



Stratégies des Opérateurs pour la Recharge des Véhicules Électriques en Espaces Publics avec un Comportement Piloté par l'Utilisateur

Alix Dupont

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Operator Strategies for Electric Vehicle Charging in Public Areas with User-Driven Behavior

Soutenue publiquement le 25/02/2025 devant le jury composé de :

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Résumé

Les Véhicules Électriques (VEs) sont considérés comme une solution clé pour décarboner le secteur du transport. Cependant, les capacités de recharge des Infrastructures de Recharge pour VEs (IRVEs) existantes restent limitées. Augmenter la puissance disponible sur un site entraîne des coûts significatifs pour le réseau électrique, tandis que l'installation d'un grand nombre de bornes est également onéreuse pour les opérateurs. Ainsi, la recharge des VEs doit souvent être gérée dans un environnement contraint et congestionné, ce qui peut nuire à la qualité de service pour les usagers. Dans ce contexte, cette thèse vise à éclairer les prises de décision des acteurs de la recharge publique. Un point de vue décentralisé est adopté, où les décisions des usagers sont prises de manière individuelle. Dans une première partie, ces décisions sont modélisées par un jeu de file d'attente, permettant de comprendre et anticiper les niveaux de congestion aux différentes IRVEs. Dans le cadre d'une recharge standard (typiquement 7 kW à 22 kW) et opportuniste, l'occupation des IRVEs est modélisée par un système de file d'attente avec perte. Une approximation sur l'énergie moyenne reçue par un VE est proposée et la qualité de cette approximation est mesurée théoriquement et numériquement. La congestion impacte la probabilité de blocage et la puissance individuelle de charge. L'interaction qui en résulte entre les usagers est modélisée par un *jeu de potentiel*. L'unicité d'un *équilibre de Wardrop* est démontrée, caractérisée, et associée à des bornes sur le *prix de l'anarchie*. Dans le cadre d'un besoin urgent de recharge rapide, l'occupation aux IRVEs est représentée par des files d'attente sans perte. La congestion à une IRVE affecte le temps d'attente pour la recharge. Les résultats montrent que les usagers tendent à utiliser davantage les IRVEs attractives (par leur puissance de charge ou leur emplacement) qu'il ne le serait dans un système centralisé, où les choix de recharge sont décidés par une entité centrale plutôt que par les usagers eux-mêmes. Cependant, l'inefficacité du système décentralisé reste modérée. Dans une seconde partie, des schémas de tarification en temps réel sont explorés, via une approche *bi-niveau*. Ces schémas visent à atténuer les effets de congestion, et tiennent compte des couplages dans le temps. Dans le cadre d'une IRVE limitée par le nombre de points de charge et proposant une recharge standard, la politique de tarification proposée impose un coût aux usagers dépassant une durée de stationnement prédéfinie. L'occupation est modélisée par une *chaîne de Markov en temps continu*, partiellement contrôlée par la tarification. La dynamique de la tarification dépend du niveau d'occupation instantané, de l'heure de la journée, ou d'une combinaison des deux. Les résultats numériques indiquent que l'intégration du niveau d'occupation instantané n'améliore pas significativement les performances. Enfin, la possibilité de réduire ponctuellement la puissance agrégée à une ou plusieurs IRVEs en échange d'une rémunération est examinée. Un menu de niveaux de puissance de charge est proposé aux usagers. Les prix pour chaque option sont ajustés dynamiquement, tenant compte de la variation des prix de l'électricité et de la puissance agrégée (quand elle est impactante). Dans ce cadre, le problème *bi-niveau* est transformé en un PLMNE (Programmation Linéaire Mixte en Nombres Entiers). Les résultats numériques donnent des intuitions sur la manière dont un opérateur de charge répondrait face à des incitatifs de diminution ponctuelle de puissance agrégée.

Mots-clés: véhicule électrique, jeu de file d'attente, chaîne de Markov, tarification

Abstract

Electric Vehicles (EVs) are considered a key solution for decarbonizing the transportation sector. However, the charging capacity of existing Charging Station Pools (CSPs) remains limited. Increasing the available power at a site incurs significant costs for the electrical grid, while installing a large number of charging points is also expensive for operators. Consequently, EV charging often has to be managed in constrained and congested environments, which can negatively affect the quality of service for users. In this context, this thesis aims to support decision-making for public charging stakeholders. A decentralized perspective is adopted, where users make individual charging decisions. In the first part, these decisions are modeled using a queueing game, allowing the analysis and anticipation of congestion levels at various CSPs. In the case of standard (typically 7 kW to 22 kW) and opportunistic charging, CSP occupancy is modeled using a loss queueing system. An approximation of the average energy received by an EV is proposed, with its quality assessed both theoretically and numerically. Congestion affects the blocking probability and individual charging power. The resulting user interactions are modeled as a *potential game*. The uniqueness of a *Wardrop equilibrium* is demonstrated, characterized, and associated with bounds on the *price of anarchy*. For urgent fast-charging needs, CSP occupancy is represented by lossless queueing systems. Congestion at a CSP impacts user waiting times. In a framework where users are differentiated by their cost functions and impact on congestion (based on energy needs), the existence of a *stochastic user equilibrium* is proven. Results show that users tend to overuse attractive CSPs (due to high charging power or location) compared to a centralized system, where charging choices are decided by a central entity rather than by the users themselves. However, the inefficiency of the decentralized system remains moderate. In the second part, real-time pricing schemes are explored using a *bi-level* approach. These schemes aim to mitigate congestion effects while accounting for temporal coupling. For a CSP limited by the number of charging points and offering standard charging, the proposed pricing policy imposes a fee on users exceeding a predefined parking duration. Occupancy is modeled using a *continuous-time Markov chain*, partially controlled by the pricing policy. The pricing dynamics depend on instantaneous occupancy, time of day, or a combination of both. Numerical results indicate that incorporating instantaneous occupancy into pricing does not significantly improve performance. Finally, the possibility of temporarily reducing the aggregate power at one or more CSPs in exchange for compensation is examined. A menu of charging power levels is offered to users, with prices for each option dynamically adjusted based on electricity price fluctuations and aggregate power constraints (when impactful). In this context, the bi-level problem is formulated as a MILP (Mixed Integer Linear Programming). Numerical results provide insights into how a charging operator might respond to incentives for temporary reductions in aggregate power.

Keywords: electric vehicle, queueing game, Markov chain, pricing

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Chapter 1

Introduction

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This chapter outlines the context and challenges behind this thesis on electric vehicle charging management. Section 1.1 introduces the background of electric vehicle charging. Section 1.2 discusses the issues related to the integration of electric vehicles, along with a brief review of the approaches explored in the literature. In Section 1.3, the scenarios examined in this thesis are summarized, highlighting the associated contributions, followed by an overview of the thesis structure. Finally, Section 1.4 lists the publications related to this thesis.

1.1 Electric vehicles charging: background and current context

1.1.1 Electric vehicles

Definition of an electric vehicle

A vehicle is a very broad term. It refers to any machine used for transporting people or goods [Oxford, 2024], such as cars, buses, trucks, boats or planes. As a consequence, the terminology Electric Vehicles (EVs) is also very vast and refers to any type of vehicle which is powered by one or several electric motors [IPCC, 2018]. In particular, there are several types of EVs that are rechargeable including: (i) Battery Electric Vehicles (BEVs), which refers to EVs which propulsion is entirely electric; (ii) Range Extended Electric Vehicles (REEVs), which in addition can use an internal combustion engine to recharge the battery; (iii) Plug-in Hybrid Electric Vehicles (PHEVs), referring to EVs which propulsion is both electric and thermal; and (iv) Fuel Cell Electric Vehicles (FCEVs), which energy is provided in particular by hydrogen [EEA, 2022]. In this thesis, EVs refer in general to light-duty on-roads BEVs that are cars. This thesis

integrates charging behaviors that better align with the user charging behavior of these types of EVs, such as the users willingness to pay for charging and/or parking, their charging need and parking duration.

Environmental impacts

In Europe, the transportation sector is responsible for 29% of the total greenhouse emissions [EEA, 2024]. EVs are seen as a key solution to decarbonize the transportation system, and thus reduce greenhouse gas emissions. For example, to promote the integration of EVs, sales of Internal Combustion Engine Vehicle (ICEV) will be forbidden in Europe by 2035 [European Commission, 2022]. To assess to which extent an EV is more beneficial to the environment than its equivalent ICEV, a brief review of the Life Cycle Analysis (LCA) of an EV, with a focus on the comparison with an ICEV, is presented in this section. LCA consists in assessing the environmental impact of an EV throughout its lifetime. It includes several stages, (i) the mining and processing of materials for manufacturing the EV, particularly the battery; (ii) the production of the EV; (iii) in-use energy consumption; and (iv) end-of-life management [EEA, 2022]. The LCA of an EV depends on many factors, such as the type of battery, its weight, lifetime and energy consumption, the user's driving style, and the electricity generation for charging the battery. Several metrics must be taken into account when assessing the impact of EVs, such as climate change, but also human health (e.g., air pollution, noise pollution and human toxicity), and ecosystem impacts (e.g., freshwater ecotoxicity, terrestrial ecotoxicity and terrestrial acidification potential) [Reichmuth et al., 2022]. Figure 1.1 shows some comparison examples between EVs and ICEVs.

Raw materials and production The mining and processing of raw materials stage has a greater impact on the environment for EVs than for ICEVs [EEA, 2022]. EVs require more raw materials than ICEVs, particularly for the manufacture batteries. Mining and processing these raw materials, such as cobalt and nickel, requires a lot of energy, which is generally generated from fuel combustion, as well as water. Moreover, the mining of rare-earth elements generates water pollution and human health toxicity as they contain radioactive substances.

In-use stage EVs can offer environmental benefits over ICEVs in terms of greenhouse gas emissions due to the in-use phase. However, the overall impact depends on two sub-stages: (1) the production, transportation, and distribution of energy (well-to-tank), and (2) the energy consumption within the vehicle (tank-to-wheel) [Ramachandran and Stimming, 2015]. The main advantage of EVs comes from the TTW stage, where emissions are zero. The extent of greenhouse gas emission reductions from the use of EVs compared to ICEVs strongly depends on the electricity generation mix for charging them [Woo et al., 2017]. When the electricity generation mix during charging is mainly composed of nuclear and renewable sources, like in Europe [EMBER, 2024, European Council, 2024], the reduction in greenhouse gas emission compared to ICEVs during the in-use phase is significant, as shown in Figure 1.1.

In addition to greenhouse gas emissions, the impact on human health must also be considered. EVs are advantageous due to their zero tailpipe emissions and lower noise pollution compared to ICEVs. However, reduced noise may lead to an increase in car accidents. Local emissions from braking, tire wear, and road abrasion still occur, with the weight of the EVs – typically higher than that of ICEVs due to their batteries – contributing to increased tire-road friction. On the

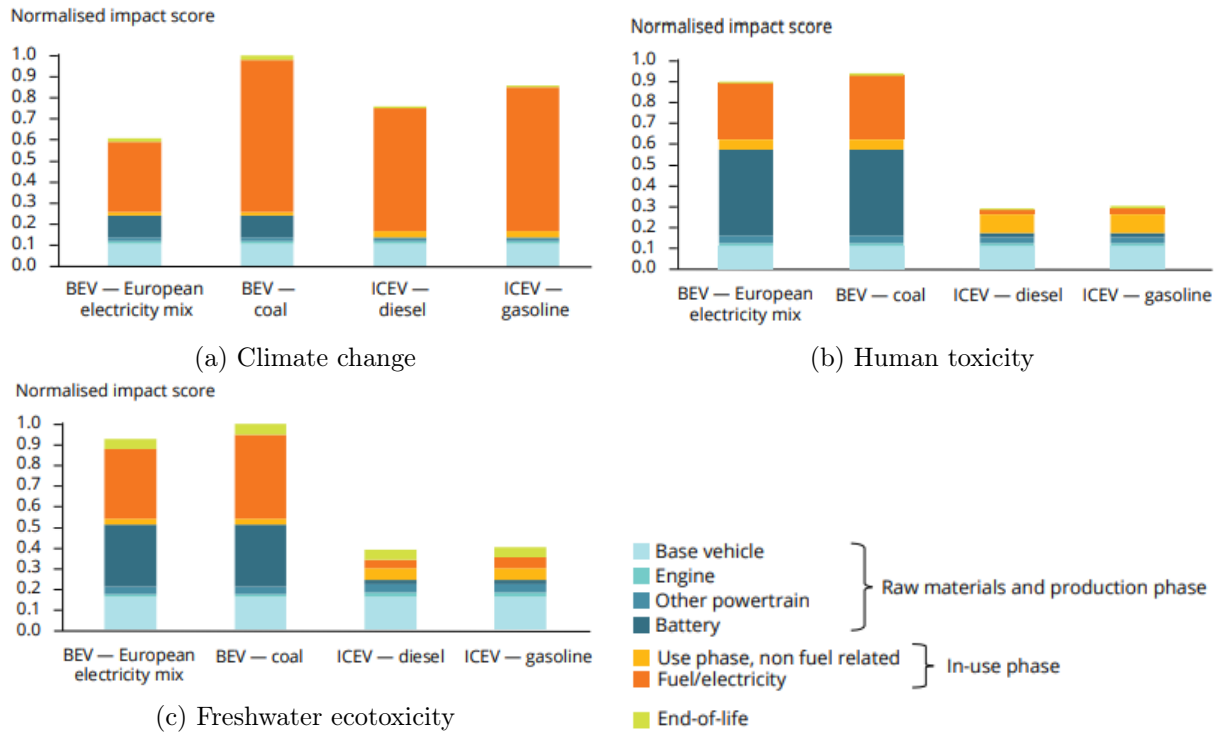


Figure 1.1: Example comparison between BEVs and ICEVs impacts. Source: [EEA, 2022] and [Hawkins et al., 2013]. *BEVs is beneficial in terms of greenhouse gas emissions compared to ICEV, given that the electricity mix for charging the BEVs is sufficiently decarbonized. However, BEVs degrades the quality of freshwater and human health compared to ICEV, mostly due to the extraction of the raw material necessary to the construction of the battery.*

other hand, regenerative braking in EVs reduces brake wear.¹

Other factors, such as the sedentary lifestyle associated with vehicle use and land use for parking, also affect both EVs and ICEVs similarly. The space required for parking contributes to urban sprawl, impacting biodiversity, as highlighted in the French online article [Bigo, 2022].

The end-of-life stage As depicted in Figure 1.1, the end-of-life stage impact is not significant compared to the other stages, both for ICEVs and EVs. In addition, it can be reduced for EVs since batteries can be reused, for example in grid storage applications to store intermittent energy production such as solar and wind. Moreover, future research on battery recycling is expected to improve the process and decrease its impact on the environment.

To summarize, although EVs have a greater impact on human health and the ecosystem compared to ICEVs, their utilization enables to reduce the greenhouse gas emissions, compared to ICEVs, provided that the electricity production is sufficiently low-carbon. For this reason, the longer the in-use stage of an EV and the more decarbonized the electricity used for charging, the more beneficial the EVs are in terms of greenhouse emissions compared to ICEVs. Moreover, the climate advantages of EVs over ICEVs can be further enhanced through smart charging management. For example, charging can be prioritized during periods of low-carbon electricity

¹ See for example [Pod Point: How Does Regenerative Braking Work?](#).

production, such as when renewable energy generation is high, or optimized to preserve battery from rapid degradation.

History and recent development

This section gives a brief overview of the past evolution of the EVs and their recent growing development.

Beginning of electric vehicles The history of the EV development goes back to the 1830s. The first EV was designed in 1834 by Thomas Davenport [S.R., 1979], and the first rechargeable EV was designed in 1859 by Gaston Planté. During the late 19th century, advancements in research led to scientific progress in EV technology. As a result, EVs gained popularity during the second half of the century. For example, the EV named *La jamais contente* broke the speed record in 1899 by exceeding 100 km/h [Le Monde, 2019], and 38% of the car market in the United States are electric [Britanica, 2024] in the beginning of the 20th century, mostly taxis.

First half of the 20th century: the shift from electric to combustion vehicles In the 1910s, and in particular with the first world war, the EVs have given way to the development of thermal vehicles due to the cost-efficiency, autonomy and faster speed of the ICEVs compared to EVs [Britanica, 2024]. Moreover, the development of EVs requires investments in the charging infrastructures, and in the whole electrical system, which was a brake for the development of EVs. During most of the 20th century, EVs remain only a back-up solution, in cases of hydrocarbons non-availability, for example during the 2nd world war (or during the oil crisis of 1973). Car manufacturers have favored the development of ICEV because the construction process was already well-established.

Second half of the 20th century: a renew of interest In the end of the 20th century, EVs have seen a renew of interest due to environmental factors, mostly air pollution in urban areas. For example, in 1966, the US congress recommend the construction of ecological vehicles, and in 1996, General Motors commercialized the EV1, in response to the California state policy in America of Zero Emission Vehicle (ZEV) [Sperling and Gordon, 2009] in order to reduce pollution in urban areas. However, this renew of interest remained low, in particular due to high manufacture costs. In the beginning of the 21st century, EVs have emerged as an important solution to reduce Global Warming [European Commission, 2022]. To address range limitations, hybrid EVs were developed first. Later, significant advancements in battery capacity led to the emergence of BEVs as well.

Today's growing development of electric mobility Due to the benefits of EVs compared to ICEVs for global warming reduction, EV is seen by global authorities as the future of mobility. For example, the EU agreed to end the sale of new CO₂ emitting cars in Europe by 2035 [European Commission, 2022]. The development of EVs is exponential, starting from the last decade, and worldwide, as can be seen in Figure 1.2. Moreover, the adoption of EVs is expected to continue growing according to the existing policies and measures around the world. In particular, about 33% of the light-duty vehicles should be electric by 2035 [IEA, 2024].

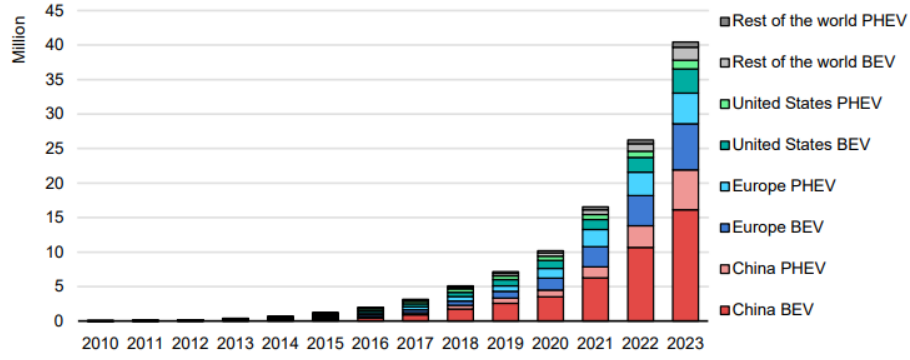


Figure 1.2: Global electric car stock trends, 2010-2023. Source: [IEA, 2024].

Electric vehicles and charging infrastructures deployment: the case of France

To follow up the increasing number of EVs on the road, significant investments are required across all stakeholders in the charging ecosystem, starting with the infrastructures expansion. The deployment of charging infrastructures in France offers a clear example.

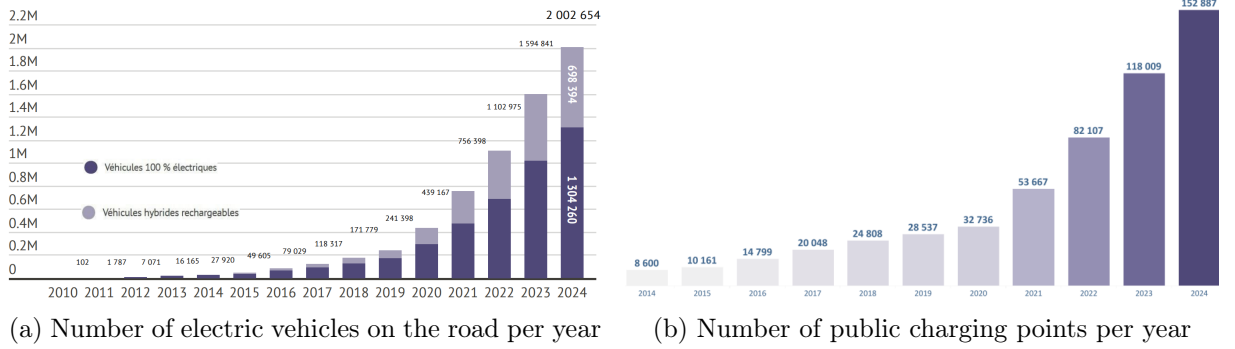


Figure 1.3: Evolution of the electric vehicles and charging points deployment in France. Source: [Avere-France, 2022a, Avere-France, 2022b].

Figure 1.3 shows the evolution over the past decade in the number of EVs and charging points, i.e., a parking space with the possibility to charge the EV (the definition will be provided below). This infrastructure rollout is essential for the successful integration of EVs and aligns with the growth trend of EV adoption.²

As of 2024, there are about 1.5 millions EVs on the road and 150,000 CPs in France, resulting in an average of one CP for every 10 EVs. This ratio matches the maximum recommended by the European Commission [European Commission, 2021].

1.1.2 Charging infrastructures: local and physical power rate limitations

Charging stations infrastructure description: a tree architecture

Definition of the terms There is no universal agreement on the terminology used to describe the various components of the charging infrastructure. The following terms will be used

² A shortage of charging points increases the risk of EVs running out of battery, contributing to the well-known phenomenon of *range anxiety*. This, in turn, can slow down the adoption of EVs. More details on this issue will be discussed in Section 1.2.

throughout this thesis. Following the European Alternative Fuels Observatory (EAFO) definitions [European Alternative Fuels Observatory, 2019], a Charging Station Pool (CSP), or *charging pool* for short, is a set of Charging Stations (CSs) at a specific location, linked by the same electricity delivery point, and operated by a unique charging operator. A CS is a physical object with one or more Charging Points (CPs), sharing a common user identification interface. A Charging Point (CP) is a parking space with the possibility to charge from a CS. Each CS at the CSP is associated with several CPs, in general one or two, but it can be more. The number of CPs at each CS is limited by the cable length, from the CS to the EVs. CSs, and consequently CPs can be *single-phase* or *triple-phase*. A connector is the physical interface between the CS and the EV, i.e., a two-side cable. There can be several connectors for each CP because some EVs are not compatible with some types of connector, but only one EV can charge at a time.^{3,4}

Power constraints Similarly to the charging infrastructure architecture, there are physical charging power constraints at different points in the charging infrastructure. First of all, the aggregate charging power at the CSP is limited at the delivery point, by the electrical wiring in the distribution network or transformer capacity. Depending on the type of infrastructure, there can be additional intermediary power constraints between the delivery point and the CSs, in a tree-like structure. In particular, each CS is constrained in the aggregated power it can deliver to its associated CPs. Figure 1.4 provides illustrative scenarios on a simplified CSP architecture without any additional charging power constraint between the CSs and the delivery point.

In practice, charging power constraints occur to each electrical phase individually, i.e., each individual conductor. As a result, depending on the CSP configuration, phase imbalances can occur. However, these technicalities being neglected in the models developed in this thesis, no further details are provided. In fact, only the limitation in power at the delivery point is considered in this thesis. In the scenario examples of Figure 1.4, the charging power is equally shared among the different "nodes" in the infrastructure. Many other charging mechanisms could be considered, for example, prioritizing EV users that arrived before, or more complex mechanisms. Voluntarily controlling the charging power rate at a charging pool in order to maximize a given objective function (for example to minimize the electricity bill) is called *smart charging* and is more and more used in practice.⁵

Power levels and plug types

AC/DC charging, speed categories and type of utilization The electricity must be delivered to the EV battery with Continuous Current (DC), whereas the current in the electrical network is an Alternative Current (AC). At a certain stage of the charging process, the current needs to be changed from AC to DC using an AC/DC converter. In AC charging modes, this conversion takes place within the EV via an internal converter. To minimize energy losses during vehicle operation, the internal converter must be lightweight, which inherently limits its conversion speed. In DC charging modes, the AC/DC conversion is operated inside the CS, making the current delivered to the EV already in DC mode. Since the AC/DC converter is located at the CS, it can be larger and heavier than an internal EV converter, allowing faster conversion. As a consequence, in general DC charging is faster than AC charging. According to the European Alternative Fuels Observatory (EAFO) [EAFO, 2023] convention, CPs (or

³ This is explained for example in the [Alternative Fuels Data Center](#) website.

⁴ In other documents, a *charging pool* may be referred to as a *charging area*, or a *charging location*. Similarly, a charging station may be called *electric vehicle charging station*, or *electric vehicle supply equipment*.

⁵ For example, the french company [Dreev](#) employs intelligent algorithms to control charging.

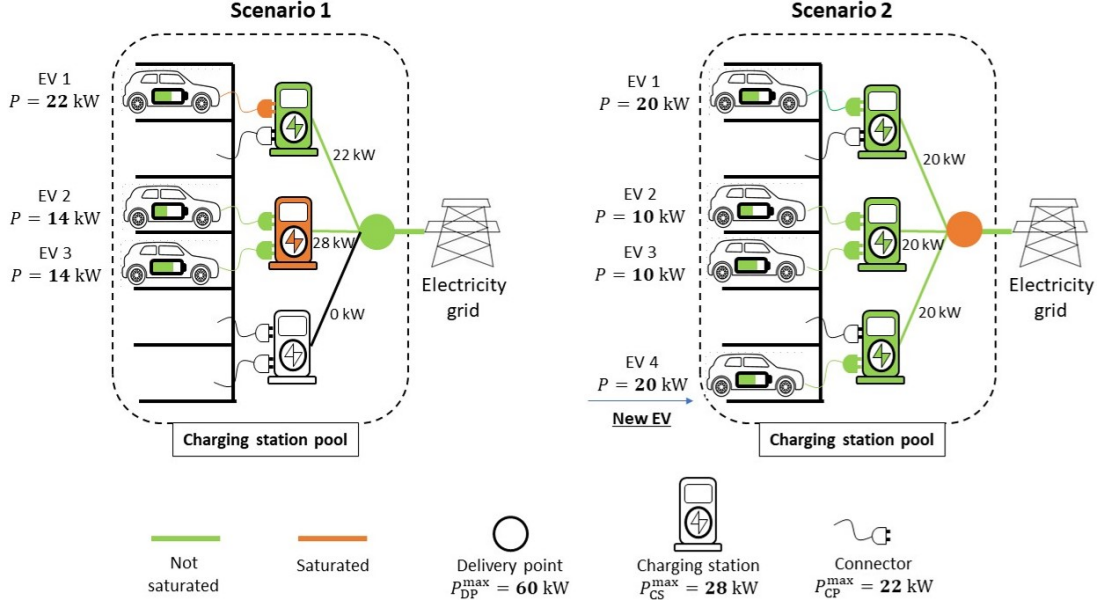


Figure 1.4: Schematic diagram example of power constraints at a charging pool infrastructure for two scenarios. The charging pool is equipped with 3 CSs, each of them equipped with 2 CPs, each of them equipped with 1 connector. P is the instantaneous charging power at the corresponding CP; and $P_{CP}^{\max}$, $P_{CS}^{\max}$ and $P_{DP}^{\max}$ are respectively the maximum power that can pass through each connector (cable), CS and at the delivery point. *Scenario 1*: EV 2 and EV 3 are charging at a power strictly lower (14 kW) than the connector capacity (22 kW) due to the limited power capacity of the CS (28 kW); *Scenario 2*: due to the arrival of EV 4, the total aggregated charging capacity of the charging pool is saturated by the delivery point capacity, which leads to lower charging power for EV 1 (from 22 kW to 20 kW), and EV 2 and 3 (from 14 kW to 10 kW).

equivalently CSs) are categorized as a function of the charging modes, namely category 1 for AC charging and category 2 for DC charging. Both categories are divided into subcategories depending on the charging speed (for AC and DC charging) and whether the charging is single-phase or triple-phase (for AC charging). For more details, see [EAFO, 2023]. Depending on these factors, the range of charging speed for EVs is very vast, going from about 1 kW for home charging to 300 kW a fast public CSs (it can go until about 1 MW for electric truck, but truck charging is out of the scope of this thesis), as shown in Table 1.5.

In this thesis, home charging is not considered, although being the most frequent charging place [AFIREV and Avere-France, 2024], due to the charging behavior that is different to the one at public places. For example, home-charging is less time-restricted compared to public spaces, as parking durations are generally longer. Consequently, typical charging speed considered in this thesis is above 3 kW. In the following, the mention of *public charging* refers to any charging location that is neither at home nor at the workplace.

In addition to home charging, it can also take place at workplaces, malls, on-street or off-street parking spaces, urban service stations, and on highways. As shown in Figure 1.6, public CPs are more present in parking and near shopping centers. The place where charging occurs is strongly correlated to the charging speed. For places where the EV users tend to stay a long

⁶ Approximated charging duration for a battery of 60 kWh, from a state of charge of 20% to 80%

Charging speed	Charging mode	Charging duration ⁶	Charging place
1 kW	AC single-phase	36h	Home
3 kW	AC single-phase	12h	Public / home / workplace
7 kW	AC single-phase	5h15	Public / home / workplace
11 kW	AC single-phase / DC	3h30	Public / workplace
22 kW	AC triple-phase	1h45	Public / workplace
24 kW	DC	1h30	Public
50 kW	DC	45min	Public
150 kW	DC	15min	Public
300 kW	DC	7.5min	Public

Table 1.5: Some typical charging speed, their associated current modes and places of installation. Note that these values correspond to the theoretical charging speed, without considering any EV battery limitations.

duration, the CSs installed are more likely to have a small charging speed capacity. On the contrary, for example on highways or at service stations, fast CSs are installed because the EV users are more impatient.

There is no consensus on the terminologies for *fast* and *standard* charging power [Wolbertus and van den Hoed, 2020]. In this thesis, standard charging speed corresponds to charging power between 3 kW and 24 kW, and fast charging speed is any charging power above 24 kW. In France, more than 80% of the public CP offers standard charging mode [Avere-France, 2022b].

EV battery charging speed limitation and aging The EV battery also has a nominal charging speed capacity. For example, the Nissan LEAF (40 kWh model) [Nissan, 2014] and the BMW i3 (42 kWh model)⁷ charging speed is limited to 50 kW. The actual charging power delivered to the EVs is the minimum between the speed capacity of the CPs, described in Table 1.5, and the speed capacity of the EV battery [Wolbertus and van den Hoed, 2020].⁸ In addition, charging speed is also influenced by environmental factors such as the ambient temperature [Trentadue et al., 2018]. Note that charging may cause battery degradation, in particular when charging occurs at a high power rate. To limit such effects, some charging protocols, such as *Constant Current - Constant Voltage* (CC-CV), are implemented to lower the charging speed depending on the State of Charge (SoC), in particular when it approaches the battery storage capacity [Tomaszewska et al., 2019].⁹

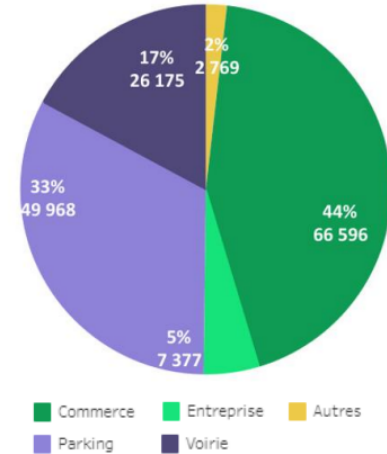


Figure 1.6: Public Charging points location distribution in France. Source: [Avere-France, 2022b].

⁷ See for example the BWN i3 Pod Point description.

⁸ EV users lacking this knowledge are in general less satisfy with the quality of charging service, as shown in the french survey [AFIREV and Avere-France, 2024].

⁹ It is recommended to slower the charging from a state of charge of approximately 80% for AC charging [Mussa et al., 2017].

1.1.3 Electric vehicle charging ecosystem/stakeholders

This section presents the ecosystem surrounding EV charging and the stakeholders involved, each of whom plays a role in ensuring the efficient operation of public charging infrastructures.

The electricity system in a nutshell

Before being supplied to the EVs, electricity is first produced, either from non-renewable sources (e.g., nuclear or coal-fired) or renewable sources (e.g., wind, water or solar). The electricity produced is connected to either the transportation network, or to the distribution network, depending on the generation capacity of the facility. For high-capacity production, such as nuclear or coal-fire plants, hydroelectric dams, offshore wind turbines, massive solar farms or onshore wind farms, the electricity is directed to the transmission network. For lower-capacity production, also called *decentralized production*, such as solar panels or wind turbines, the generated electricity is directly injected into the distribution network. Electricity in the transportation network is transported at very high voltages to reduce losses, as it covers long distances.¹⁰ Once the electricity gets closer to the consumers, the voltage is stepped down by a transformer and distributed to the consumers, including the CSPs, via the distribution network.

EV charging stakeholders

For public charging, there are many stakeholders in the EV charging ecosystem, even without considering the infrastructure manufacture.

Charging point operator The Charging Point Operator (CPO) occupies a central role in the EV charging management. It is responsible for ensuring the CSs operates reliably at all times. The CPO decides for example the charging dispatch policy within the CSP. However, the CPO is not the only actor in the EV charging ecosystem. Besides monitoring proper operation of the charging equipments, the CPO works jointly with many stakeholders.

Electricity provider The CPO, sometimes together with the site owner, must establish a contract with the electricity supplier. The electricity supplier can either be a producer, such as the french utility EDF,¹¹ or an entity buying electricity from the electricity markets¹² and selling it to the CPOs. It is responsible for supplying at all times the electricity at the CSs. This contract will establish in particular the per-unit electricity cost the CPO incurs, and the aggregate charging power capacity available at the CSP.

Transmission and distribution system operators The Transmission System Operator (TSO) and the Distribution System Operator (DSO) collaborate to ensure the safe and reliable transportation and distribution of electricity to consumers.¹³ Due to the shared use of a single

¹⁰ See [RTE website about the electricity transport operation](#) (in french) for more detailed explanations.

¹¹ See [EDF website](#).

¹² Among various electricity markets, notable examples include the *SPOT market*, which enables participants to buy or sell electricity one day in advance, and the *Real-Time market*, for intra-day exchanges addressing production or consumption forecast errors. For further explanations, see the [Iso new england: Day-Ahead and Real-Time Energy Markets website](#).

¹³ In France, the TSO is [RTE](#), and the DSO is mainly [Enedis](#), which cover 95% of the french territory, as stated [this Enedis document](#). See [RTE website: "The role of the distribution system operators"](#) for more detailed explanation about this collaboration between these two entities in France.

network infrastructure, these entities often operate as monopolies but are typically subject to regulations, for example ensuring non-discriminatory access to networks for all stakeholders. As their names suggest, the TSO supervises the balance between electricity supply and demand at the transmission level, while the DSO is responsible for deploying and maintaining local assets for the distribution network. When discussing system-wide operations, these entities are referred to as the System Operator (SO).

In the context of EV charging, at a local level, the electricity provider must establish agreements with the DSO to set up conditions in terms of charging power capacity at the delivery point of the CSP. The available power capacity depends on the existing infrastructure. Intuitively, higher charging power capacity incurring higher costs, due to the need for network upgrades and associated expenses [Andrianesis and Caramanis, 2020].

On a broader scale, EV charging across a region or country can contribute to grid stability by providing services to the TSO. For example, EV can support ancillary services such as frequency regulation by charging or discharging the battery based on a signal sent by the TSO. The concept of occasionally discharging the vehicle's battery is called V2G (Vehicle-To-Grid) and is increasingly being implemented in practice.¹⁴ This service is typically utilized at home, where longer parking durations provide greater flexibility for managing charging and discharging. These consideration are not investigated in this thesis, but are mentioned in the concluding chapter 6.5 as perspectives.

Service providers Concerning the interactions with the EV users, roaming operators and e-Mobility Service Providers (eMSPs) enable to make the use of the CSs more convenient. More particularly, roaming operators provide inter-operability of the charging with different CPOs.¹⁵ It enables EV users to use charging infrastructures of several CPOs. eMSPs are responsible for the interaction between the EV user and the CPO / roaming operators.¹⁶ They design for example mobile applications to make the payment more convenient, or offering guidance suggestion toward the closer CSP. The eMSP also designs board interfaces, and the pricing policy. Figure 1.7 gives a schematic diagram of the main stakeholders in the charging management. Note that one entity can play several roles. In this thesis, CPOs, eMSPs and Roaming operators are not distinguished, and are all referred to as one actor: the Charging Service Operator (CSO).

Charging pricing mechanisms

The charging or parking fees paid by EV users are redistributed among the various stakeholders in the charging ecosystem, described in the previous section. Specifically, the eMSP compensates the CSO,¹⁷ enabling it to cover the costs of electricity delivery. These costs include not only the price of electricity but also network taxes paid to both the TSO and DSO, which account for the operation and investment in the electricity network.¹⁸

Since EV charging is a new market, many forms of pricing have been implemented. When occupying a CP, the EV users use two-resources: a parking space and charging power. Due to the potential lack of CPs in some area, reservation or/and parking pricing mechanisms have

¹⁴For example, the french company [Dreev](#) offers V2G services.

¹⁵In Europe, [Gireve](#) is one of the most important roaming operator.

¹⁶See for example [eMSP Driivz website](#).

¹⁷In France, the average price paid by the eMSP to the CPO is 0.67€/kWh for AC charging, and 0.46€/kWh for DC charging, as stated in [Avere-France, 2022b].

¹⁸In France, this tax is called the TURPE (Tariff for Use of the Public Electricity Grid), and represents about 30% of the overall electricity price, see [Enedis website](#) for a more detailed explanation.

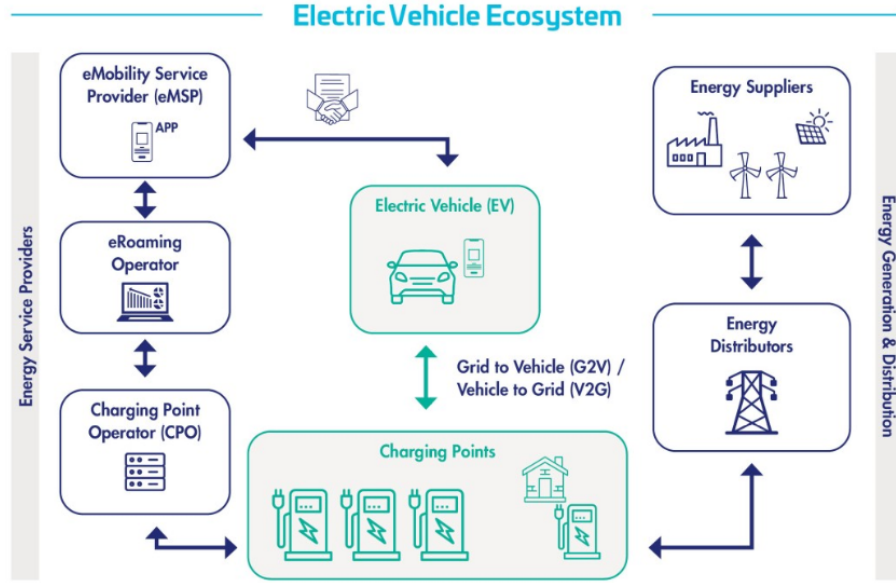


Figure 1.7: Electric vehicle ecosystem diagram. Source: Econg charger.

emerged in addition to charging pricing.¹⁹ Moreover, charging price can depend on the charging rate, because higher charging power incurs more expensive infrastructures, higher assets aging, and potential grid management complexities. For these reasons, charging pricing has been a fast-developing subject for study, leading to sometimes complex pricing policies. A brief review of the pricing structure includes per-minute price, per-kWh price, an admission price, or a mixture of these, all dependent of the quality of service levels provided by the CPO. Moreover, pricing mechanisms can have a dynamical structure, depending on many factors such as the electricity cost, the time of the day, or the instantaneous occupation level at a CSP. As a consequence, EV users claim a clearer pricing structure [AFIREV and Avere-France, 2024]. In response, the European Union provided some recommendation to make pricing structure clearer for the EV users [AFIR, 2023].

1.2 Arising challenges and literature review

As previously discussed, the rapid adoption of EVs has attracted numerous stakeholders to the EV charging operation market. This transition requires significant investments in both charging infrastructure and the electrical grid to meet the growing demand. However, due to the substantial costs involved, the available charging capacity may fall short of the demand, leading to quality of service degradation.

At the consumers level, EV users may experience a long waiting time if all CPs are occupied upon arrival. In addition, simultaneous charging by multiple users can reduce the individual charging power delivered to each EV due to limited capacity in power at the CSPs, as described previously in Figure 1.4. From users' perspective, these effects may create frustration or increase the well known *range anxiety* [Pevec et al., 2020], i.e., the fear to be out of energy. In some cases, users may abandon a CSP altogether and seek alternatives if CPs remain unavailable, resulting in a loss of revenue and potentially damaging the CSP's reputation.

¹⁹ See for example [Charge Point tariffs](#) or [Tesla tariffs](#), which include a parking fee for EV users staying parked for a long duration.

At the electricity network level, high aggregate power demand during peak periods can create congestion on the distribution network, resulting in losses or assets degradation like the transformer [Andrianesis and Caramanis, 2020].²⁰

These challenges highlight the need for effective EV charging operation management, which has become a focus of numerous studies in the literature of CSP management. The use-cases can be divided into two categories, reflecting the inherent differences in EV users behavior: (1) CSP with standard charging mode, or (2) CSP with fast charging mode. At CSPs offering standard charging, charging sessions can last several hours. During this time, EV users often engage in other activities while their vehicle charges. In this type of charging, the EV users choice of CSP is also directed by the on-site activities, and charging is opportunistic for some EV users. Due to the potential long parking duration at these types of CSPs, EV users seeing all CPs occupied upon arrival do not wait for a CP freeing up, but seek for other alternatives. Moreover, potentially longer parking duration than the minimum charging duration, i.e., when constantly charging at the maximum power rate, offers flexibility for the CSO to punctually lower the instantaneous charging power for various reasons such as overload at the transformer level, or high electricity costs. Challenges involve lost EVs because all the CP are occupied upon arrival, or to deliver enough energy to the EVs individually at their departure time, even when the charging demand is high. At CSPs with fast charging mode, user behavior differs significantly. Charging duration is often a matter of minutes, such as at service stations (see Table 1.5 for typical charging duration). As a result, EV users usually stay near their vehicle while charging, and leave as soon as their vehicle has finished charging. If all CPs are occupied upon arrival, a CP might become available shortly. In addition, charging is generally the primary purpose of users visit. Consequently, users are often willing to wait if no CP is available upon arrival (given that the waiting line is reasonable). Moreover, users may be flexible with the location. Challenges often include potential long waiting time before charging, or long travel duration to reach the CSP. Table 1.8 summarises criterias of satisfaction of the different actors in the charging ecosystem for fast and standard charging modes.

1.2.1 Charging pools with fast charging modes

In the context of CSPs with fast charging modes, research perspectives can be categorized as either local, focusing on the management of a single CSP, or macroscopic, addressing the operation of a network of CSPs. From a macroscopic perspective, additional challenges include optimizing the location of CSPs and minimizing the distance EV users must travel to access them.

Single charging pool

When determining the appropriate sizing of a specific CSP, simulations of daily charging behaviors of EV users enable to predict charging demand at a given site [Zhang et al., 2014, Liu et al., 2022, Yang et al., 2018]. These simulations help assess whether the public charging infrastructure has an adequate number of CPs and sufficient power capacity. The sizing must strike a balance: it should be sufficient to maintain a reasonable waiting time [Yang et al., 2018] but avoid excessive capacity that would lead to unnecessary investment costs. In the short term, waiting time minimization can be addressed through energy pricing strategies [Zengin et al.,

²⁰ At a macroscopic perspective, i.e., on the transportation network side, high power consumption also creates difficulties in balancing the electricity supply and demand, which causes high electricity cost, or even black-outs [Sforna and Delfanti, 2006]. However, this thesis adopts a more local perspective and does not take such considerations into account.

	Types of charging	
	Fast charging	Standard charging
EV users flexibility	Choice of CSP Energy received	Possibility to not charge Energy received Parking duration
EV QoS criteria	Waiting time Travel distance	Blocking Energy received Charging duration
CSO criteria	Profit Balking users	Profit Blocking
SO criteria		Electricity cost Power consumption System services
Levers	Sizing Energy pricing CSP location EV to CSP assignment	Sizing Pricing mechanisms Charging scheduling EV to CP assignment

Table 1.8: Type of utilization and challenges between CSPs with fast charging rates and standard charging rates. *'Balking users' differ from blocked users in the sense that they judge the waiting time is too long so that they seek for other alternatives, as opposed to 'blocked' where there is no waiting. There are more criteria for the SO in standard charging compared to fast charging because EV users are considered more flexible in their consumption due to their potential longer parking duration than charging duration.*

2017]. By penalizing EV users with high energy demands, these strategies incentivize users to charge the amount necessary, thereby freeing up CPs more quickly and increasing the overall number of EVs that can be served [Wang et al., 2018].

Network of charging pools

When analyzing a network of CSPs, a macroscopic perspective is adopted. The nature of the CSO decision-making process determines whether this perspective is *centralized* (decisions are made by a unique central entity) or *decentralized* (decisions are made by each CSO individually). In a centralized approach, typically led by public authorities, the focus is on maximizing the welfare of EV users. A key consideration in this context is the optimal placement of CSPs [Xiao et al., 2018]. CSPs should be distributed to ensure sufficient geographic coverage, minimizing the distance EV users must travel to access charging facilities. Additionally, spatial variations in charging demand and the interdependence of CSPs within close proximity must be accounted for. In the short term, mechanisms such as real-time EV assignment to CSPs can further enhance the quality of service for EV users. For example, mobile applications can direct EVs to CSPs with lower instantaneous demand, considering both the distance to the CSPs [Said et al., 2013, Guo et al., 2017] and traffic conditions [Malandrino et al., 2015, Zhou et al., 2021].²¹ Alternatively, pricing can be implemented to guide EV users in selecting CSPs so as to minimize the number of users deterred by long waiting time [Zhang et al., 2018]. In contrast, a decentralized approach reflects

²¹ There is extensive literature on the topic of the coupled power-traffic congestion. Interested readers can refer to Benoit Sohet's thesis [Sohet, 2021].

a competitive environment in which CSOs operate independently, each seeking to maximize their individual profit. In such cases, CSOs must consider the pricing strategies of their competitors and adapt accordingly to attract EV users [Zhao et al., 2020, Yuan et al., 2015, Zavvos et al., 2022].

1.2.2 Charging pools with standard charging modes

Maximize utilization

As previously described in the context of CSPs with standard charging modes, if all the CPs are occupied upon arrival, an EV will immediately seek for alternatives. Maximizing utilization at a CSP involves minimizing the number of *blocked*²² EV users. To address this issue, various approaches exist.

Menu-based Menu-based approach consists in agreeing, upon arrival of an EV user, on a parking duration at a predetermined price [Hao et al., 2022, Meng et al., 2024, Abdalrahman and Zhuang, 2022]. An other approach is to consider priority users, for example the users who have taken out a subscription, and design admission policy for non-priority users based on occupation levels in order to minimize the number of blocked priority-users [Legros and Fransoo, 2024].

Notion of overstay *Overstay* refers to the period between the end of charging and the time the EV user vacates the CP. This period creates negative externalities by preventing CPs from their primary purpose: charging. Overstay pricing [Zeng et al., 2021, Biswas et al., 2016, Song and Madawala, 2023] aims to discourage EV users from occupying CPs longer than necessary. In addition, mechanisms have been proposed to address overstaying. For example, the authors in [Zeng et al., 2019] suggest an interchange mechanism where an overstaying EV is unplugged—either manually or automatically—at a given operational cost, allowing an EV plugged but idle to begin charging. This approach is particularly relevant for CSPs where each charging station can serve one EV at a time, even if multiple EVs are plugged in.

Quality of service

With the consideration of a constrained capacity in power at the CSP level, the focus is on the Quality of Service (QoS) among the EV users that have found an available CP. The notion of QoS is imprecise and may be different between EV users. It may be in terms of charging duration [Lu et al., 2023, Wu et al., 2021], if the EV user is flexible with parking duration but not the required energy, or conversely, or in terms of both in the energy received and the charging duration [Ghosh and Aggarwal, 2018]. The authors in [Danner and Meer, 2021] suggest several measures on the QoS, differentiating for example the percentage of the energy received compared to the required one (if any), and the State of Charge (SoC), i.e., the energy received relatively to the battery capacity. In most of the works, it is related to percentage of energy delivered compared to the requested energy (if any) [Arias et al., 2023, Zhang et al., 2016], or the proportion of EV users received the requested energy at departure [Khaksari et al., 2021]. The main lever for maximizing the QoS are charging mechanisms, meaning deciding how much power is delivered to each charging EV. Considering coupling constraints at different levels in

²²The term ‘blocked’ is imbued with the literature of queueing theory.

the charging infrastructure, assigning to each arriving EV at the CSP a specific CP is another important solution [Arias et al., 2023, Zhang et al., 2016].

Electrical system point of view

Another perspective is that of a System Operator (SO). When considering dynamic electricity costs, reflecting the tensions between the electricity demand and supply, some studies focus on minimizing them by dynamically adjusting charging prices [Zeng et al., 2019, Zeng et al., 2021, Lu et al., 2023]. In such cases, the integration of a stationary battery allows electricity to be stored during periods of low prices and used later during periods of high prices, as explored in [Ghosh and Aggarwal, 2018]. Other objectives are to minimize the aggregate power consumption at a CSP [Wu et al., 2021, Fu et al., 2023], as excessive power demand can strain transformer capacity or damage the electricity grid. From a broader perspective, some studies investigate the potential of CSPs to provide services to the electricity grid, such as frequency regulation [Liu et al., 2022], or align charging operations with CO2 emission signals [Dumont et al., 2023].

1.3 Contributions and organization of the thesis

In this thesis, user behavior, such as deciding whether to visit a Charging Station Pool (CSP) or which CSP to select, is modeled through individual agents whose choice is guided by their own welfare. This approach enables the quantification of user preferences across different options and provides insight into how their decisions are made and influenced by mechanisms such as pricing. This thesis is divided into two parts. Part I is composed of three chapters (including an introductory one), and focuses on anticipating EV user choices through a queueing game theoretical framework. Part II is composed of two chapters and aims at designing offline optimal time-varying pricing strategies, taking into account dynamic charging demand and electricity costs to reflect the true impact of users on system operations.

- Chapter 2 provides a gentle reminder on queueing theory and queueing game theory, and its application to EV charging.
- Chapter 3 studies how the EV users naturally distribute, i.e., without any incentives, among two CSPs that are geographically close, e.g., on the same site. The two CSPs are differentiated in their power capacities, both at the CSP level and at each CP level, and the number of CPs. The innovation of this chapter lies in the study of a game in which congestion arises both from the blocking probability, i.e., when all the CP at a CSP are occupied upon arrival, and from the potential reductions in individual charging power if many EV charge at the same time and at the same CSP. In a charging scheduling policy where power is equally distributed among the charging EVs, an approximation on the individual expected energy received by EV users is provided, and evaluated through numerical simulations. Another part of this chapter focuses on the price of anarchy, a metric quantifying the loss in energy delivered when letting EV users choosing the CSP by themselves, instead of assigning them. The contributions include a proposed approximation for the expected energy received by an EV at its chosen CSP, with its accuracy evaluated both theoretically and numerically. Additionally, the exact blocking probability is incorporated into the EV users' decision-making process (unlike in prior work [Dupont et al., 2021]). Furthermore, the model accounts for varying maximum power rates deliverable by individual CPs within different CSPs.

- Chapter 4 studies how the EV users naturally distribute among CSP with fast charging modes with a more macroscopic point of view, incorporating the additional distance necessary for going to a CSP. Congestion increases the waiting time of the EV users, which influence their choice of CSP. The main contribution of this chapter is the demonstration of the existence of a *stochastic equilibrium* (this notion is precised in Chapter 4) in which users have a heterogeneous impact on congestion due to their varying energy needs. A sensitivity analysis of the EV users choices of CSP and congestion is analyzed with respect to the main parameters, such as the overall charging demand, or the level of users rationality.
- Chapter 5 focus on the CPs availability for a given CSP. At the times the charging demand is high, a significant proportion of EV users deprived with a CP because all of them are currently being occupied upon arrival. Therefore, the proposed solution is to design a parking pricing so as to shorten the parking duration of EV users at the CSP in order to reduce the number of blocked EV users, and thus increasing the overall energy delivered. Several pricing policies are compared, depending on their dynamics, i.e., dynamic in the instantaneous number of EVs at the CSP, in time, or both. This is the first work that formalizes and analyze such dynamics in the parking price for EV charging. The solutions are compared and analyzed numerically.
- Chapter 6 adopts of system point of view, i.e., considering dynamic electricity costs. A menu-based approach is adopted, with the idea to offer QoS-differentiated options to the arriving users, each at a specific price. This work differs from prior studies by offering constant-power charging options that ensure quality of service. The price evolves dynamically according to the electricity cost, and are optimized according to two objectives: the social welfare, and the profit generated. The possibility to arbitrarily lower the aggregate power consumption compared to the 'nominal one' at the CSP, in order to provide service to the distribution network operator, is also investigated.

1.4 Publications related to this thesis

The work carried out during this thesis has led to the scientific contributions below.

Journal articles:

- [Dupont et al., 2023a] Dupont, A., Hayel, Y., Jiménez, T., Beaude, O., & Breal, J. B. (2023). A loss queueing game for electric vehicle charging performance evaluation. *Performance Evaluation*, 161, 102350. <https://doi.org/10.1016/j.peva.2023.102350>
- [Dupont et al., 2024] Dupont, A., Breal, J. B., Hayel, Y., & Andrianesis, P. (2024). Optimal price menu design of electric vehicle charging stations providing demand response. *Electric Power Systems Research*, 235, 110798. <https://doi.org/10.1016/j.epsr.2024.110798>

InProceeding articles:

- [Dupont et al., 2023b] Dupont, A., Hayel, Y., Jiménez, T., Breal, J. B., Payen, R., & Beaude, O. (2023, October). Optimal Pricing for Electric Vehicle Parking Duration under Uncertainty. In *2023 IEEE PES Innovative Smart Grid Technologies Europe (ISGT EUROPE)* (pp. 1-5). IEEE. <https://ieeexplore.ieee.org/document/10407699>

- [Dupont et al., 2021] Dupont, A., Hayel, Y., Jiménez, T., Beaudé, O., & Wan, C. (2021). Coupled queueing and charging game model with energy capacity optimization. In *Performance Engineering and Stochastic Modeling: 17th European Workshop, EPEW 2021, and 26th International Conference, ASMTA 2021, Virtual Event, December 9–10 and December 13–14, 2021, Proceedings* (pp. 325-344). Springer International Publishing. https://doi.org/10.1007/978-3-030-91825-5_20

Some congress participations:

- Dupont, A., Hayel, Y., Jimenez, T., Beaudé, O., & Wan, C. (2021). Coupled queueing and charging game model with power capacity optimization. In *PGMO days 2021*. <https://hal.science/hal-04434954/document>
- Dupont, A., Hayel, Y., Jiménez, T., Wan, C., & Beaudé, O. (2022, February). Modélisation stochastique de la Qualité de service de charge pour véhicules électriques en compétition. In *23ème congrès annuel de la Société Française de Recherche Opérationnelle et d'Aide à la Décision (ROADEF 2022)*. <https://hal.science/hal-03596196/>
- Dupont, A., Hayel, Y., Jiménez, T., Wan, C., & Beaudé, O. (2022, June). Stochastic Modeling of the Service Quality for Electric Vehicles in Competition at two Charging Stations. In *International Electric Vehicle Symposium & Exhibition (EVS) 35*. https://hal.science/hal-04434729v1/file/EVS35_FV.pdf
- Dupont, A., Hayel, Y., Jimenez, T., Breal, J. B., Wan, C., Beaudé, O., & Payen, R. (2023, February). Optimal Pricing for Electric Vehicle Parking Duration at a Charging Station based on a Queueing Model. In *24ème congrès annuel de la Société Française de Recherche Opérationnelle et d'Aide à la Décision (ROADEF 2023)*. <https://hal.science/hal-04434810/document>
- Dupont, A., Hayel, Y., Jimenez, T., Breal, J. B., Wan, C., Beaudé, O., & Payen, R. (2022). Parking duration optimization for electric vehicles in a loss queueing system. In *PGMO days 2022*. <https://hal.science/hal-04434890/>
- Dupont, A. (2023). Optimal Price Menu Design of Electric Vehicle Charging Stations. In *PGMO days 2023*. <https://hal.science/hal-04434929/document>
- Dupont, A., Hayel, Y., Jiménez, T., Breal, J. B., & Payen, R. (2024). Dynamic Parking Pricing for Electric Vehicles at public Charging Stations. In *PGMO days 2024*. <https://hal.science/hal-04974036>

Part I

Unobservable queueing-and-charging games between electric vehicle users for charging pools selection

Chapter 2

A reminder of queueing game theory and its application to vehicle charging

2.1 Queueing theory: a framework to estimate charging pools occupancy

2.1.1 Main notions of queueing theory

Queueing theory, introduced in the context of telephone networks [Erlang, 1909], is a mathematical framework for analyzing systems where resources are shared among users. It is used in various domains, including service facilities, telecommunications, and more recently electric vehicle charging stations. A queueing system is characterized by factors such as the arrival process, the service rate, or the resource allocation, which are compactly described by the Kendall notation $A/S/c/K/N/D$, introduced in [Kendall, 1953]. This notation specifies:

- **A [Arrival process]**: distribution of inter-arrival time between customers. Example are:
 - M: independent and exponentially distributed (M stand for Markovian, and arrival time follow a *Poisson process*);
 - G: General distribution with no specific assumption;
- **S [Service time]**: the distribution of time required to serve a customer (the same notations as for the arrival process hold);
- **c [Service capacity]**: the number of servers;
- **K [Queue capacity]**: total number of customers allowed in the system, including those being served ($K \geq c$);
- **N [Calling population]**: the total pool of potential customers;
- **D [Queue's discipline]**: the rule for serving customers. Examples are:
 - FIFO (First In First Out): serve customers in order of arrival;
 - LIFO (Last In First Out): serve the most recent arrival first;
 - PS (Processor Sharing): divide service equally among customers;
 - PQ (Priority queueing): assign priority to certain customer classes.

