruling out dynamic effects allows us to abstract from this thorny issue, but could yield misleading results if dynamic effects are present. To alleviate this concern, in Section 1.6.3 we propose a modified version of our estimators, robust to dynamic effects up to a pre-specified treatment lag.

In what follows, all equalities and inequalities involving random variables are required to hold almost surely. Finally, for any random variable observed at the two time periods  $(X_1, X_2)$ , let  $\Delta X = X_2 - X_1$  denote the change of  $X$  from period 1 to 2.

## 1.2.2 Assumptions

We make the following assumptions.<sup>3</sup>

**Assumption 1** (*Parallel trends*) For all  $d_1 \in \mathcal{D}_1$ ,  $E(\Delta Y(d_1)|D_1 = d_1, D_2) = E(\Delta Y(d_1)|D_1 = d_1)$ .

Assumption 1 is a parallel trends assumption, requiring that  $\Delta Y(d_1)$  be mean independent of  $D_2$ , conditional on  $D_1 = d_1$ .

**Assumption 2** (*Bounded treatment, Lipschitz and bounded potential outcomes*)

1.  $\mathcal{D}_1$  and  $\mathcal{D}_2$  are bounded subsets of  $\mathbb{R}$ .
2. For all  $t \in \{1, 2\}$  and for all  $(d, d') \in \mathcal{D}_t^2$ , there is a random variable  $\bar{Y} \geq 0$  such that  $|Y_t(d) - Y_t(d')| \leq \bar{Y}|d - d'|$ , with  $\sup_{(d_1, d_2) \in \text{Supp}(D_1, D_2)} E[\bar{Y}|D_1 = d_1, D_2 = d_2] < \infty$ .

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<sup>3</sup>Throughout the paper, we implicitly assume that all potential outcomes have an expectation.

Assumption 2 ensures that all the expectations below are well defined. It requires that the set of values that the period-one and period-two treatments can take be bounded. It also requires that the potential outcome functions be Lipschitz (with a unit-specific Lipschitz constant). This will automatically hold if  $d \mapsto Y_2(d)$  is differentiable with respect to  $d$  and has a bounded derivative.

Finally, for estimation and inference we assume we observe an iid sample with the same distribution as  $(Y_1, Y_2, D_1, D_2)$ :

**Assumption 3** (*iid sample*) We observe  $(Y_{i,1}, Y_{i,2}, D_{i,1}, D_{i,2})_{1 \leq i \leq n}$ , that are independent and identically distributed vectors with the same probability distribution as  $(Y_1, Y_2, D_1, D_2)$ .

Importantly, Assumption 3 allows for the possibility that  $Y_1$  and  $Y_2$  (resp.  $D_1$  and  $D_2$ ) are serially correlated, as is commonly assumed in DID studies (see Bertrand et al., 2004).

### 1.2.3 Building-block identification result

Assumption 1 implies the following lemma, our building-block identification result.

**Lemma 1** *If Assumption 1 holds, then for all  $(d_1, d_2) \in \mathcal{D}_1 \times \mathcal{D}_2$  such that  $d_1 \neq d_2$  and  $P(S|D_1 = d_1) < 1$ ,*

$$\begin{aligned} TE(d_1, d_2|d_1, d_2) &:= E \left( \frac{Y_2(d_2) - Y_2(d_1)}{d_2 - d_1} \middle| D_1 = d_1, D_2 = d_2 \right) \\ &= E \left( \frac{\Delta Y - E(\Delta Y|D_1 = d_1, S = 0)}{d_2 - d_1} \middle| D_1 = d_1, D_2 = d_2 \right). \end{aligned}$$

**Proof:**

$$\begin{aligned} &E(Y_2(d_2) - Y_2(d_1)|D_1 = d_1, D_2 = d_2) \\ &= E(\Delta Y|D_1 = d_1, D_2 = d_2) - E(\Delta Y(d_1)|D_1 = d_1, D_2 = d_2) \\ &= E(\Delta Y|D_1 = d_1, D_2 = d_2) - E(\Delta Y(d_1)|D_1 = d_1, D_2 = d_1) \\ &= E(\Delta Y|D_1 = d_1, D_2 = d_2) - E(\Delta Y|D_1 = d_1, S = 0) \\ &= E(\Delta Y - E(\Delta Y|D_1 = d_1, S = 0)|D_1 = d_1, D_2 = d_2), \end{aligned}$$

where the second equality follows from Assumption 1. This proves the result  $\square$

Intuitively, under Assumption 1 the counterfactual outcome evolution switchers would have experienced if their treatment had not changed is identified by the outcome evolution of stayers with the same period-one treatment. If a unit's treatment changes from two to five, we can recover its counterfactual outcome evolution if its treatment had not changed, by using the average outcome evolution of all stayers with a baseline treatment of two. Then, a DID estimand comparing switchers' and stayers' outcome evolutions

identifies  $E(Y_2(d_2) - Y_2(d_1)|D_1 = d_1, D_2 = d_2)$ , and we can scale that effect by  $d_2 - d_1$  to identify a slope rather than an unnormalized effect.

Note that in a canonical DID design where  $\mathcal{D}_1 = 0$  and  $\mathcal{D}_2 \in \{0, 1\}$ , Lemma 1 only applies to  $(d_1, d_2) = (0, 1)$ ,  $\text{TE}(0, 1|0, 1)$  reduces to the ATT, and the estimand reduces to the canonical DID estimand comparing the outcome evolutions of treated and untreated units. Note also that in a design where all units are untreated at period one,  $d_1 = 0$ , and

$$\text{TE}(0, d_2|0, d_2) = E\left(\frac{Y_2(d_2) - Y_2(0)}{d_2} \middle| D_2 = d_2\right),$$

an effect closely related to the  $\text{ATT}(d|d)$  effect in Callaway and Sant’Anna (2021). Thus, the effects we consider are extensions of those effects to applications with a treatment continuous at all periods.

Lemma 1 shows that under Assumption 1,

$$(d_1, d_2) \mapsto \text{TE}(d_1, d_2|d_1, d_2)$$

is identified. Of course, one may be interested in other parameters, like

$$(d, d') \mapsto \text{TE}(d, d') := E\left(\frac{Y_2(d) - Y_2(d')}{d - d'}\right),$$

a function which, unlike  $(d_1, d_2) \mapsto \text{TE}(d_1, d_2|d_1, d_2)$ , applies to the entire population rather than to specific subpopulation that depend on  $(d_1, d_2)$ . Alternatively, one could also be interested in the average marginal effect

$$E(Y_2'(D_2)).$$

What is the appeal of  $\text{TE}(d_1, d_2|d_1, d_2)$  with respect to those other parameters? Conditional on  $D_1 = d_1, D_2 = d_2$ ,  $Y_2(d_2)$  is observed, so estimating  $\text{TE}(d_1, d_2|d_1, d_2)$  only requires estimating  $Y_2(d_1)$ , switchers’ counterfactual outcomes if their treatment had not changed. By definition,  $Y_1(d_1)$  is observed. If the data contains a third period 0 and the treatment of some units does not change from period 0 to 1, then  $Y_0(d_1)$  is also observed for some switchers and stayers. Then, as explained in further details in Section 1.6.2, one can placebo-test Assumption 1, by comparing the outcome evolutions of switchers and stayers from period 0 to 1. This shows that  $\text{TE}(d_1, d_2|d_1, d_2)$  is identified under a placebo-testable parallel-trends assumption. On the other hand, estimating  $\text{TE}(d, d')$  requires estimating, for most units, *two* unobserved counterfactual outcomes. This cannot be achieved under a placebo-testable assumption as we only observe *one* potential outcome at each date. When  $D_1 = 0$ , Callaway and Sant’Anna (2021) propose a “strong parallel-trends” assumption under which the dose-response function, a parameter closely related

to  $TE(0, d')$  are identified, but their “strong parallel-trends” assumption is not placebo testable. Similarly, estimating  $E\left(\frac{Y_2(D_2) - Y_2(d')}{D_2 - d'}\right)$  requires estimating  $Y_2(d')$ , which cannot be achieved under a placebo-testable assumption because  $Y_1(d')$  is not observed for all units. As  $Y_2'(D_2) = \lim_{d' \rightarrow D_2} \frac{Y_2(D_2) - Y_2(d')}{D_2 - d'}$ , the same issue applies to  $E(Y_2'(D_2))$ .

Variability in  $TE(d_1, d_2|d_1, d_2)$  across values of  $(d_1, d_2)$  conflates a dose-response relationship that may be of economic interest, and a selection bias due to the fact that units with different period one and two treatments may have heterogeneous treatment effects (Callaway and Sant’Anna, 2021). Moreover, Lemma 1 shows that estimating  $TE(d_1, d_2|d_1, d_2)$  requires estimating the values of two conditional expectations with respect to continuous variables, at points  $D_1 = d_1, D_2 = d_2$  and  $D_1 = d_1$ . Unless one is willing to make parametric functional-form assumptions, the resulting estimator will converge at a slower rate than the standard  $\sqrt{n}$ - parametric rate. For these two reasons, in this paper we focus on averages of the slopes  $TE(d_1, d_2|d_1, d_2)$ , that can be estimated non-parametrically at the standard  $\sqrt{n}$ - parametric rate, and we do not focus on the function  $(d_1, d_2) \mapsto TE(d_1, d_2|d_1, d_2)$ .

Finally, our DID estimands compare switchers and stayers with the same period-one treatment. Instead, one could propose estimands comparing switchers and stayers, without conditioning on their period-one treatment. To recover the counterfactual outcome trend of a switcher going from two to five units of treatment, one could use a stayer with treatment equal to three at both dates. On top of Assumption 1, such estimands rest on two supplementary conditions:

- (i)  $E(\Delta Y(d)|D_1 = d) = E(\Delta Y(d))$ .
- (ii) For all  $(d, d') \in \mathcal{D}_1^2$ ,  $E(\Delta Y(d)) = E(\Delta Y(d'))$ .

(i) requires that all units experience the same evolution of their potential outcome with treatment  $d$ , while Assumption 1 only imposes that requirement for units with the same baseline treatment. Assumption 1 may be more plausible: units with the same period-one treatment may be more similar and more likely to be on parallel trends than units with different period-one treatments. (ii) requires that the trend affecting all potential outcomes be the same: to rationalize a DID estimand comparing a switcher going from two to five units of treatment to a stayer with treatment equal to three,  $E(\Delta Y(2))$  and  $E(\Delta Y(3))$  should be equal. Rearranging, (ii) is equivalent to

$$E(Y_2(d) - Y_2(d')) = E(Y_1(d) - Y_1(d')) : \tag{1.1}$$

the treatment effect should be constant over time, a strong restriction on treatment effect heterogeneity. Assumption 1, on the other hand, does not impose any restriction on treatment effect heterogeneity, as it only restricts one potential outcome per unit.

## 1.3 Estimating the average of switchers' slopes

### 1.3.1 Target parameter

In this section, our target parameter is

$$\delta_1 := E \left( \frac{Y_2(D_2) - Y_2(D_1)}{D_2 - D_1} \middle| S = 1 \right), \quad (1.2)$$

the average of the slopes of switchers' potential outcome functions, between their period-one and their period-two treatments. Hereafter,  $\delta_1$  is referred to as the Average of Slopes (AS).

The AS is a local effect: it only applies to switchers, and it measures the effect of changing their treatment from its period-one to its period-two value, not of other changes of their treatment. Still, the AS can be used to point or partially identify the effect of other treatment changes under shape restrictions. First, assume that the potential outcomes are linear: for  $t \in \{1, 2\}$ ,

$$Y_t(d) = Y_t(0) + B_t d,$$

where  $B_t$  is a slope that may vary across units and may change over time. Then,  $\delta_1 = E(B_2 | S = 1)$ : the AS is equal to the average, across switchers, of the slopes of their potential outcome functions at period 2. Therefore, for all  $d \neq d'$ ,

$$E(Y_2(d) - Y_2(d') | S = 1) = (d - d')\delta_1 :$$

under linearity, knowing the AS is sufficient to recover the ATE of any uniform treatment change among switchers. Of course, this only holds under linearity, which may not be a plausible assumption. Assume instead that  $d \mapsto Y_2(d)$  is convex. Then, for any  $\epsilon > 0$ ,

$$E(Y_2(D_2 + \epsilon) - Y_2(D_2) | S = 1) \geq \epsilon \delta_1.$$

Accordingly, under convexity one can use the AS to obtain lower bounds of the effect of changing the treatment from  $D_2$  to larger values than  $D_2$ . For instance, in Fajgelbaum et al. (2020), one can use this strategy to derive a lower bound of the effect of increasing tariffs' to even higher levels than those decided by the Trump administration. Under convexity, one can also use the AS to derive an upper bound of the effect of changing the treatment from  $D_1$  to a lower value than  $D_1$ . And under concavity, one can derive an upper (resp. lower) bound of the effect of changing the treatment from  $D_2$  (resp.  $D_1$ ) to a larger (resp. lower) value.<sup>4</sup> Importantly, the AS is identified even if those linearity or convexity/concavity conditions fail.

<sup>4</sup>See D'Haultfœuille et al. (2023) for bounds of the same kind obtained under concavity or convexity.

But those non-placebo testable conditions are necessary to use the AS to identify or bound the effects of alternative policies.

### 1.3.2 Identification

To identify the AS, we use a DID estimand comparing switchers and stayers with the same period-one treatment. This requires that there be no value of the period-one treatment  $D_1$  such that only switchers have that value, as stated formally below.

**Assumption 4** (*Support condition for AS identification*)  $P(S = 1) > 0, P(S = 1|D_1) < 1$ .

Assumption 4 implies that  $P(S = 0) > 0$ , meaning that there are stayers whose treatment does not change. While we assume that  $D_1$  and  $D_2$  are continuous, we also assume that the treatment is persistent, and thus  $\Delta D$  has a mixed distribution with a mass point at zero.

To identify the AS, we also start by assuming that there are no quasi-stayers: the treatment of all switchers changes by at least  $c$  from period one to two, for some strictly positive  $c$ .

**Assumption 5** (*No quasi-stayers*)  $\exists c > 0: P(|\Delta D| > c|S = 1) = 1$ .

We relax Assumption 5 just below.

**Theorem 1** *If Assumptions 1-5 hold,*

$$\delta_1 = E \left( \frac{\Delta Y - E(\Delta Y|D_1, S = 0)}{\Delta D} \middle| S = 1 \right).$$

If there are quasi-stayers, the AS is still identified. For any  $\eta > 0$ , let  $S_\eta = 1\{|\Delta D| > \eta\}$  be an indicator for switchers whose treatment changes by at least  $\eta$  from period one to two.

**Theorem 2** *If Assumptions 1-4 hold,*

$$\delta_1 = \lim_{\eta \downarrow 0} E \left( \frac{\Delta Y - E(\Delta Y|D_1, S = 0)}{\Delta D} \middle| S_\eta = 1 \right).$$

If there are quasi-stayers whose treatment change is arbitrarily close to 0 (i.e.  $f_{|\Delta D||S=1}(0) > 0$ ), the denominator of  $(\Delta Y - E(\Delta Y|D_1, S = 0))/\Delta D$  is close to 0 for them. On the other hand,

$$\begin{aligned} & \Delta Y - E(\Delta Y|D_1, S = 0) \\ &= Y_2(D_2) - Y_2(D_1) + \Delta Y(D_1) - E(\Delta Y(D_1)|D_1, S = 0) \\ &\approx \Delta Y(D_1) - E(\Delta Y(D_1)|D_1, S = 0), \end{aligned}$$

so the ratio's numerator may not be close to 0. Then, under weak conditions,

$$E\left(\left|\frac{\Delta Y - E(\Delta Y|D_1, S = 0)}{\Delta D}\right|\middle|S = 1\right) = +\infty.$$

Therefore, we need to trim quasi-stayers from the estimand in Theorem 1, and let the trimming go to 0, as in Graham and Powell (2012) who consider a related estimand with some quasi-stayers. Accordingly, with quasi-stayers the AS is irregularly identified by a limiting estimand.

### 1.3.3 Estimation and inference

With no quasi-stayers,  $E((\Delta Y - E(\Delta Y|D_1, S = 0))/\Delta D|S = 1)$  can be estimated in three steps. First, one estimates  $E(\Delta Y|D_1, S = 0)$  using a non-parametric regression of  $\Delta Y_i$  on  $D_{i,1}$  among stayers. Second, for each switcher, one computes  $\widehat{E}(\Delta Y|D_1 = D_{i,1}, S = 0)$ , its predicted outcome evolution given its baseline treatment, according to the non-parametric regression estimated among stayers. Third, one lets

$$\widehat{\delta}_1 := \frac{1}{n_s} \sum_{i:S_i=1} \frac{\Delta Y_i - \widehat{E}(\Delta Y|D_1 = D_{i,1}, S = 0)}{\Delta D_i},$$

where  $n_s = \#\{i : S_i = 1\}$ .

To estimate  $E(\Delta Y|D_1, S = 0)$ , we consider a series estimator based on polynomials in  $D_1$ ,  $(p_{k,K_n}(D_1))_{1 \leq k \leq K_n}$ . We make the following technical assumption.

**Assumption 6** (*Conditions for asymptotic normality of AS estimator*)

1.  $D_1$  is continuously distributed on a compact interval  $I$ , with  $\inf_{d \in I} f_{D_1}(d) > 0$ .
2.  $E[\Delta Y^2] < \infty$  and  $d \mapsto E[\Delta Y^2|D_1 = d]$  is bounded on  $I$ .
3.  $P(S = 1) > 0$  and  $\sup_{d \in I} P(S = 1|D_1 = d) < 1$ .
4. The functions  $d \mapsto E[(1 - S)\Delta Y|D_1 = d]$ ,  $d \mapsto E[S|D_1 = d]$  and  $d \mapsto E[S/\Delta D|D_1 = d]$  are four times continuously differentiable.
5. The polynomials  $d \mapsto p_{k,K_n}(d)$ ,  $1 \leq k \leq K_n$ , are orthonormal on  $I$  and  $K_n^{12}/n \rightarrow +\infty$ ,  $K_n^7/n \rightarrow 0$ .

Point 3 is a slight reinforcement of Assumption 4. In Point 5,  $K_n^{12}/n \rightarrow \infty$  requires that  $K_n$ , the order of the polynomial in  $D_1$  we use to approximate  $E(\Delta Y|D_1, S = 0)$ , goes to  $+\infty$  when the sample size grows, thus ensuring that the bias of our series estimator of  $E(\Delta Y|D_1, S = 0)$  tends to zero.  $K_n^7/n \rightarrow 0$  ensures that  $K_n$  does not go to infinity too fast, thus preventing overfitting.

**Theorem 3** *If Assumptions 1-3 and 5-6 hold,*

$$\sqrt{n} \left( \widehat{\delta}_1 - \delta_1 \right) \xrightarrow{d} \mathcal{N}(0, V(\psi_1)),$$

where

$$\psi_1 := \frac{1}{E(S)} \left\{ \left( \frac{S}{\Delta D} - E \left( \frac{S}{\Delta D} \middle| D_1 \right) \frac{(1-S)}{E[1-S|D_1]} \right) [\Delta Y - E(\Delta Y | D_1, S = 0)] - \delta_1 S \right\}.$$

Theorem 3 shows that without quasi-stayers, the AS can be estimated at the  $\sqrt{n}$ -rate, and gives an expression of its estimator's asymptotic variance. With quasi-stayers, we conjecture that the AS cannot be estimated at the  $\sqrt{n}$ -rate. This conjecture is based on a result from Graham and Powell (2012). Though their result applies to a broader class of estimands, it implies in particular that with quasi-stayers,

$$\lim_{\eta \downarrow 0} E \left( \frac{\Delta Y - E(\Delta Y | S = 0)}{\Delta D} \middle| S_\eta = 1 \right)$$

cannot be estimated at a faster rate than  $n^{1/3}$ . The estimand in the previous display is closely related to our estimand

$$\lim_{\eta \downarrow 0} E \left( \frac{\Delta Y - E(\Delta Y | D_1, S = 0)}{\Delta D} \middle| S_\eta = 1 \right)$$

in Theorem 2, and is equal to it if  $E(\Delta Y | D_1, S = 0) = E(\Delta Y | S = 0)$ . Then, even though the assumptions in Graham and Powell (2012) differ from ours, it seems reasonable to assume that their general conclusion still applies to our set-up: here as well, owing to  $\delta_1$ 's irregular identification, this parameter can probably not be estimated at the parametric  $\sqrt{n}$ -rate with quasi-stayers. This is one of the reasons that lead us to consider, in the next section, another target parameter that can be estimated at the parametric  $\sqrt{n}$ -rate with quasi-stayers.

## 1.4 Estimating a weighted average of switchers' slopes

### 1.4.1 Target parameter

In this section, our target parameter is

$$\begin{aligned} \delta_2 &:= E \left( \frac{|D_2 - D_1|}{E(|D_2 - D_1| | S = 1)} \times \frac{Y_2(D_2) - Y_2(D_1)}{D_2 - D_1} \middle| S = 1 \right) \\ &= \frac{E(\text{sgn}(D_2 - D_1)(Y_2(D_2) - Y_2(D_1)) | S = 1)}{E(|D_2 - D_1| | S = 1)} \\ &= \frac{E(\text{sgn}(D_2 - D_1)(Y_2(D_2) - Y_2(D_1)))}{E(|D_2 - D_1|)}. \end{aligned}$$



$\delta_2$  is a weighted average of the slopes of switchers' potential outcome functions from their period-one to their period-two treatments, where slopes receive a weight proportional to switchers' absolute treatment change from period one to two. Accordingly, we refer to  $\delta_2$  as the Weighted Average of Slopes (WAS).

$\delta_2 = \delta_1$  if and only if

$$\text{cov}\left(\frac{Y_2(D_2) - Y_2(D_1)}{D_2 - D_1}, |D_2 - D_1| \middle| S = 1\right) = 0 : \quad (1.3)$$

the WAS and AS are equal if and only if switchers' slopes are uncorrelated with  $|D_2 - D_1|$ .

Economically, the AS and WAS serve different purposes. As discussed above, under shape restrictions on the potential outcome function, the AS can be used to identify or bound the effect of other treatment changes than the actual change switchers experienced from period one to two. The WAS cannot serve that purpose, but under some assumptions, it may be used to conduct a cost-benefit analysis of the treatment changes that took place from period one to two. To simplify the discussion, let us assume in the remainder of this paragraph that  $D_2 \geq D_1$ . Assume also that the outcome is a measure of output, such as agricultural yields or wages, expressed in monetary units. Finally, assume that the treatment is costly, with a cost linear in dose, uniform across units, and known to the analyst: the cost of giving  $d$  units of treatment to a unit at period  $t$  is  $c_t \times d$  for some known  $(c_t)_{t \in \{1,2\}}$ . Then,  $D_2$  is beneficial relative to  $D_1$  if and only if  $E(Y_2(D_2) - c_2 D_2) > E(Y_2(D_1) - c_2 D_1)$  or, equivalently,

$$\delta_2 > c_2 :$$

comparing  $\delta_2$  to  $c_2$  is sufficient to evaluate if changing the treatment from  $D_1$  to  $D_2$  was beneficial.

## 1.4.2 Identification

Let  $S_+ = 1\{D_2 - D_1 > 0\}$ ,  $S_- = 1\{D_2 - D_1 < 0\}$  and

$$\begin{aligned} \delta_{2+} &:= \frac{E(Y_2(D_2) - Y_2(D_1)|S_+ = 1)}{E(D_2 - D_1|S_+ = 1)}, \\ \delta_{2-} &:= \frac{E(Y_2(D_1) - Y_2(D_2)|S_- = 1)}{E(D_1 - D_2|S_- = 1)}. \end{aligned}$$

Hereafter, units with  $S_+ = 1$  are referred to as "switchers up", while units with  $S_- = 1$  are referred to as "switchers down". Thus,  $\delta_{2+}$  is the WAS of switchers up, and  $\delta_{2-}$  is the WAS of switchers down. One has

$$\begin{aligned} \delta_2 &= \frac{P(S_+ = 1|S = 1)E(D_2 - D_1|S_+ = 1)}{E(|D_2 - D_1||S = 1)} \delta_{2+} \\ &+ \frac{P(S_- = 1|S = 1)E(D_1 - D_2|S_- = 1)}{E(|D_2 - D_1||S = 1)} \delta_{2-}. \end{aligned} \quad (1.4)$$

To identify  $\delta_{2+}$  (resp.  $\delta_{2-}$ ) we use DID estimands comparing switchers up (resp. switchers down) to stayers with the same period-one treatment. This requires that there be no value of  $D_1$  such that some switchers up (resp. switchers down) have that baseline treatment while there is no stayer with the same baseline treatment, as stated in Point 1 (resp. 2) of Assumption 7 below.

**Assumption 7** (*Support conditions for WAS identification*)

1.  $0 < P(S_+ = 1)$ , and  $0 < P(S_+ = 1|D_1)$  implies that  $0 < P(S = 0|D_1)$ .

2.  $0 < P(S_- = 1)$ , and  $0 < P(S_- = 1|D_1)$  implies that  $0 < P(S = 0|D_1)$ .

**Theorem 4** 1. If Assumptions 1-2 and Point 1 of Assumption 7 hold,

$$\delta_{2+} = \frac{E(\Delta Y - E(\Delta Y|D_1, S = 0)|S_+ = 1)}{E(\Delta D|S_+ = 1)} \quad (1.5)$$

$$= \frac{E(\Delta Y|S_+ = 1) - E\left(\Delta Y \frac{P(S_+ = 1|D_1)}{P(S = 0|D_1)} \frac{P(S = 0)}{P(S_+ = 1)} \middle| S = 0\right)}{E(\Delta D|S_+ = 1)}. \quad (1.6)$$

2. If Assumptions 1-2 and Point 2 of Assumption 7 hold,

$$\delta_{2-} = \frac{E(\Delta Y - E(\Delta Y|D_1, S = 0)|S_- = 1)}{E(\Delta D|S_- = 1)} \quad (1.7)$$

$$= \frac{E(\Delta Y|S_- = 1) - E\left(\Delta Y \frac{P(S_- = 1|D_1)}{P(S = 0|D_1)} \frac{P(S = 0)}{P(S_- = 1)} \middle| S = 0\right)}{E(\Delta D|S_- = 1)}. \quad (1.8)$$

3. If Assumptions 1-2 and Assumption 7 hold,

$$\delta_2 = \frac{E[\text{sgn}(\Delta D) (\Delta Y - E(\Delta Y|D_1, S = 0))]}{E[|\Delta D|]} \quad (1.9)$$

$$= \frac{E[\text{sgn}(\Delta D)\Delta Y] - E\left[\Delta Y \frac{P(S_+ = 1|D_1) - P(S_- = 1|D_1)}{P(S = 0|D_1)} P(S = 0) \middle| S = 0\right]}{E[|\Delta D|]}. \quad (1.10)$$

Point 1 of Theorem 4 shows that  $\delta_{2+}$ , the WAS of switchers-up, is identified by two estimands, a regression-based and a propensity-score-based estimand. Point 2 of Theorem 4 shows that  $\delta_{2-}$ , the WAS of switchers down, is identified by two estimands similar to those identifying  $\delta_{2+}$ , replacing  $S_+$  by  $S_-$ . Finally, if the conditions in Point 1 and 2 of Theorem 4 jointly hold, it directly follows from (1.4) that  $\delta_2$ , the WAS of all switchers, is identified by a weighted average of the estimands in Equations (1.5) and (1.7), and by a weighted average of the estimands in Equations (1.6) and (1.8). Those weighted averages simplify into the expressions given in Point 3 of Theorem 4. Point 3 of Theorem 4 also implies that  $\delta_2$  is

identified by the following doubly-robust estimand:

$$\frac{E \left[ \left( S_+ - S_- - \frac{P(S_+=1|D_1) - P(S_-=1|D_1)}{P(S=0|D_1)} (1 - S) \right) (\Delta Y - E(\Delta Y|D_1, S = 0)) \right]}{E[|\Delta D|]}. \quad (1.11)$$

### 1.4.3 Estimation and inference

The regression-based estimands identifying  $\delta_{2+}$  and  $\delta_{2-}$  can be estimated following almost the same steps as in Section 1.3.3. Specifically, let

$$\begin{aligned} \widehat{\delta}_{2+}^r &:= \frac{\frac{1}{n_+} \sum_{i:S_{i+}=1} (\Delta Y_i - \widehat{E}(\Delta Y|D_1 = D_{i,1}, S = 0))}{\frac{1}{n_+} \sum_{i:S_{i+}=1} \Delta D_i} \\ \widehat{\delta}_{2-}^r &:= \frac{\frac{1}{n_-} \sum_{i:S_{i-}=1} (\Delta Y_i - \widehat{E}(\Delta Y|D_1 = D_{i,1}, S = 0))}{\frac{1}{n_-} \sum_{i:S_{i-}=1} \Delta D_i}, \end{aligned}$$

where  $n_+ = \#\{i : S_{i+} = 1\}$  and  $n_- = \#\{i : S_{i-} = 1\}$ , and where  $\widehat{E}(\Delta Y|D_1, S = 0)$  is the series estimator of  $E(\Delta Y|D_1, S = 0)$  defined in Section 1.3.3 of the paper. Then, let

$$\widehat{w}_+ = \frac{\frac{n_+}{n} \times \frac{1}{n_+} \sum_{i:S_{i+}=1} \Delta D_i}{\frac{n_+}{n} \times \frac{1}{n_+} \sum_{i:S_{i+}=1} \Delta D_i - \frac{n_-}{n} \times \frac{1}{n_-} \sum_{i:S_{i-}=1} \Delta D_i},$$

and let

$$\widehat{\delta}_2^r = \widehat{w}_+ \widehat{\delta}_{2+}^r + (1 - \widehat{w}_+) \widehat{\delta}_{2-}^r$$

be the corresponding estimator of  $\delta_2$ .

We now propose estimators of the propensity-score-based estimands identifying  $\delta_{2+}$  and  $\delta_{2-}$  in Equations (1.6) and (1.8). Let  $\widehat{P}(S_+ = 1) = n_+/n$  (resp.  $\widehat{P}(S_- = 1) = n_-/n$ ,  $\widehat{P}(S = 0) = (n - n_s)/n$ ) be an estimator of  $P(S_+ = 1)$  (resp.  $P(S_- = 1)$ ,  $P(S = 0)$ ). Let  $\widehat{P}(S_+ = 1|D_1)$  (resp.  $\widehat{P}(S_- = 1|D_1)$ ,  $\widehat{P}(S = 0|D_1)$ ) be a non-parametric estimator of  $P(S_+ = 1|D_1)$  (resp.  $P(S_- = 1|D_1)$ ,  $P(S = 0|D_1)$ ) using a series logistic regression of  $S_{i+}$  (resp.  $S_{i-}$ ,  $1 - S_i$ ) on polynomials in  $D_1$   $(p_{k,K_n}(D_1))_{1 \leq k \leq K_n}$ . We make the following technical assumption.

**Assumption 8** (Conditions for asymptotic normality of propensity-score WAS estimator)

1.  $D_1$  is continuously distributed on a compact interval  $I$ , with  $\inf_{d \in I} f_{D_1}(d) > 0$ .
2.  $E[\Delta Y^2] < \infty$  and  $d \mapsto E[\Delta Y^2|D_1 = d]$  is bounded on  $I$
3.  $0 < E[S_+] < 1$ ,  $0 < E[S_-] < 1$ ,  $E[S] > 0$  and  $\sup_{d \in I} E[S|D_1 = d] < 1$ .
4. The functions  $d \mapsto E[\Delta Y(1-S)|D_1 = d]$ ,  $d \mapsto E[S|D_1 = d]$ ,  $d \mapsto E[S_+|D_1 = d]$  and  $d \mapsto E[S_-|D_1 = d]$  are four times continuously differentiable.

5. The polynomials  $d \mapsto p_{k, K_n}(d)$ ,  $k \leq 1 \leq K_n$  are orthonormal on  $I$  and  $K_n = Cn^\nu$  where  $1/10 < \nu < 1/6$ .

Let

$$\widehat{\delta}_{2+}^{ps} := \frac{\frac{1}{n_+} \sum_{i: S_{i+}=1} \Delta Y_i - \frac{1}{n-n_s} \sum_{i: S_i=0} \Delta Y_i \frac{\widehat{P}(S_+=1|D_1=D_{i1})}{\widehat{P}(S=0|D_1=D_{i1})} \frac{\widehat{P}(S=0)}{\widehat{P}(S_+=1)}}{\frac{1}{n_+} \sum_{i: S_{i+}=1} \Delta D_i}$$

$$\widehat{\delta}_{2-}^{ps} := \frac{\frac{1}{n_-} \sum_{i: S_{i-}=1} \Delta Y_i - \frac{1}{n-n_s} \sum_{i: S_i=0} \Delta Y_i \frac{\widehat{P}(S_-=1|D_1=D_{i1})}{\widehat{P}(S=0|D_1=D_{i1})} \frac{\widehat{P}(S=0)}{\widehat{P}(S_-=1)}}{\frac{1}{n_-} \sum_{i: S_{i-}=1} \Delta D_i},$$

and let

$$\widehat{\delta}_2^{ps} = \widehat{w}_+ \widehat{\delta}_{2+}^{ps} + (1 - \widehat{w}_+) \widehat{\delta}_{2-}^{ps}$$

be the corresponding estimator of  $\delta_2$ . Let

$$\psi_{2+} := \frac{1}{E(\Delta D S_+)} \left\{ \left( S_+ - E(S_+|D_1) \frac{(1-S)}{E(1-S|D_1)} \right) (\Delta Y - E(\Delta Y|D_1, S=0)) - \delta_{2+} \Delta D S_+ \right\}$$

$$\psi_{2-} := \frac{1}{E(\Delta D S_-)} \left\{ \left( S_- - E(S_-|D_1) \frac{(1-S)}{E(1-S|D_1)} \right) (\Delta Y - E(\Delta Y|D_1, S=0)) - \delta_{2-} \Delta D S_- \right\}$$

$$\psi_2 := \frac{1}{E(|\Delta D|)} \left\{ \left( S_+ - S_- - E(S_+ - S_-|D_1) \frac{(1-S)}{E(1-S|D_1)} \right) \right. \\ \left. \times (\Delta Y - E(\Delta Y|D_1, S=0)) - \delta_2 |\Delta D| \right\}.$$

**Theorem 5** 1. If Assumptions 1-3 and 6 hold,

$$\sqrt{n} \left( (\widehat{\delta}_{2+}^r, \widehat{\delta}_{2-}^r)' - (\delta_{2+}, \delta_{2-})' \right) \xrightarrow{d} \mathcal{N}(0, V((\psi_{2+}, \psi_{2-})')).$$

and

$$\sqrt{n} \left( \widehat{\delta}_2^r - \delta_2 \right) \xrightarrow{d} \mathcal{N}(0, V(\psi_2)).$$

2. If Assumptions 1-3 and 8 hold,

$$\sqrt{n} \left( (\widehat{\delta}_{2+}^{ps}, \widehat{\delta}_{2-}^{ps})' - (\delta_{2+}, \delta_{2-})' \right) \xrightarrow{d} \mathcal{N}(0, V((\psi_{2+}, \psi_{2-})')).$$

and

$$\sqrt{n} \left( \widehat{\delta}_2^{ps} - \delta_2 \right) \xrightarrow{d} \mathcal{N}(0, V(\psi_2)).$$

Based on (1.11), we can also estimate  $\delta_2$  using the following doubly-robust estimator:

$$\widehat{\delta}_2^{dr} = \frac{\sum_i \left( S_{i+} - S_{i-} - \frac{\widehat{P}(S_+=1|D_1=D_{i1}) - P(S_{i-}=1|D_1=D_{i1})}{P(S_{i+}=0|D_1=D_{i1})} (1 - S_i) \right) (\Delta Y_i - \widehat{E}(\Delta Y_i|D_1 = D_{i1}, S_i = 0))}{\sum_i |\Delta D_i|}.$$

This estimator has an important advantage. While our regression-based (resp. propensity-score-based) estimator is nominally non-parametric, in practice it requires choosing a polynomial order to estimate  $E(\Delta Y|D_1, S = 0)$  (resp.  $P(S_+ = 1|D_1)$  and  $P(S_- = 1|D_1)$ ), and the rate conditions on  $K_n$  in Assumptions 6 (resp. 8) do not give specific guidance on the choice of this tuning parameter. With the doubly-robust estimator above, one can choose this tuning parameter in a data-driven manner, using cross-validation (CV). Results in Section 4 of Andrews (1991) imply that a series estimator of a nonparametric regression model with a polynomial order chosen by CV is optimal: the ratio of its mean-squared error and that of an oracle estimator using the best polynomial order given the sample size converges to one. Then, as  $D_1$  is a scalar variable, series estimators of  $E(\Delta Y|D_1, S = 0)$ ,  $P(S_+ = 1|D_1)$ , and  $P(S_- = 1|D_1)$  with CV-chosen polynomial orders converge at a rate strictly faster than  $n^{1/4}$ , as long as one assumes that those nuisance functions are twice differentiable. Then, we conjecture that one can show, following arguments similar to those in Farrell (2015), that  $\hat{\delta}_2^{dr}$  with CV-chosen polynomial orders in the estimation of the nuisance functions is  $\sqrt{n}$ -consistent, with asymptotic variance  $V(\psi_2)$ .

Finally, we now show that under some assumptions, the asymptotic variance of the WAS estimator is lower than that of the AS estimator.

**Proposition 1** *If Assumption 1 holds,  $(Y_2(D_2) - Y_2(D_1))/(D_2 - D_1) = \delta$  for some real number  $\delta$ ,  $V(\Delta Y(D_1)|D_1, D_2) = \sigma^2$  for some real number  $\sigma^2 > 0$ ,  $D_2 \geq D_1$ , and  $\Delta D \perp\!\!\!\perp D_1$ ,*

$$\begin{aligned} V(\psi_1) &= \sigma^2 \left[ \frac{E(1/(\Delta D)^2|S = 1)}{P(S = 1)} + \frac{(E(1/\Delta D|S = 1))^2}{P(S = 0)} \right] \\ &\geq \sigma^2 \frac{1}{(E(\Delta D|S = 1))^2} \left[ \frac{1}{P(S = 1)} + \frac{1}{P(S = 0)} \right] = V(\psi_2), \end{aligned}$$

*with equality if and only if  $V(\Delta D|S = 1) = 0$ .*

Of course, the constant treatment effect and the homoscedasticity assumptions underlying Proposition 1 are strong, but one often has to make strong assumptions to be able to rank estimators' variances. The question then is whether this ranking still holds in real-life applications, where those assumptions are unlikely to hold. Put differently, all models are wrong but some are useful, and the question is whether Proposition 1 is useful. In our empirical application, we find that the variance of  $\hat{\delta}_1$  is indeed much larger than that of  $\hat{\delta}_2^{dr}$ , as predicted by Proposition 1.

## 1.5 Instrumental-variable estimation

There are instances where the parallel-trends condition in Assumption 1 is implausible, but one has at hand an instrument satisfying a similar parallel-trends condition. For instance, one may be interested in

estimating the price-elasticity of a good's consumption, but prices respond to supply and demand shocks, and therefore do not satisfy Assumption 1. On the other hand, taxes may not respond to supply and demand shocks and may satisfy a parallel-trends assumption.

### 1.5.1 Notation and assumptions

Let  $(Z_1, Z_2)$  denote the instrument's values at period one and two and  $Z_t$  be the support of  $Z_t$ . For any  $z \in Z_1 \cup Z_2$ , let  $D_1(z)$  and  $D_2(z)$  respectively denote the unit's potential treatments at periods 1 and 2 with instrument  $z$ . Let  $SC = 1\{D_2(Z_2) \neq D_2(Z_1), Z_2 \neq Z_1\}$  be an indicator equal to 1 for switchers-compliers, namely units whose instrument changes from period one to two and whose treatment is affected by that change in the instrument.

We replace Assumption 1 by the following assumption.<sup>5</sup>

**Assumption 9** (*Reduced-form and first-stage parallel trends*) For all  $z \in Z_1$ ,

1.  $E(Y_2(D_2(z)) - Y_1(D_1(z)) | Z_1 = z, Z_2, D_1) = E(Y_2(D_2(z)) - Y_1(D_1(z)) | Z_1 = z, D_1)$ .
2.  $E(D_2(z) - D_1(z) | Z_1 = z, Z_2, D_1) = E(D_2(z) - D_1(z) | Z_1 = z, D_1)$ .

Point 1 of Assumption 9 requires that  $Y_2(D_2(z)) - Y_1(D_1(z))$ , units' outcome evolutions in the counterfactual where their instrument does not change from period one to two, be mean independent of  $Z_2$ , conditional on  $Z_1$  and  $D_1$ . Unlike Assumption 1, this condition imposes some restrictions on treatment effect heterogeneity, and the goal of conditioning on  $D_1$  is to minimize the stringency of those restrictions. To see this, note that the two following conditions are sufficient for Point 1 of Assumption 9 to hold:

$$E(Y_2(D_1(z)) - Y_1(D_1(z)) | Z_1 = z, Z_2, D_1) = E(Y_2(D_1(z)) - Y_1(D_1(z)) | Z_1 = z, D_1) \quad (1.12)$$

$$E(Y_2(D_2(z)) - Y_2(D_1(z)) | Z_1 = z, Z_2, D_1) = E(Y_2(D_2(z)) - Y_2(D_1(z)) | Z_1 = z, D_1). \quad (1.13)$$

(1.12) requires that  $Y_2(D_1(z)) - Y_1(D_1(z))$ , units' outcome evolutions in the counterfactual where their instrument and their treatment does not change from period one to two, be mean independent of  $Z_2$ , conditional on  $Z_1$  and  $D_1$ . Thanks to the conditioning on  $D_1$ , (1.12) is a standard parallel trends assumption that does not impose any restriction on treatment effect heterogeneity, like Assumption 1. If  $D_1$  was not conditioned upon, (1.12) would require parallel trends among units with different baseline treatments, which implicitly assumes homogeneous treatment effects over time, as discussed in Section 1.2. (1.13), on the other hand, is a restriction on treatment effect heterogeneity across units. Essentially, it requires that switching the treatment from  $D_1(Z_1)$  to  $D_2(Z_1)$ , the natural treatment change happening over

<sup>5</sup>Note that with our notation where potential outcomes do not depend on  $z$ , we also implicitly impose the usual exclusion restriction.

time even without any change in the instrument, has an effect on the outcome that is mean independent of  $Z_2$  conditional on  $Z_1$  and  $D_1$ . Importantly, note that Point 1 of Assumption 9 is placebo testable, by comparing the outcome evolutions of instrument-switchers and instrument-stayers before instrument-switchers experience a change of their instrument. Finally, Point 2 of Assumption 9 requires that units' treatment evolutions under  $Z_1$  be mean independent of  $Z_2$ , conditional on  $Z_1$  and  $D_1$ . Because  $D_1$  is conditioned upon, this parallel trends condition is equivalent to a sequential exogeneity assumption (see Robins, 1986, Bojinov et al., 2021).

Point 1 of Assumption 9 is related to identifying assumptions previously proposed in the literature. de Chaisemartin (2010) and Hudson et al. (2017) also consider IV-DID estimands, in classical designs with two periods and a binary instrument that turns on for some units at period two. Both papers introduce a “reduced-form” parallel trends assumption similar to Point 1 of Assumption 9, but without noting that it imposes restrictions on effects' heterogeneity, even in the simple designs considered by those papers.

We also make the following assumptions.

**Assumption 10** (*Monotonicity and strictly positive first-stage*) i) For all  $(z, z') \in \mathcal{Z}_2^2$ ,  $z \geq z' \Rightarrow D_2(z) \geq D_2(z')$ , and ii)  $E(|D_2(Z_2) - D_2(Z_1)|) > 0$ .

i) is a monotonicity assumption similar to that in Imbens and Angrist (1994). It requires that increasing the period-two instrument weakly increases the period-two treatment. This condition is plausible when the instrument is taxes and the treatment is prices, as is the case in our application. ii) requires that the instrument has a strictly positive first stage.

**Assumption 11** (*Bounded instrument, Lipschitz and bounded reduced-form potential outcomes and potential treatments*)

1.  $\mathcal{Z}_1$  and  $\mathcal{Z}_2$  are bounded subsets of  $\mathbb{R}$ .
2. For all  $t \in \{1, 2\}$  and for all  $(z, z') \in \mathcal{Z}_t^2$ , there is a random variable  $\bar{Y} \geq 0$  such that  $|Y_t(D_t(z)) - Y_t(D_t(z'))| \leq \bar{Y}|z - z'|$ , with  $\sup_{(z_1, z_2) \in \text{Supp}(Z_1, Z_2)} E[\bar{Y} | Z_1 = z_1, Z_2 = z_2] < \infty$ .
3. For all  $t \in \{1, 2\}$  and for all  $(z, z') \in \mathcal{Z}_t^2$ , there is a random variable  $\bar{D} \geq 0$  such that  $|D_t(z) - D_t(z')| \leq \bar{D}|z - z'|$ , with  $\sup_{(z_1, z_2) \in \text{Supp}(Z_1, Z_2)} E[\bar{D} | Z_1 = z_1, Z_2 = z_2] < \infty$ .

Assumption 11 is an adaptation of Assumption 2 to the IV setting we consider in this section.

**Assumption 12** (*iid sample*) We observe  $(Y_{i,1}, Y_{i,2}, D_{i,1}, D_{i,2}, Z_{i,1}, Z_{i,2})_{1 \leq i \leq n}$ , that are independent and identically distributed with the same probability distribution as  $(Y_1, Y_2, D_1, D_2, Z_1, Z_2)$ .

## 1.5.2 Target parameter

In this section, our target parameter is

$$\delta_{IV} := E \left( \frac{|D_2(Z_2) - D_2(Z_1)|}{E(|D_2(Z_2) - D_2(Z_1)| | SC = 1)} \times \frac{Y_2(D_2(Z_2)) - Y_2(D_2(Z_1))}{D_2(Z_2) - D_2(Z_1)} \Big| SC = 1 \right).$$

$\delta_{IV}$  is a weighted average of the slopes of compliers-switchers' period-two potential outcome functions, from their period-two treatment under their period-one instrument, to their period-two treatment under their period-two instrument. Slopes receive a weight proportional to the absolute value of compliers-switchers' treatment response to the instrument change.  $\delta_{IV}$  is just equal to the reduced-form WAS effect of the instrument on the outcome, divided by the first-stage WAS effect of the instrument on the treatment. With a binary instrument, such that  $Z_1 = 0$  and  $Z_2 \in \{0, 1\}$ , our IV-WAS effect coincides with that identified in Corollary 2 of Angrist et al. (2000), in a cross-sectional IV model.

We could also consider a reduced-form AS divided by a first-stage AS. The resulting target is a weighted average of the slopes  $\frac{Y_2(D_2(Z_2)) - Y_2(D_2(Z_1))}{D_2(Z_2) - D_2(Z_1)}$ , with weights proportional to  $\frac{D_2(Z_2) - D_2(Z_1)}{Z_2 - Z_1}$ . It seems more natural to us to weight compliers-switchers' slopes by the absolute value of their first-stage than by the slope of their first-stage.<sup>6</sup>

## 1.5.3 Identification

Let  $S^I = 1\{Z_2 - Z_1 \neq 0\}$ ,  $S^I_+ = 1\{Z_2 - Z_1 > 0\}$ , and  $S^I_- = 1\{Z_2 - Z_1 < 0\}$ .

**Assumption 13** (*Support conditions for IV-WAS identification*)

1.  $0 < P(S^I_+ = 1)$ , and  $0 < P(S^I_+ = 1 | Z_1, D_1)$  implies that  $0 < P(S^I = 0 | Z_1, D_1)$ .
2.  $0 < P(S^I_- = 1)$ , and  $0 < P(S^I_- = 1 | Z_1, D_1)$  implies that  $0 < P(S^I = 0 | Z_1, D_1)$ .

**Theorem 6** *If Assumptions 9-11 and 13 hold,*

$$\delta_{IV} = \frac{E [\text{sgn}(\Delta Z) (\Delta Y - E(\Delta Y | Z_1, D_1, S^I = 0))]}{E [\text{sgn}(\Delta Z) (\Delta D - E(\Delta D | Z_1, D_1, S^I = 0))]} \quad (1.14)$$

$$= \frac{E [\text{sgn}(\Delta Z) \Delta Y] - E \left[ \Delta Y \frac{P(S^I_+ = 1 | Z_1, D_1) - P(S^I_- = 1 | Z_1, D_1)}{P(S^I = 0 | Z_1, D_1)} P(S^I = 0) \Big| S^I = 0 \right]}{E [\text{sgn}(\Delta Z) \Delta D] - E \left[ \Delta D \frac{P(S^I_+ = 1 | Z_1, D_1) - P(S^I_- = 1 | Z_1, D_1)}{P(S^I = 0 | Z_1, D_1)} P(S^I = 0) \Big| S^I = 0 \right]}. \quad (1.15)$$

The regression-based (resp. propensity-score-based) estimand identifying  $\delta_{IV}$  is just equal to the regression-based (resp. propensity-score-based) estimand identifying the reduced-form WAS effect of

<sup>6</sup>If the first-stage effect is homogenous and linear, the weights in the IV-AS effect reduce to one, and one recovers a standard AS effect. However, linearity and homogeneity of the first-stage effect are strong assumptions.



the instrument on the outcome controlling for  $D_1$ , divided by the regression-based (resp. propensity-score-based) estimand identifying the first-stage WAS effect controlling for  $D_1$ .

### 1.5.4 Estimation and inference

Let

$$\widehat{\delta}_{IV}^r = \frac{\frac{1}{n} \sum_{i=1}^n \text{sgn}(\Delta Z_i) \left( \Delta Y_i - \widehat{E}(\Delta Y | Z_1 = Z_{i,1}, D_1 = D_{i,1}, S^I = 0) \right)}{\frac{1}{n} \sum_{i=1}^n \text{sgn}(\Delta Z_i) \left( \Delta D_i - \widehat{E}(\Delta D | Z_1 = Z_{i,1}, D_1 = D_{i,1}, S^I = 0) \right)}, \quad (1.16)$$

where  $\widehat{E}(\Delta Y | Z_1, D_1, S^I = 0)$  and  $\widehat{E}(\Delta D | Z_1, D_1, S^I = 0)$  are series estimators of  $E(\Delta Y | Z_1, D_1, S^I = 0)$  and  $E(\Delta D | Z_1, D_1, S^I = 0)$  defined analogously to the series estimator in Section 1.3.3.

