



**HAL**  
open science

# New structure learning algorithms and evaluation methods for large dynamic Bayesian networks

Ghada Trabelsi

► **To cite this version:**

Ghada Trabelsi. New structure learning algorithms and evaluation methods for large dynamic Bayesian networks. Machine Learning [cs.LG]. Université de Nantes; Ecole Nationale d'Ingénieurs de Sfax, 2013. English. NNT: . tel-00996061

**HAL Id: tel-00996061**

**<https://theses.hal.science/tel-00996061v1>**

Submitted on 26 May 2014

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Thèse de Doctorat

## Ghada TRABELSI

*Mémoire présenté en vue de l'obtention du  
grade de Docteur de l'Université de Nantes  
sous le label de l'Université de Nantes Angers Le Mans*

**Discipline : Informatique**

**Laboratoire : Laboratoire d'informatique de Nantes-Atlantique (LINA)**

**REsearch Group on Intelligent Machines (REGIM)**

**Ecole Nationale d'ingénieurs de Sfax, university of Sfax**

**Soutenue le 13 décembre 2013**

**École doctorale : 503 (STIM)**

**Thèse n° :**

## New structure learning algorithms and evaluation methods for large dynamic Bayesian networks

### JURY

- Rapporteurs : **M<sup>me</sup> Nahla BEN AMOR**, Professeur des Universités, University of Tunis  
**M. Ioannis TSAMARDINOS**, Professeur des universités, University of Crete
- Examineurs : **M. Marc GELGON**, Professeur des Universités, University of Nantes  
**M. Sylvain PIECHOWIAK**, Professeur des Universités, University of Valenciennes et Hainaut-Cambrésis
- Invité : **M. Mounir BEN AYED**, Maître Assistant, University of Sfax
- Directeur de thèse : **M. Philippe LERAY**, Professeur des universités, University of Nantes
- Co-directeur de thèse : **M. Adel. M. ALIMI**, Professeur des Universités, University of Sfax

Declaration of originality: I hereby declare that this thesis is based on my own work and has not been submitted for any other degree or professional qualification, except when otherwise stated. Much of the work presented here has appeared in previously published conference and workshop papers. The chapters discussing the Benchmarking dynamic Bayesian Networks structure learning, DMMHC approach and evaluating its performance on real data are based on [100, 101]. Where reference is made to the works of others, the extent to which that work has been used is indicated and duly acknowledged in the text and bibliography.

Ghada TRABELSI BEN SAID (November, 2013)

# DEDICATION

There are a number of people without whom this thesis might not have been written, and to whom I am greatly indebted.

I would first like to express my gratitude to all members of the jury, Sylvain PIECHOWIAK, Marc GELGON, Nahla BEN AMOR, Ioannis TSAMARDINOS who consented to assess my thesis document.

I would like to thank my supervisor, Pr. Adel M. AIIMI, for his guidance and support throughout this study. as well as my co-supervisor Dr. Mounir BEN AYED who believed in me and provided me with beneficial comments and questions that were vital for the completion of the thesis. I owe him a lot.

I have been very fortunate for having Pr. Philippe LERAY as my research advisor. Many thanks to Pr. Leray for introducing me to an interesting research area and inviting me to work in France, where my thesis was partially achieved. I would like to express my heartfelt gratefulness for his guidance and support. I am really lucky to learn from the best. My collaboration with him has helped me grow not only as a researcher but also as an individual.

I am indebted to Dr. Nahla BEN AMOR and Dr. Pierre-Henri WUILLEMIN who kindly accepted to have interesting discussions with me as thesis monitoring committee.

Special acknowledgement to Dr. Amanullah YASIN, Dr. Hela LTIFI and PHD. student Mouna BEN ISHAK for their generous support and help during the programming and writing phases.

I would like to thank my parents deeply as they have supported me all the way since the beginning of my studies. They have been understanding, infinitely loving and provided me with their prayers that have been my greatest strength all these years.

This thesis is dedicated also to my dear husband who has been a great source of motivation and inspiration for me and I thank him for giving me new dreams to pursue.

Finally, this thesis is dedicated to my sisters, friends, and colleagues for their love, endless support and encouragement. It is dedicated to all those who believe in the richness of learning.

# Abbreviations and Acronyms

AIC	Akaike Information Criterion
BIC	Bayesian Information Criterion
BD	Bayesian Dirichlet
BDe	Bayesian Dirichlet Equivalent
BN	Bayesian Network
CPC	set of Parents Children candidates
$CPC_t$	set of Parents Children candidates in t time slice
$CP_{t-1}$	set of Parent candidates in t-1 time slice
CPDAG	Complete Partially Directed Acyclic Graph
$CC_{t+1}$	set of Children candidates in t+1 time slice
DAG	Directed Acyclic Graph
DBN	Dynamic Bayesian Network
DMMHC	Dynamic Max Min Hill Climbing
DMMPC	Dynamic Max Min Parents Children
DSS	Decision support System
EM	Expectation Maximization
GS	Greedy Search
$G_0$	Initial network
$G_{\rightarrow}$	Transition network
HBN	Hierarchical Bayesian Network
IC, PC	Inductive/Predictive Causation
KDD	knowledge Data Discovery
KL-divergence	Kullback-Leibler divergence
MB	Markov Blanket
MCMC	Monte Carlo Markov Chain
MDL	Minimum Description Length
MIT	Mutual Information test
MMH	Max Min Heuristic
MMHC	Max Min Hill Climbing
MMPC	Max Min Parents Children

MWST	Maximum Weight Spanning Tree
$Ne_0$	set of neighborhood in time slice 0
$Ne_+$	set of neighborhood in time slice $t \geq 1$
PDAG	Partially Directed Acyclic Graph
$PDAG_k$	Partially Directed Acyclic Graph with knowledge
SA	Simulated Annuling
SHD	Structural hamming Distance
SHD(2-TBN)	Structural hamming Distance for 2-TBN
SGS	Spirtes, Glymour & Scheines
RB	Reseau Bayésien
RBD	Reseau Bayésien Dynamique
2-TBN	two-slices Temporal Bayes Net

# Résumé

La plupart des techniques de fouille des données utilisent des algorithmes qui fonctionnent sur des données fixes non temporelles. Cela est dû à la difficulté de la construction de ces modèles au niveau de l'exploitation des données temporelles (que certains qualifient comme complexes) qui sont délicates à manipuler. Ainsi peu des travaux proposent des algorithmes standards pour l'extraction des connaissances à partir de données multidimensionnelles et temporelles contrairement aux données statiques.

Dans plusieurs domaines tels que celui de la santé les données concernant un individu sont saisies à des moments différents d'une manière plus ou moins périodiques. Ainsi, dans ce domaine le temps est un paramètre important. Nous avons cherché à simplifier cette dimension (le temps) en utilisant un ensemble de techniques complémentaires de fouilles des données.

Dans ce contexte, la présente thèse présente deux contributions : l'extraction des modèles de connaissances en utilisant les réseaux bayésiens dynamiques (RBD) comme étant une technique de fouille de données et l'évaluation de ces modèles temporels (dynamiques).

L'apprentissage de structure dans les Réseaux Bayésiens (RB) est un problème d'optimisation combinatoire. En ce qui concerne la dépendance temporelle, les réseaux bayésiens sont changés par les réseaux bayésiens dynamiques. L'intégration du terme " temporel " rend l'apprentissage de structure dans les RBD plus compliqué. Nous sommes ainsi obligés de chercher l'algorithme le plus adéquat et le plus rapide pour résoudre ce problème.

Dans les développements récents, les algorithmes d'apprentissage de structure dans les modèles graphiques temporels basés sur les scores sont utilisés par plusieurs d'équipes de chercheurs dans le monde entier. Néanmoins, il existe quelques algorithmes d'apprentissage de structure des RB qui incorporent la méthode de recherche locale qui permet un meilleur passage à l'échelle. Cette méthode de recherche locale combine les algorithmes d'apprentissage basés sur les scores et ceux basés sur les contraintes. Cette méthode hybride, peut être utile pour être appliquée sur les réseaux bayésiens dynamiques pour l'apprentissage de structure. Nous avons pu par cette méthode de rendre l'apprentissage de structure dans les RBD plus pratique et diminuer l'espace de recherche de solutions dans les RBD qui est qualifié par



sa complexité à cause de la temporalité. Nous avons aussi proposé une approche pour l'évaluation des modèles graphiques temporels (construits lors de l'étape de l'apprentissage de structure).

L'apprentissage de structure pour les RB statiques est bien étudié par plusieurs groupes de recherches. Nombreuses approches concernant ce sujet sont proposées et l'évaluation de ses algorithmes se fait par l'utilisation des différents standards Benchmarks et mesures d'évaluations.

A notre connaissance, toutes les études qui s'intéressent à l'apprentissage de structure pour les RBDs utilisent ses propres réseaux, techniques d'évaluations et indicateurs de comparaison. Ainsi l'accès aux bases de données et aux codes sources de ces algorithmes n'est pas toujours possible.

Pour résoudre ce problème, nous avons proposé une nouvelle approche pour la génération des grands benchmarks standards pour les RBD. Cette approche se base sur le Tiling avec une métrique permettant d'évaluer la performance des algorithmes d'apprentissage de structure pour les RBDs de type 2-TBN.

Nous avons obtenu des résultats intéressants. Le premier fournit un outil de génération des larges benchmarks pour l'évaluation des réseaux bayésiens dynamique. Le deuxième, consiste à une nouvelle mesure pour évaluer la performance des algorithmes d'apprentissage de structure. Comme troisième résultat, nous avons montré dans la partie expérimentale que les algorithmes DMMHC développés sont plus performants que les autres algorithmes d'apprentissage de structure pour les RBDs existants.

# Abstract

Most of data mining techniques use algorithms that work on fixed non-temporal data. This is due to the difficulty of the construction of these models in exploiting temporal data (described as complex) which are difficult to handle. A small number of works that proposed algorithms to extract knowledge from temporal and multi-dimensional data differently from the static ones.

In many fields such as health, data about an individual are captured at different times on a more or less regular basis. Thus, in this field, time is an important parameter. Using a set of complementary data mining techniques, we tried to solve this persisting problem.

In this context, this thesis presents two fundamental contributions: the extraction of knowledge models using dynamic bayesian networks (DBNs) as a data mining technique and techniques for the evaluation of these temporal (dynamic) models.

The structure learning in the bayesian networks (BNs) is a combinatorial optimization problem. Regarding the time dependence, the Bayesian networks are changed by Dynamic Bayesian Networks (DBNs). The inclusion of the term "Time" makes the learning structure more complicated. We are thus obliged to seek for the most appropriate algorithm and the fastest way to solve this problem.

In recent developments, the structure learning algorithms in temporal graphical models based on the scores are used by many research teams around the world. However, there are some BNs structure learning algorithms that incorporate local search methods that provide a better scalability. This local search method is hybrid, i.e it combines the learning algorithms based on the scores and the learning algorithms based on constraints. This method can be useful to be applied to the Dynamic Bayesian Networks for learning structure. Relying on this method we tried to make the learning structure of DBNs more convenient and reduce its complexity caused by temporality.

The other problem that arises is the evaluation of these temporal graphical models (built by the structure learning step). The structure learning for static BN has been well studied by several research groups. Many approaches have been proposed on this topic. The evaluation of their algorithms is done by the use of different standards Benchmarks and evaluation measures.

As far as we know, all studies on DBNs structure learning use their own net-

works and indicators of comparison. In addition, the access to the datasets and to the source code is not always possible. To solve this problem, we proposed a new approach for the generation of large standard benchmarks for DBNs which is based on the Tiling with a metric used to evaluate the performance of structure learning algorithms for DBNs (2-TBN).

We obtained interesting results, first we provided a tool for benchmarking dynamic Bayesian network structure learning algorithms. Second, we proposed a novel metric for evaluating the structure learning algorithms performance. As a third result, we showed in the experimental results that the DMMHC algorithms achieve better performance than the other existing structure learning for DBNs.

# Contents

<b>1</b>	<b>Introduction</b>	<b>19</b>
1.1	Research context . . . . .	20
1.2	Thesis overview . . . . .	22
1.3	Publications . . . . .	23
<b>I</b>	<b>State of the art</b>	<b>25</b>
<b>2</b>	<b>Bayesian Network and structure learning</b>	<b>27</b>
2.1	Introduction . . . . .	28
2.2	Bayesian Network definition . . . . .	28
2.2.1	Description example [75] . . . . .	28
2.2.2	Basic concepts . . . . .	30
2.2.3	Bayesian Network definition . . . . .	32
2.2.4	Markov equivalence classes for directed Acyclic Graphs . . . . .	33
2.3	Bayesian network structure learning . . . . .	35
2.3.1	Constraint-Based Learning . . . . .	36
2.3.2	Score-Based Learning . . . . .	37
2.3.3	Hybrid method or local search . . . . .	39
2.3.4	Learning with large number of variables . . . . .	41
2.4	Evaluation of Bayesian Network structure learning algorithms . . . . .	42
2.4.1	Evaluation metrics . . . . .	42
2.4.2	Generating large Benchmarks . . . . .	45
2.5	Conclusion . . . . .	46
<b>3</b>	<b>Dynamic Bayesian Networks and structure learning</b>	<b>49</b>
3.1	Introduction . . . . .	50
3.2	Dynamic Bayesian Network Definition . . . . .	50
3.2.1	Representation . . . . .	50
3.2.2	Different forms of K-TBN . . . . .	51
3.3	Dynamic Bayesian Network structure learning . . . . .	55
3.3.1	Principle . . . . .	55

3.3.2	Structure learning approaches for 2-TBN and k-TBN models	55
3.3.3	Structure learning approaches for simplified "k-TBN and 2-TBN" models	56
3.3.4	Structure learning approaches for other forms of DBN models	57
3.4	Evaluation of DBN structure learning algorithms	58
3.4.1	Benchmarks	58
3.4.2	Evaluation metric	59
3.5	Conclusion	60
<b>II</b>	<b>Contributions</b>	<b>63</b>
<b>4</b>	<b>Benchmarking dynamic Bayesian networks</b>	<b>65</b>
4.1	Introduction	66
4.2	Generation of large k-TBN and simplified k-TBN	67
4.2.1	Principle	67
4.2.2	Tiling the initial model	67
4.2.3	Tiling the transition model	68
4.3	Evaluation of k-TBN generated by data and prior knowledge	69
4.3.1	Principle	69
4.4	Validation examples	72
4.4.1	2-TBN Benchmark generation	72
4.4.2	SHD for 2-TBN	74
4.5	Conclusion	75
<b>5</b>	<b>Dynamic Max Min Hill Climbing (DMMHC)</b>	<b>77</b>
5.1	Introduction	79
5.2	Dynamic Max Min Parents and children DMMPC	79
5.2.1	Naïve $\overline{DMMPC}$ (neighborhood identification)	80
5.2.2	Optimized $\overline{DMMPC}$ (Neighborhood identification)	81
5.2.3	Toy example (naive $\overline{DMMPC}$ vs optimised $\overline{DMMPC}$ )	83
5.2.4	Symmetrical correction	84
5.3	Dynamic Max Min Hill-Climbing DMMHC	85
5.4	DMMHC for simplified 2-TBN	87
5.5	Time complexity of the algorithms	88
5.6	Related work	90
5.7	Conclusion	90
<b>6</b>	<b>Experimental study</b>	<b>95</b>
6.1	Introduction	96
6.2	Experimental protocol	96

<i>CONTENTS</i>	13
6.2.1 Algorithms . . . . .	96
6.2.2 Benchmarks . . . . .	97
6.2.3 Performance indicators . . . . .	97
6.3 Empirical results and interpretations on complete search space . . .	98
6.3.1 Initial validation . . . . .	98
6.3.2 DMMHC versus dynamic GS and SA . . . . .	99
6.3.3 Naive versus optimized DMMHC . . . . .	102
6.4 Empirical results and interpretations on simplified search space . . .	104
6.4.1 Simplified 2-TBN with small benchmark . . . . .	104
6.4.2 Simplified 2-TBN with large benchmark . . . . .	107
6.5 Comparison between all DMMHC versions . . . . .	108
6.6 Conclusion . . . . .	110
<b>7 Conclusion</b>	<b>111</b>
7.1 Summary . . . . .	112
7.2 Applications . . . . .	113
7.3 Limitations . . . . .	114
7.4 Issues for future researches . . . . .	115
<b>A Bayesian Network and Parameter learning</b>	<b>117</b>
A.1 From completed data . . . . .	117
A.1.1 Statistical Learning . . . . .	117
A.1.2 Bayesian Learning . . . . .	118
A.2 From incomplete data . . . . .	118
A.2.1 Nature of the missing data and their treatment . . . . .	118
A.2.2 EM algorithm . . . . .	119
<b>B Benchmarking the dynamic bayesian network</b>	<b>121</b>
B.1 BNtiling for 2-TBN implementation . . . . .	121
B.2 Generating large Dynamic Bayesian Networks . . . . .	122
<b>C DMMHC: Dynamic Max Min Hill Climbing</b>	<b>123</b>
C.1 Proofs of propositions used by DMMHC . . . . .	123
C.1.1 Proof proposition 5.2.2: . . . . .	123
C.1.2 Proof proposition 5.4.1: . . . . .	125



# List of Tables

2.1	The values of the features . . . . .	30
4.1	Generated benchmarks for Dynamic Bayesian Network . . . . .	73
6.1	Characteristics of 2T-BNs ( $G_0$ , $G_{\rightarrow}$ ) generated from 6 usual static BNs (Asia, Alarm, Hailfinder, Win95pts, Andes, Link) . . . . .	97
6.2	SHD comparison with existing static structure learning approaches ([34],[22]), for Alarm, Hailfinder and Link benchmarks with 5000 samples. . . . .	98
6.3	Average±standard deviation of SHD obtained by GS and DMMHC in $G_0$ . . . . .	99
6.4	Average±standard deviation of SHD obtained by GS and DMMHC in $G_{\rightarrow}$ . . . . .	100
6.5	Average±standard deviation running time for DMMHC and GS in $G_0$	100
6.6	Average±standard deviation running time for DMMHC and GS in $G_{\rightarrow}$	100
6.7	Average±standard deviation of SHD obtained by SA and DMMHC in $G_{\rightarrow}$ . . . . .	101
6.8	Average±standard deviation SHD for naive and optimised DMMHC in $G_{\rightarrow}$ . . . . .	103
6.9	Average±standard deviation running time for naive and optimised DMMHC in $G_{\rightarrow}$ . . . . .	103

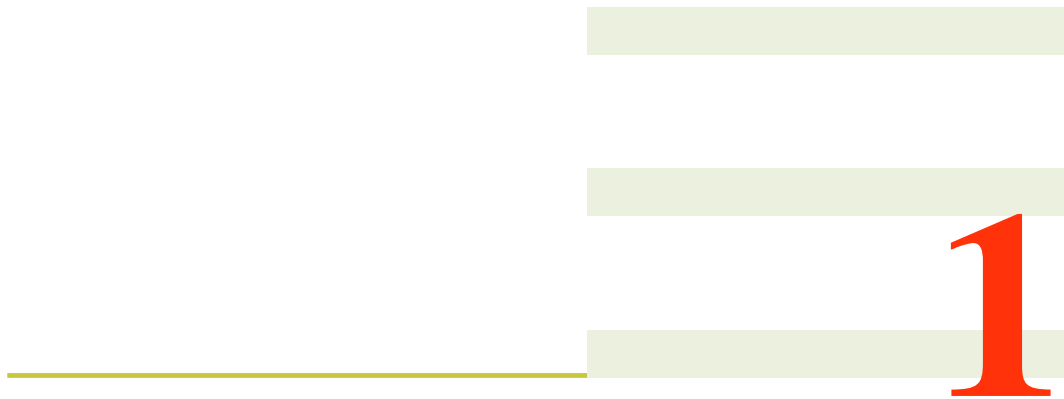




# List of Figures

1.1	Thesis overview and interdependencies between chapters . . . . .	22
2.1	The different concepts presented in chapter 2 . . . . .	29
2.2	Example of Bayesian Network . . . . .	30
2.3	Patterns for paths through a node . . . . .	32
2.4	(a) Example of DAG, (b) the skeleton relative to the DAG and (c) an example of a PDAG [70] . . . . .	34
2.5	Orientation rules for patterns [69] . . . . .	35
3.1	A structure of 2-TBN Bayesian network . . . . .	51
3.2	An example of 2-TBN Bayesian network [110] . . . . .	52
3.3	An example of k-TBN visualization of a second-order DBN (k=3) using t = 3 time-slices . . . . .	53
3.4	Simplified k-TBN . . . . .	54
4.1	An example output of the initial model generation algorithm. The generating network is ASIA benchmark [59] shown in the top left of the figure. . . . .	68
4.2	An example output of the 2-TBN generation algorithm. The generating network is ASIA benchmark [59] shown in the top left of figure a). a) The output of the initial network consists of three tiles of Asia with the addition of several intraconnecting edges shown with the dashed edges. b) The transition network output together with several added interconnecting edges are shown with the dashed red edges. . . . .	73
4.3	Examples of structural Hamming distance with or without temporal correction. A first 2-TBN and its corresponding PDAG and corrected PDAG_k are shown in (a). (b) and (c) show another 2-TBN and their corresponding PDAG, and the structural Hamming distance with the first model . . . . .	75
5.1	Conditional dependence. (a) divergence case. (b) convergence case . . . . .	82
5.2	The extended Umbrella DBN . . . . .	83
5.3	Example trace of naive $\overline{DMMPC}$ . . . . .	85

5.4	Example trace of optimised $\overline{DMMPC}$ . . . . .	86
5.5	Example trace of simplified $\overline{DMMPC}$ . . . . .	88
6.1	Average SHD vs running time obtained by GS and DMMHC with respect to sample size in initial graph . . . . .	101
6.2	Average SHD(DMMHC) vs SHD(GS) with respect to sample size in transition graph . . . . .	101
6.3	Average SHD(DMMHC) vs SHD(SA) with respect to sample size in transition graph . . . . .	102
6.4	Average SHD vs running time obtained by naive and optimised DMMHC with respect to sample size in transition graph . . . . .	104
6.5	Network reconstruction results for GS algorithm of Banjo tool. The used benchmark is the biological dataset 20 variables and 2000 observations . . . . .	105
6.6	Network reconstruction results for simplified 2-TBN DMMHC. The used benchmark is the biological dataset 20 variables and 2000 observations. . . . .	106
6.7	Network reconstruction results for MIT-based global, MIT-based MMB, Banjo [110] and DMMHC from Cyanothecce genetic benchmarks . . . . .	108
6.8	Network reconstruction results . . . . .	109
6.9	Complexity of all DMMHC versions . . . . .	110
A.1	Example of BN structure . . . . .	118



# Introduction

## Sommaire

---

- 1.1 Research context . . . . . 20
- 1.2 Thesis overview . . . . . 22
- 1.3 Publications . . . . . 23

## 1.1 Research context

Time is an important factor in several domains as medicine, finance and industry. It is one of the important problems in machine learning. Machine Learning is the study of methods for programming computers to learn. Computers are used to perform a wide range of tasks, and for most of these it is relatively easy for programmers to design and implement the necessary software. However, there are many tasks for which this is difficult or impossible. These can be divided into four general categories.

- Tasks for which there exist no human experts.
- Tasks where human experts exist, but where they are unable to explain their expertise.
- Tasks with rapidly-changing phenomena.
- Tasks where applications that need to be customized for each user separately.

Machine learning addresses many of the same research questions as the fields of statistics, data mining, but with differences of emphasis. Statistics focuses on understanding the phenomena that have generated the data, often with the goal of testing different hypotheses about those phenomena. Data mining seeks to find patterns in the data that are understandable by people [26].

Most of studies use data mining techniques for the training of fixed data and the extraction of the static knowledge models. Among these models we find the probabilistic graphical model, commonly used in probability theory, statistics, Bayesian statistics and machine learning.

*Graphical models are a marriage between probability theory and graph theory. They provide a natural tool for dealing with two problems that occur throughout applied mathematics and engineering, uncertainty and complexity, and in particular they are playing an increasingly important role in the design and analysis of machine learning algorithms. Fundamental to the idea of a graphical model is the notion of modularity, a complex system is built by combining simpler parts. Probability theory provides the glue whereby the parts are combined, ensuring that the system as a whole is consistent, and providing ways to interface models to data. The graph theoretic side of graphical models provides both an intuitively appealing interface by which humans can model highly-interacting sets of variables as well as a data structure that lends itself naturally to the design of efficient general-purpose algorithms [32].*

Bayesian networks [78, 48, 52] are a kind of the probabilistic graphical models. They are frequently used in the field of Knowledge from Data Discovery (KDD). A Bayesian network is a directed acyclic graph whose nodes represent variables and directed arcs denote statistical dependence relations between variables, and a probability distribution is specified over these variables. Dynamic systems model-

ing has resulted an extension of the BN called Dynamic Bayesian networks (DBN) [23, 73]. In order to exploit relational models from time-series data (called discovery task), it is natural to use a dynamic Bayesian network (DBN). DBN is a directed graphical model whose nodes are across different time slices, to learn and reason dynamic systems. DBNs model a temporal process using a conditional probability distribution for each node and a probabilistic transition between time slices [113].

Learning a BN from observational data is an important problem that has been studied extensively during the last decade. The construction of these models can be achieved either by using expertise, or from data. Many studies have been conducted on this topic, leading to different approaches:

- Methods for the discovery of conditional independence in the data to reconstruct the graph
- Methods for optimizing an objective function called score
- Methods for the discovery of local structure around a target variable to reconstruct the global structure of the network

Most of the studies have been limited to the structure learning of Bayesian networks in the static cases. There are a few algorithms for DBN structure learning [35, 50]. Most of these algorithms use a conventional method based on scores. Indeed, the addition of the time dimension further complicated the search space of solutions.

In the remainder of this thesis, we focus on the interest of hybrid approaches (the local search identification and global optimisation) in the dynamic case, which could more easily take into account the temporal dimension of our models.

Moreover, we noticed that the static BN structure learning is a well-studied domain. Many approaches have been proposed and the quality of these algorithms has been studied over a range of different standard networks and methods of evaluation. To our knowledge, all studies about DBN structure learning use their own benchmarks and techniques for evaluation. The problem in the dynamic case is that we don't find previous works that provide details about used networks and indicators of comparison. In addition, access to the datasets and the source code is not always possible. In this thesis, we also focus on solving the evaluation problem in dynamic case.

Our contribution consists to answer these issues:

1. Can we prove that the hybrid learning allows scaling in the dynamic case?
2. How can we evaluate this approach to justify the efficiency and scalability of our approach?

## 1.2 Thesis overview

The structure of the thesis is organized around three intertwined topics, see Figure 1.1.

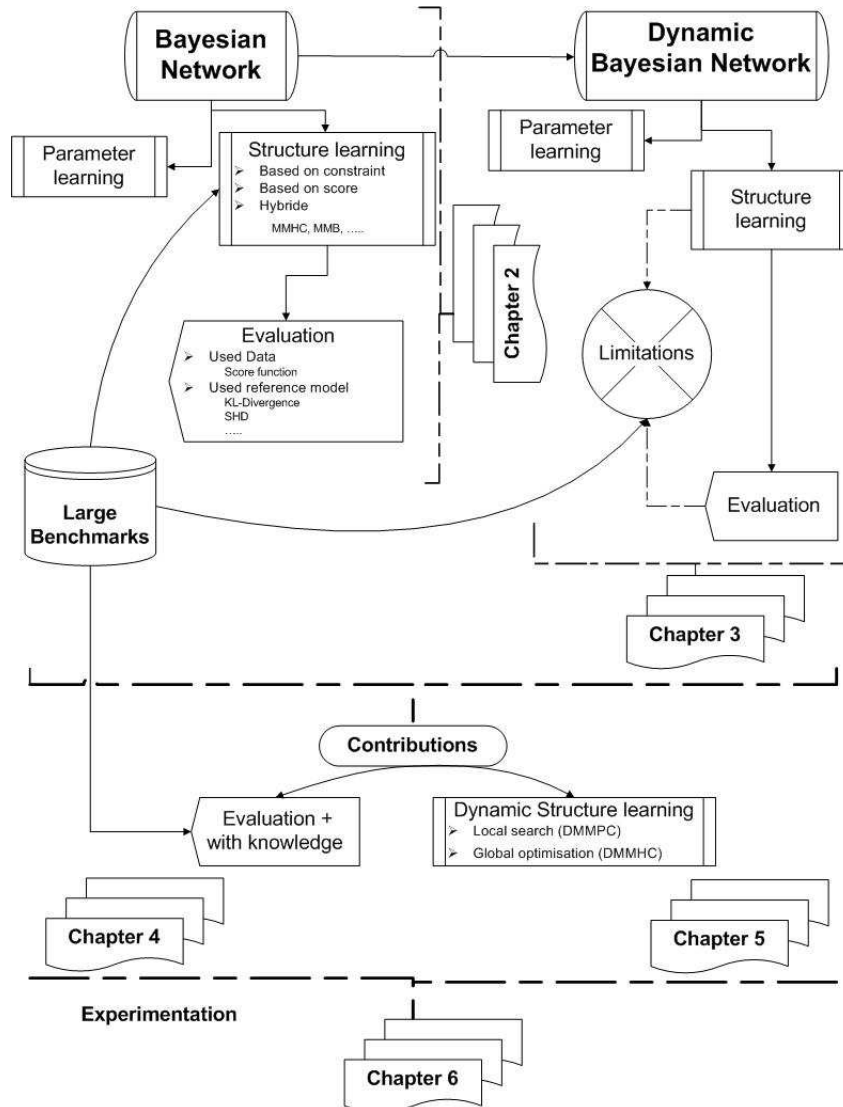


Figure 1.1: Thesis overview and interdependencies between chapters

Chapters 2 and 3 review the scientific background and establish the terminology required to discuss the structure learning for Bayesian Networks, thus providing the basis for the subsequent chapters of this thesis. It starts off by reminding some of the basic notations and definitions that are commonly used in the Bayesian Network literature.

Moreover, In chapter 3, we present the difference between the static Bayesian networks and their extension called dynamic bayesian networks when the time dimension is added. The two chapters are end up with an overview of the existing approaches for benchmarking static and dynamic bayesian networks and evaluating the structure learning algorithms in literature.

Chapter 4 is dedicated for our contributions. Initially, we show a description of our Benchmarking algorithms. The first one generates large dynamic bayesian with (2-TBN) form. The second is an extension of the structural hamming distance algorithm for the evaluation in the dynamic case. We also provide some experimental validation to justify the performance and utility of our first contributions.

Chapter 5 describes our new algorithm for DBN structure learning based on hybrid method. In this chapter, we give three (naïve, optimized and simplified) versions of our algorithm with the different necessary demonstrations. The chapter ends with the Time complexity of the algorithms and a toy example that presents the construction of structure learning proposed by our algorithms applied on simple Network.

Chapter 6 gives the different experiments achieved to show the performance of DMMHC algorithm in many levels as the quality and computational complexity of the algorithm for learning structure. In this chapter, we present the experimental protocol used in this thesis project. After that, we try to represent the obtained results and to compare the results given by the GS and SA algorithms with those given by naïve and optimized versions of our DMMHC. Also an interpretation and discussion of these results are achieved. In addition, we present a comparison between our simplified DMMHC and the structure learning algorithms applied on a reduced search space (2-TBN without intra-slice edges). Additional evidence of the scalability of DMMHC is found in the final section of this chapter.

Chapter 7 concludes the thesis summarizing the major results that we obtained. In addition, we tried to identify some perspectives for future research.

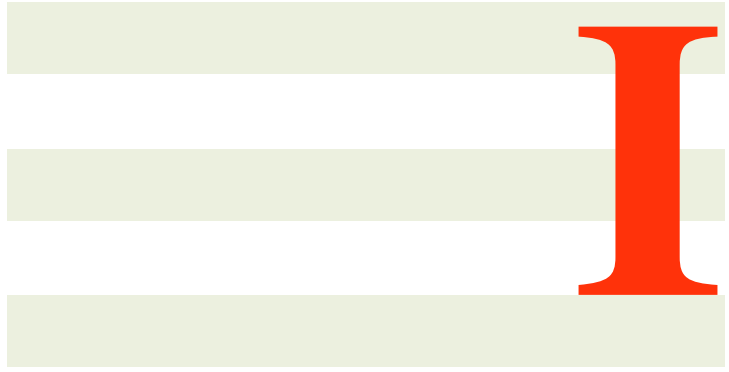
## 1.3 Publications

The following parts of this work have previously been published in different international journals and conferences:

- The contribution presented in chapter 4 was summarized in an article published in the proceedings of ICMSAO 2013 conference [100].
- The contribution presented in chapter 5 was summarized in an article published in the proceedings of IDA 2013 conference [101].
- Other application contributions not described in this thesis, were made by our research group in REGIM lab (Tunisia), have been the object of two scientific publications in Data Mining Workshops (ICDMW), 2012 IEEE 12th International Conference [63] and International Journal of Advanced Research in Artificial Intelligence [64]. In these works, we discussed the use of different data mining techniques as the dynamic Bayesian Networks and 3D visualization in Dynamic Medical Decision Support System.







## State of the art



# Bayesian Network and structure learning

## Sommaire

<b>2.1</b>	<b>Introduction</b>	<b>28</b>
<b>2.2</b>	<b>Bayesian Network definition</b>	<b>28</b>
2.2.1	Description example [75]	28
2.2.2	Basic concepts	30
2.2.3	Bayesian Network definition	32
2.2.4	Markov equivalence classes for directed Acyclic Graphs	33
<b>2.3</b>	<b>Bayesian network structure learning</b>	<b>35</b>
2.3.1	Constraint-Based Learning	36
2.3.2	Score-Based Learning	37
	Greedy search algorithm	38
	Simulated annealing algorithm (SA)	39
2.3.3	Hybrid method or local search	39
2.3.4	Learning with large number of variables	41
<b>2.4</b>	<b>Evaluation of Bayesian Network structure learning algorithms</b>	<b>42</b>
2.4.1	Evaluation metrics	42
	Score-based method	43
	Kullback-Leibler divergence-based method	43
	Sensitivity and specificity-based method	44
	Distance-based method	44
2.4.2	Generating large Benchmarks	45

## 2.1 Introduction

Knowledge representation and reasoning from these representations have given birth for many models. Probabilistic graphical models, and more specifically the Bayesian networks, initiated by Judea Pearl in the 1980s, have proven to be useful tools for the representation of uncertain knowledge, and reasoning from incomplete information. Afterwards many studies such as [78, 59, 49, 52] introduced Bayesian probabilistic reasoning formalism.

In this thesis we are interested in Bayesian network learning. Learning Bayesian network consists of two phases: parameter learning and structure learning. There are three types of approaches for the structure learning: methods based on identification of conditional independence, methods based on the optimization of a score and hybrid methods (cf. Figure 2.1).