The default queue capacity and calling population are $+\infty$, and the default queue discipline is FIFO. Unless specified otherwise, these default characteristics apply. For example, an M/M/c queue represents exponentially distributed inter-arrivals and service times, c servers, unlimited queue capacity and customer population, with a FIFO discipline.

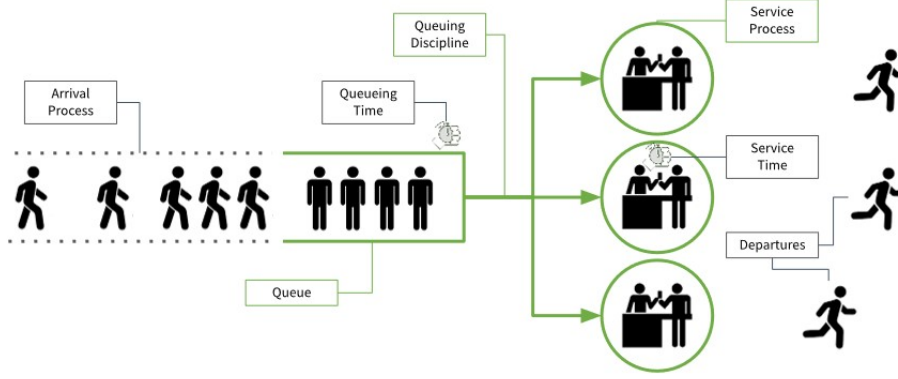


Figure 2.1: Schematic diagram of a queueing system. Source: [agility at scale](#). Customers arrive according to a given stochastic process. If all the servers are occupied upon arrival, the arriving customer goes in the waiting line, if the waiting line capacity is not reached. Otherwise, the customers immediately leave upon arrival. Once a server becomes available, the next customer in the waiting queue is served, according to the queue discipline. At the end of its service time, a customer leaves.

Figure 2.1 provides an illustration of a queueing system. In queueing theory, the *service time* of a customer corresponds to the duration of the service, the *waiting time* is the duration waiting before the service starts, and the *sojourn time* is the overall duration spent in the system, that is, the sum of the two.

Limiting behavior of a queueing process At the start of a queueing process, the system begins with an initial state, such as zero customers. Determining the probability distribution of the system's state at a specific time can be obtained solving a system of linear equations, provided that it can be written as a Markov chain, where the dimension of the system's state is reasonable. It is however analytically intractable for most systems. To address this, queueing analysis often focuses on the system's stationary regime, which describes the long-term behavior of the system when it has stabilized. In this regime, the probabilities associated with different states become time-invariant, as stated below.

Definition 2.1 (stationary regime). A stationary regime of a queueing process is a probability distribution over the possible states (e.g. the number of customers being currently served and waiting) which does not change over time.

The time-invariant property of the stationary regime enables, for specific systems (e.g., M/M/c, M/M/c/c, M/M/c/K), to derive analytical expressions of the limiting behavior over time of the queueing process. In some cases, these results generalize to systems with general inter-arrivals or service time distributions, such as M/G/1 [Pollaczek, 1930], G/M/1 [Kendall, 1953] or M/G/c/c systems. The stationary regime is particularly valuable as it simplifies the analysis of the system's behavior and provides insights into key performance metrics, such as utilization, waiting time, and system occupancy.

For an introduction to queueing theory, readers may refer to the book [Ross, 2014], which covers probability and Markov chains, or [Adan and Boucherie, 2015], which focuses on the queueing systems part.

2.1.2 Application to vehicle parking and charging

User arrivals and departures at a CSP are inherently stochastic. Queueing theory is therefore a valuable tool for estimating occupancy levels, capturing the probability distribution of the number of users parking and/or charging. Since users behavior at CSPs shares similarities with that in conventional parking lots (without CPs), we provide context for the application of queueing theory in both scenarios below.

Conventional parking lots In parking lot occupancy modeling, the service time corresponds to the parking duration, and the number of servers is the number of parking spaces. Queueing theory has been widely used for this purpose and has demonstrated accuracy in predicting parking occupancy levels [Tavafoghi et al., 2019]. A detailed review of queueing theory applications for parking is available in [MAI and CUONG, 2021]. Typically, the arrival process of vehicles is modeled as a Poisson process, while parking duration (service time) is often assumed to follow an exponential distribution, though other distributions may be more accurate, as stated in [Tavafoghi et al., 2019]. Various queueing models have been used:

- M/M/c/c: assumes that vehicles leave the system immediately if no parking space is available upon arrival [Xiao et al., 2018, Dowling et al., 2020];
- M/M/c or M/M/c/K: accounts for vehicles circling the lot, waiting for a parking space to become available [Ratliff et al., 2016, Gong et al., 2013]. These models may incorporate the possibility of reneging, where users leave the lot after waiting too long [Larson and Sasanuma, 2010].

Charging pools with standard charging modes At CSPs with standard charging modes (e.g., 7 kW to 24 kW), charging durations are long (see Table 1.5), and EV users typically engage in other activities while charging. As a result, a user may not want to wait if all the CPs are occupied upon arrival, due to the potential long parking duration of others. In this setting, CPs serve both parking and charging purposes, and CSPs occupancy can be modeled as two interdependent virtual queues [Aveklouris et al., 2019, Bayram, 2022]:

- *charging queue*: represents EVs actively charging. The arrival time corresponds to that of the CSP, and the service time depends on the user’s energy needs, the CP’s power capacity, and the operator’s scheduling policy.
- *fully-charged queue*: represents EVs that remain parked after charging is complete. The arrival time corresponds to the end of charge time (given that the users leave after charging ends), and the service time is the difference between user’s parking and charging duration.

These queues interact dynamically. In particular, the arrival rate at the charging queue depends on the parking queue occupancy, as a fully occupied CSP leads to blocked arrivals. Moreover, the arrival rate at the parking queue depends on the charging queue occupancy and charging durations. This scenario is explored in Chapters 3 and 5.

Figure 2.2 shows datasets of charging and parking duration, available at [yvenn-amara github](#), where parking duration refers to the total duration spent at the CPs. The parking duration is strictly greater than the charging duration if the user leaves after charging ends, and is equal

otherwise. Such data give useful information for determining the service time distribution over the users population, a necessary input for determining performance metrics using queueing theory.

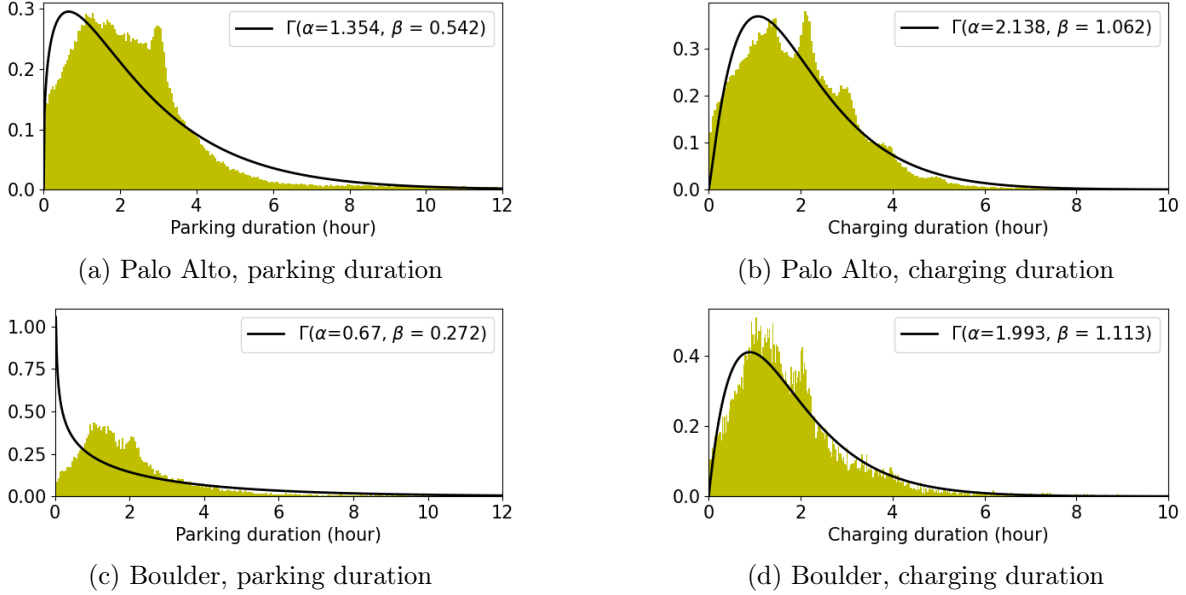


Figure 2.2: Histogram of parking and charging duration for standard charging modes (3 kW and 7 kW), along with the optimal gamma distribution match, in two cities in the USA: Palo Alto, California (about 250 000 sessions) and Boulder, Colorado (about 20 000 sessions).

In the context of limited power capacity at a CSP with processor-sharing charging scheduling, [Aveklouris et al., 2019] investigates approximations for the probability that a user departs with a fully charged EV. Ignoring the blocking probability of arriving users, [Danner and Meer, 2021] compares various charging scheduling, such as FIFO and LIFO, particularly in terms of energy received.

Charging pools with fast charging points At CSPs with fast CPs, EV users typically leave as soon as charging is complete, as charging is the primary purpose of their visit. Unlike in standard charging modes, EV users are willing to wait if all CPs are occupied upon arrival also because charging durations are short (minutes rather than hours). In this context:

- queueing model: The queue represents the number of EVs either charging or waiting. The number of servers corresponds to the number of CPs.
- service times: These depend on energy needs, power capacities, and dispatch policies, similar to standard charging modes.
- queue types: Common models include M/G/c or M/G/c/K, where the latter accounts for users leaving due to excessive waiting time, or space constraints [Zhang et al., 2018, Xiao et al., 2020, Meng et al., 2024].

Typical power dispatch used in the context of fast charging is FIFO. This scenario is explored in Chapter 4, where CSP occupancy is modeled as a M/G/c queue. Figure 2.3 illustrates the differences between queueing models for standard and fast charging modes. In this thesis, the terms service/charging time and parking time are used interchangeably when applicable.

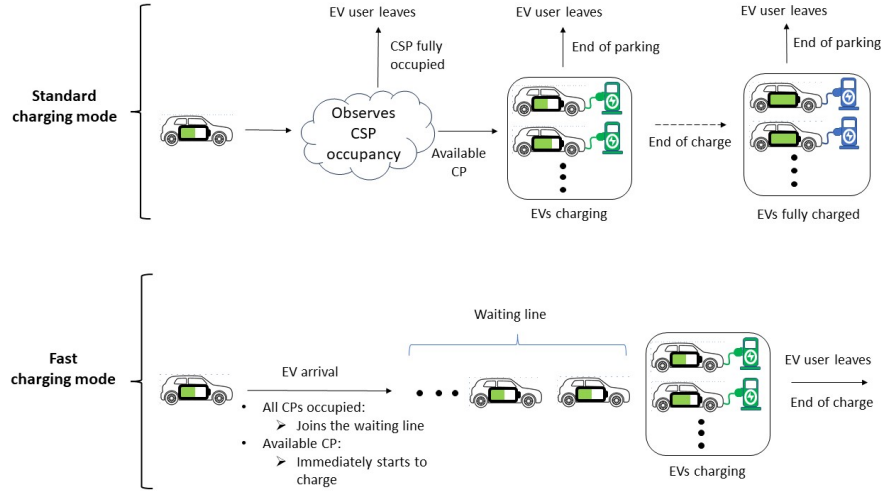


Figure 2.3: Illustration of the queueing systems used in this thesis for charging pools with standard and fast charging mode. For standard charging modes, EV users leave when their parking duration ends, which can be shorter or longer than the duration necessary to fulfill the required energy. For fast charging modes, EV users leave as soon as their charging ends.

2.2 Queueing game theory: a framework for EV users behavior modeling

In queueing theory, the entities entering the system to be processed are referred to as *jobs*. These jobs are typically customers but can also represent other elements. In this thesis, the jobs – or customers – are the EV users, referred to simply as users for brevity, and will be called as such moving forward.

2.2.1 Queueing game theory: a non-atomic approach

In the descriptions above, user decisions are not taken into account. Queueing games study the behavior of users in systems where queues form due to limited resources, and how users' choices and system performance affect each other. For example, a user may decide whether to join a queue based on the waiting time, which in turn is impacted by such decisions. As emphasis in the book [Hassin and Haviv, 2003], queueing games are broadly categorized into two subfields: *observable* queues, where users have information about the queue state before making decisions, and *unobservable* queues, where such information is unavailable.¹

Observable queues

Such situations typically lead to threshold-based strategies, as first described in [Naor, 1969]. For instance, users may decide to join a queue if and only if the number of users already in the system is below a certain threshold. In such settings, user interactions arise because the waiting time of one user is influenced by the decisions of others. However, an individual user's strategy is generally not impacted by the strategies of other users due to its knowledge about the system

¹ The authors in [Shone et al., 2013] showed that, under the basic M/M/1 queueing system, more users tend to join the system in the unobservable case (comparatively to the *observable* case) when the utility for joining the queue is high, and conversely when the valuation for joining the queue is low.

state. This type of queueing system will be analyzed in Chapter 5, where users are subject to a fee varying according to the system's level of congestion.

Unobservable queues

When users lack information about the system's state at the time of their decision, they must base their choices on the possible states of the system. Since the state of the queue(s) is influenced by user decisions, individuals need to anticipate the behavior of others before making their own choice. This results in a classical game-theoretic setting. In this framework, decision-makers are referred to as *players*, their choices as *strategies*, and their outcomes as *utilities*, *payoffs*, or *costs*, depending on the context. Typical components in unobservable queueing games are:

- players: the users;
- strategies: for instance whether to join or avoid queues, leaving a queue after joining (reneging), or timing their arrival;
- costs: the waiting time, and possible fees.

Sometimes, a value representing a user's satisfaction for being served (e.g., charged in the context of EV charging) is included in their decision-making criteria. In this thesis, *utility* will refer to the satisfaction value (if applicable), *welfare* to the utility minus the cost, and *social welfare* to the total of all users' welfare (or the social cost if no satisfaction value is considered).

Random costs and PASTA property Due to the stochastic nature of arrival time and service durations, waiting time is random, and analysis typically focus on expected costs.² When the arrival process follows a Poisson distribution—a common assumption in queueing theory—the *Poisson Arrivals See Time Averages* PASTA property holds: "the fraction of arrivals who find (observe or see) the process in a state is equal to the corresponding fraction of time" [Wolff, 1982]. As a consequence, since none of the users have an information about the system's state, their estimated probability of encountering the system in a specific state A equals the long-run fraction of time the system remains in state A , corresponding to the stationary regime of the queueing process.

Non-atomic users In *unobservable* queueing games, a common assumption is that the action of a single user has a negligible impact on the overall welfare of the user population. This assumption is reasonable when the population is very large. Such games are referred to as *non-atomic games* in the literature [Aumann and Shapley, 2015].³ In this framework, only the collective behavior of a proportion of users affects the costs experienced by others. The concept of *Wardrop equilibrium* [Wardrop, 1952], originally introduced for traffic assignment problems involving many vehicles, describes how users behave in such systems (i.e., with non-atomic users). It identifies a distribution of user choices where no individual has an incentive to unilaterally deviate from her choice: if she deviates, her welfare for the other alternatives is lower than the chosen one. Formal definitions of non-atomic congestion games and equilibria are reviewed in [Wan, 2013] (in french). Figure 2.4 illustrates the Wardrop equilibrium concept. In general, a Wardrop equilibrium may not exist or may not be unique.

² More complex users behavior modeling can be considered, such as incorporating one aversion to risk or cognitive bias [Kahneman and Tversky, 2013].

³ In measure theory, an atom refers to a set which as a strictly positive measure, and no other set can have a smaller measure. Analogously, in non-atomic games, none of the users (i.e., player) is an atom: the measure, or weight, of a single user over the all population is 0.

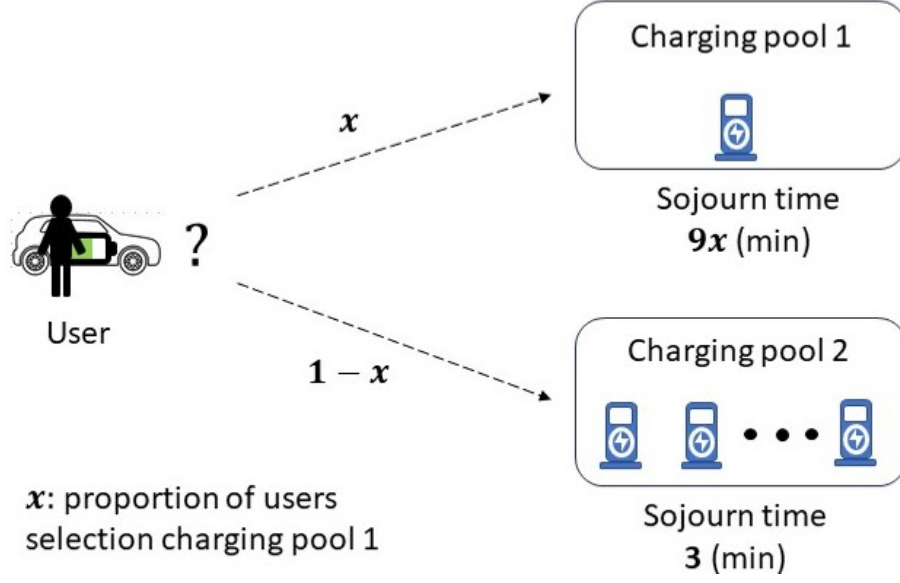


Figure 2.4: Illustrative diagram of a queueing game in the context of vehicle charging. The diagram represents a scenario where users decide between two charging pools for their vehicles charging. The proportion of users choosing Charging Pool 1 is denoted by x . Charging pool 1: offers very fast charging but has only a single CP. As a result, the sojourn time (total time spent at the station) is highly sensitive to the proportion of users selecting this pool. Charging pool 2: provides slower charging but has a large number of CPs, making the sojourn time independent of the proportion of users choosing it. In this illustrative example, a Wardrop equilibrium is achieved when $1/3$ of the users population choose charging pool 1, and $2/3$ charging pool 2. At equilibrium: the sojourn time at Pool 1 is $9 \times 1/3 = 3$ minutes, and also 3 minutes at Pool 2. To minimize social cost – the sum of the users’ sojourn time – only $1/6$ of the users should select Pool 1, resulting in a lower average cost of 2 minutes and 45 seconds. However, such distribution of users’ choice is not stable since users selecting CSP 2 have an incentive to choose CSP 1 instead (sojourn time of 1.5 min at CSP 1 against 3 min at CSP 2).

Remark 2.2. While a Wardrop equilibrium is individually optimal, i.e., no user has any incentive to change her choice given the other users’ decision, it is typically suboptimal from a social welfare/cost perspective. Users often overutilize queues that are more attractive but highly sensitive to congestion, compared to what would be optimal for social welfare, as illustrated in Figure 2.4.

Such games will be explored in Chapters 3 and 4.

2.2.2 Application to vehicle users charging behavior

In the context of EV charging, the points of view adopted are generally macroscopic, focusing on users CSP selection among several, as in Figure 2.4.

Unobservable queueing games for charging pool selection

Although unobservable queueing games have been extensively studied (see [Hassin and Haviv, 2003] for a review), only a few works [Liu et al., 2023, Yuan et al., 2015] have applied this concept to EV charging. The context of charging stations with fast charging modes closely resembles

standard queueing games: EV users select a CSP from a finite set and must wait if all CPs are occupied upon arrival.

In this framework, the cost function includes not only the waiting time but also the entire sojourn time at the chosen CSP, as illustrated in Figure 2.4. However, most research on congestion at CSPs (and queues in general) assumes that only the waiting time is affected by congestion. In some cases, such as processor-sharing scheduling, charging duration can also be congestion-sensitive, but this aspect is less commonly explored.

However, most research on congestion at CSPs (and queues in general) assumes that congestion affects only the waiting time.

An additional characteristic of queueing games in the context of CSPs is that users may have heterogeneous cost functions. These differences arise from factors such as their locations at the time of selecting a CSP or charging costs that are based on their energy received. The following description provides some studies that incorporate such heterogeneity and their associated queueing modeling:

- the authors in [Liu et al., 2023] model a network of CSPs as M/D/c queues, assuming all EVs require the same amount of energy, resulting in deterministic service times;
- in [Yuan et al., 2015], an approximation of the mean waiting time in M/G/N queues is used to model congestion at two CSPs;
- conversely, the focus in [Zhang et al., 2018] switches to social welfare maximization rather than individual welfare. Their study examines two CSPs modeled as M/M/c/K queues, with the objective of minimizing the number of balking EVs (due to long waiting lines).

For CSPs with standard charging modes, no corresponding game models exist in the literature. In such cases, congestion can primarily be represented by the probability of being blocked if all CPs are occupied upon arrival.

Alternative models for waiting time at charging pools

In most queueing systems, the waiting time is convex with respect to arrival rates, as in M/M/c or M/G/1 queues. This implies that the marginal negative externality of an additional user joining the queue increases with the mass of the population joining the queue. To model the waiting time at CSPs, alternative functions to standard queueing theory waiting time are used:

- *bureau of public roads* function: introduced by [Sheffi, 1985], and used in [Zhou et al., 2021].
- Davidson function: introduced by [Davidson, 1966], and employed mostly in the coupled electrical and transportation system literature, as in [Wei et al., 2017, Zhou et al., 2018, Lai et al., 2022, Feng et al., 2023, Yousuf et al., 2024].⁴
- linear functions: applied in [Zavvos et al., 2022, Li et al., 2022].

When considering system dynamics, such as time-varying charging demands, atomic games (involving a finite number of users) often provide a more appropriate framework [Malandrino et al., 2015, Guo et al., 2017, Xu et al., 2017]. At any given time, the number of users competing for charging slots is relatively small, making this approach particularly suitable.

⁴The Davidson function was primarily used for modeling traffic-flow congestion, and is inspired from queueing theory.

Conventional parking lots

In a context similar to CSPs, some studies [Ratliff et al., 2016, Sun et al., 2016] explore congestion games in conventional parking lots. Congestion arises when all parking spaces are occupied, forcing drivers to circle around looking for an available spot. However, this scenario differs slightly from CSPs due to the typically larger number of parking spaces compared to CPs at CSPs, as well as the fact that parking is a necessity for drivers, whereas charging may be opportunistic for some users.

2.3 Conclusion

This thesis integrates EV users behavior at CSPs with constrained power capacities, therefore forming physical or virtual queues. Queueing theory plays a key role to describe congestion at the CSPs, especially in Part I, composed of Chapters 3 and 4.

- Chapter 3 explores unobservable queueing games where EV users choose between two independent CSPs with standard charging modes. Independence refers to the decoupling of power capacity constraints and the lack of redirection for blocked users between CSPs. User interactions arise from the probability of being blocked from a chosen CSP and the reduction in individual charging power at a CSP due to simultaneous charging by multiple EVs.
- Chapter 4 examines unobservable queueing games for CSPs with fast charging modes. In this context, user interactions are influenced by a potential waiting time before charging when all CPs are occupied upon arrival.
- Chapter 5 focuses on observable queueing games in the same context as Chapter 3, but emphasizes CSP pricing strategies designed to reduce the blocking probability for EV users. Performance metrics, such as the proportion of blocked EVs, are derived using Markov chains rather than queueing theory.
- Chapter 6 investigates pricing strategies that account for user decisions among several options at a CSP. While congestion is described using criteria other than queueing theory, the study shares similarities with observable queueing games as it incorporates user decision-making and the impact of individual choices on the welfare of subsequent users.

Chapter 3

A loss queueing game at two pools with standard charging mode

This chapter is based on the following paper:

[Dupont et al., 2023a] Dupont, A., Hayel, Y., Jiménez, T., Beaude, O., & Breal, J. B. (2023). A loss queueing game for electric vehicle charging performance evaluation. *Performance Evaluation*, 161, 102350. <https://doi.org/10.1016/j.peva.2023.102350>

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Notation	Signification
N_i^{cp}	Number of charging points at CSP $i \in \{1, 2\}$
α_i	Power sizing coefficient ($\in [0, 1]$)
p_i^{max}	Maximum charging rate at CSP i (in kW)
b^{capa}	Battery energy storage capacity (in kWh)
λ	EV charging demand frequency (in h^{-1})
SoC_{arr}	Random state of charge of the battery when arriving (in $[0, 1]$)
ν	Inverse of the mean parking duration of a user (in h^{-1})
μ	Inverse of the mean energy need of a user (in kWh^{-1})
D	Random parking duration (in h)
E^{req}	Random energy needed for a fully charged battery (in kWh)
$\tilde{E}_i$	Approximation of the random energy received at CSP i (in kWh)
$\tilde{z}_i$	Approximation of the average number of charging EVs at CSP i
B_i	Blocking probability at CSP i
θ	Probability that a user chooses CSP 1
θ_{eq}	Probability that a user chooses CSP 1 at equilibrium of $\tilde{\Gamma}$
$\tilde{u}$	Utility function of users

Table 3.1: Main parameter notations used in this chapter. *Red*: CSP parameters; *Blue*: EV users parameters.

3.1 Introduction

Sites like malls are increasingly equipped with CSPs to provide charging opportunities for users while they engage in on-site activities. Some sites offer fast charging to attract customers, while others install CSPs with standard charging modes, better suited for locations where parking durations are long.

At a given site, there may be several CSPs, and users interested in charging decide which one to use. To design the infrastructure appropriately or other management decisions, it is essential to anticipate how users will naturally distribute themselves among the CSPs. While studies such as [Yuan et al., 2015, Liu et al., 2022] address this question, they focus on fast charging modes. For standard charging modes, as described in Chapters 1 and 2, users typically do not wait for an available CP if all are occupied upon arrival. This behavior aligns with loss systems, where users renege on charging. System with lossed users was studied, for example in [Zhang et al., 2019] for fast charging, or [Aveklouris et al., 2019] for conventional parking, but waiting is considered in these case. Indeed, fast charging or parking aligns with necessary needs. In [Aveklouris et al., 2019], no waiting is considered, and the users quality of service is analyzed using blocking probabilities. However, user behavior is not taken into account.

This chapter considers a scenario where EV users arrive at a site equipped with one or two public CSPs. Similar to [Aveklouris et al., 2019], the number of EVs charging or fully charged (but still occupying a CP) at a CSP is modeled as a queueing system. At each CSP, the aggregate power capacity is shared among charging EVs in a Processor Sharging way (detailed in Section 3.2). This power depends on some physical local grid constraints or on the contract between the CSO or the site manager and the electricity supplier. In the case of two CSP, users choose upon arrival one CSP among the two for parking and charging. In our context, charging is seen as an opportunity: the EV users take advantage of the possibility to charge, but it is not their primary reason for visiting the site. Consequently, the parking duration of each EV user is independent

The energy E^{req} is modeled by an exponential random variable with parameter μ .¹ The EVs parking duration D at the chosen CSP is random and follows an exponential distribution with parameter ν , which is in accordance with studies on the parking duration at a mall [Shen et al., 2019, Parmar et al., 2021]. In fact, in our model the charging service is an opportunity for the EV customer whose first aim is to go to the shopping center for another kind of service (restaurant, cinema, or other activities). Thus, the parking duration D is supposed to be independent of the energy level required E^{req} in order to get a fully charged battery.

At each instant, among all EVs parked at a CSP, some of them are charging and some are not. Indeed, some EVs can stay plugged at the end of their charge. As the main goal of the EV users is not to charge their EV but to get a service in the commercial center, the redirection of rejected EVs from one CSP to the other is not considered.

Charging station characteristics The CSP has $N^{\text{cp}} > 0$ parking spaces; each one of them being equipped with one CP. It means that only one EV can charge at one time at each CP. At time t , Z_t denotes the number of charging EVs and C_t the number of EVs that are parked but not charging at this time because their battery is fully charged. Then, the total number of EVs at the CSP at time t is $Q_t := Z_t + C_t$.

Charging station limitations and charging scheduling rule by processor sharing The CSP can deliver the same maximum power $p^{\text{max}} > 0$, which depends on the physical characteristics of the chargers as stated in [Wolbertus and van den Hoed, 2020]. Then the potential global available power at the CSP is given by $N^{\text{cp}} \times p^{\text{max}}$. Typically, the global available power is limited and the parameter $\alpha \in]0, 1]$ is the proportion of the potential global available power that can be drawn from the CSP. This proportion determines the number $\lfloor \alpha N^{\text{cp}} \rfloor$ of CPs that can draw simultaneously the maximum power level p^{max} at CSP i .

At each CSP, the limited available power is shared among the EVs charging at the CSP according to a *processor sharing* mechanism. This type of charging scheduling policy is already used by some CSOs in practice like Izivia in France.² The charging scheduling policy plays a crucial role in the satisfaction of EV users [Danner and Meer, 2021] because it determines the instantaneous power received by each EV. When the number of charging EVs Z_t at the CSP at a given instant t is above $\lfloor \alpha N^{\text{cp}} \rfloor$, the maximum power p^{max} is equally shared among all EVs within the CSP. In turn, at each time t , the instantaneous power p delivered to each charging EV at the CSP depends on Z_t through the following equation:

$$p(Z_t) = \begin{cases} p^{\text{max}} \frac{\min(Z_t, \alpha N^{\text{cp}})}{Z_t} & \text{if } Z_t > 0, \\ p^{\text{max}} & \text{if } Z_t = 0. \end{cases} \quad (3.1)$$

The instantaneous power received by an EV in the case $Z_t = 0$ is mathematically defined although it does not have any physical meaning. Figure 3.3 illustrates on examples how the processor sharing policy given by Equation (3.1) works.

It is clear from our context that, for each of the CSP, the two-dimensional process (Q_t, Z_t) is a Markov process. EV users determine which CSP to join in order to maximize the expected energy

¹ In practice, E^{req} is in general not drawn from a exponential distribution: the possible values are bounded by the maximum battery capacity among the EV population, about 125 kWh (see for example [ev-database](#)). However, this assumption is standard in queueing theory as it simplifies the analysis, and is used in many studies on EV charging [Yang et al., 2018, Zhang et al., 2019, Xiao et al., 2020].

² See [Izivia website](#) for a commercial description of the power scheduling used by Izivia.

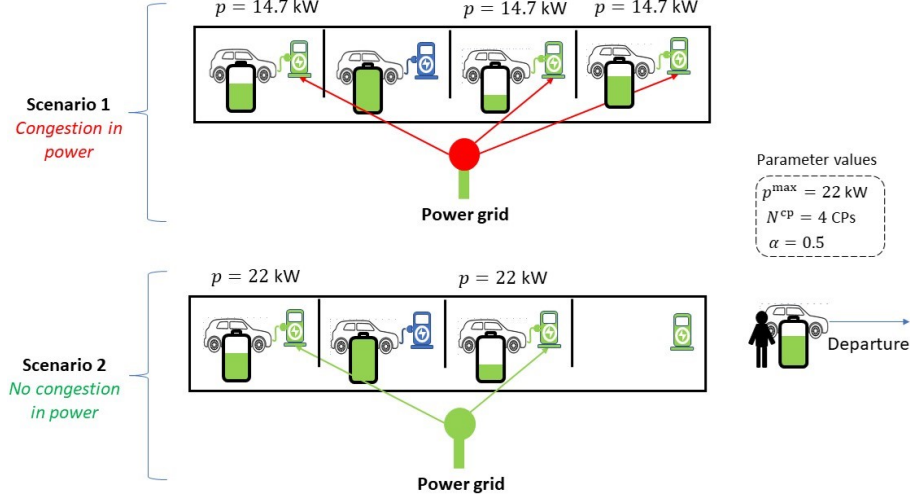


Figure 3.3: Illustration of the *processor sharing* charging scheduling policy. There are 4 charging points at the pool, but at maximum two EVs can simultaneously charge at the maximum power rate $p^{\max}$ (because $\alpha = 0.5$). Scenario 1. There are 3 EVs charging. They must share the aggregated power capacity ($\alpha N^{\text{cp}} p^{\max} = 42 \text{ kW}$). According to the processor sharing rule, given by Equation (3.1), each EV charge with $p = 42/3 = 14.7 \text{ kW}$. Scenario 2. One of the 3 EV leaves. There are only 2 remaining EVs charging: they can charge at the maximum power rate $p^{\max} = 22 \text{ kW}$.

received in steady state of the Markov process (Q_t, Z_t) before leaving. Below, an approximation of the expected energy received is presented.

3.2.2 Approximation of the expected number of charging vehicles based on Erlang loss formula

The Poisson Arrivals See Time Averages (PASTA) property [Wolff, 1982] applied to our setting states that an EV arriving at the CSP at any random instant sees the state (q, z) of the Markov process with probability $\pi(q, z)$ where π is the stationary distribution of the Markov process

(Q_t, Z_t) . This stationary distribution π , as well as the expected number $\sum_{q=0}^{N^{\text{cp}}} \sum_{z=0}^q z \pi(q, z)$ of

charging EVs, cannot be computed explicitly due to the complexity of the system. In [Aveklouris et al., 2019], an approximation $\bar{z}$ of the expected number of charging EVs is proposed, which depends in the effective arrival rate λ^e , i.e., the rate at which EVs find an available parking space (EV arriving and not being rejected). In our previous work [Dupont et al., 2021], the effective arrival rate was approximated by $\lambda^e \approx \min(\lambda, \nu N^{\text{cp}})$ in order to have explicit solutions for the analysis. However, this approximation can be inaccurate for some values of λ . The exact effective arrival rate is given by:

$$\lambda^e = \lambda \times (1 - B), \quad (3.2)$$

where $B := \frac{\rho^{N^{\text{cp}}}}{\sum_{j=0}^{N^{\text{cp}}} \frac{\rho^j}{j!}}$ is the *blocking probability* of an Erlang-B queue with load $\rho := \frac{\lambda}{\nu}$. Indeed,

the number of EVs at the CSP corresponds to an M/M/ $N^{\text{cp}}/N^{\text{cp}}$ queue. Then, $(1 - B)$ is the probability that an EV is accepted, i.e., finds an available parking space, at the CSP. Figure 3.4

illustrates the dependency of the expected effective arrival frequency λ^e with respect to the expected arrival frequency λ .

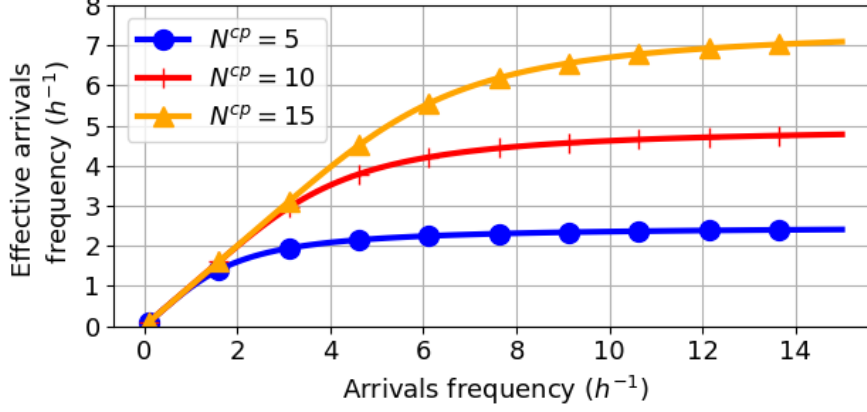


Figure 3.4: Expected effective arrival frequency λ^e (users joining the charging pool) with respect to the expected arrival frequency λ (charging demand), for different number of charging point N^{cp} , and with $\nu^{-1} = 2$ (hours). As stated below in Lemma 3.6, the effective arrivals λ^e increases with the arrival frequency λ . For low values of λ , $\lambda^e \approx \lambda$ because the blocking probability B is negligible (few users arrive at the charging pool). For high values of λ , the expected effective arrival frequency converges to $N^{cp} \nu$. The demonstration is shown in Proposition A.2 in Appendix A.

The approximation $\bar{z}$ of the expected number of charging EVs given by [Aveklouris et al., 2019] is the unique solution of the following equation:

$$\begin{cases} \bar{z} = \lambda^e \mathbb{E} \left[\min \left(D, \frac{E^{\text{req}}}{p(\bar{z})} \right) \right], \\ 0 \leq \bar{z} \leq N^{cp}, \end{cases} \quad (3.3)$$

where p is the power allocation function for the CSP, see (3.1).

The charging process of an EV is stopped when the maximum capacity of its battery is reached or when the EV leaves the CP (therefore without a fully charged battery). Then the charging duration of an EV is expressed as the minimum between the parking duration and the time to fully charge the EV's battery.

Following the result of Proposition 1 in [Dupont et al., 2021], the explicit expression of the approximation of the expected number of charging EVs at the CSP is obtained by solving the fixed-point equation (3.3).

Proposition 3.1. *In stationary regime of the Markov process (Q_t, Z_t) , the expected number $\bar{z}$ of charging EVs at the CSP can be approximated by:*

$$\bar{z} = \begin{cases} \frac{\lambda^e}{\nu + p^{\max} \mu} & \text{if } \frac{\lambda^e}{\nu + p^{\max} \mu} < \alpha N^{cp}, \\ \frac{\lambda^e - p^{\max} \mu \alpha N^{cp}}{\nu} & \text{else.} \end{cases} \quad (3.4)$$

Proof of Proposition 3.1 Let us solve the fixed-point equation (3.3): $f(z) = z$, with

$$\begin{cases} f(z) = z \\ 0 \leq z \leq N^{\text{cp}}, \end{cases} \quad (3.5)$$

with $f(z) = \lambda^e \mathbb{E} \left[\min \left(D, \frac{E^{\text{req}}}{p(z)} \right) \right]$, or more explicitly:

$$f(z) = \begin{cases} \frac{1-B}{\nu + p^{\max} \mu} & \text{if } z \leq \alpha N^{\text{cp}}, \\ \frac{1-B}{\nu} \times \frac{z}{z + \frac{p^{\max} \mu}{\nu} \alpha N^{\text{cp}}} & \text{if } z > \alpha N^{\text{cp}}. \end{cases}$$

The following two cases arise:

- Case $\frac{\lambda^e}{\nu + p^{\max} \mu} \leq \alpha N^{\text{cp}}$: the unique solution z^* to equation $f(z) = z$ is:

$$z^* = \frac{\lambda^e}{\nu + p^{\max} \mu}.$$

In this case, z^* is smaller than αN^{cp} .

- Case $\frac{\lambda^e}{\nu + p^{\max} \mu} > \alpha N^{\text{cp}}$: the unique solution z^* to equation $f(z) = z$ is:

$$z^* = \frac{\lambda(1-B) - p^{\max} \mu \alpha N^{\text{cp}}}{\nu}.$$

In this case, z^* is greater than αN^{cp} . ■

Proposition 3.1 states that the expression of the average number of charging EVs takes two different forms. The first one, given by the first line of Equation (3.4), corresponds to the case where the power utilization rate $\frac{\lambda^e}{\nu + p^{\max} \mu}$ at the CSP is sufficiently low compared to the resource αN^{cp} available at the CSP, such that every charging EV charge at CP maximum power $p^{\max}$. The second one corresponds to the opposite case, where the power utilization rate $\frac{\lambda^e}{\nu + p^{\max} \mu}$ at the CSP is high, so that all EVs charge at a lower power than the maximum one $p^{\max}$, see Equation (3.1) for the power allocation formula. The closed form expression $\bar{z}$ allows us to get a closed form expression of an approximation $p(\bar{z})$ of the expected power received by an EV at the CSP. This enables to express another important metric of the system which is the expected energy received by each EV at the CSP.

3.2.3 Approximation of the expected energy received during the parking duration

The effective energy received E by an EV which has found an available parking space is determined by the minimum between the need in energy E^{req} and the potential cumulative power received over the parking duration D :

$$E = (1-B) \times \min \left(E^{\text{req}}, \int_D p(Z(t)|Z(t) > 0) dt \right). \quad (3.6)$$

It is difficult to derive an explicit formula for the expected energy received by an EV user prior to finding an available CP, or even to compute it numerically. Although the expected number of EV users upon arrival can be obtained by solving the balance equations of the Markov chain (Q_t, Z_t) , it may evolve during the charging session, as illustrated below in Figure 3.5. Let us provide an approximation $\tilde{E}(n)$ of the expected energy received at the CSP. $\tilde{E}(n)$ is the energy received at the CSP considering a constant number n of charging EVs. In this case, the power received during the charging duration is constant equal to $p(n)$, with $p(\cdot)$ the instantaneous power received by an EV defined in Equation (3.1); See Figure 3.5 for an illustration of this approximation. Therefore, the potential cumulative power $\int_D p(Z(t)|Z(t) > 0)$ received over the parking duration in Equation (3.6) is simplified to $D \times p(n)$, so that:

$$\tilde{E}(n) = (1 - B) \times \min(E^{\text{req}}, D \times p(n)).$$

$\tilde{E}(n)$ then follows an exponential distribution with parameter $\frac{p(n)\mu + \nu}{p(n) \times (1 - B)}$. The following Lemma 3.2 gives an approximation of the expected energy received.

Lemma 3.2. *The approximate expected energy received $\mathbb{E}[\tilde{E}(\bar{z})]$ at the CSP for any EV, is given by:*

$$\mathbb{E}[\tilde{E}(\bar{z})] = \begin{cases} \frac{p^{\max}}{p^{\max} \alpha N^{cp}} \times (1 - B) & \text{if } \lambda^e < \alpha N^{cp}(\nu + p^{\max} \mu), \\ \frac{p^{\max} \mu + \nu}{\lambda} & \text{else.} \end{cases} \quad (3.7)$$

The reasoning and computations leading to this formula are presented in Section A.3 of Appendix A. In what follows, the propositions involving mainly technical computations are presented in the Appendix.

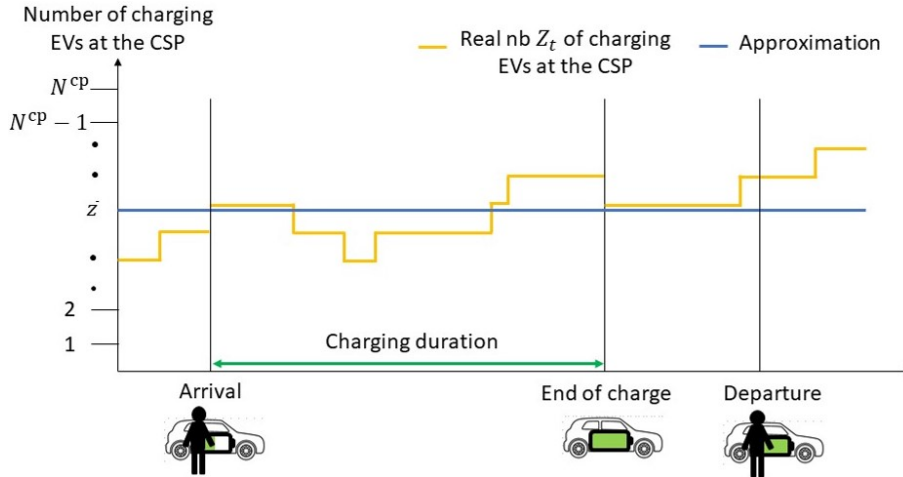


Figure 3.5: Illustrative diagram of the approximate expected number of EVs charging at the CSP during a charging session. *In the approximation, the number of charging EVs is supposed constant equal to the expected number $\bar{z}$ of charging EVs in stationary regime.*

Section 3.2.4 numerically measures the quality of this approximation. The following Lemma 3.3 gives a theoretical bound for the accuracy of the approximation on the expected energy received, given by formula (3.7). Let δ be the absolute difference between the effective energy received given by Equation (3.6) and the corresponding approximation by an EV at the CSP.

Lemma 3.3. *The error δ at CSP i , defined by the absolute value of the difference between the effective energy received of an EV during its parking duration and the approximation given by Equation (3.4) is bounded by:*

$$\delta \leq \frac{1}{\mu} \left(\frac{\min(1, \alpha N^{\text{cp}})}{\min(1, \alpha N^{\text{cp}}) + \beta} - \frac{\alpha}{\alpha + \beta} \right) \times (1 - B),$$

with $\beta = \frac{\nu}{p^{\text{max}} \mu}$.

Proof of Lemma 3.3 According to Equations (3.1) and (A.4), the energy received when being accepted to the chosen CSP is upper (resp. lower) bounded by the energy received when there is only 1 (resp. N^{cp}) charging EVs. In this case, the power drawn by an EV is $p^{\text{max}} \min(1, \alpha N^{\text{cp}})$ (resp. $p^{\text{max}} \alpha$). Therefore,

$$\begin{aligned} \delta &\leq (\mathbb{E}[\tilde{E}(1)] - \mathbb{E}[\tilde{E}(N^{\text{cp}})]) \times (1 - B) \\ &= \left(\frac{p^{\text{max}} \min(1, \alpha N^{\text{cp}})}{p^{\text{max}} \min(1, \alpha N^{\text{cp}}) \mu + \nu} - \frac{p^{\text{max}} \alpha}{p^{\text{max}} \alpha \mu + \nu} \right) \times (1 - B) \\ &= \frac{1}{\mu} \left(\frac{\min(1, \alpha N^{\text{cp}})}{\min(1, \alpha N^{\text{cp}}) + \frac{\nu}{p^{\text{max}} \mu}} - \frac{\alpha}{\alpha + \frac{\nu}{p^{\text{max}} \mu}} \right) \times (1 - B). \end{aligned}$$

■

The bound on the error δ is the difference between the highest $\bar{\delta}$ and the lowest expected energy an EV can draw $\underline{\delta}$, with:

$$\bar{\delta} := \frac{p^{\text{max}} \min(1, \alpha N^{\text{cp}})}{p^{\text{max}} \min(1, \alpha N^{\text{cp}}) \mu + \nu} \times (1 - B), \quad (3.8)$$

and

$$\underline{\delta} := \frac{p^{\text{max}} \alpha}{p^{\text{max}} \alpha \mu + \nu} \times (1 - B). \quad (3.9)$$

Even if this bound is simple to obtain, as it is based on extreme cases, it gives an explicit expression.

The bound given in Lemma 3.3 is strictly decreasing and convex with respect to the power sizing coefficient α for $\alpha > \frac{1}{N^{\text{cp}}}$. In particular, when $\alpha = 1$, i.e., the EVs always get served with the maximal power p^{max} , this bound is equal to zero, that means that the expected energy received at the CSP given by Equation (A.4) is the effective energy received. When the system is highly congested in terms of the number of parked EVs, i.e., when the utilization rate $\rho = \lambda/\nu$ is high which implies a blocking probability B close to 1, the error is tight. Note also that the error δ is bounded by the expected energy needed μ^{-1} because the energy received is always lower than the energy needed.

Here the Quality of Service (QoS) is defined as the expected energy received during the parking duration. Other criteria could be used such as the probability to leave with a fully charged battery [Dupont et al., 2021, Aveklouris et al., 2019]. The following Lemma 3.4 gives a relation between the expected energy received during the parking duration and the probability to leave with a fully charged battery, considering a constant number of charging EVs during the parking duration.

Lemma 3.4. *The probability $\mathbb{P}(F)$ to leave the CSP with a fully charged battery considering a constant number n of charging EVs during the charging duration is approximated by:*

$$\mathbb{P}(F) = \frac{\mathbb{E}[\tilde{E}(n)]}{\mathbb{E}[E^{\text{req}}]}. \quad (3.10)$$

Proof of Lemma 3.4 Assuming an EV has been accepted to CSP i , its probability to leave with a fully charged battery is the probability that its charging duration $\frac{E^{\text{req}}}{p(n)}$ is less than its parking duration D , both following exponential distributions of parameter respectively $p(n)\mu$ and ν . Multiplied by the probability to be accepted at the CSP, it gives:

$$\begin{aligned} \mathbb{P}(F) &= \mathbb{P}\left(\frac{E^{\text{req}}}{p(n)} < D\right) \times (1 - B) \\ &= \mu \times \frac{p(n) \times (1 - B)}{p(n)\mu + \nu} \\ &= \frac{1}{\mathbb{E}[E^{\text{req}}]} \times \mathbb{E}[\tilde{E}(n)]. \end{aligned}$$

■

The approximate expected energy received $\mathbb{E}[\tilde{E}]$, given by Equation (A.4), at the CSP is the average energy needed for a fully charged battery multiplied by the probability to leave with a fully charged battery. Lemma 3.4 therefore states that the EV user satisfaction is very similar whereas she wants to leave with a fully charged battery with the highest probability possible or with the more energy possible.

3.2.4 Numerical simulations

The proposed model was simulated using the *Simpy* package, which is a Python discrete-event simulation framework.³ Inter-arrival duration, parking duration and amount of energy needed for a fully charged battery are randomly generated following exponential distributions as assumed in the model in Section 3.2. Each EV user chooses a CSP to join upon arrival at the given site. If the chosen CSP is fully occupied, the EV user leaves without charging. Otherwise, the EV user joins the CSP and immediately starts to charge according to the power allocation rule given by Equation 3.1. After each event at a CSP (arrival, end of charge or departure of an EV), the current available power, as well as the number of available parking spaces are updated.

Table 3.6 gives the main parameter values used for the simulations. In [Venegas et al., 2021], the battery storage capacities among the most popular EVs is given, with an average of about 50kWh. In [Smart and Schey, 2012], the most frequent charging levels upon arrival is between 50% and 60%, against between 40% and 50% in [Hu et al., 2019]. Therefore, an initial SoC of 50% is assumed in this scenario, which leads to an average amount of energy needed for a fully charged battery of 25 kWh. The maximum power each CP can deliver to an EV at the CSP (11 kW) is in line with the range of typical charging levels for EV parking at supermarkets or shopping centers [Jessica et al., 2014].

3.2.4.1 Global statistics

The power sizing α , which depends mainly on the charging infrastructure but also on electrical contracts, is equal to 0.6 at each CSP, i.e., 60% of the number of CPs can deliver simultaneously

³ See [Izivia, 2024] for the documentation.

$\lambda = 6$	EVs arrive in average every 10 minutes.
$\mu = 1/25$	EVs need in average 25 kWh to have a fully charged battery.
$N^{\text{CP}} = 10$	Number of parking spaces at the CSP.
$p^{\text{max}} = 11$	Maximum power at each CP (in kW).

Table 3.6: Parameter values.

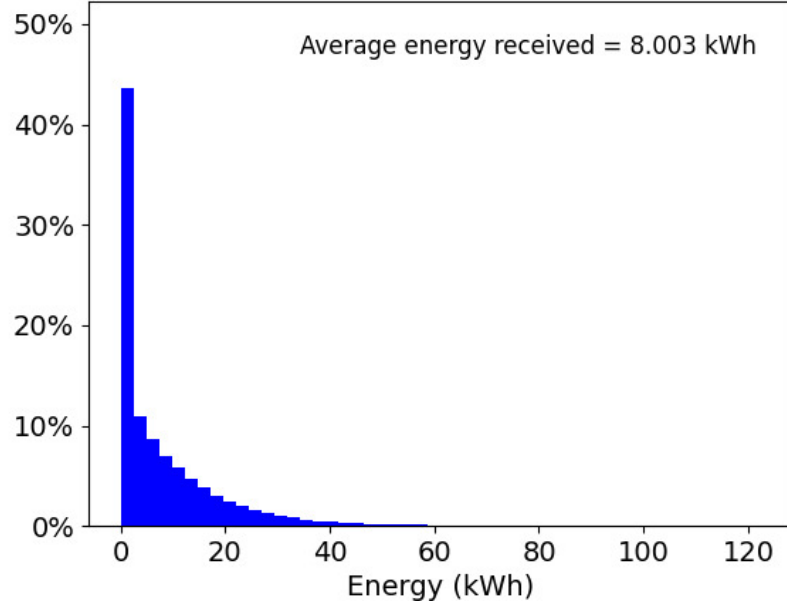


Figure 3.7: Histogram of the energy received per EV. *The high proportion (about 30%) of rejected EVs from the CSP indicates that the number of parking spaces is not enough compared to the utilization rate, i.e the frequency of arrivals times the parking duration.*

the maximum instantaneous power p^{max} , which is a typical power sizing decision for standard charging modes (< 43 kW).