Let us define

$$\widehat{\delta}_{IV}^{ps} = \frac{\frac{1}{n} \sum_{i=1}^n \text{sgn}(\Delta Z_i) \Delta Y_i - \frac{1}{n} \sum_{i: S_i^I = 0} \Delta Y_i \frac{\widehat{P}(S_+^I = 1 | Z_1 = Z_{i,1}, D_1 = D_{i,1}) - \widehat{P}(S_-^I = 1 | Z_1 = Z_{i,1}, D_1 = D_{i,1})}{\widehat{P}(S^I = 0 | Z_1 = Z_{i,1}, D_1 = D_{i,1})}}{\frac{1}{n} \sum_{i=1}^n \text{sgn}(\Delta Z_i) \Delta D_i - \frac{1}{n} \sum_{i: S_i^I = 0} \Delta D_i \frac{\widehat{P}(S_+^I = 1 | Z_1 = Z_{i,1}, D_1 = D_{i,1}) - \widehat{P}(S_-^I = 1 | Z_1 = Z_{i,1}, D_1 = D_{i,1})}{\widehat{P}(S^I = 0 | Z_1 = Z_{i,1}, D_1 = D_{i,1})}}, \quad (1.17)$$

where  $\widehat{P}(S_+^I = 1 | Z_1, D_1)$  (resp.  $\widehat{P}(S_-^I = 1 | Z_1, D_1)$ ,  $\widehat{P}(S^I = 0 | Z_1, D_1)$ ) is a series logistic regression estimator of  $P(S_+^I = 1 | Z_1, D_1)$  (resp.  $P(S_-^I = 1 | Z_1, D_1)$ ,  $P(S^I = 0 | Z_1, D_1)$ ) defined analogously to the series logistic regression estimators in Section 1.4.3.

For any variable  $X$ , let

$$\begin{aligned} \delta_X &= E \left[ \text{sgn}(\Delta Z) (\Delta X - E(\Delta X | Z_1, D_1, S^I = 0)) \right] \\ \psi_X &= \frac{1}{E(|\Delta Z|)} \left\{ \left( S_+^I - S_-^I - E(S_+^I - S_-^I | Z_1, D_1) \frac{(1 - S^I)}{E(1 - S^I | Z_1, D_1)} \right) \right. \\ &\quad \left. \times (\Delta X - E(\Delta X | Z_1, D_1, S^I = 0)) - \delta_X |\Delta Z| \right\}. \end{aligned}$$

Then, let

$$\psi_{IV} = \frac{\psi_Y - \delta_{IV} \psi_D}{\delta_D}.$$

Under technical conditions similar to those in Assumptions 6 and 8, one can show that

$$\begin{aligned} \sqrt{n} \left( \widehat{\delta}_{IV}^r - \delta_{IV} \right) &\xrightarrow{d} \mathcal{N}(0, V(\psi_{IV})), \\ \sqrt{n} \left( \widehat{\delta}_{IV}^{ps} - \delta_{IV} \right) &\xrightarrow{d} \mathcal{N}(0, V(\psi_{IV})). \end{aligned}$$

## 1.6 Extensions

In this section, we return to the case where the treatment, rather than an instrument, satisfies a parallel-trends condition. Combining the extensions below with the IV case is possible.

### 1.6.1 More than two time periods

In this section, we assume the representative unit is observed at  $T > 2$  time periods. Let  $(D_1, \dots, D_T)$  denote the unit's treatments and  $\mathcal{D}_t = \text{Supp}(D_t)$  for all  $t \in \{1, \dots, T\}$ . For any  $t \in \{1, \dots, T\}$ , and for any  $d \in \mathcal{D}_t$  let  $Y_t(d)$  denote the unit's potential outcome at period  $t$  with treatment  $d$ . Finally, let  $Y_t$  denote their observed outcome at  $t$ . For any  $t \in \{2, \dots, T\}$ , let  $S_t = 1\{D_t \neq D_{t-1}\}$  be an indicator equal to 1 if the unit's treatment switches from period  $t - 1$  to  $t$ . Let also  $S_{+,t} = 1\{D_t > D_{t-1}\}$  and  $S_{-,t} = 1\{D_t < D_{t-1}\}$ . We assume that the assumptions made in the paper, rather than just holding for  $t = 1$  and  $t = 2$ , actually hold for all pairs of consecutive time periods  $(t - 1, t)$ . For instance, we replace Assumption 1 by:

**Assumption 14** (*Parallel trends*) For all  $t \geq 2$ , for all  $d \in \mathcal{D}_{t-1}$ ,  $E(\Delta Y_t(d) | D_{t-1} = d, D_t) = E(\Delta Y_t(d) | D_{t-1} = d)$ .

Assumption 14 requires that  $E(\Delta Y_t(d) | D_{t-1} = d, D_t = d')$  be constant across  $d'$ : groups of units with the same period- $t - 1$  treatment but different period- $t$  treatments all have the same expected outcome evolution in the counterfactual where their period- $t - 1$  treatment would not have changed. Importantly, note that because Assumption 14 is conditional on  $D_{t-1}$ , it cannot be "chained" across pairs of time periods: it requires parallel trends over pairs of consecutive time periods, not over the entire duration of the panel. To preserve space, we do not restate our other assumptions with more than two periods.

Let

$$\begin{aligned} \delta_{1,t} &= E \left( \frac{Y_t(D_t) - Y_t(D_{t-1})}{D_t - D_{t-1}} \middle| S_t = 1 \right), \\ \delta_{2,t} &= \frac{E(\text{sgn}(D_t - D_{t-1})(Y_t(D_t) - Y_t(D_{t-1})))}{E(|D_t - D_{t-1}|)}. \end{aligned}$$

Let

$$\begin{aligned} \delta_1^{T \geq 3} &= \sum_{t=2}^T \frac{P(S_t = 1)}{\sum_{k=2}^T P(S_k = 1)} \delta_{1,t}, \\ \delta_2^{T \geq 3} &= \sum_{t=2}^T \frac{E(|\Delta D_t|)}{\sum_{k=2}^T E(|\Delta D_k|)} \delta_{2,t} \end{aligned}$$

be generalizations of the AS and WAS effects to applications with more than two periods. Note that in line with the spirit of the two effects, we propose different weights to aggregate the AS and WAS across

time periods. For the AS, the weights are just proportional to the proportion of switchers between  $t - 1$  and  $t$ . For the WAS, the weights are proportional to the average absolute value of the treatment switch from  $t - 1$  to  $t$ .

**Theorem 7** *If Assumption 14 and generalizations of Assumptions 2-5 to more than two periods hold,*

$$\delta_1^{T \geq 3} = \sum_{t=2}^T \frac{P(S_t = 1)}{\sum_{k=2}^T P(S_k = 1)} E \left( \frac{\Delta Y_t - E(\Delta Y_t | D_{t-1}, S_t = 0)}{\Delta D_t} \middle| S_t = 1 \right).$$

**Theorem 8** *If Assumption 14 and generalizations of Assumptions 2 and 7 to more than two periods hold,*

$$\begin{aligned} \delta_2^{T \geq 3} &= \sum_{t=2}^T \frac{E(|\Delta D_t|)}{\sum_{k=2}^T E(|\Delta D_k|)} \frac{E(\text{sgn}(\Delta D_t) (\Delta Y_t - E(\Delta Y_t | D_{t-1}, S_t = 0)))}{E(|\Delta D_t|)} \\ &= \sum_{t=2}^T \frac{E(|\Delta D_t|)}{\sum_{k=2}^T E(|\Delta D_k|)} \frac{E[\text{sgn}(\Delta D_t) \Delta Y_t] - E \left[ \Delta Y_t \frac{P(S_{+,t}=1|D_{t-1}) - P(S_{-,t}=1|D_{t-1})}{P(S_t=0|D_{t-1})} P(S_t = 0) \middle| S_t = 0 \right]}{E(|\Delta D_t|)}. \end{aligned}$$

Theorems 7 and 8 are straightforward generalizations of Theorems 1 and 4 to settings with more than two time periods.

Let

$$\begin{aligned} \psi_{1,t} &= \frac{1}{E(S_t)} \left\{ \left( \frac{S_t}{\Delta D_t} - E \left( \frac{S_t}{\Delta D_t} \middle| D_{t-1} \right) \frac{(1 - S_t)}{E[1 - S_t | D_{t-1}]} \right) [\Delta Y_t - E(\Delta Y_t | D_{t-1}, S_t = 0)] - \delta_{1,t} S_t \right\}, \\ \psi_{2,t} &= \frac{1}{E(|\Delta D_t|)} \left\{ (S_{+,t} - S_{-,t} - E(S_{+,t} - S_{-,t} | D_{t-1})) \frac{(1 - S_t)}{E(1 - S_t | D_{t-1})} (\Delta Y_t - E(\Delta Y_t | D_{t-1}, S_t = 0)) - \delta_{2,t} |\Delta D_t| \right\}. \end{aligned}$$

After some algebra, one can show that the influence function of the AS estimator with several periods is

$$\psi_1^{T \geq 3} := \frac{\sum_{t=2}^T (P(S_t = 1) \psi_{1,t} + (\delta_{1,t} - \delta_1^{T \geq 3}) (S_t - P(S_t = 1)))}{\sum_{t=2}^T P(S_t = 1)}, \quad (1.18)$$

while the influence function of the WAS estimators with several periods is

$$\psi_2^{T \geq 3} := \frac{\sum_{t=2}^T E(|\Delta D_t|) \psi_{2,t} + (\delta_{2,t} - \delta_2^{T \geq 3}) (|\Delta D_t| - E(|\Delta D_t|))}{\sum_{t=2}^T E(|\Delta D_t|)}. \quad (1.19)$$

Importantly, those influence functions allow the unit's treatments and outcomes to be arbitrarily serially correlated.

## 1.6.2 Placebo tests

With several time periods, one can test the following condition, which is closely related to Assumption 14:

**Assumption 15** (*Testable parallel trends*) For all  $t \geq 3, t \leq T$ , for all  $d \in \mathcal{D}_{t-1}$ ,  $E(\Delta Y_{t-1}(d)|D_{t-2} = D_{t-1} = d, D_t) = E(\Delta Y_{t-1}(d)|D_{t-2} = D_{t-1} = d)$ .

To test that condition, one can compute a placebo version of the estimators described in the previous subsection, replacing  $\Delta Y_t$  by  $\Delta Y_{t-1}$ , and restricting the sample, for each pair of consecutive time periods  $(t-1, t)$ , to units whose treatment did not change between  $t-2$  and  $t-1$ . Thus, the placebo compares the average  $\Delta Y_{t-1}$  of the  $t-1$ -to- $t$  switchers and stayers, restricting attention to  $t-2$ -to- $t-1$  stayers. If one finds that from  $t-2$ -to- $t-1$ ,  $t-1$ -to- $t$  switchers and stayers are on parallel trends, this lends credibility to Assumption 14.

Assumption 14 can only be placebo tested among  $t-2$ -to- $t-1$  stayers. Then, as a robustness check one may restrict the estimation of  $\delta_1$  and  $\delta_2$  to  $t-2$ -to- $t-1$  stayers, to ensure that effects are only estimated in a subsample for which the identifying assumption can be placebo tested. The resulting estimator relies on the following identifying assumption:

$$\forall t \geq 3, t \leq T, d \in \mathcal{D}_{t-1} : E(\Delta Y_t(d)|D_{t-2} = D_{t-1} = d, D_t) = E(\Delta Y_t(d)|D_{t-2} = D_{t-1} = d),$$

the exact analogue of Assumption 15 but one period ahead.

### 1.6.3 Estimators robust to dynamic effects up to a pre-specified treatment lag.

Importantly, the robustness check in the previous section also yields an estimator robust to dynamic effects up to one treatment lag. If units' current and first treatment lag affect their current outcome, our  $t-1$ -to- $t$  estimators in the subsample of  $t-2$ -to- $t-1$  stayers are unbiased for effects of the current treatment on the outcome under the following assumption:

$$\forall t \geq 3, t \leq T, d \in \mathcal{D}_{t-1} : E(Y_t(d, d) - Y_{t-1}(d, d)|D_{t-2} = D_{t-1} = d, D_t) = E(Y_t(d, d) - Y_{t-1}(d, d)|D_{t-2} = D_{t-1} = d).$$

Similarly, if one wants to allow for effects of the first and second treatment lags on the outcome, one just needs to restrict the estimation sample to  $t-3$ -to- $t-1$  stayers. However, the more robustness to dynamic effects one would like to have, the smaller the estimation sample becomes.

## 1.7 Application

**Data and research questions.** We use the yearly 1966-to-2008 panel dataset of Li et al. (2014), covering 48 US states (Alaska and Hawaii are excluded). In view of the long duration of this panel, it is important to keep in mind that our estimators only assume parallel trends across pairs of consecutive

years, not over the panel's entire duration. For each state×year cell  $(i, t)$ , the data contains  $Z_{i,t}$ , the total (state plus federal) gasoline tax in cents per gallon,  $D_{i,t}$ , the log tax-inclusive price of gasoline, and  $Y_{i,t}$ , the log gasoline consumption per adult. Our goal is to estimate the effect of gasoline taxes on gasoline consumption and prices, and to estimate the price-elasticity of gasoline consumption, using taxes as an instrument. Instead, Li et al. (2014) jointly estimate the effect of gasoline taxes and tax-exclusive prices on consumption, using a TWFE regression with two treatments. Between each pair of consecutive periods, the tax-exclusive price changes in all states, so this treatment does not have stayers and its effect cannot be estimated using the estimators proposed in this paper. Thus, our estimates cannot be compared to those of Li et al. (2014).

**Switching cells, and how they compare to the entire sample.** Let  $S$  be the set of switching  $(i, t)$  cells such that  $Z_{i,t} \neq Z_{i,t-1}$  but  $Z_{i',t} = Z_{i',t-1}$  for some  $i'$ . The second condition drops from the estimation seven pairs of consecutive time periods between which the federal gasoline tax changed, thus implying that all states experienced a change of their tax.  $S$  includes 384 cells, so effects of taxes on gasoline prices and consumptions can be estimated for 19% of the 2,016 state×year cells for which  $Z_{i,t} - Z_{i,t-1}$  can be computed. Table 1.1 below compares some observable characteristics of switchers and stayers. Switchers seem slightly over-represented in the later years of the panel:  $t$  is on average 2.5 years larger for switchers than for stayers, and the difference is significant. On the other hand, switchers are not more populated than stayers, and their gasoline consumption and gasoline price in 1966 are not significantly different from that of stayers. Thus, there is no strong indication that the cells in  $S$  are a very selected subgroup.

**Distribution of taxes.** As an example, the top panel of Figure 1.1 below shows the distribution of  $Z_{g,1987}$  for 1987-to-1988 stayers, while the bottom panel shows the distribution for 1987-to-1988 switchers. The figure shows that there are many values of  $Z_{g,1987}$  such that only one or two states have that value, so  $Z_{g,1987}$  is close to being continuously distributed. Moreover, all switchers  $g$  are such that

$$\min_{g': Z_{g',1988} = Z_{g',1987}} Z_{g',1987} \leq Z_{g,1987} \leq \max_{g': Z_{g',1988} = Z_{g',1987}} Z_{g',1987}.$$

Thus, Assumption 4 seems to hold for this pair of years. (1987, 1988) is not atypical. While  $Z_{i,t}$  varies less across states in the first years of the panel, there are many other years where  $Z_{i,t}$  is close to being continuously distributed. Similarly, almost 95% of cells in  $S$  are such that  $\min_{g': Z_{i',t} = Z_{i',t-1}} Z_{i',t-1} \leq Z_{i,t-1} \leq \max_{g': Z_{i',t} = Z_{i',t-1}} Z_{i',t-1}$ . Dropping the few cells that do not satisfy this condition barely changes the results presented below.

**Distribution of tax changes.** Figure 1.2 below shows the distribution of  $Z_{i,t} - Z_{i,t-1}$  for the 384 cells in  $S$ . The majority experience an increase in their taxes, but 38 cells experience a decrease. The average value of  $|Z_{i,t} - Z_{i,t-1}|$  is equal to 1.61 cents, while prior to the tax change, switchers' average gasoline price is equal to 112 cents: our estimators leverage small changes in taxes relative to gasoline prices. Finally,  $\min_{(i,t) \in S} |Z_{i,t} - Z_{i,t-1}| = 0.05$  : some switchers experience a very small change in their taxes.

Table 1.1: Comparing switchers and stayers

Dependent Variables:	$t$	Adult Population	$\log(\text{quantity})_{1966}$	$\log(\text{price})_{1966}$
Constant	1,986.7 (0.2739)	3,691,608.0 (577,164.0)	-0.5161 (0.0210)	3.471 (0.0054)
$\mathbf{1}\{Z_{i,t} \neq Z_{i,t-1}\}$	2.481 (0.7519)	39,588.0 (320,342.1)	-0.0099 (0.0096)	0.0014 (0.0029)
N	2,016	2,016	2,016	2,016

Notes: The table show the results of regressions of some dependent variables on a constant and an indicator for switching cells. The standard errors shown in parentheses are clustered at the state level.

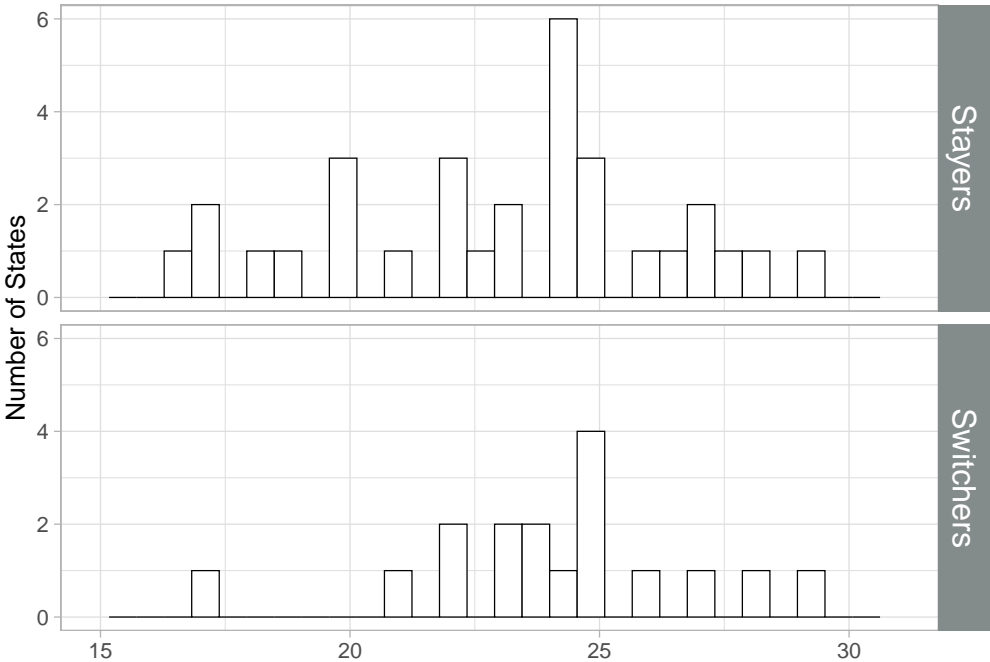


Figure 1.1: Gasoline tax in 1987 among 1987-to-1988 switchers and stayers

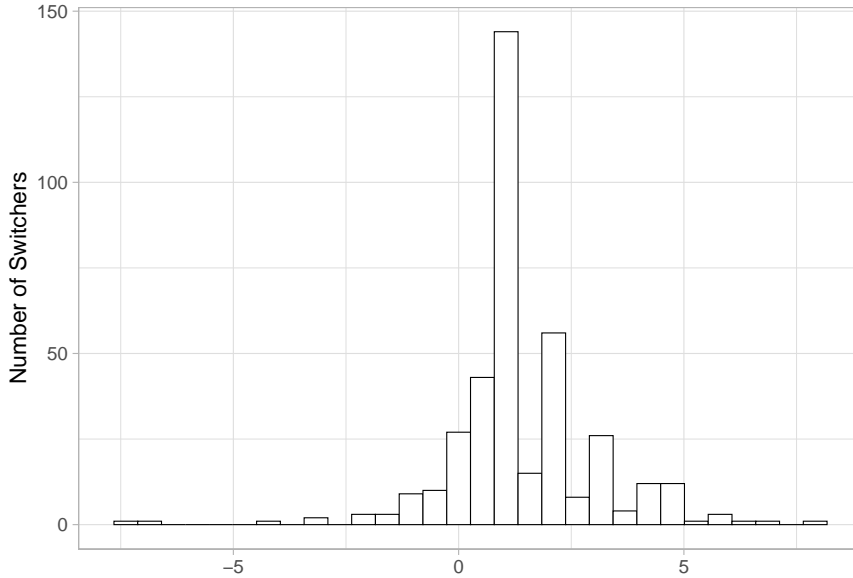


Figure 1.2: Distribution of tax changes between consecutive periods

**Reduced-form and first-stage AS and WAS estimates.** Table 1.2 below shows the AS and doubly-robust WAS estimates of the reduced-form (Panel A) and first-stage (Panel B) effects of taxes on quantities and prices. We follow results from Section 1.5, and estimate the reduced-form and the first-stage controlling for lagged prices  $D_{t-1}$ , to ensure that the resulting IV estimator is robust to heterogeneous effects over time. Reduced-form and first-stage estimators where  $D_{t-1}$  is not controlled for are not very different, but controlling for  $D_{t-1}$  reduces the standard error of the first-stage estimator. In Column (1), the estimators are computed using a polynomial of order 1 in  $(Z_{t-1}, D_{t-1})$  to estimate  $E(\Delta Y_t | Z_{t-1}, D_{t-1}, S_t = 0)$ ,  $E(\Delta D_t | Z_{t-1}, D_{t-1}, S_t = 0)$ , and the propensity scores  $P(S_{+,t} = 1 | Z_{t-1}, D_{t-1})$ ,  $P(S_{-,t} = 1 | Z_{t-1}, D_{t-1})$ , and  $P(S_t = 0 | Z_{t-1}, D_{t-1})$ . In Column (2), a polynomial of order 2 is used in those estimations. 10-folds cross-validation selects a polynomial of order two for  $E(\Delta D_t | Z_{t-1}, D_{t-1}, S_t = 0)$ , and a polynomial of order one for all the other conditional expectations. Thus, polynomials of order 1 and 2 are in line with those selected by cross validation. Standard errors clustered at the state level, computed following (1.18) and (1.19), are shown below the estimates, between parentheses. All estimations use 1632 ( $48 \times 35$ ) first-difference observations: 7 periods have to be excluded as they do not have stayers. Finally, the last line of each panel shows the p-value of a test that the AS and WAS effects are equal. In Panel A Column (1), the AS estimate indicates that increasing gasoline tax by 1 cent decreases quantities consumed by 0.55 percent on average for the switchers. That effect is significant at the 5% level, but it becomes smaller and insignificant when one uses a quadratic model to estimate  $E(\Delta Y_t | Z_{t-1}, D_{t-1}, S_t = 0)$ . The WAS estimates are slightly lower than, but close to, the AS estimates, and they are significant irrespective of the polynomial order used in the estimation. As predicted by Proposition 1, the standard errors of the WAS estimators are almost 3 times smaller than that of the AS estimators. Equality tests that the AS and WAS

effects are equal are not rejected. In Panel B, the AS estimates of the first-stage effect are insignificant. Importantly, this implies that an IV-AS estimator of the price elasticity of gasoline consumption cannot be used: this estimator does not have a significant first stage. The WAS estimates are significant, and they indicate that if gasoline tax increases by 1 cent on average, prices increase by around 0.5 percent on average for the switchers. Again, the differences between the AS and WAS effects of taxes on prices are insignificant.

Table 1.2: Effects of gasoline tax on quantities consumed and prices

	(1) Linear model	(2) Quadratic model
<b>Panel A: Reduced-form effect of taxes on quantities consumed</b>		
AS	-0.0055 (0.0027)	-0.0034 (0.0032)
WAS	-0.0038 (0.0010)	-0.0034 (0.0011)
Observations	1,632	1,632
<b>P-value</b>	0.4482	0.9974
<b>Panel B: First-stage effect of taxes on prices</b>		
AS	0.0042 (0.0024)	0.0047 (0.0025)
WAS	0.0056 (0.0009)	0.0056 (0.0008)
Observations	1,632	1,632
<b>P-value</b>	0.4729	0.6798

Notes: All estimators in the table are computed using the data of Li et al. (2014). Panel A (resp. B) shows the AS and doubly-robust WAS estimates of the reduced-form (resp. first-stage) effect of taxes on quantities (resp. prices). All estimates control for the lag of prices. In Column (1), estimates are computed using a polynomial of order 1 in  $(Z_{t-1}, D_{t-1})$  to estimate  $E(\Delta Y_t | Z_{t-1}, D_{t-1}, S_t = 0)$  and the propensity scores  $P(S_{+,t} = 1 | Z_{t-1}, D_{t-1})$ ,  $P(S_{-,t} = 1 | Z_{t-1}, D_{t-1})$ , and  $P(S_t = 0 | Z_{t-1}, D_{t-1})$ . In Column (2), estimates are computed using a polynomial of order 2 in those estimations. Standard errors clustered at the state level, computed following (1.18) and (1.19) are shown below the estimates, between parentheses. All estimations use 1632 ( $48 \times 35$ ) first-difference observations: 7 periods have to be excluded as they do not have stayers. Finally, the last line of each panel shows the p-value of a test that the AS and WAS effects are equal.

**Placebo analysis.** Table 1.3 below shows placebo AS and doubly-robust WAS estimates of the reduced-form and first-stage effects. The placebo estimators are analogous to the actual estimators, but they replace  $\Delta Y_t$  by  $\Delta Y_{t-1}$ , and they restrict the sample, for each pair of consecutive time periods  $(t-1, t)$ , to states whose taxes did not change between  $t-2$  and  $t-1$ . The placebo WAS estimates are small and



insignificant, both for quantities and prices. The placebo AS estimates are larger for quantities, but they are insignificant, and less precisely estimated. This placebo analysis shows that before switchers change their gasoline taxes, switchers' and stayers' consumption of gasoline and gasoline prices do not follow detectably different evolutions. As a robustness check, we reestimate the AS and WAS in the placebo subsample, to ensure that effects are estimated in a subsample for which the identifying assumption can be placebo tested, and also because in that subsample estimators remain valid if the first lag of taxes affect current gasoline prices and quantities. WAS reduced-form effects are very close to those in Table 1.2. WAS first-stage effects are 25 to 35% smaller, though they are still positive and highly significant.

Table 1.3: Placebo effects of gasoline tax on quantities consumed and prices

	(1) Linear model	(2) Quadratic model
<b>Panel A: Reduced-form placebo effect of taxes on quantities consumed</b>		
AS	0.0039 (0.0035)	0.0055 (0.0036)
WAS	0.0001 (0.0017)	0.0012 (0.0017)
Observations	1,059	1,059
<b>Panel B: First-stage placebo effect of taxes on prices</b>		
AS	0.0006 (0.0056)	0.0009 (0.0053)
WAS	0.0014 (0.0017)	0.0013 (0.0015)
Observations	1,059	1,059

Notes: The table shows the placebo AS and doubly-robust WAS estimates of the reduced-form and first-stage effects of taxes on quantities and prices. The estimators and their standard errors are computed as the actual estimators, replacing  $\Delta Y_t$  by  $\Delta Y_{t-1}$ , and restricting the sample, for each pair of consecutive time periods  $(t-1, t)$ , to states whose taxes did not change between  $t-2$  and  $t-1$ .

**IV-WAS estimate of the price-elasticity of gasoline consumption.** Table 1.4 shows doubly-robust IV-WAS estimates of the price-elasticity of gasoline consumption. As the instrument's first stage is not very strong and the sample effectively only has 48 observations, asymptotic approximations may not be reliable for inference. In line with that conjecture, we find that the bootstrap distributions of the three estimators in Table 1.4 are non-normal, with some outliers. Therefore, we use the percentile bootstrap for inference, clustering the bootstrap at the state level. Reassuringly, these confidence

intervals have nominal coverage in simulations tailored to our application.<sup>7</sup> The IV-WAS estimates are negative, significant, and larger than -1, though their confidence intervals contain -1. We compare those estimates to a 2SLS-TWFE estimator, computed via a 2SLS regression of  $Y_{i,t}$  on  $D_{i,t}$  and state and year fixed effects, using  $Z_{i,t}$  as the instrument. The 2SLS-TWFE coefficient is equal to -1.0836 (bootstrap confidence interval=[-2.1207, -0.4405]), which is 60% larger in absolute value than the IV-WAS estimate in Column (1), and almost 80% larger than that in Column (2), though the 2SLS-TWFE coefficient does not significantly differ from the two IV-WAS estimates (P-value = 0.320 and 0.232, respectively). Interestingly, the confidence interval of the 2SLS-TWFE coefficient is almost 80% wider than that of the IV-WAS coefficient in Column (1) and 27% wider than that in Column (2), thus showing that using a more robust estimator does not always come with a substantive precision cost.

Table 1.4: IV estimators of the price-elasticity of gasoline consumption

	(1) Linear model	(2) Quadratic model
IV-WAS	-0.6773 [-1.2101,-0.2622]	-0.6130 [-1.3183,-0.0004]
Observations	1,632	1,632

Notes: The table shows doubly-robust IV-WAS estimates of the price-elasticity of gasoline consumption, computed using the data of Li et al. (2014). Bootstrap confidence intervals are shown below the estimates. They are computed with 500 bootstrap replications, clustered at the state level.

## 1.8 Conclusion

We propose new difference-in-difference (DID) estimators for continuous treatments. We assume that between pairs of consecutive periods, the treatment of some units, the switchers, changes, while the treatment of other units, the stayers, does not change. We propose a parallel trends assumption on the outcome evolution of switchers and stayers with the same baseline treatment. Under that assumption, two target parameters can be estimated. Our first target is the average slope of switchers' period-two potential outcome function, from their period-one to their period-two treatment, referred to as the AS. Our second target is a weighted average of switchers' slopes, where switchers receive a weight proportional to the absolute value of their treatment change, referred to as the WAS. Economically, the AS and WAS serve different purposes, so neither parameter dominates the other. On the other hand, when it comes to

<sup>7</sup>Here is the DGP used in our simulations. We estimate TWFE regressions of  $Y_{i,t}$  on state and year fixed effects and  $Z_{i,t}$ , and of  $D_{i,t}$  on state and year fixed effects and  $Z_{i,t}$ . We let  $\hat{\gamma}_i^Y + \hat{\lambda}_t^Y + \hat{\beta}^Y Z_{i,t} + \epsilon_{i,t}^Y$  and  $\hat{\gamma}_i^D + \hat{\lambda}_t^D + \hat{\beta}^D Z_{i,t} + \epsilon_{i,t}^D$  denote the resulting regression decompositions. In each simulation, the simulated instrument is just the actual instrument, while the simulated outcomes and treatments are respectively equal to  $Y_{i,t}^s = \hat{\gamma}_i^Y + \hat{\lambda}_t^Y + \hat{\beta}^Y Z_{i,t} + \epsilon_{i,t}^{Y,s}$ , and  $D_{i,t}^s = \hat{\gamma}_i^D + \hat{\lambda}_t^D + \hat{\beta}^D Z_{i,t} + \epsilon_{i,t}^{D,s}$ , where the vector of simulated residuals  $(\epsilon_{g,1}^{Y,s}, \dots, \epsilon_{g,T}^{Y,s}, \epsilon_{g,1}^{D,s}, \dots, \epsilon_{g,T}^{D,s})$  is drawn at random and with replacement from the estimated vectors of residuals  $((\epsilon_{g',1}^Y, \dots, \epsilon_{g',T}^Y, \epsilon_{g',1}^D, \dots, \epsilon_{g',T}^D))_{g' \in \{1, \dots, G\}}$ . Thus, the first-stage and reduced-form effects, the correlation between the reduced-form and first-stage residuals, and the residuals' serial correlation are the same as in the sample.

estimation, the WAS unambiguously dominates the AS. First, it can be estimated at the parametric rate even if units can experience an arbitrarily small treatment change. Second, under some conditions, its asymptotic variance is strictly lower than that of the AS estimator. Third, unlike the AS, it is amenable to doubly-robust estimation. In our application, we use US-state-level panel data to estimate the effect of gasoline taxes on gasoline consumption. The standard error of the WAS is almost three times smaller than that of the AS, and the two estimates are close.

We also consider the instrumental-variable case, as there are instances where units experiencing/not experiencing a treatment change are unlikely to be on parallel trends, but one has at hand an instrument such that units experiencing/not experiencing an instrument change are more likely to be on parallel trends. Then, we propose widely applicable IV-DID estimators, that are robust to heterogeneous effects over time but impose some restrictions on effects' heterogeneity across units.

## Chapter 2

# Estimating heterogeneous peer effects with partial population experiments

with Pauline Rossi and Zheng Wang

The standard linear-in-means model of peer effects assumes that the endogenous peer effect parameter is homogeneous. We relax this assumption by allowing individuals to respond differently to the outcomes of other group members depending on the identity of these members. Specifically, we distinguish peer effects *within* members sharing the same identity and peer effects *between* members of different identities. We propose a simple methodology to identify and estimate the model using partial population experiments (i.e. designs in which only some individuals in a group are eligible for treatment) with variation in the share of eligible individuals across groups. We discuss two cases: randomized experiments and differences-in-differences. The estimation procedure builds on the Generalized Method of Moments. We are able to quantify the direct effect on the eligibles, the indirect effect on the non-eligibles and the population multipliers. We apply our methodology to study peer effects in school attendance using a conditional cash transfer program targeting the poor in Mexico.

### 2.1 Introduction

In the field of peer effects, applied economists often use the linear-in-means model. The model postulates that individuals are influenced by the average outcome in a given reference group. This is a way to formalize the concept of conformism: people incur a cost when their behavior is different from the average behavior. An important parameter in this model is the *endogenous peer effect*, which captures the change

in the outcome of an individual in response to a change in the group average outcome. The endogenous peer effect is assumed to be homogeneous, in the sense that (i) all individuals in a reference group respond to the change in the same way, and (ii) all individuals have the same weight in the group average.

This assumption fails to account for complex group dynamics, such as polarization, social distinction, or role models. Sociologists have long documented that the relevant reference group may consist of several sub-groups who do not necessarily interact in a symmetric way. Some individuals may put more weight on their own sub-group; other individuals may put more weight on the other sub-groups, either emulating them or opposing them. These dynamics depend on what defines the identity of these sub-groups, their number and relative size, as well as the social hierarchy.

In this paper, we relax the homogeneity assumption by allowing individuals to respond differently to changes in the outcomes of other group members depending on the identity of these members. We distinguish peer effects *within* members sharing the same identity and peer effects *between* members of different identities. We propose a methodology to identify and estimate these parameters using experiments in which only a share of the population is eligible for treatment. These designs are called partial population experiments. We discuss two cases: (i) controlled experiments, where the treatment is randomly allocated to groups; and (ii) natural experiments, more specifically differences-in-differences, where the treatment is allocated to different groups at different times and the common trend condition holds.

First, we show that we can identify within and between peer effects when the share of eligible individuals varies across groups; in theory, three values are enough for identification. Second, we propose a simple estimation procedure building on the Generalized Method of Moments (GMM). Third, we consider several extensions. The first extension allows the between and within parameters to vary across sub-groups. The second extension considers designs where the identity that matters for the heterogeneous peer effect dimension is orthogonal to treatment eligibility. The last extension shows how to include covariates to improve the plausibility of the common trend assumption.

Estimating endogenous peer effects is particularly important from a policy evaluation perspective because they generate population multipliers. The treatment is a shock affecting the behaviors of eligible individuals; then, non-eligibles respond to the change in eligibles' behaviors; and the shock propagates further (between and within sub-groups) until a new equilibrium is reached. Our estimates are useful to quantify (i) the *direct effect* of the policy on the eligibles, (ii) the *indirect effect* of the policy on the non-eligibles and (iii) the population multipliers, i.e. by how much the direct and indirect effects are amplified by social interactions between and within sub-groups. In contrast, research designs based on the random allocation of peers cannot separately identify whether individuals are influenced by the

behavior of their peers (endogeneous peer effects) or by the characteristics of their peers (exogeneous peer effects). Separating both effects is crucial since only the endogenous component gives rise to population multipliers.

We apply our methodology to explore peer effects in school attendance using a conditional cash transfer program in Mexico – Progresa. This program has been extensively studied: it was randomly allocated across villages and it targeted poor households. Previous research finds a positive effect on school attendance of the eligibles and a positive spillover effects on school attendance of the non-eligibles belonging to the same group, defined as grade  $\times$  village. For instance, Lalive and Cattaneo (2009) estimate a direct effect of 3*p.p.*: in the absence of social interactions, school attendance among the poor should increase by 3 percentage points in treated villages. They estimate an homogeneous endogenous peer effect parameter of 0.5: the individual probability of attending school increases by 5 percentage points when the average attendance in a group increases by 10 percentage points. When we allow for heterogeneity, we estimate a “between” peer effect of 0.31, a “within” peer effect of 0.24, and a direct effect of 5*p.p.*. Yet, we cannot reject the null that all those parameters are zero.

Other potential applications cover important topics in education (e.g. estimate peer effects in graduation rates using scholarships targeting some categories of students and variation across majors), labor (e.g. estimate peer effects in parental leave take-up using collective agreements targeting some professions and variation across neighborhoods), health (e.g. estimate peer effects in contraception take-up using rules specific to minors and variation across classes), political economy (e.g. estimate peer effects in support for local authorities using public infrastructure devoted to the elderly or to young children and variation across neighborhoods), and crime (e.g. estimate peer effects in criminal activities using interventions targeting at-risk youth and variation across classes). Our methodology is adequate in settings where the non-eligibles are affected by the treatment only through changes in the eligibles’ outcomes. By assumption, we rule out any direct effect on the non-eligibles.<sup>1</sup>

Our paper is at the intersection of two streams of literature. First, it relates to the literature using partial population experiments to identify the endogenous peer effect parameter in linear-in-means models (Moffitt et al. (2001), Bobonis and Finan (2009), Brown and Laschever (2012), Hirano and Hahn (2010)). We contribute to this literature by allowing for heterogeneous parameters. Relaxing the homogeneity assumption is important because this assumption restricts the shape of the population multipliers: they have to be the same for eligibles and non-eligibles, and they have to be linear in the share of eligibles ( $s$ ). For instance, the population multiplier is equal to  $2s$  in Lalive and Cattaneo (2009) This restriction

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<sup>1</sup>For instance, in the case of Progresa, this assumption could be violated if cash transfers were shared with non-poor households, or if they caused inflation in the village: the treatment would directly affect the non-eligibles’ budget constraint. Lalive and Cattaneo (2009) provide arguments to rule out these possibilities.

implies that the total effect on the eligibles and the total effect on the non-eligibles should be linear in  $s$ , a prediction which is not always verified in the data. By allowing for different “between” and “within” parameters, we can rationalize diverse empirical patterns: convex or concave, increasing or decreasing, depending on the relative magnitudes and on the signs of the “between” and “within” parameters. In the case of Progresa, we find that the total effects are increasing and slightly concave, and that the multiplier is larger for the eligibles than for the non-eligibles.

Second, it relates to the literature on the estimation of spillover effects in experiments. Previous research has studied how the outcome of an individual is affected by the treatment status of her peers in a non-parametric way (Hudgens and Halloran (2008), Tchetgen and VanderWeele (2012)). In particular, Vazquez-Bare (2023) discusses the case of heterogeneous spillover effects in a flexible framework, allowing the treatment status of peers to have heterogeneous effects depending on the characteristics of the peers. The interpretation of the parameters in this approach is reduced-form, in the sense that they capture different mechanisms: endogenous peer effects, exogenous peer effects, general equilibrium effects through prices, etc. In contrast, our approach focuses on a specific mechanism. We can only study a subset of the questions explored in the literature. However, for these questions, we have a structural interpretation of the parameters and we learn something fundamental about the drivers of behaviors. We learn about how individuals weigh other members of their reference groups, whether they imitate some members in particular, or try to distinguish themselves from other members. Our results are useful to deepen our understanding of social influence, in particular to study dynamics going beyond plain conformism.

At this intersection, we are aware of two other papers. The first one is Masten (2018) that considers a linear-in-means models with random coefficients on the endogenous variable. This model implies that individuals respond differently to the average behavior of their peers. On the contrary, in our model, individuals respond differently to their peers' behavior depending on their peers' identity. The second one is Arduini et al. (2020). We consider the same linear-in-means model as them but we develop a different estimation strategy. Our strategy does not require the share of eligibles to be exogenous to the treatment and is therefore compatible with natural experiments.

The remainder of the paper is organized as follows. Section 2.2 introduces notations, presents our baseline model and discusses our main assumptions. Section 2.3 explains our identification strategy. Section 2.4 presents our estimation results. Section 2.5 details our extensions. Finally, section 2.6 deals with our application : Progresa.

## 2.2 Set-up, notations and main assumptions

We start by describing the set-up with randomized treatment assignment and then turn to the difference-in-difference.

### 2.2.1 Randomized experiment

We consider a setting inspired by Moffitt et al. (2001). We suppose that  $G$  reference groups are observed. Group  $g$  contains  $n_g$  units (indexed by  $i$ ); and a share  $s_g$  of those units are eligible for a binary treatment, denoted  $D_g$ . Some groups receive the treatment while others don't. Let  $E_{ig}$  be the binary variable indicating whether or not unit  $i$  in group  $g$  is eligible for the treatment and let  $D_{ig}$  be a binary variable indicating whether or not unit  $i$  has received the treatment. We suppose perfect compliance. None of the ineligible individuals may receive the treatment, i.e.  $P(D_{ig} = 1 | E_{ig} = 0) = 0$ . And if group  $g$  is treated, all of the eligible units receive the treatment, i.e.  $P(D_{ig} = 1 | E_{ig} = 1, D_g = 1) = 1$ . Let  $n_g^E = \sum_{i=1}^{n_g} E_{ig} = s_g n_g$  and  $n_g^N = \sum_{i=1}^{n_g} (1 - E_{ig}) = n_g - n_g^E$  be the number of units that are respectively eligible and non-eligible for the treatment in group  $g$ . We consider the case where eligibility  $E_{ig}$  is *not* randomly determined: eligibility reflects an observable characteristic, that we call "identity". In our application that evaluates the effects of Progresa, a reference group corresponds to a grade level associated to a village, the treatment corresponds to Progresa's conditional cash transfer that was randomly allocated across villages, and eligibility status is determined by a threshold on the income distribution.

We start from the commonly used linear-in-means (LIM) model of social interactions (Manski (1993), Blume et al. (2011), Bramoullé et al. (2009)), defined as

$$y_{ig} = \alpha_g + x'_{ig}\eta_0 + z'_g\gamma + \delta_0 D_{ig} + \theta_0 \times \frac{1}{n_g} \sum_{i=1}^{n_g} y_{ig} + \varepsilon_{ig} \quad (2.1)$$

where  $y_{ig}$  is individual  $i$ 's scalar outcome,  $z_g$  is a vector of attributes characterising individual  $i$ 's reference group  $g$ , including peer characteristics within the group,  $x_{ig}$  and  $\varepsilon_{ig}$  are respectively individual  $i$ 's observed (resp. unobserved) attributes that directly affect  $y_{ig}$ . If  $\theta_0 \neq 0$ , this model expresses an endogenous peer effect: individual  $i$ 's outcome varies with the average outcome of her peers in group  $g$ . It is standard to rule out explosive trajectories by assuming that  $|\theta_0| < 1$ . The vector  $(\eta_0, \delta_0)$  captures the exogenous effects while  $\alpha_g$  captures the correlated effects, as defined by Manski (1993). Moffitt et al. (2001) shows that all parameters, and thus all different types of effects, are identified as long as  $\delta_0 \neq 0$ .

We extend the standard model by allowing for heterogeneous endogenous peer effects. Individuals may be differently influenced by their peers' outcomes depending on whether they share the same "identity" or not. In the following, for any variable  $w$ , we denote the average among eligibles and the



average among non-eligibles as follows:  $\bar{w}_g^E = \frac{1}{n_g^E} \sum_{i \in \mathcal{E}_g} w_{ig}^E$  and  $\bar{w}_g^N = \frac{1}{n_g^N} \sum_{i \in \mathcal{O}_g} w_{ig}^N$  where  $n_g^E = \sum_{i=1}^{n_g} E_{ig}$  and  $n_g^N = n_g - n_g^E$ . We make the following assumptions.

**Assumption 1 (Linear-in-means model)** *Within each group  $g \in \{1, \dots, G\}$  of the i.i.d sample, the outcome of individual  $i$  is defined as*

$$y_{ig} = E_{ig} y_{ig}^E + (1 - E_{ig}) y_{ig}^N$$

with

$$y_{ig}^E = \theta_0^w s_g \bar{y}_g^E + \theta_0^b (1 - s_g) \bar{y}_g^N + \delta_0 D_{ig} + e_{ig}^E \quad (2.2)$$

$$y_{ig}^N = \theta_0^b s_g \bar{y}_g^E + \theta_0^w (1 - s_g) \bar{y}_g^N + e_{ig}^N \quad (2.3)$$

The parameters of interest are  $\delta_0$ ,  $\theta_0^b$  and  $\theta_0^w$ , with  $\delta_0 \in \mathbb{R}^*$  and  $\theta_0^b, \theta_0^w \in (-1, 1)$ .  $\delta_0$  represents the direct effect of the treatment on the eligible individuals that receive it. It is assumed to be homogeneous across groups.  $\theta_0^w$  is the coefficient for the endogenous peer effects from peers who share the same identity as the individual while  $\theta_0^b$  is the one for endogenous peers from peers of a different identity. For a given group  $g$ , when  $\bar{y}_g^E$  and  $\bar{y}_g^N$  both increase by 1, the outcome of an eligible unit will increase by  $\theta_0^w s_g + \theta_0^b (1 - s_g)$  and the outcome of a non-eligible unit will increase by  $\theta_0^b s_g + \theta_0^w (1 - s_g)$ . These quantities are convex combinations of the within and between endogenous peer effects, weighted by the share of the eligible units. To illustrate the interpretation of  $(\theta_0^w, \theta_0^b)$ , consider a scenario where  $s_g = 0.5$ , indicating an equal number of eligible and non-eligible units within a group. Under this condition, a simultaneous increase in  $\bar{y}_g^E$  and  $\bar{y}_g^N$  by respectively  $\Delta^E$  and  $\Delta^N$  results in an increase in an eligible individual's outcome by  $\frac{1}{2}(\theta_0^w \Delta^E + \theta_0^b \Delta^N)$ . This demonstrates that  $(\theta_0^w, \theta_0^b)$  quantifies the relative impact of within-group and between-group peer effects. Conversely, when  $\theta_0^w = \theta_0^b$ , the increase in an eligible unit's outcome depends linearly on the group composition, reflecting a stronger response to peers' average outcomes when they constitute a larger proportion of the group. For conciseness, the residual terms  $e_{ig}^E$  and  $e_{ig}^N$  capture all the individual, exogenous and correlated effects. Because our focus is on  $\theta_0^b$  and  $\theta_0^w$ , we do not express explicitly all these effects in our baseline model. One may think of these terms, for example, as

$$e_{ig}^E = \alpha_g^E + x'_{ig} \eta_0^E + z'_g \gamma_0^E + \varepsilon_{ig}^E$$

$$e_{ig}^N = \alpha_g^N + x'_{ig} \eta_0^N + z'_g \gamma_0^N + \varepsilon_{ig}^N$$

Two features of this baseline model have to be emphasized at this stage. First, eligibility to the treatment corresponds to the “identity” that matters when evaluating the heterogeneity of peer effects. In our Progresa application for instance, we evaluate how the action of a child coming from a poor (resp. relatively rich) family is impacted by the actions of her peers who also come from the same economic

background and by the actions of her peers coming from a different economic background. In Section 2.5, we consider a setting where identity is orthogonal to treatment eligibility. In the Progres application, for example, one may want to consider gender as a more relevant identity to model the complex social mechanisms regarding a child's decision to drop-out as boys may be more influenced by the actions of other boys than by the actions of girls. Second, by allowing for heterogeneous peer effects, we may uncover social mechanisms beyond the conventional LIM model, such as polarisation. This mechanism happens when  $\theta_0^b < 0$  and  $\theta_0^w > 0$ . For instance, a disruptive child might have a negative influence on his friends' attitudes towards school while prompting other children to exert more effort.

From Assumption 1, we can average respectively  $y_{ig}^E$  and  $y_{ig}^N$  among eligible (resp. non-eligible) units in group  $g$  to get

$$\begin{aligned}\bar{y}_g^E &= \theta_0^w s_g \bar{y}_g^E + \theta_0^b (1 - s_g) \bar{y}_g^N + \delta_0 D_g + \bar{e}_g^E \\ \bar{y}_g^N &= \theta_0^b s_g \bar{y}_g^E + \theta_0^w (1 - s_g) \bar{y}_g^N + \bar{e}_g^N\end{aligned}$$

After some development, we get the following reduced forms

$$\begin{aligned}\bar{y}_g^E &= \frac{\theta_0^b (1 - s_g)}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_g^E \\ &+ \frac{1 - \theta_0^w (1 - s_g)}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_g^E \\ &+ \delta_0 \left( 1 + s_g \cdot \frac{\theta_0^w - (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \right) D_g\end{aligned}\tag{2.4}$$

$$\begin{aligned}\bar{y}_g^N &= \frac{1 - \theta_0^w s_g}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_g^N \\ &+ \frac{\theta_0^b s_g}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_g^E \\ &+ \frac{\delta_0 \theta_0^b s_g}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} D_g\end{aligned}\tag{2.5}$$

Then, we specify the treatment assignment across groups.

