



# Development and study of coadaptive interactive systems

Amaury Dechaux

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# THÈSE POUR OBTENIR LE GRADE DE DOCTEUR DE L'UNIVERSITÉ DE MONTPELLIER

En Système Automatique et Micro-Électronique

École doctorale Information, Structures et Systèmes

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Development and study of coadaptive interactive systems

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le 22 Octobre 2024

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*Then, undoubtedly, such captives would consider the truth to be nothing but the shadows.*

Plato, *Peri politeías*, 428 B.C.

1. *At the beginning God created the sky and the earth.*
2. *No.*
3. *That's not it.*
4. *Let's start from the beginning.*
5. *At the start there was God.*
6. *And nothing else.*
7. *There could be nothing else because, for that there be anything else, it should have been created by God.*
8. *But God had not created anything yet.*
9. *Because it was the beginning.*
10. *At the beginning there was therefore no sky nor earth.*
11. *There was not even the idea of sky nor the idea of earth in God's mind.*
12. *There could only be what God had created.*
13. *But God had not created the idea of sky nor the idea of earth yet.*
14. *He had not even created the idea of an idea.*
15. *He had not even created the idea to create.*
16. *There must have been a time like that ?*
17. *No ?*
18. *Why God would He have created ? God doesn't need that. God has no need.*
19. *Because God is God.*
20. *And for God to need, it is necessary first that He create the need.*
21. *That He might possibly do, naturally.*
22. *Because God is God.*
23. *But if God create the need, He stops being God.*
24. *Because a God with needs is not God.*
25. *God knew all that because He is the supreme intelligence.*
26. *He knew all that but what's good being God if you can't do anything ?*
27. *So he decided to take the risk.*
28. *And He did well.*
29. *So, voila :*
30. *At the beginning God created Contradiction.*
31. *That, that was a good idea.*
32. *Now, it can go.*
33. *Let's go.*

F. Cavanna, *Les écritures*, 1982

# Summary of thesis

The concept of coadaptation is a fairly recent introduction to the literature on Human Machine Interaction highlighting the fact that, when a user interact with a complex system, both the user and the system adapt to each other in order to maximize their joint performance in some task.

Historically, most examples of coadaptive processes correspond to a sequential optimization procedure where user and system adapt to each other but not at the same time. In recent years however, the introduction of artificial intelligence in a variety of interactive systems has brought examples of concurrent coadaptive processes under which both human and machine continuously adapt to each other while performing some task. However, unlike machines which possess almost endless adaptive capabilities, adaptability in humans is an eminently constrained process in multiple aspects. In this thesis, we investigate the limits of human adaptability in the context of interactive system design, notably from the point of view of behavioral biases or the idea that, unlike machine, not all behavioral adaptation in humans are equivalent from the perspective of cognitive costs.

First, we studied an example of "naive" coadaptation applied to the control of a supernumerary robotic finger. By naive, we mean a coadaptive system with few prior assumptions such as we do not postulate on any particular method of control for the robotic finger and derive learning equation from a simple linear model. In particular, we studied how, in the case where multiple channels of communication are available to perform this control, a coadaptation between the user and the system might allow to achieve different control schemes adapted to the context in which a task is performed.

Second, we studied the reaction times of humans to lateralized stimulus and actions using visual, tactile and visuo-tactile stimulus. We show that while multimodal cueing increase performance for non-lateralized stimulus, it surprisingly does not help when the stimulus are lateralized.

Third, we illustrate the decision process internal to a user in this context with a state-of-the-art computational model of human behavior based on the theory of Active Inference. In particular, we studied the prediction of this model in relation to the Sensory Response Compatibility (SRC) phenomenon and show that this model can reproduce, in approximate form, various experimental observations pertaining to SRCs in general and the Simon effect in particular.

Finally, we designed and studied two interactive systems pertaining to the two types of user-augmentation that may be associated with a machine. An "extended motor plant" first, where we designed an embodied control interface for a drone with a view to allow the user to control this drone "like their own body", and an "extended sensory plant", where we designed a tactile radar allowing its user to "remotely touch objects" using vibro-tactile feedback of distance information measured by capacitive sensors.

We report on the demonstration of our embodied control interface in an open event

where members of the public were invited to try the setup in virtual reality as well as its subsequent transposition to a quadcopter drone and discuss the perspective and future research directions for such embodied drone control schemes. Similarly, we report on the evaluation of our prototype of tactile radar in a controlled experiment where the users were tasked with finding and identifying various objects solely relying on the information provided by the device.

We conclude this manuscript with a summary of our contributions as well as a more general discussion on the concept and perspective of coadaptation, notably in relation to the recent introduction and rapid development of artificial intelligence.

# Résumé des travaux de thèse

Le concept de coadaptation est une introduction assez récente dans la littérature sur les interactions homme machine (IHMs) mettant en lumière le fait que la relation entre les systèmes interactifs et leurs utilisateurs est une relation d'adaptation mutuelle. En effet, si l'utilisateur d'un système s'adapte continuellement par apprentissage, la machines aussi s'adapte à son utilisateur, par exemple par des options de customisation.

Historiquement, la plupart des exemples de processus coadaptatif dans le cadre des IHMs correspondent à des processus d'optimisation séquentiels où l'utilisateur et la machine s'adaptent l'un à l'autre à tour de rôle. Ces dernières années, l'introduction de l'intelligence artificielle dans le domaine public amène de plus en plus d'exemples de processus coadaptatifs concurrents, où l'homme et la machine s'adaptent l'un à l'autre de manière continue à travers leurs interactions. Néanmoins, en comparaison des machines dont les capacités d'adaptation sont pratiquement illimitées, l'adaptabilité des utilisateurs est un processus largement contraint sous de multiples aspects. Dans cette thèse, nous explorons les limites de l'adaptabilité des humains dans le contexte des IHMs et ce notamment du point de vue des biais comportementaux ou l'idée que toutes les adaptations comportementales chez l'humain ne sont pas équivalentes et peuvent entrer en conflit avec d'autres comportements appris dans notre vie de tous les jours.

Premièrement, nous étudions un exemple de processus coadaptatif concurrent "naïf" appliqué au contrôle d'un doigt robotique surnuméraire. Par naïf, nous faisons référence à un processus coadaptatif dans lequel nous ne choisissons pas a priori une méthode de contrôle particulière. En particulier, nous étudions comment l'homme et la machine s'adaptent ensemble dans le cas où des perturbations viendraient à rendre plus difficile l'exécution de la tâche en fonction de la méthode de contrôle utilisée.

Deuxièmement, nous étudions les performances des humains dans une tâche demandant de réagir à des stimuli visuels, tactiles et visuo-tactiles en se basant sur leur position dans l'espace.

Par la suite, nous modélisons le processus de décision interne à l'utilisateur dans ce contexte en nous basant sur la théorie de l'Active Inference et montrons que ce modèle peut reproduire, de façon approximative, diverses observations expérimentales en lien avec le phénomène de compatibilité stimulus réponse en général et en particulier l'effet de Simon.

Enfin, nous traitons le design des système interactifs en développant deux prototypes en lien avec les deux principaux types d'augmentation permis par l'utilisation des machines. Une extension du système moteur de l'utilisateur en premier lieu, où nous réalisons une interface de contrôle pour drone permettant à celui ci de contrôler les déplacements du drone dans l'espace "comme son propre corps" en nous basant à la fois sur des données posturales et sur les mouvements de la tête. Nous reportons l'évaluation, dans un évènement ouvert où des membres du public étaient invités à essayer le dispositif en réalité virtuelle et à l'aide d'un drone. En second lieu, nous étudions une extension du système sensoriel de l'utilisateur en réalisant un prototype de radar tactile permettant à l'utilisateur de "toucher des objets à distance" via un retour tactile de mesures de distance obtenues par des senseurs capacitifs. Nous reportons l'évaluation de ce prototype dans une expérience contrôlée où

les participants avaient pour objectif de trouver et d'identifier divers objets en se basant uniquement sur les informations du radar tactile.

Nous finissons ce manuscrit par un résumé des contributions de ces travaux et avec une discussion générale sur les perspectives offertes par la coadaptation, notamment au regard du rapide développement de l'intelligence artificielle.

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## General introduction

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When we ask ourselves what distinguishes our species, *Homo Sapiens*, from the others that populate the Earth, one of the answer we can come up with is our unrivalled ability to use tools [Johnson-Frey, 2003]. Unlike animals, who are known to use tools for certain activities [Povinelli and Bering, 2002, Blaisdell et al., 2006, Penn and Povinelli, 2007], our relationship with our tools goes far beyond what can be observed in the animal kingdom. Among the many unique features of this relationship, humans use tools to make tools, they may also use a variety of tools jointly for the same task, or collaborate to use them. If this relationship is indeed very special in relation to animals, it's not so special if we consider the different *Homo* species that once walked the Earth. Three million years ago, our ancestors were already skilled tool users, and the large number of artefacts they left behind testifies to a deep understanding of what we would today call the "laws of nature" in general, and Newtonian mechanics in particular. Although this natural ability for high-grade *causal reasoning* [Wolpert, 2003, Woodward, 2011, Gärdenfors and Lombard, 2018, Gärdenfors and Lombard, 2020] is therefore not a unique feature of *Homo Sapiens*, our tools, unlike those left by our ancestors, possess a novel attribute: they don't need us to power them. Machine is the term we use to globally designate these very special tools, whose energy source - from wind and water to electricity and oil - is detached from our bodies. Threading the naturalistic needle, it is clear that the *Machina* genus (should we say an order, a reign?) is highly adapted to its environment and this thanks to a harsh selective pressure that, in typical Darwinian fashion, drives the various machine species to become stronger, faster, safer - in short, more useful to their human users. While it is clear that machines have evolved and adapted over the three millennia or so of their existence, what about their partner in this relationship? Nowadays, there are few people whose daily activities are not carried out with the help of one or more machines, and it could be said that to be a member of our modern society, you need to possess a certain degree of mastery in the use of machines. *Co-evolution* [Lovelock and Margulis, 1974, Thompson, 1989] is the term coined to refer to the natural process where two or more species adapt and evolve in unison so that after millions of years, one can no longer exist, or only with difficulty, without the other. Our environment is full of examples of co-evolution, such as birds and trees or insects and flowers. If we look at a modern city, it's clear that *Homo* and *Machina* are "co-evolving", each being a fundamental part of the other's ecological niche. However, as the *Machina* genus is not really a genus, the term *Coadaptation* is probably more appropriate to describe this relationship.

In a thesis published in 1990 [Mackay, 1990], W.E Mackay argues, as did several authors before him [Orlikowski and Baroudi, 1989], that in the traditional approach to interactive systems engineering, which tends to separate the study of users and machines, a fundamental aspect of the process underlying the interaction between them is irretrievably lost. He coined the term coadaptation (at least I couldn't find an earlier reference in this context) to refer to this fundamental aspect in order to "*emphasize the dual nature of the phenomenon that people adapt to technology (reacting to it) and adapt it to meet their needs (proactively changing it)*". With the emergence of artificial intelligence in the public domain, thanks in particular to a rapid reduction in the need for and cost of computing power, this emphasis is, thirty years on, probably more justified than ever. Interestingly however, it seems the term coadaptation, although extremely relevant today, did not really find its public in the scientific community. A quick search on Google scholar for the terms "coadaptation" yields only about a few hundreds articles relating to Human-Machine Interactions. Compared to the millions of articles published every year in fields where this concept seems most relevant, such as Human-Robot Interaction or Brain-Computer Interfaces, it seems that the community is more inclined to use other terms. Although we can only conjecture as to the reasons for this, it seems to me that they are, in part, to be found in a fundamental lack of clarity around this concept. While the idea of a human-machine relationship so intimate as to verge on *symbiosis* [Sawaragi, 2005, Sanchez et al., 2009, Huang et al., 2021a, Inga et al., 2023, Picard et al., 2023] may seem deeply enticing for obvious reasons, it is unclear *what is* coadaptation, both from a phenomenological (*what*) and operational (*why*) perspective. For example, consider these two definitions found in the literature. In [van Zoelen et al., 2021], the definition of coadaptation between a human and a robot is as follows :

*"A process in which at least two parties change their behavior and/or mental models concurrently as a consequence to changes in task or team situation while collaborating with each other".*

A slightly different definition is presented in [Alahmad and Robert Jr, 2021] in the context of a Cognitive Computing System (AI assistant) :

*"We define co-adaptation as the series of activities that a user and a system engage in simultaneously to make the system fit the user."*

These two definitions raise a number of questions. First of all, it should be noted that these authors propose a very different vision of the process. Whereas [van Zoelen et al., 2021] treats coadaptation as a by-product of the collaboration between a human and a machine (task completion requires a simultaneous change in behavior and mental model), [Alahmad and Robert Jr, 2021] present coadaptation as a collaborative process in itself (human being and machine cooperate so that the system adapts to the user). Secondly, while [Alahmad and Robert Jr, 2021] propose a clear directionality in the coadaptive process (although the user participates in this process, it is the system that must adapt to the user, which is implicitly assumed to be relatively constant), [van Zoelen et al., 2021] proposes a definition where human and machine are on an equal footing (they are both anonymous "parties" in this process). Thirdly, the two authors implicitly propose very different notions of temporality for a coadaptive process. In [van Zoelen et al., 2021], since coadaptation is the result of a change in the team's task and/or situation, it is reasonable to assume that the time constant of the process is related to the time constant associated with these changes. The faster the task or situation changes, the "more" coadaptation will be required on both sides to dynamically

adapt to these changes. In [Alahmad and Robert Jr, 2021], on the contrary, since the aim of the coadaptive process is for the system to adapt to the user, its time constant is linked to the time constant associated with the user's changes.

Of course, given the complexity of the process conceptualized by the term coadaptation, it is to be expected that a definition will be ambiguous on a number of points. It seems, however, that at least two different notions of coadaptation coexist in the literature. One inspired by its application in the context of human-computer interfaces [Rezazadeh et al., 2012, Hahne et al., 2015, Couraud et al., 2018, De Santis, 2021, Madduri et al., 2022, Yeung et al., 2022], and in particular Brain-Computer Interfaces [Blankertz and Vidaurre, 2009, Vidaurre et al., 2011, Perrin, 2012, Orsborn et al., 2012, Merel et al., 2013, Shenoy and Carmenta, 2014, Thomas et al., 2014, Mattout et al., 2015, Ehrlich and Cheng, 2018] where the notion that a system (decoder) must fit the user is central to the field [Lotte et al., 2007, Shende and Jabade, 2015], and another inspired in its application in the context of Human Machine Interactions [Gallina et al., 2015, Alahmad and Robert Jr, 2021, Maharmeh and Tahboub, 2023, Shen et al., 2024], and notably Human Robot Interactions [Kahn et al., 2008, Hu et al., 2023, Kalinowska et al., 2023, Inga et al., 2023, Liu, 2024] where the emergence of joint behavior is essential to the realization of complex tasks requiring both human and machine intervention [Bauer et al., 2008, Ajoudani et al., 2018].

## 1.1 Research question

This thesis will focus on the second of these two definitions, where:

*"Co-adaptation is the series of activities that a user and a system engage in simultaneously to make the system fit the user."*

However, it will be less concerned with "simultaneity" and concentrate on the idea that an interactive system must "fit to its user". This "user-centric" approach to interactive system design, ubiquitous since the earliest days of machines, is fairly self-explanatory. Since machines can be considered as "descendants" of tools, designed to help their users accomplish various tasks, they must not add to the "burden" of performing the task itself [Parasuraman and Riley, 1997]. The problem of assessing the "adaptive burden" that an interactive system represents for its human user is an important issue at the heart of interactive system design. Depending on the application, terms such as difficulty [Park and Kim, 2008, Lucchese et al., 2022b, Lucchese et al., 2022a], intuitiveness [Niwa et al., 2010, Patterson, 2017, Blackler et al., 2019, Satogata et al., 2022] or ease-of-use [Venkatesh and Davis, 1996, Hess et al., 2014, Kurniawan et al., 2022] help to capture the "amount of adaptation" required of the user to use an interactive system.

A tool can be viewed as an extension of its user's motor-system and studies have shown that, in some sense, our brains treat the use of tool as such [Cardinali et al., 2012, Canzoneri et al., 2013, Martel et al., 2019]. Similarly to tools, many machine can be viewed as extension of their user's motor-system. This is for example the case of supernumerary robotic limbs (SRLs; [Prattichizzo et al., 2021, Yang et al., 2021]), systems design to augment the body of their users by offering them additional "body part" (legs, arms, fingers) designed to extend their capabilities. Contrary to tools however, many machines also include a feedback, allowing them to transmit information back to their user. In this respect, they can also be viewed

as extension of their users sensory-system, allowing them to access environmental information that would not be accessible with their "natural" sensory-system. Sensory augmentation and substitution devices [Bach-y Rita and Kercel, 2003, Schmidmaier, , Kristjánsson et al., 2016, Macpherson, 2018, Eagleman and Perrotta, 2023] are typical examples of such use of machines.

In order for a machine to "fit" its user, it is therefore paramount that the control loop of the system, that is, the actions needed as inputs and the feedback obtained in return, integrate within the natural sensory-motor loop which we use to monitor and control our own body. In general, this involves three fundamental steps. *Data collection*, where human behaviour is tested and analyzed, often in simplified settings such as choice reaction tasks. *Modelization*, where the collected data are converted into an approximate model of the user [Card, 1981, Laird et al., 1987, John and Kieras, 1996, Kieras and Meyer, 1997, Anderson et al., 2004] against which the performance of various design choices can be compared in simulation [Trafton et al., 2013, Cassenti and Schaefer, , Held et al., 2021]. And finally, *prototyping*, where the interactive system is finally engineered and evaluated in user-studies to validate those choices. Historically, this general "fitting process" was performed by human designers, however the recent advances in machine learning and statistics allow to automate it to some degree, such as some system nowadays have the ability to automatically adapt to their users, either during the execution of a task [Hahne et al., 2015, Ehrlich and Cheng, 2018, Satogata et al., 2022, Reddy et al., 2022], or based on an interaction history [Broad et al., 2018, De Santis et al., 2018, Jonnavittula and Losey, 2021].

## 1.2 Structure of the thesis

In this manuscript, the three steps of this general process are treated under various perspectives. Given the broad scope of the work presented here, I only give a summary of the thematic and contributions presented in the following chapters in this general introduction. Additionally, a dedicated and contextualized literature review can be found in the introduction of each of chapter.

We start the interactive system design loop in chapter I, where we study an example of "naive" coadaptation between a user and a supernumerary limb (6th finger). By naive, we mean a coadaptive system with few prior assumptions such as we do not postulate on any particular method of control and derive the learning equations from a simple linear model [Müller et al., 2017]. We let human and machine coadapt in a supervised task and analyze the results, showing notably that coadaptation improves the robustness of a system to perturbations.

Chapter II is dedicated to data collection. Through a choice reaction task, we study the reaction times of humans to lateralized stimulus and actions using visual, tactile and visuotactile stimulus. We show that, while multimodal cueing increase performance for non-lateralized stimulus, it surprisingly does not help when the stimulus are lateralized. This result is important with respect to the design of a machine's feedback since it implies that some sensory modalities are more adapted than others to transmit a given information (spatial here).

Modelization follows in Chapter III, where we study a computational model (discrete

time active inference; [Smith et al., 2022]) of the sensory response compatibility phenomenon (SRC; [Kornblum et al., 1990]) and show that this model can reproduce, in approximate form, various experimental observations pertaining to SRC in general and the Simon effect [Simon and Wolf, 1963] in particular.

In Chapters IV and V we prototype and evaluate interactive systems. An extended motor plant in chapter IV, with a view to allow a user to control a drone "like ones own body" [De Vignemont, 2011] using a combination of postural and head motion data. We report on the evaluation, via an open event where members of the public were invited to try the setup, of this prototype in virtual reality and subsequently when transposed to a commercial quadcopter drone. In Chapter V we design an extended sensory plant, a tactile radar [Cassinelli et al., 2006] allowing a user to "remotely touch objects" via a vibro-tactile feedback of distance information measured by capacitive sensors. In particular, this prototype embed a denser network of sensors and tactile transducers, in comparison with similar devices found in the literature, with the aim to enable its user to rely on active tactile sensing to explore the environment. The system is then evaluated in a controlled experiment where the users are tasked to find and recognize various objects solely relying on the device.

Finally, we present a general discussion on the perspective of coadaptation, notably in relation to the recent introduction and rapid development of artificial intelligence in the public domain, and highlight some limits of the approach presented in this manuscript.



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# A coadaptive approach to supernumerary finger control

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## Preamble

Most of the systems we daily interact with impose severe constraints on how we, as users, can accomplish the task for which they are designed. In the case of a car, for example, the relationship between our actions on the pedals and steering wheel and the resulting change in the car's speed and direction correspond to a fixed map that the driver must learn. While humans are undoubtedly the animals with the greatest adaptive potential, and this thanks to a brain with an ability to make sense of the world surrounding us which far exceeds that of any other animal, it is fairly intuitive that this ability does not free us from physical constraints. The most obvious of these constraints are the limits imposed by our bodies, such as the strength of our muscles, the length of our bones, the energy stored in our cells, and so on. These constraints are fairly well known, at least in a statistical sense, and are obviously included in the design process of all interactive systems. In other words, no car will ever ask us to press pedals with superhuman strength. However, these limits are only considered on average and, for a number of tasks, it is necessary for the machine to adapt to its specific user. Such adaptive behavior on the part of interactive systems can be quite trivial; allowing the user to change the position of the seat in a car, for example, can be viewed as a form of coadaptation between the user and the machine. The development of statistical methods, however, enables to take this relaxation of the parameters of the aforementioned map to a higher level. While modifying the relationship between the force applied on the pedal and the car's acceleration is today a "privilege" that only a few users with in-depth engineering knowledge can afford, machine learning typically promises to extend such adaptation to all of them, and so mostly irrespective of their prior knowledge. In this first chapter, we study a simple example of such a "machine learning-based" approach in the context of supernumerary robotic limbs, a peculiar class of wearable systems aiming to extend their users' motor capabilities by providing them with additional limbs.

## 2.1 Introduction

Supernumerary Robotic Limbs, or SRL, are wearable devices designed to augment the sensory-motor capabilities of humans [Prattichizzo et al., 2021, Yang et al., 2021]. Augmentation of the human body can refer to many functions depending on the application. For example, many systems focus on assisting a worker in an industrial environment, by alleviating the physical strain represented by performing certain tasks in uncomfortable positions [Parietti and Asada, 2014, Shin et al., 2015], protecting a worker from potentially dangerous dynamics [Davenport et al., 2012], providing additional safety [Parietti et al., 2015, Parietti and Asada, 2016] or simply to provide a "third hand" when it is required [Abdi et al., 2016, Sasaki et al., 2017, Nakabayashi et al., 2017]. These systems therefore exist in many forms (arms; [Zhang et al., 2022]; legs; [Khazoom et al., 2020]; fingers [Wu and Asada, 2014b]), and with widely varying capabilities, for example in term of strength [Véronneau et al., 2020] or dexterity [Huang et al., 2021b]. In the specific case of supernumerary robotic fingers (SRF), artificial fingers designed to be worn on the hand, numerous studies have focused on their use as rehabilitation devices. For example, to rehabilitate prehension after a loss of the hand mobility resulting from a brain injury [Ort et al., 2015, Hussain et al., 2017a, Hussain et al., 2017c, Salvietti et al., 2021]. In another area of interest, other studies have examined the potential of SRF to augment hand capabilities, in particular to enable a user to grasp larger objects than they can with their natural, non-augmented hand [Wu and Asada, 2014a, Prattichizzo et al., 2014b, Prattichizzo et al., 2014a]. There are mainly two designs for both applications, the most popular of the two being wrist-mounted SRF. These are usually operated by cables with coils on the wrist [Wu and Asada, 2014b, Hussain et al., 2017c], although a few electric motor-driven designs have been proposed [Wu and Asada, 2014a, Hussain et al., 2015]. Wrist-mounted SRF are particularly useful in rehabilitation scenarios, as they tend to offer a stronger grip and can work in coordination with the hand or arm, which act as a purely passive "pinch support". Conversely, the "mirror thumb" design [Cunningham et al., 2018, Kieliba et al., 2021], where the SRF acts as a 6th finger located on the ulnar side of the augmented hand, is found more often in "augmentative" scenarios, not least because their position close to the natural fingers, combined with their relatively small size, allows to envision dexterous synergies between the robotic and natural fingers. While rehabilitative SRF have enjoyed a degree of success in recent years, with one commercialized model [Danielle, 2018, Kieliba et al., 2021], this is less the case for augmentative SRF, which are arguably still at an early stage of development. This state of affairs can be explained, in part, by the very different nature of the problem posed by augmentative versus rehabilitative technologies. As previous authors have noted [Leigh et al., 2018, Prattichizzo et al., 2021, Eden et al., 2022] augmentative technologies, and SRL in particular, can represent "additional degrees of freedom" for their human users. These degrees of freedom being ideally controlled in addition to the other natural limbs.

### 2.1.1 The control of SRL

This problem poses a major challenge in terms of how control of an SRL can be transferred to its human user. By their very nature, human-machine interfaces can be described as "parasitic", since they require a bodily structure to act as an "information source". Of course, the ability of humans to learn and adapt their behavior means that a wide variety of solution may be envisioned. Although parasitic by our definition, the cost, functional or cognitive, induced by controlling an SRL can, in some cases, be reduced to almost nothing. Over the year, a number of authors have been interested in developing taxonomies to classify

the variety of control interfaces used in the literature [Leigh et al., 2018, Prattichizzo et al., 2021, Eden et al., 2022]. In particular, an important "dimension" along which control systems for SRL can be distributed corresponds to the level of autonomy given to the robotic limb. At the lowest level of autonomy, we find "volitional" or "explicit" systems. That is, a control interface where the user must, through voluntary actions, generate commands for the system. For an example applied to SRF, [Hussain et al., 2017a] proposed a wrist-mounted supernumerary finger whose flexion/extension is triggered by pressing a button on a "ring interface" worn on the participant's non-augmented hand. Similar approaches can be found in [Cunningham et al., 2018] involving movement of the user's feet and fingers or [Ort et al., 2015] based on movement of the non-augmented hand. Electromyographic (EMG) interfaces are also a popular choice for voluntary control schemes. In [Hussain et al., 2016] for example, the controller of an SRF is implemented as a state machine whose transitions are triggered by EMG-based gesture recognition of the non-augmented hand, while in [Salviotti et al., 2021] the robotic finger is controlled via an electrode placed on the user's eyebrow. Of course, a low level of autonomy does not mean that the user enjoys total freedom over the state of the SRL at any given moment. In general, the user of such systems has control over the position of a point on a low-dimensional manifold embedded in the SRL state space, so that the potentially large number of DoFs included in the system can be reduced to a smaller number from the user's point of view. In other words, the user controls the position of an SRL end-effector rather than each individual DoF [Saraiji et al., 2018, Penalosa et al., 2018].

In addition to purely voluntary control, many authors have been interested in SRL controllers allowing to offload a more significant part of the work to the robotic limb. This greater decision-making role can be implemented in several ways. For example in [Bonilla and Asada, 2014], a wearable robotic arm cooperates with a user in a screwing task via a predictive controller based on image recognition. Similarly in [Bretan et al., 2016], the SRL and its user synchronize their behaviour based on the rhythm of a piece of music. Halfway between autonomous SRL and those based on volitional control, pseudo-mapping strategies, also known as limb mapping [Tong and Liu, 2021], propose to control SRL on a "leader-follower" basis. In [Parietti and Asada, 2014] and [Srinivas et al., 2015] for example, the movement of the user's arm is associated with a movement of the SRL, so that man and machine can cooperate in a variety of tasks. It is important to note that for these pseudo-mapping strategies, the user's movements on which the control is based are not "directed" at the machine, as in volitional control schemes, but at the task that human and machine are jointly performing. There is therefore no additional cost for the SRL control from the user's point of view, unlike for volitional schemes. To the author's knowledge, there is no example in the literature on SRF incorporating high-level task understanding similar to the example mentioned above, but pseudo-mapping strategies have been studied in depth in a series of papers by F.Y. Wu and H. Asada [Wu and Asada, 2014a, Wu and Asada, 2014b, Wu and Asada, 2016]. Their control method, which they describe as based upon "bio-artificial synergies" [Santello et al., 2016], rely on the prediction of an appropriate posture for the SRF based on the behavior of the natural fingers. Notably in [Wu and Asada, 2014b], they showed that the large synergy between the fingers involved in everyday grasping tasks, where 80% of the variance in the fingers' joint angle can be reduced to two principal components [Santello et al., 1998], could be similarly observed in 7-finger grasping tasks. Based on this correlation, they investigate a regression-based control method to predict the optimal position of the robotic finger according to its "natural synergies" with the other natural fingers during grasping.

Given the wide range of tasks we perform with our hands on a daily basis, it can be expected that no single control method will perform optimally in all cases. Notably, while an explicitly controlled SRL offers the user finer control over the artificial limb, potentially allowing for a wider range of use, the "substitutive" nature of the interface [Kieliba et al., 2021], which requires the user to perform a motor action with another limb, presents a number of disadvantages. Firstly, such an approach makes learning to use the SRL a time-consuming and cognitively demanding process, as users have to learn the correspondence between a motor action and the robotic finger's movements [Shafti et al., 2021]. A related problem is that the control of the robotic limb can be quite "unnatural". Previous studies on the control of prosthesis have shown that the "sense of embodiment" (SoE; [Niedernhuber et al., 2018]; [Zbinden et al., 2022]), or the vicarious sensation that a prosthesis is a legitimate part of one's body, is promoted by control schemes that mimic the "natural" control, in term of actions and sensations, of the body part [De Vignemont, 2011, Makin et al., 2017]. The sense of embodiment has been linked to a number of positive rehabilitation outcomes, including improved postural stabilization [Imaizumi et al., 2016], better motor planning [Gouzien et al., 2017] but also better user evaluation [Bekrater-Bodmann, 2021]. In this respect, a study by [Segura Meraz et al., 2018] has shown that the control of an SRF can similarly induce changes in the user's body schema. Similarly, [Hussain et al., 2017b] showed via fMRI imaging that the use of an SRF recruits similar motor region in the brain as voluntary control of the natural fingers.

In summary, when a device is designed for a variety of tasks or for tasks demanding precise control, voluntary control scheme must rely on dexterous, "high value", body structure such as the feet [Cunningham et al., 2018], the hands [Ort et al., 2015] or the voice [House et al., 2009]. Given that the final aim of a SRL is to augment the natural abilities of their users, the choice of a body structure to base the control on is therefore a difficult trade-off between performance, easiness to learn and use, and the resulting cost in term of functionality [Hussain et al., 2017c, Leigh et al., 2018].

### 2.1.2 Coadaptation as a negotiation

Although many definitions of the term coadaptation have been proposed [van Zoelen et al., 2021, Alahmad and Robert Jr, 2021], a number of authors use this term to refer to a process where human and machine "negotiate" on an optimal control method given the task they jointly have to perform [Perrin, 2012, Müller et al., 2017, Perdakis and Millan, 2020]. In other words, human and machine jointly adapt their behavior through learning such as the resulting control interface can be optimized from the perspective of both partners. For SRL, such approach can be linked to Iterative Learning Control Strategies (ILCS; [Bristow et al., 2006, Gregg et al., 2014]) where the behavior of the robotic limb is optimized iteratively on the basis of a control error. This approach is commonly used to generate and optimize the previously mentioned leader-follower schemes [Llorens-Bonilla et al., 2012, Parietti and Asada, 2016, Wu and Asada, 2016] but can also be employed in voluntary control [Ha et al., 2010]. A comparative study by [Zhang et al., 2015] showed the advantages of such an approach for an ankle exoskeleton, where ICLS provides superior performance to non-adaptive equivalents. It should be noted, however, that these are examples of sequential or iterative coadaptation. That is, human and machine adapt their behavior to each other but not at the same time. Typically, while the user adapts his behavior during the execution of a task involving an SRL, the robotic limb only learns on the data sets generated during this execution after the task has been completed. Concurrent coadaptation, where human and

machine learn jointly during the execution of a task, has been previously studied mainly in Brain Computer Interfaces (BCI), for example in [Vidaurre et al., 2011] or [Ehrlich and Cheng, 2018], where the parameters of the decoder/controller are optimized in parallel with the task execution. Closer to our purpose, [Hu et al., 2023] have investigated the use of concurrent coadaptation for an EMG based interface used to control a robotic arm. They show that such an approach could be used to improve the robustness of the interface to perturbations, such as an unexpected shift of the EMG electrodes, compared with a conventional (non-adaptive) approach.

### 2.1.3 Objectives

In this exploratory study, we further investigate this "robustness to perturbation" property of concurrent coadaptive schemes and apply it to the control of a mirror thumb SRF with two DoFs. More specifically, we are interested in the potential of coadaptation in the context of a multimodal interface. In a first phase of our experiment, we provided a range of communication channels between the human and the machine and let them negotiate on an optimal control scheme through concurrent coadaptation. In a second phase, we repeated this process but additionally introduced perturbations, in the form of secondary tasks, to study how they would affect the resulting control scheme. In the following sections, we first present the system and the coadaptation algorithm, based on [Müller et al., 2017], used in the experiment. We then present the "posture matching task" we asked our participant to perform with the SRF and the perturbations we introduced during its execution before presenting and discussing our findings.

## 2.2 Methods

### 2.2.1 Robotic Finger

This study was designed around a SRF (see figure 2.2) with two DoFs designed by EXII [exi, ]. This mirror-thumb SRF, similar to previous designs [Cunningham et al., 2018, Kieliba et al., 2021], was attached on the back of the right hand and held in place with straps. The two DoF are driven by two electric motors controlled with pulse width modulation (PWM) between a fully extended position where the finger is pointing along the ulnar axis in the coronal plane and a fully curled position where the SRF folds around the palm to point along the radial axis. The second degree of freedom allows the SRF to rotate around its base, from a maximum rotation of  $35^\circ$ , with the finger resting in the ulnar-distal quadrant and a minimum rotation of  $-35^\circ$ , with the finger resting in the ulnar-proximal quadrant. Hereafter we will refer to these two DoFs as the finger's *flexion* ( $\theta$ ) and *rotation* ( $\phi$ ) respectively.

The control signal (PWM) was generated using a micro-controller (Arduino 33 BLE) that was affixed along the user's forearm. This micro-controller was also used to collect various data from the user.

### 2.2.2 Multimodal data collection

To enable the system and its user to converge on the most convenient control method according to context and preference, various sensors were distributed along the user's right arm (summarized in figure 2.1). Based on the modalities commonly used in the literature,

|         | Sensor Position            | Measurements          | Sampling |
|---------|----------------------------|-----------------------|----------|
| Fingers | Thumb & Index              | Fingertip Pressure    | 64Hz     |
|         |                            | Flexion (Curl)        |          |
| IMU     | Forearm                    | Acceleration (3 axis) | 64Hz     |
|         |                            | Rotation (3 axis)     |          |
| EMG     | Extensor Digitorum         | EMG Activity          | 2048Hz   |
|         | Flexor Digitorum Profundus |                       |          |
|         | Biceps                     |                       |          |
|         | Triceps                    |                       |          |

Figure 2.1: The sensors used as control inputs for the supernumerary finger. To enable the system to adapt to the preference of the user and to the context of use, various control modalities were used in parallel. These correspond to the flexion and fingertip pressure of both thumb and index, arm movements and the electromyographic activity of four muscles.

we chose to monitor the movements of the user's (natural) fingers and arm, as well as the electromyographic activity of four muscles. For the fingers' movement, we developed a data glove which, via pressure and flexion sensors (see figure 2.2), measured the flexion of the user's index finger and thumb, as well as the pressure exerted on the fingertip. These sensors were operated by the same micro-controller used to generate the PWM controlling the robotic finger and sampled at 64Hz.

To record the participant's arm movement, we used the inertial measurement unit (IMU) embedded in the micro-controller affixed to the user's forearm. Acceleration and orientation information were similarly sampled at 64Hz.

Finally, we used a commercial electromyographic acquisition device (Delsys system) to monitor the user's muscle activity. The electromyographic electrodes were positioned to record the activity of extensor digitorum (ED), flexor digitorum profundus (FD), biceps (Bc) and triceps (Tc). Electromyographic data were sampled at 2048Hz.

## Processing

The data collected from the user's right arm were pre-processed before being fed into a classifier allowing to infer the user's movement intention for the supernumerary finger.

Pressure and flexion data (4 channels) collected on the index and thumb, as well as the accelerometric and gyrometric information (6 channels) were all downsampled to 16Hz by averaging them over 4 points. For the EMGs, we extracted their envelop using a numerical band-pass filter (Butterworth, 3rd order,  $f_c = [0.1, 20]$ ), as is it commonly done for EMG processing [Simao et al., 2019, Esposito et al., 2023]. The resulting envelopes were then downsampled to 16Hz by averaging them over 128 points.

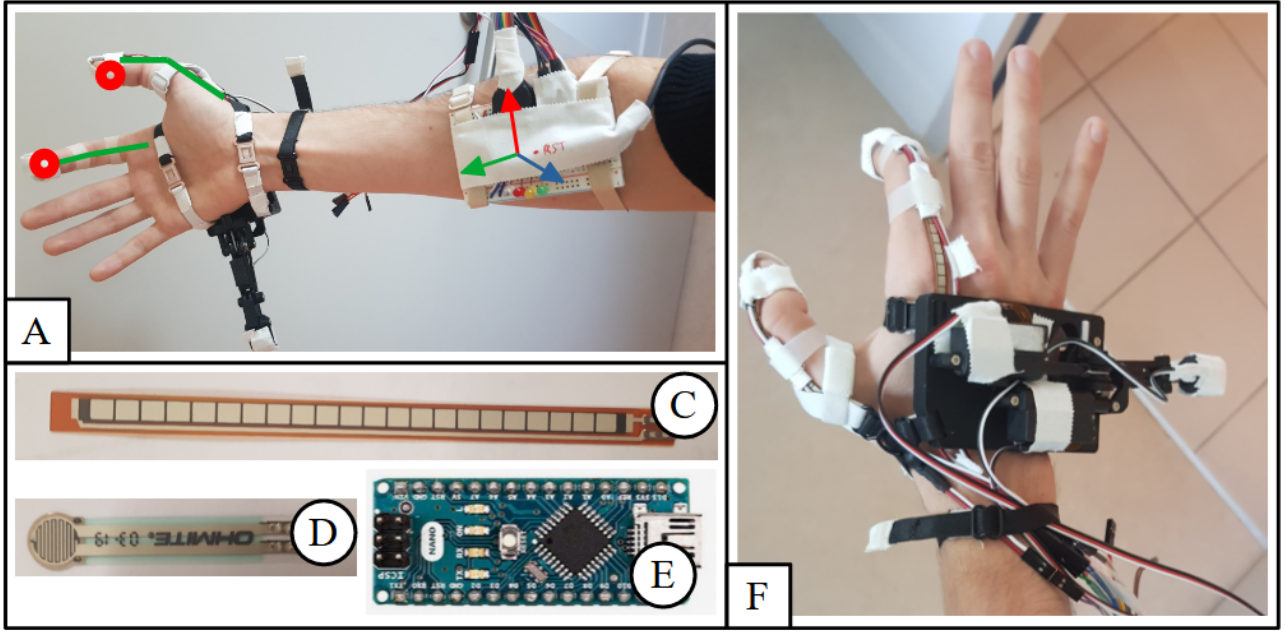


Figure 2.2: The supernumerary robotic finger and the data glove. A. The device is worn on the right arm with the robotic finger affixed on the back of the user's right hand (F). The green bars and red circles highlight the position of the two flexion (C) and pressure (D) sensors monitoring the right thumb and index of the user. The basis mark the position of the micro-controller (E) embedding the IMU. Note that the EMG electrodes used to record the muscle activity are not shown here.

Finally, every 125ms (8Hz), the last two data points from each channel were normalized and concatenated in a vector ( $2(4 + 6 + 4) = 28$  points total), hereafter referred to as a *frame*. This frame was then classified using an hyperplane classifier, according to the procedure detailed below, to control the movements of the robotic finger.

### 2.2.3 A simple coadaptation procedure

The classifier used with the device was based on the algorithm presented by [Müller et al., 2017] which was specifically designed on the basis of a model of a coadaptive process between two learning agents jointly attempting to maximize their ability to communicate through a noisy channel. The algorithm, detailed below, was in our case extended to two dimensions and is used to identify, among the sensors distributed along the user's arm and fingers, those which are most suited to control the supernumerary finger depending on both the user's preferences and on the context of use.

#### Communication through a noisy channel

The basic model can be summarized as such. Two learning agents, a human ( $\mathcal{H}$ ) and a machine ( $\mathcal{M}$ ) want to communicate with each other. It is assumed here that the communication from  $\mathcal{M}$  to  $\mathcal{H}$  is perfect and the goal is to optimize the communication from  $\mathcal{H}$  to  $\mathcal{M}$ .

The challenge facing  $\mathcal{H}$  and  $\mathcal{M}$  here is that the symbol they exchange,  $y \in \{-1, 1\}$ , cannot be transmitted directly but must first be encoded into a signal  $x$  that can be transmitted through some communication channel, with crucially no a priori rule as to the encoding of a symbol into a signal. In addition, the communication channel from  $\mathcal{H}$  to  $\mathcal{M}$  is noisy and

$\mathcal{M}$  therefore only have access to an altered version of the signal originally emitted by  $\mathcal{H}$ . The common goal of the partners is thus to find a encoding/decoding procedure such as to minimize the discrepancy between the symbol emitted by  $\mathcal{H}$ ,  $y$ , and the symbol decoded by  $\mathcal{M}$ ,  $\hat{y}$ . To achieve this goal,  $\mathcal{H}$  and  $\mathcal{M}$  engage in a supervised interaction, where both of them know of the symbol they are trying to exchange beforehand, and progressively learn an optimal set of parameters for the encoding ( $\mathcal{H}$ ) and decoding ( $\mathcal{M}$ ) functions.

First,  $\mathcal{M}$  randomly chooses a symbol  $y \in \{-1, 1\}$ , which we'll refer to in the later as the supervisory symbol, and communicates it to  $\mathcal{H}$  with this communication being assumed perfect. To encode this supervisory symbol into a signal  $x$  that can be sent through the channel,  $\mathcal{H}$  employs a linear transformation with parameters  $w_{\mathcal{H}}$  such as the emitted signal is :

$$x = w_{\mathcal{H}} y \quad (2.1)$$

In other words,  $\mathcal{H}$  transmits  $x = w_{\mathcal{H}}$  when  $y = 1$  and  $x = -w_{\mathcal{H}}$  when  $y = -1$ . Since the channel is noisy however,  $\mathcal{M}$  receives an altered version of the signal,  $\tilde{x}$ , such as :

$$\begin{aligned} \tilde{x} &= x + \epsilon \\ \epsilon &\sim \mathcal{N}(0, \Sigma) \end{aligned} \quad (2.2)$$

where  $\epsilon$  is assumed to be additive Gaussian noise with covariance  $\Sigma$ . Upon receiving this signal,  $\mathcal{M}$  decodes it using its own linear transformation with parameters  $w_{\mathcal{M}}$ , such as the estimated symbol,  $\hat{y}$ , is :

$$\hat{y} = w_{\mathcal{M}}^T \tilde{x} \quad (2.3)$$

Finally, this estimate is sent back to  $\mathcal{H}$ , this transmission being once again assumed perfect, such as both  $\mathcal{H}$  and  $\mathcal{M}$  can update their internal parameters,  $w_{\mathcal{H}}$  and  $w_{\mathcal{M}}$  respectively, with a view to minimize the discrepancy between the emitted symbol  $y$  and the decoded symbol  $\hat{y}$ .

### Optimization of the machine's internal parameters

From the machine's point of view, the discrepancy between the transmitted and decoded symbol can be written as a quadratic error function with expectation :

$$\begin{aligned} \mathbb{E}[(\hat{y} - y)^2] &= \mathbb{E}[(w_{\mathcal{M}}^T \tilde{x} - y)^2] \\ &= \mathbb{E}[(w_{\mathcal{M}}^T (w_{\mathcal{H}} y + \epsilon) - y)^2] \\ &= (w_{\mathcal{M}}^T w_{\mathcal{H}} - 1)^2 y^2 + w_{\mathcal{M}}^T \Sigma w_{\mathcal{M}} \end{aligned} \quad (2.4)$$

This expected error naturally decomposes into a *common loss*,  $C[w_{\mathcal{M}}, w_{\mathcal{H}}, y] = (w_{\mathcal{M}}^T w_{\mathcal{H}} - 1)^2 y^2$ , the component of the expected error depending both on the parameterization of the transformation used by  $\mathcal{H}$  and  $\mathcal{M}$ , and a *private loss*  $P[w_{\mathcal{M}}] = w_{\mathcal{M}}^T \Sigma w_{\mathcal{M}}$ , which depends solely on  $w_{\mathcal{M}}$ . Heuristically, the common loss scores the discrepancy between the decoded symbol  $\hat{y}$  and the emitted symbol  $y$  in the absence of noise, while the private loss measures the sensibility of the machine decoder to the noise on the channel.

Computing the gradient of the error ( $L[w_{\mathcal{M}}, w_{\mathcal{H}}, y] = C[w_{\mathcal{M}}, w_{\mathcal{H}}, y] + P[w_{\mathcal{M}}]$ ) w.r.t  $\mathcal{M}$ 's internal parameters, we get an update rule for a simple stochastic gradient descent scheme :

$$w_{\mathcal{M}} \leftarrow w_{\mathcal{M}} - \eta \cdot \frac{\nabla_{w_{\mathcal{M}}} L}{\|\nabla_{w_{\mathcal{M}}} L\|} \quad (2.5)$$

$$\nabla_{w_{\mathcal{M}}} L = (w_{\mathcal{M}}^T w_{\mathcal{H}} - 1)y^2 w_{\mathcal{H}} + \Sigma w_{\mathcal{M}}$$

where  $\nabla_{w_{\mathcal{M}}}$  denotes gradient w.r.t to  $w_{\mathcal{M}}$  and  $\eta$  a learning rate. Of course,  $\mathcal{H}$ 's internal parameters,  $w_{\mathcal{H}}$ , as well as the noise covariance matrix,  $\Sigma$ , are unknown to  $\mathcal{M}$ . One can however provide an estimator for each of them. First via an online least-square procedure on the prediction error for  $\mathcal{H}$ 's internal parameters, noting  $\hat{w}_{\mathcal{H}}$   $\mathcal{M}$ 's estimate of the encoding parameters  $w_{\mathcal{H}}$ , we have :

$$\hat{w}_{\mathcal{H}} \leftarrow \hat{w}_{\mathcal{H}} + \eta y \frac{\tilde{x} - \hat{w}_{\mathcal{H}} y}{\|\tilde{x} - \hat{w}_{\mathcal{H}} y\|} \quad (2.6)$$

where  $\eta$  is a learning rate and  $sgn$  the sign function. Given this estimate,  $\mathcal{M}$  can approximate the noise,  $\epsilon$ , affecting the emitted signal to progressively estimate its covariance  $\Sigma$ . With each new couples,  $\tilde{x}$  and  $y$ , we have the update rule :

$$\begin{aligned} \hat{\epsilon} &= \tilde{x} - \hat{w}_{\mathcal{H}} y \\ \hat{\Sigma} &\leftarrow (1 - \beta)\hat{\Sigma} + \beta \hat{\epsilon} \hat{\epsilon}^T \end{aligned} \quad (2.7)$$

where  $\hat{\epsilon}$  is  $\mathcal{M}$ 's estimate of the noise affecting the signal emitted by  $\mathcal{H}$ ,  $\hat{\Sigma}$  the estimate of its covariance and  $\beta \in [0, 1]$  an update rate. Given those estimates,  $\mathcal{M}$  can estimate the gradient of the loss w.r.t its internal parameters as :

$$\hat{\nabla}_{w_{\mathcal{M}}} L = (w_{\mathcal{M}}^T \hat{w}_{\mathcal{H}} - 1)y^2 \hat{w}_{\mathcal{H}} + \hat{\Sigma} w_{\mathcal{M}} \quad (2.8)$$

In summary, every time  $\mathcal{M}$  receives a signal  $\tilde{x}$  and given the supervisory symbol  $y$ , its internal parameters,  $w_{\mathcal{M}}$ ,  $\hat{w}_{\mathcal{H}}$  and  $\hat{\Sigma}$ , are updated as (omitting gradient normalization for clarity) :

$$\begin{aligned} \hat{\epsilon} &= \tilde{x} - \hat{w}_{\mathcal{H}} y \\ \hat{w}_{\mathcal{H}} &\leftarrow \hat{w}_{\mathcal{H}} + \eta y \hat{\epsilon} \\ \hat{\Sigma} &\leftarrow (1 - \beta)\hat{\Sigma} + \beta \hat{\epsilon} \hat{\epsilon}^T \\ w_{\mathcal{M}} &\leftarrow w_{\mathcal{M}} - \eta [(w_{\mathcal{M}}^T \hat{w}_{\mathcal{H}} - 1)y^2 \hat{w}_{\mathcal{H}} + \hat{\Sigma} w_{\mathcal{M}}] \end{aligned} \quad (2.9)$$

This process allows  $\mathcal{M}$  to progressively learn the mapping  $\hat{y} = w_{\mathcal{M}} \cdot \tilde{x}$  between the signals emitted by  $\mathcal{H}$  and the symbols such as to both maximize the "compatibility" of the resulting decoder to the initially unknown encoder used by  $\mathcal{H}$  (the common loss) and minimize the sensibility of this decoder to the Gaussian noise affecting the communication channel (the private loss).

### 2.2.4 Control of the supernumerary finger

In the process we describe above, the symbols,  $y$ , exchanged by  $\mathcal{H}$  and  $\mathcal{M}$  correspond to commands for the supernumerary finger. The symbol  $y = 1$  is mapped to a small increase in either the flexion or rotation of the robotic finger while  $y = -1$  is mapped to a small decrease. Furthermore, to enable the user to maintain the finger in a given posture, we supplemented the decoding function with a threshold (i.e dead-band) such that the decoded symbol is :

$$\hat{y} = \begin{cases} \text{sgn}(w_{\mathcal{M}}^T \tilde{x}) & \text{if } |w_{\mathcal{M}}^T \tilde{x}| > \gamma_0 \\ 0 & \text{otherwise} \end{cases} \quad (2.10)$$

where  $\gamma_0$  is a threshold on the absolute value of the dot product between  $w_{\mathcal{M}}$  and  $\tilde{x}$  under which the decoded symbol is 0 (i.e no movement). This threshold is kept constant for all participants in our evaluation study and was hand-tuned during preliminary tests with the device. In this equation, the signals emitted by  $\mathcal{H}$ ,  $\tilde{x} \in \mathbb{R}^{28}$ , correspond to the frames collected from the user we introduced in section 2.2.2.

Since the 6th finger possesses two DoF, we use two classifiers in parallel to decode the signals emitted by  $\mathcal{H}$ . Thereafter, we note  $y_\theta$  and  $y_\phi$ , the symbols pertaining to the supernumerary finger's flexion ( $\theta$ ) and rotation ( $\phi$ ) respectively. The parameters of the decoder used by  $\mathcal{M}$  to infer the symbol emitted by  $\mathcal{H}$  will be noted  $w_{\mathcal{M}}^\theta$  and  $w_{\mathcal{M}}^\phi$  respectively and similarly for its estimates of the encoder used by  $\mathcal{H}$  for each of the two DoFs, noted  $\hat{w}_{\mathcal{H}}^\theta$  and  $\hat{w}_{\mathcal{H}}^\phi$  respectively. Importantly, while  $\mathcal{M}$  uses a specific transformation for each of the DoF, the signals on which the classification is based are the same and correspond to the frames collected from the user.

In other words, the robotic finger's posture,  $p = [\theta, \phi]$ , is updated in response to the frame  $\tilde{x}$  collected from the user as :

$$p \leftarrow p + \hat{y} \Delta_p$$

$$\hat{y} = \begin{cases} \text{sgn}(w_{\mathcal{M}}^T \tilde{x}) & \text{if } |w_{\mathcal{M}}^T \tilde{x}| > \gamma_0 \\ 0 & \text{otherwise} \end{cases} \quad (2.11)$$

where  $\Delta_p$  is a constant controlling the increase/decrease in angle resulting from the symbols,  $\hat{y} = [\hat{y}_\theta, \hat{y}_\phi]$ , decoded by  $\mathcal{M}$ . This constant was hand tuned so that it takes about 2 seconds to cover the whole range of motion in both DoFs.

## 2.3 Posture Matching Task

In order to study the ability of the coadaptive system to adapt to the conditions under which a task is performed, we designed a preliminary experiment in which 7 participants (5 males, 2 females, aged 20–26, mean 25) took part. The task required the participants to reach a target flexion ( $\theta$ ) and rotation ( $\psi$ ) angle, thereafter termed *target posture* ( $p_t = [\theta_t, \psi_t]$ ), with the 6th finger. Wearing the device on their right arm, the participants sat comfortably on a chair in front of a computer screen onto which was reported the robotic finger's posture as a cursor as well as the target (see section 2.3.1 and figure 2.3). We simply instructed them to perform the task as best as they could and gave them no information as to the system inner workings. The "posture matching" task was performed in various conditions designed to assess different aspects of the coadaptive process.

During the preliminary tests of the experimental procedure, we brought a few "task specific" adjustments to the coadaptive procedure with a view to better its performance, these adjustments are detailed below.

### 2.3.1 Supervisory labels

A strong requirement of the algorithm we overviewed in 2.2.3 is the assumption that the communication from  $\mathcal{M}$  to  $\mathcal{H}$  is perfect such as the supervisory labels can be transmitted without error. In the posture matching task we perform here, this supervisory label is obtained as the sign of the error between current and target posture, such as :

$$y_\tau^\theta = \begin{cases} \text{sgn}(\theta_t - \theta_\tau) & \text{if } \theta_t - \theta_\tau > \Delta_p \\ 0 & \text{otherwise} \end{cases} \quad (2.12)$$

where  $y_\tau^\theta$  is the supervisory symbol for the flexion at time  $\tau$ ,  $\theta_t$  and  $\theta_\tau$  the target and current flexion respectively (similarly for the rotation  $\phi$ ) while  $\Delta_p$  is the increase/decrease in flexion. This supervisory symbol is then matched with the frame  $\tilde{x}_\tau$  collected from the user at time  $\tau$  to perform the learning as presented in 2.9 (discarding the cases where  $y_\tau = 0$ ). A perfect communication between  $\mathcal{M}$  and  $\mathcal{H}$  imply that the participant has, at any time, perfect information about the target posture as well as the supernumerary finger current posture, such as there is no ambiguity with regard to the symbol they are expected to emit. At an early development stage, we chose to present the target posture to the participant via a photography of the 6th finger in the correct configuration and tasking them to reproduce it. We quickly realized, however, that presenting the task in this fashion was, to some extent, incompatible with the assumption of a perfect communication for the supervisory signal. Firstly, because the presentation of the target as a photograph requires a certain amount of interpretation from the participant to match a 2D image to a 3D posture for the 6th finger. A misinterpretation leading to a discrepancy between the supervisory signal, or what  $\mathcal{M}$  expect the participant to emit, and what is actually being transmitted by the participant. Similarly, since the supernumerary finger is affixed onto the back of the participant's right hand during the experiment, it was sometimes difficult for them to visually assess the current posture of the 6th finger and its movement, leading to discrepancies between what  $\mathcal{M}$  is expecting of the participant's movement intention and the participant's "real" intention.

Because of these incompatibilities, we chose to provide the participant with a visual representation of the supernumerary finger's posture as well as of the target. During the experiment, the posture of the robotic finger was represented as a cursor on a screen placed

in front of the participant, the target was similarly displayed (figure 2.3). The vertical axis corresponded to the rotation of the robotic finger while the horizontal axis corresponded to the flexion. Thanks to this visual feedback, the participants were able to understand the objective more clearly, thus resolving the discrepancy between  $\mathcal{M}$ 's expectations and the participant real intention. However, the inclusion of this visual representation means that during the experiment participants were more focused on controlling a cursor representing the posture of the supernumerary finger than on controlling it directly (even though there is a 1:1 correspondence between the two).

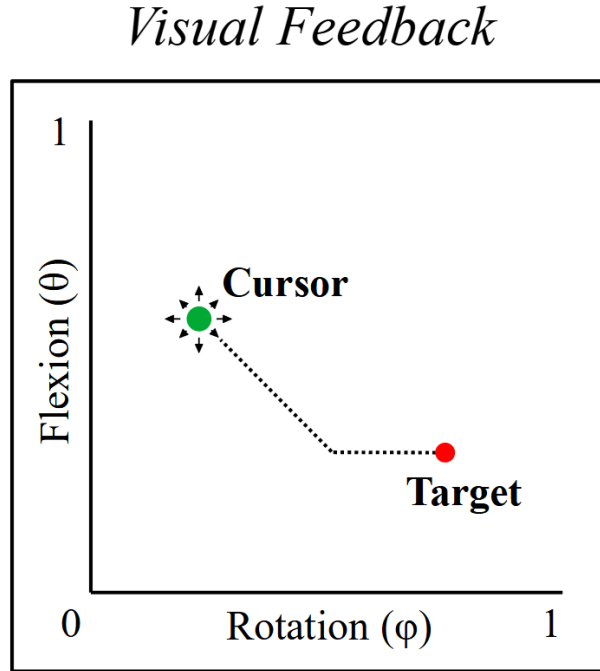


Figure 2.3: During the experiment, the participants were provided with a visual feedback of the SRF current posture as a green cursor, the target was similarly represented in red. Note that, given that the command to the SRF can only be  $+1$ ,  $-1$  or  $0$  in either axes, the participant can only move this cursor along diagonals and vertical/horizontal lines.

### 2.3.2 Feedback bias

After some preliminary tests with the system, we were concerned that the relatively erratic behavior of the robotic finger at an early stage in the optimization process might be destabilizing for the participant. For this reason, we chose to introduce a positive bias in the behavior of the supernumerary finger during training. This bias has the effect that the current posture of the robotic finger is slightly drawn to the target posture with a strength modulated by a bias coefficient  $\alpha \in [0, 1]$  such as the update rule for the movement of the 6th finger is :

$$p_c \leftarrow [(1 - \alpha) \hat{y} + \alpha y] \Delta_{PWM} + p_c \quad (2.13)$$

where  $\hat{y}$  is the estimate obtained via classification (see 2.2.3) and  $y$  the supervisory symbol. Previous studies have shown that positively biased feedback can improve classification accuracy in poorly performing BCI users [Barbero and Grosse-Wentrup, 2010, Alimardani

et al., 2014]. It has been hypothesized that a positive bias reduces the perceived lack of control and randomness of the feedback and is therefore helpful to focus on the task. Another concern was that the user would abandon a preferred control method before the machine could learn an appropriate decoder to match it. In this respect, biased feedback, by reducing the perceived volatility in this task, could encourage participants to persist with a given strategy, in line with previous studies on the exploration-exploitation trade-off in a volatile environment [Cohen et al., 2007, Doya, 2008, Findling et al., 2019].

### 2.3.3 Dynamic Learning Rate

The model on which our co-adaptive system is based is fundamentally sequential. The signals emitted by  $\mathcal{H}$  in response to a supervisory symbol from  $\mathcal{M}$  are assumed to be temporally uncorrelated apart of the underlying parameterization of the encoding function. Similarly, the supervisory symbol offered by  $\mathcal{M}$  are assumed to be independent and identically distributed (iid.). However, our task violates this assumption since both the sequence of supervisory symbols and of frames collected from the participants have a temporal structure. Reaching a target posture implies some relatively long sequences of repetitions of a given command and it is evidently not possible for the participant to emit a signal  $x$  at one frame and  $-x$  125ms later on the next, and so irrespective of the supervisory symbol emitted by  $\mathcal{M}$ . To take this into account, we assumed that the participant's strategy would become more pronounced as they moved further away from the target, and modulated the learning rate  $\eta$  in 2.9, on the basis of this information. The learning rate for each of the axis was independently modulated via a sigmoid such as :

$$\eta_{\theta,\tau} = \eta_0 \sigma[\nu(\theta_t - \theta_\tau) - \mu] \quad (2.14)$$

where  $\eta_0$  is the base learning rate,  $\sigma(x)$  the sigmoid function,  $\theta_t$  the target flexion,  $\theta_\tau$  the robotic finger's flexion at time  $\tau$  and  $\eta_{\theta,\tau}$  the effective learning rate for the frame  $\tilde{x}_\tau$  collected at time  $\tau$  (equivalently. for the rotation replacing  $\theta$  by  $\phi$ ). The sharpness and offset parameters,  $\nu$ ,  $\mu$ , were hand tuned to 15 and 0.3 respectively. Heuristically, examples where the participant is further away from the target are weighted more than examples where the distance is lower (independently between the two axes).

## 2.4 Experimental Procedure

### 2.4.1 Trials, Blocks and Sessions

In the following, we will refer to the presentation of a target posture to the participant and the subsequent attempt by the participant to reach it as a trial. Each trial began with the presentation of the target to the participant and ended when the robotic finger posture matched the target posture within a small margin, or when 30 seconds had elapsed. Trials in which participants succeeded in reaching the target were labeled as "success" and as "failure" otherwise.

At the end of each trial, the robotic finger was manually re-positioned on the target so that participants began each trial in the posture they would have been in should they have successfully completed the previous trial. All participants started the first trial in a "neutral posture", with the robotic finger fully extended ( $\theta = 1$ ) and centered ( $\phi = 0.5$ ).

Trials were grouped into blocks of five, each block corresponding to one of three experimental conditions : "Adaptive", "Non-Adaptive" and "Evaluation", differing in the set of hyperparameters used for the learning and control.

### Adaptive blocks

During the adaptive blocks, the base learning rates  $\eta_0$  and  $\beta_0$  controlling the size of the update to  $\mathcal{M}$ 's internal parameters and estimates were hand tuned to  $10^{-2}$ . This relatively small value was obtained by trial and error during preliminary tests. In addition, the bias coefficient was set to 0.2.

### Non-Adaptive blocks

For Non-Adaptive blocks, the bias coefficient was similarly set to 0.2. Conversely however, both learning rates were set to 0. In this case, there is therefore no adaptation of the robotic finger to the participant's input.

### Evaluation blocks

Finally, Evaluation blocks are designed to evaluate the performance of the participant in controlling the 6th finger given a prior adaptation (or not) on the part of the machine. In this condition, both learning rates, as well as the bias coefficient, are set to 0.

## 2.4.2 Sessions

Each participant took part in two sessions, with each session being designed to test one of our hypotheses. In the first session, hereafter referred to as the "Free Session", the participants performed the task without any constraints. In other words, their right arm and hand were both free and they could perform any movement of their choice. Conversely, during the second session, referred to hereafter as the "Constrained Session", the participants were required to reach the target configuration while performing a secondary, "perturbation" task described below.

Each session started with a calibration during which we asked the participants to make random movements with their right arm and right hand fingers. The data obtained during this step were then used to normalize the frame collected in the following trials before their classification. We additionally initialized the machine estimates during this calibration. For all participants, the noise covariance estimate ( $\hat{\Sigma}$ ) was set to identity while both the estimate for the parameters of the user's encoder ( $\hat{w}_{\mathcal{H}}$ ) and the parameters of the machine decoder ( $w_{\mathcal{M}}$ ) were randomly sampled from a Gaussian distribution with mean 0 and variance 1.

### Free Session

The Free Session, performed first, was designed to compare the participants' ability to control the supernumerary finger with and without adaption from the machine. After a calibration of the system, half of the participants (4) performed a block of 5 trials in the adaptive condition, while the other half (3) performed 5 trials in the non-adaptive condition. Finally, all participants performed a block of 5 trials in the evaluation condition. Crucially,  $\mathcal{M}$ 's internal parameters,  $w_{\mathcal{M}}$ ,  $\hat{w}_{\mathcal{H}}$  and  $\hat{\Sigma}$ , were preserved from one block of trials to the next. In other words, during the five trials of the evaluation block, the four participants who

previously performed in the adaptive condition were controlling the finger via a decoder optimized to their inputs. While the other three, who performed the previous block in the non-adaptive condition, had to cope with the randomly generated decoder we introduced during the calibration (figure 2.4 - Top Left).

### Constrained Session

The Constrained Session, performed after the Free Session, was designed to evaluate the ability of an adaptive system to adapt to the different conditions under which a task might be performed. During this session, the participants were instructed to perform the posture matching task while performing a secondary, perturbation, task designed to interfere with the data collected from the participants.