Figures 3.7, 3.8 and 3.9 show the histograms of respectively the energy received per EVs among all EVs, the energy received per EV among the accepted EVs, and the average power drawn per EV. The parameters of the CSP and the EV characteristics are as shown in Table 3.6. In addition, the average parking duration is 2 hours, which is based on the results from [Parmar et al., 2021]. This value is in accordance with the context of an average parking duration at a shopping center. Since the previous theoretical results obtained correspond to long-run metrics, about 60000 EVs are simulated. Some EVs stay parked for a small amount of time or lack only little energy to have a fully charged battery, which explain that some accepted EVs receive little energy during their parking duration, as seen in Figures 3.7 and 3.8. For example, about 45% of accepted EVs stopped charging because their battery is fully charged. However, even among the EV users that found an available parking space, the average energy received per EV is about 11.5 kWh which is around half of the average energy needed for a fully charged battery which is 25 kWh. This shows that also a high proportion of EVs only received a small proportion of the energy needed for a fully charged battery. Figure 3.8 shows that among the EV users that found an available parking space, the distribution of the energy received is similar to the

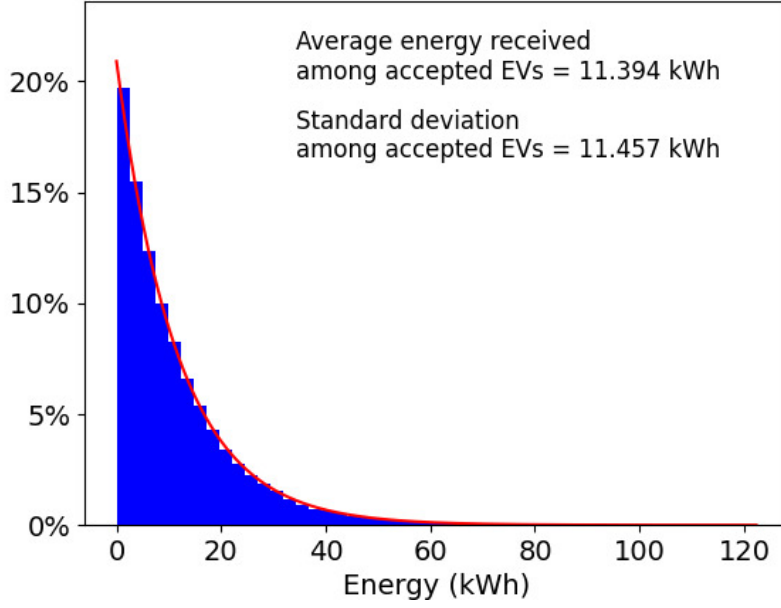


Figure 3.8: Histogram of the energy received per EV among accepted EVs. *The distribution of the energy received among the accepted EVs is very similar to the exponential distribution (the curve in red), which validates the approximation given in the proof of Proposition 3.2.*

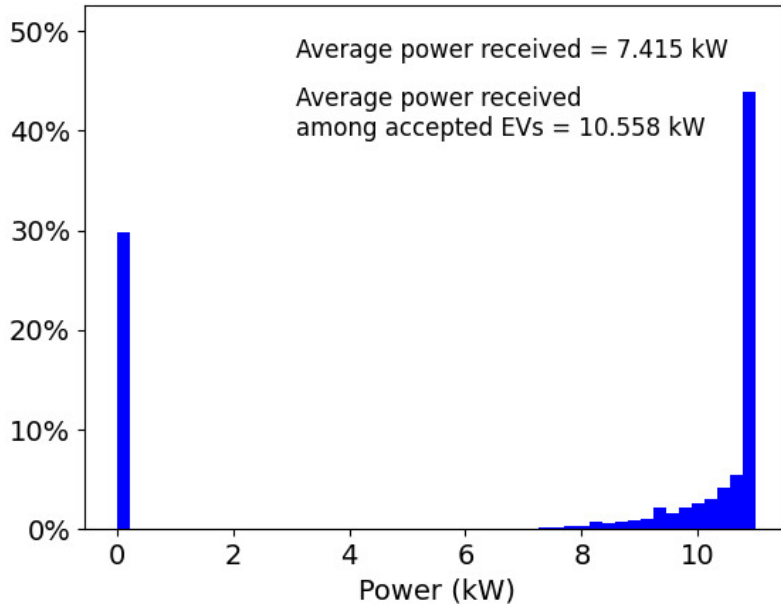


Figure 3.9: Histogram of the average power received per EV. *The CSP is well designed in terms of available power at the CSP level, as a majority of EVs received the maximum power of 11 kW or almost.*

exponential distribution, which is our approximation, given in Section 3.2. Indeed, the average energy received among the accepted EVs (about 11.39 kWh) is very close to the corresponding standard deviation (about 11.46 kWh), which is one of the main characteristics of the exponential distribution. When an EV is rejected, its average power drawn is considered to be 0 kW. Figure 3.9 therefore highlights the high proportion of EV users rejected from the CSP, which is about 30.2% against 30.0% theoretically. This is inline with the fact that the *blocking probability* is exact considering the stationary regime of the original Markov chain (Q_t, Z_t) . The average power received by an EV is about 10.5 kW, which is close to the maximum power level of 11 kW. This illustrates that, in our use-case, most of the time the number of charging EVs is sufficiently low such that all EVs get the maximum power that can be delivered by the CPs.

3.2.4.2 Approximation of energy received per EV: power sizing effect

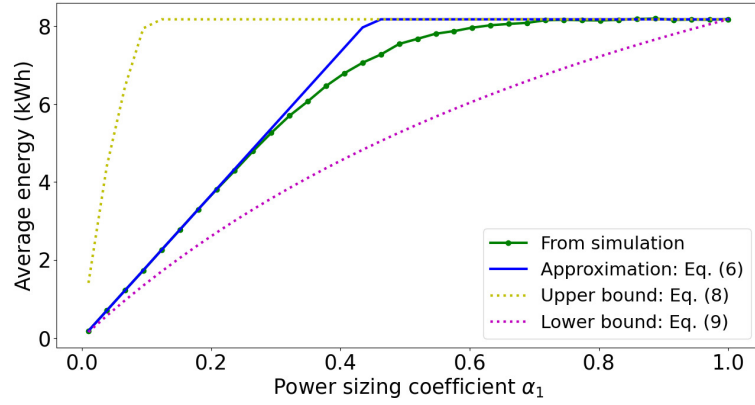


Figure 3.10: Energy received per EV in average with respect to the proportion α of the number of parking spaces that can charge simultaneously with maximum power $p^{\max} = 11$ kW. Setting $\alpha \approx 0.7$ is sufficient to maximize the expected energy received by the EVs they can charge with the maximum power of 11 kW.

In the following, each numerical value is an average taken over 50 simulations of 1000 (virtual) hours each, corresponding to about 6000 EV arrivals for each simulation. The average energy and power received by the EVs through simulation and the corresponding approximate theoretical values determined in Section 3.2 are illustrated next in Figures 3.10 and 3.11. Lower and upper bounds are also illustrated. These metrics are shown with respect to the power sizing coefficient α , and when the average parking duration also equals to 2 hours. α determines the available instantaneous power at the CSP, e.g. there is no power constraint at the level of the CSP when $\alpha = 1$. Setting this parameter correctly enables a CSO to know the optimal trade-off between the cost of infrastructure and electrical contract and the QoS for the EV users.

The expected energy received per EV is closely related to the probability for an EV to leave with a fully charged battery (see Lemma 3.4). The latter is plotted in [Aveklouris et al., 2019] and has a very similar shape as Figure 3.10. It validates the accuracy of the energy received per EV from the simulations. The average power received per EV is illustrated on Figure 3.11. The theoretical values of the average power are obtained by using the allocation power formula 3.1 considering the average number $\bar{z}$ of charging EVs given by Equation (3.4). For α sufficiently high (about 0.7), there is enough power at the CSP such that the EVs that found an available parking

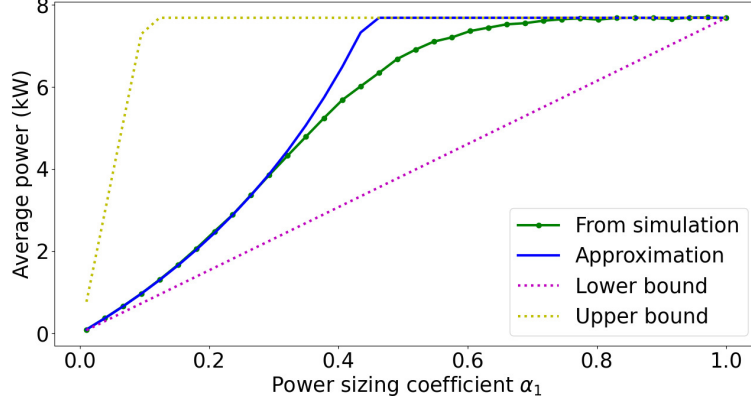


Figure 3.11: Average power received per EV in average with respect to the proportion of the number of parking spaces that can charge simultaneously with maximum power $p^{\max} = 11$ kW. Seeing Figure 3.10, there is a nonlinear relation between the average energy received and the average power received.

space draw the maximum power of 11 kW. Appendix A (Section A.2) presents a theoretical analysis of appropriate choices of the power sizing coefficient α , with a particular focus on its dependence on the arrival frequency λ . This question may be relevant for a CSO aiming to design properly the CSP. Note that Figure 3.11 shows the average power for all the EVs, which is strictly lower than 11 kW because of the rejected EV users. For this use-case, we observe that our theoretical expression of the average power is very close to the simulated one.

In our scenario, the error between the theoretical and the simulated average energy received by an EV (as well as the average power), is the highest when α is around 0.45. In this case, the difference in terms of energy received is about 10%, which represents about 0.8 kWh. Note that this energy corresponds to about 5 km of autonomy with a standard EV with an average consumption of 0.15 kWh/km for urban trips [Al-Wreikat et al., 2021]. This value is high knowing that the average distance when going shopping is around 4 km [US department of transportation, nd]. However, the power sizing coefficient is usually designed around 0.6. In this case, the theoretical approximations is more accurate: it corresponds to a difference between the theory and the practice of less than 1 km, which is more acceptable.

In both cases, for the energy received and the average power received per EV, the lower and upper bounds get more narrow for high values of α .

3.2.4.3 Effect of the parking duration on the approximation of energy received per EV

Figures 3.12 and 3.13 show the average energy and power received by an EV, obtained by simulation and by analytical formula given above, depending on the average parking duration of an EV in the system. Note that in our setting the upper bound is exactly equal to the theoretical values. Indeed, in the approximation the average number of charging EVs, given by Equation (3.4) is sufficiently low such that all charging EVs charge at maximum power $p^{\max}$. The average power, and therefore the average energy, received by the EVs is very close to the upper bound.

As seen in Figure 3.12, the average energy received per EV increases with respect to the average parking duration for low values of the latter and decreases for high average parking

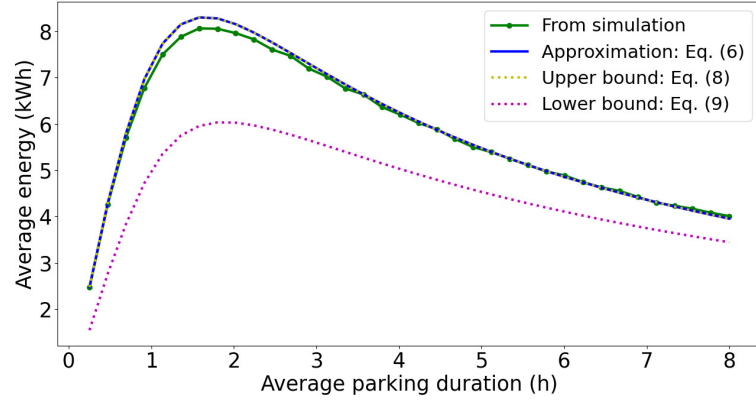


Figure 3.12: Energy received per EV in average with respect to the average parking duration ν^{-1} . An average parking duration of approximately 1h45 optimizes the trade-off (in terms of energy delivered) between allowing an EV sufficient time to charge and freeing up space promptly for incoming EVs.

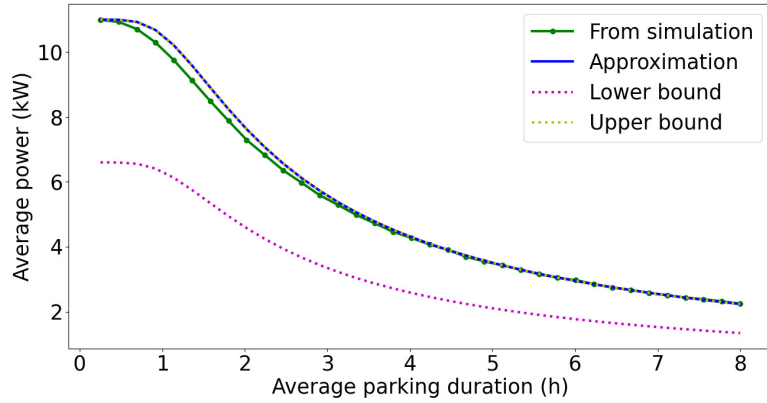


Figure 3.13: Average power drawn per EV in average with respect to the average parking duration ν^{-1} . The decreasing average power received per EV is due to the increasing blocking probability.

duration. As seen in this figure, the average energy received per EV increases with respect to the average parking duration for low values of the latter and decreases for high average parking duration. Considering low average parking duration, the probability for an EV to face a full CSP is negligible. However, as the parking duration is low, EVs do not have the time to get a lot of energy. In the opposite case, for high average parking duration, the EVs that entered the CSP have more time to fill in their battery. Also, the proportion of fully charged EVs is very high, so only a small proportion of EVs are actually charging, which enables them the charge at maximum power $p^{\max}$. However, the proportion of rejected EVs is very high in this case and only a very small proportion of EVs can find an available parking space, which reduces the expected energy received per EV. The unique optimal trade-off between these two situations is an average parking duration of about 1h45 hours, which is inline with average parking duration at a shopping center [Parmar et al., 2021]. Several external factors can influence the parking duration of the EVs. Legal authorities such as a CSO can impose pricing mechanisms that encourage EV users to

reduce the time parked in order to improve the quality of service. Recall that in our context the parking duration does not depend on the energy received.

Assuming now that there are two CSPs instead of a single one. the EV users are rational, they will join the CSP that enables to maximize the expected energy received during its parking duration. However, the expected energy received by an EV at a CSP depends on the proportion of EV users choosing this CSP. This congestion effect, both in terms of the available power, see Equation (3.1), and in terms of the limited number of parking spaces at the chosen CSP, creates an interaction between EV users.

3.3 Scenario with two charging pools: a queueing game framework

We extend in this section the previous scenario with two CSPs at the site. Upon arrival, an EV user chooses between the two CSPs according to both the instantaneous expected power and the number of parking spaces at each CSP. Indeed, if the chosen CSP is fully occupied at time of arrival, the EV user receives no energy as she goes to a parking space without a CP. All the notations of the previous section 3.2 are indexed by the corresponding CSP.

3.3.1 User charging behavior

The two CSPs are supposed to be used by the same population of EVs. The EV arrival time to each CSP $i \in \{1, 2\}$ follow a common Poisson process with rate $\lambda_i := \theta_i \lambda$, where θ_i is the probability that an EV chooses to join CSP i . As there are only two CSPs, (θ_1, θ_2) is fully characterized by the probability that an EV chooses to join CSP i , also denoted θ . This decision is taken before joining a CSP, without knowing any information about CSPs status, i.e., the number of charging EVs and parked EVs. This assumption is standard, corresponding to the class of *unobservable queueing games* [Hassin and Haviv, 2003].⁴ The case with information is left for a future extension.⁵

In practice, EV users are self-interested and if they have the choice between several CSPs, they will choose the one where they can expect maximum energy. In the most simple case, EV users determine which CSP to join without any influence from their private information: their SoC upon arrival or parking duration; and without knowing any information about the CSPs occupation level. Their decision is only based on the expectation of the energy received at each CSP. The energy received E_i at CSP i depends on the stationary distribution of the number of EVs charging which is not explicitly computable. Therefore, the energy received E_i at the CSP is approximated by considering that any new arriving EV observes the system in stationary regime (following the PASTA property of queueing systems [Wolff, 1982]) and then, in average, $\bar{z}_i$ charging EVs at CSP i . Thus, the expectation of the energy received for any new EV at the CSP is approximated by $\mathbb{E}[\tilde{E}_i(\bar{z}_i)]$.

EV users are interacting through a congestion effect (both for the physical access to parking space and power received), and are considered as players in a non-cooperative game setting. Moreover, this scenario is modeled as a non-observable queueing game [Naor, 1969] in normal form, i.e., a queueing game where the EV users have no information on the CSP status, i.e the number of charging and fully charged EVs at the CSP, before making their choice.

⁴ See Chapter 2 for further explanations.

⁵ See Chapter 6.5 for some perspectives of this thesis.

3.3.2 Equilibria analysis

A possible very large number of EVs interact and there is no specific number of players, so a nonatomic game [Aumann and Shapley, 2015] setting is suitable to model our non-cooperative system. In such game frameworks, the decision of a single EV user is assumed to be negligible, i.e., it has no impact on the decision of the other EVs. This type of game has been widely used in transportation networks and is related to the concept of *Wardrop equilibrium* [Schmeidler, 1973]. This is also typically the case in queueing game models [Hassin and Haviv, 2003]. Let $\tilde{\Gamma}$ denote the game defined by the 3-tuple $\tilde{\Gamma} = \{\mathcal{N}, \Delta, \tilde{u}\}$, where:

- $\mathcal{N} := [0, 1]$ is the set of players, which are the EV users;
- $\Delta := [0, 1]$ is the set of mixed-strategies, which corresponds to the probability to choose CSP 1;
- $\tilde{u} : \tilde{\Delta}^2 \mapsto \mathbb{R}_+$ is the utility (payoff) function of an EV that chooses action CSP 1 with probability θ and it is defined by:

$$\forall(\theta, \bar{\theta}) \in \tilde{\Delta}^2, \tilde{u}(\theta, \bar{\theta}) = \theta \mathbb{E}_{\bar{\theta}}[\tilde{E}_1] + (1 - \theta) \mathbb{E}_{1-\bar{\theta}}[\tilde{E}_2], \quad (3.11)$$

where $\bar{\theta}$ (resp. $1 - \bar{\theta}$) denotes the probability used to choose CSP 1 (resp. CSP 2) by any other player in the population assuming that the players are indistinguishable. $\mathbb{E}_{\bar{\theta}}[\tilde{E}_i]$, $\bar{\theta} \in [0, 1]$, $i \in \{1, 2\}$, is the approximation of the expected energy received by an EV at CSP i when EV users choose this CSP with probability $\bar{\theta}$. Note that based on the assumption of an infinite number of players, the approximation of the expected energy $\mathbb{E}_{\bar{\theta}}[\tilde{E}_i]$ received for a particular EV at the CSP depends only on the probability $\bar{\theta}$. In realistic scenarios, EV users will choose the CSP based on the exact expected energy received, not the approximate one. However, defining the game $\tilde{\Gamma}$ with the approximation enables to find analytic results. The quality of the approximation of some metrics of the game $\tilde{\Gamma}$, in comparison of the game Γ for which EV users choose the CSP according to the exact energy received, will be analyzed through numerical simulations in Section 3.3.4.

The solution concept of such non-cooperative games with indistinguishable infinitely many players is equilibria, which correspond to strategy profiles for which there is no interest for any player to unilaterally deviate [Correa and Stier-Moses, 2011]. An equilibrium is therefore a natural candidate for the EVs distribution, i.e., an allocation of EVs to the CSPs, one can empirically observe. The EVs being indistinguishable, they are supposed to behave equally, therefore symmetric equilibria are studied, i.e., equilibria where all EVs follow the same strategy. A symmetric mixed-strategy equilibrium is formally a constant function from $[0, 1]$ to $[0, 1]$, where the interval $[0, 1]$ corresponds to the mass of players. To simplify the notation, it is assimilated to a probability value $\tilde{\theta}_{\text{eq}} \in [0, 1]$.

Definition 3.5 (Symmetric mixed-strategy equilibrium). *A symmetric mixed-strategy equilibrium is a profile of identical mixed-strategies $\tilde{\theta}_{\text{eq}}$ for which no EV user has an incentive to deviate:*

$$\forall \theta \in [0, 1], \quad \tilde{u}(\theta, \tilde{\theta}_{\text{eq}}) \leq \tilde{u}(\tilde{\theta}_{\text{eq}}, \tilde{\theta}_{\text{eq}}). \quad (3.12)$$

The effective arrival rate at CSP i is denoted λ_i^e , and follows the same formula given by Equation (3.2), replacing λ by $\lambda \times \theta_i$. The following Lemma (3.6) is essential in order to derive some properties of the game $\tilde{\Gamma}$ as well as of the game Γ for which EV users choose the CSP according to the exact expected energy received.

Lemma 3.6 ([Yao and Shanthikumar, 1987]). *The effective arrival rate λ_i^e , $i \in \{1, 2\}$, increases with respect to the arrival rate λ_i .*

The result of Lemma (3.6), proven in [Yao and Shanthikumar, 1987], states that the higher the number of EV users who choose CSP i , the more EV users will enter CSP i .

One way to determine an equilibrium is to look for the distribution of the EV users decision such that the expected energy received by an EV is the same at each CSP. Such computations can be difficult in general. Another approach based on the existence of a potential function is described in the next section.

3.3.2.1 A potential function approach

A variant $\tilde{\Gamma}_{\text{pure}} = \{\mathcal{N}, \tilde{\mathcal{A}}, \tilde{u}(\cdot, \cdot)\}$ of the previous game $\tilde{\Gamma}$ is introduced in this section, where $\mathcal{A} := \{CS1, CS2\}$ is the set of actions. In the game $\tilde{\Gamma}_{\text{pure}}$, each EV user chooses a pure action (CS 1 or CSP 2), instead of a mixed strategy in the game $\tilde{\Gamma}$. The utility of each EV user depends on her pure action and on the proportion of EV users choosing the same action, i.e., the same CSP. As described below, the game $\tilde{\Gamma}_{\text{pure}}$ admits a potential function which enables to prove the uniqueness of an equilibrium. Several EVs to CSPs allocations, i.e., maps from the mass $[0, 1]$ of EVs to the set of actions $\{1, 2\}$, can lead to the same proportion of EV users choosing the first CSP, i.e., $\mathcal{L}(S)$ where $\mathcal{L}$ is the Lebesgue measure and S the set of EV users choosing the first CSP. Because the utility of EV users considers only the proportion of EV users choosing each CSP, by abuse of language, an equilibrium is a proportion of EV users choosing the first CSP. An equilibrium in pure strategy of the game $\tilde{\Gamma}_{\text{pure}}$ is defined as follows.

Definition 3.7 (Pure-strategy equilibrium). *A pure strategy equilibrium is a proportion $\theta_{\text{eq}}^{\text{pure}}$ of EVs choosing the first CSP such that:*

$$\begin{cases} \mathbb{E}_{\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_1] \geq \mathbb{E}_{1-\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_2] & \text{if } \theta_{\text{eq}}^{\text{pure}} > 0, \\ \mathbb{E}_{\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_1] \leq \mathbb{E}_{1-\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_2] & \text{if } \theta_{\text{eq}}^{\text{pure}} < 1. \end{cases}$$

The game $\tilde{\Gamma}_{\text{pure}}$ is equivalent to the game $\tilde{\Gamma}$ in the sense that the equilibria of both games, based on Definitions 3.5 and 3.7, are identical, as stated in Proposition 3.8.

Proposition 3.8. *The set of equilibria for the game $\tilde{\Gamma}_{\text{pure}}$ and $\tilde{\Gamma}$ is identical.*

Proof of Proposition 3.8

- Assume $\tilde{\theta}_{\text{eq}} > 0$ and $\mathbb{E}_{\tilde{\theta}_{\text{eq}}}[\tilde{E}_1] < \mathbb{E}_{1-\tilde{\theta}_{\text{eq}}}[\tilde{E}_2]$. Therefore, $\tilde{u}(0, \tilde{\theta}_{\text{eq}}^{\tilde{\Gamma}}) > \tilde{u}(\tilde{\theta}_{\text{eq}}^{\tilde{\Gamma}}, \tilde{\theta}_{\text{eq}}^{\tilde{\Gamma}})$, which contradicts the equilibrium definition of the game $\tilde{\Gamma}$. Therefore, $\mathbb{E}_{\tilde{\theta}_{\text{eq}}}[\tilde{E}_1] \geq \mathbb{E}_{1-\tilde{\theta}_{\text{eq}}}[\tilde{E}_2]$. With the same reasoning, the second condition of Definition 3.7 holds. It gives that the set of equilibria of the game $\tilde{\Gamma}$ is included in that of the game $\tilde{\Gamma}_{\text{pure}}$.
- Now let $\theta_{\text{eq}}^{\text{pure}}$ be an equilibrium of the game $\tilde{\Gamma}_{\text{pure}}$, and let $\theta \in [0, 1]$.
 - Assume $\theta_{\text{eq}}^{\text{pure}} \in]0, 1[$. From Definition 3.7 of an equilibrium of the game $\tilde{\Gamma}_{\text{pure}}$, $\mathbb{E}_{\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_1] = \mathbb{E}_{1-\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_2]$, which gives:

$$\begin{aligned} \tilde{u}(\theta, \theta_{\text{eq}}^{\text{pure}}) &= \theta \mathbb{E}_{\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_1] + (1 - \theta) \mathbb{E}_{1-\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_2] \\ &= \theta_{\text{eq}}^{\text{pure}} \mathbb{E}_{\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_1] + (1 - \theta_{\text{eq}}^{\text{pure}}) \mathbb{E}_{1-\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_2] \\ &= \tilde{u}(\theta_{\text{eq}}^{\text{pure}}, \theta_{\text{eq}}^{\text{pure}}). \end{aligned}$$

In particular, $\tilde{u}(\theta, \theta_{\text{eq}}^{\text{pure}}) \leq \tilde{u}(\theta_{\text{eq}}^{\text{pure}}, \theta_{\text{eq}}^{\text{pure}})$, which implies that $\theta_{\text{eq}}^{\text{pure}}$ is an equilibrium of the game $\tilde{\Gamma}$.

- Now assume $\theta_{\text{eq}}^{\text{pure}} = 0$. From the definition 3.7 of an equilibrium of the game $\tilde{\Gamma}_{\text{pure}}$, $\mathbb{E}_{\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_1] \leq \mathbb{E}_{1-\theta_{\text{eq}}^{\text{pure}}}[\tilde{E}_2]$. Then:

$$\begin{aligned} \tilde{u}(\theta, \theta_{\text{eq}}^{\text{pure}}) &= \tilde{u}(\theta, 0) = \theta \mathbb{E}_0[\tilde{E}_1] + (1 - \theta) \mathbb{E}_1[\tilde{E}_2] \\ &\leq \theta \mathbb{E}_1[\tilde{E}_2] + (1 - \theta) \mathbb{E}_1[\tilde{E}_2] \\ &= \mathbb{E}_1[\tilde{E}_2] = \tilde{u}(0, 0) = \tilde{u}(\theta_{\text{eq}}^{\text{pure}}, \theta_{\text{eq}}^{\text{pure}}), \end{aligned}$$

which implies that $\theta_{\text{eq}}^{\text{pure}}$ is an equilibrium of the game $\tilde{\Gamma}$.

- The same reasoning as for $\theta_{\text{eq}}^{\text{pure}} = 0$ holds if $\theta_{\text{eq}}^{\text{pure}} = 1$.

It gives that $\forall \theta \in [0, 1]$, $\tilde{u}(\theta, \theta_{\text{eq}}^{\text{pure}}) \leq \tilde{u}(\theta_{\text{eq}}^{\text{pure}}, \theta_{\text{eq}}^{\text{pure}})$ which implies that $\theta_{\text{eq}}^{\text{pure}}$ is by definition an equilibrium of the game $\tilde{\Gamma}$. The set of equilibria of the game $\tilde{\Gamma}_{\text{pure}}$ is included in the one of the game $\tilde{\Gamma}$. ■

Note that the game $\tilde{\Gamma}_{\text{pure}}$ is a particular type of game, it is a congestion game [Rosenthal, 1973]: the utility of the player depends only on the proportion of players choosing the same pure action (CS in our context). It therefore falls within the class of potential games [Monderer and Shapley, 1996] because the utility function is the same for all EVs. As a consequence, first, a pure equilibrium always exists (through the use of the potential function). Second, a potential function P_o associated to the game $\tilde{\Gamma}_{\text{pure}}$ can be defined as follows:

$$P_o(\theta) = \int_0^\theta \mathbb{E}_x[\tilde{E}_1] dx + \int_0^{1-\theta} \mathbb{E}_x[\tilde{E}_2] dx. \quad (3.13)$$

It turns out that the equilibria of the game $\tilde{\Gamma}_{\text{pure}}$ [William H, 2001], and then of the game $\tilde{\Gamma}$ (see Proposition 3.8), are the maximizers of the potential function P_o . As the potential function P_o is differentiable, a standard algorithm such as the gradient-based descent enables one to find an equilibrium numerically.

3.3.2.2 Uniqueness and characterization of equilibria

Due to the complicated expression of the utility function, and in particular that of the effective arrival rate λ_i^e at each CSP i (see Equation (3.2)), obtaining an explicit form of the equilibria is not possible. However, some properties of the equilibria can be described, in particular due to the increasing effective arrival rate λ_i^e , $i \in \{1, 2\}$, with respect to λ_i , given by Lemma (3.6).

Next Proposition 3.9 proves the uniqueness of the equilibrium, and is a direct consequence of the monotonicity of λ_i^e with respect to λ_i .

Proposition 3.9 (Uniqueness). *The non-cooperative game $\tilde{\Gamma}$ has a unique equilibrium denoted $\tilde{\theta}_{\text{eq}}$.*

Proof of Proposition 3.9 As stated in Proposition 3.8, the set of equilibria of the games $\tilde{\Gamma}$ and $\tilde{\Gamma}_{\text{pure}}$ are equivalent. As the set of maximum points of the potential function P_o is identical to the set of equilibria defined previously by Equation (3.13) of the game $\tilde{\Gamma}_{\text{pure}}$, as shown in

[William H, 2001], it is also identical to the set of equilibria of the game $\tilde{\Gamma}$. P_o is differentiable and its derivative is $P'_o(\theta) = \mathbb{E}_\theta[\tilde{E}_1] - \mathbb{E}_{1-\theta}[\tilde{E}_2]$. Because $x \mapsto \mathbb{E}_x[\tilde{E}_i]$, $i \in \{1, 2\}$ strictly decreases, it is clear that P'_o strictly decreases, which implies that P_o is strictly concave on $[0, 1]$. P_o has therefore a unique maximum point, the equilibrium, which concludes the proof. ■

As in Proposition 3.10, this result relies on the decreasing approximate expected energy received with respect to λ_i . Based on this property, uniqueness directly follows, as originally shown by [Beckmann et al., 1956]. It also holds for the exact expected energy received and thus ensures uniqueness of the equilibrium in the game Γ , where EV users' utility is based on this exact value. In many systems in which agents interact, empirical distributions of agents to their possible actions are observed. The uniqueness of the equilibrium enables to predict the empirical configuration of EVs to CSPs in a decentralized system, i.e., when the EVs are the decision-maker and individually choose their best strategy.

In some particular configuration settings, the equilibrium is deterministic: all EV users choose the same CSP, i.e., $\tilde{\theta}_{\text{eq}} \in \{0, 1\}$. In this case, the chosen CSP gives a higher expected energy compared to the other CSP, even if all EV users choose it. Proposition 3.10 states that this situation does not occur if the maximum power each CP can deliver at each CSP is equal.

Proposition 3.10. *If the maximum power that the CPs can deliver at each CSP is equal, i.e., $p_1^{\max} = p_2^{\max}$, then the equilibrium $\tilde{\theta}_{\text{eq}} \in]0, 1[$.*

Proof of Proposition 3.10 Suppose that $p_1^{\max} = p_2^{\max}$ and $\tilde{\theta}_{\text{eq}} = 0$. Then, because $x \mapsto \mathbb{E}_x[\tilde{E}_2]$ strictly decreases, $\tilde{u}(\tilde{\theta}_{\text{eq}}, \tilde{\theta}_{\text{eq}}) = \tilde{u}(0, 0) = \mathbb{E}_1[\tilde{E}_2] < \mathbb{E}_0[\tilde{E}_2] = \mathbb{E}_0[\tilde{E}_1] = \tilde{u}(1, 0)$, which contradicts the definition of equilibrium, given by 3.5. The reasoning is the same in the opposite case: $\tilde{\theta}_{\text{eq}} = 1$. Therefore, $p_1^{\max} = p_2^{\max} \Rightarrow \tilde{\theta}_{\text{eq}} \in]0, 1[$. ■

Note that the first principle of Wardrop equilibrium can also be observed in queueing games [Hassin and Haviv, 2003]. Thus, if the equilibrium is inner, i.e., $\tilde{\theta}_{\text{eq}} \in]0, 1[$, the EV users' utility is the same for both CSP: $\mathbb{E}_{\tilde{\theta}_{\text{eq}}}[\tilde{E}_1] = \mathbb{E}_{1-\tilde{\theta}_{\text{eq}}}[\tilde{E}_2]$.

The proof of the previous proposition relies on the decreasing property of the expected energy received at the CSP with respect to the probability that EV users choose CSP i . This property is also true if we consider the exact energy received, because the effective arrival rate λ_i^e strictly increases in λ_i . Therefore, the previous Proposition 3.10 is also true for the game Γ where the exact energy received is considered in the EV users' utility.

As seen in Section 3.3.2.1, the unique equilibrium can be found numerically using standard gradient algorithms. In some special cases, studied below in detail, it is even possible to obtain an explicit formula for the equilibrium. Let us consider the following equation in $\theta_i \in [0, 1]$ for each CSP i :

$$\theta_i \lambda \left(1 - \frac{\frac{(\theta_i \frac{\lambda}{\nu})^{N_i^{\text{cp}}}}{N_i^{\text{cp}}!}}{\sum_{j=0}^{N_i^{\text{cp}}} \frac{(\theta_i \frac{\lambda}{\nu})^j}{j!}} \right) = (\nu + p_i^{\max} \mu) \alpha_i N_i^{\text{cp}}. \quad (3.14)$$

The left term of Equation (3.14) is exactly the effective arrival rate λ_i^e at CSP i . The strict monotonicity of λ_i^e with respect to $\lambda_i := \theta_i \lambda$, and therefore with respect to θ_i also, gives the uniqueness of the solution to Equation (3.14).

For each CSP $i \in \{1, 2\}$, a parameter $\tilde{\theta}_i^{\text{thr}}$ is defined as follows:

$$\tilde{\theta}_i^{\text{thr}} = \begin{cases} \text{unique solution of Equation (3.14)} & \text{if } \lambda \left(1 - \frac{\frac{(\theta_i \frac{\lambda}{\nu})^{N_i^{\text{cp}}}}{N_i^{\text{cp}}!}}{\sum_{j=0}^{N_i^{\text{cp}}} \frac{(\theta_i \frac{\lambda}{\nu})^j}{j!}}\right) > (\nu + p_i^{\text{max}} \mu) \alpha_i N_i^{\text{cp}} \\ +\infty & \text{otherwise.} \end{cases} \quad (3.15)$$

The threshold $\tilde{\theta}_i^{\text{thr}}$ is an important metric that characterizes the capacity of the CSP to offer the maximum instantaneous power p_i^{max} to the EVs. When $\tilde{\theta}_i^{\text{thr}} < +\infty$, i.e., there is a solution to Equation (3.14), the latter $\tilde{\theta}_i^{\text{thr}}$ is a threshold such that if the proportion of EVs is above this threshold, EVs charge at a instantaneous power strictly lower than p_i^{max} . Observe that this threshold $\tilde{\theta}_i^{\text{thr}}$ increases with $\alpha_i N_i^{\text{cp}}$ (recall that $\lfloor \alpha_i N_i^{\text{cp}} \rfloor$ is the number of parking spaces that can offer simultaneously the maximum power rate p_i^{max}). Moreover, if the power sizing coefficient α_i is sufficiently high ($\alpha_i > \frac{\nu}{\nu + p_i^{\text{max}} \mu}$), $\tilde{\theta}_i^{\text{thr}} = +\infty$: EVs charge at maximum rate p_i^{max} at the CSP regardless of the probability θ_i with which EV users choose CSP i .

The following Propositions 3.11 and 3.12 below are particular cases of parameters such that the equilibrium can be determined explicitly. Proposition 3.11 deals with the case when the capacity of both CSPs to offer the maximum power $p_i^{\text{max}}, i \in \{1, 2\}$, parameterized by $\tilde{\theta}_i^{\text{thr}}, i \in \{1, 2\}$, is low.

Proposition 3.11 (Low resource in power). *Assume that the following conditions hold:*

$$\tilde{\theta}_1^{\text{thr}} < \frac{p_1^{\text{max}} \alpha_1 N_1^{\text{cp}}}{p_1^{\text{max}} \alpha_1 N_1^{\text{cp}} + p_2^{\text{max}} \alpha_2 N_2^{\text{cp}}} \quad \text{and} \quad \tilde{\theta}_2^{\text{thr}} < \frac{p_2^{\text{max}} \alpha_2 N_2^{\text{cp}}}{p_1^{\text{max}} \alpha_1 N_1^{\text{cp}} + p_2^{\text{max}} \alpha_2 N_2^{\text{cp}}}. \quad (3.16)$$

Then the equilibrium $\tilde{\theta}_{\text{eq}}$ is given by:

$$\tilde{\theta}_{\text{eq}} = \frac{p_1^{\text{max}} \alpha_1 N_1^{\text{cp}}}{p_1^{\text{max}} \alpha_1 N_1^{\text{cp}} + p_2^{\text{max}} \alpha_2 N_2^{\text{cp}}}.$$

Moreover, for each CSP i , $p_i(\tilde{\theta}_{\text{eq}}) < p_i^{\text{max}}$ and the utility $\tilde{U}_{\text{eq}}$ at equilibrium for each EV is:

$$\tilde{U}_{\text{eq}} = \frac{p_1^{\text{max}} \alpha_1 N_1^{\text{cp}} + p_2^{\text{max}} \alpha_2 N_2^{\text{cp}}}{\lambda}.$$

The proof of Proposition 3.11 is presented in Section A.3 of Appendix A. The idea is to find the unique probability $\tilde{\theta}_{\text{eq}}$ such that the utilities for selecting each CSP are the same.

Note that when conditions (3.16) hold, $\tilde{\theta}_{\text{eq}} \in]0, 1[$, which means that at equilibrium both CSPs are chosen by EV users. Therefore, as stated in Proposition 3.10, at equilibrium, EV users expect the same approximate energy at both CSPs: $\mathbb{E}_{\tilde{\theta}_{\text{eq}}}[\tilde{E}_1] = \mathbb{E}_{1-\tilde{\theta}_{\text{eq}}}[\tilde{E}_2]$. At equilibrium, the probability with which EV users choose the CSP is exactly the proportion of the available power $p_i^{\text{max}} \alpha_i N_i^{\text{cp}}$ at the CSP in relation to the total available power $p_1^{\text{max}} \alpha_1 N_1^{\text{cp}} + p_2^{\text{max}} \alpha_2 N_2^{\text{cp}}$ at both CSPs. Moreover, the EVs users' utility at equilibrium is exactly the ratio between the total available power $p_1^{\text{max}} \alpha_1 N_1^{\text{cp}} + p_2^{\text{max}} \alpha_2 N_2^{\text{cp}}$ and the frequency of arrivals λ at both CSPs.

In the exact opposite case, when the capacity of both CSPs to offer the maximum power $p_i^{\text{max}}, i \in \{1, 2\}$, parameterized by $\tilde{\theta}_i^{\text{thr}}, i \in \{1, 2\}$, is high, it is also possible to precisely characterize the equilibrium, as stated in the following Proposition 3.12.

Proposition 3.12 (High resource in power). *Assume that the following condition holds:*

$$\min(\tilde{\theta}_1^{\text{thr}}, \tilde{\theta}_2^{\text{thr}}) = +\infty. \quad (3.17)$$

Then,

1. if

$$\begin{cases} \frac{\frac{\rho_1^{N_1^{cp}}}{N_1^{cp}!}}{\sum_{j=0}^{N_1^{cp}} \frac{\rho_1^j}{j!}} \geq 1 - \eta, \\ \frac{\frac{\rho_2^{N_2^{cp}}}{N_2^{cp}!}}{\sum_{j=0}^{N_2^{cp}} \frac{\rho_2^j}{j!}} \geq 1 - \frac{1}{\eta}, \end{cases} \quad (3.18)$$

with $\eta = \frac{p_2^{\max}(\nu + p_1^{\max}\mu)}{p_1^{\max}(\nu + p_2^{\max}\mu)}$, then $\tilde{\theta}_{eq}$ is the unique solution of:

$$\eta \sum_{i=0}^{N_1^{cp}-1} \sum_{j=0}^{N_2^{cp}} \frac{(1-\theta)^i \theta^j \rho^{i+j}}{i!j!} = \sum_{i=0}^{N_2^{cp}-1} \sum_{j=0}^{N_1^{cp}} \frac{\theta^i (1-\theta)^j \rho^{i+j}}{i!j!}. \quad (3.19)$$

Moreover, $\mathbb{E}_{\tilde{\theta}_{eq}}[\tilde{E}_1] = \mathbb{E}_{1-\tilde{\theta}_{eq}}[\tilde{E}_2]$;

2. otherwise, $\mathbb{E}_{\tilde{\theta}_{eq}}[\tilde{E}_1] \neq \mathbb{E}_{1-\tilde{\theta}_{eq}}(E_2)$,

$$\tilde{\theta}_{eq} = \begin{cases} 0 & \text{if } p_1^{\max} < p_2^{\max}, \\ 1 & \text{if } p_1^{\max} > p_2^{\max}, \end{cases}$$

and

$$\tilde{U}_{eq} = \begin{cases} \frac{p_2^{\max}}{\nu + p_2^{\max}\mu} \times \left(1 - \frac{\frac{\rho_2^{N_2^{cp}}}{N_2^{cp}!}}{\sum_{j=0}^{N_2^{cp}} \frac{\rho_2^j}{j!}}\right) & \text{if } p_1^{\max} < p_2^{\max}, \\ \frac{p_1^{\max}}{\nu + p_1^{\max}\mu} \times \left(1 - \frac{\frac{\rho_1^{N_1^{cp}}}{N_1^{cp}!}}{\sum_{j=0}^{N_1^{cp}} \frac{\rho_1^j}{j!}}\right) & \text{if } p_1^{\max} > p_2^{\max}. \end{cases}$$

The proof of Proposition 3.12 is presented in Section A.3 of Appendix A. The idea is that in the case of Proposition 3.12, EVs can charge at the maximum charging power rate ($p_1^{\max}$ or $p_2^{\max}$, depending on the selected CSP), regardless of the strategy profile of the EV population. As a result, in this case, congestion occurs only in relation to the blocking probability. This simplifies the formula for the expected energy received at a CSP.

In the first case of the previous Proposition 3.12, the two CSPs have similar characteristics in terms of the number of parking spaces N_i^{cp} , or in terms of the maximum instantaneous power $p_i^{\max}$ each CP can deliver to EVs, at CSP i . The approximation of the expected energy at both CSPs is then equal. The second case of Proposition 3.12 corresponds to the opposite case, where the two CSPs have very different characteristics. In this case, at equilibrium all EVs choose the same CSP because its capacity to offer a high energy is much greater than that of the other CSP.

This section focuses on a decentralized system in which EVs individually choose their best strategy (the CSP to join) without taking into account the externalities of their actions. For instance, before choosing the CSP, an arriving EV user at the site does not consider that she will deprive the next arriving EVs of a parking space, or lower the current charging power of the other charging EVs at the chosen CSP. This system is not always optimal with respect to social welfare. A study of the comparison between the decentralized system and the centralized system is shown in next section by studying the Price of Anarchy (PoA) of our system.

3.3.3 Price of anarchy

In a centralized system where the CSO has no information on the EVs (their SoC at arrival or parking duration) nor the CSPs occupancy, but can assign each EV user a CSP, the CSO may seek to maximize user satisfaction, measured by the expected energy received. In this case, the optimal utility $\tilde{U}_{\text{opt}}$ of an EV user is defined as:

$$\tilde{U}_{\text{opt}} = \max_{\theta \in [0,1]} \tilde{u}(\theta, \theta). \quad (3.20)$$

Similarly to equilibrium, it is difficult to obtain an explicit formula for the optimal utility $\tilde{U}_{\text{opt}}$ except in some specific cases. The Price of Anarchy (PoA) [Roughgarden, 2005] is a well-known metric in game theory that quantifies the inefficiency of a decentralized system compared to its centralized counterpart. It is defined as the ratio between the optimal utility $\tilde{U}_{\text{opt}}$ and the utility at unique equilibrium $\tilde{U}_{\text{eq}}$:

$$\widetilde{PoA} = \frac{\tilde{U}_{\text{opt}}}{\tilde{U}_{\text{eq}}}. \quad (3.21)$$

Note that the utility is strictly positive so that $\widetilde{PoA}$ is well defined. In general, when there exists possibly several equilibria, the lowest utility among the ones at equilibria is taken in Definition (3.21) of $\widetilde{PoA}$. Note that $\widetilde{PoA}$ is always greater than 1 because a centralized scheduler can reproduce the equilibrium, i.e., the EVs to CSPs distribution in decentralized system. The PoA provides a metric for the quality of the equilibrium in terms of social welfare (sum of user utilities). The closer $\widetilde{PoA}$ is to 1, the better the decentralized system is because the corresponding utility is close to the optimal one. The next Proposition 3.13 gives a sufficient condition such that the decentralized system is as efficient as the centralized system, i.e., $\widetilde{PoA} = 1$, as well as upper bounds on the price of anarchy.

Proposition 3.13 (Upper bounds). *The PoA verifies the following properties:*

1. Suppose Conditions (3.16) hold, then: $\widetilde{PoA} = 1$.
2. Suppose Conditions (3.17) hold, then:

$$\widetilde{PoA} \leq \frac{\min(1, \frac{N_1^{cp} + N_2^{cp}}{\rho})}{1 - \frac{\rho^{N_m^{cp}}}{N_m^{cp}!} \frac{1}{\sum_{j=0}^{N_m^{cp}} \frac{\rho^j}{j!}}},$$

with $m = \arg \min(N_1^{cp}, N_2^{cp})$.

The proof of Proposition 3.13 is provided in Section A.3 of Appendix A and primarily relies on a technical analysis of the utility functions. Proposition 3.13 shows that the decentralized system is optimal in terms of social welfare when the capacity for both CSPs to deliver the corresponding maximum instantaneous power $p_i^{\max}$ is poor (part (i) of Proposition 3.13). In the opposite case (part (ii) of Proposition 3.13), EV users tend to overuse the CSP that offers the highest maximum instantaneous power $p_i^{\max}$, creating a high blocking probability at this CSP. However, when the number of parking spaces at both CSPs is high, i.e., the blocking probabilities are low, this overuse does not have a great impact on the utility of EV users at equilibrium. In general, $\widetilde{PoA}$ remains relatively low, indicating that the decentralized system studied in this chapter is efficient. In other words, the selfish behavior of individual EV users

does not significantly impact overall EV welfare. This is because the blocking probability or the energy received by future arriving EVs is not greatly affected by the individually optimal choices of each EV user. This tendency is clarified in the next section 3.3.4 on a similar use-case as in Section 3.2.4.

Theoretical results on equilibrium analysis and its comparison with the optimal configuration in a centralized system have been studied through the PoA. In order to compare them with an empirical analysis, statistical results from a simulation of the model are shown in Section 3.3.4.

3.3.4 Numerical simulations

In this section, numerical experiments are provided with the scenario with two CSPs. The parameter values of the EV users are the same as in the case with a single CSP i , shown in previous Section 3.2.4. Table 3.14 presents the parameter values of CSP 1 and 2. Note that there is an asymmetry between the two CSPs: one has a small number of CPs (equivalently parking spaces) but with a higher power capacity.

$N_1^{\text{cp}} = 10$ $N_2^{\text{cp}} = 4$	Number of parking spaces at each CSP.
$p_1^{\text{max}} = 11$ $p_2^{\text{max}} = 22$	Maximum power at each CP (in kW).
$\alpha_1 = 0.6$ $\alpha_2 = 0.6$	Maximum power at each CP (in kW).

Table 3.14: Parameter values of the two CSPs.

3.3.4.1 Equilibrium analysis

The equilibrium using the exact expected energy received, computed from the simulations, is shown to compare it with the approximation proposed in the model. This equilibrium is denoted by θ_{eq} . In order to compute the equilibrium (a probability with which each EV joins each CSP) from the simulations, a verification is performed on whether the equilibrium is inner ($\theta_{\text{eq}} \in]0, 1[$) or not. If the equilibrium is inner, a dichotomy algorithm is used to find the proportion of EV users choosing each CSP such that the expected energy received at both CSPs is equal. As a stopping criteria of the dichotomy algorithm, a difference of 0.01 kWh between the expected energy received at the CSPs is considered. This condition is met in about 10 iterations.

Figure 3.15 shows the exact equilibrium θ_{eq} (green curve), and the equilibrium $\tilde{\theta}_{\text{eq}}$ (blue curve), see Definition 3.5, of the game $\tilde{\Gamma}$, i.e, considering the theoretical approximation on the expected energy received, with respect to the average parking duration. Recall that the EV users parking duration is exogenous in the scenario.

For low values of the average parking duration, all EV users choose to go to CSP 2 because the blocking probability is negligible and they can charge at a higher power than at CSP 1. However, when the average parking duration increases, the blocking probability at CSP 2 increases as well, so that some EV users choose CSP 1 instead of CSP 2.

In Figures 3.16, 3.17 and 3.18, the exact metrics (green curves) computed through the simulations at the exact equilibrium θ_{eq} , as well as the theoretical approximations (blue curves) at the approximate equilibrium $\tilde{\theta}_{\text{eq}}$, are shown with respect to the average parking duration ν^{-1} . According to the figures, the approximations are very accurate. For instance, the approx-

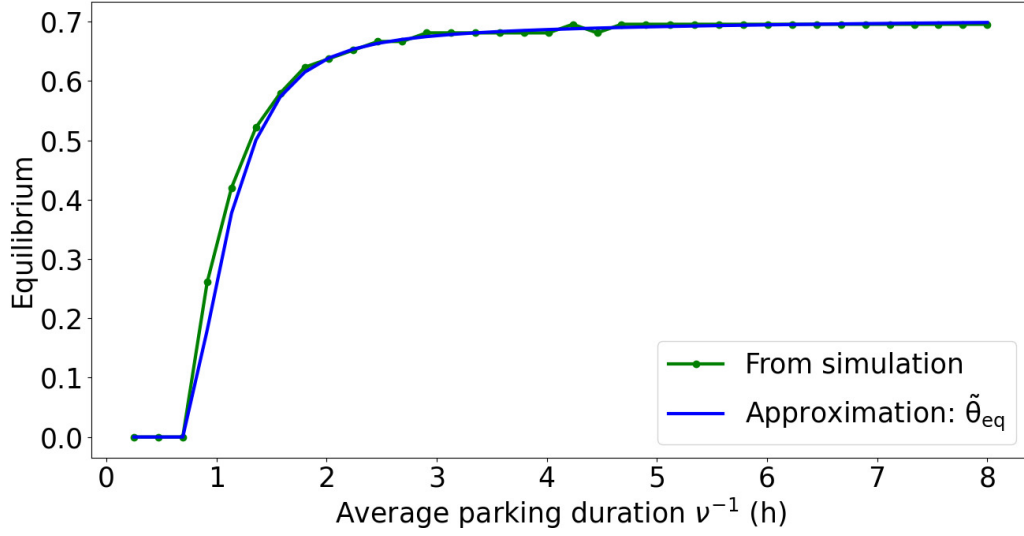


Figure 3.15: Equilibrium with respect to the average parking duration ν^{-1} . A higher proportion of EV users tends to choose CSP 1 when the average parking duration increases: the probability to be rejected at CSP 2 is then very high due to a lack of parking spaces.

imate energy received at equilibrium $\tilde{\theta}_{eq}$ differs by about 5% from the exact energy received at equilibrium θ_{eq} and for an average parking duration of 2 hours.

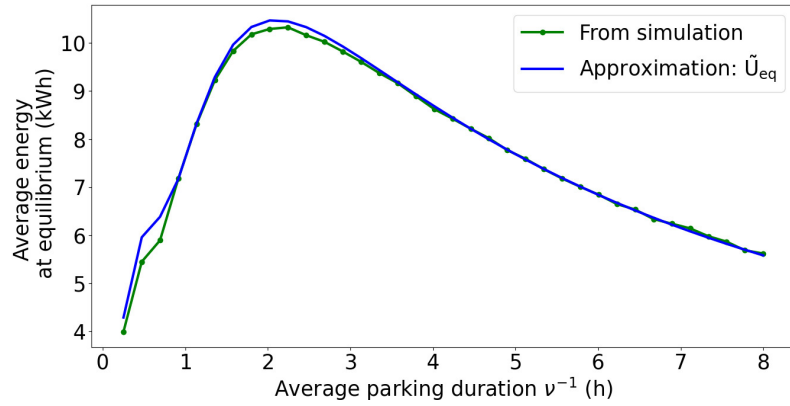


Figure 3.16: Average energy received per EV at equilibrium with respect to the average parking duration. An optimal trade-off in term of average parking duration is observed (about 2h in our use-case). In order to maximize the energy delivered, users should have a sufficient length of time (i.e., parking duration) to obtain a consequent energy, but at the same time a low probability to be rejected.

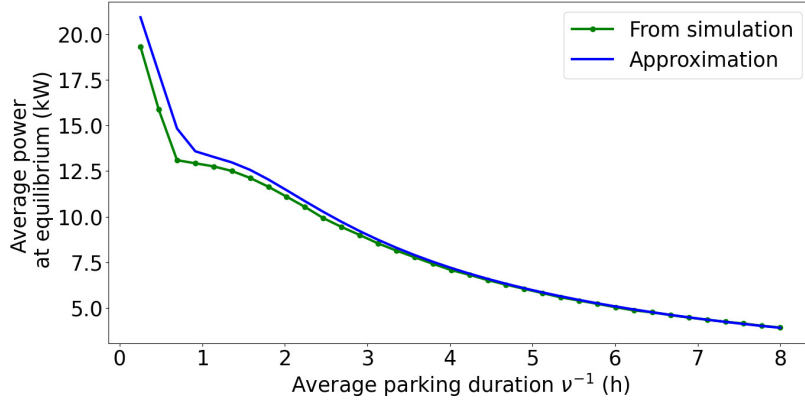


Figure 3.17: Average power received per EV in average and at equilibrium with respect to the average parking duration. *With an increasing average parking duration, the average power received by an EV decreases although the average power received by an accepted EVs increases.*

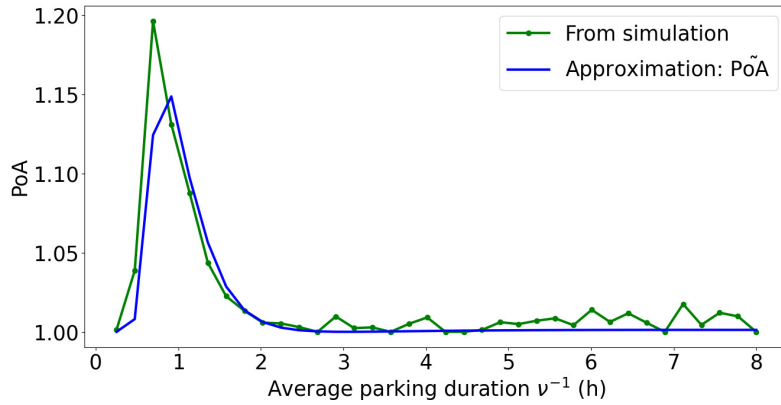


Figure 3.18: Price of anarchy with respect to the average parking duration. *When the average parking duration is less than 2 hours, in proportion, the EV users tend to overuse CSP 2 when they act individually, which lowers the quality of charging service.*

3.3.4.2 Comparison between the centralized and the decentralized system through the price of anarchy

As explained in Section 3.3.3, the PoA enables to measure the inefficiency of the equilibrium, i.e., when EVs individually choose the best CSPs for themselves, in comparison of the optimal EV distribution, i.e., when a centralized scheduler choose the CSP for the EV users in order to maximize the average energy received per EV. Recall that the PoA is defined as the ratio between the expected energy received at optimum and at equilibrium, and is defined by Equation (3.21). Figure 3.18 shows the PoA computed with the approximations with respect to the average parking duration ν^{-1} .

In general, the proportion of EV users choosing CSP 2 is higher than that at optimum. Indeed, the maximum power drawn at CSP 2 (22 kW) is greater than that at CSP 1 (11 kW). When choosing CSP 2, EV users create negative externalities by increasing the probability for the next

arriving EVs to be blocked from CSP 2, which has only 4 parking spaces.

The equilibrium becomes more inefficient when one CSP is more sensitive to congestion than the other. When parking duration is very short, neither CSP is significantly affected by the proportion of users choosing them, as congestion remains low. Conversely, when the average parking duration is very high, both CSPs become highly sensitive to user distribution due to the increased blocking probability. However, when the parking duration is around 1 hour, CSP 2 is more sensitive to congestion than CSP 1 because of its high blocking probability (only 4 parking spaces). As a result, in this case, the equilibrium is approximately 20% less efficient than the optimum, as shown in Figure 3.18.

The fluctuations in the green curve result from the randomness of user arrivals, energy needs, and parking durations. These effects would theoretically diminish with a larger number of simulations.

3.4 Conclusion

This chapter deals with a decision-making problem in which EV users choose to join one of two charging pools (CSPs) with finite capacity in terms of number of Charging Points (CPs). Congestion effects occur at two levels: access to the CSP and share of the limited available power at the CSP. Therefore, a loss queueing game framework is studied in which the players are the EV users. Each EV user chooses the CSP to join by maximizing its expected energy received during its parking duration. The latter does not depend on the charging time. In this work, the symmetric mixed-strategy equilibrium is unique. Moreover, characterizations as well as closed form expressions for some parameter values on the utility at equilibrium are given. Finally, theoretical bounds on the price of anarchy are obtained. Theoretical approximations on the expected energy received and power levels drawn from the CPs are validated through numerical simulations for realistic parameter values.

This study enables one to predict the performance of the charging service such as the average energy received in a competitive scenario. The equilibrium analysis can help a charging point operator that aims to optimize the design of a CSP infrastructure in terms of the maximum power available at the CSP level or the number of installed CPs. It also shows the usefulness of encouraging EV users to reduce their parking duration to provide a better overall quality of charging service.