**Assumption 2 (Randomized Experiment)** *The group level treatment  $D_g$  is randomly assigned, i.e.*

$$(\bar{e}_g^N, \bar{e}_g^E, s_g) \perp\!\!\!\perp D_g\tag{2.6}$$

Assumption 2 states that treatment is randomly assigned. In particular, groups receive the treatment independently of their share of eligible units. We also impose a restriction on the support of  $D_g$  conditional

on the share of eligible units  $s_g$ . Let  $\mathcal{S} \subseteq (0, 1]$  be the support of shares that are observed in the sample,

**Assumption 3 (Common Support)** For each  $s \in \mathcal{S}$ ,

$$0 < P(D_g = 1|s) < 1 \quad (2.7)$$

Assumption 3 states that for any share of eligible units that is observed in the population of reference groups, there exist some groups that are treated and some that are not.

## 2.2.2 Natural Experiment

The second type of designs we consider are natural experiments. It is a panel extension of the previous subsection. The  $G$  groups may be observed over  $T$  time periods indexed by  $t$ . Group  $g$  treatment status for period  $t$  is given by  $D_{gt}$ . Let  $E_{igt}$  be the eligibility status of individual  $i$  in group  $g$  at time  $t$ . For  $k \in \{E, N\}$ ,  $\bar{y}_{gt}^k = (n_{gt}^k)^{-1} \sum_{i=1}^{n_{gt}^k} y_{igt}^k$  where  $n_{gt}^k$  is the number of individuals whose "identity" is  $k$  at time period  $t$ .

**Assumption 1' (Linear-in-means model with panel data)** Within each group  $g$  in  $\{1, \dots, G\}$  present in an i.i.d panel, for each time period  $t \in \{1, \dots, T\}$ , the outcome of an individual  $i$  is defined as

$$y_{igt} = E_{igt}y_{igt}^E + (1 - E_{igt})y_{igt}^N$$

with

$$y_{igt}^E = \theta_0^w s_{gt} \bar{y}_{gt}^E + \theta_0^b (1 - s_{gt}) \bar{y}_{gt}^N + \delta_0 D_{gt} + e_{igt}^E \quad (2.8)$$

$$y_{igt}^N = \theta_0^b s_{gt} \bar{y}_{gt}^E + \theta_0^w (1 - s_{gt}) \bar{y}_{gt}^N + e_{igt}^N \quad (2.9)$$

Assumption 1' is the counterpart of Assumption 1 in a panel data context. Note that we rule out time dependence, in the sense that former own outcomes and former peers' outcomes do not directly influence current outcomes. As before, we can average the outcomes of eligible and non-eligible units in group  $g$  for each time period  $t$  and develop to get the following reduced forms

$$\begin{aligned} \bar{y}_{gt}^E &= \frac{\theta_0^b (1 - s_{gt})}{1 - \theta_0^w + s_{gt} (1 - s_{gt}) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_{gt}^N \\ &+ \frac{1 - \theta_0^w (1 - s_{gt})}{1 - \theta_0^w + s_{gt} (1 - s_{gt}) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_{gt}^E \\ &+ \delta_0 \left( 1 + s_{gt} \cdot \frac{\theta_0^w - (1 - s_{gt}) [(\theta_0^w)^2 - (\theta_0^b)^2]}{1 - \theta_0^w + s_{gt} (1 - s_{gt}) [(\theta_0^w)^2 - (\theta_0^b)^2]} \right) D_{gt} \end{aligned} \quad (2.10)$$

$$\begin{aligned}
\bar{y}_{gt}^N &= \frac{1 - \theta_0^w s_{gt}}{1 - \theta_0^w + s_{gt}(1 - s_{gt}) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_{gt}^N \\
&+ \frac{\theta_0^b s_{gt}}{1 - \theta_0^w + s_{gt}(1 - s_{gt}) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_{gt}^E \\
&+ \frac{\delta_0 \theta_0^b s_{gt}}{1 - \theta_0^w + s_{gt}(1 - s_{gt}) [(\theta_0^w)^2 - (\theta_0^b)^2]} D_{gt}
\end{aligned} \tag{2.11}$$

For the rest of the paper, we suppose that  $T = 2$ . However, the natural experiment setting we consider could potentially be extended to multiple periods. In this setting, treatment assignment may not be random but we make other restrictions on the distribution of the data.

**Assumption 2' (Stable Shares)** For all  $(g, t) \in \{1, \dots, G\} \times \{1, 2\}$ ,

$$s_{gt} = s_g$$

Assumption 2' states that, for each group, the share of eligible units does not change from period 1 to period 2. It implies in particular that the composition of groups is not affected by the treatment. This assumption is thus credible in designs where the time span between the 2 periods is short.

**Assumption 3' (Treatment Distribution)**

$$D_{g1} = 0 \text{ a.s. and for all } s \in \mathcal{S}, 0 < P(D_{g2} = 1|s) < 1 \tag{2.12}$$

Assumption 3' states, first, that no group is treated at the initial period. Second, for all the values of shares of eligible units that are observed in the population, there are some treated groups and some control groups.

**Assumption 4' (Conditional Common Trends)** For any  $k \in \{NE, E\}$ ,

$$E[\bar{e}_{g2}^k - \bar{e}_{g1}^k | s_g, D_{g2} = 1] = E[\bar{e}_{g2}^k - \bar{e}_{g1}^k | s_g, D_{g2} = 0] \tag{2.13}$$

Assumption 4' is a common trend assumption conditional on the share of eligible units. Intuitively, we form pairs of treated and control groups with the same share of eligibles. For each pair, we assume that, in the absence of the treatment, the average change in the aggregate outcome among eligible units in treated groups would have been the same as the average change in the aggregate outcome among eligible units in control groups. We make the same assumption regarding the average change in aggregate outcome among non-eligible units. Note that the value of the conditional trend may be different for the eligible and the non-eligible sub-populations.

## 2.3 Identification

In this section, we show how one can identify the different parts that contribute to the overall effect of the policy.

### 2.3.1 Intuition

Figure 2.1 illustrates how the initial shock (the treatment received by the eligibles) propagates to the whole group through social interactions.  $\delta_0$  is the *direct effect* of the treatment, loosely defined as the first step in the causal chain (the effect of the treatment on the eligibles before any social interaction takes place).  $\delta_0\theta_0^b$  is the *indirect effect* of the treatment, loosely defined as the second step in the causal chain (the response of the non-eligibles to changes in the eligibles' outcomes before any other social interaction takes place).  $M^E(s)$  and  $M^N(s)$  are the *population multipliers* for the eligible and non-eligible individuals, loosely defined as the third step in the causal chain (the propagation of the initial shock through social interactions). With these definitions, the direct effect and the indirect effect are independent of population structure, while the intensity of the population multipliers depends on the population structure, as illustrated in Figure 2.2. Indeed,  $s$  influences  $M$  in two ways:

- Through the initial shock: the strength of the shock is proportional to the share of eligible individuals
- Through the propagation: the shock is amplified by social interactions in a non-linear way that depends on the relative magnitudes of  $\theta_0^b$  and  $\theta_0^w$ . Intuitively, if  $\theta_0^w > \theta_0^b$  (within peer effects are more intense than between peer effects), the amplification is stronger in homogeneous groups (low  $s$  and high  $s$ ) than in mixed groups ( $s \approx \frac{1}{2}$ ). Conversely, if  $\theta_0^w < \theta_0^b$ , the amplification is weaker in homogeneous groups than in mixed groups.

As formally shown in the next section, the treatment effects and the population multipliers for the eligible and the non-eligible individuals can be written as follows:

$$\tau^E(s) = \delta_0 M^E(s) \text{ and } \tau^N(s) = \delta_0 \theta_0^b M^N(s)$$

and

$$M^E(s) = 1 + sP^E(s) \text{ and } M^N(s) = sP^N(s)$$

where  $P^E(s)$  and  $P^N(s)$  capture the strength of the propagation in the eligible and non-eligible sub-populations. These functions have a U-shape when  $\theta_0^w > \theta_0^b$  and an inverted-U shape when  $\theta_0^w < \theta_0^b$ . They are different in the eligible and non-eligible sub-populations because their situations are not symmetric: e.g. a low  $s$  implies a lot of “between” interactions for eligibles and a lot of “within” interactions for non-eligibles. By contrast, in the homogeneous case where  $\theta_0^w = \theta_0^b = \theta$ , these functions are identical

for the eligibles and the non-eligibles, and they do not depend on  $s$ : we have  $P^E = P^N = \frac{1}{1-\theta}$ . As a consequence, the population multiplier is linear in  $s$ .

The shapes of  $\tau^E(s)$  and  $\tau^N(s)$  are informative about  $\delta_0, \theta_0^b, \theta_0^w$ . As an illustration, Figure 2.3 plots  $\tau^E(s)$  and  $\tau^N(s)$  as a function of  $s$  in the case when  $\theta_0^w > \theta_0^b$  and in the case when  $\theta_0^w < \theta_0^b$ . We can get an intuition of the identification by looking at the limits:

$$\begin{array}{ccc} \tau^E(s) \rightarrow \delta_0 & \tau^E(s) \rightarrow \frac{\delta_0}{1-\theta_0^w} & \tau^N(s) \rightarrow \frac{\delta_0 \theta_0^b}{1-\theta_0^w} \\ s \rightarrow 0 & s \rightarrow 1 & s \rightarrow 1 \end{array}$$

First, looking at eligibles in groups with a very low fraction of eligibles is informative about the direct effect. Second, looking at eligible individuals in groups with a very high fraction of eligible individuals is informative about “within” peer effects. Third, looking at non-eligibles in groups with a very high fraction of eligibles is informative about “between” peer effects. In practice, we observe few eligibles in groups with low  $s$  and few non-eligibles in groups with high  $s$ . That is why our procedure exploits the whole distribution of  $s$ , and not only the limits. The next section derives the formulas for  $\tau^E(s)$  and  $\tau^N(s)$  and discusses more formally the identification in the case of randomized experiments and in the case of natural experiments.

## 2.3.2 Randomized Experiment

This subsection presents how the treatment effects and population multipliers can be recovered in the randomized experiment setting described in section 2.1.

**Proposition 1** *Provided Assumptions 1, 2 and 3 hold and all the mentioned conditional expectations are well-defined, we have*

$$\begin{aligned} \tau^E(s) &= \text{E} [\bar{y}_g^E | s_g = s, D_g = 1] - \text{E} [\bar{y}_g^E | s_g = s, D_g = 0] \\ &= \text{E} \left[ \frac{D_g - \text{Pr}(D_g = 1)}{\text{Pr}(D_g = 1)(1 - \text{Pr}(D_g = 1))} \cdot \bar{y}_g^E \middle| s_g = s \right] \end{aligned} \quad (2.14)$$

$$= \delta_0 \left( 1 + s \cdot \frac{\theta_0^w - (1-s) [(\theta_0^w)^2 - (\theta_0^b)^2]}{1 - \theta_0^w + s(1-s) [(\theta_0^w)^2 - (\theta_0^b)^2]} \right) \quad (2.15)$$

and

$$\begin{aligned} \tau^N(s) &= \text{E} [\bar{y}_g^N | s_g = s, D_g = 1] - \text{E} [\bar{y}_g^N | s_g = s, D_g = 0] \\ &= \text{E} \left[ \frac{D_g - \text{Pr}(D_g = 1)}{\text{Pr}(D_g = 1)(1 - \text{Pr}(D_g = 1))} \cdot \bar{y}_g^N \middle| s_g = s \right] \end{aligned} \quad (2.16)$$

$$= \frac{\delta_0 \theta_0^b s}{1 - \theta_0^w + s(1-s) [(\theta_0^w)^2 - (\theta_0^b)^2]} \quad (2.17)$$

Hence,

$$P^E(s) := \frac{\theta_0^w - (1-s) [(\theta_0^w)^2 - (\theta_0^b)^2]}{1 - \theta_0^w + s(1-s) [(\theta_0^w)^2 - (\theta_0^b)^2]} \quad (2.18)$$

$$P^N(s) := \frac{1}{1 - \theta_0^w + s(1-s) [(\theta_0^w)^2 - (\theta_0^b)^2]} \quad (2.19)$$

**Proof:** see Appendix

Proposition 1 provides closed-form solutions for the total average treatment effects on the average outcome within the eligible (respectively non-eligible) sub-population, for a given share of eligible individuals  $s$ , as well as for the strength of the propagation shock. The difference of conditional expectations between treated and untreated groups can be expressed as a single conditional moment of a weighted outcome, as shown by equations (2.14) and (2.16), whose weights are based on a transformation process à la Abadie (2005). We rewrite the treatment effects in this way because we want to replace two conditional expectations by one. This will allow us to combine the different conditional expectations into a single unconditional expectation, as explained in the estimation section. This alternative formulation leads to some natural conditional moment conditions, as expressed by the following corollary.

**Corollary 1 (Conditional Moment Conditions - Randomized Experiment)** *Let  $I$  be an interval on  $\mathbb{R} \setminus \{0\}$  and  $\lambda_0 = (\delta_0, \theta_0^w, \theta_0^b) \in \Theta := I \times (-1, 1) \times (-1, 1)$ , be the true value of the parameters. For any  $s \in (0, 1)$ , provided Assumptions 1, 2 and 3 hold and all the mentioned conditional moments are well-defined,*

$$\mathbb{E} [u^{R,E}(\bar{y}_g^E, \bar{y}_g^N, D_g, s_g; \lambda_0) | s_g = s] = \begin{pmatrix} \mathbb{E} [u^{R,E}(\bar{y}_g^E, D_g, s_g; \lambda_0) | s_g = s] \\ \mathbb{E} [u^{R,NE}(\bar{y}_g^N, D_g, s_g; \lambda_0) | s_g = s] \end{pmatrix} = 0 \quad (2.20)$$

where for all  $\lambda \in \Theta$ ,

$$u^{R,E}(\bar{y}_g^E, D_g, s_g; \lambda) = \omega^R(D_g) \bar{y}_g^E - \frac{\delta(1 - \theta^w(1 - s_g))}{1 - \theta^w + s_g(1 - s_g) [(\theta^w)^2 - (\theta^b)^2]}$$

$$u^{R,NE}(\bar{y}_g^N, D_g, s_g; \lambda) = \omega^R(D_g) \bar{y}_g^N - \frac{\delta \theta^b s_g}{1 - \theta^w + s_g(1 - s_g) [(\theta^w)^2 - (\theta^b)^2]}$$

$$\text{and } \omega^R(D_g) = \frac{D_g - P(D_g = 1)}{P(D_g = 1)(1 - P(D_g = 1))}$$

**Proof:** It is an immediate consequence of Proposition 4 since the right-hand sides of equations (2.15) and (2.17) are functions of  $s$ .

### 2.3.3 Natural Experiment

In this subsection, we show that similar results are obtained in the natural experiment design. In the following, for all  $k \in \{E, N\}$ , we use the conventional notation for first differenced variables:

$$\Delta \bar{y}_g^k = \bar{y}_{g2}^k - \bar{y}_{g1}^k$$

**Proposition 2** *Suppose Assumptions 1', 2', 3', and 4' hold and that all the mentioned conditional expectations are well-defined. Then, we have*

$$\begin{aligned} \tau^E(s) &= \mathbb{E} [\Delta \bar{y}_g^E | s_g = s, D_g = 1] - \mathbb{E} [\Delta \bar{y}_g^E | s_g = s, D_g = 0] \\ &= \mathbb{E} \left[ \frac{D_{g2} - \Pr(D_{g2} = 1 | s_g = s)}{\Pr(D_g = 1 | s_g = s)(1 - \Pr(D_g = 1 | s_g = s))} \cdot \Delta \bar{y}_g^E | s_g = s \right] \end{aligned} \quad (2.21)$$

$$= \delta_0 \left( 1 + s \cdot \frac{\theta_0^w - (1-s) [(\theta_0^w)^2 - (\theta_0^b)^2]}{1 - \theta_0^w + s(1-s) [(\theta_0^w)^2 - (\theta_0^b)^2]} \right) \quad (2.22)$$

and

$$\begin{aligned} \tau^N(s) &= \mathbb{E} [\Delta \bar{y}_g^N | s_g = s, T_g = 1] - \mathbb{E} [\Delta \bar{y}_g^N | s_g = s, D_g = 0] \\ &= \mathbb{E} \left[ \frac{D_{g2} - \Pr(D_{g2} = 1 | s_g = s)}{\Pr(D_{g2} = 1 | s_g = s)(1 - \Pr(D_{g2} = 1 | s_g = s))} \cdot \Delta \bar{y}_g^N | s_g = s \right] \end{aligned} \quad (2.23)$$

$$= \frac{\delta_0 \theta_0^b s}{1 - \theta_0^w + s(1-s) [(\theta_0^w)^2 - (\theta_0^b)^2]} \quad (2.24)$$

Proposition 2 is the counterpart of Proposition 1 in the natural experiment setting. There are two main differences. First, the outcomes of interest here are the first differenced aggregate outcomes within the eligible and non-eligible subpopulations. Second, the weighting function  $\omega^{DiD}$  is now a function of both the treatment assignment variable and the share of eligible individuals in the group. In this design, treatment assignment is allowed to be correlated with the share of eligible units. Once again, immediate conditional moment conditions arise, as stated by the following corollary.

**Corollary 2 (Conditional Moment Conditions - Natural Experiment)** *Let  $I$  be an interval on  $\mathbb{R} \setminus \{0\}$ . Let  $\lambda_0 = (\delta_0, \theta_0^w, \theta_0^b)$  be the true value of the parameters with  $\lambda_0 \in \Theta := I \times (-1, 1) \times (-1, 1)$ . For any  $s \in (0, 1)$ , provided Assumptions 1', 2', 3' and 4' hold and all the mentioned conditional moments are well-defined,*

$$\mathbb{E} [u^{DiD}(\Delta \bar{y}_g^E, \Delta \bar{y}_g^N, D_{g2}, s_g; \lambda_0) | s_g = s] = \begin{pmatrix} \mathbb{E} [u^{DiD,E}(\Delta \bar{y}_g^E, D_{g2}, s_g; \lambda_0) | s_g = s] \\ \mathbb{E} [u^{DiD,NE}(\Delta \bar{y}_g^N, D_{g2}, s_g; \lambda_0) | s_g = s] \end{pmatrix} = 0 \quad (2.25)$$



where for all  $\lambda \in \Theta$ ,

$$u^{DiD,E}(\Delta\bar{y}_g^E, D_{g2}, s_g; \lambda) = \omega^{DiD}(D_{g2}, s_g)\Delta\bar{y}_g^E - \frac{\delta(1 - \theta^w(1 - s_g))}{1 - \theta^w + s_g(1 - s_g)[(\theta^w)^2 - (\theta^b)^2]}$$

$$u^{DiD,NE}(\Delta\bar{y}_g^N, D_{g2}, s_g; \lambda) = \omega^{DiD}(D_{g2}, s_g)\Delta\bar{y}_g^N - \frac{\delta\theta^b s_g}{1 - \theta^w + s_g(1 - s_g)[(\theta^w)^2 - (\theta^b)^2]}$$

$$\text{and } \omega^{DiD}(D_{g2}, s_g) = \frac{D_{g2} - P(D_{g2} = 1|s_g)}{P(D_{g2} = 1|s_g)(1 - P(D_{g2} = 1|s_g))}$$

**Proof:** immediate consequence from Proposition 2

### 2.3.4 Sufficient Conditions for Identification

The following proposition provides some sufficient conditions to ensure the identification of  $\lambda_0$ , the vector of the true parameters, quantifying the direct treatment effect and the within and between endogenous effects.

**Proposition 3 (Sufficient Conditions for Identification of  $\lambda_0$ )** *Provided  $\delta_0 \neq 0$  and  $\theta_0^b \neq 0$ , if Assumptions 1, 2 and 3 hold OR if Assumptions 1', 2', 3' and 4' hold and if one of the two following conditions is satisfied*

1. *there exist at least 2 shares  $s_1, s_2 \in (0, 1)$ ,  $s_1 \neq s_2$  that have positive probability mass and for which both  $\tau^N$  and  $\tau^E$  are well-defined*
2. *there is a continuum of shares  $I_s \subseteq (0, 1]$  such that  $P(s \in I_s) > 0$  and both  $\tau^N(s)$  and  $\tau^E(s)$  are well-defined, for any  $s \in I_s$*

then  $\lambda_0$  is the unique vector defined on  $\Theta$  that satisfies equation (2.20)

**Proof:** see Appendix

## 2.4 Estimation

In this section, we show how the vector of true parameters  $\lambda_0 := (\delta_0, \theta_0^w, \theta_0^b)$  can be estimated. Both the randomized and natural experiment settings lead to conditional moment restrictions of the form

$$E[u(Z_g; \lambda_0)|s_g] = 0$$

where  $u(Z_g; \lambda)$  is a 2-dimensional vector of known functions of the i.i.d random vector of observed variables  $Z_g$  and  $\lambda \in \Theta$ .

When we want to estimate these conditional moments, we may not have enough groups for a given value of  $s$ , especially when  $s$  is continuous. To overcome this issue, we combine the different conditional moments into a single unconditional moment using the law of iterated expectations. In particular, we consider the following unconditional moment restrictions

$$E[w(s)u(Z_g; \lambda_0)] = 0$$

where  $w$  is a  $k \times 2$  matrix, with  $k \geq 3$ , whose entries are functions of  $s_g$ . In the application, we set arbitrarily that, for any  $s \in (0, 1]$

$$w(s) = \begin{pmatrix} s & 0 \\ 0 & s\mathbf{1}(s < 1) \\ s^2 & 0 \\ 0 & s^2\mathbf{1}(s < 1) \end{pmatrix}$$

We picked this form as it provides 4 moments conditions to recover 3 coefficients, it is a vector of polynomials transformations of  $s$  and it puts a weight equal to 0 on groups whose share of eligible individuals is 1 for the moment conditions that are related to the non-eligible sub-population. We consider the class of Generalized Method of Moments (GMM) estimators of the form

$$\hat{\lambda} := \arg \min_{\lambda \in \Theta} \left( \frac{1}{G} \sum_{g=1}^G m(Z_g, \lambda, p) \right)' \hat{W} \left( \frac{1}{G} \sum_{g=1}^G m(Z_g, \lambda, p) \right) \quad (2.26)$$

with  $m(Z_g, \lambda, p) = \frac{1}{G} \sum_{g=1}^G w(s_g) \hat{u}(Z_g, \lambda, p)$  where  $\hat{u}$  is a 2-dimensional vector of known functions of observed characteristics  $Z_g$ , of a vector of parameters  $\lambda = (\delta, \theta^w, \theta^b)$  and of a propensity score  $p$ , which is function of  $s$ .  $\hat{W}$  is a non-negative definite, symmetric matrix such that  $\hat{W} \xrightarrow{p} W$ .

### 2.4.1 Randomized Experiment

In the randomized experiment setting, the propensity score function is supposed to be known and independent of the share of eligible units, so that for any  $s \in \mathcal{S}$ ,  $p(s) = P(D = 1)$ . In this particular design,

$$\begin{aligned} m^R(Z, \lambda, p) &= m^R(Z, \lambda) \\ &= w(s)u^R(Z, \lambda) \\ &= w(s) \left\{ \omega^R(D)\bar{Y} - \frac{\delta}{\phi(s, \lambda)} \begin{pmatrix} 1 - (1-s)\theta^w \\ s\theta^b \end{pmatrix} \right\} \end{aligned}$$

with  $\bar{Y} = (\bar{y}^E, \bar{y}^N)$ ,  $\omega^R(D) = \frac{D}{P(D=1)} - \frac{1-D}{1-P(D=1)}$  and

$$\phi(s, \lambda) = 1 - \theta^w + s(1-s) [(\theta^w)^2 - (\theta^b)^2]$$

Then, we have the following result

**Proposition 4 (GMM Estimator in the Randomized Setting)** *Let  $(Z_g)_{g=1, \dots, G}$  be an i.i.d sample of  $G$  groups, where  $Z_g = (\bar{y}_g^E, \bar{y}_g^N, D_g, s_g)$ . Supposing that*

1. *Assumptions 1, 2 and 3 hold and identification of  $\lambda_0$  is ensured*
2.  *$\Theta := I_1 \times [-K, K] \times [-K, K]$  where  $I_1$  is a compact subset of  $\mathbb{R}$  with  $0 \notin I_1$  and  $K \in (0, 1)$*
3.  *$E[|\bar{y}_g^E|^2] < +\infty$  and  $E[|\bar{y}_g^N|^2] < +\infty$*

Then

$$\hat{\lambda}^* = \begin{pmatrix} \delta^* \\ \theta^{w*} \\ \theta^{b*} \end{pmatrix} := \arg \min_{\lambda \in \Theta} \left( \frac{1}{G} \sum_{g=1}^G m^R(Z_g, \lambda) \right)' \hat{V}_G^{-1} \left( \frac{1}{G} \sum_{g=1}^G m^R(Z_g, \lambda) \right) \quad (2.27)$$

where  $\hat{V}_G \xrightarrow{p} V_0 := E[m^R(Z, \lambda_0)m^R(Z, \lambda_0)']$  is a consistent estimator of  $\lambda_0$ , whose asymptotic distribution is

$$\sqrt{G}(\hat{\lambda}^* - \lambda_0) \xrightarrow{d} \mathcal{N}\left(0, [M_0' V_0^{-1} M_0]^{-1}\right) \quad (2.28)$$

with  $\lambda_0 := (\delta_0, \theta_0^w, \theta_0^b)$  and  $M_0 := E\left[\frac{\partial}{\partial \lambda} m^R(Z, \lambda_0)\right]$

**Proof:** see Appendix

## 2.4.2 Natural Experiment

In the natural experiment setting, we consider

$$m^{DiD}(Z, \lambda, \hat{\mu}_D) = w(s) \left\{ \hat{\omega}^{DiD}(Z_g, \hat{\mu}_D) \bar{Y} - \frac{\delta}{\phi(s, \lambda)} \begin{pmatrix} 1 - (1-s)\theta^w \\ s\theta^b \end{pmatrix} \right\}$$

with

$$\hat{\omega}^{DiD}(Z, \hat{\mu}_D) = \frac{D_2}{\hat{\mu}_D(s)} - \frac{1-D_2}{1-\hat{\mu}_D(s)}$$

and for any  $s \in \mathcal{S}$ ,  $\hat{\mu}_D(s)$  is a non-parametric estimator of  $\mu_D(s) = E[D_2|s]$  based on a series logistic regression of  $D_2$  on polynomials in  $s$ , denoted  $(p_{k, K_g}(s))_{1 \leq k \leq K_g}$ . We make the following technical assumption

**Assumption 5' (Technical Conditions for Asymptotic Normality of the 2-step GMM)**

1.  $s$  is continuously distributed on  $\mathcal{S}$  and  $\inf_{x \in \mathcal{S}} f_s(x) > 0$ , with  $f_s$  the pdf of  $s$
2.  $E[|\bar{y}^E|] < +\infty$ ,  $E[|\bar{y}^N|] < +\infty$
3.  $s \mapsto E[|\bar{y}^E| | s]$  and  $s \mapsto E[|\bar{y}^N| | s]$  are bounded on  $\mathcal{S}$
4. The functions  $s \mapsto E[\bar{Y}D_2 | s]$ ,  $s \mapsto E[\bar{Y}(1 - D_2) | s]$  and  $s \mapsto E[D_2 | s]$  are four times continuously differentiable
5. The polynomials  $s \mapsto p_{k, K_G}(s)$ ,  $1 \leq k \leq K_G$ , are orthonormal on  $\mathcal{S}$  and  $K_G = C.G^\nu$  with  $\frac{1}{10} < \nu < \frac{1}{6}$

**Proposition 5 (GMM Estimator in the Natural Experiment Setting)** Let  $(Z_g)_{g=1, \dots, G}$  be an i.i.d sample of  $G$  groups, where  $Z_g = (\Delta \bar{y}_g^E, \Delta \bar{y}_g^N, D_{2g}, s_g)$ . Let

$$\hat{V}_G \xrightarrow{p} V_0 := E \left[ \{m^{DiD}(Z, \lambda_0, \mu_D) + \alpha(Z)\} \{m^{DiD}(Z, \lambda_0, \mu_D) + \alpha(Z)\}' \right]$$

with

$$\alpha(Z) = -w(s) \left( \frac{\mu_1(s)}{\mu_D(s)^2} + \frac{\mu_0(s)}{(1 - \mu_D(s))^2} \right) (D_2 - \mu_D(s))$$

where, for any  $s \in \mathcal{S}$ ,  $\mu_1(s) = E[\bar{Y}D_2 | s]$  and  $\mu_0(s) = E[\bar{Y}(1 - D_2) | s]$ . Let

$$\tilde{m}^{DiD}(Z, \lambda_0) = \frac{\partial}{\partial \lambda'} m^{DiD}(Z, \lambda_0, \mu_D)$$

which does not depend on the propensity score. Supposing that

1. Assumptions 1', 2', 3', 4' and 5' hold and identification  $\lambda_0$  is ensured
2.  $\Theta := I_1 \times [-K, K] \times [-K, K]$  where  $I$  is a compact subset of  $\mathbb{R}$  with  $0 \notin I$  and  $K \in (0, 1)$

Then

$$\hat{\lambda}^* := \arg \min_{\lambda \in \Theta} \left( \frac{1}{G} \sum_{g=1}^G m^{DiD}(Z_g, \lambda, \hat{\mu}_D) \right)' \hat{V}_G^{-1} \left( \frac{1}{G} \sum_{g=1}^G m^{DiD}(Z_g, \lambda, \hat{\mu}_D) \right) \quad (2.29)$$

is a consistent estimator of  $\lambda_0$  whose asymptotic distribution is

$$\sqrt{G}(\hat{\lambda}^* - \lambda_0) \xrightarrow{d} \mathcal{N} \left( 0, [M_0' V_0^{-1} M_0]^{-1} \right)$$

with  $\lambda_0 := (\delta_0, \theta_0^w, \theta_0^b)$  and  $M_0 := E[\tilde{m}^{DiD}(Z, \lambda_0)]$

**Proof :** see Appendix

Despite a non-parametric first stage, our estimator converges at parametric rate even in the natural experiment design. Note that this estimation procedure can be easily adapted to allow for the inclusion of covariates in the model, as explained in B.4.

## 2.5 Extensions

### 2.5.1 Identity-specific “within” and “between” influences

Our baseline model imposes that  $\theta_0^w$  and  $\theta_0^b$  are homogeneous for eligible and non-eligible units. Yet, one may want to allow for different “within” peer effects among eligible and among non-eligible individuals as well as asymmetric “between” influences between eligible and non-eligible individuals. We thus consider the more flexible scenario with two distinct “within” parameters,  $\theta_0^{w,E}$  and  $\theta_0^{w,N}$ , and two distinct “between” parameters,  $\theta_0^{b,E,N}$  and  $\theta_0^{b,E,N}$ , defined as follows

**Assumption 1\* (Linear-in-means model - 5 parameters)** *Within each group  $g \in \{1, \dots, G\}$  of the i.i.d sample, the outcome of individual  $i$  is defined as*

$$y_{ig} = E_{ig}y_{ig}^E + (1 - E_{ig})y_{ig}^N$$

with

$$y_{ig}^E = \theta_0^{w,E} s_g \bar{y}_g^E + \theta_0^{b,E,N} (1 - s_g) \bar{y}_g^N + \delta_0 D_{ig} + e_{ig}^E \quad (2.30)$$

$$y_{ig}^N = \theta_0^{b,N,E} s_g \bar{y}_g^E + \theta_0^{w,N} (1 - s_g) \bar{y}_g^N + e_{ig}^N \quad (2.31)$$

By allowing for these asymmetries in peer effects, new social mechanisms can be identified, such as the role model phenomenon. It would arise when, for instance,  $\theta_0^{b,N,E} > 0$  and  $\theta_0^{b,E,N} = 0$ . In B.2, we show how that our identification and estimation results can be easily adapted to this refinement. Although we demonstrate that our GMM estimator is asymptotically normal, our simulations show that the asymptotic variance is very large. We explain this imprecision by the difficulty the estimator has in distinguishing the “between” effect of the non-eligible individuals on the eligible individuals from the direct effect of the treatment.

### 2.5.2 Identity orthogonal to treatment eligibility

In this extension, we suppose that the identity that matters in social interactions is orthogonal to eligibility to treatment. In the context of Progresá, we may suppose that boys (resp. girls) are more influenced by

the actions of their male (resp. female) peers than by their female (resp. male) peers. Peers' gender is orthogonal to eligibility, which is only based on households' income. We consider the following LIM model

**Assumption 1† (Linear-in-means model - orthogonal identity)** *For a group  $g \in \{1, \dots, G\}$  of the i.i.d sample, the outcome of individual  $i$  is defined as follow*

$$y_{ig} = y_{ig}^M M_{ig} + y_{ig}^F (1 - M_{ig})$$

$$y_{ig}^M = \theta_0^{w,M} s_g^M \bar{y}_g^M + \theta_0^{b,M,F} (1 - s_g^M) \bar{y}_g^F + \delta_0 D_{ig} E_{ig} + e_{ig}^M \quad (2.32)$$

$$y_{ig}^F = \theta_0^{b,F,M} s_g^M \bar{y}_g^M + \theta_0^{w,F} (1 - s_g^M) \bar{y}_g^F + \delta_0 D_{ig} E_{ig} + e_{ig}^F \quad (2.33)$$

with  $M_{ig} \in \{0, 1\}$  equals 1 if the individual is from the "male" identity and  $E_{ig} \in \{0, 1\}$  equals 1 if the individual is eligible to the treatment.  $s_g^M := \bar{M}_g$ , i.e. it is the share of "male" individuals in the peer group.

In this design, both "male" and "female" individuals can be treated. In B.3, we show that our identification and estimation strategies can be adapted to assess the different components of the total treatment effect, within each "community". The main adjustment resides in the need to compare treated and control groups that have the same proportion of "male", eligible "male" and eligible "female" individuals.

### 2.5.3 Including Covariates

One may want to include covariates in the model to improve the plausibility of the common trends assumption in the case of a natural experiment setting. In B.4, we provide alternative moment conditions that are based on a new conditional common trends assumption. Intuitively, we compare treated and control groups that have the same share of eligible units and that experience the same evolution of their observed covariates from period 1 to 2. Then, we assume that, in the absence of the treatment, the average change in the aggregate outcome among eligible units in treated groups would have been the same as the average change in the aggregate outcomes among eligible units in control groups. We make the same assumption regarding the change in group average outcome among non-eligible units.

## 2.6 Application: Conditional cash transfers in Mexico (Progresa)

Progresa is a conditional cash transfer (CCT) program introduced in Mexico in 1997 and aimed at developing the human capital of poor households. The program conditioned cash payments on children regularly attending school and health checkups at clinics. The program was means tested, with a two-step targeting procedure. First, the poorest villages were identified using socio-economic characteristics in census data. Second, within a village, the poorest households were identified using a specific survey

collecting data on assets and demographic composition. Only the poorest households were eligible for the program.

Progresa has been widely studied for two reasons. First, it was one of the earliest CCT implemented at a large scale; since then, similar programs spread around the world. Second, the implementation of the program was experimental during the first 18 months. In the spring 1998, among the 506 poorest villages, 320 were randomly chosen to participate in the program and eligible households started receiving transfers. The program was extended to the 186 control villages at the end of 1999, and then gradually to a larger set of villages. The randomization was exploited by several studies to estimate the short-term impact on education (see Parker and Todd (2017) for a review). Most studies focus on eligible households. A few others also estimate the treatment effect on the non-eligibles, taking advantage of the fact that post-program evaluation surveys interviewed all households, including non-eligible households, in treated and control villages.

Of particular interest for us is a study by Lalive and Cattaneo (2009) who use Progresa as a partial-population design to study peer effects in school attendance. They define the reference group as all children living in the same village who have reached the same grade level. Using the interaction between treatment and share of kids from eligible households as an instrument for average group attendance in October 1998, they estimate an endogenous peer effect parameter of  $\theta = 0.5$  (95% CI=[0;1]). This implies that increasing the average attendance in a child's group by 10 percentage points will raise her likelihood to attend school by 5 percentage points. The population multiplier is equal to  $\frac{1}{1-\theta}s = 2s$ . Furthermore, they estimate a direct effect of  $\delta_0 = 0.03$  (95% CI=[0;0.06]), which represents a 4% increase compared to the control mean of 0.69.

We use the same data and implement our procedure allowing for heterogeneous  $\theta$ . Figure 2.4 plots the distribution of  $s$ , by treatment status, supporting the validity of the common support condition. Table 2.1 provides some descriptive statistics on peer groups, that support the correct randomization of the treatment. In total, we observe 1,798 peer groups, in 496 villages. Based on our simulations, the number of observations is enough to provide us consistent estimates of the parameters and of their associated standard errors in our baseline scenario. However, it is too small to add several pre-program control variables, as in Lalive and Cattaneo (2009), or to consider a model with four distinct parameters for the endogenous peer effects. We only add the number of kids in the peer group in a specification, as we believe it is more reasonable to compare groups of the same size. Estimates are reported in Table 2.2.

We find a "between" peer effect of  $\hat{\theta}^b = 0.31$ , relatively close to the homogeneous  $\theta$  estimated by Lalive and Cattaneo (2009). This makes sense because their instrumental variable strategy exploits the response of non-eligible children to the introduction of Progresa in the peer group. In other words, their

identification of  $\theta$  comes from “between” interactions. However, our estimate is noisy and we cannot reject the null hypothesis that the poor children have no influence on non-poor children. We get an estimate of the “within” peer effects  $\hat{\theta}^w$  that is equal to 0.24. Once again, we cannot reject the null that students are not influenced by their peers’ actions. Our estimates suggest that conformism is the main social mechanism at play in this context. For example, suppose that one poor child drops out of school in a group of 10 children (the average number of peers in our sample). In the rest of the group, the likelihood that a child attends school decreases by  $\frac{1}{10} \times 0.24 = 2.4$  percentage points among the poor.<sup>2</sup> This is the effect *ceteris paribus*, looking at the response of a given child and holding the attendance of other children constant. These magnitudes can be compared with the direct effect of Progresá. We find that  $\hat{\delta} = 0.048$ .

Our estimate of  $\delta_0$  is greater than the estimate of Lalive and Cattaneo (2009) (although it belongs to their confidence interval). However, it is insignificant while Lalive and Cattaneo (2009)’s not. This can potentially be explained by the fact that we include all their control variables in our specification. The fact that our estimate is greater than Lalive and Cattaneo (2009)’s may suggest that, in this context, the homogeneity assumption leads to an amplification bias. Our explanation is the following:  $\theta$  is estimated through “between” interactions; since  $\hat{\theta}^b > \hat{\theta}^w$ , the homogeneity assumption leads to overestimating the strength of peer effects, and consequently to overestimating the total effect when  $s$  is very high and “within” interactions matter a lot; therefore, the procedure underestimates  $\delta_0$  in order to fit the data (recall that  $\tau^E(s = 1) = \frac{\delta_0}{1 - \theta_0^w}$ ).

In terms of policy evaluation, Figure 2.5 represents the total effects and the population multipliers as a function of  $s$ . Figure (a) plots the estimates of  $\tau^E$  and  $\tau^N$ . In groups with a very low share of poor children ( $s \rightarrow 0$ ), Progresá raises school attendance by 4.7 percentage points for the (very rare) poor children and this is not enough to trigger any response by non-poor children. As the fraction of poor increases, the effects on poor and non-poor increase in a slightly concave way. In groups with a very high share of poor children ( $s \rightarrow 1$ ), Progresá raises school attendance by 6.2 percentage points for poor children and by 2 percentage points for the (very rare) non-poor children. The population multipliers are equal to  $\frac{1}{1 - \theta^w} = 1.32$  when  $s = 1$ . The dotted line plots the total effect of the policy:  $s\tau^E + (1 - s)\tau^N$ . Magnitudes range from 0 when  $s = 0$  to 6.2 percentage points when  $s = 1$ . The total effect for the average  $s$  in our sample (roughly 70%) is equal to 5.9 p.p. for eligible children and 1.4p.p. for non-eligible children. We conclude that the policy effect is mildly magnified by peer effects.

To better understand why the total effects are slightly concave, Figures (b) and (c) plot the population

<sup>2</sup>In this example, the total effect is independent of  $s$  because two effects cancel out: when  $s$  is large, (i) a change in the average attendance of poor children matters more, and (ii) the average attendance of poor children changes less in response to one poor dropping out. Going back to structural equations, the change in  $y_i^E$  is equal to  $\theta_0^w s \frac{-1}{10s}$  and the change in  $y_i^N$  is equal to  $\theta_0^b s \frac{-1}{10s}$ .



multipliers,  $M(s)$ , for the non-eligibles and for the eligibles, respectively. Recall that the population multipliers are the products of two components: the first one,  $s$ , captures the strength of the initial shock; the second one,  $P(s)$ , may be not linear in  $s$  and captures the propagation of the shock. In the case of Progresa,  $P(s)$  has an almost linear shape because  $\hat{\theta}^w$  and  $\hat{\theta}^b$  are not very different. This application does not allow us to fully illustrate the potential of our methodology.

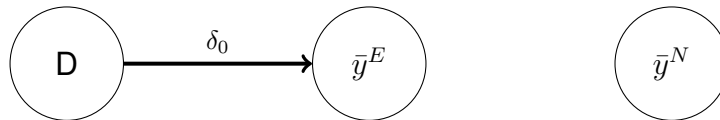
## 2.7 Conclusion

We propose a simple methodology to estimate heterogeneous endogenous peer effects using partial population experiments. We discuss the case of randomized experiments and differences-in-differences. The procedure requires the following conditions: (i) there is no direct effect of the treatment of the non-eligibles; (ii) there is sufficient variation in the share of eligibles across groups and common support in the distribution by treatment status; (iii) there is a sufficient number of groups. Intuitively, we match treated and control groups with a similar share of eligibles. For each pair, we estimate the treatment effect on the eligibles and the treatment effect on the non-eligibles. The relationship between the treatment effects and the share of eligibles is informative about the propagation of the initial shock through peer effects. Our methodology provides an estimate of (i) the endogenous peer effects “between” the sub-populations of eligibles and non-eligibles; (ii) the endogenous peer effects “within” each sub-population; (iii) the direct effect of the policy; (iv) the population multipliers. These estimates are useful in two ways. First, they help us understand how social influence works: who is influenced by whom, and how much. Second, we can compute the policy effect and decompose the total effect into a direct effect and an indirect effect generated by peers. Models with homogeneous peer effects strongly restrict the relationship between the policy effects and the share of eligibles, while our model with heterogeneous peer effects is less restrictive. The estimation procedure relies on GMM. It is easy to implement with standard statistical software and is not computationally intensive. Therefore, we think that our methodology has the potential to be used broadly by applied economists interested in peer effects and/or policy evaluation.

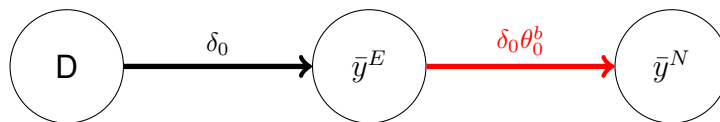
## Figures and Tables

Figure 2.1: Decomposition of Total Treatment Effects

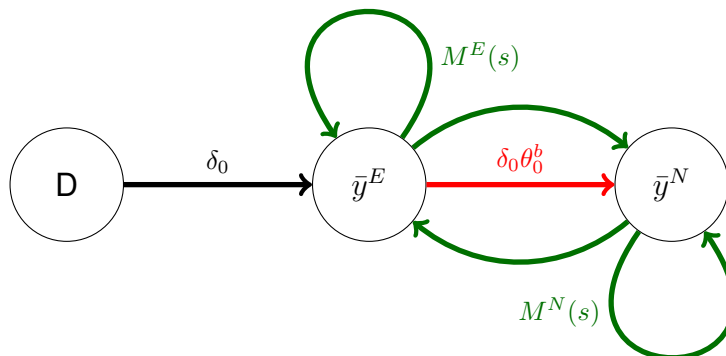
**Step 1** : direct treatment effect on eligible individuals



**Step 2** : indirect treatment effect of non-eligible individuals



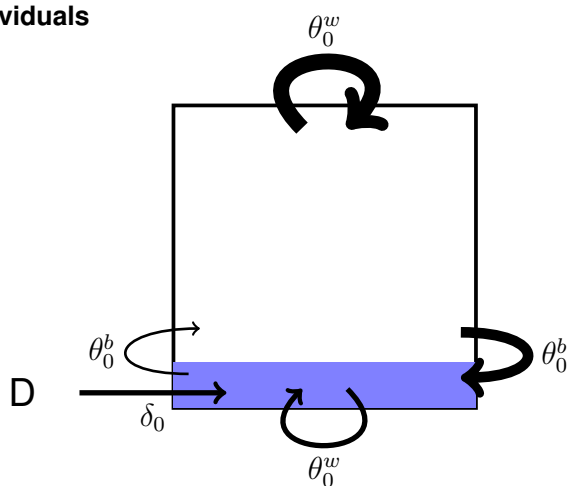
**Step 3** : amplification of initial effects through social interactions



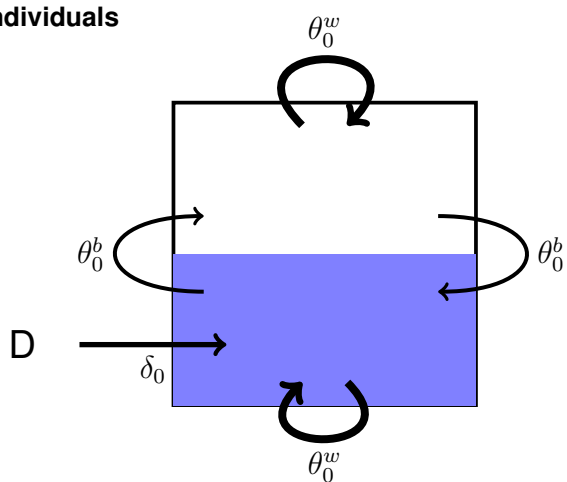
*Note:* The figure illustrates how the initial shock to eligible individuals propagates through peer effects. First, the eligible individuals respond to the treatment:  $y^E$  changes by  $\delta_0$  (direct effect). Second, the non-eligible individuals respond to the change in  $y^E$ :  $y^N$  changes by  $\delta_0 \theta_0^b$  (indirect effect). Third, these initial changes lead to a cascade process, in which eligible and non-eligible individuals keep responding to each other (in proportion to  $\theta_0^b$ ) and also respond to other members of their own sub-group (in proportion to  $\theta_0^w$ ).

Figure 2.2: Population Multipliers by Population Structure

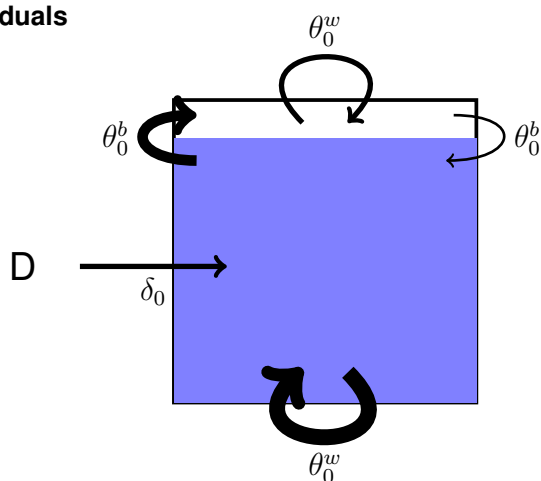
**Small share of eligible individuals**



**Medium share of eligible individuals**

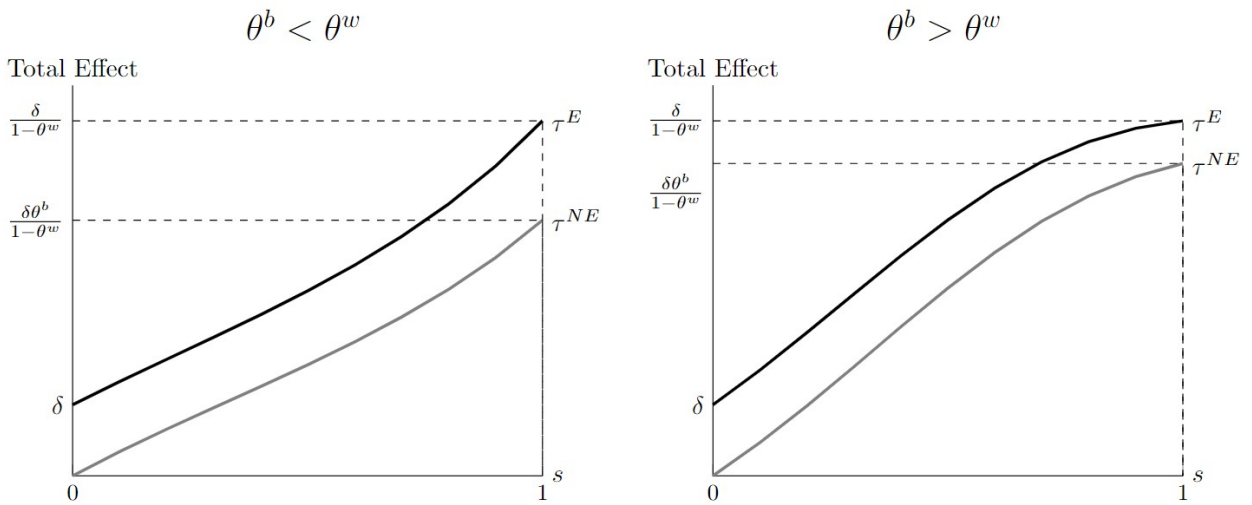


**High share of eligible individuals**



*Note:* The figure illustrates how the process intensity varies with the share of eligible individuals,  $s$ , represented in blue. First, the initial shock mechanically affects more individuals in the group when  $s$  is higher. Second, the propagation depends on  $s$ : arrows originating from the blue area are proportional to  $s$  while arrows originating from the white area are proportional to  $(1 - s)$ . In this illustration, we consider the case when  $|\theta_0^w| > |\theta_0^b|$  and we vary the size of the arrows accordingly (arrows within blue area and within white area are thicker than arrows between blue and white areas).