To highlight the benefits of structure learning algorithms, we have to evaluate the quality of the Bayesian networks obtained by these learning algorithms. Many evaluation metrics are used in research. Some of them are characterized by the use of data as the score-based method. Some others are characterized by the use of a reference model. In our context, we are interested in the evaluation techniques using a reference model with a large number of variables.

This chapter reviews basic definitions and notations of classical Bayesian network and conditional independence. Section 2.2 introduces some notations and definitions of BN. Section 2.3 provides an overview of the static Bayesian networks structure learning. Based on this background, Section 2.4 is devoted to the evaluation of Bayesian networks structure. Finally, in this chapter we present the existing approaches used to evaluate these structure learning algorithms on large benchmarks.

## 2.2 Bayesian Network definition

### 2.2.1 Description example [75]

The presence or absence of a disease in a human being has a direct influence on whether a test for that disease turns out positive or negative. We would use Bayes' theorem (cf. theorem 2.2.1) to compute the conditional probability of an individual to have a disease when a test for the disease turns out to be positive.

Let's consider the situation where several features are related through inference chains. For example, whether or not an individual has a history of smoking has a

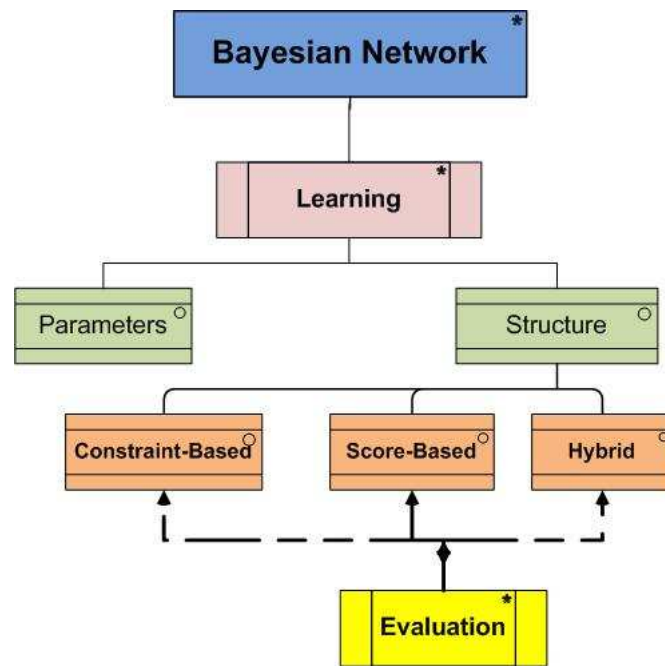


Figure 2.1: The different concepts presented in chapter 2

direct influence both on whether or not that individual has bronchitis and on whether or not that individual has lung cancer. In turn, the presence or absence of each of these diseases has a direct influence on whether or not the individual experiences fatigue.

Also, the presence or absence of lung cancer has a direct influence on whether or not a chest X-ray is positive. We would want to determine, for example, the conditional probabilities of both of bronchitis and of lung cancer when it is known that the individual smokes, is fatigued, and has a positive chest X-ray. Yet bronchitis has no direct influence (indeed no influence at all) on whether a chest X-ray is positive. Therefore, these conditional probabilities cannot be computed using a simple application of Bayes' theorem. There is a straightforward algorithm for computing them, but the probability values it requires are not ordinarily accessible;

Bayesian networks were developed to address these difficulties. By exploiting conditional independencies entailed by influence chains, it is able to represent a large instance in a Bayesian network using little space. We are also often able to perform probabilistic inference among the features in an acceptable period of time. In addition, the graphical nature of the Bayesian networks gives a much better intuitive grasp of the relationships among the features.

Figure 2.2 shows a Bayesian network representing the probabilistic relationships among the above discussed features. The values of the features in that network are represented in table 2.1

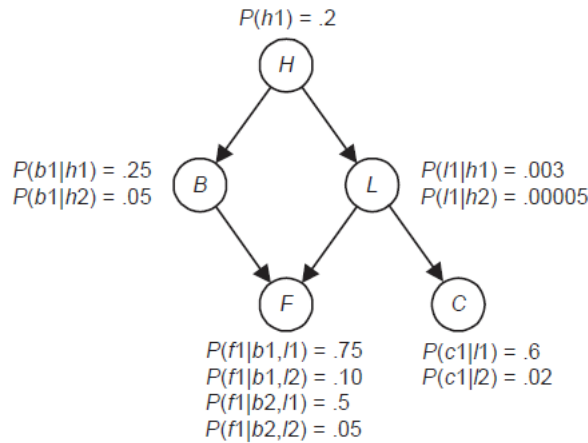


Figure 2.2: Example of Bayesian Network

Feature	Value	When the Feature Takes this Value
H	h1	There is a history of smoking
	h2	There is no history of smoking
B	b1	Bronchitis is present
	b2	Bronchitis is absent
L	l1	Lung cancer is present
	l2	Lung cancer is absent
F	f1	Fatigue is present
	f2	Fatigue is absent
C	c1	Chest X-ray is positive
	c2	Chest X-ray is negative

Table 2.1: The values of the features

### 2.2.2 Basic concepts

In this section, we give some basic concepts related to the Bayesian network used in our approach:

**Definition 2.2.1** Let  $X$  and  $Y$  be two sets of random variables, given that the probability of  $Y$  is different from zero, the conditional probability of  $X$  is defined by:

$$P(X|Y) = \frac{P(X; Y)}{P(Y)} \tag{2.1}$$

**Theorem 2.2.1** (Bayes' theorem) [7]

Let  $X$  and  $Y$  two sets of random variables, the Bayes' theorem determines the conditional probability of  $X$  given  $Y$  based on the probabilities of  $X$ ,  $Y$  and  $Y$  given  $X$ , with the following formula:

$$P(X|Y) = \frac{P(Y|X).P(X)}{P(Y)} \tag{2.2}$$

**Definition 2.2.2** (*Independence*)

Let  $X$  and  $Y$  two sets of random variables,  $X$  and  $Y$  are independent, denoted  $\langle X \perp Y \rangle$ , if and only if  $P(X|Y) = P(X)$ .

**Definition 2.2.3** (*Conditionally independent*)

Let  $X$ ,  $Y$ , and  $Z$  be any three subsets of random variables.  $X$  and  $Y$  are said to be conditionally independent given  $Z$  (noted  $Ind_p(X;Y | Z)$ ) if  $P(X | Y, Z) = P(X | Z)$  whenever  $P(Y, Z) > 0$ .

**Definition 2.2.4** (*directed graph*)

A directed graph  $G$  can be defined as an ordered pair that consists of a finite set  $V$  of nodes and an irreflexive adjacency relation  $E$  on  $V$ . The graph  $G$  is denoted as  $(V, E)$ . For each  $(x, y) \in E$  we say that there is an arc (directed edge) from node  $x$  to node  $y$ . In the graph, this is denoted by an arrow from  $x$  to  $y$  and  $x$  and  $y$  are called the start point and the end point of the arrow respectively. We also say that node  $x$  and node  $y$  are adjacent or  $x$  and  $y$  are neighbors of each other and node  $y$  and node  $x$  are adjacent or  $y$  and  $x$  are neighbors of each other.  $x$  is also called a parent of  $y$  and  $y$  is called a child of  $x$ . By using the concepts of parent and child recursively, we can also define the concept of ancestor and descendent. We also call a node that does not have any parent a root node. By irreflexive adjacency relation we mean that for any  $x \in V$ ,  $(x, x) \notin E$ , i.e., an arc cannot have a node as both its start point and end point.

**Definition 2.2.5** (*Path*)

A path in a directed graph is a sequence of nodes from one node to another using the arcs.

**Definition 2.2.6** (*directed path and cycle*)

A directed path from  $X_1$  to  $X_n$  in a DAG  $G$  is a sequence of directed edges  $X_1 \rightarrow X_2 \dots \rightarrow X_n$ . The directed path is a cycle if  $X_1 = X_n$  (i.e. it begins and ends at the same variable).

**Definition 2.2.7** (*A directed acyclic graph*)

A DAG is a Directed Acyclic (without cycles) Graph (See Figure 2.4.a).

**Definition 2.2.8** (*d-separation*)

Let  $S$  be a trail (that is, a collection of edges which is like a path, but each of whose edges may have any direction) from node  $u$  to  $v$ . Then  $S$  is said to be  $d$ -separated by a set of nodes  $Z$  if and only if (at least) one of the following holds:

1.  $S$  contains a chain,  $x \leftarrow m \leftarrow y$ , such that the middle node  $m$  is in  $Z$ ,
2.  $S$  contains a fork,  $x \leftarrow m \rightarrow y$ , such that the middle node  $m$  is in  $Z$ , or



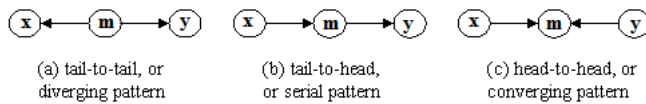


Figure 2.3: Patterns for paths through a node

3.  $S$  contains an inverted fork (or collider),  $x \rightarrow m \leftarrow y$ , such that the middle node  $m$  is not in  $Z$  and no descendant of  $m$  is in  $Z$ .

The graph patterns of "tail-to-tail", "tail-to-head" and "head-to-head" are shown in Figure 2.3. Thus  $u$  and  $v$  are said to be  $d$ -separated by  $Z$  if all trails between them are  $d$ -separated. If  $u$  and  $v$  are not  $d$ -separated, they are called  $d$ -connected.

**Definition 2.2.9** (*D-separation for two sets of variables*)

Two sets of variables  $X$  and  $Y$  are  $d$ -separated by  $Z$  in a graph  $G$  (noted  $Dsep_G(X; Y \mid Z)$ ) if and only if  $\forall u \in X$  and  $\forall v \in Y$ , every trail from  $u$  to  $v$   $d$ -separated by  $Z$ .

### 2.2.3 Bayesian Network definition

**Definition 2.2.10** (*Markov condition*) [78]

The Markov condition can be stated as follows: Each variable  $X_i$  is conditionally independent of all its no-descending, knowing the state of his parents,  $Pa(X_i)$ .

$$Ind_p(X_i; NoDesc(X_i) \mid Pa(X_i)). \quad (2.3)$$

$$P(X_i \mid Pa(X_i); NoDesc(X_i)) = P(X_i \mid Pa(X_i)) \quad (2.4)$$

**Definition 2.2.11** (*Bayesian Network*) [75]

Let  $P$  be a joint probability distribution of the random variables in some set  $V$ , and  $G = (V, E)$  be a DAG. We call  $(G, P)$  a Bayesian network if  $(G, P)$  satisfies the Markov condition.  $P$  is the product of its conditional distributions in  $G$ , and this is the way  $P$  is always represented in a Bayesian network.

A Bayesian Network  $B=(G, \theta)$  is defined by:

- $G=(V, E)$  directed graph without circuit whose vertices are associated with a set of random variables  $X = X_1, \dots, X_n$ , there is a bijection between  $X$  and  $V$  (i.e.  $X_i \longleftrightarrow V_i$ ) and  $E$  is the set of arcs representing the conditional independence between variables,
- $\theta = P(X_i \mid Pa(X_i))$ , all probabilities of each node  $X_i$  conditional on the state of its parents  $Pa(X_i)$  in  $G$ .

Pearl et al. [78] have also shown that the Bayesian networks allow us to represent compactly the joint probability distribution over all variables:

$$P(X_1, X_2, \dots, X_n) = \prod_{i=1}^n P(X_i | Pa(X_i)) \quad (2.5)$$

This decomposition of a global function, into a product of local terms depending only on the same node and its parents in the graph, is a fundamental property of Bayesian networks.

**Theorem 2.2.2** [93]

*In a BN, two nodes are  $d$ -separated by  $Z$ , they are also conditionally independent by  $Z$ .*

$$Dsep_G(X; T|Z) = Ind_P(X; T|Z) \quad (2.6)$$

The graphical part of the Bayesian network indicates the dependencies (or independence) between variables and provides a visual tool for knowledge representation to make it more comprehensible to its users. In addition, the use of probabilities allows taking into account the uncertainty in quantifying the dependencies between variables.

## 2.2.4 Markov equivalence classes for directed Acyclic Graphs

There may be many DAGs associated to BNs that determine the same dependence model. Thus, the family of all DAGs with a given set of vertices is naturally partitioned into Markov-equivalence classes, with each class being associated with a unique independence model (See Figure 2.4.c).

**Definition 2.2.12** (Markov Equivalence) [77, 75, 104]

*Two DAGs  $G_1$  and  $G_2$  on the same set of nodes are Markov Equivalent if for every three mutually disjoint subsets  $A, B, C \subseteq V$ ,  $Desp_{G_1}(A, B | C) \Leftrightarrow Desp_{G_2}(A, B | C)$ .*

**Theorem 2.2.3** (Verma and Pearl, 1991) [109]

*Two DAGs are equivalent if and only if they have the same skeleton and the same  $V$ -structures.*

**Definition 2.2.13** (Patterns) [69]

*The pattern for a partially directed graph  $G$  is the partially directed graph which has the identical adjacencies as  $G$  and which has an oriented edge  $A \rightarrow B$  if and only if there is a vertex  $C \notin adj(A)$  such that  $A \rightarrow B$  and  $C \rightarrow B$  in  $G$ . Let pattern( $G$ ) denote the pattern for  $G$ . A triple  $(A, B, C)$  is an unshielded collider in  $G$  if and only if  $A \rightarrow B, C \rightarrow B$  and  $A$  is not adjacent to  $B$  ( $V$ -structure). It is easy to show*

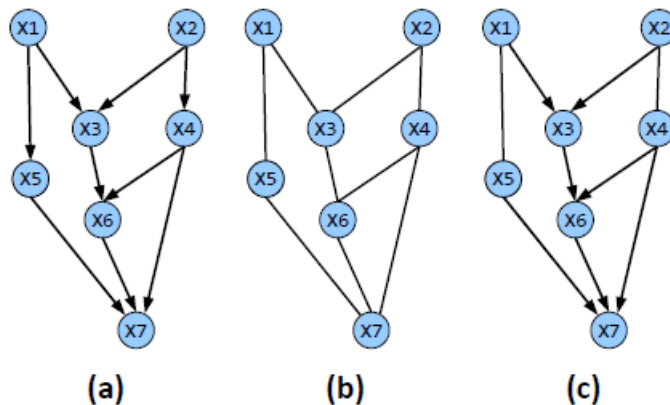


Figure 2.4: (a) Example of DAG, (b) the skeleton relative to the DAG and (c) an example of a PDAG [70]

*that two directed acyclic graphs have the same pattern if and only if they have the same adjacencies and same unshielded colliders.*

**Theorem 2.2.4** (Verma and Pearl, 1991) [109]

*Two directed acyclic graphs  $G$  and  $G'$  are Markov equivalent if and only if  $\text{pattern}(G) = \text{pattern}(G')$ .*

**Definition 2.2.14** (Completed PDAG (CPDAG))

*The completed PDAG corresponding to an equivalence class is the PDAG consisting of a directed edge for every compelled edge in the equivalence class, and an undirected edge for every reversible edge in the equivalence class. An arc is said to be reversible if its reversion leads to a graph which is equivalent to the first one. An arc is said to be compelled if it is not reversible.*

Several researchers, including [109, 69, 14], present rules based algorithms that can be used to implement DAG-to-CPDAG. The idea of these implementations is as follows. First, they undirect every edge in a DAG, except for those edges that participate in a v-structure. Then, they repeatedly apply one of a set of rules that transform undirected edges into directed edges. Meek proves that the transformation rules are sound and complete. That is, once no rule matches on the current graph, that graph must be a completed PDAG [69] (see Figures 2.5 R1, R2 and R3). We can notice that Chickering proposed an optimized implementation for this phase (PDAG determination) [14]. A CPDAG of graph  $G$  is the representation of all graphs equivalent to  $G$ .

**Definition 2.2.15** (Dependency model) [69]

*A dependency model is a list  $M$  of conditional independence statements of the form  $A \perp B | S$  where  $A$ ,  $B$ , and  $S$  are disjoint subsets of  $V$*

**Definition 2.2.16** (*complete causal explanation*) [69]

A directed acyclic graph  $G$  is a complete causal explanation of  $\mathbf{M}$  if-and only if the set of conditional independence facts entailed by  $G$  is exactly the set of facts in  $\mathbf{M}$ .

Meek proposed some orientation rules to find a partially directed graph whose adjacencies are the same as any complete causal explanation for  $\mathbf{M}$  and whose edges are directed if and only if every complete causal explanation for  $\mathbf{M}$  has the edges oriented.

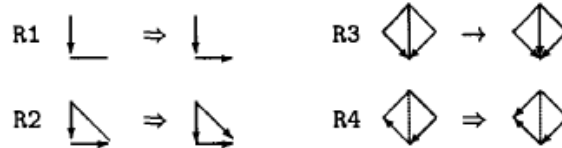


Figure 2.5: Orientation rules for patterns [69]

Let  $\mathbf{K}$  be a set of background knowledge.  $\mathbf{K}$  is a pair  $(\mathbf{F}, \mathbf{R})$  where  $\mathbf{F}$  is the set of directed edges which are forbidden,  $\mathbf{R}$  is the set of directed edges which are required.

**Definition 2.2.17** (*CPDAG $_K$  with Knowledge*)

A CPDAG $_K$  of graph  $G$  representing all equivalent graphs of  $G$  and consistent with a set of knowledge  $\mathbf{K}$  defining the priori.

When proposing a first algorithm to determine the PDAG of a given graph, [69] Meek also proposed a way to take into account prior background knowledge.

This solution is decomposed into three phases. The first phase consists in determining the PDAG. As we mentioned before, this step can be solved by keeping the skeleton of the given DAG, and its V-structures, and then applying recursively a set of three rules  $R_1$ ,  $R_2$ , and  $R_3$  (Figure 2.5) in order to infer all the edge orientations compatible with the initial DAG.

The second phase consists in comparing this PDAG with the prior knowledge. If some information are conflicting, the algorithm turns out to be an error. The final step consists in iteratively adding the prior knowledge (edges) not present in the PDAG and applying again the previous recursive orientation rules in order to infer all the new edge orientations induced by the addition of the prior knowledge.

Meek demonstrates that another rule  $R_4$  (figure 2.5) is needed in order to complete the three previous ones when we add the prior knowledge for the model.

## 2.3 Bayesian network structure learning

In recent years there has been a growing interest in the structure learning of the Bayesian networks from data [42, 77, 11, 75, 104]. There are three approaches for

finding a structure. The first approach poses learning as a constraint satisfaction problem. In this approach, we try to identify the properties of conditional independence among the variables in the data. This is usually done using a statistical hypothesis test, like the  $\chi^2$  test. We then build a network that exhibits the observed dependencies and independencies.

The second approach poses learning as an optimization problem. We start by defining a statistically motivated score that describes the fitness of each possible structure from the observed data. The learner’s task is then to find a structure that maximizes the score. In general, this is an NP-hard problem [11], and thus we need to resort to heuristic methods. Although the constraint satisfaction approach is efficient, it is sensitive to failures in independence tests. Thus, the common opinion is that the optimization approach is a better tool for learning structure from a little amount of data.

The third approach, named hybrid methods, can solve this problem by the combination of the two previous ones. Local search methods are dealing with local structure identification and global model optimization constrained with these local information. These methods are able to scale distributions with more than thousands of variables.

Generally, people face the problem of learning BNs from training data in order to apply BNs to real-world applications. Typically, there are two categories in learning BNs, one is to learn BN parameters when a BN structure is known, and another is to learn both BN structures and parameters. We notice that, learning BN parameters can be handled with complete and incomplete data. This kind of learning is not as complicated as learning a BN structure. In this thesis, we focus on structure learning with complete data, as we recommend to read appendix A before reading this section.

### 2.3.1 Constraint-Based Learning

The idea of constraint-based structure learning is to know how to construct the structure of Bayesian network if we can perform independence test ( $A \perp B|C$ ).

Constraint-based methods employ the conditional independence tests to first identify a set of conditional independence properties, and then attempts to identify the network structure that best satisfies these constraints corresponding to a CPDAG.

Fast [29] describes the constraint identification as ”the process of learning the skeleton and separating sets from the training data. Due to limited size of the training data, this process is inherently error-prone. The goal of constraint identification is to efficiently identify the independence assertions while minimizing the number of constraints that are inaccurate. Constraint identification algorithms appearing in

the literature can be differentiated by three different design decisions: (1) the type of independence test used, (2) the ordering heuristic and other algorithmic decisions, and (3) the technique used to determine the reliability of the test”.

This first family of structure learning approaches for BN, often called search under constraints, is derived from the work of Pearl’s team and Spirtes’ team. The most popular constraint-based algorithms are IC [78], SGS and PC [77, 104] algorithms. Three of them try to d-separate all the variable pairs with all the possible conditional sets whose sizes are lower than a given threshold. They are based on the same principle:

- build an undirected graph containing the relationships between variables, from conditional independence tests
- detect the V-structures (also using conditional independence tests)
- propagate the orientation of some arcs with meek’s rules in section 2.2.4
- randomly direct some other edges in the graph without changing its CPDAG
- take into account the possible artificial causes due to latent variables.

One problem with constraint-based approaches is that they are difficult to reliably identify the conditional independence properties and to optimize the network structure [67].

The constraint-based approaches lack an explicit objective function and they do not try to directly find the global structure with maximum likelihood. So they do not fit in the probabilistic framework.

This kind of approach rely on quite heavy assumptions such as the faithfulness and correctness of the independence tests. In contrast, score-based algorithms search for the minimal model among all Bayesian networks.

### 2.3.2 Score-Based Learning

Unlike the first family of methods that construct the structure with the use of conditional independence between variables, this approach tries to define scoring function that evaluates how well a structure matches the data and to identify the graph with the highest score. These approaches are based on several scores such as BIC [87], AIC [2], MDL [57], BD [17], BDe [42]. All these scores are approximation of the marginal likelihood  $P(D|G)$  [18].

The score-Based methods are feasible in practice if the score should be locally decomposable. This score is expressed as the sum of local scores at each node. There is also the problem of search space of bayesian networks to find the best structure. The main problem with these approaches is that, it is NP-hard to compute the optimal structure using bayesian scores [11]. There are several algorithms for structure learning based on BN scores as MWST [15], K2 [17], and GS. Some proposed algorithms work in a small space such as trees [15], polytrees [15] and

hypertrees [94].

In this section, we present details of the Greedy search (GS) and Simulated Annealing (SA) as we will compare them later with our algorithms because their extensions in dynamic case and their code sources (softwares) are available on the internet.

**Greedy search algorithm** This algorithm (algorithm 1) follows the heuristic problem solving of making the locally optimal choice at each step with the hope of finding a global optimum. It is initialized by a network  $G_0$ . It collects all possible simple graph operations (e.g., edge addition, removal or reversal) that can be performed on the network without violating the constraints (e.g., introducing a cycle), we use for this the Generate neighborhood algorithm (algorithm 2). Then, it picks the operation that increases the score of the network the most (This step can be repeated if the network can still be improved or the maximal number of interactions haven't been reached).

---

#### Algorithm 1 $GS(G_0)$

---

**Require:** initial graph ( $G_0$ )

**Ensure:** BN structure ( $DAG$ )

---

```

1:  $G \leftarrow G_0$ 
2:  $Test \leftarrow True$ 
3:  $S \leftarrow Score(G,D)$ 
4: while  $Test=True$  do
5:    $N \leftarrow Generate\_neighborhood(G, \emptyset)$ 
6:    $G_{max} = \arg \max_{F \in N} Score(F,D)$ 
7:   if  $Score(G_{max},D) > S$  then
8:      $G \leftarrow G_{max}$ 
9:      $S \leftarrow Score(G_{max},D)$ 
10:  else
11:     $Test \leftarrow False$ 
12:  end if
13: end while
14: return the DAG  $G$  found

```

---



---

#### Algorithm 2 $Generate\_neighborhood(G,G_c)$

---

**Require:** current DAG ( $G$ ); undirected graph of constraints ( $G_c$ )

**Ensure:** set of neighborhood DAGs ( $N$ )

---

```

1:  $N \leftarrow \phi$ 
2: for all  $e \in G$  do
3:    $N \leftarrow N \cup (G \setminus \{e\})$  % delete_edge(e)
4:   if  $acyclic(G \setminus \{e\} \cup invert(e))$  then
5:      $N \leftarrow N \cup (G \setminus \{e\} \cup invert(e))$  % invert_edge(e)
6:   end if
7: end for
8: for all  $e \in G_c$  And  $e \notin G$  do
9:   if  $acyclic(G \cup \{e\})$  then
10:     $N \leftarrow N \cup (G \cup \{e\})$  % add_edge(e)
11:   end if
12: end for

```

---

**Simulated annealing algorithm (SA)** This is another generic metaheuristic for the global optimization problem of locating a good approximation to the global optimum of a given function in a large search space. It can be used when the search space is discrete. the goal of this method is to find an acceptable good solution in a fixed amount of time, rather than the best possible one.

the SA algorithm can be seen as the previous GS algorithm where we replace step 8 by a softer comparison where we allow score decrease in the first iterations of the algorithm and we become more and more strict over the iterations.

### 2.3.3 Hybrid method or local search

---

#### Algorithm 3 MMHC( $D$ )

---

**Require:** Data ( $D$ )

**Ensure:** BN structure ( $DAG$ )

---

```

1:  $G_c \leftarrow \phi$ 
2:  $G \leftarrow \phi$ 
3:  $S \leftarrow 0$ 
   % Local identification
4: for all  $X \in \mathbf{X}$  do
5:    $CPC_X = MMPC(X, D)$ 
6: end for
7: for all  $X \in \mathbf{X}$  And  $Y \in CPC_X$  do
8:    $G_c \leftarrow G_c \cup (X, Y)$ 
9: end for
   % Greedy search (GS) optimizing score function in DAG space
10:  $G \leftarrow GS(G_c)$ 
11: return the DAG  $G$  found

```

---



---

#### Algorithm 4 MMPC( $T, D$ )

---

**Require:** target variable ( $T$ ); Data ( $D$ )

**Ensure:** neighborhood of  $T$  ( $CPC$ )

---

```

1:  $ListC = \mathbf{X} \setminus \{T\}$ 
2:  $CPC = MMPC(T, D, ListC)$ 
   % Symmetrical correction
3: for all  $X \in CPC$  do
4:   if  $T \notin MMPC(X, D, \mathbf{X} \setminus \{X\})$  then
5:      $CPC = CPC \setminus \{X\}$ 
6:   end if
7: end for

```

---

Local search algorithms are hybrid BN structure learning methods dealing with local structure identification and global model optimization constrained with these local information.

Several local structure identifications have been proposed. They were dedicated to discover the candidate Parent-Children (PC) set of a target node such as the Max-Min Parent Children (MMPC) algorithm [102] or the Markov Blanket (MB) i.e. parents, children and spouses, of the target node [105, 85]. If the global structure



identification is the final goal, Parent-Children identification is sufficient in order to generate a global undirected graph which can be used as a set of constraints in the global model identification. For instance, the recent Max-Min Hill-Climbing algorithm (MMHC) (cf. algorithm 3) proposed by Tsamardinos [104] combines the local identification provided by Max-Min Parent Children (MMPC) algorithm and a global greedy search (GS) where the neighborhood of a given graph is generated with the following operators: `add_edge` is restricted to edges discovered in the local search identification phase (if the edge belongs to the set of constraints and if the resulting is acyclic DAG) (see algorithm 3), `delete_edge` and `invert_edge` (if the resulting is acyclic DAG) (see algorithm 2).

The MMPC local structure identification, described in Algorithm 4, is decomposed into two tasks, the neighborhood identification itself ( $\overline{\text{MMPC}}$ ), completed by a symmetrical correction ( $X$  belongs to the neighborhood of  $T$  if the opposite is also true). The neighborhood identification ( $\overline{\text{MMPC}}$ ), described in Algorithm 5, uses the Max-Min Heuristic defined in Algorithm 6 in order to iteratively add (forward phase) in the candidate Parent-Children set (neighborhood) of a target variable  $T$  the variable the most directly dependent on  $T$  conditionally to its current neighborhood (line 1 in algorithm 6). This procedure can potentially add some false positives which are then deleted in the backward phase. Dependency is measured with an association measurement function  $Assoc$  like  $\chi^2$ , mutual information or  $G^2$ .

---

**Algorithm 5**  $\overline{\text{MMPC}}(T, D, ListC)$ 


---

**Require:** target variable ( $T$ ); Data ( $D$ ); List of potential candidates ( $ListC$ )

**Ensure:** neighborhood of  $T$  ( $CPC$ )

---

```

1:  $CPC = \emptyset$ 
   % Phase I: Forward
2: repeat
3:    $\langle F, assocF \rangle = \text{MaxMinHeuristic}(T, CPC, ListC)$ 
4:   if  $assocF \neq 0$  then
5:      $CPC = CPC \cup \{F\}$ 
6:      $ListC = ListC \setminus \{F\}$ 
7:   end if
8: until  $CPC$  has not changed or  $assocF = 0$  or  $ListC = \emptyset$ 
   % Phase II: Backward
9: for all  $X \in CPC$  do
10:  if  $\exists S \subseteq CPC$  and  $assoc(X; T|S) = 0$  then
11:     $CPC \setminus \{X\}$ 
12:  end if
13: end for

```

---



---

**Algorithm 6**  $\text{MaxMinHeuristic}(T, CPC, ListC)$ 


---

**Require:** target variable ( $T$ ); current neighborhood ( $CPC$ ); List of potential candidates ( $ListC$ )

**Ensure:** the candidate the most directly dependent to  $T$  given  $CPC$  ( $F$ ) and its association measurement ( $AssocF$ )

---

```

1:  $assocF = \max_{X \in ListC} \min_{S \subseteq CPC} Assoc(X; T|S)$ 
2:  $F = \text{argmax}_{X \in ListC} \min_{S \subseteq CPC} Assoc(X; T|S)$ 

```

---

### 2.3.4 Learning with large number of variables

Bayesian network learning is a useful tool for exploratory data analysis. However, applying Bayesian networks to the analysis of large-scale data, consisting of thousands of variables, is not straight forward because of the heavy computational burden in learning and visualization. Some studies have focused on this problem to solve it through different approaches.

Friedman et al [33] introduce an algorithm that achieves faster learning from massive datasets by restricting the search space. Their iterative algorithm named "Sparse Candidate" restricts the parents of each variable to belong to a small subset of candidates. They then search for a network that satisfies these constraints. The learned network is then used to select better candidates for the next iteration. This algorithm is based on 2 steps: the first step is named restrict based on data  $D$  and  $B_{n-1}$  network, the selected candidates for  $X_i$ 's parents include  $X_i$ 's current parents. This requirement (restrict step) implies that the winning network  $B_n$  is a legal structure in the  $n+1$  iteration. Thus, if the search procedure at the second step named maximize also examines this structure, it must return a structure that scores at least as well as  $B_n$ . The stopping criteria for the algorithm is when  $\text{Score}(B_n) = \text{Score}(B_{n-1})$ .

Steck and Jaakkola [96] look at active learning in domains with a large number of variables. They developed an approach to unsupervised active learning which is tailored to large domains. The computational efficiency crucially depends on the properties of the measure with respect to which the optimal query is chosen. The standard information gain turns out to require a committee size that scales exponentially with the size of the domain (the number of random variables). Also, they propose a new measure, which they term "average KL divergence of pairs" (KL2). It emerges as a natural extension to the query by the committee approach [30]. The advantages of this approach are illustrated in the context of recovering (regulatory) network models. The regulatory network involves 33 variables, 56 edges, and each variable was discretized to 4 states.

In 2006, Hwang and et al [44] proposed a novel method for large-scale data analysis based on hierarchical compression of information and constrained structural learning, i.e., hierarchical Bayesian networks (HBNs). The HBN can compactly visualize global probabilistic structure through a small number of hidden variables, approximately representing a large number of observed variables. An efficient learning algorithm for HBNs, which incrementally maximizes the lower bound of the likelihood function, is also suggested. They propose a two-phases learning algorithm for hierarchical Bayesian networks based on the above decomposition. In the first phase, a hierarchy for information compression is learned. After building the hierarchy, they learn the edges inside a layer when necessary by

the second phase. The effectiveness of their method is demonstrated by the experiments on synthetic large-scale Bayesian networks and a real-life microarray dataset. All variables were binary and local probability distributions were randomly generated. In this work, they show the results on two scale-free and modular Bayesian networks, consisting of 5000 nodes. Training datasets having 1000 examples.

Tsamardinos et al. [104] present an algorithm for the Bayesian network structure learning, called Max-Min Hill-Climbing (MMHC) described in the previous section. They show the ability of MMHC to scale up to thousands of variables. Tsamardinos brought the evidence of the scalability of MMHC by a reported experiment in Tsamardinos et al. [102] (for example MMPC was run on each node and reconstructed the skeleton of a tiled-ALARM network with approximately 10,000 variables from 1000 training instances).

De Campos et al [20] propose a new any-time exact algorithm using a branch-and-bound (B&B) approach with caches. Scores are computed during the initialization and a poll is built. Then, they perform the search over the possible graphs iterating over arcs. Although iterating over orderings is probably faster, iterating over arcs allows us to work with constraints in a straight forward way. Because of the B&B properties, the algorithm can be stopped at any-time with a best current solution found so far and an upper bound to the global optimum, which gives a kind of certificate to the answer and allows the user to stop the computation when she believes that the current solution is good enough. They show also empirically the advantages of the properties and the constraints, and the applicability of their presented algorithm that integrates parameter and structural constraints with large data sets (up to one hundred variables), that cannot be handled by other current methods (limited to around 30 variables), in a way to guarantee global optimality with respect to the score function.

## 2.4 Evaluation of Bayesian Network structure learning algorithms

### 2.4.1 Evaluation metrics

In the previous sections, we presented some bayes Net structure learning algorithms. In this section we present methods that evaluate the quality of the Bayesian network obtained by the structure learning algorithms.

There are two scenarios for the evaluation of a structure learning algorithm:

- Given a theoretical BN  $B_0 = (G_0 ; \theta_0)$  and data  $D$  generated from the BN, the measures evaluate the quality of the algorithm by comparing the quality of the learned graph  $B = (G, \theta)$  and that of the theoretical network  $B_0$  with the

use of data

- the measure evaluates the quality of the algorithm by comparing the structure  $G$  of the learned graph and the structure  $G_0$  of the theoretical graph.

To this end we notice that it would be better to compare the equivalence classes given by the original and learned BN. A BN obtained from the data is identified by its equivalence class. The best BN is obtained if we find that the CPDAG of the generated network is equal to that of the learned BN. So, all evaluation metrics must use the equivalence class to compare between the BNs for better evaluation. There are several measures proposed in the literature for the evaluation of structure learning algorithms [76].

### Score-based method

To evaluate a structure learning algorithm, many studies use a comparison between the score function of the learned graph and the original graph. The learning algorithm is good if  $S(B,D) \approx S(G_0,D)$  where  $S$  is a score among the scores given in the previous section 2.3.2.

