



Essays on Metals : Sustainability and Markets

Maylis Peyret

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Soutenue par

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À mes grands-mères,

« Tout doit être construit. Un arbre comme un corps humain, un corps humain comme une cathédrale. » – Henri Matisse

« En tout chaos, il y a un cosmos. En tout désordre, un ordre secret. » – Carl Jung

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Introduction

Sustainability in resource markets hinges on the delicate interplay between supply and demand, where economic, technological, and environmental factors converge to shape market equilibriums. Following the definition proposed by the Brundtland Commission (1987), sustainability refers to meeting the needs of the present without compromising the ability of future generations to meet their own needs—a principle that, when applied to resource markets, requires balancing current extraction and use with long-term availability and environmental constraints (Krausmann et al., 2009; Bleischwitz et al., 2012). While the Brundtland definition underscores the environmental and intergenerational dimensions of sustainability, this thesis adopts a more focused perspective, analyzing sustainability exclusively in economic terms.

Specifically, we assess whether long-term interactions between supply and demand can produce stable, non-explosive equilibrium paths for quantities and prices. Environmental, social, and governance (ESG) considerations—such as carbon footprints, water usage, or social license to operate—are not directly integrated into the model. This choice reflects a deliberate scope limitation: the objective is to disentangle the structural economic mechanisms underpinning long-term resource dynamics. ESG concerns remain essential and are widely addressed in complementary research (e.g. Sovacool et al., 2020; Gulley et al., 2019; Mancini et al., 2013), but fall outside the analytical lens adopted here, which centers on systemic market behavior and economic feasibility over time.

This thesis positions itself at the intersection of two important bodies of literature: the classical resource economics tradition, which provides the theoretical backbone for analyzing long-term depletion dynamics, and the rapidly expanding energy transition literature, which underscores the accelerating demand for critical metals in clean technologies. By bridging these two domains, the thesis aims at offering a holistic framework to understand metal markets under the structural transformations of the 21st century.

Before delving into the specific dynamics of metal markets, it is important to clarify the terminology used throughout this thesis. *Metals* are chemical elements characterized by properties such as electrical and thermal conductivity, ductility, and malleability. They are typically solid at room temperature—except for mercury—and can form metallic bonds that allow for deformation without breaking. *Minerals*, by contrast, are naturally occurring inorganic compounds with a defined chemical composition and crystalline structure. Metals are usually extracted from metal-bearing minerals embedded in rock matrices, known as mineral deposits, which vary in geological characteristics and metal content. The concentration of metal—referred to as *ore grade*—can range from several percent

to fractions of a percent. Some metals, like copper, iron, or bauxite (aluminum ore), are extracted from relatively abundant primary deposits, while others—such as cobalt, indium, and tellurium—are typically obtained as co-products or by-products of the extraction of more abundant metals, such as copper or nickel. This by-product nature often constrains their supply response to rising demand.

It is also important to distinguish metallic commodities—tradable goods derived from metals, typically in the form of cathodes, ingots, concentrates, or alloys—from non-metallic mineral commodities, such as sand, phosphates, or clays, which lack the conductive and malleable properties that define metals. Metallic commodities are priced on international markets and can be subject to industrial specifications and market segmentation. Contrary to the common perception of metals as homogeneous commodities, they exhibit significant heterogeneity. Horizontally, metals differ by type and use (for example, copper vs. lithium); vertically, even within the same metal, markets can be segmented by purity levels, physical forms (cathodes, ingots, powders), and processing stages. This heterogeneity shapes both economic valuation and market structure and has direct implications for modeling supply constraints and forecasting price dynamics.

Metals are considered non-renewable resources because their formation in the Earth's crust occurs over geological timescales far exceeding human planning horizons. Once extracted and dispersed, they cannot be replenished in any meaningful timeframe. However, unlike fossil fuels, metals are not destroyed through use and can, in principle, be recycled indefinitely—a key distinction that profoundly influences the economics of their consumption. While depletion raises the marginal cost of extraction, it does not imply absolute scarcity in the long term as it does for hydrocarbons. Instead, metals can gradually shift from being sourced predominantly through primary extraction to increasingly relying on secondary supply streams through recycling. This unique feature of metals introduces a delayed but potentially powerful adjustment mechanism that can alleviate resource constraints and price pressures over time—a dynamic explored in depth in this thesis.

The concept of depletion in mineral economics does not refer to the physical disappearance of metals, but rather to the progressive increase in the economic, energetic, and environmental costs of extracting them. While metals themselves are not destroyed through use and can theoretically be recycled indefinitely, the quality of available ore bodies declines as high-grade¹, easily accessible deposits are exhausted. This phenomenon—known as economic depletion—leads to deeper mining, more complex processing, and greater environmental externalities. As Tilton (2003) emphasizes in "Is

¹ High-grade ores are mineral deposits with relatively high concentrations of the target metal, typically above 1% for copper and over 60% for iron. These grades vary by metal and over time, depending on technological advances and market conditions (Tilton and Guzmán, 2016; Mudd and Jowitt, 2018).

Mineral Depletion a Threat?", the central issue is not the exhaustion of physical quantities, but whether the rising marginal costs of extraction will constrain future supply or drive prices upward in a way that harms economic activity. In this view, depletion is primarily a question of economic viability, not physical scarcity.

A central distinction in mineral economics—fundamental to the approach adopted in this thesis—is that between reserves and resources. Reserves denote the quantities that are economically recoverable with current technology at prevailing prices; they are therefore an economic and dynamic concept that expands or contracts as prices, extraction costs, and technologies evolve. By contrast, resources represent the broader geological endowment that could become economically recoverable at some price and thus capture a long-run physical–technological constraint rather than an economic one. This distinction clarifies that depletion operates simultaneously on these two planes—tightening short-run reserves while gradually pushing extraction toward the limits set by the broader resource base.

Over time, the process of depletion can be mitigated or exacerbated by various forces: technological progress, exploration success, recycling development, and substitution. For instance, the discovery of new deposits, improvements in extraction techniques, or the ability to recycle secondary materials more efficiently can delay the impacts of depletion (Tilton and Lagos, 2007). Conversely, rising demand, especially for metals produced as by-products, may outpace these compensating mechanisms, leading to structural scarcity (Vidal et al., 2018). Depletion is therefore best understood as a dynamic and context-dependent process, shaped by both natural endowments and economic incentives.

Given their pivotal role in industrial processes and emerging technologies, metals' status as finite and non-renewable resources raises unique sustainability challenges. These challenges are magnified by the accelerating global demand for metals, driven by technological innovation, economic growth, and, increasingly, the transition to low-carbon energy systems.

Indeed, the transition to a clean energy system represents a profound shift from the traditional reliance on hydrocarbon resources, with significant implications for global metal markets. Clean energy technologies, such as solar photovoltaic (PV) systems, wind turbines, and electric vehicles (EVs), require substantially more mineral resources than their fossil fuel-based counterparts. For instance, a typical electric car requires six times the mineral inputs of a conventional car, and an onshore wind plant needs nine times more mineral resources than a gas-fired power plant². Since 2010, the average

² IEA, 2021, The Role of Critical Minerals in Clean Energy Transitions, IEA, Paris <https://www.iea.org/reports/the-role-of-critical-minerals-in-clean-energy-transitions>

mineral intensity³ of new power generation capacity has increased by 50%, driven by the growing share of renewable energy.

Metals, broadly defined, are naturally occurring elements that are typically solid, conductive, and malleable. When traded on global markets, they are referred to as metal commodities, with prices set by international exchanges and influenced by industrial demand, technological development, and geopolitical conditions. For the purposes of economic analysis, it is useful to distinguish between three broad categories of metals. First, *industrial metals*—such as copper, aluminum, zinc, and steel—are produced and consumed in large volumes and are central to construction, manufacturing, and infrastructure. Second, *technical metals*—including cobalt, lithium, and rare earth elements (REEs)—are produced in smaller quantities but are vital for advanced technologies, particularly in batteries, electronics, and renewable energy systems. Third, the *platinum group metals* (PGMs), such as platinum, palladium, and rhodium, are used in catalytic converters, chemical industries, and jewelry, and are typically high-value and supply-constrained. Each group presents distinct economic characteristics and strategic challenges, from supply chain concentration to price volatility.

The specific mineral requirements vary depending on the technology. Lithium, nickel, cobalt, manganese, and graphite are essential for batteries, determining their performance, lifespan, and energy density. Rare earth elements play a pivotal role in permanent magnets, which are indispensable for wind turbines and EV motors. Copper and aluminum are critical for electricity networks, with copper serving as a cornerstone for all electricity-related technologies.

Beyond sheer volume, the diversity of metals required to support technological systems is expanding. Whereas 20th-century industrialization relied primarily on a handful of base metals, today's energy and digital transitions depend on an increasingly complex array of elements. As Vidal et al. (2013, 2021) highlight, modern technological systems exhibit growing material diversity—meaning that a greater number of distinct metals are needed per unit of technological output—and material co-dependence, whereby specific technologies depend on the simultaneous availability of several critical elements. This marks a structural shift: from bulk-intensive, relatively flexible systems to fine-tuned technological architectures that are often vulnerable to the disruption of a single supply chain. The diversification of

³ Mineral intensity refers to the quantity of mineral resources required per unit of output or activity in a specific process, technology, or system. In the context of energy systems, it measures how much of various metals or minerals are needed to build and operate clean energy technologies compared to their conventional fossil fuel-based counterparts. An increase in mineral intensity means that newer technologies, like renewable energy system, require more metals per unit of power generation capacity or per unit of end-use product (e.g., an electric vehicle) than older, traditional technologies (like gas-fired plants or internal combustion engine vehicles). This metric is crucial for assessing the demand implications for specific minerals as energy transitions gather pace.

metal demand thus not only adds complexity to global supply networks but also increases the strategic significance of a broader range of elements whose production is often concentrated or environmentally challenging.

As the energy transition accelerates, the demand for these metals is set to rise dramatically, positioning the energy sector as a major driver of mineral markets. Historically, the energy sector accounted for a relatively small share of mineral demand. As of the early 2010s, fossil fuel-based power generation and energy infrastructure represented less than 10% of total copper and aluminum demand, with the vast majority driven by construction, transportation, and consumer goods sectors (World Bank, 2017). However, clean energy technologies are now emerging as the fastest-growing segment of demand. In scenarios aligned with the Paris Agreement, clean energy technologies could account for over 40% of global demand for copper and rare earth elements, 60-70% for nickel and cobalt, and nearly 90% for lithium within the next two decades. EVs and battery storage have already surpassed consumer electronics as the largest consumers of lithium and are on track to overtake stainless steel as the primary end user of nickel by 2040 (IEA 2021).

As a result, copper, essential for electricity conduction, is projected to nearly double in demand by 2030, reaching approximately 50 million metric tons annually, fueled by its critical role in power grids, wind turbines, and electric vehicles (IEA, 2021). Each kilometer of high-voltage power line requires about 1 ton of copper, and global investments in grid expansion are expected to exceed \$400 billion annually by 2030 (Copper Alliance, 2021; IEA, 2021). Wind turbines also consume significant quantities of copper, with a single onshore turbine requiring 2-4 tons and offshore turbines using up to 10 tons. Similarly, low-carbon power generation is set to triple its mineral requirements by 2040, with offshore wind and solar photovoltaic (PV) systems driving the demand.

Lithium-ion batteries, the dominant technology for grid storage and electric vehicles (EVs), are central to the energy transition. A typical EV battery requires approximately 8 kilograms of lithium, 14 kilograms of cobalt, 35 kilograms of nickel, and 70 kilograms of graphite. Demand for these materials is expected to rise significantly: lithium demand is projected to grow over 40-fold by 2040, while graphite, cobalt, and nickel demands are anticipated to increase by 20 to 25 times (IEA, 2021). EVs, at the forefront of decarbonizing transportation, are expected to see global sales rise from over 10 million units in 2022 to 40 million by 2030 (IEA, 2023). This surge in EV adoption accelerates the need for battery metals as well as lightweight materials such as aluminum to enhance energy efficiency.

Rare Earth Elements (REEs), including neodymium and dysprosium, are also vital to the transition, as they are integral to the permanent magnets used in wind turbines and EV motors. Demand for REEs is projected to triple by 2040 (IEA, 2021). Beyond transportation and energy storage, integrating

renewables into power grids necessitates significant upgrades to transmission and distribution systems. These upgrades, coupled with the electrification of industrial processes—such as the use of electric arc furnaces for steel production and electric heat pumps—are driving increased demand for metals like copper, aluminum, and specialty alloys (World Bank, 2020).

The demand trajectories for these resources are subject to significant variability due to technological and policy uncertainties. For instance, cobalt demand could rise from six to thirty times current levels depending on the evolution of battery chemistries and climate policy commitments (IEA, 2021). Similarly, rare earth elements may experience a three- to sevenfold increase in demand, contingent upon wind turbine designs and the strength of policy support. The largest variability stems from the stringency of climate policies, with global suppliers facing the pressing question of whether the world will adopt scenarios consistent with the Paris Agreement.

However, the energy transition is not the sole driver of this surge in mineral demand. The digital transformation, characterized by the rapid growth of data centers, artificial intelligence (AI), and other advanced technologies, is another significant contributor. For example, data centers currently account for less than 1% of global copper demand, but this share is expected to grow to 6-7% by 2050, adding an estimated 3.4 million metric tons to annual copper consumption⁴. This projection aligns with findings by Sovacool et al. (2020), who emphasize that the expansion of digital infrastructure—particularly AI, cloud computing, and smart grids—will generate increasing pressures on copper, aluminum, and rare earths. Similarly, Gulley et al. (2019) note that technological advances in connectivity and automation are increasing the demand for specialty metals, especially neodymium and dysprosium used in magnet and sensors. Werner et al. (2020) highlight that miniaturization and automation in electronics are raising the rare earth intensity of consumer and industrial devices, contributing to a projected doubling of global demand for these elements between 2020 and 2035.

Economic development, particularly in emerging economies, represents a third major driver of mineral demand—distinct from and often underestimated relative to energy and digital transitions. As urbanization and industrialization continue, the infrastructural and technological needs of developing nations are expanding rapidly, fueling demand of construction materials, transportation systems, consumer electronics, and energy infrastructure. Between 2020 and 2050, the global urban population is expected to grow by more than 2.5 billion people, with 90% of this increase concentrated in Asia and Africa (UN, 2018). This urban expansion entails massive investments in roads, housing, water

⁴ “BHP warns AI growth will worsen copper shortfall”, Financial Times, 15 September 2024. <https://www.ft.com/content/da407b47-4133-470a-9574-508cee43e107>

systems, and electrical grids, all of which are highly metal-intensive. For instance, copper use in construction, which already accounts for roughly 30% of global copper demand (Vidal et al., 2021), could grow significantly as cities expand and electrify. BHP forecasts a 70% increase in global copper demand by 2050, of which only a portion is attributed to the low-carbon transition⁵. The share of copper demand linked to the energy transition is expected to rise from 7% today to 23% by mid-century, while the digital sector's share could grow to 6%, leaving the remainder (over 70%) driven by economic development. According to Vidal et al. (2021), meeting the basic material needs of low- and middle-income countries—i.e. bringing them to infrastructure and service levels comparable to current OECD averages—would require cumulative copper consumption of over 2.7 billion metric tons between 2015 and 2050. This alone approaches the upper bound of known copper resources under current technologies and prices.

While demand for metals is accelerating at a sustained pace, the supply side faces significant constraints, moving much more slowly to adapt to the new dynamics of energy transition, digital transformation, and the growth of emerging economies. Supply chains for these materials are fraught with long lead times, declining resource quality, environmental and social scrutiny, and high geographical concentration, all of which create vulnerabilities that could hinder the rapid scaling of production needed to meet rising demand.

One of the most pressing challenges is the lengthy timeline required to develop new mining projects. For copper, it takes on average 10 to 16 years to bring a project from discovery to first production, due to complex permitting, financing, and construction phases (IEA, 2021; S&P Global, 2022). For other base and critical metals, the timeline can vary. Nickel and lithium projects typically range from 6 to 10 years, while rare earth projects often exceed 10 years due to environmental and geopolitical constraints (IEA, 2021). Beyond the mine itself, supporting infrastructure such as roads, power lines, or railways is often needed to transport output from remote sites to ports or processing facilities. In regions lacking existing logistics networks, this can add several years to project timelines. This slow pace raises critical questions about the supply side's ability to respond swiftly to surging demand. Companies that delay investment until deficits materialize risk creating prolonged periods of market tightness and price volatility, disrupting the smooth progression of energy transitions. This situation is reminiscent of the commodity super-cycle that culminated in the early 2010s, which had spurred a wave of investments in mining capacity.

⁵ “BHP expects copper demand to grow by 1 million metric tons a year until 2035”, Reuters, 30 September 2024, https://www.reuters.com/markets/commodities/bhp-expects-copper-demand-grow-by-1-mln-metric-tons-year-until-2035-2024-09-30/?utm_source=chatgpt.com

Yet since then, despite sustained growth in global resource inventories, exploration success has remained limited and investment levels have not returned to their previous heights. As noted by Guj and Schodde (2025), resource expansion over the past decade has been driven primarily by the progressive delineation of existing deposits, not new discoveries—underscoring the fragility of the supply response if prices were to spike again.

Compounding this challenge is the declining quality of existing resources, particularly in key producing regions. Chile, the world’s leading producer of mined copper—accounting for approximately 27% of global extraction—illustrates this trend vividly. Over the course of the 20th century, the average copper concentration in quarried rock fell from around 4% to 1%, according to estimates by Schodde (2010), and has continued to decline to roughly 0.5% today. In Chile alone, average ore grades have dropped by 30% over the past 15 years. Lower-grade ores require significantly more energy and resources (water, chemical inputs) to extract usable metal content, increasing production costs, greenhouse gas emissions, and mine waste volumes. This trend of declining marginal productivity further exacerbates the difficulties of meeting demand efficiently and sustainably.

In addition, environmental, social, and governance (ESG) concerns have become central to mineral supply chains. The production and processing of critical minerals often come with significant environmental and social costs, including high water use, habitat destruction, and adverse effects on local communities. For example, over half of today’s lithium and copper production occurs in regions facing high water stress. These challenges are compounded by growing scrutiny from consumers and investors, who increasingly demand sustainably and responsibly sourced minerals. However, achieving higher ESG standards often slows project development further, potentially leaving supply unable to meet demand.

Finally, climate risks present yet another major supply-side challenge. Many mining regions, such as Australia, China, and parts of Africa, are exposed to extreme heat, flooding, and water scarcity, jeopardizing the reliability of future supplies. Lithium and copper production are particularly vulnerable to water stress, which could increase competition for water resources in arid regions, further complicating operations.

The imbalance between rapidly increasing demand and the inherently slow supply response creates significant potential for heightened price volatility. When robust and persistent demand growth is met with supply constraints, the resulting disequilibrium places substantial upward pressure on prices, often leading to pronounced and unpredictable fluctuations. This phenomenon is particularly pronounced in the context of global metal markets, which are characterized by deep interconnections and extensive supply chain dependencies.

Metal markets are uniquely exposed to such volatility due to the highly concentrated nature of their supply chains, even more so than those of fossil fuels. According to the IEA, the top three producing countries account for over three-quarters of global output for critical minerals such as lithium, cobalt, and rare earth elements. For example, the Democratic Republic of Congo produces approximately 70% of the world's cobalt, while China accounts for 60% of rare earth element production. The concentration is even more pronounced in processing activities, with China refining nearly 90% of rare earth elements, 50-70% of lithium and cobalt, and 35% of nickel. These high levels of concentration increase exposure to risks such as geopolitical tensions, trade restrictions, and localized disruptions, all of which can have immediate and widespread effects on global markets.

The global and interconnected nature of metal markets amplifies the impacts of such disruptions. A supply chain disturbance in one region can rapidly propagate across international markets, triggering significant price volatility and supply shortages. For instance, restrictions on rare earth element exports in a key producing country could disrupt production across industries relying on these inputs, including electric vehicles, renewable energy infrastructure, and advanced technologies. The structural characteristics of metal markets exacerbate these risks: unlike fossil fuels, where strategic reserves and diversified production often mitigate supply shocks, metals lack comparable mechanisms to buffer against rapid changes in market conditions.

These fluctuations undermine investor confidence by introducing uncertainty about the long-term profitability of large-scale mining and refining projects—especially given the long lead times, high capital intensity, and exposure to policy and environmental risks inherent to such investments. While rising prices generally signal opportunities and should encourage investment, the volatility itself can be a deterrent, particularly if investors anticipate that high prices may not be sustained or may trigger political backlash, regulatory tightening, or social opposition. As pointed out by Radetzki et al. (2008), the cyclical nature of commodity markets often leads to hesitant investment behavior, where decision-makers delay project initiation until price trends appear durable. This hesitancy can delay capacity expansion, perpetuating supply-demand imbalances and reinforcing the very volatility that discourages investment—a feedback loop that can trap the market in a cycle of underinvestment and instability.

Considering these persistent challenges—supply-demand imbalances, heightened price volatility, and delayed investment in production capacity—, concerns have increasingly shifted toward the systemic implications of constrained mineral availability. This has given rise to the concept of *criticality* in mineral markets. This notion reflects the recognition that certain minerals play an indispensable role in economic and technological development while being subject to heightened risks of supply disruptions. The concept of criticality emerges at the intersection of economic importance and supply vulnerability,

addressing the systemic risks posed by these materials to industries, economies, and national security. The European Union has acted upon this strategic challenge and introduced in March 2023 the Critical Raw Materials Act (CRMA), which seeks to bolster European autonomy in critical mineral supply chains. The CRMA sets ambitious targets, aiming to derive 10% of annual consumption from domestic extraction, 40% from refining, and 25% from recycling by 2030. In terms of metrics, researchers have scrutinized the effectiveness of tools like the Herfindahl–Hirschman Index (HHI) in assessing supply risks associated with strategic metals. Bucciarelli, Hache and Mignon (2025)’s analysis of 33 metals from 1995 to 2021 reveals that changes in HHI values more significantly influence prices in less concentrated markets, challenging traditional thresholds used to identify high-risk situations. Such findings suggest that existing metrics may underestimate risks in these markets, highlighting the need for more nuanced tools that reflect the realities of global supply chains.

While the concept of criticality provides valuable insights into supply chain vulnerabilities and the strategic importance of certain materials, this thesis does not frame exclusively its analysis through the lens of criticality. The criticality framework, which has gained significant traction in recent years, particularly in policy circles, often emphasizes short-term concerns such as supply bottlenecks and geopolitical risks. However, it is less suited to the objectives of this thesis, which focuses on the long-term structural dynamics of metal markets, including cumulative availability, technological progress, and sustained economic growth. In the academic literature, the criticality approach itself has been subject to substantial methodological critiques. Frenzel et al. (2017) notably argue that many current assessments suffer from a lack of theoretical rigor and fail to align with classical risk analysis. They emphasize that criticality is not an inherent property of a material but rather a contextual and consumer-specific measure of supply risk and economic vulnerability. Existing studies can sometimes oversimplify complex supply chain realities, ignore the full range of potential disruption scenarios, and rely on arbitrarily weighted indicators rather than empirically grounded relationships.

Beyond the limitations of the criticality framework, several physical and economic indicators commonly used to assess metal availability also present some weaknesses when applied to questions of long-term sustainability. Among these, the reserves-to-production (R/P) ratio is frequently cited as a simple proxy for the number of years of supply remaining at current extraction rates. However, as Eggert (2011) and the U.S. Geological Survey emphasize, this indicator provides only a static snapshot that does not account for the dynamic evolution of both reserves and production over time. Reserves increase through exploration, technological innovation, and changing economic conditions, while production levels fluctuate in response to demand shifts, supply chain developments, and macroeconomic cycles. For example, the copper R/P ratio has oscillated between 30 and 40 years over the past two decades, reflecting the balance between continuous extraction and expanding reserves.

The same applies to nickel and lithium, where recent demand surges have compressed the ratio without necessarily indicating imminent depletion.

Other physical indicators—such as crustal abundance and ore grades—offer incomplete pictures of sustainability. According to the academic literature, crustal abundance hardly captures the concentration of economically viable deposits (Rudnick and Gao, 2003; Skinner, 1986), while declining ore grades can either signal depletion or simply reflect the ability to profitably exploit lower-grade resources thanks to technological progress (Tilton and Guzmán, 2016). Economic indicators, such as extraction costs and long-term price trends, also lack predictive reliability. Historical analyses show that extraction costs have generally decreased due to technological improvements, and long-term real prices for most metals do not exhibit clear upward trends (Tilton, 2002). Finally, structural constraints such as by-product dependency (APS-MRS, 2011) and technical barriers to separating metals from complex ores further complicate the assessment of availability based on any single indicator. Taken together, these measures provide useful, but fundamentally insufficient, insights into the sustainability of metal consumption. They fail to capture the dynamic feedback loops, resource depletion trajectories, and the role of technological progress over time. These limitations reinforce our choice to adopt a systemic perspective centered on long-term equilibrium dynamics rather than short-term criticality assessments.

Instead, our work examines the broader interplay between resource availability, demand trends, and technological progress to provide an understanding of metal market behavior. By shifting the focus from specific supply risks to systemic market dynamics, the analysis contributes to understanding how structural and technological shifts can, in the long run, shape the sustainability of metal markets in the context of global energy transitions and economic development.

The thesis examines the long-term sustainability of metal markets excluding short-term disruptions such as geopolitical tensions, supply chain bottlenecks, and temporary demand shocks. While these factors do significantly influence market behavior, they are beyond the scope of this analysis which centers on structural and systemic forces shaping the trajectory of metal demand and supply. This approach aligns with the principles of exhaustible resource (ER) economics, which has provided valuable insights into the long-term predictability of supply and demand trends. As Pindyck (1978b) argues, the very term "exhaustible" is somewhat misleading, and he advocates for replacing it with "non-renewable," since the notions of reserves and depletion are not strictly geological but fundamentally economic. This view reflects a broader tension between economists and geologists: where the latter tend to define availability based on physical stocks, the former emphasize how

economic variables—such as prices, technologies, and costs—determine what is exploitable. This perspective underpins our modeling approach and will be revisited throughout the thesis.

It is important to explicitly state that this thesis deliberately abstracts from geopolitical disruptions, trade policy shocks, and supply chain bottlenecks. While these factors have significant real-world implications and often dominate short- and medium-term market analyses, they fall outside the long-run scope of this work. The objective here is to focus mainly on structural, long-term dynamics that shape metal markets under the assumptions of open, functioning global trade systems and uninterrupted supply chains. This abstraction allows us to isolate the fundamental interactions between resource availability, demand growth, and technological progress, without conflating them with episodic disruptions or politically induced supply constraints.

This thesis is structured in five interconnected chapters, each building upon the previous one to explore the long-term sustainability of metal markets from both theoretical and applied perspectives. Chapter 1 provides a review of the relevant literature, tracing the evolution of economic thinking on resource depletion, market equilibrium, and supply constraints, and highlighting the methodological approaches that inform this work. Chapter 2 introduces a stylized analytical model of metal markets, grounded in a cumulative availability-based supply function and a CES demand function, designed to capture the structural interactions between long-term demand and supply. Chapter 3 applies this model to the case of copper, calibrating it with empirical data to assess the feasibility and stability of projected demand-supply equilibria over the coming decades. Chapter 4 extends the analytical framework by incorporating a secondary supply mechanism through recycling, examining its potential to relieve pressure on primary resources and moderate long-run price trajectories. Finally, Chapter 5 shifts the focus to shorter time horizons, adopting an econometric approach to investigate price dynamics and improve forecasting accuracy, thus complementing the long-term structural analysis with a more granular perspective on market behavior.

Chapter 1 provides an overview a well-established tradition in exhaustible resource economics, which seeks to understand how finite resource availability and structural demand trends interact to shape market equilibria over time. Building on this foundation, a wide array of models has been developed to explore different aspects of resource use, ranging from optimal depletion paths and sustainability conditions to the effect of uncertainty, technological change, and market imperfections. The literature originates with Harold Hotelling's seminal 1931 paper, which laid the foundation for economic analyses of exhaustible resources by predicting that the net price of a non-renewable resource should rise at the rate of interest under optimal extraction. This insight gave rise to an influential line of research exploring the implications of scarcity, substitution, and intertemporal resource allocation (see,

among others, Solow (1974), Stiglitz (1974), and Dasgupta and Heal (1974)). In subsequent decades, additional realism was introduced into resource market modeling (Pindyck (1978, 1980), Kemp and Long (1980), Arrow and Chang (1982), Gaudet (2007); Gilbert (1979) and Devarajan and Fisher (1981)). More recently, an emerging strand of literature has used stylized but tractable functions—such as logistic depletion curves for supply and CES (Constant Elasticity of Substitution) functions for demand—calibrated with empirical data to explore the long-run behavior of resource markets under different policy or technological scenarios. Notable examples include the work of Castillo and Eggert (2020) and Vidal et al. (2018, 2021), who provide flexible frameworks to assess the physical and economic feasibility of projected extraction pathways.

Chapter 2 builds on this literature and develops a stylized theoretical framework to assess long-term equilibrium of metal markets, focusing on the interplay between supply and demand dynamics. On the supply side, the model introduces a Cumulative Availability Curve (CAC), which reflects the total recoverable quantity of a resource as a function of price. This concept builds on the work of Tilton (2003) and later adapted by Castillo and Eggert (2020). It captures both the physical and economic constraints of extraction, illustrating how rising costs reduce the volume of economically accessible material. By introducing a price-responsive CAC and pairing it with a logistic production profile, this thesis captures the potentially non-linear evolution of annual supply as resource exhaustion sets in. In doing so, it contributes to a more realistic and flexible modeling of long-term supply behavior, consistent with recent calls in the literature for better integration of economic and geological constraints (Vidal et al., 2018; Eggert, 2010).

On the demand side, we use a Constant Elasticity of Substitution (CES) demand function, which bridges short-term and long-term demand behaviors. The CES specification is particularly well suited to capture the structural shifts that define the 21st-century metal demand landscape—most notably the green energy transition and the continued economic development of emerging economies. Building on recent empirical insights (e.g. World Bank, 2017; IEA, 2021), we incorporate scenario-based projections for renewable energy deployment, electric vehicle diffusion, and infrastructure needs, allowing the model to distinguish between different drivers of metal consumption and their evolution over time. This detailed structuring of demand not only reflects heterogeneity across end uses but also allows us to explore how varying policy trajectories, technological choices, and macroeconomic dynamics shape long-term needs. In doing so, the CES demand block fills a gap in the resource economics literature, where demand is often represented in overly stylized or aggregated forms, and rarely linked explicitly to energy transition pathways or sector-specific needs.

Coupling this demand function with a supply block grounded in the cumulative availability curve (CAC) enables a dynamic and internally consistent representation of market equilibria. The resulting framework captures both the geological and technological limits on supply and the evolving structure of demand, providing a robust analytical tool for assessing the sustainability of metal markets under structural transformation.

The equilibrium price emerges from the interaction between these supply and demand functions. In this thesis, we define sustainability as the existence of a *Non-Explosive, Non-Zero* (henceforth, NENO) equilibrium path for both prices and volumes — that is, one in which neither prices nor quantities tend toward zero, nor do they diverge toward infinity. This criterion reflects a dynamically stable and structurally viable market trajectory over the long term.⁶ The NENO criterion proposed here offers a structural, diagnostic tool for assessing long-run viability under empirical constraints. This approach aligns more closely with applied sustainability thinking, such as the Brundtland Commission's widely adopted definition — meeting the needs of the present without compromising the ability of future generations to meet their own needs (Brundtland Report, 1987). While the Brundtland view emphasizes the balance between present use and long-term availability, our framework makes this operational by requiring that price and volume trajectories remain economically viable and physically feasible over time. This complements broader perspectives on resource sustainability in material flow analysis and environmental economics (Krausmann et al., 2009; Bleischwitz et al., 2012; Vidal et al., 2018).

In **Chapter 3**, this theoretical framework is parametrized on copper data. Copper, often referred to as "Dr. Copper"⁷ due to its reputation as an economic indicator, is a widely used metal that plays a central role in economic growth, digital transformation, and the energy transition. Moreover, data availability for copper is comparatively higher than for other metals, enabling robust calibration and parametrization.

To assess the compatibility of future copper demand with long-term supply constraints, we construct four forward-looking scenarios. The first serves as a "business-as-usual" benchmark, in which countries—particularly developing economies—continue to rely heavily on fossil fuels without

⁶ This definition diverges from traditional approaches in resource economics, as reviewed in the first chapter, which generally fall into two main streams. The first is rooted in *intertemporal optimization*, where sustainability is assessed through the lens of efficient depletion paths that maximize discounted utility or profit over time (Hotelling, 1931; Solow, 1974; Dasgupta & Heal, 1974). The second emphasizes *intergenerational equity*, often operationalized through the Hartwick rule, which holds that investing all resource rents in reproducible capital can sustain constant consumption across generations (Hartwick, 1977; Asheim, 1994).

⁷ Golding, B. and Golding, S.D., 2017. Doctor Copper and King Coal. *Metals, Energy and Sustainability: The Story of Doctor Copper and King Coal*, pp.1-20.

significantly adjusting their energy mix to mitigate greenhouse gas emissions. Building on this baseline, we then incorporate the material requirements implied by various energy transition pathways developed by the International Energy Agency (IEA). These scenarios reflect different assumptions about the pace of electrification, the deployment of renewable energy, and investment in low-carbon infrastructure, thereby capturing a wide spectrum of plausible futures. Specifically, we consider the Stated Policies Scenario (STEPS), the Announced Pledges Scenario (APS), and the Net Zero Emissions by 2050 Scenario (NZE), each of which embodies a distinct level of ambition in pursuing clean energy goals⁸. These pathways imply markedly different trajectories for copper demand, as technologies such as wind turbines, solar panels, electric vehicles, and smart grids significantly increase the metal intensity of energy systems.

By comparing these divergent pathways, the model allows us to assess the ability of currently known copper deposits to meet future demands and the implications of falling short.

Our numerical results are explicitly simulations and not forecasts. While the simulations rely on robust data and credible foresights, their purpose is to explore potential dynamics rather than predict precise future outcomes.

The model explores how supply-side technological progress can alleviate price pressures. Theoretically, price increases incentivize exploration and the development of new extraction technologies, leading to greater resource availability. By quantifying the extent to which technological advances can offset demand-driven price increases, the analysis provides insights into the robustness of copper markets under various conditions. This nuanced approach underscores the importance of technological innovation in ensuring market sustainability while accommodating the increasing resource demands of a transitioning global economy.

Chapter 4 extends our analytical model to incorporate secondary supply from recycling in settings where refined secondary metal is commercially indistinguishable from primary output, so that supply consists of two streams clearing with a single price. Demand retains the CES structure developed earlier, and primary supply still follows the cumulative-availability, logistic-depletion logic. Secondary supply is modeled as a price-responsive, state-dependent flow governed by a stochastic state variable, following a finite-state Markov process whose transition probabilities encode policy ambition,

⁸ The Stated Policies Scenario (STEPS) reflects current policy settings and announced commitments, assuming only partial implementation of climate targets. The Announced Pledges Scenario (APS) outlines a trajectory aligned with the Paris Agreement, aiming to limit global warming to well below 2°C while ensuring universal energy access and improving air quality. The Net Zero Emissions by 2050 Scenario (NZE) is the most ambitious, describing a narrow but achievable path to reach net-zero CO₂ emissions globally by 2050 through rapid deployment of clean technologies and behavioral changes.

technological progress, and path dependence. This choice lets us map energy transition scenarios into different recycling infrastructure scale-up scenarios.

Analytically, we show how introducing a price-responsive, state-dependent recycling stream reshapes adjustment dynamics in metal markets. As recycling infrastructure matures, total supply becomes more elastic—secondary flows respond more readily to price signals—so the market’s reaction to a given demand impulse is progressively smaller. The system also becomes less vulnerable to primary-side disruptions: a larger secondary share cushions the pass-through from supply shocks to prices and quantities, decoupling aggregate consumption from primary-driven volatility. Because the recycling state evolves through a path-dependent (Markovian) process, periods of high prices or strong policy support leave a persistent imprint on capacity, generating hysteresis that outlives the initial shock. In parallel, structural (baseline) demand growth is increasingly absorbed by secondary flows rather than being transmitted one-for-one into higher prices or new mining. Taken together, these mechanisms move the market from rigid and shock-prone to more responsive and resilient—though with diminishing returns as product durability, scrap availability, and institutional frictions impose upper bounds on the buffering role of recycling.

We then proceed to calibrate our model to copper markets, keeping the same energy transition scenarios, STEPS, APS, NZE, and BAU. We show that the share of secondary supply rises meaningfully in policy-driven pathways and lags in a no-transition world. Penetration is highly sensitive to policy ambition here; yet it remains structurally bounded by product durability and the expansion of in-use stocks. On prices, introducing a secondary stream flattens the effective supply curve and delivers steep but non-explosive paths in transition scenarios, consistent with a NENO equilibrium. The no-transition counterfactual underlines how weak secondary expansion against unconstrained demand produces explosive dynamics. In all cases, recycling functions as a buffer rather than a substitute for mining, lowering peaks, shortening the half-life of shocks, and easing the adjustment borne by primary supply. Depletion outcomes converge across scenarios because geology and cumulative availability cap long-run extraction; recycling mainly moderates the path (prices and interim pressures) rather than the ultimate volume mined. Sensitivity exercises confirm that results hinge on the price responsiveness of secondary supply: stronger responsiveness amplifies the buffering role of recycling, while weaker responsiveness shifts more adjustment to primary extraction. Relative to a no-recycling benchmark, secondary supply materially narrows the demand–supply gap and, importantly, reduces the model’s sensitivity to demand-elasticity assumptions—making long-run outcomes less fragile to uncertainty in those parameters.

Finally, the chapter derives the theoretical conditions for price convergence when markets are not in a “single good, single price” setting, establishing when primary and secondary prices do converge via arbitrage versus remain durably segmented under cost-based or capacity-constrained pricing. Illustratively, metals like aluminum (and, to a degree, lead or zinc) often approximate single-price integration, while nickel classes, copper cathode vs. scrap, and several specialty metals exhibit durable dual-price equilibria shaped by form, purity, and capacity frictions. In the general case, convergence rests on three pillars: secondary output must scale with price as fast as demand adjusts; saturated secondary capacity must be sufficient to meet aggregate demand; and substitution must be symmetric across streams—otherwise persistent price spreads remain. Relaxing baseline assumptions, we find that when primary prices are bounded, convergence hinges on secondary capacity at the price cap—so mature, high-recycling systems can approach a single price. By contrast, with trendless or volatile prices convergence is only episodic, and under cost-anchored secondary pricing segmentation persists with a bounded discount, parity remaining a knife-edge exception.

Chapter 5 turns to more short-term dynamics in metal markets. It adopts an econometric approach, applying machine learning techniques to forecast copper prices. This empirical analysis complements the forward-looking scenarios by addressing the challenge of improving forecast accuracy in the presence of short-term fluctuations. By combining traditional econometric models with advanced predictive algorithms, the chapter demonstrates the value of hybrid methods for enhancing the robustness of price projections.

This chapter contributes to the literature by addressing several limitations of traditional econometric approaches and proposing an integrated framework that combines the strengths of both econometrics and machine learning in forecasting metal prices. Classical econometric models, while well-grounded in economic theory, often rely on some restrictive linear assumptions and struggle to capture the non-linear relationships that characterize commodity markets.

At the same time, the use of futures prices as benchmarks or predictors has shown notable limitations. While futures markets incorporate some forward-looking information, their predictive power often remains weak over medium- to long-term horizons. As highlighted in recent work (e.g., Stuermer, 2018; Jacks & Stuermer, 2020), structural disconnections between paper markets and physical fundamentals can reduce the informational content of futures, limiting their usefulness for long-term market analysis and planning.

To address these challenges, Chapter 5 adopts a hybrid modeling strategy that brings machine learning techniques and econometric insights in tandem, i.e., using the same sets of exogenous variables. By using tree-based algorithms such as XGBoost, the model can flexibly approximate non-linearities and

complex interactions between variables, without imposing strong parametric assumptions. At the same time, the inclusion of economically meaningful features—such as mine capacity utilization rates, production volumes, stock levels, and global demand proxies—ensures that the model remains grounded in the structural realities of the market.

Our contribution is twofold here. First, it highlights the potential of supply-side granular data, often overlooked in price forecasting exercises, to improve predictive performance. In particular, mine capacity utilization serves as a high-frequency indicator of operational constraints, helping to bridge the gap between macroeconomic conditions and real-world production dynamics. Second, it shows how hybrid models can respond more effectively to structural shifts in the market—such as technological transitions, shifts in policy expectations, or supply disruptions—by adapting to new patterns without requiring full model re-specification.

These results offer a path forward for improving the robustness of price forecasts in metal markets. They also provide a practical tool for market analysts and policymakers who require reliable forward-looking indicators to guide investment decisions, infrastructure planning, or resource policy. By anchoring predictions in both economic fundamentals and data-driven learning, this chapter contributes to a growing literature that seeks to modernize forecasting practices in resource economics.

CHAPTER 1: Literature Review: Current State of Research in the Economics of Metals

The economics of metals lie at the intersection of resource sustainability and market dynamics, making it a critical field for understanding the long-term viability of industrial and economic growth. Metals, as exhaustible resources, play a foundational role in modern economies, and their extraction, consumption, and pricing raise pressing questions about intertemporal decision-making, market efficiency, and environmental sustainability. This literature review provides an overview of the theoretical, empirical, and methodological advancements in the economics of metals.

The review begins by revisiting Harold Hotelling's (1931) seminal contribution, which laid the foundation for resource economics through an intertemporal framework for analyzing the extraction of exhaustible resources. Building on the theoretical richness of Hotelling's work, the first section discusses both its core insights and the subsequent extensions made by economists in the 1970s and 1980s. While Hotelling's approach remains influential, its empirical limitations have spurred alternative frameworks and critiques, which are also addressed.

The second section examines the “peak models” introduced by M. King Hubbert (1956) and popularized in subsequent works such as the Meadows Report (1972). These models, which emphasize the finite nature of resources and predict bell-shaped production curves, represent a more empirical, albeit less theoretically grounded, approach. This section explores both the strengths and significant critiques of peak models, including their inability to account for market forces, technological progress, and substitution effects.

The third section introduces the Cumulative Availability Curve (CAC) approach, a paradigm heralded by John E. Tilton (1996) that usefully integrates economic considerations into assessments of resource availability. By linking resource availability to price levels, the CAC methodology offers a valuable tool for understanding the interplay between geology, technology, and market behavior. However, it also faces criticism for its reliance on static assumptions and limited integration of economic dynamics. Recent efforts to refine the CAC framework, that influence our modelling in the following chapters, particularly those by Castillo and Eggert (2020), are discussed in detail.

Finally, the review turns to market models that explore the role of demand and short-term price fluctuations in resource markets. These models, which incorporate insights from econometrics and modern statistical methods, provide a more nuanced understanding of the cyclical and stochastic

nature of commodity prices. The section highlights advancements in price cycle modeling and the identification of demand-driven and supply-specific price shocks, particularly in the context of metals.

Together, these sections lay the groundwork for the thesis by situating it within the broader discourse on resource sustainability and market dynamics. They highlight the evolution of theoretical frameworks, the empirical challenges faced by resource economists, and the methodological innovations that continue to shape the field. By critically engaging with these perspectives, this literature review not only underscores the complexity of metal economics but also identifies key gaps and opportunities that the following chapters will address.

I. Theoretical Foundations: Hotelling and the Intertemporal Economics of Resource Extraction

The study of exhaustible resources in an economic framework owes much to Harold Hotelling's seminal 1931 paper, *The Economics of Exhaustible Resources*. Building on earlier insights, particularly those of L.C. Gray (1914), Hotelling introduced a formal intertemporal approach to resource extraction. Gray's work had already highlighted the concept of "scarcity rent", emphasizing the trade-off resource owners face between extracting now and preserving resources for future revenue.

Hotelling's contribution was groundbreaking for its use of advanced mathematical tools to analyze resource extraction. By employing calculus to model the problem, he derived the famous result that, under competitive conditions, the net price (price minus marginal extraction cost) of an exhaustible resource should grow at the rate of interest. This insight formalized the relationship between resource scarcity, pricing, and intertemporal decision-making, providing a framework that remains central to the economics of non-renewable resources. By elevating the discussion from qualitative observations to a rigorous quantitative science, Hotelling laid the groundwork for subsequent advancements in resource economics, anchoring the study of exhaustible resources in a unified theoretical model.

A recent reassessment by Gaspard, Missemer, and Mueller (2024) further contextualizes Hotelling's contributions, emphasizing the applied nature of his mathematical economics and its focus on addressing concrete social and economic problems. They situate the famous Hotelling rule—the principle of resource price growth at the rate of interest—within a broader intellectual project shaped by his Georgist inclinations and concern for public welfare. Far from being a purely theoretical abstraction, Hotelling's 1931 model was part of a broader inquiry into optimal resource allocation under different competitive structures and policy regimes. The authors underscore the dual role of Hotelling's work as both a theoretical exploration and a practical guide for policymakers, grounded in his interdisciplinary engagement with mathematics, economics, and institutional reform. By examining

archival materials, they reveal how Hotelling aimed to balance abstraction and empirical relevance in his approach to natural resource economics. This nuanced perspective enriches our understanding of Hotelling's work, situating it as a foundational yet deeply practical contribution to the study of non-renewable resource economics.

1. Hotelling's Rule: Intertemporal Optimization of Resource Extraction

Hotelling (1931) studies the optimal behavior of a raw materials producer. To this end, he equates a natural resource production site (such as hydrocarbons or metal ores) to an asset whose return should correspond to that of financial markets (simplified here to the long-term interest rate). Unlike a financial asset, however, a natural resource deposit does not pay interest or dividends. Consequently, its return can only be linked to an increase in the price of the extracted raw material. In Hotelling's framework, this price depends on the producer's supply behavior, who arbitrates between producing today at the current price or producing tomorrow at a higher price. Hotelling thus analyzes the supply of raw materials within a dynamic (or intertemporal) framework.

The intertemporal framework of Hotelling's model is rich with several important aspects. First, it introduces the notion of opportunity cost (often but incorrectly referred to as the *Hotelling rent*) into the reasoning as a critical economic trade-off, unique to non-renewable resources. In standard theory, a firm in a competitive market produces until its marginal production cost equals the market price. In extractive industries, however, Hotelling observes that this production cost should include an additional opportunity cost arising from the following fact: producing one more unit today reduces the stock of non-renewable resources available for the future, thereby limiting future production. This opportunity cost is equal to the net present value of the loss of future profits due to producing one marginal unit of a non-renewable resource today. This loss reflects either a reduction in the stock of available resources (as Hotelling suggested) or a deterioration in the quality of future reserves (since the metal content of extracted rocks tends to decline over the life of the mine), making future production more expensive.

In other words, if owners of non-renewable resources follow Hotelling's rule—that is, they extract and sell these resources over time to maximize their net present value given the interest rate—they extract the resource more quickly as its price rises (due to scarcity or declining reserve quality⁹) and leave less of it for the future.

⁹ For analyses on the sequencing of resource extraction, Herfindahl (1967) establishes that the optimal order minimizes extraction costs, favoring the exploitation of lower-cost deposits first, under the assumption of competitive markets and constant marginal costs. Hartwick (1978) builds on this by showing that technological progress can alter this sequence by

Notably, demand is not considered in Hotelling's intertemporal modeling: the producer observes a price based on market conditions (raw material stock and discount rate) and adjusts their extraction rate solely based on these parameters. This approach is justified by the assumption that short-term market fluctuations (resulting from supply-demand interactions) do not significantly affect the long-term net value of the resource (see Section IV below on market models). The standard version of Hotelling's model also assumes fixed extraction costs and no technical progress in extraction, assumptions we will revisit.

These intuitions can be formalized in a simple but illustrative way. Let $\bar{y}$ be the stock of raw materials, y_t the production of resources in a given year t , $G(y_t)$ the net present value of profits from extraction over the mine's lifetime, p_t the marginal profit (equal to the market price minus the opportunity cost mentioned earlier and extraction costs) in year t , and γ the private discount rate (initially assumed to equal the long-term interest rate). Hotelling's model is also based on several major assumptions: a) the total resource stock is fully known from the outset; b) marginal extraction costs are constant; c) the market is perfectly competitive (producers have no significant market power to influence the resource's market price; they are price takers); d) storage costs are negligible, so the extracted resource is sold immediately on the market.

In Hotelling's model, the mining producer seeks to maximize their intertemporal profit $G(y_t) = \sum_{t=0}^T \frac{p_t y_t}{(1+\gamma)^t}$, subject to the resource extraction constraint $\sum_{t=0}^T y_t = \bar{y}$.

This constraint is what makes the primary producer unique in Hotelling's framework: it embodies the principle mentioned earlier that extracting more today implies extracting less tomorrow. By defining the utility function $u(y_t) = \int_0^{y_t} p(t) dy$, the objective becomes maximizing $\sum_{t=0}^T \frac{u(y_t)}{(1+\gamma)^t}$.

Since the stock $\sum_{t=0}^T y_t$ is constant, the extremum of intertemporal utility is reached if and only if $u'(y_t) = p(y_t)$.

making higher-cost reserves economically viable earlier. Drury (1982) highlights the influence of price fluctuations and policy interventions, demonstrating that market incentives can lead to deviations from a purely cost-driven extraction order. Hanson (1980) incorporates uncertainty, arguing that imperfect knowledge about reserve sizes and future prices leads to risk-hedging behaviors, where producers diversify extraction across deposits. Cairns and Lasserre (1985) extend these findings by modeling the interplay between market dynamics, substitution effects, and demand, showing that the optimal sequence also depends on broader economic conditions and the competitive structure of the market.

This is where Hotelling's rule emerges: along the optimal extraction path (from the perspective of maximizing intertemporal utility), producers are indifferent between extracting the resource or leaving it in the ground, based on the expected appreciation in the value of the resource.

In this framework, certain assumptions about the shape of $u(y_t)$ are required. First, $u(y_t)$ is assumed to be monotonically increasing, ensuring that extracting more of the resource yields greater utility, consistent with $p(y_t) > 0$. Second, $u(y_t)$ is typically concave, reflecting diminishing marginal utility or revenue as the quantity extracted increases, a common characteristic in resource economics. Third, the utility function is non-negative ($u(y_t) \geq 0$), as it reflects the cumulative value derived from selling the resource. Finally, the derivative $u'(y_t) = p(y_t)$, representing the price function, is generally assumed to be decreasing with y_t , consistent with the scarcity value of the resource and basic supply-demand dynamics.

While these properties are not explicitly constrained in Hotelling's original model, they are implicit in ensuring the model's consistency and practical relevance. For certain applications, linear or quadratic forms for $u(y_t)$ are often adopted for analytical convenience, but the general insights of the model remain valid under these broader assumptions.

Finally, to determine the resource price, Hotelling defines:

- p_0 , the initial price of the resource at the start of extraction activity;
- $\bar{p}$, the price toward which the resource price converges as stocks are depleted, and the extracted quantity decreases toward zero.

It should be noted that this latter consideration relies on the assumption that such a price $\bar{p}$ exists. Specifically, Hotelling assumes that the extraction horizon is finite¹⁰, meaning there exists a time T , such that $y_T = 0$. Thus $\bar{p} = p_0(1 + \gamma)^T$, from which, more generally $p_t = p_0(1 + \gamma)^t$.

This formula, based on numerous assumptions, indicates that the mining resource producer sets their production path so that their net profit (equal to the market price minus extraction costs and the

¹⁰ This assumption of a finite extraction horizon was later challenged—with significant consequences—in some of the literature inspired by Hotelling's model during the 1970s and 1980s. Notable contributions include Dasgupta and Heal (1974), who explored the implications of infinite resource availability under varying technological and economic conditions, and Solow (1974), who examined the role of substitutability and technological progress in extending the effective lifespan of resources. Stiglitz (1974) further analyzed conditions under which resource constraints might be alleviated through innovation and substitution. These works opened the door to a broader understanding of resource dynamics beyond Hotelling's initial assumptions. We will explore this branch of the literature in the following subsections.

opportunity cost associated with the reduced future stock, as mentioned above) increases at a rate equal to the interest rate, year by year. This is the condition for optimizing their profit and the return on their investment.

In other words, when the stock of extractable resources is known and fixed, the optimal extraction path, from the perspective of intertemporal utility, implies that in Hotelling's framework, producers are indifferent between extracting the resource or leaving it in the ground, based on the expected appreciation in the value of the resource.

If opportunity and extraction costs are constant, the producer only extracts resources if the market price increases at a rate at least equal to the interest rate. This is the simplest version of Hotelling's rule, often referred to in the literature as the *r-percent rule* (where the private discount rate is equated to the long-term interest rate, r). The intuition is that the discounted profit of a unit of resource extracted from the ground must be the same across all periods: there is no gain from shifting extraction from one period to another. For the present value of the net price (market price minus extraction costs) to be equal across all periods, the undiscounted value must grow precisely at a rate equal to the interest rate.

In this framework, if *ex-ante* demand remains stable year to year, production decreases monotonically over time.

A few remarks are already useful regarding this simple yet intuition-rich model.

First, it is notable that demand is not accounted for in Hotelling's intertemporal modeling: the producer observes a price determined by market conditions (resource stock and discount rate) and adjusts their extraction rate based solely on these parameters. This approach is justified by the assumption that short-term market fluctuations (driven by the interaction between supply and demand) do not significantly affect the long-term net value of the resource (see Section IV below on market models).

The standard version of Hotelling's model also assumes fixed extraction costs and no technical progress in extraction, assumptions that we will revisit.

Hotelling's reasoning is that of an economist, as price plays a direct role in influencing agents' decisions. In a more developed version of the model (see below), it is possible to account for the effects of technological progress, which reduces extraction costs (affecting p_t) and/or increases the stock $\bar{y}$ of economically exploitable resources (we will return to this point). The model raises the question of a potential difference between the firm's discount rate (γ) and society's social discount rate. It also

reflects the specificities of the extractive sector (both because p_t includes opportunity costs and because total production is capped at $\bar{y}$).

The model also incorporates the concept of the opportunity cost of producing a non-renewable resource, which, by definition (see above), depends on the chosen production level at time t . It is the existence of this opportunity cost—covered by the market price in addition to the marginal cost of extraction—that explains why the price of traded raw materials is set to always be higher than the marginal cost of extraction. From this, Hotelling concludes that there is no risk of overexploitation of mineral resources: a price higher than the marginal production cost implies lower demand than would exist in a standard market where equilibrium is achieved at a price equal to marginal production costs alone.

In terms of the production profile over time, Hotelling predicts an asymmetric bell-shaped trajectory, with production accelerating to a rapidly reached maximum, followed by a decline in the extraction rate. While an intersection between this curve and the time axis is not explicitly addressed, it can be inferred from the assumption of a finite time horizon T , at which point the resource is completely exhausted¹¹.

2. Extensions to Hotelling's Framework in the 1970s and 1980s

Following Solow's (1974) remarkable article on Hotelling¹², numerous contributions emerged in subsequent years enriching and extending the original framework. These works addressed key limitations and introduced new dimensions to the study of non-renewable resource markets. A significant portion of this literature can be framed as variations of the "cake-eating" approach, which considers the optimal consumption of a finite or dynamically evolving resource stock. These models

¹¹ Some extensions of Hotelling's framework relax the assumption of a finite extraction horizon and instead consider infinite time horizons for resource depletion. This approach introduces significant mathematical complexity, as it requires solving for equilibrium paths where extraction occurs indefinitely, subject to intertemporal constraints and discounting. Models with infinite horizons often emphasize sustainability or steady-state dynamics and are more relevant when resources are perceived as vast or when technological progress continually lowers extraction costs (see Dasgupta and Heal, 1974). However, these models face practical limitations, as they abstract from real-world uncertainties such as geological constraints, technological shifts, and policy interventions that inevitably shape finite resource extraction.

¹² Solow (1974) extends Hotelling's framework by analyzing the role of substitutability and technological progress in resource economics. He demonstrates that the depletion of non-renewable resources need not constrain economic growth if adequate substitutes for these resources exist or if technological progress continually offsets the diminishing availability of high-quality reserves. By relaxing the assumption of fixed resource constraints, Solow emphasizes the importance of adaptive economic mechanisms, such as innovation and the development of alternatives, in mitigating the long-term impacts of resource scarcity. His work laid the foundation for integrating sustainability considerations into resource and growth models.

introduce critical augmentations to Hotelling's original formulation, addressing issues such as unknown or growing reserves, the trade-offs inherent in stochastic dynamics, and intergenerational equity.

In what follows, we do not aim for exhaustive coverage. Instead, we will examine major extensions to Hotelling's model across several themes: the dynamics of extraction costs, the role of uncertainty (including the exploration of unknown reserves), adjustments to the discount rate to account for risk, the absence of storage dynamics, and the impact of cartelization on resource extraction and pricing. Each subsection will highlight how these themes build on or diverge from the foundational principles laid out by Hotelling, often through the lens of the "cake-eating" problem and its various formulations.

a. Extraction Costs

In Hotelling's framework, marginal extraction costs are assumed to be fixed, meaning that technological progress and resource depletion are excluded from the analysis. However, real-world resource extraction costs are dynamic, shaped by opposing forces: technological progress, which reduces average marginal costs, and depletion, which increases them. These dynamics have significant implications for the net marginal profit p_t —defined as the market price minus extraction costs and opportunity costs—and, therefore, for the optimal extraction path.

Technological innovation reduces marginal extraction costs by increasing efficiency, thereby raising the net marginal profit p_t . Schmitz (2005) estimates that total factor productivity in the iron ore industry in Canada and the United States rose by 51% between 1981 and 1995, illustrating how advancements can significantly lighten the cost structure of mining industries. However, Hotelling's original model assumes fixed marginal extraction costs over time and therefore does not explicitly account for such cost-reducing mechanisms. These innovations represent an extension of the framework, showing how technological progress can delay cost-driven price increases and sustain higher profits over time.

The mining industry is unique in its mechanisms for reducing costs, which represent a departure from Hotelling's assumption of fixed marginal extraction costs over time. West (2011) identifies three key dynamics that drive cost reduction in the sector. First, rapid advancements in extractive technologies, characterized by high innovation rates, make previously uneconomical deposits viable. Second, techniques that enable more efficient ore recovery, often referred to as "high volume, lower cost" methods, allow producers to extract more ore from the same volume of rock, thereby prioritizing efficiency. Finally, extending existing mines is generally more cost-effective than developing new ones, ensuring continued efficiency gains even as accessible deposits become increasingly scarce. These cost-

reducing mechanisms illustrate the dynamic nature of mining operations, which contrasts with the static cost assumptions in Hotelling's original framework.

However, these cost-reducing mechanisms are counterbalanced by the long-term effects of depletion, as emphasized by Herfindahl (1967). As higher-quality deposits are exploited first, producers must increasingly rely on lower-quality ores, leading to a rise in marginal extraction costs over time. Heal (1976), Solow and Wan (1976), and Weitzman (1975) further formalize this relationship, demonstrating that depletion-related cost increases can eventually outweigh the benefits of technological progress, particularly in settings where reserve quality is heterogeneous. Hartwick (1978) builds on this by highlighting how the interplay between technological progress and depletion dynamics shapes the trajectory of resource prices, which often follow a U-shaped pattern over the long term. These studies collectively argue that while technological innovation can delay cost increases, depletion exerts a persistent upward pressure on extraction costs as cumulative production grows.

This interplay between technological progress and depletion introduces significant non-linearities into extraction paths, challenging the constant cost assumptions in Hotelling's original framework. While Hotelling assumes fixed marginal costs, these extensions reveal that real-world extraction is influenced by dynamic factors that alter the net marginal profit p_t . Empirical studies, such as those by Schmitz (2005) and Mudd (2007), further confirm that while technological progress often mitigates the immediate impact of depletion, the long-term balance between these forces remains an open question.

Mudd (2007) provides a compelling case study of Australian copper mining, showing that "the massive decline in copper ore grades is not due to the depletion of high-quality deposits but is instead the direct result of innovation, which has transformed previously worthless waste rock into valuable ore." This finding challenges the common assumption that declining ore grades necessarily lead to higher costs or prices.

Over time, the cumulative exploitation of resources tends to increase costs due to the declining quality of remaining reserves. Slade (1982) and Gaudet (2007) emphasize that as higher-grade ores are extracted first, producers are left with lower-quality reserves, leading to rising marginal extraction costs—which is considered as a direct manifestation of geological depletion. Devarajan and Fisher (1981) extend Hotelling's model by focusing on the interaction between reserve levels and extraction costs over time. They argue that extraction costs are not constant, as assumed in the standard framework, but instead increase as reserves diminish. This is particularly relevant for low-grade deposits, where extraction becomes more expensive as higher-grade reserves are depleted. Their model highlights that cost increases are not linear, introducing non-linearities into resource pricing and extraction trajectories. These findings align with Slade's (1982) observation of U-shaped price paths,

where early extraction benefits from lower costs, but subsequent depletion pressures lead to higher marginal costs.

This effect is evident in historical trends. For example, copper mines in the early 20th century extracted rocks with approximately 4% copper ore content, whereas today, ores with less than 1% content are commonly mined¹³. This decline in ore quality underscores the long-term cost pressures caused by depletion, as producers must process more material to extract the same amount of metal.

Stollery (1983), in his theoretical work on global nickel prices, demonstrates that declining ore quality is a critical determinant of price. He argues that relaxing the assumption of homogeneous reserve quality significantly improves the predictive power of Hotelling's model, particularly over longer time horizons.

Empirical studies reveal varying magnitudes for technological progress and depletion as opposing forces. Lasserre and Ouellette (1988), examining asbestos extraction in Canada between 1953 and 1982, found that productivity gains from technological progress (+76%) were partially offset by depletion-related productivity losses, resulting in an overall sector productivity gain of only 13%. Similarly, Barnett and Morse (1963) observe a 78% decline in extraction costs for minerals over the period 1870-1957, suggesting that technological progress often outweighs depletion effects in the long run. However, the balance between these forces varies over the life of a mine. Slade (1982), in her analysis of major metals and fossil fuels between 1870 and 1978, identifies a U-shaped price trajectory: technological progress dominates in the early stages, reducing costs and prices, but eventually, the unfavorable effects of depletion drive prices upward. In contrast, Buñuel (2001), analyzing copper prices from 1910 to 1996, finds a more complex dynamic. According to Buñuel, technological progress dominated until the late 1930s, leading to a steady decline in real prices. Between 1938 and 1976, depletion effects began to outweigh innovation, resulting in rising prices, before a renewed decline in the 1980s driven by another wave of technological progress.

These dynamics challenge Hotelling's assumption of fixed extraction costs and suggest that the net marginal profit p_t does not grow at a constant rate as predicted by $p_t = p_0(1 + \gamma)^t$. Instead, the interplay between technological progress and depletion introduces non-linearities into the extraction path. Moreover, heterogeneity in reserve quality, as highlighted by Herfindahl (1967), requires

¹³ Schodde, R., 2010, March. The key drivers behind resource growth: an analysis of the copper industry over the last 100 years. In *MEMS Conference Mineral and Metal Markets over the Long Term, Joint Program with the SME Annual Meeting, Phoenix, USA* (Vol. 3).

adjustments to Hotelling's optimal extraction rule. Producers tend to prioritize higher-quality deposits first, leading to sequencing effects that influence extraction costs and the price trajectory over time.

b. Uncertainty

Mining exploration is inherently risky: substantial expenditures may yield no significant results. For example, the average cost per discovery in the mining industry—calculated as the ratio of total exploration spending to the number of discoveries—was \$238 million in constant 2016 dollars, a threefold increase from \$86 million in 2007¹⁴. This dramatic rise highlights the escalating financial risks associated with uncovering viable deposits, driven by factors such as the increasing depth of deposits, exploration in remote locations, and the declining availability of high-grade ore. Industry players may adopt strategic behavior, waiting for a competitor to invest in exploration, thereby revealing public information about deposit locations, before deciding whether to concentrate their efforts in the same area. In this context, investment in mining exploration risks being suboptimal, as noted by Peterson (1978) and Stiglitz (1976), due to these informational challenges. Peterson highlights how firms may delay exploration investments to free ride on the information revealed by competitors' discoveries, reducing overall exploratory activity. Stiglitz further examines how the public good nature of geological information exacerbates underinvestment, as firms are reluctant to bear the high costs of exploration when the benefits of discoveries can be indirectly shared with competitors.

Beyond informational issues, a key source of uncertainty concerns the actual quantity of metal reserves available (at a specific site or globally) and its implications for a Hotelling-style model. Gilbert (1979), Heal (1976), and Loury (1978) analyze extraction trajectories under reserve uncertainty. Gilbert (1979) explores how uncertainty about reserve sizes influences extraction strategies, emphasizing that producers tend to adopt more conservative depletion paths to mitigate the risk of unexpectedly exhausting their reserves too soon. His work demonstrates that the pace of extraction slows as uncertainty grows, reshaping the traditional trajectories envisioned by Hotelling. Heal (1976) examines the interplay between uncertainty and resource exhaustion, focusing on how stochastic processes can be incorporated into Hotelling's model. Heal's analysis highlights the importance of optimal adjustment in response to uncertain reserve sizes and shows that conservative extraction paths are a rational response to unpredictable resource availability. Loury (1978) extends the discussion by incorporating the strategic dimension of exploration. He models how firms balance the high costs of

¹⁴ Recent Trends and Outlook for Global Exploration, Schodde 2017, MinEx Consulting (https://minexconsulting.com/wp-content/uploads/2019/04/Exploration-Trends-and-Outlook-PDAC-Presentation_compressed.pdf)

exploration with the potential benefits of increasing reserves, showing that the exploration process itself is often inefficient due to underinvestment. Loury's work underscores the role of economic incentives in driving exploration and links this behavior directly to the level of reserve uncertainty. Their shared intuition is that producers adopt a more conservative extraction strategy to avoid unexpected and premature resource exhaustion. In other words, if the metal resource is of uniform quality and fixed in size but unknown in quantity, the owner extracts it at a slower rate than if the stock size were known with certainty.

Pindyck (1980) extends Hotelling's model by incorporating exploration to reduce uncertainty and accumulate reserves. Under the assumptions of constant extraction costs and risk-neutral firms, Pindyck suggests that reserve uncertainty does not affect the expected price dynamics, and Hotelling's (1931) r -percent rule still applies. However, if extraction costs depend on the reserve level, reserve uncertainty significantly affects price variation. Pindyck shows that a sudden decrease in reserve levels leads to a sharper rise in extraction costs, in absolute terms, than the cost reduction associated with an equivalent increase in reserve levels (e.g., from the discovery of a new deposit). As a result, producers are incentivized to accelerate production to mitigate the overall cost increase associated with reserve uncertainty. Consequently, resource prices start lower at the beginning of the extraction period and rise over time.

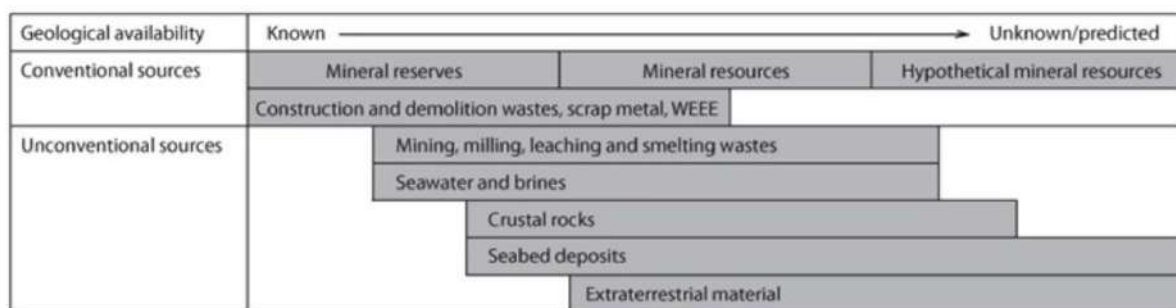
The economic analysis of the 1980s thus suggested that the production profile for minerals is likely more complex than the gradual decline envisioned by Hotelling. If additional reserves exist elsewhere, the price pressure from resource scarcity, as envisaged by Hotelling, incentivizes exploration and the discovery of new deposits. Assuming some exploration efforts are successful, the resulting supply increase would reduce prices and extend the trajectory of Hotelling's price path.

Rather than treating reserves as a fixed, finite stock depleting over time (as in Hotelling's model), Pindyck (1978) argues that producers do not possess pre-determined reserves but must develop them through exploration. Pindyck views reserves as being created and depleted by producers responding to economic incentives. He advocates replacing the term "exhaustible" with "non-renewable," since the concepts of reserves and depletion are ultimately economic, not geological or physical. This introduces a recurring tension between economists and geologists: economists often regard exploitable reserves of primary metals as not strictly determined by nature but as variables dependent on various economic parameters.

Pindyck assumes that "potential reserves" are unlimited but that repeated exploratory efforts yield smaller discoveries over time due to depletion effects. Under these constraints, resource producers must simultaneously determine their optimal rates of exploration and production (and thus depletion).

He derives the optimal intertemporal strategy for exploration and production under both competitive and monopolistic market structures. When the initial reserve endowment is low, the resource price curve takes on a U-shape: production initially increases as reserves grow due to exploration efforts but eventually declines as exploration activity and discovery rates decrease. Early in the exploitation process, geological exploration is intense to build a sufficient reserve base, which lowers extraction costs. Over time, exploration is adjusted, balancing the trade-off between continued investment in exploration to reduce marginal extraction costs and delaying exploration to postpone its associated expenses. The key parameters driving this trade-off are the initial reserve level and the resource depletion rate.