Figure 2.3: Treatment effects on eligibles and non-eligibles



Note: The figure plots the total effect on the eligibles  $\tau^E(s)$ , in black, and the total effect on the non-eligibles  $\tau^N(s)$ , in gray, as a function of  $s$ . The graph on the left illustrates the case when  $\theta^b < \theta^w$  and the graph on the right illustrates the case when  $\theta^b > \theta^w$ . As shown in Section 2.3,  $\tau^E(s) = \delta M^E(s)$  and  $\tau^N(s) = \delta\theta^b M^N(s)$ , where:

$$M^E(s) = 1 + sP^E(s), M^N(s) = sP^N(s), P^E(s) = \frac{\theta^w - (1-s)[(\theta^w)^2 - (\theta^b)^2]}{1 - \theta^w + s(1-s)[(\theta^w)^2 - (\theta^b)^2]}, P^N(s) = \frac{1}{1 - \theta^w + s(1-s)[(\theta^w)^2 - (\theta^b)^2]}$$

Figure 2.4: Progresa - Common Support Assumption

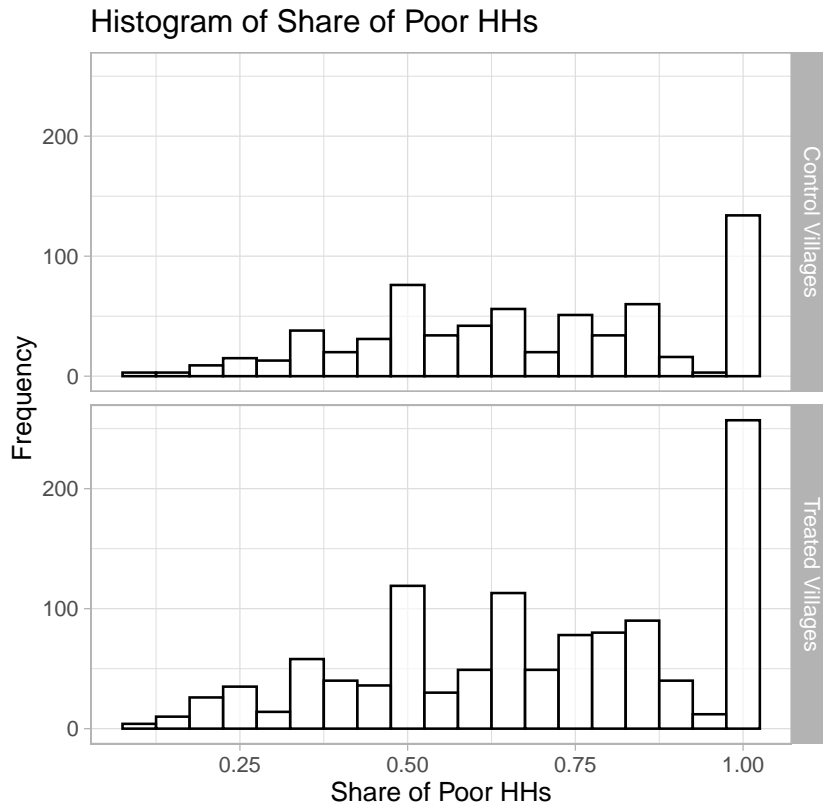
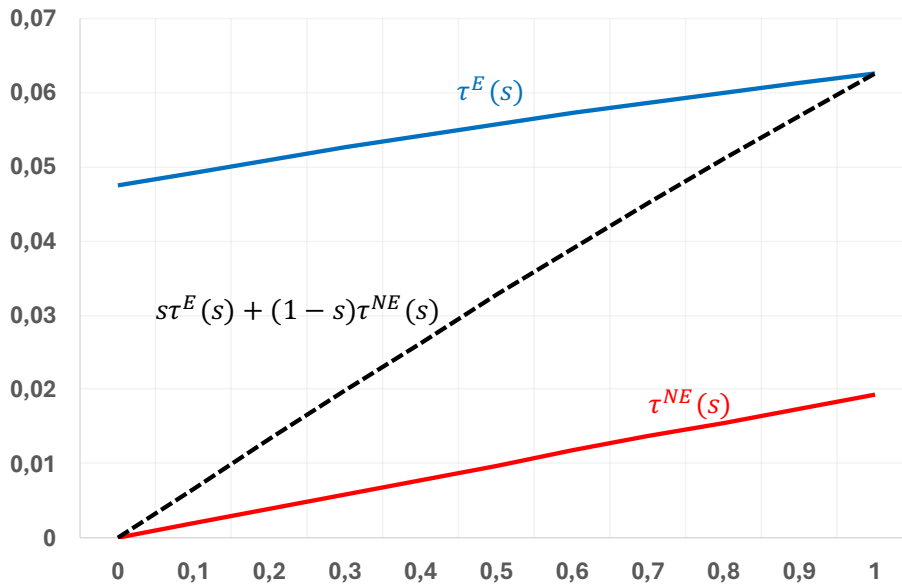
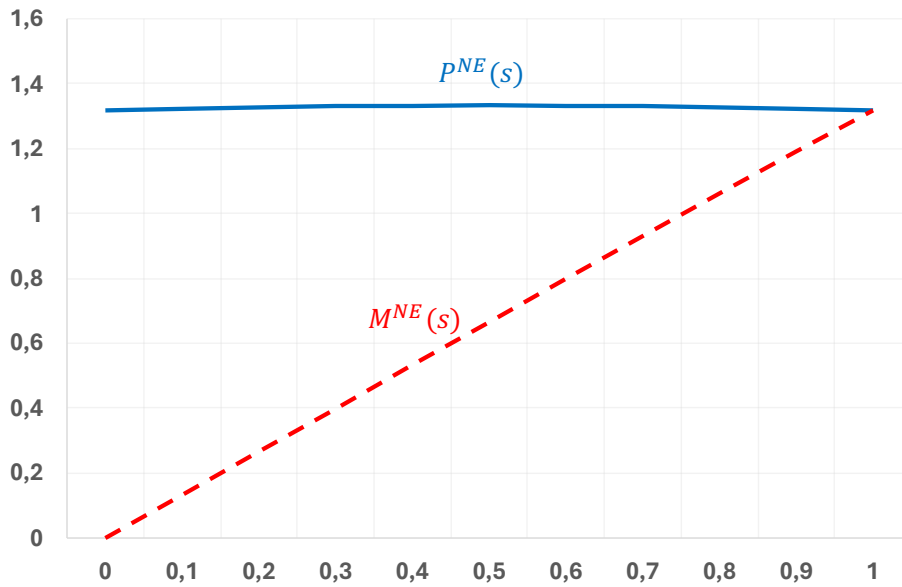


Figure 2.5: Results - Progresa



(a) Total effect on eligibles and non-eligibles



(b) Population multipliers for non-eligibles

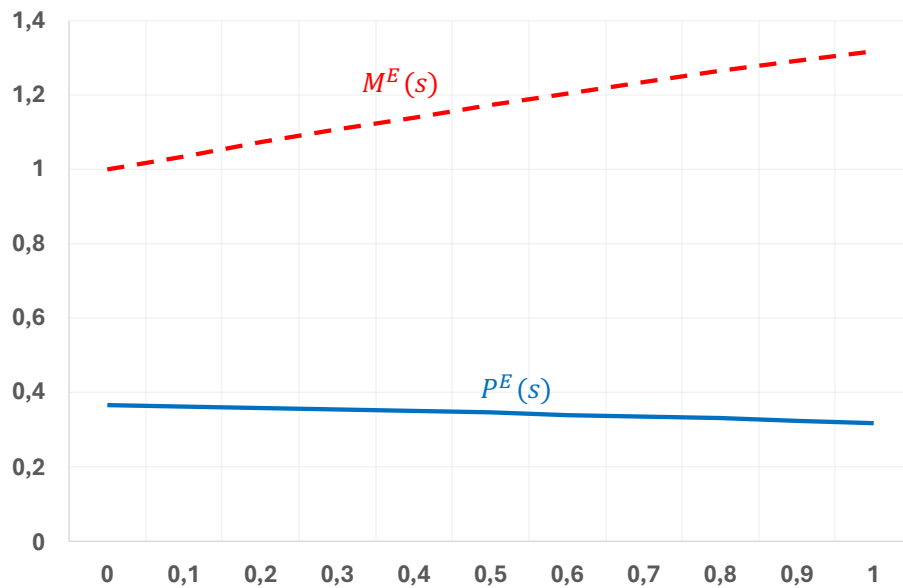
Note: The figure shows the total effects and population multipliers based on the Progresa estimates reported in Table 2.2.

Graph (a) plots the total effect on the eligibles  $\tau^E(s)$ , in black, and the total effect on the non-eligibles  $\tau^N(s)$ , in gray, as a function of  $s$ . The dotted line plots the total effect of the policy in the peer group.

Graph (b) plots the population multiplier  $M(s)$ , in dashed lines, and the propagation function  $P(s)$ , in solid lines. See formulas in the note below Figure 2.3.

Figure 2.5: Results - Progesa

(c) Population multipliers for eligibles



Note: The figure shows the total effects and population multipliers based on the Progesa estimates reported in Table 2.2. Graph (c) plots the population multiplier  $M(s)$ , in dashed lines, and the propagation function  $P(s)$ , in solid lines. See formulas in the note below Figure 2.3.

Table 2.1: Descriptive Statistics - Progesa

	Control Peer Groups	Treated Peer Groups
N	658	1140
Average Peer Group Size	8.72 (6.08)	8.31 (5.74)
Share of Poor	0.68 (0.24)	0.69 (0.24)
Share Grade 4	0.25 (0.43)	0.25 (0.43)
Share Grade 5	0.24 (0.43)	0.24 (0.43)
Share Grade 6	0.25 (0.43)	0.25 (0.43)
Share of Males	0.51 (0.23)	0.51 (0.23)

Note: this table describes the 658 peer groups (a grade in a village) that were treated and the remaining 1,140 that were not. On average, a peer group has 8.31 members in treated villages and 8.71 members in control villages.

Table 2.2: Estimation results - Progresa

	(1)	(2)
$\hat{\theta}^b$	0.463 (0.366)	0.308 (0.225)
$\hat{\theta}^w$	0.352 (0.592)	0.241 (0.635)
$\hat{\delta}$	0.037 (0.027)	0.048 (0.031)
Control	No	Yes
# peer groups	1,798	1,798
# villages	496	496

*Note:* The table reports the estimates of our three parameters of interest: the “between” peer effect parameter  $\theta^b$ , the “within” peer effect parameter  $\theta^w$ , and the direct effect  $\delta$ . We implement the procedure described in section 2.4 using Progresa evaluation data. Standard errors are clustered at the village level. Specification (1) is the basic specification while in Specification (2), we control for the number of kids in the peer group.

## Chapter 3

# Elderly Home Care Market and Spousal Informal Care Supply

This project develops and estimates a demand and supply model for the elderly personal and social home care in France. It is aimed at understanding how the provision of informal care by spouses of dependent elderly individuals responds to a change in the access to formal personal and social home care. On the demand side, households make decisions on their consumption, on the informal care provision by the spouse and on the use of formal personal care. On the supply side, two types of firms are modeled: home care and support services firms and self-employed care assistants. Despite this horizontal differentiation, all the services that are provided are homogeneous. The model is estimating using the *CARE-Ménages* survey and administrative matched employer-employee data. On the demand side, preliminary results suggest that, all other things equal, the informal care provision of male and older caregiving spouses is more responsive to a variation of the price of formal care. On the supply side, organizations providing formal personal and social home care appear to operate under constant marginal costs.

### 3.1 Introduction

Population aging, declining fertility rates, rising female workforce participation, and the diversification of family structures are reshaping how the care of the frail elderly is managed. Traditionally, families and close relatives have been the primary source of support for seniors who are no longer able to perform daily tasks on their own. However, these socio-demographic shifts are reducing the availability of informal family care while placing greater strain on the remaining caregivers. In response, developed countries—where these changes are the most pronounced—have seen the rise of a formal long-term care (LTC)



market. Services such as nursing homes and in-home care now cater to the growing demand for elderly care, tailored to the individual needs. The focus of this project is on the Personal and Social <sup>1</sup> Home (PSH) care sector, as it may serve as a substitute for informal family care. Seniors and their informal family carers determine how many hours of PSH care to purchase from PSH formal care providers and how much informal care to provide themselves. On the informal care side, the focus is on the spouses of dependent seniors, who have been regularly identified as the main informal caregivers (Bertogg and Strauss (2020) for review).

The goal of the project consists in understanding how a change in access to formal PSH care affects the provision of informal care by the caring spouses of dependent seniors. Public intervention plays a significant role in ensuring access to formal LTC. As the elderly population in need of care continues to grow, there is increasing debate about how to develop more financially sustainable policies to organize and fund this care. While the provision of care has been shown to deteriorate the informal carers' health (e.g. Hiel et al. (2015), Wagner and Brandt (2018), Barbosa et al. (2020)), improving access to formal PSH care could possibly meet these objectives. The services that home care assistants provide not only enhance the well-being of dependent seniors but can also ease the burden on their relatives. Being less affected, the caring spouses may have less costly healthcare needs and/or delay their entry into dependency. A robust evaluation of any reforms on the LTC market must consider their impacts on both the demand for and supply of professional care, as well as the well-being of informal caregivers.

The main contribution of the project is the development of an analytical framework to identify optimal policies for organizing the PSH care sector. A supply and demand model is built and estimated. To our knowledge, it is the first project that accounts for the supply. On the demand side, households consisting of a dependent senior and their spouse jointly decide on their consumption of formal care and the amount of informal care provided by the spouse. The senior's well-being is influenced by both types of care, which are treated as substitutes. Households are heterogeneous in observed characteristics (gender of the dependent elderly, social position, income, etc.). They are also heterogeneous in unobserved preferences. On the supply side, the model accounts for some horizontal differentiation of formal personal and social care providers. Households can choose between paying for personal and social care services from a firm or hiring a self-employed home care assistant. Whatever their choice, the services that are provided are supposed to be identical.

The identification and estimation of this supply and demand model exploit the specific features of the French market. While the national government defines the regulatory framework for personal and

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<sup>1</sup>Following the WHO/OECD definitions (Barber et al. (2021)), personal LTC services provide help with activities of daily living (ADL) such as eating, bathing, washing, dressing, getting in and out of bed, getting to and from the toilet and managing incontinence. Social LTC consists of assistance services that enable a person to live independently. It relates to help with instrumental ADL such as shopping, laundry, cooking, performing housework, managing finances, etc.

assistance care services, local authorities known as *départements* manage its operation. Any person over the age of 60 in France who experiences a loss of autonomy and who wants to remain in the community can partially finance their non-medical home care needs through the *Allocation Personnalisée d'Autonomie à domicile* (APA), a means- and needs-tested allowance, associated with a 50% tax credit on the remaining out-of-pocket expenses not covered by the APA. Each *département* manages the administration of the APA and determines funding based on its financial resources and political priorities, which leads to exogeneous variation in out-of-pocket costs for households across different *départements*. While the *département* is the relevant unit for market regulation, the market is defined at the *arrondissement* level—a smaller administrative division within each *département*—which typically spans a geographic area of up to 15 km in urban settings and 30 km in rural areas. This scale better reflects the operational range of personal care services. During the study period, market entry was unrestricted. Given that the production of PSH care services require minimal capital investment, that it relies on low-skilled labor and that there is no vertical differentiation, a market of perfect competition (within a type of provider) is assumed.

The structural parameters of the demand model are estimated using maximum likelihood, drawing on data from the *CARE-Ménages* survey, that was implemented in 2015. This representative survey of the French population over 60 living in the community provides detailed information on the human and financial assistance seniors receive for daily activities, whether through family or formal services. The preliminary estimated parameters can be used to assess the cross-elasticity of the provision of informal care with respect to the out-of-pocket price of formal PSH care. They suggest that, all other things being equal, male and older caregiving spouses are more responsive to a variation in the price of formal PSH care. The structural parameters of the supply model are estimated using two-stage least squares (2SLS), with data on firm-reported employee information from the French matched employer-employee administrative database, data on the prices set by the *départements* from the *CARE-Ménages*, and publicly available socio-demographic data at the *arrondissement* level. Preliminary results reveal that personal and social home care services operate under constant marginal costs.

## **Related Literature**

This project contributes to the literature on the substitution mechanisms between formal and informal care. Previous studies, such as Van Houtven and Norton (2004) and Bonsang (2009), have estimated substitution elasticities between these two types of care. However, they do not explore how policies that alter access to the formal care market impact both the consumption of formal care and the provision of informal care. More recent research has examined these mechanisms by exploiting exogenous shocks

that reduce the cost of professional PSH care, such as changes in public subsidies (Løken et al. (2017), Shen (2021), Massner and Wikström (2023)) or migration shocks that increase the supply of professional care assistants (Frimmel et al. (2020)). While these studies focus on specific shocks, they do not allow for ex-ante evaluations of alternative counterfactual policies—an approach this study aims to address. The closest related work is Perdrix and Roquebert (2020), which estimates a substitution elasticity between formal and informal PSH care in France using the same CARE survey and the decentralized nature of the French market. However, their analysis is limited to the price effect on seniors who already use formal care. In contrast, our study accounts for all seniors who require care, and includes households that do not consume formal care nor informal care. Additionally, unlike their work, our model incorporates heterogeneous personal care and assistance service providers, offering a more comprehensive view of the market dynamics. Finally, they don't have results on the specific informal care provision of spouses.

This project is also related to the literature evaluating the effects of different formal care subsidy policies through a structural demand model that accounts for the informal care provided by family members. This literature includes studies developing dynamic models of interactions between the senior's household and an adult child's household. At each period, these two households decide on their consumption and savings. The child decides on the allocation of their time between work, informal care, and leisure. Mommaerts (2015) and Ko (2022) use such models to determine the mechanisms leading to the observed low demand for long-term care insurance contracts. Barczyk and Kredler (2018) uses a model of this type to evaluate policies subsidizing informal or formal care. However, these studies, focusing on the American market, define formal care as being institutional care. The focus of this project is on PSH care sector, where the substitution mechanisms between formal and informal cares appear clearer. Chen and Lin (2022) is an exception for this type of model, as it explicitly considers personal and social home care services. However, this study aims at examining the effects of a decrease in the cost of formal care on the labor market participation of the senior's children, which is not the objective of our project. Unlike all studies in this field of literature, our project considers a static framework, and the partial equilibrium in PSH care market is modeled.

In terms of methodology, this project draws on the literature that models the childcare market, which shares many similarities with the Long-Term Care market. Professional carers can substitute parental care. Firms in this sector face minimal fixed costs and primarily rely on labor to provide their services. Public authorities intervene in the market through subsidy and regulatory policies that are highly similar to those in the LTC market. The key issues surrounding the organization of both markets are identical: reducing the burden on informal caregivers to increase their labor supply, promoting higher-quality care to enhance child development or improve the condition of the elderly, and limiting public expenditures.

The work most closely related to this project is the article by Berlinski et al. (2024), which models the childcare market within a static framework. In this model, parents decide on their level of consumption, the number of hours of informal care they provide, and their labor supply, taking into account their potential hourly wage and the availability of professional childcare. The market is assumed to be characterized by monopolistic competition with firms differentiated both vertically and horizontally. This project adapts this model to the PSH care market, accounting for the French specificities and available data. For instance, financial resources of a dependent elderly household are assumed to be exogenous, as household members are considered retired.

The remainder of the chapter is organized as follows. Section 2 explains the institutional setting. Section 3 presents the model. Section 4 describes the data that are used and provides a quantitative overview of the French PSH care market. Section 5 presents the empirical approach and the results. Section 6 discusses future developments.

## **3.2 Institutional Setting**

In France, the LTC sector and its financing can be analyzed along 2 dimensions: the type of care and the location where the care is provided. The first dimension separates medical care from personal care and social care. The location can either be the senior's home or in a specialized residential facility.

### **3.2.1 French LTC Providers**

The most prevalent type of residential care facilities in France are *Établissements d'Hébergement pour Personnes Âgées Dépendantes* (EHPADs). These medical nursing homes account for over 85% of the total capacity of residential facilities for elderly individuals. EHPADs provide a comprehensive range of services, combining personal, social, and medical care. The primary caregivers in EHPADs are paramedical staff, such as certified nursing assistants and registered nurses, who typically work in collaboration with a part-time physician and a psychologist. The majority of EHPAD residents suffer from a significant loss of autonomy (Bozio et al. (2016)). The remaining share of residential facilities offer hybrid models where elderly residents live independently in private accommodations but still benefit from collective personal, social care services, and communal amenities. For elderly individuals who remain at home, medical care is generally provided by independent self-employed nurses or nurses employed by a home-care nursing service. Although these nurses should technically only provide only medical care, they frequently perform personal care as well. Personal and social care services are primarily delivered by organizations called *Services d'Aide et d'Accompagnement à Domicile* (SAADs), whose

care assistant employees help with both Activities of Daily Living (ADLs) and Instrumental Activities of Daily Living (IADLs). SAADs supply approximately 75% of home care in France (Libault (2019)). Most SAADs are private non-profit organizations (60%), with the remainder comprising 11% public and 29% for-profit firms. SAADs operate under one of two legal regimes. The first one is the *autorisation* regime. It is granted by the *département*. The SAADs that are *autorisés* are priced every year by departmental councils, based on past year total costs and output. The price of an hour of care is set to ensure that the SAAD does not make any profit. A SAAD can otherwise fall under the *agrément* regime. It is granted by the representative of the national authority in the *département*. The SAADs that are *agrées* set their own prices, but can only change their rates within a regulated range once a year. As an alternative, elderly individuals can directly hire a home care worker. Self-employed home care assistants are free to set their own prices. Finally, regardless of whether they live at home or in residential facilities, 75% of dependent elderly individuals receive personal and social care from family caregivers, predominantly their spouses or children (Bozio et al. (2016)).

The typical trajectory of a French senior entering dependency begins with remaining at home. As their level of dependency increases and requires constant supervision, they typically transition into an EHPAD, where they spend their final years. However, this transition is contingent on the senior's ability to finance significant out-of-pocket expenses, which are not fully covered by public funding.

### **3.2.2 LTC Funding**

Whether care is provided at home or in a residential facility, all medical services related to loss of autonomy are fully covered by the national universal health insurance system, through the Long-Term Care scheme. In contrast, the funding of personal and social care is more complex. *Départements* (local administrative units) are responsible for the management of the medico-social care sectors. Metropolitan France is divided into 95 *départements*. Each *département* is governed by a council elected by universal suffrage which sets local policy for personal and social care, regulates SAADs, and manages the primary allowance for financing care for the elderly: the *Allocation Personnalisée d'Autonomie* (APA). The APA is a social allowance that is available to all individuals living in France that are aged 60 or over and who require assistance with daily activities. The APA can be used to finance personal and social care, either at home or in a residential facility. Since this project focuses exclusively on home care, only the at-home aspect of the APA will be discussed. Once an elderly person applies for the APA, the *département* sends a medico-social team to assess the applicant's level of dependency using the national AGGIR grid, which evaluates the individual's ability to perform 10 ADLs and 7 IADLs. Based on this evaluation, a dependency score (GIR score) is assigned, ranging from 1 (most dependent, requiring constant supervision) to 6

(least dependent, self-sufficient). Those classified in GIR levels 1–4 are eligible for the APA. As the eligible elderly individual remains at home, the medico-social team, in collaboration with the senior and family caregivers, creates a "care plan" that outlines the senior's specific needs. This plan specifies the number of hours of personal and social care required, as well as any needs for day-care services or home modifications. The plan also identifies the SAAD or self-employed home care assistant chosen to provide these services. The care plan is then converted into a monetary amount, based on the *département's* pricing grid, with a national ceiling for care costs that varies according to the individual's GIR score. Each *département* sets its own reference prices for services, often applying a standardized rate for self-employed personal carers and SAADs under the *agrément* regime. However, these reference prices may differ from actual market rates. For SAADs under the *autorisation* regime, the *département* sets individualized APA rates that match the actual cost of services. Finally, the senior's co-payment rate is calculated, based on their individualized income. In 2015, seniors with monthly incomes below €739 paid no co-payment, while those earning more than €2,945 had a co-payment rate of 0.9. Between these thresholds, the co-payment rate increases linearly with income. The senior's out-of-pocket cost is determined by multiplying the co-payment rate by the total care plan amount, and adding the difference between the market price and the reference price set by the *département* multiplied by the number of hours of care provided (which is equal to 0 for a SAAD *autorisé*). Seniors can also claim a 50% tax credit on total out-of-pocket expenses. If a senior is ineligible for the APA, they may seek financial assistance from their pension fund for home cleaning services, although these plans are generally less generous than the APA. In such cases, seniors are still eligible for a 50% tax credit on personal and social care expenses. The following formula summarizes the out-of-pocket price  $p_s^{OOP}$  an elderly individual faces for an hour of personal and social care provided by a SAAD or self-employed care assistant  $s$ , based on their level of disability and individualized monthly income  $R$ :

$$p_s^{OOP} = \begin{cases} \frac{1}{2} \times (c(R)t_s + (p_s - t_s)) & \text{if APA recipient \& care plan ceiling not reached} \\ \frac{1}{2} \times p_s & \text{otherwise} \end{cases} \quad (3.1)$$

where  $c(\cdot)$  denotes the co-payment rate, which is a function of the senior's monthly income,  $p_s$  represents the market price of service  $s$ , and  $t_s$  is the rate set by the *département* for service  $s$  under the APA care plan.

### 3.3 Model

The market of the elderly home personal and social care market is supposed to be composed of households and firms.

#### 3.3.1 Households

Households are composed of a frail elderly and their spouse. Together, they "jointly" decide the type of care provider they prefer (hiring a self-employed home care worker (*DE*) or purchasing services from a SAAD (*SP*)), the spouse's provision of informal care and the consumption of formal care. For a given mode of care  $s \in \{DE, SP\}$ , the household's utility function is defined as

$$U_s = h(F, I)^{\alpha_h} c^{\alpha_c} \ell^{\alpha_\ell} \quad (3.2)$$

where  $I$  denotes the daily amount of informal care provided,  $F$  the daily amount of formal care consumed,  $c$  the daily consumption of a numeraire and  $\ell$  the spouse's "free" time. It is supposed that  $\alpha_c + \alpha_\ell + \alpha_h = 1$ . The function  $h$  represents the amount of care received by the frail elderly. In the model, it takes the form

$$h(F, I) = F + I + 1 \quad (3.3)$$

Since the spouse and professional carer perform the same task, formal and informal care are considered as being perfect substitutes. A baseline unit of care is assumed to ensure that the total care received is never zero. The parameters  $\alpha_h$ ,  $\alpha_c$  and  $\alpha_\ell$  are parameters that determine the relative importance of care, consumption, and leisure in the utility function. Observed and unobserved heterogeneity in characteristics and in preferences are included into the  $\alpha$  parameters. More specifically,  $\alpha$  is such that

$$\begin{aligned} \alpha_\ell &= \frac{\exp(v_\ell)}{1 + \sum_{k \in \{\ell, c\}} \exp(v_k)} \\ \alpha_c &= \frac{\exp(v_c)}{1 + \sum_{k \in \{\ell, c\}} \exp(v_k)} \\ \alpha_h &= \frac{1}{1 + \sum_{k \in \{\ell, c\}} \exp(v_k)} \end{aligned}$$

where  $(v_\ell, v_c)$  follows a distribution  $\mathcal{N}(\mu, \Sigma)$  and whose mean parameter  $\mu$  may depend on household's observed characteristics. For instance, we may expect that the elderly's frailty is severe, the greater is  $\alpha_h$ . If the elderly's spouse is fairly old, we may expect two things. First, the valuation of free time by the spouse will be more important. Second, formal care will be relatively more efficient than informal care. We may thus expect a relatively greater  $\alpha_\ell$  and/or a relatively lower  $\alpha_c$ . The household has some daily

income  $R$ , which is supposed to be exogenous as both the elderly and the spouse are supposed to be retired. They face the following constraints:

$$p_s^{OOP} F + c = R \quad (3.4)$$

$$\ell + I = 18 \quad (3.5)$$

$$I \leq 16 \quad (3.6)$$

$$c > 0, I \geq 0, F \geq 0 \quad (3.7)$$

Equation (3.4) corresponds to the budget constraint. Equation (3.5) corresponds to the spouse's daily total time endowment. Constraint (3.6) corresponds to the maximal amount of daily informal care that can be provided. It is assumed that spouses have at least two hours of "free time" per day. (3.7) states that all quantities must be positive. From this utility function, the household's care "demand" function is

$$(I^D, F^D) = \left\{ \begin{array}{ll} (16, 0) & \text{if } \begin{array}{l} e^{v_\ell} \leq \frac{2}{17} \\ e^{v_c} \geq \frac{R}{17p_s^{OOP}} \end{array} \\ (0, 0) & \text{if } \begin{array}{l} e^{v_\ell} \geq 18 \\ e^{v_c} \geq \frac{R}{p_s^{OOP}} \end{array} \\ (16, F^*(16)) & \text{if } \begin{array}{l} e^{v_\ell} \geq \frac{2p_s^{OOP} e^{v_c}}{R + 17p_s^{OOP}} \\ e^{v_c} \leq \frac{R}{17p_s^{OOP}} \end{array} \\ (0, F^*(0)) & \text{if } \begin{array}{l} e^{v_\ell} \geq \frac{18p_s^{OOP}(1 + e^{v_c})}{R + p_s^{OOP}} \\ e^{v_c} \leq \frac{R}{p_s^{OOP}} \end{array} \\ (I^*(0), 0) & \text{if } \begin{array}{l} \frac{2}{17} \leq e^{v_\ell} \leq 18 \\ e^{v_c} \geq \frac{R(1 + e^{v_\ell})}{p_s^{OOP} 19} \end{array} \\ (I^*(F^*), F^*(I^*)) & \text{otherwise} \end{array} \right.$$

with

$$F^*(x) = \frac{\alpha_h}{\alpha_h + \alpha_c} \frac{R}{p_s^{OOP}} - \frac{\alpha_c}{\alpha_h + \alpha_c} (x + 1)$$

$$I^*(x) = \frac{\alpha_h}{\alpha_h + \alpha_\ell} 18 - \frac{\alpha_\ell}{\alpha_h + \alpha_\ell} (x + 1)$$



Households choose the type of formal care they prefer, between hiring a self-employed care assistant and buying care services from a SAAD. We assume that the services that are provided by a SAAD and an independent home care assistant are the same. Thus, the differentiation between the two types of services is only horizontal. We define a "real price" for each mode, that accounts for unobserved extra costs. For service providers, this "real cost" of an hour of care corresponds to the out-of-pocket price  $p_{SP}^{OOP}$ . For direct employment, this "real cost" also includes the administrative burden of declaring a worker. This burden is modeled by an unobserved random variable  $e^\xi$ , with  $\xi \sim \mathcal{N}(\mu_\xi, 1^2)$ . It is assumed that  $\xi \perp (v_\ell, v_c)$ . A household is assumed to choose direct employment over service provision if the marginal "real cost" of an hour of care from a service provider is strictly greater than the "real marginal cost" of direct employment, i.e. formally

$$p_{SP}^{OOP} > p_{ED}^{OOP} + \exp(\xi)$$

### 3.3.2 Firms

On the supply side, a different cost function is modeled for the SAADs, indexed by  $SP$ , and the self-employed care assistants, indexed by  $DE$ . SAADs are firms that have multiple employees who visit different seniors throughout the day according to a variable daily schedule. The distinction between SAADs *autorisés* and SAADs *agrés* is not made as, apart from the way their market price is set, they are exactly the same. The services that provided by SAADs and self-employed care assistants are the same, there is no vertical differentiation. However, they are treated differently as the costs incurred by the senior and their family are different. First, SAADs and self-employed carers are not subsidized the same way, as the *départements* set different prices. Second, the senior and their family incur an additional administrative cost when declaring the self-employed carer. Although these two markets are supposed to be distinct, they share common characteristics. First, firms are assumed to face no fixed costs. This assumption is quite realistic as the only production costs are the employees' working hours and travel expenses (Haut Conseil de la Famille, de l'Enfance et de l'Âge (2020), page 77). Secondly, there are no entry barriers. This assumption is also relatively weak as obtaining an approval is relatively easy to get from the representatives of the national authority in the *département*. Except for allowing new entrants, the representative of the national authority is not in charge of regulating the personal and social care local market and is not responsible for the funding of the APA. Moreover, no qualification is needed to become a care assistant. Labor market frictions are supposed to be limited. Thirdly, there are many firms in the market, and none is large enough to influence prices alone. This assumption is also quite credible given the large number of firms per *arrondissement*. All these assumptions make it plausible that the

<sup>2</sup>The variance of  $\xi$  is set to 1 to ensure identification

market operates under conditions of perfect competition. On the market for the care mode  $s \in \{SP, DE\}$ , a firm  $f$  is going to maximize the hours of formal care it produces for a given price  $p^s$ , i.e.

$$F_f^{s,*}(p^s) = \arg \max_F p^s \cdot F - C_f^s(F)$$

where  $C_f^s(F)$  denote the cost associated to the production of  $F$  hours of formal EHC for firm  $f$  of type  $s$ . The price  $p^{s,*}$  on the market satisfies the market clearing condition

$$D^s(p^{s,*}) = \sum_{f=1}^{N_f^s} F_f^{s,*}(p^{s,*})$$

where  $N_f^s$  is the total number of firms on the market. We suppose that

$$C_f^s(F) = \zeta_s^f + (W'\lambda^s + \nu^s + \epsilon_f^s)F + \frac{\kappa^s}{2}F^2 \quad (3.8)$$

where  $W$  is a set of observed factors influencing costs,  $\nu^s$  corresponds to the unobserved market factors influencing costs and  $\zeta_s^f, \epsilon_f^s$  are unobserved firm-specific factors influencing costs. Then,

$$p^{s,*} = \frac{\partial}{\partial F} C_s^f(F_f^{s,*}) = \kappa^s F_f^{s,*} + W'\lambda^s + \nu^s + \epsilon_f^s \quad (3.9)$$

Supposing that  $E[\epsilon_f^s] = 0$  and that  $N_f^s$  is large (which is consistent with perfect competition), then taking the average over all firms, we get, using the law of large numbers,

$$p^{s,*} = \kappa^s \frac{D^s(p^{s,*})}{N_f^s} + W'\lambda^s + \nu^s \quad (3.10)$$

The equilibrium price,  $p^{s,*}$ , is thus a function of the average demand per firm.

## 3.4 Data

The data used in this project are the results of the *CARE - Ménages* survey, a subset of the *Base Tous Salariés*, the French employer-employee dataset as well as publicly available socio-demographic data at the *arrondissement* level.

### 3.4.1 CARE - Ménages

The *Capacités, Aides et REssources des seniors* (CARE) surveys were conducted in October 2015 by the French Directorate for Research, Studies, Evaluation and Statistics (Drees). They aimed at better

understanding the living conditions of individuals aged 60 and over. One part of the CARE surveys covers seniors living in residential care facilities. We only consider the second part that covers the seniors living in the community : the *CARE - Ménages* survey. This *CARE - Ménages* survey is, itself, divided into two sections. First, around 11,000 seniors were surveyed on their living conditions, on their relationships with those around them, on their difficulties in performing daily activities and on the financial and human assistance they receive to overcome these difficulties. Their self-assessment of difficulties in performing certain daily activities allowed for the estimation of a "subjective" GIR (Groupe Iso-Ressources) for each senior. Second, all the informal carers that were cited by respondents were surveyed on their relationship with the senior, on the type and amount assistance they provide, as well as on their health condition and on the impact their informal care have on their professional, social and emotional life. The survey results of the "Senior" section were matched with fiscal and social data from INSEE. Additionally, the DREES requested each department to provide the names of their SAADs *autorisés* as well as their list of reference prices for self-employed personal and social carers and SAADs *agrés* as well as the rates they set for their SAADs *autorisés*. This information allows us to know relatively precisely the price paid by a senior for an hour of home care.

Table 3.1 presents some raw data from the *CARE - Ménages* database, with and without the weights that make the sample representative of the French population over 60. The proportion of individuals aged 75 and over is overrepresented in the sample, as is the proportion of dependent individuals. Seniors in the sample are also poorer than the general senior population, which could be explained by the fact that there are more single individuals. Table 3.2 provides information on the consumption of formal PSH care and on the provision of informal care, according to the senior's marital status and their estimated level of dependence. It appears that, when the senior lives with a partner, the partner is by far the primary caregiver: the informal care provided by the partner is nearly 7 times greater than that provided by all the children combined. In the absence of a partner, the informal care provided by children is greater but still does not, on average, reach the level of care provided by a partner. A greater reliance on formal care is also observed in the absence of a partner. The estimated GIR estimated based on seniors' self-assessment of their difficulties is imperfect (some seniors evaluated as GIR 5 and 6 still receive APA), but it nonetheless appears to be a good proxy for the actual GIR, as the share of APA recipients and the average amount of care received do increase with the self-assessed level of dependency.

Table 3.1: *CARE-Ménages* - Summary Statistics

		Unweighted	Weighted
Women	Share	60%	54%
Share in a relationship	Share	50%	64%
Age	mean	76.3	72.17
	med.	77	70
	Q1	68	65
	Q3	84	78
GIR 1		1%	0.2%
GIR 2		6%	1.5%
GIR 3	Share	6%	1.5%
GIR 4		20%	6.8%
GIR 5		11%	6.3%
GIR 6		56%	83.7%
APA recipient	Share	18%	4.5%
Ind. monthly income	mean	1,753	2,015
	med.	1,473	1,686
	Q1	1,096	1,241
	Q3	2,036	2380
Number of obs.		10,126	13,727,058

*Note: this table presents statistics on the individuals in the CARE-Ménages database, with and without the weights that make the sample representative of the French population over 60. Lecture notes: in the sample, 33% of the individuals are assessed to be dependent (i.e. have an estimated GIR score between 1 and 4). In the French population aged 60 or more, the share of dependent individuals is estimated to be around 10%.*

Table 3.2: Care by level of dependency and relationship status

<b>"Subjective" GIR score</b>	<b>Seniors in a relationship</b>					
	1	2	3	4	5	6
Number of observations	52	262	225	727	514	3296
Share of APA recipients	61.5%	50.8%	38.2%	17.7%	9.14%	1.52%
Share receiving informal care	65.4%	75.2%	78.7%	62.2%	44%	12.3%
Share receiving care from spouse	59.6%	62.6%	65.3%	54.1%	35.2%	9.83%
Average daily hours of care from children (if any)	1.25	0.70	0.42	0.33	0.24	0.05
Average daily hours of care from spouse (if any)	7.19	5.66	4.23	2.03	1.06	0.274
Average daily hours of PSH care for APA recipients	1.01	0.86	0.60	0.46	0.40	0.37

<b>"Subjective" GIR score</b>	<b>Single or widowed seniors</b>					
	1	2	3	4	5	6
Number of observations	50	340	380	1252	662	2366
Share of APA recipients	66%	68.5%	61.6%	37.7%	22.1%	6.5%
Share receiving informal care	60%	61.2%	58.4%	45.1%	34.1%	17.7%
Average daily hours of care from children (if any)	3.38	2.97	1.85	0.88	0.63	0.27
Average daily hours of PSH care for APA recipients	1.72	1.36	0.90	0.78	0.65	0.65

*Lecture note: in the CARE-Ménages database, there are 727 seniors that are in a couple and whose answers to questions regarding the abilities to perform daily activities led them to be assessed as having a GIR 4. 17.7% of them are APA recipients. For those who receive care from their spouse, their spouse provides on average 2.03 hours of care per day.*

From the "Carer" section of the *CARE-Ménages* database, several patterns emerge when looking at spouses identified as caregivers. First, Table 3.3 reveals that a significant proportion of caregiving spouses report that providing care affects their health. This proportion increases with the age of the spouse and is between one and a half to two times higher among female caregivers compared to male caregivers. However, nearly three-quarters of caregivers do not view their caregiving as leading to sacrifices, with the vast majority seeing it as a pleasant moment of sharing, and almost all consider it as a normal behaviour. Therefore, it seems that providing care is a voluntary and desired choice, even though it impacts caregivers' health. On the one hand, these descriptive statistics tend to support the modeling choices. One could alternatively see the optimization problem as being the one from the spouse's perspective. On the other hand, these statistics may suggest that spouses do not fully internalize the costs of the care they provide. While they feel in poorer health, spouses face a higher risk of becoming dependent themselves. They may also require more medical care, which is more expensive than LTC costs. One-fifth of men and nearly one-third of women caregivers express a desire for some or more respite. While nearly a quarter of spouses providing care at age 80 or more manage to get moments of respite through the use of PSH care, policies aimed at improving access to non-medical formal home care could be a lever to encourage households to take decisions that are more optimal in terms of inter-temporal welfare.

For the estimation of the demand side of the model, the considered sample consists of seniors from the survey who are in a couple, with a 'subjective' GIR score between 1 and 4 (i.e. that are dependent). Table 3.4 describes the sample. Nearly 30% of seniors receive neither assistance from their spouse nor professional care. 40% rely solely on informal care from their spouse, while 12.2% receive only professional help. Lastly, about 15% of seniors benefit from both informal care from their spouse and professional assistance simultaneously. When spouses provide care, they spend an average of 5 hours and 45 minutes per day doing so. In contrast, households that receive professional care get approximately 50 minutes of daily assistance on average. The out-of-pocket cost for one hour of formal home care is, on average, €3.27 for households purchasing services from a home care provider (SAAD) and €2.70 for those hiring an independent home care worker.

Table 3.3: Health and feelings among caring spouses, by gender and age

		Men		Women	
		Share	95% CI	Share	95% CI
<b>Caring affects health</b>					
	Spouse age : 60-69	13.7%	[9.30 ; 18.1]	29.4%	[25.0 ; 33.8]
	Spouse age : 70-79	19.7%	[15.0 ; 27.3]	36.3%	[30.5 ; 42.1]
	Spouse age : 80+	27.7%	[22.8 ; 32.7]	41.7%	[34.2 ; 49.1]
<b>Wish for some or more respite</b>					
	Spouse age : 60-69	27.0%	[20.4 ; 33.6]	32.2%	[27.2 ; 37.2]
	Spouse age : 70-79	21.9%	[15.8 ; 28.1]	30.4%	[24.1 ; 36.6]
	Spouse age : 80+	15.5%	[10.8 ; 20.1]	36.9%	[28.6 ; 45.2]
<b>Respite thanks to PSH care</b>					
	Spouse age : 60-69	8.3%	[4.14 ; 12.4]	3.9%	[1.69 ; 6.07]
	Spouse age : 70-79	28.6%	[21.6 ; 35.5]	11.5%	[7.01 ; 16.1]
	Spouse age : 80+	23.7%	[18.1 ; 29.3]	27.1%	[19.0 ; 35.2]
<b>Agreement to "I feel that caring..."</b>					
<b>"...leads to sacrifices"</b>	No	72.5% [70.3 ; 74.6]			
	Yes, a little	17.9% [16.0 ; 19.7]			
	Yes, a lot	9.66% [8.24 ; 11.1]			
<b>"...is a time of connection"</b>	No	11.1% [9.58 ; 12.6]			
	Yes, a little	31.8% [29.5 ; 34.0]			
	Yes, a lot	57.1% [54.8 ; 59.5]			
<b>"... is normal"</b>	No	1.48% [0.09 ; 2.05]			
	Yes, a little	9.73% [8.31 ; 11.2]			
	Yes, a lot	88.8% [87.3 ; 90.3]			
<b>Number of observations:</b>		1,556			

Lecture Notes: 27.7% of male caring spouses that are 80 or more affirm that caring affects their health. 88.8% of caring spouses declare fully agree that caring is normal.

Table 3.4: Subsample for estimation of the demand - summary statistics

Variable	Shares			
Woman	42.6 %			
Former white collar	13.6 %			
"Subjective" GIR : 1	4.0 %			
"Subjective" GIR : 2	18.1 %			
"Subjective" GIR : 3	16.6 %			
"Subjective" GIR : 4	61.3 %			
Care received from spouse	56.0 %			
Care received from spouse and formal PSH care	14.3 %			
Consumption of formal PSH care	26.5 %			
Formal care from a SAAD (if formal care received)	71.4 %			
At least one child lives "in the same region"	81.7 %			
Variable	1 <sup>st</sup> Quartile	Median	Mean	3 <sup>rd</sup> Quartile
Daily hours of care from spouse (if care received)	2	3.5	5.75	8.09
Daily hours of formal PSH care (if care received)	0.428	0.658	0.809	1.02
Age	70	78	77.32	84
Spouse's age	68	77	75.19	83
Daily Income	59.6	75.5	86.4	99.5
Out-of-pocket price - SAAD	1.28	2.59	3.27	4.68
Out-of-pocket price - Self-employed care assistant	1.50	2.42	2.70	3.62
<b>Number of observations:</b>	<b>1,129</b>			

*Lecture note: 16.6% of the seniors in the sample used for the estimation of the demand are assessed to have a GIR 3, based on their answers to the questions on their difficulties to perform activities of daily livings. On average, the out-of-pocket price for an hour of formal care from a SAAD is 3.27 euros for a senior in the sample.*

The prices for one hour of professional care from SAADs in an *arrondissement* are inferred from the pricing grids of the *départements*, that were provided as supplementary material in the *CARE-Ménages* survey. Prices set by independent home care workers are obtained from the *Base Tous Salariés* (BTS), which is the French administrative matched employer-employee database. For each employee, the database includes information such as job type and qualifications, start and end dates of the pay period, number of salaried hours, employment conditions (full-time, part-time), and the remuneration that was paid. The *BTS-Postes* section provides details about the job position, individual employee characteristics (e.g. age, gender), and some information about the employer, including its sector of activity. Employees



hired directly by households are also recorded in this database, which allows for the retrieval of all employment contracts for independent home care assistants. The gross hourly wage of the home care assistant is considered as the price per hour of care. Additionally, the BTS database provides data on the number of SAAD providers and independent home care workers by district. Table 3.5 outlines the sample used to estimate the supply equations. On average, each *arrondissement* has 78 SAADs employing 19 home care assistants, along with approximately 800 contracts for independent home care workers. These figures tend to support the assumption that the PSH care market operates under conditions of perfect competition. Additional publicly available socio-demographic data have been added to the sample. *Arrondissements* exhibit considerable heterogeneity in terms of population density and demographics. In one-quarter of metropolitan French *arrondissements*, there are at least 1.5 individuals over 65 for every teenager, whereas in another quarter, there is less than one individual over 65 for each person under 20.

Table 3.5: Subsample for the estimation of the supply - summary statistics

	<b>Variable</b>	<b>1<sup>st</sup> Quartile</b>	<b>Median</b>	<b>Mean</b>	<b>3<sup>rd</sup> Quartile</b>
	Market price	20.48	21.06	21.01	22.02
	Regulated rate - SAADs <i>agrés</i>	17.80	18.87	18.99	19.96
SAAD	N. firms	32	61	78.31	100
	N. paid hours of labor	11,840	15,747	16,052	19,780
	N. care assistants (FTE)	14.43	18.87	19.32	23.33
	Market price	12.48	13.2	13.31	13.95
Self-employed	Regulated rate	11.06	11.93	11.52	12.33
	N. firms	337	578	791.51	991
	N. paid hours of labor	282	324	332.48	359
	<b><i>Arrondissements</i> - Variable (reference year : 2014)</b>				
	Population Density	45.21	80.51	489.77	168.83
	Unemployment Rate	11.50 %	13.20 %	13.34 %	14.90 %
	Share of pop. over 75	8.60 %	10.20 %	10.43 %	12.10 %
	Aging Index (pop. over 65 / pop. below 20)	68	83	88.41	105
	Share of people over 75 living alone	35.6 %	37.6 %	37.55 %	39.40 %
	Poverty rate among people over 75	8.0 %	9.9 %	10.24 %	12.0 %
	Median monthly income	20,880	21,630	22,008	22,688
<b>Number of <i>arrondissements</i></b>			286		

Notes: This table contains information on the formal PSH care market at the *arrondissement* level, in 2015. Lecture notes: the median number of SAADs in an *arrondissement* is 61. The median unemployment rate in an *arrondissement*, in 2014, is 13.20%

## 3.5 Estimation

### 3.5.1 A supply with constant marginal costs

One of the main findings of the project is that firms providing PSH care face constant marginal costs. In other words, the price set by these firms does not depend on the demand they face. The supply equation is estimated at the *arrondissement* level, distinctly for SAADs and independent home care workers. Equation (3.10) is approximated with

$$p_m^{s,*} = \kappa^s \bar{F}_m^s + W'_m \lambda^s + \nu_m^s \quad (3.11)$$

where  $\bar{F}_m^s$  is the average demand (or supply equivalently, by the clearing market condition) per firm for the mode of care  $s$  in market  $m$ . For the self-employed home care assistants, a firm is equivalent to a contract in the *BTS* database. The production per firm is thus directly observed. For SAADs, the average supply per firm is unknown as the hours in the contracts of care workers both include the hours spent providing care as well as the hours of commuting between customers. Consequently, the average demand per firm is estimated using the APA care plans from the full *CARE-Ménages* database. To infer the total demand of care from SAADs, the observed demand of observation  $i$  living in *arrondissement*  $d$  is weighted by

$$\tilde{w}_{id} = w_{id} \times \frac{P_d}{\sum_{j=1}^{n_d} w_{jd}}$$

where  $w_{id}$  is the original weight of individual  $i$  in the *CARE* sample,  $P_d$  is the population of people over 60 in *arrondissement*  $d$  and  $n_d$  is the number of observations living in *arrondissement*  $d$  observed in *CARE*. In some *arrondissements*, there is no senior from the *CARE-Ménages* survey that consumes formal care from SAADs. The demand is thus inferred from the total number of hours worked by care assistants in SAADs, using the *BTS* data. The relationship between the logarithm of the estimated demand and the logarithm of the total number of hours worked by personal carers, in an *arrondissement*, is close to being linear, as shown by Figure C.1. This linear fit is used as a proxy for the demand of SAADs in all the *arrondissements*.