Future directions include incorporating heterogeneity in users' utility functions, similarly to [Du and Gong, 2016] in the context of conventional parking lots. For example, some users may prefer a CSP closer to their on-site activities, which can vary from one user to another. Moreover, an extension of this work for more than two CSPs could generalize the results obtained. Another perspective is to consider users' bounded rationality, reflecting biases or imperfect knowledge about CSPs [Nakayama et al., 2001]. These aspects will be explored further in Chapter 4, in the context of fast charging modes instead of standard charging.

Chapter 4

A lossless queueing game for fast charging with bounded rational users

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4.1 Introduction

This chapter focuses on CSPs with fast charging mode, where EV users need in energy is indispensable. In such setting, congestion occurs through the waiting time at the CSPs. The potential waiting time at a CSP is influenced by the choice of the other EV users. For example, if all the CPs are occupied at the time an EV user arrives, it creates some waiting time which degrades the EV users satisfaction.

Similarly to the previous Chapter 3, a solution concept is to find some type of equilibrium, where EV users choice selection are such that no EV user has interest in changing its choice. Some studies suggests real-time recommendation for the CSP choice selection by the EV users [Malandrino et al., 2015, Guo et al., 2017], which enables to account for time-coupling. The

Notation	Signification
$\mathcal{I}$	Set of CSPs
N_i^{cp}	Number of CPs at CSP i
p_i^{max}	Charging power rate capacity per CP at CSP i (in kW)
p_i^{agg}	Aggregated power capacity at CSP i (in kW)
x_i	Position of CSP i
f_{ev}	Distribution of EV users population
λ	EV charging demand frequency (in h^{-1})
γ	EV users rationality parameter
Ω_x	The map of user origins/destinations and of CSPs
Ω_c	Set of EV charging features
x	EV origin-destination
c	EV charging features
$\mathcal{C}$	EV users cost function (output values are in €)
$\tilde{\mathcal{C}}$	EV users perceived cost function (output values are in €)
λ_i	Arrival frequency at CSP i
Ψ	Approximated expected waiting time function (in hour)
π	EV users choice profile

Table 4.1: Main parameter notations used in this chapter. *Red*: CSP parameters; *Blue*: EV users parameters.

idea is to provide fairness to the EV users because the recommendations are such that all EV users have interest in following the recommendation. Some other studies focus on a two-level interaction, with EV users making the CSP choice selection at the lower level, and CSO making the choice of prices of infrastructure at the upper level [Yuan et al., 2015, Xu et al., 2017, Zhou et al., 2021, Zavvos et al., 2022]. Among these studies, except for [Zhou et al., 2021], EV users are assumed to be perfectly rational, and do not have any bias in their choice. However, in practice drivers may have cognitive limitation and bias, and are heterogeneous for example in their value of time, as shown in [Nakayama et al., 2001]. Therefore, the perfect rationality assumption may create a gap between the models results and EV users actual decisions. Moreover, none of these studies incorporates the EV users differentiated in their energy need, which has an influence on their choice. For example, intuitively EV users with higher energy need may select CSPs with higher charging rates. This consideration makes notable changes because the energy need among EV users selecting a given CSP affects their charging duration, which has an impact on the waiting time of future EV users arrivals.

This chapter focuses on the forecast of EV users choice of CSP among a set of CSP, with given charging characteristics, including the power rates and number of CPs, and prices. Figure 4.2 illustrates the scenario considered. In a short-term perspective, the charging price is generally not influenced by the EV users choice of CSP. In this chapter, prices are static: congestion-based pricing strategies are not considered. Considering that, at a particular instant, EV users with a charging need represents a small fraction of the drivers on the road, the traffic can be considered independent of these EV users choice of CSP, as in [Yuan et al., 2015].

Section 4.2 presents the mathematical model of the EV users choices criteria. Section 4.3 presents the formulation of the interaction between the EV users. Section 4.4 shows some numerical illustrations, and Section 4.5 draws a conclusion of this work.

In this chapter, the mathematical notions and notation of Definition 4.1 are used.

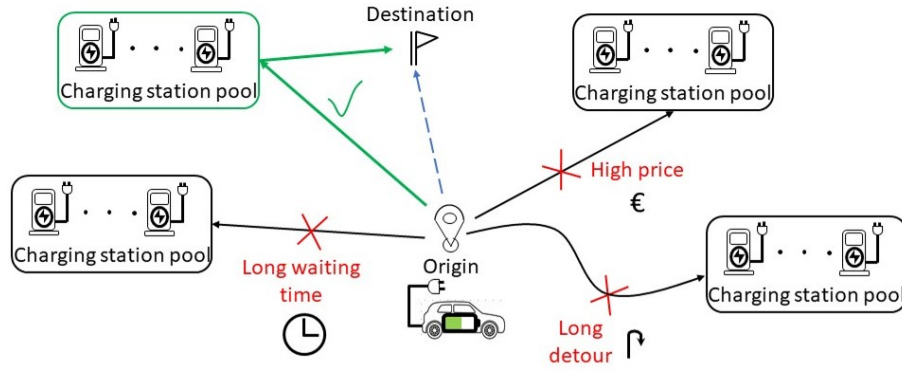


Figure 4.2: Illustrative diagram of the scenario and users choice criteria.

Definition 4.1 (Notations). Let us define the following sets and functions.

- The set $\mathcal{S}^{|\mathcal{I}|}$:

$$\mathcal{S}^{|\mathcal{I}|} := \{y \in \mathbb{R}_+^{|\mathcal{I}|} \mid \sum_{i=1}^{|\mathcal{I}|} y_i = 1\},$$

represents the set of probability distribution over the set $\mathcal{I}$ of CSPs, of size $|\mathcal{I}|$, and will be used in particular for representing the set of EV users stochastic choices.

- The set $\mathcal{F}(A, B)$ of functions from A to B .
- The set $\mathcal{C}^k(A, B)$ ($\in \mathcal{F}(A, B)$) of functions from $(A, \|\cdot\|_A)$ to $(B, \|\cdot\|_B)$ that are k ($\in \mathbb{N}$) times continuously differentiable. In particular, $\mathcal{C}^0(A, B)$ represents the set of continuous functions from $(A, \|\cdot\|_A)$ to $(B, \|\cdot\|_B)$.¹
- $\text{Lip}_L(A, B) \in \mathcal{F}(A, B)$ is the set of function from A to B that are L -Lipschitz:

$$\forall a_1, a_2 \in A, h \in \text{Lip}_L(A, B), \quad \|h(a_2) - h(a_1)\|_B \leq L \|a_2 - a_1\|_A,$$

meaning that the increase/decrease of a function between two point is not too abrupt.

- The L^2 norm $\|\cdot\|_2$ on $\mathbb{R}^n$:

$$\forall y \in \mathbb{R}^n, \quad \|y\|_2 = \left(\sum_{i=1}^n y_i^2 \right)^{\frac{1}{2}},$$

which represents the conventional length.

- The L^∞ norm $\|\cdot\|_\infty$ on $\mathcal{F}(A, \mathbb{R}^n)$:

$$\forall f \in \mathcal{F}(A, \mathbb{R}^n), \quad \|f\|_\infty = \sup_{y \in A} \max_{i \in \llbracket 1, n \rrbracket} |f_i(y)|,$$

with $|\cdot|$ the absolute value, and $\llbracket 1, n \rrbracket := [1, \dots, n]$ the set of integers from 1 to n .

¹ A set $(A, \|\cdot\|_A)$ is sometimes written A for short, when the associated metric is implicit.

4.2 Model preliminaries

4.2.1 Scenario of EV users charging demand

Charging stations Let Ω_x be a bounded domain in $\mathbb{R}^2$, in which there is a finite set $\mathcal{I}$ of CSPs. Each CSP $i \in \mathcal{I}$ is located at position $x_i \in \Omega_x$, and equipped with a set of CPs delivering the same nominal charging power $p_i^{\max}$. Considering the aggregated charging power capacity p_i^{agg} at each CSP i , there are:

$$N_i^{\text{cp}} := \left\lfloor \frac{p_i^{\text{agg}}}{p_i^{\max}} \right\rfloor,$$

CPs at each CSPs that can simultaneously charge an EV at the maximum power $p_i^{\max}$. For simplicity, it is assumed that p_i^{agg} is a multiple of the nominal charging power $p_i^{\max}$ of the CPs.² Moreover, the charging scheduling considered here is First In First Out (FIFO). As a consequence, only N_i^{cp} EVs can charge simultaneously, rendering potential additional CPs ineffective, as shown in Figure 4.3 on an illustrative example. It is therefore considered that there are only N_i^{cp} CPs at CSP i .

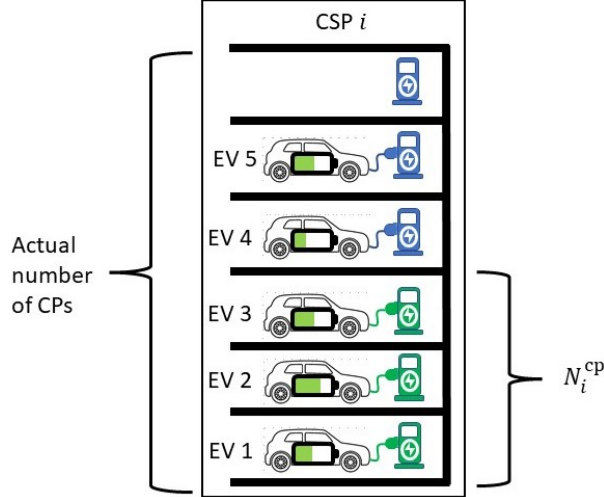


Figure 4.3: Illustrative scheme of the FIFO scheduling. In this example, there are 6 CPs at CSP i . However, due to the aggregated power limitation, only 3 CPs can charge simultaneously at the maximum power $p_i^{\max}$. Thus only the 3 first EV arrived (EV 1, 2 and 3) can charge. The others (EV 4 and 5) are waiting. The EV users leave as soon as their required energy is reached, and the fourth EV will start to charge straight away, etc.

EV users At random instants, some EV users in the domain Ω_x aim to charge at one of the $|\mathcal{I}|$ CSPs, and are characterized by their initial position, their destination, their energy need and maximum charging power capacity.³ More precisely, the EV users charging demand time follows a homogeneous Poisson process of parameter λ (homogeneous account for the fact that the frequency of arrivals is constant over time). The EV users trip features are denoted $x := (x_o, x_d)$,

² Otherwise, given the FIFO power dispatch, when there are $N_i^{\text{cp}} + 1$ EVs charging at CSP i , the last arrived EV can not charge at the maximum power $p_i^{\max}$.

³ The charging power actually delivered can be lower than the nominal charging power of the CP due to EV battery physical constraints.

compound on an origin location x_o and destination x_d , and are distributed according to a joint probability distribution $f_x(\cdot)$ on Ω_x^2 . Finally, the charging features $c = (c_e, c_p)$, with c_e the energy need and c_p the maximum charging power of the EV, are distributed according to a probability distribution $f_c(\cdot)$ on a bounded set $\Omega_c \subset \mathbb{R}_+^2$.^{4,5} The joint distribution of all the EV users features, namely their trip and charging features, are drawn from a joint distribution $f_{ev}(\cdot)$ on $\Omega_x^2 \times \Omega_c$.

All these characteristics are random from the point of view of an outside analysis, but are known by the EV users themselves. Each EV user, at the time of appearance, chooses a CSP where to charge, depending for example on the position of the CSPs and the waiting time at the CSPs before starting to charge, which will be developed below.

4.2.2 EV users choice of charging station pool: a non-atomic congestion game

As the number of CPs at each CSP may be insufficient compared to the charging demand (as explained in the previous section), EV users have to wait before starting to charge if all the N_i^{cp} CPs are currently being occupied upon arrival. As a consequence, the waiting time at each CSP for a given EV user depends especially on the "number" of EV users going to each CSP, i.e. their choice of CSP. Moreover, with the expected future huge number of EV users on the road [IEA, 2024], the influence of a single EV users on the waiting time of all the other EV users is negligible. In this type of situation, non-atomic games [Aumann and Shapley, 2015] is an appropriate mathematical framework. Without loss of generality, the set $\Omega_x^2 \times \Omega_c$ of EV users⁶ is of mass 1:

$$\int_{\Omega_x^2} \int_{\Omega_c} f_{ev}(dx, dc) = 1.$$

Cost function

The cost function $\mathcal{C}_{(x,c)}(i, \pi)$ of any EV user $(x, c) \in \Omega_x^2 \times \Omega_c$ incorporates: (i) the **travel cost**, corresponding to the detour for going to the chosen CSP; (ii) the **sojourn cost**, composed of the eventual *waiting duration* before starting to charge plus the *charging duration*; (iii) and the **charging cost** which depends on the per energy unit charging price at the CSPs. In the following, the terminologies waiting time *vs* waiting duration, and service time *vs* charging time *vs* charging duration have the same meaning and are used interchangeably. Similarly to [Zhou et al., 2021], an additional **noise** is incorporated in the EV users perceived cost function, modeling the imperfect rationality of the EV users (or non-observable factors from an outside analysis) [Train, 2009]. Thus, from a system perspective, each EV user (x, c) chooses a CSP i with a probability $\pi_{(x,c)}(i) \in [0, 1]$. The total perceived cost function $\tilde{\mathcal{C}}_{(x,c)}(\cdot)$ of an EV user (x, c) depends on her choice, i.e. a CSP i , and the choice probability π of all the other EV users:

$$\forall i \in \mathcal{I}, \forall \pi \in \mathcal{F}(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}), \quad \tilde{\mathcal{C}}_{(x,c)}(i, \pi) = \gamma \mathcal{C}_{(x,c)}(i, \pi) + \varepsilon_i, \quad (4.1)$$

with ε_i a realization of a normalized Gumbel random variable, and $\gamma > 0$ representing the EV users rationality level, such that $\{\varepsilon_i, i \in \mathcal{I}\}$ is *i.i.d.*. The *i.i.d.* condition of the noise means that the positive or negative bias at a given CSP is independent from the bias at the other CSPs.

⁴ The bound for the energy need corresponds to the maximum battery storage capacity of any EV, about 125 kWh, see for example [ev-database](#) for a list of EVs with their respective battery storage capacities.

⁵ Other charging characteristics could be considered without affecting the model and associated results, such as the initial state of charge and battery storage capacity, which has an influence on the charging rate in practice.

⁶ By language abuse, $\Omega_x^2 \times \Omega_c$ denotes both the set of EV users characteristics, and the set of EV users, which is common in the non-atomic games literature [Wan, 2013].

The Gumbel distribution is a standard assumption for modeling the users' bounded rationality and corresponds to the logit model [Train, 2009].⁷

4.2.3 Occupancy level at the charging pools: a queueing system perspective

Arrival process at the charging station pools

Given a choice profile $\boldsymbol{\pi}$, the probability $\theta_i(\boldsymbol{\pi})$ that an EV user chooses CSP i is:

$$\theta_i(\boldsymbol{\pi}) := \int_{\Omega_c} \int_{\Omega_x^2} \pi_{(x,c)}(i) f_{\text{ev}}(dx, dc), \quad (4.2)$$

with $\pi_{(x,c)}$ the strategy of player (x, c) in the choice profile $\boldsymbol{\pi}$. Therefore, the arrival times of the EV users at CSP i follow a Poisson process of parameter $\lambda_i(\boldsymbol{\pi}) := \lambda \times \theta_i(\boldsymbol{\pi})$.

Service time

Let $T_i^S[\boldsymbol{\pi}]$ be the charging duration of an EV user randomly taken at CSP i , and, for any charging characteristics $c := (c_e, c_p)$, $\Phi(c, i)$ be the charging duration at CSP i . Assuming for simplicity constant power rate during the charging session, the charging duration is the required energy over the charging power:⁸

$$\forall c = (c_e, c_p), \quad \Phi(c, i) := \frac{c_e}{\min(p_i^{\max}, c_p)}.$$

The mean $\mathbb{E}[T_i^S[\boldsymbol{\pi}]]$ and square mean charging duration $\mathbb{E}[(T_i^S[\boldsymbol{\pi}])^2]$ at CSP i depend on the choice profile $\boldsymbol{\pi}$ of the EV users population through:

$$\begin{aligned} \mathbb{E}[T_i^S[\boldsymbol{\pi}]] &= \begin{cases} \frac{1}{\theta_i(\boldsymbol{\pi})} \int_{\Omega_c} \int_{\Omega_x^2} \Phi(c, i) \pi_{(x,c)}(i) f_{\text{ev}}(dx, dc) & \text{if } \theta_i(\boldsymbol{\pi}) > 0 \\ 0 & \text{else,} \end{cases} \\ \mathbb{E}[(T_i^S[\boldsymbol{\pi}])^2] &= \begin{cases} \frac{1}{\theta_i(\boldsymbol{\pi})} \int_{\Omega_c} \int_{\Omega_x^2} \Phi^2(c, i) \pi_{(x,c)}(i) f_{\text{ev}}(dx, dc) & \text{if } \theta_i(\boldsymbol{\pi}) > 0 \\ 0 & \text{else.} \end{cases} \end{aligned} \quad (4.3)$$

Waiting time

Each EV user is assumed to leave as soon as the required energy c_e is fulfilled. As a consequence, the number of EV users at each CSP i follows a M/G/ N_i^{cp} queue, where G stands for a General charging duration distribution. There does not exist any analytical expression of the probability distribution of the waiting time $T_i^W[\boldsymbol{\pi}]$ of a M/G/ N_i^{cp} queue in stationary regime [Kingman, 2009]. However, there exists an approximation $\Psi(i, \boldsymbol{\pi}) \approx \mathbb{E}[T_i^W[\boldsymbol{\pi}]]$ for the mean waiting time in stationary regime [Nozaki and Ross, 1978], given by:⁹

$$\Psi(i, \boldsymbol{\pi}) = \begin{cases} \frac{\rho_i(\boldsymbol{\pi})^{N_i^{\text{cp}}} \mathbb{E}[(T_i^S[\boldsymbol{\pi}])^2]}{2 \mathbb{E}[T_i^S[\boldsymbol{\pi}]] (N_i^{\text{cp}} - \rho_i(\boldsymbol{\pi}))^2 \times \Psi_0(i, \boldsymbol{\pi})} & \text{if } \rho_i(\boldsymbol{\pi}) < N_i^{\text{cp}} \\ +\infty & \text{else,} \end{cases} \quad (4.4)$$

⁷ Although the results extend to other distributions (as long as it is continuous *w.r.t.* the Lebesgue measure), the explicit expression of Equation (4.11) below provides analytical tractability, simplifying the proofs.

⁸ It can be generalized to any continuous function with respect to the charging features c . In practice, the charging power rate decreases when the state of charge approaches the fully charged battery [Wei et al., 2018].

⁹ This approximation is exact in the special case of an exponential charging duration distribution.

with $\rho_i(\pi) := \lambda_i(\pi) \mathbb{E}[T_i^S[\pi]]$ the utilization rate of CSP i , and

$$\Psi_0(i, \pi) := (N_i^{\text{cp}} - 1)! \times \sum_{k=0}^{N_i^{\text{cp}}-1} \frac{\rho_i(\pi)^k}{k!} + \frac{\rho_i(\pi)^{N_i^{\text{cp}}}}{(N_i^{\text{cp}} - 1)!(N_i^{\text{cp}} - \rho_i(\pi))}$$

a factor related to the probability that an arriving EV user has to wait in the case of an exponential distribution of the service time at CSP i (in a $M/M/N_i^{\text{cp}}$ queue). Figure 4.4 shows some waiting time values with respect to the frequency of arrivals at a given CSP. If the utilization rate is too high (i.e. $\geq N_i^{\text{cp}}$), the expected waiting time will eventually become arbitrary large.

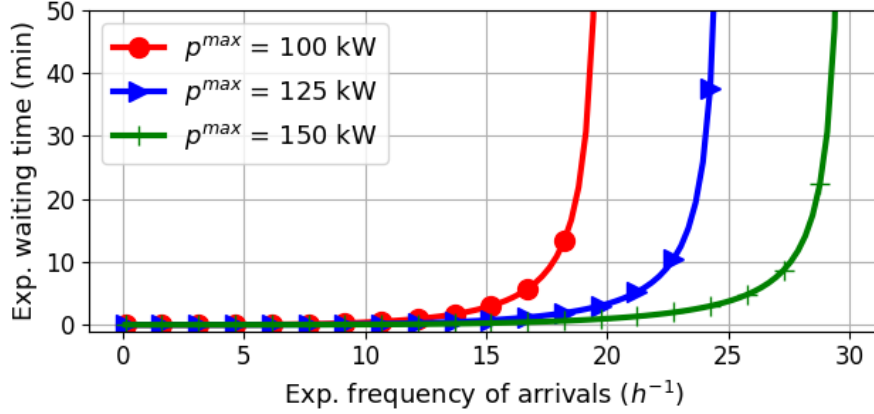


Figure 4.4: Example of approximated expected waiting time, given by Equation (4.4), at a charging station pool i with respect to the expected frequency of arrivals λ_i . $N_i^{\text{cp}} = 5$ CPs at the CSP, and expected energy need of 25 kWh. The mean waiting duration is very close to 0 when the utilization rate $\rho_i(\cdot)$ is significantly lower than the number N_i^{cp} of CPs. Conversely, the mean charging duration $\Psi(i, \cdot)$ explodes to $+\infty$ when the utilization rate $\rho_i(\cdot)$ approaches the number N_i^{cp} of CPs, here for $\lambda_i(\cdot)$ approaching 20, 25, and 30 h^{-1} for respectively a p_i^{max} of 100, 125, and 150 kW.

4.2.4 The cost function of the EV users

The travel cost

For each EV user with origin-destination $x := (x_o, x_d)$, the additional distance covered for going to CSP i is measured by the function $d : \Omega_x^2 \times \mathcal{I} : \mathbb{R}_+$.¹⁰

$$d(x_o, x_d, i) = \|x_o - x_i\|_2 + \|x_d - x_i\|_2 - \|x_o - x_d\|_2. \quad (4.5)$$

Note that the distance $d(\cdot)$ is positive because $\|x_o - x_d\|_2$ is the minimal distance between the origin x_o and the destination x_d .

The travel cost $C^T(\cdot)$ is the dissatisfaction, measured in €, associated with the additional distance $d(\cdot)$:

$$C^T(x_o, x_d, i) = (a_d + \frac{a_t}{d^{\text{speed}}}) d(x_o, x_d, i),$$

¹⁰The distance function $d(\cdot)$ can be generalized to any function, for example incorporating the network of roads, or the traffic jam, as long as it is continuous in x , and exogenous (independent from congestion).

with a_d (in €/km) and a_t (€/h) real positive coefficients, representing the monetary equivalence respectively to the energy consumption and time [Sanders et al., 2015], and $\bar{d}^{\text{speed}}$ the average driving speed on the map Ω_x . In the following, $C^T(x_o, x_d, i)$ is written $C^T(x, i)$ for short, except when it is necessary to specify the influence of x_o or x_d .

The sojourn cost

The sojourn cost at a CSP is the total duration spent at this CSP, and incorporates both the waiting duration and the charging duration. The sojourn cost $C^S(\cdot)$ of an EV user with charging characteristics c is:

$$C^S(c, i, \pi) := a_t (T_i^W[\pi] + \Phi(c, i)). \quad (4.6)$$

The charging cost

For an energy charged e (kWh), and per-unit charging price a_i^{ch} , the charging cost incurred by an EV user is:¹¹

$$C^C(c_e, i) := e \times a_i^{\text{ch}}. \quad (4.7)$$

Total cost

For an EV user (x, c) , the total cost $C(x, c, i)$ incurred for going to CSP i is a weighted sum of the *travel cost*, the *sojourn cost* and the *charging cost*. Let $C : \Omega_x^2 \times \Omega_c \times \mathcal{I}$ be the random cost function in stationary regime incurred by an EV user, defined by:

$$\begin{aligned} \forall x \in \Omega_x^2, \forall c \in \Omega_c, \forall i \in \mathcal{I}, \forall \pi \in \mathcal{F}(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}), \\ C(x, c, i, \pi) := C^T(x, i) + a_t (T_i^W[\pi] + \Phi(c, i)) + C^C(c_e, i). \end{aligned} \quad (4.8)$$

Upon making the choice of the CSP, the EV users do not have information about the current CSP occupation levels, and thus the potential waiting time at the CSPs. Without this information, their estimation of the occupation level at the different CSPs is made at the stationary regime of the system. In addition, their cost C being random (because of the random waiting duration $T_i^W[\pi]$), the EV user minimizes their (approximated) expected cost function. Thus, the total cost function of an EV user (x, c) is:

$$\begin{aligned} \forall i \in \mathcal{I}, \forall \pi \in \mathcal{F}(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}), \\ \mathcal{C}_{(x,c)}(i, \pi) := C^T(x, i) + a_t (\Psi(i, \pi) + \Phi(c, i)) + C^C(c_e, i), \end{aligned} \quad (4.9)$$

with $\Psi(i, \pi)$ the approximated expected waiting time at CSP i , given by Equation (4.4).

Remark 4.2. *The interaction between the EV users lies in the expected waiting duration. The other metrics depend only on one's own characteristics.*

4.3 Equilibrium with bounded rationality

4.3.1 Definition of an equilibrium with bounded rational users

A Wardrop equilibrium is a profile of choice of CSP such that no EV users has any incentive to deviate from its choice [Correa and Stier-Moses, 2011]. Thus, in our setting where EV users

¹¹This can be generalized to other CSO pricing strategies that are Lipschitz continuous with respect to the energy need c_e , such as a piece-wise linear pricing in the energy, or an additional admission fee.

have a bounded rationality, an equilibrium π^* is a profile of probability distribution over the set of CSPs [Zhou et al., 2021]:

$$\forall (x, c) \in \Omega_x^2 \times \Omega_c, \forall i \in \mathcal{I}, \quad \pi_{(x,c)}^*(i) := \mathbb{P}(\tilde{\mathcal{C}}_{(x,c)}(i, \pi) = \min_{j \in \mathcal{I}} \tilde{\mathcal{C}}_{(x,c)}(j, \pi)). \quad (4.10)$$

Given the Gumbel distribution (see Equation (4.1)) and Equation (4.10), Definition 4.3 provides a definition of an equilibrium in our use-case.

Definition 4.3 (Stochastic equilibrium [Sheffi, 1985]). *An equilibrium π^* is a profile of probability distribution over the set $\mathcal{I}$ of CSPs, such that:*

$$\forall (x, c) \in \Omega_x^2 \times \Omega_c, \quad \forall i \in \mathcal{I}, \quad \pi_{(x,c)}^*(i) = \frac{e^{-\gamma \mathcal{C}_{(x,c)}(i, \pi^*)}}{\sum_{j \in \mathcal{I}} e^{-\gamma \mathcal{C}_{(x,c)}(j, \pi^*)}}. \quad (4.11)$$

The approximated waiting time at the CSPs can be deduced from the equilibrium given in Definition 4.3. Given such waiting time, one can deduce a corresponding Wardrop equilibrium in pure strategy, that is the deterministic choice by each EV user (x, c) and noise ϵ to a CSP, by selecting the CSP minimizing their perceived cost given by Equation (4.1).

4.3.2 Existence of an stochastic equilibrium

Assumption 4.4 (Queueing system not overloaded). *Let $\Phi^{\max} := \max_{\substack{c \in \Omega_c \\ i \in \mathcal{I}}} \Phi(c, i)$ be the maximum possible service time at any CSP. It is assumed that the following condition holds.¹²*

$$\lambda \times \Phi^{\max} < \sum_{i \in \mathcal{I}} N_i^{\text{cp}}. \quad (4.12)$$

Assumption 4.4 states that there are enough CPs in total in the system such that the waiting time at the CSP does not explodes to infinity, as illustrated in Figure 4.4, and is a standard assumption in the queueing literature [Adan and Boucherie, 2015]. The following Lemma 4.5 states that for any distribution of choices made by the EV users, there exists at least one CSP which is not overloaded, i.e. the waiting time is not infinite.

Lemma 4.5. *There exists $K > 0$ such that:*

$$\sup_{\pi \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})} \min_{i \in \mathcal{I}} \Psi(i, \pi) < K.$$

The idea of the proof is to show that if none of the waiting time are upper bounded, there exists some probability distribution over the set of CSPs such that at least one CSP is overloaded, which contradicts Assumption 4.4.

Proof. By contradiction. Assume Lemma 4.5 is False. Therefore, there exists a sequence $(\pi^n)_{n \in \mathbb{N}}$ of choice profiles in $\mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})$, such that:

$$\lim_{n \rightarrow +\infty} \min_{i \in \mathcal{I}} \Psi(i, \pi^n) = +\infty.$$

¹²One could consider a less restrictive assumption by considering the expected, maximum charging duration over all the CSP, over the EV charging characteristics.

Using the expression of $\Psi(i, \pi^n)$, given in Equation (4.4), and that the mean service time is bounded by $\Phi^{\max}$, one has that at the limit over n , any CSP i is overloaded:

$$\forall i \in \mathcal{I}, \quad \lim_{n \rightarrow +\infty} \lambda \times \int_{\Omega_x^2} \int_{\Omega_c} \Phi(c, i) \pi_{(x,c)}^n(i) f_{\text{ev}}(dx, dc) \geq N_i^{\text{cp}}.$$

Summing over the set of CSPs (a finite sum), one has:

$$\sum_{i \in \mathcal{I}} \lim_{n \rightarrow +\infty} \lambda \times \int_{\Omega_x^2} \int_{\Omega_c} \Phi(c, i) \pi_{(x,c)}^n(i) f_{\text{ev}}(dx, dc) \geq \sum_{i \in \mathcal{I}} N_i^{\text{cp}}. \quad (4.13)$$

The following gives an upper bound of the limit over n :

$$\begin{aligned} & \sum_{i \in \mathcal{I}} \lim_{n \rightarrow +\infty} \lambda \times \int_{\Omega_x^2} \int_{\Omega_c} \Phi(c, i) \pi_{(x,c)}^n(i) f_{\text{ev}}(dx, dc) \\ & \leq \lambda \times \sum_{i \in \mathcal{I}} \lim_{n \rightarrow +\infty} \int_{\Omega_x^2} \int_{\Omega_c} \Phi^{\max} \pi_{(x,c)}^n(i) f_{\text{ev}}(dx, dc) \\ & \leq \lambda \times \Phi^{\max} \lim_{n \rightarrow +\infty} \int_{\Omega_x^2} \int_{\Omega_c} \sum_{i \in \mathcal{I}} \pi_{(x,c)}^n(i) f_{\text{ev}}(dx, dc) \\ & = \lambda \times \Phi^{\max} \lim_{n \rightarrow +\infty} \int_{\Omega_x^2} \int_{\Omega_c} 1 f_{\text{ev}}(dx, dc) \\ & = \lambda \times \Phi^{\max}, \end{aligned}$$

using the fact that $\sum_{i \in \mathcal{I}} \pi_{(x,c)}^n(i) = 1$ because $\pi_{(x,c)}^n \in \mathcal{S}^{|\mathcal{I}|}$.

Using the previous inequality and Equation (4.13), it gives:

$$\lambda \times \Phi^{\max} \geq \sum_{i \in \mathcal{I}} N_i^{\text{cp}},$$

which contradicts Assumption 4.4. $\square$

In the following, it is assumed, for mathematical technicality, that the distribution $f_{\text{ev}}(\cdot)$ of EV users is continuous with respect to the Lebesgue measure over the set $\Omega_x^2 \times \Omega_c$. However, all the following results holds in the case when $f_{\text{ev}}(\cdot)$ is discrete or semi-discrete distribution, with proofs following similar arguments. In practice, the set of energy needs is continuous, as it can be any value between 0 and the maximum possible battery storage capacity. The set of maximum charging power c_p is by nature discrete, as there exists a finite number of EV types.

Now, let us define the following function $g[\cdot] : \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}) \rightarrow \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})$, such that:

$$\forall (x, c) \in \Omega_x^2 \times \Omega_c, \quad \forall i \in \mathcal{I}, \quad g[\pi]_i(x, c) := \frac{e^{-\gamma \times (C^T(x, i) + a_t(\Psi(i, \pi) + \Phi(c, i)) + C^C(c_e, i))}}{\sum_{j \in \mathcal{I}} e^{-\gamma \times (C^T(x, j) + a_t(\Psi(j, \pi) + \Phi(c, j)) + C^C(c_e, j))}}. \quad (4.14)$$

The well definition of the function $g[\cdot]$ is shown in in Lemma 4.6 and relies on Lemma 4.5. The function $g[\cdot]$ plays an important role in the demonstration of the existence of a Stochastic Equilibrium (SE). Given a choice profile π , a waiting time vector $\Psi(i, \pi)$ is induced for each i . $g[\pi]$ gives the optimal choice profile of EV users in response to the waiting time vector $\Psi(i, \pi)$. Intuitively, a SE is a stable choice profile, that is, such that the choice profile induced by the

waiting time vector $\Psi(i, \pi)$ is π again. Mathematically, the definition of a SE in the sense of Definition 4.3 is equivalent to finding the following fixed point equation:

$$\begin{cases} g[\pi] = \pi \\ \pi \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}). \end{cases} \quad (4.15)$$

Lemma 4.6. *For all $\pi \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})$, the function $g[\pi](\cdot)$ is well defined.*

Proof. By Lemma 4.5, there exists $K > 0$ such that for any $\pi \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})$,

$$\sup_{j \in \mathcal{I}} e^{-\gamma a_t \Psi(j, \pi)} \geq e^{-\gamma a_t K}.$$

Thus, for any $\pi \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})$ and $(x, c) \in \Omega_x^2 \times \Omega_c$, one gets:

$$\sum_{j \in \mathcal{I}} e^{-\gamma \times (C^T(x, j) + a_t (\Psi(j, \pi) + \Phi(c, j)) + C^C(c_e, j))} > e^{-\gamma \times (\|C^T\|_\infty + a_t (K + \|\Phi\|_\infty) + \|C^C\|_\infty)}. \quad (4.16)$$

Therefore, the denominator of $g[\pi]$ is always strictly positive, making the function $g[\pi](\cdot)$ well defined on $\Omega_x^2 \times \Omega_c$, which concludes the proof. $\square$

The following lemma provides the continuity of the function $g[\cdot]$

Lemma 4.7. *The function $g[\cdot]$ is continuous from $\mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})$ to $\mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})$, with respect to the norm $\|\cdot\|_\infty$.*

Proof. It turns out that for all $i \in \mathcal{I}$, the function $\rho_i(\cdot) : \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}) \rightarrow \mathbb{R}_+$, defined in Equation (4.4), is continuous with respect to the norm $\|\cdot\|_\infty$. Recalling that $a_t > 0$, we observe that for any $\pi \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})$ such that $\rho_i(\pi) \geq N_i^{\text{cp}}$ for a certain $i \in \mathcal{I}$, one has:

$$e^{-a_t \gamma \Psi(\pi, i)} = 0. \quad (4.17)$$

Let $\pi \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})$ and let $\{\pi^n\}_{n \in \mathbb{N}}$ be a sequence in $\mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})$ converging to π with respect to $\|\cdot\|_\infty$. The aim is to show that for any $i \in \mathcal{I}$, the sequence $\{g[\pi^n]_i\}_{n \in \mathbb{N}}$ converges to $g[\pi]_i$ with respect to $\|\cdot\|_\infty$. Let $i \in \mathcal{I}$.

- If $\rho_i(\pi) < N_i^{\text{cp}}$, then by the continuity of the map $\rho_i(\cdot)$ and of $\Psi(\cdot, i)$ in its domain, we deduce that

$$\lim_{n \rightarrow \infty} e^{-\gamma a_t \Psi(\pi^n, i)} = e^{-\gamma a_t \Psi(\pi, i)}. \quad (4.18)$$

- Now assume that $\rho_i(\pi) \geq N_i^{\text{cp}}$. By the continuity of $\rho_i(\cdot)$, we have $\lim_{n \rightarrow \infty} \rho_i(\pi^n) \geq N_i^{\text{cp}}$. By the definition of $\Psi(\pi, i)$, one has:

$$\lim_{n \rightarrow \infty} \Psi(i, \pi^n) = +\infty.$$

By previous equality, one derives

$$\lim_{n \rightarrow \infty} e^{-\gamma a_t \Psi(\pi^n, i)} = e^{-\gamma a_t \Psi(\pi, i)} = 0. \quad (4.19)$$

Observing from Lemma 4.5, there exists $i \in \mathcal{I}$ such that $e^{-\gamma a_t \Psi(\pi, i)} > 0$, then by the continuity of the map,

$$y \in \mathbb{R}_+^{|\mathcal{I}|} \setminus \{0^{|\mathcal{I}|}\} \mapsto \frac{y_i e^{-\gamma \times (C^T(x, i) + a_t \Phi(c, i) + C^C(c_e, i))}}{\sum_{j \in \mathcal{I}} y_j e^{-\gamma \times (C^T(x, j) + a_t \Phi(c, j) + C^C(c_e, j))}}, \quad (4.20)$$

and by (4.18) and (4.19), one has for any $i \in \mathcal{I}$ that $\{g[\pi^n]_i\}_{n \in \mathbb{N}}$ converges to $g[\pi]_i$ with respect to $\|\cdot\|_\infty$. This concludes the proof.

□

Let us define the constant $L_\gamma > 0$ by

$$L_\gamma := \gamma \max\left(\left\|\frac{\partial C^T(\cdot)}{\partial x}\right\|_\infty, a_t \left\|\frac{\partial \Phi(\cdot)}{\partial c}\right\|_\infty\right) \times \left(e^{2\gamma \times (\|C^T\|_\infty + a_t(K + \|\Phi\|_\infty) + \|C^C\|_\infty)} + e^{4\gamma \times (\|C^T\|_\infty + a_t(K + \|\Phi\|_\infty) + \|C^C\|_\infty)}\right). \quad (4.21)$$

It is shown in the following lemma 4.8 that for any choice profile π in $\text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega^c, \mathcal{S}^{|\mathcal{I}|})$, $g[\pi]$ is also in $\text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega^c, \mathcal{S}^{|\mathcal{I}|})$. Intuitively, it means that the choice probabilities between two EV users in response to a vector of waiting time are similar if their origin-destination and charging features are close to each others.

Lemma 4.8. *The set $\{g[\pi] \mid \pi \in \text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega^c, \mathcal{S}^{|\mathcal{I}|})\}$ is included in $\text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega^c, \mathcal{S}^{|\mathcal{I}|})$.*

The idea of the proof is to show that, for any choice profile π in $\text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega^c, \mathcal{S}^{|\mathcal{I}|})$, the derivatives of $g[\pi]$ are bounded by the constant L_γ .

Proof. We want to show that for any $\gamma \in \mathbb{R}_+^*$ and $\pi \in \text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega^c, \mathcal{S}^{|\mathcal{I}|})$, $g[\pi]$ is Lipschitz continuous with Lipschitz constant L_γ . Since the map d is in $C^1(\Omega \times \mathcal{I})$ and Φ in $C^1(\Omega_c \times \mathcal{I})$, by composition $g[\pi]$ is differentiable on $\Omega_x^2 \times \Omega_c$ for any $\pi \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|})$. For any $i \in \mathcal{I}$ and $(x, e) \in \Omega_x^2 \times \Omega_c$, one has:

$$\begin{aligned} \partial_x g[\pi]_i(x, e) = & \gamma \frac{\frac{\partial C^T(\cdot, i)}{\partial x} e^{-\gamma \times (C^T(c_x, i) + a_t(\Psi(i, \pi) + \Phi(e, i)) + C^C(c_e, i))}}{\left(\sum_{j \in \mathcal{I}} e^{-\gamma \times (C^T(x, j) + a_t(\Psi(j, \pi) + \Phi(c_e, j)) + C^C(c_e, j))}\right)} \\ & + \gamma \frac{e^{-\gamma \times (C^T(x, i) + a_t(\Psi(i, \pi) + \Phi(e, i)) + C^C(c_e, i))} \sum_{j \in \mathcal{I}} \frac{\partial C^T(\cdot, j)}{\partial x} e^{-\gamma \times (C^T(x, j) + a_t(\Psi(j, \pi) + \Phi(e, j)) + C^C(c_e, j))}}{\left(\sum_{j \in \mathcal{I}} e^{-\gamma \times (C^T(x, j) + a_t(\Psi(j, \pi) + \Phi(e, j)) + C^C(c_e, j))}\right)^2}. \end{aligned}$$

Using that $\Psi \geq 0$ and that for any $j \in \mathcal{I}$,

$$|e^{-\gamma \times (C^T(x, j) + a_t(\Psi(j, \pi) + \Phi(e, j)) + C^C(c_e, j))}| \leq e^{\gamma \times (\|C^T\|_\infty + a_t \|\Phi\|_\infty + \|C^C\|_\infty)},$$

and using inequality (4.16), previous equality becomes, for any $i \in \mathcal{I}$,

$$\|\partial_x g[\pi]_i\|_\infty \leq \gamma \left\|\frac{\partial C^T(\cdot)}{\partial x}\right\|_\infty \left(e^{2\gamma \times (\|C^T\|_\infty + a_t(K + \|\Phi\|_\infty) + \|C^C\|_\infty)} + e^{4\gamma \times (\|C^T\|_\infty + a_t(K + \|\Phi\|_\infty) + \|C^C\|_\infty)}\right). \quad (4.22)$$

Similar arguments give:

$$\|\partial_c g[\pi]_i\|_\infty \leq \gamma a_t \|\Phi'\|_\infty \left(e^{2\gamma \times (\|C^T\|_\infty + a_t(K + \|\Phi\|_\infty) + \|C^C\|_\infty)} + e^{4\gamma \times (\|C^T\|_\infty + a_t(K + \|\Phi\|_\infty) + \|C^C\|_\infty)}\right). \quad (4.23)$$

By (4.22) and (4.23) and the definition of L_γ in (4.21), one derives that $g[\pi]$ is in $\text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega^c, \mathcal{S}^{|\mathcal{I}|})$. □

Note that in the extreme case when $\gamma = 0$, i.e. the EV users are completely irrational, $g[\pi]$ is constant, as given by the definition of $g[\cdot]$ in Equation (4.14). This is coherent with the bound $L_0 = 0$ of the derivative of $g[\pi]$.

The following theorem is central in this chapter, and states that there exists a fixed point solution to $g[\cdot]$ in $\text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega^c, \mathcal{S}^{|\mathcal{I}|})$.

Theorem 4.9. *There exists a SE π^* in the sense of definition 4.3.*

The proof consists in showing that there exists a solution to Equation (4.15), and relies on the Schauder's fixed point theorem of the map $g[\cdot]$ in the Banach space $\mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|I|})$, endowed with the norm $\|\cdot\|_\infty$.

Proof. Using Ascoli-Arzelà theorem, one can show that the set $\text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|I|})$ is a compact subset of $\mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|I|})$ with respect to the norm $\|\cdot\|_\infty$.¹³ Moreover, $\text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|I|})$ is convex, and, according to lemma 4.8, $\{g[\pi] \mid \pi \in \text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|I|})\} \in \text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|I|})$. Moreover, according to lemma 4.7, $g[\cdot]$ is continuous.

Therefore, $g[\cdot]$ is a continuous function from a convex compact set to itself. Thus, according to the Schauder's fixed point theorem, $g[\cdot]$ admits a fixed point in $\text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|I|})$.¹⁴ It means that Equation (4.15) admits a solution, which concludes the proof. $\square$

Given Theorem 4.9, there exists a SE in the sense of Definition 4.3. Although there is no guarantee of uniqueness, the following numerical study suggests that there exists a unique SE, which is important in order to be able to anticipate the congestion levels at the different CSPs.

4.4 Numerical experiments

4.4.1 Use-case description

In the use-case study, the map Ω_x is a square of 10 km over 10 km. The coordinate scale is one unit equals one kilometer (km), with the (0,0) coordinate at the lower left of the map. The EV users' origin and destination positions are finite and uniformly distributed over Ω_x , as shown in Figure 4.5.

Their energy need are uniformly distributed over 10, 20, 30 and 40 kWh. To make the analysis and interpretation easier, no limit is set in the charging power on the side of the EVs, i.e., $c_p := 350$ kW. The other parameter values are shown in Table 4.6.

Description	Notation	Value
Distance value coefficient (€/km)	$(a_d + \frac{a_t}{\bar{d}^{\text{speed}}})$	0.075
Time value coefficient (€/h)	a_t	30
Average driving speed (km/h)	$\bar{d}^{\text{speed}}$	30
Rationality parameter	γ	1
Charging demand frequency (h^{-1})	λ	80

Table 4.6: User parameter values.

The features of the three CSPs are also shown in Figure 4.5. CSP 1 and 3 are symmetrical in their position, and have opposite charging characteristics: high charging rates but few CPs for CSP 1, and low charging rate but many CPs for CSP 3. The number of CPs and the charging speed of CSP 2 is in-between, but CSP 2 benefits from a very convenient location. The aggregated

¹³The idea is to show the equicontinuity of $\text{Lip}_{L_\gamma}(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|I|})$. See for example [this document](#) for a statement of the Ascoli-Arzelà theorem and the notion of equicontinuity.

¹⁴Schauder's theorem: *Let A be a compact convex subset of a Banach space and g a continuous map of A into itself. Then g has a fixed point.* See for example [\[Morris, 1975\]](#) for more details.

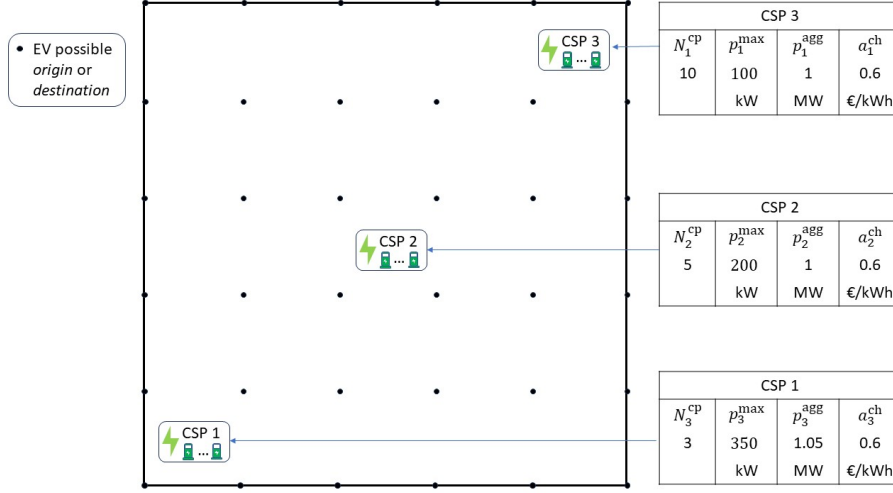


Figure 4.5: EV possible origins and destinations, and charging station pools position and resource in power

power capacity is approximately the same for each of the CSP. Although the quality of service, i.e. charging speed, is different among the CSPs, the charging price is set uniformly among the CSPs in order to highlight the influence of the other features (the position, the charging speed and the number of CPs) on the EV users choice.

The SE, characterized by the fixed point Equation (4.15), is searched numerically using the *optimize.root* Python package *scipy*, with a tolerance of the waiting time at SE of 10^{-5} hour, i.e. 0.036 seconds. In order to stabilize the convergence of the algorithm, the waiting time at the CSPs was upper bounded by 1 hour.¹⁵ In all the following figures, the metrics are displayed at a SE π^* in the sense of Definition 4.3.

4.4.2 Some metrics considered

The following metrics are mathematically formalized and are shown numerically below.

Expected detour The expected detour among EV users going to CSP i is:

$$\forall \pi \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}), \quad \bar{d}_i(\pi) := \frac{1}{\theta_i(\pi)} \times \int_{\Omega_x} \int_{\Omega_c} d(x, i) f_{\text{ev}}(dx, dc). \quad (4.24)$$

Expected travel cost The expected travel cost $\bar{C}_i^T(\pi)$ among EV users going to CSP i is related to the expected detour:

$$\forall \pi \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}), \quad \bar{C}_i^T(\pi) := \frac{1}{\theta_i(\pi)} \times \int_{\Omega_x} \int_{\Omega_c} C^T(x, i) \pi(i) f_{\text{ev}}(dx, dc). \quad (4.25)$$

Expected energy need The expected energy need $\bar{E}_i(\pi)$ among EV users going to CSP i is:

$$\forall \pi \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}), \quad \bar{E}_i(\pi) := \frac{1}{\theta_i(\pi)} \times \int_{\Omega_x} \int_{\Omega_c} c_e \pi(i) f_{\text{ev}}(dx, dc). \quad (4.26)$$

¹⁵ Otherwise, it creates a *yo-yo* effect, where EV users switch their choice in mass due to very high or very low waiting time.

Expected service time The expected service time $\bar{\Phi}_i(\boldsymbol{\pi})$ among EV users going to CSP i is related to their expected energy need:

$$\forall \boldsymbol{\pi} \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}), \quad \bar{\Phi}_i(\boldsymbol{\pi}) := \frac{1}{\theta_i(\boldsymbol{\pi})} \times \int_{\Omega_x} \int_{\Omega_c} \Phi(c, i) \boldsymbol{\pi}(i) f_{\text{ev}}(dx, dc). \quad (4.27)$$

Expected charging cost The expected charging cost $\bar{C}_i^C(\boldsymbol{\pi})$ among EV users going to CSP i is also related to their expected energy need:

$$\forall \boldsymbol{\pi} \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}), \quad \bar{C}_i^C(\boldsymbol{\pi}) := \frac{1}{\theta_i(\boldsymbol{\pi})} \times \int_{\Omega_x} \int_{\Omega_c} C^C(c_e, i) \boldsymbol{\pi}(i) f_{\text{ev}}(dx, dc). \quad (4.28)$$

Expected total EV cost The expected total EV cost $\bar{C}_i(\boldsymbol{\pi})$ among EV users going to CSP i is:

$$\forall \boldsymbol{\pi} \in \mathcal{C}^0(\Omega_x^2 \times \Omega_c, \mathcal{S}^{|\mathcal{I}|}), \quad \bar{C}_i(\boldsymbol{\pi}) := \frac{1}{\theta_i(\boldsymbol{\pi})} \times \int_{\Omega_x} \int_{\Omega_c} \mathcal{C}_{(x,c)}(i, \boldsymbol{\pi}) \boldsymbol{\pi}(i) f_{\text{ev}}(dx, dc). \quad (4.29)$$

4.4.3 Performance evaluation at equilibrium

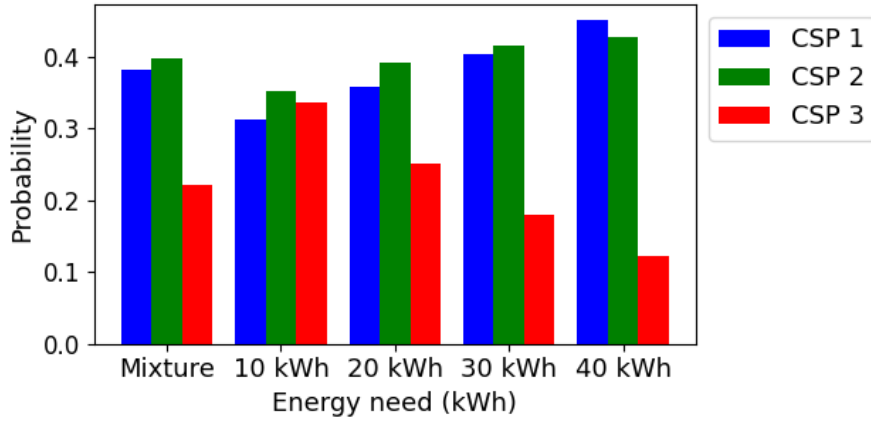
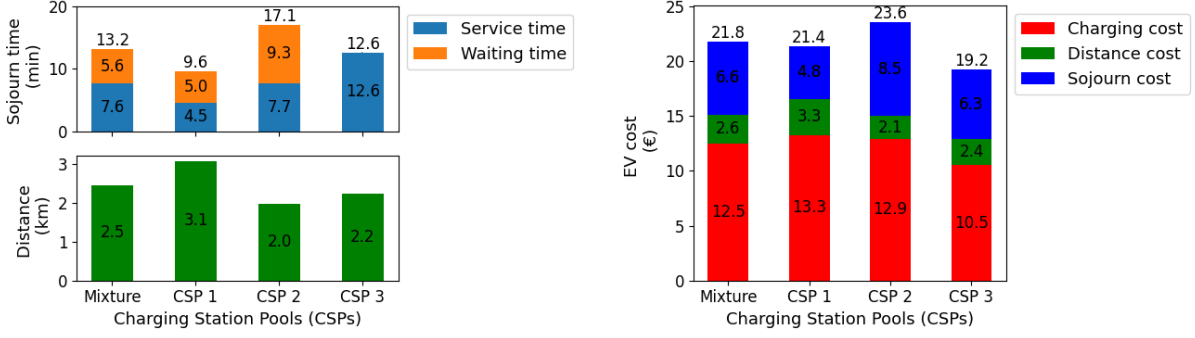


Figure 4.7: Probability of charging pools selection, overall in the EV population and per energy need.

Figure 4.7 shows the probability selection of the CSPs by the EV users. Except for the EV users with an energy need of 40 kWh, CSP 2 attracts more EV users, as it benefits from its convenient location, i.e. at the middle of the map. Between CSP 1 and CSP 3, which are symmetrical geographically, EV users prefer in general CSP 1. Although having approximately the same capacity in terms of aggregated charging power (1.05 MW for CSP 1 and 1 MW for CSP 3), CSP 1 enables faster charging speed. Indeed, CSP 1 tends to attract EV users with higher energy need (e.g. 40 kWh), because these EVs are more sensitive to the charging speed, due to their potential very long charging duration.

Figure 4.8a shows, for each CSP, the expected detour for the EV users choosing it, given by Equation (4.5), and the sojourn duration, given by Equation (4.6). The expected service time is higher for CSPs with lower charging speed. As small proportion of EV users choose CSP 3 (about 20 % of them, see Figure 4.7), the waiting time is low in expectation (only a few seconds).



(a) EV users expected sojourn time $\Psi(i, \pi^*) + \bar{\Phi}_i(\pi^*)$ at each charging station pools and overall, and detour length from them. (b) EV users expected total cost $\bar{\mathcal{C}}_i(\pi^*)$ at each charging station pools, and overall in the EV population.

Figure 4.8: EV users different costs values for joining the charging pools.

The distance between CSP 3 and the EV users choosing it is short because only the EV users which initial travel close to CSP 3 choose it. Otherwise this choice of CSP is not profitable due to the long service time (only 100 kW). As the EV users tend to prefer CSP 2 (approximately 40 % of them, see Figure 4.7), it creates high congestion at this CSP (9.3 minutes of waiting). This congestion is dissuasive, which implies that it is not profitable to choose it when the distance is long. This, along with the central position of CSP 2, makes the detour for EV users going to CSP 2 especially short (2 km in expectation). On the opposite, CSP 1 enables short service time, i.e. charging duration, implying reasonable sojourn times (less than 10 minutes). Thus, some EV users, in particular the ones with higher energy need, are willing to cover longer distance to reach CSP 1.

Figure 4.8b shows the expected total cost of the EV users, overall in the EV population, i.e., by a randomly taken EV user regardless of her choice, and at each CSP. The figure shows that the situation is more beneficial for the EV users choosing CSP 3, with an total expected cost of 19.2 €. This is due to the difference in the population of EV users choosing CSP 3. As described below with Figure 4.9, EV users choosing CSP 3 tend to have lower energy need, which make their charging cost lower, i.e. 10.5 € in contrast to 12.5 € overall in the EV population. Another key observation is that EV users choosing CSP 2, which benefit from the more convenient location, incur higher cost in total (23.6 €) in contrast to the overall total cost in the EV population (21.8 €), in particular due to the long waiting time caused by congestion. In addition, in the configuration setting of this use-case, the distance cost is relatively low because the superficies of the map is reasonable (100 km²), but sufficient to influence the choice of the EV users, as described previously. The charging cost, however, takes an important part on the EV total cost, making it a potential key factor for influencing the EV users choices.

Figure 4.9 shows, for each CSP, the proportion of energy need and expected energy need among EV users choosing the CSP. The difference in the EV users choice between the different energy need is mostly due to the charging speed at the CSPs. EV users choosing CSP 3 tend to have lower needs in energy due to the low charging speed at this CSP (100 kW) making the charging duration too high for high needs in energy. The EV users with a high need in energy therefore report their choice to the other two CSPs.