Figure 1.1: Spectrum of Mineral Availability: From Known Reserves and Resources to Hypothetical Resources



This diagram categorizes the sources of mineral resources based on their geological availability, ranging from known reserves and resources to hypothetical or predicted resources. It differentiates between conventional sources, such as mineral reserves, mineral resources, and recycled materials (e.g., construction waste, scrap metal, and WEEE), and unconventional sources, including mining and smelting wastes, seawater, crustal rocks, seabed deposits, and extraterrestrial materials. The gradient from left to right reflects increasing uncertainty or predictive nature of these resources, illustrating the potential for future supply depending on technological advancements and exploration efforts.

Pindyck's findings explain why real prices for many non-renewable resources have declined over time, contrary to the steadily increasing price trajectory predicted by Hotelling. For example, the real price of bauxite declined before the cartelization of the global bauxite market, which can be attributed to significant increases in proven reserves that allowed production to grow steadily. Later in the resource exploitation process (or from the outset if initial reserves are abundant), prices rise at a constant rate over time, as predicted by Hotelling. However, the introduction of exploratory activity reduces the rate of price increase.

Beyond reserve uncertainty, uncertainties in demand responsiveness to observed price changes raise specific challenges. In some cases, these do not fundamentally undermine Hotelling's model or its main conclusion (i.e., resource prices increase at the interest rate). Weinstein and Zeckhauser (1975) examine demand uncertainty using a discrete-time model in which current demand is known, but future demand is uncertain and revealed at the start of each period. Assuming zero extraction costs and risk-neutral producers, they find that the expected price in a competitive market still increases at the interest rate.

Pindyck (1980) further explores the effect of demand levels and their variations on the validity of Hotelling's conclusions. He models market demand as uncertain, evolving stochastically over time with shocks that are independent across periods. While current demand may be known with precision, future demand is surrounded by uncertainty, with stochastic variance increasing with the time horizon. Additionally, Pindyck assumes that available reserves evolve stochastically, either increasing or decreasing relative to current levels. Producers may thus discover that available reserves are greater or smaller than originally expected.

In this world, where demand and reserve levels evolve as random walks, Pindyck concludes that observed market prices also follow a stochastic trajectory. Pindyck also addresses several questions regarding market behavior in terms of expected value and shortcomings of the simplified Hotelling approach: does uncertainty lead producers in competitive or monopolistic markets to behave more conservatively than they would in a world without uncertainty? Do competitive markets exploit resources at socially optimal rates in the presence of uncertainty? What are the implications of uncertainty for exploration, both to reduce uncertainty itself and as a mechanism to accumulate reserves?

In Pindyck's model, producers have complete information about the current resource market but lack knowledge of future demand and reserve levels. However, as stochastic fluctuations occur continuously over time, producers can adapt dynamically. Consequently, stochastic fluctuations only alter the expected price and extraction rate when production costs or exploration productivity are non-linear in fluctuating variables. With constant average production costs, future prices still follow Hotelling's r -percent rule. However, uncertainty affects the initial production and price levels and the pace of production decline.

Exploration serves both to reduce uncertainty by gathering information and to accumulate reserves. Pindyck shows that exploration should only be used to reduce stochastic reserve fluctuations if production costs depend on reserve levels. When exploration is used to accumulate reserves, its optimal timing and intensity are shaped by how stochastic fluctuations affect the average productivity of exploratory effort.

Similarly, Levhari and Liviatan (1977) examine the optimal depletion of resources when future demand conditions are uncertain. They introduce a probabilistic framework in which producers adjust extraction rates based on expected shifts in demand, showing that uncertainty about future demand can lead to more conservative extraction strategies in the short term. Their findings also suggest that higher levels of uncertainty amplify the value of preserving reserves, effectively lowering initial extraction rates.

c. Adjusting the Discount Rate for Risk

The discount rate (γ) plays a central role in Hotelling's model, as it governs the rate at which resource prices are expected to rise over time. In the base model, the interest rate is typically assumed to be constant. However, it can be adjusted to account for risk premiums, leading to a higher effective discount rate ($\gamma_{ra} > \gamma$) (see Copeland et al., 2005, p. 157). Young and Ryan (1996) extend Hotelling's model by incorporating risk considerations, finding positive risk premiums for copper and zinc in their study of various metal commodities. This adjustment steepens the price trajectory compared to the original Hotelling scenario: the higher the level of operational risk, the steeper the price trajectory over time. This reflects the market's compensation for uncertainty in resource availability or project viability.

Furthermore, Gaugler (2015) suggests that an increase in interest rates can cause a one-time upward price adjustment. The intuition is that higher interest rates raise the cost of capital, endangering the viability of mines whose net present value (NPV) becomes negative following the rate hike. Such events reduce supply and lead to price jumps. This reasoning is particularly relevant in the current context of rapid monetary tightening in Western economies, the most significant in the past forty years. Revisiting this framework under contemporary conditions could yield valuable insights.

Several studies have expanded upon or contextualized these insights. Hartwick (1978) examines the implications of varying discount rates on resource extraction strategies, showing that rising interest rates can accelerate the depletion of reserves as producers prioritize near-term profits over long-term resource availability. Pindyck (1980) incorporates stochastic processes into Hotelling's model, exploring the effects of uncertainty in interest rates on resource prices. His findings suggest that stochastic variability in discount rates introduces nonlinearities in price trajectories, particularly when coupled with reserve uncertainty or cost-dependent extraction dynamics.

Weitzman (1998) offers a broader perspective on the discount rate dilemma, emphasizing its intergenerational implications for resource management. He highlights that the choice of discount rate critically affects long-term resource trajectories, particularly for non-renewable resources with significant environmental or societal impacts. More detail is given in subsection I.2.f. below.

Slade and Thille (2009) analyze risk and uncertainty in resource markets, focusing on how producers adjust their extraction strategies in response to interest rate volatility. Their work underscores the importance of incorporating risk management into extraction decisions, particularly in industries characterized by high capital intensity and long investment horizons.

Farzin (1992) examines how time preferences and public policy influence resource depletion, introducing the idea that societal discount rates may differ from private ones. He argues that a higher societal discount rate (reflecting risk aversion or intergenerational equity concerns) could lead to faster depletion, as it prioritizes near-term gains over long-term sustainability. Conversely, lower discount rates encourage producers to align with Hotelling's optimal extraction paths, delaying depletion. His findings also highlight how regulatory interventions, such as taxes or subsidies, can effectively alter the implicit discount rate used by producers.

d. Storage Dynamics

Levhari and Pindyck (1981) explore how contextual factors cause the price trajectory of non-renewable resources to deviate from the r -percent growth path predicted by Hotelling. They emphasize that many resources are durable, meaning their demand depends more on the stock of the resource in circulation than on production flows. As a result, demand is influenced not only by the current price level but also by expected price changes (e.g., "capital gains" or "capital losses"). This introduces dynamics specific to durable resources, which differ fundamentally from the flow-based demand of non-durable resources like oil or gas.

Levhari and Pindyck (p. 366) provide examples of durable resources, such as "diamonds, gold, silver, and other precious metals," while noting that "copper and other non-precious metals also have durable aspects." They distinguish between partially durable resources, which depreciate over time, and perfectly durable goods, which maintain their utility indefinitely. For partially durable resources, demand depends on both the immediate utility provided and the expected future value of the stock. This distinction is critical in understanding how storage behavior influences price trajectories.

Their analysis shows that Hotelling's rule applies to partially or fully durable resources in a competitive market but fails in monopolistic settings. Even in competitive markets, the r -percent rule does not necessarily imply a steady price increase. Instead, under competitive conditions, if the marginal cost of production rises with production rates, prices initially decrease as the stock in circulation grows. Over time, as the stock diminishes, prices rise again, and eventually asymptotically decline to zero after production ceases. This results in a general U-shaped price trajectory for many durable resources, a trend often observed empirically.

Several complementary studies provide additional perspectives on the role of stock dynamics in resource pricing. Slade (1982) emphasizes the role of stock management in durable resource markets. While her primary focus is on the interplay between technological progress and depletion, she highlights how storage behavior—especially for resources with durable properties—can amplify or

delay observed price changes. For example, when prices are expected to rise, market participants may hold onto resources as a form of speculative investment, further affecting the trajectory of prices over time. Gaudet (2007) extends the discussion by formally modeling the impact of storage and stockpiling on resource markets. He shows how market expectations and inventory management for durable resources lead to deviations from Hotelling's predictions. Gaudet's findings underscore that storage behavior interacts with both flow-based demand and durability, creating feedback loops that shape price trajectories. Hartwick (1978) examines monopolistic resource markets, specifically addressing how durability and stockpiling behavior influence intertemporal extraction decisions. His work highlights the strategic use of stockpiles in non-competitive settings, where monopolists may intentionally withhold resources to manipulate future prices. This insight complements Levhari and Pindyck's conclusion that Hotelling's rule breaks down in monopolistic markets, particularly for durable resources. Pindyck (1980) introduces stochastic price and demand dynamics into the Hotelling framework, showing how expectations about future price fluctuations influence stockpiling behavior. His findings demonstrate that resource durability intensifies the role of market expectations in price setting, as producers and consumers alike factor potential "capital gains" or "capital losses" into their decisions to hold or release stock.

e. Cartelization

Pindyck (1978) investigates the effects of cartelization on the extraction rates and pricing of non-renewable resources, focusing on two prominent examples: CIPEC (Intergovernmental Council of Copper Exporting Countries, *Conseil Intergouvernemental des Pays Exportateurs de Cuivre*), which accounted for one-third of copper production in the 1970s, and the International Bauxite Association (IBA). His analysis highlights how cartel behavior can significantly alter resource pricing trajectories, deviating from those predicted by competitive markets.

For bauxite, Pindyck identifies a key feature: the existence of a "limit price." Below \$15.60 per ton, demand for bauxite remains relatively inelastic, as aluminum production heavily depends on it. However, beyond this price, producing aluminum from alternative sources becomes more economical, making the demand for bauxite almost infinitely elastic. Within the inelastic range, bauxite demand is closely tied to aluminum demand, with short-term price elasticities significantly lower in absolute value than long-term elasticities. Pindyck predicts that as the price approaches \$15.60, bauxite demand would fall sharply, establishing \$15.60 per ton as the effective limit price for the resource. Despite this price sensitivity at higher levels, bauxite is relatively abundant. Estimated reserves in the 1970s were sufficient to sustain production for nearly 300 years at then-current consumption rates. In this context, Pindyck demonstrates that depletion plays a minimal role in price determination during the first 30

years, whether in a competitive market or under monopoly conditions, as competitive margins remain small relative to production costs. Under monopoly, however, the cartel can set the price close to the limit price, maximizing profits in each period while largely ignoring future production. In contrast, the competitive price starts lower but increases very slowly over time. Pindyck argues that the gains from cartelization in the bauxite market are substantial and, in some cases, even greater than those observed in the oil market.

Copper, by contrast, exhibits markedly different dynamics from bauxite or oil, primarily due to the significant volumes traded on the secondary market (i.e., scrap-based production). This secondary market reduces the adjustment lags that might otherwise benefit monopolistic or competitive producers. While the supply and demand for bauxite adapt slowly to price changes—allowing cartels to achieve significant short-term gains—secondary copper supply reacts quickly to price variations, limiting the potential for cartelization to generate large profits. Pindyck concludes that the responsiveness of the secondary copper market dampens the effectiveness of cartel strategies in this sector.

Hotelling's (1931) seminal work on resource pricing included a discussion of monopolistic behavior, offering early insights into the dynamics of cartelized markets. In a monopoly, Hotelling demonstrated that the resource owner could manipulate the price trajectory to maximize profits by extracting the resource more slowly compared to a competitive market. This slower extraction rate results in higher prices and allows monopolists to capture a greater share of resource rents. Pindyck's analysis builds on this foundation, emphasizing that monopolists—such as cartels—can set prices closer to the limit price in each period, largely disregarding the intertemporal trade-offs that would otherwise constrain competitive producers.

Several complementary studies provide deeper insights into the dynamics of cartel behavior and stock management in resource markets. Stiglitz (1976) provides a theoretical framework for understanding how cartels maximize profits by exploiting market power. His work is particularly relevant for explaining Pindyck's findings about the ability of bauxite cartels to maintain prices near the limit price. Salant (1976) examines the stability of cartels over time and how resource depletion affects their pricing strategies. His findings support Pindyck's conclusion that cartels can sustain higher prices in the short term by coordinating extraction rates but face challenges in maintaining stability over time, especially when alternative suppliers or substitutes emerge. Hartwick (1978) analyzes monopolistic behavior in resource markets, particularly the intertemporal trade-offs faced by cartels. He highlights how durability and the ability to store resources influence price trajectories, reinforcing Pindyck's findings about the constraints faced by cartels in durable resource markets, such as copper. Kemp and Long

(1984) delve into the conditions for cartel stability, particularly in markets with heterogeneous producers. Their analysis complements Pindyck's discussion of CIPEC and IBA, illustrating how cartel behavior is shaped by the interplay of producer heterogeneity and market structure.

Offering a different approach, Long (2011) highlights the role of strategic interactions in resource extraction, extending the traditional Hotelling framework to multi-agent settings. By employing game theory, Van Long distinguishes between cooperative scenarios, where resource owners coordinate extraction for long-term efficiency, and non-cooperative ones, where competition often leads to overexploitation. He also incorporates complexities like asymmetric information, stochastic reserve sizes, and externalities, showing how these factors exacerbate inefficiencies in non-cooperative contexts.

Long's work aligns with the "cake-eating" literature by exploring how shared resources are optimally consumed under strategic interdependence. His analysis enriches resource economics by integrating dynamic pricing mechanisms and environmental costs, offering valuable insights into sustainable resource management and the challenges of aligning individual incentives with socially optimal outcomes.

The insights gained from the extensions to Hotelling's model underscore the intricate trade-offs involved in managing non-renewable resources. While much of the literature focuses on optimizing resource extraction to maximize intertemporal surplus from a producer's perspective, a growing body of work shifts attention to the broader implications of resource use for societal welfare. This pivot introduces the concept of intergenerational equity, addressing how the consumption of exhaustible resources today impacts future generations. The following subsection explores this critical dimension, examining how frameworks rooted in sustainability and equity reshape traditional economic approaches to resource management.

f. Intergenerational Equity

While Hotelling's model focuses on maximizing intertemporal surplus for resource owners, it inherently assumes a utilitarian perspective, emphasizing profit maximization over the resource's lifespan. However, a significant body of literature critiques this approach for its neglect of intergenerational equity—the ethical consideration of how resource use impacts future generations.

Intergenerational equity frameworks often adopt a social welfare function that balances the utility of current and future generations. Solow (1974) introduced the concept of "sustainable paths," arguing that resource allocation should ensure a consistent standard of living over time. This idea was extended by Hartwick (1977), who proposed the famous Hartwick Rule: if the rents from exhaustible resource

extraction are reinvested in productive capital, sustainability can be achieved. Heal (1998) further formalized these ideas, exploring how equity considerations can be embedded into resource management policies.

Another significant contribution comes from Dasgupta and Heal (1974), who analyzed resource depletion under a utilitarian social welfare framework, demonstrating the conditions under which optimal extraction paths align with sustainability goals. Their work highlights the importance of substitutability between natural and man-made capital, arguing that technological progress can mitigate resource constraints if capital investments offset resource depletion.

One key difference from Hotelling's approach lies in the treatment of discount rates. While Hotelling assumes that the private discount rate reflects the opportunity cost of capital, intergenerational models often incorporate lower, socially determined discount rates to avoid disproportionately favoring the present over the future. Ramsey (1928) provided an early foundation for this perspective, emphasizing the ethical dimensions of discounting when considering intergenerational well-being. Weitzman (1998) built on this by advocating for declining discount rates over time, reflecting the uncertainty in valuing future generations' welfare.

Mathematically, intergenerational equity introduces additional constraints into the optimization problem. Instead of maximizing the resource owner's surplus, the objective function may include terms reflecting sustainability or intergenerational fairness. For instance, Chichilnisky (1996) proposed a framework that balances the interests of current and future generations by weighting welfare over infinite time horizons. These models often result in extraction paths that differ significantly from Hotelling's predictions, typically favoring slower extraction to mitigate depletion risks for future generations.

Empirical applications of these theories highlight their policy relevance. Pezzey (1992) demonstrated how incorporating intergenerational equity constraints can lead to policies that prioritize resource conservation. Similarly, Pezzey and Toman (2012) examined how sustainability principles can guide optimal resource use under economic and ecological constraints. These works underscore the normative dimensions of resource economics, challenging the purely profit-driven assumptions of classical models and advocating for a broader, more inclusive framework for managing non-renewable resources.

3. Empirical Challenges to Hotelling Models: Technology, Substitution, and Demand Effects

The empirical relevance of Hotelling's model and its extensions has been a subject of debate. At its core, Hotelling's framework suggests that producers of non-renewable resources, facing a finite stock, should conserve reserves, restrict supply, and allow prices to rise at the rate of interest to maximize intertemporal profits. However, empirical studies that attempt to test these predictions have yielded inconsistent results, raising questions about the model's applicability to real-world data, particularly when its core assumptions—such as fixed reserves, no technical progress, and absence of uncertainty—are relaxed.

Numerous empirical studies over the years have attempted to validate Hotelling's model by analyzing long-term price trends for non-renewable resources. McRae (1978), for instance, examined price data for five minerals over the period 1922-1974 and concluded that their prices evolved more like a random walk than the systematic growth predicted by Hotelling. Similarly, Smith (1981), in a broader study of extractive industries from 1900 to 1973, reported a general decline in real prices across these sectors, challenging the notion of rising rents over time. Lee et al. (2006), analyzing price trajectories of non-renewable commodities between 1870 and 1990, found them to be stationary around a deterministic trend with structural breaks, adding another layer of complexity to the discussion. Gaudet (2007) further complicates the picture, observing that price variation for mineral resources during the 20th century was approximately centered around zero, not the rate of interest as Hotelling suggested.

Livernois (2009) provides a comprehensive review of 34 empirical studies testing Hotelling's rule, concluding that the data does not support the model's predictions in a consistent or robust manner. Similarly, works by Hart and Spiro (2011), Gaugler (2015), Atewamba and Nkuiya (2017), and Karp (2017), which test specific extensions or components of the Hotelling framework, reach similar conclusions, often pointing to significant deviations between theoretical predictions and observed price behavior. Earlier studies, including those by Farrow (1985), Heal and Barrow (1980), Tilton (1999), and Cuddington and Nülle (2014), also fail to confirm the hypothesis of rising resource prices implied by Hotelling's model.

One critical factor influencing these empirical results is the treatment of real (inflation-adjusted) commodity price series. Svedberg and Tilton (2006), for example, revisited the long-term price trends of copper using adjusted deflators to correct biases in the U.S. Producer Price Index and other standard deflators, which often overestimate inflation. They found that while unadjusted deflators suggested a long-term downward trend in resource prices, corrected deflators indicated no clear trend or even a

slight upward trajectory. Although this adjustment provides new insights, it does not resolve the broader issue of empirical validation for Hotelling's rule.

Berck and Roberts (1996) further complicate the discussion by showing that modeling price series as trend-stationary predicts significant price increases consistent with Hotelling's theory. However, when the same data is modeled using time-differentiation techniques, the predictions of long-term price increases become much less certain.

Krautkraemer (1998), in his influential meta-analysis, examines the long-term price behavior of non-renewable resources, emphasizing the lack of empirical support for the steadily increasing price trajectory predicted by Hotelling. He argues that new technologies, resource discoveries, and recycling have historically counteracted the effects of depletion, keeping real resource prices relatively stable or even declining over extended periods. Krautkraemer notes that the observed trends often reflect a balance between technological innovation, which reduces costs and enhances resource availability, and the economic and physical pressures of depletion, which push costs upward over time. His work underscores the importance of understanding these opposing forces to interpret empirical results.

In addition to technological progress, Krautkraemer also highlights the role of substitutes and shifts in resource demand. For instance, as specific resources become scarcer or more expensive, economic incentives drive the adoption of alternatives or the development of more efficient technologies, further dampening the upward pressure on prices. This perspective aligns with Nordhaus (1974), who critiques simplistic depletion models by emphasizing the role of substitutes and economic adaptation. Nordhaus argues that resource scarcity alone does not necessarily constrain economic growth, as market mechanisms often lead to the substitution of scarce resources with more abundant or cost-effective alternatives. His analysis highlights the dynamic interplay between resource availability, technological innovation, and economic incentives, challenging the deterministic outcomes often associated with Hotelling's model.

Krautkraemer's findings also resonate with the broader empirical literature on long-term resource price trends. He points out that many studies using modern econometric techniques have failed to detect a statistically significant upward trend in real resource prices over time. For example, the historical price series for many mineral commodities, when adjusted for inflation, show stability or even a gradual decline since the mid-19th century. This pattern is typically attributed to the mitigating effects of technological progress, exploration, and resource substitution, which offset the cost pressures associated with depletion. Krautkraemer concludes that while Hotelling's model provides valuable theoretical insights, its empirical applicability is limited without accounting for these dynamic market adjustments.

Parallel to these findings, empirical studies have also questioned the causal mechanisms assumed in Hotelling's framework and its extensions. Recent econometric techniques, including vector autoregression (VAR) and Granger causality tests, have examined whether resource producers behave more as price-makers than price-takers. Studies by Ahumada and Cornejo (2014), Chai et al. (2011), Klotz et al. (2014), and Papiez et al. (2014) suggest that producers' supply behaviors can significantly influence market prices, particularly in non-competitive markets, which challenges the standard assumption of exogenous price determination.

While much of the literature focuses on hydrocarbons, recent studies have also examined metals. Stuermer (2018) and Jacks and Stuermer (2020) analyzed price data for copper, lead, tin, and zinc over the period 1870-2013. Their findings suggest that only the supply behaviors of copper and tin producers exert significant long-term influence on market prices. Furthermore, these studies highlight the critical role of demand—a factor largely ignored in Hotelling's original model—in shaping price trajectories. For instance, the rapid economic growth of China in recent decades has driven a substantial increase in the prices of most metallic commodities, underscoring the importance of demand-side dynamics in modern resource markets.

In conclusion, while the Hotelling framework and its extensions provide a rich theoretical foundation for analyzing non-renewable resource markets, their empirical validation remains elusive. The observed price dynamics of non-renewable resources often diverge from theoretical predictions, influenced by factors such as technological progress, resource discoveries, recycling, and demand fluctuations. These discrepancies underline the need for continued refinement of resource models to better account for the complex realities of global markets. Further discussions on these new considerations, including the role of demand, are explored in Section IV below.

II. Peak Models: Empirical Appeal, but Without Theoretical Precision

The concept of peak resource production emerged prominently in the mid-20th century, with Hubbert's (1956) pioneering work predicting that the production of resources like oil would follow a bell-shaped curve over time. This framework gained widespread attention during periods of geopolitical tension, such as the 1973 oil crisis, when resource scarcity and environmental concerns intensified. Hubbert's approach prioritized empirical observations of production and reserves, sidestepping economic considerations such as price dynamics, technological progress, and substitution effects.

Peak models gained further prominence with the publication of the *Meadows Report* (*The Limits to Growth*, 1972), which expanded the idea of finite resources to global sustainability challenges, linking

resource depletion to population growth and environmental constraints. These models resonated in political and environmental discourse, providing a clear narrative of inevitable scarcity. However, their deterministic assumptions and limited integration of economic mechanisms, such as price signals and market responses, have drawn significant criticism for their lack of predictive accuracy and theoretical depth.

Despite the well-known limitations of peak models and their mixed empirical record, they continue to occupy a central place in both academic and policy discussions on resource depletion. As highlighted by Riondet et al. (2023), the Hubbert model has “acquired a vast notoriety as a conceptual approach to resource depletion” and remains widely applied to a variety of mining elements, especially in the context of the energy transition. Its simplicity, its historical success in specific cases (notably U.S. conventional oil), and its strong presence in public-facing debates explain why it persists as a reference point. For this reason, even if the present thesis ultimately departs from Hubbert-style models in favor of more economic and dynamic approaches, it is necessary to review the Hubbert framework, clarify its assumptions, and document why its predictions often fail to hold in mineral markets.

This section examines the evolution of peak models, from Hubbert’s foundational work to the broader applications and critiques that followed. It highlights their empirical strengths while addressing their methodological limitations, setting the stage for more dynamic and economically grounded approaches to resource sustainability.

1. Hubbert’s Approach (1956)

In his influential 1956 paper “Nuclear Energy and the Fossil Fuels,” M. King Hubbert, a geologist at Shell, sought to address the long-term availability of energy resources in the United States. Presented at a conference of the American Petroleum Institute in San Antonio, the paper made the now-famous prediction that U.S. oil production would peak between 1965 and 1970—an estimate that, in hindsight, closely aligned with observed production patterns.

Hubbert’s argument was based on the observation that fossil fuel extraction tends to follow a life cycle: rapid growth during discovery and early exploitation, a peak when about half of the recoverable resource has been extracted, followed by a decline. Using historical production data from regions such as Illinois and Ohio, where production curves of oil and gas appeared to trace a bell shape, Hubbert proposed that this empirical pattern could be applied to the entire continental United States. He estimated the total recoverable reserves of crude oil and used a symmetric logistic function to model cumulative production over time. From this, he derived the expected date of peak production.

Importantly, Hubbert did not claim that the bell-shaped curve was a universal law or that it applied equally to all resources in all contexts. His model made no pretense of capturing the economic determinants of extraction—prices, costs, technological advances, or policy shifts were not part of the equation. Rather, the model was meant as a geophysical forecast based on known reserves and current production trends. It was a deliberately simplified tool, aiming to spark awareness about the finite nature of fossil resources and the need to transition to alternative energy sources—nuclear energy being, in his view, the most promising candidate at the time.

Subsequent commentators and some policy circles later generalized Hubbert’s empirical model into a broader claim about all non-renewable resources, interpreting the bell curve as a natural and inevitable law of depletion. This misinterpretation gave rise to “peak oil” discourses that often oversimplified the dynamics of extraction. In contrast, economists have long argued that resource depletion is not governed solely by physical availability, but by market forces and innovation. Hubbert’s approach—while prescient in its short-term forecasts—lacked a theoretical foundation to endogenize how scarcity, price signals, and technological change shape extraction paths over time.

Whereas Hotelling’s (1931) model was theoretically elegant but empirically elusive, Hubbert’s framework was empirically grounded but theoretically narrow. Both leave important questions open, and particularly in Hubbert’s case, a rigorous economic interpretation of resource behavior requires going beyond static reserve estimates to consider dynamic feedback mechanisms between supply, demand, and price.

2. The Meadows Report (1972)

The Meadows Report, officially titled *“The Limits to Growth”* and published in 1972 by a team of MIT researchers for the Club of Rome, radically altered perceptions of resource depletion and provided fertile ground for Hubbert’s bell-curve model.

Using simulation models, the report explored the long-term consequences of global population and economic growth trends, natural resource consumption, and pollution. It warned that if exponential growth trends continued unchecked, planetary limits would be reached within a century, leading to a sudden and uncontrollable decline in population and industrial capacity.

Before the report, resource discussions often emphasized continuous and increasing exploitation, with implicit faith in technological innovation to resolve future shortages. The Meadows team challenged this view, emphasizing the finite nature of resources and the risk of eventual exhaustion if extraction rates continued to grow.

The report popularized the bell-curve production model, extending Hubbert’s ideas to various natural resources, including metals and fossil fuels. According to this model, production grows rapidly when resources are abundant and easy to extract, peaks as depletion sets in and declines as extraction becomes increasingly difficult and costly. This perspective highlighted the need for producers to consider sustainability and prudent resource management.

3. Peak Models Variants

Modeling these intuitions, the so-called “peak models”, also known as bell-curve resource production models, rely on a single key variable: Ultimate Recoverable Resources (URR). URR represents the total recoverable mineral resources, both discovered and undiscovered, within an economy. These models estimate production trajectories using mathematical functions like $S_t = \frac{URR}{b} e^{-\frac{1}{2}\left(\frac{t-t_{peak}}{b}\right)^2}$, where S_t represents production at time t , and t_{peak} is the estimated year of peak production. The production peak is reached when $S_{t_{peak}} = \frac{URR}{b\sqrt{2\pi}}$.

In this framework, variables such as price, technology, or exploration costs are considered irrelevant for analyzing long-term resource depletion. Demand is assumed to remain non-decreasing per capita, ensuring it does not constrain the production function. These strong and often debatable assumptions form the foundation of peak models (Tilton, 2018).

Despite criticisms, peak models produce seemingly robust predictions because variations in URR do not significantly alter t_{peak} (Northey et al., 2014; Sverdrup et al., 2014). Applied to copper, recent literature suggests shortages within the next 20 to 30 years, regardless of URR assumptions (Bardi and Pagani, 2007; Prior et al., 2012; Laherrère and de Sousa, 2010). However, the historical record has very regularly been contradicting such dire predictions, as demonstrated by the relatively constant reserves-to-production ratio for copper, observed over 120 years by the US Geological Survey¹⁵. The next subsection addresses this issue in some more detail.

4. Serious Critiques and Limitations: Fixed Reserves, Static Demand, and Neglect of Technological Progress

The critiques of peak models have been extensively debated by proponents of economic frameworks, such as Hotelling’s, which incorporate extraction costs and opportunity costs. These critiques highlight

¹⁵ <https://internationalcopper.org/sustainable-copper/about-copper/cu-demand-long-term-availability/>

significant methodological limitations and question the validity of peak models' underlying assumptions.

A primary critique of peak models is their conflation of geological availability with economic availability. While geological estimates of reserves and resources are inherently uncertain, peak models treat these figures as fixed and unchanging, effectively ignoring the economic processes that determine resource accessibility. For instance, the models fail to consider that technological advances and changing market conditions can transform previously uneconomical deposits into viable reserves (May et al., 2012; Meinert et al., 2016; Wellmer and Scholz, 2018). This rigid treatment of resource availability disregards the dynamic interplay between geology, technology, and economics, resulting in overly pessimistic projections.

The exclusion of technological progress is another critical flaw. Historical evidence demonstrates that technological advancements have consistently expanded the economically recoverable reserves of many resources. Innovations in extraction, processing, and recycling technologies have not only mitigated depletion but, in many cases, have led to cost reductions and supply expansions (Kharitonova et al., 2013). For example, advancements in mining technology have allowed the exploitation of lower-grade ores and previously inaccessible deposits, directly challenging the deterministic predictions of peak models. By failing to incorporate such mechanisms, peak models overlook a fundamental driver of resource availability and underestimate the resilience of resource markets.

Moreover, peak models often ignore the declining material intensity of economies as they develop. As countries industrialize and transition to service-oriented economies, their reliance on raw materials tends to decrease. This shift, coupled with improvements in material efficiency and recycling rates, reduces the overall demand for primary resources (Criqui, 2013; Crowson, 2011; Ericsson and Söderholm, 2013). By assuming constant or growing per capita demand, peak models oversimplify the relationship between economic development and resource consumption, leading to exaggerated predictions of scarcity.

Nordhaus (1974) provides a critical perspective, arguing that scarcity-induced constraints on resources often drive innovation and substitution. Markets are adaptive, pivoting toward alternative materials, technologies, and processes when faced with rising resource costs. This dynamic capacity of markets to mitigate depletion challenges the static assumptions of peak models. Nordhaus further highlights the importance of understanding resource substitution, which can alleviate the pressures of scarcity long before physical exhaustion occurs.

Empirical evidence also undermines the predictive accuracy of peak models. Sverdrup and Ragnarsdottir (2016), for example, predicted platinum depletion by 2022. However, despite annual production of 2,000 tons, reserves declined by only 2.8% between 2007 and 2017. Bardi and Pagani (2007) estimated that gallium production had peaked in 2002 at 85 tons, yet by 2014, production had reached nearly 450 tons, with potential production capacity estimated at 2,100 tons annually. Such failed predictions highlight the limitations of peak models in accounting for technological progress and resource substitution.

The case of copper further illustrates these shortcomings. Despite recurring concerns about "peak copper" since the early 20th century, global reserves have remained relatively stable. The US Geological Survey (USGS, 2018) notes that the reserves-to-production ratio—a metric often used to gauge resource longevity—has remained between 30 and 50 years for over a century. This stability reflects the continuous discovery of new deposits and the reclassification of resources into reserves, driven by advancements in exploration and extraction technologies. Moreover, estimates of undiscovered copper deposits suggest a potential additional 3.5 billion metric tons, further dispelling fears of imminent scarcity.

Peak models' reliance on Ultimate Recoverable Resources (URR) as the sole determinant of production profiles also faces criticism. While URR estimates underpin the bell-shaped production curve, their reliance on fixed assumptions about geological stocks makes them inflexible to changes in technology or economic conditions. Studies have shown that even substantial variations in URR assumptions produce only minor shifts in the estimated year of peak production (Northey et al., 2014; Sverdrup et al., 2014). This apparent robustness belies the models' inability to incorporate dynamic feedback mechanisms, such as price signals or technological shifts, which are central to resource markets.

Finally, peak models often fail to recognize that markets may not provide reliable scarcity signals. Henckens et al. (2016) argue that resource prices do not always reflect scarcity due to market imperfections, externalities, and regulatory interventions. For example, environmental costs associated with mining and processing are often excluded from market prices, leading to underpricing and inefficient resource use. These distortions undermine the capacity of market signals to warn of impending shortages, prompting some proponents to advocate for physical stock assessments as a more reliable measure of resource availability (Ali et al., 2017; Northey et al., 2014).

Given these critiques, peak models face significant methodological and conceptual challenges. While they offer a simple framework for understanding resource depletion, their rigid assumptions and poor empirical track record limit their utility as predictive tools. As Tilton (2018) observes, peak models often present a pessimistic view of resource availability that is unsupported by historical trends and

economic theory. For a more accurate understanding of resource dynamics, models must account for the adaptive nature of markets, the impact of technological progress, and the interplay between economic and geological factors.

III. An Alternative Paradigm: The Cumulative Availability Curve (CAC)

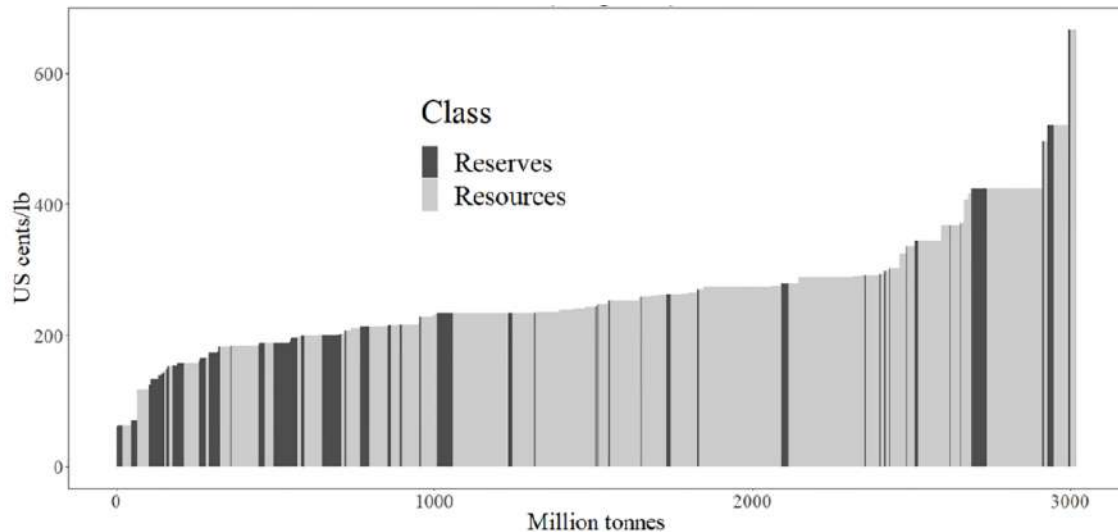
In response to the limitations of peak models, the cumulative availability curve (CAC) emerged as a more flexible and economically grounded framework for analyzing resource depletion. Championed by Tilton and his collaborators, the CAC methodology shifts the focus from rigid production trajectories to the dynamic relationship between resource availability and cost, incorporating both geological constraints and market adaptability. Unlike peak models, CAC-based approaches emphasize how technological advancements, economic incentives, and policy interventions can influence the long-term availability of resources.

This section introduces the CAC methodology, which mainly stems from the research of John E. Tilton. It examines the CAC key insights, and discusses critiques of its application, illustrating how it addresses many of the shortcomings associated with peak models. By integrating economic and geological perspectives, the CAC represents a significant step forward in the study of resource sustainability.

1. Concept: Mapping Geological Stocks with Economically Available Resources

The Cumulative Availability Curve (CAC) of a non-renewable resource is a graphical representation that links a given price of the resource to the total economically exploitable global stock at that price. Unlike the traditional supply curve in economics, which depicts the flow of goods offered on the market during a specific period (usually one year) as a function of price, the CAC represents a stock available over the long term. It expresses how the quantity that is economically recoverable expands as prices rise (Tilton and Lagos, 2007; Tilton et al., 2018). Importantly, the CAC builds on the distinction between reserves, the subset of the resource base that is economically recoverable at current prices, and resources, the broader geological endowment that could become recoverable at higher prices or with better technology. Like the traditional supply curve, the CAC assumes that all other determinants of resource availability—such as exploration costs, production costs, and technology levels—are fixed aside from price.

Figure 1.2: Cumulative Availability Curve (CAC) for Copper (Castillo and Eggert, 2020)



For example, the CAC for copper (from Castillo and Eggert, 2020; see Figure 1.2 above) illustrates the quantity of copper recoverable globally at different price points. The data includes reserves—quantities both discovered and economically exploitable—and resources, which also incorporate known but currently uneconomical deposits and undiscovered but potentially exploitable quantities.

This methodology is particularly valuable for forecasting the sustainability of global metal demand. It allows researchers to differentiate between the economic effects of three key factors influencing demand sustainability:

1. Geological determinants: These define the shape of the CAC curve, such as ore grades and mining depths, which influence future extraction costs.
2. Economic determinants: These govern the speed at which global demand increases, such as global per capita growth, the industrial development of emerging economies, the metal intensity of the low-carbon transition, and recycling policies.
3. Technological advancements: These shift the CAC curve outward over time by reducing extraction and production costs.

The first group of factors shapes the curve, the second governs the pace of movement along the curve, and the third shifts the curve outward, reflecting technological progress. These elements integrate two forces already identified in Hotelling's framework: the upward pressure of depletion and the downward pressure of technological progress.

2. Insights from CACs: Geological Cost Structures, By-Product Discontinuities, and Long-Run Resource Sustainability

The CAC for a resource, as the one presented in Figure 1.2 above, is often smooth and monotonically increasing. However, not all resources follow this pattern. Some metals, particularly those that are by-products of primary metal production, present unique challenges. For example, cobalt, gallium, and lithium are often co-produced during the extraction of primary metals like copper, aluminum, and complex silicates. If global demand for these co-products surpasses the supply derived from primary metal production, their extraction costs would increase sharply, introducing discontinuities in the CAC.

CACs have been developed for various resources, including oil (Aguilera et al., 2009), lithium (Yaksic and Tilton, 2009), thorium (Jordon et al., 2015), and copper (Castillo and Eggert, 2020). Constructing these curves involves estimating production costs for unexploited locations, a challenging but achievable task requiring careful empirical work and scenario analysis. Nevertheless, these calculations rely on data that may not always be publicly available or reliable, particularly when provided by producers with strategic interests.

The CAC methodology is particularly useful for assessing the sustainability of global metal demand. Unlike future-dependent models, the CAC's shape is determined by geological factors rooted in the past, enabling relatively objective calculations. For example, combined CAC and global demand evolution analyses have reasonably projected that lithium demand could remain sustainable over the century, even under optimistic demand and conservative supply assumptions (Yaksic and Tilton, 2009). However, caution is always warranted given the potential for unexpected geological or technological developments. Recent studies have further refined and expanded the CAC methodology, particularly in the context of metals. Mudd (2007) highlights the long-term trends in declining ore grades for metals like copper, emphasizing how these shifts shape the CAC and its implications for sustainability. Tilton and Guzmán (2016) explore the CAC's potential to guide mineral resource policy and assess long-term market sustainability. Bardi (2014) critiques peak production models and underscores how CAC-based approaches offer a more nuanced understanding of resource depletion over time. Henckens et al. (2016) highlight the limitations of market mechanisms in signaling resource scarcity, reinforcing the importance of CAC models in capturing both geological and economic dimensions of metal availability. Cuddington and Zellou (2013) integrate CAC methodology into economic models to explore price trends and the possibility of "super cycles" in resource markets, while Eggert (2011) examines the real-world applicability of CAC, considering complexities like recycling and technological advancements. Cunningham (2003) demonstrates how the CAC can be applied to specialty metals like tantalum, illustrating its adaptability to various contexts. Collectively, these works validate the CAC's

utility in evaluating resource availability while emphasizing the importance of ongoing refinements to address economic and technological dynamics.

3. Limits of the CAC Approach

Despite its utility, the CAC paradigm does not enjoy unanimous support as a comprehensive framework for assessing mineral resource exhaustion. Key critiques include the following:

1. **Market Signals and External Costs:** Critics argue that markets may fail to provide adequate signals to compensate for resource depletion. External social and environmental costs of mining are often not internalized, and price trends do not always clearly signal resource scarcity. For example, price trajectories do not systematically differ between geologically abundant and rare minerals (Henckens et al., 2016; Segura-Salazar and Tavares, 2018).
2. **Overestimation of Technological Compensations:** Some critiques suggest that the CAC framework may overestimate the extent to which technological progress can compensate for depletion. While technology has historically expanded recoverable reserves, it is not a panacea for all resource constraints (Gordon et al., 2007; Humphreys, 2013).
3. **Purely Accounting-Based and Partial Equilibrium Nature:** More fundamentally, the CAC is a purely accounting-based tool that lacks the economic behavioral modeling present in frameworks like Hotelling's. It does not incorporate optimizing behavior by agents or market dynamics. Moreover, it is a partial equilibrium analysis that overlooks feedback loops, such as how price increases due to depletion might curtail demand, incentivize substitution, and encourage recycling or technological advancements like carbon nanotubes. These feedback mechanisms are potentially crucial for analyzing the sustainability of global demand for a metal.
4. **Limited Applicability to Co-Product Metals:** The CAC is best suited to metals that are produced as primary products. For co-product or by-product metals—such as molybdenum (typically extracted alongside copper) or cobalt (a frequent co-product of nickel and copper mining)—the assumptions underlying the CAC do not fully capture the unique supply dynamics. In these cases, the availability of the metal is determined not by its own price or cost structure, but by the economics of the primary metal. As a result, the CAC may underestimate or misrepresent the responsiveness of supply to market conditions. An integrated modeling approach that

explicitly considers the linkage between primary and co-product extraction would better reflect the true constraints and market behavior associated with these materials¹⁶.

Nordhaus (1974) provides a salient critique of such static approaches, emphasizing that markets are dynamic and adaptive. Rising prices often trigger innovation, substitution, and alternative supply chains, mechanisms not captured in the CAC framework. For instance, Nordhaus argues that scarcity constraints historically lead to technological breakthroughs and shifts in consumption patterns, challenging the deterministic view of resource exhaustion implied by static models.

The CAC offers valuable insights into the geological determinants of resource availability and provides a basis for projecting resource sustainability under current conditions. However, its static nature and exclusion of market dynamics limit its scope. While the CAC integrates some elements of Hotelling's framework, such as depletion and technological progress, it fails to capture the dynamic interplay of prices, demand, and innovation. As such, it serves as a complementary tool rather than a standalone paradigm for analyzing the long-term sustainability of global metal demand.

4. Evolving the CAC: Integrating Demand, Substitution, and Updated Resource Data (Castillo and Eggert, 2020)

In an article aimed at bridging two paradigms that often stand in stark opposition, Castillo and Eggert (2020) examine in detail the two primary methodologies used to evaluate mineral depletion: peak models and the application of the Cumulative Availability Curve (CAC). Both approaches share common ground on several key points: they acknowledge that supply and extraction costs are the primary drivers of long-term depletion and recognize the inherent uncertainty in geological information.

However, Castillo and Eggert (2020) identify areas of contention and critique within each paradigm. Peak models, for instance, are based on the premise that rising extraction costs eventually lead to a decline in production over time. This is often exemplified by the decline in ore grades, used as a proxy

¹⁶ A) Co-product metals rely heavily on the extraction rates of the primary metals with which they are associated. Their availability is largely controlled by the demand and price dynamics of the primary metal, rather than by the co-product's own market conditions. This dependency means that the production of co-products cannot adjust freely in response to their own prices or demand.

B) While primary metals can theoretically expand or contract production based on price signals, co-product metals lack this flexibility. Even if the co-product's price rises, its production remains constrained by the extraction levels of the primary metal. This makes it difficult to model the supply of co-product metals using the CAC, as it does not account for the influence of the primary metal's production cycle.

C) The CAC assumes that supply can increase as demand and prices rise. However, for co-product metals, high demand alone may not lead to increased supply unless there is also high demand for the primary metal. This interdependence makes it challenging for the CAC to model supply adjustments in response to the co-product's own market conditions.

for increasing production costs and environmental impacts. While proponents of peak models emphasize the importance of extraction costs in determining future mineral availability, they are criticized for their limited consideration of evolving demand patterns and the uncertain conversion of resources into reserves over time.

Conversely, the CAC approach provides a more generalized perspective on mineral availability but is criticized for its reliance on fixed geological stocks and its lack of attention to changing economic conditions. Critics argue that treating reserves and resources as fixed stocks neglects the dynamic nature of mineral deposits and their potential for future exploration and discovery.

In response to these critiques, Castillo and Eggert (2020) propose refinements to the CAC methodology to reconcile these divergent perspectives on the sustainability of mineral markets. Their proposed modifications focus primarily on refining the analysis of both demand and supply aspects.

From the demand side, the authors suggest incorporating evolving per capita demand as countries develop and accounting for substitution effects as prices rise. This involves analyzing historical patterns of per capita metal consumption and considering the convergence hypothesis, which posits that less developed regions may approach the consumption levels of more developed regions over time.

On the supply side, the authors advocate for a more nuanced understanding of reserves, resources, and undiscovered deposits. They propose adjustments to the CAC to reflect evolving information over time and to account for the uncertain conversion of resources into reserves. Additionally, the article emphasizes the importance of factoring in externalities such as environmental and social impacts when assessing extraction costs. These supply-side considerations could benefit from further elaboration, as the discussion in Castillo and Eggert (2020) is somewhat succinct in this regard.

In conclusion, Castillo and Eggert (2020) provide a comprehensive overview of ongoing debates about mineral depletion and propose a methodological framework to reconcile differing perspectives. They acknowledge the complexities involved in accurately assessing future mineral availability but emphasize the importance of continued research in this area.

The present description, while concise, serves as an introduction to the approach that underpins our analytical model. A more comprehensive and detailed examination of this methodology will be provided in the subsequent chapter (Chapter 1, Section I.1.b), where its foundational role in our analysis will be fully elaborated.

IV. Market Models and Short-Term Price Dynamics

Theoretical frameworks, including those proposed by Hotelling and Hubbert, as well as their extensions, have often omitted the role of demand as a critical driver of resource price evolution. In contrast, analytical approaches that account for demand mechanisms within models inspired by Hotelling have a longer history. Pindyck (1980), for example, incorporated stochastic demand into a Hotelling framework, revealing how demand uncertainty and random fluctuations in reserves affect price trajectories. This early consideration of demand dynamics has laid the groundwork for more recent developments in econometric and empirical modeling of commodity prices, which increasingly emphasize the interplay between short-term fluctuations and long-term trends.

The renewed interest in these "market models" can be attributed to two factors. First, the sharp rebound in commodity prices at the turn of the century highlighted the need for tools capable of explaining price volatility in response to demand and supply shocks. Second, the availability of advanced econometric and statistical techniques has facilitated a closer examination of the correlation between short-term phenomena and longer-term price dynamics.

1. Reintroducing Demand into Hotelling's Legacy

Pindyck's (1980) work remains a seminal contribution, illustrating the stochastic evolution of market demand and reserves over time. By introducing random walks into demand and reserve levels, Pindyck demonstrated that market prices would follow similarly stochastic trajectories. This perspective highlighted the tenuous link between short-term price fluctuations and long-term trends, underscoring the need for models capable of capturing transient demand and supply shocks.

Heal (1976) extended these insights by exploring the cyclicity of resource prices in response to scarcity, while Slade (1982) incorporated technological progress into extraction costs, showing how it interacts with demand dynamics. More recently, Gaudet (2007) emphasized the importance of distinguishing between permanent and transitory components of price movements to understand cyclical behavior.

2. Commodity Price Cycles and Metal's Volatility

The study of price cycles for metals gained momentum in the early 2000s, coinciding with the dramatic increase in energy and commodity prices between 1998 and 2001. These models typically decompose price movements into transient and permanent components, identifying cycles at short (economic cycles), medium (8-20 years), and long (super-cycles lasting several decades) time scales.

For metals, price volatility is particularly pronounced. Baffes and Kabundi (2023) show that the variance in metal prices is approximately twice as high as that for energy or agricultural commodities, with short- and medium-term shocks contributing significantly to this volatility. Temporary disruptions, such as recessions (e.g., the 2007-2009 financial crisis) or supply chain accidents (e.g., the 2019 Vale dam collapse in Brazil¹⁷), have an outsized effect on metals compared to other commodities.

3. Explaining Price Shocks: Demand Versus Supply Drivers

A growing body of literature seeks to identify the drivers of price shocks. Kilian's (2009) structural vector autoregressive (SVAR) model introduced a framework to disentangle demand- and supply-side shocks. His approach, which employs sign restrictions to identify the relative importance of different shock types, has become a standard in analyzing commodity price fluctuations.

For metals, Stuermer (2018) and Jacks and Stuermer (2020) emphasize the dominant role of global demand shocks, particularly in contrast to hydrocarbons, where supply-side shocks often play a larger role. Their findings reveal that the influence of demand-specific shocks has grown over time, driven by factors such as the rapid industrialization and urbanization of emerging economies, particularly China during the 2000s.

Complementing this approach, Wets and Rios (2015) developed a structural econometric model to analyze price dynamics in copper markets, distinguishing short-term fluctuations from medium- and long-term trends. Their approach addressed the challenge of disentangling transient shocks, such as mining strikes or speculative trading, from durable structural factors like technological advancements or global economic growth. By separating these influences, their model provided a nuanced understanding of how copper prices respond to both immediate disruptions and broader market shifts. Their study revealed that short-term dynamics, driven by factors such as inventory adjustments, speculative trading, and macroeconomic announcements, contribute significantly to copper price volatility. These fluctuations, while frequent and impactful in the short term, have minimal influence on long-term price levels, which are shaped by fundamental drivers like resource availability and industrial demand. The model demonstrated that copper's dual role as an industrial input and financial asset amplifies these short-term dynamics, making them a persistent feature of the market. While Wets and Rios (2015) primarily focused on copper, they suggested their framework could be applied to other commodities with sufficient data. However, the lack of comprehensive datasets, particularly for historical price series and inventories, remains a limitation for broader application. Despite this, their

¹⁷ <https://pages.marketintelligence.spglobal.com/Vale-dam-burst-metals-and-mining.html>

findings underscore the recurring nature of short-term volatility in copper markets and the importance of distinguishing it from long-term trends.

4. Advances in Empirical and Econometric Approaches

Building on these foundational studies, more recent works have refined econometric techniques to better capture the unique characteristics of metal prices. Kriechbaumer et al. (2014) introduced wavelet analysis to decompose price fluctuations across multiple time scales, enhancing the detection of short-term volatility and medium-term trends. Similarly, Khalifa et al. (2011) applied Fourier transformations to identify cyclical patterns in copper prices, isolating seasonal trends from underlying economic drivers.

GARCH and Markov-switching models, as employed by Hammoudeh and Yuan (2008) and Mensi et al. (2014), remain dominant in forecasting short-term price volatility. However, these models' reliance on historical price data often limits their ability to incorporate economic fundamentals. Recent studies, such as those by Goss and Avsar (2013) and Cortazar and Eterovic (2010), highlight the importance of incorporating macroeconomic variables and interdependencies between metals to enhance predictive accuracy.

* * *

The literature on resource economics, particularly concerning metals, has evolved significantly over time, reflecting the growing complexity of market dynamics and the challenges of forecasting long-term and medium-term price behavior. Hotelling's (1931) seminal work provides a robust theoretical foundation for understanding non-renewable resource pricing, emphasizing intertemporal optimization and the role of scarcity. However, as subsequent sections have shown, empirical applications of Hotelling's model often struggle to align with observed data, particularly when key assumptions, such as fixed costs or certainty in reserves, were relaxed. The reinvigoration of this framework by economists in the 1970s and 1980s introduced important extensions, including the incorporation of uncertainty, the role of technological progress, and the interplay between reserve levels and extraction costs.

In contrast, peak models, popularized by Hubbert (1956) and later adapted in works such as the Meadows Report (1972), offer an empirically driven but overly simplistic view of resource extraction dynamics. By reducing the future of resource availability to geological limits, these models ignore economic, technological, and demand-side factors. Critiques of peak models highlight the lack of

predictive precision of these models and inability to account for market feedback mechanisms, making them a limited tool for addressing contemporary questions about resource sustainability.

The Cumulative Availability Curve (CAC) represents a promising paradigm shift, focusing on the economic availability of resources as a function of price. This approach introduces a more nuanced framework for analyzing resource sustainability by integrating geological factors with economic considerations. Despite its promise, the CAC approach is not without limitations, particularly its reliance on static assumptions about reserves and its partial equilibrium perspective. Castillo and Eggert's (2020) refinements to the CAC methodology offer a valuable bridge, addressing some critiques by incorporating demand dynamics and acknowledging the role of technological and economic shifts over time.

Market models, emphasizing short-term price dynamics, further expand the analytical landscape. These models delve into the interplay of demand shocks, supply disruptions, and macroeconomic factors, providing a more granular understanding of price cycles and volatility. Advances in econometric techniques, such as Structural Vector Autoregressive (SVAR) models, refine the ability to decompose price changes into transitory and permanent components, underscoring the importance of global demand as a critical driver of metal prices.

In sum, the literature reflects a rich interplay between theoretical models and empirical observations. While theoretical frameworks like Hotelling's offer invaluable insights into long-term dynamics, empirical approaches—whether through peak models, CAC methodologies, or market models—highlight the importance of integrating economic, technological, and demand-side considerations to capture the complexity of metal markets. The challenges of forecasting prices and ensuring sustainable resource management remain at the forefront of resource economics, requiring continued innovation in analytical and methodological approaches.

Building on the insights from this literature review, Chapter 2 develops a stylized analytical model, focusing on the sustainability of the global balance between supply and demand for metals in the long term. This chapter addresses the pressing question of whether resource markets can achieve a stable equilibrium under the combined pressures of economic growth, technological innovation, and environmental constraints. By developing an analytical model that integrates theoretical insights from the Hotelling framework, critiques of peak models, and the CAC paradigm, this chapter aims to provide a comprehensive understanding of how supply and demand dynamics shape the future of metal markets. In Chapter 3, the theoretical model is calibrated using data for copper.

CHAPTER 2: An Analytical Model of Long-Term Metal Global Market Sustainability

This chapter develops the analytical model we use to study the long-run behavior of metal markets. We focus on metals whose primary supply can be represented by a cumulative availability curve (CAC)—that is, where cumulative recoverable quantities rise monotonically with price and annual extraction maps to a depletion path. This domain covers most base, industrial, and precious metals (copper, aluminum, lead, zinc, tin, nickel, magnesium, tungsten, silver, gold, titanium). By contrast, alloys (e.g., steel, bronze), industrial by-/co-products (e.g., cobalt), and rare earth elements fall outside our scope because their supply does not admit an independent CAC representation; excluding them does not narrow the model’s relevance in volumetric terms.

Within this scope, we set out the model’s theoretical foundations by separating demand and supply (Section I). The demand function captures the economic drivers of metal consumption; the supply side formalizes resource availability and extraction constraints via the CAC and an associated depletion schedule. We then derive the annual equilibria implied by their interaction and establish the conditions for a non-explosive, non-zero (NENO) equilibrium—i.e., prices neither collapse to zero nor diverge without bound, and traded quantities do not converge to zero (Section II). Throughout, we use “sustainable” to mean able to sustain NENO dynamics over the horizon: the market clears without explosive prices or vanishing volumes, given finite resources and plausible technological progress.

Sustainability requires (i) sufficient demand responsiveness—prices must elicit measurable demand adjustment so pressure does not simply compound; (ii) disciplined demand growth, so cumulative extraction does not outrun the resource base implied by the CAC over the horizon; (iii) supply-side progress that expands recoverable quantities—linear gains or one-off discoveries mainly buy time, while sustained, faster-than-demand improvements provide durable relief; and (iv) the depletion feedback embedded in the CAC, whereby approaching the availability limit tightens annual supply, raises prices, and tempers quantities. Intuitively, the CAC’s sigmoidal shape yields three phases—early scarcity, mid-stage relief as availability accelerates with price, and late-stage re-tightening—within which the market can remain on a NENO trajectory when these conditions hold (derivations in the appendix in Section III).

I. Foundations of the Model: CES Demand and a Depletion-Constrained Supply

This subsection outlines the two core functions of our model: the demand function and the supply function. The demand function is structured around price elasticity, substitution effects, and a time-

dependent baseline level of demand, capturing how consumption evolves and responds to price changes. The supply function is based on cumulative resource availability and extraction dynamics, representing the constraints and potential of long-term supply. Together, these functions set the foundation for analyzing market behavior and NENO equilibrium conditions in metal markets.

1. Demand with CES and a Time-Varying Baseline of Structural Drivers

In our model, the demand for a given metal responds to price through a CES function, capturing both price sensitivity and substitution flexibility. This form allows us to model how changes in price influence metal consumption across various applications. Thus, the demand for the metal $Q_{d,t}$ at time t is expressed as:

$$Q_{d,t} = B(t) \cdot P_t^{\sigma-1}$$

Where:

- $Q_{d,t}$ is the total quantity of the metal demanded in year t ,
- P_t is the market price of the metal in year t ,
- σ is the elasticity of demand¹⁸,
- $B(t)$ is a time-varying baseline parameter incorporating macroeconomic and structural factors such as GDP growth, population, sectoral shifts, technological progress, or policy incentives¹⁹,

¹⁸ All parameters in the demand function above are metal-specific, acknowledging that each metal's demand dynamics are distinct. Since the 1970s, the empirical literature has largely agreed that the price of a metal commodity itself is a more significant explanatory variable than the industrial properties of the metal. This led some studies from 1975 to 2002 to assume that all metals were interchangeable and that a single demand function, characterized by a unique price elasticity, could define demand levels for all metals based on price. However, Evans and Lewis (2002) highlighted the econometric limitations of this approach and demonstrated that, despite strong substitution between certain industrial metals, each traded metal is characterized by a unique demand function. See Evans, M. and Lewis, A.C., 2002. *Is there a common metals demand curve?* Resources Policy, 28(3-4), pp.95-104.

¹⁹ As in Georgentalis, S., Nutting, J. and Phillips, G., 1990. Relationship between price and consumption of metals. Materials science and technology, 6(2), pp.192-195.

For most metals—and especially for large industrial metals like copper—demand has historically grown alongside price. This trend is largely due to underlying economic growth factors, such as increases in industrial production, infrastructure development, and technological expansion, which have driven demand for these metals regardless of price increases. However, in this model, we focus on iterative short-term adjustments to achieve equilibrium each year. For this reason, our demand function is decreasing with price, capturing the immediate sensitivity of demand to price changes rather than long-term growth effects. This approach allows us to model the short-run price elasticity of demand, reflecting how consumers or industries may reduce metal usage in response to rising prices over a short period.

The parameter $B(t)$ is not a static constant but a dynamic factor reflecting long-term trends, such as shifts in metal intensity per unit of GDP or per capita income. This adjustment aligns the demand model with the evolving macroeconomic environment. For instance, over the long term, industrialization, urbanization, technological progress as well as energy transition and digitalization can alter the baseline level of metal consumption, requiring $B(t)$ to capture these changes.

While the demand function incorporates short-term elasticity effects through P_t , its structure also accommodates long-term dynamics via $B(t)$. Thus, our model bridges short- and long-term perspectives by integrating price responsiveness and evolving structural factors. This dual focus ensures that the function remains appropriate for assessing long-term market equilibrium, even though its immediate adjustments are driven by price fluctuations.

In this framework, σ should be understood as a generalized demand elasticity: it captures both the intrinsic price sensitivity of final uses and the degree to which consumers and industries can substitute away from the metal toward alternative materials or technologies. In other words, σ reflects the aggregated propensity of the economy to reduce metal use when prices rise, whether by efficiency improvements, redesign, or outright material substitution.

While it is plausible that technological advancements or shifts in material preferences could alter this parameter in the long term, we assume that the elasticity σ remains constant over time for several reasons. First, σ captures intrinsic features of the demand system, such as the ability to substitute the metal with competing materials and the fundamental price sensitivity of final uses. These characteristics evolve only when there are major technological or industrial discontinuities, not gradually with consumption levels. Empirically, this is supported by studies such as Fernandez (2018), who finds that although metal demand elasticities display heterogeneity across countries, they do not exhibit any systematic time trend and no dependence on the level of consumption for a given market. Second, allowing this parameter to vary dynamically would introduce unidentifiable degrees of freedom and risk conflating the intrinsic price responsiveness of metal demand with the exogenous structural shifts in material intensity and efficiency embedded in $B(t)$. Finally, fixing this parameter aligns our approach with existing literature (see below), where similar models have been developed under the assumption of constant substitution elasticities over time (Nordhaus, 1994; Van der Werf, 2008; Vidal et al., 2013; Hund et al., 2023). This provides a consistent framework for comparison with prior empirical and theoretical studies.

Overall, our demand function articulates two features: non-linearity and price elasticity. In modeling demand for metals, non-linear relationships are crucial for accurately representing the dynamics of

resource consumption. These non-linearities reflect how demand does not always respond proportionally to price changes, particularly when resources become scarce, or prices shift rapidly. For instance, if copper becomes scarce, prices may rise sharply, leading to reduced demand. Yet this reduction is often not straightforward, as it may limit the capacity to meet critical needs, such as electricity conductance (Sverdrup et al., 2014)²⁰. These inherent non-linearities necessitate flexible modeling approaches that reflect demand responses to dynamic market conditions.

Several studies support this non-linear modeling approach. The CES (Constant Elasticity of Substitution) function is an extension that accommodates varying substitution elasticities between inputs, as initially introduced by Christensen, Jorgenson, and Lau (1973). This function has been widely applied; for example, Pindyck (1979) used the CES function in energy demand modeling, capturing interfuel substitution when elasticity is partial rather than perfect. Methodologies like cointegration and error-correction models have also been used to capture long-term equilibrium relationships while allowing for short-term adjustments. Jaunky (2012), for instance, employed cointegration to analyze aluminum demand in ‘rich economies’, associating it with factors like industrial output and infrastructure investment. Similarly, Labson and Crompton (1993) blended intensity-of-use and cointegration approaches to assess responsiveness of demand for steel, copper, lead, zinc, and tin to structural changes. Structural and panel data models address these complexities by integrating sectoral details and feedback loops. Chen and Graedel (2012) developed a dynamic material flow model for iron, aluminum, copper, and zinc, incorporating recycling rates and technological change, while Wårell and Olsson (2011) used panel data to analyze demand for steel across OECD and non-OECD countries, highlighting demand pattern differences between these groups.

We focus on how metal demand adjusts to price fluctuations by employing a price elasticity model with a functional form that directly links commodity prices and quantities demanded. As Dahl (1993) demonstrated for energy markets, elasticity is essential for predicting how demand responds to price changes, particularly in resource markets.

The existing literature on the demand for metals often uses log-linear relationships between consumption and prices, incorporating elasticity parameters to capture consumption responses.

²⁰ Sverdrup et al. (2014) design a COPPER system dynamics model that integrates various components (mining, trade, pricing mechanisms, population dynamics, societal use, waste, and recycling) to analyze global copper supply. Four methods—burn-off rates, peak discovery warnings, Hubbert’s production model, and the COPPER model itself—are used to estimate sustainability and resource horizons, providing a comprehensive framework for assessing long-term copper availability. Sverdrup, H.U., Ragnarsdottir, K.V. and Koca, D., 2014. On modelling the global copper mining rates, market supply, copper price and the end of copper reserves. *Resources, Conservation and Recycling*, 87, pp.158-174.

Nutting (1977), following Hughes (1972)²¹, already observed a stable inverse relationship between metal prices and consumption levels, estimating demand elasticity for metals like copper, nickel, and zinc. Further works by Jacobson and Evans (1985)²² and Georgentalis et al. (1990)²³ supports the idea of a stable demand curve across metals and over long timescales.

Other empirical works challenge price as the main determinant of consumption, though emphasizing income as a primary driver. The "intensity of use" hypothesis suggests that metal demand correlates more closely with a country's income level, particularly over long-term trends, rather than with price alone. Tilton (1996) and Fanyu and Tilton (1999)²⁴ found that national income often explains changes in material demand, reflecting structural transformations within economies.

While these latter studies offer valuable insights into long-term trends, our analysis focuses on short-term demand adjustments. Accordingly, we use a CES demand function, which captures the elasticity of demand with respect to price and accommodates substitution between metals based on material properties and cost-effectiveness. This approach seems relevant for metals in manufacturing, where material choice often depends on both properties and price.

Moreover, the CES function aligns well with scenarios of rapid price changes or technological advancements, where short-term demand adjustments are essential. Through this model, we try to address some gaps in the literature by focusing on demand responsiveness in shorter time frames, where elasticity remains a critical factor. This, in our view, complements the "intensity of use" models.

In scenarios where demand for this metal is expected to grow such as those involving renewable energy technologies or infrastructure expansion, the CES form allows us to see how price-sensitive this demand might be. Higher values of σ imply that demand could be moderated by substituting other materials if prices rise, whereas lower values suggest that price increases would not significantly alter demand. For technology-dependent metals with high elasticity of substitution, such as cobalt, there is an important interaction between elasticity and demand growth. When a metal has high substitution elasticity (close to 1), it means that demand for the metal is highly responsive to price changes. As

²¹ Studies on cross-section data for 14 metals (aluminum, copper, chromium, gold, iron, lead, niobium, nickel, platinum, silver, tin, titanium, vanadium and zinc) in 1972 and 1977.

²² Study of panel data for 16 metals (tellurium, antimony, magnesium, lead, zinc, cast iron, aluminium, cadmium, cobalt, copper, gold, mercury, nickel, selenium, silver and tin) from 1961 to 1980.

²³ Study of panel data for 14 metals (aluminium, copper, chromium, gold, iron, lead, neobium, nickel, platinum, silver, tin, titanium, vanadium and zinc) from 1975 to 1983.

²⁴ Estimation of log-linear demand function applied to aluminium, copper, lead, nickel, tin, and zinc. Tilton, J.E. and Fanyu, P., 1999. Consumer preferences, technological change, and the short run income elasticity of demand. *Resources Policy*, 25, pp.87-109.

prices rise, consumers and industries can more easily substitute other materials or technologies, reducing their reliance on that metal. Due to this high elasticity, demand growth is moderated. Unlike metals with lower elasticity, where demand might grow steadily or exponentially even as prices rise, demand for highly elastic metals is likely to increase more slowly or even stabilize as prices increase. This is especially relevant for technology-dependent metals, where substitution possibilities are more abundant. High elasticity effectively makes sustained exponential demand growth relatively more unlikely: as prices rise, demand adjusts downward more readily, avoiding the kind of sustained high demand growth seen in less substitutable metals. This moderating effect on demand interacts directly with the supply side. In particular, since Herfindahl (1967) shows that extraction sequences naturally exhaust the lower-cost deposits first, rising prices gradually reflect the move toward higher-cost deposits, meaning that elastic demand helps contain the speed at which the system progresses through increasingly expensive segments of the resource base. In doing so, high elasticity contributes to preventing rapid escalation in both demand and resource depletion.

2. Supply from the Inverse CAC, Modeled as Logistic with Resource Depletion

The supply side of the model builds on the CAC, originally formalized by Tilton and co-authors. The CAC maps the cumulative quantity of metal that becomes economically recoverable to the marginal extraction cost required to bring that quantity into production. The CAC provides a convenient way to formalize the distinction between resources and reserves: as prices rise, cost thresholds increase and a larger portion of the underlying resource base becomes economically viable. Thus, the CAC is best interpreted as a long-run stock availability schedule²⁵.