For the first perturbation task, participants were instructed to hold a relatively heavy hammer (1.5kg) in their right hand. We chose a hammer for three reasons. Firstly, the fact that a hammer is heavy means that participants would require more force to move the arm, increasing their muscle activity in comparison to the case where the hand is free and therefore affecting the collected EMG. Secondly, the fact that the weight of a hammer is concentrated away from the participants' grasping point means that holding it in any position other than upright is relatively uncomfortable due to the resulting torque. This inclines the participants to prefer movement where the hammer is upwards, thus affecting the orientation and acceleration measured by the IMU affixed to their forearm. Finally, since holding a hammer requires a relatively strong grasp, we also expect the participant index and thumb to be relatively less mobile than in the Free session.

For the second perturbation task, participants were instructed to balance a small ball on a plate held with both hands. As for the hammer, we chose this task because it imposed strong restriction on the type of motion the participant could perform. In particular, balancing the ball implies that part of the participants' arm movement will be directed towards this task rather than to control the robotic finger. For this reason, we expect that this secondary task would force participant to rely less on arm movement in comparison to both the Free session and under the first perturbation task.

In the second session, after calibration of the system, all participants performed a block of five trials in the Adaptive condition holding the hammer (perturbation task I) after which half of the participants (3) performed a block in the Adaptive condition while balancing the ball (perturbation task II), while the other half did the same in the Non-Adaptive condition. Finally, all participants performed an evaluation block while balancing the ball. Here again,  $\mathcal{M}$ 's internal parameters were preserved from one block of trial to the next over the course of the Constrained session (figure 2.4 - Top Right).

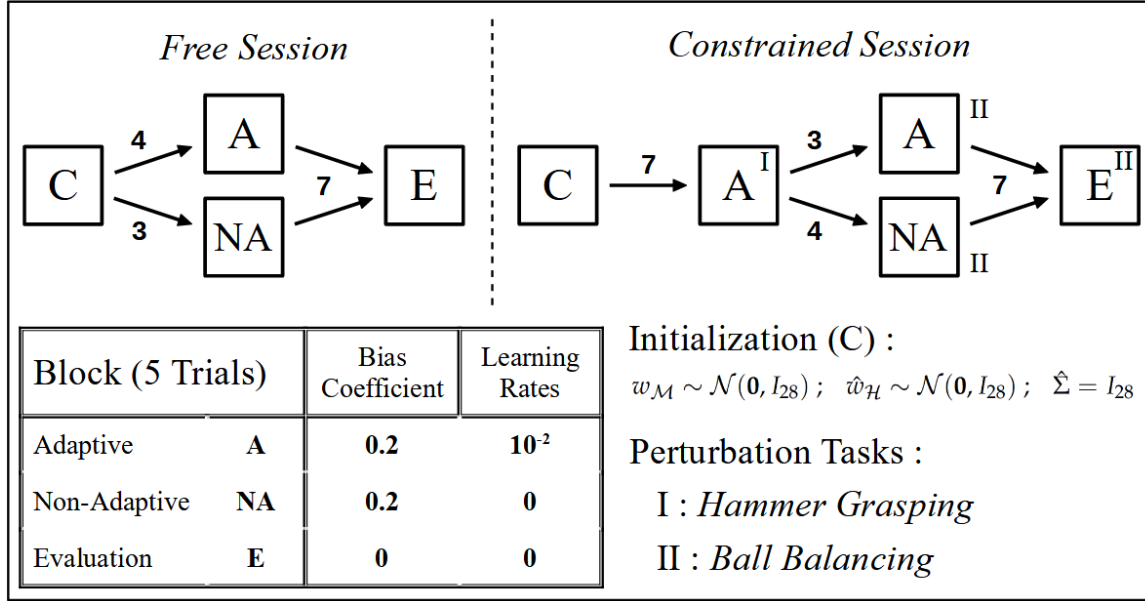


Figure 2.4: The experimental protocol. The participants performed the posture matching task in two sessions, each divided in blocks (2 blocks for the Free session and 3 blocks for the Constrained session). All 7 participants started both sessions with a calibration (C) during which we randomly initialized the system's estimates. Afterwards, they performed each blocks (5 trials) of the session in the conditions they were randomly assigned to, where each conditions differ by the bias coefficient and learning rates set for the system. The arrows are annotated with the number of participants assigned to each condition. For the Constrained session, the participant were instructed to perform a secondary, perturbation task, in parallel with the posture matching task. Each block in the constrained session is annotated with its corresponding perturbation task (I for the hammer grasping and II for the ball balancing). Crucially, the calibration is the only time at which  $\mathcal{M}$ 's parameters were modified, its estimates are thus preserved from one block of trials to the next over the course of a session.

## 2.5 Results - Performance

### 2.5.1 Session 1 : Free Session

To analyze the results obtained during the Free Session, the participants were distributed into two groups according to the condition in which they performed the first block of the session. The four participants who performed in the Adaptive condition were assigned to the Adaptive group, while the three participants who performed in the Non-Adaptive condition were assigned to the Non-Adaptive group. This enabled us to compare the performance of participants who were given the opportunity to coadapt with the system in order to control the supernumerary finger with participants who had to performed this control using a randomly generated decoder.

### 2.5.2 Measure of performance

We first examined the decoding accuracy and the trajectories generated by the participants in both groups during the evaluation block. Starting with the decoding accuracy, figure 2.5 shows the confusion matrix between the estimate,  $\hat{y}$ , and the supervision,  $y$ , averaged over the two joints and over the participants in each group during the evaluation block.

| Decoding Accuracy<br>(Free Session – Evaluation Block) |                 |                    |           |
|--------------------------------------------------------|-----------------|--------------------|-----------|
| Group                                                  | <i>Estimate</i> | <i>Supervision</i> |           |
|                                                        |                 | <b>+1</b>          | <b>-1</b> |
| Adaptive                                               | <b>+1</b>       | 83%                | 69%       |
|                                                        | <b>-1</b>       | 16%                | 31%       |
| Non-Adaptive                                           | <b>+1</b>       | 80%                | 86%       |
|                                                        | <b>-1</b>       | 20%                | 14%       |

Figure 2.5: The decoding accuracy during the evaluation block of the Free session for the participants (4) of the Adaptive group and for the participants (3) of the Non-Adaptive group. The accuracy reported within each cell is obtained by averaging over participants and control axes (flexion and rotation).

As can be observed, the classification accuracy is fairly low and so irrespective of the group. It should however be noted that classification accuracy is obviously a secondary metric here since the posture matching task did not specifically require our participants to maximize this quantity but rather to reach the target posture. As such, we were mostly interested by the trajectories generated by the participants in each groups given starting and target posture. Figure 2.6 shows examples of such trajectories generated during the evaluation block.

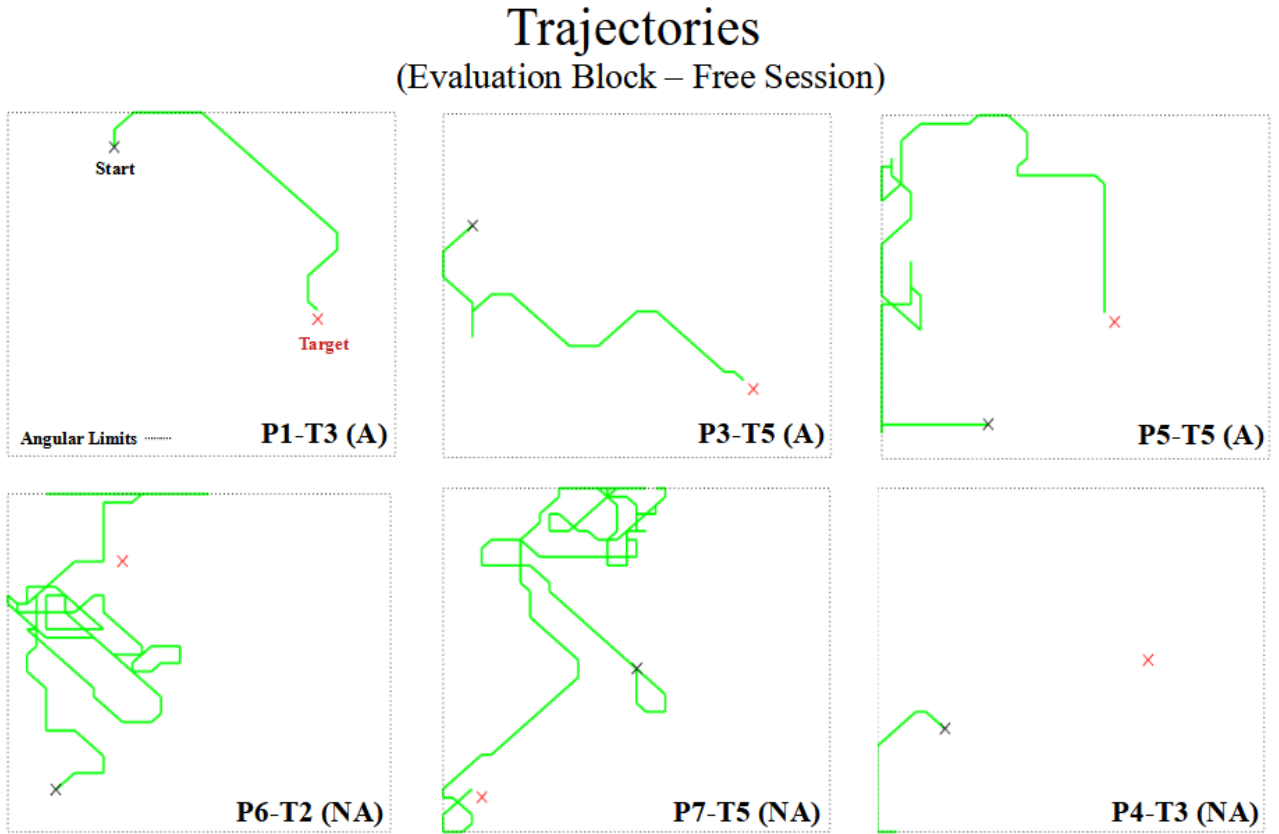


Figure 2.6: Examples of trajectories generated by the participants during the Evaluation block of the Free Session. The starting posture is represented by a black cross while a red cross represent the target posture. Dotted lines represent angular limits for the robotic finger’s flexion (vertical axis) and rotation (horizontal axis). The source participant and trial is identified by an index in the bottom right corner. For example, the top left graph (annotated P1-T3) correspond to the trajectory generated by participant 1 in the third trial (over 5) of the evaluation block. In brackets the group to which the participant belong is indicated, with A representing the Adaptive and NA the Non-Adaptive group respectively.

### Interactivity

A review of the trajectories generated by the participants in each group motivates a few measures of performance. Firstly, looking at the trajectory shown in the bottom right corner of figure 2.6, it is fairly obvious that the participant is unable to control the motion of the robotic finger and thus spent most of the duration of the trial in the bottom left corner of the graph (which corresponds to angular limits). This is because, for this participant belonging to the Non-Adaptive group, the randomly generated decoder introduced during the calibration induced a strong bias towards the symbol  $\hat{y} = [-1 - 1]$ . As such, all frames collected from this participant are here classified as  $-1$  in both axes. Motivated by this example, our first measure of performance correspond to the (Shannon) entropy,  $H[\hat{y}]$ , computed over the symbols decoded during the evaluation block, which we refer to as the *interactivity* metric below. Heuristically, this simply measures the ability of the participant to cross the decision boundary induced by the parameterization of the machine decoder  $w_{\mathcal{M}}$ , in either axes, and so irrespective of the supervisory label.

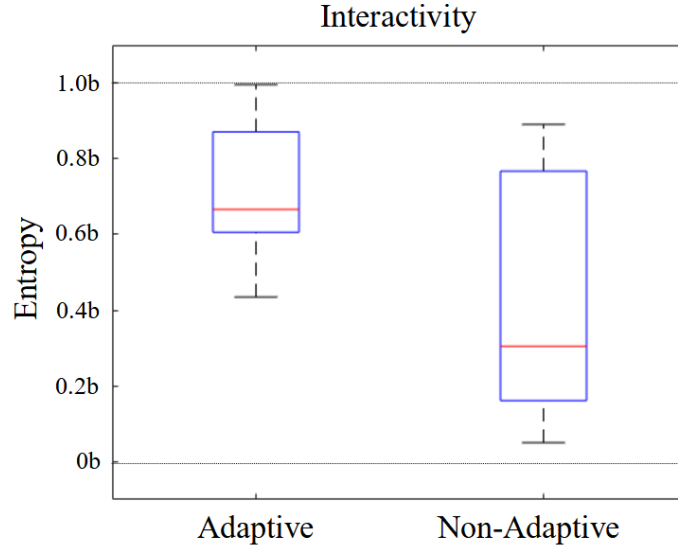


Figure 2.7: (Top) The distribution of Shannon entropy of decoded symbols ( $H[\hat{y}]$ ), or interactivity, observed over each participant and each axis during the evaluation block of the Free Session. Values are shown in bits, separately for the participants of the Adaptive and Non-Adaptive groups.

With respect to this measure, participants in the Adaptive group yielded a mean entropy of  $0.71b \pm 0.18b$  (median =  $0.66b$ ) for the symbols, in either axes, decoded in trials performed during the evaluation block (figure 2.7). However, participants in the Non-Adaptive group yielded a mean entropy of  $0.41b \pm 0.34b$  (median =  $0.30b$ ). This result shows that, thanks to the machine adaptation in the previous block, it was on average "easier" for the participants of the Adaptive group to cross the decision boundary induced by the parameterization of the machine decoder than for the participants of the Non-Adaptive group who had to cope with the randomly generated decoder we introduced during the calibration.

### Relative Increase in Integral Cost (RIIC)

For our second measure of performance, we investigated how "efficient" the participants in both group were in completing the posture matching task. For this, we postulated that our participants primary objective during the task was to reduce the distance between the cursor and the target displayed on the screen in front of them. At each time-step, the error is thus computed as :

$$e_{\tau} = ||p_t - p_{\tau}|| \quad (2.15)$$

where  $p_t$  is the target and  $p_{\tau}$  the posture at time  $\tau$  respectively while  $||.||$  denotes the Euclidean norm. Summing this error over the course of each trial, we obtain the total error

$$I = \sum_{\tau=0}^T ||p_t - p_{\tau}|| \quad (2.16)$$

where  $T$  is the trial's duration. Since our participants were assigned randomly generated targets in each trial, we normalized this integral cost by the optimal-cost-to-go, or the total cost along the optimal trajectory  $p_{\tau}^*$  which minimize the error at each  $\tau$  (see figure 2.8).

## Optimal Trajectories

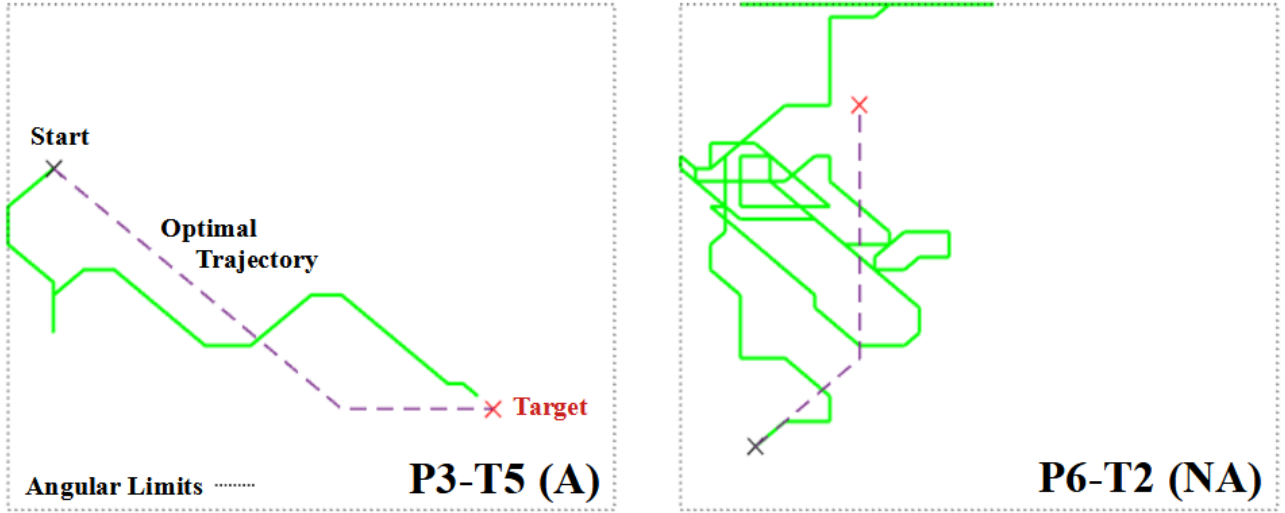


Figure 2.8: The error integrated along the trajectory generated by a participant in a trial is compared to the optimal-cost-to-go, or the error integrated along the optimal trajectory (shown here as a dashed line) given starting and target posture in a trial.

Given this optimal cost-to-go  $I^*$  the relative increase in integral cost (RIIC) is computed as :

$$\delta_I = \frac{I - I^*}{I^*} \geq 0 \quad (2.17)$$

Heuristically, larger RIIC indicates worse performance with a  $\delta_I$  of 0 indicating optimal control of the supernumerary finger with respect to the error.

Based on this metric, the participants of the Adaptive group performed better on average than the participant of the Non-Adaptive group. This can be seen on figure 2.9 which shows the cumulative density function (CDF) of the RIIC observed in each trials separately for the participants in the Adaptive and Non-Adaptive group. As expected, this difference in performance is much more marked in the A/NA blocks, where the participant of the Adaptive group, benefiting from the concurrent machine learning, yielded an average RIIC of  $9.66 \pm 8.69\text{sd}$  (median = 6.18) far superior to that of the Non-Adaptive group, which yielded an average RIIC of  $60.20 \pm 73.29\text{sd}$  (median = 27.18) over all trials or  $35.21 \pm 34.29\text{sd}$  (median = 24.20) after the removal of 2 trials (over 15) outlying beyond 2sd. The difference in performance was less important in the subsequent Evaluation block, where the performance of the participants in the Adaptive group decreased, yielding an average RIIC of  $23.41 \pm 25.3\text{sd}$  (median = 12.87), while the participant in the Non-Adaptive group performed better on average, yielding an average RIIC of  $29.94 \pm 22.46\text{sd}$  (median = 24.92).

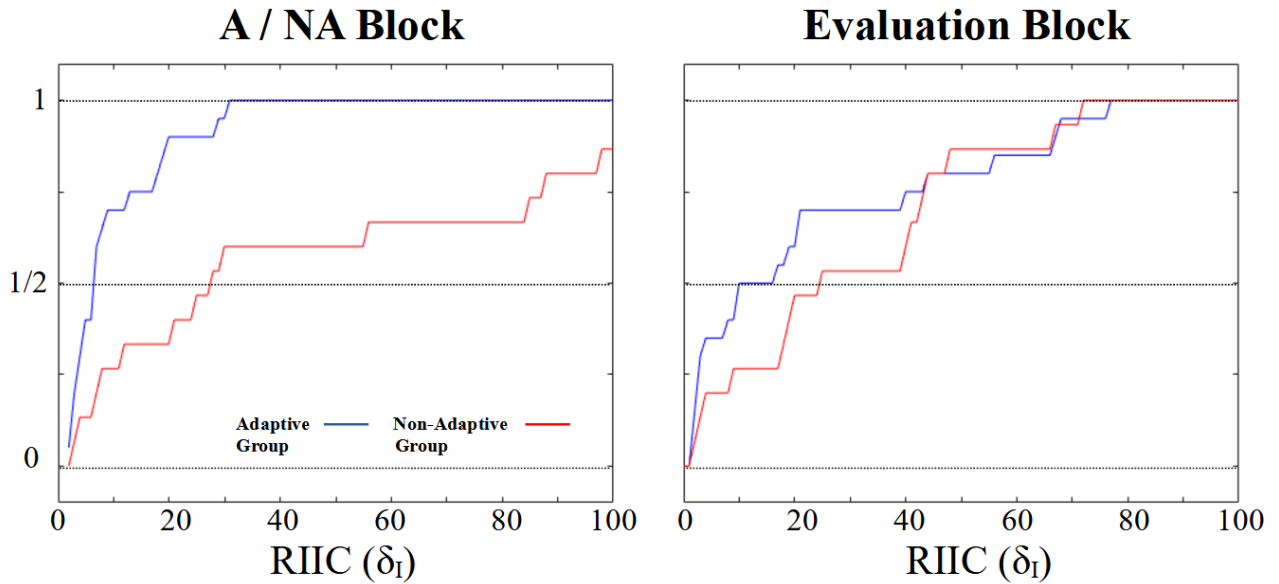


Figure 2.9: The cumulative density function (CDF) of the relative increase in total cost (RIIC) for the trials performed by the participants in the Adaptive/Non-Adaptive (A/NA; left) and Evaluation (right) block respectively. The CDF are represented separately for the  $4 \times 5 = 20$  trials performed by the 4 participants in the Adaptive group (in blue) and for the  $3 \times 5 = 15$  trials performed by the 3 participants in the Non-Adaptive group (in red). Note that, on the left graph representing CDF in the A/NA block, the CDF associated with the Non-Adaptive group (in red) does not reach 100% because of two outliers generated by two different participants at  $\delta_I = 212.7$  and  $\delta_I = 232.6$ .

A Wilcoxon ranksum test which showed that the difference in performance between the two groups was significant in the A/NA block ( $p < 0.004$ ,  $z = -2.88$  over all trials,  $p < 0.015$ ,  $z = -2.45$  after elimination of outliers), but not in the Evaluation block ( $p = 0.3$ ,  $z = -1.05$ ).

### 2.5.3 Session 2 : Constrained Session

The analysis of the participants' performance was similarly conducted for the trials performed during the Constrained Session. We divided the participants in two groups according to the condition in which they performed the second block of the session. The three participants which performed in the Adaptive condition were assigned to the Adaptive group while the four participants which performed in the Non-Adaptive condition were assigned to the Non-Adaptive group. Given that all participants performed the first block of the session in the Adaptive condition while performing the first perturbation task (holding the hammer) but only half of them, belonging to the Adaptive group, were given the opportunity to re-adapt the control to the second perturbation task (balancing the ball) this allowed us to compare the ability of a coadaptive system to adapt to the different conditions under which a task, here the posture matching task, can be performed.

Figure 2.10 shows examples of trajectory generated by the participants in each group during the evaluation block of the Constrained session.

## Trajectories

(Evaluation Block – Constrained Session)

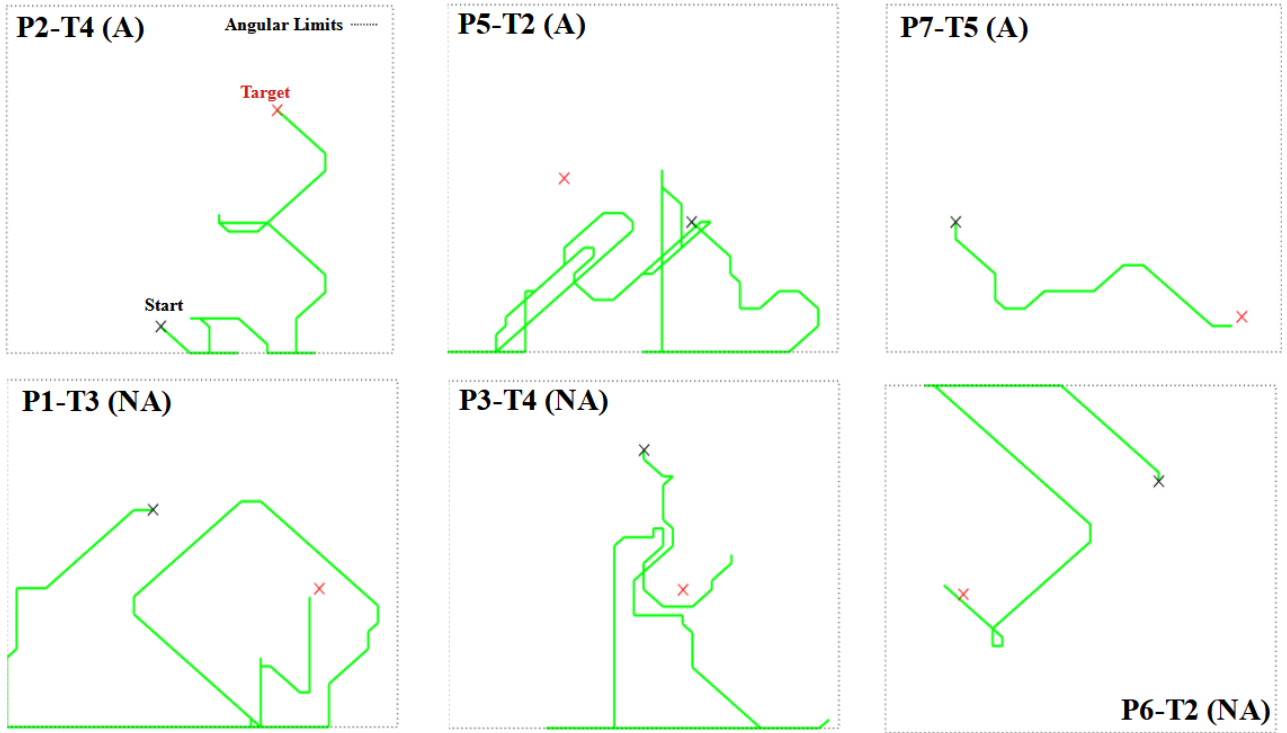


Figure 2.10: Examples of trajectory generated by the participants during the Evaluation block of the Constrained Session. The group to which the participant belong is indicated in brackets, with A for Adaptive and NA for Non-Adaptive.

### Interactivity

With respect to the interactivity metric introduced in the previous section, the average interactivity increased for both groups in comparison to the Free session. As shown in figure 2.11, the average interactivity was found to be of  $0.94b \pm 0.05sd$  (median =  $0.93b$ ) across the participants of the Adaptive group which is higher than during the Free session ( $0.71b \pm 0.18b$ ). Similarly, the average interactivity was found to be of  $0.59b \pm 0.21sd$  (median =  $0.63b$ ) across the participants of the Non-Adaptive group, once again higher in comparison to the Free session ( $0.41b \pm 0.34b$ ). For both groups this difference between the two sessions may be explained by the fact that the participants are now more familiar with the system, each of them having previously performed 20 trials in the posture matching task at the start of the evaluation block of the Constrained session. It is thus possible that the participant fined tuned their strategy in order to more easily cross the decision boundary induced by the parameterization of the machine's decoder. In addition, compared to the Free session in which the participants of the Adaptive group coadapted with the system for only 5 trials while the participant of the Non-Adaptive group were not given this opportunity, in the Constrained session, both groups were given the opportunity to coadapt with the system (10 trials in total for the participant of the Adaptive group and 5 trials for the participants of the Non-Adaptive group).

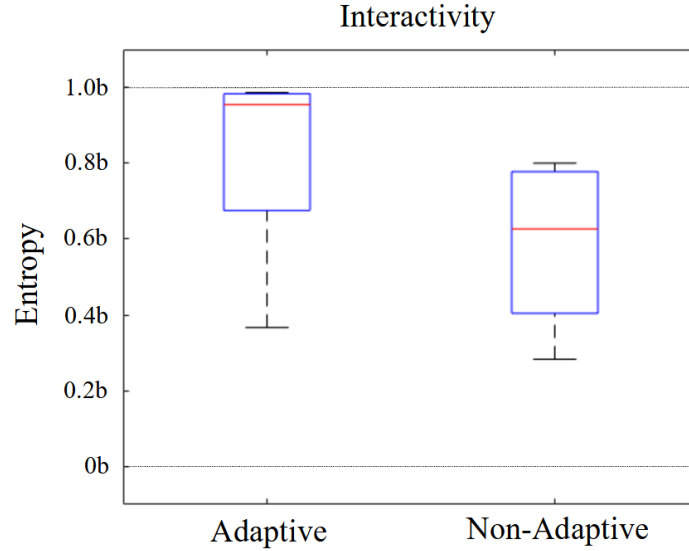


Figure 2.11: The interactivity (Shannon entropy of decoded symbols) during the evaluation block of the Constrained session. Values are shown in bits separately for the participant of the Adaptive and Non-Adaptive group.

Given the results obtained in the Free session, where a coadaptation with the system clearly benefited the interactivity, it is therefore probable that part of the difference in observed interactivity between the two sessions can be explained by their difference in term of the number of "coadapting trials" performed by each group.

### Relative Increase in Total Costs

With regard to the RITC metric, the trend observed in the Free session is reproduced in the Constrained session. Figure 2.12 shows the CDF of observed RITCs for all the trials performed in each of the 3 blocks making up the Constrained session.

The participants of the two groups performed similarly in the first block of the session, with the  $3 \times 5 = 15$  trials performed by the Adaptive group yielding an average RIIC of  $6.61 \pm 3.40\text{sd}$  (median = 5.10), while the  $4 \times 5 = 20$  trials performed by the participants of the Non-Adaptive group yielded an average of  $10.4 \pm 13.89\text{sd}$  (median = 5.72) decreasing to  $7.56 \pm 5.77\text{sd}$  (median = 5.32) after the elimination of one outlier beyond 4sd. This similar performance is to be expected given that the participants of both groups performed this block in the Adaptive condition (i.e with concurrent machine learning). On the other hand for the second block, only the participants of the Adaptive group were given the opportunity to "re-adapt" with the system under the second perturbation task, while the participants of the Non-Adaptive group kept the same decoder trained during the first block which was performed under the first perturbation task. A large difference in performance can be observed in this second block, with the participants in the Adaptive group yielding an average RITC of  $5.82 \pm 3.89\text{sd}$  (median = 4.31) while the participants in the Non-Adaptive group yielded an average of  $18.77 \pm 23.42\text{sd}$  (median = 10.24), decreasing to  $11.51 \pm 6.73\text{sd}$  (median = 9.36) after the elimination of two outliers (generated by two different participants) beyond 2sd.

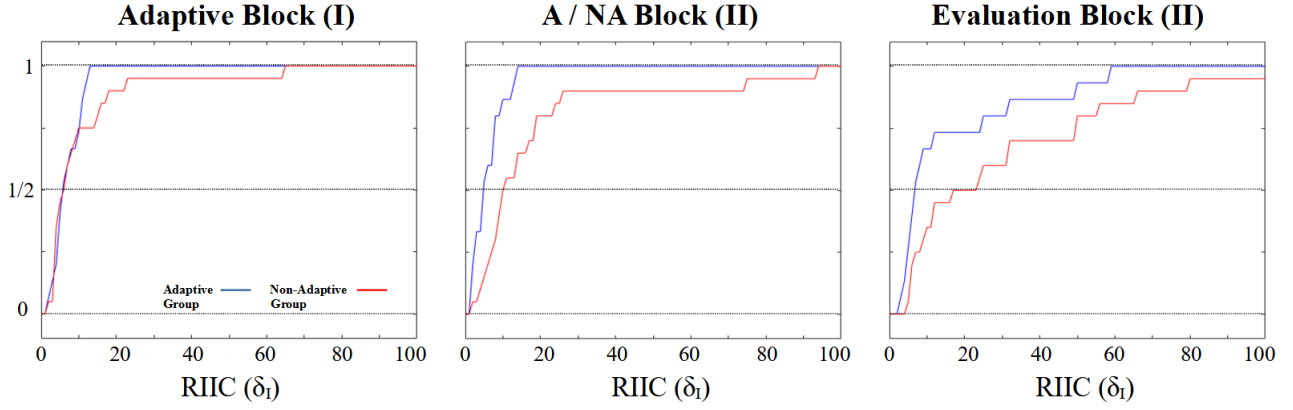


Figure 2.12: The cumulative density function of observed RIICs for the trials performed in each of the three blocks composing the second session. The data are plotted separately for the three participants of the Adaptive group in blue, and for the four participants of the Non-Adaptive group in red. Note that both groups performed the first block of the session (Adaptive Block (I) on the left) with concurrent machine learning and it is only by the second block (A/NA Block (II) in the center) that the conditions starts to differ between the two groups. Each block is labeled with the perturbation task the participant was asked to perform in parallel with the posture matching task, (I) for the hammer gripping and (II) for the balancing task (section 2.4.2). In the right-most graph (Evaluation Block (II)) the CDF for the Non-Adaptive group does not reach 100% because of an outlier at  $\delta_I = 258.9$ .

Finally, in the subsequent Evaluation block, this difference in performance decreased, with the participant of the Adaptive group yielding an average of  $16.46 \pm 17.59\text{sd}$  (median = 7.90) while the participant of the Non-Adaptive group yielded an average of  $38.82 \pm 56.46\text{sd}$  (median = 21.4) decreasing to  $27.23 \pm 23.08\text{sd}$  (median = 17.65) after the removal of one outlier around 4sd.

With regard to the statistical significance of these differences, a Wilcoxon ranksum test showed that participants of both groups performed similarly during the Adaptive block ( $p = 0.95, z = 0.05$  over all trials,  $p < 0.75, z = 0.31$  after outlier removal). For the second block (A/NA block), participants of the Adaptive group performed significantly better than those of the Non-Adaptive group ( $p < 0.005, z = -2.95$  over all trials,  $p < 0.008, z = -2.65$  after outlier removal). Finally, in the last Evaluation block, while the difference in performance reaches significance considering all trials ( $p < 0.04, z = -2.65$ ), the elimination of one outlier renders this difference barely significant ( $p = 0.056, z = -1.91$ ).

## 2.5.4 Performance Improvement

During our analysis of the results, we noted that the participant of the Adaptive group seemed to improve over the course of the Evaluation block in both of the sessions. To investigate this, we compared the participants' performance between the first three and last two trials they performed during an evaluation block. A participant "improvement index" is computed as :

$$\gamma = 1 - \frac{\bar{\delta}_{L2}}{\bar{\delta}_{F3}} \quad (2.18)$$

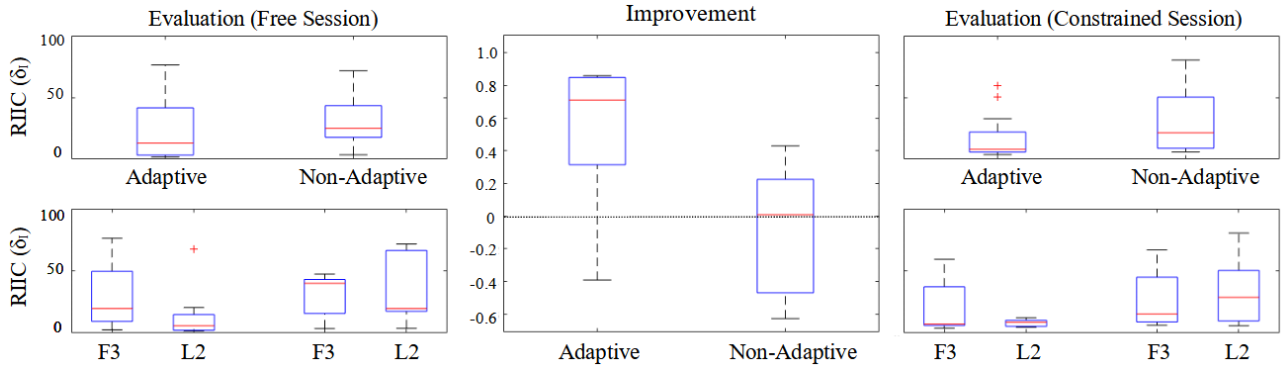


Figure 2.13: (Center) The improvement index of participants between the first three and last two trials of the evaluation block in both sessions. (Left/Right - Top) The distributions of observed RIIC for the trial performed by the participant of the Adaptive and Non-Adaptive group during the evaluation block of the Free session (on the left) and of the Constrained session (on the right). (Left/Right - Bottom) Those same distributions subdivided between the first three (F3) and last two (L2) trials performed during the evaluation block. All four plot share a same scale.

Where  $\bar{\delta}_{F3}$  and  $\bar{\delta}_{L2}$  correspond to the mean RIIC of the first three and last two trials performed by a participant respectively. An improvement index of one indicates that the participant performance in the last two trials was close to optimal with respect to the distance between the current posture and the target while a 0 or negative index indicate a similar or worse performance in the last two trials compared to the first three. Figure 2.13 (center) shows the distribution of improvement index of the two groups of participants, aggregating the index computed for the trials performed in the Evaluation block of both Free and Constrained sessions.

Considering both sessions, participants belonging to the Adaptive group presented a mean improvement index of  $0.52 \pm 0.42sd$  (median = 0.71), or  $0.68 \pm 0.24sd$  (median = 0.77) removing one outlier beyond 2sd (an interesting case which we further discuss below), while participants of the Non-Adaptive group yielded an average of  $-0.06 \pm 0.38$  (median = 0.01).

After confirming the normality of the data (Kolmogorov-Smirnoff and Shapiro-Wilk tests), a two sample t-test showed that, while this difference in improvement is barely not significant considering all participant of the Adaptive group [ $t(6) = 2.18$ ,  $p = 0.07$ ,  $d = 1.35$ ], removing one outlier and re-testing under a jackknife resampling (permutations) of the participants of the Non-Adaptive group yield significance in all cases [ $t(5) = 3.70 \pm 0.31$ ,  $p = 0.014 \pm 0.005$ ,  $d = 2.27 \pm 0.19$ ]. In addition, the average improvement index of the participants in the Non-Adaptive group was not significantly different from zero [ $t(6) = -0.41$ ,  $p = 0.69$ ], meaning that, on average, participants in the Non-Adaptive group did not improve in both sessions.

### Outlying participant in the Adaptive group

Examining the improvement index of the participant belonging to the Adaptive group, we noted that only one of them did not improve between the first three and the last two trials of the evaluation block. In fact, this participant performed significantly worse, yielding an improvement index of -0.40 during the evaluation block of the Free Session. To understand why this participant did not perform like the others, we noted that our posture matching task required the participant to control not one but two DoFs of the supernumerary finger.

We hypothesized that this result might therefore be explained by some large "interference" between the classifiers used for each DoF. Indeed, since we base the decoding of movement intention on the same signal  $\tilde{x}$  for both axes, it is possible that a coadaptation with the system yields decoder parameters that strongly correlate the symbols decoded between the two axes (i.e. the participant can only control the supernumerary finger along the "diagonals").

For each participant of the Adaptive group, we first computed the correlations between the estimated encoder parameters  $\hat{w}_{\mathcal{H}}$  in both axes. This correlations coefficient (Pearson's  $r$ ) is computed as :

$$\rho_{\mathcal{H}} = \frac{\hat{w}_{\mathcal{H}}^{\theta} \cdot \hat{w}_{\mathcal{H}}^{\phi}}{||\hat{w}_{\mathcal{H}}^{\theta}|| ||\hat{w}_{\mathcal{H}}^{\phi}||} \quad (2.19)$$

where  $\hat{w}_{\mathcal{H}}^{\theta}$  is the estimated encoder parameter vector for the flexion and  $\hat{w}_{\mathcal{H}}^{\phi}$  for the rotation. We also computed this correlation between the decoder parameters  $w_{\mathcal{M}}$  for both axes ( $\rho_{\mathcal{M}}$ ). Both of these correlation coefficient may however be explained by the correlation between supervisory labels during the trials performed in the previous adaptive condition. In the limit case where  $y_{\tau}^{\theta} = y_{\tau}^{\phi} \forall \tau$  during training it is clear that the decoders in both axes would converge to be equal (or opposite of each other when  $y_{\tau}^{\theta} = -y_{\tau}^{\phi} \forall \tau$ ), yielding a correlation of 1 in the limit. We therefore also computed the correlation coefficient between supervisory labels as :

$$\rho_L = r(y^{\theta}, y^{\phi}) = \frac{\sum_{\tau} (y_{\tau}^{\theta} - \bar{y}_{\tau}^{\theta})(y_{\tau}^{\phi} - \bar{y}_{\tau}^{\phi})}{\sqrt{\sum_{\tau} (y_{\tau}^{\theta} - \bar{y}_{\tau}^{\theta})^2 \sum_{\tau} (y_{\tau}^{\phi} - \bar{y}_{\tau}^{\phi})^2}} \quad (2.20)$$

where  $y_{\tau}^{\theta}$  is the supervisory label for the flexion and  $y_{\tau}^{\phi}$  for the rotation. Finally, although a participant might exhibit a high correlation between encoder/decoder parameters after a coadaptation with the system, this does not mean that they were unable to dissociate the control in both axes. For a simple counter-example, consider a hypothetical strategy where a participant distributes the control of both axes between two "encoding channel". Emitting for example the signal  $x = [w_1 \cdot y_{\theta} \ w_2 \cdot y_{\phi}]^T$  in response to a supervisory symbol  $[y_{\theta}, y_{\phi}]$ . Here, it is clear that, as per the procedure we present in 2.2.3 and assuming that the supervisory symbols are independently and identically distributed (i.i.d.), this participant would exhibit a correlation of 0 between both the encoder and decoder parameters in the two axes. Now consider that this participant alternatively "switches" the encoding channel used for each axes. That is, it first emits  $x_{\tau} = [w_1 \cdot y_{\theta} \ w_2 \cdot y_{\phi}]^T$ , then  $x_{\tau} = [w_2 \cdot y_{\theta} \ w_1 \cdot y_{\phi}]^T$ , followed by  $x_{\tau} = [w_1 \cdot y_{\theta} \ w_2 \cdot y_{\phi}]^T$  and so on. In this case, it is clear that this participant would yield a correlation of 1 between the encoding and decoding parameters (again assuming that supervisory symbols are i.i.d.) since the optimal least square estimate for both axes is  $\hat{w}_{\mathcal{H}}^{\theta} = \hat{w}_{\mathcal{H}}^{\phi} = \frac{1}{2}[(w_1 + w_2) (w_1 + w_2)]$ . For these two examples, however, it is quite intuitive that the correlation coefficient computed on the decoder output ( $\hat{y} = w_{\mathcal{M}}^T \tilde{x}$ ) in both axes is 0, meaning this participant controls the two axes independently of each other. We therefore also studied the correlations between decoder outputs, computing  $\rho_P$  as :

$$\rho_P = r(w_{\mathcal{M}}^{\theta} \cdot \tilde{x}, w_{\mathcal{M}}^{\phi} \cdot \tilde{x}) \quad (2.21)$$

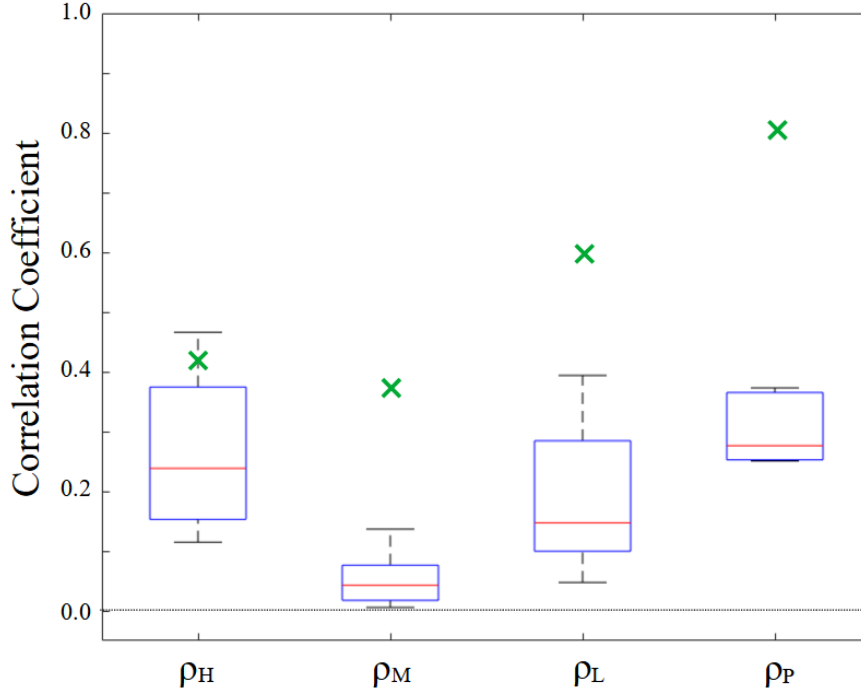


Figure 2.14: The distributions of (absolute) correlations coefficients (Pearson's  $r$ ; see the main text for details) computed for each participant belonging to the Adaptive group in both sessions. The coefficients associated to the outlying participant with regard to the improvement index are highlighted with a green cross.

The absolute value of these four correlation coefficients are shown in figure 2.14 in which we highlight the outlying participant with a green cross. As can be seen, this participant is a clear outlier with respect to the correlations observed between decoding hyperplanes ( $\rho_M = 0.32$  versus group mean of  $\rho_M = 0.05 \pm 0.13$ , median = 0.05) and decoder outputs ( $\rho_P = 0.83$  versus group mean of  $\rho_P = 0.26 \pm 0.11$ , median = 0.25). A result which might be explained by the larger correlation observed between the supervisory labels under which the training was performed ( $\rho_L = 0.62$  versus group mean of  $\rho_L = 0.19 \pm 0.12$ , median = 0.15). In other words, for this participant, the decoder parameters in both axes coupled such that they effectively lost a DoF (i.e. can only move along diagonals) under the coadaptive process.

## 2.6 Results - Adaptation of control strategies

Given the superior performance of the participants in the Adaptive group in both sessions, we now turn to the "strategies" they employed to perform the posture matching task. Here, we refer to the estimate  $\hat{w}_H$  resulting from a participant's coadaptation with the system as the participant's "encoding hyperplane". Note that, since the posture matching task required to control two DoFs, we have two encoding hyperplanes (flexion and rotation) for each participant. However, as these encoding hyperplanes are qualitatively similar to each other, they are combined and analyzed together here. For example, figure 2.15 shows the (flexion) encoding hyperplane of a participant after the Adaptive block in the Free session.

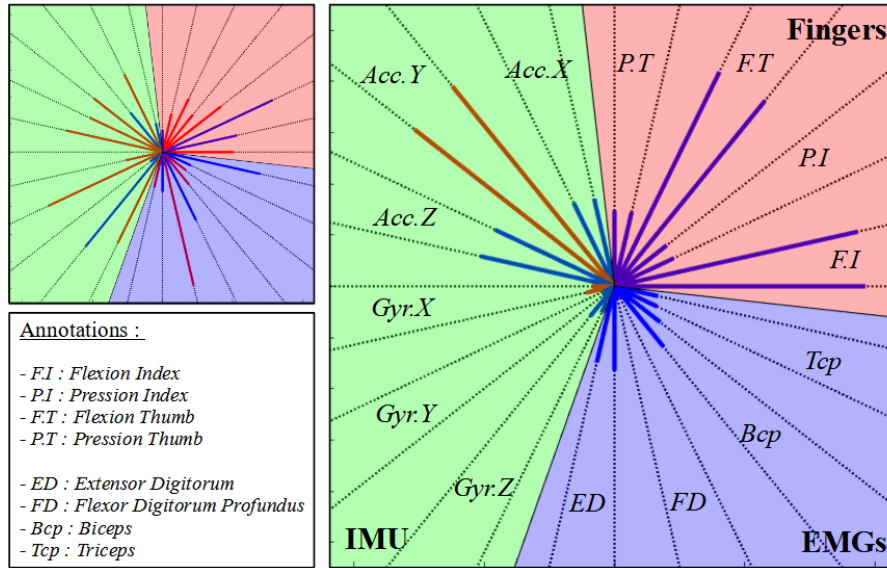


Figure 2.15: (Right) The flexion encoding hyperplane ( $\hat{w}_{\mathcal{H}}^{\theta}$ ) for a participant in the Adaptive group after the Adaptive block of the Free session. Each element of the  $28 \times 1$  parameter vector, hereafter referred to as a weight, is represented radially with a line whose length is proportional to its magnitude and whose color reflects its sign (blue for negative and red for positive). The graph is divided into three regions whose color is based on the source of the data corresponding to the weight: red for the sensors attached to the fingers, green for the accelerometer and gyroscope of the IMU, blue for the four muscles whose EMGs were recorded (see section 2.2.2 for details). (Top left) The initial state of the encoding hyperplane, before coadaptation, after its random initialization during the calibration. (Bottom left) The labels used to annotate the sensor corresponding to each weight. Note that, since the frames are constructed by concatenating two points per sensor, the weights always come in pairs (often with similar values).

### 2.6.1 Encoding strategies

In this section, we speak of a participant's encoding hyperplane as the result of some, conscious or not, "strategy" they employed to control of supernumerary finger. Of course, this hyperplane, obtained via the least square estimator in 2.6, simply reflects the correlations between the participant "behavior", as measured by the various sensors distributed along their arm, and the supervisory symbol. Nevertheless, most participants did report that they had some control strategy. For example, one of the participants, when asked about the system after the experiment, reported that he noticed a reaction of the supernumerary finger to a motion of his natural fingers and tried to base his control on this relationship. A participant's encoding hyperplane may therefore be interpreted as reflecting, to some extent, of their control strategy.

Figure 2.16 illustrates the encoding hyperplane resulting from coadaptation with the system in the three Adaptive blocks of the experiment. One block in the Free session (4 "coadapting participants") and two in the Constrained session with the first performed under perturbation task I (hammer grasping; 7 coadapting participants) and the second under perturbation task II (ball balancing; 3 coadapting participants).

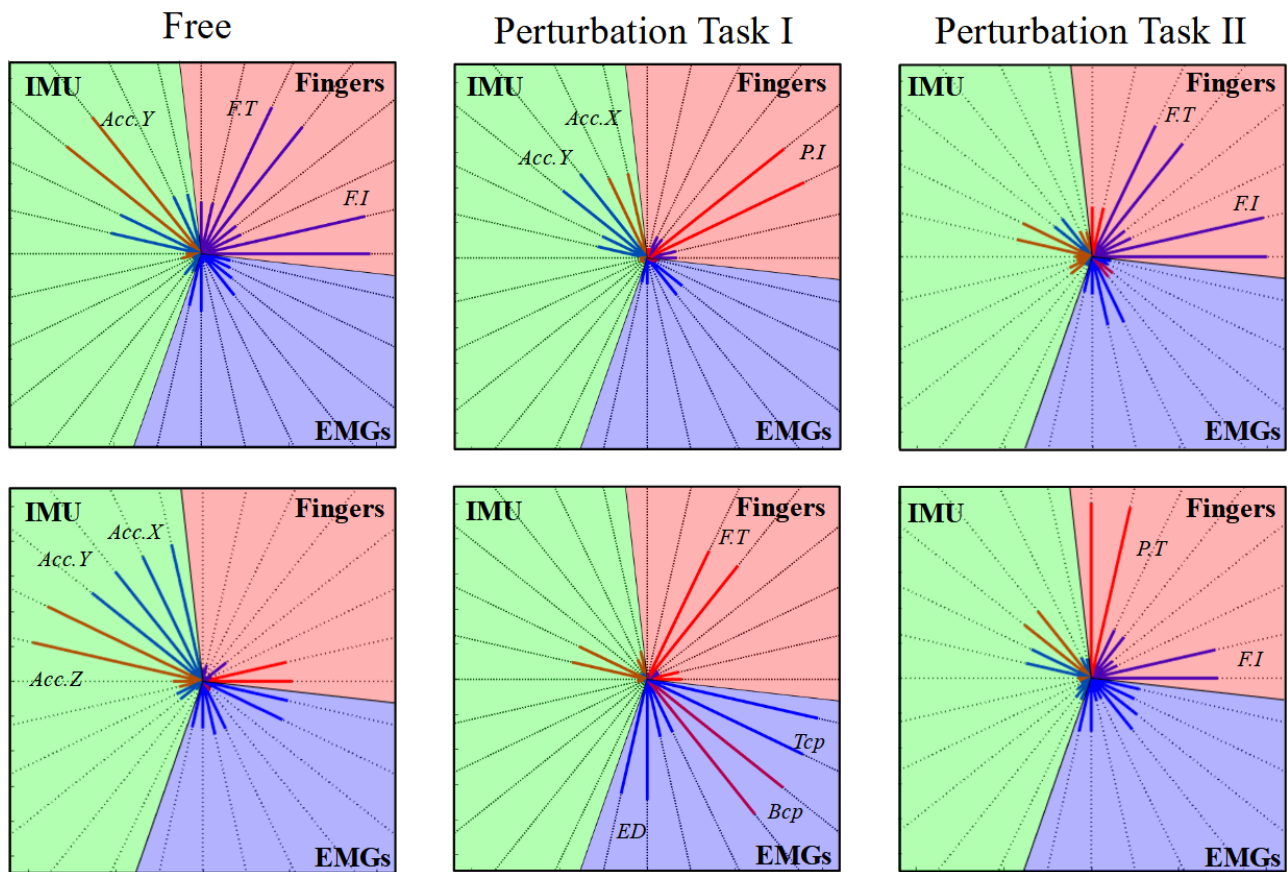


Figure 2.16: A selection of participants' encoding hyperplanes illustrating the strategies they used to control the supernumerary finger in the three Adaptive blocks of the experiment. The Adaptive block of the Free session on the left, the first Adaptive block of the Constrained session in the center (performed under the perturbation task I) and the second Adaptive block of the Constrained session (performed under the perturbation task II) on the right.

Examining the participant's encoding hyperplane after the Adaptive block in the Free session (2.16, left), two strategies seem to dominate. Under the first strategy, the participants controlled the motion of the supernumerary finger via a flexion/extension of their fingers (thumb and index). This can be seen by the large weight associated to the data coming from the flexion sensors attached to those fingers (2.16, top-left). Another strategy they employed was to map a desired motion of the finger to a movement of their right arm as indicated by the large weight associated to the data coming from the accelerometer attached to the base of their forearm (2.16, bottom-left).

The strategies employed are quite different under the first perturbation task, where the participant had to performed the posture matching task while holding a hammer. First of all, we see the emergence of a "pressure-based" strategy that was not observed during the Free session. Such strategy is illustrated in the top-center graph of figure 2.16, where we observe a large response for the pressure sensor attached to the tip of the participant index. Interestingly, among the participants who used this strategy, only the pressure sensor attached to the index was selected, and rarely the thumb. This may be explained by the way a hammer is held, where the index fingertip is naturally in contact with the handle whereas the thumb fingertip is not. Additionally, this "grasping perturbation" might also be introducing an asymmetry between the mobility of the thumb and index. Reviewing the participant encoding hyperplanes, we see that, while some of them yielded a large response

for the flexion of the thumb (figure 2.16, bottom-center), none did the same for the flexion of the index (figure 2.18, left). Additionally, as the weight of the hammer increases the force required to perform movement with the arm, we see the appearance of "EMG-based" strategies (figure 2.16, bottom-center) more consistently than during the Free session, where only few large response for the EMGs (for example figure 2.16, bottom-left) were observed.

Finally, for the second perturbation task, where the participants had to perform the posture matching task while balancing a ball on a plate they held with both hands, we mainly observed finger-based strategies. Similarly to the Free session, a large response for the flexion of both index and thumb are commonly found among the encoding hyperplanes (2.16, top-right) but a few participants also employed "mixed strategies" involving flexion of a finger, usually the index, and pressure for the other, usually the thumb, as is shown in figure 2.16, bottom right. A notable difference between the results obtained under the second perturbation task and the result reviewed above is the disappearance of arm movement-based strategies for most participants. This may be explained by the balancing task requiring the arm to be relatively stable and therefore interfering with arm movement-based control of the robotic finger.

## 2.6.2 Adaptation of the control strategies to the perturbations

We base the comparison of the strategies used by participants in the three conditions under which they performed the posture matching task (Free, perturbation task I, perturbation task II) on each channel contribution to the effect size of symbol on signal.

Assuming, in accordance with our theoretical model of communication (2.2.3), that the operator emits  $\tilde{x} = w_{\mathcal{H}} \cdot y + \epsilon$  in response to a supervisory label  $y$  (where  $\epsilon \sim \mathcal{N}(0, \Sigma)$ ), this effect size is a non-linear function of the magnitude and orientation of a vector between the mean of two Gaussian distributions ( $P(\tilde{x}|y = 1) = \mathcal{N}(w_{\mathcal{H}}, \Sigma)$ ,  $P(\tilde{x}|y = -1) = \mathcal{N}(-w_{\mathcal{H}}, \Sigma)$ ). If we additionally assume that the noise covariance is isotropic (i.e.  $\Sigma = \sigma I_{28}$ ), the effect size only depends on the magnitude of the vector of encoding parameters. We thus compute the "relative contribution to effect size"  $c_i$  of a channel  $i$  with associated weight  $(w_{\mathcal{H}})_i$  as

$$c_i = 1 - \frac{\|w'_{\mathcal{H}}\|}{\|w_{\mathcal{H}}\|} \quad (2.22)$$

where  $\|\cdot\|$  is the vector norm and  $w'_{\mathcal{H}}$  is the encoding hyperplane where we set  $(w_{\mathcal{H}})_i = 0$ . In other words, we are comparing the magnitude of the encoding vector with and without considering channel  $i$  and consider it channel  $i$ 's contribution to the effect size. Since we have two weights per sensors, we first average them two before computing the contribution of a sensor. Note that, due to the non-linear nature of the vector norm, summing the contribution of all channels to the effect size does not yield 1. Assuming an equal contribution of each channel yields a contribution of  $c_i = 1 - \sqrt{13}/\sqrt{14} = 3.6\%$  per channel.

For example, figure 2.17 shows the distribution of contribution to effect size for each channel across the participants of the Adaptive group (computed on the machine's estimate  $\hat{w}_{\mathcal{H}}$ ) during the Free session). This distribution aggregates the contributions computed on both the flexion ( $\hat{w}_{\mathcal{H}}^{\theta}$ ) and rotation ( $\hat{w}_{\mathcal{H}}^{\phi}$ ) encoding hyperplanes, meaning we have here  $4 \times 2 = 8$  "contribution samples" per channel.

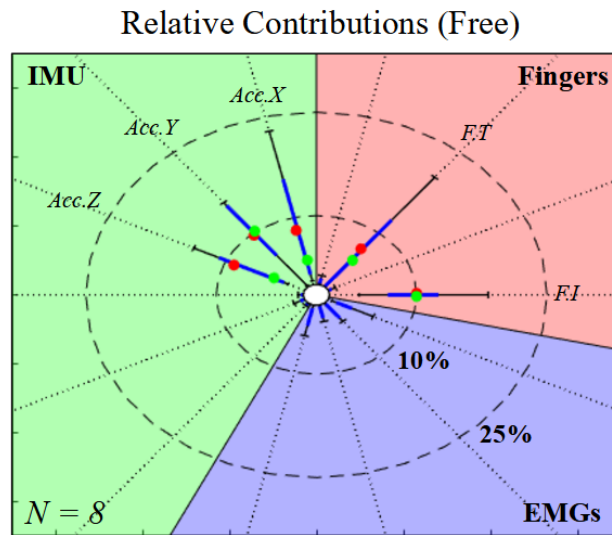


Figure 2.17: The distribution of relative contributions to the effect size of each channel in the Free session across participants and DoF (flexion and rotation). This graph (and the one presented thereafter) is constructed as a radial boxplot with the inter-quartile range in blue and green/red dots marking the median and mean contribution, respectively. For clarity, these are only shown for the channels whose third quartile contribution is above 5%. The central white patch marks the 0% contribution while outer dashed circles marks the 10% and 25% contributions respectively. The number of data samples used to construct the graph is indicated in the bottom left.

This distribution, obtained from the encoding hyperplane collected during the Free session, was then compared with the distributions obtained in the Constrained session to study the adaptation of the participant strategies to the perturbations introduced by the secondary tasks.

### Adaptation to perturbation task I

Starting with the first perturbation task (hammer grasping), figure 2.18 shows the distribution of contributions for each channel during the Free session (on the left; also shown in figure 2.17) and under the first perturbation task (on the right). Note that, since all participants performed the first perturbation task while coadapting with the machine, the distribution on the right contains  $7 \times 2 = 14$  "contribution samples" per channels.

As noted in the previous section, performing the posture matching task under the first perturbation task induced an adaptation from the participants. In particular, the fact that the fingers are comparatively less mobile than during the Free session means that finger movement-based strategies are now less viable. This effect can be observed for the index, which "transitioned" from a flexion-based strategy (high contribution of *FI* in 2.18 - left) to a pressure-based strategy (high contribution of *PI* in 2.18 - right) but not for the thumb whose contribution in flexion and pressure (*FT* and *PT*) remains similar between the two blocks. The higher force required to move the arm under the first perturbation task, in comparison to the Free session, also increased the system reliance on the participants' EMG. While the contribution of EMG sensors was relatively marginal after the coadaptation during the Free session (2.18 - left), such that EMG channels were never found among the top 3 contributors

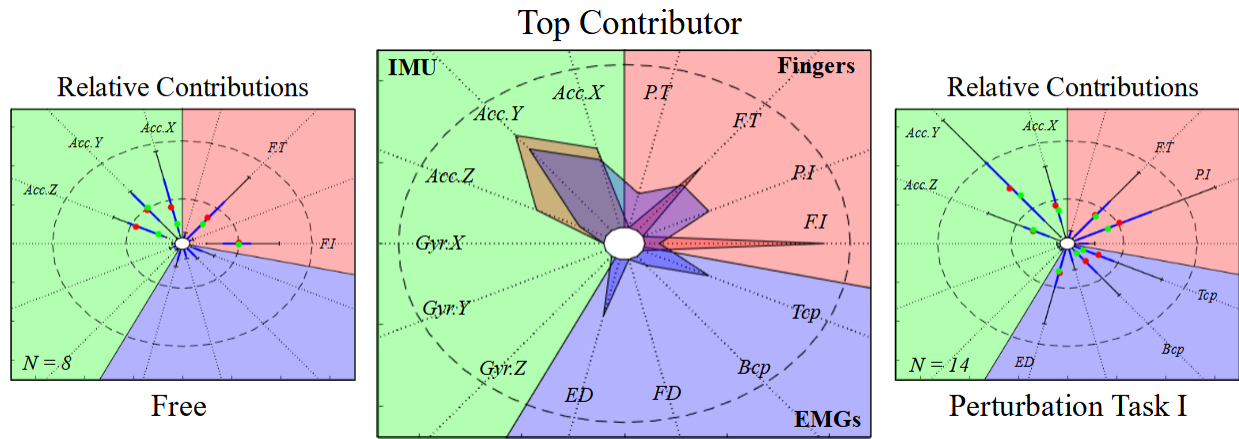


Figure 2.18: The distribution of contribution to effect size for each channel in the first session (left) and under the first perturbation task (right). The center graph shows the number of times a given channel is found among the top 3 contributors to effect size (red for the Free session and blue for the first perturbation task). The outer dashed circle marks 1, meaning a channel is always found among the top 3 contributors in both the flexion and rotation encoding hyperplane while the inner white patch marks the 0, meaning a channel is never found among the top 3 contributors. Note that, since the weight for a given channel come in pair, we first average them so they are not counted twice.

for any of the participant in either the flexion or rotation (2.18 - center), this contribution largely increased under the first perturbation task.

It should however be noted that the contributions of the EMG channels are relatively inconsistent across participants, as can be seen by the low median and mean contribution for EMG channels (2.18 - right). This inconsistent contribution of EMGs may be interpreted as the participants not specifically integrating their muscle activity into their control strategy. Under this hypothesis, the observed contribution of EMG could be the result of the participants using an arm movement-based strategy, which induced a correlation between the electromyographic activity and supervisory labels for some but not all participants. Finally, an interesting effect of the first perturbation task on arm movement-based strategies is the increased contribution of the accelerometer in the Y axis. In comparison to the Free session, where the contribution of the accelerometer was roughly evenly distributed between the three axes (2.18 - left), most of this contribution concentrates in the Y axis under the first perturbation task (2.18 - right). This effect might be explained by the fact that, given the position of the IMU on the arm of the participant, the Y axis was aligned with the gravity vector. Given the relatively slow paced movement performed by the participants in the posture matching task, most of the information contained in the signals measured by the IMU is its orientation relative to this vector. It is therefore possible that holding a hammer prompted the participant to perform arm movement with a reduced range of orientation in order to maintain the hammer upward, thus reducing the available information on the X and Z axes which would be predominantly orthogonal to the gravity vector under such movement.