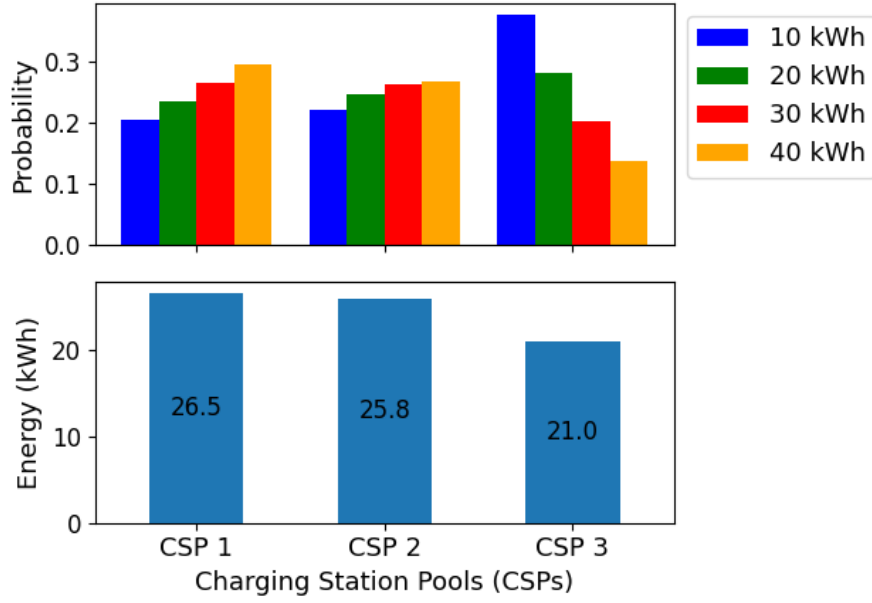


Figure 4.9: Proportion of energy need and expected energy need $\bar{E}_i(\pi^*)$ among EV users choosing a CSP, for each CSP.

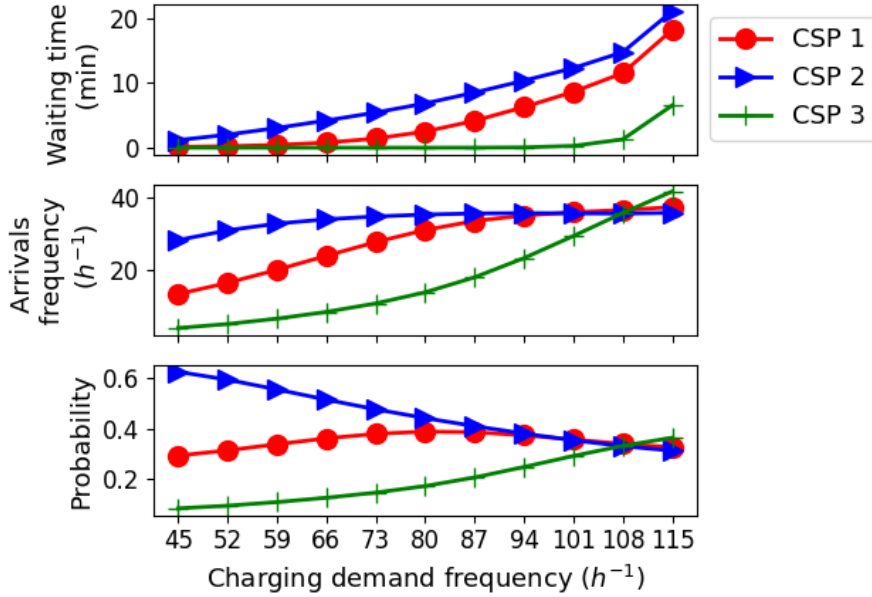


Figure 4.10: EV users choices π^* , arrivals frequency $\lambda_i(\pi^*)$ and waiting time $\Psi(i, \pi^*)$ at each charging station pool with respect to the charging demand frequency λ .

4.4.4 Sensitivity analysis of the charging demand and users rationality

Figure 4.10 shows the EV users choices, the arrivals frequency and the waiting time at each CSP, with respect to the EV charging demand frequency, or equivalently, the EV population mass in need for charging. When the EV charging demand frequency is low, there is very few congestion,

making the waiting time low. In these situations, EV users tend to prefer CSP 2 because of its convenient location, and CSP 1 because of the fast charging rates of its CPs. With the increasing EV charging demand frequency, the EV users tend to choose more equally the three CSPs. In cases of high charging demand, the waiting time increase in such a way that it becomes the main choice factor, relatively to the service time, the travel and charging costs. CSP 2 reaches its utilization capacity (about 40 arrivals per hour) at charging demand frequency of 1 per minutes. Even with higher overall demand than this, almost no additional EV users joins CSP 2. In the extreme case of high overall demand, there are even more EV users selecting CSP 3, even though the charging speed is low, and the location is inconvenient. It can be explained by the lower energy need among EV users joining CSP 3 in expectation, making the service time faster (than with an expected energy need of 25 kWh).

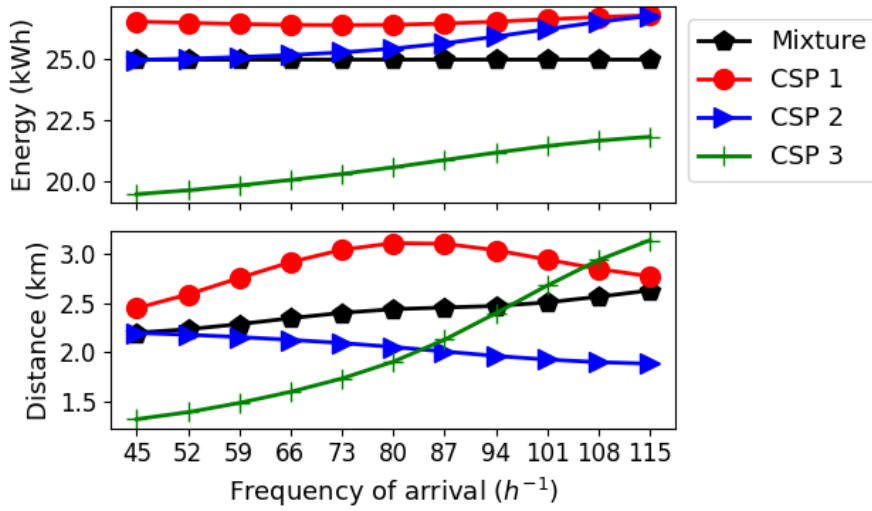


Figure 4.11: Expected energy need $\bar{E}_i(\pi^*)$ and detour $\bar{d}_i(\pi^*)$ from the CSPs among the EV users choosing each CSP.

Figure 4.11 shows the EV features among the ones choosing each CSP.¹⁶ As seen in Figure 4.9, EV users with higher charging needs tend to select CSP 1 or 2. However, with the increasing charging demands, the congestion at CSP 2 becomes more dissuasive relatively to the other CSPs, and this CSP stays interesting mostly for the EV users with higher charging needs, making the expected energy need at this CSP increasing. The same effect occurs for CSP 3. Another key observation is that the distance covered by the EV users increases with the charging demand. The waiting time becoming a major factor in the EV choices, it is more advantageous to make higher detour in order to avoid CSPs that are congested. In particular, the waiting time being short at CSP 3 relatively to the other CSPs, EV users are willing to cover higher distance to charge at CSP 3. However, with the increasing congestion at CSP 2, CSP 2 is only of interest for EV users with short detour for going to this CSP. The distance covered among EV users going to CSP 1 and 3 and the probability to select them follow the same tendency. With the increasing charging demand, it becomes advantageous to make a detour to go to CSP 1 and 3 in order to avoid congestion at CSP 2, making EV users shifting their choice from CSP 2 to CSP 1 and 3.

¹⁶ It can be observed that the expected energy need among EV users choosing a CSP increases with respect to the arrivals frequency for each CSP, although the overall charging needs in the EV population remain constant (25 kWh). This behavior is possible and depends on the EV users to CSP distribution.

When the charging demand cross about 80 h^{-1} , the waiting time at CSP 1 also becomes very affecting, so that EV users shift their choice to CSP 3 instead of CSP 1.

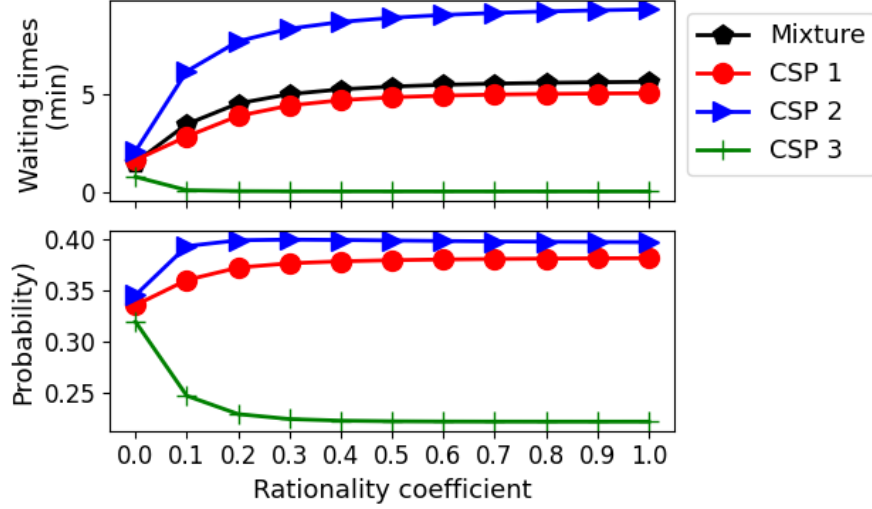


Figure 4.12: EV users' waiting time $\Psi(i, \pi^*)$ and selection probability π^* with respect to the rationality coefficient γ .

Figure 4.12 shows the EV users waiting time and selection probability with respect to their rationality. When the rationality coefficient is close to 0, EV users chose a CSP uniformly, without any consideration of the cost incurred. When their rationality increases, their probability selection converges to the equilibrium without any noise in their cost functions. One can observe that the overall waiting time at the CSPs increases with the rationality of the EV users. The reason is that the waiting time function, given in Equation (4.4) is convex in the charging demand frequency. Therefore, in term of overall waiting time, it is optimal to uniformly distribute the EV users to the different CSPs, instead of concentrating the EVs toward a group of CSPs, which is what happens with rational EV users (they mostly go to CSP 1 and 2).

Figure 4.13 shows the EV users total expected cost and expected detour for going to the chosen CSP. When EV users are uniformly distributed towards the different CSPs, i.e. when the rationality coefficient approaches 0, the detour is the same for CSP 1 and 3, because of the symmetry of their location. When the EV users are more rational, they choose more individual optimal CSP, and therefore lower their distance covered. As a consequence, the total EV cost decreases with the rationality coefficient, meaning that the decrease in the covered distance compensates the increasing waiting time, shown in Figure 4.13, of the CSPs. Another element is that the EV users also tend to select more optimal CSP in terms of the service time, taking into consideration their energy need.

4.5 Conclusion

This work analyzes a queuing game among EV users seeking fast charging, where actions are the choice of a CSP with fast charging mode. The interaction arises from potential waiting time if all CPs at the chosen CSP are occupied upon arrival. In this game, each EV user has a different impact on the congestion (determined by their charging needs), and determines their individual optimal CSP based on charging costs, detour distance, waiting and charging time, and

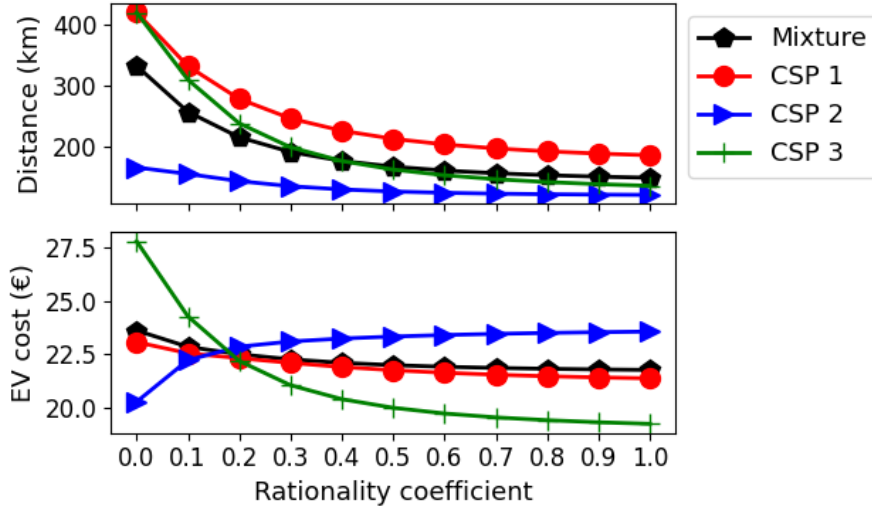


Figure 4.13: EV users total expected cost $\bar{C}_i(\pi^*)$ and expected detour $\bar{d}_i(\pi^*)$ from the chosen CSP with respect to the rationality coefficient γ .

a noise term capturing perception errors. The analysis demonstrates that, assuming each CSP can meet total charging demand, a stochastic equilibrium exists. Numerical results reveal that CSPs with more favorable locations tend to experience greater congestion, resulting in higher overall user costs at these stations. Additionally, numerical results show that the expected total cost experienced among the EV population tend to decrease when the EV users are more rational.

Future work is directed to explore conditions, if any, for the uniqueness of equilibrium, as observed numerically. Another direction is to investigate the design of incentives such as charging price adjustments, in order to optimize social welfare. Additionally, a decentralized perspective could be examined, focusing on competition among charging station operators.

Part II

Pricing to alleviate congestion at charging stations

Chapter 5

Parking pricing for increasing the charging points utilization

This chapter is an extension of the following paper:

[Dupont et al., 2023b] Dupont, A., Hayel, Y., Jiménez, T., Breal, J.B., Payen, R., Beaude, O. (2023). “Optimal Pricing for Electric Vehicle Parking Duration under Uncertainty”. In: *2023 IEEE PES Innovative Smart Grid Technologies Europe (ISGT EUROPE)*, pp. 1–5. <https://ieeexplore.ieee.org/document/10407699>

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Notation	Signification
N^{cp}	Number of CPs at the unique CSP
p^{ch}	Charging rate at a CP (in kW)
a^{park}	Parking price (in €/h)
d^{thr}	Parking duration threshold below which parking is free of charge (in hour)
$f_{\text{soc}}(\cdot)$	Probability density function of the EVs battery capacity and energy at arrival
b^{capa}	EV battery energy storage capacity (in kWh)
e^0	Energy at arrival (in kWh)
d	Parking duration of a user (hour)
ν_i	Inverse of the mean parking duration (in h^{-1})
λ	Expected arrivals frequency at the CSP (in h^{-1})
σ_i	Probability that an EV user belongs to class i
$\bar{\pi}$	Stationary distribution of the Markov chain on the possible states in the <i>time-constant system</i> case
π_t^{in}	Stationary distribution of the Markov chain on the possible states at time t in the <i>time-varying system</i> case
$n(\mathbf{s})$	Number of EV users at state $\mathbf{s}$
$\tilde{\theta}_i$	Probability function that a class- i user aim to join the CSP for a given parking pricing
$\bar{W}_i$	Expected welfare function of class- i EV users (output values are in €)
$\hat{W}_i$	Perceived welfare function of class- i EV users (output values are in €)

Table 5.1: Main parameter notations used in this chapter. *Red*: CSP parameters/variables; *Blue*: EV users parameters.

5.1 Introduction

Context and motivation Parking pricing has been widely used in the context of conventional parking lots, i.e. not equipped with a CS. In the context of CPs, some similarities exist when the CPs are used for both charging and parking. At CPs with standard charging modes, the EV users do some other activities while the EV is parked at the CSP due to the long charging duration.¹ Moreover, the parking duration can be entirely independent of the charging duration if charging is opportunistic, meaning EV users visit the site primarily for other activities rather than charging their vehicle. As a consequence, their parking duration might be longer than their charging duration, thereby occupying a CP without charging for some duration. As for conventional parking lots, if many users stay parked for a long duration, it may deprive future arrivals from accessing a CP. This effect creates an opportunity loss for CSO, and create frustration or dissatisfaction for the EV users. Figure 5.2 illustrate from simulation the influence of the parking duration on the number of EVs charging at the CSP at a random point in time. It therefore would be beneficial for both the CSO and the EV users to incentivize EV users to stay parked for a shorter duration.

¹ See Table 1.5 for typical charging duration examples.

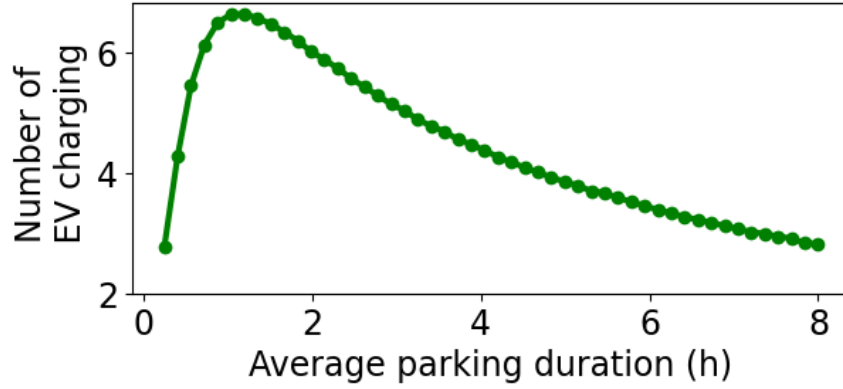


Figure 5.2: Average number of EV charging at a CSP at a random point in time, with respect to their average parking duration. *The results are obtained from simulation, with EV users arriving every 5 minutes at a CSP equipped with 10 CPs with a 11 kW charging rate, and no limit in aggregate power capacity. EV users do not waiting in case when all the CPs are occupied upon arrival. When EV users tend to stay parked for a long duration (more than 1h30 in this use-case example), it decreases the number of EV charging at the CSP, for two reasons: (1) the CSP is often fully occupied, depriving other EV users from joining the CSP; and (2) most of the EV users at the CSP are not charging because they stay parked longer than the duration necessary for charging the EV.*

Static pricing In the context of conventional parking lots, pricing schemes effectiveness have been studied in the literature. It has been shown that a parking price that increase proportionally with the parking duration, i.e. an *hourly pricing basis*, yields better performances than a simple entrance fee [Keren and Hadad, 2021]. Indeed, an entrance fee has the opposite effect to that intended: it encourage users to stay a longer duration, in order make their spawning profitable. However, hourly pricing is less effective than *progressive pricing* [Ornelas et al., 2023], i.e. a piece-wise linear price structure, because it enables to specifically target the users staying for a long duration. Most studies on parking fees analyze static pricing schemes, i.e. that does not depend either on the occupation level nor the time of the day. For example, the authors in [Biswas et al., 2016] propose the design of a static penalty scheme, using a learning method (UCB algorithm [Auer, 2002]), for EV users staying at the CP while their battery has finished charging, in order to reduce overstay at the CPs.

Menu-based dynamic pricing Some other studies [Hao et al., 2022], [Zeng et al., 2021] propose dynamic price-menus in an online fashion, in which EV users select a departure time among a menu. However, the opportunity cost from an overstaying EV user is either quantified arbitrarily [Zeng et al., 2021], or it is assumed that the EV users know in advance their parking duration and that they will respect the option selected [Hao et al., 2022], although in practice, some users deal with substantial uncertainty in their parking duration.

5.1.1 Scenario description

The EV users arrivals/departures at a given CSP are modeled as a queueing system, composed of charging and fully charged EVs. EV users are clustered into classes depending on their parking

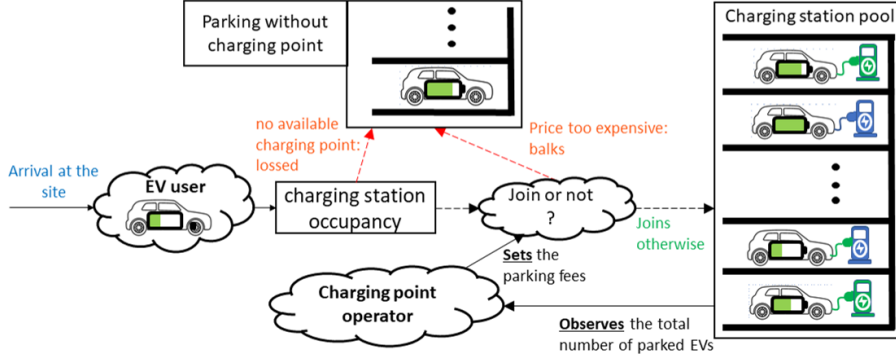


Figure 5.3: Illustrative scheme of the scenario.

duration. The occupancy of the CSP is modeled as a Continuous-Time Markov Chain (CTMC), where each state correspond to a number of EV users at the CSP for each class. Upon arrival, the EV users directly see if there is an available CP: there is no "cruising".² They either join the CSP, i.e. go charging at one CP in the CSP, if they are willing to and there is an available CP, or go somewhere else to park otherwise, in which case they are considered lost from the system perspective. See Figure 5.3 for an illustrative scheme of the scenario. In this chapter, the impact of congestion on the charging power rate is neglected, or equivalently, there is no limit in the aggregate power capacity at the CSP. Indeed, this chapter focuses on the availability of the CPs. Similarly to [Ornelas et al., 2023], a progressive parking pricing is set by the CSO, with the aim to be dissuasive for the EV users with long parking duration to join the CSP in order to increase the utilization of the CPs. This parking pricing policy is applied in two cases: (i) *time-constant system*, where the parameter values are static in time; and (ii) *time-varying system*, with time-varying parameter values, such as the frequency of EV arrival at the site.³ In the *time-constant system* case, the EV users arrival flow at the CSP is constant throughout the considered period. In this setting, we analyze the performance of an occupation-dependent pricing, i.e. dependent on the current occupancy level at the CSP, which is already used in practice [Tesla, 2024], as opposed to a static pricing scheme. In the *time-varying system* case, EV users frequency of arrivals/departure can evolve throughout the day. In this setting, occupation-dependent pricing, time-dependent pricing, or a mixture of both is analyzed. The main novelty of this study include: (1) an occupation-dependent progressive pricing scheme without complete information about the state of the system; and (2) a system with time-varying parameters with the incorporation of dynamic pricing schemes.

5.1.2 Theoretical results on dynamic pricing in queues

Pricing models in queues follow the setting of [Naor, 1969] for observable queues, where customers upon arrival at a single-server queue choose to join it or to balk, depending on the pricing policy. With multi-servers and differentiated-customers in service time,⁴ it is shown in [Bergquist and Elmachetoub, 2023] that static pricing with an admission threshold gives similar performances

² For example, at on-street parking with many spots, the drivers don't see if there is an available spot near by, and thus "cruise for parking". This case is not considered here as the CPs are supposed sufficiently close to each other.

³ The associated Markov chain in the *time-constant system* (resp. *time-varying system*) case is called *time-homogeneous* (resp. *time-inhomogeneous*) CTMC in the literature.

⁴ In our case, a server corresponds to a CP and the service time is the parking duration.

than dynamic pricing in terms of revenue and congestion, and gets closer to optimality when the number of servers is high. Similar insights holds for loss systems, i.e. when a customer is lost if all the servers are occupied upon arrival,⁵ and single-class customers [KS et al., 2022, Besbes et al., 2021] or multi-class [Paschalidis and Tsitsiklis, 2000]. In addition, it is shown in [Besbes et al., 2021] that the optimal dynamic price, in regards to the profit, must increase with respect to the number of occupied servers. In application to vehicle parking with multi-class vehicle users, it is shown in [Legros and Fransoo, 2024] that knowledge about only the total number of vehicles parked at the parking lot is sufficient (from the CSO perspective), i.e. knowledge about the number of vehicles for each class does not increase much the parking manager revenue. When the frequency of arrivals is time-inhomogeneous, the analysis of optimality is more complicated, and heuristic approaches are relevant, as in [Abdalrahman and Zhuang, 2022] which use reinforcement learning for CSP occupancy management. Our methodology mainly differs from these works in that the pricing policy has to be made without knowledge about the class of the EV users, i.e. their expected parking duration in our context.

5.1.3 Organization of the chapter

In this chapter, Section 5.2 presents the modeling of the CSP and the EV users charging behavior, i.e., decision to go charging at the CSP or not. In Section 5.3, the Markov chain associated with the *time-constant system* case is described, as well as the corresponding pricing optimization formulation. Similarly, Section 5.4 describes the *time-varying system* case. Finally, Section 5.5 draws a conclusion of the work and provides some future research directions.

5.2 Model preliminaries

We consider a public site, e.g. a mall, in which the parking lot is equipped with a CSP composed of N^{CP} CPs. Upon arriving at the site, if there is an available CP at the CSP, a user decides whether to charge or not at the CSP depending on the parking price in addition to the charging power and price. Otherwise, the arriving EV user goes directly to a parking space elsewhere: there is no waiting queue.

5.2.1 EV users characteristics

EV users parking duration

The EV users arriving at the site and interested in charging are divided into I classes, depending on their parking duration. As in [Dowling et al., 2020, Zhang et al., 2018], the parking duration D_i of EV class $i \in \mathcal{I}$ ($= \{1, \dots, I\}$) is random and exponentially distributed with mean ν_i^{-1} (the mean of a random variable exponentially distributed fully characterizes its distribution). Similarly to [Biswas et al., 2016, Legros and Fransoo, 2024], the parking duration is uncertain, even for the EV users themselves. However, each user knows their expected parking duration ν_i^{-1} , i.e. their class.

Energy received during a charging session

The battery storage capacity of an EV, i.e. the maximum energy that can be stored in the EV battery, is denoted b^{capa} (in kWh), and depends in practice on the battery type and aging [Xiong

⁵ The CSP occupation corresponds to a loss system in this chapter.

et al., 2020]. The energy already stored in the battery⁶ upon arrival is denoted e^0 (in kWh) and depends on the previous trips charging decisions [Liu et al., 2022], and on exogenous environment factors impacting the energy consumption of the EVs [Yi and Bauer, 2017]. In this work, upon arrival at the site, the battery capacity b^{capa} and the energy at arrival e^0 are drawn from a joint distribution $f_{\text{soc}}(\cdot)$ on $\mathbb{R}_+^2$, which, for notational simplicity, and without loss of generality, are assumed independent from the parking duration, as in [Biswas et al., 2016]. The maximum energy that can be received during the charging session is therefore the difference between the battery capacity b^{capa} and the energy at arrival e^0 . For any parking duration d , battery capacity b^{capa} , and energy at arrival e^0 , the function $\phi^{\text{rec}}(\cdot)$ determines the energy received (in kWh) during the charging session, and is given by:

$$\phi^{\text{rec}}(d, b^{\text{capa}}, e^0) = \min(p^{\text{ch}} \times d, b^{\text{capa}} - e^0),$$

where $p^{\text{ch}} \in \mathbb{R}_+$ (in kW) is the charging power rate at a CP, is the same for each CP at the CSP and is constant during the whole charging session.⁷ The energy delivered to the EV grows linearly with the parking duration until departure or until the battery capacity is reached.

Utility for charging

The utility function represents the EV user surplus for charging at the CSP, taking into account the per energy unit charging price a^{ch} (€/kWh) at the CSP. In this work, this surplus is represented by a utility function that is quadratic with respect to the energy received during the charging session.⁸ More precisely, for a given parking duration d , battery capacity b^{capa} , and energy at arrival e^0 , the utility function $U^{\text{ch}}(\cdot)$ is:

$$U^{\text{ch}}(d, b^{\text{capa}}, e^0) = (\alpha(b^{\text{capa}}, e^0) - a^{\text{ch}}) \times \left(\phi^{\text{rec}}(d, e^0) - \frac{1}{2} \beta(b^{\text{capa}}, e^0) \phi^{\text{rec}}(d, e^0)^2 \right), \quad (5.1)$$

where $\alpha(b^{\text{capa}}, e^0)$ and $\beta(b^{\text{capa}}, e^0)$ are positive coefficients, representing the EV users willingness to charge, which can be calibrated using statistical data as in [Lu et al., 2023]. Intuitively, $\alpha(b^{\text{capa}}, e^0)$ (resp. $\beta(b^{\text{capa}}, e^0)$) decreases (resp. increases) with the energy at arrival e^0 , as EV users urgency to charge tends to be greater when the current state of charge gets lower.

5.2.2 Parking pricing policy

The parking pricing structure is piece-wise linear with respect to the parking duration as in [Ornelas et al., 2023]. More specifically, the parking cost $C^{\text{park}}(\cdot)$ for an EV user with parking duration d is:

$$C^{\text{park}}(d, a^{\text{park}}, d^{\text{thr}}) = a^{\text{park}} (d - d^{\text{thr}}) \mathbb{1}_{d > d^{\text{thr}}}, \quad (5.2)$$

where a^{park} (€/h) and d^{thr} (hour) are non-negative coefficients, set by the CSO. There is no parking fee for the EV users who stay parked less than the pre-defined duration threshold d^{thr} .

⁶ Sometimes referred as the *state of charge*.

⁷ In practice, the charging power for EV charging decreases as the SoC approaches the battery capacity, due to battery physical considerations [Wei et al., 2018]. However, it is recommended to avoid reaching full battery capacity in order to preserve the battery's longevity [Xiong et al., 2020]. b^{capa} can therefore be defined as the maximum state of charge above which charging degrades the battery, in which case charging stops when the state of charge reaches b^{capa} and a constant charging power over the charging session is a realistic approximation.

⁸ A common assumption in the EV literature [Alsabbagh and Ma, 2020], or more generally in the smart grid literature [Samadi et al., 2010], is the concavity of the utility with respect to the energy received. Other concave utility function could also be considered, such as those with an exponential shape, as in [Jie et al., 2021].

Otherwise, the parking fee grows linearly with the parking duration exceeding d^{thr} , at the rate a^{park} . The free of charge parking duration threshold d^{thr} enables to penalize only the EV users with a long parking duration, and is commonly used in practice at CSPs.⁹

5.2.3 EV users' total surplus for charging

For an EV with characteristics $(d, b^{\text{capa}}, e^0)$ and a parking pricing $(a^{\text{park}}, d^{\text{thr}})$, the welfare $W(\cdot)$ of the EV users for joining the CSP is the difference between the utility $U^{\text{ch}}(\cdot)$ for charging, and the cost $C^{\text{park}}(\cdot)$ for parking:

$$W(d, b^{\text{capa}}, e^0, a^{\text{park}}, d^{\text{thr}}) := U^{\text{ch}}(d, b^{\text{capa}}, e^0) - C^{\text{park}}(d, a^{\text{park}}, d^{\text{thr}}).$$

The welfare $W(\cdot)$ represents an EV user's total surplus when considering the cost for both charging, which is included in the utility function $U^{\text{ch}}(\cdot)$ given by Equation (5.1), and parking. Without loss of generality, the welfare for going to a parking space elsewhere is set to 0. From the point of view of the EV users, the energy at arrival e^0 is known, but not the parking duration, as explained in Section 5.2.1. Thus, the EV users consider their welfare for charging at the CSP through its expectation, as in [Legros and Fransoo, 2024], with respect to their random parking duration. The expected welfare $\bar{W}_i(\cdot)$ of a class- i EV user is the difference between the expected utility for charging $\bar{U}_i^{\text{ch}}(b^{\text{capa}}, e^0) := \mathbb{E}_{D_i}[U(D_i, e^0)]$ and the expected cost for parking $\bar{C}_i^{\text{park}}(a^{\text{park}}, d^{\text{thr}}) := \mathbb{E}_{D_i}[C^{\text{park}}(D_i, a, d^{\text{thr}})]$:

$$\bar{W}_i(b^{\text{capa}}, e^0, a^{\text{park}}, d^{\text{thr}}) = \bar{U}_i^{\text{ch}}(b^{\text{capa}}, e^0) - \bar{C}_i^{\text{park}}(a^{\text{park}}, d^{\text{thr}}). \quad (5.3)$$

The expected utility $\bar{U}_i^{\text{ch}}(b^{\text{capa}}, e^0)$ does not depend on the parking price, and its expression is given in Appendix B.5 by Proposition B.10. The expected parking cost for a class- i EV user can be computed using the memoryless property of the exponential law, and its expression is given by:¹⁰

$$\bar{C}_i^{\text{park}}(a^{\text{park}}, d^{\text{thr}}) = a^{\text{park}} \frac{1}{\nu_i} e^{-\nu_i d^{\text{thr}}}. \quad (5.4)$$

This expression plays an important role as it determines how the parking pricing influences the decision to join or not to join the CSP. This decision is further described in the next section.

5.2.4 EV users charging behavior: join or balk

Binomial discrete choice models [Train, 2009] are used for incorporating the perception errors of the EV users into their choice modeling of whether to join or not the CSP. The perceived welfare of the EV users for joining the CSP is:

$$\tilde{W}_i(b^{\text{capa}}, e^0, a^{\text{park}}, d^{\text{thr}}) := \gamma \bar{W}_i(b^{\text{capa}}, e^0, a^{\text{park}}, d^{\text{thr}}) + \varepsilon,$$

where $\bar{W}_i(\cdot)$ is the expected welfare given by Equation (5.3), ε is the realization of a normalized logistic distribution, and γ is a non-negative coefficient representing the rationality of the EV users. Given this, if a CP at the CSP is available, the probability $\hat{\theta}_i(\cdot)$ (from the CSO perspective) that a class- i EV with battery capacity b^{capa} and energy at arrival e^0 joins the CSP is:¹¹

$$\begin{aligned} \hat{\theta}_i(b^{\text{capa}}, e^0, a^{\text{park}}, d^{\text{thr}}) &= \mathbb{P}(\tilde{W}_i(b^{\text{capa}}, e^0, a^{\text{park}}, d^{\text{thr}}) \geq 0) \\ &= \frac{\exp(\gamma \bar{W}_i(b^{\text{capa}}, e^0, a^{\text{park}}, d^{\text{thr}}))}{1 + \exp(\gamma \bar{W}_i(b^{\text{capa}}, e^0, a^{\text{park}}, d^{\text{thr}}))}. \end{aligned}$$

⁹ See for example [ChargePlace Scotland Charge Point tariffs](#).

¹⁰ The computation details are given in Appendix B.1.

¹¹ The expression of the probability to join the CSP comes from the cumulative distribution function of the logistic distribution.

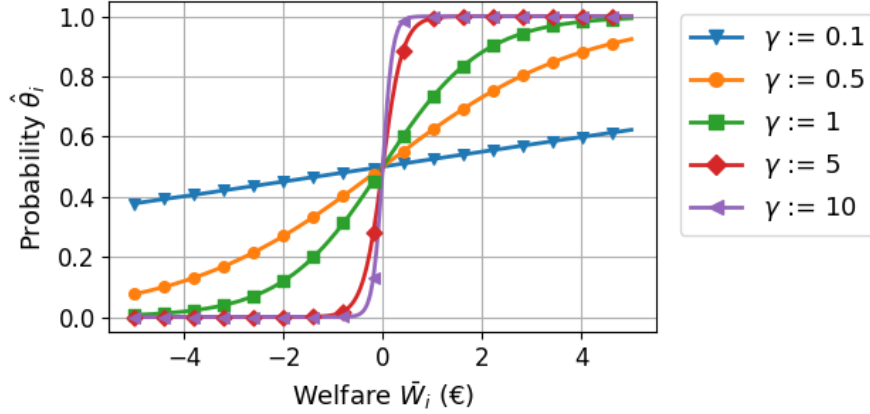


Figure 5.4: Probability that an EV user is willing to join the CSP with respect to her expected welfare, for different values of γ . A rationality parameter $\gamma = +\infty$ means that the EV users join the CSP if and only if their expected welfare $\bar{W}_i(\cdot)$ is positive. Conversely, $\gamma = 0$ implies that the EV users are willing to join the CSP with probability 1/2 regardless the parking price.

Figure 5.4 shows the sensitivity of the willingness to join the CSP with respect to the expected welfare and the rationality parameter γ . In particular, if the users are perfectly rational ($\gamma = +\infty$), one can characterize, for a given parking pricing $(a^{\text{park}}, d^{\text{thr}})$, the set of EV users characteristics (b^{capa}, e^0) such that the perceived welfare is positive.¹² In this case, it directly enables the computation of the mixture probability $\theta_i(a^{\text{park}}, d^{\text{thr}})$ that a class- i EV user joins the CSP, which is expressed in the general case as:

$$\theta_i(a^{\text{park}}, d^{\text{thr}}) = \int_{\mathbb{R}_+^2} \hat{\theta}_i(b^{\text{capa}}, e^0, a^{\text{park}}, d^{\text{thr}}) f_{\text{soc}}(db^{\text{capa}}, de^0). \quad (5.5)$$

5.3 Dynamic parking price in a stationary system

This section presents the *time-constant system* case, where all parameter values are constant in time (average arrival frequency, EV users expected parking duration and electricity cost). The EV users arrival times at the site follow a homogeneous Poisson process, i.e. the frequency of arrivals is constant over time, with parameter λ . Upon arrival, each EV user belongs to a class $i \in \mathcal{I}$ with a constant probability σ_i .

5.3.1 Occupation-dependent parking pricing

The CSO observes the total number n of EV users at the CSP, referred to as the *occupancy level*, yet not the class of each EV user, as no information about their parking duration is given. The coefficients a_n^{park} and d_n^{thr} can therefore be adjusted dynamically by the CSO, as a function of the occupancy level n .

¹² This characterization is investigated in the previous work [Dupont et al., 2023b] in the case of homogeneous battery capacity b^{capa} across the user population.

5.3.2 A time-homogeneous Markov chain

The number of EV users at the CSP of each class and at each time t is described by the multi-dimensional CTMC $\mathcal{M}^h := \{X^h(t) := (X_1^h(t), X_2^h(t), \dots, X_n^h(t)), t \in [0, +\infty[\}$, defined by:

- the set of states $\mathbb{S} := \{\mathbf{s} = (s_1, \dots, s_I), \forall i \in \{1, \dots, I\}, s_i \geq 0, \text{ and } n(\mathbf{s}) \leq N^{\text{cp}}\}$, with s_i the number of class- i EV users at the CSP, and $n(\mathbf{s}) := \sum_{i=1}^I s_i$ the total number of EV users at the CSP, i.e., the *occupancy level*;¹³
- its initial probability distribution $\pi_0^h : \mathbb{S} \rightarrow [0, 1]$, with the following notation: $\forall t \in [0, +\infty[, \forall \mathbf{s} \in \mathbb{S}, \pi_t^h(\mathbf{s}) := \mathbb{P}(X(t) = \mathbf{s})$;
- the generator matrix G , which coefficients are the transition rates:

$$\forall \mathbf{s}, \tilde{\mathbf{s}} \in \mathbb{S}, \quad G[\mathbf{s}, \tilde{\mathbf{s}}] := \begin{cases} \lambda \sigma_i \theta_i(a_{n(\mathbf{s})}^{\text{park}}, d_{n(\mathbf{s})}^{\text{thr}}) & \text{if } \tilde{s}_i = s_i + 1 \text{ and } \tilde{s}_j = s_j \quad \forall j \neq i \\ \nu_i s_i & \text{if } \tilde{s}_i = s_i - 1 \text{ and } \tilde{s}_j = s_j \quad \forall j \neq i \\ 0 & \text{else.} \end{cases}$$

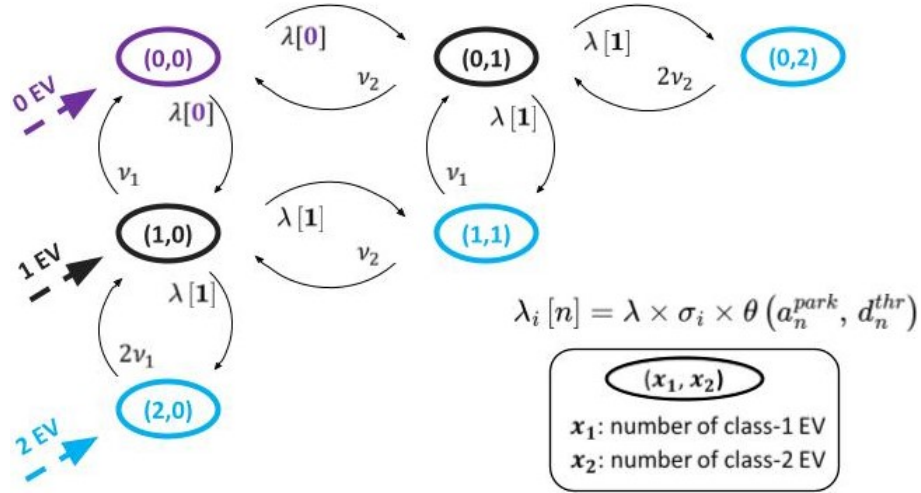


Figure 5.5: The Markov chain $\mathcal{M}^h$ and its transition rates, in an example with 2 EV classes and 2 CPs. The CSO has only knowledge about the color of the state, i.e. the total number of EVs at the CSP, but not the exact state, i.e. their class. In this example, the occupation-dependent pricing can take 3 different values, that are fixed by the CSO: when there are 0, 1 or 2 EV users at the CSP.

Figure 5.5 illustrates the transition rates between two states of the CTMC $\mathcal{M}^h$, partially controlled by the occupation-dependent parking pricing. An important property of the CTMC $\mathcal{M}^h$ is that any state can be reached by any other state in a finite time with a positive probability, as can be seen in Figure 5.5. This property is called *irreducibility*, and is the key for demonstrating the existence and uniqueness of a stationary distribution, stated by the following proposition.

¹³Throughout this chapter, the term *occupancy level* refers to the total number of EV users the the CSP.

Proposition 5.1. *The Markov chain $\mathcal{M}^h$ has a unique stationary distribution $\hat{\pi}$, given by the unique solution of the following equation:*

$$\begin{cases} G\pi = 0, \\ \pi \in [0, 1]^{|\mathbb{S}|}, \\ \sum_{s \in \mathbb{S}} \pi(s) = 1. \end{cases} \quad (5.6)$$

Proof. Let us show that $\mathcal{M}^h$ is irreducible. Let s and $\tilde{s}$ be two states, and let us show that $s := (s_1, \dots, s_I)$ and $s' := (s'_1, \dots, s'_I)$ communicates. For any $i \in [1, \dots, I]$, the probability to go from s_i to $s_i + 1$ or $s_i - 1$ in any time is positive. Therefore, the probability to go from s_i to s'_i in any time is also positive. Repeating this reasoning for all i yields that the probability to go from state s to state s' is also positive. Therefore, $\mathcal{M}^h$ is irreducible and has a unique stationary distribution, denoted $\hat{\pi}$.

As $\frac{\partial \pi_t^h}{\partial t} = G\pi_t^h$ is the first order differential equation of π_t^h , and that any stationary distribution does not evolve over time, i.e. $\frac{\partial \pi_t^h}{\partial t} = 0$, $\hat{\pi}$ is the unique solution of $G\pi = 0$. $\square$

The probability distribution $\hat{\pi}$ is actually independent from the initial condition π_0^h . Proposition 5.2 gives an analytical expression of $\hat{\pi}$ in the particular case when the parking price is static. This result has already been shown in the book [Ross, 2012], where efficient methods for computing the formula of Equation (5.7) in regards to the computation time is discussed.¹⁴

Proposition 5.2 ([Ross, 2012]). *If the parking price strategy is static, i.e. $\forall n, n' \in \{1, \dots, N^{\text{cp}} - 1\}$, $a_n = a_{n'}$ and $d_n^{\text{thr}} = d_{n'}^{\text{thr}}$, the stationary probability distribution $\hat{\pi}(\cdot)$ is:*

$$\forall s \in \mathbb{S}, \quad \hat{\pi}(s) = \frac{\prod_{i=1}^I \frac{\rho_i^{s_i}}{n_i!}}{\sum_{s' \in \mathbb{S}} \prod_{i=1}^I \frac{\rho_i^{s'_i}}{s'_i!}}, \quad (5.7)$$

with, for all $i \in \mathcal{I}$, $\rho_i := \lambda \sigma_i \theta_i(a_{n(s)}^{\text{park}}, d_{n(s)}^{\text{thr}}) \times \nu_i^{-1}$ the utilization rate of the class- i EV users.

Equation (5.7) can be found using the *local balance equations* of the Markov chain $\mathcal{M}^h$.¹⁵ However, it is not possible in general to find an analytical expression of the stationary distribution when the price is dynamic. Nevertheless, the stationary distribution $\hat{\pi}$ can be numerically found by solving the linear equation given by Equation (5.6).

5.3.3 Charging profit rate maximization in stationary regime

Operator's expected profit The per-unit electricity price incurred by the CSO is denoted c^{elec} (€/kWh). Its value depends on the contract between the CSO and the electricity provider, and is assumed to be constant over the day for the *time-constant system* case. The profit

¹⁴ When the number of states becomes large, the sum in the denominator may take a significant time to be computed. Moreover, if the number of CPs is large, the product inside the sum can be difficult to compute numerically due to the factorial and the exponential factors.

¹⁵ Solving the *local balance equations* is a standard method to obtain an analytical expression of the stationary regime of a Markov chain. It is a set of linear equations describing that in stationary regime, each pair of connected states s_i and s_j is locally balanced: the rate at which the chain transitions from s_i to s_j must be equal to the rate at which it transitions from s_j to s_i .

generated from charging an EV with parking duration d , battery capacity b^{capa} and energy e^0 is the difference between the revenue and the cost:

$$\Gamma^h(d, b^{\text{capa}}, e^0) := (a^{\text{ch}} - c^{\text{elec}}) \times \phi^{\text{rec}}(d, b^{\text{capa}}, e^0). \quad (5.8)$$

The expected charging profit $\hat{\Gamma}_i^h$ from a new class- i EV arrival can be derived from Equation (5.8) through:

$$\hat{\Gamma}_i^h = (a^{\text{ch}} - c^{\text{elec}}) \int_{\mathbb{R}_+^2} \bar{\phi}_i^{\text{rec}}(b^{\text{capa}}, e^0) f_{\text{soc}}(db^{\text{capa}}, de^0), \quad (5.9)$$

where $\bar{\phi}_i^{\text{rec}}(b^{\text{capa}}, e^0) = p^{\text{ch}} \frac{1}{\nu_i} e^{-\nu_i \frac{b^{\text{capa}} - e^0}{p^{\text{ch}}}}$ is the expected energy received during a charging session, computed in Appendix B.4. Note that the expected charging profit $\hat{\Gamma}_i^h$ for charging a given EV increases linearly with the charging power p^{ch} , which can be of interest for a CSO aiming to choose which type of charger to install at a CSP but is out of the scope of this work. The expected charging profit per time unit in stationary regime depends on the probability distribution $\hat{\pi}(\cdot)$ through:

$$\bar{\Gamma}^h := \sum_{\mathbf{s} \in \mathbb{S}} \hat{\pi}(\mathbf{s}) \sum_{i \in \mathcal{I}} \lambda \sigma_i \theta_i(a_{n(\mathbf{s})}^{\text{park}}, d_{n(\mathbf{s})}^{\text{thr}}) \times \hat{\Gamma}_i^h. \quad (5.10)$$

The factor $\lambda \sigma_i \theta_i(\cdot, \cdot)$ represents the rate at which a class- i EV joins the CSP, and can be controlled through the parking price policy.

Assuming the CSO has information about the distribution of the population characteristics, namely $\{\sigma_i, i \in \mathcal{I}\}$ and $f_{\text{soc}}(\cdot)$, it can compute the expected charging profit rate, given by Equation (5.10). The parking pricing strategy is not used to directly generate revenue from it. By partially controlling the arrival rates for each EV class, the parking pricing policy enables to control the departure rate of the EV users, by discouraging the EV users with a long parking duration to join the CSP. As a consequence, the parking pricing strategy enables to indirectly increase the profit generated from charging the EVs, referred to as the *charging profit*, by increasing the rate at which the EV users join the CSP. The corresponding optimization problem $\mathcal{P}_{\text{occ-dyn}}^h$ is formulated as follows:

$$\mathcal{P}_{\text{occ-dyn}}^h : \max_{\mathbf{a}^{\text{park}}, \mathbf{d}^{\text{thr}}} \bar{\Gamma}^h \quad \text{given by (5.10)} \quad (5.11)$$

$$\text{s.t.} \quad \left. \begin{array}{l} 0 \leq a_n^{\text{park}} \leq a^{\text{park}, \max} \\ 0 \leq d_n^{\text{thr}} \leq d^{\text{thr}, \max} \end{array} \right\} \forall n \in \{0, \dots, N^{\text{cp}} - 1\}. \quad (5.12)$$

Constraint (5.12) enables a_n^{park} and d_n^{thr} to take reasonable values. Moreover, as $\bar{\Gamma}^h$ is continuous in $\mathbf{a}^{\text{park}}$ and $\mathbf{d}^{\text{thr}}$, there exists a solution to Problem (5.11). In order to measure the significance of an occupation-dependent parking price, two auxiliary problems are introduced, namely:

- $\mathcal{P}_{\text{static}}^h$, for which the parking price is static, i.e. the variables a^{park} and d^{thr} are scalars. In this case, this model can be generalized to any type of parking duration distribution;¹⁶
- $\mathcal{P}_{\text{free}}^{\text{in}}$, for which the parking price is set as $a^{\text{park}} \equiv 0$ (there is no optimization).

¹⁶ M/G/N/N queues stationary probability distribution on the occupancy levels depends on the service time distribution only through its mean [Cohen, 2012].

5.3.4 Some other metrics

In this section, some important metrics are introduced, and are analyzed in Section 5.3.5. All these metrics are computed in the stationary state of the system, and can be derived from the stationary distribution $\dot{\pi}(\cdot)$.

Blocking probability By the PASTA property [Wolff, 1982], the probability $\mathbb{P}^{\text{h,blocked}}$ that an EV user upon arrival is blocked is the proportion of time the CSP is fully occupied:

$$\mathbb{P}^{\text{h,blocked}} = \sum_{\mathbf{s} \in \mathbb{S}} \dot{\pi}(\mathbf{s}) \mathbb{1}_{\{n(\mathbf{s})=N^{\text{cp}}\}}. \quad (5.13)$$

Joining probability For each class i , the probability $\mathbb{P}_i^{\text{h,join}}$ that an EV user from class i joins the CSP upon arrival is calculated as the product of the probability that a CP is available upon arrival and the probability that the user is willing to join the CSP:

$$\mathbb{P}_i^{\text{h,join}} = (1 - \mathbb{P}^{\text{h,blocked}}) \times \mathbb{P}_i^{\text{h,acc}}, \quad (5.14)$$

with $\mathbb{P}_i^{\text{h,acc}}$ the probability that an EV user is willing to join the CSP:

$$\mathbb{P}_i^{\text{h,acc}} = \frac{1}{(1 - \mathbb{P}^{\text{h,blocked}})} \sum_{\mathbf{s} \in \mathbb{S}} \dot{\pi}(\mathbf{s}) \theta_i(a_{n(\mathbf{s})}^{\text{park}}, d_{n(\mathbf{s})}^{\text{thr}}) \mathbb{1}_{\{n(\mathbf{s}) < N^{\text{cp}}\}}. \quad (5.15)$$

Given Equation (5.14), one can compute the probability $\mathbb{P}^{\text{h,join}}$ that an EV user, randomly taken, joins the CS:

$$\mathbb{P}^{\text{h,join}} = (1 - \mathbb{P}^{\text{h,blocked}}) \sum_{i \in \mathcal{I}} \sigma_i \mathbb{P}_i^{\text{h,join}}. \quad (5.16)$$

Mean number of EV users at the CSP The mean number $\bar{N}^{\text{h,park}}$ of EV users at the CSP is:

$$\bar{N}^{\text{h,park}} = \sum_{\mathbf{s} \in \mathbb{S}} \dot{\pi}(\mathbf{s}) n(\mathbf{s}). \quad (5.17)$$

Mean number of EV users charging / fully charged at the CSP The expected total number of EV users fully charged, or equivalently, not charging, at the CSP is given by:

$$\bar{N}^{\text{full}} := \sum_{\mathbf{s} \in \mathbb{S}} \dot{\pi}(\mathbf{s}) \sum_{i \in \mathcal{I}} s_i \mathbb{P}_i(F|\mathbf{s}), \quad (5.18)$$

with:

$$\mathbb{P}_i(F|\mathbf{s}) = \frac{1}{\theta_i(a_{n(\mathbf{s})}^{\text{park}}, d_{n(\mathbf{s})}^{\text{thr}})} \int_{\mathbb{R}_+^2} e^{-\nu_i \frac{b^{\text{capa}} - e^0}{p^{\text{ch}}}} \hat{\theta}_i(b^{\text{capa}}, e^0, a_{n(\mathbf{s})}^{\text{park}}, d_{n(\mathbf{s})}^{\text{thr}}) f_{\text{soc}}(db^{\text{capa}}, de^0),$$

the probability that a class- i EV user that joined the CSP in state $\mathbf{s} \in \mathbb{S}$ is fully charged.¹⁷ The expected number $\bar{N}^{\text{ch}} = \bar{N}^{\text{park}} - \bar{N}^{\text{full}}$ of EV charging at the CSP can directly be deduced from Equations (5.17) and (5.18) because the EVs parked at the CSP are either charging or fully charged.

¹⁷ The computation details are given in Appendix B.2.

5.3.5 Numerical results and analysis

In this section, some numerical illustrations of the optimal parking pricing in the optimization problems $\mathcal{P}_{\text{occ-dyn}}^h$ and $\mathcal{P}_{\text{static}}^h$, and the corresponding metrics given in Section 5.3.4, are provided.

Test case description

Parameters		EV class indices			
Description	Notation	1	2	3	4
Exp. parking duration (hour)	ν_i^{-1}	0.5	1.5	3	5
Proportion	σ_i	0.25	0.25	0.25	0.25

Table 5.6: EV classes characteristics.

Description	Notation	Possible values						
Energy at arrival (in kWh)	e^0	5	10	15	20	25	30	35
Battery capacity (in kWh)	b^{capa}	50						
Linear utility parameter	α	0.66	0.63	0.60	0.56	0.53	0.49	0.46
Quadratic utility parameter	β	0.01	0.01	0.01	0.01	0.01	0.01	0.02
Proportion		0.7	0.13	0.2	0.2	0.2	0.13	0.07

Table 5.7: EV overall charging characteristics distribution.

Description	Notation	Value
Number of CP	N^{cp}	5
Charging power (kW)	p^{max}	22
Electricity cost (€/kWh)	c^{elec}	0.2
Charging price (€/kWh)	a^{ch}	0.4

Table 5.8: Charging pool parameter values.

Tables 5.6 to 5.8 respectively provide the EV classes parking duration, their charging characteristics, and the CSP parameter values. The number of states of the CTMC $\mathcal{M}^h$ increasing exponentially with the number of CPs, only 5 are considered. 4 EV classes are considered in order to represent significant heterogeneity in the users' planned parking duration. The charging characteristics, shown in Table 5.7, are the same for each EV classes. Each EV user's rationality parameter γ is set to 10 (recall that Figure 5.4 shows the influence of γ on the EV users behavior). Each optimization problem considered is solved using L-BFGS-B algorithm¹⁸ [Zhu et al., 1997] in Python 3.7 *scipy* package. The initial variable a_n^{park} and d_n^{thr} are respectively set to 1 (€/h) and 1 (hour) for each CSP occupancy level n , and $a^{\text{park,max}}$ and $d^{\text{thr,max}}$ are respectively set to 10 (€/h) and 10 (hour). There are in average 1 EV arrival every 12 minutes, i.e. $\lambda = 5$. A user, randomly taken, stays parked in average about 2h30 at the site. These parameters are set so that the CSP is highly congested, making a parking price potentially interesting to relieve

¹⁸L-BFGS-B algorithm is a quasi-Newton method, which relies on an approximation of the second derivative of the objective function in order to limit the computation time, and extends the L-BFGS algorithm for constrained optimization.

the congestion. The utility parameters in Table 5.7 are set such that the utility for charging decreases with the energy at arrival, representing the correlation between the energy at arrival and the urgency to charge. The initial states of charge goes from 10% to 70% of the battery capacity, with a distribution centered on an initial state of charge of 40%, as found in [Hu et al., 2019]. The charging price and electricity cost in Table 5.8 correspond to typical pricing in practice,¹⁹ and the charging power correspond to a triple-phase AC charging, which is standard in public places.²⁰

Optimal parking pricing

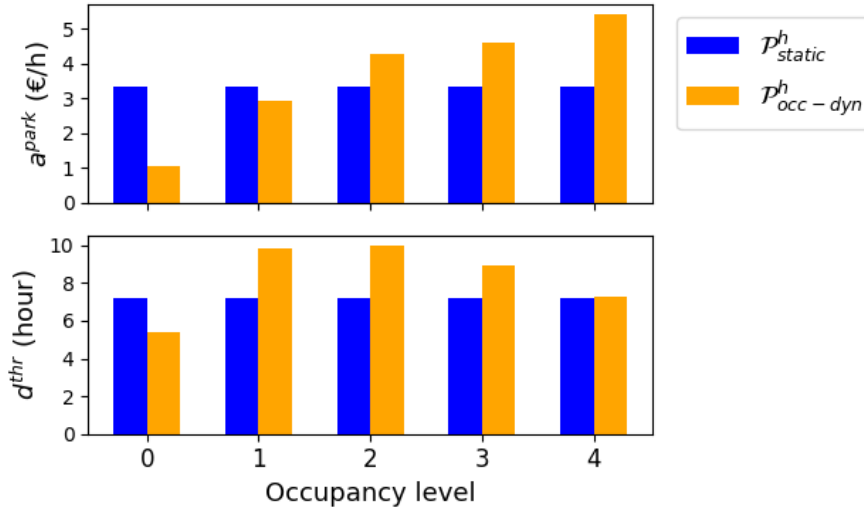


Figure 5.9: Optimal parking pricing of $\mathcal{P}_{\text{static}}^h$ and $\mathcal{P}_{\text{occ-dyn}}^h$.