In Tilton's formulation, the CAC gives the marginal extraction cost associated with producing a cumulative quantity (see Figure 2.1.a.). For our purposes, we invert this function: the *market price* becomes the argument of the cumulative availability function (x-axis), and the corresponding *recoverable quantity* becomes the output (y-axis)²⁶:

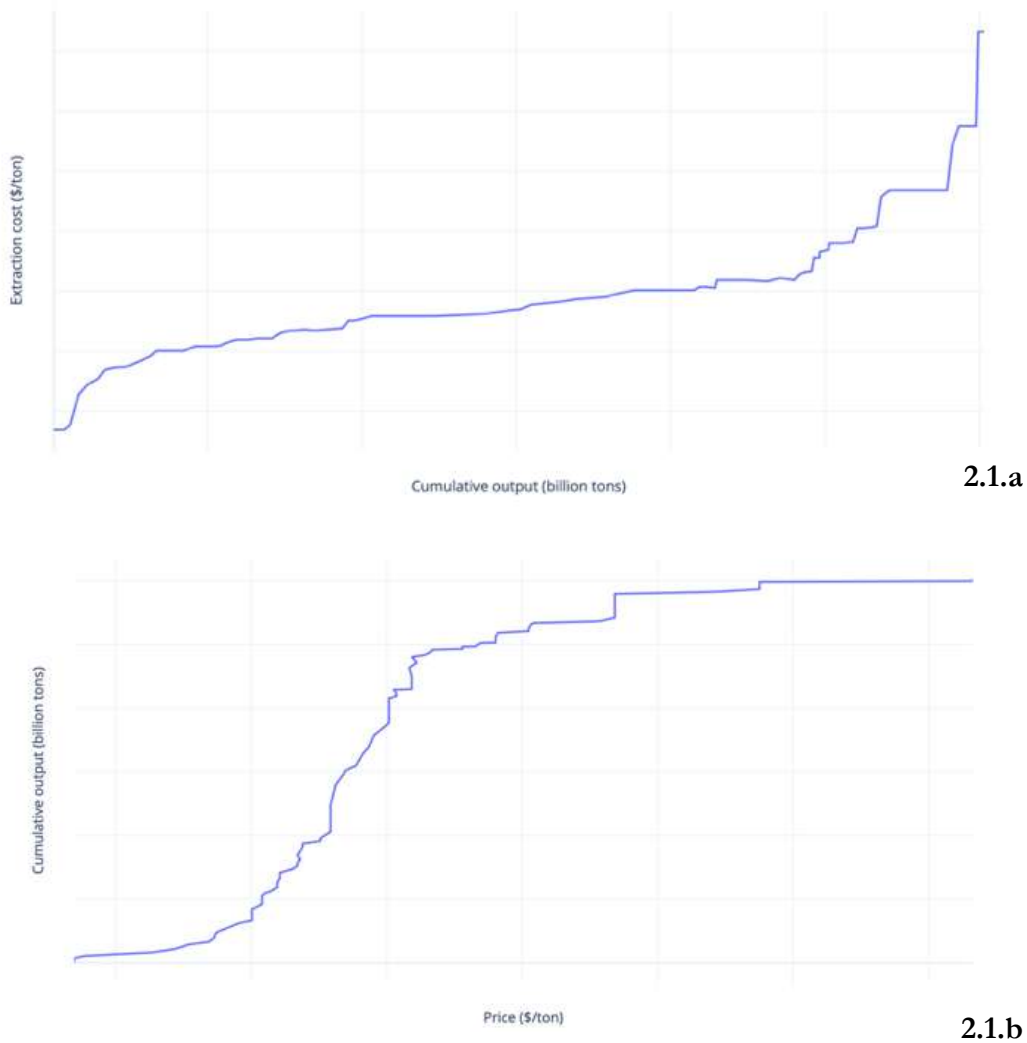
²⁵ A common critique of resource supply models is the assumption of "myopia," where producers are alleged to insufficiently consider the long-term implications of their decisions. In economic literature, "myopia" typically refers to a high pure rate of time preference or an undervaluation of future outcomes relative to present ones, as discussed by Bommier and Gollier (2001). Their work highlights how such discounting can lead to suboptimal long-term decisions, particularly in contexts like resource management and environmental policies. Our model does not explicitly model intertemporal optimization or strategic supplier behavior. Instead, it incorporates forward-looking dynamics at the aggregate market level, ensuring that it avoids the extreme form of myopia where future outcomes are entirely disregarded. This distinction aligns with Bommier and Gollier's (2001) perspective, where insufficient—but not absent—consideration of the future defines myopic behavior. The CAC framework mitigates this concern by embedding long-term economic and geological constraints into the resource availability function.

²⁶ Using the inverse of Tilton's function also has practical advantages for empirical modeling. While the original CAC requires fitting cost distributions that may involve non-standard or unstable statistical parameters, the inverted formulation

$$c_{max}(P) = CAC^{-1}(P)$$

Where $c_{max}(P)$ now represents the total stock of metal globally available²⁷ as a function of the price P . We assume, consistent with competitive-market reasoning, that extraction costs form the lower bound of prices, so that using price rather than cost does not alter the economic meaning of the curve.

Figure 2.1: The Cumulative Availability Curve (2.1.a) Relating Extraction Costs to Cumulative Output, and Its Inverse (2.1.b) Expressing Cumulative Output as a Function of Price



Source: Castillo and Eggert (2020)²⁸ and authors' computations

allows the use of a sigmoid function—such as a logistic curve—that is well-behaved, easily parameterized, and stable across a wide range of assumptions. This enhances both tractability and robustness when calibrating the model to real-world data.

²⁷ i.e. not fully recovered yet.

²⁸ Castillo, E. and Eggert, R., 2020. Reconciling diverging views on mineral depletion: a modified cumulative availability curve applied to copper resources. *Resources, Conservation and Recycling*, 161, p.104896.

Empirically, inverses CACs for metals such as copper (Castillo and Eggert, 2020) exhibit a sigmoid shape (see Figure 2.1.b above). We therefore model $c_{max}(P)$ using a sigmoid function of the form:

$$c_{max}(P) = \frac{L}{1 + e^{-\kappa(P-P_m)}}$$

Where:

- L is the asymptotic recoverable resource²⁹,
- κ controls the steepness of the sigmoid curve,
- P_m is the midpoint price where availability grows more rapidly.

The distinction between L and $c_{max}(P)$ is crucial: L represents the total recoverable quantity of the resource under current geological knowledge and technology, while $c_{max}(P)$ is the subset of this amount that becomes economically viable at a given price level P . The function thus translates geological potential into economically accessible supply, conditional on price. Additionally, $c_{max}(P)$ should not be interpreted as reserves: it represents a long-run, price-conditioned upper bound on what could eventually become reserves if exploration, development, and investment responded fully to sustained price levels. Actual reserves at any point in time are typically a smaller, operationally constrained subset of $c_{max}(P)$.

The use of a sigmoid function is justified by the observation that resource availability typically exhibits three phases, which encompass the same phenomena as Tilton's CAC that we previously described in the literature review:

- During the initial stages of extraction, prices are low, and recoverable quantities are similarly low. This stage (left part of the graph) reflects a market that is not yet mature, characterized by minimal extraction activities and the tapping of only the most accessible resources. In this situation, changes in price exert little influence on the quantities that can be recovered because the market operates under conditions of abundant accessible resources and low marginal costs. Since the initial stages of extraction involve resources that are geographically concentrated, easily reachable, and technologically simple to exploit, price fluctuations do not yet provide a

²⁹ In some studies—though not in our analysis— L can also incorporate an estimated fraction of extrapolated unknown deposits, accounting for potential future discoveries or resources that may become accessible with technological advancements. This extended definition provides an upper boundary for the total recoverable resource base, but for our purposes, L is limited to known reserves and resources.

strong incentive to expand recovery efforts or invest in more advanced extraction technologies. Consequently, the cumulative availability remains relatively inelastic to price movements during this phase.

- As prices increase, the curve transitions into the middle portion of the graph, which displays significant growth in recoverable quantities. This steep ascent indicates a more mature market; wherein rising prices incentivize investments in extraction technologies and the development of previously uneconomic resources. This phase represents an extraction stage, marked by heightened extraction activities and the economic viability of a larger pool of resources.
- Eventually, as prices continue to rise, the curve levels off in the right section of the graph. This flattening signifies a market where most easily recoverable resources have been exploited. At this stage, further price increases yield diminishing returns in terms of additional recoverable quantities, reflecting the challenges and costs associated with accessing more difficult or lower-quality deposits.

In this context, depletion refers to the progressive reduction in accessible, economically viable resources as cumulative extraction increases over time. Depletion occurs when the most readily accessible and cost-effective deposits are exhausted, pushing extraction efforts toward resources that are more challenging and expensive to access. This process is reflected in the right portion of the sigmoid curve, where the incremental gains in recoverable quantities diminish despite rising prices. Depletion does not imply that resources are entirely exhausted but rather that remaining quantities are increasingly costly to extract, requiring advanced technology, higher prices, or significant investment to make continued extraction viable. This dynamic is a central component of long-term resource availability of metals and plays a critical role in shaping the price and supply landscape in mature resource markets.

The sigmoid curve used here that we derive from inverting Tilton's CAC finds strong echoes in existing literature on resource extraction and depletion. Such S-shaped curves, which are initially convex and later concave, effectively capture the progression from abundant, easily accessible resources to more challenging and costly extraction phases. The literature provides various interpretations of this phenomenon. Krautkraemer (1998) discusses the impact of rising extraction costs over time, leading to a leveling-off in production rates and reinforcing the sigmoid shape's applicability. Stuermer (2018) also notes that resource constraints over time lead to a slowing in extraction growth, consistent with an S-shaped path where early abundance transitions to scarcity effects. Lastly, Loury (1978) highlights this S-curve dynamic in his optimal extraction models, where rising marginal extraction costs cause a convex-to-concave shift as resources are depleted.

These references, which were already reviewed in the first chapter, demonstrate the relevance of a sigmoid curve for modeling resource extraction paths, supporting the approach taken in this study.

The c_{max} function represents a cumulative quantity, not a yearly flow. To translate it into annual supply $Q_{s,t}$, we introduce a logistic depletion function, which reduces yearly supply as cumulative extraction approaches the cumulative availability limit c_{max} . Accordingly, the annual supply function is expressed as:

$$Q_{s,t} = \frac{c_{max}(P_{t-1})}{1 + e^{\gamma\left(\frac{c_t}{c_{max}(P_{t-1})} - \theta\right)}}$$

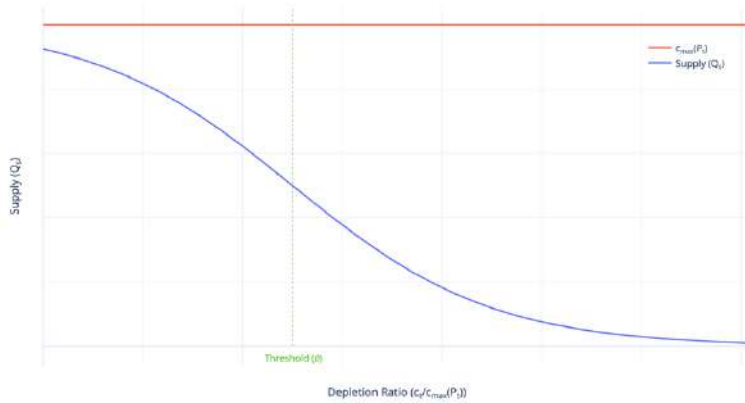
Where:

- $Q_{s,t}$ is the annual quantity of metal supplied in year t ,
- P_{t-1} is the market price of the metal in year $t - 1$ ³⁰,
- c_t represents the cumulative extraction of the metal up to year t , i.e., $c_t = \sum_{\tau=0}^{t-1} Q_{s,\tau}$,
- γ is a parameter controlling the steepness of supply decline, it governs how sensitively the annual supply $Q_{s,t}$ responds to increases in cumulative extraction c_t , representing scarcity effects,
- θ represents the inflection point at which marginal annual supply begins to decrease significantly as cumulative extraction c_t approaches the cumulative availability limit $c_{max}(P_t)$. At this point, the remaining recoverable resources become progressively more difficult and costly to extract, leading to a sharper decline in the annual supply curve.

The impact on this supply curve of different values of θ and γ is displayed in the Appendix.

³⁰ We use the lagged price P_{t-1} rather than the contemporaneous price P_t in the supply function to reflect the delayed nature of production responses in metal markets. Metal extraction involves significant operational inertia: firms adjust output based on past price signals due to the time required for investment decisions, permitting, labor allocation, and physical production. This lag captures the empirical reality that mining supply reacts to recent market conditions rather than instantaneously. Moreover, using P_{t-1} avoids simultaneity bias in the model by ensuring that supply is predetermined when the market-clearing condition $Q_{d,t}(P_t) = Q_{s,t}(P_{t-1})$ is solved endogenously for P_t , allowing for an adequate equilibrium determination. This choice also aligns with industry practice, where production adjustments are often based on price expectations formed using recent trends. Finally, since the cumulative availability function $c_{max}(P)$ reflects the idea that more resources become recoverable as prices increase, linking it to P_{t-1} ensures that long-term resource expectations evolve with the historical price environment rather than being overly reactive to short-term fluctuations.

Figure 2.2: Logistic Supply Curve Based on $c_{max}(P_t)$



Reading note: As cumulative extraction c_t grows closer to c_{max} (the 'Depletion Ratio' increases), the annual supply $Q_{s,t}$ decreases, capturing the dynamics of resource scarcity over time

A key motivation for adopting a logistic form for $Q_{s,t}$ is the distinction between resources, long-run availability, and actual reserves. Whereas L represents the geological endowment (resources) and c_{max} the economically accessible share of that endowment at a given price, reserves are the short-run operational stock that producers can mobilize with existing mines and already-developed projects. As cumulative extraction increases, the gap between reserves and c_{max} narrows: easily accessible reserves are progressively depleted, and the remaining portion requires increasingly costly, lower-grade, or technically demanding deposits to be converted into new reserves. This *reserve scarcity* means that annual supply cannot scale linearly with cumulative availability, even when prices remain constant. Instead, short-run supply becomes gradually more constrained as the stock of undeveloped low-cost reserves diminishes. A logistic depletion function provides a parsimonious way to capture this tightening: supply is relatively unconstrained when reserve scarcity is low but declines sharply as cumulative extraction approaches the upper bound c_{max} .

Other options, like a linear or proportional decline, would not capture the non-linear impact of scarcity as cumulative extraction nears the resource limit. Exponential functions, while allowing for rapid decay, can lead to overly steep declines that may not reflect the actual dynamics of resource extraction. In his review on nonrenewable resource scarcity, Krautkraemer (1998) examines the limitations of linear and constant-rate models for modeling resource extraction. He argues that these simpler approaches fail to capture the progressively increasing costs and challenges associated with resource depletion. Linear models, for instance, imply a steady, predictable decline in extraction, which doesn't align with the real-world scenario where resources become harder to access, requiring greater economic and physical efforts over time. Krautkraemer suggests that models incorporating non-linear dynamics—such as logistic forms—are better suited to representing the reality of resource scarcity. Pindyck (1978) also highlights the drawbacks of linear and exponential models in his study on the optimal extraction of

finite resources. He points out that exponential models, while capturing rapid decay, often overestimate the speed of decline in resource availability, resulting in projections that don't accurately reflect observed extraction patterns. Realistically, scarcity effects tend to develop more gradually, with the early stages of resource extraction benefiting from relative abundance. Pindyck thus supports the use of models like the logistic function, which allow for a more nuanced, gradual transition from resource availability to scarcity. In their foundational work on resource depletion, Dasgupta and Heal (1974) explore the economic principles underpinning extraction paths and resource scarcity. They argue that linear or proportional declines are insufficient for capturing the intensifying effects of depletion on extraction costs. As resources become scarcer, extraction becomes increasingly costly, an effect linear models often fail to reflect. Dasgupta and Heal advocate for extraction paths that reflect this non-linear relationship, which logistic functions can effectively represent, as they account for the early ease of extraction followed by steeply rising challenges as resources dwindle.

Studies that leverage a sigmoid-shaped cumulative availability function also use a logistic supply function. Slade (1982), Loury (1978), and Stuermer (2018) support the use of this form for representing long-term scarcity in metals. These works collectively validate the logistic approach as an effective model for depicting the realistic, non-linear decline associated with resource depletion.

To summarize, the logistic function provides a framework which encompasses a gradual decline with an accelerated reduction near depletion limits, suitable for capturing scarcity impacts over time. The logistic function accounts for supply constraints by progressively slowing supply growth as cumulative extraction approaches the cumulative availability limit. This setup reflects the reality that, over time, supply becomes less elastic due to factors such as resource depletion, higher extraction costs, and the physical limitations of mobilizing new production. Thus, while the cumulative availability level c_{max} , as a function of the price, assumes an initially flexible supply, the logistic function introduces a gradual tightening, mirroring real-world conditions where significant capacity expansion encounters increasing constraints as resource availability declines.

This supply function $Q_{s,t}$, when paired with the demand function, sets the basis for annual equilibrium conditions in the model.

II. Dynamics of Price, Demand, and Depletion: Deriving Annual Equilibria and Long-Run Outcomes

In this subsection, we derive the annual equilibrium price as determined by the interaction between the demand and supply functions previously described. By setting these functions in balance, we identify the conditions under which the market reaches a sustainable state, where prices and quantities

follow non-explosive, non-zero (NENO) dynamics, providing insights into the factors that drive sustainability in metal markets.

Some mechanisms in the model are structural assumptions (e.g., supply adjustment occurs with a one-period lag; the CES demand block allows demand to respond to price). Others—such as whether price increases effectively generate substantial demand reductions—are quantitative outcomes that depend on parameters and scarcity dynamics. We highlight this distinction to avoid interpreting built-in mechanisms as results of the model.

1. A Dynamic Equilibrium Framework: Price Adjustments under Demand Growth and Depletion Constraints

The equilibrium captures how market price adjusts each year to balance the quantity demanded and supplied, given the constraints of cumulative resource availability and the sensitivity of demand to price. The equilibrium condition in any given year t is defined by the equality between the demand and supply functions:

$$Q_{d,t} = Q_{s,t}$$

Where:

- $Q_{d,t}$ is the quantity of metal demanded in year t , as defined by the CES demand function,
- $Q_{s,t}$ is the quantity of metal supplied in year t , derived from cumulative availability limit and influenced by cumulative extraction.

On the supply side, instead of solving for P_t directly, we assume that the cumulative availability c_{max} at time t depends on the price level from the previous period, P_{t-1} , which reflects how past market conditions influence current resource availability. In real-world metal markets, resource availability is not only determined by instantaneous price levels but also by the cumulative effects of past market conditions. Price fluctuations influence long-term investment decisions, technological progress in extraction methods, and the pace at which resources are developed or abandoned.

By linking cumulative availability $c_{max}(P_t)$ to the previous period's price P_{t-1} , we introduce a dynamic lagged effect that captures how past prices influence current supply potential. This approach captures several key economic mechanisms. First, investment in resource development involves significant lead times, meaning that current availability is influenced by past price signals rather than immediate fluctuations. Second, as extraction progresses, remaining resources become more difficult to access,

and the ability to expand supply depends on whether past high prices spurred sufficient investment in new deposits. Third, technological progress driven by sustained periods of high prices plays a crucial role in expanding effective resource availability over time, but such innovations take time to materialize, further justifying the use of lagged price effects. Finally, producers' extraction decisions are often based on expectations formed by recent price trends, making it more realistic to model cumulative availability as a function of past rather than current prices. By introducing a dynamic adjustment mechanism, this formulation avoids simultaneity issues and provides a more accurate representation of how resource markets evolve over time.

We therefore obtain:

$$B(t).P_t^{\sigma-1} = \frac{c_{max}(P_{t-1})}{1 + e^{\gamma\left(\frac{c_t}{c_{max}(P_{t-1})} - \theta\right)}}$$

Where:

- $B(t).P_t^{\sigma-1}$ represents the CES demand function, with $B(t)$ as a time-varying scaling parameter and σ the elasticity of substitution,
- $\frac{c_{max}(P_{t-1})}{1 + e^{\gamma\left(\frac{c_t}{c_{max}(P_{t-1})} - \theta\right)}}$ represents the annual supply evaluated based on the price from the previous period, with the logistic depletion function reflecting scarcity effects as cumulative extraction c_t approaches the cumulative available limit c_{max} .

Thus, the equilibrium price P_t each year is then determined dynamically by solving the equation:

$$P_t = \left(\frac{c_{max}(P_{t-1})}{B(t). \left(1 + e^{\gamma\left(\frac{c_t}{c_{max}(P_{t-1})} - \theta\right)} \right)} \right)^{\frac{1}{\sigma-1}}$$

This expression shows that the equilibrium price P_t depends on:

- The total recoverable amount $c_{max}(P_{t-1})$ as a function of price from the previous period,
- The cumulative extraction c_t relative to the total recoverable amount $c_{max}(P_{t-1})$, which drives scarcity,
- The elasticity of substitution σ , which controls how responsive demand is to price changes.

This equilibrium framework highlights how the model responds to resource constraints over time. As cumulative extraction increases, the available annual supply declines, requiring prices to adjust upward to balance demand. Conversely, if technological advances or other factors increase the total recoverable amount, prices could stabilize or decline, reflecting expanded supply possibilities.

This dynamic equilibrium condition forms the basis for exploring yearly interactions between demand and supply, establishing a path-dependent trajectory for prices and quantities as resource scarcity impacts availability over time.

2. Stability Conditions for Non-Explosive, Non-Zero Equilibria

In each year, the equilibrium price and quantity for the metal reflect not only the immediate balance of demand and supply but also the cumulative effects of past extraction and price adjustments. Here, we focus on the iterative adjustments that characterize this demand-supply interaction over time.

- 1) The CES demand function's price elasticity σ introduces flexibility in demand, allowing it to adjust downward as prices increase. However, when core demand $B(t)$ is high—such as during periods of rapid economic growth or significant shifts toward metal-intensive technologies like renewable energy infrastructures and electric vehicles—this elasticity may not sufficiently offset price increases, particularly if viable substitutes are unavailable. In such cases, demand pressures on prices can remain strong, potentially leading to sustained price increases over time.³¹
- 2) Each year's extraction contributes to cumulative extraction c_t , which gradually reduces the remaining resource availability as defined by c_{max} . As c_t approaches the cumulative availability limit, the annual supply $Q_{s,t}$ becomes more constrained, raising pressure on the equilibrium price P_t , all else being equal, to adjust upwards in subsequent periods.
- 3) The equilibrium path is dynamic insofar as each year's equilibrium price and quantity set the foundation for the following year's dynamics. High prices in early years can reduce demand, therefore slowing cumulative extraction and easing future supply constraints. Conversely, if early demand is high and cumulative extraction increases rapidly, supply scarcity intensifies, leading to steeper price increases in later periods.

³¹ The exact analytical expressions of the derivatives involved here are computed further in the text.

In this context, even if prices increase, the market can still be considered as sustainable (in our “NENO” definition) for as long as resources are not depleted faster than their long-term recoverability allows, and prices remain within levels that do not lead to severe economic disruptions³².

Indeed, reaching a NENO equilibrium path requires certain conditions:

- 1) A sufficiently high elasticity in the CES demand function (i.e., a larger σ) enables demand to adjust in response to price increases. When demand is highly elastic, rising prices lead to a quick reduction in demand, which helps ease pressure on supply and supports the process of market convergence. In contrast, if demand is perfectly inelastic, market convergence would be hindered because demand would remain unchanged regardless of price increases. Prices could rise indefinitely if supply becomes constrained, leading to potentially unstable price dynamics rather than convergence. This inflexibility prevents the usual demand-side adjustments that help balance supply and demand, ultimately affecting the market’s ability to reach a stable relationship. Thus, elasticity acts as a stabilizing factor, allowing the market to gradually move toward a balanced trajectory between supply and demand.
- 2) When demand grows too rapidly, annual extraction rises quickly and cumulative extraction accelerates. This pushes the system closer to the upper bound $c_{\max}(P)$, tightening supply and exerting upward pressure on prices. Higher prices, in turn, trigger stronger demand responses in the CES block—reducing consumption in the long run but potentially amplifying short-run volatility. The interaction between these two effects creates a feedback loop: fast demand growth accelerates depletion, depletion raises prices, and rising prices induce sharp demand adjustments. In this setting, *convergence* refers to the ability of the market to settle on a NENO long-run trajectory for both prices and quantities, rather than entering explosive paths where depletion dominates the adjustment process. For convergence to occur, demand growth must be sufficiently moderate so that cumulative extraction does not approach the availability boundary too quickly; otherwise, the steep scarcity effects embedded in the supply function push prices upward at a rate incompatible with a stable equilibrium.
- 3) If technological advancements increase the recoverable amount within the CAC or lower extraction costs, this enables additional supply at lower prices, alleviating pressure on price increases. While not a necessary condition for market convergence, such progress can ease the

³² Since precise thresholds for what constitutes “too rapid” depletion or “unmanageable” price levels can vary by context, we will further explore these criteria in the following sections to establish a clearer understanding of how sustainability can be facilitated over the long run.

process by supporting the sustainability of the demand-supply interaction, as it helps offset cumulative extraction effects and slows down resource depletion.

- 4) When cumulative extraction c_t reaches a significant fraction of the cumulative availability limit c_{max} , the model naturally approaches a limit where further extraction becomes economically and physically unfeasible. This built-in constraint prevents indefinite growth in extraction, leading prices to stabilize as supply diminishes. Consequently, the market gradually converges to a balanced state where demand and supply reach a sustainable interaction.

These four factors of convergence will now be addressed in further detail to demonstrate how they provide boundaries within which the system's dynamics can stabilize, helping prices and quantities settle on sustainable trajectories over time.

a. Demand Elasticity as a Stabilizing Force

To analyze the impact of demand elasticity on sustainability in the market equilibrium, we examine the derivative of the demand function with respect to price P_t with different values of the elasticity of substitution σ . This elasticity determines how sensitive demand is to changes in price, with higher values indicating greater flexibility for demand reduction as prices rise. Given the CES demand function $Q_{d,t} = B(t) \cdot P_t^{\sigma-1}$, the elasticity of demand with respect to price $\epsilon_{Q,P}$ is:

$$\epsilon_{Q,P} = \frac{\partial Q_{d,t}}{\partial P_t} \times \frac{P_t}{Q_{d,t}} = \sigma - 1$$

When σ is less than 1 (relatively inelastic demand), the derivative of demand with respect to price is positive. This implies that demand does not decrease substantially as prices rise. This inelastic response can contribute to system unsustainability, as rising prices do not reduce demand sufficiently to relieve supply pressures, leading to sustained upward price trends in the face of resource scarcity.

When σ is greater than 1 (relatively elastic demand), the derivative of demand with respect to price becomes negative and increasingly steep as σ rises. This elasticity allows demand to decrease significantly in response to price increases, providing a somewhat stabilizing effect on the system. As prices rise, the reduction in demand alleviates pressure on supply, slowing cumulative extraction and mitigating scarcity-driven price spikes.

Thus, sustainability of the market equilibrium is enhanced when $\sigma > 1$, allowing price increases to moderate demand and reduce scarcity pressures (and reciprocally). This highlights the importance of

maintaining a sufficiently high elasticity in the demand function to ensure sustainability within the model's iterative equilibrium framework.

b. Initial Demand Growth Drives the Speed of Depletion

Given the supply function based on the cumulative availability limit, rapid demand growth implies that cumulative extraction may approach c_{max} too quickly for prices and supply to adjust smoothly. To illustrate this mechanism, it is useful to examine how cumulative extraction responds mechanically to the initial demand level and its growth rate.

As a diagnostic device—and abstracting from short-run supply rigidities—we can approximate the cumulative extraction path by the cumulative demand path. Let demand grow at a rate g , so that $Q_{d,t} = Q_{d,0}(1 + g)^t$.

Cumulative extraction then follows: $c_t = \sum_{\tau=0}^{t-1} Q_{d,\tau} = \sum_{\tau=0}^{t-1} Q_{d,0}(1 + g)^\tau = Q_{d,0} \frac{(1+g)^t - 1}{g}$.

This expression makes explicit how strongly cumulative extraction depends on three parameters: the initial demand level $Q_{d,0}$, the growth rate g , and the time horizon t . A higher initial demand raises cumulative extraction from the outset, while a higher growth rate g accelerates extraction exponentially through compounding effects. This diagnostic highlights why high demand growth can push the system rapidly toward the cumulative availability frontier, triggering the scarcity-driven price increases discussed earlier.

c. Supply-Side Technological Progress as an Expanding Resource Frontier

Analyzing the impact of technological progress on market sustainability involves assessing how changes in cumulative availability limit affect $Q_{s,t}$, and equilibrium price P_t . We extend the model by introducing an exogenous term $\Delta(t)$, which increases the cumulative availability limit c_{max} over time. While the CAC already embeds a baseline level of technological adaptation through its relationship to price, $\Delta(t)$ captures external advances—such as improved mining technologies, better geological mapping, or new resource discoveries—that expand the recoverable resource base independently of market price.

We consider three stylized forms of $\Delta(t)$, each reflecting a different pattern of progress:

- **Linear Growth ($\Delta(t) = \delta t$).** Linear growth represents a steady annual increase in available resources. While this reduces scarcity pressures and tempers the pace of price increases, it is generally insufficient to fully offset exponential demand growth. As shown in the Appendix,

linear growth can only stabilize prices if the growth rate δ is sufficiently large relative to the initial demand $Q_{d,0}$ and its growth rate g . In practice, unless δ is unrealistically high, demand eventually outpaces the linear increase in supply, leading to resumed upward pressure on prices.

- **Exponential Growth ($\Delta(t) = \delta \times e^{\beta t}$).** Exponential growth is a more optimistic scenario, reflecting accelerating innovation in exploration and extraction. In this case, supply expands at an increasing rate over time. We find that if the acceleration rate β exceeds the demand growth rate g , then cumulative supply growth eventually outpaces demand, stabilizing or even lowering equilibrium prices over the long term. The critical threshold for stability is $\beta > \ln(1 + g)$, ensuring that resource availability expands faster than cumulative demand. Under these conditions, the model produces bell-shaped price trajectories, with a peak followed by a decline—a dynamic particularly relevant in high-demand scenarios.
- **Discrete Jump ($\Delta(t) = \Delta \times \mathbb{1}_{\{t \geq T\}}$).** Finally, we model technological breakthroughs or major discoveries as a discrete step increase in the availability limit at a given time T . These "jumps" provide immediate relief from scarcity, lowering prices temporarily and delaying the approach to the depletion limit. However, in the absence of continued progress, their effect is transient: price trajectories resume their upward path as cumulative extraction catches up. Thus, jumps are useful for buying time, but sustained exponential progress is needed to ensure long-term sustainability.

These results suggest that only exponential forms of technological progress are capable of structurally reversing scarcity-driven price trajectories in high-demand scenarios. Linear growth and discrete jumps can delay depletion or smooth price paths temporarily, but they do not fundamentally alter long-run dynamics unless paired with other mitigating mechanisms such as recycling or demand management.

The analytical derivations for each functional form and corresponding sustainability conditions are presented in the Appendix (section III).

d. Depletion, Reserve Scarcity, and Resource Scarcity as Joint Constraints

When cumulative extraction c_t reaches a significant fraction of the cumulative availability limit c_{max} before the latter has had time to adjust, the model approaches a zone where further extraction becomes increasingly difficult and costly. Prices then rise significantly due to scarcity effects, leading to demand

destruction³³, and a natural slowdown in extraction. This mechanism prevents the system from following an indefinitely growing path and instead pushes it toward a NENO-type interaction between supply and demand.

As c_t rises and approaches c_{max} , the logistic term forces $Q_{s,t}$ downward. This is the first scarcity channel, capturing *reserve scarcity*, measured by the depletion ratio $c_t/c_{max}(P)$: the exhaustion of economically available reserves³⁴.

A second scarcity channel becomes relevant as the price-responsive cumulative availability c_{max} itself approaches the geological limit L . The ratio $c_{max}(P)/L$ captures resource scarcity, i.e., how close economic availability is to the known physical endowment. When this ratio is high, even large price increases can no longer unlock significant additional reserves.

When both channels are simultaneously tight, the system becomes extremely constrained: annual supply shrinks, the price must rise steeply to equilibrate demand, and the economy is pushed onto a high-price, low-volume, but non-explosive long-run path.

3. Nonlinear Phases of Adjustment: Interactions Between Price, Demand, and Resource Scarcity

For the equilibrium price P_t to reflect resource depletion, three conditions must be satisfied. First, as cumulative extraction c_t approaches the cumulative availability $c_{max}(P)$, annual supply, $Q_{s,t}$ declines. Second, the demand function must exhibit price responsiveness, i.e., the elasticity of substitution σ must differ from 1 to ensure that prices adjust in response to supply constraints. Third, market information must be transparent, so that expectations about future scarcity are reflected in current prices.

³³ Demand destruction refers to the reduction in consumption resulting from sustained high prices, particularly when prices increase to levels that make certain uses economically unviable. While metals typically exhibit relatively inelastic demand, with long-run price elasticities estimated at -0.7 for aluminum, -0.4 for copper, and below or equal to -0.2 for tin and zinc (Federal Reserve Bank of Dallas, 2014; Stuermer, M., 2017. Industrialization and the demand for mineral commodities. *Journal of International Money and Finance*, 76, pp.16-27.), prolonged price increases can still lead to significant changes in consumption patterns, including efficiency improvements or the substitution of alternative materials. This phenomenon was notably observed during the 2000s commodities boom, when sharp and sustained price rises led to adjustments in demand for several key metals (Helbling, T. (2008). Commodities Boom: Riding a Wave. *Finance & Development*, 45(1))

³⁴ Indeed, when $c_t \ll c_{max}(P)$, the term $\frac{c_t}{c_{max}(P)}$ is small, so $Q_{s,t} \approx c_{max}(P)$, and the resource constraint does not heavily impact supply. But as c_t approaches $c_{max}(P_t)$, the exponential term $e^{\gamma(\frac{c_t}{c_{max}(P)} - \theta)}$ grows large, pushing $Q_{s,t}$ toward zero, reflecting severe supply constraints as the resource becomes nearly exhausted.

We focus here on the first condition³⁵, which requires a precise understanding of how cumulative extraction evolves as a function of price. This relationship is captured through the cumulative availability function $c_{max}(P)$, which plays a central role in modeling the depletion process and shaping long-run supply trajectories.

As previously discussed in section I.2., we model the cumulative availability as a sigmoid function:

$$c_{max}(P) = \frac{L}{1 + e^{-\kappa(P-P_m)}}$$

The sigmoidal relationship reflects the non-linear nature of resource accessibility in response to price changes. Because $c_{max}(P)$ grows at different rates depending on the price level, the depletion ratio $c_t/c_{max}(P)$ also evolves differently depending on the stage of resource availability.

Figure 2.3: Comparison of Cumulative Availability and Cumulative Demand Growth Rates



Early stage (left): Cumulative demand grows more rapidly than cumulative availability, reflecting initial scarcity pressures. Middle stage: Cumulative availability expands quickly, temporarily outpacing demand growth, which moderates the ratio of demand to availability. Later stage (right): As the cumulative availability function flattens, cumulative demand continues to increase, leading to a renewed acceleration in the ratio of demand to availability.

Stage 1: $c_{max}(P)$ is in the left (relatively flat part) of the sigmoid curve

³⁵ The second condition, which pertains to the role of demand elasticity in maintaining a NENO equilibrium, has already been examined in subsection I.2.a.i.

The third condition, which concerns the availability and propagation of market information and its role in stabilizing market dynamics, is not modeled explicitly in our long-term framework. However, it remains an important consideration for the overall consistency of the equilibrium logic. As such, Chapter 5 provides a complementary empirical perspective on this theoretical consideration by exploring short-term market price behavior and its relationship with economic fundamentals, offering preliminary insights into how price signals may or may not reflect real-time shifts in supply and demand.

In the left flat region, namely in early stages of extraction, P is relatively low, so $c_{max}(P)$ remains close to its initial value $c_{max}(P) \approx \frac{L}{1+e^{\kappa P_m}}$. As previously stated, this region represents a low sensitivity of availability to price changes, where only low-cost resources are recoverable. Small increases in P lead to minimal changes in $c_{max}(P)$ because the curve of the cumulative availability function is relatively flat here. Since $c_{max}(P)$ does not expand significantly, cumulative extraction c_t approaches the availability limit faster in this region. As prices rise due to scarcity, demand reduction alone does not sufficiently slow down c_t 's approach to $c_{max}(P)$. Mathematically, $\frac{dc_{max}(P)}{dP} \approx 0$ for $P < P_m - \frac{1}{\kappa}$. Therefore, cumulative extraction accelerates toward the availability limit because the availability curve remains essentially fixed, and depletion effects dominate without additional expansion in $c_{max}(P)$. Accordingly, the price P rises and leads to Stage 2.

Stage 2: $c_{max}(P)$ is in the middle (steep part) of the sigmoid curve

In the middle part of the sigmoid curve, P is near P_m , where $c_{max}(P)$ is highly responsive to changes in P . This region, with rising prices, drives the most significant expansion in cumulative availability. As prices increase due to scarcity, $c_{max}(P)$ grows rapidly in response, delaying the approach of c_t to the availability limit. The increase in $c_{max}(P)$ due to price hikes in this steep region may temporarily outpace cumulative extraction growth, thus widening the gap between $c_{max}(P)$ and c_t . This expansion provides a temporary buffer against depletion. Mathematically, in this middle region:

$$\frac{dc_{max}(P)}{dP} = \frac{L\kappa e^{-\kappa(P-P_m)}}{(1 + e^{-\kappa(P-P_m)})^2}$$

The derivative is maximized around $P = P_m$, indicating the steepest expansion rate. The middle steep part of the sigmoid curve is characterized by price stabilization as the expanding resource availability absorbs some of the scarcity pressure. The speed at which c_t approaches $c_{max}(P)$ slows down significantly. With cumulative availability expanding quickly, the balance of supply and demand shifts favorably, so the scarcity effect diminishes. In an equilibrium setting, the market can clear at lower prices because the supply constraint is less binding. Thus, in the middle steep part, price increases are moderated significantly, and in some cases, prices may even start to decrease if the rate of resource expansion outpaces demand pressures.

Nonetheless, as $c_{max}(P)$ expands rapidly, more specifically at a greater speed than c_t , more metal is readily available and therefore extracted. Eventually, as the metal becomes massively extracted, this draws the CAC limit closer to the right end of the sigmoid curve, that we now address. This leads to Stage 3.

Stage 3: $c_{max}(P)$ is in the right (flat part) of the sigmoid curve

In the right flat region, prices are relatively high, and most of the recoverable resources have already been extracted. $c_{max}(P)$ asymptotically approaches its maximum L , and further price increases have minimal impact on cumulative availability. Accordingly, cumulative extraction c_t again accelerates toward the availability limit. The scarcity-driven price increases lose their ability to expand resources effectively, leading to depletion effects reasserting dominance. Mathematically, $\frac{dc_{max}(P)}{dP} \approx 0$ for $P > P_m + \frac{1}{\kappa}$. Here, $c_{max}(P)$ levels off, and additional extraction leads to rapid depletion as the resource base is near saturation³⁶.

To sum up: a) in the left flat part: Limited responsiveness of $c_{max}(P)$ to price. Depletion effects dominate, leading to fast accumulation of c_t relative to $c_{max}(P)$ and steep price increases; b) in the middle steep part: High responsiveness of $c_{max}(P)$ to price increases. Resource expansion delays c_t 's approach to $c_{max}(P)$, stabilizing prices and slowing depletion; c) in the right flat part: Diminishing responsiveness of $c_{max}(P)$ to price. Depletion effects re-emerge strongly as cumulative extraction approaches a saturated availability limit, leading to rapid price escalation as scarcity pressures return³⁷.

4. Conditions for Long-Term Market Sustainability: Price Signals, Demand Elasticity, and Technological Progress

In this section, we have developed a detailed framework to understand the dynamics of metal markets by modeling the interaction between demand, supply, and cumulative resource availability. Our analysis

³⁶ When prices exceed $P_m + 1/\kappa$, the cumulative availability curve (CAC) enters the flat region of the sigmoid curve, where the majority of economically viable resources have already been accessed. At this point, further price increases yield only marginal gains in accessible resources. This reflects diminishing returns on price-driven resource expansion, with high-cost resources becoming dominant. Extractive industries face reduced profitability from additional extraction as higher prices no longer correspond with significant increases in available resources. Companies must focus on cost-efficiency, productivity improvements, and technological innovations to unlock remaining high-cost reserves. As the resource base is constrained, marginal projects or lower-quality reserves may become economically unviable. Market consolidation may occur, with larger firms benefiting from economies of scale, advanced extraction technologies, and greater capital access, while smaller producers struggle. This leads to higher barriers to entry for new competitors. Furthermore, the high prices incentivize innovation in extraction methods (e.g., deep-sea mining, bio-mining) and increased exploration, although these innovations have long lead times and high costs. Additionally, industries may shift focus towards resource recycling and alternative materials to reduce reliance on primary extraction, as high prices prompt interest in secondary resources and substitutes for critical inputs.

³⁷ In some cases, the supply side may be relatively price inelastic. This can mirror the impacts of various factors, such as geological constraints, technological limitations, regulatory hurdles, or capital constraints. Certain mining operations may take years to scale up due to permitting processes or the need for new infrastructure. If firms are unable to adjust supply quickly enough to match demand, it results in a lag in production response, keeping prices elevated longer than anticipated. In the long run, sustained high prices could lead to demand destruction, where consumers or industries seek alternatives or reduce consumption due to unaffordability, leading to a potential correction in market dynamics. Furthermore, if investments do eventually lead to increased supply, this may result in a sharp decline in prices.

highlights how price, cumulative extraction, and technological progress jointly shape the path-dependent equilibrium in resource markets over time.

A critical result of the analysis is the dependence of equilibrium dynamics on the responsiveness of demand to price changes, represented by the elasticity of substitution σ .

We also showed how technological progress influences market sustainability. We modeled technological advancements as an exogenous component $\Delta(t)$ added to the cumulative availability limit. Different forms of technological progress—linear growth, exponential growth, and discrete jumps—were analyzed, revealing distinct effects on price dynamics.

By defining cumulative availability as a sigmoid function of price, we captured the non-linear response of resource availability to price changes. This sigmoidal relationship introduces three distinct phases in price-resource dynamics.

Finally, we established conditions for market sustainability, defined by non-explosive, non-zero paths (NENO) for prices and volumes over the long-term. Prices need to reflect depletion effects accurately, demand elasticity needs to be sufficiently high to moderate extraction rates, and sufficient technological progress is required on the supply-side to enhance resource availability at a pace that keeps up with or exceeds demand growth. When these conditions are met, the market avoids explosive price dynamics and maintains a sustainable interaction between supply and demand over time.

Building on this theoretical framework, we turn next to a numerical application focused on copper—a metal with high industrial demand and relatively low elasticity of substitution.

III. Appendix: Derivations of Technological Progress in the Model

Technological progress in extractive industries can occur independently of price increases, driven by factors such as regulatory changes, environmental policies, and proactive R&D. For instance, innovations in mining techniques, like seismic testing improvements, have enhanced efficiency without being directly linked to price fluctuations (World Economic Forum, 2017). Similarly, the emergence of unconventional oil sources, such as shale oil, has been driven by technological advances rather than price incentives (IMF, 2016). In the copper sector, continuous technological advancements can mitigate the impacts of resource depletion, demonstrating that efficiency gains can occur without price-driven motivations (Mitra, 2019).

Let's suppose technological improvements effectively increase c_{max} over time by lowering extraction costs or accessing additional resources. Let's define $c_{max}(P_{t-1}, t) = CAC^{-1}(P_{t-1}) + \Delta(t)$ where $\Delta(t)$ represents the additional resources made available through technology.

- Linear Growth in $\Delta(t)$

First, we consider $\Delta(t) = \delta t$. This represents a steady, linear increase in resource availability each year, in absolute terms. Therefore, the annual supply function becomes:

$$Q_{s,t} = \frac{CAC^{-1}(P_{t-1}) + \delta t}{1 + e^{\gamma\left(\frac{c_t}{c_{max}(P_{t-1}, t)} - \theta\right)}}$$

Each year, as δt increases, the total recoverable amount expands linearly, thus gradually slowing the effect of cumulative extraction c_t on the denominator. This keeps $Q_{s,t}$ higher for longer, thereby reducing the scarcity-driven upward pressure on prices. Thus, linear growth alleviates scarcity effects, though it may not fully offset them if demand grows faster than the linear increase in availability.

Indeed, let D_t the cumulative demand up to year t grow at a rate g , so that:

$$D_t = \sum_{\tau=0}^{t-1} Q_{d,\tau} = Q_{d,0} \frac{(1+g)^t - 1}{g}$$

To determine if linear growth in $\Delta(t)$ sufficiently offsets cumulative demand, we need the cumulative availability $c_{max}(P_{t-1}, t)$ to grow at least as fast as D_t . This requires $D_t \leq c_{max}(P_{t-1}, t)$, or more explicitly $Q_{d,0} \frac{(1+g)^t - 1}{g} \leq CAC^{-1}(P_{t-1}) + \delta t$.

For high values of t , the exponential growth in demand outpaces the linear growth in the availability of supply if $g > 0$. Specifically, if the demand growth rate exceeds zero, linear growth in $\Delta(t)$ is generally insufficient to offset scarcity indefinitely, as demand grows faster than the recoverable amount.

Sustainability requires $\delta t \approx Q_{d,0} \frac{(1+g)^t - 1}{g} - CAC^{-1}(P_{t-1})$ which reveals that a non-zero g is not sustainable unless $\Delta(t)$ grows at least as quickly as demand, i.e. faster than linearly.

This analytical setup shows that linear growth in $\Delta(t)$ only temporarily delays scarcity effects if demand grows at a positive rate. For sustained stability, $\Delta(t)$ must grow faster than linearly, suggesting that exponential growth or significant discrete jumps are more effective for long-term scarcity mitigation.

- Exponential Growth in $\Delta(t)$

We now consider $\Delta(t) = \delta \times e^{\beta t}$. This exponential growth allows $\Delta(t)$ to increase more rapidly over time in absolute terms, for instance in line with accelerating improvements in technology or resource extraction efficiency. The annual supply function becomes:

$$Q_{s,t} = \frac{CAC^{-1}(P_{t-1}) + \delta \times e^{\beta t}}{1 + e^{\gamma \left(\frac{c_t}{c_{max}(P_{t-1}, t)} - \theta \right)}}$$

Exponential growth provides an increase in the total recoverable amount as $\delta e^{\beta t}$ rises. This expansion can significantly delay the effect of cumulative extraction c_t on annual supply, resulting in a stabilizing effect on prices. This form of growth can even fully offset demand growth, especially if β is high, allowing for a steady or even declining price environment as supply keeps pace with or surpasses demand pressures. Indeed, the stability condition for cumulative demand growth now becomes:

$$Q_{d,0} \frac{(1+g)^t - 1}{g} \leq CAC^{-1}(P_{t-1}) + \delta e^{\beta t}$$

For high values of t , the initial values $Q_{d,0}$ and $CAC^{-1}(P_{t-1})$ become less significant compared to the exponential growth terms. We can focus on the main growth terms of each expression. For cumulative demand, we have $D_t \approx Q_{d,0}(1+g)^t$, and for cumulative availability $c_{max}(P_{t-1}, t) \approx \delta e^{\beta t}$. To ensure that cumulative availability keeps up with or exceeds cumulative demand as t grows, we need: $Q_{d,0}(1+g)^t \leq \delta e^{\beta t}$. Hence: $\frac{Q_{d,0}(1+g)^t}{e^{\beta t}} \leq \delta$

For high values of t , the term $(1 + g)^t / e^{\beta t}$ approaches zero if $e^{\beta t}$ grows faster than $(1 + g)^t$. This means we need $(1 + g)^t < e^{\beta t}$, hence $\ln(1 + g) < \beta$.

Thus, for cumulative availability $c_{max}(P_{t-1}, t)$ to grow faster than cumulative demand D_t over time, the growth rate β of $\Delta(t)$ must satisfy $\beta > \ln(1 + g)$. This condition ensures that exponential growth in availability due to technological progress can keep pace with or exceed the exponential growth in cumulative demand.

To analyze the effect of exponential technological growth on *price*, we substitute $c_{max}(P_{t-1})$ by $c_{max}(P_{t-1}, t) = CAC^{-1}(P_{t-1}) + \delta e^{\beta t}$. The equilibrium price becomes:

$$P_t = \left(\frac{CAC^{-1}(P_{t-1}) + \delta e^{\beta t}}{B(t) \cdot \left(1 + e^{\gamma \left(\frac{c_t}{c_{max}(P_{t-1}, t)} \right)^{-\theta}} \right)} \right)^{\frac{1}{\sigma-1}}$$

Where $B(t)$ encompasses the level of baseline demand at time t . Over time, as t increases, the term $\delta e^{\beta t}$ grows exponentially. Consequently, $\delta e^{\beta t}$ becomes significantly large relative to $CAC^{-1}(P_{t-1})$. We can therefore approximate $CAC^{-1}(P_{t-1}) + \delta e^{\beta t} \approx \delta e^{\beta t}$. Moreover, with $\delta e^{\beta t}$ growing faster than cumulative extraction c_t , the term $\frac{c_t}{c_{max}(P_{t-1}, t)}$ decreases over time, reducing the impact of the exponential term in the denominator. As a result, $P_t \approx \left(\frac{\delta e^{\beta t}}{B(t)} \right)^{\frac{1}{\sigma-1}}$.

If $\beta > g > 0$, the expansion of $c_{max}(P, t)$ reduces scarcity pressure on P_t , stabilizing or even lowering prices over time, as $Q_{s,t}$ can meet demand without requiring significant price increases³⁸.

In this regard, exponential increases in $B(t)$ can keep pushing demand higher than the technologically enhanced supply, creating upward pressure on prices even with rapid supply-side growth.

As seen in Chapter 3, this framework applies well to metals like copper, which has significant structural demand drivers and relatively moderate elasticity of substitution.

³⁸ In order for prices P_t to decline over time, the exponent $\frac{\beta}{\sigma-1}$ must be negative. This implies that if $\sigma > 1$, then β must be negative, meaning that cumulative availability would need to shrink over time, which is not feasible for declining prices. Conversely, if $\sigma < 1$, prices can naturally decline if β is sufficiently large, as long as $\delta e^{\beta t}$ grows faster than B .

- **Non monotonic change in $\Delta(t)$ (jump)**

Lastly, we also consider $\Delta(t) = \Delta \times \mathbb{1}_{\{t \geq T\}}$. This represents a “jump” effect, introducing a discrete increase at time T . This models a sudden breakthrough that makes a significant additional quantity available instantly, such as one-off technological leap or discovery of new resources. With such a jump, the supply function becomes:

$$Q_{s,t} = \frac{CAC^{-1}(P_{t-1}) + \Delta \times \mathbb{1}_{\{t \geq T\}}}{1 + e^{\gamma \left(\frac{c_t}{c_{max}(P_{t-1}, t)} - \theta \right)}}$$

Before T , the supply function follows the standard form with no additional resources available. After T , the additional Δ quantity from the jump effectively increases the CAC, expanding annual supply. Namely:

$$Q_{s,t} = \begin{cases} \frac{CAC^{-1}(P_{t-1})}{1 + e^{\gamma \left(\frac{c_t}{CAC^{-1}(P_{t-1})} - \theta \right)}} & \text{for } t < T \\ \frac{CAC^{-1}(P_{t-1}) + \Delta}{1 + e^{\gamma \left(\frac{c_t}{CAC^{-1}(P_{t-1}) + \delta} - \theta \right)}} & \text{for } t \geq T \end{cases}$$

The sudden increase in availability at T can provide an immediate easing effect on scarcity, allowing $Q_{s,t}$ to increase substantially in the years following the jump. This stabilizes prices more abruptly, which can delay scarcity-driven price rises.

The equilibrium price P_t is set by balancing the demand $Q_{d,t}$ with the adjusted supply $Q_{s,t}$. From the equilibrium condition $B(t)P_t^{\sigma-1} = Q_{s,t}$, we get $P_t = \left(\frac{Q_{s,t}}{B(t)} \right)^{\frac{1}{\sigma-1}}$. Since $Q_{s,t}$ has increased due to the jump in availability δ , the equilibrium price P_t adjusts downward. However, this drop in price may encourage higher demand, depending on the elasticity σ , as the market takes advantage of the temporarily higher supply.

In the years immediately following T , the impact of the availability jump on equilibrium quantities and prices is strongest, as the market adjusts to the sudden influx of supply. Over time, cumulative extraction c_t continues to grow, and the scarcity pressure gradually returns, raising prices once again unless further supply expansions or technological advances occur. The price effect is stabilizing rather than permanently deflationary. While the jump temporarily reduces prices, sustained technological growth would be needed to maintain these lower prices in the long term.

CHAPTER 3: Copper Under Constraint: Long-Term Price and Supply Dynamics in the Face of Growing Demand

The critical role of copper in enabling the energy transition—through its applications in electrification, renewable energy technologies, and grid infrastructure—has made its long-term availability and price dynamics a subject of growing analytical and policy concern. The objective of this chapter is to apply the analytical model developed for metals in general in Chapter 2 to the empirical case of copper, with the aim of simulating the evolution of volumes and prices under contrasting transition scenarios as set by the International Energy Agency (IEA). The analysis also explicitly integrates core structural factors shaping copper markets in the 21st century: long-run demand trajectories, geological constraints, technological shifts, and the feedback between physical scarcity, price, and unmet needs.

This chapter develops a dynamic scenario-based approach in which copper prices emerge from the interaction of several endogenous market forces. The model incorporates *ex ante* demand projections aligned with global electrification and digitalization trends, drawing from scenario pathways by the IEA (2021, 2023) and Elshkaki et al. (2016). It also introduces the notion of unmet needs, defined as the share of demand that remains unfulfilled due to capacity constraints or price-induced rationing—an element not always addressed in the empirical literature. Resource depletion and supply-side bottlenecks are accounted for through the construction of updated Cumulative Availability Curves, using contemporary assessments of global copper resources (Castillo & Eggert, 2020; Mudd & Jowitt, 2018). Finally, the model captures how tightening physical constraints trigger non-linear price responses, consistent with the findings of Fizaine and Court (2016) and Northey et al. (2014) and reflecting a growing recognition of feedback loops between scarcity and market behavior.

In doing so, this chapter contributes to and extends a well-established empirical literature on copper markets. Historical analyses (Tilton, 1996; Radetzki, 2009; Crowson, 2011) emphasize the cyclical behavior of copper prices, shaped by long investment lags, substitution effects, and adaptive technological change. More recent quantitative studies (Stuermer, 2018; Northey et al., 2014) incorporate resource exhaustion and supply-side dynamics into long-run projections but often do so without modeling how unmet demand or feedback effects emerge from tightening availability. The present approach endogenizes demand satisfaction and its price implications within a coherent scenario framework.

The model adopts the revised resource inventories proposed by Castillo and Eggert (2020) and Mudd and Jowitt (2018) as a baseline for long-term CAC construction, anchoring the supply side in the most recent geological data. Demand paths are derived from major global energy and infrastructure outlooks

(IEA, 2021, 2023; Elshkaki et al., 2016; World Bank, 2020), allowing for a consistent mapping of how structural trends and policy choices affect pressure on copper markets over time.

The purpose of this chapter is to use the theoretical framework developed in Chapter 2 so as to internally consistent and thus plausible trajectories under a variety of possible futures regarding energy transition policies. By linking price formation to both the structural drivers of demand and the physical limits of supply, the analysis sheds light on how potential mismatches between needs and availability might translate into unsustainable price trajectories. Accordingly, our numerical results are explicitly simulations and not forecasts. While the simulations rely on robust data and credible foresights, their purpose is to explore potential dynamics rather than predict precise one precise future outcome.

I. Data Sources for Global Copper Supply and Demand

The application of our analytical model to copper relies on a detailed calibration process grounded in data from authoritative sources. On the demand side, we use projections from the International Energy Agency (IEA)’s report “The Role of Critical Minerals in Clean Energy Transitions”³⁹, which translates policy goals into metal needs under three distinct energy transition policy scenarios: the Stated Policies Scenario (STEPS), the Announced Pledges Scenario (APS), the Net Zero Emissions by 2050 Scenario (NZE). We consider a counterfactual scenario where renewable energy shares remain static, and economic growth in developing economies is unconstrained by energy transition requirements. These scenarios, which are described in greater detail below, reflect various trajectories for technological progress and economic development.

Our approach seeks to move beyond the concept of criticality—the focus of the IEA’s report, which assesses the availability of materials essential for strategic objectives—and instead aims to adopt a more long-term and aggregated perspective on copper demand. We examine how shifts in demand for copper, driven by renewable energy systems, electric vehicles, and broader economic growth, shape the dynamics of copper market and their long-term sustainability, while also analyzing how these dynamics can be influenced by the various parameters incorporated in our model.

Past price data from the London Metal Exchange (LME) is included for illustrative purposes, facilitating a comparison between the model's outputs and historical trends. However, it is important

³⁹ IEA (2021), The Role of Critical Minerals in Clean Energy Transitions, IEA, Paris <https://www.iea.org/reports/the-role-of-critical-minerals-in-clean-energy-transitions>, Licence: CC BY 4.0

to note that the primary focus of this analysis is not on reproducing past price trajectories but rather on simulating potential long-term dynamics based on supply and demand hypotheses.

II. Calibration of the Analytical Model to Global Copper Market Dynamics

1. Calibrating the CES Demand Function

We apply our CES demand function specifically calibrated to capture the dynamics of copper demand. The CES structure allows us to reconcile short-run price sensitivity with long-run scenario-driven shifts in underlying requirements linked to energy transition pathways and economic development.

a. Structural Drivers and Scenario-Specific Projections

To operationalize the CES demand function in a forward-looking setting, we redefine the time-varying demand shifter $B(t)$ so that it incorporates three components: a) structural demand needs, b) scenario-specific projections based on alternative energy policy pathways, and c) the accumulation of unmet needs arising whenever market constraints prevent part of the planned demand for materializing.

We follow the IEA's Critical Minerals Outlook (2025) and consider four distinct policy environments:

- Stated Policies Scenario (STEPS)⁴⁰: This scenario represents the policies and measures that governments have already implemented or announced as of today. It reflects a moderate level of commitment to the energy transition, with policy actions taken to reduce emissions but without necessarily reaching ambitious climate goals. In this scenario, metal demand grows due to increased infrastructure and renewable energy installations, but the pace of change remains relatively incremental.
- Announced Pledges Scenario (APS)⁴¹: Introduced in 2021 by the IEA, the APS reflects a pathway where all climate targets and pledges announced by governments as of August 2024 are fully achieved on time. It includes both 2030 targets and longer-term net zero ambitions, whether they are legally binding. The APS assumes a faster deployment of low-carbon technologies, electrification, and infrastructure development, driving strong metal demand. While some residual emissions may remain, offset by land use or forestry, the APS outlines the deep energy system changes required if all announced pledges are met in full.

⁴⁰ <https://www.iea.org/reports/global-energy-and-climate-model/stated-policies-scenario-steps>

⁴¹ <https://www.iea.org/reports/global-energy-and-climate-model/announced-pledges-scenario-aps>

- Net Zero Emissions by 2050 (NZE)⁴²: This *backcasting* scenario is the most ambitious, targeting net-zero CO₂ emissions by 2050. It incorporates policies designed to keep global temperature increases within 1.5°C, in line with recommendations from the Intergovernmental Panel on Climate Change (IPCC). NZE involves accelerated adoption of renewables, substantial infrastructure investments, and enhanced efficiency in material use, ultimately pushing metal demand to its highest levels due to intensive low-carbon technology deployment. Material efficiency measures are critical in this scenario, reducing the net metal content in goods and infrastructure where possible.
- Business as Usual (BAU): This scenario assumes a continuation of current trends without any significant increase in the share of renewables in the energy mix. Additionally, it assumes that the economic growth of developing economies remains unaffected by energy transition requirements⁴³. Metal demand in this scenario is driven primarily by economic growth rather than a shift toward renewables, as the transition to a low-carbon energy system remains limited. The focus here is on maintaining conventional energy infrastructure, leading to lower growth in renewable-related metal demand but higher reliance on traditional industrial growth.

b. Constructing the Projected Requirement Function $R_{i,t}$

For each scenario i and year t , we define a projected requirement function $R_{i,t}$ capturing the amount of copper needed to meet both clean energy deployment plans and conventional economic growth.

$$R_{i,t} = \alpha_{i,fossil,t} \beta_{i,t} X_t + \alpha_{wind,t} \delta_{i,wind} X_t + \alpha_{sun,t} \delta_{i,sun} X_t + \delta_{i,EV} \Gamma_t + \delta_{i,E} E_t + \delta'_{i,GDP,t} Y_t$$

Where:

- $\alpha_{i,j,t}$ represents the share of “clean” (according to the terminology of the IEA) energy sector j in the total energy mix. This parameter reflects the composition of energy sources within each scenario, showing how the shift away from fossil fuels to renewable resources impacts metal demand,
- $\beta_{i,t}$ is the copper intensity of the fossil energy sector, capturing the quantity of copper required per unit of fossil energy consumption,

⁴² <https://www.iea.org/reports/global-energy-and-climate-model/net-zero-emissions-by-2050-scenario-nze>

⁴³ <https://www.pwc.com/gx/en/world-2050/assets/pwc-the-world-in-2050-full-report-feb-2017.pdf>

- $\delta_{i,j}$ indicates the copper intensity of clean sectors j , detailing the demand for copper from renewable energy technologies deployment, electric vehicles adoption, and grid expansion. These parameters adjust according to technological advancements that affect sector-specific copper needs,
- $X_{i,t}$ is the final energy⁴⁴ consumption, representing the broader economic consumption patterns that drive total energy demand across all sectors,
- $\Gamma_{i,t}$ reflects the demand for electric vehicles, a significant component of metal demand in the context of decarbonization efforts,
- $E_{i,t}$ denotes the expansion of new electricity networks, indicating infrastructure growth to support renewable integration and electrification,
- $\delta'_{i,GDP,t}$ captures the content of copper in non-energy sectors such as manufacturing and consumer goods,
- Y_t captures the annual economic activity in non-energy sectors (e.g., manufacturing, consumer goods), ensuring we account for copper needs outside of energy-related demand.

We do not consider other renewable sectors such as hydrogen, hydropower, or biomass, as their copper requirements are largely negligible compared to the sectors included here, and in some cases, nonexistent⁴⁵.

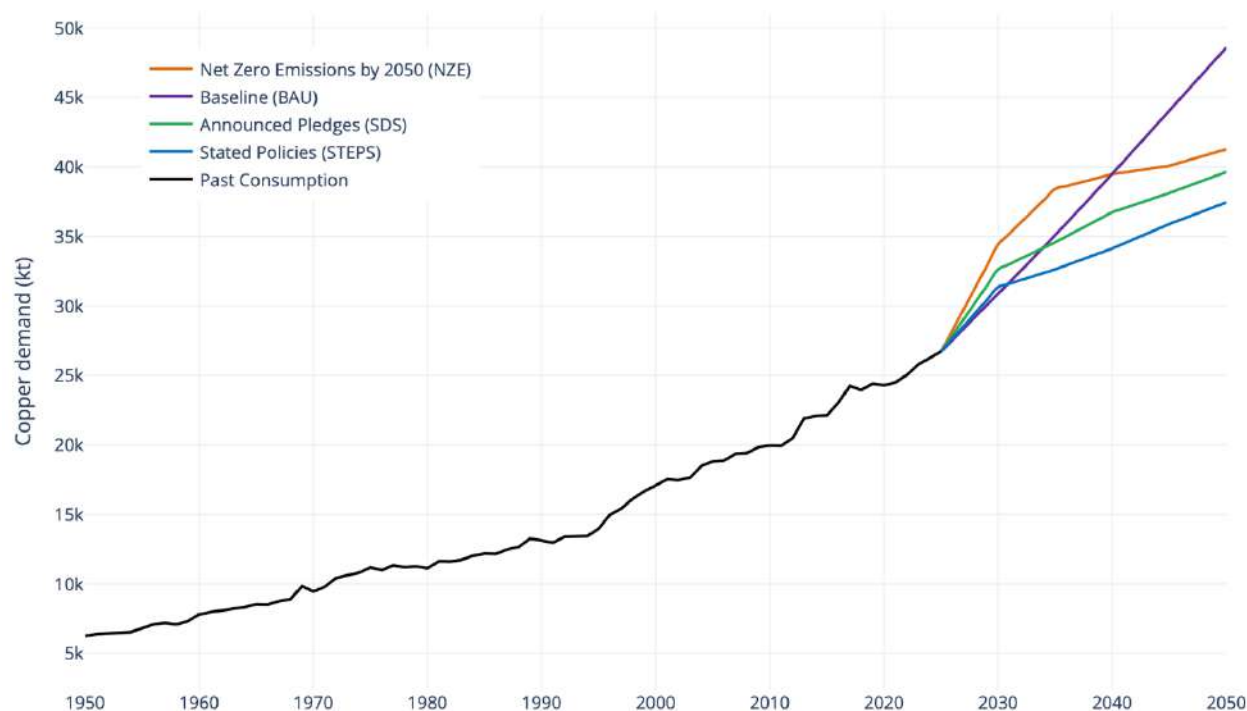
For simplicity, the projected need function $R_{i,t}$ presented above is in its aggregated form, but we do account for regional variations (mostly in speed of changes in the energy mix and the copper content of non-energy sectors). Regional disaggregation, as well as full parameter values, and sectoral shares are provided in the Appendix (Section V).

Figure 3.1 illustrates the projected requirements trajectories $R_{i,t}$ through 2050 across the four scenarios. Table 3.1 summarize their decomposition between clean technology uses and traditional sectors.

⁴⁴ Final energy is the energy available to consumers after transformation, transport and distribution. It is the energy that can be used for purposes such as heating, electricity and fuels. It is the opposite of primary energy, which is the raw energy extracted directly from nature, such as crude oil, coal, natural gas and renewable sources (solar, wind, etc.).

⁴⁵ International Energy Agency, 2021. The role of critical minerals in clean energy transitions. OECD Publishing, p.8

Figure 3.1: Global Copper Projected Requirements Evolution by 2050 under Different Energy Transition Scenarios, Compared with Historical Consumption Data



Source: LME, IEA Global Critical Minerals Outlook 2025 and author's calculations

Table 3.1: Copper Demand Scenarios (2025-2050)

<i>(in kt)</i>		STEPS	APS	NZE	BAU
Clean technologies	2025	7737	7737	7737	7737
	2050	13042	17176	19679	14131
Other sectors	2025	18980	18980	18980	18980
	2050	24409	22465	21598	34468
TOTAL	2025	26717	26717	26717	26717
	2050	37451	39641	41277	48599

Source: IEA, Global Critical Minerals Outlook 2025

In the clean energy transition scenarios—STEPS, APS, and NZE—copper demand grows steadily, driven by the increasing deployment of low-carbon technologies. The STEPS and APS trajectories are relatively close throughout the period, with APS showing slightly higher copper needs due to its more ambitious implementation of announced climate pledges. This moderate divergence reflects the

incremental acceleration in clean energy deployment expected in APS compared to the more conservative, policy-anchored assumptions of STEPS. The NZE scenario displays the highest copper requirements in the early decades, consistent with its aggressive pathway toward net zero emissions by 2050. However, the growth rate of copper needs under NZE progressively slows down after 2035. This reflects the impact of material efficiency and sobriety assumptions embedded in NZE, which partially offset the otherwise intense demand surge associated with rapid electrification and renewable energy expansion.

By contrast, the BAU (Business as Usual) scenario follows a distinct dynamic. In this scenario, copper demand from clean energy sectors remains largely stagnant over time, except for the electric vehicle segment which is assumed to grow at the G20 emerging economies' average growth rate⁴⁶. However, copper demand from 'traditional' sectors—those linked to fossil energy and general industrial uses—is extrapolated to grow in line with the OECD's GDP growth projections for G20 countries. This uncapped economic growth drives a sustained and ultimately faster increase in copper needs in the BAU scenario compared to the clean energy scenarios after the mid-2030s.

By 2050, copper needs in BAU surpass those in all other scenarios. This inversion highlights a key distinction: while clean energy transitions drive short- to medium-term copper demand, material efficiency and technological saturation can temper long-term needs in these pathways, whereas in the absence of decarbonization policies, unconstrained economic growth leads to persistent and accelerating copper demand.

Sectoral insights for each of these scenarios are available upon request. Overall, the projections indicate that the scale of copper requirements over the coming decades can largely depend on the extent and pace of the energy transition.

c. Mapping Ex-Ante Requirements to Ex-Post Demand Dynamics: Introducing Unmet Needs

Due to supply inelasticity or lack of investment, actual demand (consumption levels) may fall below $R_{i,t}$. However, demand coming from policy-committed sectors reflect mandated trajectories rather than discretionary demand. A shortfall of ex post demand therefore does not eliminate the underlying need; instead, it delays its fulfillment. When this occurs, the unmet demand for copper carries over to

⁴⁶ <https://www.oecd.org/en/data/indicators/real-gdp-long-term-forecast.html>

future years, compounding demand pressure as transition-related requirements remain unfulfilled. To capture this constraint, we introduce an unmet needs variable N_t :

$$N_t = N_{t-1} + (R_{i,t} - Q_{d,t}) \times \mathbb{1}_{\{Q_{d,t} < R_{i,t}\}}$$

Where N_t increases whenever actual consumption $Q_{d,t}$ is less than the scenario-specific requirement $R_{i,t}$. This accumulated unmet need is then incorporated into the demand function for subsequent years, adjusting the baseline demand upwards. Indeed, B is now scenario-specific, and defined as:

$$B_i(t) = A(R_{i,t} + N_t)$$

Where A is a constant scaling parameter.

The demand function $Q_{d,t} = B_i(t)P_t^{\sigma-1}$ now captures both short-term price responsiveness and long-term structural pressures. It reflects not only how demand adjusts to market prices each year, but also how underlying needs for copper to meet energy-transition and economic growth goals accumulate over time when supply falls short, exerting a “push” effect on future demand, compounding baseline requirements according to each scenario’s trajectory. This dynamic formulation ensures that delayed or constrained demand is not discarded but carried forward, leading to cumulative demand effects. This approach seems valuable for projecting the feasibility of different energy transition scenarios.

d. Price Elasticity of Copper Demand: Empirical Evidence for Moderate Inelasticity

Empirical studies have consistently found that copper demand is relatively inelastic, with price elasticity estimates typically ranging from 0.4 to 0.8.

Shojaeinia (2023) estimates the long-run price elasticity of copper demand to be around 0.73, which suggests that demand for copper is only moderately responsive to price changes, reinforcing copper’s essential role in various industrial applications.

In *"Energy Transition Metals"* (IMF, 2021) the IMF explores the price elasticity of demand for several transition metals, including copper. The IMF study finds that demand elasticity is generally low for copper, suggesting that demand decreases only slightly when prices increase. This study highlights copper’s limited substitution possibilities due to its unique properties, especially in the context of energy transition sectors.