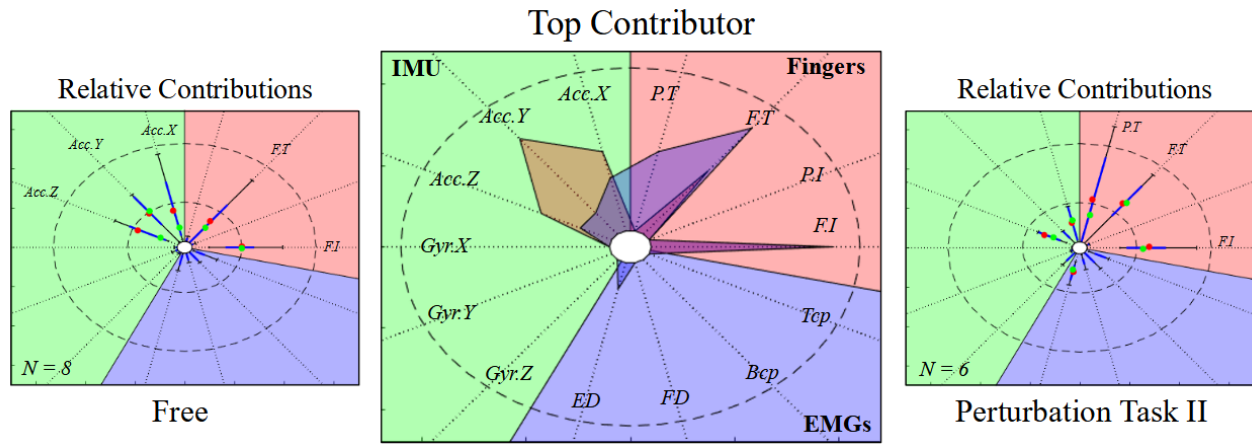


Figure 2.19: The distribution of contribution to effect size for each channel in the first session (left) and under the second perturbation task (right). The center graph shows the number of times a given channel is found among the top 3 contributor to effect size (red for the Free session and blue for the second perturbation task).

### Adaptation to perturbation task II

Under the second perturbation task, participants had to balance a small ball on a plate they held with both hands. Given such perturbation, it is clear that arm movement-based strategies are now less viable than during both the Free session and under the first perturbation task (since arm movement are now required to perform the balancing task and therefore cannot be used freely to control the supernumerary finger). Consequently, we see a sharp decrease in the contribution of the IMU under this constraint (2.19 - right) in comparison to its contribution during the Free session (2.19 - left), although IMU sensors can still be found among the top three contributor for some participants (2.19 - center). For this perturbation, it appears that most participants chose to rely on finger-based strategies, as can be seen by the relatively large and consistent contribution of the sensors attached to the participant fingers, with only the flexion for the index and either the pressure or flexion for the thumb.

## 2.7 Discussion

Many studies on the coadaptation between human and machines have focused on the "negotiation" aspect of coadaptation. That is, the generation of a control scheme through a negotiation between two adaptive agents [Müller et al., 2017, Ehrlich and Cheng, 2018]. Compared to previous studies, we focused here on the "adaptive" aspect of this process. Firstly, by providing the human-machine couple with a large number of possible communication channels and leaving them to decide which of them would be preferable for the task. And, secondly, by introducing perturbations, in the form of secondary tasks, to study how such constraints would lead to an adaptation of their behavior in the primary task.

### 2.7.1 Optimal Mappings

Given the variety of communication channels provided to the human machine couple in this experiment, it is clear that not all of them are equally "usable" to perform the posture matching task. On the machine side, noisy channels, such as the EMG here for example, are less reliable to decode movement intentions from the participant. As we have seen in

this experiment, these channels are fairly systematically pruned out during the coadaptive process, with the exception of the coadaptation performed under the first perturbation task, which can be seen as a contextual increase of the EMG signal-to-noise ratio (SNR).

On the human side, the ease of use associated with a communication channel can be decomposed in two terms relating to the two transformations linking the supernumerary finger's movement to the actions of the participant. Although the model on which we rely assumes that the linear transformation  $x = f_{\mathcal{H}}(y) = w_{\mathcal{H}} \cdot y$  mapping supervisory symbols to emitted signals is known by  $\mathcal{H}$ . This assumption does not hold in this experiment. For the signals measured by the gyroscope integrated in the IMU attached to their arm for example, the transformation mapping an arm movement to a gyrometric signal is highly non-linear and arguably difficult to learn from the participant perspective. Numerous studies in cognitive neuroscience have shown that humans learning tend to converge to simple representation of the sensory data (the Occam's razor). This "Simplicity Principle" [Chater, 1999, Chater and Vitányi, 2003, Feldman, 2016], ubiquitous in both experimental [Leeuwenberg, 1971, Restle, 1979, Little and Shiffrin, 2009] and computational [Valiant, 1984, Feldman, 2009b, Gershman and Niv, 2013] neuroscience, may explain why gyrometric information were consistently pruned out of the communication scheme by the coadaptive process in favor of sensors relating to simpler maps, such as the fingers' movement. Another related explanation for this phenomenon is the fact that the gyrometric sensors were the only purely dynamic sensors used here (i.e. only measuring changes in the IMU orientation and not its orientation per se), which, given the position of the IMU, meant that participant could not sustain prolonged emission of a given signal before reaching joint constraints. Of course, in this experiment, the participants had no feedback on the signals they emitted and could only observe the movement of the machine resulting from their classification. Nevertheless, a non-linear map between motor actions and emitted signals translates into an equally non-linear relationship between motor actions and feedback.

Another important aspect of the negotiation between  $\mathcal{H}$  and  $\mathcal{M}$  for the selection of a control method are the extrinsic costs associated to the generation of a given signal. In this experiment, two such costs can be taken in consideration. Firstly, signals can be ranked according to the energetic costs, or effort, associated to their generation by  $\mathcal{H}$ . It is known that motor control in humans follows from a close to optimal minimization of error and effort [Sparrow and Newell, 1994, Todorov and Jordan, 2002, Shadmehr et al., 2016]. In our case, it is arguably the modulation of the EMG that requires the highest effort from the participant as this modulation must be sufficient to overcome the noise affecting the signal to achieve a high SNR. Conversely, slow-paced finger and arm movements may be generated with less effort. Beyond noise, this may also explain why accelerometric data and the natural finger's flexion have been primarily selected over the EMG. The effect of effort minimization on the coadaptive process can also be appreciated when comparing the strategies employed under the first perturbation task to the strategies employed in the Free session (figure 2.18). Given the weight of the hammer we asked the participant to hold and the distribution of this weight away from the grasping point, a significant torque was induced in proportion to the angle between the hammer handle and the gravity vector. This may explain why accelerometric data in the Y direction have a higher contribution in the first perturbation task. Since the "effort manifold" over arm movement is not isotropic when the participant are holding a hammer, participant may have preferred movement with restricted reorientation of the arm, thus inducing a larger contribution in the Y axis which was "naturally" aligned with the gravity vector in our experiment. Of course, given the low number of participants, this

effect can also be explained by the inherent variability in the outcome of the coadaptation process we are studying here.

The second "type" of extrinsic cost to be considered are the one introduced by the secondary tasks we asked the participant to perform in parallel to the posture matching task. In particular for the second perturbation task, where the participants were asked to balance a ball disposed a small plate they held with both hands. The sharp decrease in contribution of accelerometric data across all three axes in this case (figure 2.19) can be explained by an incompatibility between arm-movement based control strategy of the supernumerary finger and the balancing task, thus pushing participant towards a finger-based control strategy that does not interfere with the goal of the balancing task. This effect can also be observed for the first perturbation task, where the "grasp objective" solicited the participants' natural fingers that were therefore not available to control the robotic finger.

### 2.7.2 Context dependent adaptation

Although most of our everyday interactions with machines are performed under relatively consistent context, this is not the case for a number of interactive systems. Rehabilitative and/or augmentative systems, in particular those aiming to accompany the user in its daily life over an extended period of time, are intended to be usable in a wide variety of contexts. In the literature, most of the control schemes studied will inevitably be sub-optimal in some well chosen context of use. In the case of supernumerary robotic finger for example, feet movement-based control [Cunningham et al., 2018] is inapplicable when the user is walking. Similarly, control schemes based on a wearable interface operated by the contra-lateral hand [Ort et al., 2015, Hussain et al., 2017a] will interfere with tasks that require the use of both hands. In this respect, concurrent coadaptive systems offer a mean of negotiating, online, a control method adapted to the context of use. Multi-modality is a key aspect of this adaptation, as sampling the user's activity over various body structure (feet, hands, etc...) increase the probability of finding a communication channel available for the control. Such behavior can be observed in our experiment between arm-based and finger-based control. As is shown in figure 2.18, when the coadaptive process takes place in an unconstrained context during the Free session, the participants rely on both arm and finger movements to perform the control. Conversely, in the subsequent Constrained session, a perturbation affecting one of them - the finger movement under the first perturbation task and the arm movement under the second - induced an adaptation of the control method such as the system converged to rely more on the body structures that were left free by the context of use. This is especially apparent in the second perturbation task, where arm movement-based control disappeared in favor of finger movement-based control.

In fact, the participant of the Adaptive group performed qualitatively better during the Evaluation block of the Constrained session, with an average RIIC of  $16.46 \pm 17.59\text{sd}$  (median = 7.90), than during the Free session, where the average RIIC was  $23.41 \pm 25.3\text{sd}$  (median = 12.87), even though the Evaluation block of the Constrained session included an additional difficulty since the participants were required to balance the ball. Conversely, the participants belonging to the Non-Adaptive group performed similarly between the two blocks, with an average of  $29.94 \pm 22.46\text{sd}$  (median = 24.92) during the Free session and  $27.23 \pm 23.08\text{sd}$  (median = 17.65) during the Constrained session. One might interpret this result as the added difficulty in the second session being compensated by the participants in both groups familiarizing with the system over the ten trials they performed during the

first session, thus explaining why the participants in the Non-Adaptive group managed to perform similarly between the two sessions. The increase in performance observed with the participants in the Adaptive group between the Free and Constrained session might then be explained by the coadaptive process yielding a control scheme that is "compatible" with the perturbation, thereby reducing the added difficulty induced by the perturbation task. In turn, this enabled those participants to draw on the experience gained in the first session to better their performance in the second.

### 2.7.3 Adaptive dimensionality reduction of the control space

Beyond extrinsic cost, such as error or effort, control also entails what may be referred as "cognitive" or "intrinsic cost" for the user, this cost being relatable to the dimensionality of the control space (ie it is usually more difficult for a user to control a large number of DoFs than a single one). In the literature, this "intrinsic cost of control" is commonly captured via a divergence metric (Kullback-Leibler notably in the setup of E.Todorov [Todorov, 2006, Guan et al., 2014, Busic and Meyn, 2018]) between prior, also referred to as autonomous or passive, and posterior, value-driven, behavioral dynamics. In human machine interfaces, the cost of control may also be seen as the "behavioral freedom" offered to the user when interacting with a system. In our experiment, the participant had to control two independent DoFs. The corresponding (minimum) intrinsic cost of control may therefore be computed as the divergence between a uniform prior distribution over the four possible couples of symbols (mixing UP and DOWN in the two axes) and a posterior distribution where probability mass concentrate on a specific couple. An interesting aspect of the coadaptive system we present here is that, due to the use of two independent decoders for the decoding of movement intentions, the machine is adapting both to the signals emitted by the user but also to the joint prior distribution over the supervisory symbols in both axes. In the case where supervisory symbols are completely correlated in the two axes (i.e. the task only require the participant to transmit matched, either  $\{1, 1\}$  or  $\{-1, -1\}$ , or mismatched,  $\{1, -1\}$  or  $\{-1, 1\}$ , couples of symbols to the machine) it is clear that, in the limit, one would obtain a same set of parameters for the decoders in both axes (up to a sign in the mismatched case). In such case, the coadaptive process yields a control scheme where the user effectively lost a DoF. In our experiment, this effect can be observed for the outlying participant we highlighted with regard to the improvement index (discussed in 2.5.4).

Of course, in our case, the experiment was designed so supervisory symbols in each axis would be as independent of each other as possible and therefore this effect negatively impacted the performance of this participant in the subsequent evaluation block. Nevertheless, we consider this to be an important aspect of the coadaptive process since this adaptive reduction in the dimensionality of the control space, which can be seen as the emergence of a high-level control strategy [Prattichizzo et al., 2021], is an adaptive equivalent to a fundamental design process in human machine interfacing that aims to minimize the intrinsic cost of control.

### 2.7.4 Conclusion

The results we presented here are preliminary and stronger evidence is necessary to reach some conclusion on the potential of coadaptation applied to augmentative technologies, and the extent to which the added plasticity can be exploited to improve the integration of such systems in their users' daily life. However, our findings highlight what we believe are the

main reasons why concurrent coadaptation may be considered as one of the most promising venue of research for the interfacing between human and machines and the key role multi-modality will have in this regard. To our knowledge, the interaction between these two aspects has not been explored in this context before. An important limitation of the result we present here is that the interaction between the participant and the machine in our experiment was supervised. While the simple nature of the encoder we used as model for the participant behavior meant that few examples were necessary to make the system "usable" (in a very broad sense), applying such supervised procedure to complex models incorporating millions of parameters or more, such as deep neural networks, is quite unrealistic, especially if these models need to be trained for each context of use. An interesting challenge for future research on the application of concurrent coadaptation for augmentative technologies may therefore reside in the provision of methods allowing to dispense of such supervision (for example using error related potentials [Ehrlich and Cheng, 2018]).



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## Multi-modal Interfaces and Sensory Response Compatibility

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### Preamble

In the previous chapter, we studied an example of "machine learning-based" coadaptation applied in the context of supernumerary limb control. However, our approach was rather naive. Indeed, we did not postulate on the way the participant in our study might prefer, or be able, given the constraints we imposed, to control the robotic finger. As such, we equipped them with a variety of sensors and initialized the mapping between these sensors and the movement of the robotic finger in a random fashion, letting them "negotiate" with the system on an optimal parameterization of the control interface. Given the results we obtained, for example in term of intention decoding accuracy (64% on average during the Free period), or in terms of the optimality of the trajectories generated (23.41 times the optimal cost-to-go, or RIIC, on average in the Free period), it can be argued that this naive approach would be difficult to apply in realistic settings. "Naive" here can also be understood as referring to an uninformative prior in the sense that we assumed that all possible mappings between the participant's actions, measured by the sensors distributed on their arm, and a movement of the supernumerary finger were equally likely to be optimal. Of course, if we humans are all "unique" in a certain sense, we are also all similar in the same sense. A key aspect to the process of fitting an interactive system to its user is therefore to endow this system with some "good", or informative, prior about the population its users are sampled from. Identifying commonalities between all potential users of an interactive system is therefore a key aspect of interactive systems design and thus results from behavioral psychology and cognitive neuroscience are central to the process of endowing the machine with "good" priors about its users. In this chapter, we contribute to these broad fields by studying in more detail the behavior of humans in a choice-reaction task designed to mimic the type of sensorimotor responses expected from users of a wide variety of interactive systems, in which spatialized stimulus (i.e. differing by their position in space) are to be answered to with spatialized responses). In particular, we interest ourselves in the potential of tactile interfaces to enhance the user performance in such sensorimotor tasks.

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### 3.1 Introduction

Humans interactions with their environment are modulated by the various stimuli we receive in multiple sensory modalities. This is particularly true in the case of human machine interfaces (HMI) which commonly use a combination of visual, tactile and auditory stimuli to convey information to their users. For example, many cars today are equipped with a parking assistance system which typically use a combination of visual and auditory stimuli to indicate the proximity of an obstacle on a particular side of the car. While auditory and visual modalities are the most common in human machine interactions, recent years have seen a rapid development of tactile interfaces in a variety of applications [van Erp et al., 2003, Cheng et al., 2012, McGrath et al., 2014, Brock et al., 2014, Clemente et al., 2015, Zhao et al., 2019, Zhang et al., 2020, Grushko et al., 2021, Simons et al., 2021]. For example, [Fellah and Guiatni, 2018] present a tactile flight envelop display where an aircraft's flight parameters (which are also visually available in the cockpit) are fed back to the pilot via tactile stimuli (vibrations).

It is well known that the speed and accuracy with which humans can react to various stimuli not only dependent on their physical properties [Chan and Chan, 2005, Harrar and Harris, 2005, Shelton and Kumar, 2010] and the nature of the information we extract from them [Stein and Peelen, 2015], but also on the action required in response, with certain 'compatible' stimuli-response couples enabling better performance than other 'incompatible' ones [Simon and Wolf, 1963, Proctor and Reeve, 1989, Kornblum et al., 1990, Hasbroucq, 1987, Hasbroucq and Guiard, 1992, Wang et al., 2016]. However, while the basic principles underlying the Stimulus Response Compatibility (SRC) phenomenon are nowadays relatively well understood from an epistemological perspective [Kornblum et al., 1990], it is known that SRC are, in vivo, determined by a broad variety of factors [Alluisi and Warm, 1990, Cespón et al., 2020]. As such, the extent to which the wealth of results obtained over the year in various experiments on SRC ([Proctor and Reeve, 1989, Kornblum et al., 1990, Alluisi and Warm, 1990] for a review) can be generalized beyond the laboratory to provide design guidance for the design of HMI is still unclear. In particular, while the effects of spatial SRC, where the compatibility of stimulus and response is determined by their position in space, have been demonstrated in both the visual [Simon and Wolf, 1963] and tactile [Hasbroucq and Guiard, 1992] modality, it is still unclear what can be expected from the point of view of user performance in a HMI where both modalities would be available to convey spatial information.

In this study, we investigate this issue in more details through a Choice Reaction Task (CRT) where the participant are instructed to react to stimulus presented in the visual, tactile and both visual and tactile modality in a "spatially compatible" fashion with regard to SRC effects.

#### 3.1.1 The Spatial Stimulus-Response Compatibility

In 1953, a study by Fitts and Seeger [Fitts and Seeger, 1953] showed that, in a CRT where participants had to respond to a visual stimulus by moving a stylus towards it, participants responded faster and more accurately for visual stimuli presented in the same visual hemifield as the responding hand, an effect subsequently termed the (spatial) "Stimulus-Response Compatibility" (SRC). Although, in this experiment, the position of the stimulus in space is relevant to the task (since the participant respond by moving towards it), a later

study by J.R. Simon [Simon and Wolf, 1963] showed that this effect could still be observed when the position of the stimulus is irrelevant. This particular type of spatial SRC, induced by the position of a stimulus that is irrelevant to the decision, was termed the "Simon effect". Since these studies, spatial SRC and the Simon effect have also been demonstrated for auditive [Simon and Rudell, 1967, D'Ascenzo et al., 2018, Wang et al., 2021] as well as tactile [Hasbroucq, 1987, Hasbroucq and Guiard, 1992, Medina et al., 2014] stimuli. In the case of visual stimuli, the compatibility between stimuli and responses is determined by their positions, usually left or right, relative to a point of reference in visual space [Wallace, 1971, Umiltà and Liotti, 1987, Stoffer, 1991, Lamberts et al., 1992]. For example in [Simon and Wolf, 1963], the participant responds faster to a visual cue when this cue is presented in the same (compatible) visual hemifield as the responding hand rather than the opposite (incompatible). On the other hand, studies on spatial SRC effects with tactile stimuli showed that the compatibility between stimuli and responses is determined by their relative distribution between somatosensory hemispace (i.e. in relation to the body midline; [Hasbroucq and Guiard, 1992, Medina et al., 2014]). However, most of these previous studies have focused on monomodal stimuli ([Fitts and Seeger, 1953, Simon and Wolf, 1963, Simon and Rudell, 1967, Simon et al., 1981, Hasbroucq, 1987, Umiltà and Liotti, 1987, Worringham and Kerr, 2000, D'Ascenzo et al., 2018, Mahani et al., 2019, Wang et al., 2021], see also [Proctor and Reeve, 1989, Kornblum et al., 1990] for a review), in which only one sensory modality contains task-relevant information, while a secondary sensory modality, if present, is used only as a distractor. Furthermore, these studies traditionally focus their attention on the difference in performance between compatible and incompatible stimuli-response couples. In contrast to these previous studies, we are here interested to understand how multimodality influence a participant performance (as measured by reaction time) in a task where participants are instructed to answer to spatialized stimuli and responses in a compatible fashion. This type of scenario is typical of HMI, which are generally designed to ensure that stimuli and responses are compatible to maximize user performance.

## 3.2 Methods and Hypotheses

Our experimental design was motivated by the example of a car parking assistance system. The participants sat comfortably in a chair in front of a computer screen (32 inch, 50 cm away). On this screen were represented four colored rectangles which, initially blue, turned red to indicate a commanding stimulus. These four rectangles were distributed in a square, each representing one of four possible directions : North-West (NW), North-East (NE), South-East (SE) and South-West (SW). In addition to these visual stimuli, the participants were also equipped with four vibrating motors (VC0825B002F - 225Hz), used to deliver the tactile stimuli, attached to their elbows and knees (figure 3.1 - A). The vibrating motors on the participants' left and right knees represented the NW and NE directions respectively, while the motors attached to the elbows represented the SW and SE directions. Note that, given the position of the participant, tactile stimuli were thus distributed between the front (knees) and back (elbows), while the visual stimuli were arranged vertically (top and bottom). It has however been shown that spatial SRC does not differentiate between depth and height [Zhong et al., 2018] such as both of these arrangements can be considered compatible with the same responses.

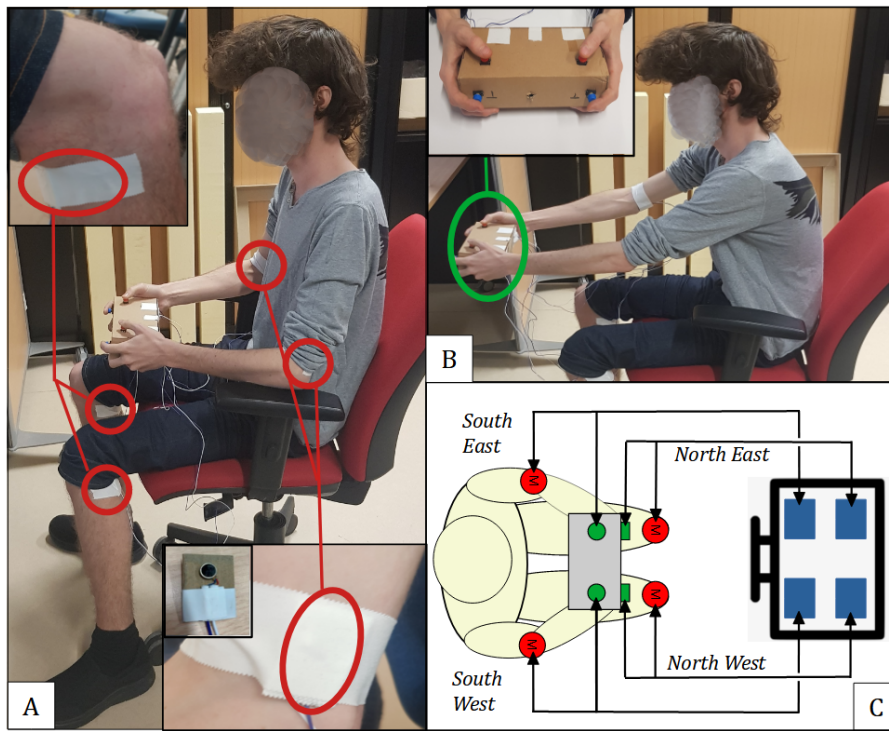


Figure 3.1: The experimental setup. (A) The participant's sat on a chair in front of a table with a screen. The vibrating motors were located on the participant's knees and elbows. (B) The position of the participant in the Tactile Far condition. In this position the participants held the button box in front of their knees along the anteroposterior axis. (B - Inset) The response box and its four buttons. (C) The mapping between the stimuli (visual or tactile), and the buttons the participants were instructed to press in response (the screen was placed on the table facing the participant).

To answer the commanding stimulus, the participants were instructed to press a button on a "response box" (figure 3.1 - B, inset) onto which were disposed four buttons resting under their thumbs and index fingers. During the experiment, each participant held the box such as the four buttons were arranged in two rows, the front buttons, resting under the participant's indexes were mapped to the NW and NE directions while the back buttons, resting under the participant thumbs, were mapped to the SW and SE directions (figure 3.1 - C).

### 3.2.1 Trials and Conditions

In all conditions, a trial (presentation of a commanding stimulus and recording of the reaction time) started with a random delay between 1250 and 2000ms before the stimulus onset. This stimulus remained active until the participant pressed the correct button as determined by the directions mentioned earlier (i.e. the NW visual or tactile stimulus was to be answered by pressing the NW button on the box, similarly for the other directions 3.1 - C). When they did, the stimulus was turned off and the reaction time, defined as the time elapsed between the stimulus onset and the button press, was recorded. We had planned to stop the trial in the eventuality where the participant did not react within 3500ms of the stimulus onset but this never occurred in practice.

### Experimental conditions

The experiment was divided into 6 blocks, each corresponding to one of five experimental conditions. To avoid the handedness of the participant having an effect on the measured reaction time, we made sure that every conditions contained an equal amount of left/right handed button presses and subsequently averaged the results.

Each participant began and ended the experiment (block 1 and 6) with 20 trials in the NOMAP condition. Each of those two blocks were further divided in two periods of 10 trials where the commanding stimuli were presented tactilely (NOMAP-Tactile) or visually (NOMAP-Visual). In this condition, the four visual or tactile stimuli were always presented all at once (i.e. at the stimulus onset, all square on the screen turned red in NOMAP-Visual or all vibrating motors turned-on in NOMAP-Tactile). The participant were instructed to answer with the right hand for half of the trials (5 in NOMAP-Visual and 5 in NOMAP-Tactile) and with the left hand for the other half. Beside this constraint, they were free to choose the responding finger. The sequence of responding hand and stimulus modality of each NOMAP block was randomized for all participants.

The remaining four blocks (block 2 to 5) in the experiment, each containing 40 trials, were performed in the Tactile, Visual, Visuo-Tactile or Far-Tactile condition. The order in which these conditions were performed by each participant was also randomized.

For the trials carried out in the Visual, Tactile or Visuo-Tactile conditions, the participants were presented with a commanding stimulus corresponding to one of the four possible directions. A visual stimulus in the Visual condition, a tactile stimulus in the Tactile condition or a matching pair of visual and tactile stimulus (synchronous onset; same direction) in the Visuo-Tactile condition. The number of trials performed under each possible directions was balanced such as participants answered to 10 stimulus for each of the four possible directions in each conditions, the order in which these directions were presented was semi-randomized such as to avoid long repetitions.

Finally, the Far-Tactile condition was exactly the same as the Tactile condition to the exception that the participants were instructed to hold the response box far away from their body such that the response buttons were spatially located beyond their knees along the antero-posterior axis (figure 3.1 - B). The choice to include this condition was motivated by preliminary tests with the setup (see next section on hypotheses).

### 3.2.2 Hypotheses

The decision process underlined by a choice reaction task is generally represented in the literature via the Drift-Diffusion Model [Ratcliff and McKoon, 2008, Roxin, 2019]. This model, inspired by the Bayesian Brain Hypothesis [Bitzer et al., 2014], postulates that humans accumulate sensory evidence in favor of a response from the sensory stream overtime and enact this response when the accumulated evidence crosses a given threshold. In this case, the reaction time is therefore primarily determined by the rate at which this sensory evidence can be accumulated. Given that the visual and tactile stimulus used in our experiment contain the same information, it can thus be expected, in line with the literature [Ernst and Banks, 2002, Berniker and Kording, 2011, van Dam et al., 2014], that the presentation of both a visual and a tactile stimulus to the participant would increase the rate at which sensory evidence can be accumulated (since there is two "sources" for this evidence), thus *shortening*

*the reaction time in the Visuo-Tactile Condition, where both visual and tactile stimulus are available, in comparison to the Visual and Tactile conditions, which involve only one modality. This is our first hypothesis (H1).*

It should be noted, however, that the choice reaction task requires our participants to first perceive (detect the stimulus) and then process (relate the detected stimulus to the appropriate response) a stimulus in order to succeed in a trial. In our daily life, it seems that we predominantly use visual information to act in the "external space", for example to grab a cup of coffee. Conversely, tactile information are predominantly used to act in the "body space", for example to swat a mosquito when we feel a pinch on our leg. Since our experiment requires the participants to press a button on the reaction box based on tactile (Tactile condition) or visual (Visual condition) information, an action which, one may reasonably argue [Wallace, 1971], belongs to the external space, we may predict that our participants will select a response faster with visual stimulus than with tactile stimulus. As such, due to this predominance of visual information, and our habituation towards visual cues, we hypothesized that *a visual stimulus would enable faster response selection based on spatial information compared to tactile stimulus*. This is our second hypothesis (H2).

Our final hypothesis comes from preliminary experiments with the setup. While trying different positions for the participant in this experiment, we noted what could be interpreted as a form of tactile Simon effect depending on the position of the response box along the antero-posterior axis. Specifically, it seemed that participants were answering faster with specific buttons when they held the response box closer to the trunk rather than closer to the knees. Since a similar effect was never mentioned in previous studies and that it seemingly contradicted our understanding of the Simon effect in the tactile modality [Hasbroucq, 1987, Medina et al., 2014], we decided to include a third hypothesis in the study. Namely, that *the reaction time to tactile stimulus would vary based on the location of the response box, which is held close to the trunk and above the lap in the Tactile condition and above the knees, further away from the trunk, in the Tactile-Far condition*. This is our third hypothesis (H3). However, as we'll show in the result section, we were unable to reproduce this effect and it is therefore unclear to us what led to this discrepancy during preliminary tests.

### 3.3 Results

24 participants (17 males, 7 females, aged 19–45, mean 26) took part in our study. The participants' handedness was tested using an Edinburgh Handedness Inventory [Robinson, 2021] and 20 of them reported being right handed while 4 reported being left handed. All of them gave informed consent prior to the experiment whose procedure was approved by the Comité d'Ethique de Euromov (IRB-UM; Approval ID: 2201F).

Prior to the data collection, a preliminary study with the setup allowed us to estimate an effect size of about 0.35 ( $\eta = 0.17$ ) for the ANOVA between the Tactile, Visual and Visuo-Tactile conditions. Based on this estimated effect size, we calculated the necessary number of participants as 15 to reach a significance level ( $\alpha = 0.05$ ) with the G-Power® software [Kang, 2021]. The additional 9 participants were included due to some concern we had with the saliency of visual stimulus and helped us ensure that our results would not differ significantly based on the luminosity conditions of the room in which the study was performed. We found no significant effect of luminosity on the results and therefore do not discuss this

aspect in the main result section (more details are available in the appendix section at the end of this chapter).

### Rejection criterion and Data processing

The collected reaction times (RT in the later) were rejected from the data analysis based on two criteria determined prior to data collection. Incorrect responses to the commanding stimulus were rejected, as well as responses occurring before 150ms and after 1000ms of stimulus onset. Overall, the proportion of rejected trials was below 2% on average across all participants and conditions, and less than 5% across all participants in each condition. In detail, the rejected trials represented 1.1% (11/960) of the trials in the NOMAP condition, 4.6% (44/960) of the trials in the Tactile condition, 3.4% (33/960) of the trials in the Visual condition, 4.4% (42/960) of the trials in the Visuo-Tactile condition and 4.9% (47/960) of the trials in the Far-Tactile condition. The remaining RT were then averaged for each subject across all trials performed in each condition.

#### 3.3.1 Hypothesis testing

The results obtained in this study are reported a per hypothesis basis below.

##### (H1) Multimodal stimuli shortens the reaction time

With regard to the first hypothesis, that a combination of visual and tactile stimuli would yield shorter reaction time in the Visuo-Tactile condition in comparison to both the Visual and Tactile conditions, we examined the participants' average RT across the four stimulus directions in the Visual, Tactile and Visuo-Tactile conditions.

The 24 participants yielded a mean RT of  $489ms \pm 77sd$  in the Visual condition,  $462ms \pm 60sd$  in the Tactile condition and  $420ms \pm 54sd$  in the Visuo-tactile condition (figure 3.2). A one-way repeated measures ANOVA (normality of the datasets was ensured by a series of Kolmogorov-Smirnoff tests) showed a significant effect of the experimental condition on the mean RT [ $F(2,46) = 20.62$ ;  $p < 10^{-7}$ ;  $\eta = 0.17$ ]. A post hoc analysis (Bonferroni corrected two-samples T-test) confirmed that participants responded significantly faster in the Visuo-Tactile condition compared to both the Visual condition [ $t(23) = 8.05$ ;  $p < 10^{-5}$ ;  $d = 1.01$ ] (by 69 ms on average) and the Tactile condition [ $t(23) = 4.07$ ;  $p < 0.05$ ;  $d = 0.71$ ] (by 27 ms on average). On the other hand, comparing the mean RT obtained in the Visual and Tactile conditions revealed no significant difference [ $t(23) = 2.08$ ;  $p > 0.1$ ;  $d = 0.32$ ].

These results can be viewed as strong evidence in favor of H1; here, it seems that multi-modality benefited the reaction time in the CRT. Although we found no significant difference between the mean RT in the Visual and Tactile conditions, one should however note that this result is to be considered in light of each (visual and tactile) stimulus saliency, we elaborate further on this point in the discussion section.

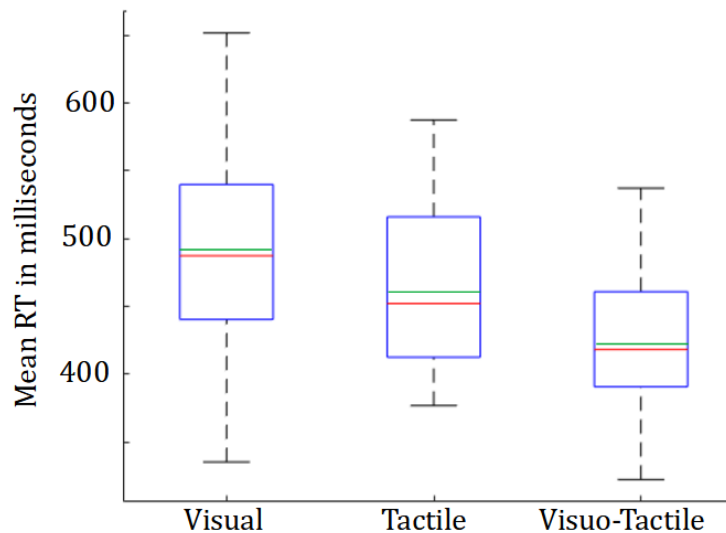


Figure 3.2: The participants' average reaction time in the Visual, Tactile and Visuo-Tactile conditions. We see a clear effect of sensory modality on the reaction time. The median average reaction time is displayed in red, the mean in green and the first and third quartiles in blue. No outliers were observed.

## (H2) Spatial information based decisions are faster for visual stimuli than for tactile stimuli

The reaction time of the participants can be decomposed in three distinct parts [Harris et al., 2004], the first correspond to the detection and localization of the stimulus in the sensory stream (*perception delay*), the second correspond to the selection of a response based on this localization (*choice delay*), while the third correspond to the enaction of the response by the participant motor system (*motor delay*). Interestingly, it is known that, in simple reaction task (a single stimulus matching to a single response thus no choice), reaction times are generally shorter for tactile stimuli than for visual stimuli [Chan and Chan, 2005, Harrar and Harris, 2005].

Given that we did not observe a significant difference between the participants' mean RT in the Visual and Tactile conditions, this suggests that the choice delay for visual stimulus was shorter than for the tactile stimuli, thereby confirming our second hypothesis. To investigate this further, we examined the participants' RT in both NOMAP conditions. Recall that, in the NOMAP conditions, participants are presented with all four stimulus at once (all four visual/tactile stimuli for the NOMAP-Visual/Tactile respectively) and are allowed to respond by pressing any buttons (with the left or right hand). The NOMAP condition is thus similar to a simple reaction task where no further information other than the detection of a stimulus is required to respond. In line with previous studies, participants reacted significantly faster [ $t(23) = -7.9$ ;  $p < 10^{-7}$ ;  $d = 1.88$ ] to the tactile stimuli ( $230ms \pm 30sd$ ) than to the visual stimuli ( $340ms \pm 60sd$ ) in the NOMAP condition, and so by 110ms on average. Assuming that the RTs obtained in the NOMAP condition are representative of the time necessary to detect a stimulus and enact a response (perception and motor delay), we can therefore estimate the time necessary to select a correct response based on the localization of the stimulus (choice delay) by subtracting the average RT in the NOMAP visual and NOMAP tactile conditions from those in the Visual and Tactile conditions respectively [Harris et al., 2004, Schweickert et al., 2010].

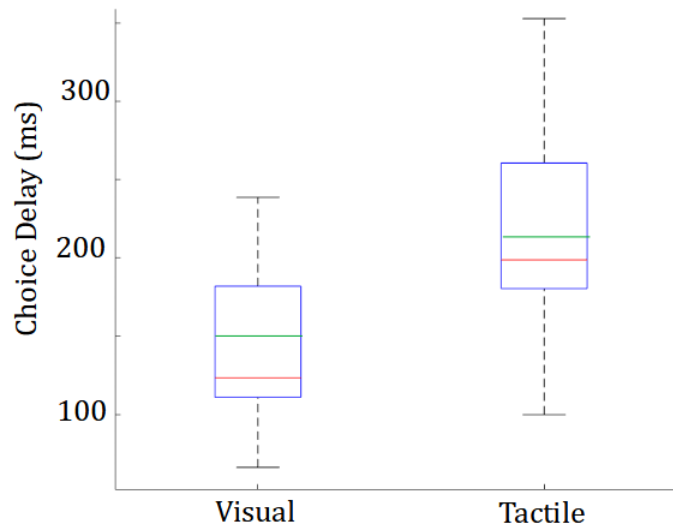


Figure 3.3: The choice delay in the visual and tactile modality. The difference of the total visual and tactile RT (figure 3.2) relative to the corresponding NOMAP RT provides an estimate of the choice delay introduced by the mapping. The median is displayed in red, the mean in green and the first and third quartiles in blue. No outliers were observed.

Proceeding in this way for each participant reveals (figure 3.3) that this estimate of the choice delay introduced by the mappings in the Visual and Tactile conditions is indeed larger for tactile ( $218 \pm 61sd$  ms) than for visual stimuli ( $149 \pm 61sd$  ms). The significance of this 69ms difference on average is confirmed by a two-samples t-test [ $t(23) = -5.77$ ;  $p < 10^{-5}$ ;  $d = 1.13$ ]. This result can be viewed as strong evidence in favor of our second hypothesis that the choice based on stimulus location is faster for visual than for tactile stimuli in this task. We further elaborate on a few rational for this result in the discussion.

### (H3) Response box location did not change response time

For a final analysis, we investigated whether, as we had apparently observed during preliminary tests, the reaction time to the tactile stimuli did indeed depend on the location of the response box.

In which space were the finger responses in the CRT coded in? One can assume that these responses are encoded in a "body-centric" coordinate system or, alternatively, the location of responses may also be coded in a referential whose center may be placed at center of the response box. In the position of the participant during the Tactile condition (figure 3.1 - A), which required them to hold the response box over their lap, the responses to the tactile stimuli were spatially compatible with respect to both these reference frames. That is, the participants were instructed to respond to the NW and NE stimuli, presented near the knee, which, given their position, are at the front of their body, with the NW and NE buttons which are located on the front of the response box. Similarly, the SW and SE stimuli, presented at the elbows and therefore behind the knee in the body-centric reference frame, were to be answered with the SW and SE buttons, on the back of the response box. Under both reference frames, the relationship between stimulus and responses is therefore the same, "front" tactile stimuli, in both body and response box centric coordinate systems, are to be answered with the front buttons (similarly for the "back").

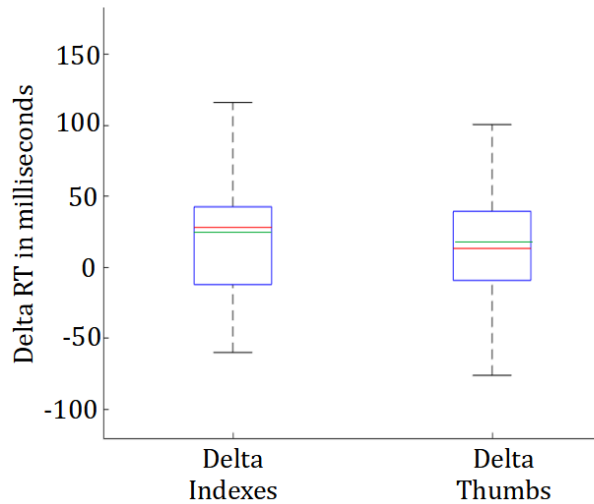


Figure 3.4: The average difference in reaction time between the front (NW/NE; actuated by the indexes) and back (SW/SE; actuated by the thumbs) button in the Far-Tactile and Tactile conditions. While we did observe a general change in average RTs between the two conditions, these changes were not different between the front and back buttons. The median is displayed in red, the mean in green and the first and third quartiles in blue. No outliers were observed.

This symmetry is broken, however, in the Far Tactile condition, in which we asked the participants to hold the response box beyond their knees (figure 3.1 - B). In this condition, the index and thumb fingers are still the ‘front’ and ‘back’ fingers in the response box centric reference frame. However, in a body centric reference, while the front buttons are still in front relative to the front stimuli (presented on the knee), the back buttons too are now in front of the front stimuli, potentially affecting the stimulus compatibility. Indeed, if we assume that the response location is coded in a body centric coordinate system, we can expect the compatibility between stimuli and responses to be more affected for the back buttons between the Tactile and Far-Tactile conditions than for the front buttons.

To investigate this question, we compared the difference in RT between the front buttons (NW/NE) and back buttons (SW/SE) in the Tactile and Far-Tactile conditions (figure 3.4). We chose to compare the difference between RT obtained for front and back button, and not directly the RT, to control for an eventual effect of discomfort, caused by stretching the arms in the Far-Tactile condition, that may affect the RT between the two conditions.

While we observed a slight delay in the reaction time for both front and back tactile RTs between the Tactile and Tactile-far conditions, these differences did not reach significance ( $[t(23) = -1.85; p > 0.05; d = 0.33]$ , and  $[t(23) = -1.35; p > 0.15; d = 0.31]$  respectively). Crucially, this delay introduced by the Tactile-Far condition (8ms on average) did not significantly vary between the front ( $25ms \pm 62sd$ ) and back ( $17ms \pm 67sd$ ) tactile RTs  $[t(23) = 0.59; p > 0.5; d = 0.07]$ . Overall, although we observed a general change in the average RT with the change in the location of the response box, probably due to the discomfort induced by the position of the participant in the Far-Tactile condition, these results could not confirm changes in stimulus response compatibility between the two conditions and did not support the third hypothesis.

## 3.4 Discussion

In this study, we focused on a CRT where participants were instructed to answer spatialized stimuli with compatible responses and investigated how the participants' performance, as measured by reaction times, might differ between visual and tactile stimuli. This choice of experimental procedure being motivated by a parking assistance system, very common in modern automobile, though with visuo-tactile rather than visuo-auditory stimuli.

It is well-known that reactions to tactile stimuli are generally faster than to visual stimuli [Ng and Chan, 2012, Harrar and Harris, 2005]. However, these reports were obtained under simple reaction tasks in which no choice was involved. A priori, we hypothesized that such results would also be observed in this study, that is, that tactile stimuli would similarly lead to better performance than visual stimuli. While we did observe a significantly shorter reaction time in the NOMAP tactile ( $230ms \pm 30sd$ ) in comparison to the NOMAP visual condition ( $340ms \pm 60sd$ ), and so by 110ms on average, this result did not extend to the case where participants were to select a response based on the localization of a stimulus. We showed here that, in fact, the additional delay in the selection of a response based on the location of a tactile stimulus, which was significantly longer than based on the location of a visual stimulus (figure 3.3), was sufficient to lead to an almost equivalent reaction time to the tactile and visual stimuli in this CRT (figure 3.2).

Several factors may have contributed to the faster decisions based on location for visual stimuli. Firstly, visual interfaces, which require us to respond to visual stimuli, are much more common in our daily lives than tactile interfaces. It is therefore likely that this habituation towards making decisions based on visual stimuli contributed to the results obtained here. This is also supported by the fact that participant made more mistakes (incorrect button press) during the Tactile condition ( $1.8 \pm 1.8sd$  errors on average) in comparison to the Visual condition ( $1.5 \pm 1.3sd$  errors on average).

Secondly, it is also likely that the results obtained are dependent on the stimulation patterns. Both the visual and tactile stimuli used in this study were square waves. That is, at the onset of the stimulus, its "value" (color or vibration intensity) changed from "zero" to a constant value until a reaction of the participant brought it back to zero. Other choice of stimulation patterns, especially for the tactile stimuli [Chouvardas et al., 2008], may yield different results.

Finally, another aspect to consider is the relative saliency of the stimuli used in this study. The saliency of a sensory stimulus is dependent on a number of factors [VanRullen, 2003, Elliott et al., 2015], for example luminance contrast [Levitt and Lund, 1997, Rodieck, 1998, Nothdurft, 2000] in the case of visual stimuli or vibration frequency and stimulated body-part [Sherrick Jr, 1953, Bolanowski et al., 1994, Gescheider et al., 2002, Chouvardas et al., 2008, Corniani and Saal, 2020] in the case of tactile stimuli. While we did test the dependence of the results in relation to different luminosity for the visual stimuli (see section 3.5) and found no significant interactions, we did not perform similar tests for the tactile stimuli and simply attempted to maximize their saliency by operating the vibratory motors at maximum intensity. We also chose to use vibrators originally designed for phones assuming that their stimulation patterns would be salient for the participants due to their habituation towards this type of tactile stimuli. That being said, we recognize that we cannot guarantee an equal saliency of the tactile and visual stimuli, since these are two different

modalities. Further studies are therefore needed to clarify the extent to which the result reported here may generalize to different visual and tactile stimuli saliency.

In the preliminary tests used to estimate the required number of participant, we observed that the reaction time was seemingly dependent on the position of the response box relative to the body along the antero-posterior axis, in contradiction with our knowledge of the Simon effect in the tactile modality. We therefore chose to include a third hypothesis and the Far Tactile condition in the study to investigate this issue further. After full data collection however, we were unable to reproduce this effect. Our results suggest that a "response-box centric" representation of the actions (button presses) was used by the participants in this study but it is known that such representations are eminently dependent on context and instructions [Umiltà and Liotti, 1987]. In this respect, it is unclear how the result presented here may evolve given different experimental designs.

Finally, this report should also be understood in light of the tested population. Most our participants were relatively young (19 to 45 years old, mean 26) and none of them reported of a major visual deficit or tactile insensitivity. It can be expected that subjects in a different age group or with deficits in one or the other sensory modality would yield different results. Similarly, most our participants reported extensive experience using computers and other information systems based primarily on visual interfaces. The selection of different groups of individuals, either without such experience or with specific training in visuo-motor tasks, such as pilots and athletes, may also influence the results. It is important to note, however, that multimodal feedback, here combining tactile and visual, is clearly beneficial for the general population [Ernst and Banks, 2002, Berniker and Kording, 2011, van Dam et al., 2014]. Our study provides further evidence of this in a CRT where participant were instructed to react based on the location of a stimulus. This result is significant for the design of HMI, in particular for scenarios where accurate and fast responses are paramount, such as when driving an automobile or piloting an aircraft.

### 3.5 Annex - Controlling for dependence on luminosity

As mentioned in the method section of this article, our preliminary analysis allowed us to estimate that 15 participants were required to obtain statistically significant results in this study. However, after starting the data collection, we recognized that the lightning conditions in which the study was performed may have an effect on the results (via a modulation of the visual stimulus saliency). To determine whether this was the case, we decided to include 9 additional participants in the study to control for a possible effect of luminosity on the results. The participants were divided into three groups corresponding to different screen luminosity : Low (6 participants), Medium (12 participants) and High (6 participants). The participant of the Low group performed the task under a mean luminosity (gray level) of  $L = 0.07$  and Weber contrast (computed between the visual stimulus and surrounding background) of  $W = 0.35$ , the Medium group performed under  $L = 0.24$  and  $W = 0.34$  while the High group performed under  $L = 0.38$  and  $W = 0.31$ . The tactile stimulus were the same for all three groups. We did not find a significant effect of group on either the average reaction time [ $F(5,10) = 0.96$ ;  $p > 0.4$ ;  $\eta = 0.14$ ], nor on the decision time [ $F(5,10) = 1.06$ ;  $p > 0.35$ ;  $\eta = 0.12$ ] and hence combined all data and treated them as a single group in this report.

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## A computational model of Stimulus Response Compatibility

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### Preamble

The last chapter focused on a fairly typical study on human cognition in which we "tested" our participants' behavior by asking them to perform a very simple sensorimotor task. Of course, the simplified and controlled settings of our experiment, necessary in order to obtain meaningful results, mean that these results may be very difficult to generalize to the rich and stochastic environments in which machines help us achieve our goals. In addition, it is quite intuitive that we only lifted the veil on a very small part of all the processes that together form human cognition. Summarizing the wealth of information provided by studies on human behavior to generate an approximate "model of users", which forms the prior endowing the coadaptive machine, is therefore another challenge facing designers of interactive systems. To paraphrase S.K. Card, one of the authors of the Model Human Processor (MHP) which can be considered the first user model, *"It is a great mistake to think that, by merely listing a miscellaneous collection of results, psychology is thereby rendered usable to support practical engineering projects. In a psychology useful for engineering design, what is important is the ability to do task analysis, calculation, and approximation."* ([Card, 1981]).

The choice of a framework in which to express such model of user is eminently dependent on the type of interactive system to be designed. Production systems, such as the MHP, are for example well suited to model computer users. In contrast, the user of a prosthesis is more often modeled as a (random) dynamical system. In this chapter, we explore this computational modelization challenge by simulating human behavior in a choice-reaction task similar to that studied in the previous chapter. In particular, we adopt the framework of Active Inference, which can be seen as an intermediate framework between production and random dynamical systems, to represent in an approximate form the phenomenon of Sensory Response Compatibility that we mentioned in the introduction to the previous chapter.

## 4.1 Introduction

Choice reaction tasks (CRT), where a participant must select an appropriate action from several options on the basis of available information, constitute one of the most fundamental experimental paradigms of the literature on human behavior. Over the years, CRT have been used by experimenters to study various aspects of the cognitive process in humans, the way our sensory system integrates information from various sensory modalities [Proctor et al., 2005, Yu and Möeller, 2014], how we use this information to take decisions [Ratcliff and McKoon, 2008, Shahar et al., 2019] or learn from these decisions for the future [Kornblum, 1973, Schumacher et al., 2005, Lee et al., 2016]. In turn, this wealth of results played a key role in guiding design choices for interactive systems, to make them easier to use, to learn, or simply to increase their "performance" in a general sense.

Among the findings of behavioral psychology with major impact on the design of interactive systems, the Sensory Response Compatibility phenomenon (SRC) is undoubtedly one of the most important, with the authors of studies around SRC regularly stressing its importance for the design of human machine interfaces [Fitts and Seeger, 1953, Vu and Proctor, 2003, Chan and Chan, 2005, Proctor et al., 2005]. The term SRC refers to the experimental fact that certain reaction tasks are more or less difficult, as measured by reaction times and error rates, given a particular combination of stimuli and responses [Kornblum et al., 1990, Alluisi and Warm, 1990, Kornblum et al., 1999]. For example, in a two-choice reaction task where participant are instructed to respond to a colored stimulus (for example *red* or *green*) by pressing a colored key (similarly a *red* or *green* key), the participant performance will be optimal (fastest response and highest accuracy) if the colored stimuli are responded to by pressing the key with matching color than under the reverse association [Simon et al., 1981].

Given this epistemic definition, the range of experimental results encompassed by the SRC phenomenon is extremely broad, including for example the Simon [Simon and Rudell, 1967, Simon and Small Jr, 1969], Stroop [Stroop, 1935, MacLeod, 1991, MacLeod, 2015] and Flanker [Eriksen and Eriksen, 1974, Sanders and Lamers, 2002] effect as particular case, and it is therefore difficult to propose a general theory for its origin in the human brain [Verbruggen et al., 2006, Leuthold, 2011, Soutschek et al., 2013, Cespón et al., 2020]. In this respect, the dimensional overlap theory and the related dual route model [Kornblum et al., 1990, Kornblum and Lee, 1995] seem relevant to explain, if only partially, many of the experimental results relating to SRC, by invoking a conflict between two responses resulting in the observed difference in performance between trials performed under compatible (matching color of stimulus and key in the previous example) and incompatible (mismatching) stimulus-response pairs. Simply put, the dimensional overlap theory begins with the simple observation that, in order to decide which response is to be enacted when presented with a stimulus, the participants in a CRT need to be able to discriminate the possible stimuli (and responses) from each other. To do this, we can assume that the various stimuli and responses involved are first "coded" in the brain according to a number of discriminating features (i.e. their color in the previous example). In this case, it is thus possible that the features allowing to discriminate between the different stimuli to be the same as, or *overlap* with, the features allowing to discriminate between the different responses. This is referred to as a *dimensional overlap* between the stimulus and response set. Given this overlap, the participant's decision, which can be viewed as the translation of a stimulus "feature code" into a response feature code, can then be influenced by a priming effect of the stimulus fea-

tures on the corresponding features of the response. In other words, when presented with a red stimulus, the participant is "naturally drawn" to respond with the red key, irrespective of what is required by the rule of the CRT. However, when this feature congruent response is incorrect, the participant must suppress this natural tendency in favor of the correct, goal-directed, response demanded by the reaction rule. Dual route models therefore postulate that the arbitration between "automated" and "goal-directed" response causes the difference in performance between feature-congruent reaction rules, where the stimulus feature code matches that of the response, and feature-incongruent reaction rules, where the stimulus feature code is different from that of the response.

The literature proposes mainly two computational representations of the dual route model: connectionist models [De Jong et al., 1994, Zorzi and Umiltá, 1995, Tagliabue et al., 2000, Spironelli et al., 2009] which represent the two routes as two paths in a network connecting stimulus feature nodes to response feature nodes, and production system-based models [Kornblum et al., 1990, Stocco et al., 2017], in which the two routes correspond to two systems that differ in their production rules. While the later allows for great flexibility in the tasks that may be simulated, connectionist models are comparatively easier to relate to observation made via electrophysiological measurements, which have received a growing amount of attention in recent years due to the key insight they offer on the processes underlying SRC effects [Christ et al., 2000, Dassonville et al., 2001, Praamstra and Oostenveld, 2003, Cespón et al., 2020]. However the canonical connectionist model of SRC, originally proposed in relation to the Simon effect [De Jong et al., 1994], is rather specialized and difficult to generalize beyond very simple tasks. It is therefore relatively ill-suited to represent human machine interactions which often involve complex series of decisions.

As a general framework for approximating human behavior, Active Inference [Da Costa et al., 2020] has proved effective in reproducing, "in-silico", a variety of psycho-neurological observations, such as addictive behaviors [Schwartenbeck et al., 2015, Miller et al., 2020], foraging [Mirza et al., 2016, Neacsu et al., 2022], curiosity [Friston et al., 2015, Kaplan and Friston, 2018, Gottlieb and Oudeyer, 2018, Schwartenbeck et al., 2019], valence [Smith et al., 2019, Hesp et al., 2021], various symptoms of psychopathy [Cullen et al., 2018, Linson et al., 2020, Maisto et al., 2021] to name a few example (see also [Friston et al., 2017a, Friston, 2018, Parr and Friston, 2019, Da Costa et al., 2020] for reviews).

Furthermore, this framework offers great flexibility in terms of the type of model to which it can be applied (continuous, [Lanillos et al., 2021, Priorelli et al., 2023]; discrete, [Da Costa et al., 2020, Smith et al., 2022]; or mixed, [Friston et al., 2017b, Çatal et al., 2021]), making it an interesting tool for framing the interaction between a user and a variety of interactive systems. Surprisingly, however, it seems that the SRC phenomenon is still missing from the list of phenomena that have previously been addressed under Active Inference. In this chapter, we therefore revisit some of the key results offered by studies on SRC using a simple model of Active Inference [Smith et al., 2022], with a view to compare its predictions with the literature and highlight the similarities between this framework and the connectionist Dual-Route model.

The following sections are structured as follows: we start by presenting the model of a simple CRT as a Partially Observable Markov Decision Process (POMDP), before briefly reviewing the equations enabling its treatment within the Active Inference framework. After this, we introduce the parameterization of the generative model allowing our simulated participant to perform a two-choice reaction task and illustrate the process with an example. Finally, we study the model's prediction in relation to SRC effects, in particular the Simon effect, before comparing its computational structure with the canonical (connectionist) Dual-Route model.

## 4.2 Partially Observable Markov Decision Processes and Choice Reaction Task

In this study, we will focus on the predictions of the Active Inference framework in relation to a two-choice reaction task expressed as a POMDP. This choice of representation is simply motivated by the fact that a CRTs, which typically involve sequences of discrete choices that are relatively independent of each other, can easily be related to the discrete structure of a POMDP.

### 4.2.1 Partially Observable Markov Decision Processes

Markov Decision Processes, or "MDP" for short, are a discrete representation of decision processes extending through time. Formally, a MDP can be expressed as a tuple  $\langle \mathcal{S}, \mathcal{V}, \mathcal{P}_s, \mathcal{R} \rangle$ , where :

$\mathcal{S} = \bigoplus_{i \in I} \mathcal{S}_i$ ,  $\mathcal{S}_i \subset \mathbb{N}$  the MDP state space describing the possible state of the "world" at each time step.

$\mathcal{A} = \bigoplus_{i \in I} \mathcal{V}_i$ ,  $\mathcal{V}_i \subset \mathbb{N}$  the similarly constructed "action space" describing the actions available to the agent navigating the MDP.

$\mathcal{P}_s \subset \{B : (\mathcal{S} \times \mathcal{V}) \rightarrow \Delta(\mathcal{S})\}$  a conditional probability distribution (likelihood) describing the probability for the world to transition from a state  $s_\tau \in \mathcal{S}$  at time step  $\tau$  to a state  $s_{\tau+1}$  under an action  $v \in \mathcal{V}$  of the agent.

$\mathcal{R} : \mathcal{S} \rightarrow \mathbb{R}$  the scalar field over  $\mathcal{S}$  describing the reward obtained by the agent given the state of the world (or  $\mathcal{R} : \mathcal{S} \times \mathcal{V} \rightarrow \mathbb{R}$ , if this reward also depends on the action).

Due to their relative simplicity, MDP have been widely used to model a variety of decision problems [Boucherie and Van Dijk, 2017]. For example, economic agents attempting to maximize their payoff in a financial system [Bäuerle and Rieder, 2009, Buongiorno and Zhou, 2017] or robots navigating in an uncertain environment [Kurniawati et al., 2011, Gopalan et al., 2017]. As such, various methods and algorithm have been proposed in the literature to study and solve MDP but also numerous generalization of the concept to more complex decision processes [White III and White, 1989, Murphy, 2000, Sanner and Boutilier, 2009, Puterman, 2014]. One of the earliest of these generalizations is the Partially Observable Markov Decision Process (POMDP) which supplement the tuple  $\langle \mathcal{S}, \mathcal{V}, \mathcal{P} \rangle$  with an outcome space,  $\mathcal{O} = \bigoplus_{i \in I} \mathcal{O}_i$ ,  $\mathcal{O}_i \subset \mathbb{N}$ , describing the range of observations that can be made by an

agent, and a set of conditional probability measure  $P_o \subset \{A : \mathcal{S} \rightarrow \Delta(\mathcal{O})\}$  describing the probability for the agent to observe an outcome  $o \in \mathcal{O}$  given the state  $s \in \mathcal{S}$  of the world.

### 4.2.2 A generative process

A simple CRT can be defined as an experimental protocol in which a participant is instructed to react to some unpredictable sensory stimulus, for example a light turning on or a beeping sound, with a particular response depending on the properties, or features, of the stimulus. For example, a typical instruction given to a CRT participant might be "when you see a red or green light turning on, answer by pressing the red or green key respectively as fast as possible". The reaction time, which is usually the dependent variable in such experiments, is then simply defined as the time elapsed between the presentation of the stimulus and the response of the participant. In addition to this basic process, it is common practice for CRT to also inform participants of their success in the task, displaying for example "success" if the participants reacted correctly to the stimulus, or "failure" if they made a mistake.

This basic process can be broken down into three important time-periods corresponding to the discrete time-steps  $t$  in its representation as a POMDP. The *preparation* time-period ( $t = 0$ ) corresponds to the time before the presentation of the stimulus, followed by the *decision* time-period ( $t = 1$ ) starting at the stimulus onset and ending with the (simulated) participant reaction, and the trial ends by a *feedback* time-period ( $t = 2$ ) in which the participant is presented with a feedback depending on its actions in the previous two periods. At both  $t = 0$  and  $t = 1$ , the agent may decide to press one of two response key or none of them (the *wait* action). Importantly then, for the participant to obtain a reward in the third time-period (the *success* feedback) it must first decide to wait at  $t = 0$  (since the stimulus is not shown yet) and then press the appropriate key, determined by the reaction rule, at  $t = 1$ . Any deviation from this and the participant is "punished" with a *failure* feedback at  $t = 2$  (in particular, waiting at  $t = 1$  is considered as a mistake).

In the following sections, the two-choice reaction task will be simulated with either two or four possible stimuli corresponding to all combinations of one or two binary features typically used in the study of SRC effects, colors (*red* or *green*) and laterality (*left* or *right*). The relationship between these stimulus  $o$  and their representation via feature codes  $s$  is then expressed as a categorical likelihood  $A \sim P(o|s)$  which, to any stimulus, associates the probability for  $o$  to be coded by  $s$ . The reaction rule is then expressed as a map between a stimulus feature code  $s$  and the appropriate response  $v$  (i.e. when the stimulus color feature is *red*, respond by pressing *key 1*), this reaction rule being assumed to be known by the participant. Finally, the transitions between time-periods in the CRT are encoded by categorical likelihoods  $B_v \sim P(s_{t+1}|s_t, v_t)$  parameterized such as to reproduce the sequence presented earlier.

An Active Inference treatment of this POMDP also requires us to provide policies to the simulated participant, where a policy can be heuristically understood as a sequence of two actions  $\pi = (v_0, v_1)$ , one for each transitions between time-step. In particular, while the first action ( $v_0$ ) enacted during the preparation time-period ( $t = 0$ ) will always be the *wait* action for all policies, the second ( $v_1$ ), which correspond to the choice of a response key in the decision period ( $t = 1$ ), will depend on the features of the stimulus. In that regard, each policy will underlie a different map between the stimulus features and the responding key such as, for a stimulus with one binary features (such as a *red* or *green* color), the participant

will be equipped with two policies where the first policy maps the *red* and *green* stimuli to *key 1* and *key 2* respectively while the second will revert this relationship (i.e. mapping *key 1* and *key 2* to the *green* and *red* stimulus respectively).

### 4.2.3 Active Inference in discrete time

Numerous detailed reviews of the equations underlying Active Inference in discrete time can be found in the literature (see for example [Smith et al., 2022]), our aim in this section is therefore not to offer a detailed description of these equations but merely to introduce the variables relevant in the simulation SRC effects. Thus, this section on Active Inference applied in discrete settings will be rather succinct and simplified.

When presenting the Active Inference framework, it is quite common to start with the definition of the generative model as a joint probability distribution over latent states  $s$ , outcomes  $o$  and policies  $\pi$ .

$$P(\tilde{o}, \tilde{s}, \pi) = P(\tilde{o}|\tilde{s})P(\tilde{s}|\pi)P(\pi) \quad (4.1)$$

where  $\tilde{o}$  and  $\tilde{s}$  denote the temporal series  $(o_0, o_1 \dots o_T)$  and  $(s_0, s_1 \dots s_T)$  from the initial time-step  $\tau = 0$  to the final time-step  $\tau = T$  (with  $T = 2$  in our case). Given the POMDP setting, this generative model can be explicitly written as :

$$\begin{aligned} P(\tilde{o}, \tilde{s}, \pi) &= P(\pi)P(s_0) \prod_{\tau=1}^T P(o_\tau|s_\tau)P(s_{\tau+1}|s_\tau, \pi) \\ &= P(\pi)D \prod_{\tau=1}^T AB_\pi \end{aligned} \quad (4.2)$$

$$\begin{aligned} P(o_\tau|s_\tau) &= \text{Cat}(A) \\ P(s_{\tau+1}|s_\tau, \pi) &= \text{Cat}(B_\pi) \\ P(s_0) &= \text{Cat}(D) \end{aligned}$$

where  $\text{Cat}$  denotes a categorical distribution. There are two important points to note here. The first is that action, previously noted  $v$ , is absent from the generative model. This is simply because the Active Inference framework adopts the ideomotor theory of action [Stock and Stock, 2004, Shin et al., 2010], that is, actions are represented via their sensory consequences in the generative model. Intuitively, instead of "pressing a key", which is an action, the agent only understands the "sensation of pressing a key", a sensation relating to muscle stretch or the pressure exerted on the fingertip for example. In other words, the action  $v_\tau$  at times-step  $\tau$  is represented via a (latent) motor state  $s_\tau$  and a motor outcome  $o_\tau$ , enabling them to be inferred (and predicted) like any other state (hence the name Active Inference). In particular, a well chosen parameterization of a policy's transition matrix  $B_\pi$  will allow us to condition the response to the stimulus ( $v_1$ ), which is simply the state of the motor system at  $\tau = 2$ , based on the stimulus features ( $s_1$ ). According to this view, each policy therefore corresponds to a "model of the evolution of the world" and the Active Inference process simply to a Bayesian Model Selection (BMS; [Çatal et al., 2019]), a model selection that is however appropriately biased so that the "winning" policy (i.e. the policy/model whose marginal

evidence is highest) is also the one that allows the agent to obtain its preferred outcomes. Adopting this BMS perspective, two components are required to obtain a posterior distribution over policies ( $P(\pi|\tilde{o})$ ) given some series of outcomes  $\tilde{o}$ . The first is a prior,  $P(\pi)$ , which is formally the role played by the Expected Free-Energy  $\mathcal{G}$ , while the second is the model evidence, or marginal likelihood,  $P(\tilde{o}|\pi)$ , which is the role played by the Free-Energy  $\mathcal{F}$ .