The average demand per firm has to be instrumented. The *département* rates of APA care plans for the SAADs *agrés* for the service providers and the rate used for independent home care workers are natural candidates. They affect directly the demand as a greater *département* rate reduces the out-of-pocket price, without influencing directly the production costs. However, these rates only varies at the *département* level. The small variation of these rates affects the precision of the estimation. This is why others variables have been considered as potential relevant instruments. The relative size of the

elderly population should naturally increase demand. This is why variables such as the proportion of the population over 75 and the aging index, which corresponds to the ratio of the population aged over 65 to those under 20, are thought as potentially relevant instrumental variables. The income level of the population, particularly for individuals over 75, may also influence the demand for formal care without directly affecting firms' production costs. Therefore, the logarithm of the median income and the share of people over 75 living below the poverty line are also included in the vector of potential instruments.

The vector  $W$  of (3.11) includes variables that are thought to directly influence firms' production costs. The variables that have been retained are the unemployment rate and the population density. They are included in the regression in a very flexible way (a polynomial of order 2 and in a logarithmic transformation for the density). The unemployment rate is a proxy of the labor market dynamism. It can provide information on the wages of care assistants. Population density can be seen as a proxy of "technology". In less dense areas, the time spent in commuting is greater so that the share of paid hours dedicated to care is lower than in dense urban areas. To select flexibly the best instrumental variables and covariates, the algorithm proposed by Chernozhukov et al. (2015) was used. Table 3.6 presents the results

Table 3.6: Supply - IV estimates

	Price - SAADs	Price - Independent home care workers
$\bar{F}^s$	-0.000219 (0.000188)	-0.00421 (0.00582)
F Statistic - 1 <sup>st</sup> stage	17.78	12.18
Selected Instrument(s)	Share of pop. over 75	Log. of median income Poverty rate among people over 75
N	286	286

*Lecture note: this table presents the coefficients associated with firm-level demand in the regression (3.11) estimated using the double machine learning method developed by Chernozhukov et al. (2018). The vector of control variables  $W$  consists of transformations of the unemployment rate and population density in the *arrondissement*. The first column refers to the SAADs market, while the second corresponds to independent workers. In both cases, it is impossible to reject the hypothesis that the theoretical coefficient is zero, implying constant marginal costs.*

For direct employment, only one instrument has been selected by the algorithm: the share of people over 75. All the covariates in  $W$  have been selected. For the service providers design, two instrumental variables were selected: the share of people over 75 that were below the poverty rate and the logarithm of the median income in the *arrondissement*. The vector of covariates include the population density, the logarithm of the population density and the square of the unemployment rate. Whether for SAADs or

independent home care assistants, the null hypothesis that prices are not influenced by the level of the average demand faced by a firm cannot be rejected at any level. In other words, it seems reasonable to assume that the marginal cost of providing home care services remains constant. While this assumption is common in the care literature (e.g., Berlinski et al. (2024)), to our knowledge, it has not been formally tested. One key implication of this result is that supply-side considerations can be disregarded when designing counterfactual policy interventions on the demand side.

### 3.5.2 A greater reaction of male caregiving spouses to a change in the out-of-pocket formal home care price

Table 3.7: Demand - Maximum Likelihood estimates

Parameters - $\alpha$		Parameters - $\alpha_\ell$		Parameters - $\alpha_c$		Parameters - $\xi$	
$\mu_\ell$	-2.38 (0.81)	$\beta_\ell^{woman}$	-0.39 (0.19)	$\beta_c^{woman}$	-0.12 (0.30)	$\mu_\xi$	0.44 (0.13)
$\mu_c$	11.93 (1.35)	$\beta_\ell^{age}$	-0.02 (0.02)	$\beta_c^{age}$	-0.04 (0.03)	$\beta_\xi^{rich}$	1.42 (0.16)
$\sigma_\ell$	2.59 (0.09)	$\beta_\ell^{spouse.age}$	0.08 (0.02)	$\beta_c^{spouse.age}$	-0.05 (0.03)		
$\sigma_c$	3.34 (0.14)	$\beta_\ell^{close.child}$	0.41 (0.22)	$\beta_c^{close.child}$	1.16 (0.34)		
$\rho$	0.52 (0.04)	$\beta_\ell^{white.collar}$	0.42 (0.48)	$\beta_c^{white.collar}$	2.10 (1.15)		

*Lecture note: this table presents the coefficients of the demand model for professional and informal PSH care for households consisting of a dependent senior and a (potentially caregiving) spouse, estimated using maximum likelihood. Lecture note: the estimated coefficient  $\beta_\ell^{spouse.age}$  is significantly positive. This implies that, all else being equal, a household with an older spouse places relatively more weight on the free time available to the spouse than on the care provided to the dependent senior or on consumption.*

The demand side of the model is estimated using maximum likelihood. The likelihood function is detailed in the appendix. In the model, the vector  $\mu$ , representing the expected values of the random utility parameters  $(\alpha_\ell, \alpha_c)$ , is assumed to depend linearly on observed household characteristics. This approach allows for some heterogeneity in household preferences to be both controlled and measured. In this framework, the senior's gender, their age, their spouse's age, whether at least one child lives in the same (broadly defined) region, and whether the man in the household held a white-collar job during his

career, are all incorporated as explanatory variables. Additionally, the expected value  $\mu_\xi$  of the variable  $\xi$ , which captures the unobserved costs of hiring an independent care worker versus using the services of a home care agency (SAAD), is modeled as a function of household income. Specifically, an indicator variable takes the value of 1 for households whose income exceeds the sample median. The convergence of the maximum likelihood estimator is verified through simulations, with results provided in the appendix. Table 3.7 presents the preliminary coefficient estimates. The results suggest that having an older spouse significantly increases the importance placed on leisure time, while having a male spouse reduces it. Households with at least one child living in the same region exhibit stronger preferences for both leisure and consumption, likely because these children themselves contribute informal care.

Based on the demand function, it is possible to derive a closed-form expression for the elasticity of informal care provision by a spouse in response to changes in the out-of-pocket price of formal home care services. For households that consume both formal and informal care, the formula is as follows:

$$\varepsilon_{p^{OOP}}^I = \left( \frac{p^{OOP}}{R} \left( \frac{18}{\alpha_\ell} - 19 \right) - 1 \right)^{-1} (> 0) \quad (3.12)$$

Knowing this elasticity across different household groups helps identify which households are more likely to adjust their informal care provision in response to a reduction in the price of formal home care services. This elasticity increases with  $\alpha_\ell$ . Therefore, households that place less value on their free time are the most responsive to price changes of formal care. Preliminary results suggest that, all else being equal, households with a male spouse are more likely to reduce their provision of informal care in response to a change of formal care price. Similarly, older couples are expected to react more strongly to changes in the price of formal care services, all other factors remaining constant. At this stage of the project, these findings are primarily qualitative.

### 3.6 Future Developments

This project is intended to be developed in several dimensions. First, the model's quality still needs to be improved. It is currently being evaluated in-sample by calculating the root mean squared error (RMSE) on predicted care quantities. Additionally, the model's performance should be assessed out-of-sample. The cross-price elasticity mentioned in the previous paragraph, which has so far been evaluated qualitatively, will also be estimated. Second, this modeling approach aims to be used for evaluating the effects of counterfactual policies, to understand how different households might respond to changes in access to the professional home care market. The first experiment would involve an ex ante assessment of certain measures from the Adaptation of Society to Aging (ASV) law. This law, passed in December 2015 and

officially implemented in March 2016, notably changed the way the copayment rate (the share of the APA care plan paid by seniors) is calculated. The rate now depends not only on household income but also on the amount of formal care consumed. Third, an interesting aspect would be to account for the health costs borne by the spouse providing informal care, which are not internalized in current evaluations of public policy. Data from the *CARE-Ménages* survey has been matched with health insurance data, offering a valuable opportunity to explore these broader costs associated with home-based dependency care.

### **3.7 Conclusion**

Caring for the elderly with dependency is one of the major challenges associated with population aging. Currently, much of this care still relies on the seniors' relatives. When a dependent senior is in a couple, it is the spouse who provides the majority of the care. Having a spouse is, in fact, the main source of inequality in access to care among seniors who become dependent. However, the spouse is often elderly as well, and providing care affects their health. By offering assistance, they increase their own risk of becoming dependent and needing healthcare. In many countries, a market for professional non-medical home care has emerged to replace the informal care provided by relatives. The goal of this project is to understand how informal care provided by the spouses of dependent seniors evolves when the conditions for accessing the formal care market change, based on household characteristics. To achieve this, a model of the supply and demand for non-medical home care is developed. On the demand side, households consisting of a dependent senior and their spouse decide on their consumption, the amount of informal daily care provided by the spouse, and the number of formal care hours purchased from home care services. The supply side is also modeled to understand how it responds to changes in demand. The model takes advantage of the specific features of the French market, which is largely decentralized. The data used comes from the *CARE-Ménages* survey, which provides information on the French population over 60, supplemented by data from the French matched employer-employee database. Preliminary estimates from the model suggest that, on the supply side, personal and social home care services face constant marginal costs. An initial interpretation of this result implies that supply is not a significant factor in assessing subsidy policies for the demand for formal personal and social home care. However, the price set by the sector may be too low, especially given that the quality of services provided is difficult to measure, whether by legislators or potential consumers. On the demand side, early results suggest that the level of informal care provided by the spouse, when the spouse is male, reacts more significantly to changes in the price of formal care, all else being equal. The same holds for households where the spouse is older, all else being equal.



# Conclusion

As the collection of information becomes less challenging thanks to new information and communication technologies and access to new datasets is easier, the evaluation of public policies is becoming more systematic. At the same time, econometric tools and procedures used to measure causal effects have significantly improved. In particular, the use of experimental and quasi-experimental designs is becoming the norm. However, the causal interpretation of results derived from conventional estimation methods associated with these evaluation designs relies on assumptions, some of which impose a homogeneity of parameter values across observations. This thesis develops several alternative estimation methods that are robust to the fact that some parameters may vary across observations.

The first chapter presents a new method within the difference-in-differences (DiD) design, which is one of the most widely used designs for evaluating the causal effects of public policies.<sup>3</sup> In its standard form, the DiD design consists in comparing the evolution of the mean of an outcome variable in a group affected by the policy (treatment group) with that of a non-affected group (control group). The identifying of the average causal effect post-treatment in the treated group hinges on a common trend assumption: the average evolution of the outcome variable in the group affected by the policy would have been the same as in the control group, had the policy not been implemented. Under this assumption, the causal effect can be estimated through a two-way fixed effects regression (group and period) on panel data. However, in designs with multiple groups and/or multiple time periods, several studies (e.g., De Chaisemartin and d'Haultfoeuille (2020), Goodman-Bacon 2021, Imai and Kim (2021), Sun and Abraham (2021), Borusyak et al. (2024)) have demonstrated that the coefficient estimated by a two-way fixed effects regression has no longer a relevant causal interpretation when the policy effects are not assumed to be homogeneous across groups and periods. Callaway et al. (2024) further shows that this negative result also holds in designs where the treatment variable is continuous (i.e. when groups experience different doses of a treatment over time). This first chapter introduces two new estimators for newly defined, valuable causal parameters within the DiD framework with a continuous treatment variable, which are robust to

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<sup>3</sup>According to the survey by De Chaisemartin and d'Haultfoeuille (2020), 20% of papers published in the *American Economic Review* between 2010 and 2012 employed a DiD design.



heterogeneous treatment effects. The identification and estimation of these two parameters rely on a novel common trend assumption: in the absence of a change in their treatment dose between the two periods, the switcher groups would have experienced, on average, the same evolution in their outcome variable as the stayer groups with the same initial treatment dose.

The second chapter presents a new method to evaluate the causal effects of a binary treatment in the presence of peer effects. In the absence of detailed information on the structure of social networks, the conventional strategy to identify both the direct effect of the treatment on treated individuals and the sign and magnitude of the social multiplier effect is to estimate a linear-in-means model within a partial population experiment. However, the standard linear-in-means model overlooks the complexity of social interactions. This second chapter adapts the linear-in-means model by allowing individuals to respond differently to the behaviors of other group members, depending on whether or not they share the same identity. The identification and estimation of the model's parameters still rely on a partial population experiment but the proportion of individuals eligible for the treatment has to be known even within groups that do not receive the treatment. The magnitude of the direct effect of the treatment on eligible individuals, the indirect effect on non-eligible individuals, and the social multiplier effects can be assessed by comparing groups that receive the treatment with those that have the exact same proportion of eligible individuals that do not receive the treatment.

The third chapter introduces a new structural model to evaluate the effects of policies that change access to the personal and social home care market for dependent elderly individuals, that allows for rich unobserved heterogeneity, within the French institutional context. On the demand side, households—comprising a dependent senior and their spouse—make decisions regarding their consumption, the informal care provided by the spouse, and the use of professional home care services. These households are heterogeneous in both their characteristics and preferences. As a result, the informal care provided by caregiving spouses of dependent seniors respond differently depending on the household. On the supply side, two types of providers are modeled: firms that offer personal and social home care services, and self-employed personal home care assistants. Within each type of care provider, firms exhibit different cost functions, which allows for a more nuanced understanding of the market dynamics and how different providers respond to policy changes.

# Appendix A

## Difference-in-Differences for Continuous Treatments and Instruments with Stayers

### A.1 Proofs

Hereafter,  $\text{Supp}(X)$  denotes the support of  $X$ . Note that under Assumption 2, one can show that for all  $(t, t') \in \{0, 1\}^2$ ,  $E(Y_t(D_{t'}))$  exists.

#### A.1.1 Theorem 1

The result is just a special case of Theorem 2, under Assumption 5  $\square$

#### A.1.2 Theorem 2

First, observe that the sets  $\{S_\eta = 1\}$  are decreasing for the inclusion and  $\{S = 1\} = \cup_{\eta > 0} \{S_\eta = 1\}$ . Then, by continuity of probability measures,

$$\lim_{\eta \downarrow 0} P(S_\eta = 1) = P(S = 1) > 0, \tag{A.1}$$

where the inequality follows by Assumption 4. Thus, there exists  $\underline{\eta} > 0$  such that for all  $\eta \in (0, \underline{\eta})$ ,  $P(S_\eta = 1) > 0$ . Hereafter, we assume that  $\eta \in (0, \underline{\eta})$ .

We have  $\text{Supp}(D_1|S_\eta = 1) \subseteq \text{Supp}(D_1|S = 1)$  and by Assumption 4,  $\text{Supp}(D_1|S = 1) \subseteq \text{Supp}(D_1|S =$

0). Thus, for all  $(d_1, d_2) \in \text{Supp}(D_1, D_2 | S_\eta = 1)$ ,  $d_1 \in \text{Supp}(D_1 | S = 0)$ , so  $E(Y_2(d_1) - Y_1(d_1) | D_1 = d_1, S = 0) = E(Y_2(d_1) - Y_1(d_1) | D_1 = d_1, D_2 = d_1)$  is well-defined. Moreover, for almost all such  $(d_1, d_2)$ ,

$$\begin{aligned} E(Y_2(d_1) - Y_1(d_1) | D_1 = d_1, D_2 = d_2) &= E(Y_2(d_1) - Y_1(d_1) | D_1 = d_1, D_2 = d_1) \\ &= E(\Delta Y | D_1 = d_1, S = 0), \end{aligned} \quad (\text{A.2})$$

where the first equality follows from Assumption 1. Now, by Point 2 of Assumption 2,  $[Y_2(D_2) - Y_2(D_1)] / \Delta D$  admits an expectation. Moreover,

$$\begin{aligned} & E \left( \frac{Y_2(D_2) - Y_2(D_1)}{\Delta D} \middle| S_\eta = 1 \right) \\ &= E \left( \frac{E(Y_2(D_2) - Y_1(D_1) | D_1, D_2) - E(Y_2(D_1) - Y_1(D_1) | D_1, D_2)}{\Delta D} \middle| S_\eta = 1 \right) \\ &= E \left( \frac{E(\Delta Y | D_1, D_2) - E(\Delta Y | D_1, S = 0)}{\Delta D} \middle| S_\eta = 1 \right) \\ &= E \left( \frac{\Delta Y - E(\Delta Y | D_1, S = 0)}{\Delta D} \middle| S_\eta = 1 \right), \end{aligned} \quad (\text{A.3})$$

where the first equality follows from the law of iterated expectations, the second follows from (A.2), and the third again by the law of iterated expectations. Next,

$$\delta_1 = \Pr(S_\eta = 1 | S = 1) E \left[ \frac{Y_2(D_2) - Y_2(D_1)}{\Delta D} \middle| S_\eta = 1 \right] + E \left[ (1 - S_\eta) \frac{Y_2(D_2) - Y_2(D_1)}{\Delta D} \middle| S = 1 \right].$$

Moreover,

$$\begin{aligned} \left| E \left[ (1 - S_\eta) \frac{Y_2(D_2) - Y_2(D_1)}{\Delta D} \middle| S = 1 \right] \right| &\leq E \left[ (1 - S_\eta) \left| \frac{Y_2(D_2) - Y_2(D_1)}{\Delta D} \right| \middle| S = 1 \right] \\ &\leq E \left[ (1 - S_\eta) \bar{Y} \middle| S = 1 \right], \end{aligned}$$

where the second inequality follows by Assumption 2. Now, by (A.1) again,  $\lim_{\eta \downarrow 0} (1 - S_\eta) \bar{Y} = 0$  a.s. Moreover,  $(1 - S_\eta) \bar{Y} \leq \bar{Y}$  with  $E[\bar{Y} | S = 1] < \infty$ . Then, by the dominated convergence theorem,

$$\lim_{\eta \downarrow 0} E \left[ (1 - S_\eta) \frac{Y_2(D_2) - Y_2(D_1)}{\Delta D} \middle| S = 1 \right] = 0.$$

We finally obtain

$$\delta_1 = \lim_{\eta \downarrow 0} E \left[ \frac{Y_2(D_2) - Y_2(D_1)}{\Delta D} \middle| S_\eta = 1 \right]. \quad (\text{A.4})$$

The result follows by combining (A.3) and (A.4)  $\square$

### A.1.3 Theorem 3

Let  $\Delta Y = Y_2 - Y_1$ ,  $\Delta D = D_2 - D_1$ ,  $\mu_1(D_1) = E[(1 - S)Y|D_1]$ ,  $\mu_2(D_1) = E[1 - S|D_1]$ . In what follows we let  $\mu(D_1) = (\mu_1(D_1), \mu_2(D_1))'$ . From Theorem 1, the parameter  $\delta_1$  is characterized by the condition:

$$0 = E \left[ \frac{S}{\Delta D} \left( \Delta Y - \delta_1 \Delta D - \frac{\mu_1(D_1)}{\mu_2(D_1)} \right) \right]$$

Define:

$$g(Z, \delta, \mu) = \frac{S}{\Delta D} \left( \Delta Y - \frac{\mu_1(D_1)}{\mu_2(D_2)} \right) - S\delta_1$$

where  $Z = (Y_1, Y_2, D_1, D_2)$ . Also define:

$$\mathcal{L}(Z, \mu, \delta_1, \tilde{\mu}) = -\frac{S}{\Delta D} \cdot \frac{1}{\tilde{\mu}_2(D_1)} \left( \mu_1(D_1) - \frac{\tilde{\mu}_1(D_1)}{\tilde{\mu}_2(D_1)} \mu_2(D_1) \right)$$

We verify conditions 6.1 to 6.3, 5.1(i) and 6.4(ii) to 6.6 in Newey (1994). Following his notation, we let  $\mu_0 = (\mu_{10}, \mu_{20})'$  and  $\delta_{10}$  represent the true parameters, and  $g(Z, \mu) = g(Z, \delta_{10}, \mu)$ .

**Step 1.** We verify condition 6.1. First, since  $S$  is binary  $E[(S - E[S|D_1])^2|D_1] = V[S|D_1] \leq 1/4$ . On the other hand,  $E[((1 - S)\Delta Y - E[(1 - S)\Delta Y|D_1])^2|D_1] \leq E[\Delta Y^2|D_1] < \infty$  by part 2 of Assumption 6. Thus, condition 6.1 holds.

**Step 2.** We verify condition 6.2. Since  $p^K(d_1)$  is a power series, the support of  $D_1$  is compact and the density of  $D_1$  is uniformly bounded below, by Lemma A.15 in Newey (1995) for each  $K$  there exists a constant nonsingular matrix  $A_K$  such that for  $P^K(d_1) = A_K p^K(d_1)$ , the smallest eigenvalue of  $E[P^K(D_1)P^K(D_1)']$  is bounded away from zero uniformly over  $K$ , and  $P^K(D_1)$  is a subvector of  $P^{K+1}(D_1)$ . Since the series-based propensity scores estimators are invariant to nonsingular linear transformations, we do not need to distinguish between  $P^K(d_1)$  and  $p^K(d_1)$  and thus conditions 6.2(i) and 6.2(ii) are satisfied. Finally, because  $p_{1K}(d_1) \equiv 1$  for all  $K$ , for a vector  $\tilde{\gamma} = (1, 0, 0, \dots, 0)$  we have that  $\tilde{\gamma}' p^K(d_1) = \tilde{\gamma}_1 \neq 0$  for all  $d_1$ . Since  $A_K$  is nonsingular, letting  $\gamma = A_K^{-1} \tilde{\gamma}$ ,  $\gamma' P^K(d_1) = \tilde{\gamma}' A_K^{-1} P^K(d_1)$  is a non-zero constant for all  $d_1$  and thus condition 6.2(iii) holds.

**Step 3.** We verify condition 6.3 for  $d = 0$ . Since  $p^K(d_1)$  is a power series, the support of  $D_1$  is compact and the functions to be estimated have 4 continuous derivatives, by Lemma A.12 in Newey (1995) there is a constant  $C > 0$  such that there is  $\pi$  with  $\|\mu - (p^K)' \pi\| \leq CK^{-\alpha}$ , where in our case  $\alpha = s/r = 4$  since the dimension of the covariates is 1 and the unknown functions are 4 times continuously differentiable. Thus, condition 6.3 holds.

**Step 4.** We verify condition 5.1(i). By part 3 of Assumption 6,  $\mu_{20}(D_1) = E[1 - S|D_1] = 1 - E[S|D_1] \geq 1 - c_M$  for some constant  $c_M > 0$ . Let  $C = 1 - c_M$ . For  $\mu$  such that  $\|\mu - \mu_0\|_\infty < C/2$ ,

$$\begin{aligned}
& |g(Z, \mu) - g(Z, \mu_0) - \mathcal{L}(Z, \mu - \mu_0, \delta_{10}, \mu_0)| \\
&= \left| \frac{S}{\Delta D} \left| \frac{\mu_1(D_1)}{\mu_2(D_1)} - \frac{\mu_{10}(D_1)}{\mu_{20}(D_1)} - \frac{1}{\mu_{20}(D_1)} \left( \mu_1(D_1) - \mu_{10}(D_1) - \frac{\mu_{10}(D_1)}{\mu_{20}(D_1)} (\mu_2(D_1) - \mu_{20}(D_1)) \right) \right| \right| \\
&\leq \frac{1}{c} \left| \frac{\mu_1(D_1)}{\mu_2(D_1)} - \frac{\mu_{10}(D_1)}{\mu_{20}(D_1)} - \frac{1}{\mu_{20}(D_1)} \left( \mu_1(D_1) - \mu_{10}(D_1) - \frac{\mu_{10}(D_1)}{\mu_{20}(D_1)} (\mu_2(D_1) - \mu_{20}(D_1)) \right) \right| \\
&\leq \frac{1}{c} \cdot \frac{2(1 + |\mu_{10}(D_1)| / |\mu_{20}(D_1)|)}{C^2} \max \{ |\mu_1(D_1) - \mu_{10}(D_1)|, |\mu_2(D_1) - \mu_{20}(D_1)| \}^2 \\
&\leq \frac{1}{c} \cdot \frac{2(1 + |\mu_{10}(D_1)| / |\mu_{20}(D_1)|)}{C^2} \|\mu - \mu_0\|_\infty^2
\end{aligned}$$

where the first inequality follows from Assumption 5 and the second inequality follows from Lemma S3 in the Web Appendix of de Chaisemartin and D'Haultfœuille (2018). Thus, condition 5.1(i) holds.

**Step 5.** We verify condition 6.4(ii). First,  $E[(1 + |\mu_{10}(D_1)| / |\mu_{20}(D_1)|)^2] < \infty$ . For power series, by Lemma A.15 in Newey (1995),  $\zeta_d(K) = \sup_{|\lambda|=d, x \in I} \|\partial^\lambda p^K(x)\| \leq CK^{1+2d}$  so setting  $d = 0$ ,

$$\zeta_0(K) \left( (K/n)^{1/2} + K^{-\alpha} \right) \leq CK \left( (K/n)^{1/2} + K^{-\alpha} \right) = C \left( \sqrt{\frac{K^3}{n}} + K^{1-\alpha} \right) \rightarrow 0$$

since  $\alpha = 4 > 1/2$ ,  $K^7/n \rightarrow 0$  and  $K \rightarrow \infty$ . Finally,

$$\sqrt{n} \zeta_0(K)^2 \left( \frac{K}{n} + K^{-2\alpha} \right) \leq C^2 \sqrt{n} K^2 \left( \frac{K}{n} + K^{-2\alpha} \right) = C \left( \sqrt{\frac{K^6}{n}} + \sqrt{\frac{n}{K^{4\alpha-4}}} \right) \rightarrow 0$$

since  $K^7/n \rightarrow 0$  and for  $\alpha = 4$ ,  $K^{4\alpha-4}/n = K^{12}/n \rightarrow \infty$ . Hence condition 6.4(ii) holds.

**Step 6.** We verify condition 6.5 for  $d = 1$  and where  $|\mu|_d = \sup_{|\lambda| \leq d, x \in I} \|\partial^\lambda \mu(x)\|$ . Since  $E[(1 + |\mu_{10}(D_1)| / |\mu_{20}(D_1)|)^2] < \infty$ ,

$$\begin{aligned}
|\mathcal{L}(Z, \mu, \delta_{10}, \mu_0)| &= \left| \frac{S}{\Delta D} \cdot \frac{1}{\mu_{20}(D_1)} \left( \mu_1(D_1) - \frac{\mu_{10}(D_1)}{\mu_{20}(D_1)} \mu_2(D_1) \right) \right| \\
&\leq \frac{1}{c(1 - c_M)} \left( 1 + \left| \frac{\mu_{10}(D_1)}{\mu_{20}(D_1)} \right| \right) |\mu|_1.
\end{aligned}$$

Next, the same linear transformation of  $p^K$  as in Step 2, namely  $P^K$  is, by Lemma A.15 in Newey (1995), such that  $|P_k^K|_d \leq CK^{1/2+2d}$ . As a result,  $(\sum_k |P_k^K|_1^2)^{1/2} \leq CK^{1+2d}$ . Then, for  $d = 1$ ,

$$\left( \sum_k |P_k^K|^2 \right)^{1/2} \left( \sqrt{\frac{K}{n}} + K^{-\alpha} \right) \leq CK^3 \left( \sqrt{\frac{K}{n}} + K^{-\alpha} \right) = C \left( \sqrt{\frac{K^7}{n}} + K^{3-\alpha} \right) \rightarrow 0$$

since  $K^7/n \rightarrow 0$  and  $K^{3-\alpha} = K^{-1} \rightarrow 0$  for  $\alpha = 4$ . Thus, condition 6.5 holds.

**Step 7.** We verify condition 6.6. Condition 6.6(i) holds for

$$\delta(D_1) = [-E[S/\Delta D|D_1]/\mu_{20}(D_1)](1, -\mu_{10}(D_1)/\mu_{20}(D_1)).$$

Because the involved functions are continuously differentiable, by Lemma A.12 from Newey (1995) there exist  $\pi_K$  and  $\xi_K$  such that:

$$E \left[ \|\delta(D_1) - \xi_K p^K(D_1)\|^2 \right] \leq \|\delta - \xi_K p^K\|_\infty^2 \leq CK^{-2\alpha}$$

and

$$E \left[ \|\mu_0(D_1) - \pi_K p^K(D_1)\|^2 \right] \leq \|\mu_0 - \pi_K p^K\|_\infty^2 \leq CK^{-2\alpha}$$

where we recall that  $\alpha = 4$ . Thus, the first part of condition 6.6(ii) follows from

$$nE \left[ \|\delta(D_1) - \xi_K p^K(D_1)\|^2 \right] E \left[ \|\mu_0(D_1) - \pi_K p^K(D_1)\|^2 \right] \leq CnK^{-16} \rightarrow 0.$$

Next,

$$\zeta_0(K)^4 \frac{K}{n} \leq C \frac{K^5}{n} \rightarrow 0$$

and finally

$$\zeta_0(K)^2 E \left[ \|\mu_0(D_1) - \pi_K p^K(D_1)\|^2 \right] \leq CK^{2-2\alpha} \rightarrow 0$$

and

$$E \left[ \|\delta(D_1) - \xi_K p^K(D_1)\|^2 \right] \leq CK^{-2\alpha} \rightarrow 0.$$

Thus, condition 6.6 holds.

By inspection of the proof of Theorem 6.1 in Newey (1994), condition 6.4(ii) implies 5.1(ii) therein, conditions 6.5 and 6.2 imply 5.2 therein, and condition 6.6 implies 5.3 therein. Then, conditions 5.1-5.3 in Newey (1994) hold, and thus by his Lemma 5.1,

$$\frac{1}{\sqrt{n}} \sum_i g(Z_i, \delta_{10}, \hat{\mu}) = \frac{1}{\sqrt{n}} \sum_i [g(Z_i, \mu_0) + \alpha(Z_i)] + o_P(1) \rightarrow_d \mathcal{N}(0, V)$$

where

$$\alpha(Z) = \delta(D_1) \begin{bmatrix} \Delta Y(1 - S) - \mu_{10}(D_1) \\ (1 - S) - \mu_{20}(D_1) \end{bmatrix} = -\frac{E\left(\frac{S}{\Delta D} \mid D_1\right)}{E[1 - S \mid D_1]}(1 - S)(\Delta Y - \mu_0(D_1))$$

and  $V = E[(g(Z_i, \mu_0) + \alpha(Z_i))(g(Z_i, \mu_0) + \alpha(Z_i))']$ . Finally note that:

$$\sqrt{n}(\hat{\delta}_1 - \delta_{10}) = \frac{n}{\sum_i S_i} \cdot \frac{1}{\sqrt{n}} \sum_i g(Z_i, \delta_{10}, \hat{\mu}) = \frac{1}{E[S]} \cdot \frac{1}{\sqrt{n}} \sum_i [g(Z_i, \mu_0) + \alpha(Z_i)] + o_P(1)$$

and the result follows defining  $\psi_1 = [g(Z_i, \mu_0) + \alpha(Z_i)]/E[S]$ .  $\square$

#### A.1.4 Theorem 4

We only prove the first point, as the proof of the second point is similar and (1.9)-(1.10) follow by combining these two points. Moreover, the proof of (1.5) is similar to the proof of Theorem 1 so it is omitted. We thus focus on (1.6) hereafter.

For all  $d_1 \in \text{Supp}(D_1|S_+ = 1)$ , by Point 1 of Assumption 7,  $d_1 \in \text{Supp}(D_1|S = 0)$ . Thus,  $E(\Delta Y|D_1 = d_1, S = 0)$  is well-defined. Then, using the same reasoning as that used to show (A.2) above, we obtain

$$E(Y_2(d_1) - Y_1(d_1)|D_1 = d_1, S_+ = 1) = E(\Delta Y|D_1 = d_1, S = 0).$$

Now, let  $\text{Supp}(D_1|S_+ = 1)^c$  be the complement of  $\text{Supp}(D_1|S_+ = 1)$ . For all  $d_1 \in \text{Supp}(D_1|S = 0) \cap \text{Supp}(D_1|S_+ = 1)^c$ ,  $P(S_+ = 1|D_1 = d_1) = 0$ . Then, with the convention that  $E(\Delta Y|D_1 = d_1, S_+ = 1)P(S_+ = 1|D_1 = d_1) = 0$ ,

$$\begin{aligned} & E(\Delta Y|D_1 = d_1, S = 0)P(S_+ = 1|D_1 = d_1) \\ &= E(Y_2(d_1) - Y_1(d_1)|D_1 = d_1, S_+ = 1)P(S_+ = 1|D_1 = d_1). \end{aligned}$$

Combining the two preceding displays implies that for all  $d_1 \in \text{Supp}(D_1|S = 0)$ ,

$$\begin{aligned} & E(\Delta Y|D_1 = d_1, S = 0)P(S_+ = 1|D_1 = d_1) \\ &= E(Y_2(d_1) - Y_1(d_1)|D_1 = d_1, S_+ = 1)P(S_+ = 1|D_1 = d_1). \end{aligned}$$

Hence, by repeated use of the law of iterated expectation,

$$\begin{aligned}
& E \left( \Delta Y \frac{P(S_+ = 1|D_1)}{P(S = 0|D_1)} \frac{P(S = 0)}{P(S_+ = 1)} \middle| S = 0 \right) \\
&= E \left( E[Y_2(D_1) - Y_1(D_1)|D_1, S_+ = 1] \frac{P(S_+ = 1|D_1)}{P(S = 0|D_1)} \frac{P(S = 0)}{P(S_+ = 1)} \middle| S = 0 \right) \\
&= E \left( E[Y_2(D_1) - Y_1(D_1)|D_1, S_+ = 1] \frac{P(S_+ = 1|D_1)}{P(S = 0|D_1)} \frac{1 - S}{P(S_+ = 1)} \right) \\
&= E \left( E[Y_2(D_1) - Y_1(D_1)|D_1, S_+ = 1] \frac{P(S_+ = 1|D_1)}{P(S_+ = 1)} \right) \\
&= E \left( E[Y_2(D_1) - Y_1(D_1)|D_1, S_+ = 1] \frac{S_+}{P(S_+ = 1)} \right) \\
&= E(Y_2(D_1) - Y_1(D_1)|S_+ = 1).
\end{aligned}$$

The result follows after some algebra.  $\square$

### A.1.5 Theorem 5

We prove the result for the propensity-score-based estimator and drop the “ps” subscript to reduce notation.

Let  $\mu_1(d) = E[S_+|D_1 = d]$ ,  $\mu_2(d) = E[1 - S|D_1 = d]$ ,  $\mu_3(d) = E[S_-|D_1 = d]$  and  $\mu_Y(D_1) = E[\Delta Y(1 - S)|D_1]$ . The logit series estimators of the unknown functions  $\mu_j(d)$  are given by  $\hat{\mu}_j(d) = \Lambda(P^K(d)' \hat{\pi}_j)$  where  $\Lambda(z) = 1/(1 + e^{-z})$  is the logit function and

$$0 = \sum_i (S_{ji} - \Lambda(P^K(D_{1i})' \hat{\pi}_j)) P^K(D_{1i})$$

for  $S_{ji}$  equal to  $1 - S_i$ ,  $S_{i+}$  or  $S_{i-}$ . Under Assumption 8, there exists a constant  $\pi_{j,K}$  that satisfies:

$$\left\| \log \left( \frac{\mu_j}{1 - \mu_j} \right) - (P^K)' \pi_{j,K} \right\|_{\infty} = O(K^{-\alpha})$$

and we let  $\mu_{j,K} = \Lambda(P^K(D_{1i})' \pi_{j,K})$ . We suppress the  $n$  subscript on  $K$  to reduce notation and let  $\mu_{ji} := \mu_j(D_{1i})$  and  $\hat{\mu}_{ji} := \hat{\mu}_j(D_{1i})$ . Under Assumption 8 part 1, Lemma A.15 in Newey (1995) ensures that the smallest eigenvalue of  $E[P^K(D_1)P^K(D_1)']$ , is bounded away from zero uniformly over  $K$ . In addition, Cattaneo (2010) shows that under Assumption 8, the multinomial logit series estimator satisfies:

$$\|\mu_{j,K} - \mu_j\|_{\infty} = O(K^{-\alpha}), \quad \|\hat{\pi}_j - \pi_{j,K}\| = O_P \left( \sqrt{\frac{K}{n}} + K^{-\alpha+1/2} \right)$$

and

$$\|\hat{\mu}_j - \mu_j\|_{\infty} = O_P \left( \zeta(K) \left( \sqrt{\frac{K}{n}} + K^{-\alpha+1/2} \right) \right)$$



where  $\zeta(K) = \sup_{d \in I} \|P^K(d)\|$ . Newey (1994) also shows that for orthonormal polynomials,  $\zeta(K)$  is bounded above by  $CK$  for some constant  $C$ , which implies in our case that  $\|\hat{\mu}_j - \mu_j\|_\infty = O_P\left(K\left(\sqrt{\frac{K}{n}} + K^{-\alpha+1/2}\right)\right)$ . Throughout the proof, we also use the fact that by a second-order mean value expansion, there exists a  $\tilde{\pi}_j$  such that:

$$\begin{aligned}\hat{\mu}_{ji} - \mu_{ji,K} &= \Lambda(P^K(D_{1i})'\hat{\pi}_j) - \Lambda(P^K(D_{1i})'\pi_{j,K}) \\ &= \dot{\Lambda}(P^K(D_{1i})'\pi_{j,K})P^K(D_{1i})'(\hat{\pi}_j - \pi_{j,K}) + \ddot{\Lambda}(P^K(D_{1i})'\tilde{\pi}_j)(P^K(D_{1i})'(\hat{\pi}_j - \pi_{j,K}))^2\end{aligned}$$

where both  $\dot{\Lambda}(z)$  and  $\ddot{\Lambda}(z)$  are bounded. We start by considering the  $\delta_{2+}$  parameter and omit the “ps” superscript to reduce notation. Recall that

$$\hat{\delta}_{2+} = \frac{1}{\sum_i \Delta D_i S_{i+}} \sum_i \left\{ \Delta Y_i S_{i+} - \Delta Y_i (1 - S_i) \frac{\hat{\mu}_{1i}}{\hat{\mu}_{2i}} \right\}.$$

Thus,

$$\sqrt{n}(\hat{\delta}_{2+} - \delta_{2+}) = \frac{1}{E[\Delta DS_+]} \cdot \frac{1}{\sqrt{n}} \sum_i \left\{ \Delta Y_i S_{i+} - \Delta Y_i (1 - S_i) \frac{\hat{\mu}_{1i}}{\hat{\mu}_{2i}} - \delta_{2+} E[\Delta DS_+] \right\} + o_P(1).$$

Define:

$$V_i = \Delta Y_i S_{i+} - \Delta Y_i (1 - S_i) \frac{\hat{\mu}_{1i}}{\hat{\mu}_{2i}} - \delta_{2+} E[\Delta DS_+].$$

Let  $\psi_{2+,i}$  be the influence function defined in the statement of the theorem. Using the identity:

$$\frac{1}{\hat{b}} - \frac{1}{b} = -\frac{1}{b^2}(\hat{b} - b) + \frac{1}{b^2\hat{b}}(\hat{b} - b)^2$$

we have, after some rearranging,

$$\begin{aligned}
\frac{1}{\sqrt{n}} \sum_i V_i &= E[\Delta DS_+] \cdot \frac{1}{\sqrt{n}} \sum_i \psi_{2+,i} \\
&\quad - \frac{1}{\sqrt{n}} \sum_i \left( \frac{\Delta Y_i(1 - S_i)}{\mu_{2i}} - \frac{\mu_{Yi}}{\mu_{2i}} \right) (\hat{\mu}_{1i} - \mu_{1i}) \\
&\quad + \frac{1}{\sqrt{n}} \sum_i (\Delta Y_i(1 - S_i) - \mu_{Yi}) \frac{\mu_{1i}}{\mu_{2i}^2} (\hat{\mu}_{2i} - \mu_{2i}) \\
&\quad - \frac{1}{\sqrt{n}} \sum_i \Delta Y_i(1 - S_i) \frac{\mu_{1i}}{\mu_{2i}^2 \hat{\mu}_{2i}} (\hat{\mu}_{2i} - \mu_{2i})^2 \\
&\quad + \frac{1}{\sqrt{n}} \sum_i \frac{\Delta Y_i(1 - S_i)}{\mu_{2i}^2} (\hat{\mu}_{1i} - \mu_{1i}) (\hat{\mu}_{2i} - \mu_{2i}) \\
&\quad - \frac{1}{\sqrt{n}} \sum_i \frac{\Delta Y_i(1 - S_i)}{\mu_{2i}^2 \hat{\mu}_{2i}} (\hat{\mu}_{1i} - \mu_{1i}) (\hat{\mu}_{2i} - \mu_{2i})^2 \\
&\quad + \frac{1}{\sqrt{n}} \sum_i \frac{\mu_{Yi}}{\mu_{2i}} (S_{i+} - \hat{\mu}_{1i}) \\
&\quad - \frac{1}{\sqrt{n}} \sum_i \frac{\mu_{Yi} \mu_{1i}}{\mu_{2i}^2} (1 - S_i - \hat{\mu}_{2i}).
\end{aligned}$$

which we rewrite as:

$$\frac{1}{\sqrt{n}} \sum_i V_i = E[\Delta DS_+] \cdot \frac{1}{\sqrt{n}} \sum_i \psi_{2+,i} + \sum_{j=1}^7 A_{j,n}$$

where each  $A_{j,n}$  represents one term on the above display. We now bound each one of these terms.

**Term 1.** For the first term, we have that:

$$\begin{aligned}
-A_{1,n} &= \frac{1}{\sqrt{n}} \sum_i \left( \frac{\Delta Y_i(1 - S_i)}{\mu_{2i}} - \frac{\mu_{Yi}}{\mu_{2i}} \right) (\hat{\mu}_{1i} - \mu_{1i}) \\
&= \frac{1}{\sqrt{n}} \sum_i \left( \frac{\Delta Y_i(1 - S_i)}{\mu_{2i}} - \frac{\mu_{Yi}}{\mu_{2i}} \right) (\hat{\mu}_{1i} - \mu_{1i,K}) \\
&\quad + \frac{1}{\sqrt{n}} \sum_i \left( \frac{\Delta Y_i(1 - S_i)}{\mu_{2i}} - \frac{\mu_{Yi}}{\mu_{2i}} \right) (\mu_{1i,K} - \mu_{1i}) \\
&= A_{11,n} + A_{12,n}.
\end{aligned}$$

Now, by a second-order mean value expansion,

$$\begin{aligned}
A_{11,n} &= \frac{1}{\sqrt{n}} \sum_i \left( \frac{\Delta Y_i(1 - S_i)}{\mu_{2i}} - \frac{\mu_{Yi}}{\mu_{2i}} \right) \dot{\Lambda}(P^K(D_{1i})' \pi_{j,K}) P^K(D_{1i})' (\hat{\pi}_K - \pi_K) \\
&\quad + \frac{1}{\sqrt{n}} \sum_i \left( \frac{\Delta Y_i(1 - S_i)}{\mu_{2i}} - \frac{\mu_{Yi}}{\mu_{2i}} \right) \ddot{\Lambda}(P^K(D_{1i})' \tilde{\pi}) (P^K(D_{1i})' (\hat{\pi}_K - \pi_K))^2 \\
&= A_{111,n} + A_{112,n}.
\end{aligned}$$

Next note that

$$|A_{111,n}| \leq \|\hat{\pi}_K - \pi_K\| \left\| \frac{1}{\sqrt{n}} \sum_i \left( \frac{\Delta Y_i(1 - S_i)}{\mu_{2i}} - \frac{\mu_{Y_i}}{\mu_{2i}} \right) \dot{\Lambda}(P^K(D_{1i})' \pi_{j,K}) P^K(D_{1i})' \right\|.$$

Now,  $\|\hat{\pi}_K - \pi_K\| = O_P\left(\left(\sqrt{K/n} + K^{-\alpha+1/2}\right)\right)$ . Let

$$U_i = (U_i^1, \dots, U_i^K)' := \left( \frac{\Delta Y_i(1 - S_i)}{\mu_{2i}} - \frac{\mu_{Y_i}}{\mu_{2i}} \right) \dot{\Lambda}(P^K(D_{1i})' \pi_{j,K}) P^K(D_{1i})'.$$

We have  $E[U_i] = E[E[U_i|D_{1i}]] = 0$  and

$$\begin{aligned} E\left[\|U_i\|^2\right] &\leq E\left[\left(\frac{\Delta Y_i(1 - S_i)}{\mu_{2i}} - \frac{\mu_{Y_i}}{\mu_{2i}}\right)^2 \|P^K(D_{1i})\|^2\right] \\ &\leq CE\left[\|P^K(D_{1i})\|^2\right] \\ &= CE\left[\text{trace}\{P^K(D_{1i})' P^K(D_{1i})\}\right] \\ &= C \times \text{trace}\left(E\left[P^K(D_{1i}) P^K(D_{1i})'\right]\right) \\ &= CK, \end{aligned} \tag{A.5}$$

since the polynomials can be chosen such that  $E\left[P^K(D_{1i}) P^K(D_{1i})'\right] = I_K$ , see Newey (1997), page 161. Hence,

$$\begin{aligned} E\left[\left\|\frac{1}{\sqrt{n}} \sum_i U_i\right\|^2\right] &= E\left[\sum_{j=1}^K \left(\frac{1}{\sqrt{n}} \sum_i U_i^j\right)^2\right] \\ &= \sum_{j=1}^K \frac{1}{n} \sum_{i,i'} E\left[U_i^j U_{i'}^j\right] \\ &= \sum_{j=1}^K \frac{1}{n} \sum_{i=1}^n E\left[U_i^{j2}\right] \\ &= E\left[\|U_1\|^2\right]. \end{aligned}$$

Therefore, by Markov's inequality,

$$A_{111,n} = O_P\left(K^{1/2} \left(\sqrt{\frac{K}{n}} + K^{-\alpha+1/2}\right)\right).$$

Next,

$$\begin{aligned}
|A_{112,n}| &\leq C\sqrt{n} \|\hat{\pi}_K - \pi_K\|^2 \frac{1}{n} \sum_i \left| \frac{\Delta Y_i(1-S_i)}{\mu_{2i}} - \frac{\mu_{Y_i}}{\mu_{2i}} \right| \|P^K(D_{1i})\|^2 \\
&= O_P \left[ \sqrt{n} \left( \frac{K}{n} + K^{-2\alpha+1} \right) E \left( \left| \frac{\Delta Y_i(1-S_i)}{\mu_{2i}} - \frac{\mu_{Y_i}}{\mu_{2i}} \right| \|P^K(D_{1i})\|^2 \right) \right] \\
&= O_P \left( \sqrt{n} K \left( \frac{K}{n} + K^{-2\alpha+1} \right) \right),
\end{aligned}$$

where the first inequality follows by Cauchy-Schwarz inequality, the second by Markov's inequality and the third by the same reasoning as to obtain (A.5). Hence,

$$A_{11,n} = O_P \left( K^{1/2} \left( \sqrt{\frac{K}{n}} + K^{-\alpha+1/2} \right) \right) + O_P \left( \sqrt{n} K \left( \frac{K}{n} + K^{-2\alpha+1} \right) \right).$$

Finally, for  $A_{12,n}$  we have that

$$E \left[ \left( \frac{\Delta Y_i(1-S_i)}{\mu_{2i}} - \frac{\mu_{Y_i}}{\mu_{2i}} \right) (\mu_{1i,K} - \mu_{1i}) \middle| D_1 \right] = 0$$

and

$$E \left[ \left\| \left( \frac{\Delta Y_i(1-S_i)}{\mu_{2i}} - \frac{\mu_{Y_i}}{\mu_{2i}} \right) (\mu_{1i,K} - \mu_{1i}) \right\|^2 \right] \leq C \|\mu_{1,K} - \mu_1\|_\infty^2 = O(K^{-2\alpha})$$

and therefore

$$A_{1,n} = O_P \left( K^{1/2} \left( \sqrt{\frac{K}{n}} + K^{-\alpha+1/2} \right) \right) + O_P \left( \sqrt{n} K \left( \frac{K}{n} + K^{-2\alpha+1} \right) \right) + O_P(K^{-\alpha}).$$

**Term 2.** This follows by the same argument as that of Term 1 and we obtain:

$$A_{2,n} = O_P \left( K^{1/2} \left( \sqrt{\frac{K}{n}} + K^{-\alpha+1/2} \right) \right) + O_P \left( \sqrt{n} K \left( \frac{K}{n} + K^{-2\alpha+1} \right) \right) + O_P(K^{-\alpha}).$$

**Term 3.** For the third term, since  $\mu_{2i}$  is uniformly bounded and  $\hat{\mu}_2$  converges uniformly to  $\mu_2$ , for  $n$  large enough

$$|A_{3,n}| \leq \sqrt{n} \|\hat{\mu}_2 - \mu_2\|_\infty^2 \frac{1}{C} \frac{1}{n} \sum_i |\Delta Y_i(1-S_i)| = O_P \left( \sqrt{n} K^2 \left( \frac{K}{n} + K^{-2\alpha+1} \right) \right).$$

**Term 4.** For the fourth term,

$$|A_{4,n}| \leq \sqrt{n} \|\hat{\mu}_1 - \mu_1\|_\infty \|\hat{\mu}_2 - \mu_2\|_\infty \frac{1}{C} \frac{1}{n} \sum_i |\Delta Y_i(1-S_i)| = O_P \left( \sqrt{n} K^2 \left( \frac{K}{n} + K^{-2\alpha+1} \right) \right)$$

**Term 5.** For the fifth term, since  $\mu_{2i}$  is uniformly bounded and  $\hat{\mu}_2$  converges uniformly to  $\mu_2$ , for  $n$  large enough

$$|A_{5,n}| \leq \sqrt{n} \|\hat{\mu}_1 - \mu_1\|_\infty \|\hat{\mu}_2 - \mu_2\|_\infty^2 \frac{1}{C} \frac{1}{n} \sum_i |\Delta Y_i (1 - S_i)| = O_P \left( \sqrt{n} K^3 \left( \left( \frac{K}{n} \right)^{3/2} + K^{-3\alpha+3/2} \right) \right).$$

**Term 6.** For the sixth term, let  $\gamma_{6,K}$  be the population coefficient from a (linear) series approximation to the function  $\mu_Y(D_1)/\mu_2(D_1)$ . Then we have that

$$\begin{aligned} A_{6,n} &= \frac{1}{\sqrt{n}} \sum_i \left( \frac{\mu_{Yi}}{\mu_{2i}} - P^K(D_{1i})' \gamma_{6,K} \right) (S_{i+} - \hat{\mu}_{1i}) + \frac{1}{\sqrt{n}} \sum_i P^K(D_{1i})' \gamma_{6,K} (S_{i+} - \hat{\mu}_{1i}) \\ &= \frac{1}{\sqrt{n}} \sum_i \left( \frac{\mu_{Yi}}{\mu_{2i}} - P^K(D_{1i})' \gamma_{6,K} \right) (S_{i+} - \hat{\mu}_{1i}) \end{aligned}$$

because the last term in the second line equals zero by the first-order conditions of the logit series estimator. Next, we have that

$$\begin{aligned} \frac{1}{\sqrt{n}} \sum_i \left( \frac{\mu_{Yi}}{\mu_{2i}} - P^K(D_{1i})' \gamma_{6,K} \right) (S_{i+} - \hat{\mu}_{1i}) &= \frac{1}{\sqrt{n}} \sum_i \left( \frac{\mu_{Yi}}{\mu_{2i}} - P^K(D_{1i})' \gamma_{6,K} \right) (S_{i+} - \mu_{1i}) \\ &\quad - \frac{1}{\sqrt{n}} \sum_i \left( \frac{\mu_{Yi}}{\mu_{2i}} - P^K(D_{1i})' \gamma_{6,K} \right) (\hat{\mu}_{1i} - \mu_{1i}) \\ &= A_{61,n} + A_{62,n}. \end{aligned}$$

Now, for  $A_{61,n}$ , we have that

$$E \left[ \left( \frac{\mu_{Yi}}{\mu_{2i}} - P^K(D_{1i})' \gamma_{6,K} \right) (S_{i+} - \mu_{1i}) \middle| D_1 \right] = 0$$

and

$$E \left[ (S_{i+} - \mu_{1i})^2 \left\| \left( \frac{\mu_{Yi}}{\mu_{2i}} - P^K(D_{1i})' \gamma_{6,K} \right) \right\|^2 \right] \leq O(K^{-2\alpha})$$

so that

$$A_{61,n} = O_P(K^{-\alpha}).$$

On the other hand, for  $A_{62,n}$ , we have that

$$|A_{62,n}| \leq \sqrt{n} \left\| \frac{\mu_Y}{\mu_2} - (P^K)' \gamma_{6,K} \right\|_\infty \|\hat{\mu}_1 - \mu_1\|_\infty = O_P \left( \sqrt{n} K^{1-\alpha} \left( \sqrt{\frac{K}{n}} + K^{-\alpha+1/2} \right) \right)$$

from which

$$A_{6,n} = O_P \left( \sqrt{n} K^{1-\alpha} \left( \sqrt{\frac{K}{n}} + K^{-\alpha+1/2} \right) + K^{-\alpha} \right).$$

**Term 7.** This follows by the same argument as that of Term 6 and we obtain

$$A_{7,n} = O_P \left( \sqrt{n} K^{1-\alpha} \left( \sqrt{\frac{K}{n}} + K^{-\alpha+1/2} \right) + K^{-\alpha} \right).$$

Collecting all the terms, it follows that under the conditions

$$\frac{K^6}{n} \rightarrow 0, \quad \frac{K^{4\alpha-6}}{n} \rightarrow \infty, \quad \alpha > 3$$

we obtain

$$\sqrt{n}(\hat{\delta}_{2+} - \delta_{2+}) = \frac{1}{\sqrt{n}} \sum_i \psi_{2+,i} + o_P(1).$$

Setting  $\alpha = 4$ , this implies

$$\frac{K^6}{n} \rightarrow 0, \quad \frac{K^{10}}{n} \rightarrow \infty.$$

These conditions are satisfied when  $K = n^\nu$  for  $1/(4\alpha - 6) < \nu < 1/6$  or in this case  $1/10 < \nu < 1/6$ .