Evans and Lewis (2002) examine the demand elasticity across several metals, finding copper's elasticity to fall in the 0.6 to 0.8 range, indicating moderately inelastic demand. This work emphasizes that, despite some degree of substitution between metals, copper's essential applications make its demand less responsive to price changes.

Roberts (1996) reports price elasticity estimates for copper demand between 0.5 and 0.7. This range reinforces findings from other studies, suggesting that while demand does respond to price increases, the effect is relatively small. Copper's role in infrastructure and electronics limits the flexibility of consumers to reduce demand significantly.

Using a panel data approach across OECD and non-OECD countries, Wårell and Olsson (2011) estimate the price elasticity of copper demand at 0.75 on average, though they note variations depending on industrial and economic contexts. This study highlights that while some substitution exists in developed economies, overall demand remains fairly inelastic across regions.

Across these studies, estimates for the price elasticity of copper demand consistently indicate a relatively inelastic demand, typically ranging from 0.5 to 0.8. These values show that copper demand tends to decrease only moderately in response to price increases, reflecting its essential use in industries with limited substitution options, such as in construction, electronics, and energy infrastructure.

For our analysis, we adopt an intermediate elasticity value of 0.65, which balances the range of available estimates and provides a reasonable approximation of copper's price responsiveness considering the available literature. This relatively inelastic nature underscores the importance of understanding price impacts in sectors heavily dependent on copper, especially under scenarios of fluctuating market prices and growing demand from energy transition requirements.

2. Calibrating the Supply Function

The supply module consists of two main elements: the cumulative availability curve $c_{max}(P_t)$, which defines the total recoverable copper at a given price level, and the logistic supply function $Q_{s,t}$, which determines annual supply as a function of cumulative extraction and price-induced availability.

a. The Cumulative Availability Curve: A Sigmoid Mapping of Recoverable Copper

We defined cumulative availability as a sigmoid function of the form:

$$c_{max}(P) = \frac{L}{1 + e^{-\kappa(P-P_m)}}$$

The function describes the total recoverable quantity of copper at or below a given price P , where L represents the asymptotic limit of recoverable resources, κ controls the steepness of the curve, and P_m denotes the price at which 50% of the total recoverable copper becomes economically viable.

The parameter L is set at 3.5 billion metric tons. This value reflects an upper-bound estimate of ultimately recoverable copper, assuming no major breakthroughs in extractive technologies. It is based on the sum of approximately 700 million tons already extracted, as reported by the USGS (2024)^{47,48,49}, and the combination of copper reserves, resources, and undiscovered deposits referenced in Castillo and Eggert (2020)⁵⁰. The selected value captures the current physical and economic limits under conservative assumptions, consistent with other studies employing modified cumulative availability curves (e.g., Mudd and Jowitt, 2018; Vidal et al., 2021).

The price midpoint P_m , at which half of L becomes economically recoverable, is set at 6,000 US\$/ton. This value corresponds to long-term marginal production costs observed over the latest two decades. Industry data from sources such as the IEA (2021), Wood Mackenzie (2025)⁵¹, and S&P Global (2022)⁵² suggest that for a large portion of marginal copper producers, breakeven costs—including capital, energy, and refining—have typically ranged between 5,500 and 6,500 US\$/ton. By choosing 6,000 US\$/ton, we place the inflection point of the availability curve at a level that reflects historical price dynamics and sectoral cost structures.

The steepness parameter κ governs the rate at which recoverable quantities increase with price. It is calibrated to match the empirical behavior of cumulative availability curves found in Castillo and Eggert (2020). Specifically, we set $\kappa = \frac{1}{38.4} \approx 0.026$, a value that ensures the function transitions from

⁴⁷ <https://pubs.usgs.gov/periodicals/mcs2020/mcs2020-appendixes.pdf>

⁴⁸ https://www.fs.usda.gov/Internet/FSE_DOCUMENTS/stelprdb5336497.pdf

⁴⁹ <https://www.usgs.gov/faqs/how-much-copper-has-been-found-world>

⁵⁰ Reserves are known and economically viable copper deposits expected to be mined first, providing the initial low-cost supply. Resources are known deposits with higher extraction costs, reflecting greater economic and geological uncertainty. These are assigned the top cost quartile within each country, extending the CAC curve at higher price levels. Undiscovered deposits are hypothetical sources with less certainty in cost and location. To represent these, we assign costs from the highest quartile of current underground mining operations. These deposits form a potential buffer against depletion, but they remain uncertain. A discovery barrier separates known resources and reserves from undiscovered deposits, emphasizing the importance of exploration and reserve expansion to ensure future supply stability. See [Graphs 1.a](#) and [1.b](#). To streamline the model, we do not adjust c_i based on resource type. Instead, the modified CAC incorporates resource-type differences by combining reserves, identified resources, and potential undiscovered deposits into a single availability framework, with their associated levels of extraction cost. This simplification enables efficient computation without reducing the model's capacity to represent supply limits effectively across different resource categories.

⁵¹ https://www.woodmac.com/reports/metals-global-copper-mine-cost-curve-37912265/?utm_source=chatgpt.com

⁵² https://cdn.ihsmarkit.com/www/pdf/0722/The-Future-of-Copper-Full-Report-14July2022.pdf?utm_source=chatgpt.com

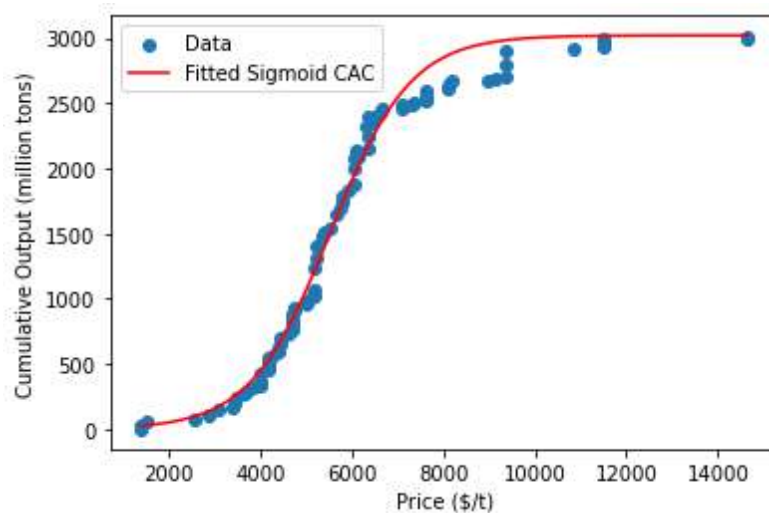
10% to 90% of LL within a price range of approximately 4,000 to 10,000 US\$/ton—a range that encompasses the typical fluctuation band of copper prices since the early 2000s. This allows the model to reflect both the historically low-price inelasticity of early resource extraction and the eventual saturation of availability as prices reach the upper bound of economically feasible exploitation.

In this section, we assume no additional technological progress over the time horizon of this model, meaning that the CAC remains constant and does not shift due to potential advances in mining technology or extraction efficiency. While this assumption may overlook future technological improvements that could lower costs or increase access to lower-grade deposits, it provides a conservative baseline for analyzing depletion risks.

The resulting function $c_{max}(P)$ is displayed in Figure 3.2 below, which illustrates the relationship between price and cumulative recoverable resources under these calibrated assumptions.

Under current market conditions—with copper prices around 9,000 US\$ per metric ton—the market appears to be operating near the upper plateau of the sigmoid curve, where additional price increases yield only marginal gains in recoverable quantities. It is important to emphasize that this curve is constructed under the assumption of unchanged mining technology, reflecting a conservative scenario with no future breakthroughs in extraction efficiency or ore processing. Moreover, the results presented here should not be interpreted as forecasts but as forward-looking simulations designed to explore possible market trajectories under defined sets of assumptions.

Figure 3.2: Estimated CAC Function for Copper



This observation is significant because it highlights the reduced responsiveness of supply to further price increases at this stage, reinforcing the idea that resource scarcity may soon become a critical constraint. As a result, price signals alone may not be sufficient to drive substantial new supply,

underscoring the need for technological breakthroughs or alternative solutions to meet future demand sustainably. This point will be addressed in greater detail in the *Discussion* section.

b. Logistic Supply Function Parameters: Capturing Depletion-Induced Decline

In the logistic supply function, the parameters γ (steepness) and θ (inflection point) govern the supply response as cumulative extraction approaches the cumulative availability curve (CAC). These parameters are essential for capturing copper's supply constraints under varying demand scenarios. Both parameters are calibrated empirically using historical copper supply and extraction data from the International Copper Study Group⁵³, specifically focusing on the period from 2001 to 2024. This period was chosen because it encapsulates significant production trends and supply constraints as global copper extraction neared the limits of identified reserves and resources. By leveraging real-world data, this approach ensures the model reflects actual copper availability patterns and aligns with observed supply limitations.

1.1. Parameter γ : Steepness of Supply Decline

The parameter γ governs how sharply the supply declines as cumulative extraction approaches the reserve limit c_{max} . A higher γ reflects a stronger decrease in supply elasticity, a steeper, more abrupt supply contraction when easily accessible deposits are exhausted.

To calibrate γ , we use production data from the International Copper Study Group (ICSG), covering the period from 2001 to 2024. This data provides annual copper production levels, which we use to track the growth of cumulative copper extraction over this period. We first calculate the cumulative copper extraction c_t from 2001 to 2024 based on the annual production data from the ICSG. Next, we examine the rate of supply growth over time. As cumulative extraction increases, we identify key periods where supply growth slows or plateaus. Notably, we look for periods where the rate of increase in copper production begins to decelerate, signaling that supply is becoming constrained by resource availability. We fit a logistic supply function to the cumulative extraction data, adjusting the steepness parameter γ to match the observed rate of decline in supply. Using production data from the ICSG, we assess how the supply growth has slowed at different stages of cumulative extraction. For example, between 2010 and 2020, copper production growth decelerated as more accessible reserves were depleted, and extraction shifted toward harder-to-reach resources. These empirical observations provide a basis for setting the value of γ .

⁵³ <https://icsg.org/press-releases/>

In logistic supply functions for resource depletion models, the steepness parameter γ typically ranges from 5 to 10. Lower values (around 5) imply a gradual supply decline as cumulative extraction approaches the resource limit, whereas higher values (closer to 10) indicate a steeper decline.

Sverdrup et al. (2014)⁵⁴ focused on modeling the global copper supply using a logistic function to represent the depletion of copper resources. They calibrated the steepness parameter γ by observing how copper production slowed as cumulative extraction approached resource limits. Sverdrup et al. suggested that values for γ typically fall within the range of 5 to 8, with the exact value depending on the resource characteristics and extraction rates. They found that a value closer to 8 provided a better fit for the observed data, especially when modeling the decline in supply as extraction nears resource constraints. This range aligns with the general characteristics of copper, where supply starts to decelerate significantly once accessible reserves are depleted and extraction shifts to more difficult-to-access resources.

This approach is supported by their empirical analysis, which examined production data from 1950 to 2010, a period during which global copper production growth began to show signs of slowing. The slowdown was especially noticeable when more challenging resources were required to meet global demand. The steepness of the decline captured by their chosen values of γ reflects the inelasticity of supply in the face of these increasing extraction costs, which is crucial for understanding the dynamics of copper availability as the resource becomes scarcer.

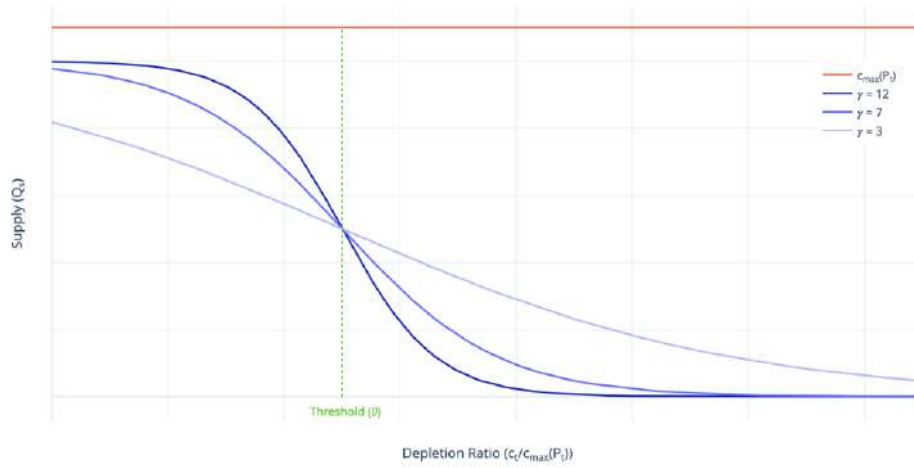
Mudd and Jowitt (2018)⁵⁵ further explored the changing role of copper in the global resource system, focusing on the dynamics of resource depletion and the logistic modeling of copper supply. They emphasized that the steepness parameter γ should reflect the way in which the supply of copper becomes increasingly constrained as extraction approaches the limits of known resources and reserves.

In their study, Mudd and Jowitt suggested a value range for γ similar to Sverdrup et al., between 5 and 10. They observed that lower values of γ were suitable for modeling resource systems where extraction slowed gradually over time, while higher values were appropriate for systems where resource constraints led to a more abrupt decline in supply as reserves were exhausted. They specifically pointed out that a value of $\gamma = 7$ aligns well with observed production rates in copper markets, offering a balance between a gradual and sharp decline in supply.

⁵⁴ Sverdrup, H.U., Ragnarsdottir, K.V. and Koca, D., 2014. On modelling the global copper mining rates, market supply, copper price and the end of copper reserves. *Resources, Conservation and Recycling*, 87, pp.158-174.

⁵⁵ Mudd, G.M. and Jowitt, S.M., 2018. Growing global copper resources, reserves and production: Discovery is not the only control on supply. *Economic Geology*, 113(6), pp.1235-1267.

Figure 3.10: Logistic Supply Curve Based on $c_{max}(P_t)$ with Different Values for γ



Their analysis of historical copper production data, spanning several decades, indicated that as the share of easily accessible reserves dwindled, copper supply growth began to decelerate significantly. This phenomenon was evident in the 1990s and early 2000s, as production shifted to lower-grade ores and more complex extraction methods. These shifts contributed to the steepening of the supply curve, necessitating the use of a higher γ value in the logistic function to accurately capture these constraints.

We estimate the γ value to be of 7, striking a balance between a moderate-to-steep decline, which aligns with the characteristics of copper as a critical industrial resource, where extraction constraints accelerate notably as availability limits are approached.

1.2. Parameter θ : Inflection Point of Supply Decline

The parameter θ defines the inflection point at which the logistic function begins to sharply reduce supply as c_t approaches $CAC(P_t)$. This threshold marks the critical level of cumulative extraction where supply constraints begin to intensify⁵⁶.

⁵⁶ The inflection point θ of the logistic supply function is not necessarily the same as the inflection point of the CAC. The CAC represents the relationship between price and total recoverable resources. Its inflection point indicates where incremental increases in price yield diminishing returns in terms of additional recoverable copper quantities. In the logistic supply function, θ represents the point in cumulative extraction at which the decline in supply availability becomes significantly steeper. It is a threshold based on cumulative extraction, not on price, and reflects practical supply constraints as cumulative production approaches total known resources. It is typically set based on economic feasibility and depletion factors, independent of the CAC's shape, because it reflects a threshold at which supply limitations intensify in response to cumulative extraction nearing known limits. The CAC inflection point, on the other hand, represents the point where the availability of copper per unit cost (or per price increase) starts to decline more sharply. This means that as the price of copper rises, additional resources become available, but at a slower rate due to higher extraction costs or technical difficulties. This inflection point is driven by economic considerations rather than cumulative extraction. Thus, the CAC's inflection point is often linked to geological and economic factors rather than directly to cumulative extraction, as it represents recoverable quantities at given price levels rather than availability based on cumulative depletion. In summary, while both inflection points relate to supply constraints, θ in the logistic supply function captures cumulative extraction thresholds, whereas the CAC's inflection point captures economic and geological limitations based on price. They serve related but distinct purposes in modeling copper availability.

The parameter θ represents the point where supply constraints become significant, marking a tipping point for resource accessibility. This parameter determines when cumulative extraction reaches levels that result in a sharper supply contraction.

Sverdrup et al. (2014) in their study on global copper supply utilized a logistic model with θ values of 0.75 for reserves and 0.6 for resources to simulate supply constraints as extraction progresses. These values align with observed patterns where extraction becomes increasingly challenging as a larger proportion of reserves and resources are depleted.

Reserves are deposits with higher economic feasibility and are generally mined first because they are accessible and have known, lower extraction costs. This makes reserves more straightforward to exploit up to a higher cumulative extraction level before significant constraints appear. Thus, a higher θ value for reserves (e.g., 0.75) reflects the fact that these deposits can sustain extraction until cumulative production reaches a high proportion of the identified reserve total.

Resources, however, are deposits with lower certainty in terms of economic feasibility and often involve higher extraction costs or logistical challenges. These factors mean that resource extraction becomes constrained at a lower cumulative extraction level. Setting θ lower for resources (e.g., 0.6) models these early constraints effectively, indicating that the inflection point for supply decline appears sooner in response to increasing cumulative extraction.

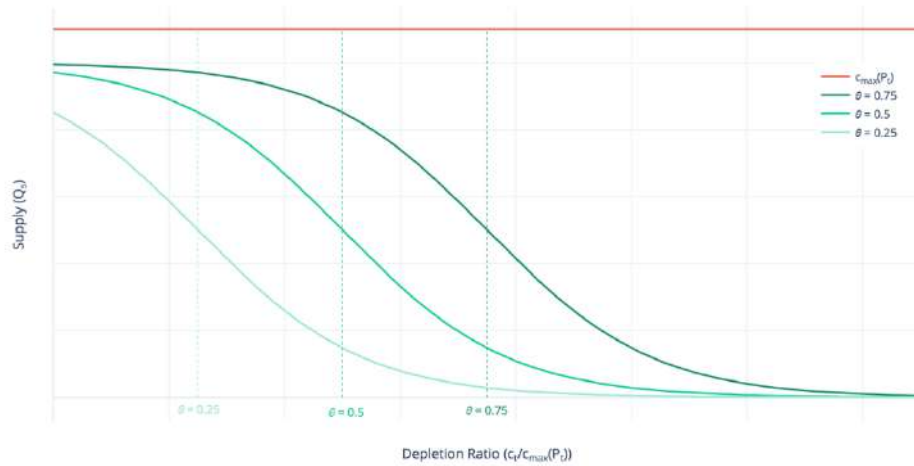
In practice, resources are developed progressively, often as reserves are depleted or when prices rise sufficiently to justify their extraction costs. By setting θ lower for resources, the model captures a realistic supply pattern, where resource deposits start facing constraints earlier than reserves, limiting their contribution to the total supply at lower cumulative production levels.

Based on estimates from Castillo and Eggert (2020)⁵⁷, who build on data from Mudd and Jowitt (2018), identified copper deposits—including both resources and reserves—are estimated at 3.020 billion metric tons, with 641 million metric tons classified as reserves. As a comparison, the most recent estimate of undiscovered copper resources, from the USGS 2024 Mineral Commodity Summaries⁵⁸, places these deposits at approximately 3.5 billion metric tons.

⁵⁷ Castillo, E. and Eggert, R., 2020. Reconciling diverging views on mineral depletion: a modified cumulative availability curve applied to copper resources. *Resources, Conservation and Recycling*, 161, p.104896.

⁵⁸ <https://pubs.usgs.gov/periodicals/mcs2024/mcs2024-copper.pdf>

Figure 3.11: Logistic Supply Curve Based on $c_{max}(P_t)$ with Different Values for θ



The total identified copper deposits are estimated to consist of approximately a fifth in reserves, a ratio that has remained fairly constant throughout the 20th and 21st centuries according to the USGS⁵⁹. As stated earlier and building on the literature, we set θ at 0.75 for reserves and 0.6 for resources. Given that reserves account for about 20% of total identified deposits, we calculate a weighted average θ of 0.72, which provides a balanced inflection point that realistically captures the accessibility limits for both resource categories.

As a side note, in traditional resource peak theory, θ often takes an arbitrary value around 50% of total resources, representing a conceptual midpoint at which production typically peaks before entering decline. In our model, we adjust θ to reflect more specific estimates for copper reserves and resources, creating a customized inflection point that aligns with known resource availability.

III. Simulated Outcomes

1. A Supply-Constrained Future: Price Surges under Inelastic Demand

Building on the projected global copper requirements outlined across various scenarios as computed by the IEA, we now assess how these hypothetical copper needs may influence demand patterns under our assumptions of demand price-elasticity and base intensity factor, as well as the responsiveness (or stickiness) of copper supply, particularly within a depletion framework. The resulting dynamics are governed by the structure of our analytical model, in which the equilibrium price P_t each year is determined by solving the following equation:

⁵⁹ <https://www.usgs.gov/centers/national-minerals-information-center/copper-statistics-and-information>

$$P_t = \left(\frac{c_{max}(P_{t-1})}{B(t) \cdot \left(1 + e^{\gamma \left(\frac{c_t}{c_{max}(P_{t-1})} - \theta \right)} \right)} \right)^{\frac{1}{\sigma-1}}$$

This expression captures how equilibrium price is jointly shaped by:

- the total recoverable amount $c_{max}(P_{t-1})$ as a function of the previous period's price;
- the level of cumulative extraction c_t relative to this recoverable amount, which reflects resource depletion and scarcity;
- the elasticity of substitution σ , which determines how sensitive demand is to price changes.

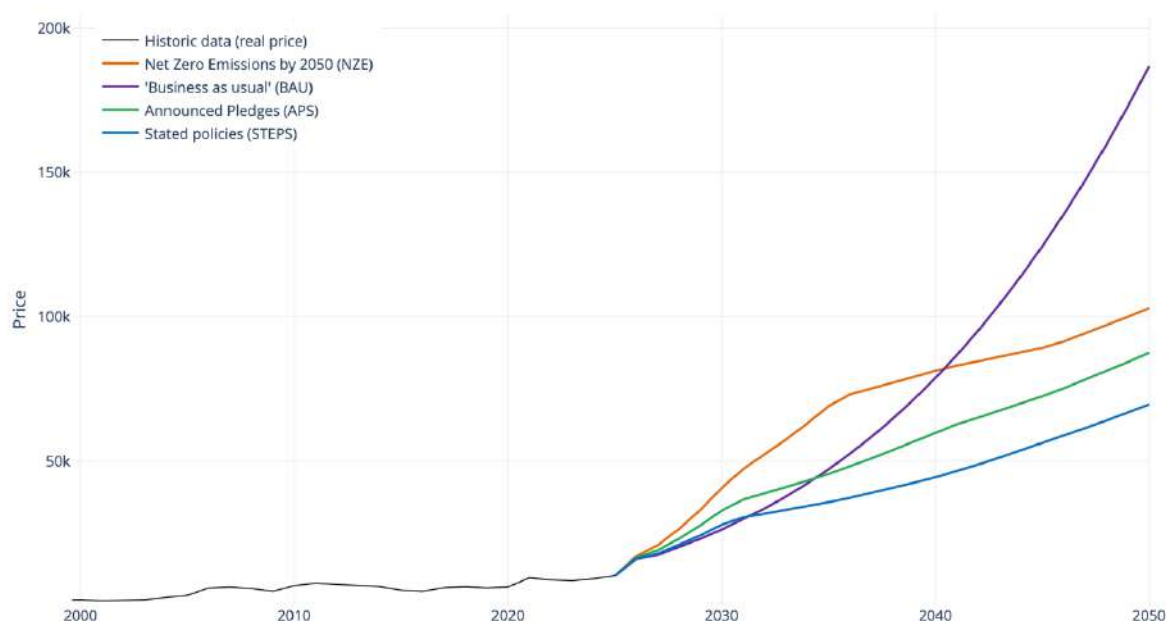
Table 3.2: Simulated Equilibrium Price Paths Over the 2025-2050 horizon (in US\$/t)

<i>(in US\$/t)</i>	STEPS	APS	NZE	BAU
2025	10187	10187	10187	10187
2030	27852	32763	40606	26195
2040	44302	59899	81229	78719
2050	69559	87485	102944	186979
2025/2050 increase (%)	583%	759%	911%	1734%

The price curves presented for each scenario in Figure 3.3 are based on projected requirements and modeled demand-supply dynamics with specific parameters, as detailed in section II.1. These assumptions create a framework where projected requirements have a significant impact on demand, which makes the latter relatively inelastic, while supply is also restricted in its responsiveness, especially as resources become more depleted.

Figure 3.3: Projected Price Curves per scenario over 2025-2050 (in US\$/t)

Simulated price curves per energy transition policy scenario under the assumption of no supply-side technological progress



Note: In these simulations, we assume supply unresponsiveness, and in particular no increase in the resource base, nor technological progress that allows for lower extraction cost.

The simulated price trajectories across the four scenarios—STEPS, APS, NZE, and BAU—reveal not only varying growth patterns but also fundamentally different equilibrium dynamics. The key insight from these projections is not the precise price levels, which remain indicative, but the shape and stability of the trajectories over the long term, particularly whether they are consistent with a non-explosive, non-zero (NENO) equilibrium.

In the STEPS and APS scenarios, prices follow a steep but contained growth path throughout the horizon. Despite significant upward pressure from increasing copper needs for clean technologies, especially in APS, prices rise at a pace that remains compatible with NENO dynamics. These trajectories suggest that, while the copper market will face sustained tension, particularly in the medium term, the system is not on an inherently destabilizing path if the demand growth is anchored in progressive and policy-driven transitions.

The NZE scenario pushes these dynamics further. It exhibits the fastest price growth, driven by an aggressive early demand surge linked to the rapid electrification and expansion of renewable infrastructure. However, the material efficiency and substitution assumptions embedded in NZE contribute to a gradual slowing of demand growth beyond 2035, which helps to moderate the long-term trajectory. The NZE pathway shows a steep profile, and illustrates an overstretched market,

calling for the pursue of technological advancements, particularly in recycling and extraction, alongside demand-side management.

By contrast, the BAU scenario reveals a fundamentally different, potentially more concerning, pattern. In the short term, particularly over the upcoming decade, BAU displays only a relatively moderate increase in prices. This is primarily due to the inertia of current consumption structures, where clean technology adoption remains limited and demand continues to follow historical patterns. However, the underlying dynamic is deceptive. As demand from traditional sectors accelerates in line with uncapped economic growth assumptions, especially coming from currently emerging economies, the price trajectory under BAU becomes explosive over the longer term, diverging sharply from a NENO equilibrium. What is particularly striking is that this explosive dynamic emerges not from the energy transition, but from its absence. The sustained growth of fossil-related and non-energy sectors—rarely the focus of critical materials discourse—places extreme pressure on copper supply in the BAU scenario.

This challenges the widespread narrative that energy transitions alone are the primary drivers of future material criticality. On the contrary, the absence of decarbonization efforts and the continuation of historical growth patterns may generate the most destabilizing market pressures for copper. Historically, the rhythm of copper consumption in the 20th century—fueled by the economic expansion of now-developed economies—resembled the demand growth seen in the BAU scenario. Yet during much of this period, technological progress in exploration, extraction, and processing kept pace with demand, allowing prices to rise moderately and remain economically sustainable. In the BAU scenario, absent further supply-side innovation, this balancing mechanism fades, resulting in unsustainable long-term price escalations.

The findings suggest that while price elasticity provides some moderation, the depletion model significantly limits supply responsiveness, underscoring the need for long-term strategies to ensure copper price stability and market sustainability.

Without significant technological breakthroughs on the supply side—whether through improved access to resources, secondary supply development, or efficiency gains—market dynamics could become increasingly unstable, particularly in non-transition scenarios where unchecked demand exerts persistent pressure.

It is worth noting that the simulated price increases, although steep over the long term, are not unprecedented when compared to recent market behavior. Between 2020 and 2025, the market price of copper surged by approximately 493%, driven by a combination of post-pandemic recovery, supply

chain tensions, and accelerated investment in clean technologies. This recent five-year price escalation provides a real-world benchmark that illustrates how quickly copper markets can react to tight supply and growing demand. While the model focuses on structural, long-term dynamics under constant technology assumptions, this historical price surge demonstrates that even sharp price increases of the magnitude projected in the simulations are within the realm of observed market behavior, particularly when supply responsiveness is constrained.

2. Depletion-Driven Saturation: When Exhaustion Caps Supply

Across all four scenarios, the simulated annual equilibrium quantity $Q_{s,t}$ remains relatively stagnant throughout 2025–2050—22 Mt per year—which mechanically reflects the rigidity of the supply block; this stagnation is common across scenarios and explains why cumulative extraction trajectories converge.

Our depletion assessment begins with an initial cumulative extraction baseline of 700,000 kt, representing the amount of copper extracted globally to date. Under each scenario, the cumulative extraction by 2050 is projected to reach approximately double this baseline. This result aligns with the CNRS prediction that "we will extract more metals in the next 30 years than since the dawn of mankind"⁶⁰. Indeed, despite varied projected requirements across scenarios, the actual cumulative extraction volumes by 2050 remain nearly equivalent⁶¹: STEPS: 1,365,493 kt; APS: 1,366,215 kt; NZE: 1,367,557 kt; BAU: 1,367,250 kt.

This result is particularly noteworthy: across all scenarios, cumulative extraction converges to nearly identical values in the long run, illustrating that the supply side is the dominant constraint on copper availability. Regardless of demand fluctuations, the limited elasticity of supply and the depletion-driven constraints effectively cap cumulative extraction at similar levels across all scenarios. This underscores a critical insight: *it is not the projected demand that primarily governs the long-run future of copper markets, but rather the supply capacity, which imposes a ceiling on the cumulative extraction.* This convergence reinforces the idea that, irrespective of differing demand pressures, the equilibrium is ultimately determined by the available supply and its constraints.

In each scenario, the cumulative availability curve threshold is reached at different points, indicating when the total known and recoverable copper becomes accessible for extraction. In other words, the

⁶⁰ <https://www.insu.cnrs.fr/fr/cnrsinfo/les-metaux-et-la-transition-energetique-quels-enjeux>

⁶¹ These cumulative extraction volumes account to around 39% of the assumed total recoverable material. This order of magnitude is intermediary between results from IFPEN (78 to 89%) and Castillo and Eggert (2020) (25% of resources). <https://www.ifpenergiesnouvelles.fr/article/cuivre-transition-energetique-metal-essentiel-structurel-et-geopolitique>

price P reaches a level such that the total amount of copper that can be extracted is economy feasible, i.e. $c_{max}(P) = L$. In STEPS, APS, and BAU, and at unchanged technological progress, all recoverable copper becomes economically accessible by 2030. Under the NZE scenario, characterized by its aggressive clean energy deployment, our simulations suggest that cumulative extraction reaches critical levels relative to available supply as early as three years after the starting point of our model horizon, i.e., around 2028⁶².

These findings suggest that high-demand scenarios in the very short term like NZE lead to faster market access to total recoverable resources, whereas relatively more moderate demand pathways delay resource accessibility. This outcome reflects the strong influence of demand-side pressures on resource exhaustion timelines, as the market reaches the price level necessary to make remaining resources economically viable earlier in high-demand scenarios.

We again emphasize that analysis, up to now, has assumed constant technological progress, meaning no significant improvements in extraction technology or cost reductions are considered. Section III.4 below will highlight the sizeable potential impact of technological advances in moderating resource constraints and altering accessibility timelines.

3. Energy Transition Pathways under Pressure: How Explosive Price Trajectories Jeopardize Scenario Attainability

Copper requirements are computed by translating the energy transition policy scenarios into clean technology deployments, which in turn determine the material needs for copper, regardless of price. In contrast, *ex post* demand, as computed in the calibration, is derived from demand-side reactions to actual market prices (in real terms), yielding unmet requirements. This gap arises because consumers are unable to afford the copper quantities set by the policies, given the prevailing market prices. Consequently, the unmet needs in each scenario highlight significant gaps between projected requirements and actual attainable supply volumes, reinforcing the notion that supply-side constraints heavily influence the attainability of these transition pathways, particularly in the absence of any additional technological progress in the extractive industries in the future.

In the STEPS scenario, unmet needs account for 33% of total projected requirements by 2050. This indicates that, although demand growth is moderate, supply limitations still prevent full attainment of

⁶² It is noteworthy that the model horizon begins in 2024 and runs independently of the real-time of writing or defense, and that the world has not, in reality, followed the NZE pathway since then, making this an illustrative counterfactual rather than a forecast.

projected requirements. With a third of projected needs unmet, the STEPS scenario suggests that while a steady transition is more attainable than aggressive pathways, resource constraints remain a challenge. Supply-side measures, including gradual expansion of recycling and secondary sourcing, are necessary to bridge this gap.

The APS scenario shows 36% of projected requirements unmet by 2050, slightly higher than STEPS due to increased clean technology demand. This highlights the difficulty of sustaining higher renewable and EV deployment rates within current supply constraints. The APS scenario could potentially be attainable with additional support for copper recycling and secondary sourcing, though the level of unmet needs underscore the urgency of enhancing supply infrastructure. Without significant policy-driven recycling initiatives, this demand gap could limit the full realization of energy transition goals.

In the NZE scenario, unmet needs rise to 40% of projected requirements, reflecting a significant shortfall. This scenario's high demand for clean technologies—particularly renewables, EVs, and grid infrastructure—places severe pressure on copper resources. Crucially, the NZE pathway relies heavily on assumptions of material efficiency and sobriety; without these, the scenario's unattainability would likely be even greater. To make this scenario feasible, extensive investment in recycling, secondary sources, and material alternatives becomes essential. Demand management strategies, such as efficiency gains and material substitutions, are not just beneficial but critical to closing resource gaps and reaching the ambitious energy transition targets embedded in NZE.

The BAU scenario also exhibits an unmet needs rate of 40%, similar in magnitude to the NZE scenario, but it reflects a fundamentally different reality. In BAU, this shortfall is not driven by aggressive energy transition policies but by the persistent, unchecked growth of traditional sectors and the inertia of current consumption patterns. Unlike NZE, which integrates material sobriety assumptions to temper long-term demand, BAU follows an explosive, unsustainable price trajectory—suggesting that the actual unmet needs could be even greater than what the model currently captures. As prices escalate beyond economically viable levels, additional market frictions and demand destruction would likely emerge, pushing the system further away from equilibrium. This underlines that supply-side constraints are not exclusive to clean energy pathways; they are structural, tied to resource finiteness, and can become just as acute—or even more so—under business-as-usual conditions.

In sum, assuming unchanged technological progress in the extractive industries, the attainability of each transition scenario is limited by substantial unmet needs, with higher-demand scenarios like NZE and BAU showing the greatest challenges. While STEPS and APS have relatively lower unmet needs, they still reveal a supply gap that highlights the importance of sustainable resource management practices.

A broader comparison with the resource-economics literature reinforces these findings. Vidal and Le Boulzec (2022), using a fully bottom-up stock-and-flow model of infrastructure, energy systems and material intensities across world regions, also conclude that future mineral supply is primarily constrained by ore grade deterioration and the approach of thermodynamic limits. Their work shows that technological progress historically offset declining ore grades, but that this compensating mechanism is weakening as metals such as copper enter the high-energy, low-grade part of the cost curve. In their framework, long-term production costs rise structurally once the practical minimum energy threshold is reached—leading to strong, sometimes explosive price dynamics unless accompanied by major efficiency gains or substitution. Our results echo this pattern: the explosive BAU trajectory corresponds to Vidal and Le Boulzec’s “post-efficiency-gains” phase, where demand grows faster than both technological progress and resource accessibility, while the more moderate STEPS/APS paths resemble their “maturation” regimes where demand growth slows and recycling begins to play a stabilizing role. Importantly, the high unmet-needs ratios we obtain in NZE and BAU are consistent with their conclusion that mineral scarcity is not driven solely by energy-transition requirements but by the cumulative pressure of all infrastructures in a world approaching resource-quality limits. This convergence strengthens the interpretation that supply-side constraints, rather than scenario-specific demand shocks, dominate long-term copper market stability.

4. Can Technological Progress in Resource Extraction Prevent Long-Term Price Explosion?

Our previous results reveal that, across all scenarios, supply is the primary constraint when future technological progress is nil, while demand’s relatively inelastic nature limits its responsiveness to price shifts. As a result, the equilibrium is governed largely by supply limitations and the rhythm of resource depletion, regardless of the aggressive or moderate nature of demand growth. This constraint-driven equilibrium may not fully capture real-world dynamics, as actual supply may adjust with market conditions. Additionally, resource availability has historically increased over time, which could ease some of the limitations suggested by our model.

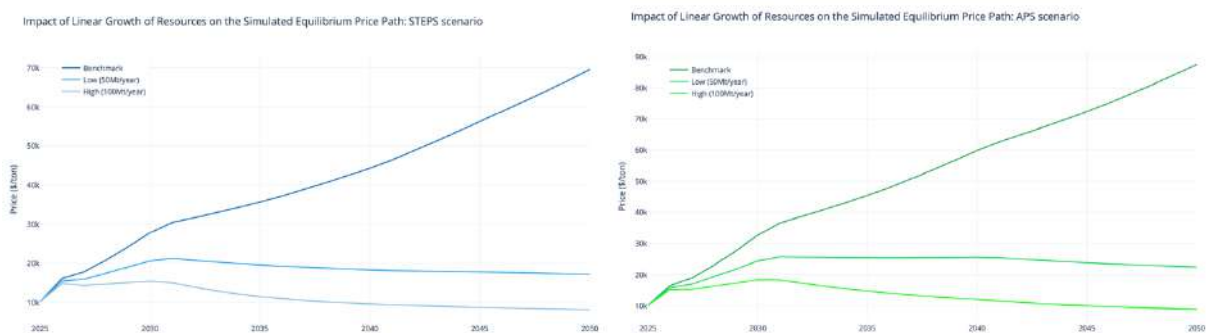
The modeling of supply-side technological progress in this section builds on the framework outlined in the analytical model, where we introduced $\Delta(t)$ as an exogenous term representing technological progress that operates independently of price signals. In the analytical section, we proposed three distinct functional forms for $\Delta(t)$, each representing a different pattern of technological progress: linear growth ($\Delta(t) = \delta t$), which represents the systematic, gradual expansion of resources observed historically, where new deposits are discovered or better extraction methods are developed at a steady pace, exponential growth ($\Delta(t) = \delta e^{\beta t}$), which is suited to scenarios where innovation compounds

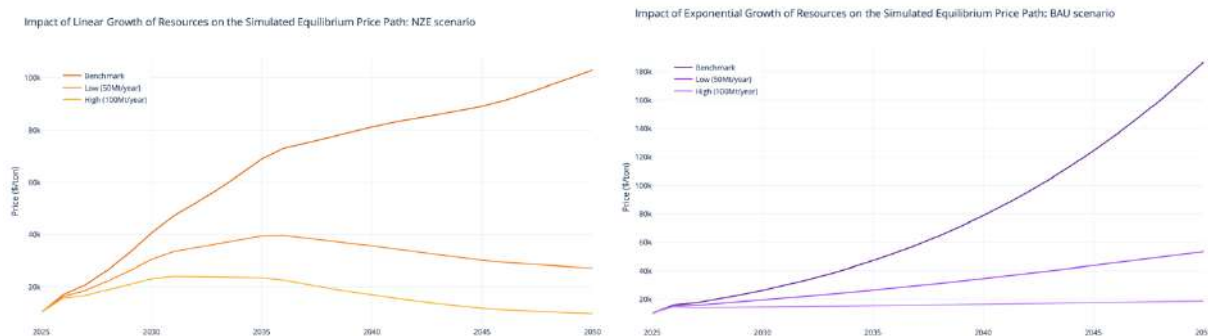
over time, such as the adoption of advanced mining technologies or significant improvements in ore recovery efficiency, and a jump function ($\Delta(t) = \delta H(t - T)$), which could represent the effect of a major technological breakthrough, such as the commercialization of a new mining technique or the discovery of a large, previously unknown deposit.

a. Linear Expansion of Resources: Steady Increases in Availability and Moderate Price Relief

We test two illustrative assumptions of linear growth in the cumulative availability limit c_{max} : first, a moderate rate of $\delta_{linear} = 50$ million metric tons (Mt)/year, and second, a more optimistic rate of $\delta_{linear} = 100$ Mt/year. These values do not correspond to production volumes but rather to annual increments in *resource availability*, as might occur through sustained exploration, improved geological mapping, or technological improvements in extraction and processing. While these increments appear large relative to current annual copper production (around 22 Mt/year), they are meant to reflect long-term processes that have historically led to significant expansions in the resource base. For instance, USGS data shows that global identified copper resources increased by over 700 Mt between 2010 and 2023, averaging roughly 50-60 Mt/year over that period (USGS, 2023). This historical context supports the plausibility of assuming linear additions in this range as part of our sensitivity analysis. Nevertheless, these values should be interpreted as scenario-specific assumptions rather than predictive estimates, designed to bound the potential effects of technological and exploratory progress on the resource frontier.

Figure 3.4: Impact of Linear Growth of Resources on the Simulated Equilibrium Price Path for Each Scenario





These results demonstrate that introducing linear growth in cumulative availability has a significant impact on price trajectories across all tested scenarios (STEPS, APS, NZE, and BAU) and for both levels of linear growth.

Under the assumption of moderate linear growth in resource availability (50 Mt/year), we observe a substantial alleviation of upward price pressures across all scenarios. In the energy transition pathways—STEPS, APS, and NZE—this moderate expansion of the resource base is sufficient to fundamentally alter the shape of the price trajectories. Notably, we identify points of inflexion where prices either stabilize or begin to decrease over time: around 2032 for STEPS and APS, and 2036 for NZE. This indicates that even a relatively conservative assumption of sustained exploration and technological improvement can shift the system toward more economically sustainable outcomes, at least in the clean energy scenarios.

For the BAU scenario, the effect is more limited, but still significant. The price increase by 2050 is contained to roughly one third of the level observed without supply-side progress. However, unlike the energy transition scenarios, the BAU trajectory does not exhibit a price reversal—prices continue to rise, albeit at a much slower and more manageable pace.

When we consider higher linear growth in resource availability (100 Mt/year), the moderating effects on prices are even more pronounced. By the end of the 2050 horizon, prices in all energy transition scenarios return close to their initial 2025 levels (around 10,000 USD/ton). In the case of STEPS, prices even fall below this starting point. The BAU scenario, while still showing upward pressure, is capped at under 20,000 USD/ton—a total price increase of just 83% over 25 years, compared to the explosive dynamics observed in the absence of supply-side technological progress.

b. Exponential Expansion: Accelerating Gains and Stronger Price Moderation

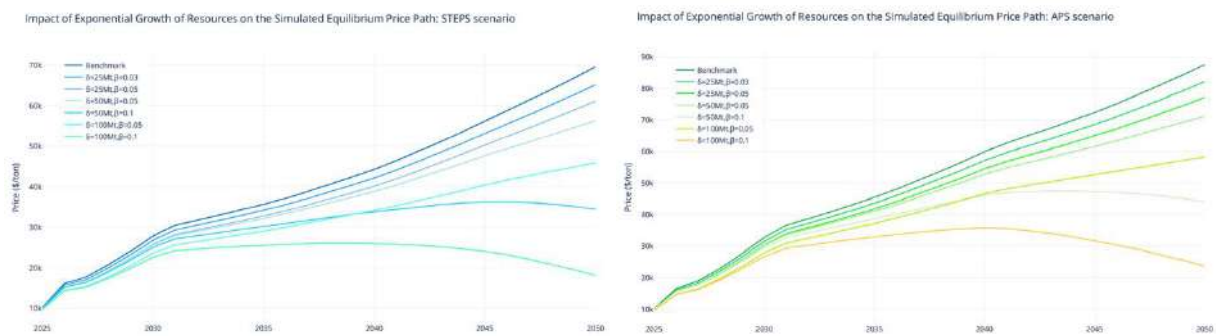
The modeled scenarios for exponential technological progress ($\Delta(t) = \delta e^{\beta t}$) explore a range of possibilities for resource growth, incorporating various combinations of initial growth rates δ and

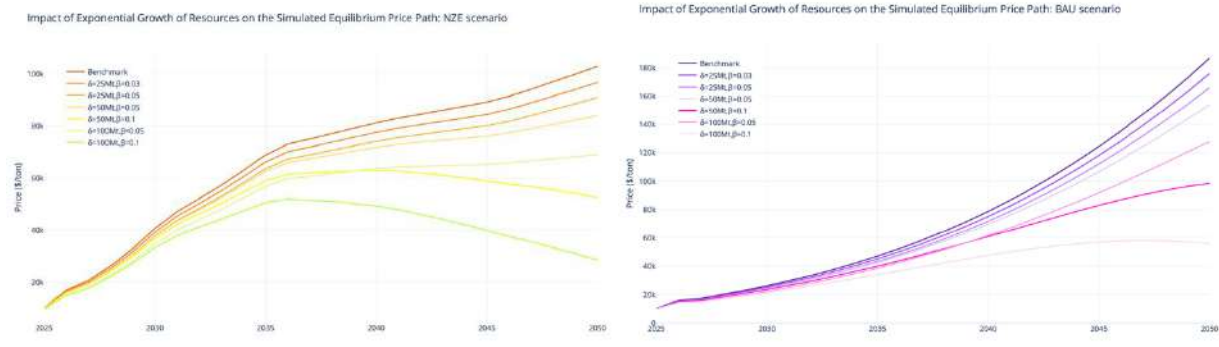
acceleration rates β . Specifically, the scenarios consider $\delta \in \{25, 50, 100\}$ Mt/year and $\beta \in \{0.03, 0.05, 0.1\}$, reflecting different levels of consistency with historical trends and plausible future developments. The results underscore the significance of these parameters in shaping price dynamics across all demand scenarios.

In all cases, higher values of β and δ result in less steep price increases, demonstrating the mitigating effect of accelerated resource growth on supply-side pressures. The role of β , in particular, is crucial in shaping the price trajectories. When $\beta=0.1$, the price paths exhibit a bell-shaped curve rather than a monotonically increasing trend. This bell-shaped behavior is observed across all scenarios, with the peak occurring around the 2040 mark. The timing of the peak varies depending on demand intensity: it occurs earlier in moderate-demand scenarios such as STEPS and APS and later in more aggressive scenarios like NZE. Remarkably, even for NZE, which represents a high-demand scenario driven by aggressive decarbonization, the price path follows a bell-shaped trend, indicating that accelerated exponential resource growth can effectively counteract sustained demand pressures.

A particularly striking result is observed when comparing $\delta=50$ Mt with $\beta=0.1$ against $\delta=100$ Mt with $\beta=0.05$. The downward pressure on prices is significantly stronger in the former case, underscoring the dominance of β as the parameter that carries more weight in shaping price dynamics. This highlights the importance of acceleration in resource availability over the magnitude of the initial annual growth rate.

Figure 3.5: Impact of Exponential Growth of Resources on the Simulated Equilibrium Price Path for Each Scenario





c. Discrete Jumps: Temporary Relief and the Limits of Breakthroughs

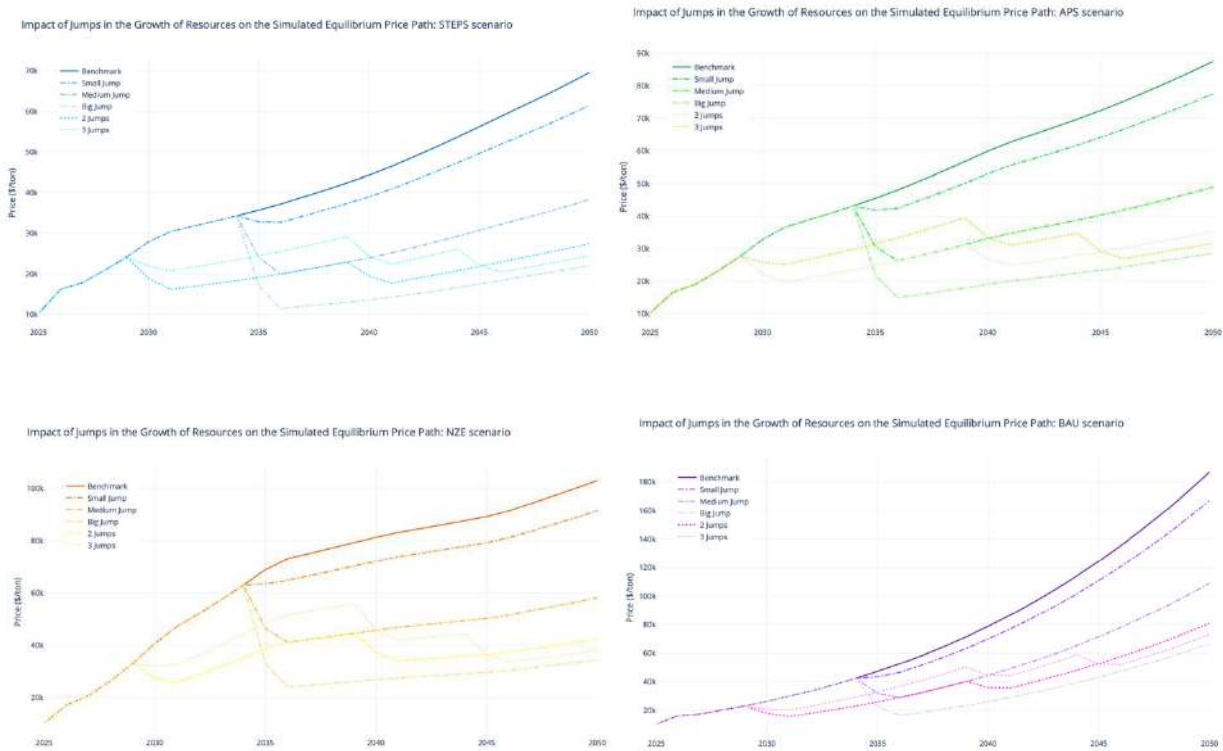
The jump scenarios simulate sudden increases in copper resources due to breakthroughs in exploration or technological advancements. We model 5 different scenarios:

- 1) A 100 Mt increase in 2035, which represents a relatively small discovery or a localized reclassification of resources. This aligns well with historical trends, as smaller deposits or technological advancements often contribute incremental gains.
- 2) A 500 Mt jump in 2035, which is more optimistic, representing a substantial technological breakthrough or discovery of a large new deposit. Historical analogs exist, such as the progressive identification of significant deposits globally, but this level of addition is less common within a single year.
- 3) A jump of 1,000Mt in 2035, which is highly optimistic, representing a transformative event like a massive breakthrough in mining or discovery of an exceptionally large deposit. While unlikely in a single year, it could occur over a decade with sustained global exploration.
- 4) Two jumps of 500Mt and 300Mt, respectively in 2030 and 2040. This scenario reflects staggered progress, with a significant early addition followed by a secondary discovery. It is realistic if global exploration and technological advancements progress steadily.
- 5) Three jumps of 300Mt each in 2030, 2040, and 2045. This scenario represents a series of more moderate discoveries or breakthroughs. It aligns with historical patterns of consistent but smaller discoveries, making it plausible.

The one-time increases can be interpreted as reflecting sudden reclassifications of resources due to major technological breakthroughs or policy-driven initiatives. Multiple jumps or staggered increases reflect more gradual, yet significant, advancements in exploration or improvements in mining efficiency. While these scenarios are less typical historically, they could represent responses

to external shocks, such as a sharp increase in demand from energy transition initiatives or significant price incentives.

Figure 3.6: Impact of Jumps in the Growth of Resources on the Simulated Equilibrium Price Path for Each Scenario



Across all scenarios, jumps in the cumulative availability limit lead to significant decreases in prices immediately following those jumps. Similarly, when there are two or three jumps, corresponding moments of price decreases are observed after each event. However, in all cases, the overall steepness of the price paths remains relatively parallel across scenarios. This indicates that while jumps provide temporary relief from demand pressure on prices, they do not fundamentally alter the underlying price dynamics. Ultimately, the price paths revert to the initial monotonous upward trend, highlighting the transient nature of the price moderation achieved through discrete resource additions.

d. Empirical Evidence on Technological Progress in Mining: Exogenous or Price-Induced?

Historical analysis by Mudd and Jowitt (2018) indicates that world copper resources grew from 1861.3 Mt in 2010 to 3533.2 Mt in 2024, achieving a compound annual growth rate (CAGR) of approximately 3.9%. This trend reflects continuous exploration efforts, reassessment of deposits, and technological improvements, despite the apparent declining trend in new discoveries. The observed increase of 1,443.4 Mt between 2010 and 2024 corresponds to net growth of 103 Mt/year, which

exceeds the 50 Mt/year growth rate used in this scenario. While conservative compared to historical trends, this growth rate reflects a systematic, price-independent expansion of resources.

The integration of 50 Mt/year linear growth added a total of 1.3 Bt of cumulative availability by 2050, resulting in a final cumulative availability limit of 4.8 Bt. This represents a modest but steady improvement in resource availability, consistent with historical observations that cumulative availability increases over time. This growth assumption also aligns with observations from the International Copper Study Group (2023)⁶³. Achieving a cumulative availability limit of almost 5 Bt by 2050 is reasonable within the context of this linear growth scenario. While historical trends suggest a higher CAGR for resources (~3.9%), the linear growth rate of 50 Mt/year represents a conservative estimate that reflects systematic, rather than breakthrough, improvements in resource identification.

A higher growth rate set at $\delta_{\text{linear}}=100$ Mt/year represents a more optimistic assumption, and assumes sustained technological progress and exploration activity, which may not fully capture regional constraints or periods of stagnation in the global mining industry.

Between 2010 and 2024, global copper resources grew from 1,861 Mt to 3,533 Mt, representing a compound annual growth rate (CAGR) of approximately 3.9%. This growth, equivalent to an annual addition of 103 Mt/year, was driven by systematic exploration, improved deposit classification, and advancements in extraction technologies. Copper reserves also increased, albeit at a slower CAGR of 1.0%, reflecting resource depletion offset by resource reclassification and new discoveries.

These trends highlight a mix of linear and exponential growth patterns in resource availability. While technological progress and exploration efforts underpin consistent growth, periods of rapid acceleration may occur due to breakthroughs in mining techniques or policy-driven exploration incentives.

Regarding the initial growth rate, historical resource additions (~103 Mt/year) align closely with $\delta=100$ Mt, making it a reasonable reflection of sustained growth. Lower values ($\delta=50$ Mt) represent conservative assumptions, potentially underestimating resource expansion unless exploration or technological progress is constrained. As for the acceleration rate, historical CAGR of 3.9% corresponds to $\beta \approx 0.04$, suggesting moderate acceleration scenarios ($\beta=0.05$) align best with observed trends. Higher acceleration ($\beta=0.1$) reflects optimistic futures driven by innovation but exceeds historical growth patterns. Scenarios with multiple jumps (e.g., Two or Three Jumps) reflect a more

⁶³ International Copper Study Group, 2023. The world copper factbook. <https://icsg.org/download/factbook2023/>

realistic and sustainable pattern of resource growth, spreading discoveries over time and providing punctual but reoccurring relief to price pressures.

The results of our sensitivity analysis highlight two insights.

First, moderate and sustained supply-side progress is sufficient to break the explosive dynamic of the baseline simulations, creating conditions that allow for non-explosive, non-zero (NENO) equilibria in all scenarios.

Second, the energy transition scenarios are particularly sensitive to improvements in resource availability, with potential for price stabilization or even reduction if technological advancements continue at historical rates or better. By contrast, the BAU scenario remains structurally more challenging: its persistent demand from traditional sectors limits the extent to which supply-side progress can fully mitigate long-term price pressures, though the trajectory becomes far less destabilizing.

Overall, this sensitivity analysis underscores the central role of technological progress and exploration in ensuring long-term copper market sustainability. Even conservative improvements in resource accessibility can substantially ease the market tensions projected under both energy transition and non-transition scenarios.

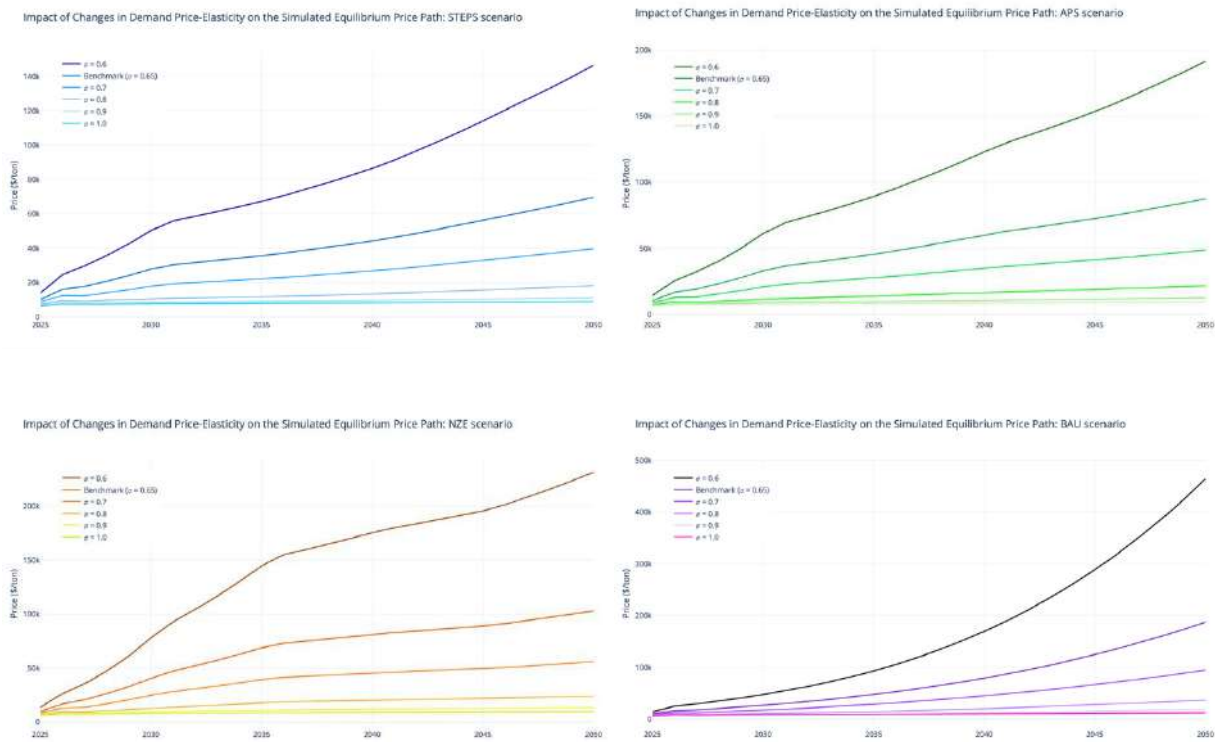
5. Demand Elasticity as the Tipping Point Between Stability and Explosive Prices

In this analysis, we test values for the elasticity of substitution (σ) in our CES framework within the range of 0.6 to 1.0. These values are chosen to reflect plausible ranges of substitution elasticity for industrial metals like copper, based on empirical findings and assumptions about the long-term adaptability of demand to price changes, as argued earlier in the calibration section (Section II.1.c). They encompass plausible substitution behaviors for industrial metals like copper. Values closer to 0 reflect inelastic demand, consistent with the limited short-term substitutability of copper in critical applications such as renewable energy infrastructure, electric vehicles, and electricity grids. These inelastic conditions align with empirical findings that suggest demand for metals essential to the energy transition is relatively unresponsive to price changes, particularly in the near term (Tilton and Guzmán, 2016; Stuermer, 2018).

The sensitivity analysis reveals that *even minor variations in demand elasticity (σ) induce substantial differences in long-term price projections*, particularly in high-demand scenarios such as NZE and BAU. Specifically, a decrease in elasticity from 0.65 to 0.60 results in markedly greater increases in projected prices compared to an equivalent increase to 0.70. This asymmetry is particularly pronounced in scenarios

characterized by aggressive decarbonization targets or unconstrained economic growth, where baseline price trajectories are already elevated.

Figure 3.7: Impact of Changes in Demand Elasticity on the Simulated Equilibrium Price Path for Each Scenario



These results underscore the asymmetric impact of elasticity changes, wherein reductions amplify cumulative price trajectories due to the exponential effects of demand insensitivity, while increases dampen price growth more gradually. The asymmetry is most pronounced in high-demand scenarios, as they exhibit greater sensitivity to changes in price responsiveness. In contrast, scenarios like STEPS and APS, which assume more moderate growth trajectories, demonstrate less asymmetric responses to elasticity changes, highlighting the stabilizing effect of more moderate baseline price growth.

The findings align with well-established results in the literature on resource economics, which highlight the nonlinear and compounding effects of elasticity changes in long-term demand models. Kilian (2009) emphasizes that small variations in elasticity can have amplified effects in high-growth scenarios due to feedback loops between price, demand, and supply constraints. Similarly, Pindyck (1980) notes that elasticity plays a critical role in shaping resource exhaustion trajectories, with lower elasticity intensifying demand pressures over time. Tilton and Guzmán (2016) discuss similar dynamics in mineral markets, where reduced elasticity exacerbates long-term resource scarcity concerns, particularly under aggressive energy transition scenarios. Stuermer (2018) extends this analysis by showing how high-growth scenarios amplify the nonlinearities inherent in resource demand models, leading to outsized sensitivity

to elasticity assumptions. Dasgupta and Heal (1979) further demonstrate that the impact of elasticity is particularly pronounced in markets characterized by high baseline demand and limited substitution possibilities, such as copper and other industrial metals. The results of this analysis are consistent with these findings, particularly in high-demand scenarios like NZE and BAU, where the compounding effects of reduced elasticity magnify demand trajectories significantly.

IV. Discussion and Conclusion: Sustainability under Constraints

1. Technological Progress in Supply: Why Abundance Does Not Guarantee Immediate Availability

The model presents a comprehensive view of how equilibrium prices and quantities are shaped by the interaction between the global demand and supply of copper over time, especially as cumulative extraction (c_t) approaches the cumulative availability curve limit c_{max} . Applying these theoretical insights to copper calibration results reveals specific trends and challenges in managing copper resources sustainably, particularly as copper demand grows in response to global energy transition and economic development.

It is important to note that this analysis focuses exclusively on primary supply dynamics and does not account for secondary supply factors, such as recycling or the development of circular economy practices (see Chapter 4).

The results of the copper calibration results reflect sharp price growth in case of minimal expansion in supply, in line with copper extraction now entering the right flat phase of the CAC. In this phase, resource constraints become more binding, and price increases yield diminishing returns in terms of new resource availability. This phase dependency has critical implications for understanding when and how copper scarcity might drive future price escalations, as well as for anticipating shifts in copper availability as cumulative extraction advances.

In this situation, demand elasticity may play a crucial role in moderating scarcity-driven price increases. In the copper calibration, using an intermediate elasticity of $\sigma = 0.65$ provides some flexibility, allowing copper demand to adjust in response to rising prices. Calibration results show that copper demand remains steady despite rising prices, indicating a lower-than-expected effective elasticity in critical sectors, such as infrastructure construction, or potentially clean technology deployment pressures. In such cases, copper demand shows inelastic, leading to sustained price pressure as demand remains high, even as resources become scarcer and supply-side bottlenecks are encountered. This outcome aligns with the model's prediction that low elasticity accelerates depletion and drives persistent price increases.

In the copper calibration, persistent scarcity-driven price growth (though it does not lead to a rapid approach toward resource limits) may nevertheless indicate that one of the convergence conditions highlighted in the Chapter 2, namely the ability of price to reflect depletion effects through supply constraints, is not met. In this scenario, long-term sustainability of the copper market could be at risk, with implications for policy and industry. Demand-side flexibility, such as enhancing substitution possibilities or improving recycling efficiency, would then become essential to managing copper scarcity and stabilizing market conditions. The calibration highlights that if market conditions and policy do not support convergence, copper may experience unsustainable price increases.

The numerical calibration also indicates that, in the absence of further technological progress in extractive industries, the attainability of energy transition policy scenarios is compromised. Meeting these targets will require either accelerated technological innovation or complementary measures such as recycling, secondary sourcing, and demand-side management to address unmet needs and support the long-term sustainability of copper resources.

Technological progress, modeled as an exogenous increase in recoverable resources, significantly alleviates long-term price pressures by expanding the cumulative availability limit over time. Whether gradual (linear), accelerating (exponential), or sudden (discrete jumps), such progress delays or reduces scarcity-driven price increases by pushing back the supply constraint. While linear growth helps moderate price trajectories in moderate-demand scenarios, only exponential growth consistently offsets rapid demand expansion.

This sharp decline in prices also underscores a limitation of the model: it overlooks the supply bottlenecks inherent in developing extractive capacities. Even if resources are abundant, their economic availability depends on multiple factors:

- 1) *Mining and Refining Capacity*: The ability to scale up extraction and processing facilities is constrained by investment, technology, and regulatory barriers. Significant expansions in capacity require long-term capital commitments, technological upgrades, and favorable regulatory environments, which are not immediately responsive to resource availability
- 2) *Operational Constraints*: Infrastructure development, labor availability, and environmental challenges often delay or limit production rates. For example, transporting ore from remote deposits may require extensive investment in rail, ports, or roads, while labor shortages or environmental restrictions can further hinder operations.
- 3) *Capital and Policy Factors*: High capital requirements and inconsistent or unstable policy environments frequently restrict how quickly extractive capacity can grow. Projects in developing countries, for

instance, may face heightened political and financial risks, therefore deterring investors and delaying development timelines.

The lack of these constraints in the model explains part of the disproportionately large price reductions observed in scenarios like STEPS and APS. These bottlenecks would moderate the price impacts of increased resource growth, particularly in the short to medium term, where capacity constraints and time lags dominate market dynamics.

Furthermore, *high-jump scenarios* represent optimistic futures with major breakthroughs in resource identification or technological development. However, these scenarios likely underestimate the challenges of scaling extractive capacity, and the time required to operationalize new resources. They implicitly assume that discovered resources can be translated directly into supply, ignoring significant bottlenecks such as infrastructure constraints, including transport and processing facilities, regulatory delays associated with permitting and environmental approvals, or economic feasibility, particularly for low-grade or technically complex deposits. These assumptions simplify the real-world dynamics of supply expansion. While resource jumps provide immediate relief to price pressures in the model, the practical translation of these resources into market supply would involve significant delays, reducing the extent of short-term price reductions and altering long-term equilibrium trajectories. Despite their limitations, these results align with Guj and Schodde (2025), who note that perceptions of impending scarcity often drive price increases, while resource expansions and capacity improvements provide downward pressure.

In scenarios with linear or exponential growth, price stabilization occurs gradually, reflecting the continuous addition of resources. Conversely, in jump-based scenarios, significant reductions in price occur at the time of resource reclassification or discovery, although these effects may diminish over time as market adjustments take place. The timing and magnitude of these jumps are critical, as earlier or larger resource additions provide greater price relief in the long term.

2. Model Simplifications and Caveats

While the model offers a structured framework for understanding the dynamics of metal demand and supply under cumulative resource constraints, it has several limitations:

- *Assumptions in Demand and Supply Functions:* The model relies on static functional forms for demand (CES) and supply (logistic), assuming fixed elasticity and responsiveness over time. However, these parameters could shift due to changes in technology, market structure, or consumer preferences, which the model does not capture. For instance, demand elasticity may

evolve as recycling technologies improve or as substitutes for copper become more viable, while supply responses might adjust due to innovations in extraction methods.

- *Some Simplification of Technological Progress and Resource Expansion:* Technological advancements are assumed to expand resource availability uniformly via the CAC, overlooking the variability in technology's impact across regions or resource types. Technological improvements could sometimes unlock entirely new resource types (e.g., deep-sea or low-grade deposits), a factor not captured by the sigmoid CAC function, which assumes gradual availability increases rather than sudden expansions or paradigm shifts.
- *Focus on Price-Driven Adjustments:* The model largely centers on market-driven, price-based adjustments, assuming efficient markets and unregulated price responses. However, policy interventions, such as subsidies, trade restrictions, or environmental regulations, could substantially alter demand, supply, or price behavior. Policies aimed at promoting recycling, setting price floors, or limiting extraction could decouple resource availability from price in ways that the model does not reflect.
- *Possibly Underestimated Elasticity in Some Specific Sectors:* While the CES function allows for flexible demand, it may underestimate elasticity in sectors with strong responses to price signals, such as consumer goods and certain manufacturing industries. In these sectors, demand could decline more sharply than expected in response to rising prices, particularly if substitutes or recycled materials are readily available. This potential for over-adjustment may lead the model to underestimate reductions in cumulative extraction and overestimate equilibrium stability, especially when sectors with high elasticity are prominent in overall demand.
- *No Recycling and Secondary Supply Dynamics:* This version of the model does not account for secondary sources like recycled copper, which could help alleviate primary extraction pressures and mitigate scarcity-driven price increases. Recycling could contribute significantly to copper supply as policies and technologies evolve, yet by excluding it, the model may overstate resource constraints and overlook a vital component of long-term equilibrium.
- *Simplified Treatment of Scarcity Signals and Market Expectations:* Assuming that prices effectively signal scarcity, the model does not account for speculative behaviors or market psychology, which can distort prices in the short term. Resource markets often experience volatility due to factors like investor speculation, geopolitical risks, or demand shocks. The model's reliance on smooth price adjustments limits its ability to capture sudden fluctuations, reducing its predictive power in volatile market conditions.

- *No Regional Variability and Heterogeneity:* The model assumes globally uniform demand and supply functions, lacking differentiation across regions where extraction costs, resource endowments, or demand growth rates may vary. This uniformity limits the model's applicability to global markets, where regional differences and trade barriers significantly impact price dynamics and resource availability.
- *No Consideration of Market Structure and Competition Effects:* The model does not account for the growing concentration of activities in the copper supply chain⁶⁴, nor does it incorporate competition effects among producers. It treats the market as perfectly competitive, implicitly assuming that extraction costs and market prices are equal, which overlooks the strategic behaviors of firms with market power. In reality, the copper supply chain involves distinct stages—extraction, processing, and distribution—each potentially dominated by a limited number of players whose pricing strategies, investment decisions, and capacity management can influence market outcomes. By ignoring these competitive dynamics, the model may underestimate the potential for price markups, supply withholding, or coordinated responses that could amplify scarcity signals and price volatility. This simplification could lead to an overly optimistic representation of market efficiency and the speed of price-based adjustments.

