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Design of future-generation public transport to reduce inequality of accessibility

Duo Wang

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Thèse de doctorat

TELECOM
SudParis



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Design of Future-generation Public Transport to Reduce Inequality of Accessibility

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Declaration

I hereby declare that this thesis represents my own work which has been done after registration for the degree of PhD at Institut Polytechnique de Paris, and has not been previously included in a thesis or dissertation submitted to this or any other institution for a degree, diploma or other qualifications.

I have read the University's current research ethics guidelines, and accept responsibility for the conduct of the procedures in accordance with the University's Committee on the Use of Human and Animal Subjects in Teaching and Research (HASC). I have attempted to identify all the risks related to this research that may arise in conducting this research, obtained the relevant ethical and/or safety approval (where applicable), and acknowledged my obligations and the rights of the participants.

Duo WANG

Abstract

The accessibility of a location measures the ease for people living there to reach surrounding opportunities, such as schools, shops, or jobs. This thesis focuses on the accessibility provided by Public Transport (PT). Conventional PT, based on fixed lines and schedules, results in an unequal distribution of accessibility: it is generally good in city centers but poor in the suburbs, which are thus car-dependent. This makes cities environmentally, economically, and socially unsustainable. Flexible mobility services (e.g., Demand-Responsive Transport (DRT)) are potentially better suited for low-density areas and can effectively complement conventional PT.

However, most methods to optimize the design and operation of flexible mobility target traditional objectives, e.g., user and operational costs, neglecting accessibility.

The **overarching aim** of this thesis is to develop novel methods for designing future-generation PT systems, including conventional and flexible PT, with the objective of reducing the inequality of accessibility distribution.

To achieve this objective, this thesis is structured around four major and distinct scientific contributions. The first and second contributions involve developing and proposing advanced strategic planning methods for public transport (PT) networks, specifically designed to minimize statistical indices measuring accessibility inequality. To this end, we propose original algorithmic solutions based on sophisticated heuristics and recent reinforcement learning techniques applied to graph structures, enabling efficient exploration of the solution space.

The third contribution marks a significant advancement by developing, to the best of our knowledge, the very first planning method dedicated specifically to the optimal allocation of a Demand-Responsive Transport (DRT) fleet, where the central objective function is the reduction of accessibility inequality indices rather than cost alone. Within this innovative contribution, we combine an accurate graphical representation of the existing conventional PT network with a mathematical model of "Continuous Approximation" for the DRT service, allowing us to model the fluidity of on-demand services. We then propose a complex bi-level optimization method: the upper level, which decides on resource allocation, is solved via an ingenious hybrid approach combining a heuristic based on ranking priority zones with Bayesian optimization to dynamically allocate the DRT fleet; simultaneously, the lower level performs the precise assignment of user trips to available vehicles.

Finally, the fourth contribution consists of designing the architecture of a novel flexible public transport system, strategically positioned as a hybrid situated between the rigidity of conventional

PT and the total flexibility of on-demand mobility. This system aims to combine the operational efficiency stemming from the strong and predictable structure of the former with the high adaptability and responsiveness of the latter. The real-time operational management of such a complex system is ensured by optimization using a receding horizon (Model Predictive Control), based on advanced Monte Carlo Tree Search algorithms, enabling robust decision-making in the face of demand uncertainty.

In summary, this thesis offers a transformative perspective on the concrete operationalization of accessibility and social equity concepts in the engineering design of public transport systems. While these two crucial aspects have been predominantly analyzed in academic literature in an "ex-post" manner (i.e., by evaluating performance after network implementation), the methods proposed here allow them to be internalized "ex-ante." This means that equity and accessibility become direct and constraining variables within the optimization objective functions themselves from the design phase, thereby ensuring that future transport networks are intrinsically fair and inclusive even before their physical deployment.

Résumé

L'accessibilité d'un lieu mesure la facilité, pour les personnes qui y résident, d'atteindre les opportunités environnantes, telles que les écoles, les commerces ou les emplois. Cette thèse s'intéresse à l'accessibilité offerte par les Transports Publics (TP). Les TP conventionnels, basés sur des lignes et des horaires fixes, entraînent une répartition inégale de l'accessibilité : elle est généralement bonne dans les centres-villes mais faible en périphérie, qui devient ainsi dépendante de la voiture. Cela rend les villes insoutenables sur les plans environnemental, économique et social.

Les services de mobilité flexible (par ex., le Transport à la Demande – TAD) semblent mieux adaptés aux zones de faible densité et peuvent efficacement compléter les TP conventionnels. Toutefois, la plupart des méthodes visant à optimiser la conception et l'exploitation de la mobilité flexible ciblent des objectifs traditionnels, tels que les coûts pour les usagers et les coûts opérationnels, en négligeant l'accessibilité.

L'objectif général de cette thèse est de développer de nouvelles méthodes pour concevoir les systèmes de TP de prochaine génération, incluant TP conventionnels et flexibles, ayant pour finalité de réduire les inégalités dans la distribution de l'accessibilité.

Pour atteindre cet objectif, cette thèse s'articule autour de quatre contributions scientifiques majeures et distinctes. Les première et deuxième contributions consistent à élaborer et à proposer des méthodes avancées de planification stratégique des réseaux de TP, spécifiquement conçues pour minimiser les indices statistiques mesurant l'inégalité d'accessibilité. Pour ce faire, nous proposons des solutions algorithmiques originales qui s'appuient sur des heuristiques sophistiquées ainsi que sur des techniques récentes d'apprentissage par renforcement appliqué aux structures de graphes, permettant une exploration efficace de l'espace des solutions possibles.

La troisième contribution marque une avancée significative en développant, à notre connaissance, la toute première méthode de planification dédiée spécifiquement à l'allocation optimale d'une flotte de véhicules de TAD, où la fonction objectif centrale est la réduction des indices d'inégalité d'accessibilité plutôt que le seul coût. Dans le cadre de cette contribution novatrice, nous combinons une représentation graphique précise du réseau de TP conventionnel existant avec un modèle mathématique d'« Approximation Continue » du service de TAD, permettant de modéliser la fluidité du service à la demande. Nous proposons ensuite une méthode d'optimisation bi-niveaux complexe : le niveau supérieur, qui décide de l'allocation des ressources, est résolu via une approche hybride ingénieuse combinant une heuristique basée sur le classement des zones prioritaires et l'optimisation bayésienne pour allouer dynamiquement la flotte de TAD ;

simultanément, le niveau inférieur effectue l'affectation précise des déplacements des usagers aux véhicules disponibles.

Enfin, la quatrième contribution consiste à concevoir l'architecture d'un nouveau système de transport public flexible inédit, positionné stratégiquement comme un hybride situé entre la rigidité du TP conventionnel et la totale flexibilité de la mobilité à la demande. Ce système vise à combiner l'efficacité opérationnelle découlant de la structure forte et prévisible du premier, avec la grande adaptabilité et la réactivité du second. La gestion opérationnelle en temps réel d'un tel système complexe est assurée par une optimisation en horizon glissant (Model Predictive Control), fondée sur des algorithmes avancés de recherche arborescente de Monte-Carlo (Monte Carlo Tree Search), permettant de prendre des décisions robustes face à l'incertitude de la demande.

En synthèse, cette thèse offre une perspective transformative sur l'opérationnalisation concrète des concepts d'accessibilité et d'égalité sociale dans la conception ingénierie des systèmes de transports publics. Alors que ces deux aspects cruciaux ont été majoritairement analysés dans la littérature académique de manière « ex-post » (c'est-à-dire en évaluant les performances après la mise en place des réseaux), les méthodes proposées ici permettent de les internaliser « ex-ante ». Cela signifie que l'équité et l'accessibilité deviennent des variables directes et contraignantes au sein même des fonctions objectifs d'optimisation dès la phase de conception, garantissant ainsi que les futurs réseaux de transport soient intrinsèquement justes et inclusifs avant même leur déploiement physique.

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List of Acronyms

AMoD Autonomous Mobility-on-Demand

CA Continuous Approximation

CDF Cumulative Distribution Function

DRT Demand-Responsive Transit

DVRP Dynamic Vehicle Routing Problem

GA Genetic Algorithm

GTFS General Transit Feed Specification

LTS Level of Traffic Stress

MCTS Monte Carlo Tree Search

MDP Markov Decision Process

ML Machine Learning

MPNN Message Passing Neural Network

NYC New York City

OD2N Online Design of Dynamic Networks

PAB Population Accessibility-based Bayesian Optimization

PT Public Transport

PTNDP Public Transport Network Design Problem

RL Reinforcement Learning

RSB Rank Score Bayesian

RSG Rank Score Greedy

RSGB Rank Score Greedy Bayesian

SAV Shared Autonomous Vehicle

SOTA State-of-the-Art

SVM Support Vector Machine

TEG Time-Expanded Graph

TLC Taxi and Limousine Commission

TSP Traveling Salesman Problem

VRP Vehicle Routing Problem

Chapter 1

Introduction

1.1 Open Challenges of Public Transport

A major challenge to urban sustainability is the inability of Public Transport (PT) to meet the mobility needs of all citizens. According to United Nations estimates, only half of the urban population has convenient access to PT [1]. The effectiveness of PT can be measured in terms of accessibility provided. PT accessibility of a certain location indicates how easy it is, in terms of travel time and/or cost, to reach the surrounding opportunities, e.g., jobs, schools, hospitals, etc. Accessibility is unequally distributed in urban regions: it is generally good in the city centers but poor in the suburbs [2]. This inequality is structurally unavoidable. Indeed, conventional PT operates on fixed routes and pre-scheduled timetables. Their coverage and frequency needs are constrained to be relatively low in sparse demand areas to prevent operator costs from exploding [3].

A modern and sustainable approach to Transport planning should not focus solely on efficiency—measured, for example, by the number of passengers served per unit time—but also address equality, ensuring that residents in peripheral and suburban areas have reasonable access to public transport and urban amenities [4]. However, current Transport systems exhibit significant spatial inequalities, which discourage suburban residents from using public Transport and instead lead to a heavy dependence on cars for daily mobility needs.

The conceptual scheme of Figure 1.1 well represents the thread underlying my research work. While egalitarian theories focus on differences among individuals, sufficientarianism in the field of transportation assumes that all individuals should attain at least a certain minimum threshold of accessibility that is considered ‘sufficient’ to satisfy their basic needs and to secure their continued well-being [5]. From a sufficientarian perspective, I defined the Minimum Accessibility Level as the threshold of accessibility required to ensure that residents can meet their basic daily activity needs, including commuting to work, accessing education, obtaining healthcare, and purchasing essential goods. Fixing a value for such minimum accessibility is an ethical and political matter [6]. For simplicity, I represent it with the horizontal line of Figure 1.1. There is a gap between the accessibility provided by conventional PT and the Minimum Accessibility Level. This gap in-

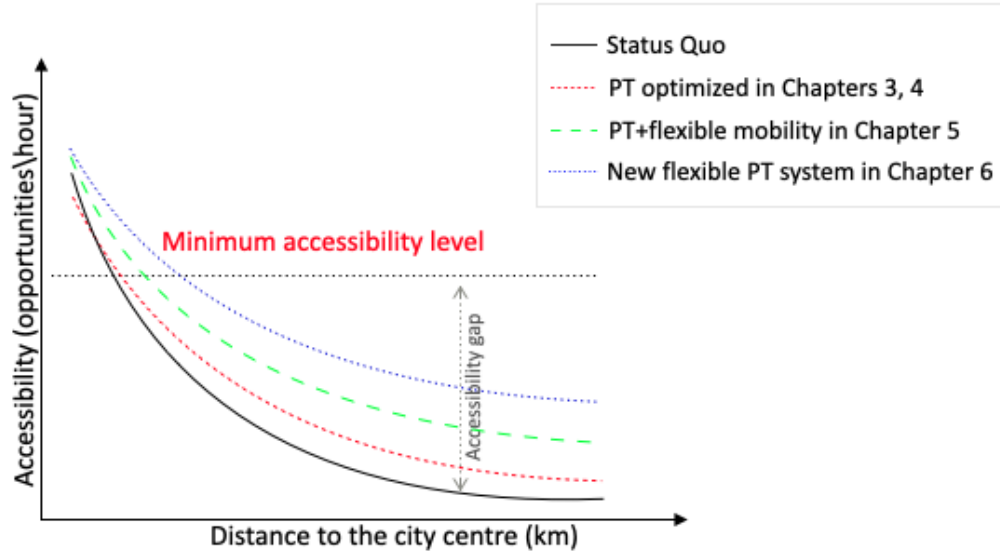


Figure 1.1: Impact of different transport optimization methods on the inequality of accessibility.

creases as the distance from the city center grows. For residents in the urban core, the gap can be addressed by walking or cycling; however, for suburban residents, such a substantial gap can only be filled by the use of private vehicles.¹

Car-dependency in suburbs is a serious barrier to sustainable urban development. Studies indicate that private vehicles are a major contributor to urban pollution [8].

1.2 Flexible Mobility: Advantage and Issues

Flexible services operate a fleet of vehicles that change their routes on-the-fly based on user requests. Flexible mobility, in fact, represents a dynamic paradigm that contrasts with traditional static public transport (PT). It diverges from fixed-route and fixed-schedule transit, which typically operates based on pre-established timetables and service patterns, offering limited adaptability to spatial and temporal variations in demand. Therefore, public transport (PT) exhibits a high degree of structural stability, whereas flexible mobility services are better able to adapt to demand.

Examples of flexible services are Demand Response Transport (DRT) (Chapter 5) or shared taxis [9]. In low demand areas, where it is not cost-feasible to deploy high-frequency, high-coverage traditional public transportation (PT), demand-responsive transportation (DRT) can com-

¹Regarding the shape of the curves in Figure 1.1, it is well known that gravity-based accessibility decreases with increasing distance from the city center. The decay pattern we adopt is similar to that in Figure [7]. The particular shape (convex, concave, irregular) might depend on the city under study and is not relevant in Figure 1.1, which is a conceptual representation.

plement PT in the first and last mile by operating as a feeder to conveniently bring passengers from/to main PT stations, reducing walking times [3, 10].

Flexible mobility presents some unexploited opportunities and open issues. Recent work [2, 11–13] quantified the impact of DRT on improving accessibility from a *descriptive* perspective. However, accessibility inequality could better addressed through a *prescriptive* approach, i.e., PT and DRT could be planned with the objective to reduce accessibility inequality, directly. For the best of my knowledge, such approaches do not exist yet. Chapters 3, 4 and 5 aim to fill this gap.

An open issue of flexible mobility is its inefficiency. Evidence from real implementations indicates that DRT services have often low vehicle occupancy, typically in the range of 1.24 - 1.38 passengers per trip [14]. This phenomenon is mainly due to the lack of *structure* in DRT routes. Such routes are indeed continuously modified to *insert* incoming requests, preventing any possible synchronization between vehicles. As a result, passenger trips are completed within a single vehicle, unlike conventional PT where transfers between multiple vehicles are possible. This operational constraint reduces the average vehicle occupancy and increases cost. Chapter 6 aims to address such inefficiency.

1.3 Goals of the PhD

The overarching aim of my PhD is to build a novel methodology to design future generation Public Transport, whose originality is that fixed and flexible services are planned together, with the objective of minimizing inequality of PT accessibility distribution. By reducing inequality, the resulting PT has the potential to reduce car-dependency and its negative consequences on environmental, economic and social sustainability of cities.

More specifically, the methods I present offer policy makers a novel planning framework that encompasses both of conventional public transport (PT) optimization decision, such as stop selection (Chapter 3) and line planning (Chapter 4), and the integration of flexible mobility to PT (Chapter 5). Our investigation into flexible mobility covers strategies for deploying demand-responsive transit (DRT) to support the first-mile and last-mile of users' trips, with particular attention to service area selection and vehicle allocation.

Moreover, my research is not limited to existing transport modes. As noted in Section 1.2, PT has a strong structure but lacks flexibility, whereas DRT exhibits the opposite characteristics. Simply combining PT and DRT into a hybrid system does not fundamentally resolve this issue. The structural limitation of PT arises from its offline design: once the network is planned, it remains fixed for an extended period and cannot be easily modified. In contrast, DRT routes are designed and modified based on each incoming request. Each new request can significantly affect existing DRT routes. Moreover, the common limitation that each request is served by a single DRT vehicle, rather than by multiple vehicles through transfers, determines inefficiency. To tackle this issue, I propose in Chapter 6 a novel transport mode that preserves the structural stability of PT while incorporating the flexibility of DRT.

1.4 Contributions

To achieve the goals of Section 1.3, my thesis provides four contributions. The first three contributions are covered in Chapters 3 – 5, in which I progressively move from conventional PT planning optimization to integrated multimodal planning. In addition, in Chapter 6, this thesis also investigates a novel transport mode, of which I demonstrate its superior efficiency compared to both conventional PT and existing flexible mobility. This suggests also a potential to reduce accessibility inequality, when applied to peripheral areas.

More specifically, the contributions are

1. **Stop Selection for Equality Improvement (Chapter 3)** Addressing the structural accessibility gap between central and peripheral areas, this stage optimizes stop selection within existing PT networks to improve accessibility equality while controlling efficiency loss. The PT network is modelled as a graph, where nodes are stops and centroids, and edges are travel or transfer links. Accessibility is computed using a gravity-based measure, and inequality is assessed via the Atkinson index. A heuristic algorithm iteratively deactivates stops and consolidates routes, with the objective of maximising the accessibility of the worst-performing $m\%$ of centroids (bottom quantile in terms of accessibility). This strategy reduces excessive service concentration in already well-served urban cores and reallocates benefits toward underserved suburbs.
2. **Reinforcement Learning–based Network Design (Chapter 4)** To overcome the limitations of static heuristic design, the second stage formulates PT network design as a sequential decision-making problem. Graph Neural Networks (Message Passing Neural Networks) embed the PT topology, while a reinforcement learning (RL) agent explores stop and route modifications. The reward function is tailored to accessibility improvements in low-accessibility areas, enabling the model to autonomously learn structural patterns that enhance equality, with better scalability and adaptability than classic meta-heuristic approaches.
3. **Integrated PT and DRT Planning (Chapter 5)** Acknowledging that conventional PT cannot cost-effectively serve low-density areas, this stage integrates DRT to serve first- and last-mile of requests. A hybrid representation combines a multi-layer PT graph for fixed services with a Continuous Approximation (CA) model for DRT operations. The Rank Score Greedy (RSG) algorithm identifies DRT service zones and allocates fleet resources according to a composite score balancing population density and existing accessibility. A bilevel optimization approach is employed: the upper level heuristically allocates DRT vehicles, while the lower level conducts transit assignment, achieving measurable reductions in accessibility inequality.
4. **Online Dynamic Network Design (Chapter 6)** While the integration of PT and DRT addresses spatial inequality in Chapter 5, this hybrid model does not fundamentally resolve the problem of the lack of flexibility in PT and the lack of structure in DRT as noted in Section 1.2. This chapter proposes a method that we call *Online Design of Dynamic Networks*

(OD2N), to introduce adaptivity into PT operations, while keep a strong network structure. The system is represented as a Time-Expanded Graph (TEG) that evolves in real time. Monte Carlo Tree Search (MCTS) is used to determine network modifications proactively, based on predicted demand, enabling routes to adapt dynamically. At the same time, via maintaining a structured network, transfers from a vehicle to another are allowed, improving demand consolidation and efficiency. This approach combines the strong structure of conventional PT with the flexibility of DRT.

The impacts of different methods on the distribution of urban accessibility are shown in Figure 1.1. The optimized PT approach, which includes stop selection (Chapter 3) and route design strategies (Chapter 4) to reduce the inequality of accessibility, results in a slight decrease in accessibility in the urban core but yields a more substantial improvement in suburban accessibility. In the city centre, the remaining gap can still be bridged through walking or cycling, whereas in suburban areas, the gap is significantly reduced. The PT + flexible mobility in Chapter 5 integrates DRT with conventional PT by identifying areas to be designated as DRT zones and allocating DRT buses for the first-mile and last-mile needs. This method markedly improves suburban accessibility, while also providing a slight enhancement in the urban core, as some trips from the city center to the suburbs can also benefit from the convenience of DRT. Furthermore, as demonstrated in Chapter 6, our new flexible PT system exhibits higher capacity than flexible mobility. It is therefore reasonable to infer that its corresponding curve, as shown in Figure 1.1, would overall lie above that of the curve of DRT + flexible mobility (although further research would be needed to validate this intuition).

1.5 Methodology

I summarize the methods that I used in my research into three categories.

- **Modelling.** In the contributions about planning the following models are used: (i) a graph-based model for conventional PT networks incorporating accessibility and inequality metrics (Section 3.2, 4.2), (ii) a hybrid multimodal framework combining a graph representing the conventional PT topology and a Continuous Approximation (CA) model for demand-responsive transport (DRT) (Section 5.2). The contribution of Section 6.3.6 deals with operational aspects. For this reason, I adopt (iii) a time-expanded graph (TEG) representation, which enables real-time adaptive network design (Section 6.2). These models capture the interdependence between transport infrastructure, population distribution, and opportunity accessibility.
- **Optimization.** Building on the above models, the thesis develops optimization strategies tailored to accessibility equality. These include a heuristic stop selection approach for service redistribution (Chapter 3), a reinforcement learning framework with graph neural network embeddings for scalable network redesign (Chapter 4) and a bilevel hybrid PT–DRT planning algorithm balancing population density and accessibility via Bayesian and Heuristic optimization (Chapter 5). Moreover, an Online Design of Dynamic Networks (OD2N)

method that combines Monte Carlo Tree Search (MCTS) with predicted demand is proposed in Chapter 6.

- **Evaluation.** The proposed models and methods are validated through extensive experiments on synthetic (Chapter 3) and real-world (Chapter 4, 5) datasets describing the distribution of population, opportunities and current PT infrastructure. In Chapter 6 I perform Monte Carlo simulation on a realistic trip request dataset. The metrics considered in our evaluation account for operational cost, user-related cost and, more importantly, accessibility. I show in the following chapters that each proposed method outperforms its corresponding state-of-the-art benchmark. Overall, this thesis provides a solid theoretical foundation and encouraging empirical evidence for optimizing the inequality of accessibility.

For what concerns accessibility, I use a gravity-based measure. This choice is discussed in Section 2.1. I adopt the same formula in Chapter 3 and Chapter 5 (formulas (3.1) and (5.13), respectively). In Chapter 4, instead, I use a minimally modified version of that formula, for computational reasons (instead of accounting for all opportunities, I only restrict the computation to those within a threshold).

1.6 Thesis Organization

The remainder of this thesis is organized as follows:

Chapter 2 – *State of the Art and Conceptual Foundations*. This chapter reviews existing studies on public transport (PT) accessibility, equality metrics, and network design, as well as the integration of flexible mobility solutions such as Demand-Responsive Transit (DRT). It discusses key theoretical concepts, measurement methodologies, and optimization approaches, highlighting research gaps that motivate the proposed contributions.

Chapter 3 – *Stop Selection for Accessibility Equality*. This chapter formulates the problem of stop selection in an existing PT network with the objective of improving the equality of accessibility distribution. It introduces the accessibility and inequality measures adopted throughout the thesis, develops a heuristic optimization framework, and evaluates the approach on synthetic case studies.

Chapter 4 – *Reinforcement Learning-based Network Design*. Extending beyond stop selection, this chapter presents a network design framework that combines Graph Neural Networks with reinforcement learning to redesign line planning. The proposed approach is applied to realistic network scenarios and compared against meta-heuristics.

Chapter 5 – *Integrated PT and DRT Planning*. This chapter develops a joint planning model integrating conventional PT with DRT to address first- and last-mile accessibility gaps. It introduces a hybrid representation of multimodal networks (graph based plus Continuous Approximation) and a bilevel optimization, where in the upper level a greedy approach or Bayesian Optimization are used to select DRT areas and decide fleet allocation, and in the lower level transit assignment is performed. I assess the effectiveness of our approach in reducing accessibility inequality in three metropolitan-scale areas.

Chapter 6 – *Online Design of Dynamic Networks (OD2N)*. This chapter proposes an online adaptive design method to dynamically adjust PT operations while preserving the network structure. It details the Time-Expanded Graph representation and the Monte Carlo Tree Search-based decision framework and evaluates the method in simulated dynamic demand conditions.

Chapter 7 – *Conclusion*. The final chapter summarizes the main contributions of the thesis and outlines directions for future research, including the potential of a new flexible PT and the integration of mobility.

Note. I wrote each chapter of this thesis to make it self-contained. While all chapters are closely interconnected and complementary, the reader should be able to follow the formulation without needing to continuously refer back to previous chapters. This editorial decision aims to ensure a smooth and uninterrupted reading. I made efforts to minimize redundancy, but only when it did not compromise the intended clarity and flow.

1.7 List of Publications

- Wang, D., Araldo, A., & El Yacoubi, M. (2025). Planning Demand-Responsive Transit to reduce inequality of accessibility. *Transportation Research Part A: Policy and Practice*, 199, 104544.
- Wang, D., Chau, M., & Araldo, A.(2025). Public Transport Network Design for Equality of Accessibility via Message Passing Neural Networks and Reinforcement Learning. 17th International Conference on Agents and Artificial Intelligence (ICAART).
- Wang, D., Araldo, A., & El Yacoubi, M. (2024), Public Transport Stop Selection to Reduce the Inequality in the Distribution of Accessibility, the European Association for Research in Transportation (hEART), Helsinki, Finland, June 18–20, 2024.
- Wang, D., Araldo, A., & El Yacoubi, M. (2024). Online design of dynamic networks. Target submission: International Conference on Automated Planning and Scheduling (ICAPS), 2026.

Chapter 2

Context and Positioning within the State of the Art

Public Transport Network Design Problems (PTNDPs) can be studied at two levels: the planning level or the operational level. At the *planning level*,¹ a PT planner decides the design of the different lines as well as their frequencies. In case DRT is integrated into PT, a planner also decides in which areas to deploy DRT and how to allocate the DRT fleet across those areas [10]. At an *operational level*, a PT operator organizes the service to match the decisions made at the planning level, e.g., by deciding on vehicle routing. Chapters 3, 4, and 5 focus on the planning level. A review of planning level PTNDPs is provided in [17]. Chapter 6 focuses on the operating level, particularly in vehicle routing.

Since the overarching aim of my work is to reduce the inequality in the distribution of accessibility, I first present the different definitions of accessibility and equality (Sec. 2.1). The literature on accessibility in relation to planning methods and operational management methods will be reviewed in Sections 2.2 and 2.3, respectively. The limitations of current public transport systems will be discussed in Section 2.4. Section 2.5 provides an overview of the different models and related studies used in *what-to* methods. Finally, I position my work within the state of the art and I briefly discuss its novelty (Section 2.6).

2.1 Accessibility and Equity

2.1.1 Accessibility

Accessibility is a concept that measures the ease for people of accessing opportunities.

The classification of accessibility indicators has been discussed in reviews [18, 19], and they are generally divided into the following categories [19]: (1) Distance-based measure: the distance (or travel time) to the nearest opportunity; (2) Isochrone or cumulative measure: the number of the

¹“” we refer to strategic and tactical level decisions [15], [16, Section 1.5.2].

opportunities that can be reached within a specified maximum distance (or travel time) threshold; (3) Gravity-based measure: the weighted sum of opportunities, where each opportunity is weighted by a distance-decay (or travel-time-decay) function that decreases with increasing travel cost, so that closer and larger opportunities contribute more to accessibility; (4) Random utility-based measure: the expected maximum utility that can be obtained via the available opportunities, typically derived from discrete choice models (e.g., logit models); (5) Diversity-based measure: the sum of different categories of opportunities (e.g., employment, shopping, healthcare, education) that can be reached within a specified maximum travel time threshold.²

This thesis adopts a gravity-based measure of accessibility for the following reasons: First, the distance-based measure (1) is sensitive only to the separation between the origin and the nearest opportunity, without accounting for the relative attractiveness or magnitude of opportunities. In other words, it implicitly assumes that only the closest opportunity matters. However, in most cases, the possibility for an individual to choose among many opportunities (not restricted to the closest one) has an intrinsic value that measure (1) fails to capture. Second, unlike the isochrone measure (2) and the diversity-based measure (5), which rely on a strict cut-off threshold, the gravity-based measure accounts for the gradual decline in accessibility as travel cost increases. This avoids the arbitrariness of threshold selection and provides a more realistic representation of spatial interaction. Finally, the gravity-based measure (3) and the random utility-based measure (4) are, in fact, mathematically equivalent [19, Section 2.5]. The gravity-based measure (3) has also demonstrated remarkable flexibility with respect to the variety of weights and impedance functions that can be incorporated [21, 22]. Moreover, it is simpler to calculate compared to (4), which requires deploying and calibrating discrete choice models, greatly complicating its adoption.

The methodology developed in this thesis is, however, general and can be replicated by replacing the gravity-based measure with any other accessibility measure, if needed.

2.1.2 Horizontal and Vertical Equity

The conceptual standpoint of this thesis integrates the vision of Rawlsian and luck egalitarians, i.e., “individuals should not be disadvantaged due to morally arbitrary circumstances beyond their control, such as being born with a physical disability, into a low-wealth family, into a specific racialized group or gender, or in a particular country” [23] (see also [24, 25]).

Horizontal equity, also named *Equality*, denotes the principle that, ideally, all individuals should enjoy access to opportunities with similar degrees of ease, regardless of their personal circumstances (such as health status, level of wealth, etc.). By contrast, *vertical equity* requires that differences in individuals’ circumstances be taken into account, such that access to opportunities is adjusted according to their different circumstances [26]. In transportation equity evaluations, Lorenz curves and Gini coefficients are commonly employed to operationalize egalitarianism, as they fundamentally measure how equal the distribution of accessibility is among individuals [2, 5, 27, 28]. Other indices are available to quantify inequality, such as the Atkinson index, Theil index, and Pietra index, although they have not been applied to accessibility distributions.

²Measure (5) is not mentioned in [18, 19] but it is discussed in [20].

Another index is the Palma index, which can be easily adapted to measure vertical inequity. Originally defined as the ratio of the income share of the richest 10% to that of the poorest 40%, it can be adapted to accessibility analysis in two distinct ways. If the ratio is computed between the aggregate accessibility of the top 10% and the bottom 40% in terms of accessibility values, the Palma index emphasizes distributional differences in accessibility, thereby expressing a notion of equality. Alternatively, if the ratio is calculated between the accessibility enjoyed by the richest 10% of the population and that of the poorest 40%, the Palma index highlights disparities in accessibility across income groups, thereby reflecting vertical equity. The same analysis could be repeated by ranking the individuals with respect to other socio-economic indicators, beyond income, making the Palma index very flexible for several vertical equality analyzes.

The results presented in this thesis concern the equality (horizontal equity) of the geographical distribution of PT accessibility (rather than vertical equity) for the following reasons: (1) My work focuses on the methodological dimension, and the specific choice of inequality indicator is not relevant from a methodological perspective. For presentation purposes, I therefore employ horizontal indicators. If the proposed method were to be applied by a transport authority, they could simply replace our inequality indicator with whichever measure is most pertinent for the case study at hand, provided that sufficient data are available to differentiate between individuals' conditions. For example, as introduced previously, the Palma index can be used either directly to measure distributional differences in accessibility (horizontal) or, if income data are available and individuals can be classified accordingly, the Palma index can also be applied to evaluate disparities in accessibility across income groups (vertical). (2) Focusing on reducing horizontal inequality indices is more compatible with environmental sustainability concerns. Indeed, to achieve environmental sustainability, the use of cars should be reduced, both for advantaged and disadvantaged individuals. Optimizing PT, as an alternative to private cars, to reduce accessibility inequality (horizontal inequity), will encourage an increasing number of people (among the entire population) to prefer PT than private cars. However, optimizing PT with a focus on vertical equity would only target specific groups, rather than the entire population. For example, if the Palma index were used to measure accessibility disparities between income groups, and PT were optimized to decrease that index, the resulting design would target the improvement of accessibility from PT to the poorest 40% only (which is desirable from a social point of view), without any effort to reduce the car usage of the rest of the population (which is undesirable from an environmental point of view).

2.2 Planning Methods

PT planning methods can be classified into *what-if* and *what-to* methods [16, p 589]. The former estimate change in performance with respect to few design alternatives. The latter, instead, aim to find the optimal design of PT according to some objective function.

2.2.1 What-if Planning Methods

For the sake of thematic conciseness, I focus the review of *what-if* methods to those tackling accessibility and inequality and, then, to those examining the impact of flexible mobility.

The authors of [2] compute accessibility scores of four different cities via open data, using the method previously proposed by [29]. The authors of [11] compute the contribution of each PT line in the overall equality of accessibility. The Level of Traffic Stress (LTS) index [30] is used by [12] to measure whether a bicycle-lane can be arranged on a route and study the changes in accessibility of Hamilton County, Ohio, U.S. before and after bicycle-lane is arranged. The authors of [13] combines datasets of microtransit ridership from two public transit agencies, transit surveys, land use data, and expert interviews in Minneapolis-St. Paul in Minnesota, USA, to conduct spatial analysis, accessibility analysis, and equity impact assessments for suburbs.