Figure 5.9 shows the optimal parking pricing of $\mathcal{P}_{\text{static}}^h$ and $\mathcal{P}_{\text{occ-dyn}}^h$. Recall that the negative externality of CPs occupancy is the probability that a future arrival cannot go charging at the CSP because all the 5 CPs are occupied by an EV (not necessarily charging), which creates negative effects in terms of profit from charging the EVs. Intuitively, the goal would be to lower the proportion of time the CSP is fully occupied. To this end, the pricing must adequately discourage EV users with longer expected parking duration to join the CSP. In the case of a static parking price, the EV users must pay approximately 0.05 € per minute of parking duration above 7 hours and 15 minutes. Clearly, this parking pricing is set to penalize class-4 EV users, staying in average 5 hour at the CSP. EV users of class 1, 2 and 3 respectively exceed the parking duration threshold d^{thr} with a probability of 0, 0.01, 0.09. On the contrary, class-4 EV users cross the parking limit with a probability 0.23, which is more dissuasive. Concerning the occupation-dynamic parking pricing, the price rate a^{park} is increasing with the occupancy level. However, it is not sufficient to conclude on the monotonicity of the expected parking cost incurred by the EV users, because the free of charge duration threshold d^{thr} is not decreasing with the occupancy level. This will be discussed below with Figure 5.10. When there is only one or two EVs at the CSP, the strategy is to set the duration threshold d^{thr} at the maximum,

¹⁹ See for example [Pod Point website: Cost of Charging an Electric Car](#) for the charging price, and [EDF website: Quel tarif EDF Pro choisir en 2024 ?](#) for typical electricity prices that are constant over the day

²⁰ See for example [Paris public charging points](#).

i.e. 10 hours, in order to target only the class-4 EV users. However, when the occupation level approaches the CSP capacity, i.e. 5 CPs, the pricing is more restrictive because both a price rate a^{park} increases and the duration threshold d^{thr} decreases with the occupancy level.

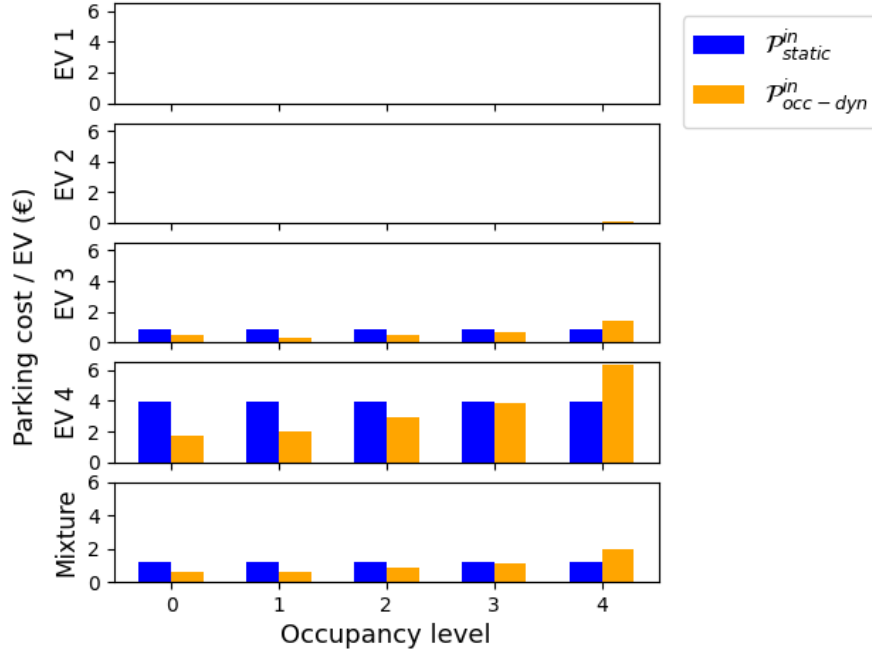
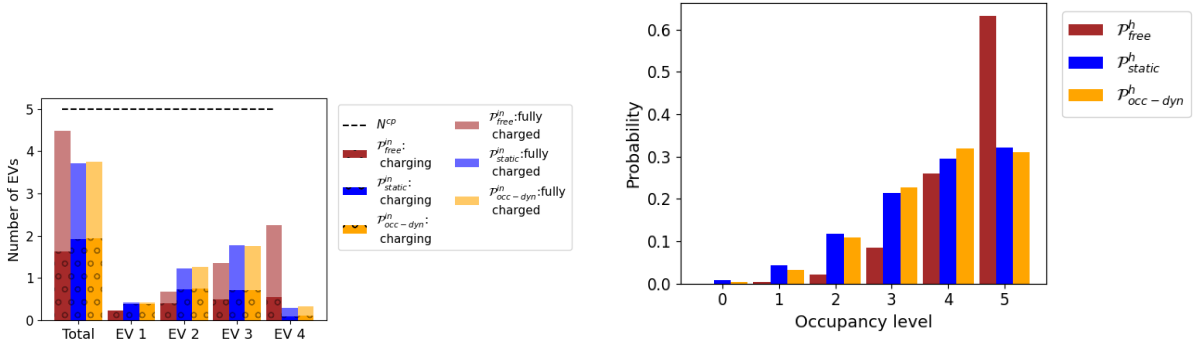


Figure 5.10: Expected parking cost C_i^{park} incurred by each EV class i , and by a randomly taken EV user (*Mixture*), with respect to the occupancy level.

Complementary to Figure 5.9, Figure 5.10 shows the expected parking cost incurred by each EV class, given by Equation (5.4), with respect to the occupancy level. The pricing actually only serves to specify the expected parking cost for each EV class. The expected parking cost is higher for EV classes with longer parking duration due to increasing parking cost structure $C^{\text{park}}(\cdot)$, given by Equation (5.2). One can observe that the strategy is to set, for each EV user, a more dissuasive parking price when the occupancy level approaches the CSP capacity. EV users of class 1 and 2 paying for parking with a probability of almost 0, the expected parking cost is close to 0 as well. Note that the expected parking cost faced by a random EV (see the graph with y-label *Mixture* in Figure 5.10) is lower for the dynamic pricing when there are 3 or less EVs at the CSP. Moreover, the overall expected parking cost faced by a random EV is 1.21 € for the static pricing, against 1 € for the dynamic pricing, meaning that the dynamic parking price enables a less restrictive parking cost.

Occupancy levels and objective value at optimum

Figure 5.11a shows the expected number of EV users parked at the CSP, given by Equation (5.17), split between the expected number of EV charging and fully charged, given by Equation (5.18). As the pricing is very dissuasive for the class-4 EV users, due to their long parking duration, only a few of them are joining the CSP, which makes their expected number significantly low compared to the case without a parking price. As there are less class-4 EV users joining the CSP with the pricing, the average parking duration of an EV user at the CSP is lower than in the case without a parking price. For this reason, there are fewer EV users at the CSP with a



(a) Expected number of EV users charging (area with circles) and fully charged (light area) at the CSP, in total and for each EV class. (b) Occupancy level probability distribution at the CSP.

Figure 5.11: Expected number of EV users at the CSP (5.11a), and occupancy level probability distribution (5.11b).

parking price (3.75 EVs) than without (4.5). On the contrary, the pricing is not too restrictive for the other classes (1, 2 and 3) and enables a better availability of the CPs, so that there are in average more EV users of class 1, 2 and 3 at the CSP with a parking price than without. For this reason, even though there are less EVs parked at the CSP in the case of a pricing, there are actually more EVs charging in average with pricing (about 2 EVs against 1.6 EVs without pricing).

Figure 5.11b shows the probability distribution of the occupancy levels. One can observe that the blocking probability, i.e. the probability that there are 5 EVs at the CSP, given by Equation (5.13), is about 0.63 without a parking price. It means that about 2 arriving EV users over 3 cannot go charging at the CSP because all the CPs are being occupied. The static and dynamic pricing enable respectively to reduce it to 32% and 31%, thus about one EV over three. In comparison to the static pricing case, in the dynamic pricing case, the percentage of time the CSP is almost, but not, fully occupied, i.e. when there are three or four EV users, is higher. This situation is optimal, because several EV users can benefit from parking and charging at the CSP without depriving a future arrival from joining the CSP.

Figure 5.12 shows how the social welfare is distributed, for the different parking pricing. The static and dynamic pricing provides respectively an increase of 17% and 18% of the charging profit, and 22% and 23% of the social welfare, which includes the revenue generated from the parking cost and the EV welfare. It shows that setting a parking price enables to significantly improve the profit of the CSP, without degrading the EV welfare. Indeed, the parking cost incurred by the EV users is compensated by the satisfaction generated from the overall availability of the CPs. However, the dynamic pricing does significantly improve the charging profit of the CSP, nor the social welfare. In addition, a dynamic pricing creates some complexity, both for the CSO for the implementation of such policy, and for the understanding of the EV users.

In this section, the parameter values are constant over time. Although a use-case with time-varying parameter values can be modeled by dividing the day into several stages, and solved independently for all the stages, it does not enable to anticipate future high or low parking demand. Next section provides a mathematical framework to optimize the parking pricing in a context of time-varying parameter values.

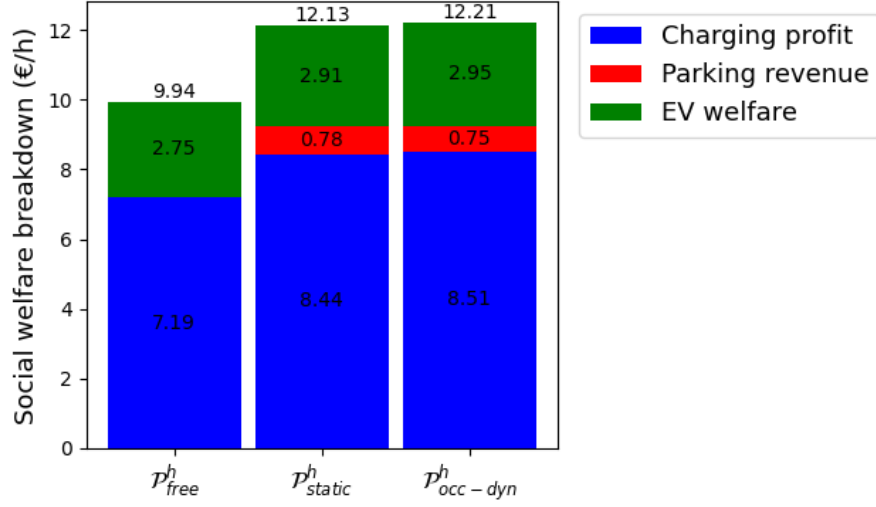


Figure 5.12: Social welfare breakdown for the different parking pricing.

5.4 Dynamic parking price in a time-dependent system

This section generalizes Section 5.3 to time-dependent frequency of EV arrivals at the site. In other words, EV users arrive at the site according to an in-homogeneous Poisson process, of parameter λ_t . At time t , the proportion $\sigma_{i,t}$ of EV users belonging to the classes is also time-dependent. λ_t and $\sigma_{i,t}$ are piece-wise constant. More specifically, let $\mathcal{T} := \{t_1, t_2, \dots, t_N\}$, with $N \in \mathbb{N}$, be a set of specific times of the day at which the frequency of arrivals evolve. Mathematically:

$$\forall t_j \in \mathcal{T}, \forall t \in [t_j, t_{j+1}], \quad \begin{cases} \lambda_t = \lambda_{t_j}, \\ \sigma_{i,t} = \sigma_{i,t_j}. \end{cases} \quad (5.19)$$

5.4.1 A dynamic in time parking pricing

In this section, the CSO adjusts the parking price coefficients $\mathbf{a}^{\text{park}}$ and $\mathbf{d}^{\text{thr}}$, not only as a function of the occupancy level, but also on the time of the day. More specifically, the parking price coefficients $a_{n,t}^{\text{park}}$ and $d_{n,t}^{\text{thr}}$ can evolve at any time $t \in \mathcal{T}$, regardless of the CSP occupancy. Increasing the cardinality of $\mathcal{T}$ improves the objective value but also increases the number of variables to be set by the CSO, which can significantly increase the computation time.

5.4.2 A time-inhomogeneous Markov chain

Similarly to Section 5.3, the number of EV users of each class at the CSP follows a continuous-time multi-dimensional and in-homogeneous Markov Chain $\mathcal{M}^{\text{in}} := \{X^{\text{in}}(t) := (X_1^{\text{in}}(t), X_2^{\text{in}}(t), \dots, X_n^{\text{in}}(t)), t \in [0, +\infty]\}$ [Vassiliou, 2022], defined by:

- the same set of states $\mathbb{S}$ as in Section 5.3;
- an initial probability distribution $\pi_0^{\text{in}} : \mathbb{S} \rightarrow [0, 1]$, with the the following notations: $\forall t \in \mathbb{R}_+, \forall s \in \mathbb{S}, \pi_t^{\text{in}}(s) := \mathbb{P}(X(t) = s)$;

- a time-dependent generator matrix G_t , which coefficients are the transition rates:

$$\forall t \in \mathbb{R}_+, \forall \mathbf{s}, \tilde{\mathbf{s}} \in \mathcal{S}, \quad G_t[\mathbf{s}, \tilde{\mathbf{s}}] := \begin{cases} \lambda_t \sigma_{i,t} \theta_i(a_{n(s),t}^{\text{park}} d_{n(s),t}^{\text{thr}}) & \text{if } \tilde{s}_i = s_i + 1 \text{ and } \tilde{s}_j = s_j \quad \forall j \neq i, \\ \nu_i s_i & \text{if } \tilde{s}_i = s_i - 1 \text{ and } \tilde{s}_j = s_j \quad \forall j \neq i, \\ 0 & \text{else.} \end{cases}$$

Contrary to Section 5.3, the time of the day also describes the state of the system. According to Equation (5.19), the generator matrix G_t is also piece-wise constant. π_t^{in} verifies the following ordinal differential equation:

$$\forall t, \quad \frac{\partial \pi_t^{\text{in}}}{\partial t} = G_t \pi_t^{\text{in}}. \quad (5.20)$$

Because of the non-commutativity of the generator matrix at two different time slots, a solution to Equation (5.20) is not trivial. π_t^{in} is therefore computed using Method 5.1.

Method 5.1: Probability distribution computation of π_t^{in}

Input: $\mathcal{T} := [t_1, \dots, t_n]$, t , π_0^{in}
Output: π_t^{in}
 $\pi \leftarrow \pi_0^{\text{in}};$
while $t_j < t$ **do**
 $\pi \leftarrow \exp(G_{t_j} \times (\min(t_{j+1}, t) - t_j)) \pi;$
 $j \leftarrow j + 1$
end
return $\pi;$

Equation (5.20) has an explicit solution when the generator matrix G_t is constant. The idea of Algorithm 5.1 is to solve Equation (5.20) recursively for each time interval such that the generator matrix is constant, using the following formula:

$$\forall t_j \in \mathcal{T}, \quad \pi_{t_{j+1}}^{\text{in}} = e^{G_{t_j} (t_{j+1} - t_j)} \pi_{t_j}^{\text{in}}.$$

Note however that the computation time of π_t increases linearly with the granularity of the time intervals $\mathcal{T}$. Moreover, compared to the *time-constant system*, where one has to solve the linear system given by Equation (5.6), the computation time using Method 5.1 can thus be significantly greater.

5.4.3 Cumulative charging profit maximization

Operator's expected profit The per-unit electricity price c_t^{elec} (€/kWh) incurred by the CSO is time-dependent. The per-unit profit $\Gamma_t^{\text{in}}(\cdot)$ generated from charging an EV with parking duration d , battery capacity b^{capa} and energy e^0 is the difference between the revenue and the cost:

$$\Gamma_t^{\text{in}}(d, b^{\text{capa}}, e^0) := \int_0^{+\infty} (a^{\text{ch}} - c_t^{\text{elec}}) \mathbb{1}_{\{t < \min(d, d^{\text{ch}})\}} dt. \quad (5.21)$$

The expected charging profit $\bar{\Gamma}_i^{\text{h}}$ from a new class- i EV arrival is deduced from Equation (5.21) through:

$$\hat{\Gamma}_{i,t}^{\text{in}} = \int_{\mathbb{R}_+^2} \left(\int_{\mathbb{R}_+} \Gamma_t^{\text{in}}(d, b^{\text{capa}}, e^0) \nu_i e^{-\nu_i d} dd \right) f_{\text{soc}}(db^{\text{capa}}, de^0). \quad (5.22)$$

The expected charging profit rate at time t can be deduced from the probability distribution π_t^{in} :

$$\bar{\Gamma}_t^{\text{in}} := \sum_{\mathbf{s} \in \mathbb{S}} \pi_t^{\text{in}}(\mathbf{s}) \sum_{i \in \mathcal{I}} \lambda \sigma_{i,t} \theta_i(a_{n(\mathbf{s}),t}^{\text{park}}, d_{n(\mathbf{s}),t}^{\text{thr}}) \times \hat{\Gamma}_{i,t}^{\text{in}}. \quad (5.23)$$

The CSO's cumulative expected charging profit optimization problem, $\mathcal{P}_{\text{fully-dyn}}^{\text{in}}$, is given by:

$$\begin{aligned} \mathcal{P}_{\text{fully-dyn}}^{\text{in}} : \quad & \max_{\mathbf{a}^{\text{park}}, \mathbf{d}^{\text{thr}}} \int_{\mathcal{T}} \bar{\Gamma}_t^{\text{in}} dt \\ \text{s.t.} \quad & \left. \begin{aligned} 0 \leq a_{n,t}^{\text{park}} \leq a^{\text{park,max}} \\ 0 \leq d_{n,t}^{\text{thr}} \leq d^{\text{thr,max}} \end{aligned} \right\} \forall t \in \mathcal{T}, \forall n \in \{0, \dots, N^{\text{cp}}\}, \end{aligned} \quad (5.24)$$

with $\bar{\Gamma}_t^{\text{in}}$ given by Equation (5.23). The time-varying parking price enables to follow appropriately the varying CP demand over the day. In order to compare the significance of time and state varying parking price, we compare problem $\mathcal{P}_{\text{occ-dyn}}^{\text{in}}$ with four other problems, namely:

- $\mathcal{P}_{\text{occ-dyn}}^{\text{in}}$, for which the parking price varies only with respect to the CSP occupancy, i.e. the variables $\mathbf{a}^{\text{park}}$ and $\mathbf{d}^{\text{thr}}$ are N^{cp} dimensional vectors;
- $\mathcal{P}_{\text{time-dyn}}^{\text{in}}$, for which the parking price varies only with respect to the time, i.e. the variables $\mathbf{a}^{\text{park}}$ and $\mathbf{d}^{\text{thr}}$ are $|\mathcal{T}|$ -dimensional vectors;
- $\mathcal{P}_{\text{static}}^{\text{in}}$, for which the parking price is fixed for the whole day regardless the CSP occupancy or the time, i.e. the variables a^{park} and d^{thr} are scalars;
- $\mathcal{P}_{\text{free}}^{\text{in}}$, for which the parking price is set as $a^{\text{park}} \equiv 0$ for the whole day (there is no optimization).

Table 5.13 summarizes the differences in dynamics between the different optimization problems.

Notation	Pricing dynamics	
	Occupancy level	Time of the day
$\mathcal{P}_{\text{fully-dyn}}$	Yes	Yes
$\mathcal{P}_{\text{time-dyn}}$	No	Yes
$\mathcal{P}_{\text{state-dyn}}$	Yes	No
$\mathcal{P}_{\text{static}}$	No	No
$\mathcal{P}_{\text{free}}$	No parking price: $\mathbf{a}^{\text{park}} \equiv 0$	

Table 5.13: Pricing dynamics for different optimization problems.

Remark 5.3. Problem (5.24) does not include any condition about the state of the system at the end of period $\mathcal{T}$. For example, if $\mathcal{T}$ represents a day and several days present similar EV arrivals/departures flows, one could consider a cycling condition $\pi_T^{\text{in}} = \pi_0^{\text{in}}$, with T the last instant in $\mathcal{T}$.

5.4.4 Some other metrics considered

Similarly to section 5.3, some metrics are defined for numerical analysis. The expression of the metrics listed below are very similar to the ones in Section (5.3), and mostly consists in replacing $\hat{\pi}$ by π_t .

Blocking probability The probability $\mathbb{P}_t^{\text{in,blocked}}$ that a user upon arrival at time t is blocked is:

$$\mathbb{P}_t^{\text{in,blocked}} = \sum_{\mathbf{s} \in \mathbb{S}} \pi_t(\mathbf{s}) \mathbb{1}_{\{n(\mathbf{s}) = N^{\text{CP}}\}}. \quad (5.25)$$

Joining probability Similarly, for each class i , the probability $\mathbb{P}_{i,t}^{\text{join}}$ that upon arrival an EV user from class i joins the CSP is the probability that a CP is available times the probability that an EV user is willing to join the CSP:

$$\mathbb{P}_{i,t}^{\text{in,join}} = (1 - \mathbb{P}_t^{\text{in,blocked}}) \times \mathbb{P}_{i,t}^{\text{in,acc}}, \quad (5.26)$$

with $\mathbb{P}_{i,t}^{\text{in,acc}}$ the probability at time t that an class- i EV user is willing to join the CSP given by:

$$\mathbb{P}_{i,t}^{\text{in,acc}} = \frac{1}{(1 - \mathbb{P}_t^{\text{in,blocked}})} \times \sum_{\mathbf{s} \in \mathbb{S}} \pi_t(\mathbf{s}) \theta_i(a_{n(\mathbf{s}),t}^{\text{park}}, d_{n(\mathbf{s}),t}^{\text{thr}}) \mathbb{1}_{\{n(\mathbf{s}) < N^{\text{CP}}\}}. \quad (5.27)$$

Moreover, the probability that a user, randomly taken, joins the CSP is given by:

$$\mathbb{P}_t^{\text{in,join}} := \sum_{i \in \mathcal{I}} \sigma_i \mathbb{P}_{i,t}^{\text{in,join}}. \quad (5.28)$$

Mean number of EV users at the CSP The mean number $\bar{N}_t^{\text{in,park}}$ of EV users at the CSP at time t is:

$$\bar{N}_t^{\text{in,park}} := \sum_{\mathbf{s} \in \mathbb{S}} \pi_t(\mathbf{s}) n(\mathbf{s}). \quad (5.29)$$

Mean number of EV users charging / fully charged at the CSP The expected total number of EV users fully charged, or equivalently, not charging, at the CSP is given by:

$$\bar{N}_t^{\text{in,full}} := \sum_{\mathbf{s} \in \mathbb{S}} \pi_t(\mathbf{s}) \sum_{i \in \mathcal{I}} s_i \mathbb{P}_{i,t}(F|\mathbf{s}), \quad (5.30)$$

with:

$$\mathbb{P}_{i,t}(F|\mathbf{s}) = \frac{1}{\theta_i(a_{n(\mathbf{s}),t}^{\text{park}}, d_{n(\mathbf{s}),t}^{\text{thr}})} \int_{\mathbb{R}_+^2} e^{-\nu_i \frac{b^{\text{capa}} - e^0}{p^{\text{ch}}}} \hat{\theta}_i(b^{\text{capa}}, e^0, a_{n(\mathbf{s}),t}^{\text{park}}, d_{n(\mathbf{s}),t}^{\text{thr}}) f_{\text{soc}}(db^{\text{capa}}, de^0).$$

The mean number $\bar{N}^{\text{in,ch}} = \bar{N}^{\text{in,park}} - \bar{N}^{\text{in,full}}$ of EVs charging at the CSP is directly deduced from Equations (5.29) and (5.30).

Overall metrics over the day For any process $x_{(\cdot)} := \{x_t, t \in [0, T]\}$, $h(x_{(\cdot)}) = \frac{1}{T} \int_0^T x_t dt$ gives the value of the corresponding metric at a random point in time if this metric is a probability, or the overall value over the day otherwise. For example, $h(\mathbb{P}_{(\cdot)}^{\text{in,blocked}})$ gives the probability, at a random point in time, that an EV user, upon arrival, is blocked; and $h(\bar{N}_{(\cdot)}^{\text{in,park}})$ gives the overall mean number of EVs at the CSP over the day.

Similarly, for any process $x_{(\cdot)}$, $\text{cum}(x_{(\cdot)}) := \int_0^T x_t dt$ gives the cumulative value of the process $x_{(\cdot)}$ over the day. For example, $\text{cum}(\bar{\Gamma}_{(\cdot)}^{\text{in}})$ gives the cumulative charging profit over the day.

Parameters		EV class indices		
Description	Notation	1	2	3
Exp. parking duration (hour)	ν_i^{-1}	1	2	5
Proportion	$\sigma_{i,t}$	0.33	0.33	0.33

Table 5.14: EV classes characteristics.

Time intervals	Avg frequency of arrivals λ_t (h^{-1})	Electricity cost c_t^{elec} (€/kWh)
08:00 - 10:00	5	0.25 (PH) ²¹
10:00 - 12:00	2	
12:00 - 14:00	10	0.15 (OPH)
14:00 - 18:00	2	
18:00 - 22:00	10	0.25 (PH)
22:00 - 08:00	1	0.15 (OPH)

Table 5.15: Average arrival frequency over the day and Time-of-Use electricity costs.

5.4.5 Numerical results and analysis

Tables 5.14 and 5.15 show respectively the EV classes characteristics, and the average number of arrivals per hour and the Time-of-Use electricity costs.²² Note that the computation time is significantly greater (more than 100 times more) than in the *time-constant system* case, in particular because the number of variables is much greater. Thus, only 3 EV classes are considered in this use-case (compared to 4 for the *time-constant system* case). Although the average arrivals frequency varies over time, the proportion of EV classes remains constant and equally distributed. The EV charging characteristic distribution is the same as in the time-homogeneous case, and is shown in Table 5.7.

Figure 5.16 shows the expected parking cost that would be faced by an EV user, randomly taken, upon arrival (a mixture between all the EV classes) if she/he joins the CSP, at each time of the day. As described in Remark 5.3, the problem $\mathcal{P}_{\text{fully-dyn}}^{\text{in}}$ and its auxiliary problems do not include any terminal condition, meaning in particular that the state at time 7:59 (the end of the period considered) is not necessarily the same as the state at time 8:00 (the beginning of the period considered), because the future after 7:59 is neglected. For $\mathcal{P}_{\text{static}}^{\text{in}}$ (blue) and $\mathcal{P}_{\text{occ-dyn}}^{\text{in}}$ (orange), the pricing policy is constant over the day (by definition), and must be a trade off between being enough dissuasive because of peak arrivals, for example during the evening, but not too much because of off-peak arrival period, for example during the mid-afternoon or the night. In $\mathcal{P}_{\text{occ-dyn}}^{\text{in}}$, although the pricing can evolve with the occupancy level, it cannot adapt to future high or low frequency of arrivals. In opposition, one can observe that for $\mathcal{P}_{\text{time-dyn}}^{\text{in}}$ (green) and $\mathcal{P}_{\text{fully-dyn}}^{\text{in}}$ (yellow), the pricing is much more dissuasive during peak arrivals. Moreover, the pricing anticipates peak and off-peak arrivals. For example, the expected cost decreases with time between 20:00 and 22:00, although the peak arrivals stops only at 22:00. The same phenomenon occurs on the opposite site, for example the expected parking cost increases from 11:00 to 12:00, although the peak arrival starts at 12:00.

²¹ Standard terminology in Time-of-Use electricity price. PH: Peak Hour; OPH: Off Peak Hour

²² See for example EDF [EDF website: Quel tarif EDF Pro choisir en 2024 ?](#) for typical Time-of-Use electricity prices

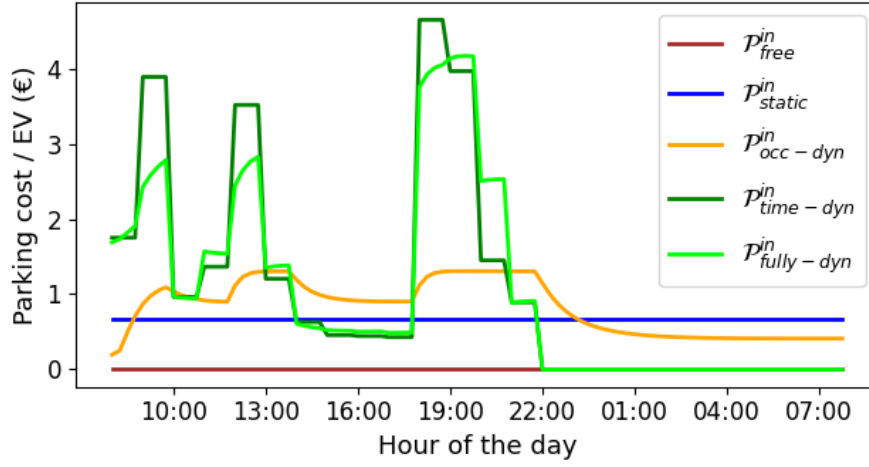


Figure 5.16: Expected parking cost over time incurred by a EV user randomly taken upon arrival if joining the CSP. *By definition, the pricing is constant over the occupation level for $\mathcal{P}_{static}^{in}$ (blue) and is null for $\mathcal{P}_{free}^{in}$ (brown). In $\mathcal{P}_{occ-dyn}^{in}$, although the pricing is constant over time (by definition), the expected parking cost follow the peak and off peak frequency of arrivals because it has an influence on the occupancy level probability distribution.*

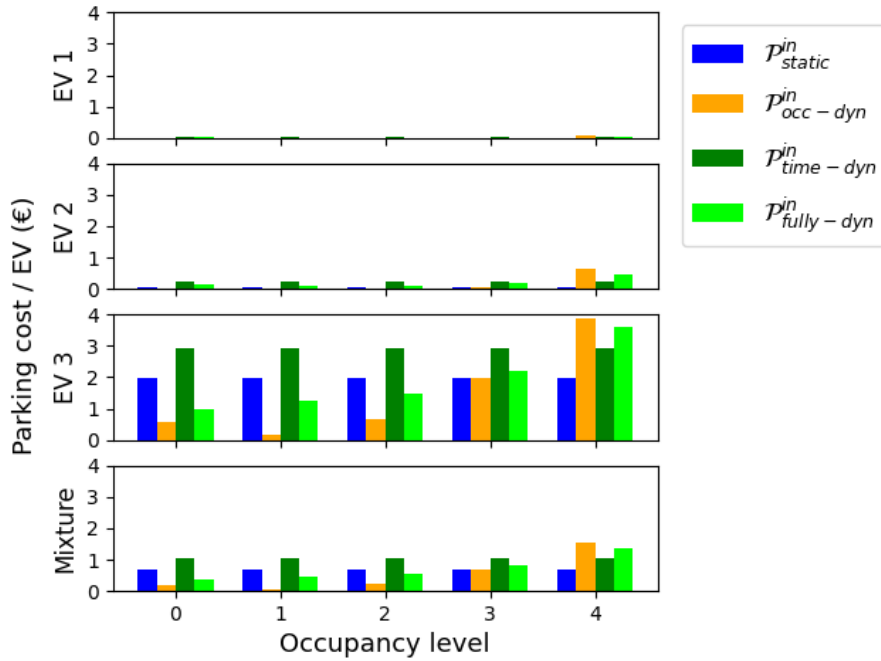


Figure 5.17: Expected parking cost incurred by each EV class, and by a mixture, at a random point in time if joining the CSP, with respect to the occupancy level. *By definition, the pricing is constant over the occupation level for $\mathcal{P}_{static}^{in}$ (blue) and $\mathcal{P}_{time-dyn}^{in}$ (green).*

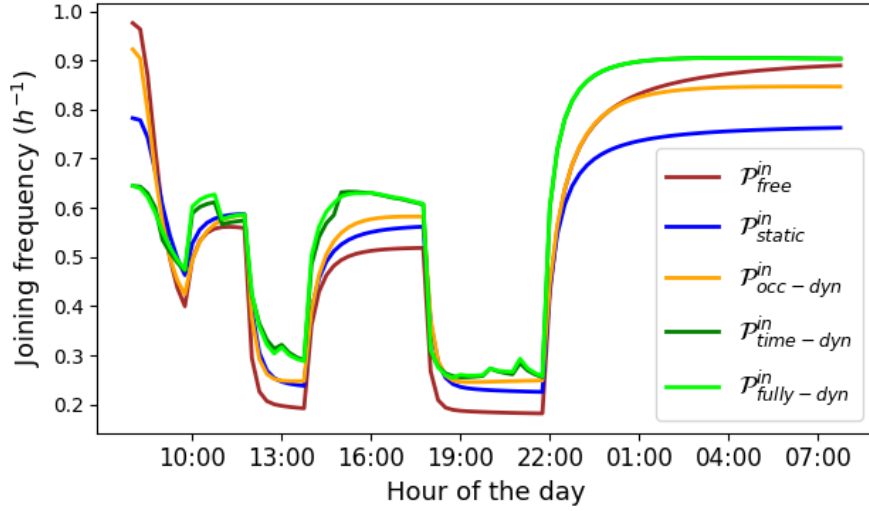


Figure 5.18: Expected frequency of EV users joining the CSP over time.

Figure 5.17 shows the expected parking cost incurred by each EV class, and by a randomly taken EV user and at a random point in time, with respect to the occupancy level. As in the homogeneous case, the expected parking cost is higher for the EV users with longer parking duration. As an example, parking in the case $\mathcal{P}_{\text{time-dyn}}^{\text{in}}$ is almost free of charge for EV class 1 (1 hour of parking in expectation), against 0.26 € for class-2 EV users (2 hours of parking in expectation) and 3 € for class-3 EV users (5 hours of parking in expectation). Note that in expectation, parking is generally more expensive for $\mathcal{P}_{\text{time-dyn}}^{\text{in}}$ and $\mathcal{P}_{\text{static}}^{\text{in}}$ compared to $\mathcal{P}_{\text{fully-dyn}}^{\text{in}}$ and $\mathcal{P}_{\text{occ-dyn}}^{\text{in}}$ when the occupation level is low, and conversely when the occupation rate is high. Indeed, the pricing for $\mathcal{P}_{\text{time-dyn}}^{\text{in}}$ and $\mathcal{P}_{\text{static}}^{\text{in}}$ cannot be dynamically adjusted with the occupation level.

Figure 5.18 shows the expected frequency of EV users joining the CSP over time, i.e. the ones both willing to join the CSP (judging that the parking pricing is reasonable) and finding an available CP, over time. One can observe that for $\mathcal{P}_{\text{time-dyn}}^{\text{in}}$ and $\mathcal{P}_{\text{fully-dyn}}^{\text{in}}$, there are always more EV users joining in expectation (except at the very beginning of the day). Indeed, as discussed previously for Figure 5.16, these pricing dynamics enable to adequately anticipate the future peak / off-peak arrivals, to adjust at all times the right proportion of long-duration EV users. It enables to keep most of the time an available CP, without discouraging too much EV users from joining the CSP. Concerning $\mathcal{P}_{\text{time-dyn}}^{\text{in}}$ and $\mathcal{P}_{\text{fully-dyn}}^{\text{in}}$, the pricing enables to increase the number of users joining the CSP during peak arrivals, but at the cost of discouraging some users from joining the CSP even at the times when there is no congestion issue, for example during the night.

Figure 5.19 shows the expected number of EV parked at the CSP (split between the number of EV charging and fully charged) at a random point in time. As in the homogeneous case, even though it decreases the total number of EV parked at the CSP in expectation, the pricing enables more EV charging in expectation. Recall that the objective of the optimization problems is to maximize the charging profit, which is highly correlated to the expected number of EV charging at the CSP. One can also observe the huge decrease in class-3 EV users (with 5 hours of parking in expectation) with the pricing, as it is very dissuasive for EV with long parking duration.

Figure 5.20 shows the social welfare breakdown, which includes the charging profit, the

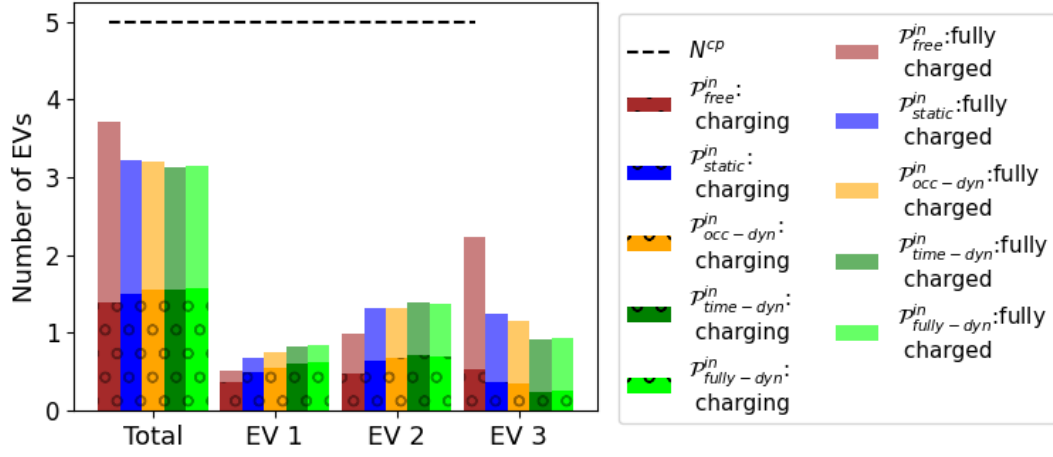


Figure 5.19: Expected number of EV charging (area with circles) and fully charged (light area) at the CSP at a random point in time, for each EV class and in total.

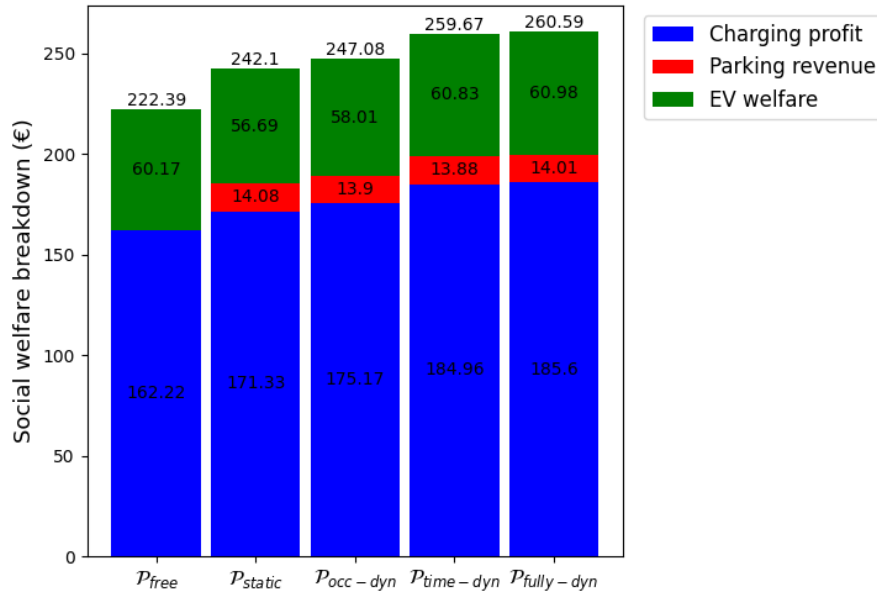


Figure 5.20: Social welfare breakdown for the different pricing dynamics. One can observe the following Pareto order between the pricing dynamics in terms of charging profit and social welfare: $P_{fully-dyn} \succ P_{time-dyn} \succ P_{occ-dyn} \succ P_{static} \succ P_{free}$.

revenue generated with the parking price, and the EV welfare, defined in Equation (5.3), for all the different pricing dynamics. The pricing in $\mathcal{P}_{\text{fully-dyn}}$ provides higher charging profit (blue area) than all the other pricing dynamics, and $\mathcal{P}_{\text{time-dyn}}$ and $\mathcal{P}_{\text{occ-dyn}}$ than $\mathcal{P}_{\text{static}}$. Indeed, the consideration of additional variables (here $\mathbf{a}^{\text{park}}$ and $\mathbf{d}^{\text{thr}}$ that are vectors of larger dimension) can only provide a higher objective function, here the charging profit. Apart from this, one can observe that pricing in $\mathcal{P}_{\text{time-dyn}}$ provides better charging profit and social welfare than $\mathcal{P}_{\text{occ-dyn}}$. However, this effect highly depends on the dynamicity of the frequency of arrivals at the site over the day. For example, if the frequency of arrivals is constant over the day, $\mathcal{P}_{\text{time-dyn}}$ does not provide any benefit compared to $\mathcal{P}_{\text{static}}$. The practical complexity of implementing a pricing mechanism that depends on the occupancy level should also be considered. Considering this, and because $\mathcal{P}_{\text{fully-dyn}}$ (resp. $\mathcal{P}_{\text{occ-dyn}}$) does not improve much the charging profit compared to $\mathcal{P}_{\text{time-dyn}}$ (resp. $\mathcal{P}_{\text{static}}$), $\mathcal{P}_{\text{time-dyn}}$ and $\mathcal{P}_{\text{static}}$ may be better solutions for a CSO to deal with congestion in CP availability. $\mathcal{P}_{\text{time-dyn}}$ increases the charging profit by 8% (resp. 11%) compared to $\mathcal{P}_{\text{static}}$ (resp. $\mathcal{P}_{\text{free}}$). In addition, $\mathcal{P}_{\text{static}}$ increases the charging profit by 3% compared to $\mathcal{P}_{\text{free}}$. Regarding EV welfare, it is only mildly affected by the pricing. Time-dynamic pricing does not reduce EV welfare, and static pricing results in only a slight decrease. Although some EV users may need to pay parking fees or choose not to use the CSP, this is offset by the increased availability of CPs.

5.5 Conclusion

In order to effectively manage a CSP in situations of potential high demand in CPs, this work suggests a threshold-based parking duration pricing strategy. The impact of this pricing strategy on EV users' willingness to charge at the CSP is modeled and analyzed. The pricing strategy is set to dissuade the EV users with longer parking duration to join the CSP, with a view to decrease the proportion of EV users blocked because all the CP at the CSP are occupied, thereby increasing the profit generated from charging the EV users. The occupancy level at the CSP is modeled as a continuous-time Markov chain incorporating a parking pricing that can evolve dynamically. Two cases are investigated. The *time-constant system* case, in which the system parameters such as the arrivals frequency, or the parking durations, are constant over time. In such a case, there exists a unique stationary probability distribution of the associated Markov chain, given by analytical expressions. The stationary charging profit maximization, and other performance metrics, are formalized. In the *time-varying system* case, the system parameters can evolve dynamically over time in a piece-wise constant manner. The cumulative expected charging profit over a given period, a day in the numerical use-case, is formalized by solving a set of systems of linear equations. In both scenarios, key metrics, such as the number of EVs charging or fully charged, are formalized, enabling numerical analysis without the need for system simulation. In the proposed use-case, the numerical results show that, in the *time-constant system* case, a static pricing enables an increase of the expected charging profit by 17% against no parking price, and that an occupation-dynamic pricing does not significantly improve the performance compared to a static pricing. When the parameter values evolve over time, similar observations hold: occupation-dynamic pricing does not significantly improve the charging profit compared to their counterpart without occupation-dynamics. However, a time-dynamic pricing significantly increases the charging profit.

Future research aims to reduce computation time for dynamic system parameters, as the current method faces exponential growth in computation time with the number of CPs, limiting its application to small CSPs. A key research direction would involve reducing the number of states,

potentially at the expense of some accuracy in the information and optimization outcomes. In particular, the framework of *Partially Observable Markov Decision Process* with online heuristic algorithms, such as the ones presented in [Silver and Veness, 2010], seems relevant for our application. An important generalization of this work involves incorporating heterogeneity in the charging rate limitation of EVs and CPs. In practice, CSPs often feature various types of CPs, each offering different nominal charging power rates. Thus, the parking pricing policy could vary with the type of CP. In such cases, the number of EV parked (and their associated class) at the CSP is not sufficient to fully describe the state of the system.

Chapter 6

Price-menu design for electric vehicle charging pools providing demand response

This chapter is largely based on the following paper:

[Dupont et al., 2024] Dupont, A., Breal, J. B., Hayel, Y., & Andrianesis, P. (2024). Optimal price menu design of electric vehicle charging stations providing demand response. *Electric Power Systems Research*, 235, 110798. <https://doi.org/10.1016/j.epsr.2024.110798>

Notation	Signification
$\mathcal{K}$	Set of charging options
p_k	Individual charging power rate for option k (in kW)
π_k	Charging price for option k (in €/kWh)
λ_t	Electricity cost at time t (in €/kWh)
d_i	Parking duration of class- i EV users (in hour)
$e_i^{\max}$	Maximum energy that can be received by a class- i EV (in kWh)
e_i^0	Energy already stored at arrival of a class- i EV users (in kWh)
θ_i	Proportion of class- i EV users ($\in [0, 1]$)
c_i	Average electricity cost during a class- i EV charging session (in €/kWh)
Π_i	CSO profit function for charging class- i (outputs are in €)
W_i	Welfare function of class- i EV users (outputs are in €)
W_i^S	Social welfare function when charging a class- i EV user (outputs are in €)

Table 6.1: Main parameter notations used in this chapter. *Red*: CSP parameters/decision variables; *Blue*: EV users parameters.

6.1 Introduction

A public CSO, with a capability to modulate its aggregate power consumption for a given period, can provide some type of remunerated demand response service to the grid operator. However, modulating the aggregate power consumption can be at the expense of the quality of service

provided to the EV users. Menu-based approach, with options differentiated by their quality of service and price, enables to account for users heterogeneity in their energy need.

6.1.1 Related works on price-menus

Such a menu typically consists of a price that can be associated with a delay in the completion of charging [Lu et al., 2023, Zeng et al., 2021, Wu et al., 2021, Mathioudaki et al., 2021] (assuming that the utility of EV users depends on the delay of their departure), and/or the energy that EV users receive [Bitar and Xu, 2017, Alinia et al., 2019, Ghosh and Aggarwal, 2018].

Considering the flexibility of EV users in the completion of charging, some works [Lu et al., 2023, Zeng et al., 2021, Wu et al., 2021] propose some type of “deadline-differentiated” pricing, i.e., a price associated with a deadline. In [Lu et al., 2023], the CSO solves a profit maximization mixed integer quadratic programming problem to determine optimal prices that incent EVs to prolong their charging sessions when the cost of electricity is low. In the same spirit, [Zeng et al., 2021] adds a linear penalty in the CSO aggregate power consumption, and uses block coordinated descent to solve a profit maximization problem that considers bounded rationality for the EV user deadline choice. In [Wu et al., 2021], the focus shifts to identifying optimal deadline value offers, and the CSO solves an optimal control problem, aiming at smoothing the aggregate power consumption that is associated with a convex cost for electricity. In [Mathioudaki et al., 2021] the charging intervals and corresponding prices are determined in an online fashion, considering a bound on the aggregate power consumption, and aiming at minimizing the overall delay in the completion of charging.

The aforementioned works [Lu et al., 2023, Zeng et al., 2021, Wu et al., 2021, Mathioudaki et al., 2021] account for the EV user flexibility in the charging deadline but not in their energy demand. Conversely, [Bitar and Xu, 2017, Alinia et al., 2019] account for the flexibility in the energy demand but consider fixed deadlines. In a rather general setting for electric power loads [Bitar and Xu, 2017], consumers define their energy demand for time intervals before their deadline, and pay a price that depends on the aggregate power consumption. Using a game theoretic setting, it is found that marginal cost pricing, combined with “earliest-deadline-first” charging, maximizes the CSO profit as well as the social welfare provided that the EV user willingness to charge is greater than the electricity cost. In [Alinia et al., 2019], several online scheduling algorithms are proposed that maximize social welfare, in which the CSO ensures a guaranteed amount of energy upon the EV arrival and until the deadline, while taking into account constraints in aggregate power and concurrent EV charging.

In [Ghosh and Aggarwal, 2018], the CSO offers a price menu that considers the EV user flexibility in both the energy demand and the deadline for charging. Electricity is sourced either from a local renewable energy supply at zero cost or through purchases from the grid. The optimal pricing is provided as a function of the outcome of the charging scheduling problem (cost minimization linear programming problem), both for the social welfare and the CSO profit maximization. Conversely, [Hao et al., 2022, Zhang et al., 2019] consider inflexible EV users in both the energy demand and the deadline. In [Hao et al., 2022], the CSO price menu consists of several options with different energy amounts and deadlines; the arriving EV users choose whether to charge or not depending on their utility and the charging price of the option that matches their requirements. The profit and social welfare maximization are formulated as a partially observable Markov decision process and solved through deep reinforcement learning.

6.1.2 Proposed approach

Most of the aforementioned works require EV users to specify or provide some type of information on their parking duration (deadline) and/or their energy demand. Apart from privacy concerns, it is not unreasonable to assume that many users may not have a specific requirement for energy, and/or there may be some uncertainty in their parking duration. Furthermore, in the deadline-differentiated pricing, there may be no guarantee on the amount of energy charged if the EV user leaves sooner than expected (also not unreasonable to assume).

For these reasons, a price menu that offers a guaranteed power rate for a certain price is suggested in this Chapter, so that EVs can choose among different power rates, thus securing a certain quality of service. The aim of this chapter is to investigate the problem of the CSO price menu design and its capability to provide demand response. We consider a CSO equipped with a large enough number of charging points, i.e., the station's capacity (in terms of charging points) does not become a limiting factor for serving EVs. We also consider EV users that view charging more as an opportunity rather than an essential need. Hence, the parking duration depends only on their on-site activities — see, e.g., [Pierce and Shoup, 2013] on the in-elasticity of the parking demand. A profit maximizing CSO would offer a menu with different prices per power rate, which can adapt to heterogeneous requirements of EV users, using information that may be either provided by the users or obtained from the CSO statistics/data collected on its EV user utility/characteristics/behavior. Assuming a certain price for offering demand response in real time, the CSO could then dynamically adjust the price menu to reduce its aggregate power consumption.

The contribution of this chapter is three-fold (1) The price menu design problem of an EV CSO that differentiates the options in the power rate, instead of the deadline, is formulated as a Mixed Integer Linear Programming (MILP) problem. (2) A setting for the provision of demand response by the CSO is proposed, given a price for reducing its aggregate power consumption over a certain time period, by adjusting the price menu in real time. (3) Insights through numerical experimentation on the construction of the optimal price menu, and the trade-offs considered in terms of the CSO profits and the social welfare under different prices for electricity and demand response are provided.

The remainder of this Chapter is organized as follows. Section 6.2 presents the preliminaries for the CSO and EV models. Section 6.3 formulates the price menu design problem, and Section 6.4 describes the provision of demand response by the CSO. Section 6.3.3 discusses a numerical experimentation, and Section 6.5 concludes and provides further research directions.

6.2 Model preliminaries

In this section, we introduce the EV CSO offered charging options (in Subsection 6.2.1), the utility function of EV users, (in Subsection 6.2.2), and we define the CSO profit (in Subsection 6.2.3) and the social welfare (in Subsection 6.2.4).

6.2.1 Operator offered charging options

The CSO offers a discrete set of K options, denoted by $\mathcal{K} = \{1, \dots, K\}$, where option k corresponds to a constant power rate p_k and a price per energy unit (kWh) π_k . For notational simplicity, we denote the set of available options, which includes option $k = 0$ corresponding to “not charging,” i.e., $p_0 = 0$ and $\pi_0 = 0$, by $\mathcal{K}^+ := \mathcal{K} \cup \{0\}$.

The CSO options are ordered from lowest power rate to the highest, i.e., $p_{k-1} < p_k, \forall k \in \mathcal{K}$, with prices that are non-decreasing with the power rate, i.e.,

$$\pi_{k-1} \leq \pi_k, \forall k \in \mathcal{K}. \quad (6.1)$$

In most of the charging stations in reality, a higher charging power is offered at a higher price, see for example [Charge Place Scotland, nd].

6.2.2 User utility

Let EV class $i \in \mathcal{I}$, where $\mathcal{I} = \{1, \dots, I\}$ is the set of EV classes, have (upon arrival) an initial State of Charge (SoC)¹ denoted by e_i^0 , and parking duration denoted by d_i . Let $e_i^{\max}$ denote the EV battery capacity, and e_i the SoC at the time of departure.

For brevity, we refer to EV i instead of EV class i . If EV i charges at a power rate p , its SoC at the time of departure is given by:

$$e_i = e_i^0 + p d_i. \quad (6.2)$$

Given any option k , EV i can charge at most $\min\{e_i^{\max} - e_i^0, p_k d_i\}$, so that the battery capacity is not exceeded. We can therefore denote the options available to EV i using a subset $\mathcal{K}_i = \{1, \dots, K_i\} \subseteq \mathcal{K}$, where K_i is defined as the highest available power rate such that the battery capacity is not exceeded given the parking duration d_i .

Let U_i denote the utility of EV i , which depends on the SoC e_i at the time of departure, and is increasing and concave in e_i [Samadi et al., 2010]. Essentially, increasing the SoC increases the EV utility but with diminishing returns (due to the concavity). From Equation (6.2), we can express the dependency of the EV utility on the power rate p using $U_i(p)$.

Let $u_i(p_k)$ denote the marginal utility of EV i choosing option k related to power p_k , i.e.,

$$u_i(p_k) := \left. \frac{\partial U_i(p)}{\partial p} \right|_{p=p_k}. \quad (6.3)$$

Given the increasing and concave utility in e_i , which also, from Equation (6.2), implies that the utility is increasing and concave in P , the marginal utility is non-negative and decreasing in k , i.e.,

$$u_i(p_{k-1}) \geq u_i(p_k) \geq 0, \quad \forall k \in \mathcal{K}_i^+. \quad (6.4)$$

Eq. Equation (6.4) simply states that the marginal utility is positive but decreasing in the power rate, hence, there are diminishing returns when choosing an option with a higher power rate.

6.2.3 Operator profit

The CSO buys electricity from the grid and sells electricity to EVs. Let λ_t be the price (per kWh) at which the CSO buys electricity at time t .² Let c_i denote the average cost (per kWh) over time that the CSO incurs when charging EV i , which is given by:

$$c_i = \frac{1}{d_i} \int_{\tau_i^0}^{\tau_i^0 + d_i} \lambda_t dt, \quad (6.5)$$

where τ_i^0 and $\tau_i^0 + d_i$ are the arrival and departure times, respectively of EV i .

¹ The term "initial SoC" is equivalent to the term "energy at arrival" used in the previous chapters.

² It can be for example the day-ahead electricity market price.

When EV i chooses option k , the CSO charges π_k (per kWh), whereas incurs a cost c_i (per kWh). Hence, the CSO profit for charging EV i choosing option k , denoted by $\Pi_i(\pi_k)$ to indicate the dependence on π_k , is then given by:

$$\Pi_i(\pi_k) = (\pi_k - c_i) p_k d_i. \quad (6.6)$$

6.2.4 Social welfare

Let $W_i(\pi_k)$ denote the welfare of EV i choosing option k , which is the difference between their utility $U_i(p_k)$, and the cost of charging at the charging station, i.e.,

$$W_i(\pi_k) := U_i(p_k) - \pi_k p_k d_i. \quad (6.7)$$

Note that we express the dependence of the welfare on the price π_k , whereas the dependence of the utility is on the associated power rate p_k .