3. Supply as the Ultimate Arbiter

The numerical calibration and sensitivity analyses presented in this chapter provide a detailed assessment of the long-term dynamics governing copper prices and cumulative extraction under different demand scenarios and supply-side assumptions.

The results show that, in the absence of supply-side technological progress, the model consistently produces steep price increases over the 2025-2050 horizon across all scenarios. In particular, the BAU scenario generates an explosive price trajectory over the long term, largely due to the persistent growth of traditional, fossil-related sectors. This result is notable because it indicates that substantial supply pressures are not limited to clean energy transition scenarios but may also emerge in non-transition pathways. In fact, the BAU scenario produces the most destabilizing price pattern, despite its limited focus on clean technologies.

In contrast, the STEPS and APS scenarios show steep but non-explosive price growth, remaining within the bounds of non-explosive, non-zero (NENO) equilibria. The NZE scenario exhibits the

⁶⁴ IEA (2025), Global Critical Minerals Outlook 2025, IEA, Paris <https://www.iea.org/reports/global-critical-minerals-outlook-2025>, Licence: CC BY 4.0

fastest price increases, especially in the early stages, driven by aggressive deployment of clean technologies. However, the embedded material efficiency and substitution assumptions in NZE contribute to a relative slowdown in demand growth beyond 2035, which helps to moderate the long-term price trajectory.

Despite the differences in price patterns across scenarios, the model consistently finds that cumulative extraction volumes converge toward similar levels by 2050, regardless of the demand pathway. The timing at which each scenario reaches the cumulative availability limit varies, but in all cases, the supply ceiling imposes a hard constraint on achievable volumes.

The analysis of unmet needs reveals that all scenarios face significant supply gaps. By 2050, 33% of projected requirements remain unmet in STEPS, 36% in APS, and 40% in both NZE and BAU. These gaps reflect the limited capacity of the supply side to respond to the required volumes, particularly under the assumption of no further technological progress. In the NZE scenario, unmet needs materialize despite the demand-side efficiency and material-sobriety assumptions embedded in this pathway (for instance: reduced material intensity of clean technologies, slower underlying demand growth compared with STEPS or BAU). In contrast, in BAU the unmet needs arise from unchecked demand growth in traditional sectors, with no efficiency improvements to moderate this trajectory.

Even moderate supply-side technological progress has a substantial effect on long-term price dynamics. When assuming linear or exponential growth in the cumulative availability limit, the system shifts away from explosive behavior and toward more stable, or even decreasing, price trajectories. The introduction of exogenous technological progress delays or mitigates scarcity-driven price increases, especially in the energy transition scenarios. In contrast, the BAU scenario remains structurally more constrained, though supply-side progress still moderates its price growth substantially.

Among the tested forms of technological progress, exponential growth scenarios produce the strongest dampening effect on prices, often leading to bell-shaped price curves with peaks occurring around 2040. Jump scenarios—representing sudden resource discoveries—produce sharp but temporary price relief, with the underlying price trend eventually returning to a steady upward path. The extent to which these jumps modify long-term dynamics depends on their magnitude and timing.

The sensitivity of the model to the elasticity of demand is also significant. Small variations in elasticity assumptions induce substantial differences in long-term price projections, particularly in high-demand scenarios such as NZE and BAU. The analysis confirms that the price trajectories are especially sensitive to reductions in elasticity, which amplify scarcity-driven price increases.

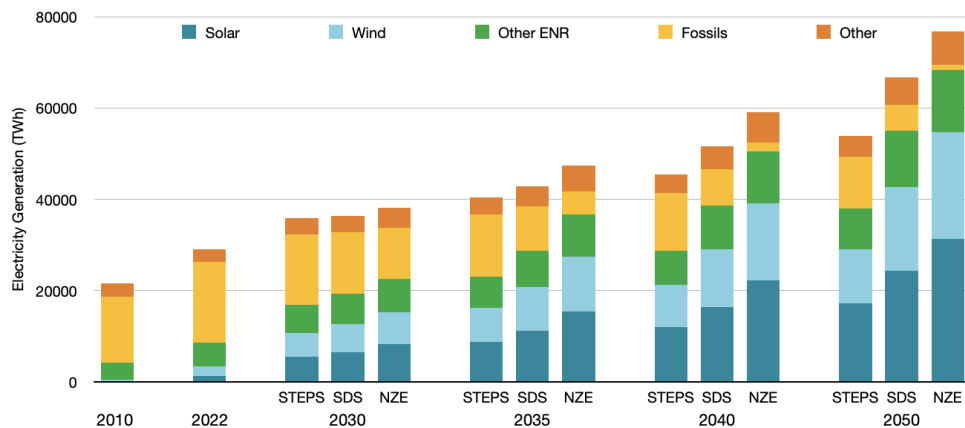
V. Appendix: Calibration of the Projected Copper Need Function

This section outlines the calibration of the projected need function, which estimates annual copper needs based on key parameters across energy sector dynamics, technological progress, and economic growth indicators⁶⁵. For each year and scenario, the function adjusts to reflect the energy mix's composition, advances in low-carbon technologies, and shifts in demand for electric vehicles, electricity grid expansion, and consumer goods. The calibration approach involves sourcing data from sector-specific studies, industry standards, and scenario-based projections to reflect the most current and relevant assumptions. Energy mix shares, for example, are calibrated based on national and international energy outlooks, while metal intensities reflect recent advances in renewable technologies and infrastructure needs. Macroeconomic indicators are adjusted according to long-term GDP and energy consumption forecasts, capturing the broader growth in energy demand and manufacturing.

1.3. Calibration of the Energy Mix Shares $[\alpha_{j,t}]_j$

The energy mix shares $\alpha_{j,t}$ represent the percentage contributions of fossil fuels (coal, oil, and natural gas) and renewable sources (solar, wind, nuclear, and bioenergy) in the total energy supply, varying across each scenario. This parameter is calibrated based on projections for each scenario, reflecting the relative shares of each energy source as they evolve toward 2050⁶⁶. We use data from the IEA. Additional details are available upon request.

Figure 3.8: Shares of Different Energy Sources in Electricity Generation Across Different Energy Transition Scenarios



Source: IEA's *The Role of Critical Minerals in Clean Energy Transitions Report* (2021)

⁶⁵ <https://www.oecd.org/environment/waste/highlights-global-material-resources-outlook-to-2060.pdf>

⁶⁶ <https://www.iea.org/reports/world-energy-outlook-2023>

1.4. Calibration of the Metal Intensities δ_j , β_t , and $\delta'_{GDP,t}$

1.4.1. Copper Intensity of the Wind Sector δ_{wind}

Wind turbines, particularly offshore installations, require a significant volume and variety of materials. Copper is indispensable in these systems, primarily for electrical wiring and generator windings, ensuring efficient energy transmission. Unlike some other materials used in turbine construction, where efficiency gains are anticipated, copper's role in turbine infrastructure is unlikely to diminish⁶⁷. While innovations may gradually reduce the demand for rare earth elements (such as neodymium, praseodymium, and dysprosium) through alternative magnet technologies, no comparable reductions are expected for copper use. The unique conductivity and durability requirements that copper fulfills make it challenging to replace or reduce without compromising performance. Therefore, in this model, the copper intensity factor for wind energy δ_{wind} is considered constant over time.

A standard onshore wind turbine of 2.5-3 MW contains roughly 8 tons of copper⁶⁸, with offshore turbines requiring up to 50% more due to extensive cabling⁶⁹. This reliance underscores copper's stable role in wind energy infrastructure, where, unlike other materials, no foreseeable efficiency gains are achievable.

1.4.2. Copper Intensity of the Solar Sector δ_{sun}

Solar photovoltaic (PV) panels rely on a variety of materials to generate and conduct electricity efficiently. The primary material in solar cells is silicon, typically manufactured as monocrystalline or polycrystalline wafers. Aluminum frames provide structural support and protection for the cells and glass layers, while copper is essential within the panel for electrical cabling and connections, enabling the transport of electricity generated by the cells. Additionally, certain conductive metals—silver, lead, tin, indium, gallium, and selenium—are used in smaller quantities due to their specific conductive properties.

In terms of material intensity, PV panels require approximately 67.9 tons of steel, 7.5 tons of aluminum, and 4.6 tons of copper per MW of installed capacity (assuming an average generation capacity of 400

⁶⁷ Carrara S., Alves Dias P., Plazzotta B. and Pavel C., Raw materials demand for wind and solar PV technologies in the transition towards a decarbonised energy system, EUR 30095 EN, Publication Office of the European Union, Luxembourg, 2020, ISBN 978-92-76-16225-4, doi:10.2760/160859, JRC119941

⁶⁸ <https://op.europa.eu/en/publication-detail/-/publication/19aae047-7f88-11ea-aea8-01aa75ed71a1/language-en>

⁶⁹ <https://www.iea.org/reports/global-critical-minerals-outlook-2024>

watts per panel)⁷⁰. While moderate efficiency improvements in aluminum and steel usage are anticipated by 2050, copper's material intensity is expected to remain constant due to the absence of viable alternatives or reductions in copper's functional requirements within PV systems.

1.4.3. Copper Intensity of Electric Vehicles δ_{EV}

Electric vehicles (EVs) are at the center of extensive research efforts, particularly around battery technology. Over the next decade, significant advancements in battery cathodes and anodes are anticipated. Manufacturers are already working to reduce cobalt content by shifting from nickel-manganese-cobalt (NMC) configurations like 6/2/2 to lower-cobalt alternatives, such as 8/1/1 or even 9/0.5/0.5 compositions. Lithium iron phosphate (LFP) batteries, though lower in energy density, are also gaining market share. Importantly, an electric vehicle requires, on average, six times more metals than a conventional car (210 versus 35 kg per vehicle⁷¹), making it a sector that is particularly influential in shaping future metal needs. These potential changes are considered in our projections up to 2030. Beyond this horizon, however, technological developments remain uncertain, so we could assume a stable battery composition after 2030, despite potential breakthroughs like solid-state and cobalt-free batteries.

Copper's role in EVs is consistent. Used extensively in wiring for motors, batteries, and charging infrastructure, copper is critical for vehicle performance. Without feasible substitutes or anticipated reductions in copper intensity⁷², δ_{EV} is considered constant in this model, reflecting copper's enduring importance in electric vehicle systems.

1.4.4. Copper Intensity of Electric Networks δ_E

Electric network cables comprise primarily copper (36%) and aluminum (63%), with trace amounts of zinc. Copper, with 60% higher conductivity than aluminum, is the preferred material for applications where performance and durability are paramount. However, aluminum's lighter weight (approximately 30% of copper's weight for equivalent volume) makes it advantageous in applications where weight constraints are critical. For instance, cables of 10mm² cross-section contain approximately 17 kg of aluminum and 32 kg of copper per kilometer, while larger 300mm² cables contain up to 510 kg of aluminum and 961 kg of copper per kilometer.

⁷⁰ Carrara S., Alves Dias P., Plazzotta B. and Pavel C., Raw materials demand for wind and solar PV technologies in the transition towards a decarbonised energy system, EUR 30095 EN, Publication Office of the European Union, Luxembourg, 2020, ISBN 978-92-76-16225-4, doi:10.2760/160859, JRC119941

⁷¹ <https://www.ica.org/reports/the-role-of-critical-minerals-in-clean-energy-transitions/executive-summary>

⁷² Ibid

The choice between copper and aluminum involves significant technical and economic trade-offs. While copper offers superior conductivity and long-term durability, its high and volatile price makes it a costly option. Aluminum, being more abundant and less expensive to produce, often provides a more stable long-term supply. Only under extreme carbon tax scenarios would aluminum's higher carbon footprint make copper more economically favorable. Yet, aluminum's shorter lifespan, especially its susceptibility to corrosion, continues to present technical challenges, complicating the substitution potential.

Given these complexities, forecasting the evolution of copper and aluminum intensities in electric cables is challenging. Thus, we adopt the IEA assumptions, which suggest a) A stable aluminum-to-copper ratio across all scenarios through 2050; b) Gradual material efficiency gains of 1% per year beginning in 2040 under the STEPS and APS scenarios and 2% per year beginning in 2050 under the Net Zero Emissions scenario. These changes are marginal and do not significantly alter the copper intensity of electric networks. These assumptions allow us to hold the copper intensity parameter for electric networks δ_E effectively constant, providing a simplified yet robust basis for our projections.

1.4.5. Copper Intensity of Fossil Fuel Technologies β_t

While we assume constant copper intensities for other sectors, the copper intensity in fossil fuel technologies β_t requires dynamic modeling due to the anticipated impact of decarbonization and energy transition efforts. As fossil fuel technologies adapt to reduce greenhouse gas emissions, the copper demand per unit of energy produced is expected to rise, driven by the need for advanced technologies and upgrades to existing infrastructure.

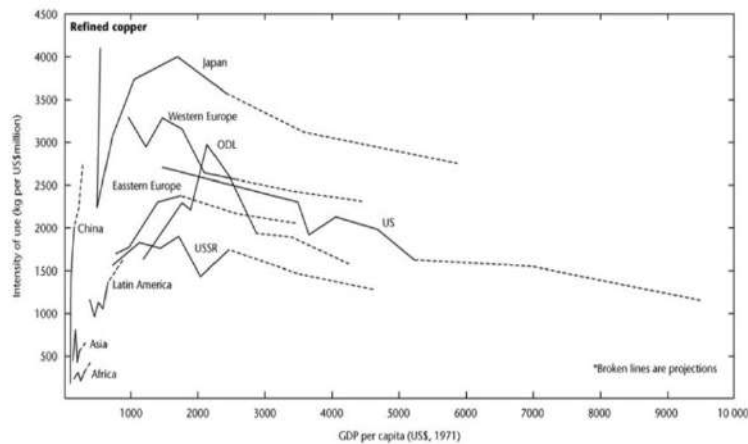
The unique electrical properties and durability of copper make it irreplaceable for the high-performance needs of fossil fuel technology upgrades. Copper's role is expected to grow as fossil fuel facilities undergo further enhancements in response to regulatory requirements and technological advancements. Thus, unlike in other sectors, we model β_t as a variable, reflecting the increasing copper demand per unit of fossil fuel energy as decarbonization and industrial efficiency efforts evolve. This approach allows us to capture copper's critical and expanding role in the energy transition within the fossil fuel sector.

1.4.6. Copper Intensity of Non-Energy Sectors $\delta'_{GDP,t}$

For the non-energy sectors, we assume that copper intensity $\delta'_{GDP,t}$ evolves similarly to the historical trends in the metal content of economic growth. This assumption rests on the notion that, in non-energy sectors, current economic development patterns reflect those observed historically in the

broader economy. As these sectors progress, they exhibit similar trends in industrialization, urbanization, and infrastructure expansion, all of which influence copper demand. This approach aligns with the established “intensity of use” hypothesis, which suggests that the metal content of economic growth generally follows an inverted-U or bell curve as economies advance.⁷³

Figure 3.9: Trends in Intensity-of-use of Refined Copper in 10 World Regions as a Function of Per-Capita GDP (referred to 1971 US\$)



Source: Malenbaum (1978)

We model this trend by setting the inflection point—where copper intensity shifts from increasing to decreasing—at a per capita GDP of \$15,000⁷⁴. Below this threshold, economies typically emphasize capital-intensive growth, resulting in higher copper demand relative to GDP, while above it, economic growth tends to favor efficiency gains and reduced metal intensity.

This differentiation captures the diverse stages of economic development globally, allowing the model to reflect varying copper consumption patterns aligned with regional economic maturity. By

⁷³ As economies grow, they move through distinct phases of metal consumption:

- During early stages of development, GDP growth is often driven by substantial investments in physical infrastructure—such as roads, buildings, and industrial facilities—resulting in high copper demand. This phase aligns with rapid urbanization and industrialization, and corresponds to high copper intensity, as predicted by neoclassical growth models like the Solow model⁷³, where capital accumulation is a primary growth driver.
- Once economies reach a certain development threshold, returns on capital accumulation diminish, and growth increasingly relies on total factor productivity (TFP) gains⁷³. Here, metal intensity begins to decline as productivity, technological advancement, and shifting consumer preferences reduce per capita metal consumption. Technological innovations enable material substitutions and efficiency improvements, reducing the copper content in goods. Additionally, as economies shift towards a higher proportion of services, per capita demand for metals decreases.

⁷⁴ This value is based on the World Bank's classification of economies by income level. This threshold is expressed by the World Bank in terms of gross national income (GNI) per capita, but to simplify our analyses and make comparisons easier, we make the approximation that GNI per capita is roughly equivalent to GDP per capita. In addition, we consider a rounded value of \$15,000 for an exact value of \$13,846 used by the World Bank. See <https://datahelpdesk.worldbank.org/knowledgebase/articles/906519>

anchoring $\delta'_{GDP,t}$ to GDP per capita, we effectively simulate how copper demand in non-energy sectors evolves alongside broader economic growth trends.

1.5. Calibration of the Final Energy Consumption X_t

Global final energy demand is projected to grow from approximately 630 exajoules (EJ) in 2022 to around 670 EJ by 2030 in the STEPS scenario, at an annual growth rate of 0.7%. Beyond 2030, continued demand growth is anticipated in emerging and developing markets, which offsets declines in advanced economies. The APS scenario, with greater emphasis on renewable deployment, anticipates a slight reduction in energy demand through 2030, while the NZE scenario projects a 1.2% annual decrease due to more rapid electrification and efficiency improvements.

1.6. Calibration of the Demand for Electric Vehicles EV_t

Electric vehicle (EV) demand is set for sustained growth, supported by environmental policies, technological advancements, and infrastructure expansion⁷⁵. Regulatory measures worldwide—such as tax incentives, subsidies, and restrictions on internal combustion engines—are making EVs a cornerstone of emissions reduction strategies. Alongside policy support, technological advances in batteries, including cost reductions and enhanced energy density, have improved EV accessibility and consumer appeal. In 2023, EVs accounted for 18% of global new vehicle registrations, with sales concentrated in China, Europe, and the United States⁷⁶. These three regions drive global EV adoption, with China alone representing nearly 60% of new EV registrations. By 2030, annual EV sales are projected to reach 20 million in the STEPS scenario, 40 million in APS, and 55 million in NZE.

The relevance of this growth for copper demand is profound. Each EV contains significantly more copper than a conventional vehicle, given copper's essential role in battery wiring, power electronics, and charging systems. As the global EV fleet expands, particularly in high-adoption regions, the cumulative copper demand grows sharply. Calibrating EV_t across scenarios thus captures this scaling effect, where accelerated EV adoption translates to a substantial increase in copper requirements, underscoring EV demand as a critical driver of future copper needs.

Post-2030, the EV market's growth builds on the trends set in the 2020s decade. In the STEPS scenario, moderate annual EV sales growth should continue, as policy support and technological advancements maintain steady adoption rates. In the APS scenario, the market accelerates further post-

⁷⁵ <https://www.iea.org/energy-system/transport/electric-vehicles>

⁷⁶ <https://www.iea.org/reports/global-ev-outlook-2024/trends-in-electric-cars>

2030 as ambitious emissions reduction policies and technological breakthroughs drive rapid expansion. In the NZE scenario, post-2030 growth is to be exponential, with near-universal EV adoption driven by strict climate policies and technological leadership, leading to the highest copper demand due to widespread infrastructure development and increased copper usage in EVs.

1.7. Calibration of the Grid Expansion E_t

In all scenarios, global electricity demand is projected to grow significantly due to population growth, rising GDP per capita, and increased electrification of end-uses⁷⁷. By 2050, electricity demand is expected to exceed current levels by over 80% in the STEPS scenario, 120% in APS, and 150% in NZE. This demand growth is primarily driven by emerging markets and developing economies, which account for nearly three-quarters of the total increase across scenarios. In addition to meeting rising demand, the grid must also support a growing share of intermittent renewable energy sources, such as solar and wind, necessitating more resilient and extensive grid infrastructure.

The expansion and reinforcement of transmission and distribution networks are critical to supporting the accelerated electrification and renewable integration envisioned in all scenarios. The global grid is expected to grow by approximately 18% by 2030 in STEPS and APS, and by 20% in NZE, with transmission and distribution networks extending by about 2 million kilometers annually until 2030 in NZE⁷⁸.

Copper plays a central role in this expansion due to its high electrical conductivity, making it indispensable for transmission lines, transformers, and distribution networks. In fact, the demand for copper in grid infrastructure alone is projected to double in the 2041-2050 period compared to 2012-2021 in the APS scenario. From an average of 5 Mt/year in 2012-2021, copper demand for grid infrastructure is anticipated to rise to 5.5 Mt/year by 2022-2030 and reach 12 Mt/year in NZE by 2041-2050⁷⁹. This growth represents a significant portion of global copper supply, underscoring the grid's structural impact on future copper requirements.

1.8. Growth of the Non-Energy Sector Y_t

For simplicity, the growth of the non-energy sector Y_t is modeled to align with OECD GDP growth projections. This calibration reflects the increasing demand for metals in non-energy sectors driven by

⁷⁷ <https://www.ica.org/reports/electricity-grids-and-secure-energy-transitions>

⁷⁸ <https://www.ica.org/reports/world-energy-outlook-2023>

⁷⁹ <https://theoregongroup.com/commodities/a-lot-more-copper-needed-to-expand-global-electricity-grid/>

infrastructure, housing, industrial production, and consumer goods—all closely linked to economic and demographic growth in consuming economies.

Given the diverse development stages globally, granular modeling by GDP per capita is essential for capturing regional variations in metal demand. Emerging economies, with faster GDP growth and lower current GDP per capita, are poised for greater metal intensity in their growth phase. For these regions, copper demand is driven by rising infrastructure and industrial needs until reaching an inflection point—around \$15,000 GDP per capita—after which copper intensity stabilizes and ultimately declines. In contrast, for developed economies, metal intensity in the non-energy sector is expected to decrease steadily, particularly under scenarios emphasizing resource efficiency and economic sobriety.

This calibration of Y_t enables a realistic projection of future metal requirements, where copper demand in the non-energy sector aligns closely with economic growth dynamics, urbanization, and industrialization patterns, especially within high-growth, emerging economies. By reflecting varied growth trajectories, Y_t captures how structural changes and regional development shifts shape the evolving demand for copper and other metals outside of energy-specific applications.

CHAPTER 4: Recycling and Secondary Supply in Metal Markets: An Extended Analytical Framework

Global demand for metals is projected to rise steadily in the coming decades, propelled by technological innovation, demographic growth, urbanization, and, more recently, the acceleration of the energy transition. This structural transformation in material use—linked to electrification, renewable infrastructure, and digital technologies—raises several risks for metal markets. In the short run, rapidly expanding demand may collide with the relative rigidity of supply, constrained by geological and technical limits, generating imbalances and forcing prices to adjust sharply to restore a non-explosive, non-zero equilibrium. Over the medium to long term, cumulative extraction may deplete available resources to the extent that, absent significant technological progress, the supply base cannot sustain projected demand.

Metals, however, are not ordinary commodities. Unlike fossil fuels, which are irreversibly consumed when used, most metals are physically durable and can be recovered from end-of-life products or industrial waste. This recyclability gives rise to secondary supply, defined here as the flow of metal reintroduced into the economy through recycling processes. Secondary metals can, to varying degrees, substitute for primary metals extracted from the earth, depending on factors such as purity requirements, collection infrastructure, and scrap availability. This distinctive feature makes metals economically and environmentally unique, and it fundamentally alters how supply constraints and price dynamics should be modeled—particularly in a long-term context where circularity is expected to expand.

At the same time, metals differ substantially in the degree to which primary and secondary outputs are commercially integrated. For many base, industrial, and precious metals—including copper, aluminum, tin, lead, zinc, magnesium, tungsten, silver, gold, and titanium—the recycled form is physically and commercially indistinguishable from the primary metal once refined. These markets are therefore characterized by a single good traded at a single price, regardless of origin. It is this category of metals that constitutes the analytical focus of the present model.

Other categories of metals do not fall within this scope. Alloys (such as steel or bronze), industrial by-products (such as cobalt, often a co-product of copper or nickel mining), and rare earth elements do not conform to a single-price setting because either their secondary streams produce differentiated outputs, or their primary supply does not follow a cumulative availability curve (CAC) formulation. These metals were already excluded in the baseline analytical model, and their exclusion here does not

narrow the general validity of the framework: in volumetric terms, the overwhelming share of global metal consumption occurs precisely in markets where the “one good, one price” assumption holds.

Against this backdrop, medium- to long-term forecasts project substantial growth of the global metal recycling market. Estimates vary widely depending on methodological scope, metals included, and value chain stages considered, but all converge on the expectation of sustained expansion. Precedence Research (2025), for example, projects the market to grow from USD 295 billion in 2024 to over USD 630 billion by 2034, driven by advancements in recycling technologies—particularly AI-enabled sorting—and rising demand for sustainable inputs. Grand View Research (2023) and Databridge Market Research (2024) estimate market sizes closer to USD 850-890 billion in 2024, with compound annual growth rates of 4-5% through the early 2030s, supported by demand from construction, automotive, and green infrastructure sectors. Several reports also emphasize the growing role of Asia-Pacific in global capacity expansion and the alignment of recycling with broader policy efforts to reduce emissions and secure raw material supply. These projections reflect not only increased scrap generation and processing but also the anticipated economic value of circular flows as metal-intensive industries adjust to sustainability targets and resource constraints.

Yet, while recent industry forecasts anticipate robust growth in the value of the global metal recycling market, Fizaine (2020) provides a more cautious assessment of physical recycling rates, particularly over the short term. His analysis suggests that despite economic and environmental incentives, most metals—especially those embedded in complex consumer products—continue to face structural barriers to higher recycling rates. This divergence between projected market value growth and stagnating material recovery underscores the importance of distinguishing between economic potential and realized outcomes in secondary supply development. It also reinforces the relevance of a dynamic, state-dependent modeling approach that captures both the promise and limitations of recycling as a structural response to long-term demand pressures.

In the initial calibration of our analytical model—focused on copper and restricted to primary supply—we find that energy transition scenarios may tend to generate steep, convex, and potentially unsustainable price trajectories in case of insufficient technological progress in extraction activities. This is especially true when cumulative extraction approaches the resource limit under constant technology assumptions, as suggested by our calibration where c_{max} is nearing the asymptotic maximum L . More precisely, the results of Chapter 3 show that primary supply alone is insufficient to meet projected copper requirements in the coming decades, even under moderate demand scenarios. Across all energy transition pathways, substantial unmet needs persist, ranging from 33% to 40% of projected requirements by 2050, while price trajectories tend to become steep, or even

explosive, in the absence of supply-side technological progress. The calibration also highlights that cumulative extraction quickly approaches the modeled resource ceiling, with convergence toward similar extraction volumes across scenarios, indicating that supply—not demand—is the binding constraint.

Overall, these findings suggest that, without additional sources of supply, primary copper markets may be structurally unable to accommodate long-term demand pressures, highlighting the extent to which secondary supply will determine whether projected needs can be met within feasible price and resource conditions.

The introduction of recycling into the analytical framework does more than add a supplementary source of material—it activates a different set of market mechanisms, feedback loops, and potential equilibria. Unlike primary extraction, which is subject to geological depletion and rising marginal costs, secondary supply operates under constraints tied to scrap availability, infrastructure maturity, and policy environments. Its inclusion in the model has the potential to redistribute adjustment dynamics across time, dampen unmet needs, and moderate price trajectories that would otherwise be unsustainable.

The contribution of this chapter is to extend the baseline model by explicitly incorporating secondary supply as an endogenous, state-contingent process. In doing so, the model seeks to move beyond static representations of recycling and capture the nonlinear, path-dependent dynamics that characterize real-world metal markets.

This chapter is structured as follows. Section I reviews the theoretical and empirical literature on secondary supply of metals and its influence on market stability, price formation, and system resilience. This review also highlights the methodological advances and limitations that inform the structure of the extended model. Section II introduces the extended analytical model for metals, focusing on the category of metals for which primary and secondary outputs are physically and commercially indistinguishable and therefore traded as a single good at a single price. Section III explores a series of stylized facts that emerge from the model and echo empirical regularities observed in metal markets—including the endogenous moderation of price pressures, regime-dependent recycling responses, state-contingent supply elasticity, hysteresis effects, and the evolving structure of demand. Section IV presents a numerical calibration based on copper market data under energy transition scenarios, assessing how secondary supply alters long-term price trajectories, cumulative extraction, and unmet demand. Section V concludes. Finally, Section VI provides an annex that turns to the case of metals where recycled and primary outputs are not perfectly substitutable, and markets display two distinct prices. It derives the conditions under which the premium between the two narrows toward convergence or remains durably segmented.

I. Recycling in Metal Economics: A Brief Overview of the Literature

This literature review synthesizes key findings from theoretical, empirical, and methodological contributions to the study of secondary supply of metals, highlighting features that motivate the modeling approach developed in the following sections.

The buffering role of recycling in moderating metal price volatility has long been recognized in resource economics. Ayres (1997) notes that metals are inherently recyclable, and their reuse can mitigate both environmental externalities and supply instability. This aligns with Moison's (1997) critique of the price-centered view of recycling: while price increases may contribute to higher recycling rates, more significant drivers often lie in the availability of scrap and technological capacity, particularly during periods of economic growth. This observation is reinforced by Söderholm and Ekvall (2020), who emphasize that recycling is governed by a complex interaction of market forces, institutional design, and technological progress.

Several studies have explored the stabilizing effects of secondary substitutability. Fleming (2011) shows that when secondary metals effectively substitute for primary supply, price volatility decreases, as the system becomes more elastic and less exposed to extraction shocks. Radetzki and Van Duyne (1985), along with Graedel et al. (2011), similarly argue that the presence of elastic secondary supply dampens the transmission of supply shocks to price fluctuations. Blomberg and Hellmer (2000) demonstrate these dynamics in the case of lead, and Reck et al. (2010) highlight that recycling reduces the amplitude of price cycles by injecting flexibility into supply.

The assumption that prices alone drive recycling uptake has been increasingly challenged. Fizaine (2020), in a cross-sectional analysis of WEEE recycling, finds that a \$1000 increase in metal price results in only a modest increase in recycling rates—approximately 1.94 percentage points. His findings underscore the importance of technical variables such as metal concentration in products, the scale of recycling infrastructure, and the relative weight of secondary supply in total consumption. These results reinforce Moison's and Söderholm and Ekvall's assertions that price incentives, while relevant, are insufficient in isolation.

Dynamic models of recycling growth often capture this limited short-run responsiveness through S-shaped or logistic adjustment trajectories. Studies by Reuter et al. (2006), Ayres and Peiró (2013), and Zhang et al. (2019) show that recycling capacity tends to follow a regime-dependent path: slow initial uptake, followed by acceleration as infrastructure matures and policy incentives scale, and eventually plateauing due to thermodynamic and economic limits. These dynamics justify using Markov processes

to model regime transitions, particularly when combined with state-dependent elasticities and nonlinear feedback.

From a methodological perspective, a range of tools has been applied to study the interaction between primary and secondary supply. Econometric approaches, including cointegration and error correction models (ECM), have been employed to assess short- and long-run relationships between primary and scrap prices (Xiarchos & Fletcher, 2009). GARCH models have proven effective at tracing volatility spillovers, revealing that secondary markets tend to dampen conditional volatility by absorbing price shocks originating in the primary market (Bollerslev, 1986; Engle, 1982). Markov-switching models (Fleming, 2011) have helped identify nonlinear regime transitions in supply behavior, demonstrating how shifts in recycling capacity coincide with abrupt changes in price dynamics.

Complementary to these econometric tools, Material Flow Analysis (MFA) and Life Cycle Assessment (LCA) provide broader systemic insights. MFA studies (Ayres, 1997; Graedel et al., 2011) quantify metal flows and scrap potential across economies, while LCA techniques (Reuter & Kojo, 2012) assess the environmental trade-offs between primary and secondary production, often revealing lower energy use and emissions for recycled metals. These tools are particularly important for calibrating long-term scenarios and informing policy designs that promote sustainable supply chains.

Despite these contributions, important gaps remain. Most existing models focus on static or linear relationships, treating price elasticity and recycling rates as either constant or externally determined. Fewer studies have addressed the implications of dynamic transitions—such as regime shifts, lagged responses, or feedback loops—in shaping long-term equilibrium behavior. In particular, the interaction between structural demand growth and the evolving maturity of the recycling sector is often overlooked.

To address these limitations, the next section introduces a dynamic analytical model that combines a Markovian framework for the evolution of recycling capacity with nonlinear supply responses. This structure lays the groundwork for a richer exploration of metal market behavior, unpacked in Section III through a series of stylized facts.

II. Revisiting the Analytical Model

Our initial framework models metal markets through the interaction of a CES demand function and a primary supply function derived from the Cumulative Availability Curve (CAC) and a logistic depletion process, with prices adjusting recursively to balance the two (see Chapter 2). In this section, we extend this framework to incorporate secondary supply from recycling, introducing additional dynamics and feedback effects on both supply and demand.

1. The Original Framework⁸⁰

We briefly remind that the original model is centered around a CES demand function, which captures the relationship between price and demand. The supply side, specifically for primary metal, is constrained by the CAC, which defines the potential for extraction based on current technology and resource availability.

The demand equation for a given metal is expressed as follows:

$$Q_{d,t} = B(t) \cdot P_t^{\sigma-1}$$

On the supply side, primary metal production is governed by the CAC, which is influenced by the price of the metal. The CAC function is defined as:

$$c_{max}(P) = \frac{L}{1 + e^{-\kappa(P-P_m)}}$$

The annual supply of primary metal is then constrained by the available quantity of metal at a given price, modeled as:

$$Q_{s,t} = \frac{c_{max}(P_{t-1})}{1 + e^{\gamma\left(\frac{c_t}{c_{max}(P_{t-1})} - \theta\right)}}$$

This supply function captures the natural limits of resource extraction, becoming more stringent as prices increase. As the quantity of the available resource decreases with cumulative extraction, the marginal cost of extraction rises, leading to slower growth in the supply of metal. This non-linear behavior in the supply curve reflects the increasing difficulty and cost of extraction as more of the metal is mined and the more easily accessible resources are depleted.

Because the model is anchored in this CAC–logistic depletion structure, it applies only to metals for which primary supply can be meaningfully represented in this way. Metals whose production does not follow a cumulative availability framework—such as alloys, industrial by-products, or co-/by-products like cobalt and rare earth elements—are therefore excluded from the scope of the model.

⁸⁰ Notations for all three equations in this subsection follow the notations introduced in Chapter 2.

2. Extending the Model to Include Recycling

Recycling introduces a second stream of supply into the market, distinct from extraction but converging on the same product. Both primary and secondary production ultimately yield a homogeneous metal that is physically and commercially indistinguishable once refined. This assumption holds for the bulk of global metal consumption—base metals such as copper, aluminum, lead, or zinc, as well as precious and industrial metals like gold, silver, tin, tungsten, or titanium. In all these cases, regardless of whether the metal is newly mined or recycled, the market trades a single good at a single price. Our analytical framework is therefore restricted to this “one good, one price” setting. Metals for which primary and secondary outputs are not perfectly substitutable, and for which markets exhibit two distinct prices, are addressed separately in the annex (section VI).

a. Demand

Accordingly, the demand function remains unchanged from the initial framework. Aggregate demand continues to be expressed as:

$$Q_{d,t} = B(t) \cdot P_t^{\sigma-1}$$

The demand function applies to the aggregate good, traded at a single price regardless of origin, and retains the same CES form as in Chapter 2 and 3, but is now defined over a market where $B(t)$, σ , and P_t reflect equilibrium against the combined availability of primary and secondary supply, thereby preserving our analytical structural while extending it to the expanded setting.

b. Primary Supply

Total supply is now decomposed into two streams: a primary supply function derived from cumulative availability and depletion dynamics, and a secondary supply function driven by recycling incentives and scrap availability.

The primary supply of metal is constrained by the available resources, which are limited by extraction capabilities. The supply of primary metal is modeled through the cumulative availability curve c_{max} , which is influenced by the market price of metal P_t :

$$Q_{s,t}^{primary} = \frac{c_{max}(P_{primary,t-1})}{1 + e^{\gamma \left(\frac{c_t}{c_{max}(P_{primary,t-1})} - \theta \right)}}$$

As previously argued, this equation reflects the reality that as extraction proceeds, the cost of extracting additional metal increases due to depletion of easily accessible resources (see Chapter 2).

c. Secondary Supply in the Extended Setting

The supply of secondary metal is influenced not only by the market price of the metal but also by macroeconomic factors, such as scrap availability and technological progress, which are often not directly captured by price alone.

A Markov process captures the dynamic nature of recycling activities, which are affected by historical states and evolving conditions in the economy. The secondary supply is modeled as:

$$Q_{s,t}^{secondary} = \eta \cdot P_t^\delta \cdot S_{t-1}$$

Where:

- $Q_{s,t}^{secondary}$ is the quantity of secondary metal supplied at time t ,
- η is a parameter reflecting recycling efficiency,
- δ is the price responsiveness of recycling,
- S_{t-1} is the state variable from the previous period $t - 1$, capturing factors like the available scrap stock, technological improvements, and macroeconomic conditions that influence recycling rates⁸¹. At low maturity, most scrap remains inaccessible: end-of-life products are not systematically dismantled, collection rates are low, and the small quantities that do circulate are absorbed almost exclusively as direct-use scrap. As systems evolve—through the expansion of recovery facilities, the adoption of sorting and refining technologies, and the establishment of regulatory frameworks—scrap stocks can increasingly be mobilized for secondary production. In highly mature states, recycling approaches a closed-loop configuration where direct-use and smelting pathways are integrated with manufacturing, collection rates are high, and yield losses are minimized. This construction means that S_{t-1} is not simply an accounting of available

⁸¹ The inclusion of S_{t-1} directly links current secondary output to the stock of metal available for recycling, which is itself determined by cumulative past primary production and dissipation rates. This formulation ensures that secondary supply cannot expand beyond the physical scrap pool, even under strong price incentives, so that growth in recycling capacity naturally becomes concave over time when primary flows are stable.

scrap stocks, but rather a state-dependent filter that governs how much of those stocks can be transformed into usable secondary copper.

The literature debates on the relative strength of price versus non-price factors in determining secondary supply, with different studies emphasizing different drivers. Many studies highlight that price is a crucial determinant in driving secondary supply, particularly because rising prices of primary metals make recycling more economically viable. Fleming (2011) and Radetzki and Van Duyne (1985) argue that higher primary metal prices create stronger incentives for recycling. As prices increase, the cost of obtaining metal from secondary sources becomes relatively more competitive compared to extraction, thus encouraging increased recycling activity. This relationship between price and secondary supply is particularly important in markets where the supply of scrap metal is sufficient and the recycling process is technologically feasible.

As stated in the literature review however, several authors temper the emphasis on price by highlighting the importance of non-price factors in shaping secondary supply. Moison (1997) and Söderholm and Ekvall (2020) argue that technological progress, scrap availability, and macroeconomic conditions often weigh more heavily than price alone. Even substantial price increases may not translate into higher recycling rates unless these enabling conditions are favorable. Fizaine (2020) provides empirical support for this view, showing that price effects on recycling rates are relatively modest and context dependent. These findings reinforce the idea that recycling dynamics arise from the interaction of price signals with structural and technological constraints—an insight that underpins the modeling approach adopted here, which explicitly accounts for both price and non-price determinants of secondary supply.

To account for these evolving and partly unpredictable dynamics, we introduce a Markov process to describe the development of secondary supply over time. Unlike a static specification, a Markov process captures the idea that recycling systems can transition between different states—such as periods of high or low recycling activity—depending on their past trajectory and external conditions. This approach is particularly suited to modeling technological progress, changes in scrap availability, and macroeconomic cycles, which are known to drive persistent shifts in recycling capacity that cannot be fully explained by price movements alone (Söderholm & Ekvall, 2020; Moison, 1997; Fizaine, 2020). The relevance of regime-based modeling is also supported by studies in metal market dynamics that highlight the occurrence of structural shifts and regime transitions in response to macroeconomic booms, technological step-changes, or scrap surges (Fleming, 2011; Radetzki & Van Duyne, 1985). Markov-switching frameworks have been successfully applied to capture such nonlinear market behaviors and the segmentation between phases of high and low recycling activity (Fleming, 2011). Additionally, historical analyses show that improvements in recycling rates and secondary supply

capacities often occur in waves rather than following smooth trajectories, driven by delayed technological adoption or policy shocks (Graedel et al., 2011; Reck et al., 2010; Söderholm & Ekvall, 2020).

Formally, we assume that the state variable S_t , which governs the non-price determinants of secondary supply, follows a Markov chain where the probability of transitioning to a new state depends only on the current state. This structure allows the model to incorporate path dependency, potential hysteresis effects, and regime-switching behaviors, where the development of recycling systems is influenced by their prior state as much as by contemporaneous price signals. By combining this probabilistic evolution of non-price drivers with the explicit price sensitivity of secondary supply, the model provides a flexible representation of metal market dynamics, better aligned with empirical observations of fluctuating recycling rates, cyclical scrap availability, and episodic technological breakthroughs.

Uncertainty is introduced because the evolution of recycling capacity depends on technological, regulatory, and investment conditions that are inherently stochastic: policy support, permitting for new smelters, global scrap trade frictions, and R&D progress. A Markov process provides a parsimonious way to represent these uncertainties without imposing arbitrary deterministic assumptions. It also allows the system to transition gradually among maturity regimes, which mirrors how real-world recycling systems develop.

We denote the recycling state at time t as S_t , which can take discrete values corresponding to different levels of recycling infrastructure, scrap mobilization capacity, and technological efficiency. State s_1 reflects an initial, underdeveloped stage of recycling infrastructure. It is characterized by limited scrap collection systems, low penetration of advanced recycling technologies, poor process efficiency, and minimal policy support. The recycling industry is fragmented and primarily driven by local initiatives or basic cost-saving incentives. Intermediate states $s_2, s_3, \dots, s_{n-1}$ represent progressively higher levels of recycling maturity. Moving through these stages typically involves expanded collection networks improving scrap availability, increased investments in recycling facilities and the development of regional processing hubs, technological upgrades that allow for better metal separation and reduction of material losses, strengthening regulatory frameworks that actively promote recycling through subsidies, extended producer responsibility (EPR) schemes, or carbon pricing incentives, as well as the industrialization of recycling as it becomes more deeply integrated into supply chains, particularly in high-growth sectors like construction, automotive, and consumer goods.

The upper state s_n represents the extrapolated physical and technological ceiling of the recycling system. It is not necessarily attainable within the considered time horizon (2025-2050) but serves as a

boundary condition. It reflects the theoretical maximum recycling rate achievable if collection systems were near-perfect, scrap flows were fully optimized, and the best available technologies were universally deployed. It may also incorporate technological breakthroughs that are not yet feasible or commercialized but that define the ultimate physical limit of material recovery.

The evolution of S_t along the discrete-time Markov chain is defined by the following transition probability:

$$\mathbb{P}(S_t = s_j \mid S_{t-1} = s_i) = p_{ij}$$

Where p_{ij} is the probability of moving from recycling state s_i to recycling state s_j between periods $t - 1$ and t . The probabilities satisfy the standard condition:

$$\sum_{j=1}^n p_{ij} = 1 \quad \forall i$$

The full set of transition probabilities is collected in a transition matrix $\mathcal{P}$. This matrix is assumed to remain constant over the projection horizon. This modelling choice stands for two reasons: 1) The drivers of transitions between recycling states—such as policy frameworks, technological improvements, supply chain integration, and market growth—tend to evolve incrementally and are path-dependent⁸². These structural features suggest that the probabilities governing state changes are stable over time rather than subject to high-frequency fluctuations; 2) The upper state s_n is not dynamically adjusted during the simulation. It is designed as a fixed ceiling extrapolated from physical laws and long-term technological possibilities. Since this ceiling is not expected to shift within the time frame considered, the structure of the transition probabilities does not require updating or recalibration during the simulation.

We assume several properties for the structure of this matrix:

⁸² The incremental and path-dependent evolution of drivers influencing transitions between recycling states has been well documented in scholarly literature. Georg (1994) provides the conceptual foundation, illustrating how environmental regulation generally advances through gradual, path-dependent processes. Söderholm & Ekvall (2020) reinforce this in the context of metal recycling, noting that industry progression is rarely abrupt and typically builds on existing infrastructure and policies. Empirical studies by Graedel & Reck (2016) and Reck & Graedel (2012) further demonstrate that system-level improvements in recycling arise cumulatively, constrained by established capacities. Eckelman, Reck & Graedel (2012) show via Markov modeling that phases of recycling advancement are tightly linked to past investments and regulatory environments. Finally, Linder & Williander (2017) confirm within the circular economy literature that organizational learning and previous investment patterns cause recycling and circular business transitions to proceed along incremental, history-dependent paths.

- High persistence: Diagonal elements p_{ii} are expected to be the highest, reflecting the strong inertia typical of industrial capacities—sectors generally maintain their current level of infrastructure and capability from one period to the next rather than abruptly switching regimes⁸³.
- Growth asymmetry: The probabilities of moving to a higher-capacity state $p_{i,i+1}$ are systematically larger than those of moving to a lower-capacity state $p_{i,i-1}$ capturing the fact that dismantling recycling capacity is unlikely, especially under long-term energy transition pressures. This assumption is grounded in both structural industry characteristics and observed market trends⁸⁴.
- Limited long-range jumps: The probabilities of transitioning directly from state s_i to a distant state s_j where $|j - i| > 1$ are close to zero, in line with the practical constraints of scaling industrial processes.
- The structure of the transition matrix $\mathcal{P}$ also captures the characteristic S-shaped trajectory of secondary supply growth widely discussed in the literature. Studies such as Reuter et al. (2006), Söderholm and Ekvall (2020), and Ayres and Peiró (2013) emphasize that the expansion of recycling capacity typically follows a sigmoidal (logistic-like) path. Early stages are characterized by slow progress due to technical barriers, limited policy support, and fragmented scrap collection systems. This is followed by an acceleration phase driven by technological breakthroughs, improved scrap flows, and stronger economic incentives. Eventually, growth slows again as the system approaches its maturity ceiling, constrained by physical recovery limits, saturation of collection infrastructures, and the diminishing returns of further investments. Empirical observations in Grand View Research (2025) also align with this trajectory. The reported long-term market trends suggest that, while the metal recycling

⁸³ This behavior has been documented in system dynamics model of remanufacturing and recycling: for instance, Papachristos (2014) shows that “competition in the market [...] forces [supply chains] into a situation where improving business and environmental performance is difficult,” due to the lock-in effects and persistent inertia of incumbent systems.

⁸⁴ The assumption that transitions toward higher-capacity recycling states occur more frequently than capacity reductions is well grounded in the structural characteristics and observed behavior of metal recycling systems. Recycling infrastructure is capital intensive and characterized by long-lived assets, leading to strong industry inertia and path dependence in capacity evolution (Reck & Graedel, 2012; Eckelman, Reck & Graedel, 2012). Regulatory pressures and sustainability policies create additional incentives to maintain or expand capacity, especially under long-term energy transition goals (Söderholm & Ekvall, 2020; Graedel & Reck, 2016). Temporary supply disruptions or economic shocks rarely cause permanent dismantling of recycling facilities (Georg, 1994). Empirical and modeling studies consistently document cumulative, incremental growth patterns in recycling capacity driven by regulatory frameworks, technological progress, and market demand (Linder & Williander, 2017; Söderholm & Ekvall, 2020). This structural growth asymmetry aligns with rapid market expansion and positive growth projections reported by organizations such as the IEA, McKinsey, and Precedence Research (see in the introduction), reinforcing the validity of assigning higher probabilities to upward capacity transitions in modeling frameworks.

industry is rapidly expanding, the pace of capacity growth is not constant. It accelerates as economies of scale and policy incentives accumulate but is expected to decelerate in later stages as the market becomes increasingly saturated and technologically optimized. To analytically reproduce this sigmoidal behavior within the Markov framework, we introduce state-dependent transition probabilities that vary non-linearly along the sequence of states. Specifically, we define the probability of remaining in the same state p_{ii} as a decreasing sigmoidal function:

$$p_{ii} = 1 - \alpha \cdot \frac{e^{\beta(i-i_m)}}{1 + e^{\beta(i-i_m)}}$$

Where:

- $\alpha \in (0,1)$ controls the maximum probability of exiting a state, effectively constraining the fastest possible transition speed,
- $\beta > 0$ governs the steepness of the transition curve, with higher values reflecting more abrupt changes around the acceleration point,
- i_m is the inflection state, corresponding to the point where the recycling capacity grows most rapidly.

The probability of moving forward by one state is defined as the complementary transition:

$$p_{i,i+1} = \alpha \cdot \frac{e^{\beta(i-i_m)}}{1 + e^{\beta(i-i_m)}} - \epsilon$$

Where ϵ is equal to the backward transition probability $p_{i,i-1}$. As stated above, backward transitions are assumed to be unlikely, except for rare adverse events, such as temporary geopolitical disturbances or logistical mismatches in scrap flows. In the absence of such shocks, the system is structurally biased towards growth, in line with observed market dynamics where capacity dismantling is uncommon.

This formalization allows to capture the empirically observed S-shaped dynamics of secondary supply growth while preserving the flexibility to explore different transition speeds and

scenarios. By embedding this structure within a Markovian framework, our model reflects the path-dependent and probabilistic nature of recycling capacity development in metal markets⁸⁵.

Having defined the secondary supply function and the Markovian evolution of recycling states, we now turn to the market clearing condition, which governs the equilibrium between total metal demand and the combined supply from both primary and secondary sources.

3. Market-Clearing with Dual Supply Streams

In this extended framework, equilibrium no longer reflects the balance between demand and primary production alone. Instead, it arises from the joint contribution of both supply streams. The market clearing condition is thus written as:

$$Q_{d,t} = Q_{s,t}^{primary} + Q_{s,t}^{secondary}$$

Which expands to:

$$B(t) \cdot P_t^{\sigma-1} = \frac{c_{max}(P_{t-1})}{1 + e^{\gamma(\frac{c_t}{c_{max}(P_{t-1})} - \theta)}} + \eta \cdot P_t^{\delta} \cdot S_{t-1}$$

And gives:

⁸⁵ The use of sigmoidal (logistic-like) transition probabilities in Markov chain models to represent secondary metal supply growth captures the widely observed S-shaped trajectory of recycling system expansion, reflecting slow initial progress, accelerated growth phases, and eventual saturation or maturity constraints. This modeling approach aligns with extensive literature in recycling dynamics, technology diffusion, and probabilistic state-transition frameworks. Recycling system development often follows a logistic growth pattern due to initial technical barriers, limited policy support, and fragmented scrap collection at early stages; this is followed by rapid capacity expansions driven by technological breakthroughs, economies of scale, and strengthened incentives; growth eventually slows as physical recovery limits and infrastructure saturation are approached (Reuter et al., 2006; Söderholm & Ekvall, 2020; Ayres & Peiró, 2013). Empirical market analyses (Grand View Research, 2025) further confirm these features in real-world metal recycling industry trends. From a technology diffusion perspective, the classic Bass diffusion model (Bass, 1969) and extensions (Mahajan et al., 1990) formalize innovation and adoption processes with S-shaped curves, incorporating nonlinear adoption rates and saturation effects. Several studies adapt such logistic dynamics within probabilistic or Markov-based transition frameworks to capture state-dependent growth trajectories in technology systems analogous to recycling infrastructure evolution (Zhang et al., 2019; Dogan et al., 2020). In Markov modeling applications, logistic or sigmoidal functions parameterize transition probabilities between discrete states to model regime switching or phased development realistically, with forward transition probabilities increasing sigmoidally and backward transitions remaining low except under rare adverse conditions. As section III will show when exploring different stylized facts, this method reflects path dependence, hysteresis, and cumulative investment effects inherent in metal recycling capacity growth (Eckelman, Reck & Graedel, 2012; Linder & Willander, 2017). Analogous approaches are common in resource and ecological modeling, where Markov chains with logistic transition probabilities have been used to model land use change, resource regeneration, and adoption of improved management regimes, reinforcing the validity of embedding nonlinear growth functions into Markov processes (Van Vliet et al., 2014). Collectively, these literatures support the conceptual and mathematical rationale for embedding sigmoidal transition probabilities within our Markov process to reproduce empirically observed S-shaped secondary metal supply growth. This framework balances flexibility in transition dynamics with realistic reflection of growth phases and capacity maturation, consistent with both empirical data and theoretical precedents in industrial ecology, economics, and technology diffusion.

$$P_t = \left(\frac{\frac{c_{max}(P_{t-1})}{1 + e^{\gamma(\frac{c_t}{c_{max}(P_{t-1})} - \theta)}} + \eta \cdot P_t^\delta \cdot S_{t-1}}{B(t)} \right)^{\frac{1}{\sigma-1}}$$

In this formulation, the market-clearing price emerges from the joint adjustment of two structurally distinct forces. The first term reflects the gradually tightening constraints of primary extraction, governed by cumulative depletion and rising costs. The second term incorporates secondary supply, whose responsiveness is mediated by the maturity of recycling infrastructure, here proxied by the state variable S_{t-1} . Together, they generate a price that balances rigid, resource-constrained extraction with path-dependent and state-contingent recycling flows.

What this structure makes explicit is that price is no longer determined solely by the physical scarcity of primary resources, as in the baseline model, but also by the evolving capacity of secondary supply to absorb shocks and meet demand. In effect, equilibrium price formation becomes the outcome of a dynamic interplay between depletion-driven scarcity and recycling-driven resilience.

III. Stylized Facts and Dynamic Properties of the Extended Model

The integration of secondary supply with Markovian dynamics brings into view a set of stylized facts and dynamic properties that were absent from the baseline model. First, recycling endogenously increases the price elasticity of total supply, shifting markets from rigid to more responsive configurations. Second, the build-up of recycling capacity progressively buffers prices against demand shocks, while third, it also reduces the sensitivity of aggregate consumption to primary supply disruptions. Fourth, because transitions across recycling regimes are path-dependent, shocks leave lasting imprints on capacity, generating hysteresis effects that outlive the initial disturbance. Fifth, as secondary metals gain market share, effective demand elasticity declines, decoupling total consumption from primary-driven price volatility. Finally, baseline demand growth increasingly falls on secondary supply rather than being fully transmitted into higher prices or additional mining. Together, these mechanisms reshape the adjustment dynamics of metal markets, from short-run shock absorption to long-term structural rebalancing.

1. From Rigid to Responsive: How Recycling Increases Total Supply Price-Elasticity

A feature of the extended model lies in the progressively intensifying influence of recycling on the overall metal market as recycling infrastructure develops. Whereas the short-run price elasticity of primary supply is low—constrained by geological, technical, and capacity limits (Miller & Martinez,

2025)—the secondary supply response can be more flexible. Its responsiveness, however, depends crucially on the state of the recycling system. Because scrap collection, processing, and re-entry into the market require sufficient infrastructure and organization, the contribution of secondary flows to total supply is modest when recycling capacity is underdeveloped but grows as the system transitions into more advanced stages.

In this framework, the limiting factor for recycled metals is not so much the price signal itself, but the interaction between market price and available infrastructure. At early stages, low scrap collection efficiency and limited processing capacity cap the physical volumes that can be mobilized, even when prices rise. As S_{t-1} evolves—reflecting cumulative investment, technological readiness, and policy support—the responsiveness of secondary supply strengthens, allowing it to absorb more of the pressure from demand shocks.

Formally, the own-price elasticity of secondary supply is:

$$\varepsilon_{s,t}^{secondary} = \frac{\partial Q_{s,t}^{secondary}}{\partial P_t} \cdot \frac{P_t}{Q_{s,t}^{secondary}} = \delta$$

This elasticity is constant at each time step for a given S_{t-1} . However, the own-price elasticity of total supply is:

$$\varepsilon_{s,t}^{total} = \left(\frac{\partial Q_{s,t}^{primary}}{\partial P_t} + \eta \cdot \delta \cdot P_t^{\delta-1} \cdot S_{t-1} \right) \cdot \frac{P_t}{Q_{s,t}}$$

The second term $\eta \cdot \delta \cdot P_t^{\delta-1} \cdot S_{t-1}$ increases as S_{t-1} progresses along the Markov chain toward higher recycling regimes. This means that the contribution of secondary supply to total supply elasticity systematically grows as the recycling system matures, i.e. $\frac{\partial \varepsilon_{s,t}^{total}}{\partial S_{t-1}} > 0$ ⁸⁶.

⁸⁶ We reasonably and parsimoniously assume that $\partial Q_{s,t}^{primary} / \partial P_{primary,t}$ is independent of S_{t-1} . This is justified by the fact that short-run primary supply elasticity is already close to zero and largely determined by geological and technical constraints, rather than by the state of recycling infrastructure. In the medium to long run, however, the relationship is more nuanced. Expanding secondary capacity can reduce the need for new mining projects, but it does not eliminate it. According to the IEA's *Recycling of Critical Minerals* report (2024), scaling up recycling could reduce mining requirements for minerals such as copper and cobalt by 25–40% by 2050. The World Economic Forum (2025) similarly underlines that while advanced recycling and circular economy strategies can offset planned mining expansions, they cannot fully absorb growing demand. Industry analyses by the International Copper Association (2021) reinforce this point, estimating that even under optimistic recycling scenarios, more than 300 new mines may be needed by 2040 to meet energy transition goals. Academic work by Li et al. (2022) confirms that higher recycling shares alleviate pressure on primary supply, but stresses that with global demand continuing to rise, investment in both primary and secondary capacity typically proceed in parallel. Finally, the OECD's *Government Support for Primary and Secondary Metal Production* report (2018) finds that financing for recycling projects increasingly complements—rather than substitutes for—investment in mining.

This expresses that the market's capacity to stabilize itself through recycling is a path-dependent, gradually emerging property of the system. Initially, when S_{t-1} is low, the elasticity of total supply is dominated by the rigidity of primary production, and the recycling response is limited. As S_{t-1} increases, the recycling sector can play a progressively larger role in absorbing price pressures, raising the overall elasticity of supply⁸⁷.

The model thus captures a key stylized fact of metal markets: the stabilizing influence of secondary supply is not fixed but rises endogenously with the maturity of recycling infrastructure. At early stages, price surges may still generate episodes of volatility because the recycling channel remains thin. Over time, however, as the system transitions into higher regimes, the amplitude and persistence of such cycles diminish, and the market converges toward a more resilient configuration with greater total supply elasticity.

2. From Volatile to Buffered: How Recycling Dampens the Price Impact of Demand Shocks

While the previous subsection showed that recycling directly increases the responsiveness of secondary supply, this section focuses on the broader market implication: as the recycling share grows, the market price itself becomes less sensitive to demand shocks. In other words, recycling not only shifts the supply curve but also reduces the volatility of equilibrium prices.

In metals markets where recycling remains marginal, the model suggests that a positive demand shock or a tightening of primary supply can lead to pronounced and rapid price increases. By contrast, the same magnitude of shock generates a smaller price response in later recycling regimes, where the recycling sector is more developed. This asymmetry is not driven by exogenous policies or external constraints but emerges endogenously from the evolving supply structure itself.

Differentiating the equilibrium condition with respect to total demand yields the primary-price sensitivity to demand:

⁸⁷ It is important to note, however, that at very early stages of recycling-system development—when S_{t-1} is small—the contribution of secondary supply to total responsiveness is negligible, while its contribution to total volume is not. As a result, the term $\eta \cdot \delta \cdot P_t^{\delta-1} \cdot S_{t-1}$ remains close to zero while $Q_{s,t}$ already incorporates a non-trivial (albeit small) secondary component. In this configuration, the total elasticity $\varepsilon_{s,t}^{total}$ can initially be lower than the elasticity of purely primary supply. This does not contradict the mechanism described below: it simply reflects the fact that recycling begins as a thin, capacity-constrained channel that adds quantity before it adds elasticity. Once $\varepsilon_{s,t}^{total}$ increases, however, the secondary elasticity term grows, and the overall mechanism described next fully applies.

$$\frac{\partial P_t}{\partial Q_{d,t}} = - \left[B(t)(\sigma - 1)P_t^{\sigma-2} - \frac{dQ_{s,t}^{primary}}{dP_{primary,t}} - \eta \cdot \delta \cdot P_t^{\delta-1} \cdot S_{t-1} \right]^{-1}$$

This expression shows that $\frac{\partial P_t}{\partial Q_{d,t}}$ decreases monotonically as S_{t-1} increases, all else being equal⁸⁸: in low-recycling regimes, prices absorb most of a demand shock, whereas in more advanced regimes secondary supply expands more readily and dampens the price response.

Empirical evidence is consistent with this endogenous asymmetry. In the early 2000s, aluminum prices were highly sensitive to demand growth—particularly from China—due to relatively limited scrap availability and constrained recycling capacity. China’s primary aluminum production grew rapidly from around 10 million tons in the mid-1990s to approximately 30 million tons by 2010, but recycling infrastructure lagged, resulting in tight secondary supply and volatile prices. By 2024, with China’s annual primary production exceeding 40 million tons (accounting for about 60% of global production) and a well-developed scrap system, demand surges have been absorbed more smoothly, muting price volatility (Transition Asia, 2025; International Aluminum Institute, 2024). This trajectory aligns with Söderholm & Ekvall (2020) and Graedel et al. (2011), who show that maturing recycling systems structurally raise supply elasticity. In contrast, metals like nickel and many minor or specialty metals, where recycling infrastructure and scrap availability remain more limited, continue to experience frequent and pronounced price spikes as primary supply constraints transmit rapidly and with high amplitude to market prices (aligned with Radetzki, 2010; IEA, 2021).

The asymmetry in shock absorption capacity is therefore a structural property of the system: total supply responsiveness is state-contingent and regime-dependent. As recycling matures, elasticity rises and volatility falls, while in early stages, the market remains vulnerable to sharp price adjustments. This dynamic non-linearity is a central implication of the model, showing how the evolving role of secondary supply reshapes price behavior endogenously.

⁸⁸ For the equilibrium to be locally stable, the slope of demand (in absolute value) must exceed the slope of aggregate supply at the point of intersection. Under this condition and only under this condition, a positive demand shock raises the primary price, and the magnitude of this price response decreases as the recycling system becomes more mature. Some qualifications refine this mechanism. First, if recycling capacity constraints bind, the effective supply curve can exhibit kinks: in such cases the price response to demand shocks may change abruptly rather than smoothly. These are discontinuities in practice, but they do not alter the general tendency for greater maturity in recycling to dampen price swings. Second, as previously argued, recycling expansion can potentially affect primary mining investment decisions. However, this interaction operates on an investment timescale rather than within the short-run comparative statics developed here, so we assume in this model that primary supply is independent from S_{t-1} , therefore $dQ_{s,t}^{primary}/dP_t$ is constant with respect to S_{t-1} .

3. From Fragile to Resilient: How Recycling Dampens Market Sensitivity to Price Shocks

A central implication of introducing recycling into the analytical framework is that the responsiveness of total demand to market price fluctuations becomes time-varying. In a market initially dominated by primary supply, shocks to extraction capacity translate directly into sharp price adjustments and large contractions in total demand. As the share of recycled material into total supply grows, however, a larger fraction of consumption is met by flows that are relatively insulated from short-run primary disturbances. This gradually weakens the effective pass-through from primary shocks to aggregate quantities, creating a progressive decoupling between price volatility and total demand responsiveness.

Since $Q_{d,t}$ is represented by a CES function, the Marshallian elasticity of demand is constant and equal to $\sigma - 1 < 0$.

We now focus on short-run adjustments within period t . We define the secondary share:

$$s_t \equiv \frac{Q_{s,t}^{secondary}}{Q_{s,t}^{total}} \in [0,1]$$

So that the short-run aggregate supply elasticity can be written as:

$$\varepsilon_{s,t}^{total} = \frac{P_t}{Q_{s,t}^{total}} \frac{\partial Q_{s,t}^{primary}}{\partial P_t} + \frac{P_t}{Q_{s,t}^{total}} \frac{\partial Q_{s,t}^{secondary}}{\partial P_t} = \varepsilon_{s,t}^{primary} (1 - s_t) + \varepsilon_{s,t}^{secondary} s_t$$

We now consider a small primary supply disturbance at time t . Primary supply $Q_{s,t}^{primary}$ depends on price P_t but also on a parameter vector $v \equiv (L, \kappa, P_m, \gamma, \theta)$. Suppose v shifts (due to capacity outage, ore-grade deterioration, regulatory disruption, etc.) so that at fixed P_t primary quantity falls by a proportional amount:

$$dQ_{s,t}^{primary}|_P = -\mu_t Q_{s,t}^{primary} \text{ with } \mu_t > 0$$

Secondary quantity is not directly affected by v in the short run.

The equilibrium condition implies:

$$\frac{\partial Q_{d,t}}{\partial P_t} dP_t = \frac{\partial Q_{s,t}^{primary}}{\partial P_t} dP_t + \frac{\partial Q_{s,t}^{primary}}{\partial v} dv + \frac{\partial Q_{s,t}^{secondary}}{\partial P_t} dP_t$$

Using $dQ_{s,t}^{primary}|_P = \frac{\partial Q_{s,t}^{primary}}{\partial v} dv = -\mu_t Q_{s,t}^{primary}$, we obtain the short-run pass-through of a proportional primary shock into the equilibrium price:

$$\frac{dP_t}{P_t} = \frac{\mu_t Q_{s,t}^{primary}}{\varepsilon_{s,t}^{primary} Q_{s,t}^{primary} + \varepsilon_{s,t}^{secondary} Q_{s,t}^{secondary} - (\sigma - 1) Q_{s,t}^{total}}$$

Hence:

$$\frac{dP_t}{P_t} = \frac{\mu_t(1 - s_t)}{\varepsilon_{s,t}^{primary}(1 - s_t) + \varepsilon_{s,t}^{secondary}s_t + |\sigma - 1|}$$

Through demand, the corresponding change in total quantity (with $Q_t \equiv Q_{d,t} = Q_{s,t}^{total}$ at the equilibrium):

$$\frac{dQ_t}{Q_t} = (\sigma - 1) \frac{dP_t}{P_t} = (\sigma - 1) \frac{\mu_t(1 - s_t)}{\varepsilon_{s,t}^{primary}(1 - s_t) + \varepsilon_{s,t}^{secondary}s_t + |\sigma - 1|}$$

Holding μ_t and $\varepsilon_{s,t}^{primary}$ fixed, both dP_t/P_t and dQ_t/Q_t are strictly decreasing in s_t . The numerator falls with $(1 - s_t)$, while the denominator rises with s_t .

As the secondary share increases (higher s_t), a given proportional primary disruption produces smaller price increases and smaller contractions in total quantity. Intuitively, the presence of a secondary block shifts the effective supply schedule outward, reducing the price pressure that a primary-only market would experience under the same shock. In the limit $s_t \rightarrow 1$, the effects of proportional primary shocks vanish, i.e. both price and quantity effects converge to zero, since primary output has negligible weight in total supply⁸⁹.

These derivations formalize the insulation role of recycling in the short run and the progressive decoupling of total demand from primary-driven price fluctuations. As s_t rises along the recycling transition, primary-driven disturbances generate progressively smaller price movements, and quantity contractions⁹⁰. While the Marshallian elasticity of demand remains invariant, the equilibrium response to shocks depends on composition through $\varepsilon_{s,t}^{secondary}$ and s_t . Market-wide responsiveness therefore

⁸⁹ Over time, however, the buffering effect of recycling reaches diminishing returns: with a given flow of primary production, secondary supply growth slows as it approaches the physical limit set by available scrap.

⁹⁰ In reduced-form terms, the shock-induced mapping $\mu_t \rightarrow (dP/P, dQ/Q)$ becomes flatter in both coordinates as s_t increases.

diminishes as recycling scales up, reflecting a gradual decoupling of total demand from primary-drive price fluctuations.

This mechanism can be observed empirically in metals where recycling has already reached a significant share of supply. In the case of aluminum, recycling now accounts for over 30-40% of annual global supply, supported by mature infrastructures and well-integrated scrap collection systems (International Aluminum Institute [IAI], 2024; Graedel & Reck, 2016). The large and relatively elastic secondary pool buffers the market against shocks originating in primary production. Empirical studies confirm that aluminum's high recycling rates dampen the pass-through of primary price volatility to total consumption (Söderholm & Ekvall, 2020; Reck & Graedel, 2012). Closed-loop and end-of-life recycling pathways further enhance resilience, allowing secondary flows to substitute for primary extraction during demand surges and thereby moderating aggregate price sensitivity over time (Ayres & Peiró, 2013).

In the case of cobalt, the share of recycled material in total supply remains relatively low, currently around 10%, leaving the vast majority of consumption dependent on primary mined supply (Cobalt Blue Holdings, 2022; Zeng et al., 2022). This limited secondary supply means that fluctuations in primary market prices more directly influence total consumption and contribute to price volatility, as recycled cobalt provides minimal buffering capacity (World Bank, 2021). Indeed, cobalt prices have historically experienced significant cyclical spikes linked to supply constraints in primary production regions, such as the Democratic Republic of Congo, which dominates cobalt supply (Alves Dias et al., 2018). While secondary cobalt supply is expected to grow substantially post-2025 as electric vehicle battery recycling scales up, the current low recycling share makes cobalt particularly vulnerable to supply shocks and price volatility compared to metals with more mature recycling systems.

4. From Short-Lived Shocks to Lasting Shifts: How Recycling Creates Hysteresis in Supply

A central feature of the extended model is its ability to move from a view of supply as purely temporary and reversible to one where past developments leave a persistent imprint. In standard commodity market models, the supply response is assumed to react only to contemporaneous price signals: when the stimulus disappears, supply is expected to contract symmetrically, returning immediately to its baseline. In contrast, the present framework embeds memory effects in two complementary ways.