What-if methods have been presented to assess the impact of flexible mobility on cost-efficiency [31] and for cost-benefit analysis [32]. I focus here on *what-if* studies that estimate the impact of flexible mobility in the distribution of accessibility. The authors of [33, 34] both implement an activity-based accessibility measure in the SimMobility simulation for analyzing the performance of automated mobility on-demand (AMoD) services. The authors of [33] conclude that introducing AMoD can improve accessibility. The authors of [34] conclude that AMoD could alleviate inequity in accessibility as it appears to benefit the disadvantaged socioeconomic groups. The authors of [35] propose a geostatistical method to compute accessibility from DRT in a data-driven fashion. Applying this method on trips simulated in MATSim, they conclude that DRT has the potential to improve accessibility in areas where conventional PT is insufficient, thus suggesting potential for reduction in inequality. The authors of [36] calculate accessibility from AMoD. They compare in MATSim simulation different scenarios of deployment and confirm the potential to reduce inequity. The authors of [37] propose a logsum-based measure of employment accessibility benefits for AMoD. The results indicate that workers living in lower density areas benefit more than workers living in high density areas from AMoD. The authors of [38] first uses a linear regression analysis to examine how different forms of accessibility correlate with socio-economic indicators (average income, unemployment rate and education level), which could inform policymakers in decisions related to deployment of shared autonomous vehicles (SAVs). The authors of [39] analyze the potential for ride-hailing services to improve employment accessibility when used as a standalone transportation mode and in conjunction with transit as a first-mile connection. They conclude that ride-hailing can improve urban accessibility, but without policies to address affordability, it risks reinforcing socio-economic disparities. The authors of [40] presents an approach for accessibility categorization in areas where there is no extensive data available.

All this previous work indicates the potential of flexible mobility to improve the accessibility distribution, provided that it is appropriately deployed in the right areas. However, due to the descriptive nature of these *what-if* methods, they do not provide a method to plan the deployment of flexible mobility.

2.2.2 What-to Planning Methods

What-to methods optimize PT according to some objective function. Most what-to approaches aim to minimize the generalized cost, such as the total of travel time plus some cost for the PT agency [10, 41–44]. In my work, I adopt the inequality of accessibility as objective, which is more complex than generalized cost. Indeed, reducing inequality of accessibility requires capturing the interrelation between the distribution of opportunities, of population, and the PT graph topology.

We now discuss some work considering accessibility [45–47]. The authors of [45] add a constraint concerning equality in a classic cost minimization problem. The authors of [46] optimize headway and stop spacing of a single PT line, considering equality. The authors of [47] use a geographically weighted regression model to estimate the local relationships between accessibility and previous PT investments, and then find an optimal allocation of 10% growth in PT investments by optimizing inequality of accessibility. The authors of [48] employ a bi-level programming model to optimize the PT network. In the upper level, the model seeks to maximize overall accessibility by strategically adding PT lines. The lower level simulates user behavior, assuming that travelers choose routes that minimize their travel time within the constraints of the PT. The authors of [49] propose an Activity-based space-time accessibility measure, and use a heuristic algorithm to optimize headways and fares of conventional PT.

None of these papers, however, consider the optimization of the accessibility distribution (this thesis tackles the reduction of inequality) in a system integrating flexible mobility.

2.3 Operational Management Methods

Operational management methods address short-term or real-time decision making in transportation systems. Such decisions may happen in normal operations (for instance the routes within flexible mobility) or during disruptions. The latter case is common in real transport system and may arise from both the demand side—such as shifts in passenger travel patterns—and the supply side, including traffic congestion, roadworks, accidents, vehicle breakdowns, or staff shortages. To adapt to unexpected events that might occur during an operation day, operational management becomes essential. Common operational management methods include stop-skipping [50–53], vehicle holding at specific stops [54–58] or rescheduling the dispatching times of trips [59, 60]. Operational management strategies faced to disruptions are not tackled in this manuscript. We thus focus in the following on operational management methods for normal operations of flexible mobility systems.

Recent literature has studied the integration of DRT and conventional PT at an operational level. For instance, the authors of [61] propose a framework to update DRT shuttle routes for the new incoming request by minimizing marginal costs, where marginal costs are the difference in total shuttle user costs before and after the new request is added to the shuttle queue. The authors of [41] formulate the problem of integration of DRT as a linear program to minimize total passenger travel time using the link transmission model [62], and find a suboptimal solution using a rolling horizon method. The authors of [63] optimize jointly conventional PT line planning and DRT fleet deployment. The authors of [64] establish fixed lines in areas where flexible mobility paths are the

most concentrated, with the objective of minimizing the total generalized cost. The authors of [65] optimize timetables and ticketing policies to enhance equity by prioritizing low-income passengers.

Chapter 6 focuses on real time vehicle routing, which has been tackled in the literature with Dynamic Vehicle Routing Problems (DVRP), where conditions such as user demand is dynamic and uncertain. DVRP algorithms are at the heart of flexible mobility services. DVRP is reviewed in [66] and [67]. In DVRP, trip requests arrive over time and the route of a fleet of vehicles is dynamically adjusted to serve those trips. Heuristics such as insertion methods [68, 69] or reoptimization methods [70] are the most popular. The optimization method of [71] is considered by many the state of the art. Such a method imposed maximum delay constraints on passenger requests to aggregate multiple requests into feasible vehicle trips. This trip-based abstraction transforms a vehicle-request assignment problem into a vehicle-trip assignment problem, thus significantly reducing computational complexity and enabling real-time high-capacity operation. The work of [72] proposes another DVRP method, based on the classic request-vehicle graph, and on MCTS. Since fleet rerouting is required at each new request, the size of the problems they can treat is quite limited (fleet size = 3, 4, 5) and serving special groups (about 140 requests per day). Our case is orders of magnitude bigger (requests are in the order of thousands/h and fleet size=40) where the approach in [72] would not work. For these reasons, we compare our method with the most prestigious SOTA in DVRP available today, able to handle such a scale [71].

However, in Chapter 6, I proposed a method for dynamic routing that goes beyond the aforementioned proposed DVRP methods. Our approach is different from traditional DVRP. DVRP methods generally suffer from the following limitations: (1) each vehicle is routed independently and (2) a user can only use one vehicle, no transfers are allowed. These characteristics are typical of shared-taxis systems and limit their capacity, far below conventional public transport.

2.4 Limitations of Current Public Transport Systems

2.4.1 Conventional PT

Conventional PT is based on predefined lines, operating under a fixed schedule. Conventional PT exhibit a strong topological structure [73] that offers users a wide range of journey options, often involving multiple lines connected through transfers. However, a key limitation is that its routes are designed offline [74] under nominal assumptions regarding traffic and demand, and these routes typically remain unchanged for many years [75]. In practice, actual operating conditions frequently deviate from these nominal assumptions, rendering such pre-defined routes inefficient.

In suburban and peri-urban areas, travel demand is often relatively low in volume and exhibits substantial variance. On the one hand, to keep the average operating cost per passenger at a sustainable level, PT in these regions typically operates at low frequencies, which results in a poor level of service [76]. On the other hand, demand with high variance indicates that actual travel needs often diverge significantly from nominal planning assumptions. Consequently, routes predetermined offline generally fail to align with the trips users actually wish to undertake. This mismatch can lead to occupancy levels so low that the energy efficiency of buses falls below that of private

cars [77]. Moreover, when disruptions, unplanned events, or road congestion occur, PT systems are often unable to respond effectively, which further diminishes user satisfaction with existing PT services [76, 78].

2.4.2 Flexible Mobility

Flexible mobility can overcome the limitations of conventional PT, by adapting vehicle routes to demands. However, a key limitation of flexible mobility systems is their lack of structure, which results in high cost of operation. The problem is that vehicle routes are dynamically adjusted in response to incoming requests [79, 80], typically without reference to any overarching master plan. As a result, flexible services fail to produce an organized network. Solutions to increase efficiency by consolidating demand around a limited set of stops and making users walk to them [81] have shown some benefits, which however remains limited. Indeed, in online flexible services, any synchronization across multiple routes is likely disrupted due to the frequent route adjustments. Therefore, while conventional PT enables users to navigate within a well-designed network that effectively consolidates trips, flexible mobility does not. As a consequence, flexible mobility is often too costly and inefficient [82].

2.4.3 PT + Flexible Mobility

A *hybrid PT system* integrates conventional public transport with Flexible services deployed as feeder operations around major PT stations [83], within predefined corridors, or in other designated areas. While such integration can indeed mitigate the inefficiency of conventional PT in suburban contexts, it still presents notable limitations: the conventional PT component is still designed offline, making it incapable of effectively serving dynamic non-stationary demand; meanwhile, the flexible services component remains largely unstructured, with the inefficiency discussed in Section 2.4.2.

2.4.4 New Flexible PT System in Chapter 6

To overcome the limitation of the aforementioned existing PT systems, Chapter 6 proposes a new flexible PT system. Such a system is in-between a shared-taxi system (flexible mobility) and a conventional PT. As in the former, it can dynamically change vehicle routes to adapt to arrival requests. As in the latter, it creates a *structured* network of bus lines, with the possibility of transfers, with the difference that our lines change continuously over time. In a certain sense, we invert the logic of DVRP: while in DVRP vehicle routes change upon request arrival, in our system, instead, bus lines are proactively modified, in order to create a rich graph ahead of time, on which future requests can be efficiently routed. To adapt to changes in request arrival, our design actions are evaluated against a prediction model of future requests. This original approach allows us to achieve higher system capacity and cost-efficiency compared to the state of the art classic DVRP-based flexible mobility approaches (Section 2.3).

2.5 Modeling Frameworks

Models for Public Transport can be divided, in simplified terms, into three categories: fully analytical models, graph-based models and detailed simulations.

Different models can be used to represent public transport supply and demand, ranging from high-level approximated analytical models up to high-fidelity agent-based simulation, which I will here very briefly review. In this thesis, I adopted a wide range of models, to reach the level of detail appropriate for the problems solved in the different chapters.

2.5.1 Fully Analytical Models

At strategic level, to make optimization tractable, high level analytical models are often used, e.g., continuous approximation [10, 41, 42] or similar models [31, 43, 84]. Such models represent PT supply and demand as a regular geometrical patterns. For example, the work of [10] (which follows [85] and [3]) abstracts the PT network as a circular structure, demand as a 2-dimensional density function, and estimates expected values of DRT trips, under strong simplifying assumptions. The advantage of these highly aggregated and simplified representations is their computational efficiency, which makes them appropriate to perform optimization on top of them. Moreover, they have minimal data requirements. Another advantage is their interpretability: the relation between decision variables (for instance, spacing between lines) and the performance objective is compactly captured by equations. This allows to reason about macroscopic trends [86]. Moreover, for strategic planning, when the system under study is stochastic and the input uncertain, an “approximate [...] solution of an idealized problem may be more useful than the [...] exact solution.” [87] These models can be employed in the initial stage of transport planning, or to study general relations between different aspects of the system under study. The high level decisions obtained from those models cannot however be directly applied to real cities, and need further refinements [88].

The limit of fully analytical models is that they can loosely describe “any city”, but at the end they describe “no city”: indeed, a PT network is always much more complex than regular geometrical shapes. In our work we focus on accessibility, which results from an intimate interplay between the actual structure of PT deployed and how population and opportunities distribute around it. Idealizing PT with simple shapes would lead to findings with limited real relevance. This is why, to capture the topology of PT and its relation to the distribution of population and opportunities, in this thesis conventional PT is modeled as a graph. On the other hand, to deal with the stochastic nature of DRT, while still capture the impact of strategic and tactical decisions, such as fleet size, allocation of fleet in different areas, we kept a high-level continuous-approximation model of DRT in Chapter 5.

2.5.2 Graph-Based Models

In the transportation domain, an alternative and widely used network representation in optimal strategy design is graph-based, which is also referred to in some studies as line-based or frequency-

based [48,49,64,89]. In this representation, vehicle routes are modeled as graph edges, with service frequencies and average travel times assumed to remain constant within a given time period. Some recent studies have proposed a run-based representation [90]. In a run graph, each run consists of nodes, and each node is characterized by two dimensions (location and time), representing a vehicle's arrival and departure at a given stop. Compared with graph-based models, run-based models provide a more dynamic and accurate representation of individual vehicle operations. However, this increased level of detail inevitably results in significantly larger graph sizes, leading to higher computational costs for path search and optimization. Therefore, how to more efficiently utilize graph-based models to improve computational performance remains an open question. The work of [91] considers the network design problem for On-Demand Multimodal Transit Systems through transfer-expanded graphs, thus resulting in significant computational improvements. Chapter 6 of this thesis presents a solution to this problem.

2.5.3 Detailed Simulation

Building a detailed simulation of the entire multimodal PT, as in [92], is very complex to develop and heavy to run together with multiple optimization iterations. Powerful agent-based simulations, such as SimMobility and MATSim [93], are also able to finely capture the behavior of individuals, but they require very detailed socio-demographic and land-use data (often unavailable), as well as difficult and time consuming set-up and calibration phases. This would overshoot the needs of preliminary findings at the planning level. The work [94] shows that shared automated vehicles (SAVs) have the potential to help solve the first-mile-last-mile transit problem when fare benefits are provided to transit users, via performing an agent-based simulation of Austin, Texas. The what-if approach of [33], [35], [36] relies on detailed simulators to analyze the impact of flexible mobility. However, such models are inappropriate for what-to approaches, which are instead the subject of this thesis.

In Chapter 6, I use a simplified simulation approach: demand is represented in a disaggregated way; i.e., each trip request is handled individually. However, since the focus of that chapter is on the routing discipline, we do not represent the behavior of agents; we make the simplifying assumption that they wish to arrive at their destination as soon as possible.

2.5.4 Modeling Approach of this Thesis

My thesis focuses on the inequality of accessibility, which is more complex than the generalized cost that is usually minimized in what-to approaches. Indeed, reducing inequality of accessibility requires understanding the interrelation between the distribution of opportunities and the population, together with the PT graph topology. For this purpose, we started from two aspects: on the one hand, we considered the integration of PT and DRT. and on the other hand, we proposed a new mode of transport that combined the advantages of PT and DRT. Regarding the former (Chapter 3 - 5), a summary of methods concerning PT design is given in Table 2.1. In this study, we integrate public transit (PT), represented with a graph-based model, and flexible mobility, modeled

Table 2.1: Work on PT design using what-to methods.

What-to method	Objective function		Modeling framework		Solution procedure		
	Generalized cost	Accessibility	Graph based	Continuous Approximation	Solver	Metaheuristic	AI
Luo and Nie (2019, [42]) Fielbaum (2020, [43]) Calabro et al. (2023, [10]) Fielbaum (2024, [95])	x			x	x		
Fielbaum (2024, [64])	x		x			x	
Dai et al. (2022, [46]) Wang and Chen (2021, [47])		x		x	x		
Tong et al. (2015, [48])		x	x		x		
Fu et al. (2022, [49])		x	x			x	
Our approach		x	x	x		x	x

through continuous approximation. The advantage of this approach is that PT systems are generally fixed and periodic; thus, graph-based modeling can more accurately capture their structural characteristics. In contrast, flexible mobility is inherently dynamic and stochastic; adopting a run-based representation would significantly increase computational costs. Continuous approximation, however, enables the estimation of key operational parameters (e.g., average trip time and service frequency) with much lower computational effort, which is sufficient for the intended analysis. By tailoring the modeling approach to the specific characteristics of different transport modes and combining them, we achieve a more efficient and realistic representation of the multimodal system. For the latter (Chapter 6), the SOTA relies on DVRP methods, in which vehicles are routed one by one (presented in Section 2.3).

It should be noted that the purpose of this chapter is not to provide an exhaustive survey of all theoretical and methodological work related to accessibility and transport analysis, but rather to introduce the minimum set of concepts that are essential for the thesis. In particular, only those accessibility measures, demographic dimensions and contextual factors that are directly used in the subsequent chapters are discussed in detail, while many other important contributions are deliberately kept in the background and referred to through citations. This choice is intended to avoid overloading the reader with information that can already be abundantly found in existing surveys [18, 19] and to keep the focus on the elements that are most relevant for the research questions addressed here.

2.6 Position of this Work within the State of the Art and Novelty

In the domain of strategic planning, to the best of my knowledge, my work is the first to provide what-to approaches to reduce the inequality of the distribution of accessibility, resulting from conventional and flexible PT systems. The specific planning strategies considered include stop selection, redesign of bus lines, and allocation of demand-responsive transport (DRT) (Chapters 3-5). Moreover, Chapter 6 introduces a novel perspective on the management of flexible mobility, by addressing the real-time routing problem through the online control of the evolution of dynamic temporal graphs. Its operational contribution is the ability to generate real-time schedules for the bus network over a forward-looking time horizon.

Chapter 3

Public Transport Stop Selection to Reduce the Inequality in the Distribution of Accessibility

3.1 Introduction

The main indicator we consider in this chapter is *accessibility*, i.e., the ease of reaching *opportunities*, such as schools, jobs, and other people, from a location via PT [96]. There is substantial inequality in the geographical distribution of accessibility in real cities [2]. Recent work attempts to include indicators of equality of public transport deployment across a territory in public transport network design.

The work [97] proposes a classic cost-minimizing PT design problem, with the addition of a constraint preventing the inequality of PT supply from exceeding a certain threshold. PT supply is a *local measure*, accounting for the number of busses passing by or stops in a location: improving supply in a location can be directly achieved by modifying PT production therein. We are instead interested in accessibility, which is not local and is thus more complex: the accessibility of a location may depend on the PT topology or opportunities in locations that may be further away but are strongly connected. Therefore, improving accessibility in a location cannot be achieved by simply changing PT production in the very same location, but always requires consideration of the territory and the PT topology in their entirety.

The work [98] designs PT lines to reduce the difference in travel times between cars and PT. They do not consider accessibility either.

More similar to us, The work [46] optimizes headway and stop spacing of a single line for equal distribution of accessibility. We, however, tackle a more realistic network with multiple lines, which prevents the use of their method.

The objective function of [99] is the sum of the accessibility of all areas, weighted by “Transit Need” (TN) coefficients, so as to prioritize accessibility improvement in high-TN areas. This sub-

mits PT design to political will by appropriately adjusting TN computation. We propose instead a method that is as objective as possible, based on geographical horizontal equality, which is technically more difficult: we cannot rely on a simple weighted sum but must always deal with the entire distribution of accessibilities.

Finding an effective way of expressing our inequality-reduction objective into an optimization algorithm is challenging. Our preliminary attempts of using inequality indicators [100] directly into the objective function resulted in poor performance. The classic max-min optimization, which in our case would mean to maximize the lowest accessibility, also was ineffective, as it tends to improve accessibility of too few locations, often remote or isolated, which then results in a weak improvement of global inequality indices and an excessive loss in overall efficiency.

We propose here a novel approach for equality-maximization PT design, consisting in maximizing some bottom quantiles of the accessibility distribution. We apply our approach to the PT stop selection problem, which we solve with a heuristic algorithm. Numerical results show significant improvement in equality, with a negligible loss of overall efficiency. Via statistically analyzing the results of the optimization algorithms, we infer some simple guidelines for equality-based PT design. Our code is available as open source [101].

3.2 Methodology

3.2.1 Model of the Public Transport Network

A PT network is modeled as a graph \mathcal{G} as (Figure 5.1). A PT line (train, metro or bus) is a sequence of stops, linked by corresponding edges. On each PT line l a number N_l of vehicles operate, traveling at speed v_{veh} . At each stop, a vehicle stays for a dwell time t_{dwell} to allow boarding and alighting. We include in t_{dwell} the time lost in acceleration and deceleration. Assume a line with stop sequence s_1, \dots, s_{K_l} and let us denote with $d(s_j, s_{j+1})$ the Euclidean distance between s_j and s_{j+1} . The time a vehicle takes to visit the entire line is $T_l = \sum_{j=1}^{K_l-1} \left(\frac{d(s_j, s_{j+1})}{v_{\text{veh}}} + t_{\text{dwell}} \right)$. As in [102] and [92], the headway and the average waiting time of line l are $H_l = 2 \cdot T_l / N_l$ and $w_l = H_l / 2$, respectively.

Graph \mathcal{G} represents PT network in a certain timeslot, within which we assume that line routes and headway values do not change. In further work, the operation of PT during a day could be represented as a sequence of graphs, one per each timeslot.

For simplicity, we consider a square study area, partitioned with a regular tessellation (no matter the shape of the tiles). The center of each tile is called *centroid*. We denote with \mathcal{V} the set of centroids. Each tile contains a certain amount of opportunities, that we associate to the respective centroid. Users start at any centroid and are willing to reach opportunities around other centroids. In the numerical results, we will consider a particular form of accessibility, called *sociality score* [29], where people are considered as opportunities.

A trip can be performed entirely by walking, or combining walking and PT. Within the PT network, a user can change from line l_1 to line l_2 , if the egress stop s in l_1 and the ingress stop s'

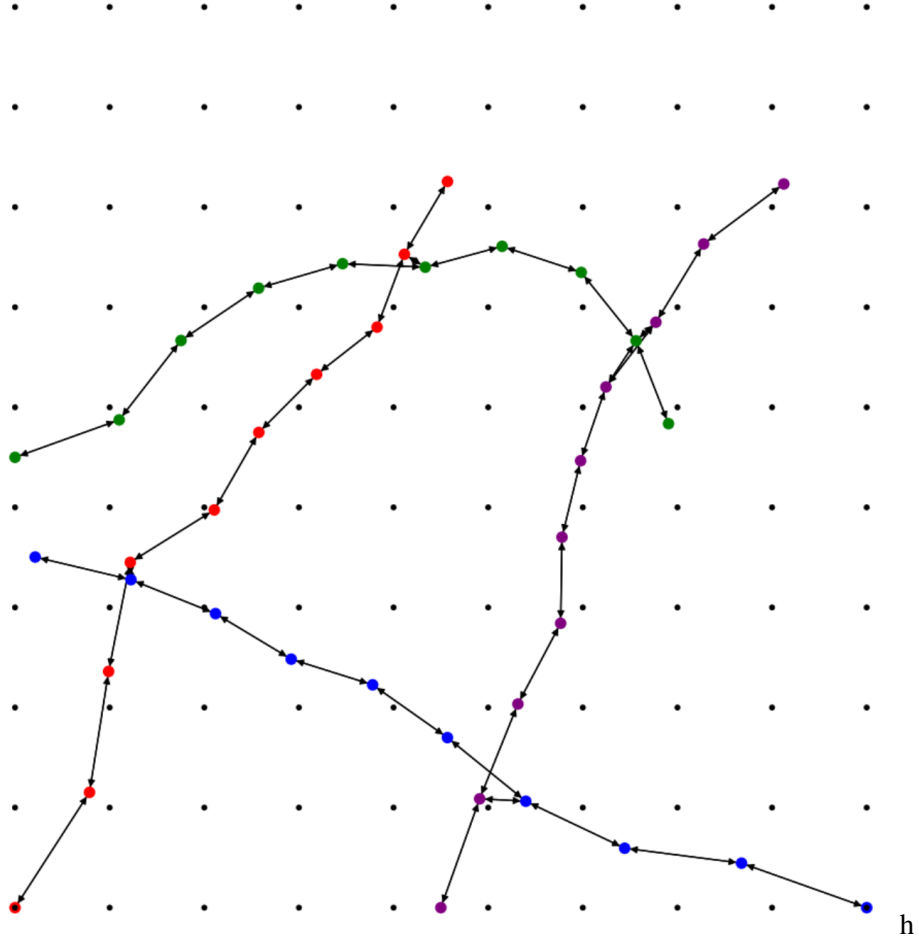


Figure 3.1: An example of PT network: 4 PT lines containing 40 stops and 100 centroids

in l_2 are within distance d_{exchange} . Multiple changes are allowed. When entering line l (directly after walking or during a change of line), a user suffers waiting time w_l . User always travels along shortest time path. More advanced PT assignments [103] could be considered without changing the spirit of this work.

3.2.2 Accessibility

The *gravity-based* definition of accessibility [96] of centroid $v \in \mathcal{V}$ is

$$\text{acc}(v) = \sum_{u \in \mathcal{V}} X(u) \cdot f(T_{v,u}) \quad (3.1)$$

where $T_{v,u}$ is the shortest time to go from v to u , $f(\cdot)$ is the impedance function and $X(u)$ is the amount of opportunities of the tile having centroid u . Choosing impedance function $f(T) = T^{-1}$ has the advantage of expressing accessibility in intuitive units of measurement, i.e. number of opportunities that can be reached per hour. When optimizing for equality, we will focus in particular on set $\mathcal{V}^{m\%}$ of the *worst $m\%$ centroids*, i.e., those with the lowest accessibility. We define the accessibility of graph \mathcal{G} as:

$$Acc(\mathcal{G}; m) = \frac{1}{|\mathcal{V}^{m\%}|} \sum_{v \in \mathcal{V}^{m\%}} acc(v) \quad (3.2)$$

which, with $m = 100$, corresponds to average accessibility $\overline{acc}(\mathcal{G}) = Acc(\mathcal{G}, 100)$.

3.2.3 Inequality index

Denoting the accessibility values of centroids v_1, \dots, v_K by y_1, \dots, y_K respectively, the Atkinson inequality index is (from [100], setting $\varepsilon = 2$):

$$Atk(\mathcal{G}) = 1 - \frac{1}{\overline{acc}(\mathcal{G})} \cdot \left(\frac{1}{K} \sum_{i=1}^k y_i^{-1} \right)^{-1} \quad (3.3)$$

The Atkinson index goes from 0 (perfect equality) to 1 (maximum inequality). When we tried to embed the inequality indices directly into the optimization, we found poor results, as such indices are poorly sensitive to PT modifications. We therefore only use (3.2) during optimization and compute the inequality index a posteriori, to check the quality of the solution. If there is data about population age, income distribution, etc., we can also calculate the vertical equity resulting from our optimization. Results did not change when we used other indicators, e.g., Theil, Gini, Pietra.

3.2.4 Heuristic algorithm

As shown in Algorithm 8, for a given PT graph \mathcal{G} , we run n independent instances of the heuristic algorithm. Each instance randomly deactivate stops. Each time a stop is deactivated, the corresponding line is consolidated: the edges incident to that stop are removed and a new edge is added between the stop before and the stop after the removed one, thus reducing travel and dwell times. We consider in the numerical results a bus network, where edge consolidation also reduces the traveled distance. The method can be generalized to rail networks, where distance would not be reduced. The algorithm might also remove, in some iterations, stops that are connected to multiple lines. If such connections were used by many shortest paths, this may generally decrease accessibility. However, the algorithm selects the best solution (Lines 9,11), which is likely to filter out solutions where “important” connections have been cut. Figure 3.6 will confirm this. Note that deactivating a stop does not necessarily mean to physically removing it, but to simply skip it during the considered timeslot.

Efficiency-based optimization (in short **Ef-Opt**) is obtained by running Algorithm 8 with $m = 100$, so that the search algorithm will tend to maximize the average accessibility. **Equality-based optimization** (in short **Eq-Opt**) is instead obtained with $m = 5$, to preferentially improve the accessibility of the worst 5% centroids. Given initial graph \mathcal{G} , we denote with $\mathcal{G}_{\text{ef}}^*$ and $\mathcal{G}_{\text{eq}}^*$ the respective results. We found that for m smaller than 5, we tend to improve accessibility of only few remote centroids, without achieving good overall inequality (poor Atkison index) and excessively degrading overall accessibility, similar to maxmin optimization. On the other hand, higher values of m would excessively penalize equality in favor of overall accessibility.

Algorithm 1: Heuristic algorithm

- 1: **Input** Public transport graph \mathcal{G} with stops \mathcal{S} .
Parameter m of the accessibility formula (3.2).
 - 2: **For** search instance $i \leftarrow 1$ to n :
 - 3: Initialize $\mathcal{G}_0 \leftarrow \mathcal{G}$ and $\mathcal{S}_0 \leftarrow \mathcal{S}$.
 - 4: **For** step $t \leftarrow 0$ to ∞ until **less than** n_{end} **stops remain active**:
 - 5: Select a random stop $s_t \in \mathcal{S}_t$ and deactivate it.
 - 6: Set $\mathcal{S}_{t+1} \leftarrow \mathcal{S}_t \setminus \{s_t\}$ and let \mathcal{G}_{t+1} the resulting PT graph.
 - 7: Compute the new accessibility: $\text{Acc}(\mathcal{G}_{t+1}; m)$
 - 8: **EndFor**
 - 9: Record $\mathcal{G}^i = \arg \max_{\tau=0}^{t+1} \text{Acc}(\mathcal{G}_\tau, m)$.
 - 10: **EndFor**
 - 11: **Return** PT graph $\mathcal{G}^* = \arg \max_{i=1}^n \text{Acc}(\mathcal{G}^i, m)$.
-

3.3 Evaluation

3.3.1 Considered scenarios

Due to city peculiarities, morphological constraints, pre-existent infrastructure and even political influences real PT networks are extremely heterogeneous and often characterized by irrationality and inefficiency. We could have considered few cities, model their PT graph and apply our optimization. However, we would have ended up in a few case studies, without assessing the generality of our approach faced with PT heterogeneity. Since the focus of this chapter is on the method, rather than specific case studies, we choose instead to synthetically generate 50 PT graphs and verify the performance is satisfying, no matter the graph at hand. We study the graphs resulting from *Ef-Opt* (baseline) and from *Eq-Opt*, which we advocate.

A synthetic graph includes multiple lines, generated via Algorithm 5, which captures the properties of real PT lines. Stop spacing increases with the distance from the center, as in [104] (Lines (4)-(5) of Algorithm 5). To achieve the clear directionality, we allow limited direction change from a line segment to the next (Lines (6)-(8) of Algorithm 5). If a stop falls outside the study area,

the algorithm restarts. Figure 5.1 gives an example of graph. We only present experiments with 8 lines, as with fewer lines trends were similar after adjusting number n of search instances of Algorithm 8: with 4 lines increasing $n > 5$ did not bring improvement in the performance, while with 8 lines improvement stops after $n = 20$ instances. We visualize a large set of synthetic PT graphs and verify they reasonably represent realistic PT networks.

Algorithm 2: Generation of a synthetic PT line.

- 1: **Input:** Range Θ of $\Delta\theta$; Maximum number S of stops;
 - 2: **Initialization:** Random initial bus stop location: s_0 ; Set $\mathcal{S} = \{s_0\}$ of bus stops; Random initial angle θ (with respect to the horizontal axis); Set the initial graph \mathcal{G} as only composed by node s_0 with no edges.
 - 3: **repeat**
 - 4: Calculate the distance x from stop s_{i-1} to the center
 - 5: Calculate the distance between s_{i-1} and s_i as $1/\beta(x)$ (see Table 3.1)
 - 6: Choose an angle $\Delta\theta$ uniformly at random from set Θ
 - 7: Calculate $\theta = \theta + \Delta\theta$
 - 8: Calculate the location of the new bus stop, at distance $1/\beta(x)$ and angle $\theta + \Delta\theta$
 - 9: Add the new stop s_i to \mathcal{S} and an edge from s_{i-1} to s_i
 - 10: **until** The number of stops in \mathcal{S} reaches S
 - 11: **Return** PT graph \mathcal{G} made of stops \mathcal{S} and the added edges
-

We set amount $X(u)$ of opportunities in the tile around centroid u . Since the considered accessibility is the sociality score, opportunities correspond to residents:

$$X(u) = C \cdot \rho(u) \quad (3.4)$$

where $C = 0.25 \text{ km}^2$ is the area of a tile, $\rho(u)$ is the population density, uniformly distributed inside the tile around u and constant within the considered timeslot. Population density follows the classic pattern [105]:

$$\rho(u) = \rho_0 \cdot e^{-\gamma d(u)} \quad (3.5)$$

where $d(u)$ is the distance between centroid u and the city center, ρ_0 and γ are hyperparameters. Stop spacing at distance x from the center is $1/\beta(x)$, where $\beta(x)$ is the stop density and, similar to population density, follows an exponential distribution from [104] (Figure 3.2). Table 3.1 reports the adopted parameter values.

3.3.2 Results

For all generated graphs $\mathcal{G}_i, i = 1..50$, we compute average accessibilities $a_i = \text{Acc}(\mathcal{G}_{i,\text{eq}}^*, 100)$ of graph $\mathcal{G}_{i,\text{eq}}^*$ optimized via *Eq-Opt*. Subsequently, we perform the same calculation using *Ef-Opt* optimization, resulting in values $b_i, i = 1..50$. The t -test on these two lists yields p -value = 0.0377,

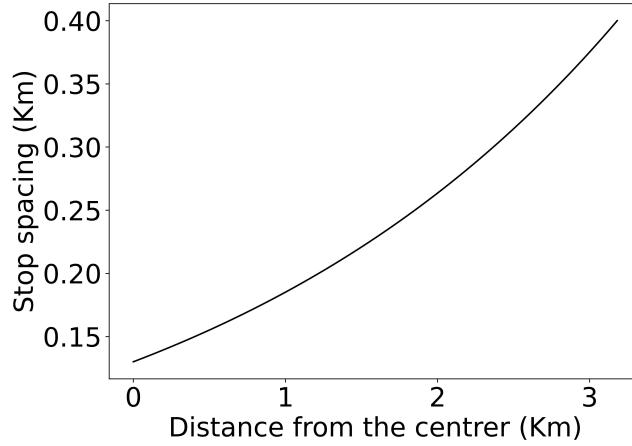


Figure 3.2: Stop spacing.