#### 4.2.4 A Bayesian Policy Selection : Approximating the marginal likelihood

Starting with the later, for any particular policy  $\pi$ , the marginal likelihood of a series of outcomes,  $P(\tilde{o}|\pi)$  can be obtained by marginalizing out latent states from the joint conditional distribution  $P(\tilde{o}, \tilde{s}|\pi)$ . Through a slight abuse of notation this marginalization can be written :

$$P(\tilde{o}|\pi) = \sum_{\tilde{s}} P(\tilde{o}, \tilde{s}|\pi) \quad (4.3)$$

In all but the simplest case, calculating this marginal is either unrealistic or impossible. In order to circumvent this problem, the agent performing Active Inference can approximate it by optimizing a bound on the log-marginal (also known as evidence), the Gibbs Free-Energy (GFE), that is a functional of a recognition density  $Q(\tilde{s}|\pi)$ , which may be viewed as the participant's beliefs about the series of latent states  $\tilde{s}$  given that the world is evolving according to  $\pi$ , and the generative model  $P(\tilde{o}, \tilde{s}|\pi)$  described above. This GFE,  $\mathcal{F}(\pi)$ , and its relationship with the evidence, can be expressed as follows :

$$\begin{aligned} -\ln P(\tilde{o}|\pi) &= -\ln \mathbb{E}_{Q(\tilde{s}|\pi)} \left[ \frac{P(\tilde{o}, \tilde{s}|\pi)}{Q(\tilde{s}|\pi)} \right] \\ &\leq -\ln \mathbb{E}_{Q(\tilde{s}|\pi)} \left[ \ln \frac{P(\tilde{s}|\tilde{o}, \pi) P(\tilde{o}|\pi)}{Q(\tilde{s}|\pi)} \right] \\ &= D_{KL}[Q(\tilde{s}|\pi) || P(\tilde{s}|\tilde{o}, \pi)] - \ln P(\tilde{o}|\pi) = \mathcal{F}(\pi) \end{aligned} \quad (4.4)$$

where  $D_{KL}$  is the Kullback-Leibler divergence. The bound from the first line to the second is obtained via Jensen's inequality (by convexity of the logarithm), while the last line highlight the role of the GFE as a bound on the log-marginal likelihood (since the divergence,  $D_{KL}[Q(\tilde{s}|\pi) || P(\tilde{s}|\tilde{o}, \pi)]$ , is always positive) with equality whenever  $Q(\tilde{s}|\pi) = P(\tilde{s}|\tilde{o}, \pi)$ . In other words, for the GFE to be a close approximation of the marginal likelihood, the agent must first optimize its beliefs about states ( $Q(\tilde{s}|\pi)$ ) such as they closely approximate the "true posterior" given the observed outcomes ( $P(\tilde{s}|\tilde{o}, \pi)$ ). In the Active Inference literature, this optimization is usually approached from the expression of the GFE as the sum of an internal energy and an entropy :

$$\mathcal{F}(\pi) = -\mathbb{E}_{Q(\tilde{s}|\pi)} [\ln P(\tilde{o}, \tilde{s}|\pi)] - H[Q(\tilde{s}|\pi)] \quad (4.5)$$

where  $\mathcal{U}(\pi) = \mathbb{E}_{Q(\tilde{s}|\pi)} [-\ln P(\tilde{o}, \tilde{s}|\pi)]$  is the internal energy (prediction error) while  $H[Q(\tilde{s}|\pi)]$  is the (Shannon) entropy of the recognition density. Noting that we minimize this functional w.r.t to the recognition density  $Q(\tilde{s}|\pi)$  the expression above require us to minimize the energy (minimize prediction error) while maximizing entropy (maximum entropy principle; [Guinasu and Shenitzer, 1985]). However, although the GFE enables us to cast a Bayesian inference problem as an optimization problem, it did not make this problem more tractable. As such, approximate Bayesian inference first start by restricting the recognition density to families of distribution for which energy and entropy can be computed easily.

### The Mean-Field Approximation and Variational Message Passing

The simplest of those approximations, inspired by statistical physics, is the Mean-Field approximation (MFA; [Han and Yang, 2019, Sakthivadivel, 2022, Zhang and Yang, 2024]) where it is assumed that the recognition density can be factorized into a number of partitions for which energy and entropy can be easily computed. In our case, the MFA is temporal, that is, we assume that  $Q(\tilde{s}|\pi)$  can be factorized as follows :

$$\begin{aligned} Q(\tilde{s}|\pi) &= \prod_{\tau} Q(s_{\tau}|\pi) \\ Q(s_{\tau}|\pi) &= \text{Cat}(s_{\tau,\pi}) \end{aligned} \quad (4.6)$$

where we introduced the notation  $s_{\tau,\pi}$  to refer to beliefs about latent states at some time-step  $\tau$  expressed as a probability vector. Under this approximation, the GFE, often referred to as the variational Free-Energy (vFE; [Winn et al., 2005, Friston et al., 2007]), simplifies considerably. For the entropy :

$$H[Q(\tilde{s}|\pi)] = \sum_{\tau} H[Q(s_{\tau}|\pi)] = - \sum_{\tau} s_{\tau,\pi} \cdot \ln s_{\tau,\pi} \quad (4.7)$$

and for the energy :

$$\mathcal{U}(\pi) = \sum_{\tau} \mathcal{U}_{\tau}(\pi) = s_{0,\pi} \cdot (-\ln A \cdot o_0 - \ln D) + \sum_{\tau > 0} s_{\tau,\pi} \cdot (\ln A \cdot o_{\tau} - \ln B_{\pi} s_{\tau-1,\pi}) \quad (4.8)$$

allowing us to express the vFE as a sum of "marginal vFE", computed individually for each time-steps, such that :

$$\begin{aligned} F(\pi) &= \sum_{\tau} F_{\tau}(\pi) \\ &= s_{0,\pi} \cdot (\ln s_{0,\pi} - \ln A \cdot o_0 - \ln D) + \sum_{\tau > 0} s_{\tau,\pi} \cdot (\ln s_{\tau,\pi} - \ln A \cdot o_{\tau} - \ln B_{\pi} s_{\tau-1,\pi}) \end{aligned} \quad (4.9)$$

where we introduced the notation  $o_{\tau}$  to denotes the observations at some time-step  $\tau$  expressed as a probability vector. Zeroing the gradient of the vFE w.r.t to  $s_{\tau,\pi}$  allows us to derive the optimal approximate marginal log-posterior  $\ln s_{\tau,\pi}^*$  as

$$\begin{aligned} \nabla_{s_{\tau,\pi}} F_{\tau}(\pi) &= 0 \\ \ln s_{\tau,\pi}^* &= -\ln A \cdot o_{\tau} - \ln B_{\pi} s_{\tau-1,\pi} - \ln B_{\pi} \cdot s_{\tau+1,\pi} \end{aligned} \quad (4.10)$$

replacing  $-\ln B_{\pi} s_{\tau-1,\pi}$  by the prior  $\ln D$  for  $\tau = 0$ . Note, however, that due to the MFA, we are only considering approximate beliefs about states immediately before ( $s_{\tau-1,\pi}$ ) and after ( $s_{\tau+1,\pi}$ ) our state of interest to obtain this approximate. One must therefore repeat this computation until convergence such as the information can "propagate" through the network, this procedure being known in the literature as the variational Message Passing algorithm (vMP; [Markovic and Kiebel, 2019, Dauwels, 2007]; see also figure 4.1). In the specific case of the Active Inference literature, we are also interested in a "biologically plausible" implementation of this computation in the brain. Thus, equation 4.10 is generally expressed as

a dynamical system performing a gradient descent on the vFE which, in our case, will be useful for approximating reaction times :

$$\begin{aligned} s_{\tau,\pi} &= \sigma(v_{\tau,\pi}) \\ \dot{v}_{\tau,\pi} &= -\nabla_{s_{\tau,\pi}} F_{\pi,\tau} = -\nabla_{s_{\tau,\pi}} [\ln s_{\tau,\pi}^* - \ln s_{\tau,\pi}] \\ \nabla_{s_{\tau,\pi}} F_{\pi,\tau} &= \ln s_{\tau,\pi} - \ln A_{\tau} \cdot o_{\tau} - \ln B_{\pi} s_{\tau-1,\pi} - \ln B_{\pi} \cdot s_{\tau+1,\pi} \end{aligned} \quad (4.11)$$

where the auxiliary variable  $v$  is often related to the membrane potential driving the firing rates in the underlying neural substrate [Friston et al., 2017a, Parr et al., 2019, Smith et al., 2022] and  $\sigma(x)$  the soft-max function enforcing normalization.

Finally, due to the known weakness of the temporal MFA, which tends to overestimate the precision of posterior beliefs, most implementations of Active Inference in POMDP use a slightly "altered" formulation of the vMP, introduced as a Marginal Message Passing (MMP) in [Parr et al., 2019], such as the gradient in 4.10 is instead taken with respect to the marginal Free-Energy at each time-step:

$$\begin{aligned} F(\pi) &= \sum_{\tau} F_{\tau}(\pi) \\ F_{\tau}(\pi) &= s_{\tau,\pi} \cdot (\ln A \cdot o_{\tau} + \frac{1}{2}(\ln B_{\pi} s_{\tau-1,\pi} - \ln B'_{\pi} s_{\tau+1,\pi}) - \ln s_{\tau,\pi}) \end{aligned} \quad (4.12)$$

Thus, the gradient descent is finally expressed as :

$$\begin{aligned} s_{\tau,\pi} &= \sigma(v_{\tau,\pi}) \\ \dot{v}_{\tau,\pi} &= \gamma (\ln A \cdot o_{\tau} + \frac{1}{2}(\ln B_{\pi} s_{\tau-1,\pi} + \ln B'_{\pi} s_{\tau+1,\pi}) - \ln s_{\tau,\pi}) \end{aligned} \quad (4.13)$$

where  $B'_{\pi}$  is the transpose of  $B_{\pi}$  with normalized column. For completeness, we have also introduced the parameters  $\gamma$  controlling the time constant of the gradient descent, which is fixed to 1/56 in the following simulations.

In comparison to the vMP and other, more complex propagation schemes, such as the well-known belief propagation algorithm (Bethe approximation and Bethe Free-Energy; [Weller et al., 2014]; see also [Schwöbel et al., 2018] for its use in Active Inference) or the generalized message passing scheme (Kikuchi approximation and Kikuchi Free-Energy; [Mohri, 2013]) the MMP may be viewed as an attempt to retain the "biological plausibility" of a vMP while obtaining results closer to those obtained with a belief propagation.

### Approximating the marginal likelihood : Key points

As the technical approach adopted in the last section for the sake of brevity may obscure what has been achieved here, let us quickly summarize the key points before moving on to the "other term", the prior over policies expressed via their Expected Free-Energy, in the BMS.

As mentioned in section 4.2.2, the participant in a CRT is expected to respond to some particular sensory stimulus  $o$  with an appropriate response  $v$ , the "appropriateness" of a response being defined by the reaction rule. With this objective in mind, the gradient descent on the marginal Free-Energy (equation 4.13) can be viewed as achieving two essential things. First, at convergence, the simulated participant's beliefs ( $Q(\tilde{s}|\pi)$ ) closely match the intractable Bayes optimal posterior ( $P(\tilde{s}|\tilde{o}, \pi)$ ). Considering  $o$  as a pattern in the sensory stream, this allows the participant to code this pattern as per a number of features  $s$  (for example the stimulus color). Second, given this "feature code" the participant may then, using its policies as models, simulate different decisions and their outcomes. In other words,  $Q(\tilde{s}|\pi)$  contains information about the past of the agent but also about its future should it follow the policy  $\pi$ . Heuristically, this can be viewed as the agent "simulating", in parallel, all possible responses to the stimulus it is presented with, thus enabling it to search among the "possible futures" for the most rewarding (this is the role of the Expected Free-Energy). In particular, it is therefore important to distinguish between time-steps in the generative process, noted  $t$ , and time-steps in the generative model, noted  $\tau$ . While the former correspond to the "physical time", ticking forward over the course of the trial, the latter corresponds to the "temporal location" of a belief. For example, given the time  $t$  of the generative process, a belief  $Q(s_\tau|\pi)$  correspond to a prediction about the future whenever  $\tau > t$  (predictive inference) and a prediction about the past/present whenever  $\tau \leq t$  (postdictive inference). In particular, this distinction will be important for the computation of the Expected Free-Energy, which, as we'll see in the next section, only considers beliefs about the future.

Returning to the BMS, the agent may now tract equation 4.12 to obtain the MFE a particular policy, which is now a close approximation of the log-marginal likelihood  $\ln P(\tilde{o}|\pi)$  required to compare them as model. Given this approximate evidence, we turn to the computation of a policy's Expected Free-Energy (EFE) which formally act as the prior in the BMS.

#### 4.2.5 Prior over Policies, Habits and Expected Free-Energy

The Expected Free-Energy may be loosely referred to as the "Free-Energy of the future" since it depends solely on the posterior predictive density (beliefs about future states) and has a similar functional form to the GFE. Its role as a prior over policies in the Active Inference formalization of behavior is however a subject of debates, a number of authors arguing that there exist more natural ways to compute such a Free-Energy of the future [Millidge et al., 2021] or what principle motivates this role beyond the usual "reductio ad absurdum" argument that a Free-Energy minimizing agent is mandated to believe that it will minimize its Free-Energy in the future [Friston et al., 2015]. We leave these considerations aside here to being with the expression of the EFE as a literal "expected Gibbs Free-Energy", similar in expression to the GFE in equation 4.4 :

$$\begin{aligned}
G(\pi) &= -\mathbb{E}_{\vec{Q}(\tilde{o}, \tilde{s}|\pi)} \left[ \ln \frac{\vec{P}(\tilde{o}, \tilde{s}|\pi)}{\vec{Q}(\tilde{s}|\pi)} \right] \\
&= -\mathbb{E}_{\vec{Q}(\tilde{s}, \tilde{o}|\pi)} [\ln \vec{P}(\tilde{s}|\tilde{o}, \pi) - \ln \vec{Q}(\tilde{s}|\pi)] - \mathbb{E}_{\vec{Q}(\tilde{o}|\pi)} [\ln \vec{P}(\tilde{o}|\pi)] \\
&\approx -\mathbb{E}_{\vec{Q}(\tilde{s}, \tilde{o}|\pi)} [\ln \vec{Q}(\tilde{s}|\tilde{o}, \pi) - \ln \vec{Q}(\tilde{s}|\pi)] - \mathbb{E}_{\vec{Q}(\tilde{o}|\pi)} [\ln \vec{P}(\tilde{o}|\mathcal{C})] \\
&= -\mathbb{E}_{\vec{Q}(\tilde{o}|\pi)} [D_{KL}[\vec{Q}(\tilde{s}|\tilde{o}, \pi) || \vec{Q}(\tilde{s}|\pi)]] - \mathbb{E}_{\vec{Q}(\tilde{o}|\pi)} [\ln \vec{P}(\tilde{o}|\mathcal{C})]
\end{aligned} \tag{4.14}$$

where we have introduced the notation  $\vec{Q}(\tilde{o}, \tilde{s}|\pi) = \prod_{\tau>t} P(o_\tau|s)Q(s_\tau|\pi)$  (equivalently  $\vec{P}$ ) to denote predictive densities (i.e. defined only over future states and observations). The approximate sign between the second and third lines highlights the "key move" [Friston and Ao, 2012, Friston et al., 2017a, Parr et al., 2022] allowing goal-directed behavior under Active Inference, that is, the substitution of what may be loosely referred to as a policy's "future evidence" ( $\ln \vec{P}(\tilde{o}|\pi)$ ) by the preference ( $\ln P(\tilde{o}|\mathcal{C})$ ) which, heuristically, encodes the marginal probabilities of each outcome under the agent goals (i.e. what the agent wants to observe in its future). In other words, instead of asking the question, "*given my goals, what is my best choice of policy to pursue among my available choice?*", the agent turns the problem on its head and ask, "*given that I achieved my goal in the future, what is the likelihood I pursued this policy among all that are available to me?*". The second "trick", as important as the first, replaces the "true" predictive posterior  $\vec{P}(\tilde{s}|\tilde{o}, \pi)$  with its variational approximation  $\vec{Q}(\tilde{s}|\tilde{o}, \pi)$ , which has for notable effect to reverse the sign in the divergence. In other words, whereas the minimization of the marginal Free-Energy involved minimizing the divergence between approximate and true posterior, minimizing the Expected Free-Energy entails to *maximize* the divergence between the (predictive) approximate and its (future) posterior (i.e. the agent seeks not only preferred outcomes but also those which, once obtained, will maximally change its beliefs). This term is called the "epistemic value" of a policy in the Active Inference literature [Parr et al., 2022].

Conveniently, under the temporal mean-field approximation, the EFE can be decomposed as a sum of marginal EFEs computed for each future time-step  $\tau > t$ , such as :

$$\begin{aligned}
G(\pi) &= \sum_{\tau>t} G_{\pi,\tau} \\
G_{\pi,\tau} &= D_{KL}[Q(o_\tau|\pi) || P(o_\tau|\mathcal{C})] + \mathbb{E}_{Q(s_\tau|\pi)} [H[P(o_\tau|s_\tau)]] \\
&= As_{\tau,\pi} \cdot [\ln A s_{\tau,\pi} - \ln C] - \text{diag}(A^T \ln A) \cdot s_{\tau,\pi}
\end{aligned} \tag{4.15}$$

where we rewrote the marginal EFE in the risk-ambiguity form in the second line by analogy with the way it is numerically computed in the last line. The vector  $C = P(o_\tau|\mathcal{C})$  encodes the agent preferences (i.e goals).

### The Expected Free-Energy : Key points

As previously mentioned in section 4.2.4, the various policies equipping the agent allow it to simulate various decision based on the inferred stimulus feature code  $s$ . Typically in a CRT, the reaction rule maps each possible stimulus to a unique response. In this case, the

agent can predict that every policies which prescribe a response to the stimulus that does not match the reaction rule will lead to a failure in the trial. Conversely, some of these policies do match the reaction rule, allowing it therefore to succeed in the task. Under this view, the EFE can be heuristically understood as scoring each of these "possible futures", obtained by simulating different behaviors, on the basis of their alignment with the agent's preferences, in this case to succeed in the trial.

Given the MFE and EFE computed for each policy, the agent can finally perform the BMS and obtain a posterior over policies, posterior which, given the bias introduced in the computation of the EFE, will assign higher probabilities to the policies that result in a success in the trial.

#### 4.2.6 Posterior over policies and action potentials

Given a policy's expected and marginal Free-Energy, we can now write the Free-Energy as a functional of beliefs about policies  $Q(\pi)$  [Parr et al., 2022].

$$\begin{aligned} F &= \mathbb{E}_{Q(\pi)}[\ln Q(\pi) - \ln P(\tilde{o}, \pi)] \\ &= \mathbb{E}_{Q(\pi)}[\ln Q(\pi) - \ln P(\tilde{o}|\pi) - \ln P(\pi)] \\ &\approx \mathbb{E}_{Q(\pi)}[\ln Q(\pi) + F(\pi) + G(\pi)] \end{aligned} \quad (4.16)$$

where the third line approximates the log-marginal likelihood of a policy by its marginal Free-Energy ( $-\ln P(\tilde{o}|\pi) \approx F(\pi)$ ) and sets the log-prior over policies as their Expected Free-Energy ( $-\ln P(\pi) \approx G(\pi)$ ). Finally, zeroing the gradient of  $F$  with respect to the beliefs about policies,  $Q(\pi)$ , yields the optimal posterior over policies :

$$\nabla_{Q(\pi)} F = 0 \Leftrightarrow Q(\pi) = \sigma(-G(\pi) - F(\pi)) \quad (4.17)$$

where  $\sigma(x)$  is again the soft-max function.

In the Active Inference literature, the prior over policies is commonly supplemented by an additional, biasing term. Noted  $E$  below, this term enters the prior over policies as a set of fixed beliefs resulting from a caching process of past behaviour, in other words, habits [Friston et al., 2016, Maisto et al., 2019, Smith et al., 2022, Markovic and Kiebel, 2019].

$$\begin{aligned} -\ln P(\pi) &= G(\pi) - \ln E \\ \nabla_{Q(\pi)} F &= 0 \Leftrightarrow Q(\pi) = \sigma(\ln E - G(\pi) - F(\pi)) \end{aligned} \quad (4.18)$$

#### Reactions and Reaction times

The Bayesian policy selection process described above is entirely internal to the agent. In order for our simulated participant to react to the stimulus, the final step is therefore to equip it with a motor system that can carry out the actions recommended by the inference process. Importantly, the ideomotor approach to action means that, through this inference process, the agent generates predictions regarding the sensations caused by its (future) actions. These predictions are generative of predictions error which can be then minimized by a reflex arc [Feldman, 2009a, Friston, 2011, Adams et al., 2013]. It is in this sense that the agents performing AIF are often said to "feel themselves move according to their predictions" more than they are "actively moving". Given the posterior over policies obtained

above, the "motosensory predictions" of the agent for the next time-step ( $\tau = t + 1$ ) are obtained via a Bayesian Model Average (BMA; [Friston et al., 2017a, Parr et al., 2021]) such as

$$Q(o_{t+1}) = \mathbb{E}_{Q(\pi)}[Q(o_{t+1}|\pi)] \quad (4.19)$$

In the latter, we refer to this predictive density over motor sensations,  $Q(o_{\tau+1})$  above, as the *action potential* since it determines the fixed points of the agent's motor system. Finally, this action potential gives rise to a prediction error, which is then minimized by a reflex arc

$$\begin{aligned} v^* &= \min_v \epsilon_o(v) \\ \epsilon_o(v) &= D_{KL}[Q(o_{t+1}) || P(o_{t+1}|v)] \\ &= o_{t+1} \cdot (\ln o_{t+1} - \ln AB(v)s_t) \end{aligned} \quad (4.20)$$

where  $B(v) \sim P(s_{\tau+1}|s_\tau, v)$  is the state transition matrix of the generative process (a function of action). We simplify this process here and replace 4.20 with a threshold on the action potential, such that :

$$\begin{aligned} v &= i \text{ if } Q(o_{t+1} = i) > .7 \Rightarrow t = t + 1 \\ &= \emptyset \text{ else } \Rightarrow t = t \end{aligned} \quad (4.21)$$

Where the second line indicates that no action is taken if the action potential never exceeds 70% probability for any particular action, in such case the process time  $t$  is kept the same ( $t = t$ ), while a generative process transition occur whenever an action has been enacted ( $t = t + 1$ ). Simply put, whenever the participant predict that it will feel a particular reaction being enacted at time  $t + 1$  with a probability above 70%, we consider this as a reaction. How "long" does it take for this to happen will be our reaction time.

Reaction time here is understood in term of a number of gradient descent steps on the marginal Free-Energy (equation 4.13). Therefore, in what follows, we refer to as a "time-unit" a small step down the marginal Free-Energy gradient while a reaction time, for example of 80 time-unit, will correspond to the number of gradient descent steps required between the presentation of the cue to the agent and the action potential crossing the 70% threshold.

A summary of the entire process can be found in figure 4.1.

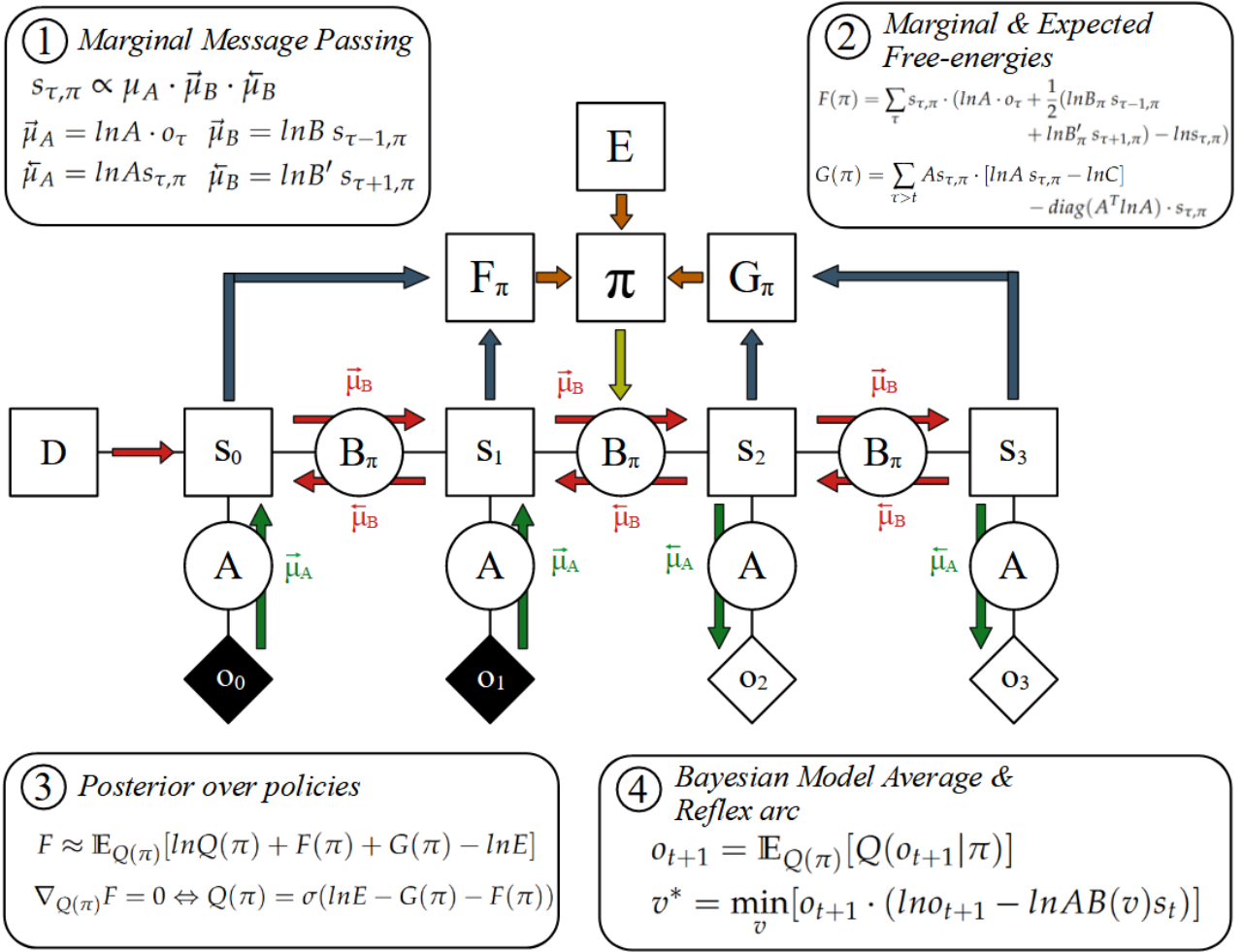


Figure 4.1: The Free-Energy minimization process on a generative model expressed as a POMDP. 1. Red and green arrows: The observation of the outcome  $o_1$  at  $t = 1$  by the agent triggers a belief update process via marginal message passing allowing to infer the maximum a posteriori distribution over past and current states ( $s_0$  and  $s_1$ ) but also over future states ( $s_2$  and  $s_3$ ) and outcomes ( $o_2$  and  $o_3$ ). 2. Blue arrows: At equilibrium, the marginal Free-Energy at each time-step is integrated to obtain the approximate marginal likelihood of outcomes given each policy ( $F_{\pi}$ ). In parallel, each policy's Expected Free-Energy ( $G_{\pi}$ ) is obtained by integrating the marginal EFE at each future time-step. 3. Orange arrows: Free-Energy and Expected Free-Energy are combined with the habits ( $E$ ) to obtain a posterior over policies. 4. Yellow arrow: A Bayesian Model Average over policies is computed to obtain sensory expectations at the next time-step, expectations carried-out by a minimization of prediction error by the motor system. The agent act and the process moves one time-step in the future at  $t = 2$ , offering a new observations  $o_2$  and the whole process repeats.

### 4.2.7 Parameterizing the generative model

To close this technical section before moving to the simulation of SRC effects, we introduce here the important parameters in the model that will be kept fixed thereafter. We then shortly illustrate the Active Inference process with an example of its application in the context of a two-choice reaction task.

#### Preferences

In such experiment, it is reasonable to assume that the participant want to "succeed" in the reaction task. As mentioned in the previous section, goal-directed behaviors in Active Inference are obtained via the parameterization of the agent's goal as a log-marginal probability distribution over outcomes, the preferences  $C \sim \ln P(\tilde{o}|\mathcal{C})$ . In practice, this can be viewed as a vector encoding the reward (more precisely value) associated with all observations that can possibly be made over the course of the trial. In the later, this reward is set to 0 for all outcomes  $o$  except for the feedback provided at time-step  $t = 2$  (feedback time period) where we set :

$$\begin{aligned}\sigma(C) &= P(o|\mathcal{C}) \\ C(o_{\text{feedback}} = \text{success}) &= 3 \\ C(o_{\text{feedback}} = \text{failure}) &= -5 \\ C(o_{\text{feedback}} = \text{none}) &= 0\end{aligned}\tag{4.22}$$

where *none* is the "neutral" feedback received by the participant during the preparation and decision phase ( $t = 0$  and  $t = 1$ ) and  $\sigma(x)$  the softmax function. Note that the agent places more importance on avoiding failure (reward of  $-5$ ) than on achieving success (reward of 3). During testing, we found that introducing this asymmetry resulted in better results with regard to SRC effects.

#### Feature codes and salencies

Importantly for our simulation, the time constant associated to the translation of a stimulus into a corresponding feature code is primarily driven by the *saliency* of the feature (note that we use this term in the computational sense of "expected entropy" or ambiguity). In other words, since a stimulus may be decomposed into an arbitrary number of features, the uncertainty regarding the value (code) of salient features is resolved faster than comparatively less salient features. In the case of a POMDP, a feature's saliency is encoded within the categorical likelihood  $A \sim P(o|s)$ , which formally describes the likelihood to observe a given pattern in the sensory stream (the outcome  $o$ ) given a particular combination of features (the hidden cause  $s$ ).

Since most CRT use relatively simple stimulus with only a few distinct features, we assume here that each feature is encoded independently of the others. That is, the likelihood  $P(o|s)$ , which given each feature code  $s$  gives a probability for a sensory pattern  $o$ , can be decomposed as a product of  $N$  "elementary likelihood" such as  $P(o|s) = \prod_{i=1}^N P(o_i|s_i)$ . For example, considering a typical stimulus set  $\mathcal{O} = \{\text{red-square}, \text{red-triangle}, \text{green-square}, \text{green-triangle}\}$ , we assume that the stimulus color  $o_1 \in \{\text{red}, \text{green}\} = \mathcal{F}_1$  and its shape  $o_2 \in \{\text{square}, \text{triangle}\} = \mathcal{F}_2$  are coded independently of each other into a color-code  $s_1 \in \mathcal{F}_1$  and a shape-code  $s_2 \in \mathcal{F}_2$ .

Simplifying further, we also assume that, for each elementary likelihood  $P(o_i|s_i)$  associated to a feature  $i$ , we have the following :

$$\begin{aligned} p &= P(o_i = k|s_i = k) \quad \forall k \in \mathcal{F}_i \\ q &= P(o_i = k|s_i = j) \quad \forall (k, j) \in \mathcal{F}_i^2, j \neq k \\ &= \frac{1 - p}{|\mathcal{F}_i| - 1} \end{aligned} \quad (4.23)$$

where  $|\mathcal{F}_i|$  is the cardinality of the set  $\mathcal{F}_i$  (number of possible values for the feature  $i$ ). In other words, the matrix representation of the elementary likelihood  $P(o_i|s_i) = \text{Cat}(A_i)$  is a square matrix of size  $n \times n$  with  $n = |\mathcal{F}_i|$ , with  $p$  along its diagonal and  $q$  everywhere else. Given these assumptions, we define the saliency  $\beta_i$  of feature  $i$  as

$$\beta_i = \frac{p}{q} \quad (4.24)$$

A visual interpretation of this parameterization along with an example of its effect on the speed at which a stimulus feature is coded is presented in figure 4.2.

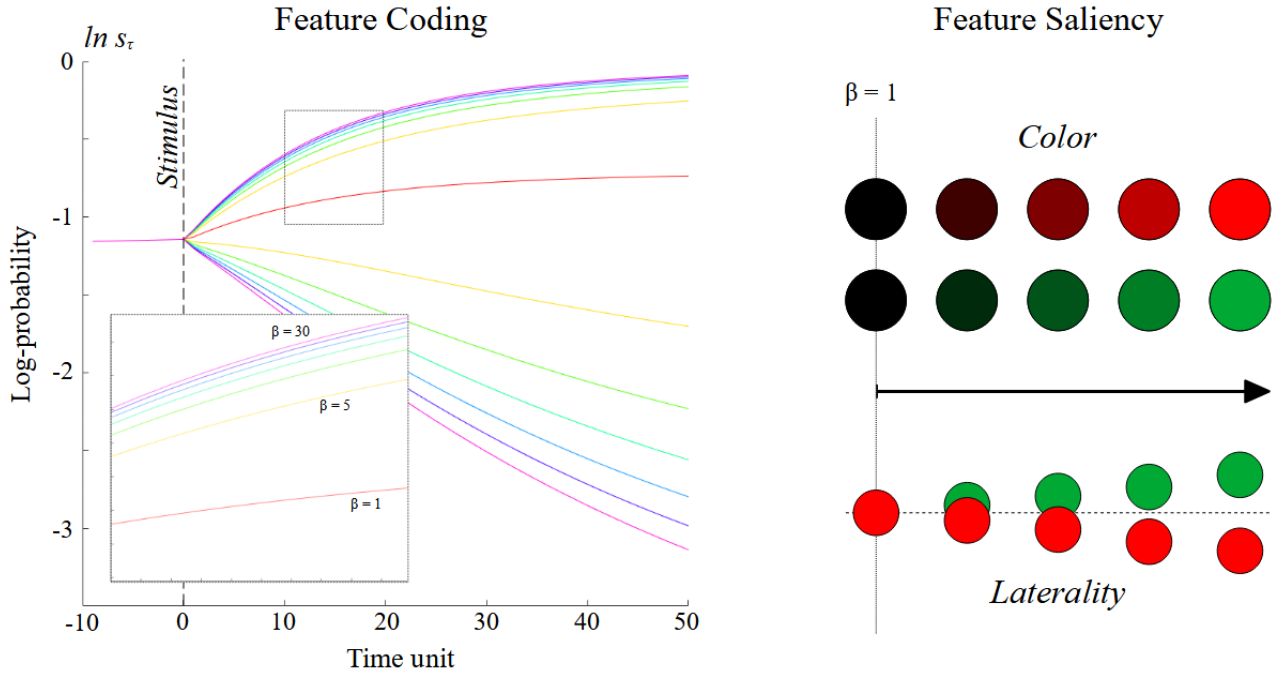


Figure 4.2: The effect of a (binary-valued) feature's saliency  $\beta$  on the time constant associated to its coding (left) and a visual interpretation of an increase in saliency for the color feature and laterality (left-right) feature we use as example in the following simulations (right). The coding of a stimulus various features is here associated to the gradient descent on the MFE allowing to obtain the approximate posterior  $Q(s) \sim P(s|o)$  which to a sensory pattern  $o$  associate the posterior probability for  $o$  to be coded by  $s$ . Here is compared the evolution of this approximate posterior over time-unit (gradient descent steps) under different saliency parameters for binary valued features ( $\beta = 1$  to  $\beta = 30$  with increments of 5; color coded). The bottom left insert is a zoom on the region enclosed by the dashed rectangle (time unit 10 to 20).

### What is the stimulus color...

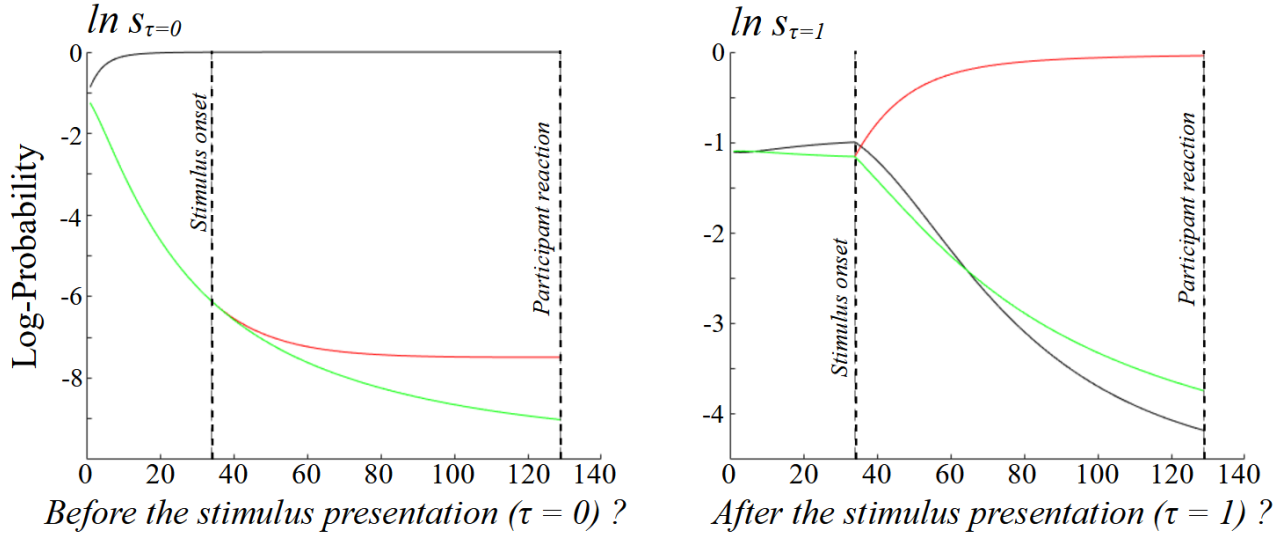


Figure 4.3: The evolution of the participant beliefs (in log-probabilities) about the stimulus color (*none* in black, *red* in red and *green* in green) over the course of the trial. On the left are shown the beliefs about the stimulus color during the preparation phase (prior to the presentation of the stimulus, its color is *none*). While on the right are shown the participants beliefs about the stimulus color during the decision phase (after being presented with the red stimulus). For both these graphs, vertical dashed line mark the transition between the trial phase triggered by a decision of the participant. The transition from the preparation to the decision phase occurs at time-unit 38, which also corresponds to the stimulus onset. Similarly, the transition from the decision to the feedback phase occurs at time-unit 120, which also correspond to the reaction of the participant (the reaction time in this example is therefore 82 time-units). Note that a time-unit is here understood as a small step down the MFE gradient.

#### An example

Using the procedure we described above, we can now simulate a trial in our CRT and investigate the prediction of the model w.r.t to a participant's "reaction time" (i.e. number of gradient descent steps between the presentation of a stimulus and a reaction) as a function of different parameterizations of the generative model. Before moving on to the SRC phenomenon, we briefly overview the simulation of a trial to give an intuition of the quantities we discuss later.

Consider a CRT where the participants are instructed to react to colored cues (stimuli), a red and a green light, by pressing one of two keys, a red and a green key. In this example, the reaction rule requires the colored cue to be answered by pressing the key with matching color. To simplify, we assume that, regardless of the reaction rule, only two policies can be followed by the participant. The participant can answer the colored cue by pressing the key with matching color or invert the relationship. We refer to these two policies as the *color-congruent* and *color-incongruent* policy respectively. The agent goal here is therefore to infer which of the two is to be followed if it wants to succeed in the trial.

To do so, the first thing required when presented with a stimulus  $o$  is to infer its color (we set  $\beta = 25$  in this example) which is formally the role of the gradient descent on the MFE.

Figure 4.3 shows how the agent beliefs about the stimulus color evolve over the course of the trial.

Importantly, note that these beliefs are "temporally locked", that is, both graphs represent the evolution of the (log) probability distribution ( $Q(s_{\tau=0})$  on the left and  $Q(s_{\tau=1})$  on the right) over the course of the whole trial ( $t = 0, 1, 2$ ). Notably for the graph on the right, which represents the participant's beliefs about the stimulus color during the decision phase ( $\tau = 1$ ), the probability distribution corresponds to a prediction about the future from time-unit 0 to 38, when the trial is still in its preparation phase and a postdiction from time-unit 38 onward, when the trial has reached the decision phase. We see that the participant is initially uncertain about the future color of the stimulus (roughly equal log-probabilities between the three possible value of a stimulus color, *none* in black, *red* in red and *green* in green) and then resolves this uncertainty after being presented with the red stimulus at time unit 38.

Following the observation of the red stimulus at time-unit 38, the two policies equipping the participant make different predictions about which of the two keys will be pressed in response. As shown in figure 4.4, the color congruent policy predicts that the red key will be pressed in response to the stimulus, while the color incongruent policy conversely predicts that the green key will be pressed.

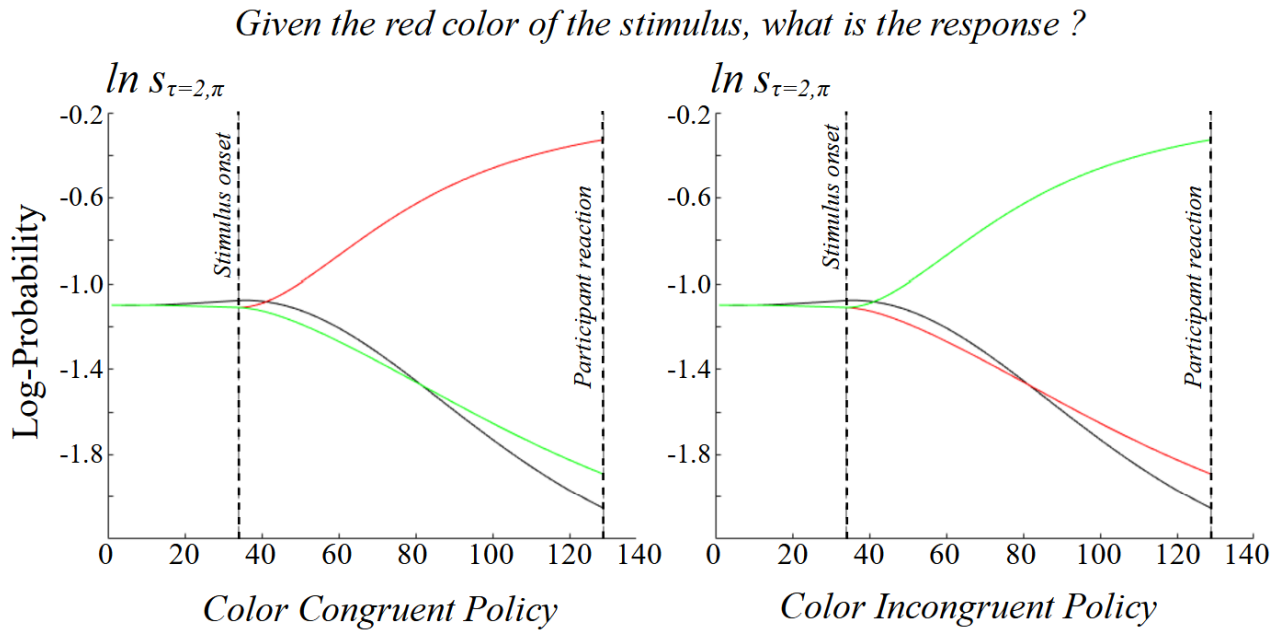


Figure 4.4: The beliefs of the participant about the state of its motor system at  $t = 2$  ( $Q(s_{\tau=2}|\pi)$ ) should it follows the color-congruent (left) or color-incongruent (right) policy. Each of these curves correspond to the log-probability associated to the *wait* action (black), *red key press* (red) and *green key press* (green). Note that, as before, these beliefs about the action enacted during the transition from  $t = 1$  to  $t = 2$  are temporally locked, such as the participant is uncertain about its future action given each of the two policies during the preparation phase ( $t = 0$ , time-unit 0 to 38; before the stimulus onset) and subsequently resolve this uncertainty after the observation of the stimulus.

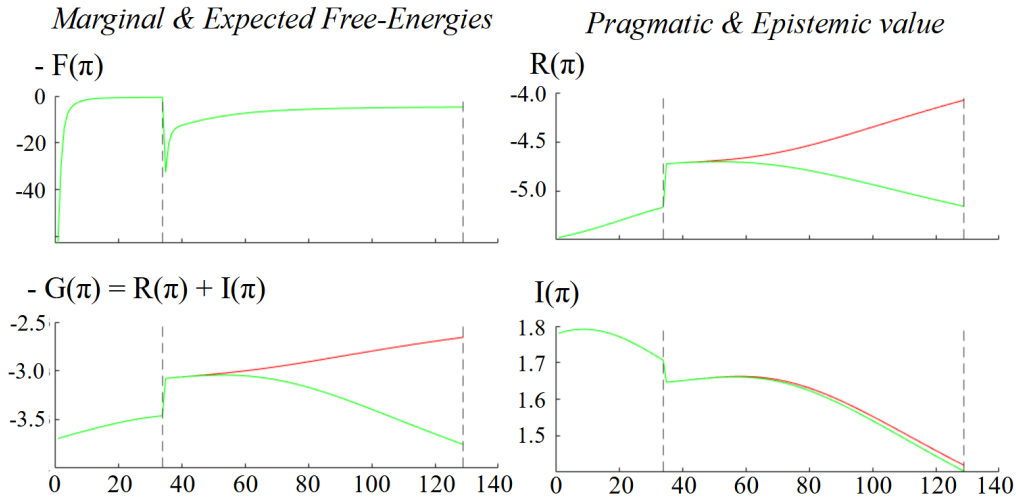


Figure 4.5: Left. The evolution of the (negative) Marginal ( $-F(\pi)$ , top) and Expected Free-Energies ( $-G(\pi)$ , bottom) under each policy (color-congruent in red and color-incongruent in green) over the course of the trial. Right. The decomposition of the Expected-Free Energy into pragmatic ( $R(\pi)$ , top) and epistemic ( $I(\pi)$ , bottom) value. Given the simplicity of the task, the Marginal Free-Energy and epistemic value are virtually the same for the two policies. The main difference between the two resides in the pragmatic value component of the Expected Free-Energy. Here the color-congruent policy, whose predicted response matches that of the reaction rule, yields the smallest Expected Free-Energy and is therefore more likely to be followed than its color-incongruent counterpart.

Since the agent knows the reaction rule, which here specifies that the red key must be pressed in response to the red stimulus, this also enables it to predict that the color-congruent policy will yield a success in the trial while, conversely, the color-incongruent policy will lead to a failure. Although we do not show these predictions here, their effects can be observed through the Expected Free-Energy associated with each policy which, in this simple example, behaves similarly to an expected value (figure 4.5).

Finally, by combining the Marginal and Expected Free-Energies for the two policies, we obtain, via equation 4.18, the approximate posterior over policies (figure 4.6) which heuristically represent the probability for any particular policy to be followed. It's important to note that, unlike beliefs about the states which are temporally locked, beliefs about policies are not, such that the graph in figure 4.6 - left represents the probability for a policy to be followed during the preparation phase from time-unit 0 to 38, and during the decision phase from time-unit 38 onward. As can be seen, due to its lower Expected-Free-Energy, the color-congruent policy is much more likely to be followed than its color-incongruent counterpart after the observation of the stimulus (and not before since both policies make equivalent predictions during the preparation phase; figure 4.4).

In turn, the action potential, which represents the probability for the simulated participant to enact any particular action at each time-step, can be obtained by a weighted average of the actions predicted by each policy given its posterior probability (the Bayesian Model Average underlined in equation 4.20).

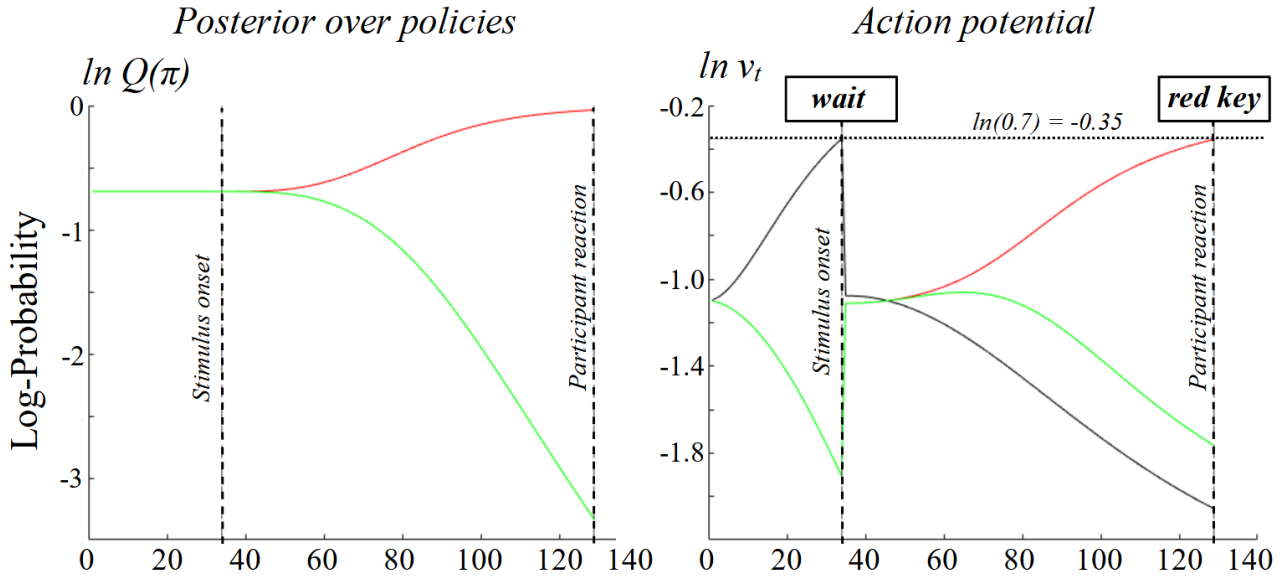


Figure 4.6: Left. The log-posterior over policies  $Q(\pi)$  (color-congruent in red and color-incongruent in green) obtain as per equation 4.18 given the two policies' Expected and Marginal Free-Energies shown in figure 4.5. Note that, conversely to beliefs about states, beliefs about policies are not temporally locked such as the two curve represent the log-posterior probability associated to each policy during the preparation phase ( $t = 0$ ; time-unit 0 to 38) and during the decision phase ( $t = 1$ ; time-unit 38 onward). Right. The evolution of the action potential (log-probability over possible actions) during the preparation phase ( $t = 0$ ; time-unit 0 to 38) and during the decision phase ( $t = 1$ ; time-unit 38 onward). For each phase, the log-probability for the participant to enact the *wait* action is shown in black, the *red key press* in red and the *green key press* in green. The top dotted line marks the decision threshold of  $\ln(0.7) = -0.35$ . For each phase, the winning action is reported in the top square.

This action potential is shown on the right of figure 4.6. As with the posterior over policies, this potential is also not temporally locked. Two decisions can be observed here, the first corresponding to the participant choice to *wait* during the preparation phase and the second corresponding to the response to the stimulus at the end of the decision phase. Here, given that the color-congruent policy is much more likely (20 to 1 in odds) to be followed than its color-incongruent counterpart it is also the response it predicts, the red-key press, that finally reaches the decision threshold of  $\ln(0.7) = -0.35$  we set, allowing the participant to move to the feedback phase to obtain its rewarding *success* feedback.

### 4.3 Congruency biases and Sensory Response Compatibility

As mentioned in the introduction of this chapter, the concept of Sensory Response Compatibility refers to a broad range of experimental observations where a participant's performance in a CRT, as measured by accuracy and reaction times, is eminently dependent on the pairings between stimulus and responses introduced by the reaction rule (we also use the term SR-map below). For a typical example, a study performed by [Simon et al., 1981], whose results are partially reported in table 4.7, showed that, given a CRT where participants are tasked to answer a red or green visual stimulus by pressing a red or green key, the accuracy and reaction time are systematically better if the reaction rule requires the participant to answer the colored stimulus by pressing the key of matching color.

| SR-Map      | Subject Average Reaction Time (ms) |           |         |
|-------------|------------------------------------|-----------|---------|
|             | Red Key                            | Green Key | Average |
| Congruent   | 698                                | 568       | 633     |
| Incongruent | 790                                | 819       | 804     |
| Difference  | 92                                 | 251       | 171     |

Figure 4.7: The average reaction times observed under congruent and incongruent SR-maps in [Simon et al., 1981]. In this experiment, the subject had to respond to a red or green colored visual stimulus by pressing a red or green colored key. As can be observed, the participant is systematically slower to answer under the incongruent SR-map, where the red (green) visual cue is to be answered by pressing the green (red) key, than under the congruent SR-map, where the color of the cue match that of the key, and so irrespective of the specific key being pressed.

To account for this seemingly inexplicable difference in performance between two reaction rules that, from a computational perspective, do not alter the difficulty of the task, the dimensional overlap theory [Kornblum et al., 1990, Kornblum et al., 1999] postulates that, because the discriminative features of the stimulus overlap with the discriminative features of the responses in this task (the visual stimuli and the response keys both can be distinguished by their color), the difference in performance is due to a priming effect of the color of the visual stimulus on the color of the response key, such that the participant is naturally drawn towards a response with the red key when presented with a red stimulus and vice versa. In this case, when the reaction rule associates stimuli and responses in a color incongruent fashion, the participant experiences an internal conflict between the color code of the stimulus and the color code of the response, resulting in a longer reaction time.

#### 4.3.1 Representing compatibility under Active Inference

Adopting the viewpoint of the dimensional overlap account of SRC, we can represent this natural tendency to answer a stimulus with a congruent response by a bias in the decision process such that it is a priori more likely that the feature code of the response match that of the stimulus. In the Active Inference framework, this is the role of the  $E$  vector (equation 4.18) corresponding to the prior bias over policies.

Given a stimulus and response sets overlapping over  $n$  (binary) features, we parameterize the participant's natural tendency to respond to the stimulus in a feature congruent manner on a feature basis. In other words, for each stimulus feature  $i$ , the simulated participant is endowed with both a feature congruent and a feature incongruent policy. The bias in favor of congruent responses for feature  $i$  is then parameterized by a scalar  $e_i$ , subsequently referred to as feature's  $i$  *congruency bias*, representing the relative log-prior bias in favor of the feature congruent policy versus the feature incongruent policy. Returning to our example based on color, a color congruency bias  $e_c = 1$  implies that it a priori  $e^{2e_c} \approx 7.4$  times more likely that the color of the response matches the color of the stimulus. This can equivalently be viewed as the simulated participant answering colored stimulus with a congruently colored response 88% of the time in its "daily life".

Finally, when the stimulus possesses more than one feature, it may also be interesting to consider the relative contribution of these features in the decision process. In the later, we refer to feature's  $i$  *prior decisional weight* as the scalar  $b_i$  modulating feature's  $i$  "biasing power" in relation to the other features. For example, considering a stimulus with two features with prior decisional weight  $b_1 = 0$  and  $b_2 = 1$  respectively, feature 2 is a priori  $e^{b_2 - b_1} = 2.71$  times more likely to predict the response to the stimulus than feature 1.

### 4.3.2 Simulating a Sensory Response Compatibility

Given this parameterization, we can now test the simulated participant's behavior in a CRT similar to [Simon et al., 1981] and whose results we showed in table 4.7. In this simple CRT, the stimulus included only one discriminative feature, a color which is assumed to have a relatively high saliency. We therefore set  $\beta_{color} = 25$  here and test the model's predictions under various congruency biases, ranging from  $e_{color} = 0$  (no bias) to  $e_{color} = 2$  (strong bias), and thus under both color-congruent and color-incongruent reaction rules. The results are shown in figure 4.8.

A clear effect of the congruency of the SR-map on simulated reaction time can be observed, with the SRC effect increasing as the congruency bias becomes larger. Furthermore, we note that the SRC effect obtained here is in fact the sum of two effects. In comparison to the "neutral case" where the congruency bias is 0 (and for which the reaction times are the same under the feature-congruent and feature-incongruent SR-map), increasing this bias both shortens the reaction time under the feature congruent SR-map, the facilitatory regime, and lengthens the reaction time under the feature incongruent SR-map, the perturbatory regime.

This change in reaction time as a function of congruency bias can easily be explained by noting that this bias formally acts as an initial condition for the posterior over policies. Figure 4.9 allows to compare the effect of this congruency bias between the three extreme cases showed in figure 4.8. That is, a trial performed under color congruent SR-map with strong color congruency bias ( $e_{color} = 2$ , left column), a trial performed under color-congruent SR-map with no congruency bias ( $e_{color} = 0$ , center column) and a trial performed color incongruent SR-map with strong color congruency bias ( $e_{color} = 2$ , right column).

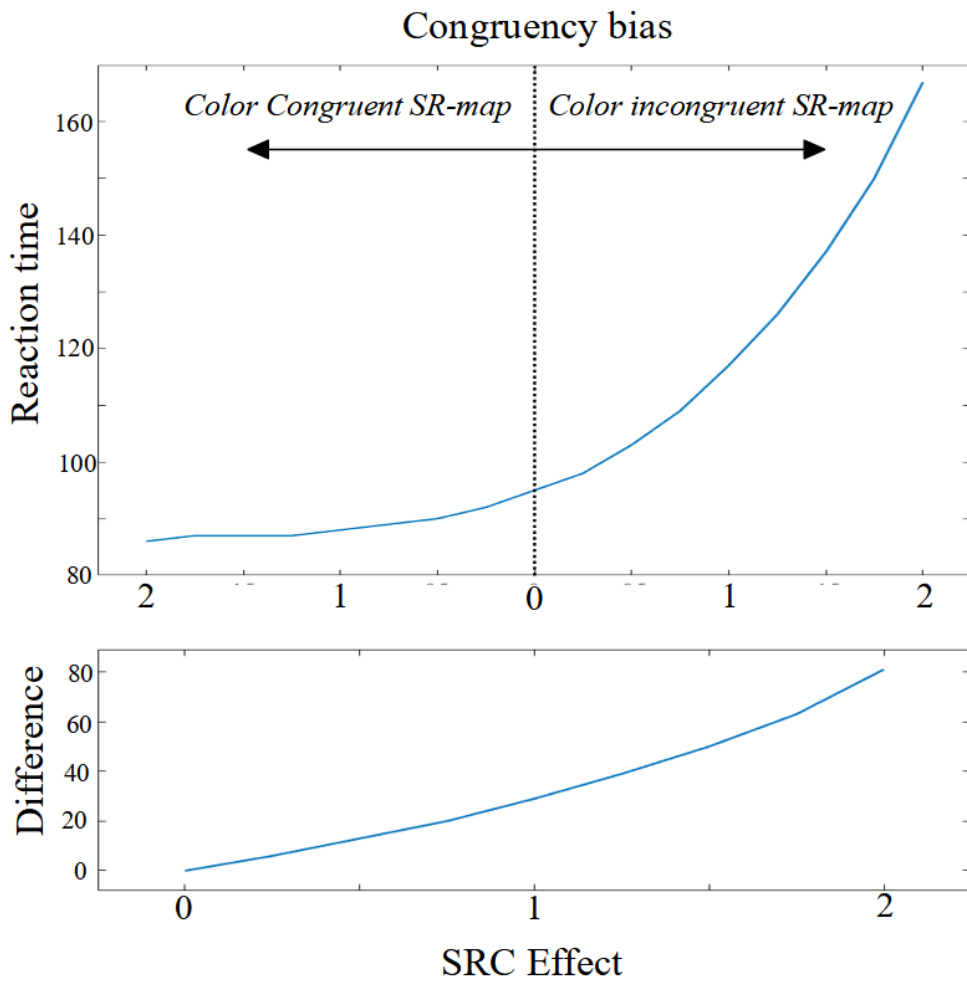


Figure 4.8: A SRC effect resulting from a prior bias over policies in the AIF framework. Top. The reaction time unit (number of gradient descent steps between the stimulus presentation and the reaction) as a function of color congruency bias under both color congruent and color incongruent SR-map. Bottom. The SRC effect (difference in performance between the feature congruent and feature incongruent SR-map) as a function of congruency bias.

For all three examples, the simulated participant was presented with the red visual stimulus and successfully performed the task by pressing the red key in the first and second cases (left and center column) and the green key in the last case (right column), resulting in a reaction time of 86, 95 and 167 time-units respectively. It is important to note that, in all the graphs in figure 4.9, the period to the left of the first vertical dashed line (time-unit 0 to 35) corresponds to the preparation phase ( $t = 0$ ). The model is parameterized such as all policies would be equivalent during this period (i.e. their EFE, shown in the top row, are exactly the same), so that the posterior over policies (center row) is initially influenced only by the congruency bias. The origin of the difference in simulated reaction time between the three cases is fairly obvious if we look at the evolution of the posterior over policies. In the first case, this posterior hardly changes over the course of the trial, such that the reaction time is mainly determined by the time required to identify the stimulus color. Conversely in the second and third cases, a large variation of the posterior over policies can be observed, a variation induced by the evolution of the policies' EFE shown immediately above.

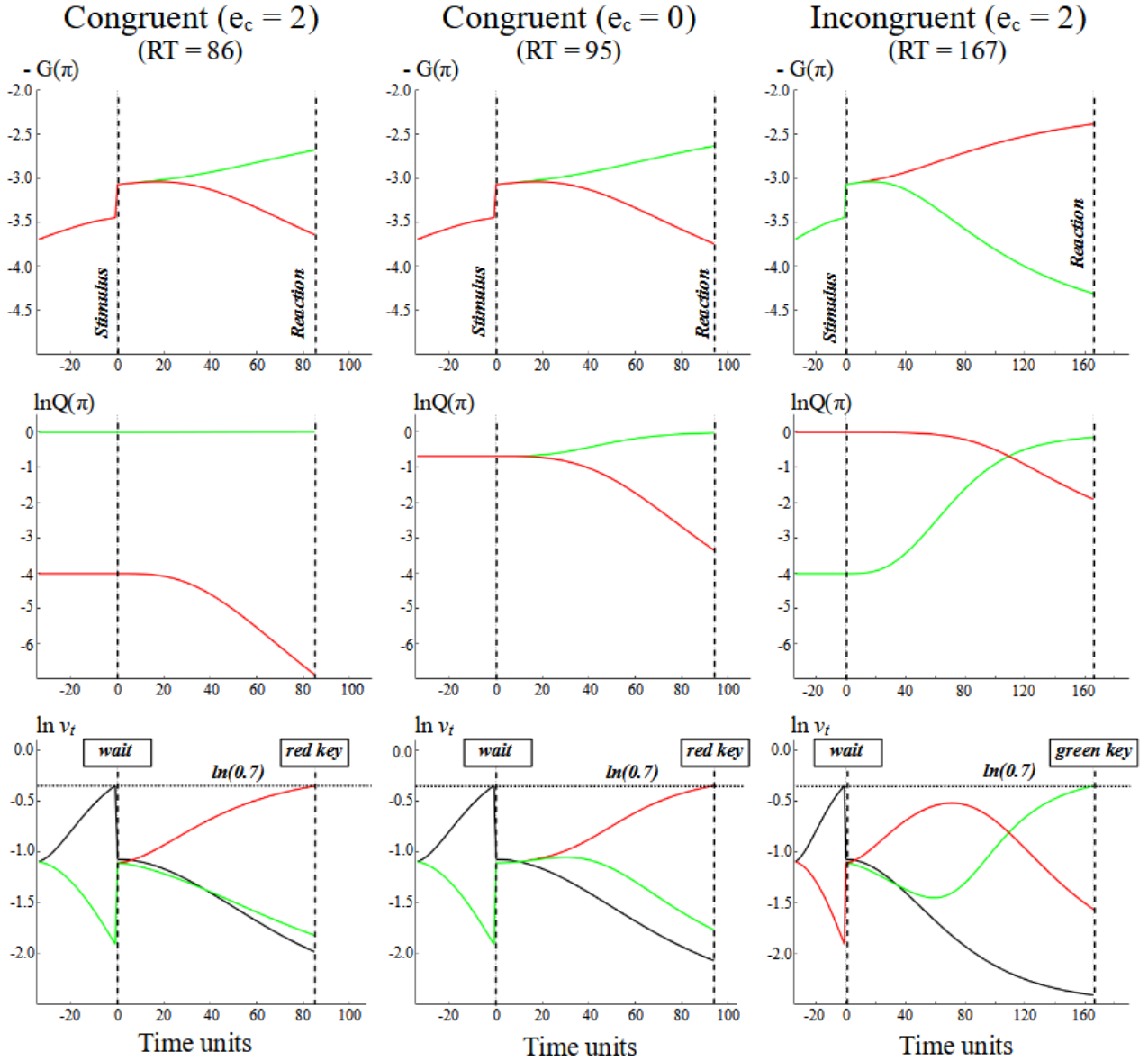


Figure 4.9: The decisional dynamics under three parameterization of the generative model. From left to right, strong congruency bias ( $e_{color} = 2$ ) under color-congruent SR-map, no bias ( $e_{color} = 0$ ) under color-congruent SR-map, strong bias ( $e_{color} = 2$ ) under color-incongruent SR-map. For all three examples, the simulated participant observed the *red* stimulus such as a success is achieved by pressing the *red*, *red* and *green* key respectively. From top to bottom. The evolution over the course of the trial of the (negative) Expected Free-Energy ( $G(\pi)$ ) associated to the color-congruent policy (green) and color incongruent policy (red). The evolution of the (log) posterior over policies ( $\ln Q(\pi)$ ) with the color-congruent policy in green and color-incongruent in red. The evolution of the action potential ( $v_t = \ln Q(o_{t+1})$ ) (*red key* in red, *green key* in green and *wait* in black). For all graphs, the first vertical dashed line marks the presentation of the stimulus to the participant while the second marks the reaction of the participant to the stimulus. See the main text for discussion.