First, following our modeling choices, the capacity to recycle evolves as a Markov process. Transitions between recycling regimes are asymmetric and sticky, so capacity tends to move upward over time and

seldom reverts. This irreversibility is an assumption about how infrastructure, investment and regulation evolve: once built, recycling capacity is rarely dismantled.

Second, second supply flows exhibit hysteresis even when capacity does not change. This is a result from the model. Flows depend on the interaction between, on the one hand, available end-of-life scrap, determined by past extraction and lifetime distributions, and, on the other hand, price incentives.

The model allows us to derive a closed-form expression for expected secondary supply under Markovian state dynamics. At any future horizon h , the expected secondary supply at time $t + h$ is, by definition, given by:

$$\mathbb{E}(Q_{s,t+h}^{secondary} \mid S_t = s_i) = \eta \cdot \mathbb{E}(P_{primary,t+h}^\delta) \cdot \mathbb{E}(S_{t+h-1} \mid S_t = s_i)$$

Since S_{t+h-1} depends on the Markov process, we can express its expectation as:

$$\mathbb{E}(Q_{s,t+h}^{secondary} \mid S_t = s_i) = \eta \cdot \mathbb{E}(P_{primary,t+h}^\delta) \cdot \sum_{j=1}^n p_{ij}^{(h-1)} \cdot s_j$$

Where $p_{ij}^{(h-1)}$ denotes the (i, j) -th entry of the matrix $\mathcal{P}^{h-1}$, representing the probability of transition from state s_i to state s_j in $h - 1$ steps. The non-linear structure of $\mathcal{P}$ governs the dynamic behavior of $\sum_{j=1}^n p_{ij}^{(h-1)} \cdot s_j$ in several ways. First, the weighting on higher-capacity states s_j increases with both the horizon h and the proximity of i to the inflection point i_m . For small h and low initial state i , the summation is dominated by terms with $j = i$ or $j = i + 1$, resulting in a modest expectation $\mathbb{E}(S_{t+h-1} \mid S_t = s_i) \approx s_i$. As h increases, the influence of multi-step forward transition grows, particularly for i near or below i_m , where the system is in the “acceleration” zone of the sigmoid. In that regime, the term $\sum_{j=1}^n p_{ij}^{(h-1)} \cdot s_j$ becomes increasingly convex in h , reflecting the growing likelihood of transitioning to higher states. In contrast, for $i \gg i_m$, the forward probabilities decline, and the transition process slows, which compresses the distribution around high but relatively stable values of s_j , yielding a concave expected path of S_{t+h-1} .

The overall shape of $\sum_{j=1}^n p_{ij}^{(h-1)} \cdot s_j$ as a function of h is thus sigmoidal, reflecting the composition of short-term inertia, mid-term acceleration, and long-term saturation. Importantly, the irreversibility embedded in the transition structure via $p_{i,i-1} = \epsilon \ll p_{i,i+1}$ ensures that once the process moves into a higher-capacity regime, it does not revert. This asymmetry implies that the expected state $\mathbb{E}(S_{t+h-1} \mid S_t = s_i)$ is strictly increasing with h , though at a decreasing rate beyond i_m , and that for

any transient price shock capable of inducing upward transitions, the supply response remains durably elevated. In other words, the structure of $\mathcal{P}$ embeds both hysteresis, via persistence and irreversibility, and path dependence, via state-contingent transition probabilities, which are fully transmitted into the dynamic behavior of secondary supply.

This hysteresis mechanism captures the lagged and asymmetric behavior of recycling supply. For example, in periods of sustained high prices, the system progressively transitions toward more advanced recycling regimes, increasing its capacity to absorb future shocks. *If prices subsequently decline, the elevated recycling capacity does not vanish.* The system retains its enhanced supply response, leading to a decoupling between short-term price dynamics and the structural supply potential built over previous periods.

Let us consider a temporary positive price shock and trace its effects on secondary supply. Suppose the primary metal price jump from a baseline level P^* to $P^* + \epsilon$ at time $t = T$, before returning immediately to P^* in the following period. In a memoryless framework, this would induce only a short-lived increase in secondary supply, followed by an immediate reversion to baseline. In contrast, under the present Markovian formulation, the recycling state S_t retains the imprint of past configurations and evolves with structural inertia.

During the shock period, the elevated price raises supply directly through the price term $P_{primary,T}^\delta$, but also indirectly by altering the transition kernel. Specifically, the one-step transition probabilities are shifted upward: the likelihood of moving forward to state s_{i+1} increases, while the probability of remaining in state s_i declines. This upward shift in the transition kernel skews the probability distribution of future states toward higher-capacity regimes. Even once the price returns to P^* , the expected recycling state remains elevated, since:

$$\mathbb{E}(S_T | \text{shock}) - \mathbb{E}(S_T | \text{no shock}) = (p_{i,i+1}^{shock} - p_{i,i+1}^{base})s_{i+1} + (p_{ii}^{shock} - p_{ii}^{base})s_i$$

Because $s_{i+1} > s_i$, $p_{i,i+1}^{shock} - p_{i,i+1}^{base} > 0$, and $p_{ii}^{shock} - p_{ii}^{base} < 0$, the difference is strictly positive. Multiplying through by the positive term $\eta(P^*)^\delta$, we obtain

$$\mathbb{E}(Q_{s,T+1}^{secondary} | \text{shock}) > \mathbb{E}(Q_{s,T+1}^{secondary} | \text{no shock})$$

Demonstrating that the supply response remains durably elevated even after the price normalizes.

This memory effect has real-world counterparts. In metals like aluminum, where the recycling industry has reached high maturity, the secondary supply remains structurally strong regardless of short-term

price fluctuations. The scrap collection systems, processing facilities, and global trade networks are sufficiently embedded to sustain recycling activity even when prices soften. In contrast, in metals where the recycling sector is less developed—such as lithium or rare earths—supply remains closely tied to current price levels, and the system exhibits limited hysteresis.

This hysteresis property therefore helps explain why secondary supply can remain elevated during price downturns, contributing to stabilization, and why long-term path dependence emerges. Markets exposed to sustained high-price phases may permanently shift into regimes of higher supply responsiveness.

In summary, by embedding Markovian evolution with asymmetric transition probabilities and upward shifts in the transition kernel under price shocks, the model formally integrates memory effects and hysteresis into the supply-side dynamics. This allows it to reproduce empirically observed features of metals markets, where secondary supply capacity is “sticky” and shaped by accumulated developments rather than by contemporaneous price signals alone.

5. From Inflationary Pressures to Supply Rebalancing: How Recycling Absorbs Baseline Demand Growth

We now examine how structural growth in baseline demand—captured by a rise in the demand shifter $B(t)$ —is absorbed across supply sources. Structural forces such as demographic expansion, industrialization, and the energy transition all drive persistent increases in $B(t)$, amplifying the scale of demand. To assess how this additional pressure is transmitted to supply, we apply the chain rule to the secondary-supply term with respect to $B(t)$:

$$\frac{\partial Q_{s,t}^{secondary}}{\partial B(t)} = \eta \cdot \delta \cdot P_t^{\delta-1} \cdot \frac{\partial P_t}{\partial B(t)} \cdot S_{t-1}$$

Under the model’s standard parameter restrictions ($\eta > 0$, $\delta > 0$, $S_{t-1} > 0$, and $\partial P_t / \partial B(t) > 0$ ⁹¹), the derivative $\partial Q_{s,t}^{secondary} / \partial B(t)$ is strictly positive. For a given state of recycling infrastructure S_{t-1} , an outward shift in baseline demand raises the equilibrium price, which in turn lifts the contemporaneous flow of secondary supply through the term $\eta \delta P_t^{\delta-1} S_{t-1}$. This captures the fact that, within a single

⁹¹ Two points justify that $\partial P_t / \partial B(t)$ is strictly positive. First, on intuitive grounds, $B(t)$ is a baseline demand shifter: raising it shifts the demand curve outward. In equilibrium, a stronger demand curve necessarily requires a higher primary price to restore balance, all else being equal. Second, the chain rule gives that $\frac{\partial P_t}{\partial B(t)} = \frac{\partial P_t}{\partial Q_{d,t}} \cdot P_t^{\sigma-1}$. As shown in section III.3 $\partial P_{primary,t} / \partial Q_{d,t} > 0$, and $P_{primary,t}^{\sigma-1}$ is positive as well, hence $\partial P_{primary,t} / \partial B(t)$.

period, the recycling sector can respond on the intensive margin to price signals, while still remaining bounded by the physical constraint imposed by the available scrap stock and processing capacity.

The magnitude of this positive effect is state-dependent. In early recycling regimes, when S_{t-1} is small and processing capacity limited, the immediate response of secondary supply is weak. Primary extraction cannot expand significantly in the short run either, because of geological and technical constraints, which yield a near-zero price elasticity (Miller & Martinez, 2025). As a result, most of the adjustment to baseline-demand growth falls on the equilibrium price, which rises sharply to clear the market. As the system matures and S_{t-1} grows, the factor $\eta\delta P_t^{\delta-1}S_{t-1}$ scales up, enabling a larger share of incremental demand to be absorbed by recycling flows within the same period. Over time, the gradual expansion of effective scrap availability and processing maturity ensures that structural demand growth is increasingly met through secondary supply, moderating reliance on new primary capacity and dampening long-run price pressures.

IV. Numerical Application on the Copper Market

Having derived the extended analytical framework with secondary supply, we now turn to its numerical application to copper. This case is of particular interest because copper represents one of the few metals that can be reasonably modeled as a single, homogeneous good with a single global price. Once refined to LME Grade A cathode (or equivalent), copper is physically and commercially indistinguishable regardless of its origin, whether from primary mining or from recycling. This characteristic simplifies the modeling exercise, allowing the CES demand function and the dual supply structure to be calibrated without introducing heterogeneity across product types or qualities.

The case of copper is also distinctive because of its large recycling potential. End-of-life copper scrap is expected to expand significantly over the coming decades: by 2030, available volumes are projected to reach nearly 19 Mt, equivalent to around 60% of global demand in the APS scenario, and by 2050, they could reach 27 Mt, or around 70% of APS demand. Although yield losses must be accounted for, these figures underscore the scale of the opportunity. With copper recycling processes already technologically mature, future gains will depend less on improvements in processing yields than on increases in collection rates, which remain uneven across regions. Historically, the share of secondary supply in total copper demand has stagnated or declined, falling from 37% in 2015 to 33% in 2023. This setback reflects a combination of lower copper prices, China's restrictions on scrap imports, higher energy and shipping costs, the temporary disruption of Covid-19, and the delayed effects of EU and US policies aimed at boosting domestic collection (IEA, 2024).

Additional structural trends shape the prospects for copper recycling. Process automation in semi-fabrication has reduced scrap generation, tightening the prompt scrap pool. At the same time, the strong growth of Chinese smelting and refining capacity has driven down treatment and refining charges (TC/RCS), squeezing smelter margins. In this context, smelters have turned increasingly to scrap, which offers greater discounts than concentrates, and new secondary smelting capacity has emerged as a response (IEA, 2024). These developments highlight that recycling in copper markets is not only a technological issue but also a function of industrial organization and market incentives.

In this chapter, however, we adopt a reduced-form treatment of scrap, capturing both prompt and end-of-life flows within the effective recycling state S_{t-1} . This abstraction is consistent with our original analytical approach: what matters is not the distinction between different scrap streams, but the aggregate responsiveness of secondary supply to prices and its cumulative buildup over time.

The section proceeds as follows. First, we present the calibration of the model to copper, with particular attention to the parameters governing secondary supply. Second, we discuss the results, including the projected share of recycled copper in total consumption, the trajectories of equilibrium prices under different transition scenarios, and the implications for cumulative depletion of copper resources. Third, we conduct sensitivity analyses to assess two questions: (i) how sensitive outcomes are to changes in the parameter δ , which captures the price-responsiveness of secondary supply; and (ii) how the inclusion of recycling modifies the results relative to the no-recycling framework of Chapter 3, where $S_t = 0$.

1. Calibration to Global Copper Dynamics

a. Primary Supply: Cumulative Availability and Logistic Decline

We retain the same two-step specification as in Chapter 3, section II.2: a cumulative availability curve (CAC) that maps price into the total recoverable stock, and a logistic depletion rule that maps the CAC and cumulative extraction into annual primary output.

The CAC determines the price-conditioned recoverable stock. It is given by:

$$c_{max}(P) = \frac{L}{1 + e^{-\kappa(P-P_m)}}$$

With the ultimately recoverable copper upper-bound $L = 3.5 \times 10^9$ t, the midpoint price, consistent with long-run marginal costs, $P_m = 6,000$ US\$/t, and the steepness parameter calibrated to match

observed CAC shapes $\kappa = 1/38.4 \approx 0.026$. We hold the CAC fixed over the horizon (i.e. no exogenous technological shift). Cumulative extraction starts from the historical stock $c_0 \approx 700\text{Mt}$.

The annual primary supply is then given by a logistic depletion function, dependent on the CAC:

$$Q_{s,t}^{primary} = \frac{c_{max}(P_{t-1})}{1 + e^{\gamma\left(\frac{c_t}{c_{max}(P_{t-1})} - \theta\right)}}$$

Where c_t is cumulative extraction up to t , and the parameters γ (steepness) and θ (inflection point as a fraction of the CAC) are empirically estimated on 2001-2024 data and kept identical to Chapter 3 ($\gamma = 7$ and $\theta = 0.72$). This yields a supply path that is initially responsive to price-induced availability but progressively declines as $c_t/c_{max}(P_t)$ approaches θ and beyond.

b. Secondary Supply: Recycling Infrastructure Maturity and Market-Price Sensitivity

Secondary supply⁹² is modeled as a function of both current prices and lagged scrap availability⁹³:

$$Q_{s,t}^{secondary} = \eta \cdot P_t^\delta \cdot S_{t-1}$$

The exponent δ is set at 0.3, consistent with the range of elasticities reported in the empirical literature on secondary copper supply and comparable base metals. Cuddington & Jerrett (2008) find that copper supply elasticity is extremely low in the short run when measured at the aggregate level but note that secondary flows are structurally more responsive. Stuermer (2018) estimates total copper supply elasticities in the 0.1-0.2 range, with secondary supply contributing the bulk of the responsiveness. More recent OECD work (2020) on critical minerals and scrap markets places secondary supply elasticities for base metals between 0.2 and 0.4 in the short run, depending on the region and dataset. Taken together, these results suggest that $\delta = 0.3$ provides a conservative yet realistic benchmark, reflecting a moderate but sub-linear price responsiveness. This aligns with the theoretical expectation

⁹² The IEA defines copper secondary supply as cathode produced by smelting and refining lower-grade (typically end-of-life) scrap that cannot enter direct-use loops. Such scrap is treated either in primary smelters (engineered for concentrates and typically constrained to ~15% scrap blending for heat balance) or in dedicated, more modern secondary smelters designed for high scrap flexibility; after smelting, the metal is refined to cathode. Owing to higher impurity content, this material trades at a discount to primary copper, yielding an upgrading margin that links throughput to prices and available capacity (IEA, *Recycling of Critical Minerals*, 2024).

⁹³ The IEA corroborates this approach in its *Recycling of Critical Minerals* report (2024), and describes how scrap generation and collection are driven by prices and the business cycle: in expansions, higher industrial output and construction/demolition raise both manufacturing and end-of-life scrap, and higher prices prompt traders to release inventory from critical scrap pools; in downturns, output and retirements fall, prices soften, and traders hoard until recovery. In the United States and Europe especially, secondary copper supply co-moves with copper prices with little lag, consistent with scrap acting as a short-run market-balancing mechanism.

that recycling output increases less than proportionally with price, owing to collection frictions, capacity constraints, and the finite nature of available scrap. Although some evidence suggests that the responsiveness of secondary copper supply to price movements has weakened in recent years—consistent with the saturation effects reported in the literature when recovery and collection systems operate near capacity—we retain a constant elasticity $\delta = 0.3$ for the purposes of calibration. This choice provides a conservative benchmark that avoids overstating the ability of secondary supply to adjust to price shocks while remaining within the range reported by recent econometric work⁹⁴.

The scaling parameter η , which captures overall efficiency and intensity of recycling at a given maturity stage, is also calibrated empirically, ensuring that the level of simulated secondary output aligns with observed historical quantities.

The variable S_{t-1} is defined as the state of recycling maturity at the beginning of period t , and it plays a central role in shaping how available scrap stocks⁹⁵ are transformed into secondary copper output, as explained previously. Following conventions in adjacent domains such as e-waste, plastics, and batteries (Reuter et al., 2006, Söderholm et Ekvall, 2020, Ayes et Peiró, 2013), the evolution of recycling infrastructure is represented through six discrete maturity states $\{s_0, s_1, \dots, s_5\}$, ranging from the complete absence of organized recycling to a theoretical upper bound of saturation. At the lowest level, state s_0 corresponds to a primitive or absent system, where no formal infrastructure exists and recovery is limited to informal practices or negligible activity. Progressing to state s_1 , recycling enters an emerging or basic phase, marked by the first elements of organized collection and the establishment of limited facilities, though overall efficiency remains low. In state s_2 , the system is best described as developing or intermediate: infrastructure expands, collection rates improve, and regulatory frameworks begin to stabilize supply chains. State s_3 reflects a mature or advanced system, where coverage is broad, sorting technologies are sophisticated, and regulatory oversight is robust. At state s_4 ,

⁹⁴ From our own estimation using monthly data for 2002-2025, we obtain $\delta \approx 0.5$, but this figure is likely upward-biased because it conflates pure price responsiveness with the gradual expansion of recycling infrastructure capacity. Moreover, the estimated δ showed a declining trend toward zero at the end of the sample (2022-2025). While this pattern is consistent with capacity saturation, the underlying estimate is biased by the omission of infrastructure effects, so we do not incorporate this declining profile into the calibration.

⁹⁵ In practice, copper scrap is heterogeneous and follows distinct pathways depending on its origin, quality, and the technological and institutional context of recycling. High-grade manufacturing scrap can be reintroduced directly into semi-fabrication processes, a route known as direct-use scrap. This pathway is relatively simple and highly efficient: the material requires little more than remelting, and when purity exceeds 99%, it can even substitute for cathode as the sole feedstock. In contrast, lower-grade or end-of-life scrap must undergo mechanical dismantling, sorting, and pyrometallurgical processing in specialized smelters or refiners. Here, recovery is more complex and costly, yields are lower, and throughput is constrained by the technical limits of both primary and secondary smelting capacity. In our calibration, we do not separate prompt and end-of-life channels: the state S_{t-1} represents the effective maturity of the recycling system and encompasses both. Prompt scrap is absorbed in the level calibration of secondary supply, so results depends on (P_t, S_{t-1}) only and the scenario ordering is unaffected. Equivalently, a constant prompt-scrap share would simply rescale net refined demand.

the system reaches an optimal or closed-loop configuration, characterized by integration with manufacturing processes, near-complete recovery of scrap, and minimal material losses. Finally, state s_5 represents a theoretical upper bound, a stylized ceiling that captures the saturation of infrastructure, the physical limits of recovery, and the diminishing returns to further investment in recycling capacity.

In 2025, this stage-based framework suggests that copper recycling may stand at stage s_3 . This corresponds to about 30% of copper supply from secondary sources and a global recycling rate near 60%, supported by advanced sorting technologies, increasing policy support, and innovations in chemical recycling (Wang et al., 2021; Glöser et al., 2013). Market evidence confirms that while infrastructure has expanded and become more sophisticated, the system is only about halfway through functioning as a closed loop: Loibl et al. (2021) emphasize ongoing losses and quality challenges, while Born et al. (2024) conclude that current recycling capacities cannot exceed meeting 50% of global copper demand. Historical analysis by Wang et al. (2021) traces stepwise progress: early stage transitions (from s_1 to s_2) occurred in the 1940s-1950s, driven by wartime scrap mobilization and post-war industrialization, while the shift from basic recycling to advanced partial loop recycling (from s_2 to s_3) took place between the 1970s and 1990s, shaped by the rise of environmental regulation and improvements in metallurgical recovery technologies. Between 2002 and 2025, improvements were steady but not transformational: material flow analyses point to incremental infrastructure growth coupled with stagnation in some regions, driven by uneven investment and the persistent quality constraints of recovered scrap (Klose et al., 2021; Wang et al. 2021).

The calibration of the transition process rests on three key parameters α , β , and ε , which we calibrate to reproduce the secondary supply trajectories of the IEA's *Recycling of Critical Minerals* report (2024). These parameters translate the broad qualitative differences in the IEA scenarios into a precise probabilistic structure: STEPS embodies slow, incremental progress; APS captures a moderate but steady acceleration; and NZE reflects the rapid, policy-driven expansion of secondary supply consistent with a full-scale energy transition.

The first parameter is α , which governs the maximum probability of exiting a given maturity stage. Intuitively, it can be read as the “speed limit” of infrastructure transformation. In the STEPS scenario, we retain $\alpha = 0.18$, as it yields a slow and incremental expansion of recycling capacity, in line with the IEA's projection of only modest secondary supply growth under current policies. In APS, where the IEA assumes more proactive regulatory support, α is calibrated at 0.30, allowing for visibly faster state transitions. In NZE, the IEA foresees an even more ambitious build-out of recycling, and $\alpha = 0.45$

allows the model to match that rapid pace of expansion, with transitions between maturity levels possibly occurring at nearly twice the speed of the STEPS baseline.

The second parameter, β , determines the steepness of the transition curve—that is, how abruptly the system accelerates once it reaches the critical middle stage of development. Here again, values are chosen to reproduce the shape of the IEA’s supply paths. In STEPS, $\beta = 1.4$ generates a gradual slope, mirroring the IEA’s expectation that recycling supply expands but without sharp inflection. In APS, $\beta = 2.4$ produces a more pronounced acceleration, reflecting the IEA’s assumption that supportive policies generate stronger momentum once recycling infrastructure reaches critical mass. In NZE, $\beta = 3.0$ creates a highly sigmoidal trajectory, directly aligned with the IEA’s assumption that ambitious circularity policies induce rapid scaling in a relatively compressed time frame.

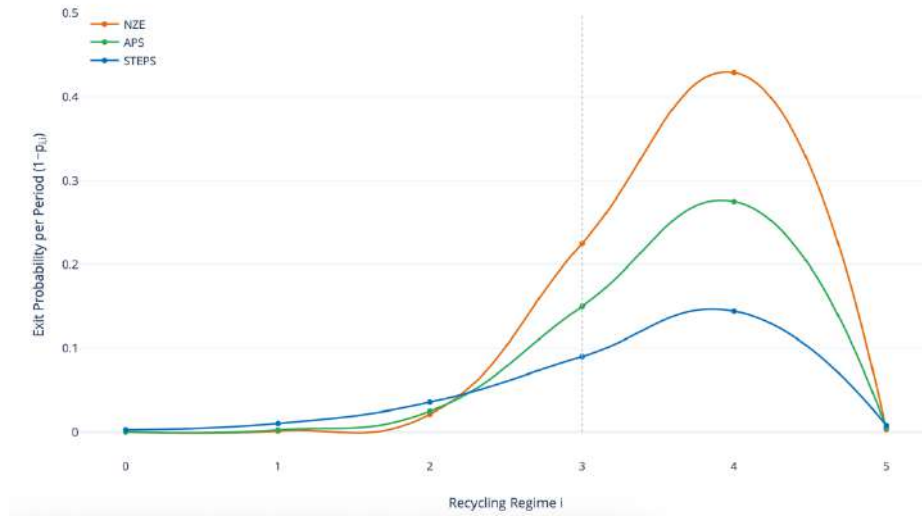
Finally, the small probability of regression, ε , is also calibrated to track the IEA’s scenario narratives, taking values between 0.008 and 0.003 depending on the scenario. This declining sequence reflects the IEA’s implicit assumption that stronger policy environments not only accelerate forward progress but also reduce the risk of setbacks. In STEPS, $\varepsilon = 0.008$ acknowledges the possibility of occasional stagnation or reversal in recycling capacity, while APS ($\varepsilon = 0.005$) and NZE ($\varepsilon = 0.003$) assume that supportive regulatory frameworks, sustained investment, and market demand make regressions increasingly unlikely.

To summarize:

<u>Scenario</u>	<u>STEPS</u>	<u>APS</u>	<u>NZE</u>
α	0.18	0.30	0.45
β	1.4	2.4	3.0
ε	0.008	0.005	0.003

Finally, we set $i_m = 3$, the midpoint of the maturity scale, since no strong evidence suggests asymmetric acceleration at earlier or later stages.

Figure 4.1: Exit Probability from Recycling Regime 1 – $p_{i,i}$ under STEPS, APS, and NZE



In the graph above, we plot the exit probability $1 - p_{i,i}$ for recycling regimes $s_{i=0,\dots,5}$ under STEPS, APS, and NZE. More ambitious scenarios reduce persistence and raise exit rates. At the lower boundary s_0 , exits are necessarily upgrades and are throttled by early-stage scrap scarcity and immature collection, so the transition hazard is low and rises only gradually. At the upper boundary s_5 , upgrades are ruled out, and the exit probability equals the rare backward move ε , encoding practical stability while allowing for minimal, plausible regressions (policy reversals, bottleneck, cyclical shocks). The vertical dotted line marks the logistic midpoint $i_m = 3$. It also indicates the copper industry’s initial recycling state at the start of our horizon, $S_{2025} = s_3$. NZE yields the highest exit probabilities across regimes (lowest diagonal mass in the transition matrix P), implying faster advancement of recycling capacity.

The “Business as Usual” (BAU) scenario, by contrast, is not parametrized through a Markov structure but through a simple extrapolation of the historical trend, assuming secondary supply continues to grow at 4.2% CAGR to 2050 (ICSG).

c. Demand: Structural Drivers, Energy Transition Scenarios, and Non-Energy Consumption

This demand function is parametrized in the same way as the original demand function introduced in Chapter 3. We apply a CES specification calibrated specifically to capture the dynamics of copper demand, in line with earlier methodologies such as those of the US Bureau of Mines (Soyster & Serali, 1981). Core global copper demand, $B_i(t)$, is defined across the four energy transition scenarios—STEPS, APS, NZE, and BAU—by combining structural demand drivers, scenario-specific policy trajectories, and the accumulation of unmet needs from prior years. In our framework, structural

demand reflects copper use in non-energy sectors such as construction, manufacturing, and consumer goods, while the scenario component accounts for clean-technology requirements (wind, solar, EVs, electricity networks) and the evolving energy mix. For each scenario, $B_i(t)$ is re-estimated annually to integrate both baseline projected requirements $R_i(t)$ and any unmet needs from previous periods N_t , ensuring that the demand block captures the persistence of transition-related pressures when supply falls short.

The demand function is parameterized with an elasticity of $\sigma = 0.65$, an intermediate value consistent with the range of empirical estimates in the literature (typically 0.5-0.8), thereby reflecting copper’s relatively inelastic nature. This captures the fact that demand decreases only moderately in response to price increases, due to copper’s essential role in construction, infrastructure, and electrical systems.

2. Results

a. The Growing Share of Secondary Supply in Total Consumption

According to the IEA’s report *Recycling of Critical Minerals* (2025), the share of secondary copper in total refined copper consumption increased steadily between 2005 and 2015 but has since stagnated at around 17-18% of global supply. This plateau reflects both structural constraints in collection and processing, and the fact that demand growth over the past decade has been driven largely by new infrastructure and consumer goods, where scrap return rates remain limited.

Our simulations, calibrated to the IEA’s scenario trajectories, reproduce this stagnation in the early 2020s and project diverging pathways, thereafter, depending on policy ambition and recycling investment. In all scenarios, the share of secondary copper begins at 19% in 2025, but its subsequent evolution differs sharply:

Table 4.1: 2025-2050 Evolution of the Share of Recycled Copper in Total Consumption (in %)

	STEPS	APS	NZE	BAU
2025	19%	19%	19%	19%
2030	20%	20%	21%	19%
2040	28%	31%	31%	23%
2050	36%	41%	42%	26%

Figure 4.2: Share of Recycled Copper in Total Consumption (in %)



In the STEPS scenario, the share rises gradually to 19.8% in 2030, 28.2% in 2040, and 36.4% in 2050. This trajectory is consistent with the IEA's benchmarks (17%, 26%, and 33% in 2030, 2040, and 2050, respectively), though our calibration suggests a somewhat faster increase in later decades.

In the APS scenario, secondary copper expands more rapidly, reaching 20.2% in 2030, 31.1% in 2040, and 40.7% in 2050. These values align closely with the IEA's projections of 18%, 28%, and 39% over the same horizons, reflecting the impact of stronger policy commitments and targeted investment in recycling infrastructure.

In the NZE scenario, secondary copper plays an even more prominent role, rising to 20.6% in 2030, 31.4% in 2040, and 41.8% in 2050. This trajectory is consistent with the IEA's forecast of 17%, 26%, and 39%, though again with slightly higher penetration by mid-century as recycling infrastructures reach advanced stages of maturity in our framework.

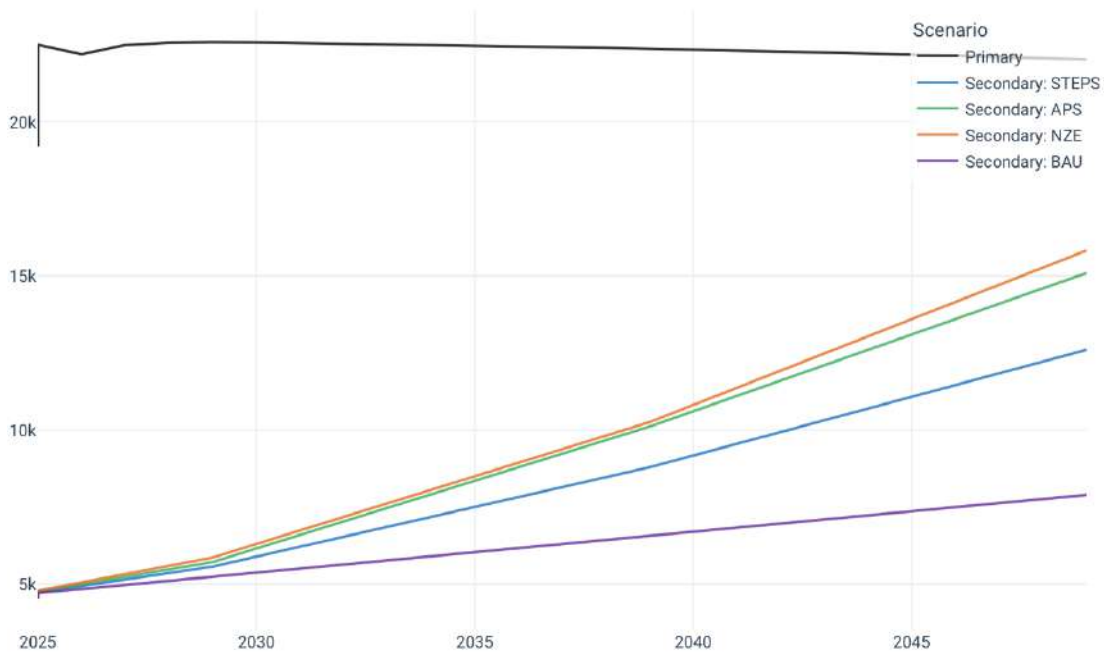
By contrast, the BAU scenario shows only modest gains: 18.8% in 2030, 22.7% in 2040, and 26.4% in 2050. In the absence of decarbonization policies and without systematic investment in recycling infrastructures, secondary copper supply remains tied to historical growth rates. This limited expansion leaves the share of recycling far below that in policy-driven scenarios and highlights the structural dependence on primary extraction in a no-transition world.

Overall, the projections highlight that the share of secondary copper in total supply is highly sensitive to the policy and investment assumptions embedded in each scenario. In STEPS, APS, and NZE, the gradual strengthening of recycling systems results in a steady increase of secondary copper's contribution, broadly consistent with the IEA's benchmarks. By contrast, in BAU the absence of transition-related policy support leads to slower progress, with recycling stabilizing at a lower share of

total consumption. These differences illustrate that future recycling penetration is not predetermined but contingent on the trajectory of energy transition policies, technological improvements, and infrastructure development⁹⁶.

For the record, Figure 4.3 below shows the evolution of primary and secondary supply in volumes. The evolution of primary supply is arguably identical across scenarios, since it is primarily governed by non-scenario specific resource availability constraints, so it is only represented by a single curve.

Figure 4.3: Primary and Secondary Supply Growth over 2025-2050



b. Price Trajectories: Stabilization through Recycling

Building on the projected global copper requirements outlined across various IEA scenarios and incorporating the policy-driven growth trajectories of secondary supply derived from the same scenarios, we now assess how these combined forces shape demand-supply interactions in our model.

⁹⁶ Nonetheless, we note that even under ambitious policies, the secondary share is structurally capped. First, when ex-ante demand grows faster than scrap availability, additions to in-use stock outpace retirements, so recycling cannot scale one-for-one with consumption. Second, product longevity imposes a hard flow constraint on the secondary pool. Many building and grid uses are designed for multi-decade service lives: the global technological lifetime of high-grade copper averages 45 years in use; infrastructure applications often reach 60 years; and copper water mains and piping commonly last 50–70 years (or even surpasses 70 years in optimal conditions). Significant legacy volumes (e.g., cabling in unused yet undemolished buildings) can remain inaccessible for more than 30 years, delaying their entry into the scrap pool. Thus, even under closed-loop conditions with perfect collection and yields, replacement cycles bound the recoverable flow; the traditionally low-margin economics of scrap collection and sorting further disincentivize investment, slowing throughput. As a result, primary supply remains necessary to cover stock growth and long replacement cycles.

On the demand side, future copper needs as anticipated by the IEA in its different scenarios are translated into effective consumption through our CES specification with calibrated elasticity and base intensity factors. On the supply side, primary production follows the depletion-constrained trajectory defined by the cumulative availability curve, while secondary production evolves according to scenario-specific recycling maturity paths. The resulting equilibrium price P_t in each year is determined by the adjustment between demand and dual-stream supply:

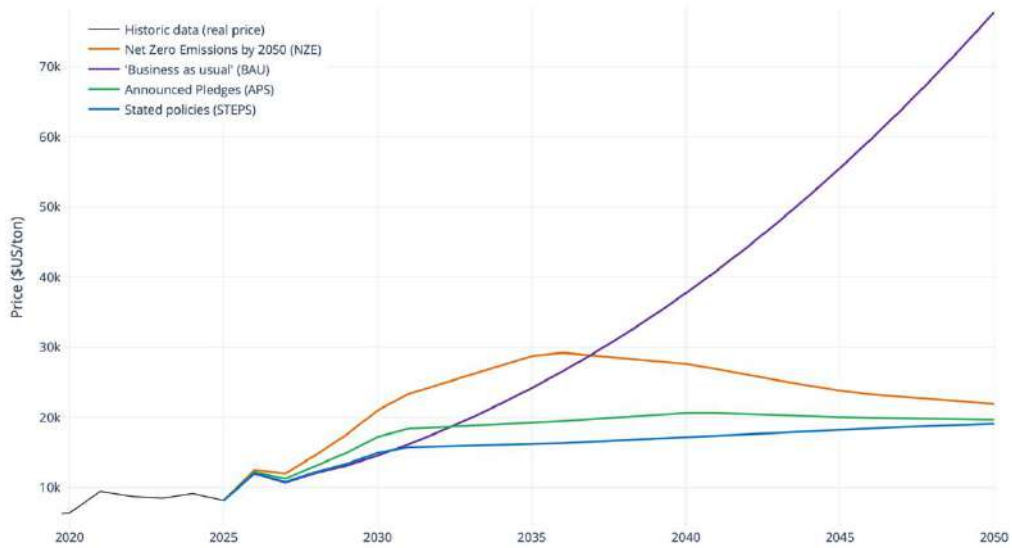
$$P_t = \left(\frac{\frac{c_{max}(P_{t-1})}{1 + e^{\gamma \left(\frac{c_t}{c_{max}(P_{t-1})} - \theta \right)}} + \eta \cdot P_t^\delta \cdot S_{t-1}}{B(t)} \right)^{\frac{1}{\sigma-1}}$$

The demand-supply interactions of the expanded model yield the following price trajectories:

Table 4.2: Simulated Equilibrium Price Paths Over the 2025-2050 horizon (in US\$/t)

<i>(in US\$/t)</i>	STEPS	APS	NZE	BAU
2025	8139	8139	8139	8139
2030	14927	17223	21009	14545
2040	17179	20626	27616	37708
2050	19072	19670	21890	77765
2025/2050 increase (%)	134%	142%	169%	855%

Figure 4.4: Projected Price Curves per Scenario over 2025-2050 (in US\$/t)



The simulated price trajectories across the four scenarios reveal not only differing growth patterns but also fundamentally distinct equilibrium dynamics. The central insight is not the absolute price levels for themselves, which remain indicative, but the trajectory and its consistency with a non-explosive, non-zero (NENO) equilibrium.

In the STEPS and APS scenarios, prices follow a steep but ultimately contained growth path. Starting from 8,139 US\$/t in 2025, prices rise to about 19,072 US\$/t in STEPS and 19,670 US\$/t in APS by 2050—an increase of 134% and 142% respectively. This reflects mounting demand for clean technologies but also the moderating effect of policy-driven improvements in recycling infrastructure. In both cases, the expansion of secondary supply plays a central role in alleviating pressure on primary production, ensuring that the market remains compatible with NENO dynamics despite sustained tension.

The NZE scenario produces the fastest price growth, consistent with the aggressive early deployment of low-carbon technologies. Prices nearly triple between 2025 and 2050, climbing to 21,890 US\$/t—an increase of 169%. The profile is particularly steep until 2035, reflecting the surge in demand for renewables, grids, and electric vehicles. Beyond this point, however, demand growth slows as efficiency measures and substitution take hold, while accelerated investment in recycling capacity provides additional relief. Together, these dynamics prevent the trajectory from becoming explosive. NZE illustrates a market under acute pressure, requiring major advances in both primary and secondary supply, but not locked into destabilization.

By contrast, the BAU scenario reveals a fundamentally different and more concerning pattern. In the short term, prices increase only moderately compared to the transition scenarios, reflecting the inertia of current consumption structures where clean technologies expand only slowly. However, this dynamic is deceptive. In the absence of decarbonization policies, demand from traditional sectors continues to expand without constraint, driven largely by the industrialization of emerging economies. At the same time, the lack of strong policy incentives means that recycling infrastructures do not experience the scale of investment seen in the transition scenarios, leaving secondary supply growth tied to historical trends. The combined effect is explosive: by 2050, prices in BAU reach nearly 77,800 US\$/t—an increase of more than 850% compared to 2025 levels. This trajectory diverges sharply from a NENO equilibrium. What is particularly striking is that this destabilizing dynamic emerges not from the push of the energy transition, but from its absence. Without coordinated investment in recycling capacity, efficiency improvements, or substitution, uncapped consumption growth places unsustainable pressure on primary supply, generating the most extreme long-term escalation in prices.

c. Depletion: Converging Long-Run Extraction Limits Despite Recycling

Our depletion assessment begins with an initial cumulative extraction baseline of 700,000 kt, representing the amount of copper extracted globally to date. By 2050, cumulative primary extraction is projected to reach 1,256,203 kt in STEPS, 1,256,556 kt in APS, 1,256,802 kt in NZE, and 1,256,178 kt in BAU. These figures indicate that across all scenarios, cumulative extraction converges to nearly identical levels in the long run. This convergence highlights the central role of supply-side constraints: regardless of differences in demand growth, the limited elasticity of primary supply and depletion dynamics impose a ceiling on the total amount of copper that can be extracted over the coming decades⁹⁷.

In practice, secondary supply acts as a relief mechanism that alleviates pressure on primary resources, yet its contribution remains insufficient to offset the scale of copper requirements associated with the energy transition. Recycling and scrap availability increase gradually and account for a rising share of consumption, but the intensity of new demand ultimately drives extraction toward the boundaries of known recoverable resources. In this sense, depletion remains fundamentally governed by geological and economic availability, with secondary supply moderating—but not fundamentally altering—the trajectory.

3. Sensitivity Analysis

a. How Market Responsiveness of Copper Recycling Shapes Price and Supply Shares

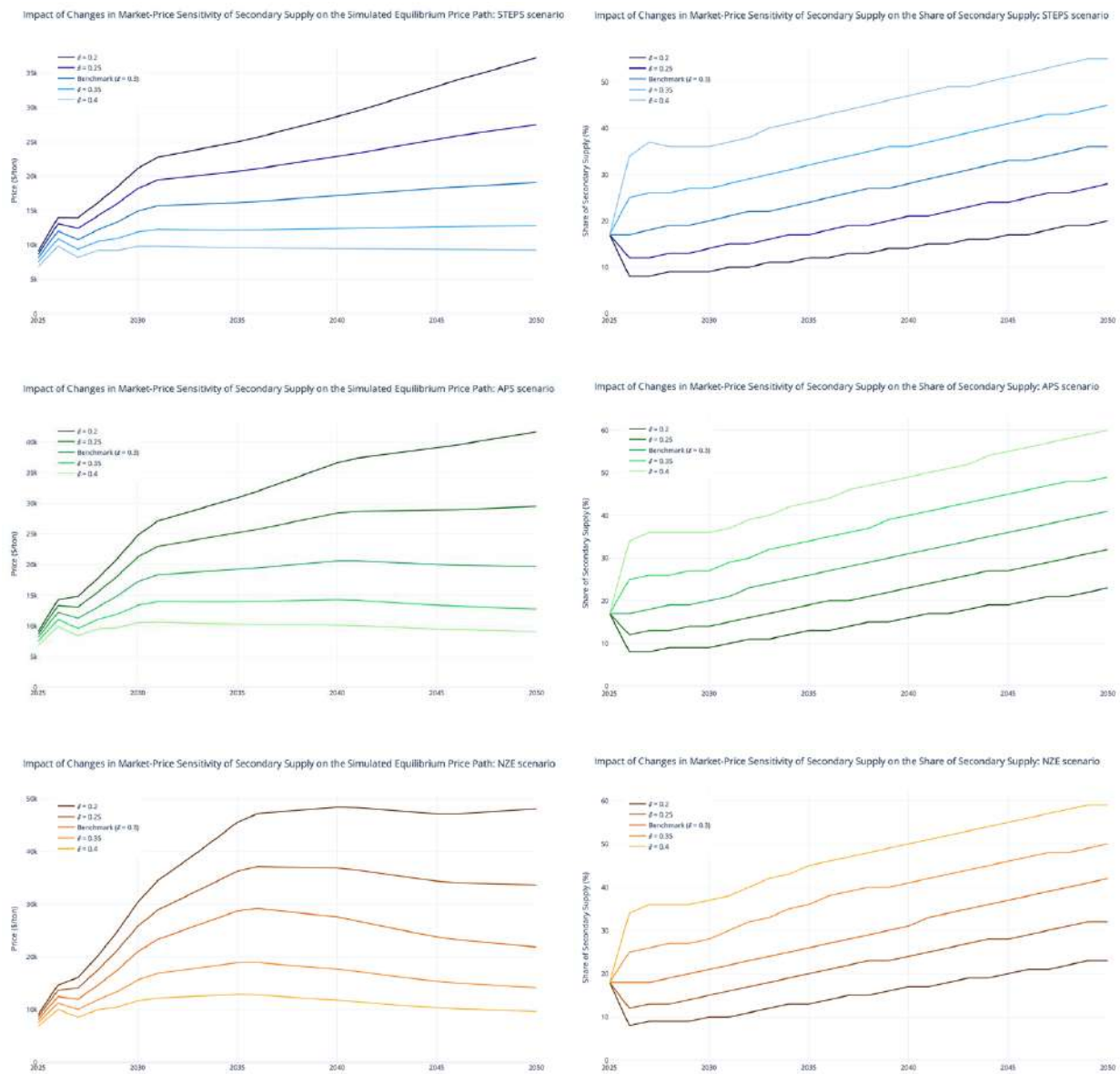
To assess the robustness of our simulations and our specification for secondary supply, we conduct a sensitivity analysis on the price elasticity parameter δ in $Q_{s,t}^{secondary} = \eta P_t^\delta S_t$.

The benchmark calibration adopts $\delta = 0.3$. We test alternative values $\{0.2, 0.25, 0.35, 0.4\}$, thereby exploring both weaker and stronger nonlinearities in the response of recycling flows to price signals. The analysis is run for the three IEA energy transition scenarios STEPS, APS, NZE. We do not extend it to the BAU case, since in that scenario secondary supply is modeled exogenously as a constant-CAGR expansion and not as an endogenous function of price.

⁹⁷ Importantly, the CAC is parametrized on a total of 3,500,000 kt of known reserves and resources, meaning that even under accelerated transition scenarios, we remain far from exhausting copper endowments—contrary to common concerns about imminent depletion.

The results show that the choice of δ has a decisive effect on the magnitude and trajectory of equilibrium prices. Lower values of δ (0.2 and 0.25) imply a weaker responsiveness of secondary supply to price incentives. In these cases, secondary flows expand more slowly, leaving a greater share of adjustment to primary extraction. A notable consequence is that, during the first decade of the simulation, the share of secondary supply actually falls below current levels. This reflects the strong surge in demand, which contrasts with the relative rigidity of recycling flows when their responsiveness to price is muted. The consequence is a markedly higher price path across all scenarios: under $\delta = 0.2$, average prices by 2030-2040 are roughly twice those observed under the benchmark calibration, and in the NZE scenario they exceed 40,000 \$/t by mid-century.

Figure 4.5: Impact of Changes in Secondary Supply Market-Price Sensitivity on the Simulated Equilibrium Price Paths and Shares of Secondary Supply for Each Scenario



Conversely, higher values of δ (0.35-0.4) make secondary supply much more sensitive to price signals. Here, even moderate price increases quickly stimulate recycling flows, easing pressure on primary supply. As a result, equilibrium prices stabilize at much lower levels: under $\delta = 0.4$, projected prices flatten out close to 10,000 USD/t from the early 2030s onward, even in the NZE scenario.

Regarding the share of secondary in meeting total consumption, the key observations are that the impact of δ on this share is also symmetric, and more importantly, that the slope of increase in the secondary share is constant across all calibrations. In other words, while the absolute level of secondary contribution varies, the rate of growth of that share through time is invariant. This outcome is coherent with the fact (demonstrated analytically in Section III.1) that the own-price elasticity of secondary supply is constant and equal to δ . The sensitivity results therefore not only confirm the consistency of our formulation but also provide an empirical illustration of its implications.

b. Benchmarking Against the No-Recycling Case: Recycling as a Buffer Rather than a Substitute for Mining

As a complementary exercise, we also examine the particular case in which $S_t = 0$ for the entire horizon. In this setup, no secondary flows are available, and the model collapses back to the framework of Chapter 3, where equilibrium relies exclusively on primary supply. It provides a natural benchmark for measuring the extent to which recycling dampens price pressures and alleviates reliance on primary extraction.

Figure 4.6: Comparison of Simulated Price Trajectories per Scenario, With and Without Secondary Supply Contribution



The comparison between the two frameworks illustrates the central role of secondary supply in moderating long-term price dynamics. By 2050, recycling reduces equilibrium prices by around 72% in APS, 79% in NZE, and about 73% in STEPS. Even in BAU, where secondary supply expands only exogenously and moderately, the gap remains very large: prices are reduced by almost 60% (from nearly 187,000 to 77,800 USD/t). In other words, the presence of recycling shifts the system from explosive trajectories to a more contained long-run stabilization, across all demand scenarios. This contrast gives empirical validity to the stylized fact highlighted in Section III.2, namely that recycling dampens the price impact of demand shocks. Formally, while $\partial P_t / \partial Q_{d,t}$ remains positive, it decreases as lagged secondary capacity S_{t-1} grows. The simulation results illustrate this mechanism in practice: when S_t is absent, the entire adjustment falls on primary supply and prices surge accordingly; when recycling is present, the responsiveness of secondary flows absorbs part of the adjustment, flattening the long-term price path.

A similar conclusion emerges from the analysis of depletion. By 2050, cumulative extraction converges to nearly identical levels across scenarios both with and without recycling, confirming that depletion dynamics cap long-run primary supply. Introducing recycling lowers cumulative primary extraction only marginally (about 0.5–0.6% relative to the no-recycling case) without altering the convergence

pattern. The main effect of recycling is therefore not to reduce the ultimate volume of copper mined, but to moderate the price trajectory along the way⁹⁸.

This moderation also effects the balance between requirements and attainable supply. When accounting for secondary supply, unmet needs decline sharply across all scenarios compared to the no-recycling case. By 2050, the share of requirements left unmet falls to 8% in STEPS, 10% in APS, 14% in NZE, and 24% in BAU, versus 33%, 36%, 40%, and 40% respectively without recycling. While recycling cannot eliminate resource scarcity, it materially alleviates the gap between projected requirements and attainable supply. In lower-demand scenarios such as STEPS and APS, the shortfall is reduced to under 10%, suggesting that recycling significantly enhances the feasibility of these transition pathways. Even in NZE, where clean technology requirements are most demanding, unmet needs are nearly halved compared to the no-recycling case. BAU remains the most constrained pathway, with unmet needs still above 20%, reflecting the materially unsustainable trajectory of unchecked demand growth. Overall, these results confirm that the main contribution of recycling is not to change the ceiling on ultimate cumulative extraction, but to moderate prices and shrink the demand-supply gap along the transition path.

Finally, recycling also reshapes the role of demand elasticity. With secondary supply, the model's sensitivity to demand elasticity is markedly reduced. Variations in σ continue to affect outcomes, and the asymmetry remains (i.e. lower elasticity amplifies scarcity pressures more strongly than higher elasticity mitigates them) but the overall spread between scenarios narrows substantially. What was a decisive driver of long-term trajectories in the primary-only framework becomes a more moderate influence once secondary supply is introduced. This shift highlights recycling's deeper role: beyond moderating scarcity directly, it also stabilizes the system's response to elasticity assumptions, making outcomes less fragile to uncertainties in the calibration of σ . Recycling therefore not only dampens

⁹⁸ This finding provides an important contrast with the IEA's (2024) assessment that a successful scale-up of recycling could lower the need for new mining activity by 25–40% by 2050 in a scenario that meets national climate pledges (APS). While the IEA frames recycling as substituting for new mine development, our model produces a much smaller, almost negligible quantitative effect. The reason lies in the different adjustment variables. In our framework, price is the central equilibrating mechanism: as demand rises, prices increase, stimulating both primary and secondary supply until cumulative availability constraints are binding. In this setting, recycling moderates the price trajectory but does not substantially reduce the total volume of mining required over the horizon. By contrast, the IEA's metric is benchmarked to ex ante requirements derived from demand forecasts, against which the expansion of primary supply is assessed. In practice, this often relies on extrapolating growth from the pipeline of new mining and refining projects. Within this framing, additional recycling can be interpreted as directly reducing the need to bring forward new projects. In our equilibrium-based model, however, unmet demand is not absorbed by recycling alone but instead reflected in higher prices and constrained primary extraction. Recycling thus plays a critical role as a price stabilizer, but not as a substitute for mine development in volumetric terms. Importantly, this result is robust even when we introduce more elastic primary supply through technological progress: additional primary flows alleviate unmet demand and reduce price tensions, but primary and secondary supply do not substitute for one another in the face of such strong baseline demand growth.

market sensitivity to price shocks but also reduces the model's sensitivity to demand responsiveness itself, reinforcing the robustness of the long-run equilibrium.

V. Conclusion

This chapter extended the original analytical framework by incorporating secondary supply as an endogenous component of the market. The analytical derivations show how the inclusion of recycling modifies several properties of the system. First, the price elasticity of total supply becomes state-dependent, increasing as recycling capacity builds up. Second, the price response to demand shocks diminishes progressively as the share of secondary flows rises. Third, the effective sensitivity of aggregate consumption to primary disruptions is reduced, since a larger fraction of total supply is provided by streams that are less directly exposed to depletion shocks. Fourth, the Markovian formulation introduces hysteresis: once recycling advances, it does not revert, and capacity carries the imprint of past market conditions. Finally, the emergence of recycling capacities allow to absorb baseline demand growth. These stylized facts, absent in the primary-only framework, highlight how secondary supply changes the dynamic adjustment process rather than the long-run ceiling on extraction.

The calibration to copper supports these theoretical properties. Relative to the no-recycling case, recycling reduces price pressures by 60–80% across transition scenarios and narrows the divergence between elasticity assumptions, thereby stabilizing outcomes. Most importantly, it drastically reduces unmet needs, making the transition scenarios more attainable. This shift is decisive: while recycling does not change the ultimate limit of cumulative primary extraction, it alters the feasibility of demand pathways by bridging a substantial portion of the gap between requirements and attainable supply. In practical terms, secondary supply transforms transition scenarios from being structurally unattainable in a primary-only framework into trajectories that, while still challenging, lie within the bounds of market feasibility.

Several caveats apply to these results. The model excludes stochastic shocks such as wars, financial crises, or abrupt technological breakthroughs, which could alter both primary and secondary supply trajectories. It also abstracts from explicit substitution between metals (for example, aluminum substituting for copper in certain applications) which could change the responsiveness of demand. Extraction technology progress is assumed to be constant rather than explicitly modeled, and secondary supply is treated at the global level without accounting for regional scrap flows, trade frictions, or heterogeneous recycling infrastructures. These simplifications are consistent with the baseline analytical framework but limit the scope of the conclusions to the stylized behavior of the system rather than precise forecasts.

In sum, the integration of recycling shows that secondary supply does not lower the long-run ceiling on extraction, but it fundamentally alters the adjustment path. It reduces price escalation, alleviates unmet needs to an extent that makes transition pathways far more attainable, and diminishes the fragility of outcomes to elasticity assumptions. The results illustrate how recycling shapes the feasibility of copper markets under transition-driven demand growth, while remaining subject to the simplifying assumptions embedded in the model.

VI. Annex: Condition Price Segmentation: When Primary and Secondary Metals Do Not Share a Unified Market

In the main body of this chapter, we restrict attention to metals where primary and secondary outputs are physically and commercially indistinguishable and therefore traded as a single good at a single price. This assumption covers the bulk of global metal consumption in volumetric terms (copper, aluminum, lead, zinc, tin, silver, gold, titanium, magnesium, tungsten, etc.).

However, some markets remain segmented: persistent price spreads exist between primary and secondary materials, reflecting differences in quality, form, processing requirements, or end-use suitability. Examples include the copper scrap discount relative to cathode, the persistent premia between nickel Class I and Class II, or the continuing segmentation between mined and recycled rare earths (He et al., 2020)⁹⁹. In such markets, a dual-price equilibrium arises, where primary and secondary metals coexist as imperfect substitutes, each with its own price trajectory.

The purpose of this annex is to extend the analytical framework to such cases. Specifically, it asks: under what conditions does the relative price of secondary metal converge toward that of primary metal, and under what conditions does segmentation persist?

The analytical development that follows formalizes the conditions under which these two prices may converge or remain segmented. It characterizes (i) the scaling rules required on the supply side for secondary flows to fully substitute for primary extraction; (ii) the substitution symmetries needed in the demand system; and (iii) the knife-edge cases in which cost-based secondary pricing could achieve parity with primary prices.

In order to study markets where the single price assumption does not hold, we introduce two distinct price levels: $P_{primary,t}$ and $P_{secondary,t}$. We refer to the higher-quality, benchmark form as “primary” and to the alternative stream as “secondary.” This terminology is meant broadly: in some markets it corresponds to newly mined versus recycled metal (e.g. copper cathode vs. scrap), while in others it reflects different product classes or qualities (e.g. nickel Class I vs. Class II).

⁹⁹ He et al. (2020) estimate the life cycle cost (LCC) of recovering rare earth elements from waste smartphones in China at about USD \$0.08/g for neodymium and USD \$0.016/g for praseodymium. As of August 2025, market prices are roughly USD \$0.09-\$0.10/g and USD \$0.12/g respectively, leaving neodymium only marginally profitable and praseodymium with a wider margin, but subject to pronounced market volatility. By contrast, cobalt (USD \$10.106/g LCC vs. ≈USD \$0.03/g market) and antimony (USD \$0.135/g vs. ≈USD \$0.009/g) are deeply uneconomic to recycle, while palladium, platinum, and beryllium have market prices (≈USD \$38/g, USD \$31/g, USD \$0.32/g) far above their extraction costs, suggesting stronger economic incentives. These figures show that, even where price spreads are favorable, REE recovery is hindered by technological complexity, market instability, and supply chain barriers—explaining why it remains scarce at scale despite strategic importance.

The relative price ratio is then defined as:

$$\lambda_t \equiv \frac{P_{secondary,t}}{P_{primary,t}} \in (0,1)$$

This ratio reflects both technological asymmetries (e.g., purity, form, performance) and economic frictions (e.g., collection costs, regulatory gaps). Presumably, in early recycling regimes, λ_t tends to be low due to quality differentials, immature infrastructure, and low elasticity of secondary supply. Our core analytical question is to determine under which does λ_t converge toward 1 over time.

Additionally, we introduce the total metal demand as a CES aggregated function over the two traded streams, primary and secondary (in the broad sense defined above):

$$Q_{d,t} = B(t) \cdot P_{primary,t}^{\sigma-1} + B'(t) \cdot P_{secondary,t}^{\rho-1}$$

Where:

- σ and ρ are composite parameters that reflect two distinct but related features of the demand system: the degree to which primary and secondary metals can be substituted for one another (i.e., the elasticity of substitution), and the responsiveness of demand for each metal type to changes in its own price (i.e., own-price elasticity). The key assumption is that primary and secondary metals are substitutes, but with different price sensitivities, as highlighted in the literature (Fleming, 2011; Radetzki & Van Duyne, 1985).
- $B(t)$ and $B'(t)$ encapsulate time-varying baseline parameters capturing macroeconomic factors like GDP growth, population changes, and technological progress.

We distinguish two pathways toward convergence: one from the quantity side—driven by the interplay between supply limits and aggregate demand—and one from the demand structure itself—shaped by the internal logic of the CES specification¹⁰⁰.

¹⁰⁰ While the formal conditions derived below are necessary, they are not empirically sufficient: real-world convergence also depends on structural features. First, the scrap stock S_{t-1} must reach and maintain a sufficiently high level—though not necessarily its absolute maximum S_n —which in practice requires institutional development, infrastructure buildup, and improved material recovery. Second, the ratio $B(t)/B'(t)$ should remain bounded away from zero and infinity over time, preventing either primary or secondary supply from dominating total demand and preserving the relevance of both inputs. Finally, persistent quality differentials between primary and secondary metals must diminish through standardization, technological upgrades, or shifts in industrial practice, so that substitutability is not constrained by physical characteristics.

- **Quantity-Side Convergence: Capacity and Scaling Conditions**

The market equilibrium with two price streams is now:

$$B(t) \cdot P_{primary,t}^{\sigma-1} + B'(t) \cdot P_{secondary,t}^{\rho-1} = Q_{s,t}^{primary} + \eta \cdot P_{primary,t}^{\delta} \cdot S_{t-1}$$

Substituting $P_{secondary,t} = \lambda_t \cdot P_{primary,t}$ yields:

$$P_{primary,t}^{\sigma-1} \cdot [B(t) + B'(t) \cdot \lambda_t^{\rho-1} \cdot P_{primary,t}^{\rho-\sigma}] = Q_{s,t}^{primary} + \eta \cdot P_{primary,t}^{\delta} \cdot S_{t-1}$$

Solving for λ_t (assuming $\rho \neq 1$ ¹⁰¹), we obtain:

$$\lambda_t = \left(\frac{\frac{Q_{s,t}^{primary}}{P_{primary,t}^{\sigma-1}} + \eta \cdot S_{t-1} \cdot P_{primary,t}^{\delta-\sigma+1} - B(t)}{B'(t)} \right)^{\frac{1}{\rho-1}}$$

In the long run, three features are key:

- $Q_{s,t}^{primary}$ follows a logistic depletion curve and is therefore bounded;
- S_{t-1} converges toward a finite upper bound S_n ;
- Parameters η , $B(t)$, and $B'(t)$ are all positive and bounded^{102,103}.

Additionally, we assume that $P_{primary,t}$ increases monotonically over time¹⁰⁴.

¹⁰¹ The case $\rho = 1$ corresponds to a Cobb-Douglas specification, implying unitary elasticity of substitution between primary and secondary metals. This parametrization would impose strong behavioral symmetry: the relative input shares remain constant in response to price changes. In practice, substitution is often constrained by quality differentials, technological lock-ins, or regulatory standards. Such frictions support assuming $\rho \neq 1$: values of $\rho < 1$ reflect limited substitutability and structural segmentation, while $\rho > 1$ corresponds to stronger competitive pressure and greater input flexibility.

¹⁰² Indeed, it represents the efficiency with which available scrap stock is converted into usable secondary metal in response to price signals. A positive value ensures that higher prices lead to an increase in secondary supply, consistent with observed recycling behavior, while boundedness reflects physical, technical, and economic constraints—such as material losses, processing limitations, and diminishing returns—that prevent all available scrap from being fully or instantly recoverable. Allowing η to be unbounded would imply implausibly large or infinite secondary supply responses, violating both real-world feasibility and the internal stability of the model.

¹⁰³ Since they reflect structural demand intensities for primary and secondary metals that evolve over time but remain constrained by technological saturation, substitution limits, and material requirements, ensuring the CES demand function remains economically and mathematically well-behaved.

¹⁰⁴ The assumption of a monotonically increasing primary metal price path should be viewed as a stylized long-term trend rather than a literal description of historical price series. Metal prices are known to fluctuate due to business cycles, speculative dynamics, and temporary supply-demand imbalances. However, over multi-decade horizons, structural drivers such as declining ore grades, increasing energy and environmental costs, and sustained demand growth—amplified by industrialization and the energy transition—tend to exert

As $P_{primary,t} \rightarrow \infty$, the primary supply term vanishes relative to demand, so convergence depends on the asymptotic balance:

$$f(t) \equiv \frac{\eta \cdot S_n \cdot P_{primary,t}^{\delta-\sigma+1} - B(t)}{B'(t)}$$

For $\lambda_t = (f(t))^{\frac{1}{\rho-1}}$ to converge toward 1, two cumulative conditions must hold¹⁰⁵:

(1) *Supply-Demand Elasticity Parity*

$$\delta = \sigma - 1$$

Secondary supply must scale at the same rate as demand under rising primary prices. If $\delta > \sigma + 1$, secondary overshoots and distorts equilibrium; if $\delta < \sigma + 1$, it lags behind, preventing convergence ($f(t)$ converges toward $\lim_{t \rightarrow \infty} -\frac{B(t)}{B'(t)} < 0$, leaving λ_t is undefined).

(2) *Long-Run Capacity Alignment*

$$\eta \cdot S_n = B(t) + B'(t)$$

Secondary supply at saturation must be sufficient to meet aggregate demand at equilibrium.

Taken together, these two cumulative relationships describe the conditions, on the quantity side of the model, under which the relative price ratio λ_t approaches unity over time. In other words, convergence requires that the rate at which secondary supply scales with rising primary prices matches the price sensitivity of demand, and that long-run secondary capacity is sufficient to meet aggregate demand at equilibrium.

This mechanism is well illustrated by aluminum, which in practice falls within the general one-price setting analyzed in the main text. High recyclability, long-lived scrap availability, and efficient collection systems sustain a large secondary base. With an estimated 75% of all aluminum ever produced still in

upward pressure on real prices. Thus, the monotonicity assumption captures the dominant long-run trajectory, abstracting from short-term volatility to focus on the asymptotic behavior of the system.

¹⁰⁵ Other combinations can also deliver $f(t) \rightarrow 1$: e.g., bounded $P_{primary,t}$ with $\eta S_n \bar{P}^{\delta-\sigma+1} = B_\infty + B'_\infty$, or time-varying $\delta(t)$ and $\eta(t)S_{t-1}$ such that $\eta(t)S_{t-1}P^{\delta(t)-\sigma+1} = B(t) + B'(t) + o(1)$. In trending-price calibrations with $\delta < \sigma - 1$, however, $f(t)$ tends to be a negative constant, so the canonical pari $\delta = \sigma - 1$ and $\eta S_n = B + B'$ is asymptotically required for convergence.

use and a global recycling efficiency near 76% (IAI, 2025), aluminum markets approximate the conditions for convergence, even if temporary segmentation can occur.

- **Demand-Side Convergence: Symmetry of Substitution**

On the demand side, convergence depends on the internal logic of the CES aggregator. Recall that the CES specification determines how strongly relative price changes shift demand shares between “primary” and “secondary” streams.

Substituting $P_{secondary,t} = \lambda_t P_{primary,t}$ into the demand equation, we obtain:

$$Q_{d,t} = P_{primary,t}^{\sigma-1} [B(t) + B'(t) \lambda_t^{\rho-1} P_{primary,t}^{\rho-\sigma}]$$

This expression shows that the weight of each component depends not only on λ_t but also on the relative magnitude of ρ and σ . We can distinguish three cases:

- If $\rho = \sigma$, the demand function is scale-invariant in prices. Both streams retain comparable weight, and $\lambda_t \rightarrow 1$ is structurally feasible.
- If $\rho > \sigma$, the secondary term grows disproportionately as $P_{primary,t}$ rises, which is only sustainable if $\lambda_t^{\rho-1} \rightarrow 0$, i.e convergence fails.
- If $\rho < \sigma$, the secondary component contracts too quickly, forcing $\lambda_t^{\rho-1}$ to diverge to maintain balance, which again contradicts convergence.

Hence a third necessary condition for price convergence:

(3) *Substitution symmetry between inputs*

$$\rho = \sigma$$

This ensures that neither stream is structurally favored as prices evolve, allowing relative prices to converge under stable substitution patterns.

The importance of this condition is visible in markets like nickel, where Class I (battery-grade) and Class II (ferronickel or NPI) serve distinct applications. Even if prices were to converge mechanically, persistent substitution asymmetry prevents full integration of demand.

- **Extended and Alternative Cases**

The three conditions derived above (elasticity parity, capacity alignment, and substitution symmetry) define the benchmark setting under which long-run price convergence can occur. These results rely on strong assumptions: that primary prices rise monotonically and without bound, that demand expands accordingly, and that secondary systems scale smoothly. In practice, metal markets often deviate from such idealized dynamics. To evaluate robustness, it is useful to consider alternative configurations where price dynamics or system constraints relax the baseline requirements.

Case 1: Bounded Primary Prices

Under the assumption that $P_{primary}$ increases monotonically, convergence requires $\delta = \sigma - 1$, ensuring that secondary supply scales exactly with the price-driven contraction of demand as $P_{primary}$ grows without limit. If instead primary prices converge toward an upper bound $\bar{P}$ (due to technological breakthroughs, demand stabilization, substitution pressures, or policy interventions like price caps and strategic stockpiles), then the strict scaling condition can be relaxed.

However, primary metal prices may exhibit long-run boundedness (e.g., price caps, strategic stockpiles).

In this case, what matters is not the growth rate but the absolute capacity of secondary supply at the limiting price level:

$$\eta S_n \bar{P}^{\delta - \sigma + 1} = B(t) + B'(t)$$

Even if $\delta \neq \sigma - 1$, convergence is feasible as long as secondary capacity, adjusted by the final price, is sufficient to close the demand gap. The emphasis thus shifts from relative scaling to level adequacy.

This “bounded price” regime is consistent with metals in mature industrial ecosystems where demand has plateaued and recycling systems are entrenched. Lead provides a strong example: 45–60% of refined output comes from secondary supply (mainly recycled batteries), and this share stabilizes long-run equilibrium without requiring precise scaling rules. Importantly, lead markets fall within the general “one good, one price” setting, since primary and secondary outputs are fully substitutable once refined. Zinc shows a more moderate version of this pattern, with about 13% of refined output coming from secondary sources (ILZSG 2024), while smelters’ widespread integration of recycled inputs helps dampen price fluctuations. Here too, zinc markets fit the single-price framework of the main model, since recycled zinc is commercially indistinguishable from primary metal. In both cases, the prevalence of recycling supports long-term stability and convergence within the one-price setting.

Case 2: Primary Prices With No Clear Trend

The baseline conditions above relied on the assumption that primary prices either increase monotonically or converge toward a finite upper bound, which made it possible to derive long-run convergence analytically. In reality, primary metal prices often defy such deterministic paths: they may fluctuate cyclically, display sharp reversals, or remain volatile over extended periods due to macroeconomic shocks, speculative pressures, policy changes, or supply disruptions.

Under such conditions, strict convergence cannot be guaranteed. Instead, the price ratio λ_t may approach unity only transiently or locally, over finite intervals when market conditions align, before diverging again. These temporary episodes of convergence reflect regime-dependent dynamics rather than a permanent integration of the two streams¹⁰⁶.

The likelihood of such local convergence depends on several structural parameters. A higher η (secondary supply efficiency) amplifies the responsiveness of recycling to price shifts, facilitating convergence when profitability is high. The state variable S_{t-1} acts as a binding capacity constraint: when large volumes of end-of-life material are available and recycling infrastructures are well-developed, secondary output can respond more elastically to shifts in demand, enabling closer price alignment. Conversely, when S_{t-1} is low, either due to limited scrap generation or weak recycling capacity, secondary markets are rigid, and convergence is unlikely. Finally, the recycling price-responsiveness parameter δ matters¹⁰⁷: if it is close to 1, secondary output adjusts nearly proportionally with primary prices, which increases the chance of short-term convergence. If $\delta < 1$, the response is muted, and price gaps tend to persist.