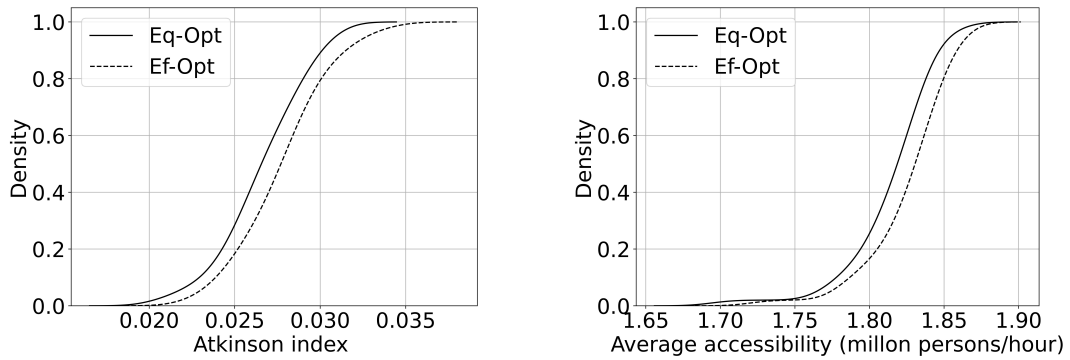


Figure 3.3: Cumulative Distribution Functions (CDFs) of Atkinson index and average accessibility after *Eq-Opt* and *Ef-Opt*

Table 3.1: Scenario parameters

Parameter	Value
PT networks	4500 m \times 4500 m
Number of synthetic graphs	50
Number of PT lines per graph	8
Number of PT stops per line	10
Number of centroids	100
Distance	
between two nearest centroids	500 m
maximum distance d_{exchange} that user can walk to change line	300 m
Average speed	
walking [106]	60 m min ⁻¹
PT	300 m min ⁻¹
Dwell time	1 min
Stop density at distance x from the center (β_0 and γ' are such that spacing is 1/130m in the center and 1/400m at the extremum of the study area, in line with [104])	$\beta(x) = \beta_0 \cdot e^{-\gamma' \cdot d(x)}$
Parameters of Algorithm 8	
m	5 or 100
n	20
M_{end}	10
number of PT overall stops for termination n_{end}	10
Parameters of Algorithm 5	
Θ	$[-\pi/8, \pi/8]$
S	80
Parameters of Formula (3.5) [107]	
ρ_0	36000 people/km ²
γ	0.01 km ⁻¹

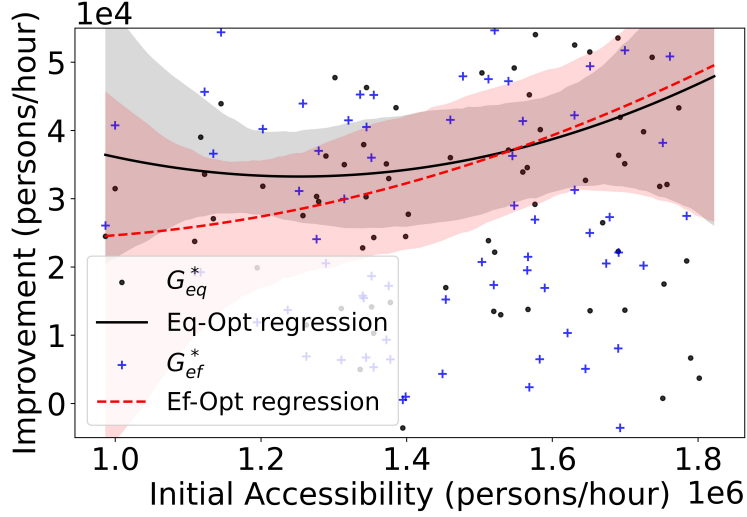


Figure 3.4: Change in accessibility on one exemplary graph.

indicating a significant difference in average accessibility between *Eq-Opt* and *Ef-Opt*. A similar *t*-test on the lists of Atkinson inequality indices yields p -value = 0.0005, indicating that the reduction of inequality obtained via *Eq-Opt* is very significant. For each graph, we compute the two indices above and we report their Cumulative Distribution Function (CDF) over the 50 graphs in Figure 3.3. Atkinson’s inequality is consistently lower with *Eq-Opt* than with *Ef-Opt*, at the cost of a slight decrease in average accessibility. This is a trade-off between efficiency and equality. If we place excessive emphasis on efficiency, the resulting design may be very unequal.

Figure 3.4 focus on one exemplary graph \mathcal{G}_i , chosen randomly. Each dot corresponds to a centroid u . The x -coordinate indicates its accessibility $acc(u)$ (before optimization), while the y -coordinate indicates the improvement in accessibility obtained in $\mathcal{G}_{i,eq}^*$ and in $\mathcal{G}_{i,ef}^*$, i.e., the number of additional persons per hour that is possible to reach with respect to the initial graph \mathcal{G}_i . Quadratic regression lines and 95% confidence intervals are also shown. Observe that *Eq-Opt* concentrates the improvement within disadvantaged centroids, i.e., those with low accessibility in \mathcal{G}_i , at the cost of a slight decrease in accessibility for the advantaged centroids.

Next, we consider two features of each bus stop: distance D_s from the center and number nbl_s of bus lines reachable within 300 m. For each value of D , we take the 100 stops having D_s closest to D and calculate what proportion of them were skipped. Figure 3.5 shows that the closer the distance it from the center, the higher the probability that *Eq-Opt* skips bus stops. The significant higher skipping probability within 1Km from the city center means that in the initial graphs, where spacing is generated as in real deployments [104], stop density is excessive and is not useful to accessibility. This confirms that need of de-densifying stop distribution, already raised by [104].

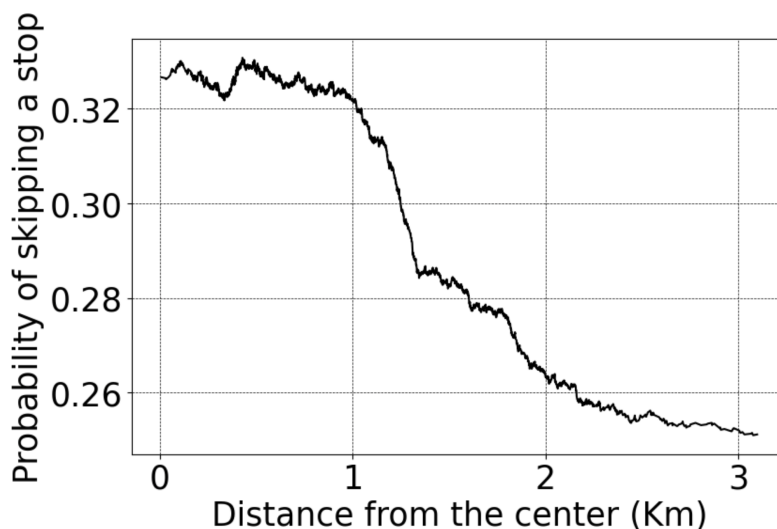


Figure 3.5: Relation between the probability of skipping a stop and distance from the center by *Eq-Opt*

Figure 3.6 plots feature nbl_s (gray bars) per each stop s , and the probability of skipping a stop is calculated in the same way as in Figure 3.5. Figure 3.6 shows *Eq-Opt* and *Ef-Opt* both tend not to skip stops that are close to other bus lines. Indeed, skipping such stops would strongly reduce the possibility of line change, thus cutting out paths that may greatly contribute to accessibility. Figure 3.7 shows that it is generally preferable, for both efficiency and equality, to maintain higher stop density close to the center.

3.4 Conclusions

We proposed a method to embed the inequality of the distribution of accessibility into the decisions within a what-to PT planning strategy. We applied it to the stop selection problem. Numerical results show that a significant reduction in the inequality of accessibility is achievable, which encourages the pursuit of future work applying such a method to other dimensions of PT planning [61]. By systematically applying the proposed optimization procedure to a large ensemble of synthetic network topologies, this chapter also provides statistical evidence on how PT stop selection affects spatial inequality in accessibility. Analyzing the distribution of outcomes across many different graphs shows that optimizing stop locations can consistently reduce inequality, rather than producing effects that are specific to a single network configuration. The focus of my work is methodological and not use-case driven, which is why we prefer to provide evidence on a large dataset of synthetic networks. Our proposed method opens up a line of research for extending accessibility-driven stop design to more realistic urban settings, where additional constraints and richer representations of travelers' behavior and temporal dynamics can be explicitly modeled. We

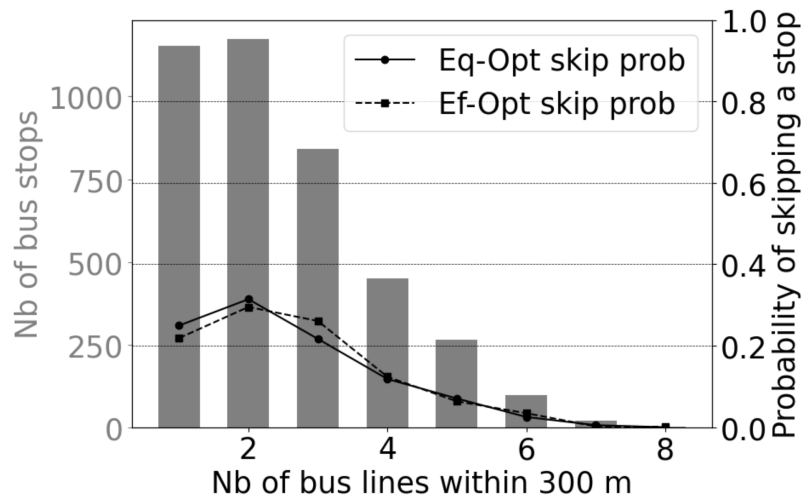


Figure 3.6: Relation between the probability of skipping a stop and the number of bus lines nearby (Note $d_{exchange} = 300\text{ m}$ is the maximum distance users walk to change line, see Table 3.1)

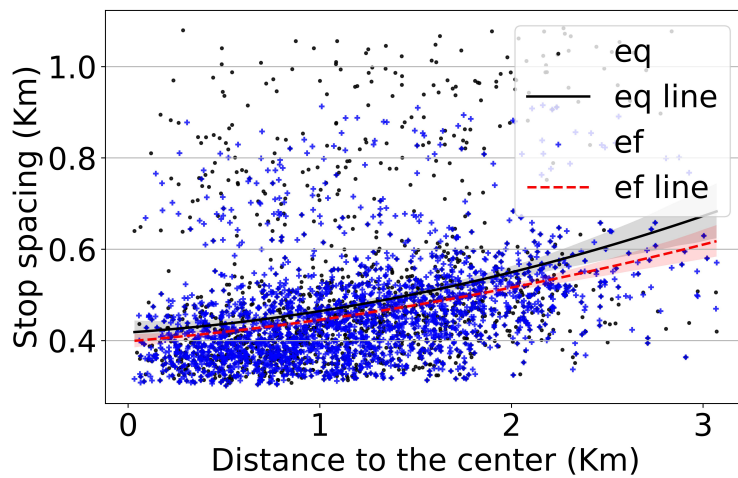


Figure 3.7: Relation between the distance to the nearest stop and the distance to the center

did not do this already in this manuscript, for obvious lack of time.

Note that, as already discussed in Chapter 2, the scope of conventional PT planning alone is structurally limited in its ability to fully address accessibility inequality. For this reason, the remainder of this thesis focuses on how conventional PT can be combined with flexible mobility services to further mitigate these inequalities, building on the insights gained from the synthetic experiments in this chapter.

Chapter 4

Public Transport Network Design for Equality of Accessibility via Message Passing Neural Networks and Reinforcement Learning

4.1 Introduction

Existing Public Transport (PT) is less and less adequate to satisfy mobility needs of the people, in a context of urban sprawl [108]. The United Nations estimate that only “1/2 of the urban population has convenient access to PT” [109]. Building more and more PT lines to keep pace with urban sprawl, using traditional planning objectives, has proved to be ineffective.

PT operators generally design PT lines with the purpose of maximizing overall efficiency, measured in terms of generalized cost (which takes into account travel times and costs for the operators), the number of kilometers traveled, or the number of passengers transported. This has resulted in the unequal development of PT within urban areas. The level of service offered by PT is often satisfactory in city centers and poor in the suburbs. In transportation, there is a consensus that it is not sustainable for people in the suburbs to travel by car, as private cars are the largest polluters, comprising 60.6% of all transport [110]. Therefore, suburban populations depend on their private cars to perform their daily activities [111, 112]. As an example, the modal share of the car in the city center of Prague is double that of the modal share in the city center. The dependence on private cars has negative economic, social, and environmental impacts [113, Section 2.2], which are common to different cities across the world. For example, 61% of EU road transport CO_2 comes from cars. jobseekers without a car have 72% fewer chances of finding a job in Flanders, etc. Therefore, a sufficient condition to achieve sustainability is to improve the level of service of public transport where it is currently poor. We propose in this work to set geographical equality of PT level of service as the main design objective. We focus in this chapter on the PT *accessibility* metric, which

measures the ease (in terms of time and/or monetary cost) of reaching Points of Interest (PoIs) via PT. To improve geographical equality, we prioritize increasing PT accessibility in underserved areas.

A trivial strategy to do so would be to place more stops and lines in underserved areas. However, this may not be the most efficient way to increase accessibility there. Indeed, the ability to reach PoIs might be increased even more by improving PT network close to other nodes, possibly far away, that may enable convenient changes with other important lines. In general the PT network extends the inter-dependencies between locations and PoIs far beyond those that are in physical proximity. Therefore, to reduce the inequality of the distribution of accessibility it is always required to take the entire PT graph into consideration, rather than just around the local areas where we want improvement. This makes our problem particularly challenging.

Most cities already have an existing PT network, and the need to build lines from scratch is very limited. Re-designing the whole PT network is also not an option as it would lead to major costs to the operators. For this reason, in this work, we assume a core PT network (e.g., metro) that does not change, and we only tackle the design of some bus lines, which comes at a limited infrastructure cost. By focusing on bus network design only, we aim to achieve important reduction of accessibility inequality with relatively low expenses for the operator.

The contribution of this chapter is a novel approach to PT network design, in which the non-trivial inter-dependencies involved into the accessibility metrics are captured via a Message Passing Neural Network (MPNN) [114–116] and a Deep Reinforcement Learning (RL) agent. While MPNN and RL have been used to solve canonical optimization problems on graphs, to the best of our knowledge we are the first to use them for PT network design. To reduce inequality, we propose a simple yet effective approach, consisting in using quantiles of the accessibility metrics as objective function.

Numerical results in a scenario inspired by Montreal show that our method effectively reduces accessibility inequality, more effectively than metaheuristics classically used for PT design. This improvement is due to the capability of the MPNN to capture the structure of the PT network and its relation with the PoIs, while metaheuristics do not learn any dependencies and restrict themselves in randomly exploring the space of designs.

4.2 Model

4.2.1 Model of Territory and of Public Transport

We partition the study area with a regular tessellation (here we adopt square tiles, but any regular shape can be used), as in Figure 4.1. The center of each tile is called *centroid* (blue points in Figure 4.1). The study area also contains Points of Interest (PoIs) often called "opportunities" in the literature about accessibility. PoIs can be shops, jobs, schools, restaurants, etc. PoIs are depicted as green points in Figure 4.1.

As illustrated in Figure 4.1, our model of PT is composed of:

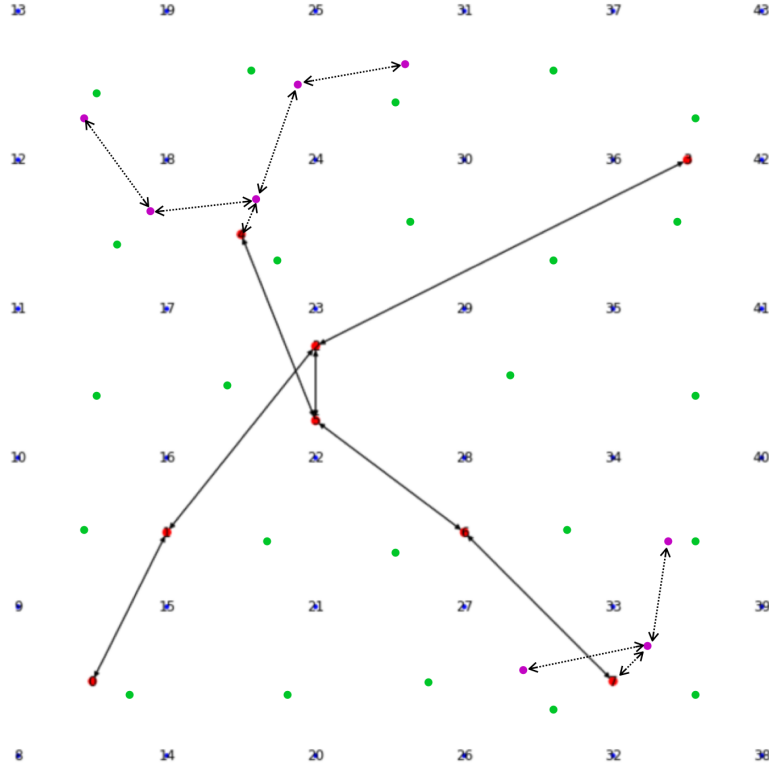


Figure 4.1: Model of Public Transit: PT graph \mathcal{G} has 2 metro lines (red points represent metro stations) and 2 bus lines (purple points represent bus stops), in addition, the blue points are the centroids, and the green points are the points of interest.

1. Metro lines and metro stations (red points).
2. Bus lines and bus stops (purple points).

Changing metro lines is very costly and time consuming. On the other hand, redesigning bus lines requires much less infrastructure cost and can be done in shorter time. In this chapter, we only focus on redesigning bus lines and keep metro lines unchanged.

We model PT as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{L})$, where \mathcal{V} is the set of nodes, composed of centroids, \mathcal{B} is the set of candidate nodes and PT stops $\mathcal{V} = \mathcal{C} \cup \mathcal{B} \cup \overline{\mathcal{B}}$, \mathcal{E} is the set of edges, and \mathcal{L} is the set of k lines. Any PT line l (metro line or bus line) is a sequence of PT stops, linked by edges $e \in \mathcal{E}$. Each edge has a weight, which represents the time used by a vehicle to go from a PT stop to another. A PT line l also has a headway t_l , which is the time between two vehicle departures in the same direction. Since we only optimize bus lines, headway t_l of any metro line remains unchanged. For metro lines, headway t_l can be obtained from real data. Instead, since we build bus lines, we need to calculate ourselves headway t_l for any bus line l . Once we decide the sequence of bus stops

composing l , we can get the total length of the line d_l , to go from the first to the last stop. We assume number N_l of buses deployed on line l is fixed in advance. In this case, the cost is always the same since the total fleet size ($= kN_l$) is fixed. Denoting with s_b the bus speed, headway t_l is:

$$t_l = \frac{d_l}{s_b \cdot N_l}. \quad (4.1)$$

In reality, the headway could also be a bit higher, due to the time spent by the bus at the terminal before starting the next run.

As in Figure 4.1, we include in \mathcal{G} the set of centroids \mathcal{C} and the set of points of interest \mathcal{P} . We also include edges (in the two directions) between any centroid and all PT stops, between any point of interest and all PT stops, and between any centroid and any point of interest. Note that \mathcal{V} is defined as the set of nodes, \mathcal{C} is a subset of \mathcal{V} .

For a trip from centroid c to point of interest poi , a traveler can choose between different modes of travel. For example, a traveler could simply walk to poi at speed s_w or walk from centroid c to a PT stop (metro station or bus stop), go via PT to another stop, and from there walk to the destination poi . If the stops of two lines are close enough, we will also add the edge between two stops to indicate that passengers can transfer by walking. We consider an average waiting time $t_p/2$ at this station. We assume that travelers always take the shortest path, i.e., the one that allows to arrive at destination with the least time.

4.2.2 Accessibility

Accessibility measures the ease of reaching PoIs via PT. A simplified depiction is given in Figure 4.2. Accessibility depends on both land use (which determines where PoIs are) and the transport system (which determines the time to reach each PoI). There are several ways of mathematically defining accessibility. We define the accessibility of centroid c as:

$$acc(c) = \sum_{poi \in \mathcal{P}} \max\left(0, 1 - \frac{T_{c,poi}}{T_{\max}}\right), \quad (4.2)$$

where $T_{c,poi}$ is the shortest travel time from centroid c to point of interest poi and T_{\max} is a predefined threshold for travel time (e.g., 30 mins). Intuitively, $acc(c)$ measures the number of PoIs that can be reached by individuals departing from centroid c , within time T_{\max} . Such PoIs are weighted by the time to reach them, so that the closer a PoI, the more it contributes to accessibility. Our definition is a combination of two classic definitions of accessibility, namely the *isochrone* and *gravity-based* [19]. The purely isochrone definition of accessibility has the issue of counting all PoIs the same, despite the difference in travel time to reach time. On the other hand, the purely gravity based definition of accessibility, factors in all PoIs, even those that would require a prohibitive travel time. By combining the two aspects, we solve the aforementioned limitations. Note that accessibility is agnostic to demand: it does not describe where people *currently* go but it measures where they *potentially* go [19]. When modifying the PT structure, we modify this “potential

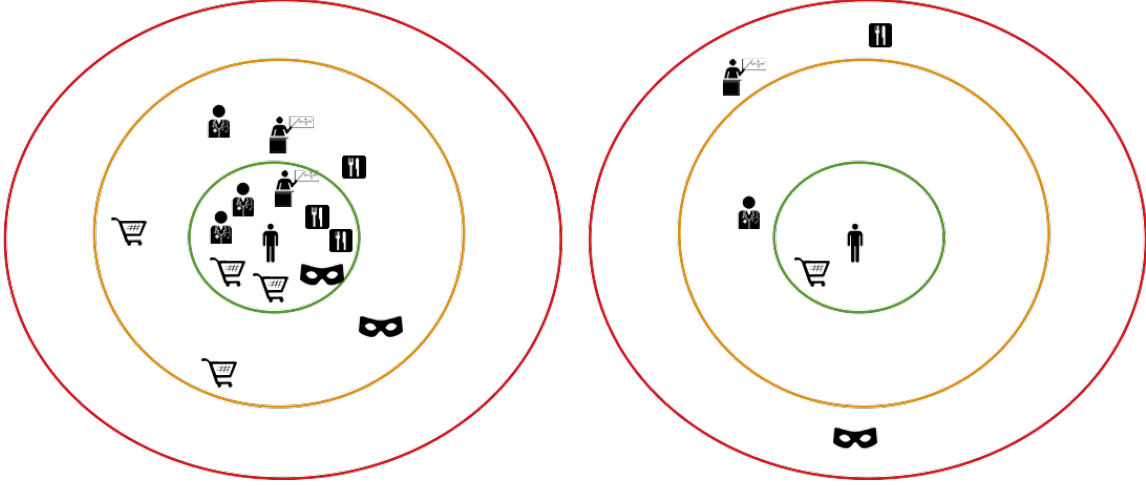


Figure 4.2: Accessibility example: the location on the left enjoys high accessibility as, departing from it, one can reach many PoIs in little time. On the right, instead, accessibility is poor: few PoIs are reachable and high travel times are required. The left and right locations are typical of city centers and suburbs, respectively.

of mobility” and thus we also impact where people *will go*. For this reason, mixing current demand with the definition of accessibility would be misleading, as current demand is implicitly invalidated by PT design actions.

We define the global accessibility of graph \mathcal{G} as

$$acc(\mathcal{G}) = \sum_{c \in \mathcal{C}} acc(c). \quad (4.3)$$

Classic efficiency-based optimization of PT would aim to maximize $acc(\mathcal{G})$. Instead, our aim is to reduce the inequality in the geographical distribution of accessibility, and thus we choose to consider the accessibility of the centroids that suffer from the worst accessibility, as we aim to concentrate improvement in such zones.

One could be tempted to apply max-min optimization, trying to maximize the lowest accessibility in the territory. However, when we tried that, we obtained poor results. Indeed, we were ending up improving areas that were remote and often uninhabited. Often, the improvement was enjoyed by too few locations. We therefore propose to maximize some bottom quantile of the accessibility distribution. To the best of our knowledge, this simple yet effective idea has not been explored so far. We define the following accessibility metric, related to the q th quantile:

$$acc^q(\mathcal{G}) = \sum_{c \in \mathcal{C}^q} acc(c), \quad (4.4)$$

where \mathcal{C}^q defines the set containing the $q\%$ of centroids with the least accessibility. Note that $acc(\mathcal{G}) = acc^{100}(\mathcal{G})$.

4.2.3 Problem Definition

Let us consider a PT graph \mathcal{G} and a set \mathcal{B} of n_b *candidate bus stops*. Set \mathcal{B} is contained in set \mathcal{V} of nodes of \mathcal{G} . Set $\overline{\mathcal{B}} = \mathcal{V} \setminus \mathcal{C} \setminus \mathcal{B}$ is the set of *non candidate stops*, i.e., the ones that will not be used to create the new lines.

In broad terms, we consider the problem of the PT operator to design k bus lines $\{l_1, \dots, l_k\}$ (where number k is fixed a-priori), passing by these n_b bus stops, in order to reduce inequality of accessibility. Any stop in \mathcal{B} may be already part of pre-existing lines or not. The problem at hand may emerge in case a PT operator wishes to build additional bus lines, passing by stops \mathcal{B} . Another case is when a PT operator wishes to redesign current bus lines, while reusing current bus stops.

In quantitative terms, we wish to find graph \mathcal{G}^* , which is as \mathcal{G} but also contains additional lines l_1, \dots, l_k , such that $acc^q(\mathcal{G})$ is maximized. Specially, if we define:

$$x_{i,j} = \begin{cases} 1, & \text{if bus stop } j \text{ in line } l_i. \\ 0, & \text{if bus stop } j \text{ not in line } l_i. \end{cases} \quad (4.5)$$

We aim to solve the following optimization

$$\max_{\sigma(l_1), \sigma(l_2), \dots, \sigma(l_k)} acc^q(\mathcal{G}), \quad (4.6)$$

subject to the following constraint:

$$\sum_{i=1}^k x_{i,j} = 1, \forall j = 1, 2, \dots, n_b. \quad (4.7)$$

$$\sum_{j=1}^{n_b} x_{i,j} \geq 2, \forall i = 1, 2, \dots, k. \quad (4.8)$$

According to the values of $x_{i,j}$, we can define each line:

$$l_i = \{x_{i_1}, \dots, x_{i_{n_i}}\}, \forall i = 1, 2, \dots, k. \quad (4.9)$$

Calculating accessibility needs to find shortest path in current PT graph, which makes impossible to give specific formulation of accessibility. Therefore, this problem belongs to the integer programming of a black-box function. Constraint 4.7 ensures that each candidate stop is assigned to a line. With this constraint, we are forbidding a node to be part of multiple lines. However, this limitation can be easily removed by “duplicating” the same real stop into multiple candidate stops in our model. Constraint 4.8 means that each line has at least two bus stops. In addition to determining which stops each line contains based on the value of $x_{i,j}$ and Constraint 4.9, the permutation function $\sigma(l_i)$ (in Formulation 4.6) is also needed to determine the order of the stops in each line l_i .

Note that maximizing the accessibility of the bottom quantiles means, indeed, to improve the accessibility of the poorest locations. Observe that, selecting only the location with the worst

accessibility as the objective function generally returned, in our preliminary experiments, not reasonable results, as it concentrates all the optimization effort to just few, possibly very remote, locations, where improving accessibility is anyways hopeless. Rather than optimising for one extreme outlier, we focus on a set of relatively disadvantaged areas. Within this set, the algorithm naturally favours those locations where accessibility can meaningfully increase, instead of “wasting” effort on isolated zones whose accessibility is structurally hard to improve. Moreover, the bottom-quantile group is not fixed: as the network changes and some places improve, different locations may enter or leave this group. In that sense, the optimisation target adapts to the evolving accessibility distribution, rather than being statically tied to one predefined poorest point.

4.3 Resolution Method based on Graph Reinforcement Learning

We decompose our problem (§4.2.3) as a bi-level optimization (see Sec. 9.2 in [117] and Sec. 9.1.3.3 of [118]): in the upper level, we partition the candidate bus stops in k subsets. In the lower level, each subset will be transformed into a line, deciding the order of the stops. The upper level evaluates the quality of the solution, under the constraint (enforced in the lower level) that the order of the stops within each line must be the one producing the smallest amount of kilometers traveled.

4.3.1 Markov Decision Process Formulation

Let us denote by \mathcal{G} the initial graph, i.e., the one when no new bus lines have yet been added. We model the upper level problem as the following Markov Decision Problem (MDP):

- **States:** A state is a partition $S = (l_1, \dots, l_k)$, where $l_i = \{b_{i_1}, \dots, b_{i_{n_i}}\}$ is the set of bus stops assigned to line l_i .¹ Each bus stop is assigned to a single line. Given any state S , we build the line corresponding set l_i of stops, $i = 1, \dots, k$. To transform set l_i into a line, we need to decide the order in which the stops in l_i will be visited. Such an order is calculated via a heuristic (Section 4.3.4). Given state $S = (l_1, \dots, l_k)$ and having defined the lines corresponding to $l_i, i = 1, \dots, k$, we add to graph \mathcal{G} the edges corresponding to those lines and obtain a new graph $\mathcal{G}(S)$.
- **Actions:** At each step, our optimization agent shifts a bus stop b_i from its current line l_o to a target line l_t . The action is defined by a tuple $a = (b_i, l_t)$. The state changes from S to S' : state S' is equal to S , except for line l_o , which becomes $l_o = l_o \setminus \{b_i\}$, and for l_t , which becomes $l_t = l_t \cup \{b_i\}$. The action of changing a bus stop to its own line is not permitted. Observe that ours is a Deterministic MDP [119], i.e., the arrival state S' can be deterministically calculated from the departing state S and action a .

¹For the simplicity of notation, we use the same symbol l_i to denote a line and also the set of stops assigned to it.

- **Rewards:** The instantaneous reward collected when applying action $a = (b_i, l_t)$ on state S , is

$$r(S, a) = acc^q(\mathcal{G}(S')) - acc^q(\mathcal{G}(S)), \quad (4.10)$$

where parameter q must be chosen in advance.

- **Policy:** During training, our agent follows an ε -greedy policy. At test time, actions are chosen greedily with respect to the Q-values, but our agent continues to explore with a random action each time it finds a local optimum.

The sizes of the state space and the action space are $k \cdot n_b$, considering a matrix with k lines and n_b bus stops that can represent any state and action.

4.3.2 High-level View of the Optimization Approach

Due to the large size of the action and state spaces, enumerating all the states and actions and learning a Q -function that takes those states and actions directly as input is hopeless. Therefore, as is common in graph-related optimization tasks, we resort to a Message Passing Neural Network (MPNN) [114–116, 120]. Via an MPNN, we embed each node in a low-dimensional Euclidean space. Such representation captures the “role” of that node within the graph, based on the direct or indirect connections with the other nodes. The process of node embedding is thus able to capture the structure of a graph so that the RL agent can make decisions that take this structure into account.

A MPNN takes as input a PT graph $\mathcal{G}(S)$ of the current state S and then outputs the Q value for each action. Next, the Greedy Policy performs an action according to the Q values. At last, the reward, which is the change in the accessibility metric, helps to update the parameters of MPNN. The following section will introduce MPNN in more detail. Every time we shift a bus stop from a bus line to another, a reasonable method of deleting and inserting bus stops in a line is presented in Section 4.3.4.

4.3.3 Message Passing Neural Network

Let us associate each candidate stop b with a feature vector x_b . Let us denote by X the matrix of the feature vectors of all stops and by M_{adj} the adjacency matrix, where element (i, j) is 1 if there is a line in which bus stop b_i comes right before b_j , and 0 otherwise.

A MPNN calculates a vector μ_b , called *embedding*, for each candidate bus stop b . Embedding μ_b is a function of the feature vector x_b of features, matrix X , and the adjacency matrix M_{adj} .

In the following, all vectors like Θ_j denote parameters that are learned during training. The embedding of bus stop b is initialized as

$$\mu_b^0 = f(x_b, \Theta_1) \in \mathbb{R}^n. \quad (4.11)$$

The vector of edge embeddings, one for each edge, is initialized as

$$w^0 = g(M_{\text{adj}}, X, \Theta_2) \in \mathbb{R}^m. \quad (4.12)$$

Let us denote with $\mathcal{N}(b)$ the neighboring bus stops of bus stop b . Note that since the bus lines change from a state S to another, $\mathcal{N}(b)$ may also change for bus stop b .

The information is then shared with each of the nodes' neighbors through T rounds of messages. The message passed to candidate stop b at round $t + 1$ is:

$$m_b^{t+1} = M(\mu_b^t, \{\mu_u^t\}_{u \in \mathcal{N}(b)}, \{w_{ub}^t\}_{u \in \mathcal{N}(b)}, \Theta_3^t) \in \mathbb{R}^{n'}, \quad (4.13)$$

where w_{ub}^t is the embedding of the edge between bus stop u and bus stop b after t iterations.