An advantage of this approach is that it takes into account Markov equivalence to evaluate the structure. this metric gives the best results if there is large learned dataset.

However, the use of score functions to evaluate the learning structure algorithms can produce some problems. For example, if the data are limited, it is possible to obtain a graph  $G$  with the same score as  $G_0$ , but that is not in the same equivalence class.

### Kullback-Leibler divergence-based method

The Kullback-Leibler (KL) divergence is a fundamental equation of information theory that quantifies the proximity of two probability distributions [86]. It is a non-symmetric measure of the difference between the probability distributions of a learned network  $P$  and the probability distributions of a target network  $Q$ . Specifically, the Kullback-Leibler divergence of  $Q$  from  $P$ , denoted  $D_{KL}(P | Q)$ , is a measure of the information lost when  $Q$  is used to approximate  $P$ : KL measures the expected number of extra bits required to encode samples from  $P$  when using a code based on  $Q$ , rather than using a code based on  $P$ . Typically  $P$  represents the "true" distribution of data, observations, or a precisely calculated theoretical distribution. The measure  $Q$  typically represents a theory, a model, a description, or an approximation of  $P$ .

If the variables are discrete, this divergence is defined by:

$$D_{KL}(P | Q) = \sum_{x \in \mathcal{X}} P(x) \log \frac{P(x)}{Q(x)} \quad (2.7)$$

$D_{KL}$  is non-negative ( $\geq 0$ ), not symmetric in  $P$  and  $Q$ , zero if the distributions match exactly, it can potentially equal infinity.

This metric is important when the network is used for inference. However, the KL-divergence is not directly penalized for extraneous edges and parameters [104]. There are situations where the KL divergence has limitations. In fact, the computational complexity is exponential to the number of variables. In this case, the principle of sampling can be used to reduce this complexity.

### Sensitivity and specificity-based method

Sensitivity and specificity are statistical measures of the performance of a binary classification test, also known as classification function in statistics [29].

Sensitivity measures the proportion of actual positives which are correctly identified. Specificity measures the proportion of negatives which are correctly identified.

Given the graph of the theoretical network  $G_0 = (V, E_0)$  and the graph of the learned network  $G = (V, E)$ , The sensitivity-specificity-based method begins by calculating the following indices:

- TP (true positive) = number of edges present in both  $G_0$  and  $G$
- TN (true negative) = number of edges absent in both  $G_0$  and  $G$
- FP (false positive) = number of edges present in  $G$ , but not in  $G_0$
- FN (false negative) = number of edges absent in  $G$ , but not in  $G_0$

Then, the sensitivity and specificity can be calculated as follows:

$$\begin{aligned} \text{Sensitivity} &= \frac{TP}{TP+FN} \\ \text{Specificity} &= \frac{TN}{TN+FP} \end{aligned}$$

These measures are easy to calculate. They are often used in the literature. However, the differences in orientation between the two graphs in the same equivalence class are counted as errors.

### Distance-based method

Tsamardinos et al. [104] have proposed an adaptation of the usual Hamming distance between graphs taking into account the fact that some graphs with different orientations can be statistically indistinguishable. As graphical models of independence, several equivalent graphs will represent the same set of dependence / independence properties. These equivalent graphs (also named Markov or likelihood equivalent graphs) can be summarized by a partially DAG (PDAG) (section 2.2.4). This new structural Hamming distance (SHD) compares the structure of the

PDAG of the learned and the original networks as described in algorithm 7 in order to compare only the orientations that are really statistically distinguishable.

As in the sensitivity and specificity, the advantage of this approach is the simplicity of calculation. This method also takes into account the Markov equivalence and it can run with a large number of variables.

As mentioned before, structure learning is a difficult task. Some works propose to use prior knowledge in order to limit the search space, for instance by declaring some forbidden or required edges in the final graph [21]. When dealing with PDAGs, the previous SHD measure takes into account only the information from the learning data, forgetting that some orientations have been provided by prior knowledge.

---

**Algorithm 7** Structural Hamming distance [104]

---

**Require:** Learned PDAG H; Original PDAG G

**Ensure:** SHD value

```

1:  $SHD = 0$ 
2: for all edge E different between H and G do
3:   if (E is missing in H) or (E is extra in H) or (E is incorrectly oriented in H) then
4:      $SHD = SHD + 1$ 
5:   end if
6: end for

```

---

## 2.4.2 Generating large Benchmarks

As mentioned in section 2.4, the evaluation tasks need a reference model (is either a known graph or a randomly generated one). To evaluate the structure learning algorithms we must have different benchmarks that we use to construct the learned structures from data and compare them to the reference model. In this thesis we focus on the large benchmarks.

The constraint-based learning and score-based learning approaches are usually validated on benchmark models with a small number of variables (e.g., less than 100 variables). In literature, there are few standard benchmarks with a large size that are not very used in research. The most known large datasets are Andes (223 nodes) [16]; Pigs(442 nodes)(Created by Claus S. Jensen on the basis of a data base from Soren Andersen (Danske Slagterier, Axeltorv Copenhagen)); Link (724 nodes) [49] and Munin (1041 nodes) [3].

As existing BN benchmarks are limited in their number of variables, some researchers proposed generating BNs by controlling their size and/or complexity. We can decompose these works into two families. The first allows to randomly generate large benchmarks. The seconds permits to generate large benchmarks from known graphs.

As an example of the first family Ide et al.'s work [45], it presented methods for the generation of random BN by generating uniformly distributed samples of

directed acyclic graphs. They develop a uniform generation of multi-connected and single-connected networks for a given number of nodes. Some studies use the generation of random graphs. These kinds of generation have some disadvantages. These graphs can have some repeated substructure. In addition some characteristics can be missed.

Generating a very large BN randomly is not very realistic. In many large applications, the global model can be decomposed in coherent repeated subgraphs. In the second family, they chose the reference model as multi-copies of the original standard model. Tsamardinos et al, in [95], have proposed a novel algorithm and software for the generation of an arbitrary large BN (e.g., graphical models representation and joint probability distributions) by tiling smaller real-world known networks (tiles). The complexity of the final model is controlled by two parameters : the number of tiling  $n$  and the connectivity parameter  $c$  which determines the maximum number of connections between one node and the next tile. In literature, to solve the problems given by generation of random BN, most of the studies [104, 22, 39] use tiling to generate large benchmarks and use them for the validation of their learning algorithms. In these works, they use tiling of small standard Benchmarks as (Asia, Alarm and Hailfinder) by tiling the original structure 3, 5 and 10 times.

As far as we know, the tiling technique is the best and most useful algorithm for simulating large BNs. In fact, during our research, we have found that the Tiling was used in many studies, such as to compare different Markov Blanket learning algorithms in large (5000 variables) networks [105, 102]; or to compare many BN learning algorithms in [104, 6, 56, 39], and to examine the time efficiency and quality of a local BN learning algorithm to reconstruct local regions in a large (10000 variable) network in [104].

## 2.5 Conclusion

In this chapter, we proceeded by presenting the concepts of Bayesian networks, then we reviewed the different structure learning methods of these models. We also addressed the question of evaluation for these learning algorithms.

We have seen that the hybrid method for learning the BN perform better than others techniques based on constraints or on score on the level of scalability and complexity of algorithms.

Section 2.4 presented an overview of the evaluation methods. The score-based method takes into account the equivalence of Markov during the evaluation. But if the data were reduced, it would be difficult to distinguish two different networks by their scores. Other methods based on reference model are also able to take into

account Markov equivalence, though the calculation is costly when the number of variables is important.

In the next chapter we will introduce an extension of the Bayesian networks named Dynamic Bayesian Network. This kind of model is a special Bayesian Network, which is used with dynamic stochastic process models [73].





# Dynamic Bayesian Networks and structure learning

## Sommaire

---

<b>3.1</b>	<b>Introduction</b>	<b>50</b>
<b>3.2</b>	<b>Dynamic Bayesian Network Definition</b>	<b>50</b>
3.2.1	Representation	50
3.2.2	Different forms of K-TBN	51
	2-TBN	51
	k-TBN	52
	Simplified k-TBN	53
	Other DBN forms	54
<b>3.3</b>	<b>Dynamic Bayesian Network structure learning</b>	<b>55</b>
3.3.1	Principle	55
3.3.2	Structure learning approaches for 2-TBN and k-TBN models	55
3.3.3	Structure learning approaches for simplified "k-TBN and 2-TBN" models	56
3.3.4	Structure learning approaches for other forms of DBN models	57
<b>3.4</b>	<b>Evaluation of DBN structure learning algorithms</b>	<b>58</b>
3.4.1	Benchmarks	58
3.4.2	Evaluation metric	59
<b>3.5</b>	<b>Conclusion</b>	<b>60</b>

---

## 3.1 Introduction

It is natural to use a dynamic Bayesian network (DBN), a directed graphical model whose nodes are across different time slices, to learn and reason about dynamic systems. DBN model is a temporal process and use a conditional probability distribution for each node and a probabilistic transition between time slices.

It is generally assumed that a DBN model structure and parameters do not change over time, i.e., the model is time-invariant [73]. Learning a DBN model from data aims to find the best model of the unknown probability distribution underlying the temporal process on the basis of a random sample of a finite size, i.e. the learning data.

As the static case, the learning dynamic Bayesian network consists of two phases: learning of the parameters and the learning of the structure. In the dynamic context most of the structure learning methods for DBN are based on the optimization of a score and deal with small dimension of benchmarks. Some recent works use hybrid methods (local search identification and global optimisation). However, this kind of methods deals with large dimension of benchmarks on a "simplified" search space.

As mentioned in Chapter 2, all studies try to evaluate their structure learning algorithms through some evaluation techniques. Many studies tried to know the goodness of their methods with the help of a scoring criterion, which is not the best way as we presented in section 2.4.1.

This chapter deals with basic definitions and notations of dynamic (temporal) Bayesian networks. Section 2.2 introduces some DBN notations and definitions. Section 2.3 provides an overview of the dynamic Bayesian networks structure learning. And as in the previous chapter, section 2.4 presents some techniques for the evaluation of Bayesian networks structure. Also, in this section we present what the existing approaches used to evaluate these structure learning algorithms on small and large Benchmarks. Finally, we try to enumerate the different limitations of the structure learning and evaluation.

## 3.2 Dynamic Bayesian Network Definition

### 3.2.1 Representation

A dynamic Bayesian network (DBN) is a probabilistic graphical model devoted to represent sequential systems. More precisely, a DBN defines the probability distribution of a collection of random variables  $\mathbf{X}[t]$  where  $\mathbf{X} = \{X_1 \dots X_n\}$  is the set of variables observed along discrete time  $t$  [23, 73].

In this work, we consider the special class of DBNs mentioned in the previous paragraph, namely the 2-Time slice Bayesian Networks (2T-BN) (cf. Figure 3.1).

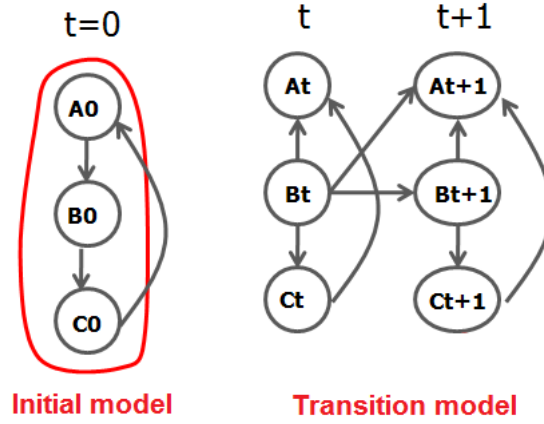


Figure 3.1: A structure of 2-TBN Bayesian network

### 3.2.2 Different forms of K-TBN

#### 2-TBN

To represent a DBN, we don't need the entire unrolled DBN. This kind of model represents a process that is stationary and Markovian. (1) Stationary: the node relationships within a time-slice  $t$  and the transition function from time-slice  $t$  to time-slice  $t+1$  do not depend on  $t$ . Therefore, we only need the initial time-slice; (2) Markovian: the transition function depends only on the immediately-preceding time-slice and not on any previous time-slices (e.g., no arrows go from time-slice  $t$  to time-slice  $t+2$ ). Therefore, the set of nodes in a time-slice  $d$ -separate the past from the future.

A 2T-BN is a DBN which satisfies the Markov property of order 1  $\mathbf{X}[t-1] \perp \mathbf{X}[t+1] \mid \mathbf{X}[t]$ . As a consequence, a 2T-BN is described by a pair  $(M_0, M_{\rightarrow})$ .

$M_0$  (**initial model**) is a BN representing the initial joint distribution of the process  $P(\mathbf{X}[t=0])$  and consisting of a direct acyclic graph (DAG)  $G_0$  containing the variables  $\mathbf{X}[t=0]$  and a set of conditional distributions  $P(X_i[t=0] \mid pa_{G_0}(X_i))$  where  $pa_{G_0}(X_i)$  are the parents of the variable  $X_i[t=0]$  in  $G_0$ .

$M_{\rightarrow}$  (**transition model**) is another BN representing the distribution  $P(\mathbf{X}[t+1] \mid \mathbf{X}[t])$  and consisting of a DAG  $G_{\rightarrow}$ , containing the variables in  $\mathbf{X}[t] \cup \mathbf{X}[t+1]$  and a set of conditional distributions  $P(X_i[t+1] \mid pa_{G_{\rightarrow}}(X_i))$  where  $pa_{G_{\rightarrow}}(X_i)$  are the parents of the variable  $X_i[t+1]$  in  $G_{\rightarrow}$ , parents which can belong to time  $t$  or  $t+1$ .  $M_{\rightarrow}$  is a two slice temporal Bayesian network (2TBN) that defines the transition

model  $p(\mathbf{X}_t|\mathbf{X}_{t-1})$  as follows:

$$p(\mathbf{Z}_t|\mathbf{Z}_{t-1}) = \prod_{i=1}^N p(Z_t^i|Pa(Z_t^i)) \quad (3.1)$$

The joint probability distribution for a sequence of length  $T$  can be obtained by unrolling the 2TBN network:

$$p(\mathbf{Z}_{1:T}) = \prod_{t=1}^T \prod_{i=1}^N p(Z_t^i|Pa(Z_t^i)) \quad (3.2)$$

An example of 2-TBN structure, as one dynamic Bayesian network form, is presented in figure 3.2.

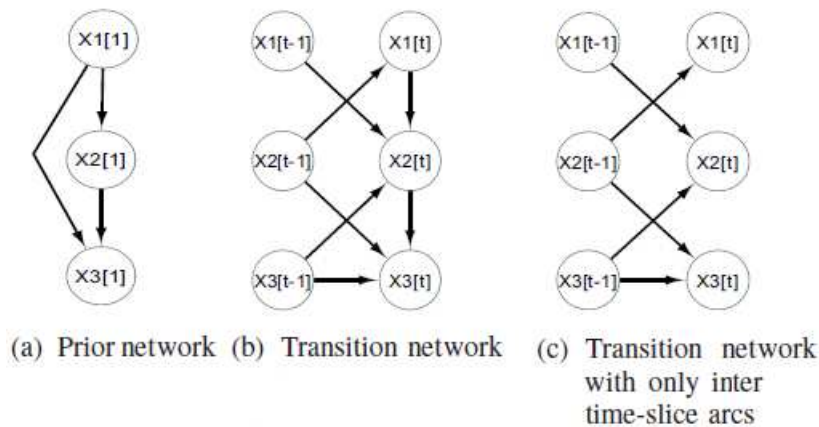


Figure 3.2: An example of 2-TBN Bayesian network [110]

### k-TBN

Murphy's definition of DBNs is sufficient for modeling first-order Markov processes. However, there are others extensions for modeling temporal processes. It is possible to model  $k-1^{th}$ -order Markov processes. This extension can be useful when modeling physiological processes at a small time-scale. For example when devising a DBN model that catches the dynamics of the human cardiovascular system for every 50[ms] [47]. In such temporal models, assuming a first-order process is not sufficient enough.

For  $M_0$  (**initial model**) is a DBN representing the initial joint distribution of the process  $P(\mathbf{X}[t = 0], \dots, \mathbf{X}[t = k - 2])$ .

For  $M_{\rightarrow}$  (**transition model**), the extension of Murphy's definition to the following:  $\mathbf{B}$  is not defined as a 2-TBN, but as a  $k$ -TBN that defines the transition model

$p(Z_t|Z_{t-1}, Z_{t-2}, \dots, Z_{t-k})$  for a  $k-1^{th}$ -order Markov process.

$$p(Z_t|Z_{t-1}, Z_{t-2}, \dots, Z_{t-k}) = \prod_{i=1}^N p(Z_t^i|Pa(Z_t^i)) \quad (3.3)$$

The equation 3.3 is essentially the same as equation 3.1, but for the set of parents which is not restricted to nodes in the previous or current time-slice, but can also contain nodes in time-slices further in the past. In this definition, the joint distribution for a sequence of length T can be obtained by unrolling the k-TBN and then multiplying all the CPTs. Again this equation is similar to equation 3.2.

$$p(Z_{1:T}) = \prod_{t=1}^T \prod_{i=1}^N p(Z_t^i|Pa(Z_t^i)) \quad (3.4)$$

**Definition 3.2.1** (*Temporal arc*)

A temporal arc is an arc between a parent node and a child node with an index that denotes the temporal order or time-delay  $k > 0$ . A parent and a child node can be the same node.

An example of second-order DBN according to the k-TBN visualization is shown in figure 3.3 This example of DBN is relatively simple, but imagine specifying a DBN model with many variables and different temporal orders per time-slice.

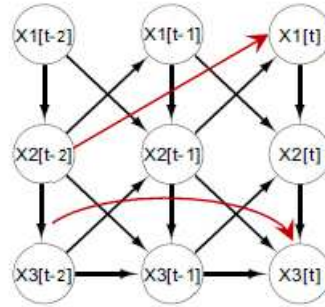


Figure 3.3: An example of k-TBN visualization of a second-order DBN ( $k=3$ ) using  $t = 3$  time-slices

**Simplified k-TBN**

In general, the transition networks may contain both inter and intra time slice arcs, as we presented in the previous figures 3.2 and 3.3. But some works have further restricted the transition network to contain only inter-time slice arcs, as illustrated in figures 3.2.c and 3.4 [27, 114, 110]. These models only capture the time delayed interactions between the variables while overlooking all the instantaneous interactions. The applicability of DBN models with only inter-time slice arcs largely depends on the nature of the interactions, and the time granularity, i.e., the

data sampling rate. Researchers use this kind of model in many fields as biological systems [27], genetics [110], nuclear translocation and turnover of the regulatory protein [83].

This kind of model is restricted in such a way that it contains only inter-time slice arcs that provides an important algorithmic advantage: there exist polynomial time algorithms for learning this class of DBN. So, with this model we can reduce the space search which is NP-hard in the DBN structure learning due primarily to temporal dimension.

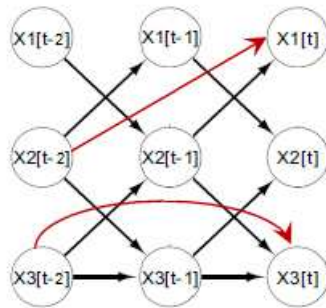


Figure 3.4: Simplified k-TBN

### Other DBN forms

There are many studies using other kinds of DBN. Dynamic Bayesian networks (DBN) are a class of graphical models that has become a standard tool for modeling various stochastic time-varying or non-stationary phenomena [62].

The first kind is named non-Stationary DBN. In this model, an important assumption of the traditional DBN structure learning is that the data are generated by a stationary process, an assumption that is not true in many important settings. Non-stationary dynamic Bayesian networks represent a new framework for studying problems in which the structure of a network is evolving over time [62].

Robinson et al. [84] present also this class of graphical model called a non-stationary dynamic Bayesian network, in which the conditional dependence structure of the underlying data-generation process is allowed to change over time.

Wang and al. [113] propose another form (autoDBN) to learn DBNs with changing structures from multivariate time series. In auto-DBN, segmentation of time series is achieved first through detecting geometric structures transformed from time series, and then model regions are found from the segmentation by designed finding strategies.

Song and al. [91] propose time-varying dynamic Bayesian networks (TV-DBN) for modeling the structurally varying directed dependency structures underlying non-stationary biological/neural time series.

### 3.3 Dynamic Bayesian Network structure learning

The existing techniques for learning DBNs are mostly straightforward extensions of the techniques for learning BNs [73]. In this thesis, we only focus on the structure learning in DBN. Before explaining the structure learning in a DBN, We mentioned that to achieve parameter learning, we can use the techniques discussed in Appendix A. In this case, we just comment on two points that are applicable specifically to dynamic systems: (1) Parameters must be tied across time-slices, so we can model sequences of unbounded length; (2) The parameters,  $\pi$ , for  $P(X_1)$  are usually taken to represent the initial state of the dynamic system [73].

#### 3.3.1 Principle

As in Murphy’s definition, the structure learning of a DBN has some assumptions must be respected:

- Learning the intra-slice connectivity, which must be a DAG,
- Learning the inter-slice connectivity, which is equivalent to the variable selection problem, since for each node in slice  $t$ , we must choose its parents from slice  $t-1$ .
- If we assume the intra-slice connections to be fixed, this means that structure learning for DBNs reduces to feature selection (simplified search space). If the system is fully observed (complete data), we can apply standard feature selection algorithms, such as forward or backwards stepwise selection, or the leaps and bounds algorithm [73].

When the system is partially observed (incomplete data), the structure learning becomes computationally intensive. One practical approach is the structural EM (SEM) algorithm [31].

#### 3.3.2 Structure learning approaches for 2-TBN and k-TBN models

Friedman et al. [32] have shown that this task can be decomposed in two independent phases: learning the initial graph  $G_0$  as a static BN structure with a static dataset corresponding to  $\mathbf{X}[t = 0]$  and learning the transition graph  $M_{\rightarrow}$ , with another ”static” dataset corresponding to all the transitions  $\mathbf{X}[t] \cup \mathbf{X}[t + 1]$ . They then proposed to apply usual score-based algorithms such as greedy search (GS) in order to find both graphs. Learning dynamic Bayesian network structures provides a main mechanism for identifying conditional dependencies in time-series data.

[35] developed another evolutionary approach for 2T-BN structure learning. In this work, The DBN structure learning based on BOA consists of two parts. The first part is to obtain the structure and parameters of the DBN in terms of a good



solution, and the second part is to produce new groups according to the obtained DBN structure. Wang and al. [112] also look at using evolutionary computation in 2T-BN structure learning by incorporating sampling methods.

Some other research work in structure learning propose their own algorithm for learning in such domains as the medicine, biology, geometry. Rajapakse et al. [82] use the dynamic Bayesian networks (DBN) and a Markov chain to model fMRI time-series and thereby determine temporal relationships of interactions among brain regions. Experiments on synthetic fMRI data demonstrate that the performance of the DBN is comparable to Granger causality mapping (GCM) in determining the structure of linearly connected networks.

Pena et al. [81] study cross-validation as a scoring criterion for learning dynamic Bayesian network models that can be well generalized. they argue that cross-validation is more suitable than the Bayesian scoring criterion as one of the most common interpretations of generalization.

Aydin and al. [5] present a secondary structure prediction method that employs dynamic Bayesian networks and support vector machines. This work is applied on biology.

All these approaches are validated on small benchmark models (with about 10 variables). In a more general context, due to inherent limitations of score-based structure learning methods, all these methods will have a very high complexity if the number of variables increases.

### 3.3.3 Structure learning approaches for simplified "k-TBN and 2-TBN" models

Dojer [27] offers an algorithm for learning an optimal network structure. It works in polynomial time for both MDL and BDe scores. His method is addressed to the case when the benchmark is small and there is no need to examine the acyclicity of the graph. According to Dojer, the second assumption is satisfied in the DBN case. In fact, as he mentioned in his work, A DBN describes a stochastic evolution of a set of random variables over discretized time. Therefore conditional distributions refer to random variables in neighboring time points. The acyclicity constraint is relaxed, because the "unrolled" graph is always acyclic.

In 2009, Wilczynski et al. [114] implemented and ran the previous Dojer's algorithm. He presented in a BNFinder software, which allows the Bayesian network reconstruction from experimental data. It supports dynamic Bayesian networks.

Vinh et al. dealt particularly with the problem of learning the globally optimal structure of a dynamic Bayesian network (DBN) [111]. They said that the MIT (Mutual Information Test) is better for evaluating the goodness-of-fit of the DBN structure than the other popular scoring metrics, such as BIC/MDL, K2 and BD,

and the well-known constraint-based PC algorithm.

Vinh et al. [110] propose another score based algorithm without equicardinality assumptions named MIT-global. Also, they propose another contribution in this work consist of a hybrid method with local Blanket identification (MIT-MMMB). They show that the local search MIT-MMMB and global-MIT have similar results better than advanced score based algorithm (simulated annealing). The MIT based on MMB try to identify the Markov Blanket MB but in a restricted subclass of 2T-BNs named simplified 2-TBN (cf. figure 3.2.c) or kT-BNs (cf. figure 3.3). This assumption permits to simplify both the MB discovery and the global optimization but it is not able to identify the intra-time dependencies.

### 3.3.4 Structure learning approaches for other forms of DBN models

Tucker et al. have developed an evolutionary algorithm which exploits certain characteristics of the MTS process in order to generate good networks as quickly as possible. They compare this algorithm to other standard learning algorithms that have traditionally been used for static Bayesian networks but adapted for DBNs [108].

The work of Wyncoop and al.[115] addresses the problem of learning dynamic Bayesian network (DBN) models to support the reinforcement of learning. It focuses on the learning regression tree (context-specific dependence) models of the conditional probability distributions of the DBNs. In this study, they introduce a regression tree algorithm in which each leaf node is modeled as a finite mixture of deterministic functions. This mixture is approximated via a greedy set cover.

In the work of Wang et al. [113], the auto-DBN model permits a segmentation of time series achieved first through detecting geometric structures transformed from time series, and then the model regions are found from the segmentation by designed finding strategies; in each found model region, a DBN model is established by existing structure learning methods; Finally, a model revisiting is developed to refine model regions and improve DBN models. These techniques provide a special mechanism to find accurate model regions and discover a sequence of DBNs with changing structures, which are adaptive to changing relations between multivariate time series.

Robinson et al. [84] define the non-stationary DBN model, present an MCMC sampling algorithm to efficiently learn the structure of an nsDBN and the times of non-stationarities (transition times) under different assumptions, and demonstrate the effectiveness of the algorithm on simulated data.

## 3.4 Evaluation of DBN structure learning algorithms

All works for structure learning in literature develop an experimental study to present the goodness of their DBN structure learning algorithms. There are popular tools for the evaluation of the learning from time-series data. In this section we present the existing methods to evaluate structure learning algorithms for the DBN.

### 3.4.1 Benchmarks

The first step to evaluate the performance of these algorithms is to assemble different standard benchmarks. In Pena's work [81], the authors involve learning databases of different sizes sampled from a partial model of the transcriptional regulatory network of *Saccharomyces cerevisiae*. The model involves 30 transcription or (variables) factors and 56 interactions (dependences) between them.

In [50], the authors ran experiments with their DBN learning approach in the coffee task [9] (containing 6 variables), Taxi task [25] (containing 10 variables), and a simplified autonomous guided vehicle (AGV) task [38] (containing 20 variables).

Gao et al. [35] select DBNs that have 8 variables to match the database as an optimization object. They use the database from BNT Structure Learning Package [61].

Rajapakse et al. [82] illustrate their method with experiments on synthetic data as well as on two real fMRI datasets from fMRI Data Center, Dartmouth College (fMRIDC, 2004) (containing 5 variables).

Wang et al. [113] use a real data set, it is about the interest rates of Australia, France, UK and US in 240 months from July 1980 to June 2000 [65] with three variables. Another real data set is about daily stock prices of ten aerospace companies from January 1988 to October 1991 [1] with six variables.

Song [91] conducted 3 experiments using synthetic data, gene expression data and EEG signals. they generate 8 different anchor transition matrices  $A_1^t : : : A_8^t$ , each of which corresponds to a random graph of node size  $n=50$  and average in degree of 2 (they have also experimented with  $n = 75$  and 100 which provides similar results).

Finally in Hwang's work [43], the writers gathered video data recorded at several intersections and used them to detect accidents at different intersections which have different traffic flows and intersection designs. A total of 70 video scripts are used in the experiments, containing 33 accidents, 10 situations similar to accidents, and 27 normal situations. The model studied in this work contains 10 variables.

Although all the works interested in DBN structure learning generally use small benchmarks size, there are some other recent works as try to improve this disadvantage of the DBN structure learning. They use large benchmarks to run their

algorithms.

Dojer [27] tested the time complexity of learning an optimal dynamic BN on microarray experiments data of Arabidopsis thaliana ( $\approx 23\,000$  genes). The running time computed was about 48 hours for the MDL score and 170 hours for the BDe score.

In order to judge the performance of their software, [114] have compared it to the Banjo library. As a realistic dataset, we have chosen the dataset attached as an example to the Banjo package, consisting of 20 variables and 2000 observations, published by Smith et al. [90].

Vinh et al [111] employ several synthetic data sets generated by different data generation schemes that have been used in some previous studies. As a realistic number of samples for microarray data, they generated data sets of between 30 and 300 samples. With the ground-truth network available, they count the number of true positive (TP), false positive (FP), true negative (TN) and false negative (FN) edges, and report two network quality metrics, namely sensitivity and specificity.

In 2012, Vin et al. [110] proposed a study where the experimental study has been conducted with about 1595 variables.

### 3.4.2 Evaluation metric

We remark in this study that, like the static BNs, the structure learning for DBNs studies use the score functions or comparison between graphs to evaluate the performance of algorithms. Contrary to the static BN, that has many metrics used for evaluation as we mentioned in section 2.4. Also, in DBNs studies, we didn't find standard metrics that can be used as the SHD in the static case.

[50] compared their active learning scheme with passive learning. In this case, they use the BIC and BDe scores to detect most of the refinements necessary to learn the true DBN model.

[35] used fitness function for a DBN, there exist two structure measure scales, namely, Bayesian Dirichlet metric (BD) and Bayesian information metric (BIC). In their paper, they use BD to measure network.

[82] used the square error  $e^2$  between the true connectivity structure  $C = \{c_{ij}\}$  and the estimated structure  $\phi = \{\phi_{ij}\}$  is measured by the square error between the elements of the matrices defining the structures:

$$e^2 = \frac{1}{2n^2} \sum_{i=1}^n \sum_{j=1}^n (c_{ij} - \phi_{ij})^2 \quad (3.5)$$

the elements  $c$  and  $\phi$  were scaled to the range  $[0,1]$  for comparison purposes with other approaches

Song et al evaluate the performance using an F1 score, which is the harmonic

mean of sensitivity and scores call-function in retrieving the true time-varying network edges [91].

In [43] they used the fitness function as a metric to evaluate the performance of population for the given problem. Fitness function, therefore, evaluates the suitability of a dynamic Bayesian network structure.

As we mentioned above most of the works use score functions for the evaluation step. However, there are a few works that use large datasets to evaluate their approaches. They use some other techniques of evaluation. Dojer and Wilczynski [27, 114] use computation running time as measure. Vinh et al. in [111] use three features: the percent sensitivity, the percent imprecision and the running time. Vinh et al. in [110] use node degree or connectivity degree distribution and runtime to compare the performance of their algorithms to others algorithms.

We remark that all these studies used the running time as a common metric for the evaluation. This is explained by the fact that in high dimension it is not enough to show the reliability of system but the time is an important factor for the evaluation process.

### 3.5 Conclusion

In this chapter, after presenting the key concepts of Dynamic Bayesian network, we reviewed the different structure learning studies of these models in literature. We also addressed the topic of how the different works proceeded to evaluate their learning algorithms.

In section 2.4, we focused on presenting an overview of the evaluation methods. The important conclusion reached in this study, is that the structure learning in the dynamic Bayesian Network has many disadvantages. This conclusion was given by many points:

Firstly, most of the works use networks with a small number of variables. This choice is explained by the fact that all works for learning structure are based on score and we showed previously that these methods of learning can't achieve a good performance when we increase the number of variables. we noted in previous section that the score function takes into account the equivalence of Markov in the evaluation. If an algorithm returns a graph  $G$  equivalent to  $G_0$ , both structures have the same score. This advantage is also a disadvantage, since if there is little data, it is possible to obtain a graph  $G$  with the same score  $G_0$ , but that is not in the same equivalence class. We think that this choice was taken by all these studies because of all the limitations mentioned before.

Secondly, all the studies about the DBN structure learning use their own benchmarks and techniques for evaluation. The problem in the dynamic case is that we

don't find previous works that provide details about the used networks and indicators of comparison. In addition, the access to the datasets and the source code is not always possible.

The last point, concerns the way of dealing with high dimension. The existing works, such as Vinh et al. [110], consider a restricted subclass of 2T-BN (eg. they learn only the inter-slice connectivity and ignore the intra-slice connectivity).

In the remaining chapters we will describe our contributions at the level of learning the DBN structure using the hybrid methods and we developed some techniques to evaluate these learning algorithms. These techniques are standard, and can be used with any other technique and on any standard large Benchmark. Also, they allow the amelioration of the evaluation quality with the insertion of a priori knowledge.





## Contributions







# 4

---

## Benchmarking dynamic Bayesian networks

### Sommaire

---

<b>4.1</b>	<b>Introduction</b>	<b>66</b>
<b>4.2</b>	<b>Generation of large k-TBN and simplified k-TBN</b>	<b>67</b>
4.2.1	Principle	67
4.2.2	Tiling the initial model	67
4.2.3	Tiling the transition model	68
<b>4.3</b>	<b>Evaluation of k-TBN generated by data and prior knowledge</b>	<b>69</b>
4.3.1	Principle	69
	PDAG with knowledge	70
	SHD(2T-BN) advantages	71
<b>4.4</b>	<b>Validation examples</b>	<b>72</b>
4.4.1	2-TBN Benchmark generation	72
	Implementation	72
	Toy example	72
4.4.2	SHD for 2-TBN	74
	Implementation	74
	Toy example	74
<b>4.5</b>	<b>Conclusion</b>	<b>75</b>

---

## 4.1 Introduction

During the last two decades, there has been an increasing interest in the Bayesian Network (BN) formalism [78, 42]. The BNs have been recognized as one of the most successful complete and consistent formalisms for the acquisition and representation of knowledge and for reasoning from incomplete and/or uncertain data. This success is due to the following factors: (1) their mathematical bases are rigorously justified; (2) they deal in an innate way with uncertainty (modeled as a joint probability distribution); (3) they are understandable (graphical representation); and (4) they take advantage of locality both in knowledge representation and during inference.

Learning the graphical part (i.e. the structure) of these models from data is an NP-hard problem. Many studies have been conducted on this subject [12, 11, 13]. Most of these works and their result interpretations use standard networks and common performance indicators, such as the approximation of the marginal likelihood of the obtained model or comparison of the resulting graph to the original graph given in benchmarking tasks with the help of the Structural Hamming Distance (SHD) proposed by Tsamardinos [104].