Empirical evidence illustrates this mechanism in metals that fall within the general one-price setting of the main model but nonetheless experience fluctuating spreads. In the aluminum market, secondary

¹⁰⁶ Transient convergence may arise through several mechanisms. First, a sudden spike in P_{primary} (e.g., from geopolitical risk or concentrate shortages) can shift demand toward recycled metal in substitutable applications; because secondary supply cannot expand proportionately given scrap, capacity, or cost constraints, the additional demand pushes $P_{\text{secondary}}$ upward, narrowing the price gap. Second, a sharp fall in P_{primary} can redirect demand away from recycled metal and compress recycling margins—especially when $P_{\text{secondary}}$ is anchored to recovery costs and adjusts only slowly downward—so that secondary output contracts but prices decline more gradually, lifting λ_t . Third, during periods of relative stability, when P_{primary} fluctuates within a narrow range and secondary costs remain anchored, the two price series may remain closely aligned, creating a de facto converged regime for the duration.

¹⁰⁷ For tractability, this version of the model treats δ as a constant parameter. In reality, δ is better understood as a state-dependent variable that varies with the relative price spread. When the gap between primary and secondary price is large, recycling is highly profitable, and supply tends to be more responsive. As the spread narrows, incentives weaken, and responsiveness declines. Modeling δ as a function of λ_t (e.g., $\delta(\lambda_t) = \delta_0 + \delta_1(1 - \lambda_t)$) would therefore capture the self-limiting nature of convergence. This specification is consistent with evidence of price linkages and spillovers from primary to scrap markets, implying that arbitrage incentives tied to spreads condition secondary market behavior, as documented for copper and precious metals (Aruga & Managi, 2011; Hammoudeh et al. 2010).

prices typically track primary prices closely under stable conditions, thanks to material homogeneity and efficient recycling systems (Fisher and Owen, 1981). Yet during stress episodes—such as energy price shocks or abrupt demand shifts—the price gap widens as recycling costs decouple from primary production costs (IAI, 2024). Similarly, in the copper market, the spread between cathode and scrap prices varies over time with scrap collection efficiency, smelter availability, and arbitrage opportunities. Periods of near-convergence do occur—particularly under strong demand and smooth supply chains—but they are temporary, embedded within a broader pattern of recurrent segmentation (ICSG, 2022; Aruga and Managi, 2011).

In sum, when primary prices lack a clear trend, long-run analytical convergence cannot be established. Nevertheless, markets may exhibit episodic partial integration, where secondary and primary prices move closely together for limited periods, without eliminating structural segmentation over the long run.

Case 3: Secondary Prices are Cost-Based

A different analytical regime emerges when secondary prices are assumed to be anchored in marginal recovery costs rather than determined by market clearing¹⁰⁸. Let the secondary price be determined by a simple cost-plus technology:

$$P_{secondary,t} = c_0 + \frac{1}{\zeta} \cdot \frac{Q_{s,t}^{secondary}}{S_{t-1}}$$

¹⁰⁸ There are several motivations for a cost-based approach of recycled metal prices. First, secondary supply is capacity- and stock-constrained. The short run quantity of recoverable material is bounded by the legacy stock S_{t-1} and by processing capacity. When the supply side is effectively fixed, or highly inelastic, while downstream buyers face limited short-run substitution across grades, prices tend to reflect marginal recovery costs rather than a frictionless market-clearing of elastic supply and demand. Second, quality heterogeneity and asymmetric information are acute in scrap markets. Metals arrive in diverse forms and impurity profiles, often requiring sorting, blending, and yield-uncertain processing. Where verification is costly and contracts are bilateral, administered prices—fee schedules, discounts, and premia tied to assay bands—are common. These arrangements weaken instantaneous price discovery and anchor effective transaction prices to processing cost schedules rather than to a single competitive spot price. Third, the recycling chain mixes a service component with a commodity component. Much of the value added is the *service* of collection, pre-processing, environmental handling, and de-pollution. In industries with high fixed/overhead costs and regulated operating standards (EPR/take-back schemes, permitting, waste handling), pricing frequently follows cost-plus or formula pricing to ensure cost recovery and stable throughput, especially when volumes are volatile. Fourth, transaction structures often reference the primary benchmark but incorporate administrative discounts/premia and floors that are effectively cost-based. In practice, many contracts index to LME/COMEX (or to refined premia) and then apply fixed handling fees, treatment/processing charges, impurity penalties, and logistics surcharges. The indexation transmits some primary-price information, but the net realized price for secondary metal is capped/floored by cost components. Finally, inventory and queueing frictions reinforce cost anchoring. With finite storage and batch processing, plants manage queues by adjusting fees and acceptance thresholds rather than continuously clearing at a single market price; effective prices thus move discretely with cost shocks and utilization, not smoothly with every demand perturbation.

Where $c_0 > 0$ captures baseline collection, sorting, and processing costs, and $\zeta > 0$ governs the slope of marginal recovery costs. The relative price ratio then becomes:

$$\lambda_t = \frac{c_0}{P_{primary,t}} + \frac{\eta}{\zeta} \cdot P_{primary,t}^{\delta-1}$$

When $0 < \delta < 1$, which is economically most likely since secondary output typically responds less than proportionally to price because of capacity limits, frictions, and because of the finite nature of S_{t-1} , both terms of λ_t are decreasing with $P_{primary,t}$. If primary prices drift upward without bound, λ_t collapses toward 0: secondary prices rise in levels but fall ever further behind in relative terms. Convergence is therefore ruled out.

If $\delta < 1$ but primary prices are bounded, λ_t does not vanish but stabilizes within a finite band. This implies a weak form of segmentation: the spread persists but does not widen indefinitely.

Only in the knife-edge case $\delta = 1$ can λ_t converge to a constant η/ζ . Full parity would then require $\eta = \zeta$, i.e. exact offset between secondary responsiveness and marginal recovery costs. In practice, such conditions are highly unlikely: recycling systems typically face diminishing returns as scrap quality declines and processing costs rise, so that $\eta/\zeta < 1$ and λ_t settles strictly below unity. In this case, the relative price ratio is given by $\lambda_t = \frac{c_0}{P_{primary,t}} + \frac{\eta}{\zeta}$. Imposing parity $\lambda_t = 1$ would yield the following parity price:

$$P^* = \frac{c_0}{1 - \eta/\zeta}$$

This expression makes two points explicit, parity requires $\eta < \zeta$ otherwise P^* is undefined, and even when $\eta < \zeta$, P^* is typically very large because the denominator is small while c_0 reflects sizable, fixed collection and processing costs. In practice, markets are unlikely to reach such levels: demand contraction, substitution, policy constraints, or the effective exhaustion of recyclable flows would cap $P_{primary,t}$ well before P^* . Hence the parity price is best viewed as a theoretical construct; under cost anchoring, persistent (bounded) segmentation is the empirically relevant outcome even in metals with high recycling penetration.

Empirical evidence supports this interpretation. Even in metals with mature recycling systems—such as aluminum or copper, which generally fall within the one-price setting analyzed in the main text—episodes of price differentials between primary and secondary material remain recurrent. In aluminum, secondary ingot prices usually track primary closely, but discounts widen during periods of high energy

costs or demand shocks (IAI, 2024). In copper, the spread between cathode and scrap has been shown to fluctuate with collection efficiency, smelter bottlenecks, and regional arbitrage opportunities (ICSG, 2022; Aruga & Managi, 2011). These recurring spreads underline that, even in highly recycled metals, cost-based factors and frictions can prevent full price parity from being sustained over time.

In conclusion, the cost-based pricing regime highlights the structural limits of secondary markets as substitutes for primary supply. Unless secondary responsiveness scales perfectly with price and cost escalation is precisely offset—a knife-edge condition unlikely to hold—long-run convergence cannot be sustained. Instead, secondary markets exhibit persistent, bounded price differentials reflecting scrap scarcity, quality heterogeneity, and rising marginal costs. This explains why, even in metals with high recycling penetration, secondary outputs act as stabilizing complements but rarely achieve full price integration with primary production.

The extended framework developed in this annex does not treat convergence between primary and secondary metal prices as an automatic outcome. Instead, it identifies the formal conditions—elasticity parity, long-run capacity alignment, and substitution symmetry—under which convergence is feasible and shows how relaxing these assumptions often leads to persistent segmentation.

In practice, outcomes differ sharply across metals. In integrated and mature markets, such as aluminum, the structural conditions can be approximated, making near-convergence empirically plausible. By contrast, in segmented or path-dependent markets, such as rare earths or nickel, at least one condition typically fails, sustaining a dual-price equilibrium.

This annex therefore complements the main body of the chapter. While the core model applies to metals traded under a single unified price, the conditional convergence framework explains why some markets remain durably segmented and clarifies the thresholds that would need to be met for integration to occur.

CHAPTER 5: Beyond the Futures: Evaluating Copper Price Dynamics through Structural and Machine Learning Models

This chapter investigates the drivers and dynamics of copper prices over the medium term¹⁰⁹, using both error-correction models (ECM) and machine learning (ML) approaches. While forecasting accuracy remains a practical objective, the deeper analytical purpose of this chapter is to assess the extent to which market prices reflect fundamental economic signals—especially those related to supply-demand balance, inventory levels, and the gradual depletion of copper resources. This echoes one of the convergence conditions established in the theoretical model developed in Chapter 2: that the long-term sustainability of metal markets requires market prices to transmit meaningful information about underlying scarcity and physical constraints.

We therefore ask whether medium-term price forecasting models, working with monthly data, can shed light on the ability of copper prices to embody such signals. Opting for a dual approach—combining ECM and ML modeling—allows for a more comprehensive understanding of price formation, which traditional models alone may overlook. This duality is particularly relevant in a market like copper, where long-term structural pressures and short-term fluctuations coexist and interact.

The recent econometric literature on metal prices has focused on understanding the underlying factors affecting price volatility, supply-demand equilibrium, and price elasticity in response to economic cycles (Stuermer, 2018; Pindyck, 1999; Tilton, 1996). Studies often aim to capture the influence of macroeconomic indicators, resource constraints, and technological advancements on price behavior. To address these questions, researchers have employed a variety of econometric methods and, more recently, machine learning techniques that capture nonlinear relationships and complex data structures.

In the short run, many studies emphasize high-frequency models (*i.e.*, using daily or intradaily data) to capture immediate price responses to market shocks or temporary disruptions in supply and demand. Models like ARIMA, GARCH, and structural break tests are commonly used to account for these fluctuations (Kilian, 2009; Bollerslev, 1986). Earlier contributions such as Gilbert (1979) laid the

¹⁰⁹ In this chapter, the *medium term* refers to forecast horizons spanning several months, using monthly data. While such timeframes may seem long from the viewpoint of financial market participants focused on short-term volatility, they are more appropriately categorized as short to medium term in economic analysis. This horizon allows us to capture the adjustment mechanisms between supply and demand fundamentals, beyond day-to-day noise, while remaining responsive to cyclical dynamics and recent shocks. This time horizon is also well suited to the needs of manufacturers buying metals on wholesale markets.

groundwork for understanding price volatility in commodity markets, relying on more conventional time series and structural approaches.

By contrast, long-run analyses in metal pricing often turn to resource economics models, most notably those inspired by Hotelling's (1931) seminal work. These models, as described in Chapter 1, explore sustainable price trajectories under the assumption of finite resources, rational extraction behavior, and intertemporal optimization. Numerous extensions have been proposed to address some of the original model's limitations. Pindyck (1980) and Heal (1976) introduce uncertainty in reserve estimates and demand, showing how this affects optimal extraction paths and price volatility. Slade (1982) incorporates technological progress and extraction cost dynamics, resulting in U-shaped price paths rather than monotonically increasing ones. Devarajan and Fisher (1981) analyze the role of extraction taxes, while Dasgupta and Heal (1974) explore the conditions under which competitive resource depletion leads to socially optimal outcomes. Loury (1978) emphasizes the importance of exploration incentives under uncertainty, showing that expectations about future discoveries can fundamentally alter extraction and pricing strategies. More recent works such as Crowson (2011) and Henckens et al. (2016) emphasize the role of geological scarcity and the pace of depletion in driving long-term prices, while Gilbert (1979) and Stuermer (2018) explore the interaction between macroeconomic demand cycles and commodity price trajectories. Together, these models and extensions highlight how structural trends in extraction costs, resource quality, and economic development shape long-term price dynamics.

Medium-term price analysis, which this study addresses, bridges the gap between these short- and long-run approaches. It captures cyclical and structural changes in the market without assuming convergence to a long-run equilibrium. The medium-term perspective is particularly relevant to understanding whether observed prices reflect adjustments in response to market fundamentals—such as shifts in inventories, production constraints, and economic activity—and whether they serve as informative signals about the state of resource pressure or depletion. For producers, monthly price signals inform realistic budgeting and strategic extraction decisions within a manageable planning horizon. For consumers, a forecasting tool with monthly frequency enables a relatively precise monitoring of purchase decisions, aligning with their operational needs on an infra-annual basis. Finally, governments can leverage monthly forecasts to better estimate revenue ad valorem royalty taxes, enhancing fiscal planning and financial forecasting capabilities throughout the year.

The forecasting of copper prices has traditionally leveraged futures markets as benchmarks for market expectations (Fama & French, 1987; Alquist et al., 2013). However, futures prices on the copper market often seem to carry limited independent information. As we show, they tend to track spot prices closely and may primarily respond to spot market dynamics rather than offering genuinely forward-looking insights. Their effectiveness as predictive tools is further

called into question during volatile periods when self-fulfilling dynamics and speculative activity tend to amplify price movements rather than reflect fundamental drivers. This dual role of futures prices—as both reflections of and potential drivers of market conditions—may raise concerns about their reliability as forward-looking indicators and their capacity to encode long-term scarcity signals.

The choice to apply both ECM and ML approaches reflects the need to capture complex relationships within the copper market, where cointegrated variables may have stable long-term relationships but require flexible modeling to detect nonlinear patterns. The ECM is chosen for its ability to model equilibrium relationships¹¹⁰ and adjust for short-term deviations. In contrast, the ML model adds predictive power by capturing complex, nonlinear interactions that may be missed by traditional econometric models. Both models utilize the same set of independent variables, selected based on a thorough review of the literature and the analytical modeling developed in previous chapters. This ensures that differences in model performance are interpretable in terms of their structure, rather than their informational content.

We therefore develop a forecasting framework comprising an Error Correction Model (ECM) and an XGBoost approach. The ECM leverages the time-dependent nature of copper market dynamics, capturing medium-term equilibrium relationships while modeling short-term adjustments driven by deviations from equilibrium. The XGBoost model adopts a more agnostic approach, free from temporal constraints, focusing on uncovering non-linear interactions between market variables and short-term shocks. This methodological distinction allows the models to complement each other. The ECM explicitly incorporates the structured role of time and interdependence between spot and futures markets, making it well-suited for identifying equilibrium dynamics and gradual adjustments. The XGBoost model emphasizes explanatory variables such as market disruptions, macroeconomic indicators, and event-driven dynamics, providing an alternative perspective unencumbered by assumptions about time trends or temporal dependencies.

Our preliminary analysis demonstrates that while futures markets can play some role in reflecting market sentiment during turbulent periods, their predictive power is often limited to specific contexts. By integrating the ECM and XGBoost approaches, we aim to assess not only forecasting performance but also the degree to which copper prices encode fundamental signals related to resource scarcity and economic equilibrium. In doing so, we explore whether copper prices, over medium-term horizons, carry signals that correspond to underlying supply-demand fundamentals and resource pressure—an

¹¹⁰ In case of some multicollinearity among explanatory variables, ECM models can handle cointegration relationships among variables (Engle & Granger, 1987).

essential requirement for the convergence condition of price-informative equilibria derived in our theoretical model.

The results reveal the presence of two different cointegrating relationships within the Error Correction Model (ECM), indicating the existence of stable long-term interactions between copper prices, production levels, stock levels, and broader economic indicators. The contrasting speed of adjustment coefficients highlights distinct underlying mechanisms: one coefficient points to a gradual correction process, suggesting that market disequilibria may persist over extended periods, while the other reveals a more immediate response to deviations, reflecting the influence of short-term market sentiment and external shocks.

The performance comparison between the ECM and the XGBoost model underscores the trade-off between interpretability and predictive power. While the ECM provides clear insights into the economic relationships driving price movements, its predictive accuracy remains limited, particularly when confronted with rapid, non-linear shifts in market conditions. In contrast, the XGBoost model, though less transparent and easily interpretable, demonstrates superior forecasting performance across all tested horizons, with RMSE values consistently lower than those of the ECM, particularly for short-term predictions. This enhanced predictive ability stems from the model's capacity to capture complex, non-linear interactions between variables without relying on pre-defined structural assumptions.

The futures market analysis further illustrates the nuanced role of futures prices as predictors of copper spot prices. Our findings suggest that futures prices primarily mirror spot prices rather than providing independent predictive information. The correlation between spot prices and futures remains exceptionally high across all periods, with futures prices offering limited incremental explanatory power in low-volatility contexts.

Together, these results shed light on the complex nature of copper price formation, where medium-run adjustment mechanisms rooted in fundamental drivers coexist with short-term market-driven dynamics. The relative strengths and weaknesses of the ECM, the XGBoost model, and the futures market as forecasting tools for manufacturers offer valuable insights into the mechanisms underlying price movements and, more importantly, into the capacity of market prices to reflect underlying scarcity and equilibrium conditions in the context of long-term sustainability.

We begin this chapter with a review of the literature, situating the research within the broader context of commodity price modeling. The second section evaluates the informational content of copper futures prices, comparing market expectations with realized spot price movements through correlation analyses and Granger causality tests. Rather than treating futures as forecasting benchmarks, we assess their ability to reflect forward-looking scarcity signals. This empirical finding motivates the need to turn to explicit modeling approaches to better capture the structural and cyclical factors shaping prices.

The third section describes the dataset, justifies the selection of variables, and examines statistical properties such as correlation patterns, seasonality, and structural breaks. The fourth section introduces the specification, estimation, and interpretive focus of the Error Correction Model (ECM), emphasizing how the coefficients of the equilibrium relationships and short-term adjustment dynamics reflect underlying supply-demand fundamentals. The fifth section presents the XGBoost algorithm, detailing the methodological choices, optimization steps, and feature importance analysis, with particular attention to the temporal stability of these signals and their alignment with economic fundamentals. The sixth section conducts a comparative analysis of the ECM, the XGBoost model, and futures-based forecasts, discussing the trade-offs between interpretability, flexibility, and the capacity of each model to recover price signals consistent with the theoretical conditions for market convergence. Finally, the chapter concludes by reflecting on what these empirical insights reveal about copper market functioning in light of long-term sustainability and resource depletion concerns, and by outlining potential avenues for future research.

I. Fragmented Forecasting Traditions: From Statistical Accuracy to Structural Blind Spots in Copper Price Models

The forecasting of metal commodity prices, particularly in standardized markets, has been a well-developed area of research. Consumers, producers, and investors each rely on these forecasts for distinct purposes—production planning, procurement strategies, and portfolio management, respectively. These requirements have fueled the development of various econometric and mathematical models to capture the unique cyclical and economic dynamics of metals like copper.

In the medium term, time series models have historically formed the basis for copper price forecasting. Dooley and Lenihan (2005), for instance, used autoregressive models to capture trends in monthly copper prices, demonstrating that even simple models could track basic price movements. However, as forecasting needs evolved, these basic approaches were increasingly augmented with techniques that allowed for a more granular analysis of price patterns. Kriechbaumer et al. (2014) integrated wavelet analysis to study price fluctuations across multiple time scales, which improved detection of short-term volatility and medium-term trends. Khalifa et al. (2011) applied Fourier transformations to decompose gold, silver, and copper prices into cyclical components, isolating seasonal trends that impact price movements.

Alongside these methods, model-based forecasting approaches emerged to embed structured economic relationships into predictive frameworks. Goss and Avsar (2013) demonstrated the value of incorporating macroeconomic drivers into forecasting models. Optimization algorithms, such as the swarm intelligence approach applied by Ma et al. (2013), further refined these models by optimizing parameters in response to evolving market dynamics. In parallel, multicommodity models developed

by Cortazar and Eterovic (2010) accounted for the interdependencies between metals, suggesting that copper prices often correlate with those of other metals due to shared demand and resource constraints.

Recent literature on copper price forecasting has been dominated by computing processes, such as Generalized Autoregressive Conditional Heteroskedasticity (GARCH) and Markov switching models, which primarily adopt univariate approaches (Hammoudeh and Yuan, 2008; Mensi et al, 2014; Liu et al, 2008; Alizadeh and Nokimos, 2004; Tse and Tsui, 2001). While these models are effective for capturing stochastic and cyclical fluctuations in copper prices, they often lack the ability to capture structural fundamentals such as supply constraints, inventory levels, and macroeconomic trends—factors that are crucial for assessing whether prices reflect the underlying state of the market.

To overcome some of these limitations, more recent research has turned to model averaging and time-varying parameter techniques to account for structural shifts and uncertainty in model specification. Buncic and Moretto (2015) apply a dynamic model averaging and selection (DMA/DMS) framework to monthly copper returns, incorporating a wide set of predictors including inventories, the convenience yield, and global risk appetite. Similarly, Li and Li (2015) focus on volatility forecasting for copper futures, demonstrating that model averaging techniques—especially OLS-based weighted averages—can significantly reduce forecast uncertainty and improve performance across various loss functions.

Another stream of literature integrates artificial intelligence and machine learning tools to capture nonlinearity and interaction effects that are difficult to model in traditional frameworks. Lasheras et al. (2015), for instance, compare ARIMA models with two types of artificial neural networks and find that both neural architectures outperform ARIMA in forecasting COMEX copper spot prices. Likewise, Liu et al. (2017) use decision tree learning methods to forecast copper prices, confirming the ability of tree-based models to exploit hidden patterns in historical price behavior. More broadly, Charlot and Marimoutou (2014) develop a multivariate Hidden Markov Decision Tree model to analyze structural shifts in volatility and co-movement across energy and metal markets, illustrating the value of regime-switching machine learning methods in commodity price modeling.

Multivariate forecasting frameworks that incorporate macroeconomic and financial variables have also gained attention in recent years. Gargano and Timmermann (2014) show that commodity price indexes can be meaningfully predicted using variables such as industrial production, investment-to-capital ratios, and commodity currencies, with multivariate specifications improving forecast accuracy across horizons. These findings echo the calls made by Goss and Avsar (2013) for a more explicit integration of fundamental economic signals into forecasting models.

In parallel, earlier theoretical models rooted in resource economics have provided the foundation for understanding how scarcity and depletion affect price behavior. Hotelling's foundational work (1931) established the link between non-renewable resource pricing and interest rates, providing a basis for subsequent research on price dynamics over time. Pindyck (1980) and Heal (1976) explored the impact of scarcity and economic cycles on resource prices. Henckens et al. (2016) highlighted the importance of considering resource depletion in price models. Slade (1982) incorporated technological progress into the analysis of extraction costs, showing that price paths could be U-shaped rather than strictly increasing. Dasgupta and Heal (1974) examined optimal extraction under competitive markets. Loury (1978) introduced exploration behavior under uncertainty, and Devarajan and Fisher (1981) modeled the effects of taxation on extraction incentives. More recently, Henckens et al. (2016) emphasized physical depletion constraints, while Crowson (2011) examined price responsiveness under cost-tiered resource availability. Together, these extensions inform the theoretical underpinnings of models like the one presented in Chapter 2, where resource depletion interacts with supply responsiveness and economic signaling.

Although daily or intra-daily forecasting models, such as Kilian's (2009) structural vector autoregressive (SVAR) models, capture immediate market reactions and short-term volatility, they employ different analytical tools suited to high-frequency noise and short-lived trends. Conversely, long-horizon approaches often reference Hubbert's (1956) peak theory or Loury's (1978) insights on resource depletion, examining price trajectories over decades under assumptions of finite availability. Due to the distinct requirements of these horizons, we do not dwell extensively on these models here. Instead, our focus remains on medium-run price dynamics, measured at the monthly level, which are crucial for understanding the pace and nature of adjustment between physical fundamentals and market signals.

In summary, while a variety of models have been developed to forecast copper prices, many existing approaches either focus too narrowly on short-term volatility or fail to capture the underlying economic signals that are critical to understanding how prices reflect the state of the supply-demand balance. In this chapter, we adopt a dual approach combining econometric and machine learning methods, not only to enhance predictive capabilities, but more fundamentally to assess the informational value of copper prices in relation to equilibrium signals and resource pressure—core concerns for long-term sustainability.

II. Copper Futures: Limits of Market-Based Expectations

1. When Futures Follow the Spot: Testing the Forward-Looking Role of Copper Futures

The informational role of futures prices in commodity markets is often assumed rather than empirically confirmed. In principle, futures markets aggregate available information to form expectations about future spot prices, reflecting anticipated shifts in supply, demand, and inventory dynamics. In practice, however, the ability of futures prices to signal structural imbalances or approaching resource constraints remains uncertain. This section examines whether copper futures offer meaningful insights into market dynamics beyond what is already embedded in contemporaneous spot prices.

We focus on the relationship between copper spot prices and 3-month lagged futures prices over the period from January 2001 to September 2024. As in the remainder of this chapter, we center the analysis on the recent period from September 2022 to August 2024, with volatility regimes defined using rolling standard deviations of spot prices.

A preliminary correlation analysis reveals that the relationship between spot prices and 3-month lagged futures prices is notably strong in raw levels, with a correlation coefficient of 0.8417 across the full sample. However, this relationship largely disappears when examining first differences, suggesting that futures prices do not consistently anticipate short-term variations. Volatility stratification reinforces this finding: in high-volatility periods (top 10% of the rolling standard deviation), the correlation increases to 0.9250; in low-volatility periods, it remains high (0.8295), but the first-difference correlation turns negative at -0.4241, hinting at a possible mechanical correction mechanism rather than forward-looking behavior.

Contemporaneous futures and spot prices exhibit a near-perfect correlation (>0.99) under all market conditions. This synchronicity likely reflects arbitrage efficiency but also calls into question the independent informational role of futures prices. If futures prices move almost identically to spot prices, they may serve more as mirrors of current market sentiment than as vehicles for anticipating fundamental transitions.

2. A Weak Signal in Calm Waters: Evaluating the Predictive Value of Futures Across Market Volatility Regimes

To evaluate the degree to which futures prices contain distinct information, we conduct Granger causality tests and forecasting comparisons using XGBoost. These are not used here to select the “best

predictor,” but rather to understand whether futures prices encode meaningful forward-looking signals relevant to equilibrium adjustment and resource pressure.

Granger causality tests reveal a statistically significant causal link at lag 1 for 3-month futures (F-statistic = 14.15, p-value = 0.0002), but no significance at longer lags. This suggests futures prices may capture some immediate sentiment effects but lack persistent signaling power. Non-lagged futures prices show no causal relationship, further reinforcing the idea that they merely mirror spot dynamics.

Using a consistent XGBoost framework with time series train-test splits, we test the incremental value of including futures prices. When added alongside lagged spot prices, 3-month lagged futures reduce RMSE from 614.42 to 532.31. However, when futures are used alone, RMSE rises sharply to 1472.92, indicating that their weak signal requires anchoring in observed price history. In high-volatility periods, their standalone RMSE improves (300.43), but this likely reflects short-term speculation or hedging adjustments rather than deeper fundamental insight.

In low-volatility regimes, their contribution collapses. The addition of lagged futures prices reduces RMSE only slightly (859.37 to 811.79) and using them alone leads to a deteriorated RMSE of 1099.71. This asymmetry—where futures gain relevance only in unstable environments—suggests that they respond to heightened risk perception, not to underlying structural shifts in the market.

3. Beyond Arbitrage: What the Limits of Futures Pricing Mean for Long-Term Resource Signaling

These findings illustrate the context-dependent role of copper futures prices. In volatile periods, they appear to aggregate short-term expectations or speculative positioning, sometimes even influencing spot prices. In calmer markets, however, they offer little information beyond what is already observable. This inconsistency challenges their use as systematic indicators of the supply-demand equilibrium or depletion trajectories.

From a sustainability perspective, this limitation is critical. As established in the theoretical framework (Chapter 2), one condition for long-term sustainability is that market prices must reflect underlying fundamentals—particularly the tension between resource exhaustion and demand growth. If futures prices fail to encode that tension except in exceptional conditions, they cannot be relied upon to guide adaptive behavior across time.

This motivates the use of explicit modeling strategies: the following sections present two contrasting approaches that attempt to retrieve fundamental signals directly. The first—an error correction model (ECM)—tests for stable, equilibrium-based relationships between prices and key economic variables.

The second—a machine learning model (XGBoost)—aims to detect latent, nonlinear interactions that may escape structural frameworks but still reveal systematic dynamics. Together, these methods offer a clearer picture of whether copper prices are consistent with the kind of informational efficiency required for long-term equilibrium adjustment.

To evaluate whether these models can capture signals consistent with market fundamentals, we begin by introducing the dataset and variable choices that underpin the analysis. This empirical foundation is critical: if copper prices are to reflect the supply-demand balance or resource-related pressures in a meaningful way, the inputs to our models must themselves be rooted in relevant economic dynamics. The next section therefore details the construction of the dataset, the rationale behind the selection of explanatory variables, and the statistical properties—such as correlations, seasonality, and potential structural breaks—that inform the modeling choices to follow.

III. Building an Empirical Framework to Test Price Signaling

1. Selecting Relevant Variables: From Market Fundamentals to Structural Indicators

The copper settlement price established on the London Metal Exchange (LME) is considered to effectively represent the global copper price (Gilbert, 1996). The LME serves as the primary international market for key industrial non-ferrous metals, including aluminum, aluminum alloys, copper, lead, nickel, tin, zinc, and silver. Most studies on copper prices and metal commodities utilize LME data as the reference market due to its influence and comprehensive trading volumes (e.g., Cuddington and Jerrett, 2008; Radetzki, 2006; Fattouh and Mahadeva, 2013). The LME is globally recognized by producers and consumers of non-ferrous metals as a central hub for spot, futures, and options trading in these commodities, making it crucial for both market participants and researchers studying price dynamics in metals (Gilbert, 2010; Geman, 2007; Hammoudeh et al, 2010)¹¹¹.

We use monthly data, which offer distinct advantages for analyzing medium-term price volatility in copper markets. Daily data, while capturing high-frequency fluctuations, often reflect short-term noise that can obscure underlying medium-term trends relevant to supply-demand adjustments. Conversely, quarterly data, typically used for longer-term analyses, may smooth over important infra-annual dynamics essential for understanding cyclical changes within the year. Monthly data strike a balance,

¹¹¹ The LME accounts for approximately 40-50% of the world's copper trading, including both spot and futures contracts, while the Shanghai Futures Exchange (SHFE) accounts for about 30-40%. The Chicago Mercantile Exchange (CME) generally represents around 10-20% of copper trading, primarily focusing on futures contracts. Smaller regional markets typically engage only in spot trading of non-ferrous metals.

enabling us to investigate whether market prices adjust over medium-run horizons in response to fundamental signals—particularly those linked to supply, demand, and potential structural constraints.

To evaluate whether copper prices reflect economic fundamentals—as required for sustainable market adjustment—we select key variables that capture structural dynamics on both the supply and demand side.

Variations in production levels¹¹² are calculated using the world primary supply of *refined copper*, rather than copper mine production^{113,114}. Several studies also modelled the choice of refined copper production over mine production. For instance, Stuermer (2018) uses refined copper production as a primary indicator when examining long-term commodity price cycles, emphasizing its relevance in capturing shifts in industrial demand. Additionally, Jacks and Stuermer (2020) highlight the predictive value of refined metal supply for short-term price fluctuations in global markets. In the context of supply-demand equilibrium modeling, Ghoshray (2011) employs refined copper production to better understand inventory dynamics and their impact on prices. These references collectively underscore the relevance of refined copper production in commodity price forecasting, validating its use in our analysis.

¹¹² The inclusion of production levels as an exogenous variable in analyzing copper prices is well-supported by existing literature, as previously addressed in the literature review. Stuermer (2018) highlights the influence of production changes on the supply-demand balance, demonstrating how fluctuations in production directly impact metal prices. Pindyck (1980) further explores how production constraints shape price dynamics in exhaustible resource markets, while Gilbert (1979) underscores the role of production and inventory adjustments in stabilizing commodity prices in response to supply and demand shifts. Additionally, Slade (1982) analyzes long-term trends in resource prices, showing that production limitations and technological advancements influence price trajectories. Henckens et al. (2016) add to this by examining how production and extraction rates affect resource availability, contributing to price stability in commodity markets. Together, these studies establish production levels as a crucial factor in understanding price behavior in the copper market.

¹¹³ <https://icsg.org/wp-content/uploads/Table1.pdf>

¹¹⁴ Firstly, refined copper is the form actively traded and used by industries, making it more relevant for analyzing price dynamics. As a fully processed, standardized product, refined copper is immediately usable in manufacturing, unlike unrefined mine output, which contains impurities. Secondly, focusing on refined copper captures the entire supply chain—mining, smelting, and refining—providing a more complete view of factors affecting copper availability, such as operational efficiencies and technological advancements. Thirdly, refined copper production data provide a clearer assessment of the balance between supply and demand, as refined copper is closer to the final product purchased by consumers. This proximity to end-use stages allows refined production figures to more accurately reflect real-time market needs and shifts in industrial demand, which play a direct role in price formation. Moreover, refined copper production closely correlates with inventory levels, acting as a buffer or indicator of surplus and shortage. When inventories are low relative to refined copper output, supply shortages are likely to drive prices up, while higher inventories suggest that supply is meeting or exceeding demand. This close relationship between refined copper production, inventories, and demand provides valuable insights into price pressures and market behavior that cannot be observed through raw mine production data alone. Finally, refined copper data are more frequently updated and accessible, improving analysis reliability and facilitating more timely forecasting. Using refined copper production data also helps address potential collinearity issues, particularly as the model includes mine capacity utilization rate as a separate explanatory variable. Refined copper production and mine capacity utilization capture different aspects of the copper supply chain—refined copper production reflects the actual output available for consumption, while mine capacity utilization indicates how efficiently mining capacities are being used. This distinction reduces the risk of redundancy in the model, improves interpretability, and allows for clearer insights into market dynamics.

We include the Purchasing Managers' Index (PMI) from the primary copper-consuming economies, specifically China (57% of the global refined copper consumption volume in 2023¹¹⁵), along with the United States (8%¹¹⁶), and the European Union (13%¹¹⁷). Rising PMI values indicate anticipations of higher industrial activity over a few months' horizon and, consequently, heightened demand for copper. Conversely, a declining PMI often signals economic slowdowns, potentially leading to decreased copper demand and downward pressure on prices (World Bank, 2019¹¹⁸; S&P Global, 2021¹¹⁹). Additionally, incorporating these indices may also capture supply-side dynamics in the longer run, as both China and, to a lesser extent, the US serve as industrial producers and refiners of copper (China and the US respectively account for 42% and 3.3% of refined copper production in 2023¹²⁰).

Data on copper stock levels, both globally and on major exchanges, provide additional insight into the supply-demand balance and is integral to the theory of storage¹²¹. Stock levels play a pivotal role in commodity price dynamics, influencing both market volatility and price stability.¹²² We use as an aggregate variable the sum of all inventories from the three marketplaces (LME, SHFE, and COMEX) instead of considering them separately¹²³.

¹¹⁵ <https://www.statista.com/statistics/693466/distribution-of-global-refined-copper-consumption-by-region/>

¹¹⁶ <https://www.statista.com/statistics/1180230/refined-copper-consumption-distribution-by-world-region/>

¹¹⁷ <https://www.coppercouncil.org/iwcc-press-release-short-term-forecasts-for-copper-oct-2024>

¹¹⁸ World Bank (2019). "Commodity Markets Outlook: Metals and Precious Metals." Highlights the positive correlation between global manufacturing PMI and copper prices.

¹¹⁹ S&P Global (2021). "PMI Data and Copper Price Trends." Provides analysis on the global PMI index for copper-using manufacturers and its predictive power for copper price trends.

¹²⁰ <https://pubs.usgs.gov/periodicals/mcs2024/mcs2024.pdf>

¹²¹ According to the theory of storage, inventory levels play a critical role in determining commodity prices. High inventory levels may indicate an oversupply, exerting downward pressure on prices, while low inventory levels can signal potential shortages, driving prices up. The theory also posits that the costs associated with holding inventory, such as storage fees and opportunity costs, influence market prices and participant behavior. Therefore, understanding inventory trends is essential for accurately modeling price movements. Pindyck (1990) highlights how inventories contribute to short-term commodity price dynamics, emphasizing their role in amplifying or dampening price volatility depending on market conditions. Fama and French (1987) also demonstrate that inventory fluctuations impact the shape of the futures curve, as lower inventories tend to increase the convenience yield, leading to backwardation. More recently, Alquist and Kilian (2010) explore the predictive power of inventories for commodity price movements, reinforcing the relevance of inventory monitoring in price modeling frameworks.

¹²² Gilbert (1979) examines how inventory adjustments help stabilize prices in response to fluctuations in supply and demand, emphasizing that stock levels can serve as a buffer against market shocks. Geman and Smith (2013) expand on this through the theory of storage, demonstrating that inventory levels on the LME affect volatility and price trends for base metals, including copper. Additionally, Pindyck (1980) and Slade (1982) discuss how stock levels interact with production constraints to moderate price fluctuations over time, while Stuermer (2018) also highlights stock levels as a critical factor in meeting demand during periods of supply disruption. These studies underscore the importance of stock levels in forecasting copper prices by accounting for the moderating effects of inventory on market stability.

¹²³ First, aggregating inventories provides a comprehensive view of the total available supply, capturing the overall market dynamics and better reflecting the collective influence of inventory levels on copper prices. This approach helps smooth out the high volatility that individual inventory levels may exhibit due to local market conditions or specific demand fluctuations. Second, the interconnected nature of copper markets means that changes in inventory levels in one marketplace can influence prices and trading behavior in others. By summing the inventories, the analysis acknowledges these relationships and recognizes that price dynamics are often shaped by combined market forces. Third, the total inventory can serve as an indicator of market sentiment. High combined inventories may suggest bearish sentiment

The mine capacity utilization rate measures how effectively existing mining resources are being utilized. High-capacity utilization rates suggest that mines are operating near their full potential, impacting the availability of copper in the market. Conversely, low utilization rates may signal operational setbacks or external disruptions, such as labor strikes or unforeseen production issues¹²⁴. The capacity utilization rate is also influenced by seasonal factors, such as annual maintenance or weather-related challenges, which can impact production cycles. For instance, Blomberg and Harris (1995) emphasize how external shocks like strikes or adverse weather conditions can create temporary price pressures by disrupting production. Additionally, Jacks and Stuermer (2020) illustrate how variations in utilization rates, combined with seasonal production patterns and inventory adjustments, influence both short- and long-term commodity price dynamics. A detailed seasonality analysis follows later in this chapter, examining the temporal patterns and their implications for copper price forecasting.

The ore grade is the quality of ore being mined, i.e. the copper content in quarried rocks. It directly affects production costs and overall supply. Higher-grade ore typically results in more efficient extraction and lower production costs, while lower-grade ore can increase operational expenses and reduce profitability, impacting supply levels and copper prices. The impact of ore grade on metal prices has been widely documented in the literature. Tilton and Lagos (2007) argue that declining ore grades drive up production costs, ultimately impacting market prices due to increasing marginal costs of extraction. Likewise, Mudd (2010) and Crowson (2011) emphasize that declining ore quality can amplify supply constraints and increase the volatility of metal prices as miners adjust output based on available reserves. Eggert et al. (2016) further demonstrate that ore grade is a key factor in long-term commodity price forecasting, as it shapes the cost structure and supply curve for metals. These studies underscore the importance of ore concentration in understanding the price behavior of metal commodities over time.

Changes in interest rates can significantly influence investment decisions within the copper market. According to Hotelling's (1931) seminal theory, the price path of a non-renewable resource, such as copper, is fundamentally linked to the prevailing interest rate. Lower interest rates reduce the opportunity cost of extracting resources in the present rather than conserving them for future sale, potentially stimulating investment in mining operations and increasing supply. Additionally, lower rates

(oversupply), while low inventories can indicate bullish sentiment (potential shortages), influencing market participants' decisions¹²³. In summary, summing all inventories from the three marketplaces allows for a robust understanding of how inventory levels influence copper pricing, recognizing the interconnectedness of the markets and enhancing the overall effectiveness of the model.

¹²⁴ For instance, in 2017, a 43-day strike at Chile's Escondida mine—the world's largest copper mine—led to a sharp drop in global copper supply, creating price pressures. Similar events occurred in 2019 at Peru's Las Bambas mine, where protests and labor disruptions blocked transport routes, further tightening supply and impacting prices. In 2020, Zambia's Konkola Copper Mines faced production halts due to labor disputes, affecting supply in the African and Asian markets.

can lead to heightened demand from industries that consume copper, as reduced borrowing costs make capital investments more attractive, thereby enhancing production capacity and consumption (Pindyck, 1980; Slade, 1982). This interaction highlights the importance of interest rates not only for short-term investment behavior, but also for understanding whether copper prices adjust consistently with intertemporal extraction incentives—an essential feature in any market where depletion and future value are linked.

We choose not to include some variables in our models, for the following reasons:

Exchange rates (see Deghanhi, 2018) are not included as exogenous variables as they may increase the risk of multicollinearity with other variables that are also sensitive to economic conditions or international trade dynamics. Our analysis prioritizes fundamental factors that directly impact copper prices, such as supply-demand dynamics, production levels, and inventory.

Global economic indicators like GDP growth, industrial production, and construction activity are excluded as they provide indirect and lagged signals for copper demand, that often overlap with more sector-specific indicators such as PMIs. Their exclusion allows the model to focus on direct, copper-specific factors, enhancing interpretability and avoiding redundancy, thus supporting a more targeted approach.

Technological advancements are already implicitly accounted for within our existing variables, as they influence both supply and demand dynamics. On the supply side, technological improvements affect production efficiency, ore processing, and mine capacity utilization. On the demand side, advancements in industries reliant on copper (such as electronics, renewable energy, and electric vehicles) drive changes in copper consumption. Thus, the impact of technological progress is captured through our selected supply and demand indicators, making additional variables for technology unnecessary.

Geopolitical events are excluded because they are unpredictable, and their inclusion would shift focus from core, measurable fundamentals, like supply-demand dynamics. However, our data series are corrected for dummy variables related with idiosyncratic factors (see below).¹²⁵

The data used in this chapter are monthly from January 2001 to September 2024. For the London Metal Exchange (LME) price (hereafter *Price*), we use the monthly average copper spot prices in current

¹²⁵ As for other commodities, despite their high correlation with copper prices (Dehghani 2018), variables like silver, nickel, aluminum, and diverse oil prices (OPEC, WTI, and Brent) are excluded to maintain a focus on copper-specific fundamentals. Including them could dilute the model's emphasis on supply-demand dynamics directly relevant to copper, introduce redundancy, and increase multicollinearity, complicating interpretation. Additionally, these variables often reflect broader market or geopolitical conditions rather than long-term copper price drivers, so their exclusion supports a more parsimonious and interpretable model.

US dollars per ton. As for production data (hereafter *Production*), we consider monthly worldwide primary production of refined copper in thousand tons provided by the International Copper Study Group (ICSG). The worldwide mining capacity utilization rate (hereafter *Mine capacity*), also provided by the International Copper Study Group (ICSG), is the ratio of worldwide production of copper from mines to worldwide mining capacity in percentage. The PMIs in the manufacturing sector are provided by Standard & Poor's global.¹²⁶ The sum of inventories held on major marketplaces (hereafter *Stock*) stems from public data from the London Metal Exchange (LME), Shanghai Foreign Exchange (SHFE), and Commodities Exchange (COMEX), expressed in thousands of tons. The Ore grade concentration (hereafter *Ore Grade*) is provided by the International Energy Agency and refers to the ore concentration in major Chilean mine (which represents close to half of the world copper production¹²⁷), expressed as a percentage. This data serves as a proxy for the worldwide trend in copper concentration in quarried rocks, as comprehensive global data are challenging to obtain. The monthly interest rate (hereafter *Interest rate*) is the Federal Reserve main interest rate, expressed in percentage.

2. Correlation Analysis, Lagged Relationships, and Collinearity Issues

Table 5.1 presents the correlation coefficients, and their associated p-values for each variable.

Table 5.1: Results from Correlation Analysis

Variable	Correlation Coefficient	p-value	Result
Production	0.588589	2.62e-27	Strong positive correlation. Statistically significant.
PMI China	0.211367	3.87e-04	Weak positive correlation. Statistically significant.
PMI US	0.389948	1.57e-11	Moderate positive correlation. Statistically significant.
PMI EU	0.284059	1.48e-06	Moderate positive correlation. Statistically significant.
Stock	-0.454715	1.36e-15	Moderate negative correlation. Statistically significant.
Mine capacity	-0.601459	9.72e-29	Strong negative correlation. Statistically significant.
Ore grade	-0.798339	9.73e-63	Very strong negative correlation. Statistically significant.
Interest rate ¹²⁸	0.061778	3.05e-01	Very weak positive correlation. Not statistically significant in the short run.

¹²⁶ Caixin Manufacturing PMI (hereafter PMI China), United States Manufacturing PMI (hereafter PMI US), HCOB Eurozone Manufacturing PMI (hereafter PMU EU)

¹²⁷ <https://www.ica.org/data-and-statistics/charts/average-copper-ore-grade-in-chile-by-production-route-2005-2019>

¹²⁸ The interest rate variable exhibits a weak positive correlation (0.062) with copper prices, suggesting it may have minimal influence on prices in the short run. Nevertheless, interest rates were included in our list of variables due to their theoretical relevance in resource economics, as outlined above. Interest rates tend to exert a delayed effect on the underlying supply and demand dynamics, which do not have an immediate impact on prices. This apparent temporal disconnect highlights

We explore the dynamics of these correlations over time, assessing potential lagged effects. Accordingly, we examine the correlation coefficients at 60-month lags (i.e. 5 years) and their significance. This examination provides insights into how the relationships between the variables evolve over time and the persistence of their effects^{129,130}.

We turn to some preliminary analysis of collinearity among explanatory variables. The results of the Variance Inflation Factor (VIF) test indicate significant multicollinearity among the independent variables in this analysis¹³¹. Beyond their technical capacity to handle multicollinearity and non-stationarity, models like the ECM are particularly useful for examining whether copper prices respond to disequilibrium in key structural relationships—such as mismatches between supply and demand—

the complexity of the relationships involved and emphasizes the importance of examining these variables further in subsequent sections of this study.

¹²⁹ The results indicate that price maintains a consistently positive correlation with *production* throughout the entire lag order horizon. This suggests that fluctuations in production levels are closely and durably linked to price movements, reinforcing the significance of supply dynamics in the copper market.

The correlation analysis reveals that the decline in *ore concentration* exerts a persistent effect on copper prices, remaining significant for approximately 55 months. This enduring relationship indicates that reductions in ore concentration have a lasting impact on market prices. As ore quality decreases, the implications for extraction costs and overall supply dynamics become more pronounced, which can lead to sustained upward pressure on prices over time.

Additionally, the correlation between price and *Purchasing Managers' Indices (PMIs)* holds for around 15 lags. This observation aligns with economic theory, emphasizing that demand-side dynamics tend to exhibit less persistence over time. As economic activity fluctuates, it directly impacts the demand for copper, albeit with a lagged effect.

Copper prices show a negative correlation with *stock* and *mine capacity* for nearly 45 periods, though this correlation exhibits a decreasing trend. This indicates that while higher stock and mine capacity levels are associated with lower prices, the relationship weakens as time progresses, suggesting that initial adjustments in supply and inventory have a more significant impact on prices, which diminishes over time.

The correlation between mine capacity and copper prices becomes significant again but positive after 57 lags. This transition underscores the idea that higher capacity utilization can be interpreted as a sign of greater operational efficiency and readiness to meet supply demands. When mining operations are effectively utilized, it typically indicates that producers are actively responding to rising demand for copper. This responsiveness fosters market confidence, as it suggests that supply can adjust to match increasing consumption. As a result, the market may react positively, driving copper prices upward. This dynamic illustrates how the interplay between production capabilities and market demand influences pricing behavior over the long term.

¹³⁰ A noteworthy finding is the negative correlation between price and the Federal Reserve's interest rate, observed from month 16 to month 29. This delayed effect underscores the nature of interest rates influencing supply and demand dynamics in the copper market, reinforcing the idea that the impact of monetary policy on prices is not immediate. After a certain lag period, the effects of lower interest rates on investment and consumption may become more pronounced, leading to increased demand for copper. For instance, lower interest rates can stimulate construction, manufacturing, and infrastructure projects, all of which require copper. Consequently, this delayed reaction reflects the time it takes for changes in interest rates to influence broader economic conditions and, in turn, copper demand.

The correlation between the interest rate and copper prices gets significant again after 41 monthly lags but turns positive, can be explained through several mechanisms. Initially, a negative correlation may occur because rising interest rates typically increase borrowing costs, which can dampen economic activity and reduce demand for copper in the short term. However, as time progresses, the economic landscape can shift. All this is in line for the time dynamics of monetary policy, see Avalos, F., Igan, D., Manea, C. and Moessner, R., 2023. *Monetary policy, financial conditions and real activity: is this time different?* (No. 80). Bank for International Settlements.

¹³¹ Specifically, variables such as Production (VIF = 116.38), PMI China (VIF = 303.69), PMI US (VIF = 210.92), PMI EU (VIF = 220.41), and Mine Capacity (VIF = 521.57) exhibit extremely high VIF values, suggesting redundancy and potential instability in the model due to their strong intercorrelations. While Stock (VIF = 7.52) falls below the typical threshold for concern, it still merits attention. In contrast, the Interest Rate (VIF = 2.85) does not indicate any multicollinearity issues.

thereby helping to assess the informational content of market prices. ECM is particularly suited for situations where multiple time series exhibit cointegration, meaning that, although individual series may be non-stationary, a linear combination of these series is stationary (Hamilton, 2020, p.571). An ECM model captures both short-term and long-term dynamics of relationships, enabling an analysis of how shocks to one variable may impact others over time.

3. Decomposing the Series: Structural Features, Seasonality, and Exogenous Shocks

a. Regime Shifts and Long-Term Drifts: Statistical Evidence of Trend Behavior

Table 5.2 summarizes, for each variable, their mean, standard deviation, minimum, 25th percentile, median, 75th percentile, and maximum values, overtime July 2001-August 2024:

Table 5.2: Summary Statistics for Copper Price and Explanatory Variables

	<i>Price (\$)</i>	<i>Prod. (kt)</i>	<i>PMI Chine</i>	<i>PMI US</i>	<i>PMI EU</i>	<i>Stock (kt)</i>	<i>Mine cap (%)</i>	<i>Ore grade</i>	<i>Interest rate (%)</i>
<i>Mean</i>	6,197	1,457	51.2	52.6	51.0	490	83.8	0.76	1.63
<i>Std</i>	2344	244	2.50	5.38	5.17	286	5.0	0.18	1.80
<i>Min</i>	1,377	1,048	35.7	32.4	33.4	75.7	66.3	0.55	0.05
<i>25%</i>	4,818	1,260	50.0	49.6	47.7	250	80.3	0.64	0.13
<i>50%</i>	6,688	1,402	50.6	52.6	51.6	454	83.6	0.71	1.01
<i>75%</i>	7,955	1,647	52.0	56.5	54.6	626	86.9	0.83	2.40
<i>Max</i>	10,237	1,963	59.2	64.7	63.4	1,317	98.3	1.25	5.50

The trend analysis of copper prices from 2001 to 2024 indicates a significant upward temporal trend over the long run, with a slope of 19.26. This consistent upward movement reflects a general increase in demand and possibly supply constraints within the copper market. The monthly percentage change in copper prices is approximately 0.88%. The standard deviation of the percentage change is 6.3363, indicating significant volatility in copper prices. Trend analysis on the percentage change in copper prices suggests that, although copper prices are generally rising, the pace of this increase has been slowing down over the latest years.¹³²

¹³² The trend analysis of monthly production reveals that the rate of production growth is rising. However, the magnitude of this increase remains relatively small compared to total production levels (the average monthly production growth is around 0.3%, and the average yearly production growth is around 2.5%). These moderate growth figures are consistent with the structurally inelastic nature of metal production we studied in the first chapter, where supply constraints—such as long lead times, regulatory hurdles, and finite resource availability—limit rapid expansion in response to demand increases. The trend analysis of stock levels from 2001 to 2024 indicates a slight downward trend in stock levels. This negative slope suggests that stock levels have been gradually decreasing over the analyzed period, indicating a potential tightening in inventory management practices or an increased demand relative to supply.

As for ore grade, the average concentration of copper in quarried rocks has been decreasing by approximately 0.205% per month from 2001 to 2024. This decline signifies that less copper is being extracted from each ton of ore, which is concerning for the overall sustainability of copper production. A declining trend in ore grade may reflect several underlying issues within the mining industry¹³³.

b. Recurring Patterns in the Monthly Data: Detecting and Interpreting Seasonality

The most prominent seasonal component is observed in the monthly increase in copper prices, which exhibits a seasonal strength of 0.8054. This high value indicates a strong seasonal pattern influencing the percentage changes in copper prices. It suggests that fluctuations in prices are significantly driven by seasonal factors, highlighting the importance of understanding these dynamics when forecasting future price movements¹³⁴.

c. What the Residuals Reveal: Structural Shocks and Hidden Dependencies Beyond Seasonality

The analysis involves two key statistical tests: the Jarque-Bera test for normality and the Ljung-Box test for autocorrelation. The Jarque-Bera test assesses whether the residuals from the seasonal decomposition are normally distributed. The results indicate that the residuals for most variables (excluding Stock) exhibit significant non-normality, as evidenced by extremely low p-values (generally below 0.05). Production, all PMI indicators, ore grade, mine capacity, and interest rate demonstrate strong deviations from normality¹³⁵. The Ljung-Box test evaluates the presence of autocorrelation within the residuals. The Ljung-Box test results reveal significant autocorrelations for all tested variables at all lags (10, 15, 20), with p-values consistently below 0.05. This indicates that the residuals

The trend analysis for mining capacity does not display strong temporal trend for this variable.

¹³³ These may include the depletion of high-grade ore reserves, which are becoming increasingly rare and costly to extract. Additionally, stricter environmental regulations and operational challenges may hinder miners' ability to access and process higher-quality ores. As the concentration of copper in mined ore decreases, it can lead to higher extraction costs and lower profitability for mining operations, making it more challenging to meet the rising global demand for copper.

¹³⁴ The mine capacity demonstrates a seasonal strength of 0.4551. This indicates that seasonal factors play a substantial role in the utilization of mine capacity, suggesting that operational adjustments may correspond to seasonal demand or supply cycles. The ore grade displays an extremely weak seasonal strength of 0.0006, indicating that seasonal fluctuations do not significantly impact the variability in ore quality. As expected, this finding suggests that ore grade stability is driven more by geological factors than by seasonal trends.

¹³⁵ The Stock variable, with a p-value of 0.211, does not significantly reject the null hypothesis of normality, suggesting its residuals may approximate a normal distribution. The potential normality of the residuals for the stock time series can be attributed to several factors. A straightforward relationship with predictors, consistent demand or inventory practices, and reduced volatility may lead to fewer outliers and a more normal distribution. Additionally, a larger dataset enhances this approximation due to the Central Limit Theorem. If the seasonal decomposition model indicates that these residuals are normally distributed, it suggests the model effectively explains the variation in stock levels by capturing seasonal patterns and separating predictable fluctuations from the data. Stable demand and supply dynamics, a lack of significant structural breaks, and a homogeneous dataset further support the model's effectiveness in capturing the essential dynamics of stock levels, indicating that the influences on stock levels are less complex or volatile compared to other variables.

from these models exhibit systematic patterns over time, suggesting that the seasonal decomposition does not fully capture the dynamics of the variables.

The results of the Jarque-Bera and Ljung-Box tests reveal significant non-normality and autocorrelation in the residuals derived from the seasonal decomposition of the variables. This reinforces the idea that copper price behavior cannot be fully explained by deterministic or seasonal forces alone, underscoring the need for models that account for both structural relationships and evolving market shocks. The presence of these patterns suggests that other underlying factors, in addition to seasonality, are influencing the behavior of the variables analyzed. Consequently, these findings justify the need for more complex econometric modeling to fully account for the dynamics at play and improve the understanding of the factors driving changes in these series.

d. Discontinuities and Endogenous Shocks: Breakpoints, Disruptions, and Autocorrelation Features

The analysis of *structural breaks* within our time series data from 2001 to 2024 relies on the Bai-Perron test across all variables to identify significant shifts in trends (Bai and Perron, 1998). The test results were compared with the Chow test for individual verification, though the Bai-Perron test was preferred due to its adaptability for detecting breaks without specifying exact dates in advance. Several periods emerge as particularly influential, where multiple variables experienced structural breaks¹³⁶.

Anomalies are data points that deviate significantly from the expected patterns in the dataset. In commodity markets, anomalies may arise due to economic crises, unexpected demand surges, supply chain disruptions, or other market shocks. Detecting these anomalies allows us to distinguish typical fluctuations from events that could significantly impact model accuracy and interpretability. To detect anomalies in the dataset, we used the Grubbs' Test. Grubbs' test is a statistical method specifically designed to identify outliers in univariate datasets under the assumption of normality. We perform Grubbs' test individually for each variable in our data, iteratively removing any identified outliers until

¹³⁶ Notably: **2004-2006:** The rapid industrial expansion of China during this period significantly impacted global metal demand. China's demand for copper and other metals drove global production adjustments, affecting stock levels, production metrics, and PMI indicators. This intense industrialization phase led to the most substantial shifts across production and economic activity metrics. **2008-2009 Financial Crisis:** During August 2008 and January 2009, the data shows structural breaks across multiple variables. The global financial crisis led to sharp contractions in industrial demand and impacted metal prices significantly. These structural breaks highlight the crisis's impact on production levels, stock management, and related PMI indicators. **COVID-19 Pandemic (April and September 2020):** The onset and progression of the COVID-19 pandemic induced considerable disruptions in production, supply chains, and market demand. Structural breaks detected during this period indicate the volatility across stock levels, production rates, and PMI indices, reflecting the broader economic shock. **October 2022:** The largest number of breaks occurred in October 2022, likely reflecting continued economic adjustments post-pandemic, supply chain challenges, inflationary pressures, and geopolitical tensions impacting the metals market. These shifts influenced multiple metrics simultaneously, signaling a period of high volatility and reactivity across the market.

no further anomalies were detected. Applying Grubbs' test across all variables reveals some anomalies¹³⁷. To further examine anomalies within our dataset, we applied a Z-test. The Z-test identifies data points as outliers if their values deviate significantly from the mean (typically more than 3 standard deviations). The Z-test results in similar findings across various variables¹³⁸. These anomalies reflect real historical events rather than data errors, which could provide valuable information to our model regarding market responses to economic crises. Retaining these data points in the analysis may help the model better capture economic shocks.

We use a time series of disruptive events in copper mining; each associated with a dummy variable indicating the presence (1) or absence (0) of a disruption during a given month. These disruptions encompass a variety of occurrences, such as mine closures, economic crises, strikes, production starts, site discoveries, expansions, and geopolitical events. Notable disruptions include mine-specific events (key strikes as in Escondida in 2006; production starts as in Antamina in 2001, Chuquicamata in 2010, Cobre Panama in 2019; or various expansions of major mines (e.g., Escondida, Ertzberg, and Collahuasi), economic crises, scandals and market manipulations (as in the Chinese reserve case in 2006). These events are integral to the dataset as they serve to parametrize forecasting models by reflecting periods of significant supply-side shocks in the global copper market. The dummy variables facilitate the identification and quantification of the impact of these disruptions on copper production and price dynamics.

Autocorrelation analysis reveals the persistence of past values on current levels across key variables.¹³⁹ *Partial autocorrelation analysis* confirms that most variables have significant short-term dependencies (2-3 lags), with limited long-term influences. An exception is production, which exhibits cyclical patterns

¹³⁷ Four anomalies were detected for the PMI China series, aligning with periods of significant economic events: February 2020 (Value: 35.7), reflecting the economic impact of the COVID-19 pandemic; November 2008 (Value: 38.8) and December 2008 (Value: 41.2), during the global financial crisis; and April 2008 (Value: 59.2), showing an unusually high PMI, due to specific economic events in China at that time. One anomaly was detected for the PMI US series in January 2009 (Value: 32.4), also corresponding with the economic downturn during the global financial crisis. No anomalies were detected in the other variables, namely Price, Production, PMI EU, Stock, Mine Capacity, Ore Grade, and Interest Rate.

¹³⁸ PMI China: Detected outliers include April 2008 (59.2), November 2008 (38.8), December 2008 (41.2), and February 2020 (35.7). These dates correspond with significant economic events, such as the global financial crisis and the COVID-19 pandemic. PMI US: Outliers identified from December 2008 to April 2009 (values between 32.4 and 36.3), highlighting an extended period of economic downturn during the financial crisis. PMI EU: Outliers detected in December 2008 to March 2009 and in April 2020 (values between 33.4 and 34.4), associated with both the global financial crisis and the impact of COVID-19. Mine Capacity: Three outliers in early 2023 (values between 0.663 and 0.673), which may indicate recent industry-specific supply issues or production variability. No outliers detected in Price, Production, Stock, Ore Grade, and Interest Rate variables.

¹³⁹ Copper prices exhibit autocorrelation up to 16 lags, indicating cyclical patterns influenced by long-term supply-demand balances. Production shows a longer memory effect (23 lags), reflecting industry cycles and operational constraints. PMI indexes (China, US, EU) display shorter memory effects (7-8 lags), highlighting their responsiveness to immediate economic conditions. Stock levels persist up to 13 lags due to inventory management practices, while mine capacity utilization (16 lags) reflects operational adjustments. Ore grade (22 lags) demonstrates long-term influence due to gradual quality decline, and interest rates (14 lags) show moderate persistence, impacting financing conditions and copper demand.

at 12- and 24-lag intervals, indicating periodic fluctuations in supply dynamics. This suggests that capturing the cyclical nature of supply is essential for understanding whether and how price adjustments reflect real underlying constraints—especially in a market where physical depletion and capacity limits may emerge over time.

Full details, including graphical analysis and uncertainty measures, are available upon request.

In summary, the variables selected in this chapter—covering refined production, industrial activity, inventories, capacity utilization, ore grade, and interest rates—reflect both short-term market conditions and deeper structural forces shaping the copper market. These indicators are not only relevant for forecasting purposes but also essential for assessing whether observed price movements align with the economic fundamentals required for meaningful adjustment. By modeling the interaction between prices and these core variables, the following sections aim to determine whether copper prices carry the type of information that markets would need to reflect scarcity, coordinate investment and extraction, and support long-term resource sustainability.

Building on this empirical foundation, we now turn to the selection and specification of two complementary modeling approaches designed to evaluate whether copper prices respond to the underlying structural forces identified above.

4. Two Paths to Test Price Signaling: Econometric Structure Using an Error Correction Model (ECM) versus Nonlinear Flexibility with an XGBoost algorithm

This section introduces the models selected to assess how well copper price dynamics reflect underlying supply-demand fundamentals. Rather than pursuing forecasting performance as an end in itself, our goal is to evaluate whether different modeling approaches capture the types of structural signals that a well-functioning market should embed—particularly in the context of long-term resource sustainability.

Forecasting econometric models that were initially tested include linear regression, ARIMA, ARDL, and VAR. While these models provided useful insights, they exhibited limitations in capturing the complex, dynamic relationships in copper price movements. Linear models struggled with non-linear interactions, ARIMA faced challenges with structural breaks, ARDL offered flexibility but insufficient accuracy, and VAR failed to incorporate long-term equilibrium relationships. These limitations highlighted the need for a modeling approach that integrates both short-term dynamics and long-term relationships while also accommodating non-linear patterns and economic shocks.

The ECM is chosen for its dual capacity to handle short-term fluctuations and long-term equilibrium adjustments. Key factors justifying this selection include:

- **Handling Non-Stationarity:** The copper price series and many of the explanatory variables (e.g., production, stocks, interest rates) exhibited non-stationary behavior, with stationarity achieved only after differencing. An ECM is inherently designed for such data, allowing for robust analysis without compromising the underlying economic theory.
- **Cointegration Among Variables:** The presence of cointegration relationships among key indicators justified the use of an ECM, which ensures that any deviation from the long-term equilibrium is corrected over time. This is particularly relevant in commodity markets, where supply-demand imbalances are often transient.
- **Incorporation of Structural Breaks:** As described above, the copper market has experienced significant structural shifts (e.g., the 2008 financial crisis, COVID-19 pandemic), necessitating a model that can adapt to these changes. The ECM framework allows for the inclusion of dummy variables to account for these breaks, ensuring more reliable forecasts.
- **Autocorrelation and Partial Autocorrelation Analysis:** The ACF and PACF results demonstrated significant autocorrelations at longer lags, reinforcing the need for a model capable of capturing both short-term dependencies and long-term trends. The ECM's structure, with both short-run adjustment coefficients and long-run equilibrium relationships, addresses this requirement effectively.
- **Multicollinearity Considerations:** The Variance Inflation Factor (VIF) test revealed significant collinearity among the explanatory variables. An ECM mitigates these concerns by focusing on cointegrated relationships rather than independent predictors, maintaining the integrity of the model.

Additionally, the ECM's ability to provide interpretable economic insights into how each variable influences copper prices over time was crucial, particularly given the need to balance academic rigor with practical applicability in the metals market.

In parallel, XGBoost is selected for its strengths in handling non-linear relationships and large feature spaces, addressing limitations faced by traditional econometric models. The choice of XGBoost was driven by:

- **Flexibility in Capturing Non-Linear Interactions:** Unlike linear models, XGBoost excels at identifying complex patterns between copper prices and economic indicators such as PMI, production levels, and stock changes, even when these relationships are highly non-linear and dynamic.
- **Robustness to Noise and Missing Data:** Given the volatility in commodity markets, XGBoost's ability to handle noisy data while maintaining accuracy was a significant advantage.

- Regularization for Overfitting Prevention: The inclusion of L1 (Lasso) and L2 (Ridge) regularization within the XGBoost framework ensures that the model remained stable across various forecasting horizons despite the inherent volatility of copper prices.
- Technical Superiority Over Other Machine Learning Models: While Ridge regression managed multicollinearity, it struggled with capturing complex non-linear relationships, and Lasso regression, though effective in feature selection, often oversimplified the model by excluding important predictors. Elastic Net balanced regularization and feature selection but remained limited by linear assumptions. Random Forest captured non-linear trends but lacked the subtlety required for medium-term copper price forecasting. In contrast, XGBoost's iterative boosting mechanism corrected errors from previous iterations, achieving higher accuracy and stability, making it the most suitable choice for this study.
- Data Processing Consistency: XGBoost utilized the same dataset as the ECM, ensuring comparability of results. While the ECM captured time dependencies through cointegration, XGBoost focused on immediate economic drivers without imposing a temporal structure, reflecting the market's sensitivity to recent events rather than historical trends.
- Exclusion of Time-Dependence in XGBoost: This choice stemmed from the assumption that copper prices in the short to medium term are more influenced by current market conditions than by historical patterns. By avoiding time-based constraints, XGBoost provided a flexible platform to explore the impact of contemporary economic indicators, making it a suitable counterpart to the ECM's time-sensitive structure.

To summarize, the integration of ECM and XGBoost in this study offers a comprehensive approach to copper price forecasting. The ECM helps identify whether price movements are anchored in structured, equilibrium-based relationships with fundamental variables—such as production, inventories, and demand proxies—offering a test of whether the market behaves in ways compatible with theoretical conditions for convergence and adjustment. The XGBoost algorithm complements this with a flexible, data-driven lens capable of detecting complex, nonlinear responses—allowing us to explore whether copper prices reflect more subtle, adaptive interactions between economic variables and short-term shocks, even in the absence of explicit structural modeling.

Further details on the tested models, statistical justifications, and technical configurations are available upon request.

We begin with the Error Correction Model, which offers a theory-driven framework to evaluate whether copper prices adjust in response to deviations from structural equilibria—particularly those

involving production, demand, inventories, and cost-related variables. The next section details the specification, estimation strategy, and interpretive insights of the ECM.

IV. Error Correction Model: Structural Specification, Dynamics, and Forecasting Insights

1. Model Specification

a. Preliminary Diagnostics: Stationarity Test, Lag Selection, Cointegration Analysis

To estimate an ECM, ensuring that all series are stationary is essential, as non-stationary data can lead to spurious regressions. The Augmented Dickey-Fuller test is employed to assess stationarity, revealing that copper price, production¹⁴⁰, mine capacity utilization¹⁴¹, and ore grade exhibit non-stationarity due to long-term trends in global demand, technological advancements, and resource depletion. Differencing these series resolves this issue, with ore grade requiring second differencing due to its persistent decline over time¹⁴².

Determining the optimal lag length allows capturing data dynamics without overfitting. Using AIC, BIC, FPE, and HQIC, a lag of one is selected, indicating that immediate past values have the most significant influence on current copper prices—consistent with the copper market's sensitivity to recent supply-demand changes¹⁴³.

¹⁴⁰ The price and production time series for copper are non-stationary in levels, as both exhibit upward trends over time, reflecting the expansion of the market and the overall growth in global demand. This upward trajectory can be attributed to various factors, including increasing industrial applications of copper, population growth, and economic development in emerging markets, all of which contribute to a sustained rise in consumption and production capacity. As a result, the underlying time series data for prices and production is influenced by these long-term trends, leading to non-stationarity in their levels.

¹⁴¹ The non-stationarity of the mine capacity utilization rate reflects long-term structural changes in the mining industry, driven by technological advancements, regulatory shifts, and global market conditions. Innovations in extraction technologies can improve operational efficiency, potentially increasing utilization rates, while sectoral expansions or contractions introduce fluctuations over time. Additionally, growing social activism and protest movements around mining operations, often fueled by environmental and social concerns such as land rights, pollution, and ecosystem impacts, contribute to this volatility. Operational disruptions from protests or regulatory interventions can reduce utilization rates, highlighting the industry's sensitivity to both economic and social pressures. This dynamic environment underscores the need for econometric modeling techniques that can account for these structural changes and evolving challenges.