Embedding μ_b^{t+1} of candidate stop b by message:

$$\mu_b^{t+1} = U(\mu_b^t, m_b^{t+1}, \Theta_4^t) \in \mathbb{R}^n, \quad (4.14)$$

M and U are, respectively, the message and update functions at round t . After T rounds of message passing, a prediction is produced by some Readout function, R . In our case, the prediction is the set of Q-values corresponding to the actions of the changing bus stops and their target line:

$$\{Q(S, a)\}_{a \in \mathcal{A}} = R(\{\mu_b^T\}_{b \in \mathcal{B}}, \Theta_5), \quad (4.15)$$

where \mathcal{A} is the set of actions, and \mathcal{B} is the set of bus stops. Note that μ_b^T is calculated via Formula 4.13 and Formula 4.14, thus varies for different states S . Message function M , Update function U , and Readout function R , as well as the embedding functions f and g , are all neural network layers with learnable weights $\{\Theta_1, \Theta_2, \{\Theta_3^t\}_t, \{\Theta_4^t\}_t, \Theta_5\}$. Every time our agent takes an action and receives a reward r . The One-Step Q-learning loss is:

$$\text{Loss}(S, a) = (\gamma \cdot \max_{a'} Q(S', a') + r - Q(S, a))^2, \quad (4.16)$$

where γ is a Discount factor. We update learnable weights via Stochastic Gradient Descent of the loss function 4.16. [121] also used the same loss to address the combination problem on the graph. From [122], the total time complexity of the forward step and the backward step of MPNN is $O(T \cdot (|\mathcal{V}| + |\mathcal{E}|))$ for a sparse matrix of the PT graph with an unchanged embedding vector dimension. Combined with action space is $k \cdot n_b$ in Section 4.3.1, the time cost of each step is linearly related to the sum of the number of nodes and edges in the graph \mathcal{G} .

4.3.4 Sorting Algorithm

Recall that state S is a partition (l_1, \dots, l_k) of the set \mathcal{B} of candidate nodes. State S just establishes to which line each candidate node belongs. However, to transform any set l_i into a line, a certain ordering of its stops must be established.

In this section, we will introduce the method of determining the order of bus stops. This optimal method should also maximize our accessibility objective function. However, since it is

a Traveling Salesman Problem, defining such a function seems utterly complex or incurs a very high computational cost. Considering that the order must be determined every time the RL agent selects a new bus stop subset, we decided to use the shortest path algorithm as a proxy for the maximum accessibility path algorithm. We understand that this approach is suboptimal as we optimize distances, but accessibility is a measure calculated over the whole graph. Nevertheless, using this algorithm optimizes the headway of our bus lines and thus provides good accessibility with low computational costs.

4.3.5 Reinforcement Learning Equality Algorithm

Given the number of bus lines k and the PT network \mathcal{G} , we propose a Reinforcement Learning Equality algorithm (Algorithm 5) to plan k bus lines efficiently in order to optimize the accessibility objective function $acc^q(\cdot)$ ($q = 20, 50, 100$). Our base idea is to obtain a better state-action function Q by updating the MPNN network. Note that only when our objective function value is better than before will we update our MPNN network. We still explore different actions (and thus different graph configurations) for each step, including those that decrease the score with respect to the last graph obtained. We just remove from exploration those graphs that are worse than the initial one. If $acc^q(\mathcal{G})$ has not changed in the past certain iterations (e.g., 5 iterations), the episode will be ended. Algorithm 5 terminates when a certain time threshold (e.g., 1 h) is exceeded. In fact, the initial graph \mathcal{G} of our algorithm can differ for each episode. Therefore, it has the ability to learn across different PT networks.

4.4 Evaluation

4.4.1 Considered Scenario

To showcase our agent, we consider a simplified version of Montreal. Note that we do not aim to set up a realistic mobility scenario with all the necessary details for transport planning. This would indeed require years of effort by specialized transport consulting companies, and it is out of our scope. Here, our intention is to show that the proposed approach is effective from a methodological and algorithmic point of view. For this reason, any cost-benefit analysis of our resulting PT network in Montreal, and any detailed environmental assessment, is out of scope for this algorithmic focused chapter. It was observed that one could replace accessibility with cost, kilometers traveled, or any other transport related metric. However, we chose to focus on the most difficult metric among those used in common PT design problems, as optimizing accessibility quantiles requires the model to be able to deeply capture the inter-relation between the graph and its surrounding environment, as well as spatial inequalities. This is the focus and interest of this chapter.

From the General Transit Feed Specification (GTFS) data of Montreal ([123]), we take the station locations and the sequence of stations for all lines. We assume the PT operators wish to keep the metro (subway) network as it is but wish to build bus lines in order to reduce the inequality of the geographical distribution of accessibility. Therefore, set \mathcal{B} of candidate stops

Algorithm 3: Online Reinforcement Learning Equality algorithm.

Data: Number of lines k , initial graphs \mathcal{G} , quantile q

Result: The best accessibility acc_{best}^q

```
1 while Running time threshold is not exceeded do
2   Initialization: Randomly partition the bus stops between the lines to obtain initial
   state  $S$ ;
3   Sort the lines (§4.3.4), and update  $\mathcal{G}$ ;
4   Set  $acc_{best}^q = acc^q(\mathcal{G})$ ;
5   while  $acc^q(\mathcal{G})$  has not changed in the past 5 iterations do
6     Predict  $Q = MPNN(\mathcal{G}, S)$  (4.15);
7     Update the state with action  $a = \arg \max_a Q(S, a)$ ;
8     Sort each line  $l_i$  (§4.3.4);
9     Calculate headway  $t_{l_i}$  via (4.1) for each  $l_i$ ;
10    Update graph  $\mathcal{G}$  with the new bus lines and headway ;
11    Compute  $acc^q(\mathcal{G})$  (see (4.4)) ;
12    if  $acc_{best}^q > acc^q(\mathcal{G})$  then
13      Set  $acc_{best}^q = acc^q(\mathcal{G})$ ;
14      Compute loss via (4.16);
15      Update learnable weights  $\{\Theta_1, \Theta_2, \{\Theta_3^t\}_t, \{\Theta_4^t\}_t, \Theta_5\}$  of MPNN by Gradient
      Descent;
16    else
17      Continue;
```

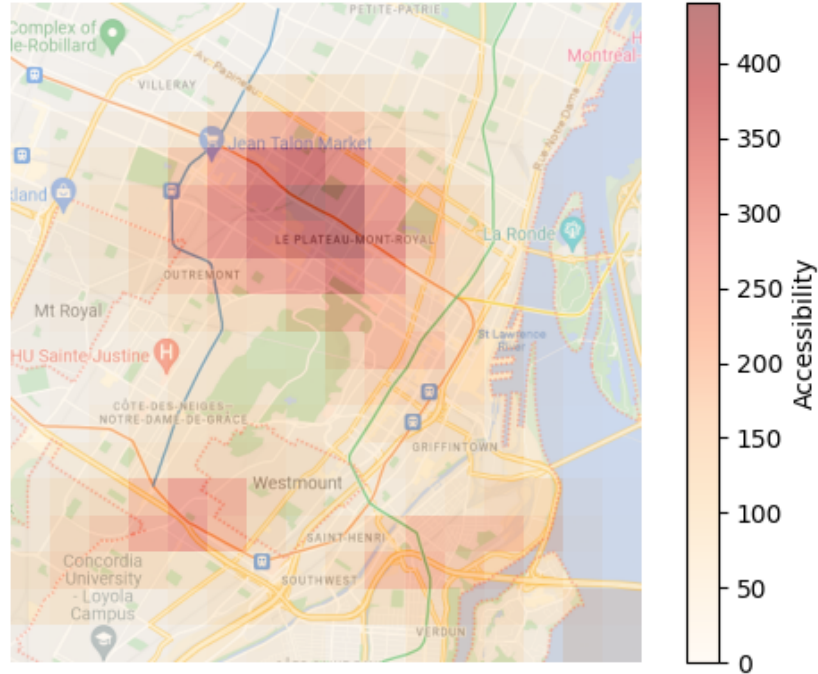


Figure 4.3: Accessibility of Montreal Metro network.

consists of all bus stops, while set \bar{B} of non-candidate stops consists of the metro stops. Figure 4.3 shows the distribution of accessibility resulting from the initial PT graph \mathcal{G} of Montreal, which we assume consists of only the current lines. These assumptions may correspond to the case in which the PT operator wishes to completely redesign bus lines (so that we can remove all bus lines from our initial graph \mathcal{G}). The metro network is composed of 4 lines of different sizes. It mostly covers the center of Montreal, which delineates our environmental boundaries. In our case, bus stop locations can change throughout the year. We wanted to check that our method conveys good results, regardless of the set of stops. This is the reason our results are shown for 100 different instances of the problem, each with a different set of stops. The real locations of bus stops in Montreal can be easily added to the training set as instances. To establish preliminary results, we first limit the number of bus stops to an arbitrarily low number of 72. We generate the bus stops as follows. We tessellate the territory with a regular grid. Note that this tessellation is not necessarily the same as the one described in Section 4.2.1. Within each cell of such a grid, a bus stop is created at a random location within the cell. By generating bus stop locations in this way, we ensure the

uniformity of the bus stops on a larger scale. Consequently, all areas of the territory have access to the public transport network in all regions. We extract points of interest from OpenStreetMap [124] in combination with the Overpass API and assign the point of interest number in their area to the centroids. For the points of interest, we select some of the main amenities in a city (schools, hospitals, police stations, libraries, cinemas, banks, restaurants, and bars). Some of these amenities present a distribution that is relatively uniform over the map (e.g., schools), while others have higher densities in popular streets (e.g., restaurants). Scenario parameters are in Table 6.1.

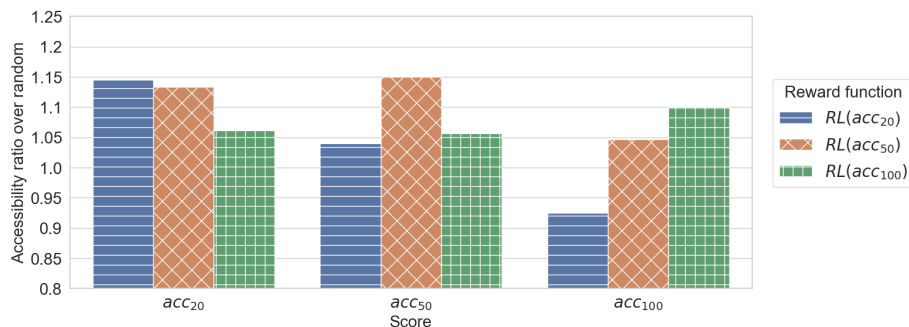


Figure 4.4: Accessibility ratio ($\frac{acc_a(RL)}{acc_b(random)}$), $a, b = 20, 50$ or 100) via our Reinforcement Learning Equality algorithm (Algorithm 5) against the random search algorithm. x-axis shows different metrics of the Random search algorithm, and the labels show the different metrics used by RL methods.

Table 4.1: Scenario parameters.

Parameter	Value
Number of bus stops n_b	72
Number of lines k	3
Maximum accessibility time T_{max}	30 minutes
Walking speed s_w [125]	4.5 km/h
Bus speed s_b [126]	28km/h
Fleet size per line N_l	10
Distance between adjacent centroids	1km
Discount factor γ	0.95

4.4.2 Baselines

We compare the performance of our algorithm to two baselines. To compare the algorithms, we define a maximum running time of 1-hour for each method and check which bus line deployment yields the best result. Specifically, 1 hour is the training time for our algorithm; our testing time is

too short and can be ignored. For Baselines, they keep looking for better graphs within one hour. After one hour, we simply stop searching and output the current optimal graph of Baselines.

4.4.2.1 Random Search Algorithm

Random states of the form $S = (l_1, \dots, l_k)$ are generated, each corresponding to a random partition of the set \mathcal{B} of candidate stops. At every generated state $S = (l_1, \dots, l_k)$, each set l_i is sorted as described in Section 4.3.4 to generate the corresponding lines. Then the corresponding graph \mathcal{G}^{rnd} is constructed, and accessibility $acc^q(\mathcal{G}^{\text{rnd}})$ is computed. This process is repeated until the running time threshold is exceeded. The largest value of $acc^q(\mathcal{G}^{\text{rnd}})$ found at that point is then returned. This random search algorithm has the advantage over our algorithm of visiting very diverse states without taking time for computing node embeddings, rewards, etc. The number of states visited by this algorithm is much larger than the number visited by our approach, within the same running time threshold.

4.4.2.2 Genetic Algorithm

The second baseline is a genetic algorithm (Algorithm 11). It is a popular metaheuristic to solve combinatorial problems, and PT design problems in particular [17]. In transit network design, state-of-the-art algorithms are generally metaheuristic. There are many flavors of GA for transit network design. Our GA algorithm is an effort to reproduce those. However, due to difficult reproducibility of such methods, we cannot claim our GA algorithm is exactly the same as the one in the state of the art. We are sure that we did our best to improve our GA algorithm, in the effort to have it represent the state of the art. We adopt $acc^q(\mathcal{G})$ as the fitness function. Next, the design of the evolutionary functions must be done carefully so that the new generations inherit good genes from their parents. Therefore, the most important function that must be modified for that case is the crossover function. More popular approaches, such as Order Crossover 1 (OX1), preserve the relative order of the states and thus their structure. They are used in similar problems, like the Traveling Salesman Problem (TSP) [127] and the Vehicle Routing Problem (VRP) [128]. Children can benefit from advantageous orderings of the parent nodes, as from our experience, close nodes (sorted by the *SORT* function) are often good for increasing accessibility.

4.4.3 Results

For the first comparison, we train our Reinforcement Learning Equality agent and test it on the same bus stop distribution. The random search baseline runs with different accessibility metrics. In Figure 4.4, the different colors correspond to different accessibility metrics adopted as rewards for our RL algorithm. Figure 4.4 shows that our approach outperforms the random baseline by 10%-15% when trained and tested on the same accessibility metric (e.g., the agent optimizes acc_{20} with its reward function and is tested on the same metric). Our algorithm also improves on other accessibility metrics compared to the one it has been trained on by an average ratio of 5% over the baseline. Only the agent trained on acc_{20} performs worse than the random baseline. Because

Algorithm 4: Genetic algorithm.

- 1 **Input:** Size n_{pop} of the population, number of lines n_{lines} , number of parents n_{par} per generation, timeout t_{limit} of the benchmark, probability P_{mut} of an attribute to mutate ;
Result: The best accessibility over all generations acc^{best} .
 - 2 **Initialization:** Initialize $pop_0 = (I_1, \dots, I_{n_{pop}})$ at generation 0 as individuals with random partitions of the bus stops between the lines. Sort the lines of each individual. Compute acc_0^{best} ;
 - 3 **while** *Running time threshold is exceeded* **do**
 - 4 Select n_{par} best $parents = Tournament(pop, acc_{20})$;
 - 5 **while** pop_i is filled with n_{pop} children. **do**
 - 6 $p_1, p_2 = Sample(parents, 2)$;
 - 7 $c = OX(p_1, p_2)$;
 - 8 $c = Mutate(c, P_{mut})$;
 - 9 Sort the lines of each child $c = SORT(c)$;
 - 10 $pop_i.Append(c)$;
 - 11 Compute $acc_i^{best} = \max(acc^q(pop))$;
-

when we optimize PT with equality (acc_{20}) as the optimization goal, we may lose some efficiency (acc_{100}). Planning bus lines needs to make a trade-off between equality and efficiency. It should be noted that the balance between equality and efficiency is a “political” choice that the transport planner should make, and it is not possible to define the “best” value of the quantile. A practical option for the transport planner could be to obtain different networks, with different quantiles, and then compare them a posteriori to check which one is the most adapted to the scenario at hand. In any case, our method is agnostic to the “political” choice of $q\%$.

Our agent is also able to apply to different bus stop distributions. We assume that each bus stop remains in the same grid cell, but its position is changed for different distributions. Now we select acc^{20} as our accessibility metric for training and testing on a set of 100 PT graphs with different bus stop distributions. In Fig. 4.5, we plot the value of the CDF (y-axis) of acc_{20} improvement values.

$$R = \frac{acc(algo) - acc(random)}{acc(random)}, \quad (4.17)$$

where $acc(random)$ is the optimized acc^{20} value from the random search, and $acc(algo)$ is the optimized acc^{20} value from Algorithm 5 or GA. The higher the value of R (x-axis), the better the Algorithm 5 or the GA is compared to the random baseline. We observe in both cases that R follows a normal distribution. Using this data, we conduct a t-test with the null hypothesis (H_0) that $mean(R) = 0$. We run the t-test and obtain a T-statistic value of 17. From the p-value $p = 3 * 10^{-31}$, we can reject H_0 with confidence greater than 99.9%. Thus, we confirm that our approach works better than a random baseline on graphs with different bus stop distributions. Similarly, the Genetic

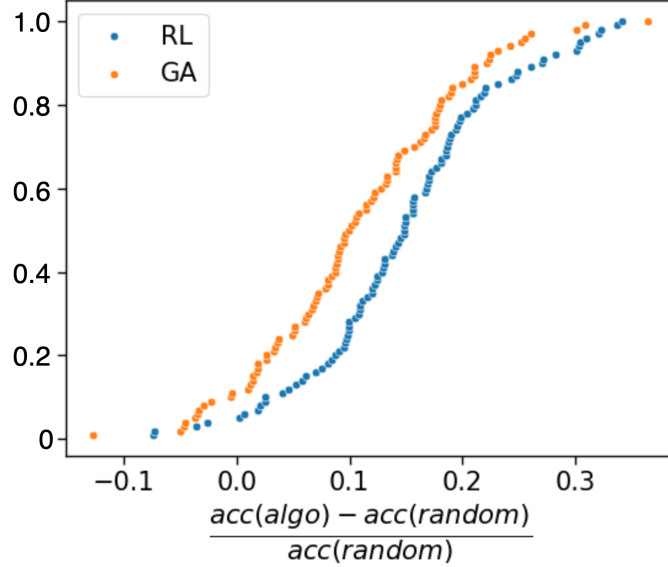


Figure 4.5: CDF of acc_{20} improvements of different algorithms comparing with random baseline.

Algorithm performs better than the random baseline. Using a similar procedure to compare our Reinforcement Learning Equality algorithm and the Genetic algorithm, we show that our approach performs better on different test graphs with confidence $> 99.9\%$ (the T-statistic value is 8).

We next test the performance of our algorithm on the actual geography of Montreal. Considering acc^{20} as the objective function, our algorithm focuses more on optimizing those areas with the 20% worst accessibility. We also calculate the value of improvements compared to the Random Baseline ($acc_{RL}^{20} - acc_{Random}^{20}$) for each centroid with the lowest accessibility in the initial graph. Figure 4.6 shows that our approach performs better than the random baseline in these areas; the improvement in most of these areas exceeded 40%.

4.5 Conclusion

In this chapter, we propose an approach that combines Message Passing Neural Networks (MPNN) and Reinforcement Learning (RL) to optimize bus line design. The objective is to reduce the inequality in the distribution of accessibility provided via Public Transport (PT). Our results showed that MPNN and RL are more effective than commonly used metaheuristics, as they can capture the PT graph structure and learn the dependencies between lines and the distribution of Points of Interest, whereas metaheuristics restrict themselves to a random exploration of the solution space.

In future work, we will test how our method generalizes to different metro networks, realistically modeled via open data, and how it scales when the problem involves many more bus stops than those considered here. In fact, different transit modes can be represented by different graphs.

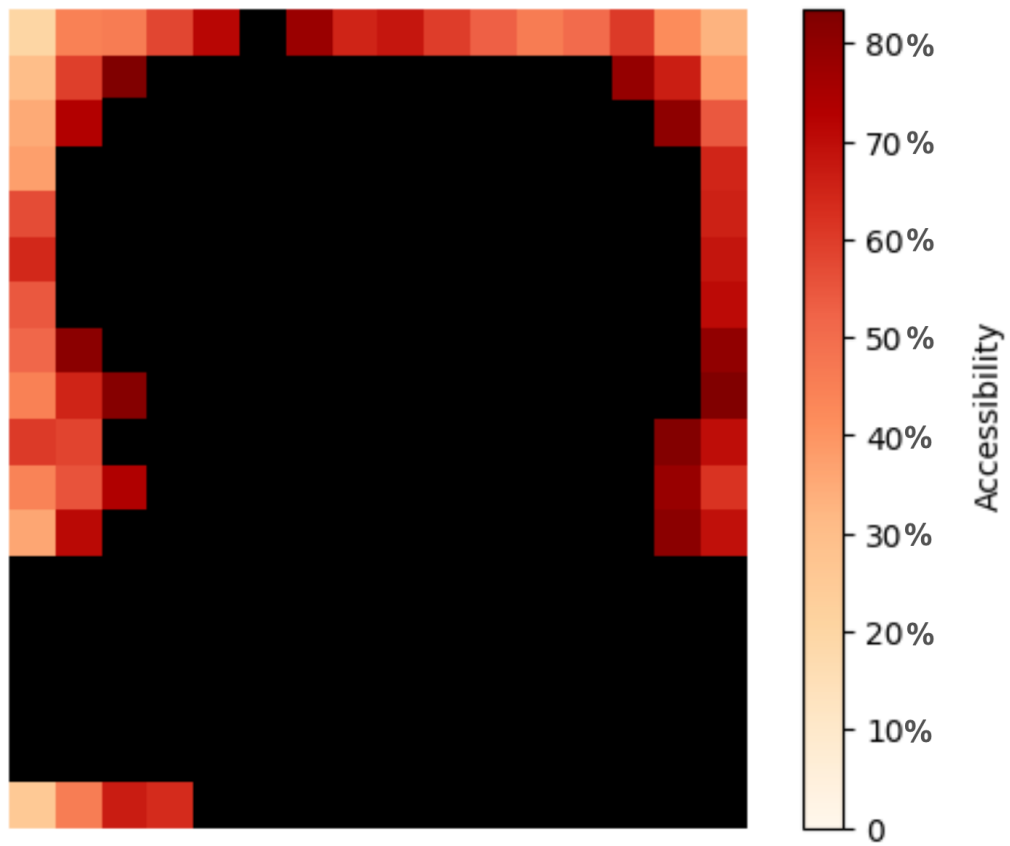


Figure 4.6: Heatmap of percentage improvement of accessibility (%) via Reinforcement Learning Equality algorithm against the random baseline on underserved areas (The darker red areas indicate that our approach improves more obviously than the random baseline in these areas. Black areas with the best 80% accessibility are not considered).

We can connect these graphs to one another to form a multilayer graph. Then, our method can be applied with modifications to the multilayer graph. Of course, for metro networks, once the network infrastructure is put in place, it cannot be easily modified, as it can be with bus lines. Results from the algorithm would thus require a thorough “political evaluation” of the outcomes of the algorithm before implementation. For example, We can set different weights so that the algorithm prefers to modify bus lines rather than metro lines. Moreover, we expect even better results when allowing the same bus stop to be part of multiple bus lines (which is not the case in this work).

Since there are no available methods for planning PT networks with the aim to reduce accessibility inequality, we only compared here our proposed method to a metaheuristic commonly used for transport problems. However, we could potentially devise more sophisticated methods to perform the same optimization task and compare their performance with our proposed message-passing approach. For the lack of time, we left this for future work.

The main takeaway from this chapter is that combining MPNN and RL is promising for solving PT network design during the strategic planning phase.

Chapter 5

Planning Demand-Responsive Transit to reduce inequality of accessibility

5.1 Introduction

The *accessibility* of a location measures the ease with which its residents can reach surrounding opportunities (e.g., schools, shops, or jobs). Here, we study Public Transit (PT) accessibility, i.e., how many opportunities per unit of time one can reach by using PT. Accessibility is unequally distributed in urban regions [2]: it is generally good in the city centers and areas close to main public transport corridors, and poor in the suburbs, which are thus car-dependent. This makes cities environmentally, economically, and socially unsustainable [113, 129]. Minimizing the inequality of the accessibility distribution in a territory is thus crucial and is the central focus of this study.

Poor accessibility in the suburbs is a chronic problem of conventional public transportation, which is based on fixed routes with predetermined schedules. Offering a good service requires an extremely large number of lines, stops, and frequencies. Considering the low population density, the cost per-capita would be prohibitive for a PT agency.

By dynamically building routes that adapt to the observed user requests, Demand-Responsive Transit (DRT) is better suited for low density areas [3]. DRT is also a promising complement to conventional PT [10] for serving the first-mile and last-mile for suburbs. Autonomous vehicles are expected to reduce costs, thus making DRT a feasible option for PT operators [43]. DRT is already deployed in real cities, often by private companies (e.g., [130–132]) that make appropriate agreements with transport authorities [133]. However, to the best of our knowledge, such deployments are not planned having a precise quantification of the reduction of accessibility inequality in mind. Even in the literature, most of the proposed planning strategies for DRT aim to minimize generalized costs, including both user and agency related costs [10, 42]. Limited attention has been paid to the quantitative assessment of how DRT allocation strategies influence the inequality of accessibility at the city-wide level. Moreover, there is a lack of research that formulates DRT allo-

cation strategies grounded in clearly defined inequality indicators of accessibility. This study seeks to address this gap in the literature and addresses the deployment of DRT in the first and last mile.

Three aspects make this problem challenging. *First*, accessibility is the result of intricate and latent dependencies between different parts of the PT network. Therefore, a local change may affect accessibility of remote areas. Such dependencies are coupled in a non-trivial fashion with the topology of the PT network. *Second*, there is a complex demand-supply dependence, since the performance of DRT impacts the number of users that choose to use it, which in turn impacts the performance. *Third*, when considering accessibility inequality, one should not only examine the geographical distribution of accessibility, but also population density. This is important in order to prevent DRT planning decisions from being heavily driven by few people who have chosen to live in isolated areas. Population and accessibility distributions may be contrasting.

Our main contributions are the following:

- We combined a Continuous Approximation (CA) model of DRT and a graph model of conventional PT. CA allows to efficiently estimate the average performance indicators of DRT. On the other hand, the graph model of conventional PT represents the topology of the network that already exists in a city, thus capturing the geographical dependencies in the accessibility computation. This makes the findings obtained with our model relevant for the city under study. We obtain a single multilayer graph describing a multimodal PT, containing both DRT and conventional PT, in which accessibility can be computed.
- We propose an optimization method to support DRT planning, at the strategic level, in an urban conurbation. The method suggests in which areas to deploy DRT and with how many vehicles. The objective is to reduce the spatial inequality of the accessibility distribution. To solve the demand-supply circular dependency, our optimization procedure is bilevel: the upper level decides how many DRT vehicles to deploy in each area, while the lower level iteratively solves a transit assignment problem, to estimate how travelers distribute in the multimodal network.

Numerical results in scenarios representing three different cities show that our method effectively reduces accessibility inequality and shows superior performance when compared with state-of-the-art Bayesian optimization algorithms. We release our code to run our strategy and reproduce the analysis as open source [134]. A simplified version of our code for more demonstration purposes is also available as open source [135].

The chapter is organized as follows: Section 5.2 introduces the a multilayer graph which combines a Continuous Approximation model of DRT and a graph model of conventional PT. Section 5.3 formulates the problem of optimizing inequality of accessibility as an integer programming model. To solve it, we propose a bi-level optimization algorithm: the upper level addresses the allocation of Demand-Responsive Transit (DRT) resources, while the lower level updates the multi-layer network via a Traffic Assignment procedure. Section 5.4 compares its performance with the state-of-the-art Bayesian optimization method across three different cities. Finally, Sections 5.5 and Sections 5.6 discuss future perspectives and conclude the chapter, respectively.

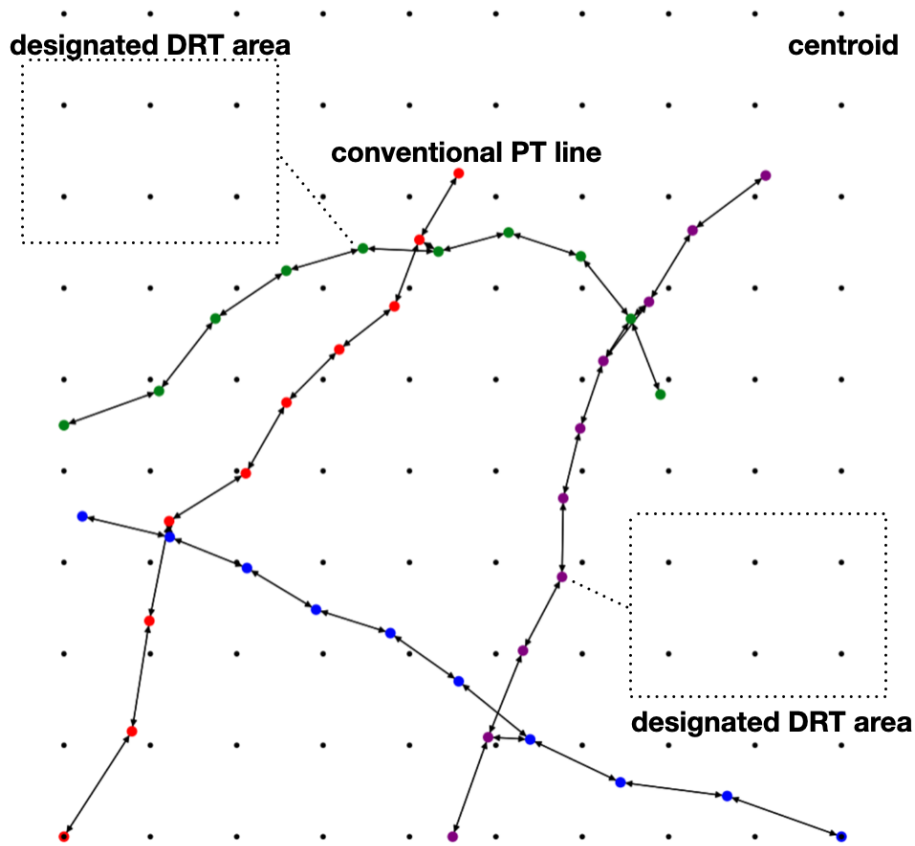


Figure 5.1: Multimodal Public Transit

5.2 Model

We study a multimodal urban transit system, as in Fig. 5.1:

1. Conventional PT, consisting of metro, train or bus lines, with regular routes and schedules.
2. Demand-Responsive Transit (DRT), provided by bus, acting as a feeder for conventional PT and serving the First Mile and Last Mile.

Fig. 5.2 presents the types of trips a user can perform in the considered PT system. A user can walk to the PT station or choose to go to the PT station via a DRT bus from the origin. Symmetrically, the “last mile” from a PT station to the user’s destination can be performed by walking or DRT. A user can also walk directly from origin to destination. We assume a user always chooses the combination of modes with which they can go from origin to destination.

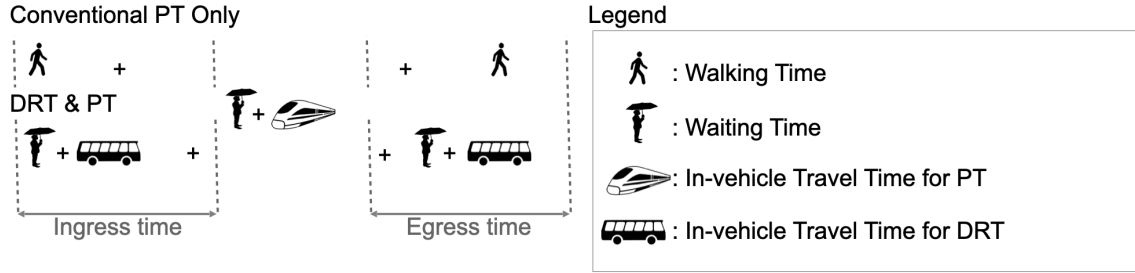


Figure 5.2: Components of the access, egress and waiting time for the case with conventional PT only and the case with DRT.

We now describe the model that we use as a base for RSG optimization. After explaining the tessellation of the study area (see Sec. 5.2.1), we first explain how we model conventional PT (see Sec. 5.2.2) and DRT (see Sec. 5.2.3) separately and how we merge such models into a single multilayer graph (see Sec. 5.2.4). We then explain how we generate the demand (see Sec. 5.2.5). Table of notation is in 8.1.

5.2.1 Tessellation of the study area

We partition the study area with a regular tessellation. In the experiments, we adopt square tiles of 1 Km^2 , but any other regular shape could be employed. The center of each tile is called *centroid*. We denote by $\mathcal{C} := \{c_1, \dots, c_m\}$ the set of all m *centroids* in the study area. By slight abuse of notation, we will sometimes refer to “tile c_i ” to denote the “tile around centroid c_i ”. We will thus use “tile” and “centroid” interchangeably when this does not create ambiguity.

Opportunities are distributed in the tiles. They can be shops, restaurants, schools, etc. In the experiments. We assume tiles are small enough so that the distance between any point in a tile and the corresponding centroid in the center is negligible. We can thus approximately assume all trips generate from a centroid and end at another centroid. In some cases, approximating the destination to the closest centroid may overestimate the time of some trips and underestimate the time of other trips. These overestimated and underestimated times statistically compensate each other. The same argument applies to the origin. Note that, as in several studies on accessibility [29], we prefer regular tessellation to standard zoning. Regular tessellation is preferred in various studies due to its ability to simplify complex data and provide a clearer interpretation [136]. This will allow simple development of the CA model (Equations 5.2-5.8).