Dynamic Bayesian networks (DBNs) are a general and flexible model class for the representation of complex stochastic temporal processes [73]. Some structure learning algorithms have been proposed, adapting principles already used in "static" BNs. Comparing these algorithms is a difficult task because the evaluation techniques and/or the reference networks used differ across research works. The evaluation of these algorithms is also often restricted to networks with a small number of variables. However, with the static BNs, the evaluation was carried out using a large number of variables.

In this chapter, we highlight our two major contributions: section 4.2 presents an algorithm for generating large 2-TBN networks (which could be used as standard 2-TBN benchmarks) using the tiling approach (see section 2.4.2) in the dynamic case; section 4.3 presents an algorithm for the evaluation of a 2-TBN structure learning algorithm adapting the SHD measure (section 2.4.1) no more correct with temporal networks. Finally, we show in section 4.4 some validation examples to prove the advantages of our methods. Our work can be a useful tool for benchmarking any 2-TBN structure learning algorithm in a common framework.

## 4.2 Generation of large k-TBN and simplified k-TBN

### 4.2.1 Principle

In order to generate a large k-TBN or a simplified model, we propose generating two models  $M_0$  and  $M_{\rightarrow}$  from an initial static benchmark BN  $M$ .

First we use the Tsamardinos' work for the generation of realistic large bayesian networks by tiling (section 2.4.2). We have used tiling approach because in the case of k-TBN ( $M_0$  and  $M_{\rightarrow}$ ) can be seen as a model obtained by one first tiling to generate  $M_0$  from  $M$  and another one to generate  $M_{\rightarrow}$  from  $M_0$  when it is assumed that all the "intra slices" dependencies (dependencies between the variables in the same time slice) can be found at any time and the "inter slices" dependencies (dependencies between the variables in the successive time slices) can be repeated all the time.

In the case simplified k-TBN, we start with an empty model  $M$ . The tiling algorithm prevents the construction of "intra slices" dependencies and allows the "inter slices" dependencies. Our method consists in generating a large initial model  $M_0$  and its conditional probability distribution by tiling  $n$  copies of the initial model  $M$ . Then we use tiling again to generate  $M_{\rightarrow}$  and the transition probability distribution by tiling  $k$  copies of  $M_0$ .

Our suggestion is described in Algorithm 8. The complexity of the final k-TBN can be controlled by changing the number of tiling copies and the intra-connectivity  $c_i$  (used to generate  $M_0$ ) or the temporal connectivity  $c_t$  (used to generate temporal edges).

---

#### Algorithm 8 Generation of large k-TBNs (TileBN for kT-BN)

---

**Require:** BN  $M$ , number of copies  $n$ , intra-connectivity  $c_i$ , temporal connectivity  $c_t$

**Ensure:** Return initial  $M_0$  and transition models  $M_{\rightarrow}$

- 1:  $M_0 = \text{bn\_tiling}(M, n, c_i)$
  - 2:  $M_{\rightarrow} = \text{bn\_tiling}(M_0, k, c_t)$
- 

### 4.2.2 Tiling the initial model

In this first step of generation, the algorithm needs to enter a Bayesian network BN  $M$  with a number  $n$  of tiles desired as an input and  $\text{BN}_1; \dots; \text{BN}_n$  and an integer-valued connectivity parameter  $c_i$  controlling the number of introduced edges as an output of the algorithm. The algorithm returns the initial model characterized by a large Bayesian network consisting of tiles with the new edges between tiles. The joint probability distribution of the tiles is preserved. Also, our algorithm adds the number of level used for tiling as an optional parameter (cf. figure 4.1) (i.e. we give

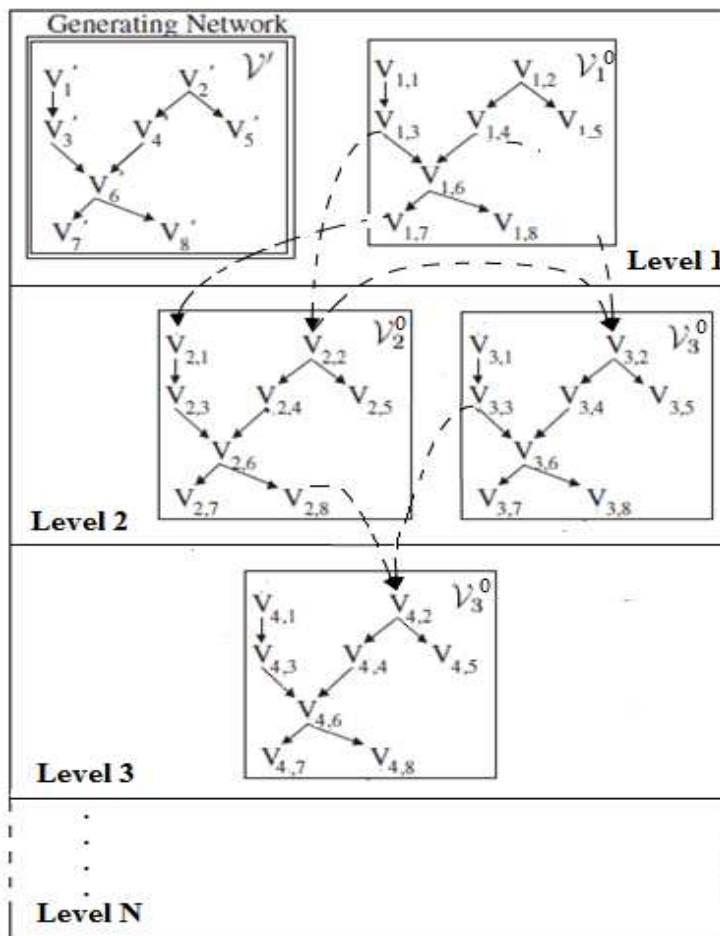


Figure 4.1: An example output of the initial model generation algorithm. The generating network is ASIA benchmark [59] shown in the top left of the figure.

$N$  as number of levels and for each level we give the number of tiles  $n$ , and as result we have automatically a large network with  $N*n$  tiles).

There are two changes in the case of the simplified  $k$ -TBN to construct the initial model: (1) the initial model  $M$  is an empty graph; (2) and the connectivity parameter  $c_i$  is equal to zero. With these constraints, we are sure that the initial model  $M_0$  couldn't contain intra-slice edges.

### 4.2.3 Tiling the transition model

The second step is to generate the transition model. It is applied for  $k$ -TBN and simplified  $k$ -TBN in the same way. The algorithm needs to enter the initial model as input. It provides  $k$  copies (i.e.  $c_t = k$  for example,  $c_t = 2$  when the model is 2-TBN) of the initial graph with interconnections between them. These interconnections are the temporal relationships between nodes in  $t$  and  $t+1$  time slices.

The interconnections must satisfy the property requirements of the tile algorithm in static and temporal cases. The tiles are topologically sorted according to their index and time, i.e., tile  $N_{j-1}$  is lower in the order than tile  $N_j$  and tile  $N_j^t$  is lower

in the order than tile  $N_j^{t+1}$ . Just as in the static case, TileBN for 2-TBN uses the number of the randomly selected interconnecting edges between the network in t-1 time slice and network in t time slice. TileBN for 2-TBN may add the edge  $V_{p,k}^{t-1} \rightarrow V_{q,j}^t$  if the following four requirements are satisfied:

1.  $p \neq q$ , i.e., the edge goes from a variable to another belonging to a tile downstream (not allowed any cycle) [106]. This requirement is always checked because  $V_{p,k}$  is in time slice t-1 and  $V_{q,j}$  is in time slice t.
2.  $Pa_j = \emptyset$ , i.e., the node  $V_j^t$  of the generating network that corresponds to the endpoint of the edge  $V_{t-1,j}$  is a minimal node in the generating network (it has no parents) [106].
3. If there exists another edge  $V_{p',k'}^t \rightarrow V_{q,j}^t$  to the same tile q then  $j = j'$ , i.e., for each tile q only one minimal node can receive tile interconnecting edges. The second and third requirements will be used to determine the probabilistic properties of the output network [106].
4. All the interconnections go from a variable in t-1 to a variable in t (not allowed to invert any temporal edges).

We notice that the first three requirements are satisfied in the static case, i.e., we use these proprieties to construct the initial model. However, the last requirement is used when the algorithm links between  $BN_t$  and  $BN_{t+1}$ .

The DBN definition is unrolled for  $k + 1$  time-slices where k denotes the temporal order of the Markov process. The DBN needs to be unrolled for this number of time-slices, because in the resulting unrolled network every CPD that needs to be learned will be represented with at least one copy (see appendix B algorithm 15).

## 4.3 Evaluation of k-TBN generated by data and prior knowledge

### 4.3.1 Principle

As 2-TBNs are defined by two graphs  $G_0$  and  $G_{\rightarrow}$ , we propose to evaluate the structural difference between the theoretical 2-TBN and a learned one by the pair of the structural Hamming distance for the corresponding initial and transition graphs as described in algorithm 9.

As seen in section 2.4 (SHD method), taking into account Markov equivalence by comparing PDAGs is important for the BN structure learning evaluation, but it's not sufficient for DBNs. Some temporal information (a priori knowledge) is used for 2-TBN structure learning and can be lost by reasoning with PDAGs.

In the case of 2-TBN, two different models are learnt. The first one  $M_0$  doesn't model temporal information, so the usual Tsamardinos' SHD (cf. section 2.4. algorithm 7) can be used.

The transition model  $M_{\rightarrow}$  represents the dependency relations between nodes of the same slice  $t$  or between the nodes of slices  $t$  and  $t + 1$ . In fact, here we have an important background (temporal) knowledge, edges between time slices are directed from  $t$  to  $t + 1$ . We then propose to adapt the Tsamardinos' SHD in order to deal with this additional knowledge as proposed in definition 2.2.17 for BNs. One temporal correction is applied for each PDAG in order to obtain a corrected  $PDAG_k$  compatible with the prior knowledge. The structural Hamming distance is then computed between these  $PDAG_k$ s.

---

**Algorithm 9** Structural Hamming distance for 2-TBNs
 

---

**Require:** Learned PDAG  $H_0$ ; Learned PDAG  $H_{\rightarrow}$ ; Original PDAG  $G_0$ ; Original PDAG  $G_{\rightarrow}$

**Ensure:** SHD values for initial and transition graphs

- 1:  $H_k = H_{\rightarrow}$
  - 2:  $G_k = G_{\rightarrow}$   
   % calculate  $SHD_0$
  - 3:  $SHD_0 = SHD(H_0, G_0)$   
   % Temporal correction for  $G_{\rightarrow}$
  - 4: Select randomly an undirected temporal edge from  $G_{\rightarrow}$
  - 5: Orient this temporal edge in  $G_k$
  - 6: Recursively apply the Meek rules in  $G_k$
  - 7: If there exist any unprocessed temporal edge then repeat 4, 5, 6.  
   % Temporal correction for  $H_{\rightarrow}$
  - 8: Select randomly an undirected temporal edge from  $H_{\rightarrow}$
  - 9: Orient this temporal edge in  $H_k$
  - 10: Recursively apply the Meek rules in  $H_k$
  - 11: If there exist any unprocessed temporal edge then repeat 8, 9, 10.  
   % Calculate  $SHD_{\rightarrow}$
  - 12:  $SHD_{\rightarrow} = SHD(H_k, G_k)$   
   % calculate SHD in 2-TBN
  - 13:  $SHD = (SHD_0, SHD_{\rightarrow})$
- 

### PDAG with knowledge

The PDAG definition (see definition.2.2.12) is not sufficient when the DAGs are 2T-BN. As we mentioned before, a 2T-BN contains intra-connections and inter-connections. The second type of edges can be not oriented and therefore lose some information when we search the Markov Equivalence class (PDAG). However, we know that all temporal edges are oriented as the direction of time, i.e. the temporal edge is from a node in  $t$  to a node  $t+1$ . So, with this knowledge, we can orient some other edges in PDAG and we named this new PDAG with temporal correction  $PDAG_k$ .

To establish this  $PDAG_k$  building method we inspired by the temporal correction from the solution proposed by Meek to solve the following problem that there is a complete causal explanation for  $M$  that are the causal relationships common to every

complete causal explanation consistent with respect to the background knowledge  $K$ ?

To construct this  $PDAG_k$  graph we have to follow three steps. (1) Construct the PDAG, for this step we have used the Meek approach optimized by Chickering (cf. section 2.2.4); (2) insert the temporal knowledge  $K$ , using the solution proposed by Meek to insert the background knowledge (the set of directed edges which are forbidden, the set of directed edges which are required). (3) apply Meek's four orientation rules (described in definition 2.2.17)

**Proposition 4.3.1** (*PDAG with temporal knowledge*) [100]

*Two 2T-BN structures are said to be equivalent if the set of distributions and temporal constraints that can be represented with one of those structures is identical to the set of distributions and temporal constraints that can be represented with the other.*

We notice that, we can use the same principal to calculate the SHD for k-TBN, but we have to respect the definitions of initial and transition networks.

### SHD(2T-BN) advantages

As we previously presented, the SHD for 2T-BN is a measure inspired from SHD metric in the static case. So, all the SHD advantages are advantages characterizing the SHD(2T-BN). In addition, the new SHD has other advantages when it is used in the temporal (dynamic) case.

SHD advantages in the static context:

- It uses a reference model to evaluate contrary to the evaluation techniques that use data. These are not reliable when there is little data;
- It is defined by the distance between two PDAGs and not between two DAGs; With this method the problem of differences in orientation between the two graphs in the same equivalence class are counted as errors is removed;
- It uses the hamming distance, which is simple to compute. It can maintain its performance when the number of variables is large;
- It doesn't use certain assumptions or conditions in calculating;

SHD advantages in the temporal context:

- It inserts the priori knowledge;
- It can run with large Dynamic Bayesian network;
- It uses the equivalence class to evaluate, contrary to other metrics used in structure learning DBN studies (mentioned in section 3.4.2) as the method based on sensitivity and specificity;
- It uses the temporal correction to construct  $PDAG_k$  (see section 2.2.17);



## 4.4 Validation examples

### 4.4.1 2-TBN Benchmark generation

#### Implementation

The TileBN algorithm is implemented in Mathworks Matlab and is published as part of the Causal Explorer system <sup>1</sup>. The input BN is specified in HUGIN format, and a simple parser was written to read HUGIN BNs in a custom Matlab format.

The measure of evaluation used by many BN learning algorithm is plotted over several networks. Specifically, the following networks were used in the analysis, listed in order to increase the number of variables (number of variables given in parenthesis): Asia(8), Child (20), Insurance (27), Mildew (35), Alarm (37), Barley (48), and HailFinder (56). These BN learning algorithms are compared to different networks that contain 1, 3, 5, and 10 tiles of the original networks mentioned previously.

For our implementation, we decided that the initial network  $M$  is provided in Hugin <sup>2</sup> format readable by several BN software such as Genie/Smile <sup>3</sup> or Matlab toolboxes such as Causal Explorer <sup>4</sup>.

As we mentioned previously, our method is inspired from tiling BN of Tsamardinos. So, we implement our method on the same tool used by tiling approach which is matlab (*causal explorer*) but we used some other tools as genie/smile. The implementation details of our method are shown in appendix B.

As Causal Explorer also proposes an implementation of `bn_tiling()`, this function allows generating a tiled standard network consisting of  $N$  copies of tiles with given connectivity parameter. We implemented our 2-TBN benchmark generation in Matlab using this function, and added another function in order to export an unrolled 2-TBN model in Hugin format. This exported 2-TBN function can then be used by several softwares for data generation and structure learning.

#### Toy example

Figure 4.2 illustrates our algorithm with ASIA [59] generating network, tiled 3 times for the initial model, with a maximum connectivity equal to 3. As we can see, we are now able to generate realistic 2-TBNs with very large domains by choosing any static and well-known benchmarks and controlling the complexity by increasing the number of tiles.

We recommend the reader to consult our Web page for Dynamic Bayesian

---

1. [http://www.dsl-lab.org/causal\\_explorer](http://www.dsl-lab.org/causal_explorer)

2. <http://www.hugin.com/>

3. <http://genie.sis.pitt.edu/>

4. [http://www.dsl-lab.org/causal\\_explorer/index.html](http://www.dsl-lab.org/causal_explorer/index.html)

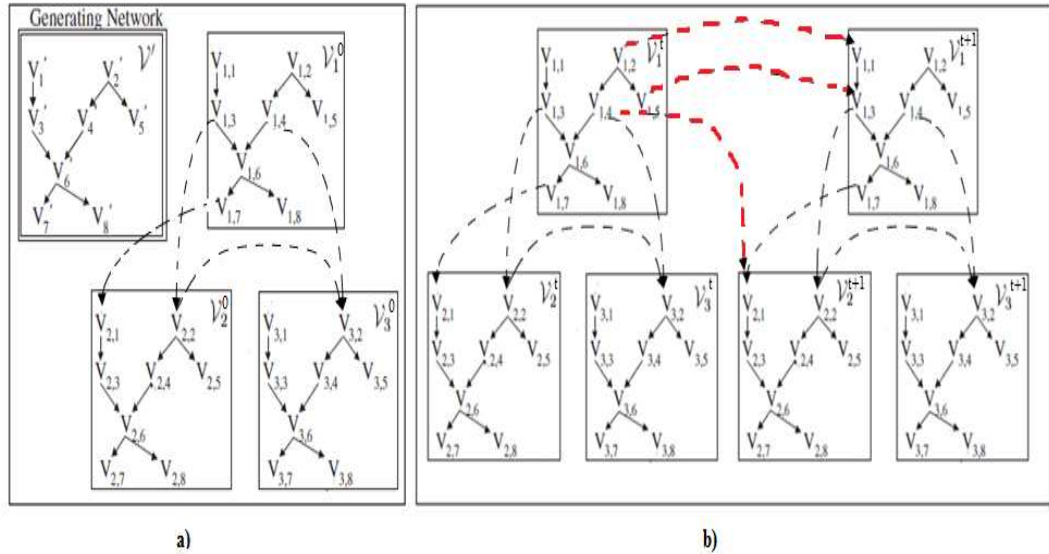


Figure 4.2: An example output of the 2-TBN generation algorithm. The generating network is ASIA benchmark [59] shown in the top left of figure a). a) The output of the initial network consists of three tiles of Asia with the addition of several intraconnecting edges shown with the dashed edges. b) The transition network output together with several added interconnecting edges are shown with the dashed red edges.

Networks	# vars	# edges	Temporal edges	# states (min-Max)	# parents (min-Max)	# neighbors (min-Max)
Asia_ $G_0$	8	8		2-2	0-2	1-4
Asia_ $G_{\rightarrow}$	16	21	5	2-2	0-3	1-5
Alarm_ $G_0$	37	46		2-4	0-4	1-6
Alarm_ $G_{\rightarrow}$	74	110	18	2-4	0-4	1-7
Hailfinder_ $G_0$	56	66		2-11	0-4	1-7
Hailfinder_ $G_{\rightarrow}$	112	156	24	2-11	0-4	1-17
Win95pts_ $G_0$	76	112		2-2	0-7	1-10
Win95pts_ $G_{\rightarrow}$	156	256	41	2-2	0-7	1-10
Andes_ $G_0$	223	338		2-2	0-6	1-12
Andes_ $G_{\rightarrow}$	446	820	144	2-2	0-6	1-12
Link_ $G_0$	724	1125		2-4	0-3	1-17
Link_ $G_{\rightarrow}$	1448	2530	280	2-4	0-4	1-17

Table 4.1: Generated benchmarks for Dynamic Bayesian Network

Network benchmarking (<https://sites.google.com/site/dynamicbenmhrking/>). We present all standard large benchmarks for DBN that were constructed by our method for temporal tiling (see table 4.1).

## 4.4.2 SHD for 2-TBN

### Implementation

We have implemented the SHD measure algorithm in a static case as described in section 2.3.3. This algorithm is considered as the reference for evaluation in many studies for DBN structure learning. Moreover, this implementation allows to convert DAG into PDAG based on Chikering approach. We have also implemented our new algorithms (SHD(2-TBN) and DAG to PDAG<sub>k</sub>) to evaluate the structure learning dynamic BN algorithm with the insertion the temporal correction.

We also implemented these algorithms in our structure learning platform in C++ using Boost graph<sup>5</sup> and ProBT<sup>6</sup> libraries. Experiments were carried out on a dedicated PC with Intel(R) Core(TM) 2.20 Ghz,64 bits architecture, 4 Gb RAM memory and under Windows 7.

### Toy example

In Figure 4.3, we show the interest of the temporal correction proposed for the SHD in section 2.4. We can notice that the PDAG corresponding to each 2-TBN loses some temporal information by un-orienting some temporal edges (resp. 1 and 2 for 2-TBN<sub>1</sub> and 2-TBN<sub>2</sub>).

Applying the structural Hamming distance without correction gives us a distance equal to 2 between transition graphs related to 2-TBN<sub>0</sub> and 2-TBN<sub>1</sub> (example (b) SHD calculated between PDAG<sub>0</sub> and PDAG<sub>1</sub>). This distance takes into account the missing edge between  $C_{t+1}$  and  $D_{t+1}$  and the missing orientation of the temporal edge between  $D_t$  and  $D_{t+1}$ . This distance increases to 4 between 2-TBN<sub>0</sub> and 2-TBN<sub>2</sub> (example (c) SHD calculated between PDAG<sub>0</sub> and PDAG<sub>2</sub>) because of the 2 modifications (one missing edge and one added), but also the 3 missing orientations in 2-TBN<sub>0</sub>.

The application of our temporal correction orients the temporal edges in the corrected PDAG<sub>0</sub>, PDAG<sub>1</sub> and PDAG<sub>2</sub>, we respectively orient 1, 2 and 0 (caused by V-structures in 2-TBN<sub>2</sub>) more edges in the transition graph PDAG<sub>k</sub> related to 2-TBN<sub>0</sub>, 2-TBN<sub>1</sub> and 2-TBN<sub>2</sub>.

Applying the SHD with correction gives us a distance equal to 1 between transition graphs related to PDAG<sub>k0</sub> and PDAG<sub>k1</sub>, which corresponds to the "true" missing edge, and a distance equal to 2 between transition graphs related to PDAG<sub>0</sub> and PDAG<sub>1</sub> (example (b)).

Applying the structural Hamming distance with correction gives us a distance equal to 1 between transition graphs related to PDAG<sub>k0</sub> and PDAG<sub>k2</sub>, which cor-

5. <http://www.boost.org/>

6. <http://www.probayes.com/index.php>

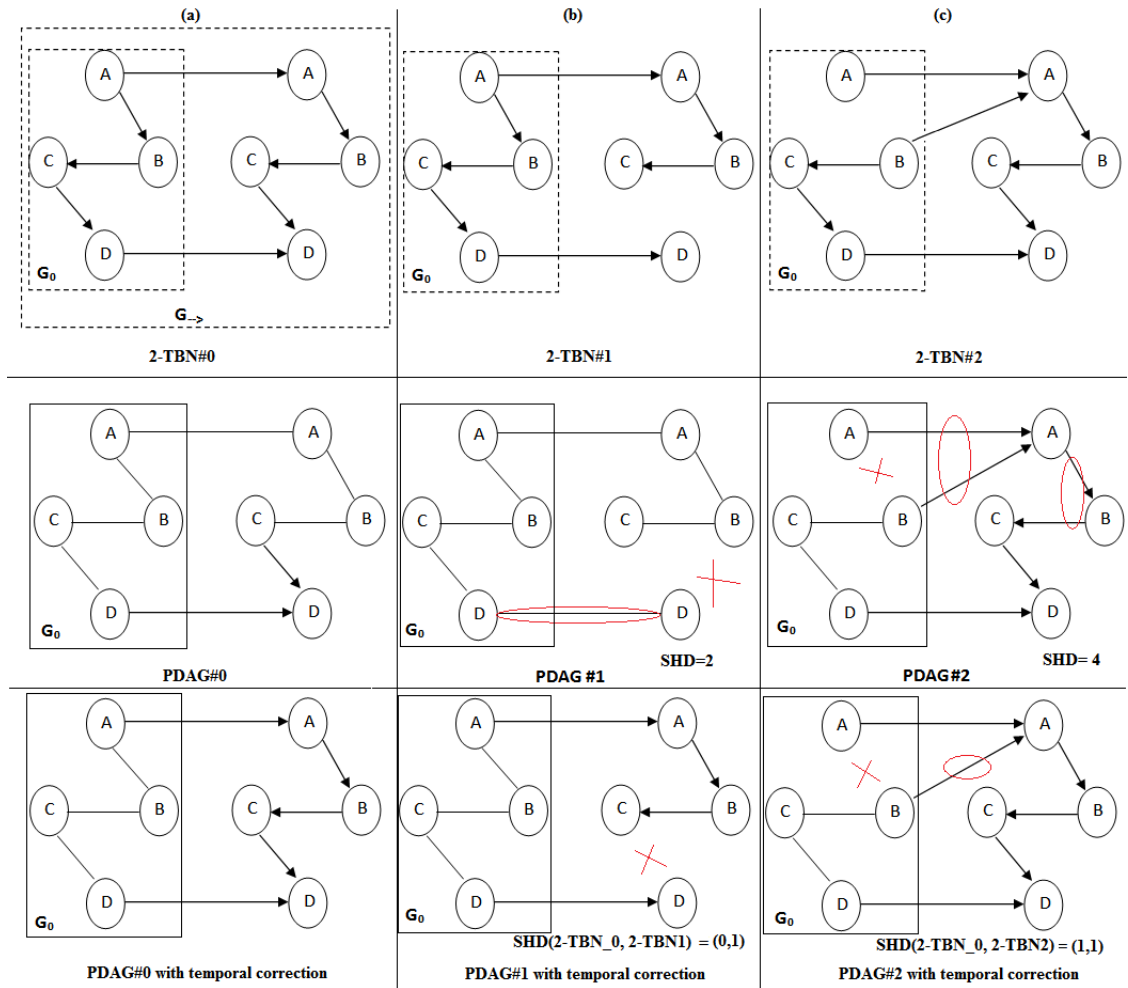


Figure 4.3: Examples of structural Hamming distance with or without temporal correction. A first 2-TBN and its corresponding PDAG and corrected PDAG<sub>k</sub> are shown in (a). (b) and (c) show another 2-TBN and their corresponding PDAG, and the structural Hamming distance with the first model

responds to the "true" missing edge, and a distance equal to 4 between transition graphs related to PDAG<sub>0</sub> and PDAG<sub>2</sub> (example (c)).

As we can see in these toy examples, that our SHD with temporal correction is better in term of structural comparison of dynamic bayesian networks. The improvement is given by the integration of knowledge (temporal knowledge) in our metric.

## 4.5 Conclusion

We focused in this chapter on providing tools for benchmarking dynamic Bayesian network structure learning algorithms. Our first contribution is a 2-TBN generation algorithm inspired from the Tiling technique proposed by [95]. Our algorithm is able to generate large and realistic 2-TBNs that can, then, be used for sampling datasets. These datasets can then feed any 2-TBN structure learning algorithm.

Our second contribution is a novel metric for evaluating the performance of these structure learning algorithms, by correcting the Structural Hamming distance proposed by [104] in order to take into account temporal background information.

We have proposed a website by providing some 2-TBNs benchmarks (graphs and datasets) in order to afford common evaluation tools for every researcher interested in 2-TBN structure learning<sup>7</sup>.

---

7. <https://sites.google.com/site/dynamicbenmharking/>

# Dynamic Max Min Hill Climbing (DMMHC)

## Sommaire

<b>5.1</b>	<b>Introduction</b>	<b>79</b>
<b>5.2</b>	<b>Dynamic Max Min Parents and children DMMPC</b>	<b>79</b>
5.2.1	Naïve $\overline{DMMPC}$ (neighborhood identification)	80
	Initial model	80
	Transition model	80
5.2.2	Optimized $\overline{DMMPC}$ (Neighborhood identification)	81
	Initial model	81
	Transition model	81
5.2.3	Toy example (naive $\overline{DMMPC}$ vs optimised $\overline{DMMPC}$ )	83
5.2.4	Symmetrical correction	84
<b>5.3</b>	<b>Dynamic Max Min Hill-Climbing DMMHC</b>	<b>85</b>
<b>5.4</b>	<b>DMMHC for simplified 2-TBN</b>	<b>87</b>
	Neighborhood identification	87
	Symmetrical correction	88
	DMMHC	88
<b>5.5</b>	<b>Time complexity of the algorithms</b>	<b>88</b>
	Naive DMMHC	89
	Optimised DMMHC	89
	Simplified DMMHC	89
<b>5.6</b>	<b>Related work</b>	<b>90</b>

**5.7 Conclusion** ..... **90**



## 5.1 Introduction

In this chapter, we focused on DBN structure learning with local search methods, by adapting the MMHC algorithm proposed by [104], one of the "state of the art" of this family algorithms. We believe that these local search algorithms can easily take into account the temporal dimension. Our objective is to propose a general structure learning algorithm that can work on 2-TBN models (with large search space) and use large benchmarks for validation tests.

We first present a version of DMMHC which is a simple adaptation of MMHC algorithm on 2-TBN models in section 5.2.1. Then, we optimized our approach by the use of temporal constraints in section 5.2.2. In addition we defined another version of the DMMHC that can run on simplified k-TBN models in section 5.4. We described, the complexity of each version of the DMMHC in section 5.5 and enumerated the advantages and disadvantages of our approach compared to others existing approaches in literature.

## 5.2 Dynamic Max Min Parents and children DMMPC

In 2T-BN models, temporality is limited by the first order Markov assumption. We think that local search algorithms can easily take into account this temporal constraint. We propose here a new DBN structure learning algorithm inspired from local search methods, by adapting the MMHC algorithm described in the previous section.

As seen in section 3.2, the 2-TBN structure is defined by two independent graphs  $G_0$  and  $G_{\rightarrow}$ . Inspired from the MMHC algorithm detailed in section 2.3.3, our Dynamic MMHC algorithm proposes to identify independently these graphs by applying a greedy search algorithm (adapted by [32] for 2T-BN) constrained with local informations. This information is provided by the identification of the neighborhood  $Ne_0$  (resp.  $Ne_{\rightarrow}$ ) of each node in  $G_0$  (resp.  $G_{\rightarrow}$ ).

By mimicking the procedure decomposition of MMHC described in section 2.3.3, our local structure identification DMMPC will be decomposed into two tasks: the neighborhood identification itself ( $\overline{\text{DMMPC}}$ ) completed by a symmetrical correction. We notice here that because of the non-symmetry of temporality, our local structure identification will be able to automatically detect some directed parent or children relationships if corresponding variables do not belong to the same time slice.

The following sub-sections described separately each of the proposed procedures, from the inner ones (the naive  $\overline{\text{DMMPC}}$ , the optimised  $\overline{\text{DMMPC}}$  and DMMPC) to the final one (DMMHC).



### 5.2.1 Naïve $\overline{\text{DMMPC}}$ (neighborhood identification)

Our  $\overline{\text{DMMPC}}$  algorithm consists of two phases detailed in Algorithm 10 dedicated to the identification of the neighborhood  $Ne_0$  (resp.  $Ne_+$ ) of a target variable  $T$  in  $G_0$  (resp.  $G_{\rightarrow}$ ).

**Initial model**  $\mathbf{X}[0]$  and  $\mathbf{X}[1]$  respectively denote the variables  $\mathbf{X}$  for  $t = 0$  and  $t = 1$ . We remind that the 2T-BN model is first-order Markov. Hence, it is possible that the neighborhood  $Ne_0$  of a variable  $T$  in  $\mathbf{X}[0]$  can belong to  $\mathbf{X}[0]$  and  $\mathbf{X}[1]$ .

Let us define  $CPC_0$  the parents or children of  $T$  in slice 0 and  $CC_1$  the children of  $T$  in slice 1.

In  $\overline{\text{DMMPC}}$ , we propose to use the static MMPC algorithm with the candidate variables  $ListC_0 = \mathbf{X}[0] \cup \mathbf{X}[1] \setminus \{T\}$  in order to identify  $Ne_0$ .  $Ne_0$  contains variables included in  $ListC_0$ . We notice that, in this case, the MaxMinHeuristic function returns the maximum over all variables of the minimum association with  $T$  relative to  $Ne_0$  (found), and the variable that belongs to  $ListC_0$  that achieves the maximum. Because of temporal information, we will then be able to separate  $Ne_0 = CPC_0 \cup CC_1$ .

**Transition model** In the same way,  $\mathbf{X}[t-1]$ ,  $\mathbf{X}[t]$  and  $\mathbf{X}[t+1]$  respectively denote the variables  $\mathbf{X}$  for times  $t-1$ ,  $t$  and  $t+1$ .

$Ne_+$  of a variable  $T$  in  $\mathbf{X}[t]$  can belong to  $\mathbf{X}[t-1]$ ,  $\mathbf{X}[t]$  and  $\mathbf{X}[t+1]$ . So let us define  $CPC_t$  the parents or children of  $T$  in slice  $t$ ,  $CC_{t+1}$  the children of  $T$  in slice  $t+1$  and  $CP_{t-1}$  the parents of  $T$  in slice  $t-1$ .

We propose to use the static MMPC algorithm with the candidate variables  $ListC = \mathbf{X}[t-1] \cup \mathbf{X}[t] \cup \mathbf{X}[t+1] \setminus \{T\}$  in order to identify  $Ne_+$ . We notice that, in this case, the MaxMinHeuristic function returns the maximum over all variables of the minimum association with  $T$  relative to  $Ne_+$  (found), and the variable that belongs to  $ListC$  that achieves the maximum. Because of temporal information, we will then be able to separate  $Ne_+ = CPC_t \cup CC_{t+1} \cup CP_{t-1}$ .

---

#### Algorithm 10 $\overline{\text{DMMPC}}(T, D)$

---

**Require:** target variable ( $T$ ); Data ( $D$ )

**Ensure:** neighborhood of  $T$  in  $G_0$  ( $Ne_0$ ) and in  $G_{\rightarrow}$  ( $Ne_+$ )

---

```

% search  $Ne_0$  of  $T$  in  $t = 0$ 
1:  $ListC_0 = \mathbf{X}[0] \setminus \{T\} \cup \mathbf{X}[1]$ 
2:  $Ne_0 = \overline{\text{MMPC}}(T, D, ListC_0)$ 
% search  $Ne_+$  of  $T$  in  $t > 0$ 
3:  $ListC = \mathbf{X}[t-1] \cup \mathbf{X}[t] \setminus \{T\} \cup \mathbf{X}[t+1]$ 
4:  $Ne_+ = \overline{\text{MMPC}}(T, D, ListC)$ 

```

---

### 5.2.2 Optimized $\overline{DMMPC}$ (Neighborhood identification)

The purpose of the following subsection is to redefine the previous method by reducing the computational complexity result of section 5.2.1. To this end, we relied on some properties:

- 2-TBN model is a first Markov order: the neighborhoods of a node in  $t$  time slice belongs to 3 time slices ( $t-1, t, t+1$ ).
- The orientations of all temporal edges are from  $t-1$  to  $t$  time slices or from  $t$  to  $t+1$  time slices.
- The identification of these neighborhoods can be done in 3 steps: identification of  $CPC_t$  then  $CP_{t-1}$  then  $CC_{t-1}$  (this property is proposed by this thesis, we give its proof later in the appendix C)

**Initial model** As mentioned in section 3.2.2, the initial model is a static BN representing the initial joint distribution of the process  $P(X[t = 0])$  and consisting of a direct acyclic graph (DAG)  $G_0$  containing the variables  $X[t = 0]$ .