By an analogous argument, we can show that under the same conditions

$$\sqrt{n}(\hat{\delta}_{2-} - \delta_{2-}) = \frac{1}{\sqrt{n}} \sum_i \psi_{2-,i} + o_P(1)$$

and the result follows by a multivariate CLT. Finally, notice that letting  $\mu_{1-}(d) = E[S_- | D_1 = d]$  and  $\hat{\mu}_{ji-} = \hat{\mu}_{1-}(D_{1i})$ , and using that  $\text{sgn}(\Delta D_i) = S_{i+} - S_{i-}$  and  $|\Delta D_i| = \Delta D_i(S_{i+} - S_{i-})$ , after some simple manipulations:

$$\hat{\delta}_2 = \frac{1}{\sum_i |\Delta D_i|} \sum_i \left\{ \Delta Y_i(S_{i+} - S_{i-}) - \Delta Y_i(1 - S_i) \left( \frac{\hat{\mu}_{1i-} - \hat{\mu}_{1i-}}{\hat{\mu}_{2i}} \right) \right\}$$

which is analogous to  $\hat{\delta}_{2+}$  replacing  $S_{i+}$  by  $(S_{i+} - S_{i-})$  and the denominator by  $\sum_i |\Delta D_i|$ . Thus, under the same conditions

$$\sqrt{n}(\hat{\delta}_2 - \delta_2) = \frac{1}{\sqrt{n}} \sum_i \psi_{2,i} + o_P(1)$$

where  $\psi_{2,i}$  is defined in the statement of the theorem  $\square$

### A.1.6 Proposition 1

If  $D_2 \geq D_1$  and  $\Delta D \perp\!\!\!\perp D_1$ ,

$$\begin{aligned}\psi_1 &= \frac{1}{E(S)} \left\{ \left( \frac{S}{\Delta D} - E \left( \frac{S}{\Delta D} \right) \frac{(1-S)}{E[1-S]} \right) [\Delta Y - E(\Delta Y|D_1, S=0)] - \delta_1 S \right\}, \\ \psi_2 &= \frac{1}{E(\Delta D)} \left\{ \left( S - E(S) \frac{(1-S)}{1-E(S)} \right) \times (\Delta Y - E(\Delta Y|D_1, S=0)) - \delta_2 \Delta D \right\}.\end{aligned}$$

If  $(Y_2(D_2) - Y_2(D_1))/(D_2 - D_1) = \delta$ , then  $\delta_1 = \delta_2 = \delta$ , and  $\Delta Y = \Delta Y(D_1) + \Delta D\delta$ , so after some algebra the previous display simplifies to

$$\begin{aligned}\psi_1 &= \frac{1}{\Delta D} \left( \frac{S}{E(S)} - \frac{(1-S)}{E[1-S]} \frac{\Delta D}{E(S)} E \left( \frac{S}{\Delta D} \right) \right) \times (\Delta Y(D_1) - E(\Delta Y(D_1)|D_1, S=0)). \\ \psi_2 &= \frac{1}{E(\Delta D)} \left( S - (1-S) \frac{E(S)}{1-E(S)} \right) \times (\Delta Y(D_1) - E(\Delta Y(D_1)|D_1, S=0)).\end{aligned}$$

Then, under Assumption 1,

$$E(\psi_1|D_1, D_2) = E(\psi_2|D_1, D_2) = 0.$$

Then, using the law of total variance, the fact that  $V(\Delta Y(D_1)|D_1, D_2) = \sigma^2$ , and some algebra,

$$\begin{aligned}V(\psi_1) &= E(V(\psi_1|D_1, D_2)) \\ &= \sigma^2 E \left( \left[ \frac{\frac{S}{\Delta D} - \frac{1-S}{1-E(S)} E \left( \frac{S}{\Delta D} \right)}{E(S)} \right]^2 \right) \\ &= \sigma^2 \left[ \frac{E(1/(\Delta D)^2|S=1)}{P(S=1)} + \frac{(E(1/\Delta D|S=1))^2}{P(S=0)} \right],\end{aligned}$$

and

$$\begin{aligned}V(\psi_2) &= E(V(\psi_2|D_1, D_2)) \\ &= \sigma^2 E \left( \left[ \frac{S - (1-S) \frac{E(S)}{1-E(S)}}{E(\Delta D)} \right]^2 \right) \\ &= \sigma^2 \frac{1}{(E(\Delta D|S=1))^2} \left[ \frac{1}{P(S=1)} + \frac{1}{P(S=0)} \right].\end{aligned}$$

The inequality follows from the convexity of  $x \mapsto x^2$ , the convexity of  $x \mapsto 1/x$  on  $\mathbb{R}^+ \setminus \{0\}$  and  $\Delta D|S = 1 \in \mathbb{R}^+ \setminus \{0\}$ , Jensen's inequality, and  $x \mapsto x^2$  increasing on  $\mathbb{R}^+$ , which together imply that

$$E(1/(\Delta D)^2|S = 1) \geq (E(1/\Delta D|S = 1))^2 \geq \frac{1}{(E(\Delta D|S = 1))^2}.$$

Finally, Jensen's inequality is strict for strictly convex functions, unless the random variable is actually constant. The last claim of the proposition follows.

### A.1.7 Theorem 6

The parameter  $\delta_{IV}$  can be written as:

$$\delta_{IV} = \frac{E[\text{sgn}(\Delta Z) (Y_2(D_2(Z_2)) - Y_2(D_2(Z_1))) | SC = 1]}{E[|D_2(Z_2) - D_2(Z_1)| | SC = 1]}.$$
 (A.6)

The regression-based estimand is:

$$\frac{E[\text{sgn}(\Delta Z) (\Delta Y - E(\Delta Y|Z_1, S^I = 0, D_1))]}{E[\text{sgn}(\Delta Z) (\Delta D - E(\Delta D|Z_1, S^I = 0, D_1))]}.$$

Following previous arguments, the conditional expectations are well-defined under Assumption 13. For the denominator,

$$\begin{aligned} & E[\text{sgn}(\Delta Z) (\Delta D - E(\Delta D|Z_1, S^I = 0, D_1))] \\ &= E[\text{sgn}(\Delta Z) (D_2(Z_2) - D_2(Z_1))] + E[\text{sgn}(\Delta Z) (D_2(Z_1) - D_1(Z_1) - E(\Delta D|Z_1, S^I = 0, D_1))] \\ &= E[\text{sgn}(\Delta Z) (D_2(Z_2) - D_2(Z_1))] \end{aligned}$$

because

$$\begin{aligned} & E[\text{sgn}(\Delta Z) (D_2(Z_1) - D_1(Z_1) - E(\Delta D|Z_1, S^I = 0, D_1))] \\ &= E\{E[\text{sgn}(\Delta Z) (D_2(Z_1) - D_1(Z_1) - E(\Delta D|Z_1, S^I = 0, D_1)) | Z_1, Z_2, D_1]\} \\ &= E\{\text{sgn}(\Delta Z) (E(\Delta D(Z_1)|Z_1, Z_2, D_1) - E(\Delta D(Z_1)|Z_1, S^I = 0, D_1))\} \\ &= 0, \end{aligned}$$



by Assumption 9. On the other hand,

$$\begin{aligned}
E[\text{sgn}(\Delta Z)(D_2(Z_2) - D_2(Z_1))] &= E[\text{sgn}(\Delta Z)(D_2(Z_2) - D_2(Z_1)) | D_2(Z_2) \neq D_2(Z_1)] \\
&\quad \times P(D_2(Z_2) \neq D_2(Z_1)) \\
&= E[|D_2(Z_2) - D_2(Z_1)| | SC = 1] P(SC = 1),
\end{aligned}$$

where the last equality follows from monotonicity (Assumption 10) and the definition of switchers-compliers.

Next, the numerator is:

$$\begin{aligned}
&E[\text{sgn}(\Delta Z)(\Delta Y - E(\Delta Y | Z_1, S^I = 0, D_1 = 0))] \\
&= E[\text{sgn}(\Delta Z)(Y_2(D_2(Z_2)) - Y_1(D_1(Z_1)) - E(\Delta Y | Z_1, S^I = 0, D_1 = 0))] \\
&= E[\text{sgn}(\Delta Z)(Y_2(D_2(Z_2)) - Y_2(D_2(Z_1)))],
\end{aligned}$$

using the parallel trends assumption as before. Then,

$$\begin{aligned}
&E[\text{sgn}(\Delta Z)(Y_2(D_2(Z_2)) - Y_2(D_2(Z_1)))] \\
&= E[\text{sgn}(\Delta Z)(Y_2(D_2(Z_2)) - Y_2(D_2(Z_1))) | SC = 1] P(SC = 1),
\end{aligned}$$

and thus, in view of (A.6),

$$\frac{E[\text{sgn}(\Delta Z)(\Delta Y - E(\Delta Y | Z_1, S^I = 0, D_1))]}{E[\text{sgn}(\Delta Z)(\Delta D - E(\Delta D | Z_1, S^I = 0, D_1))]} = \delta_{IV}.$$

For the propensity-score estimand, notice that

$$\begin{aligned}
&\frac{E[\text{sgn}(\Delta Z)(\Delta Y - E(\Delta Y | Z_1, S^I = 0, D_1))]}{E[\text{sgn}(\Delta Z)(\Delta D - E(\Delta D | Z_1, S^I = 0, D_1))]} \\
&= \frac{E[\text{sgn}(\Delta Z)\Delta Y] - E[\text{sgn}(\Delta Z)E(\Delta Y | Z_1, S^I = 0, D_1)]}{E[\text{sgn}(\Delta Z)\Delta D] - E[\text{sgn}(\Delta Z)E(\Delta D | Z_1, S^I = 0, D_1)]}.
\end{aligned}$$

Then, using  $\text{sgn}(\Delta Z) = S_+^I - S_-^I$ , the law of iterated expectations and Assumption 9,

$$\begin{aligned}
E[\text{sgn}(\Delta Z)E(\Delta D|Z_1, S^I = 0, D_1)] &= E\left[(S_+^I - S_-^I)E\left(\frac{\Delta D(1 - S^I)}{P(S^I = 0|Z_1, D_1)}\middle|Z_1, D_1\right)\right] \\
&= E\left[E(S_+^I - S_-^I|Z_1, D_1)E\left(\frac{\Delta D(1 - S^I)}{P(S^I = 0|Z_1, D_1)}\middle|Z_1, D_1\right)\right] \\
&= E\left[E\left(\frac{\Delta D(1 - S^I)E(S_+^I - S_-^I|Z_1, D_1)}{P(S^I = 0|Z_1, D_1)}\middle|Z_1, D_1\right)\right] \\
&= E\left[\frac{\Delta D(1 - S^I)E(S_+^I - S_-^I|Z_1, D_1)}{P(S^I = 0|Z_1, D_1)}\right] \\
&= E\left[\Delta D\frac{E(S_+^I - S_-^I|Z_1, D_1)}{P(S^I = 0|Z_1, D_1)}P(S^I = 0)\middle|S^I = 0\right] \\
&= E\left[\Delta D\frac{P(S_+^I = 1|Z_1, D_1) - P(S_-^I = 1|Z_1, D_1)}{P(S^I = 0|Z_1, D_1)}\middle|S^I = 0\right] \\
&\quad \times P(S^I = 0),
\end{aligned}$$

as required. The same argument replacing  $\Delta D$  by  $\Delta Y$  completes the proof  $\square$

### A.1.8 Theorem 7

Using the same steps as in the proof of Theorem 1, one can show that for all  $t \geq 2$ ,

$$\delta_{1t} = E\left(\frac{Y_t - Y_{t-1} - E(Y_t - Y_{t-1}|D_{t-1}, S_t = 0)}{D_t - D_{t-1}}\middle|S_t = 1\right).$$

This proves the result  $\square$

### A.1.9 Theorem 8

The proof is similar to that of Theorem 7, and is therefore omitted.



## Appendix B

# Estimating heterogeneous peer effects with partial population experiments

### B.1 Proofs

#### B.1.1 Reduced forms in the randomized experiment design

Let's start from the aggregated model, based on Assumption 1

$$\begin{aligned}\bar{y}_g^E &= \theta_0^w s_g \bar{y}_g^E + \theta_0^b (1 - s_g) \bar{y}_g^N + \delta_0 D_g + \bar{e}_g^E \\ \bar{y}_g^N &= \theta_0^b s_g \bar{y}_g^E + \theta_0^w (1 - s_g) \bar{y}_g^N + \bar{e}_g^N\end{aligned}$$

Plugging-in the expression of  $\bar{y}_g^E$  into the one of  $\bar{y}_g^N$ , we get

$$\bar{y}_g^N = \frac{\theta_0^b s_g}{1 - \theta_0^w s_g} (\theta_0^b (1 - s_g) \bar{y}_g^N + \delta_0 D_g + \bar{e}_g^E) + \theta_0^w (1 - s_g) \bar{y}_g^N + \bar{e}_g^N$$

Rearranging terms, we get

$$[1 - \theta_0^w + s_g(1 - s_g)((\theta_0^w)^2 - (\theta_0^b)^2)] \bar{y}_g^N = \theta_0^b s_g (\delta_0 D_g + \bar{e}_g^E) + (1 - \theta_0^w s_g) \bar{e}_g^N$$

Hence,

$$\begin{aligned}\bar{y}_g^N &= \frac{1 - \theta_0^w s_g}{1 - \theta_0^w + s_g(1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_g^N \\ &\quad + \frac{\theta_0^b s_g}{1 - \theta_0^w + s_g(1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_g^E \\ &\quad + \frac{\delta_0 \theta_0^b s_g}{1 - \theta_0^w + s_g(1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} D_g\end{aligned}$$

Now, plugging-in the reduced form expression of  $\bar{y}_g^N$  into  $\bar{y}_g^E$  and developping, we get

$$\begin{aligned}\bar{y}_g^E &= \frac{1 - \theta_0^w(1 - s_g)}{1 - \theta_0^w + s_g(1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_g^E \\ &\quad + \frac{\theta_0^b(1 - s_g)}{1 - \theta_0^w + s_g(1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_g^N \\ &\quad + \frac{\delta_0(1 - \theta_0^w(1 - s_g))}{1 - \theta_0^w + s_g(1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} D_g \\ &= \frac{\theta_0^b(1 - s_g)}{1 - \theta_0^w + s_g(1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_g^N \\ &\quad + \frac{1 - \theta_0^w(1 - s_g)}{1 - \theta_0^w + s_g(1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \bar{e}_g^E \\ &\quad + \delta_0 \left( 1 + s_g \cdot \frac{\theta_0^w - (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]}{1 - \theta_0^w + s_g(1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \right) D_g\end{aligned} \tag{B.1}$$

### B.1.2 Proposition 2

Suppose Assumptions 1, 2 and 3 are satisfied. For any  $k \in \{E, N\}$  and any  $s \in S$ ,

$$\tau^k(s) = \mathbb{E} [\bar{y}_g^k | D_g = 1, s_g = s] - \mathbb{E} [\bar{y}_g^k | D_g = 0, s_g = s]$$

This quantity exists as long as Assumption 3 holds and the relevant conditional expectations are well-defined. Let's show first that

$$\tau^k(s) = \mathbb{E} [\omega^R(D_g) \bar{y}_g^k | s_g = s] \text{ with } \omega^R(D_g) = \frac{D_g - P(D_g = 1)}{P(D_g = 1)(1 - P(D_g = 1))}$$

$$\begin{aligned}
\mathbb{E} [\omega^R(D_g)\bar{y}_g^k | s_g = s] &= \mathbb{E} \left[ \frac{D_g - P(D_g = 1 | s_g = s)}{P(D_g = 1 | s_g = s)(1 - P(D_g = 1 | s_g = s))} \bar{y}_g^k | s_g = s \right] \\
&= P(D_g = 1 | s_g = s) \mathbb{E} \left[ \frac{D_g - P(D_g = 1 | s_g = s)}{P(D_g = 1 | s_g = s)(1 - P(D_g = 1 | s_g = s))} \bar{y}_g^k | s_g = s, D_g = 1 \right] \\
&\quad + P(D_g = 0 | s_g = s) \mathbb{E} \left[ \frac{D_g - P(D_g = 1 | s_g = s)}{P(D_g = 1 | s_g = s)(1 - P(D_g = 1 | s_g = s))} \bar{y}_g^k | s_g = s, D_g = 0 \right] \\
&= \frac{P(D_g = 1)(1 - P(D_g = 1))}{P(D_g = 1)(1 - P(D_g = 1))} \mathbb{E} [\bar{y}_g^k | s_g = s, D_g = 1] \\
&\quad - \frac{(1 - P(D_g = 1))P(D_g = 1)}{P(D_g = 1)(1 - P(D_g = 1))} \mathbb{E} [\bar{y}_g^k | s_g = s, D_g = 0] \\
&= \mathbb{E} [\bar{y}_g^k | s_g = s, D_g = 1] - \mathbb{E} [\bar{y}_g^k | s_g = s, D_g = 0] \\
&= \tau^k(s)
\end{aligned}$$

The second equality is obtained using the law of iterated expectations and Assumption 2, in particular the fact that  $D_g \perp\!\!\!\perp s_g$ . The third and fourth equalities are algebra. Now,

$$\begin{aligned}
\mathbb{E} [\bar{y}_g^E | s_g = s, D_g] &= \frac{\theta_0^b(1-s)}{1 - \theta_0^w + s(1-s) [(\theta_0^w)^2 - (\theta_0^b)^2]} \mathbb{E} [\bar{e}_g^N | s_g = s] \\
&\quad + \frac{1 - \theta_0^w(1-s_{gt})}{1 - \theta_0^w + s_{gt}(1-s_{gt}) [(\theta_0^w)^2 - (\theta_0^b)^2]} \mathbb{E} [\bar{e}_g^E | s_g = s] \\
&\quad + \delta_0 \left( 1 + s_g \cdot \frac{\theta_0^w - (1-s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]}{1 - \theta_0^w + s_g(1-s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \right) D_g
\end{aligned}$$

Since, by Assumption 2,  $D_g \perp\!\!\!\perp (\bar{e}_g^E, \bar{e}_g^N)$ . It follows that

$$\tau^E(s) = \delta_0 \left( 1 + s_g \cdot \frac{\theta_0^w - (1-s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]}{1 - \theta_0^w + s_g(1-s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \right)$$

Similarly, one can show that

$$\tau^N(s) = \frac{\delta_0 \theta_0^b s_g}{1 - \theta_0^w + s_g(1-s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]}$$

Finally, equations (2.18) and (2.19) are immediately identified, based on the definition of the population multipliers.

### B.1.3 Proposition 2

From Assumption 1', using the same steps as in Appendix A.1, we get

$$\begin{aligned}\bar{y}_{gt}^E &= \frac{\theta_0^b(1-s_{gt})}{1-\theta_0^w+s_{gt}(1-s_{gt})[(\theta_0^w)^2-(\theta_0^b)^2]}\bar{e}_{gt}^N \\ &\quad + \frac{1-\theta_0^w(1-s_{gt})}{1-\theta_0^w+s_{gt}(1-s_{gt})[(\theta_0^w)^2-(\theta_0^b)^2]}\bar{e}_{gt}^E \\ &\quad + \frac{\delta_0(1-\theta_0^w(1-s_{gt}))}{1-\theta_0^w+s_{gt}(1-s_{gt})[(\theta_0^w)^2-(\theta_0^b)^2]}D_{gt} \\ \bar{y}_{gt}^N &= \frac{1-\theta_0^w s_{gt}}{1-\theta_0^w+s_{gt}(1-s_{gt})[(\theta_0^w)^2-(\theta_0^b)^2]}\bar{e}_{gt}^N \\ &\quad + \frac{\theta_0^b s_{gt}}{1-\theta_0^w+s_{gt}(1-s_{gt})[(\theta_0^w)^2-(\theta_0^b)^2]}\bar{e}_{gt}^E \\ &\quad + \frac{\delta_0\theta_0^b s_{gt}}{1-\theta_0^w+s_{gt}(1-s_{gt})[(\theta_0^w)^2-(\theta_0^b)^2]}D_{gt}\end{aligned}$$

Using Assumptions 2' and the first part of 3', we get

$$\begin{aligned}\Delta\bar{y}_g^E &= \frac{\theta_0^b(1-s_g)}{1-\theta_0^w+s_g(1-s_g)[(\theta_0^w)^2-(\theta_0^b)^2]}(\bar{e}_{g2}^N-\bar{e}_{g1}^N) \\ &\quad + \frac{1-\theta_0^w(1-s_g)}{1-\theta_0^w+s_g(1-s_g)[(\theta_0^w)^2-(\theta_0^b)^2]}(\bar{e}_{g2}^E-\bar{e}_{g1}^E) \\ &\quad + \frac{\delta_0(1-\theta_0^w(1-s_g))}{1-\theta_0^w+s_g(1-s_g)[(\theta_0^w)^2-(\theta_0^b)^2]}D_{g2} \\ \Delta\bar{y}_g^N &= \frac{1-\theta_0^w s_g}{1-\theta_0^w+s_g(1-s_g)[(\theta_0^w)^2-(\theta_0^b)^2]}(\bar{e}_{g2}^N-\bar{e}_{g1}^N) \\ &\quad + \frac{\theta_0^b s_g}{1-\theta_0^w+s_g(1-s_g)[(\theta_0^w)^2-(\theta_0^b)^2]}(\bar{e}_{g2}^E-\bar{e}_{g1}^E) \\ &\quad + \frac{\delta_0\theta_0^b s_g}{1-\theta_0^w+s_g(1-s_g)[(\theta_0^w)^2-(\theta_0^b)^2]}D_{g2}\end{aligned}$$

Then, since there are supposed to be well-defined, we can take the conditional expectations of these

quantities with respect to  $(D_{g2}, s_g)$ , we get

$$\begin{aligned} \mathbb{E} \left[ \Delta \bar{y}_g^E \middle| D_{g2}, s_g \right] &= \frac{\theta_0^b (1 - s_g)}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \mathbb{E} \left[ \bar{e}_{g2}^N - \bar{e}_{g1}^N \middle| D_{g2}, s_g \right] \\ &+ \frac{1 - \theta_0^w (1 - s_g)}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \mathbb{E} \left[ \bar{e}_{g2}^E - \bar{e}_{g1}^E \middle| D_{g2}, s_g \right] \\ &+ \frac{\delta_0 (1 - \theta_0^w (1 - s_g))}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} D_{g2} \end{aligned}$$

$$\begin{aligned} \mathbb{E} \left[ \Delta \bar{y}_g^N \middle| D_{g2}, s_g \right] &= \frac{1 - \theta_0^w s_g}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \mathbb{E} \left[ \bar{e}_{g2}^N - \bar{e}_{g1}^N \middle| D_{g2}, s_g \right] \\ &+ \frac{\theta_0^b s_g}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} \mathbb{E} \left[ \bar{e}_{g2}^E - \bar{e}_{g1}^E \middle| D_{g2}, s_g \right] \\ &+ \frac{\delta_0 \theta_0^b s_g}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]} D_{g2} \end{aligned}$$

Now,

$$\mathbb{E} \left[ \Delta \bar{y}_g^E \middle| D_{g2} = 1, s_g \right] - \mathbb{E} \left[ \Delta \bar{y}_g^E \middle| D_{g2} = 0, s_g \right] = \frac{\delta_0 (1 - \theta_0^w (1 - s_g))}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]}$$

$$\mathbb{E} \left[ \Delta \bar{y}_g^N \middle| D_{g2} = 1, s_g \right] - \mathbb{E} \left[ \Delta \bar{y}_g^N \middle| D_{g2} = 0, s_g \right] = \frac{\delta_0 \theta_0^b s_g}{1 - \theta_0^w + s_g (1 - s_g) [(\theta_0^w)^2 - (\theta_0^b)^2]}$$

using Assumption 4'. Then, using the same steps as in the proof for Proposition 4, one can show that, for any  $k \in \{E, N\}$  and any  $s \in \mathcal{S}$ ,

$$\tau^k(s) = \mathbb{E} \left[ \omega^{DiD}(D_{g2}, s_g) \Delta \bar{y}_g^k \middle| s_g = s \right] \text{ with } \omega^{DiD}(D_{g2}, s_g) = \frac{D_{g2} - P(D_{g2} = 1 | s_g)}{P(D_{g2} = 1 | s_g) (1 - P(D_{g2} = 1 | s_g))}$$



### B.1.4 Proposition 3

Let  $s_1, s_2$  be two elements of  $(0, 1)$ , such that  $s_1 \neq s_2$ . Whether Assumptions 1, 2 and 3 hold together or Assumptions 1', 2', 3' and 4' hold together, we have the following conditions

$$\tau^E(s_1) = \frac{\delta_0 (1 - \theta_0^w (1 - s_1))}{1 - \theta_0^w + s_1(1 - s_1) [(\theta_0^w)^2 - (\theta_0^b)^2]} \quad (\text{B.2})$$

$$\tau^N(s_1) = \frac{\delta_0 \theta_0^b s_1}{1 - \theta_0^w + s_1(1 - s_1) [(\theta_0^w)^2 - (\theta_0^b)^2]} \quad (\text{B.3})$$

$$\tau^E(s_2) = \frac{\delta_0 (1 - \theta_0^w (1 - s_2))}{1 - \theta_0^w + s_2(1 - s_2) [(\theta_0^w)^2 - (\theta_0^b)^2]} \quad (\text{B.4})$$

$$\tau^N(s_2) = \frac{\delta_0 \theta_0^b s_2}{1 - \theta_0^w + s_2(1 - s_2) [(\theta_0^w)^2 - (\theta_0^b)^2]} \quad (\text{B.5})$$

where equations (B.2) and (B.4) are written in the reduced form, as derived in (B.1). Note that for any  $s \in (0, 1)$  and for any  $(x_1, x_2) \in (-1, 1) \times (-1, 1)$ ,

$$1 - x_1 + s(1 - s) [(x_1)^2 - (x_2)^2] > 0$$

Dividing (B.3) by (B.2) and (B.5) by (B.4), we get

$$\frac{\tau^N(s_1)}{\tau^E(s_1)} = \frac{\theta_0^b s_1}{1 - \theta_0^w (1 - s_1)} \quad (\text{B.6})$$

$$\frac{\tau^N(s_2)}{\tau^E(s_2)} = \frac{\theta_0^b s_2}{1 - \theta_0^w (1 - s_2)} \quad (\text{B.7})$$

which can be expressed as new moment conditions

$$(1 - \theta_0^w (1 - s_1)) \tau^N(s_1) - \theta_0^b s_1 \tau^E(s_1) = 0 \quad (\text{B.8})$$

$$(1 - \theta_0^w (1 - s_2)) \tau^N(s_2) - \theta_0^b s_2 \tau^E(s_2) = 0 \quad (\text{B.9})$$

Solving the system composed of (B.8) and (B.9), provided  $s_1 \neq s_2$ , leads to

$$\begin{aligned} & \begin{cases} \theta_0^b = \frac{(1 - \theta_0^w(1 - s_1))\tau^N(s_1)}{s_1\tau^E(s_1)} \\ \theta_0^w = \frac{\tau^N(s_2) - s_2\tau^E(s_2)\theta_0^b}{\tau^N(s_2)(1 - s_2)} \end{cases} \\ \Rightarrow & \begin{cases} \theta_0^b = \frac{(1 - \theta_0^w(1 - s_1))\tau^N(s_1)}{s_1\tau^E(s_1)} \\ \theta_0^w = \frac{s_1\tau^E(s_1)\tau^N(s_2) - s_2\tau^E(s_2)\tau^N(s_1)}{s_1\tau^E(s_1)(1 - s_2)\tau^N(s_2) - (1 - s_1)\tau^N(s_1)s_2\tau^E(s_2)} \end{cases} \\ \Rightarrow & \begin{cases} \theta_0^w = \frac{s_1\tau^E(s_1)\tau^N(s_2) - s_2\tau^E(s_2)\tau^N(s_1)}{s_1(1 - s_2)\tau^E(s_1)\tau^N(s_2) - (1 - s_1)s_2\tau^N(s_1)\tau^E(s_2)} \\ \theta_0^b = \frac{\tau^N(s_1)\tau^N(s_2)(s_1 - s_2)}{s_1(1 - s_2)\tau^E(s_1)\tau^N(s_2) - (1 - s_1)s_2\tau^N(s_1)\tau^E(s_2)} \end{cases} \end{aligned}$$

Hence,  $\theta_0^b$  and  $\theta_0^w$  are identified. As a consequence,  $\delta_0$  is also identified based on (B.2), (B.4), (B.3) or (B.5)

### B.1.5 Proposition 4

Let

$$\hat{\lambda}^* = \begin{pmatrix} \delta^* \\ \theta^{w*} \\ \theta^{b*} \end{pmatrix} := \arg \min_{\lambda \in \Theta} \left( \frac{1}{G} \sum_{g=1}^G w(s_g) u^R(Z_g, \lambda) \right)' \hat{V}_G^{-1} \left( \frac{1}{G} \sum_{g=1}^G w(s_g) u^R(Z_g, \lambda) \right)$$

with

$$\begin{aligned} Z_g &:= (\bar{y}_g^E, \bar{y}_g^N, D_g, s_g) \\ u^R(Z, \lambda) &= \omega^R(D)\bar{Y} - \frac{\delta}{\phi(s, \lambda)} \begin{pmatrix} 1 - (1 - s)\theta^w \\ s\theta^b \end{pmatrix} \end{aligned}$$

and  $\hat{V}_G \xrightarrow{p} V_0 := E \left[ (w(s)u^R(Z, \lambda_0)) (w(s)u^R(Z, \lambda_0))' \right]$ . According to Theorem 2.6 of Newey and McFadden (1994), if

1.  $V_0$  is positive semi-definite and  $E [w(s)u^R(Z, \lambda)] = 0$  only if  $\lambda = \lambda_0$
2.  $\Theta$  is compact
3.  $w(s)u^R(Z, \lambda)$  is continuous at each  $\lambda \in \Theta$  almost surely
4.  $E \left[ \sup_{\lambda \in \Theta} \|w(s)u^R(Z, \lambda)\| \right] < +\infty$

Then,  $\hat{\lambda}^*$  is a consistent estimator of  $\lambda_0$ .

By assumptions, Conditions 1 and 2 are satisfied (1 holds for instance when the conditions of Proposition 3 are met). Since  $u^R$  is differentiable with respect to  $\lambda$  on  $\Theta$ , it is also continuous with respect to  $\lambda$ . So Condition 3 is also satisfied. Finally, since for  $k \in \{E, N\}$ ,  $E[|\bar{y}^k|] < +\infty$  and  $s \in (0, 1]$ , there exists  $\kappa \in \mathbb{R}^+$  such that for any  $\lambda \in \Theta$ ,

$$\frac{\delta}{\phi(s, \lambda)} \times \max(s\theta^b, 1 - (1-s)\theta^w) < \kappa$$

which implies that  $E[\sup_{\lambda \in \Theta} \|w(s)u^R(Z, \lambda)\|] < +\infty$  so Condition 4 is also satisfied. As a conclusion,  $\hat{\lambda}^*$  is a consistent estimator of  $\lambda_0$ . Then, according to Theorem 3.4 of Newey and McFadden (1994), if

1.  $\lambda_0 \in \text{Int}(\Theta)$
2.  $w(s)u^R(Z, \lambda)$  is continuous differentiable in a neighborhood of  $\lambda_0$ , almost surely
3.  $E[w(s)u^R(Z; \lambda_0)] = 0$  and  $E[\|w(s)u^R(Z, \lambda_0)\|^2] < +\infty$
4.  $E[\sup_{\lambda \in \Theta} \|\frac{\partial}{\partial \lambda'} w(s)u^R(Z, \lambda)\|^2] < +\infty$
5.  $M_0'V_0^{-1}M_0$  is non-singular with  $M_0 := E\left[w(s)\frac{\partial}{\partial \lambda'} u^R(Z, \lambda_0)\right]$

Then,

$$\sqrt{G}(\hat{\lambda}^* - \lambda_0) \xrightarrow{d} \mathcal{N}\left(0, [M_0'V_0^{-1}M_0]^{-1}\right)$$

By assumption, Condition 1 holds. Since  $u^R(\cdot, \lambda)$  is twice differentiable with respect to  $\lambda$  on  $\Theta$ , it is continuously differentiable, so condition 2 holds as well. Since, for  $k \in \{E, N\}$ ,  $E[|\bar{y}^k|^2] < +\infty$  and  $s \in (0, 1]$ , then Assumption 3 holds as well. Points 4 and 5 hold as well, due to the restriction on the distribution of  $\bar{y}^k$  and from the definitions of  $S$  and  $\Theta$ .

### B.1.6 Proposition 5

Let  $Z_g := (\bar{Y}_g^E, \bar{Y}_g^N, D_g, s_g)$  and

$$m^{DiD}(Z, \lambda, p) = w(s) \left\{ \omega(Z, p)\bar{Y} - \frac{\delta}{\phi(s; \lambda)} \begin{pmatrix} 1 - (1-s)\theta^w \\ s\theta^b \end{pmatrix} \right\}$$

where

$$w(s) = \begin{pmatrix} s & 0 \\ 0 & s\mathbf{1}(s < 1) \\ s^2 & 0 \\ 0 & s^2\mathbf{1}(s < 1) \end{pmatrix}, \bar{Y} = \begin{pmatrix} \bar{Y}^E \\ \bar{Y}^N \end{pmatrix}$$

$$\omega(Z, p) = \frac{D}{p(s)} - \frac{1-D}{1-p(s)}$$

and

$$\phi(s; \lambda) = 1 - \theta^w + s(1-s)((\theta^w)^2 - (\theta^b)^2)$$

Let  $\mu_D(s) = E[D|s]$ . The logit series estimator of the unknown function  $\mu_D$  is given by  $\hat{\mu}_D(s) = \Lambda(P^K(s)'\hat{\pi})$  where  $\Lambda(z) = 1/(1+e^{-z})$  is the logit function,  $P^K(s)$  is the vector of the  $K$  first powers of  $s$  (with a constant) and

$$0 = \sum_{g=0}^G (D_g - \Lambda(P^K(s_g)'\hat{\pi}))P^K(s_g)$$

Under Assumption 5', there exists a constant  $\pi_K$  that satisfies:

$$\left\| \log \left( \frac{\mu_D}{1-\mu_D} \right) - (P^K)'\pi_K \right\|_{\infty} = O(K^{-\alpha})$$

and we let  $\mu_{D,K}(s_g) = \Lambda(P^K(s_g)'\pi_K)$ . We suppress the  $G$  subscript on  $K$  to reduce notation. Under Assumption 5', Lemma A.15 in Newey (1997) ensures that the smallest eigenvalue of  $E[P^K(s)P^K(s)']$ , is bounded away from zero uniformly over  $K$ . In addition, Cattaneo (2010) shows that under Assumption 5', the multinomial logit series estimator satisfies:

$$\|\mu_{D,K} - \mu_D\|_{\infty} = O(K^{-\alpha}), \quad \|\hat{\pi} - \pi_K\| = O_P \left( \sqrt{\frac{K}{G}} + K^{-\alpha+1/2} \right)$$

and

$$\|\hat{\mu}_D - \mu_D\|_{\infty} = O_P \left( \zeta(K) \left( \sqrt{\frac{K}{G}} + K^{-\alpha+1/2} \right) \right)$$

where  $\zeta(K) = \sup_{s \in \mathcal{S}} \|P^K(s)\|$  and  $\alpha = d/r = 4$  where  $d$ , the degree of continuous differentiability of  $\mu_D$ , is supposed to be equal to 4 and  $r$ , the number of covariates, is equal to 1. Newey (1994) also shows that for orthonormal polynomials,  $\zeta(K)$  is bounded above by  $CK$  for some constant  $C$ . Notice that, in the case without covariates, since  $s \in (0, 1]$ ,  $\zeta(K) \leq K$ , which implies that  $\|\hat{\mu}_D - \mu_D\|_{\infty} = O_P \left( K \left( \sqrt{\frac{K}{G}} + K^{-\alpha+1/2} \right) \right)$  Throughout the proof, we also use the fact that by a second-order mean value expansion, there exists a  $\tilde{\pi}$  such that:

$$\begin{aligned} \hat{\mu}_D(s_g) - \mu_{D,K}(s_g) &= \Lambda(P^K(s_g)'\hat{\pi}) - \Lambda(P^K(s_g)'\pi_K) \\ &= \dot{\Lambda}(P^K(s_g)'\pi_K)P^K(s_g)'(\hat{\pi} - \pi_K) + \ddot{\Lambda}(P^K(s_g)'\tilde{\pi})(P^K(s_g)'(\hat{\pi} - \pi_K))^2 \end{aligned}$$

where both  $\dot{\Lambda}$  and  $\ddot{\Lambda}$  are bounded.

Let  $\mu_1(s) = E[\bar{Y}D|s]$  and  $\mu_0(s) = E[\bar{Y}(1-D)|s]$ . Under Assumption 5' and based on Lemma A.12 in Newey (1997), for any  $i \in \{0, 1\}$ , there exists constant population coefficients  $\pi_{i,K}$  that satisfies:

$$\|\mu_i - (P^K)' \pi_{i,K}\|_\infty = O(K^{-\alpha})$$

We consider the following optimisation problem

$$\hat{\lambda} = \arg \min_{\lambda \in \Theta} \left( \frac{1}{G} \sum_{g=1}^G m^{DiD}(Z_g, \lambda, \hat{\mu}_D) \right)' \hat{W} \left( \frac{1}{G} \sum_{g=1}^G m^{DiD}(Z_g, \lambda, \hat{\mu}_D) \right)$$

The associated first order condition is

$$\left( \frac{1}{G} \sum_{g=1}^G \frac{\partial}{\partial \lambda} m^{DiD}(Z_g, \hat{\lambda}, \hat{\mu}_D) \right)' \hat{W} \left( \frac{1}{G} \sum_{g=1}^G m^{DiD}(Z_g, \hat{\lambda}, \hat{\mu}_D) \right) = 0$$

Let's notice, at this stage, that

$$\frac{\partial}{\partial \lambda} m^{DiD}(Z, \lambda, p) = \frac{\partial}{\partial \lambda} m^{DiD}(Z, \lambda)$$

and define

$$\hat{M}(\lambda) = \frac{1}{G} \sum_{g=1}^G \frac{\partial}{\partial \lambda} m^{DiD}(Z_g, \lambda)$$

Using a Taylor expansion, we get

$$\sqrt{G}(\hat{\lambda} - \lambda_0) = - \left( \hat{M}(\hat{\lambda})' \hat{W} \hat{M}(\hat{\lambda}) \right)^{-1} \hat{M}(\hat{\lambda})' \hat{W} \left( \frac{1}{\sqrt{G}} \sum_{g=1}^G m^{DiD}(Z_g, \lambda_0, \hat{\mu}_D) \right)$$

Let's first show that

$$\frac{1}{\sqrt{G}} \sum_{g=1}^G m^{DiD}(Z_g, \lambda_0, \hat{\mu}_D) = \frac{1}{\sqrt{G}} \sum_{g=1}^G \{m^{DiD}(Z_g, \lambda_0, \mu_D) + \alpha(Z_g)\} + o_P(1)$$

with

$$\alpha(Z) = -w(s) \left( \frac{\mu_1(s)}{\mu_D(s)^2} + \frac{\mu_0(s)}{(1-\mu_D(s))^2} \right) (D - \mu_D(s))$$

First, notice that, for any fixed  $a \in (0, +\infty)$ , and for any  $x$ ,

$$\left( \frac{a}{x} - \frac{1-a}{1-x} \right) = \left( \frac{a}{x_0} - \frac{1-a}{1-x_0} \right) - \left( \frac{a}{x_0^2} + \frac{1-a}{(1-x_0)^2} \right) (x - x_0) + \left( \frac{a}{x_0^2 x} - \frac{1-a}{(1-x_0)^2(1-x)} \right) (x - x_0)^2$$

Using this property and rearranging terms, we get

$$\begin{aligned}
\frac{1}{\sqrt{G}} \sum_g m^{DiD}(Z_g, \lambda_0, \hat{\mu}_D) &= \frac{1}{\sqrt{G}} \sum_{g=1}^G \{m^{DiD}(Z_g, \lambda_0, \mu_D) + \alpha(Z_g)\} \\
&\quad - \frac{1}{\sqrt{G}} \sum_g w(s_g) \left( \frac{\bar{Y}_g D_g - \mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\bar{Y}_g(1 - D_g) - \mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) (\hat{\mu}_D(s_g) - \mu_D(s_g)) \\
&\quad + \frac{1}{\sqrt{G}} \sum_g w(s_g) \left( \frac{\bar{Y}_g D_g}{\mu_D(s_g)^2 \hat{\mu}_D(s_g)} - \frac{\bar{Y}_g(1 - D_g)}{(1 - \mu_D(s_g))^2 (1 - \hat{\mu}_D(s_g))} \right) (\hat{\mu}_D(s_g) - \mu_D(s_g))^2 \\
&\quad + \frac{1}{\sqrt{G}} \sum_g w(s_g) \left( \frac{\mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) (D_g - \hat{\mu}_D(s_g)).
\end{aligned}$$

which we can rewrite as:

$$\frac{1}{\sqrt{G}} \sum_g m^{DiD}(Z_g, \lambda_0, \hat{\mu}_D) = \frac{1}{\sqrt{G}} \sum_g \{m^{DiD}(Z_g, \lambda_0, \mu_D) + \alpha(Z_g, \mu_D)\} + \sum_{j=1}^3 A_{j,G}$$

where each  $A_{j,G}$  represents one term on the above display. We now bound each one of these terms.

**Term 1.** For the first term, we have that:

$$\begin{aligned}
-A_{1,G} &= \frac{1}{\sqrt{G}} \sum_g w(s_g) \left( \frac{\bar{Y}_g D_g - \mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\bar{Y}_g(1 - D_g) - \mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) (\hat{\mu}_D(s_g) - \mu_D(s_g)) \\
&= \frac{1}{\sqrt{G}} \sum_g w(s_g) \left( \frac{\bar{Y}_g D_g - \mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\bar{Y}_g(1 - D_g) - \mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) (\hat{\mu}_D(s_g) - \mu_{D,K}(s_g)) \\
&\quad + \frac{1}{\sqrt{G}} \sum_g w(s_g) \left( \frac{\bar{Y}_g D_g - \mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\bar{Y}_g(1 - D_g) - \mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) (\mu_{D,K}(s_g) - \mu_D(s_g)) \\
&= A_{11,G} + A_{12,G}.
\end{aligned}$$

Now, by a second-order mean value expansion,

$$\begin{aligned}
A_{11,G} &= \frac{1}{\sqrt{G}} \sum_g w(s_g) \left( \frac{\bar{Y}_g D_g - \mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\bar{Y}_g(1 - D_g) - \mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) \dot{\Lambda}(P^K(s_g)' \pi_K) P^K(s_g)' (\hat{\pi}_K - \pi_K) \\
&\quad + \frac{1}{\sqrt{G}} \sum_g w(s_g) \left( \frac{\bar{Y}_g D_g - \mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\bar{Y}_g(1 - D_g) - \mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) \ddot{\Lambda}(P^K(s_g)' \tilde{\pi}) (P^K(s_g)' (\hat{\pi}_K - \pi_K))^2 \\
&= A_{111,G} + A_{112,G}.
\end{aligned}$$

Next, letting

$$f(\bar{Y}_g, s_g) = \frac{\bar{Y}_g D_g - \mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\bar{Y}_g(1 - D_g) - \mu_0(s_g)}{(1 - \mu_D(s_g))^2} = \begin{pmatrix} f_1(\bar{Y}_g, s_g) \\ f_2(\bar{Y}_g, s_g) \end{pmatrix}$$

we get

$$\|A_{111,G}\| \leq \|\hat{\pi}_K - \pi_K\| \left\| \frac{1}{\sqrt{G}} \sum_g w(s_g) f(\bar{Y}_g, s_g) \dot{\Lambda}(P^K(s_g)' \pi_K) P^K(s_g)' \right\|.$$

Now,  $\|\hat{\pi}_K - \pi_K\| = O_P\left(\left(\sqrt{K/G} + K^{-\alpha+1/2}\right)\right)$ . Let

$$U_g = (U_g^1, \dots, U_g^K)' := w(s_g) f(\bar{Y}_g, s_g) \dot{\Lambda}(P^K(s_g)' \pi_K) P^K(s_g)'.$$

and

$$\begin{aligned} \|U_g\|^2 &= \text{trace}(\dot{\Lambda}(P^K(s_g)' \pi_K)^2 P^K(s_g) f(\bar{Y}_g, s_g)' w(s_g)' w(s_g) f(\bar{Y}_g, s_g) P^K(s_g)') \\ &= \dot{\Lambda}(P^K(s_g)' \pi_K)^2 \underbrace{(w(s_g) f(\bar{Y}_g, s_g))' w(s_g) f(\bar{Y}_g, s_g)}_{1 \times 1} \text{trace}(P^K(s_g) P^K(s_g)') \\ &= \dot{\Lambda}(P^K(s_g)' \pi_K)^2 (s^2(1 + s^2) (\|f_1(\bar{Y}_g, s_g)\| + \mathbf{1}(s < 1) \|f_2(\bar{Y}_g, s_g)\|)) \text{trace}(P^K(s_g) P^K(s_g)') \\ &\leq \kappa \text{trace}(P^K(s_g) P^K(s_g)') \end{aligned}$$

where  $\kappa$  is a bound of  $\|f_1(\bar{Y}_g, s_g)\| + \|f_2(\bar{Y}_g, s_g)\|$  (well-defined from Assumption 5'). We have  $E[U_g] = E[E[U_g|s_g]] = 0_{4 \times K}$  and

$$\begin{aligned} E[\|U_g\|^2] &\leq E[\kappa \text{trace}(P^K(s_g) P^K(s_g)')] \\ &= \kappa E[\text{trace}(P^K(s_g) P^K(s_g)')] \\ &= \kappa \times \text{trace}(E[P^K(s_g) P^K(s_g)']) \\ &= \kappa K, \end{aligned} \tag{B.10}$$

since the polynomials can be chosen such that  $E[P^K(s_g) P^K(s_g)'] = I_K$ , see Newey (1997), page 161.