5.2.2 Graph model of conventional PT

We model conventional PT as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ (see Figure 5.1), \mathcal{V} is the set of all nodes and \mathcal{E} is the set of all edges. \mathcal{G} is composed of all conventional PT lines. We will refer to them as “metro lines”, for brevity, but the model does not change if a part of those lines is served via buses.

Set $\mathcal{S} \subset \mathcal{V}$ is the set of all stations and set $\mathcal{E}_s \subset \mathcal{E}$ is the set of all edges of metro lines. A line l has a headway t_l , which is the time between two consecutive vehicles in the same direction. We approximate the average waiting time for line l as $w_l = t_l/2$ as in [92]. Line l is a sequence of K_l stations, s_1, \dots, s_{K_l} , linked by edges. Let us denote with $t(s_i, s_{i+1})$ the time spent by a vehicle going from station s_i to the next station s_{i+1} . To allow boarding and alighting at station s_i , the vehicle stops for a dwell time t_{s_i} . If a passenger is at a transfer station s_i and changes from PT line l to line l' , we consider an average waiting time $w_{l'}$ at station s_i .

We also include in \mathcal{V} the set of centroids \mathcal{C} . We also include in \mathcal{E} the set \mathcal{E}_c of all edges between to neighbor centroids, and the set \mathcal{E}_d of edges between any centroid and all stations. Each edge $e \in \mathcal{E}_c \cup \mathcal{E}_d$ represents the possibility to walk with the time cost t_e . Time cost t_e is calculated by: $t_e = d_e/v_{\text{walking}}$, where d_e is the Euclidean distance between two nodes of edge e , and v_{walking} is the walking speed. Note that we are studying a geographic graph, and each node has its location coordinates, so the Euclidean distance between any nodes can be calculated. Therefore, in the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, we have $\mathcal{V} = \mathcal{C} \cup \mathcal{S}$ and $\mathcal{E} = \mathcal{E}_s \cup \mathcal{E}_c \cup \mathcal{E}_d$.

For a trip from centroid c_i and c_j , the traveler can choose between different modes of travel. For example, travelers could perform the entire trip by walking at speed v_{walking} or they could walk from the origin centroid c_i to a conventional PT station, go via conventional PT to another station, and from there walk to the destination centroid c_j . Travelers always select the shortest time path between the origin centroid and the destination centroid (for brevity, we will call it “shortest path”). This is more accurate than [10, 42] in which shortest distance path is selected.

Note that graph \mathcal{G} represents a PT network in a certain time slot, e.g. 1 hour, within which we can assume that line routes and headway values do not change. To obtain a planning over a day, the optimization we will present here should be repeated in the different time-slots.

5.2.3 Continuous Approximation (CA) model of DRT

Due to financial constraints, there is a limitation on the total number of buses that can be deployed for DRT services. Using more buses means more expenses, such as repair costs and more employee salaries. Because of this, only selected *designated DRT areas* can be served via DRT. Vehicles will depart from a selected conventional PT station at regular times to pick up travelers in the designated DRT area. As a result, passengers in the service area will travel as shown in Figure 5.2. A passenger may wait until a DRT bus arrives (waiting time) and spend some time in the bus (in-vehicle travel-time) to the conventional PT station. Symmetrically, to reach the final destination from an egress PT station, a passenger must walk or use another DRT service, if available.

The DRT model is shown in Figure 5.3. For a generic designated DRT area A associated to conventional PT station s_A , a DRT bus departs from s_A and proceeds to point s_1 , without performing any pick up/drop off between s_A and s_1 . Starting from s_1 , the bus serves each tile c_i in sequence and then returns to station s_A via point s_1 .

Our DRT model is based on Sec. 4.4 of [10] and relies on two assumptions:

- A DRT bus can only travel along the horizontal and the vertical direction following a no-backtracking policy.

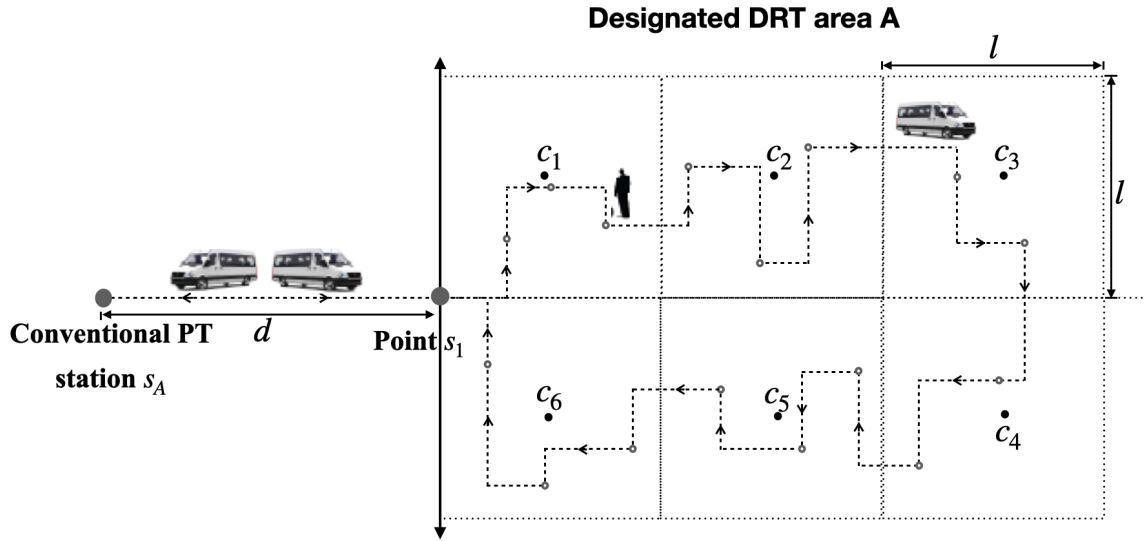


Figure 5.3: Model of Demand Response Transit (DRT).

- Demand is uniformly distributed within each tile, but it changes from tile to tile (see Sec. 5.2.5).¹

Obviously, in reality, these aspects might be more complicated. However, simplifying assumptions are common (and necessary) in any strategic level decision making process, like the one we tackle in this chapter. As shown in real world examples, going from high-level strategic decisions to detailed plans [137], the findings we get thanks to those simplifying assumptions are an important guidance step toward realistic designs, where findings are adapted to the specific use case at hand, via appropriate methods (e.g. agent based models), which are outside the scope of this chapter.

We extended the model of [10] in the following terms:

- We combine a CA for DRT in the first and last mile with a graph representation of conventional PT
- We do not just model population densities, but also opportunity densities.
- We allow the tiles contained in a DRT area to have different population and opportunity densities. We consequently allow the bus to have different number of pickups and dropoffs in each tile.
- We allow the main PT station associated to each DRT area to be outside of the area.

¹If tiles are sufficiently small this assumption is not problematic. In any case, the granularity of demand locations is limited by available data that, for privacy reason, usually aggregate over the space. As an example, the open dataset that we use in the numerical section (Table 5.1) are aggregated at a 1Km² level.

We now derive the equations of the Continuous Approximation of DRT. Let time $h(A)$ denote the DRT service headway, which means a new DRT bus leaves from the starting station s_A every time $h(A)$, e.g., if $h(A) = 5$ mins, there a DRT bus will leave from the starting station s_A every 5 mins.

Referring to Figure 5.3, we denote with K the number of tiles in area A , with l the side of each tile. Let x_A be the number of DRT buses allocated to area A . Let v_{DRT} be the DRT bus speed, τ_s the time lost per stop (assuming a single passenger served per stop) and τ_T the dwell time of the DRT bus at the terminal s_A . Let d the distance between stop s_A and point s_1 .

The following proposition allows, for any allocation of DRT buses, to compute $h(A)$. This result will be later used for transit assignment.

Proposition 1. *Let $n(A)$ denote the number of pickups and dropoffs requested within area A , generated during time $h(A)$. Thus, $n(A)$ is the number of requests to be served by one bus. Let $\phi_i^{DRT, in}$ denote the number of passengers per unit of time, having tile c_i as destination, and using DRT for their last mile. Let $C_{DRT}(A)$ be the cycle time, i.e., the expected time required to complete a cycle of a DRT bus, which corresponds the time between two departures of the same bus.*

The relation between those quantities is expressed by:

$$C_{DRT}(A) = h(A) \cdot x_A \quad (5.1)$$

$$n(A) = h(A) \cdot l^2 \left(\sum_{c_i \in A} \phi_i^{DRT, out} + \sum_{c_i \in A} \phi_i^{DRT, in} \right) \quad (5.2)$$

$$\begin{aligned} h(A) \cdot x_A &= \frac{Kl}{v_{DRT}} \cdot \frac{h(A) \cdot l^2 (\sum_{c_i \in A} \phi_i^{DRT, out} + \sum_{c_i \in A} \phi_i^{DRT, in})}{h(A) \cdot l^2 (\sum_{c_i \in A} \phi_i^{DRT, out} + \sum_{c_i \in A} \phi_i^{DRT, in}) + 1} \\ &\quad + \left(\frac{l}{3v_{DRT}} + \tau_s \right) \cdot h(A) \cdot l^2 \left(\sum_{c_i \in A} \phi_i^{DRT, out} + \sum_{c_i \in A} \phi_i^{DRT, in} \right) \\ &\quad + \frac{4l}{3v_{DRT}} + \frac{2d}{v_{DRT}} + \tau_T \end{aligned} \quad (5.3)$$

Proof. To prove the proposition, we will compute $C_{DRT}(A)$ in two ways: first, considering the point of view of multiple buses, second, considering the point of view of a single bus.

Point of view of multiple buses. The equation relating $h(A)$ and $C_{DRT}(A)$ is straightforward:

$$h(A) = \frac{C_{DRT}(A)}{x_A}$$

For example, if on average, each bus takes 1 hour to complete its cycle, and we have 10 buses, then the headway $h(A)$ is 6 minutes.

Point of view of a single bus. We now calculate the number of requests $n(A)$. To ensure the stability of the system (i.e., no degenerating accumulation of unserved requests), on average, each bus needs to serve a number of requests equal to those generated during time $h(A)$. Since l is the side of a tile, each tile has an area of l^2 , and the number of pickups and dropoffs in designated DRT

area A is the following sum over all tiles c_i contained in designated DRT area A :

$$n(A) = h(A) \cdot l^2 \left(\sum_{c_i \in A} \phi_i^{\text{DRT, out}} + \sum_{c_i \in A} \phi_i^{\text{DRT, in}} \right) \quad (5.4)$$

The two terms in the above sum correspond to the number of pickups and dropoffs, respectively.

To calculate cycle time $C_{DRT}(A)$, we first need to estimate length $CL_{DRT}(A)$, which is the expected value of the distance traveled within the entire area A . Such length is represented by the circle in Figure 5.3 and measures how many Km the DRT bus travels from the moment it leaves point s_1 to the moment it comes back. The formula for $CL_{DRT}(A)$ is (refer to 8.2 for details):

$$CL_{DRT}(A) = Kl \frac{n(A)}{n(A)+1} + n(A) \cdot \frac{l}{3} + \frac{4l}{3} \quad (5.5)$$

Therefore, the expected time required to complete a cycle $C_{DRT}(A)$ is:

$$C_{DRT}(A) = \frac{CL_{DRT}(A) + 2d}{v_{DRT}} + \tau_s \cdot n(A) + \tau_T \quad (5.6)$$

where v_{DRT} is the DRT bus' speed, τ_s is the time lost per stop (assuming a single passenger served per stop) and τ_T is the terminal dwell time of the DRT bus at the terminal s_A . Term $2d$ accounts for the distance travelled from s_A to point s_1 and back.

Taking Eqn. (5.5) into Eqn. (5.6), we get:

$$\begin{aligned} C_{DRT}(A) &= \frac{CL_{DRT}(A) + 2d}{v_{DRT}} + \tau_s \cdot n(A) + \tau_T \\ &= \frac{\left(Kl \frac{n(A)}{n(A)+1} + n(A) \cdot \frac{l}{3} + \frac{4l}{3} \right) + 2d}{v_{DRT}} + \tau_s \cdot n(A) + \tau_T \\ &= \frac{Kl}{v_{DRT}} \cdot \frac{n(A)}{n(A)+1} + \left(\frac{l}{3v_{DRT}} + \tau_s \right) \cdot n(A) + \frac{4l}{3v_{DRT}} + \frac{2d}{v_{DRT}} + \tau_T \end{aligned} \quad (5.7)$$

Final step. Note that the only variable in Eqn. (5.7) is $n(A)$, and the other parameters are fixed. Via Eqn. (5.1), we can express the left size of the above equation in terms of $h(A)$ and x_A . We then replace $n(A)$ in the right side via Eqn. (5.2). By doing so, we get the relationship equation between headway $h(A)$, number of DRT buses x_A , and demand flows $\phi_i^{\text{DRT, out}}$ and $\phi_i^{\text{DRT, in}}$:

$$\begin{aligned} h(A) \cdot x_A &= \frac{Kl}{v_{DRT}} \cdot \frac{h(A) \cdot l^2 (\sum_{c_i \in A} \phi_i^{\text{DRT, out}} + \sum_{c_i \in A} \phi_i^{\text{DRT, in}})}{h(A) \cdot l^2 (\sum_{c_i \in A} \phi_i^{\text{DRT, out}} + \sum_{c_i \in A} \phi_i^{\text{DRT, in}}) + 1} \\ &\quad + \left(\frac{l}{3v_{DRT}} + \tau_s \right) \cdot h(A) \cdot l^2 \left(\sum_{c_i \in A} \phi_i^{\text{DRT, out}} + \sum_{c_i \in A} \phi_i^{\text{DRT, in}} \right) \\ &\quad + \frac{4l}{3v_{DRT}} + \frac{2d}{v_{DRT}} + \tau_T \end{aligned}$$

□

The previous Proposition gives the quantitative relation between headway $h(A)$, number of DRT buses x_A , and demand flows $\phi_i^{\text{DRT, out}}$ and $\phi_i^{\text{DRT, in}}$. For example, once we calculate demand flows, we can directly get the average headway $h(A)$ obtained by deploying x_A buses in area A , via Eqn. (5.3). Then, we can compute the cycle time $C_{DRT}(A)$ via Eqn. (5.1).

In the last part of section 3.3, we will introduce how to calculate the expected ingress time $T_{in}(c_i)$ at centroid $c_i \in A$, i.e., the time it would take for a user at tile c_i from the moment they requested a trip to the moment in which they arrive at the designated station s_A . Symmetrically, we define the egress time. Assume designated DRT area A has $2 \times \frac{K}{2}$ tiles ($K=6$ in Figure 5.3). For centroid $c_i \in A$ let $n(c_i)$ denote the number of pickups and dropoffs requested within tile c_i between the departure of a DRT bus and the next. The expected value of ingress and egress access time for centroid c_i is:

$$T_{in}(c_i) = T_{out}(c_i) = \underbrace{\frac{h(A)}{2}}_{(1)} + \underbrace{\frac{\frac{n(c_i)}{2} + \sum_{j=i+1}^K n(c_j)}{n(A)} \cdot \frac{CL_{DRT}(A) + \tau_s \cdot n(A)}{v_{DRT}}}_{(2)} + \underbrace{\frac{d}{v_{DRT}}}_{(3)} \quad (5.8)$$

The three components (1), (2) and (3) of Eqn. (5.8) calculate the average waiting time for the DRT bus, the average in-vehicle time from c_i to s_1 , and the average in-vehicle time from s_1 to s_A , respectively (as shown in Figures 5.2 and 5.3). For component (1), since the headway is $h(A)$, i.e., there will be a new DRT bus every $h(A)$, the average waiting time is $\frac{h(A)}{2}$. We now explain the calculation method of component (2). First, $\frac{CL_{DRT}(A) + \tau_s \cdot n(A)}{v_{DRT}}$ is the expected time used by a DRT bus running within the DRT area. Because the number of pickups and dropoffs $n(c_i)$ within different tiles c_i in the considered designated DRT area varies from a tile to another, the calculation formula differs from [3]. The fraction of pickups and dropoffs demand in tile c_i is $\frac{n(c_i)}{n(A)}$. Therefore $\frac{n(c_i)}{n(A)} \cdot \frac{CL_{DRT}(A) + \tau_s \cdot n(A)}{v_{DRT}}$ is the average time the bus takes passing through tile c_i . A passenger who gets on the bus at c_i needs on average to go through half of tile c_i , so they will experience fraction $\frac{n(c_i)}{2n(A)}$ of stops within c_i . Then, such a passenger will go through all the subsequent tiles, which contain fraction $\frac{\sum_{j=i+1}^K n(c_j)}{n(A)}$ of pickups and dropoffs (Fig. 5.3). Component (3) is just the average in-vehicle time from s_1 to s_A .

5.2.4 Multilayer graph

We now include in \mathcal{G} the set \mathcal{E}_{DRT} of all DRT edges, therefore, we have $\mathcal{V} = \mathcal{C} \cup \mathcal{S}$ and $\mathcal{E} = \mathcal{E}_s \cup \mathcal{E}_c \cup \mathcal{E}_d \cup \mathcal{E}_{DRT}$. The edges in set \mathcal{E}_{DRT} are between any centroid in any designated DRT area A and the corresponding conventional PT station s_A (Fig. 5.3). Their values represent the expected ingress and egress times (Eqn. (5.8)). By optimizing the number x_A of buses deployed in designated DRT area A , we change the values of such DRT edges.

5.2.5 Demand model

The last part of section 5.2, we will introduce how to calculate demand flow $\phi_i^{\text{DRT, out}}$, $\phi_i^{\text{DRT, in}}$ for each tile c_i based on our multilayer graph \mathcal{G} . We now explain how we obtain them for a certain time-slot. Let ρ_i be the population density in tile c_i and *Trip* the trip rate, i.e., the percentage of residents performing any trip in that time-slot. The trip density ϕ_i originating in centroid c_i is

$$\phi_i = \text{Trip} \cdot \rho_i. \quad (5.9)$$

Let σ_j be the amount of opportunities in tile c_j . Note that σ_j represents the ‘‘attractiveness’’ of c_j . Adopting a singly constrained gravity model (formulas (1) and (2) in [138]), the trip density ϕ_{ij} from centroid c_i to c_j is :

$$\phi_{ij} = \phi_i \cdot \frac{\sigma_j \cdot e^{-\beta T(c_i, c_j)}}{\sum_{k=1}^M (\sigma_k \cdot e^{-\beta T(c_i, c_k)})} \quad [\text{trips/hour}] \quad (5.10)$$

being β a constant called dispersion parameter and $T(c_i, c_j)$ the shortest time to go from c_i to c_j on the multilayer graph \mathcal{G} . Eqn. 5.10 captures the fact that travelers tend to go to places with many reachable opportunities in a short time. Observe that, in order to go from c_i to c_j , the shortest path might use DRT in the first mile, or in the last mile, or in both, or in none of them. To account for this, let us assume that c_i is inside a designated DRT area A and that s_A is the associated conventional PT station (Fig. 5.1). We introduce the indicator function $\mathbb{I}_{(c_i, c_j)}^{s_A, \text{first}}$, which is 1 if the shortest path from c_i to c_j passes by s_A (a passenger would enter conventional PT via s_A). Symmetrically, indicator function $\mathbb{I}_{(c_j, c_i)}^{s_A, \text{last}}$ is 1 if the shortest path from c_j to c_i passes by s_A (a passenger would exit conventional PT from s_A to reach c_k via DRT). Otherwise, such indicators are 0. The numbers of DRT users originating and directed to centroid c_k are, respectively (see definitions in page 69)

$$\phi_i^{\text{DRT, out}} = \sum_{c_j \notin A} \phi_{i,j} \cdot \mathbb{I}_{(c_i, c_j)}^{s_A, \text{first}} \quad (5.11)$$

$$\phi_i^{\text{DRT, in}} = \sum_{c_j \notin A} \phi_{j,i} \cdot \mathbb{I}_{(c_j, c_i)}^{s_A, \text{last}} \quad (5.12)$$

5.3 DRT Planning Strategy

We first introduce accessibility, which is at the base of our approach, and the inequality indicator we will use to evaluate the quality of our method. We then describe our optimization procedure.

5.3.1 Accessibility and inequality

The accessibility of a centroid measures how well it is connected to the surrounding opportunities (schools, people, businesses). We select the classic *gravity-based* definition of accessibility [139],

which can be intended as the number of opportunities reachable per unit of time. The accessibility of centroid $c_i \in \mathcal{C}$ is

$$acc(c_i) = \sum_{c_j} (f(T(c_i, c_j)) \cdot \sigma_j) \quad (5.13)$$

where $T(c_i, c_j)$ is the shortest time to go from c_i to c_j , $f(\cdot)$ is the impedance function and σ_j is the amount of opportunities of the tile having centroid c_j . There are many different impedance functions, we choose $f(T) = T^{-1}$ as our impedance function. The accessibility using T^{-1} as impedance function represents the number of opportunities that can be reached in a unit of time, which is easier to interpret. In broad terms, a large $acc(c_i)$ indicates that the centroid c_i is well connected to opportunities. Let \mathcal{P} be the set of all individuals in the study area. For all individuals $p \in \mathcal{P}$ resident in tile c_i , we define their accessibility as

$$acc(p) = acc(c_i) \quad (5.14)$$

To quantify the benefits of our RSG strategy, we measure the inequality of the distribution of accessibility, before and after our optimization. We quantify inequality via the well-known Atkinson index on the set of individuals accessibility values $\{acc(p) | \text{individual } p \in \mathcal{P}\}$. The Atkinson inequality index is (setting $\varepsilon = 2$, from [100, 140]):

$$Atk(\mathcal{G}) = 1 - \left(\frac{\overline{acc}(\mathcal{G})}{|\mathcal{P}|} \sum_{p \in \mathcal{P}} \frac{1}{acc(p)} \right)^{-1} \quad (5.15)$$

where $\overline{acc}(\mathcal{G}) = \frac{1}{|\mathcal{P}|} \sum_{p \in \mathcal{P}} acc(p)$ is the average individual accessibility. The Atkinson index goes from 0 (perfect equality) to 1 (maximum inequality).

5.3.2 Accessibility inequality reduction problem

Let us consider a PT graph \mathcal{G} and a set $\mathcal{A} := \{A_1, \dots, A_{n_G}\}$ of n_G candidate areas where DRT can be allocated. Due to limited resources, the total number of DRT buses that can be allocated is fixed to N . In broad terms, we consider the problem of the PT operator to deploy DRT, which is a vector $\mathbf{x} = (x_1, \dots, x_{n_G})$ of n_G natural numbers. Each number $x_k, k = 1, \dots, n_G$, is the number of buses allocated in area A_k . Let $\mathcal{G}(\mathbf{x})$ denotes the multilayer graph built as in Sec. 5.2.4, adding DRT deployment according to allocation \mathbf{x} . We aim to solve the following optimization

$$\max_{\mathbf{x}} Atk(\mathcal{G}(\mathbf{x})) \quad (5.16)$$

subject to the following constraints:

$$\sum_{k=1}^{n_G} x_k = N, \quad (5.17)$$

$$x_k \in \mathbb{N}_0, k = 1, 2, \dots, n_G. \quad (5.18)$$

Performing an exhaustive search is infeasible, as the size of the entire solution space is $\binom{N+n_G-1}{n_G-1}$ by the calculation method from [141] and computing the accessibility of each solution is time-consuming in large networks. Our problem is NP-hard (as we show in 8.3). For this reason, we propose a heuristic to solve our problem

5.3.3 Rank Score Greedy (RSG) allocation strategy

We propose a Rank Score Greedy (RSG) planning strategy (Algorithm 5) to allocate DRT resources efficiently, in order to reduce accessibility inequality.

Our basic idea is to identify the regions that need large accessibility improvements and deploy DRT there. If DRT is already deployed in that region, the number of DRT buses is increased. To find the regions in need of improvement, we need a score that quantifies such a need, taking into account both population and accessibility. Intuitively, the regions needing improvements are those that suffer from low accessibility and where many people live. We thus need a score function that matches this intuition in order to guide our planning strategy. To build such a score, we order tiles from the least populated to the most. Let us denote $Rank(\rho_i)$ the position of centroid c_i in the aforementioned list. Separately, we also order tiles from the least accessibility to the most, and denote $Rank(acc(c_i))$ the position of centroid c_i for accessibility rank list. For each centroid $c_i \in \mathcal{C}$, the score of centroid c_i :

$$Score(c_i) = \alpha \cdot Rank(\rho_i) + (1 - \alpha) \cdot (|\mathcal{C}| - Rank(acc(c_i))), \quad (5.19)$$

where α is a constant that weights the importance given to accessibility in the score with respect to population. If the population of c_i is large and the accessibility is low, the score will be large, denoting that c_i “needs” accessibility improvement, as it will benefit many people. Recall that DRT is not deployed within a single tile, but in designated DRT areas with K tiles each, as shown in Figure 5.3. Therefore, we need to associate a score with each potential designated DRT area A :

$$SCORE(A) = \frac{1}{K} \sum_{c_i \in A} Score(c_i) \quad (5.20)$$

A high score for the designated DRT area A means that a large population suffers from low accessibility, and that by deploying DRT there (or by increasing DRT busses), we can potentially reduce overall inequality.

Our approach is a bilevel optimization procedure (see Sec. 9.2 in [117] and Sec. 9.1.3.3 of [118]). The upper level, Algorithm 5, performs planning of the DRT fleet over the study area, iteratively assigning DRT busses to the designated areas with the highest score. At each iteration, the lower level, Algorithm 8, is invoked to solve the transit assignment. This is crucial because increasing the number of DRT busses in a designated DRT area A_i reduces ingress and egress

travel times $T_{in}(c_i), T_{out}(c_i)$ for passengers originating or directed to any tile c_i contained in the area A_i (see Eqn. (5.3)). However, this encourages more passengers to choose DRT, which in turn increases the aforementioned ingress and egress travel times. The transit assignment implemented in Algorithm 8 is static and deterministic (travelers always deterministically choose the shortest time path), which makes our computation simple and computationally efficient. Algorithm 8 alternatively updates (i) traveler flows $\phi_i^{DRT,out}, \phi_i^{DRT,in}$ entering or exiting the designated DRT area A and (ii) travel times until convergence is observed (flows do not change by more than 5%).

Algorithm 5: Rank Score Greedy (RSG) planning strategy

- 1: **Input** Multilayer graph \mathcal{G} with stations \mathcal{S} and centroids $\mathcal{C} = \{c_1, \dots, c_M\}$.
 The limited number of DRT buses N .
 The set of all centroids' demand density: $\Phi = \{\phi_1, \dots, \phi_M\}$.
 - 2: **Initialization**
 - 3: Partition the study region in a set \mathcal{A} of candidate designated DRT areas
 - 4: Set the center point s_0 in the left side of each $A \in \mathcal{A}$ (Fig. 5.3)
 - 5: Associate to each $A \in \mathcal{A}$ conventional PT stop s_A that is closest to s_0
 - 6: Compute all travel times $T(c_i, c_j)$ and shortest paths $P(c_i, c_j)$ for all origin-destination pairs (c_i, c_j) assuming no DRT.
 - 7: Compute all demand $\phi_{i,j}$ for all pairs (c_i, c_j) using Eqn. (5.10).
 - 8: **For** step $i \leftarrow 1$ to N :
 - 9: For each centroid $c_i \in \mathcal{C}$, calculate $Score(c_i)$ via Eqn. 5.19.
 - 10: For each area $A \in \mathcal{A}$, calculate $SCORE(A)$ via Eqn. 5.20.
 - 11: Find the area A_i with the largest $SCORE(A)$.
 - 12: Update the number of DRT buses $n_{A_i} = n_{A_i} + 1$.
 - 13: Update multilayer graph \mathcal{G} via Algorithm 8.
 - 14: **End For**
 - 15: **Return** Updated multilayer graph \mathcal{G} .
-

Algorithm 6: Transit assignment within a designated DRT area A

- 1: **Input** Multilayer graph \mathcal{G} .
Area A deployed DRT.
Number of DRT buses x_A for area A .
 - 2: **While flows are not stable** (either $\sum_{i=1}^K \phi_i^{\text{DRT, out}}$ or $\sum_{i=1}^K \phi_i^{\text{DRT, in}}$ change by more than 5%):
 - 3: **For** each tile c_i inside designated DRT area A :
 - 4: Update $\phi_i^{\text{DRT, out}}$ via Eqn. (5.11).
 - 5: Update $\phi_i^{\text{DRT, in}}$ via Eqn. (5.12).
 - 6: **End For**
 - 7: Taking $\phi_i^{\text{DRT, out}}$, $\phi_i^{\text{DRT, in}}$ ($\forall c_i \in A$) and x_A into Eqn. (5.3) to find $h(A)$.
 - 8: **If** $h(A) > 0$:
 - 9: **For** each tile c_i inside designated DRT area A :
 - 10: Compute $T_{in}(c_i)$ and $T_{out}(c_i)$ from Eqn. (5.8).
 - 11: Update DRT edge (c_i, s_A) ($(s_A, c_i) \in \mathcal{E}_{DRT}$) with value $T_{in}(c_i)$ ($T_{out}(c_i)$) in graph \mathcal{G} .
 - 12: Find all the shortest path value $T(c_i, c_j)$, $\forall c_i, c_j$.
 - 13: Update demand density $\phi_{i,j}$ and $\phi_{j,i}$ $\forall c_i \notin A$ via gravity model (Eqn. (5.10)).
 - 14: **End For**
 - 15: **ELSE:**
 - 16: Exit the algorithm.
 - 17: **End For**
 - 18: **Return** Updated graph \mathcal{G} with new DRT edges set \mathcal{E}_{DRT} .
-

5.4 Evaluation

5.4.1 Considered scenario

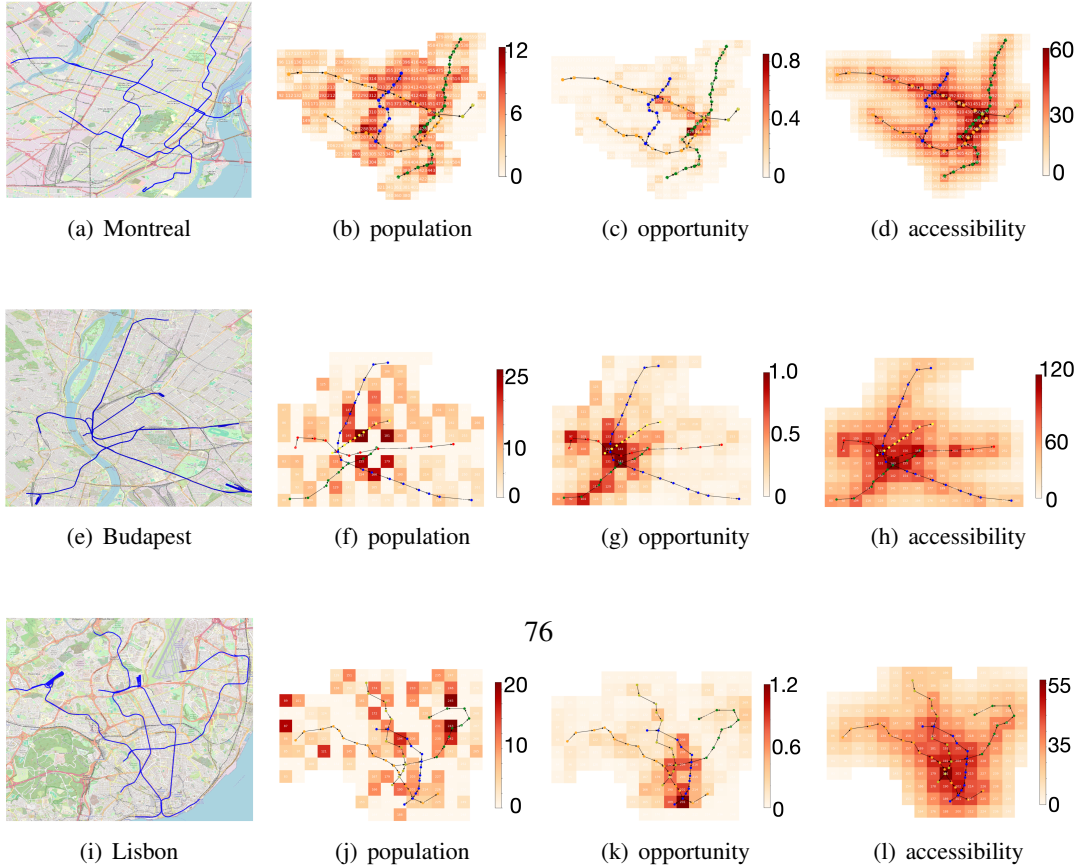


Figure 5.4: OpenStreetMap, population (K ppl./ km^2), opportunity (K opps./ km^2), and initial ac-

We build the simplified model of three cities, which are Montreal, Budapest and Lisbon. We do not include in our analysis fixed-line buses, for simplicity. Fixed buses are identical to metro, from the modeling point of view. So adding them would only imply more time for pre-processing the data and running the optimization, but no additional methodological complexity.