EV i aims at maximizing her welfare, hence, the optimal choice, k_i^* , is given by:

$$k_i^* = \arg \max_{k \in \mathcal{K}_i^+} W_i(\pi_k). \quad (6.8)$$

Using the concavity of the utility function and the price ordering Equation (6.1), one can show that for any set of options the welfare increases (in k) until the option that corresponds to the optimal choice for the EV user, and then decreases. Hence, one can find the optimal choice, with a simple search among ordered options (from the lowest power rate to the highest power rate) until the welfare begins to decrease.

The *social welfare*, $W_i^S(\pi_k)$, i.e., the sum of the CSO profit and the welfare of EV i choosing option k , is given by:

$$W_i^S(\pi_k) := \Pi_i(\pi_k) + W_i(\pi_k) = U_i(p_k) - c_i p_k d_i, \quad (6.9)$$

where we added the dependence on π_k , which naturally associates to the power rate p_k . The next section offers insights into user choices and the pricing that maximizes either the CSO's profit or social welfare through an example.

6.2.5 Illustrative example: single price, single class, quadratic utility

Consider a single class of EVs (i.e., a homogeneous fleet of vehicles), with a quadratic utility function [Samadi et al., 2010] given by:³

$$U(P) = \alpha [P d - \frac{1}{2} \beta (P d)^2], \quad (6.10)$$

where index i is dropped for brevity, and α , β , are positive scalars that parameterizes EV willingness to charge while ensuring an increasing utility for practical power rates. From Equation (6.10), the marginal utility is given by:

$$u(P) = \alpha d (1 - \beta P d), \quad (6.11)$$

i.e., decreasing in the power rate.

³ Similarly to Chapter 5.

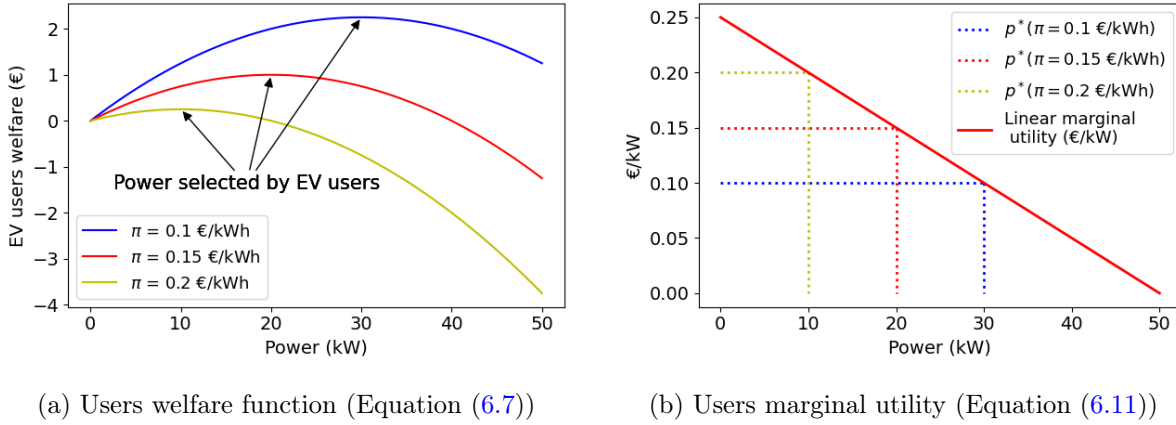


Figure 6.2: Users welfare function and marginal utility, for $\alpha = 0.25$ (€/kWh), $\beta = 0.02$ (kWh⁻¹) and $d = 1$ (hour). The optimal charging power is such that the associated marginal utility equals the price (see Figure 6.2b). The users optimal power rate decreases with the price, as formalized by Equation (6.12).

Consider a CSO setting the same price π for all options, and that EVs can choose any power rate (i.e, there are infinitely many options). From Equation (6.7), the power rate P^* maximizing the EV welfare (at which the marginal utility equals πd) is:

$$P^* = \frac{\alpha - \pi}{\alpha \beta d}, \quad (6.12)$$

where it is assumed that $\pi \leq \alpha$. Note that when $\pi \geq \alpha$, then $P = 0$, since the marginal utility given by Equation (6.11) is always lower than the price (πd).

Figure 6.2 gives an illustration of the users welfare function and marginal utility, associated with the power rate P^* , given by Equation (6.12), maximizing their welfare. Given P^* , the optimal power rate decreases in the price (which derives from the concavity of the utility function). A profit-maximizing CSO considers the standard trade-off between increasing the price but selling a lower quantity (here the power). We assume that the users coefficient α is greater than the average electricity price c (otherwise, the CSO optimal profit and EV welfare are 0). Replacing the optimal power rate P^* in Equation (6.6), and solving for the CSO profit-maximizing price, π_{Π}^* , we get:

$$\pi_{\Pi}^* = \frac{\alpha + c}{2}. \quad (6.13)$$

Unsurprisingly, the profit maximizing price π_{Π}^* is greater than c (with the assumption that $\alpha \geq c$).

Let π_W^* denote the price that maximizes the social welfare — see Equation (6.9). One obtain:

$$\pi_W^* = c, \quad (6.14)$$

at which the CSO profit is zero.⁴ Indeed, from a social welfare point of view, the price should be the lowest possible (equal to the electricity cost), so that it encourages EV users to charge more

⁴ A similar result was demonstrated in [Ghosh and Aggarwal, 2018], where the options correspond to an energy requirement and a deadline. In this scenario, the charging pool is supplied by a renewable and intermittent energy source (in addition to purchasing electricity) and is equipped with a stationary battery. The study shows that the charging price maximizing social welfare corresponds to the actual cost incurred by the system to serve newly arriving users.

and increase their utility and thus social welfare. From the point of view of a profit-maximizing CSO, the price should be higher than the cost of electricity, at the expense of a lower charging rate. Replacing the optimal prices, π_{Π}^* and π_W^* , in Equation (6.12), we obtain:

$$P_{\Pi}^* = \frac{1}{2}P_W^* = \frac{1}{2}\left(\frac{\alpha - c}{\alpha \beta d}\right), \quad (6.15)$$

i.e., EVs would charge at half the rate when the CSO maximizes the profits. In the next section, the optimal pricing strategy is examined for the general case previously described: heterogeneous users, a finite set of options, and a specific price assigned to each option.

6.3 Price menu design problem

In this section, we consider the price menu design problem of a CSO who has information on the distribution of EV users among a finite set of classes, $\mathcal{I}$. These classes and their weight over the population, denoted by θ_i for EV (class) i , can be obtained using data on the EV user behavior — see e.g., [Lu et al., 2023].

In what follows, we consider the price menu design problem under two objectives: (i) maximization of the expected CSO profit (in Subsection 6.3.1), and (ii) maximization of the expected social welfare (in Subsection 6.3.2). Notably, the two problems can be solved for each time period separately (no time coupling constraints), i.e., the CSO can update the price menu at each time period, assuming without loss of generality an hourly granularity (although the offered price when an EV arrives will be binding for that EV for its entire parking duration).

6.3.1 Operator profit maximization

The CSO expected profit is given by:

$$\mathbb{E}_{\theta}[\Pi_i(\pi_{k_i^*})] = \sum_{i \in \mathcal{I}} \theta_i (\pi_{k_i^*} - c_i) p_{k_i^*} d_i, \quad (6.16)$$

where k_i^* is the optimal choice of EV (class) i that is given by Equation (6.8). Hence, the CSO profit maximization problem is given by:

$$\begin{aligned} \max_{\pi, \mathbf{k}^*} \quad & \mathbb{E}_{\theta}[\Pi_i(\pi_{k_i^*})] \text{ given by Equation (6.16),} \\ \text{s.t.} \quad & \text{Equation (6.1) and Equation (6.8).} \end{aligned} \quad (6.17)$$

The optimal choice, k_i^* , for EV user i can be modeled using a binary variable, $a_{i,k}$, $\forall i \in \mathcal{I}$, $k \in \mathcal{K}_i^+$, denoting whether EV i chooses option k (value 1, otherwise 0). The expected profit is thus given by:

$$\mathbb{E}_{\theta}[\Pi_i] = \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i^+} \theta_i a_{i,k} (\pi_k - c_i) p_k d_i, \quad (6.18)$$

where we require that only one option is chosen, i.e.,

$$\sum_{k \in \mathcal{K}_i^+} a_{i,k} = 1, \quad \forall i \in \mathcal{I}. \quad (6.19)$$

Similarly to [Lu et al., 2023], Equation (6.8) can then be reformulated using the big M method as follows:

$$W_i(\pi_k) + M^W \sum_{m \neq k} a_{i,m} \geq W_i(\pi_{\ell}), \quad \forall i \in \mathcal{I}, k, \ell \in \mathcal{K}_i^+, \ell \neq k, \quad (6.20)$$

where M^W is a positive and sufficiently large number. With Equation (6.19), constraint Equation (6.20) ensures that $a_{i,k_i^*} = 1$. The idea of this constraint is that for k such that $a_{i,k} = 0$, the inequality is verified because the lhs ⁵ second term takes a very large value (i.e., M^W). For the unique k such that $a_{i,k} = 1$, the lhs second term becomes equal to zero, which enforces option k to be the optimal option in terms of EV welfare maximization.

The objective function in Equation (6.18) contains a product of a binary variable $a_{i,k}$ and a non-negative continuous variable π_k . Using standard big M linearization techniques [Fortet, 1960], we can replace the product by a new, non-negative, continuous variable, say $b_{i,k}$, $\forall i \in \mathcal{I}$, $k \in \mathcal{K}_i$, so that Equation (6.18) becomes:

$$\mathbb{E}_\theta[\Pi_i] = \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} \theta_i (b_{i,k} - a_{i,k} c_i) p_k d_i, \quad (6.21)$$

and add the following constraints:

$$b_{i,k} \leq a_{i,k} M^\pi, \quad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, \quad (6.22a)$$

$$b_{i,k} \geq 0, \quad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, \quad (6.22b)$$

$$b_{i,k} \leq \pi_k, \quad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, \quad (6.22c)$$

$$b_{i,k} \geq \pi_k - (1 - a_{i,k}) M^\pi, \quad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, \quad (6.22d)$$

where M^π is a positive and sufficiently large number. Constraints Equation (6.22) ensure that the variables $b_{i,k}$ take the value π_k when $a_{i,k} = 1$ and 0 otherwise. Specifically, constraints Equation (6.22a) and Equation (6.22b) enforce $b_{i,k} = 0$ when $a_{i,k} = 0$, whereas Equation (6.22a) becomes redundant when $a_{i,k} = 1$. Constraints Equation (6.22c) and Equation (6.22d) enforce $b_{i,k} = \pi_k$ when $a_{i,k} = 1$, and they become redundant otherwise.

Summarizing, the CSO expected profit maximization problem, $\Pi_{\max}^{\text{CSO}}$ is given by:

$$\begin{aligned} \Pi_{\max}^{\text{CSO}} : \quad & \max_{\pi, \mathbf{a}, \mathbf{b}} \mathbb{E}_\theta[\Pi_i] \text{ given by Equation (6.21),} \\ & \text{s.t. Equation (6.1), Equation (6.7), Equation (6.19), Equation (6.20), and Equation (6.22).} \end{aligned} \quad (6.23)$$

Problem Equation (6.23) is a MILP problem.

6.3.2 Social welfare maximization

Using the aforementioned binary variable $a_{i,k}$, the expected social welfare is given by:

$$\mathbb{E}_\theta[W_i^S] = \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} \theta_i a_{i,k} [U_i(p_k) - c_i p_k d_i]. \quad (6.24)$$

For practical purposes, in order to ensure a non-negative profit for the CSO (who sets the prices), we require from Equation (6.21) that:

$$\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} \theta_i (b_{i,k} - a_{i,k} c_i) p_k d_i \geq 0, \quad (6.25)$$

where $b_{i,k}$ are described by constraints Equation (6.22).

⁵ lhs = left hand side.

Table 6.3: EV class parameters.

e_i^0 / d_i	Class i : K_i				Utility Parameters	
	1 h	2 h	3 h	4 h	α_i	β_i
10 kWh	1: 4	2: 4	3: 4	4: 3	0.425	0.017
20 kWh	5: 4	6: 4	7: 2	8: 2	0.35	0.021
30 kWh	9: 4	10: 2	11: 1	12: 1	0.275	0.027

Summarizing, the expected social welfare maximization problem, $\mathbf{W}_{\max}^S$, is as follows:

$$\begin{aligned} \mathbf{W}_{\max}^S : \quad & \max_{\pi, a} \quad \mathbb{E}_{\theta}[W_i^S] \text{ given by Equation (6.24),} \\ & \text{s.t. Equations (6.1), (6.7), (6.19), (6.20), (6.22) and (6.25).} \end{aligned} \quad (6.26)$$

Problem (6.26) is also a MILP.

6.3.3 Numerical test case

In this section, we provide a numerical experimentation of the optimal price menu design problem. In Subsection 6.3.3.1, we describe the test case (CSO and EVs). In Subsection 6.3.3.2, we present the results from the profit and social welfare maximization problems.

6.3.3.1 Test case description

CSO menu We consider the provision of 4 power rates for charging: $p_1 = 2.5$ kW, $p_2 = 5$ kW, $p_3 = 7.5$ kW, and $p_4 = 10$ kW, i.e., $K = 4$. Note that we consider “low” charging rates [Wolbertus and van den Hoed, 2020] in our numerical illustrations to emphasize the fact that EVs view charging more as an opportunity than as an absolute need (which would make them less flexible or direct them to fast charging stations). However, our analysis remains relevant for higher charging rates.

EV characteristics We consider that all EVs have a battery capacity of 50 kWh, and can charge between 20% and 80%. EV classes are determined by their initial SoC and parking duration. We consider 3 different initial SoCs: 10 kWh, 20 kWh, and 30 kWh. We also consider 4 different parking durations: 1h, 2h, 3h, and 4h. Hence, in total, there are 12 EV classes, assumed to be equally distributed. EVs arrive at the charging pool with a constant rate of 120 vehicles per hour, from 7am to 8pm. They share the same quadratic utility function as in the illustrative example of Section 6.2.5: $U_i(p) = \alpha_i [p d_i - \frac{1}{2} \beta_i (p d_i)^2]$. In order to ease the exposition and interpretation of the results, we assume that scalars α_i and β_i depend only on the initial SoC, thus keeping the number of EV classes contained. Table 6.3 summarizes the 12 EV classes, the highest (in power rate) option available (K_i) for EV class i , and utility parameters. Big M values are set at 15 for M^W and 0.5 for M^π .

All problems were solved using CPLEX 22.1.1.0 solver.

6.3.3.2 Profit and social welfare maximization results

For convenience, we shall refer to the outcome of the CSO expected profit maximization problem Equation (6.23), $\Pi_{\max}^{\text{CSO}}$, as the “Base Case,” and to the expected social welfare maximization problem Equation (6.24), $\mathbf{W}_{\max}^S$, as the “Benchmark.”

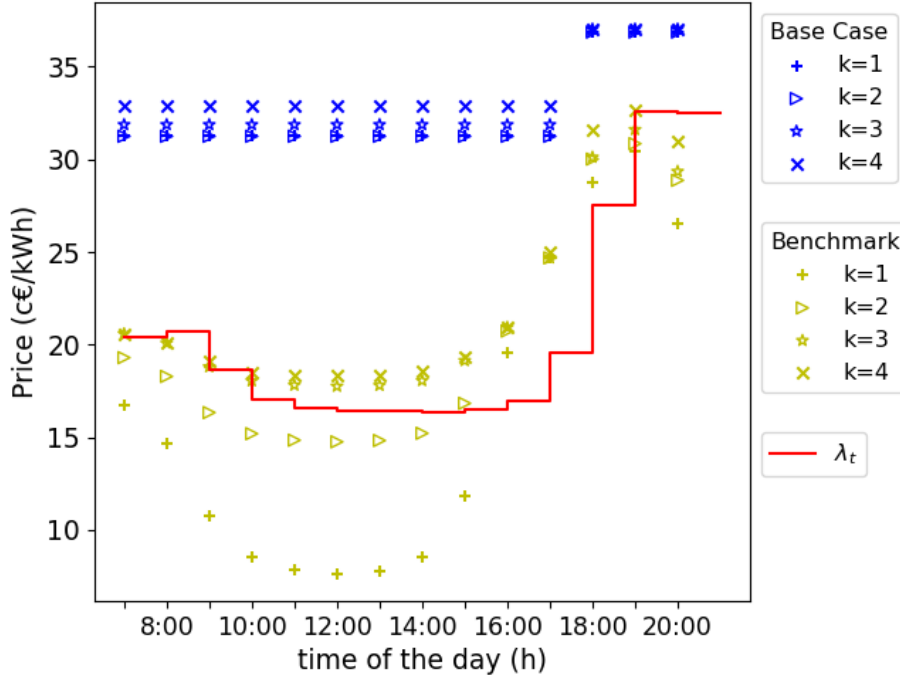


Figure 6.4: Optimal price menu, Base Case and Benchmark.

Figure 6.4 presents the optimal price menu for the Base Case and the Benchmark, as well as the electricity price (parameter), λ_t , for comparison purposes. We observe that the Base Case optimal price menu offers higher charging prices than the electricity price (maximizing CSO profit). For the most part of the day (from 7am to 5pm), when the electricity price is low (around 20 cents per kWh), the prices in the Base Case menu remain stable (spanning around 2 cents from option 1 to option 4). When the electricity price increases (from 6pm to 8pm), the prices in the menu increase and remain practically stable for all options. The shape of the Base Case menu indicates that, when the CSO maximizes its profits, the prices of the menu are not very sensitive to small changes in the electricity price. This is attributed to the fact that the price menu considers also the utility of the EV users, whose optimal choices are discrete; hence, a small increase in the offered price might result in the EVs choosing a lower power rate or even not charging. Consider for example the case of EV class 5, which chooses option 2 at 5 pm. Note that this class could choose any option, however, it chooses option 2 and not the higher power rate option 3, even though the price difference (between options 2 and 3) is only 0.625 cents; hence, this small difference in the price is enough to discourage EV class 5 from choosing option 3. The Benchmark optimal price menu in Figure 6.4 exhibits a larger span during the day, with a trend of a “weighted moving average” of the electricity cost, and some prices well below the electricity price that attract EV users with a low marginal utility. As shown in the example in Section 6.2.5, the price gap between the Base Case and the Benchmark becomes narrower when electricity prices are high, here between 6 p.m. and 8 p.m.

Figure 6.5 illustrates the average (over the entire day) power chosen per EV class for the Base Case and the Benchmark. Unsurprisingly, the Base Case average power is lower for all EV classes, due to the higher charging prices — recall Figure 6.4 compared to the Benchmark. For both, in general, the higher the parking duration and/or the higher the initial SoC, the lower the average power (due to lower marginal utilities). In fact, EV classes 9–12, with a high initial

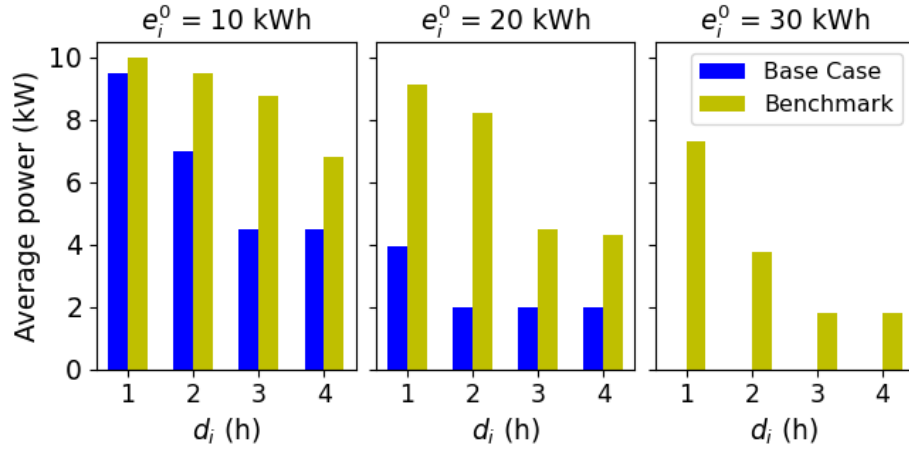


Figure 6.5: Average power chosen during the day for each EV class, Base Case and Benchmark.

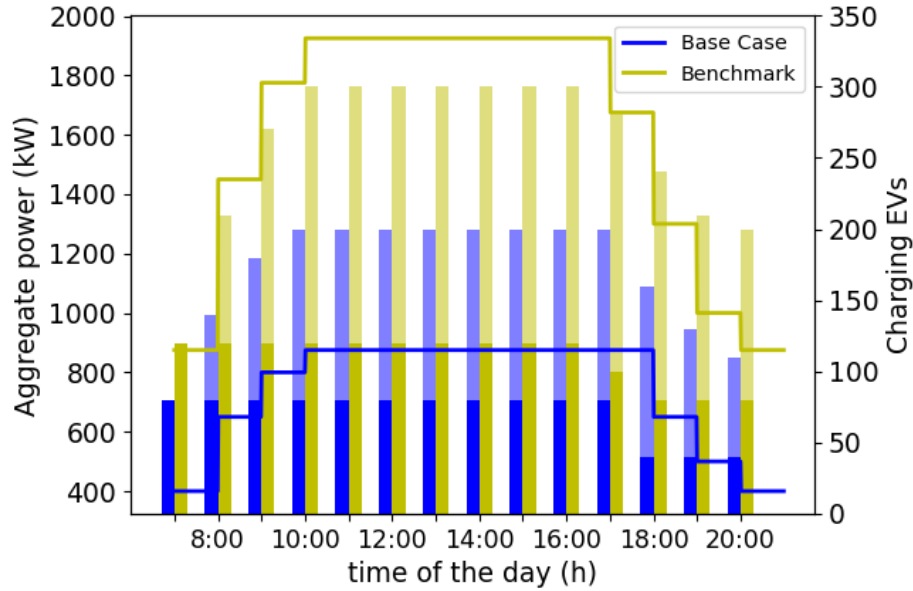


Figure 6.6: Aggregate power consumption (left axis, line) and number of EVs deciding to charge (right axis, columns, lower part: EVs arriving at the hour; upper part EVs remaining from previous hours).

SoC (30 kWh) refrain from charging in the Base Case, since their marginal utility (of 27.5 cents per kWh) is below the lowest price in the menu (that ranges from 31 to 37 cents per kWh).

In Figure 6.6, we illustrate the aggregate power consumption and the number of EVs deciding to charge, for the Base Case and the Benchmark. Clearly, the Benchmark charges more EVs compared to Base Case, and at lower prices — see Figure 6.4 — thus resulting in a significantly higher power consumption, peaking at 1,925 kW, whereas the Base Case peaks at 875 kW. Recall that the average EV power consumption is also higher for the Benchmark — see Figure 6.5 for the daily average — due to the lower prices compared to the Base Case. We elaborate further on the aggregate consumption shape of Figure 6.6 and illustrate in Figure 6.7, the hourly profit and social welfare, we observe that the Benchmark maximum social welfare (2519.22 € for the entire day) is achieved with zero profits for the CSO, and maximum welfare for the EV users.

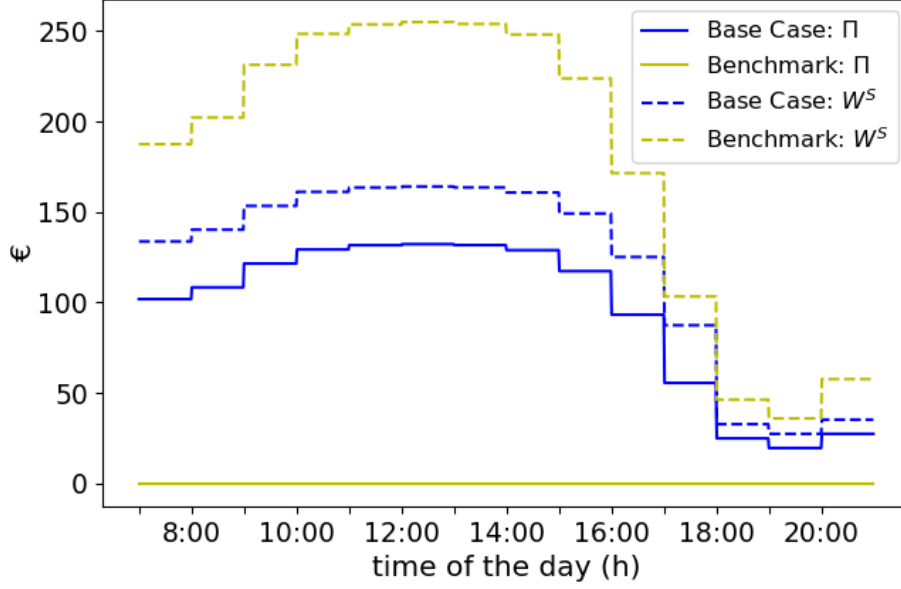


Figure 6.7: Hourly profit and social welfare, Base Case and Benchmark.

In the Base Case, the observed social welfare (821.58 € lower than the Benchmark) is primarily attributed to the CSO profits.

6.4 Operator demand response

In this section, we consider the case of a CSO providing demand response, by reducing the aggregate power consumption by a certain amount, during specific time periods. This reduction can be viewed as the provision of a reserve product (to be used interchangeably with demand response) — or flexibility as is often mentioned in the literature — *w.r.t.* its “usual” power consumption. Indeed, there is a long debate on how to determine such a “base case,” or whether this is even possible. However, for the purposes of this study, we can employ the outcome of the CSO profit maximization problem as the “usual consumption,” upon which any reduction can be calculated. Alternatively, one can think of a day-ahead commitment on a certain power consumption (considering an expected profit maximization problem and the resulting power consumption given the “optimal price menu”) and a “real-time adjustment” of the price menu to reduce the actual consumption by a certain amount. In what follows, we consider the problem for the provision of demand response through a certain reduction in the aggregate consumption during a certain time period.

6.4.1 Problem formulation

For clarity, we introduce the notation in the multi-period problem, whose horizon is represented by the set $\mathcal{T} = \{1, \dots, T\}$, assuming without loss of generality that t refers to an hourly period.

Let N_t denote the number of EV arrivals at time t , of expectation $\bar{N}_t$, and with weights per class i at hour t , $\theta_{i,t}$. For brevity, we define parameter $n_{i,t} = \bar{N}_t \theta_{i,t}$ representing the expected number of vehicles of class i arriving at time t .

The price menu is denoted by $\pi_{k,t}$, i.e., the prices can change every hour, assuming without loss of generality that the power rates, p_k , are parameters that do not change every hour. Still,

for each time period, prices should be ordered as in Equation (6.1), i.e.:

$$\pi_{k-1,t} \leq \pi_{k,t}, \quad \forall k \in \mathcal{K}, t \in \mathcal{T}. \quad (6.27)$$

The binary variables $a_{i,k,t}$ denote whether EV i , arriving at time t , chooses option k , with:

$$\sum_{k \in \mathcal{K}_i^+} a_{i,k,t} = 1, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \quad (6.28)$$

The optimal choice of EV i , that is k_i^* such that $a_{i,k_i^*,t} = 1$, arriving at time t is given by the following constraints:

$$U_i(p_k) - \pi_{k,t} p_k d_i + M^W \sum_{m \neq k} a_{i,m,t} \geq U_i(p_\ell) - \pi_{\ell,t} p_\ell d_i, \quad (6.29)$$

$$\forall i \in \mathcal{I}, k, \ell \in \mathcal{K}_i^+, \ell \neq k, t \in \mathcal{T},$$

where we replaced the EV welfare by its definition — see Equation (6.7).

Similarly to Equation (6.22), the continuous variables $b_{i,k,t}$ verify the following conditions:

$$\left. \begin{aligned} 0 &\leq b_{i,k,t} \leq a_{i,k,t} M^\pi \\ \pi_{k,t} - (1 - a_{i,k,t}) M^\pi &\leq b_{i,k,t} \leq \pi_{k,t} \end{aligned} \right\} \forall i \in \mathcal{I}, k \in \mathcal{K}_i, t \in \mathcal{T}. \quad (6.30)$$

The aggregate CSO expected profit over the time horizon, using Equation (6.21) and parameter $n_{i,t}$, is given by:

$$\hat{\Pi}_{\mathcal{T}} = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} n_{i,t} (b_{i,k,t} - a_{i,k,t} c_i) p_k d_i. \quad (6.31)$$

Let $\mathcal{T}_R = \{\tau_1, \dots, \tau_2\}$ be the set of time periods during which the CSO reduces the aggregate power consumption by a certain amount denoted by P_t^R , from the aggregate power consumption, denoted by P_t^Π that pertains to the solution of Problem Equation (6.23), which can be solved separately for each time period.

Let indicator (parameter) $\delta_{i,t',t}$ denote whether an EV of class i arriving at time t' is connected at time t , where $t \geq t'$ — i.e., for as long as its parking duration d_i . Let variable $\hat{P}_t$ denote the expected aggregate power consumption, given by:

$$\hat{P}_t = \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} \sum_{t'=1}^t n_{i,t'} a_{i,k,t'} p_k \delta_{i,t',t}, \quad \forall t \in \mathcal{T}. \quad (6.32)$$

The reduction of the power consumption for the provision of demand response is then enforced through:

$$\hat{P}_t \leq P_t^\Pi - P_t^R, \quad \forall t \in \mathcal{T}_R. \quad (6.33)$$

Summarizing, the CSO demand response problem, given a price λ_t^R for reducing the power consumption during hour t , is now as follows:

$$\begin{aligned} \mathbf{DR}_{\lambda^R}^{\text{CSO}} : \quad & \max_{\pi, a, b, P^R} [\hat{\Pi}_{\mathcal{T}} \text{ given by Equation (6.31)}] + \sum_{t \in \mathcal{T}_R} \lambda_t^R P_t^R, \\ & \text{s.t. Equation (6.27) – Equation (6.29), (6.32), and (6.33),} \end{aligned} \quad (6.34)$$

with $P_t^R \geq 0, \forall t \in \mathcal{T}_R$. Problem (6.34) is a MILP problem.

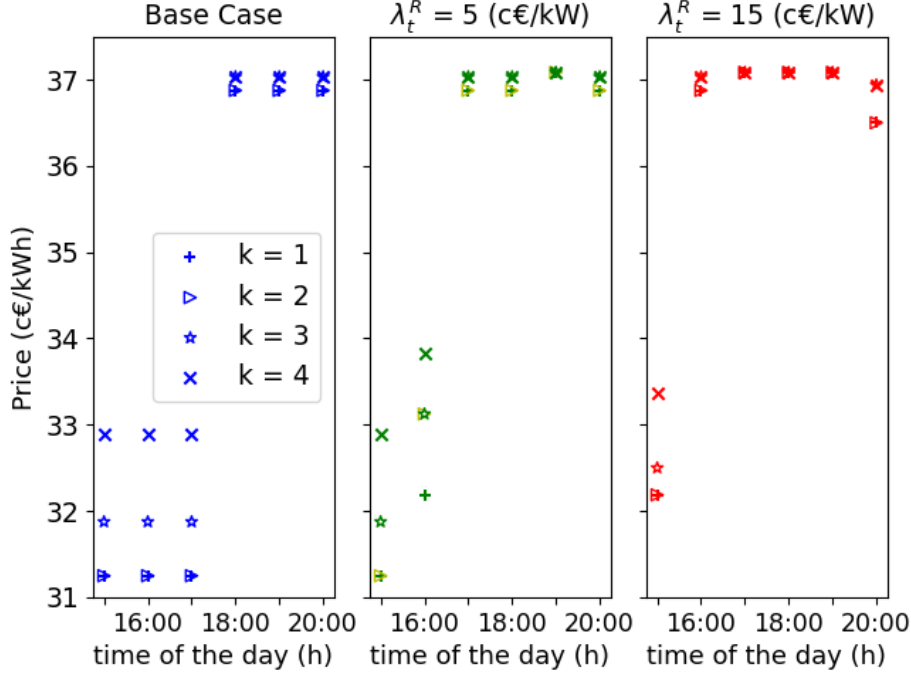


Figure 6.8: Optimal price menu (3pm to 8pm): Base Case and provision of demand response (two reserve prices).

So far, we have considered the expected number of arrivals per class, $n_{i,t}$, and calculated expected profits and power consumption. One may think that the provision of demand response, i.e., the reduction in power, should be enforced with a higher probability. A simple way would be to replace the expected number of arrivals $n_{i,t}$ in constraint (6.32) by an appropriately-defined “worst-case” number of arrivals, during the periods $\mathcal{T}_R$ (potentially even earlier). Indeed, in a practical setting, the CSO can re-solve the problem, after each hour, and define the price menu dynamically every hour, with the information available up to that hour. However, due to space considerations, we will refrain from such settings in this first work. Future work will be directed to more elaborate robust optimization formulations and appropriate uncertainty sets as well as more dynamic settings that employ the information that becomes available during the day.

6.4.2 Numerical results

Using the solution of the Base Case as a “baseline” for the CSO aggregate consumption, we numerically solve the demand response problem (6.34), with a 5-hour reserve period between 4pm and 9pm. We consider two cases for the reserve price, λ_t^R (constant for the entire 5-hour period): 5 and 10 cents per kW. Other than that, the parameter values are the same as in Subsection 6.3.3.1.

In Figure 6.8, we present the price menu for the Base Case, and for the two values of the reserve price. For reasons that will soon become apparent, we present the 5 hours affected (4pm, 5pm, 6pm, 7pm, 8pm) as well as the previous hour (3pm). In Figure 6.9, we illustrate the aggregate power consumption, for the Base Case and the two reserve prices. Considering first the lower reserve price (Figure 6.8, middle), we observe that the main change *w.r.t.* the Base Case refers to hours 4pm and 5pm, when the prices increase (they span between 32 and 34 cents at 4pm, and reach 37 cents at 5pm). The reduction in the aggregate power consumption

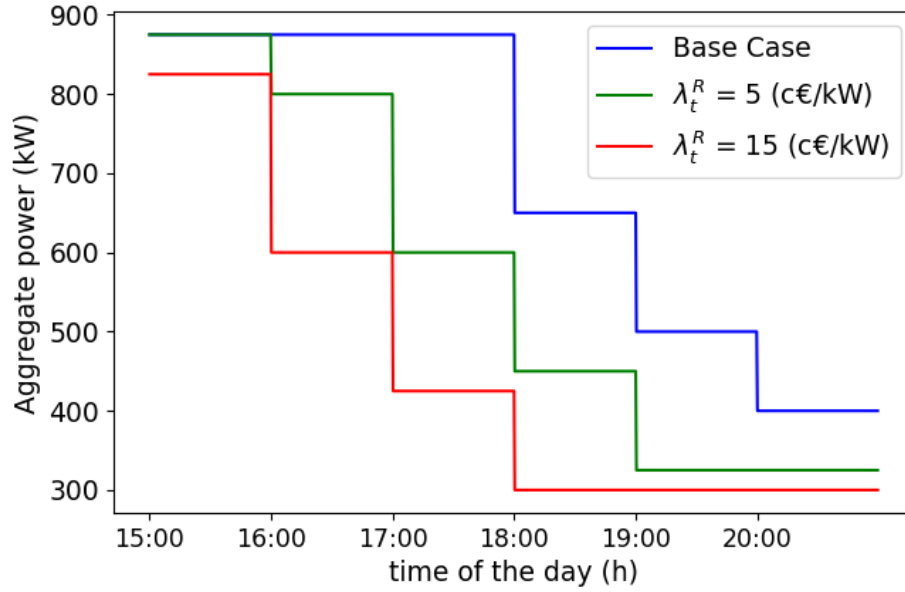


Figure 6.9: Aggregate power consumption (3pm to 8pm): Base Case and provision of demand response (two reserve prices).

(Figure 6.9, green line) is 75 kW at 4pm and 275 kW at 5pm. Note that the price menu at 6pm is the same with the Base Case, however, we still see a reduction by 200 kW, which is due to the lower amount of EVs deciding to charge in the previous hours (we remind that EVs may charge for 4 hours). The reduction drops to 175 kW at 7pm and 75 kW at 8pm because the Base Case aggregate power consumption is already low at these hours. Considering next the higher reserve price (Figure 6.8, right), we observe that the prices increase earlier (slightly at 3pm, and then reach around 37 cents) achieving a higher reduction in the aggregate power consumption (Figure 6.9, red line), which amounts to 50 kW at 3pm, 275 kW at 4pm, 450 kW at 5pm, 350 kW at 6pm, 200 kW at 7pm, and 100 kW at 8pm. The reduction at 3pm — before the beginning of the reserve period — is explained by the fact that a lower number of EVs that decide to charge at 3pm affects the aggregate power consumption in the hours that follow — recall the EV parking duration of up to 4 hours.

In Figure 6.10, we illustrate the CSO profit for the Base Case and the two reserve prices, which includes the revenue for providing demand response. We observe that at 3pm, the Base Case and the lower reserve price yield the same profit, whereas the higher reserve price yields lower profit (see the higher prices in Figure 6.8, right). This occurs in anticipation of the reserve period (as we saw in Figure 6.8, right; and Figure 6.9, red line), which is advantageous when the reserve price is high, e.g. 15 c€/kW. Indeed, the profit of the higher reserve price (red column) is the highest in all other hours. The profit of the lower reserve price (green column) becomes higher than the Base Case at 5pm and remains higher until 8pm.

Figure 6.11 shows the breakdown of the total social welfare over the entire day into CSO profit, revenue from providing demand response, and EV welfare. The total CSO profit for the Base Case is 1,323.45 €. The total profit when providing demand response increases by 16.04 € (1.21%) for the low reserve price, and by 128.41 € (9.70%) for the high reserve price. The total social welfare over the entire day for the Base Case is 1697.64 €. Accounting for the revenues from demand response, the total social welfare decreases by 56.69 € (3.34%) for the low reserve price, and by 131.70 € (7.76%) for the high reserve price. The total welfare of the EVs over the

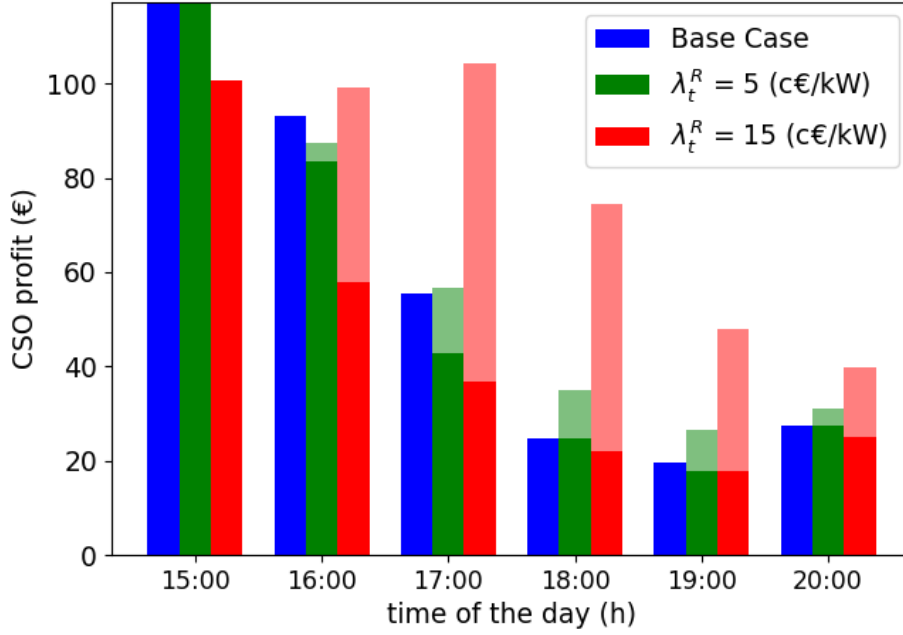


Figure 6.10: CSO profit (lower part of column: profit for energy; upper part: revenue from demand response).

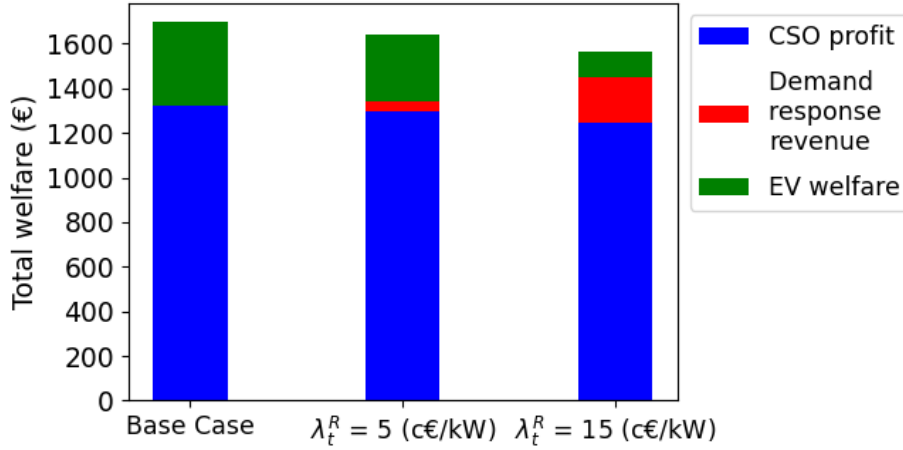


Figure 6.11: Total social welfare (over the entire day) breakdown: CSO profit, revenue from providing demand response, and EV welfare (Base Case and two reserve prices).

entire day, which is 374.19 € for the Base Case, decreases to 301.46 € for the low reserve price, and to 114.08 € for the high reserve price. Summarizing, a higher reserve price results in higher CSO profits, lower EV welfare, and lower total social welfare.

6.5 Conclusions and further research

In this work, we considered a CSO serving EVs that are flexible in their energy demand but not in their parking duration, offering several power rates at different prices (price menu). Arguably, this CSO model could fit well public charging stations, considering the future EV adoption,

where charging would be viewed more as an opportunity (e.g., in a public parking station), and charging point availability is unlikely to be a limiting factor. In this context, we formulated the optimal price menu design problem as a MILP problem, considering both a profit maximizing CSO and (as a benchmark) social welfare maximization. We further accounted for the provision of demand response by considering an adjustment of the price menu (in real time) to reduce the aggregate power consumption and benefit from a certain price for remuneration (as a type of reserve deployment). Our numerical experimentation illustrates the construction of the optimal price menu, the trade-offs considered for the CSO profit and the social welfare, and the price menu adjustment to provide demand response under different (reserve) prices.

Future research is directed to account for uncertainty in the parking duration and the EV user utility through robust optimization approaches and the construction of appropriate data-driven uncertainty sets. A complementary approach is to incorporate online pricing to adjust for unexpected charging demand in real-time. More elaborate price menus which might also relate to the EV state of charge, as well as CSO-EV contracts for lowering the charging rate in case of demand response provision, are also interesting directions for further research. In this work, the 'baseline' (power consumption without demand response) is set as the outcome of the profit maximizing optimization problem. In practice, it is difficult to assess the baseline, and different methods exists.⁶ A focus on the comparison between different methods for assessing the 'baseline' in the context of EV charging is a new and crucial research direction. Finally, establishing the link between the construction of the price menus and the spatiotemporal marginal costs for delivering electricity at the charging points is key for efficient EV charging, including aggregator managed EVs [Yanikara et al., 2020], in an adaptive manner that accounts for the impact on the grid and its assets [Andrianesis and Caramanis, 2019, Andrianesis et al., 2022].

⁶ See for example section 7.2.1 in the [NEBEF 3.3 réglementation](#) (in french).

Conclusion

Context and approach adopted

In the context of the energy transition, Electric Vehicles (EVs) are considered a key solution for decarbonizing the transportation sector. However, developing and maintaining charging infrastructure entails significant costs, both for the electrical network and the physical charging facilities. To address these challenges, efficient EV charging management is essential to ensure a high quality of service while operating within limited resource capacities.

Part I of this thesis focuses on understanding EV user behavior in decentralized systems, where users act independently to maximize their individual welfare. In environments with limited charging power and a finite number of Charging Points (CPs), an EV user quality of service, and therefore choice, depends on the choices made by the other users, as it impacts the remaining resource available. To model and analyze such systems, the framework of queueing games is leveraged.

Part II explores how user behavior can be influenced through real-time pricing strategies to optimize the use of charging infrastructure. These mechanisms are designed to optimize system performance by aligning user behavior with system objectives. Chapter 5 focuses on maximizing CP utilization through parking pricing mechanisms, while Chapter 6 examines a price-menu design to maximize either profit or social welfare. In both cases, pricing is applied at a single Charging Station Pool (CSP) to manage congestion by charging higher fees to users who consume more resources, reflecting the negative externalities they impose on the system.

Achievements and scientific contributions

Part I examines queueing games where EV users seeking charging choose one CSP among several. Using queueing game theory and equilibrium concepts such as *Wardrop equilibrium* (Chapter 3) or *Stochastic user equilibrium* (Chapter 4), the analysis quantifies how user decisions influence social welfare. In particular, congestion degrades the quality of service through the blocking probability and expected energy received during a charging session under standard charging modes (Chapter 3), and waiting time under fast charging modes (Chapter 4).

Chapter 3 provides, in a loss queueing system, an approximation and bounds of the expected energy received by an EV (Lemma 3.2), and bound on the error of this approximation (Lemma 3.3), inspired by the queueing theory analysis in [Aveklouris et al., 2019], which enables to obtain a close-form expression. Using results from potential games, the existence and uniqueness of an equilibrium are demonstrated, and characterised (Propositions 3.11 and 3.12).

In lossless queueing systems, Chapter 4 extends the analysis to account for heterogeneous energy needs, making cost functions depend on more than just the proportion of users selecting the same resource (here CSPs). This goes beyond the framework of potential games. In this

framework, Theorem 4.9 establishes the existence of an equilibrium in this context, although uniqueness is not theoretically guaranteed.

Theoretical bounds on the price of anarchy (in Chapter 3) and numerical observations show that users tend to overuse the CSP providing high valuation, i.e. high power rate (Chapter 3) or convenient location (Chapter 4), creating negative externality on the overall users welfare. Such observation is in line with the theoretical results obtained in [Anily and Haviv, 2022] for loss queueing systems where congestion occurs through the blocking probability (as in Chapter 3), and [Gilboa-Freedman et al., 2013] for standard M/M/1 queues (lossless queueing systems as in Chapter 4).

Building on this foundation, the Part II of this thesis explores offline dynamic pricing mechanisms to address resource constraints and manage congestion at CSPs. Resource limitations manifest in two forms: the availability of CPs (Chapter 5), and the power capacity of the charging infrastructure (Chapter 6). In Chapter 5, the pricing mechanisms target parking usage, addressing the externality where one EV's occupation of a CP potentially prevents another from accessing one. In Chapter 6, pricing is tied to the power rate chosen by EVs, mitigating the externality caused by high aggregate power demand, which can strain the distribution network (although not explicitly modeled because the point of view is that of the CSO).

In Chapter 5, and similarly to Chapter 3, the CSP occupancy is modeled as a *continuous-time Markov chain*. Users are differentiated in particular in their expected parking duration, and their choice of joining or not the CSP depends on the parking pricing. Thus, the arrival and departure rates that are controled through dynamic parking pricing, with the aim of lowering the blocking probability in order to increase the profit generated from charging the EVs. For static system parameters (Section 5.3), the existence and uniqueness of the Markov chain's stationary regime is shown in Proposition 5.1. For dynamic system parameters (Section 5.4), reflecting time-varying charging demand, the state of the Markov chain is derived iteratively, using Method 5.1. While standard Markov decision processes cannot be applied due to incomplete information about parking durations, unlike other similar works such as [Legros and Fransoo, 2024], no guarantee is provided about the optimality of the parking pricing. However, numerical analysis shows that the incorporation of instantaneous occupancy into pricing does not significantly improve the overall utilization of the CPs. These results are in line with the theoretical studies on dynamic pricing in queue, such as in [Paschalidis and Tsitsiklis, 2000, Besbes et al., 2021].

Dynamic energy pricing is next investigated in Chapter 6, where the EV users can choose between different power rates at different prices. Conversely to Chapter 5, the blocking probability is neglected, assuming that there are enough CPs, and resource is determined in terms of charging power at the CSP level instead of the number of CPs. Using the framework of *bi-level optimization*, in particular inspired by the method used in [Lu et al., 2023], the initial problems were transformed into MILPs, as detailed in Section 6.3. This optimization problem is extended to consider the possibility to temporaly lower the aggregate power consumption when necessary due to exogeneous network congestion issues. Numerical analysis is conducted, and show in particular that profit maximizing pricing is not very sensitive to small changes in the electricity cost.

Finally, this thesis also contributes by applying theoretical frameworks (queueing game theory, Markov chains, bi-level optimization) to the context of EV charging. Notably, it offers models of user behavior across various scenarios, including standard charging, fast charging, and different user decision-making criteria.

Guidance for stakeholders in public electric vehicle charging

The analysis in Part I shows that users tend to overuse attractive CSPs, such as those offering high charging power or convenient locations. From the perspective of a public authority or a Charging Station Operator (CSO) with regional control, aiming to maximize user welfare, it may be beneficial to encourage users to spread more evenly across CSPs. One possible approach is to offer more attractive pricing at less popular CSPs.

Chapter 5 focuses on a CSO managing a CSP with the following characteristics:

- the CPs provide standard charging modes (e.g., 3 kW to 22 kW);
- the CSP is located at a site where users primarily visit for other activities and charge their vehicles while parked, often resulting in parking durations longer than charging durations;
- the CSP has relatively few CPs compared to demand, leading to frequent situations where users cannot charge because all CPs are occupied upon arrival.

In this context, a CSO may be interested in designing an appropriate parking pricing mechanism in order to maximize CP utilization. For a concrete set-up of the parking pricing, the analysis conducted in Chapter 5 requires prior statistical analysis. This involves collecting data on the potential charging demand (i.e., the expected number of EV users arriving at the site per hour), distributions of parking durations and energy needs. Information about users' charging behavior, such as their estimated parking duration upon arrival or willingness to pay, should also be assessed. This data can be collected, for example, through surveys. While the optimal pricing policy depends on these inputs, general insights are provided in Sections 5.3.5 and 5.4.5. For scenarios where charging demand remains stable throughout the day (Section 5.3), results indicate that static pricing — remaining constant all day — is nearly as effective as more complex pricing policies that depend on occupancy levels. However, if occupancy-dependent pricing is applied, prices should increase as more EVs occupy the CSP. In cases where charging demand and other factors (e.g., parking durations, electricity costs) vary throughout the day (Section 5.3), adjusting pricing based on the time of day significantly improves outcomes. Similarly to the previous scenario, it does not seem necessary to make pricing depend on occupancy levels.

Chapter 6 introduces a framework for menu-based pricing with constant power levels for the entire charging session. From the user's perspective, this approach ensures a predictable quality of service, regardless of parking duration. Numerical results indicate that time-based pricing with one or two variations throughout the day is sufficient to optimize profit. From the perspective of the electrical grid, this study also sets the basis for collaboration between network operators and CSOs in managing power consumption, which will become more and more necessary ([Enedis, nd]). Furthermore, the study emphasizes the importance of a fair distribution of welfare (see Figure 6.11), ensuring that user quality of service and charging costs are not excessively compromised.

Limitation and research perspectives

These past few years of work on this thesis have led me to reflect on how to make the proposed models more realistic and explore alternative methods to address them. They have also inspired many ideas for future research directions building on this work. Below, an overview of these reflections and perspectives is shared.

More accurate charging pool power constraints modeling

As explained in Chapter 1, constraints occur at various levels within a CSP infrastructure. For instance, while a CSP may face constraints on aggregate power consumption, similar constraints apply to individual charging stations, each consisting of multiple charging points (as illustrated in Figure 1.4). One approach could be to include additional variables or states to describe CSP occupancy, tracking which charging point is in use. This raises interesting questions, such as how to guide users in choosing a charging point within the CSP and how to design incentives to influence their choices, as in [Zhang et al., 2016, Arias et al., 2023]. Accurately modeling CSP constraints is crucial for better predicting the charging power delivered to EVs, and thus evaluating the quality of service for users at a local level.

More efficient optimization

The dynamic pricing methods proposed in Part II evaluate the entire planning horizon at once, leading to a large number of variables and significantly increasing computation times. Future work could focus on reducing these computation times, making it possible for instance to extend the analysis to CSPs with more CPs in the parking optimization context discussed in Chapter 5. One potential solution is to adopt a more myopic approach, focusing on shorter time windows and dynamically in an online manner. While this may reduce the accuracy of the information and optimization outcomes, it could significantly improve scalability (for instance extending the analysis to CSPs with more CPs in the parking optimization context discussed in Chapter 5).

Combine parking and charging management

In Part II, pricing is employed to address excessive parking duration in Chapter 5, neglecting coupling charging power constraints, and conversely in Chapter 6. Most studies in the literature also focus on one of the two aspects individually. However, developing pricing and/or other mechanisms to address both issues simultaneously represents a crucial research direction. This was for instance explored in [Hao et al., 2022] using the framework of Partially Observable Markov Decision Processes (POMDP) combined with Deep Neural Networks, an approach that may be energy-intensive [Yang et al., 2017]. One of the key challenges in this area is designing incentives to guide users' charging behavior without creating pricing policies that are too complex.⁷

Learning the system parameters

User choices, such as where to charge, are heavily influenced by their level of knowledge. As discussed in Chapter 2, a key assumption in the analysis conducted in Part I (and in most games in general) is that users are aware of the cost incurred by their choice, and also aware of the choices made by the other users. While this assumption is partially relaxed in Chapter 4 by introducing bounded rationality, it remains to be shown whether user behavior indeed converges to the equilibrium described. This challenge is even greater in queueing games, where users make decisions based on the system's long-term behavior, which may not reflect conditions at the start of the observed time window. Additionally, the quality of service is inherently stochastic due to the random nature of user arrivals and departures. A key research direction is to study how users adapt their choices based on past experiences without prior knowledge of system

⁷ As highlighted in the introductory Chapter 1 (end of Section 1.1.3, the complexity of pricing policies is one of the main sources of dissatisfaction and a significant barrier to EV adoption [AFIREV and Avere-France, 2024].

parameters. One potential approach is to apply reinforcement learning algorithms, as explored in [Sohet et al., 2020].

Another critical question is the time required for the potential convergence to an equilibrium. If the convergence time is long, system dynamics, such as variations in charging demand throughout the day, must be incorporated into the analysis.

Learning also applies to the CSO. A CSO may not initially know all the parameters necessary for the pricing optimizations discussed in Chapters 5 and 6. While surveys and statistical analyses can help estimate these parameters, such methods have limitations, as user responses in surveys may differ from their actual behavior [Hufnagel and Conca, 1994]. An alternative approach is to adopt learning algorithms for the CSO to learn user behavior dynamically while simultaneously adjusting optimization variables, as explored in works like [Biswas et al., 2016]. This dual learning dynamic — where users are also adapting their choices — presents an important avenue for future research.

Observable vs. unobservable queues: implications for EV charging

User choices, such as where to charge, are significantly influenced by real-time information about the occupancy levels at various CSPs, as shown in [Shone et al., 2013]. Assessing the impact of the information provided to users on the negative externalities of individual choices represents promising direction for future research. In the queueing framework, this involves comparing *observable* and *unobservable* queues, as briefly mentioned in Chapter 2. In Part I, *unobservable* queueing systems were analysed. For instance, in the context of EV users seeking fast charging (as analyzed in Chapter 4), an appropriate question arises: is providing real-time information about CSP waiting times more beneficial for the social welfare? This issue is challenging, as waiting time may fluctuate between a user’s decision and her actual arrival time at the CSP. Some studies, such as [Yuan et al., 2015], assume users have no access to real-time information, while others, like [Malandrino et al., 2015, Guo et al., 2017], explore scenarios where such information is provided. However, little work has been done to directly compare these two approaches.

Another critical aspect is determining the type and granularity of information to share to users. For example, is it more effective to inform users only about the number of EVs currently charging at a CSP, without revealing the queue length? Such non-monetary incentives could serve as alternatives to standard pricing mechanisms and are feasible to implement in practice.

Charging scheduling policies

Assessing charging scheduling policies, coupled with user behavior, is another promising research direction. For instance, a LIFO (Last In, First Out) dispatch policy offers an illustrative case. Under this policy, new arrivals do not create negative externalities for future users, as they are given priority. As stated in [Hassin and Haviv, 2003], this approach leads to social optimality [Hassin, 1986]. In Chapter 3 investigate Processor Sharing (PS) dispatch policy, while Chapter 4 assumes FIFO scheduling. However, no direct comparison analysis between the two were conducted.