Importantly then, because the computation of this functional requires to make predictions about the trial evolution under a particular policy (notably whether that policy, when pursued, results in a success or a failure in the trial), its time constant is necessarily longer than the time constant associated to the feature integration process (perceiving the stimulus color). Indeed, for the EFE to vary, the information obtained through this feature integration needs to be propagated from beliefs about the present to predictions about the future. This longer time constant can be seen in the top row of graphs showing the EFE associated to the color-congruent (green) and color-incongruent (red) policies. While the stimulus is presented at time-unit 0 (vertical dashed line), thus starting the feature integration process allowing to identify its red color, the EFE of the two policies only starts to diverge roughly 10 time-units later, corresponding to the time necessary for the sensory information to propagate from current to future states.

An interesting result can be seen by looking at the action potential in the third case (bottom-right graph), where a participant with a strong color-congruency bias performed the reaction task under a color-incongruent SR-map. Because of its strong bias in favor of color-congruent responses, we see here that the red-key press is, in a sense, "automatically activated" by the observation of the red stimulus. Note that this automated activation also occurs when this participant performs the reaction task under the color-congruent SR-map (bottom-left graph), which explains why the probability associated to the red-key and green-key press immediately starts to diverge after the presentation of the stimulus, whereas it takes roughly 10 time-units (the EFE time-constant) for this to occur in the absence of bias (bottom center plot). Such automated activation can in fact be observed "in-vivo" in SRC studies and is one of the key results motivating the Dual-route model of the SRC phenomenon. However, as this partial activation of the incorrect response is more commonly associated to the Simon effect, let's first move to a simulation of this particular case before examining this aspect in more detail.

#### 4.3.3 The effect of irrelevant features : the Simon effect under Active Inference

The simulation presented in the previous section can be linked to the "proper" SRC effect, that is, a difference in performance obtained where the overlapping feature between stimuli and responses is relevant to the task (this is sometimes referred to as a *type II SR-ensemble*; [Kornblum et al., 1990]). Indeed, in [Simon et al., 1981], the participants are specifically instructed to answer the red / green stimulus with the red / green key (or vice versa). The colors of stimulus and keys are therefore features that *need* to be encoded for the participant to perform the task correctly. It is however possible to obtain an SRC effect involving a feature that is completely irrelevant to the task, the term "irrelevant" being understood here as the value of this feature for a stimulus being completely uncorrelated with the subsequent response. A well known example of this is the Simon effect [Simon and Rudell, 1967, Simon and Small Jr, 1969, Simon, 1969, Cespón et al., 2020] where the irrelevant feature corresponds to the position (we call it the "laterality" feature hereafter), usually left or right, of stimuli and responses relative to each other.

The Simon effect was first reported by J.R. Simon and A.P. Rudell in the auditory modality [Simon and Rudell, 1967] and in the visual modality three years later [Simon and Craft, 1970]. For an "in vivo" example of this phenomena, we can in fact refer to the same study we previously took as example of SRC effects based on color congruency [Simon et al., 1981].

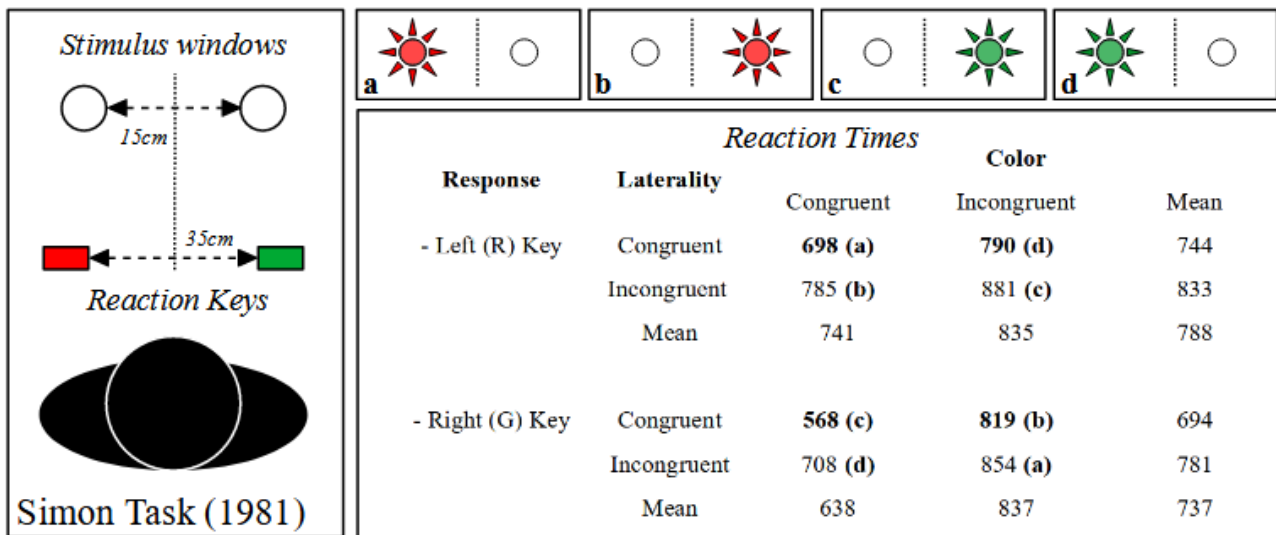


Figure 4.10: The effect of an irrelevant feature on the decision process. Left. The apparatus in [Simon et al., 1981], two colored keys are disposed to the left and right of the participant body midline. Colored visual stimuli are provided via two windows disposed above each key. Depending on the condition, participants are instructed to answer stimulus in a color-congruent or color-incongruent fashion and so irrespective of the laterality (window) of the stimulus (the four possible stimuli are shown at the top). Right. The mean reaction time decomposed by responding key and feature (color and laterality) congruency. A clear SRC effect is observed for both features. In bold are the mean reaction time took as illustration in the previous section.

In this study, color-congruency was not the only manipulated experimental variable, the participants were also tested under both laterality-congruent and laterality-incongruent SR-maps. Put differently, in addition to colors, the protocol called for the visual stimuli and response keys to be distributed to the left and right of the participant body midline (see figure 4.10) and, while the positions of response keys were fixed (red key on the left and green key on the right), the red and green stimuli could appear on either side of the participants' body midline. In this case, therefore, we have two overlapping features, a task relevant color feature and a task irrelevant laterality feature. Reviewing the results the authors obtained, reproduced in figure 4.10, it is clear that a SRC effect is occurring for both of them.

Similarly to the dimensional overlap theory, most (but not all; see for example [Hasbroucq and Guiard, 1991, Umiltà and Nicoletti, 2016]) accounts of the Simon effect postulate that the difference in reaction time observed for the laterality results from a conflict between the spatial code of the stimulus and that of the response [Wallace, 1971, Umiltà and Liotti, 1987, Umiltà et al., 1999, Hommel, 1993b, Hommel, 2011b, Hasbroucq and Guiard, 1991]. In particular, the fact that SRC effects can be observed for the laterality feature that is irrelevant to the task is seen as major evidence for a largely "automatic" feature coding process. Indeed, for a conflict to occur between spatial codes, it must be assumed that these codes are formed and this irrespective of their relevance for the subsequent decision [Nicoletti et al., 1984, Umiltà and Nicoletti, 2016, Cespón et al., 2020]. We can therefore apply the same feature integration procedure to all features, irrespective of their relevance.

Adding the laterality feature to our model, we can replicate the Simon effect in the Active Inference framework by testing the reaction time of our simulated participant with the same combination of color and laterality as tested in our example study. Note that, as with the participants in [Simon et al., 1981], the simulated participant knows that the stimulus laterality is irrelevant and that the reaction rule only consider the stimulus's and key's color. The natural tendency of the simulated participant to answer the stimulus in a color-congruent and laterality-congruent fashion is here represented via two congruency biases,  $e_C$  and  $e_L$ , each representing the relative bias in favor of a color-congruent versus color-incongruent policy and laterality-congruent versus laterality-incongruent policy (giving a total of four policies in the participant's generative model). Assuming that both features have similar saliency, we set  $\beta_C = \beta_L = 25$ , and start with  $b_C = b_L = 0$ , meaning that color and laterality have the same relevance for the decision on average.

Table 4.11 shows the results obtained when both color and laterality congruency biases are set to 2 while figure 4.12 sweeps through different combinations of color and laterality congruency biases.

| <i>Simulated Reaction Times</i> |              |             |             |
|---------------------------------|--------------|-------------|-------------|
| <b>Laterality</b>               | <b>Color</b> |             | <b>Mean</b> |
|                                 | Congruent    | Incongruent |             |
| Congruent                       | 87           | 95          | 91          |
| Incongruent                     | 95           | 167         | 131         |
| Mean                            | 91           | 131         | 127         |

Figure 4.11: The simulated reaction time obtained under a congruency bias of  $e_C = e_L = 2$  for both laterality and color features.

In trend, the simulation is in accordance with the results found in the literature, that is, the shortest reaction times are obtained when the participant responds to the stimulus in both a color and laterality congruent fashion and longest with this response is both color and laterality incongruent (figure 4.10). Interestingly, the model also predicts largely non-linear interactions between the SRC effects induced by the two features. Heuristically, the feature to which the participant responds congruently is, to some extent, able to compensate for the SRC effect induced by the feature to which one respond in a non-congruent fashion, so that, overall, the difference in reaction time between congruent and non-congruent SR-maps is much smaller than when a feature is taken in isolation (figure 4.8 in the previous section). We also note that the effect of congruency bias is perfectly symmetrical between the two features, and so independent of their relevance to the task.

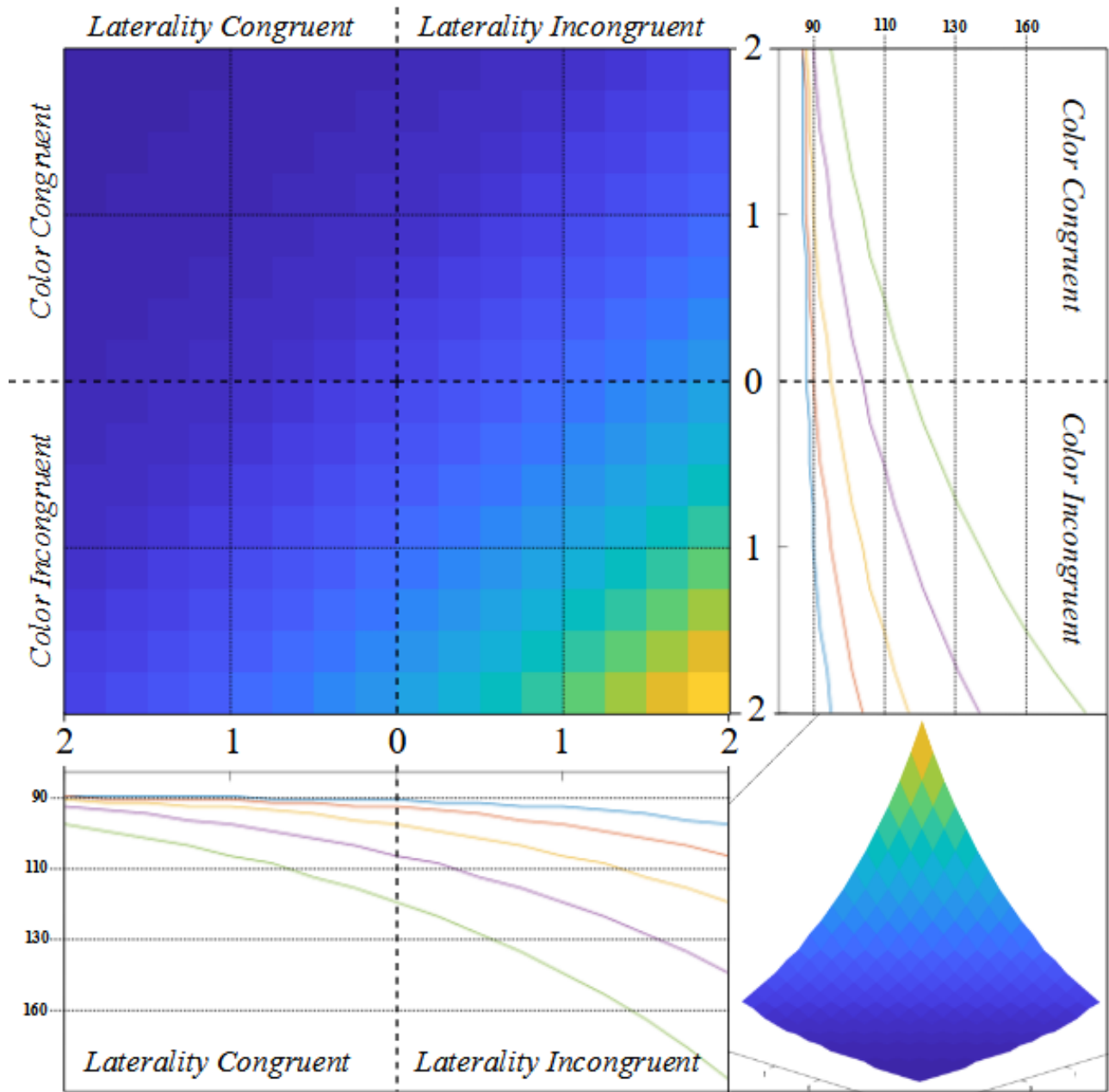


Figure 4.12: The effect of different combination of congruency bias on the reaction time obtained under feature (laterality and color) congruent and feature incongruent SR-map. The reaction time obtained are displayed as a heatmap in the top left while numerical reaction times are reported in the bottom (respectively right) by slicing the map along the horizontal (respectively vertical) axis. In the bottom, the blue curve correspond to reaction time obtained under color congruent (CC) SR-Map setting  $e_C = 2$  (red CC  $e_C = 1$ , yellow neutral  $e_C = 0$ , purple color incongruent (CI)  $e_C = 1$ , green CI  $e_C = 2$ ) for varying laterality congruency bias and SR-Map (LC  $e_L = 2$  from the right to the left LI  $e_L = 2$ ), the right plot is obtained in the same fashion inverting laterality and color. The bottom right insert shows the heatmap in three dimensions, highlighting its symmetry.

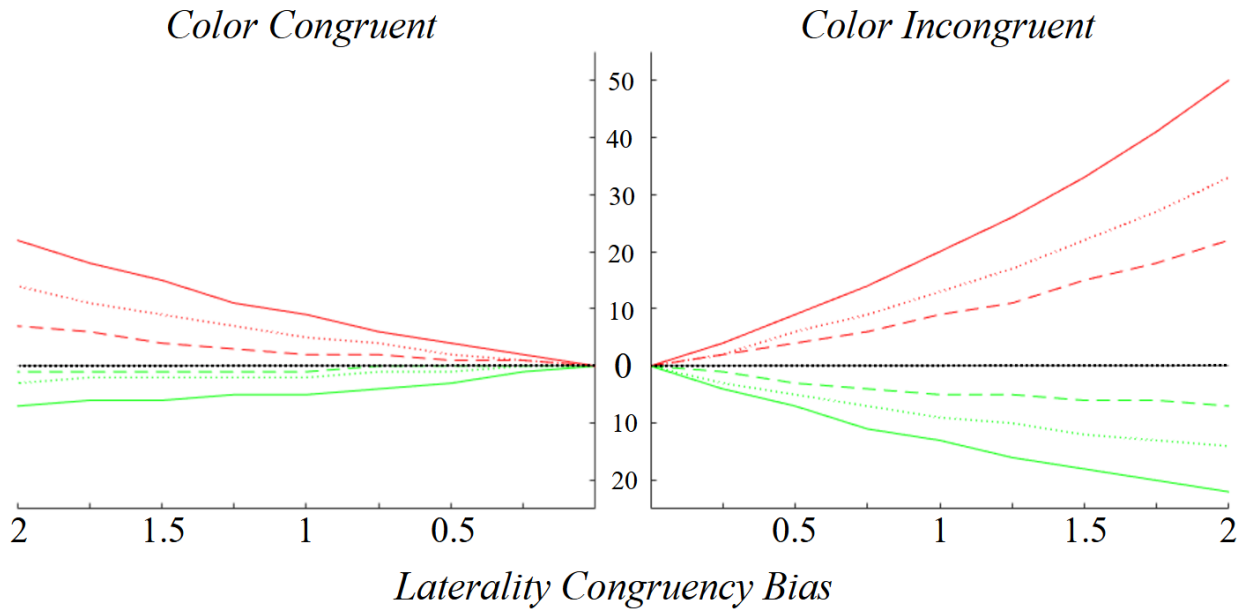


Figure 4.13: The facilitatory (green) and perturbatory (red) effect of the laterality on reaction time as a function of laterality and color congruency bias ( $\beta_C = 25$ ,  $\beta_L = 25$ ,  $b = 0$ ). The two graphs are obtained by subtracting the reaction time obtained under a laterality neutral condition ( $e_L = 0$ ) with the reaction time under a laterality congruent (green curves) and laterality incongruent (red curves) conditions. Solid curves are obtained under neutral color congruency bias ( $e_C = 0$ ), dotted curves under  $e_C = 1$  and dashed curves under  $e_C = 2$ . Note the compensatory effect of color congruency on the Simon effect induced by laterality, such that higher color congruency biases systematically yields smaller effect of laterality.

#### 4.3.4 Facilitatory and Perturbatory regime of the Simon effect

Although we have already noted the existence of facilitatory and perturbatory regimes when simulating SRC effects based on a task-relevant feature in the last section, in the literature, these two regimes are more commonly studied in relation to the Simon effect. A number of studies have shown that the Simon effect "in-vivo" can similarly be decomposed into a facilitatory and perturbatory regimes. That is, in relation to the reaction time obtained in a "laterality neutral" condition, in which the stimuli are all shown in the center, participants consistently yield a shorter reaction time in laterality congruent trials and longer reaction time in laterality incongruent trials [Simon and Craft, 1970, Hommel, 1993a, Lu and Proctor, 1995, Wühr and Ansorge, 2005, Burle et al., 2005]. This result can also be observed in simulation. Figure 4.13 shows the difference in simulated reaction time between a laterality neutral condition (ie setting  $e_L = 0$ ) and laterality congruent / incongruent conditions.

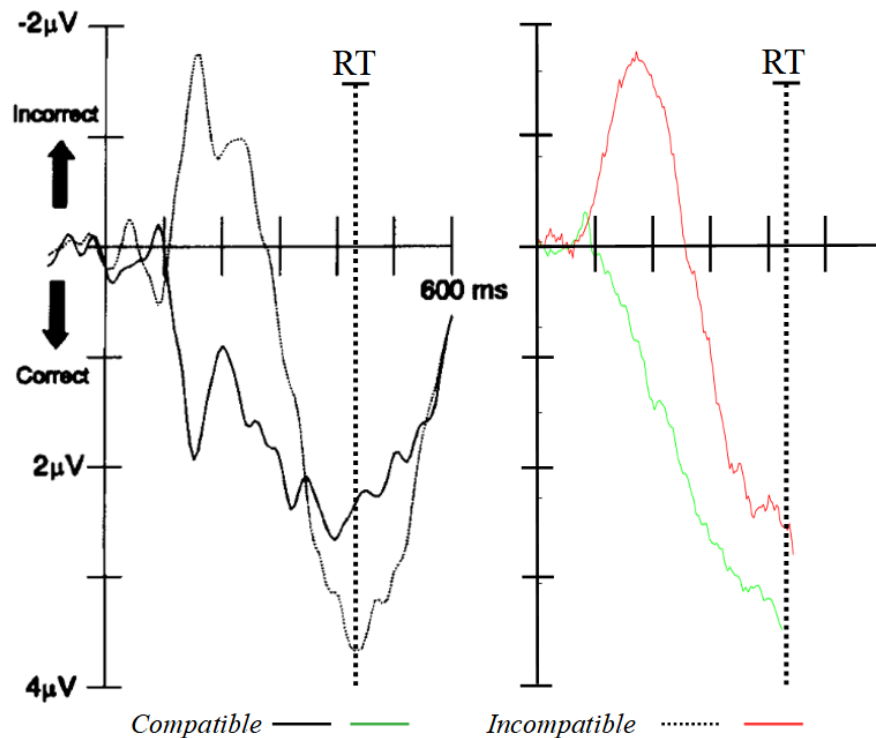
Interestingly, for the highest values of the laterality congruency bias, the perturbatory effects are consistently larger than the facilitatory effects. In this regard, it should be noted that previous reports on these two regimes generally mention that their effects are of similar size [Wallace, 1971, Simon and Craft, 1970, Hommel, 1993a, Lu and Proctor, 1995, Wühr and Ansorge, 2005]. Compared with these reports, this result is therefore rather peculiar. Indeed, given the large effect of laterality on reaction time in a Simon task, one might assume the existence of relatively large biases in favor of laterality congruent responses, since large biases lead to large differences in reaction time. However, we see here that large biases also yields asymmetries between the facilitatory and perturbatory components of these dif-

ferences. Of course, this is only a simulation result and this discrepancy may be attributed to the necessary simplifications and underlying assumptions. However, this may also be viewed as evidence that the Simon effect cannot be solely attributed to biases in favor of laterality congruent responses. In fact, many authors have warned about understanding the Simon effect through the prism of "natural tendencies" alone, and many studies provide evidence that other processes are at play [Hedge and Marsh, 1975, Hommel, 1993a, Valle-Inclán, 1996b, Valle-Inclán and Redondo, 1998, Rubichi et al., 2000]. For example, some authors have proposed that the "conflict" explaining the longer reaction times in laterality incongruent trials might not only be due to a priming effect of the laterality of a stimulus on the laterality of a response but also to a side-effect of the instruction given to the participant [Hasbroucq and Guiard, 1991]. In other words, they argue that, when a participant is instructed to answer a "red" or "green" stimulus with a "left" or "right" key (such as in our example study) these colors "acquire" a new meaning (i.e. "red" also means "left"), such that it is at the stimulus encoding stage that the conflict occur and not at the response selection stage as we put it here. In general, it is known that many cognitive processes are involved in the overall Simon effect (attentional shifts; [Simon, 1969, Stoffer and Yakin, 1994, Umiltà and Nicoletti, 2016]; callosal transfers [Di Stefano et al., 1992, Wascher et al., 2001, McIntosh and Mehring, 2017]; see also [Cespón et al., 2020] for a review). Interestingly, studies comparing facilitatory and perturbatory effects in the closely related Stroop task, where participants are instructed to pronounce the color of a word expressing a color (i.e. pronouncing "green" when presented with the word "red" written in green), have shown that the facilitatory component ("red" written in red) is much smaller than its perturbatory counterpart ("green" written in red) for adults, whereas they are about the same size for children [Wright and Wanley, 2003] which would make sense given our results here assuming adults have larger bias in favor of reading words than children (noting however that perturbatory effects are roughly the same size for both populations, which is not what we get here).

#### 4.3.5 Partial response activations and the Dual route model

As we mentioned, while some authors have argued that the Simon effect may be partly due to interactions at the stimulus encoding stage [Hasbroucq and Guiard, 1991], the most popular hypothesis place it at the response selection stage [Kornblum et al., 1990, De Jong et al., 1994, Zorzi and Umiltà, 1995, Tagliabue et al., 2000, Spironelli et al., 2009, Stocco et al., 2017, Cespón et al., 2020]. This is the "response conflict" or *Dual-Route* model of the Simon effect. One reason for the popularity of this model in the community is that studies have shown that the laterality-congruent response is partially activated when participants perform a Simon task under a laterality-incongruent SR-map. This partial response activation is observed in mainly two ways, either via electromyographic monitoring, under which one observe an early subthreshold EMG activity in the limb ipsilateral to the stimulus [Zachay, 1991, Burle et al., 2002, Hasbroucq et al., 2009, Cohen and van Gaal, 2014] or via Lateralized Readiness Potentials (LRPs; [De Jong et al., 1994, Valle-Inclán, 1996a, Eimer, 1997, Vallesi et al., 2005, Wang et al., 2019]), under which a transiently higher activity is observed in the precentral motor cortex of the hemisphere contralateral to the stimulus (an example of such potential, adapted from [Eimer, 1997], is provided in figure 4.14 - left).

## Lateralized Readiness Potentials



(Left) Adapted from : "The lateralized readiness potential as an on-line measure of automatic response activation in SR compatibility situations." E. Martin. (figure 2). *Advances in Psychology*. Vol. 118. North-Holland, 1997. 51-73.

Figure 4.14: Left. A *Lateralized Readiness Potential* observed in vivo in [Eimer, 1997] under compatible (solid curve) and incompatible (dotted curve) conditions (two choices Simon task). Right. A reproduction based on (Gaussian noised) simulation data following the double difference method (compatible in green and incompatible in red), taking action potential as neural activity. The dotted line on the right was added to mark the average reaction time (averaged between both conditions) observed in the original experiment (427ms vs 443ms), our simulation does not go beyond this point. The curve obtained in simulation were scaled (matching max negative activation and the zero between simulation and experimental curves) and realigned (matching the 4% RT difference observed in the original experiment). Readiness potentials (RPs; [Deecke et al., 1976]; [Deecke, 2007]) are motor related negative potentials observed over the central motor cortex during a movement whose onset largely precede the muscle activity (by up to half a second; [Shibasaki and Hallett, 2006]), as such they are commonly used as indicator of the early stage of a response activation. Note that this example of LRP is contaminated by visual integration and is abnormally large (see [Eimer, 1997] for details).

As the name suggests, dual route models propose that a stimulus is integrated and translated into a response along two routes in the brain, these two routes differing from each other in two key aspects. The first "automated route", generally assumed to be faster and less adaptive than its counterpart, is responsible for the automated activation of laterality congruent responses when presented with a lateralized stimulus (although most authors stay agnostic with respect to why this route should be fast; [Hommel, 2011a]). In contrast, the "controlled route", slower and adaptive, takes into account the decision context (the reaction rule) to formulate a goal directed response.

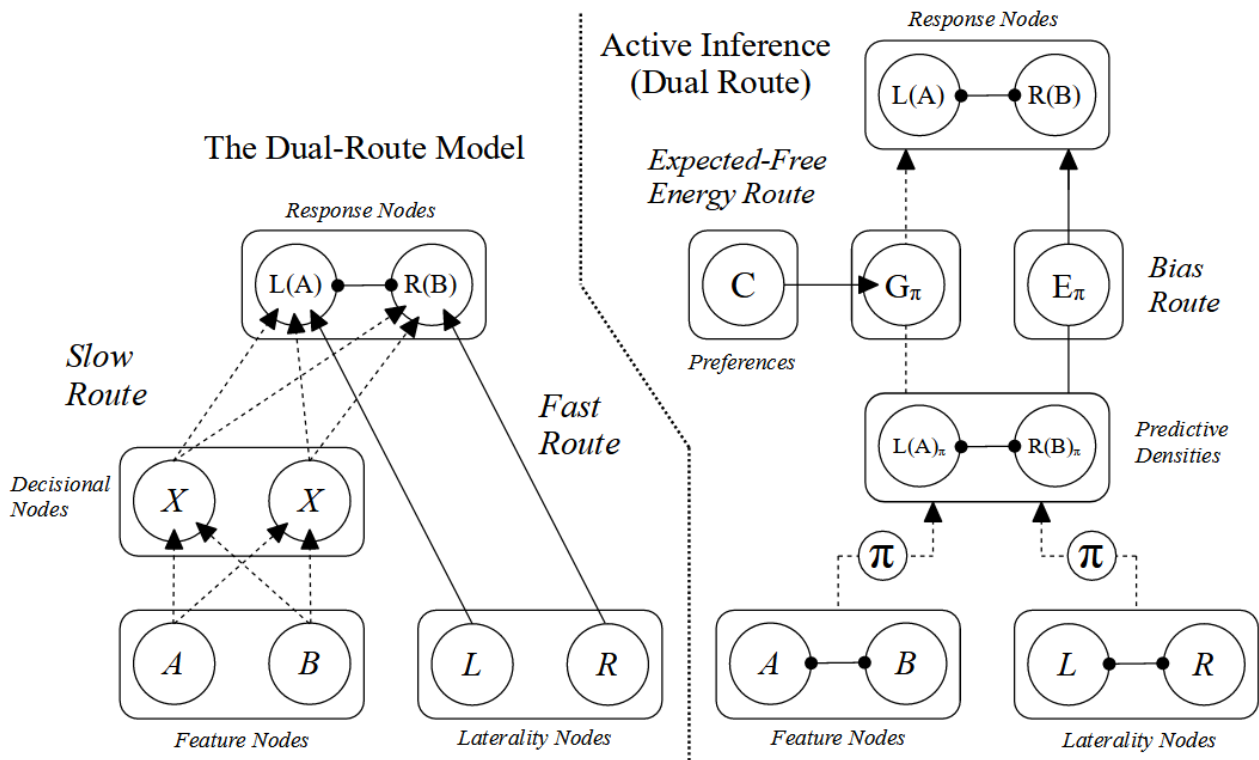


Figure 4.15: Left. The connectionist Dual route model presented in [De Jong et al., 1994]. The fast route correspond to the automated activation of response node by laterality nodes via connections with fixed weight (solid lines). Conversely in the slow route, the activation of response node by feature node is mediated by intermediate decisional nodes where the connections have goal dependent weight (one shot Hebbian learning; dashed lines). Right. The Active inference model simplified and restructured as a dual route. Each policy  $\pi$  underline a particular set of connections from feature and laterality nodes to predictive nodes, among which policy specific response nodes. The activation of response nodes by these policy response nodes is mediated by a fixed weight connection depending on the bias in favor of the policy on one side (Bias Route; solid lines) and by a goal dependent connection through the policy's Expected Free-Energy (Expected Free-Energy Route; dashed line). In both figures, horizontal lines between nodes represent lateral inhibitory connections (link with negative weight in the connectionist model and softmax normalization to probability distribution in the Active Inference model).

In this case, it is therefore possible for the responses produced by these routes to conflict when the goal directed response requires one to answer in a laterality incongruent fashion. Indeed, because the automated route is non-adaptive, the responses it produces are always laterality congruent, and so irrespective of the reaction rule, explaining why a conflict may occur. Furthermore, because it is fast, these laterality congruent responses always precede the goal-directed responses produced by the controlled route, explaining why these responses can be transiently observed using electrophysiological measures before they are eventually suppressed in favor of a slower, goal-directed, response in laterality incongruent trials. Although various computational implementations of the dual route model exist, many authors adopt the connectionist model proposed by [De Jong et al., 1994] which is reproduced in figure 4.15 - left.

Although the computational structure of the connectionist Dual route and the Active Inference model are quite different, it is fairly straightforward to rewrite the equations of Active Inference to highlight that the action potential  $v_\tau = Q(o_{\tau+1})$  is expressed as a sum of two activation similarly obtained along an automated and a goal-directed route. Recall that the action potential is obtained by a Bayesian Model Average over policies (equation 4.19) such that :

$$Q(o_{t+1}) = \mathbb{E}_{Q(\pi)}[Q(o_{t+1}|\pi)] \quad (4.25)$$

In addition, the approximate posterior over policies,  $Q(\pi)$ , is given by :

$$Q(\pi) = \sigma(E_\pi - F_\pi - G_\pi) \quad (4.26)$$

Where  $\sigma(x)$  is the softmax function,  $E_\pi$  is a vector of log prior bias,  $F_\pi$  a vector of marginal Free-Energy and  $G_\pi$  a vector of Expected Free-Energy, all of size  $n \times 1$  where  $n$  is the number of policies.

Ideally, in order to "mimic" the Dual-Route activation, we would prefer for this action potential to be expressed as a sum of an automated activation  $v_E$  resulting from prior biases on one hand and a goal directed activation  $v_G$  resulting from the marginal and Expected Free-Energies on the other. Such that the action potential  $v$  can be expressed as follows :

$$v \propto v_E + v_G \quad (4.27)$$

Following this intuition, we obtain the following expression for the automated and goal directed responses, noting  $\mathcal{G}_\pi = -F_\pi - G_\pi$  and  $v_\pi = Q(o_{t+1}|\pi)$  for conciseness :

$$\begin{aligned} v &= \frac{1}{\mathcal{Z}}[v_E + v_G] \\ v_E &= \sum_{\pi} \mathcal{K}_{\pi} \sigma(E_{\pi})_{\pi} v_{\pi} & v_G &= \sum_{\pi} \mathcal{K}_{\pi} \sigma(\mathcal{G}_{\pi})_{\pi} v_{\pi} \\ \mathcal{Z} &= \frac{(\sum_{\pi} e^{E_{\pi}})(\sum_{\pi} e^{\mathcal{G}_{\pi}})}{\sum_{\pi} e^{E_{\pi} + \mathcal{G}_{\pi}}} & \mathcal{K}_{\pi}^{-1} &= \sum_{\pi'} e^{E_{\pi'} - E_{\pi}} + \sum_{\pi'} e^{\mathcal{G}_{\pi'} - \mathcal{G}_{\pi}} \end{aligned} \quad (4.28)$$

where  $\mathcal{Z}$  and  $\mathcal{K}_{\pi}$  are two normalization terms that are independent and dependent on  $\pi$  respectively and  $\sigma(E_{\pi})_{\pi}$  (respectively  $\sigma(\mathcal{G}_{\pi})_{\pi}$ ) the softmaxed vector  $E_{\pi}$  (respectively  $\mathcal{G}_{\pi}$ ) evaluated at  $\pi$ . Note that, unlike the dual route model, these two activations are not perfectly independent because of the normalization  $\mathcal{K}_{\pi}$  which includes both goal directed ( $\mathcal{G}_{\pi}$ ) and automated ( $E_{\pi}$ ) terms (this is due to the softmax normalization used to obtain the posterior over policies).

To conclude, figure 4.16 shows examples of partial activations obtained in a laterality incongruent, color congruent, reaction rule under a few combinations of laterality and color congruency biases (the bottom three examples are sampled from figure 4.12 while the top example reproduces the dual route model, where automated activation is determined solely by laterality, by setting  $b_C = -32$ ), it also shows the decomposition of the action potential into goal directed and automated activation.

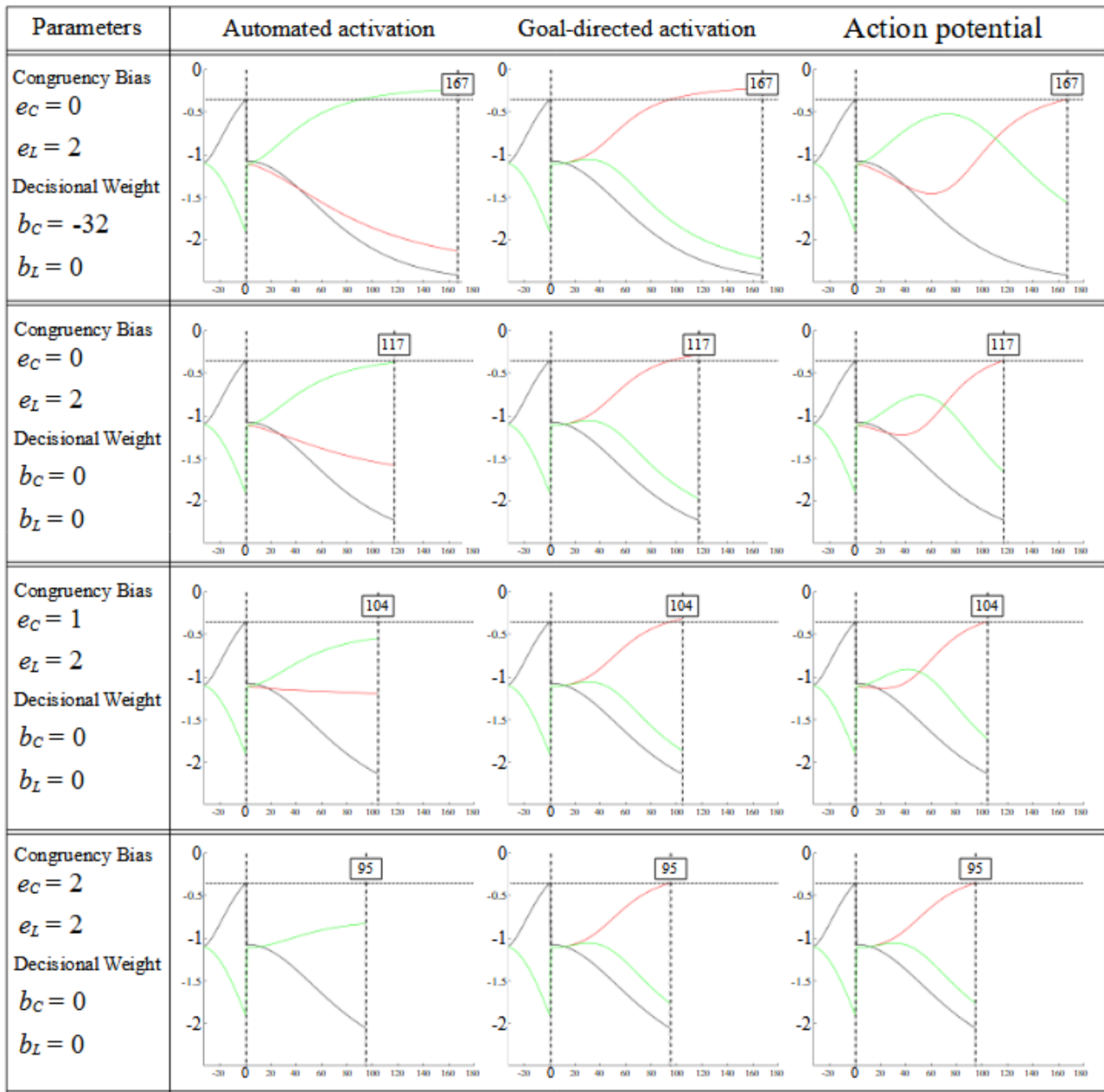


Figure 4.16: The automated ( $a_E$ ; first column) and goal-directed ( $a_G$ ; second column) components of the action potential ( $a$ ; third column) under various combination of color/laterality congruency biases ( $e_C$ ,  $e_L$ ) and prior decisional weights ( $b_C$ ,  $b_L$ ). All these examples correspond to a color-congruent, laterality-incongruent trial where the simulated participant is presented with a red stimulus on the right and expected to press the red key on the left. For all plots, the stimulus onset and the reaction are marked with dashed lines (reaction times are reported on top) while the black, red and green curves correspond to the log-probability associated to the *wait*, *left-red key* and *right-green key* action respectively. The top example aims to reproduce the activation obtained under the connectionist Dual route model where only laterality contributes to the automated activation while the other three illustrate these activations under the parameter sets studied in previous sections. We see here the compensatory effect of the color-congruency on the laterality-incongruency when the participant is biased both in color and laterality congruency (bottom half plots), explaining the reduction in the magnitude of the Simon effect observed in the previous sections.

## 4.4 Discussion

In this chapter, we reviewed the predictions of an Active Inference model in relation to a well-known phenomenon of the literature on human behavior, the Sensory Response Compatibility (SRC). We showed that, with a well chosen parameterization of the generative model, important results of the literature on SRC could be reproduced with relatively high fidelity, notably the Simon effect with its facilitatory and perturbatory components, as well as the partial response activation that is typically observed using electrophysiological measurements in studies on SRC. We have shown that this behavior could be explained by the strong similarities between the computational structure of an Active Inference model, in particular from the point of view of the role of prior biases over policies (habits), with the connectionist Dual route model that is the typical computational model used to reproduce SRC effects "in silico".

These results are only partial and there is many more aspects to both SRC and Active Inference that we have not covered here. In particular, we did not cover sequential effects in SRC [Stöcker et al., 2003, Chen and Melara, 2009, Miles and Vu, 2021], choosing to omit learning in these equations to focus on the foundations of the theory, nor have we covered the interactions of these results with other parameters commonly found in the Active Inference framework, such as the preference distribution or the Expected Free-Energy precision, to focus on prior biases. As such, these results must be taken with caution and it is likely that different choices of parameters, as well as other numerical implementations of the underlying equations, would yield results different from those presented here.

It is well known that the natural tendency prism we have adopted here cannot be considered as sole responsible for SRC effects [Hasbroucq and Guiard, 1991, Umiltà and Nicoletti, 2016, Cespón et al., 2020], indeed, numerous experiments prove that other processes are at play. These processes are however eminently dependent on the specifics of the stimulus and response sets and thus difficult to address in general. In particular for the Simon effect, studies have shown that the automated response evoked by the laterality of a stimulus decay overtime [Hommel, 1993b, Hommel, 1994], such as pre-cueing the future position of a stimulus a few hundreds milliseconds before its appearance, is enough to dissipate the effect of laterality on the participant performance [Umiltà and Liotti, 1987]. In the connectionist dual-route model, this decay is explicitly included via a decay constant damping overtime the contribution of spatial codes in the decision process [Zorzi and Umiltà, 1995] a parameter that is not present in the Active Inference model we studied here. Furthermore, it is known that the laterality, or more generally the spatial information relating to a stimulus, are integrated along a different pathway than other features (the *what* and *where* pathways; [Kraus and Nicol, 2005, Ahveninen et al., 2006, de Haan and Cowey, 2011]), which implies that the integration of spatial features is carried out relatively independently of the others. Although the parameterization introduced in section 4.2.7 decoupled, to some extent, the integration of the laterality and color features used in this simulation, the marginal message passing scheme employed in the Active Inference framework still produces significant interactions between them, such that they cannot be considered truly independent here. In this respect, a question remains as to the modifications to the model we studied here may be most suited to encompass a wider range of results relating to SRC effects.

Although a partial account, the dimensional overlap theory [Kornblum et al., 1990, Kornblum et al., 1999], or "natural tendency", does provide a simple rational for the most robust results of the literature on SRC, which are also the most relevant for the design of interactive systems. In this respect, we showed here that the Active Inference framework could be used as a intermediate model between the connectionist Dual route, whose simplicity makes it difficult to apply in general, and full-fledged integrated model of cognition, whose global approach can make them difficult to manipulate and modify.

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### An embodied control scheme for drones

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#### 5.1 Preamble

In the last two chapters, we focused on the processes by which the designer of an interactive system can endow the machine with "good" priors on the type of user it might interact with. First with a study on human behavior, where we highlighted some properties of the cognitive processes underlying decision making in humans, and then with a computational model allowing to "summarize" these and other experimental observations to formulate an approximate "model of user".

A common theme between these two chapters was to highlight that decision-making in humans is strongly influenced by "external factors" that are not related to the task we are performing. The phenomenon of Stimulus Response Compatibility is a good example of such biases in the decision process. From a purely computational perspective, a permutation in the mapping between stimuli and responses in a choice reaction task does not alter the difficulty of the task the participant is performing. Yet numerous experiments have shown that our performance can depend strongly on the relationship between stimuli and responses. Priors, biases, habits or tendencies are some of the terms we use to refer to the fact that the choices we make in the present are strongly influenced, through learning, by the choices we have made in the past and their consequences. Although our biases are highly dependent on our individual experience, a common factor to this individual experience is that the human brain's interaction with its environment is always mediated by the human body [Wilson, 2002]. Given that our bodies are similar it is not surprising then that the design of an interactive system, particularly when that system aims to help us perform tasks that we can "naturally" perform with that body, often attempts to reproduce in approximate form the relationship between sensory inputs and motor outputs that we experience when using our body to perform a similar task. This approach is particularly true for interactive systems in which the user is "embodied" and where the machine acts as an artificial body for its user.

In this chapter, we follow this approach and apply it to the control of a drone, a class of interactive systems in which the user is often embodied as they receives feedback on the task in "first-person view", i.e. seeing the world through the drone's front facing camera.

## 5.2 Introduction

The "Sense of Embodiment", or SoE [De Vignemont, 2011], refers to the vicarious sensation of ownership and control we feel towards our body [Longo et al., 2008, Glenberg, 2010]. Studies have shown that what we define as "our body" is plastic however and that humans can temporarily "embody" (feel a sense of embodiment towards) objects which are not natural part of their body. One of the best examples of this phenomenon is the well-known rubber hand illusion, [Kammers et al., 2009], but experiments have also shown that human may embody the tools they use [Weser and Proffitt, 2021, Bell and Macuga, 2022].

In the context of human machine interactions, this phenomenon has been particularly studied in rehabilitative technologies [Pazzaglia and Molinari, 2016], and notably for prostheses [Murray, 2008, Imaizumi et al., 2016, Gouzien et al., 2017]. More recently, many authors have focused on the SoE in virtual reality [Riva, 2008, Tham et al., 2018], where the user's interactions with the virtual environment is mediated by an avatar, in which the user is "incarnated", and which acts as one's "virtual body" [Kiltner et al., 2012, Spanlang et al., 2014, Shin, 2018, Kim et al., 2020].

Although there is no unified theory of embodiment, some authors have previously proposed that : *"E is embodied if and only if some properties of E are processed in the same way as the properties of one's body."* [De Vignemont, 2011]. For example, the author illustrate this view by stating that, in the rubber hand experiment, the location of the rubber hand can be internally represented in an "external" representation frame, such as any other objects, or in an "embodied" representation frame, such as any of one's legitimate body part. In relation to this definition of embodiment, we are here specifically interested in its relationship with control. We consider an interactive system to be under "embodied control" if its underlying task is performed in the same way as with the user's body. In other words, the relations between the user's motor actions and their sensory consequences when using the machine are the same as, or very similar to, the case where the user relies on their own body to perform the same task. For example, a "pick and place" task can be performed with our hands, where we feel the object in our palm, the stretch of our muscular fibers and see the object moving in space through our own eyes, all these sensations being correlated to each other under some particular, "embodied", generative model. Conversely, we may also perform this task using a robotic hand. If this robot is controlled via a joystick for example, the sensations are obviously very different, we do not feel the object in our palm and, while we can still feel the muscular stretch associated to our control of the joystick, the correlations between those "motor sensations" and the observation of the object moving in space are very different.

Various interactive systems enable us to efficiently perform tasks that we can naturally perform with our bodies. In this respect, the control of a drone can easily be likened to a "locomotion task" from the user's point of view. Previous studies on locomotion tasks in virtual environments (VE) have shown that active locomotion, where the user's uses their own body to move, and the resulting availability of body-based information during movement, benefit the spatial updating process we human rely on to track the evolution of the surrounding space and the object populating it [Ruddle, 2013]. In particular, "leaning interfaces", where the motion in the VE depends on the movement of the user's body center of mass, such as the Joyman [Pettré et al., 2011, Marchal et al., 2011] or the NaviBoard [Nguyen-Vo et al., 2019], have been shown to improve spatial perception and orientation [Klatzky et al., 1998, Harris et al., 2014], to reduce the number of collisions with the envi-

ronment [Zielasko et al., 2020] but also to reduce the user's mental workload [Nguyen-Vo et al., 2019] in comparison to handheld controllers. Users also tend to report that active locomotion is more immersive, engaging and enjoyable [Langbehn et al., 2015, Nguyen-Vo et al., 2019, Zielasko and Riecke, 2021, Vezzani et al., 2023], although this largely depend on implementation [Pittman and LaViola Jr, 2014, Kitson et al., 2015] and even though such interface may lead to poorer performance from a practical point of view.

Leaning interfaces were originally developed for 2D locomotion tasks, where the user navigates on a ground plane, and it was for a time unclear if the benefits they bring in this context could be extended to 3D locomotion tasks typical of drones. Firstly because, by virtue of being a novel degree of freedom (DoF), it is unclear how body based information resulting from active locomotion can be related to a change in altitude. Secondly, orientation in a 3D locomotion task involve three axes of rotation which compose in a non-commutative fashion. Because of this nonholomicity, orienting oneself in 3D space is therefore a notoriously more difficult problem than what humans are used to do in 2D space.

For these reasons, a number of studies have focused on extending the leaning interface paradigm to 3D locomotion tasks. For example, an upper body leaning based interface is introduced in [Schulte et al., 2016a] where a seated user controls the yaw and pitch of a simulated dragon in a "dragon riding" task by leaning left/right and forward/backward respectively. Although they do not compare their interface with a standard controller, participants reported some difficulties achieving changes in altitude as well as increased motion sickness. A similar method based on upper-body leaning was proposed in [Miehlbradt et al., 2018] for the control of a fixed wing drone. Here the (seated) users were offered control over the pitch and roll of a simulated airplane which, for fixed wing aircraft, results in the system flying up/down and left/right respectively. The authors compared their method with standard RC control in a "fly through the waypoints" task and reported better performance for the leaning interface. They also demonstrated the usability of their setup for the control of a quadcopter (fixing the forward velocity to liken it to a fixed wing aircraft) in a follow-up experiment. With a similar approach, the author of the "Fly-Jacket" [Rognon et al., 2018] reported that participants felt slightly better agency over the drone motion, more naturalness in the way it is operated and less discomfort over a standard handheld controller. Although, in their study, no significant improvement in task performance was observed for the leaning interface.

While the aforementioned studies only allow the user to control 2 degrees of freedom (yaw/pitch or roll/pitch) in a fixed velocity paradigm for the locomotion task, a few studies have also investigated the control of 4 DoF. The "FlyingHead" [Higuchi and Rekimoto, 2013] is a position-based drone control paradigm where the user's head position (in 3D space) is mapped in a 1:N fashion to the position of the drone, the yaw being mapped in a 1:1 fashion. Participants involved in a comparative "search and picture task" (i.e. find an object in the environment and take a picture of it) with the setup reported it as more intuitive and enjoyable than with a handheld controller. The authors also noted an improvement in performance with lower task completion time. An extensive comparative study [Pittman and LaViola Jr, 2014] on the control of quadcopters included three standing leaning flying interface (among five control paradigms), the "Head Translation", "Head Rotation" and "Modified Flying Head". The "Head Translation" paradigm they propose associates the position of the user's head in relation to some reference point to the drone's speed. In this case, the users can step forward/backward and to the side to induce a motion of the drone at constant ve-

locity. Similarly for vertical movement, the users have to tiptoe or crouch in order to change the altitude of their head. The approach for the Head Rotation paradigm is similar, this time mapping the user's head tilt forward and sideways to a velocity in the four cardinal directions. The final "Modified Flying Head" paradigm is the same as the Head Translation but also include a 1:1 mapping between the user's body rotation and the drone for the yaw. For all three interfaces, they reported that the participant performance in a fly through the way points task was poorer than with a handheld controller and, interestingly, the handheld controller was also preferred by the participants in their study. To explain this result, they hypothesized that the low resolution of the head mounted display they used, coupled to a number of calibration problem with the interface, meant that most participants found the experience less enjoyable than when using a controller.

In this preliminary study, we investigate a new control paradigm for 3D locomotion tasks with a view to an application to the control of (quadcopter) drones. In particular, we are here interested in two themes. Firstly, the intuitiveness of an embodied control scheme, that is, given a drone control interface that closely mimics the way we use our bodies to move in our daily life, can a naive user quickly learn to take control of the drone without prior instructions. Secondly, a number of human machine interfaces rely on what can be referred to as "elicited responses", that is, automated motor or physiological responses from the user given some sensory context. For example, a number of studies have sought to measure changes in skin conductivity, known as electrodermal activity, to obtain information about a user's emotional states (for example trust [Morris et al., 2017]; anxiety [Kritikos et al., 2019] or arousal [Greco et al., 2016]). We call this physiological response "elicited" in the sense that the user has no control over this automated response resulting from a particular sensory stimulus and may not be aware that it is occurring. Another example closer to our purpose is that of variations in brain activity caused by exposure to a sensory stimulus. In the case of the well known P300-BCi for example [Fazel-Rezai et al., 2012], the presentation of a particular stimulus, such as a character in brain-to-text applications, provokes a variation in the brain activity (an event related potential; [Sutton et al., 1967, Fabiani et al., 1987, Luck, 2012]) that can be correlated to the user's intention. Once again, this response is elicited by some sensory context and escapes both the user's awareness and control. We hypothesized that a similar approach could be used in the context of locomotion tasks, specifically for the 2D movement on a ground plane, and explore this idea here.

In the first section we introduce the embodied control interface we designed and explain the underlying approach. In the second section we discuss some of the results and feedback we obtained during the presentations of this prototype during an open event where members of the public were invited to try out the setup in virtual reality (VR). Finally, we discuss the transposition of the prototype to the control of a commercial quadcopter drone.

### 5.3 An embodied control method for a 3D locomotion task

We built a virtual environment in which the user can control a virtual drone in 3 dimensions. The environment was designed in Unity with the pluggin SteamVR to handle the rendering of the environment in a HTC Vive headset. The user inputs, which we detail below, were processed in Matlab before being fed back into Unity via a local TCP-IP server.

The drone control interface was decomposed into motion and orientation (yaw) on a ground plane, typical of 2D locomotion tasks, and for which we designed a standing leaning interface similar to previous design [Nguyen-Vo et al., 2019, Zielasko et al., 2020] and the control of the altitude.

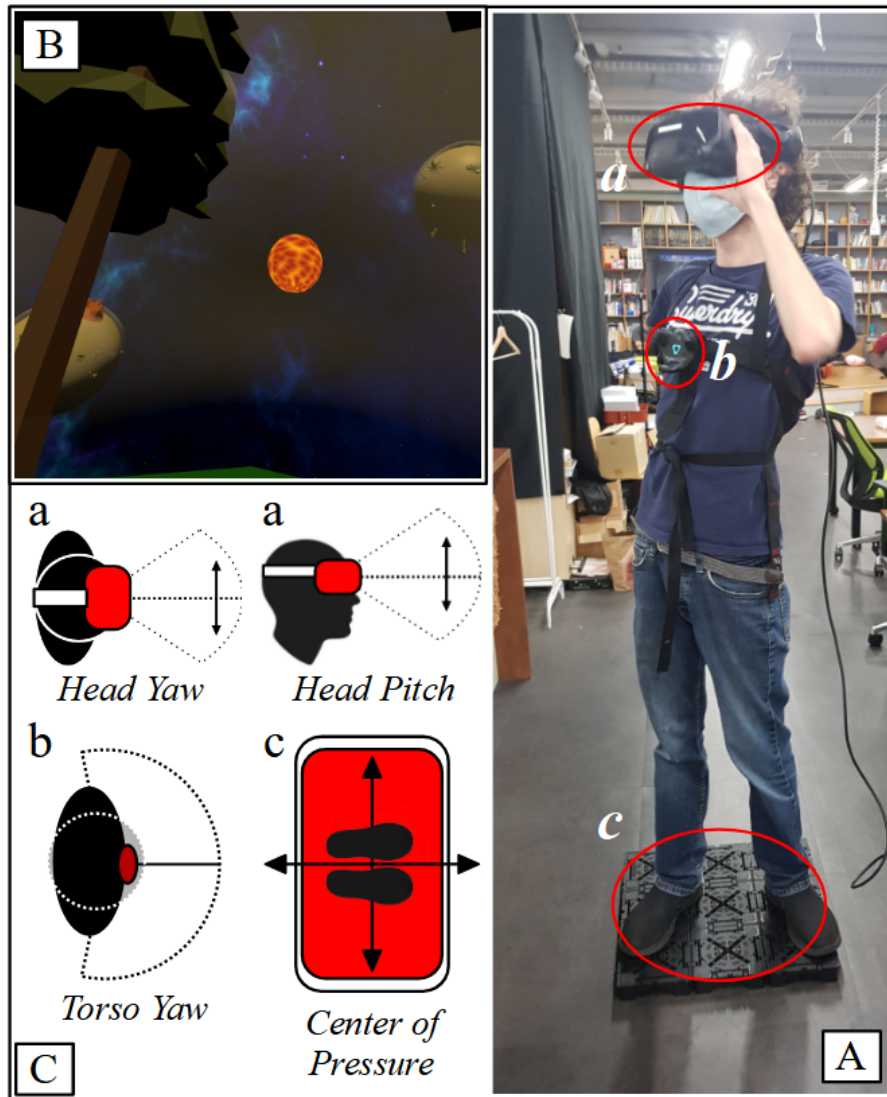


Figure 5.1: The setup. A. The user, wearing a HTC Vive head mounted display (HMD), is standing on a force platform. In addition, a tracker is fixed onto his / her plexus. B. The user view inside the HMD. For the public demonstration, we designed a "space themed" environment the user can freely explore in 3 dimensions. C. The postural data collected from the user around which the control interface is designed. The HMD (a) allows to retrieve the user's head yaw and pitch, a tracker (b) measure the torso's yaw while the force platform (c) measure the position of the Center of Pressure.

### 5.3.1 Standing Leaning Interface

It is known that locomotive tasks in human involve precise control of the Center of Pressure (CoP). This is the case, for example, with walking, where the first phase of the gait cycle is an imbalance in the direction of movement resulting from a shift of the CoP [Kharb et al., 2011, Abbass and Abdulrahman, 2013]. Given the importance of locomotive tasks in our everyday life and the CoP control they involve, we hypothesized that the simple intention to move in the virtual environment would cause the user's CoP to move slightly in the intended direction of motion. Following this hypothesis, we captured the position of the CoP via a force platform manufactured by our laboratory on which the user stands (figure 5.1) in order to detect these displacements. However, it is known that the CoP is never perfectly still when humans are simply standing and sway around some position [Benda et al., 1994, Latash et al., 2003]. Thus, to dissociate noise in the CoP position from a displacement induced by a movement intention, we assumed that the distribution of the CoP position at rest could be approximated by a bi-variate Gaussian ( $\mathcal{N}_{sway}(\mu_s, \Sigma_s)$ ), which we refer to as the sway distribution hereafter, with unknown mean and covariance matrix. To estimate these parameters, we simply asked the user to stand relaxed on the force platform and recorded the position of their CoP every few hundreds milliseconds before fitting a Gaussian to these data (assuming the samples are i.i.d). Given the fitted sway distribution, for a measured position of the CoP on the force platform  $cop = (x, y)$ , the CoP shift vector,  $c\vec{o}p$ , is obtained as :

$$c\vec{o}p = cop - \mu_s \quad c = ||c\vec{o}p|| \quad \vec{c} = \frac{c\vec{o}p}{||c\vec{o}p||} \quad (5.1)$$

where  $cop$  is the measured position  $(x, y)$  of the CoP on the force platform,  $\mu_s$  is the mode of the sway distribution and  $c\vec{o}p$  the CoP displacement vector ( $c$  and  $\vec{c}$  denoting its magnitude and direction respectively below). The mapping between the measured CoP shift vector,  $c\vec{o}p$ , and the force applied to the drone in the ground plane,  $\vec{F}_g$ , is then obtained via a shifted sigmoidal mapping such as :

$$\vec{F}_g = \frac{F_{max}}{1 + e^{-\alpha \cdot (c - \gamma \sigma_s(\vec{c}))}} \vec{c} \quad (5.2)$$

where  $\sigma_s(\vec{c})$  is the standard deviation of the sway distribution measured along the CoP displacement vector (figure 5.2 - bottom left).  $F_{max} = 15$ ,  $\alpha = 0.7$  and  $\gamma = 3.5$  are three hand-tuned parameters controlling the maximal force applied to the drone, the response sharpness and the magnitude of the CoP displacement vector at which the applied force is half the maximum respectively (figure 5.2 - bottom right). Finally, the motion of the drone resulting from the application of  $\vec{F}_g$  was simulated via numerical integration of a second order differential equation corresponding to a point of mass  $m = 10$  with a friction  $D = 8$  (figure 5.2 - top right). We chose a relatively high mass and friction to smooth small perturbations in the measured position of the CoP.

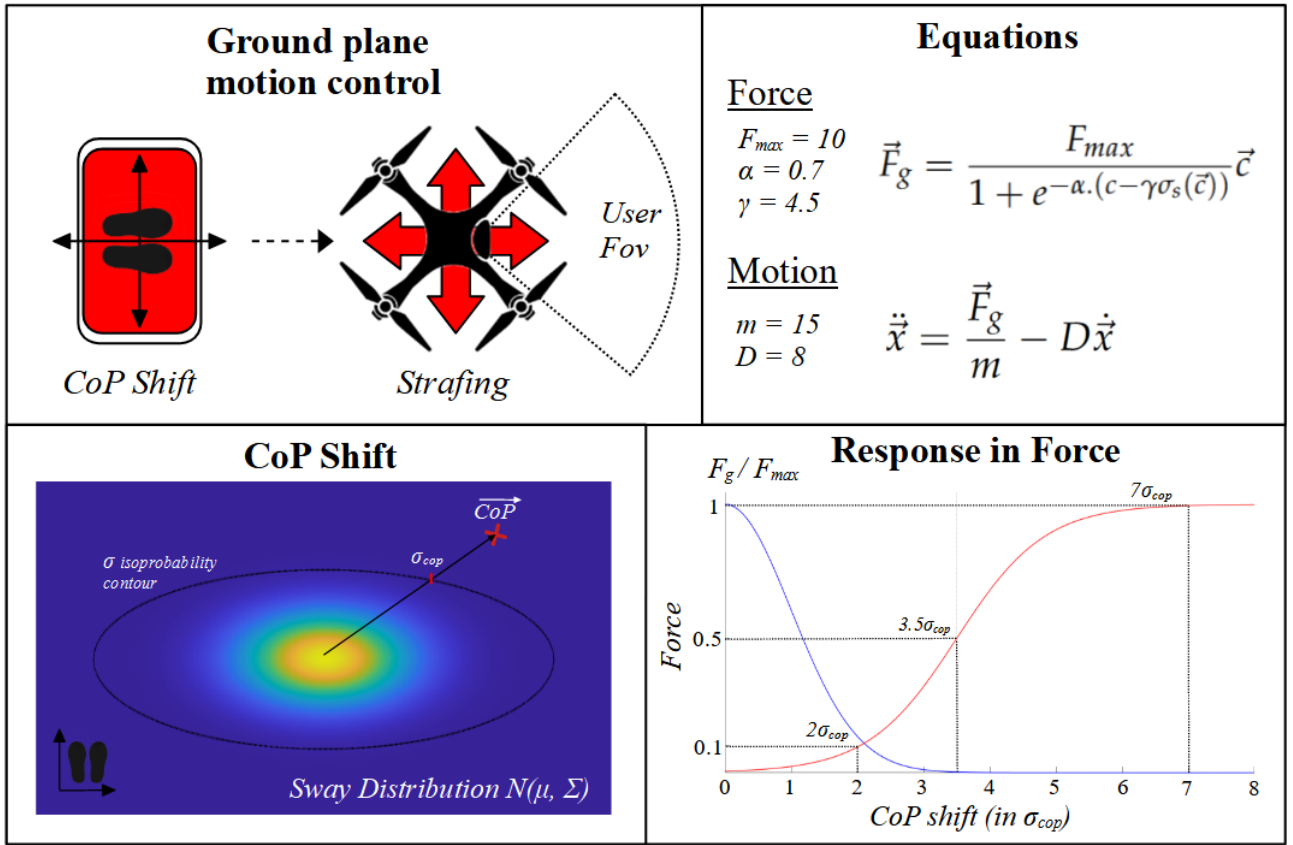


Figure 5.2: Top-Left. The standing leaning interface allowing the user to control the drone motion in the ground plane (strafe) via a shift of their CoP. Bottom-Left : The measure of the standard deviation of the CoP sway distribution ( $\sigma_{cop} = \sigma_s(\vec{c})$ ) along the CoP displacement vector ( $c\vec{op}$ ). Bottom-Right : The response of the system due to a shift of the CoP. In blue is represented the CoP sway distribution while in red is represented the response in force (both curve are normalized to take values between 0 and 1). Given the parameters used, the applied force is 10% of the maximum at  $2\sigma_{cop}$  (95% confidence), half the maximum at  $3.5\sigma_{cop}$  ( $\gamma$  in equation 5.2) and maximal at roughly  $7\sigma_{cop}$ . Top-Right. A summary of the equations underlying the control interface, the motion of the drone in the ground plane ( $\vec{x}$ ) resulting from the application of the force  $\vec{F}_g$  is simulated via numerical integration of a 2nd order ODE.