In this case, it is not needful to identify the temporal dependences (with  $X[t = 1]$ ) which concern only the transition model. For this reason, we propose in this optimized version using the static MMPC algorithm with the candidate variables  $ListC_0 = X[0] \setminus \{T\}$  (instead of  $X[0] \cup X[1] \setminus \{T\}$ ) in order to restrict this step to the identification of the neighborhoods  $Ne_0$  of a variable  $T$  in this specific time slice  $X[0]$ .

**Transition model** In this case,  $X[t-1]$ ,  $X[t]$  and  $X[t+1]$  respectively denote the variables  $X$  for times  $t-1$ ;  $t$  and  $t+1$ . Let us define  $CPC_t$  the parents or children of  $T$  in slice  $t$ ,  $CC_{t+1}$  the children of  $T$  in slice  $t + 1$  and  $CP_{t-1}$  the parents of  $T$  in slice  $t - 1$ . According to the proposition 5.2.1 (the proof of this proposition given by the appendix C), contrary to the naive method we will separate the set of  $Ne_+$  on  $CPC_t$ ,  $CC_{t+1}$  and  $CP_{t-1}$  of a variable  $T$  in  $X[t]$ . Our algorithm tries to find the candidate parents and/or children through separate steps.

In the first step, we start by a forward phase that searches for the candidates that belong to  $CPC_t$  using the static MMPC algorithm with the candidate variables  $ListC_t = X[t] \setminus \{T\}$ . We notice that, the MaxMinHeuristic function returns the maximum over all variables of the minimum association with  $T$  relative to  $CPC_t$  (found), and  $X$  the variable that belongs to  $ListC_t$  that achieves the maximum. The second Backward phase will reveal if there is any independence between every  $S$  variable in  $CPC_t$  and  $T$  knowing all subsets in  $CPC_t$ , and if it is true, this phase will remove  $S$  from the  $CPC_t$  as a false positive.

In the second step, we start by a forward phase that searches the candidates that belong to  $CP_{t-1}$  using the static MMPC algorithm with the candidate variables

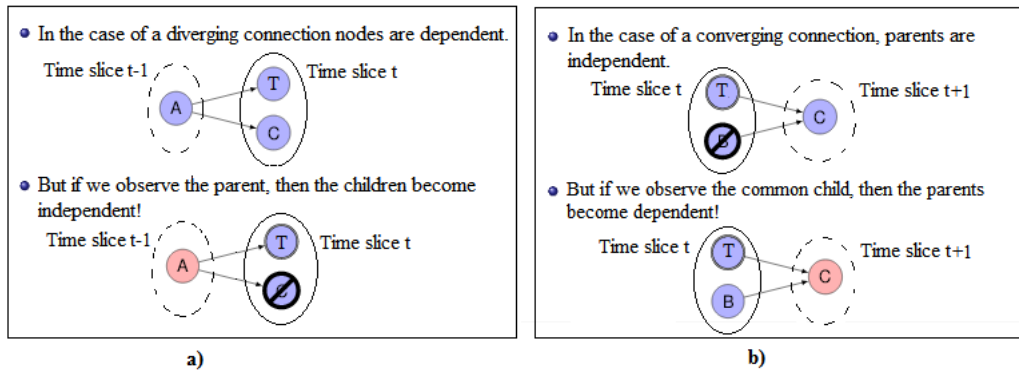


Figure 5.1: Conditional dependence. (a) divergence case. (b) convergence case

$ListC_{t-1} = X[t-1]$ . We notice that, the MaxMinHeuristic function returns the maximum over all variables of the minimum association with T relative to  $CPC_t \cup CP_{t-1}$  (found), and X the variable that belongs to  $ListC_{t-1}$  that achieves the maximum. The second phase will discover if there are an independence between every variable  $A \in CP_{t-1}$  or  $C \neq T \in CPC_t$  and T knowing all subset in  $CPC_t \cup CP_{t-1}$  (divergence case), and if it is true, this phase will remove A or C respectively from the  $CP_{t-1}$  or  $CPC_t$  as a false positive dependence (cf. figure 5.1.a).

In the third step, we start by a forward phase that searches the candidates that belong to  $CC_{t+1}$  using the static MMPC algorithm with the candidate variables  $ListC_{t+1} = X[t+1]$ . We notice that, the MaxMinHeuristic function returns the maximum over all variables of the minimum association with T relative to  $CPC_t \cup CC_{t+1}$  (found), and X the variable that belongs to  $ListC_{t+1}$  that achieves the maximum. The second phase will discover if there are an independence between every C variable in  $CC_{t+1}$  and T knowing all subsets in  $CPC_t \cup CC_{t+1}$  (convergence case), and if it is true, this phase will remove C from the  $CC_{t+1}$  as a false positive dependence (cf. figure 5.1.b).

**Proposition 5.2.1** *Let T be a target variable and D the learning data, if we consider having an infinite amount of data or results of statistical tests given by an oracle, we claim that our OptimisedDMMPC algorithm will identify the same neighborhood than the Naive one, in a reduced number of iterations.*

In the  $\overline{DMMPC}$  algorithm, there are two principal phases (Forward and Backward). We can show that the number of iterations in Forward and Backward steps are smaller for our optimised algorithm. This reduction was obtained by some optimisations (in red in the following equations) used in algorithm 11 and proved in appendix C.

$$Forward\_NaiveDMMPC(\max_{X \in V_t \setminus TUV_{t+1}UV_{t-1}} MinAssoc(X; T | CPC_t \cup CC_{t+1} \cup CP_{t-1})) \quad (5.1)$$

$$\simeq Forward\_optimisedDMMPC(\max_{X \in V_t \setminus TUV_{t+1}UV_{t-1}} MinAssoc(X; T | CPC_t \cup CC_{t+1} \cup CP_{t-1})) \quad (5.2)$$

$$\simeq CPC_{t\_found} \cup CC_{t+1\_found} \cup CP_{t-1\_found} \quad (5.3)$$

Where

$$\text{Forward\_optimisedDMMPC}(\max_{X \in V_t \setminus T} \text{MinAssoc}(X; T | CPC_{t\_found})) = CPC_{t\_found}$$

$$\text{Forward\_optimisedDMMPC}(\max_{X \in V_{t+1}} \text{MinAssoc}(X; T | CPC_{t\_found} \cup CC_{t+1\_found}))$$

$$= CC_{t+1\_found}$$

$$\text{Forward\_optimisedDMMPC}(\max_{X \in V_{t-1}} \text{MinAssoc}(X; T | CPC_{t\_found} \cup CP_{t-1\_found}))$$

$$= CP_{t-1\_found}$$

And

$$\text{Backward\_NaiveDMMPC}(\text{Assoc}_{X \in CPC_t \cup CC_{t+1} \cup CP_{t-1}}(X; T | S \subset CPC_t \cup CC_{t+1} \cup CP_{t-1})) \quad (5.4)$$

$$\simeq \text{Backward\_optimisedDMMPC}(\text{Assoc}_{X \in CPC_t \cup CC_{t+1} \cup CP_{t-1}}(X; T | S \subset CPC_t \cup CC_{t+1} \cup CP_{t-1})) \quad (5.5)$$

$$\simeq CPC_t \cup CC_{t+1} \cup CP_{t-1} \quad (5.6)$$

Where

$$\text{Backward\_optimisedDMMPC}(\text{Assoc}_{X \in CPC_{t\_found}}(X; T | S \subset CPC_{t\_found})) = CPC_t$$

$$\text{Backward\_optimisedDMMPC}(\text{Assoc}_{X \in CC_{t+1\_found}}(X; T | S \subset CPC_{t\_found} \cup CC_{t+1\_found}))$$

$$= CC_{t+1}$$

$$\text{Backward\_optimisedDMMPC}(\text{Assoc}_{X \in CP_{t-1\_found}}(X; T | S \subset CPC_{t\_found} \cup CP_{t-1\_found}))$$

$$= CP_{t-1}$$

### 5.2.3 Toy example (naive $\overline{\text{DMMPC}}$ vs optimised $\overline{\text{DMMPC}}$ )

In this section, we provide an example trace of  $\overline{\text{DMMPC}}$  with a toy example, the extended Umbrella network shown in figure 5.2.

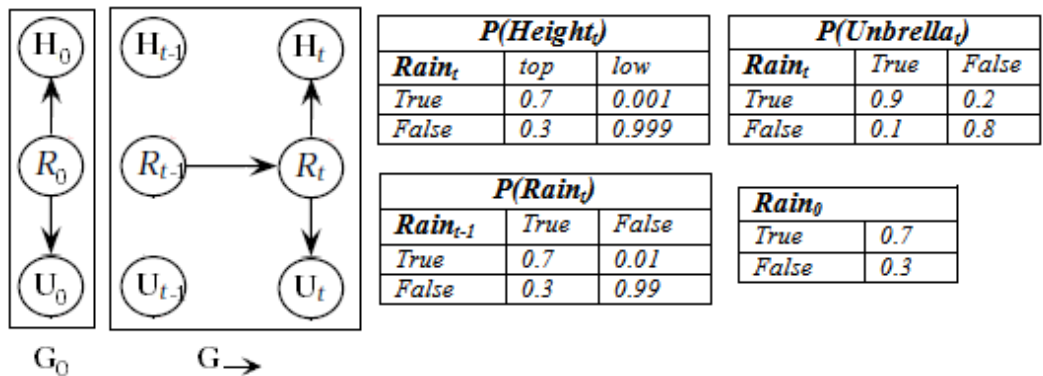


Figure 5.2: The extended Umbrella DBN

The dataset  $D$  is sampled from this network, with 20.000 time sequences of 8 time slices. The  $\text{Assoc}$  function used for independence measurement is the  $\chi^2$  test with  $\alpha = 0.05$ .

Figure 5.3 describes how the naive approach deals with each iteration involved in  $Ne_+$  identification for target node  $R_t$ . We can see that the naive  $\overline{DMMPC}$  progressively discovers the right neighborhood and is able at the end to separate the candidate parents (in blue) in slice  $t - 1$ , the candidate parents or children (in red) in slice  $t$  and the candidate children (in green) in slice  $t + 1$ .

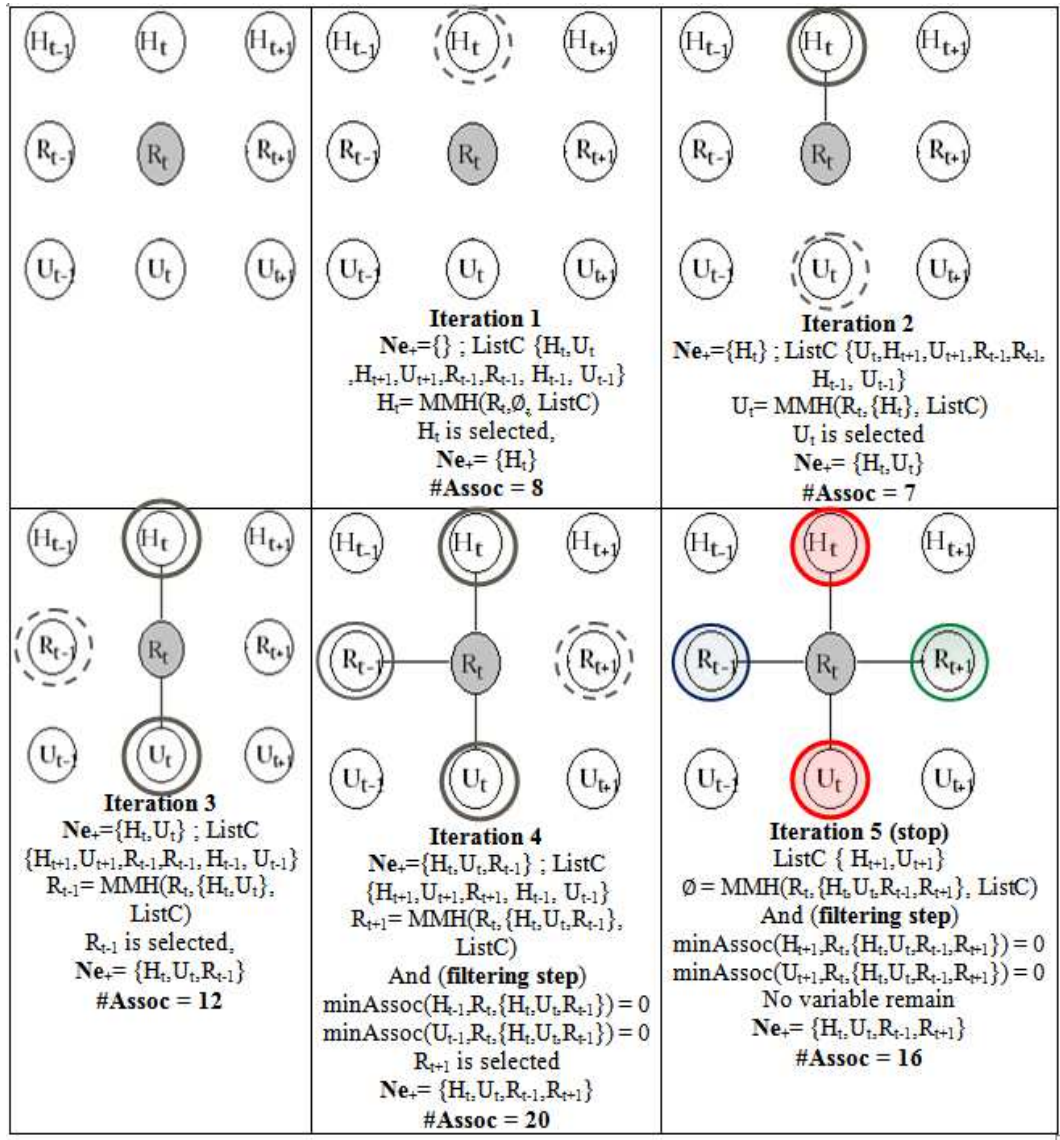
As proposed by [104], two optimizations are performed in order to control the algorithm complexity (estimated here by the number of calls to the *Assoc* function). The first, in MaxMinheuristic (*MMH*) corresponds to the fact that one part of the *Assoc* calls at iteration has already been computed in the previous iteration and can be saved. This optimization can be seen in iteration 2 where only 7 *Assoc* calls have been performed. The same optimization can be achieved in all the next iterations. The second optimization corresponds to the fact that if one variable in *ListC* reaches a minimal *Assoc* value equal to zero for a given conditioning set, then it will get this minimal association for all the other iterations. We can then discard this variable from *ListC* and save some other *Assoc* calls in the next iterations. This "filtering step" is illustrated in iteration 4 where  $H_{t-1}$  and  $U_{t-1}$  get a minimal association equal to zero and are discarded of *ListC* in iteration 5.

In figure 5.4 we show that the optimized  $\overline{DMMPC}$  can discover the right neighborhood in  $CPC_t$ ,  $CP_{t-1}$  and  $CC_{t+1}$  for target node  $R_t$ . In the previous paragraph, we mentioned that two optimizations are performed in order to control the algorithm complexity of the naïve method. In this optimized method (section 5.2.2), we added another optimization. To search for the neighborhood in  $CPC_t$  for target node  $R_t$ , we only need to use the variables in the  $t$  time slice ( $ListC_t$ ). So, the number of the *Assoc* calls is minimized. This optimization can be seen in iterations 1 and 2 where respectively only 2 and 1 *Assoc* calls have been computed. However, in the naïve method the iterations 1 and 3 make respectively 8 and 7 *Assoc* calls. We can see this optimization in all iterations involved in  $CP_{t-1}$  and  $CC_{t+1}$  identification for the target node  $R_t$ .

## 5.2.4 Symmetrical correction

As its static counterpart, DMMPC algorithm described in algorithm 12 has to perform a symmetrical correction. Because of the non-symmetry of temporality, we have to adapt this correction. When  $t = 0$ , we have to apply the symmetrical correction on  $CPC_0$  and  $CC_1$  separately because for all  $X \in CC_1$ ,  $X$  doesn't belong to slice  $t = 0$  but in slice  $t = 1$  and its temporal neighborhoods are given by  $Ne_+(X)$ . The DMMPC algorithm needs as inputs the result of (naïve or optimized)  $\overline{DMMPC}$ . In the case of initial model, the symmetrical correction is as follows.

1. Divide the  $Ne_0$  into two subsets  $CPC_0$  and  $CC_1$
2. For all  $X \in X[0]$  and  $X \in CPC_0(T)$ , find if  $T \notin CPC_0(X)$  then  $X$  is removed

Figure 5.3: Example trace of naive  $\overline{DMMPC}$ 

from  $CPC_0(T)$ .

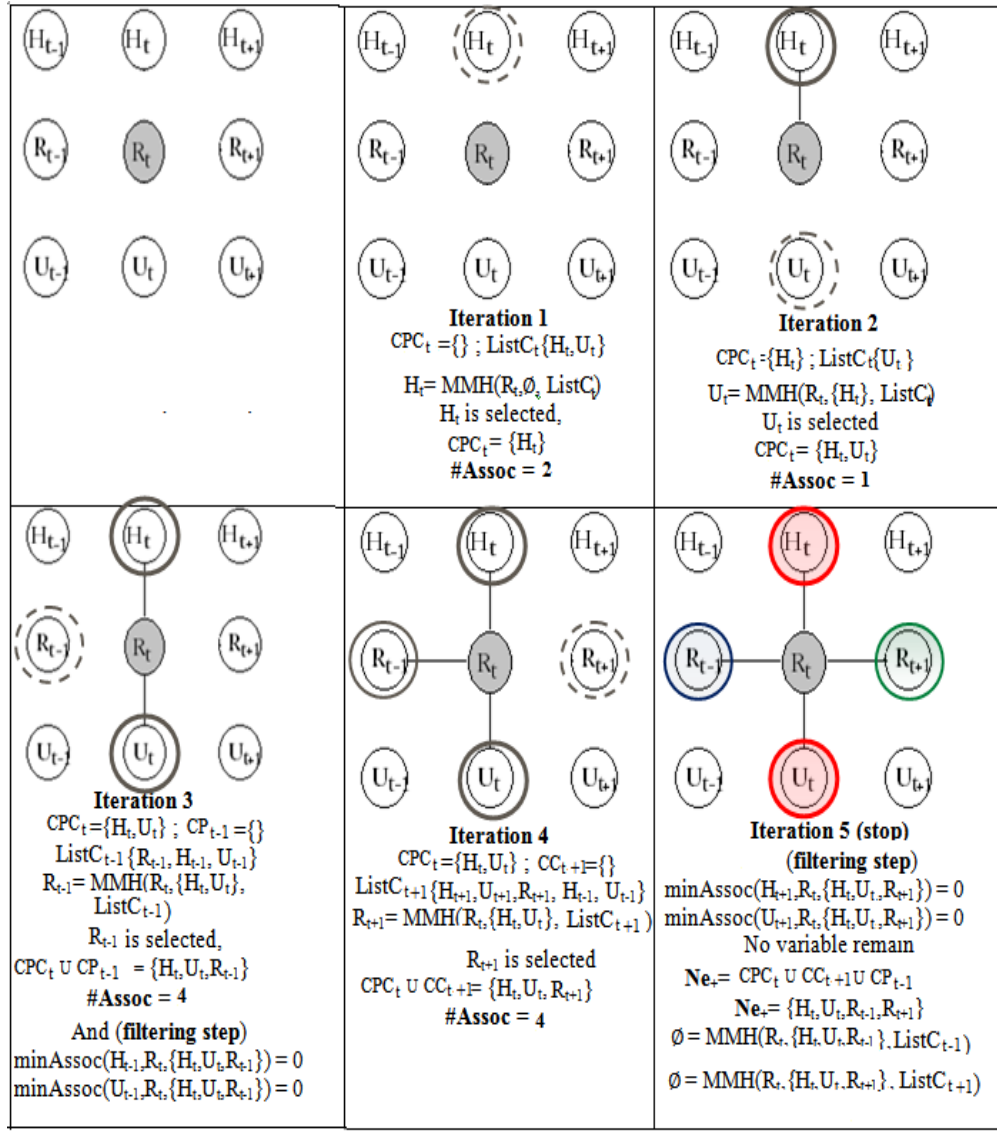
3. For all  $X \in X[1]$  and  $X \in CC_1(T)$ , find if  $T \notin Ne_+(X)$  then  $X$  is removed from  $CC_1(T)$ . It uses  $Ne_+$ , because  $X$  belongs to the time slice  $1 > 0$  and the set of all parents and children of  $X$  exists in  $Ne_+$ .

In the case of transition model, the symmetrical correction is as follows.

1. The separation of the subsets  $CPC_t, CC_{t+1}$  and  $CP_{t-1}$  is not used.
2. For all  $X \in Ne_+(T)$ , finds if  $T \notin Ne_+(X)$  then  $X$  is removed from  $Ne_+(T)$ .
3. Divide the  $Ne_+$  into three subsets  $CPC_t, CP_{t-1}$  and  $CC_{t+1}$  depending on their membership to the time slices respectively  $t, t-1, t+1$ .

## 5.3 Dynamic Max Min Hill-Climbing DMMHC



Figure 5.4: Example trace of optimised  $\overline{DMMPC}$ 

This phase is dedicated to the global model optimization. Our Dynamic MMHC algorithm described in Algorithm 13 proposes to identify independently the graphs given by the DMMPC phase by applying a greedy search algorithm (adapted by [32] for a 2-TBN) constrained with local information. This information is provided by the identification of the neighborhood  $Ne_0$  (resp.  $Ne_+$ ) of each node in  $G_0$  (resp.  $G_{\rightarrow}$ ).

As its static counterpart, DMMHC will consider adding an edge during the greedy search, if and only if the starting node is in the neighborhood of the ending node.  $G_0$  learning only concerns the variables in slice  $t = 0$ , so we can restrict the `add_edge` operator only to variables found in a  $CPC_0$  of another variable.

$G_{\rightarrow}$  is a graph with variables in slices  $t - 1$  and  $t$  but this graph only describes the temporal dependencies between  $t - 1$  and  $t$  and "inner" dependencies in  $t$ . So we also restrict our operators in order to consider adding edges with these constraints

and we don't authorize reversing the temporal edges.

## 5.4 DMMHC for simplified 2-TBN

In simplified kT-BN , the temporality is constrained by the k order Markov assumption. There are two cases for this model when  $k=2$  and  $k \geq 3$ . The first case is the simplified k-TBN. It allows representing the transition network with only the inter-connectivity between  $t-1$  and  $t$  time slices and we can apply the first order Markov assumption in this model. We used this case in our work DMMHC for simplified 2-TBN. The second case represents the transition network with the inter-connectivity between  $t-s$  and  $t$  time slices where  $k > s$ .

We think that our DMMHC algorithm can easily take into account these temporal constraints. We propose here an extension of DMMHC named simplified DMMHC for 2-TBN structure learning.

As seen in section 3.2, the 2-TBN simplified structure focuses only on  $G_{\rightarrow}$ . This kind of model is limited to contain only inter time slice arcs. Inspired by DMMHC algorithm, our new simplified DMMHC algorithm proposes to identify  $G_{\rightarrow}$  graph by applying the different phases of DMMHC shown in the previous sections.

**Neighborhood identification** In the first phase of neighborhood identification, we need only to identify the  $Ne_+$  for all variables  $T$  in  $t$  time slice. As we previously defined the set of  $Ne_+(T) = CP_{t-1}(T) \cup CPC_t(T) \cup CC_{t+1}(T)$ . In the simplified 2-TBN we have many properties to define (figure 5.5):

1. we don't need to identify the  $CPC_t(T)$  candidates because we are interested in the inter connectivity only.
2. we don't need to identify the  $CC_{t+1}(T)$  candidates because if  $X \in CC(T) \Rightarrow T \in CP(X)$  where  $X$  in  $t$  time slice (the temporal symmetrical correction result).
3.  $Ne_+(T) = CP_{t-1}(T) \cup CPC_t(T) \cup CC_{t+1}(T)$ . We notice that with the use of the properties 1 and 2  $\Rightarrow Ne_+(T) = CP_{t-1}(T)$ .
4. To identify all  $X$  in  $CP_{t-1}(T)$ , we need to compute the  $MaxMinAssoc(X; T|CP_{t-1} \cup CPC_t)$ . We suggested below the proposition 5.4.1 to facilitate the computation (the proof of this proposition given by the appendix C):
5. The Backward phase of DMMHC allows to detect any false positives depending on the structure. But after the Forward phase we musn't find any one. So we can remove the Backward phase in DMMHC simplified.

**Proposition 5.4.1** *Let  $V_{t-1}$  the set of variables for time slice  $t-1$  and  $X \in V_{t-1}$ .*

$$\max_X \min Assoc(X; T|Ne_+(T)) = \max_X \min Assoc(X; T|CP_{t-1}) = \max_X Assoc(X; T|\emptyset) \quad (5.7)$$



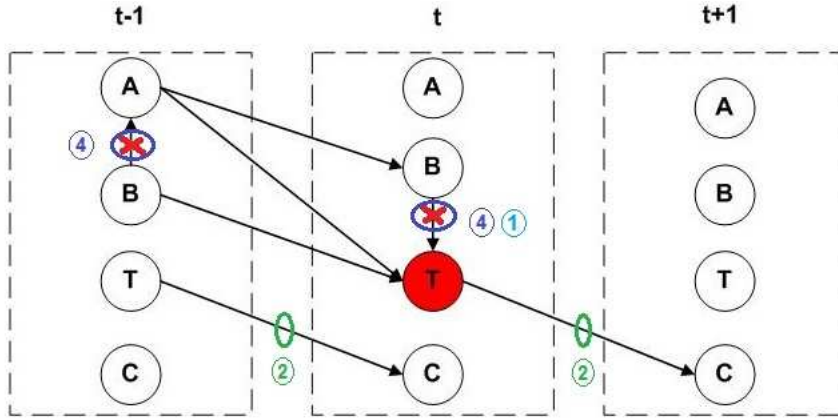


Figure 5.5: Example trace of simplified  $\overline{DMMPC}$

**Symmetrical correction** Contrarily to its static and dynamic counterpart, the simplified DMMPC algorithm doesn't need to perform a symmetrical correction. (1) in this case, DMMHC doesn't look for any  $CPC_t$  candidates. (2) Because of the non-symmetry of temporality, we have to adapt this correction. But, we assume that in propriety (2) in neighborhood identification the  $CC_{t+1}(T)$  is defined by  $CP_{t-1}(X)$  where  $X \in CC_{t+1}(T)$ .

**DMMHC** In the naïve and optimized cases, DMMHC applies a greedy search algorithm constrained with local informations. In the naïve and optimized DMMPC, contrary to the edges existing in intra-time slice, all temporal edges are oriented by DMMPC. DMMHC uses the results of DMMPC as inputs and orients the undirected edges. In the case of simplified 2-TBN model, we need only to identify the temporal edges, so all edges in the graph are oriented by the DMMPC algorithm and we don't need to use the DMMHC to orient other edges. However we need the DMMHC to delete the false temporal edges.

## 5.5 Time complexity of the algorithms

In this section, we calculated the time complexity of our algorithm. First we remind how the time complexity of MMPC was calculated in the static case. According to [104] the number of independence tests for all variables with the target conditioned on all subsets of CPC (target parents and children set) is bound by  $O(|V|.2^{|CPC|})$ , where  $|V|$  represents the number of all variables in the Bayesian network and  $|CPC|$  is the number of all variables in the parents/children target set. The overall cost of identifying the skeleton of the BN is  $O(|V|^2 2^{|PC|})$ , where PC is the largest set of parents and children overall variables in  $V$ .

**Naive DMMHC** In our dynamic (temporal) case, as for the static, we first identify the number of tests in  $\overline{DMMPC}$ . In this part we have two cases to present (i.e.  $t=0$  and  $t>0$ ). For  $t=0$ ,  $\overline{DMMPC}$  will calculate the association of all variables in time slices  $t=0$  and  $t=1$  with target in slice  $t=0$  conditioned on all subset of  $Ne_0$  (in the worst case). Thus, the number of tests in the case  $t=0$  is bounded by  $O(|2V|.2^{|Ne_0|})$ , where  $V$  is the set of all variables in a time slice. For  $t>0$  case, the number of tests is bounded by  $O(|3V|.2^{|Ne_+|})$ , because  $\overline{DMMPC}$  calculates the association of all variables in three time slices  $t, t-1, t+1$  with the target in slice  $t$  conditioned on all subsets of  $Ne_+$  (in the worst case).

We sum up, the total number of tests in both phases at  $t = 0$  and  $t > 0$  is bounded respectively by  $O(|2V|.2^{|Ne_0|})$  and  $O(|3V|.2^{|Ne_+|})$ . Thus, the total number of tests in both phases is bounded by  $O(|3V|.2^{|Ne_+|})$ . The overall cost of identifying the skeleton of the initial and transition models in dynamic Bayesian network (i.e., calling DMMPC with all targets in  $G_0$  and  $G_{\rightarrow}$ ) is  $O(|3V|^2.2^{|Ne|})$ , where  $Ne$  is the largest set of neighborhood (in  $t$  and  $t+1$  and  $t-1$ ) over all variables in the time slice  $t$ .

**Optimised DMMHC** For the optimized DMMPC, we start with  $t=0$ , DMMPC will calculate the association of all variables in time slices  $t=0$  with target in slice  $t=0$  conditioned on all subsets of  $CPC_0$  (in the worst case). Thus, the number of tests in the case  $t=0$  is bounded by  $O(|V|.2^{|CPC_0|})$ , where  $V$  is the set of all variables in a time slice. For  $t \geq 1$  case, the number of tests is bounded by  $O(|V|.2^{|CPC_t|} + |V|.2^{|CPC_t|+|CPC_{t-1}|} + |V|.2^{|CPC_t|+|CC_{t+1}|})$ , because DMMPC calculates the association of all variables in  $t$  or in  $t-1$  or  $t+1$  with the target in slice  $t$  conditioned on all subsets respectively of  $CPC_t$  or  $CPC_t \cup CP_{t-1}$  or  $CPC_t \cup CC_{t+1}$  (in the worst case).

To recapitulate, the total number of tests in both phases at  $t = 0$  and  $t > 0$  is bounded respectively by  $O(|V|.2^{|CPC|})$  and  $O(|V|.2^{2|CPC|})$ , where  $CPC$  is the largest set of neighborhood in one time slice ( $t$  or  $t+1$  or  $t-1$ ) over all variables in  $t$  (i.e.  $|Ne| \simeq 3*|CPC|$ ). Thus, the total number of tests in both phases is bounded by  $O(|V|.2^{2|CPC|})$ . The overall cost of identifying the skeleton of initial and transition models in dynamic Bayesian network (i.e., calling DMMPC with all targets in  $G_0$  and  $G_{\rightarrow}$ ) is  $O(|V|^2.2^{2|CPC|})$ .

**Simplified DMMHC** The simplified DMMHC deals with the transition model only.  $\overline{DMMPC}$  calculated the association of all variables in time slice  $t-1$  with target in slice  $t$  conditioned on empty set (in all cases). Thus, the number of tests in the case  $t$  is bounded by  $O(N)$ , where  $N$  is the set of all variables in a time slice.

We assume that, since we don't need to use the symmetrical correction and DMMHC, the overall cost of identifying the skeleton of transition model in dynamic Bayesian network (i.e., calling  $\overline{DMMPC}$  with all targets in  $G_{\rightarrow}$ ) is  $O(N^2)$ .

## 5.6 Related work

Many approaches for structure learning in DBN [32, 107, 108, 35, 112] are validated on benchmark models with about 10 variables. In a more general context, due to inherent limitations of the score-based structure learning methods, all these methods will have a very high complexity if the number of variables increases.

With the help of a local search, DMMHC is able to limit the search space in the final global optimization (GS). In this way, we can theoretically work in high dimensions like MMHC with static BNs. We can say that our approach outperforms the other approaches based on score, because contrary to these last kind of approaches, DMMHC can deal with large dimension and on complete search space.

Another way to deal with scalability is to restrict the class of 2T-BN by only considering parents of  $X_i[t]$  in time slice  $t-1$  (for 2T-BN) or any previous time slice (for kT-BN).

Dojer [27] proposes a score based method named polynomial time algorithm for learning this class of DBN (with BDe and MDL scores). Vinh et al. [111] proposed another polynomial time algorithm in the same context (equicardinality requirement) with other scoring function (MIT). Also, the authors proposed another contribution in this work consisting of a hybrid method with local Blanket identification (MIT-MMMB).

DMMHC and MIT based MMMB are both hybrid methods with local search and global optimization. When DMMHC tries to identify the candidate parents/children CPC set of a target variable in a 2T-BN, MIT based MMMB tries to identify the (more complex) Markov Blanket MB but in a restricted subclass of 2T-BNs. On the one hand, this assumption allows to simplify both MB discovery and global optimization. On the other hand, contrary to DMMHC, MIT based MMMB is not able to identify the intra-time dependencies. So, the hybrid method does exist in literature and it can deal with large dimension but it is executed on simplified search space. In addition, we have proposed another version of DMMHC which can work on this kind of models.

## 5.7 Conclusion

In this chapter, we proposed a new 2T-BN structure learning algorithm dealing with large networks. Inspired from the MMHC algorithm proposed by [104] for static BNs, our DMMHC algorithm is a local search algorithm dealing with local structure identification (DMMPC) and global model optimization constrained with this local information.

We have shown that the local structure identification can easily take into account the temporal dimension in order to provide some additional information about tem-

porality that can then be used with a greedy search in order to learn the global structure, the initial model  $G_0$ , and the transition one  $G_{\rightarrow}$ . As far as we know, no method capable to learn 2-TBN models with such approaches has been proposed previously.

We also presented an optimisation for our DMMHC named optimised DMMHC. It is a local improvement of our algorithm. We think that our scalability can be improved during the local structure identification using temporality constraints in DMMPC in order to decrease the number of *Assoc* calls and consequently decrease the running time that we will demonstrate in the next chapter dealing with our experimentation.

Furthmore, in this chapter, we suggested a new version of DMMHC, which work on simplified 2-TBN without intra-connectivity edges. We compared all the versions of DMMHC (naive DMMHC, optimised DMMHC and simplified DMMHC) for 2T-BN structure learning in terms of complexity and showed that we are able to decrease the complexity with the use of temporal constraints.