Although scheduling policies have been extensively studied for quality of service purposes in EV charging (e.g., [Khaksari et al., 2021, Danner and Meer, 2021]), exploring how such policies interact with user behavior and decisions presents an interesting avenue for further research.

Reservation mechanism and priority users

Another promising research direction involves integrating reservation mechanisms into the system. For instance, in scenarios with limited CP availability, as discussed in Chapter 5, incorporating the ability for users to reserve a CP introduces complex yet intriguing challenges. For example, users with urgent charging needs could benefit from the option to reserve a CP in advance. This raises important research questions, such as: how should the admission policy for non-reserved users be scheduled to ensure CP availability for reserved users, as explored in [Legros and Fransoo, 2024]? Or how should reservation prices be designed to balance fairness and system efficiency? Similar considerations apply to power reservations.

Additionally, these questions extend to scenarios involving priority users, such as those with subscriptions. Offering guarantees to regular customers through priority mechanisms or subscription-based services could enhance user satisfaction and loyalty, making this a highly valuable a CSO.

From a single-level game to a multi-level optimization

Part I presents a basis for other numerous future research directions. The equilibrium analysis presented in this section focuses on optimization from the perspective of EV users.

As mentioned above, this work can be extended broader optimization frameworks that incorporate the objectives of Charging Station Operators (CSOs) or central planners. These stakeholders may aim to optimize their own strategies and levers. Different scenarios can be considered: a decentralized system, where CSOs compete (e.g., [Yuan et al., 2015, Zhou et al., 2021, Zavvos et al., 2022]), or a centralized system, where a single entity operates as a monopoly (e.g., [Malandrino et al., 2015]). Moreover, additional considerations than users quality of service can be taken into account, such as the grid operation efficiency (asset degradation, power losses, etc.).

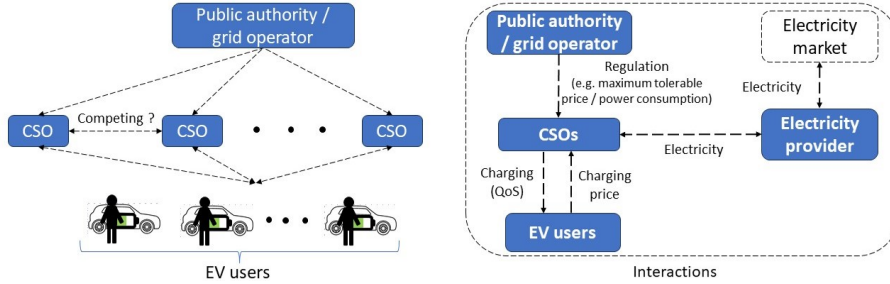


Figure 1: Interactions between the central authority (e.g. public, or grid operator), the charging station operators and the electric vehicle users. *Several scenarios are possible: (1) CSOs cooperate (own by the single entity, public or private); or (2) compete for: (i) grid services if any, and (ii) attracting EV users. If the CSO are not owned by the public authority, they also may be subject to regulations aiming to optimize the system welfare.*

Figure 1 illustrates scenario examples where the different actors aim to maximize their own objective: the public authority aims to maximize the system welfare, the grid operator aims to maximize grid operation efficiency, the CSOs (if private) aim to maximize their profit, and the users aim to maximize their own welfare (as examined in Part I). For example, [Sohet et al., 2021] examines a tri-level optimization in this context, with strategic behavior of a single CSO (assuming the strategy of the other CSOs is fixed and exogenous), and without considering competition between the CSOs.

For broader EV charging integration into the electricity system, aggregators emerge as intermediaries between the grid operator and CSOs (as shown in Figure 1). These aggregators combine contributions from multiple participants, including other energy consumers. This perspective adds another layer of complexity to the analysis and is briefly discussed in the next section: *Electric vehicle participation to frequency ancillary services through V2G*.

Electric vehicle participation to frequency ancillary services through V2G

A final perspective proposed here is to use EVs as mobile batteries to support the grid through frequency ancillary services, through the Vehicle-to-Grid (V2G) concept. In this setup, EV batteries are charged or discharged to regulate frequency in the electricity grid (closely related to balance supply and demand), as briefly described in the introductory chapter 1 (Section 1.1.3).⁸ In practice, V2G technology is still in its early stages and faces numerous challenges. On the technical side, charging stations must be compatible with the latest communication protocols, particularly ISO 15118-20. From the user perspective, it is essential to minimize any negative impact on the quality of service, address concerns about battery degradation, and offer sufficiently attractive incentives. Additionally, this type of service is best suited for locations where EVs remain parked for long duration, such as workplaces or homes.

Apart from that, this perspective is related to the topics discussed Chapter 6, but with some differences. Unlike Chapter 6, where the application is on reducing power consumption, the goal here is to participate in ancillary services for grid stability. Additionally, the role of aggregators must be incorporated into the analysis. To enable effective EV participation in frequency regulation, a significant amount of aggregated power is required. Aggregators achieve this by combining contributions from multiple participants (not only EV users but also other energy consumers). They may either directly control EV charging/discharging or collaborate with CSOs by sending signals to adjust the charging process. This system requires multi-level optimization:

- **Aggregator level:** maximize profits from participating in frequency regulation markets while compensating CSOs or EV users for their services;
- **CSO level** (if EV charging/discharging not directly controlled by the aggregator): maximize their profit (similarly to Chapter 6);
- **EV user level:** tradeoff between financial compensation from frequency regulation services and quality of service (e.g., energy received at departure as in Chapter 3) and consideration of battery degradation from discharging.

For instance, the authors in [Amamra and Marco, 2019] have explored these issues, assuming that the aggregator directly control the EV charging. Similarly to the previous section (*From a single-level game to a multi-level optimization*), the involvement of multiple actors introduces additional complexity to the system. In particular, competition arises among aggregators in the frequency regulation markets.

⁸ These types of service are typically managed by the transmission system operator (RTE in France). See for example [Providing frequency ancillary services](#) website for further explanation on frequency ancillary services participation.

List of Accronymes

AC	Alternative Current
CPO	Charging Point Operator
CP	Charging Point
CSO	Charging Station Operator
CSP	Charging Station Pool / Charging Pool
CS	Charging Station
CTMC	Continuous Time Markov Chain
DC	Direct Current
DSO	Distribution System Operator
EV	Electric Vehicle
FIFO	First In First Out
ICEV	Internal Combustion Engine Vehicle
LCA	Life Cycle Analysis
LIFO	Last In First Out
MILP	Mixed Integer Linear Programming
PASTA	Poisson See Time Averages
PoA	Price of Anarchy
PS	Processor Sharing
QoS	Quality of Service
SE	Stochastic Equilibrium
SoC	State of Charge
SO	System Operator
TSO	Transportation System Operator

Appendix A

Proofs and additional remarks about Chapter 3 framework

This appendix provides additional comments on the effective arrival rate at a charging pool and its influence toward the congestion in power (the fact that the individual charging power decreases if many EVs are simultaneously charging).

A.1 Convergence of the effective arrival in an Erlang B queueing system

This section provides general remarks on an Erlang-B system.

Let λ be the arrival rate, and ν be the service time of an Erlang-B queue (M/M/c/c), and $\rho := \lambda/\nu$ the corresponding utilization rate. Let λ^e be the effective arrival rate in the Erlang-B queue:

$$\lambda^e := \lambda \times (1 - B(\rho)),$$

with $B(\rho) := \frac{\rho^c}{c!} \times \left(\sum_{j=0}^c \frac{\rho^j}{j!} \right)^{-1}$ the *blocking probability* of the Erlang-B queue. Note that, in Chapter 3 framework, $c = c$ and ν^{-1} is the expected parking duration of an EV.

Lemma A.1. $\lim_{\rho \rightarrow +\infty} B(\rho) = 1$.

Proof. Let us denote $\tilde{B}(\rho) = B(\frac{1}{\rho})$. One has: $\lim_{\rho \rightarrow +\infty} B(\rho) = \lim_{x \rightarrow 0} \tilde{B}(\rho)$.

$$\begin{aligned} \tilde{B}(\rho) &= \frac{\frac{1}{\rho^c c!}}{\sum_{j=0}^c \frac{1}{\rho^j j!}} \\ &= \frac{1}{c! \sum_{j=0}^c \frac{\rho^{c-j}}{j!}} \\ &= \frac{1}{1 + \sum_{j=0}^{c-1} \frac{c!}{j!} \rho^{c-j}} \\ &= \frac{1}{1 + Pol(\rho)}, \end{aligned}$$

with $Pol(\rho) := \sum_{j=0}^{c-1} \frac{c!}{j!} \rho^{c-j}$ a polynom of degree c such that $Pol(0) = 0$. Therefore, using the DL of $x \mapsto \frac{1}{1+x}$ in 0, it follows:

$$\begin{aligned}\tilde{B}(\rho) &= 1 - Pol(\rho) + o(Pol(\rho)) \\ &= 1 - Pol(\rho) + o(\rho) \\ &= 1 - \sum_{j=0}^{c-1} \frac{c!}{j!} \rho^{c-j} + o(\rho) \\ &= 1 - c\rho + o(\rho),\end{aligned}$$

with $o(\rho)$ such that $\lim_{\rho \rightarrow 0} \frac{o(\rho)}{\rho} = 0$. In particular, $\lim_{\rho \rightarrow +\infty} B(\rho) = \lim_{\rho \rightarrow 0} \tilde{B}(\rho) = 1$. □

Lemma A.1 states that when the inter-arrival times between two arrivals approaches 0 (there is a continuous flow of arrivals), then the blocking probability approaches to 1. This results enables to establish the following proposition A.2, which provides the existence and expression of the limit of λ^e when λ converges to $+\infty$.

Proposition A.2. $\lim_{\lambda \rightarrow \infty} \lambda^e = \nu c$.

The proof relies on analytical manipulation of the blocking probability $B(\rho)$.

Proof. Similarly to Lemma A.1, let us denote $\tilde{B}(\rho) = B(\frac{1}{\rho})$. Given Lemma A.1, there exists a limit to $\tilde{B}(\cdot)$ as ρ tends to 0, and the value of this limit is 1. Thus, we extend by continuity $\tilde{B}$ in 0 with $\tilde{B}(0) = 1$. Now, we do a *series expansion* of $\tilde{B}$ in 0_+ :

$$\tilde{B}(\rho) = \tilde{B}(0) + \rho \tilde{B}'(0) + o(\rho),$$

where $\lim_{\rho \rightarrow 0} \frac{o(\rho)}{\rho} = 0$: $o(\rho)$ converges to 0 strictly faster than linearly. Let us characterize $\tilde{B}'(0)$.

- Suppose $c \geq 2$.

$$\begin{aligned}\tilde{B}'(\rho) &= \left(\frac{\frac{\rho^{-c}}{c!}}{\sum_{j=0}^c \frac{\rho^{-j}}{j!}} \right)' \\ &= \frac{\frac{-\rho^{-(c+1)}}{(c-1)!} \times (\sum_{j=0}^c \frac{\rho^{-j}}{j!}) - (\sum_{j=1}^c \frac{\rho^{-(j+1)}}{(j-1)!}) \times \frac{\rho^{-c}}{c!}}{(\sum_{j=0}^c \frac{\rho^{-j}}{j!})^2} \\ &= \frac{-\frac{\rho^{-(2c+1)}}{c!(c-1)!} - \frac{\rho^{-2c}}{(c-1)!(c-1)!} - \beta_1 \rho^{-2c+1} - \dots + \frac{\rho^{-(2c+1)}}{c!(c-1)!} + \frac{\rho^{-2c}}{c!(c-2)!} + \gamma_1 \rho^{-2c+1} + \dots}{\frac{\rho^{-2c}}{c!c!} + \zeta_1 \rho^{-2c+1} + \dots} \\ &= \frac{\frac{1}{c!(c-2)!} + \gamma_1 \rho + \dots - \frac{1}{(c-1)!(c-1)!} - \beta_1 \rho - \dots \rho^{-2c+1}}{\frac{1}{c!c!} + \zeta_1 \rho + \dots},\end{aligned}$$

with β_i and ζ_i positive real coefficients (which value is not impacting).

In particular, one has:

$$\begin{aligned}
\tilde{B}'(0) &= \frac{\frac{1}{c!(c-2)!} - \frac{1}{(c-1)!(c-1)!}}{\frac{1}{c!c!}} \\
&= \frac{c!c!}{c!(c-2)!} - \frac{c!c!}{(c-1)!(c-1)!} \\
&= c(c-1) - cc \\
&= -c.
\end{aligned}$$

- Suppose $c = 1$.

$$\begin{aligned}
\tilde{B}'(\rho) &= \left(\frac{\rho^{-1}}{1 + \rho^{-1}} \right)' \\
&= \frac{-\rho^{-2} \times (1 + \rho^{-1}) - (-\rho^{-2}) \times \rho^{-1}}{(1 + \rho^{-1})^2} \\
&= \frac{-\rho^{-2} - \rho^{-3} + \rho^{-3}}{\rho^{-2} + 2\rho^{-1} + 1} \\
&= \frac{-1}{1 + 2\rho + \rho^2}.
\end{aligned}$$

Therefore,

$$\tilde{B}'(0) = -1 = -c.$$

We then have that $\forall c \geq 1, \tilde{B}(0) = -c$. Since $\tilde{B}(0) = 1$, it gives:

$$\tilde{B}(\rho) = 1 - c\rho + o(\rho),$$

or equivalently:

$$B(\rho) = 1 - \frac{c}{\rho} + o\left(\frac{1}{\rho}\right).$$

Finally,

$$\begin{aligned}
\lambda^e(\rho) &= \nu \rho (1 - B(\rho)) \\
&= \nu \rho \left(1 - \left(1 - \frac{c}{\rho} + o\left(\frac{1}{\rho}\right)\right)\right) \\
&= \nu \rho \left(\frac{c}{\rho} + o\left(\frac{1}{\rho}\right)\right) \\
&= \nu c + o(1).
\end{aligned}$$

This concludes the proof. □

In the paper [Dupont et al., 2021], which precedes the work presented in Chapter 3 and builds upon [Aveklouris et al., 2019], the effective arrival rate is approximated as:

$$\lambda^e \approx \min(\lambda, \nu c). \tag{A.1}$$

According to Lemma 3.6, λ^e increases with λ . Therefore, the approximation A.1 is an upper bound of λ^e . Given previous proposition A.2, this upper bound is tight for λ approaching $+\infty$. In particular, the effective arrivals frequency cannot exceed the ratio between the number of servers and the expected service time. Next section A.2 focuses, in the framework of Chapter 3, on the condition on the arrivals frequency such that the individual charging power rate of an EV is constrained by the aggregate power capacity at the charging pool.

A.2 Dependency of power congestion on system parameters

This section provides some insights about how congestion impact the individual charging power delivered to an EV.

Let us keep the same notations as in Chapter 3. In particular, λ is the arrival rate at the charging pool (not necessarily joining because of the *blocking probability*), $p^{\max}$ is the individual maximum charging power rate, and $\alpha \in]0, 1]$ is the sizing coefficient describing the aggregate power capacity at the charging pool, and N^{cp} is the number of charging points ($:= c$ regarding previous section). Moreover, ν is the departure rate, and μ the end of charge rate. Recall that when the aggregate charging power at a charging pool reaches the charging pool capacity, power is equally shared among the EV users.

As described in Chapter 3, it is assumed that an arriving EV sees the expected number of charging EVs in stationary regime of the Markov chain described in Chapter 3 as well. Therefore, Proposition 3.1 (Chapter 3) states in particular that:

$$\bar{z} > \alpha N^{\text{cp}} \Leftrightarrow \lambda(1 - B(\lambda/\nu)) > \alpha N^{\text{cp}} \times (\nu + p^{\max} \mu).$$

In other words, there is some power congestion, i.e., the individual charging power is strictly lower than $p_i^{\max}$ if and only if the following inequation is verified:

$$\begin{cases} \lambda(1 - B(\frac{\lambda}{\nu})) > \alpha N^{\text{cp}}(\nu + p^{\max} \mu), \\ (\lambda, \alpha) \in \mathbb{R}_+ \times [0, 1]. \end{cases} \quad (\text{A.2})$$

The following helps to characterize the values of λ and α such that the inequation (A.2) is verified, i.e., there is power congestion.

First, using that $\lambda(1 - B) := \lambda^e < \nu N^{\text{cp}}$, it yields that:

$$A = [\frac{\nu}{\nu + p^{\max} \mu}, 1],$$

is the set of power sizing coefficients α such that there is no congestion at the charging pool, regardless the arrivals frequency λ , as state in Proposition A.4 below.

A.2.1 Influence of the power sizing coefficient

Here we describe the solution to inequation (A.2) with a λ fixed.

The following proposition A.3 gives a sufficient and necessary condition on the power sizing α such that there is no power congestion at the charging pool.

Proposition A.3. *There exists a unique $\alpha_0 \notin A$ such that:*

$$\alpha < \alpha_0 \Leftrightarrow \bar{z} > \alpha N^{\text{cp}}.$$

Moreover, α_0 verifies the following formula:

$$\alpha_0 = \frac{\lambda(1 - B(\frac{\lambda}{\nu}))}{c(\nu + p^{\max} \mu)}.$$

Proposition A.3 is direct regarding the inequation (A.2). Following the expression of α_0 , ν^{-1} (average parking duration) has an ambiguous effect. Once large, it increases the *blocking probability* but also reduces the average number of EVs on load. In other words, if users stay parked a long duration in expectation, then most of the users will no have access to a charging points, but the ones who has access benefit from a high charging power rate, since most of the EV parked are fully charged. This is coherent with Figure 3.12 showing the expected energy received by an EV users with respect to ν^{-1} .

A.2.2 Influence of the users arrivals rate

Here we describe the solution to inequation (A.2) with a α fixed.

Proposition A.4. : *The following results hold:*

- if $\alpha \in A$, then $\forall \lambda \in \mathbb{R}_+$, $\bar{z} \leq \alpha N^{cp}$.
- if $\alpha \notin A$, then $\exists! \lambda_0(\alpha) \in \mathbb{R}_+$ such that:

$$\bar{z}(\lambda) \leq \alpha N^{cp} \Leftrightarrow \lambda \leq \lambda_0(\alpha)$$

Proposition A.4 is an immediate consequence on the increase of effective arrivals rate λ^e with respect to λ , given by Lemma 3.6 (Chapter 3), and the limit of λ^e when $\lambda \mapsto +\infty$, given by Proposition A.2. It states that is particular that if α is not sufficiently high ($\notin A$), there exists a unique threshold $\lambda_0(\alpha)$ such that there is some power congestion if $\lambda \geq \lambda_0(\alpha)$.

However, due to the complicated expression of the effective arrival rate λ^e , it is not possible to obtain an explicit expression of λ . Using the *implicit function theorem*, local information about the threshold $\lambda_0(\alpha)$ can be obtained.

Proposition A.5. *Let $f : ([0, 1], \mathbb{R}_+) \mapsto \mathbb{R}$ such that $f(\alpha, \lambda) = \lambda(1 - B(\lambda/\nu) - \alpha N^{cp}(\nu + p^{\max} \mu))$. By definition of λ_0 , $f(\alpha, \lambda_0(\alpha)) = 0$. Let $\alpha_0 \notin A$. Then, for α in a neighborhood of α_0 :*

$$\lambda_0(\alpha) = \lambda_0(\alpha_0) + (\alpha - \alpha_0) \times \frac{N^{cp}(\nu + p^{\max} \mu)}{1 - B - \lambda \frac{\partial B}{\partial \lambda}} + o(\alpha - \alpha_0), \quad (\text{A.3})$$

where $\lim_{\alpha \rightarrow \alpha_0} o(\alpha - \alpha_0) = 0$ with a speed more than linearly.

Proof. $\frac{d\lambda_e}{d\lambda} \neq 0$ by assumption. Therefore, according to the implicit function theorem, there exist a function ϕ such that $f(\alpha, \phi(\alpha)) = 0$ in a neighborhood of α_0 , verifying:

$$\frac{\partial \phi}{\partial \alpha} = \frac{\frac{\partial f}{\partial \alpha}}{\frac{\partial f}{\partial \lambda}}.$$

- Derivative with respect to α : $\frac{\partial f}{\partial \alpha} = N^{cp}(\nu + p^{\max} \mu)$.
- Derivative with respect to λ :

$$\begin{aligned} \frac{\partial f}{\partial \lambda} &= \frac{\partial \lambda_e}{\partial \lambda} \\ &= \frac{\partial(\lambda - \lambda B(\frac{\lambda}{\nu}))}{\partial \lambda} \\ &= 1 - B(\frac{\lambda}{\nu}) - \lambda \frac{\partial B(\frac{\lambda}{\nu})}{\partial \lambda}. \end{aligned}$$

Therefore,

$$\phi'(\alpha) = \frac{c(\nu + p^{\max} \mu)}{1 - B(\frac{\lambda}{\nu}) - \lambda \frac{\partial B(\frac{\lambda}{\nu})}{\partial \lambda}}.$$

Note that $\phi'_\lambda(\alpha)$ is independent of α . Moreover, $\phi_\lambda(\alpha_0) = \lambda_0$, which gives the final result. $\square$

Note that the relation between the effective arrival rates $\lambda_0^e(\alpha_0)$ is linear (direct consequence of the inequation (A.2)). For small values of λ , the blocking probability B is approximatively equal to 0. Therefore, $\lambda_0(\alpha)$ is also approximatively linear. This is coherent with the equation (A.3).

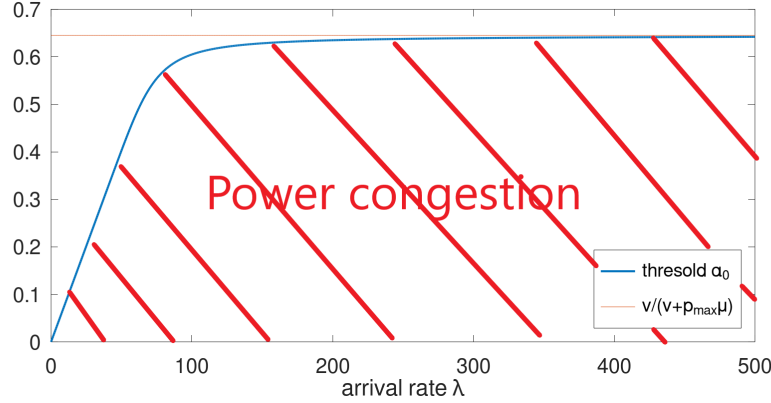


Figure A.1: Solution set (λ, α) of Inequation A.2 for specific parameter values. *In this example, $\frac{\nu}{\nu + p^{\max} \mu} \approx 0.65$. Therefore, is $\alpha > 0.65$, there is no power congestion, regardless the arrivals frequency λ .*

Figure A.1 illustrates the area (λ, α) such that there is some power congestion. This can help a charging station operator aiming to design properly the power sizing coefficient α .

A.3 Proofs

EV characteristics As the approximation of the energy received by an EV follows an exponential distribution with parameter $\frac{p(n) \mu + \nu}{p(n) \times (1 - B)}$, its expectation is:

$$\begin{aligned} \mathbb{E}[\tilde{E}(n)] &= \frac{p(n) \times (1 - B)}{p(n) \mu + \nu} \\ &= \frac{p^{\max} \min(n, \alpha N^{\text{cp}}) \times (1 - B)}{p^{\max} \min(n, \alpha N^{\text{cp}}) \mu + n \nu}. \end{aligned} \quad (\text{A.4})$$

Replacing in Equation (A.4) n by the expected number of charging EVs $\bar{z}$ leads to Equation

(3.7):

$$\begin{aligned}
\mathbb{E}[\tilde{E}(\bar{z})] &= \frac{p^{\max} \min(\bar{z}, \alpha N^{\text{cp}}) \times (1 - B)}{p^{\max} \min(\bar{z}, \alpha N^{\text{cp}}) \mu + \bar{z} \nu} \\
&= \begin{cases} \frac{p^{\max}}{p^{\max} \mu + \nu} \times (1 - B) & \text{if } \lambda^e < \alpha N^{\text{cp}}(\nu + p^{\max} \mu) \\ \frac{p^{\max} \alpha N^{\text{cp}} \times (1 - B)}{p^{\max} \alpha N^{\text{cp}} \mu + \frac{\lambda^e - p^{\max} \mu \alpha N^{\text{cp}}}{\nu} \nu} & \text{else} \end{cases} \\
&= \begin{cases} \frac{p^{\max}}{p^{\max} \mu + \nu} \times (1 - B) & \text{if } \lambda^e < \alpha N^{\text{cp}}(\nu + p^{\max} \mu) \\ \frac{p^{\max} \alpha N^{\text{cp}} \times (1 - B)}{\lambda^e} & \text{else.} \end{cases} \\
&= \begin{cases} \frac{p^{\max}}{p^{\max} \mu + \nu} \times (1 - B) & \text{if } \lambda^e < \alpha N^{\text{cp}}(\nu + p^{\max} \mu) \\ \frac{p^{\max} \alpha N^{\text{cp}}}{\lambda} & \text{else.} \end{cases}
\end{aligned}$$

■

EV characteristics Note that Conditions (3.16) are equivalent to the following system:

$$\begin{cases} \mathbb{E}_{1-\tilde{\theta}_2^{\text{thr}}}(\tilde{E}_1) \leq \mathbb{E}_{\tilde{\theta}_2^{\text{thr}}}(\tilde{E}_2), \\ \mathbb{E}_{1-\tilde{\theta}_1^{\text{thr}}}(\tilde{E}_2) \leq \mathbb{E}_{\tilde{\theta}_1^{\text{thr}}}(\tilde{E}_1). \end{cases}$$

Therefore, $\mathbb{E}_{\tilde{\theta}_{\text{eq}}}(\tilde{E}_1) = \frac{p_1^{\max} \alpha_1 N_1^{\text{cp}}}{\tilde{\theta}_{\text{eq}} \lambda}$ and $\mathbb{E}_{1-\tilde{\theta}_{\text{eq}}}(\tilde{E}_2) = \frac{p_2^{\max} \alpha_2 N_2^{\text{cp}}}{(1 - \tilde{\theta}_{\text{eq}}) \lambda}$ and $\tilde{\theta}_{\text{eq}}$ verifies the following equation:

$$\begin{aligned}
E_{\tilde{\theta}_{\text{eq}}}(\tilde{E}_1) = E_{1-\tilde{\theta}_{\text{eq}}}(E_2) &\Leftrightarrow \frac{p_1^{\max} \alpha_1 N_1^{\text{cp}}}{\tilde{\theta}_{\text{eq}} \lambda} = \frac{p_2^{\max} \alpha_2 N_2^{\text{cp}}}{(1 - \tilde{\theta}_{\text{eq}}) \lambda} \\
&\Leftrightarrow \tilde{\theta}_{\text{eq}} = \frac{p_1^{\max} \alpha_1 N_1^{\text{cp}}}{p_1^{\max} \alpha_1 N_1^{\text{cp}} + p_2^{\max} \alpha_2 N_2^{\text{cp}}}.
\end{aligned}$$

The value of the utility function $\tilde{U}_{\text{eq}}$ at equilibrium follows by applying Equation (3.11):

$$\begin{aligned}
\tilde{U}_{\text{eq}} &= \tilde{u}(\tilde{\theta}_{\text{eq}}, \tilde{\theta}_{\text{eq}}) = \tilde{\theta}_{\text{eq}} \mathbb{E}_{\tilde{\theta}_{\text{eq}}}[\tilde{E}_1] + (1 - \tilde{\theta}_{\text{eq}}) \mathbb{E}_{1-\tilde{\theta}_{\text{eq}}}[\tilde{E}_2] \\
&= \tilde{\theta}_{\text{eq}} \frac{p_1^{\max} \alpha_1 N_1^{\text{cp}}}{\tilde{\theta}_{\text{eq}} \lambda} + (1 - \tilde{\theta}_{\text{eq}}) \frac{p_2^{\max} \alpha_2 N_2^{\text{cp}}}{(1 - \tilde{\theta}_{\text{eq}}) \lambda} \\
&= \frac{p_1^{\max} \alpha_1 N_1^{\text{cp}} + p_2^{\max} \alpha_2 N_2^{\text{cp}}}{\lambda}.
\end{aligned}$$

■

EV characteristics

1. Recall that, by Equation (A.4),

$$\forall i \in \{1, 2\}, \forall \theta_i \in [0, 1], \mathbb{E}_{\theta_i}[\tilde{E}_i(\bar{z}_i)] = \frac{p_i^{\max} \min(\bar{z}_i, \alpha_i N_i^{\text{cp}}) \times (1 - B_i)}{p_i^{\max} \min(\bar{z}_i, \alpha_i N_i^{\text{cp}}) \mu + n \nu}, \text{ with}$$

$B_i := \frac{\frac{\rho_i^{N_i^{\text{cp}}}}{N_i^{\text{cp}}!}}{\sum_{j=0}^{N_i^{\text{cp}}} \frac{\rho_i^j}{j!}}$ the *blocking probability*, and that, by Equation (3.4),

$$\forall i \in \{1, 2\}, \bar{z}_i = \begin{cases} \frac{\lambda_i^e}{\nu + p_i^{\max} \mu} & \text{if } \frac{\lambda_i^e}{\nu + p_i^{\max} \mu} < \alpha_i N_i^{\text{cp}}, \\ \frac{\lambda_i^e - p_i^{\max} \mu \alpha_i N_i^{\text{cp}}}{\nu} & \text{else,} \end{cases}$$

with $\lambda_i^e = \lambda(1 - B_i)$ the effective arrival rate. As stated in the formula of the threshold $\hat{\theta}_i$ given by Equation (3.15), $\min(\hat{\theta}_1^{\text{thr}}, \hat{\theta}_2^{\text{thr}}) = +\infty$ implies that $\forall i \in \{1, 2\}, \forall \theta_i \in [0, 1], \frac{\lambda_i^e}{\nu + p_i^{\max} \mu} < \alpha_i N_i^{\text{cp}}$, and therefore, $\bar{z}_i < \alpha_i N_i^{\text{cp}}$. It means that the approximation of the expected energy received at the CSP is:

$$\begin{aligned} \forall i \in \{1, 2\}, \forall \theta_i \in [0, 1], \mathbb{E}_{\theta_i}[\tilde{E}_i] &= \frac{p_i^{\max} \bar{z}_i \times (1 - B_i)}{p_i^{\max} \bar{z}_i \mu + \bar{z}_i \nu}, \\ &= \frac{p_i^{\max}}{\nu + p_i^{\max} \mu} \times \left(1 - \frac{\frac{(\frac{\theta_i \lambda}{\nu})^{N_i^{\text{cp}}}}{N_i^{\text{cp}}!}}{\sum_{j=0}^{N_i^{\text{cp}}} \frac{(\frac{\theta_i \lambda}{\nu})^j}{j!}}\right). \end{aligned}$$

Moreover, the equilibrium $\tilde{\theta}_{\text{eq}}$ is the solution of

$$\mathbb{E}_{\theta}[\tilde{E}_1] = \mathbb{E}_{1-\theta}[\tilde{E}_2], \quad (\text{A.5})$$

if it exists. The condition for which there is a solution to (A.5) is:

$$\begin{cases} \mathbb{E}_1[\tilde{E}_1] \leq \mathbb{E}_0[\tilde{E}_2], \\ \mathbb{E}_0[\tilde{E}_1] \geq \mathbb{E}_1[\tilde{E}_2], \end{cases}$$

which is equivalent to (3.18):

$$\begin{aligned} \begin{cases} \mathbb{E}_1[\tilde{E}_1] \leq \mathbb{E}_0[\tilde{E}_2], \\ \mathbb{E}_0[\tilde{E}_1] \geq \mathbb{E}_1[\tilde{E}_2], \end{cases} &\Leftrightarrow \begin{cases} \frac{p_1^{\max}}{\nu + p_1^{\max} \mu} \times \left(1 - \frac{\frac{\rho_1^{N_1^{\text{cp}}}}{N_1^{\text{cp}}!}}{\sum_{j=0}^{N_1^{\text{cp}}} \frac{\rho_1^j}{j!}}\right) \leq \frac{p_2^{\max}}{\nu + p_2^{\max} \mu}, \\ \frac{p_2^{\max}}{\nu + p_2^{\max} \mu} \times \left(1 - \frac{\frac{\rho_2^{N_2^{\text{cp}}}}{N_2^{\text{cp}}!}}{\sum_{j=0}^{N_2^{\text{cp}}} \frac{\rho_2^j}{j!}}\right) \leq \frac{p_1^{\max}}{\nu + p_1^{\max} \mu}, \end{cases} \\ &\Leftrightarrow \begin{cases} \frac{\frac{\rho_1^{N_1^{\text{cp}}}}{N_1^{\text{cp}}!}}{\sum_{j=0}^{N_1^{\text{cp}}} \frac{\rho_1^j}{j!}} \geq 1 - \frac{p_2^{\max}(\nu + p_1^{\max} \mu)}{p_1^{\max}(\nu + p_2^{\max} \mu)}, \\ \frac{\frac{\rho_2^{N_2^{\text{cp}}}}{N_2^{\text{cp}}!}}{\sum_{j=0}^{N_2^{\text{cp}}} \frac{\rho_2^j}{j!}} \geq 1 - \frac{p_1^{\max}(\nu + p_2^{\max} \mu)}{p_2^{\max}(\nu + p_1^{\max} \mu)}. \end{cases} \end{aligned}$$

Equation (A.5), i.e., $\mathbb{E}_\theta[\tilde{E}_1] = \mathbb{E}_{1-\theta}[\tilde{E}_2]$, equivalent to Equation (3.19):

$$\begin{aligned}
\mathbb{E}_\theta[\tilde{E}_1] &= \mathbb{E}_{1-\theta}[\tilde{E}_2] \\
&\Leftrightarrow \frac{p_1^{\max}}{\nu + p_1^{\max}\mu} \times \left(1 - \frac{\frac{(\frac{\theta_1\lambda}{\nu})^{N_1^{\text{cp}}}}{N_1^{\text{cp}}!}}{\sum_{j=0}^{N_1^{\text{cp}}} \frac{(\frac{\theta_1\lambda}{\nu})^j}{j!}}\right) = \frac{p_2^{\max}}{\nu + p_2^{\max}\mu} \times \left(1 - \frac{\frac{(\frac{\theta_2\lambda}{\nu})^{N_2^{\text{cp}}}}{N_2^{\text{cp}}!}}{\sum_{j=0}^{N_2^{\text{cp}}} \frac{(\frac{\theta_2\lambda}{\nu})^j}{j!}}\right) \\
&\Leftrightarrow \frac{p_2^{\max}(\nu + p_1^{\max}\mu)}{p_1^{\max}(\nu + p_2^{\max}\mu)} \times \frac{\sum_{j=0}^{N_1^{\text{cp}}-1} \frac{(\frac{\theta\lambda}{\nu})^j}{j!}}{\sum_{j=0}^{N_1^{\text{cp}}} \frac{(\frac{\theta\lambda}{\nu})^j}{j!}} = \frac{\sum_{j=0}^{N_2^{\text{cp}}-1} \frac{(\frac{(1-\theta)\lambda}{\nu})^j}{j!}}{\sum_{j=0}^{N_2^{\text{cp}}} \frac{(\frac{(1-\theta)\lambda}{\nu})^j}{j!}} \\
&\Leftrightarrow \eta \times \sum_{j=0}^{N_1^{\text{cp}}-1} \frac{(\frac{\theta\lambda}{\nu})^j}{j!} \times \sum_{i=0}^{N_2^{\text{cp}}} \frac{(\frac{(1-\theta)\lambda}{\nu})^i}{i!} = \sum_{i=0}^{N_2^{\text{cp}}-1} \frac{(\frac{(1-\theta)\lambda}{\nu})^i}{i!} \times \sum_{j=0}^{N_1^{\text{cp}}} \frac{(\frac{\theta\lambda}{\nu})^j}{j!}.
\end{aligned}$$

This concludes the first part of Proposition 3.12.

2. If there is no solution to (A.5), $\tilde{\theta}_{\text{eq}} \in \{0, 1\}$ and $p_1^{\max} \neq p_2^{\max}$. Suppose $\tilde{\theta}_{\text{eq}} = 0$. By definition of an equilibrium:

$$\begin{aligned}
\forall \theta \in [0, 1], \tilde{u}(\theta, 0) &= \theta \mathbb{E}_0[\tilde{E}_1] + (1 - \theta) \mathbb{E}_1[\tilde{E}_2] \\
&\leq \mathbb{E}_1[\tilde{E}_2] = \tilde{u}(0, 0).
\end{aligned}$$

Therefore $\mathbb{E}_0[\tilde{E}_1] \leq \mathbb{E}_1[\tilde{E}_2]$. In other words, if all EV users choose CSP 2 at equilibrium, the approximation of the expected energy received at CSP 2 is greater than the expected energy received at CSP 1 at equilibrium. Moreover, as there is no solution to (A.5), $\mathbb{E}_0[\tilde{E}_1] \neq \mathbb{E}_1[\tilde{E}_2]$ and therefore $\mathbb{E}_0[\tilde{E}_1] < \mathbb{E}_1[\tilde{E}_2]$. Recall that $\forall i \in \{1, 2\}, x \mapsto \mathbb{E}_x[\tilde{E}_i]$ decreases, which implies that $\mathbb{E}_0[\tilde{E}_1] < \mathbb{E}_0[\tilde{E}_2]$. Moreover,

$$\mathbb{E}_0[\tilde{E}_1] < \mathbb{E}_0[\tilde{E}_2] \Leftrightarrow \frac{p_1^{\max}}{\nu + p_1^{\max}\mu} < \frac{p_2^{\max}}{\nu + p_2^{\max}\mu} \Leftrightarrow p_1^{\max} < p_2^{\max},$$

and

$$\tilde{U}_{\text{eq}} = \tilde{u}(\tilde{\theta}_{\text{eq}}, \tilde{\theta}_{\text{eq}}) = \tilde{u}(0, 0) = \mathbb{E}_1[\tilde{E}_2] = \frac{p_2^{\max}}{\nu + p_2^{\max}\mu} \times \left(1 - \frac{\frac{\rho^{N_2^{\text{cp}}}}{N_2^{\text{cp}}!}}{\sum_{j=0}^{N_2^{\text{cp}}} \frac{\rho^j}{j!}}\right).$$

The same reasoning for $\tilde{\theta}_{\text{eq}} = 1$ concludes the second part of Proposition 3.12. ■

EV characteristics

1. Note that if Conditions (3.16) hold, an immediate functional analysis shows that the utility function $\theta \mapsto \tilde{u}(\theta, \theta)$ strictly increases for θ in $[0, \tilde{\theta}_1^{\text{thr}}[$, is constant for θ in $[\tilde{\theta}_1^{\text{thr}}, 1 - \tilde{\theta}_2^{\text{thr}}]$ and then strictly decreases for θ in $]1 - \tilde{\theta}_2^{\text{thr}}, 1]$. Therefore, $\forall \theta \in [\tilde{\theta}_1^{\text{thr}}, 1 - \tilde{\theta}_2^{\text{thr}}]$, $\tilde{U}_{\text{opt}} = \tilde{u}(\theta, \theta) = \frac{(p_1^{\max}\alpha_1 N_1^{\text{cp}} + p_2^{\max}\alpha_2 N_2^{\text{cp}})}{\lambda}$. Using Proposition 3.11 which states that the utility at equilibrium is $\tilde{U}_{\text{eq}} = \frac{(p_1^{\max}\alpha_1 N_1^{\text{cp}} + p_2^{\max}\alpha_2 N_2^{\text{cp}})}{\lambda}$, it holds that $\tilde{U}_{\text{opt}} = \tilde{U}_{\text{eq}}$, which gives $\widetilde{PoA} = 1$.

2. Firstly,

$$\begin{aligned}
 \tilde{U}_{\text{eq}} &\geq \max_{i \in \{1,2\}} \mathbb{E}_1[\tilde{E}_i] \\
 &= \max_{i \in \{1,2\}} \left(\frac{p_i^{\max}}{\nu + p_i^{\max} \mu} \right) \times \left(1 - \frac{\frac{\rho^{N_i^{\text{cp}}}}{N_i^{\text{cp}}!}}{\sum_{j=0}^{N_i^{\text{cp}}} \frac{\rho^j}{j!}} \right) \\
 &\geq \max_{i \in \{1,2\}} \left(\frac{p_i^{\max}}{\nu + p_i^{\max} \mu} \right) \times \min_{j \in \{1,2\}} \left(1 - \frac{\frac{\rho^{N_j^{\text{cp}}}}{N_j^{\text{cp}}!}}{\sum_{k=0}^{N_j^{\text{cp}}} \frac{\rho^k}{k!}} \right).
 \end{aligned} \tag{A.6}$$

In order to obtain the first inequality, the following fact was used: the utility at equilibrium is greater than the minimal utility for both CSPs, and the latter is reached when all EVs go to the same CSP.

Secondly,

$$\begin{aligned}
 \tilde{U}_{\text{opt}} &= \max_{\theta \in [0,1]} \theta \frac{p_1^{\max} \mu}{\nu + p_1^{\max} \mu} \left(1 - \frac{\lambda_1^e(\theta)}{\lambda_1} \right) + (1 - \theta) \frac{p_2^{\max} \mu}{\nu + p_2^{\max} \mu} \left(1 - \frac{\lambda_2^e(1 - \theta)}{\lambda_2} \right) \\
 &\leq \max_{\theta \in [0,1]} \max_{i \in \{1,2\}} \frac{p_i^{\max} \mu}{\nu + p_i^{\max} \mu} \left(1 - \frac{\lambda_i^e(\theta_i)}{\lambda_i} \right) \\
 &= \max_{i \in \{1,2\}} \max_{\theta \in [0,1]} \frac{p_i^{\max} \mu}{\nu + p_i^{\max} \mu} \left(1 - \frac{\lambda_i^e(\theta_i)}{\lambda_i} \right) \\
 &\leq \max_{i \in \{1,2\}} \frac{p_i^{\max} \mu}{\nu + p_i^{\max} \mu}.
 \end{aligned}$$

Since λ_i^e increases with λ_i , one has $\lambda_i^e \leq \nu N^{\text{cp}}$ (to obtain the first inequality). Therefore,

$$\tilde{U}_{\text{opt}} \leq \min\left(1, \frac{N_1^{\text{cp}} + N_2^{\text{cp}}}{\rho}\right) \times \max_{i \in \{1,2\}} \left(\frac{p_i^{\max} \mu}{\nu + p_i^{\max} \mu} \right). \tag{A.7}$$

(A.6) with (A.7) gives:

$$\begin{aligned}
 \widetilde{PoA} &\leq \frac{\min\left(1, \frac{N_1^{\text{cp}} + N_2^{\text{cp}}}{\rho}\right) \times \max_{i \in \{1,2\}} \left(\frac{p_i^{\max} \mu}{\nu + p_i^{\max} \mu} \right)}{\max_{i \in \{1,2\}} \left(\frac{p_i^{\max} \mu}{\nu + p_i^{\max} \mu} \right) \times \min_{j \in \{1,2\}} (1 - \mathbb{P}_1(R_j))} \\
 &= \frac{\min\left(1, \frac{N_1^{\text{cp}} + N_2^{\text{cp}}}{\rho}\right)}{\min_{j \in \{1,2\}} (1 - \mathbb{P}_1(R_m))} \\
 &= \frac{\min\left(1, \frac{N_1^{\text{cp}} + N_2^{\text{cp}}}{\rho}\right)}{1 - \mathbb{P}_1(R_m)},
 \end{aligned}$$

where $m = \arg \min(N_1^{\text{cp}}, N_2^{\text{cp}})$.

■

Appendix B

Proof of the propositions of Chapter 5

B.1 Expected parking cost computation

Proposition B.6. *The expected parking cost for an EV class i is:*

$$\bar{C}_i^{\text{park}}(a^{\text{park}}, d^{\text{thr}}) = a^{\text{park}} \frac{1}{\nu_i} \mathbb{P}_i(P),$$

with $\mathbb{P}_i(P) := e^{-\nu_i d^{\text{thr}}}$ the probability that an EV user from class i pays for parking.

Proof. Let $a^{\text{park}} \in \mathbb{R}_+$ and $d^{\text{thr}} \in \mathbb{R}_+$.

Using the memory-less property of the exponential distribution, the expected cost $\mathbb{E}_{D_i}[C^{\text{park}}(D_i)]$ can be computed as follows:

$$\begin{aligned} \mathbb{E}_{D_i}[C^{\text{park}}(D_i)] &:= \mathbb{E}_{D_i}[a^{\text{park}} (D_i - d^{\text{thr}}) \mathbb{1}_{D_i > d^{\text{thr}}}] \\ &= a^{\text{park}} \mathbb{E}_{D_i}[(D_i - d^{\text{thr}}) | D_i > d^{\text{thr}}] \mathbb{P}(D_i > d^{\text{thr}}) \\ &= a^{\text{park}} \times \mathbb{E}_{D_i}[D_i] \mathbb{P}(D_i > d^{\text{thr}}) \\ &= a^{\text{park}} \frac{1}{\nu_i} e^{-\nu_i d^{\text{thr}}}. \end{aligned}$$

□

B.2 Stationary probability that an EV user at the CSP is fully charged

Proposition B.7. *The expected total number of EV users fully charged at the CSP is given by:*

$$\bar{N}^{\text{full}} := \sum_{\mathbf{s} \in \mathbb{S}} \hat{\pi}(\mathbf{s}) \sum_{i \in \mathcal{I}} s_i \mathbb{P}_i(F | \mathbf{s}),$$

with:

$$\mathbb{P}_i(F | \mathbf{s}) = \frac{1}{\theta_i(a_{n(\mathbf{s})}^{\text{park}}, d_{n(\mathbf{s})}^{\text{thr}})} \int_{\mathbb{R}_+^2} e^{-\nu_i \frac{b^{\text{capa}} - e^0}{p^{\text{ch}}}} \hat{\theta}_i(b^{\text{capa}}, e^0, a_{n(\mathbf{s})}^{\text{park}}, d_{n(\mathbf{s})}^{\text{thr}}) f_{\text{soc}}(d b^{\text{capa}}, d e^0),$$

the probability that a class- i EV user that joined the CSP in state $\mathbf{s} \in \mathbb{S}$ is fully charged.

Proof. The fraction of EV fully charged at the CSP among class- i EVs with a battery b^{capa} and energy at arrival e^0 is the probability $\mathbb{P}_i(F|b^{\text{capa}}, e^0)$ that an EV with these characteristics leaves with an fully charged battery [Aveklouris et al., 2019]:

$$\begin{aligned}\mathbb{P}_i(F|b^{\text{capa}}, e^0) &= \int_{\frac{b^{\text{capa}} - e^0}{p^{\text{ch}}}}^{+\infty} \nu_i e^{-\nu_i t} dt \\ &= e^{-\nu_i \frac{b^{\text{capa}} - e^0}{p^{\text{ch}}}}.\end{aligned}$$

Given this, denoting $\hat{\theta}_i(b^{\text{capa}}, e^0, a^{\text{park}}, d^{\text{thr}})$ the probability that a class- i EV user with battery capacity b^{capa} and energy at arrival e^0 joined the CSP given the parking price $(a_{n(\mathbf{s})}^{\text{park}}, d_{n(\mathbf{s})}^{\text{thr}})$, the probability that a class- i EV user that joined the CSP in state $\mathbf{s} \in \mathbb{S}$ is fully charged is given by:

$$\mathbb{P}_i(F|\mathbf{s}) := \frac{1}{\theta_i(a_{n(\mathbf{s})}^{\text{park}}, d_{n(\mathbf{s})}^{\text{thr}})} \int_{\mathbb{R}_+^2} e^{-\nu_i \frac{b^{\text{capa}} - e^0}{p^{\text{ch}}}} \hat{\theta}_i(b^{\text{capa}}, e^0, a_{n(\mathbf{s})}^{\text{park}}, d_{n(\mathbf{s})}^{\text{thr}}) f_{\text{soc}}(db^{\text{capa}}, de^0) \quad (\text{B.1})$$

Then, using Equation (B.1) with $\mathbb{P}_i(F|\mathbf{s})$ weighted by the state probabilities concludes the proof. $\square$

B.3 Lemma

The following lemma is used for the computation of the expected energy received during a charging session, and therefore for the computation of the utility function $U^{\text{ch}}(\cdot)$.

Lemma B.8. *For any $z \in \mathbb{R}_+$, knowing the parking duration is below z , the expected parking duration and the squared expected parking duration are given by:*

$$\mathbb{E}_{D_i}[D_i \mathbb{1}_{D_i \leq z}] = \frac{1}{\nu_i} (1 - e^{-\nu_i z}) - z e^{-\nu_i z}, \quad (\text{B.2})$$

and

$$\mathbb{E}_{D_i}[D_i^2 \mathbb{1}_{D_i \leq z}] = -e^{-\nu_i z} z^2 + \frac{2}{\nu_i} \left(\frac{1}{\nu_i} - \left(\frac{1}{\nu_i} + z \right) e^{-\nu_i z} \right). \quad (\text{B.3})$$

Proof. Using Integration by Parts (IPP), it yields:

$$\begin{aligned}\mathbb{E}_{D_i}[D_i \mathbb{1}_{D_i \leq z}] &= \mathbb{E}_{D_i}[D_i (1 - \mathbb{1}_{D_i > z})] \\ &= \mathbb{E}_{D_i}[D_i] - \mathbb{E}_{D_i}[D_i \mathbb{1}_{D_i > z}] \\ &= \mathbb{E}_{D_i}[D_i] - \mathbb{E}_{D_i}[D_i | D_i > z] \frac{1}{\mathbb{P}(D_i > z)} \\ &= \mathbb{E}_{D_i}[D_i] - (\mathbb{E}_{D_i}[D_i] + z) \frac{1}{\mathbb{P}(D_i > z)} \\ &= \frac{1}{\nu_i} - \left(\frac{1}{\nu_i} + z \right) e^{-\nu_i z}\end{aligned}$$

Similarly, also using IPP, it yields:

$$\begin{aligned}
& \mathbb{E}_{D_i}[D_i^2 \mathbb{1}_{D_i \leq z}] \\
&= \int_{t=0}^z \nu_i e^{-\nu_i t} t^2 dt \\
&= [-e^{-\nu_i t} t^2]_0^z + \frac{2}{\nu_i} \int_{t=0}^z \nu_i e^{-\nu_i t} t dt \\
&= -e^{-\nu_i z} z^2 + \frac{2}{\nu_i} \mathbb{E}_{D_i}[D_i \mathbb{1}_{D_i \leq z}] \\
&= -e^{-\nu_i z} z^2 + \frac{2}{\nu_i} \left(\frac{1}{\nu_i} - \left(\frac{1}{\nu_i} + z \right) e^{-\nu_i z} \right).
\end{aligned}$$

□

Lemma B.8 relies on IPP and the memory-less property of the exponential distribution.

B.4 Expected energy received during a charging session

Proposition B.9. *The expected energy received $\mathbb{E}_{D_i}[\phi^{\text{rec}}(D_i, b^{\text{capa}}, e^0)]$ during a charging session of an EV randomly taken within class i is given by:*

$$\mathbb{E}_{D_i}[\phi^{\text{rec}}(D_i, b^{\text{capa}}, e^0)] = p^{\text{ch}} \frac{1}{\nu_i} (1 - e^{-\nu_i d^{\text{ch}}}), \quad (\text{B.4})$$

with $d^{\text{ch}} = \frac{b^{\text{capa}} - e^0}{p^{\text{ch}}}$ the time for an EV with battery capacity b^{capa} and state of charge at arrival e^0 to get fully charged.

Note that the expected energy received depends in particular on the probability that the EV leaves with an unfully charged battery, that is the probability $\mathbb{P}(D_i \leq d^{\text{ch}}) = 1 - e^{-\nu_i \frac{b^{\text{capa}} - e^0}{p^{\text{ch}}}}$ that the parking duration D_i is lower than the time d^{ch} to get fully charged.

Proof. Using Equation (B.2) applied to $z = d_{\text{ch}}$, and the exponential probability distribution of the parking duration D_i , the following equalities hold:

$$\begin{aligned}
& \mathbb{E}_{D_i}[\phi^{\text{rec}}(D_i, e^0)] \\
&= p^{\text{ch}} \times \mathbb{E}_{D_i}[\min(D_i, d_{\text{ch}})] \\
&= p^{\text{ch}} \times (\mathbb{E}_{D_i}[D_i \mathbb{1}_{D_i \leq d_{\text{ch}}}] + \mathbb{E}_{D_i}[d_{\text{ch}} \mathbb{1}_{D_i > d_{\text{ch}}}]) \\
&= p^{\text{ch}} \times \left(\frac{1}{\nu_i} (1 - e^{-\nu_i d_{\text{ch}}}) - d_{\text{ch}} e^{-\nu_i d_{\text{ch}}} + d_{\text{ch}} \mathbb{P}(D_i > d_{\text{ch}}) \right) \\
&= p^{\text{ch}} \times \left(\frac{1}{\nu_i} (1 - e^{-\nu_i d_{\text{ch}}}) - d_{\text{ch}} e^{-\nu_i d_{\text{ch}}} + d_{\text{ch}} e^{-\nu_i d_{\text{ch}}} \right) \\
&= p^{\text{ch}} \times \frac{1}{\nu_i} (1 - e^{-\nu_i d_{\text{ch}}}).
\end{aligned}$$

□

B.5 Expected utility for charging

Let us denote $d^{\text{ch}} = \frac{b^{\text{capa}} - e^0}{p^{\text{ch}}}$ the time for an EV with battery capacity b^{capa} and state of charge at arrival e^0 to get fully charged.

Proposition B.10. *The expected utility of an EV user of class i for charging is:*

$$\begin{aligned} \bar{U}_i^{\text{ch}}(e^0) &:= (\alpha_{e^0} - a^{\text{ch}}) \mathbb{E}_{D_i}[\phi^{\text{rec}}(D_i, b^{\text{capa}}, e^0)] \\ &\quad - \beta_{e^0} \mathbb{E}_{D_i}[\phi^{\text{rec}}(D_i, b^{\text{capa}}, e^0)^2], \end{aligned} \quad (\text{B.5})$$

with:

- $\mathbb{E}_{D_i}[\phi^{\text{rec}}(D_i, b^{\text{capa}}, e^0)]$ the expected energy received during the parking duration, given by Equation (B.4);
- $\mathbb{E}_{D_i}[\phi^{\text{rec}}(D_i, b^{\text{capa}}, e^0)^2] := (p^{\text{ch}})^2 \times \left(2 \frac{1}{\nu_i} (1 - e^{-\nu_i d_{\text{ch}}}) + d_{\text{ch}} e^{-\nu_i d_{\text{ch}}} (d_{\text{ch}} - 3) \right)$ the expected squared energy received;
- and $d^{\text{ch}} = \frac{b^{\text{capa}} - e^0}{p^{\text{ch}}}$ the time for an EV with battery capacity b^{capa} and state of charge at arrival e^0 to get fully charged.

Proof. The formula given in Equation (B.5) simply relies on the linearity of the expectation. One has to compute the expected energy received, and the expected squared energy received.

Similarly, using Equation (B.3) applied to $z = d_{\text{ch}}$, the following equalities hold:

$$\begin{aligned} &\mathbb{E}_{D_i}[\phi^{\text{rec}}(D_i, b^{\text{capa}}, e^0)^2] \\ &= (p^{\text{ch}})^2 \times \mathbb{E}_{D_i}[(\min(D_i, d_{\text{ch}}))^2] \\ &= (p^{\text{ch}})^2 \times (\mathbb{E}_{D_i}[D_i^2 \mathbb{1}_{D_i \leq d_{\text{ch}}}] + \mathbb{E}_{D_i}[d_{\text{ch}}^2 \mathbb{1}_{D_i > d_{\text{ch}}}]) \\ &= (p^{\text{ch}})^2 \times \left(2 \frac{1}{\nu_i} (1 - e^{-\nu_i d_{\text{ch}}}) - 3 d_{\text{ch}} e^{-\nu_i d_{\text{ch}}} + d_{\text{ch}}^2 e^{-\nu_i d_{\text{ch}}} \right) \\ &= (p^{\text{ch}})^2 \times \left(2 \frac{1}{\nu_i} (1 - e^{-\nu_i d_{\text{ch}}}) + d_{\text{ch}} e^{-\nu_i d_{\text{ch}}} (d_{\text{ch}} - 3) \right). \end{aligned}$$

□

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