### 5.3.2 Orientation in the ground plane

After a review of previous studies on locomotion tasks in VR, it appears that a 1:1 correspondence between the user's body rotation and the drone's rotation is a widely used, and perhaps the most appropriate, approach to embodied control [Higuchi and Rekimoto, 2013, Rognon et al., 2018, Hashemian et al., 2020]. In our case however, such an approach would be unsuitable given the calibration procedure needed to estimate the sway distribution (the users are required to keep their feet static on the platform). Thus for simplicity, we chose a control in velocity for the drone's yaw based on the user's head orientation (i.e. a left/right rotation of the user's head results in a rotation at constant velocity in the same direction for the drone), an approach previously explored in the context of 2D locomotion tasks [Beckhaus et al., 2005, Kitson et al., 2017]. However, compared to previous implementation of this paradigm, we investigated how to make the relationship between the head's and the drone's rotation as "natural" as possible for the user. In particular, since the user is

embodied in the drone, we studied the relationship between gaze, head and torso rotation during gaze shifts.

Many study have interested themselves in the relationship between eye, head and torso rotation during gaze shift, both in humans [Warabi, 1977, Freedman, 2008] and in animals [Blakemore and Donaghy, 1980, McCluskey and Cullen, 2007]. In particular, these studies shows that, although the eyes have a range of motion of about  $50^\circ$  in all directions [Lappi, 2016], they rarely rotate beyond  $30^\circ$  in relation to the head [Bahill et al., 1975, Land and Tatler, 2009]. It is known that the relative contribution of eyes, head and torso rotation in large gaze transitions is modulated by a number of factors [Freedman, 2008, Kim et al., 2010]. In particular, we based ourselves on an extensive study by [Sidenmark and Gellersen, 2019], which examined these relative contributions in the specific case of gaze shifts in VR. Assuming that the drone's contribution to a gaze shift should be similar to that of the user's torso, we chose an approximate threshold of  $25^\circ$  of the user's head rotation in the left/right directions to initiate a rotation of the drone. This value corresponds to the gaze target azimuth at which the torso begins to contribute significantly ( $>10\%$  based on [Sidenmark and Gellersen, 2019]) to the gaze transition. Noting that, due to the restricted field of view in VR, rotations of the head tend to contribute more to gaze shift than in natural settings, which we aim to reproduce [Pfeil et al., 2018]. We decided to increase this threshold by  $5^\circ$  to account for the fact that the onset of torso rotation as a function of gaze target azimuth reported in [Sidenmark and Gellersen, 2019] might occur earlier due to the greater contribution of head movement in VR.

It should be noted however that these studies focused on these relative contributions at rest, without underlying locomotion task. In the case of locomotion tasks, studies on the relationship between eyes, head and torso rotations show that a change in direction is invariably preceded by a gaze shift in the future direction of movement [Imai et al., 2001, Hicheur et al., 2005]. This gaze shift is itself immediately followed by a reorientation of the head in the same direction (a reorientation whose onset is about the same as the onset of the gaze shift), and then by the torso. In other words, these studies show that the coupling between eyes, head and torso rotation during locomotion is much tighter than at rest. Following these experimental results, we chose to modulate the relationship between the head rotation and the drone's yaw velocity as a function of drone's speed. Such as the relationship between the two is expressed as :

$$\dot{\phi}_d = \frac{\beta}{1 + e^{-\alpha \cdot (|\phi_h| - \mu_v)}} \quad (5.3)$$

where  $\dot{\phi}_d$  is the drone's rotation velocity (in the direction of the head),  $|\phi_h|$  the (absolute) yaw of the user's head and  $\alpha = 0.3$ ,  $\beta = 15.s^{-1}$  and  $\mu_v$  three parameters controlling the response sharpness, the maximum yaw velocity and a threshold on the head yaw angle at which the rotation velocity is half the maximum respectively. This threshold  $\mu_v$  also depends on the drone's (forward) velocity such as :

$$\mu_v = \frac{\mu_{min} - \mu_{max}}{1 + e^{v_{th} - v_d}} + \mu_{max} \quad (5.4)$$

where  $\mu_{max} = 45$  and  $\mu_{min} = 25$  correspond to the maximal and minimal value of the offset  $\mu_v$ ,  $v_d$  is the drone velocity and  $v_{th}$  an offset set to  $7m.s^{-1}$ . At low or zero speed, this

corresponds to a head rotation range of up to  $30^\circ$  on either side before the drone starts to rotate in yaw. Conversely at higher speed, the relationship between the head's yaw and drone's yaw velocity is much tighter, with a range of about  $12^\circ$  (figure 5.3).

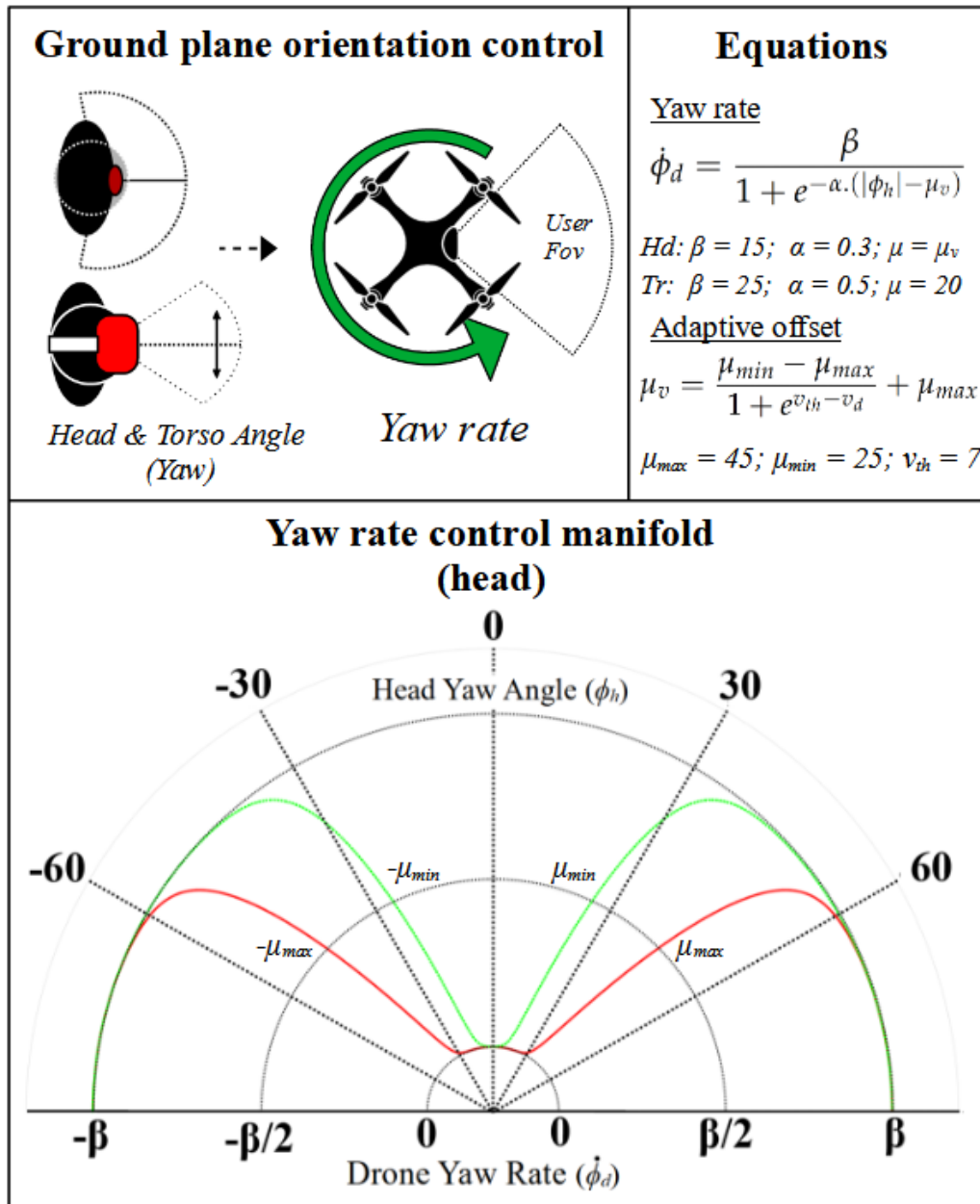


Figure 5.3: Top-Left. The control interface allowing the user to control the drone's orientation (yaw) based on posture (head and torso yaw angles). The equation parameterizing the interface and their parameter sets are summarized on the top-right. The equations are evaluated separately for the head (*Hd*) and torso (*Tr*) with the appropriate parameters and then summed. Bottom. The adaptation of the (head-based) control interface based on the drone velocity. At low or zero speed (red curve), the user's head can rotate by about  $30^\circ$  in either direction before the drone reacts by rotating in the same direction. Conversely, as the user gains (forward) speed this range starts to contract, to reach about  $12^\circ$  when the speed is above  $7m.s^{-1}$  (green curve).

In addition to this "head rotation based control", we measured the participant torso rotation using a tracker (figure 5.1) and used it as an additional method to control the drone's yaw velocity. This was motivated in particular by a previous study on 2D locomotion in VR [Miehlbradt et al., 2018] which showed that a number of users, when given the choice, preferred to adopt a torso-based method of control for the yaw. Unlike the control based on head rotation however, the mapping between the torso rotation and the drone yaw velocity is kept constant irrespective of speed, with a tight range of about  $10^\circ$  ( $\alpha = 0.5, \beta = 25.s^{-1}, \mu_v = 20$  in equation 5.3). The drone's yaw velocity due to head and torso rotation are then summed so that the highest yaw velocity can be achieved when rotating both head and torso together.

Finally, as the drone's motion is generated via a numerical simulation of a mass-damp system with relatively high mass, we were concerned that users might experience difficulties with inertia when turning during motion. To alleviate these problems, we chose to also apply yaw changes to the drone's momentum vector. That is, when the user is changing direction, the drone velocity stays constant in the user's reference frame.

### 5.3.3 Elevation control

For the control of the drone altitude, we first explored the possibilities offered by gaze-based control, a control method sometimes referred to as the "Where-You-Look-Is-Where-You-Go" paradigm [Subramanian et al., 2021]. However, a well-known problem of this method, referred to in the literature as the "Midas Touch" [Velichkovsky et al., 1997, Meena et al., 2017, Elmadjian and Morimoto, 2021], is that not every gaze shift can be associated to an intention to move. Numerous approaches have been proposed to solve this problem, that is, to differentiate between a "natural gaze", unrelated to an intention to move, and those that can be associated with a movement intention [Mohan et al., 2018]. However, given that this paradigm would only be used for the control of the drone's altitude, we decided to work around the Midas Touch problem rather than directly solving it. Noting that the user can already express a movement intention via a displacement of their CoP, we chose to rely on the previously implemented control for the forward/backward direction to offer the possibility to change the drone altitude.

For this, we chose to rotate the "ground plane", in which the user control the drone's motion via a shift of their CoP, up or down depending on the user's head pitch. In other words, a user lifting the head up (down) while leaning forward moves up (down) in the virtual environment according to an elevation angle. Again with the aim of making this control as natural as possible, we chose to make the relationship between head pitch and elevation angle asymmetrical with respect to zero, so that lowering the head by a certain angle has a greater effect than raising it by the same angle. As with the yaw control, this choice motivated by studies on the relationship between eye and head pitch in vertical gaze shifts. In this case, studies show that the head tends to contribute less to gaze shifts for targets with negative elevation than for target with positive elevation [Kim et al., 2010, Sidenmark and Gellersen, 2019]. We therefore hypothesized that highly negative head pitches would be less natural to the user than highly positive ones and reproduced this asymmetry in the relationship between the user's head pitch and the resulting drone's elevation angle. For a pitch of the user's head measured as  $\theta_h$ , the elevation angle,  $\theta_d$ , is computed as :

$$\sin(\theta_d) = \frac{1}{e^{-\alpha(\theta_h - \mu)}} \quad (5.5)$$

Where  $\alpha$  and  $\mu$  are two parameters controlling the response sharpness and the user's head pitch at which the elevation angle is of  $45^\circ$  (half the maximum of  $90^\circ$ ). As mentioned these two parameters were tuned differently for negative ( $\alpha_- = -0.4$ ,  $\mu_- = -18$ ) and positive ( $\alpha_+ = 0.3$ ,  $\mu_+ = 30$ ) head pitch angles (figure 5.4).

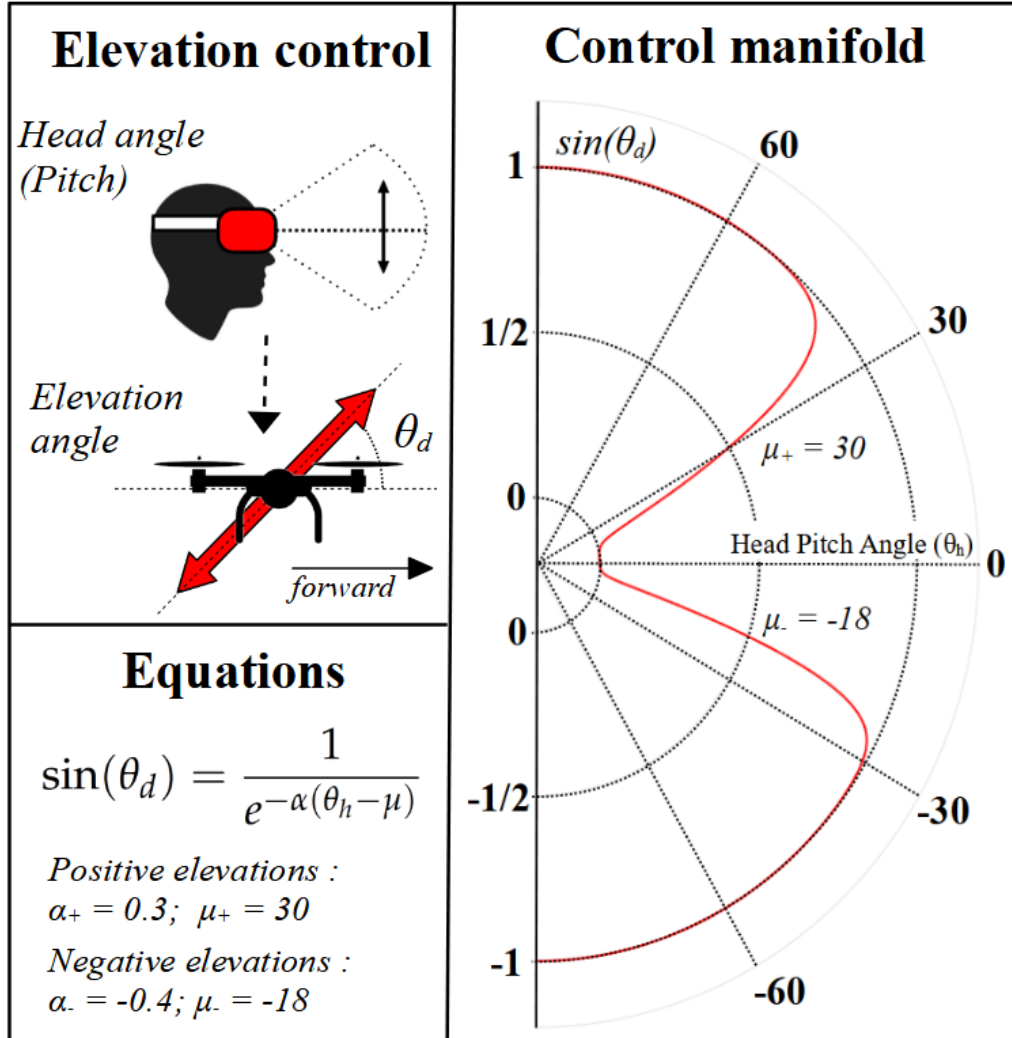


Figure 5.4: Top-Left. The control interface allowing the user to control the drone elevation angle ( $\theta_d$ ) via the head pitch angle ( $\theta_h$ ). Bottom-Left. The control of the elevation angle is asymmetric about the zero with two different parameter sets used for positive and negative elevations. Right. These parameters were hand tuned such as, when the user's head pitch is between  $+12^\circ$  and  $-8^\circ$ , the drone moves forward/backward when the user leans forward/backward ( $\sin(\theta_d) = 0$ ). Above and below these values, the elevation angle increase such that, for head pitch angle above  $40^\circ$  and below  $-30^\circ$  ( $\sin(\theta_d) = 1$ ), the drone moves up and down (or down and up for negative head pitch) when the user leans forward and backward.

Overall, this method of control gives to the user control over 4 DoF for the drone, two of which can be controlled independently, the drone's left and right strafing motion and its yaw, and two correlated, its forward and backward motion with its elevation.

## 5.4 Demonstrations and some preliminary results

The prototype was presented at the final evaluation of the Erato Project during which members of the public could try out the system. As we were interested in the intuitiveness of our control interface, we gave participants no instructions other than to "freely explore the virtual environment without trying too hard to reverse engineer the system they were interacting with", and offered to answer any questions about the inner workings of the system after the participant had tried it out.

The event, which enjoyed a relatively widespread publicity in Japan, attracted a wide range of profiles from the public. From students reporting to be very familiar with VR technologies to older people who, although interested, never had any experience with VR. A few young children (10-12 years' old) also experimented with the setup. Before each demonstration, we warned the user that VR is known to induce motion sickness in some case, and asked them to close their eyes and remove the VR headset should they feel any discomfort during the tryout. To promote engagement, these tryouts were performed in a "space themed" virtual environment (VE) with various planets of different sizes and ecosystems (forests, deserts, mountains; randomly generated using freely available assets; see figure 5.1) placed far apart from each other so the user were incentivized to explore and travel in the environment.

Due to the nature of the event, we were unable to collect data about the participants and simply discussed their experience orally as well as monitored them during their trial.

### 5.4.1 Demonstration at an open event

Most of about forty participants were visibly able to take control of the system fairly quickly (from a few seconds to a few minutes) in order to reach self selected point of interest, and so regardless of age or prior experience. After the tryouts, which lasted around 15 minutes per participant depending on the attendance, many of them reported their experience as enjoyable and immersive, most notably children, in line with previous reports on user experience with similar systems [Langbehn et al., 2015, Nguyen-Vo et al., 2019, Zielasko and Riecke, 2021, Vezzani et al., 2023]. We take these reports as an encouraging sign with regard to the intuitiveness of our interface since no participant required further instructions on how to operate the system.

Due to the structure of the VE, we mainly observed two types of trajectories, "travels", where the participant were navigating from one planet to another, an activity which, due to the great distances, could take upwards of one to two minutes, and "explorations" where, upon reaching some planet, participants attempted to perform orbiting motions to observe them from different angles. Travels were by far the easiest type of movement and every participants were able to cover long distances in the VE. Interestingly, traveling trajectories were rarely straight lines from A to B. For most of the trajectories we observed, participants only started to move towards a point of their interest before changing their minds and re-orient the drone towards some other points. This may be seen as a sign of low cognitive workload induced by the control scheme during travels, such as the participants were able to focus on their experience rather than on controlling the drone (a known advantage previously highlighted in similar setups; [Nguyen-Vo et al., 2019]).

In contrast, exploratory trajectories, which mainly consisted of orbiting a given planet at low speed, appeared much less natural. Most users reported difficulties in fine-tuning the drone's position and orientation. This result was to be expected however, since visual exploration is a type of task where the control interface of the drone, based on head motion, largely interfere with the experience of the participant. In addition, it is known that precise control is generally more difficult with standing leaning interfaces than with handheld controllers [Pittman and LaViola Jr, 2014]. For such trajectories, it is likely that a different approach to ours would be more suitable, for example by replacing the control in force by a control in velocity, more precise by nature, for the standing leaning interface at low speeds.

A very common problem with standing leaning interface is the difficulty to "find the zero" [Pittman and LaViola Jr, 2014, Hashemian et al., 2020]. However, it is interesting to note that, from our observations and oral reports, participants did not seem to have difficulties with this task. This may be explained in part by the fact that our calibration method based on the CoP sway at rest required user to stand in a relaxed fashion. We hypothesized that this pose is fairly familiar and therefore easier to return to. A related problem we did observe however is the high sensibility of the system to movements of the user's feet on the force platform. Although small adjustments were possible after the calibration, too large of a shift in the CoP position at rest due to a movement of the feet necessitated a re-calibration of the system, a well-known problem with standing leaning interfaces [Pittman and LaViola Jr, 2014, Nguyen-Vo et al., 2019].

#### 5.4.2 Transposition to a commercial VTOL-drone

To validate our approach beyond a well controlled virtual environment, we also transposed the control scheme onto a commercial, Vertical Take-Off and Landing (VTOL), quadcopter (DJI Tello Ryze). The interface was kept exactly the same as the one used to control the virtual drone (figure 5.1 - A). We used the virtual drone velocity in the 3 axis (scaled down by a constant factor) as velocity command for the real drone, setting a limit magnitude for the velocity vector of  $10 \text{ cm.s}^{-1}$ . The yaw rate was mapped in a 1:1 fashion between the virtual and real drone. Finally, we retrieved the video stream supplied by the small front facing camera of the drone and fed it into the HMD (figure 5.5).

As might be expected, controlling the real drone proved much more difficult than the virtual one. We therefore decided against including this demonstration in the public event and relied on in-house tests with members of our laboratory to evaluate the setup. The control interface was evaluated in a standard "fly through the way points" (FTW) task in which users were asked to fly the drone through hoops (figure 5.5 - C,D) spaced from each others by about five meters and fixed one to two meters above the ground.

The big difference between the difficulties associated with controlling the virtual drone and its real counterpart can be explained by a few factors. Firstly, from a technical point of view, the front facing camera of the drone used in this preliminary experiment produced a 2D video stream of relatively poor quality (720p-30FPS), making distances much harder to appreciate than in the 3D rendered VE. In addition, the fairly restricted field of view of the camera ( $82.6^\circ$ ) meant that no visual feedback was available for objects immediately to the side of the drone. While this didn't pose a problem in open environments, both these aspects made the FTW task extremely cognitively demanding.

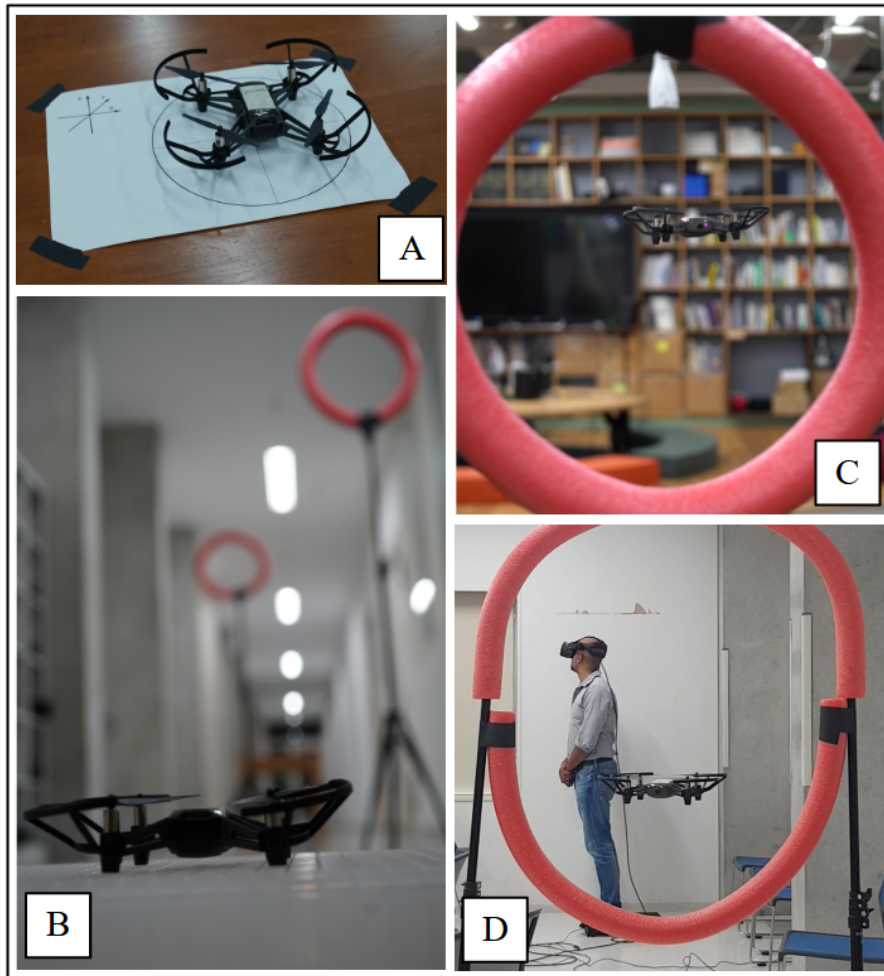


Figure 5.5: Transposition to a quadcopter drone. A. The quadcopter used with the control interface is a DJI Tello Ryze equipped with a front facing camera. B,C. The interface was evaluated in a "fly through the way points task" where the participant were instructed to fly the drone through a series of hoops. D. A participant (seen in the background) flying the drone through one of the hoops using the embodied control interface.

This may be explained by the fact that, due to the low FOV, users were losing visual feedback on the hoops position well before passing through them (depending on the approach, the hoop disappeared from the video stream up to 50cm before it was crossed). To perform the FTW task correctly, it was therefore necessary to track the drone's position relative to the hoops based solely on an estimate of the drone's trajectory obtained from the video stream (i.e. mainly based on optical flows). While it is known that humans are fairly accurate in such task [Kirschen et al., 2000, Redlick et al., 2001, Warren et al., 2001], it is probable that the low quality and the 2D nature of the video stream largely hindered this ability.

Another problem we quickly identified with our setup was the fact that, although the drone, and therefore its front facing camera, had a fixed pitch, controlling the drone elevation requires the user to tilt the head up and down. While we expected some reduction in the Sense of Embodiment induced by the interface due to this discrepancy, the mismatch between the sensory feedback resulting from the head motion (neck muscle stretch) and the absence of change in the visual feedback was in fact very confusing with regard to our sense of the drone's position and orientation. This effect was largely more pronounced in case of pure elevation control (i.e. a change in the drone elevation without forward motion), which

in our control interface is achieved when the head pitch angle is above  $40^\circ$  and below  $-25^\circ$  (in which case leaning forward make the drone move up or down depending on the sign of the head pitch). Meaning that successfully performing the FTW task required careful planning to avoid such motions as much as possible.

Because of these technical aspects, we decided to suspend the evaluation of our control interface on a real drone until a more suitable device became available. Indeed, we feared that the results obtained with the current version would reflect more about the technical limitations of the drone rather than the validity of our control method.

## 5.5 Discussion

Although we are yet to proceed with a proper experimental evaluation of control interface, the presentation of the device in a public event allowed us to obtain valuable feedback with regard to our approach.

Firstly, with respect to our initial hypothesis, that the simple intention to move would cause an implicit movement of the CoP without the participants being necessarily aware of it. It is not clear whether this happened during the demonstration. Indeed, most participants seemed to quickly become aware of the relationship between the position of their CoP and their movement in the virtual environment such that, if an initially involuntary movement of the CoP did occur, it was quickly accentuated by voluntary postural change, with most participants choosing to lean strongly forward in order to gain additional speed. Interestingly however, when asked about the inner workings of the system, not all participant were able to clearly explain how they were able to control the drone. While this is an encouraging sign with regard to the "implicit" form of control we sought after in this exploratory study, it is clear that the combination of our instructions (we specifically asked our participants to not think too much about this question) combined to a lack of technical knowledge about the sensor technologies used in VR also played a role in this.

Nevertheless, one appreciable effect of our control interface, both in the virtual environment and with the real drone, was the "reflex preservation" that occurred when the users felt in danger of colliding with an object. Intuitively, a very natural reaction of the users to an object in the (real or virtual) environment getting uncomfortably close is to lean in the opposite direction in an attempt to put distance between them and this object. Obviously, these reactions had a clear effect on the position of their CoP such that both real and virtual drone immediately followed by moving away from the object. A phenomenon which has been reported in a number of studies on leaning interface similar to ours [Zielasko et al., 2020, Hashemian et al., 2020]. We may consider this phenomenon as a sign, albeit a small one, of the Sense of Embodiment induced by the control interface, as the users reacted to a potential collision between the drone and its environment in the same way as to a potential collision between their body and its environment. Moreover, the fact that such natural behavior is useful in our setup since the user can, without training, react appropriately to an imminent collision is an encouraging sign of its intuitiveness.

With regard to our approach in designing an "embodied control interface" for a drone, having the opportunity to have members of the public tryout the setup highlighted a number of potential improvements. First, as mentioned in the previous section, this demonstra-

tion enabled us to identify two very different dynamics involved in drone control, which we previously referred to as "travel" and "exploratory" trajectories. While the results were qualitatively good for all participants for travelling trajectories, many of them did report that exploratory motions, which requires finer control over of the drone motion, felt difficult to achieve. A report which may be taken as evidence that the intuitiveness of the control scheme was sub-par in such situations. Numerous studies on standing leaning interfaces report that, although such interface often, but not always, yield better result from the point of view of user experience [Pittman and LaViola Jr, 2014, Schulte et al., 2016b, Zielasko et al., 2020], they often time yield worse performance than standard controller interfaces from a practical standpoint [Pittman and LaViola Jr, 2014, Kitson et al., 2015] and it is worth noting that the interfaces that achieve good performance in both areas, such as the Flying Head [Higuchi and Rekimoto, 2013], often sacrifice in scalability to real world scenarios. Although we have to some extent taken this aspect into account by modulating the parameters of the control interface in relation to the drone speed, since travel motion is characterized by generally higher velocity than exploratory motion, these results tend to indicate that better performance could be achieved if we designed entirely different control methods for the two types of motion, while ensuring smooth transition between them depending on the context of use. To our knowledge such approach, sometimes referred to as *homotopic switching* in the Human Robot Interaction literature [Evrard and Kheddar, 2009, Anderson et al., 2014], are yet to be explored in the context of (standing or seated) leaning interfaces. In this regard, this might be an interesting direction to explore in the future.

In conclusion, although the results presented here are only preliminary, studies have shown [Menshchikov et al., 2019] that the proficient use of handheld controller for flying drones require extensive training. In this respect, the fact that all participants were able to quickly understand and take control of the setup, even though the simulated drone dynamics were familiar but not trivial, is an encouraging sign for the potential of such control schemes in the future.

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## A Tactile Radar for Active Sensing of Proximal Objects

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### 6.1 Preamble

In the previous chapter, we designed an "embodied" control interface for a drone, in which the user was incarnated, and for which we aimed to mimic the "natural" relationships between the user's motor actions and sensory feedback in locomotion tasks. In this application, we focused on the "control" part of the interactive system and examined how to endow the user with control over the drone's position in a virtual or physical space in an intuitive and efficient manner. In such a "control-centric" view to the design of an interactive system, the machine can be seen as an extension of the user's motor system, enabling them to gain agency over states, in this case the drone's position and orientation, that are not covered by the "natural" motor system provided by their body. However, while many machines can be seen as augmenting their user's motor system, some are designed with the aim of extending their user's sensory system. Indeed, if machines can be endowed with motor capabilities that far exceed those of the human body, they can also be endowed with sensory capabilities that similarly go beyond what is afforded by our sensory system, enabling them to perceive what we cannot such as a radio-telescope or a medical scanner.

These systems, thus designed under a more "perception-centric" perspective, involve different challenges in their design process. In these applications, the aim is to present the user with a summary of the information available to the machine in an intuitive and accurate way. Similarly to the supernumerary robotic limbs we over-viewed in the first chapter that can be understood as endowing the user with artificial muscles and bones, sensory augmentation devices can be viewed as endowing their user with artificial skin, eyes or ears, thereby augmenting their sensory capabilities.

In this chapter we prototype and study such a sensory augmentation system, a tactile radar, which aims to endow its user's skin with a "new functionality", i.e. to touch object "at a distance" thanks to a combination of capacitive distance sensors and tactile transducers (vibrating motors). This work has been presented at the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS - 2024).

## 6.2 Introduction

Most of today's human machine interfaces rely on visual feedback to convey device and environment related information to their users. Sight enables humans to perceive information with great precision, but it also comes with certain constraints. First, sight is highly directional, which means that we can only process a small part of our environment at a time, with this part diminishing as the precision required increases [Fulton, 2005, Poletti et al., 2013]. Indeed the fovea, the region at the center of the retina that possesses the highest accuracy, only represents about 2% of the total surface of the retina. Secondly, sight impairment is the most common type of disability, with an estimated 1.1 billion people worldwide [Steinmetz et al., 2021] suffering from some form of visual deficit, ranging from mild visual impairment to blindness. Finally, common visual interfaces (screens) offer only two dimensions for encoding a task feedback that may be multi-dimensional. Tactile feedback is therefore seen as a promising modality to compensate for some of these constraints, and tactile displays are therefore receiving increasing attention in human-machine interfaces as the complexity of the machines around us increases [Kaczmarek et al., 1991, Fella and Guatni, 2018].

Tactile interfaces have been used with great success for *sensory substitution* to compensate for visual [Bach-y Rita and Kercel, 2003, Bach-y Rita et al., 2003, Gallo et al., 2010] auditive [Cieřla et al., 2022] and vestibular [Tyler et al., 2003] impairments. In general, Tactile-Visual Sensory-Substitution (TVSS) devices use matrix of tactile transducers (electrical or mechanical) to represent the image captured by a camera on the back [Bach-y Rita et al., 1969], the abdomen [Kaczmarek, 2000] or the tongue [Danilov and Tyler, 2005]. Beyond sensory substitution, tactile interfaces are also commonly used with limb prostheses [Saunders and Vijayakumar, 2011, Hernandez Arieta et al., 2008] in order to feedback tactile and proprioceptive information to the user. This reduces the cognitive load induced by the control and it has been reported to increase the sense of "ownership" of the prosthesis [Hernandez Arieta et al., 2008, Saunders and Vijayakumar, 2011, Gouzien et al., 2017].

In this study, we propose a new tactile transducer, a *tactile radar*, which does not aim to substitute a sense but rather add a functionality (proximity sensing) to our skin. This radar uses high grade capacitive sensors to detect the proximity of objects near the user's limb and convey this information to the limb's skin in the form of vibratory feedback via a vibratory motor array placed below the capacitive sensors. A number of devices with similar aim can be found in the literature. For example, head-mounted tactile radar [Gilliland and Schlegel, 1994, Kerdegari et al., 2014], such as the "Tactile-Helmet" [Cassinelli et al., 2006] or the "Proximity-Hat" [Berning et al., 2015], are designed to be worn on the user's head and to provide a proximity warning, in the form of a tactile stimulus, when the user's head is at risk of colliding with the environment. Similarly, the "Spider Sense" suit [Mateevitsi et al., 2013, Mateevitsi, 2018] is a system which proposes to cover the whole body in sensors and tactile transducers and enabling, for example, its users to navigate closed environments in the dark.

In comparison to these devices, our proposal is two-folds. Firstly, in terms of sensor technology, the aforementioned prototypes all rely either on laser or ultrasonic distance sensors to sample the surrounding environment. The device we propose here, on the other hand, is based on capacitive sensors which, although arguably more difficult to manipulate due to their non-directional nature, offer unique advantages in comparison to other technologies (see section 6.3.1). Secondly, previous devices have focused on providing the greatest

range of  $360^\circ$  to cover the environment surrounding the user. To this extent, the number of measurements provided is therefore relatively sparse spatially. In particular, this severely limits the ability of the user to identify the object being detected by the sensors but also to detect small objects. Typically, object smaller than the sensor's spatial pitch can be missed or detected only intermittently.

In contrast, the device we are proposing here is designed for close proximity detection with greater precision, enabling the user to estimate the shape of an object roughly 10 cm in size and the detection of objects of even smaller dimension (we work with 2cm coins here). For this, the device is based on a denser array of sensors and tactile transducers than previous prototypes of tactile radar. This denser layout of sensors (every 3cm along the forearm) and stimulators (every 1.5cm) is of particular interest to us as it could, theoretically, enable the users to detect objects smaller than the sensor resolution (smaller than the sensor size). Indeed, a sufficiently dense and non-directional sensor network can provide multiple measurements for the same object at different (but nearby) locations on the body, enabling the user to rely on sensory gradients and active movements to enhance their object detection capabilities. Similarly to how we actively detect and explore objects (like a toy) with our hands by moving it inside our palms. We hypothesize that such a high-density tactile radars can enable object detection that is similar to direct touch (without actual touch), and that they can be very useful in enabling better spatial awareness in cluttered scenarios such as industries (for example in aircraft manufacturing or oil refineries) or in low-visibility environments. However, it remains unclear how users can employ such high density tactile radar and whether they can intuitively and efficiently use it like their skin. The work presented here is centered around this question.

The next sections are structured as follow. We begin by presenting the design of our prototype of tactile radar before presenting the results obtained in three user studies. The first examined the accuracy with which the device enables its users to localize an object of about 2 cm. The second study quantified the user ability to spatially localize and recognize the shape of an unseen object using the device. Finally, in the third study, we used a real-object touch paradigm to compare the spatial accuracy obtained with our device with the spatial accuracy of real touch (on the forearm) in a similar task. We close this work with a discussion on the results and the strategies employed by users to accomplish these tasks.

## 6.3 Methods

### 6.3.1 Device

Our *Tactile Radar* consists of an array of 8 flexible capacitive cells ( $3.5 \times 3.5$  cm; designed by Fogale nanotech), layered with an array of 16 vibratory motors (VC0825B002F from Vybionics - 225Hz) (figure 6.1 - A and C). Cubic pieces of foam are sandwiched between each vibrator and the capacitive sensor array to isolate the motor and prevent its vibrations to propagate to the capacitor array and the nearby vibrators. Overall, this gives the device dimensions of 30 by 5 cm, and a weight of 150 grams. This includes the velcro straps enabling the user to attach the radar to his/her limb. In this study, the device was attached to and operated on the user's forearm.

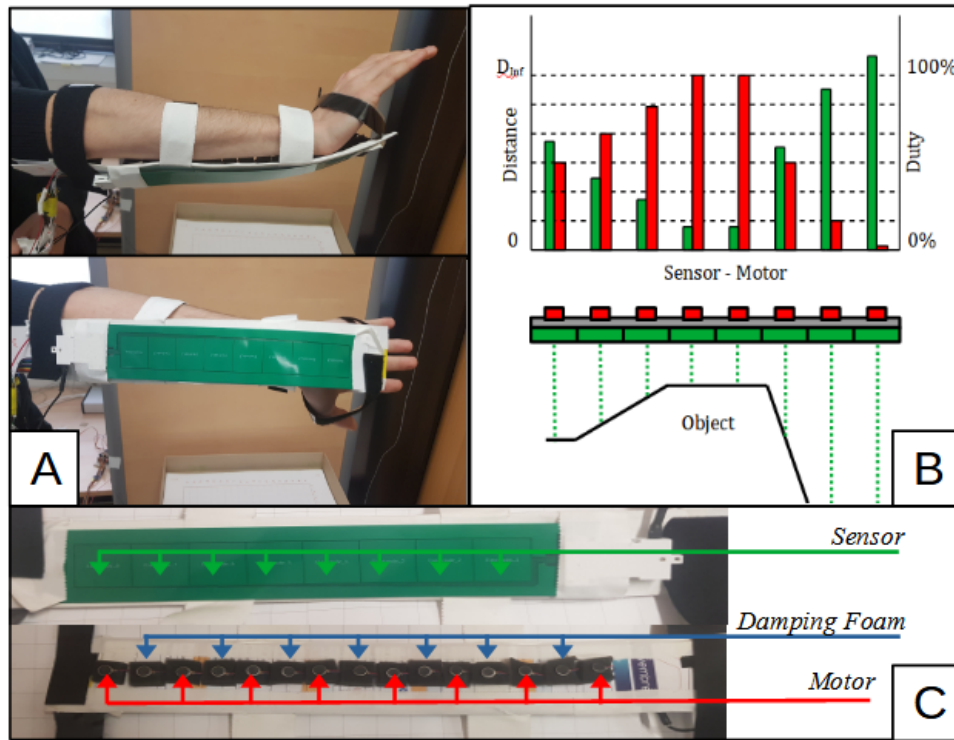


Figure 6.1: The Tactile Radar. A. The device is strapped on the user dominant arm, with the motor array in contact with the skin and the sensors facing away from the limb. B. The distance measured by each sensor is used to modulate the power fed to the motor located directly beneath it (an average is used for motors located in-between two cells). C. Top - Front view of the device showing the capacitive sensor array. Bottom - Back view of the device showing the array of 16 vibratory motors reporting the distance measured to the user.

Given they allow to reach sub-nanometer precision at relatively low cost, capacitive sensors are commonly used in the industry for high precision measurements. A capacitive cell consists in a metallic sheet connected to a known sinusoidal potential. When this metallic sheet is brought in proximity to a conductive object, i.e any metal but also moist material such as biological tissue, concrete, brick, plaster walls and wood (unless very dry), a small capacitance is induced between the two. Among other things, the value of this capacitance is a function of the distance between the sensor and the object of interest (the capacitance decreasing linearly as the distance increases). Since most objects are grounded by virtue of being in contact with the earth, what we are effectively creating is a low pass filter at the sensor input. By studying the effect of this filter on the known sensor potential, we can measure the value of the capacitance created and, using some approximation relative to the shape and size of the object, estimate the distance between the sensor and the object. In comparison to laser and ultrasonic devices, capacitive sensors suffer from a relatively smaller range as the capacitance quickly becomes too small to be measured at distances of the order of two meters. On the other hand, they are non-directional, which means that the device does not need to directly face the object of interest to detect it. This can be an interesting property, as it implies that this technology cannot suffer for intermittent detection. In addition, capacitive sensors are unique in the world of sensory augmentation in the sense that they easily allow for saliency modulation. That is, connecting any object to the same sinusoidal potential as the sensor (a process known as “guarding”) has the effect of making this object “invisible” to the device. Similarly, connecting an object to a positive or negative potential allows to

decrease or increase its detectability respectively (i.e. the object will appear closer/further away). This effectively allows to modulate which object can or cannot be detected and how “salient” they should be for the device, something which is very difficult to achieve with other type of sensor technology. In this study, this property is used to ensure that only the object of interest (metallic coins in Experiment 1 and shapes in Experiment 2) can be detected.

The distance estimate from each capacitive cell is then transmitted to a micro-controller (Arduino Nano 33BLE) which modulates the vibratory amplitude of the motor directly below the cell as a function of the measured distance (or an average between cells for the motors located between two cells). The user is thus provided with a vibration whose amplitude is inversely proportional to the distance between a specific part of the forearm and the object. We chose to modulate the duty cycle of the vibrating motors between 6 different intensities (figure 6.1 - B), with a stronger vibration indicating a shorter distance and vice versa.

### 6.3.2 Participants

22 participants (12 in the first two experiments and 10 in the third experiment) took part in our evaluation study, which comprised three experiments. All experiments were conducted in accordance with the principles of the Declaration of Helsinki. The participants gave informed consent prior to the experiment and all three experiments were approved by the local ethics committee in Laboratoire d’Informatique, Robotique et Microélectronique de Montpellier (LIRMM, Approval ID: 2201F). Volunteers were orally introduced to the working of the device before it was attached to their right arm. This oral description was the only information given to each participant about the device and none of them had any previous experience with a similar device prior to the evaluation.

### 6.3.3 Experiment 1 – Localizing small objects

For the first experiment, the participants stood in front of an elevated table (figure 6.2 - A) onto which was disposed a checkered sheet of paper (of 2x2 cm cells) serving as the search area. The rows of the search area (Y axis) were indexed from A to R (top to bottom), while its columns (X axis) were indexed from 1 to 30 (left to right). As the sheet of paper and table were made of dielectric material, they were invisible to the capacitive sensors.

In each trial of Experiment 1, the participant were instructed to locate 3 metallic objects (figure 6.2 - B) hidden underneath the search area such that they were not visible to the participant. Participants had to “scan” the search area by moving their arm over it in order to localize the objects only through the tactile feedback they received from the device. Each participant performed eight trials with the three target positions drawn from a set of 12 possible configurations. In order to answer, the participants were instructed to place paper discs on the search area to indicate where they estimated the hidden targets to be positioned. This task had no time limit and the participant were allowed to adjust their answers (relocate the paper discs) as many times as they wished. Once they felt confident in their answers, we recorded the indexes of the cells the participant had marked during the search.

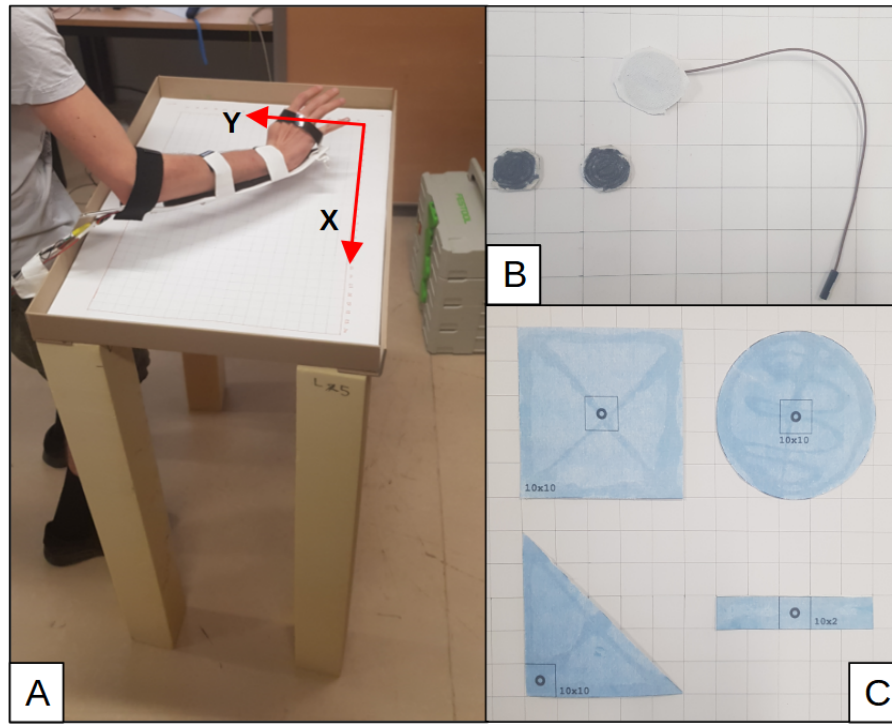


Figure 6.2: The experimental setup A. The participant scan the search area with the haptic radar in order to find the marker/shapes hidden beneath the checkered sheet. B. The metallic coins used in Experiment 1. C. The 4 shapes used in Experiment 2.

#### 6.3.4 Experiment 2 – Identifying simple shapes

The second experiment was carried-out using the same setup as Experiment 1. The experimental procedure was similar to the first to the exception that the participants were tasked with locating and identifying a unique shape hidden in the search area. This shape, made of aluminum foil, was semi-randomly selected from 4 possible shapes (a 10x10cm square, a circle with 10cm diameter, a 10x10cm square-triangle and a 2x10cm bar, figure 6.2 - C). All participants again completed eight trials, with only one shape presented each time, and they experienced all shapes at least once. The participants were provided with paper versions of the 4 different shapes and were instructed to place what they estimated to be the hidden shape in the location (and orientation) where they estimated it to be hidden. As in Experiment I, the participants were given unlimited time to explore the search area and we recorded their responses (i.e. the shape they selected as well as its position and orientation) once they felt confident in their answers.

#### 6.3.5 Experiment 3 – Estimating a position from touch

Finally, Experiment 3 was designed to measure the accuracy with which humans are able to localize a tactile stimulation onto their forearm and, consequently, to the accuracy with which they may relate that position on the forearm to a position in external space. Although previous studies have examined the resolution of tactile perception by humans [Boring, 1942, Weber, 1996], these estimates are known to be heavily dependent on a wide range of factors (for example the part of the body being stimulated [Green, 1982] or the physical properties of the stimulus [Chouvardas et al., 2008]). We therefore decided to include this last experiment in order to provide a baseline specific to our experimental settings

(stimulus locations and properties). This enabled us to formulate some estimate of the proportion of the errors in Experiment 1 that was the result of the “natural” inaccuracy in tactile localization and the proportion that should be attributed to the device.

For this third experiment, the participants were instructed to place their right arm in an opaque cardboard box (figure 6.5 - A). Atop the box and in view of the participant, were marked 9 positions spaced by 2cm and numbered from 1 to 9 (starting approximately at the wrist and ending a little before the elbow). Each participant took part in 15 localization trials in which the experimenter selected one of the nine positions at random and applied a tactile stimulus onto the participant skin immediately below the selected position using the same vibratory motor as employed for the device. Since the participant could not see their arm inside the box they could only feel where the tactile stimulus was applied. If needed, the participants were allowed to move the arm during the application of the stimulus in order to better localize it, similarly to the first two experiments. Importantly, although the participants could move their arm, the position at which the stimulus was applied remained fixed in relation to the box. As for experiments 1 and 2, the participant could take as much time (and arm movement) as they wished before answering, by pointing on the box the position which they estimated to be directly above the location where the stimulus was applied by the experimenter. After 15 trials, the error between the position randomly selected by the experimenter and the participant answer was analyzed to estimate the localization error when attempting to relate a tactile stimulus to a position in external space and so when the exact point of stimulation is not visible (as was the case in the first two experiments).

## 6.4 Results

### 6.4.1 Experiment 1 - Localization error for small objects

To evaluate the participants’ performance in Experiment 1, each cell containing a hidden marker (hereafter referred as the ground truth) was paired with the closest cell selected by the participants (hereafter referred as the response) without repetition (i.e. each response was matched with a single ground truth). This procedure was carried out for every trials and participants. Finally, couples whose localization error (distance between the ground truth and response) was more than 3 times the standard deviation computed across all the couples were considered outliers and excluded from the analysis. These outliers represented less than 2% of the data.

Overall, the participants exhibited an average localization error of  $4.21\text{cm} \pm 1.26\text{sd}$  and, in term of accuracy, the average standard deviation of this error (computed for each participant) was observed to be  $2.11\text{cm} \pm 0.65\text{sd}$ . Interestingly, as can be seen on figure 6.3 - A, the participant were more accurate along the rows (Y) with an average error of  $2.13\text{cm} \pm 0.63\text{sd}$  than along the column (X), with an average error of  $3.03\text{cm} \pm 1.25\text{sd}$ . The significance of this difference in accuracy was confirmed by a two-samples t-test [ $t(244) = -4.14$ ;  $p < 10^{-4}$ ;  $d = 0.367$ ]. We also observed a slight positive bias (0.36cm) along the Y axis, indicating that participants tend to respond closer to their bodies, which however did not reach significance [ $t(244) = 1.84$ ;  $p = 0.06$ ;  $d = 0.12$ ]. In contrast, we observed a significant positive bias (0.97cm) along the X axis, indicating that participants tend to answer to the right of the ground truth [ $t(244) = 3.72$ ;  $p < 10^{-3}$ ;  $d = 0.24$ ].

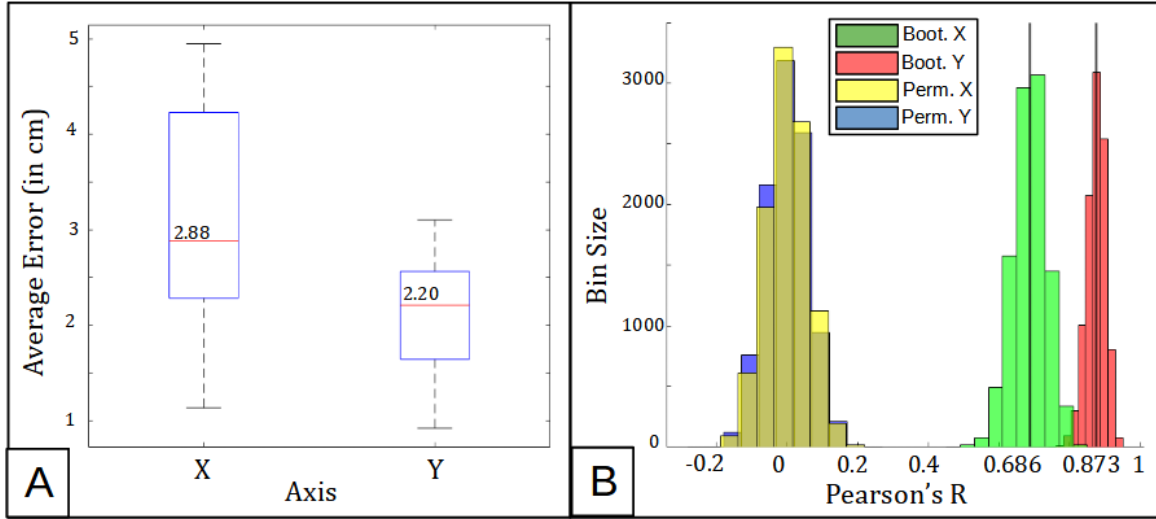


Figure 6.3: Performance in Experiment 1. A. The distribution of observed localization error for each of the 12 possible marker configurations. The red lines indicate the median across participants while the box edges show the 25th and 75th percentile respectively. The error bars indicate the 5th and 95th percentile of the data. B. The distribution of correlation coefficient (Pearson's R) between ground truth and answers (X along the columns and Y along the rows) for couples obtained by bootstrapping (Boot. in red and green) and by permutations (Perm. in blue and yellow). The black line marks the correlation coefficient computed from the experiment data. Total bin size is 10000.

To analyze the significance of the participants' performance, we performed a Bootstrap / Permutation test to compare the distribution of correlation coefficient (Pearson's R) between the couples of ground truth and response observed in our experiment with the distribution of correlation coefficient obtained via a random pairing. Figure 6.3 - B, shows the distribution of correlation coefficient between the ground truth and response (computed individually along the columns - X, and rows - Y) computed over 10000 bootstrapping of the couples (sampling with replacement of the couples, in red and green respectively) and over 10000 permutation of the couples (sampling with replacement of individual response/ground truth, in blue and yellow respectively). With a significant p-value of  $p < 10^{-4}$ , the limit of the test, this test demonstrated that the participants were able to use the device to localize the targets in their reaching space well beyond randomness.

#### 6.4.2 Experiment 2 - Identification and localization error for shapes

To assess the participants' performance in the second experiment, we defined a score measuring the overlap between the shape hidden beneath the search area (ground truth) and the shape positioned by the participant onto the search area (response) :

$$Score = 100 * \frac{T \cap A}{T \cup A} \quad (6.1)$$

with  $T$ ,  $A$  the ground truth and participant response, respectively, and  $T \cap A$ ,  $T \cup A$ , the area of the intersection and union of the ground truth with the response respectively (see figure 6.4 - A). A score of 0 indicates that the ground truth and response do not overlap at all, while a score of 100 indicates perfect overlap.

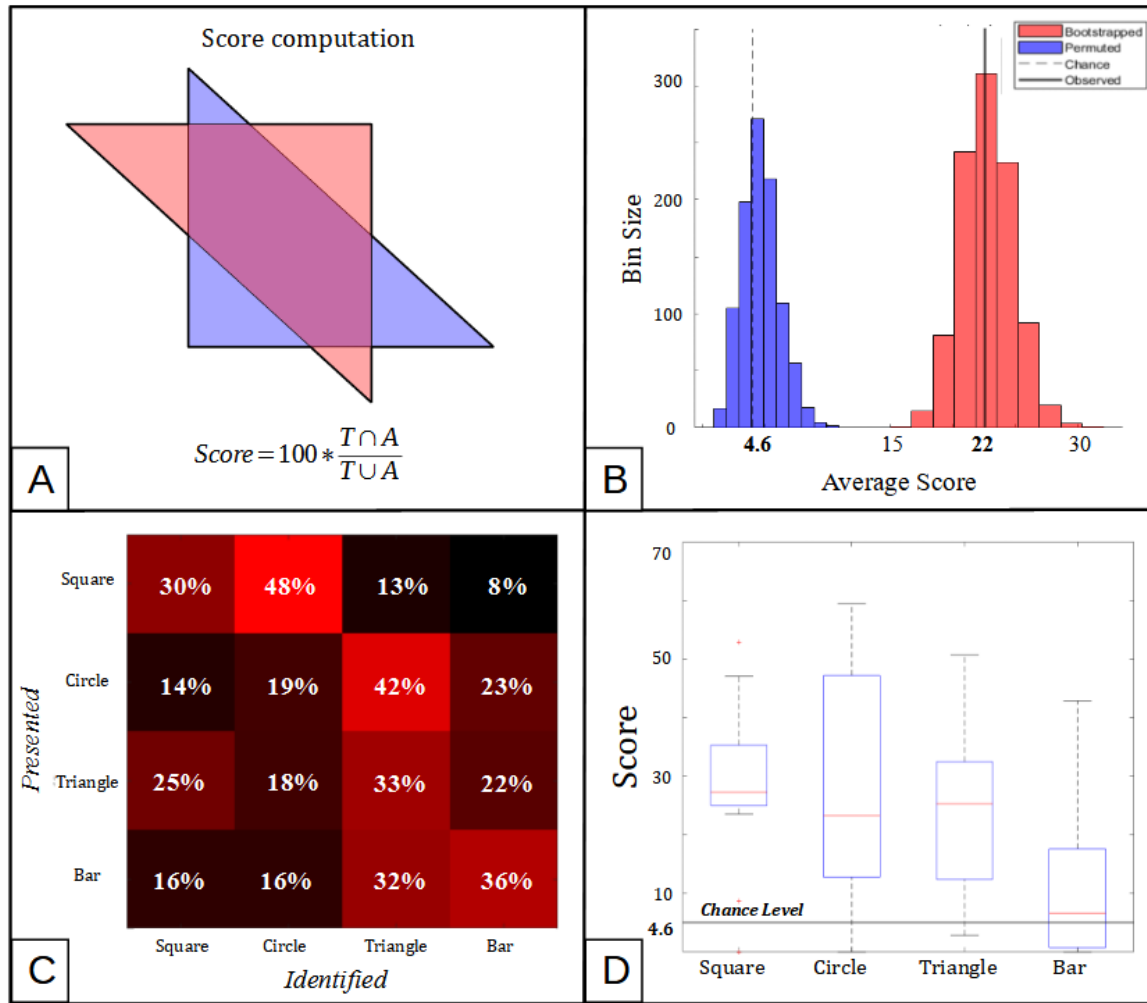


Figure 6.4: Performance in Experiment 2. A. The performance score in the task is computed as the area of the intersection of the response and ground truth shapes (in violet) divided by the area of their union (bold black line),  $S = 47$  for this example. B. The distribution of average error for bootstrapped couples of ground truth / response (Bootstrapped in red) and permuted couples (Permuted in blue), total bin size is 1000. C. The confusion matrix between shows the presented (in column) and identified (in line) shapes. D. The distribution of scores computed individually for each shape presented during the experiment, a score of 0 indicates that the ground truth and response do not overlap at all, while a score of 100 indicates perfect overlap.

The participants' score varied heavily depending on the hidden shape (figure 6.4 - C,D), from an average score of  $11 \pm 12\text{sd}$  with the bar to an average of  $28 \pm 14\text{sd}$  with the square. This is understandable since, of all the shapes, the bar had the smallest surface area, thus misidentifying it for any other shape will lead to a score that cannot exceed 40 (when the bar is contained within the square triangle). Figure 6.4 - C shows the confusion matrix, with the presented shapes in rows and the shape they selected by the participants in the columns. The square shape covered the most surface and was mainly confused with the circle (48%) which had similar surface coverage. Similarly, both square and circle are symmetric given a  $90^\circ$  rotation, which is not the case for the triangle and the bar. As a result, the bar and square triangle add a layer of difficulty in comparison to the square and circle, in the sense that their orientation is also important for scoring.

It should be noted that the performance score we use here is a very strict measure that dip quickly with small error in the localization. Thus, to assess the extent to which the scores we observed were significant, we decided to compare the participants score with chance level using the same Bootstrap/Permutation test as in Experiment 1. We permuted the ground truth and response shape as well as bootstrapped them as couples to estimate the variability of our results (figure 6.4 - B). By repeating this process 1000 times, we can see that our participants, even with an apparently low average score of  $22 \pm 16sd$  across all shapes, performed way beyond chance level ( $p < 10^{-3}$ , again the limit of our test) which was around 4.6 with our measure of performance.

The average score across participants were also significantly different from chance level for each of the 4 shapes; for the square [ $t(11) = 6.10$ ;  $p < 10^{-4}$ ;  $d = 1.97$ ]; the circle [ $t(11) = 4.07$ ;  $p = 0.02$ ;  $d = 1.32$ ]; the triangle [ $t(11) = 5.31$ ;  $p < 10^{-3}$ ;  $d = 1.97$ ]; and the bar [ $t(11) = 2.22$ ;  $p < 0.048$ ;  $d = 0.88$ ].

### 6.4.3 Experiment 3 - Accuracy in estimating a position from touch

To assess the accuracy with which participants were able to localize the tactile stimulation on their arm, we calculated the distance between the position identified by the participant (response) and the position selected by the experimenter (ground truth).

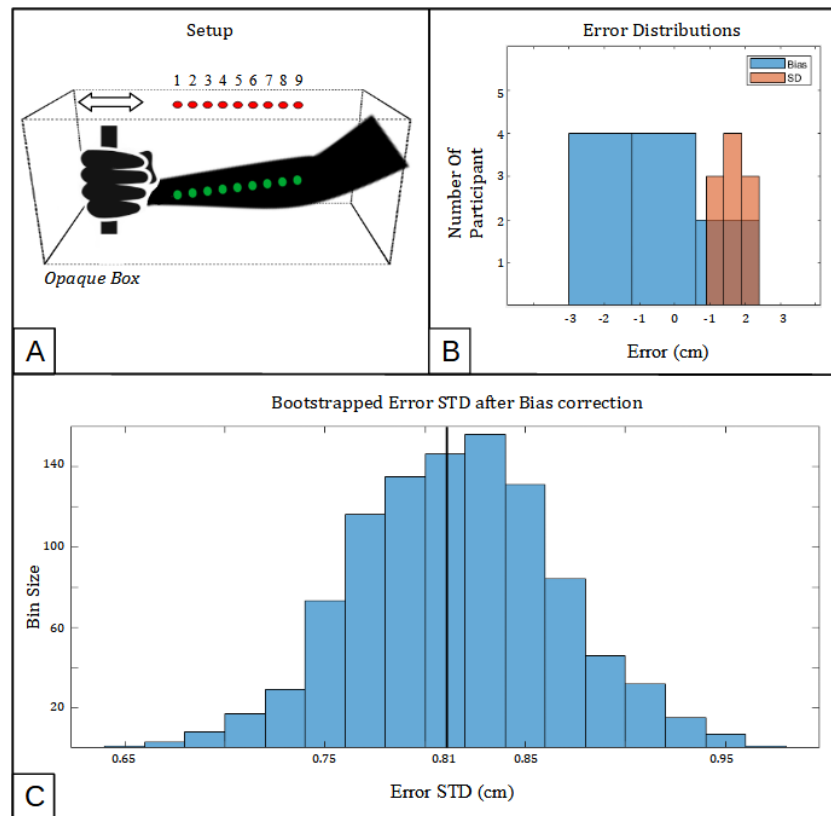


Figure 6.5: Performance in Experiment 3. A. The participant were asked to point on the box (red mark) the position closest to the location chosen by the experimenter to apply the tactile stimulus (green mark on the arm). B. The distribution of the mean and standard deviation of error exhibited by each participant across all 15 trials. C. The distribution of the standard deviation of error exhibited by a thousand bootstrapping of the participants' answers after bias correction. The obtained average standard deviation for the error is 0.81cm.