¹⁴² Copper ore concentration is often subject to long-term trends driven by factors such as resource depletion and technological advancements. As high-grade ore deposits are extracted, mining operations may increasingly rely on lower-grade ores, resulting in a downward trend in average ore concentration. This persistent downward trend indicates that the series is integrated of order one. However, if the rate of decline in ore concentration accelerates due to resource depletion or changes in mining practices, this can create a second level of integration, necessitating a second differencing to remove both the trend and any resulting cyclical patterns.

¹⁴³ The fact that a lag of one is sufficient suggests that the relationship between the copper price and its influencing factors is primarily immediate. In other words, current values of supply and demand variables have a significant effect on the copper price in the next period, indicating that market participants react quickly to new information. In the context of our monthly data, a lag order of one suggests that the previous month's information is sufficient to explain current price movements, indicating that immediate past values are particularly influential. Metal commodities like copper are known for

Importantly, the Johansen cointegration test confirms the presence of two stable structural relationships among key variables, suggesting that copper prices are statistically linked to fundamental market forces over time—a precondition for prices to convey meaningful information about resource scarcity and demand conditions. Statistical evidence supports selecting a cointegration rank of 2 as it balances capturing significant long-term relationships without adding unnecessary complexity. This choice reflects stable equilibrium interactions, likely driven by core economic factors like production and stock levels, which are critical in shaping copper price dynamics.¹⁴⁴

Two cointegrating relationships are frequently employed in analyzing long-term equilibria, as demonstrated by Johansen and Juselius (1990), who emphasize the importance of economic interpretability when selecting cointegration ranks. Gonzalo (1994) similarly highlights that a limited number of vectors allows for more meaningful economic conclusions. Empirical studies on commodity markets, such as Ghoshray (2011), Sari, Hammoudeh, and Ewing (2009), and Ewing and Thompson (2007), further illustrate the use of multiple cointegrating vectors to capture stable long-term interactions between key variables. These works underscore the importance of selecting an appropriate cointegration rank that aligns with both statistical significance and economic theory.

Therefore, the choice of a cointegration rank of 2 in this analysis offers a focused and interpretable model that captures essential long-term interactions within the copper market while avoiding the risks of overfitting and complexity associated with additional cointegrating relationships. Further technical details and statistical outputs are available upon request.

b. Final Model Formulation and Error Correction Mechanism

The chosen ECM specification is therefore given by the equation below:

$$\Delta Price_t = \alpha + \alpha_0 \Delta Price_{t-1} + \sum_j \alpha_j \Delta X_{j,t-1} + \gamma_1 ECT_{1,t-1} + \gamma_2 ECT_{2,t-1} + \epsilon_t$$

Where:

- $\Delta Price_t$ is the monthly log-difference of copper price,
- α is the intercept, capturing any constant drift in the differenced copper price,

their price volatility, influenced by various factors such as geopolitical events, changes in demand from key industries, fluctuations in production, and shifts in inventory levels. A one-period lag highlights the fact that the market is responsive to these sudden changes, which is essential in a volatile environment.

¹⁴⁴ Including a third cointegrating vector, while statistically supported, could introduce noise and reduce the interpretability of the model by incorporating weaker relationships, thereby diluting economic insights.

- $\alpha_0 \Delta Price_{t-1}$ reflects the influence of the previous period's change in copper price on the current period's change, capturing short-term autoregressive dynamics,
- $\sum_j \alpha_j \Delta X_{j,t-1}$ captures the immediate impact of changes in each of the exogenous variables (Production, PMI indexes, Stock, Mine capacity, Ore grade, Interest rate) from the previous period,
- γ_1 and γ_2 are the speed-of-adjustment coefficients for the two cointegration relationships,
- $ECT_{1,t-1}$ and $ECT_{2,t-1}$ are the error correction terms for the two cointegrating relationships at time $t - 1$, calculated from the long-term cointegration vectors,
- ϵ_t is the error term.

The Error Correction Term ECT can be expressed as:

$$ECT_{i,t-1} = Price_{t-1} - \sum_j \beta_{i,j} X_{t-1,j} - c$$

Where $X_{t-1,j}$ are the exogenous variables at time $t - 1$ and $\beta_{i,j}$ are the coefficients for each exogenous variable in the first and second cointegration relationships, and c is the deterministic term. ECT_{t-1} captures the deviation from the long run equilibrium. If $ECT_{t-1} \neq 0$, this means the system is out of equilibrium, and the ECM will adjust in subsequent periods to bring the system back to equilibrium.

This equation defines the structural relationship among prices and fundamentals. Deviations from this equilibrium captured by the error correction term ECT_{t-1} indicate a temporary imbalance that the system will adjust over time.

2. Estimation Results and Economic Interpretation

a. Long-Term Cointegration Coefficients: Structural Drivers of Copper Prices

Table 5.3: Long-Term Cointegration Coefficients

Coefficient type	Variable	First cointegration relationship	Second cointegration relationship
Long-term coefficients ($\beta_{i,j}$)	Production	1.69e-15	-1.855
	Mine capacity	1265.68	3115.23
	PMI China	-60.4633	-1.2327
	PMI US	-13.9026	0.0798
	PMI EU	76.6219	1.3746
	Stock	-10.3288	-10.3908
	Ore grade	16368.4	550.217
	Interest rate	15.2746	1.3022

The coefficient associated with the increase in *production levels* in the second cointegration relationship (-1.855) indicates that as the pace of copper production increases, we can indeed expect copper prices to decline significantly. This observation aligns with the fundamental economic principle that when production growth outpaces demand growth, prices are likely to fall. In essence, if the increase in production levels exceeds the corresponding rise in demand, the market experiences an oversupply situation, which exerts weights on prices. This dynamic highlights the delicate balance between supply and demand in the copper market: a stronger growth in production serves to alleviate the equilibrium between demand and supply, ultimately resulting in lower prices, all else being equal.¹⁴⁵

As concerns the *mine capacity utilization rate*, the strongly positive coefficients for both cointegration relationships (1265.68 and 3115.23) indicate that even a small increase in the mine capacity utilization rate difference, whether due to a deceleration in its decline or rebound in utilization, correlates with a significant rise in copper prices.¹⁴⁶

As concerns the *PMI indexes*, in the first cointegration relationship, the coefficient of -60.4633 means that, in the long run, a 1 unit increase in the PMI China index, signaling greater optimism in the Chinese manufacturing sector, is associated with a decrease in the price of copper by \$60.46.¹⁴⁷ As for the

¹⁴⁵ The positive coefficient associated with the increase in *production levels* in the first cointegration relationship (1.69e-15) is quantitatively very small. This likely reflects the specific nature of the first cointegration relationship in the model, which demonstrates more intricate dynamics when interacting with Purchasing Managers' Index (PMI) indices and stock levels (see below). These factors may overshadow the structural relationship between prices and production.

It is important to note that the negative coefficient associated with production in the short term does not contradict the overall trend of both production levels and prices growing together in the long term. While increased production may lead to immediate price declines when supply outpaces demand, the long-term trajectory typically reflects simultaneous growth in both production and prices due to sustained economic expansion.

¹⁴⁶ This positive relationship underscores the market's perception in the long term that higher capacity utilization signals greater operational efficiency and supply readiness. When mines are utilized more effectively, it typically suggests that producers are responding to rising demand, bolstering market confidence. This aligns with findings from Crowson (2011), who documented that increased utilization rates in copper production often coincide with periods of rising prices due to greater demand-driven activity. In the short run, a lower utilization rate often signals operational disruptions, such as strikes, maintenance issues, or other unforeseen events that hinder production. These disruptions can create a supply shock in the market, leading to increased upward pressure on prices. When utilization rates drop due to operational accidents or labor disputes, the immediate consequence is a reduction in the effective supply of copper available to meet ongoing demand. As a result, market participants may react by bidding up prices in anticipation of tighter supply conditions, consistent with the observations of Tilton (1996), who emphasized the role of short-term disruptions in driving price spikes. Jacks and Stuermer (2020) further illustrate this relationship by showing how variations in mine capacity utilization influence copper price trends, particularly during demand surges or periods of constrained supply. When utilization rates rise in response to growing demand, prices tend to follow suit due to the market's expectations of increased consumption and sustained growth in industrial activity. This scenario contrasts with the short-term dynamic, where operational disruptions dominate. Nonetheless, the positive long-term relationship found in this analysis reflects the structural nature of the copper market, where capacity utilization rates serve as a signal of both current production activity and anticipated demand growth.

¹⁴⁷ This negative relationship might reflect a scenario where higher PMI in China could lead to expectations of increased supply, reducing copper prices. It could be that a booming Chinese manufacturing sector leads to higher production (and potentially inventory) of copper, driving prices down. In the second relationship, the coefficient of -1.2327 suggests a similar negative relationship, although of a lesser extent.

American PMI, the coefficient is smaller compared with the one associated to the Chinese PMI. This is consistent with the fact that refining activities in the US are more than 10 times smaller than in China (China produced 10,021.4 thousand metric tons of copper in 2020 vs. 905.2 for the US¹⁴⁸)¹⁴⁹. When it comes to the Eurozone PMI, a 76.6219 coefficient in the first cointegration relationship suggests that a 1 unit increase in the EU PMI leads to a \$76.62 increase in copper prices. This is a strong positive relationship, indicating that optimism in the European manufacturing sector is closely linked to rising copper prices, possibly due to higher anticipated demand for copper in European production. A positive sign in this relationship is consistent with the fact that the EU has a very limited supply position in the copper industry, as opposed to China and the US. In the second relationship, the coefficient of 1.3746 suggests a similar positive relationship, although of a lesser extent.

For *stocks*, both coefficients in the cointegration relationships are negative, one being larger than the other (-0010.3 vs -001.04).¹⁵⁰

For *the Ore grade variable*, both coefficients in the cointegration relationships are strongly positive (16368.4 and 550.35). Since we are estimating the model with the second difference of the Ore grade time series, this means that an increase in the rate of change of ore concentration would yield a significant price increase on the market. This confirms the market's sensitivity to signals of geological scarcity—an important condition for long-term sustainability, as price responsiveness to declining ore grades suggests that depletion is partially priced in¹⁵¹.

Regarding the *Interest rate variable*, the estimated coefficients from the two cointegration relationships indicate a modest and inconsistent association with copper prices. In the first relationship, a 1 percentage point increase in the Federal Reserve's interest rate is associated with a copper price increase of approximately \$15.27. In the second, the effect is negligible, around \$1.30. These small and uneven effects suggest that, over the medium run, interest rates may play only a limited role in shaping

¹⁴⁸ <https://www.statista.com/statistics/1241269/world-annual-refined-copper-production-by-country/>

¹⁴⁹ In the second relationship, the coefficient of 0.0798 suggests a very weak, almost negligible positive relationship, where an increase in PMI US leads to a slight increase in copper prices. This may reflect a more nuanced or secondary influence on prices, perhaps through demand for refined copper products. Weak demand-side effects are consistent with the nature of a long-term relationship.

¹⁵⁰ The first coefficient suggests that for every 1,000 tons increase in stock levels, copper prices are expected to decrease by approximately \$10.30 in the long run. This negative relationship indicates that higher stock levels lead to lower prices, likely due to increased supply and therefore greater availability of copper. The average absolute movement of stock levels from one month to the other is 41,510 tons, yielding an opposite price movement of \$427.6. This demonstrates the sensitivity of copper prices to variations in supply. The interpretation for the second coefficient is similar.

¹⁵¹ The quality of the time series data used in this analysis is not optimal in terms of levels due to the difficulty in obtaining comprehensive data. Consequently, we utilize data from several large Chilean copper mines, which effectively imitates the worldwide trend. While this approach does not present econometric issues since we are focusing on the second difference, it limits the interpretability of the coefficient values. Therefore, we refrain from further interpretation of these coefficients, as they may not adequately reflect the magnitude of observed dynamics.

structural price movements in the copper market¹⁵². Given these dynamics, our empirical findings thus suggest that interest rate movements are more likely to affect copper prices in the short run rather than through sustained long-term impacts. Short-term adjustments often occur due to announcement effects, where market participants quickly reassess demand expectations based on interest rate changes. For instance, a sudden rate increase might prompt an immediate recalibration in the copper market as investors adjust their expectations considering perceived economic conditions. However, these adjustments tend to be short-lived, driven by temporary expectations rather than a fundamental shift in long-term supply-demand equilibrium. Thus, while interest rates can influence immediate market conditions and price fluctuations, their muted long-term impact in our empirical results may reflect specific structural and macroeconomic conditions within the studied period that depart somewhat from traditional theoretical expectations. These findings align with the literature presented in Chapter 1, though they suggest that Hotelling’s framework may be less predictive in contexts where external factors, such as regulatory barriers or unconventional monetary policies, introduce additional layers of complexity in commodity markets. This muted structural impact is notable given Hotelling’s prediction that interest rates guide resource extraction timing. Our finding suggests that other constraints—such as regulatory rigidities, short-term financial conditions, or geopolitical shocks—may increasingly dominate extraction and price formation, reducing the role of pure intertemporal optimization.

b. Short-Term Adjustment Dynamics: Immediate Responses to Market Shocks

As concerns the *Price variable*, the -0.356 coefficient suggests that if copper price increased by one unit in the previous period, it tends to decrease in the current period by about 0.356 unit. This negative coefficient indicates a reversion to the recent price trend, suggesting the presence of short-horizon correction mechanisms.

¹⁵² The theoretical expectation, as per Hotelling’s (1931) model, would be that rising interest rates could affect copper prices by increasing the opportunity cost of holding reserves, potentially encouraging accelerated extraction and influencing prices. However, our empirical results for the 2001-2024 period indicate a subdued long-term relationship between interest rates and copper prices, pointing to several structural factors that may modulate or diminish this impact

Firstly, the structural inelasticity of copper supply—characterized by slow output growth even amid rising demand—limits the responsiveness of supply to changes in borrowing costs. Mining operations face inherent constraints such as regulatory challenges and geological limitations in ore deposits, which hinder supply adjustments in response to interest rate changes. Consequently, higher interest rates may not significantly slow down supply, making it an unlikely channel for upward price adjustments on the supply side, in contrast with theoretical expectations.

Secondly, while the Hotelling model implies a link between resource prices and interest rates, the demand for copper and interest rates do not show a consistently correlated relationship over the long term. Within our timeframe (2001-2024), the relationship has been complicated by atypical interest rate movements influenced by various macroeconomic events, including financial crises, shifts in monetary policy, and fluctuating global economic conditions. Demand for copper is driven by industrial production, construction, and technological advancements, which do not always align with interest rate fluctuations, leading to a complex and variable relationship over extended periods.

Table 5.4: Short-Term Adjustment Coefficients

Coefficient type	Variable	Value
Adjustment coefficients (α_j)	Price	-0.356
	Production	1.042
	Mine capacity	-166.054
	PMI China	-39.33
	PMI US	0.267
	PMI EU	-24.71
	Stock	-41.013
	Ore grade	-13473.2
	Interest rate	-214.373

As concerns the *Production variable*, the positive coefficient (1.042) suggests that an increase in copper production in the previous period has an immediate positive impact on copper price, possibly indicating that higher production is a response to rising demand, which helps sustain prices in the short term. Given that our "Production" series specifically measures refined copper production rather than raw extraction volumes, this positive impact may further reflect that higher refined output is closely aligned with rising demand for processed copper. This could suggest a degree of tightness in the refined copper market, with refined production acting as a more direct and responsive indicator of market conditions than raw extraction, thereby capturing demand-driven price adjustments more accurately.

As concerns the *Mine capacity utilization rate variable*, the coefficient of -166.054 hints that a 1 pp increase in the mine capacity utilization rate would lead to a decrease of the price by \$166.054.¹⁵³

The contrasting responses of the *PMI indicators* in the short term can be attributed to various economic and market dynamics at play in the respective regions¹⁵⁴.

¹⁵³ As preemptively mentioned in the previous section on the long-term coefficients, the short-term relationship between copper market price and mine capacity utilization rate is opposite to the long-term one. Indeed, in the short term, when utilization rates decline—often due to incidents such as strikes, maintenance issues, or other unforeseen disruptions—the effective supply of copper diminishes, creating a supply shock in the market. In such cases, market participants may react by bidding up prices in anticipation of tighter supply conditions. This immediate response contrasts with the long-term perspective, where rising utilization rates signal improved operational efficiency and readiness to meet growing demand, thereby supporting higher copper prices.

¹⁵⁴ The significant negative coefficient for PMI China (-39.33) suggests that increases in manufacturing activity might be signaling overcapacity or potential market corrections. If Chinese manufacturers ramp up production but face weaker demand domestically or internationally, it can lead to anticipatory pricing behaviors, where traders expect prices to fall due to potential oversupply. In contrast, the slight positive coefficient for PMI US (0.267) indicates that a stronger US manufacturing sector can be associated with a healthy demand environment. The US economy might be experiencing growth or recovery, leading to increased demand for raw materials, including copper. This positive signal could reflect the market's optimism regarding consumption patterns and infrastructure spending in the US, which would support higher copper prices. The negative coefficient for the EU PMI (-24.71) might reflect regional economic challenges or uncertainty. If European manufacturers are not experiencing the same levels of growth as their US counterparts, increases in EU PMI could be perceived as insufficient to support copper prices, leading to downward pressure instead.

The coefficient for *Stock*, expressed in tons, indicates a negative relationship with copper prices, suggesting that for every 1,000 tons of copper added to inventory, prices are expected to decrease by approximately \$41.013. This relationship underscores the market's perception that rising inventories signal excess supply conditions, which can exert downward pressure on copper prices, particularly in the short term. As inventories accumulate, market participants may anticipate a surplus in supply relative to demand, leading to a decrease in prices as buyers adjust their expectations in response to potential oversupply.¹⁵⁵

The large negative coefficient for *ore concentration*, measured at -13,473.2, underscores the significant cost and supply implications associated with declining ore grades. As the concentration of ore decreases, the extraction process becomes more costly and inefficient, leading to potential slowdowns in overall supply. This increased cost burden can exert significant upward pressure on copper prices in the short term.¹⁵⁶

Lastly, the significant negative coefficient for *the FED interest rate* (-214.373) indicates a pronounced relationship where increases in US interest rates correlate with a substantial short-term decline in copper prices. This dynamic can be understood through several interconnected mechanisms¹⁵⁷.

Overall, the mixed responses indicate that while Chinese manufacturing activity might reflect supply-side issues, the US PMI could be signaling demand-side strength, resulting in a slightly positive short-term effect on copper prices. The divergence highlights the complexities of global supply chains and how regional economic conditions can lead to varying impacts on commodity markets.

¹⁵⁵ The short-term adjustment coefficient for stocks is approximately four times larger than the corresponding coefficient in the first cointegration relationship. This significant difference highlights the immediate impact that changes in inventory levels can have on copper prices in the short term compared to the long-term equilibrium established by the cointegration relationship.

While the long-term relationship may reflect broader market dynamics, such as supply and demand trends and operational efficiency, the larger short-term coefficient indicates that fluctuations in stock levels can lead to more pronounced price reactions. In essence, this suggests that traders are particularly sensitive to changes in inventories, adjusting their pricing strategies quickly in response to perceived shifts in supply availability.

¹⁵⁶ Furthermore, the scale of the ore concentration metric contributes to the magnitude of this coefficient, reflecting substantial shifts in the underlying data. The convex declining profile of ore concentration illustrates the increasing difficulty and cost associated with extracting copper as grades diminish. This scarcity narrative is particularly important, as it highlights how declining ore quality can lead to fears of insufficient supply in the future.

Thus, while lower ore concentrations signal long-term challenges for production viability, they also evoke concerns about scarcity that can impact market perceptions and pricing. The immediate short-term effects are encapsulated in this negative relationship, where increased extraction costs, driven by declining ore quality, lead to tighter supply conditions and contribute to price increases in the copper market.

¹⁵⁷ Firstly, higher interest rates typically strengthen the US dollar, as they attract foreign capital seeking higher returns on investments. A stronger dollar makes copper, which is priced in dollars, more expensive for non-dollar-denominated buyers. Consequently, as the cost of copper rises for international purchasers, demand may diminish, leading to downward pressure on prices in the global market. Additionally, increased interest rates often signal tighter monetary policy, which can have broader economic implications. Higher borrowing costs can dampen investment and consumption, slowing economic growth. For sectors reliant on copper, such as construction and manufacturing, reduced activity can lead to lower demand for copper, further exacerbating the price decline. Moreover, the negative coefficient reflects market participants' expectations; as interest rates rise, traders may anticipate reduced economic activity and lower demand for commodities like copper. This sentiment can trigger preemptive selling in the market, causing prices to drop even further.

c. Speed of Adjustment: Dual Equilibrium Correction Mechanisms

Table 5.5: Speed of Adjustment Coefficients

Coefficient type	First cointegration relationship	Second cointegration relationship
Speed of adjustment coefficients (γ_i)	-0.0336652	-1.90392

Both speed of adjustment coefficients are negative, which aligns with theoretical expectations. Negative coefficients indicate that when there is a deviation from the long-term equilibrium, adjustments work to reduce this gap, moving the system back toward equilibrium.

The first coefficient (-0.0336652) suggests a rather gradual adjustment back to equilibrium, using monthly data. Specifically, when there is a deviation from the long-term equilibrium, approximately 3.37% of the gap is closed each period, hinting towards 2.5 years necessary to fully correct the deviation. This slower adjustment may reflect the complexities, and time lags inherent in the copper market, where prices do not respond instantaneously to changes in underlying supply and demand dynamics. The small magnitude of this coefficient indicates that factors influencing price adjustments—such as operational delays, market sentiment, or broader economic conditions—can prolong the time it takes for prices to realign with fundamental values.

In contrast, the second coefficient (-1.90392) indicates a much faster adjustment back to equilibrium, suggesting that approximately 190.39% of the deviation is corrected in a single period. This rapid correction indicates that, in some cases, the market responds strongly to deviations, potentially due to quick reactions from market participants or external conditions influencing copper prices. The magnitude suggests that the adjustment may momentarily *overshoot* the equilibrium level, reflecting the heightened sensitivity of the copper market to certain short-term shocks. This may be a result of traders quickly recalibrating their expectations in response to new information or abrupt changes in the economic environment, temporarily pushing prices past the equilibrium before stabilizing.

The contrasting speed of adjustment coefficients reveal different dynamics at play within the copper market. The first coefficient implies that while long-term relationships may remain stable, short-term fluctuations can persist for extended periods before adjustments take effect. Conversely, the second coefficient that the market reacts strongly and immediately to certain deviations, indicating that short-term market sentiment and external shocks can quickly drive price corrections.

Several studies on copper and other metal prices have reported speed of adjustment coefficients that generally align with our first coefficient. In their study examining the relationship between copper prices and exchange rates, Byrne et al (2013) found speed of adjustment coefficients ranging from

–0.10 to –0.30, indicating that approximately 10% to 30% of the disequilibrium is corrected each month. These values suggest a moderate adjustment speed, which is relatively close to the first coefficient observed in this analysis. In a study investigating the long-term relationship between gold prices and various macroeconomic variables, Zhang and Wei (2010) reported adjustment coefficients around –0.05, implying that 5% of the deviation is corrected per period (working with daily data). This gradual adjustment rate is consistent with the first coefficient in our model, highlighting a slower realignment process.

The weights assigned to the explanatory variables in each respective cointegration relationship further illuminate the nature of these dynamics¹⁵⁸.

The presence of two cointegrating relationships in our model suggests that both a slow and a fast adjustment dynamic are at play simultaneously. The first coefficient reflects a gradual adjustment back to equilibrium, likely driven by long-term structural factors such as production capacity and supply constraints. The second coefficient, with its rapid adjustment rate, suggests a faster response, potentially driven by short-term market reactions to demand shocks or changes in investor sentiment.

This dual dynamic aligns rather well with the values observed in the literature, which report adjustment speeds closer to, but generally higher than, our first coefficient. These studies similarly capture moderate adjustment rates for structural, long-term equilibria, indicating that our findings are consistent with established models while also providing unique insights into short-term volatility in the copper market.

The dual adjustment speeds observed in this model—one gradual and one immediate—mirror the coexistence of structural and reactive forces in copper markets. This supports the idea that while prices are partially anchored in fundamentals, they also incorporate fast, potentially sentiment-driven corrections. Such a dual dynamic must be accounted for when assessing whether market signals are consistent with sustainable adjustment to resource constraints.

d. Residual Diagnostics: Model Validity and Distributional Properties

The analysis of ECM residuals for copper prices reveals several key diagnostic properties. Skewness is close to zero (–0.0028), suggesting a symmetrical residual distribution, while kurtosis (1.8206) indicates

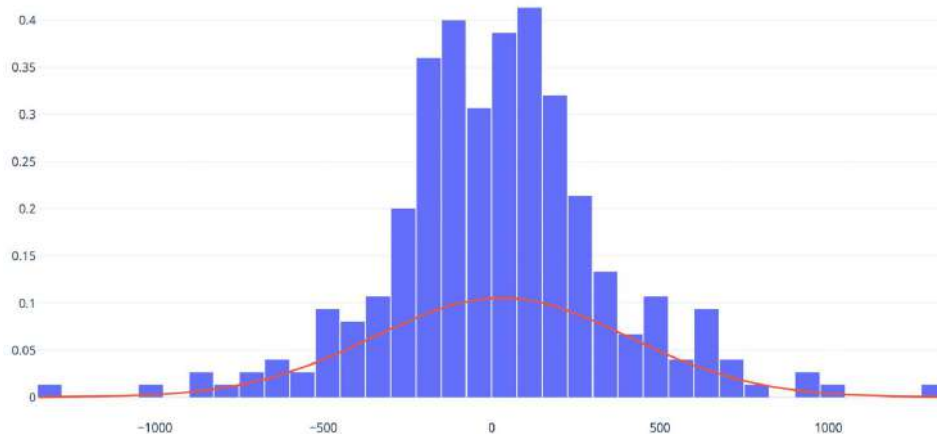
¹⁵⁸ Production has a very small weight in the first relationship (1.69e-15) but a substantial negative weight in the second (–1.855), suggesting that while refined copper output fluctuations have limited impact on the long-term equilibrium but can significantly influence short-term prices. Mine capacity demonstrates a strong positive influence in both relationships, emphasizing its critical role in determining copper prices. The weights for PMIs highlight varying impacts across economies. The negative effect of China’s PMI in the first relationship suggests that a contraction in Chinese manufacturing may put downward pressure on copper prices. Ore grade exhibits substantial weights in both relationships, underscoring its significance: heightened variations in ore concentrations correlate positively with long-term price levels and contribute to short-term adjustments.

a platykurtic shape with lighter tails and fewer outliers than a normal distribution. Statistical tests confirm that the residuals are stationary (ADF test), uncorrelated (Ljung-Box test), and homoscedastic (Breusch-Pagan test). While the Jarque-Bera test formally rejects normality, additional diagnostics such as the Shapiro-Wilk and Kolmogorov-Smirnov test yield mixed results.

From a practical standpoint¹⁵⁹, the residuals' symmetry, low skewness, and absence of strong autocorrelation or heteroskedasticity suggest that the ECM provides stable and interpretable estimates. The departure from strict normality does not undermine the model's validity, particularly given that the Central Limit Theorem ensures reliable inference for sufficiently large samples. Moreover, such deviations are common in applied macroeconomic and commodity data, especially when structural shifts and volatility clustering are present. This position is consistent with standard econometric practice, which recognizes that non-normality in residuals does not undermine inference when the sample size is large and residuals are homoscedastic and uncorrelated (Hamilton, 1994, p. 571; Enders, 2014). In commodity price modeling in particular, such departures from normality are common and often reflect structural features of the data rather than model misspecification (Pindyck and Rotemberg, 1990).

In this context, the residual diagnostics confirm that the ECM adequately captures the adjustment dynamics of copper prices, allowing us to draw meaningful conclusions about the informational role of price signals in this market.

Figure 5.1: Residuals of the ECM specification



¹⁵⁹ While classical assumptions like residual normality are standard in econometric modeling, the goal here is not statistical refinement for its own sake, but rather the ability to identify structural price adjustment consistent with equilibrium-based dynamics in the copper market.

3. Forecasting Insights: Price Adjustment Speeds and the Limits of Structural Predictability

In this section, we evaluate the forecasting performance of the Error Correction Model (ECM) for copper prices. The following assessment of the ECM's forecasting performance does not seek to optimize predictive accuracy as an end. Rather, the objective is to use the model's structural parametrization to gain insights into the underlying mechanisms driving copper price movements. This performance analysis also provides a foundation for comparing ECM forecasts with more commonly used predictive tools, such as futures prices, thereby offering a broader perspective on the effectiveness and informational value of different forecasting approaches in the copper market.

Figure 5.2 illustrates the observed copper prices from January 2022 to September 2024, alongside several sets of monthly forecasts generated by the ECM model. The sets of forecasts were produced for a range of months, beginning in September 2022 and extending through June 2024, all with the same time horizon, August 2024. This means that the September 2022 set of forecasts includes 24 forecast points while the June 2024 set consists of only three forecast points. For each forecasting month, the ECM was estimated using only the information available at that specific time.

Figure 5.2: Actual Copper Prices vs Historical Forecast Iterations (ECM)



Four forecast horizons are assessed. Full sets include all forecast points from each start date until August 2024, ranging from 3 to 24 months depending on the starting point. The 6-month sets focus on the first six months after each forecast date, capturing short- to medium-term accuracy. The 3-month sets consider only the first three months, reflecting short-term precision. Finally, next-month sets assess the following month's price after each forecast date.

Root Mean Squared Error (RMSE) values¹⁶⁰ show a clear pattern: shorter horizons yield lower errors.¹⁶¹ This trend underscores the ECM's strength in short-term forecasting, while its accuracy decreases with longer horizons. These results highlight the model's effectiveness in capturing immediate market dynamics and suggest the potential need for more flexible models for long-term predictions. Further details on RMSE computations and forecast intervals are available upon request.

V. Machine Learning Approach: XGBoost Algorithm

The XGBoost section presents the methodology, optimization process, and forecasting performance assessment for the machine learning model applied to copper prices. While the ECM relies on structural relationships rooted in economic theory, XGBoost offers a data-driven, flexible approach, enabling the detection of intricate interactions and patterns among the selected explanatory variables.

In this section, we first outline the methodological principles guiding the use of XGBoost, including the model's tree-based structure and the rationale for using time-series cross-validation. We then describe the optimization process, detailing the hyperparameter tuning and the selection of final model specifications. Finally, we assess the model's forecasting performance across various forecast horizons, comparing its results with those of the ECM and evaluating its ability to track medium-term copper price dynamics. This analysis ultimately contributes to the broader objective of assessing how well copper prices incorporate economic fundamentals over time—and whether a non-parametric model like XGBoost can reveal dynamics that may elude structurally-constrained approaches such as the ECM.

1. Model Specification and Theoretical Justification

The XGBoost algorithm, short for eXtreme Gradient Boosting, is an ensemble learning technique based on the gradient boosting framework, designed to improve the predictive accuracy and robustness of machine learning models (Chen and Guestrin, 2016). It has become a widely used tool in empirical economics and applied forecasting, including in commodity markets (e.g., Liu et al., 2017; Lasheras et al., 2015). At its core, XGBoost constructs an ensemble of weak learners, typically decision trees, in a

¹⁶⁰ We choose RMSE as the evaluation metric because it measures the average magnitude of the forecasting errors in the same units as the copper price, making the results easily interpretable. RMSE penalizes larger errors more heavily than other metrics like Mean Absolute Error, which is particularly relevant in commodity price forecasting where large deviations can have significant economic implications. Additionally, RMSE is widely used in time series analysis and forecasting literature, facilitating comparison with existing studies. Its sensitivity to large errors helps in assessing the robustness of the forecasting model, especially when evaluating different forecast horizons.

¹⁶¹ The full sets average 697.34, the 6-month sets 510.20, the 3-month sets 423.99, and the next-month sets 233.82, representing 38.7% of the copper price standard deviation (604.84) over the same period.

sequential manner. Each tree is trained to correct the residual errors of the preceding ones, thereby minimizing the loss function iteratively.

Formally, the XGBoost model for a given dataset with n observations and m features can be represented by $\hat{y}_i = \sum_{k=1}^K f_k(x_i)$, where f_k denotes the k -th tree, and x_i is the feature vector of the i -th observation.

The objective function in XGBoost combines a loss function $L(\theta)$, which measures the difference between the predicted values and the actual targets, and a regularization term $\Omega(f)$, which penalizes model complexity to prevent overfitting. The regularization term includes both L1 (Lasso) and L2 (Ridge) penalties, ensuring a balance between bias and variance.

This objective function is then expressed as $\mathcal{O} = \sum_{i=1}^n L(y_i, \hat{y}_i) + \sum_{k=1}^K \Omega(f_k)$ where y_i represents the actual value.

XGBoost optimizes the objective function using a second-order Taylor expansion, where the loss function is approximated by its first and second derivatives with respect to the predictions. For each iteration, the algorithm computes the gradients $g_i = \frac{\partial L(y_i, \hat{y}_i)}{\partial \hat{y}_i}$ and the Hessians $h_i = \frac{\partial^2 L(y_i, \hat{y}_i)}{\partial \hat{y}_i^2}$, which guide the optimization process by determining the direction and step size for updates.

The tree construction process in XGBoost differs from traditional decision tree algorithms such as CART (Classification and Regression Trees) which typically select splits based on simple impurity metrics like Gini index or information gain¹⁶². Instead, XGBoost employs a greedy algorithm that selects the best split based on a gain function. This gain function, calculated as $Gain = \frac{1}{2} \left[\frac{(\sum g_i)^2}{\sum h_i + \lambda} \right] - \gamma$, where λ and γ are regularization parameters, ensures that each split maximizes the reduction in the loss function while penalizing complex trees.

A critical feature of XGBoost is its ability to handle missing values, utilize column subsampling, and incorporate early stopping to prevent overfitting. It also employs a shrinkage factor, or learning rate η , which scales the contribution of each tree, allowing the model to refine its predictions gradually.

XGBoost offers a flexible, data-driven approach that does not rely on pre-specified functional forms or strict assumptions about data distribution. Its iterative boosting mechanism is analogous to refining regression coefficients through successive approximations, while the regularization terms serve as a

¹⁶² Traditional decision tree algorithms like CART select splits using impurity measures such as the Gini index (used in classification tasks to measure how mixed the classes are in a node) or information gain (based on entropy reduction). These methods evaluate splits locally without accounting for regularization or second-order loss curvature, unlike XGBoost's gradient-based gain function.

form of model selection akin to penalized regression techniques. The algorithm's capacity to capture complex, non-linear interactions among features makes it particularly suitable for economic data characterized by volatility and structural breaks, such as commodity prices. This capacity is especially important in metal markets like copper, where prices may reflect short-term expectations, speculation, or event-driven shifts that are not easily captured by cointegration-based models.

2. Model Calibration and The Stability of Structural Signals in Copper Price Dynamics

To maintain consistency with the Error Correction Model (ECM), extensive feature engineering is intentionally avoided. Using the same dataset for both models eases comparability in results, allowing a more straightforward evaluation of their forecasting capabilities without introducing discrepancies from differing feature sets.¹⁶³

To ensure the model's predictive consistency, *cross-validation methods* tailored for time series data are employed. Rolling window cross-validation, with a fixed-size training set and advancing test set, mirrors live forecasting environments by adapting to changes over time. Expanding window cross-validation, which increases the training set size at each iteration, enhances the model's ability to capture long-term trends. Both methods provide stable RMSE values across various timeframes, indicating reliable forecasting performance and generalization capabilities. A detailed explanation of rolling and expanding window cross-validation methods implemented in this study is available upon request.

Hyperparameter Tuning. To improve upon the baseline performance of the XGBoost model, we performed an extensive hyperparameter optimization using GridSearch with 3-fold time series cross-validation. The optimization process tested 2,916 parameter combinations and resulted in a 9.4% reduction in the RMSE—from 402.29 with untuned default settings to 364.32 with the selected configuration¹⁶⁴. This improvement was primarily driven by finer learning rates, deeper trees, and a more balanced regularization setup. This result suggests that, although tuning enhances predictive performance, the model's fundamental advantage lies in its capacity to adapt to changing market conditions without relying on predefined structural assumptions.

Feature Importance Analysis and Stability. In the context of this thesis, interpreting feature importance is also essential for understanding whether the copper price reacts systematically to the same economic

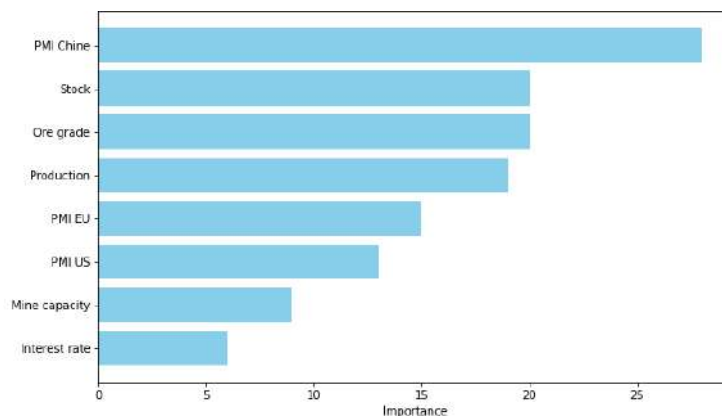
¹⁶³ The absence of a time-dependent component in the XGBoost model is a deliberate choice, complementing the ECM's focus on long-term temporal adjustments. XGBoost's agnostic approach to temporal dependencies captures immediate market shocks and non-linear relationships, while the ECM models structured temporal dynamics. This dual approach balances short-term volatility analysis with long-term trend forecasting.

¹⁶⁴ The tuned parameters include: `learning_rate = 0.03`, `max_depth = 6`, `n_estimators = 300`, `subsample = 0.9`, `colsample_bytree = 0.8`, `lambda = 1`, `alpha = 0.1`. Full details on parameter grids and intermediate scores are available upon request.

fundamentals over time—or whether its drivers shift in ways that suggest market instability, sentiment, or context-dependence. Evaluating feature importance not only offers insights into the economic drivers influencing price dynamics but also facilitates a meaningful comparison with the Error Correction Model (ECM).

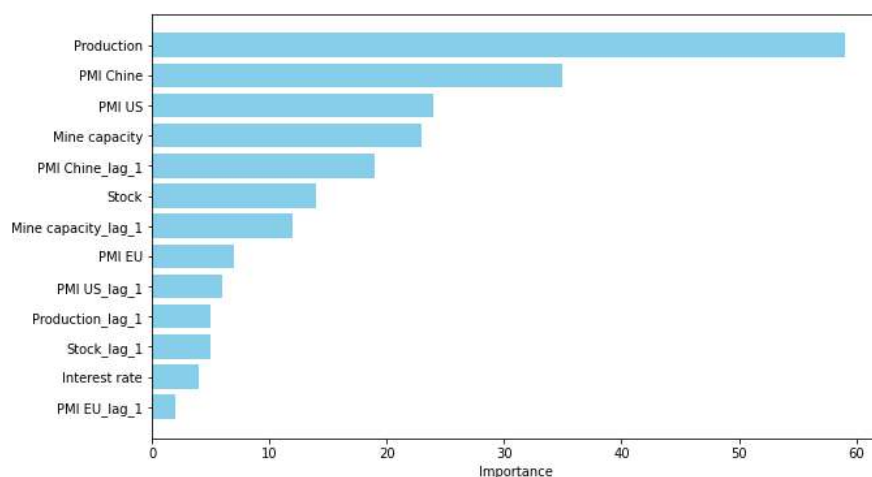
A preliminary computation of feature importance was based on the model optimized using the parameters detailed in the previous section. The results, shown in Figure 5.3, indicate that the PMI Index for China is the most influential feature. Stock levels and ore grades are also key contributors, as inventory levels signal market supply conditions, and ore grades affect production costs. Production levels, PMI indicators for the EU and US, mine capacity, and interest rates further shape market expectations and price movements, though their relative influence varies.

Figure 5.3: Feature Importance of the Exogenous Variables in Forecasting Copper Prices with XGBoost Model



We extend the analysis by incorporating the first lag of each exogenous variable. This allows us to assess whether historical values contribute significantly to current price predictions. The inclusion of lagged variables yields new importance scores, with lagged PMI China and lagged mine capacity emerging as notable contributors, as shown in Figure 5.4.

Figure 5.4: Feature Importance of the Exogenous Variables and Their First Lag

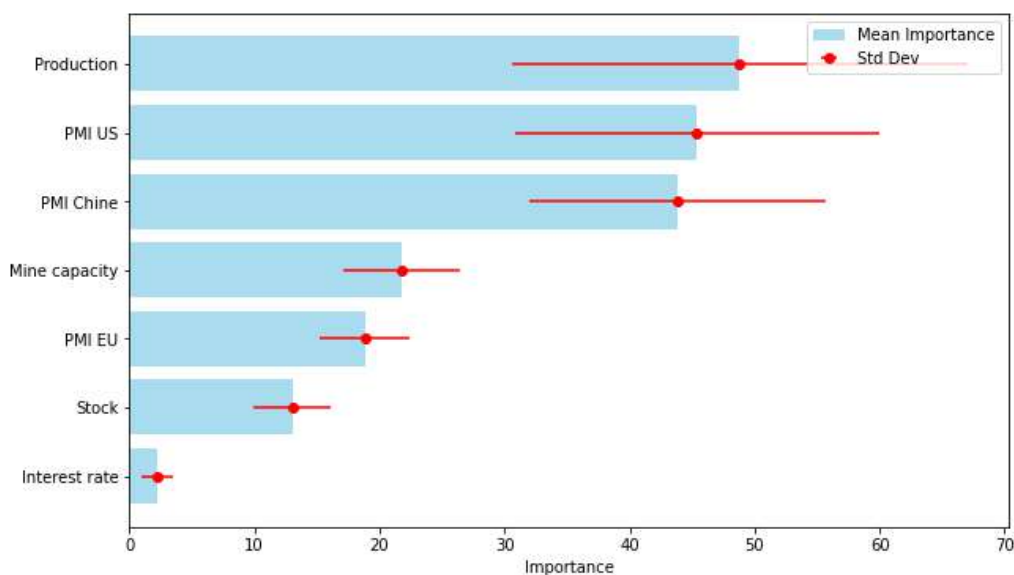


We further refine our analysis by examining feature importance from January 2022 onward. This recent period captures the latest economic conditions and geopolitical developments. As shown in Figure 5.5, this time-windowed analysis reveals a notable shift: production becomes the dominant feature, surpassing PMI China, while ore grade loses significance due to its lack of variation during this period.

This variability raises a critical issue for using machine learning to assess long-term sustainability: if the variables driving price changes fluctuate significantly over time, it may indicate that prices respond more to shifting expectations or noise than to enduring structural fundamentals¹⁶⁵. In contrast to the ECM's fixed equilibrium relationships, XGBoost reveals a more adaptive—yet potentially unstable—mapping between price and fundamentals.

The sensitivity of feature importance in machine learning models like XGBoost arises from their flexible, adaptive structure. Unlike econometric models that impose strict relationships between variables, machine learning algorithms continuously adjust their internal parameters to minimize prediction errors. While this flexibility allows them to capture complex, non-linear interactions, it also means that the relative contribution of features can change dramatically depending on the training data. Consequently, a variable deemed highly influential in one period might become less relevant in another, posing challenges for analysts seeking stable, interpretable insights over time.

Figure 5.5: Feature Importance of the Exogenous Variables from January 2022 to August 2024, Including Standard Deviations



¹⁶⁵ To evaluate the stability of feature importance, we implement a 5-fold time series cross-validation approach. This method ensures that each model iteration is trained on past data and tested on future data, preserving the temporal structure and preventing data leakage. This analysis reveals significant variability in feature importance, particularly for production and mine capacity, whose importance fluctuates across different time intervals. Such instability is a critical challenge in machine learning models, as it complicates the interpretation of economic drivers and reduces the reliability of policy or investment decisions based on these models.

This section highlights the need for rigorous stability assessments in machine learning-based forecasting. While XGBoost performs well in capturing short-term, non-linear patterns, its interpretative reliability depends heavily on the stability of feature importance across different timeframes. The comparison with ECM coefficients, which maintain structural consistency due to their econometric foundations, further emphasizes this point. To enhance the robustness of machine learning models, it is essential to continually evaluate feature importance stability and consider ensemble approaches or hybrid models that balance flexibility with interpretability.

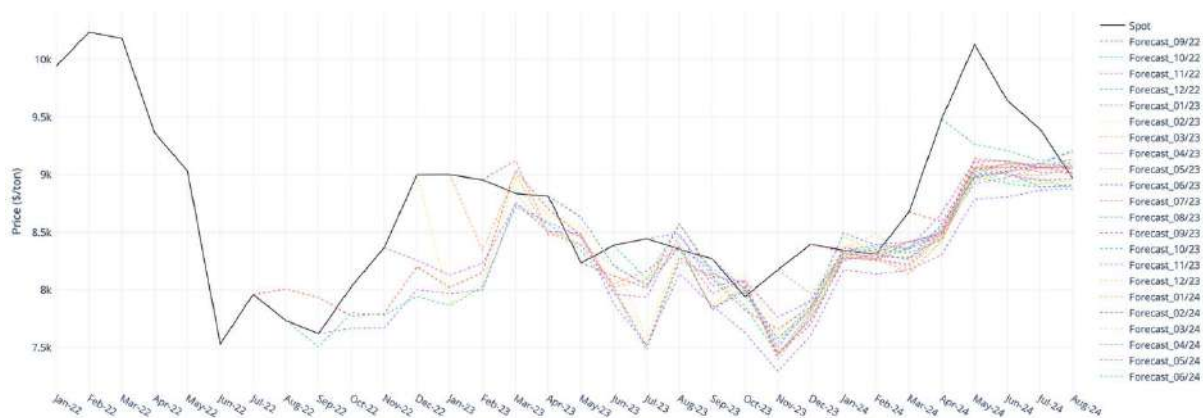
Further computational details, including full parameter settings, cross-validation procedures, and raw feature importance outputs, are available upon request.

3. Forecasting Results Reveal the Reactive and Non-Structural Nature of Copper Price Movements

In this section, we evaluate the forecasting performance of the XGBoost model for copper prices. As for the ECM, the following of the XGBoost model's forecasting performance does not aim to maximize predictive accuracy as an end. Instead, the objective is to evaluate how a non-parametric, machine learning-based model performs when applied to copper price forecasting, particularly in comparison to the structurally parametrized ECM priorly assessed.

Figure 5.6 illustrates the observed copper prices from January 2022 to September 2024, alongside several sets of monthly forecasts generated by the XGBoost model. These forecasts cover the same time range, from September 2022 to June 2024, with a common target horizon of August 2024, and were produced using only the information available at each forecast date, ensuring comparability with the ECM results and replicating the kind of forward-looking uncertainty that market participants face.

Figure 5.6: Actual Copper Prices vs Historical Forecast Iterations (XGBoost)



As in the ECM analysis, the model's performance is assessed primarily at the end of the time window, where forecasts were produced using only the information available at each forecast start date. The categorization of forecast sets follows the same logic: full sets include all forecast points up to August

2024; the 6-month sets consider the first six months of forecasts; the 3-month sets the first three months; and the next-month sets the immediate next month's forecast.

The RMSE results reveal a consistent pattern across horizons. The full sets exhibit an RMSE of 199.17 (33.0% of the copper price standard deviation), with a clear downward trend as the forecast horizon shortens. The 6-month sets yield a slightly higher RMSE of 204.22 (33.8%), while the 3-month sets show an RMSE of 194.92 (32.3%). The next-month forecasts achieve the best performance, with an RMSE of 164.77 (27.3%), indicating the model's capacity to adapt efficiently to recent market conditions.

The relatively stable performance across these different horizons suggests that the model captures underlying patterns in copper price movements with consistent accuracy. The RMSE values for the full, 6-month, 3-month, and next-month sets represent 2.29%, 2.35%, 2.25%, and 1.90%, respectively, of the average copper price of \$8,682 observed during this period.

Lastly, we compute the RMSE for the average models, which are obtained by averaging forecast points within each set. The results are 191.75 (31.8%) for the full sets, 185.11 (30.7%) for the 6-month sets, and 172.01 (28.5%) for the 3-month sets. While modest, these improvements suggest that even in a highly flexible model, copper price dynamics contain a degree of short-horizon noise that averaging can help absorb—highlighting again the trade-off between precision and interpretability.

Overall, the XGBoost model demonstrates good forecasting performance, with a notable advantage in short-term predictions. Further details on RMSE computations and forecast intervals are available upon request.

4. Insights from Comparing the ECM and XGBoost models: Structural Anchoring vs. Reactive Adaptation in Copper Price Dynamics

The comparison of forecast performance between the ECM and the XGBoost model reveals both similarities and key differences, offering insights into the underlying mechanisms driving copper price dynamics. Both models demonstrate satisfactory performance in short-term forecasts, with accuracy decreasing as the forecast horizon extends. However, XGBoost consistently outperforms the ECM across all horizons, particularly for the next-month predictions, where it achieves a notably lower RMSE. This difference likely stems from XGBoost's flexibility in capturing non-linear relationships and interactions among explanatory variables without relying on predefined economic relationships.

The ECM, rooted in economic theory, captures long-term equilibrium relationships and short-term adjustments based on historical interactions between variables. Its comparatively lower performance in short-term predictions suggests that copper prices are heavily influenced by immediate market conditions and external shocks, which are more readily captured by a machine learning model like

XGBoost. In contrast, the relatively consistent performance of the XGBoost model across different horizons indicates that the drivers of copper prices are not solely governed by historical trends or structural relationships but also by dynamic factors that evolve rapidly in response to market events.

This divergence in model performance highlights a broader insight: copper prices appear to embed both structural signals and reactive dynamics. The ECM captures the former but misses the latter; XGBoost captures the latter, but at the cost of structural clarity. The metal market appears to reflect a combination of structural supply-demand fundamentals and shorter-term speculative or event-driven factors. The ECM's reliance on past equilibrium relationships may underestimate the weight of these transient dynamics, while XGBoost, free from theoretical constraints, captures such patterns more effectively.

These findings lead to the next step in our analysis: assessing the forecasting performance of copper futures. The futures market, often regarded as a forward-looking pricing tool, provides a distinct perspective on price expectations by aggregating market participants' anticipations about future supply, demand, and potential shocks. By comparing futures prices with our ECM and XGBoost forecasts, we not only evaluate predictive performance but also continue probing a deeper question: do market prices reflect the kinds of structural fundamentals that would support long-term coordination of resource extraction, consumption, and investment?

VI. Price Predictability and Information Value: Comparing ECM, XGBoost, and Futures

1. The Trade-Off Between Interpretability and Predictive Accuracy

In forecasting copper prices, two fundamental objectives often come into tension: interpretability and predictive accuracy. Interpretability refers to the model's ability to explain the influence of various factors—such as production, stock levels, and macroeconomic indicators—on copper price movements. This quality is crucial for economists and industry stakeholders who seek to understand the mechanisms behind price fluctuations and base their decisions on well-defined economic relationships. Predictive accuracy, in contrast, measures the model's ability to generate reliable forecasts, irrespective of the clarity of its internal workings. High predictive accuracy ensures dependable forward-looking insights, especially relevant for understanding whether price signals reflect underlying supply-demand conditions or are distorted by short-term shocks.

The tension between predictive accuracy and interpretability reflects a broader epistemological divide. Breiman (2001) frames it as the difference between theory-driven and algorithm-driven modeling. Mullainathan and Spiess (2017) highlight how machine learning excels at prediction tasks in high-dimensional settings but falls short of causal inference. Athey (2019) offers a synthesis, suggesting that ML can enhance, rather than replace, structural models—particularly when exploring complex,

nonlinear economic behavior. This chapter adopts a similar stance: by comparing an ECM and XGBoost, we aim not just to forecast copper prices, but to understand the extent to which those prices reflect underlying economic fundamentals.

2. RMSE Comparison and Forecasting Performance for Copper prices (2022-2024)

a. Structural Interpretability and Long-Term Signals: The Strength of the ECM

The ECM offers a transparent, theory-grounded representation of copper price dynamics by modeling long-term relationships between spot prices and macroeconomic indicators. Its coefficients provide direct insights into how variables such as production, mine capacity utilization, and PMI indices affect copper prices over time. The ECM analysis reveals several stable, long-term relationships, with production levels and Chinese PMI exerting the strongest influence on copper prices. The estimated error correction terms indicate that prices revert to their equilibrium level relatively quickly following a supply or demand shock, consistent with the expected behavior of commodity markets. This model's ability to explicitly model these adjustments, combined with its consistency across different sample periods, makes it a valuable tool for assessing whether price dynamics are consistent with the structural equilibria implied by fundamental market forces.

However, this interpretability comes at the cost of reduced predictive accuracy. Over the test period from September 2022 to August 2024, the ECM's RMSE values demonstrate clear limitations. The next-month forecasts, which represent the model's strongest performance, yield an average RMSE of 233.82, corresponding to 38.7% of the copper price's observed standard deviation. For longer horizons, the RMSE increases significantly, exceeding 697.34 for full forecast sets. This performance gap reflects the ECM's reliance on linear relationships and its sensitivity to structural market changes, particularly when demand or supply conditions deviate from historical patterns.

The ECM's limited short-horizon predictive power highlights the potential instability or contextual dependence of structural price relationships—even when cointegration is statistically confirmed. Its coefficients are estimated under the assumption that structural economic relationships remain stable over time. Yet, the copper market's dynamics are increasingly influenced by factors like geopolitical uncertainty, technological changes, and speculative activity—elements that traditional econometric frameworks struggle to accommodate.

b. Predictive Power and Adaptive Learning: The Strength of the XGBoost model

In contrast, the XGBoost model exhibits significantly higher predictive performance across all forecast horizons. Its next-month forecasts yield an RMSE of 164.77, corresponding to only 27.3% of the copper price's standard deviation, outperforming the ECM by over 29%. Similar improvements are

observed across the 3-month, 6-month, and full forecast sets, with RMSE values remaining relatively stable between 194.92 and 204.22. The machine learning model thus delivers more reliable forecasts, regardless of the horizon considered. This improved accuracy stems from XGBoost's capacity to capture non-linear interactions and adaptive market responses. The model's flexible, data-driven approach allows it to respond to shifts in production, demand, and external factors without imposing restrictive theoretical assumptions. By leveraging feature interactions and dynamic patterns in the data, XGBoost succeeds where the ECM struggles to adapt.

However, the interpretability of XGBoost forecasts remains limited. The feature importance analysis conducted in the previous section underscores this issue: the relative importance of variables such as production, PMI indices, and stock levels fluctuates significantly across different time periods. For example, production importance increases substantially when focusing on the period starting in January 2022, while ore grade loses all relevance due to data invariability. Such variations complicate the identification of stable price drivers over time, which may limit the model's ability to inform long-term policy or investment decisions based on structural fundamentals.

c. Comparative Predictive Power: RMSEs Across Models and Times Horizons

A direct comparison of RMSE values for the ECM and XGBoost forecasts reveals a clear advantage for the ML model in predictive accuracy. Across all forecast horizons, the RMSEs of the XGBoost model are between 25% and 30% lower than those of the ECM. The next-month forecasts illustrate this disparity, with the XGBoost RMSE (164.77) significantly outperforming the ECM RMSE (233.82). The 3-month, 6-month, and full sets show similar patterns, albeit with progressively smaller differences as the horizon lengthens.

However, when comparing these models to the predictive power of futures prices, the results become more nuanced. Futures-based predictions exhibit mixed performance depending on market conditions. During high-volatility periods, incorporating 3-month lagged futures prices into an XGBoost model significantly reduces RMSE, suggesting that in uncertain conditions, futures markets may temporarily integrate expectations about fundamental imbalances—but this signal is neither stable nor reliable across regimes. In calm markets, however, the futures-based model performs poorly, providing no meaningful improvement over lagged spot prices alone.

The Granger causality tests further reinforce this volatility-dependent behavior. Futures prices Granger-cause spot prices only at the first lag and only during high-volatility periods. In these contexts, futures appear to serve as forward-looking indicators, potentially reflecting speculative adjustments or expectations of supply-demand imbalances. In low-volatility conditions, however, futures fail to improve forecasting accuracy, suggesting that their predictive power is context-dependent.

d. Economics Insights from Forecasting Performance: Do Prices Reflect Fundamentals or Shifting Expectations?

The ECM's comparatively weak predictive accuracy indicates that the relationships between supply, demand, and prices may be more non-linear and time-varying than classical econometric models assume. The fluctuating feature importance scores in the XGBoost model corroborate this hypothesis, demonstrating that the relative influence of economic indicators changes over time, likely reflecting evolving market dynamics.

The futures market results further underscore this conclusion. Futures prices, expected to provide reliable forward-looking information, only exhibit predictive power in periods of heightened volatility. This supports the hypothesis that copper prices blend structural fundamentals with speculative, expectation-driven dynamics—making it difficult to interpret them as consistent signals of market scarcity or equilibrium. In calm markets, prices adhere to historical patterns and stable economic relationships, making ECM-based forecasts relatively more accurate. In turbulent conditions, speculative forces and evolving expectations dominate, requiring more flexible, machine-learning-based approaches.

The futures market's time-inconsistent predictive performance suggests that market participants should be cautious when relying solely on futures prices as forecasting tools. The futures market for copper seems to serve as a reliable indicator of market expectations only in uncertain contexts, when speculative behavior and risk adjustments come to the fore. In stable periods, futures prices provide little added value beyond the information already embedded in lagged spot prices.

These findings raise important considerations for sustainability assessments: if price signals are unstable and context-dependent, they may not reliably guide long-term decisions around extraction, investment, or policy design in resource markets.

VII. Structural Anchoring and Reactive Signals in Copper Price Formation: Conclusions from Econometric and Machine Learning Models

This chapter has investigated medium-term copper price formation by contrasting a traditional econometric framework—the Error Correction Model (ECM)—with a flexible, data-driven machine learning algorithm (XGBoost), and by examining the predictive signals embedded in copper futures over the 2022-2024 period. From this multi-pronged approach emerge several critical insights that clarify the mechanisms behind copper price dynamics and challenge the idea that futures markets offer the most reliable guide to price expectations.

A key outcome is the persistent outperformance of XGBoost relative to the ECM across most forecast horizons. Because XGBoost operates without structural constraints, it more readily captures short-term fluctuations, nonlinearities, and abrupt shifts, whether driven by macroeconomic changes, speculative adjustments, or short-lived disruptions. In doing so, it highlights that a substantial share of copper price volatility may stem from market behaviors not easily absorbed by theory-based models.

By contrast, the ECM explicitly models long-term cointegrated relationships and short-run corrections that bring prices back toward structural equilibrium. This specification is deeply rooted in resource economics: copper supply adjusts only gradually due to geological, technical, and regulatory lags, while demand reacts to macroeconomic signals such as industrial output and purchasing managers' expectations. The two cointegrating relationships identified in this study confirm that copper prices maintain stable links to variables like world refined production, ore grade, and mine capacity utilization. Yet, the ECM's weaker performance over short horizons suggests that price dynamics frequently deviate from equilibrium in the face of rapid shocks or sentiment-driven episodes.

From an economic perspective, the strength of the ECM lies in its interpretability: its coefficients reflect identifiable, long-term supply and demand fundamentals. In contrast, the machine learning approach demonstrates how real-world market dynamics—especially in short- to medium-term contexts—may be governed by evolving expectations, short-lived imbalances, or nonlinear feedback mechanisms that structural models may overlook.

With respect to futures contracts, our findings advise caution. While futures markets do convey meaningful information during high-volatility episodes, their predictive power is conditional and often weak outside such regimes. This performance variability calls into question the idea that futures prices consistently offer forward-looking guidance. Despite their value as real-time sentiment indicators, copper futures appear less reliable for anticipating medium-term structural shifts. Relying on them in isolation—for budgeting, hedging, or procurement planning—could therefore misrepresent underlying market risks.

Taken together, the divergent results between the ECM and XGBoost provide a more nuanced understanding of copper price formation. The ECM reaffirms that structural fundamentals—production capacity, inventory levels, ore quality, and macroeconomic indicators—still anchor price dynamics in the long run, even if the correction path is sometimes gradual and at other times abrupt. Meanwhile, XGBoost's higher predictive accuracy reveals the market's responsiveness to short-term shocks and nonlinear adjustments, highlighting a persistent tension between structural forces and reactive volatility.

Ultimately, these insights support a broader conclusion: that copper markets—like many commodity markets—operate at the intersection of slow-moving fundamentals and fast-moving events.

Understanding price dynamics thus requires methodological pluralism. Forecasting models grounded in economic theory offer transparency and structural insight; machine learning tools, meanwhile, are better suited to detect emergent patterns and rapid shifts. A comprehensive understanding of price behavior, particularly in the context of long-term sustainability and resource policy, demands the combination of both.

Conclusion

This thesis has sought to advance our understanding of the long-term sustainability of metal markets by focusing on their structural economic dynamics. The guiding question has been whether the interaction of demand growth, depletion constraints, and technological change can give rise to sustainable, non-explosive equilibrium trajectories for prices and quantities. In doing so, the work has highlighted that metals occupy a singular place among commodities: they are finite and costly to extract, yet durable and recyclable, unlike hydrocarbons which are irreversibly consumed, or agricultural goods which are renewable within seasonal cycles. This duality—exhaustibility combined with durability—creates market behaviors that are not easily assimilated to those of other raw materials.

These dynamics point to a distinctive temporality in metals. Primary extraction is slow-moving, tied to long investment cycles and declining ore grades, while secondary supply evolves in a path-dependent manner. Once recycling infrastructures are developed, they tend not to disappear even if prices soften. Markets thus acquire a form of memory: past phases of high investment or policy support leave lasting imprints on future capacity. This hysteresis differentiates metals from hydrocarbons, where supply cycles can be adjusted more flexibly, and from agricultural commodities, where renewal is rapid and reversible. Metal markets are therefore defined as much by history as by current conditions, making path dependence central to their long-run adjustment.

The broader comparison with other commodity markets reinforces this singularity. In oil, scarcity is felt through the definitive consumption of each barrel; in agriculture, renewal occurs naturally through seasonal cycles; in metals, scarcity manifests through depletion of ore quality and rising extraction costs but is partially offset by recycling. Prices in oil are strongly geopolitical, while in agriculture they are often shaped by weather and harvest cycles. Metal prices, by contrast, hinge on the interplay of depletion, technological progress, and the maturation of recycling systems. This makes their equilibria more fragile in the short run, yet potentially more resilient in the long run if circularity gains traction.

A central contribution of this thesis is to show that what ultimately constrains metals is not the exhaustion of physical stocks, but the pace at which primary supply can be mobilized. Depletion here is less a question of running out than of encountering progressively tighter limits on flows: as ore grades decline and costs rise, the capacity of the system to scale up extraction lags behind the growth of demand. This is why, across scenarios, cumulative production converges toward similar long-term levels, not because the earth's crust is emptying, but because geological and technical frictions place a ceiling on the flow of new supply. This structural insight is general and applies across metals: geology and costs ultimately set the outer boundary of supply. What varies, however, is the path by which

markets approach this ceiling. In the case of copper, recycling alters the adjustment process, moderating price pressures, reducing unmet needs, and tempering the sensitivity of outcomes to demand elasticity. These findings illustrate more broadly how secondary supply can reshape market dynamics, while also reminding us that the magnitude of its influence differs markedly across metals depending on their recyclability, co-production characteristics, and infrastructure maturity.

However, metals as a class are heterogeneous, and this heterogeneity is itself central to sustainability. Bulk industrial metals such as copper, aluminum, or zinc are characterized by large, globally integrated markets where recycling plays a stabilizing role. By contrast, many critical or specialty metals—cobalt, rare earths, indium—are produced as by-products, their supply determined by the economics of primary host metals, not their own price signals. In such cases, the CAC framework is less effective, and the stabilizing influence of recycling remains limited. This diversity in market structures underscores why metals cannot be treated as a homogeneous category. Their economic behaviors depend on whether they are primary or co-product, abundant or scarce, easily recycled or difficult to recover. This typology is not only descriptive but structural: it determines which metals can realistically moderate scarcity through recycling and which remain intrinsically vulnerable to volatility and bottlenecks.

Additionally, it is important to recognize the simplifications of the present framework. The model is deterministic, excluding stochastic shocks such as wars, financial crises, or abrupt technological breakthroughs. It does not account for substitution across metals, even though aluminum may replace copper in some uses, or lithium chemistries may shift within batteries. Extraction technologies are assumed to progress at a steady rate, whereas breakthroughs are often discontinuous. Recycling is modeled at the global scale, without accounting for regional scrap flows, trade restrictions, or geopolitical frictions. These caveats remind us that the results are not forecasts but structured simulations, designed to reveal underlying mechanisms rather than predict precise outcomes.

The empirical chapters partially address this gap by examining how copper prices behave in real markets subject to frictions and shocks. They show that while prices do reflect elements of the underlying scarcity conditions, they are also shaped by short-term volatility, speculative behavior, and nonlinear feedbacks. Futures markets, econometric models, and machine learning techniques each capture part of this dynamic, but none fully reconcile the structural signal of depletion with the transient noise of market activity. This reinforces the central insight derived from the analytical model, that sustainability in resource markets requires that prices transmit scarcity constraints, yet in practice this signal is often obscured by disruptions and feedbacks that lie outside the scope of deterministic modeling.

Taken together, the essays in this thesis argue that the sustainability of metal markets rests on a paradox. On the one hand, depletion imposes hard constraints: extraction cannot expand indefinitely, and long-term cumulative production is capped by geological and technical availability. On the other hand, the durability and recyclability of metals prevent this constraint from translating into absolute scarcity. The addition of secondary supply to primary extraction reshapes adjustment paths: it smooths price trajectories, reduces unmet needs, and buffers markets against volatility. Sustainability lies in the balance between geology and circularity: between finite deposits and the gradual build-up of secondary flows.

This balance is not self-evident. It reflects the interplay of market prices, technological change, policy choices, and the slow accumulation of infrastructure. These mechanisms together condition the ability of metals to accommodate the structural shifts of the energy and digital transitions. Metals differ from other commodities in this respect: finite yet not destroyed in use; recyclable, though subject to technological, economic, and institutional frictions; global, yet with supply dynamics that remain strongly path dependent. Understanding this singular configuration of depletion at the frontier, and circularity in the core is key to assessing the long-term economic sustainability of metal markets.

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RÉSUMÉ

Cette thèse étudie la soutenabilité au sens économique des marchés de métaux à l'horizon de la transition bas carbone. Elle combine un modèle analytique de long terme et des applications empiriques sur le cuivre, pour évaluer si les interactions entre une demande croissante et des contraintes d'offre permettent des trajectoires de prix non explosives. Le chapitre 1 rappelle la littérature sur les ressources épuisables et notamment les métaux, de Hotelling (1931) jusqu'aux approches les plus récentes. Notre modèle théorique (chapitre 2) articule une courbe de disponibilité cumulative (*Cumulative Availability Curve* - CAC) avec une fonction de demande CES pour établir les conditions théoriques spécifiques de soutenabilité des marchés des métaux. Appliqué au cuivre (chapitre 3), le modèle montre que les contraintes d'offre compromettent la faisabilité des trajectoires de transition énergétique avant même 2050 : la rareté économique précède la rareté physique, du moins en l'absence de progrès technologique. L'intégration du recyclage dans le modèle (chapitre 4) montre que l'offre secondaire amortit les tensions sur les prix et limite la destruction de demande, mais ne compense pas la croissance structurelle de la consommation : elle joue un rôle de « tampon » plutôt que de substitution. Enfin, l'analyse empirique (chapitre 5) confirme les limites des contrats *futures* comme outil prospectif et propose deux autres modèles (ECM et XGBoost) qui suggèrent que les prix du cuivre réagissent plus aux anticipations qu'à l'équilibre offre-demande immédiat. La thèse conclut que la soutenabilité des marchés des métaux ne réside pas dans un épuisement physique imminent mais dans des tensions économiques croissantes, exprimées par les prix mais modulées sensiblement par l'innovation technologique et la régulation publique.

MOTS CLÉS

Soutenabilité économique ; Marchés des métaux ; Transition bas carbone ; Rareté ; Recyclage ; Prix ; Anticipations

ABSTRACT

This dissertation examines the economic sustainability of metal markets in the context of the low-carbon transition. It combines a long-term analytical model with empirical applications to copper, assessing whether interactions between rising demand and supply constraints allow for non-explosive price trajectories. Chapter 1 reviews the literature on exhaustible resources — particularly metals — from Hotelling (1931) to the most recent approaches. Our theoretical model (Chapter 2) links a Cumulative Availability Curve (CAC) with a CES demand function, establishing the specific theoretical conditions under which metal markets can be considered sustainable. When applied to copper (Chapter 3), the model shows that supply constraints threaten the feasibility of energy transition pathways even before 2050: economic scarcity precedes physical scarcity, at least in the absence of technological progress. The integration of recycling (Chapter 4) demonstrates that secondary supply mitigates price pressures and limits demand destruction but cannot offset the structural growth of consumption — acting more as a buffer than a substitute. Finally, the empirical analysis (Chapter 5) highlights the limitations of futures contracts as forward-looking tools and proposes two alternative models (ECM and XGBoost) suggesting that copper prices respond more to expectations than to immediate supply-demand balance. The dissertation concludes that the sustainability of metal markets does not hinge on imminent physical depletion, but rather on rising economic tensions — reflected in prices yet strongly mediated by technological innovation and public regulation.

KEYWORDS

Economic sustainability ; Metal markets ; Low-carbon transition ; Scarcity ; Recycling ; Prices ; Expectations