**Algorithm 11**  $\overline{\text{DMMP}}\text{C}(\text{Optimised}(T, D))$ **Require:** target variable ( $T$ ); Data ( $D$ )**Ensure:** neighborhood of  $T$  in  $G_0$  ( $Ne_0$ ) and in  $G_+$  ( $Ne_+$ )

---

```

% search  $Ne_0$  of  $T$  in  $t = 0$ 
1:  $ListC_0 = \mathbf{X}[0] \setminus \{T\}$ 
2:  $Ne_0 = \overline{\text{MMP}}\text{C}(T, D, ListC_0)$ 
% search  $Ne_+$  of  $T$  in  $t > 0$ 
3:  $CPC_t = \emptyset$  % parents children candidates for T in slice t
4:  $CP_{t-1} = \emptyset$  % only parents candidates for T in slice t-1
5:  $CC_{t+1} = \emptyset$  % only children candidates for T in slice t+1
% search  $Ne_+$  of  $T$  in  $t, t + 1$  and  $t - 1$ 
6:  $ListC_t = \mathbf{X}[t] \setminus \{T\}$ 
7:  $ListC_{t+1} = \mathbf{X}[t+1] \setminus \{T\}$ 
8:  $ListC_{t-1} = \mathbf{X}[t-1] \setminus \{T\}$ 
% search  $CPC_t$ 
% Phase I: Forward
9: repeat
10:  $\langle F, assocF \rangle = \text{MaxMinHeuristic}(T, CPC_t, ListC_t)$ 
11: if  $assocF \neq 0$  then
12:    $CPC_t = CPC_t \cup \{F\}$ 
13:    $ListC_t = ListC_t \setminus \{F\}$ 
14: end if
15: until  $CPC_t$  has not change or  $assocF = 0$  or  $ListC_t = \emptyset$ 
% Phase II: Backward
16: for all  $X \in CPC_t$  do
17:   if  $\exists S \subseteq CPC_t$  and  $assoc(X; T|S) = 0$  then
18:      $CPC \setminus \{X\}$ 
19:   end if
20: end for
% search  $CP_{t-1}$ 
% Phase I: Forward
21: repeat
22:  $\langle F, assocF \rangle = \text{MaxMinHeuristic}(T, CPC_t \cup CP_{t-1}, ListC_{t-1})$ 
23: if  $assocF \neq 0$  then
24:    $CP_{t-1} = CP_{t-1} \cup \{F\}$ 
25:    $ListC_{t-1} = ListC_{t-1} \setminus \{F\}$ 
26: end if
27: until  $CP_{t-1}$  has not change or  $assocF = 0$  or  $ListC_{t-1} = \emptyset$ 
% Phase II: Backward
28: for all  $X \in CPC_t \cup CP_{t-1}$  do
29:   if  $\exists S \subseteq CPC_t \cup CP_{t-1}$  and  $assoc(X; T|S) = 0$  then
30:      $CP_{t-1} \setminus \{X\}$  or  $CPC_t \setminus \{X\}$ 
31:   end if
32: end for
% search  $CC_{t+1}$ 
% Phase I: Forward
33: repeat
34:  $\langle F, assocF \rangle = \text{MaxMinHeuristic}(T, CPC_t \cup CC_{t+1}, ListC_{t+1})$ 
35: if  $assocF \neq 0$  then
36:    $CC_{t+1} = CC_{t+1} \cup \{F\}$ 
37:    $ListC_{t+1} = ListC_{t+1} \setminus \{F\}$ 
38: end if
39: until  $CC_{t+1}$  has not change or  $assocF = 0$  or  $ListC_{t+1} = \emptyset$ 
% Phase II: Backward
40: for all  $X \in CC_{t+1}$  do
41:   if  $\exists S \subseteq CPC_t \cup CC_{t+1}$  and  $assoc(X; T|S) = 0$  then
42:      $CC_{t+1} \setminus \{X\}$ 
43:   end if
44: end for
45:  $Ne_+ = CPC_t \cup CP_{t-1} \cup CC_{t+1}$ 

```

---

**Algorithm 12** DMMPC( $T, D$ )**Require:** target variable ( $T$ ); Data ( $D$ )**Ensure:** neighborhood of  $T$  in  $G_0$  ( $Ne_0$ ) and  $G_{\rightarrow}$  ( $Ne_+$ )

---

```

1:  $Ne_0 = \overline{\text{DMMPC}}(T, D).Ne_0$ 
2:  $Ne_+ = \overline{\text{DMMPC}}(T, D).Ne_+$ 
   % symmetrical correction  $Ne_0$  of  $T$  in  $t = 0$ 
3:  $CPC_0 = Ne_0 \cap \mathbf{X}[0]$ 
4:  $CC_1 = Ne_0 \cap \mathbf{X}[1]$ 
5: for all  $X \in CPC_0$  do
6:   if  $T \notin \overline{\text{DMMPC}}(X, D).Ne_0$  then
7:      $CPC_0 = CPC_0 \setminus \{X\}$ 
8:   end if
9: end for
10: for all  $X \in CC_1$  do
11:   if  $T \notin \overline{\text{DMMPC}}(X, D).Ne_+$  then
12:      $CC_1 = CC_1 \setminus \{X\}$ 
13:   end if
14: end for
15:  $Ne_0 = CPC_0 \cup CC_1$ 
   % symmetrical correction  $Ne_+$  of  $T$  in  $t > 0$ 
16: for all  $X \in Ne_+$  do
17:   if  $T \notin \overline{\text{DMMPC}}(X, D).Ne_+$  then
18:      $Ne_+ = Ne_+ \setminus \{X\}$ 
19:   end if
20: end for
21:  $CPC = Ne_+ \cap \mathbf{X}[t]$ 
22:  $CC = Ne_+ \cap \mathbf{X}[t + 1]$ 
23:  $CP = Ne_+ \cap \mathbf{X}[t - 1]$ 

```

---

**Algorithm 13** DMMHC( $D$ )**Require:** Data  $D$ **Ensure:**  $G_0$  and  $G_{\rightarrow}$ 


---

```

   % Restrict
   % Construction initial model  $G_0$ 
1: for all  $X \in \mathbf{X}[0]$  do
2:    $CPC_X = \overline{\text{DMMPC}}(X, D).CPC_0$ 
3:    $CC_X = \overline{\text{DMMPC}}(X, D).CC_1$ 
4: end for
   % Greedy search (GS)
5: Only try operator add.edge  $Y \rightarrow X$  if  $Y \in CPC_X$ 
   % Construction transition model
6: for all  $X \in \mathbf{X}[t]$  do
7:    $CPC_X = \overline{\text{DMMPC}}(X, D).CPC$ 
8:    $CC_X = \overline{\text{DMMPC}}(X, D).CC$ 
9:    $CP_X = \overline{\text{DMMPC}}(X, D).CP$ 
10: end for
   % Greedy search (GS)
11: Only try operator add.edge  $Y \rightarrow X$  if  $Y \in CPC_X$  and  $\{X, Y\} \in \mathbf{X}[t]$ 
12: Only try operator add.edge  $X \rightarrow Y$  if  $Y \in CC_X$  and  $X \in \mathbf{X}[t]$  and  $Y \in \mathbf{X}[t + 1]$ 
13: Don't try operator reverse.edge  $X \rightarrow Y$  if  $Y \in CC_X$  and  $X \in \mathbf{X}[t]$  and  $Y \in \mathbf{X}[t + 1]$ 

```

---

---

**Algorithm 14**  $\overline{\text{DMMPCsimplified}}(T, D)$ 


---

**Require:** target variable ( $T$ ); Data ( $D$ )

**Ensure:** neighborhood of  $T$  in  $G_{\rightarrow}$  ( $Ne_+$ )

---

```

% search  $Ne_+$  of  $T$  in  $t > 0$ 
1:  $CP_{t-1} = \emptyset$  % only parents candidates for T in slice t-1
   % search  $Ne_+$  of  $T$  in  $t - 1$ 
2:  $ListC_{t-1} = \mathbf{X}[t-1] \setminus \{T\}$ 
   % search  $CP_{t-1}$ 
   % Phase I: Forward
3: repeat
4:    $\langle F, assocF \rangle = \text{MaxAssoc}(T, \{\}, ListC_{t-1})$ 
5:   if  $assocF \neq 0$  then
6:      $CP_{t-1} = CP_{t-1} \cup \{F\}$ 
7:      $ListC_{t-1} = ListC_{t-1} \setminus \{F\}$ 
8:   end if
9: until  $CP_{t-1}$  has not change or  $assocF = 0$  or  $ListC_{t-1} = \emptyset$ 
10:  $Ne_+ = CP_{t-1}$ 

```

---



# 6

---

## Experimental study

### Sommaire

---

<b>6.1</b>	<b>Introduction</b>	<b>96</b>
<b>6.2</b>	<b>Experimental protocol</b>	<b>96</b>
6.2.1	Algorithms	96
6.2.2	Benchmarks	97
6.2.3	Performance indicators	97
<b>6.3</b>	<b>Empirical results and interpretations on complete search space</b>	<b>98</b>
6.3.1	Initial validation	98
6.3.2	DMMHC versus dynamic GS and SA	99
6.3.3	Naive versus optimized DMMHC	102
<b>6.4</b>	<b>Empirical results and interpretations on simplified search space</b>	<b>104</b>
6.4.1	Simplified 2-TBN with small benchmark	104
6.4.2	Simplified 2-TBN with large benchmark	107
<b>6.5</b>	<b>Comparison between all DMMHC versions</b>	<b>108</b>
<b>6.6</b>	<b>Conclusion</b>	<b>110</b>

---



## 6.1 Introduction

The experimental study presented in this chapter covered many points that presented in the previous chapters of our contributions. In this chapter, we were justified our theoretical ideas by real experiments. First, in section 6.2, we described our experimental protocol. We explained the steps used to prepare the necessary elements to run the experiments concerning the structure learning algorithms, the small and large benchmarks used for evaluation and finally the different metrics used to compare between different techniques presented in this thesis.

In section 6.3, we performed some experiments with our methods and others techniques existing in literature on the same context, and we showed the different results with their corresponding interpretations.

After that, in section 6.4 we mainly focused on the application of simplified DMMHC to one biological and genetic task on simplified search space. We compared our DMMHC algorithms with others tools available in the internet as Banjo, BNFinder, etc... Finally, section 6.5 is dedicated to a scalability comparison of all versions of DMMHC.

## 6.2 Experimental protocol

### 6.2.1 Algorithms

We have implemented the Greedy Search (GS) for 2T-BN as described in section 3.2. This algorithm is considered as the reference algorithm for DBN structure learning. We have also implemented our proposal, DMMHC, described in chapter 5. We can notice here that our algorithm uses a constrained greedy search, whereas the original MMHC algorithm proposes to use a Tabu search.

We can notice that the MMHC implementation available in our platform employs a chi-square test based on the  $\chi^2$  statistics. Our implementation also uses a standard formula to compute the degree of freedom instead of the heuristic proposed in the original publication [103]. These differences may also explain some of the discrepancy found when we are trying to reproduce the results from the literature.

In the greedy search, we used the BIC score function.  $\chi^2$  independence test is used with  $\alpha = 0.05$ .

Experiments were carried out on a dedicated PC with Intel(R) Core(TM) 2.20 Ghz, 64 bits architecture, 4 Gb RAM memory and under Windows 7.

Networks	# vars	# edges	Temporal edges	# states (min-Max)	# parents (min-Max)	# neighbors (min-Max)
Asia_ $G_0$	8	8		2-2	0-2	1-4
Asia_ $G_{\rightarrow}$	16	21	5	2-2	0-3	1-5
Alarm_ $G_0$	37	46		2-4	0-4	1-6
Alarm_ $G_{\rightarrow}$	74	110	18	2-4	0-4	1-7
Hailfinder_ $G_0$	56	66		2-11	0-4	1-7
Hailfinder_ $G_{\rightarrow}$	112	156	24	2-11	0-4	1-17
Win95pts_ $G_0$	76	112		2-2	0-7	1-10
Win95pts_ $G_{\rightarrow}$	156	256	41	2-2	0-7	1-10
Andes_ $G_0$	223	338		2-2	0-6	1-12
Andes_ $G_{\rightarrow}$	446	820	144	2-2	0-6	1-12
Link_ $G_0$	724	1125		2-4	0-3	1-17
Link_ $G_{\rightarrow}$	1448	2530	280	2-4	0-4	1-17

Table 6.1: Characteristics of 2T-BNs ( $G_0$ ,  $G_{\rightarrow}$ ) generated from 6 usual static BNs (Asia, Alarm, Hailfinder, Win95pts, Andes, Link)

## 6.2.2 Benchmarks

Contrary to the static BN, evaluating a DBN structure learning algorithm is more difficult. One reason is the unavailability of standard benchmarks, except for instance some reference networks with a small number of variables (less than 10), such as Umbrella and Water<sup>1</sup>.

In chapter 4, we provided tools for benchmarking dynamic Bayesian network structure learning algorithms by proposing a 2T-BN generation algorithm, able to generate large and realistic 2T-BNs from existing static BNs. We used these tools to generate 2T-BNs from 6 well-known static BNs (Asia, Alarm [8], Hailfinder [49], Win95pts, Andes [16] and Link). Details about these networks are given in table 6.1

For each of these 2T-BNs, we have respectively generated 2.000, 5.000 and 10.000 sequences of length equal to 6, which corresponds to datasets of size 2.000, 5.000 and 10.000 for  $G_0$  structure learning and 5x2.000, 5x5.000 and 5x10.000 for  $G_{\rightarrow}$  structure learning. Each data generation was also repeated 3 times for each network.

## 6.2.3 Performance indicators

The studies about DBN structure learning use different indicators to argue about the reliability of their proposals.

In chapter 4, we proposed a new metric to evaluate the performance of these structure learning algorithms, by correcting the existing Structural Hamming distance in order to take into account temporal background information. As 2T-BNs

1. <http://www.cs.huji.ac.il/galel/Repository/Datasets/water/water.html>

Networks	GS[34]	GTT[22]	GS( $G_0$ )	MMHC[34]	DMMHC( $G_0$ )
Alarm	47	44	40	24	27
Hailfinder	<b>114</b>	48	54	<b>88</b>	46
Link				687	650

Table 6.2: SHD comparison with existing static structure learning approaches ([34],[22]), for Alarm, Hailfinder and Link benchmarks with 5000 samples.

are defined by two graphs  $G_0$  and  $G_{\rightarrow}$ , the distance between one theoretical 2T-BN and the learnt one is defined as the pair of the structural Hamming distances for initial and transition graphs.

The running time was also measured. Experiments were canceled when computations were not completed within 48 hours.

## 6.3 Empirical results and interpretations on complete search space

### 6.3.1 Initial validation

To carry out comparisons with other the existing algorithms, Murphy [74] lists over 50 software packages available for different applications of Bayesian networks. However, as we mentioned in chapter III, there are few free softwares able to infer the structure of static and dynamic Bayesian networks from data, which are:

- Banjo package<sup>2</sup>: Bayesian Network Inference with Java Objects
- Bayes Net Toolbox [72] for Matlab with an extension for dynamic Bayesian networks inference using MCMC.
- BNFinder package [114]: exact and efficient method for learning Bayesian networks

Learning the initial model for our proposed algorithm is very similar to the static structure learning. As we created our 2T-BN benchmarks from usual static BNs, we can compare the results of our implementations to static ones. [34] provides one comparative study involving GS and MMHC for Alarm, Hailfinder and Link benchmarks with 5000 samples, with BDeu score (and similar results for BIC score). [22] provides SHD results for Alarm and Hailfinder with 5000 samples for another structure learning algorithm, Greedy Thick Thinning (GTT) [19], a two-phase hill-climbing heuristic algorithm.

Table 6.2 summarizes the structure Hamming distance obtained for 3 different hill-climbing algorithms and 2 implementations of MMHC, in about the same contexts (5000 samples). We can observe that our implementation gives similar results as concurrent ones for Alarm and Link benchmarks. Some strange results occur

2. <http://www.cs.duke.edu/amink/software/banjo/>

Networks	DMMHC $G_0$			GS $G_0$		
	2000	5000	10000	2000	5000	10000
Asia	<b>6±2</b>	<b>5±1</b>	<b>4±1</b>	<b>5±2</b>	<b>4±2</b>	<b>2±1</b>
Alarm	<b>33±3</b>	<b>27±3</b>	<b>22±3</b>	43±7	40±3	35±5
Hailfinder	<b>48±1</b>	<b>46±1</b>	<b>40±1</b>	55±1	54±7	48±5
Win95pts	<b>86±3</b>	<b>74±3</b>	<b>64±1</b>	102±11	93±5	85±15
Andes	<b>132±11</b>	<b>110±9</b>	<b>94±3</b>	-	-	-
Link	<b>698±13</b>	<b>650±8</b>	<b>598±10</b>	-	-	-

Table 6.3: Average±standard deviation of SHD obtained by GS and DMMHC in  $G_0$ 

for Hailfinder benchmark. [34] reports SHD results equal to 114 (resp. 88) for GS (resp. MMHC) and our implementation obtains 54 (resp. 46) when [22] obtains 48. A deeper study was conducted in order to understand this phenomenon. The definition of the SHD in the original publication is different than the one employed here. It is defined as the number of operations "add/remove undirected edge", "add/remove arrowhead of an edge" required to make two CPDAGs become the same. Thus, if CPDAG  $G_1$  contains a directed edge and CPDAG  $G_2$  does not contain the edge their SHD is increased by 2: one operation to insert the missing edge and one operation to direct it. In this thesis, the SHD metric employed would increase by 1 instead. This does not present a problem with the empirical evaluation but it is better to note it. It could also explain why there is a discrepancy in the results on the Hailfinder network between the published SHD obtained by MMHC on the same network and the one reproduced by our used SHD.

### 6.3.2 DMMHC versus dynamic GS and SA

Tables 6.3 and 6.4 present the average results obtained by GS and DMMHC algorithms with respect to the sample sizes. This table describes the results for the initial model and transition model corresponding to six (small and large) benchmarks (Asia, Alarm, Hailfinder, Win95pts, Andes, Link). We can notice that for every benchmark, DMMHC algorithm obtains better SHD than GS. except for Asia where the results are similar for  $G_0$ .

Table 6.5 shows the running time measured for DMMHC and GS algorithms for each experimental context. We can observe that DMMHC overperforms GS running time. This situation is really significant even for benchmarks with a small number of variables. We can then notice that, unlike GS, DMMHC is able to provide results in a decent time for large benchmarks (Andes and Link).

Figure 6.1 presents the average results of SHD and running time obtained by GS and DMMHC algorithms with respect to the sample sizes. This figure describes the results for the initial model corresponding to six (small and large) benchmarks (Asia, Alarm, Hailfinder, Win95pts, Andes, Link).

Networks	DMMHC $G_{\rightarrow}$			GS $G_{\rightarrow}$		
	2000	5000	10000	2000	5000	10000
Asia	<b>4±0</b>	<b>4±0</b>	<b>4±0</b>	19±2	22±6	22±5
Alarm	<b>35±2</b>	<b>29±1</b>	<b>26±3</b>	115±6	107±2	101±3
Hailfinder	<b>46±1</b>	<b>45±1</b>	<b>45±1</b>	129±9	122±12	118±8
Win95pts	<b>153±0</b>	<b>153±0</b>	<b>153±0</b>	238±20	224±18	217±23
Andes	<b>155±0</b>	<b>134±2</b>	<b>124±3</b>	-	-	-
Link	<b>804±60</b>	<b>720±45</b>	<b>589±12</b>	-	-	-

Table 6.4: Average±standard deviation of SHD obtained by GS and DMMHC in  $G_{\rightarrow}$

Networks	DMMHC $G_0$			GS $G_0$		
	2000	5000	10000	2000	5000	10000
Asia	<b>0,8±0</b>	<b>1±0,3</b>	<b>29±3</b>	4±0.3	5±0,5	112±2
Alarm	<b>12±1</b>	<b>17±1</b>	<b>22±2</b>	293±40	343±20	430±26
Hailfinder	<b>20±1</b>	<b>37±5</b>	<b>46±1</b>	1070±161	995±162	1318±222
Win95pts	<b>30±0</b>	<b>43±6</b>	<b>63±10</b>	3492±328	5153±229	6545±1961
Andes	<b>593±51</b>	<b>603±26</b>	<b>775±87</b>	-	-	-
Link	<b>1153±122</b>	<b>1525±153</b>	<b>1836 ±112</b>	-	-	-

Table 6.5: Average±standard deviation running time for DMMHC and GS in  $G_0$

Networks	DMMHC $G_{\rightarrow}$			GS $G_{\rightarrow}$		
	2000	5000	10000	2000	5000	10000
Asia	<b>126±11</b>	<b>409±40</b>	<b>831±58</b>	260±8	917±68	1564±308
Alarm	<b>346±65</b>	<b>383±16</b>	<b>911±64</b>	1448±545	2206±272	5094±1338
Hailfinder	<b>755±170</b>	<b>1629±35</b>	<b>2145±144</b>	5225±1058	20789±3121	40210±6335
Win95pts	<b>792±145</b>	<b>1736±46</b>	<b>4143±87</b>	35318±3760	72653±6912	102851±7012
Andes	<b>12996±131</b>	<b>23755±623</b>	<b>62149±3883</b>	-	-	-
Link	<b>68156±2963</b>	<b>149773±3310</b>	<b>263624±7518</b>	-	-	-

Table 6.6: Average±standard deviation running time for DMMHC and GS in  $G_{\rightarrow}$

Figure 6.2 compares the results of SHD(2-TBN) in dynamic GS and DMMHC for a transition graph corresponding to benchmarks used before. For all the sample sizes 2000, 5000 and 10000, the DMMHC always outperforms GS for 2-TBN in terms of quality (construction transition network) (cf. Figure 6.2). Also, we notice that GS couldn't deal with large dimension benchmarks. The obtained results are only for Asia, Alarm, Hailfinder and Win95pts datasets. However, DMMHC could be run on all benchmarks of different complexities with reasonable time whether for the initial network (cf. figure 6.1) or the transition network (cf. figure 6.4).

In figure 6.3 between the results of SHD(2-TBN) in SA algorithm, provided by Banjo tools, and those of the DMMHC for transition graph corresponding to benchmarks used before. For all sample sizes 2000, 5000 and 10000, the DMMHC outperforms SA in terms of quality (construction transition network) (cf. Figure 6.3). We also remark that SA couldn't deal with two benchmarks (Hailfinder and Link). The obtained results are only for Asia, Alarm, Win95pts and Andes datasets. However, DMMHC could be run on all benchmarks with different complexities

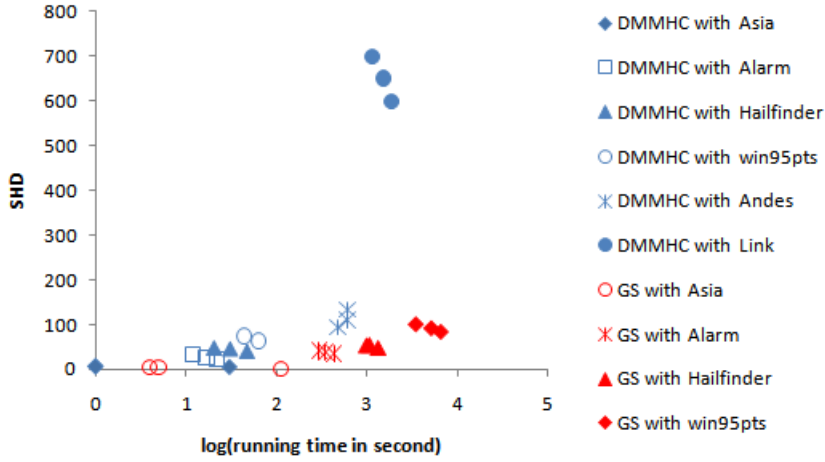


Figure 6.1: Average SHD vs running time obtained by GS and DMMHC with respect to sample size in initial graph

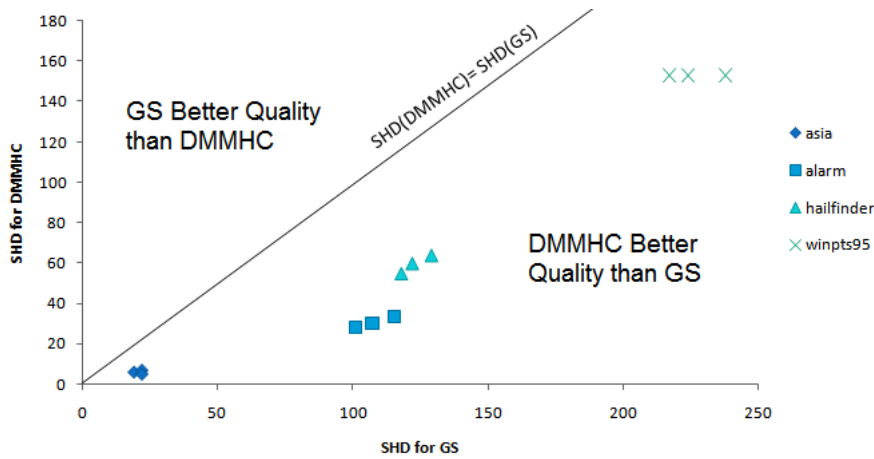


Figure 6.2: Average SHD(DMMHC) vs SHD(GS) with respect to sample size in transition graph

Networks	DMMHC $G_{\rightarrow}$			SA $G_{\rightarrow}$		
	2000	5000	10000	2000	5000	10000
Asia	4±0	4±0	4±0	17±0	14±2	3±0
Alarm	35±2	29±1	26±3	118±5	112±3	76±7
Hailfinder	46±1	45±1	45±1	-	-	-
Win95pts	153±0	153±0	153±0	266±20	264±4	256±13
Andes	155±0	134±2	124±3	592±10	563±24	551±19
Link	804±60	720±45	589±12	-	-	-

Table 6.7: Average±standard deviation of SHD obtained by SA and DMMHC in  $G_{\rightarrow}$

with reasonable time whether for the initial network (cf. figure 6.1) or the transition network (cf. figure 6.4).

We also notice that SA algorithm couldn't deal with hailfinder and link 6.7, the first execution (hailfinder benchmark) has stopped because the maximum number

of states that a variable can assume is limited to 7 in banjo tools however, Hailfinder dataset contains variables with more than 7 states. The second execution (link benchmark) has stopped because Banjo has run out of available memory (link contains about 1500 variables with more than 10000 sequences).

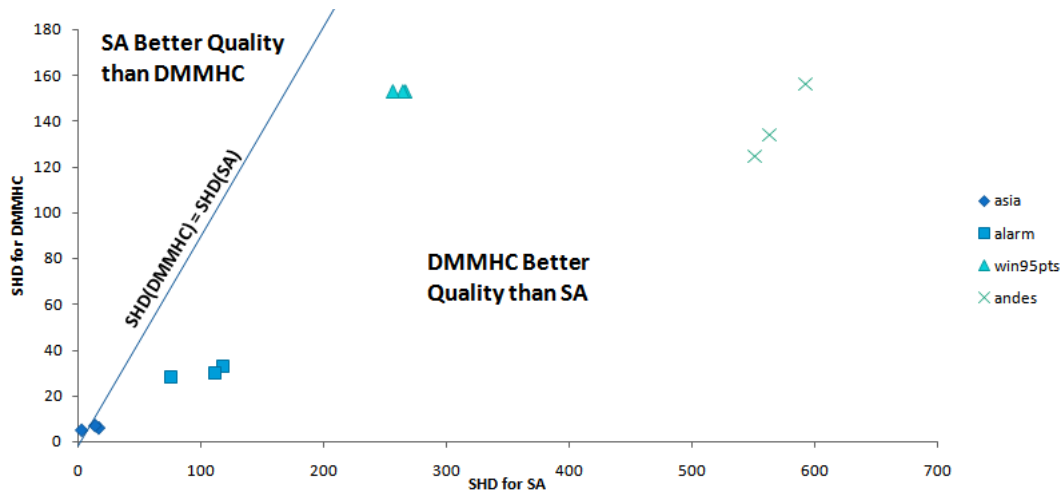


Figure 6.3: Average SHD(DMMHC) vs SHD(SA) with respect to sample size in transition graph

From these results, we can see that the DMMHC is an efficient algorithm for 2T-BN structure learning. The quality of the learned structure is better than for our reference algorithm (2T-BN greedy search) with a better scalability: the results are obtained in a lower running time. In addition, the DMMHC can successfully manage high dimensional benchmarks such as 2T-BN generated from Andes and Link (cf. figure 6.9).

The comparison with the existing works is a difficult task because the works about 2T-BN structure learning deal with specific benchmarks or a specific evaluation metric, as reported in [100].

It is the first time that the existing static benchmarks have been extended for 2T-BN structure learning. It is also the first time that the Structural Hamming distance has been used for 2T-BN, with a temporal correction as described in [100]. For these reasons, comparisons with the existing results obtained by concurrent 2T-BN structure learning algorithms are not possible. We intend to disseminate our benchmarks and performance indicators in order to propose a unified evaluation framework for 2T-BN structure learning.

### 6.3.3 Naive versus optimized DMMHC

In this part of experimentations we present only the results obtained by the naïve and optimized DMMHC in the transition model. In fact, we remark that there is not a big difference or change between the results of the naïve DMMHC and the



Networks	naiveDMMHC $G_{\rightarrow}$			optimisedDMMHC $G_{\rightarrow}$		
	2000	5000	10000	2000	5000	10000
Asia	<b>4±0</b>	<b>4±0</b>	<b>4±0</b>	<b>6±2</b>	<b>7±0</b>	<b>5±2</b>
Alarm	<b>35±2</b>	<b>29±1</b>	<b>26±3</b>	<b>33±0</b>	<b>30±2</b>	<b>28±2</b>
Hailfinder	<b>46±1</b>	<b>45±1</b>	<b>45±1</b>	64±4	60±8	55±2
Win95pts	<b>153±0</b>	<b>153±0</b>	<b>153±0</b>	<b>153±0</b>	<b>153±0</b>	<b>153±0</b>
Andes	<b>155±0</b>	<b>134±2</b>	<b>124±3</b>	<b>156±2</b>	<b>134±1</b>	<b>125±2</b>
Link	<b>804±60</b>	<b>720±45</b>	<b>589±12</b>	<b>815±48</b>	728±56	<b>591±24</b>

Table 6.8: Average±standard deviation SHD for naive and optimised DMMHC in  $G_{\rightarrow}$

Networks	naiveDMMHC $G_{\rightarrow}$			optimisedDMMHC $G_{\rightarrow}$		
	2000	5000	10000	2000	5000	10000
Asia	126±11	409±40	831±58	<b>62±10</b>	<b>202±21</b>	<b>419±26</b>
Alarm	346±65	383±16	911±64	<b>216±36</b>	<b>312±8</b>	<b>638±23</b>
Hailfinder	<b>755±170</b>	<b>1629±35</b>	<b>2145±144</b>	743±87	2549±76	4695±118
Win95pts	792±145	1736±46	4143±87	<b>488±8</b>	<b>1137±60</b>	<b>2968±384</b>
Andes	12996±131	23755±623	62149±3883	<b>10607 ±476</b>	<b>19090±2405</b>	<b>35832±3160</b>
Link	68156±2963	149773±3310	336879±12589	<b>63401±2596</b>	<b>141325±5967</b>	<b>283819±10221</b>

Table 6.9: Average±standard deviation running time for naive and optimised DMMHC in  $G_{\rightarrow}$

optimized DMMHC in the initial model. This is explained by the use of a small number of variables which makes the difference between the number of calls to the assoc function in the naïve and optimized approaches limited.

Table 6.8 presents a comparison of SHD(2T-BN) average results obtained by the naive and the optimised DMMHC algorithms with respect to the sample sizes. This table describes only the results for the transition model corresponding to six (small and large) benchmarks (Asia, Alarm, Hailfinder, Win95pts, Andes, Link). We can notice that for every benchmark, the two versions of DMMHC algorithm obtain in almost all the cases similar SHD results. Apart from, for Hailfinder benchmarks, the naïve DMMHC algorithm obtains better SHD than the optimized version with a difference in interval [8, 15].

Similarly, Table 6.9 shows the running time measured for the naive and optimized DMMHC algorithms for each experimental context. We can observe that the optimized version outperforms the naïve one in terms of running time. This situation is really significant even for benchmarks with a small number of variables. But, we can also notice that the more the number of variables increases, the running time differences between the naïve and the optimized versions are important. We can then notice that, unlike the GS, the two versions of the DMMHC are able to provide results in an acceptable time for large benchmarks (Andes and Link).

In figure 6.4, we present all SHD(2-TBN) results obtained for the transition model in terms of both time and quality. We remark that the results of SHD for the naive and the optimized DMMHC are similar in most of the several benchmarks with different complexity and sample sizes. However, the optimized DMMHC out-



performs the naïve DMMHC in terms of running time. In addition, we can see that the results of the naïve and the optimized DMMHC are better than those of the dynamic GS in terms of both accuracy (SHD(2-TBN)) and complexity (running time).

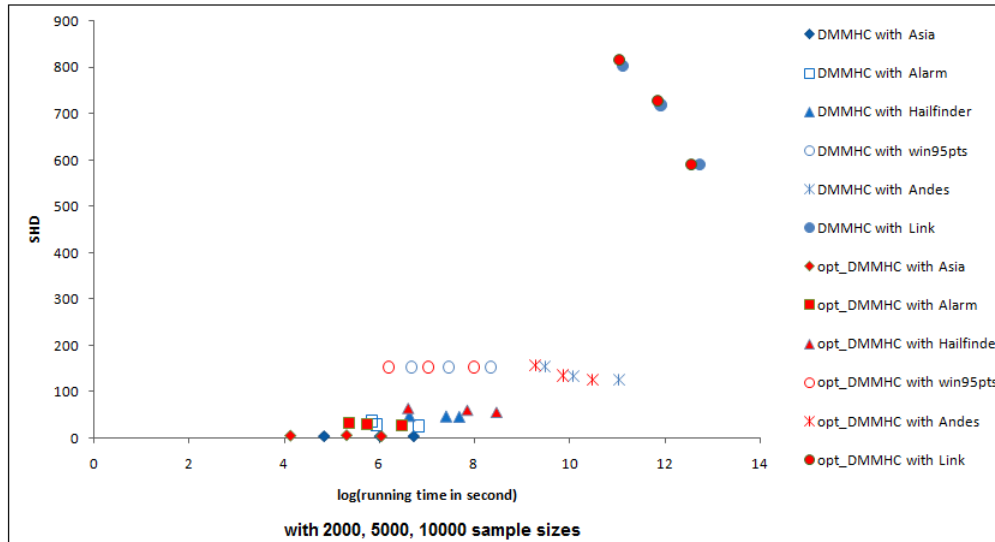


Figure 6.4: Average SHD vs running time obtained by naïve and optimised DMMHC with respect to sample size in transition graph

We have demonstrated in sections 5.2.3 and 5.5 that, the optimized version of the DMMHC performs the calls of assoc function and minimizes the complexity of the algorithm. And we have demonstrated these optimizations theoretically. Now, we try to prove them with experimentally. According to the two previous paragraphs, we can summarize that, the optimized version permits to learn large model in a shorter running time than other algorithms for DBN structure learning. In addition, this optimization maintains the quality of the algorithm for the learning structure. We present this improvement in tables 6.8, 6.9 and figure 6.4 showing the different results of SHD(2T-BN) for transition graph given by dynamic GS, naïve and optimised DMMHC.