Hence,

$$\begin{aligned} E\left[\left\|\frac{1}{\sqrt{G}} \sum_g U_g\right\|^2\right] &= E\left[\text{trace}\left(\left(\frac{1}{\sqrt{G}} \sum_g U_g\right)' \left(\frac{1}{\sqrt{G}} \sum_g U_g\right)\right)\right] \\ &= \frac{1}{G} \sum_{g,g'} E[\text{trace}(U_g' U_{g'})] \\ &= \frac{1}{G} \sum_{g,g'} \text{trace}(E[U_g' U_{g'}]) \\ &= \frac{1}{G} \sum_g \text{trace}(E[U_g' U_g]) \end{aligned}$$

$$\begin{aligned} E \left[ \left\| \frac{1}{\sqrt{G}} \sum_g U_g \right\|^2 \right] &= \frac{1}{G} \sum_g E \left[ \|U_g\|^2 \right] \\ &= E \left[ \|U_1\|^2 \right]. \end{aligned}$$

Therefore, by Markov's inequality,

$$A_{111,G} = O_P \left( K^{1/2} \left( \sqrt{\frac{K}{G}} + K^{-\alpha+1/2} \right) \right).$$

Next,

$$\begin{aligned} \|A_{112,G}\| &\leq \tilde{\kappa} \sqrt{G} \|\hat{\pi}_K - \pi_K\|^2 \frac{1}{G} \sum_g \|w(s_g) f(\bar{Y}_g, s_g)\| \|P^K(s_g)\|^2 \\ &= O_P \left[ \sqrt{G} \left( \frac{K}{G} + K^{-2\alpha+1} \right) E \left( \|w(s_g) f(\bar{Y}_g, s_g)\| \|P^K(s_g)\|^2 \right) \right] \\ &= O_P \left( \sqrt{G} K \left( \frac{K}{G} + K^{-2\alpha+1} \right) \right), \end{aligned}$$

where  $\tilde{\kappa}$  is a bound on  $\ddot{\Lambda}$ . The first inequality follows by Cauchy-Schwarz inequality, the second by Markov's inequality and the third by the same reasoning as to obtain (B.10). Hence,

$$A_{11,G} = O_P \left( K^{1/2} \left( \sqrt{\frac{K}{G}} + K^{-\alpha+1/2} \right) \right) + O_P \left( \sqrt{G} K \left( \frac{K}{G} + K^{-2\alpha+1} \right) \right).$$

Finally, for  $A_{12,G}$  we have that

$$E \left[ w(s_g) \left( \frac{\bar{Y}_g D_g - \mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\bar{Y}_g(1 - D_g) - \mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) (\mu_{D,K}(s_g) - \mu_D(s_g)) \middle| s_g \right] = 0$$

and

$$E \left[ \left\| w(s_g) \left( \frac{\bar{Y}_g D_g - \mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\bar{Y}_g(1 - D_g) - \mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) (\mu_{D,K}(s_g) - \mu_D(s_g)) \right\|^2 \right] \leq \kappa \|\mu_{D,K} - \mu_D\|_\infty^2 = O(K^{-2\alpha})$$

and therefore

$$A_{1,G} = O_P \left( K^{1/2} \left( \sqrt{\frac{K}{G}} + K^{-\alpha+1/2} \right) \right) + O_P \left( \sqrt{G} K \left( \frac{K}{G} + K^{-2\alpha+1} \right) \right) + O_P(K^{-\alpha}).$$



**Term 2.** For the second term, since  $\mu_D$  is uniformly bounded and  $\hat{\mu}_D$  converges uniformly to  $\mu_D$ , for  $G$  large enough

$$|A_{2,G}| \leq \sqrt{G} \|\hat{\mu}_D - \mu_D\|_\infty^2 \cdot \kappa_2 \cdot \frac{1}{G} \sum_g \|\bar{Y}_g\| = O_P \left( \sqrt{G} K^2 \left( \frac{K}{G} + K^{-2\alpha+1} \right) \right).$$

where  $\kappa_2$  is a bound on  $\left| \frac{1}{\mu_D(s_g)^2 \hat{\mu}_D(s_g)} + \frac{1}{(1 - \mu_D(s_g))^2 (1 - \hat{\mu}_D(s_g))} \right|$

**Term 3.** For the third term, let  $\gamma_K$  be a  $2 \times K$  matrix of population coefficients from a (linear) series approximation to the  $(2 \times 1)$  function  $\mu_1(s)/\mu_D(s)^2 + \mu_0(s)/(1 - \mu_D(s))^2$ . Then we have that

$$\begin{aligned} A_{3,G} &= \frac{1}{\sqrt{G}} \sum_g \left( \left( \frac{\mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) - \gamma_K P^K(s_g) \right) (D_g - \hat{\mu}_D(s_g)) \\ &\quad + \frac{1}{\sqrt{G}} \sum_g \gamma_K P^K(s_g) \times (D_g - \hat{\mu}_D(s_g)) \\ &= \frac{1}{\sqrt{G}} \sum_g \left( \left( \frac{\mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) - \gamma_K P^K(s_g) \right) (D_g - \mu_D(s_g)) \end{aligned}$$

because the last term in the second line equals zero by the first-order conditions of the logit series estimator. Next, we have that

$$\begin{aligned} &\frac{1}{\sqrt{G}} \sum_g \left( \left( \frac{\mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) - \gamma_K P^K(s_g) \right) (D_g - \hat{\mu}_D(s_g)) \\ &= \frac{1}{\sqrt{G}} \sum_g \left( \left( \frac{\mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) - \gamma_K P^K(s_g) \right) (D_g - \mu_D(s_g)) \\ &\quad - \frac{1}{\sqrt{G}} \sum_g \left( \left( \frac{\mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) - \gamma_K P^K(s_g) \right) (\mu_D(s_g) - \hat{\mu}_D(s_g)) \\ &= A_{31,G} + A_{32,G}. \end{aligned}$$

Now, for  $A_{31,G}$ , we have that

$$E \left[ \left( \left( \frac{\mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) - \gamma_K P^K(s_g) \right) (D_g - \mu_D(s_g)) \middle| s_g \right] = 0$$

and by Lemma A.12 from Newey (1997)

$$E \left[ (D_g - \mu_D(s_g))^2 \left\| \left( \frac{\mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\mu_0(s_g)}{(1 - \mu_D(s_g))^2} \right) - \gamma_K P^K(s_g) \right\|^2 \right] \leq O_P(K^{-2\alpha})$$

since the involved functions are continuously differentiable. Hence,

$$A_{31,G} = O_P(K^{-\alpha}).$$

On the other hand, for  $A_{32,G}$ , we have that

$$\begin{aligned} |A_{32,G}| &\leq \sqrt{G} \left\| \left( \frac{\mu_1(s_g)}{\mu_D(s_g)^2} + \frac{\mu_0(s_g)}{(1-\mu_D(s_g))^2} \right) - \gamma_K P^K(s_g) \right\|_{\infty} \|\hat{\mu}_D - \mu_D\|_{\infty} \\ &= O_P \left( \sqrt{G} K^{1-\alpha} \left( \sqrt{\frac{K}{G}} + K^{-\alpha+1/2} \right) \right) \end{aligned}$$

based on Assumption to be defined. Hence,

$$A_{3,G} = O_P \left( \sqrt{G} K^{1-\alpha} \left( \sqrt{\frac{K}{G}} + K^{-\alpha+1/2} \right) + K^{-\alpha} \right).$$

Collecting all the terms, it follows that under the conditions

$$\frac{K^6}{G} \rightarrow 0, \quad \frac{K^{4\alpha-6}}{G} \rightarrow \infty, \quad \alpha \geq 4$$

we obtain

$$\frac{1}{\sqrt{G}} \sum_g m^{DiD}(Z_g, \lambda_0, \hat{\mu}_D) = \frac{1}{\sqrt{G}} \sum_{g=1}^G \{m^{DiD}(Z_g, \lambda_0, \mu_D) + \alpha(Z_g)\} + o_P(1)$$

Setting  $\alpha = 4$ , this implies

$$\frac{K^6}{G} \rightarrow 0, \quad \frac{K^{10}}{G} \rightarrow \infty.$$

These conditions are satisfied when  $K = G^\nu$  for  $1/(4\alpha - 6) < \nu < 1/6$  or in this case  $1/10 < \nu < 1/6$ .

Then, by a standard multivariate CLT, we get that

$$\frac{1}{\sqrt{G}} \sum_g m^{DiD}(Z_g, \lambda_0, \hat{\mu}_D) \xrightarrow{d} \mathcal{N}(0, \Omega)$$

where

$$\Omega = E \left[ (m^{DiD}(Z, \lambda_0, \mu_D) + \alpha(Z)) (m^{DiD}(Z, \lambda_0, \mu_D) + \alpha(Z))' \right]$$

Now, since  $\left\| \frac{\partial}{\partial \lambda} m^{DiD}(Z, \lambda) \right\|$  is a bounded quantity in any neighbourhood of  $\lambda_0$ ,

$$\hat{M}(\hat{\lambda}) \xrightarrow{p} M_0 = E \left[ \frac{\partial}{\partial \lambda} m^{DiD}(Z, \lambda_0) \right]$$

Supposing that  $\hat{W} \xrightarrow{p} W$  and  $M_0' W M_0$  is invertible, then by Slutsky's lemma,

$$\sqrt{G}(\hat{\lambda} - \lambda_0) \xrightarrow{d} \mathcal{N}(0, (M_0'WM_0)^{-1}M_0'W\Omega WM_0(M_0'WM_0)^{-1})$$

## B.2 Identity-specific “within” and “between” influences

In this section, we consider the more general LIM model that allows for 2 different “within” endogenous peer effects for eligible and non-eligible individuals. They are denoted by  $\theta^{w,E}$  and  $\theta^{w,N}$ . Moreover, we suppose that influences may be non-symmetric between eligible and non-eligible individuals. Let  $\theta^{b,N,E}$  be the “between” endogenous effects from eligible individuals on non-eligible individuals and  $\theta^{b,E,N}$  be the “between” endogenous effects from non-eligible individuals on eligible individuals. We remind the

**Assumption 1\* (Linear-in-means model - 5 parameters)** *Within each group  $g \in \{1, \dots, G\}$  of the i.i.d sample, the outcome of individual  $i$  is defined as*

$$y_{ig} = E_{ig}y_{ig}^E + (1 - E_{ig})y_{ig}^N$$

with

$$\begin{aligned} y_{ig}^E &= \theta_0^{w,E} s_g \bar{y}_g^E + \theta_0^{b,E,N} (1 - s_g) \bar{y}_g^N + \delta_0 D_{ig} + e_{ig}^E \\ y_{ig}^N &= \theta_0^{b,N,E} s_g \bar{y}_g^E + \theta_0^{w,N} (1 - s_g) \bar{y}_g^N + e_{ig}^N \end{aligned}$$

### B.2.1 Identification

Averaging, within group  $g$ , the outcome variables for eligible and non-eligible individuals, we get the following expressions

$$\begin{aligned} \bar{y}_g^E &= \theta_0^{w,E} s_g \bar{y}_g^E + \theta_0^{b,E,N} (1 - s_g) \bar{y}_g^N + \delta_0 D_g + \bar{e}_g^E \\ \bar{y}_g^N &= \theta_0^{b,N,E} s_g \bar{y}_g^E + \theta_0^{w,N} (1 - s_g) \bar{y}_g^N + \bar{e}_g^N \end{aligned}$$

Then, plugging-in the expression of  $\bar{y}_g^E$  into the one of  $\bar{y}_g^N$ , we get

$$\bar{y}_g^N = \frac{\theta_0^{b,N,E} s_g}{1 - \theta_0^{w,E} s_g} \left( \theta_0^{b,E,N} (1 - s_g) \bar{y}_g^N + \delta_0 D_g + \bar{e}_g^E \right) + \theta_0^{w,N} (1 - s_g) \bar{y}_g^N + \bar{e}_g^N$$

Rearranging terms, we get

$$\left[1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g(1 - s_g) \left( \theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N} \right)\right] \bar{y}_g^N = \theta_0^{b,N,E} s_g (\delta_0 D_g + \bar{e}_g^E) + (1 - \theta_0^{w,E} s_g) \bar{e}_g^N$$

Hence,

$$\begin{aligned} \bar{y}_g^N &= \frac{1 - \theta_0^{w,E} s_g}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g(1 - s_g) \left( \theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N} \right)} \bar{e}_g^N \\ &\quad + \frac{\theta_0^{b,N,E} s_g}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g(1 - s_g) \left( \theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N} \right)} \bar{e}_g^E \\ &\quad + \frac{\delta_0 \theta_0^{b,N,E} s_g}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g(1 - s_g) \left( \theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N} \right)} D_g \end{aligned}$$

Now, plugging-in the reduced form expression of  $\bar{y}_g^N$  into  $\bar{y}_g^E$  and developing, we get

$$\begin{aligned} \bar{y}_g^E &= \frac{1 - \theta_0^{w,N} (1 - s_g)}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g(1 - s_g) \left( \theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N} \right)} \bar{e}_g^E \\ &\quad + \frac{\theta_0^{b,E,N} (1 - s_g)}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g(1 - s_g) \left( \theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N} \right)} \bar{e}_g^N \\ &\quad + \frac{\delta_0 (1 - \theta_0^{w,N} (1 - s_g))}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g(1 - s_g) \left( \theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N} \right)} D_g \\ &= \frac{1 - \theta_0^{w,N} (1 - s_g)}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g(1 - s_g) \left( \theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N} \right)} \bar{e}_g^E \\ &\quad + \frac{\theta_0^{b,E,N} (1 - s_g)}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g(1 - s_g) \left( \theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N} \right)} \bar{e}_g^N \\ &\quad + \delta_0 \left( 1 + s_g \cdot \frac{\theta_0^{w,E} - (1 - s_g) \left[ \theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N} \right]}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g(1 - s_g) \left( \theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N} \right)} \right) D_g \end{aligned} \tag{B.11}$$

We then get the following proposition

**Proposition 6** *Provided Assumptions 1\*, 2 and 3 hold and all the mentioned conditional expectations*

are well-defined, we have

$$\begin{aligned}
\tau^E(s) &= \mathbb{E}[\bar{y}_g^E | s_g = s, D_g = 1] - \mathbb{E}[\bar{y}_g^E | s_g = s, D_g = 0] \\
&= \mathbb{E}\left[\frac{D_g - \Pr(D_g = 1 | s_g = s)}{\Pr(D_g = 1 | s_g = s)(1 - \Pr(D_g = 1 | s_g = s))} \cdot \bar{y}_g^E | s_g = s\right] \\
&= \delta_0 \left(1 + s_g \cdot \frac{\theta_0^{w,E} - (1 - s_g) [\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N}]}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g (1 - s_g) (\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N})}\right) \quad (\text{B.12})
\end{aligned}$$

and

$$\begin{aligned}
\tau^N(s) &= \mathbb{E}[\bar{y}_g^N | s_g = s, T_g = 1] - \mathbb{E}[\bar{y}_g^N | s_g = s, D_g = 0] \\
&= \mathbb{E}\left[\frac{D_g - \Pr(D_g = 1 | s_g = s)}{\Pr(D_g = 1 | s_g = s)(1 - \Pr(D_g = 1 | s_g = s))} \cdot \bar{y}_g^N | s_g = s\right] \\
&= \frac{\delta_0 \theta_0^{b,N,E} s_g}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g (1 - s_g) (\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N})} \quad (\text{B.13})
\end{aligned}$$

Hence,

$$P^E(s) := \frac{\theta_0^{w,E} - (1 - s_g) [\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N}]}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g (1 - s_g) (\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N})} \quad (\text{B.14})$$

$$P^N(s) := \frac{1}{1 - s_g \theta_0^{w,E} - (1 - s_g) \theta_0^{w,N} + s_g (1 - s_g) (\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N})} \quad (\text{B.15})$$

Similarly, we could state the analogous proposition to proposition 2 in the 5 parameter scenario, provided Assumptions 1\*, 2', 3' and 4' hold.

We also have the following proposition

**Proposition 7 (Sufficient Conditions for Identification of  $\lambda_0 := (\delta_0, \theta^{w,E}, \theta^{w,N}, \theta^{b,N,E}, \theta^{b,E,N})$ )** *Provided  $\delta_0 \neq 0$  and  $\theta_0^b \neq 0$ , if Assumptions 1\*, 2 and 3 hold and if one of the two following conditions is satisfied*

1. *there exist at least 3 shares  $s_1, s_2, s_3 \in (0, 1)$ ,  $s_1 \neq s_2 \neq s_3$ , that have positive probability mass and for which both  $\tau^N$  and  $\tau^E$  are well-defined*
2. *there is a continuum of shares  $I_s \subseteq (0, 1]$  such that  $P(s \in I_s) > 0$  and both  $\tau^N(s)$  and  $\tau^E(s)$  are well-defined, for any  $s \in I_s$*

then  $\lambda_0$  is the unique vector defined on  $\Theta$  that satisfies the moment condition

$$\mathbb{E} [\tilde{u}(\bar{y}_g^E, \bar{y}_g^N, D_g, s_g; \lambda_0) | s_g = s] = \begin{pmatrix} \mathbb{E} [\tilde{u}^E(\bar{y}_g^E, D_g, s_g; \lambda_0) | s_g = s] \\ \mathbb{E} [\tilde{u}^N(\bar{y}_g^N, D_g, s_g; \lambda_0) | s_g = s] \end{pmatrix} = 0 \quad (\text{B.16})$$

where for all  $\lambda \in \Theta$ ,

$$\begin{aligned} \tilde{u}^E(\bar{y}_g^E, D_g, s_g; \lambda) &= \omega^R(D_g, s_g) \bar{y}_g^E - \frac{\delta(1 - \theta^{w,N}(1 - s_g))}{1 - s_g \theta^{w,E} - (1 - s_g) \theta^{w,N} + s_g(1 - s_g) (\theta^{w,E} \theta^{w,N} - \theta^{b,N,E} \theta^{b,E,N})} \\ \tilde{u}^N(\bar{y}_g^N, D_g, s_g; \lambda) &= \omega^R(D_g, s_g) \bar{y}_g^N - \frac{\delta \theta^{b,N,E} s_g}{1 - s_g \theta^{w,E} - (1 - s_g) \theta^{w,N} + s_g(1 - s_g) (\theta^{w,E} \theta^{w,N} - \theta^{b,N,E} \theta^{b,E,N})} \end{aligned}$$

$$\text{and } \omega^R(D_g, s_g) = \frac{D_g - P(D_g = 1 | s_g)}{P(D_g = 1 | s_g)(1 - P(D_g = 1 | s_g))}$$

**Proof:** let  $s_1, s_2, s_3$  be three distinct elements of  $(0, 1)$ , such that  $s_1 \neq s_2 \neq s_3$ . Supposing that Assumptions 1\*, 2 and 3 hold, we have the following conditions

$$\tau^E(s_1) = \frac{\delta_0 (1 - \theta_0^{w,N}(1 - s_1))}{1 - s_1 \theta_0^{w,E} - (1 - s_1) \theta_0^{w,N} + s_1(1 - s_1) (\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N})} \quad (\text{B.17})$$

$$\tau^N(s_1) = \frac{\delta_0 \theta_0^{b,N,E} s_1}{1 - s_1 \theta_0^{w,E} - (1 - s_1) \theta_0^{w,N} + s_1(1 - s_1) (\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N})} \quad (\text{B.18})$$

$$\tau^E(s_2) = \frac{\delta_0 (1 - \theta_0^{w,N}(1 - s_2))}{1 - s_2 \theta_0^{w,E} - (1 - s_2) \theta_0^{w,N} + s_2(1 - s_2) (\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N})} \quad (\text{B.19})$$

$$\tau^N(s_2) = \frac{\delta_0 \theta_0^{b,N,E} s_2}{1 - s_2 \theta_0^{w,E} - (1 - s_2) \theta_0^{w,N} + s_2(1 - s_2) (\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N})} \quad (\text{B.20})$$

$$\tau^E(s_3) = \frac{\delta_0 (1 - \theta_0^{w,N}(1 - s_3))}{1 - s_3 \theta_0^{w,E} - (1 - s_3) \theta_0^{w,N} + s_3(1 - s_3) (\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N})} \quad (\text{B.21})$$

$$\tau^N(s_3) = \frac{\delta_0 \theta_0^{b,N,E} s_3}{1 - s_3 \theta_0^{w,E} - (1 - s_3) \theta_0^{w,N} + s_3(1 - s_3) (\theta_0^{w,E} \theta_0^{w,N} - \theta_0^{b,N,E} \theta_0^{b,E,N})} \quad (\text{B.22})$$

Note that for any  $s \in (0, 1)$  and for any  $(x_1, x_2, x_3, x_4) \in (-1, 1)^4$ ,

$$1 - x_1 s - x_2(1 - s) + s(1 - s) [x_1 x_2 - x_3 x_4] > 0$$

Dividing (B.18) by (B.17) and (B.20) by (B.19), we get

$$\frac{\tau^N(s_1)}{\tau^E(s_1)} = \frac{\theta_0^{b,N,E} s_1}{1 - \theta_0^{w,N} (1 - s_1)} \quad (\text{B.23})$$

$$\frac{\tau^N(s_2)}{\tau^E(s_2)} = \frac{\theta_0^{b,N,E} s_2}{1 - \theta_0^{w,N} (1 - s_2)} \quad (\text{B.24})$$

which can be expressed as new moment conditions

$$(1 - \theta_0^{w,N} (1 - s_1)) \tau^N(s_1) - \theta_0^{b,N,E} s_1 \tau^E(s_1) = 0 \quad (\text{B.25})$$

$$(1 - \theta_0^{w,N} (1 - s_2)) \tau^N(s_2) - \theta_0^{b,N,E} s_2 \tau^E(s_2) = 0 \quad (\text{B.26})$$

Solving the system composed of (B.25) and (B.26), provided  $s_1 \neq s_2$ , leads to

$$\begin{cases} \theta_0^{w,N} = \frac{s_1 \tau^E(s_1) \tau^N(s_2) - s_2 \tau^E(s_2) \tau^N(s_1)}{s_1(1 - s_2) \tau^E(s_1) \tau^N(s_2) - (1 - s_1) s_2 \tau^N(s_1) \tau^E(s_2)} \\ \theta_0^{b,N,E} = \frac{\tau^N(s_1) \tau^N(s_2) (s_1 - s_2)}{s_1(1 - s_2) \tau^E(s_1) \tau^N(s_2) - (1 - s_1) s_2 \tau^N(s_1) \tau^E(s_2)} \end{cases}$$

Hence,  $\theta_0^{b,N,E}$  and  $\theta_0^{w,N}$  are identified. As a consequence,  $\delta_0$  is also identified based on (B.21), (B.22) and one equation from (B.17) to (B.20) can be used to identify the 3 other parameters.

## B.2.2 Estimation

The true vector of parameters  $\lambda_0 := (\delta_0, \theta_0^{w,E}, \theta_0^{w,N}, \theta_0^{b,N,E}, \theta_0^{b,E,N})$  can be estimated via an adapted version of our Generalized Method of Moments (GMM) estimators. Let  $Z_g := (\bar{y}^E, \bar{y}^N, s_g, D_g)$ ,

$$\hat{\lambda}^* := \arg \min_{\lambda \in \Theta} \left( \frac{1}{G} \sum_{g=1}^G m(Z_g, \lambda, \hat{p}) \right)' \hat{V}_G^{-1} \left( \frac{1}{G} \sum_{g=1}^G m(Z_g, \lambda, \hat{p}) \right) \quad (\text{B.27})$$

with

$$m(Z_g, \lambda, \hat{p}) = w(s_g) \left[ \left( \frac{D_g}{\hat{p}(s_g)} - \frac{1 - D_g}{1 - \hat{p}(s_g)} \right) \begin{pmatrix} \bar{y}^E \\ \bar{y}^N \end{pmatrix} - \frac{\delta}{\phi(s_g, \lambda)} \begin{pmatrix} 1 - (1 - s_g) \theta^{w,N} \\ s \theta^{b,N,E} \end{pmatrix} \right]$$

and

$$\phi(s_g, \lambda) = 1 - \theta^{w,E} s_g - \theta^{w,N} (1 - s_g) + s_g (1 - s_g) (\theta^{w,E} \theta^{w,N} - \theta^{b,N,E} \theta^{b,E,N})$$

$w(s_g)$  is a  $k \times 2$  matrix of functions of  $s_g$  with  $k \geq 5$ .  $\hat{p}(s_g)$  is a series logit estimate of  $\mu_D(s_g) := P(D_g =$

$1|s_g)$ . Finally,

$$\hat{V}_G \xrightarrow{p} V_0 := E [m^{DiD}(Z, \lambda_0, \mu_D)m^{DiD}(Z, \lambda_0, \mu_D)']$$

with

$$m^{DiD}(Z, \lambda_0, \mu_D) = m(Z, \lambda_0, \mu_D) - w(s) \left( \frac{E[\bar{Y}D|s]}{\mu_D(s)^2} + \frac{E[\bar{Y}(1-D)|s]}{(1-\mu_D(s))^2} \right) (D - \mu_D(s))$$

and  $\bar{Y} = \begin{pmatrix} \bar{y}^E & \bar{y}^N \end{pmatrix}'$ . We get the following result

**Proposition 8 (GMM Asymptotic Properties)** *Provided Assumptions 1\*, 2, 3 and technical conditions hold,*

$$\sqrt{G}(\hat{\lambda}^* - \lambda_0) \xrightarrow{d} \mathcal{N}\left(0, [M_0'V_0^{-1}M_0]^{-1}\right) \text{ with } M_0 := E \left[ \frac{\partial m}{\partial \lambda}(Z, \lambda_0) \right]$$

In practice, standard errors can be estimated via bootstrap.

### B.2.3 Simulations

We evaluate our estimation procedure for the 3 and 5 parameter scenarios via simulations. The following Data Generating Process. An i.i.d sample of  $G$  groups is simulated. For each group  $g$ , a vector  $Z_g = (\Delta \bar{y}_g^E, \Delta \bar{y}_g^N, s_g, D_g, \Delta X_g^E, \Delta X_g^N)$  is drawn such that

$$\begin{cases} \bar{y}_g^E &= (\alpha_g^E + \beta^E X_g^E) + \theta_0^w s_g \bar{y}_g^E + \theta_0^b (1 - s_g) \bar{y}_g^N + \delta_0 D_g \\ \bar{y}_g^N &= (\alpha_g^N + \beta^N X_g^N) + \theta_0^b s_g \bar{y}_g^E + \theta_0^w (1 - s_g) \bar{y}_g^N \end{cases}$$

with  $(\beta^E, \beta^N) = (-1, -2)$  and

$$s_g \sim \mathcal{U}_{(0,1]}$$

$$D_g \sim \mathcal{B}(1/2)$$

$$\begin{pmatrix} \alpha_g^E \\ \alpha_g^N \\ X_g^E \\ X_g^N \end{pmatrix} \sim \mathcal{N}(\mu_{\alpha, X}, \Sigma_{\alpha, X})$$

300 simulations were performed. Figures B.1 and B.2 show that having 1,000 or 10,000 groups is not enough to obtain consistent estimators in the 5 parameter scenario. It appears that  $\hat{\theta}^{b, N, E}$  and  $\hat{\theta}^{w, E}$  converge relatively fast while  $\hat{\theta}^{b, E, N}$ ,  $\hat{\theta}^{w, N}$  and  $\hat{\delta}$  are quite imprecise. Global convergence is assured for sample of a million groups, as shown by Figure B.3. We explain this slow convergence, coming from a



large asymptotic variance, by the difficulty to disentangle the endogenous peer effects from non-eligible to eligible individuals,  $\theta_0^{b,E,N}$ , from the direct effect of the treatment  $\delta_0$ . In our baseline scenario with 3 parameter, convergence is faster. Asymptotic normality is almost reached with 1,000 groups, as shown by Figure B.4.

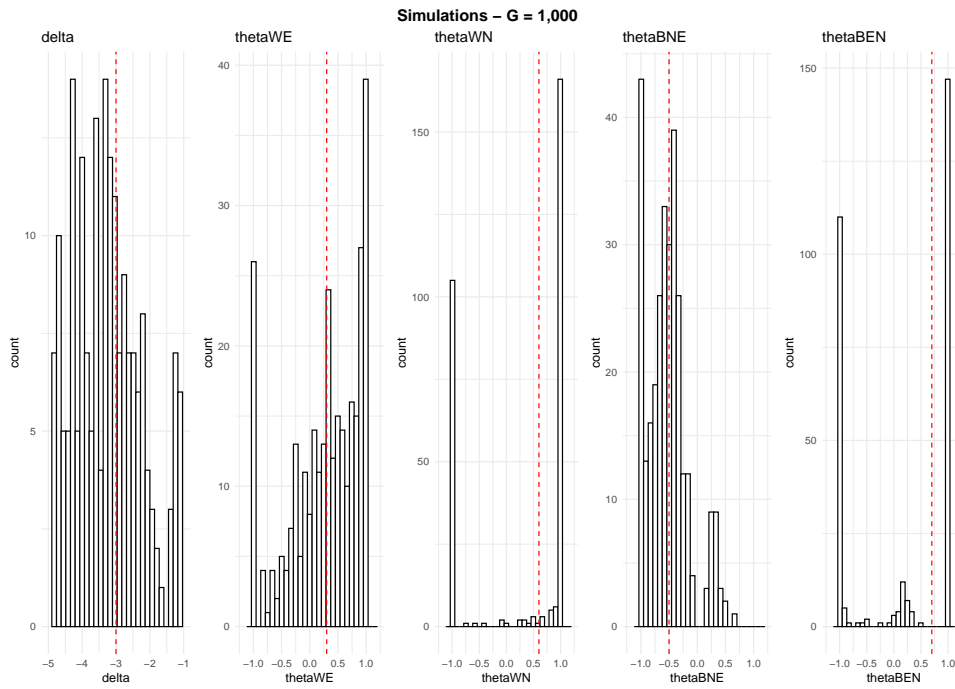


Figure B.1: Simulations - 4 endogenous peer effects parameters - 1,000 groups

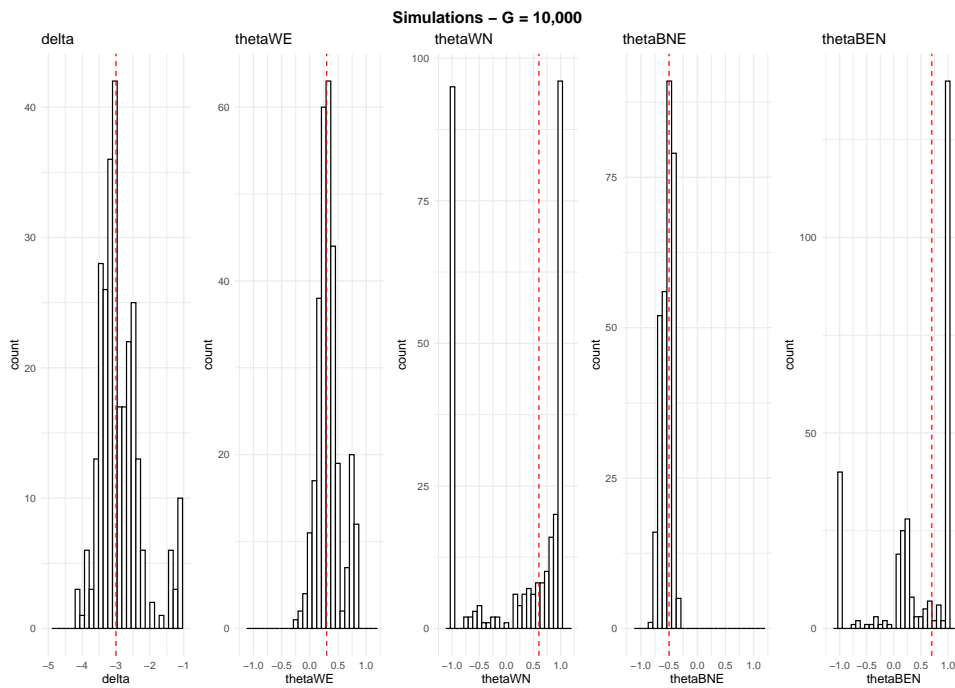


Figure B.2: Simulations - 4 endogenous peer effects parameters - 10,000 groups

*Note:* the dotted red line represents the true values of the coefficients, in order  $\delta_0$ ,  $\theta_0^{w,E}$ ,  $\theta_0^{w,N}$ ,  $\theta_0^{b,N,E}$  and  $\theta_0^{b,E,N}$

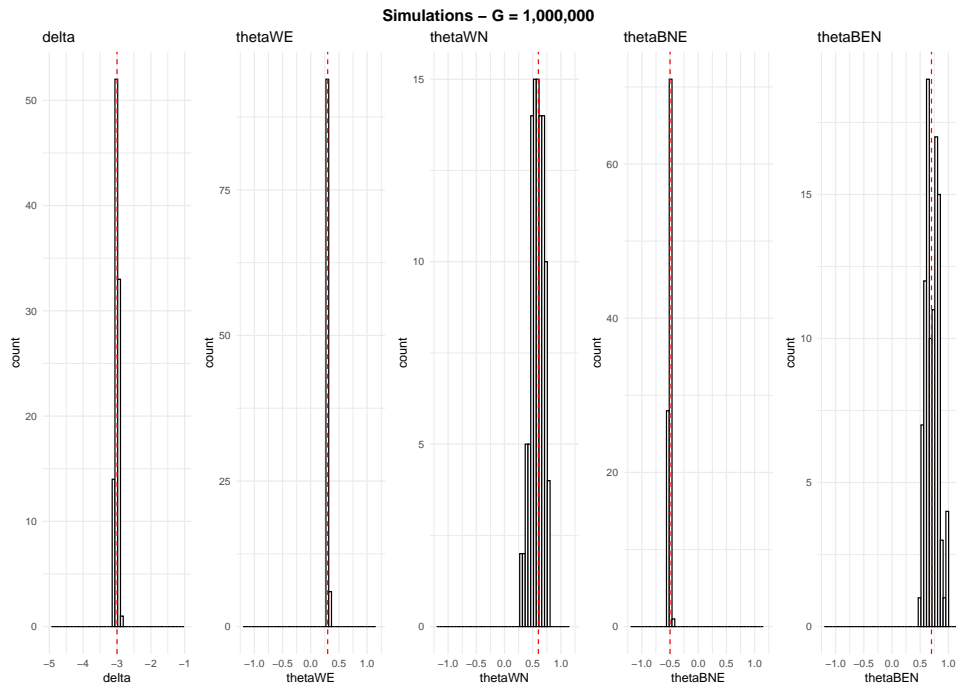


Figure B.3: Simulations - 4 endogenous peer effects parameters - 1,000,000 groups

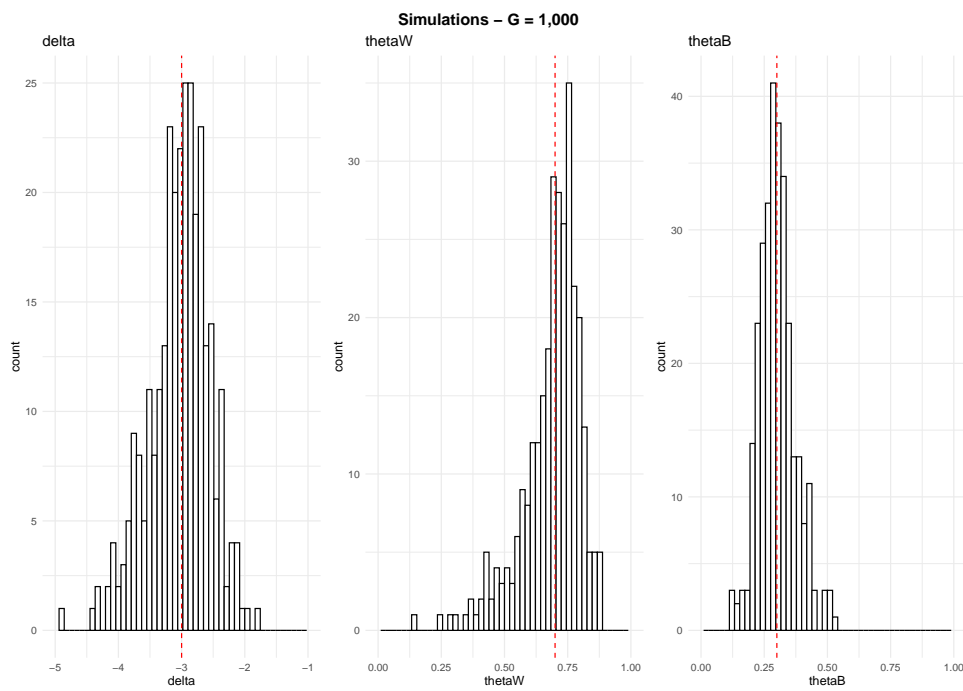


Figure B.4: Simulations - 2 endogenous peer effects parameters - 1,000 groups

*Note:* the dotted red line represents the true values of the coefficients.

### B.3 Orthogonality between Eligibility and Identity

In this extension, we suppose that the identity that matters in social interactions is orthogonal to eligibility. For instance, in the context of Progresa, we may suppose that boys (resp. girls) are more influenced by the actions of their male (resp. female) peers than by their female (resp. male) peers. Gender, which the relevant identity for social interactions, is orthogonal to being eligible to Progresa, since eligibility is only based on households' income. Formally, here is the model that we consider in this extension

**Assumption 1† (Linear-in-means model - orthogonal identity)** For a group  $g \in \{1, \dots, G\}$  of the i.i.d sample, the outcome of individual  $i$  is defined as follow

$$y_{ig} = y_{ig}^M M_{ig} + y_{ig}^F (1 - M_{ig})$$

$$y_{ig}^M = \theta_0^{w,M} s_g^M \bar{y}_g^M + \theta_0^{b,M,F} (1 - s_g^M) \bar{y}_g^F + \delta_0 D_{ig} E_{ig} + e_{ig}^M \quad (\text{B.28})$$

$$y_{ig}^F = \theta_0^{b,F,M} s_g^M \bar{y}_g^M + \theta_0^{w,F} (1 - s_g^M) \bar{y}_g^F + \delta_0 D_{ig} E_{ig} + e_{ig}^F \quad (\text{B.29})$$

with  $M_{ig} \in \{0, 1\}$  equals 1 if the individual is from the "male" identity and  $E_{ig} \in \{0, 1\}$  equals 1 if the individual is eligible to the treatment.  $s_g^M := \bar{M}_g$ , i.e. it is the share of "males" in the peer group.

Averaging, within group  $g$ , the outcome variables for "male" and "female" individuals, we get the following expressions

$$\bar{y}_g^M = \theta_0^{w,E} s_g^M \bar{y}_g^M + \theta_0^{b,M,F} (1 - s_g^M) \bar{y}_g^F + \delta_0 s_g^{E,M} D_g + \bar{e}_g^M$$

$$\bar{y}_g^F = \theta_0^{b,F,M} s_g^M \bar{y}_g^M + \theta_0^{w,F} (1 - s_g^M) \bar{y}_g^F + \delta_0 s_g^{E,F} D_g + \bar{e}_g^F$$

where  $s_g^{E,M} = \frac{\sum_{i=1}^{n_g} E_{ig} M_{ig}}{\sum_{i=1}^{n_g} M_{ig}}$  and  $s_g^{E,F} = \frac{\sum_{i=1}^{n_g} E_{ig} (1 - M_{ig})}{\sum_{i=1}^{n_g} (1 - M_{ig})}$  are, respectively, the share of "males" (resp. "females") that are eligible to the treatment. Then, plugging-in the expression of  $\bar{y}_g^F$  into the one of  $\bar{y}_g^M$ , we get

$$\bar{y}_g^M = \theta^{w,M} s_g^M \bar{y}_g^M + \frac{\theta_0^{b,M,F} (1 - s_g^M)}{1 - \theta_0^{w,F} (1 - s_g^M)} \left( \theta_0^{b,F,M} s_g^M \bar{y}_g^M + \delta_0 s_g^{E,F} D_g + \bar{e}_g^F \right) + \delta_0 s_g^{E,M} D_g + \bar{e}_g^M$$

Rearranging terms, we get

$$\left[ 1 - s_g^M \theta_0^{w,M} - (1 - s_g^M) \theta_0^{w,F} + s_g^M (1 - s_g^M) \left( \theta_0^{w,M} \theta_0^{w,F} - \theta_0^{b,M,F} \theta_0^{b,F,M} \right) \right] \bar{y}_g^M$$

$$= \delta_0 \left( \theta_0^{b,M,F} s_g^{E,F} (1 - s_g^M) + (1 - \theta_0^{w,F} (1 - s_g^M)) s_g^{E,M} \right) D_g + \theta_0^{b,M,F} (1 - s_g^M) \bar{e}_g^F + \left( 1 - \theta_0^{w,F} (1 - s_g^M) \right) \bar{e}_g^M$$

Hence,

$$\begin{aligned}\bar{y}_g^M &= \frac{1 - \theta_0^{w,F}(1 - s_g^M)}{1 - s_g^M \theta_0^{w,M} - (1 - s_g^M) \theta_0^{w,F} + s_g^M (1 - s_g^M) (\theta_0^{w,M} \theta_0^{w,F} - \theta_0^{b,M,F} \theta_0^{b,F,M})} \bar{e}_g^M \\ &+ \frac{\theta_0^{b,M,F} (1 - s_g^M)}{1 - s_g^M \theta_0^{w,M} - (1 - s_g^M) \theta_0^{w,F} + s_g^M (1 - s_g^M) (\theta_0^{w,M} \theta_0^{w,F} - \theta_0^{b,M,F} \theta_0^{b,F,M})} \bar{e}_g^F \\ &+ \frac{\delta_0 (s_g^{E,F} \theta_0^{b,M,F} (1 - s_g^M) + s_g^{E,M} (1 - \theta_0^{w,F} (1 - s_g^M)))}{1 - s_g^M \theta_0^{w,M} - (1 - s_g^M) \theta_0^{w,F} + s_g^M (1 - s_g^M) (\theta_0^{w,M} \theta_0^{w,F} - \theta_0^{b,M,F} \theta_0^{b,F,M})} D_g\end{aligned}$$

Now, plugging-in the reduced form expression of  $\bar{y}_g^M$  into  $\bar{y}_g^F$  and developing, we get

$$\begin{aligned}\bar{y}_g^F &= \frac{\theta_0^{b,F,M} s_g^M}{1 - s_g^M \theta_0^{w,M} - (1 - s_g^M) \theta_0^{w,F} + s_g^M (1 - s_g^M) (\theta_0^{w,M} \theta_0^{w,F} - \theta_0^{b,M,F} \theta_0^{b,F,M})} \bar{e}_g^M \\ &+ \frac{(1 - \theta_0^{w,M} s_g^M)}{1 - s_g^M \theta_0^{w,M} - (1 - s_g^M) \theta_0^{w,F} + s_g^M (1 - s_g^M) (\theta_0^{w,M} \theta_0^{w,F} - \theta_0^{b,M,F} \theta_0^{b,F,M})} \bar{e}_g^F \\ &+ \frac{\delta_0 (s_g^{E,F} (1 - \theta_0^{w,M} s_g^M) + s_g^{E,M} \theta_0^{b,F,M} s_g^M)}{1 - s_g^M \theta_0^{w,M} - (1 - s_g^M) \theta_0^{w,F} + s_g^M (1 - s_g^M) (\theta_0^{w,M} \theta_0^{w,F} - \theta_0^{b,M,F} \theta_0^{b,F,M})} D_g\end{aligned} \quad (\text{B.30})$$

We then make the following "adapted" assumptions

**Assumption 2† (Randomized Experiment)** *The group level treatment  $D_g$  is randomly assigned, i.e.*

$$(\bar{e}_g^F, \bar{e}_g^M, s_g^M, s_g^{E,F}, s_g^{E,M}) \perp\!\!\!\perp D_g \quad (\text{B.31})$$

Assumption 2† states that treatment is randomly assigned. In particular, groups receive the treatment independently of their share of eligible units for both identities and of their share of "males" in the group. We also impose a restriction on the support of  $D_g$  conditional on the share of eligible units  $s_g$ . Let  $S \subseteq (0, 1]$  be the support of shares that are observed in the sample,

**Assumption 3† (Common Support)** *For each  $(s_g^M, s_g^{E,F}, s_g^{E,M}) \in S^M \times S^{E,F} \times S^{E,M}$ ,*

$$0 < P(D_g = 1 | s_g^M, s_g^{E,F}, s_g^{E,M}) < 1 \quad (\text{B.32})$$

Assumption 3† states that for any share of "males", of eligible "males" and of eligible "females" that is observed in the population of reference groups, there exist some groups that are treated and some that are not.

**Proposition 9** *Provided Assumptions 1†, 2† and 3† hold and all the mentioned conditional expectations are well-defined, we have*

$$\begin{aligned}
\tau^M(s^M, s^{E,F}, s^{E,M}) &= \mathbb{E}[\bar{y}^M | s^M, s^{E,F}, s^{E,M}, D = 1] - \mathbb{E}[\bar{y}^M | s^M, s^{E,F}, s^{E,M}, D = 0] \\
&= \mathbb{E}\left[\frac{D - \Pr(D = 1 | s^M, s^{E,F}, s^{E,M})}{\Pr(D = 1 | s^M, s^{E,F}, s^{E,M})(1 - \Pr(D = 1 | s^M, s^{E,F}, s^{E,M}))} \cdot \bar{y}^M \Big| s^M, s^{E,F}, s^{E,M}\right] \\
&= \frac{\delta_0 (s^{E,F} \theta_0^{b,M,F} (1 - s^M) + s^{E,M} (1 - \theta^{w,F} (1 - s^M)))}{1 - s^M \theta_0^{w,M} - (1 - s^M) \theta_0^{w,F} + s^M (1 - s^M) (\theta_0^{w,M} \theta_0^{w,F} - \theta_0^{b,M,F} \theta_0^{b,F,M})} \quad (\text{B.33})
\end{aligned}$$

and

$$\begin{aligned}
\tau^F(s^M, s^{E,F}, s^{E,M}) &= \mathbb{E}[\bar{y}^F | s^M, s^{E,F}, s^{E,M}, D = 1] - \mathbb{E}[\bar{y}^F | s^M, s^{E,F}, s^{E,M}, D = 0] \\
&= \mathbb{E}\left[\frac{D - \Pr(D = 1 | s^M, s^{E,F}, s^{E,M})}{\Pr(D = 1 | s^M, s^{E,F}, s^{E,M})(1 - \Pr(D = 1 | s^M, s^{E,F}, s^{E,M}))} \cdot \bar{y}^F \Big| s^M, s^{E,F}, s^{E,M}\right] \\
&= \frac{\delta_0 (s^{E,F} (1 - \theta^{w,M} s^M) + s^{E,M} \theta^{b,F,M} s^M)}{1 - s^M \theta_0^{w,M} - (1 - s^M) \theta_0^{w,F} + s^M (1 - s^M) (\theta_0^{w,M} \theta_0^{w,F} - \theta_0^{b,M,F} \theta_0^{b,F,M})} \quad (\text{B.34})
\end{aligned}$$

In the following, for conciseness, we define, for any  $g \in \{1, \dots, G\}$ ,  $s_g := (s_g^M, s_g^{E,F}, s_g^{E,M})$  and we let  $\lambda_0^O := (\delta_0, \theta^{w,F}, \theta^{w,M}, \theta^{b,F,M}, \theta^{b,M,F})$

**Proposition 10 (Sufficient Conditions for Identification of  $\lambda_0^O := (\delta_0, \theta^{w,F}, \theta^{w,M}, \theta^{b,F,M}, \theta^{b,M,F})$ )** *Provided  $\delta_0 \neq 0$ , if Assumptions 1†, 2† and 3† hold and if one of the two following conditions is satisfied*

1. *there exist at least 4 different vectors of shares  $(s_1^M, s_1^{E,F}, s_1^{E,M})$ ,  $(s_2^M, s_2^{E,F}, s_2^{E,M})$ ,  $(s_3^M, s_3^{E,F}, s_3^{E,M})$  and  $(s_4^M, s_4^{E,F}, s_4^{E,M}) \in (0, 1)^3$  that have positive probability mass and for which both  $\tau^M$  and  $\tau^F$  are well-defined*
2. *there is a continuum of shares  $I_s \subseteq [0, 1] \times [0, 1] \times [0, 1]$  such that  $P(s \in I_s) > 0$  and both  $\tau^F(s)$  and  $\tau^M(s)$  are well-defined, for any  $s \in I_s$*

then  $\lambda_0^O$  is the unique vector defined on  $\Theta$  that satisfies the moment condition

$$\mathbb{E}[u^O(\bar{y}_g^M, \bar{y}_g^F, D_g, s_g; \lambda_0^O) | s_g = s] = \begin{pmatrix} \mathbb{E}[u^{O,M}(\bar{y}_g^M, D_g, s_g; \lambda_0^O) | s_g = s] \\ \mathbb{E}[u^{O,F}(\bar{y}_g^F, D_g, s_g; \lambda_0^O) | s_g = s] \end{pmatrix} = 0 \quad (\text{B.35})$$

where for all  $\lambda \in \Theta$ ,

$$u^{O,M}(\bar{y}_g^M, D_g, s_g; \lambda) = \omega^O(D_g, s_g) \bar{y}_g^M - \frac{\delta_0 \left( s_g^{E,F} \theta_0^{b,M,F} (1 - s_g^M) + s_g^{E,M} (1 - \theta^{w,F} (1 - s_g^M)) \right)}{1 - s_g^M \theta_0^{w,M} - (1 - s_g^M) \theta_0^{w,F} + s_g^M (1 - s_g^M) \left( \theta_0^{w,M} \theta_0^{w,F} - \theta_0^{b,M,F} \theta_0^{b,F,M} \right)}$$

$$u^{O,F}(\bar{y}_g^F, D_g, s_g; \lambda) = \omega^O(D_g, s_g) \bar{y}_g^F - \frac{\delta_0 \left( s_g^{E,F} (1 - \theta^{w,M} s_g^M) + s_g^{E,M} \theta^{b,F,M} s_g^M \right)}{1 - s_g^M \theta_0^{w,M} - (1 - s_g^M) \theta_0^{w,F} + s_g^M (1 - s_g^M) \left( \theta_0^{w,M} \theta_0^{w,F} - \theta_0^{b,M,F} \theta_0^{b,F,M} \right)}$$

$$\text{and } \omega^O(D_g, s_g) = \frac{D_g - P(D_g = 1 | s_g)}{P(D_g = 1 | s_g)(1 - P(D_g = 1 | s_g))}$$

## B.4 Inclusion of Covariates

In this section, we show how the model can be adapted, in the natural experiment setting, so as to make the common trends assumption more plausible.