For each city, we take the station locations, the sequence of stations of all lines, the dwell times at each station and the travel time between a station and the other from the General Transit Feed Specification (GTFS) data of Cities [123, 142]. We then get the headway of each metro line during peak hours from [143]. We also take the gridded population of the city from [142]. We extract opportunities, corresponding to Points of Interest (POIs) from [124], which have been shown to be a good proxy of area attractiveness [144]. We perform this extraction via the [145]. We assign to the centroids the number of POIs within their respective tile. For the POIs, we select some of the main amenities in a city (schools, hospitals, police stations, libraries, cinemas, banks, restaurants, and bars). Fig. 5.4 shows the population, opportunities and initial accessibility of each tile (size: 1 km^2) in different cities.

In order to filter out non urbanized regions, we do not include in the study areas the centroids that are located more than 5 km far from a metro line and the tiles with 0 population. The scenario parameters are in Table 5.1.

Table 5.1: Scenario parameters

Name	Parameter	Value	Reference
headway for PT lines	t_l	4, 5, 7, 7.5 min	GTFS data [123]
average waiting time for PT lines	w_l	0.5, 1, 2 min	Eqn. 2 of [92]
walking speed	$v_{walking}$	4.5 km/h	Google Maps
average trip rate per peak hour	$Trip_{peak}$	0.16 trips/person/hour	[138] and [10]
PT share rate	$Trip_{PT}$	12.4%	[146]
time period considered			
peak hour	t_{peak}	6h30 to 8h30 and 17h00 to 20h30	[10]
off-peak hour	$t_{off-peak}$	8h30 to 17h00	[10]
dispersion parameter of Eq. 5.10	β	0.12	[138]
side of a tile	l	1 km	[142]
number of tiles in a DRT area	K	6	-
time lost per DRT pickup or dropoff	τ_s	32 s	[10]
terminal dwell time	τ_T	1 min	[10]
DRT bus speed	v_{DRT}	25 km/h	[10]
weight for the score of Eqn. (5.19)	α	0, 0.25, 0.5, 0.7, 15	-

5.4.2 Demand generation

Let $Trip_{peak}$ and $Trip_{off-peak}$ denote the trips/person/h in peak and off-peak time, respectively. From [138], the average trip rate $Trip_{mean}$ is 1.32 trips/person/day. Assume that there are only trips during peak times and off-peak times in a day. In [10], peak hours are: 6h30 to 8h30 and 17h00 to 20h30 (t_{peak} is 5.5 hours in total), and Off-peak hours are: 8h30 to 17h00 ($t_{off-peak}$ is 8.5 hours in total). And the ratio of average trip rate in one peak hour $Trip_{peak}$ and one off-peak hour $Trip_{off-peak}$ is 10:3.

$$Trip_{peak} \cdot t_{peak} + Trip_{off-peak} \cdot t_{off-peak} = Trip_{mean} \quad (5.21)$$

$$Trip_{peak} : Trip_{off-peak} = 10 : 3 \quad (5.22)$$

Therefore, $Trip_{peak} = 0.16 \text{ trips/person/hour}$ is obtained by calculation. $Trip_{peak}$ also includes the trips generated by using private cars. Since we are considering the comparison of DRT and Montreal’s conventional public transportation, we need to multiply $Trip_{peak}$ by the PT share rate $Trip_{PT} = 12.4\%$ [146] to get the trip rate using PT. We replace this value of trip rate into Eqn. (5.9)-(5.12).

5.4.3 Baselines

We did not find in the literature any approach that combines a graph that represents the real topology of a conventional PT network and a Continuous Approximation (CA) model of DRT. We did not either find any approach to optimize DRT planning to reduce inequality of accessibility. A straightforward comparison with the state-of-the-art is thus necessarily nonexistent. For this reason, to show the relevance of the performance of RSG, we compare it with three different baselines, which are based on advanced optimization methods. The fleet size values range from 0 to 200.

No matter how the upper-level algorithm chooses to allocate DRT buses in designed areas, the lower-level Algorithm 8 is needed to perform transit assignment and update the graph. However, transit assignment is time consuming, rendering learning methods that require extensive training data, as well as metaheuristics like genetic algorithms, impractical. Given allocation vector \mathbf{x} , transit assignment Algorithm 8 updates $\mathcal{G}(\mathbf{x})$ first. Then, we calculate the Atkinson index $Atk(\mathcal{G}(\mathbf{x}))$. The relation between \mathbf{x} and $Atk(\mathcal{G}(\mathbf{x}))$ might be very complex and unknown. In this situations, $Atk(\mathcal{G}(\mathbf{x}))$ can be regarded as a *black box function*, whose evaluation in any point is very time consuming. In such cases, Bayesian optimization has recently been considered the most appropriate method, as it allows to efficiently trade off exploration and exploitation, thus limiting the number of needed function evaluations [147]. Bayesian Optimization cannot be used directly to optimize functions over high dimensional variables, which is the case of \mathbf{x} . It is thus necessary to reduce the dimension of \mathbf{x} . In other words, a subset \mathcal{A}' of areas \mathcal{A} needs to be selected in advance and Bayesian Optimization will be employed to allocate the fleet therein. Algorithm 7 shows how

Bayesian optimization finds an allocation \mathbf{x}^* , after inputting a pre-selected areas \mathcal{A}' .² We propose three different algorithms based on the different methods of preselecting \mathcal{A}' :

- Population Accessibility based Bayesian (PAB) optimization: We have

$$Atk(\mathcal{G}) = 1 - \left(\frac{\overline{acc}(\mathcal{G})}{|\mathcal{P}|} \sum_{p \in \mathcal{P}} \frac{1}{acc(p)} \right)^{-1} = 1 - \left(\frac{\overline{acc}(\mathcal{G})}{|\mathcal{P}|} \sum_{c_i \in \mathcal{C}} \frac{\rho_i}{acc(c_i)} \right)^{-1}$$

from Eqn. (5.15), where the last equality holds because all ρ_i individuals living in tile c_i enjoy the same accessibility $acc(c_i)$ (see (5.14)). Therefore, the contribution of an area A to Atkinson can be measured by $\sum_{c_i \in A} \frac{\rho_i}{acc(c_i)}$. Guided by this finding, we select in \mathcal{A}' the $\varpi \in \mathbb{N}$ areas with the largest value $\sum_{c_i \in A} \frac{\rho_i}{acc(c_i)}$.

- Rank Score based Bayesian (RSB) optimization: We calculate $SCORE(A)$ of all areas $A \in \mathcal{A}$, based on the initial graph \mathcal{G} . We then select in \mathcal{A}' the $\varpi \in \mathbb{N}$ areas with the largest value $SCORE(A)$.
- Rank Score Greedy based Bayesian (RSGB) optimization: We first run our algorithm, RSG (Sec. 5.3.3), with the entire fleet size. RSG will select a list of areas. We take this list as our pre-selected \mathcal{A}' and use Bayesian Optimization (Algorithm 7) to optimize the allocation of buses within \mathcal{A}' .

Value $\varpi \in \mathbb{N}$ is a hyperparameter. We set it to 10. Larger numbers led to high computation time, as typical for Bayesian Optimization.

5.4.4 Results

We first study the performance of RSG with different $SCORE(A)$ (different α of Eqn. (5.19)) in Montreal. When α increases, the weight of the population relative to accessibility increases. As a consequence, the PT structure resulting from our optimization will tend to have better $\overline{acc}(\mathcal{G})$ to the detriment of Atkinson, i.e., the optimization will favor efficiency over equality. Therefore, the choice of α requires a trade-off between efficiency and equality. When $\alpha = 0$, the optimization algorithm only considers areas with low accessibility and does not consider population at all. It usually selects areas with low accessibility but small population, which makes its efficiency very low compared with other α cases. When $\alpha = 1$, the optimization algorithm only considers areas with large population and always allocates DRT buses to the areas with the largest population, which will increase inequality. Therefore, these two extreme cases will not be considered in the following.

Fig. 5.6 shows that hyperparameter α should be set ≤ 0.5 . It is clear that the Population Accessibility based Bayesian optimization method is outperformed by the others. Our proposed method Rank Score Greedy (RSG) shows competitive performance with respect to the Bayesian

²We use the default values of the <https://bayesian-optimization.github.io/BayesianOptimization/2.0.3/>.

Algorithm 7: Bayesian Optimization with pre-selected areas set \mathcal{A}'

- 1: **Input** Objective function $-Atk(\mathcal{G}(\mathbf{x}))$
 \mathcal{A}' is the subset of \mathcal{A}
Number of initial samples $n_{\text{init}} = 5$
Hyperparameter κ (we use $\kappa = 2.576$, i.e. 99% confidence.)
 - 2: Initialize dataset $\mathcal{H} = \emptyset$
 - 3: Determine search space $\mathcal{D} = \{\mathbf{x} = (x_1, \dots, x_{n_{\mathcal{G}}}) \mid x_i = 0, \forall i \in \{i \mid A_i \notin \mathcal{A}'\}\}$
 - 4: **for** $i = 1$ to n_{init} **do**
 - 5: Sample \mathbf{x}_i randomly from search space \mathcal{D}
 - 6: Compute $\mathcal{G}(\mathbf{x}_i)$ via taking \mathbf{x}_i and initial graph \mathcal{G} into Algorithm 8
 - 7: Evaluate $y_i = -Atk(\mathcal{G}(\mathbf{x}_i))$
 - 8: Add (\mathbf{x}_i, y_i) to \mathcal{H}
 - 9: **end for**
 - 10: **Repeat**
 - 11: Fit a surrogate model (Gaussian Process) on dataset \mathcal{H}
 - 12: Let $\mu(\mathbf{x})$ and $\sigma(\mathbf{x})$ be its mean and standard deviation.
 - 13: Select next point \mathbf{x}_i by maximizing acquisition function:
$$\mathbf{x}_i = \arg \max_{\mathbf{x}} [\mu(\mathbf{x}) + \kappa \cdot \sigma(\mathbf{x})]$$
 - 14: Compute $\mathcal{G}(\mathbf{x}_i)$ via taking \mathbf{x}_i and initial graph \mathcal{G} into Algorithm 8
 - 15: Evaluate $y_i = -Atk(\mathcal{G}(\mathbf{x}_i))$
 - 16: Add (\mathbf{x}_i, y_i) to \mathcal{H}
 - 17: **Until** Running time exceeds time T, e.g., T = 1 h
 - 18: $\mathbf{x}^* \leftarrow \arg \max_{(\mathbf{x}_i, y_i) \in \mathcal{H}} y_i$
 - 19: **return** \mathbf{x}^* and $-Atk(\mathbf{x}^*)$
-

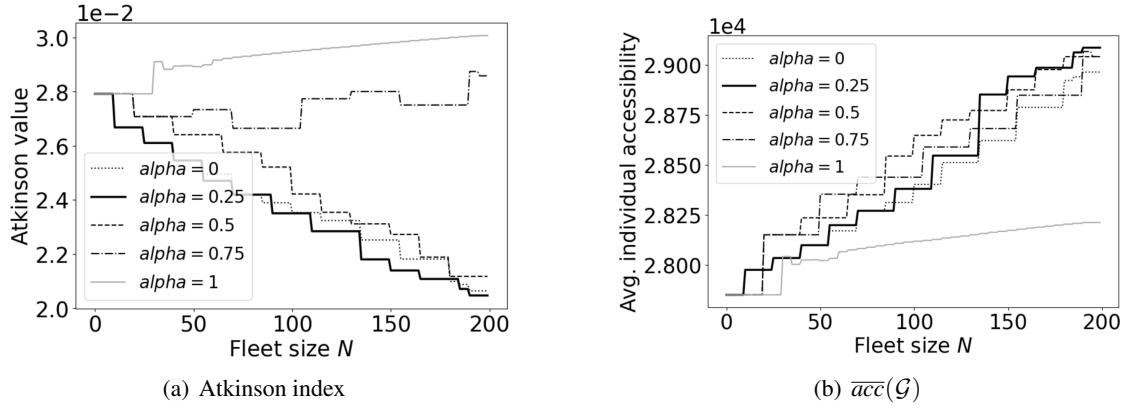


Figure 5.5: The Atkinson index and $\overline{acc}(\mathcal{G})$ of RSG with different SCORE(A) (different α) in Montreal.

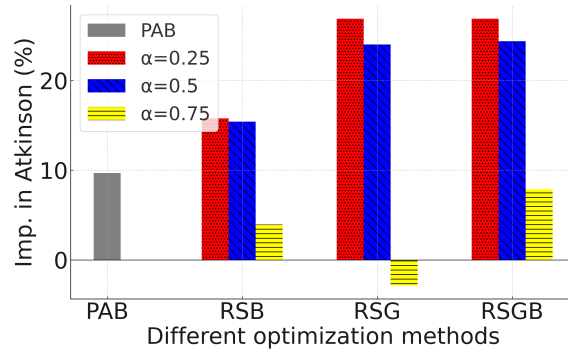
Table 5.2: Improvement in terms of different indexes in different cities (RSG with $\alpha = 0.25$ and fleet size = 200 buses)

Name	Montreal	Budapest	Lisbon	Reference
Atkinson	26.98%	27.36%	21.43%	[100]
Theil	27.04%	26.49%	22.37%	[100]
Pietra	10.37%	8.29%	10.33%	[100]
Palma	8.63%	7.46%	8.55%	[148]

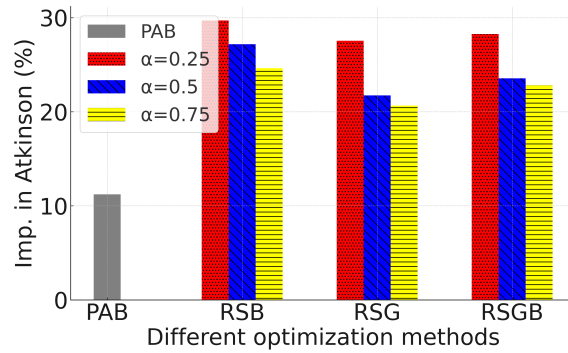
Optimization-based baselines (namely RSB and RSGB). Observe, however, that the computation time of RSG is ≈ 5 minutes, versus ≈ 1 hour for RSB and RSGB. Since RSGB is a method based on RSG, and further optimization is performed using the results of RSG as the initial starting point, the effect should theoretically be no worse than that of RSG. However, it can be seen from the Fig. 5.6 that the improvement is negligible, which means that RSG has found a good solution. If we consider scenarios with more conventional PT lines, larger fleet sizes, in larger areas, the advantage of RSG in terms of computation cost may be very significant. On the other hand, if computation cost is not an issue, RSB and RSGB are also feasible choices. To recap, our proposed RSG effectively reduced inequality and at the same time shows excellent computational efficiency.

The efficiency of RSG is also confirmed when using inequality indicators different from Atkinson's (Table 5.2). This indicates that RSG reduces the gap of accessibility among individuals.

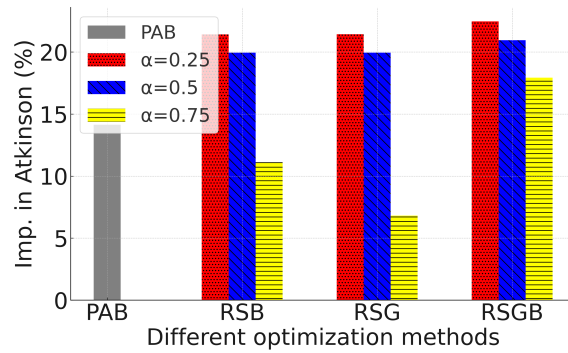
Fig. 5.7 and 5.8 explain the origin of the inequality reduction brought by RSG ($\alpha = 0.25$) in different cities. In these figures, we compare the accessibility of each tile and each individual,



(a) Montreal



(b) Budapest



(c) Lisbon

Figure 5.6: Improvement in Atkinson indices after optimization by different algorithms in different cities with fleet size = 200 buses (running time of PAB, RSB and RSGB are **1 hour**, and running time of RSG is **5 mins**).

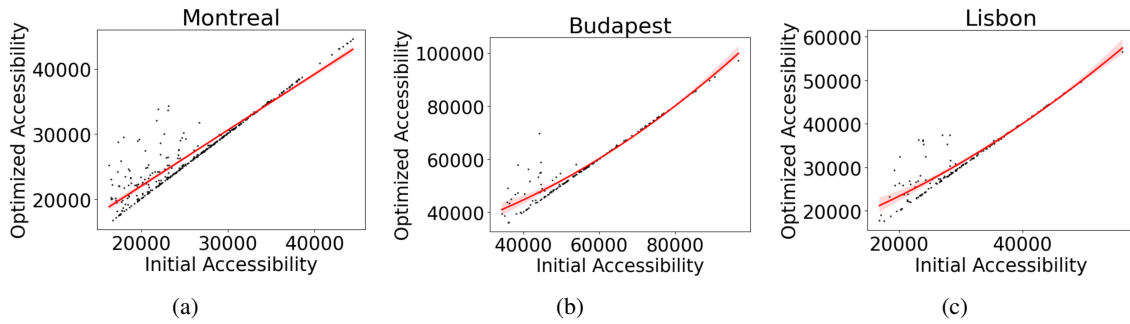


Figure 5.7: Distribution of accessibility ($opp./h$) with the regression lines (RSG, with $N = 200$). Each point represents a tile.

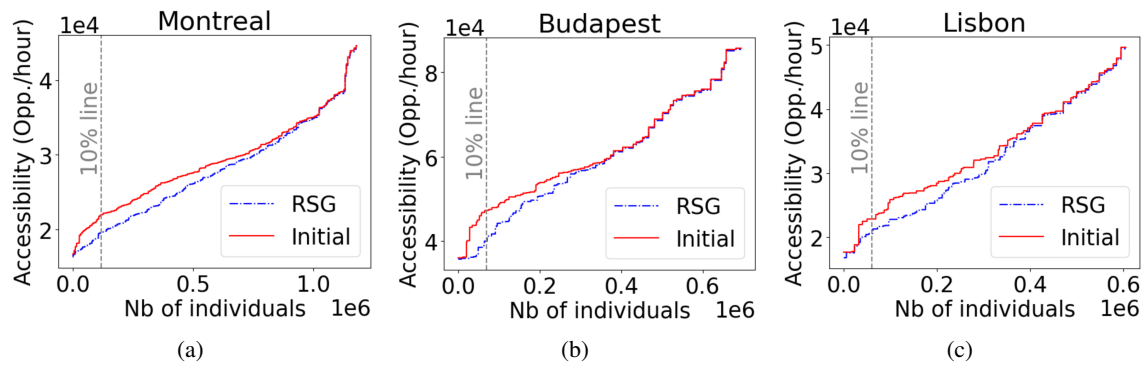


Figure 5.8: Accessibility of each individual from small to large (RSG with $N = 200$).

Table 5.3: Information about DRT services in each area in Montreal

Area	Number of DRT buses	Headway $h(A)$ (mins)	Expected cycle time $C_{DRT}(A)$ (mins)	Average number of people per bus
A_1	20	2	53	12
A_2	15	2	41	8
A_3	15	2	35	4
A_4	15	3	60	16
A_5	15	3	49	12
A_6	20	2	32	2
A_7	20	2	42	9
A_8	25	2	50	13
A_9	15	2	40	10
A_{10}	15	2	29	2
A_{11}	20	2	35	4

before and after the introduction of DRT. RSG improves tiles and individuals that suffered from low accessibility with conventional PT only (those are the ones on the left of the x-axis of Fig. 5.7 and 5.8). These results confirm that the score functions we designed in Eqn. (5.19)-(5.20) are an effective guidance for the DRT bus allocation of Algorithm 5.

Finally, the changes observed in Montreal are taken as a case study. Fig. 5.9 shows all the areas deploying DRT after optimization by the RSG ($\alpha = 0.25$) model with 200 DRT buses. We observe that RSG can potentially allocate buses everywhere. However, as wished, it ends up deploying buses in only a few areas, which evidently are those that need most of the improvement. We also observe that the DRT gives priority to suburban areas. Detailed information on each area after the deployment of DRT can be found in Table 5.3. Furthermore, although we do not limit *a priori* bus capacity, the resulting bus allocation does not significantly increase bus occupation, with a maximum of not more than 16 people per bus in Table 5.3.

5.5 Perspectives

While the present work deals with optimization at the planning level, in our future work we will also test the performance of our design in operation, using [149, 150]. It will also be interesting to tackle, together with DRT deployment decisions, a deeper re-design of conventional PT lines, possibly removing lines that contribute less to accessibility equality [11].

From the methodological point of view, our optimization is guided by a score that combines population and accessibility of the designated areas. While the structure of this score is fixed *a priori* (based on our trial and error experiments), we will investigate the possibility to “learn” optimal scores via Machine Learning (ML). By appropriately engineering features, ML would allow to consider complex local information (e.g. graph connectivity, topological or socio-economic data), which would be too difficult to capture with scoring functions constructed *a priori*, and which could automatically adapt to each city.

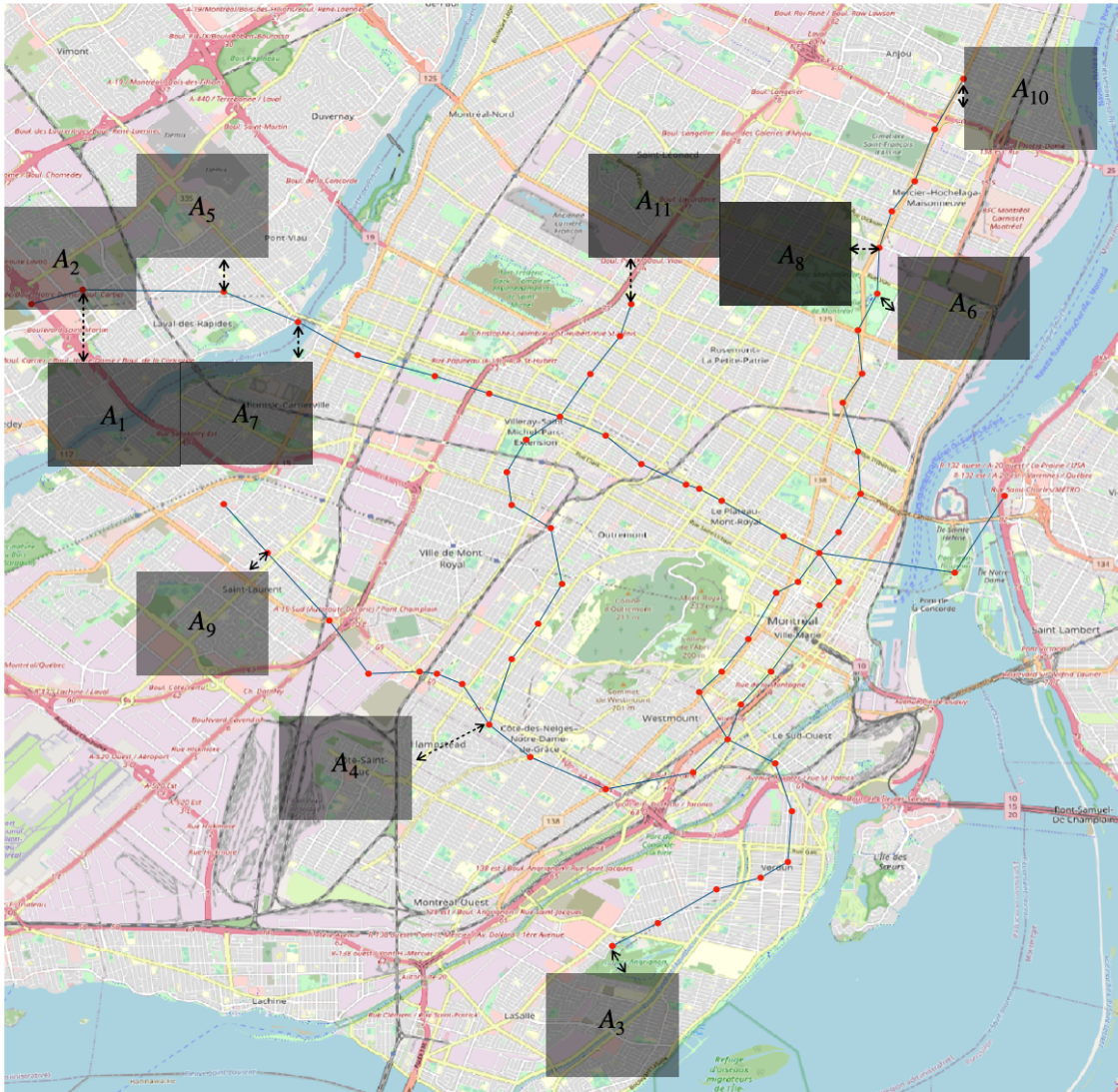


Figure 5.9: Areas with DRT obtained via RSG ($\alpha = 0.25$, $N=200k$) in Montreal. *The darker the color of the area, the more DRT buses are deployed in that area (exact allocated fleet per area are in Table 5.3).*

5.6 Conclusion

We presented a methodology to plan DRT in order to reduce accessibility inequality among individuals in an urban region. We combine a graph model of conventional PT, capable of capturing the characteristics of currently existing PT, with a Continuous Approximation (CA) model of DRT. We combine the two models in a single multilayer graph, on which we compute accessibility indicators. We propose a bilevel optimization method, where the upper level allocates DRT buses in designated areas scattered around the city and the lower level solves transit assignment. Numerical results shows strong improvement in the accessibility for those who were suffering from low accessibility with conventional PT, thereby reducing inequality.

We believe the method presented here is a first step toward optimizing accessibility inequality when planning DRT, thus exploiting the full potential of DRT in fighting against car dependency.

Chapter 6

Online Design of Dynamic Networks

6.1 Introduction

Network design methods are generally limited to static networks. However, many real-world networks are dynamic; e.g., friend relationships on social networks are added or deleted over time. These dynamics can be captured by Temporal Graphs (TGs), which can change over time. TGs have been adopted to model transport systems [151, 152], social networks [153], recommendation systems [154], and many others [155–157].

The state of the art in dynamic networks is of *an analytic nature*, i.e., it focuses on characterizing and modeling the evolution of TGs. Temporal Graph Neural Networks have achieved promising results in the problems of node classification [158], edge classification [159], and temporal link prediction [160]. However, in many practical problems, such as transport service design, a *prescriptive approach is needed*; i.e., we need to *design* dynamic networks to optimize certain goals, rather than simply analyze them through unsupervised or supervised learning models.

The novelty of this chapter is that we focus on the problem of *designing* the evolution of TGs, faced with a random and time-varying environment. As with real-world problems, we assume that the events generated by the environment are not known in advance. Facing unknown environments, we need to continuously make the most favorable decisions, in real time, based on predictions built upon previous experience. To the best of our knowledge, this chapter is the first to tackle the *design* of TGs with unknown incoming events. Our main contributions are the following:

- We introduce the problem of *designing* dynamic Temporal Graphs (TGs), while previous work has focused only on their *analysis*.
- We adapt Monte Carlo Tree Search (MCTS) to the design of dynamic networks, modeled as Temporal Graphs (TGs) in a stochastic environment. The main challenge involved in this adaptation is to maintain a strong network structure while allowing for its evolution. Indeed, in classic MCTS, actions (adding network arcs, in our case) would be applied directly at the moment they are decided. That approach would, however, greatly reduce the usefulness

of the network under construction. Indeed, for a network to be useful, its structure should be known in advance so that some activities of interest, e.g., computing shortest paths, can be *planned* on that structure. A dynamic network built with classic MCTS would prevent any activities from being planned, as it would fail to provide a network structure and keep it available in the near future. To overcome this limitation, we modify MCTS by introducing a look-ahead buffer B : at time t , via MCTS we make network design decisions that will take effect at time $> t + B$. The quality of those decisions is evaluated via a prediction model of the environment. Our approach allows for planning activities on the network designed between t and $t + B$, while at the same time allowing the network to adapt to the evolution of the environment.

- We apply our approach to three different problems, with a particular focus on one of them, related to the transport domain, where a stochastic sequence of trip requests needs to be served by a fleet of vehicles. The state of the art for solving such a problem consists of Dynamic Vehicle Routing Problem (DVRP) algorithms, which insert incoming user requests into appropriately modified single vehicle routes, resulting in a set of vehicle trajectories that lack structure. Indeed, any planned user trip involving transferring from one vehicle to another would be disrupted by DVRP-like approaches because of the continuous modifications to vehicle routes due to the insertion of new requests. On the contrary, with our method, we build a *network* of bus lines that has a *structure*, i.e., on which complex user trips can be planned and preserved, despite the dynamic evolution of the network. Therefore, users can navigate such a network and make transfers from one line to another. Using a dataset of real trip requests in New York, we show that our approach can serve almost double the requests compared to state-of-the-art (SOTA) Dynamic Vehicle Routing Problem (DVRP) methods, requiring only a small fleet.

6.2 Problem Definition and Solution Method

6.2.1 Time-expanded Graph

Let $\mathcal{G}_{\text{substr}} = (\mathcal{V}, \mathcal{E}_{\text{substr}})$ be a graph, which we call *the substrate graph*, where \mathcal{V} is the set of nodes and $\mathcal{E}_{\text{substr}} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. If there exists an edge $(v, v') \in \mathcal{E}_{\text{substr}}$, then it is possible to go directly from node v to node v' , taking a certain time $w_{u,u'} \geq 0$.

A time-expanded edge is a tuple $e = (v, v', t)$ that represents the possibility of transitioning from node v to v' , leaving at the time instant t and arriving at time $t' = t + w_{v,v'}$, which we call *the departure time* and *arrival time* of e , respectively. Let \mathcal{E} be a set of time-expanded edges. We define $\mathcal{G} = (\mathcal{V}, \mathcal{E}_{\text{substr}}, \mathcal{E})$ a Time-Expanded Graph (TEG). TEG \mathcal{G} represents the entire set of possible transitions from a node to another, together with the time instants in which they are possible.

6.2.2 Decision Process

Let Env be a marked point process [161, Def. 2.1.2], which we call the environment, that generates events over time. Env may, for instance, generate perturbations of weights $w_{v,v'}$ of the substrate graph. Or Env may generate requests. A context $cx(t)$ evolves over time, which may contain, for instance, the values of the weights of the substrate graph or the set of events generated by the environment that have not yet been treated by the system. Finally, let $\mathcal{G}(t)$ denote the Time-Expanded Graph (TEG) constructed at time t . Such a graph may also contain edges departing at time instants later than t . The state is a pair $st(t) = (cx(t), \mathcal{G}(t))$.

We impose a *lookahead* buffer $B > 0$ into $\mathcal{G}(t)$, i.e., graph $\mathcal{G}(t)$ must already contain all the evolution of the TEG until instant $t + B$. In other words, in any other graph $\mathcal{G}(t + \varepsilon), \forall \varepsilon > 0$, there will not be any edge $e = (v_e, v'_e, t_e)$ with $t_e < t + B$ that was not already present in $\mathcal{G}(t)$. The introduction of the lookahead buffer in our formulation is crucial in our setting, since it preserves *structure* in the dynamic network we build: since at time t the evolution of the network up to time $t + B$ is already established, it is possible to *plan* activities *on top of* the dynamic network we are constructing, e.g., planning routes on it. For instance, in the transport system case that we analyze in §6.3, it is important to build the TEG representing bus routes in advance, so that the journeys planned by users can be actually performed according to their plan, without imposing en-route modifications of such journeys.

Design actions to modify TEG $\mathcal{G}(t)$ cannot be taken at any instant, but only at specific instants, which we call *action instants*. Such instants are determined by the specific problem at hand, and examples will be given in §6.3 and §6.4.4. An action $a(t)$ at action instant t consists of extending TEG $\mathcal{G}(t)$ by adding time-expanded edges with departure time $> t + B$. TEG $\mathcal{G}(t)$ remains the same until the next action instant t' . As time passes from t to t' , the environment generates events $ev(t''), t'' \in [t, t']$ that will affect the context $cx(t)$. Such a modification generates a reward $r(a(t), cx(t), \mathcal{G}(t))$. The lookahead buffer imposes that an action at time t can only add a time-expanded edge whose departure is after $t + B$. Let $\mathcal{A}(t, \mathcal{G}(t))$ denote the set of possible actions at instant t , which generally depends on time t in which we take actions and the current TEG $\mathcal{G}(t)$.

The setting we are describing is thus a Time-Dependent Markov Decision Process. Different from standard Markov Decision Processes (MDP), in which, in general, actions are taken at time instants regularly stated, in our case instead we allow the next instant where an action can be taken to depend on current state, as in semi-Markov Decision Models [162].