## 6.4 Empirical results and interpretations on simplified search space

### 6.4.1 Simplified 2-TBN with small benchmark

Our aim is to provide an algorithm which could be used for different applications of a large dynamic Bayesian network reconstruction. In order to assess the performance of our software, we have first compared it to the Banjo library. As a realistic dataset, we have chosen the dataset attached as an example to the Banjo

package, consisting of 20 variables and 2000 observations, published by Smith et al. [90] (available in BNFinder tutorial<sup>3</sup>). The results of both techniques are discussed below:

We notice that in this part of work both techniques (simplified DMMHC and GS (used in Banjo tool)) are used as inputs of the minimum and maximum Markov lag, and in this example both are equal to 1, which means that no links between nodes of Markov lag 0 are allowed (i.e. we use the simplified 2-TBN "no intra-slice edges" in model).

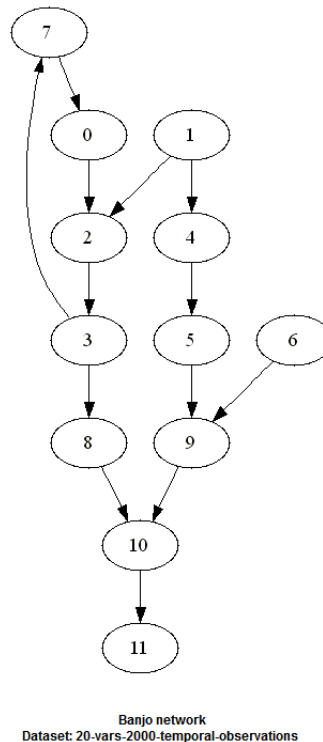


Figure 6.5: Network reconstruction results for GS algorithm of Banjo tool. The used benchmark is the biological dataset 20 variables and 2000 observations

Banjo uses a setting file where we can change any parameter (or input) for algorithms. We modified the provided settings file to allow a run (of 15 minutes), Greedy searcher as structure learning algorithms and made a few other changes. The result of this algorithm is presented in figure 6.5.

we note that, this figure presents a path ( $7 \rightarrow 0 \rightarrow 2 \rightarrow 3 \rightarrow 7$ ) which is not a cycle, but shows temporal dependence between two nodes in different time slices (For example  $7 \rightarrow 0 \iff 7_{t-1} \rightarrow 0_t$  and  $3 \rightarrow 7 \iff 3_{t-1} \rightarrow 7_t$ ).

As mentioned before, the DMMHC uses  $\chi^2$  independence test with  $\alpha = 0.05$ . Also, it uses greedy search GS algorithm if it can give a better network score. The GS uses the BIC score function. The result of this algorithm is presented in figure 6.6.

3. <http://bioputer.mimuw.edu.pl/software/bnf/tutorial.php>

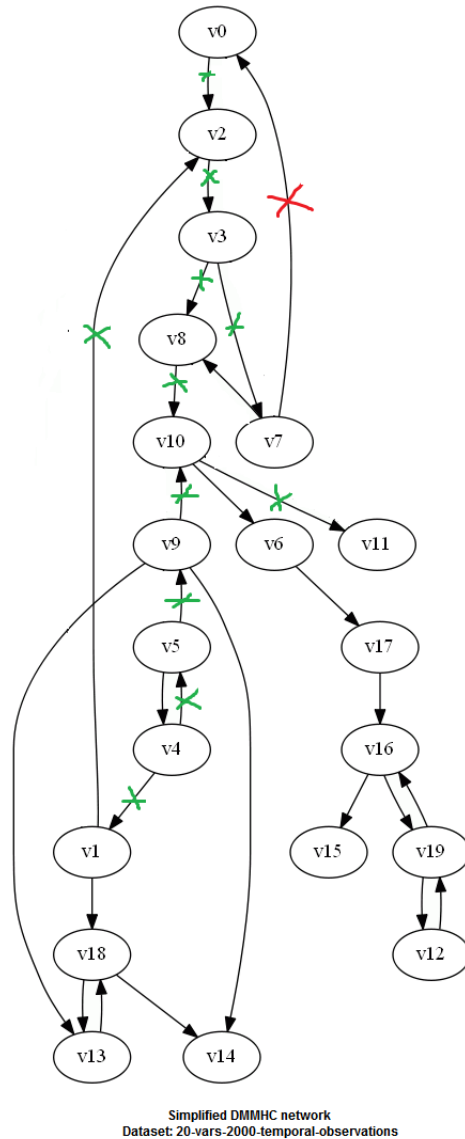


Figure 6.6: Network reconstruction results for simplified 2-TBN DMMHC. The used benchmark is the biological dataset 20 variables and 2000 observations.

Figures 6.5 and 6.6 show that the DMMHC and the Banjo tool are able to identify the same temporal edges (12 edges, the green cross present the same temporal edges identified by simplified DMMHC and GS in Banjo), there is only one edge (from 0 to 7 presented by the red cross) is inversed. Besides, the simplified DMMHC discover other temporal edges (for 12,...,19 nodes) but we can not prove if these edges are truly or falsely identified because we don't find the generating (original) network with which we can compare the other learned graphs. We think that the comparison between both techniques is not precise, however we can say that the DMMHC is able to discover a 2-TBN model by simplifying the search space.

## 6.4.2 Simplified 2-TBN with large benchmark

### Cyanothece genetic network

The second test for our simplified DMMHC algorithm on a large dimension as the cyanobacterial genetic network was of great interest. For more details of this dataset it is recommended to read [97, 98, 110].

Vinh et al. [110] used transcriptomic data from two previous studies [97, 98]. They analyzed the same set of 1595 genes which presents a fold-change superior to 2.5 states for one variable. In addition, they also included a variable that represents the relevant environmental condition, namely light, that takes binary values (L/D). Gene expression data were discretized using 3-state quantile discretization. After the changes made on the data set by Vinh, we obtained a set of 1595 columns and 24 sequences.

For this large network, the two methods (DMMHC and Vinh's algorithm) set the significance level of  $\alpha = 0.9999$ . Every method took a reasonably small amount of time to learn this network, i.e., Banjo was set to run for 1 hour and DMMHC was set to run for 2 hours. The networks learned by the MIT-based global and local algorithms are presented in figure 6.7(a,b) [110]. The networks learned by DMMHC and Banjo algorithms are presented in figure 6.7(c,d). DMMHC generated a small network and was able to identify many edges. We think that this result is attributed to the small number of sequences and it is insignificant to learn 1595 variables with very small number of sequences.

We remark that in the overall network structure the majority of nodes have only a few connections and a small number of hubs (i.e. node with many connections) that have many connections. The node degree in a scale-free network is calculated according to a powerlaw distribution,  $P(x) \propto x^{-\gamma}$ .

For the networks reconstructed by the MIT-based global and local MMMB algorithms, the scale parameters, estimated via linear regression on the log-log node degree distribution plot, are respectively  $\gamma_1 = 2.33$  and  $\gamma_2 = 2.35$ , falling well within the typical range. Furthermore, the coefficients of determination are respectively  $R_1^2 = 0.90$  and  $R_2^2 = 0.93$ , indicate very good fits. The node degree distributions of various reconstructed networks are shown in figure 6.8.

However, for the networks reconstructed by DMMHC, the scale parameter is  $\gamma=1.7$  and the determination coefficient is  $R^2 = 0.95$ . The Banjo reconstructed network is presented in Figure 6.7(c). It exhibits a radically different structure as compared to the networks reconstructed by the other algorithms in figure 6.8. A linear regression on the log-log node degree distribution plot yields  $\gamma = 1.41$  and  $R^2 = 0.358$ , clearly indicating a large deviation from a scale-free topology [110]. In addition, Vinh plots the degree distribution for a random graph with the same number of nodes and connections as in the Banjo reconstructed network. It is noted that

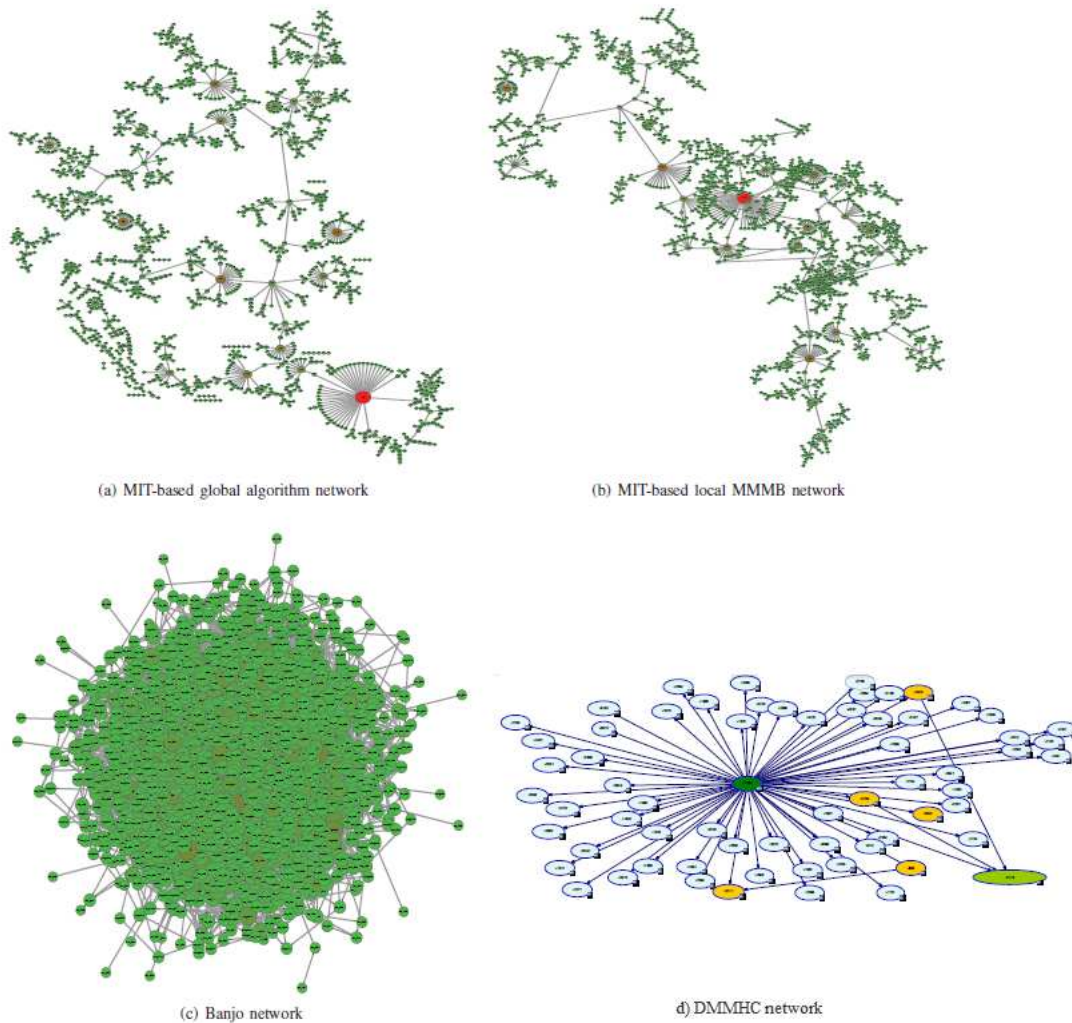


Figure 6.7: Network reconstruction results for MIT-based global, MIT-based MMB, Banjo [110] and DMMHC from Cyanothecce genetic benchmarks

the node degree distributions of the Banjo network and the random graph are very similar.

## 6.5 Comparison between all DMMHC versions

An evidence of the scalability of DMMHC is found in the previous experiments (section 6.3.2, 6.3.3 and 6.4.1). We gave a summary of all these experiments in figure 6.9. Naive, optimized and simplified DMMPCs were run on each node and reconstructed the skeleton of a tiled-LINK networks with approximately more than one thousand variables from the 10,000 training instances using the same hardware as above in respectively 4 days, 3 days and 1 day of a single-CPU time.

Figure 6.9 illustrates the ability of the DMMHC to scale up to thousands of variables. We present in this figure, the behavior or the variation of running time

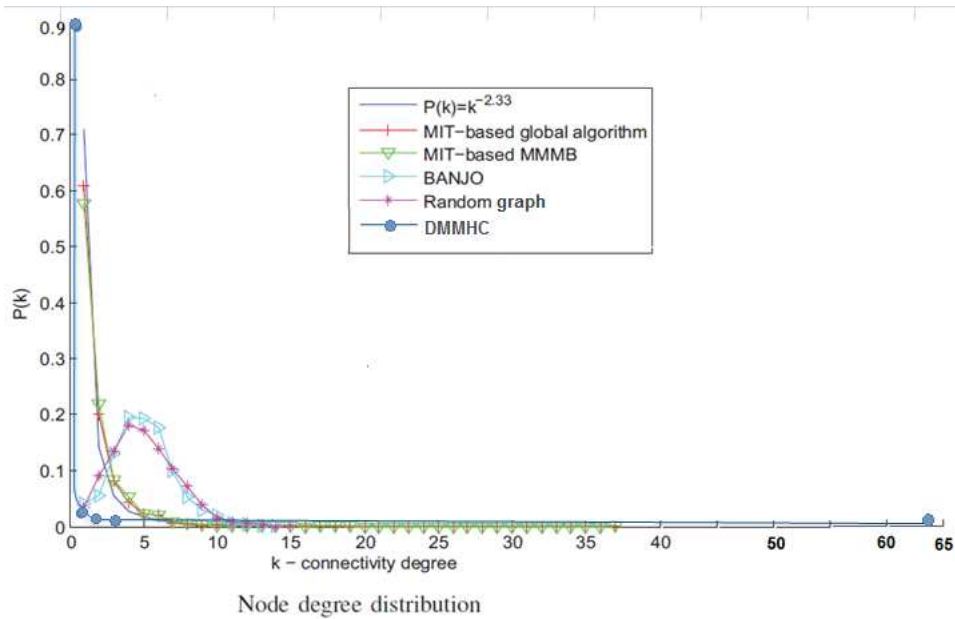


Figure 6.8: Network reconstruction results

with the number of variables on different benchmarks for all versions of DMMHC. In this figure, the X axis presents the number of variables and the Y axis presents the logarithmic function of running time.

The complexity variation of all DMMHC versions is different from the exponential function. These results were demonstrate theoretically in section 5.5, and now, we demonstrate this advantage experimentally. We calculated the different Asymptote equations eq1, eq2 and eq3 respectively for the naive DMMHC, the optimised DMMHC and the simplified DMMHC variations. We noticed that the asymptote equations are different from the exponential function.

$$\text{equation1} : \ln(T) = a_1N + b_1 = 0,125N + 650 \quad (6.1)$$

$$\text{equation2} : \ln(T) = a_2N + b_2 = 0,125N + 600 \quad (6.2)$$

$$\text{equation3} : \ln(T) = a_3N + b_3 = 0,125N + 450 \quad (6.3)$$

where T is the running time and N the number of variables.

We would like to remind that, in section 5.5, we said that the complexity of the optimized DMMHC is lower than that of the naive DMMHC and the simplified DMMHC complexity is lower than that of the optimized DMMHC one. This remark is well presented in figure 6.9.

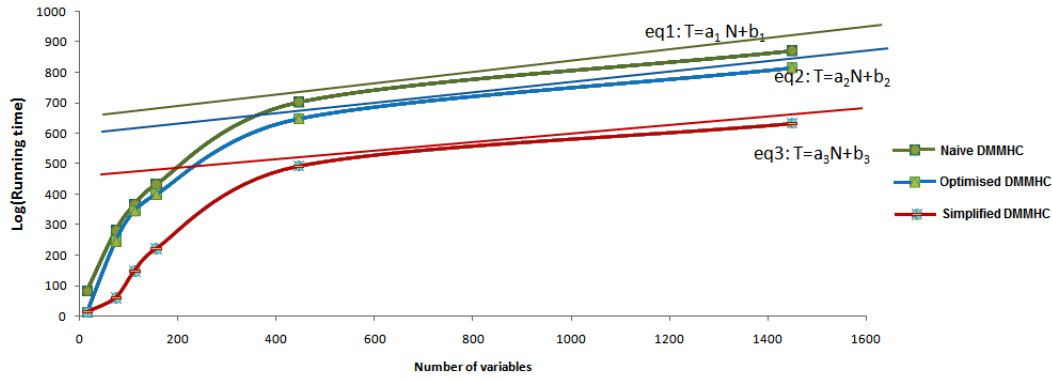


Figure 6.9: Complexity of all DMMHC versions

## 6.6 Conclusion

The experimental results, provided in this chapter, show that the proposed DMMHC algorithms achieve a better performance than the other existing structure learning for DBN such as Greedy Search or the Simulated Annuling algorithms.

The proposed approach was tested through real and standard benchmarks created by our evaluation techniques for a dynamic case. Also, we validated the DMMHC on a real biology field (Cyanothecce genetic network). Nevertheless, the main problem is that there is no commonly accepted benchmark that can be used to learn with a large number of instances (i.e. Cyanothecce genetic benchmark contains only 24 instances) that can lead to more rigorous results.



# Conclusion

## Sommaire

---

<b>7.1 Summary</b> . . . . .	<b>112</b>
<b>7.2 Applications</b> . . . . .	<b>113</b>
<b>7.3 Limitations</b> . . . . .	<b>114</b>
<b>7.4 Issues for future researches</b> . . . . .	<b>115</b>

---



## 7.1 Summary

In the different sections of this thesis work, we discussed in (the chapter 2 and chapter 3), the problem of representation the Probabilistic graphical Models (PGM), especially in the Bayesian networks and their extension Dynamic BN.

In **chapter 2**, we present the key concepts of Bayesian networks, structure learning methods of these models and all techniques used to evaluate these learning algorithms. We concluded in this chapter that the hybrid method for learning the BN performed better than the techniques based on constraints or those based on score. We also noted that most of the studies use the methods based on the score for evaluation, although this kind of method has disadvantages when the data size is reduced and the calculation is costly when the number of variables is important.

In the same way, in **chapter 3**, we present the key concepts of the dynamic bayesian network and the different structure learning studies of these models. We also addressed to a discussion of the different elaborated works to evaluate their DBN structure learning algorithms.

For these approaches, there are many weaknesses as they use their own evaluation Benchmarks and techniques to evaluate. In addition, we don't find any online available structure learning algorithm software for DBN, that can be compared to our methods. The important conclusions that can be drawn from this study is that the most of the studies use networks with small number of variables and score based techniques for evaluation. This makes the comparison between our study and other works more difficult.

In the chapters 4 and 5, we tried to solve the problems mentioned in the previous chapters. In **chapter 4**, we focused on solving the problem of evaluation for the temporal graphical model as the DBN and especially the 2T-BN models. We also introduced, two contributions: the first is a large 2-TBN generation algorithm inspired from the Tiling technique; the second is a novel metric to evaluate the performance of DBN structure learning algorithms. It is inspired from the Structural Hamming distance used in the static case, but it takes into account temporal background information.

In **chapter 5**, we developed a novel approach for DBN structure learning named DMMHC. It is a hybrid method that uses a local search algorithm dealing with local structure identification (DMMPC) and global model optimization constrained with the local information (DMMHC). We demonstrated that this approach can easily take into account the temporal dimension in order to provide some additional information about temporality that can then be used with a greedy search in order to learn the global structure the initial model  $G_0$  and the transition one  $G_{\rightarrow}$ . As far as we know, no method able to learn dynamic network models with such approaches has been proposed previously.

To analyze and validate the merits of our proposed and used methods, we carried out some experiments, detailed in **chapter 6**, on standard large benchmarks constructed with our method described in chapter 4. These benchmarks are derived from the standard benchmarks used in the evaluation of structure learning methods in a static case as Asia, alarm, hailfinder ...

The results have shown the significance of our novel algorithms for dynamic Bayesian network structure learning on the level of scalability, complexity, running time and accuracy (quality of construction models).

## 7.2 Applications

Since the sixties, saving the multi-dimensional and temporal data has increased in many fields. "Time" is an important concept in medical care [79, 89]. Several studies done in the medical field for the evaluation of the nosocomial infections (IN) predominance and the factors associated to its presence [37, 40]. In our future studies, we are interested in solving this problem through the prediction of the prognostic patient's cases. Our system, named Decision Support System Nosocomial Infection (**DSS-NI**), enables to predict their cases relying on the patient's history and medical treatment during his hospital stay. This work is important since physicians complain about incomplete and irregular data structure, which makes their job more difficult. As a result, the predictions are judged to be poor in question of the extracted knowledge [55, 60, 66, 99].

Nosocomial infections (NIs) continue to be a major problem. It causes high morbidity and mortality. Consequently, it increases the length of hospitalization and the cost of treatment. The supervision of NIs is an essential part of the infection control program. Therefore, prevention is better than cure for our patients. Successful infection control measures depend on education at the level of healthcare workers (HCWs), surveillance of the prevalent microorganisms and their antimicrobial resistance patterns, and an efficient interdisciplinary collaboration. The supervision activities are part of infection control efforts to improve the quality of hospital care. Most of the research on the controls of the NI concentrate on the acquisition of this supervision. Since a hospitalization may end up with the patient's death.

In the decision support analysis initialized by Morton and Keen [88, 53], most of the studies used in this domain are limited to the static aspect as the static Bayesian Networks [4, 68]. This is insufficient for a dynamic decision making as in the medical field. The dynamic processes which advance over time, known as Markov decision processes, are used as the basis for prediction models. The techniques such as decision-trees [71], neural networks [116], association rule, Hidden Markov models [28, 46] and (dynamic) Bayesian networks [10, 32, 66, 79], developed by the

artificial intelligence community, have become popular prediction models. Markov models have been used in various medical applications. For instance, in medical artificial intelligence (AI) Markov models often have a complex and multivariate state representation. The models are represented as a dynamic Bayesian network DBN [79], in this thesis we justify our choice to use this existing technique to provide Markov models with multivariate and complex state descriptions of medical data. Our objective is to extract models that can identify temporal patterns, predict the daily prognostic of the patient's state and test clinical hypotheses. Furthermore, our models should be able to detect the future probable presence of a nosocomial infection at ICU for two levels 1) the processes regarding the eventual outcome of ICU (to die or to survive), 2) and the processes related to the development of the patient's state (acquisition of NI) during of his/her hospitalization period.

The results of this thesis are relevant to these processes and demonstrate the induction, inference, evaluation, and use of these models in practice data concerning patients admitted to the ICU of Habib Bourguiba hospital (Sfax, Tunisia). The Intensive Care Unit of the CHU Habib Bourguiba of Sfax contains 22 beds in 11 boxes of 2 beds. Each box is thus made up of 2 beds separated by a care zone containing all that a nurse needs (compress, gloves, serum, drugs, etc.) to deal with his/her 2 patients. A nurse is responsible for the care of 2 patients. The majority of the patients admitted in ICU are known as "heavy" because they require a 24h/24h care. They are often connected to machines (artificial respirator, electrocardiogram, electric syringe, etc.) and/or connected to catheters (venous catheters, urinary probe, thoracic drain...). These patients are often in a very delicate health situation. They are vulnerable to new germs entering their bodies. For each appearance of infection either nosocomial or not, it is necessary to send a blood sample to the laboratory to carry out an antibiogram. According to the antibiogram result, an antibiotherapy is prescribed. The problem of the antibiotherapy is that a germ can be sensitive to the antibiotic during the first period and resisting a few weeks or months later. This sensitivity can be different from an individual to another [64].

### 7.3 Limitations

Despite our multiple attempts to take into account all interactions that can occur between the DBN structure learning and evaluation techniques, we still face a few difficulties to justify the performance of our techniques. In our experiments we compared our algorithms to only the GS and the SA algorithms. This is not enough to show the utility of our approach applied on large benchmarks.

As a second limitation, we should point out that the DBN and more generally BN also led to create a decision extension called influence diagrams (ID) [54] that

can solve the decision problems. It is possible to distinguish the variables on which it is possible to act, with respect to other contextual variables. Thus we can act on the costs of each decision or utility with respect to each context. In this thesis, we don't have enough time to achieve this final part of the study.

The third limitation, is that in our experiment we did not use real data. For two reasons: (1) First we didn't have enough time to make experiments using data generated from standard benchmarks and data from a real dataset. (2) The medical database provided by Habib Bouguiba hospital is not well prepared for use in experiments due to the small base size (less than 1000 cases and less than 100 variables). However, we need thousands of variables to apply our learning algorithms.

## 7.4 Issues for future researches

Our main immediate perspective is to introduce some local improvements on our algorithm. Our framework offers several opportunities for future research. Among this, we believe that all the theoretical properties and experimentation results should be extended. We have identified several perspective ideas:

1. Our next step in this direction is the adaptation of the structural Hamming distance in order to take into account any background knowledge (forbidden edges, required ones, partial ordering, ...).
2. We think that the quality of the structure reconstruction can be improved during the global optimization using a more evolved meta-heuristic such as a Tabu search instead of a standard hill-climbing algorithm.
3. We plan to extend our dynamic approach with the use of others local identification methods such as PCD algorithm [80] that can correct it under faithfulness assumption.
4. Develop the structure learning algorithm for duration graphical model (DGM) relying on what we have reached in this thesis.
5. Extend our approach for learning the dynamic models to a decisional task with the use of influence diagram (ID) and dynamic decision models (DDM).
6. Apply the algorithms developed before for a decision support task in the medical field. Indeed, the currently developed system, for ICU Habib Bourguiba hospital of Sfax, to predict the occurrence of nosocomial infections (NI) is based on the dynamic bayesian network. This system gives only the scores for predicting occurrence of IN without proposing any decision. Our major goal is to develop a decision support system to fight against the NI in the ICU of Habib Bourguiba Hospital of Sfax.





# Bayesian Network and Parameter learning

Here we are looking for estimating the probability distributions from available data. The estimation of the probability distributions, parametric or not, is a vast and complex subject. We describe here the most commonly used methods in the Bayesian networks, as the data are complete, for more information, we advise to read [51, 41].

## A.1 From completed data

### A.1.1 Statistical Learning

In the case where all variables are observed, the easiest and most used method is the statistical estimate. It estimates the probability of an event by the frequency of occurrence of the event in the database. This approach, called maximum likelihood (ML). It is defined by:

$$\hat{P}(X_i = x_k | Pa(X_i) = x_j) = \frac{N_{i,j,k}}{\sum_k (N_{i,j,k})} \quad (\text{A.1})$$

Where  $N_{i,j,k}$  is the number of events in the data base, for which the variable  $X_i$  is in the state  $x_k$  and its parents are in the configuration  $x_j$ .

## A.1.2 Bayesian Learning

The Bayesian estimation follows a different principle. It consists in finding the most likely parameter theta as the data were observed using the a priori knowledge over the parameters (e.g. maximum a posteriori (MAP) estimation). If we assume that each  $P(X_{i,j} | Pa(X_i))$  follows a multinomial distribution, the conjugate distribution follows a Dirichlet distribution with the parameters  $\alpha_{i,j,k}$

$$\hat{\theta}_{i,j,k}^{MAP}(X_i = x_k | Pa(X_i) = x_j) = \frac{N_{i,j,k} + \alpha_{i,j,k} - 1}{\sum_k (N_{i,j,k} + \alpha_{i,j,k} - 1)} \quad (\text{A.2})$$

$\alpha_{i,j,k}$  are the parameters of the Dirichlet distribution associated with the prior probability  $P(X_i = x_k | Pa(X_i) = x_j)$ .

## A.2 From incomplete data

### A.2.1 Nature of the missing data and their treatment

The real-world data usually contains missing entries. We need to deal with incomplete data sets that looks like the following (example A.1):

X1	X2	X3	X1	X2	X3
1	1	1	2	1	1
2	?	1	?	1	1
1	2	1	?	?	?

where ? indicates missing values.

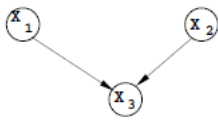


Figure A.1: Example of BN structure

To deal with the missing values, we need to make the following assumptions:

- MCAR (Missing Completely At Random): the actual value of X and the event X-is-missing are independent on data.
- MAR (Missing At Random): the actual value of X and the event X-is-missing are conditionally independent given other observed variables.
- NMAR (Not Missing At Random): the actual value of X and the event X-is-missing are conditionally independent given other observed and missing variables.

The situations MCAR and MAR are the easiest to solve, because the observed data contain all the information necessary to estimate the distribution of the missing data.

When the missing data are MCAR, the earliest and simplest approach is the analysis of complete examples. This method tried to estimate the parameters from completed data sets of all the examples fully observed in the observed data.

Many methods attempt to estimate the parameters of a model from data in MAR. For example, we can cite the sequential updating [92], Gibbs sampling [36], and the expectation maximization algorithm (EM) [58].

### A.2.2 EM algorithm

One algorithm for parameters learning from incomplete data: The expectation-maximization (EM) algorithm. Developed in the Statistics community (Dempster et al. 1977)[24]. Adapted for Bayesian networks by Lauritzen (1994) [58].

It is an iterative algorithm.

- Starts with an initial estimation  $\theta_0$ .
- At each iteration  $t$ :
  - Expectation: Complete the data set based on  $\theta_t$ .
  - Maximization: Re-estimate parameters using the completed data set, obtaining  $\theta_{t+1}$ .







# Benchmarking the dynamic bayesian network

## B.1 BNtiling for 2-TBN implementation

Our approach is based on five steps: (1) generating a standard BN; (2) tiling the BN; (3) conversting the BN to DBN with the determined number of time slices; (4) constructing the initial and transition models of DBN; (5) generating of data for the new DBN; the benchmarking for 2T-BN process is described as follows:

1. Building a Bayesian network with HUGIN format (.net)
2. Generating data file from the network with .dat format used the geniesmile tools.
3. Converting the generated data on matlab format and save them in a data file (.mat)

```
data=data_converter_hugin('./Data/network.dat', './Data/network.net');
```

4. Use the "bn.tiling" process to generate the new network R1 with N copies of the original network

```
network_filename='./Data/network.net';  
dataset_filename='./Data/data.mat';  
nodes=bn_tiling(network_filename, dataset_filename, nbr_variables*N, 2);
```

Remark:  $N$  represents the number of copies "tile" in  $R1$ . This command does not give  $R1$  in HUGIN format, so we have to create a method that allows the construction of the new network with new transformations.

```
data=data_converter_hugin('./Data/data.dat', './Data/network.net');
```

5. Extract all the characteristics of  $R1$  and nodes (name, parents, table of probabilities)
6. Generate the Dynamic Bayesian network  $R2$  from network  $R1$ .
  - Building the DBN with  $K$  time slices:

```
Hugin_DBN(nodes, N, K);
```

- Building the initial and transition model by :

```
Hugin_Net(nodes,1); % the initial model
Hugin_Net(nodes,2); % the transition model
```

7. Generate the new database from  $R2$  dynamic network used the *geniesmile* tools.

## B.2 Generating large Dynamic Bayesian Networks

---

### Algorithm 15 Generating large Dynamic Bayesian Networks

---

**Require:** DAG  $M$ , number of copies ( $n$ ), number of time slices ( $k$ )

**Ensure:** Return initial and transition models ( $M_{i0}$ ,  $M_{i\rightarrow}$ ), and DBN  $B_i$  with  $N$  time slices

- 1:  $B_i$  is empty graph
  - 2: Generate Dataset  $D$  for  $M$
  - 3: Create a large Bayesian network  $M_i$  with  $n$  copies of  $M$  by tiling
  - 4:  $M_{i0} = M_i$
  - 5: Generate Dataset  $D_i$  for  $M_i$ 
    - % add priori Knowledges (Temporal dependencies)
  - 6: Create  $M_{i\rightarrow}$  with 2 copies of  $M_i$  by tiling
  - 7: 2-TBN model = ( $M_{i0}$ ,  $M_{i\rightarrow}$ )
  - 8: **for all**  $i \geq 0$  and  $i \leq k$  **do**
  - 9:   add  $M_{i\rightarrow}$  to  $B_i$
  - 10: **end for**
  - 11: Return  $B_i$
-



# DMMHC: Dynamic Max Min Hill Climbing

## C.1 Proofs of propositions used by DMMHC

### C.1.1 Proof proposition 5.2.2:

Let  $V_i$ ,  $V_{i+1}$  et  $V_{i-1}$  be the variable sets respectively of time slices  $i$ ,  $i+1$  and  $i-1$ . Let  $Ne_+$  be the set of candidates parents/children of variable  $T \in V_i$  (noted  $Ne_+ = CPC_i \cup CC_{i+1} \cup CP_{i-1}$ ).

#### 1. $\forall X \in V_i \setminus T$

Let  $p$  a be path from  $T$  to  $X$  through the node  $m \in V_{i+1}$  and  $k$  its child ( $k \in V_{i+1}$ )

Because of temporality, there is at least one V-structure in the path  $p$  and  $m$  is its well (middle node), so  $p$  contains at least one convergence:  $X \rightarrow m \leftarrow T$ .