The average error between response and ground truth was found to be  $1.74\text{cm} \pm 1.59\text{sd}$  across participants. This represents about 61% of the error observed along the columns (X) and about 80% of the error along the rows (Y) in Experiment 1. Overall, most participants (8 of 10) exhibited a negative bias (towards the hand) in their estimation of the position but some (2 of 10) exhibited a positive bias (oriented towards the elbow). Figure 6.5 shows the distribution of bias (A) and standard deviation (B) for all participants. Bootstrapping the answers given by each participant after bias correction (C) shows that the participants were able to identify the position selected by the experimenter based on the tactile stimulus with an accuracy (i.e standard deviation) of  $0.81\text{cm} \pm 0.05\text{sd}$  on average. Given that, on average, the accuracy of participants' responses in Experiment 1 was  $2.11\text{cm} \pm 0.65\text{sd}$ . This result suggests that almost 38% of the localization error in Experiment 1 was due to natural inaccuracies in the estimation of the position of an invisible source of a tactile stimulus by humans.

## 6.5 Discussion

In this study, we designed a tactile radar with a view to enable its users to "touch object at a distance" in their reaching space. In comparison to previous design, this prototype included capacitive, non-directional, distance sensors combined to a denser array of sensors and tactile transducers. The device was then evaluated in two user studies where we quantified the ability of our participant to take advantage of this non-directionality and of the denser layout to first localize small objects (whose size of 2cm was slightly below the size of the capacitive cells of 3x3cm) and then to discriminate between simple shapes.

The first experiment showed that the participants could locate the objects using the radar with an absolute average error of  $3.03\text{cm} \pm 1.25\text{sd}$  along the columns (X) and  $2.13\text{cm} \pm 0.63\text{sd}$  along the lines (Y). It is interesting to note that these values are close to or lower than the size of the individual capacitive cells (3x3 cm). This was arguably possible because our participants were able to integrate movement related gradients in the vibratory feedback to improve the precision of their responses as we initially hypothesized. It is also interesting to note that, during this first experiment, the performance was uneven along the X and Y axis (red and green distributions in figure 6.3 - B) while the distribution at chance level was the same for both axes (blue and yellow distributions in 6.3 - B). This difference in performance cannot therefore be explained by the greater number of cells available in the X axis (columns; 30 cells) compared to the Y axis (rows; 18 cells). One explanation for this result probably lies in the configuration of the experiment. When our participants faced the search area with their arm extended over it (figure 6.2 - A), the tactile radar was oriented roughly along the Y axis. Conversely, it was fairly uncomfortable for our participants to rotate their arm such as to orient the tactile radar along the X axis. Consequently, while the position of the target along the Y-axis could be estimated by feeling the gradient in vibration amplitude along their forearm, our participants had to primarily use arm movements and movement related gradients in vibration amplitude to estimate the positions of the objects along the X axis. This result thus suggests that the participants were better at estimating the position of the object when offered feedback on its distance to their arm at various points.

Similarly, the positive biases we observed for both the X (0.97cm) and Y (0.36cm) axes may also be explained by the position of the participant during the experiment. For the Y axis, the positive direction points towards the participant. Consequently, cells with a higher

Y coordinate are easier to reach. For the X axis, the positive direction points to the right of the participant and, given that all of our participants wore the device on their right arm, the cells with higher X coordinates were also easier to reach.

The second experiment showed that participants were relatively successful in detecting and identifying shapes based on the information provided by the device. While localizing an object is possible by estimating the vibration gradients in relation to movements, identifying a shape additionally requires our participant to detect its edges and invariably remember and integrate gradient information over time. Although, overall, it appears that the participants struggled to identify and localize the hidden shape (with an average score of 30, 19, 33 and 36 for the square, triangle, circle, bar respectively), several issues need to be considered here.

Firstly, a closer examination of the confusion matrix (figure 6.4 - C) shows that the participants were, in fact, able to identify the object size fairly well. The bar, which was the shape with the smallest surface area, was mainly identified as the bar over the four possible shapes (36%). In contrast, the square, whose surface area is similar to that of the circle and was the shape with the greatest surface area, was misidentified as the circle in almost 50% of the trials but rarely (8%) with the bar. The fact that the circle was often (42%) misidentified with the square triangle may also be indicative of the fact that the participants were able to roughly detect the edges of the hidden shape, since these shapes were the only two whose edges were not aligned with the grid. Secondly, it should be noted that the shapes presented to the participants were relatively small in comparison to the capacitive cells, making a difference in size difficult to recognize and possibly blurring the exact location of the shape edges. Furthermore, our participants received no prior training with the device and had to work without feedback throughout all experiments (i.e. we never disclosed the correct answers before the end of the experiments), as we preferred not to influence their performance through learning. In this regard, it has been shown that training can greatly improve the ability of a participant to recognize simple geometric shape using TVSS [Bermejo et al., 2015]. It can therefore be hypothesized that the results reported here can be improved by providing the participants with feedback on their mistakes in order for them to adapt to the device.

Finally, the results reported in the first two experiments should also be understood in light of the results obtained in the third. This last experiment, performed in similar conditions, showed that the accuracy of our participants in relating a tactile stimulus on the forearm to a position in space was on average of 0.81cm. This result is close to the accuracy observed in the first experiment, where the standard deviation of the localization error of the hidden markers was found to be of 0.63cm in the lines (Y axis) and of 1.25cm in the column (X axis). This tends to indicate that a large part of the localization error observed in Experiment 1 may be explained by the natural imprecision of humans when attempting to relate a tactile stimulus to a position in space.

Overall, although these results show that the participants were able to identify shapes above chance level using our prototype, further research is needed to accurately evaluate this ability and the extent to which it may be improved through training. In addition, the device we presented here is far from optimal. For example, it is known that different tactile receptors on the skin (the Pacinian's and Meissner's corpuscle, or the Ruffini's and Merkel's endings for examples) exhibit different sensitivity depending on the tactile stimu-

lation frequency as well as different rates of desensitization under a continuous stimulation [Kaczmarek et al., 1991]. In the current implementation, due to the use of vibratory motors, the changes in proximity to an object result in correlated modulations of the frequency and intensity of the vibrations. Our prototype thus involve a number of different sensory receptors on the skin in the feedback of distance information. It is to be expected that different stimulus patterns and/or intensities would lead to different result and the question of an optimal feedback profile given our application is still an open question [Chouvardas et al., 2008].

This study is the first to propose capacitive sensors as a valid substitution to the laser and ultrasonic devices that are more commonly used in tactile radar applications. Although the non-directional nature of this sensor technology could be considered a drawback given our application, this did not seem to bother our participants and, in fact, enabled them to easily localize small objects by following vibratory amplitude gradients that were consequently available over the whole search area (and not solely when a sensor is pointed directly at the object). The ability to modulate the saliency of different surfaces to the device and its user is also an interesting aspect of this sensor technology. In this study, this allowed us to ensure that every object other than the marker and shapes (the table, walls and even the participant him/herself) were undetectable by the sensors. In a broader context, this ability to modulate "at will" the saliency of various object to the sensor by simply connecting them to a small potential can be very useful to provide contextualized information (a lifeline guiding the user to an exit in case of an emergency) or to highlight "points of interest" (a door handle) thus making the use of such devices considerably easier.

In conclusion, this study suggests of the great potential of capacitive sensors as well as densely arrayed sensors and stimulators as a research direction for devices aiming to augment the spatial awareness of their users, which can be crucial for the safety of workers in the cluttered environments commonly encountered in heavy industries. Although the presented prototype is still naive and not optimized, our investigations show that users can detect and localize objects well with the device, and first results show promises in regard to shape recognition. Finally, we note that the prototype presented here use collocated sensors and stimulators (i.e the sensor and the stimulator it modulates are in the same location on the user's skin). An interesting venue of research lies in the extension of this paradigm to non-collocated configurations, for example by distributing the sensors over the body of a car to provide the driver with information they may not access visually.



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### Closing Discussion

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In this thesis, we have approached the concept of coadaptation from a perspective primarily found in the literature on adaptive interfaces [Alahmad and Robert Jr, 2021]. The problem to "fit" an interactive system to its user is however quite vast and, depending on the application, is expressed in different terms that underlie different conceptualizations of the relationship between the user and the system. In this respect, the work presented here is largely centered around the view that a machine can be considered as an extension of the user's sensory-motor system, or simply body.

Although fairly widespread in the literature, this view was in our case motivated in part by our work on the application of coadaptation in the context of supernumerary robotic limbs, which we presented in chapter I. In this seminal work, we showed how coadaptation could be applied to dynamically adapt the control scheme of an interactive system to the context in which a task is being performed. Although this study was performed in a simplified and controlled setting, we notice the parallel between the contextualized negotiation between the user and the supernumerary finger and the everyday "negotiations" our brains and bodies engage in. Our behaviour adapts to our body's capabilities, while our body adapts to our needs.

With this parallel in mind, we moved on to a study on sensory motor processes in humans, the "brain-body" loop, a work presented in the second chapter of this manuscript. We showed that, in a choice reaction task involving tactile and/or visual stimulus where the reaction was conditioned on the stimulus position in space, the speed at which humans make decision based on the position of a stimulus depends on the modality in which it is presented. In particular, we have shown that spatial information presented tactilely are slower to integrate than when the same information is presented visually, a surprising result considering the fact that the reaction time to a tactile stimulus is known to be faster than to a visual stimulus.

In chapter III, we continued this work by studying a computational model of the phenomenon of Sensory Response Compatibility (SRC). Although many authors had interested themselves in such model before us, we proposed to an alternative model based on the theory of Active Inference. We showed that this Bayesian framework could reproduce, in

approximate form, a number of experimental observations relating to SRC, in particular the partial response activation observed via electro-physiological monitoring of participants in SRC studies. We explained this by relating the structure of the model to the connectionist Dual Route model, one of the computational model proposed in the literature to explain these observations.

These experimental and computational treatments of the issue to fit an interactive system to its user were complemented with a "practical approach" in the last two chapters. Firstly, in chapter IV, we presented a design for an "embodied" control interface for a drone. In this case, we were interested in how the control interface could be designed to mimic the relationship between motor actions and sensory feedback the user experience in locomotion tasks. In particular, we examined the relationships between eyes, head and torso rotations at rest and during movement, and attempted to reproduce these relationships between the user's posture and the drone rotation. In addition, we hypothesized that a Center-of-Pressure (CoP) based control interface could take advantage of small shifts in the user's CoP elicited by the intention to move, and we incorporated this hypothesis in the design process. Although we are yet to experimentally validate our approach, we reported on encouraging preliminary results obtained in an open event where members of the public were invited to try out the setup. In particular, the fact that users did not require any instructions on the means by which to control the drone in virtual reality suggests the intuitiveness of the proposed control interface.

Finally, in chapter V, we reported on the design and evaluation of a "tactile radar" aimed at enabling its user "touch an object at a distance" using a combination of capacitive distance sensors and tactile transducers (vibratory motors). Compared with designs previously proposed in this application context, our prototype included a greater density of transducers and sensors, and employed a different type of sensor technology (capacitive instead of laser or ultrasonic sensors). We demonstrated that, although it might be assumed that due to their non-directional nature, capacitive sensors are less suited than other sensor technologies for this type of application, the error of a user using the device to locate small objects could in fact be explained, in large part, by the natural inaccuracies of humans when we attempt to relate a position in "body space", in which the tactile stimuli are presented, to a position in external space, in which the small objects were hidden. Although performance in a shape identification task was fairly low, we also showed some preliminary evidence that the greater density of transducers and sensors benefited the accuracy of participants.

## 7.1 Limitations and Perspectives

In this manuscript, our approach to the challenge of fitting an interactive system to its user has led us to treat the problem from different perspectives, from a naive, "machine learning-based" approach using stochastic gradient descent in Chapter I, to a more practical approach to interactive system design in Chapters IV and V, and a study and computational modeling of human behavior in Chapters II and III. We already discussed some of the limitations of the works presented in the previous chapters. Rather than revisiting these discussions to summarize them here, here I will rather discuss the limitations of the very idea to fit an interactive system to a user, leading a more general discussion on the perspective offered by viewing our relationship with interactive systems as a *coadaptation*.

An obvious partitioning of a "human-machine system" consists in dividing it into three sub-systems, the human, the machine and the environment they share. A task can then be heuristically defined as a modification of this environment such as its state match some preferred state. Our bodies are eminently limited and these limits can be viewed as constraints on our ability to shape the world around us. In this respect, tools enable us to overcome some of these limitations. A hammer, for example, allow us to impact this world with a greater force than permitted by our fists, a saw, to cut through materials that we cannot split using our strength alone. Tools, however, rely on the energy provided by our body and only enable us to use this energy more efficiently to shape our environment. On the other hand, machines, by relying on their own power source, also enable us to exceed our bodies' limits in term of the effort we can develop for extended periods of time. However, because they use an external energy source, machines, unlike tools, also have dynamics of their own which "escape" our control. As the purpose of machines is to help us shape our environment, it is therefore necessary to define a "language" through which we can communicate our intentions, our preferred state for this environment, such as we hold some form of agency over these dynamics that we do not control. However, over the years, machines have become increasingly complex, and the number of degrees of freedom (DoF) they incorporate has grown to the point where it quickly became unreasonable to expect us to communicate an intention over all these DoF. There are therefore two challenges when trying to fit an interactive system to a user. The first, on which we have focused in this manuscript, concerns the definition of a "good" language through which the user can communicate their intention to the system. "Good" may be understood as a language that is both unambiguous, an idea that can be linked, for example, to the first chapter of this manuscript where we let the user and the system negotiate a communication method maximizing the information they exchange over a noisy channel. But also intuitive, or easy to learn, an idea that can be linked to the fourth chapter, where we sought to use the "natural language" of locomotion, based on the motion of the center of pressure, to communicate the user's movement intention to the system. Conversely, the second challenge relates to the selection, among all the degrees of freedom embedded in the system, which ones do we expect the user to express an intention for? We touched on this "DoF reduction" problem, in the context of supernumerary robotic limbs (SRL) in chapter I. SRL are highly complex systems and it is extremely rare to give the user control over all the DoF they incorporate. In our case, this was possible since the supernumerary finger we used was extremely simple and has only two degrees of freedom. In general, however, high-level control strategies must be employed such as the user is offered agency over the system as a whole rather than over each individual DoF. There are therefore "inexpressible" intentions in the language defined for the interactive system, intentions that lie outside the low-dimensional manifest embedded in the system's state space and over which the user is endowed with agency. This reduction in dimensionality and the existence of inexpressible intentions can be linked to the concept of "automation" [Parasuraman and Riley, 1997, Parasuraman et al., 2000], a term which, in some sense, refers to all situations in which the system takes decisions for the task without the user intervention.

Fitting an interactive system to its user therefore implies that those automated decisions, taken by the system without the user input, also match the user intention. For most of the interactive systems we interact with, automation, which can also be seen as a *value alignment problem* [Hadfield-Menell et al., 2016, Christian, 2021, Mechergui and Sreedharan, 2023] since we expect the system's utility to match the user's utility, is fairly straightforward. While there are many ways in which we may wish to modify our environment to more closely match our subjective tastes, there are also "objectively undesirable" states for this environ-

ment. For example, although there are an infinite number of destinations, and therefore trajectories, that the driver of a car may wish to reach, it is reasonable to assume that none of these trajectories has the car colliding at high speed in some environmental feature. Similarly, while there are many activities, and therefore trajectories, that a user may wish to perform with a prosthetic leg, it is reasonable to assume that none of these trajectories includes an unexpected fall on the ground.

As machines become more and more complex, enabling them to assist us in increasingly complex tasks, a fundamental problem begins to emerge. Fitting an interactive system to its user implies that there exists "something" that can be fitted. In a Bayesian Game framework [Harsanyi, 1967, Barron, 2024] and in particular the ad-hoc coordination game which formalizes the problem faced by an agent required to cooperate with others about whom it knows very little [Stone et al., 2010, Albrecht and Ramamoorthy, 2013, Albrecht and Ramamoorthy, 2015, Barrett et al., 2017, Rahman et al., 2021], fitting the user correspond to the inference process by which the system characterizes its user's *Harsanyi type* [Harsanyi, 1967, Böge and Eisele, 1979, Brandenburger and Dekel, 1993]. A type that can be heuristically considered as a set of parameters defining the user's utility and admissible policies and, more generally, as the "program" underlying the user's behavior. This idea can be related to chapter III, where the Active Inference based computational model we studied was such a program. Given this approximate model of the user behavior, a machine can fit it to its user in order to efficiently collaborate with them on the basis of what it has observed of their behavior (consider chapter II for example).

However, utility, cost, reward or payoff functions are only "convenient" tools, the main problem for an agent is to define its value function [Bellman, 1954, Schal, 1979, Friston and Ao, 2012], which determines its motion and allows it to efficiently achieve its goals. When the achievement of these goals depends on mutual coordination with another agent, however, this value generally depends on the agent's actions and on those of its partner. In this case, the two agents can be seen as engaging in a form of "mind-reading" [Malle et al., 2001, Frith and Wolpert, 2004, Barrett et al., 2010, Lombard and Gärdenfors, 2023], both of them trying to "get into each other's head" and infer what is the likely course of action the other might follow, a probability distribution that is determined by their respective Harsanyi types. This inference process, and the resulting adaptation, is easily transposable to human-machine interactions. To use a car, a driver needs to understand to some extent how it works, such as the correspondence between the angular position of the steering wheel and the car's trajectory. In other words, they are required to infer the car's Harsanyi type. Similarly, to help the driver in reaching their destination, the car (or the designer of the car) is required, to some extent, to understand how they function, for example the amount of force that may be exerted on the steering wheel by the user's arm, and so, once again, it must infer the user's Harsanyi type. It is however interesting to note that here, the "goal" of both driver and car is not really to explain each other's behaviour, but their own. Driver and car both believe they are a "driver" and a "car", however they do not know what it means to "behave" like a driver and a car and thus have to learn it through their interactions. And so even though they share the common prior that a driver has a destination and that a car can help them in reaching it. As such, it is less about *utility*, or goals, than it is about *value*, or motion.

The example of a car, I believe, represents well the dynamics of the human machine relationship, the coadaptive process, as well as the limits that exist to the idea that a system must adapt to its user. The user of a car is a driver, but a car is the system whose user is a driver,

and if neither driver nor car *commit*, a priori, to a behavior that the other can adapt to, it is fairly obvious that both of them will never get anywhere. In turn, this commitment shapes the value function of the partner and fixes, in part, its own behavior, which the other can adapt to. This Stackelberg (leader-follower and vice-versa, [Li and Sethi, 2017]) game goes on until both car and driver hold some precise beliefs about their respective value, i.e. what it means to *behave* like a car and a driver, at which point deviating from this self-assigned behavior may be viewed as going against their own utility. Indeed, there are limits to the idea of fitting a system to its user and this simply because, in some cases, the user him/herself does not know what it implies to behave like a user.

This process, I believe, can be related to the theory of *mindshaping* [Zawidzki, 2013, Zawidzki, 2016] proposed by cognitive scientists to explain the fact that our "mind-reading abilities" often exceed what would be theoretically achievable based on available evidence alone. They argue that humans coordinate with each other neither by simulating the thinking of their conspecifics using their own as basis, nor are endowed with some innate psychological theory (the *simulation theory* and *theory theory* respectively; [Nichols and Stich, 2003, Gerrans, 2009]), but by constructing and enforcing shared normative behaviors ([Finnegan, 2023] ch.2 s2.1), intuitively a form of "role-playing" where the role to be played is that of a fictitious agent. Our ability to coordinate is thus not the result of our ability to predict each other, but to co-create and conform to idealized versions of ourselves that are made to coordinate.

According to this view, values and beliefs are not consequences of pre-existing, exogenous utilities that the coordinating agents strive to accommodate, but are in fact discovered throughout their interactions [Ross and Stirling, 2023] and the motivation for praxeological (i.e. purposeful) behaviour is that of epistemic gains. By becoming predictable for you by conforming to what you expect of me, you become predictable for me by conforming to what I expect of you, enabling us to coordinate without relying too heavily on the difficult process of reading each other's minds. Such behavior, driven by epistemic rather than pragmatic utility, can also be related to the theory of Active Inference and the Free-Energy Principle [Friston et al., 2015], which proposes to consider adaptive behavior through the sole lenses of surprise (i.e. negative marginal evidence and, more generally, entropy; [Friston and Ao, 2012]) minimization and under which the adaptive agent strive to provide a rational for its *own* behavior, as reported by its sensory afferent, above anything else [Ramstead et al., 2020]. Consider [Sajid et al., 2021] for example, where a surprise-minimizing agent is placed in a utility-free environment (i.e. no costs nor rewards) and engages in a rich exploration of this environment, motivated solely by the need to discover "who it is", to finally conform to the answer it came up with to this question. We might also draw a parallel between this and endogenous reinforcement learning [Bereby-Meyer and Erev, 1998, Börgers and Sarin, 2000], where the utility is determined by the learning agent itself, in contrast with the exogenous payoffs (i.e. fixed and determined by the environment) generally employed in reinforcement learning.

Finally, it is also interesting to note, in relation to our introduction, that endogenous reinforcement learning is historically the way by which learning games (repeated games played by two learning agents; [Lakshmivarahan and Narendra, 1982]) have been related to evolutionary games (game theoretic model of the evolution of two or more populations interacting with each other; [Weibull, 1997]). Indeed, it can be shown that, under an appropriately constructed continuous-time limit [Börgers and Sarin, 1997], the dynamics of the interactions of two agents adapting to each other through endogenous reinforcement learn-

ing, converge to the continuous time replicator equation [Taylor, 1979], thus bridging the gap, at least conceptually, between coadaptation and coevolution [Tuyls et al., 2002].

## 7.2 Conclusion

Although machines can be seen as descendants of tools, it can be argued that the independence they acquire every day, in a number of cases, blurs the line between the role of the user and the role of the tool in the human machine relationship.

This blurring can be seen in one of the definition of coadaptation we presented in the introduction, where [van Zoelen et al., 2021] defined coadaptation by referring to anonymous "parties" rather than a machine and a user. While considering human and machine on an equal ground in their relationship is eminently justifiable in light of the systems endowed with "human-like" intelligence and capabilities we may come to interact with, now and in the future. It also implies that the difficult challenge of balancing this relationship is now more pressing than ever. Consider for example a predictive policing systems providing "tips" to a police officer regarding the area where a crime is likely to occur [Jouvenal, 2016, Degeling and Berendt, 2018], or an automated profiling system advising border guards on which individual should be searched [Ferraris et al., 2015]. Many authors argue that although, in both these tasks, the final decision is supposed to fall onto the user, since he/she is liable for the consequences of that decision (i.e investigating the area or stop and search the individual highlighted by the system), users of these systems often find themselves doing little more than simply "rubber-stamping" the system's decisions as they come [Degeling and Berendt, 2018, Wagner, 2019, Gugliotta et al., 2023, Musco Eklund, 2023]. And thus, in such case, it can be argued that the historical relationship between users and systems has been reversed, the user finding himself or herself in the position of a "useful tool" enabling the system to enforce its decisions.

Should we consider the concept of coadaptation under the light of the mindshaping theory of collaboration, for example defining it as, "the process by which human and machine jointly create, endow and enforce role for each other", it is clear that part of the challenge posed by coadaptation lies in constructing a fitting role for the user. But we, as the systems with which we interact, can play an infinite number of them and thus an equivalent challenge lies in constructing an appropriate role for the machines with which we collaborate. With the risk that, should our trust in these systems inadvertently turns to faith, the rapid gains in independence of machines come to the cost of our own.

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## Bibliography

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- [exi, ] Exii design inc. <https://exiii-design.com/>.
- [Abbass and Abdulrahman, 2013] Abbass, S. J. and Abdulrahman, G. (2013). Kinematic analysis of human gait cycle. *Al-Nahrain Journal for Engineering Sciences*, 16(2):208–222.
- [Abdi et al., 2016] Abdi, E., Burdet, E., Bouri, M., Himidan, S., and Bleuler, H. (2016). In a demanding task, three-handed manipulation is preferred to two-handed manipulation. *Scientific reports*, 6(1):21758.
- [Adams et al., 2013] Adams, R. A., Shipp, S., and Friston, K. J. (2013). Predictions not commands: active inference in the motor system. *Brain Structure and Function*, 218:611–643.
- [Ahveninen et al., 2006] Ahveninen, J., Jääskeläinen, I. P., Raij, T., Bonmassar, G., Devore, S., Hämäläinen, M., Levänen, S., Lin, F.-H., Sams, M., Shinn-Cunningham, B. G., et al. (2006). Task-modulated “what” and “where” pathways in human auditory cortex. *Proceedings of the National Academy of Sciences*, 103(39):14608–14613.
- [Ajoudani et al., 2018] Ajoudani, A., Zanchettin, A. M., Ivaldi, S., Albu-Schäffer, A., Kosuge, K., and Khatib, O. (2018). Progress and prospects of the human–robot collaboration. *Autonomous Robots*, 42:957–975.
- [Alahmad and Robert Jr, 2021] Alahmad, R. and Robert Jr, L. P. (2021). Capturing the complexity of cognitive computing systems: co-adaptation theory for individuals. In *Proceedings of the 2021 on Computers and People Research Conference*, pages 93–95.
- [Albrecht and Ramamoorthy, 2013] Albrecht, S. V. and Ramamoorthy, S. (2013). Ad hoc coordination in multiagent systems with applications to human-machine interaction. In *Proceedings of the 2013 international conference on Autonomous agents and multi-agent systems*, pages 1415–1416.
- [Albrecht and Ramamoorthy, 2015] Albrecht, S. V. and Ramamoorthy, S. (2015). A game-theoretic model and best-response learning method for ad hoc coordination in multiagent systems. *arXiv preprint arXiv:1506.01170*.
- [Alimardani et al., 2014] Alimardani, M., Nishio, S., and Ishiguro, H. (2014). Effect of biased feedback on motor imagery learning in bci-teleoperation system. *Frontiers in systems neuroscience*, 8:52.

- [Alluisi and Warm, 1990] Alluisi, E. A. and Warm, J. S. (1990). Things that go together: A review of stimulus-response compatibility and related effects. *Advances in psychology*, 65:3–30.
- [Anderson et al., 2004] Anderson, J. R., Bothell, D., Byrne, M. D., Douglass, S., Lebiere, C., and Qin, Y. (2004). An integrated theory of the mind. *Psychological review*, 111(4):1036.
- [Anderson et al., 2014] Anderson, S. J., Walker, J. M., and Iagnemma, K. (2014). Experimental performance analysis of a homotopy-based shared autonomy framework. *IEEE Transactions on Human-Machine Systems*, 44(2):190–199.
- [Bach-y Rita et al., 1969] Bach-y Rita, P., Collins, C. C., Saunders, F. A., White, B., and Scadden, L. (1969). Vision substitution by tactile image projection. *Nature*, 221(5184):963–964.
- [Bach-y Rita and Kercel, 2003] Bach-y Rita, P. and Kercel, S. W. (2003). Sensory substitution and the human-machine interface. *Trends in cognitive sciences*, 7(12):541–546.
- [Bach-y Rita et al., 2003] Bach-y Rita, P., Tyler, M. E., and Kaczmarek, K. A. (2003). Seeing with the brain. *International journal of human-computer interaction*, 15(2):285–295.
- [Bahill et al., 1975] Bahill, A. T., Adler, D., and Stark, L. (1975). Most naturally occurring human saccades have magnitudes of 15 degrees or less. *Investigative Ophthalmology & Visual Science*, 14(6):468–469.
- [Barbero and Grosse-Wentrup, 2010] Barbero, Á. and Grosse-Wentrup, M. (2010). Biased feedback in brain-computer interfaces. *Journal of neuroengineering and rehabilitation*, 7:1–4.
- [Barrett et al., 2010] Barrett, H. C., Cosmides, L., and Tooby, J. (2010). Coevolution of cooperation, causal cognition and mindreading. *Communicative & integrative biology*, 3(6):522–524.
- [Barrett et al., 2017] Barrett, S., Rosenfeld, A., Kraus, S., and Stone, P. (2017). Making friends on the fly: Cooperating with new teammates. *Artificial Intelligence*, 242:132–171.
- [Barron, 2024] Barron, E. N. (2024). *Game theory: an introduction*. John Wiley & Sons.
- [Bauer et al., 2008] Bauer, A., Wollherr, D., and Buss, M. (2008). Human-robot collaboration: a survey. *International Journal of Humanoid Robotics*, 5(01):47–66.
- [Bäuerle and Rieder, 2009] Bäuerle, N. and Rieder, U. (2009). Mdp algorithms for portfolio optimization problems in pure jump markets. *Finance and Stochastics*, 13:591–611.
- [Beckhaus et al., 2005] Beckhaus, S., Blom, K. J., and Haringer, M. (2005). A new gaming device and interaction method for a first-person-shooter. *Proceedings of the Computer Science and Magic*, 2005.
- [Bekrater-Bodmann, 2021] Bekrater-Bodmann, R. (2021). Factors associated with prosthesis embodiment and its importance for prosthetic satisfaction in lower limb amputees. *Frontiers in neurorobotics*, 14:604376.
- [Bell and Macuga, 2022] Bell, J. D. and Macuga, K. L. (2022). Are tools truly incorporated as an extension of the body representation?: Assessing the evidence for tool embodiment. *Psychonomic Bulletin & Review*, 29(2):343–368.

- [Bellman, 1954] Bellman, R. (1954). The theory of dynamic programming. *Bulletin of the American Mathematical Society*, 60(6):503–515.
- [Benda et al., 1994] Benda, B. J., Riley, P. O., and Krebs, D. E. (1994). Biomechanical relationship between center of gravity and center of pressure during standing. *IEEE Transactions on Rehabilitation Engineering*, 2(1):3–10.
- [Bereby-Meyer and Erev, 1998] Bereby-Meyer, Y. and Erev, I. (1998). On learning to become a successful loser: A comparison of alternative abstractions of learning processes in the loss domain. *Journal of mathematical psychology*, 42(2-3):266–286.
- [Bermejo et al., 2015] Bermejo, F., Di Paolo, E. A., Hüg, M. X., and Arias, C. (2015). Sensorimotor strategies for recognizing geometrical shapes: A comparative study with different sensory substitution devices. *Frontiers in psychology*, 6:679.
- [Berniker and Kording, 2011] Berniker, M. and Kording, K. (2011). Bayesian approaches to sensory integration for motor control. *Wiley Interdisciplinary Reviews: Cognitive Science*, 2(4):419–428.
- [Berning et al., 2015] Berning, M., Braun, F., Riedel, T., and Beigl, M. (2015). Proximityhat: a head-worn system for subtle sensory augmentation with tactile stimulation. In *Proceedings of the 2015 ACM International Symposium on Wearable Computers*, pages 31–38.
- [Bitzer et al., 2014] Bitzer, S., Park, H., Blankenburg, F., and Kiebel, S. J. (2014). Perceptual decision making: drift-diffusion model is equivalent to a bayesian model. *Frontiers in human neuroscience*, 8:102.
- [Blackler et al., 2019] Blackler, A., Desai, S., McEwan, M., Popovic, V., and Diefenbach, S. (2019). Perspectives on the nature of intuitive interaction. *Intuitive Interaction Research and Application*, pages 19–39.
- [Blaisdell et al., 2006] Blaisdell, A. P., Sawa, K., Leising, K. J., and Waldmann, M. R. (2006). Causal reasoning in rats. *Science*, 311(5763):1020–1022.
- [Blakemore and Donaghy, 1980] Blakemore, C. and Donaghy, M. (1980). Co-ordination of head and eyes in the gaze changing behaviour of cats. *The Journal of physiology*, 300:317.
- [Blankertz and Vidaurre, 2009] Blankertz, B. and Vidaurre, C. (2009). Towards a cure for bci illiteracy: machine learning based co-adaptive learning. *BMC Neuroscience*, 10(Suppl 1):P85.
- [Böge and Eisele, 1979] Böge, W. and Eisele, T. (1979). On solutions of bayesian games. *International journal of game theory*, 8:193–215.
- [Bolanowski et al., 1994] Bolanowski, S. J., Gescheider, G. A., and Verrillo, R. T. (1994). Hairy skin: psychophysical channels and their physiological substrates. *Somatosensory & motor research*, 11(3):279–290.
- [Bonilla and Asada, 2014] Bonilla, B. L. and Asada, H. H. (2014). A robot on the shoulder: Coordinated human-wearable robot control using coloured petri nets and partial least squares predictions. In *2014 IEEE international conference on robotics and automation (ICRA)*, pages 119–125. IEEE.

- [Börger and Sarin, 1997] Börger, T. and Sarin, R. (1997). Learning through reinforcement and replicator dynamics. *Journal of economic theory*, 77(1):1–14.
- [Börger and Sarin, 2000] Börger, T. and Sarin, R. (2000). Naive reinforcement learning with endogenous aspirations. *International Economic Review*, 41(4):921–950.
- [Boring, 1942] Boring, E. G. (1942). Sensation and perception in the history of experimental psychology.
- [Boucherie and Van Dijk, 2017] Boucherie, R. J. and Van Dijk, N. M. (2017). *Markov decision processes in practice*. Springer.
- [Brandenburger and Dekel, 1993] Brandenburger, A. and Dekel, E. (1993). Hierarchies of beliefs and common knowledge. *Journal of economic theory*, 59(1):189–198.
- [Bretan et al., 2016] Bretan, M., Gopinath, D., Mullins, P., and Weinberg, G. (2016). A robotic prosthesis for an amputee drummer. *arXiv preprint arXiv:1612.04391*.
- [Bristow et al., 2006] Bristow, D. A., Tharayil, M., and Alleyne, A. G. (2006). A survey of iterative learning control. *IEEE control systems magazine*, 26(3):96–114.
- [Broad et al., 2018] Broad, A., Murphey, T., and Argall, B. (2018). Learning models for shared control of human-machine systems with unknown dynamics. *arXiv preprint arXiv:1808.08268*.
- [Brock et al., 2014] Brock, A., Kammoun, S., Macé, M., and Jouffrais, C. (2014). Using wrist vibrations to guide hand movement and whole body navigation. *i-com*, 13(3):19–28.
- [Buongiorno and Zhou, 2017] Buongiorno, J. and Zhou, M. (2017). Multicriteria forest decisionmaking under risk with goal-programming markov decision process models. *Forest Science*, 63(5):474–484.
- [Burle et al., 2002] Burle, B., Possamaï, C.-A., Vidal, F., Bonnet, M., and Hasbroucq, T. (2002). Executive control in the simon effect: an electromyographic and distributional analysis. *Psychological research*, 66:324–336.
- [Burle et al., 2005] Burle, B., van den Wildenberg, W., and Ridderinkhof, K. R. (2005). Dynamics of facilitation and interference in cue-priming and simon tasks. *European Journal of Cognitive Psychology*, 17(5):619–641.
- [Busic and Meyn, 2018] Busic, A. and Meyn, S. (2018). Ordinary differential equation methods for markov decision processes and application to kullback–leibler control cost. *SIAM Journal on Control and Optimization*, 56(1):343–366.
- [Canzoneri et al., 2013] Canzoneri, E., Ubaldi, S., Rastelli, V., Finisguerra, A., Bassolino, M., and Serino, A. (2013). Tool-use reshapes the boundaries of body and peripersonal space representations. *Experimental brain research*, 228:25–42.
- [Card, 1981] Card, S. K. (1981). The model human processor: A model for making engineering calculations of human performance. In *Proceedings of the Human Factors Society Annual Meeting*, volume 25, pages 301–305. SAGE Publications Sage CA: Los Angeles, CA.
- [Cardinali et al., 2012] Cardinali, L., Jacobs, S., Brozzoli, C., Frassinetti, F., Roy, A. C., and Farnè, A. (2012). Grab an object with a tool and change your body: tool-use-dependent changes of body representation for action. *Experimental Brain Research*, 218:259–271.

- [Cassenti and Schaefer, ] Cassenti, D. N. and Schaefer, K. E. Robotics operator manager act-r model and validation.
- [Cassinelli et al., 2006] Cassinelli, A., Reynolds, C., and Ishikawa, M. (2006). Augmenting spatial awareness with haptic radar. In *2006 10th IEEE International Symposium on Wearable Computers*, pages 61–64. IEEE.
- [Çatal et al., 2019] Çatal, O., Nauta, J., Verbelen, T., Simoens, P., and Dhoedt, B. (2019). Bayesian policy selection using active inference. *arXiv preprint arXiv:1904.08149*.
- [Çatal et al., 2021] Çatal, O., Verbelen, T., Van de Maele, T., Dhoedt, B., and Safron, A. (2021). Robot navigation as hierarchical active inference. *Neural Networks*, 142:192–204.
- [Cespón et al., 2020] Cespón, J., Hommel, B., Korsch, M., and Galashan, D. (2020). The neurocognitive underpinnings of the simon effect: An integrative review of current research. *Cognitive, Affective, & Behavioral Neuroscience*, 20:1133–1172.
- [Chan and Chan, 2005] Chan, K. W. and Chan, A. H. (2005). Spatial s–r compatibility of visual and auditory signals: implications for human–machine interface design. *Displays*, 26(3):109–119.
- [Chater, 1999] Chater, N. (1999). The search for simplicity: A fundamental cognitive principle? *The Quarterly Journal of Experimental Psychology: Section A*, 52(2):273–302.
- [Chater and Vitányi, 2003] Chater, N. and Vitányi, P. (2003). Simplicity: a unifying principle in cognitive science? *Trends in cognitive sciences*, 7(1):19–22.
- [Chen and Melara, 2009] Chen, S. and Melara, R. D. (2009). Sequential effects in the simon task: Conflict adaptation or feature integration? *Brain research*, 1297:89–100.
- [Cheng et al., 2012] Cheng, A., Nichols, K. A., Weeks, H. M., Gurari, N., and Okamura, A. M. (2012). Conveying the configuration of a virtual human hand using vibrotactile feedback. In *2012 IEEE Haptics Symposium (HAPTICS)*, pages 155–162. IEEE.
- [Chouvardas et al., 2008] Chouvardas, V. G., Miliou, A. N., and Hatalis, M. K. (2008). Tactile displays: Overview and recent advances. *Displays*, 29(3):185–194.
- [Christ et al., 2000] Christ, S., Falkenstein, M., Heuer, H., and Hohnsbein, J. (2000). Different error types and error processing in spatial stimulus-response-compatibility tasks: behavioural and electrophysiological data. *Biological psychology*, 51(2-3):129–150.
- [Christian, 2021] Christian, B. (2021). *The alignment problem: How can machines learn human values?* Atlantic Books.
- [Cieśła et al., 2022] Cieśła, K., Wolak, T., Lorens, A., Mentzel, M., Skarżyński, H., and Amedi, A. (2022). Effects of training and using an audio-tactile sensory substitution device on speech-in-noise understanding. *Scientific Reports*, 12(1):3206.
- [Clemente et al., 2015] Clemente, F., D’Alonzo, M., Controzzi, M., Edin, B. B., and Cipriani, C. (2015). Non-invasive, temporally discrete feedback of object contact and release improves grasp control of closed-loop myoelectric transradial prostheses. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, 24(12):1314–1322.

- [Cohen et al., 2007] Cohen, J. D., McClure, S. M., and Yu, A. J. (2007). Should i stay or should i go? how the human brain manages the trade-off between exploitation and exploration. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 362(1481):933–942.
- [Cohen and van Gaal, 2014] Cohen, M. X. and van Gaal, S. (2014). Subthreshold muscle twitches dissociate oscillatory neural signatures of conflicts from errors. *Neuroimage*, 86:503–513.
- [Corniani and Saal, 2020] Corniani, G. and Saal, H. P. (2020). Tactile innervation densities across the whole body. *Journal of Neurophysiology*, 124(4):1229–1240.
- [Couraud et al., 2018] Couraud, M., Cattaert, D., Paclet, F., Oudeyer, P.-Y., and De Rugy, A. (2018). Model and experiments to optimize co-adaptation in a simplified myoelectric control system. *Journal of neural engineering*, 15(2):026006.
- [Cullen et al., 2018] Cullen, M., Davey, B., Friston, K. J., and Moran, R. J. (2018). Active inference in openai gym: A paradigm for computational investigations into psychiatric illness. *Biological psychiatry: cognitive neuroscience and neuroimaging*, 3(9):809–818.
- [Cunningham et al., 2018] Cunningham, J., Hapsari, A., Guilleminot, P., Shafti, A., and Faisal, A. A. (2018). The supernumerary robotic 3 rd thumb for skilled music tasks. In *2018 7th IEEE International Conference on Biomedical Robotics and Biomechatronics (Biorob)*, pages 665–670. IEEE.
- [Da Costa et al., 2020] Da Costa, L., Parr, T., Sajid, N., Veselic, S., Neacsu, V., and Friston, K. (2020). Active inference on discrete state-spaces: A synthesis. *Journal of Mathematical Psychology*, 99:102447.
- [Danielle, 2018] Danielle, C. (2018). The third thumb. <https://www.daniclodedesign.com/thethirdthumb>. Accessed: 2024-03-22.
- [Danilov and Tyler, 2005] Danilov, Y. and Tyler, M. (2005). Brainport: an alternative input to the brain. *Journal of integrative neuroscience*, 4(04):537–550.
- [Dassonville et al., 2001] Dassonville, P., Lewis, S. M., Zhu, X.-H., Ugurbil, K., Kim, S.-G., and Ashe, J. (2001). The effect of stimulus–response compatibility on cortical motor activation. *Neuroimage*, 13(1):1–14.
- [Dauwels, 2007] Dauwels, J. (2007). On variational message passing on factor graphs. In *2007 IEEE international symposium on information theory*, pages 2546–2550. IEEE.
- [Davenport et al., 2012] Davenport, C., Parietti, F., and Asada, H. H. (2012). Design and biomechanical analysis of supernumerary robotic limbs. In *Dynamic Systems and Control Conference*, volume 45295, pages 787–793. American Society of Mechanical Engineers.
- [de Haan and Cowey, 2011] de Haan, E. H. and Cowey, A. (2011). On the usefulness of ‘what’ and ‘where’ pathways in vision. *Trends in cognitive sciences*, 15(10):460–466.
- [De Jong et al., 1994] De Jong, R., Liang, C.-C., and Lauber, E. (1994). Conditional and unconditional automaticity: a dual-process model of effects of spatial stimulus-response correspondence. *Journal of Experimental Psychology: Human Perception and Performance*, 20(4):731.

- [De Santis, 2021] De Santis, D. (2021). A framework for optimizing co-adaptation in body-machine interfaces. *Frontiers in Neurorobotics*, 15:662181.
- [De Santis et al., 2018] De Santis, D., Dzialecka, P., and Mussa-Ivaldi, F. A. (2018). Unsupervised coadaptation of an assistive interface to facilitate sensorimotor learning of redundant control. In *2018 7th IEEE International Conference on Biomedical Robotics and Biomechatronics (Biorob)*, pages 801–806. IEEE.
- [De Vignemont, 2011] De Vignemont, F. (2011). Embodiment, ownership and disownership. *Consciousness and cognition*, 20(1):82–93.
- [Dechaux et al., 2022] Dechaux, A., Haytam-Mahsoub, M., Kitazaki, M., Lagarde, J., and Ganesh, G. (2022). Multi-sensory feedback improves spatially compatible sensori-motor responses. *Scientific Reports*, 12(1):20253.
- [Deecke, 2007] Deecke, L. (2007). Bereitschaftspotential as an indicator of movement preparation in supplementary motor area and motor cortex. In *Ciba Foundation Symposium 132-Motor Areas of the Cerebral Cortex: Motor Areas of The Cerebral Cortex: Ciba Foundation Symposium 132*, pages 231–250. Wiley Online Library.
- [Deecke et al., 1976] Deecke, L., Grözing, B., and Kornhuber, H. (1976). Voluntary finger movement in man: Cerebral potentials and theory. *Biological cybernetics*, 23(2):99–119.
- [Degeling and Berendt, 2018] Degeling, M. and Berendt, B. (2018). What is wrong about robocops as consultants? a technology-centric critique of predictive policing. *Ai & Society*, 33:347–356.
- [Di Stefano et al., 1992] Di Stefano, M., Sauerwein, H. C., and Lassonde, M. (1992). Influence of anatomical factors and spatial compatibility on the stimulus-response relationship in the absence of the corpus callosum. *Neuropsychologia*, 30(2):177–185.
- [Doya, 2008] Doya, K. (2008). Modulators of decision making. *Nature neuroscience*, 11(4):410–416.
- [D’Ascenzo et al., 2018] D’Ascenzo, S., Lugli, L., Baroni, G., Guidotti, R., Rubichi, S., Iani, C., and Nicoletti, R. (2018). Visual versus auditory simon effect: A behavioural and physiological investigation. *Quarterly Journal of Experimental Psychology*, 71(4):917–930.
- [Eagleman and Perrotta, 2023] Eagleman, D. M. and Perrotta, M. V. (2023). The future of sensory substitution, addition, and expansion via haptic devices. *Frontiers in Human Neuroscience*, 16:1055546.
- [Eden et al., 2022] Eden, J., Bräcklein, M., Ibáñez, J., Barsakcioglu, D. Y., Di Pino, G., Farina, D., Burdet, E., and Mehring, C. (2022). Principles of human movement augmentation and the challenges in making it a reality. *Nature Communications*, 13(1):1345.
- [Ehrlich and Cheng, 2018] Ehrlich, S. K. and Cheng, G. (2018). Human-agent co-adaptation using error-related potentials. *Journal of neural engineering*, 15(6):066014.
- [Eimer, 1997] Eimer, M. (1997). The lateralized readiness potential as an on-line measure of automatic response activation in sr compatibility situations. In *Advances in Psychology*, volume 118, pages 51–73. Elsevier.

- [Elliott et al., 2015] Elliott, L., Mortimer, B., Cholewiak, R., Mort, G., Zets, G., Pomranky-Hartnett, G., Pettitt, R. W., and Wooldridge, R. (2015). Salience of tactile cues: An examination of tactor actuator and tactile cue characteristics. *Aberdeen Proving Ground (MD): Army Research Laboratory (US)*.
- [Elmadjian and Morimoto, 2021] Elmadjian, C. and Morimoto, C. H. (2021). Gazebar: Exploiting the midas touch in gaze interaction. In *Extended abstracts of the 2021 CHI conference on human factors in computing systems*, pages 1–7.
- [Eriksen and Eriksen, 1974] Eriksen, B. A. and Eriksen, C. W. (1974). Effects of noise letters upon the identification of a target letter in a nonsearch task. *Perception & psychophysics*, 16(1):143–149.
- [Ernst and Banks, 2002] Ernst, M. O. and Banks, M. S. (2002). Humans integrate visual and haptic information in a statistically optimal fashion. *Nature*, 415(6870):429–433.
- [Esposito et al., 2023] Esposito, D., Centracchio, J., Bifulco, P., and Andreozzi, E. (2023). A smart approach to emg envelope extraction and powerful denoising for human–machine interfaces. *Scientific Reports*, 13(1):7768.
- [Evrard and Kheddar, 2009] Evrard, P. and Kheddar, A. (2009). Homotopy switching model for dyad haptic interaction in physical collaborative tasks. In *World haptics 2009-third joint EuroHaptics conference and symposium on haptic interfaces for virtual environment and teleoperator systems*, pages 45–50. IEEE.
- [Fabiani et al., 1987] Fabiani, M., Gratton, G., Karis, D., Donchin, E., et al. (1987). Definition, identification, and reliability of measurement of the p300 component of the event-related brain potential. *Advances in psychophysiology*, 2(S1):78.
- [Fazel-Rezai et al., 2012] Fazel-Rezai, R., Allison, B. Z., Guger, C., Sellers, E. W., Kleih, S. C., and Kübler, A. (2012). P300 brain computer interface: current challenges and emerging trends. *Frontiers in neuroengineering*, page 14.
- [Feldman, 2009a] Feldman, A. G. (2009a). New insights into action–perception coupling. *Experimental brain research*, 194:39–58.
- [Feldman, 2009b] Feldman, J. (2009b). Bayes and the simplicity principle in perception. *Psychological review*, 116(4):875.
- [Feldman, 2016] Feldman, J. (2016). The simplicity principle in perception and cognition. *Wiley Interdisciplinary Reviews: Cognitive Science*, 7(5):330–340.
- [Fellah and Guiatni, 2018] Fellah, K. and Guiatni, M. (2018). Tactile display design for flight envelope protection and situational awareness. *IEEE transactions on haptics*, 12(1):87–98.
- [Ferraris et al., 2015] Ferraris, V. et al. (2015). Border control: a new frontier for automated decision making and profiling? In *Profiling Technologies in Practice. Applications and Impact on Fundamental Rights and Values*, pages 89–126. Wolf Legal Publishers.
- [Findling et al., 2019] Findling, C., Skvortsova, V., Dromnelle, R., Palminteri, S., and Wyart, V. (2019). Computational noise in reward-guided learning drives behavioral variability in volatile environments. *Nature neuroscience*, 22(12):2066–2077.

- [Finnegan, 2023] Finnegan, C. (2023). *Coordinating minds: mindshaping, communication and technology*. PhD thesis, University College Cork.
- [Fitts and Seeger, 1953] Fitts, P. M. and Seeger, C. M. (1953). Sr compatibility: spatial characteristics of stimulus and response codes. *Journal of experimental psychology*, 46(3):199.
- [Freedman, 2008] Freedman, E. G. (2008). Coordination of the eyes and head during visual orienting. *Experimental brain research*, 190:369–387.
- [Friston, 2011] Friston, K. (2011). What is optimal about motor control? *Neuron*, 72(3):488–498.
- [Friston and Ao, 2012] Friston, K. and Ao, P. (2012). Free energy, value, and attractors. *Computational and mathematical methods in medicine*, 2012(1):937860.
- [Friston et al., 2017a] Friston, K., FitzGerald, T., Rigoli, F., Schwartenbeck, P., and Pezzulo, G. (2017a). Active inference: a process theory. *Neural computation*, 29(1):1–49.
- [Friston et al., 2016] Friston, K., FitzGerald, T., Rigoli, F., Schwartenbeck, P., Pezzulo, G., et al. (2016). Active inference and learning. *Neuroscience & Biobehavioral Reviews*, 68:862–879.
- [Friston et al., 2007] Friston, K., Mattout, J., Trujillo-Barreto, N., Ashburner, J., and Penny, W. (2007). Variational free energy and the laplace approximation. *Neuroimage*, 34(1):220–234.
- [Friston et al., 2015] Friston, K., Rigoli, F., Ognibene, D., Mathys, C., Fitzgerald, T., and Pezzulo, G. (2015). Active inference and epistemic value. *Cognitive neuroscience*, 6(4):187–214.
- [Friston, 2018] Friston, K. J. (2018). Active inference and cognitive consistency. *Psychological inquiry*, 29(2):67–73.
- [Friston et al., 2017b] Friston, K. J., Rosch, R., Parr, T., Price, C., and Bowman, H. (2017b). Deep temporal models and active inference. *Neuroscience & Biobehavioral Reviews*, 77:388–402.
- [Frith and Wolpert, 2004] Frith, C. D. and Wolpert, D. M. (2004). *The neuroscience of social interaction: Decoding, imitating, and influencing the actions of others*. Oxford University Press New York.
- [Fulton, 2005] Fulton, J. T. (2005). Processes in biological vision. *Available on the Internet: URL: <http://www.4colorvision.com>*.
- [Gallina et al., 2015] Gallina, P., Bellotto, N., and Di Luca, M. (2015). Progressive co-adaptation in human-machine interaction. In *2015 12th International Conference on Informatics in Control, Automation and Robotics (ICINCO)*, volume 2, pages 362–368. IEEE.
- [Gallo et al., 2010] Gallo, S., Chapuis, D., Santos-Carreras, L., Kim, Y., Retornaz, P., Bleuler, H., and Gassert, R. (2010). Augmented white cane with multimodal haptic feedback. In *2010 3rd IEEE RAS & EMBS International Conference on Biomedical Robotics and Biomechatronics*, pages 149–155. IEEE.
- [Gärdenfors and Lombard, 2018] Gärdenfors, P. and Lombard, M. (2018). Causal cognition, force dynamics and early hunting technologies. *Frontiers in Psychology*, 9:87.

- [Gärdenfors and Lombard, 2020] Gärdenfors, P. and Lombard, M. (2020). Technology led to more abstract causal reasoning. *Biology & Philosophy*, 35(4):40.
- [Gerrans, 2009] Gerrans, P. (2009). Imitation and theory of mind. *Handbook of neuroscience for the behavioral sciences*, 2:905–922.
- [Gershman and Niv, 2013] Gershman, S. J. and Niv, Y. (2013). Perceptual estimation obeys occam’s razor. *Frontiers in psychology*, 4:53824.
- [Gescheider et al., 2002] Gescheider, G. A., Bolanowski, S. J., Pope, J. V., and Verrillo, R. T. (2002). A four-channel analysis of the tactile sensitivity of the fingertip: frequency selectivity, spatial summation, and temporal summation. *Somatosensory & motor research*, 19(2):114–124.
- [Gilliland and Schlegel, 1994] Gilliland, K. and Schlegel, R. E. (1994). Tactile stimulation of the human head for information display. *Human Factors*, 36(4):700–717.
- [Glenberg, 2010] Glenberg, A. M. (2010). Embodiment as a unifying perspective for psychology. *Wiley interdisciplinary reviews: Cognitive science*, 1(4):586–596.
- [Gopalan et al., 2017] Gopalan, N., Littman, M., MacGlashan, J., Squire, S., Tellex, S., Winder, J., Wong, L., et al. (2017). Planning with abstract markov decision processes. In *Proceedings of the International Conference on Automated Planning and Scheduling*, volume 27, pages 480–488.
- [Gottlieb and Oudeyer, 2018] Gottlieb, J. and Oudeyer, P.-Y. (2018). Towards a neuroscience of active sampling and curiosity. *Nature Reviews Neuroscience*, 19(12):758–770.
- [Gouzien et al., 2017] Gouzien, A., de Vignemont, F., Touillet, A., Martinet, N., De Graaf, J., Jarrasse, N., and Roby-Brami, A. (2017). Reachability and the sense of embodiment in amputees using prostheses. *Scientific Reports*, 7(1):4999.
- [Greco et al., 2016] Greco, A., Valenza, G., Citi, L., and Scilingo, E. P. (2016). Arousal and valence recognition of affective sounds based on electrodermal activity. *IEEE Sensors Journal*, 17(3):716–725.
- [Green, 1982] Green, B. G. (1982). The perception of distance and location for dual tactile pressures. *Perception & Psychophysics*, 31(4):315–323.
- [Gregg et al., 2014] Gregg, R. D., Rouse, E. J., Hargrove, L. J., and Sensinger, J. W. (2014). Evidence for a time-invariant phase variable in human ankle control. *PloS one*, 9(2):e89163.
- [Grushko et al., 2021] Grushko, S., Vysocký, A., Heczko, D., and Bobovský, Z. (2021). Intuitive spatial tactile feedback for better awareness about robot trajectory during human–robot collaboration. *Sensors*, 21(17):5748.
- [Guan et al., 2014] Guan, P., Raginsky, M., and Willett, R. M. (2014). Online markov decision processes with kullback–leibler control cost. *IEEE Transactions on Automatic Control*, 59(6):1423–1438.
- [Gugliotta et al., 2023] Gugliotta, L. et al. (2023). Will ai ‘subtly’ take over decision-making in the eu migration context? warnings and lessons from etias and vis. *Jean Monnet Network on EU Law Enforcement Working Paper Series*.

- [Guiasu and Shenitzer, 1985] Guiasu, S. and Shenitzer, A. (1985). The principle of maximum entropy. *The mathematical intelligencer*, 7:42–48.
- [Ha et al., 2010] Ha, K. H., Varol, H. A., and Goldfarb, M. (2010). Volitional control of a prosthetic knee using surface electromyography. *IEEE Transactions on Biomedical Engineering*, 58(1):144–151.
- [Hadfield-Menell et al., 2016] Hadfield-Menell, D., Russell, S. J., Abbeel, P., and Dragan, A. (2016). Cooperative inverse reinforcement learning. *Advances in neural information processing systems*, 29.
- [Hahne et al., 2015] Hahne, J. M., Dähne, S., Hwang, H.-J., Müller, K.-R., and Parra, L. C. (2015). Concurrent adaptation of human and machine improves simultaneous and proportional myoelectric control. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, 23(4):618–627.
- [Han and Yang, 2019] Han, W. and Yang, Y. (2019). Statistical inference in mean-field variational bayes. *arXiv preprint arXiv:1911.01525*.
- [Harrar and Harris, 2005] Harrar, V. and Harris, L. R. (2005). Simultaneity constancy: detecting events with touch and vision. *Experimental Brain Research*, 166:465–473.
- [Harris et al., 2014] Harris, A., Nguyen, K., Wilson, P. T., Jackoski, M., and Williams, B. (2014). Human joystick: Wii-leaning to translate in large virtual environments. In *Proceedings of the 13th ACM SIGGRAPH international conference on virtual-reality continuum and its applications in industry*, pages 231–234.
- [Harris et al., 2004] Harris, J. A., Thein, T., and Clifford, C. W. (2004). Dissociating detection from localization of tactile stimuli. *Journal of Neuroscience*, 24(14):3683–3693.
- [Harsanyi, 1967] Harsanyi, J. C. (1967). Games with incomplete information played by “bayesian” players, i-iii. *Management science*, 14(3):159–182.
- [Hasbroucq, 1987] Hasbroucq, T. (1987). Preliminary evidence for body-centered coding of tactile motor events. *Perceptual and Motor Skills*, 64(2):631–634.
- [Hasbroucq et al., 2009] Hasbroucq, T., Burle, B., Vidal, F., and Possamaï, C.-A. (2009). Stimulus-hand correspondence and direct response activation: An electromyographic analysis. *Psychophysiology*, 46(6):1160–1169.
- [Hasbroucq and Guiard, 1991] Hasbroucq, T. and Guiard, Y. (1991). Stimulus-response compatibility and the simon effect: toward a conceptual clarification. *Journal of Experimental Psychology: Human Perception and Performance*, 17(1):246.
- [Hasbroucq and Guiard, 1992] Hasbroucq, T. and Guiard, Y. (1992). The effects of intensity and irrelevant location of a tactile stimulation in a choice reaction time task. *Neuropsychologia*, 30(1):91–94.
- [Hashemian et al., 2020] Hashemian, A. M., Lotfaliei, M., Adhikari, A., Kruijff, E., and Riecke, B. E. (2020). Headjoystick: Improving flying in vr using a novel leaning-based interface. *IEEE Transactions on Visualization and Computer Graphics*, 28(4):1792–1809.
- [Hedge and Marsh, 1975] Hedge, A. and Marsh, N. (1975). The effect of irrelevant spatial correspondences on two-choice response-time. *Acta psychologica*, 39(6):427–439.

- [Held et al., 2021] Held, M., Borst, J., Unni, A., and Rieger, J. (2021). Utilizing act-r to investigate interactions between working memory and visuospatial attention while driving. In *Proceedings of the Annual Meeting of the Cognitive Science Society*, volume 43.
- [Hernandez Arieta et al., 2008] Hernandez Arieta, A., Dermitzakis, K., Damian, D., Lungarella, M., and Pfeifer, R. (2008). Sensorymotor coupling in rehabilitation robotics. *Serv Robot Appl. Edited by Yoshihiko Takahashi (Ed.). INTECH Open Access Publisher*, pages 21–36.
- [Hesp et al., 2021] Hesp, C., Smith, R., Parr, T., Allen, M., Friston, K. J., and Ramstead, M. J. (2021). Deeply felt affect: The emergence of valence in deep active inference. *Neural computation*, 33(2):398–446.
- [Hess et al., 2014] Hess, T. J., McNab, A. L., and Basoglu, K. A. (2014). Reliability generalization of perceived ease of use, perceived usefulness, and behavioral intentions. *MIS quarterly*, 38(1):1–28.
- [Hicheur et al., 2005] Hicheur, H., Vieilledent, S., and Berthoz, A. (2005). Head motion in humans alternating between straight and curved walking path: combination of stabilizing and anticipatory orienting mechanisms. *Neuroscience letters*, 383(1-2):87–92.
- [Higuchi and Rekimoto, 2013] Higuchi, K. and Rekimoto, J. (2013). Flying head: a head motion synchronization mechanism for unmanned aerial vehicle control. In *CHI'13 Extended Abstracts on Human Factors in Computing Systems*, pages 2029–2038.
- [Hommel, 1993a] Hommel, B. (1993a). Inverting the simon effect by intention: Determinants of direction and extent of effects of irrelevant spatial information. *Psychological Research*, 55(4):270–279.
- [Hommel, 1993b] Hommel, B. (1993b). The relationship between stimulus processing and response selection in the simon task: Evidence for a temporal overlap. *Psychological research*, 55(4):280–290.
- [Hommel, 1994] Hommel, B. (1994). Effects of irrelevant spatial sr compatibility depend on stimulus complexity. *Psychological research*, 56(3):179–184.
- [Hommel, 2011a] Hommel, B. (2011a). Attention and spatial stimulus coding in the simon task: A rejoinder to van der lubbe and abrahamse (2010). *Acta psychologica*, 136(2):265–268.
- [Hommel, 2011b] Hommel, B. (2011b). The simon effect as tool and heuristic. *Acta psychologica*, 136(2):189–202.
- [House et al., 2009] House, B., Malkin, J., and Bilmes, J. (2009). The voicebot: a voice controlled robot arm. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, pages 183–192.
- [Hu et al., 2023] Hu, X., Song, A., Zeng, H., Wei, Z., Deng, H., and Chen, D. (2023). Bridging human-robot co-adaptation via biofeedback for continuous myoelectric control. *IEEE Robotics and Automation Letters*.
- [Huang et al., 2021a] Huang, H. H., Si, J., Brandt, A., and Li, M. (2021a). Taking both sides: seeking symbiosis between intelligent prostheses and human motor control during locomotion. *Current opinion in biomedical engineering*, 20:100314.

- [Huang et al., 2021b] Huang, Y., Ivanova, E., Eden, J., and Burdet, E. (2021b). Identification of multiple limbs coordination strategies in a three-goal independent task. *IEEE Transactions on Medical Robotics and Bionics*, 4(2):348–351.
- [Hussain et al., 2017a] Hussain, I., Meli, L., Pacchierotti, C., and Prattichizzo, D. (2017a). A soft robotic supernumerary finger and a wearable cutaneous finger interface to compensate the missing grasping capabilities in chronic stroke patients. In *2017 IEEE World Haptics Conference (WHC)*, pages 183–188. IEEE.
- [Hussain et al., 2015] Hussain, I., Salviati, G., Meli, L., Pacchierotti, C., Cioncoloni, D., Rossi, S., and Prattichizzo, D. (2015). Using the robotic sixth finger and vibrotactile feedback for grasp compensation in chronic stroke patients. In *2015 IEEE International Conference on Rehabilitation Robotics (ICORR)*, pages 67–72. IEEE.
- [Hussain et al., 2017b] Hussain, I., Santarnecchi, E., Leo, A., Ricciardi, E., Rossi, S., and Prattichizzo, D. (2017b). A magnetic compatible supernumerary robotic finger for functional magnetic resonance imaging (fmri) acquisitions: Device description and preliminary results. In *2017 International Conference on Rehabilitation Robotics (ICORR)*, pages 1177–1182. IEEE.
- [Hussain et al., 2016] Hussain, I., Spagnoletti, G., Salviati, G., and Prattichizzo, D. (2016). An emg interface for the control of motion and compliance of a supernumerary robotic finger. *Frontiers in neurorobotics*, 10:18.
- [Hussain et al., 2017c] Hussain, I., Spagnoletti, G., Salviati, G., and Prattichizzo, D. (2017c). Toward wearable supernumerary robotic fingers to compensate missing grasping abilities in hemiparetic upper limb. *The International Journal of Robotics Research*, 36(13-14):1414–1436.
- [Imai et al., 2001] Imai, T., Moore, S. T., Raphan, T., and Cohen, B. (2001). Interaction of the body, head, and eyes during walking and turning. *Experimental brain research*, 136:1–18.
- [Imaizumi et al., 2016] Imaizumi, S., Asai, T., and Koyama, S. (2016). Embodied prosthetic arm stabilizes body posture, while unembodied one perturbs it. *Consciousness and cognition*, 45:75–88.
- [Inga et al., 2023] Inga, J., Ruess, M., Robens, J. H., Nelius, T., Rothfuß, S., Kille, S., Dahlinger, P., Lindenmann, A., Thomaschke, R., Neumann, G., et al. (2023). Human-machine symbiosis: A multivariate perspective for physically coupled human-machine systems. *International Journal of Human-Computer Studies*, 170:102926.
- [John and Kieras, 1996] John, B. E. and Kieras, D. E. (1996). Using goms for user interface design and evaluation: Which technique? *ACM Transactions on Computer-Human Interaction (TOCHI)*, 3(4):287–319.
- [Johnson-Frey, 2003] Johnson-Frey, S. H. (2003). What’s so special about human tool use? *Neuron*, 39(2):201–204.
- [Jonnavittula and Losey, 2021] Jonnavittula, A. and Losey, D. P. (2021). Learning to share autonomy across repeated interaction. In *2021 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 1851–1858. IEEE.
- [Jouvenal, 2016] Jouvenal, J. (2016). The new way police are surveilling you: Calculating your threat ‘score’. *The Washington Post*, 10.