6.2.3 Monte Carlo Tree Search for Dynamic Network Design

Since random events arrive over time, our goal is to build a TEG online in order to adapt to such events, with the aim to maximize collected rewards. Action space $\mathcal{A}(t, st(t))$, consisting of the possible time-expanded edges that can be added, might be too big for Q-learning methods. In such cases, Monte Carlo Tree Search (MCTS) is preferable. MCTS converges to the optimal policy

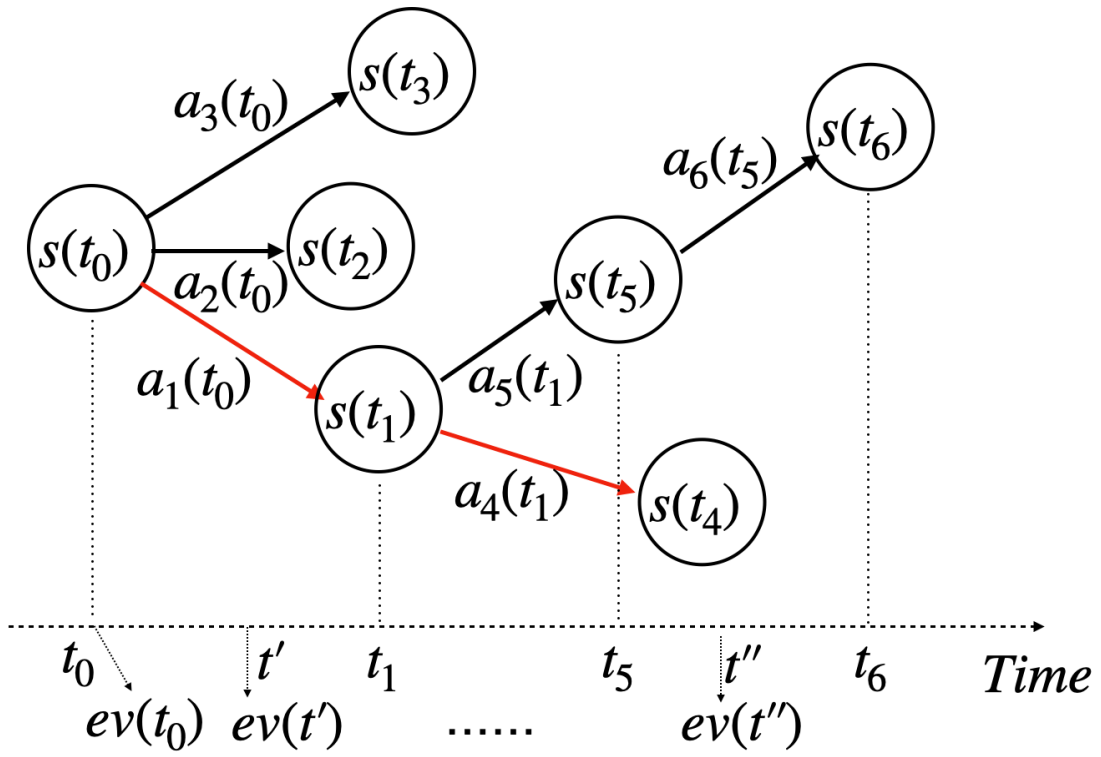


Figure 6.1: An example of search tree $Tree(st(t_0))$. Bold red lines represent actions that are actually chosen and actuated. Actions can be taken only at t_0, \dots, t_6 , while events can occur at any time.

(i.e., the sequence of action which collects the most reward) as the agent collects more and more experience [163, Theorem 6].

MCTS constructs a search tree $Tree(s)$, starting from current state $s = st(t) = (cx(t), \mathcal{G}(t))$ as root. $Tree(s)$ is a decision tree, each node represents a new state resulting from an action. $Tree(s)$ is created by running n simulations. In each simulation, a sequence of random events (e.g., trip requests) is generated via a prediction model, mimicking the statistical characteristics of the environment. Moreover, time advancement is also simulated. Fig. 6.1 shows an example of $Tree(s)$. Each node on the tree represents state $st(t_i) = (cx(t_i), \mathcal{G}(t_i))$.

Suppose we are at time t_0 and we need to take an action, guided by MCTS. To do so, we have to construct a tree via simulations. At the beginning of each simulation, we have $Tree(st(t_0))$, which is the result from previous simulations. Each simulation includes the following four steps (*selection, expansion, rollout, backpropagation*). *Selection*: Starting from the root $st(t_0) = (cx(t_0), \mathcal{G}(t_0))$, an action is chosen in $\mathcal{A}(t_0, \mathcal{G}_0)$ via selecting the child node in the tree with the largest Upper Confidence Bound applied to trees (UCT) [164, Figure 1]. The action deterministically induces another node $st(t_1) = (cx(t_1), \mathcal{G}(t_1))$, where t_1 is the next time instant and $\mathcal{G}(t_1)$ is the graph obtained by applying that action on graph $\mathcal{G}(t_0)$ (Fig. 6.1). The reward of the action is estimated, based on a simulated sequence of events between t_0 and t_1 , together with the modifications induced on the context, which would thus go from $cx(t_0)$ to $cx(t_1)$. The selection process ends until that it finds a node which can be expanded. for instance, node $st(t_4)$ of Fig. 6.1. We then make an *expansion*: The expandable node usually has some actions that have never been selected in previous simulations, so the expansion process will randomly select one of the actions and initialize the child node accordingly. We then begin a *rollout*: Starting from this child node, we select a sequence of actions uniformly at random. At the same time, we record the sum of simulated rewards in rollout process as the cumulative reward starting from the leaf node. The rollout ends after taking a certain number of random actions (which must be decided as a hyperparameter). In *backpropagation*, for all nodes passed through the selection and expansion process, their number of visits will increase by one, and their new cumulative rewards will be updated. We repeat n simulations, similarly.

Observe that the n simulations occur along a “virtual time”, i.e., they are calculated while we are at time t_0 in the real system. While we are at state $st(t_0) = (cx(t_0), \mathcal{G}(t_0))$, we wish to use MCTS to make the next decision. MCTS will start from root node $st(t_0)$ and perform the simulations explained above. Note that the actions taken in the simulations have no effect in the real system: the graph in the real system remains $\mathcal{G}(t_0)$ although during simulation many other graphs are evaluated. After MCTS finishes, we can apply in the real system the most promising action, which is the one corresponding to a child of the root associated with the highest simulated reward.

Since we do not advance the actual time during the MCTS simulations, we cannot get new events from the real environment Env . Therefore, we need to generate a virtual environment Env' , which corresponds to a prediction model based on the historical data of environment Env . An example on how to build Env' will be given in §6.3. The events required for the MCTS simulations process all come from this virtual environment Env' .

Algorithm 8: Online Design of Dynamic Networks (OD²N)

- 1: **Input** Substrate graph $\mathcal{G}_{\text{substr}} = (\mathcal{V}, \mathcal{E}_{\text{substr}})$
Starting time t_0 .
Ending time t_1 .
Lookahead buffer B .
 - 2: **Initialization**
 - 3: Current time $t = t_0$.
 - 4: Initial context $cx(t) = \emptyset$.
 - 5: Initial TEG $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}_{\text{substr}}, \mathcal{E}(t) = \emptyset)$.
 - 6: Initial state $st(t) = (cx(t), \mathcal{G}(t))$.
 - 7: Initialize $\mathcal{E}(t)$ by adding random edges (while satisfying system constraints)
 - 8: **repeat**
 - 9: Let real world time t advance.
 - 10: **if** Environment Ev generates event ev in t **then**
 - 11: Update context $cx(t)$
 - 12: Collect corresponding reward $r(ev(t), cx(t), \mathcal{G}(t))$.
 - 13: **end if**
 - 14: **if** Time instant t is an *action instant* (§6.2.2) **then**
 - 15: Evaluate available actions $\mathcal{A}(t, s(t))$, which consist of adding time-expanded edges with departure later than $t + B$.
 - 16: Choose action $a(t) \in \mathcal{A}(t, s(t))$ by running MCTS (§ 6.2.3).
 - 17: Apply action $a(t)$, i.e., add the corresponding edge to $\mathcal{G}(t)$.
 - 18: Evaluate reward $r(a(t), cx(t), \mathcal{G}(t))$ obtained via action $a(t)$.
 - 19: **end if**
 - 20: **until** $t > t_1$.
 - 21: **Return** contexts $cx(t)$ or data required for different problems.
-

6.2.4 Online Design of Dynamic Networks (OD²N) Algorithm

We propose Algorithm 8 to design dynamic networks. Our basic idea is to use MCTS introduced in Section 6.2.3 to choose actions, then update the state of the network based on these actions, and record changes during state transitions. It should be noted that Algorithm 8 is an online method, and we can set ending time $t_1 = \infty$, which means that we will never terminate the algorithm to provide network designing plans in real time.

6.3 Specialization of the Method for the Online Bus Network Design

We now apply our general framework to design a futuristic public transport system. **The problem is to design a network of bus lines that dynamically evolves, to adapt to a sequence of unknown user trip requests. Such design must be performed in real time, while the network being constructed is operated.** Observe that serving a sequence of unknown user trip requests could also be framed as a Dynamic Vehicle Routing Problem (DVRP), where incoming requests are associated

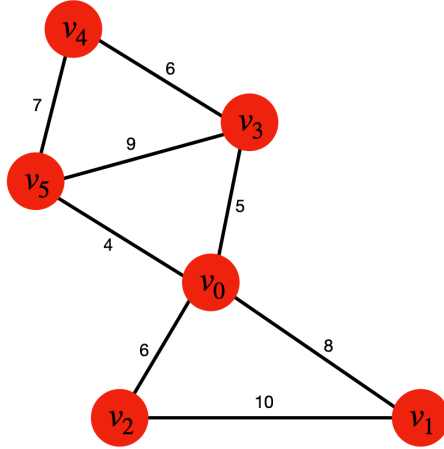


Figure 6.2: Substrate graph $\mathcal{G}_{\text{substr}} = (\mathcal{V}, \mathcal{E}_{\text{substr}})$.

to vehicles, whose routes are adjusted on the fly. As already discussed in Section 2.3, DVRP is a *solution* employed to route a fleet of shared taxis for low density demand, whose vehicle routes are extended independently. We tackle instead a system comparable with conventional public transport, which can serve high density demand via building a structured network of bus lines, where transfers from a vehicle to another are possible. The difference with conventional public transport is that, in our system, (i) bus lines evolve over time and (ii) are designed in real time.

To apply the method of §6.2, we now map state $st(t) = (cx(t), \mathcal{G}(t))$, environment Env , action $a(t)$, and reward $r(a(t), st(t))$ to elements specific to the transport problem at hand.

6.3.1 Time-Expanded Graph $\mathcal{G}(t)$

In the case considered here, substrate graph $\mathcal{G}_{\text{substr}} = (\mathcal{V}, \mathcal{E}_{\text{substr}})$ consists of sets \mathcal{V} of potential bus stops and $\mathcal{E}_{\text{substr}} \subseteq \mathcal{V} \times \mathcal{V}$ of edges (Fig. 6.2). Edge $(v, v') \in \mathcal{E}_{\text{substr}}$ indicates that a bus can travel directly from stop v to stop v' , taking time $w_{v,v'}$. Observe that an edge in \mathcal{E} generally corresponds to a sequence of multiple roads. We further introduce a set \mathcal{B} of N buses.

Let us now detail the elements of Time-Expanded Graph (TEG) $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}_{\text{substr}}, \mathcal{E}(t))$. Set $\mathcal{E}(t)$ contains all the possible transitions that users can choose before time $t + B$. To be more precise, time-expanded edge $e = (v, v', t) \in \mathcal{E}(t)$ means the following: if $v' \neq v$, edge e represents that there is a bus departing from bus stop v to bus stop v' at time t (red lines and blues lines in Fig. 6.3); if $v' = v$, edge e represents that a user begin to wait in a bus stop v at time t until a bus arrives (green lines in Fig. 6.3). For example, in Fig. 6.3, when we schedule that bus 1 to arrive at stop v_0 at 9:22 and bus 2 to arrive at v_0 at 9:24, we automatically add an edge (green line) from $(9:22, v_0)$ to $(9:24, v_0)$, which allows users us to transfer from bus 1 to bus 2, waiting for 2 minutes.

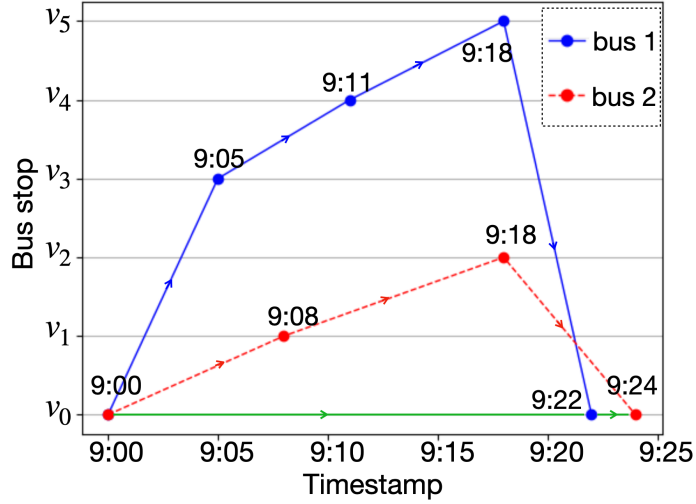


Figure 6.3: TEG of bus schedule (2 buses).

Let t denote the current real-world time and $\mathcal{G}(t)$ the TEG available at time t . All edges departing before $t + B$ must be already present in $\mathcal{G}(t)$. In other words, the schedule in the interval $[t, t + B]$ of all buses has been already planned at time t and it cannot be changed anymore.

6.3.2 Environment Env

Environment Env is a process that generates events. An Event $ev(t)$ in this case represents a new coming request $d = (v, v', t)$ at time t , where $v, v' \in \mathcal{V}$ are the origin and destination nodes respectively, t is the departure time of a passenger.

6.3.3 Context $cx(t)$

Routing request d into TEG $\mathcal{G}(t)$ means to calculate a time-expanded *path*, defined as $p = (v, t, e_1, \dots, e_n, v')$, where e_1, \dots, e_n is a set of time-expanded edges in $\mathcal{G}(t)$. Path p must satisfy the following conditions:

$$t + w_{v, v_1} \leq t_1 \tag{6.1}$$

$$e_j = (v_j, v_{j+1}, t_j) \in \mathcal{E}, \quad \forall j = 1, \dots, n \tag{6.2}$$

$$t_j + w_{v_j, v_{j+1}} \leq t_{j+1}, \quad \forall j = 1, \dots, n-1 \tag{6.3}$$

$$v_1 = v, \quad v_{n+1} = v' \tag{6.4}$$

The arrival time of p is

$$t'(p) = t'_n + w_{v'_n, v'}. \tag{6.5}$$

Let $\mathcal{P}(v, v', t, \mathcal{G})$ be the set of paths in \mathcal{G} starting at node $v \in \mathcal{V}$ at time t and arriving at node $v' \in \mathcal{V}$. The shortest path is

$$p^*(v, v', t) \in \arg \min_{p \in \mathcal{P}(v, v', t, \mathcal{G})} t'(p). \quad (6.6)$$

We assume that request $d = (v, v', t)$ is served along shortest path $p^*(d) = p^*(v, v', t)$.

However, there might be requests for which no paths exist in graph $\mathcal{G}(t)$ at time t . Context $cx(t)$ in this case is the list of requests not yet served at time t . Observe that requests in $cx(t)$ might have the chance to be served in some later graphs $\mathcal{G}(t'), t' > t$, if appropriate time-expanded edges will be added.

6.3.4 Action $a(t)$

TEG $\mathcal{G}(t)$ is composed of a set of time-expanded subgraphs, each representing the movement of a bus (Fig. 6.3). Time-expanded subgraph $\mathcal{S}\mathcal{G}_i(t)$ of bus $i \in \mathcal{B}$ is composed of a sequence of edges, describing its movement. Designing a bus network amounts to designing such subgraphs, by taking actions consisting in adding time-expanded edges to each subgraph over time.

An action $a(t) \in \mathcal{A}(t, st(t))$ available at time t consists in extending subgraph $\mathcal{S}\mathcal{G}_{i^*(t)}$ of a certain bus $i(t) \in \mathcal{B}$, by adding a time-expanded edge $e_{a(t)} = (v', v'', t + B)$, where v' is the arriving stop of the last edge of bus $i^*(t)$. As explained in §6.2.2, actions cannot be taken at any instants, but only at some specific instants, which we call *action instants*. After action $a(t) \in \mathcal{A}(t, st(t))$ is taken, and the subgraph of bus $i^*(t)$ expanded, we let the real world time advance until the next action instant, and we repeat the process.

The details about how to select the bus $i^*(t)$ to extend and how to determine the action instants are as follows: Let $st(t) = (cx(t), \mathcal{G}(t))$ be the current state. For simplicity, our action always extend the bus i with the shortest schedule. In particular, for any bus $i \in \mathcal{B}$, let $e_i^{\text{last}}(t) = (v, v', t')$ the last edge, i.e., the edge of subgraph $\mathcal{S}\mathcal{G}_i$ with the latest departure time. Let $\tau_i(t)$ the arrival time of such an edge. Observe that we need to ensure at any moment that $\tau_i(t) \geq t + B$. Let bus $i^*(t) = \arg \min_{i \in \mathcal{B}} \tau_i(t)$ be the bus with the shortest schedule. We extend its route before the others. If $\arg \min_{i \in \mathcal{B}} \tau_i(t)$ does not return a single bus, we break ties choosing $i^*(t)$ randomly among them. In the process of repeatedly extending the subgraph $\mathcal{S}\mathcal{G}_i$, each action instant is determined by the shortest schedule $\min_{i \in \mathcal{B}} \tau_i(t)$ and buffer B : $t_{\text{action}} = \min_{i \in \mathcal{B}} \tau_i(t) - B$.

6.3.5 Reward

By applying action $a(t_0)$ to state $st(t_0)$, we induce a transition to state $st(t_1)$, as explained in §6.2.3, which we denote as

$$cx(t_0), \mathcal{G}(t_0) \xrightarrow{a(t_0)} cx(t_1), \mathcal{G}(t_1). \quad (6.7)$$

As time goes from t_0 to t_1 , environment Env generates new requests. According to the definition of context in §6.3.3, context $cx(t_1)$ is the list of requests not yet served at time t_1 . Specifically, it

includes two parts. One part is some requests in $cx(t_0)$ that still cannot be served in $\mathcal{G}(t_1)$, and the other part is the requests generated from t_0 to t_1 that cannot be served by $\mathcal{G}(t_1)$. The reward is defined as:

$$r(a(t_0), st(t_0)) = |cx(t_0)| - |cx(t_1)|. \quad (6.8)$$

where $|\cdot|$ counts the number of elements in a list. The higher the reward, the more the requests action $a(t_0)$ allows satisfying.

6.3.6 Online Bus Network Design Process

In order to select which of the actions available in $\mathcal{A}(t, st(t))$ should be taken, we run MCTS, as explained in §6.2.3. All the necessary definitions have been instantiated in the previous sections, allowing us running Algorithm 8 to route the available fleet of buses. This algorithm will design the bus schedules to serve as many incoming requests, unknown in advance, as possible.

6.3.7 Requests Generation Model

As explained in §6.2.3, MCTS requires simulating future possible evolution trajectories of the system, so that one can choose the next action that will likely induce the most desirable evolution. To perform such simulation, we need a model Env' of the environment, which generates events (requests in this case) that are statistically similar to those generated by the real environment Env . We can fit Env' on historical data of real-world environment Env . In the transport case we are tackling here, model Env' can be trained on historical dataset \mathcal{D} of previously observed trips. Requests are a spatial-temporal marked point process [165]. Therefore, Env' must be able to mimic the spatial patterns (where origins and destinations of trips are distributed) and temporal patterns (rate of requests).

1. *Learning temporal patterns.* For each day in the past, we first count the number of requests generated at different timeslots. We train a support-vector machine (SVM) to predict the request counts of all timeslots in the next day.

2. *Learning spatial patterns.* We count the origin and destination of all past requests and build an OD matrix $M = \{a_{uv}\}_{u,v \in \mathcal{V}}$ where a_{uv} is the fraction of observed trips that went from origin u to destination v . The value of a_{ij} can be interpreted as the probability that, given any trip request, its origin will be u and its destination v .

When, within MCTS, we use Env' to generate simulated requests, we do as follows. We first predict the number of future requests via the SVM at each timeslot. For each of them, we randomly select the origin and the destination based on probabilities $a_{u,v}$. Note that we deliberately adopted this simple prediction model Env' , so as to provide a conservative indication of the performance of the proposed system. Indeed, already with this simple model, §6.4 will show that the proposed method outperforms SOTA DVRP methods. One could then replace this simple Env' with any of the advanced demand prediction models available in the literature, and expect even better performance, which is however outside the scope of this work.

6.4 Numerical Results

In this section we extensively evaluate the performance of the propose method, mainly focusing on the application of the real-time design of a bus network. To show the generality of the method, we also apply it beyond the transport domain in the last subsection 6.4.4.

6.4.1 Considered Scenario

We apply our method to design a bus network online in a scenario in Manhattan, where the environment consists of a sequence of trip requests from end-users. All the relevant numerical parameters are reported in Table 6.1, together with the literature supporting their choice. We consider a period between 9h00 and 13h00, since our focus is the performance of the proposed method, rather than thoroughly analyzing an urban mobility scenario. For the lack of space, we keep the analysis of an urban mobility scenario over an entire day as future work.

To test our method, we replay a randomly selected subset of real taxi requests from a public dataset [166]. Manhattan has 67 taxis zones. We find the centroid of each taxi zone, and use these centroids as candidate bus stops. Therefore, the substrate graph is $\mathcal{G}_{\text{substr}} = (\mathcal{V}, \mathcal{E}_{\text{substr}})$, where \mathcal{V} is the set of all centroids. Graph $\mathcal{G}_{\text{substr}}$ is a fully connected graph. Weight $w_{u,v}$ of edge $(u, v) \in \mathcal{G}_{\text{substr}}$ is the time (in minutes) to travel between nodes, i.e., $w_{u,v} = d(u, v) / v_{\text{bus}}$, where $d_{u,v}$ is the distance between $u, v \in \mathcal{V}$ and v_{bus} is the average speed of the bus. We introduce set \mathcal{B} of N buses, with random initial positions.

We run Alg. 8 on a laptop with an Intel Core i7-12700KF. In the current setting, the time to decide how to extend the route of each vehicle is about 40 seconds. For problems with larger fleet sizes (even if, again, thousands is not the scale that we target), larger servers can help, together with parallelization techniques, succesfully applied to MCTS. For example, 'update context $cx(t)$ ' (line 11, Alg. 8) is the most time-consuming part, because it requires to run the shortest path algorithm to check whether each request has been served. Parallelization techniques could be used on big computation cluster running on the network operator premises, and a large number of requests could be routed.

6.4.2 Performance

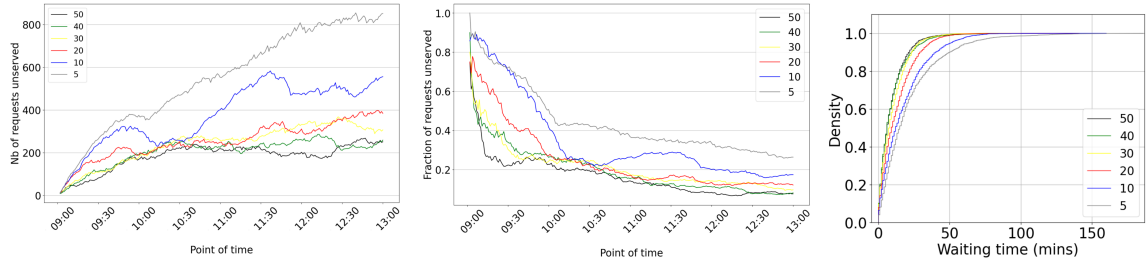
A good service should be capable of providing high service rate (high number of served requests) while using few resources (small fleet size), and providing good user experience, by minimizing trip times (average waiting time + average in-vehicle time). Therefore, **service rate**, **fleet size** and **average trip time (average waiting time + average in-vehicle time)** are the main measurements to compare the quality of different transport systems. Observe that when a user trip involves multiple vehicles, i.e., at least one transfer, the waiting time is the sum of the waiting times for each vehicle.

The performance of our method with different fleet sizes is shown in Figure 6.4 and Table 6.2. We did not consider thousands of buses, because we target efficient transport, where a small fleet,

Table 6.1: Scenario parameters

Parameter	Value
Number of candidate bus stops	67
Fleet size N	5, 10, 20, 30, 40, 50
Speed of a bus v_{bus}	17.3 km/h [167]
Speed of a private car v_{car}	11.4 km/h [167]
Walking speed v_{walk}	4.3 km/h [168]
Rollout termination number N_{end}	5
Lookahead buffer B	30 mins
Timeslot for training model Env' (§6.3.7)	1 minute
Training set \mathcal{D} (§6.3.7)	All request data from [166] in February 2024
Tested scenario (§6.3.7)	20% request data from [166], 9:00-13:00 March 1, 2024 (to get faster results)

intelligently routed, can already serve relatively high demand. Indeed, we showed that we can serve 90% of demand with just 40 buses, which requires much less than the multiple thousand taxis registered in New York. The advantage of our method is to scale down from fleets of thousands to fleets of tens, for efficiency and environmental reasons. Upon arrival, a request is placed in a list of unserved requests. Figure 6.4-(a) shows the number of such requests over time. For any fleet size, the number of unserved requests increases significantly in the period close to 9:00. This is because our system has just started, and we initialize TEG $\mathcal{G}(t)$ randomly (Alg. 8) at $t = 9:00$, which means that bus schedule is randomly planned in until time $t + B = 9:30$. After the system runs for a sufficient amount of time, the real-time number of unserved requests no longer increases significantly. Figure 6.4 (b) shows the percentage of unserved requests out of all requests received



(a) Number of unserved requests. (b) Fraction of requests unserved.(c) ECDF of the waiting time.

Figure 6.4: Real-time MCTS results with different fleet sizes from 9:00 to 13:00.

Table 6.2: Comparison of results with different fleet sizes

Fleet size	Service rate	Avg. Trip time (mins)	Avg. Waiting time (mins)
5	73.58%	27.81	16.01
10	82.55%	27.75	15.76
20	87.82%	29.68	15.03
30	90.25%	29.93	14.20
40	91.83%	31.79	14.63
50	92.31%	32.20	13.89

Table 6.3: Comparison with SOTA DVRP method (40 buses)

	Max. Waiting time (mins)	Service rate	Avg. Trip time (mins)	Avg. Waiting time (mins)
SOTA	20	34.37%	23.01	4.16
SOTA	30	49.70%	24.60	4.73
Our method	20	63.94%	29.46	9.35
Our method	30	91.77 %	31.62	14.43

so far. Unserved requests go below 8% with a sufficiently large fleet size. It should be noted that, in any dynamic bus system, it is impossible to serve all requests. There can be requests whose origin and destination are very far or in remote places. This kind of trips might be rejected also in the transport systems currently deployed in the reality (and the user would be better-off taking their car). Public transport needs indeed to balance user satisfaction and efficiency, and it cannot thus provide a service tightly tailored to very specific user needs. As expected, a larger fleet size serves more requests and reduces waiting time (Table 6.2 and Fig. 6.4 (c)). The trend of the average trip time of Table 6.2 may seem instead counter intuitive. However, it can be easily explained: the increase of the service rate obtained via increasing the fleet size implies serving more requests, and in particular those that were difficult to serve with smaller fleet sizes. Such requests often require longer O-D distances and longer travel times, which obviously increase the average trip time. A more detailed quantitative analysis of this trade-off will be presented later in Figure 6.5.

We now compare the performance of our method with three benchmarks, considering the same sequence of incoming requests. Observe that our service is in-between a shared taxi service and a conventional bus network. Therefore, the three benchmarks we selected are (i) the real taxi service, (ii) a SOTA DVRP method (representing the optimized shared taxi service) and (iii) a SOTA conventional bus network.

Table 6.4: Comparison with SOTA static bus network (40 buses)

Number Of lines	Buses per line	Service rate	Avg. Trip time (mins)	Avg. Waiting time (mins)
4	10	16.7%	10.42	1.48
10	4	49.6%	17.14	4.82
20	2	72.5%	32.16	16.38
40	1	80.3 %	56.22	41.13
Our method	1	91.8 %	31.79	14.63

Comparison with real TLC Trip Record Data Since we are using TLC Trip Record Data from [166], and there are around 13,000 yellow taxis operating in Manhattan [169], we first calculated the average time it takes for all requests between 9:00 and 13:00 to be actually completed by yellow taxis, which is 12.80 minutes. Table 6.2 and Table 6.5 show the difference in performance of our method with different fleet sizes. The average trip time is between 27 and 33 minutes, depending on the fleet size. Although this time is longer than the one of the real yellow taxis (12.8 minutes), it is still a time compatible with trips in urban areas (in particular commuting trips [170, Fig. 1]). Importantly, we achieve this performance by only using 50 buses for serving 20% of request data from [166], it is far fewer than the 2600 (20% of 13,000) taxis.

Comparison with SOTA DVRP method In Table 6.3, we compare our results with the algorithm in [71], largely known as state-of-the-art (SOTA) and adopted still now in very recent work [171]. We run an open source implementation [172]. A maximum waiting time should be pre-selected as a hyperparameter of the algorithm [71]. We run both our Alg. 8 and the algorithm of [71] with a fleet size of 40 and no seat capacity constraints. Observe that the goal of SOTA [71] is to optimize an on-demand “high-capacity” ride-sharing, in which the route of each vehicle is adjusted independently, by adding some new requests as they come. In such a system, vehicle routes *are not structured* and are incrementally modified to ‘insert’ each request into vehicle routes. Users cannot make transfers from one vehicle to another. Our method allows instead to run a public transport system that does not exist yet, which is dynamic, as ride-sharing, but at the same time builds a structured *network of bus lines*, where transfers from a line to another are enabled. In this sense, we design a dynamic network (representing bus schedule) and we let users find their best journey within that schedule. Our system is similar to conventional public transport, and offers much more capacity (higher service rate) than SOTA ride-sharing. Our approach nearly doubles the service rate, with an average trip time loss of only about 5 minutes Table 6.3.

Comparison with a SOTA conventional bus network [171] Observe that the conventional bus network is computed offline and is not changed in real time. We first build offline the optimized set of conventional bus lines via the method of [171], based on the requests of February 2024, which correspond to the trainig dataset for our prediction model Env' (see §6.3.7 and Table 6.1). The performance of the requests of the test set (March 2024) on such a conventional bus network is then measured. For both our method and the one of [171], fleet size = 40. Table 6.4 shows that, by increasing the number of lines, the conventional bus system improves the service rate, which is due

Table 6.5: Performance with different fleet sizes

Fleet size	Avg. Occupation (requests/bus)	Avg. Stretch (taxi)	Avg. Stretch (walking)	Avg. Number of transfers
5	24	1.80	0.68	1.24
10	14	1.56	0.61	1.79
20	9	1.90	0.72	1.85
30	7	2.35	0.89	1.59
40	6	2.62	0.99	1.68
50	5	2.83	1.07	1.50

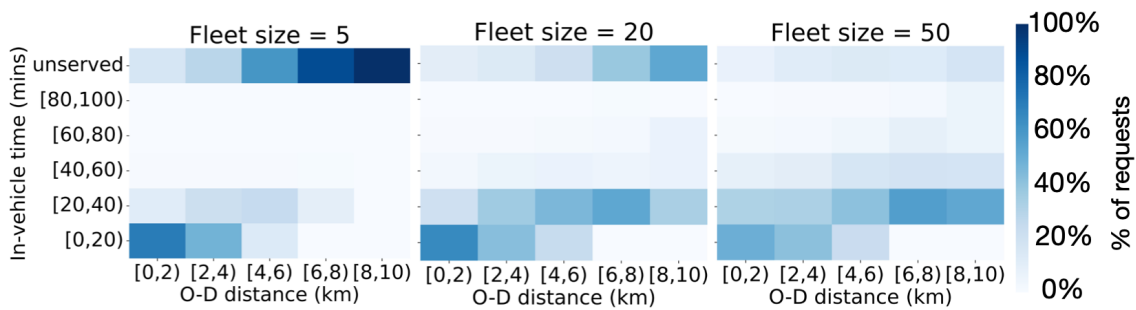


Figure 6.5: Fraction of requests served with a certain in-vehicle time over total requests, per each O-D distance interval.

to increase spatial coverage. However, since resources (i.e., fleet) are unvaried, average trip time increases. Moreover, due to the reduction in the number of buses assigned to each line, waiting time increases. Differently from the conventional bus network, our network of lines evolves over time, and is built in real time. Our dynamic model can achieve a higher service rate (91.8%) and a shorter average trip time while using the same 40 buses.

To summarize, our proposed service can meet higher service rate with smaller fleet size without excessively degrading trip time.

6.4.3 Analysis of practical aspects

We further analyze the service obtained with our method, taking into account practical considerations.