- (a) If  $m \notin CC_{i+1}$  and  $k \notin CC_{i+1}$  then the path  $p$  is blocked by  $CC_{i+1} \Rightarrow \text{Indp}(X ; T | CC_{i+1})$
- (b) If  $m \in CC_{i+1}$  then the path  $p$  is active by  $CC_{i+1}$  and we have  $\text{dep}(X ; T | m)$  but this dependance is indirect between  $X$  and  $T$ , so it will be removed in Phase II (Backward) of our algorithm and hence  $\Rightarrow \text{Indp}(X ; T | CC_{i+1})$
- (c) If  $m \notin CC_{i+1}$  and  $k \in CC_{i+1}$  then there is at least one V-structure in the path  $p'$  and  $k$  is its middle node  $X \rightarrow k \leftarrow T \Rightarrow \text{Indp}(X ; T | CC_{i+1})$

Let  $p$  be a path from  $T$  to  $X$  through the node  $m \in V_{i-1}$

- (a) If  $m \in CP_{i-1}$  then, because of temporality, there is at least a divergence in the path  $p$ , so the path  $p$  is blocked by  $CP_{i-1} \Rightarrow \text{Indp}(X ; T \mid CP_{i-1})$   
Hence  $\forall X \in V_i \setminus, \text{Min}_{(Z \subseteq CP_{i-1})} \text{Assoc}(X ; T \mid Z) = 0$

we conclude that  $\forall Z \subseteq V_{i+1} \Rightarrow \text{Indp}(X ; T \mid Z) \Rightarrow \text{Indp}(X ; T \mid CC_{i+1})$

Hence  $\forall X \in V_i \setminus, \text{Min}_{(Z \subseteq CC_{i+1})} \text{Assoc}(X ; T \mid Z) = 0$

Hence  $\text{Max}_{X \in V_i \setminus T} \text{Min}_{(Z \subseteq Ne_+)} \text{Assoc}(X ; T \mid Z)$

$= \text{Max}_{X \in V_i \setminus T} \text{Min}_{(Z \subseteq CPC_i \cup CC_{i+1} \cup CP_{i-1})} \text{Assoc}(X ; T \mid Z)$

$= \text{Max}_{X \in V_i \setminus T} \{ \text{Min}_{(Z \subseteq CPC_i)} \text{Assoc}(X ; T \mid Z), \text{Min}_{(Z \subseteq CC_{i+1})} \text{Assoc}(X ; T \mid Z), \text{Min}_{(Z \subseteq CP_{i-1})} \text{Assoc}(X ; T \mid Z) \}$

$= \text{Max}_{X \in V_i \setminus T} \{ \text{Min}_{(Z \subseteq CPC_i)} \text{Assoc}(X ; T \mid Z), 0, 0 \}$

$= \text{Max}_{X \in V_i \setminus T} \text{Min}_{(Z \subseteq CPC_i)} \text{Assoc}(X ; T \mid Z)$

## 2. $\forall X \in V_{i+1}$

Our model is the first Markov-order  $P(X_t \mid X_{1:t-1}) = P(X_t \mid X_{t-1})$

Hence  $\text{Max}_{X \in V_i \setminus T} \text{Min}_{(Z \subseteq Ne_+)} \text{Assoc}(X ; T \mid Z)$

$= \text{Max}_{X \in V_i \setminus T} \text{Min}_{(Z \subseteq CPC_i \cup CC_{i+1} \cup CP_{i-1})} \text{Assoc}(X ; T \mid Z)$

$= \text{Max}_{X \in V_i \setminus T} \text{Min}_{(Z \subseteq CPC_i \cup CC_{i+1})} \text{Assoc}(X ; T \mid Z)$

## 3. $\forall X \in V_{i-1}$

Let  $p$  be a path from  $X$  to  $T$  through the node  $m \in V_{i+1}$  and  $K$  its child ( $k \in V_{i+1}$ )

Because of temporality, there is at least one V-structure in the path  $p$  and  $m$  is its well (middle node), so  $p$  contains at least one convergence:  $X \rightarrow m \leftarrow T$ .

- (a) If  $m \notin CC_{i+1}$  et  $k \notin CC_{i+1}$  then the path  $p$  is blocked by  $CC_{i+1} \Rightarrow \text{Indp}(X ; T \mid CC_{i+1})$
- (b) If  $m \in CC_{i+1}$  then the path  $p$  is active by  $CC_{i+1}$  and we have  $\text{dep}(X ; T \mid m)$  but this dependance is indirect between  $X$  and  $T$ , so it will be removed in Phase II (Backward) of our algorithm and hence  $\Rightarrow \text{Indp}(X ; T \mid CC_{i+1})$
- (c) If  $m \notin CC_{i+1}$  et  $k \in CC_{i+1}$  then there is at least one V-structure in the path  $p'$  and  $k$  is its middle node  $X \rightarrow k \leftarrow T \Rightarrow \text{Indp}(X ; T \mid CC_{i+1})$

We conclude that  $\forall Z \subseteq V_{i+1}$  we have  $\text{Indp}(X ; T \mid Z) \Rightarrow \text{Indp}(X ; T \mid CC_{i+1})$

Hence  $\forall X \in V_i \setminus, \text{Min}_{(Z \subseteq CC_{i+1})} \text{Assoc}(X ; T \mid Z) = 0$

Hence  $\text{Max}_{X \in V_{i-1} \setminus T} \text{Min}_{(Z \subseteq Ne_+)} \text{Assoc}(X ; T \mid Z)$

$= \text{Max}_{X \in V_{i-1} \setminus T} \text{Min}_{(Z \subseteq CPC_i \cup CC_{i+1} \cup CP_{i-1})} \text{Assoc}(X ; T \mid Z)$

$= \text{Max}_{X \in V_{i-1} \setminus T} \{ \text{Min}_{(Z \subseteq CPC_i)} \text{Assoc}(X ; T \mid Z), \text{Min}_{(Z \subseteq CC_{i+1})} \text{Assoc}(X ; T \mid Z), \text{Min}_{(Z \subseteq CP_{i-1})} \text{Assoc}(X ; T \mid Z) \}$

$$\begin{aligned}
& |Z\} \\
& = \text{Max}_{X \in V_{t-1}} \{ \text{Min}_{(Z \subseteq CPC_t)} \text{Assoc}(X; T | Z), 0, \text{Min}_{(Z \subseteq CP_{t-1})} \text{Assoc}(X; T | Z) \} \\
& = \text{Max}_{X \in V_{t-1}} \text{Min}_{(Z \subseteq CPC_t \cup CP_{t-1})} \text{Assoc}(X; T | Z)
\end{aligned}$$

### C.1.2 Proof proposition 5.4.1:

As we know, in the Forward phase of DMMHC we need to calculate the assoc function. Function  $\text{Assoc}(X; T|Z)$  is an estimate of the strength of association (dependency) of  $X$  and  $T$  given  $Z$ . To identify all  $X$  in  $CP_{t-1}(T)$ , we need to compute the  $\text{Assoc}(X; T|CP_{t-1} \cup CPC_t)$  (see algorithm 12).

- in this case we don't have  $CPC_t(T)$  candidates. So, we mustn't find any false positives depending on the structure  $\text{Assoc}(X; T|CP_{t-1} \cup CPC_t) = \text{Assoc}(X; T|CP_{t-1})$ .
- we mustn't find any intra-connectivity in  $t-1$  time slice. So,  $\text{Assoc}(X; T|CP_{t-1} \cup CPC_t) = \text{Assoc}(X; T|\emptyset)$  where  $X \in V_{t-1}$  and  $V_{t-1}$  is the set of all variables in  $t-1$  time slice.

$$\max_{X \in V_{t-1}} \min \text{Assoc}(X; T|Z) = \max_{X \in V_{t-1}} \min_{S \subseteq Z} \text{Assoc}(X; T|S) \quad (\text{C.1})$$

$$= \max_{X \in V_{t-1}} \min_{S \subseteq CP \cup CPC} \text{Assoc}(X; T|S) \quad (\text{C.2})$$

$$= \max_{X \in V_{t-1}} \min_{S \subseteq \emptyset} \text{Assoc}(X; T|S) \quad (\text{C.3})$$

$$= \max_{X \in V_{t-1}} \text{Assoc}(X; T|\emptyset) \quad (\text{C.4})$$



# Bibliography

- [1] Statlib repository. <http://lib.stat.cmu.edu/>;  
<http://www.liacc.up.pt/ltorgo/Regression/stock.tgz>. 58
- [2] H. Akaike. Statistical predictor identification. *Ann. Inst. Statist. Math.*, 22:203–217, 1970. 37
- [3] S. Andreassen, F.V. Jensen, S.K. Andersen, B. Falck, U. Kjærulff, M. Woldbye, A.R. Sørensen, A. Rosenfalck, and F. Jensen. MUNIN — an expert EMG assistant. In *Computer-Aided Electromyography and Expert Systems*, chapter 21. Elsevier Science Publishers, 1989. 45
- [4] A. Antonucci and M. Zaffalon. Decision-theoretic specification of credal networks: A unified language for uncertain modeling with sets of Bayesian networks. *International Journal of Approximate Reasoning*, 49:345–361, 2008. 113
- [5] Z. Aydin, A. Singh, J. Bilmes, and W.S Noble. Learning sparse models for a dynamic Bayesian network classifier of protein secondary structure. *BMC Bioinformatics*, 12:154, 2011. 56
- [6] M. Bartlett and J. Cussens. Advances in Bayesian network learning using integer programming. In *Proceedings of the 29th Conference on Uncertainty in Artificial Intelligence (UAI 2013)*. AUAI Press, 2013. 46
- [7] T. Bayes. An essay towards solving a problem in the doctrine of chances. *Philosophical Transactions of the Royal Society*, 53:370–418, 1763. 30
- [8] I. Beinlich, G. Suermondt, R. Chavez, and G. Cooper. The alarm monitoring system: A case study with two probabilistic inference techniques for belief networks. In *Proceedings of the 2nd European Conference on AI and Medicine*, pages 247–256, Berlin, 1989. Springer-Verlag. 97
- [9] C. Boutilier, R. Dearden, and M. Goldszmidt. Exploiting structure in policy construction. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pages 1104–1113, 1995. 58
- [10] T. Charitos, Van der Gaag L.C, Visscher S., Schurink K.A.M., and Lucas P.J.F. A dynamic Bayesian network for diagnosing ventilator-associated pneumonia in icu patients. *Expert Systems with Applications*, 36:1249–1258, 2009. 113



- [11] D. Chickering. Learning Bayesian networks is np-complete. In *InD. Fisher and H. Lenz (Eds.), Learning from data: Artificial intelligence and statistics V*. Springer-Verlag, 1996. [35](#), [36](#), [37](#), [66](#)
- [12] D. Chickering, D. Geiger, and D.E. Heckerman. Learning Bayesian networks is NP-hard. Technical Report MSR-TR-94-17, Microsoft Research Technical Report, 1994. [66](#)
- [13] D. Chickering, C. Meek, and D. Heckerman. Large-sample learning of Bayesian networks is np-hard. *Journal of Machine Learning Research*, 5:1287–1330, 2004. [66](#)
- [14] D.M. Chickering. Learning equivalence classes of Bayesian-network structures. *Journal of Machine Learning Research*, 2:445–498, 2002. [34](#)
- [15] C. Chow and C. Liu. Approximating discrete probability distributions with dependence trees. *IEEE Transactions on Information Theory*, 14(3):462–467, 1968. [37](#)
- [16] C. Conati, A. S. Gertner, K. VanLehn, and M. J. Druzdzel. On-line student modeling for coached problem solving using Bayesian networks. In *Proceedings of the Sixth International Conference on User Modeling (UM-96)*, pages 231–242, 1997. [45](#), [97](#)
- [17] G. Cooper and E. Hersovits. A Bayesian method for the induction of probabilistic networks from data. *Maching Learning*, 9:309–347, 1992. [37](#)
- [18] Chickering D. and Heckerman D. Efficient Approximation for the Marginal Likelihood of Incomplete Data given a Bayesian Network. In *UAI'96*, pages 158–168. Morgan Kaufmann, 1996. [37](#)
- [19] Denver Dash and Marek Druzdzel. A robust independence test for constraint-based learning of causal structure. In *Proceedings of the Nineteenth Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-03)*, pages 167–174, San Francisco, CA, 2003. Morgan Kaufmann. [98](#)
- [20] C.P. de Campos, Z. Zeng, and Q. Ji. Structure learning of Bayesian networks using constraints. In *Proceedings of the 26 th International Conference on Machine Learning, Montreal, Canada*, pages 113–120, 2009. [42](#)
- [21] Luis M. de Campos and Javier G. Castellano. Bayesian network learning algorithms using structural restrictions. *International Journal of Approximate Reasoning*, pages 233–254, 2007. [45](#)
- [22] M. de Jongh and M.J. Druzdzel. A comparison of structural distance measures for causal Bayesian network models. *Recent Advances in Intelligent Information Systems, Challenging Problems of Science, Computer Science series*, pages 443–456, 2009. [15](#), [46](#), [98](#), [99](#)

- [23] T. Dean and K.A. Kanazawa. A model for reasoning about persistence and causation. *Artificial Intelligence*, 93(2):1–27, 1989. [21](#), [50](#)
- [24] A. Dempster, N. Laird, and D. Rubin. Maximum likelihood from incomplete data via the em algorithm. *Journal of the Royal Statistical Society*, 39:1–38, 1977. [119](#)
- [25] T. Dietterich. Hierarchical reinforcement learning with the maxq value function decomposition. *Journal of Artificial Intelligence Research*, 13:227–303, 2000. [58](#)
- [26] T. G. Dietterich. Machine learning. In *Nature Encyclopedia of Cognitive Science*, London: Macmillan, 2003. [20](#)
- [27] N. Dojer. Learning Bayesian networks does not have to be np-hard. In Springer-Verlag Berlin Heidelberg, editor, *MFCS 2006, LNCS 4162*, pages 305–314, 2006. [53](#), [54](#), [56](#), [59](#), [60](#), [90](#)
- [28] R. Donat. *Modélisation de la fiabilité et de la Maintenance à partir de modèles Graphiques probabilistes*. Thèse de doctorat, INSA de Rouen, France, 2009. [113](#)
- [29] A. S. FAST. *Learning the structure of Bayesian networks with constraint satisfaction*. Phd, University of Massachusetts Amherst, 2010. [36](#), [44](#)
- [30] Y. Freund, H.S. Seung, E. Shamir, and N. Tishby. Selective sampling using the query by committee algorithm. *Machine Learning*, 28:133–168, 1997. [41](#)
- [31] N. Friedman. The Bayesian structural em algorithm. In *Proceedings of the 14th Conference on Uncertainty in Artificial Intelligence*, pages 129–138, San Francisco, 1996. Morgan Kaufmann. [55](#)
- [32] N. Friedman, K. Murphy, and S. Russell. Learning the structure of dynamic probabilistic networks. In Gregory F. Cooper and Serafín Moral, editors, *Proceedings of the 14th Conference on Uncertainty in Artificial Intelligence (UAI-98)*, pages 139–147, San Francisco, 1998. Morgan Kaufmann. [20](#), [55](#), [79](#), [86](#), [90](#), [113](#)
- [33] N. Friedman, I. Nachman, and D. Peér. Learning Bayesian network structure from massive datasets: The "sparse candidate" algorithm. In *Proceeding UAI'99 Proceedings of the Fifteenth conference on Uncertainty in artificial intelligence*, pages 206–215, 1999. [41](#)
- [34] José A. Gámez, Juan L. Mateo, and José M. Puerta. Learning Bayesian networks by hill climbing: efficient methods based on progressive restriction of the neighborhood. *Data Mining and Knowledge Discovery*, 22:106–148, 2011. [15](#), [98](#), [99](#)

- [35] S. Gao, Q. Xiao, Q. Pan, and Q. Li. Learning dynamic Bayesian networks structure based on Bayesian optimization algorithm. *Advances in Neural Networks, Computer Science, Springer Berlin / Heidelberg*, 4492:424–431, 2007. [21](#), [55](#), [58](#), [59](#), [90](#)
- [36] S. Geman and D. Geman. Stochastic relaxation gibbs distributions, and the Bayesian restoration of images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 6(6):721–741, 1984. [119](#)
- [37] M.F. Geyik, C. Ayaz, M.K. Çelen, and C. ÜSTÜN. Surveillance of nosocomial infections in dicle university hospital: a ten-year experience. *Turk J Med Sci*, 38(6):587–593, 2008. [113](#)
- [38] M. Ghavamzadeh and S. Mahadevan. Continuous-time hierarchical reinforcement learning. In *Proceedings of the International Conference on Machine Learning*, pages 186–193, 2001. [58](#)
- [39] J.A. Gomez, J.L. Mateo, and J.M. Puerta. Learning Bayesian networks by hill climbing: efficient methods based on progressive restriction of the neighborhood. *Data Mining and Knowledge Discovery*, 22(1-2):106–148, 2011. [46](#)
- [40] Kallel H., Bouaziz M., Ksibi H., Chelly H., Hmida C.B., Chaari A., and Rekik N. Prevalence of hospital-acquired infection in a tunisian hospital. *Journal of Hospital Infection*, 59:343–347, 2005. [113](#)
- [41] D. Heckerman. A tutorial on learning with Bayesian network. In *Michael I. Jordan, editor, Learning in Graphical Models*, pages 301–354, Boston, 1998. [117](#)
- [42] D. Heckerman, D. Geiger, and D. Chickering. Learning Bayesian networks: The combination of knowledge and statistical data. *Machine Learning*, 20:197–243, 1995. [35](#), [37](#), [66](#)
- [43] J.W. Hwang, Y.S. Lee, and Cho S.B. Structure evolution of dynamic Bayesian network for traffic accident detection. In *IEEE Congress on Evolutionary Computation*, pages 1655–1671, 2011. [58](#), [60](#)
- [44] K. Hwang, B. Kim, and B. Zhang. Learning hierarchical Bayesian networks for large-scale data analysis. In *CONIP 2006*, pages 670–679, 2006. [41](#)
- [45] Jaime S. Ide and Fabio G. Cozman. Random generation of Bayesian networks. In *Brazilian Symp. on Artificial Intelligence*, pages 366–375. Springer-Verlag, 2002. [45](#)
- [46] Georges J. and Luciano M. Prediction of financial time series with timeline hidden Markov experts and anns. *Wseas Transactions on Business and Economics*, 4(9):140–146, 2007. [113](#)

- [47] Hulst J. Modeling physiological processes with dynamic Bayesian networks. Technical report, Faculty of Electrical Engineering, Mathematics and Computer Science, 2006. [52](#)
- [48] C.S. Jensen and A. Kong. Blocking Gibbs sampling for linkage analysis in large pedigrees with many loops. Research Report R-96-2048, Department of Computer Science, Aalborg University, Denmark, 1996. [20](#)
- [49] C.S. Jensen and A. Kong. Blocking Gibbs sampling for linkage analysis in large pedigrees with many loops. Research Report R-96-2048, Department of Computer Science, Aalborg University, Denmark, 1996. [28](#), [45](#), [97](#)
- [50] A. Jonsson and A. Barto. Active learning of dynamic Bayesian networks in Markov decision processes. *Springer, SARA 2007, LNAI 4612*, 19:273–284, 2007. [21](#), [58](#), [59](#)
- [51] M. Jordan. Learning in graphical models. In *Academic Publishers, Dordrecht, The Netherlands*, 1998. [117](#)
- [52] M.I. Jordan. Graphical models. *Statistical Science (Special Issue on Bayesian Statistics)*, 19:140–155, 2004. [20](#), [28](#)
- [53] P. Keen and M. Scott. An organizational perspective. *Decision Support, Systems Addison-Wesley Publishing Company*, 1978. [113](#)
- [54] U.B. Kjærulff and A.L. Madsen. *Bayesian Networks and Influence Diagrams*. Springer Science+Business Media, LLC., 2008. [114](#)
- [55] W.A. Knaus, D.P. Wagner, and J. Lynn. Short term mortality predictions for critically ill hospitalised adults. *Science and ethics.*, 254:389–394, 1991. [113](#)
- [56] k. Kojima, E. Perrier, S. Imoto, and S. Miyano. Optimal search on clustered structural constraint for learning Bayesian network structure. *Journal of Machine Learning Research*, 11:285–310, 2010. [46](#)
- [57] W. Lam and F. Bacchus. Using causal information and local measures to learn bayesian networks. In *Proceedings of the 9th Conference on Uncertainty in Artificial Intelligence*, pages 243–250. Morgan Kaufman Publishers, 1993. [37](#)
- [58] S. Lauritzen. The em algorithm for graphical association models with missing data. *Computational Statistics and Data Analysis*, 19:191–201, 1995. [119](#)
- [59] S. Lauritzen and D. Spiegelhalter. Local computations with probabilities on graphical structures and their application to expert systems. *Royal statistical Society B*, 50:157–224, 1988. [17](#), [28](#), [68](#), [72](#), [73](#)
- [60] K.L. Lee, D.B. Pryor, F.E. Harrell, H.M. Califf, V.S. Behar, and W.L. Floyd. Predicting outcome in coronary disease: statistical models vs. expert clinicians. *Am J Med*, 80(4):553–560, 1986. [113](#)

- [61] P. Leray and O. Francois. Bnt structure learning package : Documentation and experiments. Technical report, Laboratoire PSI - INSA Rouen, 2005. [58](#)
- [62] H. Lähdesmäki and I. Shmulevich. Learning the structure of dynamic Bayesian networks from time series and steady state measurements. *Mach Learn, Springer Science and Business Media*, 71(2-3):185–217, 2008. [54](#)
- [63] H. Ltifi, M. B. Ayed, G. Trabelsi, and A. M. Alimi. Using perspective wall to visualize medical data in the intensive care unit. In *Workshops at IEEE International Conference on Data Mining ICDM2012*, pages 72–78, 2012. [23](#)
- [64] H. Ltifi, G. Trabelsi, M. B. Ayed, and A. M Alimi. Dynamic decision support system based on Bayesian networks -application to fight against the nosocomial infections. *International Journal of Advanced Research in Artificial Intelligence*, 1:22–29, 2012. [23](#), [114](#)
- [65] E. Maharaj. A pattern recognition of time series using wavelets. In *15th Computational Statistics Conference of the International Association of Statistical Computing*, Berlin, 2002. [58](#)
- [66] A.J. Marcel, G.T. Babs, and J.F. Peter. Dynamic Bayesian networks as prognostic models for clinical patient management. *Journal of Biomedical Informatics*, 41:515–529, 2008. [113](#)
- [67] D. Margaritis. *Learning Bayesian networks model structure from data*. Phd thesis, Carnegie Mellon University, Pittsburgh, 2003. [37](#)
- [68] M. Correa, Bielza C., and Pamies-Teixeira J. Comparison of Bayesian networks and artificial neural networks for quality detection in a machining process. *Expert Systems with Applications*, 36:7270–7279, 2009. [113](#)
- [69] Christopher Meek. Causal inference and causal explanation with background knowledge. In *Proceedings of the Eleventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-95)*, pages 403–410. Morgan Kaufmann, 1995. [17](#), [33](#), [34](#), [35](#)
- [70] M. Ben Messaoud. *SemCaDo: an approach for serendipitous causal discovery and ontology evolution*. Thèse de doctorat, University of Nantes, 2012. [17](#), [34](#)
- [71] E.M. Mugambi, A. Hunter, G. Oatley, and L. Kennedy. Polynomial-fuzzy decision tree structures for classifying medical data. *Knowledge-Based Systems*, 17:81–87, 2004. [113](#)
- [72] K.P. Murphy. Bayes net toolbox, technical report. Technical report, MIT Artificial Intelligence Laboratory, 2002. [98](#)
- [73] K.P. Murphy. *Dynamic Bayesian Networks: Representation, Inference and Learning*. Phd, University of California, Berkeley, 2002. [21](#), [47](#), [50](#), [55](#), [66](#)

- [74] K.P. Murphy. Software packages for graphical models, Bayesian networks. In *Bull.Int.Soc.Bayesian Anal*, page 14, 2007. 98
- [75] R.E. Neapolitan. *Learning Bayesian Networks*. Pearson Education, 2004. 11, 27, 28, 32, 33, 35
- [76] H.T. Nguyen. *Réseaux bayésiens et apprentissage ensembliste pour l'étude différentielle de réseaux de régulation génétique*. Thèse de doctorat, Université Nantes, 2012. 43
- [77] Spirtes P., Glymour C., and Scheines R. *Causation, prediction, and search*. Springer-Verlag, 1993. 33, 35, 37
- [78] J. Pearl. *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference*. Morgan Kaufmann Publishers, 1988. 20, 28, 32, 33, 37, 66
- [79] L. Peelen, N.F. De Keizer, E. De Jonge, R.J. Bosman, A. Abu-Hanna, and N. Peek. Using hierarchical dynamic Bayesian networks to investigate dynamics of organ failure in the intensive care unit. *journal of Biomedical Informatics*, 43(2):273–286, 2010. 113, 114
- [80] J.M. Pena, J. Bjorkegren, and J. Tegnér. Scalable, efficient and correct learning of Markov boundaries under the faithfulness assumption. In *Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, volume 3571, pages 136–147. Springer Berlin Heidelberg, 2005. 115
- [81] M.J. Pena, J. Bjorkegren, and J. Tegnér. Learning dynamic Bayesian network models via cross-validation. *Pattern Recognition Letters*, 26:2295–2308, 2005. 56, 58
- [82] J.C. Rajapakse and J. Zhou. Learning effective brain connectivity with dynamic Bayesian networks. *NeuroImage*, 37:749–760, 2007. 56, 58, 59
- [83] Ramsey, S.A. and Klemm, S.L. and Zak, D.E. and Kennedy, Kathleen A. and Thorsson, Vestein and Li, Bin and Gilchrist, Mark and Gold, E.S. and Johnson, C.D. and Litvak, V. and Navarro, G. and Roach, J.C. and Rosenberger, Carrie M. and Rust, A.G. and Yudkovsky, N. and Aderem, A. and Shmulevich, I. Uncovering a macrophage transcriptional program by integrating evidence from motif scanning and expression dynamics. *PLoS Comput Biol*, 4(3):e1000021, 2008. 54
- [84] J.W. Robinson and A.J. Hartemink. Learning non-stationary dynamic Bayesian networks. *Journal of Machine Learning Research*, 11:3647–3680, 2010. 54, 57
- [85] S. Rodrigues de Moraes and A. Aussem. A novel scalable and data efficient feature subset selection algorithm. In *Proceedings of the European conference on Machine Learning and Knowledge Discovery in Databases - Part*



- II*, ECML PKDD '08, pages 298–312, Berlin, Heidelberg, 2008. Springer-Verlag. 39
- [86] Kullback S. and Leibler R.A. On information and sufficiency. *Annals of Mathematical Statistics*, 22(1):79–86, 1951. 43
- [87] G. Schwartz. Estimating the dimension of a model. *Annals of Statistics*, 6:461–464, 1978. 37
- [88] M. Scott. Computer based support for decision making. In *Management decision systems*, Harvard University, Boston, MA, USA, 1971. 113
- [89] Y. Shahar. Dimensions of time in illness: an objective view. *Ann Intern Med*, 132:45–53, 2000. 113
- [90] V.A. Smith, j. Yu, T.V. Smulders, A.J. Hartemink, and E.D. Jarvis. Computational inference of neural information flow networks. *PLoS Comput Biol*, 2(11):e161, 2006. 59, 105
- [91] Le Song, Mladen Kolar, and Eric P. Xing. Time-varying dynamic Bayesian networks. 54, 58, 60
- [92] D. Spiegelhalter and S. Lauritzen. Sequential updating of conditional probabilities on directed graphical structures. *Networks*, 20:579–605, 1990. 119
- [93] P. Spirtes. Conditional independence in directed cyclic graphical models for feedback, technical report cmu-phil-54. Technical report, Department of Philosophy, Carnegie Mellon University, Pittsburgh, 1994. 33
- [94] N. Srebro. Maximum likelihood bounded tree-width Markov networks. In *Artificial Intelligence*, pages 504–511, 2001. 38
- [95] A. R. Statnikov, I. Tsamardinos, and C.F. Aliferis. An algorithm for generation of large Bayesian networks. Technical Report Technical Report DSL-03-01, Department of Biomedical Informatics, Discovery Systems Laboratory, Vanderbilt University, 2003. 46, 75
- [96] H. Steck and T. S. Jaakkola. Unsupervised active learning in large domains. In *Proceedings of the Eighteenth Conference on Uncertainty in Artificial Intelligence (UAI-02)*, pages 469–476, 2002. 41
- [97] J. Stockel, E.A. Welsh, M. Liberton, Kunnvakkam, R. Aurora, and H.B. Pakrasi. Global transcriptomic analysis of cyanothecce 51142 reveals robust diurnal oscillation of central metabolic processes. In *Proceedings of the National Academy of Sciences*, volume 105, pages 6156–6161, 2008. 107
- [98] J. Toepel, E. Welsh, T.C. Summerfield, H.B. Pakrasi, and L.A. Sherman. Differential transcriptional analysis of the cyanobacterium cyanothecce sp. strain atcc 51142 during lightdark and continuous-light growth. *Journal of Bacteriology*, 190(11):3904–3913, 2008. 107

- [99] G. Trabelsi, M. Ben Ayed, and A.M. Alimi. Système d'extraction des connaissances à partir des données temporelles basé sur les réseaux bayésiens dynamiques. In *RNTI E19 Extraction et Gestion des Connaissances EGC*, pages 241–246, Hammamet, Tunisie, JAN 2010. cépadués edition. [113](#)
- [100] G. Trabelsi, P. Leray, M. Ben Ayed, and A.M. Alimi. Benchmarking dynamic Bayesian network structure learning algorithms. In *the 5th International Conference on Modeling, Simulation and Applied Optimization*, pages 1–6, Hammamet, Tunisie, 2013. IEEEExplorer. [2](#), [23](#), [71](#), [102](#)
- [101] G. Trabelsi, P. Leray, M. Ben Ayed, and A.M. Alimi. Dynamic MMHC : a local search algorithm for dynamic Bayesian network structure learning. In *The Twelfth International Symposium on Intelligent Data Analysis (IDA 2013)*, pages 392–403. Springer LNCS, 2013. [2](#), [23](#)
- [102] I. Tsamardinos, C.F. Aliferis, and A.R. Statnikov. Time and sample efficient discovery of Markov blankets and direct causal relations. In *Proceedings of ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, KDD*, 36(4):673–678, 2003. [39](#), [42](#), [46](#)
- [103] I. Tsamardinos and G. Borboudakis. Permutation testing improves Bayesian network learning. In *European Conference on Machine Learning and Principles and Practice of Knowledge Discovery in Databases (ECML-PKDD 2010)*, 2010. [96](#)
- [104] I. Tsamardinos, L. E. Brown, and C. F. Constantin, F. Aliferis. The max-min hill-climbing Bayesian network structure learning algorithm. *Mach. Learn.*, 65(1):31–78, 2006. [33](#), [35](#), [37](#), [40](#), [42](#), [44](#), [45](#), [46](#), [66](#), [76](#), [79](#), [84](#), [88](#), [90](#)
- [105] I. Tsamardinos, F. Aliferis Constantin, and A.R. Statnikov. Algorithms for large scale Markov blanket discovery. In *16th International FLAIRS Conference*, pages 376–380, 2003. [39](#), [46](#)
- [106] I. Tsamardinos, A.R. Statnikov, L. E. Brown, and F. Aliferis Constantin. Generating realistic large Bayesian networks by tiling. In *Proceedings of the 19th International Florida Artificial Intelligence Research Society Conference (FLAIRS)*, pages 592–597, 2006. [69](#)
- [107] A. Tucker and X. Liu. Extending evolutionary programming methods to the learning of dynamic Bayesian networks. In *Proceedings of the Genetic and Evolutionary Computation Conference*, pages 923–929. Morgan Kaufmann, 1999. [90](#)
- [108] A. Tucker, X. Liu, and A. Ogden-Swift. Evolutionary learning of dynamic probabilistic models with large time lags. *International Journal of Intelligent Systems*, 16(5):621–646, 2001. [57](#), [90](#)



- [109] T. Verma and J. Pearl. Equivalence and synthesis of causal models. In *Proceedings of the Sixth Conference on Uncertainty in Artificial Intelligence*, pages 220–227, San Francisco, 1991. Morgan Kaufmann. [33](#), [34](#)
- [110] N.X. Vinh, M. Chetty, R. Coppel, and P. Wangikar. Local and global algorithms for learning dynamic Bayesian networks. In *IEEE 12th International Conference on Data Mining (ICDM2012)*, pages 685–694, 2012. [17](#), [18](#), [56](#), [53](#), [54](#), [57](#), [59](#), [60](#), [61](#), [107](#), [108](#)
- [111] N.X Vinh, M. Chetty, R.L. Coppel, and P.P. Wangikar. Polynomial time algorithm for learning globally optimal dynamic Bayesian network. In *ICONIP 2011, Shanghai, China*, volume 7064, pages 719–729. Springer, 2011. [56](#), [59](#), [60](#), [90](#)
- [112] H. Wang, K. Yu, and H. Yao. Learning dynamic Bayesian networks using evolutionary mcmc. In *Proceedings of the International Conference on Computational Intelligence and Security*, pages 45–50, 2006. [56](#), [90](#)
- [113] K. Wang, J. Zhang, and F. Shen. Adaptive learning of dynamic Bayesian networks with changing structures by detecting geometric structures of time series. *Knowl Inf Syst*, 17:121–133, 2008. [21](#), [54](#), [57](#), [58](#)
- [114] B. Wilczynski and N. Dojer. Bnfinder: exact and efficient method for learning Bayesian networks. *bioinformatics*, 25(2):286–287, 2009. [53](#), [56](#), [59](#), [60](#), [98](#)
- [115] M. Wynkoop and T. Dietterich. Learning mdp action models via discrete mixture trees. *Springer-Verlag Berlin Heidelberg, ECML PKDD*, pages 597–612, 2008. [57](#)
- [116] J. Xu, H. Ge, X. Zhou, J. Yan, Q. Chi, and Z. Zhang. Prediction of vascular tissue engineering results with artificial neural networks. *journal of Biomedical Informatics*, 38:pp417–421, 2005. [113](#)



# Thèse de Doctorat

**Ghada TRABELSI**

**Nouveaux algorithmes d'apprentissage et méthodes d'évaluation pour les réseaux bayésiens dynamiques de grande dimension**

**New structure learning algorithms and evaluation methods for large dynamic Bayesian networks**

## Résumé

Les réseaux bayésiens dynamiques (RBD) sont une classe de modèles graphiques probabilistes qui est devenu un outil standard pour la modélisation de divers phénomènes stochastiques variant dans le temps. A cause de la complexité induite par l'ajout de la dimension temporelle, l'apprentissage de la structure DBN est une tâche très complexe. Les algorithmes existants sont des adaptations des algorithmes d'apprentissage de structure pour les RB basés sur score mais sont souvent limitées lorsque le nombre de variables est élevé. Une autre limitation pour les études d'apprentissage de la structure des RBD, ils utilisent leurs propres Benchmarks et techniques pour l'évaluation. Le problème dans le cas dynamique, nous ne trouvons pas de travaux antérieurs qui fournissent des détails sur les réseaux et les indicateurs de comparaison utilisés. Nous nous concentrons dans ce projet à l'apprentissage de la structure des RBD et ses méthodes d'évaluation avec respectivement une autre famille des algorithmes d'apprentissage de la structure, les méthodes de recherche locale, et une nouvelle approche de génération des grandes standard RBD et un métrique d'évaluation. Nous illustrons l'intérêt de ces méthodes avec des résultats expérimentaux.

## Mots clés

Réseaux Bayésiens Dynamiques, les Modèles 2-TBN, apprentissage de structure, scalability, les méthodes de recherche locale, Benchmarking.

## Abstract

Dynamic Bayesian networks (DBNs) are a class of probabilistic graphical models that has become a standard tool for modeling various stochastic time-varying phenomena. Probabilistic graphical models such as 2-Time slice BN (2TBNs) are the most used and popular models for DBNs. Because of the complexity induced by adding the temporal dimension, DBN structure learning is a very complex task. Existing algorithms are adaptations of score-based BN structure learning algorithms but are often limited when the number of variables is high. Another limitation of DBN structure learning studies, they use their own benchmarks and techniques for evaluation. The problem in the dynamic case is that we don't find previous works that provide details about used networks and indicators of comparison. We focus in this project on DBN structure learning and its methods of evaluation with respectively another family of structure learning algorithms, local search methods, known by its scalability and a novel approach to generate large standard DBNs and metric of evaluation. We illustrate the interest of these methods with experimental results.

## Key Words

Dynamic Bayesian networks, 2-TBN models, structure learning, scalability, local search methods, Benchmarking.