- [Kaczmarek, 2000] Kaczmarek, K. A. (2000). Electrotactile adaptation on the abdomen: Preliminary results. *IEEE Transactions on Rehabilitation Engineering*, 8(4):499–505.
- [Kaczmarek et al., 1991] Kaczmarek, K. A., Webster, J. G., Bach-y Rita, P., and Tompkins, W. J. (1991). Electrotactile and vibrotactile displays for sensory substitution systems. *IEEE transactions on biomedical engineering*, 38(1):1–16.
- [Kahn et al., 2008] Kahn, P. H., Freier, N. G., Kanda, T., Ishiguro, H., Ruckert, J. H., Severson, R. L., and Kane, S. K. (2008). Design patterns for sociality in human-robot interaction. In *Proceedings of the 3rd ACM/IEEE international conference on Human robot interaction*, pages 97–104.
- [Kalinowska et al., 2023] Kalinowska, A., Pilarski, P. M., and Murphey, T. D. (2023). Embodied communication: How robots and people communicate through physical interaction. *Annual Review of Control, Robotics, and Autonomous Systems*, 6:205–232.
- [Kammers et al., 2009] Kammers, M. P., de Vignemont, F., Verhagen, L., and Dijkerman, H. C. (2009). The rubber hand illusion in action. *Neuropsychologia*, 47(1):204–211.
- [Kang, 2021] Kang, H. (2021). Sample size determination and power analysis using the g\* power software. *Journal of educational evaluation for health professions*, 18.
- [Kaplan and Friston, 2018] Kaplan, R. and Friston, K. J. (2018). Planning and navigation as active inference. *Biological cybernetics*, 112(4):323–343.
- [Kerdegari et al., 2014] Kerdegari, H., Kim, Y., Stafford, T., and Prescott, T. J. (2014). Centralizing bias and the vibrotactile funneling illusion on the forehead. In *Haptics: Neuroscience, Devices, Modeling, and Applications: 9th International Conference, EuroHaptics 2014, Versailles, France, June 24-26, 2014, Proceedings, Part II 9*, pages 55–62. Springer.
- [Kharb et al., 2011] Kharb, A., Saini, V., Jain, Y., and Dhiman, S. (2011). A review of gait cycle and its parameters. *IJCEM International Journal of Computational Engineering & Management*, 13(01).
- [Khazoom et al., 2020] Khazoom, C., Caillouette, P., Girard, A., and Plante, J.-S. (2020). A supernumerary robotic leg powered by magnetorheological actuators to assist human locomotion. *IEEE Robotics and Automation Letters*, 5(4):5143–5150.
- [Kieliba et al., 2021] Kieliba, P., Clode, D., Maimon-Mor, R. O., and Makin, T. R. (2021). Robotic hand augmentation drives changes in neural body representation. *Science robotics*, 6(54):eabd7935.
- [Kieras and Meyer, 1997] Kieras, D. E. and Meyer, D. E. (1997). An overview of the epic architecture for cognition and performance with application to human-computer interaction. *Human-Computer Interaction*, 12(4):391–438.
- [Kiltani et al., 2012] Kiltani, K., Groten, R., and Slater, M. (2012). The sense of embodiment in virtual reality. *Presence: Teleoperators and Virtual Environments*, 21(4):373–387.
- [Kim et al., 2020] Kim, C.-S., Jung, M., Kim, S.-Y., Kim, K., et al. (2020). Controlling the sense of embodiment for virtual avatar applications: methods and empirical study. *JMIR Serious Games*, 8(3):e21879.

- [Kim et al., 2010] Kim, K. H., Reed, M. P., and Martin, B. J. (2010). A model of head movement contribution for gaze transitions. *Ergonomics*, 53(4):447–457.
- [Kirschen et al., 2000] Kirschen, M. P., Kahana, M. J., Sekuler, R., and Burack, B. (2000). Optic flow helps humans learn to navigate through synthetic environments. *Perception*, 29(7):801–818.
- [Kitson et al., 2017] Kitson, A., Hashemian, A. M., Stepanova, E. R., Kruijff, E., and Riecke, B. E. (2017). Comparing leaning-based motion cueing interfaces for virtual reality locomotion. In *2017 IEEE Symposium on 3d user interfaces (3DUI)*, pages 73–82. IEEE.
- [Kitson et al., 2015] Kitson, A., Riecke, B. E., Hashemian, A. M., and Neustaedter, C. (2015). Navichair: Evaluating an embodied interface using a pointing task to navigate virtual reality. In *Proceedings of the 3rd acm symposium on spatial user interaction*, pages 123–126.
- [Klatzky et al., 1998] Klatzky, R. L., Loomis, J. M., Beall, A. C., Chance, S. S., and Golledge, R. G. (1998). Spatial updating of self-position and orientation during real, imagined, and virtual locomotion. *Psychological science*, 9(4):293–298.
- [Kornblum, 1973] Kornblum, S. (1973). Sequential effects in choice reaction time: A tutorial review. *Attention and performance IV*, pages 259–288.
- [Kornblum et al., 1990] Kornblum, S., Hasbroucq, T., and Osman, A. (1990). Dimensional overlap: cognitive basis for stimulus-response compatibility—a model and taxonomy. *Psychological review*, 97(2):253.
- [Kornblum and Lee, 1995] Kornblum, S. and Lee, J.-W. (1995). Stimulus-response compatibility with relevant and irrelevant stimulus dimensions that do and do not overlap with the response. *Journal of Experimental Psychology: Human Perception and Performance*, 21(4):855.
- [Kornblum et al., 1999] Kornblum, S., Stevens, G. T., Whipple, A., and Requin, J. (1999). The effects of irrelevant stimuli: 1. the time course of stimulus–stimulus and stimulus–response consistency effects with stroop-like stimuli, simon-like tasks, and their factorial combinations. *Journal of Experimental Psychology: Human Perception and Performance*, 25(3):688.
- [Kraus and Nicol, 2005] Kraus, N. and Nicol, T. (2005). Brainstem origins for cortical ‘what’ and ‘where’ pathways in the auditory system. *Trends in neurosciences*, 28(4):176–181.
- [Kristjánsson et al., 2016] Kristjánsson, Á., Moldoveanu, A., Jóhannesson, Ó. I., Balan, O., Spagnol, S., Valgeirsdóttir, V. V., and Unnthorsson, R. (2016). Designing sensory-substitution devices: Principles, pitfalls and potential 1. *Restorative neurology and neuroscience*, 34(5):769–787.
- [Kritikos et al., 2019] Kritikos, J., Tzannetos, G., Zoitaki, C., Pouloupoulou, S., and Koutsouris, D. (2019). Anxiety detection from electrodermal activity sensor with movement & interaction during virtual reality simulation. In *2019 9th International IEEE/EMBS Conference on Neural Engineering (NER)*, pages 571–576. IEEE.
- [Kurniawan et al., 2022] Kurniawan, I. A., Mugiono, M., and Wijayanti, R. (2022). The effect of perceived usefulness, perceived ease of use, and social influence toward intention to use mediated by trust. *Jurnal Aplikasi Manajemen*, 20(1):117–127.

- [Kurniawati et al., 2011] Kurniawati, H., Du, Y., Hsu, D., and Lee, W. S. (2011). Motion planning under uncertainty for robotic tasks with long time horizons. *The International Journal of Robotics Research*, 30(3):308–323.
- [Laird et al., 1987] Laird, J. E., Newell, A., and Rosenbloom, P. S. (1987). Soar: An architecture for general intelligence. *Artificial intelligence*, 33(1):1–64.
- [Lakshmivarahan and Narendra, 1982] Lakshmivarahan, S. and Narendra, K. S. (1982). Learning algorithms for two-person zero-sum stochastic games with incomplete information: A unified approach. *SIAM Journal on Control and Optimization*, 20(4):541–552.
- [Lamberts et al., 1992] Lamberts, K., Tavernier, G., and d’Ydewalle, G. (1992). Effects of multiple reference points in spatial stimulus-response compatibility. *Acta psychologica*, 79(2):115–130.
- [Land and Tatler, 2009] Land, M. and Tatler, B. (2009). *Looking and acting: vision and eye movements in natural behaviour*. Oxford University Press.
- [Langbehn et al., 2015] Langbehn, E., Eichler, T., Ghose, S., von Luck, K., Bruder, G., and Steinicke, F. (2015). Evaluation of an omnidirectional walking-in-place user interface with virtual locomotion speed scaled by forward leaning angle. In *Proceedings of the GI Workshop on Virtual and Augmented Reality (GI VR/AR)*, pages 149–160.
- [Lanillos et al., 2021] Lanillos, P., Meo, C., Pezzato, C., Meera, A. A., Baioumy, M., Ohata, W., Tschantz, A., Millidge, B., Wisse, M., Buckley, C. L., et al. (2021). Active inference in robotics and artificial agents: Survey and challenges. *arXiv preprint arXiv:2112.01871*.
- [Lappi, 2016] Lappi, O. (2016). Eye movements in the wild: Oculomotor control, gaze behavior & frames of reference. *Neuroscience & Biobehavioral Reviews*, 69:49–68.
- [Latash et al., 2003] Latash, M. L., Ferreira, S. S., Wieczorek, S. A., and Duarte, M. (2003). Movement sway: changes in postural sway during voluntary shifts of the center of pressure. *Experimental brain research*, 150:314–324.
- [Lee et al., 2016] Lee, J. C., Beesley, T., and Livesey, E. J. (2016). Sequential effects and sequence learning in a three-choice serial reaction time task. *Acta Psychologica*, 170:168–176.
- [Leeuwenberg, 1971] Leeuwenberg, E. L. (1971). A perceptual coding language for visual and auditory patterns. *The American journal of psychology*, pages 307–349.
- [Leigh et al., 2018] Leigh, S.-w., Agrawal, H., and Maes, P. (2018). Robotic symbionts: Interweaving human and machine actions. *IEEE Pervasive Computing*, 17(2):34–43.
- [Leuthold, 2011] Leuthold, H. (2011). The simon effect in cognitive electrophysiology: a short review. *Acta psychologica*, 136(2):203–211.
- [Levitt and Lund, 1997] Levitt, J. B. and Lund, J. S. (1997). Contrast dependence of contextual effects in primate visual cortex. *Nature*, 387(6628):73–76.
- [Li and Sethi, 2017] Li, T. and Sethi, S. P. (2017). A review of dynamic stackelberg game models. *Discrete & Continuous Dynamical Systems-B*, 22(1):125.
- [Linson et al., 2020] Linson, A., Parr, T., and Friston, K. J. (2020). Active inference, stressors, and psychological trauma: A neuroethological model of (mal) adaptive explore-exploit dynamics in ecological context. *Behavioural brain research*, 380:112421.

- [Little and Shiffrin, 2009] Little, D. R. and Shiffrin, R. (2009). Simplicity bias in the estimation of causal functions. In *Proceedings of the annual meeting of the cognitive science society*, volume 31.
- [Liu, 2024] Liu, Y. (2024). Worker-centric human-robot co-adaptation in construction.
- [Llorens-Bonilla et al., 2012] Llorens-Bonilla, B., Parietti, F., and Asada, H. H. (2012). Demonstration-based control of supernumerary robotic limbs. In *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 3936–3942. IEEE.
- [Lombard and Gärdenfors, 2023] Lombard, M. and Gärdenfors, P. (2023). Causal cognition and theory of mind in evolutionary cognitive archaeology. *Biological Theory*, 18(4):234–252.
- [Longo et al., 2008] Longo, M. R., Schüür, F., Kammers, M. P., Tsakiris, M., and Haggard, P. (2008). What is embodiment? a psychometric approach. *Cognition*, 107(3):978–998.
- [Lotte et al., 2007] Lotte, F., Congedo, M., Lécuyer, A., Lamarche, F., and Arnaldi, B. (2007). A review of classification algorithms for eeg-based brain–computer interfaces. *Journal of neural engineering*, 4(2):R1.
- [Lovelock and Margulis, 1974] Lovelock, J. E. and Margulis, L. (1974). Atmospheric homeostasis by and for the biosphere: the gaia hypothesis. *Tellus*, 26(1-2):2–10.
- [Lu and Proctor, 1995] Lu, C.-H. and Proctor, R. W. (1995). The influence of irrelevant location information on performance: A review of the simon and spatial stroop effects. *Psychonomic bulletin & review*, 2:174–207.
- [Lucchese et al., 2022a] Lucchese, A., Digiesi, S., and Mummolo, C. (2022a). Analytical-stochastic model of motor difficulty for three-dimensional manipulation tasks. *Plos one*, 17(10):e0276308.
- [Lucchese et al., 2022b] Lucchese, A., Mummolo, G., Digiesi, S., and Mummolo, C. (2022b). Agent’s motor performance: an index of difficulty-based model. *IFAC-PapersOnLine*, 55(10):347–352.
- [Luck, 2012] Luck, S. J. (2012). Event-related potentials. *APA handbook of research methods in psychology, Vol. 1.*, page 523–546.
- [Mackay, 1990] Mackay, W. E. (1990). *Users and customizable software: A co-adaptive phenomenon*. PhD thesis, Massachusetts Institute of Technology.
- [MacLeod, 1991] MacLeod, C. M. (1991). Half a century of research on the stroop effect: an integrative review. *Psychological bulletin*, 109(2):163.
- [MacLeod, 2015] MacLeod, C. M. (2015). The stroop effect. *Encyclopedia of color science and technology*, pages 1–6.
- [Macpherson, 2018] Macpherson, F. (2018). Sensory substitution and augmentation: An introduction. *Proceedings of the British Academy, Oxford*, page 1 – 42.
- [Madduri et al., 2022] Madduri, M. M., Yamagami, M., Millevolte, A. X., Li, S. J., Burckhardt, S. N., Burden, S. A., and Orsborn, A. L. (2022). Co-adaptive myoelectric interface for continuous control. *IFAC-PapersOnLine*, 55(41):95–100.

- [Mahani et al., 2019] Mahani, M.-A. N., Bausenhart, K. M., Ahmadabadi, M. N., and Ulrich, R. (2019). Multimodal simon effect: a multimodal extension of the diffusion model for conflict tasks. *Frontiers in human neuroscience*, 12:507.
- [Maharmeh and Tahboub, 2023] Maharmeh, E. M. and Tahboub, K. (2023). Human machine co-adaptation using co-adaptive policy gradient algorithm. In *2023 International Conference on Control, Automation and Diagnosis (ICCAD)*, pages 1–6. IEEE.
- [Maisto et al., 2021] Maisto, D., Barca, L., Van den Bergh, O., and Pezzulo, G. (2021). Perception and misperception of bodily symptoms from an active inference perspective: Modelling the case of panic disorder. *Psychological Review*, 128(4):690.
- [Maisto et al., 2019] Maisto, D., Friston, K., and Pezzulo, G. (2019). Caching mechanisms for habit formation in active inference. *Neurocomputing*, 359:298–314.
- [Makin et al., 2017] Makin, T. R., De Vignemont, F., and Faisal, A. A. (2017). Neurocognitive barriers to the embodiment of technology. *Nature Biomedical Engineering*, 1(1):0014.
- [Malle et al., 2001] Malle, B. F., Moses, L. J., and Baldwin, D. A. (2001). *Intentions and intentionality: Foundations of social cognition*. MIT press.
- [Marchal et al., 2011] Marchal, M., Pettré, J., and Lécuyer, A. (2011). Joyman: A human-scale joystick for navigating in virtual worlds. In *2011 IEEE Symposium on 3D User Interfaces (3DUI)*, pages 19–26. IEEE.
- [Markovic and Kiebel, 2019] Markovic, D. and Kiebel, S. J. (2019). An active inference perspective on habit learning. *Conference on Cognitive Computational Neuroscience*.
- [Martel et al., 2019] Martel, M., Cardinali, L., Bertonati, G., Jouffrais, C., Finos, L., Farnè, A., and Roy, A. (2019). Somatosensory-guided tool use modifies arm representation for action. *Scientific reports*, 9(1):5517.
- [Mateevitsi et al., 2013] Mateevitsi, V., Haggadone, B., Leigh, J., Kunzer, B., and Kenyon, R. V. (2013). Sensing the environment through spidersense. In *Proceedings of the 4th augmented human international conference*, pages 51–57.
- [Mateevitsi, 2018] Mateevitsi, V. A. (2018). *Supporting Navigation with a Torso Wearable Tactile Display*. PhD thesis, University of Illinois at Chicago.
- [Mattout et al., 2015] Mattout, J., Perrin, M., Bertrand, O., and Maby, E. (2015). Improving bci performance through co-adaptation: applications to the p300-speller. *Annals of physical and rehabilitation medicine*, 58(1):23–28.
- [McCluskey and Cullen, 2007] McCluskey, M. K. and Cullen, K. E. (2007). Eye, head, and body coordination during large gaze shifts in rhesus monkeys: movement kinematics and the influence of posture. *Journal of neurophysiology*, 97(4):2976–2991.
- [McGrath et al., 2014] McGrath, B., Rupert, A., Stanney, K., and Hale, K. (2014). Tactile displays: from the cockpit to the clinic. In *Advances in Cognitive Engineering and Neuroergonomics*, volume 11.
- [McIntosh and Mehring, 2017] McIntosh, J. R. and Mehring, C. (2017). Modifying response times in the simon task with transcranial random noise stimulation. *Scientific reports*, 7(1):15633.

- [Mechergui and Sreedharan, 2023] Mechergui, M. and Sreedharan, S. (2023). Goal alignment: A human-aware account of value alignment problem. *arXiv preprint arXiv:2302.00813*.
- [Medina et al., 2014] Medina, J., McCloskey, M., Coslett, H., and Rapp, B. (2014). Somatotopic representation of location: evidence from the simon effect. *Journal of Experimental Psychology: Human Perception and Performance*, 40(6):2131.
- [Meena et al., 2017] Meena, Y. K., Cecotti, H., Wong-Lin, K., and Prasad, G. (2017). A multimodal interface to resolve the midas-touch problem in gaze controlled wheelchair. In *2017 39th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*, pages 905–908. IEEE.
- [Menshchikov et al., 2019] Menshchikov, A., Ermilov, D., Dranitsky, I., Kupchenko, L., Panov, M., Fedorov, M., and Somov, A. (2019). Data-driven body-machine interface for drone intuitive control through voice and gestures. In *IECON 2019-45th Annual Conference of the IEEE Industrial Electronics Society*, volume 1, pages 5602–5609. IEEE.
- [Merel et al., 2013] Merel, J. S., Fox, R., Jebara, T., and Paninski, L. (2013). A multi-agent control framework for co-adaptation in brain-computer interfaces. *Advances in Neural Information Processing Systems*, 26.
- [Miehlbradt et al., 2018] Miehlbradt, J., Cherpillod, A., Mintchev, S., Coscia, M., Artoni, F., Floreano, D., and Micera, S. (2018). Data-driven body-machine interface for the accurate control of drones. *Proceedings of the National Academy of Sciences*, 115(31):7913–7918.
- [Miles and Vu, 2021] Miles, J. D. and Vu, K.-P. L. (2021). Individual response–effect congruencies modulate subsequent stimulus–response compatibility effects. *The American Journal of Psychology*, 134(1):31–43.
- [Miller et al., 2020] Miller, M., Kiverstein, J., and Rietveld, E. (2020). Embodying addiction: A predictive processing account. *Brain and cognition*, 138:105495.
- [Millidge et al., 2021] Millidge, B., Tschantz, A., and Buckley, C. L. (2021). Whence the expected free energy? *Neural Computation*, 33(2):447–482.
- [Mirza et al., 2016] Mirza, M. B., Adams, R. A., Mathys, C. D., and Friston, K. J. (2016). Scene construction, visual foraging, and active inference. *Frontiers in computational neuroscience*, 10:56.
- [Mohan et al., 2018] Mohan, P., Goh, W. B., Fu, C.-W., and Yeung, S.-K. (2018). Dualgaze: Addressing the midas touch problem in gaze mediated vr interaction. In *2018 IEEE International Symposium on Mixed and Augmented Reality Adjunct (ISMAR-Adjunct)*, pages 79–84. IEEE.
- [Mohri, 2013] Mohri, T. (2013). Cluster variation method. *Jom*, 65:1510–1522.
- [Morris et al., 2017] Morris, D. M., Erno, J. M., and Pilcher, J. J. (2017). Electrodermal response and automation trust during simulated self-driving car use. In *Proceedings of the human factors and ergonomics society annual meeting*, volume 61, pages 1759–1762. SAGE Publications Sage CA: Los Angeles, CA.

- [Müller et al., 2017] Müller, J. S., Vidaurre, C., Schreuder, M., Meinecke, F. C., Von Büna, P., and Müller, K.-R. (2017). A mathematical model for the two-learners problem. *Journal of neural engineering*, 14(3):036005.
- [Murphy, 2000] Murphy, K. P. (2000). A survey of pomdp solution techniques. *environment*, 2(10).
- [Murray, 2008] Murray, C. D. (2008). Embodiment and prosthetics. In *Psychoprosthetics*, pages 119–129. Springer.
- [Musco Eklund, 2023] Musco Eklund, A. (2023). Rule of law challenges of ‘algorithmic discretion’ & automation in eu border control: a case study of etias through the lens of legality. *European journal of migration and law*, 25(3):249–274.
- [Nakabayashi et al., 2017] Nakabayashi, K., Iwasaki, Y., and Iwata, H. (2017). Development of evaluation indexes for human-centered design of a wearable robot arm. In *Proceedings of the 5th International Conference on Human Agent Interaction*, pages 305–310.
- [Neacsu et al., 2022] Neacsu, V., Convertino, L., and Friston, K. J. (2022). Synthetic spatial foraging with active inference in a geocaching task. *Frontiers in Neuroscience*, 16:802396.
- [Ng and Chan, 2012] Ng, A. W. and Chan, A. H. (2012). Finger response times to visual, auditory and tactile modality stimuli. In *Proceedings of the international multiconference of engineers and computer scientists*, volume 2, pages 1449–1454.
- [Nguyen-Vo et al., 2019] Nguyen-Vo, T., Riecke, B. E., Stuerzlinger, W., Pham, D.-M., and Kruijff, E. (2019). Naviboard and navichair: Limited translation combined with full rotation for efficient virtual locomotion. *IEEE transactions on visualization and computer graphics*, 27(1):165–177.
- [Nichols and Stich, 2003] Nichols, S. and Stich, S. P. (2003). *Mindreading: An integrated account of pretence, self-awareness, and understanding other minds*. Oxford University Press.
- [Nicoletti et al., 1984] Nicoletti, R., Umiltà, C., and Ladavas, E. (1984). Compatibility due to the coding of the relative position of the effectors. *Acta Psychologica*, 57(2):133–143.
- [Niedernhuber et al., 2018] Niedernhuber, M., Barone, D. G., and Lenggenhager, B. (2018). Prostheses as extensions of the body: Progress and challenges. *Neuroscience & Biobehavioral Reviews*, 92:1–6.
- [Niwa et al., 2010] Niwa, M., Okada, S., Sakaguchi, S., Azuma, K., Iizuka, H., Ando, H., and Maeda, T. (2010). Detection and transmission of “tsumori”: an archetype of behavioral intention in controlling a humanoid robot. In *Proceeding of 20th International Conference on Artificial Reality and Telexistence (ICAT2010)*, pages 197–201.
- [Nothdurft, 2000] Nothdurft, H.-C. (2000). Salience from feature contrast: additivity across dimensions. *Vision research*, 40(10-12):1183–1201.
- [Orlikowski and Baroudi, 1989] Orlikowski, W. J. and Baroudi, J. J. (1989). Is research paradigms: method versus substance.
- [Orsborn et al., 2012] Orsborn, A. L., Dangi, S., Moorman, H. G., and Carmena, J. M. (2012). Closed-loop decoder adaptation on intermediate time-scales facilitates rapid bmi performance improvements independent of decoder initialization conditions. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, 20(4):468–477.

- [Ort et al., 2015] Ort, T., Wu, F., Hensel, N. C., and Asada, H. H. (2015). Supernumerary robotic fingers as a therapeutic device for hemiparetic patients. In *Dynamic Systems and Control Conference*, volume 57250, page V002T27A010. American Society of Mechanical Engineers.
- [Parasuraman and Riley, 1997] Parasuraman, R. and Riley, V. (1997). Humans and automation: Use, misuse, disuse, abuse. *Human factors*, 39(2):230–253.
- [Parasuraman et al., 2000] Parasuraman, R., Sheridan, T. B., and Wickens, C. D. (2000). A model for types and levels of human interaction with automation. *IEEE Transactions on systems, man, and cybernetics-Part A: Systems and Humans*, 30(3):286–297.
- [Parietti and Asada, 2016] Parietti, F. and Asada, H. (2016). Supernumerary robotic limbs for human body support. *IEEE Transactions on Robotics*, 32(2):301–311.
- [Parietti and Asada, 2014] Parietti, F. and Asada, H. H. (2014). Supernumerary robotic limbs for aircraft fuselage assembly: body stabilization and guidance by bracing. In *2014 IEEE International Conference on Robotics and Automation (ICRA)*, pages 1176–1183. IEEE.
- [Parietti et al., 2015] Parietti, F., Chan, K. C., Hunter, B., and Asada, H. H. (2015). Design and control of supernumerary robotic limbs for balance augmentation. In *2015 IEEE International Conference on Robotics and Automation (ICRA)*, pages 5010–5017. IEEE.
- [Park and Kim, 2008] Park, S. and Kim, M. (2008). Test of validity of fitts’ index of difficulty as a measure of task difficulty. *Perceptual and motor skills*, 107(3):901–914.
- [Parr and Friston, 2019] Parr, T. and Friston, K. J. (2019). Generalised free energy and active inference. *Biological cybernetics*, 113(5):495–513.
- [Parr et al., 2021] Parr, T., Limanowski, J., Rawji, V., and Friston, K. (2021). The computational neurology of movement under active inference. *Brain*, 144(6):1799–1818.
- [Parr et al., 2019] Parr, T., Markovic, D., Kiebel, S. J., and Friston, K. J. (2019). Neuronal message passing using mean-field, bethe, and marginal approximations. *Scientific reports*, 9(1):1889.
- [Parr et al., 2022] Parr, T., Pezzulo, G., and Friston, K. J. (2022). *Active inference: the free energy principle in mind, brain, and behavior*. MIT Press.
- [Patterson, 2017] Patterson, R. E. (2017). Intuitive cognition and models of human–automation interaction. *Human factors*, 59(1):101–115.
- [Pazzaglia and Molinari, 2016] Pazzaglia, M. and Molinari, M. (2016). The embodiment of assistive devices—from wheelchair to exoskeleton. *Physics of life reviews*, 16:163–175.
- [Penaloza et al., 2018] Penaloza, C., Hernandez-Carmona, D., and Nishio, S. (2018). Towards intelligent brain-controlled body augmentation robotic limbs. In *2018 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, pages 1011–1015. IEEE.
- [Penn and Povinelli, 2007] Penn, D. C. and Povinelli, D. J. (2007). On the lack of evidence that non-human animals possess anything remotely resembling a ‘theory of mind’. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 362(1480):731–744.
- [Perdikis and Millan, 2020] Perdikis, S. and Millan, J. d. R. (2020). Brain-machine interfaces: a tale of two learners. *IEEE Systems, Man, and Cybernetics Magazine*, 6(3):12–19.

- [Perrin, 2012] Perrin, M. (2012). *Coadaptation cerveau machine pour une interaction optimale: application au P300-Speller*. PhD thesis, Université Claude Bernard-Lyon I.
- [Pétré et al., 2011] Pettré, J., Siret, O., Marchal, M., de la Riviere, J.-B., and Lecuyer, A. (2011). Joyman: an immersive and entertaining interface for virtual locomotion. In *SIGGRAPH Asia 2011 Emerging Technologies*, pages 1–1.
- [Pfeil et al., 2018] Pfeil, K., Taranta, E. M., Kulshreshth, A., Wisniewski, P., and LaViola Jr, J. J. (2018). A comparison of eye-head coordination between virtual and physical realities. In *Proceedings of the 15th ACM Symposium on Applied Perception*, pages 1–7.
- [Picard et al., 2023] Picard, A., Mualla, Y., Gechter, F., and Galland, S. (2023). Human-computer interaction and explainability: Intersection and terminology. In *World Conference on Explainable Artificial Intelligence*, pages 214–236. Springer.
- [Pittman and LaViola Jr, 2014] Pittman, C. and LaViola Jr, J. J. (2014). Exploring head tracked head mounted displays for first person robot teleoperation. In *Proceedings of the 19th international conference on Intelligent User Interfaces*, pages 323–328.
- [Poletti et al., 2013] Poletti, M., Listorti, C., and Rucci, M. (2013). Microscopic eye movements compensate for nonhomogeneous vision within the fovea. *Current Biology*, 23(17):1691–1695.
- [Povinelli and Bering, 2002] Povinelli, D. J. and Bering, J. M. (2002). The mentality of apes revisited. *Current Directions in Psychological Science*, 11(4):115–119.
- [Praamstra and Oostenveld, 2003] Praamstra, P. and Oostenveld, R. (2003). Attention and movement-related motor cortex activation: a high-density eeg study of spatial stimulus-response compatibility. *Cognitive brain research*, 16(3):309–322.
- [Prattichizzo et al., 2014a] Prattichizzo, D., Malvezzi, M., Hussain, I., and Salvietti, G. (2014a). The sixth-finger: a modular extra-finger to enhance human hand capabilities. In *The 23rd IEEE International Symposium on Robot and Human Interactive Communication*, pages 993–998. IEEE.
- [Prattichizzo et al., 2021] Prattichizzo, D., Pozzi, M., Baldi, T. L., Malvezzi, M., Hussain, I., Rossi, S., and Salvietti, G. (2021). Human augmentation by wearable supernumerary robotic limbs: review and perspectives. *Progress in Biomedical Engineering*, 3(4):042005.
- [Prattichizzo et al., 2014b] Prattichizzo, D., Salvietti, G., Chinello, F., and Malvezzi, M. (2014b). An object-based mapping algorithm to control wearable robotic extra-fingers. In *2014 IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, pages 1563–1568. IEEE.
- [Priorelli et al., 2023] Priorelli, M., Maggiore, F., Maselli, A., Donnarumma, F., Maisto, D., Mannella, F., Stoianov, I. P., and Pezzulo, G. (2023). Modeling motor control in continuous-time active inference: a survey. *IEEE Transactions on Cognitive and Developmental Systems*.
- [Proctor and Reeve, 1989] Proctor, R. W. and Reeve, T. G. (1989). *Stimulus-response compatibility: An integrated perspective*. Elsevier.

- [Proctor et al., 2005] Proctor, R. W., Tan, H. Z., Vu, K.-P. L., Gray, R., and Spence, C. (2005). Implications of compatibility and cuing effects for multimodal interfaces. In *Foundations of augmented cognition (Proceedings of the 11th Annual Conference on Human-Computer Interaction International 2005)*, pages 3–12.
- [Puterman, 2014] Puterman, M. L. (2014). *Markov decision processes: discrete stochastic dynamic programming*. John Wiley & Sons.
- [Rahman et al., 2021] Rahman, M. A., Hopner, N., Christianos, F., and Albrecht, S. V. (2021). Towards open ad hoc teamwork using graph-based policy learning. In *International conference on machine learning*, pages 8776–8786. PMLR.
- [Ramstead et al., 2020] Ramstead, M. J., Kirchhoff, M. D., and Friston, K. J. (2020). A tale of two densities: Active inference is enactive inference. *Adaptive behavior*, 28(4):225–239.
- [Ratcliff and McKoon, 2008] Ratcliff, R. and McKoon, G. (2008). The diffusion decision model: theory and data for two-choice decision tasks. *Neural computation*, 20(4):873–922.
- [Reddy et al., 2022] Reddy, S., Levine, S., and Dragan, A. (2022). First contact: Unsupervised human-machine co-adaptation via mutual information maximization. *Advances in Neural Information Processing Systems*, 35:31542–31556.
- [Redlick et al., 2001] Redlick, F. P., Jenkin, M., and Harris, L. R. (2001). Humans can use optic flow to estimate distance of travel. *Vision research*, 41(2):213–219.
- [Restle, 1979] Restle, F. (1979). Coding theory of the perception of motion configurations. *Psychological Review*, 86(1):1.
- [Rezazadeh et al., 2012] Rezazadeh, I. M., Firoozabadi, M., Hu, H., and Golpayegani, S. M. R. H. (2012). Co-adaptive and affective human-machine interface for improving training performances of virtual myoelectric forearm prosthesis. *IEEE Transactions on affective computing*, 3(3):285–297.
- [Riva, 2008] Riva, G. (2008). From virtual to real body: virtual reality as embodied technology. *J. Cyber Ther. Rehabil*, 1:7–22.
- [Robinson, 2021] Robinson, J. (2021). Edinburgh handedness inventory. In *Encyclopedia of autism spectrum disorders*, pages 1600–1604. Springer.
- [Rodieck, 1998] Rodieck, R. W. (1998). *The first steps in seeing*. Sinauer Associates.
- [Rognon et al., 2018] Rognon, C., Mintchev, S., Dell’Agnola, F., Cherpillod, A., Atienza, D., and Floreano, D. (2018). Flyjacket: An upper body soft exoskeleton for immersive drone control. *IEEE Robotics and Automation Letters*, 3(3):2362–2369.
- [Ross and Stirling, 2023] Ross, D. and Stirling, W. C. (2023). Mindshaping, conditional games, and the harsanyi doctrine. *CEAR Working Paper*.
- [Roxin, 2019] Roxin, A. (2019). Drift–diffusion models for multiple-alternative forced-choice decision making. *The Journal of Mathematical Neuroscience*, 9(1):5.
- [Rubichi et al., 2000] Rubichi, S., Nicoletti, R., Umiltà, C., and Zorzi, M. (2000). Response strategies and the simon effect. *Psychological Research*, 63:129–136.

- [Ruddle, 2013] Ruddle, R. A. (2013). The effect of translational and rotational body-based information on navigation. *Human walking in virtual environments: Perception, technology, and applications*, pages 99–112.
- [Sajid et al., 2021] Sajid, N., Ball, P. J., Parr, T., and Friston, K. J. (2021). Active inference: demystified and compared. *Neural computation*, 33(3):674–712.
- [Sakthivadivel, 2022] Sakthivadivel, D. A. (2022). Magnetisation and mean field theory in the ising model. *SciPost Physics Lecture Notes*, page 035.
- [Salviotti et al., 2021] Salviotti, G., Franco, L., Tschiersky, M., Wolterink, G., Bianchi, M., Bicchieri, A., Barontini, F., Catalano, M., Grioli, G., Poggiani, M., et al. (2021). Integration of a passive exoskeleton and a robotic supernumerary finger for grasping compensation in chronic stroke patients: The softpro wearable system. *Frontiers in Robotics and AI*, 8:661354.
- [Sanchez et al., 2009] Sanchez, J. C., Mahmoudi, B., DiGiovanna, J., and Principe, J. C. (2009). Exploiting co-adaptation for the design of symbiotic neuroprosthetic assistants. *Neural Networks*, 22(3):305–315.
- [Sanders and Lamers, 2002] Sanders, A. and Lamers, J. (2002). The eriksen flanker effect revisited. *Acta Psychologica*, 109(1):41–56.
- [Sanner and Boutilier, 2009] Sanner, S. and Boutilier, C. (2009). Practical solution techniques for first-order mdps. *Artificial Intelligence*, 173(5-6):748–788.
- [Santello et al., 2016] Santello, M., Bianchi, M., Gabiccini, M., Ricciardi, E., Salviotti, G., Praticchizzo, D., Ernst, M., Moscatelli, A., Jörntell, H., Kappers, A. M., et al. (2016). Hand synergies: Integration of robotics and neuroscience for understanding the control of biological and artificial hands. *Physics of life reviews*, 17:1–23.
- [Santello et al., 1998] Santello, M., Flanders, M., and Soechting, J. F. (1998). Postural hand synergies for tool use. *Journal of neuroscience*, 18(23):10105–10115.
- [Saraiji et al., 2018] Saraiji, M. Y., Sasaki, T., Kunze, K., Minamizawa, K., and Inami, M. (2018). Metaarms: Body remapping using feet-controlled artificial arms. In *Proceedings of the 31st Annual ACM Symposium on User Interface Software and Technology*, pages 65–74.
- [Sasaki et al., 2017] Sasaki, T., Saraiji, M. Y., Fernando, C. L., Minamizawa, K., and Inami, M. (2017). Metalimbs: Multiple arms interaction metamorphism. In *ACM SIGGRAPH 2017 Emerging Technologies*, pages 1–2.
- [Satogata et al., 2022] Satogata, R., Kimoto, M., Fukuchi, Y., Okuoka, K., and Imai, M. (2022). *q*-mapping: Learning user-preferred operation mappings with operation-action value function. *IEEE Transactions on Human-Machine Systems*, 52(6):1090–1102.
- [Saunders and Vijayakumar, 2011] Saunders, I. and Vijayakumar, S. (2011). The role of feed-forward and feedback processes for closed-loop prosthesis control. *Journal of neuroengineering and rehabilitation*, 8:1–12.
- [Sawaragi, 2005] Sawaragi, T. (2005). Dynamical and complex behaviors in human-machine co-adaptive systems. *IFAC Proceedings Volumes*, 38(1):94–99.
- [Schal, 1979] Schal, M. (1979). On dynamic programming and statistical decision theory. *The Annals of Statistics*, pages 432–445.

- [Schmidmaier, ] Schmidmaier, M. Sensory substitution systems. In *Media Informatics Advanced Seminar on Multimodal Human-Computer Interaction*.
- [Schulte et al., 2016a] Schulte, A., Donath, D., and Lange, D. S. (2016a). Design patterns for human-cognitive agent teaming. In *Engineering Psychology and Cognitive Ergonomics: 13th International Conference, EPCE 2016, Held as Part of HCI International 2016, Toronto, ON, Canada, July 17-22, 2016, Proceedings 13*, pages 231–243. Springer.
- [Schulte et al., 2016b] Schulte, B., Bimberg, C. P., Hertel, J., Neu, F., Heidenreich, P., Bruder, G., and Lubos, P. (2016b). Evaluation of two leaning-based control methods in a dragon riding interface. In *Proceedings of the GI-Workshop VR/AR*, pages 109–116.
- [Schumacher et al., 2005] Schumacher, E. H., Hendricks, M. J., and D’Esposito, M. (2005). Sustained involvement of a frontal–parietal network for spatial response selection with practice of a spatial choice–reaction task. *Neuropsychologia*, 43(10):1444–1455.
- [Schwartenbeck et al., 2015] Schwartenbeck, P., FitzGerald, T. H., Mathys, C., Dolan, R., Wurst, F., Kronbichler, M., and Friston, K. (2015). Optimal inference with suboptimal models: addiction and active bayesian inference. *Medical hypotheses*, 84(2):109–117.
- [Schwartenbeck et al., 2019] Schwartenbeck, P., Passecker, J., Hauser, T. U., FitzGerald, T. H., Kronbichler, M., and Friston, K. J. (2019). Computational mechanisms of curiosity and goal-directed exploration. *elife*, 8:e41703.
- [Schweickert et al., 2010] Schweickert, R., Fisher, D. L., and Goldstein, W. M. (2010). Additive factors and stages of mental processes in task networks. *Journal of Mathematical Psychology*, 54(5):405–414.
- [Schwöbel et al., 2018] Schwöbel, S., Kiebel, S., and Marković, D. (2018). Active inference, belief propagation, and the bethe approximation. *Neural computation*, 30(9):2530–2567.
- [Segura Meraz et al., 2018] Segura Meraz, N., Sobajima, M., Aoyama, T., and Hasegawa, Y. (2018). Modification of body schema by use of extra robotic thumb. *Robomech Journal*, 5(1):1–8.
- [Shadmehr et al., 2016] Shadmehr, R., Huang, H. J., and Ahmed, A. A. (2016). A representation of effort in decision-making and motor control. *Current biology*, 26(14):1929–1934.
- [Shafti et al., 2021] Shafti, A., Haar, S., Mio, R., Guilleminot, P., and Faisal, A. A. (2021). Playing the piano with a robotic third thumb: assessing constraints of human augmentation. *Scientific reports*, 11(1):21375.
- [Shahar et al., 2019] Shahar, N., Hauser, T. U., Moutoussis, M., Moran, R., Keramati, M., Consortium, N., and Dolan, R. J. (2019). Improving the reliability of model-based decision-making estimates in the two-stage decision task with reaction-times and drift-diffusion modeling. *PLoS computational biology*, 15(2):e1006803.
- [Shelton and Kumar, 2010] Shelton, J. and Kumar, G. P. (2010). Comparison between auditory and visual simple reaction times. *Neuroscience and medicine*, 1(01):30–32.
- [Shen et al., 2024] Shen, H., Kneare, T., Ghosh, R., Alkie, K., Krishna, K., Liu, Y., Ma, Z., Petridis, S., Peng, Y.-H., Qiwei, L., et al. (2024). Towards bidirectional human-ai alignment: A systematic review for clarifications, framework, and future directions. *arXiv preprint arXiv:2406.09264*.

- [Shende and Jabade, 2015] Shende, P. M. and Jabade, V. S. (2015). Literature review of brain computer interface (bci) using electroencephalogram signal. In *2015 International Conference on Pervasive Computing (ICPC)*, pages 1–5. IEEE.
- [Shenoy and Carmena, 2014] Shenoy, K. V. and Carmena, J. M. (2014). Combining decoder design and neural adaptation in brain-machine interfaces. *Neuron*, 84(4):665–680.
- [Sherrick Jr, 1953] Sherrick Jr, C. E. (1953). Variables affecting sensitivity of the human skin to mechanical vibration. *Journal of Experimental Psychology*, 45(5):273.
- [Shibasaki and Hallett, 2006] Shibasaki, H. and Hallett, M. (2006). What is the bere-itschaftspotential? *Clinical neurophysiology*, 117(11):2341–2356.
- [Shin et al., 2015] Shin, C.-Y., Bae, J., and Hong, D. (2015). Ceiling work scenario based hardware design and control algorithm of supernumerary robotic limbs. In *2015 15th International Conference on Control, Automation and Systems (ICCAS)*, pages 1228–1230. IEEE.
- [Shin, 2018] Shin, D. (2018). Empathy and embodied experience in virtual environment: To what extent can virtual reality stimulate empathy and embodied experience? *Computers in human behavior*, 78:64–73.
- [Shin et al., 2010] Shin, Y. K., Proctor, R. W., and Capaldi, E. J. (2010). A review of contemporary ideomotor theory. *Psychological bulletin*, 136(6):943.
- [Sidenmark and Gellersen, 2019] Sidenmark, L. and Gellersen, H. (2019). Eye, head and torso coordination during gaze shifts in virtual reality. *ACM Transactions on Computer-Human Interaction (TOCHI)*, 27(1):1–40.
- [Simao et al., 2019] Simao, M., Mendes, N., Gibaru, O., and Neto, P. (2019). A review on electromyography decoding and pattern recognition for human-machine interaction. *Ieee Access*, 7:39564–39582.
- [Simon, 1969] Simon, J. R. (1969). Reactions toward the source of stimulation. *Journal of experimental psychology*, 81(1):174.
- [Simon and Craft, 1970] Simon, J. R. and Craft, J. L. (1970). Effects of an irrelevant auditory stimulus on visual choice reaction time. *Journal of experimental psychology*, 86(2):272.
- [Simon and Rudell, 1967] Simon, J. R. and Rudell, A. P. (1967). Auditory sr compatibility: the effect of an irrelevant cue on information processing. *Journal of applied psychology*, 51(3):300.
- [Simon et al., 1981] Simon, J. R., Sly, P. E., and Vilapakkam, S. (1981). Effect of compatibility of sr mapping on reactions toward the stimulus source. *Acta psychologica*, 47(1):63–81.
- [Simon and Small Jr, 1969] Simon, J. R. and Small Jr, A. (1969). Processing auditory information: interference from an irrelevant cue. *Journal of Applied Psychology*, 53(5):433.
- [Simon and Wolf, 1963] Simon, J. R. and Wolf, J. D. (1963). Choice reaction time as a function of angular stimulus-response correspondence and age. *Ergonomics*, 6(1):99–105.
- [Simons et al., 2021] Simons, M. F., Digumarti, K. M., Le, N. H., Chen, H.-Y., Carreira, S. C., Zaghoul, N. S., Diteesawat, R. S., Garrad, M., Conn, A. T., Kent, C., et al. (2021). B: ionic glove: a soft smart wearable sensory feedback device for upper limb robotic prostheses. *IEEE robotics and automation letters*, 6(2):3311–3316.

- [Smith et al., 2022] Smith, R., Friston, K. J., and Whyte, C. J. (2022). A step-by-step tutorial on active inference and its application to empirical data. *Journal of mathematical psychology*, 107:102632.
- [Smith et al., 2019] Smith, R., Parr, T., and Friston, K. J. (2019). Simulating emotions: An active inference model of emotional state inference and emotion concept learning. *Frontiers in psychology*, 10:489395.
- [Soutschek et al., 2013] Soutschek, A., Müller, H. J., and Schubert, T. (2013). Conflict-specific effects of accessory stimuli on cognitive control in the stroop task and the simon task. *Experimental psychology*.
- [Spanlang et al., 2014] Spanlang, B., Normand, J.-M., Borland, D., Kiltner, K., Giannopoulos, E., Pomés, A., González-Franco, M., Perez-Marcos, D., Arroyo-Palacios, J., Muncunill, X. N., et al. (2014). How to build an embodiment lab: achieving body representation illusions in virtual reality. *Frontiers in Robotics and AI*, 1:9.
- [Sparrow and Newell, 1994] Sparrow, W. and Newell, K. (1994). Energy expenditure and motor performance relationships in humans learning a motor task. *Psychophysiology*, 31(4):338–346.
- [Spironelli et al., 2009] Spironelli, C., Tagliabue, M., and Umiltà, C. (2009). Response selection and attention orienting: A computational model of simon effect asymmetries. *Experimental Psychology*, 56(4):274–282.
- [Srinivas et al., 2015] Srinivas, S., Virk, G. S., and Haider, U. (2015). Multipurpose super-numerary robotic limbs for industrial and domestic applications. In *2015 20th International Conference on Methods and Models in Automation and Robotics (MMAR)*, pages 289–293. IEEE.
- [Stein and Peelen, 2015] Stein, T. and Peelen, M. V. (2015). Content-specific expectations enhance stimulus detectability by increasing perceptual sensitivity. *Journal of Experimental Psychology: General*, 144(6):1089.
- [Steinmetz et al., 2021] Steinmetz, J. D., Bourne, R. R., Briant, P. S., Flaxman, S. R., Taylor, H. R., Jonas, J. B., Abdoli, A. A., Abrha, W. A., Abualhasan, A., Abu-Gharbieh, E. G., et al. (2021). Causes of blindness and vision impairment in 2020 and trends over 30 years, and prevalence of avoidable blindness in relation to vision 2020: the right to sight: an analysis for the global burden of disease study. *The Lancet Global Health*, 9(2):e144–e160.
- [Stocco et al., 2017] Stocco, A., Murray, N. L., Yamasaki, B. L., Renno, T. J., Nguyen, J., and Prat, C. S. (2017). Individual differences in the simon effect are underpinned by differences in the competitive dynamics in the basal ganglia: An experimental verification and a computational model. *Cognition*, 164:31–45.
- [Stock and Stock, 2004] Stock, A. and Stock, C. (2004). A short history of ideo-motor action. *Psychological research*, 68:176–188.
- [Stöcker et al., 2003] Stöcker, C., Sebold, A., and Hoffmann, J. (2003). The influence of response–effect compatibility in a serial reaction time task. *The Quarterly Journal of Experimental Psychology Section A*, 56(4):685–703.
- [Stoffer, 1991] Stoffer, T. H. (1991). Attentional focussing and spatial stimulus-response compatibility. *Psychological research*, 53(2):127–135.

- [Stoffer and Yakin, 1994] Stoffer, T. H. and Yakin, A. R. (1994). The functional role of attention for spatial coding in the simon effect. *Psychological research*, 56:151–162.
- [Stone et al., 2010] Stone, P., Kaminka, G., Kraus, S., and Rosenschein, J. (2010). Ad hoc autonomous agent teams: Collaboration without pre-coordination. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 24, pages 1504–1509.
- [Stroop, 1935] Stroop, J. R. (1935). Studies of interference in serial verbal reactions. *Journal of experimental psychology*, 18(6):643.
- [Subramanian et al., 2021] Subramanian, M., Park, S., Orlov, P., Shafti, A., and Faisal, A. A. (2021). Gaze-contingent decoding of human navigation intention on an autonomous wheelchair platform. In *2021 10th International IEEE/EMBS Conference on Neural Engineering (NER)*, pages 335–338. IEEE.
- [Sutton et al., 1967] Sutton, S., Tueting, P., Zubin, J., and John, E. R. (1967). Information delivery and the sensory evoked potential. *Science*, 155(3768):1436–1439.
- [Tagliabue et al., 2000] Tagliabue, M., Zorzi, M., Umiltà, C., and Bassignani, F. (2000). The role of long-term-memory and short-term-memory links in the simon effect. *Journal of Experimental Psychology: Human perception and performance*, 26(2):648.
- [Taylor, 1979] Taylor, P. D. (1979). Evolutionarily stable strategies with two types of player. *Journal of applied probability*, 16(1):76–83.
- [Tham et al., 2018] Tham, J., Duin, A. H., Gee, L., Ernst, N., Abdelqader, B., and McGrath, M. (2018). Understanding virtual reality: Presence, embodiment, and professional practice. *IEEE Transactions on Professional Communication*, 61(2):178–195.
- [Thomas et al., 2014] Thomas, E., Daucé, E., Devlaminck, D., Mahé, L., Carpentier, A., Munos, R., Perrin, M., Maby, E., Mattout, J., Papadopoulos, T., et al. (2014). Coadapt p300 speller: optimized flashing sequences and online learning. In *6th International Brain Computer Interface Conference*.
- [Thompson, 1989] Thompson, J. N. (1989). Concepts of coevolution. *Trends in Ecology & Evolution*, 4(6):179–183.
- [Todorov, 2006] Todorov, E. (2006). Linearly-solvable markov decision problems. *Advances in neural information processing systems*, 19.
- [Todorov and Jordan, 2002] Todorov, E. and Jordan, M. I. (2002). Optimal feedback control as a theory of motor coordination. *Nature neuroscience*, 5(11):1226–1235.
- [Tong and Liu, 2021] Tong, Y. and Liu, J. (2021). Review of research and development of supernumerary robotic limbs. *IEEE/CAA Journal of Automatica Sinica*, 8(5):929–952.
- [Trafton et al., 2013] Trafton, J. G., Hiatt, L. M., Harrison, A. M., Tamborello, F. P., Khemlani, S. S., and Schultz, A. C. (2013). Act-r/e: An embodied cognitive architecture for human-robot interaction. *Journal of Human-Robot Interaction*, 2(1):30–55.
- [Tuyls et al., 2002] Tuyls, K., Lenaerts, T., Verbeeck, K., Maes, S., and Manderick, B. (2002). Towards a relation between learning agents and evolutionary dynamics. In *Proceedings of BNAIC*, pages 21–22.

- [Tyler et al., 2003] Tyler, M., Danilov, Y., and Bach-y Rita, P. (2003). Closing an open-loop control system: vestibular substitution through the tongue. *Journal of integrative neuroscience*, 2(02):159–164.
- [Umiltà and Liotti, 1987] Umiltà, C. and Liotti, M. (1987). Egocentric and relative spatial codes in sr compatibility. *Psychological research*, 49(2):81–90.
- [Umiltà and Nicoletti, 2016] Umiltà, C. and Nicoletti, R. (2016). Attention and coding effects in s–r compatibility due to irrelevant spatial cues. In *Attention and performance XI*, pages 457–471. Routledge.
- [Umiltà et al., 1999] Umiltà, C., Rubichi, S., and Nicoletti, R. (1999). Facilitation and interference components in the simon effect. *Archives italiennes de Biologie*, 137(2):139–149.
- [Valiant, 1984] Valiant, L. G. (1984). A theory of the learnable. *Communications of the ACM*, 27(11):1134–1142.
- [Valle-Inclán, 1996a] Valle-Inclán, F. (1996a). The locus of interference in the simon effect: an erp study. *Biological Psychology*, 43(2):147–162.
- [Valle-Inclán, 1996b] Valle-Inclán, F. (1996b). The simon effect and its reversal studied with event-related potentials. *International Journal of Psychophysiology*, 23(1-2):41–53.
- [Valle-Inclán and Redondo, 1998] Valle-Inclán, F. and Redondo, M. (1998). On the automaticity of ipsilateral response activation in the simon effect. *Psychophysiology*, 35(4):366–371.
- [Vallesi et al., 2005] Vallesi, A., Mapelli, D., Schiff, S., Amodio, P., and Umiltà, C. (2005). Horizontal and vertical simon effect: different underlying mechanisms? *Cognition*, 96(1):B33–B43.
- [van Dam et al., 2014] van Dam, L. C., Parise, C. V., and Ernst, M. O. (2014). 10 modeling multisensory integration. *Sensory integration and the unity of consciousness*, page 209.
- [van Erp et al., 2003] van Erp, J. B., Veltman, A., and van Veen, H. A. (2003). A tactile cockpit instrument to support altitude control. In *Proceedings of the human factors and ergonomics society annual meeting*, volume 47, pages 114–118. Sage Publications Sage CA: Los Angeles, CA.
- [van Zoelen et al., 2021] van Zoelen, E. M., van den Bosch, K., Rauterberg, M., Barakova, E., and Neerincx, M. (2021). Identifying interaction patterns of tangible co-adaptations in human-robot team behaviors. *Frontiers in Psychology*, 12:645545.
- [VanRullen, 2003] VanRullen, R. (2003). Visual saliency and spike timing in the ventral visual pathway. *Journal of Physiology-Paris*, 97(2-3):365–377.
- [Velichkovsky et al., 1997] Velichkovsky, B., Sprenger, A., and Unema, P. (1997). Towards gaze-mediated interaction: Collecting solutions of the “midas touch problem”. In *Human-Computer Interaction INTERACT’97: IFIP TC13 International Conference on Human-Computer Interaction, 14th–18th July 1997, Sydney, Australia*, pages 509–516. Springer.
- [Venkatesh and Davis, 1996] Venkatesh, V. and Davis, F. D. (1996). A model of the antecedents of perceived ease of use: Development and test. *Decision sciences*, 27(3):451–481.

- [Verbruggen et al., 2006] Verbruggen, F., Notebaert, W., Liefoghe, B., and Vandierendonck, A. (2006). Stimulus-and response-conflict-induced cognitive control in the flanker task. *Psychonomic Bulletin & Review*, 13:328–333.
- [Véronneau et al., 2020] Véronneau, C., Denis, J., Lebel, L.-P., Denninger, M., Blanchard, V., Girard, A., and Plante, J.-S. (2020). Multifunctional remotely actuated 3-dof supernumerary robotic arm based on magnetorheological clutches and hydrostatic transmission lines. *IEEE Robotics and Automation Letters*, 5(2):2546–2553.
- [Vezzani et al., 2023] Vezzani, L., Strada, F., Praticò, F. G., and Bottino, A. (2023). The walkingseat: A leaning interface for locomotion in virtual environments. In *International Conference on Virtual Reality and Mixed Reality*, pages 21–39. Springer.
- [Vidaurre et al., 2011] Vidaurre, C., Sannelli, C., Müller, K.-R., and Blankertz, B. (2011). Machine-learning-based coadaptive calibration for brain-computer interfaces. *Neural computation*, 23(3):791–816.
- [Vu and Proctor, 2003] Vu, K.-P. L. and Proctor, R. W. (2003). Naïve and experienced judgments of stimulus—response compatibility: implications for interface design. *Ergonomics*, 46(1-3):169–187.
- [Wagner, 2019] Wagner, B. (2019). Liable, but not in control? ensuring meaningful human agency in automated decision-making systems. *Policy & Internet*, 11(1):104–122.
- [Wallace, 1971] Wallace, R. J. (1971). Sr compatibility and the idea of a response code. *Journal of experimental psychology*, 88(3):354.
- [Wang et al., 2016] Wang, H., Liu, N., Zou, G., Li, H., Zeng, H., Chen, J., and Chen, Q. (2016). The simon effect based on the egocentric and allocentric reference frame. *Attention, Perception, & Psychophysics*, 78:427–436.
- [Wang et al., 2019] Wang, L., Chang, W., Krebs, R. M., Boehler, C. N., Theeuwes, J., and Zhou, X. (2019). Neural dynamics of reward-induced response activation and inhibition. *Cerebral Cortex*, 29(9):3961–3976.
- [Wang et al., 2021] Wang, X., Du, S., Zhang, K., and Du, F. (2021). An auditory hand-proximity effect: The auditory simon effect is enhanced near the hands. *Psychonomic Bulletin & Review*, 28:853–861.
- [Warabi, 1977] Warabi, T. (1977). The reaction time of eye-head coordination in man. *Neuroscience letters*, 6(1):47–51.
- [Warren et al., 2001] Warren, W. H., Kay, B. A., Zosh, W. D., Duchon, A. P., and Sahuc, S. (2001). Optic flow is used to control human walking. *Nature neuroscience*, 4(2):213–216.
- [Wascher et al., 2001] Wascher, E., Schatz, U., Kuder, T., et al. (2001). Validity and boundary conditions of automatic response activation in the simon task. *Journal of Experimental Psychology: Human Perception and Performance*, 27(3):731.
- [Weber, 1996] Weber, E. H. (1996). *EH Weber on the tactile senses*. Psychology Press.
- [Weibull, 1997] Weibull, J. W. (1997). *Evolutionary game theory*. MIT press.
- [Weller et al., 2014] Weller, A., Tang, K., Jebara, T., and Sontag, D. A. (2014). Understanding the bethe approximation: When and how can it go wrong? In *UAI*, pages 868–877.

- [Weser and Proffitt, 2021] Weser, V. U. and Proffitt, D. R. (2021). Expertise in tool use promotes tool embodiment. *Topics in cognitive science*, 13(4):597–609.
- [White III and White, 1989] White III, C. C. and White, D. J. (1989). Markov decision processes. *European Journal of Operational Research*, 39(1):1–16.
- [Wilson, 2002] Wilson, M. (2002). Six views of embodied cognition. *Psychonomic bulletin & review*, 9:625–636.
- [Winn et al., 2005] Winn, J., Bishop, C. M., and Jaakkola, T. (2005). Variational message passing. *Journal of Machine Learning Research*, 6(4).
- [Wolpert, 2003] Wolpert, L. (2003). Causal belief and the origins of technology. *Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, 361(1809):1709–1719.
- [Woodward, 2011] Woodward, J. (2011). A philosopher looks at tool use and causal understanding. *Tool use and causal cognition*, page 18.
- [Worringham and Kerr, 2000] Worringham, C. J. and Kerr, G. K. (2000). Proprioception and stimulus-response compatibility. *The Quarterly Journal of Experimental Psychology: Section A*, 53(1):69–83.
- [Wright and Wanley, 2003] Wright, B. C. and Wanley, A. (2003). Adults’ versus children’s performance on the stroop task: Interference and facilitation. *British Journal of Psychology*, 94(4):475–485.
- [Wu and Asada, 2014a] Wu, F. and Asada, H. (2014a). Supernumerary robotic fingers: an alternative upper-limb prosthesis. In *Dynamic Systems and Control Conference*, volume 46193, page V002T16A009. American Society of Mechanical Engineers.
- [Wu and Asada, 2014b] Wu, F. Y. and Asada, H. (2014b). Bio-artificial synergies for grasp posture control of supernumerary robotic fingers.
- [Wu and Asada, 2016] Wu, F. Y. and Asada, H. H. (2016). Implicit and intuitive grasp posture control for wearable robotic fingers: a data-driven method using partial least squares. *IEEE Transactions on Robotics*, 32(1):176–186.
- [Wühr and Ansorge, 2005] Wühr, P. and Ansorge, U. (2005). Exploring trial-by-trial modulations of the simon effect. *The Quarterly Journal of Experimental Psychology Section A*, 58(4):705–731.
- [Yang et al., 2021] Yang, B., Huang, J., Chen, X., Xiong, C., and Hasegawa, Y. (2021). Supernumerary robotic limbs: A review and future outlook. *IEEE Transactions on Medical Robotics and Bionics*, 3(3):623–639.
- [Yeung et al., 2022] Yeung, D., Guerra, I. M., Barner-Rasmussen, I., Siponen, E., Farina, D., and Vujaklija, I. (2022). Co-adaptive control of bionic limbs via unsupervised adaptation of muscle synergies. *IEEE Transactions on Biomedical Engineering*, 69(8):2581–2592.
- [Yu and Möeller, 2014] Yu, J. and Möeller, K. (2014). Investigating multimodal displays: reaction times to visual and tactile modality stimuli. In *The 15th International Conference on Biomedical Engineering: ICBME 2013, 4th to 7th December 2013, Singapore*, pages 480–483. Springer.

- [Zachay, 1991] Zachay, A. (1991). Diskrete und kontinuierliche informationsverarbeitungsmodelle zur erklärung von reiz-reaktions-inkompatibilitäten: Evidenz für einen antwortkonflikt beim simon-effekt. *Unpublished master's thesis, Tübingen, Germany*.
- [Zawidzki, 2013] Zawidzki, T. W. (2013). *Mindshaping: A new framework for understanding human social cognition*. MIT Press.
- [Zawidzki, 2016] Zawidzki, T. W. (2016). Mindshaping and self-interpretation. In *The Routledge Handbook of Philosophy of the Social Mind*, pages 495–513. Routledge.
- [Zbinden et al., 2022] Zbinden, J., Lendaro, E., and Ortiz-Catalan, M. (2022). Prosthetic embodiment: systematic review on definitions, measures, and experimental paradigms. *Journal of NeuroEngineering and Rehabilitation*, 19(1):37.
- [Zhang et al., 2020] Zhang, C., Sahoo, D. R., Pearson, J., Robinson, S., Holton, M. D., Hopkins, P., and Jones, M. (2020). Active pinscreen: Exploring spatio-temporal tactile feedback for multi-finger interaction. In *22nd International Conference on Human-Computer Interaction with Mobile Devices and Services*, pages 1–11.
- [Zhang et al., 2015] Zhang, J., Cheah, C. C., and Collins, S. H. (2015). Experimental comparison of torque control methods on an ankle exoskeleton during human walking. In *2015 IEEE international conference on robotics and automation (ICRA)*, pages 5584–5589. IEEE.
- [Zhang et al., 2022] Zhang, Q., Li, C., Jing, H., Li, H., Li, X., Ju, H., Tang, Y., Zhao, J., and Zhu, Y. (2022). Rtsras: A series-parallel-reconfigurable tendon-driven supernumerary robotic arms. *IEEE Robotics and Automation Letters*, 7(3):7407–7414.
- [Zhang and Yang, 2024] Zhang, Y. and Yang, Y. (2024). Bayesian model selection via mean-field variational approximation. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, page qkad164.
- [Zhao et al., 2019] Zhao, K., Rayar, F., Serrano, M., Oriola, B., and Jouffrais, C. (2019). Vibrotactile cues for conveying directional information during blind exploration of digital graphics. In *Proceedings of the 31st Conference on l'Interaction Homme-Machine*, pages 1–12.
- [Zhong et al., 2018] Zhong, Q., Xiong, A., Vu, K.-P. L., and Proctor, R. W. (2018). Vertically arrayed stimuli and responses: transfer of incompatible spatial mapping to simon task occurs regardless of response-device orientation. *Experimental Brain Research*, 236:175–185.
- [Zielasko et al., 2020] Zielasko, D., Law, Y. C., and Weyers, B. (2020). Take a look around—the impact of decoupling gaze and travel-direction in seated and ground-based virtual reality utilizing torso-directed steering. In *2020 IEEE Conference on Virtual Reality and 3D User Interfaces (VR)*, pages 398–406. IEEE.
- [Zielasko and Riecke, 2021] Zielasko, D. and Riecke, B. E. (2021). To sit or not to sit in vr: Analyzing influences and (dis) advantages of posture and embodied interaction. *Computers*, 10(6):73.
- [Zorzi and Umiltá, 1995] Zorzi, M. and Umiltá, C. (1995). A computational model of the simon effect. *Psychological Research*, 58(3):193–205.