### B.4.1 Identification

Let  $\Delta X_g$  denote a vector of observed changes of some of group  $g$ 's exogenous characteristics and let  $\mathcal{X}$  denote the support of  $\Delta X_g$ . Let's suppose Assumption 4' does not hold but, instead, the following condition is satisfied

**Assumption 3" (Extended Conditional Common Trends)** For any  $k \in \{E, NE\}$ ,

$$E[\bar{e}_{g2}^k - \bar{e}_{g1}^k | s_g, \Delta X_g, D_{g2} = 1] = E[\bar{e}_{g2}^k - \bar{e}_{g1}^k | s_g, \Delta X_g, D_{g2} = 0]$$

Intuitively, we are going to compare pairs of treated and control groups with the same share of eligible units and the same evolution of the covariates  $X$  from period 1 to 2. Then, we assume that, in the absence of the treatment, the change in the average aggregate outcome among eligible units in treated groups would have been the same as the change in the average aggregate outcome among eligible units in control groups. We make the same assumption regarding the change in group average outcome among non-eligible units. Finally,

**Assumption 4" (Extended Conditional Common Support)** For every  $(g, t) \in \{1, \dots, G\} \times \{1, 2\}$ ,

$$D_{g1} = 0 \text{ a.s and for all } s \in S \text{ and } x \in \mathcal{X}, 0 < P(D_{g2} = 1 | s, x) < 1$$

Then, the previous results can be adapted to this new context

**Proposition 11 (Conditional Moment Conditions - Natural Experiment with Covariates)** Let  $I$  be an interval on  $\mathbb{R} \setminus \{0\}$ . Let  $\lambda_0 = (\delta_0, \theta_0^w, \theta_0^b)$  be the true value of the parameters with  $\lambda_0 \in \Theta := I \times (-1, 1) \times (-1, 1)$ . For any  $s \in (0, 1)$ , provided Assumptions 1', 2', 3" and 4" hold and all the mentioned conditional moments are well-defined,

$$E[u^{DiDX}(Z_g; \lambda_0) | s_g = s, \Delta X_g = x] = \begin{pmatrix} E[u^{DiDX,E}(Z_g; \lambda_0) | s_g = s, \Delta X_g = x] \\ E[u^{DiDX,NE}(Z_g; \lambda_0) | s_g = s, \Delta X_g = x] \end{pmatrix} = 0$$



where  $Z_g := (\Delta \bar{y}_g^E, \Delta \bar{y}_g^N, D_{g2}, s_g, \Delta X_g)$  and for all  $\lambda \in \Theta$ ,

$$u^{DiDX,E}(Z_g; \lambda) = \omega^{DiDX}(D_{g2}, s_g, \Delta X_g) \Delta \bar{y}_g^E - \frac{\delta(1 - \theta^w(1 - s_g))}{1 - \theta^w + s_g(1 - s_g)[(\theta^w)^2 - (\theta^b)^2]}$$

$$u^{DiDX,NE}(Z_g; \lambda) = \omega^{DiDX}(D_{g2}, s_g, \Delta X_g) \Delta \bar{y}_g^N - \frac{\delta \theta^b s_g}{1 - \theta^w + s_g(1 - s_g)[(\theta^w)^2 - (\theta^b)^2]}$$

$$\text{and } \omega^{DiDX}(D_{g2}, s_g, \Delta X_g) = \frac{D_{g2} - P(D_{g2} = 1 | s_g, \Delta X_g)}{P(D_{g2} = 1 | s_g, \Delta X_g)(1 - P(D_{g2} = 1 | s_g, \Delta X_g))}$$

**Proof:** Provided they are well-defined, the conditional expectations of  $\Delta \bar{y}_g^E$  and  $\Delta \bar{y}_g^N$  with respect to  $(D_{g2}, s_g, \Delta X_g)$  are

$$\begin{aligned} \mathbb{E} \left[ \Delta \bar{y}_g^E \middle| D_{g2}, s_g, \Delta X_g \right] &= \frac{\theta_0^b(1 - s_g)}{1 - \theta_0^w + s_g(1 - s_g)[(\theta_0^w)^2 - (\theta_0^b)^2]} \mathbb{E} \left[ \bar{e}_{g2}^N - \bar{e}_{g1}^N \middle| D_{g2}, s_g, \Delta X_g \right] \\ &+ \frac{1 - \theta_0^w(1 - s_g)}{1 - \theta_0^w + s_g(1 - s_g)[(\theta_0^w)^2 - (\theta_0^b)^2]} \mathbb{E} \left[ \bar{e}_{g2}^E - \bar{e}_{g1}^E \middle| D_{g2}, s_g, \Delta X_g \right] \\ &+ \frac{\delta_0(1 - \theta_0^w(1 - s_g))}{1 - \theta_0^w + s_g(1 - s_g)[(\theta_0^w)^2 - (\theta_0^b)^2]} D_{g2} \end{aligned}$$

$$\begin{aligned} \mathbb{E} \left[ \Delta \bar{y}_g^N \middle| D_{g2}, s_g, \Delta X_g \right] &= \frac{1 - \theta_0^w s_g}{1 - \theta_0^w + s_g(1 - s_g)[(\theta_0^w)^2 - (\theta_0^b)^2]} \mathbb{E} \left[ \bar{e}_{g2}^N - \bar{e}_{g1}^N \middle| D_{g2}, s_g, \Delta X_g \right] \\ &+ \frac{\theta_0^b s_g}{1 - \theta_0^w + s_g(1 - s_g)[(\theta_0^w)^2 - (\theta_0^b)^2]} \mathbb{E} \left[ \bar{e}_{g2}^E - \bar{e}_{g1}^E \middle| D_{g2}, s_g, \Delta X_g \right] \\ &+ \frac{\delta_0 \theta_0^b s_g}{1 - \theta_0^w + s_g(1 - s_g)[(\theta_0^w)^2 - (\theta_0^b)^2]} D_{g2} \end{aligned}$$

Now,

$$\mathbb{E} \left[ \Delta \bar{y}_g^E \middle| D_{g2} = 1, s_g, \Delta X_g \right] - \mathbb{E} \left[ \Delta \bar{y}_g^E \middle| D_{g2} = 0, s_g, \Delta X_g \right] = \frac{\delta_0(1 - \theta_0^w(1 - s_g))}{1 - \theta_0^w + s_g(1 - s_g)[(\theta_0^w)^2 - (\theta_0^b)^2]}$$

$$\mathbb{E} \left[ \Delta \bar{y}_g^N \middle| D_{g2} = 1, s_g, \Delta X_g \right] - \mathbb{E} \left[ \Delta \bar{y}_g^N \middle| D_{g2} = 0, s_g, \Delta X_g \right] = \frac{\delta_0 \theta_0^b s_g}{1 - \theta_0^w + s_g(1 - s_g)[(\theta_0^w)^2 - (\theta_0^b)^2]}$$

using Assumption 3". Finally, using the same steps as in the proof for Proposition 4, one can show that, for any  $k \in \{E, N\}$ ,

$$\begin{aligned}
\mathbb{E} [\omega^{DiDX}(D_{g2}, s_g, \Delta X_g) \Delta \bar{y}_g^E | s_g, \Delta X_g] &= \mathbb{E} \left[ \frac{D_{g2} - P(D_{g2} = 1 | s_g, \Delta X_g)}{P(D_g = 1 | s_g, \Delta X_g)(1 - P(D_g = 1 | s_g, \Delta X_g))} \Delta \bar{y}_g^k | s_g, \Delta X_g \right] \\
&= \frac{P(D_{g2} = 1 | s_g, \Delta X_g)(1 - P(D_{g2} = 1 | s_g, \Delta X_g))}{P(D_{g2} = 1 | s_g, \Delta X_g)(1 - P(D_{g2} = 1 | s_g, \Delta X_g))} \mathbb{E} [\Delta \bar{y}_g^k | s_g, \Delta X_g, D_{g2} = 1] \\
&\quad - \frac{(1 - P(D_{g2} = 1 | s_g, \Delta X_g))P(D_{g2} = 1 | s_g, \Delta X_g)}{P(D_{g2} = 1 | s_g, \Delta X_g)(1 - P(D_{g2} = 1 | s_g, \Delta X_g))} \mathbb{E} [\Delta \bar{y}_g^k | s_g, \Delta X_g, D_{g2} = 0] \\
&= \mathbb{E} [\Delta \bar{y}_g^k | s_g, \Delta X_g, D_{g2} = 1] - \mathbb{E} [\bar{y}_g^k | s_g, \Delta X_g, D_{g2} = 0]
\end{aligned}$$

## B.4.2 Estimation

We consider

$$m^{DiD}(\tilde{Z}, \lambda, \hat{\mu}_D^X) = \tilde{w}(s, \Delta X) \left\{ \hat{\omega}^{DiDX}(\tilde{Z}, \hat{\mu}_D^X) \bar{Y} - \frac{\delta}{\phi(s, \lambda)} \begin{pmatrix} 1 - (1-s)\theta^w \\ s\theta^b \end{pmatrix} \right\}$$

where  $\tilde{w}(s, \Delta X)$  is a  $r \times 2$  matrix of functions of  $(s, \Delta X)$  such that  $r \geq 4$ ,

$$\hat{\omega}^{DiDX}(\tilde{Z}, \hat{\mu}_D^X) = \frac{D}{\hat{\mu}_D^X(s, \Delta X)} - \frac{1-D}{1 - \hat{\mu}_D^X(s, \Delta X)}$$

and  $\hat{\mu}_D^X(s, \Delta X)$  is a non-parametric estimator of  $\mu_D^X(s, \Delta X) = E[D_2 | s, \Delta X]$  based on a series logistic regression of  $D_2$  on a power series of  $(s, \Delta X)$ . To prove asymptotic normality, the technical condition on  $K_G$  in Assumption 5' - point 5 has to be adapted, by requiring that  $K_G$  goes at a slower rate to infinity, so as to avoid overfitting. The greater the dimension of  $\Delta X$ , the slower will be the rate. Then, one can show that

**Conjecture 1 (GMM Estimator in the Natural Experiment Setting with Covariates)** *Let  $(\tilde{Z}_g)_{g=1, \dots, G}$  be an i.i.d sample of  $G$  groups, where  $\tilde{Z}_g = (\Delta \bar{y}_g^E, \Delta \bar{y}_g^N, D_{2g}, s_g, \Delta X_g)$ . Let*

$$\hat{V}_G \xrightarrow{p} \tilde{V}_0 := \mathbb{E} \left[ \left\{ m^{DiD}(\tilde{Z}, \lambda_0, \mu_D^X) + \alpha(\tilde{Z}) \right\} \left\{ m^{DiD}(\tilde{Z}, \lambda_0, \mu_D^X) + \alpha(\tilde{Z}) \right\}' \right]$$

with

$$\alpha(\tilde{Z}) = -\tilde{w}(s, \Delta X) \left( \frac{\tilde{\mu}_1(s, \Delta X)}{\mu_D^X(s, \Delta X)^2} + \frac{\tilde{\mu}_0(s, \Delta X)}{(1 - \mu_D^X(s, \Delta X))^2} \right) (D_2 - \mu_D^X(s, \Delta X))$$

where, for any  $s \in \mathcal{S}$ ,  $\tilde{\mu}_1(s, \Delta X) = E[\bar{Y} D_2 | s, \Delta X]$  and  $\tilde{\mu}_0(s, \Delta X) = E[\bar{Y}(1 - D_2) | s, \Delta X]$ . Let

$$m^{\tilde{DiD}}(\tilde{Z}, \lambda_0) = \frac{\partial}{\partial \lambda} m^{DiD}(\tilde{Z}, \lambda_0, \mu_D^X)$$

which does not depend on the propensity score. Supposing that

1. Assumptions 1', 2', 3', 4' and 5' hold and identification  $\lambda_0$  is ensured

2.  $\Theta := I_1 \times [-K, K] \times [-K, K]$  where  $I$  is a compact subset of  $\mathbb{R}$  with  $0 \notin I$  and  $K \in (0, 1)$

Then

$$\hat{\lambda}^* := \arg \min_{\lambda \in \Theta} \left( \frac{1}{G} \sum_{g=1}^G m^{DiD}(\tilde{Z}_g, \lambda, \hat{\mu}_D^X) \right)' \hat{V}_G^{-1} \left( \frac{1}{G} \sum_{g=1}^G m^{DiD}(\tilde{Z}_g, \lambda, \hat{\mu}_D^X) \right) \quad (\text{B.36})$$

is a consistent estimator of  $\lambda_0$  whose asymptotic distribution is

$$\sqrt{G}(\hat{\lambda}^* - \lambda_0) \xrightarrow{d} \mathcal{N} \left( 0, \left[ \tilde{M}_0' \tilde{V}_0^{-1} \tilde{M}_0 \right]^{-1} \right)$$

with  $\lambda_0 := (\delta_0, \theta_0^w, \theta_0^b)$  and  $\tilde{M}_0 := \mathbf{E} \left[ m^{\tilde{D}iD}(\tilde{Z}, \lambda_0) \right]$

## Appendix C

# Elderly Home Care Market and Spousal Informal Care Supply

### C.1 Figures

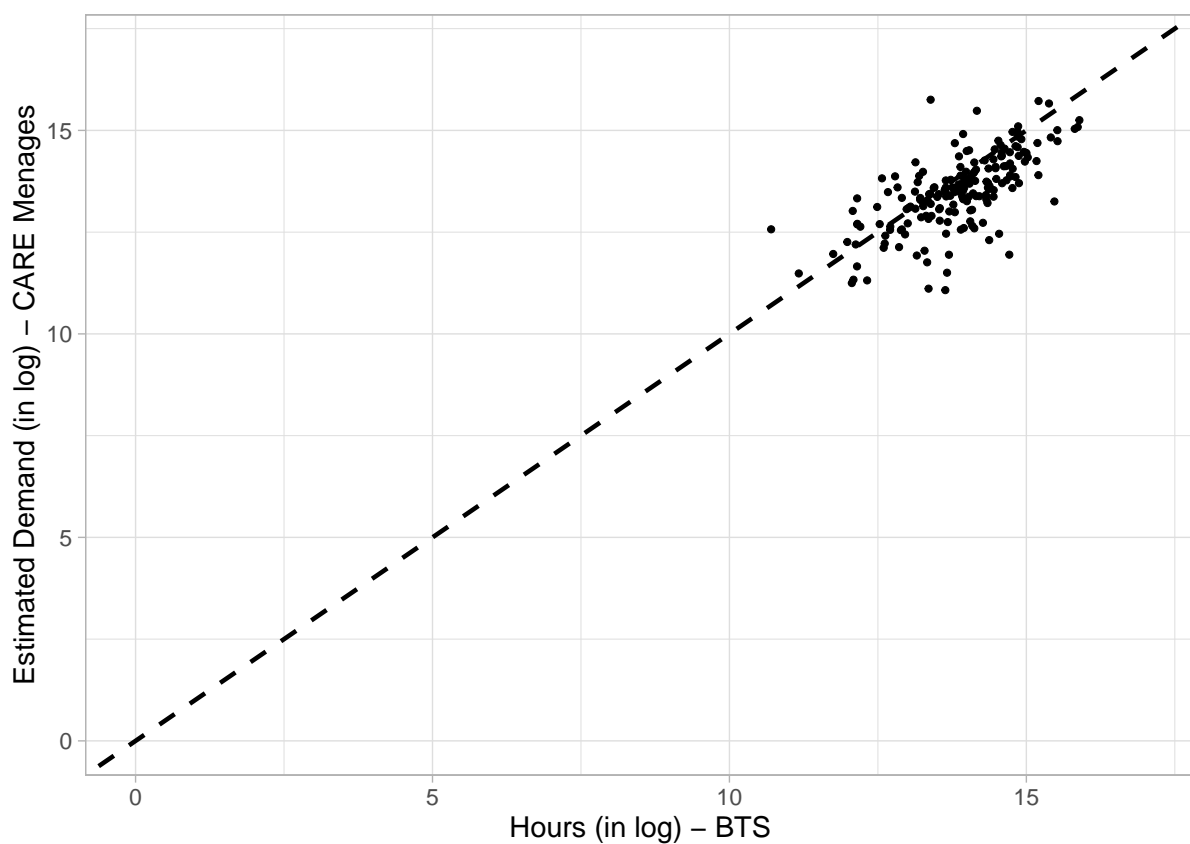


Figure C.1: Linear fit between the total contract hours of home care workers employed by a SAAD and the estimated demand for home care hours for SAADs based on data from the CARE-Ménages survey

## C.2 Likelihood Formula

The likelihood function associated to the  $\mathcal{X}_n$  i.i.d sample of  $n$  observations and the 15 possible events that are described in Table C.1 is:

$$L(\theta) = \prod_{i=1}^n \prod_{j=1}^{15} P(E_j | \theta)^{\mathbb{I}(E_j \text{ is observed for } i)}$$

where  $\theta$  is the vector of parameters of dimension  $6 + 2 \times m + p$  where  $m$  is the dimension of the vector of observed variables influencing the preference parameters while  $p$  is the dimension of the vector of observed variables that may influence the unobserved costs of hiring a self-employed home care worker.

$j$	Event $E_j$	Marginal Density
1	$p_{SP}^{OOP} \leq p_{DE}^{OOP}$ $0 < I < 16, F > 0$	$f_{v_\ell, v_c} \left( \log \left( \frac{18-I}{F+I+1} \right), \log \left( \frac{R-p_{SP}^{OOP} F}{p_{OOP}^{OOP}(F+I+1)} \right) \right)$
2	$p_{SP}^{OOP} \leq p_{DE}^{OOP}$ $0 < I < 16, F = 0$	$f_{v_\ell} \left( \log \left( \frac{18-I}{I+1} \right) \right) \times \left( 1 - F_{v_c   v_\ell = \log \left( \frac{18-I}{I+1} \right)} \left( \log \left( \frac{R}{p_{SP}^{OOP}(I+1)} \right) \right) \right)$
3	$p_{SP}^{OOP} \leq p_{DE}^{OOP}$ $I = 16, F > 0$	$F_{v_\ell   v_c = \log \left( \frac{R-p_{SP}^{OOP} F}{p_{SP}^{OOP}(16+F+1)} \right)} \left( \log \left( \frac{2}{16+F+1} \right) \right) f_{v_c} \left( \log \left( \frac{R-p_{SP}^{OOP} F}{p_{SP}^{OOP}(16+F+1)} \right) \right)$
4	$p_{SP}^{OOP} \leq p_{DE}^{OOP}$ $I = 16, F = 0$	$\int_{-\infty}^{\log \left( \frac{2}{16+1} \right)} \int_{\log \left( \frac{R}{p_{SP}^{OOP}(16+1)} \right)}^{+\infty} f_{v_\ell, v_c}(x, y) dx dy$
5	$p_{SP}^{OOP} \leq p_{DE}^{OOP}$ $I = 0, F > 0$	$\left( 1 - F_{v_\ell   v_c = \log \left( \frac{R-p_{SP}^{OOP} F}{p_{SP}^{OOP}(F+1)} \right)} \left( \log \left( \frac{18}{F+1} \right) \right) \right) f_{v_c} \left( \log \left( \frac{R-p_{SP}^{OOP} F}{p_{SP}^{OOP}(F+1)} \right) \right)$
6	$p_{SP}^{OOP} \leq p_{DE}^{OOP}$ $I = 0, F = 0$	$\int_{\log(18)}^{+\infty} \int_{\log \left( \frac{R}{p_{SP}^{OOP}} \right)}^{+\infty} f_{v_\ell, v_c}(x, y) dx dy$
7	$p_{SP}^{OOP} > p_{DE}^{OOP}$ $0 < I < 16, F > 0, SP$	$f_{v_\ell, v_c} \left( \log \left( \frac{18-I}{F+I+1} \right), \log \left( \frac{R-p_{SP}^{OOP} F}{p_{OOP}^{OOP}(F+I+1)} \right) \right) \left( 1 - F_\xi \left( \log(p_{SP}^{OOP} - p_{DE}^{OOP}) \right) \right)$
8	$p_{SP}^{OOP} > p_{DE}^{OOP}$ $I = 16, F > 0, SP$	$F_{v_\ell   v_c = \log \left( \frac{R-p_{SP}^{OOP} F}{p_{SP}^{OOP}(16+F+1)} \right)} \left( \log \left( \frac{2}{16+F+1} \right) \right) f_{v_c} \left( \log \left( \frac{R-p_{SP}^{OOP} F}{p_{SP}^{OOP}(16+F+1)} \right) \right) \times \left( 1 - F_\xi \left( \log(p_{SP}^{OOP} - p_{DE}^{OOP}) \right) \right)$

<b>9</b>	$p_{SP}^{OOP} > p_{DE}^{OOP}$ $I = 0, F > 0, SP$	$\left(1 - F_{v_\ell v_c=\log\left(\frac{R-p_{SP}^{OOP}F}{p_{SP}^{OOP}(F+1)}\right)}\left(\log\left(\frac{18}{F+1}\right)\right)\right) f_{v_c}\left(\log\left(\frac{R-p_{SP}^{OOP}F}{p_{SP}^{OOP}(F+1)}\right)\right)$ $\times (1 - F_\xi(\log(p_{SP}^{OOP} - p_{DE}^{OOP})))$
<b>10</b>	$p_{SP}^{OOP} > p_{DE}^{OOP}$ $0 < I < 16, F > 0, DE$	$\int_{-\infty}^{\log(p_{SP}^{OOP} - p_{DE}^{OOP})} f_{v_\ell, v_c}\left(\log\left(\frac{18 - I}{F + I + 1}\right), \log\left(\frac{R - (p_{DE}^{OOP} + \exp(\xi))F}{(p_{DE}^{OOP} + \exp(\xi))(F + I + 1)}\right)\right)$ $\times f_{\xi \xi < \log\left(\frac{R}{F} - p_{DE}^{OOP}\right)}(x) dx$
<b>11</b>	$p_{SP}^{OOP} > p_{DE}^{OOP}$ $I = 16, F > 0, DE$	$\int_{-\infty}^{\log(p_{SP}^{OOP} - p_{DE}^{OOP})} F_{v_\ell v_c=\log\left(\frac{R - (p_{DE}^{OOP} + \exp(\xi))F}{(p_{DE}^{OOP} + \exp(\xi))(16 + F + 1)}\right)}\left(\log\left(\frac{2}{16 + F + 1}\right)\right)$ $\times f_{v_c}\left(\log\left(\frac{R - (p_{DE}^{OOP} + \exp(\xi))F}{(p_{DE}^{OOP} + \exp(\xi))(16 + F + 1)}\right)\right) f_{\xi \xi < \log\left(\frac{R}{F} - p_{DE}^{OOP}\right)}(x) dx$
<b>12</b>	$p_{SP}^{OOP} > p_{DE}^{OOP}$ $I = 0, F > 0, DE$	$\int_{-\infty}^{\log(p_{SP}^{OOP} - p_{DE}^{OOP})} F_{v_\ell v_c=\log\left(\frac{R - (p_{DE}^{OOP} + \exp(\xi))F}{(p_{DE}^{OOP} + \exp(\xi))(16 + F + 1)}\right)}\left(\log\left(\frac{2}{16 + F + 1}\right)\right)$ $\times f_{v_c}\left(\log\left(\frac{R - (p_{DE}^{OOP} + \exp(\xi))F}{(p_{DE}^{OOP} + \exp(\xi))(16 + F + 1)}\right)\right) f_{\xi \xi < \log\left(\frac{R}{F} - p_{DE}^{OOP}\right)}(x) dx$
<b>13</b>	$p_{SP}^{OOP} > p_{DE}^{OOP}$ $0 < I < 16, F = 0$	$f_{v_\ell}\left(\log\left(\frac{18 - I}{I + 1}\right)\right) \times \left(1 - F_{v_c v_\ell=\log\left(\frac{18 - I}{I + 1}\right)}\left(\log\left(\frac{R}{p_{SP}^{OOP}(I + 1)}\right)\right)\right)$ $(1 - F_\xi(\log(p_{SP}^{OOP} - p_{DE}^{OOP})))$ $+ \int_{-\infty}^{\log(p_{SP}^{OOP} - p_{DE}^{OOP})} \left(1 - F_{v_c v_\ell=\log\left(\frac{18 - I}{I + 1}\right)}\left(\log\left(\frac{R}{(p_{DE}^{OOP} + \exp(\xi))(I + 1)}\right)\right)\right)$ $\times f_{v_\ell}\left(\log\left(\frac{18 - I}{I + 1}\right)\right) f_\xi(x) dx$
<b>14</b>	$p_{SP}^{OOP} > p_{DE}^{OOP}$ $I = 16, F = 0$	$\int_{-\infty}^{\log\left(\frac{2}{16 + 1}\right)} \int_{\log\left(\frac{R}{p_{SP}^{OOP}(16 + 1)}\right)}^{+\infty} f_{v_\ell, v_c}(x, y) dx dy$ $(1 - F_\xi(\log(p_{SP}^{OOP} - p_{DE}^{OOP})))$ $+ \int_{-\infty}^{\log(p_{SP}^{OOP} - p_{DE}^{OOP})} \int_{-\infty}^{\log\left(\frac{2}{16 + 1}\right)} \int_{\log\left(\frac{R}{(p_{DE}^{OOP} + \exp(\xi))(16 + 1)}\right)}^{+\infty} f_{v_\ell, v_c}(x, y)$ $f_\xi(z) dx dy dz$
<b>15</b>	$p_{SP}^{OOP} > p_{DE}^{OOP}$ $I = 0, F = 0$	$\int_{\log(18)}^{+\infty} \int_{\log\left(\frac{R}{p_{SP}^{OOP}}\right)}^{+\infty} f_{v_\ell, v_c}(x, y) dx dy$ $(1 - F_\xi(\log(p_{SP}^{OOP} - p_{DE}^{OOP})))$ $+ \int_{-\infty}^{\log(p_{SP}^{OOP} - p_{DE}^{OOP})} \int_{\log(18)}^{+\infty} \int_{\log\left(\frac{R}{(p_{DE}^{OOP} + \exp(\xi))}\right)}^{+\infty} f_{v_\ell, v_c}(x, y)$ $f_\xi(z) dx dy dz$

Table C.1: Marginal Densities - Likelihood Function

### C.3 Simulations

Simulations were carried out to check that the maximum likelihood estimator is consistent and asymptotically normal. An i.i.d sample of 1,000 observation was simulated. For each observation, values were drawn for:

- daily income  $R$ , generated from a Gamma distribution with shape 13 and scale 2.8, truncated at 96;
- market price for SAADs  $p_{SP}$ , generated from a Gamma distribution with shape 3 and scale 2, truncated at 10;
- market price for independent home care workers  $p_{DE}$ , generated from a uniform distribution between 12 and 15;
- a binary variable  $D$ , that is generated as the sum of two independent Bernoulli trials with probabilities 0.3 and 0.4;
- a continuous variable  $X$ , drawn from a standard normal distribution;
- another binary variable  $Z$ , that is generated as the sum of two independent Bernoulli trials with probabilities 0.5 and 0.2;
- unobserved heterogeneity terms  $v_\ell$  and  $v_c$ , drawn from a bivariate gaussian with mean  $\mu = (\beta_\ell^1 + \beta_\ell^X X + \beta_\ell^D D, \beta_c^1 + \beta_c^X X + \beta_c^D D)$  and variance-covariance matrix  $\Sigma$
- unobserved heterogeneity term  $\xi$  drawn from a gaussian distribution with expectation  $\mu_\xi = \beta_\xi^1 + \beta_\xi^Z Z$  and variance 1

Based on these simulated data, the vector of parameters  $\alpha$  is computed. Out-of-pocket prices are also generated using the APA copayment rate formula. The number of hours of formal care consumed and the number of hours of informal care provided, both obtained by maximizing the utility function presented in Section 2, are calculated. The simulated dataset is then used to verify the convergence of the estimator. Table C.2 shows the values of the estimates along with the associated analytical standard errors. The maximum likelihood estimator considered is shown to be convergent and asymptotically normal, with all parameters identified.

Table C.2: Simulations - Estimates

Parameter	True value	Estimate	Standard error
$\beta_\ell^1$	0	-0.2067	0.1570
$\beta_c^1$	1	0.8578	0.1322
$\sigma_\ell$	3	3.0021	0.1214
$\sigma_c$	2	1.9143	0.0596
$\rho$	0.4	0.3871	0.0394
$\beta_\xi^1$	-1	-0.7831	0.1920
$\beta_\ell^X$	-2	-2.0232	0.1379
$\beta_\ell^D$	1	1.1057	0.1719
$\beta_c^X$	3	2.7929	0.1025
$\beta_c^D$	-1	-0.7979	0.1133
$\beta_\xi^Z$	1	1.2051	0.2047





# Bibliography

- A. Abadie. Semiparametric difference-in-differences estimators. *The review of economic studies*, 72(1): 1–19, 2005.
- D. W. Andrews. Asymptotic optimality of generalized cl, cross-validation, and generalized cross-validation in regression with heteroskedastic errors. *Journal of Econometrics*, 47(2-3):359–377, 1991.
- J. D. Angrist and G. W. Imbens. Two-stage least squares estimation of average causal effects in models with variable treatment intensity. *Journal of the American statistical Association*, 90(430):431–442, 1995.
- J. D. Angrist and J.-S. Pischke. The credibility revolution in empirical economics: How better research design is taking the con out of econometrics. *Journal of economic perspectives*, 24(2):3–30, 2010.
- J. D. Angrist, G. W. Imbens, and D. B. Rubin. Identification of causal effects using instrumental variables. *Journal of the American statistical Association*, 91(434):444–455, 1996.
- J. D. Angrist, K. Graddy, and G. W. Imbens. The interpretation of instrumental variables estimators in simultaneous equations models with an application to the demand for fish. *The Review of Economic Studies*, 67(3):499–527, 2000.
- T. Arduini, E. Patacchini, and E. Rainone. Treatment effects with heterogeneous externalities. *Journal of Business & Economic Statistics*, 38(4):826–838, 2020.
- R. Backhouse and B. Cherrier. *Becoming applied: The transformation of economics after 1970*. Department of Economics, University of Birmingham, 2014.
- S. L. Barber, K. van Gool, S. Wise, M. Wood, Z. Or, A. Penneau, R. Milstein, N. Ikegami, S. Kwon, P. Bakx, et al. *Pricing long-term care for older persons*. World Health Organization, 2021.
- F. Barbosa, G. Voss, and A. Delerue Matos. Do european co-residential caregivers aged 50+ have an increased risk of frailty? *Health & Social Care in the Community*, 28(6):2418–2430, 2020.

- D. Barczyk and M. Kredler. Evaluating long-term-care policy options, taking the family seriously. *The Review of Economic Studies*, 85(2):766–809, 2018.
- S. Berlinski, M. M. Ferreyra, L. Flabbi, and J. D. Martin. Childcare markets, parental labor supply, and child development. *Journal of Political Economy*, 132(6):2113–2177, 2024.
- A. Bertogg and S. Strauss. Spousal care-giving arrangements in europe. the role of gender, socio-economic status and the welfare state. *Ageing & Society*, 40(4):735–758, 2020.
- M. Bertrand, E. Duflo, and S. Mullainathan. How much should we trust differences-in-differences estimates? *The Quarterly Journal of Economics*, 119(1):249–275, 2004.
- L. E. Blume, W. A. Brock, S. N. Durlauf, and Y. M. Ioannides. Identification of social interactions. In *Handbook of social economics*, volume 1, pages 853–964. Elsevier, 2011.
- G. J. Bobonis and F. Finan. Neighborhood peer effects in secondary school enrollment decisions. *Review of Economics and Statistics*, 91:695–716, 2009.
- I. Bojinov, A. Rambachan, and N. Shephard. Panel experiments and dynamic causal effects: A finite population perspective. *Quantitative Economics*, 12(4):1171–1196, 2021.
- E. Bonsang. Does informal care from children to their elderly parents substitute for formal care in europe? *Journal of health economics*, 28(1):143–154, 2009.
- K. Borusyak, X. Jaravel, and J. Spiess. Revisiting event-study designs: robust and efficient estimation. *Review of Economic Studies*, page rdae007, 2024.
- A. Bozio, A. Gramain, C. Martin, and A. Masson. Quelles politiques publiques pour la dépendance? *Notes du conseil d'analyse économique*, 35(8):1–12, 2016.
- Y. Bramoullé, H. Djebbari, and B. Fortin. Identification of peer effects through social networks. *Journal of econometrics*, 150(1):41–55, 2009.
- Y. Bramoullé, H. Djebbari, and B. Fortin. Peer effects in networks: A survey. *Annual Review of Economics*, 12(1):603–629, 2020.
- K. M. Brown and R. A. Laschever. When they're sixty-for: Peer effects and the timing of retirement. *American Economic Journal: Applied Economics*, 4:90–115, 2012.
- B. Callaway and P. H. Sant'Anna. Difference-in-differences with multiple time periods. *Journal of econometrics*, 225(2):200–230, 2021.

- B. Callaway, A. Goodman-Bacon, and P. H. Sant'Anna. Difference-in-differences with a continuous treatment. Technical report, National Bureau of Economic Research, 2024.
- C. Capuano. *Que faire de nos vieux? Une histoire de la protection sociale de 1880 à nos jours*. Presses de Sciences Po, 2018.
- M. D. Cattaneo. Efficient semiparametric estimation of multi-valued treatment effects under ignorability. *Journal of Econometrics*, 155(2):138–154, 2010.
- G. Chamberlain. Multivariate regression models for panel data. *Journal of econometrics*, 18(1):5–46, 1982.
- K.-M. Chen and M.-J. Lin. Understanding adult children's labor supply responses to parents' long-term care needs. *Available at SSRN 4051859*, 2022.
- V. Chernozhukov, C. Hansen, and M. Spindler. Instrumental variables estimation with very many instruments and controls. 2015.
- V. Chernozhukov, D. Chetverikov, M. Demirer, E. Duflo, C. Hansen, W. Newey, and J. Robins. Double/debiased machine learning for treatment and structural parameters, 2018.
- C. de Chaisemartin. A note on instrumented difference in differences. 2010.
- C. de Chaisemartin and X. D'Haultfœuille. Fuzzy differences-in-differences. *The Review of Economic Studies*, 85(2):999–1028, 2018.
- C. de Chaisemartin and X. D'Haultfœuille. Difference-in-differences estimators of intertemporal treatment effects. *arXiv preprint arXiv:2007.04267*, 2023a.
- C. de Chaisemartin and X. D'Haultfœuille. Two-way fixed effects and differences-in-differences with heterogeneous treatment effects: A survey. *Econometrics Journal*, 26(3):C1–C30, 2023b.
- C. de Chaisemartin and X. D'Haultfœuille. Two-way fixed effects and differences-in-differences estimators in heterogeneous adoption designs. *arXiv preprint arXiv:2405.04465*, 2024.
- C. De Chaisemartin and X. d'Haultfoeuille. Two-way fixed effects estimators with heterogeneous treatment effects. *American economic review*, 110(9):2964–2996, 2020.
- C. de Chaisemartin, X. D'Haultfœuille, and G. Vazquez-Bare. Difference-in-differences estimators with continuous treatments and no stayers. 2023.
- X. D'Haultfœuille, S. Hoderlein, and Y. Sasaki. Nonparametric difference-in-differences in repeated cross-sections with continuous treatments. *Journal of Econometrics*, 234(2):664–690, 2023.

- P. D. Fajgelbaum, P. K. Goldberg, P. J. Kennedy, and A. K. Khandelwal. The return to protectionism. *The Quarterly Journal of Economics*, 135(1):1–55, 2020.
- M. H. Farrell. Robust inference on average treatment effects with possibly more covariates than observations. *Journal of Econometrics*, 189(1):1–23, 2015.
- A. Finkelstein, S. Taubman, B. Wright, M. Bernstein, J. Gruber, J. P. Newhouse, H. Allen, K. Baicker, and t. Oregon Health Study Group. The oregon health insurance experiment: evidence from the first year. *The Quarterly journal of economics*, 127(3):1057–1106, 2012.
- W. Frimmel, M. Halla, J. Paetzold, and J. Schmieder. Health of elderly parents, their children’s labor supply, and the role of migrant care workers. 2020.
- A. Goodman-Bacon. Difference-in-differences with variation in treatment timing. *Journal of econometrics*, 225(2):254–277, 2021.
- B. S. Graham and J. L. Powell. Identification and estimation of average partial effects in “irregular” correlated random coefficient panel data models. *Econometrica*, 80(5):2105–2152, 2012.
- D. S. Hamermesh. Six decades of top economics publishing: Who and how? *Journal of Economic literature*, 51(1):162–172, 2013.
- J. A. Hausman and W. K. Newey. Nonparametric estimation of exact consumers surplus and deadweight loss. *Econometrica: Journal of the Econometric Society*, pages 1445–1476, 1995.
- Haut Conseil de la Famille, de l’Enfance et de l’Âge. Emplois et services à domicile pour les personnes Âgées en perte d’autonomie. Report by the Conseil de l’Âge, April 2020. URL [https://www.strategie.gouv.fr/sites/strategie.gouv.fr/files/atoms/files/hcfea\\_conseil\\_de\\_l\\_age\\_rapport\\_emplois\\_et\\_services\\_a\\_domicile\\_vf\\_avril\\_2020-2.pdf](https://www.strategie.gouv.fr/sites/strategie.gouv.fr/files/atoms/files/hcfea_conseil_de_l_age_rapport_emplois_et_services_a_domicile_vf_avril_2020-2.pdf).
- J. J. Heckman and R. Pinto. Unordered monotonicity. *Econometrica*, 86(1):1–35, 2018.
- J. J. Heckman and E. Vytlacil. Structural equations, treatment effects, and econometric policy evaluation 1. *Econometrica*, 73(3):669–738, 2005.
- L. Hiel, M. A. Beenackers, C. M. Renders, S. J. Robroek, A. Burdorf, and S. Croezen. Providing personal informal care to older european adults: should we care about the caregivers’ health? *Preventive medicine*, 70:64–68, 2015.
- K. Hirano and J. Hahn. Design of randomized experiments to measure social interaction effects. *Economics Letters*, 106(1):51–53, 2010.

- S. Hoderlein and H. White. Nonparametric identification in nonseparable panel data models with generalized fixed effects. *Journal of Econometrics*, 168(2):300–314, 2012.
- M. G. Hudgens and M. E. Halloran. Toward causal inference with interference. *Journal of the American Statistical Association*, 103(482):832–842, 2008.
- S. Hudson, P. Hull, and J. Liebersohn. Interpreting instrumented difference-in-differences. *Metrics Note*, Sept, 2017.
- K. Imai and I. S. Kim. On the use of two-way fixed effects regression models for causal inference with panel data. *Political Analysis*, 29(3):405–415, 2021.
- G. Imbens and Y. Xu. Lalonde (1986) after nearly four decades: Lessons learned. *arXiv preprint arXiv:2406.00827*, 2024.
- G. W. Imbens and J. D. Angrist. Identification and estimation of local average treatment effects. *Econometrica*, 62(2):467–475, 1994.
- G. W. Imbens, D. B. Rubin, and B. I. Sacerdote. Estimating the effect of unearned income on labor earnings, savings, and consumption: Evidence from a survey of lottery players. *American economic review*, 91(4):778–794, 2001.
- A. Ko. An equilibrium analysis of the long-term care insurance market. *The Review of Economic Studies*, 89(4):1993–2025, 2022.
- R. Lalive and M. A. Cattaneo. Social interactions and schooling decisions. *The Review of Economics and Statistics*, 91(3):457–477, 2009. URL <http://www.jstor.org/stable/25651352>.
- R. J. LaLonde. Evaluating the econometric evaluations of training programs with experimental data. *The American economic review*, pages 604–620, 1986.
- S. Li, J. Linn, and E. Muehlegger. Gasoline taxes and consumer behavior. *American Economic Journal: Economic Policy*, 6(4):302–342, 2014.
- D. Libault. Grand âge, le temps d’agir. *La documentation française*, 2019.
- K. V. Løken, S. Lundberg, and J. Riise. Lifting the burden: Formal care of the elderly and labor supply of adult children. *Journal of Human Resources*, 52(1):247–271, 2017.
- C. F. Manski. Identification of endogenous social effects: The reflection problem. *The review of economic studies*, 60(3):531–542, 1993.

- P. Massner and J. Wikström. Does it matter who cares? formal vs. informal care of the elderly. *Working Paper*, 2023.
- M. A. Masten. Random coefficients on endogenous variables in simultaneous equations models. *The Review of Economic Studies*, 85(2):1193–1250, 2018.
- E. Miguel and M. Kremer. Worms: identifying impacts on education and health in the presence of treatment externalities. *Econometrica*, 72(1):159–217, 2004.
- R. A. Moffitt et al. Policy interventions, low-level equilibria, and social interactions. *Social dynamics*, 4(45-82):6–17, 2001.
- M. Mogstad and A. Torgovitsky. Identification and extrapolation of causal effects with instrumental variables. *Annual Review of Economics*, 10(1):577–613, 2018.
- C. Mommaerts. Long-term care insurance and the family. 2015.
- W. K. Newey. The asymptotic variance of semiparametric estimators. *Econometrica*, 62(6):1349–1382, 1994.
- W. K. Newey. Convergence rates for series estimators. In G. Maddala, P. Phillips, and T. Srinivasan, editors, *Advances in Econometrics and Quantitative Economics: Essays in Honor of Professor C. R. Rao*. Basil Blackwell, 1995.
- W. K. Newey. Convergence rates and asymptotic normality for series estimators. *Journal of econometrics*, 79(1):147–168, 1997.
- W. K. Newey and D. McFadden. Large sample estimation and hypothesis testing. *Handbook of econometrics*, 4:2111–2245, 1994.
- S. W. Parker and P. E. Todd. Conditional cash transfers: The case of progresa/oportunidades. *Journal of Economic Literature*, 55(3):866–915, 2017.
- E. Perdrix and Q. Roquebert. Does an increase in formal care affect informal care? evidence among the french elderly. 2020.
- J. Robins. A new approach to causal inference in mortality studies with a sustained exposure period-application to control of the healthy worker survivor effect. *Mathematical modelling*, 7(9-12):1393–1512, 1986.
- K. Shen. Who benefits from public financing of home care for low-income seniors? Technical report, Working paper, Harvard University. <https://scholar.harvard.edu/files/kshen> . . . , 2021.

- L. Sun and S. Abraham. Estimating dynamic treatment effects in event studies with heterogeneous treatment effects. *Journal of econometrics*, 225(2):175–199, 2021.
- E. J. T. Tchetgen and T. J. VanderWeele. On causal inference in the presence of interference. *Statistical methods in medical research*, 21(1):55–75, 2012.
- D. L. Thistlethwaite and D. T. Campbell. Regression-discontinuity analysis: An alternative to the ex post facto experiment. *Journal of Educational psychology*, 51(6):309, 1960.
- C. H. Van Houtven and E. C. Norton. Informal care and health care use of older adults. *Journal of health economics*, 23(6):1159–1180, 2004.
- G. Vazquez-Bare. Identification and estimation of spillover effects in randomized experiments. *Journal of Econometrics*, 237(1):105237, 2023.
- M. Wagner and M. Brandt. Long-term care provision and the well-being of spousal caregivers: An analysis of 138 european regions. *The Journals of Gerontology: Series B*, 73(4):e24–e34, 2018.



**Titre:** Évaluation Robuste des Politiques Publiques à des Effets Hétérogènes

**Mots clés:** Microéconométrie, Inférence causale, Différences de différences, Effets de pairs, Vieillesse

**Résumé:** Cette thèse développe de nouveaux designs méthodologiques pour évaluer des politiques publiques dont les effets causaux sont hétérogènes et/ou complexes. Le premier chapitre développe deux nouveaux estimateurs de paramètres causaux importants dans le cadre d'un design de différence-de-différences avec une variable de traitement continue. Dans sa version de base, l'étude observationnelle repose sur un panel d'une multitude de groupes observés sur deux périodes. Les groupes sont potentiellement déjà tous traités à la première période, à des doses différentes. Entre les deux périodes, certains groupes, appelés switchers, connaissent un changement dans la dose de traitement qu'ils reçoivent. À l'inverse, certains groupes, appelés stayers, gardent la même dose de traitement sur les deux périodes. Le premier paramètre considéré, appelé AS, correspond à la pente moyenne de la fonction « variable de résultat potentielle » de 2<sup>de</sup> période des switchers, entre leur dose de traitement à la 1<sup>ère</sup> période et celle à la 2<sup>de</sup> période. Le second paramètre, appelé WAS, est une version pondérée de la moyenne précédente, où chaque switcher reçoit un poids proportionnel à la valeur absolue de l'évolution de sa dose de traitement entre les deux périodes. L'identification et l'estimation de ces deux paramètres reposent sur une hypothèse de tendance commune inédite : en l'absence d'un changement de leur dose de traitement entre les deux périodes, les groupes switchers auraient connu, en moyenne, la même évolution de leur variable de résultat que les groupes stayers ayant la même dose de traitement initial. Les résultats sont adaptés à un design avec variable instrumentale. Ils sont utilisés pour estimer l'élasticité-prix de la consommation d'essence. Le deuxième chapitre développe une nouvelle méthode d'estimation des effets d'un traitement binaire dans un design avec des effets de pair endogènes hétérogènes. Le modèle linéaire en moyennes conventionnel est modifié de telle

sorte à ce que les individus puissent réagir différemment aux actions des membres de leur groupe, selon qu'ils partagent ou non la même identité qu'eux. L'identification et l'estimation des paramètres du modèle reposent sur une expérience en population partielle, c'est-à-dire sur l'observation d'un panel de groupes d'individus au sein duquel la part d'individus éligibles au traitement binaire (qui est connue pour chaque groupe) varie d'un groupe à l'autre. La procédure d'estimation repose sur la méthode des moments généralisés. La méthode développée permet de quantifier l'effet direct du traitement sur les individus éligibles, l'effet indirect sur les individus non-éligibles et les multiplicateurs sociaux. La méthodologie est employée pour évaluer les effets de Progesa, un programme mexicain de transfert d'argent conditionnel ciblant les élèves issus de familles défavorisées et visant à réduire l'abandon scolaire. Le troisième chapitre modélise le marché français de l'aide à domicile non-médicalisée pour personnes âgées dépendantes. Du côté de la demande, des ménages, constitué d'un senior dépendant et de son conjoint, prennent des décisions concernant leur consommation, l'aide informelle fournie par le conjoint et le recours des aides à domicile professionnelles. Du côté de l'offre, deux types d'entreprises sont modélisés : des services d'aide et de soins à domicile et des aides à domicile indépendants. Le modèle est estimé à l'aide de l'enquête CARE-Ménages et de la Base Tous Salariés. Le modèle est utilisé pour identifier les conjoints des personnes âgées dépendantes les plus susceptibles de réagir à une modification des conditions d'accès à l'aide à domicile formelle. Les premiers résultats suggèrent, côté demande, que, toutes choses égales par ailleurs, les conjoints aidants de sexe masculin et plus âgés réagissent davantage à une variation du prix de l'aide formelle. Côté offre, les services d'aide à domicile font face à des prix constants.

**Title:** Robust Evaluation of Public Policies to Heterogeneous Effects

**Keywords:** Microeconometrics, Causal Inference, Difference-in-differences, Peer Effects, Aging

**Abstract:** This thesis develops new methodological designs to evaluate public policies with heterogeneous and/or complex causal effects. The first chapter introduces two new estimators of valuable causal parameters within a difference-in-differences design with a continuous treatment variable. In its basic version, the observational study relies on a panel of numerous groups observed over two periods. The groups may already all be treated in the first period, albeit at different doses. Between the two periods, some groups, called switchers, experience a change in the dose of treatment they receive. In contrast, some groups, called stayers, keep the same treatment dose over both periods. The first considered parameter, called AS, corresponds to the average slope of the "potential outcome" function in the second period for switchers, between their treatment dose in the first period and that in the second period. The second parameter, called WAS, is a weighted version of the previous average, where each switcher receives a weight proportional to the absolute value of the change in their treatment dose between the two periods. The identification and estimation of these two parameters rely on a novel common trend assumption: in the absence of a change in their treatment dose between the two periods, the switcher groups would have experienced, on average, the same evolution in their outcome variable as the stayer groups with the same initial treatment dose. The results are adapted to a design with instrumental variables. They are used to estimate the price elasticity of gasoline consumption. The second chapter develops a new method for estimating the effects of a binary treatment in a design with heterogeneous endogenous peer effects. The conventional linear-in-means model is modified so that

individuals can respond differently to the actions of group members, depending on whether or not they share the same identity. The identification and estimation of the model's parameters rely on a partial population experiment, that is, the observation of a panel of groups of individuals within which the proportion of individuals eligible for the binary treatment (known for each group) varies from one group to another. The estimation procedure is based on the generalized method of moments. The method allows to assess the magnitude of the direct effect of the treatment on eligible individuals, the indirect effect on non-eligible individuals, and of the social multipliers. The methodology is used to evaluate the effects of Progreso, a Mexican conditional cash transfer program targeting students from disadvantaged families that aimed at reducing school dropout. The third chapter models the French elderly personal and social home care market. On the demand side, households, consisting of a dependent senior and their spouse, make decisions regarding their consumption, the informal care provided by the spouse, and the use of professional home care services. On the supply side, two types of providers are modeled: firms providing personal and social home care and self-employed personal home care assistants. The model is estimated using the CARE-Ménages survey and the Base Tous Salariés database. The model is used to identify the spouses of dependent elderly individuals most likely to respond to a change in access conditions to formal home care. Preliminary results suggest, on the demand side, that, all other things being equal, male and older caregiving spouses are more responsive to a variation in the price of formal care. On the supply side, home care providers face constant marginal costs.