We initially do not set any limit on the bus capacity. After we have the solution from our method, we compute the occupancy, and we can thus understand what type of vehicle (buses, minivans, etc.) would be appropriate. For this reason, it is important to calculate the occupancy by the method in the Appendix 8.4. Table 6.5 shows the average occupation of a bus with different

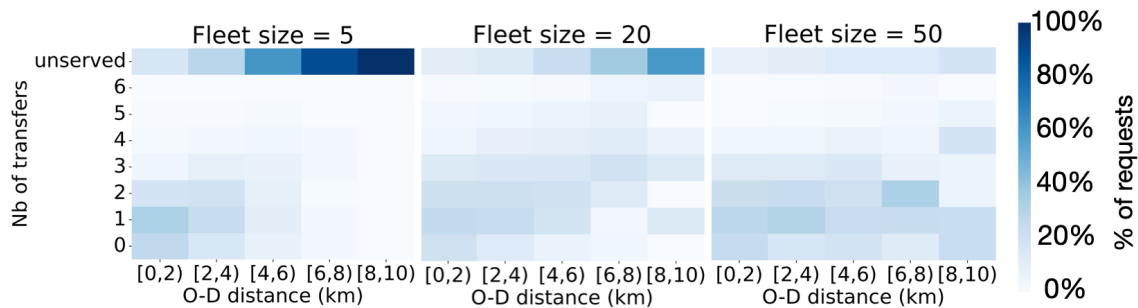


Figure 6.6: Fraction of requests experiencing certain number of transfers over total requests, per each O-D distance interval.

fleet sizes. It is interesting to note that our method adapts to a small fleet, as it is able to increase the level of *sharing*, i.e., more users share a bus trip. As fleet sizes increases, the average occupation decreases, which suggests that the kind of vehicles that should be adopted might change (from large buses to minibuses). These observations can guide a transport operator to dimension the service and the type of adopted vehicles.

In Figure 6.5 we study how our system serves trips of different length. We divide both O-D distance and in-vehicle time into five intervals. For each O-D distance interval, we count how many requests were issued by users, and we then calculate the fraction of such requests served with an in-vehicle time between 0 and 20 minutes, between 20 and 40 minutes, etc. The color scale indicates such fraction. for example, the bottom left square color represents

$$\frac{\text{served req. with OD dist.} \in [0, 2\text{km}) \ \& \ \text{trav.time} \in [0, 20\text{min})}{\text{All requests issued with O-D distance} \in [0, 2\text{km})}.$$

Fig. 6.5 shows that more trips are served with higher fleet size, in particular “long” trips (high O-D distance), which are all unserved with 5 buses. Unserved long trips might require paths longer than $B = 30$ minutes (which is the maximum path length allowed into the system), and might be served with higher lookahead buffer B .

Figure 6.6 represents the number of transfers per trip and is obtained in the same way as Figure 6.5. We observe that, with a larger fleet size, more transfers are possible, as the network of bus lines is denser. In a real system, it would be reasonable to limit the number of transfers, which negatively impact user experience. We could easily adapt our MCTS approach to the case in which users accept to do at most 1 or 2 transfers: it would suffice to consider as “served” users that can go from origin to destination with no more than those transfers and keep in the “unserved” list all the other users. In this way, the MCTS algorithm would collect rewards only for the trips respecting the “maximum transfers constraint” and would learn to adjust bus lines accordingly.

We have to consider that travelers place a significant value on the *reliability* of the service (i.e., in simple term the ability to arrive on time with that service). In conventional PT systems, reliability is associated with service *regularity*, i.e., the fact that schedules repeat every day at the same times.

However, by construction, flexible modes are not based on a regular service. Despite this fact, flexible modes are more and more used and appreciated by travelers [173]. This indicates that travelers operate a trade-off between service efficiency and regularity [174]. This makes the service proposed here a potential solution, accepted by travelers. In future work, we could “tune” our system to explore the efficiency/regularity trade-off: while our objective function is for the moment only focused on efficiency, we could introduce the following modifications: (1) incorporating an additional term into the objective function to quantify the daily periodic variations of the TEG, thereby improving reliability by adjusting its weight; and (2) restricting the operating regions of certain vehicles, which effectively reduces the size of the action and state spaces, thus promoting more regular operations. Overall, the incorporation of regularity into the objective function does not alter the fundamental architecture of our model and algorithm. Further exploration in this direction is promising, but it lies beyond the scope of this work.

6.4.4 Application to other problems

We apply our method to the problem of **switching among configurations of a complex system of Heavy Lift Launch Vehicles** [175] In this case, substrate graph $\mathcal{G}_{\text{substr}} = (\mathcal{V}, \mathcal{E}_{\text{substr}})$ consists of set \mathcal{V} of potential configurations and $\mathcal{E}_{\text{substr}} \subseteq \mathcal{V} \times \mathcal{V}$ of edges. An edge represents the possibility to switch between two configurations. When we decide on an action, which is to switch configurations, the environment Env gives us a certain cost suffered from the system (opposite of reward r), which depends on its current configuration. Contest $cx(t)$ is the accumulated cost up to t . Having defined states, actions and reward, we can use our method to design the evolution of configuration switches of the complex system, in real time, with the aim to minimize accumulated stochastic cost.

The authors of [175] used a Time-Expanded Graph (TEG) to store the cost of all state transitions of a complex system and chose the optimal state-path from an exhaustive enumeration of all possible paths. The Eq.(7) of [175] show that $\#paths = C^T$, where C is the number of states, and T is the time horizon. The size of their problem is very small: $C = 4$, $T = 4$, and $C^T = 256$ paths. We run an experiment with the same problem as [175], with a much larger state space, i.e., $C = 20$, $T = 20$. In this case, they would need to evaluate $\#paths = 20^{20} (\approx 10^{26})$ paths, which is infeasible. Our method can instead find satisfying solutions in just about 16 seconds. We test 20 different demand scenarios, obtaining an average improvement of 29% with respect to the baseline (consisting in the best static system state).

K-server problem The k-server problem is a well-known online optimization problem. K servers are located on a graph, requests arrive one by one at some nodes. For each request, one server should go to the request’s location to serve it. The goal is to minimize the total distance that the servers travel. Every incoming request needs to be processed immediately, which means our lookahead buffer $B = 0$. In this case, our algorithm predicts future requests through the request generation model and determines the best action currently through Monte Carlo Tree Search. We compare our algorithm with the commonly used Greedy algorithm for the k-server problem, which refers to moving the nearest server to the request location each time. In the experiment, we consider

a square area from (0,0) to (100,100), initialize $k = 3$ servers, and generate 100 requests following a spatial-temporal Poisson process (most requests appear in a central region, while others are scattered uniformly across the map. The time between requests is drawn from an exponential distribution with $\lambda = 1$). The results show that our algorithm has an average of 14.18% less distance traveled by the servers, compared to the greedy algorithm (average value of 50 repeated tests).

6.5 Conclusion

In this chapter, we introduced the novel problem of designing temporal graphs. Our framework allows building networks able to proactively adapt to a stochastic environment and adjust their evolution in order to keep satisfying performance. We formalized a general rolling horizon decision process and a Monte Carlo Tree Search-based solution method. We showcased the potential of our approach in a transport context, by designing a futuristic fully dynamic network of bus lines that is able to adapt to a stochastic and unknown request arrival. Our designed network outperforms state-of-the-art Dynamic Vehicle Routing Problem (DVRP) algorithms. We also applied our method to two cases beyond the transport domain. While so far dynamic networks have been studied with analytic approaches, this work opens new research venues on prescriptive approaches, aimed at designing dynamic networks.

Chapter 7

General Conclusions and Future Work

7.1 General Conclusions

This thesis has been driven by a major challenge to urban sustainability: **The inability of public transport systems to adequately satisfy the mobility needs of all people, wherever they live.** In this light, the inequality in the geographical distribution of accessibility is an appropriate indicator to assess the quality of urban public transport systems. Accordingly, a more specific and policy-relevant research question arose: **How can PT systems be optimized to reduce inequality in accessibility?** To address this question, this thesis first considered optimization strategies commonly applied to conventional PT at the planning level, namely stop selection (Chapter 3) and line design (Chapter 4). In Chapter 3, which constituted the first contribution of this PhD project, I conducted a comparative analysis of two approaches: one classically aimed at achieving overall efficiency and the other targeted instead in reducing accessibility inequality. The findings demonstrated that stop selection inherently involved a trade-off between efficiency and equality, but a substantial reduction in accessibility inequality could be achieved in conventional PT without excessively penalizing efficiency. In Chapter 4, I introduce an approach that integrates Message Passing Neural Networks with Reinforcement Learning to optimize bus line design. I evaluated the proposed method on tens of generated transport datasets and compared its performance against commonly used metaheuristics. The experimental results demonstrated that the proposed method outperformed commonly used metaheuristics, as it was able to learn the dependencies between PT lines and the spatial distribution of opportunities, and to exploit the structural features of the PT network within a given territory.

Conventional PT cannot effectively handle requests from suburban areas with relatively low population density. Indeed, to achieve satisfactory frequency and coverage, the PT operator would incur excessive costs. To address this issue, this thesis focuses on flexible mobility as a complement to public transport (PT). Chapter 5 focused on integrating DRT with PT, specifically addressing the following questions: **'How should DRT service areas be selected?'** and **'How many DRT busses should be allocated to each selected area?'** Chapter 5 developed an approach for planning DRT

with the objective of reducing accessibility inequality within urban areas. The approach integrated a graph representation of conventional PT, which reflected existing network characteristics, with a Continuous Approximation model of DRT. These two components are merged into a multilayer graph model that is used to compute accessibility indicators. To optimize fleet allocation, a bilevel optimization was formulated: the upper level determined the deployment of DRT vehicles across selected urban zones, while the lower level addressed the corresponding transit assignment problem. In terms of the optimization procedure, this thesis proposes four algorithms, each combining Bayesian optimization with a rank-score-based greedy method in a distinct way. Their performance was evaluated in three different cities: Montreal, Lisbon, and Budapest. The experimental results demonstrated notable improvements in accessibility for individuals previously disadvantaged in conventional PT systems, thereby contributing to a reduction in inequality. This work is a first step toward systematically addressing accessibility inequality in DRT planning and highlights the potential of DRT in fighting against car dependency.

Finally, this thesis goes beyond the existing transport modes by exploring the possibility of combining the structural characteristics of PT with the flexibility of demand-responsive services and proposes a concept of flexible PT in Chapter 6. First, a time-expanded graph was employed to represent the future schedules of multiple busses. A general rolling-horizon decision process was then formalized to systematically capture the temporal evolution of the time-expanded graph. To address decision-making for future bus routing, a solution based on Monte Carlo Tree Search has been proposed. To adapt to incoming requests, a buffer was incorporated into the rolling-horizon framework, while the MCTS-based method leveraged a generative model to simulate future requests, in order to proactively adjust the routes. In the evaluation phase, I compared the performance of my proposed method with state-of-the-art DVRP algorithms, a benchmark representing optimal conventional PT, and real-world taxi services in New York City. The results confirmed that my proposed service was capable of achieving a higher service rate with fewer vehicles while maintaining acceptable travel times.

In summary, this thesis has investigated novel methodologies for the design of next-generation public transport systems, with a specific focus on reducing inequality in accessibility. By developing novel models and optimization approaches at the strategic and operational levels, my work contributes to advancing the theoretical foundations of equitable transport design. The findings demonstrate that accessibility inequality can be significantly reduced without prohibitive increases in cost, thus contributing to environmentally, economically, and socially sustainable urban development. The frameworks proposed in my thesis also provide policymakers with innovative methodological approaches to designing and evaluating mobility strategies for a more sustainable city.

7.2 Future Work

Several research avenues naturally arise from my previous work. In the context of conventional PT optimization, we provide preliminary evidence that Message Passing Neural Networks are effective

in extracting structural information from PT graphs to support decision-making (Chapter 4). However, the impact of incorporating additional stop-level features, such as economic factors associated with the surrounding area, on performance remains to be further explored. Future research could investigate the integration of socioeconomic factors, such as the spatial distribution of income levels, demographic age structures, and patterns of land use and housing development, into the modeling framework. We hypothesize that a suitably designed Graph Neural Network would outperform metaheuristic approaches significantly more, as the latter are inherently unable to capture and learn the complex relationships between these factors and the underlying PT graph structure. Moreover, the scalability of the proposed algorithm requires further evaluation in large real-world cities; for example, the Paris metropolitan area, which has approximately 12,000 bus stops. A preliminary strategy for such large-scale applications is as follows: first, train the model on smaller PT graphs; then directly test it on major cities such as Paris and New York, and compare its performance against metaheuristic baselines. This approach of “learn on smaller instances” and “test on bigger instances” has recently shown some notable successes in the literature [176].

In the integration of DRT and PT (Chapter 5), a ranking score was used to determine the allocation of DRT service areas. This score is defined as a weighted sum of the population rank and the accessibility rank within each area. I expect that by building more advanced indicators that incorporate machine learning techniques, to capture intricate dependencies between socio-demographic information, land use patterns, and PT network topology, the outcome of the proposed optimization could be improved and, more importantly, better adapted to the scenarios under study. Moreover, the factors captured by such indicators could offer a more transparent and interpretable explanation of the design choices made by the optimization. Furthermore, in our experiments across different cities, only metro lines were considered, while bus lines were not included. The impact of incorporating bus lines needs to be tested for future research. However, our framework is comprehensive, as bus lines can be represented exactly the same way as metro lines within the proposed graph-based model. Therefore, the integration of bus lines would not require any extension of the theoretical framework but only an increase in the computational cost of the optimization algorithm. This might require, however, much smarter algorithmic techniques.

The flexible PT system (Chapter 6) has demonstrated superior efficiency in handling requests compared to existing flexible mobility solutions and conventional PT. However, its effectiveness in reducing inequality of accessibility still requires further empirical validation. Moreover, since the system’s decision-making relies on real-time predictions of future requests, the impact of employing different predictive models on its performance remains to be investigated. How to integrate the proposed model with DRT and PT remains an important open problem. A natural idea is to represent all PT lines together with DRT routes using a periodic time-expanded graph. The main advantage of this idea is that it does not require additional modeling framework for PT and DRT separately, as both can be captured within the time-expanded graph. However, a potential drawback is that the resulting TEG may become extremely large, which could significantly slow down decision making. Therefore, the development of appropriate acceleration techniques constitutes an important direction for future research. Behavioural aspects such as habit formation and predictability are also important when assessing new public transport concepts. In conventional PT

systems, this reliability is closely linked to schedule regularity and the repetition of services at fixed times. By design, flexible modes do not offer this kind of regularity. However, recent pilots show that such services are increasingly used [177] and appreciated, which suggests that travellers are willing to trade some schedule regularity for improvements in efficiency [174]. This is also consistent with stated-preference evidence on the joint valuation of travel time and reliability. Our proposed service builds on this trade-off and is therefore a plausible candidate for user acceptance. An interesting avenue for future research, beyond the scope of this thesis, is to investigate to what extent the efficiency gains brought by such services are sufficient to modify existing travel habits over the longer term.

Finally, detailed simulation is essential to bridge the gap between theory and real-world implementation.

7.3 Long-term perspectives

All the thesis is developed under assuming that vehicle technology is the one available today. This reinforces the realism of the study. However, in the future big technological disruptions are expected to come. Vehicle automation, in particular, is expected to further reduce staffing costs. This will very likely change the way PT is planned. The methods presented here are still valid if we consider automated vehicles (only cost coefficients are to be changed), but I expect the results to be much different than those presented here. The nature of the optimized PT could be strongly impacted by automation: the role of fixed conventional PT might be reduced in favor of flexible mobility, due to its ease and cost-efficiency to be implemented with automated vehicles. More equality could be achieved, thanks to this reduced impact of costs. On the other hand, automation will also impact private mobility. Private automated cars would likely be more attractive and convenient for the users, in particular allowing them to perform their activities during their travel. This factor could reduce the relative attractiveness of PT, and consequently reduce the benefits coming from our proposed PT optimization. These aspects, related to human behavior, deserve future research, also including appropriate regulations to limit the negative consequences of private vehicle automation.

The assumption of the methods presented in the various chapters of this thesis is that planning and operational decisions are all made by a single authority, pursuing some objective related to social welfare. In reality, mobility services result from a complex system of different stakeholders, with different (and often contrasting) interests. In real scenarios, transport authorities (e.g., Île de France Mobilité) set up a Public Service Obligation contract [178]. Under this contract, the authorities provide funding, invite private transport operators to bid, and evaluate the services of such operators using a unified set of indicators. To make the methods presented in this thesis actionable, some work is still needed to integrate them into the process of Public Service Obligation. For instance, our models could be run by the authority, to calculate what accessibility distribution (and the related equality) it is possible to achieve via certain mobility services, under a given budget constraint. This would set the bar in the competition between private operators and ensure equality

of accessibility will not be neglected in the final public service provided to the user. However, how to translate the results of our optimizations to a set of clear constraints to be embedded into the Public Service Obligation contracts (or the set of criteria to choose among different private operators' projects) remains an open problem. This problem is challenging due to the multi-criteria nature of public transport planning, where accessibility inequality should be made an important criterion, but should also be integrated with other (also important) criteria, such as overall efficiency, public acceptability, environmental impact.

Another aspect that would require further investigation is the impact of land-use on the outcome of our methods and, conversely, the impact of the outcome of our methods on land-use. Indeed, accessibility results from two inter-dependent aspects: the distribution of accessibility and the transport system. This thesis focuses on the latter only and assumes the former is exogenously given. In reality, this is not true. Indeed, a better (e.g., more equal) distribution of accessibility can result from land-use interventions, rather than transport system optimization, i.e., by re-organizing where opportunities are distributed. Understanding in which cases it is more efficient to optimize land-use or to optimize the transport system to achieve a certain accessibility outcome remains an open problem.

On the other hand, the change in accessibility resulting from the proposed transport system optimization may have important consequences for land-use, which were neglected in this manuscript. A higher level of accessibility and equality can encourage people to relocate to suburban areas. However, this phenomenon can, in turn, lead to urban sprawl, which implies that more land is used for the development of low-density and dispersed residential areas. This outcome contradicts the objectives of compact city regulations [179], which seek to limit urban expansion by promoting higher density development. As a consequence, more land would be made artificial, resulting in losses in biodiversity [180]. Moreover, more kilometers traveled would be needed (even when they are served by PT), resulting in increased emissions. Therefore, the advances achieved by our proposed optimization methods should be accompanied by appropriate regulatory frameworks to mitigate their potential negative impacts. Such regulatory frameworks are outside the scope of my research and competence, and inter-disciplinary studies combining Engineering, Public Policies, Law, and Economics would be needed.

Moving forward, close collaboration among academia, industry, and policy-makers will be essential. I believe that the theoretical foundations and experimental results established in this work can provide a solid basis and clear direction for subsequent research.

Chapter 8

Appendix

8.1 Table of Notation

Symbol	Description
\mathcal{G}	A PT graph
\mathcal{V}	the set of all nodes
\mathcal{E}	the set of all edges
\mathcal{S}	the set of all stations
\mathcal{C}	the set of all centroids
\mathcal{E}_s	the set of all edges of metro lines
\mathcal{E}_c	the set of all edges between to neighbor centroids
\mathcal{E}_d	the set of edges between any centroid and all stations
\mathcal{P}	the set of all individuals in the study are
\mathcal{A}	the set of all candidate areas where DRT can be allocated
\mathcal{A}'	the set of pre-selected areas where DRT can be allocated
$n_{\mathcal{G}}$	the total number of areas in \mathcal{A}
m	total number of centroids in study area
c_i	centroid i ($1 \leq i \leq m$)
s_i	station i
l	a metro line
t_l	headway of line l
w_l	average waiting time for line l
$t_{(s_i, s_{i+1})}$	time spent by a vehicle going from station s_i to the next station s_{i+1}
t_{s_i}	the dwell time a vehicle stops at station s_i
e	an edge
t_e	time cost of an edge e
v_{walking}	walking speed

Continued on next page

Symbol	Description
s_A	the service station of DRT area A
$h(A)$	the difference between the departure time of each DRT vehicle and the next one from station s_A
$x_A; \mathbf{x}$	the number of DRT buses allocated in area A and the allocation vector
$C_{DRT}(A)$	the expected time required to complete a cycle of a DRT bus
$n(A)$	number of pickups and dropoffs requested within area A generated in time $h(A)$
$\phi_i^{\text{DRT, out}}$	the number of passengers per unit of time that originate in tile c_i and that use DRT for their first mile
$\phi_i^{\text{DRT, in}}$	the number of passengers per unit of time having tile c_i as destination and using DRT for their last mile
l	the side of a tile
l^2	the area of a tile
K	the total number of tiles in an area
d	the distance shown in Figure. 5.3
τ_s	the time lost per stop
τ_T	the terminal dwell time of the DRT bus at the terminal s_A
v_{DRT}	DRT bus' speed
$T_{in}(c_i)$	the expected ingress time of centroid c_i
$T_{out}(c_i)$	the expected egress time of centroid c_i
$n(c_i)$	the number of pickups and dropoffs requested within tile c_i
ρ_i	the population density in tile c_i
ϕ_i	trip density originating in centroid c_i
σ_j	the amount of opportunities in tile c_j
$T(c_i, c_j)$	the shortest time to go from c_i to c_j
\mathcal{D}	the search space of Bayesian Optimization

8.2 Calculation of Circle Length $CL_{DRT}(A)$

We now derive Eqn. (5.5). Let $n(A)$ be the number of customers served per cycle by the DRC vehicle. Each customer $1 \leq p \leq n(A)$ has their position (u_p, v_p) , uniformly distributed in area A with K tile. The length of the area is $\frac{Kl}{2}$, and the width of the area is $2l$.

The maximum distance the bus can travel along the horizontal line is: $u_{max} = \max u_1, u_2, \dots, u_{n_A}$. The expectation of u_{max} is

$$\begin{aligned}
E(u_{max}) &= \int_0^{\frac{Kl}{2}} P(u_{max} \geq t) dt = \int_0^{\frac{Kl}{2}} (1 - P(u_{max} \leq t)) dt \\
&= \int_0^{\frac{Kl}{2}} \left(1 - \prod_{p=1}^{n(A)} P(u_p \leq t)\right) dt \\
\text{Assuming unif. distr. in the area} &= \int_0^{\frac{Kl}{2}} \left(1 - \left(\frac{2t}{Kl}\right)^{n(A)}\right) dt = \frac{Kl}{2} \cdot \frac{n(A)}{n(A)+1}.
\end{aligned}$$

So the horizontal expected journey length (to go and come back) is $2E(u_{max}) = Kl \cdot \frac{n(A)}{n(A)+1}$.

Let v be the random variable indicating the vertical distance between any pair of customers within the upper or lower half of the service area. We have

$$E(v) = \frac{1}{l^2} \int_0^l \int_0^l |v_p - v_{p'}| dv_p dv_{p'} = \frac{2}{l^2} \int_0^l \int_0^{v_p} (v_p - v_{p'}) dv_{p'} dv_p = \frac{1}{l^2} \int_0^l v_p^2 dv_p = \frac{l}{3}.$$

Let v' indicate the vertical distance between point s_0 and the first or last customer in the schedule; we have $E(v') = \int_0^l v_p dv_p = \frac{l}{2}$. Let v'' indicate the vertical distance between the last customer served in the upper half and the first customer served in the lower half. We have $E(v'') = l$.

Hence, we have

$$\begin{aligned}
CL_{DRT}(A) &= 2E(u_{max}) + (n(A) - 2) \cdot E(v) + 2E(v') + E(v'') \\
&= Kl \frac{n(A)}{n(A)+1} + (n(A) - 2) \cdot \frac{l}{3} + 2 \frac{l}{2} + l = Kl \frac{n(A)}{n(A)+1} + n(A) \cdot \frac{l}{3} + \frac{4l}{3}.
\end{aligned}$$

The reason for $(n(A) - 2) \cdot E(v)$ is that there are $n(A)$ edges connecting all $n(A)$ customers in sequence, but we need to remove the edge between the last and first customer, as well as the edge between the last customer served in the upper half and the first customer served in the lower half, which are accounted for separately.

8.3 Proof of NP-hardness of (5.16)-(5.18)

Given a set of n_G areas, we can allocate a number $x_i = n_i$ of DRT busses to each area A_i with a bound N , i.e., $x_i \in \{0, 1, \dots, N\}$. We associate a weight w_i to area A_i . We aim to solve the following optimization problem.

$$\max_{x_1, x_2, \dots, x_{n_G}} \text{Atk}(\mathcal{G}((x_1, x_2, \dots, x_{n_G}))) \quad (8.1)$$

subject to the following constraints:

$$\sum_{k=1}^{n_G} x_i \cdot w_i \leq N \quad (8.2)$$

$$x_i \in \mathbb{N}_0, \quad i = 1, 2, \dots, n_G \quad (8.3)$$

$$w_i = 1, \quad i = 1, 2, \dots, n_G \quad (8.4)$$

The constraints of our problem are similar to those in an integer bounded quadratic knapsack problem, which is a generalization of a quadratic knapsack problem (QKP - Ch.12 of [181]). However, our objective function is more complex than the quadratic form of the QKP. Since QKP is NP-complete, even if all weights are the same (as in our case (8.4)), our problem is strongly NP-hard.

8.4 Average Occupation of a Bus (Section 6.4.3)

We measure the average in-vehicle time $t_{\text{in-vehicle}}$, the total number of served requests N_{served} , and the considered time interval $t_{\text{simulation}} = t_1 - t_0$. We also know fleet size N . The Average occupation of a bus is: $N_{\text{occupation}} = \frac{N_{\text{served}}}{N \cdot t_{\text{simulation}}} \cdot t_{\text{in-vehicle}}$.

Factor $\frac{N_{\text{served}}}{t_{\text{simulation}}}$ represents the rate of requests (req/min) the system is supposed to pickup. The average rate of requests picked up by one bus is thus $\frac{N_{\text{served}}}{N \cdot t_{\text{simulation}}}$. Requests $\frac{N_{\text{served}}}{N \cdot t_{\text{simulation}}}$ picked up at t will leave at $t + t_{\text{in-vehicle}}$ on average, but the bus can also pick up the same number $\frac{N_{\text{served}}}{N \cdot t_{\text{simulation}}}$ of requests at $t + t_{\text{in-vehicle}}$ on average. If a bus starts running at time t_0 , during this time interval $[t_0, t_0 + t_{\text{in-vehicle}}]$, a bus can generally pick up $\frac{N_{\text{served}}}{N \cdot t_{\text{simulation}}} \cdot t_{\text{in-vehicle}}$ requests, and after $t_0 + t_{\text{in-vehicle}}$, the number of pickup requests and dropoff requests is equal on average, so the occupation of the bus will remain the same on average.

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Titre : Conception des transports publics de nouvelle génération pour la réduction des inégalités d'accessibilité

Mots clés : Transports publics ; Accessibilité ; Égalité ; Mobilité flexible

Résumé : L'accessibilité d'un lieu mesure la facilité, pour les personnes qui y résident, d'atteindre les opportunités environnantes, telles que les écoles, les commerces ou les emplois. Cette thèse s'intéresse à l'accessibilité offerte par les Transports Publics (TP). Les TP conventionnels, basés sur des lignes et des horaires fixes, entraînent une répartition inégale de l'accessibilité : elle est généralement bonne dans les centres-villes mais faible en périphérie, qui devient ainsi dépendante de la voiture. Cela rend les villes insoutenables sur les plans environnemental, économique et social.

Les services de mobilité flexible (par ex., le Transport à la Demande – TAD) semblent mieux adaptés aux zones de faible densité et peuvent efficacement compléter les TP conventionnels. Toutefois, la plupart des méthodes visant à optimiser la conception et l'exploitation de la mobilité flexible ciblent des objectifs traditionnels, tels que les coûts pour les usagers et les coûts opérationnels, en négligeant l'accessibilité. L'objectif général de cette thèse est de développer de nouvelles méthodes pour concevoir les systèmes de TP de prochaine génération, incluant TP conventionnels et flexibles, ayant pour finalité de réduire les inégalités dans la distribution de l'accessibilité.

Pour atteindre cet objectif, cette thèse s'articule autour de quatre contributions scientifiques majeures et distinctes. Les première et deuxième contributions consistent à élaborer et à proposer des méthodes avancées de planification stratégique des réseaux de TP, spécifiquement conçues pour minimiser les indices statistiques mesurant l'inégalité d'accessibilité. Pour ce faire, nous proposons des solutions algorithmiques originales qui s'appuient sur des heuristiques sophistiquées ainsi que sur des techniques récentes d'apprentissage par renforcement appliqué aux structures de graphes, permettant une exploration efficace de l'espace des solutions possibles.

La troisième contribution marque une avancée significative en développant, à notre connaissance, la toute première méthode de planification dédiée spécifiquement à l'allocation optimale d'une flotte de véhicules de TAD, où la fonction objectif centrale est la réduction des indices d'inégalité d'accessibilité plutôt que le seul coût. Dans le cadre de cette contribution

novatrice, nous combinons une représentation graphique précise du réseau de TP conventionnel existant avec un modèle mathématique d'« Approximation Continue » du service de TAD, permettant de modéliser la fluidité du service à la demande. Nous proposons ensuite une méthode d'optimisation bi-niveaux complexe : le niveau supérieur, qui décide de l'allocation des ressources, est résolu via une approche hybride ingénieuse combinant une heuristique basée sur le classement des zones prioritaires et l'optimisation bayésienne pour allouer dynamiquement la flotte de TAD ; simultanément, le niveau inférieur effectue l'affectation précise des déplacements des usagers aux véhicules disponibles.

Enfin, la quatrième contribution consiste à concevoir l'architecture d'un nouveau système de transport public flexible inédit, positionné stratégiquement comme un hybride situé entre la rigidité du TP conventionnel et la totale flexibilité de la mobilité à la demande. Ce système vise à combiner l'efficacité opérationnelle découlant de la structure forte et prévisible du premier, avec la grande adaptabilité et la réactivité du second. La gestion opérationnelle en temps réel d'un tel système complexe est assurée par une optimisation en horizon glissant (Model Predictive Control), fondée sur des algorithmes avancés de recherche arborescente de Monte-Carlo (Monte Carlo Tree Search), permettant de prendre des décisions robustes face à l'incertitude de la demande.

En synthèse, cette thèse offre une perspective transformative sur l'opérationnalisation concrète des concepts d'accessibilité et d'égalité sociale dans la conception ingénierie des systèmes de transports publics. Alors que ces deux aspects cruciaux ont été majoritairement analysés dans la littérature académique de manière « ex-post » (c'est-à-dire en évaluant les performances après la mise en place des réseaux), les méthodes proposées ici permettent de les internaliser « ex-ante ». Cela signifie que l'équité et l'accessibilité deviennent des variables directes et contraignantes au sein même des fonctions objectifs d'optimisation dès la phase de conception, garantissant ainsi que les futurs réseaux de transport soient intrinsèquement justes et inclusifs avant même leur déploiement physique.

Title : Design of Future-generation Public Transport to Reduce Inequality of Accessibility

Keywords : Public Transport ; Accessibility ; Equality ; Flexible Mobility

Abstract : The accessibility of a location measures the ease, for people leaving there, to reach surrounding opportunities, such as schools, shops, or jobs. This thesis focuses on accessibility provided by Public Transport (PT). Conventional PT, based on fixed lines and schedules, results in an unequal distribution of accessibility : it is generally good in city centers but poor in the suburbs, which are thus car-dependent. This makes cities environmentally, economically, and socially unsustainable. Flexible mobility services (e.g., Demand-Responsive Transport (DRT)) are potentially better suited for low-density areas and can effectively complement conventional PT. However, most methods to optimize the design and operation of flexible mobility target traditional objectives, e.g., user and operational costs, neglecting accessibility.

The **overarching aim** of this thesis is to develop novel methods for designing future-generation PT systems, including conventional and flexible PT, whose objective is to reduce the inequality of accessibility distribution.

To achieve this objective, this thesis is structured around four major and distinct scientific contributions. The first and second contributions involve developing and proposing advanced strategic planning methods for public transport (PT) networks, specifically designed to minimize statistical indices measuring accessibility inequality. To this end, we propose original algorithmic solutions based on sophisticated heuristics and recent reinforcement learning techniques applied to graph structures, enabling efficient exploration of the solution space.

The third contribution marks a significant advancement by developing, to the best of our knowledge, the very first planning method dedicated specifically to the optimal allocation of a Demand-Responsive Transport (DRT) fleet, where the central objective function is the reduction of accessibility inequality indices rather than cost alone. Within this innovative contribution, we combine an accurate graphical representation of

the existing conventional PT network with a mathematical model of "Continuous Approximation" for the DRT service, allowing us to model the fluidity of on-demand services. We then propose a complex bi-level optimization method : the upper level, which decides on resource allocation, is solved via an ingenious hybrid approach combining a heuristic based on ranking priority zones with Bayesian optimization to dynamically allocate the DRT fleet ; simultaneously, the lower level performs the precise assignment of user trips to available vehicles.

Finally, the fourth contribution consists of designing the architecture of a novel flexible public transport system, strategically positioned as a hybrid situated between the rigidity of conventional PT and the total flexibility of on-demand mobility. This system aims to combine the operational efficiency stemming from the strong and predictable structure of the former with the high adaptability and responsiveness of the latter. The real-time operational management of such a complex system is ensured by optimization using a receding horizon (Model Predictive Control), based on advanced Monte Carlo Tree Search algorithms, enabling robust decision-making in the face of demand uncertainty.

In summary, this thesis offers a transformative perspective on the concrete operationalization of accessibility and social equity concepts in the engineering design of public transport systems. While these two crucial aspects have been predominantly analyzed in academic literature in an "ex-post" manner (i.e., by evaluating performance after network implementation), the methods proposed here allow them to be internalized "ex-ante." This means that equity and accessibility become direct and constraining variables within the optimization objective functions themselves from the design phase, thereby ensuring that future transport networks are intrinsically fair and inclusive even before their physical deployment.